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GENERALIZED PSEUDO-EXCITATION METHOD AND ITS APPLICATION FOR VIBRATION CONTROL OF WIND/SEISMIC RESISTANT BUILDINGS

by

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BSc., MSc.

A Dissertation for the Degree of Doctor of Philosophy

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of

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2000
To my wife
DECLARATION

I hereby declare that the dissertation entitled "Generalized Pseudo-Excitation Method and Its Application for Vibration Control of Wind/Seismic Resistant Buildings" is original unless otherwise acknowledged in the text. The material, either in whole or in part, has not been submitted for other degrees previously.

SIGNED

______________
Wen Shou ZHANG
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ABSTRACT

This dissertation presents a generalized pseudo-excitation method for random vibration analysis of buildings with supplemental discrete control devices and its application to various types of vibration control problems of wind/seismic resistant buildings.

The generalized pseudo-excitation method is actually a combination of the complex modal superposition method and the pseudo-excitation method. The proposed method can naturally retain all cross-correlation terms between closely spaced modes of vibration in structural response and accurately handle the non-orthogonal structural damping due to the installation of discrete control devices. The method also provides a convenient way of determining internal force response of a building. The principle and algorithm of the generalized pseudo-excitation method is first applied to a wind-excited tall building with active tendon devices and a wind-excited tall building with a light appendage at its top. The numerical examples show that for the tall building with a light appendage under alongwind excitation, the classical random-vibration-based modal superposition method (the SRSS method) may underestimate or overestimate the building response. The installation of active tendon devices may alter the natural frequencies and increase the modal damping ratio of the building so significantly that the building is no longer lightly damped or possesses the orthogonal damping property.

To pursue the application of modern control algorithm and the closed form solution for wind-excited tall buildings with active control devices, the cross-spectral density matrix of alongwind excitation on a tall building is factorized. The generalized pseudo-excitation method is then applied to find the closed form solution
for wind-induced response of the building implemented by active control devices with Linear Quadratic Gaussian (LQG) controllers. The effectiveness of active control devices with LQG controllers in reducing wind-induced vibration of tall buildings is investigated. The advantages and disadvantages of this approach are discussed in the dissertation.

The generalized pseudo-excitation method is then extended to investigate modal properties and seismic response of steel frames with connection dampers. The connection dampers are modeled as rotational springs and rotational dampers in parallel. The dissertation derives the mass, stiffness, and damping matrices for the beam element with connection dampers using a combination of the finite element method and the direct stiffness method. After the equations of motion of the system are established, the complex modal analysis is carried out to determine the modal properties of steel frames with connection dampers and the generalized pseudo-excitation method is used to determine seismic response. The parametric studies on the example frame with and without connection dampers show that there is an optimal damper damping coefficient for a given mode of vibration and a given fixity factor of the frame. With the optimal damper damping coefficient, the modal damping ratio of the frame can be significantly increased and the seismic response, including lateral displacement, shear force, and bending moment, can be considerably reduced to the level smaller than those of the frame with rigid connections.

The generalized pseudo-excitation method is finally extended and applied to vibration control of adjacent buildings under earthquake. The viscoelastic dampers, the fluid dampers, and the active tendon devices are respectively used to link the adjacent buildings together for control of seismic response. The viscoelastic dampers are passive energy dissipation devices, mathematically represented by the Voigt
model. The fluid dampers that operate on the principle of fluid flow through orifices specially shaped are also passive energy dissipation devices but they are defined by the Maxwell model. The active tendons with damper-structure interaction but without active control algorithm and the active control devices with LQG controllers are also investigated respectively. The dissertation derives the equations of motion for earthquake-excited adjacent buildings connected by the viscoelastic dampers, the fluid dampers, and the active tendon devices, respectively. The dissertation also derives the closed form solution for seismic response of adjacent buildings with LQG controllers. The dynamic characteristics of the control device-adjacent building system are determined through the complex modal analysis. The generalized pseudo-excitation method is used to find the seismic response of the system. Extensive parametric studies are performed to assess the effectiveness of control devices and to identify the beneficial control parameters using the generalized pseudo-excitation method. It is found that if control parameters are selected properly, the modal damping ratios of the adjacent buildings can be significantly enhanced and the seismic responses can be considerably reduced. It is also found that if active control force is limited to a certain level, the effectiveness of passive control devices is almost the same as that of active control devices.

Toward the real application of vibration control of adjacent buildings, dynamic characteristic and harmonic response of adjacent buildings connected by fluid dampers are also experimentally investigated using model buildings and fluid damper. Two building models are constructed as two three-story shear buildings of different natural frequencies. Model fluid damper connecting the two buildings is designed as linear viscous damper of which damping coefficient can be adjusted. The two buildings without fluid dampers connected are first tested to obtain their
individual dynamic characteristics and response to harmonic excitation. The tests are then carried out to determine modal damping ratios of the adjacent buildings connected by the fluid damper, from which optimal damper damping coefficient and location for achieving the maximum modal damping ratio are found. The measured modal damping ratios and harmonic responses of the building-fluid damper system are finally compared with those from the individual buildings. The comparison shows that the fluid damper of proper parameter can significantly increase the modal damping ratio and tremendously reduce the dynamic response of both buildings.
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LIST OF SYMBOLS

\(\alpha\) \hspace{1cm} \text{exponent for mean wind velocity profile}

\(\alpha_i, \beta_j\) \hspace{1cm} \text{coefficients in pseudo displacement}

\(\Lambda\) \hspace{1cm} (i) \text{coefficient matrices of active tendon devices}

(ii) \text{complex eigenvalue matrix}

\(\omega\) \hspace{1cm} \text{circular frequency (rad/s)}

\(\Omega\) \hspace{1cm} \text{circular frequency matrix}

\(\rho\) \hspace{1cm} \text{density}

\(v\) \hspace{1cm} \text{fixity factor}

\(\theta\) \hspace{1cm} \text{inclination angle}

\(\psi\) \hspace{1cm} \text{left eigenvector}

\(\Psi\) \hspace{1cm} \text{left modal matrix}

\(\Phi\) \hspace{1cm} \text{modal matrix}

\(\gamma\) \hspace{1cm} \text{modal participation factor vector}

\(\tau\) \hspace{1cm} \text{normalized feedback gain}

\(\varepsilon\) \hspace{1cm} \text{normalized loop gain}

\(\varphi\) \hspace{1cm} \text{phase angle}

\(\dot{\varphi}\) \hspace{1cm} \text{right eigenvector}

\(\Phi\) \hspace{1cm} \text{right modal matrix}

\(\sigma_{\alpha_j}\) \hspace{1cm} \text{standard deviation acceleration response of the } j\text{th floor}

\(\sigma_{\nu_j}\) \hspace{1cm} \text{standard deviation displacement response of the } j\text{th floor}

\(\sigma_{\sigma_j}\) \hspace{1cm} \text{standard deviation force response of the } j\text{th floor}
\( \omega_i \)  fundamental frequency
\( \omega_{dij} \)  damped modal frequency
\( \omega_{g}, \xi_g \)  characteristics of an earthquake
\( p_{ij} \)  coherency function
\( \omega_j \)  jth angular frequency (rad/s)
\( \phi_j \)  jth modal vector
\( \xi_j \)  jth model damping ratio
\( \Psi_j \)  matrix of measured mode shapes
\( \eta_j \)  number of tendons in the jth story
\( \psi_j \)  vector of measured mode shapes
\( \mu_j, \nu_j \)  coefficients in pseudo velocity
\( \delta_{jk} \)  Kronecker delta
\( \Delta t \)  time interval
\( \ast \)  conjugation
\( \Theta \)  null matrix
\( A \)  cross section area
\( A \)  state matrix
\( a_1, a_2, a_3 \)  control gains
\( a_i \)  constants
\( A_i \)  tributary area of the jth floor
\( B \)  input matrix
\( b_j \)  external damping coefficient
\( C \)  damping matrix
\( C^a \)  generalized damping coefficient matrix
$C_i$ constant

$C_d$ (i) damper damping coefficient
(ii) drag coefficient

$c_{di}$ damping coefficient of the viscoelastic damper at the ith floor

$c_i$ internal damping coefficient

$D$ coefficient matrices of active tendon devices

$e$ index vector of inertial forces

$E$ location matrix

$E$ Young's modulus

$E[]$ ensemble average

$f(t), p(t)$ stationary random excitation vector

$f^o(t), f'^o(t)$ generalized excitation vector

$H$ (i) coefficient matrices of active tendon devices
(ii) location matrix

$H(\omega)$ complex frequency response function matrix

$h(\tau)$ unit pulse response function

$H_f(\omega)$ complex frequency response function

$I$ identity matrix

$i$ imaginary unit.

$I$ second moment of area

$J$ performance index

$K$ stiffness matrix

$K_0$ ground surface drag coefficient

$K_c$ state feedback gain matrix
$K_{d1}$  storage stiffness  \\
$K_{d2}$  loss stiffness  \\
$k_i$  stiffness coefficient of the viscoelastic damper at the $i$th floor  \\
$K_F$  estimator gain matrix  \\
$k_j$  shear stiffness  \\
$K_t$  tendon stiffness  \\
$L$  length of the beam  \\
$L(\omega)$  lower triangular matrix  \\
$L_j(\omega)$  $j$th column of $L(\omega)$  \\
$m_j^*$  generalized mass  \\
$M$  mass matrix  \\
m_j  mass  \\
n  (i) constant  \\
(ii) frequency, in hertz  \\
$N$  number of degrees of freedom  \\
P  damper force  \\
P_0  amplitude of the damper force  \\
p_r  proportional constant  \\
q  dimension of modal subspace  \\
Q  state weighting matrix  \\
$Q(\omega)$  internal force response  \\
$q(t)$  state vector  \\
$\dot{q}(t)$  state estimator  \\
$q_j(t)$  control force from the $j$th controller
\( R \)  
(i) constant matrix  
(ii) state weighting matrix  

\( r \)  
integer  

\( R_l \)  
loop gain  

\( R_o^{-1} \)  
feedback gain  

\( R_{xx}(\tau) \)  
autocorrelation function matrix  

\( s \)  
eigenvalue  

\( S_0 \)  
intensity of an earthquake  

\( S_{f_1r} \)  
auto-spectral density matrix for \( f'(t) \)  

\( S_{f(t)} \)  
auto-spectral density function matrix for \( f(t) \)  

\( S_{g(o)} \)  
power spectral density function for \( \ddot{x}_g(t) \)  

\( s_j \)  
jth complex eigenvalue  

\( S_{pp}(\omega) \)  
auto-spectral density function matrix for \( p(t) \)  

\( S_{xx}(\omega) \)  
auto-spectral density function matrix for \( x(t) \)  

\( S_{zz}(\omega) \)  
auto-spectral density matrix for \( z(t) \)  

\( r \)  
matrix transposition  

\( t \)  
time  

\( t_r \)  
duration  

\( u \)  
damper piston motion  

\( u(t) \)  
(i) control force vector  
(ii) displacement vector of the hydraulic rams of the servomechanism  

\( u_0 \)  
fraction velocity of the wind  

\( u_0 \)  
amplitude of piston displacement  

\( U_j(t) \)  
displacement of a hydraulic ram of the servomechanism
\( v(t) \) measurement noise vector

\( V_{10} \) reference mean wind velocity at 10 meters above the ground

\( V_g \) mean velocity at the gradient height

\( V_j \) wind velocity at the jth floor

\( V_j(t) \) electric voltage

\( w(t) \) unit zero-mean Gaussian white noise vector

\( W_j \) wind force at the jth floor

\( x(\omega) \) pseudo displacement

\( x(t) \) displacement vector

\( X_2, Y_2 \) Riccati matrix

\( \ddot{x}_g(t) \) ground acceleration

\( x_j \) displacement of jth floor

\( z(t), z_j(t) \) generalized coordinate vector

\( z_g \) gradient height

\( z_j \) height of the jth floor
CHAPTER ONE

INTRODUCTION

1.1 RESEARCH MOTIVATION

The increasing population, fast growing economy, limited land available, and community’s demands for modern life lead to more and more tall buildings built around the world. These modern tall buildings, using innovative structural systems and high strength materials, are slender and low damped. They are thus susceptible to dynamic action of fluctuating wind if these buildings are located in strong wind regions. Excessive wind-induced vibration of tall buildings may cause structural failure, discomfort to occupants, damage to curtain wall, and malfunction of equipment inside the building. The most recent demonstration of such consequences is Typhoon York that attacked Hong Kong on 16 September 1999.

Earthquake excitation is another major concern of structural engineers when designing buildings located in a seismic zone. Earthquakes are one of nature’s great hazards to human being on this planet. Throughout historic time, they have caused the destruction of countless cities and villages on nearly every continent. The earthquake occurring on 17th January 1995 at Kobe, which is classified as a moderate seismic region in Japan, was the first densely populated and well-industrialized city to bear the full brunt of a high-magnitude earthquake since World War II. The main lesson of Kobe event is that heavily populated modern cities on less active faults need to reconsider their earthquake risk and improve their buildings and structures’ earthquake resistance. This lesson is invaluable for Hong Kong that is a heavily populated modern city in a moderate seismic zone.
Buildings in a modern city are also often built closely to each other or form a complex due to limited land available. These buildings, in most cases, are separated without any structural connections or are connected only at the ground level. Hence, wind-resistant or earthquake-resistant capacity of each building mainly depends on itself. If the separation distances between adjacent buildings are not sufficient, mutual pounding may occur during an earthquake, as observed in the 1985 Mexico city earthquake and the 1989 Loma Prieta earthquake.

Therefore, how to mitigate wind-induced and earthquake-induced vibration of a single building and/or a group of buildings, attracts many researchers and engineers around the world. The traditional approach to environmental hazard mitigation is to introduce more conservative designs so that buildings are better able to resist large imposed loads. This approach, however, can be untenable both technologically and economically. The advanced approach may be the use of structural vibration control technology, including passive, active, and semi-active control systems.

Structural vibration control means regulating the structural characteristics of mass, damping, and stiffness using a system of control forces to ensure a desirable level of performance of a building. A passive control system does not require an external power source for operation and utilizes the motion of structure to develop control forces. Passive control devices, such as base isolation systems, viscoelastic dampers, tuned mass dampers, and fluid dampers, have been installed in many real buildings against either strong wind or earthquake. (Soong and Constantinou 1994; Soong and Dargush 1997; Housner et al. 1997).

In comparison with passive control systems, active control is a relatively new area of research and technological development. However, there was a rapid development of active structural control with many practical applications during the
past two decades. An active control system typically requires a large power source for operation of electrohydraulic or electromechanical actuators that supply control forces to a building. Control forces are developed based on feedback from sensors that measure the excitation and/or the response of the structure. Active mass dampers and active tendon systems are some of the active control devices being developed and tested both in the laboratory, and in some cases, in actual structural applications. (Soong 1990; Soong and Constantinou 1994; Housner et al. 1997).

Semi-active control system is a compromise between passive and active control systems. A Semi-active system requires only a small amount of external power source for operation and control forces. Control forces are developed based on feedback from sensors that measure the excitation and/or the response of the structure. The magnitude of control forces is adjusted by changing the parameters of a control system, such as its damping. Such examples are the variable orifice dampers, variable friction damper system, and magnetorheological (MR) dampers (Symans and Constantinou 1999).

In the analysis of buildings against environmental loads such as winds and earthquakes, dynamic loads are often modeled as stochastic processes represented in the frequency domain by their power spectral density functions. The basic theory for random vibration has been well established, and various methods for analyzing such problems have been developed. However, the analysis of a complex structure with thousands of DOF (degrees of freedom) implemented with hundreds of discrete dampers under random excitation is still a challenging task for most engineering applications. This is because a modern building with control devices may possess closely spaced modes of vibration and non-classical damping properties, and extensive parametric studies are often need to be carried out to find optimal values of
control devices. Therefore, it is important to develop efficient and effective methods for determining the response of buildings under environmental loads with reasonable computational efforts and high accuracy.

1.2 OBJECTIVES

The main objectives of this dissertation are thus:

1. To develop a generalized pseudo-excitation method for predicting dynamic characteristics and providing the exact solutions for buildings with control devices under environmental loads by extending the pseudo excitation method developed by Lin et al. (1994) for buildings without control devices. The generalized pseudo-excitation method should be a kind of CQC (Complete Quadratic Combination) method to naturally include the cross-correlation terms between the participant modes in the response calculation. The generalized pseudo-excitation method should be able to accurately treat non-classical damping properties of a building with control devices. The generalized pseudo-excitation method should also be able to efficiently handle a complex structure of thousands of degrees of freedom with discrete control devices.

2. To use the generalized pseudo-excitation method to investigate the performance of a single building with active tendon devices under wind excitation: to conduct an extensive parametric study to find optimum parameters of control devices for the facilitation of design of the control devices: and to carry out a comparison between the proposed method and the SRSS method (Square Root of Sum of Squares).

3. To apply the generalized pseudo-excitation method to steel frames with semi-
Chapter One: *Introduction*

rigid joints under earthquake excitation: to explore how to add energy
dissipation materials at the connection between the end plate and column
flange or between the angle and member flange to enhance earthquake resistant
capacity of steel frames; and to investigate the general performance of steel
frames with energy dissipation materials at semi-rigid connections.

4. To extend the generalized pseudo-excitation method to adjacent buildings
linked by viscoelastic dampers or fluid dampers subject to earthquake; to
evaluate the performance of passive dampers for mitigating seismic responses
of both buildings; and to perform sensitivity studies and parametric studies of
passive dampers to find optimal design parameters.

5. To pursue the application of modern control algorithm for wind-excited tall
buildings with active control devices and earthquake-excited adjacent
buildings with active control devices: to find the closed form solutions for such
complex systems: and to evaluate the effectiveness of active control devices
using the closed form solutions.

6. To develop a testing facility and carry out experimental investigation on
adjacent buildings linked by fluid dampers to physically demonstrate the
effectiveness of fluid dampers for mitigating building responses.

1.3 SCOPE OF THE WORK

The contents of this dissertation consist of ten chapters.

- Chapter one gives a brief introduction of motivation and scope of the present
research.

- Chapter two presents an extensive literature review on the related topics,
which cover traditional analytical methods for buildings under random excitation.
vibration control of tall buildings under wind excitation, steel frames with semi-rigid joints under earthquake excitation, vibration control of adjacent buildings under earthquake excitation, and control algorithms.

- Chapter three introduces the principle of generalized pseudo-excitation method for random vibration analysis of buildings with supplemental discrete control devices. Before that, the principle and application of the pseudo-excitation method for buildings without control devices under either wind excitation or earthquake excitation are given.

- Chapter four applies the generalized pseudo-excitation method to wind-excited single tall building with active tendon devices as well as wind-excited single tall building with a light appendage at its top. The numerical examples show that for the tall building with a light appendage under alongwind excitation, the classical random-vibration-based modal superposition method (the SRSS method) may underestimate or overestimate the building response. The installation of active tendon devices may alter the natural frequencies and increase the modal damping ratios of the building so significantly that the building is no longer lightly damped or possesses the orthogonal damping property. The active tendon devices used in the study are of electrohydraulic servomechanism.

- Chapter five derives the mass, stiffness, and damping matrices for the beam element with connection dampers at its ends using a combination of the finite element method and the dynamic stiffness method. The connection dampers are modeled as rotational springs and rotational dampers in parallel. After equations of motion of the system are established, the complex modal analysis is carried out to determine the modal properties of steel frames with connection dampers and the
generalized pseudo-excitation method is then used to determine seismic response. The parametric studies on the example frame with and without connection dampers show that there is an optimal damper damping coefficient for a given mode of vibration and a given fixity factor of the frame. With the optimal damper damping coefficient, the modal damping ratio of the frame can be significantly increased and the seismic response, including lateral displacement, shear force, and bending moment, can be considerably reduced to a level smaller than those of the frame with rigid connections.

- Chapter six extends the generalized pseudo-excitation method to vibration control of adjacent buildings under earthquake. The viscoelastic dampers and fluid dampers are respectively used to link the adjacent buildings together for control of seismic response. The viscoelastic dampers are passive energy dissipation devices, mathematically represented by the Voigt model. The fluid dampers that operate on the principle of fluid flow through orifices specially shaped are also passive energy dissipation devices but they are defined by the Maxwell model. This chapter derives the equations of motion for earthquake-excited adjacent buildings connected by the viscoelastic dampers and fluid dampers respectively. The dynamic characteristics of the damper-adjacent building system are determined through the complex modal analysis. The generalized pseudo-excitation method is used to find the solution for the seismic response of the system. Extensive parametric studies are performed to assess the effectiveness of the dampers and to identify beneficial damper parameters.

- Chapter seven pursues the application of modern control algorithm and the closed form solution for wind-excited tall buildings with active control devices. The cross-spectral density matrix of alongwind excitation on a tall building is factorized.
Chapter One: Introduction

The generalized pseudo-excitation method is applied to find the closed form solution for wind-induced response of the building implemented by active control devices with linear quadratic Gaussian (LQG) controllers. The effectiveness of active control devices with LQG controllers in reducing wind-induced vibration of tall buildings is investigated. The advantages and disadvantages of this approach are pointed out in this chapter.

• Chapter eight discusses how to use active control devices to link adjacent buildings for mitigating the response of the adjacent buildings under earthquake excitation. First, the active tendon devices and passive control algorithm used by Yang (1982) and Samali et al (1985a) for single building are extended to adjacent buildings. The performance of the active control devices with modern linear quadratic Gaussian (LQG) controllers is then investigated. The dynamic characteristics of the control device-adjacent building system are determined through the complex modal analysis. The generalized pseudo-excitation method is used to find the closed form solution for the seismic response of the whole system with LQG controllers. Extensive parametric studies are performed to assess the effectiveness of the control devices and to identify beneficial control parameters. It is found that if the control parameters are selected properly, the modal damping ratios of the adjacent buildings can be significantly enhanced and the seismic responses can be considerably reduced.

• Chapter nine experimentally investigates the dynamic characteristics and harmonic response of adjacent buildings connected by fluid dampers using model buildings and fluid dampers. Two building models are constructed as two three-story shear buildings of different natural frequencies. Model fluid damper connecting the
two buildings is designed as linear viscous damper of which damping coefficient can be adjusted. The two buildings without fluid dampers connected are first tested to obtain their individual dynamic characteristics and response to harmonic excitation. The tests are then carried out to determine modal damping ratios of the adjacent buildings connected by the fluid damper, from which optimal damper damping coefficient and location for achieving the maximum modal damping ratio are found. The measured modal damping ratios and harmonic responses of the building-fluid damper system are finally compared with those from the individual buildings. The comparison shows that the fluid damper of proper parameter can significantly increase the modal damping ratio and tremendously reduce the dynamic response of both buildings.

- Chapter ten summarizes the main conclusions obtained in the dissertation and provides some recommendations for further studies.
CHAPTER TWO

LITERATURE REVIEW

2.1 VIBRATION ANALYSIS OF STRUCTURES UNDER RANDOM EXCITATION

Environmental loads such as earthquakes and strong winds are often modeled as random processes represented in the frequency domain by their power spectral density functions. If a structure involves too many degrees of freedom, the mode-superposition method is usually adopted for determining the dynamic response of the structure under random excitation. Two modal combination methods are applicable to this scheme: the CQC (Complete Quadratic Combination) method and the SRSS (Square Root of the Sum of Squares) method (Clough and Penzien 1975). Traditional algorithms of CQC and SRSS under earthquake and wind excitations are now reviewed in the subsequent two sections, respectively.

2.1.1 Structural Response to Earthquake Excitation

Consider the equation of motion of an elastic structure subjected to ground acceleration $\ddot{x}_g(t)$, with its power spectral density function $S_x(\omega)$ given.

$$M\dddot{x}(t) + C\dot{x}(t) + Kx(t) = -Me \ddot{x}_g(t) \quad (2-1)$$

in which $M$, $C$ and $K$ are the $N \times N$ mass, damping and stiffness matrices of the structure. $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$ are the displacement, velocity and acceleration vectors, respectively. $e$ is an $N$-dimensional index vector of inertial forces. For a complex structure, $N$ is very large, and the mode superposition method is usually used. To this
end, the eigenproblem is first solved.

\[ K\Phi = M\Phi\Omega^2, \quad \Phi^T M\Phi = I \]  \hspace{1cm} (2-2)

in which \( I \) is an identity matrix. Then, the structural displacement vector \( x \) is decomposed into

\[ x(t) = \Phi z(t) = \sum_{j=1}^{q} \phi_j z_j(t) \]  \hspace{1cm} (2-3)

Finally, equation (2-1) can be reduced into

\[ \ddot{z}(t) + C^0\dot{z}(t) + \Omega^2 z(t) = -\gamma \ddot{x}_g(t) \]  \hspace{1cm} (2-4)

in which

\[ C^0 = \Phi^T C \Phi, \quad \gamma = \Phi^T M e. \]  \hspace{1cm} (2-5)

If the structure related to the damping matrix \( C \) is proportionally damped, equation (2-4) can be reduced and uncoupled into the \( q \) single-degree-of-freedom equations.

\[ \ddot{z}_j(t) + 2\xi_j \omega_j \dot{z}_j(t) + \omega_j^2 z_j(t) = -\gamma_j \ddot{x}_g(t) \]  \hspace{1cm} (2-6)

in which \( \xi_j \) and \( \omega_j \) are the \( j \)th modal damping ratio and angular frequency respectively; and \( \gamma_j \) is the \( j \)th modal participation factor.

The solution of equation (2-6) is

\[ z_j(t) = -\sum_{\tau=0}^{t} \gamma_j h(\tau)\ddot{x}_g(t - \tau)d\tau \]  \hspace{1cm} (2-7)

in which \( h(\tau) \) is the impulse response function, and so

\[ x(t) = -\sum_{j=1}^{q} \phi_j \int_{-\infty}^{t} \gamma_j h(\tau)\ddot{x}_g(t - \tau)d\tau \]  \hspace{1cm} (2-8)

Its correlation matrix is
\[ R_{x\tau}(\tau) = E[x(t)x^T(t+\tau)] \]
\[
= \sum_{j=1}^{N} \sum_{k=1}^{N} \gamma_j \gamma_k \phi_j^T \phi_k \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{s_j s_k}(\tau+\tau_1-\tau_2)h(\tau_1)h(\tau_2)d\tau_1 d\tau_2
\] (2-9)

Transforming the above equation into the frequency domain gives

\[ S_{x\tau}(\omega) = \sum_{j=1}^{N} \sum_{k=1}^{N} \gamma_j \gamma_k H_j^*(\omega)H_k(\omega)\phi_j^T \phi_k^T S_q(\omega) \] (2-10)

in which \( H_j(\omega) = (\omega^2 - \omega'^2 + i2\xi_j \omega \omega')^{-1} \).

Equation (2-10) is the traditional CQC method for computing the spectral density matrix of structural displacement responses under earthquake excitation. It is exact because all the cross-modal terms have been included. When both \( N \) and \( q \) are big, the computational efforts required are very large because equation (2-10) involves the double summation operations. For example, when \( q=10 \), the operation after the summation symbols must be repeated 100 times, in other words \( 100 N \)-dimensional vector multiplication operations are required. Therefore, for engineering computations, the following SRSS method, by neglecting the cross-modal terms, is widely used.

\[ S_{x\tau}(\omega) \approx \sum_{j=1}^{N} \gamma_j^2 |H_j(\omega)|^2 \phi_j^T \phi_j^T S_q(\omega) \] (2-11)

Equation (2-11) is acceptable for most structures with light damping and sparsely spaced participant frequencies. However, for the structures of special structural systems and the structures with either passive or active control devices, the SRSS method may not be proper because of their special dynamic characteristics. The generalized pseudo-excitation method developed in this dissertation may have to be resorted.

2.1.2 Structural Response to Wind Excitation
The equation of motion of a structure subjected to wind gust excitations can be written as

\[ M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t) = Rp(t) \] (2-12)

in which \( p(t) \) is an \( m \)-dimensional stationary random excitation vector with the power spectral density matrix \( S_{pp}(\omega) \) given, which include the spatial and temporal coherence between multi point excitations. \( R \) is an \( N \times m \) matrix consisting of 0 and 1 which expands the \( m \)-dimensional vector \( p(t) \) into the \( N \)-dimensional vector \( f(t) \). Various fluctuating wind speed (or force) spectra, for different wind directions, will lead to different \( S_{pp}(\omega) \), but the equation itself is quite general.

Assume the degree of freedom of the structure under consideration, denoted as \( N \), is very high, so that the mode-superposition scheme is adopted to reduce the computational effort. Based on the first \( q \) (\( q \ll N \)) normalized modes \( \Phi \), the structural displacement vector \( x \) can be expressed by the normal co-ordinates vector \( z \) as

\[ x(t) = \Phi z(t) \] (2-13)

and equation (2-12) can be accordingly transformed into

\[ \ddot{z}(t) + C''z(t) + \Omega^2 z(t) = f''(t) \] (2-14)

in which

\[ C'' = \Phi^T C \Phi \] (2-15)

\[ f''(t) = \Phi^T f(t) = \Phi^T Rp(t) \] (2-16)

Because \( p(t) \) has been assumed to be a stationary random excitation vector, if the system is linear, \( f'(t) \) and \( z(t) \) are also stationary random vectors. Therefore, the cross-spectral density matrix of \( z(t) \) can be found from the power spectral density matrix \( f'(t) \) based on equation (2-14), i.e.
\[ S_{zz}(\omega) = H^*(\omega)S_{rr'}(\omega)H^T(\omega) \]  \hspace{1cm} (2-17)

in which
\[ H(\omega) = (-\omega^2 I + \Omega^2 + i\omega C^n)^{-1} \]  \hspace{1cm} (2-18)

\[ S_{rr'}(\omega) \] is the power spectral density matrix of \( f(t) \). From equations (2-13) and (2-16), one obtains
\[ S_{xx}(\omega) = \Phi S_{zz}(\omega)\Phi^T \]  \hspace{1cm} (2-19)

\[ S_{rr'}(\omega) = \Phi^T S_{ff}(\omega)\Phi = \Phi^T R_{pp}(\omega)R^T \Phi \]  \hspace{1cm} (2-20)

Substituting equations (2-17) and (2-20) into equation (2-19) yields
\[ S_{xx}(\omega) = \Phi H^*(\omega)\Phi^T S_{ff}(\omega)\Phi H^T(\omega)\Phi^T \]  \hspace{1cm} (2-21)

or
\[ S_{xx}(\omega) = \Phi H^*(\omega)\Phi^T R_{pp} R^T(\omega)\Phi H^T(\omega)\Phi^T \]  \hspace{1cm} (2-22)

If \( C \) is a proportionally damped matrix, both \( C^n \) and \( H(\omega) \) will be diagonal, and equation (2-22) can be reduced into the following form:
\[ S_{xx}(\omega) = \sum_{i=1}^{n} \sum_{k=1}^{n} H_i^*(\omega)H_k(\omega)\phi_i^T R_{pp}(\omega) R^T \phi_k \]  \hspace{1cm} (2-23)

Equation (2-23) is the traditional CQC method for computing the spectral density matrix of structural displacement responses under wind excitation. For large systems, the computational effort for equation (2-23) is very considerable, and so the cross-correlation terms between the participating modes are often neglected under the assumption that the participating natural frequencies are sparsely spaced and the system is lightly damped. Thus, the CQC method based on equation (2-23) is further reduced into to the SRSS method based on equation (2-24)
\[ S_{xx}(\omega) = \sum_{i=1}^{n} |H_i(\omega)|^2 \phi_i^T R_{pp}(\omega) R^T \phi_i \]  \hspace{1cm} (2-24)
The traditional algorithms are difficult to be widely used in general engineering computations because enormous computational effort is required for complex structures. This is even for the traditional SRSS method. To overcome this problem, a pseudo-excitation method was developed by Lin and the writer (1994) for uncontrolled structures, which will be described in Chapter 3. Furthermore, if the structures are passively or actively controlled, the generalized pseudo-excitation method presented in this dissertation should be used.

2.2 VIBRATION CONTROL OF WIND-EXCITED TALL BUILDINGS

The use of high strength materials and advanced design procedures has resulted in very flexible and low damped tall buildings. They may experience excessive deflections and accelerations under strong winds. Even though most buildings and structures do not have safety problems during strong winds, wind-excited vibration may cause discomfort to occupants, damage to curtain wall, or malfunction of equipment inside the building. Hence, it is of practical interest to develop effective structural control techniques to suppress building vibrations due to fluctuating wind loads.

There has been rapid development of structural control with many practical applications during the last two or three decades. The concepts of structural control, including passive, active, semi-active and hybrid structural control methods have been growing in acceptance and a number of control devices have been implemented to real structures throughout the world. A structural control system has the basic configuration as shown schematically in Fig. 2.1.
(a) Conventional Structure

(b) Structure with Passive Energy Dissipation

(c) Structure with Active Control

(d) Structure with Hybrid Control

Figure 2.1 Structure with Various Control Schemes
(Spencer and Soong, 1999)
(e) Structure with Semi-Active Control

Figure 2.1 (cont.) Structure with Various Control Schemes
(Spencer and Soong, 1999)

Structural control means regulating the structural characteristics of mass, damping, and stiffness using a system of control forces to ensure a desirable level of performance. A passive control system does not require an external power source for operation and utilizes the motion of structure to develop the control forces. Passive control devices, such as metallic dampers, friction dampers, viscoelastic dampers, viscous fluid dampers, tuned mass dampers, and tuned fluid dampers, can not increase the energy in a passively controlled structural system and are thus inherently stable.

In comparison with passive control systems, active control is a relatively new area of research and technological development. However, there has been rapid development of active structural control with many practical applications during the past two decades. An active control system typically requires a large power source for operation of electrohydraulic or electromechanical actuators which supply control forces to the structure. Control forces are developed based on feedback from sensors.
that measure the excitation and/or the response of the structure. These forces can be used to both add and dissipate energy in the structures. Active mass dampers, active mass drivers and active tendon systems are some of the devices being developed and tested both in the laboratory and in some cases in actual structural applications.

Semi-active control system is a compromise between passive and active control systems. A Semi-active system requires only a small amount of external power source for operation and control forces. Control forces are developed based on feedback from sensors that measure the excitation and/or the response of the structure. The magnitude of control forces is adjusted by changing a parameter of the system, such as its damping. Such examples are the variable orifice dampers, variable friction damper system and magnetorheological (MR) dampers.

A hybrid system consists of an active control system and a passive control system and thus increases the performance and robustness of the control system. Hybrid mass damper (HMD) is a combination of a passive tuned mass damper and an active control actuator and is the most common control device employed in full-scale civil engineering applications.

For passive control, viscoelastic dampers as an energy dissipation device have been studied and tested by many researchers (Caldwell 1986; Nielsen et al. 1994; Lai et al. 1995; Shen and Soong 1995; Mahmoodi and Keel 1986; Tsai and Lee 1993; Kasai and Higgins 1998; Nakamura and Kaneko 1998; Iwata et al. 1998). Viscoelastic dampers can be used to increase the overall damping of buildings and were first installed in 1969 on the Twin World Trade Center Towers, New York (Mahmoodi 1969; Mahmoodi et al. 1987). In 1982, viscoelastic dampers were utilized in the Columbia Sea First Building in Seattle against wind-induced vibrations (Keel and Mahmoodi 1986). Similar applications of viscoelastic dampers
were made to the Two Union Square in Seattle in 1988 (Soong and Dargush 1997) and the Chien-Tan railroad station roof, Taipei in 1994. A combination mechanism of a friction damping device and a viscoelastic damping device has also been investigated recently (Tsiatas and Daly 1994).

Tuned mass dampers (TMDs) as an energy absorbing device can be also used to increase the damping in the main structure. Tuned mass dampers, one of the oldest structural vibration control devices (Den Hartog 1947), have been employed with fairly good results of reducing the wind-induced response of tall buildings (McNamara 1977; Kareem 1983; Xu 1996; Xu et al. 1992a, 1992b, 1992c; Xu and Kwok 1994; Kwok and Samali 1995). A number of TMDs have been installed in tall buildings for response control against primarily wind-induced vibrations. Successful examples include two TMDs installed on the Centerpoint Tower in Sydney (ENR 1971; Kwok and MacDonald 1987); the TMDs installed in the CN tower of Toronto, the Citicorp Center of New York, and the John Hancock Tower of Boston (Petersen 1980; ENR 1977).

Like TMDs, tuned liquid dampers (TLDs), including sloshing dampers (TSDs) and liquid column dampers (TLCDs), are another class of dynamic vibration absorbers and have been used primarily for suppressing wind-induced vibrations of tall structures. (Sun et al. 1998b; Wakahara et al. 1992; Qu et al. 1993; Tschanz et al. 1997; Tamura 1995; Tamura et al. 1995; Kwok et al. 1991; Samali et al. 1992; Balendra et al. 1995; Zhang and Zhang 1993; Zhang et al. 1993).

The research on active structural control did not begin until 1972 when Yao (1972) attempted to stimulate interest in the use of active structural control to suppress the vibration of civil structures subjected to wind and earthquake excitations. To date, many investigations on active control systems of structures have

Active mass damper, active tendon systems and active aerodynamic appendage have been investigated by many researchers. Roorda (1975) examined some of the concepts involved in active damping of structures by feedback controls by way of an analysis of a tendon control scheme for tall flexible structures. Yang and Giannopoulos (1978) developed an approach to solve the active control problem for continuous structures using the transfer matrix technique. Two types of tendon control, i.e., the force tendon and the moment tendon, were considered for a cantilever beam-type structure under the excitation of the stochastic wind loads. Yang and Samali (1983) investigated the control of tall buildings implemented by an active mass damper or an active tendon control system in along-wind motion. Control of coupled lateral-torsional motion of tall buildings with active mass dampers was investigated by Samali et al. (1985b). It was found that the advantage of active mass damper over a passive one is that active system is capable of reducing the acceleration responses substantially. Lund (1980) and Chang and Soong (1980a) studied the possibility of enhancing TMD effectiveness with an added active control capability. A comparative study between active tendons and active mass dampers was made by Abdel-Rohman and Leipholz (1983). It was shown that the active tendons, when applied to a building subjected to self-excited wind forces, provided more efficient control than an active tuned mass damper. Mackriell (1997) investigated the active control of the first mode of vibration of two slender, wind-loaded buildings. An algorithm using acceleration feedback was used in their study. Abdel-Rohman (1987) studied the feasibility of active control by considering various
aspects as the time delay effect on the controlled response, the effectiveness of the control mechanism in suppressing the vibration, the effect of changing the structural parameters due to the feedback control forces, and the size of actuators required to achieve the designed control law. Three mechanisms: active tendons, active tuned mass dampers, and the active aerodynamic appendage were investigated and compared. Pantelides and Cheng (1990) developed a method of scalar index for determining the optimal locations of active tendons for seismic structures on the basis of the concept of degree of controllability for free vibrations. Xu (1996) proposed a method for selecting design parameters of active mass dampers and estimating motion reduction of wind-excited tall buildings, based on aeroelastic model tests of uncontrolled tall buildings. Ikeda (1997) discussed the effect of weighting a stroke of an active mass damper in the linear quadratic regulator problem for a single-degree-of-freedom undamped structure controlled by an active mass damper. Nagashima and Shinozaki (1997) devised a systematic design procedure and an algorithm for variable gain feedback control of buildings with active mass damper (AMD) systems. Adhikari and Yamaguchi (1997) discussed how to eliminate the interaction effect from the Active Tuned Mass Damper (ATMD) to the building when sliding mode control (SMC) scheme was applied to control the vibration of tall buildings with an ATMD installed at the top floor. Chang and Soong (1980b), Klein and Salhi (1980) and Soong and Skinner (1981) studied the application of aerodynamic appendages to reduce the vibration of tall buildings under strong winds for the comfort of occupants. Other control devices, such as pulse generator (Masri et al. 1980, 1981; Miller et al. 1988; Safford and Masri 1974) and gyroscope (Murata and Ito 1971) have also been proposed and investigated.
Chapter Two: Literature Review

Most of the above analyses employed the traditional algorithms and needed considerable computational efforts. Some analyses were carried out by transfer matrix method. The transfer matrix method dispenses with the intermediate step of computing the mode shapes and deals directly with the final structural response. However, it can only be used to regular and periodic structures. In this dissertation, much more efficient analytical methods will be developed for passively or actively controlled structures under either wind excitation or earthquake excitation.

Experimental studies of active mass dampers and active tendons in the laboratory or in the field are particularly important since their feasibility and applicability to real structures are needed to be evaluated in practice. Roorda (1980) presented the results involving the use of tendons performing on a series of small-scale models which included a cantilever beam, a king post truss and a vertical cantilever column. Soong et al. (1987) and Chung et al. (1986, 1988, 1989) carried out comprehensive experimental studies of tendon control using a carefully designed, fabricated, and calibrated structural model under seismic-type loading supplied by a shaking table. Time delay compensation, robustness of control algorithms, modeling errors and structure-controller interactions were investigated. Soong et al. (1981) conducted a wind tunnel experimental study of aerodynamic appendages using a scaled tall building model. A small metal appendage, placed on the top floor, was activated by a solenoid coupled to an analog control circuit operating on an optimal feedback algorithm. The concept of aerodynamic appendages was demonstrated to be feasible.

Active control technique has also been put into practice. Kyobashi Seiya Building shown in Fig. 2.2 was built in 1989 and is the first building in the world to have an active control system (Kobori 1994; Sakamoto et al. 1994). Another
example employed HMD system is Sendagaya INTES building in Tokyo in 1991, as shown in Fig. 2.3. To date, there have been over 20 buildings that have employed active control strategies for wind-induced motion control (Spencer and Sain 1997).

Figure 2.2 Kyobashi Seiwa Building and AMD (Spencer and Soong, 1999)

Figure 2.3 Sendagaya INTES Building (Spencer and Soong, 1999)
2.3 STEEL FRAMES WITH SEMI-RIGID CONNECTIONS

Conventional analysis and design of steel frames neglect the actual behavior of connections. Instead, beam-to-column connections are assumed either fully rigid or ideally pinned. Design and analysis methods for steel or reinforced concrete structures have been developed under these simplified assumptions. Although these assumptions simplify analysis and design procedures, the predicted response of the frame may not be realistic because in reality the connections in these frames are neither "rigid" nor "pinned". Most real beam-to-column connections used in steel frames are semi-rigid in that rotational discontinuity exists between connected beam and column. The different types of connections that are commonly used fill the entire flexibility spectrum from flexible "hinge like" connections to semi-rigid connections and then to rigid connections. The term semi-rigid is commonly used to denote the non-ideal connection behavior between pinned and fully rigid. Provisions in various design codes of practice have been made for the design of steel frames with semi-rigid connections. AISC-ASD specification (AISC 1989) classifies its connections in three types: rigid, simple and semi-rigid connections, while AISC-LRFD (AISC 1994) specification reduces this classification to two types: FR (fully restrained) and PR (partially restrained) connections. The Eurocode 3 (ECS 1992), as in AISC-ASD code, has three types of connections. Increasing connection stiffness usually leads to an increase of frame strength, but in exceptional cases, it may result in a slight loss of strength (Ackroyd and Gerstle 1983).

The primary flexibility of a steel beam-to-column connection is its rotational deformation \( \theta \) caused by the in-plane bending moment \( M \). A very large number of tests on connections have been conducted since the early 1900s to study the behavior
characteristics of isolated connections (Leon and Shin 1995). Considerable research has been carried out to access the actual behavior of steel frames accounting for the effect of flexibility. Much experimental data for various types of connections such as T-stub, the end plate, the top end seated angle, the angle web etc. have been obtained. In practice, almost all types of connections exhibit some nonlinear M-θ characteristics (See Fig. 2.4). To represent the nonlinear M-θ curves of semi-rigid connections, a number of mathematical models have been developed for static problems. These include linear model (Lightfoot and LeMessurier 1974), bilinear model (Sugimoto. and Chen 1982), piecewise linear model (Razzaq 1983; Vinnakota 1982), polynomial model (Frye and Morris 1975), cubic B-spline model (Cox 1972), two-parameter power model (Krishnamurthy et al. 1979), three-parameter power

![Figure 2.4 M/θ Comparisons for Various Connection Types (Simões, 1996)](image-url)
model (Colson and Louveau 1983) exponential model (Lui and Chen 1986; Chisala 1999; Wu and Chen 1990), and others (Ang and Morris 1984; Yee and Melchers 1986). Correspondingly, methods of analysis for frames with semi-rigid connections have been proposed by many researchers. This includes linear analysis (Rathbun 1936), bifurcation analysis and semi-bifurcation analysis (Jaspart 1988; Ho and Chan 1991) and second order nonlinear analysis (Lui and Chen 1988; Al-Bermani and Kitipornchai 1992; Anderson and Benterkia 1991).

Compared with the static analysis, the research on dynamic analysis of semi-rigid jointed frames is relatively limited. In reality, the vibration and dynamic behaviour of a semi-rigid jointed frame is quite different from a rigid or pinned jointed frame. On the other hand, the effect of the nonlinear behaviour of semi-rigid joints on the structural response is more significant under dynamic loads. In order to describe the hysteretic M-θ behaviour of flexible joints under dynamic and cyclic loading, a bilinear model has been used by Sivakumarana (1983) and a trilinear M-θ model has been used by Moncarz and Gerstle (1981). Al-Bermani et al. (1994) and Zhu et al. (1995) proposed a bounding-line model simulating the connection behaviour under dynamic and cyclic loading and studied the structural response of a number of frame structures under cyclic and seismic loading conditions. Experiments have been conducted to investigate the effect of flexibility in the beam-column connections. Popov and Stephen (1972) made an experimental study of joint behaviour under cyclic loading. Analytical and experimental studies on frames with nonlinear connections under cyclic loading have been conducted by Stelmack et al (1986), using the analytical model developed by Moncarz and Gerstle (1981). A trilinear M-θ relation has been used to simulate the hysteresis loops of a flexible
connection in their studies. They tested two steel frames, a two-story one-bay frame and an one-story two-bay frame, as well as isolated top and seat angle connections used in the frame test. Nader and Astaneh (1991) made a test on a single story steel structure mounted on a shaking table under base excitation. The connections of steel structure could be changed from flexible to semi-rigid and then to rigid. They found that while the response of the rigid structure was almost elastic, the response of the semi-rigid structure had showed more inelastic hysteresis response. A comparison between the response of the three structures (flexible, semi-rigid, and rigid) subjected to a base excitation was made in their study. It was found that as the stiffness of the connection increased, the base shear resulting from the same ground motion increased, while the corresponding lateral drift did not increase in a similar manner. Experimental results in one loading case showed that maximum base shear that prevailed for the rigid structure was about 2.5 times larger than the maximum base shear that developed in the flexible structure, but the maximum lateral drift that took place in the flexible structure was only 30% more than the maximum lateral drift that took place in the rigid structure. Possibility of tuning the connection’s stiffness which can optimize the distribution of moment between connected elements and magnitude of base shear can result in the optimal design. Investigation from their study showed that flexible and semi-rigid structures had considerable potential for resisting earthquake.

