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Department of Electrical Engineering
The Hong Kong Polytechnic University

COORDINATED POWER CONTROL OF UNIFIED POWER FLOW
CONTROLLER AND ITS APPLICATION FOR ENHANCING
DYNAMIC POWER SYSTEM PERFORMANCE

Fang Wanliang
B.Sc., M.Sc.

Thesis submitted for the degree of
Doctor of Philosophy in Electrical Engineering

Hong Kong, June 1999
Abstract of thesis entitled:

Coordinated power control of Unified Power Flow Controller
and its application for enhancing dynamic power system performance

submitted by

Fang Wanliang
B.Sc., M.Sc.

for the degree of

Doctor of Philosophy in Electrical Engineering

at The Hong Kong Polytechnic University in

June 1999
Abstract

This thesis focuses on reporting my research study on a problem area relating to use of Unified Power Flow Controller (UPFC) for coordinating load flow in power systems so as to enhance their static and dynamic performance by having more secure and economical operation and higher dynamic stability margin. UPFC is considered as one of the most promising devices for implementing the Flexible AC Transmission System (FACTS) concept. Although development of UPFC is still on an infant stage, probing into its impact on power system operation is actively pursued and significant effort has been devoted to put it forward as a practical FACTS device and as a challenging academic research object. In order to consider UPFC as a basic power system element, it has to be involved in associated load flow computation essentially for power system control analysis and operational planning. An up front problem for design engineers is therefore pointing to a need to modify existing load flow program so as to accommodate interactions of UPFCs. A lot of research output start coming out but their computational efficiency are not high enough. In this regard, I propose two methods to perform the UPFC embedded load flow calculation to cater for two different types of application. The first one caters for analyzing direct control of load flow on transmission lines with embedded UPFCs. In this type of problem, active and reactive power of the lines, as well as the magnitude of bus voltages are priori given. The load flow solution can then be obtained and enables the UPFC parameters to be determined with a significantly improved computational efficiency. The second one works in contrary to the first one by which parameters of UPFCs are given before hand and the load flow calculation is performed for conforming a feasible operation. It can be regarded as an indirect load flow control calculation which is useful in planning stage for incorporating UPFC into existing system and/or carrying out various optimization processes. The next major contribution of my research project reported on the thesis relates to determination of optimal location of UPFCs which is practically important when coming to decision-making for installing and implementing the devices. As an extension on the use of the load flow calculation model, I develop an optimizing technique for identifying optimal location of UPFCs using Augmented Lagrange Multipliers method. At last but not the least, I develop a dynamic model of UPFCs embedded power system for
which dynamic performance analysis is carried out. Coordinated power control strategy is derived to show that UPFC can play an important role in mitigating power system oscillations and enhance dynamic stability margin of the system.
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Dedicated to My Beloved Family
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<th>Description</th>
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<tbody>
<tr>
<td>AEP</td>
<td>American Electric Power</td>
</tr>
<tr>
<td>BT</td>
<td>Boosting Transformer of UPFC</td>
</tr>
<tr>
<td>BFGS</td>
<td>Broyden-Flech-Goldfarb-Shanno Algorithm</td>
</tr>
<tr>
<td>ET</td>
<td>Exciting Transformer of UPFC</td>
</tr>
<tr>
<td>EMTP</td>
<td>Electric Magnetic Transient Program</td>
</tr>
<tr>
<td>EPRI</td>
<td>Electric Power Research Institute</td>
</tr>
<tr>
<td>FACTS</td>
<td>Flexible Alternative Current Transmission System</td>
</tr>
<tr>
<td>GTO</td>
<td>Gate Turn-Off thyristor</td>
</tr>
<tr>
<td>HVDC</td>
<td>High Voltage Direct Current</td>
</tr>
<tr>
<td>ICC</td>
<td>In-phase Current Control</td>
</tr>
<tr>
<td>IVC</td>
<td>In-phase Voltage Control</td>
</tr>
<tr>
<td>LTC</td>
<td>Load Tap Change</td>
</tr>
<tr>
<td>LOC</td>
<td>Line Optimisation Control</td>
</tr>
<tr>
<td>NRLD</td>
<td>Newton-Raphson Load Flow</td>
</tr>
<tr>
<td>OPF</td>
<td>Optimal Load Flow</td>
</tr>
<tr>
<td>ORPP</td>
<td>Optimal Reactive Power Planing</td>
</tr>
<tr>
<td>QCC</td>
<td>Quadrature Current Control</td>
</tr>
<tr>
<td>QVC</td>
<td>Quadrature Voltage Control</td>
</tr>
<tr>
<td>ShC</td>
<td>Shunt Compensation</td>
</tr>
<tr>
<td>SMIB</td>
<td>Single Machine Infinite Bus</td>
</tr>
<tr>
<td>SPWM</td>
<td>Sinusoid Pulse Width Magnitude</td>
</tr>
<tr>
<td>SVC</td>
<td>Static Var Compensation</td>
</tr>
<tr>
<td>TCSC</td>
<td>Thyristor Controlled Serious Compensation (or Capacitor)</td>
</tr>
<tr>
<td>TCPS</td>
<td>Thyristor Controlled Phase Shifter</td>
</tr>
<tr>
<td>TDS</td>
<td>Time Domain Simulation</td>
</tr>
<tr>
<td>TNA</td>
<td>Transient Network Analysisor</td>
</tr>
<tr>
<td>UPFC</td>
<td>Unified Power Flow Controller</td>
</tr>
<tr>
<td>UTC</td>
<td>Unload Tap Change</td>
</tr>
<tr>
<td>VSC</td>
<td>Voltage Source Converter</td>
</tr>
</tbody>
</table>
Nomenclature

\( A^* \)  
the conjugation of complex number \( A \)

\( \arg(A) \)  
the phase angle of complex number \( A \)

\( b_{ij} \)  
the shunt susceptance of the transmission line connected bus \( i \) and \( j \)

\( B_{ij} \)  
the mutual susceptance between node \( i \) and \( j \)

\( c_{dc} \)  
the capacitance of the UPFC DC capacitor

\( g_{ij} \)  
the shunt conductance of the transmission line connected bus \( i \) and \( j \)

\( G_{ij} \)  
the mutual conductance between node \( i \) and \( j \)

\( I_1 \)  
the injected current of the bus \( i \)

\( I_1 \)  
the active component of the current extracted from power system by shunt converter of UPFC

\( I_q \)  
the reactive component of the current extracted from power system by shunt converter of UPFC

\( \text{Im}(A) \)  
the imagine part of complex number \( A \)

\( I_{q_{max}} \)  
the maximum of \( I_q \)

\( k \)  
iteration times or operating condition identification

\( m_B \)  
the modulation index of UPFC series converter

\( m_E \)  
the modulation index of UPFC shunt converter

\( P_{Gi} \)  
the active power of the generator at bus \( i \)

\( P_{Li} \)  
the active power of the load at bus \( i \)

\( \bar{P}_l \)  
active power extracted by UPFC from bus \( l \)

\( \bar{P}_m \)  
active power extracted by UPFC from bus \( m \)

\( \Delta P \)  
the vector of active power mismatch

\( Q_{Gi} \)  
the reactive power of the generator at bus \( i \)

\( Q_{Li} \)  
the reactive power of the load at bus \( i \)

\( \bar{Q}_l \)  
reactive power extracted by UPFC from bus \( l \)

\( \bar{Q}_m \)  
reactive power extracted by UPFC from bus \( m \)

\( \Delta Q \)  
the vector of reactive power mismatch
\( \text{Re}(A) \) the real part of complex number \( A \)
\( \| R \|_\infty \) the infinite norm of the vector \( R \), which is equal to \( \max_i |r_i| \)

\( r_{ij} \) the series resistance of the transmission line connected bus \( i \) and \( j \)

\( S \) the sensitivity row vector

\( S_{Gi} \) complex power of the generator at bus \( i \)

\( S_{Li} \) complex power of the load at bus \( i \)

\( t^{(k)} \) the vector of turn ratios of the LTC transformer under the \( k \)th operating condition

\( T \) the vector of turn ratios of the UTC transformer in Chapter 5

\( T^* \) the optimal control vector in Chapter 6

\( U_i \) voltage magnitude of bus \( i \)

\( U_T \) the voltage phasor inserted into transmission line by the series converter of UPFC

\( U_{T_{\text{max}}} \) the maximum of \( U_T \)

\( U \) the vector of control variables in Chapter 3

\( \Delta U \) the vector of bus voltage magnitude revision

\( v_{dc} \) the voltage across the DC capacitor of UPFC

\( V_{Bt}^r \) the output voltage of UPFC series converter referred to the AC side

\( V_{Et}^r \) the output voltage of UPFC shunt converter referred to the AC side

\( X \) the vector of state variables in Chapter 3

\( x_{ij} \) the series inductance of the transmission line connected bus \( i \) and \( j \)

\( y_{lm} \) shunt complex admittance of the transmission line connected bus \( l \) and \( m \)

\( Y_{ij} \) the mutual admittance between node \( i \) and \( j \)

\( Z_{lm} \) series complex impedance of the transmission line connected bus \( l \) and \( m \)

\( \delta_B \) the phase angle of the UPFC series converter control wave

\( \delta_E \) the phase angle of the UPFC shunt converter control wave

\( \delta_i \) voltage phase angle of bus \( i \)

\( \delta_{ij} \) the difference of the voltage phase angle between bus \( i \) and bus \( j \) (\( \delta_{ij} = \delta_i - \delta_j \))
$\Delta \delta$  the vector of bus voltage phase angle revision

$\sigma$  the parameter of penalty function in Chapter 5

$\sigma_{ij}$  the real part of the $i$th eigenvalue of the state matrix at the $j$th operating condition

$\phi$  the power factor controlled by UPFC

$\phi_T$  the phase angle of the $U_T$

$\phi_{T_{\text{max}}}$  the maximum of $\phi_T$

the set of the operating area of UPFC

$\Lambda$  the vector of Lagrange multipliers corresponding to equality constraints

$M$  the vector of Lagrange multipliers corresponding to two-side inequality constraints

$\Omega$  the vector of Lagrange multipliers corresponding to one-side inequality constraints

Unless specified otherwise, unit for the above symbols are given in p.u.
Chapter 1

INTRODUCTION

1.1 General

The electric utility industry is undergoing rapid changes world-wide, including structural reformation of the electricity market, flexible power control by innovative power electronic technology and enhancement of man-machine interface in the operation of the Energy Management Systems (EMS). Rationale for these changes can be obvious as a need to create market condition for bringing in competition among the electric utilities so as to foster incentive for increasing efficiency of electric energy production and distribution, and to offer a lower-price, higher-quality and more secure product. In the past, electric power systems were relatively simple and were designed to be self-contained; power exportation and importation were rare. Nowadays, wheeling of electrical energy among utilities becomes a common phenomenon following deregulation of the electric power industry. Power is required to be transported through defined transmission line corridors. Conventional power system network can no longer meet the future projection of the required power control flexibility.

On the energy utilisation point of view, electricity is still being regarded as a mighty means of energy carrier. In fact, level of electricity consumption can reflect the standard of living of a modern city. However, as a need of improvement of our society in large, people are paying more and more concerns on the environmental impact brought along with advancement of the technology. A dilemma appears that on one end demand of electricity seems never can be satisfied and on the other end expansion on the power system network is facing with more and more pressure due to its environmental concern. Coupling with the present economic down turn, fully utilisation of existing power system capacity is more preferable and affordable than new construction. It implies a need for utilising the network to its ultimate loading limit which in turn asks for a higher degree of control in order to maintain a certain level of security and reliability of the system operation.
In other words, future power system development depends much on how to make it more flexibly to meet all sort of control requirements. Hence, the term Flexible AC Transmission Systems (FACTS) arises as a concept which will shape development of power systems in the coming decades. Dr. N. G. Hingorani [46] advocated FACTS as a consolidation of application power electronic technologies to effect control on the three basic electrical parameters of transmission line namely, the impedance, bus voltage and phase angle at both ends of the line. Essentially contribution of this research study is poised to investigate some of its control aspects along the line to make power system operating in a more flexible, secure and economic way.

1.2 Background of study

1.2.1 Why FACTS?

Conventional power system, especially the transmission network, is said to be inflexible. Although, excitation control by AVR, mechanical power input by governor, and var adjustment by tap-change on transformer are typical available on the generation side but they have inadequate effect on load flow control along the transmission network. In terms of load flow control in the transmission network, operator cannot do much except turning on and off the circuit breaker installed at two ends of the transmission line and so conditioning the operating conditions a bit. The parameters and configuration of network are almost fixed and the system is virtually dynamically uncontrolled. The network is slow in response to contingency cases and is difficult to cope with system load flow control required in terms of speed and versatility. Load flow within the network basically obeys Ohm’s law and Kirchoff’s laws and is termed as free load flow [47]. The mechanical switched/controlled equipment simply cannot match with the trend of fast on-line decision making from the modern Energy Management Systems (EMS). Thanks for advancement of power electronic technology, experience suggests that power flow control can be made more flexible. In fact, development of power electronic technology in the last two decades provide some fruitful achievements [20] on areas like, HVDC transmission, static var
compensation, and other fast speed electronic switching devices. Nowadays, large power valve of GTO can stand for high voltage more than 5kV and current rating of thousands of amperes. They can act as high speed with low loss switching devices. The rapidly developed power electronics technology in fact opens a new era for power system control and lays the foundation for making power system more flexibly controllable.

1.2.2 Basic concept of FACTS

The operational principle of FACTS can be illustrated by the following simple transmission line model:

$$P = \frac{U_1 U_2}{x} \sin \delta_{12}$$  \hspace{1cm} (1-1)

where $P$ is the active power transferred through the transmission line;
$U_1$ and $U_2$ are the voltage magnitude of terminals at both ends;
$\delta_{12}$ is the phase difference between the two terminal ends;
$x$ is the impedance of the transmission line.

In conventional power systems, the parameter $x$ is not controllable. Operator can only adjust $U_1$, $U_2$ and $\delta_{12}$ in order to control the load flow. Unfortunately, the variation range of $U_1$, $U_2$ is small normally. Furthermore, for secure system operation, sufficient transmission margin must be reserved at all times to accommodate nearly instantaneous redistribution of power flow required as a result of unexpected power system disturbance. This margin must be adequate to maintain system stability during and after the disturbance. Hence, the steady-state value of $\delta_{12}$ must be less than half pi. From equation (1-1), the limitation of transmission capability can be determined as:

$$P_{\text{max}} = \frac{U_{1\text{max}} U_{2\text{max}}}{x}$$  \hspace{1cm} (1-2)
For relatively short transmission line, \( x \) is not too large and so the loading limit obtained from equation (1-2) may be larger than its thermal limit. But for a long distance transmission, \( x \) is large enough to make the limit much less than the transmission line's thermal limit and so impedance \( x \) becomes crucial for considering the maximum transmission power capability. Thus, in conventional power systems where power carrying capacity of the line cannot be fully utilised, reducing its parameter \( x \) so as to raise the \( P_{\text{max}} \) is a method commonly termed as series compensation. Unfortunately this method can cause spontaneous resonance oscillation in the power system and is regarded as slow in response due to its mechanical switching operation.

Equation (1-1) shows the basic FACTS control dimension from which it can be seen that the transmission power can be influenced by the three parameters: voltage magnitude, impedance and voltage angle difference. In essence, FACTS devices all attempt to adjust these parameters. There are a variety of FACTS for controlling one or more of these three parameters. For example, Static Var Compensation (SVC) consists of combination of fixed capacitors or reactors, thyristor switched capacitors and thyristor [8] for acting as a controllable parallel susceptance (capacitive or inductive). It mainly serves to influence the magnitude of voltage by producing or consuming reactive power. Thyristor Controlled Phase Regulator (TCPR) only changes the phase angle difference between the two ends of the transmission line while Thyristor Controlled Series Compensation (TCSC) directly modulates impedance of transmission line. Unified Power Flow Controller (UPFC) is then identified as one which can control the three parameters at the same time. In fact, it consolidates effect of the TCSC, ASVC, TCPR and Thyristor Controlled Tap Changing (TCTC). As a summing-up, Fig.1-1 illustrates the function of the main FACTS devices.

1.2.3 Operational needs of FACTS

FACTS devices are characterised by their fast response, absence of inertia and minimum maintenance requirement. There are three level of speed response which require different
Fig. 1-1 Power flow control and stability improvement in AC systems

operation treatment. The first level of time response is in a range of minutes such as steady state load flow control. It can normally be met by mechanical switching of reactive power elements or even by rescheduling generation. For load flow control during transition between different operating conditions and possibly at time with equipment overloading, it requires response time in a range of seconds. General mechanical switching equipment cannot cope with the situation in this case. For cases of dynamic and transient analysis, load flow control needs more fast response time probably below 100 millisecond. This response can only be achieved by FACTS. There are other operational needs of FACTS such as in case unacceptable voltage condition occurs after outage or restructuring of power system. It needs a fast response to cater for the voltage instability. Transient stability can be a critical concern for determining the limit of power transmission over a
long distance or for interconnection of weak systems. Fast response FACTS devices are feasible in these cases to increase the loading limit of the system.

1.2.4 Practical type of FACTS device

Since intercalation of the FACTS concept, practical implementation of FACTS is gradually taking place. The first FACTS device is on Static Var Compensation (SVC) side and has been in service for nearly twenty yeas already [8]. NGH-Damper was installed at the 500 KV transmission line of Southern California Edison in 1985. Three Thyristor Controlled Series Compensation (TCSC) projects have been working successfully in USA since 1991. After all, they can be regarded as the first generation of FACTS. Recently installed FACTS equipment makes use of large rating (4500 V to 6000 V and 4000 A to 6000 A) gate turn-off thyristors (GTOs) for use as large power converters and three-phase sinusoid voltage sources [5]. Some typical FACTS devices implemented are listed in the following:

- Static Var Compensation (SVC)
- Static Condenser (STATCON)
- Static Synchronous Series Compensator (SSSC)
- Thyristor Controlled Series Compensation (TCSC)[42]
- Thyristor Controlled Phase Shifter (TCPS)[44]
- Thyristor Controlled Series Reactor (TCSR)
- Thyristor Controlled Break Resistor (TCBR)[25]
- Thyristor Controlled Break Capacitor (TCBC)
- NGH-Damper (N. G. Hingorani Damper)
- Phase Angle Regulator (PAR)[57]
- Interface Power Controller (IPC)[17]
- Solid-State Current Limiter (SSCL)
- Solid-State Circuit Break (SSCB)
- Improved Load Tap Changing Transformer (ILTCT)[57]
- Dynamic Voltage Limiter (DVL)
• Super-conducting Energy Storage System (SEMS)
• Series Power Flow Controller (SPFC)
• Unified Power Flow Controller (UPFC)

1.2.5 Unified Power Flow Controller

As mentioned in 1.2.2, UPFC [36] has been incorporated as a member of the FACTS family. EPRI and Westinghouse took the first initiative to develop UPFC and the first UPFC in the world is demonstrated at AEP's Inez [71]. The ongoing development on UPFC consists of a shunt Voltage Source Converter (STATCOM) rated at ±160 MVA to provide ±150 MVAR reactive power support and 50 MW real power through the DC link in full UPFC mode of operation. The series VSC is rated at ±160 MVA to offer phase shifting and/or series compensation. Both the STATCOM and VSC consist of a 160 MVA voltage sourced, multi-pulse and harmonic neutralized GTO converter with magnetic interface. The installation has been completed after the end of 1997 [1]. It is reported that there is another prototype UPFC in France. Basic structure of UPFC is shown in Fig. 1-2.

It consists of shunt (exciting) and series (boosting) transformers connected by two GTO converters and a DC circuit represented by the capacitor. Obviously UPFC is developed through combining Controlled Series Compensation with GTO converter (GTO-CSC) and Phase Shifting Transformer (PST) with Static Var Generator (SVG). Operation of the shunt converter involves drawing a control current from transmission line. One component of the current ($I_r$) is automatically determined by the requirement to balance the real power of the series converter. The remaining component of the current ($I_q$) is reactive and can be set to any desired level (capacitive or inductive) within the operational range of the converter. The series converter controls the magnitude and angle of the voltage ($U_T$) injected in series with the transmission line. This voltage injection intends to influence the flow on the line. Active power can freely flow in either direction between the AC terminals of the two converters through the common DC link. Since the UPFC is a passive element, the active power extracted from system by one converter and returned to the system by another converter. For the reactive power, the situation is different. Each converter can generate or absorb reactive power at its own AC output terminal. However
the two converters cannot internally exchange reactive power through the common DC link. In fact, this explains the basic difference between the UPFC and the PST in that the UPFC reactive power injected into the transmission line through the series branch need not pass through the shunt branch.

From the above description, we can see that the UPFC is a multifunction power flow controller because of its unique feature to simultaneously or selectively control all three parameters voltage, phase angle and line impedance. So it has the potential benefits for power flow control, loop flow control, load sharing among parallel corridors, enhancement of transient stability, mitigation of system oscillations and voltage regulation. UPFC is the most creative and the most innovative FACTS device at present and deeply exploiting its application is a significant research work.
1.3 Literature review

Since UPFC was brought to attention in 1991, it has been widely recognized as the most promising FACTS device. In the following context, a literature review helps identify various aspects of development and research activities of UPFC since its advocate in 1995.

1.3.1 Operating principle of UPFC

The qualification for describing the basic operating principle of UPFC certainly belongs to its inventor Dr. L. Gyugyi who proposed the concept of UPFC [36]. Based on load flow control relationships and experience of operation for SVC, TCSC and TCPS, the following four operating modes of UPFC were identified.

- Terminal voltage regulation: by keeping the angle of the series voltage $U_T$ (see also Fig.1-2) in phase with the terminal voltage $U_P$ and thus change only the magnitude of $U_P$.
- Combined line series compensation with terminal voltage regulation: by defining $U_T$ as the sum of the phasor $U_c$ and $U_d$. $U_c$ is perpendicular to the line current and $U_d$ is in phase with $U_P$.
- Combined phase angle regulation with terminal voltage regulation: by defining $U_T$ as the sum of the phasor $U_a$ and $U_d$. Assuming the angle to be shifted is $\alpha$, the magnitude of $U_a$ is equal to $2U_s\sin(\alpha/2)$ and the phase angle of $U_a$ is equal to $\pm(\alpha+\pi)/2$. $U_d$ is again in phase with $U_P$.
- Combined terminal voltage regulation and series line compensation and phase angle regulation: by synthesizing $U_T$ from the above three individually controlled phasors, i.e. let $U_T$ be the sum of $U_c$, $U_d$ and $U_a$.

The above summary showed that the voltage phasor and the control concept still basically came from the conventional SVC, TCSC and TCPS. Consequently, the concept of controlling power flow on transmission line directly was revealed in [37] by Dr. L. Gyugyi et al. UPFC was able to maintain prescribed controllable real power $P$ and reactive power
\( Q \) in the line independently. Within the new concept, the conventional terms of series compensation and phase shifter etc become irrelevant. The UPFC simply controls the magnitude and angular position of the injected voltage \( U_T \) in real time so as to maintain or vary the real and reactive power in the line so as to satisfy load demand and system operation conditions. The generalized \( PQ \) controller was proposed and the detailed comparison between UPFC and traditional TCSC and TPAR were carried out and their dynamic performance was investigated in that paper. Also, it indicated that fast dynamic response of the UPFC would reduce effect of power system oscillations arising on the line. Hence, it is important to understand the essence of UPFC in controlling load flow \( P \) and \( Q \) and how to choose their setting strategically by continuous closed-loop feedback control.

As the UPFC started to emerge from paper design to full-scale power system implementation, more concrete problems linked to the power electronics aspects were encountered. Basic control, sequencing and protection philosophies governing the operation of the UPFC were described in [5], which came from the same work group as [37]. The fundamental operating constraints of UPFC were discussed in that paper. The attention mainly focused on the problem associated with the design of UPFC for which its basic operating modes were classified as shunt converter and series converter. There are two control modes for the reactive component of the current in the shunt converter. One is Var control mode and another one is automatic voltage control mode. In the series converter, there are four modes including direct voltage mode, phase angle shift emulation mode, line impedance emulation mode and automatic load flow control mode according to the way of determining the series voltage \( U_T \). There are two other especial operating modes, stand-alone and alternative modes, which allow either of the two converters to operate independently by disconnecting their common DC terminals or splitting the capacitor bank. In this case, the shunt converter becomes a STATCOM and the series converter a SSSC. The problem of identifying the UPFC operation limit, UPFC under line fault conditions and UPFC for damping power system oscillation are studied by using Transient Network Analysisor (TNA). Line Optimization Control (LOC) becomes a feasible concept to guide the UPFC operation when it is operating in the mode of automatic load flow control.
Although the UPFC has many other possible operating modes, it is anticipated that the shunt converter will generally be operated in automatic voltage control mode and the series converter will typically be in automatic power flow control mode. The work reported in the above three pieces of literature are forerunners of UPFC research providing fundamental knowledge for one who expects to understand UPFC – hitherto the most complex and the most versatile FACTS device.

1.3.2 Planning consideration of UPFC

Since the world's first UPFC implementation was tailored made for AEP, a set of normal and contingency power system operating conditions were introduced in Inez Area of AEP system [43]. They showed that the UPFC could provide functional flexibility to address many issues, such as voltage depression and/or thermal overload in several single contingency conditions facing the Inez area transmission system. In a planning stage, major equipment ratings of UPFC including maximum shunt current, maximum shunt voltage, maximum series current, maximum series inserted voltage and maximum DC power transfer have to be considered. AEP developed a load flow model to determine these parameters but the model was not convenient for operations and routine planning purposes. The impact of the UPFC on their concrete power system was studied though a bit rough. A more detailed study for the UPFC equipment size and operating constraints was reported in [28]. It was a further development of [43] but having minimum line-side voltage of the UPFC as a new constraint. Since real power flow control was very important during both steady state and transient operation, the objective stated in this paper was to maximize the real power transfer of the UPFC embedded line. It showed that by solving the nonlinear programming problem, the UPFC operating parameters could be determined. An analytical formula was derived to describe the relationship between the real and reactive power flow on the line making use of the inserted series and shunt voltages and a straightforward methodology for defining a control strategy of the UPFC. A load flow model for covering three types of FACTS devices was proposed in [30]. As in the planing process, further qualitative attributes such as type, capacity and location of FACTS device would have to be known, performing a complete load flow calculation was considered a must to reveal feasible solution. The concept of security regions was used to
compare the impact of various FACTS devices on power system behavior as shown in [18]. Scalar measures of steady state performance of a power system with FACTS devices were used to quantify this impact. Such measures were obtained by solving a parametric Optimal Power Flow (OPF) problem within the constraints of the security region. The concept of an ideal FACTS device was introduced as a means to establish a theoretical upper bound on the performance of any realizable FACTS. The basic augmentation, the security region of a network with a FACTS controller always enclosed the region without FACTS, was the underlying rationale of the paper. The main use of the method proposed in that paper was to choose the optimal type of FACTS device.

1.3.3 Load flow analysis on power systems with UPFCs

Load flow analysis on power system with FACTS device such as TCST, SVC and TCPS had been relatively well studied. The methods of load flow computation with FACTS devices can be classified in two categories. One is to decouple the FACTS device from power system network by transferring the equivalent voltage/current source as node injection power. It is called node power injection method. Another one is simultaneously finding the solution of the system nodal power equations with incorporation of those for the FACTS device. It is called unified iteration solution method. There are different merits and demerits in the two methods. Recently, Genetic Algorithm is used to solve optimal power flow control problems with UPFC in [38]. UPFC is regarded as a continuous phase shift device to regulate both angle and magnitude of its branch voltage and based on which the node injection power caused by the UPFC is derived. Coupled with the fact that both of the above methods can be applicable to UPFC; the unified iteration method was used in [66] and a model of Generalized Power Flow Controller was proposed in [69]; attempt has been seen to deal with all the FACTS device by a generalized model. C. R. Fuerte-Esquivel and E. Acha did a good job in developing an algorithm suitable to calculate load flow of power system with various FACTS devices [9,10,11]. In [10], a set of nonlinear equations was introduced to describe each UPFC. Traditional network equations (one fictitious node added for each UPFC) together with these equations were simultaneously solved by Newton algorithm. The solution variables included magnitudes and angles of all bus voltages and the UPFC controllable parameters. The initial values of the UPFC control
parameters were firstly evaluated by simplifying the UPFC equations. During iteration, once some parameters violated its limitation, these parameters could be set to their limit values. Case studies showed that this method has strong convergent characteristics. However, there are still a few shortcomings in this algorithm. If multiple UPFCs are considered, the order of Jacobian matrix is largely increased in size (one UPFC will incur 9 orders increased by 7 equations of UPFC and one fictitious node). This probably reduces chance of having convergent results. In addition, the method of assessing the initial values of the UPFC parameters may not ensure they are in the vicinity of solutions. Especially, when the control goal, prescribed line active power and reactive power commands are far away from the free flow, the phenomenon may become common. However, there is drawback in other method, i.e. after load flow convergence, the computed parameters of the UPFC may result to a violated solution. Suppose that the scheduled control targets, line active and reactive power as well as the magnitude of bus voltage are not suitable, during iteration one or two parameters of the UPFC will certainly go beyond their limit. In this case, even you set these parameters at their bound value, the available solution can still not be determined. In fact, the control ability for line flow and bus voltage is not infinite in respect to a fixed UPFC capacity. The convergence is possible for an appropriate objective only. The method putting forward in [9] is not the same as [10] at all. FACTS devices are all described by their transfer admittance matrix. The convergence characteristics are doubtful because there are no available initial values provided for arbitrary control targets. The more number of FACTS devices are used, the larger the probability of divergence. As an application in reference [10], the model is directly incorporated into an OPF program [24]. A steady state model is put forward in [4]. The method to calculate load flow by its steady state model of UPFC can simply be improved. Taking the E-bus at where the UPFC exciting transformer connected as a PQ bus and B-bus at where the boosting transformer connected as a PV bus is very troublesome for computing the four control parameters of the UPFC and it has not much additional benefit. The definition of PQ and PV bus by this method contradicts with the operational characteristic of UPFC. Actually the active power and reactive power injected into the B-bus as well as the magnitude of E-bus voltage are regulated by UPFC. Therefore taking B-bus as PQ bus and E-bus as PV bus is very convenient for determining the four control parameters of UPFC.
1.3.4 Digital Simulation of UPFC

The earlier simulation studies are carried out in [37] and the main purpose is to validate the function of UPFC in steady state. The UPFC control system is divided functionally into internal and external controls. The internal controls operate the two converters so as to produce the commanded series injected voltage and draw the desired shunt reactive current. The external controls are responsible for generating the demands for the series voltage and the shunt current. J. Y. Liu and Y. H. Song reported their simulation studies for UPFC in [32,33]. The Sinusoidal Pulse Width Modulation (SPWM) control scheme is used for both shunt and series converter of UPFC and simulated by using EMTP. The PI-type control is used as the internal controller of UPFC. In [45], the shunt converter is controlled by a Hysteresis Current Forced (HCF) scheme and the series converter by PWM and simulated using PSCAD/EMTDC program. The steady state simulation of the control function of UPFC demonstrated that UPFC could successfully control the active and reactive power flow on the transmission line and can almost maintain constant DC bus voltage and pass active power bi-directionally. In simulation of dynamic performance of UPFC, PI-type control is used as the internal controller of UPFC too. The speed deviation of generator is taken as the feedback signal to drive the modulation ratio of PWM. The inserted series voltage by UPFC is controlled as either Quadrature Voltage Controller (QVC) or In-phase Voltage Controller (IVC) and the simulation exhibits that the real power flow in the UPFC with QVC is considerable less than the IVC. This is a major advantage of the QVC over IVC, as the exciter converter can provide more shunt reactive compensation, if required. The same result is pointed out in [62].

