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VIBRATION-BASED DAMAGE DETECTION
OF TALL BUILDING STRUCTURES

by

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B.Eng., M. Phil.

A Thesis for the Degree of Doctor of Philosophy

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September 2004
To my wife, Chun-mei Zhang
DECLARATION

I hereby declare that this thesis entitled "Vibration-Based Damage Detection of Tall Building Structures" has not been, either in whole or in part, previously submitted for a degree in this or any other institution, and the work presented in this thesis is original unless otherwise acknowledged in the text.

SIGNED

______________________________
Xian-tong ZHOU
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ABSTRACT

This dissertation is concerned with vibration-based seismic damage detection of tall building structures, based on the measurement dynamic signals directly, rather than on structure modeling and numerical simulation.

For the purpose of studying earthquake response behaviours and vibration-based seismic damage detection, a scaled tall building model structure simulating one typical high-rise residential building in Hong Kong is elaborately fabricated and tested on shaking table. Several earthquake records simulating different soil site condition are designed and exerted on the model structure. The excitation magnitude of such earthquakes is enhanced successively. Following each level of earthquakes, visual inspection on incurred seismic damage is conducted and white-noise exciting tests are performed, providing the basis for the subsequent damage detection.

Generally speaking, vibration-based damage identification usually needs to develop a mechanics model of the investigated structure, for deriving the required dynamic characteristics. However, it is a laborious work to develop a mechanics model for highly complex building structure, which also cannot fulfill immediate post-earthquake damage assessment requirement. Further, apart from modal identification error, structure model introduces additional modeling error, which increases uncertainty greatly in model-based damage identification. Therefore, a variety of model-free damage identification or evaluation approaches, based on the measured signals directly, are developed in this study.

Firstly, an approach of damage identification is developed, employing both excitation and response data under white-noise exerting. Considering the availability of excitation information recorded during shaking table tests and possible modal
identification errors, the identification is based on the computed frequency response functions (FRFs) directly, rather than on ultimate modal parameters derived from FRFs. Taking obtained FRFs as input, neural networks are constructed for damage location and severity identification. However, high dimensionality of FRF hinders the neural network training convergence. Using principal component analysis (PCA) technique, feature extraction on the FRFs is executed, where the functionality of PCA for information compression as well as measurement noises filtering is investigated in detail. This part of study mainly explores damage identification potential of using non-frequency or non-mode shape dynamic characteristics, when both excitation and response data are available.

From practical point of view, only response signals can be measured in real building structure, whereas excitation information is usually unavailable. As a result, FRFs can not be obtained. Approaches of output-only modal identification utilizing only white-noise excited response for the tall building model structure are developed, based on which such modal parameters as frequency and mode shape are calculated. Thereafter, using these estimated dynamic parameters, damage indicators or damage indices are constructed and compared for their performances on seismic damage location and intensity recognition.

For damage identification of building structures under earthquake attacking, it is more important to evaluate incurred damage, shortly after earthquake occurs. Accordingly, earthquake records themselves should be utilized directly for post-earthquake assessment, avoiding the necessary of extra modal testing. Eventually, the emphasis of this study is placed on exploring the potential of earthquake records - based damage evaluation. The evaluation is at overall and almost real-time senses.
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CHAPTER 1

INTRODUCTION

1.1 RESEARCH MOTIVATION

Civil engineering infrastructures deteriorate with time and continuously accumulate damage during their service life due to natural hazards such as earthquakes, storms, fires, long-term fatigue and corrosion. Undetected damage might cause a catastrophic structural failure and lead to the loss of human lives. From the viewpoint of the serviceability and safety of structures, an important issue is to detect the structural damage before failure occurs. Therefore, monitoring the health and assessing the damage of civil infrastructure systems is cost-effective and necessary if catastrophic structural failures are to be prevented and proper decisions made on repairs and on partial replacements in case of demolition. Information on damage may be utilized to make a decision on whether or not repair, partial replacement or demolition is necessary after severe natural hazards or long-term usage.

For building structures, one of the most difficult problems to manage is the identification and evaluation of both the expected and post-earthquake structural damage. An assessment of the potential damage is fundamental for defining mitigation procedures and risk management strategies. Recent natural hazards such as the Kobe earthquake in Japan and the Chi-Chi earthquake in Taiwan have also highlighted the
need for post-earthquake damage assessments of civil engineering structures, especially building structures, and the widespread implications of this for society. Hong Kong is located on the South China Sea plate. The nearest active tectonic plate boundary is that of the Philippine sea plate and the South China Sea plate between Taiwan and the Philippines. Despite being relatively far away from this active boundary, historical records have indicated that Hong Kong did experience moderate earthquakes as far back as 1874 and 1918. Many researchers also have quantified the seismicity level in recent years and concluded that Hong Kong is a region of moderate seismic risk (Lee et al., 1998; Wong et al., 1998). However, for historical reasons, no provisions for seismic activities have been made in Hong Kong. This makes facilities and building structures in Hong Kong vulnerable to damage should an earthquake occur. As a major financial centre and one of the most densely populated cities in the world, any damage to critical facilities and residential buildings may have serious social and economic consequences. A very large number of old, tall residential buildings are still in service and are in urgent need of attention. Seismic provisions are also needed for forthcoming structures to mitigate possible seismic hazards. And more importantly, a feasible seismic damage identification and post-earthquake evaluation strategy should be developed for the large numbers of tall buildings that currently exist, in case an earthquake or any other catastrophic disaster occurs.

Despite much recent research and some successful applications in recent years, assessing the damage that could occur in complex structures such as tall buildings remains a challenging task for civil engineers. No investigation has been made on the
methodologies for detecting damage specific to large-scale, flexible tall buildings. For heavily redundant structures, the global modal parameters may be insensitive to damage inflicted in some structural members. This leads to an ill-conditioned inverse problem for the kind of identification method to be used. Stimulated by these deficiencies, the aim of the present study is to develop vibration-based damage detection approaches of practical significance for tall building structures.

1.2 RESEARCH OBJECTIVES

This thesis is devoted to studying the vibration-based seismic damage identification of tall building structures. This includes information on the location(s) of the damage, an identification of the severity, as well as an overall evaluation of the post-earthquake damage. The main objectives of this research are:

(1) To propose a model-free concept of identifying seismic damage in tall building structures. All of the methods that have been developed so far for identifying seismic damage are based on measured excitation and response signals, or on response data only.

Developing models in structural mechanics is a time-consuming task, especially for large-scale tall building structures of great complexity. The uncertainty generated from modeling further disrupts the process of identifying damage. Although the present investigation is carried out on a scaled model of a building, the proposed model-free concept avoids the need for structure structural modeling and should pave
the way for implementing a strategy to detect damage in real tall buildings. All of the approaches to damage recognition that are subsequently developed in this study are derived from the model-free concept and are completely based on the measurement data, without the inclusion of any structural information.

(2) To explore the potential of directly using the intermediate frequency response function, rather than modal parameters (such as frequency, mode shape, and so forth) derived from the frequency response function, to identify damage. The dissertation is also devoted to developing a data compression and feature extraction method for dealing with large amounts of measurement information.

Unlike the ultimately identified modal parameters, the frequency response function over the resonant frequency range contains information on all modal parameters that should be fully utilized to detect damage. However, the frequency response function is also comprised of much unwanted noise or non-essential information, which should be eliminated when recognizing damage. For the purpose of identifying seismic damage, a data compression and feature extraction method based on principal component analysis technique is developed and employed to cope with the frequency response function.

(3) To develop output (response)-only approaches to estimating modal parameters and compare their efficacy.

In most measurements of field ambient vibration for large-scale civil engineering
structures, the excitations are usually unavailable because of the inconvenience of
testing and only the responses can be measured at specified measurement points (also
called \textit{measurement degrees of freedom}). Correspondingly, vibration-based damage
identification techniques should develop output (response)-only approaches to
estimating modal parameters, using only response data measured from field testing.
The present study will investigate and compare several output-only identification
algorithms on the measured white-noise response acceleration signals of the tall
building model structure.

(4) To propose real time post-earthquake damage assessment strategies, based on
the measured earthquake records themselves, rather than on any other additional
modal testing or measurements of ambient vibration.

Most of the possible damage in building structures is caused by accidental events,
such as earthquakes or terrorist attacks. Due to abruptness of such types of damage,
evaluating the overall damage shortly after the occurrence of the event is important
and practical. The post-assessment assists in the making of reasonable judgments on
the remaining serviceability of the structure and of proper decisions on retrofitting.
However, both the excitation and response of such unexpected events are often
non-stationary and short in duration. This leads to an applicability problem for the
identification algorithms of conventional FFT-derived modal parameters in
implementing carrying out vibration-based damage detection. This study stresses real
time post-earthquake evaluation strategies by resorting to joint time-frequency
analysis algorithms or super-resolution spectrum analysis techniques, rather than to conventional modal identification approaches.

1.3 LAYOUT OF THE THESIS

This dissertation is comprised of eight chapters. The research is carried out in an evolving style, which means that some chapters are trying to solve hurdles that were introduced in the previous chapter(s).

Chapter 1 introduces the motivation for the present research and makes clear the objectives to be pursued in this PhD project.

Chapter 2 contains a review of the relevant literature that covers three relatively independent subjects. These subjects include general vibration-based damage detection methodologies, output-only approaches to estimating modal parameters and specific methods for detecting damage in buildings. The first subject is on general vibration-based methods of identifying damage. These methods include the most popular strategies, such as examining changes in structural frequency, mode shape, modal flexibility, etc.; and some other advanced solutions, such as method based on the frequency response function, methods related to neural networks and genetic algorithms, and so forth. It should be noted that all of the abovementioned methods are non-specific and have already been applied to a variety types of civil structures. The second subject is on output-only modal identification techniques. Some of the most recently developed approaches are reviewed. The last subject is on some existing
methods of detecting damage that are specific for buildings, including identifying member-level damage and assessing the severity of the overall damage.

Chapter 3 focuses on descriptions of a model structure of a scaled tall building, which simulates a typical high-rise residential building structure in Hong Kong. The model structure is the research object of this PhD project. First, the prototype structure and the counterpart of the fabrication of the model structure are introduced. Then, shaking table tests performed on the scaled model are addressed in detail, including the testing procedure and specifications of excitation type. Finally, this chapter will present the seismic damage incurred in the model structure and the results of a visual inspection of the damage. These results will provide baselines for the subsequent chapters on studies on identifying damage.

From the perspective of damage identification objective and measurement signals used, the emphasis of the subsequent chapters in this dissertation can be divided into two parts. The first part is comprised of Chapters 4 and 5, which use measurement data under white-noise excitation for locating the damage and identifying its severity. The second part is made up of Chapters 6 and 7, which stress the strategy of real time post-earthquake evaluation, by directly using earthquake excitation and response.

Chapter 4 explores the potential of combining the neural network technique with information on the frequency response function computed using the traditional FFT method for the purpose of identifying damage. This approach avoids the need to
estimate such modal parameters as frequency, mode shape, and so forth, the identification of which usually involves numerical errors. Nevertheless, the amount of information on the frequency response function is too large to be directly used in neural network training. A data compression and feature extraction scheme is developed, based on the principal component analysis technique. In particular, the ability of principal component analysis to retain the original modal information involved in the frequency response function as well as to filter unwanted measurement noise are intensively investigated.

Chapter 5 provides an alternative approach to identifying the locations and severity of seismic damage, by using data on measurements of white-noise excitations. The most prominent discrimination with Chapter 4 is that: excitation signals are assumed to be unavailable in shaking table tests (although they are measured in tests) and only response signals are used for estimating the modal parameters. Two output-only modal identification algorithms, called the complex mode indicator function (CMIF) method and stochastic subspace identification (SSI) method, are implemented and results of their identification are discussed and compared with those of the frequency response function based method, which belongs to a class of the input-output modal identification approach. From the results of the comparison, the accuracy and reliability of the output-only methods of identification can be validated. Then, a preferred method that yields results closer to those of the frequency response function method is chosen as the modal identification algorithm for all cases of damage. Based on the results of the output-only modal identification, a variety of
damage indices are constructed and compared to identify the seismic damage to the model building. In most tests of in-situ field ambient vibration, the excitations exerted on structures are unknown or difficult to measure; therefore, the proposed approach in this chapter is of great practical significance.

Chapter 6 presents a joint time-frequency analysis strategy on earthquake records, to evaluate the overall post-earthquake damage. In order to eliminate the influence of earthquake excitations, the seismic pulse response is first formulated. Based on this, several joint time-frequency analysis algorithms are employed and compared. The analysis in this chapter is conducted entirely with reference to the time-frequency energy spectrum; thus the modal parameter values cannot be estimated. Accordingly, the assessment of the overall post-earthquake damage carried out in this chapter is qualitative, not quantitative.

Chapter 7 proposes an evolving time-frequency analysis strategy using the earthquake records themselves to evaluate the overall post-earthquake damage. Similarly, the analysis is also based on the seismic pulse response derived at the beginning of Chapter 6. Super-resolution spectrum analysis and approaches to estimating modal parameters are developed, based on which modal frequency of the model structure at each time instant can be identified. By examining the identified instantaneous frequency sequence during an earthquake, seismic damage can be evaluated quantitatively. As well, the destructive mechanism of certain earthquakes on the building can also be illuminated. This chapter stresses the real time and
quantitative identification of post-earthquake damage.

Chapter 8 summarizes the main conclusions achieved in this thesis from studying the identification of damage using white-noise measurement data and from studying the overall evaluation of post-earthquake damage by directly using earthquake records. The potential of applying the developed approaches on real building structures is also explored.
CHAPTER 2

LITERATURE REVIEW

2.1 VIBRATION-BASED DAMAGE DETECTION

Interests in the ability to monitor a structure and detect damage at the earliest possible stage are pervasive throughout the civil, mechanical and aerospace engineering communities. Many damage-detection methods are either visual or localized experimental, such as acoustic or ultrasonic methods, magnet field methods, radio-graphs, eddy-current methods or thermal filed methods. All of these experimental techniques require that the vicinity of the damaged region(s) or the portions of the structure being inspected are readily accessible. Subjected to these limitations, these experimental methods can detect damage on or near the surface of the structure. The need for additional global damage detection methods that can be applied to complex structures, such as large-scale cable-supported bridges and tall building structures, has led to the development of methods that examine changes in the vibration characteristics of the structure.

With the use of system identification concepts, measurements of dynamic modal characteristics can be used to diagnose structural damage. The basic idea is that modal parameters (notably frequencies, mode shapes and their derivations) are functions of the physical properties of the structure (mass, damping and stiffness). Therefore, changes in the physical properties will cause changes in the modal properties. Many of the attempts that have been made to monitor structural integrity
using changes in modal parameters through vibration measurements and corresponding methods have become commonplace in the past decades. Most methods were initially developed in the field of aerospace engineering.

As mentioned previously, the field of damage identification is very broad and encompasses both local and global methods. The subsequent literature survey will be limited to global methods that are used to infer damage from changes in the vibration characteristics of the structure. Mainly according to the kinds of experimental modal data used, methods of detecting damage have been broadly classified into five categories in the review: methods based on measured natural frequencies; methods based on measured frequencies and mode shapes; methods based on measured frequency response functions; methods based on measured strain modes; and other advanced methods including both neural networks and genetic algorithms. The widely evolved damage evaluation algorithms include: modal strain energy procedure; sensitivity procedure parameter optimization procedure; modal curvature procedure; sub-matrix perturbation procedure; eigenstructure assignment procedure; residual force procedure; optimal matrix updating procedure; minimum rank updating procedure; flexibility procedure; damage index procedure; Bayesian probabilistic procedure; pattern recognition procedure; neural network techniques; and genetic algorithms.

An alternative system of classification for damage-identification methods, as presented by Rytter (1993), defines four levels of identifying damage, as follows:

- Level 1: Determination that damage is present in the structure
- Level 2: Determination of the geometric location of the damage
- Level 3: Quantification of the severity of the damage
Level 4: Prediction of the remaining service life of the structure

Level 4 predictions are generally categorized with the fields of fracture mechanics, fatigue life analysis or structural design assessment and, as such, are not addressed in the following survey of the literature on structural vibration or modal analysis.

2.1.1 Based on Changes in Structure Frequency

The number of studies on detecting damage using shifts in natural frequencies is quite large. Lifshitz and Rotem (1969) presented what may be the first journal article to propose detecting damage via measurements of vibration.

Stubbs et al. (1990) discussed a method for identifying damage that relates changes in the resonant frequencies to changes in member stiffness using a sensitivity relation. The relationship among the normalized changes in squared frequencies \( \{z\} \), the reduction in fractional elemental stiffness \( \{\alpha\} \), and the reductions in fractional elemental mass is given by

\[
\{z\} = [F]\{\alpha\} - [G]\{\beta\}
\]  

(2.1)

where \([F]\) and \([G]\) are the sensitivities of the changes in frequency to the changes in elemental stiffness and mass magnitudes, respectively.

Sophia and Garrett (1995) presented a technique for identifying localized reductions in the stiffness of a structure using measurements of natural frequency. The sensitivities of the eigenvalues to localized changes in the stiffness were developed as a set of underdetermined equations. Using numerical simulation, a 10%
to 90% localized reduction in stiffness was successfully identified in a 10-storey, two-bay steel frame. The method was also verified using test data from an aluminum cantilever beam.

Morassi and Rovere (1997) discussed a diagnostic strategy for identifying localized damage in a multi-storey steel frame from measurements of frequency. The diagnostic technique was based on criterion for optimality where the stiffness distribution of a chosen configuration of the system was updated such that frequencies related to shear-type modes closely match with the measured ones at a certain level of deterioration.

Danilo and Fabrizio (1999) evaluated the possibility of using natural frequencies to assess structural integrity. They addressed the problem of understanding when it was sufficient to measure and used only natural frequencies, thus avoiding the need to measure modal shapes. Damage was represented as a decrease in the stiffness of beams or of beam assemblies and linear behaviour before and after damage was considered. Finally, the paper considered the applications of the procedure developed to both analytical and experimental data.

2.1.2 Based on Changes in Frequency and Mode Shape

West (1984) presented what is possibly the first systematic use of information on mode shape to locate structural damage without the use of a prior finite element model (FEM). The author utilized the modal assurance criteria (MAC) to determine the level of correlation between modes from a test of the body flap of an undamaged Space Shuttle Orbiter and modes from a test of the flap after it had been exposed to acoustic loading. The mode shapes were partitioned using various schemes, and the change in MAC across the different partitioning techniques was used to localize the
structural damage.

Yuen (1985) examined changes in the mode shape and mode-shape-slope parameters by computing

$$\left\{ \phi^* \right\} = \left\{ \phi^d \right\}_i - \left\{ \phi^u \right\}_i \quad \omega_i^d - \omega_i^u$$

The changes in these parameters were simulated for a reduction in stiffness in each structural element. The predicted changes were then compared to the measured changes to determine the location of the damage. The author identified the need for an ortho-normalization process to examine higher mode shapes.

Rizos et al. (1990) developed an analytical model for the vibration of a beam with an open crack. The sections on either side of the crack are considered to be standard slender beams in transverse vibration. The condition of compatibility between the two sections is derived based on the crack-strain-energy function. The result is a system of equations for the frequencies and mode shapes in terms of the length and position of the cracks.

Ko et al. (1994) presented a method that uses a combination of MAC, Coordinate MAC (COMAC) and sensitivity analysis to detect damage in steel-framed structures. The sensitivities of the analytically derived mode shapes to particular damage conditions were computed to determine which DOF are most relevant. The authors then analyzed the MAC between the measured modes from the undamaged structure and the measured modes from the damaged structure to select which mode pairs to use in the analysis. Using the modes and DOF selected with the
above criteria, the COMAC is computed and used as an indicator of damage. The results demonstrate that particular mode pairs can indicate damage, but when all mode pairs are used, the indication of damage is masked by modes that are not sensitive to the damage.

Lam et al. (1995) defined a mode shape normalized by the change in the natural frequency of another mode as a 'damage signature'. The damage signature is a function of crack location but not of crack length. They analytically computed a set of possible signatures by considering all possible states of damage. The measured signatures were matched to a damage state by selecting which of the analytical signatures gave the best match to the measurements using the MAC.

Hjelmstad and Shin (1996) developed a crack damage detection and localization algorithm based on system identification using the finite element model and the measured mode shape response of a cantilever beam. A modal parameter estimation algorithm was used in conjunction with an adaptive parameter grouping scheme to localize damage from sparse, noisy measurements. Then, a statistical damage index based on the identified mode shape was established through a Monte Carlo simulation of the baseline structure.

2.1.3 Methods Based on Modal Flexibility

An alternative to using structural frequency and mode shape to obtain spatial information on changes in vibration is using their derivatives, such as modal flexibility.

The measured flexibility matrix is estimated from the mass-normalized measured mode shapes and frequencies as
\[ [G] \approx [\Phi][\Lambda][\Phi]' \] (2.3)

The formulation of the flexibility matrix is approximate due to the fact that only the first few modes of the structure (typically the lowest-frequency modes) are measured.

Aktan et al. (1994) proposed the use of measured flexibility as a 'condition index' to indicate the relative integrity of a bridge. Pandey and Biswas (1994) presented a damage-detection and damage location identification method based on changes in the measured flexibility. Toksoy and Aktan (1994) computed the measured flexibility of a bridge and examine the cross-sectional deflection profiles with and without a baseline data set. They observed that anomalies in the deflection profile can indicate damage even without a baseline data set.

Mayes (1995) used the measured flexibility to locate damage from the results of a modal test on a bridge. He also proposed a method for using measured flexibility as the input for a damage-detection method that evaluates changes in the load-deflection behaviour of a spring-mass model of the structure.

Doebling and Peterson (1997) presented a method for computing a statically complete structural flexibility matrix from a dynamically measured flexibility matrix. The method required the solution of linear systems of equations only. The method was derived and applied to both numerically and experimentally measured matrices of flexibility, and the improved accuracy of static flexibility over dynamically measured flexibility was demonstrated.
Wang et al. (2000) performed a sensitivity analysis of modal flexibilities on simulated damage of Tsing Ma Bridge. A numerical computation of the mass-normalized mode shapes and natural frequency matrix based on the finite element model, then of the modal flexibility matrix was approximated by Equation (2.4). Finally, a damage indicator, named the percentage difference in modal flexibility between the intact and damaged structure, was configured to identify the location of the damage.

\[
P_{FM_{i}} = \frac{|F_{ii}^{d} - F_{ii}^{u}|}{F_{ii}^{u}}
\]  

(2.4)

where the subscript \( ii \) denotes the diagonal term of the modal flexibility matrix, and the superscripts \( d \) and \( u \) denote the damaged and undamaged structure, respectively. It was proven that, for some cases of damage, using even the first five modes was sufficient to detect the location(s) of the damage.

2.1.4 Methods Based on Mode Shape Curvature/Strain Mode Shape

Another class of damage identification method uses the derivative of the mode shape of the structure to estimate changes in the dynamic characteristics of the structure.

Pandey et al. (1991) demonstrated that absolute changes in mode shape curvature could be a good indicator of damage for the FEM beam structures they
considered. The curvature values were computed from the displacement mode shape using the central difference approximation for mode $i$ and DOF $q$

$$
\phi''_{q,i} = \frac{\phi_{q-1,i} - 2\phi_{q,i} + \phi_{q+1,i}}{h^2}
$$

(2.5)

where $h$ is the length of each of the two elements between the DOF $(q-1)$ and $(q+1)$.

Wang et al. (2000) performed a sensitivity analysis of the rate of change in the modal curvature on structural damage to the Tsing Ma Bridge. A damage index for $j$th mode is defined as

$$
\text{Index}_j(i) = \frac{|C''_d(j,i) - C''_u(j,i)|}{\sum_i |C''_d(j,i) - C''_u(j,i)|}
$$

(2.6)

where $i$ represents the segment number along the longitudinal direction of the bridge. The superscripts $u$ and $d$ denote the undamaged and damaged structure, respectively. $C_j(i)$ is the $j$th modal curvature calculated by

$$
C_j(i) = \frac{\phi_j(i-1) - 2\phi_j(i) + \phi_j(i+1)}{2l_i^2}
$$

(2.7)
where $\phi_j(i-1)$, $\phi_j(i)$ and $\phi_j(i+1)$ are the values of the $j$th mode vector at the $(i-1)$th, $i$th and $(i+1)$th nodes, respectively. Finally, the damage index defined by Equation (2.7) is normalized to yield a so-called $Z$-value as follows

$$Z_i = \frac{\text{index}(i) - M(\text{index})}{\sigma(\text{index})}$$

(2.8)

where $M$ and $\sigma$ are the mean and standard deviation of the index sequence. Compared with the modal flexibility index defined above, the $Z$-value index was found to be less sensitive to the damage that had occurred to the bridge.

Chance et al. (1994) found that numerically calculating curvature from mode shapes resulted in unacceptable errors. Instead, they used measured strains to measure curvature directly, which dramatically improved results. Chen and Swamidas (1994) found that strain mode shapes facilitated the location of a crack in a cantilever plate using FEM simulation.

Nwosu et al. (1995) evaluated changes in strain resulting from the introduction of a crack in a tribular T-joint. They found these changes to be much greater than any shifts in frequency and to be measurable even at a relatively large distance from the crack.

### 2.1.5 Methods Based on the Frequency Response Function (FRF)

Law et al. (1992) developed sensitivity from a formulation based on the change in the FRF at any point, rather than just at the resonances. In practice, many points of the FRF around the resonances are taken, and a least-squares fit is used to determine
the changes in the physical parameters. This method requires both a before- and after-damaged FRF and a physical model relating the damage parameter to a physical parameter such as stiffness.

Wu et al. (1992) identified the damage in a three-storey building model by selecting the first 200 points of the frequency response function as input to a BP neural network. Chaudhry and Ganino (1994) used measured FRF data over a specified frequency range as input to a BP neural network to identify the presence and severity of delamination in debonded beams.

Juan and Dyke (2000) presented and experimentally verified a new technique to identify damage based on changes in the component transfer functions of the structure, or the transfer functions between the floors of a structure. Multiple locations of damage can be identified and quantified using this approach.

2.1.6 Methods based on Neural Network Technique

Neural networks have been promoted as universal function approximators for functions of arbitrary complexity. In recent years, there has been increasing interest in using neural networks to estimate and predict the extent and location of damage in complex structures.

Wu et al. (1992) used a back-propagation neural network to identify damage in a three-storey building modeled by a two-dimensional ‘shear building’ driven by earthquake excitation. The damage was modeled by reducing member stiffness by 50% to 75%. The neural network was used to identify the map from the Fourier transform of acceleration data to the level of damage in each of the members. The
first 200 points in the fast Fourier transform (FFT) (0 Hz to 20 Hz) were used as network inputs. A network architecture with one hidden layer and 10 hidden nodes was selected, and 42 training cases were used.

Worden et al. (1993) used a back-propagation neural network to identify damage in a 20-member framework structure. Damage was modeled by completely removing one of the structural members. The neural network was used to identify the map from static strain data to a subjective measure of the damage. The strain in eight members was used for input, and there was one output for each of the eight members where damage was to be identified. A three-hidden-layer design with 12,12 and 8 hidden nodes, respectively, was chosen. The network was trained on data generated by an FEM and tested on an experimental model of the same geometry. When applied to experimental data, the system was mostly able to identify the location of the damage. Nevertheless, there were misclassifications of frequency owing partially to the large size of the test data.

Stephens and Vanluchene (1994) used a back-propagation network to identify damage in multi-storey buildings. The damage was modeled by introducing actual cracks into a concrete model of the structure. The network was used to identify the map from three indices of empirical damage to a qualitative scale of damage in the structure. The three indices were measures of maximum displacement, cumulative energy dissipated in the building, and degradation of stiffness.

Klenke and Paez (1996) used two probabilistic techniques to detect damage in an aerospace housing component. The first technique used a probabilistic neural network (PNN). The second technique used a probabilistic pattern classifier (PPC) of the authors' design. Both methods attempted to ascertain the existence of damage,
but neither attempted to quantify the extent of the location of the damage. Lloyd et al. (1999) applied probabilistic neural networks to detect change points in mechanical systems, where networks were utilized to approximate the multivariate density of the training data.

Suh et al. (2000) presented a hybrid Neuro-genetic technique, to identify the location and depth of a crack on a structure. Feed-forward multi-layer neural networks trained by back-propagation were used to learn the input-output relation of the structural system. In addition, with this neural network, a genetic algorithm was used to identify the location and depth of the crack, minimizing the difference from the measured frequencies.

This concludes the review of the literature on damage detection techniques based on modal parameters. The subsequent section is focused on modern modal parameters estimation approaches, which utilize only structural response data.

2.2 OUTPUT-ONLY MODAL IDENTIFICATION

2.2.1 Overview

The vibration-based structural damage detection or health monitoring system relies on modal characteristics. Therefore, the feasibility and accuracy of modal identification is an important presupposition to ensuring the robust implementation of damage identification. The application of system identification to vibrating structures has yielded a research domain in mechanical engineering known as experimental modal analysis. A modal model, consisting of eigenfrequencies,
damping ratios and mode shapes, is identified from data on vibration. Conventional methods of identifying modal parameters depend on input (excitation)-output (response) data and on an FFT-based spectrum analysis technique, where the modal parameters are found by fitting a model to the so-called frequency response function, a function relating excitation and response. There are many textbooks that give an extensive overview of methods of estimating input-output modal parameters (Ewins 1995; Heylen et al. 1995; Maia et al. 1997; Nuno and Julio 1997; Ewins 2000) and they are not addressed in the following review of the literature on modal identification.

With the increasing requirement to detect damage in real-life structures or develop health monitoring systems, field ambient vibration measurements have become obligatory due to difficulty of being excited of large-scale civil structures. Testing for ambient vibration utilizes natural excitation exerting, such as wind on building structures, ocean waves on offshore structures and traffic loading on bridge structures, which are usually of small amplitude and difficult to measure. As a result, only ambient response data are available in field measurements of ambient vibration, which poses a great challenge for the input-output modal identification technique. Recently, output (response)-only approaches to modal identification have been raised in some studies. They provide a promising solution to estimating structural dynamic parameters in cases where only ambient responses can be measured. However, most of these approaches have been applied to laboratory testing structures and long-span bridges, rather than to building structures. From the mathematical point of view, output-only approaches to estimating modal parameters can be categorized into the time-domain method and frequency-domain method. Stochastic subspace identification (SSI) represents a typical time-domain output-only method of modal
identification, and a complex mode indicator function (CMIF) represents a typical frequency-domain counterpart.

2.2.2 Stochastic Subspace Identification Method

Stochastic subspace identification is a time-domain technique. The method is derived from system state-space model theory and from the Hankel matrix constructed by correlation functions of measured response data. Some recent applications for estimates of response-based modal parameters can be found in various types of structures.

Abdelghani et al. (1998) presented a detailed study comparing methods of subspace identification, which apply to identifying state space models of flexible structures. The methods include an eigensystem realization algorithm with observer/Kalman filter Markov parameters computed from input/output data (ERA/OM), the robust version of the numerical algorithm for identifying subspace state space systems, and a refined version of the past outputs scheme of the multiple-output error state space (MOESP) family of algorithms. The comparison was performed by simulating experimental data using the five-mode reduced model of the NASA mini-mast flexible structure.

Bosse et al. (1998) presented an online modal parameter estimation algorithm using subspace methods. The underlying theory and implantation of hardware were described. The algorithm was applied to both the model and the experimental data for a 4-meter laboratory truss structure. An experimental evaluation of this algorithm demonstrated that the technique accomplished the objective of tracking multiple modes of a complex dynamical system using multiple sensors.
Katayama and Picci (1999) solved the stochastic realization problems for a
discrete-time stationary process with an exogenous input, and presented methods for
identifying subspace. They derived the state equations of the optimal predictor of the
future outputs in terms of the state vectors and the future inputs. The state vector was
chosen by using the canonical correlation analysis of past and future conditioned on
the future inputs. Four stochastic subspace identification algorithms were then
derived.

Shi *et al.* (2000) investigated a modal extraction strategy with a subspace
method for mechanical structures in operating conditions. Two methods for
determining system matrices were presented using a block Hankel matrix of raw data
and covariants, respectively. The performances of the methods were then critically
evaluated for a cantilever beam. The results of the experiment showed that the
methods were as accurate as the classical modal estimation methods.

Peeters and Roeck (2001) presented a review of the output-only modal
identification technique, where both time-domain and frequency-domain methods
were addressed. Regarding the time-domain implementation, two methods were
presented. The first was called the time-domain covariance-driven method, where
impulse response and output covariance (of a system excited by white noise) were
expressed as a function of the system parameters. The second, called the covariance-
driven stochastic subspace identification method, identified a stochastic state-space
model from output-only data.

### 2.2.3 Complex Mode Indicator Function Method

The complex mode indicator function method is based on the decomposition of
the response spectra matrix and, thus, is a frequency domain technique. It has been proven to be advantageous in computation simplicity and being mathematically straightforward. The method has received an increased interests in output-only modal parameter estimation.

Shih et al. (1989) introduced the concept of complex mode indicator function (CMIF) and its application in estimating parameters of spatial domain. The concept of CMIF was developed by performing a singular value decomposition (SVD) of the frequency response function matrix at each spectral line. The CMIF identified modes by showing the physical magnitude of each mode and the damped natural frequency for each root. The results showed that CMIF is a simple and efficient method of identifying the modes of the complex system. Nevertheless, the SVD was executed on the frequency response function matrix, rather than on the response spectrum matrix. Therefore, strictly speaking, the proposed method was not an output-only modal identification technique.

Brincker et al. (2000) introduced a modal identification method from ambient responses by using frequency domain decomposition. By introducing a singular value decomposition of the spectral density function matrix, the response was separated into a set of freedom systems of single degree, each corresponding to an individual mode. The diagonal elements of the decomposed singular value matrix constructed complex mode indicator functions. The method was applied on test data of a frame structure. The results showed that close modes can be identified with high accuracy even in the case of strong noise contamination of the signals.

