EVALUATING FLEXIBLE CAPACITY STRATEGY UNDER DEMAND UNCERTAINTY

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Certification of Originality

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Abstract

Flexible capacity strategy (FCS) has been well adopted in different industries, but limited analytical studies have investigated it and almost all of them focus on monopoly or duopoly. None of them has investigated firms' decisions when facing multiple competitors and two strategies simultaneously. Moreover, there is an absence of research on addressing the issue of flexibility degree, which means to what extent FCS can be fully exploited. Furthermore, the value of long term FCS has not been studied in existing research. To fill these research gaps, this thesis investigates FCS from different perspectives with uncertain demand.

First, this thesis identifies five possible production strategies to evaluate long term FCS with consideration of the production cost structure. By conducting a comprehensive series of comparative analyses between different strategies, this thesis evaluates long term FCS and provides the optimal production strategies under different costing environments. It is shown that FCS can benefit or damage a firm's profit.

Second, this thesis constructs a two-strategy asymmetric oligopoly competition model consisting of r firms with FCS and s firms with in-flexible capacity strategy (IFCS) under demand uncertainty. This thesis characterizes capacity and production decisions of each firm at equilibrium. The results verify that all the flexible firms make the same decisions at equilibrium, and so do all the in-flexible firms. It is shown that production cost is one of the key factors affecting whether a firm should adopt FCS or not.

Third, this thesis further investigates the endogenous flexibility of FCS in an oligopoly model by allowing firms to freely switch their strategies to maximize their profits. The results show that two strategies may always coexist under some conditions regardless of the number of firms. It is shown that the strategies that eventually survive in a market are insensitive to the total number of firms under certain environments but are sensitive under other environments. This result is further extended to a perfect competition environment. A practical approach is proposed to determine at equilibrium the exact numbers of firms adopting FCS and IFCS under any given demand distribution.

Last but not least, this thesis probes into flexibility degree to quantify the performance of FCS in competition. This thesis develops a duopoly competition model in which two firms compete with each other with different flexibility degrees. The results characterize the equilibrium of the competition and show that a firm with a higher flexibility degree always secures a higher profit when the capacity costs are identical in the two firms.

The research results highlight the strategic importance of the concept of FCS, provide insights on successful implementation of FCS, and propose suggestions to avoid the potential risk or damage of FCS.

Table of Contents

List of Tables

List of Figures

Chapter 1
Introduction

1.1 Motivation and Background

To enhance competitiveness and hedge against demand uncertainties, *chase strategy* has been a prevailing operation strategy in real business in the past two decades. The chase strategy enables a firm to better match supply and demand by adjusting its production level. Various operational ways of the chase strategy include varying workforce size by hiring or laying off workers, varying production rates through overtime or idle time, using part-time workers, subcontracting, deploying multi-trained employees, setup time reduction and design for manufacturing.

A number of successful businesses in the real world have demonstrated the advantages of the chase strategy, such as Anheuser-Bsch (Heizer and Render, 2008), Snapper's mower (Heizer and Render, 2008) and Dell (Magretta, 1998). Some data also indicate that the chase strategy has been widely used in different industries. In the 1990s, roughly 90% of U.S. business and 95% of Fortune 500 firms used some forms of temporary employments (Kucera, 2009). A survey showed that the chase strategy was preferred in 19 industries and the modified chase strategy was preferred in 12 industries over a total of 42 industries (Buxey, 2005). However, there are still a number of firms adopting the traditional *level strategy* that maintains a stable production level in firms. These firms argue that the level strategy is advantageous to ensure good quality of product and employee loyalty (Colvin, 2009). According to a number of empirical studies, both chase strategy and level strategy coexist in many industries, if not all. For example, in a study covering industries of electronics, machinery and automotive suppliers in 7 countries, roughly 44% of 211 firms equipped volume flexibility, which is the nature of chase strategy from strategic perspective (Hallgren and Olhager, 2009). By using the data from machinery and machine tool industries in Taiwan, the adoption ratio of volume flexibility is about 28% (23 firms out of a total 83 firms) (Chang et al., 2003).

According to a survey conducted in 2004, chase strategy is the main strategy in 19 industries over a total of 42 industries (Buxey, 2005). These industries include ice cream, beer, petrol and oils, greetings and seasonal cards, refrigerators, etc. Meanwhile, level strategy is the main strategy in industries of motor vehicle batteries, color television sets, cricket balls and motor car radiators. From here, one can see that when demand fluctuation is larger, an industry favors the chase strategy more.

Moreover, some research and surveys have shown that an increasing number of firms are using a *mixed strategy* with a combination of the chase strategy and the level strategy, e.g., ABB Motors in Sweden, in which 70% orders are made-to-stock and 30% orders are made-to-order (Bengtsson and Olhager, 2002). Another example is Nike's repackaging facility in Memphis. Nike employed 120 permanent employees and 60 to 225 temporary employees by Norrell Service (Kucera, 2009). A diversity of capacity strategies makes competition more complex and it is more difficult for firms to choose the optimal strategy and evaluate its effectiveness. Such complexity of competition gives rise to the following questions. (1) What is the long term impact of each strategy on each individual firm and on the entire market? (2) Which strategy is the optimal under different costing and competition environments? (3) What will be the market like in the face of strategy competition among multiple firms? (4) How should the mixed strategy be evaluated against the chase strategy and level strategy? (5) How do we distinguish the strategy differences between firms adopting the mixed strategy?

This study aims to evaluate the chase strategy from a few perspectives in various environments to address the above research questions. The chase strategy is considered as a flexible capacity strategy (FCS) and the traditional level strategy is considered as an in-flexible capacity strategy (IFCS) throughout this thesis. The research results are able to evaluate the effectiveness of FCS and provide management suggestions for successful implementation of FCS under different environments. FCS equips firms with the ability to postpone their production until knowing the real demand by keeping the capacity greater than or equal to the production level. Such ability enables firms: (1) to avoid any production waste under demand uncertainty; (2) to be in a favorable position by having more flexibility to adjust their throughput in severe competition. Figure 1.1 illustrates firms' decision-making process sequentially with FCS and IFCS. In the capacity decision stage, all the firms decide their capacity amounts to maximize their expected profits throughout the entire decision-making process. In the production decision stage, firms adopting IFCS have to make production decisions before knowing the actual demand while firms adopting FCS postpone their production decisions until knowing the actual demand. In other words, firms with FCS have flexibility in adjusting their production levels. In the pricing stage, all the firms compete on quantity in the same market, i.e., Cournot competition. The market price of the product is determined by the product demand and the total product quantity in the market.

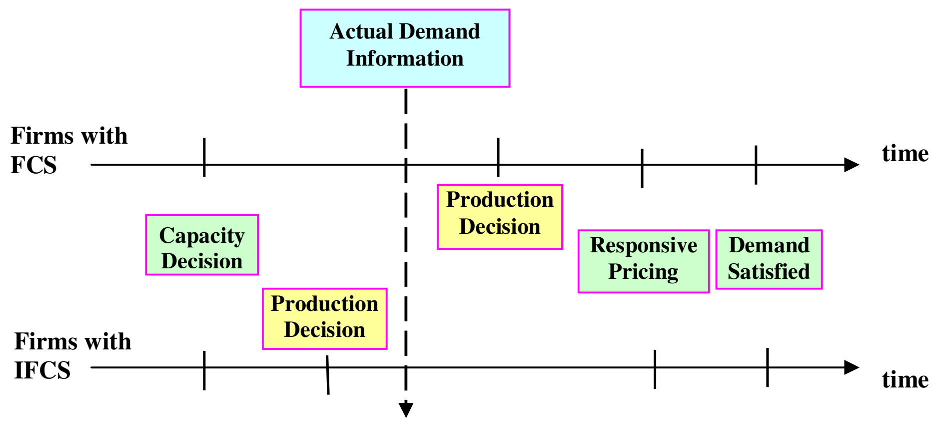


Figure 1.1: Decision-making process of firms with FCS/IFCS.

To explain the complexity of adopting FCS in various industries, four crucial issues should be taken into account as listed below.

1.1.1 Evaluation of long term FCS

The first crucial point is to evaluate long term FCS, which has been a debatable issue for FCS adoption in real business (Colvin, 2009). Considering different production capabilities that are reflected by the production cost structure in different market periods, firms can perform FCS differently. This motivates us to investigate long term FCS and identify its effectiveness. In each market period, a firm has to seek the most effective investments to augment its profit based on the existing production capability. The existing production capability is determined by several factors, such as the technology adopted, the equipment used, the organizational features, and the components of cost. On the other hand, in the long run spanning across a few market periods, making investments to improve the existing production capability is also a practical way to augment a firm's profit. With application of FCS, a firm needs to consider the interplay between the FCS complication and the existing production capability that is reflected by its production cost structure. This makes the evaluation of long term FCS crucial to successful implementation of FCS.

1.1.2 Competition environment

The second crucial issue is the competition environment that involves different numbers of firms in different industries. In a competitive environment consisting of a few firms with either FCS or IFCS, each firm has to compete with other firms in the same market. Facing multiple rivals with the same or different strategy, each firm needs to consider not only its decisions alone, but also the interplay among firms. Furthermore, every decision of each individual firm affects the market price and the resulting expected profit of each firm. Besides the coexistence of two strategies, FCS and IFCS, there are a few other factors influencing product sales and profits of firms. These factors include the number of firms adopting each strategy, the capacity costs of the two strategies, the production cost, and the market profit potential. Understanding in what ways these factors affect the profit of each firm is key to making the right decisions for firms in severe competition.

1.1.3 Quick change in strategy

The third key point is quick change in firms' strategies. It is natural that firms always seek for the most effective means to maximize their profits. Facing globalization and fierce competition, if a firm thinks the other strategies are helpful to augment its profit, and then it changes its current strategy. Therefore, firms compete with one another not only in operational decisions, but also in strategy choice. This strategy competition continues until all the firms cannot increase their profits by switching strategies and the market reaches a stable status at equilibrium. Understanding of this stable status of the market is helpful to predict the development of a market and choose the optimal strategies under various environments.

1.1.4 Flexibility degree

The fourth crucial consideration is flexibility degree, which means to what extent FCS can be fully exploited. In fact, both FCS and IFCS are two extremes of FCS implementation, one is full implementation and the other is none at all. However, there is a large space between these two extremes actually. In real business, many firms adopt a strategy called the mixed strategy, which is a combination of these two extremes. This means many firms implement FCS partially to different extents. This partial implementation of FCS can be due to various reasons. These reasons include changes of economic environments, political reasons, employee moral guarantee, labor, administrative regulars, technical problems, demand uncertainty, organization changes, and regional differences. All these factors cause different extents of FCS implementation and they distinguish firms' actual strategies. Therefore, how to quantify flexibility degree is key to evaluating FCS and explaining the variety of FCS performance in the real world.

1.2 Research Objectives

Based on the motivation introduced above, the research objectives of this study are as follows.

1. Evaluating the long term FCS with consideration of production cost curves;
2. Investigating the oligopoly competition involving multiple flexible and in-flexible firms, and identifying the competition equilibrium;
3. Characterizing the endogenous flexibility of FCS among multiple firms;
4. Formulating the mixed strategy and investigating the flexibility degree.

1.3 Research Problem

To comprehensively investigate FCS, we first discuss the nature of FCS in this section. Then, to address the effects of the aforesaid four factors on decisions of firms, i.e., evaluation in the long run, competition environments, quick change in strategies, and flexibility degree, we investigate them in four chapters respectively in this thesis.

1.3.1 Nature of FCS

To understand the nature of the chase strategy from the strategic research perspective, we have reviewed the research agenda on manufacturing flexibility by Gerwin (1993). In his study, manufacturing flexibility is classified into severn categories, including four market-oriented categories and three process-oriented categories. Particularly, he points out that volume flexibility is derived from aggregate product demand uncertainty and its adaptive methods include high capacity limits and subcontracting. Comparing the chase strategy and the concept of volume flexibility, we conclude that volume flexibility is the nature of the chase strategy from strategic research perspective. In other words, the chase strategy is detailed operations to manifest volume flexibility from the perspective of operations management.

1.3.2 Evaluation of long term FCS

To evaluate long term FCS, we consider the total production cost structure in the model. The total production cost structure reflects the long term impact of the production capability on products. Some research has shown that the long run total production cost structure is associated with the technology flexibility in varying output levels under demand fluctuations. A higher level of technology flexibility leads to a flatter marginal production cost. Therefore, examination of FCS in the long run can be conducted by testing FCS and IFCS under environments with different technology flexibilities. Comparative analyses of the expected profits under different scenarios provide the pros and cons of FCS in the long run.

In this study, technology flexibility is measured by flexible level. Flexible level is reflected by an endogenous variable in a quadratic total production cost function. The means to improve technology level is expressed as a flexible technology investment in this thesis. Flexible technology investments can be made in various operations such as using advanced technology and upgrading the equipment of plants. The study establishes five possible production strategies comprising decisions on using FCS and flexible technology investment. Each strategy is carried out by a decision-making operation process embracing technology level, capacity amount, production quantity and price setting. By conducting a comprehensive series of comparative analyses between different strategies, the study also evaluates FCS under different costing and technology level environments.

1.3.3 An asymmetric oligopoly model

To emphasize the impact of the competition environment on firms' decisions and profits, an asymmetric oligopoly model is established consisting of r flexible firms and s in-flexible firms. All the firms, both flexible and in-flexible, compete in the same market with the same market price that is determined by the product demand and total product quantity in the market. Both FCS and IFCS strategies are carried out by a decision-making process, which is composed of capacity planning, production procedures, and market pricing.

1.3.4 Endogenous flexibility of FCS among n firms

To investigate the impacts of firms' strategy changes on individual decisions of each firm and the entire market, we study the endogenous flexibility of FCS in a market involving totally n firms. In a competition model with n firms, the firms are allowed to freely choose and switch their strategies to augment their profits. With strategy changes, firms alter their optimal decisions in each stage. Furthermore, the market structure also changes when firms switch their strategies. Therefore, there is an unstable period in which firms compete with each other by seeking the optimal strategies. This unstable period continues until all the firms can no longer augment their profits by switching strategies and by this time the market reaches a stable status. We name such a stable market "Final Equilibrium" throughout this thesis. The eventual surviving strategies, as well as the exact number of firms adopting each strategy constitute the Final Equilibrium.

1.3.5 Duopoly competition model with different flexibility degrees

Regarding the issue of flexibility degree, this study establishes a duopoly competition model in which two firms compete with each other with FCS of different flexibility degrees scaling from zero to 100%. Flexibility degree is defined as the percentage of the difference between a firm's production upper bound (total capacity) and production lower bound (guaranteed or unchanged production level) over its total capacity. It reflects the extent to which FCS is exploited. A percentage zero represents the IFCS situation while a percentage of 100 represents the FCS situation in the aforementioned oligopoly model. Any other percentage between 0 and 100 represents the mixed strategy under which a firm's flexibility capability varies between IFCS and fully FCS.

1.4 Originality of the Study

According to the literature review to be given in Chapter 2, the four objectives in this study have not been addressed in the literature.

- There is no research to investigate the flexibility in manufacturing in the long run with consideration of production cost curves.

- There is no research to study the flexibility in manufacturing in an asymmetric oligopoly market involving two strategies and multiple firms simultaneously.

- The endogenous flexibility in a market involving multiple firms has not been addressed in previous research.

- The widely used mixed strategy has not been investigated by scholars from the modeling perspective. Further, there is no research to investigate the flexibility degree of FCS which reflects partial implementation of FCS.

Therefore, this study is original in the research on manufacturing flexibility.

1.5 Results of the Study

1.5.1 Evaluating long term FCS

The study formulates five possible production strategies comprising decisions on using FCS and flexible technology investment. For each strategy, the optimal operational decisions are calculated. With comparative analyses between different strategies, we show how market uncertainty, production cost structure, operation timing, and investment costing environments affect a firm's strategic decisions. The results show that there are no sequential effects of the above two investments. We also illustrate how flexible technology and flexible capacity affect a firm's profit under fluctuating demands. The results point out that flexible technology investment earns for a firm the same or a higher profit, whereas flexible capacity investment can be beneficial or harmful to a firm's profit. Moreover, we prove that more flexibility does not guarantee a higher profit. We also identify the environments in which each possible strategy combination can be the optimal strategy, i.e., no flexibility at all, only flexible capacity, only flexible technology, and both flexible technology and flexible capacity.

1.5.2 FCS in an asymmetric oligopoly competition model

Focusing on FCS in an asymmetric oligopoly competitive market involving r flexible firms and s in-flexible firms under demand uncertainty, we characterize the equilibrium of an asymmetric oligopoly competition. We find that firms adopting the same strategy make the same decisions and obtain the same profit regardless of the number of firms adopting each of two strategies, the strategy adopted, demand uncertainties, and costing environments. We prove that depending on the costing environment, the optimal strategy can be either FCS or IFCS. Further, contrary to the intuition that increasing costs are always harmful to a firms' profit, we find that firms adopting FCS can benefit from an increasing production cost when there are enough in-flexible firms existing in the market. Moreover, previous research of FCS on monopoly, duopoly, and symmetrical oligopoly are shown to be special cases of our model.

1.5.3 Endogenous flexibility of FCS in a competitive market with n firms

The characterization of the endogenous flexibility of FCS in a competitive market with a total of *n* firms yields the surviving strategies after strategy competition. It further mathematically justifies that only effective strategies can survive in a market with profit potential. We find that the surviving strategies after strategy competition are insensitive to the total number of firms under certain environments but are significantly sensitive under other environments. The technical conditions of the classification of the costing environment are provided. We theoretically prove that perfect competition is only a special case of oligopoly competition when the total number of firms tends to infinity. Moreover, the study proposes an approach to practically determine the exact numbers of flexible and in-flexible firms in the market with endogenous flexibility of FCS under any given demand distribution and any given number of firms. The theoretical justification is also provided. Numerical examples are used to demonstrate the approach.

1.5.4 FCS in a duopoly competition model with flexibility degree

Considering the flexibility degree of FCS, we establish a general mathematical model that can be used to simulate a full FCS, a mixed strategy and a zero FCS. The pattern of the optimal decisions with a certain flexibility degree is identified. Based on the conclusions of the monopoly model, we establish a duopoly competition model in which two firms with their respective flexibility degrees varying from zero to 100%. It is proved that two firms with the same flexibility degrees make the same optimal decisions under demand uncertainty. In an asymmetric duopoly model, the relationship between two firms' optimal capacities is largely restricted by their flexibility degrees. The mathematical conditions are provided. We characterize the Nash equilibrium of the competition. Numerical examples show that a firm's capacity increases as its flexibility degree increases, but decreases as the rival's flexibility degree increases.

1.6 Flowchart of the Thesis

A flowchart of this thesis is shown in Figure 1.2. Relevant studies are reviewed to find out the research gaps in Chapter 2. The system features are described in Chapter 3. Chapter 4 evaluates long term FCS with consideration of cost. Chapter 5 discusses an asymmetric oligopoly competition model under demand fluctuations and characterizes the equilibrium of the competition. Chapter 6 further investigates endogenous flexibility of the asymmetric oligopoly competition model. The issue of flexibility degree is discussed in a duopoly competition model in Chapter 7. Finally, Chapter 8 presents some conclusions and recommendations for future research. All proofs are included in the Appendix-I.

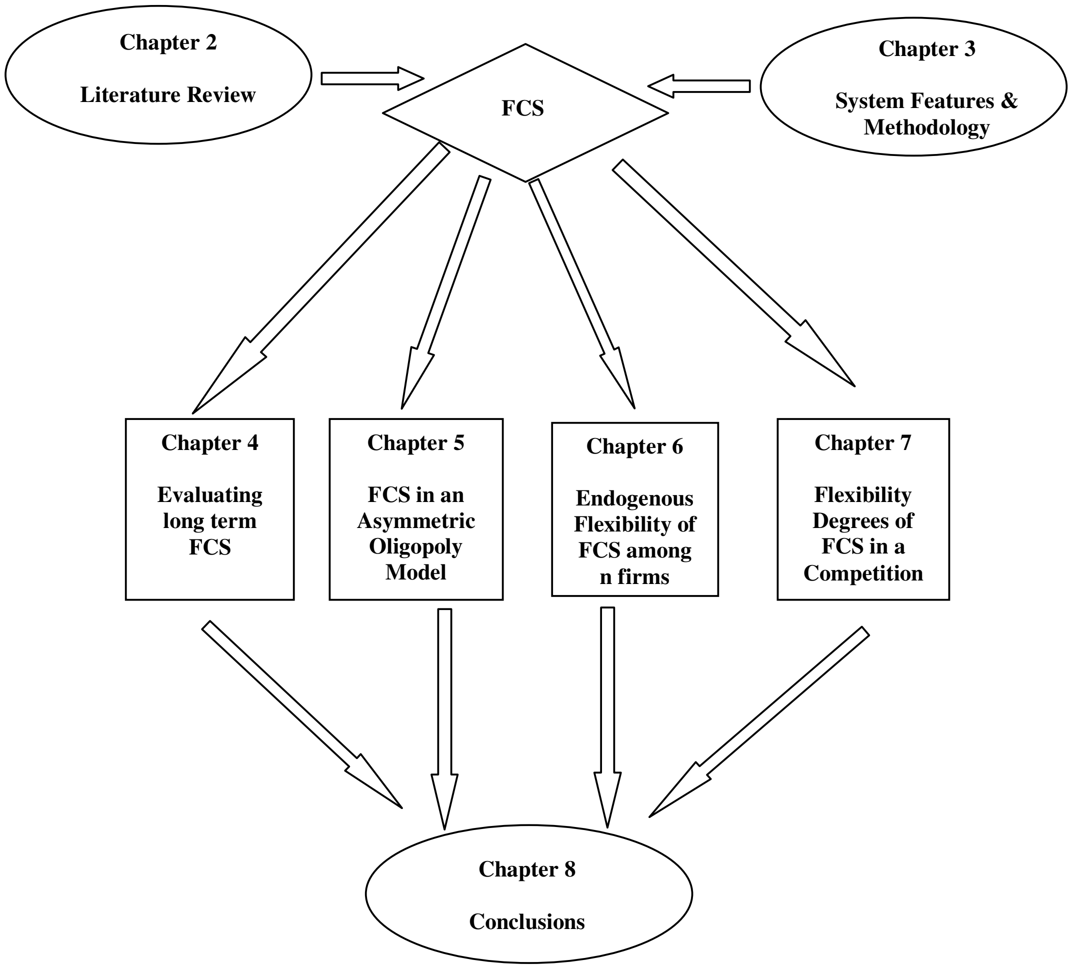


Figure 1.2: Flowchart of the Thesis.

Chapter 2

Literature Review

In the literature, there are some studies investigating flexibility in firms manufacturing process to hedge against demand uncertainty. Most of these studies focused on monopoly model to capture the features of flexibility in manufacturing; limited studies addressed the flexibility in a duopoly model to consider the effects of competition; and studies to examine and evaluate the flexibility in a symmetric oligopoly model are also scarce. Moreover, we realize that there is an absence in studying the flexibility in manufacturing in an asymmetric oligopoly model involving two strategies and multiple competitors simultaneously. All these studies assume that production cost structure is unchanged during the manufacturing process. On the other hand, a few other studies investigated firms' abilities to hedge against the demand uncertainty by improving the production cost structure in the long run. These studies focused on the relationship between technology level, production cost structure and firms' ability to hedge against demand uncertainty spanning a few market periods.

This chapter is divided into three sections. Section 2.1 reviews relevant studies about manufacturing flexibility under uncertain demand without considering the effects of production technology level in the long run. Section 2.2 focuses on research about manufacturing flexibility in a competition. Section 2.3 discusses previous research that focuses on relationship between technology level and firms' capability to hedge against demand uncertainty. In Section 2.4, some research gaps are discussed.

2.1 Research on Manufacturing Flexibility

Slack (1987) studies the manufacturing flexibility in form of empirical observations. With respect to manufacturing flexibility on theoretical research and applied work, Gerwin (1993) establishes an agenda for flexibility studies from a strategic perspective. He indicates the value in studying generic flexibility strategies, the flexibility dimensions, methods of delivery, and ways of evaluating and changing the flexibility of a process. The first stream investigates the effects of flexibility when a firm makes its decisions under demand uncertainty.

One type of flexibility in supply chains can be referred to as early or late differentiation, which is embedded in the real case of Hewlett-Packard's (HP) distribution centre (DC) localization strategy for Deskjet-plus printer division (Lee et al., 1993). In order to respond to demand uncertainty with large variety in different countries, the factory makes some design changes to the product. It produces a generic product without the power supply module and manual. The items of this generic product are shipped to non-US distribution centres and finished localization there. This DC-localization strategy results in 18% reduction of the total inventory investment. Such design for localization enables the firm to have a flexible production process to delay the customization until needed. Lee and Tang (1997) present a classification of possible design changes in the production and distribution process that leads to delayed product differentiation. They provide an analysis on determining the optimal operation sequences in order to achieve the optimal operational performance. Based on the case of HP, Aviv and Federgruen (2001a) examine the effects of environment on postponement strategy implication, i.e., under which environment, postponement will yield the major cost reduction. They consider multi-item inventory systems with random and seasonally fluctuating demands. There is a two-phase product-distribution process in their model. The generic products are manufactured in the first phase, whereas differentiating options and features are finished in the second phase. They investigate the benefits of various delayed product differentiation strategies, as well as the trade-off between the capacity and inventory investments. Aviv and Federgruen (2001b) study the postponement strategy in a Bayesian framework with unknown parameters of demand distribution. Ruetze (2006) shows that postponement strategy is adopted widely in different industries.

Besides the strategy to postpone differentiation, product flexibility (or resource flexibility) is another type of strategy about flexibility in supply chains. Such strategy enables a firm to keep a trade-off between the cost of flexibility and the ability to hedge against the variety of demand. Acquiring such flexibility, a firm can use a flexible resource to manufacture all products, while other dedicated resources can only be converted into dedicated products.

Focusing on evaluating the benefits of using a flexible production technology in flexible manufacturing systems (FMSs), Fine and Freund (1990) develop a monopoly product-flexible capacity investment model. A firm's decision making process can be divided into two stages. Capacity decisions are made in the first stage before demand information revelation. In the second stage, the firm decides on the product quantities under capacity constraints after observing the actual demand. There is a trade-off between the cost of acquiring flexibility and the benefits provided by flexibility under uncertain demand. In their model, the pricing effect is implicitly considered by a concave revenue function. Following Fine and Freund's (1990) model, Van Mieghem (1998) takes into account the role of price and cost mix differentials in a firm's optimal decisions under uncertain demand. Using multi-dimensional newsvendor problem model with exogenous prices, they investigate the optimal strategy of flexible resource investment for a two-product firm. Their analysis highlights the importance of price and cost mix differentials when a firm makes decisions. They point out that investing in flexible resources is advantageous under multivariate demand uncertainty.

After Fine and Freund (1990) and Van Mieghem (1998), a number of researchers extend the research on resource flexibility along various directions. One direction of extension is to consider flexible production process. Both models in Fine and Freund (1990) and Van Mieghem (1998) analyze the situation that a totally flexible plant can process all products. Jordan and Graves (1995) turn to limited flexibility versus total flexibility by considering the relationship between products and plants in a single-stage model. Through manufacturing process, flexibility enables a plant to produce a subset of products. They develop three principles for guiding investments of flexibility: (1) try to equalize the capacity to which each product is directly connected; (2) try to equalize the total expected demand to which each plant is directly connected; and (3) try to create a circuit(s) that encompasses as many plants and products as possible. Graves and Tomlin (2003) extend the work of Jordan and Graves (1995) to a general multi-stage model. They propose a flexibility measure, and show that increasing this measure provides higher protection against supply-chain inefficiencies. Another direction of extension from Fine and Freund (1990) and Van Mieghem (1998) is to take into account price setting problem with different timing. Considering flexibility under uncertainty, Jones and Ostroy (1984) suggest that there will be opportunities to act after further information is received and current actions can influence either the attractiveness or the availability of different future actions. Therefore, it is plausible that timing difference between information and operations makes firms have different abilities to respond to uncertainty.

Van Mieghem and Data (1999) propose the concept of pricing postponement strategy. They propose six possible postponement strategies and present comprehensive analyses of them with a two-stage model. In the model, firms make three decisions: capacity investment, production quantity and price. The strategies differ in the timing of the operational decisions relative to the demand revelation. They show that compared to production postponement, price postponement makes the investment and production decisions relatively insensitive to demand uncertainty. They also consider a postponement strategy which makes ex-post decisions on price and production. Such price and production postponement strategy is adopted by Anupindi and Jiang (2008).

Following Van Mieghem and Data (1999), some other studies take account of ex-post price setting into the model on resource flexibility. Bish and Wang (2004) incorporate ex-post price setting consideration into Van Mieghem's (1998) model. The firm makes its resource investment under an uncertain demand in the first stage. In the second stage, the firm allocates its resource and sets price constrained by its earlier resource investment when the demand curves are realized. With exogenous demand realizations, they show that the flexible resource investment follows a threshold policy. Similar to Bish and Wang (2004), Chod and Rudi (2005) analyze the effects of resource flexibility and responsive pricing in a monopoly model under demand uncertainty. However, they focus on characterizing the key drivers of flexible resources by demand variability and correlation. They show that the optimal capacity of the flexible resource is always increasing in both demand variability and demand correlation. Both of their models do not consider production costs.

2.2 Research on Competition

The second stream analyzes the competition game on a variety of strategic decisions. In the literature on economics, plenty of studies have focused on firms' price competition and quantity competition. The former is referred to as Bertrand price competition model, or Bertrand-Edgeworth model, whose original reference is Bertrand (1883). The latter is referred to as Cournot quantity competition model, which can be traced back to Cournot (1838). Edgeworth (1897) takes into account capacity constraints in the Bertrand model. The existence of equilibrium is concerned by academics since payoff functions become discontinuous in such game. Dasgupta and Maskin (1986) establish two existence theorems for mixed-strategy equilibrium in games with discontinuous payoff functions.

2.2.1 Competition on price

Kreps and Scheinkman (1983) develop a two-stage duopoly model in which firms make their capacity decisions in the first stage, and compete on price in Bertrand fashion in the second stage. We refer to this model as K-S model. They identify conditions under which Cournot competition and Bertrand competition are coinciding with the unique equilibrium Cournot outcome. The conditions established in their research have been widely adopted in the game-theoretic literature within operations management. In contrast to the results of the K-S model, Davidson and Deneckere (1986) argue that the Cournot outcome is unlikely to emerge in the model in which firms decide capacities before engaging in Bertrand price competition. They argue that the results of K-S model depend critically on the assumption of how demand is rationed when the lower-priced firm cannot meet the market demand. Instead, they propose an alternative rationing rule. Both models discussed above assume deterministic demands, i.e., there is no demand uncertainty in the markets.

Hviid (1990) reformulates the K-S model by allowing sequential capacity and price choices under demand uncertainty. He assumes that both capacity and price decisions are made before a firm observes the demand realization following a uniform distribution. Different sequential choice rules give rise to various types of two-stage games. The study analyzes the consequences of uncertainty in various types of models. Particularly, Hviid (1991) focuses on price competition, with capacity constrained in a duopoly model under demand uncertainty. He demonstrates that no pure strategy Nash equilibrium exists in the price competition stage. Additionally, if capacity is endogenous and chosen before prices, this result always holds no matter if firms can observe the real demand information. This study may indicate that price competition modeled as a subgame of a two-stage model, in which firms decide capacity simultaneously followed by pricing competition, is not always a good approximation. Reynolds and Wilson (2000) investigate the effects of demand fluctuations on firms' price competition in the K-S model. Firms make their capacity decisions before observing demand whereas they set prices after demand is revealed. They show that if variation of demand exceeds a threshold, a symmetric equilibrium in pure strategies for capacity is absent. All the models discussed above on firms' competition assume that the firms compete on price after bringing productions to the market.

2.2.2 Competition on quantity

There are some studies in the literature on Cournot quantity competition model. Saloner (1987) establishes a Cournot model with two production periods without demand uncertainty. Firms choose output simultaneously in the first period. In the second period, the output becomes common knowledge and firms make decisions to determine how much more to produce before market clearance. He shows a continuum of equilibria including the Cournot and the Stackelberg outcomes. Pal (1991) generalizes Saloner's (1987) model by allowing cost differences across production periods. He finds that the continuum of equilibria vanishes for any cost differential. If cost in the second period is slightly smaller than that in the first period, there are multiple leader-follow equilibria. Gabszewicz and Poddar (1997) study a two-stage model where firms make capacity decision at the first stage and production decision at the second stage under demand uncertainty. Firms compete on quantity in the market. They prove the existence of a symmetric subgame perfect equilibrium at which firms are in excess capacity compared with the capacity they choose in the Cournot certainty equivalent game. Maggi (1996) considers two firms' investment competition under demand uncertainty.

He describes asymmetric equilibria under some general conditions. In equilibrium, one firm takes an early investment and the other firm follows a wait-and-see strategy. The emergence of asymmetric versus symmetric ex-post outcome demands on the comparison between the expected and the actual market profitability. If the market profitability is close to, or lower than, expected, firms end up with asymmetric sizes. If the market is highly profitable, then firms end up in symmetric position.

2.2.3 Manufacturing flexibility in competitions

In the literature, limited studies investigate flexibility in a competition environment. Rőller and Tombak (1990, 1993) investigate the effects of choosing different technologies in a multi-firm Cournot competition game under deterministic demand. Firms choose one type of technologies, flexible or inflexible, in the first stage and decide on the production quantities in the second stage under a Cournot competition. They show that, in equilibrium, firms more like to adopt flexible technology in a larger and/or more concentrated market. Capacity decision and demand uncertainty are not considered in their model. Boyer and Moreaux (1997) extend the study to take into account volatility and market size effects on acquiring flexible technology. Incorporating capacity investment competition into Rőller and Tombak's (1993) model, Goyal and Netessine (2007) develop a duopoly model where each firm makes three decisions: technology choice (product-flexible or product-dedicated), capacity investment and production quantities. In each decision stage, firms play a simultaneous-move non-cooperative game with complete information. They reveal the role of competition on firms investing in flexible technology under demand uncertainty. They find that flexible and dedicated technologies may coexist in equilibrium. All models discussed here concern the product flexibility (or resource flexibility), which enables firms to manufacture multiple products with a flexible resource. Incorporating the price setting problem into the flexibility competition model, Anupindi and Jiang (2008) investigate duopoly models where firms make capacity, production and price decisions in a market with uncertain demands. Capacity investment is always made ex-ante demand realization, whereas price decision is always set ex-post demand realization. The flexibility enables a firm to postpone its production decision until the actual demand is revealed. They characterize the set of equilibria in a symmetric duopoly model under a general demand structure. In addition, they investigate the stochastic-order properties of capacity and profit of flexible firms under demand with a higher variability. Furthermore, they show that the strategic equivalence of price and quantity competition among flexible firms.

Competitions in all of these researches are non-cooperative. Taking into account the relationship between competitors, i.e., cooperative and non-cooperative, Stuart (2005) considers a model with price competition following inventory decisions. He uses the biform game formalism of Brandenburger and Stuart (2004) to model the non-cooperative inventory competition and the cooperative price competition. His analysis gives rise to two scenarios. When there is no demand uncertainty, the inventory decision is equivalent to the capacity decision in Cournot competition. When there is demand uncertainty, the result is equivalent to that of Cournot competition under some conditions on the demand curve.

Another type of flexibility in a competitive market can be found in Anand and Girotra (2007). They analyze supply chain configurations, i.e., early or delayed differentiation, in a Cournot competition environment with clearance strategies. They analyze firms' choices of supply chain configuration in terms of quantities sold, profits, consumer surplus, and welfare. Normalizing all production costs to zero, they show that delayed differentiation is not the preferred supply chain configuration to respond to demand uncertainty under competition.

All the above models, discussed about flexibility competition, are static models. With the consideration of dynamic effects, Gaimon (1989) investigates the effects of new technology on a duopoly dynamic model where firms choose open-loop or close-loop strategy over time. Firms' competition can be achieved by acquiring new technology or scrapping existing capacity. He shows that under close-loop strategies there is a more restricted acquisition of new technology and a larger reduction of existing capacity than those under open-loop strategies.

Empirically, there are a number of studies investigating flexibility in various industries. Focusing on printed circuit-board plants in Europe, Suarez et al. (1996) show that different manufacturing flexibilities coexist in the same industry. Similar conclusion is drawn by Chang et al. (2003) in a study investigating machinery and machine tool industries in Taiwan. Vickery et al. (1997) identify that flexibility is one of key dimensions in the furniture industry. Caniato et al., (2009) study the Italian luxury industry. They show that the adoption of flexibility varies largely in different industries. Stratton and Warburton (2003) explore the strategic integration of agile and lean supply in apparel industry. Their results show that both flexible and in-flexible strategies coexist in the apparel industry. Similar studies include Toni and Meneghetti (2000), Brun and Castelli (2008) and Sen (2008).

2.2.4 Other relevant research on flexibility

There are some other relevant papers investigating the role of flexibility in supply chains. These papers characterize the relationship between sellers and buyers. Eppen and Iyer (1997) analyze backup agreements in fashion buying and evaluate the value of upstream flexibility for fashion merchandising. Their results indicate that backup arrangements can have an impact on expected profits and may increase the committed quantity. Deneckere et al. (1997) investigate the relationship between demand uncertainty and price maintenance in a system consisting of one manufacturer and two competitive retailers. Their results show that profit of the manufacturer and inventory at equilibrium under resale price maintenance environment are higher than those under market-clearing environment. Tsay and Lovejoy (1999) focus on quantity flexibility contracts in a supply chain. They propose local policies, which dictate the necessary actions to support flexibility promised to a customer by a supplier. Lariviere and Porteus (2001) examine a simple supply-chain contract governed by a price-only contract. They find that the manufacturer's profit and sales quantity are increasing in market size, whereas the wholesale price is dependent on the pattern of the growth of the market. Barnes-Schuster et al. (2002) analyze the effects of options in a buyer-supplier system by using a two-period model. They illustrate how flexibility is provided by options, to increase profits of both the supplier and the buyer. With consideration of supply chain risk, Tang and Tomlin (2008) investigate 5 different stylized flexibility models. Their results show that most of the benefit is obtained at low levels of flexibility. They also conclude that to mitigate supply chain risk, a firm does not need to invest in a high degree of flexibility.

2.3 Relationship between Technology Level and Production Cost Structure

A few previous studies have investigated relationship between flexible technology investment and total production cost structure. Stigler (1939) investigates production and distribution features. He assumes that production costs consist of fixed costs (representing the return on a fixed "plant") and variable costs (day labor, materials, fuel, etc.). He refers to the attributes of the production cost curve that reflects the production cost structure. Cost curves determine the manner of output to respond to demand fluctuations as "flexibility of operation". Marschak and Nelson (1962) formalize the discussion about "flexibility of operation". They persuasively argue that Stigler's (1939) notion of "flexibility of operation" relates inversely to the curvature of the total production cost, or the slope of marginal production cost. Level of "flexibility of operation" is the lowest when the average production cost rises precipitously around the minimum and marginal production cost is steep. Level of "flexibility of operation" is higher when average production cost becomes flatter and the marginal production cost is less steep. It demonstrates that the minimum average production cost varies inversely with the level of "flexibility of operation". Such relationship between the minimum average production cost and level of "flexibility of operation" indicates that system internal organization, which determines the level of "flexibility of operation" influences the unit production cost. A simulation model in Nelson (1968) displays such interactions among system internal organization, cost structure and demand fluctuation. The simulation results implicate different effects of a variety of factors on system design and control.

Empirically, Barzel (1964) examines the relationship between production function and technical change in the steam-power industry. Zarnowitz (1956) studies technology and price structure in general interdependence system and compares different models. Ghali (2003) examines the slope of marginal cost in different industries.

Following the theoretical ideas in Stigler (1939) and Marschak and Nelson (1962), Mills (1984) investigates the effect of demand fluctuation on firms' endogenous flexibility in a competitive model. It assumes that there is a trade-off between endogenous firm flexibility and static-efficiency. To outline explicit features of cost structures, he proposes a quadratic form of the total production cost curve with an endogenous flexibility variable. He establishes some properties of the competitive equilibria under demand fluctuation.

A few studies investigate technology flexibility from different aspects based on Mills' (1984) technology flexibility formulation. Mills and Schumann (1985) show that it is possible for a firm with higher minimum average production cost to compete with other firms by flexibly adjusting its production levels in a competitive market under fluctuating demands. With inventory holding cost consideration, Fraser (1984) shows that a firm can adjust either output or inventory to buffer demand fluctuations. Incorporating the responsive price issue into Mills' (1984) model, Fluet and Phaneuf (1997) prove that price adjustment results in a flatter marginal cost curve due to application of flexible technique; while endogenous technique choice enables a firm to hedge against uncertainty with less price variations and more quantity variations. Using quadratic total production cost function, Rőller (1990) shows that trade-off exists within functional flexibility and size and slope properness of applicable region in an empirical study of Bell system. However, all these previous studies focus on production decision stage only. The decision-making process staged as capacity-production-pricing has not been addressed in the studies discussed above.

2.4 Research Gaps

The review of the previously most relevant studies shows that there are a few outstanding issues not studied sufficiently, or no relevant theoretical framework has been developed yet.

- There is not sufficient research to investigate the flexibility in manufacturing in the long run which has been a debatable issue in adopting FCS in real business.

- There is a research absence to study the flexibility in manufacturing in an asymmetric oligopoly market involving two strategies and multiple firms simultaneously. In such case, the interplay among multiple firms and different strategies are overlooked.

- The endogenous flexibility in a market involving multiple firms has not been addressed in the previous studies. Therefore, the interplay between firms' switching strategies has not been investigated. Furthermore, the study of endogenous flexibility is helpful to understand in what environments, how many firms are willing to use each of two strategies, and so the eventual equilibrium of a market can be studied.

- There is no research to investigate the flexibility degree of flexible capacity strategy which reflects partial implementation of FCS. Investigation of flexibility degree can also embody the mixed strategy widely used in reality between fully implementation of FCS and IFCS.

This present study can fill these four research gaps by investigating flexibility in manufacturing from four perspectives which are discussed in Chapters 4-7 respectively.

Chapter 3

System Features and Methodology

Based on the background in Chapter 1 and the literature review of relevant research in Chapter 2, this chapter aims to provide a general description of flexible capacity strategy (FCS), including its concept and operational stages. Notations, assumptions, function definitions, and mathematical methods adopted throughout the thesis are also provided in this chapter.

This chapter is divided into seven sections. Section 3.1 gives the notations, assumptions and some functions definitions used throughout the thesis. Section 3.2 gives a general description of the concept of FCS. Section 3.3 discusses three operational stages of FCS. In Section 3.4, the cost structures adopted in this thesis are discussed. Mathematical methods are presented in Section 3.5.

3.1 Notations, Assumptions and Function Definitions

3.1.1 Index sets

Assuming there are r flexible firms and s in-flexible firms in a model, the total number of firms is $n = r + s$. This situation is referred to as *n=(r, s)* throughout the thesis. Define $\Omega = \{1,2,...,n\}$ to be the index set of all firms, $\Omega^F = \{1,2,...,r\}$ to be the index set of flexible firms, and $\Omega^N = \{r+1, r+2,...,n\}$ to be the index set of in-flexible firms.

3.1.2 Variables

The notations of variables and parameters used throughout the thesis are defined as follows.

Π : the expected profit ;

π : the ex-post expected profit; k : capacity amount;

q : production quantity; p : market price; Q : total production quantity in the market;

$C_c(k)$: total capacity cost for capacity amount k ;

$C_p(q)$: total production cost for production quantity q ;

α : the realization of the uncertain demand which is assumed to follow a general distribution with mean μ , cumulative distribution function $F(\cdot)$ and probability density function $f(\cdot)$;

$p(\alpha, Q)$: responsive price which is a function of α and Q .

Subscripts are used to represent some characteristics of variables or parameters. These subscripts are listed below.

i: index of a firm; *e*: quantity at equilibrium; *b*: best response quantity .

3.1.3 Assumptions

To formulate the interaction between total production quantity in the market, i.e., Q , and market price p , responsive price is addressed by using inverse demand function, in which the market product price is affected by the total product quantity in the market, i.e., $p = p(\alpha, Q)$.

Market clearance rule is adopted in the model and it assumes that all products are sold in the market and firms do not hold back products to affect the price. Under this assumption, the product market price is actually determined by all products produced by all firms, i.e., $Q = \sum_{i \in \Omega} q_i$.

3.1.4 Function definitions

To facilitate the presentation, we define four functions which are used throughout the thesis.

Function 1: $L(x)=\int_x^\infty (\alpha - x) f(\alpha) d\alpha$, $x \ge 0$. It can be proved that $L(x)$ is strictly decreasing and convex in x, and $L(x) \in (0, \mu]$ for $x \ge 0$ whenever $\overline{F}(x) > 0$ for all $x \ge 0$.

Function 2: $X(C)$ is defined to be the inverse function of $L(x)$, i.e., $L(X(C)) = C$. Then $X(C)$ is strictly decreasing in $C \in (0, \mu]$.

Function 3: $G(x)=\int_x^\infty (\alpha - 2x) f(\alpha) d\alpha$. With the assumption of $xf(x) < 2\overline{F}(x)$ for all $x \ge 0$, it can be proved that $G(x)$ is strictly decreasing in x.

Function 4: Suppose $xf(x) < 2\overline{F}(x)$ for all $x \ge 0$, define $Y(C)$ to be the inverse function of $G(x)$, i.e., $G(Y(C)) = C$. Then $Y(C)$ is strictly decreasing in C.

To interpret the meaning of these four functions, we consider a model in which the inverse demand function is $p(\alpha, x) = (\alpha - x)^+$ where x is the production quantity. Therefore, the expected product price is $L(x)=\int_x^\infty (\alpha - x) f(\alpha) d\alpha$, which is Function 1. For Function 2, $X(C)$ is the inverse function of $L(x)$. The expected revenue is $R(x)=\int_0^\infty x(\alpha - x)^+ f(\alpha) d\alpha = \int_x^\infty x(\alpha - x) f(\alpha) d\alpha$. The first-order derivative is $R^{(1)}(x)=\int_x^\infty (\alpha - 2x) f(\alpha) d\alpha = G(x)$ (Function 3), which is the marginal revenue. For Function 4, $Y(C)$ is the inverse function of $G(x)$.

The assumption of $xf(x) < 2\overline{F}(x)$, where $\overline{F}(x) = 1 - F(x)$, for all $x \ge 0$, is adopted and discussed by previous studies (Van Mieghem and Dada, 1999; Anupindi and Jiang, 2008). Detailed discussions are provided by Anupindi and Jiang (2008). Some common distributions readily satisfy this assumption. The results verify that this assumption is reasonable in real business.

3.2 Concept of Flexible Capacity Strategy (FCS)

Flexible capacity investment refers to a firm's ability to adjust its production level to respond different demands under capacity constraint, i.e., by keeping its production level within its capacity. A firm without investing in flexible capacity has a stable production level that equals its capacity amount because there is no need to invest in excess capacity. According to the capacity strategy of firms, they are categorized into flexible firms and in-flexible firms. Figure 3.1 illustrates a firm's operations decision-making process sequentially with and without flexible capacity investment, respectively. The details of each decision-making stage are discussed in Section 3.3.

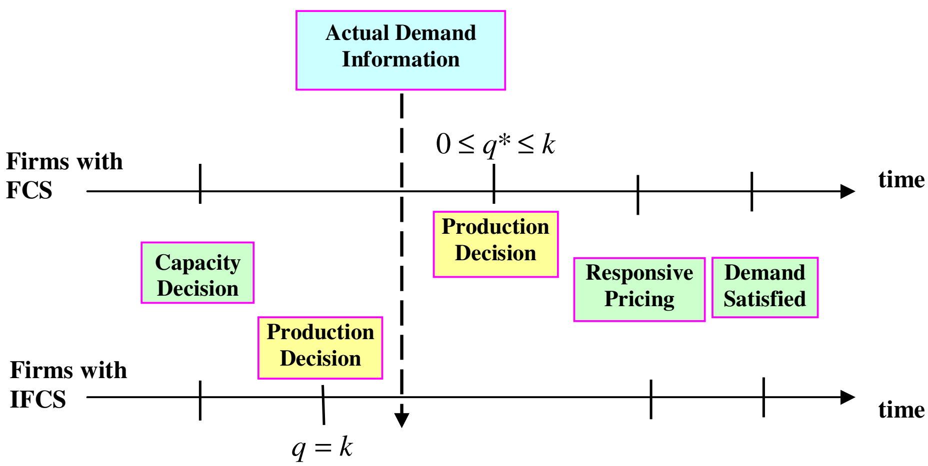


Figure 3.1: Decision-making process of firms with/without FCS.

3.3 Three-Stage Decision-Making Process of FCS

3.3.1 Capacity decision stage

At capacity decision stage, all firms, flexible and in-flexible, aim at maximizing their expected profits by determining the optimal capacity amounts. For both flexible and in-flexible firms, their capacity decisions can be formulated in a common formulation as below:

$$\text{Max} \quad \Pi(k_i) = \int_0^\infty (q_i p(\alpha, Q) - C_P(q_i)) f(\alpha) d\alpha - C_C(k_i),$$

$$\text{s.t.} \quad k_i \geq 0, \; i \in \Omega.$$

k_i is the capacity amount of firm *i*. The production quantity q_i of firm *i* is determined at the production decision stage.

3.3.2 Production decision stage

At the production decision stage, restrained by individual capacity, firms make their individual production decisions within an allowable range to maximize its ex-post profit.

For flexible firms, they have ability to postpone their production decision until knowing the actual demand. Therefore, they are able to determine the optimal production quantity between zero and its full capacity. This operation can be formulated as below:

$$\text{Max} \quad \pi_i(q_i) = q_i p(\alpha, Q) - C_P(q_i),$$

$$\text{s.t.} \quad 0 \leq q_i \leq k_i, \; i \in \Omega^F.$$

For in-flexible-capacity firms, they have to make the production decision before knowing the actual demand. Since there is no need to invest in excess capacity and therefore, in-flexible firms produce at their full capacity, i.e., $q_i = k_i$, for all $i \in \Omega^N$.

3.3.3 Pricing stage

At the pricing stage, firms compete in quantity in the same market, which is also referred to as Cournot competition. Under quantity competition, all firms compete with each other in the same market at the same price which is determined by the total production quantity.

3.4 Cost Structures

3.4.1 Capacity cost

To acquire volume flexibility, a firm needs to adopt some adaptive operations. Multi-trained employees, advanced manufacturing technology and design for manufacturing are top three practices for firms acquiring volume flexibility (Hallgren and Olhager, 2009). Besides these three practices, set-up time reduction has also attracted significant attention by firms with volume flexibility (e.g., Hallgren and Olhager, 2009). To carry out these practices, firms have to afford some additional investments. In this paper, we assume C_F and C_N are the unit capacity costs of flexible and in-flexible firms respectively. Further, we assume $C_F \geq C_N > 0$. The linear function is adopted for total capacity cost calculation. Total capacity cost for a certain capacity amount k is defined as $C_c(k)$. According to Van Mieghem and Dada (1999), all results for linear capacity cost functions can be extended to convex capacity cost functions.

3.4.2 Total production cost

Considering the planning period, the total production cost is discussed in two scenarios.

(1) In the long run, we adopt a quadratic function as $C_P(q_i) = \beta q_i + \frac{q_i^2}{2\gamma}$, $i \in \Omega$.

(2) In the intermediate or short run, we adopt a linear function as $C_P(q_i) = \beta q_i$, $i \in \Omega$.

Parameter $\beta \geq 0$ is assumed to be the same for all firms. β is also the unit production cost in the short run or medium run.

3.5 Mathematical Methods

Here we adopt the backward induction method to identify mathematically the best actions for achieving the desire results. Backward induction is the process of reasoning backwards along the time line to determine the optimal actions. It starts from the end of the decision making process where a decision maker identifies all possible decisions that might be made at this point of time for all known situation ns and determines what corresponding actions to be taken to deal with the situations. With reference to the derived information on decisions and actions, the decision maker can derive the next decisions and actions in the same manner for next earlier time towards the beginning of the problem. This process continues backwards until the first decision at the beginning of the problem. At this moment, the decision maker has determined the best actions for every possible situation at every point of time.

Specifically, in a monopoly model, the problem can be viewed as a sequential optimization problem of the firm. We solve it by starting from the problem of price setting in the market, then deciding on the optimal production quantity, and finally deciding on the capacity investment. In a duopoly model or an oligopoly model, differing from the monopoly model, a firm needs to consider the action of the other firms at each decision-making stage, rather than its own decision only. We solve the competition to find the Nash equilibrium.

Chapter 4

Evaluation of FCS with Consideration of Total Production Cost Structure

In this chapter, the long-term effect of FCS is examined with consideration of production cost structure. It has been pointed out by some research that the improvement of the total production cost structure can be achieved by adopting flexible technology (e.g., Stigler, 1939; Marschak and Nelson, 1962; Mills, 1984; Fluet and Phaneuf, 1997). Therefore, the examination of long term FCS is conducted by comparing it with flexible technology adoption. In this chapter, five possible production strategies in terms of FCS and flexible technology investment are established in a monopoly model. A production strategy consists of the decisions about flexible technology and flexible capacity, i.e., whether or not to invest in each of them. With flexible capacity investment, a firm is able to postpone its production decision until knowing the actual demand. For the same product quantity, a firm with flexible technology investment is able to reduce the total production cost. Each production strategy can be carried out by a decision-making operation process with a stage sequence of either technology-capacity-production-pricing or capacity-technology-production-pricing. Regardless of the stage sequence adopted, both technology and capacity decisions are made before knowing the real demand of each market period to determine a firm's production cost structure and production capability. By comparing a firm's profit under different production strategies, a comprehensive understanding of FCS in the long-run can be achieved.

This chapter is organized in three sections. Section 4.1 first describes the system features followed by the demonstration of details of flexible technology investment and flexible capacity investment respectively. Section 4.2 proposes five possible production strategies with different investment decisions in terms of flexible technology and flexible capacity. The optimal quantities of decision variables and the optimal profit of a firm under each production strategy are calculated. In Section 4.3, a few comparisons between different productions strategies are made to evaluate FCS in the long run and find out the interplay between FCS and flexible technology investment.

4.1 Two Aspects of Production Strategy

4.1.1 Flexible capacity investment

Following the discussion of FCS operations decision-making process in Chapter 3, we use specific cost structures and inverse demand function in this chapter to evaluate long term FCS. The additive linear demand inverse function is adopted, i.e, $p(\alpha, Q) = (\alpha - Q)^+$ (e.g., Van Mieghem and Dada, 1999; Aviv and Federgruen 2001). The linear capacity cost function and quadratic total production cost function are adopted in this chapter.

4.1.2 Relationship between cost structure and flexible technology

The relationship between cost structure and technology flexibility facing demand fluctuations is demonstrated in Figure 4.1, which is proposed by Stigler (1939) and used in a few subsequent studies (e.g., Mills 1984). In Figure 4.1, indices 1 and 2 represent two different cost structures with different marginal costs, MC_i, and average costs, AC_i. The minimum average costs for the two cost structures occur at the same output. Stigler (1939) and a few other studies (e.g., Marschak and Nelson, 1962; Mills, 1984) show that the cost structure with the flattest slope of marginal cost indicates a higher level of technology flexibility in adjusting the throughput in each production period. The means and the involved efforts to improve the existing technology level are referred to flexible technology investment in this chapter. There are a various operational options to achieve such flexible technology investments, such as using advanced technology or upgrading equipment (Stigler, 1939; Mills, 1984). By doing so, a firm is able to improve its total production cost structure.