1.3.5 Improvement of Transient Stability Using UPFC

Effect of UPFC on transient stability margin enhancement on a longitudinal system is analyzed in [60]. The transmission system is represented by a two voltage-source and a two Π-section. The UPFC model is located in the middle of the transmission line. The series converter of UPFC is depicted by a series voltage source \( U_r \angle \phi_r \). The function of the shunt converter is divided into two parts. The reactive compensation effect is represented by a controllable susceptance \( B_r \) and the active power, which balances the
active power transmitted by series converter, is represented by a current source. In this model \( U_r, \phi_r \) and \( B_r \) are independent controllable parameters. The analytical formula of the current through the transmission line is derived by the model and expressed by several not physical significant factors. Furthermore the undefined area in \( U_r \) vs. \( \phi_r \) plane is plotted by numerical computation. The authors indicated that on one of the boundaries of the undefined area, the transmission system between the voltage source reaches its static power transmission stability limit and the terminal voltage of UPFC collapses on the other boundary. It serves as a good yardstick for judging the control range of UPFC. Unfortunately, the concrete formulas of these factors are not given in the paper making it cumbersome to use and difficult to understand. The steady state and dynamic model of UPFC is originally formulated in [4]. PWM scheme is used to control the two converters of UPFC. Through detailed derivation, the output of the converter in AC side is expressed by an ideal voltage source whose magnitude and phase angle are both controllable by the modulation ratio and reference wave phase angle of the PWM scheme. A set of differential equations which describe the relationship of the two voltage sources and the currents in the two coupled transformers of the UPFC are established in stator reference frame. In steady state, the DC voltage across the DC capacitor is constant. The mathematical paraphrase for this statement is that the differential equation of DC voltage with respect to time becomes an algebraic equation in steady state implying that the steady state model of UPFC can be obtained. The extended Park’s transformation matrix is used to transfer the reference frame from abc to odq in the paper. By carefully derivation I think this is dispensable. The more detailed discussion for this question will be given in the later chapter. Nevertheless the paper has provided a method to develop the mathematical model of UPFC systematically. The reference [62] investigates the mechanism of the three control methods of UPFC, namely In-phase Voltage Control (IVC), Quadrature Voltage Control and Shunt Compensation (ShC), for improving transient stability of power system and examines the voltampere rating by the three control methods in respect of its utilization. The study is based on a Single Machine Infinite Bus (SMIB) system with a UPFC. Proportion control is used as the internal controller of UPFC and the deviation of rotor angle, i.e. \( \Delta \delta = \delta - \delta_0 \) is taken as feedback signal. This study assumes that the UPFC takes action after the fault is cleared only. Since the deviation of rotor angle is chosen as
its feedback signal, the first swing performance of the system is ameliorated. The same authors study the improvement for transient stability by UPFC based on fuzzy logical controller in [62]. In the study, three controls, namely IVC, QVC and ShC, are coordinated by their fuzzy logical controller. The system configuration, the model of UPFC and the feedback signal are all the same as those of the previous paper but the control effect is much better than that in the first paper. Not only the first swing but also oscillation are improved by UPFC. Another decomposing method for the series voltage inserted by UPFC is used in [45]. The current through the transmission line is taken as reference phasor and the series voltage inserted by UPFC is split into two components. The one is in phase with the current and the other quadrature to the current. The current through transmission line is regarded as local quantity and it can be readily measured at anywhere of the power system. The In-phase Current Controller (ICC) and Quadrature Current Controller (QCC) actually make the series voltage having an effect as an impedance inserted in the transmission line, which varies in the fourth quadrant of the impedance plane. It explains why the series voltage of the UPFC can control the line flow by modulating the line impedance. By this decomposing method, ICC and QCC are designed in SMIB system and both controllers are equipped with integral control. The magnitude of the UPFC terminal voltage is chosen as the control signal of the ICC and the active power through the line and the generator slip as the QCC. Although, the ICC and QCC working individually and concurrently is studied, better effect of improving transient stability by coordinating both the ICC and QCC together is inferred from the study.

1.4 Objective and Scope of the Studies

As reviewed from the background information, FACTS is a concept covering a widespread of application of power electronic technology by which loading limit of existing power systems can be increased. On the one hand, UPFC is generally accepted as one of the most novel, versatile and promising FACTS controllers, but on the other hand, it is the most expensive and complex setup. Hence, the main contribution of this study is geared towards in finding a balance between the two concerns by devising optimum design techniques for implementation of the UPFC in view of its operational and planning aspects. The objectives of the project can be stated as:
• to develop load flow model of power system incorporating UPFCs

• to analyze load flow control by using UPFCs

• to design optimal location for installing UPFCs

• to apply UPFC for damping power system oscillation and enhancing dynamic power system performance.

The work done is summarized as follows:

1.4.1 Development of the load flow model of power system with UPFCs

Development of the model and static representation of UPFC in power system are essentially required for studying on load flow control, power system network planning and optimizing power system operation. The effect of UPFC is described as nodal injected power and four expressions associated with this injection in terms of real power and reactive power are derived. Based on these expressions of nodal power injection, existing conventional Newton-Raphson Load Flow (NRLF) technique can readily be extended to cover the power system with multiple UPFCs. For the purpose of controlling load flow, the sensitivity of branch power with respect to the parameters of UPFC is derived. Based on this sensitivity vector, any scalar control objective can be realized. By means of the generalized converse, the envisaged methods have potential applications for solving problems with multiple objectives.

1.4.2 Setting of parameters of UPFC for load flow control

In the above studies, parameters of UPFC are assumed to be given and based on which the load flow solution is found. The load flow control is realized by adjusting the parameters of UPFC guided by the sensitivity vector. A more convenient approach is devised for controlling the voltage magnitude of the bus, at which the shunt transformer of the UPFC is connected, and the load flow through the transmission line, in which the series
transformer of the UPFC is embedded. By this method, the active power and reactive power as well as the bus voltage can be prescribed simultaneously or selectively and the UPFC is described by the nodal power injection during load flow iteration. After convergence, the parameters of UPFC can be computed without iteration.

1.4.3 Optimizing location of UPFC for steady state load flow control

As indicated in the sections 1.2 and 1.3, UPFC is regarded as an effective means for regulating voltage profile and power flow in modern power system. Optimizing its location becomes a concern when coming to practical implementation stage. This study proposes a mathematical model to work out a pragmatic approach based on the augmented Lagrange multiplier method to determine optimal location for UPFCs to be installed. Investment consideration of UPFCs and real power loss of the network concerned are integrated into the objective functions. The proposed method takes into account of steady state security constraints of the power systems represented by models having large scale and non-linear mathematical properties. According to the principle of decoupling active and reactive optimisation, the active power outputs of all generators except the slack bus generator are assumed given by other strategy in this study. UPFC should work with traditional control means such as the turn ratios of transformer together so the decision variables includes not only the parameters of UPFC but also the turn ratios of transformer and the reactive power output of all generators. For increasing the robustness of the decision for the installation place, multiple operating conditions are simultaneously taken into consideration.

1.4.4 Enhancing power system dynamic stability by using UPFC

From literature review, underpinning research, though not yet well developed, suggests that UPFC can be used to enhance dynamic stability of power systems so connected. In this study, a method to coordinate UPFC with Power System Stabilizer (PSS) so as to increase the operational dynamic stability margin of power system is proposed. The work of establishing a dynamic mathematical model of UPFC is the first requirement for this research. The common type of models available in the field cannot be directly used in
complex power system because they are expressed in actual value instead of per unit format and have no systematical method to incorporate the model into complex network. Hence, I begin by deriving a dynamic mathematical model of the UPFC with which its linear differential equations can be integrated to design an overall control strategy for enhancing dynamic stability of the UPFC connected power system. The approach is to identify the eigenvalue with the largest real part value and then to minimize it by formulating it as a nonlinear optimization problem. In order to increase the robustness of the control system, different operating conditions are simultaneously considered in case study.

The original contributions and important development of this thesis will be elaborated in the above sequence as included in the following Chapters.

1.5 Publications

Arising from this research project, three papers have already been accepted in leading international journals and one is under review. In addition, one conference paper has been presented and one accepted for presentation. These papers are listed in the following:

**Referred Journal Papers Published:**


Journal paper under review:


International Conference paper published:


International Conference paper accepted for presentation:

Chapter 2

LOAD FLOW CALCULATION OF POWER SYSTEMS WITH MULTIPLE UPFCs

2.1 Introduction

In view of the fact that power systems nowadays are becoming more openly accessible, maneuverability of their power flow continues to be a general concern in the coming decades. Hopefully, power systems can be made more controllable and flexible due to development of computing and power electronic control technology. Apparently, power flow control within the transmission system still remains as one of the main hurdles. At the moment, power transmission is mainly mechanically controlled by means of, say, on-load tap changing of transformers and switching in and out of circuits which is obviously not flexible enough to cope with the future development. Since the late 1980s, a fundamental concept known as Flexible AC Transmission System (FACTS) has been put forward under which assorted FACTS devices are designed and studied for various purposes [47]. Unified power flow controller (UPFC) promptly becomes a promising device as it incorporates all three attributes for controlling the transmission power flow. Load flow computation for power systems is clearly in need for performing other essential functions such as power system analysis and planning. It is also necessary for investigating the controllability of the UPFC during steady and dynamic state analysis. However, this work involving development of a suitable algorithm for performing load flow analysis in power systems with multi-UPFCs has rarely been reported. For instance, in [4], a useful mathematical model of UPFC is developed and employed for performing the load flow control studies. As far as load flow computation is concerned, it carries with the calculation an intrinsic drawback that requires a few more pre-specified system states such as power flow of the UPFC embedded transmission line and its specified bus voltage to be regulated. However, since no one has a priori knowledge of the operational conditions of the UPFC, the pre-specified power flow and voltage values are tentative or arbitrary which might lead to situation that no meaningful result can be obtained finally. In [60], effect of
Chapter 2 Load Flow Calculation of Power Systems with Multiple UPFCs

UPFC on transient stability margin enhancement of longitudinal systems is analyzed but its effect can only be demonstrated by a case on a double-machine system. A so-called "basic control" for UPFC is designed in [27], which enables UPFC to follow the changes in reference values of active and reactive power supplied from the outer-loop control on the transmission system level. In [59], it introduces an UPFC project being developed by American Electric Power (AEP), Electric Power Research Institute (EPRI) and Westinghouse Electric Corporation (Westinghouse). Although it includes a load flow model for analyzing the steady state performance and determining the required UPFC equipment ratings, it is still not regarded as a convenience one for operational and routine planning studies. Moreover, the work done so far by those papers mentioned above all have their attention focused either on the control aspects of the UPFC itself or confined to load flow analysis on the power line in which the UPFC is embedded. However, power system operation expects to know not only how to control the load flow of the line where UPFC is embedded but also how it can control load flow on other branches of the system.

This chapter aims to present a systematic and efficient method for performing load flow calculation of a generalised power system consisting of multi-machines and multi-UPFCs. To start with, load flow equations of the entire network including the UPFCs are derived. Since the NRLF method together with its companion techniques of sparsity and optimal ordering has been proved to be successful for performing the conventional load flow computation, the same basic approach can be extended to compute the load flow algorithm of the UPFCs embedded power systems. In fact, the approach keeps the conventional NRLF method intact. In the course of performing the iteration steps, not more than four power mismatches and a few elements of the Jacobian matrix for each UPFC need to be revised. Thus the UPFCs can readily be incorporated into the traditional NRLF by including the modified items in the iteration procedure. For each UPFC, it only refers to a few elements of the Jacobian matrix and hence additional computation burden incurred is very minimal. The computational speed, accuracy and the convergence property of the approach inherit those of the conventional NRLF. Case studies for the IEEE standard 14-bus and 57-bus systems indicate that this algorithm is robust, reliable and efficient.
2.2 Modelling approach

The load flow equations of the UPFCs embedded power system are outlined in the following context.

2.2.1 Traditional power system equations

Identifying load flow in power system is basic to most other system analysis. The function of load flow analysis is to determine all the bus (node) voltages including their magnitude and phase angle for a given set of bus loads, active and reactive power generation schedule and specified bus voltage magnitude conditions. This problem is formulated as a set of nonlinear algebraic equations which can be solved by a number of mathematical methods including Newton-Raphson algorithm which is a very efficient and reliable. According to the Kirchoff’s current and voltage laws and Ohm’s law, the nodal voltage equations of power system without UPFC shown in Fig.2-1 is expressed:

\[ I_i = \sum_{j=1}^{n} Y_{ij} U_j, \quad i = 1, 2, ..., n \]  \hspace{1cm} (2-1)

First conjugate (2-1) and then multiply by \( U_i \), through simple deduction the power system nodal power equations are obtained:

\[ P_{Gi} - P_{Li} = \sum_{j=1}^{n} U_i U_j \left( G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \right) \]  \hspace{1cm} (2-2)

\[ Q_{Gi} - Q_{Li} = \sum_{j=1}^{n} U_i U_j \left( G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij} \right) \]  \hspace{1cm} (2-3)

\[ i = 1, 2, ..., n \]

where \( n \) is the total numbers of buses in the power system. \( P_{Gi} \) and \( Q_{Gi} \) are the active and reactive power injected to bus \( i \) by generator respectively. \( P_{Li} \) and \( Q_{Li} \) are the active and reactive power extracted from bus \( i \) by load respectively. \( U_i \) is the magnitude of voltage of
bus $i$. $\delta_y = \delta_i - \delta_j$ is the phase difference between bus $i$ and bus $j$. $G_y+jB_y$ denotes the element $Y_y$ of admittance matrix of system network.

$i=1, 2, ..., n$ and bus $j$ is connected to bus $i$ through branch $i-j$

![Diagram](image)

Fig. 2-1 The configuration of power system without UPFC

### 2.2.2 Steady state model of UPFC

Basic steady state model of UPFC includes representation of its series branch and shunt branch expressed by two ideal voltage sources respectively. This concept has been used to develop a number of steady state models suitable for load flow analysis. The one used in this thesis is shown in Fig. 2-2 [7] including a series branch described by an ideal voltage source and a shunt branch by an ideal current source (see also Fig. 1-2).

![Diagram](image)

Fig. 2-2 The equivalent circuit of UPFC

The mathematical relations of the UPFC parameters are given in (2-4) to (2-8) and its phasor diagram is shown in Fig. 2-3.

$$U_s = U_p + U_T$$ \hspace{1cm} (2-4)
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\[ I_s = I_p - I_q - I_t \]  \hspace{1cm} (2-5)

\[ \arg(I_q) = \arg(U_p) \pm \pi/2 \]  \hspace{1cm} (2-6)

\[ \arg(I_t) = \arg(U_p) \]  \hspace{1cm} (2-7)

\[ I_c = \text{Re}(U_T I_s^*) / U_p \]  \hspace{1cm} (2-8)

Fig. 2-3 The phasor diagram of UPFC

where \( U_p, U_s, I_p \) and \( I_s \) are the terminal voltages and currents of the UPFC. Equations (2-6) and (2-7) imply that current \( I_q \) is the reactive component and \( I_t \) is active component of the shunt ideal current source respectively. Equation (2-8) describes the couple relation between the series branch and shunt branch in terms of active power, namely the active power generated or absorbed by the series voltage source thoroughly balanced by the shunt current source. Thus the UPFC can be regarded as a passive element with no active power generation or consumption and its active power loss is negligible. There are three independent parameters, namely \( U_T, \Phi_T \) and \( I_q \), for each UPFC and \( I_t \) is restricted by equation (2-8). These three parameters can be modulated within the region \( \Gamma \) and are determined by the rated capacity of UPFC as:
\[ I = \{ U_T, \Phi_T, I_q \mid U_T \in [0, U_{T_{\text{max}}}], \Phi_T \in [0, 2\pi], I_q \in [-I_{q_{\text{max}}}, I_{q_{\text{max}}}] \} \]

### 2.2.3 Load flow equations of power system with UPFCs

For a generic case, it is assumed that UPFC is embedded in a transmission line with one end denoted as node \( l \) and the other end as node \( m \), as shown in Fig.2-4.

![Equivalent circuit of power system with UPFC](image)

Fig.2-4 Equivalent circuit of power system with UPFC

The parameter \( \rho \) describes the distance between the exciting transformer and the bus \( l \). \( \rho \) is a variable from 0 to 1. \( Z_{lm} \) and \( y_{lm} \) denote the parameters of transmission line \( l-m \). \( Y_l \) and \( Y_m \) denote the respective shunt admittance for bus \( l \) and bus \( m \) but the contribution of line \( l-m \) is excluded. Due to the embedded UPFC, the transmission line \( l-m \) has been divided into two segments: \( l-p \) and \( s-m \). The element \( p-s \) stands for the UPFC. We formulate the branch with UPFC following.

As shown in Fig.2-4, for node \( l, m, p \) and \( s \), we have the following node injection current equations:

\[ I_l = \left\{ \sum_{j=1}^{n} Y_q U_j \right\} + (G + jB) U_m - (y_{lm} + G + jB) U_l + I_p' \]  

(2-9)

\[ I_m = \left\{ \sum_{j=1}^{n} Y_q U_j \right\} + (G + jB) U_l - (y_{lm} + G + jB) U_m - I_s' \]  

(2-10)

\[ I_p = \frac{U_l - U_p}{\rho Z_{lw}} - \rho y_{lw} U_p \]  

(2-11)
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\[ I_i = \frac{U_i - U_m}{(1 - \rho)Z_{lm}} + (1 - \rho)y_{lm}U_i \]  
\[ I'_p = \frac{U_i - U_p}{\rho Z_{lm}} + \rho y_{lm}U_i \]  
\[ I'_s = \frac{U_i - U_m}{(1 - \rho)Z_{lm}} - (1 - \rho)y_{lm}U_m \]  

where: \( G + jB = 1/Z_{lm} \). It is specially noted that in (2-9) and (2-10), the item in braces is exactly equal to the situation without UPFC. Also, it is noted that node \( l \) and node \( m \) are both natural buses of power systems, but node \( p \) and node \( s \) are two terminals of the UPFC. If we can eliminate these two nodes, then the effect of UPFC will be transferred into nodal injection. In other words, Fig.2-4 can be transformed as Fig.2-5.

Fig.2-5 UPFC is represented by nodal power injection

Comparing Fig. 2-5 with Fig 2-4, it is easy to see that the effect of UPFC has been represented by the nodal power injection \( \bar{P}_l, \bar{Q}_l, \bar{P}_m \) and \( \bar{Q}_m \). We will demonstrate Fig.2-5 and derive the formulas of the nodal injections for different value of the parameter \( \rho \) in following context.
**Case One:** $\rho \to 0$

When $\rho \to 0$, this case is defined as **Case One**, that is the shunt transformer of the UPFC is directly connected on bus $l$. From Fig. 2-4, it is easy to see that $I'_p \to I_p$ and $U_p \to U_l$. The system is shown in Fig. 2-6.

![Diagram](image)

Fig. 2-6 Shunt transformer is connected to bus $l$

From Fig. 2-6 or (2-12) and (2-14):

$$\lim_{\rho \to 0} I_s = \frac{U_s - U_m}{Z_m} + y_{mn} U_t \quad (2-15)$$

$$\lim_{\rho \to 0} I'_s = \frac{U_s - U_m}{Z_{lm}} - y_{lm} U_{eq} \quad (2-16)$$

According to (2-5) and (2-15):

$$I_p = \frac{U_s - U_m}{Z_m} + y_{mn} U_t + I_s + I_t \quad (2-16)$$

After replacing $I'_p$ by $I_p$ in (2-9) and note that (2-4):

$$I_s = \left\{ \sum_{j=1}^{\nu} Y_s U_j \right\} + \left( y_{mn} + G + jB \right) U_s + I_s + I_t \quad (2-17)$$
substituting $I'_s$ of (2-14) to (2-10) and note that (2-4):

$$I'_n = \left\{ \sum_{j=1}^{n} Y_{bj} U_j \right\} - (G + jB)U_r$$  \hspace{1cm} (2-18)

$U_p$ and $U_s$ have already been eliminated. By comparing the above two equations with those of the line $l-m$ without UPFC, we can see that due to the intervention of UPFC, a modification item for nodal injection current is produced at and only at node $l$ and node $m$. That is:

$$\Delta I_l = (y_{lm} + G + jB)U_r + I_q + I_t$$  \hspace{1cm} (2-19)

$$\Delta I_m = - (G + jB)U_r$$  \hspace{1cm} (2-20)

Obtaining each conjugate for each $I_i$ ($i = 1, 2, ..., n$) and then multiply them by their correspondence voltage phasor $U_i$, the nodal power equations can be obtained. For nodal $l$:

$$U_l I'_l = U_l \sum_{j=1}^{n} (G + jB) U'_j + U_l \Delta I'_l$$  \hspace{1cm} (2-21)

where:

$$U_l \Delta I'_l = U_l U'_r \left( y_{lm} + G + jB \right) + U_l I'_q + U_l I'_r$$

From (2-6), (2-8) and note that $U_l = U_p$, above equation becomes:

$$U_l \Delta I'_l = U_l U'_r \left( y_{lm} + G + jB \right) - jU_l I_q + \text{Re} \left( U_r I'_s \right)$$  \hspace{1cm} (2-22)

From (2-15) and note that $U_s = U_l + U_r$, obtain:

$$\text{Re} \left( U_r I'_s \right) = \text{Re} \left\{ U_r \left[ \frac{U_l + U_r - U_m}{Z_{lm}} + y_{ms} (U_l + U_r) \right] \right\} \] = \]$$

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Chapter 2 Load Flow Calculation of Power Systems with Multiple UPFCs

\[ U_T U_1 \left[ (G + g_{lm}) \cos \delta_{IT} - (B + b_{lm}) \sin \delta_{IT} \right] - U_T U_m (G \cos \delta_{mT} - B \sin \delta_{mT}) + U_T^2 (G + g_{lm}) \]

where:

\[ \delta_{iT} = \delta_i - \Phi_T \quad \text{and} \quad \delta_{mT} = \delta_m - \Phi_T \]

Substitute the above equation to (2-22) and arrange it:

\[ U_i \Delta I_i^* = \tilde{P}_i + j \tilde{Q}_i \quad (2-23) \]

and:

\[ \tilde{P}_i = 2U_i U_T (G + g_{lm}) \cos \delta_{IT} - U_m U_T (G \cos \delta_{mT} - B \sin \delta_{mT}) + U_T^2 (G + g_{lm}) \quad (2-24) \]

\[ \tilde{Q}_i = U_i U_T [(G + g_{lm}) \sin \delta_{IT} - (B + b_{lm}) \cos \delta_{IT}] - U_i I_i \quad (2-25) \]

For node \( m \) we have the same deriving procedure:

\[ U_m \Delta I_m^* = U_m \sum_{j=1}^n (G_{mj} - jB_{mj}) I_j^* + U_m \Delta I_m^* \quad (2-26) \]

where:

\[ U_m \Delta I_m^* = \tilde{P}_m + j \tilde{Q}_m = -U_m U_T^* (G - jB) \quad (2-27) \]

\[ \tilde{P}_m = -U_m U_T (G \cos \delta_{mT} + B \sin \delta_{mT}) \quad (2-28) \]

\[ \tilde{Q}_m = -U_m U_T (G \sin \delta_{mT} - B \cos \delta_{mT}) \quad (2-29) \]

From above derivation, when \( \rho \to 0 \), Fig.2-4 can be transferred into Fig.2-5.

Case Two: \( \rho \to 1 \)
When $\rho \to 1$, this case is defined as Case Two, that is the series transformer of the UPFC is directly connected on bus $m$. From Fig. 2-4, it is easy to see that $I'_s \to I_s$ and $U_s \to U_m$.

The system is shown in Fig.2-7.

From Fig.2-7 or (2-11) and (2-13):

$$\lim_{\rho \to 1} I_p = \frac{U_l - U_p}{Z_{lm}} - y_{lm}U_p$$  \hspace{1cm} (2-30)

$$\lim_{\rho \to 1} I'_p = \frac{U_l - U_p}{Z_{lm}} + y_{lm}U_l$$  \hspace{1cm} (2-31)

For node $l$, its node voltage equation is (see also Fig 2-7):

$$I_l = \sum_{j=1}^{n} Y_{lj}U_j - (G + jB + y_{lm})U_l + (G + jB)U_m + I'_p$$

By replacing $I'_p$ with (2-31), the above equation can be written as:

$$I_l = \sum_{j=1}^{n} Y_{lj}U_j - (G + jB + y_{lm})U_l + (G + jB)U_m + \frac{U_l - U_p}{Z_{lm}} + y_{lm}U_l$$

note that $U_m = U_p + U_r$ and $G + jB = 1/Z_{lm}$, thus:
\[ I_r = \left\{ \sum_{j=1}^{n} Y_{ij} U_j \right\} + (G + jB)U_r \] (2-32)

Thus:
\[ U_r I_r^* = \left\{ U_r \sum_{j=1}^{n} Y_{ij} U_j^* \right\} + U_r U_t^*(G - jB) = \left\{ U_r \sum_{j=1}^{n} Y_{ij} U_j^* \right\} + \tilde{P}_r + j\tilde{Q}_r \]
then:
\[ \tilde{P}_r = U_r U_r (G \cos \delta_r + B \sin \delta_r) \] (2-33)
\[ \tilde{Q}_r = U_r U_r (G \sin \delta_r - B \cos \delta_r) \] (2-34)

For node \( m \), its node voltage equation is (see also Fig 2-7):
\[ I_m = \sum_{j=1}^{n} Y_{mj} U_j - (G + jB + y_{im}) U_m + (G + jB)U_i - I_i \] (2-35)

For convenient of derivation, the above equation can be rearranged as:
\[ I_r = \sum_{j=1}^{n} Y_{mj} U_j - (G + jB + y_{im}) U_m + (G + jB)U_i - I_m \] (2-36)

Note that \( U_r = U_m - U_r \), and from (2-5) and (2-8), we have:
\[ U_r^* I_r + \text{Re}(U_r I_r^*) + U_m^* I_m - U_r^* I_r = \]
\[ U_m^* I_m - U_m^* I_m + \text{Re}(U_r I_r^*) + U_m^* I_m - U_r^* I_r = \]
\[ U_m^* I_m + j \text{Im}(U_r I_r^*) + U_r^* I_r - U_m^* I_r = 0 \]

Substitute \( I_p \) by (2-30), the above equation can be written as:
\[ U_m^* I_s + j \text{Im}(U_I^* I_I^*) + U_m^* I_q - U_p^* \left( \frac{U_I - U_p}{Z_m} - Y_m U_p \right) = \]

\[ U_m^* I_s + j \text{Im}(U_I^* I_I^*) + j U_p I_q - \left( U_m^* - U_I^* \right) U_I (G + jB) + U_p^2 (G + jB + Y_{im}) = 0 \]  

(2-37)

Take conjugate in (2-37) and then substitute \( I_s \) by (2-36). Thus the first item of (2-37):

\[ U_m^* I_s = U_m \left( \sum_{j \in 1} Y_{mq}^* U_j^* \right) - U_m^* (G - jB + Y_{im}^*) + U_m^* (G - jB) - U_m^* I_m = \]

\[- \left( P_{net} + jQ_{net} \right) + U_m U_i^* (G - jB) - U_m^* (G - jB + Y_{im}^*) \]  

(2-38)

where: \( P_{net} + jQ_{net} = U_m I_m^* - U_m \sum_{j \in 1} Y_{mq}^* U_j^* \)  

(2-39)

The second item of (2-37):

\[ - j \text{Im}[U_I^* I_s] = - j \text{Im} \left[ U_I \sum_{j \in 1} Y_{mq}^* U_j^* - U_I U_m^* (G - jB + Y_{im}^*) + U_I U_m^* (G - jB) - U_I I_m^* \right] = \]

\[ - j \text{Im} \left[ \frac{U_I}{U_m} \left( U_m \sum_{j \in 1} Y_{mq}^* U_j^* - U_m I_m^* \right) \right] + j \text{Im} \left[ U_I U_m^* (G - jB + Y_{im}^*) - U_I U_m^* (G - jB) \right] = \]

\[ - j \left[ \frac{U_I}{U_m} (P_{net} \sin \delta_{nt} - Q_{net} \cos \delta_{nt}) \right] + j \text{Im} \left[ U_I U_m^* (G - jB + Y_{im}^*) - U_I U_m^* (G - jB) \right] \]  

(2-40)

The rest items of (2-37):

\[- j U_p I_q - (U_m - U_I) U_I^* (G - jB) + U_p^2 (G - jB + Y_{im}^*) \]  

(2-41)

Aggregate (2-38), (2-40) and (2-41):
\[(P_{\text{net}} + jQ_{\text{net}}) + U_m U_1^* (G - jB) - U_m^2 (G - jB + y_m^*) -
\]
\[j \left[ \frac{U_T}{U_m} (P_{\text{net}} \sin \delta_{mT} - Q_{\text{net}} \cos \delta_{mT}) \right] + j \Im \left[ U_T U_m^* (G - jB + y_m^*) - U_T U_1^* (G - jB) \right] -
\]
\[j U_p I_q - (U_m - U_0) U_1^* (G - jB) + U_0^* (G - jB + y_0^*) = 0 \tag{2-42} \]

By separating the real and imaginary part from (2-42):
\[P_{\text{net}} = \left( U_T^2 - 2 U_m U_T \cos \delta_{mT} \right) (G + g_{lm}) + U_T U_T^* (G \cos \delta_{IT} - B \sin \delta_{IT}) \tag{2-43} \]
\[
\frac{U_T}{U_m} \left[ Q_{\text{net}} \cos \delta_{mT} - P_{\text{net}} \sin \delta_{mT} \right] - Q_{\text{net}} = \left( U_T^2 - 2 U_m U_T \cos \delta_{mT} \right) (B + b_{lm}) +
\]
\[U_m U_T \left[ (G + g_{lm}) \sin \delta_{mT} + (B + b_{lm}) \cos \delta_{mT} \right] + U_p I_q \tag{2-44} \]

replace \(P_{\text{net}}\) in (2-44) with (2-43):
\[Q_{\text{net}} = \left( U_T \cos \delta_{mT} - U_m \right)^{-1} \times \left[ \left( U_T^2 - 2 U_T U_m \cos \delta_{mT} \right) U_T (G + g_{lm}) \sin \delta_{mT} + U_m (B + b_{lm}) \right] + \]
\[U_T^2 U_T \sin \delta_{mT} (G \cos \delta_{IT} - B \sin \delta_{IT}) + U_m^2 U_T \left[ (G + g_{lm}) \sin \delta_{mT} + (B + b_{lm}) \cos \delta_{mT} \right] + U_m U_p I_q \tag{2-45} \]

where \[U_p = \sqrt{U_m^2 + U_T^2 - 2 U_m U_T \cos \delta_{mT}}\]

Note that (2-39) it is easy to see that:
\[\bar{P}_m = \left( U_T^2 - 2 U_m U_T \cos \delta_{mT} \right) (G + g_{lm}) + U_T U_T^* (G \cos \delta_{IT} - B \sin \delta_{IT}) \tag{2-46} \]
\[\bar{Q}_m = \left( U_T \cos \delta_{mT} - U_m \right)^{-1} \times \left[ \left( U_T^2 - 2 U_T U_m \cos \delta_{mT} \right) U_T (G + g_{lm}) \sin \delta_{mT} + U_m (B + b_{lm}) \right] + \]

34
\[ U_i^2 U_j \sin \delta_{mi} \left( G \cos \delta_{iT} - B \sin \delta_{iT} \right) + U_m^2 U_j \left[ \left( G + g_{lm} \right) \sin \delta_{mj} + \left( B + b_{lm} \right) \cos \delta_{mj} \right] \]
\[ + U_m U_j I_q \}

(2-47)

The above derivation has demonstrated that Fig.2-4 can be transformed as Fig. 2-5 when \( \rho \to 1 \) and the nodal injected powers are given by (2-33), (2-34), (2-46) and (2-47).