Peeters and Roeck (2001) presented a review of the output-only modal identification technique, where both time-domain and frequency-domain methods
were addressed. Regarding the frequency-domain implementation, the complex mode indicator function method was presented by executing the singular value decomposition on the spectrum matrix

$$S_j(j\omega) = U(j\omega)\sum (j\omega)U^H(j\omega)$$  \hspace{1cm} (2.9)

where $U \in \mathbb{C}^{nd}$ is a complex unitary matrix containing the singular vectors as columns. The diagonal matrix $\sum \in \mathbb{R}^{nd}$ contains the real positive singular values in descending order. The method was already being used in the beginning of the 1980s to obtain the modes of a vibrating system subjected to natural excitation.

Thus far, vibration-based damage detection approaches and modern modal parameter identification techniques have been reviewed in detail. The remainder of this chapter is devoted to exploring studies on detecting damage in buildings.

## 2.3 DAMAGE DETECTION IN BUILDINGS

### 2.3.1 Overview

Vibration-based damage identification techniques are mainly derived from techniques of diagnosing damage mainly in aerospace structures and rotating machines. In the past decades, great efforts have been made by civil engineers to identify damage in civil structures by measuring vibrations.
Most of the existing studies on identifying vibration-based damage and health monitoring were first verified by numerical or non-in-situ experimental simulations. Generally, for some specific structures, one developed damage indicator or damage identification technique was usually more appropriate or performed better than others. The structures that have been the subject of the greatest research interest include metal and concrete beams, trusses, plates, offshore platforms, shells and steel frames, bridges and aerospace structures, and so forth. Although many successful applications have been achieved on these structures in recent years, the identification and assessment of damage in complex structures such as buildings remains a challenging task for civil engineers.

A considerable number of studies on identifying damage in beams and trusses using measurements of vibration can be found. These steel or concrete beams and trusses are usually laboratory test specimens. Damage in such types of structures are often simulated by introducing cracks (beams), connection failures (trusses) and changes in material (beams and trusses). Likewise, much effort has been paid to applications of vibration-based techniques of detecting damage in bridges. Since 1979, numerous studies involving the development and application of damage detection techniques for bridge structures have been reported. The work has been motivated to aggregate extent by several catastrophic bridge failures. Salane et al. (1981) used changes in the dynamic properties of a three-span highway bridge during a test of fatigue as a possible means of detecting structural deterioration resulting from fatigue cracks in the girders of the bridge. Spyrokos et al. (1990) performed a series of experiments on a set of beams designed to yield dynamic responses similar to those seen in actual bridges. A different damage scenario (type, location, degree) was devised for each beam, and low-level free vibration tests were performed. It was
found that a change in frequency may be an insufficient indicator of structural safety (a less than 5% change in frequency was associated with critical damage). Aktan et al. (1994) assessed the reliability of modal flexibility as an indicator of the condition of a bridge by comparing the measured flexibility to the flexibility obtained using a static load truck test. They used the measured modes to calibrate their FEM, which they then used to assess conditions when no baseline data set was available. Farrar and Cone (1995) presented the results of a damage-detection experiment performed on the I-40 bridge over the Rio Grande river in Albuquerque, New Mexico. In general, the results of their research indicated that modal frequency is not a sensitive indicator of damage, and the mode shapes were shown to be more sensitive indicators of damage. Kong et al. (1996) performed studies on ambient vibration on a scale-model steel-girder bridge in both undamaged and damaged conditions. Damage was imposed by removing a roller support under the girder. Resonant frequencies and damping ratios were measured before and after the damage.

Researchers of dynamics group at the Hong Kong Polytechnic University have conducted many studies on large-span flexible bridges in Hong Kong. First, they developed precise damage detection-oriented finite element models for the Tsing Ma Bridge, Ting Kau Bridge and Kap Shui Mun Bridge, respectively. Next, based on the developed FE models, an identification of dynamic characteristics and an analysis of modal sensitivity were conducted (Ko et al. 2000; Ni et al. 1994; Ni et al. 1999; Ni et al. 2000; Wang et al. 2000; Xue et al. 1997). Finally, a variety of damage indices based on the achieved modal parameters were developed (Chan et al. 1999; Chan et al. 2000; Ni et al. 2000; Wang et al. 2000). These indices include those for the frequency change ratio, mode shapes-based curvatures, Z-values, modal flexibility, and so forth. A hierarchical damage identification strategy was proposed for damage
alarming, localization and severity identification (Ko et al. 2001).

However, much less effort has been devoted to applying vibration-based damage recognition techniques to building structures, compared with other civil structures like beams, trusses and bridges. The reason for this may be the complexity and nonlinearity of such structures. It is also a great challenge to develop an accurate analytical model for a highly complicated building, on which model-based approaches for detecting damage rely. The following survey explores existing studies from two aspects: member-level damage identification of frame structures and overall damage evaluation of existing buildings.

This dissertation is devoted to developing both floor (region) - level damage identification and overall post-earthquake damage evaluation methods, from data measuring white noise or earthquake excitations. Tests for white noise are conducted to identify cumulative seismic damage (including the extent and location of damage), whereas earthquake simulation testing data are used to make overall assessments of post-earthquake damage.

2.3.2 Member-level Damage Identification on Framed Structures

Most previous studies related to building structures focused on multi-storey frame structures, and the majority of these applications were conducted on steel frames. The damage on these types of buildings was usually simulated by losing connections at beam-column joints or reducing inter-storey stiffness. Typically, experimental validation of the structural integrity of a building was carried out on a component level as opposed to an as-built system level, and was performed in a research mode rather in a production mode.
Hearn and Testa (1991) formed fatigue cracks in a welded steel frame. The cracks formed where the steel angles were connected to the connection plates. The authors applied their method of assessing damage, which involves examining the initial mode shapes of the structure and changes in the ratios of the resonant frequencies. Using this method, they were able to locate the damage. However, they noted that the location of damage in symmetric portions of the structure cannot be distinguished using this method. Changes in the natural frequency were relatively insensitive to damage.

Koh et al. (1995) applied a condensation method for detecting local damage in multi-storey frame buildings to a numerical simulation of a 12-storey plane-frame structure and a 6-storey steel frame structure. Damage, ranging from reductions of 10% to 45.6% in the stiffness of the storeys, was successfully identified without false indications in the undamaged floors.

Lam et al. (1995) applied a detection routine based on mode shape to a steel frame. The frame consisted of two 100mm-deep steel I-beams, 2.82m tall, connected laterally by two 150mm-deep steel I-beams at the top and midway point of the structure. The columns were welded to a steel plate at the ground and the beam-column connections were four bolted angles. Damage was simulated by bolting the top and bottom angles at a connection. The method used by the authors enabled the correct damage state to be selected from six possible states, defined as loose connections at the four beam-column joints and the two column-ground joints.

Klaus and Norris (1995) developed a set of analytical formulations for detecting damage based on modal parameters, in which no a priori knowledge of the modal characteristics of a corresponding baseline structure was required; whereas information on the geometry of the structure reflected in its mass and stiffness
distribution was needed. The characteristic equation of an arbitrary undamped MDOF system is given by

\[ (-M \omega_i^2 + K) \phi_i = 0 \]  

where \( M \) is the system mass matrix, \( K \) is the system stiffness matrix, \( \omega_i \) is the \( i \)th eigenfrequency, and \( \phi_i \) is the \( i \)th corresponding mode shape or eigenvector. Rearranging Equation (2.11) and defining the \( i \)th eigenvalue, \( \lambda_i \), as the square of the eigenfrequency yields

\[ \lambda_i = \frac{K_i}{M_i} \]  

where \( M_i \) and \( K_i \) is the \( i \)th modal mass and modal stiffness, respectively, given by

\[ M_i = \phi_i^T M \phi_i, \quad K_i = \phi_i^T K \phi_i \]  

Correspondingly, the damaged structure can be characterized by the following quantities with asterisks

\[ \lambda_i^* = \frac{K_i^*}{M_i^*} \]
Normalizing the mode shapes with respect to the modal mass leads to

$$
\phi_1^T \Delta K \phi_1^* = \lambda_i^* - \phi_1^T K \phi_1^*
$$

(2.14)

where $\Delta K$ is the change in the stiffness matrix between the damaged and the intact structure. On defining $\Delta K_j$ as the contribution of the $j$th member of $\Delta K$, Equation (2.15) can be rewritten as

$$
\sum_{j=1}^{NE} \frac{\phi_1^T \Delta K_j \phi_1^*}{\phi_1^T K \phi_1^*} = \frac{\lambda_i^*}{\phi_1^T K \phi_1^*} - 1
$$

(2.15)

where $NE$ denotes the number of structural members. The matrix $\Delta K_j$ may be written in the following form

$$
\Delta K_j = \alpha_j K_j
$$

(2.16)

where $K_j$ is the contribution of the $j$th member of the stiffness matrix of the initial system, and $\alpha_j$ is a scalar representing the relative damage to the member.

Finally, the identification of damage is converted to a problem of solving equations set
\[ \sum_{j=1}^{NE} F_j \alpha_j = Z_i \]  

(2.17)

where

\[ F_i = \frac{\phi_i^T K \phi_i}{\phi_i^T K \phi_i} \quad \text{and} \quad Z_i = \frac{\lambda_i}{\phi_i^T K \phi_i} - 1 \]  

(2.18)

This approach has conceptual clarity and has been successfully applied to identifying damage in a 10-storey shear building model. First, no initial (baseline structure) modal properties are needed; and only limited modes of modal properties are needed. Also, the location and extent of the damage can be identified simultaneously. Nevertheless, the method has some apparent drawbacks: (a) For purpose of accurately identifying damage, it is assumed that the initial (baseline structure) stiffness matrix and modal quantities of the damaged structure can be achieved with high fidelity; however, this is often challengeable in realization (b) There is a need to solve the inverse equation sets. In the case of the direct application of this approach to tall buildings, a problem arises in that the number of structural members becomes too large to be solved efficiently. (c) There is a need to obtain a complete measurement for one mode shape, although only the lowest modes of the modal properties are needed. Usually, only a few measurement points are available in real structural modal testing. Consequently, some technical posers need to be solved in tall building structures damage identification.
Prion et al. (1996) performed test of impact and ambient vibration on a four-storey steel frame with steel shear walls. Quasi-static cyclic loading was applied to the top storey to simulate earthquake excitation. Resonant frequencies, modal damping and mode shapes were calculated for the structure in both the undamaged and damaged state. It was found that changes in viscous damping did not show a consistent trend and was regarded as a poor indicator of damage.

Skjaerbaek et al. (1996) developed a procedure to locate and quantify damage in a multi-storey reinforced concrete frame structure from a single response measurement made at the top of the structure. Damage in a substructure was defined as the average relative reduction of the stiffness matrix of the substructure that reproduces the two lowest eigenvalues of the overall structure. First, the time-varying nature of the resonant frequency in a damaged concrete structure is approximated by a smoothed fit to the measured frequency-time history. The frequency-time history is estimated using ARMA models. A maximum softening damage index, $\delta_m$, is then defined as

$$\delta_m = 1 - \frac{T_o}{T_n}$$  

(2.19)

where $T_o$ is the undamaged period and $T_n$ is the maximum period calculated during the softening portion of the response.

The authors applied this method to numerical simulations of a degrading reinforced concrete frame subjected to various seismic inputs. The proposed method correctly located the damage in the structures at higher levels of excitation, but
identified undamaged areas as damaged when very low levels of excitation were studied.

Straser and Kiremidjian (1996) applied their method of identifying damage using extended Kalman filtering (EKF) and nonlinear hysteretic models to a scale-model six-storey structure that was constructed with steel plates for the floors and threaded steel rods to simulate the supporting columns.

Zhou et al. (1998) applied the vibration parameter identification technique to assess the damage to reinforced concrete frame structures. First, the locations of the damage in the structure were identified using vibratory residual vectors. Then, the physical parameters of the structure were identified by the weighted sensitivity analysis method, so that the severity of the damage in the structure was assessed.

Escobar et al. (2001) proposed a method to locate and assess damage in the structural elements of frame buildings. Damage was expressed as the loss of stiffness in the structural elements. The transformation matrix method for detecting damage in the structural elements of frame buildings was developed. This allows the magnitude of the damage to the structural elements to be located and assessed by considering the contribution of each element to the overall performance of the structure. A three-dimensional frame model with different states of simulated damage and a reinforced concrete plane model damaged using an earthquake record as excitation were studied.

Takuji (2003) presented several robust strategies in the model-based damage detection of multi-storey buildings, to overcome such problems as noise contamination, system nonlinearity, estimation errors, modeling uncertainty, and so forth. The basic idea consists of decreasing the number of physical parameters to be estimated, increasing the number of modal properties with good accuracy, enhancing sensitivity to damage, and reducing noise and nonlinear effects. The effectiveness
and limitations of these strategies were discussed through a series of shaking table tests of two small-scale test structures.

2.3.3 Overall Damage Evaluation of Existing Buildings

Most of the member-level damage severity and location identification methods have been developed from testing framed structures and are not applicable to real existing buildings. Most of the damage in these real buildings consists of structural cracks incurred by accidental impact loading such as the occurrence on an earthquake. Much more concern is paid to the overall assessment of the damage caused by cracks in buildings than to the identification of member damage. It is vital to estimate the overall seismic damage to building structures after each occurrence of an earthquake to both evaluate the loss from damage and to repair any failures in the structure.

In the past decades, many researchers have developed global damage detectors to assess the seismic damage of building structures (Powell and Allahabadi 1988; Ricles and Kosmatka 1992; Williams and Sexsmith 1995; Cobb and Liebst 1997). These damage indices are only qualitative and not attempt to identify the locations and extent of the damage. These indices fall into two broad categories: (a) strength-based damage indices; and (b) response-based damage indices, including static response and dynamic response.

2.3.3.1 The strength-based methods

Strength-based damage indices are simple and do not require a response analysis. However, the index must be calibrated against observed damage using a large database. Shiga et al. (1968) first proposed the strength-based damage indices
that were later applied by Yang et al. (1980). These indices depend on the geometry of structural elements such as the column and wall, and on their general material properties. Detailed information on structural and material models and descriptions of ground motion consistent with the site of the structure are needed. Strength-based methods usually require the development of a mathematical model of the building being investigated, and they are not related to the vibration-based damage-detection technique. Thus, a corresponding review of the literature has not been conducted here.

2.3.3.2 The response-based methods

Dipasquale and Çakmak (1990) defined global damage indices, stiffness degradation (cracks), plastic deformation (the yielding of reinforcement bars), and the combined effect of plastic deformation and stiffness degradation (the onset of structural damage), respectively, based on equivalent linear models from the vibrational parameters. In such a way, structural damage can be detected through an analysis of records of strong motion, even without any immediate need for inspection.

Gianni et al. (1996) described a system to monitor the structure of the Brunelleschi Dome, specifically, to observe the development of cracks. The data was evaluated based on an understanding of the structural behaviour of the Dome. The correlation between variations in the width of cracks and environmental loads was analyzed. A numerical procedure was developed to estimate the trend of increase in the mean width of the cracks. Finally, the results were compared to those obtained by analyzing other monitoring systems previously installed in the Dome and the historical data. Some considerations and suggestions on the use of monitoring systems in the structural preservation of historic buildings were also discussed.
Ghobarah et al. (1999) reviewed and evaluated some of the available response-based damage models and developed several response-based damage indices to evaluate damage. These response-based damage indices utilize the static and/or dynamic response of the structure, such as maximum deformation and cumulative damage. The proposed assessment of damage was based on the determined response of the structure and the performance characteristics of its members.

Ductility ratio (DR)

The ductility ratio is defined as the ratio of the maximum deformation to the yield deformation. It has been used extensively in seismic analysis to evaluate the capacity of structures undergoing inelastic deformation and to develop inelastic response spectra. As a damage index, the ductility ratio may be unsatisfactory, especially when shear distortion in joints and the pullout of beam bottom bars is anticipated. As demonstrated by experimental studies, the ductility ratio does not account for the effect of the duration and frequency of the ground motion. It is normally assumed that failure occurs when the ductility demand (response) exceeds the structural ductility (capacity) that is equal to the ratio of the ultimate deformation under a monotonic static load to the yield deformation.

Interstorey drift (ID)

The interstorey drift is the maximum relative displacement between two storeys normalized to the storey height. The percentage of damage to the structure is given by

\[
\% \text{ of damage} = 50 \times (\text{maximum inter-storey drift in percentage}) - 25 \quad (2.20)
\]
From the analysis of the test data on components and small-scale structures, it was found that an ID value smaller than 1 percent corresponds to damage to non-structural components while values of ID larger than 4 percent may result in irreparable structural damage or collapse. A collapse was considered to occur when the ID exceeds 6 percent. Similar to the damage index based on the ductility ratio, the interstorey drift did not account for the effect of cumulative damage due to repeated inelastic deformation. In addition, the relationship between damage and interstorey drift varied depending on the maximum deformation at the time of the collapse.

Slope ration (SR)

The slope ratio was taken as a measure of damage due to stiffness degradation during seismic loading. It was defined as the ratio of the slope of the loading branch of the force-displacement diagram to the slope of the unloading branch. From tests on small-scale structural systems, it was determined that SRs with values of 1.0 and 0.2 correspond to safe structural behaviour and critically damaged structures, respectively.

Flexural damage ratio (FDR)

It was suggested that the ratio of the initial stiffness to the reduced secant stiffness at the maximum displacement be used as a measure of damage. Damage indices based on extreme inelastic deformations seem to be strongly correlated so that their predictions are usually similar. The FDR index also does not account for the effects of cumulative damage caused by repeated reversals of load.

Maximum permanent drift (MPD)
Permanent drift was found to be closely related to the plastic deformations in a structural system. Some researchers (Toussi and Yao 1982; Stephens and Yao 1987) introduced a qualitative classification of damage which, among other things, included the permanent drift experienced by the building. They defined the following four levels of structural damage: (a) safe, with a storey drift not exceeding 1 percent of the storey height and no permanent drift; (b) lightly damaged, with permanent displacement of approximately 0.5 percent of the storey height; (c) damaged, with permanent displacement of 1 percent of the storey height; and (d) critically damaged, with a top-storey displacement showing some aperiodicity at the end of the record with a poor correlation between base-shear and top-level displacement. The shortcoming of the maximum permanent drift as a measure of damage is that light damage implies a maximum permanent drift of 0.5 percent or less. However, a permanent drift of 0.5 percent does not necessarily indicate light damage. A damaged non-ductile frame structure may exhibit no permanent drift, even after severe inelastic deformation.

**Normalized cumulative rotation (NCR)**

A simple measure of structural deterioration during a seismic event is the sum of all inelastic excursions experienced by the structure. The value of this measure depends on the duration and intensity of the earthquake. The normalized cumulative rotation was defined as the ratio of the sum of the inelastic rotations during half cycles to the yield rotation (Banon and Veneziano 1982). A statistical analysis of the data on beam-column elements subjected to cyclical loads showed that damage indices based only on cumulative inelastic deformation or dissipated energy may be inadequate to characterize the complex process of damage propagation and the subsequent failure in concrete members.
Low-cycle fatigue (LCF)

The theory of low-cycle fatigue was applied to the seismic analysis of structures subjected to strong ground motion to estimate the state of damage (Stephens 1985). The determination of the damage index was found to be somewhat complex and dependent on the entire response history. Moreover, the index does not account for the effect of the maximum inelastic deformation.

The authors finally used ductile and non-ductile three-storey reinforced concrete frame office buildings to illustrate the application of the damage evaluation procedure and to compare the results to a number of response-based indices. It was concluded that critical values of the $DR$, $SR$ and $FDR$ damage indices were determined from laboratory tests and field observations. Therefore, their use in the prediction of seismic damage for structures with characteristics significantly different from those used in the calibration process requires caution. Additional difficulties in the use of these damage indices related to the differences between the characteristics of the expected earthquake and those of the earthquakes used in the calibration, such as intensity, duration and frequency content.

Gambarotta and Lagomarsino (1997) proposed a damage model for the seismic response of brick masonry shear walls. The constitutive equations for the brick masonry were obtained through a homogenization procedure involving the damage model and simple damage constitutive equations for the brick layer. The constitutive model was used in a finite element analysis of the lateral response of brick masonry shear walls in-plane loaded either by cyclic horizontal actions superimposed on vertical loads or by dynamic loads, which were representative of the seismic actions. The capabilities and the validity limits of the finite element analysis obtained by the continuum approach were indicated from the simulation of experimental results.
concerning rectangular slender and squat shear walls and also by the comparison with the theoretical results from the composite model. Finally, the same shear wall was analyzed under a dynamic strong motion at the base from which the suitability of the approach for evaluating the seismic vulnerability of masonry buildings emerged.

Koy et al. (1997) dealt with the prediction of global and localized damage and with the future reliability estimation of partly damaged reinforced concrete structures under seismic excitation. Initially, a global maximum softening damage indicator was considered, based on the variation of the eigenfrequency of the first mode due to the deterioration in the stiffness and strength of the structure. Using an ARMA model, the excitation and displacement response time series were employed to identify the instantaneous softening. The proposed model was then generalized for a MDOF system. The performance of the hysteretic model was illustrated on an RC experimental structure, which is a 1:10 scaled planar 10-storey 3-bay RC frame and consists of two parallel frames working in parallel with ten uniformly distributed storey weights. The present work could predict the global and localized damage and the future estimation of reliability under seismic excitation in terms of instantaneous softening and top-storey displacement.

Elenas (2000) described the interdependency between several seismic acceleration parameters and the behaviour of the reinforced concrete frame structures in the form of correlation coefficients. The structural behaviour was expressed in the form of overall structural damage indices. As seismic acceleration parameters, the peak, spectral and energy parameters were considered. The overall structural damage was quantified as the maximum softening.
The evaluation of overall seismic damage mentioned above often does not lend itself well to rational predictions of the strength reserves and response characteristics of a structure with a specified degree of damage because: (a) future earthquakes may have different intensities, duration and frequency content; (b) buildings in other locations and recently built structures that are designed to current codes may differ significantly from the structures used to develop the damage statistics; (c) the dynamic characteristics of the population of structures included in the statistical analysis may have altered due to repairs and to the accumulation of damage from previous earthquakes; and (d) many uncertainties exist involving the structure itself, the ground conditions and the earthquake excitations.

This dissertation proposes to evaluate the overall state of damage of tall buildings after detailed localizing of damaged regions or storeys. Considering the fact that only dynamic testing was performed on the scaled tall building model and no static responses are available, the dynamic responses will be utilized to estimate the modal characteristics, based on which the overall assessment of post-earthquake damage is to be carried out. The goal is to extract information concerning damage from the history of the dynamic response and the history of the modal parameters.

2.3.3.3 Dynamic response and neural network based methods

Cabanasa et al. (1997) presented an approach to quantifying and predicting potential structural damage by measuring the ground motions of the earthquake. To relate the energy of the ground motion with the occurrence of the damage, and eventually achieve a better assessment of seismic risk, two parameters were estimated: Arias intensity (unfiltered and filtered in certain frequency ranges) and Cumulative absolute velocity (CAV). They were defined as follows:
Arias intensity

The Arias intensity was first proposed by Arias (1970). It is a measure of the capacity for damage based on the energy of the ground motion dissipated by a population of structures and can be expressed as

\[
I_y = \frac{\pi}{2g} \int_0^{+D} a_i(t)a_j(t)dt
\]  \hfill (2.21)

where \( D \) is the duration of the record and \( a_i(t) \), \( a_j(t) \) are the acceleration amplitudes of the orthogonal components \( i \) and \( j \), respectively. The Arias intensity characterizes the energy content of a strong motion record. In order to take into account the frequencies of different distributions of structures, the accelerogram can be filtered in different bands, thus obtaining for each a computed value referred to as the filtered Arias intensity.

Cumulative absolute velocity (CAV)

The CAV is defined as the area under the absolute accelerogram

\[
CAV = \int_0^{+D} |a(t)|dt
\]  \hfill (2.22)

From the above definition, the CAV can be seen as the sum of the consecutive peak-to-valley distances in the time history of velocity. Another interpretation is that it is the area under the acceleration versus the duration curve. In this way, the CAV takes into account the contribution of both the amplitude and the duration of motion.
Lus et al. (1999) addressed the system identification of linear structural systems using the earthquake-induced time histories of the structural response. They then predicted the time histories of the structural response and the damage when subjected to earthquake-induced ground motion. The proposed identification methodology was based on the Eigensystem Realization Algorithm (ERA) and on the Observer/Kalman filter Identification (OKID) approach, using general input-output data via Markov parameters. The efficiency of the proposed technique was shown by numerical examples for the case of a finite element model of an eight-storey building subjected to earthquake excitation.

Gupta et al. (2001) presented a method of estimating structural damage from a known increase in the fundamental period of a structure after the occurrence of an earthquake. The study proposed a modified Clough-Johnston single-degree-of-freedom oscillator to establish the correlations between the response functionals, in the case of a simple elasto-plastic oscillator. It was assumed that the proposed oscillator closely models the response of a given multi-degree-of-freedom system in its fundamental mode throughout the duration of the excitation. The proposed model considered the yield displacement level and ductility supply ratio-related parameter as two input parameters that must be estimated over a narrow range of the ductility supply ratio from a frequency degradation curve. This curve was identified from a set of recorded time histories of excitation and response. Useful correlations of strength and stiffness degradation with damage were obtained, in which a simple damage index based on maximum and yield displacements and ductility supply ratio was considered.

The methodology based on neural networks has already been widely applied in a vibration-based damage-recognition application, mostly carried out on beam and
truss-like structures, just as with other methods. For building structure, Mitsuru et al. (1996) presented an approach based on neural networks to detect changes in the characteristics of structure-unknown systems through simulation studies with linear as well as nonlinear models and through the application of actual data obtained from measurements of ambient vibration on a steel building. In this approach, the training of a neural network for the purpose of identification was performed using the measurements of vibration from a 'healthy' system. Subsequently, the trained network was fed comparable measurements of vibration from the same structure under different episodes of response in order to monitor the health of the structure. In addition, authors used time-domain information, rather than modal parameters to train and test the neural network, namely, the inputs to the network were the relative displacement and relative velocity of the system, and the output was the restoring force in the time series. The methodology was then applied to actual data obtained from ambient vibration measurements of a seven-storey steel building that was damaged under the strong seismic motion of the Hyogo-Ken Nanbu earthquake in 1995. The damaged frames were repaired after the earthquake, to restore the structure to the same condition it was in before the earthquake. Thus, the structure after the repair could be regarded as having recovered its structural integrity to become a 'healthy' structure. This was then used as the baseline for network training. The time-domain method needed no forced testing of vibration and modal analysis, which is especially meaningful when applied to real building structures. Furthermore, there was also no need to use a structural mechanical model. The actual data obtained from ambient vibration measurements was what was used to train a neural network.

Stefano et al. (1999) proposed a method of predicting the mechanisms of seismic damage using probabilistic neural networks that produced a Bayesian
classification. The final goal was to exploit the capabilities of neural networks for fundamental learning and generalization to obtain an estimate of the vulnerability of structural systems. The proposed method was applied on about 100 churches damaged by the Emilia Romagna in 1987 and on about 250 churches damaged by the Friuli earthquake in 1976.
CHAPTER 3

SHAKING TABLE TESTS OF A SCALED TALL BUILDING MODEL STRUCTURE

3.1 INTRODUCTION

A scaled high-rise model building was designed and fabricated by some researchers (Lam et al. 2002) from the department of civil and structural engineering of the Hong Kong Polytechnic University. The aim was to gain an understanding of the seismic resistance of typical high-rise residential buildings in Hong Kong, and, ultimately, to provide guidelines on seismic provisions. Based on this model structure, vibration-based damage detection researches for tall building structures are to be intended thereafter. For both the earthquake response study and seismic damage identification purposes, shaking table tests of the scaled model structure are to be carried out, excited by some artificially simulated earthquake records on sites with different soil conditions and random white-noise exerting respectively.

The outline of this chapter is as follows: Firstly, the prototype building is introduced. Then, the scaled building structure simulated from the prototype building is described in detail, including similitude law, fabrication procedure and the geometry specification of the finished model. The final two sections are the heart of this chapter. They provide a through description of the entire shaking table tests procedure and address the results of the visual inspection on seismic damage after the
occurrence of each level of earthquake. In particular, the inspected results provide a baseline for the subsequent chapters of studies on identifying and assessing seismic damage.

3.2 THE PROTOTYPE BUILDING

The experimental object of this research is simulated from a kind of high-rise reinforced concrete apartment building, which is common in Hong Kong and is the type of building constructed by the government of Hong Kong Special Administrative Region. This kind of apartment building structure has some particular characteristics. For example, they are usually built up to about 39 storeys with concrete-frame apartment storeys or parking area at lower storeys, while their upper bodies are made using reinforced concrete walls. Their plan layout is usually designed as symmetric “+” type, while the retract part in the top storeys is used as an equipment room. The most prominent characteristic of this kind of building is the thick transfer plate located between the upper apartment body and the lower apartment storey or parking area with a large space. The transfer plate is a transitional region, serving as load distribution element from the upper floors to the bottom parking areas. Thus it usually holds outstanding mass and stiffness.

The prototype building simulated by the scaled model is a reinforced concrete high-rise residential building structure, which represents a considerable fraction of the building stock in Hong Kong. The building is made up of a three-level podium, 34 storeys of typical floors and one storey of top plant room. In addition, the building also has some of the features common to high-rise residential buildings, including a transfer plate and podium, which supports all of the above typical floors and the plant
machine room. A typical floor plan consists of shear walls and a central core. The central core is rectangular in shape and continues all the way down to the foundation. Under the transfer plate, 12 columns are constructed to support all of the upper shear walls.

3.3 THE SCALED MODEL STRUCTURE

It is not practical to study true structural behaviour under a variety of real earthquake excitations, due to both the uncertainty of real earthquake events and the absence of instrumentation on the real building. Alternatively, a shaking table test-oriented scaled model structure has been designed and fabricated for investigating the earthquake response and performing seismic damage detection purpose. In order to simulate the prototype building well, a strict similitude law should be followed, and the model structure should be fabricated elaborately to guarantee the fulfillment of similitude law. The basic material used to build the model is to use micro-concrete which has similar mechanical properties to that of the prototype and comparatively smaller modulus of elasticity as well as lower strength.

3.3.1 Similitude Law

The design, loading and interpretation of any structural model must follow the similitude law, which relates the model to the prototype structure (Zhang 1997).

First, the design of the model should fulfill geometry similitude law, which states that the governing factor in deciding the scale factor of the model is the clear headroom of the laboratory. Because the maximum height of the model is limited to 7m, a 1:20 scale ratio is used to represent the prototype building. The overall
dimensions of the finished model are a length of 2.370m (in East-West direction), a width of 2.160m (in South-North direction) and a height of 6.515m (in upright direction). Additionally, dimensions of the structural elements like concrete beams, columns, slabs and walls were fabricated strictly according to the dimensional scale ratio as well.

Second, in the fabrication of the model building structure, one key parameter in establishing the necessary similitude relations is the modulus of elasticity of the micro-concrete used in the fabrication of the model. The specified characteristic strength of the prototype concrete is 35 MPa. Since the in-situ strength of concrete is usually higher than the specified value, the strength of concrete is taken to be 40 MPa in defining the similitude law. The characteristic strength of the micro-concrete used in the model structure is 2.5 MPa. The modulus ratios at different levels of compressive stress are listed in Table 3.1.

<table>
<thead>
<tr>
<th>R</th>
<th>0.1-0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_r )</td>
<td>0.177</td>
<td>0.176</td>
<td>0.175</td>
<td>0.169</td>
</tr>
</tbody>
</table>

Herein, \( E_r \) is the elastic modulus ratio between micro-concrete and prototype concrete and R indicates different levels of compressive stress of micro-concrete, defined respectively as follows

\[
E_r = \frac{\text{Elastic modulus of the micro-concrete}}{\text{Elastic modulus of prototype concrete}} \quad (3.1)
\]
\[ R = \frac{\text{Applied compressive stress in the micro-concrete}}{\text{Compressive strength of the micro-concrete}} \]  

(3.2)

Apart from the geometry scaled ratio and the material property similitude law, another important parameter that needs to be considered when designing the model is loading simulation, which should be based on both the self-weights of all of the prototype building structural members and the additional imposed loadings. All of these loadings at each storey of the prototype building structure are summarized in Table 3.2.

<table>
<thead>
<tr>
<th>Building floor(s)</th>
<th>Structural members</th>
<th>Finishes, service, etc</th>
<th>Imposed loading</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st floor</td>
<td>30137</td>
<td>3733</td>
<td>7729</td>
<td>41599</td>
</tr>
<tr>
<td>2nd floor</td>
<td>25823</td>
<td>3733</td>
<td>7729</td>
<td>37285</td>
</tr>
<tr>
<td>3rd floor</td>
<td>28943</td>
<td>6222</td>
<td>22536</td>
<td>57701</td>
</tr>
<tr>
<td>Transfer plate</td>
<td>67249</td>
<td>136</td>
<td>1863</td>
<td>69248</td>
</tr>
<tr>
<td>Typical floor (C04-C10)</td>
<td>8150×7</td>
<td>136×7</td>
<td>1863×7</td>
<td>10149×7</td>
</tr>
<tr>
<td>Typical floor (C11-C38)</td>
<td>8058×28</td>
<td>136×28</td>
<td>1863×28</td>
<td>10057×28</td>
</tr>
<tr>
<td>Roof plant room</td>
<td>14504</td>
<td>2672</td>
<td>1616</td>
<td>18792</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>449330</strong></td>
<td><strong>21256</strong></td>
<td><strong>106678</strong></td>
<td><strong>577264</strong></td>
</tr>
</tbody>
</table>

The above self-weights and living loadings were scaled and simulated by self-weights of the model building micro-concrete and extra artificial mass, respectively.
Table 3.3 lists the self-weight of the model in unit kN. The computation of the self-weight is based on the unit weight of the micro-concrete taken as 23 kN/m³. The first three floors have a similar self-weight, and the transfer plate floor has a suddenly increased self-weight. All of the typical floors have the same self-weight, and the roof plant room has the lightest self-weight, at 1.58 kN.