Specifically, the level of such technology flexibility can be treated as an endogenous variable, γ, in the quadratic total production cost function $C_P(q) = \beta q + q^2/(2\gamma)$, where C_p is the total production cost, q is the production quantity, and $\beta > 0$ is a constant parameter (Mills, 1984). A larger γ implies a higher technology level yielding a lower total production cost. The basic technology level without any additional investment is defined as $\gamma_N > 0$. The constraint $\gamma \geq \gamma_N$ stipulates that the adopted technology level is not lower than the basic technology level. This production cost function is first proposed by Mills (1984), and then adopted by a few subsequent studies to investigate flexible technology (e.g., Fraser, 1984; Fluet and Phaneuf, 1997). A flexible technology firm needs to determine the optimal technology level γ and incurs the technology investment cost $C_t(\gamma) = C_r(\gamma - \gamma_N)$, where $C_r > 0$ is the technology investment cost per unit level of technology flexibility.

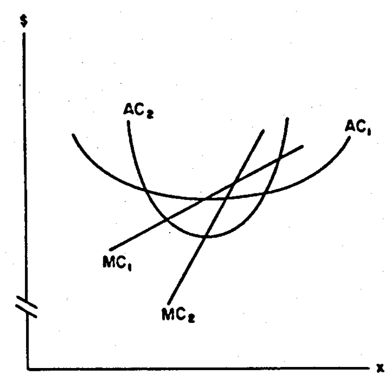


Figure 4.1: Technology Flexibility and Cost Curves.

With respect to technology investment cost function $C_t(\gamma) = C_r(\gamma - \gamma_N)$, at the current stage of this study, we simplify technology investment cost in a linear form, which reflects that: (1) a higher technology level associates with a higher investment cost (or installation cost); (2) a larger adjustment on existing technology, a higher cost occurs.

4.1.3 Production strategy

A production strategy is composed of a combination of investment decisions on flexible capacity (C) and flexible technology (T), i.e., whether or not to invest in each of them and the sequence of making the decisions. With the consideration of the investment sequence, a total of five possible production strategies are shown in Table 4.1.

C \ T	No	Yes
No	NT+NC (No flexible capacity and no flexible technology)	T-only (Flexible technology only)
Yes	C-only (Flexible capacity only)	T+C (Flexible technology followed by flexible capacity strategy) C+T (Flexible capacity followed by flexible technology strategy)

Table 4.1: Five possible production strategies.

4.2 Five Production Strategies

In this section we present the formulations of the five production strategies, followed by deriving the optimal decisions of each production strategy. The sequential decision variables of each strategy are listed in Table 4.2. Throughout this thesis, variables with superscripts N, T, C, T+C and C+T are defined as optimal values of the variables under the strategies NT+NC, T-only, C-only, T+C and C+T, respectively. To facilitate presentation, we let $\Pi_0 = \int_{\beta}^{X(C_F)} \frac{1}{4}(\alpha-\beta)^2 f(\alpha)d\alpha + \int_{X(C_F)}^{\infty} \frac{1}{4}(X(C_F)-\beta)^2 f(\alpha)d\alpha$.

Production Strategy	Sequential decision variables
NT+NC	Capacity amount
T-only	Technology level→ Capacity amount
C-only	Capacity amount→ Production quantity
T+C	Technology level→ Capacity amount→ Production quantity
C+T	Capacity amount → Technology level→ Production quantity

Table 4.2: Sequential decisions of each production strategy.

4.2.1 Production decision stage

As shown in Table 4.2, for strategies with flexible capacity investment, the production quantity is the decision variable that maximizes the *ex-post* profit after knowing the actual demand, given as follows:

$$\text{Max } \pi(q|k,\gamma,\alpha) = q(\alpha - q)^{+} - (\beta q + q^2/(2\gamma)) \text{, s.t. } 0 \le q \le k . \qquad (4.1)$$

The optimal production quantity is provided in Proposition 4.1.

Proposition 4.1 For a firm investing in flexible capacity, with any given capacity k and technology level γ, the optimal production quantity as a function of demand realization α is

$$q(\alpha|k,\gamma) = \begin{cases} 0 & \text{if } 0 \le \alpha < \beta \\ \dfrac{\gamma}{2\gamma+1}(\alpha-\beta) & \text{if } \beta \le \alpha < \beta + \dfrac{2\gamma+1}{\gamma}k \\ k & \text{if } \beta + \dfrac{2\gamma+1}{\gamma}k \le \alpha \end{cases}.$$

□

4.2.2 No flexible technology and no flexible capacity (NT+NC) strategy

Under NT+NC strategy, a firm only determines the optimal capacity to maximize its profit, as : Max $\Pi^N(k) = \int_0^\infty k(\alpha-k)^+ f(\alpha)d\alpha - (C_N + \beta + \frac{k}{2\gamma_N})k$, s.t. $k \geq 0$. (4.2)

The optimal decisions of NT+NC strategy are provided by Proposition 4.2.

Proposition 4.2 Under the NT+NC strategy, a firm's optimal production quantity equals its capacity, i.e., $q^N = k^N$, which satisfies $G(k^N) - k^N / \gamma_N = C_N + \beta$. The firm's optimal expected profit is $\Pi^N = (k^N)^2 \overline{F}(k^N) + (k^N)^2 / (2\gamma_N)$. □

4.2.3 Flexible technology only (T-only) strategy

Under T-only strategy, γ^T and k^T are formulated as follows.

Technology level decision

$$\text{Max} \quad \Pi^T(\gamma) = \int_0^\infty k(\alpha-k)^+ f(\alpha)d\alpha - (C_N + \beta + \frac{k}{2\gamma})k - C_r(\gamma - \gamma_N),$$

$$\text{s.t.} \quad \gamma \geq \gamma_N, \tag{4.3}$$

where k is the optimal solution of the following capacity decision formulation.

Capacity decision

$$\text{Max} \quad \Pi^T(k|\gamma) = \int_0^\infty k(\alpha-k)^+ f(\alpha)d\alpha - (C_N + \beta + \frac{k}{2\gamma})k \text{, s.t. } k \geq 0. \tag{4.4}$$

Proposition 4.3 Under T-only strategy with given γ_N, β, C_N and C_r,

(i) the optimal technology level is $\gamma^T = \begin{cases} \max\{\gamma_N, \ \gamma_T^*\} & \text{if } 0 < C_r \leq \frac{1}{2}(\mu - C_N - \beta)^2 \\ \gamma_N & \text{otherwise} \end{cases}$, where $\gamma_T^* = \frac{Y(C_N + \beta + \sqrt{2C_r})}{\sqrt{2C_r}}$;

(ii) if $\gamma^T = \gamma_T^*$, then the optimal production quantity equals the optimal capacity, which satisfies $q^T = k^T = Y(C_N + \beta + \sqrt{2C_r})$ and the optimal profit is $\Pi^T = (k^T)^2 \overline{F}(k^T) + C_r \gamma_N$. If $\gamma^T = \gamma_N$, then the results are the same as those under the NT+NC strategy. □

4.2.4 Flexible capacity only (C-only) strategy

Under C-only strategy, k^C and q^C formulated as below.

Capacity decision

$$\text{Max} \quad \Pi^C(k) = \int_0^\infty q((\alpha - q)^+ - \beta - \frac{q}{2\gamma_N}) f(\alpha) d\alpha - C_F k, \text{ s.t. } k \geq 0, \tag{4.5}$$

where q is the optimal solution of the following production decision formulation.

Production decision

$$\text{Max } \pi(q|k, \alpha) = q(\alpha - q)^+ - (\beta q + \frac{q^2}{2\gamma_N}), \text{ s.t. } 0 \leq q \leq k. \tag{4.6}$$

Proposition 4.4 Under C-only strategy with given γ_N, β and C_F, the optimal capacity is $k^C = \frac{\gamma_N}{1 + 2\gamma_N}(X(C_F) - \beta)$, the optimal production quantity is $q^C = q(\alpha | k^C, \gamma_N)$, and the optimal expected profit is $\Pi^C = \frac{2\gamma_N}{1 + 2\gamma_N}\Pi_0$. □

4.2.5 Flexible technology-flexible capacity (T+C) strategy

Under T+C strategy, sequential decisions γ^{T+C}, k^{T+C} and q^{T+C} are formulated as below.

Technology level decision

$$\text{Max} \quad \Pi^{T+C}(\gamma) = \int_0^\infty q((\alpha-q)^+ - \beta - \frac{q}{2\gamma}) f(\alpha) d\alpha - C_F k - C_r(\gamma-\gamma_N),$$

$$\text{s.t.} \quad \gamma \geq \gamma_N, \tag{4.7}$$

where k and q are the optimal solutions of capacity decision and production decision.

Capacity decision

$$\text{Max} \quad \Pi^{T+C}(k|\gamma) = \int_0^\infty q((\alpha-q)^+ - \beta - \frac{q}{2\gamma}) f(\alpha) d\alpha - C_F k, \quad \text{s.t. } k \geq 0. \tag{4.8}$$

Production decision

$$\text{Max} \quad \pi(q|k,\gamma,\alpha) = q(\alpha-q)^+ - (\beta q + \frac{q^2}{2\gamma}), \text{ s.t. } 0 \leq q \leq k. \tag{4.9}$$

Proposition 4.5 Under T+C strategy with given γ_N, β, C_F and C_r, the optimal technology level, capacity, production quantity and expected profit are $\gamma^{T+C} = \max\{\gamma_N, \ \gamma^*_{T+C}\}$, $k^{T+C} = \frac{\gamma^{T+C}}{1+2\gamma^{T+C}}(X(C_F)-\beta)$, $q^{T+C} = q(\alpha|k^{T+C}, \gamma^{T+C})$ and $\Pi^{T+C} = \frac{2\gamma^{T+C}}{1+2\gamma^{T+C}}\Pi_0 - C_r(\gamma^{T+C}-\gamma_N)$, respectively, where $\gamma^*_{T+C} = \frac{1}{2}(\sqrt{2\Pi_0/C_r} - 1)$. □

4.2.6 Flexible capacity-flexible technology (C+T) strategy

Under C+T strategy, the optimal k^{C+T}, γ^{C+T} and q^{C+T} are determined over time.

Capacity decision

$$\text{Max} \quad \Pi^{C+T}(k = \int_0^\infty q((\alpha-q)^+ - \beta - \frac{q}{2\gamma}) f(\alpha) d\alpha - C_r(\gamma-\gamma_N) - C_F k,$$

$$\text{s.t.} \quad k \geq 0. \tag{4.10}$$

Technology level decision

$$\text{Max} \quad \Pi^{C+T}(\gamma|k) = \int_0^\infty q((\alpha-q)^+ - \beta - \frac{q}{2\gamma}) f(\alpha) d\alpha - C_r(\gamma-\gamma_N),$$

$$\text{s.t.} \quad \gamma \geq \gamma_N. \tag{4.11}$$

Production decision

The formulation of this stage is exactly the same as that of T+C strategy.

We first test the sequential investment effect by comparing T+C strategy and C+T strategy. The difference between these two strategies is in what is determined first and the second decision is made based on the first decision. For example, flexible capacity investment can be achieved by managing workforce, such as using multi-skilled employees; flexible technology investment can be achieved by upgrading equipment, or using a new technology during the production process. Accordingly, T+C strategy means a firm choosing its technology and equipment first. Then, based on its chosen technology and equipment, a firm decides its optimal workforce. C+T strategy means that a firm determines its number of workers and type of workers, such as, high efficient or low efficient, multi-skilled or single-skilled workers. Based on this workforce management, the firm decides its technology choice and chooses its equipment.

Theorem 4.1 Under T+C strategy and C+T strategy, a firm's optimal capacity k, technology level γ, production quantity q and the optimal expected profit are exactly the same. □

Theorem 4.1 establishes that there is no sequential investment effect of a firm's optimal decisions. In other words, the same investments lead to the same decisions and profit, regardless of the sequence of the investment decisions. Therefore, a firm only needs to decide which investment(s) to make.

4.3 Strategy Comparisons

In this section we make comparisons between the five strategies to evaluate each strategy's pros and cons, with a view to understanding the relationship between flexible technology investment and flexible capacity investment, and finding out the optimal strategy.

4.3.1 Framework of strategy comparisons

Due to the equivalence between the T+C strategy and the C+T strategy, there are only four production strategies to consider: NT+NC, T-only, C-only, and T+C, as shown in Figure 4.2. Each arrow and the number next to it represent a comparison and the comparison sequence, respectively, in the following discussion. The arrows in parallel indicate that there may be some similarities between the two comparisons as the investment difference between the strategies compared is the same, i.e., comparisons between (1) and (5), and between (2) and (4). To facilitate discussion of the comparative analyses between different strategies, we define the "bound" of a strategy as: If strategy A is said to be a lower (or an upper) bound of strategy B, then for all the possible situations, the expected profit of strategy A is always not greater (or less) than the expected profit of strategy B, i.e., $\Pi^A \le \Pi^B$ (or $\Pi^A \ge \Pi^B$).

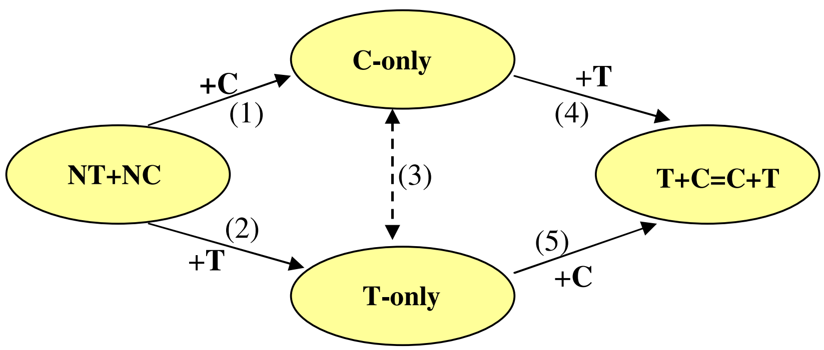


Figure 4.2: Structure of comparisons between different strategies.

4.3.2 Comparison between NT+NC strategy and C-only strategy

Comparison results between NT+NC strategy and C-only strategy are presented in Proposition 4.6.

Proposition 4.6 Given C_N, γ_N, β:

(i) For $C_N \le C_F < L(\beta)$, the expected profit of C-only strategy is strictly decreasing in C_F and $\Pi^C(C_N) \ge \Pi^C(C_F) > \Pi^C(L(\beta)) = 0$.

(ii) There exists a unique $\hat{C}_F \in [C_N,\ L(\beta))$ satisfying $\Pi^C(\hat{C}_F) = \Pi^N$; $\hat{C}_F$ is strictly increasing in $C_N \in [0,\ \mu - \beta)$;

(iii) Profit comparison between NT+NC strategy and C-only strategy is: (1) when $\hat{C}_F < C_F < L(\beta)$, $\Pi^N > \Pi^C$; (2) when $C_N \le C_F < \hat{C}_F$, $\Pi^C > \Pi^N$. □

Proposition 4.6 points out that the C-only strategy can be beneficial or harmful to a firm's expected profit under different environments. The comparison between the NT+NC strategy and the C-only strategy is shown in Figure 4.3. The comparison follows a threshold policy: there is a threshold of flexible capacity cost that leads to the same profit under the NT+NC strategy or the C-only strategy; a firm benefits from flexible capacity with a capacity cost lower than the threshold, but it incurs a loss when the capacity cost is higher than the threshold. Obviously, the C-only strategy is always better than the NT+NC strategy by avoiding excess production when $C_F = C_N$. However, as the flexible capacity cost increases, there is a trade-off between the saving from avoiding production waste and the spending on expensive capacity costs.

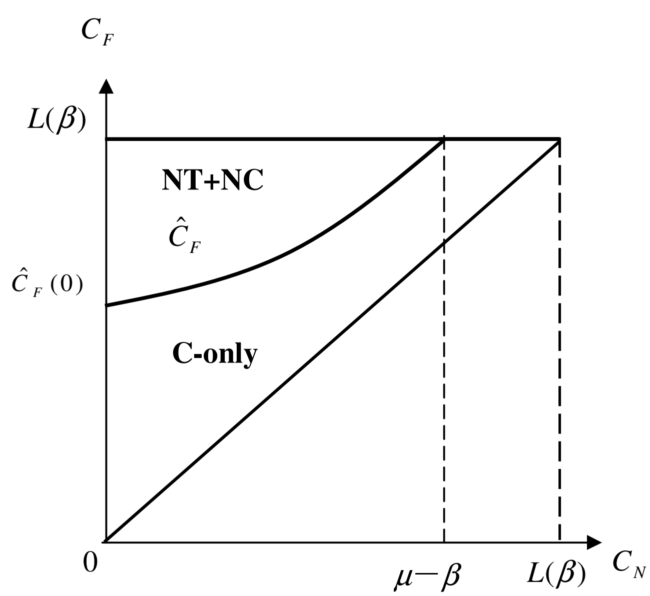


Figure 4.3: Comparison between NT+NC strategy and C-only strategy.

4.3.3 Comparison between NT+NC strategy and T-only strategy

Comparison between NT+NC strategy and T-only strategy is provided by Proposition 4.7.

Proposition 4.7 Comparing NT+NC strategy and T-only strategy, we have

(i) under T-only strategy, define $C_{rT} = \min\{\underline{C}_r,\ C_r\}$, the optimal capacity and expected profit are $k^T = Y(C_N + \beta + \sqrt{2C_{rT}})$ and $\Pi^T = (k^T)^2 \overline{F}(k^T) + C_{rT}\gamma_N$, respectively; further, if $C_r \le \underline{C}_r$, Π^T is strictly decreasing in C_r; if $C_r > \underline{C}_r$, Π^T keeps constant as $\Pi^T(\underline{C}_r)$;

(ii) the optimal decisions of NT+NC strategy can be obtained from resolving T-only strategy with modified parameter $C_r = \underline{C}_r$, and then $k^N = k^T(\underline{C}_r)$ and $\Pi^N = \Pi^T(\underline{C}_r)$;

where $0 < \underline{C}_r < \frac{1}{2}(\mu - C_N - \beta)^2$ satisfying $\dfrac{Y(C_N + \beta + \sqrt{2\underline{C}_r})}{\sqrt{2\underline{C}_r}} = \gamma_N$. □

Part (i) of Proposition 4.7 describes the pattern of a firm's expected profit under the T-only strategy. There is a threshold of the flexible technology cost resulting in the same profit for the NT+NC strategy and the T-only strategy. Only when the flexible technology cost is lower than the threshold can a firm benefit from investing in flexible technology; otherwise the firm should maintain its basic technology level. This insight of flexible technology investment enables us to draw the conclusion of the part (ii) of Proposition 4.7: the NT+NC strategy is a particular case of the T-only strategy by replacing some parameters. Specifically, the NT+NC strategy can be regarded as a lower bound for the T-only strategy. This means that the T-only strategy completely dominates the NT+NC strategy and improves profit when the technology investment cost is less than the threshold. The relationship between the NT+NC strategy and the T-only strategy is shown in Figure 4.4. The cost threshold $\underline{C}_r$ determines whether a firm should invest in flexible technology. A firm's expected profit decreases with an increase in C_r within the range $[0, \underline{C}_r]$ and remains constant as Π^N if $C_r > \underline{C}_r$.

Moreover, the optimal decision of the T-only strategy does not harm a firm's profit. The difference in the expected profit under these two strategies is $\Pi^T - \Pi^N$ as shown in Figure 4.4. While the C-only strategy can increase or decrease a firm's profit under different costing environments, the T-only strategy ensures that a firm obtains the same or a higher profit as compared with the NT+NC strategy. However, this does not mean that the T-only strategy is better than the C-only strategy as analyzed in the following section.

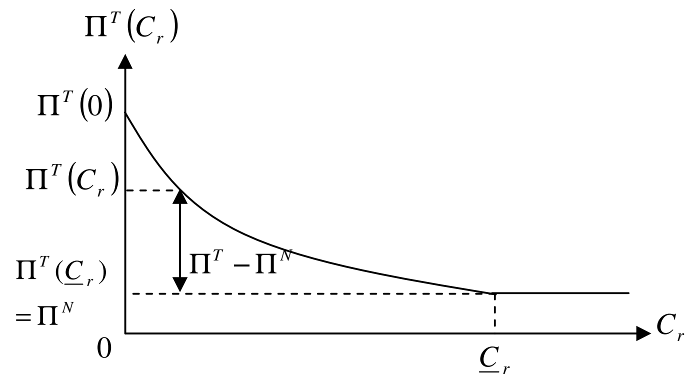


Figure 4.4: Optimal expected profit with different C_r under T-only strategy.

4.3.4 Comparison of T-only strategy and C-only strategy

Theorem 4.2 tells the optimal investment if a firm only invests in either T-only or C-only strategy.

Theorem 4.2 Given C_N, γ_N and β, the comparison between T-only and C-only strategy is:

(i) If $\Pi^C(C_N) < \Pi^T(\underline{C}_r)$, then $\Pi^C(C_F) < \Pi^T(C_r)$ for all situations;

(ii) If $\Pi^C(C_N) \geq \Pi^T(\underline{C}_r)$, then for each $C_r \in (0, C_{rm}]$, $\Pi^C(C_F) \leq \Pi^T(C_r)$ for all $C_F \in [C_N,\ L(\beta))$, where $C_{rm} = \begin{cases} 0 & \text{if } \Pi^C(C_N) \geq \Pi^T(0) \\ \overline{C}_r & \text{if } \Pi^C(C_N) < \Pi^T(0) \end{cases}$ and $\overline{C}_r \in (0, \underline{C}_r]$ satisfies $\Pi^T(\overline{C}_r) = \Pi^C(C_N)$. Moreover, for each $C_r \in (C_{rm}, \infty)$, there exists a unique $\overline{C}_F \in (C_N,\ L(\beta))$ such that $\Pi^C(C_F)\begin{cases} > \Pi^T(C_r) & \text{if } C_N \le C_F < \overline{C}_F \\ = \Pi^T(C_r) & \text{if } C_F = \overline{C}_F \\ < \Pi^T(C_r) & \text{if } \overline{C}_F < C_F < L(\beta) \end{cases}$.

Furthermore, the curve $\Pi^C(\overline{C}_F) = \Pi^T(C_r)$ is strictly increasing for $C_r \in (C_{rm}, \underline{C}_r]$ and horizontal for $C_r \in [\underline{C}_r, \infty)$. □

Theorem 4.2 provides the comparison results of the T-only and C-only strategies under various costing environments. Note that $\Pi^N = \Pi^T(\underline{C}_r)$. Only when the C-only strategy is better than the NT+NC strategy can the C-only strategy compete with the T-only strategy. There is a unique division of profit in the comparison between the T-only strategy and the C-only strategy as shown in Figure 4.5.

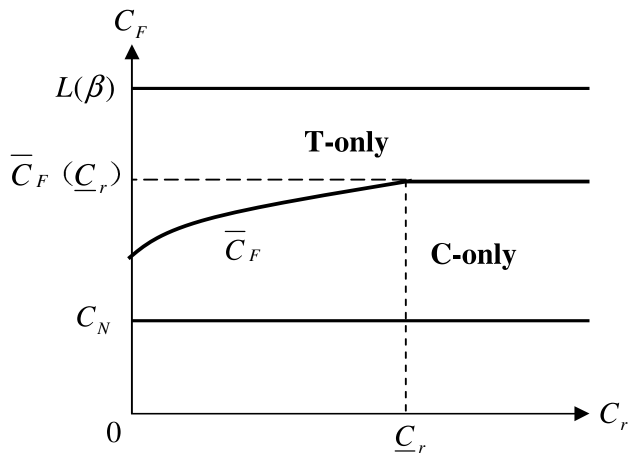


Figure 4.5: Comparison between C-only strategy and T-only strategy when $\Pi^C(C_N) > \Pi^T(\underline{C}_r)$ and $\Pi^C(C_N) \ge \Pi^T(0)$.

4.3.5 Comparison between C-only strategy and T+C strategy

The comparison between C-only strategy and T+C strategy is given in Proposition 4.8.

Proposition 4.8 Comparing C-only strategy and T+C strategy: For any given C_F,

(i) the optimal expected profit of T+C strategy $\Pi^{T+C}(C_r)$ is strictly decreasing in C_r for $0 < C_r \le \overline{C}_r^{T+C}$, and $\Pi^{T+C}(C_r) = 2\gamma_N/(1+2\gamma_N)\Pi_0$ for $C_r \ge \overline{C}_r^{T+C}$, where $\overline{C}_r^{T+C} = \dfrac{2\Pi_0}{(1+2\gamma_N)^2}$;

(ii) the C-only strategy can be reduced from T+C strategy by modifying the parameter $C_r = \overline{C}_r^{T+C}$, and then $k^C = k^{T+C}(\overline{C}_r^{T+C})$ and $\Pi^C = \Pi^{T+C}(\overline{C}_r^{T+C})$;

(iii) C-only strategy is a lower bound of T+C strategy; moreover, the increase in profit by T+C strategy relative to C-only strategy is $\delta = \dfrac{\Pi^{T+C} - \Pi^C}{\Pi^C} \le \dfrac{1}{2\gamma_N} \cdot 100\%$. □

Based on the comparative analysis, we observe from Figure 4.6 two similarities as follows: (1) the expected profit of the T+C strategy has a similar pattern to that of the T-only strategy, and (2) the similar relationship between the NT+NC strategy and the T-only strategy also exists between the C-only strategy and the T+C strategy. These observations indicate that flexible technology does not change a firm's decision-making structure, but it provides a trade-off between investment costs, total production cost reduction, and revenue changes. Moreover, compared with the C-only strategy, the largest improvement of the T+C strategy is $1/(2\gamma_N)\cdot 100\%$. However, the C-only strategy cannot always benefit a firm, nor can the T+C strategy.

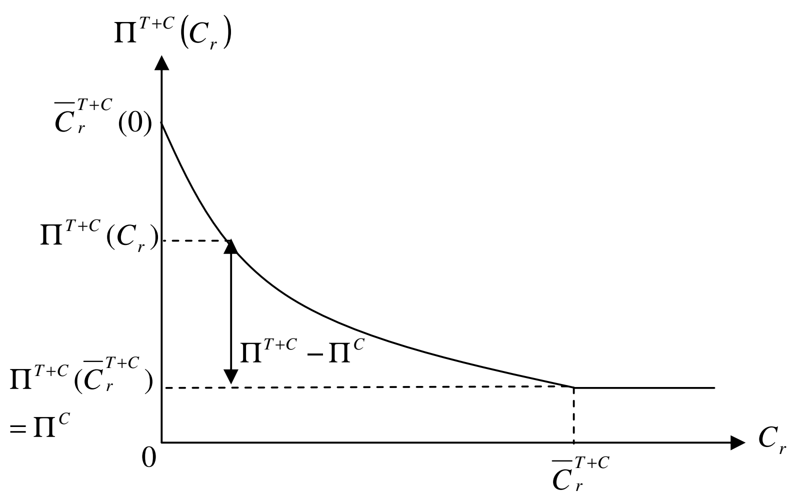


Figure 4.6: Optimal expected profit with different C_r under T+C strategy.

4.3.6 Comparison between T-only strategy and T+C strategy (equivalent to C+T strategy)

The aforementioned interpretation of the investment effects concludes that the T-only strategy dominates the NT+NC strategy, and the T+C strategy (equivalent to the C+T strategy) dominates the C-only strategy. Their expected profits can be ranked as $\Pi^N \leq \Pi^T$ and $\Pi^C \leq \Pi^{T+C} = \Pi^{C+T}$. From the strategic perspective, the optimal production strategy is either the T-only strategy or the T+C strategy (equivalent to the C+T strategy). However, we note from our results that $\Pi^N = \Pi^T$ and/or $\Pi^C = \Pi^{T+C} = \Pi^{C+T}$ under certain conditions. Under these conditions, flexible technology investment is not helpful to improve a firm's profit. This indicates that some strategies result in the same profit from different investments. To facilitate discussion, we differentiate "effective strategy" and "efficient strategy" from the strategic perspective and the operations perspective, respectively, in the following.

***Effective strategy*:** Strategy A is said to be more effective than strategy B if strategy A makes more profit than strategy B.

***Efficient strategy*:** Strategy A is said to be more efficient than strategy B if strategy A makes the same profit with a fewer number of investments than strategy B.

It is noted that under these two definitions, the efficiencies of two strategies are concerned only when the two strategies are equally effective. Therefore, it is not possible that a strategy A is more efficient and less effective than strategy B.

Based on these definitions, Theorem 4.3 and Theorem 4.4 below draw conclusions from the comparative analyses from the strategic perspective and the operations perspective, respectively.

Theorem 4.3 Define $\Delta\Pi = \Pi^{T+C} - \Pi^T$. Let $\overline{C}_F^{T+C}$ satisfy $C_r = \dfrac{2\Pi_0(\overline{C}_F^{T+C})}{(1+2\gamma_N)^2}$. A unique C_r^* satisfying $\Delta\Pi(C_r^*, \overline{C}_F^{T+C}) = 0$ exists. With a given C_N,

(i) if $0 < C_r < C_r^*$, then there exists a unique $C_F^*(C_r)$ satisfying

$$\Delta\Pi(C_r, C_F) \begin{cases} > 0 & \text{if } C_N \le C_F < C_F^*(C_r) \\ = 0 & \text{if } C_F = C_F^*(C_r) \\ < 0 & \text{if } C_F^*(C_r) < C_F \le L(\beta) \end{cases};$$

(ii) if $C_r^* \le C_r \le \underline{C}_r$, then there exists a unique $\tilde{C}_F(C_r)$ satisfying

$$\Delta\Pi(C_r, C_F) \begin{cases} > 0 & \text{if } C_N \le C_F < \tilde{C}_F(C_r) \\ = 0 & \text{if } C_F = \tilde{C}_F(C_r) \\ < 0 & \text{if } \tilde{C}_F(C_r) < C_F \le L(\beta) \end{cases};$$

(iii) if $\underline{C}_r < C_r$, then $\Delta\Pi(C_r, C_F) \begin{cases} > 0 & \text{if } C_N \le C_F < \tilde{C}_F(\underline{C}_r) \\ = 0 & \text{if } C_F = \tilde{C}_F(\underline{C}_r) \\ < 0 & \text{if } \tilde{C}_F(\underline{C}_r) < C_F \le L(\beta) \end{cases}$.

□

From the strategic perspective, Theorem 4.3 states that the most effective production strategy is either the T-only strategy or the T+C strategy (equivalent to the C+T strategy) depending on the investment costing environment. Both T-only strategy and T+C strategy include technology investment which improves the total production cost structure. This means technology investment is always preferred from the strategic perspective. On the other hand, additional investment in flexible capacity may not increase, or even damage, a firm's profit. This conclusion is similar to that of the comparison between the NT+NC strategy and the C-only strategy. The comparison results are trade-offs between cost saving from avoiding production waste and increase in revenues by controlling production, and the resulting product price, and spending on the expensive flexible capacity. The results of Theorem 3 are demonstrated in Figure 4.7.

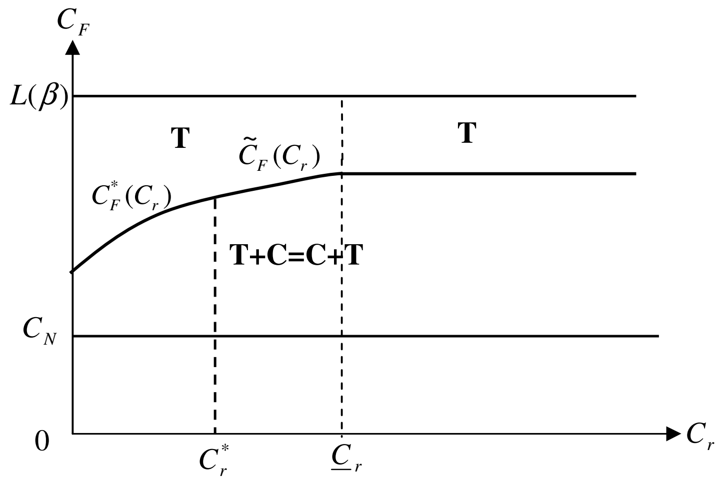


Figure 4.7: Comparison between T-only strategy and T+C strategy with given C_N.

As shown in Figure 4.7, there is a threshold C_F that determines the most effective strategy under different investment costing environments. The most effective strategy is the T-only strategy when it is above the threshold, and is the T+C strategy when it is below the threshold. Affected by both flexible capacity investment cost and flexible technology investment cost, the comparison results between the T-only strategy and the T+C strategy embrace three situations. The first situation is $C_r \in (0, C_r^*)$, in which at the division curve both the T+C and the T-only strategies improve the technology level due to the comparatively lower technology investment cost. The second situation is $C_r \in [C_r^*, \underline{C}_r)$, in which at the division curve the T-only strategy improves the technology level but the T+C strategy cannot. The third situation is $C_r \in [\underline{C}_r, \infty)$, in which at the division curve flexible technology investment is invalid for both the T-only strategy and the T+C strategy. Based on the analyses of these three situations, the most effective and efficient strategy (EES) that achieves the maximum profit using the minimum number of investments can be determined by Theorem 4.4 below from the operational perspective.

Theorem 4.4 Given C_N, γ_N, β, assume $C_N \le C_F \le L(\beta)$, following all definitions in Proposition 7.6, the most effective and efficient strategy (**EES**) is:

(i) if $0 < C_r < C_r^*$, then $\text{EES} = \begin{cases} \text{T+C strategy}, & \text{if } C_N \le C_F < C_F^*(C_r) \\ \text{T-only strategy}, & \text{otherwise} \end{cases}$;

(ii) if $C_r^* \le C_r \le \underline{C}_r$, then $\text{EES} = \begin{cases} \text{T+C strategy}, & \text{if } C_N \le C_F < \overline{C}_F^{T+C} \\ \text{C-only strategy}, & \text{if } \overline{C}_F^{T+C} \le C_F < \tilde{C}_F(C_r) \\ \text{T-only strategy}, & \text{otherwise} \end{cases}$;

(iii) if $\underline{C}_r < C_r$, then $\text{EES} = \begin{cases} \text{T+C strategy}, & \text{if } C_N \le C_F < \overline{C}_F^{T+C} \\ \text{C-only strategy}, & \text{if } \overline{C}_F^{T+C} \le C_F < \tilde{C}_F(\underline{C}_r) \\ \text{NT+NC strategy}, & \text{otherwise} \end{cases}$;

For all environments, T+C strategy equals C+T strategy. □

Theorem 4.4 is illustrated in Figure 4.8, which provides the most EES under different costing environments. Comparing Figure 4.7 and Figure 4.8, we see that the T-only strategy is equivalent to the NT+NC strategy under certain environments. An increase in flexible technology investment cost results in profit reduction until the profit equals that under the NT+NC strategy. The flexible technology investment cost at which the T-only strategy is equivalent to the NT+NC strategy is $\underline{C}_r$. A firm only improves its technology level when the unit technology investment cost is lower than $\underline{C}_r$. When the investment cost is higher than $\underline{C}_r$, the flexible technology investment of the T-only strategy is actually invalid. Regarding the T+C strategy, based on a comparison of Figure 4.7 and Figure 4.8, we can show that the T+C strategy is equivalent to the C-only strategy under some environments. The division is affected by both flexible capacity investment cost and flexible technology investment cost simultaneously. There is a decreasing curve $\overline{C}_F^{T+C}$ leading to the equivalence between the T+C strategy and the C-only strategy, as shown in Figure 4.7. In the area below the curve $\overline{C}_F^{T+C}$, a firms invests in flexible technology, while the firm maintains the basic system technology level in the area above the curve. This indicates that the T+C strategy is equivalent to the C-only strategy in the area above the curve $\overline{C}_F^{T+C}$, i.e., the flexible technology investment of the T+C strategy is not helpful to improve technology level.

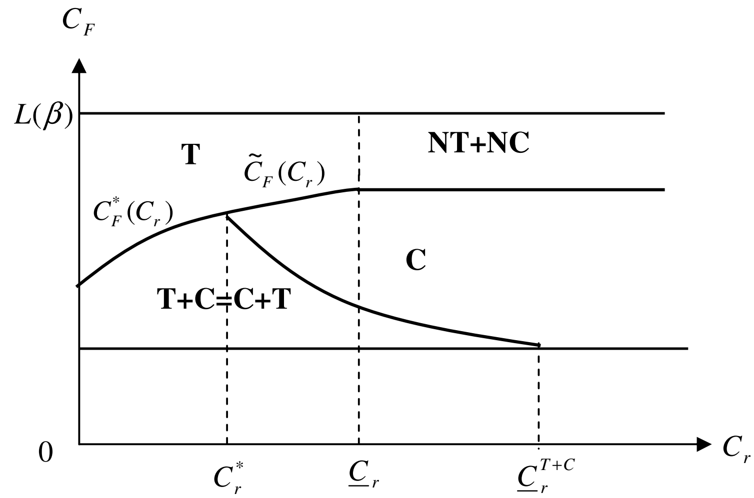


Figure 4.8: The optimal production strategy with given C_N.

We see from Figure 4.8 that the most EES can be any one of the five possible strategies, i.e., NT+NC, T-only, C-only, T+C or C+T. Under all the environments, the T+C strategy is equivalent to the C+T strategy. The results illustrate that more flexibility may not guarantee more profit. Particularly, any flexibility investment cannot increase a firm's profit within a specific area, i.e., $\{(C_r, C_F) \mid \underline{C}_r < C_r, \tilde{C}_F(\underline{C}_r) < C_F < L(\beta)\}$, in which the most EES is the NT+NC strategy; in other areas in which the most EES includes at least one type of investment, a firm benefits from flexible technology and/or flexible capacity investments.

Chapter 5

Asymmetric Oligopoly Model

In Chapter 4, we evaluate long term FCS with consideration of production cost structure. In this chapter, we focus on short term or medium term FCS in competitive environments. We construct an asymmetric oligopoly competition model consisting of *r* flexible firms and *s* in-flexible firms under demand uncertainty. The total number of firms is n, i.e., n = (r,s). Each firm carries out a decision-making operation process spanning capacity planning, production procedure and pricing stages, as discussed in Chapter 3. All firms compete on the quantity in a same market, i.e., Cournot competition. By characterizing the equilibrium, we find out interplays among decisions of multiple firms.

This chapter consists of five sections. Section 5.1 describes the assumptions adopted in the model including notations used, demand function, competition mechanism and cost structure. Section 5.2 describes a three-stage decision-making operational process of both flexible firms and in-flexible firms. It also characterizes the decision pattern of firms which adopt the same strategy. Section 5.3 characterizes the Nash equilibrium of a competition involving flexible firms and in-flexible firms. The optimal capacity amount, production quantity and the expected profit of each firm at equilibrium are provided. Section 5.4 gives the sensitivity analyses of some influential factors. The individual profits of the flexible firms and in-flexible firms are compared in Section 5.5 to evaluate the performance of FCS and IFCS in an asymmetric oligopoly competition.

5.1 Notations in an Asymmetric Oligopoly Model

Assuming there are *r* flexible firms and *s* in-flexible firms in a two-strategy multiple-firm model, the total number of firms is $n = r + s$, i.e., $n=(r, s)$. The superscript F (Flexible) and N (In-flexible) are used to specify variables of flexible and in-flexible firms, respectively. Let $Q^F = \sum_{i \in \Omega^F} q_i^F$ be the total production quantity of flexible firms; $Q^N = \sum_{j \in \Omega^N} q_j^N$ be the total production quantity of in-flexible firms, and $Q = \sum_{l \in \Omega} q_l = Q^F + Q^N$ be the total production quantity of all firms. Also, we let $k^F = \sum_{i \in \Omega^F} k_i^F$ be the total capacity of all flexible firms; $k^N = \sum_{i \in \Omega^N} k_i^N$ be the total capacity of all in-flexible firms.

5.2 Three-Stage Decision-Making Operations in Asymmetric Oligopoly Model

Following the three-stage decision-making process discussed in Chapter 3, we formulate each stage specifically under an oligopoly competition environment in a backward sequence in the following.

5.2.1 Pricing stage

Following the discussion at pricing stage in Chapters 3 and 4, the demand inverse function is $p(\alpha, Q) = (\alpha - Q)^+ = (\alpha - Q^F - Q^N)^+$.

5.2.2 Production decision stage

Similar to the discussion in Chapters 3 and 4, for each in-flexible firm $i \in \Omega^N$, the production quantity equals its capacity amount, i.e., $q_i^N = k_i^N$, for all $i \in \Omega^N$.

For each flexible firm, it aims at maximizing its ex-post profit by producing an optimal quantity under capacity constraints for different demand realizations. This can be formulated as:

$$\text{Max} \quad \pi_i^F\left(q_i^F \middle| \alpha, q_j, \forall j \in \Omega \setminus \{i\}\right) = q_i^F(\alpha - Q)^+ - \beta q_i^F,$$

$$\text{s.t.} \quad 0 \le q_i^F \le k_i^F,\ i \in \Omega^F. \tag{5.1}$$

To simply the notation, we define

$$m_i^F(q_i^F) = \pi_i^F\left(q_i^F \middle| \alpha, q_j, \forall j \in \Omega \setminus \{i\}\right) = q_i(\alpha - q_i - \sum_{j \ne i} q_j)^+ - \beta q_i. \tag{5.2}$$

The optimal production decision of each firm is provided by Proposition 5.1.

Proposition 5.1 Consider any feasible firm *i*. Suppose that α, k_j, $j \in \Omega$, and q_j^F, $j \in \Omega^F \setminus \{i\}$, are given. Then the optimal production decision q_i^{F*} of the feasible firm *i* is $q_i^{F*} = \begin{cases} 0, & x \le \beta \\ q_{ib}^F, & \beta < x \le 2k_i^F + \beta \\ k_i^F, & 2k_i^F + \beta < x \end{cases}$, where $x = \alpha - \sum_{j \ne i} q_j^F(\alpha) - k^N$ and $q_{ib}^F = \dfrac{x - \beta}{2}$. □

5.2.3 Capacity decision stage

At capacity decision stage, both flexible and in-flexible firms aim at maximizing their expected profits. Their capacity decisions can be formulated respectively as below:
For flexible firms:

$$\text{Max} \quad \Pi(k_i^F) = \int_0^\infty q_i^F((\alpha - Q^F - k^N)^+ - \beta) f(\alpha) d\alpha - C_F k_i^F,$$

$$\text{s.t.} \quad k_i^F \ge 0,\ i \in \Omega^F. \tag{5.3}$$

For in-flexible firms:

$$\text{Max} \quad \Pi(k_i^N) = \int_0^\infty q_i^N((\alpha - Q^F - k^N)^+ - \beta) f(\alpha) d\alpha - C_N k_i^N,$$

$$\text{s.t.} \quad k_i^N \ge 0,\ i \in \Omega^N. \tag{5.4}$$

Specifically, for an in-flexible firm $i \in \Omega^N$, $q_i^N = k_i^N$; for a flexible firm $i \in \Omega^F$, $q_i^F = q_i^{F*}$, $0 \le q_i^{F*} \le k_i^F$, where q_i^{F*} is the optimal production quantity to maximize the ex-post profit of the flexible firm $i \in \Omega^F$.

5.2.4 Pooling principle

To simplify the discussion of firms' decisions in this chapter, we define "pooling principle" as follows: If we say there is a pooling principle in a group, then all members in the group evenly share the risk and the profit of the group. In the following discussion in this section, to identify the decisions of each firm at equilibrium of asymmetric oligopoly competition, we characterize the decision pattern of firms which adopt the same strategy, either FCS or IFCS. We prove that firms adopting the same strategy make the same decisions regardless of the number of rivals, the strategy adopted, the demand uncertainties and costing environments. In other words, there is a general pooling principle among firms even in a market involving two strategies simultaneously.

In the following, Propositions 5.2 - 5.5 provide some characterizations of the optimal capacity decisions of flexible and in-flexible firms. Based on the results of Propositions 5.2 - 5.5, Theorems 5.1 and 5.2 state the pooling principle among flexible and in-flexible firms, respectively.

Consider any feasible firm $i \in \Omega^F$. Suppose that k_j, $j \in \Omega \setminus \{i\}$, and $q_j^F(\alpha)$, $j \in \Omega^F \setminus \{i\}$, are given. Let $A^F(k_i^F) = \Pi_i^F\left(k_i^F \middle| k_j^F,\ q_j^F(\alpha)\ \forall j \neq i,\ \text{and}\ k_l^N,\ l \in \Omega^N\right)$ be the expected profit of firm *i*. The objective of firm *i* is to maximize $A^F(k_i^F)$.

Proposition 5.2 In an oligopoly market competition with $r > 0$ flexible firms and $s \geq 0$ in-flexible firms, the optimal capacity of flexible firm $i \in \Omega^F$, i.e., k_i^{F*}, is either (i) $k_i^{F*} = 0$ and $A^{F^{(1)}}(0) \leq 0$; or (ii) $k_i^{F*} > 0$ and $A^{F(1)}(k_i^{F*}) = 0$. □

Proposition 5.3 At the equilibrium of an oligopoly market competition with $r > 0$ flexible firms and $s \geq 0$ in-flexible firms, the optimal capacities of flexible firms are either $k_i^{F*} = 0$, for all $i \in \Omega^F$; or $k_i^{F*} > 0$, for all $i \in \Omega^F$; further,

(i) $k_i^{F*} = 0$, for all $i \in \Omega^F$, is equivalent to $k^N \geq X(C_F) - \beta$;

(ii) $k_i^{F*} > 0$, for all $i \in \Omega^F$, is equivalent to $k^N < X(C_F) - \beta$. □

Theorem 5.1 At the equilibrium of an oligopoly market competition with $r>0$ flexible firms and $s\geq 0$ in-flexible firms, all flexible firms $i\in\Omega^F$ make the same capacity decision and the same production decision. That is:

(i) If $k^N \geq X(C_F)-\beta$, then $k_i^{F*}=q_i^{F*}=0$ for all $i\in\Omega^F$.

(ii) If $k^N < X(C_F)-\beta$, then $k_i^{F*}=k_e^F=\frac{1}{r}k^F>0$ for all $i\in\Omega^F$; further, we have $k^N+(r+1)k_e^F=X(C_F)-\beta$. The individual profit of each flexible firm is

$$\Pi_e^F=\frac{1}{(r+1)^2}\left(\int_{k^N+\beta}^{X(C_F)}(\alpha-k^N-\beta)^2 f(\alpha)d\alpha+\int_{X(C_F)}^{\infty}(X(C_F)-k^N-\beta)^2 f(\alpha)d\alpha\right).$$

The production decision of each flexible firm is $q_i^{F*}=q_e^F$ for all $i\in\Omega^F$,

which is presented as $q_e^F=\begin{cases}0, & 0\leq\alpha\leq\beta+k^N\\ \dfrac{\alpha-\beta-k^N}{r+1}, & \beta+k^N<\alpha\leq\beta+(r+1)k_e^F+k^N\\ k_e^F, & \beta+(r+1)k_e^F+k^N<\alpha\end{cases}$. □

It should be noted that when k^N is given, k_e^F can be uniquely determined by Theorem 5.1. Furthermore, we have corollary 5.1 as a direct consequence of Theorem 5.1.

Corollary 5.1 At the equilibrium of an oligopoly market competition with $r>0$ flexible firms and $s=0$ in-flexible firms, all flexible firms $i\in\Omega^F$ make the same capacity decision and the same production decision. That is:

(i) If $C_F\geq L(\beta)$, then $k_i^{F*}=q_i^{F*}=0$ for all $i\in\Omega^F$.

(ii) If $C_F<L(\beta)$, then $k_i^{F*}=k_e^F=\dfrac{X(C_F)-\beta}{r+1}>0$ for all $i\in\Omega^F$. The profit of each flexible firm is

$$\Pi_e^F=\frac{1}{(r+1)^2}\left(\int_{\beta}^{X(C_F)}(\alpha-\beta)^2 f(\alpha)d\alpha+\int_{X(C_F)}^{\infty}(X(C_F)-\beta)^2 f(\alpha)d\alpha\right).$$

The production decision of each flexible firm is $q_i^{F*}=q_e^F$ for all $i\in\Omega^F$, where

$$q_e^F=\begin{cases}0, & 0\leq\alpha\leq\beta\\ \dfrac{\alpha-\beta}{r+1}, & \beta<\alpha\leq X(C_F)\\ k_e^F, & X(C_F)<\alpha\end{cases}.$$ □

Consider any in-feasible firm $i \in \Omega^N$. Suppose that k_j, $j \in \Omega \setminus \{i\}$, and $q_j^F(\alpha)$, $j \in \Omega^F$, are given. Let $A^N(k_i^N) = \Pi_i^N \left(k_i^N \middle| k_j^N,\ \forall j \neq i, \text{and } k_l^F, q_l^F(\alpha)\ l \in \Omega^F \right)$ be the expected profit of firm *i*. The objective of firm *i* is to maximize $A^N(k_i^N)$.

Proposition 5.4 In an oligopoly market competition with $r \geq 0$ flexible firms and $s > 0$ in-flexible firms, the optimal capacity of in-flexible firm $i \in \Omega^N$, i.e., k_i^{N*}, is either (1) $k_i^{N*} = 0$ and $A^{N(1)}(0) \leq 0$; or (2) $k_i^{N*} > 0$, $A^{N(1)}(k_i^{N*}) = 0$ and $A^{N(2)}(k_i^{N*}) \leq 0$. □

Proposition 5.5 At the equilibrium of an oligopoly market competition with $r \geq 0$ flexible firms and $s > 0$ in-flexible firms, the optimal capacities of in-flexible firms are either (1) $k_i^{N*} = 0$, for all $i \in \Omega^N$; or (2) $k_i^{N*} > 0$, for all $i \in \Omega^N$; further, we have

(i) $k_i^{N*} = 0$, for all $i \in \Omega^N$, is equivalent to $\int_{0<v} vf(\alpha)d\alpha \leq C_N + \beta$;

(ii) $k_i^{N*} > 0$, for all $i \in \Omega^N$, is equivalent to $\int_{0<v} vf(\alpha)d\alpha > C_N + \beta$, where $v = \alpha - Q^F(\alpha) - k^N$. □

Theorem 5.2 At the equilibrium of an oligopoly market competition with $r \geq 0$ flexible firms and $s > 0$ in-flexible firms, all in-flexible firms make the same capacity decision.

(1) If $\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - sk_e^N) f(\alpha) d\alpha \leq C_N + \beta$, then $k_i^{N*} = 0$, for all $i \in \Omega^N$.

(2) If $\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - sk_e^N) f(\alpha) d\alpha > C_N + \beta$, then $k_i^{N*} = k_e^N = \frac{1}{s} k^N > 0$, for all $i \in \Omega^N$; further, we have $\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - (s+1)k_e^N) f(\alpha) d\alpha = C_N + \beta$. The individual profit of each in-flexible firm is $\Pi_e^N = (k_e^N)^2 \overline{F}(sk_e^N)$. □

Corollary 5.2 At the equilibrium of an oligopoly market competition with $r = 0$ flexible firms and $s > 0$ in-flexible firms, all in-flexible firms make the same capacity decision.

(1) If $C_N \geq \mu - \beta$, then $k_i^{N*} = 0$, for all $i \in \Omega^N$.

(2) If $C_N < \mu - \beta$, then $k_i^{N*} = k_e^N = \frac{1}{s} k^N > 0$, for all $i \in \Omega^N$; further, we have $\int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha) d\alpha = C_N + \beta$. The individual profit of each in-flexible firm is $\Pi_e^N = (k_e^N)^2 \overline{F}(sk_e^N)$. □

Theorems 5.1 and 5.2 prove that, in a two-strategy asymmetric oligopoly market, there is a pooling principle among firms adopting the same strategy: The same strategy leads to the same decisions and the same profit. This pooling principle always holds regardless of the competition environments and the number of rivals. All firms adopting the same strategy are pooling the profit and risk of this strategy. The interplay between the flexible-firm group and the in-flexible-firm group determines the profit potential allocation between the two strategies. Theorems 5.1 and 5.2 also demonstrate that, at the equilibrium of a two-strategy oligopoly market, a best way for a firm to augment its profit is to make unanimous decisions with other firms adopting the same, although in principle the firm can make decisions freely.

Theorems 5.1 and 5.2 are consistent with previous study results that focuse on duopoly model (e.g., Anand and Girotra, 2007; Anupindi and Jiang, 2007; Goyal and Netessine, 2007). However, in a duopoly model, a firm's decisions are only affected by one firm rather than a handful of flexible and in-flexible firms simultaneously. From the strategic perspective in a duopoly model, a firm faces only one strategy adopted by its rival. Unlike the duopoly model, a firm in a two-strategy oligopoly model needs to compete with multiple rivals with the same or different strategy at the same time. Therefore, the different ways of how a firm's decisions are affected by two strategies simultaneously cannot be found in a duopoly model. Our study not only generalizes previous studies, but also develops a mechanism controlling a market invisibly in a two-strategy asymmetric oligopoly market. This mechanism is also robust enough to be in a multi-strategy oligopoly competition.

After description of the general pooling principle, we conduct analysis of the detailed operations of in-flexible firms and flexible firms. For in-flexible firms, the capacity investment threshold can be presented as

$$\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - sk_e^N) f(\alpha) d\alpha = C_N + \beta .$$

Since the capacity amount equals the production quantity for in-flexible firms, $C_N + \beta$ is the constant marginal product cost (including in capacity decision and production decision stages). The left hand side of this threshold equation is the expected product price at the market. Therefore, for in-flexible firms, this threshold equation holds when marginal product cost equals the expected product price. Only when expected product price is higher than the marginal product cost, can in-flexible firms invest in capacity.

For flexible firms, at production decision stage, production is conducted only when demand is larger than $\beta + k^N$. When $\beta + k^N < \alpha \leq \beta + (r+1)k_e^F + k^N$, flexible firms' investments in capacity have no effect on their production quantities. When demand is large enough, i.e., $\beta + (r+1)k_e^F + k^N < \alpha$, the production is conducted to full capacity; any additional capacity investment creates the same production for each flexible firm. At the capacity decision stage, flexible firms' threshold to make capacity investment is $k^N = X(C_F) - \beta$, i.e.,

$$\int_{k^N + \beta}^{\infty} (\alpha - k^N) f(\alpha) d\alpha = C_F + \beta \overline{F}(k^N + \beta) .$$

It is known that $k^N + \beta$ is the minimum demand level at which flexible firms produce the product, and so the probability of a unit capacity being produced into the product is $\overline{F}(k^N + \beta)$. As a result, the expected unit cost of a product for flexible firms is $C_F + \beta \overline{F}(k^N + \beta)$. Therefore, the right hand side of this threshold equation is the expected unit product cost, whereas the left hand side of this threshold equation is the expected price when flexible firms produce the product. This indicates that this threshold equation holds when product price equals the marginal product cost. Flexible firms invest in capacity only when the product price is larger than the marginal product cost. Table 5.1 compares the costs of flexible and in-flexible firms.

	FCS	IFCS
Minimum demand to make production	$\beta + k^N$	0
Probability of one unit capacity is used to produce one unit product	$\overline{F}(\beta + k^N)$	100%
Unit production cost	β	β
Unit capacity cost	C_F	C_N
Marginal product cost	$C_F + \beta\overline{F}(k^N + \beta)$	$C_N + \beta$
Price condition to invest in capacity	$p(\alpha, Q^N) > C_F + \beta\overline{F}(k^N + \beta)$	$p(\alpha, Q^F) > C_N + \beta$

Table 5.1: Cost Comparison between FCS and IFCS.

Comparing the capacity investment conditions of in-flexible firms and flexible firms, we note that actually all firms follow the same mechanism to make capacity investment, i.e., only when expected product price is larger than the marginal product cost, do firms make the capacity investment. A difference between flexible and in-flexible firms is in the way of how they evaluate their marginal product costs and expected product prices in order to decide whether to invest in their capacities. Figure 5.1 plots the mechanism of decision-making for both flexible and in-flexible firms.

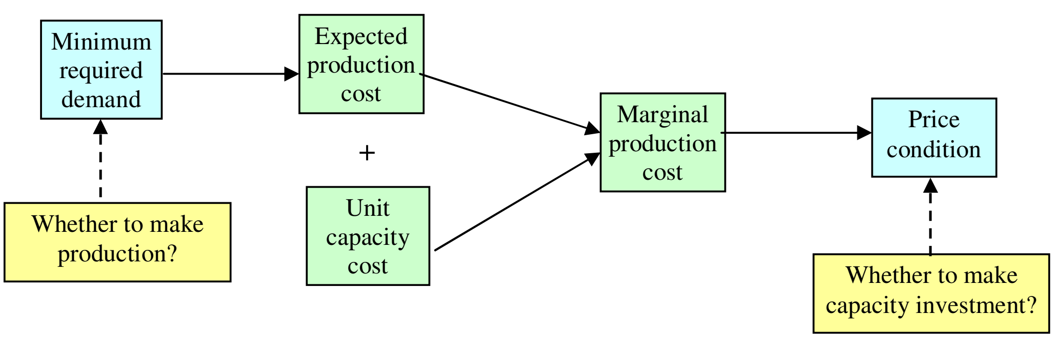


Figure 5.1: Mechanism of making decisions for both flexible and in-flexible firms.