**Case Three: \( \rho \in (0,1) \)**

As depicted in Fig.2-4, **Case Three** refers to a situation where a UPFC is positioned midway of a transmission line. Actually **Case One** and **Case Two** are the two extreme forms of **Case Three**. If \( U_p \) in Fig.2-4 is regarded as a bus voltage of the power system, obviously, **Case Three** becomes the same as **Case One**. Similarly, if \( U_s \) in Fig.2-4 is regarded as a bus voltage, **Case Three** becomes the same as **Case Two**. Of course, this will make the total number of the network nodes increased by one. In other words, **Case Three** can be considered as either **Case One** or **Case Two**.

Up to now, we have the result: for the UPFC embedded transmission line with one end denoted as node \( l \) and the other as node \( m \), the load-flow power mismatch equations can be expressed as below:

\[ \Delta P_i = P_{ci} - P_{li} - \sum_{j=1}^{n} U_j U_j \left( G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \right) \]
\[ \Delta Q_i = Q_{ci} - Q_{li} - \sum_{j=1}^{n} U_j U_j \left( G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij} \right) \]
\[ i = 1, 2, \ldots, n; \text{ but } i \neq l, m \]

(2-48)

(2-49)

\[ \Delta P_m = P_{cm} - P_{lm} - \sum_{j=1}^{n} U_j U_j \left( G_{mj} \cos \delta_{mj} + B_{mj} \sin \delta_{mj} \right) - \bar{P}_m \]

(2-50)

\[ \Delta Q_m = Q_{cm} - Q_{lm} - \sum_{j=1}^{n} U_j U_j \left( G_{mj} \sin \delta_{mj} - B_{mj} \cos \delta_{mj} \right) - \bar{Q}_m \]

(2-51)

(2-52)

(2-53)
It is obvious to see that the difference between the load flow equations with and without UPFC comes from the additional four items of modification $\vec{P}, \vec{Q}, \vec{P}_m$ and $\vec{Q}_m$.

Since the expressions of these items of modification are the analyzable formulas, the nonlinear simultaneous equations composed of (2-48) to (2-53) can be solved by the Newton-Raphson method.

2.3 Algorithm for solving load flow equations

As a convenience of describing the algorithm, a recursion formula used in the conventional NRLF is given as:

$$\begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta S \\ \Delta U/U \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$  \hspace{1cm} (2-54)

Assume that a UPFC is embedded in the line $l-m$. In the iteration process of the conventional NRLF, i.e. after forming (2-54) and before solving it, two extra tasks need to be done due to the introduction of the UPFCs. First, superimpose modification $\vec{P}, \vec{Q}, \vec{P}_m$ and $\vec{Q}_m$ into the power mismatches $\Delta P$ and $\Delta Q$ and then amend the Jacobian Matrix by adding the appropriate partial derivatives from (2-24), (2-25), (2-28) and (2-29) for Case One and (2-33), (2-34), (2-46) and (2-47) for Case Two to $H, N, J$ and $L$. The other iteration steps are exactly the same as that without UPFC. If there are other UPFCs, the same procedure is repeated and hence the method is suitable for multi-UPFC systems.

The appropriate partial derivatives can be readily derived from (2-24), (2-25), (2-28) and (2-29) for Case One and (2-33), (2-34), (2-46) and (2-47) for Case Two. The computation burden is trivial. There is no additional requirement at buses $l$ and $m$ in this method. In other words, buses $l$ and $m$ may be PQ or PV type or even the slack bus. When $l$ and $m$ are of different type, the computational burden is different. For example, if $l$ is a PV-bus, its reactive nodal power equation is excluded from the load flow equations. Thus the vector $\Delta Q$ does not include the component $\Delta Q_l$ and $\vec{Q}_l$. Hence they need not be calculated and
the relevant partial derivatives are also not required. As pointed out before, when bus \( l \) and \( m \) are both PQ-bus, at most four power mismatches need to be revised for one UPFC.

After convergence of the iteration process, all magnitude and phase angle of bus voltages are known. The terminal voltage and current of the UPFC can be obtained from the basic relationship formulas (2-3) to (2-8). The load flow calculating diagram is shown in Fig.2-8. Hence, the traditional NRLF technique is very easily be extended to cover studies on power system with UPFCs.

2.4 Case studies

Two load flow case studies on the IEEE 14-bus and 57-bus standard systems with added-on UPFCs are performed with using flat voltage start scheme and a tolerance of accuracy less than \( 10^{-7} \) p.u. of the maximum absolute mismatch of nodal power injection. Table 2-1 gives sample results on the iteration convergence processes for the respective IEEE 14-bus and 57-bus systems. These typical results persist independent to the size of the power system. In short, it shows that the conventional NRLF technique can be extended to cover multi-UPFCs with its quadratic convergence characteristic maintained. Moreover, the results indicate no evidence of impairing merits of the conventional NRLF method. In fact, the results just reinforce the theoretical deduction of the algorithm.

<table>
<thead>
<tr>
<th>Iteration sequence</th>
<th>Maximum mismatch</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>IEEE 14-bus</td>
</tr>
<tr>
<td>1</td>
<td>9.2193542D0</td>
</tr>
<tr>
<td>2</td>
<td>1.0561744D-1</td>
</tr>
<tr>
<td>3</td>
<td>7.5551399D-4</td>
</tr>
<tr>
<td>4</td>
<td>4.8933486D-8</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 2 Load Flow Calculation of Power Systems with Multiple UPFCs

Begin → Given raw data → Form the admittance matrix

Given initial value of the magnitude and phase angle of all bus voltages by the flat start scheme and $k=0$

Calculate all needed node power mismatches $\Delta P_i$ and $\Delta Q_i$ by (2-2) and (2-3)

Superimpose modification $\bar{P}_i$, $\bar{Q}_i$, $\bar{P}_m$ and $\bar{Q}_m$ into the power mismatches $\Delta P_i$, $\Delta Q_i$, $\Delta P_m$ and $\Delta Q_m$

Is the absolute value of the maximum mismatch less than tolerance $\varepsilon$?

Yes → No → Form Jacobian Matrix $H$, $N$, $J$, $L$

Amend the Jacobian Matrix by adding the appropriate partial derivatives from (2-24), (2-25), (2-28) and (2-29) for Case One and (2-33), (2-34), (2-46) and (2-47) for Case Two to $H$, $N$, $J$ and $L$

Find the solution of equations (2-54) to get $\Delta U$ and $\Delta \delta$

Revise $U$ and $\delta$ → $k \leftarrow k+1$

Calculate $U_s$, $U_p$, $I_s$, $I_p$ and $I_r$ for each UPFC by (2-3) to (2-8)

Calculate the output of generators for PV bus and slack bus

Calculate power through each branch

Stop

Fig. 2-8 The load flow computation flow chart
Example 1. IEEE 14-bus system

*Case One* is formed by having one UPFC embedded in the transmission line 13-14. Bus 13 and Bus 14 are both PQ-bus. The computation conditions: parameter $U_T$ is kept at 0.1 p.u. and $I_q$ is kept at 0.01 p.u., $\Phi_T$ is varied from 0 degree to 360 with a step size of 10 degrees. As an example, Fig.2-9 gives the curve of the real power output of the slack bus generator versus the parameter $\Phi_T$ of the UPFC. Fig.2-10 and 2-11 give the magnitude and phase angle of bus 13 and 14 voltage respectively. The abscissa denotes the $\Phi_T$ in all curves. The data of IEEE 14 bus test system are given in Appendix I.

Example 2. IEEE 57-bus system

Ten UPFCs are embedded in this power system to validate the algorithm. Table2-2 gives the position of each of the UPFCs and other associated parameters of the power system. Obviously the data are quoted just for the aim of testing the algorithm and the rationale for locating position and choosing parameters of the UPFCs is beyond the scope of the present chapter. For illustration, Fig.2-12, 2-13 and 2-14 are provided here only. The computation conditions: all parameters of UPFCs are kept constant except that $\Phi_T$ of UPFC inserted in line 23-24 is varied from 0 degree to 360 with a step size of 10 degrees. The data of IEEE 57 bus test system are given in Appendix II.

2.5 Summary

Based on exact derivation, a nodal power injection model of UPFC has been proposed in this chapter. It has already been conveniently integrated to the conventional NRLF program and provides a basic tool for calculating load flow of complex power system with multiple UPFCs. Both the theoretical analysis and numerical computation show that the algorithm is effective in terms of computational speed, accuracy, computing resources and its quadratic convergence characteristics. The computational results indicate that when the UPFC parameters $U_T$ and/or $I_q$ are set with too large values, load flow convergence is difficult due to the flat voltage starting method chosen. Practically, because of other physical limitation, such as insulation level of the apparatus in power system and the cost
Table 2-2. The position and parameters of UPFC

<table>
<thead>
<tr>
<th>bus No. and sort</th>
<th>UPFC parameters</th>
<th>form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>$m$</td>
<td>$U_T$</td>
</tr>
<tr>
<td>16 (PQ-Bus)</td>
<td>1 (slack bus)</td>
<td>0.20</td>
</tr>
<tr>
<td>10 (PQ-Bus)</td>
<td>12 (PV-Bus)</td>
<td>0.18</td>
</tr>
<tr>
<td>13 (PQ-Bus)</td>
<td>15 (PQ-Bus)</td>
<td>0.16</td>
</tr>
<tr>
<td>18 (PQ-Bus)</td>
<td>19 (PQ-Bus)</td>
<td>0.14</td>
</tr>
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<td>23 (PQ-Bus)</td>
<td>24 (PQ-Bus)</td>
<td>0.12</td>
</tr>
<tr>
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<td>27 (PQ-Bus)</td>
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</tr>
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<td>52 (PQ-Bus)</td>
<td>0.10</td>
</tr>
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<td>31 (PQ-Bus)</td>
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</tr>
<tr>
<td>38 (PQ-Bus)</td>
<td>49 (PQ-Bus)</td>
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</tr>
<tr>
<td>41 (PQ-Bus)</td>
<td>42 (PQ-Bus)</td>
<td>0.08</td>
</tr>
</tbody>
</table>

of the UPFC itself, $U_{T_{max}}$ and $I_{q_{max}}$ would not be designed with too large values. Hence, to produce a wide range using the flat voltage starting technique will not be a problem. The case studies demonstrate that the advocated approach is novel, systematic, efficient and reliable. Basically, the method does not require any fundamental change of the NRLF framework and modeling of the UPFCs can be integrated into the load flow calculation effectively. It can be seen that in this chapter, the focus is on the load flow computation. The control parameters of UPFC, $U_T$, $\varphi_T$ and $I_q$, are assumed to be given. This is an essential step for studying the impact of UPFC on power system. Since the formulas of node-injection-power are all analyzable function with reference to parameters of UPFC, they have potential for adoption on other study areas involving multi UPFCs. In fact, the model is used in chapter 3 for analyzing load flow control of UPFC and in chapter 5 for choosing the optimal location of UPFC. In order to generalize the model, the parameter $\rho$ is introduced to effect different location of UPFC at any position along a transmission line.
Fig. 2-9 The real power of slack bus generator in IEEE 14 bus system

Fig. 2-10 The voltage magnitude of bus 13 and 14 in IEEE 14 bus system

Fig. 2-11 The voltage angles of bus 13 and 14 in IEEE 14 bus system
Fig. 2-12 The real power of slack bus generator in IEEE 57 bus system

Fig. 2-13 The voltage magnitude of bus 23 in IEEE 57 bus system

Fig. 2-14 The voltage magnitude of bus 24 in IEEE 57 bus system
Chapter 3

INDIRECT LOAD FLOW CONTROL BY UPFC

3.1 Introduction

UPFC has three parameters that can be simultaneously or selectively controlled to effect on power system performance. These parameters can directly control the active power and reactive power through the transmission line, in which the series transformer of UPFC is embedded, as well as the voltage magnitude of the bus, at which the shunt transformer of UPFC is connected. These parameters can also indirectly control other electrical quantity in power system, for example, the power through other transmission line or the voltage of other bus. For the convenience of description, the former is called direct control and the latter indirect control in following context. The method of the direct control is mentioned by several scholars [4,9,10,30,66] but more research related to that of indirect control is necessary. By incorporating UPFC in an Optimal Power Flow in [24,38] is an example of the indirect control and its control objective is to optimize constraints for an overall state. In line with deregulation of the electric power industry, the need to transport power between partners through defined line corridors with or without involving other partners in the systems will become a frequent need in the future. In other words, it requires directing power flow through given corridor by means of indirect control. Other potential applications include power system contingency control. During normal operation, lines and other network equipment are loaded at typical 1/3 to 1/2 of their thermal limit. At contingency state, however, some of the elements in the system can be stressed to their limit or even beyond. If not responding effectively, a cascade trip of essential power system elements may occur which can endanger a collapse of the total system. In these cases, indirect control can be regarded as an effective means of providing fast contingency control.
Hence, it can be seen that not only direct control but also indirect control plays an important role in the context of power system load flow control. In this chapter the method of indirect control is discussed and that of the direct control is handled in the next chapter.

Qualitatively, UPFC introduces a collective dimension of control in the operation of power systems. It enables the so-called free load flow to become controllable. In this Chapter, a down to earth problem is to show how regulated UPFCs can achieve the expected control objective. The envisaged solution is to deal with the problem by using the sensitivity analysis approach of which its use in power systems is well presented in [31]. In order to protrude the effects of UPFC on power systems, the parameters of the UPFCs are chosen as the only control variables. To start with, a failure research case is briefly discussed in the next section which can show that the indirect control problem is in fact more difficult to achieve than direct control. It also explains the rationale why the sensitivity analysis is used.

3.2 A failure research case

An intuitive method for indirect control is given in the following:

\[
\begin{align*}
    h(X) &= 0 & h & \in \mathbb{R}^n \\
    f(X,U) &= 0 & f & \in \mathbb{R}^n 
\end{align*}
\]  

(3-1)

where the vector \( f \) denotes the load flow equation (see also (2-48) to (2-53)). The vector \( X \) consists of magnitudes and phase angles of bus voltages. The vector \( U \) consists of all controllable parameters of UPFC. The vector \( h \) consists of the total \( m \) control targets such as the active power through the \( m \) lines or else. Because the control is indirect, all the control targets described by \( h \) do not explicitly include the control parameters \( U \). In this case, we hope to identify the parameter vector \( U \) that satisfies the set of nonlinear algebraic equations (3-1). Normally it is expected to find the solution by using Newton method. This method has been widely used for solving direct control problems of power system with FACTS embedded [9,10,30,66].
In mathematical term, one parameter can offer one degree of freedom for power system control. Supposing there are a total of \( l \) number of UPFCs in a power system, \( m \) should not be larger than \( 3l \). Thus it is possible that the solution of (3-1) exists since the total numbers of the equations is not larger than the total numbers of unknown variables. However this is only true in mathematics, and case studies put forward in this project show that it is a failure due to diverging results at all time. The main reasons are given in the following.

- There is no general method to guarantee that these control targets are suitable for existing UPFCs in the system. In fact, the control ability of these UPFCs closely relates to the system configuration, operating condition and location at where the UPFCs are installed. So it is likely these targets can not be reached at all, i.e. no solution for (3-1).

- There is no general method to get the available initial value of \( U \). Even if the solution exists, one cannot find it.

It is worth to note that the target \( h(X) = 0 \) is a strict equality equation and the above two mentioned difficulties are insurmountable for this method. Although, exact solution cannot be reliably found by Newton method, we should find other way to get round it. Sensitivity analysis is considered a worthy trial to tackle this sole target indirect control problem.

### 3.3 Single objective control by UPFC

An indirect load flow control strategy for a single objective is set up which may involve real, reactive, or apparent power through any transmission line of interest or voltage magnitude of a certain key bus and so on.

In general, this type of problem can be represented by (3-2) and its associated load flow equations can be written compactly as (3-3):

\[
H = [h(X, U) - h_{\text{ref}}]^2 \tag{3-2}
\]

\[
\theta = f(X, U) \tag{3-3}
\]
The UPFC parameters are the only chosen control variables so as to protrude the effects of UPFC on the power system. The objective function $h$ is a scalar and, for general, $h$ can be explicit function of the UPFC parameters. There are three kinds of expectation for $h$ in practice, i.e. maximizing $h$, minimizing $h$ and making $h$ a certain value. These three kinds of expectation can all be transformed into minimization type of problem by setting the parameter $h_{goal}$ to a large positive value for maximizing $h$ or very negative value for minimizing $h$ or a certain expectation value. Therefore only minimization problem is discussed.

3.4 Sensitivity of $h$ with respect to $U$

By expanding the Taylor series upon a quiescent state $(X^{(0)}, U^{(0)})$ of $h$ and $f$:

\[
\Delta h = \left[ \frac{\partial h}{\partial X} \right] \Delta X + \left[ \frac{\partial h}{\partial U} \right] \Delta U \tag{3-4}
\]

\[
\theta = \left[ \frac{\partial f}{\partial X} \right] \Delta X + \left[ \frac{\partial f}{\partial U} \right] \Delta U \tag{3-5}
\]

$[\partial f/\partial X]$ is the Jacobian matrix of load flow equations obtainable when the load flow computation converges. Since there are not too many UPFCs in a power system, the dimension of $\Delta U$ is low, say, 3 for one UPFC. Thus the computation burden of forming the matrix $[\partial f/\partial U]$ is light and it can be readily derived from (2-24), (2-25), (2-28), (2-29), (2-33), (2-34), (2-46) and (2-47). Owing to that $h$ is a scalar, the computation burden of forming vector $[\partial h/\partial X]$ is light too and for indirect control $[\partial h/\partial U]$ is equal to zero (see also (3-11) and (3-12)). Now from (3-5), we can obtain:

\[
\Delta X = \left[ \frac{\partial X}{\partial U} \right] \Delta U \tag{3-6}
\]
where
\[
\begin{bmatrix}
\frac{\partial X}{\partial U}
\end{bmatrix} = -\begin{bmatrix}
\frac{\partial f}{\partial X} & \frac{\partial f}{\partial U}
\end{bmatrix}
\]  
(3-7)

The matrix \([\partial X/\partial U]\) is the sensitivity matrix of state variables with respect to the control variables. To avoid computing the inverse of the Jacobian matrix, it can be obtained by solving the simultaneous linear equations:

\[
\begin{bmatrix}
\frac{\partial f}{\partial X} & \frac{\partial X}{\partial U}
\end{bmatrix} = -\begin{bmatrix}
\frac{\partial f}{\partial U}
\end{bmatrix}
\]  
(3-8)

By substituting \(\Delta X\) of (3-6) to (3-4):

\[
\Delta h = \left(\begin{bmatrix}
\frac{\partial h}{\partial U} \\
\frac{\partial h}{\partial X}
\end{bmatrix} + \begin{bmatrix}
\frac{\partial h}{\partial X} & \frac{\partial X}{\partial U}
\end{bmatrix}\right) \Delta U = S \Delta U
\]  
(3-9)

Where:

\[
S = \begin{bmatrix}
\frac{\partial h}{\partial U} \\
\frac{\partial h}{\partial X}
\end{bmatrix} + \begin{bmatrix}
\frac{\partial h}{\partial X} & \frac{\partial X}{\partial U}
\end{bmatrix}
\]  
(3-10)

\(S\) denotes the sensitivity vector of \(h\) with respect to the parameters of the UPFC and it is a row vector. Please note that \([\partial h/\partial U] = \theta\) for indirect control since \(h\) is not explicit function of \(U\).

3.5 Improvement of objective

In accordance with the sensitivity, the parameters of the UPFC are adjusted and \(h\) is forced to move towards the destination \(h_{\text{goal}}\). The improvement procedure takes the following steps.
Chapter 3 Indirect Load Flow Control by UPFC

Step 1: For an arbitrary initial point, \( U^{(0)} \), by using the method proposed in chapter 2, perform the load flow calculation to obtain \( X^{(0)} \); where \( U^{(0)} \in \Gamma \) (which is defined in chapter 2) and \( X^{(0)} \in \Pi \) (a feasible set of \( X \)) and \( H = H^{(0)} \) by (3-2).

Step 2: \( K \leftarrow 1 \) and specify the revising pace \( \Delta U_j, (\forall j) \);

Step 3: Compute \( S^{(K)} \), using the method in section 3.4;

Step 4: Adjust the parameters of the UPFC so as to minimize \( H(X, U) \):

\[
U_j^{(K)} = U_j^{(K-1)} - \text{sign}[H^{(K-1)} - h_{\text{goal}}] \times S_j^{(K-1)} \times \Delta U_j \quad (\forall j)
\]

If \( U_j^{(K)} \) is beyond its boundary, set it to the boundary value;

Step 5: Calculate load flow at point \( U^{(K)} \) to get \( X^{(K)} \), \( H^{(K)} \), where \( U^{(K)} \in \Gamma \)

Step 6: If \( X^{(K)} \in \Pi \) and \( H^{(K)} \leq H^{(K-1)} \), \( K \leftarrow K+1 \) and then repeat step 3. Otherwise:

Step 7: Stop and \( U^* = U^{(K-1)} \) represents the control value of the parameter of UPFCs.

3.6 Case studies

In this investigation, three UPFCs are installed in the IEEE 57-bus case whilst two UPFCs are installed in the case of the 14-bus system respectively. In order to validate the algorithm for the load flow calculation and to analyze quantitatively the ability of UPFCs in indirect control, the range for active power control through each transmission line and transformer in the test power system has been determined individually by using the method described in this chapter. Here the parameter \( h_{\text{goal}} \) is first set to \( 10^0 \) for maximum real power and then set to \( -10^0 \) for minimal real power. Obviously the method can be used in direct control too the line embedded UPFC are also considered. Numerical computation for the two cases has been successfully carried out. As an illustration, only the first case is presented here.

There are four kinds of branch in power system and their equivalent circuit shown in Fig.3-2. Corresponding active power expressions are:

- Transmission line (see also Fig. 3-1(a))

\[
h = p_v(U_i, U_j) = \left( r^2_i + x^2_i \right)^{1/2} \left[ r^2_i U_i^2 - U_i U_j \left( r_i \cos \delta_i - x_i \sin \delta_i \right) \right] \quad (3-11)
\]
• Transformer (see also Fig. 3-1(b))

\[ h = P_y(U_i, U_r) = \left( r_y + x_y^2 \right)^{-1} \times \left[ V_y U_i^2 - T^* U_i U_r \left( r_y \cos \delta_i - x_y \sin \delta_i \right) \right] \]  \hspace{1cm} (3-12)

• Line with UPFC (see also Fig. 3-1(c))

\[ h = P_y(U_i, U_r) = \left( r_y + x_y^2 \right)^{-1} \times \left[ V_y U_i^2 - U_i U_r \left( r_y \cos \delta_i - x_y \sin \delta_i \right) \right] \]  \hspace{1cm} (3-13)

and

\[ U_r = \sqrt{U_i^2 + U_r^2 + 2 U_i U_r \cos(\Phi_r - \delta_i)} \]  \hspace{1cm} (3-14)

\[ \delta_i = \delta_i + \arctg \left( \frac{U_r \sin(\Phi_r - \delta_i)}{U_i + U_r \cos(\Phi_r - \delta_i)} \right) \]  \hspace{1cm} (3-15)

• Line with UPFC (see also Fig. 3-1(d))

\[ h = P_y(U_i, U_r) = \left( r_y + x_y^2 \right)^{-1} \times \left[ V_y U_i^2 - U_i U_r \left( r_y \cos \delta_i - x_y \sin \delta_i \right) \right] \]  \hspace{1cm} (3-16)

and

\[ U_r = \sqrt{U_i^2 + U_r^2 - 2 U_i U_r \cos(\Phi_r - \delta_i)} \]  \hspace{1cm} (3-17)

\[ \delta_i = \delta_i - \arctg \left( \frac{U_r \sin(\Phi_r - \delta_i)}{U_i - U_r \cos(\Phi_r - \delta_i)} \right) \]  \hspace{1cm} (3-18)

where \( r_y \) and \( x_y \) are branch resistance and inductance respectively. \( T \) denotes the turn ratio of transformer.
The initial operating points and forms of connection for the three UPFCs in the IEEE 57-bus system are specified in Table 3-1. The margin of active power transferable in each branch of the system by means of the UPFCs indirect control under the operating condition is determined individually. Results for only 10 out of the 80 branches are listed in Table 3-2 showing the real power transferred through the 10 branches. $P_{\text{free}}$ denotes power values obtained in the condition of free load flow with no UPFC in place and $P_{\text{inst}}$ is in the condition that the UPFCs are operating at the initial values specified in Table 3-1. The range of active power control in each branch by the UPFCs is given in Table 3-3. For example, the first line in Table 3-3 means that the active power transferred through branch 17 from bus 8 to bus 9 can be continuously controlled within the range $[1.411098, 2.181751]$ by the UPFCs. The corresponding control parameters of the UPFCs are given in Table 3-4. The uppercase L, I and J in Table 3-2 and 3-3 denote the branch number and signify the real power transferred from bus I to bus J respectively. $\phi_r$ is in degrees in Table3-1 and Table 3-4. The revising pace $\Delta U_r$ is taken the value 0.001 for all control parameters.

A phenomenon comes from the cross correlation between the parameters $U_r$ and $\phi_r$ observable during the numeral computation. In accordance with the UPFC operating principle, $U_r$ and $\phi_r$ can be changed independently. However, $\phi_r$ has to rely on $U_r$ to act
on the power system. When $U_r$ is equal to zero, $\Omega_r$ cannot have any effect on the power system control. In this case, the UPFC simply acts as a shunt compensator. Therefore the minimum value of $U_r$ is limited to those shown in Table 3-1.

The results in Table 3-2 and Table 3-3 indicate that UPFCs are able to control power flow flexibly. The indirect control to the line 73 even can inverse the active power. It implies that the less the electrical distance between the controlled branch with the UPFC, the larger the controllable active power range and this has been confirmed by the results. It has to be noted that the control capability varies as it depends significantly on the location of the UPFCs and on how the systems are operating. Since the chosen locations in Table 3-1 are used only to validate the method proposed here, this part of the problem is not dealt with at this stage. By the same token, the solution point $U^*$ is not necessarily to be global optimal practically. Nevertheless, the results carry with them the important inferences. The proposed algorithm has been proved to be reliable and efficient for indirect controlling the load flow of complex power systems with multi-UPFCs.

Table 3-1. The position and the limitation of parameter of UPFC

<table>
<thead>
<tr>
<th>Bus No. and Sort</th>
<th>UPFC parameters</th>
<th>form joined</th>
<th>UPFC No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>$m$</td>
<td>$U_r^{(0)}$</td>
<td>$\Omega_r^{(0)}$</td>
</tr>
<tr>
<td>11(PQ)</td>
<td>13(PQ)</td>
<td>0.1</td>
<td>100.0</td>
</tr>
<tr>
<td>23(PQ)</td>
<td>24(PQ)</td>
<td>0.1</td>
<td>100.0</td>
</tr>
<tr>
<td>38(PQ)</td>
<td>44(PQ)</td>
<td>0.1</td>
<td>100.0</td>
</tr>
</tbody>
</table>
Table 3-2. The load flow at point $U_T = 0$ and $U_T = U_T^{(0)}$

<table>
<thead>
<tr>
<th>L</th>
<th>I</th>
<th>J</th>
<th>$P_{f,rel}(U_T=0)$</th>
<th>$P_{mn}(U_T=U_T^{(0)})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>8</td>
<td>9</td>
<td>1.780092</td>
<td>1.880117</td>
</tr>
<tr>
<td>19</td>
<td>9</td>
<td>11</td>
<td>.129033</td>
<td>.473488</td>
</tr>
<tr>
<td>25</td>
<td>11</td>
<td>13</td>
<td>-.099249</td>
<td>.254867</td>
</tr>
<tr>
<td>34</td>
<td>14</td>
<td>15</td>
<td>-.688656</td>
<td>-.752818</td>
</tr>
<tr>
<td>35</td>
<td>14</td>
<td>46</td>
<td>.479131</td>
<td>.595779</td>
</tr>
<tr>
<td>43</td>
<td>23</td>
<td>24</td>
<td>.033511</td>
<td>.096280</td>
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<td>62</td>
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<td>44</td>
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<td>48</td>
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<td>46</td>
<td>47</td>
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<td>.595779</td>
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<td>73</td>
<td>47</td>
<td>48</td>
<td>.176112</td>
<td>.291493</td>
</tr>
</tbody>
</table>

Table 3-3. The active power control range of UPFC

<table>
<thead>
<tr>
<th>L</th>
<th>I</th>
<th>J</th>
<th>$P_{mn}$</th>
<th>$P_{max}$</th>
<th>RANGE</th>
</tr>
</thead>
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<td>9</td>
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<td>2.181751</td>
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<td>9</td>
<td>11</td>
<td>.100099</td>
<td>1.001040</td>
<td>.900941</td>
</tr>
<tr>
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<td>11</td>
<td>13</td>
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<td>1.056342</td>
<td>1.179038</td>
</tr>
<tr>
<td>34</td>
<td>14</td>
<td>15</td>
<td>-.1172565</td>
<td>-.274022</td>
<td>.898544</td>
</tr>
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<td>14</td>
<td>46</td>
<td>.045197</td>
<td>.934799</td>
<td>.889602</td>
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<td>.299879</td>
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<td>.934799</td>
<td>.889602</td>
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<tr>
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<td>48</td>
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<td>.619748</td>
<td>.872361</td>
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</tbody>
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Table 3-4 The value of control parameters of

<table>
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<th>UPFC</th>
<th>when power maximum</th>
<th>when power minimum</th>
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<td></td>
<td>$U_T$</td>
<td>$\Phi_T$</td>
</tr>
<tr>
<td>L</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0.300</td>
<td>48.4</td>
</tr>
<tr>
<td>2</td>
<td>0.300</td>
<td>59.8</td>
</tr>
<tr>
<td>3</td>
<td>0.300</td>
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</tr>
<tr>
<td>19</td>
<td>0.200</td>
<td>54.1</td>
</tr>
<tr>
<td>2</td>
<td>0.200</td>
<td>65.6</td>
</tr>
<tr>
<td>3</td>
<td>0.200</td>
<td>111.4</td>
</tr>
<tr>
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<td>59.8</td>
</tr>
<tr>
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<td>0.250</td>
<td>59.8</td>
</tr>
<tr>
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</tr>
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<td>35</td>
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<td>0.280</td>
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</tr>
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<td>0.220</td>
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<tr>
<td>3</td>
<td>0.280</td>
<td>65.6</td>
</tr>
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</table>
3.7 Indirect control problem with multiple-objective

By means of the sensitivity analysis, the indirect control problem with sole objective can be tackled with. From the results of the case study, one can see that if the control objective $h_{goal}$ belongs to the range $[P_{min}, P_{max}]$, then the control parameters of UPFC can be found by the above method. If the control objective is beyond the control scope of the system, the nearest result can be provided. Basically the method of sensitivity can be developed to cover the indirect control problem with multiple objectives. In the multiple-objective problem $S$ is a matrix instead of a vector. The column of the sensitivity matrix, whose elements have the same positive or negative sign, points to a regulating direction for corresponding parameters. But these columns, whose elements have different sign, can not directly provide a regulating direction for corresponding parameters. We can regulate the parameter whose direction is determined by corresponding column in the sensitivity matrix until no parameters can be regulated. But when there are too many objectives, it is likely no one parameter can be revised even one at a time. In this case, these multiple objectives must compromise by means of other computational technology such as transforming the multiple-objective problem into single objective one.