<table>
<thead>
<tr>
<th>Building floor(s)</th>
<th>Total self-weight (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st floor</td>
<td>3.30</td>
</tr>
<tr>
<td>2nd floor</td>
<td>2.80</td>
</tr>
<tr>
<td>3rd floor</td>
<td>3.15</td>
</tr>
<tr>
<td>Transfer plate</td>
<td>7.30</td>
</tr>
<tr>
<td>Typical floor (C04~C10)</td>
<td>0.88 x 7</td>
</tr>
<tr>
<td>Typical floor (C11~C38)</td>
<td>0.88 x 28</td>
</tr>
<tr>
<td>Roof plant room</td>
<td>1.58</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>48.90</strong></td>
</tr>
</tbody>
</table>

The self-weight of the model is 48.90 kN. The weight of the base plate on which the model is fabricated is 59.00 kN, and hence the total self-weight of the model (including self-weight of the scaled model and weight of the base plate) is 59.00 kN+48.90 kN=107.90 kN. All loadings above the shaking table is limited by the loading capacity of the shaking table at 30 tons or approximately 300.00 kN, therefore, the maximum artificial mass weight adding on the model structure is 300.00 kN-107.90 kN=192.10 kN. The artificial mass on the model is simulated by equipping steel bars in the available free space. Their distribution on each storey is summarized in Table 3.4.
<table>
<thead>
<tr>
<th>Building floor(s)</th>
<th>Total self-weight (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd floor</td>
<td>840</td>
</tr>
<tr>
<td>3rd floor</td>
<td>840</td>
</tr>
<tr>
<td>Transfer plate</td>
<td>3200</td>
</tr>
<tr>
<td>Typical floor (C04–C38)</td>
<td>265 × 35</td>
</tr>
<tr>
<td>Roof plant room</td>
<td>320 + 80</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>14555</strong></td>
</tr>
</tbody>
</table>

When similitude law with respect to geometry, material property and loading is determined, the model building structure can then be fabricated.

### 3.3.2 Model Fabrication

By using micro-concrete, steel wires and steel meshes, the 1:20 scaled model is fabricated on the reinforced concrete base plate. The reinforcements of the core walls and columns at the ground floor level are effectively lapped to the reinforcements extruded from the base plate. The mix design of the micro-concrete is targeted at achieving a compressive strength of 2.5 MPa. A set of five 70.70 mm cubes is prepared at every several stories and tested at the age of 28 days to estimate the average compressive cube strength (Zhang 1997). Figure 3.1 demonstrates the overall fabrication procedure, where polystyrene (foamed plastic) inserts are used as the formwork due to its lower density, lower strength and convenience of being cut into shape using a heated wire.
3.3.3 Description of the Finished Model Building

The model is fabricated at a rate of approximately one storey per day. Upon completion, the height of the model is 6515 mm, without taking the height of the base plate into account. On a planar view, it is 2160 mm by 2370 mm at the podium level and reduces to 2160 mm by 2200 mm at typical floors. The geometrical configurations of all of the structural members, including the beams, columns, slabs,
shear walls and core walls, are reduced directly by a scaling factor of 20.

Figure 3.2 shows a photograph of the completed model building, from which it can be seen that the accelerometers distribute nearly uniformly along the vertical direction.

Figure 3.2 Sectional view of the 1:20 scaled model building structure
Figures 3.3 and 3.4 show the elevation view of the model structure and numbering configuration of the floors. Figures 3.5 and 3.6 show the corresponding planar view.

Figure 3.3 Sectional view of the scaled model building structure
Figure 3.4 Elevations of the scaled model building structure
Figure 3.5 Planar view of each floor of the scaled model building structure
(c) Planar view of the third floor

(d) Planar view of the transfer plate

Figure 3.5 Planar view of each floor of the scaled model building (Cont’d)
Figure 3.5 Planar view of each floor of the scaled model building (Cont'd)
Figure 3.6 Planar view of roof plant room of the scaled building structure

3.4 THE SHAKING TABLE TESTS

The shaking table tests of the scaled tall building model structure were conducted at the Earthquake Engineering and Engineering Vibration Open Laboratory of the Institute of Engineering Mechanics, China Seismological Bureau, for both earthquake response analysis and seismic damage detection purposes. The remainder of this chapter is devoted to describing shaking table tests, including the specification of the shaking table parameters, the instrumentation and testing procedure, and the presentation of the visually inspected earthquake-induced damage.
3.4.1 The Specification of the Shaking Table Parameters

The shaking table used is 5 m by 5 m on plan, and is controlled by a three-directional electro-hydraulic servo control system. More details on the capacity of the shaking table are summarized in Table 3.5.

<table>
<thead>
<tr>
<th>Specification of the shaking table capacity</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum loading capacity</td>
<td>30 tons</td>
</tr>
<tr>
<td>Maximum overturning moment</td>
<td>75 tons-meter</td>
</tr>
<tr>
<td>Maximum horizontal acceleration</td>
<td>1000 cm/sec²</td>
</tr>
<tr>
<td>Maximum vertical acceleration</td>
<td>700 cm/sec²</td>
</tr>
<tr>
<td>Maximum velocity</td>
<td>60 cm/sec² for 1-dimensional tests and 30 cm/sec² for 3-dimensional tests</td>
</tr>
<tr>
<td>Maximum horizontal displacement</td>
<td>±8 cm</td>
</tr>
<tr>
<td>Maximum vertical displacement</td>
<td>±5 cm</td>
</tr>
</tbody>
</table>

3.4.2 Instrumentation for Damage Detection

A variety type of sensors, including accelerometers, displacement transducers and strain gauges, are instrumented on the model building to analyze the response of the simulated earthquakes as well as to detect seismic damage. The specification of accelerometers instrumented for earthquake-induced damage detection study is summarized in Table 3.6, where C01 refers to the base plate on which the excitations are exerted. AX and AY denote the accelerometers of the X-direction and Y-direction, respectively.
Table 3.6 Accelerometers for seismic damage detection

<table>
<thead>
<tr>
<th>Vertical installation locations</th>
<th>Signal directions to be measured</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>North-South</td>
<td>East-West</td>
</tr>
<tr>
<td></td>
<td>Horizontal installation locations</td>
<td></td>
</tr>
<tr>
<td></td>
<td>At west face</td>
<td>At east face</td>
</tr>
<tr>
<td>Roof floor (C42)</td>
<td>AX18</td>
<td>AX17</td>
</tr>
<tr>
<td>38th floor (C38)</td>
<td>AX16</td>
<td>AX15</td>
</tr>
<tr>
<td>34th floor (C34)</td>
<td>AX14</td>
<td>AX13</td>
</tr>
<tr>
<td>30th floor (C30)</td>
<td>AX12</td>
<td>AX11</td>
</tr>
<tr>
<td>25th floor (C25)</td>
<td>AX10</td>
<td>AX9</td>
</tr>
<tr>
<td>21st floor (C21)</td>
<td>AX8</td>
<td>AX7</td>
</tr>
<tr>
<td>15th floor (C15)</td>
<td>AX6</td>
<td>AX5</td>
</tr>
<tr>
<td>10th floor (C10)</td>
<td>AX4</td>
<td>AX3</td>
</tr>
<tr>
<td>Transfer plate (C04)</td>
<td>AX2</td>
<td>AX1</td>
</tr>
<tr>
<td>Base plate (C01)</td>
<td>AX20</td>
<td>AX19</td>
</tr>
</tbody>
</table>

For further clarity, the locations and directions of the accelerometers installation for seismic damage detection purpose are indicated in Figure 3.7, where circle solid dots denote the locations of the accelerometers used to detect damage, and square solid dots denote the locations of the sensors used to analyze earthquake response. It can be seen that the X-direction accelerometers are mounted at the two end-wings of the model, while the Y-direction accelerometers are mounted at the central lift well. In addition, a total of five additional accelerometers are mounted at the centre of the base plate, the floor C04, C16, C31 and the roof plant room respectively, to measure the vertical acceleration responses. The vertical-direction accelerometers are named as AZ0, AZ1, AZ2, AZ3 and AZ4.
Figure 3.7 Layout of accelerometers for seismic damage detection purpose
3.4.3 The Shaking Table Tests Procedure

Shaking table tests are conducted in an alternant style, by exerting artificially simulated earthquake records and white-noise excitations. A total of four levels of earthquake excitations with successively enhanced magnitudes are considered, including minor earthquakes, moderate earthquakes, strong earthquakes and super-strong earthquakes. Among each level of earthquakes, a set of earthquake records respectively represent rock site, medium soil site and soft soil site conditions. Between two consecutive levels of earthquake excitations, 20 minutes of white-noise excitation is applied, to provide a signal basis for the subsequent modal parameters extraction and damage identification. Due to limitation of laboratory supplying voltage for shaking table driving device, the total duration of 20 minutes of white-noise excitation is divided into six segments, each of which is about 200 seconds (over 3 minutes) in duration. It should be stressed that the excitation amplitude of the white-noise excitation is relatively small, compared with that of an earthquake excitation. Moreover, there is no new ‘earthquake’ occurring during all segments of the white-noise excitations. Therefore, it can be assumed that the model building structure remains in the same healthy state throughout all of the whole white-noise excitation shaking table tests.

The sampling rate of accelerometers for damage detection purpose is taken as 100 Hz, regardless of whether the building structure has been subjected to earthquake attacking or white-noise excitation. Totally, 17 times of earthquake excitations and the corresponding responses are recorded, among which $6 \times 5 = 30$ times of white-noise excitations and the corresponding responses are also recorded. The entire sequence of shaking table tests is summarized in Table 3.7.
<table>
<thead>
<tr>
<th>No.</th>
<th>Excitation</th>
<th>Description</th>
<th>Damage detection tests for no damage model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1~6</td>
<td>White-noise</td>
<td></td>
<td>X &amp; Y, hard soil</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>X &amp; Y, moderate-hard soil</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>X &amp; Y, soft soil</td>
</tr>
<tr>
<td>7</td>
<td>Minor-level seismic excitation</td>
<td>Earthquake simulation tests</td>
<td>X, hard soil</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td>Y, hard soil</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td>X, moderate-hard soil</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td>Y, moderate-hard soil</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td>X, soft soil</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td>Y, soft soil</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td>Visual inspection for trivial damage checking</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16~21</td>
<td>White-noise</td>
<td>Damage detection tests for slightly damaged model</td>
<td>X &amp; Y, hard soil</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>X, hard soil</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Y, hard soil</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>X &amp; Y, moderate-hard soil</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>X &amp; Y, soft soil</td>
</tr>
<tr>
<td>22</td>
<td>Moderate-level seismic excitation</td>
<td>Earthquake simulation tests</td>
<td>Visual inspection for moderate damage checking</td>
</tr>
<tr>
<td>23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27~32</td>
<td>White-noise</td>
<td>Damage detection tests for moderately damaged model</td>
<td>X, moderate-hard soil</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>X, hard soil</td>
</tr>
<tr>
<td>33</td>
<td>Strong-level seismic excitation</td>
<td>Earthquake simulation tests</td>
<td>X, soft soil</td>
</tr>
<tr>
<td>34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36~41</td>
<td>White-noise</td>
<td>Damage detection tests for severely damaged model</td>
<td>Visual inspection for complete damage checking</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>42~47</td>
<td>White-noise</td>
<td>Damage detection tests for completely damaged model</td>
<td>Signal records of super-strong-level earthquake simulation tests are corrupted due to data acquisition system problems</td>
</tr>
</tbody>
</table>
3.5 RESULTS OF VISUAL DAMAGE INSPECTION

With successively enhanced earthquakes attacking, the tested model building is subjected to seismic damage, increasing in severity from trifling, moderate, serious and, finally, to ultimate catastrophic severity. Between each level of earthquake simulation tests and subsequent white-noise tests, a visual inspection for the seismic damage incurred by that level of earthquake excitation is carried out.

3.5.1 Observation of Trifling Damage

When the tested model is subjected to minor earthquake excitations, there is no noticeable vibration. No structural damage is observed when the model is subjected to earthquake excitations representing the rock site and medium soil site conditions. However, a few fine ‘hair-line’ cracks are observed after being attacked by the soft soil site earthquake excitations. All of these very small cracks are located at the podium-level structural members.

Figure 3.8 shows one inclined crack at the South-East corner of the bulkhead. Figure 3.9 shows two almost vertical cracks at the bulkhead in one axis and horizontal cracks on the wall at the upper end of the columns. Figure 3.10 shows horizontal cracks on the wall at the upper end of one column at the third floor. Figure 3.11 shows an overall view of the first three floors after the model building was subjected to minor-level earthquakes. It can be seen that all cracks at this stage are very fine and barely noticeable. There is no any crack presence at the structural members beyond the level of the transfer plate podium.
Figure 3.8 Inclined crack at the South-East corner of the bulkhead

Figure 3.9 Cracks at the bulkheads and at the upper end of the columns
Figure 3.10  Cracks at the upper end of the third floor column

Figure 3.11  Overall view of the first three floors at the trifling damage case
3.5.2 Observation of Moderate Damage

During moderate-level earthquake excitations, small but noticeable vibrations of the model building are observed. Thereafter, the model building suffered moderate damage, with the propagation of old cracks and the appearance of new cracks. Cracks that previously appeared under minor earthquake excitations propagate with a slight increase in width. Simultaneously, several new fine 'hair-line' cracks are found at the columns below the transfer plate. Besides these old and new cracks located at or below the transfer plate podium, the most important observations made at this damage case are that new cracks began to appear in storeys above the transfer plate.

Figure 3.12 shows aggregation of the inclined crack located at the South-East corner of the bulkhead, compared with an old crack that appeared in the top right corner. Figure 3.13 shows one new crack found in one of first floor columns. And Figure 3.14 shows many more new cracks appearing above the transfer plate level.

![Image](image_url)

**Figure 3.12** Aggregation of inclined crack at the South-East corner of the bulkhead
Figure 3.13 New crack at one of the columns of the first floor

Figure 3.14 New cracks appearing above the transfer plate level (C04~C08)
3.5.3 Observation of Serious Damage

After strong-level of earthquakes attacking, the model building has accumulated to a serious-extensity of seismic damage which caused the structure nearly unrepairable. At this stage of shaking table tests, a significantly different vibration responses and incurred damage, compared with those of trifling damage and moderate damage cases, are observed. Firstly, under strong earthquake excitations, the model vibrates at a considerable magnitude. With regard to seismic damage, not only do the previous old cracks aggregate, a great number of new cracks are also observed. Furthermore, most of the new cracks appear at the middle and upper storeys, above the podium of the transfer plate.

Figure 3.15 shows the typical propagation of cracks below the transfer plate level (the first three floors).

![Image of cracks](image.jpg)

Figure 3.15 Propagation of old cracks at the columns and bulkhead of the transfer plate
Figures 3.16 to 3.19 clearly exhibit the presence of new cracks, distributed from floor C04 to the top of the model building.

Figure 3.16 One new crack appearing at the shear wall of floor C04

Figure 3.17 New cracks appearing at the shear wall between floors C09 and C10
Figure 3.18 Many new cracks appearing between floors C24 and C20

Figure 3.19 Many new cracks appearing above floor C34

It can be seen that large numbers of new cracks appear at the serious damage case, and many of them are distributed above the middle and top portions of the model structure.
3.5.4 Observation of Complete Damage

Under super strong-level of earthquake excitations, the model building vibrates significantly by swaying in the lateral directions. When subjected to such fatal earthquakes destroying, the model building suffers from a catastrophic damage, which caused the model structure approach to collapsing and completely failure in structural integrity. Almost all of the previous cracks that first appeared at trifling damage, moderate damage and serious damage cases, are aggregated at this stage. The most critical observation is that two large horizontal cracks appearing at floors C04, C10 and C13, which cut shear walls into two-halves. These cracks are caused by the substantial spalling of concrete and the buckling of the main reinforcements. Ignoring those old cracks, the new critical horizontal cracks are shown in Figures 3.20 to 3.23.

Figures 3.20 and 3.21 show wide horizontal cracks on the wall of floor C04.

Figure 3.20 Horizontal cracks on the wall of floor C04
Figures 3.21 and 3.23 show wide cracks located at floors C10 and C13, respectively.

Figure 3.22 Horizontal cracks in the wall between floors C09 and C10
Figure 3.23 Horizontal cracks in the wall between floors C13 and C14

3.5.5 Summary of AllDamages

All of the inspected seismic damage incurred from each level of earthquake attacking is summarized in Table 3.8, where N, T, M, S and C denote no damage, trifling damage, moderate damage, serious damage and complete damage cases respectively. It can be seen that the incurred seismic damage begins from the lower portion (on and beneath the transfer plate) of the model building after minor-level earthquakes, then extends to above the transfer plate portion (floors C04–C08) after moderate-level earthquakes, and finally spreads to the whole structure after strong and super-strong earthquakes. The model building is serviceable after minor and moderate level of earthquakes, and still can be repaired after strong level of earthquakes. When subjected to super strong level of earthquakes attacking, the model building loses the necessary and possibility of being repaired.
<table>
<thead>
<tr>
<th>Phases</th>
<th>Seismic wave</th>
<th>Floor No.</th>
<th>Description of cracks</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td></td>
<td>No any damage</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Small Earthquake</td>
<td>TP, C03</td>
<td>Between columns and beams</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C03</td>
<td>At the angle of south to east</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C03</td>
<td>Between columns and beams</td>
</tr>
<tr>
<td>M</td>
<td>Moderate Earthquake</td>
<td>C03</td>
<td>North to east and column</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C01</td>
<td>At columns</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C02</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>C04-C08</td>
<td>Five horizontal cracks</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C03</td>
<td>Aggravated old cracks</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TP</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>Large Earthquake</td>
<td>C03</td>
<td>At the column and transfer plate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C13-C18</td>
<td>Between the east and south walls</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C22, C27</td>
<td>One new horizontal crack</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C26, C27</td>
<td>At walls</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C32-C33</td>
<td>Horizontal</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C35, C36, C37</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>C8, C9, C10</td>
<td>Horizontal</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C4, C5, C6</td>
<td>New cracks</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C33-C37</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>C04, C05</td>
<td>At walls and transfer plate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C06, C07</td>
<td>At walls</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C09-C10</td>
<td>Horizontal</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C35, C36</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Catastrophic Earthquake</td>
<td>C34, C36, C37</td>
<td>Numerous new cracks</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C04-C09</td>
<td>Aggravated old cracks</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C04-C09</td>
<td>Nearly collapsed at C04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C10</td>
<td>Broken walls at west and north</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C14</td>
<td>Eastern outer walls broken</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C15 ~ C20</td>
<td>Between west and north</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C21-C27</td>
<td>Old and new cracks</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C35, C38</td>
<td>Numerous new cracks</td>
</tr>
</tbody>
</table>
3.6 CONCLUDING REMARKS

This chapter contains a complete description of the prototype building, the scaled model building structure, the shaking table tests and the visual inspection results of the seismic damage induced by each level of earthquakes attacking. No approaches to damage identification and overall post-earthquake damage evaluation are developed in this chapter.

Through the visual inspection on the seismic damage, it can be found that: (1). No essential damage occurs when the building structure is subjected to the micro-level of earthquakes, except for the presence of a few fine cracks on the columns and beams beneath the transfer plate; (2). The old fine cracks are enlarged and much new cracks appear around the transfer plate, after the building structure is attacked by the moderate-level of earthquakes; (3). Large numbers of new cracks begin to appear at the central floors, after the building structure is attacked by the strong-level of earthquakes; and (4). All of the old cracks are almost enlarged and countless new cracks appear at the top portion of the building structure, after it is attacked by the super strong-level of earthquakes. Conclusively, the lower portion of the building structure are the most likely to be damaged by earthquakes, and the damage extends to the central and top portions with the enhancement of earthquakes intensity. In addition, it should also be noted that the degrees of the structural inelastic or nonlinear properties drastically increase when the building structure is subjected to the strong and super strong levels of earthquakes attacking. This also generates the serious nonlinearity of the dynamic behaviours, which means that the modal parameters of the building structure greatly depend on the amplitudes of the earthquake excitations.
In accordance with the mathematical model-free damage detection strategy proposed in Chapter 1, some methods of seismic damage identification (including damage locations and severity) and overall damage assessment are to be developed in the subsequent chapters. The identification of seismic damage locations will be based on the white-noise excitations and responses. The modal parameters used for damage indications are estimated using input-output and output-only approaches, respectively. The evaluation of the overall damage severity will be directly based on the earthquake records, by employing joint time-frequency analysis algorithms and super-resolution parameters estimation methods.

The results of the visual inspection on the earthquake-induced damage provide a baseline for the studies of subsequent chapters. The proposed approaches to identifying the seismic damage will be validated by comparing their indications on the damage with the achieved results of visual inspection.
Appendix: Characteristics of Earthquake Records

In the study, three types of ground excitations, namely the earthquake records for rock site, medium soil site and soft soil site, are considered and simulated during the shaking table tests. The ground excitations are generated by inputting scaled real acceleration records, e.g., the NS component of the 1940 El Centro earthquake. Due to the accuracy limitation and dynamic effects of the shaking table, ground excitation generated by the shaking table might be different from the input and earthquake record that is targeted for a particular type of soil site. Predominant periods of ground excitations generated by the shaking table depend on the input earthquake records. Particularly, frequency content of the earthquake excitation for rock site resembles the excitation of medium soil site earthquake. The predominant periods of the original earthquake records, compressed ones and generated ones by the shaking table are listed in Table 3.9.

<table>
<thead>
<tr>
<th>Predominant period</th>
<th>Characteristics of soil site</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rock site</td>
</tr>
<tr>
<td>Real earthquake records</td>
<td>0.100s</td>
</tr>
<tr>
<td>After compression</td>
<td>0.020s</td>
</tr>
<tr>
<td>Generated by shaking table</td>
<td>0.125s</td>
</tr>
</tbody>
</table>

Peak accelerations of ground excitations simulated by the shaking table are in the range of 0.02-0.06g for minor level of earthquakes, 0.08-0.15g for moderate level of earthquakes and 0.15-0.20g for strong level of earthquakes. Additionally, a dynamic amplification factor $\beta$ is introduced to quantify the responses of the model to various levels of earthquake excitations, which is defined as
\[ \beta = \frac{A_r}{A_i} \]

where \( A_r \) represents the peak acceleration of the response and \( A_i \) is the maximum ground acceleration generated by the shaking table. Peak accelerations and the corresponding dynamic amplification factors are listed in Tables 3.10 to 3.12, for the three different levels of earthquake excitations.

<table>
<thead>
<tr>
<th>Floor level</th>
<th>Rock site</th>
<th>Medium soil site</th>
<th>Soft soil site</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( A_r )</td>
<td>( \beta )</td>
<td>( A_r )</td>
</tr>
<tr>
<td>C38</td>
<td>0.060</td>
<td>2.73</td>
<td>0.078</td>
</tr>
<tr>
<td>C34</td>
<td>0.055</td>
<td>2.50</td>
<td>0.060</td>
</tr>
<tr>
<td>C30</td>
<td>0.040</td>
<td>1.82</td>
<td>0.055</td>
</tr>
<tr>
<td>C25</td>
<td>0.060</td>
<td>2.73</td>
<td>0.060</td>
</tr>
<tr>
<td>C21</td>
<td>0.060</td>
<td>2.73</td>
<td>0.049</td>
</tr>
<tr>
<td>C15</td>
<td>0.060</td>
<td>2.73</td>
<td>0.050</td>
</tr>
<tr>
<td>C10</td>
<td>0.055</td>
<td>2.50</td>
<td>0.043</td>
</tr>
<tr>
<td>C04</td>
<td>0.040</td>
<td>1.82</td>
<td>0.038</td>
</tr>
<tr>
<td>G/F</td>
<td>0.022</td>
<td></td>
<td>0.034</td>
</tr>
</tbody>
</table>

Table 3.10 Peak accelerations (g) under minor level of earthquakes
### Table 3.11 Peak accelerations (g) under moderate level of earthquakes

<table>
<thead>
<tr>
<th>Floor level</th>
<th>Rock site</th>
<th>Medium soil site</th>
<th>Soft soil site</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A_r)</td>
<td>(\beta)</td>
<td>(A_r)</td>
</tr>
<tr>
<td>Roof</td>
<td>0.230</td>
<td>2.30</td>
<td>0.200</td>
</tr>
<tr>
<td>C38</td>
<td>0.230</td>
<td>2.20</td>
<td>0.160</td>
</tr>
<tr>
<td>C34</td>
<td>0.180</td>
<td>1.80</td>
<td>0.130</td>
</tr>
<tr>
<td>C30</td>
<td>0.110</td>
<td>1.10</td>
<td>0.100</td>
</tr>
<tr>
<td>C25</td>
<td>0.150</td>
<td>1.50</td>
<td>0.120</td>
</tr>
<tr>
<td>C21</td>
<td>0.140</td>
<td>1.40</td>
<td>0.110</td>
</tr>
<tr>
<td>C15</td>
<td>0.160</td>
<td>1.60</td>
<td>0.140</td>
</tr>
<tr>
<td>C10</td>
<td>0.130</td>
<td>1.30</td>
<td>0.110</td>
</tr>
<tr>
<td>C04</td>
<td>0.100</td>
<td>1.00</td>
<td>0.100</td>
</tr>
<tr>
<td>G/F</td>
<td>0.100</td>
<td></td>
<td>0.090</td>
</tr>
</tbody>
</table>

### Table 3.12 Peak accelerations (g) under strong level of earthquakes

<table>
<thead>
<tr>
<th>Floor level</th>
<th>Rock site</th>
<th>Medium soil site</th>
<th>Soft soil site</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A_r)</td>
<td>(\beta)</td>
<td>(A_r)</td>
</tr>
<tr>
<td>Roof</td>
<td>0.320</td>
<td>1.60</td>
<td>0.370</td>
</tr>
<tr>
<td>C38</td>
<td>0.300</td>
<td>1.50</td>
<td>0.270</td>
</tr>
<tr>
<td>C34</td>
<td>0.200</td>
<td>1.00</td>
<td>0.220</td>
</tr>
<tr>
<td>C30</td>
<td>0.170</td>
<td>0.85</td>
<td>0.160</td>
</tr>
<tr>
<td>C25</td>
<td>0.160</td>
<td>0.80</td>
<td>0.200</td>
</tr>
<tr>
<td>C21</td>
<td>0.220</td>
<td>1.10</td>
<td>0.230</td>
</tr>
<tr>
<td>C15</td>
<td>0.220</td>
<td>1.10</td>
<td>0.240</td>
</tr>
<tr>
<td>C10</td>
<td>0.290</td>
<td>1.45</td>
<td>0.210</td>
</tr>
<tr>
<td>C04</td>
<td>0.250</td>
<td>1.25</td>
<td>0.180</td>
</tr>
<tr>
<td>G/F</td>
<td>0.200</td>
<td></td>
<td>0.180</td>
</tr>
</tbody>
</table>
Time history and power spectrum of various levels (only a typical one for minor level) of earthquake excitations are shown in Figures 3.24 to 3.26.

Figure 3.24 Time history and power spectrum of a typical minor earthquake

Figure 3.25 Time history and power spectrum of moderate earthquakes
Figure 3.25 Time history and power spectrum of moderate earthquakes (Cont'd)

Figure 3.26 Time history and power spectrum of strong earthquakes
Figure 3.26 Time history and power spectrum of strong earthquakes (Cont'd)
CHAPTER 4

IDENTIFICATION OF SEISMIC DAMAGE
BASED ON FRFS AND NEURAL NETWORKS

4.1 INTRODUCTION

Based on neural network techniques and white-noise measurement data recorded in the shaking table tests, damage identification of the investigated tall building model structure is to be intensively explored in this chapter. Here, identification refers to the recognition of the overall severity as well as the approximate locations (floors) of earthquake-induced damage. The approach to identifying seismic damage developed in this chapter is to utilize white-noise measurement data and avoid establishing structure finite element model. This will greatly facilitate and simplify the damage identification implementation.

Myriad artificial neural network techniques have been employed for structural model updating and damage detection purposes. In vibration-based damage identification applications, neural networks are firstly taught to distinguish between the dynamic behaviours of the healthy and damaged structures. In most cases, this learning is supervised, and a great number of training samples are needed. This chapter deals with identifying seismic damage in the tall building structure based on training of neural network from frequency response function changes. Generally, the modal parameters used for damage detection can be frequency, mode shape, and
their derivations, such as dynamic modal flexibility. Such parameters of dynamic characteristic are usually identified from structural frequency response functions (FRFs), and the accuracy of the parameters heavily depends on the identification approaches as well as on the structural linear elasticity and consistency induced by damage. Intuitively, the doubt arises: Why not just use frequency response functions (FRFs) that simultaneously contain information on frequency, mode shape and damping ratio to directly construct indicators of healthy conditions? Recent studies also indicate that neural networks, such as the radial basis function (RBF) and multilayer back propagation (BP), can be trained on measured frequency responses of healthy and damaged specimens to recognize the actual condition of the structures (Banks et al. 1996; Thyagarajan et al. 1998; Baruch 1998; Zhang et al. 1999; Sampaio et al. 1999; Zang and Imregun 2001). Some researchers have utilized measured FRFs at discrete points over certain specified frequency ranges as input to neural network models for the purpose of identifying damage. Chaudhry and Ganino (1994) used measured FRF data over some specified frequency ranges as input to a neural network to identify the presence and extent of the delamination in debonded beams. Wu et al. (1992) identified the damage in a three-storey building model by selecting the first 200 points of FRF as input to a neural network. The use of fewer spatial locations and the selection of data points from frequency response functions may lead to the loss of important information. On the other hand, the entire points of frequency response functions involve much unwanted noise, which often conceals the information that is truly useful for damage identification. Although many studies have demonstrated the feasibility of using measured FRF data to detect damage, a very significant hurdle still remains: the size of the FRFs data, which is determined by the number of spatial response locations and the number of spectral lines, is often
too large for neural network applications. The above handicap highlights the need to find a more compact representation of the measured FRFs. In this chapter, a powerful data compression and feature extraction approach, known as principal component analysis (PCA), is employed for the compression of FRFs.

First of all, FRFs need to be computed by the classical Fourier transform technique, from the white-noise excitation and building response data.

4.2 FRFS COMPUTATION USING WHITE-NOISE MEASUREMENT DATA

According to conventional modal parameter estimation techniques, FRF is identified by performing fast Fourier transform (FFT) on excitation and response signals and taking the ratio of response Fourier transform to excitation Fourier transform. However, theoretically Fourier transform is only applicable to stationary signals, and thereby, the identification of frequency response functions under earthquake excitation and response becomes uncertainty or unreliable due to the non-stationary characteristic of earthquake records. The non-stationary property is caused by the short duration of earthquake and structure nonlinearity, especially when the building structure is subjected to a serious deterioration. Accordingly, the subsequent computation of the FRFs is performed only on white-noise measurement data. The data of earthquake records are non-stationary and thus cannot be dealt with classical modal parameter identification techniques. It is necessary to resort to some other signal processing approaches for earthquake simulation data. In the later partition of this dissertation, time-frequency spectra and super-resolution analysis are intended, to conduct an investigation of overall seismic damage.
Figure 4.1 demonstrates one procedure for computing FRF from white-noise (no damage case, test No. 1) excitation signal measured at the shaking table and the response signal measured at the top floor (C38).

(a) White-noise excitation time-history

(b) White-noise response time-history

Figure 4.1 FRF computation of one white-noise test (No.1)
(c) The identified frequency response (amplitude)

**Figure 4.1** FRF computation of one white-noise test (No.1) (Cont’d)

Figure 4.2 demonstrates another FRF computation from one earthquake (no damage case, test No. 7) excitation and response data.

(a) Earthquake excitation time-history

**Figure 4.2** FRF computation of one earthquake simulation test (No.7)
(b) Earthquake response time-history

(c) The identified frequency response function (amplitude)

Figure 4.2 FRF computation of one earthquake simulation test (No. 7) (Cont'd)

Comparing the FRF identification results from the white-noise and earthquake records data demonstrated in Figures 4.1 and 4.2, it can be seen that the FRFs of
white-noise measurement data have clear resonant frequencies, whereas FRFs of the earthquake simulation tests have become uncertain and noisy.

As addressed in Chapter 3, a 20-minute long white-noise excitation is divided into six segments, each of more than three minutes, 180 seconds and with a sampling frequency of 100 Hz, which leads to the production of more than $180 \times 100 = 18000$ sampling points in the time-domain. The FRF computing is based on performing Fourier transform on the sampling points of the excitation and response signals, where the length of one Fourier transform (FFT) is taken as 1024 and the average time for the whole duration of one white-noise test is taken as 100 for frequency response function curve smoothing. Repeating the above procedure of FRF computation from white-noise measurement data, a total of six FRFs are achieved, corresponding to six times of white-noise random vibration tests at intact and each damaged case.

Figure 4.3 illustrates two sets of FRFs corresponding to healthy and serious cases of damage respectively. Each set contains six frequency response function curves. Herein, for demonstration purpose, these FRFs are computed from response data measured at the top floor (No. 38) and excitation data measured at the shaking table in X-direction. In a similar way, the FRFs related to each damage case and from other locations of response measurement data at the respective directions are computed. All of the identified FRFs need to be compressed by the subsequent principal component analysis. And their projections on a few principal components are then taken as inputs of neural networks, for the recognition of both the global damage severity and damage distribution along the height of the model building structure.
Figure 4.3 Six FRFs of the healthy and severely damaged building structure

Each curve set of FRFs at the same damage case exhibits a small fluctuation, which results from the measurement noise effect. Clearly, the FRFs of the damaged structure differ significantly from those of healthy structure. They shift at the peak locations on the frequency axis as well as at the peaks amplitude, which indicates that both the natural frequencies and damping ratios of the structure changes due to damage.

To ensure the reliable identification of damage, the computed FRFs for damage indicator must be definite and excitation-independent. And, this chapter is aimed at identifying the cumulative seismic damage, after all the previous occurrences of earthquakes, rather than at realizing real time damage under each of earthquake excitation. Therefore, the computation of FRFs here only utilizes the white-noise excitation and response data. A detailed examination on earthquake excitations and responses to assess overall seismic damage will be made in later chapters.
4.3 FEATURE EXTRACTION OF FRFS USING PRINCIPAL COMPONENT ANALYSIS

Principal component analysis developed by Jolliffe (1986) and Bishop (1995), is a statistical technique that transforms an original set of variables into a substantially smaller set of uncorrelated variables that represent most of the information in the original set of variables (Dunteman 1989; Diamantaras and Kung 1996). Using principal component analysis, the dimensionality of original set can be reduced considerably. This data compression technique has been widely applied to virtually every substantive area including biology, medicine, chemistry, meteorology, and geology, as well as the behavioural and social sciences. Birren and Morrison (1961) conducted a principal components analysis of a correlation matrix for 11 subscales of the Wechsler Adult Intelligence Scale (WAIS) along with age and years of education completed for a sample of 933 white males and females.