The numerical study of semi-rigid frames under dynamic loading conditions have been carried out extensively. Sivakumaran (1988) presented a nonlinear dynamic analysis of steel buildings using a bilinear M-θ model with three parameters. A comparison was made to a 20 story steel building with rigid or flexible connections. The effects of the connection flexibility were discussed in his study.
Kawashima and Fujimoto (1984) investigated dynamic characteristics of steel frames with semi-rigid connections. The semi-rigid connections with energy dissipation were idealized by the rotary spring and dashpot in parallel. The mass, the damping, the stiffness matrices were derived in explicit form by means of dynamic stiffness matrix. Experiments on the vibration of a portal were conducted to verify the theoretical results. However, the instability effects were not considered and their analysis was only limited to the frequency determination of a structure in their studies. Chan (1994) extended the conventional stiffness matrix method of analysis for frame structures to dynamic and instability analysis of steel frames with semi-rigid connections under the action of dynamic loads. By incorporating the spring stiffness into the shape function, the linear stiffness, the mass and the geometric stiffness matrices were derived in his study. Shi and Atluri (1989) developed a nonlinear method for dynamic analysis of semi-rigid frames using the complementary energy principle. Lui and Lopes (1997) studied the dynamic response of simple portal semi-rigid frames using a computer model, in which the flexibility of semi-rigid connection was modeled by rotational springs with bilinear moment-rotation relationship. Connection flexibility, P-delta and column inelasticity effects were incorporated into the analyses in their studies. It was shown that connection flexibility could increase the natural period of vibration of the frames and the influence of the P-delta effect was more pronounced for semi-rigid frames than for rigid frames. The effects of damping ratio for a semi-rigid frame were also discussed. Chui and Chan (1997) investigated the influence of connection stiffness by examining the variation of vibration and deflection of a steel building connected by idealized joint types and those measured in the laboratory. They found that the rigidity of semi-rigid connections would influence the natural frequencies and the
dynamic responses of the frames. The nonlinear semi-rigid connections would also absorb vibration energy. For seismic design, semi-rigid frames are not utilized mainly due to their relative flexibility. However, recent work by several researchers such as Astaneh et al (1989) and Chan (1994) indicated that the rigid frame is not necessarily the optimum solution since flexible frames may attract lower inertial forces. The cost of constructing rigid joints is higher than that of semi-rigid joints because the rigid frame design may introduce additional fabricating cost when compared to semi-rigid construction. In general, the semi-rigid connections are preferred for low-rise multi-story frames. Kishi et al. (1996) investigated the mixed use of rigid and semi-rigid connections in a tall building. They concluded that the semi-rigid connections combining with rigid connections could be used economically for high rise building frame design.

The dynamic analyses of semi-rigid frames were usually carried out in the time domain in the above investigations. The analyses of dynamic characteristics of semi-rigid frames were very limited. The methods they used to derive the mass matrix, stiffness matrix and damping matrix for the beam element with rotational spring and damper at its ends were cumbersome and indirect. In Chapter 5, a new method is developed in this dissertation to derive the mass matrix, stiffness matrix and damping matrix for the beam element with rotational spring and damper at its ends. The possibility of adding damping materials in the connections to reduce seismic response is also proposed and investigated in Chapter 5.
2.4 VIBRATION CONTROL OF ADJACENT BUILDINGS UNDER EARTHQUAKE EXCITATION

Buildings in a modern city are often built closely to each other due to limited land available. These buildings, in most cases, are separated without any structural connections or are connected only at the ground level. Hence, earthquake-resistant capacity of each building mainly depends on itself. If the separation distances between adjacent buildings are not sufficient, mutual pounding may occur during an earthquake. In the 1985 Mexico City earthquake, pounding was present in over 40% of 330 collapsed or severely damaged buildings surveyed, and in 15% of all cases it led to collapse (Rosenblueth and Meli 1986; Bertero 1987). In the 1989 Loma Prieta earthquake, extensive building damage and collapse due to structural pounding were observed (Kasai and Maison 1997). Pounding damage was also observed in the 1964 Alaska earthquake, the 1967 Venezuela earthquake (Hanson and Degenkolb 1969), the 1971 San Fernando earthquake (Bertero and Collins 1973), the 1972 Managua earthquake (EERI 1973), the 1977 Romania earthquake (Tezcan et al. 1978), the 1977 Thessaloniki earthquake (EERI 1978), and the 1981 Central Greece earthquakes (EERI 1982).

To improve the wind and earthquake-resistant capacity of these buildings, the concept of using control devices to link adjacent buildings has been presented. The concept of coupling two tall buildings in the US to reduce the wind response was first proposed by Klein in 1972. Since then, Klein and co-workers have studied the use of dissipative links such as viscous elements, as well as semi-active devices such as cables to control the response of adjacent buildings to wind excitations. (Klein and Healy 1987; Gurley et al. 1994). The possibility of controlling the seismic response
of two or more structures and preventing mutual pounding of two or more structures has been investigated by a number of researchers. Westermo (1989) suggested using hinged links to connect two neighboring floors if the floors of adjacent buildings are in alignment. It is obvious that this system can reduce the chance for pounding, but it alters the dynamic characteristics of the unconnected buildings, enhances undesirable torsional response if the buildings have asymmetric geometry, and increases the base shear of the stiffer building. Westermo examined four specific cases: two buildings with roughly equivalent characteristics (case A), two buildings of about the same height with different frequencies (case B), a short, stiff structure connected to a tall, flexible one (case C) and a short, flexible structure connected to a tall stiff one (case D). Linkage was installed at the top of the shorter of the two structures in all four cases because lowest mode was dominant. They found that for situations where a stiff structure is linked to a flexible one, the deflections of the stiff building are small and its coupled effect is primarily to add stiffness to the more flexible building. They observed that although the coupling reduces the relative deflection difference, it does increase the absolute deflection of the stiffer of the two structures.

Kobori et al. (1988) developed bell-shaped hollow connectors to link very closely spaced adjacent buildings in a complex. The bell-shaped hollow connector is made of steel with stabilized hysteretic characteristic when the connector yields so that it can absorb vibration energy from earthquake. However, the high stiffness of the connector may significantly change the dynamic characteristics of the original buildings. The yield strength of the connector is also difficult to decide because if the yield strength is too high, the connector may not function properly but if the yield strength is too low, the energy absorbing capacity may be too small. Anagnostopoulos (1988) analyzed the effect of pounding for buildings under strong
ground motions by simplified single-degree of freedom (SDOF) model. Viscoelastic bumpers were placed at the points of probable contact to help absorb the blows of pounding. However, bumpers can still transfer some degree of impulsive loading to the structures, which was probably not anticipated in the dynamic design. Sugino et al. (1998) proposed a design strategy of passive devices consisted of a spring and damper for controlling vibration of flexible structures arranged in parallel. They modeled each distributed parameter system as a reduced order model of two-DOF and applied a Genetic Algorithm (GA) approach to the design of passive devices. The first and second vibration modes are controlled in their study.

Kageyama et al. (1994) presented an optimal frequency adjustment method and applied this method to a single building with a double frame structure. By connecting two adjacent structures with dampers at the roof, the response of both structures can be reduced simultaneously using an optimal connection damper. Two structures required to have different vibration characteristics. When the natural frequencies between both structures are optimized, the effect of vibration control is better. Otherwise, it is necessary to adjust natural frequencies of the two connected structures to an optimal level. They derived semi-optimal values for vibration control in connection buildings and presented the methods to find the optimal values from the semi-optimal values. Qi and Chang (1995) tried to apply viscous dampers to connect adjacent blocks in a large retail and entertainment facility. Because the force developed in a viscous damper is proportional to the velocity of its deformation and the relative movement between adjacent blocks caused by concrete shrinkage and temperature variation has a nearly zero velocity, shrinkage deformation and thermal expansion will not induce significant stresses to the structures due to the installation of viscous dampers. In contrast, earthquake-induced movement tends to have a high
relative velocity between adjacent blocks so that the dampers can function effectively. Using a commercial computer program, they carried out a time-history analysis of two plane frames connected by viscous dampers under the 1940 El Centro earthquake and the 1952 Taft earthquake, but the information available in their paper is very limited.

Luco and Wong (1994) modeled adjacent structures as a discrete multi-degree-of-freedom damped shear beam and obtained the optimal control forces interconnecting two structures by application of the instantaneous optimal control approach of Yang et al. (1987). They found that the damping constants would be uniform and the control forces would be proportional to the relative velocity between the attachment points in both structures if the structures were uniform. Thus, the optimal control forces could be implemented in the form of passive viscous dampers. Luco and Wong (1998) determined the optimal values for the distribution of passive dampers interconnecting two adjacent structures of different heights. The optimal damping values minimize the peak amplitudes of the transfer function for the response at the top of the taller structure in the vicinity of the first and second modes of the structure.

Xu et al. (1999) recently investigated earthquake resistant performance of adjacent buildings connected by fluid dampers. The Voigt model (Sun and Lu 1995) was employed to represent the fluid dampers. Since adjacent buildings connected by discrete fluid dampers form a non-classically damped system, a pseudo-excitation method was used in their study to determine random seismic response of the building-damper systems. The pseudo-excitation method they used, however, could not predict dynamic characteristics of the adjacent building-damper system so that the practical response spectrum method stipulated in most of seismic design codes
could not be applied to this case. In addition, the pseudo-excitation method in conjunction with modal reduction technique can provide only an approximate solution to non-classically damped systems.

Seto et al. (1991) and Mitsuta et al. (1991) explored to use an active control device to control the response of flexible structures by modeling each one of flexible structures in parallel as an undamped SDOF system. Seto and Matsumoto (1996) suggested using active actuators to connect a group of buildings to mitigate building motion and investigated how to prevent spillover problems. Yamada et al. (1994) let active actuators to generate negative stiffness so as to shift the natural frequencies of adjacent buildings away from the dominant frequency of ground motion to reduce seismic responses. They found that if the additional stiffness of joining member is positive, the period of all orders becomes shorter, but if it is negative, the period of all orders becomes longer. Both one-directional vibration and bi-directional vibration are discussed in their studies. They concluded that the control by negative stiffness reduces the response effectively.

Summarizing all the above investigations performed for the vibration control of adjacent buildings, it is noticed that they are mainly concerned with the response of structures. The dynamic characteristics of adjacent buildings were rarely studied. The analysis of dynamic characteristics is important because the retention of the natural frequencies of the unlinked buildings after the installation of the control devices is especially desirable for the adjacent buildings that have been already built and need to be strengthened. In addition, the analysis of adjacent buildings linked by passive or active control devices under earthquake excitation is quite time-consuming. In Chapter 6 and Chapter 8, the complex modal analysis is used to determine dynamic characteristics of passively and actively controlled adjacent
buildings. The generalized pseudo-excitation method is used to efficiently analyze earthquake response and hazard mitigation of adjacent buildings linked by passive or active control devices.

2.5 CONTROL ALGORITHMS

Environmental loads of practical concern for building structures, such as earthquakes and wind gusts, are random processes in nature. Building Structures are massive and heavy, and hence need large control forces and control devices. A variety of control algorithms based on different control design criteria have been applied to buildings and structures. Some are classical and some are specifically proposed for civil engineering structural control. Consider a building structure idealized by an N-degree-of-freedom system subjected to environmental loads, the equations of motion of the system can be written as

\[ M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = Ef(t) + Hu(t) \]  \hspace{1cm} (2-25)

in which \( f(t) \) is an m-dimensional external excitation vector and \( u(t) \) is a r-dimensional control force vector. \( H \) is an \( N \times r \) matrix denoting the location of \( r \) control forces, and \( E \) is an \( N \times m \) matrix defining the location of \( m \) excitations.

Equation (2-25) can be also reformulated into a first-order 2N-dimensional equation.

\[ \dot{q}(t) = Aq(t) + Gf(t) + Bu(t) \]  \hspace{1cm} (2-26)

where

\[ A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}H \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ M^{-1}E \end{bmatrix}, \quad \text{and} \quad q(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \]  \hspace{1cm} (2-27)

in which 0 and I denote the null matrix and the identity matrix of appropriate dimensions respectively.
2.5.1 Linear Quadratic Regulator (LQR) Algorithms

Linear quadratic regulator (LQR) algorithms, which are based on minimization of a quadratic performance index \( J \) including both structural response and control forces over the time interval of interest, is as follows:

\[
J = \frac{1}{2} \int_0^T [q(t)^T Q q(t) + u(t)^T R u(t)] dt \tag{2-28}
\]

in which \( Q \) is a \( 2N \times 2N \) positive semi-definite matrix, and \( R \) is a \( r \times r \) positive definite matrix, superscript \( T \) represents matrix transposition, and \( t_i \) is a duration defined to be longer than that of the external excitation. The matrices \( Q \) and \( R \) are referred to as weighting matrices whose magnitudes are assigned according to the relative importance attached to the state variables and to the control forces in the minimization procedure. If the elements of \( Q \) matrix are large, the response reduction of state vector \( q(t) \) is given priority over the control forces \( u(t) \). On the other hand, if the elements of \( R \) matrix are large, the response reduction of control force vector \( u(t) \) is given priority over the state vector \( q(t) \). Thus appropriate choice of the weighting matrices \( Q \) and \( R \) allows the designer to examine tradeoffs between control effectiveness and control energy consumption.

In the classical optimal closed loop control, it can be shown that the required control vector \( u(t) \) is given by

\[
u(t) = -R^{-1} B^T X_2(t) q(t) \tag{2-29}
\]

in which \( X_2(t) \) is the time dependent Riccati matrix. The matrix \( X_2(t) \) is symmetric and positive definite obtained from the Riccati matrix equation of the form:

\[\dot{X}_2(t) + X_2(t) A - X_2(t) B R^{-1} B^T X_2(t) + A^T X_2(t) + Q = 0, \quad X_2(t_f) = 0 \tag{2-30}\]
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The building structure considered here is a linear time invariant system and it has been found that $X_2(t)$ remains constant over a large portion of the time interval $[0,t_f]$, dropping rapidly to zero near $t_f$ (Yang et al. 1987). Hence, $X_2(t)$ can be approximated by a constant matrix and obtained by solving the approximated time invariant Riccati matrix equation

$$X_2A + A^TX_2 - X_2BR^{-1}B^TX_2 + Q = 0 \quad (2-31)$$

The external excitation term is ignored in the derivation of the Riccati matrix $X_2(t)$, so the LQR controller given by equation (2-29) is not truly optimal except that the external excitation is either zero or a white noise random process.

2.5.2 Instantaneous Optimal Control Algorithms

Instantaneous optimal control, where the performance index is now time-dependent and is minimized at every time instant $t$ for all $0 \leq t \leq t_f$. The time-dependent performance index $J(t)$ is defined by (Yang et al., 1987)

$$J(t) = q(t)^TQq(t) + u(t)^TRu(t) \quad (2-32)$$

Instantaneous optimal control algorithms are based on the fact that at any particular time $t$, the external excitation is available up to that time instant $t$. Instantaneous optimal control algorithms can be derived using open-loop, closed-loop, or open-closed loop control. Let $\Lambda$ be a $2N \times 2N$ diagonal matrix consisting of complex eigenvalues $s_j$ ($j=1, 2, \ldots, 2N$) of matrix $A$, and $\Phi$ is the $2N \times 2N$ modal matrix of corresponding eigenvectors. Then, it can be obtained from equation (2-26) as:

$$q(t) = \Phi D(t - \Delta t) + \frac{\Delta t}{2} \left[ Bu(t) + Gf(t) \right] \quad (2-33)$$

Where
\[ D(t - \Delta t) = e^{-\lambda t} \Phi^{-1} \left\{ q(t - \Delta t) + \frac{\Delta t}{2} \left[ B u(t - \Delta t) + G f(t - \Delta t) \right] \right\} \quad (2-34) \]

is a vector containing all elements evaluated at \( t-\Delta t \).

The minimization of \( J(t) \) in equation (2-32) subject to constraint (2-33) yields the following equations.

**Instantaneous optimal-open loop control:**

\[ u(t) = -\frac{\Delta t}{2} \left( \frac{\Delta t^2}{4} B^T Q B + R \right)^{-1} B^T Q \left[ \Phi D(t - \Delta t) + \frac{\Delta t}{2} G f(t) \right] \quad (2-35) \]

\[ q(t) = \left[ I - \frac{\Delta t^2}{4} B \left( \frac{\Delta t^2}{4} B^T Q B + R \right)^{-1} B^T Q \right] \left[ \Phi D(t - \Delta t) + \frac{\Delta t}{2} G f(t) \right] \quad (2-36) \]

**Instantaneous optimal closed-loop control:**

\[ u(t) = -\frac{\Delta t}{2} R^{-1} B^T Q q(t) \quad (2-37) \]

\[ q(t) = \left( I + \frac{\Delta t^2}{4} B R^{-1} B^T \right)^{-1} \left[ \Phi D(t - \Delta t) + \frac{\Delta t}{2} G f(t) \right] \quad (2-38) \]

**Instantaneous optimal closed-open-loop control:**

\[ u(t) = \frac{\Delta t}{4} R^{-1} B^T \left[ \tilde{\lambda} q(t) + \tilde{q}(t) \right] \quad (2-39) \]

\[ q(t) = \left( I - \frac{\Delta t^2}{8} B R^{-1} B^T \tilde{\lambda} \right)^{-1} \left[ \Phi D(t - \Delta t) + \frac{\Delta t^2}{8} B R^{-1} B^T \tilde{q} + \frac{\Delta t}{2} G f(t) \right] \quad (2-40) \]

in which

\[ \tilde{\lambda} = -\left( I + \frac{\Delta t^2}{8} Q B R^{-1} B^T \right)^{-1} Q \quad (2-41) \]

\[ \tilde{q}(t) = \tilde{\lambda} \left[ \Phi D(t - \Delta t) + \frac{\Delta t}{2} G f(t) \right] \quad (2-42) \]
Yang et al. (1992) also extended the instantaneous optimal control algorithms to use velocity and acceleration feedbacks because acceleration response can be more easily measured than the displacement response.

2.5.3 Linear Quadratic Gaussian (LQG) Algorithms

Environmental loads such as earthquakes and strong winds are usually modeled as stochastic process represented in the frequency domain by their power spectral density functions. When excitations are assumed random such as white or colored noise, the linear quadratic Gaussian (LQG) algorithms in the time domain and its frequency analog, H₂ algorithms are favourable. In the LQG algorithms, the statistical properties of the random excitations and measurement noises are used to design an estimator for the states of the system in the form of Kalman-Bucy filter (KBF). The advantage of LQG algorithms over the LQR algorithms, beyond improved performance, is that it does not need full state feedbacks, which requires that all states of the structures, including displacement and velocity of each degree of freedom, are available (Meirovitch 1990). In fact an LQG controller consists of an LQR compensator and a KBF state estimator. The design of the compensator can be fully separated with the design of estimator. By incorporating the feedforward link into the control algorithm, direct use of measurements of the base accelerations can be possible (Suhardjo et al. 1990). Other control algorithms include optimal polynomial algorithms (Yang et al. 1996a; Agrawal and Yang 1996, 1997; Agrawal et al. 1998), sliding mode control (Yang et al. 1995, 1996b, 1997; Wu et al. 1998), fuzzy control (Subramaniam 1996; Faravelli and Yao 1996; Casciati 1996) and neural control (Ghaboussi and Joghataie 1995).
In this dissertation, closed-form solutions are developed for LQG control based on acceleration feedback under either earthquake excitation or strong wind excitation. Both wind-excited single building and earthquake-excited adjacent buildings with control devices are investigated in details.

2.5.4 Fuzzy Control Algorithms

Traditional control systems are in general based on mathematical models that describe the control system using one or more differential equations that define the system response to its inputs. However, in many cases, the mathematical model of the control process may not exist or may be too "expensive" in terms of computer processing power and memory. Fuzzy control is a practical alternative for a variety of challenging control applications since it provides a convenient method for constructing nonlinear controllers via the use of heuristic information which may come from empirical rules. In fuzzy control, we focus on gaining an intuitive understanding of how to best control the process. then we load this information directly into the fuzzy controller.

Fuzzy controller is very simple conceptually. It has four main components: (1) The "rule-base" holds the knowledge, in the form of a set of rules, of how best to control the system. (2) The inference mechanism evaluates which control rules are relevant at the current time and then decides what the input to the plant should be. (3) The fuzzification interface simply modifies the inputs so that they can be interpreted and compared to the rules in the rule-base. And (4) the defuzzification interface converts the conclusions reached by the inference mechanism into inputs to the plant. (Passino and Stephen 1998).
To design the fuzzy controller, the control engineer must gather information on how the artificial decision maker should act in the closed-loop system. Sometimes this information can come from a human decision maker who performs the control tasks while at other times the control engineer can come to understand the plant dynamics and write down a set of rules about how to control the system without outside help. The rule-base is constructed so that it represents a human expert "in-the-loop". Generally speaking, if we load very detailed expertise into the rule-base, we enhance our chances of obtaining better performance.

Fuzzy controllers have been used extensively for control purposes. Sun & Goto (1994) use fuzzy logic to infer the optimal damping of a viscous oil damper with variable damping to control bridge vibrations. Nagarajaiah (1994) use fuzzy logic to control semiactive variable dampers, based on relative base velocity and displacement feedback. Experimental results from the use of fuzzy control on shaking table experiments are presented in Iiba et al. (1994).

2.5.5 Sliding Mode Control Algorithms

The theory of sliding mode control (SMC) was developed for robust control of uncertain non-linear systems. Its main idea is to design a controller to drive the response trajectory into the sliding surface, in which the motion of the system is stable. The error between actual and desired response is zero when the state falls on the sliding surface. Initially controls are applied such that an arbitrary initial state will be brought onto the sliding surface. Once on the sliding surface, the system is said to be in the sliding mode, and controls are applied to keep the system in the sliding mode, toward the equilibrium point. The control laws are chosen to keep the state in sliding mode and to make the closed loop system robust with respect to
parametric uncertainties. For the class of systems to which it applies, sliding controller design provides a systematic approach to the problem of maintaining stability and consistent performance in the face of modeling imprecisions.

The design of SMC involves two basic steps-design of a sliding surface and the design of a non-linear switched feedback control law. An important question in sliding mode control is how to choose the sliding surface. Krishnan et al. (1994) use two different techniques to design the sliding surface: one based on pole placement and the other based on quadratic optimization.

Sliding mode control can be used for nonlinear and hysteretic structures (Yang et al. 1994). Sliding mode control is also applied to control the vibration of tall buildings with an active tuned mass damper installed at the top floor (Adhikari and Yamaguchi 1997). Shaking table experimental verification of the SMC methods to linear and sliding-isolated building models have been conducted (Yang et al. 1996b).
CHAPTER THREE

GENERALIZED PSEUDO-EXCITATION METHOD FOR
CONTROLLED STRUCTURES UNDER RANDOM
EXCITATIONS

3.1 INTRODUCTION

Wind and earthquake may be regarded as a kind of stationary random processes. The mode-superposition method is usually adopted in random vibration analysis if the structure involves too many degrees of freedom. However, it is known that numerical random vibration analyses of complex structures still remain difficult for most engineering applications because enormous computational effort is required. To reduce the computation cost, the cross-correlation items between the participant modes are generally neglected, i.e., the CQC method is replaced by the SRSS method (Clough and Penzien 1975; Newland 1975; Wilson et al. 1981). This is acceptable only for structures with small damping and sparsely spaced participant natural frequencies. For the structures which do not meet such conditions, Lin et al. (1993, 1994) proposed a pseudo-excitation method to deal with the random vibration analysis of complex structures. The method, however, cannot predict dynamic characteristics and response of the structures with control devices because the natural modes of vibration of the corresponding undamped system are used in the pseudo-excitation method.

This chapter presents a generalized pseudo-excitation method for random vibration analysis of buildings with supplemental discrete control devices. The
generalized pseudo-excitation method is actually a combination of the complex modal superposition method and the pseudo-excitation method. The proposed method can naturally retain all cross-correlation terms between closely spaced modes of vibration in a structural response and accurately handle the non-orthogonal structural damping due to the installation of discrete control devices. The method also provides a convenient way of determining internal force response of a building.

In the following, the non-proportionally damped system is first defined. The complex modal analysis is then introduced for determining dynamic characteristics of non-proportionally damped structures. The principle of the generalized pseudo-excitation method is finally presented before a brief introduction of the pseudo-excitation method.

3.2 NON-PROPORTIONALLY DAMPED SYSTEM

Damping is a measure of structural capacity to dissipate energy and is of particular interest to the designers of high-rise buildings, where it plays an important role to help meet serviceability limit states for human comfort consideration. The estimates of damping in a structural system have intrinsic variability as a result of the complexity of damping mechanisms. In practice, the assumption of proportional damping is often invoked to utilize modal superposition method which facilitates decoupling of the equations of motion with the aid of modal matrices associated with the undamped system. The associated generalized co-ordinates are referred to as 'normal co-ordinates’. A damping matrix \( C = \alpha M \), or to the stiffness matrix \( K, C = \beta K \), or is a linear combination thereof, \( C = \alpha M + \beta K \), leads to diagonalization with normal co-ordinates. The most general proportional damping matrix \( C \) is given by
\[ C = M \sum_{j} a_{j} (M^{-1}K)^{j} \]  
(3-1)

in which \( j \) may range from \(-\infty < j < \infty \) and the summation may include as many terms as desired. A criteria specified by Caughey and O'Kelly (1965) as

\[ CM^{-1}K = KM^{-1}C \]  
(3-2)

can be used to judge whether a system is a proportionally damped system or not.

When structural damping satisfies the form of equation (3-2), the natural modes of vibration of the system are real-valued and the system is said to be classically or proportionally damped and the mode superposition method for such a system is referred to as the classical modal superposition method. Otherwise, the natural modes of vibration of the corresponding undamped system can not be used to decouple the equations of motion and such a system is called the non-proportionally or non-classically damped system.

In the dynamic analysis of structures, it is common to assume that the structure is classically damped. However, it may not be the case for a structure with supplemental discrete control devices. Consider again the controlled system described by equation (2-25), i.e.

\[ M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = Ef(t) + Hu(t) \]  
(3-3)

Suppose that the open-closed loop configuration is used in which the control force \( u(t) \) is designed to be a linear function of the measured displacement vector \( x(t) \), the velocity vector \( \dot{x}(t) \) and excitation \( f(t) \). The control force vector takes the form

\[ u(t) = a_{1}x(t) + a_{2}\dot{x}(t) + a_{3}f(t) \]  
(3-4)

where \( a_{1}, a_{2}, \) and \( a_{3} \) are respective control gains which can be time-dependent.

Substitution of equation (3-4) into equation (3-3) yields
\[ M\ddot{x}(t) + (C - a_2H)x(t) + (K - a_1H)x(t) = (E + a_3H)f(t) \] (3-5)

It is observed from equation (3-5) that the dynamic characteristics of the structure, i.e., the mass, damping and stiffness, have been modified by the control force. It is obvious that the modified structure is not a proportionally damped system any longer.

### 3.3 EIGENVALUE PROBLEMS OF NON-PROPORTIONALLY DAMPED SYSTEM

Eigenvalue problems of non-proportionally damped system can be solved by complex modal analysis (Foss 1958; Igusa et.al. 1984; Veletsos and Ventura 1986). The free vibration equation of such system can be written as

\[ M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = 0 \] (3-6)

in which \( M, C \) and \( K \) are the \( N\times N \) mass, damping and stiffness matrices of the system, respectively. Equation (3-6) can be transformed into a set of \( 2N \) first order equations:

\[ \dot{q}(t) = Aq(t) \] (3-7)

\[ A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \text{ and } q(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \] (3-8)

Since the matrix \( A \) in equation (3-8) is not symmetric, one needs to solve the following two adjoining eigenvalue problems:

\[ A\phi = s\phi \quad \text{and} \quad A^t\psi = s\psi \] (3-9)

where \( s \) is an eigenvalue, and \( \phi \) and \( \psi \) are the corresponding right eigenvector and left eigenvector, respectively. Let \( \Phi \) and \( \Psi \) be the \( 2N\times2N \) right modal matrix and left modal matrix respectively, i.e.
\[ \Phi = [\hat{\phi}_1, \hat{\phi}_2, \cdots, \hat{\phi}_{2N}] \quad \text{and} \quad \Psi = [\hat{\psi}_1, \hat{\psi}_2, \cdots, \hat{\psi}_{2N}] \]  

(3-10)

then a relation exists between \( \Phi \) and \( \Psi \)

\[ \Psi = \Phi^{-r} \]  

(3-11)

Generally, the eigenvalues of system (3-7) occur in complex conjugate pairs with corresponding complex conjugate eigenvectors, though some may be pure real. The orthogonality conditions are included in the following relations, derived from equation (3-7).

\[ \psi_\lambda^T \hat{\phi}_k = \delta_{\lambda k} \]  

(3-12)

and

\[ \psi_\lambda^T A \hat{\phi}_k = s_\lambda \delta_{\lambda k} \]  

(3-13)

where \( \delta_{\lambda k} \) is the Kronecker delta.

### 3.4 PSEUDO-EXCITATION METHOD

#### 3.4.1 Pseudo-Excitation Method Applied to Earthquake Excitation

If a stationary random excitation \( p(t) \) with power spectral density function \( S_{pp}(\omega) \) is applied to a linear system, it is known that an arbitrary response \( x(t) \) is also stationary, and its spectral density can be expressed as

\[ S_{xx} = |H|^2 S_{pp} \]  

(3-14)

This relation is illustrated in Fig. 3.1(a), in which \( H \) is the transfer function, or frequency response function. The meaning of \( H \) is shown in Fig. 3.1(b), i.e. if a unit sinusoidal excitation \( e^{i\omega t} \) is acted on the system, then the response will be \( H e^{i\omega t} \).

Obviously, if this unit sinusoidal excitation is multiplied by a factor \( \sqrt{S_{pp}(\omega)} \), then the response will also have to be multiplied by the same factor, as shown in Fig.
3.1(c), and that leads to the following relations

\[ x^*x = |x|^2 = |H|^{*}S_{pp} = S_{xx} \]  \hspace{1cm} (3-15)

\[ p^*x = \sqrt{S_{pp}}e^{-int} \cdot \sqrt{S_{pp}}He^{int} = S_{pp}H = S_{px} \]  \hspace{1cm} (3-16)

\[ x^*p = \sqrt{S_{pp}}H^{*}e^{-int} \cdot \sqrt{S_{pp}}e^{int} = H^{*}S_{pp} = S_{xp} \]  \hspace{1cm} (3-17)

in which the superscript * represents complex conjugate. The right-hand sides of the above three expressions are the conventional formulas (Newland 1975; Clough and Penzien 1975).

\[ S_{pp} \xrightarrow{H(\omega)} S_{xx} = |H|^{*}S_{pp} \hspace{1cm} p = \sqrt{S_{pp}}e^{int} \xrightarrow{H(\omega)} x = \sqrt{S_{pp}}He^{int} \]  \hspace{1cm} (a)

\[ p = e^{int} \xrightarrow{H(\omega)} x = He^{int} \hspace{1cm} p = \sqrt{S_{pp}}e^{int} \xrightarrow{H(\omega)} x_1 = \sqrt{S_{pp}}H_1e^{int} \]  \hspace{1cm} (b)

\[ x^*x = H^{*}_1\sqrt{S_{pp}}e^{-int} \cdot \sqrt{S_{pp}}H_2e^{int} = H^{*}_1S_{pp}H_2 = S_{x_1x_1} \]  \hspace{1cm} (3-18)

\[ x^*_2x_1 = H^{*}_2S_{pp}H_1 = S_{x_2x_1} \]  \hspace{1cm} (3-19)

By virtue of equations (3-15)-(3-19), the following spectral density matrix formulas can be readily established

\[ S_{xx} = x^*x \]  \hspace{1cm} (3-20)

\[ S_{px} = p^*x \hspace{1cm} S_{xp} = x^*p \]  \hspace{1cm} (3-21)
Apply the pseudo-excitation method to equation (2-1), then the pseudo-ground acceleration, according to Fig 3.1(c), would be

$$\ddot{x}_s(t) = \sqrt{S_s(\omega)} \exp(i\omega t)$$  \hspace{1cm} (3-22)

Substituting equation (3-22) into the right-hand side of equation (2-1) leads to a harmonic motion equation, its solution $\mathbf{x}(t)$ can be easily solved for in terms of the mode superposition method (see section 2.1.1), and is

$$\mathbf{x}(t) = -\sum_{j=1}^{4} \gamma_j H_j(\omega) \sqrt{S_s(\omega)} \exp(i\omega t) \phi_j = \mathbf{x}(\omega) \exp(i\omega t)$$  \hspace{1cm} (3-23)

If equation (3-23) is substituted into equation (3-20), one obtains

$$S_{xx}(\omega) = \mathbf{x}^* \mathbf{x}^T = \sum_{j=1}^{4} \sum_{k=1}^{4} \gamma_j \gamma_k H_j(\omega) H_k(\omega) \phi_j \phi_k^T \sqrt{S_s(\omega)}$$  \hspace{1cm} (3-24)

It is seen that the pseudo-excitation method based on equations (3-20) and (3-23) is mathematically identical to the conventional CQC method (Clough and Penzien 1975; Lin et al. 1993), i.e. the right-hand side of expression (3-24), in which the cross-correlation items between all participant modes have been involved. In fact, it is impossible for the pseudo-excitation method to neglect the cross-correlation items in order to provide a simplified SRSS method (Clough and Penzien 1975; Wilson et al. 1981). In other words, the pseudo-excitation method always gives the exact CQC results. That is quite different from the conventional CQC method.

From the harmonic displacement response $\mathbf{x}$, any harmonic internal force responses of interest, denoted as the vector $\mathbf{Q}$, can be readily produced. The formulas for calculating various spectral density matrices of $\mathbf{x}$ and $\mathbf{Q}$ are as follows:

$$S_{xx} = \mathbf{x}^* \mathbf{x}^T \hspace{1cm} S_{QQ} = \mathbf{Q}^T \mathbf{Q}^T$$  \hspace{1cm} (3-25)

$$S_{xQ} = \mathbf{x}^T \mathbf{Q}^T \hspace{1cm} S_{Qx} = \mathbf{Q}^T \mathbf{x}^T$$  \hspace{1cm} (3-26)
In particular, if only an internal force $f$, a stress $\sigma$, and a strain $e$ are of interest, and have been calculated based on the pseudo-harmonic displacement response $x$, then their auto-spectral density would be

$$S_f = |f|^2, \quad S_\sigma = |\sigma|^2, \quad S_e = |e|^2$$  \hspace{1cm} (3-27)

Their cross-spectral density would be, for instance

$$S_{fe} = \sigma^* e, \quad S_{xf} = xf, \quad \cdots$$  \hspace{1cm} (3-28)

Obviously, the pseudo-excitation method needs not deal with the double-summation, $\sum\sum$, operations for either auto-spectral density or cross-spectral density matrices. Therefore, this pseudo-excitation method is much more efficient than the conventional method based on equations (2-10) and (2-11).

### 3.4.2 Pseudo-Excitation Method Applied to Wind Excitation

For general buffeting problems, $S_{pp}(\omega)$ is a real symmetric matrix and so it can be decomposed, using Cholesky's scheme (Wilkinson and Reinsch 1971), into

$$S_{pp}(\omega) = L(\omega)L^T(\omega) = \sum_{j=1}^r L_j(\omega)L_j^T(\omega)$$  \hspace{1cm} (3-29)

in which $L(\omega)$ is a lower triangular matrix, and $r$ is the rank of the $m\times m$ matrix $S_{pp}(\omega)$, $r \leq m$. For such cases, $r$ pseudo-excitations have the form:

$$p_j = L_j \exp(i\omega t), \quad j = 1, 2, \cdots, r$$  \hspace{1cm} (3-30)

By using equations (2-16), (2-18) and (2-13), one obtains

$$f''_i = \Phi^T R L_i \exp(i\omega t)$$  \hspace{1cm} (3-31)

$$z_i = Hf''_i$$  \hspace{1cm} (3-32)

$$x_i = \Phi z_i$$  \hspace{1cm} (3-33)
Therefore
\[ x_j = \Phi H \Phi^T R L_j \exp(i \omega t) \quad (3-34) \]

Obviously,
\[ \sum_{j=1}^{r} x_j^* x_j^T = \Phi H^T \Phi^T S_{\phi \phi} \Phi H \Phi^T \quad (3-35) \]
or
\[ \sum_{j=1}^{r} x_j^* x_j^T = \Phi H^T \Phi^T R S_{pp} R^T \Phi H \Phi^T \quad (3-36) \]

Comparing equation (3-36) with equation (2-22) gives
\[ S_{\phi \phi} = \sum_{j=1}^{r} x_j^* x_j^T \quad (3-37) \]

For proportionally damped structures, \( H \) is a diagonal matrix, and so equation (3-36) can be reduced to
\[ x_j = \left( \sum_{k=1}^{d} H_{k} \phi_k \phi_k^T \right) R L_j \exp(i \omega t) \quad (3-38) \]

Therefore
\[ \sum_{j=1}^{r} x_j^* x_j^T = \sum_{j=1}^{r} \left( \sum_{k=1}^{d} H_{k} \phi_k \phi_k^T \right) R L_j L_j^T R^T \left( \sum_{k=1}^{d} H_{k} \phi_k \phi_k^T \right) \]
\[ = \sum_{j=1}^{r} \sum_{k=1}^{d} H_{k} \phi_k \phi_k^T R S_{pp} R^T \phi_k \phi_k^T \quad (3-39) \]

It also leads to the right-hand side of equation (2-23).

It should be noted that the pseudo-excitation method does not use equation (2-23), i.e. the right-hand side of equation (3-39). Instead it uses equations (3-38) and (3-37). Doing so is not only very convenient, but also more efficient than using the conventional method based on equation (2-23).
3.5 GENERALIZED PSEUDO-EXCITATION METHOD

From equation (3-5), the equations of motion of the building with control devices can be expressed as

\[ M\ddot{x}(t) + C_i\dot{x}(t) + K_i x(t) = E_i f(t) \]  \hspace{1cm} (3-40)

in which \( C_i, K_i \) are the \( N \times N \) damping and stiffness matrices modified by the control devices. The system described by equation (3-40) is generally not a proportionally damped system and decoupling may be accomplished utilizing a state space method.

Equation (3-40) can be reformulated into a first-order 2N-dimensional equation.

\[ \dot{q}(t) = Aq(t) + Gf(t) \]  \hspace{1cm} (3-41)

where

\[ A = \begin{bmatrix} 0 & I \\ -M^{-1}K_i & -M^{-1}C_i \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ M^{-1}E_i \end{bmatrix} \quad \text{and} \quad q(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \]  \hspace{1cm} (3-42)

3.5.1 Solution for Dynamic Characteristics

The solution of the homogeneous form of equation (3-41) can then be taken as

\[ q = \phi e^{st} \]  \hspace{1cm} (3-43)

The associated complex eigenvalue problem of equation (3-43) becomes

\[ A\dot{\phi} = s\phi \]  \hspace{1cm} (3-44)

where \( s \) is the eigenvalue and \( \phi \) is the associated eigenvector. The solution of equation (3-44) comprises a set of 2N eigenvalues and eigenvectors that exist in complex conjugate pairs (underdamped mode), or real pairs (overdamped mode). For complex conjugate pairs,

\[ \dot{\phi}_i = \dot{\phi}^*_j, \quad \text{and} \quad s_i = s^*_j, \quad (j=1,2,\ldots,N) \]  \hspace{1cm} (3-45)
The eigenvalue is usually written under the form
\[ s_j = s_{j,N}^* = -\omega_j \xi_j + i\omega_{dj} \quad (j=1,2,\cdots,N) \]  
(3-46)
in which
\[ \omega_j = |s_j|, \quad \xi_j = -\text{Re}(s_j)/|s_j| \quad \text{and} \quad \omega_{dj} = \omega_j \sqrt{1 - \xi_j^2} \]  
(3-47)

For real pairs, it is convenient to express real pairs \( s_j \) in the following form analogous to equation (3-46).
\[ s_j = -\omega_j \xi_j + \omega_{dj} \]  
(3-48)
\[ s_{j,N} = -\omega_j \xi_j - \omega_{dj} \]  
(3-49)
\[ \omega_j, \omega_{dj}, \text{and} \xi_j \text{are determined by} \]
\[ \omega_j = \sqrt{s_j s_{j,N}}, \quad \xi_j = -(s_j + s_{j,N})/(2\omega_j) \quad \text{and} \quad \omega_{dj} = \omega_j \sqrt{\xi_j^2 - 1} = (s_j - s_{j,N})/2 \]  
(3-50)
where \( \omega_j, \omega_{dj}, \text{and} \xi_j \) are the modal frequency, the damped modal frequency, and the modal damping ratio, respectively, associated with mode \( j \). The superscript * means the conjugation and i is the imaginary unit.

### 3.5.2 Response to Arbitrarily Coherent Stationary Random Excitations

Assume that the \( f(t) \) is a stationary random process and its power spectral density matrix is given as \( S_0(\omega) \). \( S_0(\omega) \) is generally a Hermitian matrix (Courant and Hilbert 1953) and can be decomposed as
\[ S_{if}(\omega) = L(\omega)D(\omega)L^T(\omega) = \sum_{k=1}^{m} d_{ik}(\omega) L_k(\omega)L_k^T(\omega) \]  
(3-51)
in which \( L(\omega) \) is the lower triangular matrix; \( D(\omega) \) is the diagonal element; \( T \) means the matrix transposition; \( L_k(\omega) \) is the k-th column of \( L(\omega) \); and \( d_{ik}(\omega) \) is the k-th
diagonal element of $D(\omega)$.

The pseudo-excitation vectors can then be constituted as

$$f_k(t) = L_k \exp(i\omega t) \quad (k=1,2,\ldots,m)$$  \hspace{1cm} (3-52)

In this way, the random buffeting analysis can be transformed into harmonic loading analysis.

The complex mode superposition method adopts the following coordinate transformation to decouple equation (3-41)

$$q_k(t) = \Phi z_k(t)$$  \hspace{1cm} (3-53)

where $z_k(t)$ is the $2N$-dimensional generalized coordinate vector and $\Phi$ is the $2N \times 2N$ complex modal matrix.

$$\Phi = [\dot{\phi}_1, \dot{\phi}_2, \cdots, \dot{\phi}_{2N}]$$  \hspace{1cm} (3-54)

By using the coordinate transformation and the orthogonality of modes, equation (3-41) can be reduced to $2N$ decoupled modal equations with the jth modal equation being

$$\dot{z}_j(t) = s_j z_j(t) + r_{n_j} \exp(i\omega t)$$  \hspace{1cm} (3-55)

in which

$$r_{n_j} = \psi_j^T G L_k$$  \hspace{1cm} (3-56)

The solution of the first-order equation (3-55) is

$$z_j(t) = \frac{r_{n_j}}{i\omega - s_j} \exp(i\omega t) \quad (j=1,2,\ldots,2N)$$  \hspace{1cm} (3-57)

Substituting equation (3-57) into equation (3-53) and comparing with the last expression of equation (3-42), one obtains

$$x_k(t) = \sum_{j=1}^{2N} \phi_j z_j(t) = \sum_{j=1}^{2N} \phi_j \frac{r_{n_j}}{i\omega - s_j} \exp(i\omega t)$$  \hspace{1cm} (3-58)
Since the 2N eigenvectors are in pairs, equation (3-58) can be reduced to

$$x_k(t) = \sum_{j=1}^{N} H_j(\omega)(i\omega\alpha_{k,j} + \beta_{k,j}) \exp(i\omega t) = x_k(\omega)\exp(i\omega t) \quad (3-59)$$

in which \(x_k(\omega)\) is called the kth pseudo displacement

$$x_k(\omega) = \sum_{j=1}^{N} H_j(\omega)(i\omega\alpha_{k,j} + \beta_{k,j}) \quad (3-60)$$

When the jth mode is an underdamped mode,

$$\alpha_{k,j} = 2 \text{Re}(\phi_j r_{k,j}) \quad \text{and} \quad \beta_{k,j} = -2 \text{Re}(\phi_j r_{k,j}^* s_j^*) \quad (3-61)$$

When the jth mode is an overdamped mode or a critically damped case

$$\alpha_{k,j} = (\phi_j r_{k,j} + \phi_j^* r_{k,j}^*) \quad \text{and} \quad \beta_{k,j} = -2(\phi_j r_{k,j}^* s_j^* + \phi_j^* r_{k,j} s_j) \quad (3-62)$$

and \(H_j(\omega)\) is the frequency response function for the jth mode.

$$H_j(\omega) = \frac{1}{\omega_j^2 - \omega^2 + i2\xi_j\omega_j \omega} \quad (3-63)$$

In a similar way, the pseudo velocity response can be obtained by

$$\dot{x}_k(\omega) = \sum_{j=1}^{N} H_j(\omega)(i\omega\mu_{k,j} + \nu_{k,j}) \quad (3-64)$$

in which when the jth mode is an underdamped mode

$$\mu_{k,j} = 2 \text{Re}(s_j \phi_j r_{k,j}) \quad \text{and} \quad \nu_{k,j} = -2\omega_j^2 \text{Re}(\phi_j r_{k,j}) \quad (3-65)$$

When the jth mode is an overdamped mode or a critically damped case

$$\mu_{k,j} = (s_j \phi_j r_{k,j} + s_j^* \phi_j^* r_{k,j}^*) \quad \text{and} \quad \nu_{k,j} = -\omega_j^2 (\phi_j r_{k,j} + \phi_j^* r_{k,j}^*) \quad (3-66)$$

From the pseudo displacement \(x_k(\omega)\), any pseudo internal force \(Q_k(\omega)\) can be easily determined following a static analysis.