3.8 Summary

The problem of controlling load flow by using UPFC is divided into direct control and indirect control as mentioned in this chapter. More research for indirect control problem is necessary. Basically, it cannot obtain solution by simultaneously finding objective equation and solving load flow equations. Based on the sensitivity analysis and the load flow model proposed in chapter 2, the method of determining the UPFC parameters for load flow control with single objective are presented in this chapter. Case studies demonstrate that this method is available for indirect control with single objective. Multiple-objective indirect control problem is roughly discussed. Although this method can be used in both direct control and indirect control, its main use is for indirect control. More efficient method for direct control is posed in the next chapter.
Chapter 4

DIRECT LOAD FLOW CONTROL BY UPFC

4.1 Introduction

The problem of direct load flow control by using UPFC is relatively well addressed in the research world. There are two questions to be answered in the direct control case. One is how to set the control parameters of UPFC to realize the control objective and another one is how to calculate the associate load flow.

It is firstly revealed that UPFC can control the active and reactive power in the line in which UPFC is embedded by regulating the line impedance, the magnitude and the phase angle difference of the voltages across the line in [36]. Furthermore the concept of direct controlling real and reactive power by regulating the series voltage is established in [37]. In the earlier stage reported in [36,37,60], simple two-bus ac inter-tie system is referred. It is suitable for studying the characteristics of UPFC for line flow control. Various issues of load flow calculation and control are studied in [4,9,10,66] when UPFC enters the complex power system. There are shortcomings in the existent method reported in open published literature. The method proposed by C. R. Fuerte-Esquivel and E. Acha [9,10,11] is an unified iteration process, i.e., the control objective function and load flow equations are simultaneously solved by iteration. However the fact, that it requires a relative precise estimation of the initial control setting for each UPFC and increases considerably the order of the Jacobian Matrix in the iteration procedure, make the algorithm virtually not suitable for extension to cover analysis on multiple UPFCs. The method proposed by A. Nabavi-Niaki and M.R.Iravani [4] is based on a reliable node-power-injection model, i.e., after load flow computation is carried out the UPFC control parameters are determined by solving the mini-scale nonlinear equations.

This chapter proposes a convenient and reliable NRLF based method to perform the load flow calculation and to determine subsequently the control setting of the UPFCs. The
envisaged method keeps the NRLF program intact and requires only little modification to the Jacobian Matrix in the iteration procedure. It can be seen, in the following context, that the power flow of the UPFC controlled transmission line together with one of the two bus voltages across it can be directly set as constant according to certain control strategy. Having obtained the load flow convergence, control setting of the UPFCs can then be determined straight away. Comparing with the mentioned above two typical method, the approach does not need to augment the Jacobian matrix and does not have the initial value for the control parameters. Case studies are successfully carried out on the IEEE 14-bus and 57-bus systems. Performance of these studies indicates that the method maintains the basic NRLF properties such as fast computational speed, high degree of accuracy and good convergence rate.

4.2 Modeling Approach

4.2.1 Power Equations of the UPFC Connected Branch

The steady state model of UPFC is shown in Fig.2-2 and the mathematical relations of the UPFC are given in (2-3) to (2-8). Consider a UPFC with its boosting transformer connected in series with a transmission line. In practice, for the convenience of maintaining equipment, UPFC should be installed in substation. Hence we can assume that the exciting transformer is connected to the bus $l$ and the two terminals of the transmission line are denoted as bus $s$ and $m$ respectively. By using the model of the UPFC illustrated in Fig.2-2 and a π-equivalent circuit of the transmission line, the branch with the UPFC connected between bus $l$ and $m$ has been modeled as shown in Fig.2-6. For the convenience of description it is shown in Fig. 4-1 again.

First we derive the expressions of power $S_{ml}$ from bus $m$ and $S_{lm}$ from bus $l$ according to the basic mathematical relations of UPFC (2-3) to (2-8). From Fig.4-1, the complex power from bus $m$ can be written as:
Fig.4-1. Equivalent circuit of a UPFC embedded transmission

\[ S_{ml} = \left( \frac{U_m - U_r - jB_{lm} U_m}{R_{lm} + jX_{lm}} \right) U_m \]

(4-1)

Note that: \( U_r = U_I + U_T \) then

\[ S_{ml} = \left( \frac{U_m - U_r - U_T - jB_{lm} U_m}{R_{lm} + jX_{lm}} \right) U_m = \]

\[ = \left( \frac{U_m - U_I}{R_{lm} + jX_{lm}} - jB_{lm} U_m \right) U_m - \left( \frac{U_T}{R_{lm} + jX_{lm}} \right) U_m \]

(4-2)

Then power \( S_{ml} \) can be expressed as:

\[ S_{ml} = P_c + jQ_c = P_f + jQ_f + \Delta S_c \]

(4-3)

where:

\[ P_f + jQ_f = \left( \frac{U_m - U_r - jB_{lm} U_m}{R_{lm} + jX_{lm}} \right) U_m \]

(4-4)

\[ \Delta S_c = P_c + jQ_c = \left( \frac{U_T}{R_{lm} + jX_{lm}} \right) U_m \]

(4-5)

From the model of UPFC in Fig.4-1, the power from bus \( l \) can be written as:
\[ S_m = P_m + jQ_m = (I_q + I_r + I_s) U_i \]  

(4-6)

From (2-6), (2-7) and (2-8):

\[ S_m = [I_q \angle (\delta_i + \pi / 2)] U_i \angle \delta_i + \left[ \frac{\text{Re}(U_r I_s^*)}{U_i} \angle \delta_i \right] U_i \angle \delta_i + I_s^* U_i \]

\[ = -jI_q U_i + \text{Re}(U_r I_s^*) + I_s^* U_i \]

(4-7)

By referring to (2-4), the phase diagram of \( U_i, U_r \) and \( U_s \) can be constructed as shown in Fig.4-2. The relationship that \( P_{im} \) is equal to \( P_{sm} \) can be established and hence \( P_{sm} \) is derived herewith.

\[ \theta = \Phi_r - \alpha - \delta_i \]

\[ U_s \]

\[ U_T \]

\[ \Phi_r - \delta_i \]

\[ \alpha \]

\[ \delta_i \]

\[ \theta \]

Reference Phase

Fig. 4-2: The phase diagram of \( U_i, U_r \) and \( U_s \)

\[ P_{sm} = \text{Re}(S_m) = \text{Re}(U_r I_s^*) + \text{Re}(U_r I_s^*) \]

(4-8)

\[ \therefore I_s^* = \frac{P_{sm} + jQ_{sm}}{U_s} \]  

(cf. Fig.4-1)  

(4-9)

\[ \therefore P_{sm} = \text{Re} \left( U_r \frac{P_{sm} + jQ_{sm}}{U_s} \right) + \text{Re} \left( U_r \frac{P_{sm} + jQ_{sm}}{U_r} \right) \]

\[ = \frac{U_r}{U_s} \left[ P_m \cos(\Phi_r - \alpha - \delta_i) - Q_m \sin(\Phi_r - \alpha - \delta_i) \right] + \frac{U_r}{U_s} \left[ P_m \cos \alpha + Q_m \sin \alpha \right] \]  

(cf. Fig 4-2)

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\[
\frac{P_{sm}}{U_s} \left[ U_s \cos(\Phi - \alpha - \delta) + U_i \cos \alpha \right] - \frac{Q_{sm}}{U_s} \left[ U_s \sin(\Phi - \alpha - \delta) - U_i \sin \alpha \right] = 0
\]

From Fig.4-2:
\[
\left[ U_s \cos(\Phi - \alpha - \delta) + U_i \cos \alpha \right] = U_s \tag{4-11}
\]
\[
\left[ U_s \sin(\Phi - \alpha - \delta) - U_i \sin \alpha \right] = 0 \tag{4-12}
\]

Thus:
\[
P_{lm} = P_{sm} \tag{4-13}
\]

Since the UPFC is a passive element, it can neither generate nor absorb active power (loss neglected) as described by (4-13). Let \( I_c \) denote the complex current through the impedance \( Z_{lm} \) as shown in Fig.4-1 and \( \Delta P_z \) as the active power consumed by the resistance of the transmission line.

Note that \( S_{ml} = P_c + jQ_c \):
\[
I_c = \left( \frac{P_c + jQ_c}{U_m} \right) - jB_m U_m \tag{4-14}
\]
\[
\Delta P_z = (I_c \cdot R_{lm}) I_c^* = \left( \frac{P_c^2 + Q_c^2}{U_m^2} + B_m^2 U_m^2 + 2B_m Q_c \right) R_{lm} \tag{4-15}
\]
\[
P_{lm} = P_{sm} = -(P_c - \Delta P_z) = \left( \frac{P_c^2 + Q_c^2}{U_m^2} + B_m^2 U_m^2 + 2B_m Q_c \right) R_{lm} - P_c \tag{4-16}
\]

(4-16) points out that the active power \( P_{lm} \) can be expressed by \( P_c \) and \( Q_c \). Whereas
\[ Q_{im} = \text{Im}(S_{im}) = \text{Im}(-jI_q U_l) + \text{Im}(I^* U_l) \]  
\[ (4-17) \]

From the model of the transmission line in Fig.4-1:

\[ I_s = -(I_z - jB_{im} U_s) = jB_{im} (U_m - I_z Z_{im}) - I_z \]
\[ = jB_{im} U_m - (1 + jB_{im} Z_{im}) I_z \]  
\[ (4-18) \]

Let:

\[ C_x + jC_Y = 1 + jB_{im} Z_{im} = (1 - B_{lm} X_{lm}) + jB_{lm} R_{lm} \]  
\[ (4-19) \]

and from (4-14), the equation (4-18) can be written as:

\[ I_s = jB_{im} (1 + C_x + jC_Y) U_m - \frac{(C_x + jC_Y) (P_c - jQ_c)}{U_m^*} = \]
\[ -[B_{im} C_Y - jB_{im} (1 + C_x)] U_m - \frac{(C_x P_c + C_Y Q_c) + j(C_Y P_c - C_x Q_c)}{U_m^*} \]  
\[ (4-20) \]

Let

\[ E_1 = C_X P_c + C_Y Q_c \]  
\[ (4-21) \]

\[ E_2 = C_Y P_c - C_X Q_c \]  
\[ (4-22) \]

\[ F_1 = B_{lm} C_Y \]  
\[ (4-23) \]

\[ F_2 = -B_{lm} (1 + C_X) \]  
\[ (4-24) \]

Then

\[ I_s = -(F_1 + jF_2) U_m - (E_1 + jE_2) / U_m^* \]  
\[ (4-25) \]
By substituting and rearranging (4-25) for $I_s$ in (4-17), obtain:

$$Q_{lm} = -I_sU_t - [(E_lU_m^* + F_lU_m^*)\sin\delta_m - (E_mU_m^* + F_mU_m^*)\cos\delta_m]U_t$$

(4-26)

where $\delta_{lm} = \delta_l - \delta_m$ is the phase angle difference between bus $l$ and $m$. Thus the reactive power $Q_{lm}$ can be expressed by $P_c$ and $Q_c$ too.

Up to now, the expressions of $S_{ml}$ and $S_{lm}$ are given by (4-3), (4-4), (4-5), (4-16) and (4-26) and $S_{lm}$ has been expressed by $S_{ml}$. From (4-3), the power in the transmission line $S_{ml}$ has been divided into two parts. The first one is free flow given by (4-4). This part of power is produced by the deviation of the voltage, including magnitude and phase angel, crossing the transmission line. The second one $\Delta S_c$ is given by (4-5) and it is caused by the series voltage $U_T$. Thereby $\Delta S_c$ the total power of the line, the active power $P_c$ and reactive power $Q_c$, are controllable and are taken as the direct control objective.

4.2.2 Load Flow Equations

Assume that for a given control strategy, the power, $S_{ml}$, on the UPFC controlled transmission line $l$-$m$, is set to constant $(P_0+jQ_0)$. By means of the substitution theorem, this branch $l$-$m$ can be detached as shown in Fig.4-3 in which $S_{ml}$ represents power coming from the bus $m$ and $S_{lm}$ from the bus $l$ respectively. For each other additional UPFC, its corresponding branch can be dealt with similarly.

After modifying all of the UPFC embedded branches, the load flow equations can be written as follows.

![Fig.4-3 The system equivalent circuit after detaching the branch l-m](image)

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\[ \Delta P_i = P_{G_i} - P_{U_i} - \sum_{j=1}^{n} U_j U_j \left( G_q \cos \delta_{ij} + B_q \sin \delta_{ij} \right) \]  
\[ \Delta Q_i = Q_{G_i} - Q_{U_i} - \sum_{j=1}^{n} U_j U_j \left( G_q \sin \delta_{ij} - B_q \cos \delta_{ij} \right) \]  
\[ i = 1, 2, \ldots, n; \text{ but } i \neq l, m \text{ and} \]  
\[ \Delta P_l = P_{G_l} - P_{U_l} - \sum_{j=1}^{n} U_j U_j \left( G_q \cos \delta_{lj} + B_q \sin \delta_{lj} \right) - P_{b_l} \]  
\[ \Delta Q_l = Q_{G_l} - Q_{U_l} - \sum_{j=1}^{n} U_j U_j \left( G_q \sin \delta_{lj} - B_q \cos \delta_{lj} \right) - Q_{b_l} \]  
\[ \Delta P_m = P_{G_m} - P_{U_m} - P_c - \sum_{j=1}^{n} U_j U_j \left( G_q \cos \delta_{mj} + B_q \sin \delta_{mj} \right) \]  
\[ \Delta Q_m = Q_{G_m} - Q_{U_m} - Q_c - \sum_{j=1}^{n} U_j U_j \left( G_q \sin \delta_{mj} - B_q \cos \delta_{mj} \right) \]  

where the \( P_c \) and \( Q_c \) are constant. The \( P_{lm} \) and \( Q_{lm} \) are given by (4-16) and (4-26) respectively.

4.3 Load Flow Computation

By comparing the equations of (4-27) to (4-32) with the corresponding conventional load flow equations, observable differences include \( P_{lm} \) in (4-29), \( Q_{lm} \) in (4-30), \( P_c \) in (4-31) and \( Q_c \) in (4-32) respectively. Since \( S_{lm} = P_c + jQ_c \) is set as constant for the given control requirement, \( S_{lm} = P_{lm} + jQ_{lm} \) can be treated as a special load varying with respect to the voltages \( U_l \) and \( U_m \). As a result, the UPFCs have already been decoupled from the system and the load flow equations (4-27) to (4-32) can be solved by standard NRLF program.
The algorithm for finding the solutions of (4-27) to (4-32) does not place any restriction on the type of bus \( l \) and \( m \), i.e. bus \( l \) and \( m \) can be PQ-bus, PV-bus or even slack bus.

When bus \( l \) is taken as a PQ-bus, \( I_q \), should be given. As seen from (4-27) to (4-32), \( I_q \) comes only from (4-30) and so it can be used to control the bus voltage \( U_l \) directly to a specified value. Hence, if bus \( l \) is a PV-bus, (4-30) is excluded from the load flow equations. Finally, if bus \( l \) is a generator bus also, \( I_q \) and \( Q_l \) can concurrently or independently keep it as a specified value.

As for bus \( m \), if it is a generator bus it certainly can be as each type of bus. If it is not the generator bus it is possible to be a PV bus by neglecting the resistance \( R_{lm} \), which is reasonable in high voltage transmission line. In (4-16) taken line resistance as zero, obtain \( P_{lm} = -P_c \). Suppose bus \( l \) is the PV bus, by means of UPFC, bus \( m \) can be controlled as PV bus too. The method is setting \( P_c \) as constant in (4-29) and (4-31) and excluding (4-30) and (4-32) from load flow equations. Thus the load flow equations can be solved by standard NRLF program. After load flow computation, the reactive power \( Q_c \), which upholds the bus \( m \) as a PV bus, can be calculated. By this approach the voltage magnitudes of the two buses both can be controlled by using UPFC but the reactive power \( Q_c \) is not an independent control objective any more. So UPFC can still control the three electrical quantities, namely the active power in the line and the magnitude of two buses.

As seen from the load flow equations (4-27) to (4-32), only two equations for each UPFC include the modification item, \( P_{lm} \) and \( Q_{lm} \). The elements of Jacobian matrix, which need to be revised, are at most five. Specially, when bus \( l \) and \( m \) are both PV bus and the line resistance is neglected, there is no need to revise the Jacobian matrix. This is important for guaranteeing the convergence.

### 4.4 Computation for the UPFC Control Setting

After load flow computation converges, the control setting of the UPFC can be computed as follows. In the case of bus \( l \) and bus \( m \) are both PV bus and bus \( m \) is not a generator bus, the node reactive power \( Q_c \) in bus \( m \) and \( Q_{lm} \) in bus \( l \) are calculated by load flow
equation (4-32) and (4-30) respectively. Then by (4-26) the parameter $I_q$ can be obtained. whereas if bus $l$ and bus $m$ are both PQ bus, $P_c$ and $Q_c$ are given and $P_f$ and $Q_f$ can be computed from (4-4). Note that $P_c$ and $Q_c$ are given and (4-3) gives:

$$\Delta S_e = P_e + jQ_e = P_c + jQ_c - P_f - jQ_f$$  \hspace{1cm} (4-33)

From (4-5):

$$U_T \angle \Phi_T = - (R_{lm} + jX_{lm}) \left( \frac{P_e + jQ_c}{U_m \angle \delta_m} \right)^*$$

Then:

$$U_T = \sqrt{(P_e^2 + Q_c^2)(R_{lm}^2 + X_{lm}^2)} \frac{1}{U_m}$$  \hspace{1cm} (4-34)

$$\Phi_T = \gamma - \beta + \delta_m$$  \hspace{1cm} (4-35)

Where: $\gamma = \arctg(Q_c / (-P_e))$ and $\beta = \arctg(X_{lm}/R_{lm})$

From (4-34) and (4-35), $U_T$ and $\Phi_T$ can be determined readily once the load flow calculation converges for the given $(P_c, Q_c)$. In case, if $U_T$ exceeds $U_{Tmax}$, the UPFC rating limit, it means that the given $(P_c, Q_c)$ are not suitable and they should be regulated as described in the following so that $U_T$ can satisfy $\Gamma$ (its definition is given in section 2.2.2; page 26). The computation flow chart is given in Fig.4-4.

From (4-33) and (4-34), it is noted that $U_T$ should not be larger than $U_{Tmax}$, i.e., the point $(P_c, Q_c)$ should fall within the circle defined by:

$$(P_e - P_f)^2 + (Q_e - Q_f)^2 \leq \frac{U_{Tmax}^2}{R_{lm}^2 + X_{lm}^2}$$  \hspace{1cm} (4-36)
Chapter 4 Direct Load Flow Control by UPFC

Fig. 4-4 The load flow computation flow chart
From (4-4) and by neglecting $R_{lm}$ and $B_{lm}$, the center of the circle $(P_f, Q_f)$ can be determined from (4-37):

$$P_f^2 + \left( Q_f - \frac{U_m^2}{X_m} \right)^2 = \left( \frac{U_r U_m}{X_m} \right)^2 \quad (4-37)$$

Since $(P_f, Q_f)$ and $U_m$ are related to $(P_c, Q_c)$, the center and radius of the circle in (4-36) are both varying as a function of $(P_c, Q_c)$. Note that if the bus $l$ is a PV-bus, $U_l$ and $X_{lm}$ are both constant. In normal operating condition, $U_m$ varies within a small range, say, from 0.95 to 1.05 in per unit. In other words, once the load flow computation is done, the dotted line circle of (4-37) in Fig.4-5 can be drawn. For each different point, $(P_c, Q_c)$, a corresponding circle of (4-36) with center, $(P_f, Q_f)$, lying on the dotted line circle of (4-37). Hence, Fig.4-5 shows (4-36) and (4-37) as a family of circles corresponding to a series of points, $(P_c, Q_c)$. Operationally speaking, the method of regulating $(P_c, Q_c)$ is to shift it into the corresponding circle area such as the shaded one in Fig.4-5.

When bus $l$ is upheld as a PV-bus by means of the UPFC, the control variable, $I_q$, is required to be determined from (4-26) by first obtaining $Q_{lm}$ from (4-30). If $I_q$ obtained is beyond its limit, it means the bus $l$ cannot be held at the specified value. In that case, let bus $l$ be a PQ-bus, fix $I_q$ at its boundary value and calculate load flow again.
4.5 Case studies

As case studies to validate the proposed method, numerical computations have been performed successfully on the standard IEEE 14-bus and 57-bus systems. For the load flow computation, a flat voltage starting method with an accuracy tolerance of less than 10^{-9} p.u. in respect of the maximum absolute mismatch of nodal power injection are adopted. For saving space, only results of the latter are provided as follows. The raw data of the basic operation condition for the system stems from [39] and are given in Appendix II.

Table 4-1 shows the basic information of the four UPFCs installed in the IEEE 57-bus system. Under specific control strategy, the table gives the required active and reactive power transferred on each UPFC controlled transmission line setting, \((P_c, Q_c)\). By keeping the apparent power \(|S_m|\) of the line 23-24 at 0.8 p.u., the load flow computation is carried out with respect to the power factor angle \((\phi)\) at 10 degrees interval for the range from 0 to 360 degree. The 36 times load flow results are summarized and plotted in the form of curves. Fig.4-6 illustrates the average convergence characteristic under a log scale. It shows that the convergence characteristic is super-linear. Fig.4-7 and Fig.4-8 show the values of \(U_{Tl}\) and \(\Phi_{Tl}\), i.e., the respective control setting of the first UPFC. Fig.4-9 to Fig.4-11 show some more sample results for the second UPFC. Further sample results such as the terminal voltages of the line 23-24 are given in Table 4-2.

<table>
<thead>
<tr>
<th>Sequence</th>
<th>The site of UPFC: Bus No. and Type</th>
<th>The value of (I_q) or (U_l)</th>
<th>The power controlled by UPFC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(l) \quad (m)</td>
<td></td>
<td>(P_c) \quad (Q_c)</td>
</tr>
<tr>
<td>1</td>
<td>23 (PQ) \quad 24 (PQ)</td>
<td>0.123</td>
<td>(S_m) \cos \phi \quad (S_m) \sin \phi</td>
</tr>
<tr>
<td>2</td>
<td>11 (PV) \quad 13 (PQ)</td>
<td>1.000</td>
<td>-0.5 \quad 0.05</td>
</tr>
<tr>
<td>3</td>
<td>38 (PV) \quad 44 (PQ)</td>
<td>1.000</td>
<td>0.3 \quad 0.03</td>
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<tr>
<td>4</td>
<td>52 (PQ) \quad 29 (PQ)</td>
<td>0.123</td>
<td>0.1 \quad 0.01</td>
</tr>
</tbody>
</table>
Fig. 4-6 The average convergence characteristic

Fig. 4-7 $U_{TI}$ versus the angle of power factor $\varphi$

Fig. 4-8 $\Phi_{TI}$ versus the angle of power factor $\varphi$
Fig. 4-9 $I_{q2}$ versus the angle of power factor $\varphi$

Fig. 4-10 $U_{T2}$ versus the angle of power factor $\varphi$

Fig. 4-11 $\Phi_{T2}$ versus the angle of power factor $\varphi$
Table 4-2 The terminal voltage of transmission line 23-24

<table>
<thead>
<tr>
<th>$P_{cl}$</th>
<th>$Q_{cl}$</th>
<th>$U_{23}$</th>
<th>$U_{24}$</th>
<th>$\delta_{23}$</th>
<th>$\delta_{24}$</th>
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<td>.800</td>
<td>.000</td>
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<td>2.717</td>
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<td>2.682</td>
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<td>1.912</td>
</tr>
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<td>.995988</td>
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4.6 Summary

This chapter has presented an effective and reliable method to perform the direct load flow control and calculation for the multiple UPFCs embedded systems. The power in the transmission line and the magnitude of the bus voltage can be directly given as constant. At one bus, this constant power is substituted with a common load and at other node the special load, whose power is varied with the two bus voltages cross the line, is used to represent the UPFC. By this method, the UPFC is decoupled from AC power system and the computation for load flow is independent of the parameters of UPFCs. The converged load flow results can then be used to determine the control settings of the UPFCs directly. Implementation of the proposed method has been carried out successfully and performance results from the case studies indicate the proposed method of load flow computation can maintain the basic NRLF properties such as fast computational speed, high degree of accuracy and good convergence rate. Furthermore, the proposed method carries with it four distinct features. Firstly, the method adapts itself to the standard NRLF program with little modification and it does not augment the Jacobian matrix. Hence, it can deal with the load flow control and calculation problems of multiple UPFCs embedded power systems effectively and it can easily incorporate the non-linear control devices of UPFCs into the conventional NRLF program. Secondly, the method can readily help demonstrate the effectiveness of the UPFCs for controlling load flow in the modern power system by regulating the control settings, $P_c$ and $Q_c$ as well as $U_l$ or $I_q$. Thirdly, this algorithm does not impose any restriction on the type of buses in which the UPFC embedded transmission line is connected. The bus voltage magnitude may be held constant by the UPFC or assumed as variable that can be determined by the load flow computation. Lastly, all control setting of the UPFCs can be determined directly without iteration and hence no need for any estimation of their initial values. This method significantly prevails over the existent method for direct controlling load flow.
Chapter 5

OPTIMISING LOCATION OF UPFC

5.1 Introduction

Optimising use of power transfer capability of transmission network is always a concern in the power supply industry. Power flow congestion in strategic routes may occur because of the inflexibility of their power control capability. Upgrading existing transmission lines or enhancing their power controllability are common practical means to achieve a better utilisation of their ultimate design limit and their precious right-of-way. Flexible AC Transmission System (FACTS) is the underpinning concept based on which promising means to effectively avoid power flow bottlenecks and to extend the loadability of existing power transmission networks are being developed. As the indication in previous chapter, UPFC is a promising FACTS device for the load flow control since it can either simultaneously or selectively control the active and reactive power flow along the line as well as its terminal voltages. This salient property has made dynamic system performance study of UPFC controlled power system a popular research topic on applications with typical assumption of pre-configured layout of UPFCs. However, when coming to implementation there is always a practical concern for finding optimal location and rating of the UPFCs but they have not been tackled extensively. As reviewed from existing literature, location of the UPFCs tends to be done empirically, if not arbitrarily, and is lacking a systematic treatment. In effect, their interest is mainly focused on how to exploit UPFCs to control load flow in static, dynamic or transient states. In reference [35], it discusses ways to determine the type, location and rating of Static Var Compensators and Thyristor Controlled Series Capacitors but their main aim is only to show how the maximum stable system loadability can be increased by using FACTS devices. Although [18,30] can further demonstrate effectiveness, roles and impact of FACTS devices on different installation sites, they are still regarded as not so suitable for use in general for solving the above-mentioned type of problem. They tend to be qualitative and lack of supporting figures.
Power flow problems in steady state operation have been well identified in interconnected power systems such as loop flows, parallel path flows and circulating flows. Especially for heavy loaded power systems, the above phenomena indulge considerable concerns on their power transfer limit being constrained. By means of the proposed direct and indirect load flow control method mentioned in the previous chapters, these problems can be eased off. For example, a so-called "electrical fence" can be constructed to prevent a loop flow or an "electrical pump" to direct power flow to bypassing the congested bottleneck. However, there are a lot of requirements or constraints during power system operation. Hence operating power system at the state that all requirements are satisfied needs the comprehensive planning on the control strategy and a systematic operational consideration.

Efficient and optimum economic operation and planning of electric power systems have always been regarded as important target to be achieved in the modern power industry. In line with development of computer and information technology, Optimal Power Flow (OPF) has been made available to deal with this challenge. Although, OPF was first discussed by Carpentier in 1962, its development is still on-going stretching to various areas like optimal economic dispatching, preventive dispatching, corrective dispatching, optimum setting for control parameters, network planning and so on.

After the concept of FACTS emerged, link up of FACTS to OPF is logical as reviewed from research output [22,23,24,38,55,65]. Series compensations and phase shifters are incorporated in traditional OPF in [22,23]. Optimal location of series capacitor or phase shifter is reported in [65], but the model is a bit immature for practical application. Their concern is mainly on making investment decision for employing the series capacitor or phase shifter and the load flow equations is only taken as constraints. Their representation is given from the work of [22]. An Optimal Reactive Power Planning (ORPP) problem covering some FACTS devices like TCSC and SVC is addressed in [55]. Its objective function takes into account of the network real power loss and the constraints are load flow equations and variables limitation. A genetic algorithm is used to solve the OPF problems with UPFC in [38]. In that paper, a UPFC model is derived as a transformer with complex turn ratio. It has a drawback that the active power balance as constraint of the
UPFC itself is neglected which is basically unacceptable. H. Ambriz-Perez et al contributes to incorporate UPFC into OPF in [24]. Also, all the studies except [65] assume that location of the devices is well defined.

This chapter proposes a systematic method to determine optimal location of the UPFCs to be installed in power systems meeting usual objectives like load demands and operational constraints, such as maximum line flows, bus voltage levels and so on. Mathematically this type of problem is treated as an OPF problem. Under the assumption that the generator active power outputs (except slack bus) are held constant, the network loss and the capital investment are incorporated as the objectives making the problem to be extended as Optimal Reactive Power Planing (ORPP). Various aspects of concern are discussed in the following.

• Objective function
The capital investment of the devices and the operating cost of the power system are the two basic factors that should be concurrently tackled in the objective function. The optimal decision is a trade off between these two factors.

• Control variables
The UPFC parameters, $U_r$, $\phi_r$ and $I_q$, are all taken as control variables. A basic criterion is that these parameters should be co-ordinated with those of the traditional control devices like AVR, transformer turn ratios etc.

• Constraints
Besides the traditional equality and inequality constraints, the steady state security constraints are also included.

• Operating conditions
A number of typical operating conditions rather than a specific one should be considered in the process for finding optimal location of the UPFCs.
Chapter 5 Optimising Location of UPFC

- **Solution method**

  Augmented Lagrange Multipliers method is adopted to convert the large-scale non-linear optimisation problem with both equality and inequality constraints into an unconstrained minimisation problem. Since convergence of the multipliers can usually be attained without the need to increase the penalty parameter to infinity thereby the method alleviates the ill-conditioning problem plagues from the Sequential Unconstrained Minimisation Technique (SUMT)[51]. The BFGS (Broyden-Flech-Goldfarb-Shanno) method [58] is used to find the solutions of the unconstrained problem.

  The detailed discussion for above points will be carried out in following context. Case studies on the standard IEEE 14-bus system shows that the method can be implemented successful and it is effective for determining optimal location of the UPFCs and for improving steady state performance of the power systems.

5.2 Modelling of UPFCs for optimising their location

The schematic layout and equivalent circuit of the UPFC have already been given by Fig.1-2 and Fig. 2-2 respectively. The load flow equations of power systems with UPFCs have been established in chapter 2.

5.2.1 Objective Function

Costing consideration of UPFC involves a trade-off between its financial cost on capital investment and marginal saving on network losses. A weighting factor, $\gamma$, defined as an investment equivalent of the active power loss in the network is used to make a compromise between these two objectives.