Principal component analysis can be viewed as a classical method of multivariate statistical analysis to achieve a reduction of data dimensionality. A small set of uncorrelated variables is much easier to understand and use in further analyses than a larger set of correlated variables. Using an orthogonal projection, the original set of variables in an $N$-dimensional space is transformed into a new set of uncorrelated variables, the so-called principal components (PCs), in a $P$-dimensional space such that $P < N$. Namely, it seeks to project the high-dimensional data into a new low-dimensional set of Cartesian coordinates $(z_1, z_2, \ldots, z_P)$. The new coordinates have the following properties: $z_1$ is the linear combination of the original coordinate $x_i (i = 1, 2, \ldots, N)$ with maximal variance, $z_2$ is the linear combination that explains most of the remaining variance and so on. If the existing
$P$-coordinates which are actually a linear combination of $N(>P)$ variables, then the first $P$ principal components will completely characterize the data and the remaining $N-P$ will be zero. The calculation can be described as follows: given the $j$th measurement data set $\{x\}_j = (x_{j1}, x_{j2}, \ldots, x_{jn})^T, j = 1, 2, \ldots, M$, where $T$ denotes transposition and $M$ is the total measurement times, one $N \times N$ dimension covariance matrix $[C]$ is formed as

$$[C] = \sum_{j=1}^{M} [x\}_j [x\}_j]^T$$  \hspace{1cm} (4.1)$$

then performs a singular value decomposition on matrix $[C]$

$$[C] = [A][\Lambda][A]^T$$  \hspace{1cm} (4.2)$$

The transformation to the principal components is then applied as

$$\{z\}_i = [A]^T (\{x\}_i - \{\bar{x}\})$$  \hspace{1cm} (4.3)$$

where $\{\bar{x}\}$ is the vector of means of the $x$-data. From the dimension reduction point of view, the PCA works by discarding those linear combinations of the data that contribute least to the overall variance or range of the data set.

An application of PCA to structural dynamic analysis is due to Hasselman and Anderson (1998), who presented a theoretical basis for dealing with the derivation of modal metrics for use in non-linear model correlations, and updating and evaluating the uncertainty of response time histories. Here, for the purpose of training time
being efficient on neural networks-based damage identification, PCA is responsible for reducing the number of spectral lines of frequency response function. In order to determine how many principal components are enough to reserve the most information on the original FRF data, the reconstruction of FRF using only the first few principal components will first be conducted. Then the results of reconstruction will be compared using a different number of principal components. The projection of the original frequency response functions matrix \([H(\omega)]_{M \times N}\), which has \(M\) rows of FRFs, each with \(N\) frequency points on the \(N\) principal components is given by

\[
[B]_{M \times N} = [H(\omega)]_{M \times N} [A]_{N \times N}
\]

(4.4)

The projection matrix \([B]\) and the principal components matrix \([A]\) can be portioned into two sub-matrices with \(P\) principal components and \(N-P\) principal components (which actually are trivial and are thus not real principal)

\[
[B]_{M \times N} = [[B_1]_{M \times P}, [B_2]_{M \times (N-P)}],
\]

\([A]_{N \times N} = [[A_1]_{N \times P}, [A_2]_{N \times (N-P)}].
\]

(4.5)

The frequency response functions matrix can now be reconstructed using the first \(P\) principal components only

\[
[I_{[H]} = [B][A]^T = [[B_1]_{M \times P}, [B_2]_{M \times (N-P)}],
\]

\([[A_1]_{N \times P}, [A_2]_{N \times (N-P)}]^T \approx [B_1]_{M \times P} [A_1]^T_{P \times N}
\]

(4.6)
In addition to using a different number of principal components to reconstruct the original FRFs, the reconstruction under a variety of noise levels is also to be carried out and compared in order to examine the noise tolerance performance of principal component analysis. The originally measured FRFs are contaminated with time-domain noise by adding a zero-mean, $\sigma$-standard deviation normal distribution series into excitation and response signals independently.

Although a visual inspection reveals that there are some shifts between the FRFs of the healthy and damaged structures, an objective recognition of the actual state of the damage based on this observation is not straightforward. The approach proposed herein is to design and train neural networks to extract changes in the frequencies and damping ratios for the identification of seismic damage. However, the size of the FRF data, both the number of measurements and the 512 spectral lines in each measurement, is prohibitively large for direct use. Therefore, a data reduction process needs to be applied. Moreover, the selection of key spectral points to represent frequency windows is not straightforward because of the large variations in the dynamic behaviour. To reduce the size of the FRF data by using principal component analysis appears to be the only way forward.

As mentioned beforehand, the most significant principal components contain those features that are dominant in most of the frequency responses. In order to determine an appropriate number of principal components that can represent the original FRFs well, the reconstruction using different numbers of principal components are conducted and compared. Firstly, five frequency response function matrices corresponding to the cases of no damage, trifling damage, moderate damage, serious damage and complete damage are generated. Each matrix has six rows consisting of six FRFs that are related to six white-noise measurement segments and
the number of columns is equal to the number of spectral lines, say, 512. Then, by
combing the above five matrices, a matrix of 30 rows x 512 columns can be yielded.
The matrix consists of 30 FRFs which represent both a healthy structure and all of
the damage cases. Finally, based on Equations (4.1) to (4.6), the FRFs are
reconstructed using 5, 13 and 30 principal components, as shown in Figures 4.4, 4.5
and 4.6, respectively. Without losing generality, the original FRF computation and
the reconstruction of FRF by using a limited number of principal components are
demonstrated for the cases of no damage case and moderate damage. In Figures 4.4,
4.5 and 4.6, the solid line denotes the original FRF curve and the dotted line denotes
the reconstructed FRF curve. It should be pointed out that both the solid line and the
dotted line plotted in the same figure are the averaged value of the six original FRFs
and the six reconstructed FRFs, respectively.

![Graphs showing FRF amplitude against frequency points for healthy and moderately damaged structures.](image)

(a) Healthy structure  
(b) Moderately damaged structure

**Figure 4.4** The original and reconstructed FRFs by using 5 principal components
Figure 4.5 The original and reconstructed FRFs by using 13 principal components

Figure 4.6 The original and reconstructed FRFs by using 30 principal components
Clearly, when only 5 principal components are used, the FRFs cannot be properly reconstructed. In Figure 4.4, the reconstructed FRFs using only 5 principal components cannot detail some of the local properties of the FRFs. Comparatively, the reconstructed FRFs using 13 principal components can more closely represent the variances in the original FRFs, which can be illustrated in Figure 4.5. The more principal components that are used, the better the reconstruction of FRFs. Figure 4.6 shows the result of reconstruction using 30 principal components, where the reconstructed FRFs are almost identical to the original ones. Considering that 13 principal components are sufficient to accounting for most of the variances in the original FRFs data, only these principal components are used for the subsequent noise-robust study and seismic damage identification.

As opposed to the most significant principal components, which contain dominant features, measurement noise uncorrelated with such global features is represented by the less significant components. In other words, reconstructing the frequency response function by using a limited number of principal components only, should accomplish data compression as well as removing most of the noise. In order to study the noise-robust property of PCA, a set of normal-distribution noise with a zero-mean and \( \sigma \)-standard deviation random are added to the raw signals in time domain directly, to simulate additional man-made measurement noise. Let \( x(t) \) denote the original time-domain signals and \( r(\sigma) \) be a sequence of random numbers representing noise, the noise contaminating scenario can then be expressed as

\[
\bar{x}(t) = x(t) + r(\sigma)
\] (4.7)
where the standard deviation of normal distribution $\sigma$ represents the noise level, taken as 0.01 and 0.05, standing for low and high levels of noise respectively. The excitation and response signals are both corrupted with such noise independently.

Figure 4.7 shows the 30 original and reconstructed FRFs of the top floor, C38, at no damage case. The original FRFs are computed by using noisy excitation and response data. And the reconstructed FRFs are achieved by using only the first 13 principal components. From the comparison of two sets of FRFs, it can be found that the original FRFs have considerable fluctuation due to noise corruption, whereas the reconstructed FRFs have much smaller fluctuation. Therefore, the first 13 principal components not only retain most of the variances in the original FRFs, but also discard the trivial variance induced by noise.

Similarly, another comparison between the 30 original FRFs and the same number of reconstructed ones is demonstrated in Figure 4.8, where a high level of human-made ‘measurement’ noise is introduced. Analogue to the case of low level of noise corruption, only the first 13 principal components are used to obtain the reconstructed FRFs. Obviously, the fluctuations of both the original FRFs and reconstructed ones become much larger than those of the case of low level noise corruption. For the original FRFs, the fluctuations are so large that they even conceal the variances in the FRFs, especially for the high-frequency band. Nevertheless, the reconstructed FRFs are likely to represent the variance details and trends. The discrimination of two sets of FRFs indicates that principal component analysis not only reduces the dimensionality size of the FRF data while remaining the most of variances, it also filters out the unwanted measurement noise. A limited number of principal components can be utilized to identify seismic damage in the building structure.
Figure 4.7 The original and reconstructed FRFs in the case of no damage and with a low level of noise corruption (with $\sigma = 0.01$)

Figure 4.8 The original and reconstructed FRFs in the case of no damage and with a high level of noise corruption (with $\sigma = 0.05$)
For utmost clarity, the potential of PCA on filtering noise is further demonstrated in Figure 4.9. Two original FRFs corresponding to a low level and high level of noise are selected and plotted in the same diagram, which is also the case for reconstructed FRFs. From the comparison of two sets of FRFs, it can be seen that the original FRFs vary dramatically with the increase of noise level, whereas the reconstructed ones remain almost consistent, over most of the frequency range. The reconstructed FRFs are insensitive and robust to noise. The above study on noise-filtering property reveals that PCA attempt to reduce the dimensionality of the original data set while reserving the most of the information as well as filtering out most of the unwanted measurement noise. Thereafter, the identification of seismic damage is to be conducted based on neural networks, which take PCA-compressed FRFs projections as inputs.
4.4 DAMAGE IDENTIFICATION BASED ON COMPRESSED FRFs AND NEURAL NETWORKS

As mentioned beforehand, neural networks need to be trained using a set of training samples. In addition, the direct use of FRFs data makes the neural network training a time-consuming process that is not applicable in cases where the determination of seismic damage must be made shortly after an earthquake has occurred. Here, the FRFs projections on a smaller number of principal components extracted by PCA are taken as inputs of neural networks instead. After training the neural networks, a set of testing samples are again fed into the trained network. From the output, predictions of seismic damage can be made and compared with the actual scenarios of damage. Figure 4.10 shows the schematic illustration of the approach to identifying seismic damage in the building structure based on neural networks and PCA-compressed FRFs.

**Figure 4.10 Illustration of neural network and FRFs based damage identification**
The overall procedure is divided into two parts: (1) the training stage; and (2) the detection stage.

Figure 4.11 illustrates the architecture of the neural network and connection at one typical node. A three-layer feed-forward network model is employed. The projections of FRFs on a few number of principal components are taken as the inputs, and one output node indicates seismic damage of the corresponding region.

![Neural network topology and connection at one typical node](image)

**Figure 4.11 Neural network topology and connection at one typical node**

The recognition of damage that is intended for discussed in this chapter includes identification of both the seismic damage severity and the location(s) or floor(s). Taking the projections of FRFs on the limited number of principal components extracted by PCA as inputs of neural networks, the severity or locations of damage can be indicated by the output(s) of the appropriately configured neural networks.
For the recognition of the global damage severity, a total of three comparative schemes are proposed for the configuration of neural network input vectors. First, 30 FRF points over the first resonant frequency range are directly taken as inputs of neural networks. One hidden layer takes 18 nodes. Apparently, in this scheme just the partial information over the specific range rather than all spectral information on the whole FRF are utilized. Second, only a limited number of extracted principal component projections that holds the most information of the original FRFs are taken as inputs. As discussed in Section 4.3, the first 13 principal components are enough to retain most of the variances in the original FRFs data. Using these few number of principal components, the reconstructed FRFs can represent the original ones well. A three-layer of feed-forward neural network model is employed and trained using the back propagation (BP) algorithm. The hidden layers take 15 nodes, and one output node represents damage severity of the specified floor. It should be pointed out that the FRFs applied in this scheme are measured from the same location, say, at typical floor C38. The last scheme of configuration of neural network for recognizing damage severity is to utilize the complete information of FRFs measured from all nine response measurement points, namely, floors C04, C10, C15, C21, C25, C30, C34, C38 and C42. Based on the each of the 13 PCs extracted from one measurement point, a total of $13 \times 9 = 117$ PCs projection are grouped together, which contains the information of each FRF at one location as well as spatial information (all nine regions along the vertical direction of the model structure). However, 117 inputs still seems too burdensome for network training. Thus, a twofold PCA strategy is proposed and PCA is executed again on the obtained 117 PCs projection. Ultimately, 30 principal components projections after twofold PCA processing are taken as inputs. The hidden layer configuration is the same as that in the first scheme.
To quantitatively define extents of five damage cases, the target output of neural networks at the training phase is specified as a definite value, whereas the actual output at the testing phase can be a permitted range, as shown in Table 4.1. A total of 1000 samples are achieved for training and testing, from the identified FRFs corrupted by artificially added time-domain noise. The 500 training samples are obtained from the first three times of white-noise random vibration tests, and another 500 testing samples are obtained from the remaining three times of white-noise tests. Therefore, the training samples are not essentially the same as the testing samples. If and only if the output value of one testing sample is located at the permitted range corresponding to the respective damage case, that testing sample can be regarded as being correctly identified.

Table 4.1 The target output and permitted actual output of neural networks

<table>
<thead>
<tr>
<th>Damage cases</th>
<th>Specified target output</th>
<th>Permitted output</th>
</tr>
</thead>
<tbody>
<tr>
<td>No damage (N)</td>
<td>0.1</td>
<td>&lt; 0.2</td>
</tr>
<tr>
<td>Trifling damage (T)</td>
<td>0.3</td>
<td>0.2~0.4</td>
</tr>
<tr>
<td>Moderate damage (M)</td>
<td>0.5</td>
<td>0.4~0.6</td>
</tr>
<tr>
<td>Serious damage (S)</td>
<td>0.7</td>
<td>0.6~0.8</td>
</tr>
<tr>
<td>Complete damage (C)</td>
<td>0.9</td>
<td>&gt; 0.8</td>
</tr>
</tbody>
</table>

Taking specified values listed in Table 4.1 as outputs, a total of three neural networks are modeled. The three neural networks are corresponding to the three schemes of seismic damage severity identification, taking direct FRFs points, principal components projections of FRFs computed from one measurement location, and principal components projections of FRFs computed from all nine measurement locations as inputs, respectively. The architectures of three neural networks are 30-
18-1, 13-15-1, and 30-18-1 nodes. The unique output node indicates the overall severity of seismic damage. It should be noted that all three proposed neural networks are for severity recognition, not for damaged locations identification.

Figure 4.12 illustrates the results of the identification of the overall damage severity in the X-direction and Y-direction from the first neural network output, where N, T, M, S and C denote testing samples of no damage, trifling damage, moderate damage, serious damage and complete damage cases, respectively.

\[\text{(a) X-direction identification results}\]

\[\text{Figure 4.12 Identification results of the direct FRFs method, where a high level of noise is introduced and 30 FRF points over the first mode frequency range are taken as inputs}\]
Figure 4.12 Identification results of the direct FRFs method, where a high level of noise is introduced and 30 FRF points over the first mode frequency range are taken as inputs (Cont’d)

It can be seen that: for the identification of either the X-direction or Y-direction, nearly half of the testing samples are beyond the prospected target region for each damage case and, thus, are not identified correctly.

Figure 4.13 illustrates the results of the identification of the overall seismic damage severity at the X-direction and Y-direction from the second neural network output, where N, T, M, S and C denote testing samples of no damage, trifling damage, moderate damage, serious damage and complete damage cases, respectively.
It can be seen that the most of testing samples are within the prospected target region, except of no damage (N) and serious damage (S) cases.

(a) X-direction identification results

Figure 4.13 Identification results of the PCA-compression method, where a high level of noise is introduced and only 13 PCs projections are taken as inputs
Figure 4.13 Identification results of the PCA-compression method, where a high level of noise is introduced and only 13 PCs projections are taken as inputs (Cont’d)
Figure 4.14 illustrates the results of the overall damage severity identification from the third neural network output, where N, T, M, S and C denote testing samples of no damage, trifling damage, moderate damage, serious damage and complete damage cases, respectively. It can be seen that almost all the testing samples are within the prospected target region. Only two or three of testing samples are beyond the target region, for serious damage and complete damage cases.

(a) X-direction identification results

Figure 4.14 Identification results of the twofold-PCA-compression method, where a high level of noise is introduced and a total of 30 PCs projections are taken as inputs

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Figure 4.14 Identification results of the twofold-PCA-compression method, where a high level of noise is introduced and a total of 30 PCs projections are taken as inputs (Cont’d)
Comparing the outputs of all three neural networks with the permitted range of output values listed in Table 4.1, it can be seen that the results of the direct FRFs method are the worst and the results of the twofold PCA method are the best. The comparison of three schemed damage severity identification results indicates that: the more information that is utilized in neural network inputs, the higher the identification accuracy (IA) is. Furthermore, in order to examine the noise influence, a high level of noise (σ = 0.05) is introduced in the direct FRFs method, and low and high levels of noise (σ = 0.01 and 0.05) are considered in the PCA and twofold PCA methods. The identification of damage severity based on the three networks are conducted again and their results are summarized in Tables 4.2 (X-direction) and 4.3 (Y-direction). It can be seen that the noise influence are negligible, especially for the twofold PCA strategy, which proves the noise-filtering functionality of PCA again.

Table 4.2 Correct identification numbers and IA for overall damage severity recognition (X-direction)

<table>
<thead>
<tr>
<th>Damage cases</th>
<th>Direct FRFs method</th>
<th>Once-PCA Method</th>
<th>Twofold PCA Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n (with σ = 0.05)</td>
<td>σ (using 13 PCs)</td>
<td>σ (using 30 PCs)</td>
</tr>
<tr>
<td>30</td>
<td>13</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>N</td>
<td>82 (100)</td>
<td>97 (100)</td>
<td>100 (100)</td>
</tr>
<tr>
<td>T</td>
<td>72 (100)</td>
<td>87 (100)</td>
<td>100 (100)</td>
</tr>
<tr>
<td>M</td>
<td>21 (100)</td>
<td>91 (100)</td>
<td>99 (100)</td>
</tr>
<tr>
<td>S</td>
<td>68 (100)</td>
<td>87 (100)</td>
<td>98 (100)</td>
</tr>
<tr>
<td>C</td>
<td>82 (100)</td>
<td>99 (100)</td>
<td>97 (100)</td>
</tr>
<tr>
<td>Σ</td>
<td>325 (500)</td>
<td>461 (500)</td>
<td>494 (500)</td>
</tr>
<tr>
<td>IA</td>
<td>65.00%</td>
<td>92.20%</td>
<td>98.80%</td>
</tr>
<tr>
<td></td>
<td>47.20%</td>
<td>90.60%</td>
<td>98.20%</td>
</tr>
</tbody>
</table>

4-29
Table 4.3 Correct identification numbers and IA for overall damage recognition (Y-direction)

<table>
<thead>
<tr>
<th>Damage cases</th>
<th>Direct FRFs method (with $\sigma = 0.05$)</th>
<th>Once-PCA Method (using 13 PCs)</th>
<th>Twofold PCA Method (using 30 PCs)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n$</td>
<td>$\sigma$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>13</td>
<td>0.01</td>
</tr>
<tr>
<td>N</td>
<td>89 (100)</td>
<td>64 (100)</td>
<td>98 (100)</td>
</tr>
<tr>
<td>T</td>
<td>73 (100)</td>
<td>56 (100)</td>
<td>89 (100)</td>
</tr>
<tr>
<td>M</td>
<td>26 (100)</td>
<td>19 (100)</td>
<td>93 (100)</td>
</tr>
<tr>
<td>S</td>
<td>71 (100)</td>
<td>45 (100)</td>
<td>91 (100)</td>
</tr>
<tr>
<td>C</td>
<td>88 (100)</td>
<td>55 (100)</td>
<td>98 (100)</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>347 (500)</td>
<td>239 (500)</td>
<td>469 (500)</td>
</tr>
<tr>
<td>IA</td>
<td>69.40%</td>
<td>47.80%</td>
<td>93.80%</td>
</tr>
</tbody>
</table>

Examining Tables 4.2 and 4.3, it can be found that the identification results of the X-direction are similar with those of the Y-direction. The identification accuracy (IA) of direct FRFs method greatly depends on the number of FRFs points used. When only 13 FRF points are taken as inputs, less than half of the testing samples cannot be identified correctly. Performing data compression and feature extraction on the FRFs by principal component analysis and then taking the projections of original FRFs on a few number of principal components as inputs, the IA is improved significantly. Just using the first 13 principal components can guarantee a satisfactory identification, even in the high-level of noise contaminating case. The twofold PCA method which utilizes the most information yields the best results.
For the purpose of identifying damaged regions (floors), a total of nine neural networks are configured, whose architecture are the same as once-PCA method in previously presented damage severity recognition. The nine neural networks are corresponding to nine measurement points. The 13 principal components projection extracted from each of nine measurement locations' FRF are taken as inputs of each of neural networks respectively. By this means, the damage occurring at the concerned floors near a particular measurement location can be indicated by the output of the respective network. Herein, the target outputs for all nine neural networks are designated as 0.1 and taken as a baseline for no damage case. All of the networks are trained with the patterns of intact building structure. Then, testing samples generated from each of the damaged patterns (trifling damage, moderate damage, serious damage and complete damage) are fed to the trained networks. Combing the outputs of all 9 networks at the same level of damage case indicates the location(s) of the damaged floor(s).

Generally speaking, a large extent of damage cannot substantially guarantee a large value of network output. However, it can be assumed that the more damage that occurs near a certain measurement location, the farther the output of the network corresponding to that location will deviate from that of the baseline one. Accordingly, for the ith (i=1, 2, 3, 4, denoting trifling, moderate, serious and complete damage, respectively) damage case, the quantified damage index $DI_i$ is finally defined as relative percentage deviation with no damage case

$$DI_i = \frac{O_{di} - 0.1}{0.1} \times 100\%$$

(4.8)
where \( O_{di} \) is the network output of the \( i \)th damage case, 0.1 denotes the output of no damage case. At the same level of damage case, the damage location(s) can be indicated by combing and examining all the \( DI_i \) sequences of nine measurement locations. The higher the \( DI_i \), the more damage will occur at the neighboring location.

A total of 2700 testing samples generated from all nine measurement locations are considered, every 300 of which are corresponding to each respective location. Figure 4.15 shows the \( DI_i \) sequences of the X-direction and Y-direction at trifling damage case. It can be seen that most of \( DI_i \) values are as low as about 1% except some singular fluctuations caused by neural network training error. To eliminate the influence of such numerical error induced by network training, the \( DI \) sequences are averaged within every 300 samples of each location and marked with red dashed line in the same plot. The averaged \( DI_i \) sequence line indicates no considerable damages occurring at trifling damage case. Figure 4.16 shows the \( DI_2 \) sequences of the X-direction and Y-direction at moderate damage case. Clearly, the averaged \( DI_2 \) values of C04, C10 and C15 (about 4% or 3%) are much greater than those of other locations (about 2%), which means the presence of seismic cracks below the floor C15. Figure 4.17 shows the \( DI_3 \) sequences of the X-direction and Y-direction at serious damage case. Most of \( DI_3 \) averaged values are beyond 4% and the averaged values of the floors near C04 and C10 are even approaching to 6%. At serious damage case, the presence of seismic cracks has been extended to the upper floors of the building structure, and the lower floors (near C04 and C10) are still the most likely to be damaged. Figure 4.18 shows the \( DI_4 \) sequences of the X-direction and Y-direction at complete damage case. The results of complete damage case are similar with those of serious damage case, except that there is a much more increasing of the averaged \( DI_4 \) values at the lower floors than at the upper floors.
Figure 4.15  $DI_1$ sequence for trifling damage case
Figure 4.16 $D_l$ sequence for trifling damage case
Figure 4.17 $D_{I3}$ sequence for serious damage case
(a) X-direction identification results

(b) Y-direction identification results

Figure 4.18 $DI_d$ sequence for complete damage case
From the results demonstrated in Figures 4.15 to 4.18 on the recognition of damaged floors, the following points can be summarized as: (1) For trifling damage case, the averaged $DI_1$ values are very low and seem almost the same along all nine measurement locations. This indicates that no notable damage occurred at this damage case. Only the presence of a few minor cracks at the columns of floor C03 and at the transfer plate location has not yielded considerable $DI_1$ values; (2) For moderate damage case, the averaged $DI_2$ values below the floor C38 exhibit a somewhat deviation from the averaged $DI_1$ values of trifling damage case. The averaged $DI_2$ values of the floor C04 and C10 are especially remarkable, which indicates more seismic cracks existing at the lower floors of the building structure; (3) For serious damage case, there are distinct increases of the averaged $DI_3$ values compared with the averaged $DI_2$ values of moderate damage case. Such increasing is particularly prominent for central-above floors, which states that much more new cracks appear at the high portion of the building structure at serious damage case; (4) For complete damage case, a prominent increase in the averaged $DI_4$ values at almost all floors is observed, compared with the averaged $DI_3$ values of serious damage case. Such increases are particularly remarkable at the central-below portion of the building structure, which indicates that many more new cracks appear along the whole building structure and that the seismic damage extent of the central-below floors expands dramatically, at complete damage case; and (5) Finally, for each damage case, the identification results of damage locations at the X-direction and Y-direction are similar. Moreover, the identified locations as well as the extents of the respective locations are approximately in accordance with the actually observed results of seismic damage described in Table 3.8 of Chapter 3, except that the minor cracks at trifling damage case have not been indicated.
It should be noted that both overall seismic damage severity and damaged location identifications are performed separately in X-direction and Y-direction, by using PCA compression and neural network techniques. As described in Table 3.6 of Chapter 3, two accelerometers are equipped at west face and east face of each floor plane respectively, to measure the response in X-direction; and one accelerometer is equipped at the centre (lift location) of the floor plane. Accordingly, an average of signals from the two accelerometers in X-direction is taken to obtain the X-direction translational response, and a difference between the two signals is taken to obtain the torsional response. The accelerometer installed in Y-direction is used to obtain the Y-direction translational response. It is observed that there is no peak in the Y-direction response spectra at the frequency points corresponding to X-direction translational modes, and vice versa. This means that the X-direction and Y-direction translational responses are separated.

4.5 CONCLUDING REMARKS

In this chapter, principal component analysis (PCA) based compression technique for the measurement data analysis and neural network based damage detection methods are proposed, to recognize the seismic damage of the scaled tall building model structure. The objective of damage identification includes not only the assessment of the overall damage severity in global sense, also the damaged region(s) recognition, by using artificial neural networks and measured FRFs data reduced via principal component analysis. Intuitively, taking the complete information of FRFs over the entire frequency axis rather than just over some specified frequency range as inputs of neural networks can ensure better identification performance due to the information completeness of entire FRFs.
However, the requirement of intensive computation makes it not feasible for effective damage identification, especially when real time implementation is intended. PCA reduces the dimensionality size of the measurement data set by extracting essential features and disregarding unwanted measurement noise, which can provide efficient training samples of neural networks. The inputs of networks take computed FRFs data over specific frequency range, or take FRFs projections on a limited number of principal components extracted by principal component analysis. The identification results of seismic damage using the above different schemes of network inputs are compared. By feeding the test samples into the trained neural networks, the outputs of networks indicate the severity of the seismic damage or the locations of the damaged floors. From the comprehensive study on the feature extraction and FRFs reconstruction using only a limited number of principal components and on principal component analysis applications to seismic damage severity recognition and localization, it can be concluded that:

1. Principal component analysis is a powerful tool for reducing the size of the frequency response functions. In the study of this chapter, the number of spectral lines for each frequency response function curve is reduced from 512 to 13. It has been proven that even when such a relatively small number of principal components is used, the PCA-compressed frequency response functions account for most of the information on the original frequency response functions. This compression and extraction functionality of principal component analysis is likely to be applicable and of great practical importance for field measurement data processing applications on real-life building structures.

2. A limited number of principal components retain most of the variance in the original frequency response functions of the building structure. In addition,
principal component analysis also filters most of unwanted measurement noise. From the simulation investigation by adding low-level and high-level of artificial ‘measurement’ noise on the time-history signals of excitation and response, it is revealed that the reconstructed FRFs using just a few principal components and the subsequent seismic damage identifications based on the extracted principal components are insensitive to noise.

3. The implementation of FRF data reduction via principal component analysis in conjunction with the use of artificial neural networks training expands the damage identification capabilities. The outputs of neural networks can successfully indicate the damage presence, locations as well as extents. Compared with the damage identification results using the same number of measured FRF points over the specified frequency range as inputs to the neural networks, the PCA-preprocessing approach proves much higher identification accuracy in global damage severity recognition, especially at the case of taking twofold PCA compressed FRFs projections as inputs of neural network. In that case, complete information on all of the instrumented accelerometers is synthesized, which guarantees the highest damage indication accuracy.

4. Besides the damage presence and severity recognition, the strategy proposed in this chapter can also identify the damaged floors (regions) in an approximate sense. The results synthesized from the outputs of a total of nine neural networks approximately agree with the actual presence and distribution of the seismic cracks. The nine networks correspond to the nine regions of the model building structure.

Regardless of above achievements, it should be noted that the study proposed in this chapter is mainly to explore the potential of principal component analysis on
data compression and feature extraction, and of neural network technique on pattern recognition.

Numerical simulation is partly employed by adding artificial ‘measurement’ noise on time-domain signals, when computing the frequency response functions. The objective of such additional noise corruption is to examine the capability of principal component analysis on filtering out the unwanted measurement noise during field testing of real building structures. Although principal component analysis is executed on the shaking table testing data of the scaled model structure, the method of data compression and feature extraction developed in this chapter is also applicable to real-life field measurement of existing building structures. The principle of data compression and feature extraction can be extended to real existing prototype buildings. Moreover, in real-life field measurements, a large number of sensors are usually equipped and this leads to the generation of a greater volume of testing data, which further appeals for information compression and extraction.

Discriminately, the FRF and neural network based methods of seismic damage recognition developed in this chapter are mainly in an academic research mode rather than in a practical mode, due to the following two assumptions: the input (excitation) signals can be measured for FRF computation, and training samples of damaged scenarios are available. Unfortunately, both of the assumptions are difficult to be fulfilled in practical applications. Firstly, the excitations exerted on real buildings are usually of ambient loading, such as wind, earth micro-vibration, and so forth. Such types of excitation signals are usually hard to be measured in real-life field testing and FRFs cannot be computed to construct neural network samples accordingly, just as what is described in this chapter. Surely, if excitation can be measured in field testing of real buildings, then FRFs can be computed and the proposed method is of
great significance. However, even if "real" excitation cannot be measured, the transfer functions, which take response of the lowest floor as input, can be obtained instead of FRFs. Secondly, for real buildings, deterioration from aging and/or sudden damage due to unexpected disasters are usually unknown beforehand and need to be identified. As a result, in real applications, training samples of neural networks for a variety of damaged scenarios are not available. Motivated by such limitations of PCA combined with the use of neural network technique on damage identification of real buildings, the subsequent chapters will resort to some alternative methods.

Aiming at being more practically meaningful than academic value of seismic damage identification methods, the remaining studies in this thesis will be focused on developing approaches which can overcome or avoid the above hurdles. With regard to the problem of excitation absence, the next chapter is devoted to developing output (response)-only modal parameters identification approaches that avoid the necessity of utilizing input (excitation) information. And then the estimated modal parameters are further used to identify seismic damage.

Actually, the majority of serious damage of real-life buildings is induced by accident disasters, such as earthquake, firing, terrorism, and so forth, rather than by deterioration from aging. Therefore, the emphasis of the remaining studies is put on the earthquake-induced damage recognition in real-time manner. Accordingly, the following Chapter 6 and Chapter 7 are devoted to developing real time approaches to recognizing the seismic damage. Therein, the approaches are directly based on the earthquake records and do not need any additional ambient vibration testing. Chapter 6 is on qualitative and overall seismic damage assessment by using joint time-frequency analysis algorithms, and Chapter 7 is on quantitative realization of seismic damage assessment by using the super-resolution methods of signal analyzing.
CHAPTER 5

OUTPUT-ONLY MODAL IDENTIFICATION USING WHITE-NOISE RESPONSES AND SEISMIC DAMAGE IDENTIFICATION

5.1 INTRODUCTION

Thus far, the approaches to estimating modal parameter have been employed, based on frequency response function (FRF). They therefore belong to input-output identification community, where both excitation and response are needed. However, cases exist where it is rather difficult or expensive to exert an artificial force on large-scale civil structures. For real tall building structures, excitations are mostly from ambient wind effects. It is practically impossible to measure this ambient excitation and the outputs are the only information that can be passed to the system identification algorithms. Therefore, modal identification techniques using only responses information are of great practical meaning for building structures and should be extensively explored. When identification is based solely on the measured response (output), things become more complicated for the following reasons:

- The excitation (input) is unknown;
- The measured response (output) is noisy;
- The response may be generated by multiple excitations.
The idea of output-only modal identification can be illustrated in the following figure.

![Diagram](image-url)

**Figure 5.1 Demonstration of output-only modal identification technique**

Generally, the unknown loads are assumed to be produced by a virtual system loaded by white noise. The white noise is not assumed to drive the structural system but the total system consisting of the real structural system and the virtual loading system. Thus, in the process of modal identification, the user identifies not only the structure itself, but might also identify some 'modes' that belongs to the virtual loading system. Also, the user might identify computational modes that appear because the signals are contaminated with noise. This means that the art of output-only modal identification is the art of identifying all modes, and then being able to separate the structural modes from the noise modes and excitation modes.