The response spectral matrix can then be obtained by

$$S_{\dot{x}x}(\omega) = \sum_{k=1}^{m} d_{kk}(\omega)x_k^*(\omega)x_k^f(\omega) \quad S_{xx}(\omega) = \sum_{k=1}^{m} d_{kk}(\omega)x_k^*(\omega)x_k^f(\omega) \quad (3-67)$$
\[ S_{ss}(\omega) = \sum_{k=1}^{m} d_{kk}(\omega) x_k^*(\omega) x_k^T(\omega), \quad S_{ss}(\omega) = \sum_{k=1}^{m} d_{kk}(\omega) x_k^*(\omega) x_k^T(\omega) \] (3-68)

\[ S_{qq}(\omega) = \sum_{k=1}^{m} d_{kk}(\omega) Q_k^*(\omega) Q_k^T(\omega) \] (3-69)

It should be pointed out that by using the generalized pseudo-excitation method, the cross-correlation terms between vibration modes can be retained. The generalized pseudo-excitation method needs not deal with the double-summation, \( \sum \sum \), operations, so it is much more efficient than the conventional method.

The standard deviation displacement response of the jth floor \( \sigma_{x_j} \) is finally evaluated from \( S_{x_j,x_j}(\omega) \) through integration.

\[ \sigma_{x_j}^2 = \int S_{x_j,x_j}(\omega) d\omega \] (3-70)

The standard deviation acceleration response of the jth floor \( \sigma_{\dot{x}_j} \) is given by

\[ \sigma_{\dot{x}_j}^2 = \int \omega^2 S_{x_j,x_j}(\omega) d\omega \] (3-71)

The standard deviation force response \( \sigma_{\ddot{u}_j} \) is evaluated from \( S_{u_j,u_j}(\omega) \) through integration

\[ \sigma_{\ddot{u}_j}^2 = \int S_{u_j,u_j}(\omega) d\omega \] (3-72)

3.6 SUMMARY

The non-proportionally damped structures have been defined and the pseudo-excitation method has been briefly introduced. The generalized pseudo-excitation method in conjunction with the complex modal analysis are then developed for investigating the dynamic characteristics and response of structures with discrete
control devices. The formulation of the method for multi-degree-of-freedom structures with discrete control devices has also been derived in this chapter. When a structure is not controlled and proportional damping is assumed, this method is equivalent to the pseudo-excitation method. When an excitation power spectral density matrix is fully coherent, the power spectral density matrix of structural response can be produced through the multiplication of response vectors which are easily obtained by means of a harmonic response analysis. When an excitation power spectral density matrix is partially coherent, it can be regarded as the sum of limited number of single-excitation power spectral density matrices. Then, superimposing the results for all the components gives the total response.

This method is mathematically accurate and computationally efficient. It is a complete CQC method because the cross-correlation terms between both the participant modes and excitations are included. It is also easy to implement on computers. The principle of the generalized pseudo-excitation method presented in this chapter will be applied to various cases of vibration control problems in the subsequent chapters.
CHAPTER FOUR

WIND-EXCITED TALL BUILDINGS WITH ACTIVE TENDON/LIGHT APPENDAGE

4.1 INTRODUCTION

As mentioned in Chapter 2 and Chapter 3, modern tall buildings may possess closely spaced modes of vibration and non-classical damping properties. However, the currently used random-vibration-based SRSS method (Square Root of Sum of Squares) for predicting dynamic response due to turbulent wind is based on the assumption that a tall building should have well-separated vibration modes and light structural damping (Davenport 1961). Wind loading codes of most countries consider only the first modal response of a wind-excited tall building (SAA 1989; Simiu and Scanlan 1996). To handle the dynamic analysis of wind-excited tall buildings with closely spaced modes of vibration and non-classical damping properties, an alternate formulation was presented by Yang and Lin (1981) and extended by Samali et al. (1985b) and Xu et al. (1992c). The formulation is based on the transfer matrix method, which dispenses with the intermediate step of computing the mode shapes and deals directly with the final structural response. However, the formulation is cumbersome and applicable mainly to regular and periodic structures.

In this chapter, the generalized pseudo-excitation method introduced in Chapter 3 is applied to wind-excited tall buildings with active tendon devices or light appendages. For a tall building with active tendon devices, its dynamic characteristics, such as damping, have been greatly modified by the active tendons

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and thus the building with control devices is no longer a proportionally damped system. For a tall building with a light appendage, it may have very closely spaced modes of vibration. Therefore, the investigation of such two problems in this chapter can examine the applicability of the generalized pseudo-excitation method and demonstrate its strength and advantages. The equations of motion of a wind-excited tall building with active tendons are first derived and expressed in the form of first order differential equations in this chapter. The complex mode analysis is then employed to determine the dynamic characteristics of the tall building with active tendons and the generalized pseudo-excitation method is then used to determine wind-induced response and examine the effectiveness of active tendons in reducing wind-induced vibration of the building. Finally, the pseudo-excitation method is applied to the tall building with a light appendage to examine the effects of the cross-correlation terms between modes of vibration through a comparison with the SRSS method.

4.2 EQUATIONS OF MOTION

For the sake of clear demonstration, the building model for the present study is an N-story linear elastic shear building as shown in Fig. 4.1(a). The mass of the building is concentrated at its floor and the stiffness is provided by its massless columns between neighboring floors. Active tendon devices are installed between neighboring floors as shown in Fig. 4.1(b). The tendon controller considered herein is of electrohydraulic servomechanism, as studied by Yang and Samali (1983). The selection of such a control system is mainly to demonstrate the advantages of the generalized pseudo-excitation method for a tall building under alongwind excitation that is hardly expressed as a function of white noise random process. For the jth
tendon controller, the sensors are placed on both the jth and j-1th floors to sense their motions:

\[ x_j'(t) = \frac{d^r x_j(t)}{dt^r}; \quad x_{j-1}'(t) = \frac{d^r x_{j-1}(t)}{dt^r} \quad (4-1) \]

in which \( x_j(t) \) is the displacement of jth floor; \( r=0, r=1 \), and \( r=2 \) correspond to the displacement, velocity, and acceleration sensors, respectively.

The sensed motions are transmitted as a feedback by the electric voltage \( V_j(t) \):

\[ V_j(t) = p_r (x_j' - x_{j-1}') \quad (4-2) \]

in which \( p_r \) is a proportional constant. The feedback voltage triggers the displacement, \( U_j(t) \), of a hydraulic ram of the servomechanism through the relation:

\[ \dot{U}_j(t) + R_t U_j(t) = \frac{R_1 V_j(t)}{R_0} \quad (4-3) \]

in which \( R_t \) and \( R_0^{-1} \) are known as the loop gain and the feedback gain, respectively.

Define the normalized loop gain \( \varepsilon \) and the feedback gain \( \tau \) as

\[ \varepsilon = \frac{R_1}{\omega_i}; \quad \tau = \frac{p_r \omega_i'}{R_0} \quad (4-4) \]

in which \( \omega_i \) is the fundamental frequency of the building without control devices.

Equation (4-3) can be rewritten as follows:

\[ \dot{U}_j(t) + \omega_i \varepsilon U_j(t) = \frac{\tau \varepsilon}{\omega_i'} (x_j' - x_{j-1}') \quad (4-5) \]

The control force is due to the elongation of the tendon, \( (x_j - x_{j-1}) \cos \theta \), and the movement of the hydraulic ram, \( U_j(t) \). Thus, if \( K_t \) denotes the tendon stiffness, the control force from the jth controller is

\[ q_j(t) = K_t [(x_j - x_{j-1}) \cos \theta + U_j(t)] \cos \theta \quad (4-6) \]
where $\theta$ is the angle between the tendon and the horizontal direction.

The equations of motion of the building with active tendon devices can be expressed as

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) + Hu(t) = Rp(t) = f(t) \quad (4-7)$$

with the auxiliary equation

$$\dot{u}(t) + Au(t) = Dx'(t) \quad (4-8)$$

where $M$, $K$ and $C$ are the mass, the stiffness and damping matrices of the system, respectively. $H$, $A$ and $D$ are the coefficient matrices of the active tendon devices; $R$ is an $N \times m$ matrix consisting of 0 and 1 which expands the $m$-dimensional vector $p(t)$ into the $N$-dimensional vector $f(t)$. $x(t)$ is the vector of displacement response; $u(t)$ is the displacement vector of the hydraulic rams of the servomechanism; $p(t)$ is an stationary random excitation vector with the cross power spectral density matrix $S_{pp}(\omega)$ being given.

Denote the mass, shear stiffness, and external damping coefficient and internal damping coefficient of the building as $m_j$, $k_j$, $b_j$, $c_j$, $(j=1,2,...,N)$, respectively. The details of each matrix can then be given as follows:

The mass matrix of the building is

$$M = \text{diag}[m_1, m_2, \cdots, m_N] \quad (4-9)$$

The damping matrix of the building is

$$C = C^e + C' \quad (4-10)$$

in which the external damping matrix is

$$C^e = \text{diag}[b_1, b_2, \cdots, b_N] \quad (4-11)$$

and the internal damping matrix is
\[
C' = \begin{bmatrix}
c_1 + c_2 & -c_2 \\
-c_2 & c_2 + c_3 & -c_3 \\
& & \ddots \\
& & & -c_{n-1} & c_{n-1} + c_n & -c_n \\
& & & & -c_n & c_n
\end{bmatrix}
\]

(4-12)

The stiffness matrix of the buildings is

\[
K = K' + K'
\]

(4-13)

in which

\[
K' = \begin{bmatrix}
k_1 + k_2 & -k_2 \\
-k_2 & k_2 + k_1 & -k_1 \\
& & \ddots \\
& & & -k_{n-1} & k_{n-1} + k_n & -k_n \\
& & & & -k_n & k_n
\end{bmatrix}
\]

(4-14)

The stiffness matrix attributed to tendons is

\[
K' = K_1 \cos^2 \theta
\]

(4-15)

in which \( \eta_j \) is the number of tendons in the jth story.

The coefficient matrices of the active tendon devices are

\[
H = K_1 \cos \theta I
\]

(4-16)

\[
\Lambda = \omega_c \varepsilon I
\]

(4-17)
4.3 ALONGWIND EXCITATION

The power spectral density of wind speed can be expressed as (Davenport 1961)

\[ S_v(n) = \frac{4K_a V_{10}^2}{n} \left( \frac{1200n}{V_{10}} \right)^2 \left[ 1 + \left( \frac{1200n}{V_{10}} \right)^2 \right]^{-\frac{4}{3}} \]  

(4-19)

in which \( n \) = frequency, in hertz (cycles per second); \( V_{10} \) is the reference mean wind velocity at 10 meters above the ground; \( K_a \) is the surface drag coefficient.

The spectral density function of the alongwind force on the \( i \)th floor is expressed as

\[ S_p(n) = 4 \left( \frac{W_i}{V_i} \right)^2 S_v(n) \]  

(4-20)

in which \( W_i \) is the wind force at the \( i \)th floor; and \( V_i \) is the wind velocity at the \( i \)th floor.

Equation (4-20) is a one-sided spectrum in the positive frequency domain.

When converted to a two-sided spectral density in \( \omega \), equation (4-20) becomes

\[ S_p(\omega) = \frac{8K_a V_{10}^2}{|\omega|} \left( \frac{W_i}{V_i} \right)^2 \left( \frac{600\omega}{\pi V_{10}} \right)^2 \left[ 1 + \left( \frac{600\omega}{\pi V_{10}} \right)^2 \right]^{-\frac{4}{3}} \]  

(4-21)

in which \( \omega \) is the frequency in radians per second. The relationship between \( W_i \) and \( V_i \) is described by
\[ W_i = \frac{1}{2} \rho A_i C_d V_i^2 \]  

(4-22)

in which \( \rho \) is the air density; \( A_i \) is the tributary area of the ith floor; and \( C_d \) is the drag coefficient. The mean velocity \( V_i \) varies with the height, which may be described by a power law

\[ V_i = V_g \left( \frac{z_i}{z_g} \right)^\alpha \]  

(4-23)

where \( z_i \) is the height of the ith floor; \( z_g \) is the gradient height; \( V_g \) is the mean velocity at the gradient height; and \( \alpha \) is a constant exponent. The \( ij \)-th element \( S_{p,p_i}(\omega) \) in the cross spectral density matrix \( S_{pp}(\omega) \), that is, the cross spectral density function between the wind force at the ith floor and the jth floor may be expressed as

\[ S_{p,p_i}(\omega) = \sqrt{S_p(\omega)S_p(\omega)} \rho_y(\omega) \]  

(4-24)

\[ \rho_y(\omega) = \exp \left( - \frac{C_1 |\omega| |H_i - H_j|}{2\pi V_g} \right) \]  

(4-25)

in which \( C_1 \) is a constant; and \( H_i \) and \( H_j \) are the heights of the ith and jth floor above the ground respectively.

The alongwind excitation spectral matrix is a symmetric matrix that can be decomposed as

\[ S_{pp}(\omega) = L(\omega)D(\omega)L^T(\omega) = \sum_{k=1}^{m} d_{kk}(\omega)L_k L_k^T(\omega) \]  

(4-26)

in which \( L \) is the lower triangular matrix; \( D \) is the diagonal element; \( T \) means the matrix transposition; \( L_k \) is the \( k \)-th column of \( L \); and \( d_{kk} \) is the \( k \)-th diagonal element of \( D \).
According to the principle of the pseudo-excitation method, the pseudo-excitation vectors can then be constituted as

\[
\mathbf{f}_k(t) = R \mathbf{L}_k(\omega) \exp(i\omega t) \quad (k=1,2,...,m)
\] (4-27)

In this way, the random buffeting analysis can be transformed into harmonic loading analysis.

### 4.4 DYNAMIC CHARACTERISTICS

Equations (4-7) and (4-8) in conjunction with equation (4-27) can be replaced by the first order differential equations of the form

\[
\mathbf{A} \dot{\mathbf{q}}_k(t) + \mathbf{B} \mathbf{q}_k(t) = \mathbf{F}_k(t) \quad (k=1,2,...,m)
\] (4-28)

in which

\[
\begin{bmatrix}
\mathbf{q}_k(t) \\
\mathbf{F}_k(t) \\
\mathbf{F}_k(t)
\end{bmatrix} =
\begin{bmatrix}
\mathbf{RL}_k(\omega) \\
0 \\
0
\end{bmatrix} \exp(i\omega t) \\
\begin{bmatrix}
\dot{\mathbf{x}}_k(t) \\
\mathbf{x}_k(t) \\
\mathbf{u}_k(t)
\end{bmatrix}
\] (4-29)

For displacement sensors.

\[
\mathbf{A} = \begin{bmatrix} M & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} C & K & H \\ -I & 0 & 0 \\ 0 & -D & \Lambda \end{bmatrix}
\] (4-30)

For velocity sensors.

\[
\mathbf{A} = \begin{bmatrix} M & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} C & K & H \\ -I & 0 & 0 \\ -D & 0 & \Lambda \end{bmatrix}
\] (4-31)

For acceleration sensors.

\[
\mathbf{A} = \begin{bmatrix} M & 0 & 0 \\ 0 & I & 0 \\ -D & 0 & I \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} C & K & H \\ -I & 0 & 0 \\ 0 & 0 & \Lambda \end{bmatrix}
\] (4-32)

in which the size of matrices \(\mathbf{A}\) and \(\mathbf{B}\) are \(3N\times3N\).
To determine the characteristics of the system, one may consider the homogeneous form of equation (4-28). Because matrices $A$ and $B$ in equations (4-30) to (4-32) are not symmetric, one needs to solve the following two adjoining eigenvalue problems:

$$ (sA + B)\hat{\phi} = 0 \quad \text{and} \quad (sA^T + B^T)\hat{\psi} = 0 $$

(4-33)

where $s$ is an eigenvalue, and $\hat{\phi}$ and $\hat{\psi}$ are the corresponding right eigenvector and left eigenvector, respectively, of the form

$$ \hat{\phi} = \begin{bmatrix} s\phi \\ \phi \\ \theta \end{bmatrix}, \quad \hat{\psi} = \begin{bmatrix} s\psi \\ \psi \\ \theta \end{bmatrix} $$

(4-34)

The solution of the eigenvalue problem comprises a set of $3N$ eigenvalues and eigenvectors that are either real or exist in complex conjugate pairs. Among these $3N$ eigenvalues and eigenvectors, there are $N$ real eigenvalues and eigenvectors attributed to active tendon devices. The $N$ real eigenvalues related to the active tendon devices are denoted by $s_{1,N}$, $(j=1,2,\ldots,N)$. The remaining $2N$ eigenvalues and eigenvectors will occur in complex conjugate pairs for an underdamped case, or in real pairs for an overdamped case. For the underdamped case,

$$ \hat{\phi}_j = \hat{\phi}_{1,N} \quad \text{and} \quad s_j = s_{1,N} \quad (j=1,2,\ldots,N) $$

(4-35)

Each eigenvalue can be expressed in the form of

$$ s_j = s_{1,N} = -\omega_j \xi_j + i\omega_h \quad (j=1,2,\ldots,N) $$

(4-36)

in which

$$ \omega_j = |s_j|, \quad \xi_j = -\Re(s_j)/|s_j| \quad \text{and} \quad \omega_h = \omega_j \sqrt{1 - \xi_j^2} $$

(4-37)

For the overdamped case, it is convenient to express real pairs $s_j$ in the following form analogous to equation (4-36).
\[ s_j = -\omega_j \xi_j + \omega_d, \quad s_{j,\chi} = -\omega_j \xi_j - \omega_d \quad (4-38) \]

in which \( \omega_j, \omega_d \) and \( \xi_j \) are determined by

\[ \omega_j = \sqrt{s_j s_{j,\chi}}, \quad \xi_j = -\frac{1}{(2\omega_j)}(s_j + s_{j,\chi}) \text{ and } \omega_d = \omega_j \sqrt{\xi_j^2} - 1 = \frac{(s_j - s_{j,\chi})}{2} \quad (4-39) \]

in which \( \omega_j, \omega_d, \) and \( \xi_j \) are the modal frequency, the damped modal frequency, and the modal damping ratio, respectively, associated with mode \( j \). The superscript * means the conjugation.

## 4.5 BUFFETING RESPONSE

To determine buffeting response of the system under alongwind excitation, the following co-ordinate transformation is adopted to decouple equation (4-28).

\[ \mathbf{q}_t(t) = \mathbf{\Phi} \mathbf{z}_t(t) \quad (4-40) \]

where \( \mathbf{z}_t(t) \) is the 3N-dimensional generalized coordinate vector and \( \mathbf{\Phi} \) is the 3Nx3N right modal matrix.

\[ \mathbf{\Phi} = [\hat{\phi}_1, \hat{\phi}_2, \ldots, \hat{\phi}_{3N}] \quad (4-41) \]

By using this transformation, equation (4-28) can be reduced to 3N decoupled modal equations with the \( j \)th modal equation being

\[ \dot{z}_{i,j}(t) - s_j z_{i,j}(t) = r_{i,j} \exp(i\omega t) \quad (4-42) \]

in which

\[ r_{i,j} = \Psi_i^T RL_k / A_i \quad (4-43) \]

\[ A_i = \Psi_i^T A \Psi_i \quad (4-44) \]

The solution of the first-order equation (4-42) is

\[ z_{i,j}(t) = \frac{r_{i,j}}{i\omega - s_j} \exp(i\omega t) \quad (j=1, 2, \ldots, 3N) \quad (4-45) \]
Substituting equation (4-45) into equation (4-40) and comparing with the last expression of equation (4-29), one obtains

\[
x_k(t) = \sum_{j=1}^{3N} \phi_j z_k(t) = \sum_{j=1}^{3N} \phi_j \frac{r_{ij}}{i\omega - s_j} \exp(i\omega t)
\]  

(4-46)

Since the 2N eigenvectors are in pairs, equation (4-46) can be reduced to

\[
x_k(t) = \left( \sum_{j=1}^{N} H_j(\omega) \left(i\omega \alpha_{s_j} + \beta_{s_j} \right) + \sum_{j=2N+1}^{3N} \phi_j \frac{r_{ij}}{i\omega - s_j} \right) \exp(i\omega t) = x_k(\omega) \exp(i\omega t)
\]  

(4-47)

in which \(x_k(\omega)\) is called the kth pseudo displacement

\[
x_k(\omega) = \sum_{j=1}^{N} H_j(\omega) \left(i\omega \alpha_{s_j} + \beta_{s_j} \right) + \sum_{j=2N+1}^{3N} \phi_j \frac{r_{ij}}{i\omega - s_j}
\]  

(4-48)

where

\[
\alpha_{s_j} = 2 \Re(\phi_j r_{ij}) \quad \text{and} \quad \beta_{s_j} = -2 \Re(\phi_j s_j^* r_{ij})
\]  

(4-49)

and \(H_j(\omega)\) is the frequency response function for the jth mode.

\[
H_j(\omega) = \frac{1}{\omega_j^* - \omega^2 + i2\xi_j \omega_j \omega}
\]  

(4-50)

In a similar way, the kth pseudo velocity response can be obtained by

\[
\dot{x}_k(\omega) = \sum_{j=1}^{N} H_j(\omega) \left(i\omega \mu_{s_j} + \nu_{s_j} \right) + \sum_{j=2N+1}^{3N} s_j \phi_j \frac{r_{ij}}{i\omega - s_j}
\]  

(4-51)

in which

\[
\mu_{s_j} = 2 \Re(s_j \phi_j r_{ij}) \quad \text{and} \quad \nu_{s_j} = -2\omega_j \Re(\phi_j r_{ij})
\]  

(4-52)

The second terms in equations (4-48) and (4-51) are attributed to the N real eigenvalues and eigenvectors of control devices and do not make significant contributions to the total responses of the buildings, and thus they can be neglected. Furthermore, the consideration of only the first q (q<<N) modes may be sufficient in
practice when calculating response. As a result, equation (4-48) can be accordingly simplified as

$$x_k(\omega) = \sum_{i=1}^{n} H_i(\omega)(i\omega \alpha_{x_i} + \beta_{x_i})$$ (4-53)

Once the pseudo displacement is determined, the pseudo internal force can be easily determined following a static analysis. For instance, the kth pseudo shear force of the buildings can be calculated by

$$Q_k(\omega) = Gx_k(\omega)$$ (4-54)

where

$$G = \begin{bmatrix}
    k_1 & & & \\
    -k_2 & k_2 & & \\
    & \ddots & \ddots & \\
    & & -k_{n-1} & k_{n-1} \\
    & & & -k_n & k_n
\end{bmatrix}$$ (4-55)

It can be readily proved that the displacement spectral density matrix $S_{x_x}(\omega)$ can be obtained by

$$S_{x_x}(\omega) = \sum_{k=1}^{m} d_{kk}(\omega) x^r_k(\omega)x^r_k(\omega). \quad S_{x_x}(\omega) = \sum_{k=1}^{m} d_{kk}(\omega) x^r_k(\omega)\dot{x}^r_k(\omega)$$ (4-56)

$$S_{x_x}(\omega) = \sum_{k=1}^{m} d_{kk}(\omega) x^r_k(\omega)x^r_k(\omega). \quad S_{x_x}(\omega) = \sum_{k=1}^{m} d_{kk}(\omega) x^r_k(\omega)\dot{x}^r_k(\omega)$$ (4-57)

$$S_{Q_x}(\omega) = \sum_{k=1}^{m} d_{kk}(\omega) Q^r_k(\omega)Q^r_k(\omega)$$ (4-58)

Finally, any standard deviation response can be evaluated through the integration of the corresponding response spectrum. For instance, the displacement response of the jth floor $\sigma_{x_j}$ is evaluated from $S_{x_j}(\omega)$ through integration.
\[ \sigma^2_{i,j} = \int_{-\infty}^{\infty} \tilde{S}_{i,j}(\omega) \, d\omega \]  \hspace{1cm} (4-59)

The standard deviation acceleration response of the jth floor \( \sigma_{i,j}(\omega) \) is given by

\[ \sigma^2_{i,j} = \int_{-\infty}^{\infty} \omega^2 \tilde{S}_{i,j}(\omega) \, d\omega \]  \hspace{1cm} (4-60)

Clearly, by using the generalized pseudo-excitation method, the cross-correlation terms between vibration modes in the random response can be retained for the non-classically damped systems whereas the computation efforts are still reasonable. The structural internal forces and control forces can be also easily determined using this method.

### 4.6 NUMERICAL EXAMPLES

#### 4.6.1 Building with Active Tendons

To illustrate the application of the generalized pseudo-excitation method, a 40-story shear building under alongwind excitation with and without active tendon devices, as shown in Figs. 4.1(a) and 4.1(b), is analyzed. The same building with and without control devices was analyzed by Yang and Samali (1983) using the transfer matrix method. The building is assumed to be composed of 40 identical floor units. Wind excitations are applied to discrete lumped masses at each floor. The structural and aerodynamic physical data for the building are as follows: the individual story height is 4 m; the lumped mass at individual floor is \( 1.29 \times 10^7 \) kg; the elastic shear stiffness in each story is \( 10^8 \) N/m; the external damping coefficient is \( 2.155 \times 10^4 \) Ns/m and the internal damping coefficient is set to zero; the wind-load tributary area
for each story $A_i$ is 192 m$^2$; the gradient height $z_g$ is 300 m; the mean wind velocity at
the gradient height $V_g$ is 44.69 m/sec; the reference mean wind velocity at 10 m
height $V_{10}$ is 11.46 m/s; the drag coefficient $C_d$ is 1.2; the air density $\rho$ is 1.23 kg/m$^3$;
the ground surface drag coefficient $K_n$ is 0.03; the exponent for the mean velocity
profile $\alpha$ is 0.4; and $C_1$ is taken as 7.7. The resulting spectrum (Fig. 4.2) portrays the
energy content of the alongwind. Each tendon has a stiffness $k_t = 5 \times 10^7$ N/m and is
installed at an inclination angle $\theta$ of 25°. Only twenty tendon controllers are used
below the 20th floor, each installed between every two neighboring floors.

For the building without active tendon devices, the first three modal
frequencies are 1.08, 3.24 and 5.39 rad/s respectively and the first three modal
damping ratios due to external damping coefficients are 0.77%, 0.26% and 0.15%.
respectively. With the active tendon devices, the first three modal damping ratios of
the controlled building are computed against normalized loop gain $\varepsilon$ and feedback
gain $\tau$ for displacement sensors, velocity sensors and acceleration sensors. Fig. 4.3
shows variations of the first three modal damping ratios of the controlled building
with normalized loop gain $\varepsilon$ for $\tau=18$ and using velocity sensors. It is seen that the
modal damping ratios increase rapidly with the increasing normalized loop gain until
the normalized loop gain reaches 8. After that, the gradients of the first three modal
damping ratios with $\varepsilon$ become small. It is also seen that with the active tendon
deVICES installed, the first three modal damping ratios of the building are
significantly increased, in particular in the first mode of vibration. Displayed in Fig.
4.3 is also the first modal damping ratio of the controlled building using the
acceleration sensors. Clearly, the use of velocity sensors is incomparably superior to
the use of acceleration sensors in terms of the achievable maximum modal damping
ratio and its sensitivity to the normalized loop gain. The results related to
displacement sensors do not appear in Fig. 4.3 because it is found that the active
tendon devices with displacement sensors cannot function properly or may lead to
response instability for the present application. Fig. 4.4 shows variations of the
modal damping ratios with the normalized feedback gain \( \tau \) for \( \varepsilon = 8 \) using either
velocity sensors or acceleration sensors. Again, active tendon devices with
displacement sensors cannot function properly and the devices with acceleration
sensors are inferior to those with velocity sensors. With the velocity sensors, the first
three modal damping ratios of the building increase rapidly with the normalized
feedback gain \( \tau \) until \( \tau \) reaches a value of about 28. After that, the modal damping
ratios decrease with the increasing normalized feedback again.

Figs. 4.5 and 4.6 depict variations of the first three modal frequencies of the
controlled building with the normalized loop gain \( \varepsilon \) for \( \tau = 18 \) and with the normalized
feedback gain \( \tau \) for \( \varepsilon = 8 \), respectively. It is seen that the first modal frequency of the
controlled building changes slightly compared with that of the uncontrolled building
if \( \varepsilon \) is larger than 8 and \( \tau \) is smaller than 18. The second and third modal frequencies
of the controlled building are, however, larger than those of the uncontrolled
building. The above results indicate that the optimal parameters may be selected as
\( \varepsilon = 8 \) and \( \tau = 18 \).

To confirm if the parameters \( \varepsilon = 8 \) and \( \tau = 18 \) are optimal, the top floor
displacement response and control force response of the building with and without
active tendon devices are computed against the normalized loop gain and feedback
gain. Figs. 4.7 and 4.8 depict variations of the standard deviation of top floor
displacement with the normalized loop gain \( \varepsilon \) for \( \tau = 18.0 \) and with the normalized
feedback \( \tau \) for \( \varepsilon = 8.0 \), respectively. Figs. 4.9 and 4.10 display variations of the standard deviation of active tendon control force at the first story with the normalized loop gain and feedback gain. All the figures indicate that velocity sensors should be used instead of acceleration sensors. This is consistent with the results drawn from the eigenvalue analysis. It is seen that the displacement response and control force are almost independent of the normalized loop gain \( \varepsilon \) when its value is larger than 2. It is also seen that when the normalized feedback gain \( \tau \) is greater than 18, the further reduction of displacement response becomes very small but the required control force is still increasing. Therefore, the beneficial parameters of the active tendon devices for this application can be selected as \( \varepsilon = 8 \) and \( \tau = 18 \).

For the optimal parameters selected herein, the first three damped modal frequencies of the controlled building are found to be 1.17, 3.74 and 6.95 rad/s, respectively. Compared with those of the uncontrolled building of 1.08, 3.24 and 5.39 rad/s, there are 8.3\%, 15.4\% and 28.9\% increments, respectively. The first three modal damping ratios of the controlled building obtained from the complex modal analysis are 32.98\%, 26.40\% and 6.37\% respectively. One can expect that the response of the buildings can be tremendously reduced. One can also see that with active tendons, the dynamic characteristics of the building are changed.

Figs. 4.11 and 4.12 show variations of the standard deviations of displacement and shear force responses with the height of building for both uncontrolled and controlled cases. Clearly, with active tendons installed, the displacement and shear force responses are significantly reduced along the height of the building. The standard deviation of top floor displacement of the uncontrolled building is 45.3 mm but with active tendons, it is reduced to 23.8 mm, leading to a 47 per cent reduction.
of the response. The standard deviations of base shear force of the building are 1832.6 kN for the uncontrolled case but 939.3 kN for the controlled case, resulting in a 49 per cent reduction of the response. In particular, the computed results show a substantial reduction of the standard deviation of top floor acceleration response of the building from 149.7 mm/s² without control to 33.0 mm/s² with control. The required control forces of active tendons to achieve the aforementioned performance are displayed in Fig. 4.13. The maximum control force occurs at the first story and becomes smaller at higher stories.

To have a further understanding of the performance of active tendons, the spectral density functions of the top floor displacement, base shear force and top floor acceleration responses are, respectively, plotted in Figs. 4.14 to 4.16 for both uncontrolled and controlled cases. It is noted that each of these spectra has a few dominant response peaks. For displacement and shear force responses, the first peak at $\omega=0.09$ rad/sec is attributed to the peak of alongwind excitation spectrum. The installation of active tendons in the building cannot alleviate this peak in the response spectra. This peak response disappears in the acceleration response spectra because the low frequency excitation contributes a little to the acceleration response. The next two peaks in the uncontrolled response spectra indicate the first two natural frequencies of the building. Clearly, these peaks are substantially suppressed by active tendons, corresponding to substantial reductions of building responses. It is also seen that for the building without the active tendons, the second mode of vibration may make a significant contribution to the acceleration response. It is, however, not true for the case with control devices.

4.6.2 Building with Light Appendage
To examine the effects of closely spaced modes of vibration, a 36-story shear building with a light appendage attached to its top is selected (see Fig.4.1(c)). No control devices are considered. The light appendage is modeled as a lumped mass system of four degrees of freedom. The structural and aerodynamic physical data for the building are the same as the above example. For the appendage, the four lumped masses are the same, each having a mass ratio of 0.02 to the lumped mass at the building floor. The stiffness of the appendage is 0.03% of the story shear stiffness of the building. The wind-load tributary area corresponding to each lumped mass of the appendage is 1% of the building wind-load tributary area for each story.

The first six natural circular frequencies of the 36-story building with the light appendage are found to be 1.154, 1.229, 3.405, 3.598, 5.224 and 5.981 rad/s respectively. Obviously, some of the natural frequencies are closely spaced. Table 4.1 lists the standard deviations of the top displacement and acceleration response of the appendage and the base shear force response of the building. The comparison of the responses obtained from the generalized pseudo-excitation method with those from the SRSS method clearly demonstrates the importance of the cross-correlation terms between modes of vibration for such a system. In this connection, the generalized pseudo-excitation method can be regarded as a kind of CQC method suggested by Wilson et al. (1981).

<table>
<thead>
<tr>
<th>Table 4.1 Response Standard Deviations of Building with Light Appendage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response</td>
</tr>
<tr>
<td>Top displacement (cm)</td>
</tr>
<tr>
<td>Top Acceleration(cm/s²)</td>
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<tr>
<td>Base shear force (kN)</td>
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</tbody>
</table>
4.7 SUMMARY

In this chapter, the complex modal analysis method and the generalized pseudo-excitation method have been applied for determining both dynamic characteristics and wind-induced response of tall buildings with either control devices or closely spaced modes of vibration. Two example buildings subject to alongwind excitation were studied: one was a tall building with a light appendage; and the other was a tall building with active tendon devices. The results show that for the building with light appendage, the SRSS method overestimated the top displacement response by 20% and underestimate the base shear force response by 12%. For the building with active tendon devices, its natural frequencies are altered compared with the same building without control and the modal damping ratios are so significantly increased that the building is no longer lightly damped structure. By using the generalized pseudo-excitation method, the wind-induced response of the building with and without control devices was computed and the results showed that if the parameters of active tendon devices were selected properly, the wind-induced responses of the building could be considerably reduced.
Figure 4.1 Structural Model of Multistory Building

Figure 4.2 Spectrum of Alongwind Speed
Figure 4.3 Variations of Modal Damping Ratios with Normalized Loop Gain

Figure 4.4 Variations of Modal Damping Ratios with Normalized Feedback Gain
Figure 4.5 Variations of Modal Frequencies with Normalized Loop Gain

Figure 4.6 Variations of Modal Frequencies with Normalized Feedback Gain
Figure 4.7 Top Floor Displacement Responses of Building vs. Normalized Loop Gain

Figure 4.8 Top Floor Displacement Responses of Building vs. Normalized Feedback Gain
Figure 4.9 Control Forces of Active Tendon at First Story vs. Normalized Loop Gain

Figure 4.10 Control Forces of Active Tendon at First Story vs. Normalized Feedback Gain
Figure 4.11 Variations of Displacement Response of Building with Height

Figure 4.12 Variations of Shear Force Response of Building with Height
Figure 4.13 Variations of Control Force of Active Tendons with Height

Figure 4.14 Spectral Density Functions of Top Floor Displacement Response
Figure 4.15 Spectral Density Functions of Base Shear Force Response

Figure 4.16 Spectral Density Functions of Top Floor Acceleration Response
CHAPTER FIVE

PASSIVE CONTROL OF EARTHQUAKE-EXCITED
STEEL FRAMES WITH SEMI-RIGID JOINTS

5.1 INTRODUCTION

As mentioned in Chapter 2, extensive research has been carried out in the past two decades to pursue the refined analysis and design of steel frames with semi-rigid connections, and such efforts have led to provisions in various design codes of practice for semi-rigid connections. To reduce earthquake-induced response, in particular displacement response, of a steel frame of bolted connections, energy dissipation materials may be placed at a connection between the end plate and column flange or between the angle and member flange. Hsu and Fafitis (1992) proposed the elastomeric bolted connections by using two elastomeric pads and a shear pin as shown in Fig. 5.1a. Some others just placed viscoelastic materials between the angles and beam flanges to maintain the original bolted connection configuration (see Fig. 5.1b). Such a bolted connection is usually modeled as a rotational spring and a rotational damper in parallel. The previous study showed that the bolted connections with energy dissipating devices provided significant improvement by reducing the lateral displacement of the frame. However, no systematic study has been carried out on this topic and no sophisticated analytical method has been developed as yet.

Thus, this chapter first derives the mass matrix, stiffness matrix and damping matrix for the beam element with rotational spring and damper at its ends using a
combination of the finite element method and the dynamic stiffness method. After
the equations of motion of steel frames with connection dampers have been
assembled, the complex modal analysis is then carried out to determine the natural
frequencies and modal damping ratios of the frame. The seismic response of the
frame is computed using the generalized pseudo-excitation method in the frequency
domain. Parametric studies are finally performed to see the effects of connection
stiffness and damper damping coefficient on the natural frequencies, modal damping
ratios, and seismic response of the frame.

5.2 FORMULATION OF ELEMENT MATRICES

Figure 5.2 shows a mechanical model for the beam with connection elements
at its ends. The rotational spring provides the stiffness of connection while the
rotational viscous damper represents the energy dissipation capacity of connection
dampers. By assuming that the beam is a Bernoulli beam and neglecting the effects
of warping, local buckling and cross sectional distortion. Chan (1994) derived the
stiffness and mass matrices for the beam with rotational springs only using the
conventional finite element method. Kawashima and Fujimoto (1984), on the other
hand, derived the stiffness, mass, and damping matrices for the beam with both
rotational springs and dampers using the dynamic stiffness method. The derivation of
the element matrices using the dynamic stiffness method is rather complicated and
difficult to be understood by engineers. However, it is also difficult to derive the
damping matrix using the finite element method. This chapter thus combines the
finite element method with the dynamic stiffness method to derive the element
matrices.
For the hybrid beam element shown in Fig. 5.2, only moments $M$ and rotational displacements $\theta$ are needed to be distinguished between global nodes $j$ and internal nodes $j' (j = 1, 2)$ as the connection elements are of zero length. Based on the results from the finite element method, the dynamic matrix relating harmonic displacement vector $d$ and harmonic force vector $f$ at the internal nodes of the beam element can be written as

$$f = Dd$$  \hspace{1cm} (5-1)

$$d = [u_1, v_1, \theta_1', u_2, v_2, \theta_2']^T$$  \hspace{1cm} (5-2)

$$f = [N_1, Q_1, M_1', N_2, Q_2, M_2']^T$$  \hspace{1cm} (5-3)

in which $D = [D_0] = K_b - \omega^2 M_b$, a 6x6 symmetric matrix; $\omega$ is the frequency in rad/s. $K_b$ and $M_b$ are the conventional stiffness and consistent mass matrices, respectively, of the beam element in the finite element method.

$$K_b = \frac{EI}{L^3} \begin{bmatrix} \frac{AL^2}{12} & 0 & 0 & -\frac{AL^2}{12} & 0 & 0 \\ 0 & 12 & 6L & 0 & -12 & 6L \\ 0 & 6L & 4L^2 & 0 & -6L & 2L^2 \\ \frac{AL^2}{12} & 0 & 0 & \frac{AL^2}{12} & 0 & 0 \\ 0 & -12 & -6L & 0 & 12 & -6L \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{bmatrix}$$  \hspace{1cm} (5-4)

$$M_b = \frac{\rho AL}{420} \begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22L & 0 & 54 & -13L \\ 0 & 22L & 4L^2 & 0 & 13L & -3L^2 \\ 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 54 & 13L & 0 & 156 & -22L \\ 0 & -13L & -3L^2 & 0 & -22L & 4L^2 \end{bmatrix}$$  \hspace{1cm} (5-5)

in which $E$, $I$, $A$, $\rho$, and $L$ are the Young’s modulus, second moment of area, cross section area, density, and length of the beam, respectively.
The dynamic matrix relating harmonic rotational displacements and harmonic moments of the connection shown in Fig. 5.2 can be written as

\[
\begin{bmatrix}
M_1' \\
M_2'
\end{bmatrix} = \begin{bmatrix}
-r_1 & -\gamma_j \\
-\gamma_j & r_j
\end{bmatrix} \begin{bmatrix}
\theta_1 \\
\theta_j
\end{bmatrix} \quad (j=1,2)
\]

in which \( r_j = k_j + i\omega c_j \); \( r_j \) is the dynamic stiffness of the connection; \( k_j \) and \( c_j \) are the rotational stiffness and rotational damper damping coefficient of the connection; and \( i \) is the imaginary unit.

Combining the two dynamic matrices, equations (5-1) and (5-6), then leads to the dynamic matrix for the hybrid beam in the form

\[
\begin{bmatrix}
M_1 \\
M_1' \\
M_2 \\
M_2' \\
Q_1 \\
Q_2 \\
N_1 \\
N_2
\end{bmatrix} = \begin{bmatrix}
r_1 & -r_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\gamma_1 & r_1 + D_{s1} & D_{s0} & 0 & D_{s2} & D_{s1} & D_{s4} & D_{s3} \\
0 & D_{s5} & r_2 + D_{o0} & -r_2 & D_{o2} & D_{o1} & D_{o4} & D_{o3} \\
0 & 0 & -\gamma_2 & r_2 & 0 & 0 & 0 & 0 \\
0 & D_{23} & D_{20} & 0 & D_{22} & D_{21} & D_{24} & D_{23} \\
0 & D_{32} & D_{30} & 0 & D_{32} & D_{31} & D_{34} & D_{33} \\
0 & D_{11} & D_{10} & 0 & D_{12} & D_{11} & D_{14} & D_{13} \\
0 & D_{43} & D_{40} & 0 & D_{42} & D_{41} & D_{44} & D_{43}
\end{bmatrix} \begin{bmatrix}
\theta_1 \\
\theta_1 \\
\theta_2 \\
\theta_2 \\
v_1 \\
v_2 \\
u_1 \\
u_2
\end{bmatrix}
\]

(5-7)

Assume that the external loads are applied at the global nodes only. \( M_1' \) and \( M_2' \) in equation (5-7) should be zero, resulting in

\[
\begin{bmatrix}
\theta_1 \\
\theta_2
\end{bmatrix} = \frac{1}{\beta} \begin{bmatrix}
r_2 + D_{o0} & -D_{s0} \\
-D_{s3} & r_1 + D_{s1}
\end{bmatrix} \begin{bmatrix}
r_1 & 0 \\
0 & r_2
\end{bmatrix} \begin{bmatrix}
\theta_1 \\
\theta_2
\end{bmatrix} - \begin{bmatrix}
D_{s2} & D_{s1} & D_{s4} \\
D_{o2} & D_{o1} & D_{o4} \\
D_{32} & D_{31} & D_{34} \\
D_{42} & D_{41} & D_{44}
\end{bmatrix} \begin{bmatrix}
v_1 \\
v_2 \\
u_1 \\
u_2
\end{bmatrix}
\]

(5-8)

in which \( \beta = (r_1 + D_{s1})(r_2 + D_{o0}) - D_{s0}D_{s3} \).

Substitution of equation (5-8) into equation (5-7) and then some manipulation yield the dynamic matrix for the hybrid element in terms of the global nodes only.
\[
\begin{bmatrix}
M_1 \\
M_2 \\
Q_1 \\
Q_2 \\
N_1 \\
N_2
\end{bmatrix}
= \begin{bmatrix}
f_1 \\
f_2 \\
f_{\theta_1} \\
f_{\theta_2} \\
u_1 \\
u_2
\end{bmatrix} = Z
\begin{bmatrix}
\theta_1 \\
\theta_2 \\
v_1 \\
v_2 \\
u_1 \\
u_2
\end{bmatrix}
\]

(5-9)

in which

\[
f_1 = \begin{bmatrix}
r_1 & 0 \\
0 & r_2
\end{bmatrix} - \frac{1}{\beta} \begin{bmatrix}
r_1 & 0 \\
0 & r_2
\end{bmatrix} \begin{bmatrix}
r_2 + D_{nn} & -D_{20} \\
-D_{10} & r_1 + D_{11}
\end{bmatrix} \begin{bmatrix}
r_1 & 0 \\
0 & r_2
\end{bmatrix}
\]

(5-10)

\[
f_2 = f_1^T = \frac{1}{\beta} \begin{bmatrix}
r_1 & 0 \\
0 & r_2
\end{bmatrix} \begin{bmatrix}
r_2 + D_{nn} & -D_{20} \\
-D_{10} & r_1 + D_{11}
\end{bmatrix} \begin{bmatrix}
D_{12} & D_{35} & D_{11} & D_{44}
\end{bmatrix}
\]

(5-11)

\[
f_3 = \begin{bmatrix}
D_{22} & D_{25} & D_{21} & D_{24} \\
D_{52} & D_{55} & D_{51} & D_{54} \\
D_{12} & D_{15} & D_{11} & D_{14} \\
D_{42} & D_{45} & D_{41} & D_{44}
\end{bmatrix}
\]

(5-12)

\[
-\frac{1}{\beta} \begin{bmatrix}
D_{23} & D_{26} \\
D_{53} & D_{56} \\
D_{13} & D_{16} \\
D_{43} & D_{46}
\end{bmatrix} \begin{bmatrix}
r_2 + D_{nn} & -D_{20} \\
-D_{10} & r_1 + D_{11}
\end{bmatrix} \begin{bmatrix}
D_{12} & D_{35} & D_{11} & D_{44}
\end{bmatrix}
\]

(5-12)

By expanding the elements of \( Z \) in series with respect to the circular frequency \( \omega \) and neglecting higher terms than the third order, the following expression is obtained

\[
Z = K_e + i\omega C_e - \omega^2 M_e
\]

(5-13)

\( K_e, \ C_e, \) and \( M_e \) are the required stiffness matrix, damping matrix and mass matrix of the hybrid element shown in Fig. 5.2, respectively. \( K_e, \ C_e, \) and \( M_e \) can be obtained in an explicit form.
\[ K_e = \begin{bmatrix}
K_0 & \frac{2(K_{11} + K_{12} + K_{22})}{L^2} & -\frac{2K_{11}}{L} & 0 & \frac{2K_{11}}{L} & \frac{2(K_{11} + K_{12} + K_{22})}{L} \\
0 & \frac{2K_{11} + K_{12}}{L} & 0 & K_0 & 0 & \frac{2K_{11} + K_{12}}{L} \\
\frac{2EI}{L} & -K_0 & 0 & 0 & K_0 & \frac{K_{12} + 2K_{22}}{L} \\
-\frac{2K_{11}}{L} & 0 & \frac{2K_{11}}{L} & 0 & 0 & \frac{2K_{11} + K_{12} + K_{22}}{L} \\
\frac{K_{12} + 2K_{22}}{L} & \frac{K_{12}}{L} & 0 & \frac{K_{12} + 2K_{22}}{L} & 0 & 2K_{22}
\end{bmatrix}
\]

in which

\[ K_0 = \frac{EA}{2EI}, \quad K_{11} = \frac{3v_1}{4 - v_1v_2}, \quad K_{12} = \frac{3v_1v_2}{4 - v_1v_2}, \quad K_{22} = \frac{3v_2}{4 - v_1v_2} \]

\[ C_e = \begin{bmatrix}
0 & \frac{2(C_{11} + C_{12} + C_{22})}{L^2} & -\frac{2C_{11}}{L} & 0 & \frac{2C_{11}}{L} & \frac{2(C_{11} + C_{12} + C_{22})}{L} \\
0 & \frac{2C_{11} + C_{12}}{L} & 0 & C_{11} & 0 & \frac{2C_{11} + C_{12}}{L} \\
0 & 0 & \frac{2C_{11} + C_{12}}{L} & 0 & 0 & \frac{2C_{11} + C_{12} + C_{22}}{L} \\
0 & \frac{C_{12} + 2C_{22}}{L} & \frac{C_{12}}{L} & 0 & \frac{C_{12} + 2C_{22}}{L} & 2C_{22}
\end{bmatrix}
\]

where

\[ C_{11} = \frac{1}{(4 - v_1v_2)^2} \left[ 4c_1(l - v_1)^2 + c_2(l - v_2)^2v_1^2 \right] \]

\[ C_{12} = \frac{4}{(4 - v_1v_2)^2} \left[ c_1v_2(l - v_1)^2 + c_2v_1(l - v_2)^2v_1^2 \right] \]

\[ C_{22} = \frac{1}{(4 - v_1v_2)^2} \left[ 4c_2(l - v_2)^2 + c_1(l - v_1)^2v_2^2 \right] \]
The mass matrix of the hybrid beam is the summation of two parts $M_k$ and $M_m$.