Essentially, a UPFC should be designed with its shunt inverter rating capacity large enough to supply the reactive current $I_q$ for upholding the bus voltage and the active current $I_f$ for satisfying requirement of the series voltage source $U_f$. For the series inverter, its rating is defined by the thermal limit of the transmission line. Hence, the cost of the UPFC can be regarded as proportional to its rating which is related to the rated voltage and
maximum current of the associated transmission line. From Fig.1-2 and the mathematical modeling equations from (2-6) to (2-8), the apparent power of the shunt inverter $S_e$ can be obtained as:

$$S_e = U_p(I_i + I_q)^* = U_p I_i + j U_p I_q = \text{Re}(U_p I_i^*) + j U_p I_q$$  \hspace{1cm} (5-1)

Since $\Phi_T$ can vary from zero to $2\pi$, the real power passing through the series inverter is at the maximum value regardless of the phase angle of $I_i$. The capacity of the shunt component of the UPFC, $|S_e|$, can be obtained as:

$$|S_e| = \sqrt{(I_{i_{\text{max}}} U_{T_{\text{max}}})^2 + I_{q_{\text{max}}}^2}$$  \hspace{1cm} (5-2)

By the same token, the capacity of the series component of the UPFC $|S_b|$ is:

$$|S_b| = |I_{q_{\text{max}}} U_{T_{\text{max}}}| = I_{q_{\text{max}}} U_{T_{\text{max}}}$$  \hspace{1cm} (5-3)

From (5-2) and (5-3), the parameter $U_{T_{\text{max}}}$ is seen to be mutually independent to $I_{q_{\text{max}}}$ as they are not related. Furthermore, it can be seen from (5-2) and (5-3) that the MVA rating of the shunt inverter is larger than that of the series inverter. In fact, $I_{q_{\text{max}}}$ represents the shunt reactive compensation function of the UPFC and $U_{T_{\text{max}}}$ together with $\Phi_T$ represent the respective shift of voltage and phase angle function of the UPFC. Hence, values denoted by the parameters $U_{T_{\text{max}}}$ and $I_{q_{\text{max}}}$ can be used to describe not only the capacity but also the function of the UPFC. Thereby the installation cost of the UPFC can be divided into the following main three parts:

$$C_{\text{UPFC}} = \sum_{i=1}^{n_u} \left( F_i + \alpha_i U_{T_{\text{max}}} + \beta_i I_{q_{\text{max}}} \right)$$  \hspace{1cm} (5-4)

where $n_u$ is the total number of lines involved in the UPFCs installation and the subscript variable $i$ represents the $i$th UPFC. The first part, $F_i$, is the installation cost mainly determined by some non-electric factors and is taken as constant here. The second part
represents the cost relating to the capacity of the series inverter and the coefficient $c_l$ ($S\times$kA/MVA) represents the per unit voltage cost determined by the maximum current of the transmission line. The third part represents the cost relating to the additional shunt reactive compensation capacity in the shunt inverter and $\beta_l$ ($S\times$kV/MVA) is per unit current cost determined by the rated voltage of the transmission line.

From the network planning point of view, it is not meaningful to assume that selected location and capacity of UPFCs are confined to one operating condition in the power system. In this research multi-operating-condition can be simultaneously taken into consideration. The cost of the network loss is obtained as:

$$C_{\text{LOSS}} = \sum_{k=1}^{N_c} \gamma^{(k)} \sum_{i=1}^{N} \left[ P_y^{(i)} + P_y^{(k)} \right]$$  \hspace{1cm} (5-5)

where $N_c$ is the total number of the system operating conditions and the superscript $k$ denotes the $k$th operating condition. $\gamma^{(k)}$ ($S/MW$) is the investment equivalent of the active power loss of the network and $P_y^{(k)} + P_y^{(k)}$ is the real power consumed by the $l$th branch of which one end is labelled by $i$ and the other end by $j$ under the $k$th operating condition. There are $n_l$ branches in the power system.

From the above analysis, the objective function can be summarised in the form of

$$C(X) = C_{\text{UPFC}} + C_{\text{LOSS}}$$  \hspace{1cm} (5-6)

where $X$ is the vector consisting of the decision variables to be defined in the following context.

### 5.2.2 Decision Variables

UPFC can make the power system control more flexible because it introduces more degree of control dimensions but these can also make the co-ordination with existing control means more complicated. In order to co-ordinate UPFC with traditional control means, a
typical overall control decision vector (5-7) including not only the parameters of the UPFCs but also those of the conventional control devices. In the optimal reactive power planing (ORPP) type of problem, the active power generation of each generator except that at slack bus is given. The transformers are differentiated as Load Tap Changing (LTC) and the Unload Tap Changing (UTC) since multi-operating-condition is concerned.

\[
X = [U_{T_{\text{max}}}, I_{q_{\text{max}}}, T^T, Q_G^{(1)}^T, U_T^{(1)}^T, I_q^{(1)}^T, \Phi_T^{(1)}^T, \ell^{(1)}^T, U^{(1)}^T, Q_G^{(2)}^T, U_T^{(2)}^T, I_q^{(2)}^T, \Phi_T^{(2)}^T, \ell^{(2)}^T, U^{(2)}^T, \ldots, Q_G^{(K)}^T, U_T^{(K)}^T, I_q^{(K)}^T, \Phi_T^{(K)}^T, \ell^{(K)}^T, U^{(K)}^T, \ldots, Q_G^{(\nu)}^T, U_T^{(\nu)}^T, I_q^{(\nu)}^T, \Phi_T^{(\nu)}^T, \ell^{(\nu)}^T, U^{(\nu)}^T]^T
\]

where sub-vectors

\[
U_{T_{\text{max}}} = [U_{T_{1_{\text{max}}}}, U_{T_{2_{\text{max}}}}, \ldots, U_{T_{n_{\text{max}}}}] ^T
\]

\[
I_{q_{\text{max}}} = [I_{q_{1_{\text{max}}}}, I_{q_{2_{\text{max}}}}, \ldots, I_{q_{n_{\text{max}}}}] ^T
\]

contain the nominal parameters of the UPFCs. Here, \(U_{TB}\) is set as a benchmark to judge whether a UPFC is necessary or not, i.e. if \(U_{T_{\text{max}}} < U_{TB}\), it means the \(l\)th UPFC is not required to be installed. Hence, the initial decisions to install a UPFC or not is changed from a 0-1 programming type of problem to one involving continuous variables. By the
same token, the discrete variables, turn ratios of transformers, can be considered simply as continuous variables as denoted in the following.

\[ T = [T_1, T_2, ..., T_{n_T}]^T \]

\( T \) is the vector of turn ratios of the UTC transformers and \( n_T \) is their total number.

\[ \ell^{(k)} = [\ell_1^{(k)}, \ell_2^{(k)}, ..., \ell_{n_t}^{(k)}]^T, \quad k=1,2, ..., n_c. \]

\( \ell^{(k)} \) is the vector of turn ratios of the LTC transformers under the \( k \)th operating condition and \( n_t \) is the total number of the LTC transformers. It is worth to note that \( T \) is independent of the change of operating conditions and it is suitable for all different operating conditions but \( \ell^{(k)} \) is regulated following the operating condition changing.

Similar treatment applies to the reactive power generation \( Q_G^{(k)} \), the actual operating parameters of the UPFCs \( U_T^{(k)}, I_q^{(k)}, \Phi_T^{(k)}, (k=1,2, ..., n_c) \) except that they may have different dimension when compared with that of \( \ell^{(k)} \).

Lastly, the vector \( U^{(k)} \) consists of the nodal voltage magnitudes and phase angles but not those for the phase angle of the slack bus voltage as \( \delta_1^{(k)} = 0 \).

\[ U^{(k)} = [U_1^{(k)}, U_2^{(k)}, \delta_1^{(k)}, U_3^{(k)}, \delta_2^{(k)}, ..., U_{n}^{(k)}, \delta_{n}^{(k)}]^T \quad k=1,2, ..., n_c \]

5.2.3 Constraints

By applying equal constraints to the load flow equations of the UPFCs controlled power system, the following equations can be obtained:

\[ P_{g}^{(i)} - P_{L}^{(i)} = \sum_{j=1}^{n} \left( U_{j}^{(i)} U_{j}^{(i)} \left[ G_{ij}^{(i)} \cos \delta_{ij}^{(i)} + B_{ij}^{(i)} \sin \delta_{ij}^{(i)} \right] + \check{P}_{j}^{(i)} \right) \]

\[ i = 2,3,...,n; \quad k = 1,2, ..., n_c \]  

(5-8)
\( Q_{L_i}^{(k)} = \sum_{j=1}^{n} \left\{ U_{j}^{(k)}U_{j}^{(k)}[G_{ij}^{(k)} \sin \delta_{ij}^{(k)} - B_{ij}^{(k)} \cos \delta_{ij}^{(k)}] + \bar{Q}_{ij}^{(k)} \right\} \)

\[ i = 1, 2, \ldots, n; \quad k = 1, 2, \ldots, n_c \quad (5-9) \]

where the supplementary items \( \bar{Q}_{ij}^{(k)} \) and \( \bar{Q}_{ij}^{(k)} \) result from embedding the \( l \)th UPFC in the transmission line with subscripts \( i \) and \( j \) denoting its two ends and they are detailed derived in chapter 2. In this research, all UPFCs are assumed that the shunt transformer is directly connected at a bus. That is:

\[ \bar{P}_{ij}^{(k)} = \]
\[ \left\{ -U_{j}^{(k)}U_{ji}^{(k)}[G_{ij}^{(k)} \cos \delta_{ij}^{(k)} - \Phi_{ji}^{(k)}] - B_{ij}^{(k)} \sin \delta_{ij}^{(k)} - \Phi_{ji}^{(k)}] + (G_{ij}^{(k)} + g_{ij}) U_{ji}^{(k)} \left[ U_{ji}^{(k)} + 2U_{ji}^{(k)} \cos \delta_{ij}^{(k)} - \Phi_{ji}^{(k)}] \right] \right\} \]
shunt transformer is connected at node \( i \).

\[ \left\{ -U_{j}^{(k)}U_{ji}^{(k)}[G_{ij}^{(k)} \cos \delta_{ij}^{(k)} - \Phi_{ji}^{(k)}] + B_{ij}^{(k)} \sin \delta_{ij}^{(k)} - \Phi_{ji}^{(k)} \right\} \]
shunt transformer is connected at node \( j \).

\[ \bar{Q}_{ij}^{(k)} = \]
\[ \left\{ U_{j}^{(k)}U_{ji}^{(k)} \left[ (G_{ij}^{(k)} + g_{ij}) \sin \delta_{ij}^{(k)} - \Phi_{ji}^{(k)}] - (B_{ij}^{(k)} + b_{ij}) \cos \delta_{ij}^{(k)} - \Phi_{ji}^{(k)} \right] - U_{ji}^{(k)}I_{ji}^{(k)} \right\} \]
shunt transformer is connected at node \( i \).

\[ \left\{ -U_{j}^{(k)}U_{ji}^{(k)} \left[ G_{ij}^{(k)} \sin \delta_{ij}^{(k)} - \Phi_{ji}^{(k)} - B_{ij}^{(k)} \cos \delta_{ij}^{(k)} - \Phi_{ji}^{(k)} \right] \right\} \]
shunt transformer is connected at node \( j \).

(5-10)

Other operational system constraints including maximum line current, bus voltage levels, limits of reactive power generation, ratios of tap changers and operating region of each UPFC are:

\[ U_{max} \geq 0 \quad (5-11) \]
\[ I_{q_{\text{max}}} \geq 0 \]  
(5-12)

\[ T_{\text{min}} \leq T \leq T_{\text{max}} \]  
(5-13)

\[ 0 \leq U_{T}^{(k)} \leq U_{T_{\text{max}}} \]  
(5-14)

\[-I_{q_{\text{max}}} \leq I_{q}^{(k)} \leq I_{q_{\text{max}}} \]  
(5-15)

\[ t_{\text{min}} \leq t^{(k)} \leq t_{\text{max}} \]  
(5-16)

\[ Q_{G_{\text{min}}}^{(k)} \leq Q_{G}^{(k)} \leq Q_{G_{\text{max}}}^{(k)} \]  
(5-17)

\[ U_{\text{min}}^{(k)} \leq U^{(k)} \leq U_{\text{max}}^{(k)} \]  
(5-18)

\[ U_{s_{\text{min}}}^{(k)} \leq U_{s}^{(k)} \leq U_{s_{\text{max}}}^{(k)} \]  
(5-19)

\[ I_{\text{min}}^{(k)} \leq I^{(k)} \leq I_{\text{max}}^{(k)} \]  
(5-20)

\[ k = 1,2,\ldots,n_c \]

where vector \( I^{(k)} \) consists of all branch currents whereas the constraints (5-19) stem from the operational principle of the UPFC [5]. Hence,

\[ U_s^{(k)} = \left[ U_{11}^{(k)}, U_{12}^{(k)}, \ldots, U_{1v}^{(k)}, \ldots, U_{1v_s}^{(k)} \right]^T \]

denotes the voltage of the point \( s \) in Fig.2-6 and the one corresponding to the \( l \)th UPFC is obtained as:

\[ U_{v_l}^{(k)} = \sqrt{(U_{1v_l}^{(k)})^2 + (U_{2v_l}^{(k)})^2 + 2U_{1v_l}^{(k)}U_{2v_l}^{(k)}\cos(\Delta^{(k)} - \Phi_{1v_l}^{(k)})} \]  
(5-21)
5.3 Solution Method

For improvement of computational efficiency, inequality constraints (5-11) to (5-20) are grouped into one-sided and two-sided inequalities. Due to that $U_{T_{\text{max}}}$ and $I_{q_{\text{max}}}$ are not constant, the two-sided inequalities (5-14) and (5-15) can be separated into two one-sided constraints:

\[
U_I^{(k)} \geq 0 \quad (5-22)
\]

\[
U_{T_{\text{max}}} - U_{I}^{(k)} \geq 0 \quad (5-23)
\]

\[
I_{q_{\text{max}}} + I_{q}^{(k)} \geq 0 \quad (5-24)
\]

\[
I_{q_{\text{max}}} - I_{q}^{(k)} \geq 0 \quad (5-25)
\]

Besides, (5-11), (5-12) and (5-22) to (5-25) are all one-sided constraints whilst (5-13) and (5-16) to (5-20) are all two-sided constraints. For convenience of illustration, the problem of optimising the location and size of the UPFCs can be rewritten as follows:

Objective: \quad \min C(\mathcal{X})

Subject to: \quad h_j(\mathcal{X}) = 0 \quad j = 1, 2, \ldots, n_h

\quad g_j(\mathcal{X}) \geq 0 \quad j = 1, 2, \ldots, n_g

\quad e_{jm} \leq e_j(\mathcal{X}) \leq e_{jM} \quad j = 1, 2, \ldots, n_e

where $h_j(\mathcal{X})$, $g_j(\mathcal{X})$ and $e_j(\mathcal{X})$ represent equality, one-sided inequality and two-sided inequality constraints whilst $e_{jm}$ and $e_{jM}$ are the lower and higher limits of the $j$th two-sided constraint.
The augmented Lagrange function can be formed as:

\[
L(X, \Lambda, M, \Omega, \sigma) = 
C(X) - \sum_{j=1}^{n} \left[ \lambda_j h_j(X) - \frac{\sigma}{2} h_j^2(X) \right] + \sum_{j=1}^{n} \mu_j \left[ \max(0, \omega_j - \sigma g_j(X)) \right]^2 - \omega_j^2 
\]

where \( \mu_j = \)

\[
\begin{align*}
\frac{\sigma}{2} \left[ e_j(X) - e_{jm} \right]^2 - \mu_j \left[ e(X) - e_{jm} \right] & \quad \text{if } \mu_j + \sigma \left[ e_{jm} - e_j(X) \right] < 0 \\
\frac{\sigma}{2} \left[ e_j(X) - e_{jm} \right]^2 - \mu_j \left[ e(X) - e_{jm} \right] & \quad \text{if } \mu_j + \sigma \left[ e_{jm} - e_j(X) \right] > 0 \\
- \frac{\mu_j^2}{2\sigma} & \quad \text{otherwise}
\end{align*}
\]

(5-26)

The vectors \( \Lambda, M, \) and \( \Omega \) consisting of \( \lambda_j, \mu_j \) and \( \omega_j \) are the Lagrange multipliers for the equality, two-sided inequality and one-sided inequality constraints respectively. \( \sigma \) represents the penalty parameter.

The iteration procedure of

\[
\min \ L(X, \Lambda^{(k)}, M^{(k)}, \Omega^{(k)}, \sigma^{(k)})
\]

(5-27)

is performed by the BFGS method and multiplier sequence \( \{ \Lambda^{(k)}, M^{(k)}, \Omega^{(k)} \} \) are generated by (5-28) to (5-30).

\[
\lambda_j^{(k+1)} = \lambda_j^{(k)} - \sigma^{(k)} \times h(X^{(k)}) \quad j = 1, 2, ..., n_n
\]

(5-28)

\[
\mu_j^{(k+1)} = \max\left[ 0, \mu_j^{(k)} - \sigma^{(k)} \times g_j(X^{(k)}) \right] \quad j = 1, 2, ..., n_g
\]

(5-29)
\[ \omega_j^{(k+1)} = \begin{cases} \omega_j^{(k)} - \sigma^{(k)} [e_j(X^{(k)}) - e_{jm}] & \text{if } \omega_j^{(k)} - \sigma^{(k)} [g_j(X^{(k)}) - e_{jm}] < 0 \\ \omega_j^{(k)} - \sigma^{(k)} [e_j(X^{(k)}) - e_{jm}] & \text{if } \omega_j^{(k)} - \sigma^{(k)} [g_j(X^{(k)}) - e_{jm}] > 0 \\ 0 & \text{otherwise} \end{cases} \]

\[ j=1,2,\ldots,n_e \]  

(5-30)

Convergence of the iteration can be determined by monitoring the residual vector \( R \) defined in (5-31) to (5-33).

\[ r_{hj} = h_j(X^{(k)}) \quad \text{for equality constraint } j \]  

(5-31)

\[ r_g = g_j(X^{(k)}) - \max[0, \, g_j(X^{(k)}) - \omega_j^{(k)}/\sigma^{(k)}] \quad \text{for one-sided constraint } j \]  

(5-32)

\[ r_u = \begin{cases} e_j(X^{(k)}) - e_{jm} & \text{if } \mu_j^{(k)} - \sigma^{(k)} [e_j(X^{(k)}) - e_{jm}] \\ e_j(X^{(k)}) - e_{jm} & \text{if } \mu_j^{(k)} - \sigma^{(k)} [e_j(X^{(k)}) - e_{jm}] \\ \mu_j^{(k)}/\sigma^{(k)} & \text{otherwise} \end{cases} \]  

(5-33)

Iteration procedure:

**Step 1:** Perform the following setting:
- residual tolerance, \( \varepsilon > 0 \);
- threshold of amplifying penalty parameter, \( \eta \in (0,1) \);
- gain of the penalty parameter, \( \rho > 0 \);
- assign initial values: \( X^{(0)}, A^{(0)}, M^{(0)}, S^{(0)} \text{ and } \sigma^{(0)} \).
- \( k = 1 \).

**Step 2:** Start from the point \( X^{(k-1)} \). Use the BFGS method to solve the unconstrained minimisation problem (5-27) and obtain the minimizer \( X^{(k)} \).
Step 3: Calculate $\|R^{(k)}\|_\infty$ by (5-33) to (5-35). If $\|R^{(k)}\|_\infty < \varepsilon$, stop the procedure and $X^{(k)}$ is the solution value, otherwise go to step 4;

Step 4: if $\|R^{(k)}\|_\infty/\|R^{(k-1)}\|_\infty > \eta$ then $\sigma^{(k+1)} = \rho \sigma^{(k)}$; else, $\sigma^{(k+1)} = \sigma^{(k)}$.

Step 5: update all multipliers by (30), (31) and (32). $k \leftarrow k + 1$, and repeat step 2.

The flow chart of the BFGS method is given in Appendix III.

5.4 Case Study

As a validation of the proposed approach, case studies are performed using the IEEE 14-bus system with details as described in the following context.

5.4.1 Loading conditions

Fig.5-1 shows the standard IEEE 14-bus system in which UPFCs are planning to be installed. Two very heavy load conditions are given in Table 5-1 for the case study. The last row of the Table 5-1 is the summation of all the load power except those at bus 3, 4 and 5, which must pass through the branch 4-7, 4-9, and 5-6. The total current is approximately 1.693 and 1.619 respectively under the operating conditions 1 and 2, which are close to the thermal limit (1.710) of the network. From the load flow without UPFC results as given in Table 5-2 and Table 5-3, the reactive power output of the generator at the slack bus exceeds its limit under condition 1. The synchronous compensators exceed their maximum capacity and the transformer between bus 5 and 6 is seriously overloaded.

5.4.2 Preliminary choice of UPFCs

By means of the simple rules:

- no more than one UPFC is required to be installed in one branch or at one bus;
• UPFC is not required in the transmission line whose impedance is relatively small (too short);

• UPFC is not required in the line that can be conveniently controlled by means of generator or synchronous compensator;

seven lines are identified as eligible for installing UPFCs. They are marked as small shadow ovals and identified by serial number in Fig.5-1 and having price values given in Table5-4.

Table 5-1. Load demand

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>( P^{(1)} )</th>
<th>( Q^{(1)} )</th>
<th>( P^{(2)} )</th>
<th>( Q^{(2)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.942</td>
<td>0.190</td>
<td>0.942</td>
<td>0.190</td>
</tr>
<tr>
<td>4</td>
<td>0.478</td>
<td>-0.039</td>
<td>0.478</td>
<td>-0.039</td>
</tr>
<tr>
<td>5</td>
<td>0.076</td>
<td>0.016</td>
<td>0.076</td>
<td>0.016</td>
</tr>
<tr>
<td>6</td>
<td>0.300</td>
<td>0.075</td>
<td>0.012</td>
<td>0.075</td>
</tr>
<tr>
<td>9</td>
<td>0.295</td>
<td>0.166</td>
<td>0.700</td>
<td>0.166</td>
</tr>
<tr>
<td>10</td>
<td>0.009</td>
<td>0.058</td>
<td>0.250</td>
<td>0.058</td>
</tr>
<tr>
<td>11</td>
<td>0.200</td>
<td>0.020</td>
<td>0.035</td>
<td>0.018</td>
</tr>
<tr>
<td>12</td>
<td>0.200</td>
<td>0.016</td>
<td>0.061</td>
<td>0.016</td>
</tr>
<tr>
<td>13</td>
<td>0.400</td>
<td>0.058</td>
<td>0.250</td>
<td>0.058</td>
</tr>
<tr>
<td>14</td>
<td>0.149</td>
<td>0.050</td>
<td>0.250</td>
<td>0.050</td>
</tr>
<tr>
<td>Sum</td>
<td>1.634</td>
<td>0.443</td>
<td>1.558</td>
<td>0.441</td>
</tr>
</tbody>
</table>

Table 5-2 The generator output before system is equipped with UPFC

<table>
<thead>
<tr>
<th>BUS</th>
<th>( P_G^{(1)} )</th>
<th>( Q_G^{(1)} )</th>
<th>( P_G^{(2)} )</th>
<th>( Q_G^{(2)} )</th>
<th>( Q_{max} )</th>
<th>( Q_{min} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.470</td>
<td>-0.5172</td>
<td>2.055</td>
<td>-0.4492</td>
<td>1.45</td>
<td>-0.45</td>
</tr>
<tr>
<td>2</td>
<td>0.800</td>
<td>0.0977</td>
<td>1.200</td>
<td>-0.0665</td>
<td>0.50</td>
<td>-0.40</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>0.4885</td>
<td>0.000</td>
<td>0.4887</td>
<td>0.40</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>0.8439</td>
<td>0.000</td>
<td>0.9644</td>
<td>1.00</td>
<td>-0.30</td>
</tr>
<tr>
<td>6</td>
<td>0.000</td>
<td>0.5132</td>
<td>0.000</td>
<td>0.3599</td>
<td>0.24</td>
<td>0.06</td>
</tr>
<tr>
<td>8</td>
<td>0.000</td>
<td>0.0814</td>
<td>0.000</td>
<td>0.1388</td>
<td>0.24</td>
<td>0.06</td>
</tr>
</tbody>
</table>

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5.4.3 Initial Setting and Penalty Parameters

All turn ratios are initially set as 1.0. The active power output of the generator at bus 2 is 1.2 and 0.8 respectively under the operating conditions 1 and 2. The active power output of the generator at the slack bus and the reactive power output of all the generators and synchronous compensators are decision variables. Their initial values are determined from (5-8) and (5-9) with all bus voltages set at 1.0 p.u. and zero phase angle. All other variables associated with the UPFC such as $U_{T_{line}}$, $j_{q_1}$, $\phi_{f_4}$ are assigned with small initial value, 0.01. The penalty parameters are set according to a clustered constraint concept to assure reasonable convergence for the iteration process. There are two extreme ways of allocating the penalty value to the constraints, i.e., with one same penalty parameter to all constraints and with each constraint having its own penalty parameter. Numerical computation experience indicates that neither of these two is practical as the former makes no differentiation among the constraints and makes convergence difficult whilst the latter appears to be too tedious and not effective for implementation. A compromised approach, as proposed in this chapter, is to classify the constraints (5-8) to (5-25) into eight categories.
Table 5-3 The branch current before system is equipped with UPFC

<table>
<thead>
<tr>
<th>i</th>
<th>j</th>
<th>$I_{y}^{(1)}$</th>
<th>$I_{y}^{(1)_{ij}}$</th>
<th>$I_{y}^{(2)}$</th>
<th>$I_{y}^{(2)_{ij}}$</th>
<th>$I_{\text{limit}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1.555</td>
<td>1.547</td>
<td>1.219</td>
<td>1.211</td>
<td>3.42</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>.929</td>
<td>.925</td>
<td>.852</td>
<td>.847</td>
<td>1.71</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>.843</td>
<td>.838</td>
<td>.867</td>
<td>.861</td>
<td>1.71</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>.793</td>
<td>.787</td>
<td>.841</td>
<td>.835</td>
<td>1.71</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>.644</td>
<td>.641</td>
<td>.658</td>
<td>.654</td>
<td>1.71</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>.131</td>
<td>.137</td>
<td>.107</td>
<td>.114</td>
<td>1.71</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>.716</td>
<td>.720</td>
<td>.826</td>
<td>.830</td>
<td>1.71</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>.435</td>
<td>.435</td>
<td>.587</td>
<td>.587</td>
<td>.65</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>.252</td>
<td>.252</td>
<td>.341</td>
<td>.341</td>
<td>.40</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>.892</td>
<td>.892</td>
<td>.666</td>
<td>.666</td>
<td>.65</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>.120</td>
<td>.120</td>
<td>.205</td>
<td>.205</td>
<td>.50</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>.200</td>
<td>.200</td>
<td>.121</td>
<td>.121</td>
<td>.50</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
<td>.370</td>
<td>.370</td>
<td>.349</td>
<td>.349</td>
<td>.50</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>.080</td>
<td>.080</td>
<td>.136</td>
<td>.136</td>
<td>.50</td>
</tr>
<tr>
<td>7</td>
<td>9</td>
<td>.450</td>
<td>.450</td>
<td>.620</td>
<td>.620</td>
<td>.65</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>.175</td>
<td>.175</td>
<td>.099</td>
<td>.099</td>
<td>.50</td>
</tr>
<tr>
<td>9</td>
<td>14</td>
<td>.217</td>
<td>.217</td>
<td>.122</td>
<td>.122</td>
<td>.50</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>.185</td>
<td>.185</td>
<td>.166</td>
<td>.166</td>
<td>.50</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>.023</td>
<td>.023</td>
<td>.058</td>
<td>.058</td>
<td>.50</td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>.093</td>
<td>.093</td>
<td>.146</td>
<td>.146</td>
<td>.50</td>
</tr>
</tbody>
</table>

Table 5-4. Costing of UPFC

<table>
<thead>
<tr>
<th>UPFC No.</th>
<th>I</th>
<th>J</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1</td>
<td>5.0</td>
<td>10.0</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>14</td>
<td>2.5</td>
<td>6.0</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>11</td>
<td>3.0</td>
<td>6.0</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>6</td>
<td>3.5</td>
<td>7.5</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>6</td>
<td>4.0</td>
<td>7.0</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
<td>12</td>
<td>3.0</td>
<td>6.4</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>13</td>
<td>3.0</td>
<td>6.5</td>
</tr>
</tbody>
</table>

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• $\sigma_1$ corresponds to all equality constraints that are load flow equations
• $\sigma_2$ associates with the constraints included in (5-11), (5-12), (5-22) to (5-25) that are relate to the parameters of UPFCs.
• $\sigma_3$ corresponds to the two-side constraints for the turn ratios of UTC (5-13).
• $\sigma_4$ corresponds to the two-side constraints for the turn ratios of LTC (5-16).
• $\sigma_5$ corresponds to the two-side constraints for the reactive power generation (5-17).
• $\sigma_6$ corresponds to the two-side constraints for the magnitude of bus voltage (5-18).
• $\sigma_7$ corresponds to the two-side constraints for the magnitude of UPFC terminal voltage (5-19).
• $\sigma_8$ correspond to the two-side constraints for the branch current (5-20)

Based on above, when numeric experiment is carried out the trial for penalty parameters are very convenient. Because when computation cannot convergence the program can point out which constraint is violated and one can modulate corresponding penalty parameter.

A threshold of amplifying penalty parameter $\eta = 0.65$ and a gain $\rho = 1.5$ are adopted to escalate all the 8 penalty parameters in the iteration procedure.

It is specially emphasized that the initial value of the penalty parameters must be selected carefully. A good choice is very important since it can have a considerable effect on the performance of the entire iteration procedure. If the initial value of these penalty parameters and their escalating scheme during iteration are not appropriate the optimization process will not be converging. Actually the scheme of increasing the penalty parameters is empirical and with different value for different scheme. This is a unfortunate feature on all methods when coming to choice of the penalty parameters. The experience obtained in our case study indicates that increasing the penalty parameters intensely prone to have the iteration divergent.

5.4.4 Calculation Results

Having defined the residuary tolerance $\varepsilon = 1.0\cdot10^{-6}$, the convergence criterion of unconstrained optimisation (5-27) can be obtained as the maximal revision increment for
decision variable less than 1.0D-9, i.e. \( \|r_k d^{(k)}\|_{\infty} \leq 10^{-10} \), where \( r_k \) is the optimal steplength of the \( k \)-th line search and \( d^{(k)} \) is the search direction produced by the BFGS method.

In the first stage of the calculation, 129 variables and 267 Lagrange multipliers are involved. The initial penalty parameters \( \sigma_1 \) to \( \sigma_8 \) are respectively taken as 5.13, 1.0, 1.0, 0.9, 0.9849, 9.0, 1.0 and 2.0. \( \gamma^{(1)}=100.0 \) and \( \gamma^{(2)}=150.0 \). Detailed results are obtained as shown in Table 5-5. From the results, the fourth to the seventh UPFC candidates can be removed as judged by the benchmark \( U_{TB}=0.1 \).

In the second stage, there are three lines remaining as suitable candidates for installing UPFC. By means of similar procedure, testing results are obtained as shown in Table 5-6. Based on the same rationale, the first UPFC candidate can be eliminated.

Finally, results for the remaining two candidates in the third stage of calculation are shown in Table 5-7. It can be seen that the steady state performance of the system under the two typical operating conditions has been improved considerably. All steady state security constraints under the two heavy load demand operating conditions are satisfied. The attempt to eliminate the candidate \( 2^x \) is made but the computation cannot converge. From the results in Table 5-7, it is easy to understand the reason is due to the two operating conditions are too heavily loaded.