5.3 Equilibrium of *n=(r, s)* Competitive Market Model

Although the decision pattern of each type of firms is known, it still needs to know the exact decisions and individual profit of each kind of firms to evaluate the performance of FCS and IFCS. This section characterizes the equilibrium of *n=(r, s)* competitive market with demand uncertainty. Let $R = \{(C_N, C_F) : 0 < C_N \le C_F < \infty\}$ be the feasible region of all C_N and C_F. According to Theorems 5.1 and 5.2, R can be divided into four regions, resulting in four cases as listed in Table 5.2.

	$k_e^N > 0$	$k_e^N = 0$
$k_e^F = 0$	Case-B	Case-A
$k_e^F > 0$	Case-D	Case-C

Table 5.2: Four possible equilibriums of an oligopoly competition.

$k^F = rk_e^F$ and $k^N = sk_e^N$ always hold for all cases. To facilitate the presentation, if $s \ge 1$, we define a function $Z : [0, \infty) \to (0, \mu]$ by $Z(x) = \int_x^\infty (\alpha - \frac{s+1}{s}x) f(\alpha) d\alpha$. So we have $Z^{(1)}(x) = \frac{1}{s}xf(x) - \frac{s+1}{s}\overline{F}(x) < 0$, and $Z(x)$ decreases as x increases. Let k_w be the unique solution satisfying $Z(k_w) = C_N + \beta$ when $0 < C_N + \beta \le \mu$.

Technically, it is not easy to verify the conditions of the above four regions and results. To pave the way to find the analytical solutions of equilibrium and their theoretical conditions, the following Propositions 5.6 - 5.9 provide the optimal capacity and expected profit of each of the flexible and in-flexible firms in each case in Table 5.1. Then, Theorem 5.3 discusses these four cases together and provides the necessary and sufficient conditions for each case, i.e., Case-A to Case-D, and their analytical solutions.

Proposition 5.6 Given $r>0$ flexible firms and $s>0$ in-flexible firms, within the area $R=\{(C_N, C_F): 0<C_N \le C_F<\infty\}$. For Case-A that $k_e^F=k_e^N=0$, we have $\Pi_e^F=\Pi_e^N=0$ and a necessary condition for Case-A is: $L(\beta) \le C_F$ and $\mu-\beta \le C_N$.

□

Proposition 5.7 Given $r>0$ flexible firms and $s>0$ in-flexible firms, within the area $R=\{(C_N, C_F): 0<C_N \le C_F<\infty\}$. For Case-B that $k_e^F=0$, $k_e^N>0$, we have $\Pi_e^F=0$, $\Pi_e^N=\frac{1}{s^2}(k^N)^2 \overline{F}(k^N)$, and (i) a necessary condition for Case-B is: $L(\beta+k_w) \le C_F$ and $C_N<\mu-\beta$; (ii) $k_e^N>0$ satisfies $\int_{sk_e^N}^{\infty}(\alpha-(s+1)k_e^N) f(\alpha) d\alpha=C_N+\beta$, $k^N=sk_e^N$. □

Proposition 5.8 Given $r>0$ flexible firms and $s>0$ in-flexible firms, within the area $R=\{(C_N, C_F): 0<C_N \le C_F<\infty\}$. For Case-C that $k_e^F>0$, $k_e^N=0$, we have $\Pi_e^F=\frac{1}{(r+1)^2}\left(\int_\beta^{X(C_F)}(\alpha-\beta)^2 f(\alpha) d\alpha+\int_{X(C_F)}^{\infty}(X(C_F)-\beta)^2 f(\alpha) d\alpha\right)$, $\Pi_e^N=0$, and (i) a necessary condition for Case-C is: $L(\beta)>C_F$ and $\mu-(C_N+\beta) \le \frac{r}{r+1}(L(\beta)-C_F)$; (ii) $k_e^F=\frac{1}{r+1}(X(C_F)-\beta)$. □

Proposition 5.9 Given $r>0$ flexible firms and $s>0$ in-flexible firms, within the area $R=\{(C_N, C_F): 0<C_N \le C_F<\infty\}$. For Case-D that $k_e^F>0$, $k_e^N>0$, we have $\Pi_e^F=\frac{1}{(r+1)^2}\left(\int_{\beta+sk_e^N}^{X(C_F)}(\alpha-sk_e^N-\beta)^2 f(\alpha) d\alpha+\int_{X(C_F)}^{\infty}(X(C_F)-sk_e^N-\beta)^2 f(\alpha) d\alpha\right)$; and $\Pi_e^N=(k_e^N)^2 \overline{F}(sk_e^N)$. The solution of Case-D is: $k_e^F=\frac{1}{r+1}(X(C_F)-\beta-sk_e^N)$ and k_e^N satisfies $C_N+\beta=\int_{sk_e^N}^{\infty}(\alpha-(s+1)k_e^N) f(\alpha) d\alpha-\frac{r}{r+1}(L(\beta+sk_e^N)-C_F)$. A necessary condition for Case-D is: $\mu-(C_N+\beta)>\frac{r}{r+1}(L(\beta)-C_F)$ and $C_F<L(\beta+k_w)$. □

Theorem 5.3 Given $r>0$ flexible firms and $s>0$ in-flexible firms, within the area $R=\{(C_N,C_F):0<C_N\le C_F<\infty\}$. At equilibrium, $k^F=rk_e^F$, $k^N=sk_e^N$, where k_e^F and k_e^N together with Π_e^F and Π_e^N in different regions of R are:

(Case-A) if $\mu-\beta\le C_N$ & $L(\beta)\le C_F$ & $C_N\le C_F$, then $k_e^F=0$, $k_e^N=0$, $\Pi_e^F=0$, $\Pi_e^N=0$;

(Case-B) if $C_N<\mu-\beta$ & $L(\beta+K_w)\le C_F$ & $C_N\le C_F$, then $k_e^F=0$, k_e^N satisfies $C_N+\beta=\int_{sk_e^N}^{\infty}(\alpha-(s+1)k_e^N)f(\alpha)d\alpha$, $\Pi_e^F=0$, $\Pi_e^N=(k_e^N)^2\overline{F}(sk_e^N)$;

(Case-C) if $\mu-(C_N+\beta)\le\frac{r}{r+1}(L(\beta)-C_F)$ & $L(\beta)>C_F$ & $C_N\le C_F$, then

$k_e^F=\frac{X(C_F)-\beta}{r+1}$, $k_e^N=0$, $\Pi_e^N=0$,

$$\Pi_e^F=\frac{1}{(r+1)^2}\left(\int_{\beta}^{X(C_F)}(\alpha-\beta)^2 f(\alpha)d\alpha+\int_{X(C_F)}^{\infty}(X(C_F)-\beta)^2 f(\alpha)d\alpha\right);$$

(Case-D) if $L(\beta+k_w)>C_F$ & $\mu-(C_N+\beta)>\frac{r}{r+1}(L(\beta)-C_F)$ & $C_N\le C_F$,

then $k_e^F=\frac{1}{r+1}(X(C_F)-\beta-sk_e^N)$, k_e^N satisfies

$$C_N+\beta=\int_{sk_e^N}^{\infty}(\alpha-(s+1)k_e^N)f(\alpha)d\alpha-\frac{r}{r+1}(L(\beta+sk_e^N)-C_F),$$

$$\Pi_e^F=\frac{1}{(r+1)^2}\left(\int_{\beta+sk_e^N}^{X(C_F)}(\alpha-sk_e^N-\beta)^2 f(\alpha)d\alpha+\int_{X(C_F)}^{\infty}(X(C_F)-sk_e^N-\beta)^2 f(\alpha)d\alpha\right),$$

$\Pi_e^N=(k_e^N)^2\overline{F}(sk_e^N)$. □

For situations of $r=0$ or $s=0$, we take the limiting cases and so Theorem 5.3 is reduced to Corollary 5.1 and Corollary 5.2. Theorem 5.3 provides analytical equilibrium solutions to a two-strategy three-decision-stage oligopoly competition. It is easy to show that the equilibrium solution in each case of the oligopoly competition is unique under the assumption $xf(x)<\overline{F}(x)$ for all $x\ge 0$. This assumption is adopted and discussed by previous studies (e.g., Van Mieghem and Dada, 1999; Anupindi and Jiang, 2008). Furthermore, contrary to a belief that flexibility is preferred by managers or researchers, the results illustrate that both flexible strategy and in-flexible strategy can be beneficial in some conditions, but otherwise harmful to firms' profits under certain conditions. Moreover, previous studies on flexible capacity strategy (Van Mieghem and Dada, 1999; Anupindi and Jiang, 2008) are particular cases of this general oligopoly model. These studies including monopoly model, duopoly models and symmetrical oligopoly models are listed in the following Table 5.3.

	r	s	n	References
Flexible monopoly model	1	0	1	Van Mieghem and Dada, 1999
In-flexible monopoly model	0	1	1	Van Mieghem and Dada, 1999
Flexible duopoly model	2	0	2	Anupindi and Jiang, 2008
In-flexible duopoly model	0	2	2	Anupindi and Jiang, 2008
Flexible vs. In-flexible duopoly model	1	1	2	Anupindi and Jiang, 2008
Symmetrical flexible oligopoly model	n	0	n	Van Mieghem and Dada, 1999
Symmetrical in-flexible oligopoly model	0	n	n	---

Table 5.3: Particular cases of asymmetric oligopoly model.

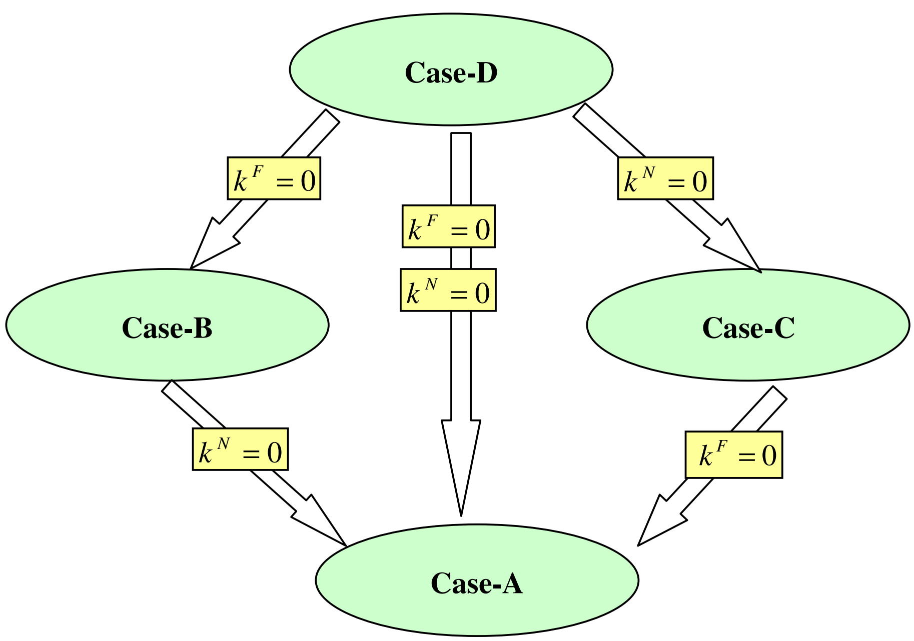


Figure 5.2: Relationships among the four cases.

In the following we further discuss the case with $r \geq 1$ and $s \geq 1$ (Theorem 5.3). Specifically, FCS and IFCS are dominant strategy within Case-C and Case-B, respectively. However, in Case-D, two strategies coexist and neither one dominates. Furthermore, in Case-D if the total capacity of flexible firms is set to zero, the solution is Case-B; similarly, if the total capacity of in-flexible firms is set to zero in Case-D, the solution is Case-C. We present the relationships between these four cases in Figure 5.2 to show that all cases can be developed from Case-D. The following is the interpretation of Figure 5.2: The partitions of four cases depend on five inter-influential factors, including production cost, flexible capacity cost, in-flexible capacity cost, number of flexible firms and number of in-flexible firms. However, if there is some interference from exogenous factors, such as lack of resources, government policy, the overseas competitors, large change in organization and new product innovation, to force flexible firms and/or in-flexible firms to set capacity zero, the equilibrium of Case-D switches over to another case.

To provide intuitive understanding of the equilibrium, Proposition 5.10 characterizes the divisions of different cases so that the equilibrium can be plotted in the following Figure 5.3.

Proposition 5.10 Given $r > 0$ flexible firms and $s > 0$ in-flexible firms, within the area $R = \{(C_N, C_F) : 0 < C_N \leq C_F < \infty\}$.

(i) The boundary between Case-B and Case-D is $C_{F1} = L(\beta + k_w)$, which is defined as Curve-1;

(ii) the boundary between Case-C and Case-D is $C_{F2} = \frac{r+1}{r} C_N - \frac{r+1}{r}(\mu - \beta) + L(\beta)$, which is defined as Curve-2;

(iii) under the assumption $\overline{F}(x) - xf(x) > 0$, in both Curve-1 and Curve-2, C_F is strictly increasing in C_N, and within $C_N \in [0, \mu - \beta]$, Curve-1 is always above Curve-2, except that they intersect at $(\mu - \beta, L(\beta))$;

(iv) further, in Curve-1 C_{F1} decreases in s with given C_N; in Curve-2 C_N decreases in r with given C_{F2}. □

Partitions of equilibrium under non-zero and zero production cost are plotted in Figure 5.3(a) and Figure 5.3(b), respectively. It can be observed that two strategies only coexist in Case-D, which indicates that strategy competition only occur in this region. The interplay between flexible firms and in-flexible firms does not exist in Regions-A to C.

For non-zero production cost situation, Curve-1 and Curve-2 represent the boundary of the Region-D in which $n = (r, s)$, as shown in Figure 5.3(a). It is interesting that Curve-1 depends only on the number of in-flexible firms (*s*) but is independent of the number of flexible firms (*r*). We consider a situation that there are more in-flexible firms entering the market but the number of flexible firms is unchanged, that is $n' = (r, s')$, where $s' > s$ and $n' - n = s' - s$. Under the situation of $n' = (r, s')$, Curve-1 goes down to Curve-1' and Curve-2 is unchanged. The area between Curve-1 and Curve-1' changes from Region-D to Region-B. In such case, in the area between Curve-1 and Curve-1', the total profit of flexible firms changes from a positive value to zero, while in-flexible firms' profits are always positive. This indicates that in this area, more in-flexible firms joining in the market enhances the competitiveness of IFCS but reduces the competitiveness of FCS. Similar characteristic can also be found in Curve-2, which is a straight line independent of the number of in-flexible firms and its slope is $\frac{r+1}{r}$. Therefore, as more flexible and/or in-flexible firms join in the market, the strategy-coexistence region (i.e., Region-D) becomes smaller with a more fierce competition.

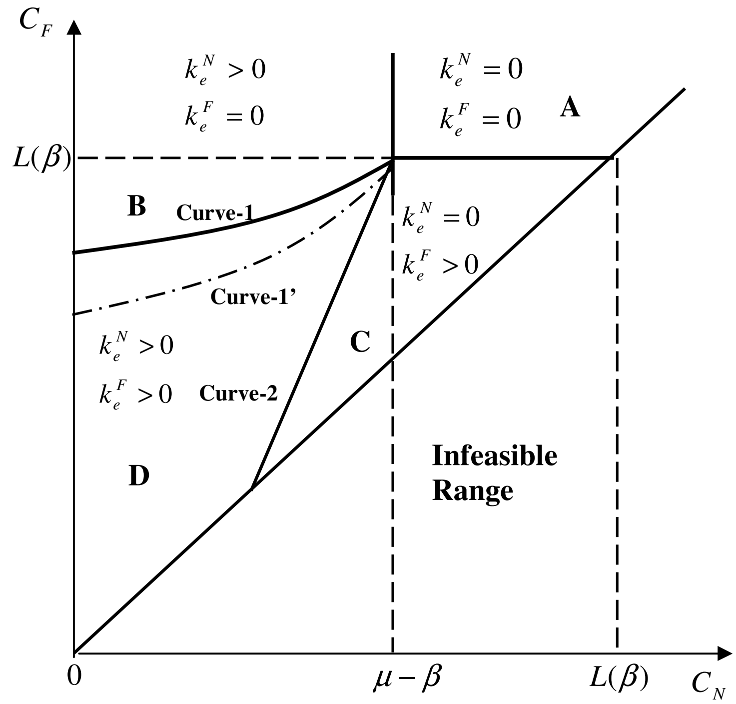


Figure 5.3(a): Partitions at equilibrium for non-zero production cost situation.

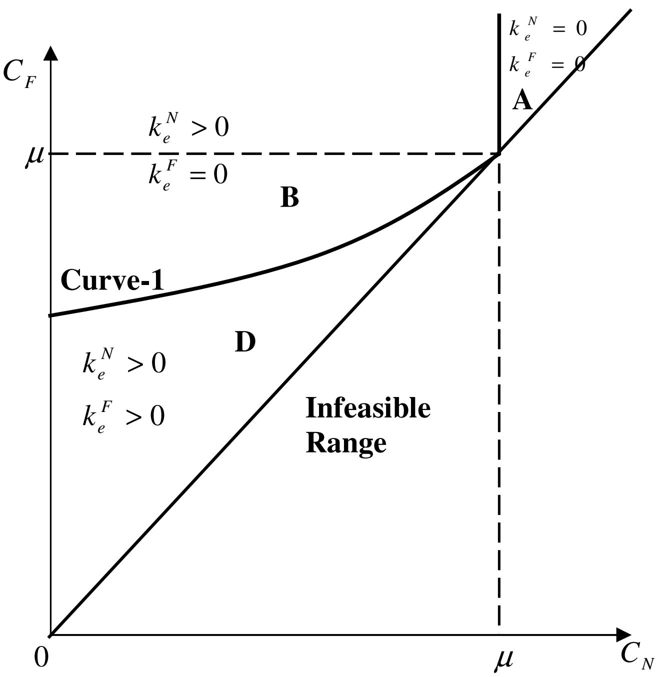


Figure 5.3(b): Partitions at equilibrium for zero production cost situation.

Figure 5.3: Partitions at equilibrium.

For zero production cost situation, i.e., $\beta = 0$, it is surprisingly noted that Curve-2 is actually the point $(C_N,\ C_F) = (\mu - \beta,\ L(\beta)) = (\mu,\ \mu)$. As a result, Case-C disappears, but other cases still exist, as shown in Figure 5.3(b). In such case, a market with profit potential can only be Case-B or Case-D. It is impossible for flexible strategy to be dominant. However, in-flexible firms always get profits for Case-B and Case-D whereas flexible firms only have limited chances to get profit. Comparing equilibrium for zero and non-zero production cost situations, it is shown that without considering production cost greatly underestimates the effectiveness of FCS. In fact, production cost is one of the key factors controlling whether a firm should adopt FCS or not.

In this chapter, we assume that for each type of strategies all firms have the same unit production cost and the same unit capacity cost, based on the following considerations:

(1) Considering the similarities between firms with the same strategy, we suppose that there is not large difference between their costs.

(2) This study focuses on the effect of the number of each type of firms. To achieve this objective, we exclude other disturbing factors by simplification. Further, theoretical results also confirm that the number of firms in each type does affect the competition equilibrium and firms' individual decisions. Interestingly, although all firms of the same type have the same unit production cost and the same unit capacity cost, their decisions at Nash equilibrium show that they cannot be treated as one firm. For example, in Case C, the total capacity and total profit of all the flexible firms depend on r, i.e.,

$$rk_e^F = \frac{r}{r+1}(X(C_F) - \beta),$$

$$r\Pi_e^F = \frac{r}{(r+1)^2}\left(\int_{\beta}^{X(C_F)} (\alpha - \beta)^2 f(\alpha)d\alpha + \int_{X(C_F)}^{\infty} (X(C_F) - \beta)^2 f(\alpha)d\alpha\right).$$

This is due to the fact that they compete against each other. At Nash equilibrium, if one firm changes its decision while the others do not, it will suffer. If they are treated as one firm, then the change of any one of them will imply the change of all firms together within the same type.

(3) There is a research absence in investigating FCS in an asymmetric oligopoly competition model in which each firm competes with two strategies simultaneously. Although it is intuitively believed that firms within the same type will make the same decisions, there is neither any theoretical proof of this result, nor this result is unique, in the literature. This research gap is settled in the thesis.

5.4 Sensitivity Analysis of Influential Factors

From the equation expressions of equilibrium solutions, five factors are extracted: production cost, flexible capacity cost, in-flexible capacity cost and numbers of flexible and in-flexible firms. In the following, we conduct sensitivity analysis of these five factors.

5.4.1 Effects of capacity costs on individual profit

Effects of capacity costs on each firm's individual expected profit in an oligopoly market are presented in Property 5.1.

Property 5.1 Given $r > 0$ flexible firms and $s > 0$ in-flexible firms, relationship between capacity costs and expected profits can be presented as follows:

(i) Flexible strategy is only effective in Region-D and Region-C; while in-flexible strategy is only effective in Region-D and Region-B.

(ii) In Region-B, $\Pi_e^F = 0$ and $\Pi_e^N > 0$,

(ii-1) Given C_F, $\dfrac{d\Pi_e^N}{dC_N} < 0$; (ii-2) Given C_N, $\dfrac{d\Pi_e^N}{dC_F} = 0$.

(iii) In Region-C, $\Pi_e^F > 0$ and $\Pi_e^N = 0$,

(iii-1) Given C_F, $\dfrac{d\Pi_e^F}{dC_N} = 0$; (iii-2) Given C_N, $\dfrac{d\Pi_e^F}{dC_F} < 0$.

(iv) In Region-D, $\Pi_e^F > 0$ and $\Pi_e^N > 0$, (iv-1) Given C_F, $\frac{d\Pi_e^N}{dC_N} < 0$; $\frac{d\Pi_e^F}{dC_N} > 0$;(iv-2) Given C_N, $\frac{d\Pi_e^F}{dC_F} < 0$; $\frac{d\Pi_e^N}{dC_F} > 0$. □

In a market with a dominant strategy, i.e., Region-B and Region-C, profits of firms adopting effective strategies are only affected by their own capacity costs, but independent of their rivals' capacity costs. However, in a strategy-coexistence market, i.e., Region-D, a firm's expected profit decreases with its own capacity cost, but increases with that of its rivals, which use the alternative strategy. As a result, a firm has to consider not only its capacity cost, but also the rivals' capacity costs. Under this situation, cutting capacity cost down does not guarantee to augment a firm's profit.

5.4.2 Effects of production costs on individual profit

Production cost effects on individual profit of each flexible firm and in-flexible firm are provided in Property 5.2.

Property 5.2 Given $r > 0$ flexible firms, $s > 0$ in-flexible firms and capacity costs $(C_N,\ C_F)$, the effects of production cost on each firm's expected profit is:

(i) In Region-A, no strategy is effective;

(ii) In Region-B, only in-flexible strategy is effective, and $\frac{d\Pi_e^N}{d\beta} < 0$;

(iii) In Region-C, only flexible strategy is effective, and $\frac{d\Pi_e^F}{d\beta} < 0$;

(iv) In Region-D, both flexible and in-flexible strategies are effective, $\frac{d\Pi_e^N}{d\beta} < 0$ and

(1) if $\frac{(s+1)\overline{F}(k^N) - k^N f(k^N)}{s} < 1$, then $\frac{d\Pi_e^F}{d\beta} > 0$;

(2) if $\frac{(s+1)\overline{F}(k^N) - k^N f(k^N)}{s} > 1$, then $\frac{d\Pi_e^F}{d\beta} < 0$. □

Intuitively, a firm's expected profit decreases with its production cost regardless of its adoptive strategy. However, the results in Property 5.2 surprisingly demonstrate that this intuition is not always true to flexible firms. In a strategy-coexistence market, the individual profit of a flexible firm increases as the production cost increases under certain conditions, which is affected by the total inflexible capacity, number of in-flexible firms and the demand distribution. In other words, flexible firms are able to augment their profits even though production cost increases. Further, making use of such advantages, flexible firms are able to enhance their competitiveness in a strategy-coexistence market with an increasing production cost. Results of Property 5.2 highlight the importance of consideration of production cost in determining whether FCS is better than IFCS or not.

5.4.3 Effects of number of flexible firms on total flexible and in-flexible capacities

In a strategy-coexistence market consisting of n firms, the effects of the number of flexible firms r on total capacity of each strategy are provided in Property 5.3.

Property 5.3 Given n firms and capacity costs $(C_N,\ C_F)$ in a strategy-coexistence market consisting of $r \geq 1$ flexible firms and $s \geq 1$ in-flexible firms where $r + s = n$. Within the range $r \in [1,\ n-1]$, we have

(i) total capacity of in-flexible firms is decreasing in r, i.e., $\frac{dk^N}{dr} < 0$;

(ii) total capacity of flexible firms is increasing in r, i.e., $\frac{dk^F}{dr} > 0$. □

The switch over of one firm's strategy will affect all other firms simultaneously. If more in-flexible firms switch their strategy to flexible strategy, total in-flexible capacity decreases but total flexible capacity increases. Therefore, one type of capacity (flexible capacity or in-flexible capacity) decreases with the other type increases and the amount of change in total capacity is less than the change of at least one type of capacity. This rule enables a market to automatically adjust any change occurred within a market and balance the weighting of each strategy to maintain a stable market without expanding infinitely.

5.4.4 Characteristics of total capacity

Property 5.4 Given (C_N, C_F), the total capacity of all firms k^T is bounded under various situations:

(i) If (C_N, C_F) is in Region-A, then $k^T = 0$.

(ii) If (C_N, C_F) is in Region-B, then k^T is decreasing in C_N, and independent of C_F, furthermore, $X(C_F) - \beta \le k^T = k^N < X(C_N + \beta)$.

(iii) If (C_N, C_F) is in Region-C, then K^T is decreasing in C_F, and independent of C_N; furthermore, $\frac{1}{2}(X(C_F) - \beta) < k^T = k^F = \frac{r}{r+1}(X(C_F) - \beta)$.

(iv) If (C_N, C_F) is in Region-D, then $\frac{r}{r+1}(X(C_F) - \beta) < k^T < X(C_F) - \beta$. □

Property 5.4 assures that the total capacity invested in the market is bracketed within the lower and upper bounds in any capacity costs. That is to say, the scale of a market with profit potential falls within a certain range. Attracted by the positive profit potential, firms are willing to make investments in the market, which results in the lower bound of total capacity investment in a market with profit potential. More and more capacity investment accumulate in the market until all profit potential has been fully explored, making total capacity investment expand to the upper bound. Property 5.4 points out that the total profit potential of a market is bounded and shared by all firms regardless of the strategies and total number of firms.

5.5 Comparison of Flexible and In-Flexible Individual Profits

According to equilibrium solution in Theorem 5.3, there is no dominant strategy in Region-D, in which both FCS and IFCS exist. To determine the most appropriate strategy, Proposition 5.11 compares the individual profits of flexible and in-flexible firms in Region-D.

Proposition 5.11 Given $r > 0$ flexible firms and $s > 0$ in-flexible firms, within Region-D, between Curve-1 and Curve-2, there exists a unique Curve-3 satisfying $\Pi_e^F(C_N, C_F) = \Pi_e^N(C_N, C_F)$; in Curve-3, C_F increases with C_N; in the area above Curve-3, denoted as Region-D1, $\Pi_e^N > \Pi_e^F > 0$; and in the area below Curve-3, denoted as Region-D2, $\Pi_e^F > \Pi_e^N > 0$. □

Proposition 5.11 proves the existence and uniqueness of the threshold that leads to the same individual profits of flexible and in-flexible firms as shown in Figure 5.4. The division of the profit comparison is determined by both flexible and in-flexible capacity costs. This division is also determined by the number of flexible firms and number of in-flexible firms. It is noted that with a low flexible capacity cost flexible strategy is always the optimal strategy in the region. However, with a low in-flexible capacity cost, the optimal strategy can be flexible or in-flexible. This indicates that the difference between the flexible capacity cost and in-flexible capacity cost determines the benefit of flexible strategy. Specifically, when all firms have the same capacity cost, i.e., $C_F = C_N$, the flexible strategy is always beneficial to a firm by avoiding production waste. For example, when a firm uses idle time or over time to acquire flexible capacity, but does not change the size or hourly pay of the workforce, then the flexible strategy gets the firm more profit. As flexible capacity cost increases, flexible firms make trade-off between the saving from avoiding production waste and the expensive capacity cost spending.

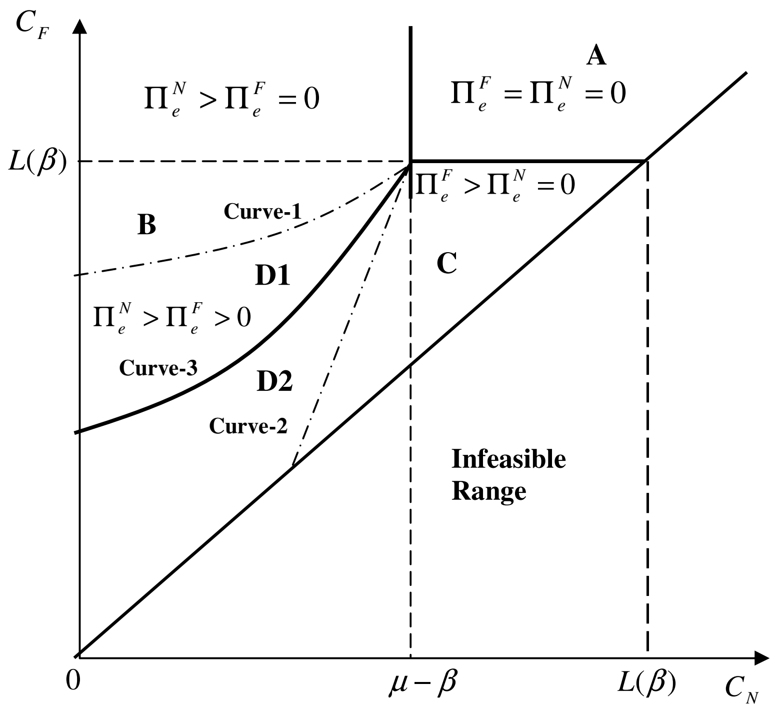


Figure 5.4: Individual profit comparison.

Chapter 6

Endogenous Flexibility of FCS in an n-Firm Competition

In Chapter 5, we have already known the equilibrium of an asymmetric oligopoly competition with *r* flexible firms and *s* in-flexible firms, i.e., n=(r, s). In this chapter, we consider the numbers of firms adopting each of two strategies, i.e., *r* and *s*, are endogenously determined in a competition with a total of *n* firms. To do so, we allow all *n* firms to freely switch their strategies to maximize individual profits until this is fully utilized. This process can be regarded as a strategy competition involving multiple players. Consequently, if no firm switches strategy (i.e., from flexible to in-flexible strategy, or vice verse), the status is defined as "**Final Equilibrium (FE)**". Among the n firms, the numbers of flexible and in-flexible firms at Final Equilibrium, i.e., $n = (r_e, s_e)$, is expressed as the endogenous flexibility of FCS in this thesis.

This chapter is divided into five sections. Section 6.1 provides the equivalent mathematical conditions for the Final Equilibrium. Section 6.2 characterizes the endogenous flexibility from strategic perspective. The results of the model are extended to perfect competition in Section 6.3. In Section 6.4, a method is proposed to practically determine the exact numbers of flexible and in-flexible firms under a given demand distribution. The theoretical justification is also provided. Section 6.5 provides the numerical examples with different demand distributions to demonstrate the method. All notations are the same as those used in Chapter 5.

6.1 Conditions of Final Equilibrium with Endogenous Flexibility

6.1.1 Conditions of final equilibrium

Given *n* firms, and $n=(r,s)$. The expected profit of each flexible firm is $\Pi_e^F(r,s)$, and of each in-flexible firm is $\Pi_e^N(r,s)$. Two possible scenarios of a firm's strategy switch exist in an *n*-firm oligopoly competition.

(1) If a firm switches from flexible strategy to in-flexible strategy, then $n=(r-1,s+1)$, the expected profit of each flexible firm will be $\Pi_e^F(r-1,s+1)$, and the expected profit of each in-flexible firm will be $\Pi_e^N(r-1,s+1)$.

(2) If a firm switches from in-flexible strategy to flexible strategy, then $n=(r+1,s-1)$, the expected profit of each flexible firm will be $\Pi_e^F(r+1,s-1)$, and the expected profit of each in-flexible firm will be $\Pi_e^N(r+1,s-1)$.

It is clear that, given *n* firms, the necessary and sufficient conditions of a Final Equilibrium at $n=(r,s)$ are $\Pi_e^F(r,s)\geq\Pi_e^N(r-1,s+1)$ and $\Pi_e^N(r,s)\geq\Pi_e^F(r+1,s-1)$, where $\Pi_e^N(-1,n+1)=0$ and $\Pi_e^F(n+1,-1)=0$.

6.2 Strategies of Endogenous Flexibility in an n-Firm Competition

Proposition 6.1 Referring to Curve-1, given total *n* firms and C_N, C_F is decreasing in *s*; referring to Curve-2, given total *n* firms and C_F, C_N is decreasing in *r*. □

Proposition 6.2 Consider Curve-1, Curve-2, Curve-4 and Curve-5 defined as follows:

Curve-1 $C_F = L(\beta + k_w)$, where k_w satisfies $\int_{k_w}^{\infty}\left(\alpha - \frac{s+1}{s}k_w\right)f(\alpha)d\alpha = C_N + \beta$;

Curve-2 $C_F = \frac{r+1}{r}C_N - \frac{r+1}{r}(\mu - \beta) + L(\beta)$, i.e., $\mu - (C_N + \beta) = \frac{r}{r+1}\left(L(\beta) - C_F\right)$;

Curve-4 $X(C_F) = X(C_N + \beta) + \beta$; Curve-5 $C_F = C_N - (\mu - \beta) + L(\beta)$.

Then we have following conclusions:

(i) Referring to each of these four curves, C_F is increasing in C_N;

(ii) there is one and only one intersection point for $C_N \in (0,\ \mu - \beta]$. The intersection point is $(C_N, C_F) = (\mu - \beta,\ L(\beta))$;

(iii) define C_{F1}, C_{F2}, C_{F4}, C_{F5} to be points on Curve-1, Curve-2, Curve-4, Curve-5, respectively, with given C_N. If $\beta > 0$, then $C_{F1} > C_{F4} > C_{F5} > C_{F2}$ for all $C_N \in (0,\ \mu - \beta)$; and, if $\beta = 0$, then $C_{F1} > C_{F4} = C_{F5} > C_{F2}$ for all $C_N \in (0,\ \mu - \beta)$. □

Proposition 6.3 Given $n = (r, s)$ and capacity costs $(C_N,\ C_F)$, we have the following conclusions about the Final Equilibrium:

(i) If $(C_N,\ C_F)$ is in Region-A, then the Final Equilibrium is obtained for any $n = (r, s)$;

(ii) If $(C_N,\ C_F)$ is in Region-B, then the Final Equilibrium is $n = (r, s) = (0, n)$;

(iii) If $(C_N,\ C_F)$ is in Region-C, then the Final Equilibrium is $n = (r, s) = (n, 0)$. □

To characterize the endogenous flexibility in an oligopoly competition, we first consider the boundaries of a market in which FCS and IFCS coexist (Region-D), i.e., Curve-1 and Curve-2. According to Proposition 6.1 and Corollaries 5.1 and 5.2, there are two respective families of Curve-1 and Curve-2 with different $(r,\ s)$ for a given number of firms n, which is shown in Figure 6.1.

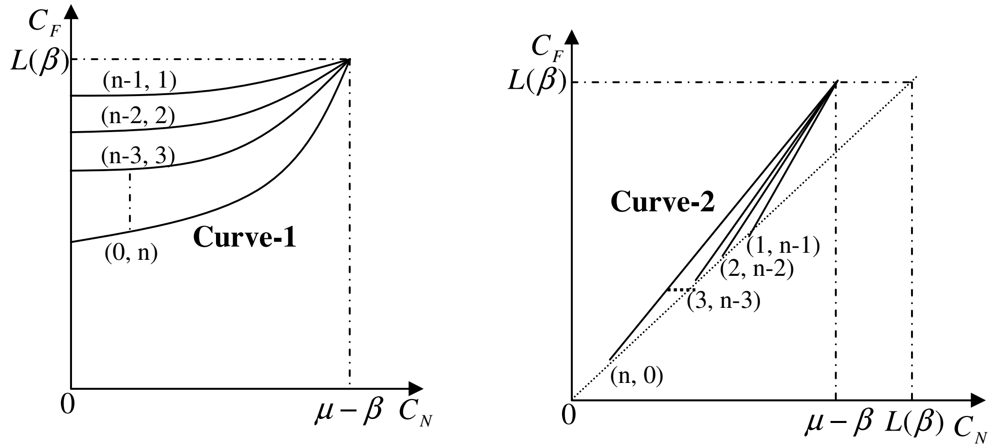


Figure 6.1: Families of Curve-1 and Curve-2.

We use $C_{F1}^{(r,s)}$ and $C_{F2}^{(r,s)}$ to represent Curve-1 and Curve-2, respectively, with respect to the combination of numbers of flexible and in-flexible firms $(r,\ s)$. According to Proposition 6.3, we know that in areas above curve $C_{F1}^{(0,n)}$ all firms transfer to in-flexible strategy at the Final Equilibrium when $C_N \in [0,\ \mu-\beta]$ for all $n \in [1,\ \infty)$; in areas on the right of curve $C_{F2}^{(n,0)}$ all firms transfer into flexible strategy at the Final Equilibrium when $C_F \in [0,\ L(\beta)]$ for all $n \in [1,\ \infty)$. In areas between curves $C_{F1}^{(0,\infty)}$ and $C_{F2}^{(\infty,0)}$, two strategies may coexist in the market at Final Equilibrium for all $n \in [2,\ \infty)$. In other areas, the endogenous flexibility is sensitive to number of firms in a market. In particular, for a given *n*, in areas above curve $C_{F1}^{(0,n)}$ all firms transfer to in-flexible strategy at the Final Equilibrium when $C_N \in [0,\ \mu-\beta]$; at areas on the right of curve $C_{F2}^{(n,0)}$ all firms transfer into flexible strategy at the Final Equilibrium when $C_F \in [0,\ L(\beta)]$; and in the area between these two curves, two strategies coexist in a market. These results are presented formally in Theorem 6.1.

Theorem 6.1 For all $n \in [1,\ \infty)$, given production cost β, within the area $\{(C_N, C_F): 0 < C_N \le C_F < \infty\}$, the Final Equilibrium can be characterized as below:

(i) in area $\mu-\beta \le C_N$ & $L(\beta) \le C_F$ & $C_N \le C_F$, for any $n=(r,s)$, two strategies lead to zero profit for all $n \in [1,\ \infty)$;

(ii) in area $0 \le C_N < \mu - \beta$ & $L(\beta + k_{w1}) \le C_F$ & $C_N \le C_F$, all firms transfer to in-flexible strategy, i.e., $n = (r,s) = (0,n)$ for all $n \in [1, \ \infty)$;

(iii) in area $\mu - (C_N + \beta) \le \frac{1}{2}(L(\beta) - C_F)$ & $L(\beta) > C_F$ & $C_N \le C_F$, all firms transfer to flexible strategy, i.e., $n = (r,s) = (n,0)$ for all $n \in [1, \ \infty)$;

(iv) in area $L(\beta + k_{w1}) > C_F$ & $\mu - (C_N + \beta) > \frac{1}{2}(L(\beta) - C_F)$ & $C_N \le C_F$, there are three sub-areas as below:

(iv-1) when $L((X(C_F) - \beta) - C_N - \beta > 0$, let $\overline{N} = \dfrac{(X(C_F) - \beta)\overline{F}(X(C_F) - \beta)}{L(X(C_F) - \beta) - C_N - \beta}$:

(iv-1-1) if $n \ge \overline{N}$, then all firms transfer to in-flexible strategies, i.e., $n = (r,s) = (0,n)$;

(iv-1-2) if $n < \overline{N}$, then at Final Equilibrium, both flexible and in-flexible firms coexist in the market;

(iv-2) when $L(X(C_F) - \beta) - C_N - \beta \le 0$ and $L(\beta) - C_F - (\mu - C_N - \beta) \le 0$, both flexible and in-flexible firms coexist in the market regardless of number of firms;

(iv-3) when $L(\beta) - C_F - (\mu - C_N - \beta) > 0$, let $\tilde{N} = \dfrac{\mu - C_N - \beta}{L(\beta) - C_F - (\mu - C_N - \beta)}$:

(iv-3-1) if $n \ge \tilde{N}$, then all firms transfer to flexible strategy, i.e., $n = (r,s) = (n,0)$;

(iv-3-2) if $n < \tilde{N}$, then at Final Equilibrium, both flexible and in-flexible firms coexist in the market.

where k_{w1} is the unique solution of the equation $\int_{k_{w1}}^{\infty} (\alpha - 2k_{w1}) f(\alpha) d\alpha = C_N + \beta$. □

In the literature, the analyses of endogenous flexibility in a competition are provided only for zero production cost situation, i.e., $\beta = 0$. For non-zero production cost situation, i.e., $\beta > 0$, even in a duopoly model, there has not been any analyses on endogenous flexibility so far due to analytical difficulties (Anupindi and Jiang, 2008). However, Theorem 5.3 in Chapter 5 concludes that the production cost is one of key factors in determining whether a firm should use FCS or not. Moreover, the zero production cost situation deviates too far from the reality so that the effectiveness of

FCS in a competition is greatly underestimated. Ways of how production cost affects endogenous flexibility are further overlooked. To fill these research gaps, we characterize the endogenous flexibility of FCS in an n-firm competition for both zero production cost and non-zero production cost situations.

Theorem 6.1 characterizes the endogenous flexibility of FCS for a variable $n \in [1, \infty)$. Figure 6.2(a) plots the results under a non-zero production cost situation, i.e., $\beta > 0$. It can be proved that curves $C_{F1}^{(0,\infty)}$ and $C_{F2}^{(\infty,0)}$ partition the area between $C_{F1}^{(0,1)}$ and $C_{F2}^{(1,0)}$ into three sub-areas. To our surprise, the endogenous flexibility of FCS in areas upper $C_{F1}^{(0,1)}$ and in areas lower $C_{F2}^{(1,0)}$ are completely insensitive to the number of firms, even though n varies from a certain number to infinity. In such case, the competition becomes a symmetric oligopoly competition eventually. In areas between curves $C_{F1}^{(0,1)}$ and $C_{F2}^{(1,0)}$, the number of firms affects the endogenous flexibility in different ways to different extends.

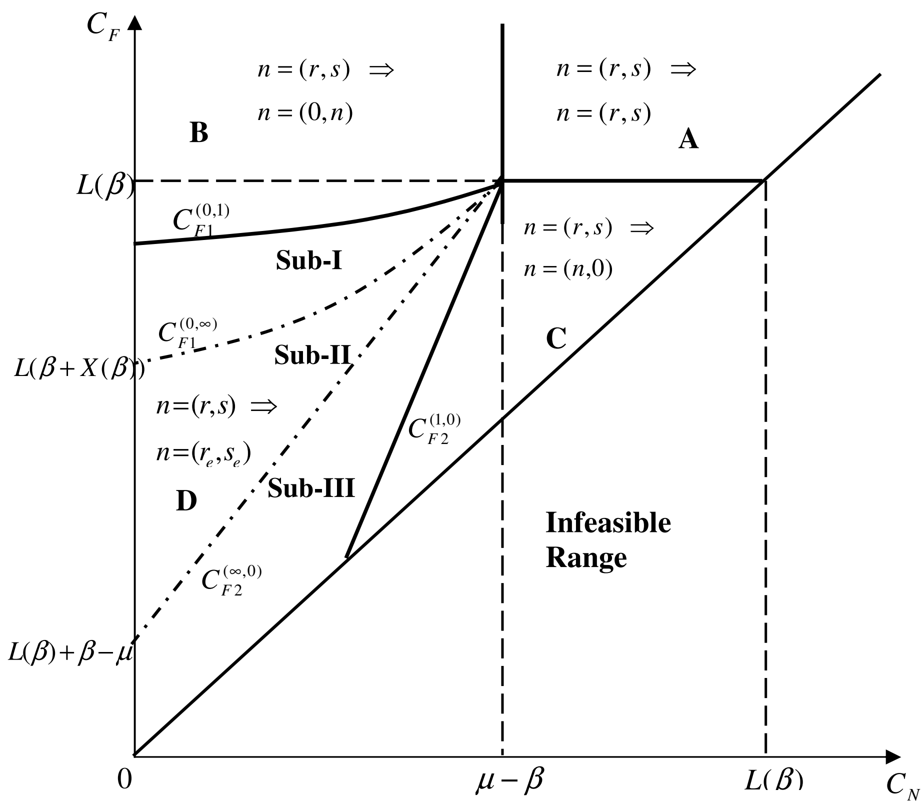


Figure 6.2(a): Final Equilibrium of non-zero production cost situation for all $n \in [1, \infty)$.

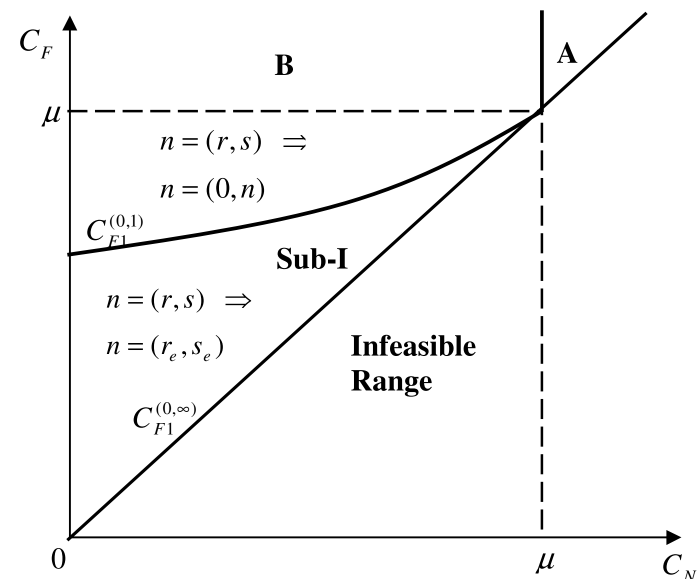


Figure 6.2(b): Final Equilibrium of zero production cost situation for all $n \in [1,\ \infty)$.

Figure 6.2: Final Equilibrium for all $n \in [1,\ \infty)$.

Particularly, in sub-area-II (corresponding to part (iv-2)), the two strategies may coexist in an oligopoly market. Although the endogenous flexibility is insensitive to the number of firms from strategic perspective, the exact numbers of flexible and in-flexible firms are affected by the number of firms. Therefore, the endogenous flexibility in this area is partially insensitive to the number of firms. In sub-area-I and sub-area-III (corresponding to part (iv-1) and (iv-3), respectively), the endogenous flexibility is sensitive to the number of firms; moreover, it may even entirely switches over in nature. For each capacity costing $(C_N,\ C_F)$ in these two areas, there is a threshold of the number of firms. When the number of firms is smaller than the threshold, two strategies may coexist in the market; as more firms join in the market and number of firms beyond the threshold, only one strategy can survive in the market. It means that the market structure entirely switches over to symmetrical oligopoly market when there are many firms in the market. It is further observed that the area of sensitive environments becomes smaller as more firms join in the market until the number of firms tends to be infinite, i.e., $n \to \infty$. In such case, no sensitive environment exists and the strategies survived after strategy competition is determined.

For zero production cost, i.e., $\beta = 0$, it is found that curves $C_{F1}^{(0,\infty)}$, $C_{F2}^{(\infty,0)}$ and $C_F = C_N$ overlap. Consequently, Region-C, sub-area-II and sub-area-III in Figure 6.2(a) disappear, whereas only region-A, region-B and sub-area-I exist, as shown in Figure 6.2(b). In such case, only Region-B and sub-area-I are regions with profit potential. As a result, there is only very limited chance for firms willing to adopt FCS, whereas there is a large chance that all firms are willing to adopt IFCS. It is further impossible that all firms adopt flexible strategy at Final Equilibrium.

For any given n, the Final Equilibrium of the competition is provided in Theorem 6.2.

Theorem 6.2 For a given *n*, (i) if $\mu - \beta \le C_N$ & $L(\beta) \le C_F$ & $C_N \le C_F$, then for any $n = (r,s)$, two strategies lead to zero profit; (ii) if $0 \le C_N < \mu - \beta$ & $L(\beta + \bar{k}_w) \le C_F$ & $C_N \le C_F$, then $n = (r,s) = (0,n)$; (iii) if $\mu - (C_N + \beta) \le \frac{n}{n+1}(L(\beta) - C_F)$ & $L(\beta) > C_F$ & $C_N \le C_F$, then $n = (r,s) = (n,0)$; (iv) if $L(\beta + \bar{k}_w) > C_F$ & $\mu - (C_N + \beta) > \frac{n}{n+1}(L(\beta) - C_F)$ & $C_N \le C_F$, then flexible and in-flexible firms may coexist in the market; where $\bar{k}_w$ is the unique solution of the equation $\int_{\bar{k}_w}^{\infty}(\alpha - \frac{n+1}{n}\bar{k}_w) f(\alpha) d\alpha = C_N + \beta$.

<u>**Proof**</u>

Following the proof of Theorem 6.1, we can get Theorem 6.2 directly. □

Theorem 6.2 characterizes the endogenous flexibility for a given *n*. For non-zero production cost situation, i.e., $\beta > 0$, the endogenous flexibility can be one of three situations under different capacity costing conditions, as shown in Figure 6.3(a), from strategic perspective. For zero production cost situation, i.e., $\beta = 0$, only two possible cases may exist after strategy competition, as shown in Figure 6.3(b). Comparing Figure 6.3(a) and Figure 6.3(b), it is noted that for non-zero production cost situation there is a region in which all firms switch to FCS, whereas such region does not exist for zero production cost situation. The comparison emphasizes that production cost creates opportunity to enhance competitiveness for flexible firms. This implication is in line with Property 5.2 in Chapter 5 that under certain environments, flexible firms may be benefited from increasing of production cost while in-flexible firms always suffer from increasing of production cost.

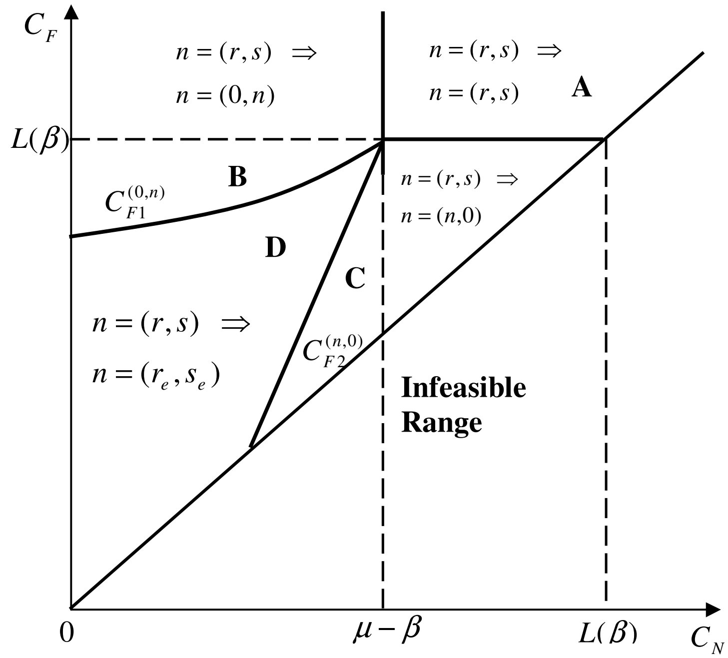


Figure 6.3(a): Final Equilibrium for non-zero production cost situation with given *n*.

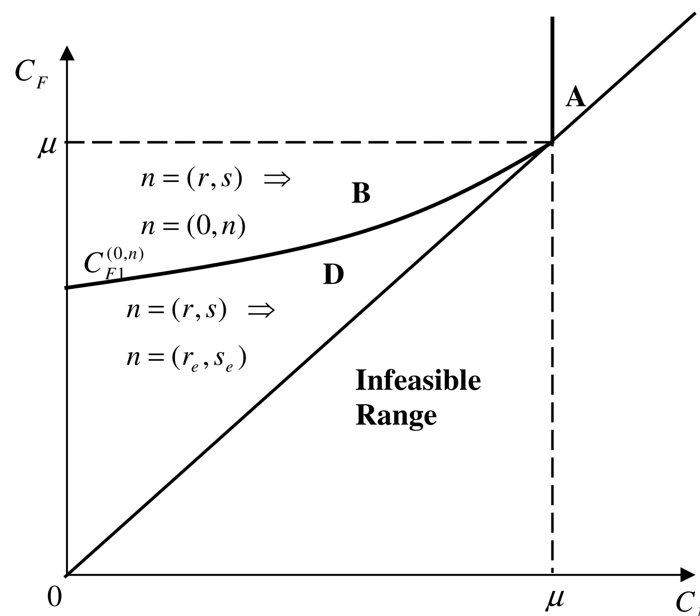


Figure 6.3(b): Final Equilibrium for zero production cost situation with given *n*.

Figure 6.3: Final Equilibrium with given *n*.

6.3 Strategies of Endogenous Flexibility in a Profit-Driven Market

Since Theorem 6.1 holds for all $n \in [1,\ \infty)$, we consider a profit-driven market in this section to examine the adaptability of our model. The profit-driven market is defined as a market in which firms have total freedom to join or quit the market, to choose or switch their capacity strategy, and there is no limitation on the number of firms in the market. Decisions of firms are absolutely driven by pursuing profit. The Final Equilibrium of such profit-driven market is expressed as "Stable Market".

Theorem 6.3 In a profit-driven market, the Stable Market can be characterized as follows within the area $\{(C_N, C_F): 0 < C_N \le C_F\}$.

(i) If $\mu - \beta \le C_N$ & $L(\beta) \le C_F$ & $C_N \le C_F$, then no firm will exist in the market eventually, i.e., $n = 0$;

(ii) If $L(X(C_F) - \beta) - C_N - \beta > 0$, i.e., above the curve $C_{F1}^{(0,\ \infty)}$, then the Stable Market stays at Case-B $n = (0, n)$, $n \to \infty$ and $\Pi_e^N(0, n) \to 0$;

(iii) If $L(\beta) - C_F - (\mu - C_N - \beta) > 0$, i.e., below the curve $C_{F2}^{(\infty,\ 0)}$, then the Stable Market stays at Case-C $n = (n, 0)$, $n \to \infty$ and $\Pi_e^F(n, 0) \to 0$;

(iv) If $L(X(C_F) - \beta) - C_N - \beta \le 0$ and $L(\beta) - C_F - (\mu - C_N - \beta) \le 0$, i.e., area between Curve-4 and Curve-5, then the Stable Market stays at Case-D $n = (r, s)$, $n \to \infty$, $s \to \infty$, $r \to \infty$, $\Pi_e^F \to 0$ and $\Pi_e^N \to 0$. □

Theorem 6.3 points out that the Stable Market is only determined by the market profit potential. It is interesting that the Stable Market in a profit-driven market is actually a perfect competition. Theorem 6.3 concludes this and there is no dominant strategy for all costing environments and both FCS and IFCS may coexist in a market, even the number of firms is infinite. We draw a conclusion that the perfect competition is actually a particular case of an oligopoly market with number of firms tending to infinity.