The output-only modal identification only needs to conduct ambient vibration testing or measurement. And the main advantages of this kind of testing are:

- Testing is cheap and fast, since the equipment for excitation is unnecessary.
- Testing does not interfere with the operation of the structure.
- The measured response is representative of the real operating conditions of the structure.
In this chapter, white-noise excitations during the shaking table tests are
assumed to be the realizations of a stochastic process and output-only modal
parameter estimation approaches that use only white-noise responses are to be
developed and employed for the subsequent identification of seismic damage. To
ensure that the modal parameters can actually be applied on seismic damage
identification, the quality and reliability of output-only modal identification are to be
examined by comparing their results with those of conventional frequency response
function technique. It should be noted that the modal identification serves only as an
analysis tool of measured response signals. Damage indices for the sizing and
localization of seismic damage using the identified modal parameters are ultimately
to be formulated and compared.

5.2 OUTPUT-ONLY MODAL IDENTIFICATION

METHODS

Traditional methods of input-output modal parameters estimation are usually
implemented by fitting a model to the so-called frequency response function, a
function relating to excitation and response. The computation of frequency response
function is based on FFT technique. Thus, it is categorized as a frequency-domain
method. According to the signal matrix on which the identification algorithms are
manipulated, the output-only approaches of modal parameter estimation can fall into
two categories: frequency-domain methods and time-domain methods. The
subsequent sections are devoted to delving into theoretical formulation of two
identification approaches: the complex mode indicator function (CMIF) method and
the stochastic subspace identification (SSI) method, corresponding to frequency-domain and time-domain techniques, respectively.

5.2.1 Description State-space Model

Both the complex mode indicator function and the stochastic subspace identification methods are derived from the structural state-space model. As is well known, the equations of motion for a finite-dimensional linear-dynamic system are a set of \( n_z \) second-order differential equations, where \( n_z \) is the number of independent coordinates. Let \( M, \zeta, K \) be the mass, damping and stiffness matrices, respectively. The state equations can be expressed in a matrix equation as

\[
M \ddot{\mathbf{x}} + \zeta \dot{\mathbf{x}} + K \mathbf{x} = \mathbf{f}(\omega, t) \tag{5.1}
\]

where \( \ddot{\mathbf{x}} \) and \( \mathbf{\omega} \) are vectors of generalized acceleration, velocity, and displacement, respectively, and \( \mathbf{f}(\omega, t) \) is the forcing function over the period of interest at certain specific locations. Equation (5.1) can be re-formulated as a first-order system of differential equations, which are more suitable for computation and can begin with the following definition

\[
\mathbf{A}_c = \begin{bmatrix} 0 & 1 \\ -M^{-1}K & -M^{-1}\zeta \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \mathbf{\omega} \\ \dot{\mathbf{x}} \end{bmatrix}, \quad \mathbf{B}_c = \begin{bmatrix} 0 \\ M^{-1}B_z \end{bmatrix}, \quad \mathbf{f}(\omega, t) = \mathbf{B}_2 \mathbf{u}(t) \tag{5.2}
\]

where \( \mathbf{A}_c \) is an \( n = 2n_z \) by \( n \) state matrix and \( \mathbf{B}_z \) is an \( n_z \) by \( r \) input influence matrix characterizing the locations and type of inputs. The integer \( r \) is the number of
inputs. Based on the above definition, the system motion Equation (5.1) can be rewritten in a more compact form as

\[ \dot{\mathbf{x}} = A_\alpha \mathbf{x} + B_\alpha \mathbf{u} \]  \hspace{1cm} (5.3)

If the response of the dynamic system is measured by the \( m \) output quantities in the output vector \( \mathbf{y}(t) \) using sensors, then a matrix output equation can be written as

\[ \mathbf{y} = C_\alpha \dot{\mathbf{x}} + C_\omega \alpha + C_\alpha \omega \]  \hspace{1cm} (5.4)

where \( C_\alpha \), \( C_\omega \) and \( C_\alpha \) are output influence matrices for acceleration, velocity and displacement, respectively. These output influence matrices describe the relationship between the vectors \( \dot{\mathbf{x}} \), \( \ddot{\mathbf{x}} \), \( \omega \) and the measurement vector \( \mathbf{y} \). Solving for \( \dot{\mathbf{x}} \) from Equation (5.1) and substituting it into Equation (5.4) yields

\[ \mathbf{y} = C_\alpha \mathbf{M}^{-1} \left[ B_\alpha \mathbf{u} - \zeta \alpha - K_\alpha \right] + C_\omega \ddot{\mathbf{x}} + C_\alpha \omega \] \hspace{1cm} (5.5)

or

\[ \mathbf{y} = \mathbf{Cx} + \mathbf{Du} \] \hspace{1cm} (5.6)

where
\[ C = [C_d - C_a M^{-1} K, 
   C_v - C_a M^{-1} \zeta], \quad D = C_a M^{-1} B_2 \]  

(5.7)

where \( C \) is an \( m \times n \) output influence matrix for the state vector \( x \) including velocity and displacement only, and \( D \) is an \( m \times r \) direct transmission matrix. Equations (5.3) and (5.6) constitute a continuous-time state-space model of a dynamical system. The order of the system is the dimension (i.e., \( n \)) of the state matrix \( A_c \) where subscript \( c \) denotes the continuous-time state-space model.

In real-life structural modal testing, the measured signals are usually digital ones discretized via A-D (analogue to digital) converters. Therefore, it is necessary to derive a discrete-time state-space model that represents the dynamical system with discretized inputs.

Assuming the initial condition \( x(t_0) \) at \( t = t_0 \) and solving for \( x(t) \) from Equation (5.3) yields

\[ x(t) = e^{A_c (t-t_0)} x(t_0) + \int_{t_0}^{t} e^{A_c (t-\tau)} B_c u(\tau) d\tau \]  

(5.8)

for \( t > t_0 \). In real-life structural measurement, let the sampling interval be denoted as \( \Delta t \). Substituting \( t = (k+1)\Delta t \) and \( t_0 = k\Delta t \) into Equation (5.8) yields

\[ x[(k+1)\Delta t] = e^{A_c \Delta t} x(k\Delta t) + \int_{0}^{\Delta t} e^{A_c \tau} B_c u(\tau) d\tau \]  

(5.9)

Assume that \( u(\tau) \) is constant between sampling times, Equation (5.9) with a constant matrix \( B_c \) becomes

5-6
\[
x[(k+1)\Delta t] = e^{A\Delta t}x(k\Delta t) + \left( \int_{\tau}^{\tau'} e^{A\tau'}d\tau' \right)B \}u(k\Delta t)
\]

(5.10)

where the variable \( \tau \) in Equation (5.9) has been changed to \( \tau' \), by letting

\[
\tau' = (k+1)\Delta t - \tau.
\]

Based on the following definition

\[
A = e^{A\Delta t}
\]

\[
B = \int_{\tau}^{\tau'} e^{A\tau'}d\tau' \right)B \}
\]

(5.11)

Equation (5.10) can be expressed in the following compact form

\[
x(k+1) = Ax(k) + Bu(k) \quad k = 0,1,2,3
\]

(5.12)

and Equation (5.6) thus becomes

\[
y(k) = Cx(k) + Du(k)
\]

(5.13)

Similarly, Equations (5.12) and (5.13) constitute a discrete-time state-space model.

### 5.2.2 Complex Mode Indicator Function (CMIF) Method

As described in section 6.2.1, the state-space model of a dynamic system can be expressed as

\[
\dot{x}(t) = Ax(t) + Bu(t)
\]

(5.14a)
\[ y(t) = Cx(t) + Du(t) \] (5.14b)

where \( x \in \mathbb{R}^{n\times1} \) is the system state variable; \( y \in \mathbb{R}^{n\times1} \) is the observation variable, usually an acceleration signal; \( u \in \mathbb{R}^{n\times1} \) is the external excitation exerted on a limited number of discrete points; and \( A \in \mathbb{R}^{m\times n} \), \( B \in \mathbb{R}^{n\times1} \), \( C \in \mathbb{R}^{m\times n} \), \( D \in \mathbb{R}^{m\times1} \) are called the system matrix, input matrix, output matrix and direct transfer matrix, respectively.

Let singular value decomposition (SVD) of the system matrix \( A \) be denoted as

\[ A = \Psi \Lambda \Psi^{-1} \] (5.15)

then the system state-space model Equation (5.14) can be rewritten as

\[ \dot{x}(t) = \Lambda x(t) + L^T u(t) \] (5.16a)

\[ y(t) = V x(t) + D u(t) \] (5.16b)

where \( L^T = \Psi^{-1} B \), \( V = C \Psi \), and the singular values of matrix \( A \) are the diagonal elements of \( \Lambda \), namely

\[ \lambda_k = -\zeta_k \omega_k + j \omega_k \sqrt{1 - \zeta_k^2} \] (5.17)

where \( \omega_k \) is the \( k \)th order of modal frequency and \( \zeta_k \) is the \( k \)th order of modal damping.
Here, we assume that the white-noise excitations of the shaking table tests are zero-mean white noise, satisfying the following condition

\[ E[u(t)] = 0, \quad R_u(r) = E[u(t + r)u^T(t)] = R_u \delta(r) \]  \hspace{1cm} (5.18)

where \( R_u \) is a constant matrix, the spectrum matrix of the input excitation can then be written in the following form

\[ S_u(s) = \int_{0}^{\infty} R_u(t)e^{-st}dt = R_u \]  \hspace{1cm} (5.19)

It can be proven that the spectrum matrix of the observation signal can be expressed as (Ljung 1999)

\[ S_y(s) = H(s)R_uH^T(s^*) \]  \hspace{1cm} (5.20)

where \( H(s) \) is the system transfer function matrix and is computed by applying Laplace transform on the state-space model

\[ H(s) = \frac{Y(s)}{u(s)} = C(sI - A)^{-1}B + D \]  \hspace{1cm} (5.21)

where \( I \) is an unit matrix and \( Y(s), u(s) \) is the Laplace transform of the observation signal and input excitation, respectively. \( H(s) \) can be further decomposed as
\[
H(s) = \sum_{k=1}^{n} \frac{s^2}{\lambda_k^2(s - \lambda_k)} \{v_k\}^T \{l_k\} 
\]  

(5.22)

where \( \{v_k\} \) is the column vector of matrix \( V \) and denotes the \( k \)th order of mode shape; \( \{l_k\} \) is the column vector of matrix \( L \) and denotes the \( k \)th order of mode participation factor. In order to obtain the estimation of power spectra of white-noise signals measured in the shaking table tests, we substitute \( H(s) \) expressed in Equation (5.22) into Equation (5.20) and can achieve

\[
S_y(s) = \left\{ \sum_{k=1}^{n} \frac{s^2}{\lambda_k^2(s - \lambda_k)} \{v_k\}^T \{l_k\} \right\} \mathbf{R_s} \left\{ \sum_{k=1}^{n} \frac{(s^*)^2}{\lambda_k^2(s - \lambda_k)} \{l_k\}^T \{v_k\} \right\} 
\]  

(5.23)

It can be seen from Equation (5.23) that when \( s = -\zeta_k \omega_k + j \omega_k \sqrt{1 - \zeta_k^2} \), \( S_y(s) \) reaches its peak value. Synthetically examining Equations (5.22) and (5.23), it also can be concluded that when \( s \rightarrow \omega_k \) and \( \omega_k \) is only corresponding to the \( k \)th order of mode, \( S_y(s) \bigg|_{s=j\omega_k} \) is determined by the \( k \)th order of mode. Therefore, if the influence of other orders of mode is ignored, \( S_y(j\omega_k) \) can then be approximated by

\[
S_y(j\omega_k) \approx \left\{ \{v_k\}^T \mathbf{R_s} \{l_k\} \right\} \mathbf{R_s} \left\{ \{l_k\}^T \{v_k\} \right\} = \{v_k\} \alpha_k \{v_k\}^T 
\]  

(5.24)

where

\[
\alpha_k = \left( \frac{\{l_k\}^T \mathbf{R_s} \{l_k\}}{(\zeta_k \omega_k)^2} \right) 
\]  

(5.25)
Herein, executing singular value decomposition on $S_j(s)$ at $s = j\omega_k$ achieves

$$S_j(j\omega_k) = U(j\omega_k)\sum(j\omega_k)[U(j\omega_k)']^r$$  \hspace{1cm} (5.26)

and

$$U(j\omega_k) = [u_1 \quad u_2 \quad \cdots \quad u_m]$$  \hspace{1cm} (5.27a)

$$\sum(j\omega_k) = \text{diag}(s_1 \quad s_2 \quad \cdots \quad s_m)$$  \hspace{1cm} (5.27b)

where $u_k \ (k = 1, 2, \Lambda, m)$ is the column vector of matrix $U(j\omega_k)$, and $s_i \ (i = 1, 2, \Lambda, m)$ is singular value sequence with a descending order. $\sum(j\omega_k)$ is a diagonal matrix comprised of singular values and called the complex mode indicator function.

If the spectrum matrix $S(j\omega)|_{\omega_0=\omega_k}$ is solely determined by a single mode, then only $s_i$ can reach the maximum; otherwise, if $S(j\omega)|_{\omega_0=\omega_k}$ is jointly influenced by $p$ modes, then the first $p$ singular values in $s_1$, $s_2$, $\cdots$, $s_m$ can reach the maximums, and the remaining ones are trivial and can be ignored. Corresponding to the non-trivial singular value $s_i$, the $i$th column vector of $U(j\omega_k)$, $u_i$, is the $i$th order of mode shape. In this means, both the modal frequency and the mode shape of the dynamic system can ultimately be identified.

### 5.2.3 Stochastic Subspace Identification (SSI) Method

Once again, combing Equations (5.12) and (5.13) achieves a discrete-time state-space model
\[ \mathbf{x}_{k+1} = \mathbf{x}(k+1) = A\mathbf{x}(k) + B\mathbf{u}(k), \quad k = 0,1,2,\Lambda \]  
\[ \mathbf{y}_k = y(k) = C\mathbf{x}(k) + D\mathbf{u}(k) \]  

(5.28a)  
(5.28b)

Let the covariance matrix of the observed response signal be defined as

\[ \mathbf{R}_i = E\{\mathbf{y}_{k+1} \cdot \mathbf{y}_k^T\} = \lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N} \mathbf{y}_{k+1} \mathbf{y}_k^T \]  

(5.29)

and the covariance matrix between the system state variable and the observed response signal be defined as

\[ \mathbf{G}_i = E\{\mathbf{x}_{k+1} \cdot \mathbf{y}_k^T\} = \lim_{N \to \infty} \sum_{k=0}^{N} \mathbf{x}_{k+1} \mathbf{y}_k^T \]  

(5.30)

The presupposition of the above definition for \( \mathbf{R}_i \) and \( \mathbf{G}_i \) is that the stochastic process \( \mathbf{x}_k \) and \( \mathbf{y}_k \) must meet the stationary condition. The duration of the response signals under ambient excitation must be long enough to meet this requirement.

Substituting Equations (5.29) and (5.30) into the state-space model Equation (5.28), the relationship between \( \mathbf{R}_i \) and \( \mathbf{G}_i \) can be derived as

\[ \mathbf{R}_i = \mathbf{CA}^{-1}\mathbf{G}_i \]  

(5.31)

Moreover, the Toeplitz matrix can be constructed using the covariance matrix of observed response signal, \( \mathbf{R}_i \).
\[
T_{\|\|} = \begin{pmatrix}
R_i & R_{i-1} & \Lambda & R_1 \\
R_{i+2} & R_i & \Lambda & R_2 \\
M & M & O & M \\
R_{3i-1} & R_{3i-2} & \Lambda & R_i
\end{pmatrix} \in R^{\nu_{\text{vec}} \times \nu_{\text{vec}}}
\]  

(5.32)

where \( \| \| \) denotes the subscription of the upper-left and upper-right elements of the Toeplitz matrix.

Substituting Equation (5.31) into (5.32) achieves

\[
T_{\|\|} = \begin{pmatrix}
CA^{i+1}G & CA^{i+2}G & \Lambda & CG \\
CA^{i}G & CA^{i-1}G & \Lambda & CAG \\
M & M & O & M \\
CA^{2i-2}G & CA^{2i-3}G & \Lambda & CA^{i-1}G
\end{pmatrix}
\]

\[
= \begin{pmatrix}
C \\
CA \\
M \\
CA^{i-1}
\end{pmatrix} \begin{pmatrix}
A^{i-1}G \\
AG \\
G
\end{pmatrix} = O_i \Gamma_i
\]  

(5.33)

where \( O_i \in R^{\nu_i \times \nu_i} \) is called the expanded observable matrix, and \( \Gamma_i \in R^{\nu_{\text{vec}} \times \nu_{\text{vec}}} \) is called the expanded controllable matrix. It has been proven that only when the rank of \( O_i \) and \( \Gamma_i \) is equal to the dimension of the system state-space model, \( n_i \), are the \( O_i \) and \( \Gamma_i \) observable and controllable (Juang 1994).

In practical ambient measurements of real building structures, the covariance matrix expressed in Equation (5.29) can be estimated from

\[
\dot{R}_i = \frac{1}{N} \sum_{k=0}^{N} y_{k+i} y_k^T
\]

(5.34)
and by substituting $\hat{R}_i$ into Equation (5.32), the estimation of Toeplitz matrix, $\hat{T}_{ij}$ of white-noise responses is finally obtained.

Performing singular value decomposition on the estimated Toeplitz matrix $\hat{T}_{ij}$

$$\hat{T}_{ij} = USV^T$$  \hspace{1cm} (5.35)

According to the property of Toeplitz matrix (Juang 1994), $U$ and $V$ are orthogonal matrices, namely, $UU^T = I$ and $VV^T = I$. Moreover, it can be known from Equation (5.33) that when the rank of $\hat{T}_{ij}$ is equal to $n$, there are only $n$ non-zero diagonal elements in matrix $S$. As a result, singular value decomposition of $\hat{T}_{ij}$ can be rewritten as

$$\hat{T}_{ij} = (U_1 \quad U_2) \begin{pmatrix} S_1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} = U_1S_1V_1^T$$  \hspace{1cm} (5.36)

where $S_1$ is an upper-left sub-matrix of $S$, corresponding to $n$ non-zero singular values; and $U_1, V_1$ are the sub-matrices constructed by the first $n$ columns of $U, V$, respectively.

Let the estimation of $\hat{O}_i$ and $\hat{\Gamma}_i$ be

$$\hat{O}_i = U_1S_1^{1/2}T$$
$$\hat{\Gamma}_i = T^{-1}S_1^{1/2}V_1^T$$  \hspace{1cm} (5.37)
where \( T \in C^{n \times n} \) can be a any non-singular matrix, serving as linear transform of the system state-space model.

It can be seen from Equation (5.33) that there exists relationship between matrix \( \hat{T}_{2(\bullet)} \) and \( \hat{T}_{\bullet \bullet} \)

\[
\hat{T}_{2(\bullet)} = \hat{O} \hat{A} \hat{\Gamma}_i
\]  
(5.38)

where \( \hat{A} \) is an estimation of the system matrix \( A \). Substituting Equation (5.37) into (5.38) can yield

\[
\hat{A} = \hat{O} \hat{\xi} \hat{T}_{2(\bullet)} \left( \hat{\Gamma}_i \right)^\dagger = S_i^{-\frac{1}{2}} U_i \hat{T}_{2(\bullet)} V_i S_i^{-\frac{1}{2}}
\]  
(5.39)

where the symbol \((\bullet)^\dagger\) denotes a pseudo-inverse operation on the matrix.

It should be noted that if only the first \( n \) diagonal elements (singular values) in \( S \) are considerable and the remaining ones are trivial, then those trivial singular values correspond to measurement noise. A system stabilization graph can be generated by trying different \( n \), which indicates how an appropriate \( n \) is selected. Compared with the complex mode indicator function method of modal parameter estimation, the implementation of the stochastic subspace identification algorithm greatly depends on the selection of state-space dimensionality and singular value orders.

The following sections describe the results of output-only modal estimation and survey their reliability and applicability on damage identification by comparing them with the results of the input-output, frequency response function based approach.
5.3 RESULTS OF OUTPUT-ONLY MODAL IDENTIFICATION

By using the two output-only modal identification approaches in frequency-domain and time-domain, CMIF and SSI, respectively, the modal frequencies and mode shapes of the tall building model structure under white-noise excitation are identified. As described in Chapter 3, there are totally 30 times of three-minute of duration white-noise excitations exerted on the model structure. Every six times of sequential white-noise responses are thought to represent the same structure health state and hence grouped together, corresponding to no damage, trifling damage, moderate damage, serious damage and complete damage cases respectively.

In subsequent sub-sections, the implementation of the CMIF and SSI methods on white-noise response-only modal identification is to be carried out thoroughly. And the results of identification are to be compared with those of the input-output modal identification approach (based on frequency response function), for validating the reliability and superiority of output-only techniques.

5.3.1 CMIF-based Modal Identification Results

The core of the CMIF modal identification technique is to estimate the spectral density matrix, $S_y(s)$, of the observed response signals, and to perform singular value decomposition on $S_y(s)$ at each frequency line $s$. There are nine measured degrees of freedom (DOFs) for the response measurement and therefore each matrix $S_y(s)$ has a dimension of 9×9. The element $S_y(s)[i,j]$ represents the cross-spectral density between the measurement locations $i$ and $j$, at frequency line $s$. Apparently, the element $S_y(s)[i,i]$ represents the auto-power spectral density of the measurement
location \( i \). For example, at no damage case, the cross-spectral density between measurement degree of freedom located at the floor C04 (the transfer plate) and measurement degree of freedom located at the top floor C38 is shown as Figure 5.2, where both the amplitude and phase of the cross-spectral density function are demonstrated.

(a) Magnitude of spectral density between floors C04 and C38

Figure 5.2 Cross-spectral density function between floors C04 and C38
(b) Phase angle of spectral density between floors C04 and C38

Figure 5.2 Cross-spectra density function between floors C04 and C38 (Cont’d)

For the computation of the spectral density matrix, the parameters listed in Table 5.1 are taken in the FFT calculation.

Table 5.1 FFT parameters for the computation of the spectral density matrix

<table>
<thead>
<tr>
<th>Sampling frequency (Hz)</th>
<th>Points of FFT</th>
<th>Frequency resolution (Hz)</th>
<th>Windowing function</th>
<th>Overlap (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>2048</td>
<td>0.024</td>
<td>Hanning</td>
<td>66.7</td>
</tr>
</tbody>
</table>
Accordingly, the frequency-domain parameter $s$ in the spectral density matrix $S_y(s)$ which is of a dimension of $9 \times 9$, can take $2048$ discrete frequency lines, $j\omega_k$, $k=1, 2, \ldots, 2048$.

![Graph showing singular value curves](image)

**Figure 5.3** Singular value curves of the spectral density function $S_y$.

By performing singular value decomposition using Equation (5.26) at each frequency line $j\omega_k$, a total of nine singular value curves can be obtained, each of which has 2048 frequency points spacing at 0.024 Hz. Figure 5.3 shows the SVD results of nine response measurement degrees at the X-direction under the first white-noise excitation (test No. 1), where the token $SV$ denotes the singular value.
Clearly, only the first singular value (SV # 1) reaches the local maximum at the first and second modes, whereas both the first (SV # 1) and the second singular value (SV # 2) reach the local maximum at the third mode. According to the discussion addressed in section 5.2.2, it can be concluded that the first and second modes are independent, and that the third mode has an interaction with higher mode(s).

Figure 5.4 shows the identification results of the CMIF method on the first three mode shapes of X-direction, at no damage case. Using the CMIF method, modal identification of all directions at all damage cases are performed as well.

![Mode Shapes](image)

(a) The 1st mode (b) The 2nd mode (c) The 3rd mode

**Figure 5.4** The first three mode shapes at the X-direction identified by the CMIF method

### 5.3.2 SSI-based Modal Identification Results

Based on the response-only signals measured at nine measurement degrees of freedom for one direction, the covariance matrix $\hat{R}_{y,y}$ can be estimated by Equation
(5.34), where \( i \) is the number of samples used to compute the correlation functions and \( m \) denotes the number of response measurement degrees. The parameters taken for the SSI modal identification of the tall building model structure are summarized in Table 5.2.

<table>
<thead>
<tr>
<th>State-space dimension ((n))</th>
<th>Measurement DOFs ((m))</th>
<th>No. of samples for correlation function computation ((i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>9</td>
<td>256</td>
</tr>
</tbody>
</table>

Accordingly, the covariance matrix \( \hat{R}^{\text{new}}_t \) has a dimension size of \( m \times m = 9 \times 9 \) and the Toeplitz matrix expressed in Equation (5.32) has a dimension size of \( m \cdot i \times m \cdot i = (9 \times 256) \times (9 \times 256) = 2304 \times 2304 \). And the element in matrix \( \hat{R}^{\text{new}}_t \), \( \hat{R}_t[j,k] \), represents the correlation function between the response measurement degrees \( j \) and \( k \). For example, the auto-correlation function of the measurement degree of freedom located at floor C04 and the cross-correlation function between the measurement degrees of freedom located at floors C04 and C38 are demonstrated in Figure 5.5. By the same means, the correlation functions for all measurement degrees of freedom are computed. Using all of these computed correlation functions, the Toeplitz matrices are finally constructed according to Equation (5.32). And the SSI modal identification is finally accomplished based on the system matrix, \( \hat{A} \). Each matrix corresponds to one time of white-noise measurement. Singular value decomposition is conducted on each Toeplitz matrix, and the corresponding modal parameters are ultimately estimated.
(a) Auto-correlation function of floor C04

(b) Cross-correlation function between floors C04 and C38

Figure 5.5 Correlation functions of the measured response signals
When all of the correlation function matrices are available, a Toeplitz matrix $\hat{T}_{kl}$ can then be constructed using Equation (5.32). An estimation of the system matrix $\hat{A}$ of Equation (5.39) can then be obtained by performing singular value decomposition on the $\hat{T}_{kl}$ using Equations (5.35) and (5.36).

The SSI is of a parametric model estimation approach. The identification result depends on the selection of system dimension (state space dimension). However, the exact state space dimension is unknown beforehand. The way to overcome this is to estimate a range of candidate state space models and then to eliminate spurious numerical poles afterwards. The information of the structural system will be contained in all the estimated models if the state space dimension is high enough. The relationship between the size of the state space dimension and the stability of identification can be indicated by one stabilization diagram, as Figure 5.6 shows. The horizontal axis is a frequency axis ranging from zero to the Nyquist frequency. The vertical axis lists the dimensions of the available state space models, ranging from 1 to the maximum state space dimension specified. The continuous solid-line curve denotes the maximum singular value decomposition of the spectral density matrices. The poles corresponding to a certain model order are compared to the poles of a one-order-lower model. If the eigenfrequency, the damping ratio and the related mode shape differences are within the preset limits (herein, 1% for eigenfrequencies, 5% for damping ratios and 2% for the modal vectors), then a mode is characterized as stable and the corresponding pole is marked with a cross. Otherwise, the mode is characterized as unstable and marked with a diagonal cross. Additionally, there exist some spurious numerical poles, which are corresponding to noise modes and are also marked with diagonal crosses. The stabilization diagram clearly shows how an
appropriate state space dimension should be selected, for identifying a stable mode. A small size of state space dimension (less than 10) only can identify the first mode (4.60 Hz) and third mode (34.78 Hz) adequately, whereas the second mode (17.22 Hz) can be identified stably only when a large size of dimension (more than 70) is used. At other frequency lines, the modes greatly depend on the selection of the system dimension size or cannot be identified at all even when using a very large size of dimension.

Figure 5.6 System stabilization diagram of the SSI method
The identification results of the SSI method on the first three mode shapes of X-direction at no damage case are shown in Figure 5.7. Using the SSI method, modal identification of all directions (X-direction, Y-direction and Torsional-direction) at all damage cases are performed as well.

![Diagram of mode shapes](image)

(a) The 1st mode    (b) The 2nd mode    (c) The 3rd mode

Figure 5.7 The first three mode shapes at X-direction identified by SSI method

### 5.3.3 Comparisons of Modal Identification Results

In order to validate the reliability and accuracy of the developed output-only modal identification approaches, the identification results of CMIF and SSI are compared with those of frequency response function based approach, which uses both excitation and response information simultaneously. Considering that the sampling frequency is taken as 100 Hz during the shaking table tests, only the first three modes can thus be estimated and compared at each direction.
In the following, the modal parameters of the X-direction translational modes and Y-direction translational modes are separately identified by the FRF, CMIF and SSI methods, respectively. As shown below, under the excitations exerted in the X-direction, the power spectra of the measured Y-direction response signals do not exhibit peaks at the frequency points corresponding to the peaks of the measured X-direction response spectra, and vice versa. This implies that the X-direction and Y-direction translational modes are uncoupled and can be separately identified using the measured responses under the excitations in the corresponding direction. The torsional modes are identified by subtracting the two measured responses at two end-wings of the respective floor in the X-direction.

5.3.3.1 Results of natural frequency identification

Modal identifications are conducted at X-direction, Y-direction and Torsional-direction respectively. Modal identifications using measurement data are thoroughly executed by means of the FRF method and the CMIF and SSI output-only methods, respectively. The identification results of the first three modal frequencies can be summarized in Tables 5.3, 5.4 and 5.5. It should be noted that each of modal frequency listed in the tables is an averaged value of identification results for six times of white-noise tests.

Table 5.3 lists the modal frequency identification results of the X-direction using the input-output method (FRF), and the output-only methods (CMIF and SSI). Both response acceleration signals measured at nine measurement points and the excitation acceleration signal measured from the shaking table are utilized for the FRF method. Comparatively, only those response acceleration signals are utilized in the CMIF and SSI methods. Similarly, modal identifications are conducted for the Y-
direction and Torsional-direction as well. It should be noted again that there are two accelerometers equipped on two end-wings of the same height level (floor) at the X-direction. Here, the two acceleration signals at the same floor are averaged for modal identification in the X-direction and their subtractions are used for modal identification in the Torsional-direction.

It can be seen from the summary tables, that the modal frequencies identified using output-only methods are very close to those identified by the input-output (FRF) method. Particularly, the error between the FRF results and the CMIF results can even be ignored. Therefore, from the perspective of modal frequencies identification, the output-only approaches can be thought to be sufficient and reliable in the modal identification of the tall building model structure subjected the exertion of white-noise excitations. And the CMIF method is superior to the SSI method.

<table>
<thead>
<tr>
<th>Case</th>
<th>1st modal frequency (Hz)</th>
<th>2nd modal frequency (Hz)</th>
<th>3rd modal frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>FRF 4.69, CMIF 4.63, SSI 4.60</td>
<td>FRF 16.80, CMIF 16.70, SSI 17.22</td>
<td>FRF 36.00, CMIF 35.63, SSI 34.78</td>
</tr>
<tr>
<td>T</td>
<td>FRF 4.68, CMIF 4.66, SSI 4.53</td>
<td>FRF 16.72, CMIF 16.66, SSI 17.62</td>
<td>FRF 35.20, CMIF 35.18, SSI 34.82</td>
</tr>
<tr>
<td>M</td>
<td>FRF 4.39, CMIF 4.35, SSI 4.32</td>
<td>FRF 15.55, CMIF 15.69, SSI 15.02</td>
<td>FRF 33.99, CMIF 33.53, SSI 34.12</td>
</tr>
<tr>
<td>C</td>
<td>FRF 2.64, CMIF 2.61, SSI 2.58</td>
<td>FRF 11.52, CMIF 11.59, SSI 12.65</td>
<td>FRF 26.08, CMIF 26.08, SSI 25.83</td>
</tr>
</tbody>
</table>
Table 5.4 Modal frequency identification results of Y-direction

<table>
<thead>
<tr>
<th>Case</th>
<th>1st modal frequency (Hz)</th>
<th>2nd modal frequency (Hz)</th>
<th>3rd modal frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FRF</td>
<td>CMIF</td>
<td>SSI</td>
</tr>
<tr>
<td>N</td>
<td>4.30</td>
<td>4.36</td>
<td>4.38</td>
</tr>
<tr>
<td>T</td>
<td>4.20</td>
<td>4.23</td>
<td>4.18</td>
</tr>
<tr>
<td>M</td>
<td>3.91</td>
<td>3.89</td>
<td>3.82</td>
</tr>
<tr>
<td>S</td>
<td>3.32</td>
<td>3.31</td>
<td>3.34</td>
</tr>
<tr>
<td>C</td>
<td>2.74</td>
<td>2.72</td>
<td>2.78</td>
</tr>
</tbody>
</table>

Table 5.5 Modal frequency identification results of Torsional-direction

<table>
<thead>
<tr>
<th>Case</th>
<th>1st modal frequency (Hz)</th>
<th>2nd modal frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FRF</td>
<td>CMIF</td>
</tr>
<tr>
<td>N</td>
<td>4.10</td>
<td>4.12</td>
</tr>
<tr>
<td>T</td>
<td>4.10</td>
<td>4.10</td>
</tr>
<tr>
<td>M</td>
<td>3.91</td>
<td>3.93</td>
</tr>
<tr>
<td>S</td>
<td>3.32</td>
<td>3.34</td>
</tr>
<tr>
<td>C</td>
<td>1.95</td>
<td>1.92</td>
</tr>
</tbody>
</table>

5.3.3.2 Results of mode shape identification

Analogously, the first several orders of mode shapes at the respective directions can be identified using input-output and output-only techniques for modal parameters estimation. Without loss of generality, the results of mode shape identification are presented only at the X-direction in the subsequent addressing of this section. As mentioned beforehand, the X-direction refers to south-north direction, at which nine response accelerometers and one excitation accelerometer are equipped. And the Y-direction refers to east-west direction. All of the accelerometers are installed at the ends of two wings of the corresponding floors.
The following figures demonstrate the results of mode shape identification using the FRF, CMIF and SSI approaches, as well as their averages comparison, under no damage, trifling damage, moderate damage, serious damage and complete damage case, respectively. Firstly, in order to examine the repeatability and consistency of identification results of each respective method, six mode shape curves identified from six segments of white-noise response data measured at the same damage case, are plotted in the same diagram. In this way, the results of the mode shapes are plotted at three different diagrams, corresponding to the FRF, CMIF and SSI modal estimation approaches, respectively. The six mode shape curves of each method are then averaged, and three averaged curves corresponding to the three methods are plotted in the last diagram for comparison. It should also be noted that each of mode shape curve is normalized to unit displacement with respect to its maximum value.