$M_k$ includes only the fixity factors, and for the case of rigid connections it coincides with the consistent mass matrix of the beam expressed by equation (5-5). $M_m$ includes both the fixity factors and the damper damping coefficients.

$$
M_c = M_k + M_m
$$

$$
\begin{bmatrix}
140 & 4Z_1(v_1, v_2) \\
2LZ_2(v_1, v_2) & 4L^2Z_1(v_1, v_2) \\
70 & 0 & 0 & 140 \\
0 & 2Z_1(v_1, v_2) & LZ_2(v_2, v_1) & 0 & 4Z_1(v_2, v_1) \\
0 & -LZ_2(v_1, v_2) & -L^2Z_1(v_1, v_2) & 0 & -2LZ_2(v_2, v_1) & 4L^2Z_2(v_2, v_1)
\end{bmatrix}
\text{Sym.}
$$

$$
\begin{bmatrix}
0 & \frac{2(M_{11} + M_{12} + M_{22})}{L^2} \\
0 & \frac{2M_{11} + M_{12}}{L} & 2M_{11} & 0 & 0 \\
0 & \frac{2(M_{11} + M_{12} + M_{22})}{L^2} & \frac{2M_{11} + M_{12}}{L} & 0 & \frac{2(M_{11} + M_{12} + M_{22})}{L^2} \\
0 & \frac{M_{12} + 2M_{22}}{L} & M_{12} & 0 & -\frac{M_{12} + 2M_{22}}{L} & 2M_{22}
\end{bmatrix}
\text{Sym.}
$$

(5-16)

where

$$
Z_1(p, q) = \frac{1}{(4 - pq)^2} (560 + 196p - 224q - 32pq + 32p^2 + 32q^2 + 50pq^2 - 55p^2q + 32p^3q^2)
$$

$$
Z_2(p, q) = \frac{1}{(4 - pq)^2} (224 + 64p - 160q - 32pq + 32p^2 + 25pq^2)
$$

$$
Z_3(p, q) = \frac{1}{(4 - pq)^2} (560 - 28p - 28q - 184pq - 64p^2 - 64q^2 + 5pq^2 + 5p^2q + 4p^3q^2)
$$

$$
Z_4(p, q) = \frac{1}{(4 - pq)^2} (392 - 100p - 128q - 38pq - 64p^2 + 55p^2q)
$$
\[ Z_s(p,q) = \frac{p^2}{(4 - pq)^2} (32 - 31q + 8q^2) \]

\[ Z_n(p,q) = \frac{pq}{(4 - pq)^2} (124 - 64p - 64q + 31pq) \]

\[ M_{11} = \frac{1}{3(4 - \nu_1 \nu_2)} \left[ 8c_1c_2 v_1 (1 - \nu_1)^2 (1 - \nu_2)^2 - 4c_1^2 (1 - \nu_1)^3 (4 - \nu_2) \right] \]

\[ - c_2 v_1 (1 - \nu_2)^3 (4 - \nu_1) \]

\[ M_{12} = \frac{4}{3(4 - \nu_1 \nu_2)} \left[ c_1 c_2 (4 + \nu_1 \nu_2) (1 - \nu_1)^2 (1 - \nu_2)^2 - c_1^2 (1 - \nu_1)^3 (4 - \nu_2) v_2 \right] \]

\[ - c_2^2 (1 - \nu_2)^3 (4 - \nu_1) v_1 \]

\[ M_{22} = \frac{1}{3(4 - \nu_1 \nu_2)} \left[ 8c_1 c_2 v_2 (1 - \nu_1)^2 (1 - \nu_2)^2 - 4c_2^2 (1 - \nu_2)^3 (4 - \nu_1) \right] \]

\[ - c_1 v_2 (1 - \nu_1)^3 (4 - \nu_2) \]

Clearly, \( \mathbf{K}_r, \mathbf{C}_r, \) and \( \mathbf{M}_r \) are all the functions of the fixity factor \( \nu \) defined as (Kawashima and Fujimoto 1984)

\[ k = \frac{3EI}{L} \frac{\nu}{1 - \nu} \quad (5-17) \]

It is seen that the fixity factor \( \nu \) relates the rotational stiffness of the connection to the flexural rigidity of the beam. It varies from zero for a pinned connection to 1.0 for a rigid connection. Apart from the fixity factor, the damping matrix and mass matrix \( \mathbf{C}_e \) and \( \mathbf{M}_e \) are also the functions of the damper damping coefficient.

### 5.3 Solution for Dynamic Characteristics

In this study, the rotational springs at connections are assumed to be linear and elastic and all members of the frame are in the linear and elastic range under earthquake excitation. The energy dissipation capacity of the bolted connection itself is considered to be included in the rotational damper already. These assumptions
indicate that the considered frame experiences only moderate or minor earthquake excitations, or because of the significant increase of energy dissipation capacity due to the installation of rotational dampers the frame is able to remain linear and elastic during an earthquake event. With these assumptions and by using the finite element method and the derived element matrices, the equation of motion of a flexibly jointed frame with rotational dampers under seismic excitation (see Fig. 5.3) can be expressed as

\[ M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = -Me \ddot{x}_g(t) \]  \hspace{1cm} (5-18)

where \( M \) and \( K \) are the mass and stiffness matrix of the frame of flexible connections, respectively; \( C \) is the total damping matrix of the frame, including structural damping as well as the damping from the rotational dampers; \( x(t) \) is the vector of relative displacement response with respect to the ground; \( \ddot{x}_g(t) \) is the ground acceleration and \( e \) is the index vector given by

\[ e = [1, 0, 0, 1, 0, 0, \ldots, 1, 0, 0]^T \]  \hspace{1cm} (5-19)

Clearly, the semi-rigid frame with connection dampers is a non-classically damped system. To facilitate the seismic design of the frame, the complex mode superposition method is employed in this study to estimate the modal damping ratios and natural frequencies.

Equation (5-18) can be replaced by the first-order 2N-dimensional equation of the form

\[ Aq + Bq = F\ddot{x}_g(t) \]  \hspace{1cm} (5-20)

where

\[ A = \begin{bmatrix} 0 & M \\ M & C \end{bmatrix}, \quad B = \begin{bmatrix} -M & 0 \\ 0 & K \end{bmatrix}, \quad F = \begin{bmatrix} 0 \\ -ME \end{bmatrix} \text{ and } q = \begin{bmatrix} \ddot{x} \\ x \end{bmatrix} \]  \hspace{1cm} (5-21)
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If the solution of the homogeneous form of equation (5-18) is taken as

\[ x(t) = \phi e^{\lambda t} \quad (5-22) \]

in which \( s \) is the eigenvalue and \( \phi \) is the associated eigenvector. The solution of the homogeneous form of equation (5-20) can then be taken as

\[ q = \begin{bmatrix} s \phi \\ \phi \end{bmatrix} e^{\lambda t} = \hat{\phi} e^{\lambda t} \quad (5-23) \]

The associated eigenvalue problem becomes

\[ (s \mathbf{A} + \mathbf{B})\hat{\phi} = \mathbf{0} \quad (5-24) \]

5.3.1 Underdamped System

The solution of equation (5-24) comprises a set of \( 2N \) eigenvalues and eigenvectors that are real or exist in complex conjugate pairs. If all the eigenvalues and eigenvectors are in complex conjugate pairs with negative real parts, the system is defined as the underdamped system. Thus, for an underdamped system, the eigenvectors and eigenvalues are

\[ \hat{\phi}_j = \hat{\phi}_j^* \quad \text{and} \quad s_j = s_j^* \quad (j=1,2,\ldots,N) \quad (5-25) \]

Each eigenvalue can be written under the form

\[ s_j = s_j^* = -\omega_j \tilde{\zeta}_j + i\omega_d \quad (j=1,2,\ldots,N) \quad (5-26) \]

in which

\[ \omega_j = |s_j|, \quad \tilde{\zeta}_j = -\text{Re}(s_j)/|s_j| \quad \text{and} \quad \omega_d = \omega_j \sqrt{1 - \tilde{\zeta}_j^2} \quad (5-27) \]

\( \omega_j, \omega_d, \) and \( \tilde{\zeta}_j \) are the modal frequency, the damped modal frequency, and the modal damping ratio, respectively, associated with mode \( j \). The superscript * means the conjugation. For the underdamped system, the value of \( \tilde{\zeta}_j \) is always less than one.
5.3.2 System with Mixed Damping

For a semi-rigid frame with connection dampers, equation (5-24) may produce some real-valued negative pairs, each associated with a real-valued eigenvector. This is because when rotational damper damping coefficient becomes very large some higher modes of vibration of the frame will be damped out. In this case, the system may be called the mixed damped system because some of the eigenvalues and eigenvectors are still in complex conjugate pairs with negative real parts. It is convenient to express real pairs \(s_j\) in the following form analogous to equation (5-26).

\[
s_j = -\omega_j \xi_j + \omega_{d_j} \quad (j=1,2,\ldots,N)
\]

\[
s_{j,\lambda} = -\omega_j \xi_j - \omega_{d_j} \quad (j=1,2,\ldots,N)
\]

in which \(\omega_j\), \(\omega_{d_j}\) and \(\xi_j\) are determined by

\[
\omega_j = \sqrt{s_j s_{j,\lambda}} \quad \xi_j = -(s_j + s_{j,\lambda})/(2\omega_j) \quad \text{and} \quad \omega_{d_j} = \omega_j \sqrt{\xi_j^2 - 1} = (s_j - s_{j,\lambda})/2
\]

The value of \(\xi_j\) is now equal to or greater than one.

5.4 SOLUTION FOR SEISMIC RESPONSE

The complex mode superposition method adopts the following coordinate transformation to decouple equation (5-20)

\[
q = \Phi z
\]

where \(z\) is the 2N-dimensional generalized coordinate vector and \(\Phi\) is the 2N×2N complex modal modal matrix.

\[
\Phi = [\dot{\phi}_1, \dot{\phi}_2, \ldots, \dot{\phi}_{2N}]
\]
By using the coordinate transformation and the orthogonality of modes, equation (5-20) can be reduced to 2N decoupled modal equations with the jth modal equation being

\[ A_j \ddot{z}_j + B_j z_j = \Phi_j^T \mathbf{F} \ddot{x}_g (t) \]  
(5-33)

or alternatively

\[ \ddot{z}_j - s_j z_j = r_j \ddot{x}_g (t) \]  
(5-34)

in which

\[ B_j = \Phi_j^T \mathbf{B} \Phi_j = -s_j A_j \]  
(5-35)

\[ r_j = \Phi_j^T \mathbf{F}/A_j = -\Phi_j^T \mathbf{M} \mathbf{E}/A_j \]  
(5-36)

Assume that the ground acceleration \( \ddot{x}_g (t) \) is a stationary random process and its power spectral density function is given as \( S_g (\omega) \). The generalized pseudo-excitation method is now used to determine the seismic response of a semi-rigid frame with connection dampers.

The pseudo-excitation is constituted for a given frequency \( \omega \) as

\[ \ddot{x}_g (t) = \sqrt{S_g (\omega)} e^{i \omega t} \]  
(5-37)

The solution of the first-order equation (5-34) to the pseudo-excitation is

\[ z_j (\omega, t) = \frac{r_j}{i \omega - s_j} \sqrt{S_g (\omega)} e^{i \omega t} \quad (j=1, 2, \ldots, 2N) \]  
(5-38)

Substituting equation (5-38) into equation (5-31) and comparing with the last part of equation (5-21), one obtains

\[ x(\omega, t) = \sum_{j=1}^{2N} \Phi_j z_j (\omega, t) = \sum_{j=1}^{2N} \Phi_j \frac{r_j}{i \omega - s_j} \sqrt{S_g (\omega)} e^{i \omega t} \]  
(5-39)
Since the eigenvector is in pairs in either underdamped system or the system with mixed damping, the $x(\omega, t)$ can be reduced to

$$x(\omega, t) = \sum_{i=1}^{N} H_i(\omega)(i\omega \alpha_+ + \beta_+) \sqrt{S_x(\omega)} \, e^{i\omega t} = x(\omega) e^{i\omega t} \quad (5-40)$$

in which $x(\omega)$ is called the pseudo displacement

$$x(\omega) = \sum_{i=1}^{N} H_i(\omega)(i\omega \alpha_+ + \beta_+) \sqrt{S_x(\omega)} \quad (5-41)$$

$H_i(\omega)$ is the frequency response function for the jth mode.

$$H_i(\omega) = \frac{1}{\omega_i^2 - \omega^2 + i2\xi_i\omega_i \omega} \quad (5-42)$$

When the jth mode is an underdamped mode.

$$\alpha_+ = 2 \text{Re}(\phi_i r_i) \quad \text{and} \quad \beta_+ = -2 \text{Re}(\phi_i r_i s_i^*) \quad (5-43)$$

When the jth mode is an overdamped mode or a critically damped case

$$\alpha_+ = (\phi_i r_i + \phi_i s_i r_i s_i) \quad \text{and} \quad \beta_+ = - (\phi_i r_i s_i + \phi_i s_i r_i s_i) \quad (5-44)$$

Similarly, the pseudo velocity response can be obtained by

$$\dot{x}(\omega) = \sum_{i=1}^{N} H_i(\omega)(i\omega \mu_i + \nu_i) \sqrt{S_y(\omega)} \quad (5-45)$$

in which when the jth mode is an underdamped mode

$$\mu_i = 2 \text{Re}(s_i \phi_i r_i) \quad \text{and} \quad \nu_i = -2\omega_i^2 \text{Re}(\phi_i r_i) \quad (5-46)$$

When the jth mode is an overdamped mode or a critically damped case

$$\mu_i = (s_i \phi_i r_i + s_i \phi_i s_i r_i s_i) \quad \text{and} \quad \nu_i = -\omega_i^2 (\phi_i r_i + \phi_i s_i r_i s_i) \quad (5-47)$$

Once the pseudo displacement is determined, the pseudo internal force, $Q(\omega)$, can be easily determined following a static analysis.

The response spectral matrix can then be obtained by
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\[ S_{xx}(\omega) = x^*(\omega)x^T(\omega) \quad S_{\alpha\alpha}(\omega) = x^*(\omega)x^T(\omega) \quad (5-48) \]
\[ S_{\alpha\alpha}(\omega) = \dot{x}^*(\omega)\dot{x}^T(\omega) \quad S_{\alpha\alpha}(\omega) = \ddot{x}^*(\omega)\ddot{x}^T(\omega) \quad (5-49) \]
\[ S_{\alpha q}(\omega) = Q^*(\omega)Q^T(\omega) \quad (5-50) \]

The standard deviation displacement response of the jth floor \( \sigma_{\xi} \) is finally evaluated from \( S_{\xi\xi}(\omega) \) through integration.

\[ \sigma_{\xi}^2 = \int_{-\infty}^{\infty} S_{\xi\xi}(\omega) d\omega \quad (5-51) \]

The standard deviation acceleration response of the jth floor \( \sigma_{\alpha} \) is given by

\[ \sigma_{\alpha}^2 = \int_{-\infty}^{\infty} \omega^2 S_{\alpha\alpha}(\omega) d\omega \quad (5-52) \]

5.5 NUMERICAL EXAMPLE

An eight story two bay semi-rigid frame with rotational dampers is selected as an example building (see Fig. 5.3) for parametric study and application. The material properties of the frame members are given in Table 5.1. The structural damping ratio is set to 1% for the first two modes of vibration of the frame without rotational dampers, following the assumption of Rayleigh damping. The stiffness of the rotational spring and the damping coefficient of the rotational damper are the same for all connections. The Kanai-Tajimi filtered white noise spectrum is used as the ground acceleration spectrum.

\[ S_g(\omega) = \frac{1 + 4\xi_g^2 \left( \frac{\omega}{\omega_g} \right)^2}{\left[ 1 - \left( \frac{\omega}{\omega_g} \right)^2 \right]^2 + 4\xi_g^2 \left( \frac{\omega}{\omega_g} \right)^2} S_n \quad (5-53) \]
in which $\omega_x$, $\xi_x$, $S_0$ may be regarded as the characteristics and the intensity of an earthquake in a particular geological location. The parameters in the ground acceleration spectrum are selected as $\omega_x=15.0$ rad/s; $\xi_x=0.65$ and $S_0=4.65\times10^4$ m$^2$/rad$^2$s$^3$. Displayed in Fig. 5.4 is the Kanai-Tajimi spectral density function with the selected parameters. It is seen that the earthquake excitation energy is mainly distributed over the range around the ground frequency.

| Table 5.1 Material Properties of Members in Example Frame |
|----------------|----------------|----------------|
| Member        | EI (N·m$^2$)  | EA (N)         | $\rho_A$ (kg/m) |
| Beam          | 1.553×10$^6$  | 2.645×10$^6$  | 101.2           |
| Column        | 2.048×10$^6$  | 7.503×10$^6$  | 287.1           |

5.5.1 Modal Frequencies and Modal Damping Ratios

The modal frequencies of the frame without rotational dampers are computed first in this study. Fig. 5.4 displays the variations of the first two modal frequencies of the frame with the fixity factor. Clearly, the stiffness of rotational springs affects the modal frequencies of the frame significantly. The smaller rotational stiffness (i.e., the smaller fixity factor) leads to the smaller modal frequencies. When the fixity factor is unit, which represents the frame of rigid connections, the first and second modal frequencies, $f_1$ and $f_2$, are 2.78 Hz and 8.73 Hz respectively. As the fixity factor reduces to 0.2, the first and second modal frequencies become 1.26 Hz and 4.48 Hz. It is interesting to see that the relationship between the modal frequency and the fixity factor is almost linear.

The modal damping ratios of the semi-rigid frame with rotational dampers for a set of fixity factors are then computed against damper damping coefficient. The
computed first and second modal damping ratios of the frame are shown in Fig. 5.5a and 5.5b. For the fixity factor equal to unit, no matter what value is the damper damping coefficient, the first and second modal damping ratios of the frame remain 0.01, i.e., the structural damping ratios set for the frame without rotational dampers. This result is expected because when the connections become rigid the rotational dampers lose their function. For the frame with flexible connections, it is seen from Figs. 5.5a and 5.5b that if the damper damping coefficient is small, the modal damping ratios of the frame are almost unchanged. Only when the damper damping coefficient exceeds a certain value, the modal damping ratios are significantly increased first and then decreased as the damper damping coefficient is further increased. Thus, there is an optimal damper damping coefficient for a given fixity factor by which the modal damping ratio of the frame is the maximum. The optimal damper damping coefficient and corresponding maximum modal-damping ratio, however, vary with the fixity factor, i.e., the connection stiffness.

The parametric study for modal properties is finally moved to the optimal damper damping coefficient and the corresponding maximum modal-damping ratio of the frame. Fig. 5.6a shows the variations of the optimal damper damping coefficient and the corresponding maximum modal damping ratio with the stiffness of the rotational springs for the first mode of vibration of the frame. Fig. 5.6b displays the variations of the corresponding first damped modal frequency with the stiffness of rotational springs. As the flexible connections become more and more stiff, the optimal damper damping coefficient required for the maximum modal damping ratio of the frame becomes larger and larger, but the achievable maximum modal damping ratio becomes smaller and smaller. Although the first damped modal frequency of the frame still increases with the increasing stiffness of the connections,
the relationship between the two quantities is no longer approximately linear. Also because of rotational dampers optimal value, the first damped modal frequency becomes larger if compared with that of the frame without rotational dampers (see Fig. 5.4).

From the above analysis, it is understood that both the connection stiffness and damper-damping coefficient affect the modal frequencies and modal damping ratios of the frame. For a given connection stiffness and a given mode of vibration, there exists an optimum value of damper damping coefficient by which the modal damping ratio of the frame reaches its maximum. The high modal-damping ratio generated by the rotational dampers of optimal value is expected to significantly bring down the seismic response of the frame.

5.5.2 Seismic Responses

Seismic response analysis is carried out to investigate the variations of seismic response of the frame with connection stiffness and rotational damper damping coefficient and to see if the optimal modal parameters identified from the modal analysis are the same as those from the seismic response analysis. Then, the effectiveness of optimal modal parameters on seismic response reduction is examined.

Fig. 5.7 depicts the variations of the standard deviation of top floor displacement response of the frame at Column C (see Fig. 5.3) with damper damping for several connection fixity factors. When the damper damping coefficient is relatively small, it does not affect the standard deviation of top displacement response but the smaller the connection stiffness (the fixity factor) is, the larger will be the standard deviation of top displacement response. For a given fixity factor,
when the damper damping coefficient increases to a certain level it starts to significantly reduce the displacement response of the frame with flexible connections. This level depends on the fixity factor: the smaller the fixity factor is, the smaller will be the level of damper damping coefficient. This is consistent with the results from the modal analysis as shown in Fig. 5.5. In particular, when the damper damping coefficient is increased beyond $1 \times 10^9$ Ns/m the top displacement response of the frame with flexible connections becomes less than that of the frame with rigid connection. This is because of the high modal damping ratios attributed to rotational dampers.

Fig. 5.8 shows the variations of the standard deviation of base shear force of the frame at Column C with damper damping coefficient for several connection fixity factors. It is seen that the smaller connection fixity factor can result in the smaller base shear force response of the frame even without rotational dampers. With rotational dampers and when damper damping coefficient is over a certain level the base shear force response can be further reduced. When the damper damping coefficient reaches its optimal value identified from the modal analysis, the base shear force response of the frame becomes the smallest. For instance, the standard deviation of base shear force of the frame at Column C with perfectly rigid connections is 10.784 N. With the flexible connections of fixity factor $\nu=0.3$ and damper damping coefficient $c=3 \times 10^9$ Ns/m, it is reduced to 2.234 N, leading to a 79% reduction of the base shear response. However, for the frame with small fixity factor, the high modes of vibration also make some contributions to the response near the optimal damper-damping coefficient, as evidenced by the small peak in Fig. 5.8.
Fig. 5.9 shows the variations of the standard deviation of base bending moment of the frame at Column C with damper damping coefficient for several connection fixity factors. It is seen that the magnitude of bending moment response depends on the value of fixity factor. Without rotational dampers or with rotational dampers but of smaller damper damping coefficient, the smaller fixity factor leads to the larger base bending moment. Until the damper damping coefficient is increased to a certain level, the further increase of damper damping coefficient will reduce the bending moment response significantly. The minimum bending moment response can be achieved at the optimal damper-damping coefficient identified from the modal analysis. The standard deviation of base bending moment of the frame with perfectly rigid connections at Column C is 31.172 N·m but with the flexible connections of fixity factor ν=0.3 and damper damping constant $c=4\times10^9$ Ns/m, it is only 9.039 N·m, leading to a 71% reduction.

From the above seismic response and modal analyses, it is found that for the studied example building, the first mode of vibration dominates the seismic response of the frame with or without rotational dampers. Thus, it is important to select the optimal damper-damping coefficient based on the first mode of vibration to achieve the maximum modal damping ratio for the first mode of vibration. The optimal value of damper damping coefficient can be determined using the complex modal analysis because the optimal values of damper damping coefficient obtained from the modal analysis are almost the same as those from the seismic response analysis.

The overall performance of the example frame under earthquake excitation is also examined in this study. Figs. 5.10 and 5.11 show the variations of standard deviation of lateral displacement and shear force, respectively, of the frame with height. It is seen from Fig. 5.10 that the lateral displacement response of the frame
with the flexible connections of $v=0.47$ but without rotational dampers are much larger than that of the frame with rigid connections at every floor. With the rotational dampers of optimal damping coefficient, the lateral displacement response of the flexible frame of $v=0.47$ is reduced tremendously and becomes much smaller than that of the rigid frame. This reduction occurs not only at the top of the frame but also at all other floor levels. For the shear force response, Fig. 5.11 demonstrates that the standard deviation of shear force response of the frame with rigid connections is slightly larger than that of the frame with flexible connections but without rotational dampers. With the rotational dampers of optimal damping coefficient, the shear force response of the flexible frame reduces further at all the levels of the frame. Thus, the overall performance of the example frame is significantly enhanced due to the installation of rotational dampers.

5.6 SUMMARY

The stiffness, mass, and damping matrices of the beam element with rotational spring and damper at its ends have been derived by using a combination of the finite element method and dynamic stiffness method. The solutions for dynamic characteristics and seismic response of the frame with flexible connections and rotational dampers have also been provided in the frequency domain using the complex modal analysis method and the generalized pseudo-excitation method. The formulation was then applied to an example frame and parametric studies were performed. The results showed that there is an optimal damper damping coefficient for a given mode of vibration and a given fixity factor of the frame. With the optimal damper damping coefficient, the modal damping ratio of the frame could be significantly increased and the seismic responses, including lateral displacement.
shear force, and bending moment, of the frame could be considerably reduced to the level smaller than those of the frame with rigid connections. It should be pointed out, however, that while the suggested linear approach in the frequency domain is convenient for the determination of modal properties and seismic response of frames with connection dampers, a nonlinear seismic analysis of the frame with nonlinear connection stiffness and geometric effects is worthwhile to be investigated in the time domain in the future. Furthermore, the experimental work may be required in the future to verify the achieved results in this study, particularly for the optimal damper damping coefficient and the maximum modal-damping ratio.
Figure 5.1 Beam-Column Connection with Energy Dissipation Devices

Figure 5.2 Beam Element with Rotational Spring and Damper at its Ends
Figure 5.3 Steel Frame with Rotational Dampers at Flexible Joints
Figure 5.4 Power Spectral Density Function of Ground Acceleration

Figure 5.5 Variations of Modal Frequencies with Fixity Factor (No Damper Case)
Figure 5.6 Variations of Modal Damping Ratio with Damper Damping Coefficient
Figure 5.7 Variations of Optimal Modal Properties with Fixity Factor
Figure 5.8 Top Floor Displacement Response of Frame at Column C vs. Damper Damping Coefficient

Figure 5.9 Base Shear Force Response of Frame at Column C vs. Damper Damping Coefficient
Figure 5.10 Base Bending Moment Response of Frame at Column C vs. Damper Damping Coefficient

Figure 5.11 Variations of Lateral Displacement Response of Frame with Height
Figure 5.12 Variations of Shear Force Response of Frame with Height
CHAPTER SIX

PASSIVE CONTROL OF EARTHQUAKE-EXCITED ADJACENT BUILDINGS

6.1 INTRODUCTION

In this chapter, the generalized pseudo-excitation method is applied to passive vibration control of adjacent buildings subject to earthquake. The viscoelastic dampers and the fluid dampers are respectively used to link adjacent buildings together for control of seismic response of both buildings. The viscoelastic dampers are passive energy dissipation devices, mathematically represented by the Voigt model in this chapter. The fluid dampers that operate on the principle of fluid flow through orifices specially shaped are also passive energy dissipation devices but they are defined by the Maxwell model. This chapter first derives the equations of motion for earthquake-excited adjacent buildings connected by the viscoelastic dampers and the fluid dampers, respectively. The second order differential equations of the system are converted into the first order differential equations. The dynamic characteristics of the control device-adjacent building system are then determined through the complex modal analysis. The generalized pseudo-excitation method is used to analyze the seismic response of the system. Extensive parametric studies are finally performed to assess the effectiveness of the control devices and to identify beneficial control parameters. The effectiveness of the Maxwell model-defined fluid dampers is compared with that of the Voigt model-defined viscoelastic dampers wherever it is possible.
6.2 ADJACENT BUILDINGS LINKED BY VISCOELASTIC DAMPERS

Viscoelastic (VE) dampers have been successfully applied in tall buildings to mitigate wind-induced vibrations (Mahmoodi et al. 1987). However, the seismic applications of VE dampers have been investigated only in the last ten years. Analytical investigations of structures with added VE dampers have been carried out by Zhang et al. (1989), Zhang and Soong (1992) and others. Their results showed that the response of buildings due to strong earthquakes could be reduced significantly. Experimental studies on steel frames and reinforced concrete frames under earthquake excitation have also been conducted (Lin et al. 1991; Bergman and Hanson 1993; Chang et al. 1995; Shen et al. 1995). The experimental results showed that seismic resistant performance of a structure could be significantly improved with added VE dampers.

Viscoelastic materials are typically made of polymers or glassy substances. The damping results from energy dissipation due to shear deformation. The typical feature of the VE-dampers is that they not only add damping but also add stiffness to the system. Actual VE material behavior is very complex, including frequency, amplitude, and temperature dependent properties (Kasai et al. 1993). In this chapter, a Voigt model (Sun and Lu 1995) is employed to represent viscoelastic dampers used to link adjacent buildings. The Voigt model is a combination of a linear and elastic spring and a viscous dashpot connected in parallel. The investigation using the Voigt model in this chapter will lay the foundation for further investigations using more accurate VE-damper model.
6.2.1 Basic Equations

To capture important characteristics of damper-connected adjacent buildings and to make the problem manageable, only the two-dimensional system consisting of two linear elastic shear buildings connected by viscoelastic dampers at each floor of the same level is considered in the present study (see Fig. 6.1). The assumption of linear elastic buildings indicates that only small and moderate seismic events are concerned. Each viscoelastic damper is represented by the Voigt model. The mass of each building is concentrated at its floor and the stiffness is provided by its massless columns. Both buildings are assumed to be subject to the same base acceleration and any effects due to spatial variations of the ground motion or due to soil-structure interactions are neglected.

Assume that a total number of degrees of freedom of two adjacent buildings are \( N \) (see Fig. 6.1), in which the number of degrees of freedom of the left building is \( L \) with its first floor designated as the first degree of freedom. \( N-L \) is then the number of degrees of freedom of the right building with its first floor designated as the \( L+1 \) degree of freedom. The equations of motion of the building-damper system can be expressed as

\[
M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = -M_x\ddot{x}_g(t) \tag{6-1}
\]

where \( M \) is the mass matrix of the adjacent buildings; \( K \) and \( C \) are the total stiffness and damping matrices of the system, respectively, including structural stiffness and damping coefficients as well as the stiffness and damping coefficients from viscoelastic dampers; \( x(t) \) is the vector of relative displacement response with respect to the ground with the left building's displacements in the first \( L \) positions and the
right building’s displacement in the last N-L positions: e is the index vector with all its elements equal to 1; and \( \ddot{x}_g(t) \) is the ground acceleration.

Denote the mass, shear stiffness, and external damping coefficient and internal damping coefficient of the adjacent buildings as \( m_i, k_i, b_i, c_i \) \((i=1,2,\ldots,N)\), respectively, and the damping coefficient and stiffness coefficient of the viscoelastic damper at the ith floor as \( c_{di} \) and \( k_{di} \), respectively. The details of each matrix in Equation (6-1) can then be given as follows:

\[
M = \text{diag}[m_1, m_2, \cdots, m_N] \tag{6-2}
\]

\[
C = C^e + C' + C^d \tag{6-3}
\]

\[
K = K^e + K^d \tag{6-4}
\]

in which the external damping matrix is

\[
C^e = \text{diag}[b_1, b_2, \cdots, b_N] \tag{6-5}
\]

The internal damping matrix is

\[
C' = \begin{bmatrix}
C_L & 0 \\
0 & C_R
\end{bmatrix} \tag{6-6}
\]

\[
C_L = 
\begin{bmatrix}
c_1 + c_2 & -c_3 \\
-c_2 & c_2 + c_3 & -c_4 \\
& & \ddots & \ddots \\
& & & c_\text{l-1} + c_\text{l} & -c_\text{1} \\
& & & -c_\text{l} & c_\text{1}
\end{bmatrix} \tag{6-7}
\]

\[
C_R = 
\begin{bmatrix}
c_\text{l-1} + c_\text{l-2} & -c_\text{l-1} \\
-c_\text{l-2} & c_\text{l-2} + c_\text{l-1} & -c_\text{l-1} \\
& & \ddots & \ddots \\
& & & c_\text{N-1} + c_\text{N} & -c_\text{N} \\
& & & -c_\text{N} & c_\text{N}
\end{bmatrix} \tag{6-8}
\]
The damping matrix attributed to the viscoelastic dampers is

\[
C^d = \begin{bmatrix}
C_{1(N-L)+(N-L)} & 0_{(N-L)+(2L-N)} & -C_{1(N-L)+(N-L)} \\
0_{1(2L-N)+(N-L)} & 0_{(2L-N)+(N-L)} & 0_{(2L-N)+(N-L)} \\
-C_{1(N-L)+(N-L)} & 0_{(N-L)+(2L-N)} & C_{1(N-L)+(N-L)}
\end{bmatrix}
\]

(if \(N<2L\)) \hspace{1cm} (6-9)

\[
C_{1(N-L)+(N-L)} = \text{diag}(c_{d1}, c_{d2}, \ldots, c_{d(N-L)})
\] \hspace{1cm} (6-10)

or

\[
C^d = \begin{bmatrix}
C_{L+L} & -C_{L+L} & 0_{L+(N-2L)} \\
-C_{L+L} & C_{L+L} & 0_{L+(N-2L)} \\
0_{(N-2L)+L} & 0_{(N-2L)+L} & 0_{(N-2L)+(N-2L)}
\end{bmatrix}
\]

(if \(N\geq2L\)) \hspace{1cm} (6-11)

\[
C_{L+L} = \text{diag}(c_{d1}, c_{d2}, \ldots, c_{dL})
\] \hspace{1cm} (6-12)

The stiffness matrix of the adjacent buildings is

\[
K^* = \begin{bmatrix}
K_L & 0 \\
0 & K_R
\end{bmatrix}
\]

\[
K_L = \begin{bmatrix}
k_{1} + k_{2} & -k_{2} & \cdots & 0 \\
-k_{2} & k_{2} + k_{3} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
-k_{L+1} & k_{L+1} & \cdots & -k_{L}
\end{bmatrix}
\]

(6-13)

(6-14)

\[
K_R = \begin{bmatrix}
k_{L+1} + k_{L+2} & -k_{L+2} & \cdots & 0 \\
-k_{L+2} & k_{L+2} + k_{L+3} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
-k_{N-1} & k_{N-1} + k_{N} & \cdots & -k_{N}
\end{bmatrix}
\]

(6-15)

The stiffness matrix attributed to the viscoelastic dampers is

\[
K^d = \begin{bmatrix}
K_{1(N-L)+(N-L)} & 0_{1(N-L)+(2L-N)} & -K_{1(N-L)+(N-L)} \\
0_{1(2L-N)+(N-L)} & 0_{(2L-N)+(N-L)} & 0_{(2L-N)+(N-L)} \\
-K_{1(N-L)+(N-L)} & 0_{(N-L)+(2L-N)} & K_{1(N-L)+(N-L)}
\end{bmatrix}
\]

(if \(N<2L\)) \hspace{1cm} (6-16)
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\[ K_{(N-L)\times(N-L)} = \text{diag}[k_{d1}, k_{d2}, \ldots, k_{dN-L}] \]  

(6-17)

or

\[
K^d = \begin{bmatrix}
K_{L\times L} & -K_{L\times L} & 0_{L\times (N-2L)} \\
-K_{L\times L} & K_{L\times L} & 0_{L\times (N-2L)} \\
0_{(N-2L)\times L} & 0_{(N-2L)\times L} & 0_{(N-2L)\times (N-2L)}
\end{bmatrix}
\]  

(if \(N \geq 2L\))  

(6-18)

\[ K_{L\times L} = \text{diag}[k_{d1}, k_{d2}, \ldots, k_{dL}] \]  

(6-19)

6.2.2 Solution for Dynamic Characteristics

The application of the response spectrum analysis method requires estimates of the dynamic characteristics of the system to be available. Since the adjacent buildings linked by viscoelastic dampers are non-classically damped systems, to facilitate the seismic design of such systems the complex mode superposition method (Hurty and Rubinstein, 1964) is employed in this study to estimate their dynamic characteristics. It should be pointed out that the complex mode superposition method has been introduced in the previous chapters. However, for the sake of clear understanding of the sensitivity study in this chapter, this method is briefly mentioned here.

For the homogeneous form of equation (6-1), a general solution can be assumed of the form

\[ x(t) = \phi e^{st} \]  

(6-20)

in which \(s\) is the eigenvalue and \(\phi\) is the associated eigenvector. Equation (6-1) can also be reformulated into a first-order \(2N\)-dimensional equation.

\[ Aq + Bq = F\ddot{x}_s(t) \]  

(6-21)

where

\[ A = \begin{bmatrix} 0 & M \\ M & C \end{bmatrix}, \quad B = \begin{bmatrix} -M & 0 \\ 0 & K \end{bmatrix}, \quad F = \begin{bmatrix} 0 \\ -ME \end{bmatrix} \text{ and } q = \begin{bmatrix} \dot{x} \\ x \end{bmatrix} \]  

(6-22)
The solution of the homogeneous form of equation (6-21) can then be taken as

\[
q = \begin{cases} 
  s\phi \\
  \phi 
\end{cases} e^{st} = \phi e^{st} \tag{6-23}
\]

The associated eigenvalue problem of equation (6-21) becomes

\[ (sA + B)\phi = 0 \] \tag{6-24}

### 6.2.2.1 Underdamped System

The solution of equation (6-24) comprises a set of 2N eigenvalues and eigenvectors that are real or exist in complex conjugate pairs. If all the eigenvalues and eigenvectors are in complex conjugate pairs with negative real parts, the system is defined as the underdamped system. For an underdamped system,

\[
\hat{\phi}_j = \phi^*_{j\infty} \quad \text{and} \quad s_j = s^*_{j\infty} \quad (j=1,2,\ldots,N) \tag{6-25}
\]

Each eigenvalue is usually written under the form

\[
s_j = s^*_{j\infty} = -\omega_j \xi_j + i\omega_{ai} \quad (j=1,2,\ldots,N) \tag{6-26}
\]

in which

\[
\omega_j = |s_j|, \quad \xi_j = -\text{Re}(s_j)/|s_j| \quad \text{and} \quad \omega_{ai} = \omega_j \sqrt{1 - \xi_j^2} \tag{6-27}
\]

\(\omega_j, \omega_{ai}, \text{and } \xi_j\) are the modal frequency, the damped modal frequency, and the modal damping ratio, respectively, associated with mode \(j\). The superscript * means the conjugation. For the underdamped system, the value of \(\xi_j\) is always less than one.

To find the sensitivity of modal frequency and modal damping ratio to the damping coefficient \(c_{dn}\) of the \(m\)th damper, the partial derivative of equation (6-24) with respect to \(c_{dn}\), is taken.

\[
\left(\frac{\partial s_j}{\partial c_{dn}} A + s_j \frac{\partial A}{\partial c_{dn}} \right)\hat{\phi}_j + (s_j A + B) \frac{\partial \hat{\phi}_j}{\partial c_{dn}} = 0 \tag{6-28}
\]
Since matrices $A$ and $B$ are symmetric, the following relationship exists by means of equation (6-24).

$$\dot{\phi}_j^\top (s_j A + B) = 0$$  \hspace{1cm} (6-29)

Multiplying equation (6-28) by $\dot{\phi}_j^\top$ and using equation (6-29) lead to

$$\frac{\partial s_j}{\partial c_{\text{dm}}} = -(s_j \phi_j^\top \frac{\partial A}{\partial c_{\text{dm}}} \phi_j)/A_j = -s_j C_{\text{pm}}/A_j \hspace{1cm} (6-30)$$

in which

$$C_{\text{pm}} = \phi_j^\top \frac{\partial A}{\partial c_{\text{dm}}} \phi_j = \phi_j^\top \frac{\partial C}{\partial c_{\text{dm}}} \phi_j \hspace{1cm} (6-31)$$

$$A_j = \phi_j^\top A \phi_j \hspace{1cm} (6-32)$$

For the underdamped system, differentiating the following relationships with respect to the $m$th damper damping coefficient $c_{\text{dm}}$

$$s_j, s_j^* = \omega_j \quad \text{and} \quad s_j + s_j^* = -2\xi_j \omega_j \hspace{1cm} (6-33)$$

and using equation (6-30) yield

$$\frac{\partial \omega_j}{\partial c_{\text{dm}}} = -\omega_j \text{Re}(\frac{C_{\text{pm}}}{A_j}) \hspace{1cm} (6-34)$$

$$\frac{\partial \xi_j}{\partial c_{\text{dm}}} = -\sqrt{1 - \xi_j^2} \text{Im}(\frac{C_{\text{pm}}}{A_j}) \hspace{1cm} (6-35)$$

By using equations (6-34) and (6-35), the sensitivity of $j$th modal frequency and $j$th modal damping ratio to the damping coefficient $c_{\text{dm}}$ of the $m$th damper can be found. In a similar way, the sensitivity of $j$th modal frequency and $j$th modal damping ratio to the stiffness coefficient $k_{\text{dm}}$ of the $m$th damper can be obtained.

$$\frac{\partial \omega_j}{\partial k_{\text{dm}}} = \xi_j \text{Re}(\frac{K_{\text{pm}}}{A_j}) - \sqrt{1 - \xi_j^2} \text{Im}(\frac{K_{\text{pm}}}{A_j}) \hspace{1cm} (6-36)$$
\[
\frac{\partial \xi_j}{\partial k_{dm}} = \sqrt{1 - \frac{\xi_j^2}{\omega_i^2}} \left[ \frac{K_{jm}}{A_j} \text{Im} \left( \frac{C_{jm}}{A_j} \right) + \sqrt{1 - \frac{\xi_j^2}{\omega_i^2}} \frac{K_{jm}}{A_j} \text{Re} \left( \frac{C_{jm}}{A_j} \right) \right]
\]  

(6-37)

in which

\[
K_{jm} = \phi_j^T \frac{\partial K}{\partial k_{dm}} \phi_j
\]

(6-38)

**6.2.2.2 System with Mixed Damping**

For the adjacent buildings connected by viscoelastic dampers, equation (6-24) may produce some real-valued negative pairs, each associated with a real-valued eigenvector, in addition to some eigenvalues and eigenvectors which are in complex conjugate pairs with negative real parts. In this case, the system may be called the mixed damped system in general, and it is convenient to express real pairs \( s_i \) in the following form analogous to equation (6-26).

\[
s_i = -\omega_i \xi_i + \omega_i d_i
\]

(6-39)

\[
s_{i-\infty} = -\omega_i \xi_i - \omega_i d_i
\]

(6-40)

in which \( \omega_i, \omega_i \) and \( \xi_i \) are determined by

\[
\omega_i = \sqrt{s_i s_{i-\infty} - \xi_i^2} = -(s_i + s_{i-\infty})/(2\omega_i) \quad \text{and} \quad \omega_i = \omega_i \sqrt{\xi_i^2 - 1} = (s_i - s_{i-\infty})/2
\]

(6-41)

The value of \( \xi_i \) is now equal to or greater than one. The sensitivity study of the modal frequency and modal damping ratio with respect to the mth damper damping coefficient \( c_{dm} \) or the mth damper stiffness coefficient \( k_{dm} \) is based on the following equations.

\[
\frac{\partial \omega_i}{\partial c_{dm}} = -\frac{\omega_i}{2} \left( \frac{C_{jm}}{A_j} + \frac{C_{i-\infty,m}}{A_{i-\infty}} \right)
\]

(6-42)

\[
\frac{\partial \xi_i}{\partial c_{dm}} = \frac{\sqrt{\xi_i^2 - 1}}{2} \left( \frac{C_{jm}}{A_j} - \frac{C_{i-\infty,m}}{A_{i-\infty}} \right)
\]

(6-43)
\[
\frac{\partial \omega_j}{\partial k_{dm}} = \frac{\zeta_j}{2} \left( \frac{K_{j,m}}{A_j} + \frac{K_{j,N,m}}{A_{j,N}} \right) + \frac{\sqrt{\zeta_j^2 - 1}}{2} \left( \frac{K_{j,m}}{A_j} - \frac{K_{j,N,m}}{A_{j,N}} \right) 
\]

(6-44)

\[
\frac{\partial \xi_j}{\partial k_{dm}} = -\frac{\sqrt{\zeta_j^2 - 1}}{2\omega_j} \left[ \sqrt{\zeta_j^2 - 1} \left( \frac{K_{j,m}}{A_j} + \frac{K_{j,N,m}}{A_{j,N}} \right) + \zeta_j \left( \frac{K_{j,m}}{A_j} - \frac{K_{j,N,m}}{A_{j,N}} \right) \right] 
\]

(6-45)

6.2.3 Solution for Seismic Response

The complex mode superposition method adopts the following coordinate

\[
q = \Phi z 
\]

(6-46)

where \( z \) is the 2N-dimentional generalized coordinate vector and \( \Phi \) is the 2N×2N complex modal matrix.

\[
\Phi = [\hat{\phi}_1, \hat{\phi}_2, \ldots, \hat{\phi}_{2N}] 
\]

(6-47)

By using the coordinate transformation and the orthogonality of modes, the equation (6-21) can be reduced to 2N decoupled modal equations with the jth modal equation being

\[
A_j \ddot{z}_j + B_j z_j = \hat{\phi}_j^T F \ddot{x}_g(t) 
\]

(6-48)

or alternatively

\[
\ddot{z}_j - s_j z_j = r_j \ddot{x}_g(t) 
\]

(6-49)

in which

\[
B_j = \hat{\phi}_j^T \Phi = -s_j A_i 
\]

(6-50)

\[
r_j = \hat{\phi}_j^T F / A_i = -\phi_j^T ME / A_i 
\]

(6-51)

Assume that the ground acceleration \( \ddot{x}_g(t) \) is a stationary random process and its power spectral density function is given as \( S_x(\omega) \). The generalized pseudo-
excitation method is now used to determine the seismic response of adjacent buildings connected by viscoelastic dampers.