In real business, a firm needs to consider other costs, such as set up costs, fixed costs, administrative costs, etc, besides capacity and production costs. So, a firm sets a profit bottom line to ensure its normal operations. As a result, a firm will not stay in a market with a profit lower than its bottom line. This rule controls a market scale so that the number of firms does not expand infinitely at whatever capacity costs are. With deep understanding of the relationship between market trends, endogenous strategy selections and cost factors, managers are able to determine appropriate strategies promptly, balance expenses and revenues, sketch a long term development plan, and avoid involving in marginal businesses.

6.4 $(r_e,\ s_e)$ at Final Equilibrium Involving Two Strategies

Although Theorem 6.1 and Theorem 6.2 fully characterize the endogenous flexibility, it is quite difficult to determine the exact numbers of flexible and in-flexible firms $(r_e,\ s_e)$ in area between curves $C_{F1}^{(0,n)}$ and $C_{F2}^{(n,0)}$. Trying to overcome such difficulties, we propose an approach to determine the exact numbers of flexible firms and in-flexible firms $(r_e,\ s_e)$ at Final Equilibrium which involves two strategies in this section. Theoretical justification of the approach is also provided.

Based on the analysis of conditions of Final Equilibrium, we define a function $D(r)$ in this section so that the conditions can be totally presented in terms of the number of flexible firms. We define $D(r) = \Pi_e^F(r, n-r) - \Pi_e^N(r-1, n-r+1)$, where $1 \le r \le n$. Therefore, the necessary and sufficient condition of the Final Equilibrium can be rewritten as: $D(r) \ge 0$ and $D(r+1) \le 0$. All curves $D(r) = 0$ where $r \in [1,\ n]$, if exist, intersect at point $(C_N,\ C_F) = (\mu - \beta,\ L(\beta))$. Given $r = r_0$, $r_0 \in [1,\ n]$, define $G(C_N,\ C_F | r_0) = D(r_0) = 0$, in terms of C_N and C_F. The existence and properties of the curve $G(C_N,\ C_F | r_0) = D(r_0) = 0$ is provided by Theorem 6.4.

Theorem 6.4 Given $r = r_0$, $r_0 \in [1,\ n]$, there exists a unique curve satisfying $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$, on which C_F increases as C_N increases; in areas above the curve $D(r_0) = 0$, we have $G(C_N,\ C_F \mid r_0) = D(r_0) < 0$; in areas below the curve $D(r_0) = 0$, we have $G(C_N,\ C_F \mid r_0) = D(r_0) > 0$. □

Based on Theorem 6.4, all curves $D(r) = 0$ where $r \in [1,\ n]$ intersect at point $(C_N,\ C_F) = (\mu - \beta,\ L(\beta))$. Specifically, consider two curves $D(r_0) = 0$ and $D(r_0 + 1) = 0$ with a given value of $r = r_0$, where $r_0 \in [1,\ n-1]$. With respect to relative positions of the two curves $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$ and $G(C_N,\ C_F \mid r_0 + 1) = D(r_0 + 1) = 0$, there are three possible situations: The curve $D(r_0) = 0$ is above, overlap and below the curve $D(r_0 + 1) = 0$. Incorporating the conditions of the Final Equilibrium, we have the conclusions in Theorem 6.5.

Theorem 6.5 Given *n* firms, for every $r_0 \in [1,\ n-1]$, consider curves $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$ and $G(C_N,\ C_F \mid r_0 + 1) = D(r_0 + 1) = 0$ within the area $\{(C_N, C_F) : C_N \le \mu - \beta\ \&\ C_F \le L(\beta)\ \&\ 0 \le C_N \le C_F\}$, then the Final Equilibrium $n = (r_e,\ s_e)$ can be categorized into one of the following five scenarios in terms of the exact numbers of flexible and in-flexible firms.

(i) In areas below curve $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$ and above curve $G(C_N,\ C_F \mid r_0 + 1) = D(r_0 + 1) = 0$, we have $n = (r_e,\ s_e) = (r_0,\ n - r_0)$;

(ii) in areas above curve $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$ and below curve $G(C_N,\ C_F \mid r_0 + 1) = D(r_0 + 1) = 0$, we have the Final Equilibrium does not obtained at $r = r_0$;

(iii) if these two curves overlaps, with respect to points on the curves, we have either $n = (r_e,\ s_e) = (r_0,\ n - r_0)$ or $n = (r_e,\ s_e) = (r_0 + 1,\ n - r_0 - 1)$;

(iv) in areas above all curves $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$, $r_0 \in [1,\ n]$, we have

$n = (r_e = 0,\ s_e = n) = (0,\ n)$;

(v) in areas below all curves $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$, $r_0 \in [1,\ n]$, we have

$n = (r_e = 0,\ s_e = n) = (n,\ 0)$. □

Theorem 6.5 shows that the equivalent condition of Final Equilibrium, i.e., $D(r) \geq 0$ and $D(r+1) \leq 0$, can be reflected by the relative position of the two curves $D(r) = 0$ and $D(r+1) = 0$. Such relationship ensures that theoretically the Final Equilibrium can be determined in terms of the exact numbers of flexible firms and in-flexible firms. Specifically, for the two curves $D(r) = 0$ and $D(r+1) = 0$, all possible situations in which these two curves have different relative positions are analyzed. As a result, the exact numbers of flexible and in-flexible firms can be determined theoretically by considering the two curves $D(r) = 0$ and $D(r+1) = 0$ for all possible values of *r*. Theorem 6.5 also concludes that the maximum number of equilibrium scenarios is $n+1$ as defined in the theorem. With application of Theorem 6.5 under a certain demand distribution, the exact numbers of flexible and in-flexible firms can be practically determined by plotting all curves $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$, where $r_0 \in [1,\ n]$. As no special assumptions about demand distributions are given, more detailed properties about the curve $D(r) = 0$ will be observed under certain demand distributions in the real decision-making operations.

6.5 Numerical Examples

As a demonstration, the proposed approach is applied to a three-firm model, i.e., $n = 3$, under some uniform and exponential distributions, respectively. Table 6.1 provides the parameters of the examples.

Distribution	PDF $f(x)$	Distribution parameters	Mean (μ)	Production cost (β)
Uniform	$f(x)=\frac{1}{b}, x\in[0,b]$	$b=40$	20	2
Exponential	$f(x)=\lambda e^{-\lambda x}, x\in[0,\infty)$	$\lambda=1$	1	0.1

Table 6.1: Parameters of numerical examples.

Applying Theorem 6.5 under these two distributions, we obtain very similar patterns of the curves $G(C_N,\ C_F|\ r_0)=D(r_0)=0$ where $r_0=1,\ 2,\ 3$, with respect to three observations: (1) all three curves intersect only at point $(C_N,\ C_F)=(\mu-\beta,\ L(\beta))$; (2) the curve $D(r)=0$ moves downward when r increases; (3) the area in which flexile and in-flexible strategies coexist is relatively small, comparing to the areas dominated by only one strategy (flexible or in-flexible). After plotting all the curves $G(C_N,\ C_F|\ r_0)=D(r_0)=0$, $r_0=1,2,3$, the endogenous flexibility can be fully determined in terms of the exact numbers of flexible and in-flexible firms.

Figure 6.4 and Figure 6.5 include all the curves $G(C_N,\ C_F|\ r_0)=D(r_0)=0$, $r_0=1,2,3$ under some uniform and exponential distributions, respectively. The Final Equilibrium of these two examples can be described as follows:

(i) With respect to points on the curves $D(r_0)=0$ ($r_0=1,\ 2,\ 3$), the Final Equilibrium stays at $n=(r_0,n-r_0)$.

(ii) In the area between $D(1)=0$ and $D(2)=0$, the Final Equilibrium stays at $3=(1,\ 2)$.

(iii) In the area between $D(2)=0$ and $D(3)=0$, the Final Equilibrium stays at $3=(2,\ 1)$.

(iv) In the area above curve $D(1)=0$, the Final Equilibrium stays at $3=(0,\ 3)$.

(v) In the area below curve $D(3)=0$, the Final Equilibrium stays at $3=(3,\ 0)$.

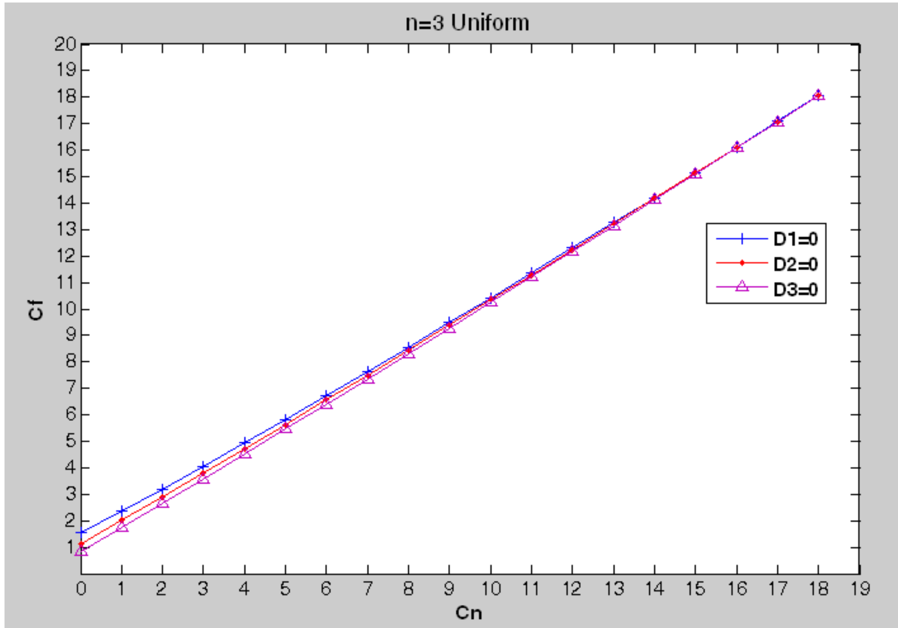


Figure 6.4: Uniform distribution example.

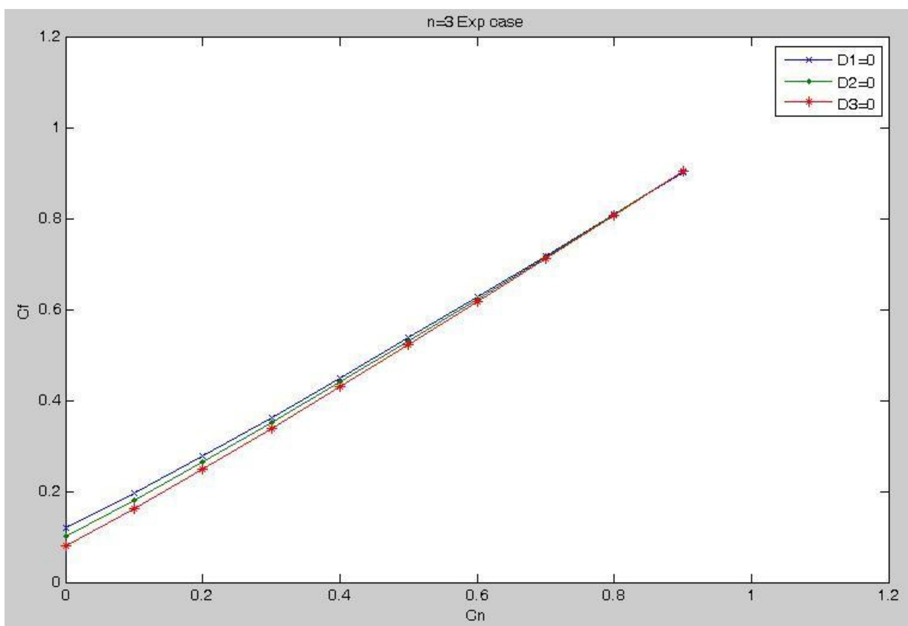


Figure 6.5: Exponential distribution example.

It is observed in the numerical examples that the curve $D(r)=0$ moves downward when r increases. This observation indicates the existence and uniqueness of pure strategy for competition involving multiple competitors. Based on this observation, we set:

Assumption A: The curve $D(r)=0$ moves downward when *r* increases.

Under Assumption A, Proposition 6.4 is obtained to characterize the endogenous flexibility.

Proposition 6.4 Given *n* firms and cost parameters, if curve $D(r)=0$ moves downward when *r* increases , then pure strategy exists at the equilibrium of the strategy competition; further, within the area $\{(C_N, C_F): 0 < C_N \le C_F\}$, we have (1) in the area above $D(1)=0$, Final Equilibrium stays at $n=(0,\ n)$; (2) in areas below $D(n)=0$, Final Equilibrium stays at $n=(n,\ 0)$; and (3) in areas below curve $D(r_0)=0$ and above curve $D(r_0+1)=0$, Final Equilibrium stays at $n=(r_0,\ n-r_0)$.

Proof

Under the assumption that the curve $D(r)=0$ moves downward when *r* increases, Proposition 6.4 can be obtained directly from Theorem 6.4. □

Under the assumption extracted from the numerical examples, Proposition 6.4 further describes the Final Equilibrium of a two-strategy oligopoly competition. It indicates endogenous flexibility follows a pattern as shown in Figure 6.6. It shows that the number of flexible firms at the Final Equilibrium can be any number from 0 to *n*. This finding emphasizes the complexity of the endogenous flexibility in an oligopoly competition; on the other hand, the pattern of endogenous flexibility reveals a regular order of the endogenous flexibility. In areas between the curves $D(1)=0$ and $D(n)=0$, for a given in-flexible capacity cost, more firms switch to flexible firm as flexible capacity cost decreases. Comparing to the areas with only one strategy, the area with two strategies coexisting is relatively narrow. This conclusion can be helpful in partially explaining that, in some industries, even though two strategies are available, all firms use the same strategy.

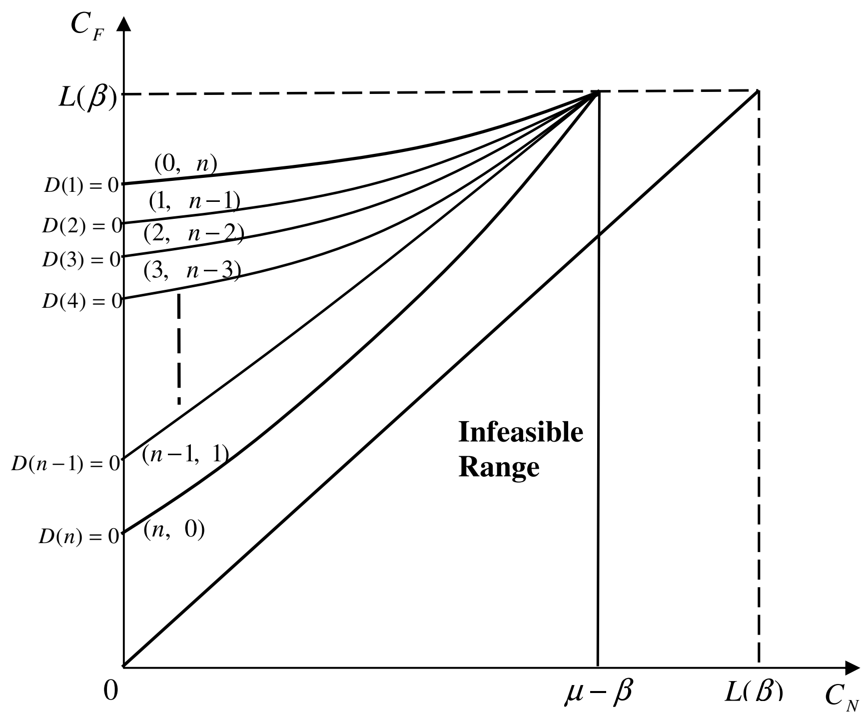


Figure 6.6: Oligopoly endogenous flexibility under Assumption A.

The conclusions of endogenous flexibility enable managers to choose the correct strategies and predict the eventual market status. Moreover, the additional profit by capacity investments can be calculated. The complexity of endogenous flexibility reminds managers to be very careful in choosing the strategy since the beginning of the competition. Further, making analysis of the current status and eventual equilibrium prevent firms from involving in marginal business.

Chapter 7

Modeling FCS with Flexibility Degree

In previous chapters, a firm's capacity strategy is either FCS or IFCS. With FCS, a firm is able to adjust its production quantity from zero to its capacity; while a firm with IFCS has to produce the quantity equal to its capacity. However, observation from the reality is that a large number of firms, if not the most, adopt a mixed strategy which is in between the two extremes: chase strategy (FCS) and level strategy (IFCS). Firms adopting the mixed strategy have the flexibility to adjust their throughput to some extent, but within limited ranges. Aiming at quantifying firms' abilities in adjustment, and distinguishing different performance of firms' FCS implementation, this chapter proposes the concept of **Flexibility Degree** to measure the FCS implementation. The flexibility degree is defined as the percentage of the difference between a firm's production upper bound (total capacity) and production lower bound (guaranteed or unchanged production level) over its total capacity. It reflects the extent to which FCS is exploited.

This chapter consists of 3 sections. Section 7.1 formulates flexibility degree in a monopoly model under demand uncertainty. With a given flexibility degree, the optimal decisions on total capacity and production quantity are derived in this section. Section 7.2 establishes a duopoly model in which two firms with different flexibility degrees compete with each other under demand uncertainty. Section 7.3 provides some numerical examples to demonstrate the theoretical results and get an intuitive understanding of the flexibility degree effects on the optimal total capacities of two firms in a competition.

7.1 Flexibility Degree Concept

7.1.1 Notations and assumptions

We follow the notations used in Chapters 5 and 6, except that the superscripts {F, N} of variables in this chapter are deleted. It is because from a general perspective, all firms are supposed to be flexible firms with their own flexibility degree; and even firms adopting IFCS can be considered as a flexible firm with zero flexibility degree. This chapter aims at figuring out the relationship between firms' decisions, their own flexibility degree and their rivals' flexibility degrees in a competition. However, according to the results of Chapters 5 and 6, the additive inverse function results in zero capacity with a large chance under various environments. Therefore, the additive inverse demand function is not the best choice to achieve the objective of study in this chapter. Instead, we adopt in this chapter the multiplicative inverse demand function, which has been widely used in the literature (e.g., Anupindi and Jiang, 2008). Specifically, $p(\alpha, Q) = \alpha(a - Q)$, where $Q = \sum_i q_i$, $i \in \Omega$, is the total production quantity in the market, and a is a large enough constant so that $a > 2k$, $a > 3\max\{k_1, k_2\}$, $a > \frac{\beta + C_F}{\mu} + \max\{k_1, k_2\}$ and $a \geq \frac{6(\beta + C_F)}{\mu}$, where k is the production capacity for monopoly case, and k_1 and k_2 are the production capacities for duopoly case.

7.1.2 Demonstration of flexibility degree concept

Inspired by the widely used mixed strategy in reality, which is actually in between the level strategy and the chase strategy, we construct a new model as shown in Figure 7.1. This figure also shows the decision variables, quantities and constraints at each decision-making stage.

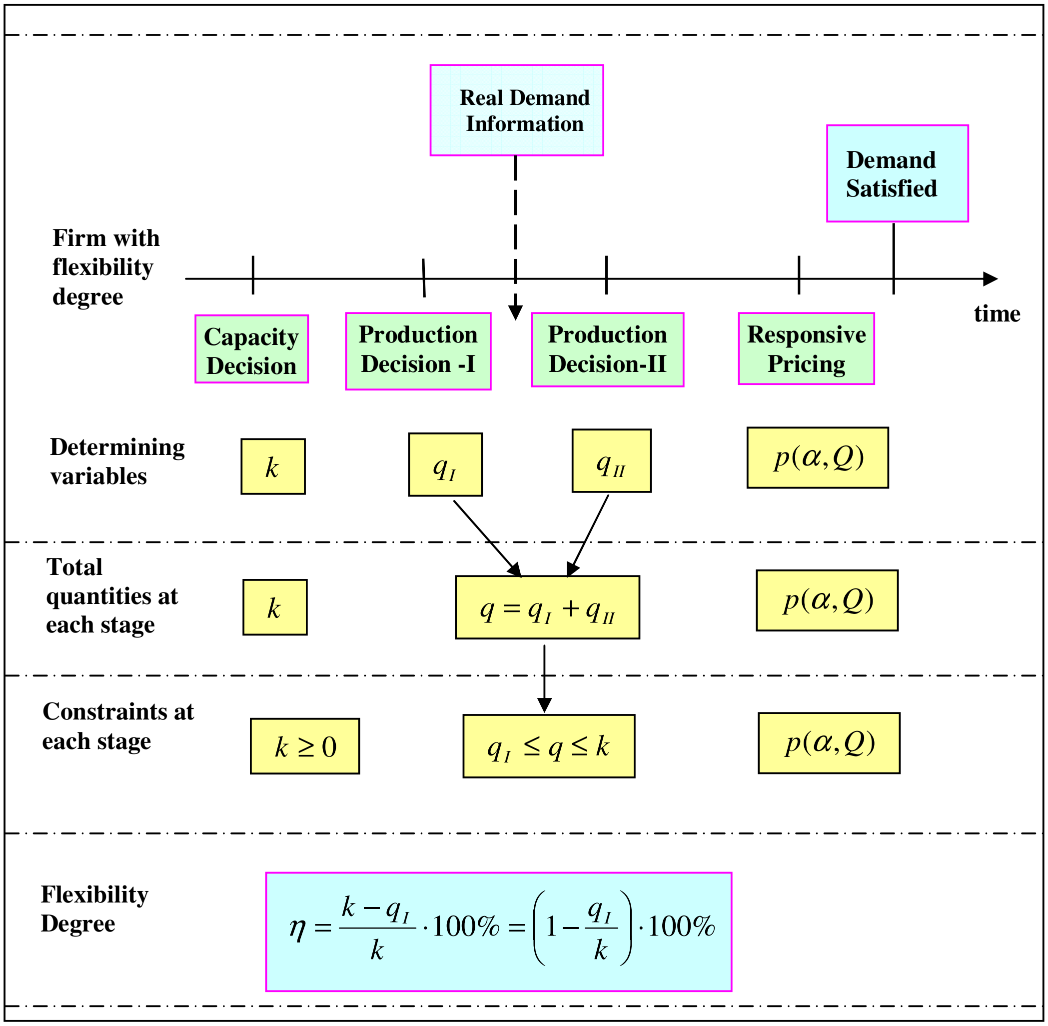


Figure 7.1: Decision-making process of a firm with flexibility degree.

As shown in Figure 7.1, the production stage is divided into two sub-stages, namely, production decision-I and production decision-II. Let q_I and q_{II} be production quantities at production decision-I and production decision-II, respectively. The total production quantity is $q = q_I + q_{II}$. At the stage of production decision-I, there is a stable production level q_I which does not change under fluctuating demand. At the stage of production decision-II, the production level q_{II} is affected by the fluctuating demand. After knowing the real demand, a firm is able to make production adjustment to maximize its ex-post profit under the constraint of the allowable capacity.

Since q_I is a stable production level, the allowable adjustment range at the production decision-II stage is $q_{II} \in [0, k - q_I]$. Accordingly, the total production quantity $q = q_I + q_{II}$ is within the range $q \in [q_I, k]$. The maximum adjustment of production level is $k - q_I$. This adjustment range reflects a firm's ability in making adjustment of its actual production level. Based on such relationship between the maximum adjustment and a firm's ability to vary its throughput, **Flexibility Degree** is defined as $\eta = \frac{k - q_I}{k} \cdot 100\% = \left(1 - \frac{q_I}{k}\right) \cdot 100\%$. Define $m = \frac{q_I}{k}$, $m \in [0,\ 1]$, to be the **In-flexibility Degree**. The flexibility degree η can be expressed in terms of the in-flexibility degree, i.e., $\eta = (1-m) \cdot 100\%$. The relationship between flexibility degree and in-flexibility degree can also be presented as $\eta + m = 1$. The definition of flexibility degree and in-flexibility degree are formally provided below.

Flexibility Degree is defined as a percentage of the difference between a firm's production upper bound (total capacity) and production lower bound (guaranteed or unchanged production level) over its total capacity.
In-Flexibility Degree is defined as a percentage of a firm's production lower bound (guaranteed or unchanged production level) over its total capacity.

Similar to the analyses in the previous chapters, mathematical formulation of this problem involves a three-stage decision-making process, which is presented in the following section.

7.2 FCS with Flexibility Degree in a Monopoly Model

In a monopoly model, a firm needs to determine its optimal capacity and optimal production quantity with a certain flexibility degree. Given a firm's in-flexibility degree $m \in [0,\ 1]$, its flexibility degree is $\eta = (1-m) \cdot 100\%$. For example, a firm and its retailers sign a contract to ensure their deal like this: the firm guarantees 80% of product supply regardless of costing or market situations; the rest 20% of product supply is determined by firm itself to respond various market situations.

7.2.1 Capacity decision stage

At capacity decision stage, a firm determines its capacity to maximize its expected profit of the whole decision-making process. The capacity is also the maximum of a firm's production ability. The capacity decision can be formulated as

$$\text{Max} \quad \Pi(k) = \int_0^\infty q(\alpha(a-q)-\beta) f(\alpha) d\alpha - C_F k, \text{ s.t. } k \geq 0, \tag{7.1}$$

where q is the optimal solution in the production decision stage.

7.2.2 Production decision stage

At the production decision stage, a firm's production quantity is bounded by $mk \leq q \leq k$. With any given demand realization α, a firm aims to maximize its ex-post profit by determining the production quantity, which is formulated as

$$\text{Max} \ \pi(q) = \alpha(a-q)q - \beta q, \text{ s.t. } mk \leq q \leq k. \tag{7.2}$$

A smaller m indicates a larger span of production quantity adjustment, and vice versa. Based on the formulations at capacity decision and production decision stages, the fully FCS discussed in Chapters 5 and 6 can be formulated by setting $q_I = 0$, i.e., $\eta = 100\%$; while IFCS can be formulated by setting $q_I = k$, i.e., $\eta = 0$. Any other percentage between 0 and 100 represents a firm's flexibility capability between IFCS and FCS. Therefore, the flexibility degree reflects the extent to which FCS is exploited by formulating the adjustment span of a firm's production decision. The optimal capacity and production quantity of a firm with a given in-flexibility degree $m \in [0, 1]$ in a monopoly model is provided by Theorem 7.1 below.

Theorem 7.1 In a monopoly model with $m \in [0,\ 1]$, we have:

(i) The optimal capacity k^* satisfies $0 < k^* < \frac{a}{2}$ and

$$\int_0^{\alpha_L} m(\alpha(a-2mk^*)-\beta) f(\alpha) d\alpha + \int_{\alpha_R}^{\infty} (\alpha(a-2k^*)-\beta) f(\alpha) d\alpha = C_F .$$

(ii) The optimal production quantity $q^* = \begin{cases} mk^*, & 0 \le \alpha < \alpha_L \\ q_b, & \alpha_L \le \alpha < \alpha_R \\ k^*, & \alpha_R \le \alpha \end{cases}$.

(iii) The optimal profit is

$$\Pi = \int_0^{\alpha_L} (mk^*)^2 \alpha f(\alpha) d\alpha + \int_{\alpha_L}^{\alpha_R} (q_b)^2 \alpha f(\alpha) d\alpha + \int_{\alpha_R}^{\infty} (k^*)^2 \alpha f(\alpha) d\alpha ,$$

where $\alpha_L = \dfrac{\beta}{a-2mk^*}$, $\alpha_R = \dfrac{\beta}{a-2k^*}$ and $q_b = \dfrac{1}{2}(a - \dfrac{\beta}{\alpha})$. □

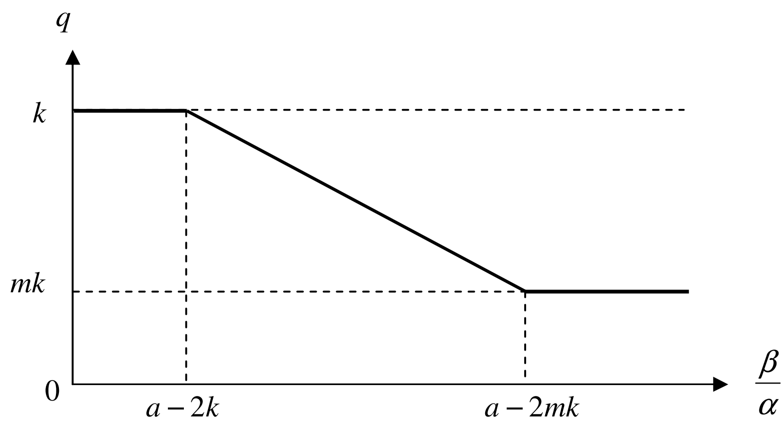


Figure 7.2: Optimal production in a monopoly model.

With in-flexibility degree m, a firm's allowable adjustment range is $mk \le q \le k$. Theorem 7.1 shows that the optimal production of the firm is a three-piece function. By using q and $\frac{\beta}{\alpha}$ as coordinates, the production pattern can be plotted in Figure 7.2. A larger α indicates a smaller value of $\frac{\beta}{\alpha}$. In particular, when $m = 1$, the production curve is a straight line $q = k$ which is the situation of no flexibility at all. When $m = 0$, a production is possible for any value within $[0,\ k]$ which is the situation of full flexibility.

7.3 FCS with Different Flexible Degrees in a Duopoly Model

In this section, we establish a duopoly model to consider the effects of competition on FCS. Subscript i is used to describe firm *i*, $i = 1,2$. The total production quantity equals the sum of product produced by the two firms, i.e., $Q = q_1 + q_2$. Therefore, with the application of the market clearance rule, the demand inverse function is $p(Q,\alpha) = \alpha(a - Q) = \alpha(a - q_1 - q_2)$. Note that $a > q_1 + q_2$ in this model.

7.3.1 Individual optimal production quantities at the production decision stage

At the production decision stage, each firm determines its own production quantity with given in-flexibility degree $m_i \in [0,1]$ and capacity $k_i \geq 0$, $i = 1,\ 2$. The formulation of each firm's production decision is:

$$\text{Max} \ \ \pi_i(q_i) = \alpha(a - q_{3-i} - q_i)q_i - \beta q_i$$
$$\text{s.t.} \ \ m_i k_i \leq q_i \leq k_i,\ i = 1,2. \tag{7.3}$$

Proposition 7.1 In a duopoly model with $0 \leq m_1, m_2 \leq 1$, the optimal production capacity of firm i, given k_1, k_2 and q_{3-i}, is $q_i^* = \begin{cases} m_i k_i, & 0 \leq \alpha < \alpha_{Li} \\ q_{ib}, & \alpha_{Li} \leq \alpha < \alpha_{Ri} \\ k_i, & \alpha_{Ri} \leq \alpha \end{cases}$ for $i = 1,\ 2$,

where $q_{ib} = \frac{1}{2}(a - \frac{\beta}{\alpha} - q_{3-i})$, $\alpha_{Li} = \frac{\beta}{a - 2m_i k_i - q_{3-i}}$ and $\alpha_{Ri} = \frac{\beta}{a - 2k_i - q_{3-i}}$ for $i = 1,\ 2$. □

7.3.2 Individual optimal capacities at the capacity decision stage

At the capacity decision stage, both firms determine their own capacities to maximize their respective profits under their flexibility restrictions. Consider firm $i \in \{1,2\}$, given $0 \le m_i \le 1$ and the rival's production quantity q_{3-i}, the capacity decision of firm i can be formulated as follows.

$$\text{Max} \quad \Pi_i(k_i) = \int_0^\infty (\alpha(a - q_{3-i} - q_i^*)q_i^* - \beta q_i^*) f(\alpha) d\alpha - C_F k_i,$$
$$\text{s.t.} \quad k_i \ge 0, \tag{7.4}$$

where q_i^* is the optimal production of firm i at the production decision stage.

By Proposition 7.1, from (7.4), the optimal expected profit at the capacity stage for firm i is

$$\Pi_i(k_i)$$
$$= \int_0^{\alpha_{Li}} m_i k_i (\alpha(a - q_{3-i} - m_i k_i) - \beta) f(\alpha) d\alpha + \int_{\alpha_{Li}}^{\alpha_{Ri}} q_{ib} (\alpha(a - q_{3-i} - q_{ib}) - \beta) f(\alpha) d\alpha$$
$$+ \int_{\alpha_{Ri}}^\infty (\alpha(a - q_{3-i} - k_i)k_i - \beta k_i) f(\alpha) d\alpha - C_F k_i. \tag{7.5}$$

Proposition 7.2 below characterizes the optimal capacity from the perspective of each individual firm.

Proposition 7.2 In a duopoly model with $0 \le m_1, m_2 \le 1$, given the production quantity of firm i's $(i \in \{1,2\})$ rival q_{3-i}, we have:

(i) Firm i's optimal capacity k_i^* satisfies $0 < k_i^* < \frac{a - q_{3-i}}{2}$ and

$$\int_0^{\alpha_{Li}} m_i (\alpha(a - q_{3-i} - 2m_i k_i^*) - \beta) f(\alpha) d\alpha + \int_{\alpha_{Ri}}^\infty (\alpha(a - q_{3-i} - 2k_i^*) - \beta) f(\alpha) d\alpha = C_F;$$

(ii) Firm i's optimal profit

$$\Pi_i = \int_0^{\alpha_{Li}} (m_i k_i^*)^2 \alpha f(\alpha) d\alpha + \int_{\alpha_{Li}}^{\alpha_{Ri}} (q_{ib})^2 \alpha f(\alpha) d\alpha + \int_{\alpha_{Ri}}^\infty (k_i^*)^2 \alpha f(\alpha) d\alpha,$$

where $q_{ib} = \frac{1}{2}(a - \frac{\beta}{\alpha} - q_{3-i})$, $\alpha_{Li} = \frac{\beta}{a - 2m_i k_i^* - q_{3-i}}$ and $\alpha_{Ri} = \frac{\beta}{a - 2k_i^* - q_{3-i}}$, $i = 1, 2$. □

From the perspective of each individual firm, Proposition 7.2 provides the optimal capacity and optimal profit a firm, given the rival's production quantity. It can be seen that each firm's decisions are affected by its rival and the interplay between the two firms is complex.

To simplify the presentation, we will drop the superscript (*) of the individual optimal production quantities and capacities when we discuss about the Nash equilibrium in the following. Furthermore, without loss of generality, we will assume $m_1 \leq m_2$ in the discussion.

7.3.3 Nash equilibrium production quantities (q_1, q_2) with given $0 \leq m_1 \leq m_2 \leq 1$

To find out the Nash equilibrium production quantities (q_1, q_2), it is necessary to compare the upper and lower bounds of the two firms' individual optimal productions. Without loss of generality, we assume $0 \leq m_1 \leq m_2 \leq 1$. It indicates that at production stage, firm 1 has a larger portion of its capacity to make adjustments of the production quantity than that of firm 2. According to the ranking of the upper and lower bounds of the two firms, a total of five possible cases may occur under the assumption $0 \leq m_1 \leq m_2 \leq 1$. Proposition 7.3 below provides the Nash equilibrium solution (q_1, q_2) in each case.

Proposition 7.3 In a duopoly model with $0 \leq m_1 \leq m_2 \leq 1$, given the capacities of the two firms $k_1 \geq 0$ and $k_2 \geq 0$, the production quantities of the two firms (q_1, q_2) at equilibrium are as follows.

(i) If $m_2k_2 \le k_2 < m_1k_1 \le k_1$, then

$$(q_1, q_2) = \begin{cases} (m_1k_1, m_2k_2), & 0 \le \alpha < \dfrac{\beta}{a - m_1k_1 - 2m_2k_2} \\ (m_1k_1, q_{2b-1}), & \dfrac{\beta}{a - m_1k_1 - 2m_2k_2} \le \alpha < \dfrac{\beta}{a - m_1k_1 - 2k_2} \\ (m_1k_1, k_2), & \dfrac{\beta}{a - m_1k_1 - 2k_2} \le \alpha < \dfrac{\beta}{a - 2m_1k_1 - k_2} \\ (q_{1b-3}, k_2), & \dfrac{\beta}{a - 2m_1k_1 - k_2} \le \alpha < \dfrac{\beta}{a - 2k_1 - k_2} \\ (k_1, k_2), & \dfrac{\beta}{a - 2k_1 - k_2} \le \alpha \end{cases}.$$

(ii) If $m_2k_2 < m_1k_1 \le k_2 < k_1$, then

$$(q_1, q_2) = \begin{cases} (m_1k_1, m_2k_2), & 0 \le \alpha < \dfrac{\beta}{a - m_1k_1 - 2m_2k_2} \\ (m_1k_1, q_{2b-1}), & \dfrac{\beta}{a - m_1k_1 - 2m_2k_2} \le \alpha < \dfrac{\beta}{a - 3m_1k_1} \\ (q_{1b-2}, q_{2b-2}), & \dfrac{\beta}{a - 3m_1k_1} \le \alpha < \dfrac{\beta}{a - 3k_2} \\ (q_{1b-3}, k_2), & \dfrac{\beta}{a - 3k_2} \le \alpha < \dfrac{\beta}{a - 2k_1 - k_2} \\ (k_1, k_2), & \dfrac{\beta}{a - 2k_1 - k_2} \le \alpha \end{cases}.$$

(iii) If $m_1k_1 \le m_2k_2 \le k_2 \le k_1$, then

$$(q_1, q_2) = \begin{cases} (m_1k_1, m_2k_2), & 0 \le \alpha < \dfrac{\beta}{a - 2m_1k_1 - m_2k_2} \\ (q_{1b-1}, m_2k_2), & \dfrac{\beta}{a - 2m_1k_1 - m_2k_2} \le \alpha < \dfrac{\beta}{a - 3m_2k_2} \\ (q_{1b-2}, q_{2b-2}), & \dfrac{\beta}{a - 3m_2k_2} \le \alpha < \dfrac{\beta}{a - 3k_2} \\ (q_{1b-3}, k_2), & \dfrac{\beta}{a - 3k_2} \le \alpha < \dfrac{\beta}{a - 2k_1 - k_2} \\ (k_1, k_2), & \dfrac{\beta}{a - 2k_1 - k_2} \le \alpha \end{cases}.$$

(iv) If $m_1k_1 \le m_2k_2 < k_1 < k_2$, then

$$(q_1, q_2) = \begin{cases} (m_1k_1, m_2k_2), & 0 \le \alpha < \dfrac{\beta}{a - 2m_1k_1 - m_2k_2} \\ (q_{1b-1}, m_2k_2), & \dfrac{\beta}{a - 2m_1k_1 - m_2k_2} \le \alpha < \dfrac{\beta}{a - 3m_2k_2} \\ (q_{1b-2}, q_{2b-2}), & \dfrac{\beta}{a - 3m_2k_2} \le \alpha < \dfrac{\beta}{a - 3k_1} \\ (k_1, q_{2b-3}), & \dfrac{\beta}{a - 3k_1} \le \alpha < \dfrac{\beta}{a - k_1 - 2k_2} \\ (k_1, k_2), & \dfrac{\beta}{a - k_1 - 2k_2} \le \alpha \end{cases}$$

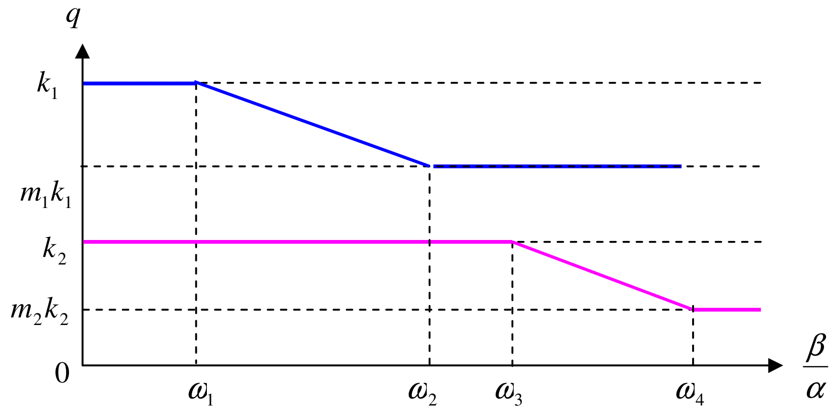
(v) If $m_1k_1 \le k_1 \le m_2k_2 < k_2$, then

$$(q_1, q_2) = \begin{cases} (m_1k_1, m_2k_2), & 0 \le \alpha < \dfrac{\beta}{a - 2m_1k_1 - m_2k_2} \\ (q_{1b-1}, m_2k_2), & \dfrac{\beta}{a - 2m_1k_1 - m_2k_2} \le \alpha < \dfrac{\beta}{a - 2k_1 - m_2k_2} \\ (k_1, m_2k_2), & \dfrac{\beta}{a - 2k_1 - m_2k_2} \le \alpha < \dfrac{\beta}{a - k_1 - 2m_2k_2} \\ (k_1, q_{2b-3}), & \dfrac{\beta}{a - k_1 - 2m_2k_2} \le \alpha < \dfrac{\beta}{a - k_1 - 2k_2} \\ (k_1, k_2), & \dfrac{\beta}{a - k_1 - 2k_2} \le \alpha \end{cases},$$

where $q_{1b-1} = \frac{1}{2}(a - m_2k_2 - \frac{\beta}{\alpha})$, $q_{1b-2} = \frac{1}{3}(a - \frac{\beta}{\alpha})$, $q_{1b-3} = \frac{1}{2}(a - k_2 - \frac{\beta}{\alpha})$, $q_{2b-1} = \frac{1}{2}(a - m_1k_1 - \frac{\beta}{\alpha})$, $q_{2b-2} = \frac{1}{3}(a - \frac{\beta}{\alpha})$ and $q_{2b-3} = \frac{1}{2}(a - k_1 - \frac{\beta}{\alpha})$. □

Proposition 7.3 provides five possible cases of the optimal productions at equilibrium. Among these five cases, Case (i) and Case (v) are symmetric; and Case (ii) and Case (iv) are symmetric. The patterns of optimal production quantities for firm 1 and firm 2, respectively, in each situation are plotted in Figure 7.3 - Figure 7.5.

Case (i) $m_2k_2 \le k_2 < m_1k_1 \le k_1$

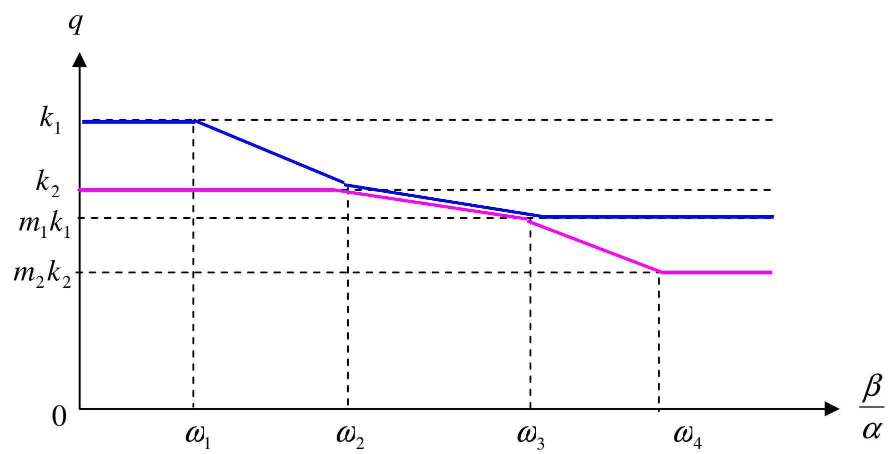


$$\omega_1 = a - 2k_1 - k_2,\ \omega_2 = a - 2m_1k_1 - k_2,\ \omega_3 = a - m_1k_1 - 2k_2,\ \omega_4 = a - m_1k_1 - 2m_2k_2$$

$$q_{1b-3} = \frac{1}{2}(a - k_2 - \frac{\beta}{\alpha}),\ q_{2b-1} = \frac{1}{2}(a - m_1k_1 - \frac{\beta}{\alpha})$$

Figure 7.3: Optimal production quantities in situation $m_2k_2 \le k_2 < m_1k_1 \le k_1$.

Case (ii) $m_2k_2 < m_1k_1 \le k_2 < k_1$

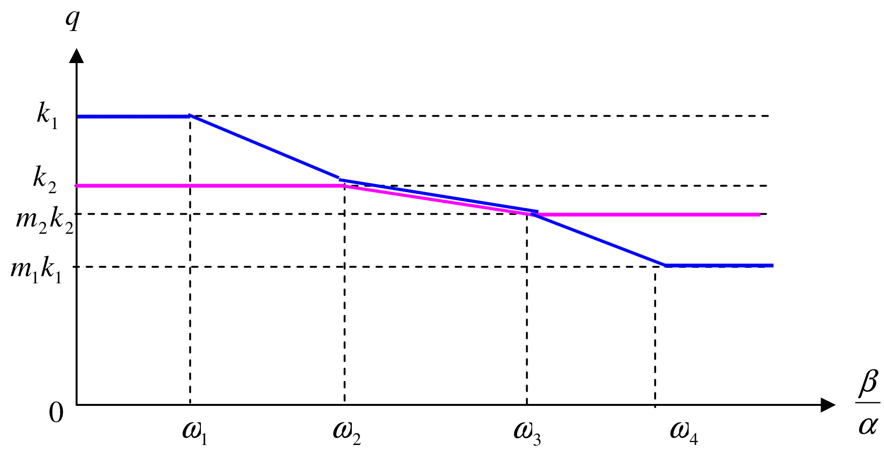


$$\omega_1 = a - 2k_1 - k_2,\ \omega_2 = a - 3k_2,\ \omega_3 = a - 3m_1k_1,\ \omega_4 = a - m_1k_1 - 2m_2k_2$$

$$q_{1b-3} = \frac{1}{2}(a - k_2 - \frac{\beta}{\alpha}),\ q_{1b-2} = \frac{1}{3}(a - \frac{\beta}{\alpha}),\ q_{2b-2} = \frac{1}{3}(a - \frac{\beta}{\alpha}),\ q_{2b-1} = \frac{1}{2}(a - m_1k_1 - \frac{\beta}{\alpha}).$$

Figure 7.4: Optimal production quantities in situation $m_2k_2 < m_1k_1 \le k_2 < k_1$.

Case (iii) $m_1k_1 \le m_2k_2 \le k_2 \le k_1$



$$\omega_1 = a - 2k_1 - k_2\,,\ \omega_2 = a - 3k_2\,,\ \omega_3 = a - 3m_2k_2\,,\ \omega_4 = a - 2m_1k_1 - m_2k_2$$

$$q_{1b-3} = \frac{1}{2}(a - k_2 - \frac{\beta}{\alpha})\,,\ q_{1b-2} = \frac{1}{3}(a - \frac{\beta}{\alpha})\,,\ q_{1b-1} = \frac{1}{2}(a - m_2k_2 - \frac{\beta}{\alpha})\,,$$

$$q_{2b-2} = \frac{1}{3}(a - \frac{\beta}{\alpha})\,.$$

Figure 7.5: Optimal production quantities in situation $m_1k_1 \le m_2k_2 \le k_2 \le k_1$**.**

As shown in Figures 7.3 - 7.5, the ranking of two firms' production upper and lower bounds affects two firms' production decisions. As reflected by the figures, production decision functions can be very different from case to case. However, the commonality of these five cases is that a firm's production function composes of three parts: the lower bound, the middle part and the upper bound. Specifically, the middle part which can be one-piece, two-piece or three-piece functions are strictly decreasing in $\frac{\beta}{\alpha}$, i.e., strictly increasing in demand realization α. It means that no matter which case, within a range of production quantities, more products are produced when demand increases, until it reaches the maximum capacity.

7.3.4 Nash equilibrium capacities $(k_1,\ k_2)$ with given $0 \le m_1 \le m_2 \le 1$

From Proposition 7.2, at Nash equilibrium, (k_1, k_2) satisfies $0 < k_i < \frac{a - q_{3-i}}{2}$ and $\Pi_i^{(1)}(k_i) = 0$, $i = 1,2$; further, (q_1, q_2) satisfies Proposition 7.3, which has 5 possible situations as illustrated in Figure 7.6 for the case of $m_2 \ne 0$.

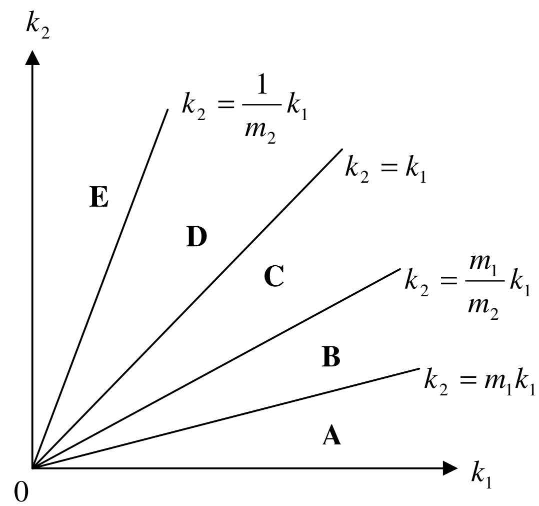


Figure 7.6: Five possible situations under duopoly equilibrium for the case of $m_2 \ne 0$.

By Propositions 7.2 and 7.3, each situation can be specifically characterized by the following Claim 7.1.

Claim 7.1 Given $0 \le m_1 \le m_2 \le 1$, there are five possible situations of the optimal capacities of two firms at equilibrium. The five situations are:

(i) Situation A $m_2 k_2 \le k_2 < m_1 k_1 \le k_1$

$$\Pi_1^{(1)}(k_1) = \int_0^{\alpha_{L2}} m_1 \alpha (a - 2m_1 k_1 - m_2 k_2 - \frac{\beta}{\alpha}) f(\alpha) d\alpha$$
$$+ \int_{\alpha_{L2}}^{\alpha_{R2}} \frac{1}{2} m_1 \alpha (a - 3m_1 k_1 - \frac{\beta}{\alpha}) f(\alpha) d\alpha + \int_{\alpha_{R2}}^{\alpha_{L1}} m_1 \alpha (a - 2m_1 k_1 - k_2 - \frac{\beta}{\alpha}) f(\alpha) d\alpha$$
$$+ \int_{\alpha_{R1}}^{\infty} \alpha (a - 2k_1 - k_2 - \frac{\beta}{\alpha}) f(\alpha) d\alpha - C_F,$$

$$\Pi_2^{(1)}(k_2)=\int_0^{\alpha_{L2}} m_2\alpha(a-m_1k_1-2m_2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha$$

$$+\int_{\alpha_{R2}}^{\alpha_{L1}} \alpha(a-m_1k_1-2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha+\int_{\alpha_{L1}}^{\alpha_{R1}}\frac{1}{2}\alpha(a-3k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha$$

$$+\int_{\alpha_{R1}}^{\infty} \alpha(a-k_1-2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha-C_F,$$

where $\alpha_{L1}=\dfrac{\beta}{a-2m_1k_1-k_2}$, $\alpha_{R1}=\dfrac{\beta}{a-2k_1-k_2}$; $\alpha_{L2}=\dfrac{\beta}{a-m_1k_1-2m_2k_2}$, $\alpha_{R2}=\dfrac{\beta}{a-m_1k_1-2k_2}$.

(ii) Situation B $m_2k_2<m_1k_1\le k_2<k_1$

$$\Pi_1^{(1)}(k_1)=\int_0^{\alpha_{L2}} m_1\alpha(a-2m_1k_1-m_2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha$$

$$+\int_{\alpha_{L2}}^{\alpha_{L1}} \frac{1}{2}m_1\alpha(a-3m_1k_1-\frac{\beta}{\alpha})f(\alpha)d\alpha$$

$$+\int_{\alpha_{R1}}^{\infty} \alpha(a-2k_1-k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha-C_F;$$

$$\Pi_2^{(1)}(k_2)=\int_0^{\alpha_{L2}} m_2\alpha(a-m_1k_1-2m_2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha$$

$$+\int_{\alpha_{R2}}^{\alpha_{R1}} \frac{1}{2}\alpha(a-3k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha$$

$$+\int_{\alpha_{R1}}^{\infty} \alpha(a-k_1-2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha-C_F;$$

where $\alpha_{L1}=\dfrac{\beta}{a-3m_1k_1}$, $\alpha_{R1}=\dfrac{\beta}{a-2k_1-k_2}$; $\alpha_{L2}=\dfrac{\beta}{a-m_1k_1-2m_2k_2}$, $\alpha_{R2}=\dfrac{\beta}{a-3k_2}$.

(iii) Situation C $m_1k_1\le m_2k_2\le k_2\le k_1$

$$\Pi_1^{(1)}(k_1)=\int_0^{\alpha_{L1}} m_1\alpha(a-2m_1k_1-m_2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha$$

$$+\int_{\alpha_{R1}}^{\infty} \alpha(a-2k_1-k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha-C_F,$$

$$\Pi_2^{(1)}(k_2)=\int_0^{\alpha_{L1}} m_2\alpha(a-m_1k_1-2m_2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha$$
$$+\int_{\alpha_{L1}}^{\alpha_{L2}}\frac{1}{2}m_2\alpha(a-3m_2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha+\int_{\alpha_{R2}}^{\alpha_{R1}}\frac{1}{2}\alpha(a-3k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha$$
$$+\int_{\alpha_{R1}}^{\infty}\alpha(a-k_1-2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha-C_F,$$

where $\alpha_{L1}=\dfrac{\beta}{a-2m_1k_1-m_2k_2}$, $\alpha_{R1}=\dfrac{\beta}{a-2k_1-k_2}$; $\alpha_{L2}=\dfrac{\beta}{a-3m_2k_2}$, $\alpha_{R2}=\dfrac{\beta}{a-3k_2}$.

(iv) Situation D $m_1k_1\le m_2k_2<k_1<k_2$

$$\Pi_1^{(1)}(k_1)=\int_0^{\alpha_{L1}} m_1\alpha(a-2m_1k_1-m_2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha$$
$$+\int_{\alpha_{R1}}^{\alpha_{R2}}\alpha\frac{1}{2}(a-3k_1-\frac{\beta}{\alpha})f(\alpha)d\alpha+\int_{\alpha_{R2}}^{\infty}\alpha(a-2k_1-k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha-C_F,$$

$$\Pi_2^{(1)}(k_2)=\int_0^{\alpha_{L1}} m_2\alpha(a-m_1k_1-2m_2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha$$
$$+\int_{\alpha_{L1}}^{\alpha_{L2}}\frac{1}{2}m_2\alpha(a-3m_2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha$$
$$+\int_{\alpha_{R2}}^{\infty}\alpha(a-k_1-2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha-C_F,$$

where $\alpha_{L1}=\dfrac{\beta}{a-2m_1k_1-m_2k_2}$, $\alpha_{R1}=\dfrac{\beta}{a-3k_1}$; $\alpha_{L2}=\dfrac{\beta}{a-3m_2k_2}$, $\alpha_{R2}=\dfrac{\beta}{a-k_1-2k_2}$.

(v) Situation E $m_1k_1\le k_1\le m_2k_2<k_2$

$$\Pi_1^{(1)}(k_1)=\int_0^{\alpha_{L1}} m_1\alpha(a-2m_1k_1-m_2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha$$
$$+\int_{\alpha_{R1}}^{\alpha_{L2}}\alpha(a-2k_1-m_2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha$$
$$+\int_{\alpha_{L2}}^{\alpha_{R2}}\frac{1}{2}\alpha(a-3k_1-\frac{\beta}{\alpha})f(\alpha)d\alpha+\int_{\alpha_{R2}}^{\infty}\alpha(a-2k_1-k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha-C_F,$$

$$\Pi_2^{(1)}(k_2) = \int_0^{\alpha_{L1}} m_2\alpha(a - m_1k_1 - 2m_2k_2 - \frac{\beta}{\alpha}) f(\alpha)d\alpha$$

$$+\int_{\alpha_{L1}}^{\alpha_{R1}} \frac{1}{2} m_2\alpha(a - 3m_2k_2 - \frac{\beta}{\alpha}) f(\alpha)d\alpha$$

$$+\int_{\alpha_{R1}}^{\alpha_{L2}} m_2\alpha(a - k_1 - 2m_2k_2 - \frac{\beta}{\alpha}) f(\alpha)d\alpha$$

$$+\int_{\alpha_{R2}}^{\infty} \alpha(a - k_1 - 2k_2 - \frac{\beta}{\alpha}) f(\alpha)d\alpha - C_F,$$

where $\alpha_{L1} = \dfrac{\beta}{a - 2m_1k_1 - m_2k_2}$, $\alpha_{R1} = \dfrac{\beta}{a - 2k_1 - m_2k_2}$; $\alpha_{L2} = \dfrac{\beta}{a - k_1 - 2m_2k_2}$,

$\alpha_{R2} = \dfrac{\beta}{a - k_1 - 2k_2}$. □

Claim 7.1 shows the complexity of the equilibrium. It seems that there are five possible equilibriums depending on the ranking of the two firms' lower and upper bounds. This gives rise to one question: Does each of the possible equilibriums exist? Therefore, how to verify these five possible equilibriums is a key to find the actual equilibrium and the resulting solutions.