<table>
<thead>
<tr>
<th>UPFC No.</th>
<th>( U_{Tmax} )</th>
<th>( U^{(1)}_T )</th>
<th>( U^{(2)}_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.190871E+0</td>
<td>0.500156E-1</td>
<td>0.190839E+0</td>
</tr>
<tr>
<td>2</td>
<td>0.149578E+0</td>
<td>0.149572E+0</td>
<td>0.663625E-1</td>
</tr>
<tr>
<td>3</td>
<td>0.217217E+0</td>
<td>0.217206E+0</td>
<td>0.346181E-1</td>
</tr>
<tr>
<td>4</td>
<td>0.138602E-1</td>
<td>0.138594E-1</td>
<td>0.169282E-5</td>
</tr>
<tr>
<td>5</td>
<td>0.206699E-3</td>
<td>0.368003E-5</td>
<td>0.315420E-4</td>
</tr>
<tr>
<td>6</td>
<td>0.134300E-5</td>
<td>0.706669E-6</td>
<td>0.126479E-5</td>
</tr>
<tr>
<td>7</td>
<td>0.650529E-1</td>
<td>0.650475E-1</td>
<td>0.766320E-5</td>
</tr>
</tbody>
</table>

Table 5-5(a) The rated \( U_T \) and its operating values
Table 5-5(b) The rated $I_{q_{\text{max}}}$ and its operating values

<table>
<thead>
<tr>
<th>UPFC No.</th>
<th>$I_{q_{\text{max}}}$</th>
<th>$I_{q}^{(1)}$</th>
<th>$I_{q}^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.82311E-4</td>
<td>-0.74882E-5</td>
<td>0.44480E-4</td>
</tr>
<tr>
<td>2</td>
<td>0.18893E+0</td>
<td>0.11191E-1</td>
<td>0.18891E+0</td>
</tr>
<tr>
<td>3</td>
<td>0.68410E-1</td>
<td>0.41466E-1</td>
<td>0.68390E-1</td>
</tr>
<tr>
<td>4</td>
<td>0.82628E-4</td>
<td>-0.10587E-4</td>
<td>0.64778E-4</td>
</tr>
<tr>
<td>5</td>
<td>0.20934E-4</td>
<td>-0.19108E-4</td>
<td>-0.96457E-5</td>
</tr>
<tr>
<td>6</td>
<td>0.16954E-4</td>
<td>-0.14662E-4</td>
<td>-0.12413E-4</td>
</tr>
<tr>
<td>7</td>
<td>0.36590E-1</td>
<td>0.36582E-1</td>
<td>0.36444E-1</td>
</tr>
</tbody>
</table>

Table 5-5(c) The operating values of $\Phi_T$

<table>
<thead>
<tr>
<th>UPFC No.</th>
<th>$\Phi_T^{(1)}$</th>
<th>$\Phi_T^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8305940E+0</td>
<td>0.1231299E+0</td>
</tr>
<tr>
<td>2</td>
<td>0.2431459E+0</td>
<td>0.3373977E+0</td>
</tr>
<tr>
<td>3</td>
<td>0.5610281E+0</td>
<td>-0.2670248E+0</td>
</tr>
<tr>
<td>4</td>
<td>0.8086522E-02</td>
<td>-0.9291807E+0</td>
</tr>
<tr>
<td>5</td>
<td>-0.2897963E+0</td>
<td>-0.5720114E+0</td>
</tr>
<tr>
<td>6</td>
<td>0.1495045E+0</td>
<td>-0.7565298E+0</td>
</tr>
<tr>
<td>7</td>
<td>0.2490455E+0</td>
<td>-0.2898110E+0</td>
</tr>
</tbody>
</table>

Table 5-6(a) The rated $U_T$ and its operating values

<table>
<thead>
<tr>
<th>UPFC No.</th>
<th>$U_{T_{\text{max}}}$</th>
<th>$U_T^{(1)}$</th>
<th>$U_T^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.266645E-4</td>
<td>0.125525E-4</td>
<td>0.176881E-4</td>
</tr>
<tr>
<td>2</td>
<td>0.169822E+0</td>
<td>0.169822E+0</td>
<td>0.635865E-1</td>
</tr>
<tr>
<td>3</td>
<td>0.177436E+0</td>
<td>0.177434E+0</td>
<td>0.517794E-1</td>
</tr>
</tbody>
</table>

Table 5-6(b) The rated $I_{q_{\text{max}}}$ and its operating values

<table>
<thead>
<tr>
<th>UPFC No.</th>
<th>$I_{q_{\text{max}}}$</th>
<th>$I_{q}^{(1)}$</th>
<th>$I_{q}^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.528269E-5</td>
<td>0.499851E-5</td>
<td>0.44049E-5</td>
</tr>
<tr>
<td>2</td>
<td>0.221808E-4</td>
<td>0.103516E-4</td>
<td>0.20347E-4</td>
</tr>
<tr>
<td>3</td>
<td>0.153254E-4</td>
<td>-0.287982E-6</td>
<td>0.14128E-4</td>
</tr>
</tbody>
</table>

91
Table 5-6(c) The operating values of $\Phi_T$

<table>
<thead>
<tr>
<th>UPFC No.</th>
<th>$\phi^{(1)}_T$</th>
<th>$\phi^{(2)}_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1618977E+01</td>
<td>0.1544490E+01</td>
</tr>
<tr>
<td>2</td>
<td>0.6841444E+00</td>
<td>0.3285766E+00</td>
</tr>
<tr>
<td>3</td>
<td>0.7705608E+00</td>
<td>0.2445872E+00</td>
</tr>
</tbody>
</table>

Table 5-7(a) The rated $U_T$ and its operating values

<table>
<thead>
<tr>
<th>UPFC No.</th>
<th>$U_{T_{\text{max}}}$</th>
<th>$U^{(1)}_{T}$</th>
<th>$U^{(2)}_{T}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.159703</td>
<td>0.159696</td>
<td>0.0146584</td>
</tr>
<tr>
<td>3</td>
<td>0.229282</td>
<td>0.229265</td>
<td>0.0007469</td>
</tr>
</tbody>
</table>

Table 5-7(b) The rated $I_{q_{\text{max}}}$ and its operating values

<table>
<thead>
<tr>
<th>UPFC No.</th>
<th>$I_{q_{\text{max}}}$</th>
<th>$I^{(1)}_{q}$</th>
<th>$I^{(2)}_{q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.455585</td>
<td>0.455489</td>
<td>0.455584</td>
</tr>
<tr>
<td>3</td>
<td>0.152626</td>
<td>0.152623</td>
<td>0.152624</td>
</tr>
</tbody>
</table>

Table 5-7(c) The operating values of $\Phi_T$ and $U_s$

<table>
<thead>
<tr>
<th>UPFC No.</th>
<th>$\Phi^{(1)}$</th>
<th>$\Phi^{(2)}$</th>
<th>$U^{(1)}_s$</th>
<th>$U^{(2)}_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.381957</td>
<td>-1.162943</td>
<td>1.09839</td>
<td>1.02322</td>
</tr>
<tr>
<td>3</td>
<td>0.599629</td>
<td>-2.319909</td>
<td>1.09423</td>
<td>1.00816</td>
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</tbody>
</table>

Table 5-7(d) Reactive power output of generators and compensators

<table>
<thead>
<tr>
<th>BUS</th>
<th>$Q^{(1)}_G$</th>
<th>$Q^{(2)}_G$</th>
<th>$Q_{\text{Gmax}}$</th>
<th>$Q_{\text{Gmin}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.387621</td>
<td>0.017278</td>
<td>1.45</td>
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</tr>
<tr>
<td>2</td>
<td>-0.088378</td>
<td>-0.174428</td>
<td>0.50</td>
<td>-0.40</td>
</tr>
<tr>
<td>3</td>
<td>0.343322</td>
<td>0.176863</td>
<td>0.40</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>0.927928</td>
<td>0.572109</td>
<td>1.00</td>
<td>-0.30</td>
</tr>
<tr>
<td>6</td>
<td>0.107163</td>
<td>0.183666</td>
<td>0.24</td>
<td>0.06</td>
</tr>
<tr>
<td>8</td>
<td>0.060004</td>
<td>0.060003</td>
<td>0.24</td>
<td>0.06</td>
</tr>
</tbody>
</table>

92
### Table 5-7(e) The turn ratio

<table>
<thead>
<tr>
<th>Tap</th>
<th>LOW</th>
<th>TOP</th>
<th>$T^{(1)}$</th>
<th>$T^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-6</td>
<td>0.95</td>
<td>1.05</td>
<td>1.000206</td>
<td>1.000206</td>
</tr>
<tr>
<td>4-7</td>
<td>0.95</td>
<td>1.05</td>
<td>1.009315</td>
<td>1.005859 *</td>
</tr>
<tr>
<td>4-9</td>
<td>0.95</td>
<td>1.05</td>
<td>0.971114</td>
<td>1.002104 *</td>
</tr>
<tr>
<td>7-8</td>
<td>0.95</td>
<td>1.05</td>
<td>1.033784</td>
<td>1.048697 *</td>
</tr>
</tbody>
</table>

* means the transformer is LTC

### Table 5-7(f) The branch current

<table>
<thead>
<tr>
<th>I</th>
<th>J</th>
<th>$I_{lim}$</th>
<th>$I^{(1)}_y$</th>
<th>$I^{(1)}_j$</th>
<th>$I^{(2)}_y$</th>
<th>$I^{(2)}_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6</td>
<td>0.65</td>
<td>0.650</td>
<td>0.649</td>
<td>0.649</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>0.65</td>
<td>0.629</td>
<td>0.612</td>
<td>0.612</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>0.40</td>
<td>0.372</td>
<td>0.349</td>
<td>0.349</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>0.50</td>
<td>0.062</td>
<td>0.062</td>
<td>0.062</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>14</td>
<td>0.50</td>
<td>0.297</td>
<td>0.172</td>
<td>0.172</td>
<td></td>
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<tr>
<td>10</td>
<td>11</td>
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<td>0.392</td>
<td>0.169</td>
<td>0.169</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3.42</td>
<td>1.639</td>
<td>1.172</td>
<td>1.146</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>1.71</td>
<td>0.984</td>
<td>0.855</td>
<td>0.825</td>
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<tr>
<td>2</td>
<td>3</td>
<td>1.71</td>
<td>0.892</td>
<td>0.863</td>
<td>0.831</td>
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</tr>
<tr>
<td>2</td>
<td>4</td>
<td>1.71</td>
<td>0.898</td>
<td>0.862</td>
<td>0.848</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1.71</td>
<td>0.678</td>
<td>0.677</td>
<td>0.665</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>1.71</td>
<td>0.146</td>
<td>0.164</td>
<td>0.157</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>1.71</td>
<td>0.972</td>
<td>0.817</td>
<td>0.818</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>0.65</td>
<td>0.214</td>
<td>0.191</td>
<td>0.191</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>0.65</td>
<td>0.184</td>
<td>0.115</td>
<td>0.113</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>13</td>
<td>0.65</td>
<td>0.318</td>
<td>0.332</td>
<td>0.323</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>9</td>
<td>0.65</td>
<td>0.621</td>
<td>0.601</td>
<td>0.605</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>0.50</td>
<td>0.407</td>
<td>0.112</td>
<td>0.111</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>0.50</td>
<td>0.025</td>
<td>0.054</td>
<td>0.053</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>0.50</td>
<td>0.149</td>
<td>0.139</td>
<td>0.139</td>
<td></td>
</tr>
</tbody>
</table>
5.5 Summary

Determining optimal locations of UPFCs is a practical concern when comes to their implementation in modern power systems. This chapter presents a model which has been proved to be successful not only for identifying the optimal location but also determining the operating parameters of UPFCs under different operating conditions. Having included the static security constraints in the model, the proposed approach can effectively be used for determining the expansion of loadability in the existing power systems. As demonstrated by the case study performed, it shows that the UPFCs can help the system meet the heavy load demand. In conclusion, the envisaged model for optimising the location and determining the rated capacity of the UPFCs has been proved to be practicable and computational efficient. So this model can be used for both network planning and operation planning.
Chapter 6

ENHANCEMENT OF DYNAMIC STABILITY OF POWER SYSTEMS BY USING UPFC

6.1 Introduction

Enhancement of dynamic performance of power systems using UPFC is an important aspect to be addressed in this thesis. It includes discussion on development of control strategy for coordinating UPFC with other conventional controllers like AVR and PSS etc as described in the following context.

6.1.1 Background

The loading limit of large interconnected power systems is often degraded due to poor damping ability of the systems especially to low frequency oscillations. Enhancing the dynamic stability of power systems has always been an interest in the power industry.

There are every sort of incentive including economic and security reasons for interconnecting power systems by means of transmission tie-lines. Low frequency oscillation in the tie-lines is not uncommon and traditional practical measure to face it is by having sufficient operational margin. The cost for doing so is in fact on reduction of the power transfer limit of the transmission system. Stability problem can also be imposed by this frequency oscillation and it may trigger a loss of synchronism of the power generators in the worse case. In the 1960s, large interconnected systems experienced growing oscillations that disrupted parallel operation of large systems [13,15]. It was discovered that the inherently weak natural damping of large and weakly coupled systems were the main cause. Moreover with installation of more and more modern fast response and large gain AVR (automatic voltage regulator) the situations of negative damping were further aggravated [3]. For the purpose of raising the transmission power ability of existent power systems, it is an important research area on how to combat growing oscillation and how to
enhance the system damping ability. The study for dynamic stability of power systems without FACTS controller is much mature when compared with that with FACTS controller. Thirty years ago, an instance of low frequency power system oscillations was reported in [50] and this damping instability incidence was corrected by reducing the gains of certain generator controllers, both governors and voltage regulators. In the years that followed, many researchers devoted great effort to this study [3,13,14,15,56,64,67] mainly by taking effect of the three types of controllers, namely, exciting, governor and power system stabilizer. The aim was to co-ordinate the controllers for increasing the dynamic stability margin as discussed in references [19,49].

The concept of FACTS provides a vigorous means to improve dynamic performance of modern power system. The study for the application of FACTS controller such as TCSC and SVC is bound to be popular continuously in the coming decade. For example, [12] discussed the analysis method for the TCSC that was installed in BPA systems. [2] discussed SVC proposed a method to increase the synchronizing and damping torque by appropriate controls. Using model analysis, [33] described a novel technique to calculate the electrical damping and synchronizing torque coefficients induced on generators through the action of FACTS stabilizers in multi-machine systems. A fundamental principle for designing FACTS controllers to damp power swing was proposed in [16] by which particular property of choosing control signal for FACTS controller was mentioned. In practical reality, the speed deviation of the machines of interest, or the control signal of PSS can readily be obtainable as these devices are next to the associated generators. In contrast with traditional PSS and AVR controllers, FACTS controllers are not situated close the generators and so short of inherent convenience in obtaining the signal associated with them. Hence, for a FACTS controller, it is desirable to make use of local signal available such as the energy function method [29] that presented a way to derive power oscillation damping control strategies for TCSC, SVC, STATCON and TCPS. All control strategies rely only on locally measurable information.

Effects of using UPFC for improving transient stability are presented in [60,61,62] and they have been proved to be significant in damping power system oscillation by employing its three dimensions of control, namely In-phase Voltage Control (IVC),
Quadrature Voltage Control (QVC) and Shunt Compensation (ShC). However, extension of the study to a highly nonlinear multi-machine system is still constrained by its need to reduce the power system as a single machine to infinite bus model. For dynamic stability study such as using TCSC and SVC to mitigate a torsional mode of oscillation, eigenvalue analysis is a popular method and widely employed [6,40,48,53,68,70]. But the study for using UPFC to enhance power system dynamic performance is only reported in [4] which is on a case to mitigate the torsional mode of power system oscillation. The research for using UPFC to enhance dynamic performance of power system is almost untouched.

6.1.2 Basic concept on stability due to small disturbance

Power system in operation has to be dynamically stable despite existence of all sort of small perturbation. From this point of view, the requirement of dynamic stability of power systems has to be more strictly observed than for transient stability of power systems. Transient stability depends not only on the cause but also on the post contingency control and response. Hence, power systems can be permitted to operate at a state that is transient unstable for a short period of time in abnormal operation condition but dynamic stability is the natural power system characteristic that does not depend of the type of fault. Moreover a lot of transient stability problem is resulted by weak damping capability on oscillations instead of first swing instability after critical fault. Therefore the dynamic stability of power system is the prerequisite for power system operation.

The several definitions related to dynamic stability are cited as below [26]

- **Steady State Operating Condition of a Power System**
  An operating condition of a power system in which all of the operating quantities that characterize it can be considered to be constant for the purpose of analysis.

- **Small Disturbance in a Power System**
  A disturbance in a power system is a sudden change or a sequence of change in one or more of the parameters of the system, or in one or more of the operating quantities. A
small disturbance is a disturbance for which the equations that describe the dynamics of the power system may be linearized for the purpose of analysis.

- **Steady State Stability of a Power System**
  A power system is steady state stable for a particular steady state operating condition if, following any small disturbance, it reaches a steady state operating condition which is identical or close to the pre-disturbance operating condition. This is also known as *Small Disturbance Stability of a Power System*.

- **Steady State Stability Limits**
  The steady state stability is a steady state operating condition for which the power system is stable under steady state but for which an arbitrarily small change in any of the operating quantities in an unfavorable direction causes the power system to lose stability. This is also known as the *Small Disturbance Stability Limit*.

### 6.1.3 Methodology of analysis on small disturbance stability of power system

In accordance with the definition of power system steady-state stability, the system oscillations have to be damped so as to allow the system to be asymptotically stable meaning that the system contains inherent forces to reduce oscillations. If there is enough inherent force to damp the oscillation following a small perturbation in power system, we say the power system is dynamically stable at this quiescent operating state as well. Otherwise, the system is unstable. But a statement declaring a power system at certain quiescent operating state to be “dynamic stable” is rather ambiguous unless their conditions are clarified first.

There are several analytical tools that have evolved and come out for the study of dynamic stability. These tools can commonly be distinguished as time domain and frequency domain analytical methods. In the time domain simulation (TDS) method, a set of nonlinear differential equations is established to describe the power systems and then directly find the solution of the equations by numerical integral technologies. Although along with the rapid development of digital computing technologies, it seems that the TDS...
method is more widely used, the computation work is still too large to analyze for large-scale power systems.

Since the dynamic stability is related to small disturbance, the system equations can be linearized about a quiescent operating state. Lyapunov's First Method defines the stability of the nonlinear system based on its linear approximation as:

A non-linear system is steady state stable in the vicinity of the equilibrium point if its linear approximation is asymptotically stable. If the linear approximation is unstable, then the nonlinear system is also unstable. If the linear approximation is stable, but not asymptotically stable, then it is impossible to assess the nonlinear system stability based on its linear approximation.

For a linear system, modern linear systems theory provides a means of evaluation of its dynamic response once a good mathematical model is developed. Eigenvalue analysis uses the standard linear, state-space form of systems equations and provides an appropriate tool for evaluating system conditions for the study of power system dynamic stability. Using the standard linear state space equation, making it possible to utilize many other analytical tools that use this same equation form. The eigenvalue analysis can compute all the exact modes of system oscillation in a single computation and can be arranged to perform a convenient parameter variation to study parameter sensitivity. It can also be used to plot root loci of eigenvalue movement in response to many different types of changes. The results of the eigenvalue calculation give both the frequency of oscillation and the damping of each frequency. Eigenvalue analysis also includes the computation of eigenvectors, which are very important quantities for analyzing the sensitivity of the control parameters in respect to each eigenvalue [52].

Based on the Lyapunov's asymptotic stability theory, i.e. all eigenvalues of the system should be positioned to the left hand half of the complex-plane as far as possible for higher stability margin, the optimization problem used in this chapter can be realized as:
Chapter 6 Enhancement of Dynamic Stability of Power Systems by Using UPFC

Objective: \( \sigma^* = \min_T \max_{i,j} [\sigma_i(T)] \) \quad i = 1,2,\ldots,n_e; \quad j = 1,2,\ldots,n_c

Subject to: \( c_L \leq T \leq c_U \) \quad (6-1)

Where the vector \( T \in \mathbb{R}^n \) comprises all the controllable parameters and constant vector \( c_L \) and \( c_U \) are the lower and upper limits of \( T \) respectively. In order to increase the robustness of the control parameters, the \( n_e \) operating conditions are simultaneously taken care of. Where \( n_e \) is the order of the system matrix in question. \( \sigma_i \) signifies the real part of the \( i \)th eigenvalue at the \( j \)th operating condition. Hence, it can be seen that the optimal solution of \( T^* \) minimizes the real part of the eigenvalue whose real part is the largest one in all the eigenvalues and in all the operating conditions concerned. In other words it forces all the eigenvalues to move toward the left hand half of the complex-plan at any operating condition \( j \). If \( \sigma^* \) is negative enough then the system is said to have ample dynamic stability margin.

A dynamic UPFC controlled power system model and its control functions are derived in section 6.2. The approach of finding the solution of the problem (6-1) is discussed in section 6.3 and the application case study is given in section 6.4.

6.2 Mathematical model of UPFC

Steady state mathematical model of UPFC can be directly cited without derivation as presented in the previous chapters. So far, there is no UPFC dynamic model perfectly developed and reported on the literature. A dynamic model of UPFC is derived in [4] but it can only be used in simple system at its present format since it is expressed by actual quantity and it cannot be directly incorporated into complex network. Hence a dynamic UPFC model is firstly derived in this section. For the convenience of illustration, Fig.6-1 and Fig.6-2 show respectively a schematic and a three-phase diagram of the UPFC developed in [4].
When sinusoid pulse width modulation (SPWM) scheme is adopted and only the fundamental frequency component under balanced operating conditions are concerned, the output voltage of the two converters of the UPFC can be equivalent to two ideal voltage sources [4]. Namely, $V_{Ea}, V_{Eb}, V_{Ec}, V_{Bb}, V_{Ba},$ and $V_{Bc}$ can be expressed by single phase.

\[ V_x = \frac{1}{2\sqrt{2}} m_x v_x \angle \delta_x \quad m_x \in [0,1] \]  \hspace{1cm} (6-2)

\[ V_b = \frac{1}{2\sqrt{2}} m_b v_x \angle \delta_b \quad m_b \in [0,1] \]  \hspace{1cm} (6-3)
where \( m_e \) and \( m_s \) denote the modulation index of the shunt and series converter respectively while \( \delta_e \) and \( \delta_s \) as the phase angles of the control wave.

In the quasi-steady-state condition, the relationship of the voltage with current in exciting transformer (ET), boosting transformer (BT) and in the valves of two side converters at the AC side of UPFC are algebraic equations. Only the equation of DC side is differential. Thus the algebraic equations can be directly written in the format of single phase. The Park transformation is used to transform the equation from three phase to single phase[4]. It is dispensable since the AC side of UPFC is really connected into the network of power system and the element UPFC is a static rather than a rotating element. From the above analysis and seeing also Fig.6-2, the equations of UPFC can be written as follows:

\[
(r_e + j\omega_l_e)I_e = V_e - V_{\delta_e} \tag{6-4}
\]

\[
(r_s + j\omega_l_s)I_s = V_s - V_{\delta_s} \tag{6-5}
\]

\[
c_{de} V_{dc}\frac{dv_{dc}}{dt} = 3\text{Re}[V_e I_e^* - V_s I_s^*] \tag{6-6}
\]

where \( r_e \) and \( r_s \) are the equivalent resistance of the transformer and the switch-on state valve conduction losses in the exciting and boosting sides respectively while \( \omega l_e \) and \( \omega l_s \) are the respective equivalent leakage reactance of the ET and BT.

All variables of the above equations are actual value and are referred to the side of the DC capacitor. Under steady state condition, \( 3\text{Re}[V_e I_e^* - V_s I_s^*] = C \) and the direct voltage of the DC capacitor keeps constant. Thus the steady state equivalent circuits of the UPFC is depicted in Fig. 6-3.
In order to convert the mathematical model (6-4), (6-5) and (6-6) into per unit form, related base quantities should be determined first.

First of all, it is worthy to note that the turn ratio of ET and BT should not be equal. There are two functions in the shunt branch of UPFC. The one offers active power to balance the active power transmitted by the series branch of the UPFC and the other one provides reactive power to support the bus voltage. From (6-4) and Fig. 6-1 the phasor diagram of $V_E^\cdot$ (referred to the AC side) can be drawn as Fig. 6-4.

Assume that $V_E^\cdot_{Et}$ is upheld at constant by the UPFC, the phasor $V_E^\cdot$ falls within the shadow area due to the limitation of the capacity of the shunt converter and transformer of UPFC.
We can see that the trajectory of maximum of $V'_{E_{\text{max}}}$ is the circle with the center $V_{E_{t}}$. If the radius of the circle is taken as $0.2 \times V'_{E_{t}}$, then $V'_{E_{\text{max}}} = 1.2 \times V'_{E_{t}}$.

From (6-5) and Fig. 6-3, note that:

$$ U_r = V_{E_{r}} + V_{B_{r}} = V_{E_{r}} + V_{B_{r}} - I_{B}Z_{B} \tag{6-7} $$

the phasor diagram of $V'_{B}$ is shown in Fig. 6-5 with neglecting the series impedance $Z_{B}$ in (6-7).

![Fig.6-5 The phasor diagram of UPFC series](image)

From Fig. 6-5, it shows that the phasor $U_{r}$ has to fall within the shadow circle and the maximum of the magnitude of $V'_{B}$ should not be too large. For example, $V'_{B_{\text{max}}} = 0.5 \times V'_{E_{t}}$.

From the above analysis, the maximum of $V'_{E}$ is much larger than the maximum of $V'_{B}$ under operation. However, under steady state operating condition and when the two modulation index $m_{e}$ and $m_{g}$ are approaching their maximal value of 1, the DC voltage on the capacitor, $v_{dc}$, is having its maximum value, $v_{dc_{\text{max}}}$. From (6-2) and (6-3), we know $V'_{E_{\text{max}}} = V'_{B_{\text{max}}}$ (referred to DC side) and the voltages referred to the AC side:

$$ V'_{E_{\text{max}}} = T_{E}V_{E_{\text{max}}} = T_{E} \left( \frac{\sqrt{2}}{4} \times 1 \times v_{dc_{\text{max}}} \right) = k_{E}V'_{E} \tag{6-8} $$

$$ V'_{B_{\text{max}}} = T_{B}V_{B_{\text{max}}} = T_{B} \left( \frac{\sqrt{2}}{4} \times 1 \times v_{dc_{\text{max}}} \right) = k_{B}V'_{E} \tag{6-9} $$
where $V_N$ is the nominal voltage of the transmission line at the point the UPFC is installed. $T_E$ and $T_B$ are the turn ratio of ET and BT respectively. $k_E$ and $k_B$ are constant coefficients with value taken as about 1.2 and 0.5 respectively (see also Fig.6-4 and Fig.6-5). Their values reflect proportionally their control capability in respect of the cost of the UPFC. From above two equations, $T_E \neq T_B$ is clear.

Because $T_E \neq T_B$, in order to make a succinct outlook, the base values of the voltage in the DC side are chosen in correspondence with the AC side as:

$$ V_{E_0} = \frac{V_N}{T_E} = \frac{V_{dc,\max}}{2\sqrt{2k_E}} \quad (6-10) $$

$$ V_{B_0} = \frac{V_N}{T_B} = \frac{V_{dc,\max}}{2\sqrt{2k_B}} \quad (6-11) $$

where $V_{E_0}$ and $V_{B_0}$ are the base value of ET and BT in the DC side respectively.

$v_{dc,\max}$ is taken as the base quantity of the voltage of the DC capacitor and the complex power's base quantity $S_r$ is determined by the network. The base values of other electrical quantities associated with the UPFC such as current and impedance can be easily derived. For clarification, the base value system is summarized below.

- The base value of complex power $S_r$: rating of the corresponding power system network
- The base value of voltage in AC side $V_N$: rated voltage of the transmission line $V_N$.
- The base value of the ET voltage in the DC side $V_{E_0}$ is chosen as in (6-10)
- The base value of the BT voltage in the DC side $V_{B_0}$ is chosen as in (6-11)
- The base value of DC voltage is taken as $v_{dc,\max}$.

Based on the above base values, there is no change of format to the algebraic equations (6-4) and (6-5) in per unit form but a little change in that for equations (6-2) and (6-3) as:
\[ V_{*E} = \frac{V_{E}}{V_{tr}} = \frac{m_e v_{de}/2\sqrt{2}}{v_{d_{max}}/2\sqrt{2} k_e} \angle \delta_e = k_e m_e v_{de} \angle \delta_e \] (6-12)

\[ V_{*S} = \frac{V_{S}}{V_{tr}} = \frac{m_s v_{de}/2\sqrt{2}}{v_{d_{max}}/2\sqrt{2} k_s} \angle \delta_s = k_s m_s v_{de} \angle \delta_s \] (6-13)

where the subscript asterisk * denotes corresponding per unit values.

For the equation (6-6), a bit of manipulation after dividing both sides by the complex power base value \( S \), is required and done as follows.

RHS:

\[ 3\text{Re}[V_{*E} I_{*E} - V_{*S} I_{*S}] / S, = \text{Re}[V_{*E} I_{*E} - V_{*S} I_{*S}] \] (6-14)

LHS:

\[ \frac{c_{de} v_{dc}}{S} \frac{dv_{de}}{dt} = \frac{2}{S} \left( \frac{1}{2} c_{de} v_{d_{max}}^2 \right) \left( \frac{v_{dc}}{v_{d_{max}}} \right) \left( \frac{d v_{dc}}{v_{d_{max}} dt} \right) = v_{dc} T_s \frac{dv_{dc}}{dt} \] (6-15)

where \( T_s \) is introduced as a time constant of the UPFC. From (6-15), \( \frac{1}{2} c_{de} v_{d_{max}}^2 = W \) is the energy stored in the DC capacitor under the voltage \( v_{d_{max}} \) and hence \( T_s = 2W / S \), can be related in (6-16):

\[ v_{dc} T_s \frac{dv_{dc}}{dt} = \text{Re}[V_{*E} I_{*E} - V_{*S} I_{*S}] \] (6-16)

The network equations are usually written in Cartesian coordination so as to combine them with differential equations later. For simplicity, equations (6-4) and (6-5) can be
Chapter 6 Enhancement of Dynamic Stability of Power Systems by Using UPFC

represented in per unit format with the subscript asterisk omitted by default as (6-17) and (6-18) respectively.

\[
\begin{bmatrix}
    r_E & -x_E \\
    x_E & r_E
\end{bmatrix}
\begin{bmatrix}
    I_{Ex} \\
    I_{Ey}
\end{bmatrix}
= 
\begin{bmatrix}
    V_{Erx} \\
    V_{Ery}
\end{bmatrix}
- 
\begin{bmatrix}
    V_{Ex} \\
    V_{Ey}
\end{bmatrix}
\quad (6-17)
\]

\[
\begin{bmatrix}
    r_B & -x_B \\
    x_B & r_B
\end{bmatrix}
\begin{bmatrix}
    I_{Bx} \\
    I_{By}
\end{bmatrix}
= 
\begin{bmatrix}
    V_{Brx} \\
    V_{Bry}
\end{bmatrix}
- 
\begin{bmatrix}
    V_{Bx} \\
    V_{By}
\end{bmatrix}
\quad (6-18)
\]

where:

\[
V_{ex} + jV_{ey} = k_e m_e v_{ac} (\cos \delta_e + j \sin \delta_e) \quad (6-19)
\]

\[
V_{bx} + jV_{by} = k_b m_b v_{ac} (\cos \delta_b + j \sin \delta_b) \quad (6-20)
\]

By substituting (6-17) and (6-18) into (6-14) we obtain:

\[
T_s \frac{dv_{ac}}{dt} = k_e m_e (I_{ex} \cos \delta_e + I_{ey} \sin \delta_e) - k_b m_b (I_{bx} \cos \delta_b + I_{by} \sin \delta_b) \quad (6-21)
\]

Up to now, the dynamic mathematical model of UPFC in per unit is expressed by (6-17) to (6-21). The variables $m_e, \delta_e, m_b, \delta_b$ and $v_{ac}$ are all differential state variables and the UPFC terminal voltage ($V_e$ and $V_b$) and current ($I_e$ and $I_b$) are algebraic state variables.