Figure 5.8 Results of the 1st mode shape and comparison at no damage case
Figure 5.9 Results of the 2nd mode shape and comparison at no damage case

Figure 5.10 Results of the 3rd mode shape and comparison at no damage case
Figure 5.11 Results of the 1st mode shape and comparison at trifling damage case

Figure 5.12 Results of the 2nd mode shape and comparison at trifling damage case
Figure 5.13  Results of the 3rd mode shape and comparison at trifling damage case

Figure 5.14  Results of the 1st mode shape and comparison at moderate damage case
Figure 5.15 Results of the 2nd mode shape and comparison at moderate damage case

Figure 5.16 Results of the 3rd mode shape and comparison at moderate damage case
Figure 5.17 Results of the 1st mode shape and comparison at serious damage case

Figure 5.18 Results of the 2nd mode shape and comparison at serious damage case
Figure 5.19 Results of the 3rd mode shape and comparison at serious damage case

Figure 5.20 Results of the 1st mode shape and comparison at complete damage case
Figure 5.21 Results of the 2nd mode shape and comparison at complete damage case

Figure 5.22 Results of the 3rd mode shape and comparison at complete damage case
Figures 5.8 to 5.22 show the results of the first three orders of mode shape identification at the X-direction. The results show that the output-only modal parameter estimation approaches (CMIF and SSI) achieve a satisfactory accuracy of identification, compared with the input-output approach. These results are sufficient and reliable for the purpose of the subsequent seismic damage identification.

By closer examination on both modal frequencies and mode shapes estimation results shown in the above tables and figures, it can also be found that the CMIF method has a relatively higher identification accuracy than the SSI approach.

5.4 SEISMIC DAMAGE IDENTIFICATION BASED ON THE IDENTIFIED MODAL PARAMETERS

Here, seismic damage identification comprises the issues involving overall damage evaluation and further damage localization. The results of both modal frequency and mode shape identification indicate that the CMIF method has a higher identification accuracy than the SSI method and the FRF method. Accordingly, modal parameters identified by the CMIF method are to be taken for the subsequent construction of the seismic damage indicator.

5.4.1 Overall Damage Severity Evaluation

Naturally, seismic damage causes degradation in the structural stiffness, and hence decreases the modal frequencies of the model building structure, when no mass changes are assumed to be taken place due to damage. However, modal frequency is belonging to a global indicator, which usually cannot indicate the local structural characteristics. Nevertheless, modal frequency can be employed to
evaluate the overall severity of seismic damage. On the other hand, the non-linearity
behaviour of the building structure becomes more and more prominent with the
enhancement of damage extent, which also causes a fluctuation of the identified
modal parameters. Thereby, the evaluation of the overall damage severity can be
accomplished either by the reduction of the modal parameters due to the degradation
of the structural stiffness, or by examining the variances in the modal parameters due
to the non-linearity behaviour of the building structure.

5.4.1.1 Damage evaluation by frequencies reduction

Based on the results of modal frequencies identification summarized in Tables
5.3, 5.4 and 5.5, the first three modes of frequencies and their degradations with the
enhancements of the seismic damage severity are graphically presented in Figure
5.23 for the X-direction. As well, the first three modes of frequencies and their
degradations for the Y-direction are presented in Figure 5.24. And Figure 5.25
demonstrates the results of the first two modes of frequencies at the Torsional-
direction.

![Graphs showing frequency changes with damage status](image)

**Figure 5.23** The first three orders of frequencies and their reduction due to
damage (X-direction)

5-38
Figure 5.24  The first three orders of frequencies and their reduction due to damage (Y-direction)

Figure 5.25  The first three orders of frequencies and their reduction due to damage (Torsional-direction)

5.4.1.2  Damage evaluation by nonlinearity indices

Structural non-linearity behaviour can alternatively be reflected by the non-consistency property of the identified modal parameters. Accordingly, another approach to evaluating the damage severity of the building structure is to be implemented by examining the fluctuations in the estimated modal frequencies. Due to the superiority of the CMIF method to the SSI method, the CMIF identification on
each of six white-noise responses under each of five damage cases (no damage, trifling damage, moderate damage, serious damage and complete damage respectively) is performed, to obtain a total of $6 \times 5 = 30$ modal frequencies. Ignoring earthquake simulation tests and plotting the first mode of modal frequencies versus the respective white-noise random vibration tests numbers (No. 1-6, 16-21, 27-32 and 36-47) can lead to the generation of Figures 5.26, 5.27 and 5.28, for the X-direction, Y-direction and Torsional-direction, respectively.

With the increasing severity of seismic damage, it can be observed that not only the modal frequency values decrease, but also the variances of six frequencies corresponding to the six white-noise tests within the same level of damage aggrandize. It can be concluded that when subjected to gradually enhanced damage, the building structure loses its stiffness, and simultaneously accumulates more and more non-linearity behaviours.

![Figure 5.26 The first mode of frequencies versus test No. (X-direction)](image-url)
Figure 5.27  The first mode of frequencies versus test No. (Y-direction)

Figure 5.28  The first mode of frequencies versus test No. (Torsional-direction)
To quantitatively estimate overall damage, the following damage severity indices are developed in this study, based on the first mode of frequency variance. The normalized average change index is defined as

$$\text{NormMeanIndex}_k = \frac{\sum_{i=1}^{M} w_{i,k} \cdot c_{i,k}}{\max(\sum_{i=1}^{M} w_{i,k} \cdot c_{i,k})_k} \quad (5.40)$$

and the normalized standard deviation index is defined as

$$\text{NormStdIndex}_k = \frac{\sum_{i=1}^{M} w_{i,k} \cdot \sigma_{i,k}}{\max(\sum_{i=1}^{M} w_{i,k} \cdot \sigma_{i,k})_k} \quad (5.41)$$

where $w_{i,k}$, $c_{i,k}$ and $\sigma_{i,k}$ are the weighted coefficient, mean value change ratio and standard deviation of the modal frequency, defined as Equations (5.42), (5.43) and (5.44) respectively.

$$w_{i,k} = m_{i,k} = \text{mean}(\phi_{i,j})_k, j = 1, 2, \Lambda , N \quad (5.42)$$

$$c_{i,k} = \left( \frac{m_{i,d} - m_{i,u}}{m_{i,u}} \right) \times 100\% \quad (5.43)$$

$$\sigma_{i,k} = \text{std}(\phi_{i,j})_k, j = 1, 2, \Lambda , N \quad (5.44)$$
where

\( M \) indicates the number of measurement points, being nine in this study;

\( N \) indicates the number of measurement times at the same damage case;

\( d \) denotes the damaged case;

\( u \) denotes the undamaged case;

\( \phi_{i,j} \) represents the normalized first mode shape value of the \( i \)th measurement point at the \( j \)th measurement.

The graphical representations of the above indices normalized with respect to the complete damage case are demonstrated in Figures 5.29, 5.30 and 5.31, for the X-direction, Y-direction and Torsional-direction, respectively.

![Graphical representation](image)

**Figure 5.29** The normalized average change index and normalized standard deviation index at X-direction
Figure 5.30 The normalized average change index and normalized standard deviation index at Y-direction

Figure 5.31 The normalized average change index and normalized standard deviation index at Torsional-direction
Combing two normalized damage severity indices indicates the overall state of seismic damage in the building structure at each respective direction. Figure 5.29 demonstrates the normalized average change index and normalized standard deviation index of the X-direction. It can be seen that at X-direction: the normalized standard deviation index aggrandizes slightly when the model building structure experiences a trifling damage, whereas both the normalized average change index and normalized standard deviation index aggrandize significantly when the building experiences a moderate damage, and such aggrandizement continues with the occurrences of the serious and complete damage. The observation on two indices change indicates that the inelastic or nonlinear properties of the model building structure begin to increase significantly when the structure incurs a moderate damage. In this respect, the moderate damage is fatal for the building at X-direction. Figure 5.30 demonstrates the normalized average change index and normalized standard deviation index of the Y-direction. The results are similar with those of the X-direction, except that both two indices aggrandize more gradually. Illuminately, the inelastic or nonlinear properties of the building increases more gradually at Y-direction. Figure 5.31 demonstrates the normalized average change index and normalized standard deviation index of the Torsional-direction. Compared with the results of the X-direction and Y-direction, the most prominent discrimination is that both the normalized standard deviation index and normalized average change index change slightly under trifling damage, moderate damage and even serious damage. The indices aggrandize drastically until complete damage occurring. It can thus be concluded that the inelastic or nonlinear properties of the building at Torsional-direction are not as apt to be changed as the cases of the X-direction and Y-direction.
When subjected to complete damage, the building loses most of its stiffness and aseismic capability.

5.4.2 Approximate Damage Localization

Using both the reductions of the identified modal frequencies and their variance can only indicate the overall severity of seismic damage. Nevertheless, the approximate damage location(s) should be identified by incorporating the mode shapes with the frequencies. In the subsequent study, a variety of damage localization indices are to be constructed and compared, based on validation with actual visual in-site inspection results. Ko et al. (2000) examined such damage indices as modal assurance criteria (MAC), mode shape based $Z$-value and modal flexibility, for damage identification of cable-supported bridges in Hong Kong.

5.4.2.1 MAC and COMAC damage index

The MAC and COMAC damage indices are defined as Equations (5.45) and (5.46) respectively.

\[
MAC(i, j) = \frac{\left(\{\phi^u\}_i \{\phi^d\}_j\right)^2}{\left(\{\phi^u\}_i \{\phi^u\}_i\right)\left(\{\phi^d\}_j \{\phi^d\}_j\right)}
\]  

(5.45)

\[
COMAC(p) = \frac{\sum_{r=1}^{N} |\phi_{pr} \varphi_{pr}|^2}{\sum_{r=1}^{N} (\phi_{pr})^2 \sum_{r=1}^{N} (\varphi_{pr})^2}
\]

(5.46)

where $i, j$ and $p$ denote the mode order, $\phi$ or $\varphi$ represents the mode shape, and subscription $u$ and $d$ indicate the undamaged and damaged cases.
The computation results of MAC and COMAC for the model building are summarized in the following tables and figures.

<table>
<thead>
<tr>
<th>Table 5.6 MAC matrix of T case</th>
<th>Table 5.7 MAC matrix of M case</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.995</td>
<td>0.096</td>
</tr>
<tr>
<td>0.103</td>
<td>0.985</td>
</tr>
<tr>
<td>0.134</td>
<td>0.058</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5.8 MAC matrix of S case</th>
<th>Table 5.9 MAC matrix of C case</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>0.089</td>
</tr>
<tr>
<td>0.098</td>
<td>0.989</td>
</tr>
<tr>
<td>0.129</td>
<td>0.064</td>
</tr>
</tbody>
</table>

Tables 5.6 to 5.9 show the MAC matrix based on the identification results of the first three modes of mode shapes, for trifling damage, moderate damage case, serious damage and complete damage case respectively. It can be seen that the diagonal elements are close to 1.0 and that the other non-diagonal elements are very small, which indicates a high dependency of the first three modes before and after being incurred earthquake damage. And there is no interaction among the identified first three modes.
Figures 5.32 and 5.33 illustrate the COMAC vector results for trifling damage, moderate damage, serious damage and complete damage cases, respectively. Compared with visual inspection results, the COMAC vector can indicate damaged locations (floors) basically, except for the floor of C10.
5.4.2.2 Mode shape based Z-value damage index

In this study, Z-value damage localization index is defined as

\[
Z_i = \frac{\text{index}(i) - M(\text{index})}{\sigma(\text{index})}
\]  \hspace{1cm} (5.47)

where

\[
\text{Index}_j(i) = \frac{|C^d_j(i) - C^u_j(i)|}{\sum_i |C^d_j(i) - C^u_j(i)|}
\]  \hspace{1cm} (5.48)

\[
C_j(i) = \frac{\phi_j(i-1) + \phi_j(i+1) - 2\phi_j(i)}{2I_i^2}
\]  \hspace{1cm} (5.49)

\(i\) and \(j\) denote the mode order and mode shape node number; \(\sigma\) and \(M\) denote the standard deviation and mean value of the index sequence. The computation results of Z-value for seismic damage indicating are demonstrated in Figures 5.34 to 5.37.

Figure 5.34. Z-value of the 1st, 2nd and 3rd mode at trifling damage case
Figure 5.35  Z-value of the 1st, 2nd and 3rd mode at moderate damage case

Figure 5.36  Z-value of the 1st, 2nd and 3rd mode at serious damage case

Figure 5.37  Z-value of the 1st, 2nd and 3rd mode at complete damage case
From Figures 5.34 to 5.37, it can be seen that: (a) for each damage case, the $Z$-values of the first, second and third modes do not give a consistent indication on seismic damage location(s); and (b) the mode shape based $Z$-values cannot conform to the results of visual inspection on seismic damage and are thus insufficient for the identification of damage locations (floors). Thereafter, an alternative index based on both mode shape and natural frequency, called modal flexibility damage index, is to be constructed for seismic damage localization.

### 5.4.2.3 Modal flexibility damage index

The dynamic modal flexibility matrix is defined as

$$ [F] = [\Phi]^T [\Lambda]^{-1} [\Phi] $$  \hspace{1cm} (5.50)

where each element can be written in the form

$$ F_{ij} = \left\{ \begin{array}{l}
\sum_r \frac{1}{\omega_r^2} \phi_{ir} \phi_{jr} \\
\sum_r \frac{1}{\omega_r^2} \phi_{ir}^2
\end{array} \right. $$ \hspace{1cm} (5.51)

For damage identification purpose, the percentage modal flexibility ($PMF$) is related to the percentage change ratio of diagonal element of $[F]$, namely defined as

$$ PMF_i = \frac{|F_{ii}^d - F_{ii}^n|}{F_{ii}^n} $$ \hspace{1cm} (5.52)
The computation results of \( PMF \) using the first three identified modal frequencies and mode shapes are shown in Figures 5.38 and 5.39, where \( T, M, S \) and \( C \) denotes the trifling, moderate, serious and complete damage cases respectively.

**Figure 5.38** \( PMF \) for trifling and moderate damage cases

**Figure 5.39** \( PMF \) for serious and complete damage cases
Clearly, the resulted PMFs correctly indicate the earthquake-induced damage locations (floors), which conforms to the results of visual inspection as described in Chapter 3. By a closer examining on Figure 5.38, it can be observed that the earthquake-induced cracks mainly concentrate on the lower floors and the transfer plate at trifling and moderate damage cases. From Figure 5.36 and 5.37, it can be concluded that such earthquake-induced cracks have gradually expanded to the whole structure at serious and complete damage cases, at which the PMF values of the upper floors (C25 to C42) also reach a considerable level.

5.5 CONCLUDING REMARKS

This chapter discusses two output-only approaches of modal parameters estimation: the stochastic subspace identification (SSI) method and the complex mode indication function (CMIF) method. Applied to the white-noise response data of the scaled model building, two output-only methods are compared with the input-output method, from the perspective of consistency and repeatability of the results. It has been proven that the developed output-only approaches achieve very close identification results with input-output method. The SSI method greatly depends on the selection of such parameters as the dimensionality of the system state-space model, the singular value order, and so forth. Comparatively, the CMIF method is much simpler and more straightforward. Moreover, the identification results of the CMIF method exhibit a better repeatability and consistency than those of the SSI method. Based on the identified natural frequencies and mode shapes using the CMIF method, a variety of damage indices are constructed, for the purpose of
evaluating the overall structural integrity and identifying the approximate damage locations (floors).

For the evaluation of inelastic or nonlinear behaviours, the configured normalized average change index and normalized standard deviation index can confidently indicate the occurrence of nonlinearity. Examining on both indices, it has been found that the nonlinear behaviours of the X-direction is the most apt to change, whereas the nonlinear behaviours of the Torsion-direction is most insusceptible. It should be noted that the proposed evaluation method herein is mainly used to detect the occurrence of structural nonlinear behaviours. For further identification on extents of such nonlinearity, the evaluation method is based on a comparison between baseline structure and damaged structure. In real applications, such nonlinearity recognitions should be implemented by developing an elaborate mechanical model and numerical simulation on each possible damage scenarios.

For the purpose of identification on damage locations (floors) or extents, some other indices than the above two nonlinearity evaluating indices are developed. Among all these damage identification indices, the modal flexibility based index, \( PMF \), is the best one for the identification of the damaged floors, and the \( COMAC \) index can also indicate the damage locations approximately. The \( Z \)-value index has not achieved a consistent and correct recognition on damage locations.

The developed output-only approaches of modal parameters estimation are of great significance for real-life applications on existing building structures, where only the ambient response measurement data are usually available. Additionally, under soft soil condition, there exists an interaction between soil and foundation of buildings. As a result, the foundation of real buildings will have translational, torsional and rock motions, which leads to a problem of multiple input and multiple
output identification. Because the output-only approaches developed in this chapter work in the spatial domain and simultaneously use measurement data from spatially distributed sensors, they automatically distinguish the modal components in different directions from multiply excited responses and identify potentially coupled modes.

The derived damage indices are independent of mathematical model of the investigated structure. Thus the approaches are applicable to real building structures of any complexity.
CHAPTER 6

JOINT TIME-FREQUENCY ANALYSIS FOR OVERALL SEISMIC DAMAGE EVALUATION

6.1 GENERAL REMARKS

Heretofore, the modal parameters estimation of the model building based on the white-noise measurement data and the identification on cumulative seismic damage have been intensively explored, by using different identification approaches and damage indices. It should be pointed out that the modal parameters estimation and identification of seismic damage are carried out based on white-noise excited responses only or on both white-noise excitations and responses simultaneously. White-noise shaking table tests are conducted between two consecutive levels of earthquake simulation tests and in a small-amplitude manner, which means that they do not lead to new damage occurrence in the building structure. The corresponding identified modal parameters indicate all of the seismic damage accumulated by all the prior earthquake events. Whatever the signals used, additional measurements of post-earthquake ambient vibration or modal testing are needed.

Intuitively, to detect the seismic-induced damage on the tall building structures, it is more impendent and practical to investigate the influences of earthquake damage on the structure in real time sense. In other words, the whole process of the evolving history of the seismic damage based on earthquake excitation and response
measurements should be fully recognized. This will provide a sufficient basis upon which to conduct the post-earthquake evaluation as well as the investigation into the destroying mechanism of the earthquakes.

Nevertheless, the identification techniques of modal parameters presented in the previous chapters are only appropriate for signals that are stationary and of relatively long duration, such as cases of white-noise excitations. For earthquake records that are of short-duration and usually non-stationary, both the input-output and output-only modal estimation approaches are inadequate. For example, Figures 6.1 to 6.3 illustrate the results of the first mode shape identification using the frequency response function (FRF) method, under three successively enhanced earthquake excitations. In each figure, the left plots use the excitation and response records of the whole duration of the earthquake for FRF computation, whereas the right ones only use the partial records of excitation and response near the peak acceleration. Clearly, the identification accuracy and consistency of the left plots are much worse than those of the right plots. This indicates that trivial portions of earthquake records can degrade the identification accuracy and reliability due to the measurement noise and non-stationary property. Therefore, the FRF method adopted in Chapter 4 is not applicable for earthquake excitation and response signals, unless a critical portion of earthquake records (near the peak acceleration) are extracted as input and output for FRF computation. Naturally, the modal estimation would be even worse, if only the earthquake response records are utilized based on the output-only modal identification approach. Figure 6.4 illustrates the identification results of the first mode shape using the output-only modal identification strategy adopted in Chapter 5. It can be seen that the identification accuracy and consistency are even worse than those obtained using the FRF method.
(a) Using complete earthquake records  (b) Using partial earthquake records

Figure 6.1 Comparison of the first mode shape identification results based on FRF method under small (minor-level) earthquakes

(a) Using complete earthquake records  (b) Using partial earthquake records

Figure 6.2 Comparison of the first mode shape identification results based on FRF method under moderate-level earthquakes
(a) Using complete earthquake records   (b) Using partial earthquake records

Figure 6.3 Comparison of the first mode shape identification results based on FRF method under large (strong-level) earthquakes

(a) Small earthquakes   (b) Moderate earthquakes   (c) Large earthquakes

Figure 6.4 The first mode shape identification results based on output-only method under three levels of earthquake excitations

6-4
In conclusion, conventional modal parameters estimated from earthquake records, whether using earthquake response or using both excitation and response, cannot be used to identify seismic damage due to the low accuracy of their estimations. Accordingly, alternative signal-processing techniques should be applied when directly identifying seismic damage based on the earthquake records, such as joint time-frequency analysis and super-resolution modal estimation.

This chapter and next are to explore the potential of real time seismic damage identification based on the earthquake records themselves, rather than on any additional post-earthquake measurements of ambient vibration. As a result, any identification activities, such as modal parameters identification and subsequent real time seismic damage evaluation are earthquake-specific. As described in Chapter 3, there are a total of 17 simulated earthquakes with various attacking intensity and site soil conditions in the shaking table testing. The damage mechanisms of all earthquakes on the tall building model structure and their induced new seismic damage evolving are to be examined in detail thereafter.

This chapter is mainly concerning on the derivation of the excitation-independent building response, called the seismic pulse response, and theoretical formulations of the joint time-frequency analysis algorithms. Practical applications of the joint time-frequency analysis methods on the derived seismic pulse responses are then presented, for the purposes of qualitative damage evaluation. Finally, comparisons and discussions on the merits and pitfalls of each time-frequency analysis method are to be made. Further work beyond these time-frequency analysis approaches is also proposed at the end of this chapter, for real time evaluation of quantitative seismic damage.
6.2 DERIVATION OF SEISMIC PULSE RESPONSE

From the spectral analysis of view, the seismic response of the building structure is simultaneously comprised of earthquake excitation components and structural response components. For damage identification purpose, the ‘pure’ structure-related responses that confidently reflect the structure-self dynamic behaviours are utilized. With regard to physical meaning, the transfer function is defined as the response of the structure under a unit excitation force, which hints that the structural response in frequency-domain can be transformed back to excitation-independent dynamic pulse response in time domain by performing a reverse-FFT on the transfer function. Accordingly, the genuine behaviours of the structure can be grasped by analyzing the obtained structure-dependent decay pulse responses.

Let \( x(t) \) denote excitation time-history and \( y(t) \) be the corresponding response signal, \( H(\omega) \) is then the transfer function of \( x(t) \) and \( y(t) \). The derivation of seismic pulse response can be summarized in the following two steps:

1. Compute transfer function \( H(\omega) \) by

\[
H(\omega) = \frac{\int y(t) \exp(-j\omega t) dt}{\int x(t) \exp(-j\omega t) dt}
\] (6.1)

where the denominator and numerator perform Fourier transform of the excitation and response signal respectively. Classically, transfer function is defined as the quotient of response Fourier transform to excitation Fourier transform.
2. Compute the seismic pulse response $h(t)$ by performing reverse-FFT on $\hat{H}(\omega)$

$$h(t) = \frac{1}{2\pi} \int_{0}^{\infty} \hat{H}(\omega) \exp(j\omega t) d\omega$$  \hfill (6.2)

It should be noted that $\hat{H}(\omega)$ is filtered version of $H(\omega)$ on certain frequency bands of interests. In this study, recognizing that the first resonant mode of the model building structure is always within the range of each earthquake excitation frequency spectra and thus can always be excited, a low-pass Butterworth filter is applied over this frequency band. Herein, $\omega_1$ is the first order of circular frequency and the obtained $h(t)$ is the first mode of free decay response of the model building structure. In the remainder of this dissertation, the terminology \textit{seismic pulse response} refers to the pulse response $h(t)$ of the building under earthquake excitations.

6.3 JOINT TIME-FREQUENCY ANALYSIS

ALGORITHMS

This section is devoted to presenting joint time-frequency analysis (JTFA) approaches and their applications on the derived seismic pulse responses.

6.3.1 The Need of JTFA

The term \textit{signals} generally refers to a function of one or more independent variables, which contain information about the behaviour or nature of some phenomenon. Common examples of the signals include electrical current, image,
speech signals, stock indexes, etc., which are all produced by some time-varying processes.

Among the number of infinite possible variables, the most important are time and frequency, because they are closely related to the everyday life of people. Based on frequency behaviours, signals can further be grouped into two categories. First is the one whose frequency contents do not change with time, such as normal engine vibration. It has been well known that the frequency behaviour of this kind of signal can be well characterized by the conventional Fourier transform. Very often, people call this type of signal a stationary signal. The second type of signals are those whose frequency contents evolve with time, such as biomedical signals, speech signals, stock indexes, and vibrations. This kind of signals is usually called a non-stationary signal. The majority of signals encountered in the real world belong to this category. Obviously, the model building responses under various levels of earthquake excitations, especially at serious ones, fall into this category. The dynamic characteristics of the building structure change during short-period of earthquake events along with the occurrence and development of cracks. Since the conventional Fourier transform does not tell how a signal’s frequency contents change with time, the classical Fourier analysis is not adequate for many real signals. The goal of this section is to systematically discuss new representations that describe a signal's behaviour in time and frequency domains simultaneously.

However, the signal’s time and frequency behaviours are usually not independent. When a signal’s time duration becomes narrower, its frequency bandwidth will become wider, and vice versa. The time duration and frequency bandwidth are arbitrarily small simultaneously, which is traditionally named uncertainty principle.
6.3.2 Theoretical Foundation: Signal, Expansion and Inner Product

From the mathematical point of view, the representations of a signal are not unique. By the expansion, a given signal can be literally represented in an infinite number of ways. In other words, given any signal \( s \) from the domain \( \psi \), where \( \psi \) can be a finite-dimension or infinite-dimension, signal \( s \) can be written in terms of a linear combination of the set of elementary functions \( \{\psi_n\}_{n \in \mathbb{Z}} \) for \( \psi \)-domain, i.e.,

\[
s = \sum_n a_n \psi_n
\]  

(6.3)

Figure 6.5 illustrates an orthogonal expansion, in which the expansion coefficients \( a_n \) are exactly the signal projection on the elementary functions.

![Diagram showing signal expansion](image)

**Figure 6.5 Demonstration of signal expansion**
The most popular example is the Fourier series that decomposes a periodic time signal as the linear combination of a set of harmonically related complex sinusoidal functions \( \exp(j2\pi n t / T) \). Namely, a signal \( s(t) \) can be expanded as so-called Fourier series

\[
s(t) = \sum_{n=-\infty}^{\infty} a_n \exp(j \frac{2\pi}{T} nt)
\]

(6.4)

where the expansion coefficients \( a_n \) are a signal's orthonormal projections on the complex sinusoidal functions \( \exp(2\pi n t / T) \), which indicates the amount of signals presented at the frequency \( 2\pi n / T \).

If the set of \( \{\psi_n\}_{n \in \mathbb{Z}} \) is complete for \( \psi \)-domain, that is, all signals \( s \in \psi \) can be expanded as in Equation (6.3), there will exist a dual set \( \{\hat{\psi}_n\} \) such that the expansion coefficients can be computed by the regular inner product, such as

\[
a_n = \langle s, \hat{\psi}_n \rangle = \sum_{k=-\infty}^{\infty} s[k] \hat{\psi}_n[k]
\]

(6.5)

In mathematical terminology, the operation in Equation (6.5) is named inner product, which is used to compute expansion coefficients \( a_n \) and is thus called transformation, where \( \hat{\psi}_n(t) \) is named the analysis function. Correspondingly, Equation (6.3) is called inverse transform and \( \psi_n(t) \) is named the synthesis function or elementary function.
Due to the short-duration and non-stationary natures of the earthquake signals of the model tall building structure, the conventional FFT and output-only modal identification techniques developed in Chapter 5 are not applicable. Therefore, it becomes imperative to resort to some appropriate joint time-frequency analysis algorithms for those earthquake excitation and response signals.

### 6.3.3 Short-time Fourier Transform

Obviously, the frequency contents of the seismic pulse response change over time, and conventional Fourier transform cannot explicitly reflect the response signal's time-varying natures due to the fact that the basis functions used in the classical Fourier analysis are not associated with any particular time instant. A simple and common way to overcome this deficiency is to add a short-length window function on the analyzed signal and to compare the signal with elementary functions that are localized in time and frequency domains simultaneously, i.e.,

$$STFT(t, \omega) = \int s(\tau) \gamma_{t,\omega}(\tau) d\tau = \int s(\tau) h(\tau - t) e^{-j\omega \tau} d\tau$$  \hspace{1cm} (6.6)

where $\gamma_{t,\omega}(t)$ is the elementary function and $h(t)$ is the window function. Equation (6.6) is a regular inner product and reflects the similarity between the original signal $s(t)$ and the elementary function $\gamma_{t,\omega}(t)$.

For discretized sampling signal, the short-time Fourier transform can be written in the following form
\[ C_{m,n} = \text{STFT}(m \Delta M, n) = \sum_{i=0} \mathcal{S}[i]h[i - m \Delta M]e^{-j\frac{2\pi i n}{N}} \]  

(6.7)

where \( N \) denotes the number of frequency bins and \( \Delta M \) denotes the time sampling interval. The STFT spectrogram is defined as the square of the STFT, as shown in the following equation

\[ SP(m \Delta M, n) = |\text{STFT}(m \Delta M, n)|^2 = \left| \sum_{i=0} \mathcal{S}[i]h[i - m \Delta M]e^{-j\frac{2\pi i n}{N}} \right|^2 \]  

(6.8)

The original signal \( s[i] \) can be represented as the weighted sum of the frequency-modulated and time-shifted function \( h[i] \)

\[ s[i] = \sum_{m} \sum_{n=0}^{N-1} C_{m,n} h[i - m \Delta M] e^{j\frac{2\pi i n}{N}} \]  

(6.9)

which is also called inverse short-time Fourier transform.

Figure 6.6 depicts the procedure of implementing the STFT. Firstly multiply the window function \( h(t) \) with the original signal \( s(t) \) and then compute the Fourier transform of the product \( s(\tau)\gamma(\tau-t) \). Because the window function usually has a short time duration, the Fourier transform reflects the signal’s local frequency properties. Finally, by moving the window function \( \gamma(t) \) and repeating the above process, how the signal’s frequency contents evolve over time can be determined.
When the window function $\gamma(t)$ is taken as a Gaussian-type function, the resulting short-time Fourier transform and reverse-transform are called the Gabor transform and the Gabor expansion respectively.

The short-time Fourier transform and STFT-based spectrogram are simple and can be computed quickly, but significantly suffers from the windowing effects.

### 6.3.4 Wigner-Ville Distribution and Cohen Class

The results achieved from the STFT are subject to the selection of window functions. This section is devoted to introducing a more general method of describing the energy distribution of the signals in the joint time-frequency domain.

As is well known, the square of the Fourier transform is called the power spectrum. According to the Wiener-Khinchin theorem (Qian and Chen 1996), the
The power spectrum can also be considered as the Fourier transform of the auto-
correlation function \( R(\tau) \)

\[
PS(t, \omega) = |S(\omega)|^2 = \int R(\tau) \exp(-j\omega\tau) d\tau
\]  
(6.10)

where the auto-correlation function \( R(\tau) \) is computed by

\[
R(\tau) = \int s(t)s(t-\tau) dt
\]  
(6.11)

Equation (6.11) is a function only of frequency, rather than of both time and
frequency, which indicates how much energy is present in frequency \( \omega \) over the
entire time period. Thus, there is no way to tell whether or not a signal’s power
spectrum changes over time.

By examining Equation (6.11), one possible way to depict a time-dependent
spectrum can be proposed to depict a time-dependent power spectrum: making the
auto-correlation function time-dependent. Accordingly, the resulting Fourier
transform of the time-dependent auto-correlation function \( R(t, \tau) \), with respect to
variable \( \tau \), is then a function of time, namely

\[
PS(t, \omega) = \int R(t, \tau) \exp(-j\omega\tau) d\tau
\]  
(6.12)
where $PS(t, \omega)$ is a time-dependent power spectrum, $R(t, \tau)$ is a time-dependent auto-correlation function, and its choice is not arbitrary.

The preceding STFT spectrogram can also be written as the Fourier transform of the instantaneous auto-correlation function $R(t, \tau)$, where

$$R(t, \tau) = \frac{1}{2\pi} \int A_r(\theta, \tau) A_r(\theta, \tau) \exp(j\theta) d\theta$$  \hspace{1cm} (6.13)

where $A_r(\theta, \tau)$ represents the ambiguity functions of signal $s(t)$ and $A_r(\theta, \tau)$ is the ambiguity function of the analysis window $\gamma(t)$.

The Wigner distribution was originally developed for the area of quantum mechanics in 1932 and was firstly introduced for signal analysis by a French scientist Ville 15 years later (Qian and Chen 1996). It is now commonly known in the signal processing community as the Wigner-Ville distribution (WVD). The WVD is defined by

$$WVD(t, \omega) = \int R(t, \tau) \exp(-j\omega \tau) d\tau$$  \hspace{1cm} (6.14)

where the instantaneous auto-correlation function is chosen to be

$$R(t, \tau) = s(t + \frac{\tau}{2}) s(t - \frac{\tau}{2})$$  \hspace{1cm} (6.15)
For a discretized sampling signal $s[i]$, the Wigner-Ville distribution can be written as

$$WVD[i,k] = \sum_{m=-r/2}^{r/2} \Re[i,m]e^{-j2\pi km/l}$$  \hspace{1cm} (6.16)$$

where the instantaneous correlation is given by

$$\Re[i,m] = z[i+m]z[i-m]$$  \hspace{1cm} (6.17)$$

and $z[i]$ is the analytical, or interpolated form of the original signal $s[i]$ (Qian and Chen 1996).