7.3.4.1 Duopoly with symmetric flexibilities $m_1 = m_2$

We consider the situation of $m_1 = m_2 = m$, $0 \le m \le 1$.

Theorem 7.2 Given $m_1 = m_2 = m$, $0 \le m \le 1$, then

(i) the optimal capacity of firm 1 and firm 2 are $k_1 = k_2 = k_e$ at equilibrium;

(ii) k_e is decreasing in $m \in [0,1]$, i.e., k_e is increasing in flexibility degree η, $\eta = 1 - m$;

(iii) $k_f \le k_e \le k_0$ where k_0 satisfies $\int_{\frac{\beta}{a-3k_0}}^{\infty} (\alpha(a - 3k_0) - \beta) f(\alpha)d\alpha = C_F$ and

$k_f = \dfrac{1}{3}(a - \dfrac{C_F + \beta}{\mu})$;

(iv) The expected profit of each firm is $\Pi_1 = \Pi_2 = \Pi_e$, where

$$\Pi_e = \int_0^{\frac{\beta}{a-3mk_e}} (mk_e)^2 \alpha f(\alpha)d\alpha + \frac{1}{9}\int_{\frac{\beta}{a-3mk_e}}^{\frac{\beta}{a-3k_e}} (a - \frac{\beta}{\alpha})^2 \alpha f(\alpha)d\alpha + \int_{\frac{\beta}{a-3k_e}}^{\infty} k_e^2 \alpha f(\alpha)d\alpha$$

and k_e satisfies

$$m\int_0^{\frac{\beta}{a-3mk_e}}(\alpha(a-3mk_e)-\beta)f(\alpha)d\alpha+\int_{\frac{\beta}{a-3k_e}}^{\infty}(\alpha(a-3k_e)-\beta)f(\alpha)d\alpha=C_F. \qquad \square$$

Theorem 7.2 provides that two firms always make the same decisions in an uncertain market as long as they have the same flexibility degree swinging from zero to 100 percent. Specifically, a higher flexibility degree leads to a larger amount of capacity of each firm at the equilibrium. When two firms both have no flexibility at all, i.e., $m_1 = m_2 = 1$, they have the lowest capacity k_0 which is still larger than zero. It means with the same flexibility degree, it is impossible that both firms do not make capacity investments.

7.3.4.2 Duopoly with asymmetric flexibility

In the following, we make analysis of each situation to determine the optimal decisions of each firm. Due to analytical complication of situations A - C, the analyses of situation D and situation E are provided first.

Proposition 7.4 Given $0 \le m_1 < m_2 \le 1$, then the optimal solution $(k_1,\ k_2)$ is not in situation E. □

Proposition 7.5 Given $0 \le m_1 \le m_2 \le 1$, then the optimal solution $(k_1,\ k_2)$ is not in situation D. □

Proposition 7.4 and Proposition 7.5 rule out situation D and situation E, which point out that if $m_1 < m_2$, then there must have $k_1 \ge k_2$. That means the firm with larger flexibility degree always make capacity investment not less than that with a smaller flexibility degree. Using the similar ways of proofs of Proposition 7.4 and Proposition 7.5 it cannot rule out any one situation among situations A - C. Focusing on the analysis of situations A - C, it is found that the optimal solutions only occurs in situation C as long as $0 \le m_1 \le m_2 \le 1$. Together with Proposition 7.4 and Proposition 7.5, the optimal solutions of capacity and production decisions of two firms only occur in situation C. This is formally presented in the following Theorem 7.3.

Theorem 7.3 Given $0 \le m_1 \le m_2 \le 1$,

(i) if $\int_{\frac{\beta}{a}}^{\infty} \alpha(a-\frac{\beta}{\alpha}) f(\alpha) d\alpha \le C_F$, then $k_{1e} = k_{2e} = 0$ and $\Pi_1 = \Pi_2 = 0$;

(ii) if $\int_{\frac{\beta}{a}}^{\infty} \alpha(a-\frac{\beta}{\alpha}) f(\alpha) d\alpha > C_F$, then $k_{1e} > 0$, $k_{2e} > 0$ and $\frac{m_1}{m_2} k_{1e} < k_{2e} < k_{1e}$;

(ii-1) the optimal productions $(q_1^* \quad q_2^*)$ are

$$(q_1^* \quad q_2^*) = \begin{cases} (m_1 k_1 \quad m_2 k_2), & \alpha \le \alpha_{L1} \\ (q_{1b-1} \quad m_2 k_2), & \alpha_{L1} < \alpha < \alpha_{L2} \\ (q_{1b-2} \quad q_{2b}), & \alpha_{L2} \le \alpha \le \alpha_{R2} \\ (q_{1b-3} \quad k_2), & \alpha_{R2} < \alpha < \alpha_{R1} \\ (k_1 \quad k_2), & \alpha_{R1} \le \alpha \end{cases}.$$

(ii-2) the optimal capacity decisions $(k_1,\ k_2) = (k_{1e},\ k_{2e})$ at equilibrium satisfy

$$\int_0^{\alpha_{L1}} m_1 \alpha(a - 2m_1 k_{1e} - m_2 k_{2e} - \frac{\beta}{\alpha}) f(\alpha) d\alpha + \int_{\alpha_{R1}}^{\infty} \alpha(a - 2k_{1e} - k_{2e} - \frac{\beta}{\alpha}) f(\alpha) d\alpha = C_F$$

and

$$\int_0^{\alpha_{L1}} m_2 \alpha(a - m_1 k_{1e} - 2m_2 k_{2e} - \frac{\beta}{\alpha}) f(\alpha) d\alpha + \int_{\alpha_{L1}}^{\alpha_{L2}} \frac{1}{2} m_2 \alpha(a - 3m_2 k_{2e} - \frac{\beta}{\alpha}) f(\alpha) d\alpha$$
$$+ \int_{\alpha_{R2}}^{\alpha_{R1}} \frac{1}{2} \alpha(a - 3k_{2e} - \frac{\beta}{\alpha}) f(\alpha) d\alpha + \int_{\alpha_{R1}}^{\infty} \alpha(a - k_{1e} - 2k_{2e} - \frac{\beta}{\alpha}) f(\alpha) d\alpha = C_F;$$

(ii-3) the optimal profits of firm 1 and firm 2 are

$$\Pi_1(k_1) = (m_1 k_1)^2 \int_0^{\alpha_{L1}} \alpha f(\alpha) d\alpha + \int_{\alpha_{L1}}^{\alpha_{L2}} q_{1b-1}^2 \alpha f(\alpha) d\alpha + \int_{\alpha_{L2}}^{\alpha_{R2}} q_{1b-2}^2 \alpha f(\alpha) d\alpha$$
$$+ \int_{\alpha_{R2}}^{\alpha_{R1}} q_{1b-3}^2 \alpha f(\alpha) d\alpha + \int_{\alpha_{R1}}^{\infty} k_1^2 \alpha f(\alpha) d\alpha;$$

$$\Pi_2(k_2) = (m_2 k_2)^2 \int_0^{\alpha_{L2}} \alpha f(\alpha) d\alpha + \int_{\alpha_{L2}}^{\alpha_{R2}} q_{2b}^2 \alpha f(\alpha) d\alpha + \int_{\alpha_{R2}}^{\infty} k_2^2 \alpha f(\alpha) d\alpha;$$

where $q_{1b-1} = \frac{1}{2}(a - m_2 k_2 - \frac{\beta}{\alpha})$, $q_{1b-2} = \frac{1}{3}(a - \frac{\beta}{\alpha})$, $q_{1b-3} = \frac{1}{2}(a - k_2 - \frac{\beta}{\alpha})$, $q_{2b} = \frac{1}{3}(a - \frac{\beta}{\alpha})$; $\alpha_{L1} = \frac{\beta}{a - 2m_1 k_{1e} - m_2 k_{2e}}$, $\alpha_{R1} = \frac{\beta}{a - 2k_{1e} - k_{2e}}$; $\alpha_{L2} = \frac{\beta}{a - 3m_2 k_{2e}}$, $\alpha_{R2} = \frac{\beta}{a - 3k_{2e}}$. □

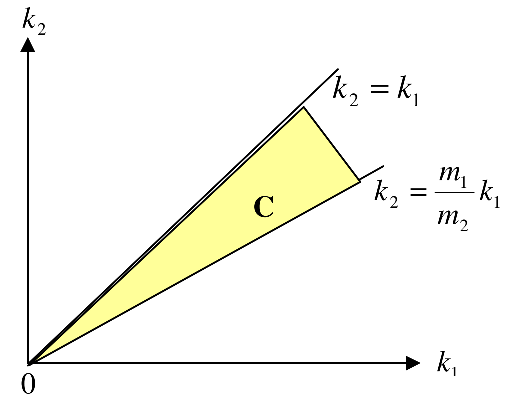


Figure 7.7: The equilibrium of the duopoly competition.

Theorem 7.3 characterizes the equilibrium of an asymmetric duopoly competition model under demand uncertainty. Given two firms' flexibility degrees $0 \le m_1 \le m_2 \le 1$, the equilibrium only occurs in Case C, i.e., $m_1 k_1 \le m_2 k_2 \le k_2 \le k_1$, as shown in Figure 7.7.

7.4 Numerical Examples

It is noted that during the proof of Theorem 7.3, we assume $a \ge 6\frac{(\beta + C_F)}{\mu}$. Under this assumption, we can assure that the unique equilibrium only occurs in Situation C. However, trying to investigate of the generality of the model, we use a numerical example that does not satisfy this assumption to testify the model and its conclusions and get an intuitive image. The basic parameters are listed in Table 7.1.

Parameter of the inverse demand function a	Production cost β	Capacity cost C_F	Demand distribution function $f(x)$
10	2	3	$f(x) = e^{-x}, x \in [0, \infty)$

Table 7.1: Parameters of numerical example.

7.4.1 Feasible solutions in situations A - E

We first test the optimal capacities of two firms under various in-flexibility degrees (m_1, m_2), as shown in Figure 7.8

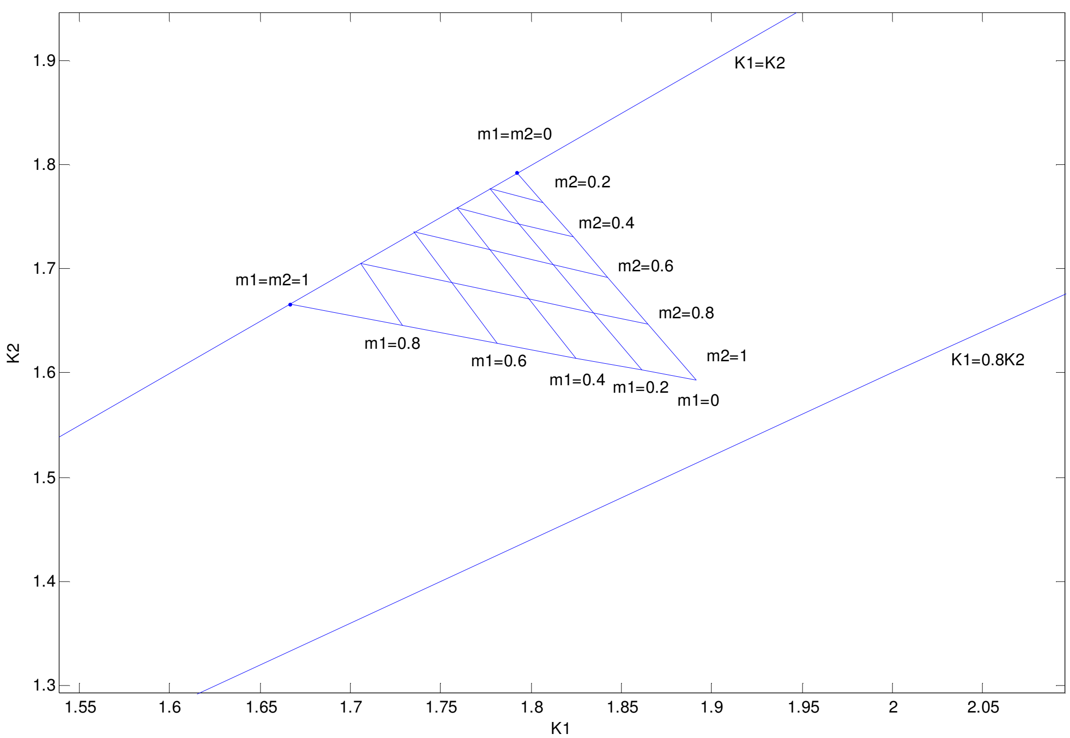


Figure 7.8: Optimal capacities (k_1, k_2) with given various (m_1, m_2).

Figure 7.8 shows the optimal capacities (k_1, k_2) with any given in-flexibility degrees (m_1, m_2). It verifies that the optimal capacities (k_1, k_2) only occur in situation C, i.e., $m_1 k_1 \le m_2 k_2 \le k_2 \le k_1$. In other situations, there is no feasible solution. Moreover, for any given pair of in-flexibility degrees (m_1, m_2), the equilibrium is unique. Solutions of the optimal capacity at equilibrium fall into a limited closed area in situation C. It can be seen that when two firms have the same flexibility, i.e., $m_1 = m_2$, their optimal capacities are in the line $k_1 = k_2$ meaning that two firms have the same capacity. Moreover, it can be observed that at equilibrium, the optimal capacity of firm 1 increases as its own flexibility degree increases, i.e., an decrease of m_1; while it decreases as the flexibility degree of firm 2 increases, i.e., a decrease of m_2. The situation of firm 2 is symmetric to that of firm 1. Therefore, as seen in Figure 7.8, when $m_1 = 0$ and $m_2 = 1$, firm 1 has the highest capacity while firm 2 has the lowest capacity. Also, it is noted that all capacities of the two firms are larger than zero. It means under this multiplicative demand structure, the market always have the profit potential. It is interesting to find that all results derived from the numerical example do not satisfy the assumption $a \geq 6\frac{(\beta + C_F)}{\mu}$, but are completely consistent with the conclusions of Theorem 7.3, which is derived under the assumption $a \geq 6\frac{(\beta + C_F)}{\mu}$. This observation verifies the significant generality of the model and its conclusions derived.

7.4.2 Effects of flexibility degrees on the optimal capacities

Given $0 \leq m_1 \leq 1$ and $0 \leq m_2 \leq 1$, Figure 7.9 plots the respect capacity of each firm.

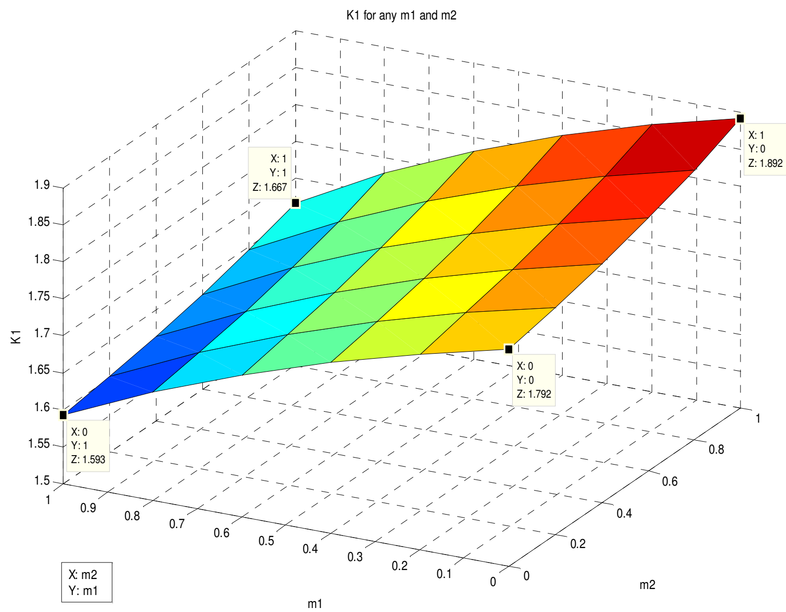


Figure 7.9(a): Firm 1's capacity k_1 under various in-flexibility degrees $(m_1,\ m_2)$.

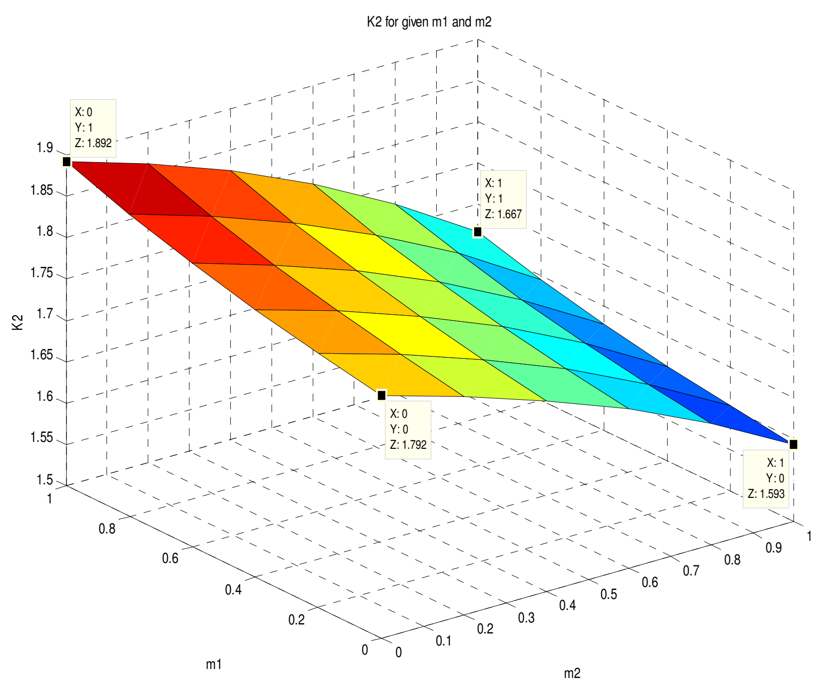


Figure 7.9(b): Firm 2's capacity k_2 under various in-flexibility degrees $(m_1,\ m_2)$.

Figure 7.9: Capacities k_1 and k_2 under various in-flexibility degrees $(m_1,\ m_2)$.

It can be seen that with any fixed m_1, $0 \le m_1 \le 1$, firm 1's capacity k_1 is increasing in m_2. It means firm 1's capacity increases as firm 2's flexibility degree $(1-m_2)$ decreases.

On the other hand, with any given m_2 , firm 1's capacity increases as it own flexibility degree $(1-m_1)$ increases. In other words, with identical capacity unit cost and production unit cost, a firm benefits from its own increasing flexibility degree and/or the rivals' decreasing flexibility degree. The highest capacity of firm 1 occurs when $m_1=0$ and $m_2=1$; while the lowest capacity of firm 1 happens when $m_1=1$ and $m_2=0$. Moreover, it is noted that with a fixed m_2, the capacity of firm 1 is only affected by m_1. Similarly, with a fixed m_1, the capacity of firm 1 is only affected by m_2. However, it can be seen that the difference between firm 1's highest capacity and lowest capacity with a fixed m_2 is larger than that with a fixed m_1. This indicates that the capacity of firm 1 is more affected by m_1 than m_2. The capacity of firm 2 under various flexibilities $(m_1,\ m_2)$ is symmetric to that of firm 1, as shown in Figure 7.9(b).

Figure 7.10 plots the total capacity of the two firms under various in-flexibility degrees $(m_1,\ m_2)$. It can be seen that with fixed in-flexibility degree m_1, the total capacity of the two firms decreases with m_2, and vice versa. Therefore, the total capacity of two firms increases with an increase in the sum of two flexibility degrees $(2-m_1-m_2)$, i.e., an decrease of m_1+m_2. This is consistent with the Figure 7.10, in which the highest total capacity occurs when $m_1=0$ and $m_2=0$, and the lowest total capacity occurs when $m_1=1$ and $m_2=1$.

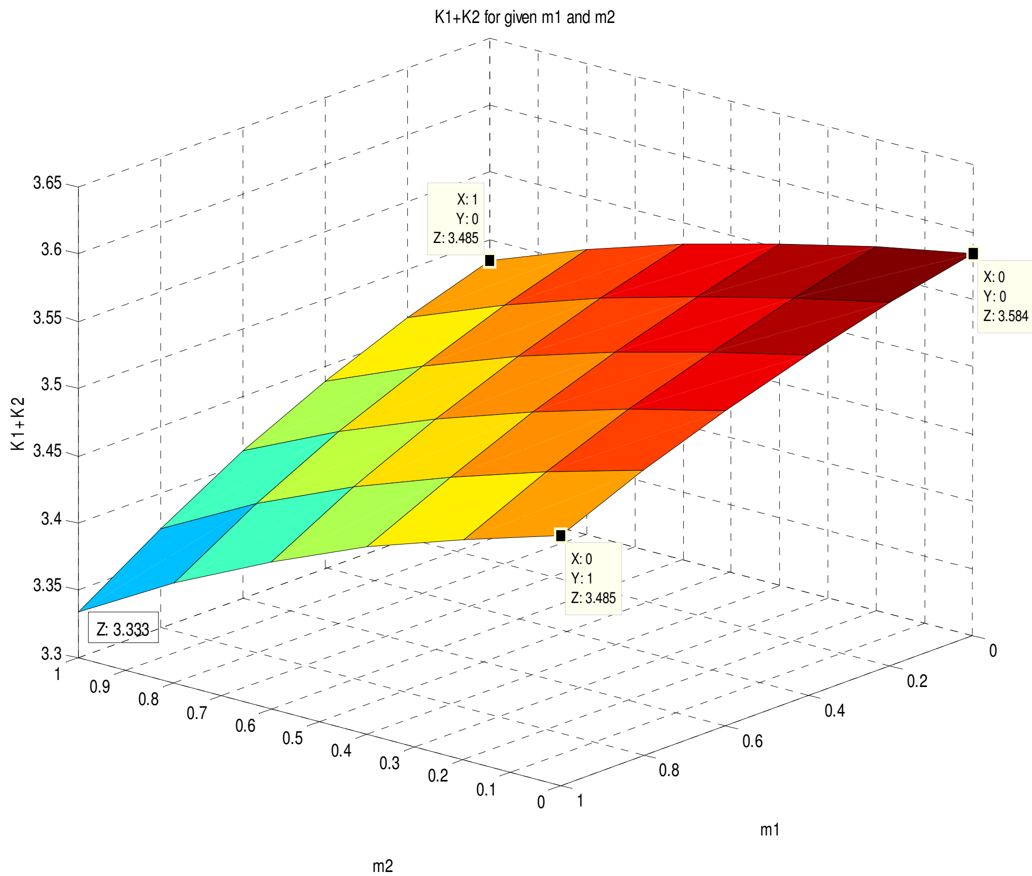


Figure 7.10: Total capacity of two firms under various in-flexibility degrees $(m_1,\ m_2)$.

Chapter 8

Conclusions

To hedge against demand uncertainty, chase strategy has been popular in real business. Using chase strategy, a firm is able to produce the optimal production level after knowing the actual demand information. However, a number of firms still advocate that the traditional level strategy enables a firm to keep a stable production level. Furthermore, an increasing number of firms adopt mixed strategy, which is in between the chase strategy and the traditional level strategy. Inspired by these actual operational strategies in reality, this thesis aims at constructing a theoretical research framework and conducting comprehensive analyses of these strategies. The prevailing chase strategy is formulated as flexible capacity strategy (FCS) while the traditional level strategy is treated as in-flexible capacity strategy (IFCS) throughout the thesis. This study investigates FCS from four different perspectives.

There are 5 sections in this chapter. Sections 8.1 - 8.4 sequentially summarize results of investigations of FCS from four aspects respectively: evaluation of long term FCS, FCS in an asymmetric oligopoly competition, endogenous flexibility of FCS in an n-firm competitive market and modeling FCS with flexibility degree. Section 8.5 suggests some the future research directions.

8.1 Evaluation of Long Term FCS

First, the study evaluates the long term FCS by considering a long term production cost structure. The improvement of the production cost structure can be achieved by increasing technology level of the existing plants, i.e., flexible technology investment. Flexible technology investment improves the total production cost structure while flexible capacity investment postpones the production quantity decision until after knowing the actual demand. In our model, a firm first chooses its production strategy that consists of investment decisions on flexible technology and flexible capacity, and then it makes the respective operations decisions. A total of five production strategies are formulated and the optimal decision variables of each strategy are solved. With the comparative analysis between different strategies, we draw the following conclusions.

1) The sequential investment effect does not exist if both flexible technology and flexible capacity are invested in.

2) More flexibility cannot guarantee more profit, which may even be worse-off under some environments.

3) Flexible technology always yields the same or a higher profit for a firm, while flexible capacity investment can be beneficial or harmful to a firm depending on the costing environment.

4) The optimal investment decision is either flexible technology or flexible capacity in different costing environments for a firm that makes only one investment.

5) The NT+NC strategy is a lower bound for the T-only strategy, and the C-only strategy is a lower bound for the T+C strategy.

6) The unique optimal strategy can be any one of the five possible strategies, i.e., NT+NC, T-only, C-only, T+C or C+T strategy, depending on the investment costing environment.

8.2 FCS in an Asymmetric Oligopoly Competition

Second, focusing on the competition factor, the study investigates FCS in a two-strategy asymmetric oligopoly competition model with demand uncertainty in a competitive market consisting of r flexible firms and s in-flexible firms. All firms compete with each other in the same market at the same price, which is determined by the demand and the total production quantity in the market. By characterizing the equilibrium of the asymmetric oligopoly competition, we draw the following significant conclusions:

1) Firms adopting the same strategy always make the same decisions at equilibrium regardless of number of firms adopting each of the two strategies.

2) We analytically identify the equilibrium of a competition consisting of *r* flexible firms and *s* in-flexible firms to demonstrate different ways of how a firm's decisions are affected by flexible and in-flexible firms in the same market simultaneously.

3) Increasing production cost damages in-flexible firms, but benefits flexible firms in a strategy-coexistent market under certain capacity costing conditions.

4) Total capacity in a market is driven by market profit potential in to a bounded range, regardless of number of firms in the competition.

5) We identify different environments in which whether FCS or IFCS is the optimal strategy.

8.3 Endogenous Flexibility of FCS in an *n*-Firm Competition

Third, to address the issue of firms switching strategies, we investigate endogenous flexibility of FCS in a competitive market involving totally *n* firms and two available strategies, FCS and IFCS. In the model, firms are able to freely choose and switch their strategies to augment their profits. When any one firm switches strategy, the profit of other firms is affected apart from its own. Furthermore, it may cause other firms to switch strategies and consequently, the entire market structure may be re-organized. Such strategy switching movements continue until no firm switches its strategy if there are no other firms making changes of their strategies. This is an equilibrium called "Final Equilibrium" in this study. A few important conclusions drawn are shown below.

1) It is found that the eventual surviving strategies are insensitive under certainty costing environments while sensitive to *n* under other costing environments. We

also identified these sensitive environments and insensitive environments. In sensitive environments, two strategies coexist when n is small, but only one strategy left in the market when n becomes large.

2) Production cost is proved to be one of the key factors in determining the eventual surviving strategies at Final Equilibrium.

3) Allowing firms to freely join in or quit the market, it is shown that the market eventually becomes a perfect competition when the number of firms tends to infinity.

4) To meet the real desire of decision-making operations, a practical approach is proposed to determine the exact numbers of flexible and in-flexible firms when demand distribution is given. The theoretical justification and numerical example demonstration have also been provided.

8.4 FCS with Flexibility Degree in a Duopoly Competition

Last but not least, aiming at quantifying firms' implementation of FCS and formulating the widely used mixed strategy, we propose the concept of flexibility degree. We further establish a duopoly competition model in which two firms compete with different flexibility degrees. A firm's production stage is composed of two sub-stages: first-production stage and second-production stage. In the first-production stage, a firm has a stable production level which is not affected by uncertain factors. In the second-production stage, the production level is adjustable under the capacity constraints. By measuring the adjustment production range over the total capacity, the flexibility degree reflects a firm's ability in adjusting production level to hedge against demand uncertainty. Therefore, the flexibility degree indicates the extent to which the FCS is implemented. By characterizing the equilibrium of an asymmetric duopoly competition model with demand uncertainty, we draw a few conclusions as follows.

1) Full FCS is a particular case of a general capacity strategy with a flexibility degree of 100%; and IFCS is a particular case of a general capacity strategy with zero flexibility degree.

2) In a symmetric duopoly model in which two firms have the same flexibility degree, two firms make the same decisions at equilibrium. Furthermore, the individual capacity and profit increases with the flexibility degree.

3) In an asymmetric duopoly model in which two firms have different flexibility degrees, the unique equilibrium is characterized with the analytical solutions. We also identify the relationship between flexibility degrees of two firms, their capacity decisions and their profits. There is an inclusive rule of the equilibrium solution.

4) Numerical results show that a firm's optimal capacity and the expected profit at equilibrium increase with its own flexibility degree, while decrease with its rival's flexibility degree. However, a firm's own flexibility degree is more powerful than its rival's flexibility degree to influence a firm's capacity and the maximum expected profit.

5) The maximum of total capacity and the maximum of total profit occur simultaneously when two firms have 100% flexibility degrees; while the minimum of total capacity and the minimum of total profit occur when two firms have zero flexibility degrees.

8.5 Future Research

Based on current findings of FCS, the following are possible future research directions.

1) Conducting empirical studies to test the applicability of the proposed practical approach in Chapter 5.

2) Exploring an oligopoly competition model with flexibility degree.

3) Considering other different forms of manufacturing flexibility and conduct empirical study so that we can find out the relationship between the nature of product and type of flexibility investment.

4) Investigating how manufacturing flexibility can be fully utilized to maximize firms' profits while simultaneously reducing various risks that may exist in a whole supply chain.

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Appendix-I

Proofs of Theorems, Propositions, Properties and Corollaries

Proofs in Chapter 4

Proposition 4.1 For a firm investing in flexible capacity, with any given capacity k and technology level γ, the optimal production quantity as a function of demand realization α is

$$q(\alpha|k,\gamma)=\begin{cases}0 & \text{if } 0\le\alpha<\beta \\ \dfrac{\gamma}{2\gamma+1}(\alpha-\beta) & \text{if } \beta\le\alpha<\beta+\dfrac{2\gamma+1}{\gamma}k \\ k & \text{if } \beta+\dfrac{2\gamma+1}{\gamma}k\le\alpha\end{cases}\cdot$$

<u>**Proof:**</u>

From (4.1), for $q\ge\alpha$, we get $\pi(q|k,\gamma,\alpha)=-q(\beta+\frac{q}{2\gamma})\le-\alpha(\beta+\frac{q}{2\gamma})=\pi(\alpha|k,\gamma,\alpha)$. So we can restrict our search for the optimal q within $0\le q\le\alpha$. Then, $\pi(q|k,\gamma,\alpha)=q(\alpha-q)-(\beta q+\frac{q^2}{2\gamma})=(\alpha-\beta)q-(1+\frac{1}{2\gamma})q^2$, which is a concave quadratic function of q with roots at 0 and $2q_0$, and attains its maximum at q_0, where $q_0=\frac{\gamma}{2\gamma+1}(\alpha-\beta)$. Note that $\alpha-q_0=\frac{\gamma+1}{2\gamma+1}\alpha+\frac{\gamma}{2\gamma+1}\beta\ge0$, implying that $q_0\le\alpha$. Thus, if $0\le\alpha<\beta$, then $q_0<0$ and $q(\alpha|k,\gamma)=0$. If $\beta\le\alpha<\beta+\frac{2\gamma+1}{\gamma}k$, then $0\le q_0<k$ and $q(\alpha|k,\gamma)=q_0$. If $\beta+\frac{2\gamma+1}{\gamma}k\le\alpha$, then $k\le q_0$ and $q(\alpha|k,\gamma)=k$.

This completes the poof of Proposition 4.1. □

Proposition 4.2 Under the NT+NC strategy, a firm's optimal production quantity equals its capacity, i.e., $q^N = k^N$, which satisfies $G(k^N) - k^N/\gamma_N = C_N + \beta$. The firm's optimal expected profit is $\Pi^N = (k^N)^2 \overline{F}(k^N) + (k^N)^2/(2\gamma_N)$.

Proof:

Since there is no flexible capacity investment, $q^N = k^N$. From (4.2), we have $\Pi^N(k) = \int_k^\infty k(\alpha - k) f(\alpha) d\alpha - (C_N + \beta + \frac{k}{2\gamma_N})k$. Therefore, $\Pi^{N(1)}(k) = G(k) - (C_N + \beta + \frac{k}{\gamma_N})$. $\Pi^{N(1)}(k=0) = \mu - C_N - \beta > 0$ and $\Pi^{N(1)}(k = \infty) = -\infty < 0$. $\Pi^{N(2)}(k) = G^{(1)}(k) - 1/\gamma_N < 0$. Therefore, $\Pi^N(k)$ is concave in k, and the unique optimal solution k^N satisfies its first-order condition, i.e., $\Pi^{N(1)}(k^N) = 0$. That is, $G(k^N) - k^N/\gamma_N = C_N + \beta$. The optimal profit can be expressed as $\Pi^N = (k^N)^2 \overline{F}(k^N) + (k^N)^2/(2\gamma_N)$.

This completes the poof of Proposition 4.2. □

Remark: In the above proof, if $C_N \geq \mu - \beta$, then $\Pi^{N(1)}(k=0) = \mu - C_N - \beta \leq 0$, and so $k^N = 0$. Therefore, in order to have a meaningful model, we assume $C_N < \mu - \beta$ in chapter 4.

Proposition 4.3 Under T-only strategy with given γ_N, β, C_N and C_r,

(i) the optimal technology level is $\gamma^T = \begin{cases} \max\{\gamma_N,\ \gamma_T^*\} & \text{if } 0 < C_r \leq \frac{1}{2}(\mu - C_N - \beta)^2 \\ \gamma_N & \text{otherwise} \end{cases}$,

where $\gamma_T^* = \dfrac{Y(C_N + \beta + \sqrt{2C_r})}{\sqrt{2C_r}}$;

(ii) if $\gamma^T = \gamma_T^*$, then the optimal production quantity equals the optimal capacity, which satisfies $q^T = k^T = Y(C_N + \beta + \sqrt{2C_r})$ and the optimal profit is $\Pi^T = (k^T)^2 \overline{F}(k^T) + C_r \gamma_N$. If $\gamma^T = \gamma_N$, then the results are the same as those under the NT+NC strategy.

Proof:

Since there is no flexible capacity investment, we always have $q^T \equiv k^T$. Following the proof of Proposition 4.2 by replacing γ_N with γ, from (4.4) we have the optimal profit for given γ is $\Pi^T(k^T|\gamma) = (k^T)^2 \overline{F}(k^T) + \frac{(k^T)^2}{2\gamma}$, where $G(k^T) = C_N + \beta + \frac{k^T}{\gamma}$ ($k^T \geq 0$ is unique for any given $\gamma > 0$). Differentiating with respect to γ, we get $\frac{dk^T}{d\gamma} = \frac{k^T/\gamma^2}{2\overline{F}(k^T) - k^T f(k^T) + 1/\gamma}$. Together with (4.3), we have $\frac{d\Pi^T(\gamma)}{d\gamma} = \frac{(k^T)^2}{2\gamma^2} - C_r$.

Furthermore, $\frac{d^2\Pi^T(\gamma)}{d\gamma^2} = -\frac{(k^T)^2}{\gamma^2} \cdot \frac{2\overline{F}(k^T) - k^T f(k^T)}{\gamma[2\overline{F}(k^T) - k^T f(k^T)] + 1} \leq 0$ for all $\gamma \geq 0$. (By $\gamma = 0$, we mean the right hand limit.)

Thus, $\Pi^T(\gamma)$ is concave and $\frac{d\Pi^T(\gamma)}{d\gamma}$ is decreasing for $\gamma \geq 0$.

(a) If $0 < C_r \leq \frac{1}{2}(\mu - C_N - \beta)^2$, then $C_N + \beta + \sqrt{2C_r} \leq \mu$. Let γ_T^* be a solution of setting $\frac{d\Pi^T(\gamma)}{d\gamma} = 0$. Then, $\gamma_T^* = \frac{k^{*T}}{\sqrt{2C_r}}$, where $G(k^{*T}) = C_N + \beta + \frac{k^{*T}}{\gamma_T^*}$. Therefore, $k^{*T} = Y(C_N + \beta + \sqrt{2C_r})$ and $\gamma_T^* = \frac{Y(C_N + \beta + \sqrt{2C_r})}{\sqrt{2C_r}} \geq 0$ (unique solution). If $\gamma_T^* > \gamma_N$, then $\gamma^T = \gamma_T^*$ and $q^T = k^T = Y(C_N + \beta + \sqrt{2C_r})$. If $\gamma_T^* \leq \gamma_N$, then $\frac{d\Pi^T(\gamma)}{d\gamma} \leq \frac{d\Pi^T(\gamma_T^*)}{d\gamma} = 0$ for all $\gamma \geq \gamma_N$. Then, $\gamma^T = \gamma_N$.

(b) If $C_r > \frac{1}{2}(\mu - C_N - \beta)^2$, then $C_N + \beta + \sqrt{2C_r} > \mu \geq G(k^T) = C_N + \beta + \frac{k^T}{\gamma}$ and so $\frac{(k^T)^2}{2\gamma^2} < C_r$. Therefore, $\frac{d\Pi^T(\gamma)}{d\gamma} < 0$ for all $\gamma \geq \gamma_N$. Thus, $\gamma^T = \gamma_N$.

Hence, the optimal γ^T under T-only strategy is $\gamma^T = \max\{\gamma_N,\ \gamma_T^*\}$. Substituting γ^T into the objective function, we have: (1) If $\gamma^T = \gamma_T^*$, then $q^T = k^T = Y(C_N + \beta + \sqrt{2C_r})$ and $\Pi^T = (k^T)^2 \overline{F}(k^T) + C_r \gamma_N$. (2) If $\gamma^T = \gamma_N$, then the results are the same as those under NT+NC strategy.

This completes the poof of Proposition 4.3. □

Proposition 4.4 Under C-only strategy with given γ_N, β and C_F, the optimal capacity is $k^C = \frac{\gamma_N}{1+2\gamma_N}(X(C_F)-\beta)$, the optimal production quantity is $q^C = q(\alpha | k^C, \gamma_N)$, and the optimal expected profit is $\Pi^C = \frac{2\gamma_N}{1+2\gamma_N}\Pi_0$.

Proof:

Based on Proposition 4.1, we get the optimal production quantity as $q(\alpha | k, \gamma_N)$. Substituting it into (4.5), we have $\Pi^C(k) = \int_\beta^{\alpha_M} \frac{\gamma_N}{2(1+2\gamma_N)}(\alpha-\beta)^2 f(\alpha)d\alpha + \int_{\alpha_M}^{\infty} k(\alpha-\beta-\frac{1+2\gamma_N}{2\gamma_N}k)f(\alpha)d\alpha - C_F k$, where $\alpha_M = \beta + \frac{2\gamma_N+1}{\gamma_N}k$. Therefore, $\Pi^{C(1)}(k) = L(\alpha_M) - C_F$, $\Pi^{C(1)}(k=0) = L(\beta) - C_F > 0$ and $\Pi^{C(1)}(k=\infty) = -C_F < 0$. $\Pi^{C(2)}(k) = -\frac{2\gamma_N+1}{\gamma_N}\overline{F}(\alpha_M) < -\frac{2\gamma_N+1}{2\gamma_N}\alpha_M f(\alpha_M) \le 0$ for all $k \ge 0$. Therefore, $\Pi^C(k)$ is concave in k, and the unique optimal solution satisfies the first-order condition, i.e., $\Pi^{C(1)}(k) = 0$. That is, $k^C = \frac{\gamma_N}{1+2\gamma_N}(X(C_F)-\beta)$, the optimal production quantity $q^C = q(\alpha | k^C, \gamma_N)$ and the optimal profit $\Pi^C = \int_\beta^{X(C_F)} \frac{\gamma_N}{2(1+2\gamma_N)}(\alpha-\beta)^2 f(\alpha)d\alpha + \int_{X(C_F)}^{\infty} \frac{1+2\gamma_N}{2\gamma_N}k^2 f(\alpha)d\alpha = \frac{2\gamma_N}{1+2\gamma_N}\Pi_0$.

This completes the poof of Proposition 4.4. □

Remark: In the above proof, if $C_F \ge L(\beta)$, then $\Pi^{C(1)}(k=0) \le 0$, and so $k^C = 0$. Therefore, in order to have a meaningful model, we assume $C_F < L(\beta)$ in this chapter.

Proposition 4.5 Under T+C strategy with given γ_N, β, C_F and C_r, the optimal technology level, capacity, production quantity and expected profit are $\gamma^{T+C} = \max\{\gamma_N, \gamma^*_{T+C}\}$, $k^{T+C} = \frac{\gamma^{T+C}}{1+2\gamma^{T+C}}(X(C_F)-\beta)$, $q^{T+C} = q(\alpha | k^{T+C}, \gamma^{T+C})$ and $\Pi^{T+C} = \frac{2\gamma^{T+C}}{1+2\gamma^{T+C}}\Pi_0 - C_r(\gamma^{T+C}-\gamma_N)$, respectively, where $\gamma^*_{T+C} = \frac{1}{2}(\sqrt{2\Pi_0/C_r}-1)$.

Proof:

Following the proof of Proposition 4.3 by replacing γ_N with γ, from (4.8) and (4.9) we get the optimal capacity $k^{T+C} = \frac{\gamma}{1+2\gamma}(X(C_F)-\beta)$, and the optimal production quantity $q^{T+C} = q(\alpha|k^{T+C},\gamma)$. From (4.7), we have $\Pi^{T+C}(\gamma) = \frac{2\gamma}{1+2\gamma}\Pi_0 - C_r(\gamma-\gamma_N)$. Furthermore, $\Pi^{T+C(1)}(\gamma) = \frac{2}{(1+2\gamma)^2}\Pi_0 - C_r$ and $\Pi^{T+C(2)}(\gamma) = -\frac{8}{(1+2\gamma)^3}\Pi_0 < 0$ for all $\gamma \geq \gamma_N$. Therefore, $\Pi^{T+C}(\gamma)$ is concave for $\gamma \geq \gamma_N$. Let γ^*_{T+C} be a solution of $\Pi^{T+C(1)}(\gamma) = 0$. We obtain $\gamma^*_{T+C} = \frac{1}{2}(\sqrt{2\Pi_0/C_r} - 1)$, and so the solution is unique. If $\gamma^*_{T+C} \geq \gamma_N$, then $\gamma^{T+C} = \gamma^*_{T+C}$. Suppose that $\gamma^*_{T+C} < \gamma_N$. $\Pi^{T+C(1)}(\gamma) = \frac{2}{(1+2\gamma)^2}\Pi_0 - C_r < \frac{2}{(1+2\gamma^*_{T+C})^2}\Pi_0 - C_r = 0$ for all $\gamma \geq \gamma_N$. Therefore, $\gamma^{T+C} = \gamma_N$. Hence, $\gamma^{T+C} = \max\{\gamma_N,\ \gamma^*_{T+C}\}$. The results follow.

This completes the poof of Proposition 4.5. □

Theorem 4.1 Under T+C strategy and C+T strategy, a firm's optimal capacity k, technology level γ, production quantity q and the optimal expected profit are exactly the same.

Proof

By formulation of T+C strategy and C+T strategy, i.e., (4.7)~(4.11), T+C strategy and C+T strategy can be expressed in a common formulation as:

$$\text{Max} \quad \Pi(\gamma,k,q(\cdot)) = \int_0^\infty q((\alpha-q)^+ - \beta - \frac{q}{2\gamma}) f(\alpha)d\alpha - C_F k - C_r(\gamma-\gamma_N),$$

$$\text{s.t.} \quad 0 \leq q(\cdot) \leq k,\ k \geq 0,\ \gamma \geq \gamma_N.$$

For C+T strategy, $\Pi^{C+T} = \max_k \max_\gamma \Pi(\gamma,k,q_o(\cdot))$; and for T+C strategy, $\Pi^{T+C} = \max_\gamma \max_k \Pi(\gamma,k,q_o(\cdot))$, where $\Pi(\gamma,k,q_o(\cdot)) = \max_{q(\cdot)} \Pi(\gamma,k,q(\cdot))$. Since $\max_k \max_\gamma \Pi(\gamma,k,q_o(\cdot)) = \max_\gamma \max_k \Pi(\gamma,k,q_o(\cdot))$ with the same optimal k and optimal γ, C+T strategy is equivalent to T+C strategy.

This completes the poof of Theorem 4.1. □

Proposition 4.6 Given C_N, γ_N, β:

(i) For $C_N \le C_F < L(\beta)$, the expected profit of C-only strategy is strictly decreasing in C_F and $\Pi^C(C_N) \ge \Pi^C(C_F) > \Pi^C(L(\beta)) = 0$.

(ii) There exists a unique $\hat{C}_F \in [C_N,\ L(\beta))$ satisfying $\Pi^C(\hat{C}_F) = \Pi^N$; $\hat{C}_F$ is strictly increasing in $C_N \in [0,\ \mu-\beta)$;

(iii) Profit comparison between NT+NC strategy and C-only strategy is: (1) when $\hat{C}_F < C_F < L(\beta)$, $\Pi^N > \Pi^C$; (2) when $C_N \le C_F < \hat{C}_F$, $\Pi^C > \Pi^N$.

<u>Proof</u>:

From Proposition 4.4, we have $\frac{d\Pi^C(C_F)}{dC_F} = -\frac{\gamma_N}{1+2\gamma_N}(X(C_F)-\beta) < 0$. Therefore, $\Pi^C(C_F)$ is strictly decreasing in C_F for $C_N \le C_F < L(\beta)$. Furthermore, $\Pi^C(C_N) \ge \Pi^C(C_F) > \Pi^C(L(\beta)) = 0$. Define $\varphi(C_N) = \Pi^C(C_F = C_N) - \Pi^N(C_N)$. With respect to C_N, we have $\frac{d\varphi(C_N)}{dC_N} = -\frac{\gamma_N}{1+2\gamma_N}(X(C_N)-\beta) + k^N$. From Proposition 4.2, it can be proved that $\int_{k^N}^{\infty}(\alpha - (2+\frac{1}{\gamma_N})k^N) f(\alpha)d\alpha > C_N + \beta$, implying that $k^N < \frac{\gamma_N}{1+2\gamma_N}(X(C_N)-\beta)$. So $\frac{d\varphi(C_N)}{dC_N} < 0$ and $\varphi(C_N)$ is strictly decreasing in $C_N \in [0,\ \mu-\beta)$. Furthermore, as $C_N \to \mu-\beta$, $\varphi(C_N) \to \Pi^C(\mu-\beta) > 0$. Therefore, $\varphi(C_N) = \Pi^C(C_N) - \Pi^N(C_N) > 0$ for all $C_N \in [0,\ \mu-\beta)$. Therefore, $\Pi^C(C_N) > \Pi^N(C_N) > 0 = \Pi^C(L(\beta))$. Since $\Pi^N(C_N)$ is independent of C_F, there exists a unique $\hat{C}_F \in [C_N,\ L(\beta))$ satisfying $\Pi^C(\hat{C}_F) = \Pi^N(C_N) = \Pi^N$. Therefore, $\Pi^N > \Pi^C$ when $\hat{C}_F < C_F < L(\beta)$; $\Pi^C > \Pi^N$ when $C_N \le C_F < \hat{C}_F$. Further, $\frac{d\Pi^N(C_N)}{dC_N} = -k^N < 0$, so we have $\frac{d\hat{C}_F}{dC_N} > 0$.

This completes the poof of Proposition 4.6. □

Proposition 4.7 Comparing NT+NC strategy and T-only strategy, we have

(i) under T-only strategy, define $C_{rT} = \min\{\underline{C}_r,\ C_r\}$, the optimal capacity and expected profit are $k^T = Y(C_N + \beta + \sqrt{2C_{rT}})$ and $\Pi^T = (k^T)^2 \overline{F}(k^T) + C_{rT}\gamma_N$, respectively; further, if $C_r \le \underline{C}_r$, Π^T is strictly decreasing in C_r; if $C_r > \underline{C}_r$, Π^T keeps constant as $\Pi^T(\underline{C}_r)$;

(ii) the optimal decisions of NT+NC strategy can be obtained from resolving T-only strategy with modified parameter $C_r = \underline{C}_r$, and then $k^N = k^T(\underline{C}_r)$ and $\Pi^N = \Pi^T(\underline{C}_r)$;

where $0 < \underline{C}_r < \frac{1}{2}(\mu - C_N - \beta)^2$ satisfying $\dfrac{Y(C_N + \beta + \sqrt{2\underline{C}_r})}{\sqrt{2\underline{C}_r}} = \gamma_N$.

<u>**Proof**</u>

Let $C_{rh} = \frac{1}{2}(\mu - C_N - \beta)^2$. If $0 < C_r \le C_{rh}$, then by Proposition 4.2, we have $\gamma^T = \max\{\gamma_N,\ \gamma_T^*\}$. Note that $\gamma_T^* = \dfrac{Y(C_N + \beta + \sqrt{2C_r})}{\sqrt{2C_r}}$. Then, $\dfrac{d\gamma_T^*}{dC_r} < 0$ and so γ_T^* is strictly decreasing in C_r. Since $\lim_{C_r \to \infty} \gamma_T^*(C_r) = \infty > \gamma_N > 0 = \gamma_T^*(C_r = C_{rh})$, there exists a unique $0 < \underline{C}_r < C_{rh}$ such that $\gamma_T^* \begin{cases} > \gamma_N & \text{if } 0 < C_r < \underline{C}_r \\ = \gamma_N & \text{if } C_r = \underline{C}_r \\ < \gamma_N & \text{if } \underline{C}_r < C_r \le C_{rh\backslash} \end{cases}$. So $\gamma_T^* \ge \gamma_N$ is equivalent to $C_r \le \underline{C}_r$. In this case, since $\dfrac{d\Pi^T(C_r)}{dC_r} = -\gamma_T^* + \gamma_N \le 0$ with equality holds if and only if $\gamma_T^* = \gamma_N$, $\Pi^T(C_r)$ is strictly decreasing for $C_r \le \underline{C}_r$. For $C_r \ge \underline{C}_r$, $\gamma^T = \gamma_N \le \gamma_T^*$ and remains constant $\Pi^T(C_r) = \Pi^N$ for $C_r \ge \underline{C}_r$, by Proposition 4.2. The results follow.

This completes the poof of Proposition 4.7. □

Theorem 4.2 Given C_N, γ_N and β, the comparison between T-only and C-only strategy is:

(i) If $\Pi^C(C_N) < \Pi^T(\underline{C}_r)$, then $\Pi^C(C_F) < \Pi^T(C_r)$ for all situations;

(ii) If $\Pi^C(C_N) \ge \Pi^T(\underline{C}_r)$, then for each $C_r \in (0, C_{rm}]$, $\Pi^C(C_F) \le \Pi^T(C_r)$ for all

$C_F \in [C_N,\ L(\beta))$, where $C_{rm} = \begin{cases} 0 & \text{if } \Pi^C(C_N) \ge \Pi^T(0) \\ \overline{C}_r & \text{if } \Pi^C(C_N) < \Pi^T(0) \end{cases}$ and $\overline{C}_r \in (0, \underline{C}_r]$

satisfies $\Pi^T(\overline{C}_r) = \Pi^C(C_N)$. Moreover, for each $C_r \in (C_{rm}, \infty)$, there exists a unique $\overline{C}_F \in (C_N,\ L(\beta))$ such that $\Pi^C(C_F) \begin{cases} > \Pi^T(C_r) & \text{if } C_N \le C_F < \overline{C}_F \\ = \Pi^T(C_r) & \text{if } C_F = \overline{C}_F \\ < \Pi^T(C_r) & \text{if } \overline{C}_F < C_F < L(\beta) \end{cases}$.

Furthermore, the curve $\Pi^C(\overline{C}_F) = \Pi^T(C_r)$ is strictly increasing for $C_r \in (C_{rm}, \underline{C}_r]$ and horizontal for $C_r \in [\underline{C}_r, \infty)$.

Proof:

(i) If $\Pi^C(C_N) < \Pi^T(\underline{C}_r)$, then, by Propositions 4.6 and 4.7, we have $\Pi^C(C_F) \le \Pi^C(C_N) < \Pi^T(\underline{C}_r) \le \Pi^T(C_r)$ for all C_F and C_r.

(ii) If $\Pi^C(C_N) \ge \Pi^T(\underline{C}_r)$, then we consider two sub-cases.

(ii-1) If $\Pi^C(C_N) \ge \Pi^T(0)$, then $\Pi^C(C_N) \ge \Pi^T(0) > \Pi^T(C_r) > 0 = \Pi^C(L(\beta))$ for all $C_r \in (0, \underline{C}_r]$ by Proposition 4.7.

(ii-2) If $\Pi^C(C_N) < \Pi^T(0)$, then $\Pi^T(0) > \Pi^C(C_N) \ge \Pi^T(\underline{C}_r)$. Since, by Proposition 4.7, $\Pi^T(C_r)$ is strictly decreasing in $(0, \underline{C}_r]$, there exists a unique $\overline{C}_r \in (0, \underline{C}_r]$ such that $\Pi^T(C_r) \begin{cases} > \Pi^C(C_N) \ge \Pi^C(C_F) & \text{if } 0 < C_r < \overline{C}_r \\ = \Pi^C(C_N) & \text{if } C_r = \overline{C}_r \\ < \Pi^C(C_N) & \text{if } \overline{C}_r < C_r \le \underline{C}_r \end{cases}$.

Therefore, for all $C_r \in (\overline{C}_r, \underline{C}_r]$, $\Pi^C(C_N) = \Pi^T(\overline{C}_r) > \Pi^T(C_r) > 0 = \Pi^C(L(\beta))$.

Let $C_{rm} = \begin{cases} 0 & \text{if } \Pi^C(C_N) \ge \Pi^T(0) \\ \overline{C}_r & \text{if } \Pi^C(C_N) < \Pi^T(0) \end{cases}$. Combining Cases (ii-1) and (ii-2), for all $C_r \in (C_{rm}, \underline{C}_r]$ $\Pi^C(C_N) > \Pi^T(C_r) > \Pi^C(L(\beta))$. Since, by Proposition 4.6, $\Pi^C(C_F)$ is strictly decreasing in $[C_N, L(\beta))$, there exists a unique $\overline{C}_F \in (C_N,\ L(\beta))$ such that

$\Pi^C(C_F)\begin{cases} >\Pi^T(C_r) & \text{if } C_N \le C_F < \overline{C}_F \\ =\Pi^T(C_r) & \text{if } C_F = \overline{C}_F \\ <\Pi^T(C_r) & \text{if } \overline{C}_F < C_F < L(\beta) \end{cases}$. Differentiating $\Pi^C(\overline{C}_F) = \Pi^T(C_r)$ with respect to C_r, we have $\frac{d\overline{C}_F}{dC_r} = \frac{\Pi^{T(1)}(C_r)}{\Pi^{C(1)}(\overline{C}_F)} > 0$. Thus, $\overline{C}_F$ is a strictly increasing function of $C_r \in (C_{rm}, \underline{C}_r]$. For all $C_r \ge \underline{C}_r$, $\Pi^T(C_r) = \Pi^T(\underline{C}_r)$, and so the results follow.