The pseudo-excitation is constituted for a given frequency \( \omega \) as

\[
\ddot{x}_g(t) = \sqrt{S_g(\omega)} e^{i\omega t}
\]  

(6-52)

The solution of the first-order equation (6-49) to the pseudo-excitation is

\[
z_j(\omega, t) = \frac{r_j}{i\omega - s_j} \sqrt{S_g(\omega)} e^{i\omega t} \quad (j=1,2,\ldots,2N)
\]  

(6-53)

Substituting equation (6-53) into equation (6-46) and comparing with the last part of equation (6-22), one obtains

\[
x(\omega, t) = \sum_{j=1}^{2N} \phi_j z_j(\omega, t) = \sum_{j=1}^{2N} \phi_j \frac{r_j}{i\omega - s_j} \sqrt{S_g(\omega)} e^{i\omega t}
\]  

(6-54)

Since the eigenvector is in pairs in either underdamped system or the system with mixed damping, the \( x(\omega, t) \) can be reduced to

\[
x(\omega, t) = \sum_{j=1}^{N} H_j(\omega)(i\omega \alpha_j + \beta_j) \sqrt{S_g(\omega)} e^{i\omega t} = x(\omega) e^{i\omega t}
\]  

(6-55)

in which \( x(\omega) \) is called the pseudo displacement

\[
x(\omega) = \sum_{j=1}^{N} H_j(\omega)(i\omega \alpha_j + \beta_j) \sqrt{S_g(\omega)}
\]  

(6-56)

\( H_j(\omega) \) is the frequency response function for the jth mode.

\[
H_j(\omega) = \frac{1}{\omega_j^2 - \omega^2 + i2\xi_j\omega_j \omega}
\]  

(6-57)

When the jth mode is an underdamped mode,

\[
\alpha_j = 2 \text{Re}(\phi_j r_j) \quad \text{and} \quad \beta_j = -2 \text{Re}(\phi_j r_j s_j^*)
\]  

(6-58)

When the jth mode is an overdamped mode or a critically damped case

\[
\alpha_j = (\phi_j r_j + \phi_j r_j s_j) \quad \text{and} \quad \beta_j = -(\phi_j r_j + \phi_j r_j s_j)
\]  

(6-59)
Similarly, the pseudo velocity response can be obtained by
\[ \dot{x}(\omega) = \sum_{j=1}^{N} H_j(\omega)(i\omega \mu_j + \nu_j) \sqrt{S_j(\omega)} \] (6-60)

in which when the jth mode is an underdamped mode
\[ \mu_j = 2\text{Re}(s_j \phi_j r_j) \quad \text{and} \quad \nu_j = -2\omega_j^2 \text{Re}(\phi_j r_j) \] (6-61)

When the jth mode is an overdamped mode or a critically damped case
\[ \mu_j = (s_j \phi_j r_j + s_j \phi_j \phi_j \phi_j r_j) \quad \text{and} \quad \nu_j = -\omega_j^2 (\phi_j r_j + \phi_j \phi_j \phi_j r_j) \] (6-62)

Once the pseudo displacement is determined, the pseudo internal force can be easily determined following a static analysis. For instance, the pseudo shear force of the adjacent buildings can be calculated by
\[ Q(\omega) = Gx(\omega) \] (6-63)

where
\[ G = \begin{bmatrix} G_L & 0 \\ 0 & G_R \end{bmatrix} \] (6-64)

and
\[ G_L = \begin{bmatrix} k_1 \\ -k_2 & k_2 \\ & \ddots \\ & & -k_{L-1} & k_{L-1} \\ & & & -k_L & k_L \end{bmatrix} \] (6-65)

\[ G_R = \begin{bmatrix} k_{L+1} \\ -k_{L+2} & k_{L+2} \\ & \ddots \\ & & -k_{N-1} & k_{N-1} \\ & & & -k_N & k_N \end{bmatrix} \] (6-66)
The mth pseudo damper force can be determined as follows

\[ f_{r_m} = k_{dm}(x_{L-m} - x_m) + c_{dm}(\dot{x}_{L-m} - \dot{x}_m) \]  

(6-67)

The response spectral matrix can then be obtained by

\[ S_{xx}(\omega) = x^r(\omega)x^r(\omega). \quad S_{\dot{x}x}(\omega) = \dot{x}^r(\omega)\dot{x}^r(\omega) \]  

(6-68)

\[ S_{\dot{x}x}(\omega) = \dot{x}^r(\omega)x^r(\omega). \quad S_{\ddot{x}x}(\omega) = \ddot{x}^r(\omega)\dot{x}^r(\omega) \]  

(6-69)

\[ S_{QQ}(\omega) = Q^r(\omega)Q^r(\omega) \]  

(6-70)

The auto-spectral density function of mth damper can be determined by

\[ S_{f_r f_r}(\omega) = f_{r_m}^r f_{r_m} \]  

(6-71)

The standard deviation displacement response of the jth floor \( \sigma_{x_j} \) is finally evaluated from \( S_{x_j x_j}(\omega) \) through integration.

\[ \sigma_{x_j}^2 = \int S_{x_j x_j}(\omega) d\omega \]  

(6-72)

The standard deviation acceleration response of the jth floor \( \sigma_{\ddot{x}_j} \) is given by

\[ \sigma_{\ddot{x}_j}^2 = \int \omega^2 S_{x_j x_j}(\omega) d\omega \]  

(6-73)

The standard deviation force response \( \sigma_{f_{r_m}} \) of the mth damper is evaluated from \( S_{f_{r_m} f_{r_m}}(\omega) \) through integration

\[ \sigma_{f_{r_m}}^2 = \int S_{f_{r_m} f_{r_m}}(\omega) d\omega \]  

(6-74)

### 6.2.4 Application to Example Buildings

For application, two 20-story buildings having the same floor elevations with viscoelastic dampers connecting two neighboring floors are used. The mass, shear
stiffness, internal damping coefficient, and external damping coefficient of the left building (also called the stiffer building, see Fig. 6.1) are uniform for all stories with the mass of $1.29 \times 10^6$ kg, the shear stiffness of $4.0 \times 10^9$ N/m, the internal damping coefficient of $3.0 \times 10^9$ N·sec/m, and the external damping coefficient of $8.0 \times 10^4$ N·sec/m. For the right building (also called the softer building), the mass, shear stiffness, internal damping coefficient, and external damping coefficient are also uniform for all stories with the same mass, internal damping and external coefficient as the left building but with the shear stiffness of $2.0 \times 10^9$ N/m only. Hence, the two buildings have the same height but the right building is less stiff than the left building. Viscoelastic dampers used to connect every two neighboring floors are assumed to have the same damper damping coefficient and damper stiffness. The Kanai-Tajimi filtered white noise spectrum is used as the ground acceleration spectrum in the computation in this study.

$$S_g(\omega) = \frac{1 + 4\xi_g^2\left(\frac{\omega}{\omega_g}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_g}\right)^2\right]^2 + 4\xi_g^2\left(\frac{\omega}{\omega_g}\right)^2}S_0 \quad (6-75)$$

in which $\omega_g$, $\xi_g$, $S_0$ may be regarded as the characteristics and the intensity of an earthquake in a particular geological location. The parameters in the ground acceleration spectrum are selected as $\omega_g = 15.0$ rad/s; $\xi_g = 0.65$ and $S_0 = 4.65 \times 10^4$ m²/rad·s³.

### 6.2.4.1 Modal Frequencies and Damping Ratios

The modal frequencies and damping ratios of each building without dampers connected are calculated. Within the frequency range between zero and 20.00 rad/s.
the first three modal frequencies of the right building are 3.02, 9.03, and 14.99 rad/s respectively. The first two modal frequencies of the left building are 4.27 and 12.77 rad/s respectively. When every two neighboring floors of the adjacent buildings are linked by the viscoelastic dampers of the same damping coefficient of $1.43 \times 10^6$ N.sec/m and the same elastic stiffness of $10^5$ N/m, the first five modal frequencies of the damper-building system calculated by the complex mode superposition method are 3.30, 3.93, 9.11, 12.68, and 15.04 rad/s respectively. Clearly, using the viscoelastic dampers to link the adjacent buildings only slightly changes the modal frequencies of the individual buildings. The retention of the natural frequencies of the unlinked buildings after the installation of the joint dampers is especially desirable for the adjacent buildings that have been already built and need to be strengthened. As to modal damping ratios, the first three modal damping ratios in the unlinked left building are calculated as 1.25%, 1.02%, and 1.33% respectively. The first two model damping ratios in the unlinked right building are computed as 0.89% and 0.72% respectively. For the linked building-damper system, the first five modal damping ratios calculated by the complex mode superposition method are 21.97%, 11.71%, 7.20%, 5.03%, and 5.04% respectively. Obviously, the viscoelastic dampers offer significant damping to the adjacent buildings. Thus, one can expect that the seismic response of the adjacent buildings will be significantly reduced.

The parameters used in the foregoing dampers are determined through a parametric study. In the parametric study, the modal frequencies and damping ratios of the building-damper system are computed against the stiffness and damping coefficient of the dampers. The beneficial stiffness $k_d$ and damping coefficient $c_d$ of the dampers can be thus found for achieving the maximum modal damping ratio and maintaining the original modal frequencies. Fig. 6.2 displays variations of the first
and second modal frequencies of the system with damper stiffness \( k_d \) for damper
damping coefficient \( c_d = 0 \) Ns/m and \( c_d = 10^6 \) Ns/m. It is seen that the modal
frequencies are almost independent of damper stiffness when the damper stiffness is
less than \( 5 \times 10^5 \) N/m. However, if the damper stiffness is beyond this value, the
second modal frequency of the system will have a rapid increase while the first
modal frequency increases moderately. It is also seen from Fig. 6.2 that the modal
frequencies may not be sensitive to damper damping coefficient \( c_d \) particularly in the
case of the use of high damper stiffness. Fig. 6.3 shows variations of the first two
modal damping ratios of the system with damper stiffness for three damper damping
coefficients. Clearly, the first two modal damping ratios of the system remain almost
constant within a damper stiffness range from zero to \( 1 \times 10^5 \) N/m. After this range,
the first modal damping ratio will decrease rapidly with the increasing damper
stiffness and the second modal damping ratio will increase first and then decrease for
the cases of non-zero damper damping coefficient. Thus, the optimal value of
damper stiffness is selected as \( 1 \times 10^5 \) N/m. Fig. 6.3 also indicates that the damper
damping coefficient affects the first two modal damping ratios of the system
significantly.

The variations of the first five modal frequencies with damper damping
coefficient are shown in Fig. 6.4 for the damper stiffness of \( 1 \times 10^5 \) N/m. All the
modal frequencies remain almost constant when the damper damping coefficient is
less than \( 1 \times 10^6 \) Ns/m. After that, the modal frequencies have a sudden change: the
three modal frequencies dominated by the softer building (\( \omega_1, \omega_3 \), and \( \omega_4 \)) become
larger while the two modal frequencies governed by the stiffer building (\( \omega_2 \) and \( \omega_4 \))
become smaller. This is an expected result for the dampers of very high damping
coefficient. The modal damping ratios in the three modes dominated by the softer building ($\omega_1$, $\omega_3$, and $\omega_4$) are depicted in Fig. 6.5a against the damper damping coefficient while the modal damping ratios in the two modes dominated by the stiffer building ($\omega_2$ and $\omega_4$) are plotted in Fig. 6.5b. It is seen that for the damper damping coefficient less than about $1 \times 10^4$ Ns/m, the dampers have no effect on the modal damping ratios, and the modal damping ratios come mainly from the buildings themselves. As the damper damping coefficient increases, all the modal damping ratios increase. However, when the damper damping coefficient is increased to certain values, the modal damping ratios dominated by the stiffer building decrease while the modal damping ratios governed by the softer building still increase and soon these modes of the softer building become overdamped modes. Therefore, for a very large damper damping coefficient, the system behaves like a lightly damped stiffer building supported by a softer building almost without motion due to very high vibration energy dissipation capability. From a practical point of view, the optimum value of the damper damping coefficient should be selected around $1.43 \times 10^6$ Ns/m, by which all the modal damping ratios of either the stiffer building or the softer building reach certain values enough to reduce the seismic modal response of the whole system.

A sensitivity study of the modal frequency and modal damping coefficient to damper stiffness and damper damping coefficient is carried out at the optimum values $c_d=1.43 \times 10^6$ Ns/m and $k_d=1 \times 10^7$ N/m. The sensitivities of the first modal frequency $\omega_1$ to changes in $c_{d_i}$ or $k_{d_i}$ ($i=1, 2, \ldots, 20$) are depicted in Fig. 6.6a in terms of $\partial \omega_1 / \partial c_{d_i}$ and $\partial \omega_1 / \partial k_{d_i}$. The sensitivities of the first modal damping ratio $\xi_1$ to changes in $c_{d_i}$ or $k_{d_i}$ ($i=1, 2, \ldots, 20$) are plotted in Fig. 6.6b by means of $\partial \xi_1 / \partial c_{d_i}$ and

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As expected, the first modal frequency and first modal damping ratio are more sensitive to the damper at the top of the buildings than others. The very small sensitivities to the dampers near the bottom of the buildings may indicate no need to install these dampers. The negative sensitivity, or gradient, of the first modal damping ratio to damper stiffness is because as the damper stiffness increases the first modal damping ratio decreases. Similar sensitivity results are found for the second modal frequency and damping ratio. Since all the sensitivity values are small even for the damper at the top of the buildings, one may conclude that the small deviations of the dampers from their optimal values ($c_d=1.43\times10^6$ N/s and $k_d=1\times10^5$ N/m) may not affect the control efficiency.

6.2.4.2 Seismic Response

Seismic response analysis is carried out to investigate variations of seismic response of the adjacent buildings with damper parameters to see if the optimal damper parameters identified from the modal analysis are the same as those from the seismic response analysis under the given earthquake excitation spectrum. Then, the effectiveness of the dampers of optimal parameters on seismic response reduction is examined. Because of the limitation of space, only a few typical figures are given in this chapter.

Figs 6.7 and 6.8 depict the variations of the top floor displacement responses of the left building and the right building, respectively, with damper stiffness for several damper damping coefficients. It is seen that the top floor displacement responses of both the left and right buildings are not affected by the damper stiffness if the damper stiffness is less than $1\times10^5$ N/m. The fact that the response mitigation is not sensitive to damper stiffness within a certain range is very helpful for the
practical application of joint dampers. The further increase of the damper stiffness from $1 \times 10^5$ N/m may reduce the seismic response of the left building as shown in Fig. 6.7, but it may increase the seismic response of the right building as shown in Fig. 6.8. If the damper stiffness is increased to a level larger than $6 \times 10^6$ N/m, the effectiveness of joint dampers deteriorates rapidly. This is because the strong damper stiffness reduces the relative velocity of the damper and hence the energy absorbing capacity from the dampers decreases. In particular, when the damper stiffness reaches a value above $1 \times 10^7$ N/m, the relative displacement and velocity between the adjacent buildings become nearly zero so that the two buildings behave as though almost rigidly connected. As a result, no matter what value of the damper damping coefficient is used, the damper totally loses its effectiveness. It is clear from the foregoing two figures that to achieve the maximum reduction of the dynamic response of both buildings, the optimum damper stiffness should be less than $1 \times 10^6$ N/m, which is the same as one found from the modal analysis. In addition, it can be seen from Figs 6.7 and 6.8 that there is an optimal damper damping coefficient between $5 \times 10^5$ Ns/m and $3 \times 10^6$ Ns/m.

To find optimal damper damping coefficient, the seismic responses including the top floor displacement response, the base shear force response, and the top floor acceleration response of both buildings are computed over a wide range of damper damping coefficient with an optimum damper stiffness of $1 \times 10^6$ N/m. Fig. 6.9 shows the variations of the top floor displacement responses of the two buildings with damper damping coefficient. Clearly, the optimum damper damping coefficient is about $1.43 \times 10^6$ Ns/m at which the displacement responses of both buildings are reduced to the minimum. With the decrease of damping coefficient from the
optimum value, the performance of the damper deteriorates gradually and as the damping coefficient approaches zero the two buildings finally return to the unlinked situation. On the other hand, if the damping coefficient increases from the optimum value, the performance of the damper also declines and as the damping coefficient becomes very large the two buildings behave as though almost rigidly connected. As a result, the top floor displacements of the two buildings become the same. Again, the optimal value of damper damping coefficient found here is the same as one found in the modal analysis.

The reason why the optimum values obtained from the modal analysis and seismic response analysis are the same is that the seismic displacement and shear force responses of both buildings are dominated by the first two modes of the system only (i.e., the first mode of the softer building plus the first mode of the stiffer building). Since the higher modes of vibration may have effects on the acceleration responses of both buildings, the optimum value of the damper damping coefficient found based on the acceleration response is slightly larger than $1.43 \times 10^9$ Ns/m.

To demonstrate the overall effectiveness of the joint dampers, the standard deviations of displacement, shear force and acceleration responses at each floor for each building with and without joint dampers are computed using the Kanai-Tajimi excitation spectrum. Fig. 6.10 shows the variations of the standard deviation of displacement response relative to the ground with the height of the buildings. The top floor displacement standard deviation of the unlinked left building is 44.7 mm but with the joint dampers installed, it is reduced to 19.3 mm, leading to a 57% reduction of the response. For right building, the top floor displacement standard deviation is 61.1 mm for the unlinked building and 22.2 mm for the linked building, resulting in a 64% reduction. The reduction of the displacement responses from the
joint dampers is also significant for other floors in either building. The standard deviations of shear force in each story for each building are plotted in Fig. 6.11. Unlike the adjacent buildings connected by hinged rigid links (Westermo 1989), the shear forces in all the stories of both buildings are reduced after installation of the joint dampers. In particular, without the joint dampers the bottom shear force standard deviation is $1.41\times10^5$ N in the left building and $9.68\times10^6$ N in the right building. With the optimum joint dampers, the base shear force standard deviation is reduced to $6.06\times10^6$ N in the left building and $3.55\times10^6$ N in the right building, leading to a 57% and a 63% reduction, respectively.

The variations of acceleration response with the building height, as shown in Fig. 6.12, are different from displacement and shear force response profiles shown in Figs 6.10 and 6.11. The acceleration response for each unlinked building does not vary monotonically with the height of the building. This is due to the contributions from higher modes of vibration (Xu et al. 1999). Clearly, the joint dampers effectively mitigate the acceleration responses not only from low modes of vibration but also higher modes of vibration, as indicated by the response curves of the linked adjacent buildings.

The values of viscoelastic damper forces required for the achievement of significant vibration reduction of the adjacent buildings are important for the design of the viscoelastic dampers and adjacent buildings. In terms of the generalized pseudo-excitation method, the viscoelastic damper forces can be easily calculated. The results are shown in Fig. 6.13 for the variations of viscoelastic damper force with the building height. The maximum damper force is $1.01\times10^5$ N at the top of the buildings.
In order to utilize VE dampers within the adjacent building system, it is necessary to formulate design guidelines and procedures, based upon knowledge gained from the above theoretical studies. The thickness of the VE material can be determined from the maximum allowable damper deformation to insure that the maximum strain in the VE material is smaller than the maximum allowable values. The design of other parameters of VE dampers, such as the number and length of viscoelastic layers, was discussed by Soong and Dargush (1997).

6.3 ADJACENT BUILDINGS LINKED BY FLUID DAMPERS

Fluid dampers that operate on the principle of fluid flow through orifices specially shaped have found more and more applications to vibration mitigation of buildings and structures (Constantinou et al. 1993). They have been recently installed into buildings and structures for mitigating earthquake- or wind-induced vibration (Constantinou and Symans 1993a). Fluid dampers are of primary interest in structural applications because they have several inherent and significant advantages: linear viscous behavior over a broad frequency range; insensitivity to temperature changes; small size in comparison to stroke and output force; easy installation; almost maintenance free; reliability and longevity (Constantinou and Symans 1993a). Furthermore, they can be manufactured less expensively to satisfy different requirements for damper parameters such as the maximum damping force, the maximum operating velocity, the maximum operating displacement, and no measurable stiffness for piston motions. For example, the fluid inertial damper used in a medical center each has the maximum damping force of 1.456 kN, the maximum displacement of 1.2 m, and the maximum velocity of 1.5 m/s, but the length and weight of the damper are only 4.4 m and 13.4 kN respectively. The fluid damper
exhibits viscoelastic fluid behavior, and the simplest model to account for this behavior is the Maxwell model, which is a combination of a linear and elastic spring and a viscous dashpot connected in series, as a result of investigation carried out by Constantinou and Symans (1993b).

### 6.3.1 Basic Equations

Consider again the same buildings in section 6.2.1 but connected by fluid dampers at each floor of the same level (see Fig. 6.14). The fluid damper commercially available consists of a cylinder and a stainless steel piston with a bronze orifice head and an accumulator. The orifice utilizes a series of specially shaped passages to alter flow characteristics with fluid speed. The research carried out by Constantinou and Symans (1993b) on the fluid damper showed that the Maxwell model proposed by Bird (1987) could be used to describe the viscoelastic fluid behavior of the fluid damper applicable to civil engineering structures.

\[ f_r + \lambda \frac{df_r(t)}{dt} = C_u \ddot{x} \]  

(6-76)

where \( f_r \) is the damper force; \( \lambda \) is the relaxation time; \( C_u \) is the damping coefficient at zero frequency; and \( \ddot{x} \) is the damper velocity. The relaxation time \( \lambda \) can be approximately regarded as the ratio of the damping coefficient at zero frequency to the spring stiffness coefficient in a damper system in which one spring and one dashpot are connected in series. However, for a real fluid damper, the relaxation time and the damping coefficient at zero frequency are identified in a slightly different way (Constantinou and Symans (1993b)).

The equations of motion of the building-damper system can be expressed as

\[ M\dddot{x}(t) + C\dot{x}(t) + Kx(t) + f_r = -Me \dddot{y}_s(t) \]  

(6-77)
with the Maxwell model

\[ f_r + \Lambda \dot{f}_r = D \dot{x} \]  

(6-78)

where \( C \) and \( K \) are the same parameters as those in equations (6-3) and (6-4) except setting \( C^d = 0 \) and \( K^d = 0 \), respectively; \( \Lambda \) and \( D \) are the relaxation time and zero-frequency damping coefficient matrices, respectively, of the fluid dampers; \( f_r \) is the damper force vector.

Denote the relaxation time and damping coefficient at zero frequency of the fluid damper at the \( i \)th floor as \( \lambda_i \) and \( C_{0i} \), respectively. The details of each matrix can then be given as follows:

The relaxation time matrix of the fluid damper is

\[ \Lambda = \text{diag}[\Lambda_{1N-1N-N-L}, 0_{12L-N_{1N-N-L}}, \ldots, \Lambda_{1N-1N-N-L}] \]  

(6-79)

in which

\[ \Lambda_{1N-1N-N-L} = \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_{N-L}] \]  

(6-80)

or

\[ \Lambda_{1N-L} = \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_L] \]  

(6-81)

The zero-frequency damping coefficient matrix of the fluid damper is

\[
D = \begin{bmatrix}
D_{1N-1N-N-L} & 0_{1N-1N-21N-N} & -D_{1N-1N-N-L} \\
0_{121N-N-1N-L} & 0_{121N-N-21N-N} & 0_{121N-N-1N-L} \\
-D_{1N-1N-N-L} & 0_{1N-1N-21N-N} & D_{1N-1N-N-L}
\end{bmatrix}
\]  

(6-83)

in which

\[ D_{1N-1N-N-L} = \text{diag}[C_{01}, C_{02}, \ldots, C_{0N-L}] \]  

(6-84)

or
in which
\[
D_{\text{L.L}} = \text{diag}[C_{0,1}, C_{0,2}, \ldots, C_{0,L}]
\]

(6-86)

### 6.3.2 Solution for Dynamic Characteristics

Since the adjacent buildings linked by fluid dampers are non-classically damped systems with unsymmetrical property matrices, the general eigenvalue analysis is employed in this study to determine the dynamic characteristics of the system. The equations of motion (equations (6-77) and (6-78)) can be replaced by an equivalent first order differential equation of the form

\[
Aq + Bq = F\ddot{x}_s(t)
\]

(6-87)

in which

\[
A = \begin{bmatrix}
M & 0 & 0 \\
0 & I & 0 \\
0 & 0 & A
\end{bmatrix}, \quad B = \begin{bmatrix}
C & K & I \\
-I & 0 & 0 \\
-D & 0 & I
\end{bmatrix}, \quad F = \begin{bmatrix}
-ME \\
0 \\
0
\end{bmatrix}
\]

and \(q = \begin{bmatrix} \dot{x} \\ x \\ f_r \end{bmatrix} \)

(6-88)

and \(I\) is the identity matrix. The matrices \(A\) and \(B\) are of size \(3N \times 3N\).

Since the matrix \(B\) in equation (6-88) is not symmetric, one needs to solve the following two adjoining eigenvalue problems:

\[
(sA + B)\dot{\phi} = 0 \quad \text{and} \quad (sA^T + B^T)\psi = 0
\]

(6-89)

where \(s\) is an eigenvalue, and \(\dot{\phi}\) and \(\psi\) are the corresponding right eigenvector and left eigenvector, respectively, of the form

\[
\dot{\phi} = \begin{bmatrix} s\phi \\ \phi \\ 0 \end{bmatrix}
\]

(6-90)
\[
\hat{\Psi} = \begin{bmatrix}
  s\psi \\
  \psi \\
  \delta
\end{bmatrix}
\]

(6-91)

The solution of each eigenvalue problem comprises a set of 3N eigenvalues and eigenvectors that are either real or exist in complex conjugate pairs. Among these 3N eigenvalues and eigenvectors, there are N real eigenvalues and eigenvectors attributed to fluid dampers. If the practical damper parameters are taken into consideration, the N real eigenvalues attributed to fluid dampers are very large in general. Some of them may be infinite if the damper relaxation time is equal to zero or there is no fluid damper at all. For this reason, the N real eigenvalues related to the fluid dampers are denoted by \( s_{j,2N} \) (j=1,2,\ldots.,N) in this study.

If the adjacent buildings linked by fluid dampers still belong to an underdamped system, the remaining 2N eigenvalues and eigenvectors (either right eigenvectors or left eigenvectors) will be in complex conjugate pairs.

\[
\hat{\phi}_j = \hat{\phi}_{j,N}^* \quad \text{and} \quad s_j = s_{j,N}^*
\]

(6-92)

Each eigenvalue can be written under the form

\[
s_j = s_{j,N}^* = -\omega_j \xi_j + i\omega_{d_j} \quad (j=1,2,\ldots,N)
\]

(6-93)
in which

\[
\omega_j = |s_j|, \quad \xi_j = -\text{Re}(s_j)/|s_j| \quad \text{and} \quad \omega_{d_j} = \omega_j \sqrt{1 - \xi_j^2}
\]

(6-94)

\( \omega_j, \omega_{d_j}, \) and \( \xi_j \) are the modal frequency, the damped modal frequency, and the modal damping ratio, respectively, associated with mode j. The superscript * means the conjugation. For the underdamped mode, the value of \( \xi_j \) is always less than one.

In some cases, which depend on the selected damper parameters, there are some real-valued negative pairs among the 2N eigenvalues. The system may be
called the mixed damped system in this situation, and it is convenient to express real
pair $s_i$ in the following form analogous to equation (6-93).

$$s_i = -\omega_i \xi_i + \omega_{d_i} \quad (j=1,2,...,N)$$  \hspace{1cm} (6-95)

$$s_{i,N} = -\omega_i \xi_i - \omega_{d_i} \quad (j=1,2,...,N)$$  \hspace{1cm} (6-96)

in which $\omega_j$, $\omega_{d_i}$ and $\xi_i$ are determined, respectively, by

$$\omega_i = \sqrt{s_i s_{i,N}}, \quad \xi_i = -(s_i + s_{i,N})/(2\omega_i) \quad \text{and} \quad \omega_{d_i} = \omega_i \sqrt{\xi_i^2 - 1} = (s_i - s_{i,N})/2$$  \hspace{1cm} (6-97)

The value of $\xi_i$ is now equal to or greater than one.

The eigenvalue analysis of a fluid damper-adjacent building system carried out
here is mainly to determine the modal damping ratio and natural frequency of the
system attributed to fluid dampers and to find optimal damper parameters for
achieving the maximum modal damping ratio. Thus, it is desirable to know the
sensitivities of the $j$th modal damping ratio and $j$th natural frequency to the $m$th
damper relaxation time or the $m$th damper damping coefficient at zero frequency.

For this purpose, the partial derivative of the first expression in equation (6-89) with
respect to the $m$th damper relaxation time $\lambda_m$ is taken, which leads to

$$\left( \begin{array}{c} \frac{\partial \hat{S}_i}{\partial \lambda_m} \hat{A} + s_i \frac{\partial \hat{A}}{\partial \lambda_m} \hat{\phi}_i + (s_i \hat{A} + B) \frac{\partial \hat{\phi}_i}{\partial \lambda_m} \end{array} \right) = 0$$  \hspace{1cm} (6-98)

Multiplying equation (6-98) by $\hat{\psi}_i^T$ and using the following relationship

$$\hat{\psi}_i^T (s_i \hat{A} + B) = 0$$  \hspace{1cm} (6-99)

one obtains

$$\frac{\partial \hat{S}_i}{\partial \lambda_m} = - (s_i \hat{\psi}_i^T \frac{\partial \hat{A}}{\partial \lambda_m} \hat{\phi}_i) / A_i = - s_i \Lambda_m / A_i$$  \hspace{1cm} (6-100)

in which
\[ \Lambda_m = \mathcal{G}_j \frac{\partial \Lambda}{\partial \lambda_m} \theta_j \]  

(6-101)

\[ \Lambda_i = \psi_i^T A \dot{\phi}_j \]  

(6-102)

For the underdamped mode, differentiating the following relationships with respect to the mth damper relaxation time \( \lambda_m \):

\[ s_j^* = \omega_j^* \quad \text{and} \quad s_j + s_j^* = -2 \xi_j \omega_j \]  

(6-103)

and using equation (6-100) yield

\[ \frac{\partial \omega_i}{\partial \lambda_m} = -\omega_i \text{Re} \left( \frac{\Lambda_{jm}}{A_i} \right) \]  

(6-104)

\[ \frac{\partial \zeta_i}{\partial \lambda_m} = -\sqrt{1 - \xi_j^2} \text{Im} \left( \frac{\Lambda_{jm}}{A_i} \right) \]  

(6-105)

These two equations can be used for the sensitivity study of the natural frequency and modal damping ratio with respect to damper relaxation time.

In a similar way, one can obtain

\[ \frac{\partial s_i}{\partial C_{0,m}} = -\left( \psi_i^T \frac{\partial B}{\partial C_{0,m}} \dot{\phi}_j \right) / A_i = s_j D_{jm} / A_i \]  

(6-106)

in which

\[ D_{jm} = \mathcal{G}_j \frac{\partial D}{\partial C_{0,m}} \phi_j \]  

(6-107)

For the underdamped mode, the sensitivities of the natural frequency and modal damping ratio with respect to zero-frequency damping coefficient \( C_{0,m} \) are as follows:

\[ \frac{\partial \omega_i}{\partial C_{0,m}} = \omega_i \text{Re} \left( \frac{D_{jm}}{A_i} \right) \]  

(6-108)
\[ \frac{\partial \xi_j}{\partial \omega_j} = \sqrt{1 - \xi_j^2} \frac{\text{Im}(D_{j,m})}{A_j} \]  \hspace{1cm} (6-109)

For the overdamped mode, one can get the following four equations for sensitivity study.

\[ \frac{\partial \omega_j}{\partial \lambda_{j,m}} = -\frac{\omega_j}{2} \left( \frac{\Lambda_{j,m}}{A_{j,m}} + \frac{\Lambda_{j,N,m}}{A_{j,N,m}} \right) \]  \hspace{1cm} (6-110)

\[ \frac{\partial \xi_j}{\partial \lambda_{j,m}} = \frac{\sqrt{\xi_j^2 - 1}}{2} \left( \frac{\Lambda_{j,m}}{A_{j,m}} - \frac{\Lambda_{j,N,m}}{A_{j,N,m}} \right) \]  \hspace{1cm} (6-111)

\[ \frac{\partial \omega_j}{\partial C_{0,m}} = \frac{\omega_j}{2} \left( \frac{D_{j,m}}{A_{j,m}} + \frac{D_{j,N,m}}{A_{j,N,m}} \right) \]  \hspace{1cm} (6-112)

\[ \frac{\partial \xi_j}{\partial C_{0,m}} = -\frac{\sqrt{\xi_j^2 - 1}}{2} \left( \frac{D_{j,m}}{A_{j,m}} - \frac{D_{j,N,m}}{A_{j,N,m}} \right) \]  \hspace{1cm} (6-113)

### 6.3.3 Solution for Seismic Response

The state space method adopts the following co-ordinate transformation to decouple equation (6-87).

\[ q = \Phi z \]  \hspace{1cm} (6-114)

where \( z \) is the 3N-dimentional generalized coordinate vector and \( \Phi \) is the 3N×3N complex modal matrix.

\[ \Phi = [\hat{\phi}_1, \hat{\phi}_2, \ldots, \hat{\phi}_N] \]  \hspace{1cm} (6-115)

By using the coordinate transformation and the orthgonality between the right eigenvectors and the left eigenvectors, equation (6-87) can be reduced to 3N decoupled modal equations with the jth modal equation being

\[ \dot{z}_j - s_j z_j = r_j \dot{x}_g(t) \]  \hspace{1cm} (6-116)

in which
\[ r_j = \psi_j^T F / A_j = -s_j \psi_j^T ME / A_j \]  

(6-117)

Assume that the ground acceleration \( \ddot{x}_g(t) \) is a stationary random process and its power spectral density function is given as \( S_g(\omega) \). The generalized pseudo-excitation method is used to determine the seismic response of adjacent buildings linked by fluid dampers. The pseudo-excitation is constituted for a given frequency \( \omega \) as

\[ \ddot{x}_g(t) = \sqrt{S_g(\omega)} e^{i\omega t} \]  

(6-118)

The solution of the first-order equation (6-116) to the pseudo-excitation is

\[ z_j(\omega, t) = \frac{r_j}{i\omega - s_j} \sqrt{S_g(\omega)} e^{i\omega t} \quad (j=1, 2, \ldots, 3N) \]  

(6-119)

Substituting equation (6-119) into equation (6-114) and comparing with the last expression of equation (6-88), one obtains

\[ x(\omega, t) = \sum_{j=1}^{3N} \phi_j z_j(\omega, t) = \sum_{j=1}^{3N} \phi_j \frac{r_j}{i\omega - s_j} \sqrt{S_g(\omega)} e^{i\omega t} \]  

(6-120)

Since the eigenvectors are in pairs for either underdamped mode or overdamped mode, equation (6-120) can be reduced to

\[ X(\omega, t) = \left( \sum_{j=1}^{N} H_j(\omega)(i\omega \alpha_j + \beta_j) + \sum_{j=3N+1}^{3N} \phi_j \frac{r_j}{i\omega - s_j} \right) \sqrt{S_g(\omega)} e^{i\omega t} = X(\omega) e^{i\omega t} \]  

(6-121)

in which \( x(\omega) \) is called the pseudo displacement; and \( H_j(\omega) \) is the frequency response function for the jth mode. When the jth mode is an underdamped mode

\[ \alpha_j = 2 \text{Re}(\phi_j r_j) \quad \text{and} \quad \beta_j = -2 \text{Re}(\phi_j r_j s_j) \]  

(6-122)

When the jth mode is an overdamped mode

\[ \alpha_j = (\phi_j r_j + \phi_{j-N} r_{j-N}) \quad \text{and} \quad \beta_j = -(\phi_j r_j s_j + \phi_{j-N} r_{j-N} s_j) \]  

(6-123)

In a similar way, the pseudo velocity response can be obtained by
\[ \dot{x}(\omega) = \left( \sum_{j=1}^{N} H_j(\omega)(i\omega \mu_j + v_j) + \sum_{i=1}^{2N} s_i \phi_i \frac{r_i}{i\omega - s_i} \right) \sqrt{S_s(\omega)} \]  

(6-124)

When the jth mode is an underdamped mode

\[ \mu_j = 2 \text{Re}(s_j \phi_j r_j) \quad \text{and} \quad v_j = -2\omega_j^2 \text{Re}(\phi_j r_j) \]  

(6-125)

When the jth mode is an overdamped mode

\[ \mu_j = (s_j \phi_j r_j + s_j \phi_j \phi_j r_j) \quad \text{and} \quad v_j = -\omega_j^2 (\phi_j r_j + \phi_j r_j) \]  

(6-126)

In practice, only the first q (q<3N) modes are needed to be included when calculating seismic response. Thus, the pseudo displacement can be simplified as

\[ x(\omega) = \left( \sum_{j=1}^{q} H_j(\omega)(i\omega \alpha_j + \beta_j) \right) \sqrt{S_s(\omega)} \]  

(6-127)

Once the pseudo displacement is determined, the pseudo internal force can be easily determined according to equations (6-63)-(6-65). The mth pseudo damper force can be derived by (6-128) as follows:

\[ f_{r_m} = C_{m,n} (\dot{x}_{1,m} - \dot{x}_m)/(1 + i\lambda_m \omega) \]

The response spectral matrix can be obtained in a similar way as discussed for the Voigt mode-defined dampers [see equations (6-68)-(6-71)].

### 6.3.4 Application to Example Buildings

For application, the same adjacent buildings subjected to the same earthquake excitation in section 6.2.4 but connected by fluid dampers at each floor of the same level (see Fig. 6.14) are used. Fluid dampers used to connect every two neighboring floors are assumed to have the same relaxation time \( \lambda \) and zero frequency-damping coefficient \( C_m \). The comparison of the effectiveness between the Voigt model-defined viscoelastic dampers and the Maxwell model defined fluid dampers will be made.
6.3.4.1 Modal Frequencies and Damping Ratios

Within the frequency range between zero and 20.00 rad/s, the first three modal frequencies of the right building without fluid dampers connected are 3.02, 9.03 and 14.99 rad/s respectively. The first two modal frequencies of the left building without fluid dampers linked are 4.27 and 12.77 rad/s respectively. When every two neighboring floors of the adjacent buildings are linked by the fluid dampers of the same relaxation time of 0.001s and the same zero frequency damping coefficient of 1.42×10⁶ N.sec/m, the first five modal frequencies of the damper-building system obtained from the complex eigenvalue analysis are 3.28, 3.93, 9.11, 12.68 and 15.04 rad/s respectively. Obviously, using the fluid dampers to link the adjacent buildings slightly changes the first modal frequency of each building but other natural frequencies remain almost unchanged. The retention of the natural frequencies of the unlinked buildings after the installation of the joint dampers is especially desirable for the adjacent buildings that have been already built and need to be strengthened.

As to modal damping ratios, the first three modal damping ratios in the unlinked left building are calculated as 1.25%, 1.02%, and 1.33% respectively. The first two model damping ratios in the unlinked right building are computed as 0.89% and 0.72% respectively. For the linked building-damper system, the first five modal damping ratios obtained from the complex eigenvalue analysis are 22.24%, 11.45%, 7.16%, 5.01%, and 5.01% respectively. Thus, one can expect that the seismic response of the adjacent buildings will be significantly reduced.

The first five natural frequencies and modal damping ratios of the adjacent buildings linked by the Voigt model-defined viscoelastic dampers were calculated in section 6.2.4. With the optimal damper stiffness of 1.0×10⁵ N/m and optimal damper
damping coefficient of $1.43 \times 10^6$ N.sec/m, the first five modal frequencies of the dampер-building system are 3.30, 3.93, 9.11, 12.68, and 15.04 rad/s respectively. The first five modal damping ratios of the system are 21.97%, 11.71%, 7.20%, 5.03%, and 5.04% respectively. Clearly, the use of fluid dampers provides almost the same damping ratios as those when viscoelastic dampers were used. Thus, we can expect that the earthquake response reduction of the damper-building system with the fluid dampers would yield similar results as the system with the viscoelastic dampers.

The relaxation time and zero-frequency damping coefficient of the fluid dampers used in the foregoing calculation are optimal parameters determined through a parametric study. In the parametric study, the modal frequencies and damping ratios of the building-damper system are computed against the relaxation time and zero-frequency damping coefficient of the dampers. The beneficial relaxation time $\lambda$ and zero-frequency damping coefficient $C_0$ of the dampers can be thus found for achieving the maximum modal damping ratio and maintaining the original modal frequencies without significant change. Fig. 6.15 displays variations of the first and second modal frequencies of the system with relaxation time $\lambda$ for zero-frequency damping coefficient $C_0=10$ Ns/m and $C_0=1.42 \times 10^6$ Ns/m. It is seen that for the very small zero-frequency damping coefficient $C_0=10$ Ns/m the modal frequencies are almost independent of relaxation time and the modal frequencies remain the same as those of the unlinked adjacent buildings. This is because the very small zero-frequency damping coefficient plus very small relaxation time indicate that the connections provided by the fluid dampers for the two buildings are very weak. For the zero-frequency damping coefficient $C_0=1.42 \times 10^6$ Ns/m, the modal frequencies are slightly changed. The modal frequencies, however, do not depend on
relaxation time until the relaxation time equals to 0.01 s. If the relaxation time is beyond this value, the modal frequencies of the system will have a rapid increase first and then a rapid decrease towards those of the unlinked adjacent buildings. This is mainly due to the large zero-frequency damping coefficient and the increase of the relaxation time.

Fig. 6.16 shows variations of the first two modal damping ratios of the system with relaxation time for three zero-frequency damping coefficients. For the very small zero-frequency damping coefficient $C_0=10$ Ns/m, the first two modal damping ratios are almost the same as those for the unlinked adjacent buildings and do not depend on the relaxation time concerned. With the higher zero-frequency damping coefficients, the first two modal damping ratios of the system are increased significantly and remain almost constant within a relaxation time range from zero to 0.001 s. Beyond this range, the first modal damping ratio will decrease rapidly with the increasing relaxation time and the second modal damping ratio will increase first and then decrease towards those of the unlinked adjacent buildings. The reason behind these observations is similar to that for the variation of natural frequencies. Furthermore, although the trend in Fig. 6.16 shows that as the relaxation time is increased to between 0.01 s and 0.1 s, the two curves intersect and the modal damping ratio is of approximately 17% for both the first and second modes of vibration, the damping ratio is too sensitive to the small change of relaxation time. Thus, from a practical point of view, the beneficial value of time relaxation is selected as 0.001 s in this study. Fig. 6.16 also indicates that the zero-frequency damping coefficient affects the first two modal damping ratios of the system significantly.
The variations of the first five modal frequencies with zero-frequency damping coefficient are shown in Fig. 6.17 for the relaxation time of 0.001 s. All the modal frequencies remain almost constant when the zero-frequency damping coefficient is less than $5 \times 10^4$ Ns/m. After that, the three modal frequencies dominated by the softer buildings ($\omega_1$, $\omega_3$, and $\omega_5$) become larger while the two modal frequencies governed by the stiffer building ($\omega_2$ and $\omega_4$) become smaller. This is because for a given relaxation time, the very large zero-frequency damping coefficient is associated with the very large spring stiffness coefficient so that the connections between the two buildings become stronger and stronger and the natural frequencies of the two buildings become closer and closer.

The modal damping ratios in the three modes dominated by the softer building ($\xi_1$, $\xi_3$, and $\xi_5$) are depicted in Fig. 6.18a against the zero frequency damping coefficient while the modal damping ratios in the two modes dominated by the stiffer building ($\xi_2$ and $\xi_4$) are plotted in Fig. 6.18b. It is seen that for the zero frequency damping coefficient less than about $1 \times 10^4$ Ns/m, the dampers have no effect on the modal damping ratios. and the modal damping ratios come mainly from the buildings themselves. As the zero frequency damping coefficient increases, all the modal damping ratios increase. However, when the damper zero-frequency damping coefficient is increased to certain values, the modal damping ratios dominated by the stiffer building decrease while the modal damping ratios governed by the softer building still increase and soon these modes of the softer building become overdamped modes. Therefore, for a very large zero frequency damping coefficient, the system behaves like a lightly damped stiffer building supported by a softer building that has almost no vibration due to its very high vibration energy dissipation.
capability. For the concerned adjacent buildings, the beneficial value of zero-frequency damping coefficient should be selected as $1.42 \times 10^6$ Ns/m so that the second mode of the system (the first mode of the stiffer building) will have the maximum value of $\xi_2$. Of course, the first two modal frequencies of the system will slightly change as seen from Fig. 6.17.