The Wigner-Ville distribution possesses many attractive properties, such as simplicity, fast and best joint time-frequency resolution of all known quadratic JTFA algorithms due to its merit of not suffering from the window problem. However, if the analyzed signal contains more than one component, the WVD method suffers from so-called cross-term interference and this severely limits the applications. One feasible way to attenuate the negative influence of the cross-term interference is to perform 2D filtering to the Wigner-Ville distribution. The result can be described as

$$\mathcal{C}[i,k] = \sum_{m=-r/2}^{r/2} \sum_n \Phi(n,m)\Re[i-n,m]e^{-j2\pi km/l}$$  \hspace{1cm} (6.18)$$

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where $\Phi[i, m]$ denotes the kernel function. Leon Cohen developed the above representation of $C[i, k]$, so it is traditionally known as Cohen's class (Cohen 1989; Cohen 1995). The Cohen's class method is more general and flexible. When the kernel function in Equation (6.18) is taken as

$$
\Phi[i, m] = \sqrt{\frac{\alpha}{4\pi m^3}} e^{-ui\kappa(m^2)}
$$

(6.19)

the corresponding yield is called the Choi-Williams distribution (CWD), where a trade-off between the cross-term interference and time-frequency resolution can be balanced by adjusting the parameter $\alpha$.

### 6.3.5 Continuous Wavelet Transform

Another alternative tool for time-frequency analysis of the seismic pulse responses employed in this chapter is continuous wavelet transform. Unlike Fourier transform, STFT and Gabor transform, where a set of harmonically related complex sinusoidal functions $\exp(j2\pi mt/T)$ are taken as elementary functions, the wavelet transform employs the dilated (scaled) and translated (time-shifted) elementary functions $\psi(t) \frac{t - b}{a}$ to measure the given signal, where the original elementary function $\psi(t)$ is known as the mother wavelet. If the centre frequency of the mother wavelet $\psi(t)$ is $\omega_0$, then the centre frequencies of its time-scaled version (or dilation) $\psi(t) \frac{t}{a}$ will be the reciprocal of $\omega_0$, say, $< \omega > = \omega_0 / a$. When the original signal
under consideration is a continuous-time function, the corresponding presentation is
called the continuous-time wavelet transform (CWT), defined as

\[
CWT(a,b) = \frac{1}{\sqrt{|a|}} \int s(t)\psi\left(\frac{t-b}{a}\right)dt \quad a \neq 0
\]  

(6.20)

where \( \psi(t) \) denotes the mother wavelet, \( a \) represents the scale index that is the
reciprocal of the frequency, and \( b \) indicates the time shifting (or translation).
Assuming that \( \psi(t) \) is centred at time zero and frequency \( \omega_0 \), then its dilation and
translation version \( \psi\left(\frac{t-b}{a}\right) \) is centred at time \( b \) and frequency \( \omega_0/a \). As a result,
the continuous wavelet transform \( CWT(a,b) \), the inner product of \( s(t) \) and \( \psi\left(\frac{t-b}{a}\right) \),
exhibits the signal local behaviour in the vicinity of \( (b, \omega_0/a) \). The square of the
quantity \( CWT(a,b) \) is named as the scalegram.

Due to its ability in local signal expressing, the CWT is efficient in
representing non-stationary signals. In the subsequent comparative study, the CWT is
to be employed for the analysis of the short-duration of seismic pulse response and
assessment of seismic damage.

Although both wavelets and STFT can be used to study a signal's local
behaviours, the selections of their elementary functions are substantially different.
For STFT-based signal decomposition, such as the Gabor expansion, any function
can be taken as the window function. For wavelet decomposition, the valid mother
wavelet has to satisfy an admissibility condition, given by
where \( \psi(\omega) \) is the Fourier transform of the mother wavelet \( \psi(t) \). Only if the \( \psi(t) \) meets the above admissibility condition, can the original signal \( s(t) \) be reconstructed by the continuous wavelet transform, \( CWT(a,b) \). The reconstruction can be expressed as

\[
s(t) = \frac{1}{C_\psi} \int \frac{1}{a^2} CWT(a,b) \psi \left( \frac{t-b}{a} \right) dadb \tag{6.22}
\]

which is named the inverse-wavelet transform, \( \hat{\psi}(t) \) is a dual function of \( \psi(t) \), usually, \( \hat{\psi}(t) = \psi(t) \).

The most prominent characteristic of wavelet transform is its multi-resolution analysis ability. The wavelet transform is unlike the STFT and Gabor transform, where all elementary functions have the same envelope and both the time and frequency resolutions of the elementary functions \( h(\tau - t) \exp(j \omega \tau) \) are fixed once the window function \( h(t) \) is chosen. Rather, the wavelet transform is accomplished by the dilation and translation of a mother wavelet, where both time and frequency resolutions of the basis function \( \psi \left( \frac{t-b}{a} \right) \) are functions of the scaling factor \( a \).

Figure 6.7 demonstrates a comparison of time-frequency resolutions between the STFT and the wavelet transform.
Figure 6.7 Comparison of the time-frequency resolutions between short-time Fourier transform and continuous wavelet transform

6.3.6 Adaptive Representation and Adaptive Spectrogram

Several approaches to joint time-frequency analysis have been described. One primary motivation for these different schemes is to improve the joint time-frequency resolution with the least amount of cross-term interference. However, time resolution and frequency resolution are often in conflict. For example, the window selection in the short-time Fourier transform and the Gabor transform, the window (long-duration or short-duration) selection is a non-trivial problem, which leads to the expense of sacrificing one property or the other. Although, the Wigner-Ville distribution does not suffer from the window problem, which possesses the best joint time-frequency resolution, the WVD has the problem of cross-term interference, which also greatly limits its applications.
Naturally, one question arises: how to make the window function more adaptive? The adaptive representation and adaptive spectrogram addressed in this section are devoted to solving this question.

In the STFT and Gabor expansion, the elementary functions \( h[i - m\Delta M]e^{j2\pi mf_iN} \) are time-shifted and frequency-modulated versions of the single prototype function \( h[i] \). To better match the analyzed signal, the adaptive representation was devised to decompose the signal \( s[i] \) as a sum of weighted linear adaptive modulated Gaussian functions

\[
s[i] = \sum_{k=0}^{N-1} A_k h_k[i]
\]  

(6.20)

where the parameter \( N \) denotes the total number of elementary functions, \( A_k \) is the weight of each individual elementary function \( h_k[i] \) which is an adaptive Gaussian function defined by

\[
h_k[i] = (\alpha_k \pi)^{-0.25} \exp\left\{ -\frac{(i-i_k)^2}{2\alpha_k} + j(2\pi \theta_k[i-i_k]) \right\}
\]  

(6.21)

which holds the three parameters: \( \alpha_k, i_k, \theta_k \). Thus, the adaptive representation is more flexible than the elementary function used in the STFT and Gabor expansion.

Correspondingly, the adaptive spectrogram method is an adaptive representation-based spectrogram, defined as

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\[ AS[i,n] = 2 \sum_{k=0}^{N-1} |A_k|^2 \exp \left\{ -\frac{(i-k)^2}{\alpha_k} - \alpha_k [n - \theta_k] \right\} \] (6.22)

So far, the theoretical foundation of dominating approaches to joint time-frequency analysis has been presented and discussed in detail. It should be stressed that some approaches are of limited applicability on specific types of signal. The most appropriate candidates among all of these approaches to joint time-frequency analysis for seismic pulse responses are to be concluded through comparative analysis. In the following descriptions, seismic pulse responses under all earthquake excitations are firstly computed. Then, a variety of methods of joint time-frequency analysis are applied and compared for the qualitative assessment seismic damage.

### 6.4 QUALITATIVE ASSESSMENT OF SEISMIC DAMAGE

Hereafter, the focus will be on applying different methods of time-frequency analysis on the derived seismic pulse responses and making comparative discussion. As described in Chapter 3, there are a total of nine minor-level earthquakes, five moderate-level earthquakes and three strong-level earthquakes. Due to the slight amplitude of small earthquakes and their neglectable destroying effects on the tall building model structure, only the first minor-level earthquake (test No. 15) is taken to represent the small earthquakes in joint time-frequency analysis of Section 6.4.2, for a qualitative assessment seismic damage.
6.4.1 Seismic Pulse Response Computation

This section describes the results of seismic pulse responses computation under different levels of earthquake excitations.

6.4.1.1 Minor earthquakes

A total of 9 minor-level of earthquakes are simulated at the beginning of the shaking table testing of the tall building model structure, numbered as tests No. 7, 8, 9, 10, 11, 12, 13, 14 and 15, respectively. According to Equations (6.1) and (6.2), the corresponding seismic pulse responses can be computed by exerting reverse-FFT method on FRFs. Figure 6.8 shows the computation results of seismic pulse responses under minor-level earthquakes (shaking table tests No. 7, 8, 9, 10, 11, 12, 13, 14 and 15). It can be seen that some of such responses fluctuate somewhat and are not of absolutely ‘decayed’ signals, which are caused by discriminate behaviours of the building responses under different earthquake excitations. The seismic pulse responses of moderate and strong earthquakes exhibit the similar phenomena.

![Seismic pulse responses of minor-level earthquakes](image)

(a) No. 7 earthquake  
(b) No. 8 earthquake

**Figure 6.8 Seismic pulse responses of minor-level earthquakes**
Figure 6.8  Seismic pulse responses of minor-level earthquakes (Cont'd)
6.4.1.2 Moderate earthquakes

(a) No. 22 earthquake

(b) No. 23 earthquake

(c) No. 24 earthquake

(d) No. 25 earthquake

(e) No. 26 earthquake

Figure 6.9 Seismic pulse responses of moderate-level earthquakes
There are a total of five moderate-level of earthquakes, numbered as tests No. 22, 23, 24, 25 and 26, respectively. Figure 6.9 shows the computation results of seismic pulse responses under moderate-level earthquake excitations.

6.4.1.3 **Strong earthquakes**

There are totally 3 strong-level of earthquake simulated at the end of shaking table tests, numbered as tests No. 33, 34 and 35, respectively.

![Seismic pulse response graphs for strong earthquakes](image)

(a) No. 33 earthquake  
(b) No. 34 earthquake  
(c) No. 35 earthquake

**Figure 6.10** Seismic pulse responses of strong-level earthquakes
Figure 6.10 shows the computation results of seismic pulse responses under strong-level earthquakes attacking (shaking table tests No. 33, 34 and 35).

6.4.2 Results of Joint Time-frequency Analysis

In this section, a variety of joint time-frequency signal analysis algorithms addressed beforehand are to be implemented on the derived seismic pulse responses. Based on the resulting joint time-frequency representation of the seismic pulse response spectrogram, post-earthquake damage evaluations are then conducted.

Taking the essential similarity of some joint time-frequency analysis algorithms into account, the applied approaches employed in the following implementation can be categorized into four groups: (1) Short-time Fourier transform class method. Different window lengths are tried, for illuminating the window effects on the time-frequency resolution. The Gabor transform actually belongs to the short-time windowed Fourier transform, provided that Gaussian function is taken as the window function; (2) Cohen class method, including Wigner-Ville distribution, Choi-Williams distribution and Cone-shaped distribution. As an improved form of Wigner-Ville distribution, the Choi-Williams distribution reduces the cross-term interference while preserving as many useful Wigner-Ville distribution properties as possible. The Choi-Williams distribution (CWD) serves as a representation of this class method; (3) Adaptive spectrogram. In adaptive transform, a signal is considered as a sum of sinusoidal functions with different Gaussian envelopes or Gaussian pulses. Adaptive spectrograms promise to provide a better time-frequency resolution; and (4) Scale spectrogram of the continuous wavelet transform.

In the subsequent section, joint time-frequency analysis on the derived seismic pulse responses of various levels of earthquake excitations are performed, using all
four classes of algorithms. Then, the efficiency and applicability of each class of methods for joint time-frequency earthquake damage analysis of the tall building model structure are to be concluded and discussed in the last section of this chapter.

6.4.2.1 Minor earthquakes

There are nine earthquakes occurring at minor level. Considering the fact that such low-intensity earthquakes lead to no essential seismic damage on the model building, only the last earthquake (test No. 15) is to be analyzed for the study of damaging influence, representing this level of earthquake.

Figure 6.11 Joint time-frequency analysis results of the No. 15 earthquake
Figure 6.11 Joint time-frequency analysis results of the No. 15 earthquake

(Cont'd)

Figure 6.11 illustrates the results of joint time-frequency analysis of the last minor-level earthquake, using the STFT, CWD, adaptive transform and continuous wavelet transform, respectively. As Figures 6.11(a), (b), (c) and (d) show, the window length (wl, number of sampling points) of the STFT greatly affects the time-frequency resolution. The elongation of the window function increases the frequency resolution, but it also scarifies the time resolution, and vice versa. Comparatively,
The CWD and adaptive spectrogram achieve a better trade-off between frequency and time resolution. Figure 6.11(g) demonstrates the time-scale relationship of the continuous morlet wavelet transform, where scale is inversely proportional to frequency.

6.4.2.2 Moderate earthquakes

There are five earthquakes occurring at moderate level. The results of joint time-frequency analysis for each earthquake event are demonstrated in Figures 6.12 to 6.16.

(a) STFT spectrogram (wl=2048)  (b) STFT spectrogram (wl=1024)

(c) STFT spectrogram (wl=512)  (d) STFT spectrogram (wl=256)

Figure 6.12 Joint time-frequency analysis results of the No. 22 earthquake
Figure 6.12 Joint time-frequency analysis results of the No. 22 earthquake

(Cont'd)

Figure 6.13 Joint time-frequency analysis results of the No. 23 earthquake
Figure 6.13 Joint time-frequency analysis results of the No. 23 earthquake

(Cont'd)
Figure 6.14 Joint time-frequency analysis results of the No. 24 earthquake
(g) Continuous wavelet transform scale spectrogram

Figure 6.14 Joint time-frequency analysis results of the No. 24 earthquake

(Cont’d)

(a) STFT spectrogram (wl=2048)  (b) STFT spectrogram (wl=1024)

(c) STFT spectrogram (wl=512)  (d) STFT spectrogram (wl=256)

Figure 6.15 Joint time-frequency analysis results of the No. 25 earthquake
Figure 6.15 Joint time-frequency analysis results of the No. 25 earthquake

(Cont'd)

Figure 6.16 Joint time-frequency analysis results of the No. 26 earthquake
Figure 6.16 Joint time-frequency analysis results of the No. 26 earthquake

(Cont'd)
Similar to the case of minor-level earthquakes, the STFT is harassed by the problem of time-frequency resolution. Additionally, examining various time-frequency spectrograms, it can be found that the test No. 24 earthquake caused the most serious damage, whereas the tests No. 22 and 23 caused no substantial damage and the tests No. 25 and 26 caused considerable damage.

6.4.2.3 Strong earthquakes

There are 3 earthquakes occurring at severe level. Their time-frequency spectrograms using a variety of algorithms are likewise demonstrated in Figures 6.17, 6.18 and 6.19.

![STFT spectrograms](image)

(a) STFT spectrogram (wl=2048)  (b) STFT spectrogram (wl=1024)

(c) STFT spectrogram (wl=512)  (d) STFT spectrogram (wl=256)

Figure 6.17 Joint time-frequency analysis results of the No. 33 earthquake
Figure 6.17 Joint time-frequency analysis results of the No. 33 earthquake

(Cont’d)

Figure 6.18 Joint time-frequency analysis results of the No. 34 earthquake
Figure 6.18 Joint time-frequency analysis results of the No. 34 earthquake
Figure 6.19 Joint time-frequency analysis results of the No. 35 earthquake
(g) Continuous wavelet transform scale-spectrogram

**Figure 6.19 Joint time-frequency analysis results of the No. 35 earthquake**

(Cont'd)

Examining Figures 6.17, 6.18 and 6.19, it can be found that the tall building model structure exhibits a serious nonlinearity under strong-level of earthquakes attacking. As a result of serious nonlinearity, the frequency band in the time-frequency spectrogram becomes wider than the previous damage cases. Among the three earthquake events at this level, the test No. 34 causes the most serious damage, which can be interpreted by Figure 6.18. Contrastively, the tests No. 33 and 35 earthquakes cause no considerable new damages on the building structure, which can be indicated by the time-frequency spectrograms in Figures 6.17 and 6.19. Therefore, the induced damages do not just depend on the intensity of earthquakes and a strong-level of earthquake may not guarantee a mortal damage. The seismic damages of the building structure are accumulated gradually with all the previous earthquakes occurring.
6.5 SUMMARY

Joint time-frequency analysis algorithms are appropriate for short-duration and non-stationary signals, with which the conventional Fourier transform usually cannot deal with efficiently. In order to eliminate the excitation effects on seismic damage evaluation, the so-called seismic pulse response is firstly proposed and derived. A variety of joint time-frequency analysis algorithms are then applied to the derived pulse response signals, based on which the post-earthquake damage evaluation can finally be conducted.

By comparing the time-frequency spectrograms of different analysis approaches, it can be found that the Choi-Williams distribution (CWD) and the adaptive spectrogram achieve the best time-frequency resolution, whereas the short-time Fourier transform (STFT) has to balance the trade-off between time resolution and frequency resolution. In this application, it is acceptable to take the window function length to be 1024 or 512 sampling points. Despite a relatively high resolution, the Choi-Williams distribution algorithm is embarrassed by cross-term influence, when the analyzed signal contains multi-frequency components simultaneously. The adaptive spectrogram avoids the above demerits, while maintaining a high time-frequency resolution. The scale spectrogram by continuous wavelet transform exhibits the best time resolution, but the scale information is reversely proportional to frequency and thus can only indirectly indicate the seismic damage. Finally, due to single-frequency component (only the first-order frequency remains after the filtering operation) of the seismic pulse response, the multi-scale analysis ability of the continuous wavelet transform cannot be fully utilized herein.
Whichever joint time-frequency analysis algorithm is used, the spectrogram proposed in this chapter only provides a qualitative evaluation of seismic damage on the model building under earthquakes attacking. The time instant(s) of occurrence and severity of earthquake-induced damage can be estimated by examining the corresponding time-frequency or time-scale spectrogram. It is no doubt worthwhile to estimate seismic damage in a quantitative sense. In other words, the further damage evaluation should be based on real modal frequency value reduction, rather than on various power spectrograms which represent the response energy. The next chapter is devoted to accomplishing such tasks, by developing a super-resolution modal identification technique.
CHAPTER 7

QUANTITATIVE REALIZATION OF SEISMIC DAMAGE ASSESSMENT BASED ON SUPER-RESOLUTION ANALYSIS

7.1 INTRODUCTION

The spectrograms of various joint time-frequency analysis algorithms exerted on the derived seismic pulse response avoid the necessary of modal parameters identification and provide an overall evaluation of seismic damage from energy respect. However, as discussed at the summary of Chapter 6, joint time-frequency analysis suffers from either low time-frequency resolution or the cross-term interference problem. Moreover, damage assessment based on the time-frequency spectrograms is limited to qualitative meanings. In a real-life post-earthquake assessment, the quantity of the induced damage must be estimated shortly after that earthquake event occurs for making proper decision on structure rehabilitation. Therefore, it is usually more urgent to make a quantitative identification of seismic damage based on the earthquake records themselves. Moreover, the estimation should be carried out in real time manner, based directly on the recorded earthquake excitations and responses, rather than on any additional post-earthquake modal testing. Many researchers have conducted much work on post-earthquake evaluation
(Shiga et al. 1968; Yang et al. 1980; Dipasquale and Cakmak 1990; Ghobarah et al. 1999). In order to quantitatively estimate seismic damage on the building structure, it is necessary to achieve an approximation of the instantaneous frequencies from the computed seismic pulse responses. By examining the changes in instantaneous frequency, the degradation in stiffness of the building is correspondingly to be indicated.

Accordingly, much more effort needs to be spent on exploring the potential of quantitative evaluation of earthquake-induced damage from the changes in the instantaneous dynamic characteristics, rather than from time-frequency energy spectra. And, in order to implement such evaluation in real time manner, the instantaneous frequencies should be estimated from earthquake records themselves, rather than from any post-earthquake modal testing or ambient vibration measurement. In addition, to eliminate the contamination of earthquake excitation frequency components on the structural instantaneous frequencies, the estimation is based on the seismic pulse response which has already been derived in Chapter 6.

Analogous to the case of joint time-frequency analysis, classical FFT-based spectrum and modal parameter estimation techniques are not apt to extracting instantaneous frequencies from the seismic pulse responses due to their short periods of duration and non-stationary property. Some alternative approaches need to be developed for estimating instantaneous frequencies from the signals of such short duration. In the following study, super-resolution spectra analysis and modal parameter estimation methods are to be developed, for performing instantaneous frequencies identification. Then, a quantitative assessment of seismic damage in real time manner is accomplished, based on the evolving history of the identified instantaneous frequencies.
7.2 THEORETICAL FORMULATION OF SUPER-RESOLUTION ANALYSIS

This section formulates a derivation of algorithms of super-resolution power spectra analysis and frequency identification for the purpose of instantaneous frequency estimation. Unlike FFT, the presented approaches assume that the investigated signals are not arbitrary, but that they should fit some certain models. Accordingly, this class of approaches is sometimes referred to as model-based methods (Kay 1987). Firstly, one general type of signal model, called auto-regressive and moving average (ARMA), is introduced. Based on the ARMA model, the specific moving average (MA) model and auto-regressive (AR) model are derived, with which the computed dynamic pulse responses fit closely. Subsequently, two algorithms of super-resolution power spectra analysis are discussed, using the derived damped sinusoids AR model. Finally, based on the same AR model as in super-resolution power spectra analysis, a model-based modal parameter estimation solution, called the Prony method (Marple 1982a), is to be presented. The super-resolution parameter estimation method is applied on the computed seismic pulse responses, to extract instantaneous frequencies of the model building structure.

7.2.1 Auto-Regressive and Moving Average (ARMA) Model

Assume that the discretized data sample $x[n]$ of one signal fits the following model (Marple 1987)

$$x[n] = -\sum_{k=1}^{p} a_k x[n-k] + \sum_{m=0}^{q} b_m w[n-m] \quad \text{for } 0 \leq n < N$$  \hspace{1cm} (7.1)
where \( b_0 = 1 \) and the sequence \( w[n] \) is the white noise with a zero-mean and variance of \( \sigma^2 \). Equation (7.1) is traditionally called the auto-regressive and moving average (ARMA) signal model.

For completeness, two special cases of ARMA, called the moving average (MA) model and auto-regressive (AR) model are briefly introduced here respectively.

### 7.2.2 Moving-Average (MA) Model

If \( a_k = 0 \) for all \( k \), Equation (7.1) reduces to

\[
x[n] = \sum_{m=0}^{q} b_m w[n-m] \quad \text{for } 0 \leq n < N
\]

(7.2)

A signal that can be represented by Equation (7.2) is called a moving average (MA) modeled signal.

### 7.2.3 Auto-Regressive (AR) Model

The second special case for the ARMA model is \( b_m = 0 \) for \( m > 0 \). Thus, the ARMA model of Equation (7.1) becomes

\[
x[n] = -\sum_{k=1}^{p} a_k x[n-k] + w[n] \quad \text{for } 0 \leq n < N
\]

(7.3)
Equation (7.3) represents an auto-regressive (AR) model. According to Equation (7.3), currently known data samples are used to predict the future data with an error of \( w[n] \). Let the predicted data be

\[
\hat{x}[n] = -\sum_{k=1}^{p} a_k x[n-k] \quad \text{for } p \leq n < N \tag{7.4}
\]

The Equation (7.4) can be written in matrix form

\[
\begin{bmatrix}
  x[p-1] & x[p-2] & \cdots & x[0] \\
  x[p] & x[p-1] & \cdots & x[1] \\
  M & M & \cdots & M \\
  x[N-2] & x[N-1] & \cdots & x[N-p+1]
\end{bmatrix}
\begin{bmatrix}
  a_1 \\
  a_2 \\
  \vdots \\
  a_p
\end{bmatrix}
= \begin{bmatrix}
  \hat{x}[p] \\
  \hat{x}[p+1] \\
  \vdots \\
  \hat{x}[N-1]
\end{bmatrix}
\tag{7.5}
\]

which is called a forward prediction. Correspondingly, there is a backward prediction that uses future data to predict the data that was previously sampled before \( p \) steps.

The AR, MA, and ARMA models cover a wide range of signals in nature. In most applications, model-based methods for power spectra and frequency analysis can be confidently applied. Practically, an appropriate model should be chosen for a specific signal. However, which of the given models best fits the specified signal is usually unknown beforehand. An important result from the Wold decomposition is that any AR or ARMA process can be represented by an MA process of possibly infinite order. Likewise, any MA or ARMA process can be represented by an AR process of possibly infinite order (Kay 1987). Therefore, if a wrong model is chosen from the above three models, a reasonable approximation by using a high enough model order can still be achieved.
Because AR model-based algorithms are better understood and more popular than their counterparts, the next super-resolution power spectra and parameters estimation are derived from this model.

7.2.4 Damped Sinusoids and AR Model Assumption

Damped sinusoids are common in applications such as noise and vibration. Herein, it is assumed that the obtained seismic pulse responses of the model building structure can be formulated as a linear combination of damped sinusoids

\[ x[n] = \sum_{k=1}^{p} C_k \exp\{i \alpha_k n\} = \sum_{k=1}^{p} C_k (z_k)^n \quad \text{for } 0 \leq n < N \quad (7.6) \]

where the parameter \( \alpha_k \) indicates the damping factor and \( C_k \) denotes the complex amplitudes. Equation (7.6) can also be written in matrix form as

\[
\begin{bmatrix}
(z_1)^0 & (z_2)^0 & \ldots & (z_p)^0 \\
(z_1)^1 & (z_2)^1 & \ldots & (z_p)^1 \\
M & M & \ldots & M \\
(z_1)^{N-1} & (z_2)^{N-1} & \ldots & (z_p)^{N-1}
\end{bmatrix}
\begin{bmatrix}
C_1 \\
C_2 \\
M \\
C_p
\end{bmatrix}
= 
\begin{bmatrix}
x[0] \\
x[1] \\
M \\
x[N-1]
\end{bmatrix}
\quad (7.7)
\]

where the matrix of the time-indexed \( z \) elements has a Vandermonde structure (Rahman and Yu 1986). It can be seen that Equation (7.7) does not seem to belong to any of the models ARMA, AR, MA. However, it has been proven that Equation (7.7) is closely related to the AR model described in Equation (7.3). And it is also discovered that \( z_k \) are actually roots of the polynomial (Kay 1987)

7-6
\[ A(z) = 1 + \sum_{k=1}^{p} a_k z^{-k} = \prod_{k=1}^{p} (1 - z_k z^{-1}) \tag{7.8} \]

where \( a_k \) are the coefficients of the regular AR model in Equation (7.3). Consequently, the procedure for super-resolution spectra analysis and finding the damped sinusoids parameters is first to compute the AR coefficients \( a_k \), then to solve the polynomial in Equation (7.8) to determine \( z_k \). Finally, the solution of the linear system in Equation (7.7) gives the complex amplitudes \( C_k \).

### 7.3 SUPER-RESOLUTION SPECTRA ANALYSIS AND PARAMETER ESTIMATION

This section is devoted to introducing the super-resolution spectra analysis and algorithms of modal parameter estimation.

Covariance method and principle component auto-regressive (PCAR) method are used to estimate power spectrum, based on the assumed signal model.

#### 7.3.1 Covariance Method

Assume that the future data is estimated by the forward prediction in Equations (7.4) and (7.5). The covariance method computes the coefficients \( a_k \) such that the error between \( x[n] \) and \( \hat{x}[n] \) is minimized.

\[
\min_{a_k} \sum_{n=p}^{N-1} \| x[n] - \hat{x}[n] \|^2 \tag{7.9}
\]
where the optimal coefficients $a_k$ are the solution of the linear system of

$$\begin{bmatrix}
x[p-1] & x[p-2] & \cdots & x[0] \\
x[p] & x[p-1] & \cdots & x[1] \\
M & M & \cdots & M \\
x[N-2] & x[N-1] & \cdots & x[N-p+1]
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
M \\
a_p
\end{bmatrix} =
\begin{bmatrix}
x[p] \\
x[p+1] \\
M \\
x[N-1]
\end{bmatrix}$$

(7.10)

The covariance method is straightforward, but it is proven to be sensitive to noise (Kay 1987).

### 7.3.2 Principal Component Auto-Regressive (PCAR)

**Method**

The covariance method only minimizes the error between $x[n]$ and $\hat{x}[n]$ for $p \leq n < N$, namely $N-p$ points, even though there are $N$ samples of $x[n]$. The PCAR method formulates the linear system as

$$\begin{bmatrix}
X_f \\
X_b
\end{bmatrix} \hat{a} =
\begin{bmatrix}
\bar{x}_f \\
\bar{x}_b
\end{bmatrix}$$

(7.11)

where $\hat{a}$ denotes the data vector $\hat{a} = [a_1, a_2, \ldots, a_p]^T$, and $\bar{x}_f$ and $\bar{x}_b$ denote the right side vectors of the forward prediction and backward prediction equations. The relationship can also be written as
\[
\begin{bmatrix}
\overset{\mathcal{Q}}{X_f} \\
\overset{\mathcal{Q}}{x_h}
\end{bmatrix}
= [x[p], x[p+1], \ldots, x[N-1], x[0], x[1], \ldots, x[N-p-1]]^T
\]  \hspace{1cm} (7.12)

In matrix form, the above relationship as can be rewritten as

\[
\begin{bmatrix}
x[p-1] & x[p-2] & \ldots & x[0] \\
x[p] & x[p-1] & \ldots & x[1] \\
M & M & M & M
\end{bmatrix}
\begin{bmatrix}
X_f \\
X_h
\end{bmatrix}
= [x[N-2], x[N-1], \ldots, x[N-p+1], x[1], x[2], \ldots, x[p], x[2], x[3], \ldots, x[p+1], M, M, M, M, x[N-p], x[N-p+1], \ldots, x[N-1]]
\]  \hspace{1cm} (7.13)

Consequently, the linear system in Equation (7.13) uses forward and backward prediction information. In this manner, extra data points and more averaging errors are achieved. Moreover, the coefficients are computed by

\[
\begin{bmatrix}
\overset{\mathcal{Q}}{a}_1 & \ldots & \overset{\mathcal{Q}}{a}_L
\end{bmatrix}
= \sum_{i=1}^{L} \frac{1}{\lambda_i} \overset{\mathcal{Q}}{v}_i X^T \overset{\mathcal{Q}}{x}
\]  \hspace{1cm} (7.14)

where

\[
\begin{align*}
X &= \begin{bmatrix} X_f \\ X_h \end{bmatrix} \quad \text{and} \quad \overset{\mathcal{Q}}{x} = \begin{bmatrix} \overset{\mathcal{Q}}{x}_f \\ \overset{\mathcal{Q}}{x}_h \end{bmatrix}
\end{align*}
\]  \hspace{1cm} (7.15)

\(\lambda_i\) denote the \(L\) largest eigenvalues of the matrix \(X\). \(\overset{\mathcal{Q}}{v}_i\) are \(L\) corresponding eigenvectors. The parameter \(L\) represents the number of complex sinusoids. Because only \(L\) principle components are used in Equation (7.14), the results obtained by
PCAR are much less sensitive to noise than the results obtained by the covariance method.

7.3.3 Prony Method for Parameter Estimation

The Prony method estimates the parameters of damped sinusoids. The following steps summarize the Prony method.

1. Apply the covariance method to compute the AR coefficients $a_k$;
2. Find the complex roots $z_k$ of the polynomial in Equation (7.10). The phase of $z_k$ indicates the frequency, and the amplitude is the damping factor; and
3. Insert $z_k$ into Equation (7.9) to solve $C_k$. The amplitude and phase of the sinusoid component $z_k$ are equal to the amplitude and phase of $C_k$, respectively.

7.4 QUANTITATIVE ASSESSMENT OF SEISMIC DAMAGE

This section presents a real time quantitative assessment of seismic damage on the tall building model structure by using super-resolution power spectra estimation and frequency analysis methods. The super-resolution analysis is performed on the building seismic pulse responses, which are computed by employing reverse-FFT technique on the frequency response functions under earthquake excitations. Depending on the duration length of the original earthquake, the obtained seismic pulse responses are separated into a limited number of segments, each of which has the same length of time-span (or the same number of sampling data points). Then, by
continuously implementing super-resolution power spectra analysis and modal parameter estimation on each of the segments, a series of instantaneous frequencies of the model building structure corresponding to one certain earthquake event are identified. Ultimately, the overall seismic damage is evaluated in quantitative and real time manner, based on the achieved instantaneous frequency series.

7.4.1 Review of the Shaking Table Tests

As described in Chapter 3, a total of four levels of earthquake excitations with successively enhanced magnitudes are exerted on the tall building model structure in shaking table tests: minor earthquake, moderate earthquake, strong earthquake and super-strong level of earthquake, respectively. Among each level of earthquakes, a set of earthquake records represent rock site, medium soil site and soft soil site conditions. Between two consecutive levels of earthquake excitations, a 20 minute-duration of white-noise excitation is applied. Compared with earthquake excitation, the amplitude of white-noise excitation is relatively small. And no new damage cracks are observed during all segments of white-noise excitations. Therefore, the tall building model structure can be considered to have remained in the same healthy state throughout each white-noise excitation test. All shaking table tests, including earthquake simulation and white-noise micro-vibrations, are summarized in Table 3.7. Earthquake records, tests No. 7-15, 22-26, and 33-35, are utilized for the following computation of the seismic pulse responses and identification the instantaneous frequencies.
7.4.2 Computation of the Seismic Pulse Responses

There are a total of 17 times of earthquake events extending from minor-level to strong-level, each of which is applied with classical FFT method for transfer functions and with reverse-FFT technique for seismic pulse responses computation.

(a) Earthquake excitation

(b) Earthquake response

(c) Transfer function

(d) Seismic pulse response

Figure 7.1. Transfer function and seismic pulse response computation
Figure 7.1 demonstrates transfer function and seismic pulse response computation procedure under one typical earthquake (test No. 24, moderate-level) attacking. It should be noted that the above computation utilizes both seismic excitation and building response signals.

In order not to lose generality, acceleration signals measured at the top of the model building structure (floor C38) in X-direction are taken as the model building responses under earthquakes attacking in transfer function computation. The building instantaneous frequencies are independent on the location at which the responses are measured.