This completes the poof of Theorem 4.2. □

Proposition 4.8 Comparing C-only strategy and T+C strategy: For any given C_F,

(i) the optimal expected profit of T+C strategy $\Pi^{T+C}(C_r)$ is strictly decreasing in C_r for $0 < C_r \le \overline{C}_r^{T+C}$, and $\Pi^{T+C}(C_r) = 2\gamma_N/(1+2\gamma_N)\Pi_0$ for $C_r \ge \overline{C}_r^{T+C}$, where $\overline{C}_r^{T+C} = \frac{2\Pi_0}{(1+2\gamma_N)^2}$;

(ii) the C-only strategy can be reduced from T+C strategy by modifying the parameter $C_r = \overline{C}_r^{T+C}$, and then $k^C = k^{T+C}(\overline{C}_r^{T+C})$ and $\Pi^C = \Pi^{T+C}(\overline{C}_r^{T+C})$;

(iii) C-only strategy is a lower bound of T+C strategy; moreover, the increase in profit by T+C strategy relative to C-only strategy is $\delta = \frac{\Pi^{T+C} - \Pi^C}{\Pi^C} \le \frac{1}{2\gamma_N} \cdot 100\%$. .

<u>**Proof**</u>**:**

(i) By Proposition 4.5, $\gamma^*_{T+C} = \frac{1}{2}(\sqrt{2\Pi_0/C_r} - 1)$ and $\Pi^{T+C}(C_r) = \frac{2\gamma^{T+C}}{1+2\gamma^{T+C}}\Pi_0 - C_r(\gamma^{T+C} - \gamma_N)$. For $0 < C_r \le \overline{C}_r^{T+C} = \frac{2\Pi_0}{(1+2\gamma_N)^2}$, we have $\gamma^*_{T+C} \ge \gamma_N$ and so $\gamma^{T+C} = \gamma^*_{T+C}$. Therefore, $\frac{d\Pi^{T+C}(C_r)}{dC_r} = -(\gamma^*_{T+C} - \gamma_N) \le 0$ with the equality holds if and only if $C_r = \overline{C}_r^{T+C}$. Thus, $\Pi^{T+C}(C_r)$ is strictly decreasing in C_r for $0 < C_r \le \overline{C}_r^{T+C}$. For $C_r \ge \overline{C}_r^{T+C}$, we have $\gamma^*_{T+C} \le \gamma_N$ and so $\gamma^{T+C} = \gamma_N$. Then, $\Pi^{T+C}(C_r) = \frac{2\gamma_N}{1+2\gamma_N}\Pi_0$, independent of C_r.

(ii) Furthermore, if $C_r = \overline{C}_r^{T+C}$, then $\gamma^{T+C} = \gamma^*_{T+C} = \gamma_N$. By Propositions 4.3 and 4.4, we have $k^{T+C} = \frac{\gamma_N}{1+2\gamma_N}(X(C_F)-\beta) = k^C$ and $\Pi^{T+C} = \frac{2\gamma_N}{1+2\gamma_N}\Pi_0 = \Pi^C$.

Therefore, T+C strategy is exactly the same as C-only strategy.

From (i) and (ii), C-only strategy is a lower bound of T+C strategy. The increase in profit by T+C strategy relative to C-only strategy is

$$\delta = \frac{\Pi^{T+C}(C_r) - \Pi^C}{\Pi^C} \le \frac{\Pi^{T+C}(C_r = 0) - \Pi^C}{\Pi^C} = \frac{\Pi_0 - 2\gamma_N \Pi_0/(1+2\gamma_N)}{2\gamma_N \Pi_0/(1+2\gamma_N)} = \frac{1}{2\gamma_N} \cdot 100\% .$$

This completes the poof of Proposition 4.8. □

Theorem 4.3 Define $\Delta\Pi = \Pi^{T+C} - \Pi^T$. Let $\overline{C}_F^{T+C}$ satisfy $C_r = \frac{2\Pi_0(\overline{C}_F^{T+C})}{(1+2\gamma_N)^2}$. A unique C_r^* satisfying $\Delta\Pi(C_r^*, \overline{C}_F^{T+C}) = 0$ exists. With a given C_N,

(i) if $0 < C_r < C_r^*$, then there exists a unique $C_F^*(C_r)$ satisfying

$$\Delta\Pi(C_r, C_F) \begin{cases} > 0 & \text{if } C_N \le C_F < C_F^*(C_r) \\ = 0 & \text{if } C_F = C_F^*(C_r) \\ < 0 & \text{if } C_F^*(C_r) < C_F \le L(\beta) \end{cases} ;$$

(ii) if $C_r^* \le C_r \le \underline{C}_r$, then there exists a unique $\tilde{C}_F(C_r)$ satisfying

$$\Delta\Pi(C_r, C_F) \begin{cases} > 0 & \text{if } C_N \le C_F < \tilde{C}_F(C_r) \\ = 0 & \text{if } C_F = \tilde{C}_F(C_r) \\ < 0 & \text{if } \tilde{C}_F(C_r) < C_F \le L(\beta) \end{cases} ;$$

(iii) if $\underline{C}_r < C_r$, then $\Delta\Pi(C_r, C_F) \begin{cases} > 0 & \text{if } C_N \le C_F < \tilde{C}_F(\underline{C}_r) \\ = 0 & \text{if } C_F = \tilde{C}_F(\underline{C}_r) \\ < 0 & \text{if } \tilde{C}_F(\underline{C}_r) < C_F \le L(\beta) \end{cases}$.

<u>Proof</u>:

In the following, we consider the effects of variations of C_F and C_r on Π^{T+C} and Π^T, so as to compare T+C strategy and T-only strategy under different environments. First, we consider the relationship between C_F and $\overline{C}_r^{T+C} = \frac{2\Pi_0(C_F)}{(1+2\gamma_N)^2}$, $C_F \in [C_N, L(\beta))$ (Proposition 4.7). Differentiating w.r.t. to C_F, we get $\frac{d}{dC_F}\overline{C}_r^{T+C} = -\frac{X(C_F)-\beta}{(1+2\gamma_N)^2} < 0$.

So, $\overline{C}_r^{T+C}$ is a strictly decreasing function of $C_F \in [C_N, L(\beta))$. Therefore, the inverse function exists, say $\overline{C}_F^{T+C}$, satisfying $C_r = \dfrac{2\Pi_0(\overline{C}_F^{T+C})}{(1+2\gamma_N)^2}$. Note that $\overline{C}_F^{T+C}$ is a strictly decreasing function of $C_r \in (0, \underline{C}_r^{T+C}]$, where $\underline{C}_r^{T+C} = \dfrac{2\Pi_0(C_N)}{(1+2\gamma_N)^2}$.

For T+C strategy: Let $a = \gamma^{T+C}$. By Proposition 4.5, $\gamma_{T+C}^* = \frac{1}{2}(\sqrt{2\Pi_0/C_r} - 1)$ and $\Pi^{T+C} = \dfrac{2a}{1+2a}\Pi_0 - C_r(a - \gamma_N)$. For T-only strategy, by Proposition 7.7 $\underline{C}_r$ satisfies $\dfrac{Y(C_N + \beta + \sqrt{2\underline{C}_r})}{\sqrt{2\underline{C}_r}} = \gamma_N$. Define $\Delta\Pi(C_r, C_F) = \Pi^{T+C} - \Pi^T$.

Case 1: $0 < C_r \le \min\{\underline{C}_r, \underline{C}_r^{T+C}\}$ and $C_N \le C_F \le \overline{C}_F^{T+C}$

For T+C strategy, following the proof of Proposition 4.8, we have $a = \gamma_{T+C}^* \ge \gamma_N$. Thus, $a = \frac{1}{2}(\sqrt{2\Pi_0/C_r} - 1)$, $\Pi_0 = \dfrac{(1+2a)^2}{2}C_r$ and $\Pi^{T+C} = (2a^2 + \gamma_N)C_r$. Then, $\dfrac{\partial a}{\partial C_F} = -\dfrac{X(C_F) - \beta}{4(1+2a)C_r}$. For T-only strategy, by Proposition 4.6, we have $\Pi^T = (k^T)^2\overline{F}(k^T) + C_r\gamma_N$, where $k^T = Y(C_N + \beta + \sqrt{2C_r})$. Then, $\Delta\Pi(C_r, C_F) = 2a^2C_r - (k^T)^2\overline{F}(k^T)$. With respect to C_F, $\dfrac{\partial\Delta\Pi}{\partial C_F} = -\dfrac{a}{1+2a}(X(C_F) - \beta) < 0$. Therefore, for any given C_r with $0 < C_r \le \min\{\underline{C}_r, \underline{C}_r^{T+C}\}$, $\Delta\Pi$ is strictly decreasing in C_F, $C_N \le C_F \le \overline{C}_F^{T+C}$. When $C_F = C_N$, T-only strategy and T+C strategy can be expressed in a common formulation as:

Max $\Pi(\gamma, k, q(\cdot)) = \int_0^\infty q((\alpha - q)^+ - \beta - \dfrac{q}{2\gamma}) f(\alpha)d\alpha - C_N k - C_r(\gamma - \gamma_N)$,

s.t. $0 \le q(\cdot) \le k$.

For T-only strategy, $\Pi^T = \max_{\gamma,k} \Pi(\gamma, k, q(\cdot) = k)$; for T+C strategy, $\Pi^{T+C} = \max_{\gamma,k} \max_{q(\cdot)} \Pi(\gamma, k, q(\cdot))$. Since T-only strategy is more restricted than T+C strategy, we have $\Pi^{T+C} \geq \Pi^T$. Therefore, $\Delta\Pi(C_r, C_N) \geq 0$.

When $C_F = \overline{C}_F^{T+C}$, then $a = \gamma^{T+C} = \gamma_N$. The T+C strategy is the same as the C-only strategy. Consider the curve $C_F = \overline{C}_F^{T+C}$, where $\overline{C}_F^{T+C}$ is a function of C_r . $\Delta\Pi(C_r, \overline{C}_F^{T+C}) = 2\gamma_N{}^2 C_r - (k^T)^2 \overline{F}(k^T)$. With respect to C_r, $\frac{d\Delta\Pi}{dC_r} = 2\gamma_N{}^2 + \frac{k^T}{\sqrt{2C_r}} > 0$.

Therefore, $\Delta\Pi$ is strictly increasing in C_r , $C_r \in (0,\ \min\{\underline{C}_r, \underline{C}_r^{T+C}\}]$. At $C_r = \min\{\underline{C}_r, \underline{C}_r^{T+C}\}$, we consider two cases:

(i) If $\underline{C}_r \leq \underline{C}_r^{T+C}$, then $C_r = \underline{C}_r$ and $C_F = \overline{C}_F^{T+C}$. We have $\gamma_T^* = \gamma_{T+C}^* = \gamma_N$, $k^T = \gamma_N \sqrt{2\underline{C}_r}$ and $\Delta\Pi(\underline{C}_r, \overline{C}_F^{T+C}) = 2\gamma_N{}^2 \underline{C}_r - 2\gamma_N{}^2 \underline{C}_r \overline{F}(\gamma_N \sqrt{2\underline{C}_r}) = 2\gamma_N{}^2 \underline{C}_r F(\gamma_N \sqrt{2\underline{C}_r}) \geq 0$.

(ii) If $\underline{C}_r > \underline{C}_r^{T+C}$, then $C_r = \underline{C}_r^{T+C}$ and $C_F = \overline{C}_F^{T+C}$. We have $C_F = C_N$. By above, we obtain $\Delta\Pi(C_r, C_N) \geq 0$. Therefore, $\Delta\Pi(\min\{\underline{C}_r, \underline{C}_r^{T+C}\}, C_N) \geq 0$.

If $C_r \to 0$ and $C_F = \overline{C}_F^{T+C}$, then $C_F = \overline{C}_F^{T+C} \to L(\beta)$ and $\Delta\Pi(C_r, \overline{C}_F^{T+C}) \to -Y^2(C_N + \beta)\overline{F}(Y(C_N + \beta)) < 0$. Therefore, when $C_F = \overline{C}_F^{T+C}$, where $\overline{C}_F^{T+C}$ is a function of C_r, there exists a unique $C_r^* \in (0,\ \min\{\underline{C}_r, \underline{C}_r^{T+C}\}]$ such that

$$\Delta\Pi(C_r, \overline{C}_F^{T+C}) \begin{cases} < 0 & \text{if } 0 < C_r < C_r^* \\ = 0 & \text{if } C_r = C_r^* \\ > 0 & \text{if } C_r^* < C_r \leq \min\{\underline{C}_r, \underline{C}_r^{T+C}\} \end{cases}$$

. Combining the results so far, we have conclusion that for $0 < C_r < C_r^*$, there exists a unique $C_F^*(C_r)$ depending on C_r, such that $\Delta\Pi = 0$. Therefore, if $0 < C_r \leq C_r^*$, then

$$\Delta\Pi(C_r, C_F) \begin{cases} > 0 & \text{if } C_N \leq C_F < C_F^*(C_r) \\ = 0 & \text{if } C_F = C_F^*(C_r) \\ < 0 & \text{if } C_F^*(C_r) < C_F \leq \overline{C}_F^{T+C} \end{cases}$$

; if $C_r^* < C_r \leq \min\{\underline{C}_r, \underline{C}_r^{T+C}\}$, then

$\Delta\Pi(C_r, C_F) \geq 0$. Now we consider $\Delta\Pi(C_r, C_F) = 0$ for $0 < C_r \leq C_r^*$ when C_N is given. Then $2a^2 C_r = (k^T)^2 \overline{F}(k^T)$, where $k^T = Y(C_N + \beta + \sqrt{2C_r})$ and $a = \frac{1}{2}(\sqrt{2\Pi_0(C_F)/C_r} - 1)$. With respect to C_r, we have $\frac{da}{dC_r} = -\frac{X(C_F) - \beta}{4\sqrt{\Pi_0(C_F)}\sqrt{2C_r}} \frac{dC_F}{dC_r} - \frac{\sqrt{\Pi_0(C_F)}}{(\sqrt{2C_r})^3}$. Consider $2a^2 C_r = (k^T)^2 \overline{F}(k^T)$, with respect to C_r, we have $2a^2 + 4aC_r \frac{da}{dC_r} = -\frac{k^T}{\sqrt{2C_r}}$. Therefore, $-\frac{a(X(C_F) - \beta)}{1+2a} \frac{dC_F}{dC_r} - a = -\frac{k^T}{\sqrt{2C_r}}$ and $\frac{dC_F}{dC_r} = \frac{k^T - a\sqrt{2C_r}}{\sqrt{2C_r}} \cdot \frac{(1+2a)}{a(X(C_F) - \beta)}$.

Since $2a^2 C_r = (k^T)^2 \overline{F}(k^T)$, $a\sqrt{2C_r} = k^T \sqrt{\overline{F}(k^T)}$. Therefore,

$$\frac{dC_F}{dC_r} = \frac{k^T - k^T\sqrt{\overline{F}(k^T)}}{k^T\sqrt{\overline{F}(k^T)}} \cdot \frac{(1+2a)}{(X(C_F) - \beta)} = \frac{1 - \sqrt{\overline{F}(k^T)}}{\sqrt{\overline{F}(k^T)}} \cdot \frac{(1+2a)}{(X(C_F) - \beta)} > 0.$$

In order to complete the analyses, we consider the following two situations.

Case 2: $\min\{\underline{C}_r, \underline{C}_r^{T+C}\} < C_r$ and $C_N \leq C_F \leq \overline{C}_F^{T+C}$

(i) If $\underline{C}_r \leq \underline{C}_r^{T+C}$, then $\underline{C}_r < C_r$ and $C_N \leq C_F \leq \overline{C}_F^{T+C}$. So, $a = \frac{1}{2}(\sqrt{2\Pi_0(C_F)/C_r} - 1) \geq \gamma_N$ and $k^T = Y(C_N + \beta + \sqrt{2\underline{C}_r})$. Therefore, $\frac{\sqrt{2\Pi_0(C_F)} - \sqrt{C_r}}{\sqrt{C_r}} \geq 1 + 2\gamma_N$ and so $\sqrt{2\Pi_0(C_F)} - \sqrt{C_r} \geq (1+2\gamma_N)\sqrt{C_r} > 2\gamma_N\sqrt{C_r}$.

k^T is independent of C_r. Therefore,

$$\begin{aligned}
\Delta\Pi(C_r, C_F) &= 2a^2 C_r - (k^T)^2 \overline{F}(k^T) + \gamma_N(C_r - \underline{C}_r) \\
&> 2a^2 C_r - (k^T)^2 \overline{F}(k^T) = \frac{1}{2}(\sqrt{2\Pi_0(C_F)} - \sqrt{C_r})^2 - 2\gamma_N{}^2 \underline{C}_r \overline{F}(\gamma_N\sqrt{2\underline{C}_r}) \\
&> 2\gamma_N^2 C_r - 2\gamma_N{}^2 \underline{C}_r \overline{F}(\gamma_N\sqrt{2\underline{C}_r}) > 0.
\end{aligned}$$

(ii) If $C_r > \underline{C}_r^{T+C}$, then $\underline{C}_r^{T+C} < C_r$ and $C_N \leq C_F \leq \overline{C}_F^{T+C}$. We have a contradiction.

Therefore, $\Delta\Pi(C_r, C_F) > 0$ if Case 2 exists.

Case 3: $\max\{C_N, \overline{C}_F^{T+C}\} < C_F < L(\beta)$

Then, for T+C strategy, $\gamma^{T+C} = \gamma_N$ and $\Pi^{T+C} = \dfrac{2\gamma_N}{1+2\gamma_N}\Pi_0(C_F)$ which is independent of C_r. Further $\dfrac{d\Pi^{T+C}}{dC_F} < 0$ for $\max\{C_N, \overline{C}_F^{T+C}\} < C_F < L(\beta)$. For T-only strategy, for $0 < C_r \le \underline{C}_r$, $\Pi^T = (k^T)^2 \overline{F}(k^T) + C_r\gamma_N$ which is independent of C_F. Further, $\dfrac{d\Pi^T}{dC_r} < 0$ within $0 < C_r \le \underline{C}_r$. Π^T keeps constant as $\Pi^T(\underline{C}_r)$ when $\underline{C}_r < C_r$. We consider three sub-cases as following.

Case-3.1 if $0 < C_r < C_r^*$, then for given C_r,

$$\Delta\Pi = \Pi^{T+C} - \Pi^T < \Pi^{T+C}(C_F = \overline{C}_F^{T+C}) - \Pi^T < 0.$$

Case-3.2 if $C_r^* \le C_r \le \underline{C}_r$, then $\dfrac{d\Pi^{T+C}}{dC_F} < 0$ and $\dfrac{d\Pi^T}{dC_r} < 0$. Therefore, $\dfrac{d\Delta\Pi}{dC_F} < 0$ for given C_r; $\dfrac{d\Delta\Pi}{dC_r} > 0$ for given C_F. As $\max\{C_N, \overline{C}_F^{T+C}\} < C_F < L(\beta)$, we check boundaries. If $C_F = \overline{C}_F^{T+C}$, then $\Delta\Pi > 0$; and if $C_F = C_N$, then $\Delta\Pi \ge 0$ as before. If $C_F \to L(\beta)$, then $\Delta\Pi < 0$. Therefore, for any given C_r, $C_r^* \le C_r \le \underline{C}_r$, there exists a unique $\tilde{C}_F$ such that $\Delta\Pi = 0$ and $\tilde{C}_F$ is a function of C_r. Consider the curve $\Delta\Pi = 0$. With respect to C_r, we have $\dfrac{\partial\Delta\Pi}{\partial C_F}\cdot\dfrac{dC_F}{dC_r} + \dfrac{\partial\Delta\Pi}{\partial C_r} = 0$, so that $\dfrac{dC_F}{dC_r} > 0$.

Therefore, $\dfrac{d\tilde{C}_F}{dC_r} > 0$.

The results are $$\Delta\Pi(C_r, C_F) \begin{cases} > 0 & \text{if } \overline{C}_F^{T+C} \le C_F < \tilde{C}_F(C_r) \\ = 0 & \text{if } C_F = \tilde{C}_F(C_r) \\ < 0 & \text{if } \tilde{C}_F(C_r) < C_F < L(\beta) \end{cases}.$$

Case-3.3 if $\underline{C}_r < C_r$, then for any given C_F, Π^T keeps constant as $\Pi^T(\underline{C}_r)$. Therefore, $\Delta\Pi = \Pi^{T+C} - \Pi^T$ keeps as constant as $\Delta\Pi(\underline{C}_r)$ with any given C_F.

This completes the proof of Theorem 4.3. □

Theorem 4.4 Given C_N, γ_N, β, assume $C_N \le C_F \le L(\beta)$, following all definitions in Proposition 4.6, the most effective and efficient strategy (**EES**) is:

(i) if $0 < C_r < C_r^*$, then $\text{EES} = \begin{cases} \text{T+C strategy}, & \text{if } C_N \le C_F < C_F^*(C_r) \\ \text{T-only strategy}, & \text{otherwise} \end{cases}$;

(ii) if $C_r^* \le C_r \le \underline{C}_r$, then $\text{EES} = \begin{cases} \text{T+C strategy}, & \text{if } C_N \le C_F < \overline{C}_F^{T+C} \\ \text{C-only strategy}, & \text{if } \overline{C}_F^{T+C} \le C_F < \tilde{C}_F(C_r) \\ \text{T-only strategy}, & \text{otherwise} \end{cases}$;

(iii) if $\underline{C}_r < C_r$, then $\text{EES} = \begin{cases} \text{T+C strategy}, & \text{if } C_N \le C_F < \overline{C}_F^{T+C} \\ \text{C-only strategy}, & \text{if } \overline{C}_F^{T+C} \le C_F < \tilde{C}_F(\underline{C}_r) \\ \text{NT+NC strategy}, & \text{otherwise} \end{cases}$;

For all environments, T+C strategy equals C+T strategy.

Proof: Following the proof of Theorem 4.3, we can get Theorem 4.4 directly. □

Proofs in Chapter 5

Proposition 5.1 Consider any feasible firm *i*. Suppose that α, k_j, $j \in \Omega$, and q_j^F, $j \in \Omega^F \setminus \{i\}$, are given. Then the optimal production decision q_i^{F*} of the feasible firm *i* is

$$q_i^{F*} = \begin{cases} 0, & x \le \beta \\ q_{ib}^F, & \beta < x \le 2k_i^F + \beta \\ k_i^F, & 2k_i^F + \beta < x \end{cases}, \text{ where } x = \alpha - \sum_{j \ne i} q_j^F(\alpha) - k^N \text{ and } q_{ib}^F = \frac{x - \beta}{2}.$$

Proof

Let $x = \alpha - \sum_{j \ne i} q_j^F - k^N$. Then by (5.2), we have $m_i^F(q_i^F) = q_i^F(x - q_i^F)^+ - \beta q_i^F$. There are three cases.

Case 1 If $x < 0$, then

$m_i^F(q_i^F) = -\beta q_i^F \le 0$, $m_i^{F(1)}(q_i^F) = -\beta \le 0$, so that $m_i^F(q_i^F)$ is decreasing as q_i^F increases. The optimal solution under this situation is $q_i^{F*} = 0$, so that $m_i^F(q_i^{F*}) = 0$.

Case 2 If $0 \le x < k_i^F$, then we have two sub-cases.

(1) If $x \le q_i^F \le k_i^F$, then $m_i^F(q_i^F) = -\beta q_i^F \le 0$, $m_i^{F(1)}(q_i^F) = -\beta \le 0$, the optimal solution under this situation is $q_i^{F*} = x$. Therefore, we can restrict the search for the optimal quantity to the range $0 \le q_i^F \le x$.

(2) If $0 \le q_i^F \le x$, then $m_i^F(q_i^F) = q_i^F(x - q_i^F) - \beta q_i^F$, $m_i^F(q_i^F)^{(1)} = x - \beta - 2q_i^F$, $m_i^{F(2)}(q_i^F) = -2 < 0$. So $m_i^F(q_i^F)$ is concave in q_i^F. By its first-order condition $m_i^{F(1)}(q_i^F) = 0$, $q_{ib}^F = \dfrac{(x-\beta)}{2}$. Therefore, we have

$$q_i^{F*} = \begin{cases} 0, & q_{ib}^F \le 0 \\ q_{ib}^F, & 0 < q_{ib}^F \le x \\ x, & x < q_{ib}^F \end{cases}, \text{ i.e., } q_i^{F*} = \begin{cases} 0, & x \le \beta \\ q_{ib}^F, & \beta < x \\ x, & x < -\beta \text{ (contradiction)} \end{cases}.$$

Therefore, we have $q_i^{F*} = \begin{cases} 0, & x \le \beta \\ q_{ib}^F, & \beta < x \end{cases}$.

Case 3 If $k_i^F \le x$, then

$m_i^F(q_i^F) = q_i^F(x - q_i^F) - \beta q_i^F$. Similar to Case 2, $m_i^F(q_i^F)$ is concave in q_i^F, and so

$$q_i^{F*} = \begin{cases} 0, & q_{ib}^F \le 0 \\ q_{ib}^F, & 0 < q_{ib}^F \le k_i^F \\ k_i^F, & k_i^F \le q_{ib}^F \end{cases}, \text{ i.e., } q_i^{F*} = \begin{cases} 0, & x \le \beta \\ q_{ib}^F, & \beta < x \le 2k_i^F + \beta \\ k_i^F, & 2k_i^F + \beta < x \end{cases}.$$

Combining Cases 1 - 3, we have $q_i^{F*} = \begin{cases} 0, & x \le \beta \\ q_{ib}^F, & \beta < x \le 2k_i^F + \beta \\ k_i^F, & 2k_i^F + \beta < x \end{cases}$.

This completes the proof of Proposition 5.1. □

Proposition 5.2 In an oligopoly market competition with $r > 0$ flexible firms and $s \ge 0$ in-flexible firms, the optimal capacity of flexible firm $i \in \Omega^F$, i.e., k_i^{F*}, is either (i) $k_i^{F*} = 0$ and $A^{F(1)}(0) \le 0$; or (ii) $k_i^{F*} > 0$ and $A^{F(1)}(k_i^{F*}) = 0$.

__Proof__

By (5.3), $A^F(k_i^F) = \Pi_i^F\left(k_i^F \middle| k_j^F,\ q_j^F(\alpha)\ \forall j \ne i, \text{ and } k_l^N,\ l \in \Omega^N\right)$

$$= \int_0^\infty q_i^{F*}((\alpha - Q^F(\alpha) - k^N)^+ - \beta) f(\alpha) d\alpha - C_F k_i^F. \qquad \text{(a5.1)}$$

According to Proposition 5.1, for any given k_i^F, the optimal q_i^{F*} is

$$q_i^{F*} = \begin{cases} 0, & x \le \beta \\ q_{ib}^F, & \beta < x \le 2k_i^F + \beta \\ k_i^F, & 2k_i^F + \beta < x \end{cases},$$

where $x = \alpha - \sum_{j \ne i} q_j^F(\alpha) - k^N = \alpha - Q^F(\alpha) - k^N + q_i^{F*}$ and $q_{ib}^F = (x-\beta)/2$. By (a5.1), the objective becomes finding the optimal k_i^{F*} to maximize $A^F(k_i^F)$. Note that

$$\alpha - Q^F(\alpha) - k^N = \begin{cases} x, & x \le \beta \\ \dfrac{x+\beta}{2} > \beta, & \beta < x \le 2k_i^F + \beta \\ x - k_i^F > k_i^F + \beta, & 2k_i^F + \beta < x \end{cases}.$$

Thus, if $x \le \beta$, then $q_i^{F*} = 0$; and if $x > \beta$, then $\alpha - Q^F(\alpha) - K^N > \beta \ge 0$. Together with (a501), we have $A^F(k_i^F) = \int_0^\infty q_i^{F*}(\alpha - Q^F(\alpha) - k^N - \beta) f(\alpha) d\alpha - C_F k_i^F$.

Let $y = q_{ib}^F = (x-\beta)/2$, then $q_i^{F*} = \begin{cases} 0, & y \le 0; \\ y, & 0 < y \le k_i^F; \\ k_i^F, & k_i^F < y. \end{cases}$

$$\begin{aligned} A^F(k_i^F) &= \int_0^\infty q_i^{F*}(2y - q_i^{F*}) f(\alpha) d\alpha - C_F k_i^F \\ &= \int_{0<y\le k_i^F} y(2y-y) f(\alpha) d\alpha + \int_{k_i^F<y} k_i^F(2y - k_i^F) f(\alpha) d\alpha - C_F k_i^F \\ &= \int_{0<y} y^2 f(\alpha) d\alpha - \int_{k_i^F<y} (y-k_i^F)^2 f(\alpha) d\alpha - C_F k_i^F. \end{aligned} \quad \text{(a5.2)}$$

Let $B(k_i^F) = \int_{0<y} y^2 f(\alpha) d\alpha$, and $C(k_i^F) = \int_{k_i^F<y} (y-k_i^F)^2 f(\alpha) d\alpha$. Note that $B(k_i^F)$ is independent of k_i^F, i.e., $\dfrac{dB(k_i^F)}{dk_i^F} = 0$. Therefore, with respect to k_i^F, by (a5.2) we have

$A^{F(1)}(k_i^F) = 2\int_{k_i^F<y} (y - k_i^F) f(\alpha) d\alpha - C_F$ and $A^{F(2)}(k_i^F) = -2\int_{k_i^F<y} f(\alpha) d\alpha \le 0$.

Therefore, $A^F(k_i^F)$ is concave. Note that $C(k_i^F) \ge 0$ and $C_F > 0$. It follows that, as $k_i^F \to \infty$, $A^F(k_i^F) \to -\infty$. Therefore, there exists an $S > 0$, such that $A^F(k_i^F) < 0$ for

all $k_i^F \geq S$. Since $A^F(0)=0$, we can restrict our search for the optimal k_i^F in $[0,S]$. Therefore, either (i) $k_i^{F*}=0$ and $A^{F(1)}(0)\leq 0$; or (ii) $k_i^{F*}>0$ and $A^{F(1)}(k_i^{F*})=0$.
This completes the proof of Proposition 5.2. □

Proposition 5.3 At the equilibrium of an oligopoly market competition with $r>0$ flexible firms and $s\geq 0$ in-flexible firms, the optimal capacities of flexible firms are either $k_i^{F*}=0$, for all $i\in\Omega^F$; or $k_i^{F*}>0$, for all $i\in\Omega^F$; further,

(i) $k_i^{F*}=0$, for all $i\in\Omega^F$, is equivalent to $k^N \geq X(C_F)-\beta$;

(ii) $k_i^{F*}>0$, for all $i\in\Omega^F$, is equivalent to $k^N < X(C_F)-\beta$.

Proof

We follow the notations in the proof of Proposition 5.2. Let $\theta = 2y - q_i^{F*} = \alpha - Q^F(\alpha) - k^N - \beta$, which is independent of i. By Proposition 5.2, there are two cases of k_i^{F*}.

In case (i) $k_i^{F*}=0$ and $A^{F(1)}(0)\leq 0$, we have $q_i^{F*}=0$ and

$$A^{F(1)}(0) = 2\int_{0<y} yf(\alpha)d\alpha - C_F \leq 0, \text{ i.e., } \int_{0<\theta}\theta\, f(\alpha)d\alpha \leq C_F. \tag{a5.3}$$

In case (ii) $k_i^{F*}>0$, $A^{F(1)}(k_i^{F*})=0$, we have

$$A^{F(1)}(k_i^F) = 2\int_{k_i^F<y}(y-k_i^F)f(\alpha)d\alpha - C_F = 0, \text{ i.e., } \frac{C_F}{2} = \int_{k_i^F<y}(y-k_i^F)f(\alpha)d\alpha. \tag{a5.4}$$

When $y>k_i^{F*}$, $q_i^{F*}=k_i^{F*}$. Therefore, $y=(\theta+k_i^{F*})/2>k_i^{F*}$ is equivalent to $\theta>k_i^{F*}$. By (a5.4), we have $\int_{k_i^{F*}<\theta}(\theta-k_i^{F*})f(\alpha)d\alpha = C_F$.

Note that $\int_{k_i^F<\theta}(\theta-k_i^F)f(\alpha)d\alpha \leq \int_{0<\theta}\theta\, f(\alpha)d\alpha - k_i^F\int_{k_i^F<\theta}f(\alpha)d\alpha$. If $\int_{k_i^F<\theta}f(\alpha)d\alpha=0$, then for any $t>0$, $0\leq \int_{0<\theta-k_i^F\leq t}(\theta-k_i^F)f(\alpha)d\alpha \leq t\int_{0<\theta-k_i^F\leq t}f(\alpha)d\alpha \leq t\int_{0<\theta-k_i^F}f(\alpha)d\alpha = 0$. So, $C_F = \int_{k_i^F<\theta}(\theta-k_i^F)f(\alpha)d\alpha = \lim_{t\to\infty}\int_{0<\theta-k_i^F\leq t}(\theta-k_i^F)f(\alpha)d\alpha = 0$, which is a contradiction.

Therefore, $\int_{k_i^F<\theta}f(\alpha)d\alpha>0$. Since $k_i^{F*}>0$, we have

$$C_F = \int_{k_i^F<\theta} (\theta - k_i^F) f(\alpha) d\alpha < \int_{0<\theta} \theta f(\alpha) d\alpha \text{, i.e., } \int_{0<\theta} \theta f(\alpha) d\alpha > C_F . \quad \text{(a5.5)}$$

Since (a5.3) and (a5.5) are contradictory to each other and independent of i, we have at equilibrium either $k_i^{F*} = 0$, for all $i \in \Omega^F$; or $k_i^{F*} > 0$, for all $i \in \Omega^F$. We discuss these two cases respectively.

Case-I $k_i^{F*} = 0$, for all $i \in \Omega^F$, then $q_i^{F*} = 0$. By (a5.3),

we have $\int_{k^N+\beta}^{\infty} (\alpha - k^N - \beta) f(\alpha) d\alpha \le C_F$, i.e., $k^N \ge X(C_F) - \beta$.

Case-II $k_i^{F*} > 0$, for all $i \in \Omega^F$, then by (a5.5),

we have $\int_{\alpha > Q^F(\alpha)+k^N+\beta} (\alpha - Q^F(\alpha) - k^N - \beta) f(\alpha) d\alpha > C_F$. Therefore,

$$\int_{\alpha > k^N+\beta} (\alpha - k^N - \beta) f(\alpha) d\alpha \ge \int_{\alpha > Q^F(\alpha)+k^N+\beta} (\alpha - k^N - \beta) f(\alpha) d\alpha$$

$$\ge \int_{\alpha > Q^F(\alpha)+k^N+\beta} (\alpha - Q^F(\alpha) - k^N - \beta) f(\alpha) d\alpha > C_F ,$$

i.e., $L(k^N + \beta) > C_F$. Equivalently, $k^N < X(C_F) - \beta$. Therefore,

(1) $k_i^{F*} = 0$, for all $i \in \Omega^F$, is equivalent to $k^N \ge X(C_F) - \beta$;

(2) $k_i^{F*} > 0$, for all $i \in \Omega^F$, is equivalent to $k^N < X(C_F) - \beta$.

This completes the proof of Proposition 5.3. □

Theorem 5.1 At the equilibrium of an oligopoly market competition with $r > 0$ flexible firms and $s \ge 0$ in-flexible firms, all flexible firms $i \in \Omega^F$ make the same capacity decision and the same production decision. That is:

(i) If $k^N \ge X(C_F) - \beta$, then $k_i^{F*} = q_i^{F*} = 0$ for all $i \in \Omega^F$.

(ii) If $k^N < X(C_F) - \beta$, then $k_i^{F*} = k_e^F = \frac{1}{r} k^F > 0$ for all $i \in \Omega^F$; further, we have

$k^N + (r+1) k_e^F = X(C_F) - \beta$. The individual profit of each flexible firm is

$$\Pi_e^F = \frac{1}{(r+1)^2} \left(\int_{k^N+\beta}^{X(C_F)} (\alpha - k^N - \beta)^2 f(\alpha) d\alpha + \int_{X(C_F)}^{\infty} (X(C_F) - k^N - \beta)^2 f(\alpha) d\alpha \right).$$

The production decision of each flexible firm is $q_i^{F*} = q_e^F$ for all $i \in \Omega^F$, which is presented as $q_e^F = \begin{cases} 0, & 0 \le \alpha \le \beta + k^N \\ \dfrac{\alpha - \beta - k^N}{r+1}, & \beta + k^N < \alpha \le \beta + (r+1)k_e^F + k^N \\ k_e^F, & \beta + (r+1)k_e^F + k^N < \alpha \end{cases}$.

Proof

We use the same notations as in the proof of Proposition 5.2. For any two flexible firms $j \ne h$, $j, h \in \Omega^F$. Without loss of generality, we assume $k_j^{F*} \le k_h^{F*}$. By (a5.5), we have $\int_{k_j^{F*} < \theta} (\theta - k_j^{F*}) f(\alpha) d\alpha = \int_{k_h^{F*} < \theta} (\theta - k_h^{F*}) f(\alpha) d\alpha$. Assume that $k_j^{F*} < k_h^{F*}$. Since $\int_{k_h^{F*} < \theta} f(\alpha) d\alpha > 0$, we have

$$\begin{aligned} \int_{k_j^{F*} < \theta} (\theta - k_j^{F*}) f(\alpha) d\alpha &\ge \int_{k_h^{F*} < \theta} (\theta - k_j^{F*}) f(\alpha) d\alpha \\ &= \int_{k_h^{F*} < \theta} \theta f(\alpha) d\alpha - k_j^{F*} \int_{k_h^{F*} < \theta} f(\alpha) d\alpha \\ &> \int_{k_h^{F*} < \theta} \theta f(\alpha) d\alpha - k_h^{F*} \int_{k_h^{F*} < \theta} f(\alpha) d\alpha = \int_{k_h^{F*} < \theta} (\theta - k_h^{F*}) f(\alpha) d\alpha. \end{aligned}$$

This is a contradiction. Hence, $k_j^{F*} = k_h^{F*}$.

Therefore, we have $k_j^{F*} = k_e^F > 0$ for all $j \in \Omega^F$. Therefore, for any feasible firm $i \in \Omega^F$, together with Proposition 5.1, we have $q_i^{F*} = \begin{cases} 0, & \theta \le 0 \\ \theta, & 0 < \theta \le k_e^F \\ k_e^F, & k_e^F < \theta \end{cases}$, where $\theta = \alpha - Q^F(\alpha) - k^N - \beta$ is independent of *i*. So all $q_i^{F*}, i \in \Omega^F$, are equal. That means $q_i^{F*} = q_e^F$ for all $i \in \Omega^F$, and we have $Q^F = rq_e^F$. Since there are *r* flexible firms, $\theta = \alpha - rq_e^F - k^N - \beta$. Therefore, $q_i^{F*} = q_e^F$ can be expressed as $q_e^F = \begin{cases} 0, & 0 \le \alpha \le \beta + k^N \\ \dfrac{\alpha - \beta - k^N}{r+1}, & \beta + k^N < \alpha \le \beta + (r+1)k_e^F + k^N \\ k_e^F, & \beta + (r+1)k_e^F + k^N < \alpha \end{cases}$. By (a5.5), we have $\int_{k^N + \beta + (r+1)k_e^F < \alpha} (\alpha - k^N - \beta - (r+1)k_e^F) f(\alpha) d\alpha = C_F$, i.e., $L(k^N + \beta + (r+1)k_e^F) = C_F$. So, we have $k^N + (r+1)k_e^F = X(C_F) - \beta$, i.e., $k_e^F = \dfrac{X(C_F) - \beta - k^N}{r+1}$. (a5.6)

We consider the individual expected profit of each flexible firm. By (a5.2) and (a5.4),

$$A^F(k_i^F) = \int_{0<y\le k_i^F} y^2 f(\alpha)d\alpha + \int_{k_i^F<y} k_i^F(2y-k_i^F)f(\alpha)d\alpha - 2k_i^F \int_{k_i^F<y}(y-k_i^F)f(\alpha)d\alpha$$

$$= \int_{0<y\le k_i^F} y^2 f(\alpha)d\alpha + \int_{k_i^F<y}(k_i^F)^2 f(\alpha)d\alpha . \qquad \text{(a5.7)}$$

Note that $y=(x-\beta)/2=\dfrac{\alpha-(r-1)q_e^F-k^N-\beta}{2}$. Then

(1) If $0\le\alpha\le\beta+k^N$, then $q_e^F=0$, $y=(\alpha-k^N-\beta)/2\le 0$.

(2) If $\beta+k^N<\alpha\le\beta+(r+1)k_e^F+k^N$, then $q_e^F=\dfrac{\alpha-k^N-\beta}{r+1}$;

$$0<y=\frac{\alpha-k^N-\beta}{r+1}\le k_e^F .$$

(3) If $\beta+(r+1)k_e^F+k^N<\alpha$, then $q_e^F=k_e^F$, $y=\dfrac{\alpha-(r-1)k_e^F-k^N-\beta}{2}>k_e^F$.

Therefore, by (a5.7) we have

$A^F(k_i^F)=\int_{k^N+\beta}^{k^N+\beta+(r+1)k_e^F}(\dfrac{\alpha-k^N-\beta}{r+1})^2 f(\alpha)d\alpha+\int_{k^N+\beta+(r+1)k_e^F}^{\infty}(k_e^F)^2 f(\alpha)d\alpha$. Together with (a5.6), we have $k^N+\beta+(r+1)k_e^F=X(C_F)$ and so

$$A^F(k_i^F)=\int_{k^N+\beta}^{X(C_F)}(\frac{\alpha-k^N-\beta}{r+1})^2 f(\alpha)d\alpha+\int_{X(C_F)}^{\infty}(k_e^F)^2 f(\alpha)d\alpha$$

$$=\int_{k^N+\beta}^{X(C_F)}(\frac{\alpha-k^N-\beta}{r+1})^2 f(\alpha)d\alpha+\int_{X(C_F)}^{\infty}(\frac{X(C_F)-k^N-\beta}{r+1})^2 f(\alpha)d\alpha$$

$$=\frac{1}{(r+1)^2}\left(\int_{k^N+\beta}^{X(C_F)}(\alpha-k^N-\beta)^2 f(\alpha)d\alpha+\int_{X(C_F)}^{\infty}(X(C_F)-k^N-\beta)^2 f(\alpha)d\alpha\right).$$

This completes the proof of Theorem 5.1. □

Corollary 5.1 At the equilibrium of an oligopoly market competition with $r>0$ flexible firms and $s=0$ in-flexible firms, all flexible firms $i\in\Omega^F$ make the same capacity decision and the same production decision. That is:

(i) If $C_F\ge L(\beta)$, then $k_i^{F*}=q_i^{F*}=0$ for all $i\in\Omega^F$.

(ii) If $C_F<L(\beta)$, then $k_i^{F*}=k_e^F=\dfrac{X(C_F)-\beta}{r+1}>0$ for all $i\in\Omega^F$. The profit of each flexible firm is

$\Pi_e^F = \frac{1}{(r+1)^2}\left(\int_\beta^{X(C_F)} (\alpha-\beta)^2 f(\alpha)d\alpha + \int_{X(C_F)}^\infty (X(C_F)-\beta)^2 f(\alpha)d\alpha\right)$. The production decision of each flexible firm is $q_i^{F*} = q_e^F$ for all $i \in \Omega^F$, where

$$q_e^F = \begin{cases} 0, & 0 \le \alpha \le \beta \\ \dfrac{\alpha-\beta}{r+1}, & \beta < \alpha \le X(C_F). \\ k_e^F, & X(C_F) < \alpha \end{cases}$$ □

Proof

A direct consequence of Theorem 5.1 □

Proposition 5.4 In an oligopoly market competition with $r \ge 0$ flexible firms and $s > 0$ in-flexible firms, the optimal capacity of in-flexible firm $i \in \Omega^N$, i.e., k_i^{N*}, is either (1) $k_i^{N*} = 0$ and $A^{N(1)}(0) \le 0$; or (2) $k_i^{N*} > 0$, $A^{N(1)}(k_i^{N*}) = 0$ and $A^{N(2)}(k_i^{N*}) \le 0$.

Proof

By (5.4), $A^N(k_i^N) = \Pi_i^N\left(k_i^N \middle| k_j^N,\ \forall j \ne i, \text{and } k_l^F, q_l^F(\alpha)\ l \in \Omega^F\right)$

$$= \int_0^\infty k_i^N(\alpha - Q^F(\alpha) - \sum_{j\ne i} k_j^N - k_i^N)^+ f(\alpha)d\alpha - (C_N+\beta)k_i^N$$

$$\le \int_0^\infty k_i^N(\alpha - k_i^N)^+ f(\alpha)d\alpha - (C_N+\beta)k_i^N$$

$$= \int_{k_i^N}^\infty k_i^N(\alpha - k_i^N) f(\alpha)d\alpha - (C_N+\beta)k_i^N = k_i^N(L(k_i^N) - \beta - C_N).$$

Let $u = \alpha - Q^F(\alpha) - \sum_{j\ne i} k_j^N$. Then,

$$A^N(k_i^N) = \int_{k_i^N<u} k_i^N(u - k_i^N) f(\alpha)d\alpha - (C_N+\beta)k_i^N. \quad \text{(a5.8)}$$

$$A^{N(1)}(k_i^N) = \int_{k_i^N<u} (u - 2k_i^N) f(\alpha)d\alpha - (C_N+\beta).$$

It is noted that as $k_i^N \to \infty$, $A^N(k_i^N) \le k_i^N(L(k_i^N) - \beta - C_N) \to -\infty$. Therefore, there exists $S_1 > 0$, such that $A^N(k_i^N) < 0$ for all $k_i^N \ge S_1$. Since $A^N(0) = 0$, to find the optimal k_i^{N*}, if any, we can restrict our search to the range $[0, S_1]$. Because $A^N(k_i^N)$ is continuous on $[0, S_1]$. Hence, there exists $k_i^{N*} \in [0, S_1]$ to maximize $A^N(k_i^N)$. Therefore, either (1) $k_i^{N*} = 0$ and $A^{N(1)}(0) \le 0$; or (2) $k_i^{N*} > 0$, $A^{N(1)}(k_i^{N*}) = 0$ and $A^{N(2)}(k_i^{N*}) \le 0$.

This completes the proof of Proposition 5.4. □

Proposition 5.5 At the equilibrium of an oligopoly market competition with $r \geq 0$ flexible firms and $s > 0$ in-flexible firms, the optimal capacities of in-flexible firms are either (1) $k_i^{N*} = 0$, for all $i \in \Omega^N$; or (2) $k_i^{N*} > 0$, for all $i \in \Omega^N$; further, we have

(i) $k_i^{N*} = 0$, for all $i \in \Omega^N$, is equivalent to $\int_{0<v} vf(\alpha)d\alpha \leq C_N + \beta$;

(ii) $k_i^{N*} > 0$, for all $i \in \Omega^N$, is equivalent to $\int_{0<v} vf(\alpha)d\alpha > C_N + \beta$, where $v = \alpha - Q^F(\alpha) - k^N$.

<u>Proof</u>

We follow the notations in the proof of Proposition 5.4. By Proposition 5.4, there are two cases.

In case (1) $k_i^{N*} = 0$ and $A^{N(1)}(0) \leq 0$, then

$$A^{N(1)}(k_i^N) = \int_{0<v} vf(\alpha)d\alpha - (C_N + \beta) \leq 0\text{, i.e., } \int_{0<v} vf(\alpha)d\alpha \leq C_N + \beta. \quad \text{(a5.9)}$$

In case (2) $k_i^{N*} > 0$ and $A^{N(1)}(k_i^{N*}) = 0$, then

$$\int_{0<v} (v - k_i^N) f(\alpha)d\alpha = C_N + \beta. \quad \text{(a5.10)}$$

By the proof of Proposition 5.4, we get $\int_{0<v} f(\alpha)d\alpha > 0$. Since $k_i^{N*} > 0$, we have

$$C_N + \beta = \int_{0<v} (v - k_i^{N*}) f(\alpha)d\alpha = \int_{0<v} vf(\alpha)d\alpha - k_i^{N*} \int_{0<v} f(\alpha)d\alpha < \int_{0<v} vf(\alpha)d\alpha,$$

$$\text{i.e., } \int_{0<v} vf(\alpha)d\alpha > C_N + \beta. \quad \text{(a5.11)}$$

Since (a5.9) and (a5.11) are contradictory to each other and independent of *i*, we have, at equilibrium, either $k_i^{N*} = 0$, for all $i \in \Omega^N$; or $k_i^{N*} > 0$, for all $i \in \Omega^N$. Therefore, at equilibrium, there are two cases.

Case-I $k_i^{N*} = 0$, for all $i \in \Omega^N$, then by (5.13) we have $\int_{0<v} vf(\alpha)d\alpha \leq C_N + \beta$.

Case-II $k_i^{N*} > 0$, for all $i \in \Omega^N$, then by (5.15) we have $\int_{0<v} vf(\alpha)d\alpha > C_N + \beta$.

Therefore, $k_i^{N*} = 0$, for all $i \in \Omega^N$, is equivalent to $\int_{0<v} vf(\alpha)d\alpha \leq C_N + \beta$;

$k_i^{N*} > 0$, for all $i \in \Omega^N$, is equivalent to $\int\limits_{0<v} vf(\alpha)d\alpha > C_N + \beta$.

This completes the proof of Proposition 5.5. □

Theorem 5.2 At the equilibrium of an oligopoly market competition with $r \geq 0$ flexible firms and $s > 0$ in-flexible firms, all in-flexible firms make the same capacity decision.

(1) If $\int\limits_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - sk_e^N) f(\alpha)d\alpha \leq C_N + \beta$, then $k_i^{N*} = 0$, for all $i \in \Omega^N$.

(2) If $\int\limits_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - sk_e^N) f(\alpha)d\alpha > C_N + \beta$, then $k_i^{N*} = k_e^N = \frac{1}{s} k^N > 0$, for all $i \in \Omega^N$; further, we have $\int\limits_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - (s+1)k_e^N) f(\alpha)d\alpha = C_N + \beta$. The individual profit of each in-flexible firm is $\Pi_e^N = (k_e^N)^2 \overline{F}(sk_e^N)$.

Proof

Following the notations in Proposition 5.5, we consider two in-flexible firms $j \neq h$, $j,h \in \Omega^N$. By (a5.10) we have $\int\limits_{0<v} (v - k_j^{N*}) f(\alpha)d\alpha = \int\limits_{0<v} (v - k_h^{N*}) f(\alpha)d\alpha$.

This implies that $(k_h^{N*} - k_j^{N*}) \int\limits_{0<v} f(\alpha)d\alpha = 0$.

To show $\int\limits_{0<v} f(\alpha)d\alpha > 0$: By (a5.10), we have $\int\limits_{0<v} (v - k_j^{N*}) f(\alpha)d\alpha = C_N + \beta$.

If $\int\limits_{0<v} f(\alpha)d\alpha = 0$, then for any $t > k_j^{N*}$, $\int\limits_{0<v\leq t} (v - k_j^{N*}) f(\alpha)d\alpha \geq -k_j^{N*} \int\limits_{0<v\leq t} f(\alpha)d\alpha \geq -k_j^{N*} \int\limits_{0<v} f(\alpha)d\alpha = 0$ and $\int\limits_{0<v\leq t} (v - k_j^{N*}) f(\alpha)d\alpha \leq (t - k_j^{N*}) \int\limits_{0<v\leq t} f(\alpha)d\alpha \leq (\alpha) \int\limits_{0<v} f(\alpha)d\alpha = 0$, implying $\int\limits_{0<v\leq t} (v - k_j^{N*}) f(\alpha)d\alpha = 0$.

So, $C_N + \beta = \int\limits_{0<v} (v - k_j^{N*}) f(\alpha)d\alpha = \lim\limits_{t\to\infty} \int\limits_{0<v\leq t} (v - k_j^{N*}) f(\alpha)d\alpha = 0$, which is a contradiction. Therefore, $\int\limits_{0<v} f(\alpha)d\alpha > 0$.

Hence, we get $k_h^{N*} = k_j^{N*}$. Therefore, we have $k_j^{N*} = k_e^N > 0$ for all $j \in \Omega^N$. Since there are s in-flexible firms, we have $k^N = sk_e^N > 0$, $v = \alpha - rq_e^F - sk_e^N$ and $\int_{0<v} vf(\alpha)d\alpha = \int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - sk_e^N) f(\alpha)d\alpha$. Therefore, by Proposition 5.5,

$k_i^{N*} = 0$, for all $i \in \Omega^N$, is equivalent to $\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - sk_e^N) f(\alpha)d\alpha \le C_N + \beta$.

$k_i^{N*} > 0$, for all $i \in \Omega^N$, is equivalent to $\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - sk_e^N) f(\alpha)d\alpha > C_N + \beta$.

If $v = \alpha - rq_e^F - sk_e^N > 0$, then $\alpha > rq_e^F + sk_e^N$. By (5.14) we have $\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - (s+1)k_e^N) f(\alpha)d\alpha = C_N + \beta$. Also, $v = u - k_i^N = \alpha - Q^F(\alpha) - k^N$, by (a5.10) we have $\int_{k_i^N < u} (u - 2k_i^N) f(\alpha)d\alpha = C_N + \beta$. Together with (a5.8), we have

$$\begin{aligned} A^N(k_i^N) &= \int_{k_i^N < u} k_i^N (u - k_i^N) f(\alpha)d\alpha - (C_N + \beta)k_i^N \\ &= \int_{k_i^N < u} k_i^N (u - k_i^N) f(\alpha)d\alpha - k_i^N \int_{k_i^N < u} (u - 2k_i^N) f(\alpha)d\alpha \\ &= \int_{sk_e^N < \alpha - rq_e^F} (k_i^N)^2 f(\alpha)d\alpha = (k_e^N)^2 \int_{sk_e^N < \alpha - rq_e^F} f(\alpha)d\alpha \end{aligned} \tag{a5.12}$$

By Theorem 5.1,
$$rq_e^F = \begin{cases} 0, & 0 \le \alpha \le \beta + sk_e^N \\ \dfrac{r(\alpha - \beta - sk_e^N)}{r+1}, & \beta + sk_e^N < \alpha \le \beta + (r+1)k_e^F + sk_e^N \\ rk_e^F, & \beta + (r+1)k_e^F + sk_e^N < \alpha \end{cases}. \tag{a5.13}$$

There are three cases as follows.

(1) If $0 \le \alpha \le \beta + sk_e^N$ and $\alpha - rq_e^F > sk_e^F$, then $rq_e^F = 0$, $\alpha - rq_e^F = \alpha > sk_e^N$, so we have $sk_e^N < \alpha \le \beta + sk_e^N$.

(2) If $\beta + sk_e^N < \alpha \le \beta + (r+1)k_e^F + sk_e^N$ and $\alpha - rq_e^F > sk_e^N$, then

$rq_e^F = \dfrac{r(\alpha - \beta - sk_e^N)}{r+1}$, $\alpha - rq_e^F = \dfrac{\alpha + r(\beta + sk_e^N)}{r+1} > sk_e^N$ and so $\alpha > sk_e^N - r\beta$.

Therefore, we have $\beta + sk_e^N < \alpha \le \beta + (r+1)k_e^F + sk_e^N$.

(3) If $\beta + (r+1)k_e^F + sk_e^N < \alpha$ and $\alpha - rq_e^F > sk_e^N$, then $rq_e^F = rk_e^F$,

$\alpha - rq_e^F = \alpha - rk_e^F > sk_e^N$, and so $\alpha > rk_e^F + sk_e^N$. Therefore, we have $\beta + (r+1)k_e^F + sk_e^N < \alpha$.

Combine these three cases, the range of α for $\alpha - rq_e^F > sk_e^F$ is $sk_e^N < \alpha$. Therefore, from (a5.12) we have $A^N(k_i^N) = (k_e^N)^2 \int_{sk_e^N}^{\infty} f(\alpha)d\alpha = (k_e^N)^2 \overline{F}(sk_e^N)$.

This completes the proof of Theorem 5.2. □

Corollary 5.2 At the equilibrium of an oligopoly market competition with $r = 0$ flexible firms and $s > 0$ in-flexible firms, all in-flexible firms make the same capacity decision.

(1) If $C_N \geq \mu - \beta$, then $k_i^{N*} = 0$, for all $i \in \Omega^N$.