Compared with the adjacent buildings linked by the Voigt model-defined viscoelastic damper, it is found that the variations of modal damping ratio and modal frequency with respect to the zero-frequency damping coefficient of fluid damper are almost the same as those with respect to the damping coefficient of viscoelastic damper. The variations of modal damping ratio and modal frequency with respect to the relaxation time of fluid damper are also almost the same as those with respect to the stiffness of viscoelastic damper when the relaxation time of fluid damper is less than 0.001 s and the stiffness of viscoelastic damper is less than $1.0 \times 10^5$ N/m. However, when the relaxation time of fluid damper is much larger than 0.001 s and the stiffness of viscoelastic damper is much larger than $1.0 \times 10^5$ N/m, the variations of modal frequency in the two cases are different: The adjacent buildings tend to be rigidly connected in the case of viscoelastic dampers but tend to be completely separated in the case of fluid dampers.

A sensitivity study of modal frequency and modal damping ratio to relaxation time and zero-frequency damping coefficient is carried out at the optimum values of $C_n$ equal to $1.42 \times 10^6$ Ns/m and $\lambda$ equal to 0.001 s. The sensitivities of the first modal frequency $\omega_1$ to changes in $\lambda$, or $C_n$ (i=1,2,...,20) are depicted in Fig. 6.19a in terms of $\partial \omega_i/\partial \lambda$, and $\partial \omega_i/\partial C_n$. The sensitivities of the first modal damping ratio $\xi_1$ to changes in $\lambda$, or $C_n$ (i=1,2,...,20) are plotted in Fig. 6.19b by means of
\( \frac{\partial \xi_1}{\partial \lambda} \) and \( \frac{\partial \xi_1}{\partial C_{\text{th}}} \). As expected, the first modal frequency and first modal damping ratio are more sensitive to the damper at the top of the buildings than others. The very small sensitivities to the dampers near the bottom of the buildings may indicate no need to install these dampers. The negative sensitivity, or gradient, of the first modal damping ratio to relaxation time is because as the relaxation time increases the first modal damping ratio decreases. Similar sensitivity results are found for the second modal frequency and damping ratio. Since all the sensitivity values are small even for the damper at the top of the buildings, one may conclude that the small deviations of the dampers from its optimal values (\( C_{\text{th}} = 1.42 \times 10^6 \) Ns/m and \( \lambda = 0.001 \) s) may not affect the control efficiency.

### 6.3.4.2 Seismic Response

Seismic response analysis is carried out to investigate variations of seismic response of the adjacent buildings with damper parameters to see if the optimal damper parameters identified from the modal analysis are the same as those from the seismic response analysis under the given earthquake excitation spectrum. Then, the effectiveness of the dampers of optimal parameters on seismic response mitigation is examined. Because of the limitation of space, only a few typical figures are given in this chapter.

Figs 6.20a and 6.20b depict the variations of the top floor displacement responses of the left building and the right building, respectively, with relaxation time for several zero frequency damping coefficients. It is seen that the top floor displacement responses of both the left and right buildings are not affected by relaxation time if the relaxation time is less than 0.001 s. The fact that the response mitigation is not sensitive to relaxation time within a certain range is very helpful for
the practical application of fluid dampers. The further increase of the relaxation time from 0.001 s may reduce the seismic response of the left building, but it will certainly increase the seismic response of the right building. In particular, when the relaxation time reaches a value above 10 s, the two buildings behave as though almost not connected. As a result, no matter what values the zero-frequency damping coefficient assumes, the damper totally loses its effectiveness. It is clear from the foregoing two figures that to achieve the maximum reduction of the dynamic response of both buildings, the optimum relaxation time should be less than 0.001 s, which is the same as one found from the modal analysis. In addition, it can be seen from Figs 6.20a and 6.20b that there is an optimal zero frequency damping coefficient between $10^5$ Ns/m and $5\times10^6$ Ns/m.

To find optimal zero frequency damping coefficient, the seismic responses including the top floor displacement response, the base shear force response, and the top floor acceleration response of both buildings are computed over a wide range of zero frequency damping coefficient with an optimum relaxation time being 0.001 s. Fig. 6.21 shows the variations of the top floor displacement responses of the two buildings with zero frequency damping coefficient. It is seen that the optimum zero frequency damping coefficient is $0.98\times10^6$ Ns/m for the left building or $1.42\times10^6$ Ns/m for the right building. The value $1.42\times10^6$ Ns/m is the same as one found from the modal analysis. With the decrease of zero frequency damping coefficient from the optimum value, the performance of the damper deteriorates gradually and as the zero frequency damping coefficient approaches to zero the two buildings finally return to the unlinked situation. On the other hand, if the zero frequency damping coefficient increases from the optimum value, the performance of the damper also
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decreases and as the zero frequency damping coefficient becomes very large the two buildings behave as though almost rigidly connected. As a result, the top floor displacements of the two buildings become the same.

To demonstrate the overall effectiveness of the fluid dampers, the standard deviations of displacement, shear force and acceleration responses at each floor for each building with and without fluid dampers are computed using the Kanai-Tajimi excitation spectrum. Fig. 6.22 shows the variations of the standard deviation of displacement response relative to the ground with the height of the buildings. The top floor displacement standard deviation of the unlinked left building is 44.7 mm but with the fluid dampers installed, it is reduced to 19.3 mm, leading to a 57% reduction of the response. For right building, the top floor displacement standard deviation is 61.1 mm for the unlinked building and 22.1 mm for the linked building, resulting in a 64% reduction. The reduction of the displacement responses from the fluid dampers is also significant for other floors in either building. The standard deviations of shear force in each story for each building are plotted in Fig. 6.23. The shear forces in all the stories of both buildings are reduced after installation of the fluid dampers. In particular, without the fluid dampers the bottom shear force standard deviation is $1.41 \times 10^7$ N in the left building and $9.68 \times 10^6$ N in the right building. With the optimum fluid dampers, the base shear force standard deviation is reduced to $6.10 \times 10^6$ N in the left building and $3.53 \times 10^6$ N in the right building, leading to a 57% and a 64% reduction, respectively.

The variations of acceleration response with the building height, as shown in Fig. 6.24, are different from displacement and shear force response profiles shown in Figs 6.22 and 6.23. The acceleration response for each unlinked building does not
vary monotonically with the building height. This is due to the contributions from higher modes of vibration (Xu et al. 1999). Clearly, the fluid dampers effectively mitigate the acceleration responses not only from low modes of vibration but also higher modes of vibration, as indicated by the response curves of the linked adjacent buildings.

The fluid damper forces are shown in Fig. 6.25 for the variations of fluid damper force with the building height. The maximum damper force is \(1.01 \times 10^5\) N at the top of the buildings. The value of fluid damper force found here is the same as the one found in viscoelastic damper.

It is worthwhile to mention that the maximum reduction levels of seismic responses of the adjacent buildings linked by the Maxwell model-defined fluid dampers are almost the same as those linked by the Voigt model-defined viscoelastic dampers for the given two buildings and ground motion in the study. This is because in the case of viscoelastic damper, the required optimal damper stiffness is quite small so that the Voigt model-defined viscoelastic damper is almost a dashpot only. In the case of fluid damper, the required optimal relaxation time is very small so that the Maxwell model-defined fluid damper also becomes almost a dashpot only. It is also worthwhile to mention that the results obtained in this study are based on the two shear buildings having the same height, mass, and damping coefficient but different column shear stiffness. For the two buildings with different mass and stiffness ratios and different heights, the effectiveness of the fluid dampers may be different. More information on this aspect can be found in Xu et al. (1999)
6.4 SUMMARY

The differential equations of motion for adjacent buildings connected by either viscoelastic dampers or fluid dampers and the formula for the sensitivity study of dampers have been derived in this chapter. The complex modal analysis and the generalized pseudo-excitation method have been used for investigating both dynamic characteristics and seismic response of damper-adjacent building system. Based on the studies on the example adjacent buildings connected by viscoelastic dampers, it was found that if damper parameters are selected appropriately, the modal frequencies of the unlinked buildings could be retained and the modal damping ratios of the system could be significantly increased and thus the earthquake-induced dynamic responses of both buildings could be considerably reduced. The optimal values of the dampers found from the modal analysis with the maximum modal damping ratios as an objective were almost the same as those determined from the seismic response analysis with the maximum seismic response as an objective.

Through the comparison between the adjacent buildings linked by the Maxwell model-defined fluid dampers and the adjacent buildings linked by the Voigt model-defined viscoelastic dampers, it was found that the Maxwell model-defined fluid dampers have almost the same effectiveness as the viscoelastic dampers under the conditions that the same adjacent buildings and ground motion are considered and the parameters of each type of damper are optimum. Some other issues related to this study, such as the optimal position of dampers, the three-dimensional vibration mitigation analysis including torsional effects, and the effect of non-stationary earthquake excitation need further investigation. The earthquake simulation test to verify the theoretical results is also desirable.
Figure 6.1 Structural Model of Adjacent Buildings with Joint Damper (Voigt Model)
Figure 6.2 Variations of Modal Frequencies with Damper Stiffness

Figure 6.3 Variations of Modal Damping Ratios with Damper Stiffness
Figure 6.4 Variations of Modal Frequencies with Damper Damping Coefficient

$k_d = 1 \times 10^6 \text{ N/m}$
Figure 6.5 Variations of Modal Damping Ratios with Damper Damping Coefficient
Figure 6.6 Sensitivities of Modal Frequency and Modal Damping Ratio
Figure 6.7 Top Floor Displacement Response of Left Building vs. Damper Stiffness

Figure 6.8 Top Floor Displacement Response of Right Building vs. Damper Stiffness
Figure 6.9 Top Floor Displacement response of Adjacent Buildings vs. Damper Damping Coefficient

Figure 6.10 Variations of Displacement Response of Adjacent Buildings with Height
Figure 6.11 Variations of Shear Force Response of Adjacent Buildings with Height

Figure 6.12 Variations of Acceleration Response of Adjacent Buildings with Height
Figure 6.13 Variations of Damper Force Response of Adjacent Buildings with Height
Figure 6.14 Structural Model of Adjacent Buildings with Joint Damper (Maxwell Model)
Figure 6.15 Variations of Modal Frequencies with Relaxation Time

Figure 6.16 Variations of Modal Damping Ratios with Relaxation Time
Figure 6.17 Variations of Modal Frequencies with Zero Frequency Damping Coefficient
Figure 6.18 Variations of Modal Damping Ratios with Zero Frequency Damping Coefficient
Figure 6.19 Sensitivities of Modal Frequency and Modal Damping Ratio
Figure 6.20 Top Floor Displacement Response of Adjacent Buildings vs. Relaxation Time
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![Graph showing the standard deviation of top floor displacement vs. zero frequency damping coefficient.](image)

\[ \lambda = 0.001 \text{s} \]

**Figure 6.21** Top Floor Displacement Response of Adjacent Buildings vs. Zero Frequency Damping Coefficient

![Graph showing the standard deviation of floor displacement vs. floor number.](image)

**Figure 6.22** Variations of Displacement Response of Adjacent Buildings with Height
Figure 6.23 Variations of Shear Force Response of Adjacent Buildings with Height

Figure 6.24 Variations of Acceleration Response of Adjacent Buildings with Height
Figure 6.25 Variations of Damper Force Response of Adjacent Buildings with Height
CHAPTER SEVEN

CLOSED FORM SOLUTION FOR WIND RESPONSE OF BUILDINGS WITH LQG CONTROLLERS

7.1 INTRODUCTION

In Chapter 4, dynamic characteristics and wind-induced response of tall buildings with active tendon devices were investigated in terms of the complex modal analysis and the generalized pseudo-excitation method. No modern active control algorithm was used in Chapter 4 for active tendon devices because wind excitation is a complex random disturbance in nature, that cannot be modeled as a white noise process. and also because the main purpose of that chapter was to test the generalized pseudo-excitation method. This chapter aims to use linear quadratic Gaussian (LQG) controllers to control wind-induced response of tall buildings under wind excitation and derive a closed form solution for wind-induced response of tall buildings to facilitate the real application of modern active control theory. In order to apply LQG optimal control to wind excited buildings, the cross-spectral density matrix of wind excitation is first factorized into a number of component excitation spectral matrices where each matrix is fully coherent and any two spectral matrices are non-coherent. The rational functions are then used to approximate the component excitation spectral matrices (Goßmann and Waller 1983; Suhardjo et al. 1992b). The linear differential equations, which play a role of linear filters, are established to relate the rational functions to a series of white noise processes. Finally, the closed form solution for actively controlled buildings with LQG controllers against wind is
derived in this chapter. The derivation of the closed form solution is naturally
fulfilled by the generalized pseudo-excitation method. By using the closed form
solution, extensive parametric studies of an example tall building actively controlled
with LQG controllers are performed in this chapter and the effectiveness and
feasibility of LQG controllers in reducing wind-induced responses of the tall
building are assessed. The optimal parameters of LQG controllers for achieving the
maximum response reduction of buildings are also identified and the problems
encountered in this approach are pointed out.

7.2 DECOMPOSITION OF WIND EXCITATION

The most important function characterizing the fluctuation of wind excitation
in the frequency domain is the cross spectral density matrix \( S_{pp}(s) \), where \( s = io \) and
\( i = \sqrt{-1}. \) This spectral matrix is the Hermitian matrix, which can be decomposed
into a number of component excitation spectral density matrices \( S_n(s) \) (Lin et al.

\[
S_{pp}(s) = L(s)L^T(s) = \sum_{n=1}^{m} L_n(s)L_n^T(s) = \sum_{n=1}^{m} S_n(s) \tag{7-1}
\]

where \( m \) is the dimension of \( S_{pp}(s) \); \( L(s) \) is the lower triangular matrix and \( L_n(s) \) is
the \( n \)-th column of \( L(s) \).

In parallel to the decomposition of the cross spectral density matrix, the wind
excitation vector \( P(t) \) with the given cross spectral density matrix \( S_{pp}(s) \) can be
decomposed as

\[
P(t) = \sum_{n=1}^{m} P_n(t) \tag{7-2}
\]
where the cross spectral density matrix of component wind excitation $P_n(t)$ is $S_n(s)$ and any two component wind excitations $P_j(t)$ and $P_k(t)$ are uncorrelated.

$$E[P_j(t)P_k^*(t + \tau)] = 0 \quad (j \neq k) \quad (7-3)$$

To relate each component wind excitation $P_n(t)$ to a vector of unit intensity white noise processes through linear filters, the rational functions are used to approximate component excitation spectra. For instance, the $jj$th element $S_{n_j}(s)$ in $S_n(s)$ can be fitted by

$$S_{n_j}(s) = H_{n_j}(-s)H_{n_j}(s) \quad (7-4)$$

where

$$H_{n_j}(s) = \frac{b_{nn} + b_{n0}s + b_{n2}s^2}{a_{nn} + a_{n0}s + a_{n2}s^2 + s} \quad (7-5)$$

Once the parameters $a_{nk}$ and $b_{nk}$ ($k=0,1,2$) are obtained by the weighted least square method (Levy 1959; Sanathanan and Koerner 1963), $H_{n_j}(s)$ becomes a wind fluctuation filter for which the input is a unit intensity white noise process and the output is the $P_{n_j}(t)$ with the spectral density function $S_{n_j}(\omega)$. As a result, $P_{n_j}(t)$ can be represented by linear differential equations of the following form:

$$P_{n_j}(t) = C_{n_n, \Gamma_{n_j}}(t) \quad (7-6)$$

$$\dot{\Gamma}_{n_j}(t) = A_{n_n, \Gamma_{n_j}}(t) + D_{n_n, w_n}(t) \quad (7-7)$$

where $w_n(t)$ is unit zero-mean Gaussian white noise and $\Gamma_{n_j}(t)$ is the state vector.

$$A_{n_n, \Gamma_{n_j}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_{n0} & -a_{nl} & -a_{n2} \end{bmatrix}, \quad D_{n_n, w_n} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad C_{n_n, \Gamma_{n_j}} = \begin{bmatrix} b_{n0} & b_{nl} & b_{n2} \\ 0 & 1 \end{bmatrix} \quad (7-8)$$
Note that in the component wind excitation $P_n(t)$, the first $n-1$ elements are zero. Let $P'_n(t)$ denote the subset of $P_n(t)$ with the first $n-1$ zero elements eliminated.

The linear differential equations about $P'_n(t)$ can be expressed as

$$P'_n(t) = C_{an} \Gamma_n(t) \quad (7-9)$$

$$\dot{\Gamma}_n(t) = A_{an} \Gamma_n(t) + D_{an} w_n(t) \quad (7-10)$$

where

$$A_{an} = \begin{bmatrix} A_{an_1} & 0 & \ldots & 0 \\ 0 & A_{an_2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & A_{an_n} \end{bmatrix}, \quad D_{an} = \begin{bmatrix} D_{an_1} \\ D_{an_2} \\ \vdots \\ D_{an_n} \end{bmatrix}$$

$$C_{an} = \begin{bmatrix} C_{an_1} & 0 & \ldots & 0 \\ 0 & C_{an_2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & C_{an_n} \end{bmatrix} \quad (7-11)$$

where $A_{an}$ is a $3(m-n+1) \times 3(m-n+1)$ matrix; $C_{an}$ is a $(m-n+1) \times 3(m-n+1)$ matrix; $D_{an}$ and $\Gamma_n(t)$ are $3(m-n+1)$ vectors; and $P'_n(t)$ is a $(m-n+1)$ vector. The wind excitation $P(t)$ can be finally assembled from the component wind excitation $P_n(t)$ as follows:

$$P(t) = C_a \Gamma(t) \quad (7-12)$$

$$\dot{\Gamma}(t) = A_a \Gamma(t) + D_a w(t) \quad (7-13)$$

where

$$A_a = \begin{bmatrix} A_{a1} & 0 & \ldots & 0 \\ 0 & A_{a2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & A_{am} \end{bmatrix}, \quad D_a = \begin{bmatrix} D_{a1} & 0 & \ldots & 0 \\ 0 & D_{a2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & D_{am} \end{bmatrix}. $$

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\[ C_n = [C'_{n1}, C'_{n2}, \ldots, C'_{nm}], \quad C_{nn} = \begin{bmatrix} 0_n \\ C'_{nn} \end{bmatrix}, \quad w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \\ \vdots \\ w_m(t) \end{bmatrix} \quad (7-14) \]

where the dimensions of matrices \( A_n, \ D_n \) and \( C_n \) are \( 3m' \times 3m' \), \( 3m' \times m \) and \( m \times 3m' \) respectively, with \( m' = m(m+1)/2; \) and \( 0_n \) is a \( (n-1) \times 3(m-n+1) \) matrix.

### 7.3 EQUATIONS OF MOTION OF SYSTEM

The structural model for the present study is an \( N \)-story linear elastic shear building as shown in Figs. 7.1a and 7.1b. The mass of the building is concentrated at its floor and the stiffness is provided by its massless columns between neighboring floors. Active control devices are installed at any desirable stories between the neighboring floors. The equations of motion of the building with active control devices can be expressed as

\[ M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t) + Hu(t) = EP(t) + Hu(t) \quad (7-15) \]

where \( M, \ K, \) and \( C \) are the mass, the stiffness, and damping matrices of the system respectively; \( H \) is an \( N \times r \) matrix denoting the location of \( r \) control devices; \( E \) is an \( N \times m \) matrix consisting of 0 and 1 which expands the \( m \)-dimensional vector \( P(t) \) into the \( N \)-dimensional vector \( \dot{f}(t) \); \( x(t) \) is the vector of displacement response; \( u(t) \) is the horizontal component of the control force; \( P(t) \) is the stationary wind excitation vector with the cross spectral density matrix \( S_{pp}(\omega) \).

Denote the mass, shear stiffness, and external damping coefficient and internal damping coefficient of the building as \( m_i, \ k_i, \ b_i, \ c_i \) \( (i=1,2,\ldots,N) \), respectively. The details of each matrix can then be given as follows:

The mass matrix of the building is
\[ M = \text{diag}[m_1, m_2, \ldots, m_N] \]  

(7-16)

The damping matrix of the building is

\[ C = C^c + C' \]  

(7-17)

where the external damping matrix is

\[ C^c = \text{diag}[b_1, b_2, \ldots, b_N] \]  

(7-18)

and the internal damping matrix is

\[
C' = \begin{bmatrix}
  c_1 + c_2 & -c_2 & & \\
- c_2 & c_1 + c_3 & -c_3 & \\
& & \ddots & \ddots \\
& & & -c_{N-1} & c_{N-1} + c_N & -c_N \\
& & & & -c_N & c_N
\end{bmatrix}
\]  

(7-19)

The stiffness matrix of the building is

\[
K = \begin{bmatrix}
k_1 + k_2 & -k_2 & & \\
- k_2 & k_1 + k_3 & -k_3 & \\
& & \ddots & \ddots \\
& & & -k_{N-1} & k_{N-1} + k_N & -k_N \\
& & & & -k_N & k_N
\end{bmatrix}
\]  

(7-20)

Equations (7-12), (7-13) and (7-15) can be reformulated into a first-order equation.

\[ \dot{q}(t) = Aq(t) + Gw(t) + Bu(t) \]  

(7-21)

where

\[
A = \begin{bmatrix}
  0 & 1 & 0 \\
-M^{-1}K & -M^{-1}C & M^{-1}EC_N \\
0 & 0 & A_N
\end{bmatrix}, \quad B = \begin{bmatrix}
  0 \\
-M^{-1}H \\
0
\end{bmatrix}, \quad G = \begin{bmatrix}
  0 \\
0 \\
D_N
\end{bmatrix}, \quad q(t) = \begin{bmatrix}
x(t) \\
\dot{x}(t) \\
\Gamma(t)
\end{bmatrix}
\]  

(7-22)
7.4 LQG CONTROLLER

In reality, the structural states, that is, displacements and velocities of buildings cannot be fully measured. The acceleration measurement is more feasible (Spencer et al. 1994). The floor accelerations of the building can be related to the displacements and velocities through the following equation.

\[ \ddot{x}(t) = -M^{-1}C\ddot{x}(t) - M^{-1}Kx(t) + M^{-1}f(t) + M^{-1}Hu(t) \]  

(7-23)

If output measurements are selected as \( m(t) = \dot{x} - M^{-1}Hu(t) - M^{-1}f(t) \), then the measured output vector \( m(t) \) can be expressed as

\[ m(t) = C_m q(t) + v_m(t) \]  

(7-24)

where \( v_m(t) \) are random signals known as measurement noises and

\[ C_m = [-M^{-1}K \quad -M^{-1}C \quad 0] \]  

(7-25)

For practical application, sensors may not be placed at every floor and thus only a subset of \( m(t) \), denoted as \( y(t) \), is used.

\[ y(t) = C_l q(t) + \nu(t) \]  

(7-26)

where the dimension of \( y(t) \) is \( N_i \ (N_i \leq N) \); \( C_l \) is a matrix obtained by eliminating the rows related to the floors without sensors in the matrix \( C_m \); and \( \nu(t) \) is the measurement noise vector of \( N_i \) dimension. If the measurement noise is regarded as the white noise of the same intensity at each measurement point and independent of each other, its covariance matrix can be expressed as

\[ E[\nu(t)\nu^T(t + \tau)] = IS_i \delta(\tau) \]  

(7-27)

where \( E \) is the expectation operator; \( I \) is the identity matrix; \( S_i \) is the intensity of noise; and \( \delta(\tau) \) is the Dirac delta function.

The LQG cost function is
\[ J = \lim_{t \to \infty} \frac{1}{2T} \mathbb{E} \left[ \int_{t_i}^{t_f} (q(t)^T Q q(t) + u(t)^T R u(t)) \, dt \right] \] (7-28)

where \( Q \) and \( R \) are positive semi-definite and positive definite weighting matrices, respectively.

A reasonable controller design for the LQG control problem can be obtained by using the linear quadratic regulator feedback gain matrix \( K \), which operates on the state estimate \( \hat{q} \) generated by the Kalman filter:

\[ u(t) = -K \hat{q}(t) \] (7-29)

\[ K = R^{-1} B^T X_2 \] (7-30)

where \( X_2 \) is the solution of the following algebraic Riccati equation.

\[ X_2 A + A^T X_2 - X_2 B R^{-1} B^T X_2 + Q = 0 \] (7-31)

The state equation for the Kalman filter (Kwakernaak and Sivan 1972) is given by

\[ \dot{\hat{q}}(t) = A \hat{q}(t) + B u(t) + K_r(y(t) - C_l \hat{q}(t)) \] (7-32)

where \( K_r \) is the estimator gain matrix given by

\[ K_r = Y_2 C_l S_l^{-1} \] (7-33)

and \( Y_2 \) is the solution of the following algebraic Riccati equation.

\[ A Y_2 + Y_2 A^T - Y_2 C_l C_l^T Y_2 S_l^{-1} + G G^T = 0 \] (7-34)

### 7.5 CLOSED FORM SOLUTIONS

#### 7.5.1 Solution for Dynamic Characteristics

Equations (7-21), (7-26), (7-29), and (7-32) can be rearranged as

\[ \dot{p}(t) = \hat{A} p(t) + F(t) \] (7-35)
where

\[ \dot{A} = \begin{bmatrix} A - BK_c & BK_c \\ 0 & A - K_r C_t \end{bmatrix}, \quad \dot{F}(t) = \begin{bmatrix} G & 0 \\ G & -K_r \end{bmatrix} \begin{bmatrix} w(t) \\ v(t) \end{bmatrix}. \]

\[ p = \begin{bmatrix} q(t) \\ e(t) \end{bmatrix}, \quad e(t) = q(t) - \dot{q}(t) \]  

(7-36)

The solution of the homogeneous form of equation (7-35) can then be taken as

\[ p(t) = \phi e^{st} \]  

(7-37)

The associated complex eigenvalue problem of equation (7-35) becomes

\[ \dot{\Phi} = s\Phi \]  

(7-38)

where \( s \) is the eigenvalue and \( \Phi \) is the associated eigenvector. The solution of equation (7-38) comprises a set of \( 2N' \) \((N' = 2N + 3m')\) eigenvalues and eigenvectors that exist in either complex conjugate pairs (underdamped mode) or real pairs (overdamped mode). For complex conjugate pairs,

\[ \phi_j = \phi_{j,N'}, \quad s_j = s_{j,N'} \quad (j = 1, 2, \ldots, N') \]  

(7-39)

The eigenvalues are usually written under the form

\[ s_j = s_{j,N'} = -\omega_j \xi_j + i\omega_{d_i} \quad (j = 1, 2, \ldots, N') \]  

(7-40)

where

\[ \omega_j = \left| s_j \right|, \quad \xi_j = -\operatorname{Re}(s_j)/\left| s_j \right|, \quad \omega_{d_i} = \omega_j \sqrt{1 - \xi_j^2} \]  

(7-41)

For real pairs, it is convenient to express real pairs \( s_i \) in the following form analogous to equation (7-40)

\[ s_i = -\omega_i \xi_i + \omega_{d_i}, \quad s_{i,N'} = -\omega_i \xi_i - \omega_{d_i} \]  

(7-42)

\( \omega_j, \omega_{d_i} \) and \( \xi_j \) are determined by

\[ \omega_j = \sqrt{s_j s_{j,N'}}, \quad \xi_j = -(s_j + s_{j,N'})/(2\omega_j), \quad \omega_{d_i} = \omega_j \sqrt{\xi_j^2 - 1} = (s_j - s_{j,N'})/2 \]  

(7-43)
where $\omega_j$, $\omega_d$, and $\xi_j$ are the modal frequency, the damped modal frequency, and the modal damping ratio, respectively, associated with mode $j$. The superscript * means the conjugation.

### 7.5.2 Solution for Wind Response

To find the closed form solution for wind response of tall buildings with LQG controllers, the pseudo-excitation method (Lin et al. 1994) is used in conjunction with the complex modal superposition method and the residue theorem. In this study, $w(t)$ and $v(t)$ are assumed to be independent and the spectral density matrix $S_{\omega\omega}$ is thus given by

\[
S_{\omega\omega} = \begin{bmatrix} I & 0 \\ 0 & S_{\omega I} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & \eta I \end{bmatrix}
\]  \hspace{1cm} (7-44)

where

\[
\eta = S_{\omega I}, \quad \eta = S_{\omega I}
\]  \hspace{1cm} (7-45)

Note that $S_{\omega\omega} = \begin{bmatrix} I & 0 \\ 0 & \eta I \end{bmatrix}$. Thus, letting $U = \begin{bmatrix} I \\ 0 \end{bmatrix}$, the pseudo-excitation vectors for the system expressed by equation (7-35) can be constituted as (Lin et al. 1994; Zhang and Xu 1999)

\[
\begin{bmatrix} w(t) \\ v(t) \end{bmatrix}_k = U_k \exp(i\omega t) \quad (k=1,2,\ldots,N_\omega+m)
\]  \hspace{1cm} (7-46)

where $U_k$ is the $k$-th column of $U$.

Considering the $k$th pseudo-excitation vector $\begin{bmatrix} w(t) \\ v(t) \end{bmatrix}_k$, equation (7-35) becomes

\[
\dot{p}_k(t) = \hat{A}_k p_k(t) + F_k \exp(i\omega t)
\]  \hspace{1cm} (7-47)
where

\[ F_k = \begin{bmatrix} G & 0 \\ G & -K_r \end{bmatrix} U_k \]  \hspace{1cm} (7-48)

To decouple equation (7-47), the following co-ordinate transformation is adopted.

\[ p_k(t) = \Phi z_k(t) \]  \hspace{1cm} (7-49)

where \( z_k(t) \) is the \( 2N' \)-dimensional generalized coordinate vector and \( \Phi \) is the \( 2N' \times 2N' \) right modal matrix, that is.

\[ \Phi = [\phi_1, \phi_2, \cdots, \phi_{2N'}] \]  \hspace{1cm} (7-50)

By using this transformation, equation (7-47) can be reduced to \( 2N' \) decoupled modal equations with the jth modal equation being

\[ \dot{z}_{j_k}(t) = s_j z_{j_k}(t) + r_{j_k} \exp(i\omega t) \]  \hspace{1cm} (7-51)

where

\[ r_{j_k} = \Psi_i^T F_k \]  \hspace{1cm} (7-52)

\( \Psi_i \) is the jth column of \( \Psi \); and \( \Psi = \Phi^{-T} \), the left modal matrix.

The solution of the first-order jth equation (7-51) to the kth pseudo excitation vector is

\[ z_{j_k}(\omega, t) = \frac{r_{j_k}}{i\omega - s_j} \exp(i\omega t) \]  \hspace{1cm} (j=1,2, \cdots,2N') (7-53)

Denoting the n-th components of \( \phi_i \) as \( \phi_{ni} \) and the n-th component of \( p_k \) as \( p_{kn} \), then the pseudo response \( p_{kn} \) is given by

\[ p_{kn}(\omega, t) = \sum_{j=1}^{2N'} \phi_{nj} z_{j_k}(\omega, t) = \sum_{j=1}^{2N'} \phi_{nj} \frac{r_{j_k}}{i\omega - s_j} \exp(i\omega t) \]  \hspace{1cm} (7-54)
Since the eigenvectors are in pairs for either underdamped mode or overdamped mode, equation (7-54) can be reduced to

\[ p_{kn}(\omega, t) = \sum_{j=1}^{\infty} H_j(\omega)(i\omega \alpha_{kn} + \beta_{kn}) \exp(i\omega t) \]  

(7-55)

where \( H_j(\omega) \) is the frequency response function for the jth mode.

\[ H_j(\omega) = \frac{1}{\omega_j^2 - \omega^2 + i2\zeta_j \omega_j \omega} \]  

(7-56)

The pseudo response \( p_{kn} \) can be the pseudo displacement response, pseudo velocity response, and pseudo acceleration response. The proper use of the pseudo displacement responses of the building can result in the pseudo shear force responses of the building. Also, the proper use of the pseudo state estimator responses in conjunction with equation (7-29) can lead to the pseudo control forces. For instance, the n-th (\( n \leq N \)) pseudo displacement, velocity, or acceleration response can be obtained from equation (7-55) if the coefficients \( \alpha_{kn} \) and \( \beta_{kn} \) are calculated by the following equations:

When the jth mode is an underdamped mode

\[ \alpha_{kn} = \begin{cases} \frac{2 \text{Re}(\phi_{kn} r_{kj})}{2 \text{Re}(s_j \phi_{kn} r_{kj})} & \text{For displacement} \\ \frac{2 \text{Re}(s_j^2 \phi_{kn} r_{kj})}{2 \text{Re}(s_j \phi_{kn} r_{kj})} & \text{For velocity} \\ \frac{2 \text{Re}(s_j^2 \phi_{kn} r_{kj})}{2 \text{Re}(s_j \phi_{kn} r_{kj})} & \text{For acceleration} \end{cases} \]  

(7-57)

\[ \beta_{kn} = \begin{cases} -\frac{2 \text{Re}(\phi_{kn} r_{kj}s_j)}{2 \omega_j^2 \text{Re}(\phi_{kn} r_{kj})} & \text{For displacement} \\ -\frac{2 \omega_j^2 \text{Re}(\phi_{kn} r_{kj})}{2 \text{Re}(\phi_{kn} r_{kj})} & \text{For velocity} \\ -\frac{2 \omega_j^2 \text{Re}(s_j \phi_{kn} r_{kj})}{2 \text{Re}(s_j \phi_{kn} r_{kj})} & \text{For acceleration} \end{cases} \]  

(7-58)

When the jth mode is an overdamped mode

\[ \alpha_{kn} = \begin{cases} \phi_{kn} r_{kj} + \phi_{n,\ldots,\infty} r_{kj,\ldots,\infty} & \text{For displacement} \\ s_j \phi_{kn} r_{kj} + s_j \phi_{n,\ldots,\infty} r_{kj,\ldots,\infty} & \text{For velocity} \\ s_j^2 \phi_{kn} r_{kj} + s_j^2 \phi_{n,\ldots,\infty} r_{kj,\ldots,\infty} & \text{For acceleration} \end{cases} \]  

(7-59)
\[
\beta_{kn} = \begin{cases} 
-(\phi_{n} r_{kn} s_{k_{n} - N} + \phi_{n_{j} - N} r_{k_{j} - N}s_{j}) & \text{For displacement} \\
-\omega_{j}^{2}(\phi_{n} r_{j} + \phi_{n_{j} - N} r_{k_{j} - N}) & \text{For velocity} \\
-\omega_{j}^{2}(s_{j} \phi_{n} r_{j} + s_{j_{N}} \phi_{n_{j} - N} r_{k_{j} - N}) & \text{For acceleration} 
\end{cases} 
(7-60)
\]

According to the principle of the pseudo-excitation method, the response spectral density of \(p_{kn}\) can then be obtained by

\[
S_{p_{kn}p_{kn}}(\omega) = p_{kn}(\omega, t)p_{kn}^{*}(\omega, t) = \sum_{i=1}^{N} \sum_{j=1}^{N} H_{i}(\omega)H_{j}^{*}(\omega)(\beta_{kn}^{2} + \omega_{j}^{2}(\alpha_{kn}^{2} - \alpha_{kn}^{2})S_{n}) 
(7-61)
\]

The variance response of \(p_{kn}\) under the \(k\)th pseudo-excitation can be evaluated as

\[
\sigma_{p_{kn}}^{2} = \int_{-\infty}^{\infty} S_{p_{kn}p_{kn}}(\omega) \, d\omega 
(7-62)
\]

The above integration in the complex plane can be accomplished using the residue theorem to have the closed form solution as

\[
\sigma_{p_{kn}}^{2} = \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{n_{i}j} u_{kn} u_{kn} + \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{n_{i}j} v_{kn} v_{kn} + \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{n_{i}j} v_{kn} v_{kn} 
(7-63)
\]

where

\[
u_{kn} = \sqrt{\frac{\pi}{2} \omega_{n} \sqrt{S_{n}}} 
\]

\[
u_{kn} = \sqrt{\frac{\pi}{2} \omega_{n} \sqrt{S_{n}}} 
(7-64)
\]

\[
\rho_{n_{i}j} = \frac{8(\xi_{n} + \gamma \xi_{j})^{2}}{(1 - \gamma^{2})^{2} + 4\xi_{n}^{2}\xi_{j}^{2}(1 + \gamma^{2}) + 4(\xi_{n}^{2} + \xi_{j}^{2})^{2}} 
(7-65)
\]

\[
\rho_{n_{i}j} = \frac{8(\xi_{n} + \gamma \xi_{j})^{2}}{(1 - \gamma^{2})^{2} + 4\xi_{n}^{2}\xi_{j}^{2}(1 + \gamma^{2}) + 4(\xi_{n}^{2} + \xi_{j}^{2})^{2}} 
(7-66)
\]

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\[ \rho_{i,j} = \frac{8\sqrt{\xi_1\xi_j}(\xi_j + \gamma\xi_j)}{(1 - \gamma^2)^2 + 4\xi_1\xi_j\gamma(1 + \gamma^2) + 4(\xi_1^2 + \xi_j^2)\gamma^2} \]  
\[ \gamma = \frac{\omega}{\omega_i} \] (7-67) (7-68)

The final variance response of \( \rho_s \) can be determined by a summation with respect to the \( N_i+m \) pseudo excitation vectors.

\[ \sigma_{p_s}^2 = \sum_{k=1}^{N_i+m} \sigma_{p_{s_k}}^2 \] (7-69)

The closed form solution derived above makes it possible to carry out extensive parametric studies and to evaluate the performance of the building with LQG controllers against wind.

### 7.6 NUMERICAL EXAMPLES

For illustration purpose, a 40-story shear building under alongwind excitation with and without active control devices, as shown in Figs. 7.1(a) and 7.1(b), is analyzed. The building is assumed to be composed of 40 identical floor units. The structural data for the building are as follows: the individual story height is 4 m; the lumped mass at individual floor is 1.29\times10^6 \text{ kg}; the elastic shear stiffness in each story is 10^6 \text{ N/m}; the damping coefficient is 2.155\times10^4 \text{ Ns/m}. Only twenty active controllers are used below the 20th floor, each installed between every two neighboring floors.

The power spectral density of wind speed can be expressed as (Simiu and Scanlan 1996)

\[ S_v(u) = \frac{200z_u^2}{V_i^4} \left( 1 + \frac{50nz_u}{V_i} \right)^{-\frac{1}{2}} \] (7-70)
in which \( n \) = frequency, in hertz (cycles per second); \( u \) is the friction velocity of the wind in meters per second; \( z_i \) is the height of the \( i \)th floor above the ground. \( V_i \) is the mean wind velocity at the \( i \)th floor.

The spectral density function of the alongwind force on the \( i \)th floor is expressed as

\[
S_p(u) = \left( \frac{W_i}{V_i} \right)^2 S_v(u) \tag{7-71}
\]

in which \( W_i \) is the wind force at the \( i \)th floor.

Equation (7-71) is a one-sided spectrum in the positive frequency domain.

When converted to a two-sided spectral density in \( \omega \), equation (7-71) becomes

\[
S_p(\omega) = \frac{1}{\pi} \left( \frac{W_i}{V_i} \right)^2 \frac{200z_iu_0^2}{V_i \left( 1 + 50 \frac{\omega z_i}{2\pi V_i} \right)^2} \tag{7-72}
\]

in which \( \omega \) is the frequency in radians per second. The relationship between \( W_i \) and \( V_i \) is described by

\[
W_i = \frac{1}{2} \rho A_i C_d V_i^2 \tag{7-73}
\]

where \( \rho \) is the air density; \( A_i \) is the tributary area of the \( i \)th floor; and \( C_d \) is the drag coefficient.

The mean velocity varies with height, which may be described by a power law

\[
V_i = V_g \left( \frac{z_i}{z_g} \right)^\alpha \tag{7-74}
\]

where \( z_g \) is the gradient height; \( V_g \) is the mean velocity at the gradient height; and \( \alpha \) is a constant exponent. The cross-spectral density function between the wind force at the \( i \)th floor and the wind force at the \( j \)th floor can then be expressed as
\[ S_{p,p_i}(\omega) = \sqrt{S_{p_i}(\omega)S_{p_i}(\omega)} \rho_{ij}(\omega) \] (7-75)

The coherency function \( \rho_{ij} \) reflects the spanwise correlation of the fluctuating wind forces at the ith and jth floors.

\[ \rho_{ij}(\omega) = \exp\left(-\frac{C_1|\omega|}{2\pi} \frac{|z_i - z_j|}{V_{10}}\right) \] (7-76)

where \( C_1 \) is a constant; and \( V_{10} \) is the mean wind velocity at 10 m above the ground.

The cross power spectral density matrix \( S_{pp}(\omega) \) of wind excitation can then be constituted from equation (7-75).

The aerodynamic physical data for the building are as follows: the wind-load tributary area for each story \( A_i \) is 192 m\(^2\); the gradient height \( z_g \) is 300 m; the mean wind velocity at the gradient height \( V_g \) is 44.69 m/sec; the reference mean wind velocity at 10 m height \( V_{10} \) is 11.46 m/s; the drag coefficient \( C_d \) is 1.2; the air density \( \rho \) is 1.23 kg/m\(^3\); the friction velocity \( u_* \) is 1.98 m/s; the exponent for the mean velocity profile \( \alpha \) is 0.4; and \( C_1 \) is taken as 7.7. Wind excitations are applied to discrete lumped masses at each floor. To make the problem manageable, the wind forces at the floors from 1 and 10 are assumed to be the same as the wind force at the floor 10. The same assumption is applied to the wind forces at the floors from 11 and 20, from 21 and 30, and from 31 to 40, respectively. As a result, the dimension of \( S_{pp}(\omega) \) is 4\( \times \)4 only. \( \eta \) in equation (7-45) is selected as 1/7. Fig. 7.2 portrays the power spectral density of the alongwind speed.

### 7.6.1 Selection of Weighting Matrices

The performance of LQG controllers depends on the weighting matrices \( R \) and \( Q \). To achieve the beneficial performance of LQG for the maximum reduction of key
structural responses of the building at a reasonable cost of external control energy. several potential weighting matrices are selected. The key structural responses are then computed in terms of the closed form solution, from which the beneficial weighting matrices can be identified. The basic configurations of the weighting matrices in this study are taken as

\[ R = 10^{-8} \mathbf{I}, \quad Q = \begin{bmatrix} Q_d & 0 \\ Q_v & 0 \end{bmatrix} \tag{7-77} \]

where the submatrices \( Q_d \) and \( Q_v \) are assigned to the displacement and velocity responses of the building, respectively; the matrix \( R \) is allocated to the control forces; and \( \mu \) is a proportional coefficient. Clearly, by varying the coefficient \( \mu \), a proper trade off between control effectiveness and control energy consumption can be achieved. Four cases are selected for finding the balanced submatrices \( Q_d \) and \( Q_v \).

These four cases are defined as follows:

Case A: \( Q_d = Q_v = \mathbf{I} \)

Case B: \( Q_d = \mathbf{I}, \quad Q_v = 0 \)

Case C: \( Q_d = 0, \quad Q_v = \mathbf{I} \)

Case D: \( Q_d = \mathbf{I}, \quad Q_v = 0.1 \mathbf{I} \)

In Case A, the two submatrices are equally assigned; in Case B, the assignment to the displacement submatrix only indicates that the displacement response reduction is maximized regardless of the velocity response reduction; Case C indicates that the velocity response reduction is maximized regardless of the displacement response reduction; and Case D implies that the displacement response reduction is given priority over the velocity response control.
For each case mentioned above, the key responses of actively controlled building are computed against the parameter $\mu$. Then, by comparing the results among all the cases the beneficial case and parameter $\mu$ can be found for achieving the maximum or beneficial response reduction of the building. Figs. 7.3, 7.4 depict the variations of the top floor displacement and base shear force responses of the building, respectively, with the parameter $\mu$. It is seen that for all cases the top floor displacement and base shear force responses of the building are rapidly reduced until $\mu$ reaches a value about $1 \times 10^7$. After that, the gradients of response reduction become small in Cases A, B, and D, but not in Case C where the further increase of $\mu$ makes the displacement and shear force responses become larger with larger control force required. Thus, Case C is disregarded in the subsequent computation. Fig. 7.5 depicts the variations of the top floor acceleration of the building with the parameter $\mu$. It is seen that for all cases the top floor acceleration responses of the building are rapidly reduced until $\mu$ reaches a value about $3 \times 10^9$. After that, the further increase of $\mu$ makes the acceleration responses become larger with larger control force required for Cases B and D. Thus, Cases B and D are also disregarded in the subsequent computation. As a result, a compromise is made in this study to select the weighting matrices from Case A. Fig. 7.6 shows how the control force at the first floor of the building varies with the parameter $\mu$ for Case A. It is seen that the control force increases with the increasing value of $\mu$. Summing up the results from Figs. 7.3, 7.4, 7.5 and 7.6, one may find that the weighting matrices in Case A are the best and the beneficial parameter $\mu$ may be selected as $3 \times 10^9$. 
7.6.2 Modal Properties

By using the weighting matrix Q in Case A, the first three natural frequencies and modal damping ratios of the actively controlled building are computed against the parameter $\mu$. The results are depicted in Figs. 7.7 and 7.8 for the natural frequencies and modal damping ratios, respectively. It is seen from Fig. 7.7 that the first three natural frequencies increase slightly with the parameter $\mu$. However, it is seen from Fig. 7.8 that the first three modal damping ratios increase significantly and monotonically with the increasing value of $\mu$. Clearly, the addition of damping to the building is due to LQG controllers. For the beneficial parameter $\mu = 3 \times 10^6$ selected above, the first three damping ratios are 28.2%, 15.3%, 17.6%, respectively, for the controlled building, compared with the first three damping ratios of 0.77%, 0.26% and 0.15%, respectively, of the building without control.

The further modal analysis of the actively controlled building with the beneficial weighting matrix Q and beneficial parameter $\mu$ selected above shows that the real parts of all eigenvalues of $\hat{A}$ are negative. Therefore, according to Lyapunov's criterion about the stability of linear systems, the present actively controlled building forms a stable system.