7.4.3 Segmentation of the Seismic Pulse Responses

Obviously, the deduced seismic pulse response has the same length of duration as the original earthquake excitation and building response. Due to the non-stationary property of earthquake excitation and the behaviours of structural non-linearity, the dynamic characteristics of the model building structure change instantaneously, depending on the earthquake level and its destroying effects on the structure. Consequently, such instantaneous sequences of the modal characteristics should be extracted for the recognition of the time-variant structural behaviours. Conventional FFT on the throughout time history is substantially averaging the energy over the whole-duration and cannot be competent for grasping such time-variant behaviours of the structure. Therefore, it is necessary to divide the seismic pulse response into a limited number of segments, each of which is individually dealt with model-based super-resolution frequency analysis.

In the following study, every 50 sampling points are grouped as one segment, and the total number of segments depends on the actual duration of the earthquake
excitation. Due to sampling frequency being 100 Hz (0.01s of the sampling interval), each segment is equivalent to 50×0.01=0.5 s in duration. The strategy of segmentation is summarized in the following Table 7.1.

<table>
<thead>
<tr>
<th>Sampling frequency</th>
<th>Sampling interval</th>
<th>Points per segment</th>
<th>Duration per segment</th>
<th>Number of segments</th>
</tr>
</thead>
<tbody>
<tr>
<td>100Hz</td>
<td>0.01s</td>
<td>50</td>
<td>0.5s</td>
<td>Depending on earthquake</td>
</tr>
</tbody>
</table>

A series of instantaneous frequencies can thus be achieved with a time step of 0.5s, based on which the time-dependent behaviours of the building structure can be monitored in real time sense. In addition, the corresponding post-earthquake real time damage severity as well as extent the structural non-linearity is to be evaluated by observing the evolving history of the instantaneous frequencies.

### 7.4.4 Results of Super-resolution Spectra Estimation

As is well known, the frequency resolution of the FFT-based spectra analysis is bounded by the number of data samples. The relationship between the number of samples and frequency resolution can be quantified by Equation (7.16)

\[
\Delta f = \frac{\text{sampling frequency}}{\text{number of samples}}
\] (7.16)
where $\Delta f$ denotes the frequency resolution, which characterizes the distinguishable, minimum frequency difference. Here, the 50 data samples are too small to generate sufficient frequency resolution when directly using the FFT-based method. Alternatively, the two model-based super-resolution power spectra analysis algorithms described in Section 7.2, the covariance method and the principal component auto-regressive method (PCAR), are employed.

Firstly, assuming that each 50 samples of seismic pulse response can be modeled as damped sinusoid and approximately fit auto-regressive (AR) model, then the future ‘pseudo’ measurement data beyond the 50 data samples are estimated by the forward prediction described in Equations (7.4) and (7.5). As a result, the small size of the data samples experience padding and extension, and become relatively larger. Then, the covariance and principle component auto-regressive method are applied for super-resolution spectra analysis.

In order to highlight the priority of model-based super-resolution analysis to FFT, two typical segments of seismic pulse responses are taken for a comparative study. Both two sets of data samples are extracted from the same seismic pulse response of one moderate-level earthquake (test No. 24): the first set is located at commencement portion and the second one is near the end.

Figure 7.2 demonstrates the results of comparative power spectra analysis on the first segment data samples, using the FFT-based method, covariance and PCAR algorithms respectively. The FFT-based power spectrum distributes the energy over a somewhat wide range near the first mode frequency leading to low frequency resolution, whereas the covariance and PCAR-based spectrum indicate clear peaks.
(a) The first segment data samples of seismic pulse response

(b) Comparison of covariance method and FFT-based power spectrum

(c) Comparison of PCAR method and FFT-based power spectrum

Figure 7.2 Comparative study on the first segment of seismic pulse response
(a) The second segment data samples of seismic pulse response

(b) Comparison of covariance method and FFT-based power spectrum

(c) Comparison of PCAR method and FFT-based power spectrum

Figure 7.3 Comparative study on the second segment of seismic pulse response
Note that the second peak of the covariance-based power spectrum in Figure 7.2(b) is an authentic resonant frequency, whereas the first one is a pseudo frequency peak, which is caused by noise effects. Figure 7.3 demonstrates the results of comparative power spectra analysis on the second segment of data samples. Similarly, there also exists one pseudo peak before the authentic one in Figure 7.3(b). The existence of the pseudo peaks implies that the covariance method is more highly sensitive to noise than the PCAR method.

7.4.5 Results of Instantaneous Frequencies Identification and Quantitative Seismic Damage Assessment

After the AR coefficients have been computed using covariance and PCAR algorithms, the Prony method is employed to estimate the frequencies of each piece of seismic pulse response segment. The procedure for estimating modal parameters by using the Prony method is addressed in Section 7.3.3. Here, considering the uncertainty of damping identification, only the natural frequencies of the model building structure are to be estimated and utilized for a quantitative evaluation of seismic damage.

Assuming that the mass of the model building structure remains unchanged after damage and that the damage-induced cracks make a reduction in structural stiffness, then the natural frequencies of the structure are directly related to structural stiffness and therefore represent the presence and severity of seismic damage. Natural frequencies identified at all time instants construct the sequences of instantaneous frequency. By examining the evolving history of the instantaneous
frequency sequences during earthquake occurrence, the evaluation of overall damage is finally accomplished.

In order to examine how the initial tremor part of earthquake excitations and responses affect the judgment of seismic damage occurring moment by observing the calculated instantaneous frequency sequences, a numerical simulation is firstly carried out herein. An earthquake excitation without initial tremor part, as shown in Figure 7.4(a), is first considered. An ideal seismic pulse response is assumed as shown in Figure 7.4(d), where the frequency is equal to 5 Hz, 4 Hz and 2 Hz corresponding to the first two seconds, time duration from 2 sec to 6 sec, and the remaining time of the pulse response, respectively. Based on this truncated earthquake excitation $f_{eq}(t)$ and the targeted pulse response $h(t)$, then an artificial "earthquake" response $Y(t)$ of the structure is generated by

$$Y(t) = \int h(\tau) f_{eq}(t-\tau) d\tau \quad (7.17)$$

The time-history of the generated response is plotted in Figure 7.4(b), and amplitude spectra of the truncated excitation and the generated response and their transfer functions are shown in Figure 7.4(c). Making use of time-frequency analysis, the instantaneous frequencies of the simulated pulse response given in Figure 7.4(d) are obtained as shown in Figure 7.4(e). As expected, the identified frequencies coincide well with the frequency contents of the pulse response. By plotting the original excitation and instantaneous frequency together as given in Figure 7.4(f), the time instant of frequency dropping (damage occurrence) can be indicated.
(a) Time-history of truncated earthquake excitation

(b) Time-history of response generated by excitation and pulse response

(c) Amplitude spectra of excitation and response, and transfer function

(d) Time-history of simulated pulse response

Figure 7.4 Results based on earthquake excitation and simulated pulse response
(c) Instantaneous frequency of simulated pulse response

(f) Comparison of original excitation and identified instantaneous frequency

Figure 7.4 Results based on earthquake excitation and simulated pulse response (Cont’d)

Then we introduce the initial tremor part in the time history of excitation and response. Low-level white-noise random signals of 10 seconds are added in front of the truncated excitation (Figure 7.4(a)) and the generated response (Figure 7.4(b)) to form earthquake excitation and response with initial tremor part, as shown in Figures 7.5(a) and 7.5(b). The corresponding time-frequency analysis results are shown in Figures 7.5(c) to 7.5(f). It is seen from Figures 7.5(c) and 7.5(e) that the amplitude spectra, transfer function, pulse response and instantaneous frequency distribution keep almost the same as those obtained from the earthquake excitation and response.
without initial tremor part. The initial tremor part has a negligible effect on the pulse response and its instantaneous frequency. In conclusion, the pulse response and instantaneous frequency can be obtained from the truncated earthquake excitation and response after deleting the initial tremor part, and the time instant of damage occurrence is determined by the identified instantaneous frequency dropping moment plus the time duration of the initial tremor part, as indicated in Figure 7.5(f).

(a) 10s-shifted earthquake excitation

(b) 10s-shifted simulated response

Figure 7.5 Results based on 10s-shifted earthquake excitation and response
(c) Amplitude spectra of excitation and response, and transfer function

(d) Pulse response obtained from 10s-shifted earthquake excitation and response

(e) Instantaneous frequency of the recomputed pulse response

Figure 7.5 Results based on 10s-shifted earthquake excitation and response

(Cont’d)
Figure 7.5 Results based on 10s-shifted earthquake excitation and response

(Cont'd)

The "accurate" results given in Figures 7.4(a) and 7.4(f) can be understood as those obtained by cutting the first 10s tremor part of the "real" earthquake excitation and response records of Figures 7.5(a) and 7.5(b). In order to examine the sensitivity of the cutting length (i.e., the selection of commence point of the truncated excitation and response records for time-frequency analysis) on the indication accuracy of damage occurrence time, the analysis is made again by intentionally cutting the first 10.5s of the excitation and response records in Figures 7.5(a) and 7.5(b). The cut excitation and response records and the corresponding time-frequency analysis results are illustrated in Figures 7.6(a) to 7.6(g).

From Figures 7.6(d) and 7.6(e), it is found that the time instant of frequency droppings are moved up with about 0.48s, compared with Figures 7.4(d) and 7.4(e). Therefore, the time instant of frequency droppings is still indicated correctly, as shown in Figures 7.6(f) and 7.6(g). This means that the identification results are not sensitive to precise selection of the truncated commence point. In the following, we conduct time-frequency analysis of the tested model structure by using appropriately
truncated seismic excitation and response signals, and the time instant of seismic
damage occurrence is determined by taking a summation of the identified
instantaneous frequency dropping moment and the time duration of the initial tremor
part.

(a) 10.5s-cut earthquake excitation

(b) 10.5s-cut simulated response

(c) Amplitude spectra of excitation and response, and transfer function

Figure 7.6 Results based on 10.5s-cut earthquake excitation and response
(d) Pulse response obtained from 10.5s-cut earthquake excitation and response

(e) Instantaneous frequency of the recomputed pulse response

(f) Truncated excitation and identified instantaneous frequency

Figure 7.6 Results based on 10.5s-cut earthquake excitation and response

(Cont’d)
(g) Comparison of results on indicating the moment of damage occurrence

Figure 7.6 Results based on 10.5s-cut earthquake excitation and response

(Cont’d)
As aforementioned, there are a total of 17 earthquake excitations throughout the shaking table testing. Tests No. 7 to 15 correspond to minor-level earthquakes that are trivial to model building damage and dynamic non-linearity. Only the results of Test No. 15 are presented here as a representative. Tests No. 22 to 26 correspond to moderate-level earthquakes, and Tests No. 33 to 35 correspond to strong-level earthquakes. The excitation time histories and identified instantaneous frequencies are illustrated in the following figures.

(a) No. 15 earthquake excitation  (b) Identified instantaneous frequencies

Figure 7.7 Seismic excitation (test No. 15) and identified instantaneous frequencies sequence of the model building structure

Figure 7.7 shows the time history of test No. 15 (minor-level earthquake) excitation and the instantaneous frequency sequence of the model structure. The first 5s initial tremor part is cut when calculating the instantaneous frequency. No significant frequency shift is observed except for some small fluctuations due to noise and numerical error, which indicates no noticeable damage.
Figure 7.8 Seismic excitation (test No. 22) and identified instantaneous frequencies sequence of the model building structure

Figure 7.8 shows the time history of test No. 22 (the first moderate-level earthquake) excitation and the instantaneous frequency sequence of the model structure. The first 50s tremor part is cut when calculating the instantaneous frequency. There is also no evident shift except larger fluctuations compared with the case of Test No. 15, which indicates that there is no damage occurring during this earthquake event.

Figure 7.9 shows the time history of test No. 23 (the second moderate-level earthquake) seismic excitation and the instantaneous frequency sequence of the model structure. The first 15s tremor part is cut in calculating the instantaneous frequency. A noticeable drop in the instantaneous frequency is observed at about 3s, which indicates the presence of seismic damage at about 18s (15+3) of the original earthquake excitation time history. It is noted that this time point is between the
second and third ground acceleration peaks. At other time instants, there is no significant frequency drop except for some fluctuations. The fluctuations are attributed to the increased nonlinearity of the structure and the non-stationary property of the response signals under this earthquake excitation.

(a) No. 23 earthquake excitation  
(b) Identified instantaneous frequencies

**Figure 7.9 Seismic excitation (test No. 23) and identified instantaneous frequencies sequence of the model building structure**

Figure 7.10 shows the time history of test No. 24 (the third moderate-level earthquake) excitation and the instantaneous frequency sequence of the model structure. The first 16s tremor part is cut in calculating the instantaneous frequency. Clearly, there is a significant drop of the instantaneous frequency at the time instant of about 5s, which indicates serious damage occurrence at the time instant of about 21s (16+5) of the original earthquake excitation time history. It is also observed that the instantaneous frequencies resume being increased after a period of drop-off and become more ‘fluctuated’, which indicate the complexity of the building structure
behaviours caused by the nonlinearity (opening and closing of cracks) and the non-stationary property of the response signals during this earthquake excitation.

(a) No. 24 earthquake excitation        (b) Identified instantaneous frequencies

Figure 7.10 Seismic excitation (test No. 24) and identified instantaneous frequencies sequence of the model building structure

(a) No. 25 earthquake excitation        (b) Identified instantaneous frequencies

Figure 7.11 Seismic excitation (test No. 25) and identified instantaneous frequencies sequence of the model building structure
Figure 7.11 shows the time history of test No. 25 (the fourth moderate-level earthquake) excitation and the instantaneous frequency sequence of the model structure. The first 22s tremor part is cut in calculating the instantaneous frequency. A noticeable drop of the instantaneous frequency is found at about 9s, which indicates the presence of seismic damage at about 31s (22+9) of the original earthquake excitation time history. Some new damages occur under the attack of this earthquake event and the time instant of damage occurrence is beyond the two ground acceleration peaks.

![Figure 7.11](image)

(a) No. 26 earthquake excitation  (b) Identified instantaneous frequencies

**Figure 7.12 Seismic excitation (test No. 26) and identified instantaneous frequencies sequence of the model building structure**

Figure 7.12 shows the time history of test No. 26 (the fifth moderate-level earthquake) excitation and the instantaneous frequency sequence. The first 45s tremor part is cut in calculating the instantaneous frequency. There is a considerable shift at the time instant of about 15s, which indicates the presence of seismic damage at about 60s (45+15) of the original earthquake excitation time history.
Synthetically analyzing the results of Figures 7.8 to 7.12, it can be found that: (1) under moderate earthquake excitations, new seismic damage presence is always between or after peak ground acceleration arriving; and (2) the distinct levels of seismic damage are induced by different soil conditions and by earthquake events that have their own spectrum frequency components and destroying mechanism on the model structure.

(a) No. 23 earthquake  
(b) No. 24 earthquake  
(c) No. 25 earthquake  
(d) No. 26 earthquake

Figure 7.13 Seismic excitation power spectra of all moderate earthquakes
In order to validate the effects of the earthquake excitations on the damage mechanism, their respective power spectra have been computed and plotted in Figure 7.13, where test No. 22 earthquake is ignored due to no significant damage occurring. The test No. 24 earthquake has the richest frequency components around the first structural resonant frequency (at this stage, about 4.4 Hz) and under 10 Hz, thus it causes the most significant damage. By contrast, the earthquake No. 23 causes the least amount of damage due to its sparse frequency components, and the earthquake No. 25 and 26 are between the above two cases.

(a) No. 33 earthquake excitation  
(b) Identified instantaneous frequencies

Figure 7.14 Seismic excitation (test No. 33) and identified instantaneous frequencies sequence of the model building structure

Figure 7.14 shows the time history of test No. 33 (the first severe-level earthquake) excitation and the instantaneous frequency sequence of the model building structure under this earthquake excitation. The first 16s tremor part is cut
when calculating the instantaneous frequency. There is no obvious drop in the instantaneous frequency, which implies that no prominent damage has occurred during this earthquake event even though it is a severe one. On the other hand, it should be noted that a considerable fluctuation exists on the instantaneous frequency series due to the nonlinearity of the structure and the non-stationary property of the response signal.

(a) No. 34 earthquake excitation  
(b) Identified instantaneous frequencies

**Figure 7.15** Seismic excitation (test No. 34) and identified instantaneous frequencies sequence of the model building structure

Figure 7.15 shows the time history of test No. 34 (the second severe-level earthquake) excitation and the instantaneous frequency sequence of the model structure. The first 25s tremor part is cut when calculating the instantaneous frequency. Evidently, there are two large deviations, at the time instants of about 3s and 9s respectively, which are corresponding to two times of damage occurring at about 28s (25+3) and 34s (25+9) of the original excitation time history, respectively.
It should be noted that the first seismic damage appears shortly after the first peak ground acceleration occurring. And the second seismic damage is immediately incurred when the model structure is subjected to the final peak ground acceleration attack of this earthquake event.

(a) No. 35 earthquake excitation (b) Identified instantaneous frequencies

Figure 7.16 Seismic excitation (test No. 35) and identified instantaneous frequencies sequence of the model building structure

Figure 7.16 shows the time history of test No. 35 (the third severe-level earthquake) excitation and the instantaneous frequency sequence of the model structure. The first 12s tremor part is cut when calculating the instantaneous frequency. There is an obvious drop of the instantaneous frequency at the time instant of about 4.5s, which indicates damage occurrence at about 16.5s (12+4.5) of the original earthquake excitation time history, shortly after the third peak ground
acceleration arriving. Due to the relatively small amplitude of this earthquake excitation, no new prominent damage has occurred afterward.

Synthetically analyzing the results of Figures 7.14 to 7.16, the conclusions for the case of severe earthquake excitations can be drawn: most of seismic damages occur between two peak accelerations or shortly after peak acceleration arriving.

(a) No. 33 earthquake  
(b) No. 34 earthquake  
(c) No. 35 earthquake

Figure 7.17 Seismic excitation power spectra of all strong earthquakes

The power spectrum of severe-level earthquakes, No. 33, 34 and 35 are also calculated to validate their effects on seismic damage due to frequency components. The results are shown in Figure 7.17. The earthquake No. 34 has the richest
frequency components around the first structural resonant frequency (about 3.8 Hz) and under 10 Hz, thus it causes the most significant damage. By contrast, the earthquake No. 33 causes the least amount of damage due to its sparse frequency components, and the earthquake No. 35 is between the above two cases.

Ignoring the white-noise tests which have a low-level of excitation amplitude and connecting all earthquake simulation tests (listed in Table 3.7) together, the relationship between the changes of the instantaneous frequencies of the model building structure and all the earthquake events can be plotted in Figure 7.18.

![Frequency change versus earthquake history](image)

**Figure 7.18** Changes of the instantaneous frequencies of the model building structure versus the time history of all earthquakes

It can be seen that the instantaneous frequencies gradually decrease with the increase in the level of earthquake excitations and the severity of the damage incurred. By contrast, the extent of fluctuation in the instantaneous frequency sequence increases with the aggravation of the structural nonlinearity behaviours, which is consistent with the indication of the two nonlinearity indices developed in Chapter 5.
7.5 SUMMARY

The quantitative and real time evaluation of the earthquake-induced damage for the scaled tall building model structure is investigated in detail. The evaluation is substantially based on an examination of the evolving history of the instantaneous frequencies, with the occurrence of the earthquakes. The proposed approach only uses the available earthquake excitation and response signals and avoids employing additional post-earthquake modal testing for recognition of the dynamic characteristics. In this way, the assessment of seismic damage can be conducted shortly after an earthquake occurs, which means a nearly real time manner and being of great significance for overall post-earthquake damage judgment of building structures.

How to extract an accurate time-dependent instantaneous frequency sequences using information only from short-duration of earthquakes is a critical issue. Naturally, the conventional FFT-based modal identification methods are inadequate due to their limited applicability to stationary signals and low frequency resolution on small data samples. As an alternative, super-resolution approaches to power spectra analysis and modal parameter estimating are developed and employed to accomplish the identification of the instantaneous frequency. In order to eliminate the earthquake excitation frequencies contamination on the structural frequencies, an excitation-independent structural decay response, called the seismic pulse response, are firstly calculated from earthquake excitation and response data. The seismic pulse response is derived by exerting a reverse-FFT on the transfer function. Then, the obtained decay responses of the model structure under earthquake excitations are discretized into a limited number of piece-wises, each of which corresponds to one time instant. Then the developed super-resolution frequency analysis algorithms are applied on each piece of data segment, to identify the instantaneous frequency at that
time instant. Eventually, based on the identified sequences of the instantaneous frequencies, a quantitative evaluation of earthquake-induced damage on the overall building structure is accomplished. It is assumed that the building mass remains unchanged under earthquake destroying, and that the time-dependent structural frequencies represent the overall stiffness of the building. Accordingly, the overall seismic damage expressed by the reduction in stiffness can be revealed by examining the fluctuations of the identified instantaneous frequency sequences.

With respect to the two signal-model-based super-resolution modal estimation algorithms (the covariance method and the PCAR method), it should be pointed out that they sometimes contain a small numerical error due to both data samples noise effects and damped sinusoids modeling error. From the comparative study on power spectra analysis, the PCAR method is found to be much less sensitive to noise than the covariance method.

From the results of instantaneous frequency identification and overall post-earthquake damage evaluation based on the identified instantaneous frequency sequences, the following points are drawn:

1. The instantaneous frequency sequences have somewhat fluctuations even though there is no damage occurring on the building structure. These fluctuations are caused by numerical error or by structural nonlinearity. The level of this nonlinearity behaviours increases considerably with the enhancement of earthquake intensity and damage severity;

2. Under low level of earthquake excitations, the model building experienced no damage;

3. Under moderate level of earthquake excitations, all damage occur only during or after the arrival of the ‘real’ earthquake (with peak acceleration), which indicates that under micro-vibrations of the shaking table (pre-earthquake), no cracks emerges and those existing cracks cannot be aggravated;
4. The model building structure exhibits the serious nonlinear and inelastic behaviours when subjected to strong level of earthquakes attacking;

5. The mass of the structure is generally assumed to be unaltered under damaged cases, and thus the degradation of the instantaneous frequency sequence can directly reflect the loss of structural stiffness. Accordingly to the strategy proposed in this chapter, the loss in the building stiffness can be quantitatively recognized by examining the reduction of the instantaneous frequencies; and

6. The same level of earthquake excitations exhibit discriminable damaging effects. The earthquakes can cause different extensity of damage, even though they are at the same level of excitation amplitude. This observation indicates that there is a distinct mechanism by which the earthquakes destroy the building structure, which depends on the current structural seismic vulnerability as well as on the properties (duration length, frequency contents, etc.) of earthquake excitations and soil conditions.
CHAPTER 8

CONCLUSIONS AND DISCUSSIONS

8.1 CONCLUSIONS

The tests, analysis, results and conclusions presented in the previous chapters can be summarized as follows.

Shaking Table Tests of the Scaled Tall Building Model Structure

A 1:20 scaled tall building model structure is simulated and fabricated, prototyped from a typical high-rise residential apartment in Hong Kong, for the purposes of earthquake-resisting capability investigation and vibration-based damage detection study. The model structure is elaborately fabricated using micro-concrete material, strictly according to similitude law. As a result, the model building simulates the prototype structure well, without much simplification. Shaking table tests are then conducted in consecutive stages with increasing intensity of earthquake excitation. Four levels of earthquake are exerted on the model building, including minor-level earthquake, moderate-level earthquake, strong earthquake and super-strong earthquake. Between two levels of earthquake attacking, white-noise excitation tests
are conducted for damage detection purpose. Simultaneously, a visual inspection on each floor of seismic damage is performed after each level of earthquake excitation.

2. Based on visual inspections, the earthquake induced damage at each stage of shaking table testing and the cumulative damage from all foregoing earthquakes attacking are logged. The results of inspections provide references and baselines for the subsequent studies on seismic damage identification and evaluation. The incurred damage begins from the lower portion (on and beneath the transfer plate) of the model building under minor-level earthquakes, then extends to the above-transfer plate portion (floors C04–C08) under moderate-level earthquake, and finally spreads to the whole structure under strong and super-strong earthquakes. From the serviceability perspective, the building structure is serviceable under minor and moderate-level of earthquakes, and the structure still can be repaired even under strong-level earthquake attacking. However, the building structure is close to collapse and completely loses the possibility of being repaired under super-strong earthquakes.

**Frequency Response Function-Based Damage Identifications**

3. A method for identifying seismic damage based on frequency response function is developed to both locate damaged floors and recognize damage severity. The proposed method is applied in conjunction with the use of neural network technique. Generally, neural network-based methods of damage identification take identified natural frequencies, mode shapes or their derivatives as inputs, to build the mapping relationship between modal parameters (damage indicators) and healthy states of the
structure. And the required modal parameters are often identified from the frequency response function, when excitation information is available. Considering the ineluctable numerical errors introduced in modal parameters estimation and information integrity of frequency response function which contains frequency, mode shape as well as damping ratio characteristics, the presented study takes frequency response function information directly as inputs of the neural networks.

**Principal Component Analysis on Frequency Response Function**

Despite many advantages of directly using frequency response function to identify seismic damage, a very significant hurdle still remains: the size of the frequency response function data, which is too large to be trained effectively by neural network. To overcome this handicap, a data compression and feature extraction approach is developed. The approach is developed from linear principal component analysis (PCA) technique, by which the dimensionality of the frequency response function data is greatly reduced, whereas the majority of modal characteristics are still retained. Taking the projection of the original frequency response function data on a limited number of extracted principal components as input, the configured neural network can be trained efficiently with the guarantee of convergence. As well as the functionality of reducing data dimensionality, the ability of PCA on filtering out measurement noise is also substantially explored. By independently adding artificial 'measurement noise' on both of the excitation and the response signals, the computed frequency response function is then coped with PCA method. The results prove the efficacy of PCA on noise filtering.
Both the severity of the overall seismic damage and the damage location(s) are identified, based on a variety of neural networks training on the frequency response function data, reduced via principal component analysis. When overall damage severity identification is prospected, frequency response function data obtained from any one of the measurement points (at the top floor in the presented study) are taken for principal component analysis and neural network training. In this case, just one neural network model is needed and just one output node is configured, which indicates the overall severity of the damage accumulated by earthquakes attacking beforehand. However, when the approximate damaged location(s) or floor(s) need to be determined, the frequency response function data from all measurement points are utilized for principal component analysis and neural network training. Moreover, the same number of neural network models as measurement points are architectured for damage localization, where each of neural network models is corresponding to one distinguishable damaged region. Additionally, a twofold principal component analysis is needed for extra spatial information compression. The achieved results indicate that the proposed methodology is applicable for both seismic damage location(s) and severity identification.

Output-only Modal Identification Approaches

Two output-only modal identification approaches, stochastic subspace identification (SSI) method and complex mode indicator function (CMIF) method, are discussed and applied to estimate the natural frequencies and mode shapes of the tall building model structure respectively. Both of the two approaches are applicable to stationary and relatively long duration of response signals. The tall building model structure
responses are sequentially recorded during the shaking table tests when subjected to earthquakes attacking and white-noise excitations. Apparently, earthquake responses are of short-duration and non-stationary, especially in severely damaged cases. Hence, the responses under earthquake excitations cannot be dealt with by using output-only parameter estimation approaches. In contrast, responses under white-noise excitations are of relative long-duration (about 3 minutes per excitation) and can be regarded as stationary signals due to their low-level amplitude. Accordingly, the presented output-only identification methods are faithfully applied to these responses.

In order to validate the reliability and accuracy of output-only modal identification approaches, the classical input-output identification (FRF-based) method is also employed to the same record of response under white-noise excitation. From the comparative studies presented in Chapter 5, it has been proven that both stochastic subspace identification and complex mode indicator function method are adequate for the purpose of seismic damage identification where only ambient vibration responses can be measured. They achieve the results very close to those using the input-output method of parameter estimation. On the other hand, from results consistency perspective, the complex mode indicator function method has been ascertained to be superior to the stochastic subspace identification method. The SSI method greatly depends on the appropriate selection of such parameters as the state-space dimension and the order of singular value, whereas the CMIF method is much simpler and more straightforward. Either the frequency or the mode shape
identification results by the CMIF method reveal a higher repeatability and consistency than those of the SSI method.

**Nonlinear Earthquake-induced Damage Evaluation**

8 For building structures, possible damage is usually caused by earthquake attacking or by any other accidental catastrophic disaster. In such cases, the damage occurs in a very short time, rather than in a form of long-term deterioration. Consequently, the evaluation of post-event overall damage is of critically significance. With respect to such types of damage, the evaluation had better rely on the records of earthquakes or of any other catastrophic attacking themselves, rather than on additional posterior modal testing. However, such records persist for a maximum of only several dozens of seconds and usually exhibit considerable nonlinear and non-stationary behaviours. This poses a great challenge to classical techniques of frequency response function identification, or to even output-only modal identification approaches. Therefore, the presented study resorts to some specific signal processing strategies to deal with these records, for instantaneous post-earthquake damage assessment. Joint time-frequency analysis algorithms. In order to eliminate seismic excitation related frequency components influence, seismic pulse decay responses under earthquake actions are firstly proposed and derived, for the subsequent analysis.

9 A variety of joint time-frequency analysis algorithms are discussed and applied on seismic pulse responses. The analysis is based on quadratic time-frequency spectra and thus at energy sense. The resulting time-frequency spectra illuminate the evolving history of the overall damage under the corresponding destruction caused
by earthquakes. A total of four classes of joint time-frequency analysis algorithms, namely, short-time Fourier transform, Cohen-class Wigner-Ville distribution, adaptive transform and continuous wavelet transform, are employed in Chapter 6. From the comparative studies, it is found that the time-frequency resolution of the short-time Fourier transform greatly depends on the selection of the length of window function, and that the adaptive transform holds the best time-frequency resolution. The Wigner-Ville distribution suffers from cross-term interference. And the wavelet transform exhibits its superiority only when multiple-resolution for low frequency and high frequency is needed.

10 As mentioned before, joint time-frequency analysis methods result in energy spectra, rather than in ultimate modal parameters. Therefore, the evaluation of overall damage is performed qualitatively. In order to assess seismic damage quantitatively, a super-resolution spectra analysis and parameter estimation scheme is proposed, to extract the instantaneous frequencies of the tall building model structure subjected to short-duration earthquake attacking. Covariance method and principal component auto-regressive (PCAR) method are introduced to accomplish super-resolution spectra analysis, where the PCAR method achieves more consistent results than the covariance method. The Prony method is employed to estimate instantaneous frequencies, which are ultimately used to perform a quantitative evaluation of earthquake-induced damage.

11 The proposed joint time-frequency analysis and the super-resolution of algorithms for estimating modal parameters are not only able to accomplish assessing of the instantaneous seismic damage, but also reveal the damaging mechanism of
earthquake attacking. The seismic damage incurred from the occurrence of a certain earthquake event depends on the spectral components of earthquake excitations as well as on soil conditions.

8.2 DISCUSSIONS AND SUGGESTIONS

In this dissertation, although all of the developed damage identification and overall damage evaluation approaches are implemented on a scaled model building structure, they can be extended to applications for real building structures as well. The kernel strategies proposed for vibration-based damage detection of building structures in this study are reviewed herein again, and some key issues that should be taken into account for future research or practical implementation on real building structures are also stressed.

Model-free Strategy of Damage Identification

The strategy of mathematical model-free structural damage identification serves as the solid foundation for all research activities in this dissertation. Both the methods of seismic damage identification (including severity and location) and of overall evaluation of post-earthquake damage aimed at achieving the goal of model-free implementation. As already well known, developing mathematical models for highly complicated building structures is a tedious and time-consuming task, which apparently cannot fulfill the requirements of real time assessment for building structure damage. Furthermore, the unavoidable modeling errors increase the uncertainty of damage identification realization. Consequently, from the practical point of view, identification methods for
seismic damage in building structures should be independent of structural mathematical models. Motivated by the above considerations, the researches in this study are devoted to developing seismic damage identification approaches that fully rely on field measurement data rather than on the mathematical model of the investigated structure. Thus, the proposed approaches are applicable for any complexity of structures. Naturally, the quality of the measurement data becomes critical instead. Therefore, in practical applications on building structures, more effort should be spent on designing a reliable and robust data acquisition system, which will ensure that the noise level of field measurement data is as low as possible.

**Ambient Vibration Measurement and Response-only Modal Identification**

If damage location(s) identification is prospected, posterior ambient vibration measurements are necessary. In addition, a sufficient number of sensors should be instrumented at multiple measurement points. The number of measurement degrees of freedom depends on the concerned discretization of structures for damage localization. However, the inputs exerted on the building structures in such tests of ambient vibration are usually natural loadings, such as wind or micro-vibrations from ground. These types of inputs are stochastic and are thus difficult to measure or even unmeasurable. As a result, only response data can be measured at respective measurement degrees of freedom. Input-output modal identification methods are not competent to extract modal parameters from these responses data and response (output)-only modal identification methods should thereby be applied. There are two alternatives available for dealing with responses data: the stochastic subspace identification (SSI) method, and the complex mode indicator function (CMIF) method. In the SSI method, several parameters need to
be determined to guarantee authentic estimation results, whereas the CMIF is relatively simple and straightforward. Therefore, the CMIF method should be preferred in real applications of seismic damage localization in building structures from ambient responses data.

**Earthquake Records and Nonlinear Time-frequency Analysis**

When it is more important and imperative to evaluate the overall post-earthquake damage, earthquake records (including excitations and responses) should be logged. Just a few sensors needed to be instrumented on the investigated structure for damage evaluation purpose. Nevertheless, due to un-predictable property of earthquake occurrence, the sensors need to be equipped permanently. This may be feasible only for the buildings of critical significance. In addition, the earthquake records are of short-duration and non-stationary, and thus cannot be analyzed by methods based on the classical Fourier transform or by output-only modal identification approaches. Joint time-frequency analysis algorithms are necessary to evaluate the overall seismic damage. Further, if quantitative instantaneous frequencies evolving with the occurrence of earthquakes and the earthquake damaging mechanism are intended, super-resolution spectra analysis and modal parameter estimation methods are needed to be employed. The principal component auto-regressive (PCAR) spectra analysis and the Prony parameters estimation methods are competent for accomplishing this task.

This concludes the dissertation.
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