6.3 Linearized model of the UPFC

For the small signal disturbance analysis, the above equations are linearized at an operational quiescent point of the UPFC. Obviously it is a load flow problem of power system with UPFC to obtain the information of the static operating point. By means of the algorithm proposed in chapters 2, 3 and 4, the load flow of power system with UPFC can
be calculated and then the parameters of the UPFC $I_q$, $U_T$ and $\Phi_T$ as well as the voltage $U_p$ and current $I_t$ in Fig. 2-2 are all known. From (2-8), $I_t$ can be obtained and the current $I_E = I_q + I_I$ through the shunt branch of UPFC can also be obtained. Thus, in Fig. 6-3, we obtain the static equilibrium of the UPFC:

$$V_b = U_T$$  \hfill (6-22)

$$V_E = V_E - I_E (r_E + jx_E) = U_I - I_E (r_E + jx_E) = U_p - I_E (r_E + jx_E)$$  \hfill (6-23)

Equations (6-24) to (6-26) show the resultant linearized dynamic model.

$$\left[ \begin{array}{c} \Delta I_{Ex} \\ \Delta I_{Ey} \end{array} \right] = \left[ \begin{array}{ccc} a_{11}^E & a_{12}^E & a_{13}^E \\ a_{21}^E & a_{22}^E & a_{23}^E \end{array} \right] \left[ \begin{array}{c} \Delta m_E \\ \Delta \delta_E \end{array} \right] + \left[ \begin{array}{cc} y_{11}^E & y_{13}^E \\ y_{21}^E & y_{23}^E \end{array} \right] \left[ \begin{array}{c} \Delta V_{E_x} \\ \Delta V_{E_y} \end{array} \right]$$  \hfill (6-24)

$$\left[ \begin{array}{c} \Delta I_{Bx} \\ \Delta I_{By} \end{array} \right] = \left[ \begin{array}{ccc} a_{11}^B & a_{12}^B & a_{13}^B \\ a_{21}^B & a_{22}^B & a_{23}^B \end{array} \right] \left[ \begin{array}{c} \Delta m_B \\ \Delta \delta_B \end{array} \right] - \left[ \begin{array}{cc} y_{11}^B & y_{13}^B \\ y_{21}^B & y_{23}^B \end{array} \right] \left[ \begin{array}{c} \Delta V_{Bx} \\ \Delta V_{By} \end{array} \right]$$  \hfill (6-25)

$$\Delta \hat{v}_{sc} = [a_1, a_2, a_3, a_4] \Delta m_{Ex}, \Delta \delta_E, \Delta m_B, \Delta \delta_B]^T +$$

$$[b_1, b_2, b_3, b_4] \left[ \begin{array}{c} \Delta I_{Ex} \\ \Delta I_{Ey} \\ \Delta I_{Bx} \\ \Delta I_{By} \end{array} \right]$$  \hfill (6-26)

The expressions of all elements of coefficient matrix can be obtained from (6-17) to (6-21) and given in Appendix IV.

6.4 Nodal network equations with the UPFC

In order to incorporate the UPFC algebraic equations (6-17) and (6-18) into the nodal network, the branch currents $I_E$ and $I_B$ and the branch voltage, $V_{Bx}$, have to be transformed to nodal values.
By means of the substitution theorem, the UPFC in Fig.6-3 can be detached as shown in Fig.6-6, in which \( I_E, I_B, V_{El} \) and \( V_{Bl} \) denote the UPFC terminal voltage and current respectively.

![Fig.6-6 The system equivalent after detaching UPFC](image)

Let \( Y \) be the admittance matrix when there is no UPFC in the branch \( lm \). By augmenting and revising \( Y, Y' \), the admittance matrix of the network as shown in Fig.6-6, is obtained. Only six elements related to the nodes \( l, m \) and a newly formed one, \( s \), are amended such that \( Y_{ll}, Y_{lm} \) and \( Y_{ml} \) become \( Y_{ls}, Y_{sm} \) and \( Y_{ml} \) and \( Y_{ns}, Y_{mn} \) and \( Y_{sn} \) are newly formed. The network equations are:

\[
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_{l-(I_E+I_B)} \\
I_m \\
I_n \\
I_B
\end{bmatrix}
= 
\begin{bmatrix}
Y_{11} & Y_{12} & \cdots & Y_{1m} & \cdots & Y_{1n} & & 0 \\
Y_{21} & Y_{22} & \cdots & Y_{2m} & \cdots & Y_{2n} & & 0 \\
\vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & 0 \\
Y_{l1} & Y_{l2} & \cdots & Y_{lm} & \cdots & Y_{ln} & & 0 \\
Y_{m1} & Y_{m2} & \cdots & Y_{mm} & \cdots & Y_{mn} & & Y_{m} \\
\vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & 0 \\
Y_{n1} & Y_{n2} & \cdots & Y_{nm} & \cdots & Y_{nn} & & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & Y_{m} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & Y_{m} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & Y_{m} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & Y_{m} & 0 \\
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_2 \\
\vdots \\
U_{l-(I_E+I_B)} \\
U_m \\
U_n \\
U_B
\end{bmatrix}
\]

\[(6-27)\]
where

\[ Y'_{ll} = Y'_{ll} - (r_{ln} + jx_{ln})^{-1} \]  \hfill (6-28)

\[ Y'_{lm} = Y'_{ml} = Y_{lm} + (r_{ln} + jx_{ln})^{-1} \]  \hfill (6-29)

\[ Y'_{ms} = Y'_{sm} = -(r_{ln} + jx_{ln})^{-1} \]  \hfill (6-30)

\[ Y'_{ss} = Y_{lm} + (r_{ln} + jx_{ln})^{-1} \]  \hfill (6-31)

From Fig. 6-6, note that

\[ U_i = V_{E_i} \]

\[ U_s = V_{E_s} + V_{E_l} \]

The nodal voltage equations of node \(l\), \(m\) and \(s\) are given in (6-32) to (6-34) respectively.

\[ I_i - (I_{E_l} + I_{E_m}) = Y'_{ll}V_{E_i} + Y'_{lm}U_m + \sum_{j=1}^{n} Y'_{lj}U_j \]  \hfill (6-32)

\[ I_m = Y'_{ml}V_{E_l} + Y'_{mm}U_m + Y'_{ms}(V_{E_s} + V_{E_l}) + \sum_{j=1}^{n} Y'_{mj}U_j \]  \hfill (6-33)

\[ I_s = Y'_{sm}U_m + Y'_{ss}(V_{E_s} + V_{E_l}) \]  \hfill (6-34)

From (6-28) to (6-31) known:

\[ Y'_{ll} + Y'_{ss} = Y_{ll} \]

\[ Y'_{lm} + Y'_{sm} = Y_{lm} \]
Note that $V_E = U_r$, we can rearrange (6-33):

$$I_m = Y_m V_{Br} + \sum_{j \neq r} Y_{j} U_j$$  \hspace{1cm} (6-35)

By adding (6-34) to (6-32), (6-36) is obtained:

$$I_i - I_k = Y_m V_{Br} + \sum_{j \neq r} Y_j U_j$$  \hspace{1cm} (6-36)

In (6-34) to (6-36), let:

$$Y_s = Y_f = Y_r$$

$$Y_m = Y_{sf} = Y_{fm}$$

$$Y_g = Y_u$$

and take $V_{Br}$ as the voltage of the fictitious node $f$. Fig.6-7 can be used to express the network after transferring the branch current $I_E$ and $I_B$ and branch voltage $V_{Br}$ into nodal injected current and nodal voltage. The fictitious node $f$ and branch $lf$, and $mf$ as well as shunt branch $fo$, $lo$ and $mo$ are added in Figure 6-7.

![Fig.6-7 UPFC terminal voltage and current as nodal voltage and current](image)

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The fictitious branch impedance and admittance can be written as:

\[
    r_{fi} + jx_{fi} = -(r_{in} + jx_{in}) \left[1 + y_{im} (r_{in} + jx_{in})\right]^{-1}
\]  (6-37)

\[
    r_{nf} + jx_{nf} = r_{in} + jx_{in}
\]  (6-38)

\[
    y_{f0} = 2y_{in} + (r_{in} + jx_{in})^{-1}
\]  (6-39)

\[
    y_{i0} = y_{in} + (r_{in} + jx_{in})^{-1}
\]  (6-40)

\[
    y_{mn} = -(r_{in} + jx_{in})^{-1}
\]  (6-41)

Hence, the following network equations can be more easily obtainable from Fig. 6-7.

\[
    \begin{bmatrix}
    I_x \\
    I_y
    \end{bmatrix}
    = \sum_{j=1}^{n} \begin{bmatrix}
    G_{ij} & -B_{ij} \\
    B_{ij} & G_{ij}
    \end{bmatrix}
    \begin{bmatrix}
    U_x \\
    U_y
    \end{bmatrix}
\]

\[\forall \ i=1,2, ..., n \text{ but } i \neq l,m,f.\]  (6-42)

When \(i=l:\)

\[
    \begin{bmatrix}
    I_{lx} \\
    I_{ly}
    \end{bmatrix}
    - \begin{bmatrix}
    I_{Ex} \\
    I_{Ey}
    \end{bmatrix}
    = \begin{bmatrix}
    G_{lf} & -B_{lf} \\
    B_{lf} & G_{lf}
    \end{bmatrix}
    \begin{bmatrix}
    V_{BEx} \\
    V_{BEy}
    \end{bmatrix}
    + \sum_{j=1}^{n} \begin{bmatrix}
    G_{lj} & -B_{lj} \\
    B_{lj} & G_{lj}
    \end{bmatrix}
    \begin{bmatrix}
    U_{jx} \\
    U_{jy}
    \end{bmatrix}
\]  (6-43)

When \(i=m:\)

\[
    \begin{bmatrix}
    I_{mx} \\
    I_{my}
    \end{bmatrix}
    = \begin{bmatrix}
    G_{mf} & -B_{mf} \\
    B_{mf} & G_{mf}
    \end{bmatrix}
    \begin{bmatrix}
    V_{BEx} \\
    V_{BEy}
    \end{bmatrix}
    + \sum_{j=1}^{n} \begin{bmatrix}
    G_{mj} & -B_{mj} \\
    B_{mj} & G_{mj}
    \end{bmatrix}
    \begin{bmatrix}
    U_{jx} \\
    U_{jy}
    \end{bmatrix}
\]  (6-44)

When \(i=f:\)

\[
    \begin{bmatrix}
    I_{xf} \\
    I_{yf}
    \end{bmatrix}
    = \begin{bmatrix}
    G_{f} & -B_{f} \\
    B_{f} & G_{f}
    \end{bmatrix}
    \begin{bmatrix}
    V_{Ex} \\
    V_{Ey}
    \end{bmatrix}
    + \begin{bmatrix}
    G_{jm} & -B_{jm} \\
    B_{jm} & G_{jm}
    \end{bmatrix}
    \begin{bmatrix}
    U_{mx} \\
    U_{my}
    \end{bmatrix}
    + \begin{bmatrix}
    G_{f} & -B_{f} \\
    B_{f} & G_{f}
    \end{bmatrix}
    \begin{bmatrix}
    V_{BEx} \\
    V_{BEy}
    \end{bmatrix}
\]  (6-45)

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where: $G_{ij} + jB_{ij}$ denoting element of the admittance matrix, $Y_{ij}$, of the power systems with UPFC excluded and $n$ as the total number of nodes.

After linearizing (6-42) to (6-45) and then substituting $\Delta I_{Ei}$, $\Delta I_{Es}$, $\Delta I_{Bi}$ and $\Delta I_{Bs}$ in (6-43) to (6-45) and (6-26) by (6-24) and (6-5) the algebraic state variables $I_E$ and $I_B$ can be eliminated and $V_{Ei}$ and $V_{Bi}$ are both nodal voltage of network.

So far the dynamic mathematical model of UPFC is completely established. It can be used in complex power systems with multiple UPFCs.

6.5 Controller of the UPFC

For normal operation of the UPFC, the shunt converter part generally operates in voltage control mode and the series converter is in power flow control mode. Also, the two voltage sources $V_E$ and $V_B$ are closely coupled with each other via the common DC capacitor internal and the power system network external of the UPFC during dynamic procedure. Thus, the active and reactive power on the line ($P$ and $Q$ in Fig.6-3), the amplitude of bus voltage ($V_{Ei}$ in Fig.6-3) and the DC voltage ($v_{dc}$ in Fig.6-2) are chosen as the feedback signals for the UPFC controller.

In order to link up the feedback signals and the control variables, $m_E, \delta_E, m_B$ and $\delta_B$ of the SPWM scheme, a local feedback matrix $F \in \mathbb{R}^{n \times n}$ is devised synthetically for the controller as proposed in this chapter. Figure 6-8 shows a block diagram of the controller and all elements of the feedback matrix $F$ and coefficients $a, b, c$ as well as the time constant $T_p$, $T_q$, $T_r$ and $T_d$ are the controllable parameters. $T_1$ to $T_4$ represent delays introduced by the SPWM controller.

The method proposed in this chapter functionally intermingles the traditional linear optimal control with the sensitivity analysis techniques so as to determine both the feedback matrix and the control parameters at the same time.
6.6 Optimization of the control parameters

In respect of equilibrium point \( X_{j0} \) corresponding to the \( j \)th operating condition, a power system can be modeled as a linear time-invariant and autonomous system represented as:

\[
\Delta \dot{X}_j = A_j(T) \Delta X_j + B_j(T) U_j \quad \forall j
\]

\[
0 = C_j \Delta X_j + D_j U_j \quad \forall j
\]

(6-46)

(6-47)

where \( X_j \in \mathbb{R}^{n_x} \) denotes as the state vector, \( U_j \in \mathbb{R}^{n_y} \) as the nodal voltage vector and \( n_j \) as the total number of UPFC employed. Coefficient matrices \( A_j \) and \( B_j \) are the function of the vector \( T \) which is formed by including all the control parameters. \( C_j \) and \( D_j \) are independent of \( T \).

Let \( \lambda_y = \sigma_y + j \omega_y \) be the \( i \)th eigenvalue of the state matrix: \( \overline{A}_j(T) = A_j(T) - B_j(T) \times D_j^{-1} C_j \).

In order to find the decision vector \( T \), a mediate variable \( \eta \) is introduced to transform problem (6-1) into a general constrained optimization problem (6-48):

Objective : \( \min \eta \)

Subject to : \( \eta \geq \sigma_y(T) \quad \forall i,j \)

\( c_L \leq T \leq c_U \)

(6-48)

By means of a penalty function method, (6-48) is further transformed into unconstraint problem (6-49). The barrier function is:

\[
\phi = \eta + \sum_{i=1}^{n_y} \sum_{j=1}^{n_j} \max\{[\sigma_y - \eta], 0\} + \varepsilon B(T)
\]

(6-49)

where the constant \( \varepsilon \) is taken as a very small positive value and \( B(T) \) is defined as:
Fig. 6-8 The Control Strategy Diagram of Unified Power Flow Controller
\[ B(T) = \sum_{i=1}^{n} \left[ (c_{ia} - r_i) + (r_i - c_{ia}) \right] \]  

(6-50)

Solution can then be obtained in the following flow chart as shown in Fig. 6-9 and the illustration for the iteration is given in Appendix X.

6.7 Case study

The New England test system including ten generators, one PSS and one UPFC is selected as a case study for the proposed dynamic performance analysis. Basic data of the system from [41] are used to devise three operating conditions in which the UPFC is located at bus 26 and PSS is attached with the #7 generator. Configuration of the system and the information related the computations are shown in Appendix V. The ZIP load model is adopted. The traditional differential equations and transfer function diagrams associated to forming \( A, B, C \) and \( D \) are given in Appendix VI to IX.

Having adopted the following setting:

Basic parameters of UPFC taken as: \( R_e = R_f = 0.0008 \), \( X_e = X_f = 0.0160 \), \( k_e = 1.2 \), \( k_f = 0.5 \), \( T_a = 5.0 \) s;

Initial value of PSS parameters as: \( T_3 = T_5 = 0.1 \), \( T_4 = T_6 = 0.05 \);

Gain pf \( K_{PSS} = 20.0 \);

Initial value of the control vector \( T^{(0)} \) associated with UPFC given in (6-51) and Table 6-1.

\[ F^{(0)} = \begin{bmatrix} 0.11 & 0.12 & 0.13 & 0.14 \\ 0.21 & 0.22 & 0.23 & 0.24 \\ 0.31 & 0.32 & 0.33 & 0.34 \\ 0.41 & 0.42 & 0.43 & 0.44 \end{bmatrix} \]  

(6-51)
Table 6-1: the initial parameters of controller

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_p$</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>$T_p=0.1$</td>
</tr>
<tr>
<td>$\alpha_q$</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>$T_q=0.1$</td>
</tr>
<tr>
<td>$V_{ext}$</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>$T_r=0.1$</td>
</tr>
<tr>
<td>$\nu_{dc}$</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>$T_d=0.1$</td>
</tr>
</tbody>
</table>

the outcome of the PSS obtained from the computation are: $T_3=0.0982$, $T_4=0.0551$, $T_5=0.0891$, $T_6=0.6450$ and $K_{PSS}=12.0771$ and the control parameters of the UPFC are obtained as shown in (6-52) and listed in Table 6-2.

$$
F^* = \begin{bmatrix}
0.1339 & 0.0988 & 0.1336 & 0.0460 \\
0.2104 & 0.2279 & 0.2302 & 0.2391 \\
0.0272 & 0.0378 & 0.6354 & 0.7234 \\
0.0307 & 0.0598 & 0.7261 & 0.4445
\end{bmatrix}
$$

(6-52)

Table 3: the selected parameters of controller

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<th></th>
<th>a</th>
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<th>c</th>
<th>$P$</th>
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<td>$\alpha_p$</td>
<td>0.4060</td>
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<td>0.2239</td>
<td>$T_p=0.07461$</td>
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<td>$\alpha_q$</td>
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<td>0.1469</td>
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<tr>
<td>$V_{ext}$</td>
<td>0.5448</td>
<td>0.2347</td>
<td>0.1091</td>
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<tr>
<td>$\nu_{dc}$</td>
<td>0.8127</td>
<td>0.3001</td>
<td>0.2175</td>
<td>$T_d=0.04693$</td>
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</table>

The iteration procedure is show in Fig.6-10. Results show that the control vector can be determined successfully by manipulating the state matrix of 99 orders and the 37 control parameters including 5 from the PSS and 32 of from the UPFC. Out of the three operating conditions, the worst damping situation arising from the condition 2 has been improved from 1.996 to -0.8341.
For all operating conditions concerned, calculate load flow to obtain the power system equilibrium point

Given the initial value $T^{(0)}$ which must be in the feasible area and iterative pace $\Delta \eta > 0$; set $k=0$

Form the coefficient matrix $C$ and $D$

Form the coefficient matrix $A(T^{(k)})$ and $B(T^{(k)})$

Find the all eigenvalues and pick up the largest real part $\sigma_{\text{max}}^{(k)}$

Let: $\eta^{(k)} = \sigma_{\text{max}}^{(k)} - \Delta \eta$

Calculate the gradient:

$$
\nabla \phi_T^{(k)} = 2 \sum_{j=1}^{n_b} \sum_{i=1}^{n_y} \max[(\sigma_y^{(i)} - \eta^{(i)}), 0, \frac{\partial \sigma_y}{\partial T} + \varepsilon \frac{\partial B(T)}{\partial T}]_{x,T^{(k)}}
$$

If $\| \nabla \phi_T^{(k)} \|_\infty \leq 10^{-3}$ then stop calculation and take $T^{(k)}$ as the solution. Otherwise carry out next step

Find: $\alpha^{(k)} = \min \phi(T^{(k)} - \alpha \nabla \phi_T^{(k)})$

Revise control vector: $T^{(k+1)} = T^{(k)} - \alpha^{(k)} \nabla \phi_T^{(k)}$

$k \leftarrow k + 1$

Fig. 6-9. Flow chart for solving decision vector $T$
6.8 Summary

The main contributions of this chapter can be concluded as follows.

1. Derive a dynamic mathematical model of UPFC in per unit form and a method for incorporating it into power system network for dynamic performance analysis.
2. Develop a control strategy for using UPFC to enhance dynamic damping of power systems effectively.
3. Formulate setting of the control parameters of UPFC by means of an optimization process, which can coordinate control of UPFC with traditional control means, such as PSS. The proposed method is proved to be relatively robust and can adapt to multi-operating-condition.

The successful case study on its application to the New England test system demonstrates that the method is feasible, effective and robust.
Chapter 7

CONCLUSIONS AND FUTURE WORK

7.1 Summary

Unified Power Flow Controller is widely accepted as the most vigorous and promising FACTS device for load flow control under both static and dynamic conditions because of its unique salient feature of manipulating the three independently controllable parameters. Contribution of this research project including development of appropriate algorithm and techniques for extending the technical know-how on operation of the UPFCs embedded power system asserts this feature. After introducing the FACTS concept, Chapter one gives a comprehensive review on the background research work and an overview of the project objectives in respect to the UPFC. A load flow model is developed as outlined in Chapter 2. Essentially, it enables load flow calculation be preformed for all subsequent analysis. Nature of the UPFC load flow control problem can be reckoned in two ways, namely, direct and indirect control. By using the sensitivity analysis, Chapter 3 presents a detail investigation on how UPFC can indirectly control the power and voltage of other part of the system associated with the UPFC controlled line. Direct control of the UPFC to effect power flow along the line it connected is investigated and presented in Chapter 4. In Chapter 5, the envisaged optimization technique is proved to be effective for finding optimal location for installation of the UPFCs. Finally, the dynamic mathematical model presented in Chapter 6 gives a versatile means for exploiting the application of the UPFC power coordination for enhancing dynamic performance of the UPFC embedded power systems with other conventional power control devices such as AVR and PSS.

In conclusion, the following comments and findings can be summarized.

1. The load flow calculation model is effective and essential for subsequent investigation on the UPFC embedded power system performance. Formulation of the model requires representation of the UPFC power control as nodal injection. The envisaged technique
Chapter 7 Conclusions and Future Work

can then readily incorporate them into the traditional NRLF program. Results of the case studies show that this method can keep the convergence characteristics and the computation speed of the traditional NRLF technique. The method has various potential applications in other related areas such as system planning and optimal design of power system with UPFCs.

2. UPFC is able to control power flow flexibly as the method developed in the thesis shows that a prescribed level of power control can be effected by direct controlling the power flow in the UPFC embedded transmission line. From the method, it shows that the control setting parameters of the UPFC are straightway obtainable from the result of the load flow calculation. Hence, the load flow computation is seen to stand free from UPFC initial states and its convergence is guaranteed. This method significantly prevails over those reported so far in the literature.

3. Although UPFC introduces new free dimension for load flow control, they are normally confined to the line it connected. The indirect control by which the power or the voltage of other transmission lines are meant to be controlled by another associated UPFC embedded line is rarely studied but thoroughly investigated in the thesis. In fact, this indirect control is proved to be useful not only in normal operating cases but also in abnormal contingencies. The envisaged method based on the sensitivity analysis is found to be unique as otherwise normal way requires specification of initial values for the UPFC to solve the simultaneous control targets equations with load flow equations is non existence. By means of this method, the parameters of UPFCs can be obtained in accordance with the indirect control targets. For sure, the method supports positively a claim that load flow control by using UPFC can enhance power system performance.

4. One important contribution as reported in this thesis is on establishing the design technique for solving some practical implementation problems like the number of UPFCs required to be installed in the system, their rating and location to be placed. The nonlinear optimization formulation technique put forward in the thesis is suitable to include capital investment and the operational cost as objective function by means
of the investment equivalent coefficient. The method has been proved to be robust by
taking the static security constraints and the multiple operating conditions into
consideration simultaneously. The control parameter setting of the UPFC is
coordinated with the conventional control means such as the reactive generation and
the turn ratios of transformers to ensure the power system operating economically and
securely. The technique of using the Augmented Lagrange Multiplier and the BFGS is
shown to be effective in solving the constrained problem under two heavily loaded
cases.

5. The per unit based UPFC dynamic mathematical model has been developed with
objective to reconcile different references experienced in the process of integrating the
UPFC dynamic model and other network elements by a carefully chosen set of base
values. Based on this dynamic model, the UPFC internal controller is designed and
shown to enhance the damping capability when facing with power system oscillations.
According to the Lyapunov's asymptotic stability theory, the envisaged optimization
model established by minimizing the largest real part of all eigenvalues of the state
matrix has been developed to coordinate the parameters of the controller with the
conventional parameters like PSS. The control strategy is derived by making use of all
the local measurable feedback signals. Case study demonstrates that the controller can
improve the system damping capability and robustness of the controller can be
enhanced by considering all differential operating conditions simultaneously.

7.2 Future work

UPFC can no doubt be regarded as a fast and flexibly load flow controller in both normal
and abnormal operating conditions. Its emergence has opened up a large portfolio of
research interest in power system operation control and power system planning. The
practical application of UPFC will have a wide and significant impact on every facet of
power system research and operation. This thesis contributes to put forward solution in
addressing some basic but essential issues and there are surely some more work to be done
as discussed in the following.
1. Having accepted UPFC as a default power system control element, the conventional preventive control scenarios can be enriched to include power system security along side with the power system economy into consideration. A new preventive control schedule can probably be extended to cover dynamic/static security assessment of a wider area. Studies of the UPFC for enhancing the power system security is seen to be a logical extension of this scope of work.

2. Research for the method of improving transient performance of power system by using UPFC is another feasible extension. Making use of the fast control response feature of the UPFC, it can be viewed as a dynamic power wheeler for directing power flow in response to large energy changing signals. Existing energy model can be revisited to incorporate the UPFC element.
References


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Appendix I The raw data of IEEE 14 bus test system for load flow computation

Table I-1 IEEE 14 bus test system network parameters

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<th>R</th>
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Table I-2 the generator outputs

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### Table I-3 Bus load

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### Appendix II The raw data of IEEE 57 bus test system for load flow computation

### Table II-1 Bus Load

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### Table II-2 The generator outputs

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<th>Bus type</th>
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### Table II-3 Shunt Capacitor

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### Table II-4 IEEE 57 bus test system network parameters

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<td>0.0138</td>
<td>14</td>
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<td>0.0235</td>
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<td>0.0712</td>
<td>0.0097</td>
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<td>-7</td>
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<td>0.0220</td>
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<td>0.0848</td>
<td>0.0109</td>
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<td>0.1580</td>
<td>0.0203</td>
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<td>-9</td>
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<td>0.1205</td>
<td>0.9400</td>
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| 23 | 10 | 0.0277 | 0.1262 | 0.0164 | 24 | -10 | 0.0000 | 0.0712 | 0.9300 |
| 25 | 11 | 0.0223 | 0.0732 | 0.0094 | 26 | -11 | 0.0101 | 0.0749 | 0.9550 |
| 27 | -11 | 0.0000 | 0.1530 | 0.9580 | 28 | 12 | 0.0178 | 0.0580 | 0.0302 |
| 29 | 12 | 0.0180 | 0.0813 | 0.0108 | 30 | 12 | 0.0397 | 0.1790 | 0.0238 |
| 31 | 13 | 0.0132 | 0.0434 | 0.0055 | 32 | 13 | 0.0269 | 0.0869 | 0.0115 |
| 33 | -13 | 0.0000 | 0.1910 | 0.8950 | 34 | 14 | 0.0171 | 0.0547 | 0.0074 |
| 35 | -14 | 0.0000 | 0.0735 | 0.9000 | 36 | -15 | 0.0000 | 0.1042 | 0.9550 |
| 37 | 18 | 0.4610 | 0.6850 | 0.0000 | 38 | 19 | 0.2830 | 0.4340 | 0.0000 |
| 39 | 20 | 0.0000 | 0.7767 | 1.0430 | 40 | 21 | 0.0736 | 0.1170 | 0.0000 |
| 41 | 22 | 0.0099 | 0.0152 | 0.0000 | 42 | 22 | 0.0192 | 0.0295 | 0.0000 |
| 43 | 23 | 0.1660 | 0.2560 | 0.0042 | 44 | -24 | 0.0000 | 1.1820 | 1.0000 |
| 45 | -24 | 0.0000 | 1.2300 | 1.0000 | 46 | -24 | 0.0000 | 0.0473 | 1.0430 |
| 47 | 25 | 0.1350 | 0.2020 | 0.0000 | 48 | 26 | 0.1650 | 0.2540 | 0.0000 |
| 49 | 27 | 0.0693 | 0.0954 | 0.0000 | 50 | 28 | 0.0418 | 0.0587 | 0.0000 |
| 51 | 29 | 0.1442 | 0.1870 | 0.0000 | 52 | 30 | 0.3260 | 0.4970 | 0.0000 |
| 53 | 31 | 0.5070 | 0.7550 | 0.0000 | 54 | 32 | 0.0392 | 0.0360 | 0.0000 |
| 55 | 32 | -34 | 0.0000 | 0.9530 | 56 | 34 | 0.0520 | 0.0780 | 0.0016 |
| 57 | 35 | 0.0430 | 0.0537 | 0.0008 | 58 | 36 | 0.0290 | 0.0366 | 0.0000 |
| 59 | 36 | 0.0400 | 0.0466 | 0.0000 | 60 | 37 | 0.0651 | 0.1009 | 0.0010 |
| 61 | 37 | 0.0399 | 0.0379 | 0.0000 | 62 | 38 | 0.0289 | 0.0585 | 0.0010 |
| 63 | 38 | 0.0412 | 0.0482 | 0.0000 | 64 | 38 | 0.1150 | 0.1770 | 0.0030 |
| 65 | -39 | 0.0000 | 1.3550 | 0.9800 | 66 | -40 | 0.0000 | 1.1950 | 0.9580 |
| 67 | 41 | 0.2070 | 0.3520 | 0.0000 | 68 | 41 | 0.0000 | 0.4120 | 0.0000 |
| 69 | 41 | 0.5530 | 0.5490 | 0.0000 | 70 | 42 | 0.2125 | 0.3540 | 0.0000 |
| 71 | 44 | 0.0624 | 1.2420 | 0.0020 | 72 | 46 | 0.0230 | 0.0680 | 0.0016 |
| 73 | 47 | 0.0182 | 0.0233 | 0.0000 | 74 | 48 | 0.0834 | 0.1290 | 0.0024 |
| 75 | 49 | 0.0801 | 0.1280 | 0.0000 | 76 | 50 | 0.1386 | 0.2200 | 0.0000 |
| 77 | 52 | 0.0762 | 0.0984 | 0.0000 | 78 | 53 | 0.1878 | 0.2320 | 0.0000 |
| 79 | 54 | 0.1732 | 0.2265 | 0.0000 | 80 | 56 | 0.1740 | 0.2600 | 0.0000 |
Appendix III Unconstrained Optimization – BFGS Method Flow Chart

Given initial value $X^{(i)}$ and set $I_d = \text{the dimension of } X$ and $K=1$

Calculate the gradient $g^{(i)} = \frac{\partial L}{\partial X_{X \times X^{(i)}}}$

Let $H^{(1)} = \text{unit matrix}$

$$d^{(k)} = -H^{(k)}g^{(k)}$$

Line search along with the direction $d^{(k)}$ to find:

$$\tau^* = \min \left\{ L \left( X^{(k)} + \tau d^{(k)} \right) \right\}$$

No

$$\|\tau^* d^{(k)}\|_\infty \leq \varepsilon ?$$

Yes

$X^{(K)} = X^{(K)}$

Stop

$X^{(K+1)} = X^{(K)} + \tau^* d^{(k)}$ and calculate the gradient $g^{(K+1)} = \frac{\partial L}{\partial X_{X \times X^{(K+1)}}}$

No

Yes

$K < I_d ?$

Calculate $H^{(K+1)} = H^{(K)} + \Delta H^{(K)}$ by (A-1)

$X^{(K+1)} \Rightarrow X^{(K)} \Rightarrow g^{(K+1)} \Rightarrow g^{(K)} \Rightarrow K=1$

$K \leftarrow K+1$

$$\Delta H^{(K)} = \left( 1 + \frac{q^{(k)\tau} H^{(k)} q^{(k)}}{p^{(k)\tau} q^{(k)}} \right) \times \frac{p^{(k)\tau} q^{(k)\tau} r^{(k)} - q^{(k)\tau} H^{(k)} q^{(k)}}{p^{(k)\tau} q^{(k)}}$$