7.6.3 Buffeting Response

To demonstrate the overall performance of the LQG controllers, the standard deviations of displacement, shear force, and control force responses at each floor of the building with and without active control devices are computed. Figs. 7.9 and 7.10 show the variations of the standard deviation of displacement and shear force response, respectively, with the height of the building. The reduction of the
responses from the LQG controllers is significant for all floors of the building. In particular, the top floor displacement standard deviation of the uncontrolled building is 52.1 mm but with the LQG controllers installed, it is reduced to 28.0 mm, leading to a 46% reduction of the response. The acceleration reduction is also significant. The top floor acceleration reduction is about 46%. For base shear force response, without control the base shear force standard deviation is $2.2 \times 10^6$ N. With the LQG controllers, the base shear force standard deviation is reduced to $1.2 \times 10^6$ N, leading to a 45% reduction. Fig. 7.11 shows the variations of control force with the building height. The maximum control force occurs at the first floor of the building, as expected.

To further enhance the understanding of buffeting response of the building with and without control devices, the spectral density functions of the top floor displacement response and top floor acceleration response of the building with and without control devices are computed and plotted in Figs. 12 and 13, respectively. It is clearly seen that all the peaks in the response spectra of the building are significantly reduced when the LQG controllers are used.

7.7 SUMMARY

In this chapter, the cross-spectral density matrix of turbulent wind forces was first factorized into a number of component excitation spectral matrices, and the rational functions were used to approximate the component excitation spectral matrices. The closed form solution for buffeting response of building with LQG controllers was then derived by using the generalized pseudo-excitation method and the residue theorem. With the derived closed form solution, extensive parametric studies were finally performed on the multi story shear building to seek optimal
weighting matrices, modal characteristics, and maximum buffeting response reductions with reasonable control forces. It was found from the example building that if weighting matrices were selected appropriately, the modal damping ratios of the system could be significantly increased and the wind-induced dynamic responses of the building could be considerably reduced.

From the computation of the example building, it was also found that even with the closed form solution, great computational efforts were still needed mainly because of the multi point wind excitations, their spatial and temporal correlation, and the great expansion of the size of the dynamic matrices due to the use of the space state approach and the LQG controllers. Further investigation should be thus conducted to further reduce the computational efforts.

Figure 7.1 Structural Model of Multistory Building
Figure 7.2 Spectrum of Alongwind Speed

Figure 7.3 Top Floor Displacement Response of Building vs. Parameter $\mu$
Figure 7.4 Base Shear Force Response of Building vs. Parameter $\mu$

Figure 7.5 Top Floor Acceleration Response of Building vs. Parameter $\mu$
Figure 7.6 Control Force Response of Building vs. Parameter $\mu$

Figure 7.7 Variations of Modal Frequencies with Parameter $\mu$
Figure 7.8 Variations of Modal Damping Ratios with Parameter \( \mu \)

Figure 7.9 Variations of Displacement Response of Building with Height
Figure 7.10 Variations of Shear Force Response of Building with Height

Figure 7.11 Variations of Control Force of Active Tendons with Height
Figure 7.12 Spectral Density Functions of Top Floor Displacement Response

Figure 7.13 Spectral Density Functions of Top Floor Acceleration Response
CHAPTER EIGHT

ACTIVE CONTROL OF EARTHQUAKE-EXCITED
ADJACENT BUILDINGS

8.1 INTRODUCTION

In Chapter 6, the generalized pseudo-excitation method is used to perform the passive vibration control analysis of adjacent buildings subject to earthquake excitation. In this chapter, the generalized pseudo-excitation method is extended to active vibration control of adjacent buildings under earthquake excitation. The active tendon devices and the corresponding control approach used in Chapter 4 for wind-excited tall buildings are used to link the adjacent buildings for control of seismic response as one measure. The active actuators with LQG controllers are then employed to link the adjacent buildings for control of seismic response as another approach. The closed form solution for the adjacent buildings with LQG controllers under earthquake excitation is also provided in this chapter. For either control approach, the dynamic characteristics of the control device-adjacent building system are determined through the complex modal analysis. The generalized pseudo-excitation method is used to analyze the seismic response of the system. Extensive parametric studies are performed to assess the effectiveness of the control devices and to identify beneficial control parameters. The comparison of vibration control performance between two control approaches is also given.
8.2 **ADJACENT BUILDINGS LINKED BY ACTIVE TENDON DEVICES**

8.2.1 **Equations of Motion of System**

Consider again the same buildings in Section 6.2.1 but they are connected by active tendon devices at each floor of the same level (see Fig. 8.1). The two buildings are assumed to have the same story height with the tendon devices being installed horizontally at each floor to connect them together. The tendon device is of an electrohydraulic servomechanism identical to that considered by Yang and Samali (1983) and used in Chapter 4 for wind-excited tall buildings. The tendon device here is regulated by two sensors mounted on the floors at the same level with one for each building. All tendon devices used herein are identical. Both buildings are supposed to be subject to the same base acceleration, and any effects due to spatial variations of the ground motion or due to soil-structure interactions are neglected.

Assume that total degrees of freedom of the two buildings are $N$ (see Fig. 8.1), in which the number of degrees of freedom of the left building is $L$ with its first floor designated as the first degree of freedom. $N-L$ is then the number of degrees of freedom of the right building with its first floor designated as the $L+1$ degree of freedom. Two sensors for the $j$th tendon controller are placed on both the $j$th and $L+j$th floors respectively to sense their motions:

$$x'_j = \frac{d'x_j}{dt'}; \quad x'_{j+1} = \frac{d'x_{j+1}}{dt'} \quad (8-1)$$

in which $x_j$ is the displacement of $j$th floor; $r=0$, $r=1$, and $r=2$ correspond to the displacement, velocity, and acceleration sensors, respectively.

The sensed motions are transmitted as a feedback by the electric voltage $V(t)$:
\[ V_j(t) = p_i(x_j^r - x_{L,j}) \]  
\[ \dot{U}_j(t) + R_i U_j(t) = \frac{R_i V_j(t)}{R_{ii}} \]

in which \( p_i \) is a proportional constant. The feedback voltage triggers the displacement, \( U_j(t) \), of a hydraulic ram of the servomechanism through the relation

The control force is due to the elongation of the tendon resulting from the relative building motion and the movement of the hydraulic ram \( U_j(t) \). Thus, if \( K_i \) denotes the tendon stiffness, the control force from the \( j \)th controller is

\[ f_j(t) = K_i[(x_j - x_{L,j}) + U_j(t)] \]

The equations of motion of the building-tendon system can then be expressed as

\[ M\ddot{x}(t) + C\dot{x}(t) + Kx(t) + HU(t) = -Me\ddot{e}(t) \]

with the auxiliary equation

\[ U + \Lambda \dot{U} = Dx' \]

where \( M \) and \( C \) are the mass and damping matrices of the adjacent buildings, respectively; \( K \) is the total stiffness matrix of the system including both structural stiffness and the tendon stiffness; \( H, \Lambda \) and \( D \) are the coefficient matrices of the active tendon devices: \( x(t) \) is the vector of relative displacement response with respect to the ground with the left building’s displacements in the first \( L \) positions and the right building’s displacements in the last \( N-L \) positions; \( U(t) \) is the displacement of a hydraulic ram of the servomechanism; \( e \) is an index vector with all its elements equal to 1; and \( \ddot{e}(t) \) is the ground acceleration.
Denote the mass, shear stiffness, and external damping coefficient and internal damping coefficient of the adjacent buildings as \( m_i, k_i, b_i, c_i \) (i=1,2,...,N), respectively, and the normalized loop gain \( \varepsilon \) and the feedback gain \( \tau \) are defined as

\[
\varepsilon = \frac{R_1}{\omega_1} \quad \tau = \frac{p_i \omega_i^\prime}{R_0}
\]  

(8-7)

in which \( \omega_1 \) is the fundamental frequency of the adjacent buildings without active tendon devices. The details of each matrix can then be given as follows:

The mass matrix of the adjacent buildings is

\[
M = \text{diag}[m_1, m_2, \cdots, m_N]
\]  

(8-8)

The damping matrix of the adjacent buildings is

\[
C = C^e + C'
\]  

(8-9)

in which the external damping matrix is

\[
C^e = \text{diag}[b_1, b_2, \cdots, b_N]
\]  

(8-10)

and the internal damping matrix is

\[
C' = \begin{bmatrix}
C_L & 0 \\
0 & \bar{C}_R
\end{bmatrix}
\]  

(8-11)

\[
C_L = \begin{bmatrix}
c_1 + c_2 & -c_1 \\
-c_2 & c_2 + c_1 & -c_1 \\
& -c_{L-1} & c_{L-1} + c_1 & -c_1 \\
& & -c_1 & c_1
\end{bmatrix}
\]  

(8-12)
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\[
C_R = \begin{bmatrix}
c_{L-1} + c_{L-2} & -c_{L-2} \\
-c_{L-2} & c_{L-2} + c_{L-3} \\
& \ddots \\
& & & -c_{N-1} & c_{N-1} + c_N & -c_N & \cdots \\
& & & & & -c_N & c_N \\
\end{bmatrix}
\]  \hspace{1cm} (8-13)

The stiffness matrix of the adjacent buildings is

\[K = K' + K^d\]  \hspace{1cm} (8-14)

\[K' = \begin{bmatrix} K_L & 0 \\ 0 & K_R \end{bmatrix}\]  \hspace{1cm} (8-15)

\[
K_L = \begin{bmatrix}
k_1 + k_2 & -k_2 \\
-k_2 & k_2 + k_3 & -k_3 \\
& \ddots \\
& & & & -k_{L-1} & k_{L-1} + k_L & -k_L \\
& & & & & & -k_L & k_L \\
\end{bmatrix}
\]  \hspace{1cm} (8-16)

\[
K_R = \begin{bmatrix}
k_{L+1} + k_{L+2} & -k_{L+2} \\
-k_{L+2} & k_{L+2} + k_{L+3} & -k_{L+3} \\
& \ddots \\
& & & -k_{N-1} & k_{N-1} + k_N & -k_N \\
& & & & & -k_N & k_N \\
\end{bmatrix}
\]  \hspace{1cm} (8-17)

The stiffness matrix attributed to tendons is

\[
K^d = K_T \begin{bmatrix} I_{(N-1)\times(N-1)} & 0_{(N-1)\times(2L-N)} & -I_{(N-1)\times(N-1)} \\ 0_{(2L-N)\times(N-1)} & I_{(2L-N)\times(2L-N)} & 0_{(2L-N)\times(N-1)} \\ -I_{(N-1)\times(N-1)} & 0_{(N-1)\times(2L-N)} & I_{(N-1)\times(N-1)} \end{bmatrix}
\]  \hspace{1cm} (if N<2L)  \hspace{1cm} (8-18)

or

\[
K^d = K_T \begin{bmatrix} I_{L-1} & -I_{L-1} & 0_{1\times(2L-N)} \\ -I_{L-1} & I_{L-1} & 0_{1\times(2L-N)} \\ 0_{(N-2L+1)\times1} & 0_{(N-2L+1)\times1} & 0_{(N-2L+1)\times1} \end{bmatrix}
\]  \hspace{1cm} (if N\geq2L)  \hspace{1cm} (8-19)
in which $I$ is the identity matrix.

The coefficient matrices of the active tendon devices are

$$H = K, I$$  \hfill (8-20)

$$\Lambda = \frac{1}{\omega_i \epsilon} \text{diag}[I_{(N-L)\times(N-L)}, 0_{(2L-N)\times(2L-N)}, I_{(N-L)\times(N-L)}] \quad \text{if } N<2L$$  \hfill (8-21)

or

$$\Lambda = \frac{1}{\omega_i \epsilon} \text{diag}[I_{L\times L}, 0_{N\times N-2L}, 0_{2L\times 2L}] \quad \text{if } N\geq2L$$  \hfill (8-22)

$$D = \eta \begin{bmatrix}
I_{(N-L)\times(N-L)} & 0_{(N-L)\times(2L-N)} & -I_{(N-L)\times(N-L)} \\
0_{(2L-N)\times(N-L)} & 0_{(2L-N)\times(2L-N)} & 0_{(2L-N)\times(N-L)} \\
-I_{(N-L)\times(N-L)} & 0_{(N-L)\times(2L-N)} & I_{(N-L)\times(N-L)}
\end{bmatrix} \quad \text{if } N<2L$$  \hfill (8-23)

or

$$D = \eta \begin{bmatrix}
I_{L\times L} & -I_{L\times L} & 0_{(N-2L+1)\times L} \\
-I_{L\times L} & I_{L\times L} & 0_{(N-2L+1)\times L} \\
0_{(N-2L+1)\times L} & 0_{(N-2L+1)\times L} & 0_{(N-2L+1)\times(2L-N)}
\end{bmatrix} \quad \text{if } N\geq2L$$  \hfill (8-24)

in which $\eta = \tau$ for displacement sensor; $\eta = \tau / \omega_i$ for velocity sensor; and $\eta = \tau / \omega_i^2$ for acceleration sensor.

### 8.2.2 Dynamic Characteristics and Seismic Response

Equations (8-5) and (8-6) can be replaced by an equivalent first order differential equation of the form

$$A\mathbf{q}(t) + B\mathbf{q}(t) = F\ddot{\mathbf{x}}_p(t)$$  \hfill (8-25)

in which

$$F = \begin{bmatrix} -M \mathbf{E} \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{q}(t) = \begin{bmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{x}(t) \\ \mathbf{U}(t) \end{bmatrix}$$  \hfill (8-26)
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\[
A = \begin{bmatrix} M & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & \Lambda \end{bmatrix}, \quad B = \begin{bmatrix} C & K & H \\ -I & 0 & 0 \\ 0 & -D & I \end{bmatrix}
\]

for displacement sensor \hspace{1cm} (8-27)

\[
A = \begin{bmatrix} M & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & \Lambda \end{bmatrix}, \quad B = \begin{bmatrix} C & K & H \\ -I & 0 & 0 \\ -D & 0 & I \end{bmatrix}
\]

for velocity sensor \hspace{1cm} (8-28)

\[
A = \begin{bmatrix} M & 0 & 0 \\ 0 & I & 0 \\ -D & 0 & \Lambda \end{bmatrix}, \quad B = \begin{bmatrix} C & K & H \\ -I & 0 & 0 \\ 0 & 0 & I \end{bmatrix}
\]

for acceleration sensor \hspace{1cm} (8-29)

The matrices \( A \) and \( B \) are of size \( 3N \times 3N \). The generalized pseudo-excitation method employed in the previous chapters, such as in Chapter 6, can be used to solve equation (8-25) for both dynamic characteristics and seismic response. The corresponding formulae are no longer presented here for the sake of brevity.

8.2.3 Application

For application, the same adjacent buildings subjected to the same earthquake excitation investigated in Section 6.2.4 are used but they are now connected by active tendon devices at each floor of the same level (see Fig. 8.1). The structural parameters used herein are the same as those used in Section 6.2.4. Each tendon has a stiffness of \( K_T = 2 \times 10^6 \text{ N/m} \).

8.2.3.1 Modal Frequencies and Damping Ratios

Within the frequency range between zero and 20.00 rad/s, the first three modal frequencies of the right building without active tendon devices connected are 3.02, 9.03 and 14.99 rad/s respectively. The first two modal frequencies of the left building without active tendon devices linked are 4.27 and 12.77 rad/s respectively. When every two neighboring floors of the adjacent buildings are linked by the same...
active tendon devices with the normalized loop gain $\varepsilon=10.0$ and the feedback gain $\tau=2.2$ with velocity sensors, the first five modal frequencies of the tendon-building system obtained from the generalized modal analysis approach are 3.48, 4.32, 9.34, 12.99 and 15.30 rad/s respectively. Obviously, using the active tendon devices to link the adjacent buildings only slightly changes the modal frequencies of the buildings. The retention of the natural frequencies of the unlinked buildings after the installation of the active tendon devices is especially desirable for the adjacent buildings that have been already built and need to be strengthened.

As to modal damping ratios, the first three modal damping ratios in the unlinked left building are calculated as 1.25%, 1.02%, and 1.33% respectively. The first two model damping ratios in the unlinked right building are computed as 0.89% and 0.72% respectively. For the linked building-tendon system, the first five modal damping ratios obtained from the generalized modal analysis approach are 9.6%, 21.9%, 6.13%, 5.14%, and 4.12% respectively. Thus, one can expect that the seismic response of the adjacent buildings can be significantly reduced.

$\varepsilon$ and $\tau$ used in the foregoing calculations are optimal parameters determined through a parametric study. In the parametric study, the modal frequencies and damping ratios of the building-tendon system are computed against $\varepsilon$ and $\tau$. The beneficial $\varepsilon$ and $\tau$ can be thus found for achieving the maximum modal damping ratio and maintaining the original modal frequencies. Fig. 8.2 displays variations of the first two modal damping ratios of the system with $\varepsilon$ for $\tau=2.2$ using either acceleration sensors or velocity sensors. The results related to displacement sensors do not appear in Fig. 8.2 because the active tendon devices with displacement sensors cannot function properly in the present application. It is seen from Fig. 8.2
that the use of velocity sensors is incomparably superior to acceleration sensors when ε is larger than 5.0. The use of active tendon devices with velocity sensors can significantly enhance the modal damping ratios in both buildings. There is a peak in the second modal damping ratio curve occurring at ε=5.8. Afterwards, the modal damping ratios remain almost constant regardless of the value of ε. From a practical viewpoint, the beneficial value of ε is selected as 10.0 for the active tendon devices with velocity sensors in this study.

The variations of the first five modal damping ratios with τ for ε=10.0 using velocity sensors are depicted in Fig. 8.3. Again, active tendon devices with displacement sensors cannot function properly and thus the corresponding results are omitted in Fig. 8.3. Results with acceleration sensors are also omitted in Fig. 8.3 because of insignificant effects compared with that with velocity sensors. It is clear that the use of velocity sensors can enhance the first five modal damping ratios of the system significantly. There is a peak in the first modal damping ratio curve occurring at τ=2.2. Therefore, the beneficial value of τ is selected as 2.2 in this study.

Fig. 8.4 and Fig. 8.5 show variations of the first five modal frequencies of the system with ε for τ=2.2 and with τ for ε=10.0, respectively, by using active tendon devices with velocity sensors. It is seen that the modal frequencies are only slightly changed with the normalized loop gain and feedback gain. This property is desirable in the application of active tendon devices into the existing buildings and structures.

8.2.3.2 Seismic Response

Seismic response analysis is carried out to investigate variations of seismic response of the adjacent buildings with tendon parameters to see if the optimal tendon parameters identified from the modal analysis are the same as those from the
seismic response analysis under the given earthquake excitation spectrum. The effectiveness of the active tendon devices of optimal parameters on seismic response mitigation is also examined at the same time.

Fig. 8.6 depicts the variations of top floor displacement responses of the left building and the right building with $\epsilon$ for $\tau=2.2$. It is seen that the use of velocity sensors gives much better results than acceleration sensors. This conclusion is the same as that found in the free vibration analysis. It is also seen that if $\epsilon$ is larger than 10.0, the top floor displacement responses of both the left and right buildings are almost not affected by the normalized loop gain $\epsilon$. The fact that the response mitigation is not sensitive to $\epsilon$ within a certain range is very helpful for the practical application of active tendon devices. The use of smaller $\epsilon$ than 10.0 may further reduce the seismic response of the left building, but it certainly increases the seismic response of the right building. When $\epsilon$ reduces to a value below 0.1, the two buildings behave as though almost not connected. The active tendon devices totally lose their effectiveness. Therefore, the optimal value for $\epsilon$ should be larger than 10, which is the same as what was found in the free vibration analysis.

To find an optimal value for $\tau$, the top floor displacement responses of both buildings are computed over a wide range of $\tau$ with an optimum value of $\epsilon$ being 10.0. Fig. 8.7 shows the variations of top floor displacement responses of the two buildings with $\tau$. It is seen that by using velocity sensors, the maximum response reduction occurs when $\tau$ is 1.1 for the left building and 2.85 for the right building. The value of 2.2 for the normalized feedback gain $\tau$ found in the modal analysis is a good compromise.
To demonstrate the overall effectiveness of the active tendon devices with velocity sensors, the standard deviations of displacement, shear force and acceleration responses at each floor for each building with and without active tendon devices are computed against the Kanai-Tajimi excitation spectrum. Fig. 8.8 shows the variations of the standard deviation of displacement response relative to the ground with the height of the buildings. The top floor displacement standard deviation of the unlinked left building is 44.7 mm but with the active tendon devices installed, it is reduced to 17.7 mm, leading to a 60% reduction of the response. For the right building, the top floor displacement standard deviation is 61.1 mm for the unlinked building and 25.9 mm for the linked building, resulting in a 58% reduction. The reduction of the displacement responses due to the active tendon devices is also significant for other floors in either building. The standard deviations of shear force in each story for each building are plotted in Fig. 8.9. The shear forces in all the stories of both buildings are reduced after installation of the active tendon devices. In particular, without the active tendon devices the bottom shear force standard deviation is \(1.41 \times 10^{-7}\) N in the left building and \(9.68 \times 10^{-9}\) N in the right building. With the optimum active tendon devices, the base shear force standard deviation is reduced to \(5.57 \times 10^{-9}\) N in the left building and \(4.12 \times 10^{-9}\) N in the right building, leading to a 60% and a 57% reduction, respectively.

The variations of acceleration response with building height, as shown in Fig. 8.10, are different from the displacement and shear force response profiles shown in Figs. 8.8 and 8.9. The acceleration response for each unlinked building does not vary monotonically with building height. This is due to the contributions from higher modes of vibration. Clearly, the active tendon devices effectively mitigate the
acceleration responses not only from low modes of vibration but also higher modes of vibration, as indicated by the response curves of the linked adjacent buildings. In particular, reductions of 58% and 40% for the top accelerations of left building and right building are achieved respectively.

The values of control forces required for the achievement of significant vibration reduction of the adjacent buildings are important for the design of the active tendon devices. In terms of the proposed mixed method, the control forces can be easily calculated. The results are shown in Fig. 8.11 for the variations of control force with the building height. The maximum control force is about \(1.1 \times 10^5\) N at the top of the buildings.

Fig. 8.12 to Fig. 8.14 show the spectral density of top floor displacement, base shear force and top floor acceleration, respectively, of adjacent buildings with and without the active tendon devices. It is noted that these peaks indicate the natural frequencies of the building. Among these peaks, the first one of the right building at \(\omega = 3.02\) rad/sec and the first one of the left building at \(\omega = 4.27\) rad/sec are dominant and are significantly mitigated by active tendon devices. The spectral peaks in the higher frequency range are also reduced.

### 8.3 ADJACENT BUILDINGS WITH LQG REGULATORS

Closed form solution for seismic response of adjacent buildings connected by hydraulic actuators with linear quadratic Gaussian (LQG) regulators is presented in this section. By using LQG regulators, the estimation and control design processes can be fully separated with the design of the estimator independent of feedback to the structure, as stated by Chen and Hsu (1995) and used in Chapter 7 for wind-excited tall buildings. The stability and performance of LQG regulators can be thus
achieved despite partial state information and noise corruption (Housner et al. 1997). This feature and its simplicity account for the popularity of LQG regulators over other regulators. Accordingly, LQG regulators are adopted in this section for controlling adjacent buildings connected by hydraulic actuators against earthquake.

Like wind excitation, earthquake excitation is also a random disturbance in nature. The statistical responses of structures with LQG regulators are usually determined in two direct ways: one is to determine the response spectral density function through transfer function first and then integrate the spectral density function to obtain the statistical response; and the other is to solve a Lyapunov equation for the covariance matrix of the response (Suhardjo et al. 1990). Since this study concerns the application of LQG regulators to adjacent buildings which involve many degrees of freedom, the aforementioned either way needs great computational efforts for finding optimal parameters of LQG problems and determining statistical responses of both buildings. Therefore, this section aims to find general yet simple closed form solution for actively controlled adjacent buildings with LQG regulators against earthquake. The derivation for closed form solution is naturally fulfilled by the generalized pseudo-excitation method and the residue theorem. The derived closed form solution is then used to perform parametric studies of adjacent buildings connected by hydraulic actuators and to assess the effectiveness of LQG regulators in reducing seismic responses of both buildings. The dynamics of actuators and their interaction with structures are not considered (Dyke et al. 1995).
8.3.1 Equations of Motion of System

Consider again the same buildings in Section 6.2.1 but connected by active hydraulic actuators at each floor of the same level (see Fig. 8.15). Both buildings are assumed to be subjected to the same base acceleration and any effects due to spatial variations of the ground motion are neglected. Both buildings and actuators are manipulated by LQG regulators.

Assume that the total number of degrees of freedom of two adjacent buildings are \( N \) (see Fig. 8.15), in which the number of degrees of freedom of the left building is \( L \) with its first floor designated as the first degree of freedom. \( N-L \) is then the number of degrees of freedom of the right building with its first floor designated as the \( L+1 \) degree of freedom. The equations of motion of the building-control device system can be expressed as

\[
M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = -M\ddot{x}_g(t) + Hu(t) \tag{8-30}
\]

where \( M, C \) and \( K \) are the mass, damping and stiffness matrices of the adjacent buildings, respectively; \( x(t) \) is the vector of relative displacement response with respect to the ground with the left building’s displacements in the first \( L \) positions and the right building’s displacements in the last \( N-L \) positions; \( e \) is the index vector with all its elements equal to 1; \( u(t) \) is a \( r \)-dimensional vector consisting of \( r \) active control forces; \( H \) is a \( N \times r \) matrix denoting the location of \( r \) actuators; and \( \ddot{x}_g(t) \) is the ground acceleration.

The details of mass, damping and stiffness matrices, \( M, C, \) and \( K \) can be found in Chapter 6, in which the mass, shear stiffness, and external damping coefficient and internal damping coefficient of the adjacent buildings are denoted as \( m, k, b, \)
and $c_i$ ($i = 1, 2, \ldots, N$), respectively. Take the Kanai-Tajimi filtered white noise spectrum as the ground acceleration spectrum in this study.

\[
S_{\nu'}(\omega) = \frac{1 + \frac{4\xi^2_g}{\omega_g^2} \left( \frac{\omega}{\omega_g} \right)^2}{\left[ 1 - \left( \frac{\omega}{\omega_g} \right)^2 \right]^2 + 4\xi^2_g \left( \frac{\omega}{\omega_g} \right)^2} S_0
\]  

(8-31)

in which $\omega_g$, $\xi_g$, $S_0$ may be regarded as the characteristics and the intensity of an earthquake in a particular geological location. This spectrum can be represented in the time domain with the following state equations:

\[
\dot{\Gamma}(t) = A_c \Gamma(t) + D_c w(t) 
\]  

(8-32)

\[
\ddot{x}_g(t) = C_c \Gamma(t) 
\]  

(8-33)

where $w(t)$ is zero-mean Gaussian white noise with intensity $S_0$ and $\Gamma(t)$ is the state of the seismic excitation model.

\[
A_c = \begin{bmatrix} 0 & 1 \\ -\omega_g^2 & -2\xi_g \omega_g \end{bmatrix} 
\]  

(8-34)

\[
C_c = \begin{bmatrix} \omega_g^2 & 2\xi_g \omega_g \end{bmatrix} 
\]  

(8-35)

\[
D_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix} 
\]  

(8-36)

Writing equation (8-30) as a state equation and then combining it with equations (8-32) and (8-33) yield the equation of motion of the system:

\[ \dot{q}(t) = Aq(t) + Gw(t) + Bu(t) \]  

(8-37)

where

\[
A = \begin{bmatrix} 0 & I & 0 \\ -M^{-1}K & -M^{-1}C - EC_c & 0 \\ 0 & 0 & A_c \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}H \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad q = \begin{bmatrix} x(t) \\ \dot{x}(t) \\ \Gamma(t) \end{bmatrix} 
\]  

(8-38)
8.3.2 LQG Controller

In reality, the structural states, that is, displacements and velocities relative to ground with each degree of freedom, cannot be fully measured. The measurement is usually limited to absolute accelerations (Yang and Li 1994; Spencer et al. 1994; Dyke et al. 1996). The absolute accelerations of adjacent buildings can be related to the relative displacements and relative velocities through the following equation.

\[ \ddot{x}_d(t) = -M^{-1}C \dot{x}(t) - M^{-1}Kx(t) + M^{-1}H u(t) \]  \hspace{1cm} (8-39)

If output measurements are selected as \( m(t) = \ddot{x}_d(t) - M^{-1}H u(t) \), then the measured output vector \( m(t) \) can be expressed as

\[ m(t) = C_m q(t) + v_m(t) \]  \hspace{1cm} (8-40)

where \( v_m(t) \) are random signals known as measurement noises and

\[ C_m = \begin{bmatrix} -M^{-1}K & -M^{-1}C & 0 \end{bmatrix} \] \hspace{1cm} (8-41)

For practical application, sensors may not be placed at every floor and thus only a subset of \( m(t) \), denoted as \( y(t) \), is used.

\[ y(t) = C_l q(t) + v(t) \]  \hspace{1cm} (8-42)

in which the dimension of \( y(t) \) is \( n (n \leq N) \). \( C_l \) is a matrix obtained by eliminating the rows related to the floors without sensors in the matrix \( C_m \). \( v(t) \) is the measurement noise vector of \( n \) dimension. If the measurement noise is regarded as the white noise of the same intensity at each measurement point and independent of each other, its covariance matrix can be expressed as

\[ E[v(t)v^t(t + \tau)] = IS_c \delta(\tau) \]  \hspace{1cm} (8-43)

where \( E \) is the expectation operator; \( I \) is the identity matrix; \( S_c \) is the intensity of noise; and \( \delta(\tau) \) is the Dirac delta function.
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The LQG cost function is

\[ J = \lim_{T_r \to +\infty} \frac{1}{2T_r} E \left[ \int_{t_0}^{t_1} (q(t)'Qq(t) + u(t)'Ru(t))dt \right] \]  
(8-44)

where \( Q \) and \( R \) are positive semi-definite and positive definite weighting matrices, respectively.

A reasonable controller design for the LQG control problem can be obtained by using the linear quadratic regulator feedback gain matrix \( K_e \) which operates on the state estimate \( \hat{q} \) generated by the Kalman filter:

\[ u(t) = -K_e\hat{q}(t) \]  
(8-45)

\[ K_e = R^{-1}B^T X_2 \]  
(8-46)

in which, \( X_2 \) is the solution of the following algebraic Riccati equation.

\[ X_2A + A^TX_2 - X_2BR^{-1}B^TX_2 + Q = 0 \]  
(8-47)

The state equation for the Kalman filter (Kwakernaak and Sivan 1972) is given by

\[ \dot{\hat{q}}(t) = A\hat{q}(t) + Bu(t) + K_e(y(t) - C_L\hat{q}(t)) \]  
(8-48)

where \( K_e \) is the estimator gain matrix given by

\[ K_e = Y_2C_L^T S_\gamma^{-1} \]  
(8-49)

\( Y_2 \) is the solution of the following algebraic Riccati equation.

\[ AY_2 + Y_2A^T - Y_2C_L^T C_L Y_2S_\gamma^{-1} + GG^T S_\eta = 0 \]  
(8-50)

8.3.3 Closed Form Solutions

The derivation of the solution for dynamic characteristics and the closed form solution for seismic response of actively controlled adjacent building with LQG controllers is similar to that in Chapter 7 for actively-controlled wind-excited tall
buildings with LQG controllers. However, for the sake of completion, these solutions are derived again.

### 8.3.3.1 Solution for Dynamic Characteristics

Equations (8-37), (8-42), (8-45) and (8-48) can be rearranged as

\[
\dot{p}(t) = \dot{A} p(t) + f(t) \quad (8-51)
\]

where

\[
\dot{A} = \begin{bmatrix}
A - BK_c & BK_c \\
0 & A - K_r C_t
\end{bmatrix}, \quad f = \begin{bmatrix}
G & 0 \\
G & -K_f
\end{bmatrix} \begin{bmatrix}
w(t) \\
v(t)
\end{bmatrix},
\]

\[
p = \begin{bmatrix}
q(t) \\
e(t)
\end{bmatrix}, \quad e(t) = q(t) - \dot{q}(t) \quad (8-52)
\]

The solution of the homogeneous form of equation (8-51) can then be taken as

\[
p(t) = \phi e^{st} \quad (8-53)
\]

The associated complex eigenvalue problem of equation (8-51) becomes

\[
\dot{A}\phi = s\phi \quad (8-54)
\]

where \(s\) is the eigenvalue and \(\phi\) is the associated eigenvector. The solution of equation (8-54) comprises a set of \(2N'\) \((N'=2N+2)\) eigenvalues and eigenvectors that exist in either complex conjugate pairs (underdamped mode) or real pairs (overdamped mode). For complex conjugate pairs,

\[
\phi_j = \phi_{j,N'}^* \quad \text{and} \quad s_j = s_{j,N'}^* \quad (j=1,2,\ldots,N') \quad (8-55)
\]

The eigenvalue is usually written under the form

\[
s_j = s_{j,N'}^* = -\omega_j \xi_j + i\omega_n \quad (j=1,2,\ldots,N') \quad (8-56)
\]

in which

\[
\omega_j = |s_j|, \quad \xi_j = -\text{Re}(s_j)/|s_j| \quad \text{and} \quad \omega_n = \omega_j \sqrt{1 - \xi_j^2} \quad (8-57)
\]
For real pairs, it is convenient to express real pairs $s_j$ in the following form analogous to equation (8-56)

$$ s_j = -\omega_j \xi_j + \omega_{d_j} \quad s_{j,N} = -\omega_j \xi_j - \omega_{d_j} \quad (8-58) $$

$\omega_j$, $\omega_{d_j}$ and $\xi_j$ are determined by

$$ \omega_j = \sqrt{s_j s_{j,N}}, \quad \xi_j = -(s_j + s_{j,N})/(2\omega_j), \quad \omega_{d_j} = \omega_j \sqrt{\xi_j^2 - 1} = (s_j - s_{j,N})/2 \quad (8-59) $$

where $\omega_j$, $\omega_{d_j}$, and $\xi_j$ are the modal frequency, the damped modal frequency, and the modal damping ratio, respectively, associated with mode $j$. The superscript * means the conjugation and $i$ is the imaginary unit.

### 8.3.3.2 Solution for Seismic Response

To find the closed form solution for seismic responses of adjacent buildings with LQG controllers, the pseudo-excitation method (Zhang and Xu 1999) is used in conjunction with the complex modal superposition method. The seismic input $w(t)$ and measurement noise vector $v(t)$ are assumed to be independent in this study. The spectral density matrix $S_{\omega}$ of both the ground excitation and the measurement noise is thus given by

$$ S_{\omega} = \begin{bmatrix} S_\omega & 0 \\ 0 & S_\omega I \end{bmatrix} = S_\omega \begin{bmatrix} 1 & 0 \\ 0 & \eta^2 I \end{bmatrix} \quad (8-60) $$

in which

$$ \eta^2 = \frac{S_{\omega}}{S_\omega} \quad (8-61) $$

Note that $S_{\omega} = S_\omega \begin{bmatrix} 1 & 0 \\ 0 & \eta I \end{bmatrix}$. Thus, letting $L = \begin{bmatrix} 1 & 0 \\ 0 & \eta I \end{bmatrix}$, the pseudo-excitation vectors for the system expressed by equation (8-51) can be constituted as (Lin et al. 1994; Zhang and Xu 1999)
\[
\begin{bmatrix}
w(t) \\
v(t)
\end{bmatrix}_k = L_k \sqrt{S_0} \exp(i\omega t) \quad (k=1,2, \ldots, n+1) \tag{8-62}
\]

where \(L_k\) is the \(k\)-th column of \(L\).

Considering the \(k\)th pseudo-excitation vector \(\begin{bmatrix} w(t) \\ v(t) \end{bmatrix}_k\), equation (8-51) becomes

\[
\dot{p}_k(t) = \dot{A} p_k(t) + F_k \sqrt{S_0} \exp(i\omega t) \tag{8-63}
\]

where

\[
F_k = \begin{bmatrix} G & 0 \\ G & -K_r \end{bmatrix} L_k \tag{8-64}
\]

To decouple equation (8-63), the following co-ordinate transformation is adopted.

\[
p_k(t) = \Phi z_k(t) \tag{8-65}
\]

where \(z_k\) is the \(2N'\)-dimensional generalized coordinate vector and \(\Phi\) is the \(2N' \times 2N'\) right modal matrix, that is.

\[
\Phi = [\phi_1, \phi_2, \ldots, \phi_{2N'}] \tag{8-66}
\]

By using this transformation, equation (8-63) can be reduced to \(2N'\) decoupled modal equations with the \(j\)th modal equation being

\[
\dot{z}_{kj}(t) = s_j z_{kj}(t) + r_{kj} \sqrt{S_0} \exp(i\omega t) \tag{8-67}
\]

where

\[
r_{kj} = \psi_j^T F_k \tag{8-68}
\]

\(\psi_j\) is the \(j\)th column of \(\Psi\); and \(\Psi = \Phi^{-1}\), the left modal matrix.

The solution of the first-order \(j\)th equation (8-67) to the \(k\)th pseudo excitation vector is
\[ z_{kj}(\omega, t) = \frac{r_{kj}}{i\omega - s_i} \sqrt{S_{jj}} \exp(i\omega t) \quad (j=1, 2, \ldots, 2N') \]  

(8-69)

Denoting the \(m\)-th components of \(\phi_i\) as \(\phi_{mj}\) and the \(m\)-th component of \(p_k\) as \(p_{km}\), then the pseudo response \(p_{km}\) is given by

\[ p_{km}(\omega, t) = \sum_{j=1}^{N'} \phi_{mj} z_{kj}(\omega, t) = \sum_{j=1}^{N'} \phi_{mj} \frac{r_{kj}}{i\omega - s_i} \sqrt{S_{jj}} \exp(i\omega t) \]  

(8-70)

Since the eigenvectors are in pairs for either underdamped mode or overdamped mode, equation (8-70) can be reduced to

\[ p_{km}(\omega, t) = \sum_{j=1}^{N} H_j(\omega)(i\omega \alpha_{km} + \beta_{km}) \sqrt{S_{jj}} \exp(i\omega t) \]  

(8-71)

in which \(H_j(\omega)\) is the frequency response function for the \(j\)th mode.

\[ H_j(\omega) = \frac{1}{\omega_i^2 - \omega^2 + i2\xi_j \omega_i \omega} \]  

(8-72)

The pseudo response \(p_{km}\) can be the pseudo displacement response, pseudo velocity response, and pseudo acceleration response. The proper use of the pseudo displacement responses of adjacent buildings can result in the pseudo shear force responses of both buildings. Also, the proper use of the pseudo state estimator responses in conjunction with equation (8-45) can lead to the pseudo control forces. For instance, the \(m\)-th \((m \leq N)\) pseudo displacement, velocity, or acceleration response can be obtained from equation (8-71) if the coefficients \(\alpha_{km}\) and \(\beta_{km}\) are calculated by the following equations:

When the \(j\)th mode is an underdamped mode

\[ \alpha_{km} = \begin{cases} 2 \text{Re}(\phi_{mj} r_{sj}) & \text{For displacement} \\ 2 \text{Re}(s_j \phi_{mj} r_{sj}) & \text{For velocity} \\ 2 \text{Re}(s_j^2 \phi_{mj} r_{sj}) & \text{For acceleration} \end{cases} \]  

(8-73)
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\[
\beta_{km} = \begin{cases} 
-2 \text{Re}(\phi_m r_k s_{i}) & \text{For displacement} \\
-2\omega^2 \text{Re}(\phi_m r_k) & \text{For velocity} \\
-2\omega^2 \text{Re}(s_i \phi_m r_k) & \text{For acceleration}
\end{cases} \quad (8-74)
\]

When the jth mode is an overdamped mode

\[
\alpha_{kn} = \begin{cases} 
\phi_m r_k + \phi_{m-N} r_{k-N} & \text{For displacement} \\
\phi_m r_k + \phi_{m-N} r_{k-N} & \text{For velocity} \\
\phi_m r_k + \phi_{m-N} r_{k-N} & \text{For acceleration}
\end{cases} \quad (8-75)
\]

\[
\beta_{kn} = \begin{cases} 
-(\phi_m r_k s_{j-N} + \phi_{m-N} r_{k-N} s_{j-N}) & \text{For displacement} \\
-\omega^2 (\phi_m r_k + \phi_{m-N} r_{k-N}) & \text{For velocity} \\
-\omega^2 (s_j \phi_m r_k + s_{j-N} \phi_{m-N} r_{k-N}) & \text{For acceleration}
\end{cases} \quad (8-76)
\]

According to the principle of the pseudo-excitation method, the response spectral density of \( p_{km} \) can then be obtained by

\[
S_{p_{km}p_{km}}(\omega) = p_{km}(\omega, t)p^*_k(\omega, t) \\
= \sum_{i=1}^{N} \sum_{j=1}^{N} H_i(\omega)H_j^*(\omega)(\beta_{i-k} \beta_{j-k} + i\omega(\alpha_{i-k}\beta_{j-k} - \alpha_{j-k}\beta_{i-k}) + \omega^2 \alpha_{i-k} \alpha_{j-k})S_{ij}
\]

\[S_{p_{km}p_{km}}(\omega) = \int_{-\infty}^{\infty} S_{p_{km}p_{km}}(\omega) \, d\omega \quad (8-78)\]

The variance response of \( p_{km} \) under the kth pseudo-excitation can be evaluated as

\[
\sigma_{p_{km}}^2 = \int_{-\infty}^{\infty} S_{p_{km}p_{km}}(\omega) \, d\omega \quad (8-79)
\]

The above integration in the complex plane can be accomplished using the residue theorem to have the closed form solution as

\[
\sigma_{p_{km}}^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{i,j} u_{km} u_{km} + \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{i,j} v_{km} v_{km} + \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{i,j} u_{km} v_{km}
\]

where

\[
u_{km} = \sqrt{\frac{\pi}{2} \omega_i S_0 \omega} \quad u_{km} = \sqrt{\frac{\pi}{2} \omega_i S_0 \omega}\]

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\[ v_{km} = \sqrt{\frac{\pi}{2}} \frac{\alpha_{km} \sqrt{S_0}}{\sqrt{\xi_i \omega_i}}, \quad \nu_{mj} = \sqrt{\frac{\pi}{2}} \frac{\alpha_{mj} \sqrt{S_0}}{\sqrt{\xi_j \omega_j}} \]  
\hspace{1cm} (8-81)

\[ \rho_{0,j} = \frac{8\sqrt{\xi_i \xi_j} (\xi_i + \gamma \xi_j) \gamma^2}{(1 - \gamma^2)^2 + 4\xi_i \xi_j \gamma (1 + \gamma^2) + 4(\xi_i^2 + \xi_j^2) \gamma^2} \]  
\hspace{1cm} (8-82)

\[ \rho_{0,l,n} = \frac{8\sqrt{\xi_i \xi_l} (\gamma^2 - 1) \gamma^2}{(1 - \gamma^2)^2 + 4\xi_i \xi_l \gamma (1 + \gamma^2) + 4(\xi_i^2 + \xi_l^2) \gamma^2} \]  
\hspace{1cm} (8-83)

\[ \rho_{1,j} = \frac{8\sqrt{\xi_i \xi_j} (\xi_i + \gamma \xi_j) \gamma^2}{(1 - \gamma^2)^2 + 4\xi_i \xi_j \gamma (1 + \gamma^2) + 4(\xi_i^2 + \xi_j^2) \gamma^2} \]  
\hspace{1cm} (8-84)

\[ \gamma = \frac{\omega_i}{\omega_i} \]  
\hspace{1cm} (8-85)

The final variance response of \( p_n \) can be determined by a summation with respect to the \( n+1 \) pseudo excitation vectors.

\[ \sigma_{p_m}^2 = \sum_{k=1}^{n} \sigma_{p_{km}}^2 \]  
\hspace{1cm} (8-86)

The closed form solution derived above makes it possible to carry out extensive parametric studies and to evaluate the performance of both buildings with LQG controllers.

### 8.3.4 Application

For application, the same adjacent buildings subjected to the same earthquake excitation in section 6.2.4 but with LQG controllers at each floor of the same level (see Fig. 8.15) are used. The parameters for structures and for the ground motion used herein are the same as the ones used in Section 6.2.4. The intensity ratio of the measurement noise to the ground motion, \( r \), in equation (8-61) is selected as 1/7.
8.3.4.1 Selection of Weighting Matrices

The performance of LQG controllers depends on the weighting matrices $R$ and $Q$. To achieve the optimal performance of LQG for the maximum reduction of key structural responses of both buildings, several potential weighting matrices are selected. The key structural responses are then computed in terms of the closed form solution, from which the best weighting matrices can be identified. The basic configurations of the weighting matrices in this study are taken as

$$R = 10^{-8} I, \quad Q = \mu \begin{bmatrix} Q_{dl} & 0 & 0 & 0 & 0 \\ 0 & Q_{dr} & 0 & 0 & 0 \\ 0 & 0 & Q_{sL} & 0 & 0 \\ 0 & 0 & 0 & Q_{sR} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(8-87)

where the submatrices $Q_{dl}$ and $Q_{dr}$ are assigned to the displacement responses of the left and right buildings, respectively; the submatrices $Q_{sL}$ and $Q_{sR}$ are assigned to the velocity responses of the left and right buildings, respectively; the matrix $R$ is allocated to the control forces; and $\mu$ is a proportional coefficient. Clearly, by varying the coefficient $\mu$, a proper trade off between control effectiveness and control energy consumption can be achieved. Four cases are selected for finding the balanced submatrices $Q_{dl}$, $Q_{dr}$, $Q_{sL}$, and $Q_{sR}$. These four cases are defined as follows:

Case A: $Q_{dl} = Q_{dr} = Q_{sL} = Q_{sR} = I$

Case B: $Q_{dl} = Q_{dr} = I, \quad Q_{sL} = Q_{sR} = 0$

Case C: $Q_{dl} = Q_{dr} = 0, \quad Q_{sL} = Q_{sR} = I$

Case D: $Q_{dl} = Q_{dr} = I, \quad Q_{sL} = Q_{sR} = 0.1I$

In Case A, the four submatrices are equally assigned; in Case B, the assignment to the two displacement submatrices only indicates that the displacement
response reduction is maximized regardless of the velocity response reduction; Case C indicates that the velocity response reduction is maximized regardless of the displacement response reduction; and Case D implies that the displacement response reduction is given priority over the velocity response control.