(2) If $C_N < \mu - \beta$, then $k_i^{N*} = k_e^N = \frac{1}{s}k^N > 0$, for all $i \in \Omega^N$; further, we have $\int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha)d\alpha = C_N + \beta$. The individual profit of each in-flexible firm is $\Pi_e^N = (k_e^N)^2 \overline{F}(sk_e^N)$.

<u>Proof</u>

By Theorem 5.2, putting $r = 0$:

If $\int_{sk_e^N < \alpha} (\alpha - sk_e^N) f(\alpha)d\alpha \leq C_N + \beta$, then $k_i^{N*} = k_e^N = 0$, for all $i \in \Omega^N$. Thus, $\mu \leq C_N + \beta$.

If $\int_{sk_e^N < \alpha} (\alpha - sk_e^N) f(\alpha)d\alpha > C_N + \beta$, then $k_i^{N*} = k_e^N = \frac{1}{s}k^N > 0$, for all $i \in \Omega^N$. Thus, $\mu \geq L(sk_e^N) = \int_{sk_e^N < \alpha} (\alpha - sk_e^N) f(\alpha)d\alpha > C_N + \beta$. Therefore,

(1) If $C_N \geq \mu - \beta$, then $k_i^{N*} = k_e^N = 0$, for all $i \in \Omega^N$.

(2) If $C_N < \mu - \beta$, then $k_i^{N*} = k_e^N = \frac{1}{s}k^N > 0$, for all $i \in \Omega^N$. Further results follow directly.

This completes the proof of Corollary 5.2. □

Proposition 5.6 Given $r > 0$ flexible firms and $s > 0$ in-flexible firms, within the area $R = \{(C_N, C_F): 0 < C_N \le C_F < \infty\}$. For Case-A that $k_e^F = k_e^N = 0$, we have $\Pi_e^F = \Pi_e^N = 0$ and a necessary condition for Case-A is: $L(\beta) \le C_F$ and $\mu - \beta \le C_N$.

Proof

We define Case-A as the case with solution $k_e^F = k_e^N = 0$. By Theorems 5.1 and 5.2, we have

$$k^N \ge X(C_F) - \beta; \tag{a5.14}$$

$$\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - sk_e^N) f(\alpha) d\alpha \le C_N + \beta, \tag{a5.15}$$

By (a5.14), we have $L(\beta) \le C_F$; by (a5.15), we have $\mu - \beta \le C_N$.

The individual expected profits of both flexible and in-flexible firms are $\Pi_e^F = \Pi_e^N = 0$.

This completes the proof of Proposition 5.6. □

Proposition 5.7 Given $r > 0$ flexible firms and $s > 0$ in-flexible firms, within the area $R = \{(C_N, C_F): 0 < C_N \le C_F < \infty\}$. For Case-B that $k_e^F = 0$, $k_e^N > 0$, we have $\Pi_e^F = 0$, $\Pi_e^N = \frac{1}{s^2}(k^N)^2 \overline{F}(k^N)$, and (i) a necessary condition for Case-B is: $L(\beta + k_w) \le C_F$ and $C_N < \mu - \beta$; (ii) $k_e^N > 0$ satisfies $\int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha) d\alpha = C_N + \beta$, $k^N = sk_e^N$.

Proof

We define Case-B as the case with solution that $k_e^F = 0$, $k_e^N > 0$. By Theorems 5.1 and 5.2, the conditions of Case-B are:

$$sk_e^N \ge X(C_F) - \beta; \tag{a5.16}$$

$$\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - sk_e^N) f(\alpha) d\alpha > C_N + \beta. \tag{a5.17}$$

The solution satisfies $\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - (s+1)k_e^N) f(\alpha) d\alpha = C_N + \beta$. (a5.18)

Since $k_e^F = 0$, we get $q_e^F = 0$. By (a5.17), we have $\mu \ge L(sk_e^N) > C_N + \beta$.

By (a5.18), $Z(sk_e^N) = \int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha) d\alpha = C_N + \beta$. (a5.19)

Therefore, by (a5.16) and (a5.19), $X(C_F)-\beta \le sk_e^N = k_w$. Thus, $C_F \ge L(k_w+\beta)$. By Theorem 5.2, the individual expected profit of in-flexible firms is $\Pi_e^N = \frac{1}{s^2}(k^N)^2\overline{F}(k^N)$ and $k^N = sk_e^N$. A necessary condition for Case-B is: $L(\beta+k_w)\le C_F$ and $C_N < \mu-\beta$. This completes the proof of Proposition 5.7. □

Proposition 5.8 Given $r>0$ flexible firms and $s>0$ in-flexible firms, within the area $R=\{(C_N,C_F): 0<C_N\le C_F<\infty\}$. For Case-C that $k_e^F>0$, $k_e^N=0$, we have $\Pi_e^F = \frac{1}{(r+1)^2}\left(\int_\beta^{X(C_F)}(\alpha-\beta)^2 f(\alpha)d\alpha + \int_{X(C_F)}^\infty (X(C_F)-\beta)^2 f(\alpha)d\alpha\right)$, $\Pi_e^N=0$, and (i) a necessary condition for Case-C is: $L(\beta)>C_F$ and $\mu-(C_N+\beta)\le \frac{r}{r+1}(L(\beta)-C_F)$; (ii) $k_e^F = \frac{1}{r+1}(X(C_F)-\beta)$.

Proof

We define Case-C as the case with solution of $k_e^F>0$, $k_e^N=0$. By Theorems 5.1 and 5.2, the conditions of Case-C are

$$k^N < X(C_F)-\beta; \tag{a5.20}$$

$$\int_{rq_e^F+sk_e^N<\alpha}(\alpha-rq_e^F-sk_e^N)f(\alpha)d\alpha \le C_N+\beta. \tag{a5.21}$$

The solution satisfies $(r+1)k_e^F = X(C_F)-\beta$, i.e., $k_e^F = \frac{1}{r+1}(X(C_F)-\beta)$. (a5.22)

By (a5.20), we have $C_F < L(\beta)$.

By (a5.21), we have $\int_{rq_e^F<\alpha}(\alpha-rq_e^F)f(\alpha)d\alpha \le C_N+\beta$. (a5.23)

By Theorem 5.1, we have $rq_e^F = \begin{cases} 0, & 0\le\alpha\le\beta \\ \frac{r(\alpha-\beta)}{r+1}, & \beta<\alpha\le\beta+(r+1)k_e^F \\ rk_e^F, & \beta+(r+1)k_e^F<\alpha \end{cases}$. Therefore,

$\alpha-rq_e^F = \begin{cases} \alpha, & 0\le\alpha\le\beta \\ \frac{\alpha+r\beta}{r+1}, & \beta<\alpha\le\beta+(r+1)k_e^F \\ \alpha-rk_e^F, & \beta+(r+1)k_e^F<\alpha \end{cases}$. Thus, $\alpha-rq_e^F>0$ for any $\alpha>0$.

Therefore, by (a5.23) we have

$$\int_0^{\beta} \alpha f(\alpha)d\alpha + \int_{\beta}^{\beta+(r+1)k_e^F} \left(\alpha - \frac{r}{r+1}(\alpha-\beta)\right) f(\alpha)d\alpha + \int_{\beta+(r+1)k_e^F}^{\infty} (\alpha - rk_e^F) f(\alpha)d\alpha \le C_N + \beta$$

i.e., $\mu - \frac{r}{r+1}(L(\beta) - C_F) \le C_N + \beta$. Hence, $\mu - (C_N + \beta) \le \frac{r}{r+1}(L(\beta) - C_F)$.

By Theorem 5.1, the individual expected profit of flexible firms is $\Pi_e^F = \frac{1}{(r+1)^2}\left(\int_{\beta}^{X(C_F)} (\alpha-\beta)^2 f(\alpha)d\alpha + \int_{X(C_F)}^{\infty} \left(X(C_F)-\beta\right)^2 f(\alpha)d\alpha\right)$. Since $k_e^N = 0$, we have $\Pi_e^N = 0$. A necessary condition for Case-C is: $L(\beta) > C_F$ and $\mu - (C_N + \beta) \le \frac{r}{r+1}(L(\beta) - C_F)$.

This completes the proof of Proposition 5.8. □

Proposition 5.9 Given $r > 0$ flexible firms and $s > 0$ in-flexible firms, within the area $R = \{(C_N, C_F) : 0 < C_N \le C_F < \infty\}$. For Case-D that $k_e^F > 0$, $k_e^N > 0$, we have $\Pi_e^F = \frac{1}{(r+1)^2}\left(\int_{\beta+sk_e^N}^{X(C_F)} (\alpha - sk_e^N - \beta)^2 f(\alpha)d\alpha + \int_{X(C_F)}^{\infty} (X(C_F) - sk_e^N - \beta)^2 f(\alpha)d\alpha\right)$; and $\Pi_e^N = (k_e^N)^2 \overline{F}(sk_e^N)$. The solution of Case-D is: $k_e^F = \frac{1}{r+1}(X(C_F) - \beta - sk_e^N)$ and k_e^N satisfies $C_N + \beta = \int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha)d\alpha - \frac{r}{r+1}(L(\beta + sk_e^N) - C_F)$. A necessary condition for Case-D is: $\mu - (C_N + \beta) > \frac{r}{r+1}(L(\beta) - C_F)$ and $C_F < L(\beta + k_w)$.

<u>Proof</u>

We define Case-D as the case with solution that $k_e^F > 0$, $k_e^N > 0$. By Theorems 5.1 and 5.2, the conditions of Case-D are

$$k^N < X(C_F) - \beta \tag{a5.24}$$

$$\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - sk_e^N) f(\alpha)d\alpha > C_N + \beta \tag{a5.25}$$

The solutions of Case-D satisfy the following (a5.26) and (a5.27).

$$sk_e^N + (r+1)k_e^F + \beta = X(C_F); \tag{a5.26}$$

$$\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - (s+1)k_e^N) f(\alpha)d\alpha = C_N + \beta. \tag{a5.27}$$

By (a5.13), we have $rq_e^F = \begin{cases} 0, & 0 \le \alpha \le \beta + sk_e^N \\ \dfrac{r(\alpha - \beta - sk_e^N)}{r+1}, & \beta + sk_e^N < \alpha \le \beta + (r+1)k_e^F + sk_e^N \\ rk_e^F, & \beta + (r+1)k_e^F + sk_e^N < \alpha \end{cases}$.

By (a5.27), we have the following three cases.

Case 1: $0 \le \alpha \le \beta + sK_e^N$ and $\alpha - rq_e^F - sK_e^N > 0$.

In this case, $rq_e^F = 0$ and $\alpha - rq_e^F - sk_e^N = \alpha - sk_e^N > 0$. Therefore, we have $sk_e^N < \alpha \le \beta + sk_e^N$ and $\alpha - rq_e^F - (s+1)k_e^N = \alpha - (s+1)k_e^N$.

Case 2: $\beta + sk_e^N < \alpha \le \beta + (r+1)k_e^F + sk_e^N$ and $\alpha - rq_e^F - sk_e^N > 0$.

In this case, $rq_e^F = \dfrac{r(\alpha - \beta - sk_e^N)}{r+1}$ and $\alpha - rq_e^F - sk_e^N = \dfrac{\alpha - (sk_e^N - r\beta)}{r+1}$. So, $\alpha \ge sk_e^N - r\beta$. Therefore, we have $\beta + sk_e^N < \alpha \le \beta + (r+1)k_e^F + sk_e^N$ and $\alpha - rq_e^F - (s+1)k_e^N = \dfrac{\alpha - (s+r+1)k_e^N + r\beta}{r+1}$.

Case 3: $\beta + (r+1)k_e^F + sk_e^N < \alpha$ and $\alpha - rq_e^F - sk_e^N > 0$.

In this case, $rq_e^F = rk_e^F$ and $\alpha - rq_e^F - sk_e^N = \alpha - rk_e^F - sk_e^N > 0$. So, $\alpha > rk_e^F + sk_e^N$. Therefore, $\beta + (r+1)k_e^F + sk_e^N < \alpha$ and $\alpha - rq_e^F - (s+1)k_e^N = \alpha - rk_e^F - (s+1)k_e^N$.

Hence, by (a5.26) and (a5.27), we have

$$C_N + \beta = \int_{sk_e^N}^{\beta + sk_e^N} (\alpha - (s+1)k_e^N) f(\alpha) d\alpha + \frac{1}{r+1} \int_{\beta + sk_e^N}^{X(C_F)} (\alpha + r\beta - (r+s+1)k_e^N) f(\alpha) d\alpha$$
$$+ \int_{X(C_F)}^{\infty} (\alpha - rk_e^F - (s+1)k_e^N) f(\alpha) d\alpha.$$

Therefore, $C_N + \beta = \int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha) d\alpha - \dfrac{r}{r+1}(L(\beta + sk_e^N) - C_F)$.

Let $R(k_e^N) = \int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha) d\alpha - \dfrac{r}{r+1}(L(\beta + sk_e^N) - C_F)$. With respect to k_e^N,

$R^{(1)}(k_e^N) = sk_e^N f(sk_e^N) - (s+1)\overline{F}(sk_e^N) + \dfrac{rs}{r+1}\overline{F}(\beta + sk_e^N) < 0$. So, $R(k_e^N)$ is decreasing as k_e^N increases. Since $k_e^F > 0$ and $k_e^N > 0$, by (a5.26) we have $0 < k_e^N = \dfrac{X(C_F) - \beta - (r+1)k_e^F}{s} < \dfrac{X(C_F) - \beta}{s}$, and so $R(\dfrac{X(C_F) - \beta}{s}) < R(k_e^N) < R(0)$.

$R(0)=\int_0^\infty \alpha f(\alpha)d\alpha-\frac{r}{r+1}(L(\beta)-C_F)=\mu-\frac{r}{r+1}(L(\beta)-C_F)$.

$R(\frac{X(C_F)-\beta}{s})=\int_{X(C_F)-\beta}^\infty(\alpha-\frac{s+1}{s}(X(C_F)-\beta))f(\alpha)d\alpha-\frac{r}{r+1}(L(\beta+X(C_F)-\beta)-C_F)$

$$=\int_{X(C_F)-\beta}^\infty(\alpha-\frac{s+1}{s}(X(C_F)-\beta))f(\alpha)d\alpha.$$ Therefore,

$\int_{X(C_F)-\beta}^\infty(\alpha-\frac{s+1}{s}(X(C_F)-\beta))f(\alpha)d\alpha<C_N+\beta<\mu-\frac{r}{r+1}(L(\beta)-C_F)$. By (a5.24), $0<k^N<X(C_F)-\beta$ and so $L(\beta)>C_F$. Therefore, we have

$$\mu-(C_N+\beta)>\frac{r}{r+1}(L(\beta)-C_F)>0, \tag{a5.28}$$

and $Z\big(X(C_F)-\beta\big)<C_N+\beta=Z(k_w)$. Therefore, $X(C_F)-\beta>k_w$,

i.e., $C_F<L(\beta+K_w)$. (a5.29)

By Theorems 5.1 and 5.2, the individual expected profit of flexible and in-flexible firms are

$\Pi_e^F=\frac{1}{(r+1)^2}\Big(\int_{\beta+sk_e^N}^{X(C_F)}(\alpha-sk_e^N-\beta)^2 f(\alpha)d\alpha+\int_{X(C_F)}^\infty(X(C_F)-sk_e^N-\beta)^2 f(\alpha)d\alpha\Big)$;

$\Pi_e^N=(k_e^N)^2\overline{F}(sk_e^N)$, respectively. Therefore, the solution of Case-D is

$k^N=sk_e^N$; k_e^N satisfies $C_N+\beta=\int_{sk_e^N}^\infty(\alpha-(s+1)k_e^N)f(\alpha)d\alpha-\frac{r}{r+1}(L(\beta+sk_e^N)-C_F)$;

$k^F=rk_e^F$; $k_e^F=\frac{1}{r+1}(X(C_F)-\beta-sk_e^N)$; A necessary condition for Case-D is: $C_F<L(\beta+k_w)$ and $\mu-(C_N+\beta)>\frac{r}{r+1}(L(\beta)-C_F)$.

This completes the proof of Proposition 5.9. □

Theorem 5.3 Given $r>0$ flexible firms and $s>0$ in-flexible firms, within the area $R=\{(C_N,C_F):0<C_N\le C_F<\infty\}$. At equilibrium, $k^F=rk_e^F$, $k^N=sk_e^N$, where k_e^F and k_e^N together with Π_e^F and Π_e^N in different regions of R are:

(Case-A) if $\mu-\beta\le C_N$ & $L(\beta)\le C_F$ & $C_N\le C_F$, then $k_e^F=0$, $k_e^N=0$, $\Pi_e^F=0$, $\Pi_e^N=0$;

(Case-B) if $C_N<\mu-\beta$ & $L(\beta+K_w)\le C_F$ & $C_N\le C_F$, then $k_e^F=0$,

k_e^N satisfies $C_N + \beta = \int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha) d\alpha$, $\Pi_e^F = 0$,

$\Pi_e^N = (k_e^N)^2 \overline{F}(sk_e^N)$;

(Case-C) if $\mu - (C_N + \beta) \le \frac{r}{r+1}(L(\beta) - C_F)$ & $L(\beta) > C_F$ & $C_N \le C_F$, then

$k_e^F = \frac{X(C_F) - \beta}{r+1}$, $k_e^N = 0$, $\Pi_e^N = 0$,

$\Pi_e^F = \frac{1}{(r+1)^2}\left(\int_{\beta}^{X(C_F)} (\alpha - \beta)^2 f(\alpha) d\alpha + \int_{X(C_F)}^{\infty} (X(C_F) - \beta)^2 f(\alpha) d\alpha\right)$;

(Case-D) if $L(\beta + k_w) > C_F$ & $\mu - (C_N + \beta) > \frac{r}{r+1}(L(\beta) - C_F)$ & $C_N \le C_F$,

then $k_e^F = \frac{1}{r+1}(X(C_F) - \beta - sk_e^N)$, k_e^N satisfies

$C_N + \beta = \int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha) d\alpha - \frac{r}{r+1}(L(\beta + sk_e^N) - C_F)$,

$\Pi_e^F = \frac{1}{(r+1)^2}\left(\int_{\beta + sk_e^N}^{X(C_F)} (\alpha - sk_e^N - \beta)^2 f(\alpha) d\alpha + \int_{X(C_F)}^{\infty} (X(C_F) - sk_e^N - \beta)^2 f(\alpha) d\alpha\right)$,

$\Pi_e^N = (k_e^N)^2 \overline{F}(sk_e^N)$.

Proof

From Propositions 5.6 - 5.9, it is noted that in Case D, if $k^N = 0$, then $k^F = \frac{r}{r+1}(X(C_F) - \beta)$, which is the same as Case-C; If $k^F = 0$, then k^N satisfies $C_N + \beta = \int_{k^N}^{\infty} (\alpha - \frac{s+1}{s} k^N) f(\alpha) d\alpha$, which is the same as Case-B.

It is easy to check that any two of the necessary conditions for Case-A to Case-D do not overlap, except Case-B and Case-C pair. In the following, we will show that even for the Case-B and Case-C pair, their necessary conditions do not overlap.

The necessary condition for Case-B is: $L(\beta + k_w) \le C_F$ and $C_N < \mu - \beta$.

The necessary condition for Case-C is: $L(\beta) > C_F$ and $\mu - (C_N + \beta) \le \frac{r}{r+1}(L(\beta) - C_F)$.

Suppose that they overlap, i.e., there exists (C_N, C_F) such that the above conditions hold. Then, $L(\beta + k_w) \le C_F \le \frac{r+1}{r} C_N - \frac{r+1}{r}(\mu - \beta) + L(\beta)$.

Let $C_{F1} = L(\beta + k_w)$ and $C_{F2} = \frac{r+1}{r} C_N - \frac{r+1}{r}(\mu - \beta) + L(\beta)$ be two functions of $C_N \in [0, \mu - \beta]$. The curves of them are called Curve-1 and Curve-2, respectively.

Recall that $\int_{k_w}^{\infty} (\alpha - \frac{s+1}{s} k_w) f(\alpha) d\alpha = C_N + \beta$. We have $\frac{dC_{F1}}{dC_N} = \frac{dC_{F1}}{dk_w} \cdot \frac{dk_w}{dC_N}$; $\frac{dC_{F1}}{dk_w} = -\overline{F}(\beta + k_w) < 0$; $\frac{dC_N}{dk_w} = -\frac{1}{s}[(s+1)\overline{F}(k_w) - k_w f(k_w)]$. So $\frac{dC_{F1}}{dC_N} = \frac{s\overline{F}(\beta + k_w)}{(s+1)\overline{F}(k_w) - k_w f(k_w)}$. Therefore, under the assumption that $\overline{F}(x) - xf(x) > 0$, we have $0 < \frac{dC_{F1}}{dC_N} = \frac{s\overline{F}(\beta + k_w)}{(s+1)\overline{F}(k_w) - k_w f(k_w)} < \frac{\overline{F}(\beta + k_w)}{\overline{F}(k_w)} \le 1$. Thus, C_{F1} is increasing in C_N, and its slope is strictly bounded above by 1.

On the other hand, $\frac{dC_{F2}}{dC_N} = \frac{r+1}{r} > 1$. Therefore, C_{F2} is also increasing in C_N, but strictly bounded below by 1.

Let $\Delta = C_{F1} - C_{F2}$ for $C_N \in [0, \mu - \beta]$. Then, $\frac{d\Delta}{dC_N} = \frac{dC_{F1}}{dC_N} - \frac{dC_{F2}}{dC_N} < 1 - 1 = 0$. Thus, Δ is decreasing in C_N. When $C_N = \mu - \beta$, $\Delta = L(\beta + k_w) - L(\beta)$. Since $\int_{k_w}^{\infty} (\alpha - \frac{s+1}{s} k_w) f(\alpha) d\alpha = \mu - \beta + \beta = \mu$, we have $k_w = 0$. So, $\Delta = 0$. Therefore, for all $C_N \in [0, \mu - \beta)$, $\Delta = C_{F1} - C_{F2} > 0$. This is a contradiction. Hence, even for the Case-B and Case-C pair, their necessary conditions do not overlap.

Therefore, any two of the necessary conditions for Case-A to Case-D do not overlap. This implies that the four conditions are necessary and sufficient conditions for Case-A to Case-D, respectively. They partition the region $R = \{(C_N, C_F) : 0 < C_N \le C_F < \infty\}$ into four parts. Hence, given *r* flexible firms and *s* in-flexible firms, we have the following conclusions on equilibrium within R:

(I) Case-A occurs if and only if $L(\beta) \le C_F$ and $\mu - \beta \le C_N$;

(II) Case-B occurs if and only if $L(\beta + k_w) \le C_F$ and $C_N < \mu - \beta$,;

(III) Case-C occurs if and only if $L(\beta) > C_F$ and $\mu - (C_N + \beta) \le \frac{r}{r+1}(L(\beta) - C_F)$;

(IV) Case-D occurs if and only if $C_F < L(\beta + k_w)$

and $\mu - (C_N + \beta) > \frac{r}{r+1}(L(\beta) - C_F)$.

This completes the proof of Theorem 5.3. □

Proposition 5.10 Given $r > 0$ flexible firms and $s > 0$ in-flexible firms, within the area $R = \{(C_N, C_F) : 0 < C_N \le C_F < \infty\}$.

(i) The boundary between Case-B and Case-D is $C_{F1} = L(\beta + k_w)$, which is defined as Curve-1;

(ii) the boundary between Case-C and Case-D is $C_{F2} = \frac{r+1}{r} C_N - \frac{r+1}{r}(\mu - \beta) + L(\beta)$, which is defined as Curve-2;

(iii) under the assumption $\overline{F}(x) - xf(x) > 0$, in both Curve-1 and Curve-2, C_F is strictly increasing in C_N , and within $C_N \in [0, \mu - \beta]$, Curve-1 is always above Curve-2, except that they intersect at $(\mu - \beta, L(\beta))$;

(iv) further, in Curve-1 C_{F1} decreases in s with given C_N ; in Curve-2 C_N decreases in r with given C_{F2} .

Proof

Comparing Case-B and Case-D, we notice that the boundary of Case-B and Case-D is the curve $C_F = L(\beta + k_w)$, which is Curve-1 in the above discussion. Similarly, comparing Case-D and Case-C, we obtain the boundary of Case-C and Case-D as the curve $C_F = \frac{r+1}{r} C_N - \frac{r+1}{r}(\mu - \beta) + L(\beta)$, which is Curve-2 in the above discussion. Thus, all the results for these two curves are still valid. In particular, within $C_N \in [0, \mu - \beta]$, Curver-1 is always above Curve-2, except that they intersect at $(C_N, C_F) = (\mu - \beta,\ L(\beta))$.

For Curv-1 $C_{F1} = L(\beta + k_w)$, where k_w is defined as the one satisfying $C_N + \beta = \int_{k_w}^{\infty} (\alpha - \frac{s+1}{s} k_w) f(\alpha) d\alpha$. $\frac{dC_{F1}}{ds} = \frac{dC_{F1}}{dK_w} \cdot \frac{dk_w}{ds}$. Given C_N, we have $\frac{dC_{F1}}{dk_w} = -\overline{F}(\beta + K_w) < 0$; $\frac{dk_w}{ds} = \frac{1}{s} \cdot \frac{k_w \overline{F}(k_w)}{(s+1)\overline{F}(k_w) - k_w \overline{F}(k_w)} > 0$. So we have $\frac{dC_{F1}}{ds} < 0$ for given C_N.

For Curve-2 $C_{F2} = \frac{r+1}{r} C_N - \frac{r+1}{r}(\mu - \beta) + L(\beta)$, given C_{F2} , we have $\frac{dC_N}{dr} = -\frac{\mu - C_N - \beta}{r(r+1)} < 0$.

This completes the proof of Proposition 5.10. □

Property 5.1 Given $r > 0$ flexible firms and $s > 0$ in-flexible firms, relationship between capacity costs and expected profits can be presented as follows:

(i) Flexible strategy is only effective in Region-D and Region-C; while in-flexible strategy is only effective in Region-D and Region-B.

(ii) In Region-B, $\Pi_e^F = 0$ and $\Pi_e^N > 0$,

(ii-1) Given C_F, $\frac{d\Pi_e^N}{dC_N} < 0$; (ii-2) Given C_N, $\frac{d\Pi_e^N}{dC_F} = 0$.

(iii) In Region-C, $\Pi_e^F > 0$ and $\Pi_e^N = 0$,

(iii-1) Given C_F, $\frac{d\Pi_e^F}{dC_N} = 0$; (iii-2) Given C_N, $\frac{d\Pi_e^F}{dC_F} < 0$.

(iv) In Region-D, $\Pi_e^F > 0$ and $\Pi_e^N > 0$,

(iv-1) Given C_F, $\frac{d\Pi_e^N}{dC_N} < 0$; $\frac{d\Pi_e^F}{dC_N} > 0$;(iv-2) Given C_N, $\frac{d\Pi_e^F}{dC_F} < 0$; $\frac{d\Pi_e^N}{dC_F} > 0$.

<u>Proof</u>

By Theorem 5.3, in Region-A and Region-B, at equilibrium, the capacity of each flexible firm is zero. Consequently the individual profit of each flexible firm is zero, regardless the decisions of other firms. Therefore, only in Region-C and Region-D, the flexible strategy leads to a positive profit. Similarly, we conclude that in-flexible strategy is only effective in Region-B and Region-D, in which in-flexible strategy results in a positive profit. In the other two regions, in-flexible strategy leads to zero profits.

In Region-B, by Theorem 5.3, we have $\Pi_e^F = 0$ and $\Pi_e^N = \frac{1}{s^2}(k^N)^2 \overline{F}(k^N)$.

k^N satisfies $C_N + \beta = \int_{k^N}^{\infty}(\alpha - \frac{s+1}{s}k^N) f(\alpha) d\alpha$. With respect to C_N, we have $1 = -\left(\frac{s+1}{s}\overline{F}(k^N) - \frac{1}{s}k^N f(k^N)\right)\cdot\frac{dk^N}{dC_N}$, so that $\frac{dk^N}{dC_N} < 0$. Therefore, given C_F, with respect to C_N, we have $\frac{d\Pi_e^N}{dC_N} = \frac{k^N}{s^2}(2\overline{F}(k^N) - k^N f(k^N))\cdot\frac{dk^N}{dC_N} < 0$. Given C_N, with respect to C_F, we have $\frac{d\Pi_e^N}{dC_F} = 0$.

In Region-C, by Theorem 5.3, we have $\Pi_e^N = 0$ and

$$\Pi_e^F = \frac{1}{(r+1)^2}\left(\int_{\beta}^{X(C_F)}(\alpha-\beta)^2 f(\alpha)d\alpha + \int_{X(C_F)}^{\infty}(X(C_F)-\beta)^2 f(\alpha)d\alpha\right).$$

Given C_N, with respect to C_F, we have $\frac{d\Pi_e^F}{dC_F} = -\frac{2(X(C_F)-\beta)}{(r+1)^2} < 0$. Given C_F, with respect to C_N, we have $\frac{d\Pi_e^F}{dC_N} = 0$.

In Region-D, by Theorem 5.3, we have

Sub-Case-1 Given C_N, with respect to C_F, by Theorem 5.3, we have

$$\frac{d\Pi_e^F}{dC_F} = \frac{\partial\Pi_e^F}{\partial X(C_F)}\cdot\frac{dX(C_F)}{dC_F} + \frac{\partial\Pi_e^F}{\partial K^N}\cdot\frac{dk^N}{dC_F}$$
$$= -\frac{2(X(C_F)-k^N-\beta)}{(r+1)^2} - \frac{2(L(k^N+\beta)-C_F)}{(r+1)^2}\cdot\frac{dk^N}{dC_F}. \quad \text{(a5.30)}$$

$$\frac{d\Pi_e^N}{dC_F} = \frac{k^N}{s^2}(2\overline{F}(k^N) - k^N f(k^N))\cdot\frac{dk^N}{dC_F}. \quad \text{(a5.31)}$$

By Theorem 5.3,

$$C_N + \beta = \int_{k^N}^{\infty}\left(\alpha - \frac{s+1}{s}k^N\right)f(\alpha)d\alpha - \frac{r}{r+1}\left(L(\beta+k^N) - C_F\right). \quad \text{(a5.32)}$$

With respect to C_F, we have

$-\left(\frac{1}{s}(\overline{F}(k^N)-k^N f(k^N))+\overline{F}(k^N)-\frac{r}{r+1}\overline{F}(k^N+\beta)\right)\cdot\frac{dk^N}{dC_F}+\frac{r}{r+1}=0$, which leads to $\frac{dk^N}{dC_F}>0$. Together with (a5.30) and (a5.31), we have $\frac{d\Pi_e^F}{dC_F}<0$ and $\frac{d\Pi_e^N}{dC_F}>0$.

Sub-Case-2 Given C_F, with respect to C_N, by Theorem 5.3, we have

$$\frac{d\Pi_e^F}{dC_N}=\frac{\partial\Pi_e^F}{\partial K^N}\cdot\frac{dk^N}{dC_N}=-\frac{2(L(k^N+\beta)-C_F)}{(r+1)^2}\cdot\frac{dk^N}{dC_N}. \tag{a5.33}$$

$$\frac{d\Pi_e^N}{dC_N}=\frac{k^N}{s^2}(2\overline{F}(k^N)-k^N f(k^N))\cdot\frac{dk^N}{dC_N}. \tag{a5.34}$$

By (a5.32), with respect to C_N, we have

$-\left(\frac{1}{s}(\overline{F}(k^N)-k^N f(k^N))+\overline{F}(k^N)-\frac{r}{r+1}\overline{F}(k^N+\beta)\right)\cdot\frac{dk^N}{dC_N}=1$, which leads to $\frac{dk^N}{dC_N}<0$.Together with (a5.33) and (a5.34), we have $\frac{d\Pi_e^F}{dC_N}>0$ and $\frac{d\Pi_e^N}{dC_N}<0$.

This completes the proof of Property 5.1. □

Property 5.2 Given $r>0$ flexible firms, $s>0$ in-flexible firms and capacity costs $(C_N,\ C_F)$, the effects of production cost on each firm's expected profit is:

(i) In Region-A, no strategy is effective;

(ii) In Region-B, only in-flexible strategy is effective, and $\frac{d\Pi_e^N}{d\beta}<0$;

(iii) In Region-C, only flexible strategy is effective, and $\frac{d\Pi_e^F}{d\beta}<0$;

(iv) In Region-D, both flexible and in-flexible strategies are effective, $\frac{d\Pi_e^N}{d\beta}<0$ and

(1) if $\frac{(s+1)\overline{F}(k^N)-k^N f(k^N)}{s}<1$, then $\frac{d\Pi_e^F}{d\beta}>0$;

(2) if $\frac{(s+1)\overline{F}(k^N)-k^N f(k^N)}{s}>1$, then $\frac{d\Pi_e^F}{d\beta}<0$.

<u>Proof</u>

Part (i) can be obtained directly by Theorem 5.3.

Region-B By Theorem 5.3, $\Pi_e^N = \frac{1}{s^2}(k^N)^2 \overline{F}(k^N)$, where k^N satisfies $C_N + \beta = \int_{k^N}^{\infty} (\alpha - \frac{s+1}{s} k^N) f(\alpha) d\alpha$. With respect to β, we have $\frac{dk^N}{d\beta} = -\frac{s}{(s+1)\overline{F}(k^N) - k^N f(k^N)} < 0$.

Therefore, by Theorem 5.3, we have $\frac{d\Pi_e^N}{d\beta} = \frac{1}{s^2} k^N [2\overline{F}(k^N) - k^N f(k^N)] \cdot \frac{dk^N}{d\beta} < 0$.

Region-C

By Theorem 5.3,

$\Pi_e^F = \frac{1}{(r+1)^2}\left(\int_{\beta}^{X(C_F)} (\alpha - \beta)^2 f(\alpha) d\alpha + \int_{X(C_F)}^{\infty} (X(C_F) - \beta)^2 f(\alpha) d\alpha \right)$. With respect to β, we have $\frac{d\Pi_e^F}{d\beta} = -\frac{2}{(r+1)^2}[L(\beta) - C_F] < 0$.

Region-D By Theorem 5.3,

$$\Pi_e^F = \frac{1}{(r+1)^2}\left(\int_{\beta + k^N}^{X(C_F)} (\alpha - k^N - \beta)^2 f(\alpha) d\alpha + \int_{X(C_F)}^{\infty} (X(C_F) - k^N - \beta)^2 f(\alpha) d\alpha \right),$$

$\Pi_e^N = \frac{1}{s^2}(k^N)^2 \overline{F}(k^N)$, where k^N satisfies

$C_N + \beta = \int_{k^N}^{\infty} (\alpha - \frac{s+1}{s} k^N) f(\alpha) d\alpha - \frac{r}{r+1}(L(\beta + k^N) - C_F)$. With respect to β, we have $\frac{dk^N}{d\beta} = -\frac{1 - \frac{r}{r+1}\overline{F}(k^N + \beta)}{\overline{F}(k^N) - \frac{r}{r+1}\overline{F}(k^N + \beta) + \frac{\overline{F}(k^N) - k^N f(k^N)}{s}} < 0$. Then we have

$$\frac{d\Pi_e^N}{d\beta} = \frac{1}{s^2} k^N [2\overline{F}(k^N) - k^N f(k^N)] \cdot \frac{dk^N}{d\beta} < 0;$$

$$\frac{d\Pi_e^F}{d\beta} = -\frac{2}{(r+1)^2}[L(k^N + \beta) - C_F] \cdot (1 + \frac{dk^N}{d\beta}).$$

$$\frac{d\Pi_e^F}{d\beta} = -\frac{2}{(r+1)^2}[L(k^N + \beta) - C_F] \cdot \frac{\left(\frac{(s+1)\overline{F}(k^N) - k^N f(k^N)}{s} - 1 \right)}{\overline{F}(k^N) - \frac{r}{r+1}\overline{F}(k^N + \beta) + \frac{\overline{F}(k^N) - k^N f(k^N)}{s}}.$$

Therefore, we have (1) if $\frac{(s+1)\overline{F}(k^N) - k^N f(k^N)}{s} < 1$, then $\frac{d\Pi_e^F}{d\beta} > 0$;

(2) if $\frac{(s+1)\overline{F}(k^N)-k^N f(k^N)}{s}>1$, then $\frac{d\Pi_e^F}{d\beta}<0$.

This completes the proof of Property 5.2. □

Property 5.3 Given *n* firms and capacity costs $(C_N,\ C_F)$ in a strategy-coexistence market consisting of $r\geq 1$ flexible firms and $s\geq 1$ in-flexible firms where $r+s=n$. Within the range $r\in[1,\ n-1]$, we have

(i) total capacity of in-flexible firms is decreasing in *r*, i.e., $\frac{dk^N}{dr}<0$;

(ii) total capacity of flexible firms is increasing in *r*, i.e., $\frac{dk^F}{dr}>0$.

Proof

For given $(C_N,\ C_F)$ in a strategy coexisting market consisting of $r\geq 1$ flexible firms and $s\geq 1$ in-flexible firms, where $r+s=n$. By Theorem 5.3, we have $\begin{cases} k^F = \frac{r}{r+1}(X(C_F)-\beta-k^N)>0 \\ k^N>0 \end{cases}$, where k^N satisfies

$$C_N+\beta=\int_{k^N}^{\infty}(\alpha-\frac{s+1}{s}k^N)f(\alpha)d\alpha-\frac{r}{r+1}(L(\beta+k^N)-C_F). \qquad \text{(a5.35)}$$

$$C_N\leq C_F<L(\beta+k_w)<L(\beta) \text{ and } \mu-(C_N+\beta)>\frac{r}{r+1}(L(\beta)-C_F). \qquad \text{(a5.36)}$$

With respect to r, $n=(r,\ n-r)$, $s=n-r$, by Theorem 5.3, we have

$$\Pi_e^F(r,n-r)=\frac{1}{(r+1)^2}\left(\int_{\beta+k^N}^{X(C_F)}(\alpha-k^N-\beta)^2 f(\alpha)d\alpha+\int_{X(C_F)}^{\infty}(X(C_F)-k^N-\beta)^2 f(\alpha)d\alpha\right)$$

$\Pi_e^N(r,n-r)=\frac{1}{(n-r)^2}(k^N)^2\overline{F}(k^N)$, where k^N is a function of *r*.

By (a5.35) we have $C_N+\beta=\int_{k^N}^{\infty}(\alpha-\frac{n-r+1}{n-r}k^N)f(\alpha)d\alpha-\frac{r}{r+1}(L(\beta+k^N)-C_F)$.

With respect to *r*, for given $C_N,\ C_F,\ \beta,\ n$, we have

$$-\left(\frac{1}{n-r}(\overline{F}(k^N)-k^N f(k^N))+\frac{1}{r+1}((r+1)\overline{F}(k^N)-r\overline{F}(\beta+k^N))\right)\frac{dk^N}{dr}$$

$= \frac{1}{(n-r)^2} k^N \overline{F}(k^N) + \frac{1}{(r+1)^2}(L(\beta + k^N) - C_F)$. Therefore, we have

$$\frac{dk^N}{dr}$$

$$= -\left(\frac{(r+1)^2 k^N \overline{F}(k^N) + (n-r)^2 (L(\beta + k^N) - C_F)}{(n-r)(r+1)^2(\overline{F}(k^N) - k^N f(k^N)) + (n-r)^2 (r+1)((r+1)\overline{F}(k^N) - r\overline{F}(\beta + k^N))} \right)$$

< 0. Therefore, k^N is decreasing in *r*, for given C_N, C_F, β, n. Note that $\frac{(r+1)}{r} k^F + k^N = X(C_F) - \beta$. We have $\frac{(r+1)}{r} \cdot \frac{dk^F}{dr} = \frac{k^F}{r^2} - \frac{dk^N}{dr} > 0$, so that $\frac{dk^F}{dr} = \frac{k^F}{(r+1)r} - \frac{r}{(r+1)} \cdot \frac{dk^N}{dr} > 0$. Therefore, k^F is increasing in *r*.

This completes the proof of Property 5.3. □

Property 5.4 Given (C_N, C_F), the total capacity of all firms k^T is bounded under various situations:

(i) If (C_N, C_F) is in Region-A, then $k^T = 0$.

(ii) If (C_N, C_F) is in Region-B, then k^T is decreasing in C_N, and independent of C_F, furthermore, $X(C_F) - \beta \le k^T = k^N < X(C_N + \beta)$.

(iii) If (C_N, C_F) is in Region-C, then K^T is decreasing in C_F, and independent of C_N; furthermore, $\frac{1}{2}(X(C_F) - \beta) < k^T = k^F = \frac{r}{r+1}(X(C_F) - \beta)$.

(iv) If (C_N, C_F) is in Region-D, then $\frac{r}{r+1}(X(C_F) - \beta) < k^T < X(C_F) - \beta$.

Proof

Region-A By Theorem 5.3, in Case-A, capacity of all firms is zero, i.e., $k^T = 0$.

Region -B By Theorem 5.3, in Case-B total capacity of all firms equals to that of all in-flexible firms, i.e., $k^T = k^N > 0$, which satisfies $C_N + \beta = \int_{k^N}^{\infty} (\alpha - \frac{s+1}{s} k^N) f(\alpha) d\alpha$.

With respect to C_N, $\frac{dk^N}{dC_N} = -\frac{s}{s\overline{F}(k^N) + (\overline{F}(k^N) - k^N f(k^N))}$. With the assumption of $\overline{F}(x) > xf(x)$, we have $\frac{dk^N}{dC_N} < 0$. Therefore, total capacity in Case-B is decreasing in

C_N , and independent on C_F . Furthermore, by condition of Case-B, we have $X(C_F)-\beta<k^N$. Therefore, $X(C_F)-\beta<k^T=k^N<X(C_N+\beta)$.

Region -C By Theorem 5.3, in Case-C, total capacity of all firms equals to that of all flexible firms, i.e., $k^T=k^F=\frac{r}{r+1}(X(C_F)-\beta)$. With respect to C_F , $\frac{dk^F}{dC_F}=-\frac{r}{(r+1)\overline{F}(X(C_F))}<0$. Therefore, total capacity in Case-C is decreasing in C_F, and independent on C_N . Moreover, since $r\geq 1$, we have $\frac{1}{2}(X(C_F)-\beta)<k^T=k^F=\frac{r}{r+1}(X(C_F)-\beta)$.

Region -D By Theorem 5.3, in Case-D, total capacity of in-flexible firms is $k^N>0$, which satisfies $C_N+\beta=\int_{k^N}^{\infty}(\alpha-\frac{s+1}{s}k^N)f(\alpha)d\alpha-\frac{r}{r+1}(L(\beta+k^N)-C_F)$; total capacity of flexible firms is $k^F=\frac{r}{r+1}(X(C_F)-\beta-k^N)>0$. So that we have total capacity of all firms is $k^T=k^F+k^N=X(C_F)-\beta-\frac{1}{r}k^F$. Therefore, we have $k^F=r(X(C_F)-\beta-k^T)$ and $k^N=(r+1)k^T-r(X(C_F)-\beta)$.

Since $k^N>0$ and $k^F>0$, we can get lower and upper boundaries of total capacity of all firms in Case-D, i.e., $\frac{r}{(r+1)}(X(C_F)-\beta)<k^T<(X(C_F)-\beta)$.

This completes the proof of Property 5.4. □

Proposition 5.11 Given $r>0$ flexible firms and $s>0$ in-flexible firms, within Region-D, between Curve-1 and Curve-2, there exists a unique Curve-3 satisfying $\Pi_e^F(C_N,C_F)=\Pi_e^N(C_N,C_F)$; in Curve-3, C_F increases with C_N; in the area above Curve-3, denoted as Region-D1, $\Pi_e^N>\Pi_e^F>0$; and in the area below Curve-3, denoted as Region-D2, $\Pi_e^F>\Pi_e^N>0$. □

Proof

By Theorem 5.3, in Case-D we have

$$C_N+\beta=\int_{k^N}^{\infty}(\alpha-\frac{s+1}{s}k^N)f(\alpha)d\alpha-\frac{r}{r+1}(L(\beta+k^N)-C_F); \tag{a5.37}$$

$$\Pi_e^F = \frac{1}{(r+1)^2}\left(\int_{\beta+k^N}^{X(C_F)} (\alpha - k^N - \beta)^2 f(\alpha)d\alpha + \int_{X(C_F)}^{\infty} (X(C_F) - k^N - \beta)^2 f(\alpha)d\alpha\right);$$

$\Pi_e^N = \frac{1}{s^2}(k^N)^2 \overline{F}(k^N)$. Note that $0 \le C_N < \mu - \beta$ in Case-D.

Let $M_e(C_N, C_F) = \Pi_e^F - \Pi_e^N$. Given $C_N \in [0, \mu - \beta)$, by Property 5.1 (iv-2), we have $\frac{d\Pi_e^F}{dC_F} < 0$, $\frac{d\Pi_e^N}{dC_F} > 0$, and so $\frac{dM_e}{dC_F} = \frac{d\Pi_e^F}{dC_F} - \frac{d\Pi_e^N}{dC_F} < 0$. Therefore, for each given C_N, M_e is decreasing in C_F. Recall that Curve-1: $C_{F1} = L(\beta + k_w)$,

$$\text{Curve-2: } C_{F2} = \frac{r+1}{r} C_N - \frac{r+1}{r}(\mu - \beta) + L(\beta), \tag{a5.38}$$

where $C_N + \beta = \int_{k_w}^{\infty} (\alpha - \frac{s+1}{s} k_w) f(\alpha) d\alpha$. When $C_F = C_{F1}$, by Property 5.1 (ii), we have $\Pi_e^F = 0$ and $\Pi_e^N > 0$. Therefore, $M_e(C_N, C_{F1}) < 0$. When $C_F = C_{F2} < C_{F1}$, clearly $\Pi_e^F > 0$. (Otherwise, $\Pi_e^F = 0$ implies $X(C_{F2}) = k^N + \beta$, i.e., $C_{F2} = L(k^N + \beta)$. Thus, $C_N + \beta = \int_{k^N}^{\infty} (\alpha - \frac{s+1}{s} k^N) f(\alpha) d\alpha$ and $k^N = k_w$, so $C_{F2} = C_{F1}$, contradiction.) On the other hand, by (5.41) and (5.42) $\mu = \int_{k^N}^{\infty} (\alpha - \frac{s+1}{s} k^N) f(\alpha) d\alpha - \frac{r}{r+1}(L(\beta + k^N) - L(\beta))$.

Let the right hand side be $V(k^N)$, $k^N \ge 0$. Then,

$V^{(1)}(k^N) = -\frac{1}{s}(\overline{F}(k^N) - k^N f(k^N)) - (\overline{F}(k^N) - \frac{r}{r+1}\overline{F}(\beta + k^N)) < 0$ implies that $V(k^N)$ is strictly decreasing. Since $V(0) = \mu$, we have $K^N = 0$. Therefore, $\Pi_e^N = 0$. Thus, $M_e(C_N, C_F) = \Pi_e^F - \Pi_e^N > 0$. Hence, when $C_N \in [0, \mu - \beta)$ is given, there exists a unique $C_F \in (C_{F2}, C_{F1})$ such that $M_e(C_N, C_F) = 0$, i.e., $\Pi_e^F = \Pi_e^N$. Clearly, when $C_N = \mu - \beta$, we can take $C_F = C_{F2} = C_{F1} = L(\beta)$ and $\Pi_e^F = \Pi_e^N$. Thus, we obtain C_F as a function of C_N so that $M_e(C_N, C_F) = 0$, $C_N \in [0, \mu - \beta]$.

Differentiating both sides w.r.t. C_N, we have $\frac{d\Pi_e^F}{dC_N} = \frac{d\Pi_e^N}{dC_N}$.

$$\frac{d\Pi_e^F}{dC_N} = -\frac{2(X(C_F)-k^N-\beta)}{(r+1)^2}\frac{dC_F}{dC_N} - \frac{2(L(k^N+\beta)-C_F)}{(r+1)^2}\cdot\frac{dk^N}{dC_N}$$
$$= -\frac{2A_1}{(r+1)^2}\frac{dC_F}{dC_N} - \frac{2A}{(r+1)^2}\cdot\frac{dk^N}{dC_N}.$$

$$\frac{d\Pi_e^N}{dC_N} = \frac{k^N}{s^2}(2\overline{F}(k^N)-k^N f(k^N))\cdot\frac{dk^N}{dC_N} = \frac{B}{s^2}\cdot\frac{dk^N}{dC_N}$$, where

$A_1 = X(C_F)-k^N-\beta > 0$, $A = L(k^N+\beta)-C_F > 0$,

$B = k^N(2\overline{F}(k^N)-k^N f(k^N)) > 0$.

So, $-\frac{2A_1}{(r+1)^2}\frac{dC_F}{dC_N} = \left(\frac{B}{s^2}+\frac{2A}{(r+1)^2}\right)\cdot\frac{dk^N}{dC_N}$ (a5.39)

By (a5.38), differentiating with respect to C_N,

we have $1 = -B_1\frac{dk^N}{dC_N} + \frac{r}{r+1}\frac{dC_F}{dC_N}$, (a5.40)

where $B_1 = \frac{1}{s}(\overline{F}(k^N)-k^N f(k^N)) + \overline{F}(k^N) - \frac{r}{r+1}\overline{F}(k^N+\beta) > 0$. Therefore, using (a5.38) and (a5.40), $\frac{dC_F}{dC_N} = \frac{\frac{B}{s^2}+\frac{2A}{(r+1)^2}}{\frac{2A_1B_1}{(r+1)^2}+\frac{r}{r+1}(\frac{B}{s^2}+\frac{2A}{(r+1)^2})} > 0$. This shows that Curve-3 is an increasing function of C_N.

Furthermore, since for each given C_N, M_e is decreasing in C_F, we obtain that in the area above Curve-3, $\Pi_e^N > \Pi_e^F > 0$, and in the area below Curve-3, $\Pi_e^F > \Pi_e^N > 0$.

This completes the proof of Proposition 5.11. □

Proofs in Chapter 6

Proposition 6.1 Referring to Curve-1, given total *n* firms and C_N, C_F is decreasing in *s*; referring to Curve-2, given total *n* firms and C_F, C_N is decreasing in *r*.

Proof

According to Proposition 5.10, we have $C_{F1}^{(1)}(s) < 0$ for given *n* and C_N.

By definition, Curve-1 is independent of *r*. Therefore, regarding Curve-1, for given *n* and C_N, C_F is decreasing in *s*.

According to Proposition 5.10, given C_{F2}, we have $\frac{dC_N}{dr} = -\frac{\mu - C_N - \beta}{r(r+1)} < 0$. By definition, Curve-2 is independent of *s*. Therefore, for given *n* and C_F, C_N is decreasing in *r*. This completes the proof of Proposition 6.1. □

Proposition 6.2 Consider Curve-1, Curve-2, Curve-4 and Curve-5 defined as follows:

Curve-1 $C_F = L(\beta + k_w)$, where k_w satisfies $\int_{k_w}^{\infty}\left(\alpha - \frac{s+1}{s}k_w\right)f(\alpha)d\alpha = C_N + \beta$;

Curve-2 $C_F = \frac{r+1}{r}C_N - \frac{r+1}{r}(\mu - \beta) + L(\beta)$, i.e., $\mu - (C_N + \beta) = \frac{r}{r+1}\left(L(\beta) - C_F\right)$;

Curve-4 $X(C_F) = X(C_N + \beta) + \beta$; Curve-5 $C_F = C_N - (\mu - \beta) + L(\beta)$.

Then we have following conclusions:

(i) Referring to each of these four curves, C_F is increasing in C_N;

(ii) there is one and only one intersection point for $C_N \in (0, \ \mu - \beta]$. The intersection point is $(C_N, C_F) = \left(\mu - \beta, \ L(\beta)\right)$;

(iii) define C_{F1}, C_{F2}, C_{F4}, C_{F5} to be points on Curve-1, Curve-2, Curve-4, Curve-5, respectively, with given C_N. If $\beta > 0$, then $C_{F1} > C_{F4} > C_{F5} > C_{F2}$ for all $C_N \in (0, \ \mu - \beta)$; and, if $\beta = 0$, then $C_{F1} > C_{F4} = C_{F5} > C_{F2}$ for all $C_N \in (0, \ \mu - \beta)$.

Proof

To distinguish each curve, we define C_{F1}, C_{F2}, C_{F4}, C_{F5} to be points on Curve-1, Curve-2, Curve-4, Curve-5, respectively, with given C_N. Then we have following discussion in terms of the slope of each curve.

Curve-1 $C_{F1} = L(\beta + k_w)$ and $\int_{k_w}^{\infty}(\alpha - \frac{s+1}{s}k_w)f(\alpha)d\alpha = C_N + \beta$. (a6.1)

With respect to C_N, we have $\frac{dC_{F1}}{dC_N} = \frac{dC_{F1}}{dk_w} \cdot \frac{dk_w}{dC_N}$; $\frac{dC_{F1}}{dk_w} = -\overline{F}(\beta + k_w) < 0$;

$\frac{dk_w}{dC_N} = -\frac{1}{\frac{s+1}{s}[\overline{F}(k_w) - \frac{1}{s+1}k_w f(k_w)]}$. So $\frac{dC_{F1}}{dC_N} = \frac{\overline{F}(\beta + k_w)}{\frac{s+1}{s}[\overline{F}(k_w) - \frac{1}{s+1}k_w f(k_w)]}$.

Therefore, under the assumption $\overline{F}(x) - xf(x) > 0$, we have $\frac{dC_{F1}}{dC_N} > 0$.

Moreover, $\frac{dC_{F1}}{dC_N} = \frac{\overline{F}(\beta + k_w)}{\frac{s+1}{s}[\overline{F}(k_w) - \frac{1}{s+1}k_w f(k_w)]} < \frac{\overline{F}(\beta + k_w)}{\overline{F}(k_w)} \le 1$. (a6.2)

Curve-2 $C_{F2} = \frac{r+1}{r}C_N - \frac{r+1}{r}(\mu - \beta) + L(\beta)$. So that $\frac{dC_{F2}}{dC_N} = \frac{r+1}{r} > 1$. (a6.3)

Curve-4 $C_{F4} = L(\beta + X(C_N + \beta))$.

Therefore, $\frac{dC_{F4}}{dC_N} = \frac{\overline{F}(\beta + X(C_N + \beta))}{\overline{F}(X(C_N + \beta))} > 0$. (a6.4)

And if $\beta > 0$, then $\frac{dC_{F4}}{dC_N} = \frac{\overline{F}(\beta + X(C_N + \beta))}{\overline{F}(X(C_N + \beta))} < 1$. (a6.5)

If $\beta = 0$, then $\frac{dC_{F4}}{dC_N} = \frac{\overline{F}(\beta + X(C_N + \beta))}{\overline{F}(X(C_N + \beta))} = 1$. (a6.6)

Curve-5 $C_{F5} = C_N - (\mu - \beta) + L(\beta)$. Therefore, $\frac{dC_{F5}}{dC_N} = 1$. (a6.7)

Therefore, we have $\frac{dC_{Fi}}{dC_N} > 0$, $i = 1, 2, 4, 5$, i.e., C_F is increasing in C_N for each curve.

Given $C_N = \mu - \beta$, then $C_{F1} = C_{F2} = C_{F3} = C_{F4} = L(\beta)$. Therefore, four curves intersect at point $(C_N, C_F) = (\mu - \beta, L(\beta))$. By (6.1) we have $L(k_w) > C_N + \beta$, so that $k_w < X(C_N + \beta)$.

Hence, $L(\beta + k_w) > L(\beta + X(C_N + \beta))$, i.e., $C_{F1} > C_{F4}$. (a6.8)

If $\beta > 0$, then by (a6.3), (a6.5) and (a6.7), $\frac{dC_{F4}}{dC_N} < 1$, $\frac{dC_{F5}}{dC_N} = 1$ and $\frac{dC_{F2}}{dC_N} > 1$, we have $\frac{dC_{F4}}{dC_N} < \frac{dC_{F5}}{dC_N} < \frac{dC_{F2}}{dC_N}$. Since these three curves have an intersection at point $(C_N, C_F) = (\mu - \beta,\ L(\beta))$, so with given C_N, $C_N \in [0,\ \mu - \beta)$, we always have

$$C_{F4} > C_{F5} > C_{F2}. \tag{a6.9}$$

By (a6.8) and (a6.9), we have $C_{F1} > C_{F4} > C_{F5} > C_{F2}$ with given C_N, $C_N \in (0,\ \mu - \beta)$.