(A-1)

where:

$$p^{(k)} = X^{(K+1)} - X^{(k)} \quad q^{(k)} = g^{(K+1)} - g^{(k)}$$

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Appendix IV The linearized equations of UPFC

- The algebraic equation in the exciting transformer side:

\[
\begin{bmatrix}
\Delta I_{E}\,_x \\
\Delta I_{E}\,_y
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23}
\end{bmatrix}
\begin{bmatrix}
\Delta m_E \\
\Delta \delta_E
\end{bmatrix} +
\begin{bmatrix}
y_{11} & y_{12} \\
y_{21} & y_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta V_{Ex} \\
\Delta V_{Ey}
\end{bmatrix}
\]

where:

\[
y_{11} = \frac{-R_E}{R_E^2 + X_E^2} = y_{22} \quad y_{12} = \frac{-X_E}{R_E^2 + X_E^2} = -y_{12}
\]

Let:

\[
E = \frac{\sqrt{2}(R_E \cos \delta_{E0} - X_E \sin \delta_{E0})}{4(R_E^2 + X_E^2)} \quad F = \frac{\sqrt{2}(R_E \sin \delta_{E0} + X_E \cos \delta_{E0})}{4(R_E^2 + X_E^2)}
\]

then:

\[
a_{11} = v_{\delta_E} E \quad a_{21} = -v_{\delta_E} F \quad a_{12} = m_E a_{11} \quad a_{13} = m_E a_{11}
\]

\[
a_{22} = -m_E a_{11} \quad a_{13} = m_E F \quad a_{23} = -m_E F
\]

- The algebraic equation in the boosting transformer side:

\[
\begin{bmatrix}
\Delta I_{B}\,_x \\
\Delta I_{B}\,_y
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23}
\end{bmatrix}
\begin{bmatrix}
\Delta m_B \\
\Delta \delta_B
\end{bmatrix} +
\begin{bmatrix}
y_{11} & y_{12} \\
y_{21} & y_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta V_{Bx} \\
\Delta V_{By}
\end{bmatrix}
\]

where:

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\[ \begin{align*}
y_{11} &= \frac{-R_a}{R_a^2 + X_a^2} = y_{22} \\
y_{12} &= \frac{-X_a}{R_a^2 + X_a^2} = -y_{12}
\end{align*} \]

Let:

\[ E = \frac{\sqrt{2} (R_a \cos \delta_{g0} - X_a \sin \delta_{g0})}{4(R_a^2 + X_a^2)} \]
\[ F = \frac{\sqrt{2} (R_a \sin \delta_{g0} + X_a \cos \delta_{g0})}{4(R_a^2 + X_a^2)} \]

then:

\[ a_{11} = \nu_{ac} E \quad a_{21} = -\nu_{ac} F \quad a_{12} = m_{g0} a_{21} \]
\[ a_{22} = -m_{g0} a_{11} \quad a_{11} = m_{g0} E \quad a_{23} = -m_{g0} F \]

- The differential equations of UPFC:

\[ \Delta \dot{v}_{\alpha} = [a_1 \quad a_2 \quad a_3 \quad a_4 \quad \Delta m_{\delta} \quad \Delta m_{\delta} \quad \Delta \delta_{\delta}] \]
\[ \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix} [\Delta I_{\delta0} \quad \Delta I_{\delta0} \quad \Delta I_{\delta0} \quad \Delta I_{\delta0}] \]

where:

\[ a_1 = k_{E} T^{-1}_u (I_{E00} \cos \delta_{E0} + I_{E00} \sin \delta_{E0}) \quad a_2 = k_{E} T^{-1}_u (I_{E00} \cos \delta_{E0} - I_{E00} \sin \delta_{E0}) \]
\[ a_3 = -k_{g} T^{-1}_u (I_{g00} \cos \delta_{g0} + I_{g00} \sin \delta_{g0}) \quad a_4 = k_{g} T^{-1}_u (I_{g00} \sin \delta_{g0} - I_{g00} \cos \delta_{g0}) \]
\[ b_1 = k_{E} T^{-1}_u m_{E0} \cos \delta_{E0} \quad b_2 = k_{E} T^{-1}_u m_{E0} \sin \delta_{E0} \]
\[ b_3 = -k_{g} T^{-1}_u m_{g0} \cos \delta_{g0} \quad b_4 = -k_{g} T^{-1}_u m_{g0} \sin \delta_{g0} \]

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Appendix V The information related with the case study in Chapter 6

![Diagram of the 10-unit 39-bus New England test system](image)

Fig.V-1 10-unit 39-bus New England test system

<table>
<thead>
<tr>
<th>Gen.No.</th>
<th>P⁽¹⁾</th>
<th>V⁽¹⁾</th>
<th>P⁽²⁾</th>
<th>V⁽²⁾</th>
<th>P⁽³⁾</th>
<th>V⁽³⁾</th>
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<td>*</td>
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<td>*</td>
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<tr>
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<td>1.0300</td>
<td>8.00</td>
<td>1.0300</td>
<td>6.00</td>
<td>1.0300</td>
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<td>0.9831</td>
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<td>7.50</td>
<td>0.9800</td>
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<td>6.32</td>
<td>0.9972</td>
<td>7.00</td>
<td>1.0000</td>
<td>7.00</td>
<td>0.9900</td>
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<td>1.0123</td>
<td>6.00</td>
<td>1.0100</td>
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### Table V-2 Load demands

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<th>( Q^{(1)} )</th>
<th>( P^{(2)} )</th>
<th>( Q^{(2)} )</th>
<th>( P^{(3)} )</th>
<th>( Q^{(3)} )</th>
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<td>10.000, 2.500</td>
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<td>1.075, 0.900</td>
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<td>2.240, 0.472</td>
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<td>2.810, 0.750</td>
<td>2.810, 0.755</td>
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<td>5.000, 1.840</td>
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<td>5.220, 1.760</td>
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### Table V-3 Generator model

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<th>Exciter</th>
<th>Governor</th>
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<td>√</td>
</tr>
<tr>
<td>2</td>
<td>classical</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>3</td>
<td>three-order</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>4</td>
<td>five-order</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>5</td>
<td>five-order</td>
<td>√</td>
<td>×</td>
</tr>
<tr>
<td>6</td>
<td>three-order</td>
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<td>×</td>
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<td>×</td>
</tr>
<tr>
<td>8</td>
<td>three-order</td>
<td>√</td>
<td>×</td>
</tr>
<tr>
<td>9</td>
<td>five-order</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>10</td>
<td>three-order</td>
<td>√</td>
<td>×</td>
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</table>
Table V-4 The power from bus 40 to 29 and voltage magnitude of bus 26 controlled by UPFC

<table>
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<tr>
<th>p(1)</th>
<th>Q(1)</th>
<th>V(1)</th>
<th>p(2)</th>
<th>Q(2)</th>
<th>V(2)</th>
<th>p(3)</th>
<th>Q(3)</th>
<th>V(2)</th>
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<td>0.25</td>
<td>1.05</td>
<td>3.0</td>
<td>0.37</td>
<td>1.05</td>
<td>4.0</td>
<td>0.50</td>
<td>1.05</td>
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Table V-5 The equilibrium points of the UPFC under three operating conditions

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<th>Condition 2</th>
<th>Condition 3</th>
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<td>mE0</td>
<td>0.9111D+00</td>
<td>0.9078D+00</td>
<td>0.8992D+00</td>
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<tr>
<td>δE0</td>
<td>-0.9532D-01</td>
<td>-0.2535D-01</td>
<td>-0.1870D+00</td>
</tr>
<tr>
<td>mB0</td>
<td>0.8509D-01</td>
<td>0.4720D+00</td>
<td>0.5897D+00</td>
</tr>
<tr>
<td>δB0</td>
<td>0.2572D+03</td>
<td>0.2505D+03</td>
<td>0.2313D+03</td>
</tr>
<tr>
<td>Vdc0</td>
<td>0.1000D+01</td>
<td>0.1000D+01</td>
<td>0.1000D+01</td>
</tr>
<tr>
<td>IEx0</td>
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<td>-0.6253D+00</td>
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<td>-0.3824D+00</td>
<td>-0.8316D+00</td>
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<tr>
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<td>-0.2819D+01</td>
<td>-0.3600D+01</td>
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<tr>
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<td>0.2142D+00</td>
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<td>0.1028D+01</td>
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<td>VEy0</td>
<td>-0.1012D+00</td>
<td>-0.3678D-01</td>
<td>-0.2113D+00</td>
</tr>
<tr>
<td>VEx0</td>
<td>-0.8607D-03</td>
<td>-0.6047D-01</td>
<td>-0.1470D+00</td>
</tr>
<tr>
<td>VBy0</td>
<td>-0.1196D-01</td>
<td>-0.1782D+00</td>
<td>-0.1743D+00</td>
</tr>
</tbody>
</table>

$\Delta \eta = 0.02 ; \varepsilon = 10^{-5}$ (the coefficient of the barrier function);

The parameters in the ZIP load model:

\[ P = P_0 \left[ a_p \left( \frac{V}{V_0} \right)^2 + b_p \left( \frac{V}{V_0} \right) + c_p \right] \]

\[ Q = Q_0 \left[ a_q \left( \frac{V}{V_0} \right)^2 + b_q \left( \frac{V}{V_0} \right) + c_q \right] \]

\[ a_p = 0.75 \quad b_p = 0.2 \quad c_p = 0.05 \quad a_q = 0.85 \quad b_q = 0.14 \quad c_q = 0.01 \]
All time constants of the detector section (including $T_1$ of PSS, $T_L$, $T_O$ and $T_v$ of UPFC controller) are taken as 10 ms and that for the PSS washout section and the SPWM ($T_1$ to $T_4$ in Fig.6-8) are taken as 20 s and 50 ms respectively. The low limit of all the time constants are taken as 20 ms.

Appendix VI Equations of Generators

The swing equations

(1) classical model ($E_q^*$=constant)

\[
\begin{bmatrix}
\Delta \delta \\
\Delta \omega
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta \omega
\end{bmatrix} +
\begin{bmatrix}
b_{21} \\
b_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta I_d \\
\Delta U_d
\end{bmatrix}
\]

\[
a_{12} = \omega_0, \quad a_{22} = \frac{-D}{2H}, \quad b_{21} = \frac{(V_{q0} + R_s I_{q0} + X_s I_{e0})}{-2H}, \quad b_{22} = \frac{(X_q - X_e) I_{q0}}{-2H}
\]

(2) Rotor with only exciting winding model:

\[
\begin{bmatrix}
\Delta \delta \\
\Delta \omega
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta \omega \\
\Delta T_r
\end{bmatrix} +
\begin{bmatrix}
b_{21} \\
b_{22} \\
b_{23}
\end{bmatrix}
\begin{bmatrix}
\Delta I_d \\
\Delta I_s \\
\Delta U_d
\end{bmatrix}
\]

where:

\[
a_{11} = \omega_0, \quad a_{21} = \frac{-D}{2H}, \quad a_{22} = \frac{-I_{q0}}{2H}, \quad a_{23} = \frac{1}{2H},
\]

\[
b_{21} = \frac{(V_{q0} + R_s I_{q0} + X_s I_{e0})}{-2H}, \quad b_{22} = \frac{(X_q - X_e) I_{q0}}{-2H}
\]
(3). Rotor with three or four windings model:

\[
\begin{bmatrix}
\Delta \delta \\
\Delta \omega
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24}
\end{bmatrix}
\begin{bmatrix}
\Delta \omega \\
\Delta E_q \\
\Delta E_d \\
\Delta T_r
\end{bmatrix} +
\begin{bmatrix}
b_{21} & b_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta I_q \\
\Delta I_d \\
\Delta U_q \\
\Delta U_d
\end{bmatrix}
\]

where:
\[
a_{11} = \omega_0, \quad a_{12} = -\frac{D}{2H}, \quad a_{13} = \frac{-I_q}{2H}, \quad a_{14} = \frac{1}{2H},
\]
\[
b_{21} = \frac{(V_{0q} + R_s I_{q0} + X_{q}^* I_{q0})}{-2H}, \quad b_{22} = \frac{V_{0q} + I_q R_s - X_{q}^* I_{q0}}{-2H}
\]

The rotor armature equations

(1). The rotor with four windings model equations:

\[
\begin{bmatrix}
\Delta E_q' \\
\Delta E_d' \\
\Delta E_q'' \\
\Delta E_d''
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24}
\end{bmatrix}
\begin{bmatrix}
\Delta E_q' \\
\Delta E_d' \\
\Delta E_q'' \\
\Delta E_d''
\end{bmatrix} +
\begin{bmatrix}
c_1 \\
\Delta U_f
\end{bmatrix} +
\begin{bmatrix}
b_{21} & b_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta I_q \\
\Delta I_d \\
\Delta U_q \\
\Delta U_d
\end{bmatrix}
\]

where:
\[
a_{11} = \frac{X_q - X_d}{T_{q0}(X_q - X_d)}, \quad a_{12} = \frac{X_d' - X_q}{T_{q0}(X_q' - X_d)}, \quad a_{13} = \frac{1}{T_{q0}} = -a_{12}, \quad b_{21} = \frac{X_q' - X_d'}{T_{q0}}
\]
\[
a_{22} = \frac{X_q' - X_q}{T_{q0}(X_q' - X_q)}, \quad a_{23} = \frac{X_q' - X_d}{T_{q0}(X_q' - X_d)}, \quad a_{24} = \frac{1}{T_{q0}} = -a_{23}, \quad b_{22} = \frac{X_q' - X_d'}{T_{q0}}
\]
\[
c_1 = \frac{1}{T_{q0}}
\]
(2). The rotor with three windings model equations:

\[
\begin{bmatrix}
\Delta \hat{E}_q \\
\Delta \hat{E}_d \\
\Delta E_d
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
\Delta E_q \\
\Delta E_d \\
\Delta E_d
\end{bmatrix} +
\begin{bmatrix}
c_1 \\
b_{31}
\end{bmatrix}
\Delta U_f +
\begin{bmatrix}
b_{22} \\
1
\end{bmatrix}
\begin{bmatrix}
\Delta I_q \\
\Delta I_d \\
\Delta U_d
\end{bmatrix}
\]

where:

\begin{align*}
a_{11} = \frac{X_q - X_d}{T_{d0}(X_q - X_d)} & \quad a_{12} = \frac{X_q - X_d}{T_{d0}(X_q - X_d)} & \quad a_{21} = \frac{1}{T_{d0}} = -a_{22} \\
\frac{X_q - X_d}{T_{d0}} & \quad a_{33} = -\frac{1}{T_{d0}} & \quad b_{31} = \frac{X_q - X_d}{T_{q0}} & \quad c_1 = \frac{1}{T_{d0}}
\end{align*}

(3). The rotor with only exciting windings model equations:

\[
T'_{d0} \Delta \hat{E}_q = -E_q' + \Delta U_f + (X_q' - X_d') \Delta I_d
\]

The voltage algebraic equations of stator armature

(1) Rotor with four or three windings:

\[
\begin{bmatrix}
\Delta I_q \\
\Delta I_d
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23}
\end{bmatrix}
\begin{bmatrix}
\Delta \omega \\
\Delta E_q \\
\Delta E_d
\end{bmatrix} +
\begin{bmatrix}
y_{11} & y_{12} \\
y_{21} & y_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta U_q \\
\Delta U_d
\end{bmatrix}
\]

where:

\begin{align*}
a_{11} = y_{11}(E_{q0}' - X_d'I_{d0}) + y_{12}(E_{q0}' + X_q'I_{q0}) & \quad a_{13} = y_{12}(E_{q0}' - X_d'I_{d0}) + y_{22}(E_{d0}' + X_q'I_{q0}) \\
a_{21} = y_{21}(E_{q0}' - X_d'I_{d0}) + y_{22}(E_{d0}' + X_q'I_{q0})
\end{align*}
\[
\begin{align*}
a_{11} &= y_{11} = \frac{-R_a}{R_a^2 + X_q'X_d'} & a_{12} &= y_{12} = \frac{-X_q'}{R_a^2 + X_d'X_q'} & a_{13} &= y_{13} = \frac{X_d'}{R_a^2 + X_d'X_q'} \\
a_{21} &= y_{21} = y_{11} \\
\end{align*}
\]

(2) Rotor with four or three windings:

\[
\begin{align*}
\begin{bmatrix}
\Delta I_q \\
\Delta I_d
\end{bmatrix}
= 
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta \omega \\
\Delta E_q'
\end{bmatrix}
+ 
\begin{bmatrix}
y_{11} & y_{12} \\
y_{21} & y_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta U_q \\
\Delta U_d
\end{bmatrix}
\end{align*}
\]

where:

\[
\begin{align*}
a_{11} &= y_{11}(E_{q0} - X_q'I_{q0}) + y_{12}X_q'I_{q0} & a_{21} &= y_{21}(E_{q0} - X_q'I_{q0}) + y_{22}X_q'I_{q0} \\
a_{12} &= y_{11} = \frac{-R_a}{R_a^2 + X_q'X_d'} & a_{22} &= y_{21} = \frac{-X_q'}{R_a^2 + X_d'X_q'} & y_{12} &= \frac{X_d'}{R_a^2 + X_d'X_q'} & y_{22} &= y_{11}
\end{align*}
\]

Appendix VII Equations of Exciter:

![Diagram of the transfer block of the IEEE type-1 exciter]

The transfer block of the IEEE type-1 exciter
\[
\begin{bmatrix}
\Delta u_f \\
\Delta u_i \\
\Delta u_o
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34}
\end{bmatrix}
\begin{bmatrix}
\Delta u_f \\
\Delta u_i \\
\Delta u_o
\end{bmatrix}
+ 
\begin{bmatrix}
b_{13} \\
b_{23} \\
b_{33}
\end{bmatrix}
\begin{bmatrix}
\Delta I_q \\
\Delta I_d \\
\Delta U_q \\
\Delta U_d
\end{bmatrix}
\]

where:

\[
a_{11} = -\frac{k_x + S_x}{T_x} \quad a_{12} = \frac{k_f}{T_f} a_{11} \quad a_{22} = \frac{-1}{T_f} \quad a_{23} = -a_{24}
\]

\[
a_{31} = \frac{1}{T_e} \quad a_{32} = \frac{k_f}{T_f T_e} \quad a_{33} = \frac{-1}{T_a} \quad b_{33} = \frac{k V_x}{T_a V_m} \quad b_{34} = \frac{k V_o}{T_a V_m}
\]

Appendix VIII Equations of PSS

The transfer block of Power System Stabilizer

\[
\begin{bmatrix}
\Delta \hat{V}_{ps} \\
\Delta \hat{V}_1 \\
\Delta \hat{V}_2 \\
\Delta \hat{V}_3
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{bmatrix}
\begin{bmatrix}
\Delta V_{ps} \\
\Delta V_1 \\
\Delta V_2 \\
\Delta V_3
\end{bmatrix} + 
\begin{bmatrix}
c_1 \\
c_2 \\
c_3 \\
c_4
\end{bmatrix}\Delta \omega
\]

where:

\[
a_{11} = -\frac{1}{T_x} \quad a_{12} = \frac{k T_x}{T_x} \left( 1 - \frac{T_3}{T_4} \right) \quad a_{13} = \frac{k T_x}{T_x} \left( 1 - \frac{T_3}{T_2} \right) \quad a_{14} = -\frac{k T_x T_3}{T_x T_2 T_4}
\]

\[
a_{22} = -\frac{1}{T_4} \quad a_{23} = \frac{1}{T_4} \left( 1 - \frac{T_3}{T_2} \right) \quad a_{24} = -\frac{T_3}{T_x T_4} \quad a_{33} = -\frac{1}{T_x}
\]

\[
a_{34} = -1 \quad a_{44} = a_{41} \quad c_1 = -a_{14} \quad c_2 = -a_{24} \quad c_3 = -a_{34} = c_4
\]

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Appendix IX Equations of Governor:

The transfer block of the hydraulic turbine and its governor

\[
\begin{bmatrix}
\Delta P_r \\
\Delta \xi \\
\Delta \mu
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
\Delta P_r \\
\Delta \xi \\
\Delta \mu
\end{bmatrix} +
\begin{bmatrix}
b_1 \\
b_2 \\
b_3
\end{bmatrix} \Delta \omega
\]

where:

\[
a_{11} = -\frac{2}{T_w} = -a_{13} \quad a_{12} = \frac{2}{T_s} \quad a_{21} = \left(\frac{1}{T_s} + \frac{k_a + k_y}{T_s}\right) \quad a_{22} = \frac{k_a}{T_s} \quad a_{32} = -\frac{1}{T_s}
\]

\[
b_1 = \frac{2k_y}{T_s} \quad b_2 = -\frac{k_y (k_a + k_y)}{T_s} \quad b_3 = -\frac{k_y}{T_s}
\]

The transfer block of the fossil turbine and its governor
When $\alpha = 1$:

\[
\begin{bmatrix}
\Delta \dot{P}_r \\
\Delta \dot{\mu}
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta P_r \\
\Delta \mu
\end{bmatrix} +
\begin{bmatrix}
b_1
\end{bmatrix}
\Delta \omega
\]

where:

\[
a_{11} = -\frac{1}{T_{CH}} = -a_{13} \quad a_{22} = -\frac{1}{T_s} \quad b_2 = \frac{k_s}{T_s}
\]

when $\alpha \neq 1$:

\[
\begin{bmatrix}
\Delta \dot{P}_r \\
\Delta \dot{\xi} \\
\Delta \dot{\mu}
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{22} & a_{22} & a_{23} \\
a_{32} & a_{33} & a_{33}
\end{bmatrix}
\begin{bmatrix}
\Delta P_r \\
\Delta \xi \\
\Delta \mu
\end{bmatrix} +
\begin{bmatrix}
b_1 \\
b_2
\end{bmatrix}
\Delta \omega
\]

where:

\[
a_{11} = -\frac{1}{T_{RH}} \quad a_{12} = \frac{1}{T_{RH}} - \frac{\alpha}{T_{CH}} \quad a_{13} = \frac{\alpha}{T_{CH}}
\]

\[
a_{22} = -\frac{1}{T_{CH}} = -a_{23} \quad a_{33} = -\frac{1}{T_s} \quad b_3 = \frac{k_s}{T_s}
\]
Appendix X  The illustration for the convergence of the flow chart Fig.6-9

- The value of $\Delta \eta$:

Since the effect of $B(T)$ in (6-49) is very clear, it is omitted in following context. Thus:

$$\phi = \eta + \sum_{j=1}^{n_1} \sum_{i=1}^{n_2} \max^2 \left[ (\sigma_j - \eta) \right]$$  \hspace{1cm} (X-1)

At the point $T^{(k)}$, assume that we have:

$$\sigma_1^{(k)} \geq \sigma_2^{(k)} \geq \sigma_3^{(k)} \geq \cdots$$ \hspace{1cm} (X-2)

$$\eta^{(k)} = \sigma_1^{(k)}$$ \hspace{1cm} (X-3)

Thus the gradient is:

$$\nabla \phi^{(k)} = [0 \hspace{0.5cm} 0 \cdots 0 \hspace{0.5cm} 1]^T$$ \hspace{1cm} (X-4)

where the zero elements are corresponding to the components of $T$ and the last element is corresponding to the mediate variable $\eta$.

Along with the direction $-\nabla \phi^{(k)}$ search for the optimal pace $\Delta \eta$, obviously all eigenvalues are kept invariant and only the mediate variable $\eta$ is shifted from $\eta^{(k)}$ to $\eta^{(k-1)}$. Assume that:

$$\eta^{(k+1)} = \eta^{(k)} - \Delta \eta$$ \hspace{1cm} (X-5)

Obviously $\Delta \eta \geq 0$
Assume that:

\[ \sigma_2^{(k)} \ll \sigma_1^{(k)} \quad \text{(X-6)} \]

Now we find the optimal value \( \Delta \eta \):

\[
\Delta \phi^{(k)} = \phi^{(k+1)} - \phi^{(k)} = \\
= (\eta^{(k)} - \Delta \eta) + \left[ \sigma_1^{(k+1)} - (\eta^{(k)} - \Delta \eta) \right]^2 - \eta^{(k)}
\]

Note that \( \eta^{(k)} = \sigma_1^{(k)} = \sigma_1^{(k+1)} \):

\[
\Delta \phi^{(k)} = \Delta \eta^2 - \Delta \eta
\quad \text{(X-7)}
\]

Obviously \( \Delta \eta \rightarrow \frac{1}{2} \) minimizes \( \Delta \phi^{(k)} \). Up to now, the assumption (X-6) can be more exactly given by (X-8):

\[
\sigma_1^{(k)} - \sigma_2^{(k)} > \frac{1}{2}
\quad \text{(X-8)}
\]

By means of the line search and with the assumptions (X-3) and (X-8), we conclude that:

\[
0 \leq \Delta \eta \leq \frac{1}{2}
\quad \text{(X-9)}
\]

If the truncate error of computer is neglected, namely the computation precise is desired, the optimal value of \( \Delta \eta \) is 1/2.
Now we assume that

$$\sigma_1^{(k)} - \sigma_2^{(k)} = \delta_i \leq \frac{1}{2}$$  \hfill (X-10)

and find the optimal value of $\Delta \eta$.

$$\Delta \phi^{(k)} = \phi^{(k+1)} - \phi^{(k)} =$$

$$= (\eta^{(k)} - \Delta \eta) + \left[\sigma_1^{(k+1)} - (\eta^{(k)} - \Delta \eta)\right]^2 + \left[\sigma_2^{(k+1)} - (\eta^{(k)} - \Delta \eta)\right]^2 - \eta^{(k)}$$

$$= \Delta \eta^2 + (- \delta_i + \Delta \eta)^2 - \Delta \eta$$  \hfill (X-11)

When

$$\Delta \eta \to \frac{1}{2 + 2[1 + 2\delta_i]}$$  \hfill (X-12)

$\Delta \phi^{(k)}$ reaches its minimum. If $\delta_i = 0$ (i.e. $\sigma_1^{(k)} = \sigma_2^{(k)}$), then $\Delta \eta = 1/4$. And if $\delta_i = 1/2$, then $\Delta \eta = 1/2$. Hence, no matter what value of the deviation of $\sigma_1^{(k)} - \sigma_2^{(k)}$ taken, we have:

$$\frac{1}{4} \leq \Delta \eta \leq \frac{1}{2}$$  \hfill (X-13)

From the above analysis, we can induce that:

$$\frac{1}{2 \times n_z} \leq \Delta \eta \leq \frac{1}{2}$$  \hfill (X-14)
The lower limit corresponds to the worst computation case, when the real part of all eigenvalues have the same value, while the upper limit corresponds to the best computation case, when $\delta = 1/2$.

Based on the above analysis, the line search can be bypassed and $\Delta \eta$ can be directly set to a concrete value satisfying (X-14). If $\Delta \eta$ is too large, the number of active eigenvalue is also large, it results to an increased the burden for computing the gradient of the barrier function. On the contrary, if $\Delta \eta$ is too small, the computation efficiency is low. Numerical experiment indicates that $\Delta \eta$ can be taken a relatively large value in the first several time iteration and $\Delta \eta$ should be gradually decreased late.

- The shift of the eigenvalue

Without losing generality, at the point $T^{(k)}$, assume that we have:

$$\sigma_2^{(k)} \leq \eta^{(k)} \leq \sigma_1^{(k)} \quad (X-15)$$

Thus the gradient of the objective:

$$\nabla \phi_T^{(k)} = 2(\sigma_1^{(k)} - \eta^{(k)}) \frac{\partial \sigma_1^{(k)}}{\partial T} \bigg|_{T=T^{(k)}} \quad (X-16)$$

$$\nabla \phi_\eta^{(k)} = 0 \quad (X-17)$$

By the line search, we obtain:

$$\alpha^{(k)} = \min \left[ \phi(T^{(k)} - \alpha^{(k)} \nabla \phi_T^{(k)}) \right]$$

and at the point $T^{(k+1)} = T^{(k)} - \alpha^{(k)} \nabla \phi_T^{(k)}$, $\sigma_1^{(k)}$ and $\sigma_2^{(k)}$ are shifted to $\sigma_1^{(k+1)}$ and $\sigma_2^{(k+1)}$ respectively. It can be affirmed that $\sigma_1^{(k+1)} < \sigma_1^{(k)}$ because the negative gradient is adopted.
as the search direction. However there are probably two shifting direction for the eigenvalue $\sigma_2^{(k)}$. One direction is that $\sigma_2^{(k)}$ is shifted toward the left on the complex plane, i.e. $\sigma_2^{(k+1)} < \sigma_2^{(k)}$ which is a favorable direction. The other one is in opposite direction, i.e. $\sigma_2^{(k+1)} > \eta^{(k)} > \sigma_2^{(k)}$. For this case, we should confirm that $\sigma_2^{(k+1)} < \sigma_1^{(k)}$. If so, the result of line search makes the largest real part of eigenvalue decrease.

From (X-15) we have:

$$\phi^{(k)} = \eta^{(k)} + \left(\sigma_1^{(k)} - \eta^{(k)}\right)^2$$

Since $\sigma_1^{(k+1)} < \sigma_1^{(k)}$, and assume that $\sigma_1^{(k+1)}$ is larger than $\eta^{(k)}$, then:

$$\phi^{(k+1)} = \eta^{(k)} + \left(\sigma_1^{(k+1)} - \eta^{(k)}\right)^2 + \left(\sigma_2^{(k+1)} - \eta^{(k)}\right)^2$$

$$\Delta \phi^{(k)} = \phi^{(k+1)} - \phi^{(k)} =$$

$$= \left(\sigma_1^{(k+1)} - \eta^{(k)}\right)^2 + \left(\sigma_2^{(k+1)} - \eta^{(k)}\right)^2 - \left(\sigma_1^{(k)} - \eta^{(k)}\right)^2 < 0$$

$$(X-18)$$

\[\begin{align*}
\left(\sigma_1^{(k+1)} - \eta^{(k)}\right)^2 + \left(\sigma_2^{(k+1)} - \eta^{(k)}\right)^2 & < \left(\sigma_1^{(k)} - \eta^{(k)}\right)^2 \\
\therefore \quad \sigma_1^{(k+1)} & < \sigma_1^{(k)} \\
\therefore \quad \sigma_2^{(k+1)} & < \sigma_1^{(k)} \quad \text{(see also Fig. X-1)}
\end{align*}\]
The above analysis has demonstrated that each iteration can reduce the value of the largest real part in all eigenvalues.

- The convergence of the computation

By means of the physical significant of the eigenvalues, we know that it is impossible to infinitely reduce the value of the largest real part in all eigenvalues. In mathematically, when the gradient $\nabla \phi^{(k)} \to 0$ at the point $T^{(k)}$, the computation is said to be convergent. In generally:

\[
\sigma_1^{(k)} \geq \sigma_2^{(k)} > \eta^{(k)}
\]  \hspace{1cm} (X-19)

\[
\sigma_i^{(k)} \ll \eta^{(k)} \quad i = 3, 4, \ldots, n_z
\]  \hspace{1cm} (X-20)
thus, at the point $T^{(k)}$ the gradient of the barrier function is

$$\nabla \phi_T^{(k)} = 2[\sigma_1^{(k)} - \eta^{(k)}] \frac{\partial \sigma_1^{(k)}}{\partial T} + 2[\sigma_2^{(k)} - \eta^{(k)}] \frac{\partial \sigma_2^{(k)}}{\partial T} \rightarrow 0 \quad (X-20)$$

From (X-20), it can be seen that: at the point $T^{(k)}$, if one expects to decrease $\sigma_1^{(k)}$ by modulating the control parameters $T^{(k)}$, the value of $\sigma_1^{(k)}$ will increase. So the convergence must occur at this point, namely the eigenvalue with the largest real part tends to shift toward left and the eigenvalue with the second largest real part tends to shift toward right on the complex plane and $\sigma_2^{(k)} = \sigma_1^{(k)}$. 