For each case mentioned above, the key responses of actively controlled adjacent buildings are computed against the parameter \( \mu \). Then, by comparing the results among all the cases the beneficial case and parameter \( \mu \) can be found for achieving the maximum or beneficial response reduction of both buildings. Figs. 8.16 and 8.17 depict the variations of the top floor displacement responses of the left and right buildings, respectively, with the parameter \( \mu \). Figs. 8.18 and 8.19 display the variations of the base shear force responses of the left and right buildings, respectively. Fig. 8.20 shows how the control force at the top of the buildings varies with the parameter \( \mu \). It is seen that for all cases the top floor displacement and base shear force responses of both buildings are rapidly reduced until \( \mu \) reaches a value about \( 2 \times 10^4 \). After that, the gradients of response reduction become small in Cases A, B, and D, but not in Case C where the further increase of \( \mu \) makes the displacement and shear force responses become larger with larger control force required. Thus, Case C is disregarded in the subsequent computations. Furthermore, if the weighting matrices in Case A are adopted, one may benefit from the response reduction of the left building but not the right building. The situation is reversed if the weighting matrices in Case B is used. As a result, a compromise is made in this study to select the weighting matrices of Case D.
8.3.4.2 Modal Properties

By using the weighting matrix Q in Case D, the first five natural frequencies and modal damping ratios of the actively controlled adjacent buildings are computed against the parameter $\mu$. The results are depicted in Figs. 8.21 and 8.22 for the natural frequencies and modal damping ratios, respectively. It is seen from Fig. 8.21 that the first five natural frequencies do not vary with the parameter $\mu$ when $\mu$ is in the range from 10 to $2 \times 10^4$. Further analysis shows that the first, third, and fifth natural frequencies of the actively controlled adjacent buildings are almost the same as the first three natural frequencies of the uncontrolled right building of 3.02, 9.03 and 14.99 rad/s, respectively. The second and fourth natural frequencies of the actively controlled adjacent buildings are almost the same as the first two natural frequencies of the uncontrolled left building of 4.27 and 12.77 rad/s, respectively.

The first five modal damping ratios of the actively controlled adjacent buildings increase with the increasing value of $\mu$ when $\mu$ is in the range from 10 to $2 \times 10^4$, as shown in Figs. 8.22a and 8.22b. Since the first, third, and fifth modes of vibration of the system are dominated by the right building and the second and fourth modes of vibration by the left building, the pattern of the curves in Fig. 8.22a is different from that in Fig. 8.22b. When $\mu$ is further increased from $2 \times 10^4$, the second modal damping ratio starts to decrease but the fourth modal damping ratio still increases until $\mu$ reaches $1 \times 10^5$. The first, third, and fifth modal damping ratios which are dominated by the right (softer) building are always increased with increasing value of $\mu$, and eventually the modes of vibration are overdamped. With the information on how the key structural response reductions and the modal properties vary with the parameter $\mu$, this study selects $\mu$ of $2 \times 10^6$ as a beneficial
value for the subsequent computation of seismic response using the derived closed
from solution.

Further modal analysis of the actively controlled adjacent buildings with the
beneficial weighting matrix $Q$ and beneficial parameter $\mu$ selected above shows that
the real parts of all eigenvalues of $\tilde{A}$ are negative. Therefore, according to
Lyapunov’s criterion about the stability of linear systems, the present actively
controlled adjacent buildings form a stable system.

8.3.4.3 Seismic Response

To demonstrate the overall performance of the LQG controllers, the standard
deviations of displacement, shear force, acceleration responses at each floor for each
building with and without active control devices are computed. The active control
devices are arranged at every floor, and no attempt is made to investigate the optimal
location and number of active control devices in this study. Figs. 8.23 and 8.24 show
the variations of the standard deviation of displacement and shear force response,
respectively, with the height of the buildings. The reduction of the responses from
the LQG controllers is significant for all floors in either building. In particular, the
top floor displacement standard deviation of the unlinked left building is 44.7 mm
but with the active control devices installed, it is reduced to 17.3 mm, leading to a
61% reduction of the response. For the right building, the top floor displacement
standard deviation is 61.1 mm for the unlinked building and 21.6 mm for the linked
building, resulting in a 65% reduction. For base shear force response, without control
the bottom shear force standard deviation is $1.41 \times 10^7$ N in the left building and
$9.68 \times 10^8$ N in the right building. With the LQG controllers, the base shear force
standard deviation is reduced to $5.36 \times 10^6$ N in the left building and $3.44 \times 10^6$ N in the right building, leading to a 62% and a 64% reduction, respectively.

The variations of acceleration response with the building height, as shown in Fig. 8.25, are different from displacement and shear force response profiles shown in Figs. 8.23 and 8.24. The acceleration response for each unlinked building does not vary monotonically with the building height. This is due to the contributions from higher modes of vibration. Clearly, the active control devices effectively mitigate the acceleration responses not only from low modes of vibration but also higher modes of vibration, as indicated by the response curves of the linked adjacent buildings. The top floor acceleration standard deviation of the unlinked left building is $1.11 \text{m/s}^2$ but with the control devices installed, it is reduced to $0.464 \text{m/s}^2$, leading to a 58% reduction of the response. For the right building, the top floor acceleration standard deviation is $0.807 \text{m/s}^2$ for the unlinked building and $0.364 \text{m/s}^2$ for the linked building, resulting in a 55% reduction.

To further enhance the understanding of seismic response of the adjacent buildings with and without control devices, the spectral density functions of the top floor displacement response, base shear force response, and top floor acceleration response of both buildings with and without control devices are computed and plotted in Figs. 8.26, 8.27 and 8.28, respectively. It is clearly seen that all the peaks in the response spectra of both buildings are significantly reduced when the LQG controllers are used. For the buildings without control, the effects of higher modes of vibration on the acceleration response are much larger than on the top displacement and base shear force responses. With the controllers, the effects of higher mode of vibration on the acceleration response are significantly reduced. These are consistent with the results from the computed response reduction.
The values of control forces required for the achievement of significant vibration reduction of the adjacent buildings are important for the design of the hydraulic actuators and adjacent buildings. In terms of the closed form solution, the control forces can be easily calculated. Fig. 8.29 shows the variations of control force with the building height. The maximum control force occurs at the top of the building, as expected, and is in the practical range. The fact that very small control forces occur at the floors near the ground indicates that the corresponding controllers can be removed.

8.4 SUMMARY

Active tendon devices and active actuators with LQG controllers have been used to link adjacent buildings for mitigation of seismic response in this chapter. The dynamic characteristics of active tendon-linked adjacent buildings were obtained by a complex modal analysis. The random seismic responses of the tendon-linked adjacent buildings were determined by the generalized pseudo-excitation method. The active tendon devices with velocity sensors used here are the same as the fluid dampers described by Maxwell model. However, because of some limitations on the control parameters such as the loop gain and the feedback gain, the performance of active tendon devices is not so good as the fluid dampers.

The advantage of hydraulic actuators with LQG controllers is that only output feedback of acceleration measurements instead of full state measurements is needed because control algorithms that are dependent on direct measurement of the displacements and velocities may be impracticable for full-scale implementations. With the derived closed form solution, extensive parametric studies can be performed on the system of many degrees of freedom to seek optimal weighting.
matrices, modal characteristics, and maximum seismic response reductions with reasonable control forces. It was found from the example buildings that if weighting matrices were selected appropriately, the modal damping ratios of the system could be significantly increased and the earthquake-induced dynamic responses of both buildings could be considerably reduced. The results of hydraulic actuators with LQG controllers is better than that of active tendon devices. The major difficulty for control designers is to choose right weighting matrices in the performance function of optimal control strategies.

A major difference between this chapter and Chapter 4 is that a Kalman filter is used in this chapter. The LQG controller overcomes the need to measure the entire state by estimating the state, using a Kalman filter. The estimated state is then used in the linear quadratic regulator (LQR). In Chapter 4, the velocities measured by velocity sensors is used directly in control strategy.
Figure 8.1 Structural Model of Adjacent Buildings with Active Tendon Devices
Figure 8.2 Variations of Modal Damping Ratios with Normalized Loop Gain

Figure 8.3 Variations of Modal Damping Ratios with Normalized Feedback Gain
Figure 8.4 Variations of Modal Frequencies with Normalized Loop Gain

Figure 8.5 Variations of Modal Frequencies with Normalized Feedback Gain
Figure 8.6 Top Floor Displacement Responses of Buildings vs. Normalized Loop Gain

Figure 8.7 Top Floor Displacement Responses of Buildings vs. Normalized Feedback Gain
Figure 8.8 Variations of Displacement Response of Adjacent Buildings with Height

Figure 8.9 Variations of Shear Force Response of Adjacent Buildings with Height
Figure 8.10 Variations of Acceleration Response of Adjacent Buildings with Height

Figure 8.11 Variations of Control Force Response of Adjacent Buildings with Height
Figure 8.12 Spectral Density of Top Floor Displacement with and without Active Tendon Devices
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Figure 8.15 Structural Model of Adjacent Buildings with LQG controllers
Figure 8.16 Top Floor Displacement Response of Left Building vs. Parameter $\mu$

Figure 8.17 Top Floor Displacement Response of Right Building vs. Parameter $\mu$
Figure 8.18 Base Shear Force Response of Left Buildings vs. Parameter $\mu$

Figure 8.19 Base Shear Force Response of Right Buildings vs. Parameter $\mu$
Figure 8.20 Control Force of Adjacent Buildings vs. Parameter $\mu$

Figure 8.21 Variations of Modal Frequencies with Parameter $\mu$
Figure 8.22 Variations of Modal Damping Ratios with Parameter $\mu$
Figure 8.23 Variations of Displacement Response of Adjacent Buildings with Height

Figure 8.24 Variations of Shear Force Response of Adjacent Buildings with Height
Figure 8.25 Variations of Acceleration Response of Adjacent Buildings with Height
Figure 8.26 Spectral Density of Top Floor Displacement with and without Active Control Devices
Figure 8.27 Spectral Density of Base Shear Force with and without Active Control Devices
Figure 8.28 Spectral Density of Top Floor Acceleration with and without Active Control Devices
Figure 8.29 Variations of Control Force of Adjacent Buildings with Height
CHAPTER NINE

EXPERIMENTAL INVESTIGATION OF ADJACENT BUILDINGS CONNECTED BY FLUID DAMPER

9.1 INTRODUCTION

In Chapters 6 and 8, passive and active controls of adjacent buildings under random earthquake excitation are analytically investigated. This chapter presents an experimental study of adjacent buildings connected by fluid dampers under harmonic excitation using two three-story building models and a linear viscous damper. Two building models were constructed as two three-story shear buildings of different natural frequencies. Model fluid damper connecting the two buildings was designed as linear viscous damper of which damping coefficient could be adjusted. The experimental arrangement, including the design of building models and the design and calibration of fluid damper model, is introduced first. The dynamic characteristics of individual buildings, obtained using system identification technique are then presented. Followed are the investigation of variations of modal damping ratio and natural frequency of the damper-building system to damper damping coefficient. The tests were carried out to determine modal damping ratios of the adjacent buildings connected by the fluid damper of different damping coefficients and at different locations. Optimal damper damping coefficient and location for achieving the maximum modal damping ratio were thus found. Finally, the measured dynamic responses of the damper-building system to harmonic excitation are compared with those of individual buildings to evaluate the
effectiveness of fluid damper and the overall performance of the damper-building system. The comparison showed that the fluid damper of proper parameter could significantly increase the modal damping ratio and significantly reduce the dynamic response of both buildings.

9.2 EXPERIMENTAL ARRANGEMENT

9.2.1 Building Models

Two building models were designed and constructed as three-story shear buildings in order to facilitate the system identification (see Fig. 9.1). The steel frame of each building consisted of three rigid plates and four flexible columns. The plates and columns were welded properly to form rigid joints. The overall dimensions of each building were measured at 1450mm×850mm×500mm. The two buildings were then welded in a tandem arrangement to a large reinforced concrete foundation that was in turn bolted into the massive concrete laboratory ground floor. The distance between the two buildings was set at 300 mm for the installation of fluid damper. The columns were made of high strength steel of 435 MPa yield stress and 200 GPa modulus of elasticity. The 9.5 mm×75 mm cross-section columns were arranged in such a way that the first natural frequency of each building was much lower in the x-direction than in the y-direction. This arrangement restricted the building motion in the y-direction and thus the building models were effectively reduced to planar frames in the x-z plane. The thickness of each steel floor (plate) was 25 mm so that the floor can be regarded as a rigid plate in horizontal, leading to a shearing type of deformation. By placing additional mass on each floor of each building, the natural frequencies of each building can be changed. To properly
simulate the inherent energy dissipation capacity of the real building, the small dashpots were installed between every two floors and the structural damping ratios of each building could be thus adjusted within a wide range.

9.2.2 Fluid Damper

One objective of this experiment is to investigate variations of modal damping ratio and natural frequency of the damper-building system with damper damping coefficient. Thus, a viscous fluid damper that can be adjusted to have varying damping coefficient within a certain range is required. Such a fluid damper is hardly found in the market and therefore a plate fluid damper was designed and manufactured in the Hong Kong Polytechnic University. The plate fluid damper mainly consisted of a plate as a piston moving in a rectangular container filled with silicone coil (see Fig. 9.2). The relative motion of the piston to the rectangular container sheared the fluid in the two gaps and provided energy dissipation. By adjusting the gaps between the plate and the top and bottom of the rectangular container and by changing oil viscosities as well, different damper damping coefficients were achieved. To obtain a linear viscous property, some openings were made on the top of the container to release the oil pressure caused by the motion of the piston.

The mechanical characteristics of the plate fluid damper were examined using the testing arrangement shown in Fig. 9.3. An electrodynamic exciter (Link Dynamic System 409) was used under displacement control and it applied a sinusoidal motion to the piston of the plate fluid damper. The force in the damper was measured by a load cell that was connected to a reaction frame. The displacement of the damper piston was measured by a displacement transducer. Based on the measured load and
displacement time histories, the mechanical characteristics of the damper could be determined. For steady-state conditions, the damper piston motion measured from the displacement transducer and the damper force measured from the load cell can be, respectively, represented by

\[ u = u_0 \sin(\omega t) \]  

\[ P = P_0 \sin(\omega t + \phi) = K_{d1} u_0 \sin(\omega t) + K_{d2} u_0 \cos(\omega t) \]

where \( u_0 \) is the amplitude of the piston displacement, \( \omega \) is the frequency of motion, \( t \) is the time, \( P_0 \) is the amplitude of the damper force, \( \phi \) is the phase angle and \( K_{d1} \) and \( K_{d2} \) are the storage stiffness and the loss stiffness, respectively. The loss stiffness can be determined from the measured damper force and displacement time histories using the following equation:

\[ K_{d2} = \frac{P_0}{u_0} \sin \phi = \frac{\int P du}{\int u^2} \]  

\[ (9.3) \]

The damper damping coefficient, the phase angle, and the storage stiffness can then be estimated by

\[ C_d = \frac{K_{d2}}{\omega} \]  

\[ \phi = \sin^{-1}\left( \frac{K_{d2} u_0}{P_0} \right) \]  

\[ K_{d1} = \frac{P_0}{u_0} \cos \phi \]  

\[ (9.4) \]

\[ (9.5) \]

\[ (9.6) \]

Typical recorded force-displacement loops are presented in Figure 9.4 at a temperature of 25°C and frequencies of 1, 2 and 10 Hz, respectively. It was found that the plate fluid damper possesses small storage stiffness and large damping coefficient when the frequency is below 2 Hz and its behavior is essentially linear.
viscous. When the frequency of damper motion is above 2 Hz, the damper exhibits a certain amount of storage stiffness and its behavior is essentially viscoelastic. Fig. 9.5 shows variations of damper damping coefficient and storage stiffness with the frequency of motion for the piston of displacement amplitude. With a frequency range of 2 to 7 Hz, damper damping coefficient remained almost constant, but as the motion frequency was further increased, the damping coefficient started to decrease. The storage stiffness, on the other hand, increased with increasing frequency of motion. In addition, the mechanical properties of the plate fluid damper were much less affected by the amplitude of motion than the frequency of motion. This was confirmed in the tests conducted at the same frequency but in different amplitudes.

From the calibration of the plate fluid damper, it can be concluded that the plate fluid damper used in the tests has linear viscous behavior when the motion frequency is less than 2 Hz and has viscoelastic behavior when the motion frequency is above 2 Hz. These mechanical properties of the plate fluid damper are similar to the commercial fluid dampers manufactured by Tayler Devices, Inc. U.S.A. and calibrated by Constantinou and Symans (1992, 1993a). The Maxwell model describing the commercial fluid dampers can be thus used for the plate fluid damper.

9.2.3 Measurement and Instrumentation

The natural frequencies of the reinforced concrete foundation were first measured using an instrumented hammer and several accelerometers after the foundation was built. The lowest natural frequency of the foundation in the x-direction (see Fig. 9.1) was measured at 450 Hz, which was much higher than the concerned natural frequencies of two building models in the same direction. The
high-frequency guaranteed that the foundation did not affect vibration tests of the buildings.

The measurement was then taken on each building to determine their natural frequencies, structural modal damping ratios, mode shapes, and responses to harmonic loading. This was achieved by installing one accelerometer on each floor in the x-direction and using the electrodynamic exciter to give the building a constant amplitude harmonic force within a wide range of excitation frequency (see Fig. 9.6). The natural frequencies and structural modal damping ratios were identified from the recorded frequency response curves whilst the mode shapes were identified from the recorded response time histories of each floor at the resonance frequencies.

The plate fluid damper was then installed on the top floor, the second floor, and the first floor, respectively, to connect two buildings together. The dynamic responses of both buildings at each floor to harmonic excitation were measured simultaneously. The damper displacement, i.e. the relative displacement between the two buildings, and the damper force were also simultaneously measured using the load cell and displacement meter (see Fig. 9.6), from which the damper damping coefficient could be directly estimated for different damper configurations used in the tests. To capture overall characteristics of the building-damper system, the external excitation harmonic force was applied to the first floor of Building I first and then to the first floor of Building II.

For each load case and each damper location, the optimal damper damping coefficient for achieving the maximum modal damping ratio of the system was sought. The building responses were recorded for the comparison with those of the buildings without damper connected.
9.3 STRUCTURAL PROPERTIES OF INDIVIDUAL BUILDINGS

The identification of the structural properties of two individual buildings without fluid damper connected was easily accomplished by the procedure described in the last section. For Building I, the mass of each floor was equal to 314.5 kg while the mass of each floor of Building II was 139.8 kg.

The measured first three natural frequencies were 2.92, 8.39 and 12.38 Hz for Building I and 4.50, 12.97 and 19.17 Hz for Building II. Clearly, the third natural frequency of Building I was very close to the second natural frequency of Building II. The first three modal damping ratios were estimated as 0.021, 0.026 and 0.027 for Building I and 0.019, 0.026 and 0.032 for Building II. The measured mode shape matrices for Buildings I and II are listed as follows and shown in Fig 9.7:

\[
\begin{align*}
\Psi_1 &= \begin{bmatrix} 1.000 & 1.000 & 1.000 \\ 1.771 & 0.416 & -1.170 \\ 2.256 & -0.787 & 0.549 \end{bmatrix}, & \Psi_2 &= \begin{bmatrix} 1.000 & 1.000 & 1.000 \\ 1.817 & 0.437 & -1.294 \\ 2.258 & -0.819 & 0.616 \end{bmatrix} 
\end{align*}
\] (9-7)

The stiffness and damping matrices of each building can be identified using the following equations:

\[
K = M \left( \sum_{i=1}^{3} \frac{\omega_i^2}{m_i} \psi_i \psi_i^T \right) M 
\] (9-8)

\[
C = M \left( \sum_{i=1}^{3} \frac{2 \xi_i \omega_i}{m_i} \psi_i \psi_i^T \right) M 
\] (9-9)

The stiffness and damping matrices identified for Buildings I and II are listed as follows:
\[
K_1 = \begin{bmatrix}
12.19 & -6.14 & 0.34 \\
-6.14 & 11.03 & -5.75 \\
0.34 & -5.75 & 5.79 \\
\end{bmatrix} \times 10^4 \text{ N/m} \tag{9-10}
\]

\[
K_2 = \begin{bmatrix}
11.23 & -5.92 & 0.26 \\
-5.92 & 11.91 & -6.30 \\
0.26 & -6.30 & 6.18 \\
\end{bmatrix} \times 10^5 \text{ N/m} \tag{9-11}
\]

\[
C_1 = \begin{bmatrix}
1002 & -341 & -43 \\
-341 & 847 & -374 \\
-43 & -374 & 577 \\
\end{bmatrix} \text{ Ns/m} \tag{9-12}
\]

\[
C_2 = \begin{bmatrix}
652 & -275 & -7 \\
-275 & 688 & -314 \\
-7 & -314 & 406 \\
\end{bmatrix} \text{ Ns/m} \tag{9-13}
\]

The identified matrices indicate that the building models used in the tests can be regarded as shear-type buildings. It is also noticed that stiffness matrices \(K_1\) and \(K_2\) given by equations (9-10) and (9-11) are slightly different from the ones given in Appendix B because of experimental errors.

### 9.4 CHARACTERISTIC OF DAMPER-BUILDING SYSTEM

#### 9.4.1 Fluid Damper at Top Floor

A fluid damper of damping coefficient of 3531 N s/m at a frequency of 3.24 Hz was used to connect two building models together at the top floor along with the line passing through the center of each building. A harmonic load of constant amplitude was applied at the first floor of Building 1, parallel to the damper, to have a sweep test from 0.5 to 20 Hz at a very low velocity. The resulting normalized frequency response curve obtained from sensor A3 (at the top of Building 1) is displayed in Fig. 9.8. The four natural frequencies of 3.24, 8.40, 12.30 and 12.65 Hz.
were identified. The harmonic load was then applied at the first floor of Building II to have another sweep test. The measured normalized frequency response curve from sensor A6 (at the top of Building II) is also shown in Fig. 9.8, from which the four natural frequencies of 3.24, 5.36, 12.65 and 19.18 Hz were identified. Among the identified six frequencies, three natural frequencies of 3.24, 8.40 and 12.30 Hz originated from Building I while the natural frequencies of 5.36, 12.65 and 19.18 Hz came from Building II. The installation of the fluid damper made the two buildings be partially coupled, as implied by the response peak of Building II at a frequency of 3.24 Hz and the response peak of Building I at a frequency of 12.65 Hz. Compared with the natural frequencies of individual buildings, one may observe that the installation of fluid damper moderately increased the first natural frequency of each building but did not affect higher natural frequencies of the buildings.

The modal damping ratios (including the structural damping ratio) were identified from these response curves as 8.5, 30.0, 5.7 and 4.5 per cent for the first, second, third, and sixth modes, respectively. The modal damping ratios in the fourth and fifth modes were hardly identified since they are closely spaced. Obviously, the installation of the fluid damper enhanced the modal damping ratios significantly, in particular, in the first and second modes of vibration. Thus, it can be expected that the dynamic response of both buildings can be significantly reduced.

From the tests, it was found that the mode shapes of the fluid damper-adjacent building system depended on the position of external excitation. For the harmonic load applied to Building I, the measured mode shape matrices for Buildings I and II at the natural frequencies of 3.24, 8.40 and 12.30 Hz are as follows:
\[ \Psi_1 = \begin{bmatrix} 1.000 & 1.000 & 1.000 \\ 1.683 & 0.481 & -1.225 \\ 2.313 & -0.777 & 0.603 \end{bmatrix}, \quad \Psi_2 = \begin{bmatrix} 0.440 & 0.226 & 0.453 \\ 0.871 & 0.308 & 0.272 \\ 1.176 & 0.184 & -0.305 \end{bmatrix} \] (9-14)

The above mode shapes were normalized based on the first floor of Building I. Compared with the mode shape matrices of Building I without fluid damper, the mode shape matrices of Building I with fluid damper changed only slightly. The associated mode shapes of Building II, however, did not always follow those of either Building I or Building II without fluid damper. The modal motion of Building II was also found to have a phase difference from that of Building I. For the harmonic load applied to Building II, the measured mode shape matrices for Buildings II and I at the natural frequencies of 5.36, 12.65 and 19.18 Hz, which were normalized based on the first floor of Building II, are as follows:

\[ \Psi_1 = \begin{bmatrix} 0.435 & 0.254 & 0.003 \\ 0.562 & -0.300 & -0.011 \\ 0.376 & 0.229 & 0.063 \end{bmatrix}, \quad \Psi_2 = \begin{bmatrix} 1.000 & 1.000 & 1.000 \\ 1.367 & 0.531 & -1.200 \\ 1.663 & -0.799 & 0.571 \end{bmatrix} \] (9-15)

Compared with the mode shapes of Building II without fluid damper, the first mode of Building II with fluid damper was moderately changed but the higher modes of vibration were only slightly altered. The associated modal shapes of Building I, however, did not follow the mode shapes of Building I without fluid damper. The associated modal motion of Building I also had a phase difference from that of Building II. The above discussion indicates that the mode shape of the fluid damper-adjacent building system depends on the position of external excitation. The coupled system studied here is no longer a classically damped system.

Similar tests were carried out on the system for different damper damping coefficients to see the variations of system modal damping ratio and natural frequency with damper damping coefficient. The different damper damping
coefficient was achieved by adjusting the gaps between the damper piston and
rectangular box and was measured during the tests of the system. Figs. 9.9(a)-9.9(d)
show the variations of the first, second, third and sixth modal damping ratios with
damper damping coefficient. It is seen that there was an optimal damper damping
coefficient around 3500 Ns/m by which the maximum modal damping ratios could
be obtained in the first and second vibration modes. The third and sixth modal
damping ratios, however, increased almost monotonically with increasing damper
damping coefficient. The fourth and fifth modal damping ratios were hardly
estimated since these two modes were coupled. It is encouraging to see that because
of the installation of fluid damper, the modal damping ratio in the first mode of
vibration could be increased to about 8.5 per cent and the modal damping ratio in the
second mode of vibration could be increased to about 30 per cent. The reason why
the second modal damping ratio can be enhanced much higher than other modes may
be related to the stiffness ratio of two buildings and the location of fluid damper.

Figs. 9.10(a)-9.10(d) display variations of the first to sixth natural frequencies
with damper damping coefficient. Apart from the second natural frequency, all other
natural frequencies remained almost constant. The second natural frequency,
however, moderately increased with increasing damper damping coefficient.

9.4.2 Fluid Damper at Second Floor

After the tests on the adjacent buildings with a fluid damper installed at the top
floor, the fluid damper was then moved to the second floor of the adjacent buildings.
Two sweep harmonic tests were then carried out for any given damper damping
coefficient. In one test, the harmonic load of constant amplitude was applied at the
first floor of Building I while in the other test, the load was applied at the first floor
of Building II. In terms of the measured frequency response curves of the two buildings and the measured displacement and force response curves of the damper, the variations of modal damping ratio and natural frequency of the system with damper damping coefficient were observed.

Figs. 9.11(a)-9.11(d) show the variation of the first, second, third and sixth modal damping ratio with damper damping coefficient. The optimal damper damping coefficient for the first vibration mode was about 3454 Ns/m but for the second mode of vibration the optimal value was about 6025 Ns/m. The achievable maximum modal damping ratio was about 7 per cent for the first mode of vibration and 20 per cent for the second mode of vibration. For the sixth mode of vibration, when damper damping coefficient was increased to more than 3400 Ns/m, the resonant response around the sixth natural frequency almost disappeared due to high damping and thus the corresponding modal damping ratio could not be estimated. The third modal damping ratio, however, increased slightly with the increasing damper damping coefficient.

Compared with the achievable maximum modal damping ratios obtained when the fluid damper was installed at the top floor, the achievable maximum modal damping ratios of the adjacent buildings with the fluid damper installed at the second floor were smaller in the first three modes of vibration. This is because the damper at the second floor was not located at the maximum amplitude of the first and second modes of the system and also because it was close to the node of the third vibration mode of the system.

Similar to the fluid damper installed at the top floor, all the natural frequencies, apart from the second natural frequency, remained almost constant within a wide
range of damping coefficient. The second natural frequency increased with the increasing damper damping coefficient.

9.4.3 Fluid Damper at First Floor

Figs. 9.12(a)-9.12(d) show the variations of the first, second, third, and sixth modal damping ratios of the system with the damping coefficient of the damper installed at the first floor of the buildings. The optimal damper damping coefficient for the first vibration mode was increased to about 6500 Ns/m, and for the second vibration mode it was increased to about 12000 Ns/m. These optimal damper damping coefficients were much higher than those obtained when the fluid damper was installed either at the second floor or the top floor. The corresponding achievable maximum damping ratio was about 4.5 per cent for the first mode and 7 per cent for the second mode, which were smaller than those achieved when the damper was installed at the top floor and the second floor. For the third and sixth modes of vibration, there also existed some optimal damper damping coefficients by which the third and sixth modal damping ratios of the system could reach about 6 and 8 per cent, respectively. Clearly, the position of fluid damper did affect the performance of the damper.

9.5 COMPARISON OF DYNAMIC RESPONSE

9.5.1 Fluid Damper at Top Floor

When a fluid damper was installed at the top floor of the adjacent buildings, its optimal damping coefficient was 3531 Ns/m for the first vibration mode, and the associated modal damping ratios in the first and second vibration modes of the
system were 8.5 and 30.0 per cent, respectively. For such a case, a harmonic load of constant amplitude was first applied at the first floor of Building I to have a sweep test from 0.5 to 20 Hz. The measured normalized frequency response curve at the top floor of Building I was compared with the measured normalized frequency response curve at the top floor of Building I without fluid damper (see Fig 9.13(a)). Clearly, because of the fluid damper the peak responses at the first and second natural frequencies of Building I without fluid damper were significantly reduced. The ratio of the first peak response of Building I with to without the fluid damper was about 1/6 and the ratio of the second peak response was over 1/2. The frequency response curve at the top floor of Building II due to the excitation at Building I is also plotted in Fig. 9.13(a). It is seen that the peak responses of Building II were much smaller than those of Building I with the fluid damper.

The harmonic load of constant amplitude was then applied to the same system at the first floor of Building II to have another sweep test from 0.5 to 20 Hz. The measured frequency response curve at the top floor of Building II with the fluid damper is plotted in Fig. 9.13(b) together with the frequency response curve at the top floor of Building II without fluid damper. It is seen that the first peak response of Building II without fluid damper was almost completely suppressed, leading to a ratio of the peak response with to without the fluid damper of about 1/18. This is consistent with the results from the modal damping ratio measurement. The second peak response of Building II without the fluid damper was also reduced significantly, resulting in a peak response ratio over 1/2. The frequency response curve at the top floor of Building I due to the excitation at Building II is also plotted in Fig. 9.13(b). Clearly, the peak responses of Building I were much smaller than those of Building II with the fluid damper.
From the above discussion, one may conclude that the fluid damper of proper damping coefficient installed at the top floor of the buildings can significantly reduce the peak responses of either building subject to harmonic loading.

9.5.2 Fluid Damper at Second Floor

For the building-damper system with a fluid damper of 2657 Ns/m damping coefficient installed at the second floor, the harmonic excitation was applied to the first floor of Building I to have a sweep test. The measured frequency response curve at the top floor of Building I is shown in Fig. 9.14(a) together with the frequency response curve of Building II at the top floor with fluid damper and the frequency response curve of Building I at the top floor without fluid damper. Clearly, because of the fluid damper, the first peak response of Building I without the fluid damper was reduced significantly. The second peak response of Building I without the fluid damper was, however, only moderately reduced. This is because the damping ratio in this mode attributed to the fluid damper was small and the position of the fluid damper was also near the node of the second mode of vibration of Building I. From Fig. 9.14(a), it is also seen that the dynamic response of Building II due to the excitation at Building I was quite small, for the most vibration energy was absorbed by the fluid damper before it passed to Building II.

The harmonic load of constant amplitude was then applied to Building II at its first floor. The measured frequency response curve at the top floor of Building II is shown in Fig. 9.14(b) together with the damping response curves at the top floor of Building I with the fluid damper at the top floor of Building II without fluid damper. It is seen that the first peak response of Building II without the fluid damper was reduced significantly and the second peak response was reduced only moderately.
The third peak response of Building II without the fluid damper was almost completely suppressed. It is also seen that the dynamic response of Building I due to the excitation at Building II was very small but the harmonic excitation at Building II did excite out the first peak response of Building I through the fluid damper.

9.5.3 Fluid Damper at First Floor

Fig. 9.15(a) shows the frequency response curves at the top floor of Building I with and without the fluid damper and the frequency response curve at the top floor of Building II with the fluid damper for the case where the first floor of Building I was excited. Fig. 9.15(b) shows the frequency response curves at the top floor of Building II with and without the fluid damper and the frequency response curve at the top floor of Building I with the fluid damper from the case where the first floor of Building II was excited. The damping coefficient of the fluid damper used was about 6500 Ns/m, which was the optimal value for the first mode of vibration of the system. It is seen from the figures that the peak responses of Building I and Building II were significantly reduced because of the installation of the fluid damper.

However, the position of the fluid damper at the first floor was far away from the maximum amplitude of the first vibration mode of each building. Thus, the effectiveness of the fluid damper installed at the first floor on the reduction of the first peak response of each building was relatively smaller, compared with the fluid damper installed at the top floor and the second floor. Nevertheless, the fluid damper installed at the first floor could reduce the second peak response of individual buildings more than the fluid damper installed at the second floor. In particular, when the fluid damper was installed at the first floor and the external excitation was also applied at the first floor, the building coupling was relatively easily identified.
For instance, from the frequency response curve of Building II with the fluid damper in Fig. 9.15(a), the first six natural frequencies of the system could be identified even though the excitation was applied at Building I. From the frequency response curve of Building I with the fluid damper shown in Fig. 9.15(b), the first peak response of Building I was almost the same as that of Building II due to the structural coupling.

Comparing the frequency response curves obtained from the three locations of the fluid damper, one may find that the optimal position of fluid damper to suppress the first and second peak responses of each building subject to harmonic loading was at the top floor.

9.6 SUMMARY

An experimental investigation has been carried out on dynamic characteristic and dynamic response of adjacent buildings connected by a fluid damper at different positions. Two model buildings were constructed as three-story shear buildings, and the fluid damper was designed as a linear viscous damper with its damping coefficient being adjustable. The natural frequencies, mode shapes, modal damping ratios, and dynamic responses to harmonic excitation of the adjacent buildings with and without the fluid damper connected were determined. The optimal damping coefficient and location of the fluid damper for achieving the maximum modal damping ratio and the maximum reduction of dynamic response of both buildings were identified. The interaction of the fluid damper with the inherent properties of the buildings was observed. The experimental results showed that the overall performance of the fluid damper-building system could be significantly enhanced using the fluid damper of proper parameter to connect the adjacent buildings.
The interaction between the adjacent buildings and fluid damper and the overall performance of the adjacent buildings linked by fluid damper observed from this experimental investigation were consistent with the analytical results presented in Chapter 6 in general. However, because no shake table facility was available during my PhD study and the duration of my PhD study was three years only, the detailed experimental verification of the theoretical results using the developed generalized pseudo-excitation method could not be performed in my PhD study but it will be certainly worthwhile to be carried out in the future.
Figure 9.1 Configuration of Adjacent Building-Fluid Damper System
Figure 9.2 Configuration of Plate Fluid Damper Model

Figure 9.3 A Calibration Set-up for Plate Fluid Damper Model
Figure 9.4 Recorded Force-Displacement Loops of Fluid Damper
Figure 9.5 Variations of Damping Coefficient and Storage Stiffness with Motion Frequency

Figure 9.6 Instrumentation Diagram
Figure 9.7 Measured Mode Shapes for Adjacent Buildings

Figure 9.8 Frequency Response Curves of Adjacent Buildings with Fluid Damper
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Figure 9.9 Variations of Modal Damping Ratio of Adjacent Buildings with Damper at Top Floor
Figure 9.10 Variations of Natural Frequencies of Adjacent Buildings with Damper at Top Floor
Figure 9.11 Variations of Modal Damping Ratio of Adjacent Buildings with Damper at Second Floor
Figure 9.12 Variations of Modal Damping Ratio of Adjacent Buildings with Damper at First Floor
Figure 9.13 Frequency Response Curves of Adjacent Buildings with Damper at Top Floor
Figure 9.14 Frequency Response Curves of Adjacent Buildings with Damper at Second Floor
Figure 9.15 Frequency Response Curves of Adjacent Buildings with Damper at First Floor
CHAPTER TEN

CONCLUSIONS AND RECOMMENDATIONS

10.1 CONCLUSIONS

In this dissertation, the generalized pseudo-excitation method for random vibration analysis of buildings with supplemental discrete control devices has been developed by combining the complex modal superposition method and the pseudo-excitation method. The proposed method can naturally retain all cross-correlation terms between closely spaced modes of vibration in a structural response and accurately handle the non-orthogonal structural damping due to the installation of discrete control devices. It is easy to be implemented into personal computers and it is efficient and powerful.

The generalized pseudo-excitation method can also provide a convenient way of determining internal force and control force responses of a building. It can also be used to derive the closed form solutions for complicated active control problems. By applying this method to vibration control of wind-excited single building and earthquake-excited adjacent buildings, the following special conclusions are drawn as follows:

1. The effects of cross-correlation terms between closely spaced modes of vibration in a structural response have been examined by considering the tall building with a light appendage at its top under alongwind excitation. The SRSS method overestimates the response of top floor displacement by as much as 20.2 per cent and underestimates the response of base shear force by as much as 11.5 per cent.
Chapter Ten: Conclusions and Recommendations

It reflects the significance of the correlation between modal responses which are neglected in the SRSS method.

2. The generalized pseudo-excitation method has been examined by analyzing a building with active tendon devices. The results obtained by the generalized pseudo-excitation method are in good agreement with the results given by Yang and Samali (1983) using the transfer matrix method. It is found that if the parameters of active tendon devices are selected properly, the wind-induced responses of the building could be considerably reduced. It is also found that the modal damping ratios of controlled building increases greatly compared with those of the uncontrolled building. For instance, the first modal damping ratio of controlled building increases about 43 times compared with the uncontrolled building.

3. The design of LQG controllers for wind-excited building with active control devices is implicitly influenced by performance target. The selection of weighting matrices is important but difficult. With the derived closed form solution, the influence of weighting matrices on the control performance could be elaborately investigated. The effectiveness and feasibility of LQG controllers for this example building are efficiently evaluated.

4. Extensive parametric and numerical studies are performed to seek the possibility of vibration control of steel frames with connection dampers. The results showed that there is an optimal damper damping coefficient for a given mode of vibration and a given fixity factor of the frame. With the optimal damper damping coefficient, the modal damping ratio of the frame could be significantly increased and the seismic responses, including lateral displacement, shear force, and bending moment, of the frame could be considerably reduced to a level smaller
than those of the frame with rigid connections. Curves on how to select an optimal damper damping coefficient to achieve maximum modal damping ratio and its corresponding frequency for a given fixity factor are given in this study.

5. The comparison of the dynamic response reductions of adjacent buildings with different control devices under ground motion is tabulated in Table 10.1. The comparison of the adjacent buildings linked by the Voigt model-defined viscoelastic dampers and the adjacent buildings linked by the Maxwell model-defined fluid dampers shows that the Maxwell model-defined fluid dampers have the same effectiveness as the viscoelastic dampers. This is because in the case of viscoelastic damper, the required optimal damper stiffness is quite small so that the Voigt model-defined viscoelastic damper is almost a dashpot only. In the case of fluid damper, the required optimal relaxation time is very small so that the Maxwell model-defined fluid damper also becomes almost a dashpot only. For active tendons, the devices with velocity feedback is also very effective but not with displacement or acceleration feedback. The comparative results of vibration reduction levels, however, depend on types of responses. The maximum response reduction has been achieved by LQG controllers among the four control devices investigated. However, it should be noted that the effectiveness of LQG controllers is quite dependent on the weighting matrices in the performance function of optimal control strategies. The selection of weighting matrices has posed a major difficulty for control designers.
Table 10.1 Comparison of Controlled Response Reduction for Different Control Devices

<table>
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<th>Viscoelastic Damper</th>
<th>Fluid Damper</th>
<th>Active Tendons</th>
<th>LQG Controllers</th>
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<td>Top Floor Displacement</td>
<td>57%</td>
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<td>61%</td>
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<tr>
<td>Top Floor Displacement</td>
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<td>64%</td>
<td>58%</td>
<td>65%</td>
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<td>(Right Building)</td>
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<tr>
<td>Base Shear Force</td>
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<td>57%</td>
<td>60%</td>
<td>62%</td>
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<td>(Left Building)</td>
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<tr>
<td>Base Shear Force</td>
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<td>(Right Building)</td>
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<tr>
<td>Top Floor Acceleration</td>
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<td>58%</td>
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<td>(Left Building)</td>
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<td></td>
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<tr>
<td>Top Floor Acceleration</td>
<td>47%</td>
<td>47%</td>
<td>40%</td>
<td>55%</td>
</tr>
<tr>
<td>(Right Building)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum Control Force</td>
<td>101.1</td>
<td>100.7</td>
<td>111.1</td>
<td>129.5</td>
</tr>
<tr>
<td>(kN)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Vibration mitigation of adjacent buildings connected with fluid dampers has also been investigated experimentally with two scaled three-story building models and a linear viscous damper. A testing facility has been setup up for the calibration of fluid dampers. The dynamic characteristics of individual buildings are obtained using system identification technique. The tests were then carried out to determine modal damping ratios of the adjacent buildings connected by the fluid damper of different damping coefficients and at different locations. Optimal damper damping coefficient and location for achieving the maximum modal damping ratio were thus found. The measured modal damping ratios and harmonic responses of the building-fluid damper system were finally compared.
with those from the individual buildings. The comparison showed that the fluid
damper of proper parameter could significantly increase the modal damping ratio
and significantly reduce the dynamic response of both buildings.

10.2 RECOMMENDATIONS

Some of the important issues related to the generalized pseudo-excitation
method, vibration control of buildings, and experimental studies needing further
research include

1. The generalized pseudo-excitation method for analyzing stationary random
response of structures needs further extension to the non-stationary random
response. Such non-stationary random response problems are of major interest in
the field of random seismic response analysis (Priestley 1967; Gasparini and
Chaudhury 1980).

2. In the present study, only the along-wind motion of buildings has been
considered. However, wind-induced vibration of buildings is usually
complicated, which includes the coupled rotational and lateral motions.
Therefore, the effectiveness of control devices for reducing the lateral-torsional
motion of wind-excited buildings needs to be investigated in the future.

3. For adjacent buildings, only the adjacent two-dimensional buildings having the
same floor elevations with same control devices connecting two neighboring
floors are studied. Some other issues related to this study, such as the optimal
position of dampers, the three-dimensional vibration mitigation analysis
including torsional effects, the effect of the type of earthquake and its
characteristics, and the effect of non-stationary earthquake excitation need
further investigation. The earthquake simulation test to verify the theoretical results is also desirable.

4. Experimental studies in this dissertation confirmed that fluid damper of proper parameter could significantly increase the modal damping ratio and significantly reduce the dynamic response of both buildings. The experimental results obtained here are only from harmonic loads applied to level one. It would be helpful to verify some of the conclusions if the excitation is applied to the second or top floor, or applied simultaneously to all floors.

5. Semi-active control devices, such as Magnetorheological (MR) dampers or electrorheological (ER) dampers, have found more and more applications to vibration mitigation of buildings and structures. The efficacy of MR/ER dampers for seismic response reduction of adjacent buildings is a subject of interest.

6. To apply LQG optimal control to the wind excited buildings, the cross-spectral density matrix of wind fluctuation must be factorized into a number of component excitation spectral matrices where each matrix is fully coherent and any two spectral matrices are noncoherent. Because the excitations must be white noise in the LQG algorithms, the approximation of spectral matrices using rational functions is needed and further research must be performed on how to enhance the accuracy of such an approximation.
APPENDIX A   PHOTOGRAPHS OF EXPERIMENTS
Figure A.1 Calibration of Plate Fluid Damper Model

Figure A.2 Adjacent Building-Fluid Damper System in Experiment
Two models of 3-story steel structure were built for testing. The model of 3-story building can be simplified as a three degree of freedom system. The stiffness of two models is the same. The column sizes are $b=75$ mm, $h=9.5$ mm, $l=450$ mm, and

$$I_x = \frac{bh^3}{12} = 5359 \text{ mm}^4 = 5.359 \times 10^{-9} \text{ m}^4$$

$$I_y = \frac{hb^3}{12} = 33984 \text{ mm}^4 = 3.340 \times 10^{-7} \text{ m}^4$$

$$k_x = \frac{4 \times 12EI_x}{l^3} = 5.646 \times 10^5 \text{ N/m}$$

$$k_y = \frac{4 \times 12EI_y}{l^3} = 3.518 \times 10^7 \text{ N/m}$$

The theoretical stiffness matrices are

$$K_1 = K_2 = \begin{bmatrix} 11.292 & -5.646 & 0.0 \\ -5.646 & 11.292 & -5.646 \\ 0.0 & -5.646 & 5.646 \end{bmatrix} \times 10^5 \text{ N/m}$$

If $m=314.5$ kg then the modal frequencies of the three story building in x and y directions are as follows:

Fig. B.1 Horizontal Cross Section of Model
Appendix B: Calculation for Testing Model

\[ f_1^{(1)} = 3.00 \text{Hz} \quad f_1^{(2)} = 23.68 \text{Hz} \]

\[ f_2^{(1)} = 8.41 \text{Hz} \quad f_2^{(2)} = 66.39 \text{Hz} \]

\[ f_3^{(1)} = 12.15 \text{Hz} \quad f_3^{(2)} = 95.93 \text{Hz} \]

For another model, let \( m = 139.8 \) kg, then the modal frequencies in \( x \) direction are obtained.

\[ f_1^{(3)} = 4.5 \text{Hz} \quad f_2^{(3)} = 12.62 \text{Hz} \quad f_3^{(3)} = 18.23 \text{Hz} \]
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