Similarly, if $\beta = 0$, then by (a6.3), (a6.5) and (a6.6), we have $\frac{dC_{F4}}{dC_N} = \frac{dC_{F5}}{dC_N} < \frac{dC_{F2}}{dC_N}$.

Therefore, $C_{F1} > C_{F4} = C_{F5} > C_{F2}$ with given C_N, $C_N \in (0,\ \mu - \beta)$.

This completes the proof of Proposition 6.2. □

Proposition 6.3 Given $n = (r, s)$ and capacity costs $(C_N,\ C_F)$, we have the following conclusions about the Final Equilibrium:

(i) If $(C_N,\ C_F)$ is in Region-A, then the Final Equilibrium is obtained for any $n = (r, s)$;

(ii) If $(C_N,\ C_F)$ is in Region-B, then the Final Equilibrium is $n = (r, s) = (0, n)$;

(iii) If $(C_N,\ C_F)$ is in Region-C, then the Final Equilibrium is $n = (r, s) = (n, 0)$;

<u>Proof</u>

By Theorem 5.3 we have following four case analyses.

Case-A $\begin{cases} L(\beta) \le C_F \\ \mu - \beta \le C_N \le C_F \end{cases}$, and $\begin{cases} k^F = 0 \\ k^N = 0 \end{cases}$. We always have $\Pi_e^N(r, s) = \Pi_e^F(r, s) = 0$, for any $n = (r, s)$, the status can be stable.

Case-B $\begin{cases} k^F = 0 \\ k^N = sk_e^N > 0 \end{cases}$.

$L(\beta + k_w) \le C_F$, and $C_N < \mu - \beta$. We always have $\Pi_e^N(r, s) > \Pi_e^F(r, s) = 0$ for any $n = (r, s)$. There aren't firms transferring from in-flexible to flexible strategy in the Case-B. Given $(C_N,\ C_F)$, according to Proposition 6.2, as firms transfer from flexible to in-flexible strategies, the new equilibrium point still stays in Region-B as *s* increases. Therefore, all firms will transfer to in-flexible firms, i.e., $n = (r, s) = (0, n)$, as the final equilibrium. Particularly, it is noted that $\Pi_e^N > \Pi_e^F = 0$ also holds in Curve-1, therefore, all firms whose costing parameters are on Curve-1 will transfer to in-flexible firms. That is to say the Curve-1 will stay at $n=(r,s)=(0,n)$, i.e., $C_F = L(\beta + k_w)$ where k_w satisfies $\int_{k_w}^{\infty}(\alpha - \frac{(n+1)}{n}k_w)f(\alpha)d\alpha = C_N + \beta$.

Case-C $\begin{cases} k^F = \frac{r}{r+1}(X(C_F)-\beta) > 0 \\ k^N = 0 \end{cases}$, $k^F = rk_e^F$; $C_N \le C_F < L(\beta)$ and $h(C_F) - \beta \le C_N$, where $h(C_F) = \mu + \frac{r}{r+1}(C_F - L(\beta))$. We always have $\Pi_e^F(r,s) > \Pi_e^N(r,s) = 0$ for any $n=(r,s)$. There aren't firms transferring from flexible to in-flexible. Given $(C_N,\ C_F)$, according to Proposition 6.2, as firms transfer from in-flexible to flexible strategies, the new equilibrium point still stays in Region-C as r increases. Therefore, all firms will transfer to FCS, i.e., $n=(r,s)=(n,0)$, as the final equilibrium. It is noted that $n=(r,s)=(n,0)$ also holds on Curve-2. Therefore, at Final Equilibrium the Curve-2 will stay at $C_F = \frac{n+1}{n}C_N - \frac{n+1}{n}(\mu - \beta) + L(\beta)$.

This completes the proof of Proposition 6.3. □

Theorem 6.1 For all $n \in [1,\ \infty)$, given production cost β, within the area $\{(C_N, C_F): 0 < C_N \le C_F < \infty\}$, the Final Equilibrium can be characterized as below:

(i) in area $\mu - \beta \le C_N$ & $L(\beta) \le C_F$ & $C_N \le C_F$, for any $n=(r,s)$, two strategies lead to zero profit for all $n \in [1,\ \infty)$;

(ii) in area $0 \le C_N < \mu - \beta$ & $L(\beta + k_{w1}) \le C_F$ & $C_N \le C_F$, all firms transfer to in-flexible strategy, i.e., $n=(r,s)=(0,n)$ for all $n \in [1,\ \infty)$;

(iii) in area $\mu - (C_N + \beta) \le \frac{1}{2}(L(\beta) - C_F)$ & $L(\beta) > C_F$ & $C_N \le C_F$, all firms transfer to flexible strategy, i.e., $n=(r,s)=(n,0)$ for all $n \in [1,\ \infty)$;

(iv) in area $L(\beta + k_{w1}) > C_F$ & $\mu - (C_N + \beta) > \frac{1}{2}(L(\beta) - C_F)$ & $C_N \le C_F$, there are three sub-areas as below:

(iv-1) when $L((X(C_F)-\beta)-C_N-\beta>0$, let $\overline{N}=\frac{(X(C_F)-\beta)\overline{F}(X(C_F)-\beta)}{L(X(C_F)-\beta)-C_N-\beta}$:

(iv-1-1) if $n\geq\overline{N}$, then all firms transfer to in-flexible strategies, i.e., $n=(r,s)=(0,n)$;

(iv-1-2) if $n<\overline{N}$, then at Final Equilibrium, both flexible and in-flexible firms coexist in the market;

(iv-2) when $L(X(C_F)-\beta)-C_N-\beta\leq 0$ and $L(\beta)-C_F-(\mu-C_N-\beta)\leq 0$, both flexible and in-flexible firms coexist in the market regardless of number of firms;

(iv-3) when $L(\beta)-C_F-(\mu-C_N-\beta)>0$, let $\tilde{N}=\frac{\mu-C_N-\beta}{L(\beta)-C_F-(\mu-C_N-\beta)}$:

(iv-3-1) if $n\geq\tilde{N}$, then all firms transfer to flexible strategy, i.e., $n=(r,s)=(n,0)$;

(iv-3-2) if $n<\tilde{N}$, then at Final Equilibrium, both flexible and in-flexible firms coexist in the market.

where k_{w1} is the unique solution of the equation $\int_{k_{w1}}^{\infty}(\alpha-2k_{w1})f(\alpha)d\alpha=C_N+\beta$.

<u>Proof</u>

To facilitate the proof, we first give expressions of a few curves.

Curve-1 $C_{F1}^{(0,n)}$ is $C_{F1}^{(0,n)}=L(\beta+\overline{k}_w)$ where $\overline{k}_w$ is the unique solution of the equation $\int_{\overline{k}_w}^{\infty}(\alpha-\frac{n+1}{n}\overline{k}_w)f(\alpha)d\alpha=C_N+\beta$.

Curve-2 $C_{F2}^{(n,0)}$ is $C_{F2}^{(n,0)}=\frac{n+1}{n}C_N-\frac{n+1}{n}(\mu-\beta)+L(\beta)$. Let $n=1$, then $C_{F1}^{(0,1)}=L(\beta+k_{w1})$ where k_{w1} is the unique solution of the equation $\int_{k_{w1}}^{\infty}(\alpha-2k_{w1})f(\alpha)d\alpha=C_N+\beta$; $C_{F2}^{(1,0)}=2C_N-2(\mu-\beta)+L(\beta)$, $C_N\in[0,\ \mu-\beta]$.

Let $n\to\infty$, then $X(C_{F1}^{(0,\infty)})=X(C_N+\beta)+\beta$; $C_{F2}^{(\infty,0)}=C_N-(\mu-\beta)+L(\beta)$. According to Proposition 6.1, referring to Curve-1, C_F is decreasing in *s* with given C_N. Therefore, given C_N, $C_{F1}^{(0,\infty)}<C_{F1}^{(0,n)}<C_{F1}^{(0,i)}<C_{F1}^{(0,1)}$, where $i\in(1,\ n)$ and $n\in[1,\ \infty)$. With regarding to Curve-2, C_N is decreasing in *r* with given C_F. Therefore, given C_F, $C_{N2}^{(\infty,0)}<C_{N2}^{(n,0)}<C_{N2}^{(j,0)}<C_{N2}^{(1,0)}$ where $j\in(1,\ n)$ and $n\in[1,\ \infty)$. Therefore, in areas

above the Curve-1 $C_{F1}^{(0,1)}$ the equilibrium is always Case-B for $C_N \in [0, \ \mu-\beta]$; in areas right of the Curve-2 $C_{F2}^{(1,0)}$, the equilibrium is always Case-C for $C_F \in [0, \ L(\beta)]$. According to Proposition 6.3, Part(i)~(iii) of Theorem 6.1 can be obtained.

In the following discussion, we analyze the conditions of Final Equilibrium occurred within the area between curves $C_{F1}^{(0,1)}$ and $C_{F2}^{(1,0)}$.

Case-B analysis

Given C_N, C_F, β, n, if its final equilibrium occurs as Case-B, then by Theorem 5.3, in Case-B, we have $L(\beta+k_w) \le C_F$, and $C_N < \mu-\beta$, where k_w satisfies $\int_{k_w}^{\infty}(\alpha-\frac{s+1}{s}k_w)f(\alpha)d\alpha = C_N+\beta$. It is noted that at Final Equilibrium, all firms on Curve-1 transfer to in-flexible firms, i.e., $n=(r,s)=(0,n)$. By $L(\beta+k_w)\le C_F$, we have $k_w \ge X(C_F)-\beta$.

Define $H(z)=\int_z^{\infty}(\alpha-\frac{s+1}{s}z)f(\alpha)d\alpha$; and $H^{(1)}(z)=\frac{1}{s}zf(z)-\frac{s+1}{s}\overline{F}(z)<0$. So, $H(z)$ is decreasing in *z*. Therefore, at the Final Equilibrium we have

$C_N+\beta=\int_{k_w}^{\infty}(\alpha-\frac{n+1}{n}k_w)f(\alpha)d\alpha \le \int_{X(C_F)-\beta}^{\infty}[\alpha-\frac{n+1}{n}(X(C_F)-\beta)]f(\alpha)d\alpha$. This is equivalent to $C_N+\beta \le L(X(C_F)-\beta)-\frac{1}{n}(X(C_F)-\beta)\overline{F}(X(C_F)-\beta)$, so that we obtain

$$\frac{1}{n} \le \frac{L(X(C_F)-\beta)-C_N-\beta}{[X(C_F)-\beta]\overline{F}(X(C_F)-\beta)}. \tag{a6.10}$$

Therefore, two situations are discussed.

Situation-(1) If $L(X(C_F)-\beta)-C_N-\beta>0$, then we have

$$n \ge \frac{(X(C_F)-\beta)\overline{F}(X(C_F)-\beta)}{L(X(C_F)-\beta)-C_N-\beta}. \tag{a6.11}$$

By Proposition 6.1, all firms will transfer to in-flexible strategies, i.e., $n=(r,s)=(0, \ n)$.

Situation-(2) If $L(X(C_F)-\beta)-C_N-\beta \le 0$, then there is a contradiction of (a6.10). So Case-B does not exist. Therefore, if final equilibrium for a given C_N, C_F, β, n

occurs as Case-B, it has following properties: (i) $L(X(C_F)-\beta)-C_N-\beta>0$; (ii) $n\ge\frac{(X(C_F)-\beta)\overline{F}(X(C_F)-\beta)}{L(X(C_F)-\beta)-C_N-\beta}$; (iii) $n=(r,s)=(0,\ n)$.

Case-C analysis

Given C_N, C_F, β, n, if its final equilibrium occurs as Case-C, then by Theorem 5.3, in Case-C, we have $C_N\le C_F<L(\beta)$ and $\mu-(C_N+\beta)\le\frac{r}{r+1}(L(\beta)-C_F)$. It is noted that at the Final Equilibrium, all firms on the Curve-2 will transfer to flexible firms, i.e., $n=(r,s)=(n,0)$. Therefore, we have $\mu-(C_N+\beta)\le\frac{n}{n+1}(L(\beta)-C_F)$ so that we have

$$\frac{1}{n}\le\frac{L(\beta)-C_F-(\mu-C_N-\beta)}{\mu-C_N-\beta}. \tag{a6.12}$$

Therefore, we have two situations to discuss.

Situation-(1) If $L(\beta)-C_F-\left(\mu-C_N-\beta\right)>0$, then

$$n\ge\frac{\mu-C_N-\beta}{L(\beta)-C_F-(\mu-C_N-\beta)}. \tag{a6.13}$$

By Proposition 6.3, all firms will transfer to F strategies, i.e., $n=(r,s)=(n,\ 0)$.

Situation-(2) If $L(\beta)-C_F-(\mu-C_N-\beta)\le 0$, then (a6.12) does not hold. Therefore, if final equilibrium for a given C_N, C_F, β, n occurs as Case-C, it has following properties: (i) $L(\beta)-C_F-(\mu-C_N-\beta)>0$; (ii) $n\ge\frac{\mu-C_N-\beta}{L(\beta)-C_F-(\mu-C_N-\beta)}$; (iii) $n=(r,s)=(n,\ 0)$.

Case-D analysis

Given C_N, C_F, β, n, if its final equilibrium occurs as Case-D, then by Theorem 5.3, in Case-D, we have two conditions as $C_N\le C_F<L(\beta+K_w)<L(\beta)$ and $\mu-(C_N+\beta)>\frac{r}{r+1}(L(\beta)-C_F)$. We discuss these two conditions respectively.

Condition-1 $C_N\le C_F<L(\beta+K_w)<L(\beta)$

By $C_F < L(\beta + k_w)$, we have $k_w < X(C_F) - \beta$, where k_w satisfies $C_N + \beta = \int_{k_w}^{\infty}\left(\alpha - \frac{s+1}{s}k_w\right)f(\alpha)d\alpha$. Noting that at the Final Equilibrium, all firms on Curve-1 transfer to in-flexible firms, i.e., $n = (r,s) = (0,\ n)$. Therefore, we have

$C_N + \beta = \int_{k_w}^{\infty}(\alpha - \frac{n+1}{n}k_w)f(\alpha)d\alpha > \int_{X(C_F)-\beta}^{\infty}[\alpha - \frac{n+1}{n}(X(C_F)-\beta)]f(\alpha)d\alpha$, i.e.,

$C_N + \beta > L(X(C_F)-\beta) - \frac{1}{n}(X(C_F)-\beta)\overline{F}(X(C_F)-\beta)$. Therefore, we have

$$\frac{1}{n} > \frac{L(X(C_F)-\beta) - C_N - \beta}{(X(C_F)-\beta)\overline{F}(X(C_F)-\beta)}. \qquad \text{(a6.14)}$$

Therefore, two situations are discussed.

Situation-(1-i) If $L(X(C_F)-\beta) - C_N - \beta > 0$, i.e., the point $(C_N,\ C_F)$ is above the Curve-4, then we have $n < \frac{(X(C_F)-\beta)\overline{F}(X(C_F)-\beta)}{L(X(C_F)-\beta) - C_N - \beta}$. (a6.15)

Situation-(1-ii) If $L(X(C_F)-\beta) - C_N - \beta \le 0$, i.e., the point $(C_N,\ C_F)$ is below the Curve-4, then (a6.14) always holds.

Condition-2 $\mu - (C_N + \beta) > \frac{r}{r+1}(L(\beta) - C_F)$

Noting that at the Final Equilibrium, all firms on Curve-2 will transfer into flexible firms, i.e., $n = (r,s) = (n,\ 0)$. Therefore, we have $\mu - \frac{n}{n+1}(L(\beta) - C_F) > C_N + \beta$,so that

$$\frac{1}{n} > \frac{L(\beta) - C_F - (\mu - C_N - \beta)}{\mu - C_N - \beta}. \qquad \text{(a6.16)}$$

Therefore, two situations are discussed.

(1) If $L(\beta) - C_F - (\mu - C_N - \beta) > 0$, i.e., the point $(C_N,\ C_F)$ is below the Curve-5, then $n < \frac{\mu - C_N - \beta}{L(\beta) - C_F - (\mu - C_N - \beta)}$. (a6.17)

(2) If $L(\beta) - C_F - (\mu - C_N - \beta) \le 0$, i.e., the point $(C_N,\ C_F)$ is above the Curve-5, then (a6.16) always holds.

By Proposition 6.2, Curve-4 is always above Curve-5, we can divide Region-D into three areas. Define **Region-D-1** is area between Curve-1 and Curve-4; **Region-D-2** is area between Curve-4 and Curve-5; **Region-D-3** is area between Curve-5 and Curve-2.

Therefore, if the final equilibrium occurs as Case-D, there are three possibilities.

(i) If the final equilibrium stays at Region-D-1, i.e., $L(X(C_F)-\beta)-C_N-\beta>0$, then at final equilibrium it must have $n<\dfrac{(X(C_F)-\beta)\overline{F}(X(C_F)-\beta)}{L(X(C_F)-\beta)-C_N-\beta}$.

(ii) If the final equilibrium stays at Region-D-3, i.e., $L(\beta)-C_F-(\mu-C_N-\beta)>0$, then at final equilibrium it must have $n<\dfrac{\mu-C_N-\beta}{L(\beta)-C_F-(\mu-C_N-\beta)}$.

(iii) If the final equilibrium stays at Region-D-2, i.e., $L(X(C_F)-\beta)-C_N-\beta\le 0$ and $L(\beta)-C_F-(\mu-C_N-\beta)\le 0$, then any combinations of $n=(r,s)$ is possible.

Sort out the analysis within the area between curves $C_{F1}^{(0,1)}$ and $C_{F2}^{(1,0)}$, then there are three possibilities.

(1) If $L(X(C_F)-\beta)-C_N-\beta>0$, then let $\overline{N}=\dfrac{(X(C_F)-\beta)\overline{F}(X(C_F)-\beta)}{L(X(C_F)-\beta)-C_N-\beta}$:

(1) if $n\ge\overline{N}$, then all firms transfer to in-flexible strategies, i.e., $n=(r,s)=(0,n)$;

(2) if $n<\overline{N}$, then at Final Equilibrium, both flexible and in-flexible firms coexist in the market;

(2) If $L(X(C_F)-\beta)-C_N-\beta\le 0$ and $L(\beta)-C_F-(\mu-C_N-\beta)\le 0$, then at Final Equilibrium, both flexible and in-flexible firms coexist in the market.

(3) If $L(\beta)-C_F-(\mu-C_N-\beta)>0$, then let $\tilde{N}=\dfrac{\mu-C_N-\beta}{L(\beta)-C_F-(\mu-C_N-\beta)}$:

(1) if $n\ge\tilde{N}$, then all firms transfer to flexible strategy, i.e., $n=(r,s)=(n,0)$;

(2) if $n<\tilde{N}$, then at Final Equilibrium, both flexible and in-flexible firms coexist in the market.

This completes the proof of Theorem 6.1. □

Theorem 6.3 In a profit-driven market, the Stable Market can be characterized as follows within the area $\{(C_N,C_F):0<C_N\le C_F\}$.

(i) If $\mu-\beta \le C_N$ & $L(\beta)\le C_F$ & $C_N \le C_F$, then no firm will exist in the market eventually, i.e., $n=0$;

(ii) If $L(X(C_F)-\beta)-C_N-\beta>0$, i.e., above the curve $C_{F1}^{(0,\ \infty)}$, then the Stable Market stays at Case-B $n=(0,n)$, $n\to\infty$ and $\Pi_e^N(0,n)\to 0$;

(iii) If $L(\beta)-C_F-(\mu-C_N-\beta)>0$, i.e., below the curve $C_{F2}^{(\infty,\ 0)}$, then the Stable Market stays at Case-C $n=(n,0)$, $n\to\infty$ and $\Pi_e^F(n,0)\to 0$;

(iv) If $L(X(C_F)-\beta)-C_N-\beta\le 0$ and $L(\beta)-C_F-(\mu-C_N-\beta)\le 0$, i.e., area between Curve-4 and Curve-5, then the Stable Market stays at Case-D $n=(r,s)$, $n\to\infty$, $s\to\infty$, $r\to\infty$, $\Pi_e^F\to 0$ and $\Pi_e^N\to 0$.

Proof

Assume that new firms are allowed to join market freely as long as the profit is positive; and quit market freely if there is no profit. Each firm joining market can choose their strategies, flexible or in-flexible. By Theorem 5.3 in Chapter 5, in Case-B, Case-C and Case-D, there is always positive profit in market. As a result, total number of firms tends to infinite, i.e., $n\to\infty$. Define $\Pi_0^N=(k^N)^2\overline{F}(k^N)$;

$$\Pi_0^F=\int_{\beta+k^N}^{X(C_F)}(\alpha-k^N-\beta)^2 f(\alpha)d\alpha+\int_{X(C_F)}^{\infty}(X(C_F)-k^N-\beta)^2 f(\alpha)d\alpha.$$

By Theorems 5.3 and 6.2, it can be concluded that

(1) If $\begin{cases} L(\beta)\le C_F \\ \mu-\beta\le C_N \end{cases}$, then the solution is $\Pi_e^N=\Pi_e^F=0$, regardless *r* and *s*. As a result, there is no firm existing in market, i.e., $n=0$.

(2) If $\begin{cases} L(\beta)\le C_F \\ 0\le C_N<\mu-\beta \end{cases}$, then the final equilibrium stays at $n=(0,n)$ with $\begin{cases} k^F=0 \\ k^N=nk_e^N>0 \end{cases}$, and $\Pi_e^N(0,n)=\frac{k_w^2}{n^2}\overline{F}(k_w)=\frac{1}{n^2}\Pi_0^N$. It can be proved that Π_0^N is bounded above by $(X(C_F)-\beta)^2\overline{F}(X(C_F)-\beta)$. Therefore, as $n\to\infty$, $\Pi_e^N(0,n)\to 0$.

(3) If $\begin{cases} C_F < L(\beta) \\ \mu - \beta \le C_N \end{cases}$, then the equilibrium stays at $n = (n,0)$ with $\begin{cases} k^F = nk_e^F \\ k^N = 0 \end{cases}$, and $\Pi_e^F(n,0) = \frac{1}{n^2}\Pi_0^F$. It can be proved that Π_0^F is bounded above by $(X(C_F) - \beta)^2 \overline{F}(\beta)$. Therefore, as $n \to \infty$, $\Pi_e^F(n,0) \to 0$.

(4) If $\begin{cases} C_F < L(\beta) \\ C_N < \mu - \beta \end{cases}$, then there are three situations.

(4-I) If $L(X(C_F) - \beta) - C_N - \beta > 0$, i.e., above Curve-4, then we have following analysis. By Theorem 6.2, Case-B and Case-D may exist. As $n \to \infty$, there are two situations: (1) $s \to \infty$, and r is finite or $r \to \infty$; (2) s is finite, and $r \to \infty$.

(4-I-1) If $s \to \infty$, and r is finite or $r \to \infty$:

As $s \to \infty$, Curve-1 will approach $C_F = L(\beta + k_w)$ where k_w satisfies $\int_{k_w}^{\infty} (\alpha - k_w) f(\alpha) d\alpha = C_N + \beta$, so that $k_w = X(C_N + \beta)$. Therefore, we have $C_F = L(\beta + X(C_N + \beta))$, i.e., $X(C_F) = X(C_N + \beta) + \beta$, which is Curve-4. Therefore, $(C_N,\ C_F)$ will eventually stay at Case-B in the Final Equilibrium.

(4-I-2) If s is finite, and $r \to \infty$:

Considering Case-D, by Theorem 5.3, $k_e^F = \frac{1}{r+1}(X(C_F) - \beta - k^N)$; $k_e^N = \frac{1}{s}k^N$;

k^N satisfies $C_N + \beta = \int_{k^N}^{\infty} (\alpha - \frac{s+1}{s}k^N) f(\alpha) d\alpha - \frac{r}{r+1}(L(\beta + k^N) - C_F)$;

$\Pi_e^F = \frac{1}{(r+1)^2}\Pi_0^F$; $\Pi_e^N = \frac{1}{s^2}\Pi_0^N$. It can be proved that Π_0^F and Π_0^N are bounded above. As $r \to \infty$ and $s \le s_0$ for some positive number s_0, we have $\Pi_e^F \to 0$ and $C_N + \beta = \int_{k^N}^{\infty} (\alpha - \frac{s+1}{s}k^N) f(\alpha) d\alpha - (L(\beta + k^N) - C_F)$. Since $k^N > 0$ and $\Pi_e^N = \frac{1}{s^2}(k^N)^2 \overline{F}(k^N) > 0$, there exists an $\bar{r}$ so that $\Pi_e^N(\bar{r}, s+1) > \Pi_e^F(r,s) \to 0$ for all $r \ge \bar{r}$. In this case, the number of in-flexible firms should increase. Continuing with this argument will eventually lead to $s > s_0$. This is a contradiction. As a result, $n \to \infty$, and Final Equilibrium occurs as Case-B, i.e., all firms become in-flexible.

(4-II) If $L(\beta)-C_F-(\mu-C_N-\beta)>0$, i.e., below Curve-5. Similar to the analysis of (4-I), we can draw the conclusion that $n\to\infty$, and Final Equilibrium occurs as Case-C, i.e., all firms become flexible.

(4-III) If $L(X(C_F)-\beta)-C_N-\beta\le 0$ and $L(\beta)-C_F-(\mu-C_N-\beta)\le 0$, i.e., area between Curve-4 and Curve-5. By Theorem 6.2, in this region, the stable status will be stayed at Case-D for any combination of $n=(r,s)$. By Theorem 1, we have in Case-D, we always have $\Pi_e^F>0$ and $\Pi_e^N>0$. Under the assumption that new firms will join the market as long as the profit is positive. Therefore, there are always new firms joining in market. We analyze these two possible situations.

(4-III-1) $s\to\infty$, and r is finite or $r\to\infty$. By Theorem 5.3, in Case-D we have

$k^F=\frac{r}{r+1}(X(C_F)-\beta-k^N)$; K^N satisfies that

$$C_N+\beta=\int_{k^N}^{\infty}(\alpha-\frac{s+1}{s}k^N)f(\alpha)d\alpha-\frac{r}{r+1}(L(\beta+k^N)-C_F).$$

The expected profit of each flexible firm is $\Pi_e^F(k^N)=\frac{1}{(r+1)^2}\Pi_0^F$.

The expected profit of each in-flexible firm is $\Pi_e^N(k^N)=\frac{1}{s^2}\Pi_0^N$. If $s\to\infty$ and $r\le r_0$ for some positive number r_0, then $\Pi_e^F(k^N)=\frac{1}{(r+1)^2}\Pi_0^F>0$, where k^N and Π_0^F are functions of $r\le r_0$. By Property 5.4, Π_0^N is bounded above. Therefore, $\Pi_e^N(k^N)=\frac{1}{s^2}\Pi_0^N\to 0$. Thus, there exists an $\bar{s}$ such that $\Pi_e^F(r+1,\bar{s})>\Pi_e^N(r,s)\to 0$ for all $s\ge\bar{s}$. In this case, the number of flexible firms should increase. Continuing with this argument will eventually lead to $r>r_0$. This is a contradiction. Therefore, $s\to\infty$ and $r\to\infty$, and hence $\Pi_e^F\to 0$ and $\Pi_e^N\to 0$.

(4-III-2) s is finite, and $r\to\infty$: Similar to the analysis of $s\to\infty$ situation, it is easy to draw the same conclusions.

Considering these four cases, we have the following conclusions within $\{(C_N,C_F):0\le C_N\le C_F\}$.

(i) If $\begin{cases} L(\beta) \le C_F \\ \mu - \beta \le C_N \end{cases}$, there is no firm existing in market, i.e., $n = 0$;

(ii) If $L(X(C_F) - \beta) - C_N - \beta > 0$, i.e., above Curve-4, then Final Equilibrium stays at Case-B $n = (0, n)$, and $n \to \infty$, $\Pi_e^N(0, n) \to 0$;

(iii) If $L(\beta) - C_F - (\mu - C_N - \beta) > 0$, i.e., below Curve-5, then Final Equilibrium stays at Case-C $n = (n, 0)$, and $n \to \infty$, $\Pi_e^F(n, 0) \to 0$;

(iv) If $L(\mu - C_N - \beta) - C_N - \beta \le 0$ and $L(\beta) - C_F - (\mu - C_N - \beta) \le 0$, i.e., area between Curve-4 and Curve-5, then Final Equilibrium stays at Case-D, $n = (r, s)$, $s \to \infty$, $r \to \infty$, $\Pi_e^F \to 0$ and $\Pi_e^N \to 0$.

This completes the proof of Theorem 6.3. □

Theorem 6.4 Given $r = r_0$, $r_0 \in [1,\ n]$, there exists a unique curve satisfying $G(C_N,\ C_F|\ r_0) = D(r_0) = 0$, on which C_F increases as C_N increases; in areas above the curve $D(r_0) = 0$, we have $G(C_N,\ C_F|\ r_0) = D(r_0) < 0$; in areas below the curve $D(r_0) = 0$, we have $G(C_N,\ C_F|\ r_0) = D(r_0) > 0$.

Proof

Let $s_0 = n - r_0$ and $r_0 \in [1,\ n-1]$. Considering two cases $n = (r_0,\ s_0)$ and $n = (r_0 - 1,\ s_0 + 1)$, we have

$$G(C_N,\ C_F|\ r_0) = D(r_0) = \Pi_e^F(r_0, n - r_0) - \Pi_e^N(r_0 - 1, n - r_0 + 1). \quad \text{(a6.18)}$$

It is noted that given totally number of *n* firms, Curve-1 and Curve-2 depend on value of $(r,\ s)$. Given C_N, then

Curve-1 $C_F = L(\beta + k_w)$, where k_w satisfies $\int_{k_w}^{\infty} (\alpha - \frac{s+1}{s} k_w) f(\alpha) d\alpha = C_N + \beta$.

Curve-2 $C_F = \frac{r+1}{r}(C_N + \beta - \mu) + L(\beta)$.

We define Curve-1a and Curve-2a to be the curves corresponding to the case $n = (r_0,\ s_0)$, and Curve-1b and Curve-2b to be the curves corresponding to the case $n = (r_0 - 1,\ s_0 + 1)$. These four curves can be presented as below.

Curve-1a $C_{F1a} = L(\beta + k_{w1a})$, where $\int_{k_{w1a}}^{\infty} (\alpha - \frac{s_0+1}{s_0} k_{w1a}) f(\alpha) d\alpha = C_N + \beta$.

Curve-2a $C_{F2a} = \frac{r_0+1}{r_0}(C_N + \beta - \mu) + L(\beta)$.

Curve-1b $C_{F1b} = L(\beta + k_{w1b})$, where $\int_{k_{w1b}}^{\infty} (\alpha - \frac{s_0+2}{s_0+1} k_{w1b}) f(\alpha) d\alpha = C_N + \beta$.

Curve-2b $C_{F2b} = \frac{r_0}{r_0-1}(C_N + \beta - \mu) + L(\beta)$.

Given *n* firms, by Proposition 6.1, referring to Curve-1, C_F is decreasing in *s,* i.e., increasing in r; referring to Curve-2, C_F is decreasing in *r*. We relax the condition that $C_F \geq C_N$ here in this section. This condition can be added back after the discussion. Therefore, given *n* firms and $C_N \in [0, \ \mu - \beta]$, $C_{F1a} > C_{F1b} > C_{F2a} > C_{F2b}$. Three sub-areas are created between Curve-1a and Curve-2b. Define these three sub-areas as: (i) Area-D-1 is the area between Curve-2b and Curve-2a; (ii) Area-D-2 is the area between Curve-2a and Curve-1b; (iii) Area-D-3 is the area between Curve-1b and Curve-1a. Figure a6.1 shows these three areas. In the following, each sub-area is discussed.

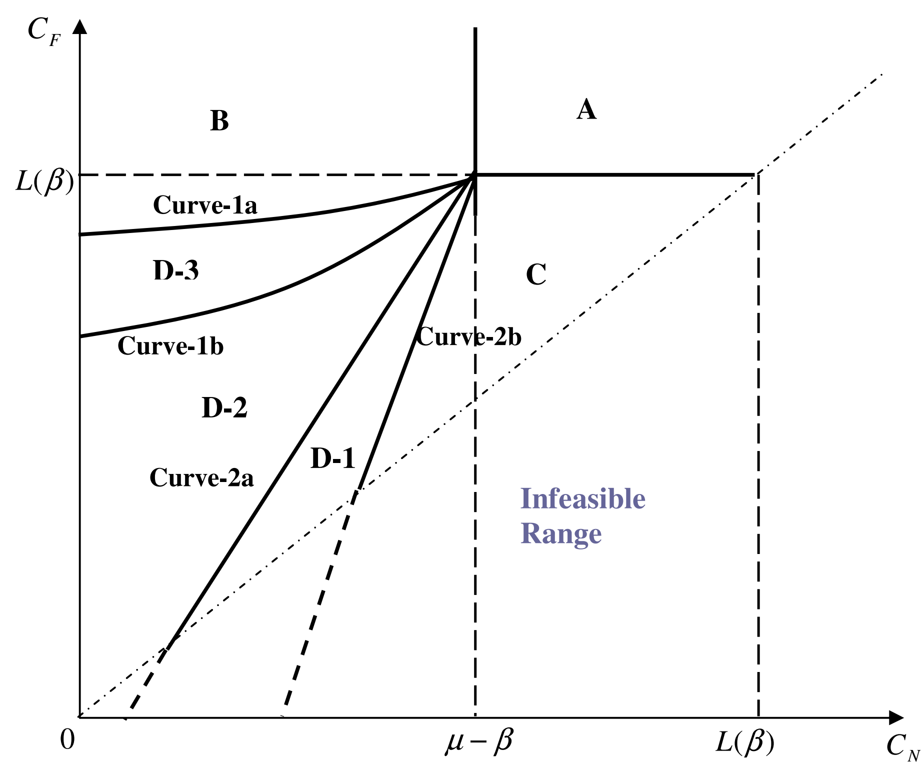

Figure a 6.1: Three areas created by Curve-1a~2b.

(i) Area-D-1

In this area, case $n=(r_0,\ s_0)$ occurs as Case-C, and case $n=(r_0-1,\ s_0+1)$ occurs as Case-D. By Theorem 5.3, we have:

In case $n=(r_0,\ s_0)$, we get $\Pi_e^F(r_0,n-r_0)=\dfrac{1}{(r_0+1)^2}\Pi_0^F(0)$; (a6.19)

In case $n=(r_0-1,\ s_0+1)$, we get $\Pi_e^N(r_0-1,n-r_0+1)=\dfrac{1}{(s_0+1)^2}\Pi_0^N(k^1)$, (a6.20)

where k^1 satisfies $C_N+\beta=\int_{k^1}^{\infty}(\alpha-\dfrac{s_0+2}{s_0+1}k^1)f(\alpha)d\alpha-\dfrac{r_0-1}{r_0}(L(\beta+k^1)-C_F)$. (a6.21)

(i-1) situation: Given C_N, with respect to C_F, by (a6.21) we have $\dfrac{dk^1}{dC_F}>0$. Together with (a6.18)~(a6.20), we have $\dfrac{dD(r_0)}{dC_F}=-\dfrac{2}{(r_0+1)^2}(X(C_F)-\beta)-\dfrac{1}{(s_0+1)^2}k^1[2\overline{F}(k^1)-k^1f(k^1)]\cdot\dfrac{dk^1}{dC_F}<0$.

(i-2) situation: Given C_F, with respect to C_N, by (a6.19) we have $\dfrac{dk^1}{dC_N}<0$. Together with (a6.18)~(a6.20), we have $\dfrac{dD(r_0)}{dC_N}=-\dfrac{1}{(s_0+1)^2}k^1[2\overline{F}(k^1)-k^1f(k^1)]\cdot\dfrac{dk^1}{dC_N}>0$.

(ii) Area-D-2

If both cases $n=(r_0,\ s_0)$ and $n=(r_0-1,\ s_0+1)$ are in Region-D-2, then the two cases occur as Case-D. We have the following discussion. By Theorem 5.3, we have $\Pi_e^F(r_0,n-r_0)=\dfrac{1}{(r_0+1)^2}\Pi_0^F(k^0)$, (a6.22)

and $\Pi_e^N(r_0-1,n-r_0+1)=\dfrac{1}{(s_0+1)^2}\Pi_0^N(k^1)$, (a6.23)

where k^0 satisfies $C_N+\beta=\int_{k^0}^{\infty}(\alpha-\dfrac{s_0+1}{s_0}k^0)f(\alpha)d\alpha-\dfrac{r_0}{r_0+1}(L(\beta+k^0)-C_F)$, (a6.24)

as the total in-flexible capacity of case $n=(r_0,\ s_0)$;

k^1 satisfies $C_N+\beta=\int_{k^1}^{\infty}(\alpha-\dfrac{s_0+2}{s_0+1}k^1)f(\alpha)d\alpha-\dfrac{r_0-1}{r_0}(L(\beta+k^1)-C_F)$, (a6.25)

as the total in-flexible capacity of case $n=(r_0-1,\ s_0+1)$.

(ii-1) situation: Given C_N, with respect to C_F, by (a6.24) and (a6.25), we have $\frac{dk^0}{dC_F} > 0$ and $\frac{dk^1}{dC_F} > 0$. Together with (a6.18), (a6.22) and (a6.23), we have

$$\frac{dD(r_0)}{dC_F} = \frac{1}{(r_0+1)^2}[-2(L(\beta+k^0)-C_F)\cdot\frac{dk^0}{dC_F} - 2(X(C_F)-\beta-k^0)]$$

$$-\frac{1}{(s_0+1)^2}k^1[2\overline{F}(k^1)-k^1 f(k^1)]\cdot\frac{dk^1}{dC_F} < 0.$$

(ii-2) situation: Given C_F, with respect to C_N, by (a6.24) and (a6.25), we have $\frac{dk^0}{dC_N} < 0$ and $\frac{dk^1}{dC_N} < 0$. Together with (a6.18), (a6.22) and (a6.23), we have

$$\frac{dD(r_0)}{dC_N} = -\frac{2}{(r_0+1)^2}(L(\beta+k^0)-C_F)\cdot\frac{dk^0}{dC_N} - \frac{1}{(s_0+1)^2}k^1[2\overline{F}(k^1)-k^1 f(k^1)]\cdot\frac{dk^1}{dC_N} > 0$$

(iii) Area-D-3

In this area, case $n=(r_0,\ s_0)$ occurs as Case-D, and case $n=(r_0-1,\ s_0+1)$ occurs as Case-B. By Theorem 5.3, we have

$$\Pi_e^F(r_0, n-r_0) = \frac{1}{(r_0+1)^2}\Pi_0^F(k^0), \tag{a6.26}$$

and $\Pi_e^N(r_0-1, n-r_0+1) = \frac{1}{(s_0+1)^2}\Pi_0^N(k^1)$, (a6.27)

k^0 satisfies $C_N+\beta = \int_{k^0}^{\infty}(\alpha - \frac{s_0+1}{s_0}k^0)f(\alpha)d\alpha - \frac{r_0}{r_0+1}(L(\beta+k^0)-C_F)$; (a6.28)

k^1 satisfies $C_N+\beta = \int_{k^1}^{\infty}(\alpha - \frac{s_0+2}{s_0+1}k^1)f(\alpha)d\alpha$. (a6.29)

(iii-1) situation: Given C_N, with respect to C_F, by (a6.28), $\frac{dk^0}{dC_F} > 0$. Note that in Case-B, $\frac{dk^1}{dC_F} = 0$. Together with (a6.18), (a6.26) and (a6.27), we have $\frac{dD(r_0)}{dC_F} < 0$.

(iii-2) situation: Given C_F, with respect to C_N, by (a6.28) and (a6.29), we have $\frac{dk^0}{dC_N} < 0$ and $\frac{dk^1}{dC_N} < 0$. Together with (a6.18), (a6.26) and (a6.27), we have

$$\frac{dD(r_0)}{dC_N} = -\frac{2}{(r_0+1)^2}(L(\beta+k^0)-C_F)\cdot\frac{dk^0}{dC_N} - \frac{1}{(s_0+1)^2}k^1[2\overline{F}(k^1)-k^1 f(k^1)]\cdot\frac{dk^1}{dC_N} > 0$$

. Considering these three sub-areas together, given *n* firms, in the areas between Curve-1a and Curve-2b, we have

(1) $\frac{dD(r_0)}{dC_F} < 0$ with given $C_N \in [0,\ \mu-\beta]$; (a6.30)

(2) $\frac{dD(r_0)}{dC_N} > 0$ with given $C_F \in [0,\ L(\beta)]$. (a6.31)

Consider the curve $D(r_0)=0$, $r_0 \in [1,\ n-1]$, given *n* firms. With respect to C_N, we have $\frac{\partial D(r_0)}{\partial C_F}\cdot\frac{dC_F}{dC_N}+\frac{\partial D(r_0)}{\partial C_N}=0$, so that $\frac{dC_F}{dC_N} = -\frac{\partial D(r_0)}{\partial C_N}\Big/\frac{\partial D(r_0)}{\partial C_F}$. (a6.32)

By (a6.30) and (a6.31), we have $\frac{dD(r_0)}{dC_F}<0$ and $\frac{dD(r_0)}{dC_N}>0$ in the areas between Curve-1a and Curve-2b. Therefore, by (a6.32) we have $\frac{dC_F}{dC_N}>0$.

To relax the condition that $C_F \ge C_N$ here in this section, we extend Curve-2a and Curve-2b to $C_N = 0$. Consider areas between Curve-1a and Curve-2b. By (a6.30) in this area, $\frac{dD(r_0)}{dC_F}<0$.

By (6.18), on Curve-2b,

$$D(r_0) = \Pi_e^F(r_0, n-r_0) - \Pi_e^N(r_0-1, n-r_0+1) = \frac{1}{(r_0+1)^2}\Pi_0^F(0) > 0;$$

on Curve-1a,

$$D(r_0) = \Pi_e^F(r_0, n-r_0) - \Pi_e^N(r_0-1, n-r_0+1) = 0 - \frac{1}{(s_0+1)^2}\Pi_0^N(k^1) < 0,$$

where k^1 satisfies $C_N+\beta = \int_{k^1}^{\infty}(\alpha - \frac{s_0+2}{s_0+1}k^1)f(\alpha)d\alpha$.

Curve-1a and Curve-2b intersect at point $(C_N,\ C_F)=(\mu-\beta,\ L(\beta))$. Note that at this point, $\Pi_e^F(r_0, n-r_0) = \Pi_e^N(r_0-1, n-r_0+1) = 0$. Therefore, we have $D(r_0)=0$. The point $(C_N,\ C_F)=(\mu-\beta,\ L(\beta))$ is a common end of curves $D(r_0)=0$, $r_0\in[1,\ n-1]$. Therefore, in area between Curve-1a and Curve-2b, there exists a unique curve which satisfies $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$.

By (a6.30), given a C_N , $D(r_0)$ is decreasing in C_F . Therefore, in areas above the curve $D(r_0)=0$, we have $D(r_0)<0$; in areas below the curve $D(r_0)=0$, we have $D(r_0)>0$.

This completes the proof of Theorem 6.4. □

Theorem 6.5 Given *n* firms, for every $r_0 \in [1,\ n-1]$, consider curves $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$ and $G(C_N,\ C_F \mid r_0+1) = D(r_0+1) = 0$ within the area $\{(C_N, C_F): C_N \le \mu-\beta\ \&\ C_F \le L(\beta)\ \&\ 0 \le C_N \le C_F\}$, then the Final Equilibrium $n=(r_e,\ s_e)$ can be categorized into one of the following five scenarios in terms of the exact numbers of flexible and in-flexible firms.

(i) In areas below curve $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$ and above curve $G(C_N,\ C_F \mid r_0+1) = D(r_0+1) = 0$, we have $n=(r_e,\ s_e)=(r_0,\ n-r_0)$;

(ii) in areas above curve $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$ and below curve $G(C_N,\ C_F \mid r_0+1) = D(r_0+1) = 0$, we have the Final Equilibrium does not obtained at $r=r_0$;

(iii) if these two curves overlaps, with respect to points on the curves, we have either $n=(r_e,\ s_e)=(r_0,\ n-r_0)$ or $n=(r_e,\ s_e)=(r_0+1,\ n-r_0-1)$;

(iv) in areas above all curves $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$, $r_0 \in [1,\ n]$, we have $n=(r_e=0,\ s_e=n)=(0,\ n)$;

(v) in areas below all curves $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$, $r_0 \in [1,\ n]$, we have $n=(r_e=0,\ s_e=n)=(n,\ 0)$.

<u>Proof</u>

By Conditions of Final Equilibrium, Parts (i)~(iii) is proved.

In areas above all curves $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$, $r_0 \in [1,\ n-1]$, by Theorem 6.3, we have for all $r_0 \in [1,\ n-1]$, $D(r_0)<0$. Therefore, $D(1) = \Pi_e^F(1,n-1) - \Pi_e^N(0,n) < 0$, which leads to that firms will use in-flexible strategy. By Theorem 6.2, the Final equilibrium will stay at $n=(r_e=0,\ s_e=n)$. This completes the proof of Part (iv).

In areas below all curves $G(C_N,\ C_F \mid r_0) = D(r_0) = 0$, $r_0 \in [1,\ n]$, by Theorem 5.3, we have for all $r_0 \in [1,\ n]$, $D(r_0) > 0$. Therefore, $D(n) = \Pi_e^F(n,0) - \Pi_e^N(n-1,1) > 0$, which leads to that firms will use flexible strategy. By Theorem 6.2, the Final equilibrium will stay at $n = (r_e = n,\ s_e = 0)$.This completes the proof of Part (v).

This completes the proof of Theorem 6.5. □

Proofs in Chapter 7

Theorem 7.1 In a monopoly model with $m \in [0,\ 1]$, we have:

(i) The optimal capacity k^* satisfies $0 < k^* < \frac{a}{2}$ and

$$\int_0^{\alpha_L} m(\alpha(a-2mk^*)-\beta)f(\alpha)d\alpha + \int_{\alpha_R}^{\infty}(\alpha(a-2k^*)-\beta)f(\alpha)d\alpha = C_F .$$

(ii) The optimal production quantity $q^* = \begin{cases} mk^*, & 0 \le \alpha < \alpha_L \\ q_b, & \alpha_L \le \alpha < \alpha_R \\ k^*, & \alpha_R \le \alpha \end{cases}$.

(iii) The optimal profit is

$$\Pi = \int_0^{\alpha_L}(mk^*)^2 \alpha f(\alpha)d\alpha + \int_{\alpha_L}^{\alpha_R}(q_b)^2 \alpha f(\alpha)d\alpha + \int_{\alpha_R}^{\infty}(k^*)^2 \alpha f(\alpha)d\alpha ,$$

where $\alpha_L = \frac{\beta}{a-2mk^*}$, $\alpha_R = \frac{\beta}{a-2k^*}$ and $q_b = \frac{1}{2}(a-\frac{\beta}{\alpha})$.

<u>Proof</u>

At the production decision stage, by (7.2), the first- and second-order derivatives of $\pi(q)$ are $\pi^{(1)}(q) = \alpha(a-2q)-\beta$, $\pi^{(2)}(q) = -2\alpha$. If $\alpha = 0$, then $\pi^{(1)}(q) = -\beta \le 0$ and we take $q^* = mk$. If $\alpha > 0$, then $\pi^{(2)}(q) = -2\alpha < 0$. So, $\pi(q)$ is concave in q and its unconstrained optimal solution is $q_b = \frac{1}{2}(a-\frac{\beta}{\alpha})$. Note that $mk \le q \le k$. Hence, the optimal production quantity $q^* = \begin{cases} mk^*, & 0 \le \alpha < \alpha_L \\ q_b, & \alpha_L \le \alpha < \alpha_R \\ k^*, & \alpha_R \le \alpha \end{cases}$, where $\alpha_L = \dfrac{\beta}{a-2mk}$ and $\alpha_R = \dfrac{\beta}{a-2k}$.

At the capacity decision stage, by (7.1) and the above results, we have

$$\Pi = \int_0^{\alpha_L} mk[\alpha(a-mk)-\beta]f(\alpha)d\alpha + \int_{\alpha_L}^{\alpha_R}(q_b)^2\alpha f(\alpha)d\alpha + \int_{\alpha_R}^{\infty} k[\alpha(a-k)-\beta]f(\alpha)d\alpha - C_F k$$

$$\Pi^{(1)}(k) = \int_0^{\alpha_L} m(\alpha(a-2mk)-\beta)f(\alpha)d\alpha + \int_{\alpha_R}^{\infty}(\alpha(a-2k)-\beta)f(\alpha)d\alpha - C_F.$$

$\Pi^{(2)}(k) = -2m^2\int_0^{\alpha_L}\alpha f(\alpha)d\alpha - 2\int_{\alpha_R}^{\infty}\alpha f(\alpha)d\alpha < 0$. Thus, $\Pi(k)$ is concave in k. Let $\alpha_0 = \dfrac{\beta}{a}$. Recall that $0 \le k < \dfrac{a}{2}$.

$$\begin{aligned}\Pi^{(1)}(0) &= \int_0^{\alpha_0} m(\alpha a-\beta)f(\alpha)d\alpha + \int_{\alpha_0}^{\infty}(\alpha a-\beta)f(\alpha)d\alpha - C_F \\ &\ge \int_0^{\alpha_0}(\alpha a-\beta)f(\alpha)d\alpha + \int_{\alpha_0}^{\infty}(\alpha a-\beta)f(\alpha)d\alpha - C_F \\ &= \mu a - \beta - C_F > 0\end{aligned}$$

As $k \to \dfrac{a}{2}$, $\Pi^{(1)}(k) \le \int_{\alpha_R}^{\infty}(\alpha(a-2k)-\beta)f(\alpha)d\alpha - C_F \to -C_F < 0$. Therefore, the optimal capacity k^* satisfies $0 < k^* < \dfrac{a}{2}$ and $\Pi^{(1)}(k^*) = 0$, i.e.,

$\int_0^{\alpha_L} m(\alpha(a-2mk^*)-\beta)f(\alpha)d\alpha + \int_{\alpha_R}^{\infty}(\alpha(a-2k^*)-\beta)f(\alpha)d\alpha = C_F$, and so

$$\Pi = \int_0^{\alpha_L}(mk^*)^2\alpha f(\alpha)d\alpha + \int_{\alpha_L}^{\alpha_R}(q_b)^2\alpha f(\alpha)d\alpha + \int_{\alpha_R}^{\infty}(k^*)^2\alpha f(\alpha)d\alpha.$$

This completes the proof of Theorem 7.1. □

Proposition 7.1 In a duopoly model with $0 \le m_1, m_2 \le 1$, the optimal production capacity of firm i, given k_1, k_2 and q_{3-i}, is $q_i^* = \begin{cases} m_i k_i, & 0 \le \alpha < \alpha_{Li} \\ q_{ib}, & \alpha_{Li} \le \alpha < \alpha_{Ri} \\ k_i, & \alpha_{Ri} \le \alpha \end{cases}$ for $i = 1,\ 2$, where $q_{ib} = \frac{1}{2}(a - \frac{\beta}{\alpha} - q_{3-i})$, $\alpha_{Li} = \frac{\beta}{a - 2m_i k_i - q_{3-i}}$ and $\alpha_{Ri} = \frac{\beta}{a - 2k_i - q_{3-i}}$ for $i = 1,\ 2$.

Proof

By (7.3), for firm i, with respect to q_i, we have $\pi_i^{(1)}(q_i) = \alpha(a - 2q_i - q_{3-i}) - \beta$ and $\pi_i^{(2)}(q_i) = -2\alpha$. If $\alpha = 0$, then $\pi_i^{(1)}(q_i) = -\beta \le 0$ and we take $q_i^* = m_i k_i$. If $\alpha > 0$, then $\pi_i^{(2)}(q_i) = -2\alpha < 0$. So, $\pi_i(q_i)$ is concave in q_i, and its unconstrained optimal solution is $q_{ib} = \frac{1}{2}(a - \frac{\beta}{\alpha} - q_{3-i})$. Note that $m_i k_i \le q_i \le k_i$. Hence, the optimal production quantity $q_i^* = \begin{cases} m_i k_i, & 0 \le \alpha < \alpha_{Li} \\ q_{ib}, & \alpha_{Li} \le \alpha < \alpha_{Ri} \\ k_i, & \alpha_{Ri} \le \alpha \end{cases}$.

This completes the proof of Proposition 7.1. □

Proposition 7.2 In a duopoly model with $0 \le m_1, m_2 \le 1$, given the production quantity of firm i's $(i \in \{1,2\})$ rival q_{3-i}, we have:

(i) Firm i's optimal capacity k_i^* satisfies $0 < k_i^* < \frac{a - q_{3-i}}{2}$ and

$$\int_0^{\alpha_{Li}} m_i(\alpha(a - q_{3-i} - 2m_i k_i^*) - \beta) f(\alpha) d\alpha + \int_{\alpha_{Ri}}^{\infty} (\alpha(a - q_{3-i} - 2k_i^*) - \beta) f(\alpha) d\alpha = C_F ;$$

(ii) Firm i's optimal profit

$$\Pi_i = \int_0^{\alpha_{Li}} (m_i k_i^*)^2 \alpha f(\alpha) d\alpha + \int_{\alpha_{Li}}^{\alpha_{Ri}} (q_{ib})^2 \alpha f(\alpha) d\alpha + \int_{\alpha_{Ri}}^{\infty} (k_i^*)^2 \alpha f(\alpha) d\alpha ,$$

where $q_{ib} = \frac{1}{2}(a - \frac{\beta}{\alpha} - q_{3-i})$, $\alpha_{Li} = \frac{\beta}{a - 2m_i k_i^* - q_{3-i}}$ and $\alpha_{Ri} = \frac{\beta}{a - 2k_i^* - q_{3-i}}$, $i = 1,\ 2$.

Proof

For firm $i \in \{1,2\}$, with respect to k_i , by (7.5) we have

$$\Pi_i^{(1)}(k_i) = \int_0^{\alpha_{Li}} m_i(\alpha(a - q_{3-i} - 2m_i k_i) - \beta) f(\alpha) d\alpha + \int_{\alpha_{Ri}}^{\infty} (\alpha(a - q_{3-i} - 2k_i) - \beta) f(\alpha) d\alpha - C_F$$

$\Pi_i^{(2)}(k_i) = -2m_i^2 \int_0^{\alpha_{Li}} \alpha f(\alpha) d\alpha - 2\int_{\alpha_{Ri}}^{\infty} \alpha f(\alpha) d\alpha < 0$. Thus, $\Pi_i(k_i)$ is concave in k_i. Let $\alpha_{0i} = \dfrac{\beta}{a - q_{3-i}}$. Note that $0 \le k_i < \dfrac{a - q_{3-i}}{2}$.

$$\begin{aligned}
\Pi^{(1)}(0) &= \int_0^{\alpha_{0i}} m_i(\alpha(a - q_{3-i}) - \beta) f(\alpha) d\alpha + \int_{\alpha_{0i}}^{\infty} (\alpha(a - q_{3-i}) - \beta) f(\alpha) d\alpha - C_F \\
&\ge \int_0^{\alpha_{0i}} (\alpha(a - q_{3-i}) - \beta) f(\alpha) d\alpha + \int_{\alpha_{0i}}^{\infty} (\alpha(a - q_{3-i}) - \beta) f(\alpha) d\alpha - C_F \\
&= \mu(a - q_{3-i}) - \beta - C_F > 0
\end{aligned}$$

As $k_i \to \dfrac{a - q_{3-i}}{2}$, $\Pi_i^{(1)}(k_i) \le \int_{\alpha_{Ri}}^{\infty} (\alpha(a - q_{3-i} - 2k_i) - \beta) f(\alpha) d\alpha - C_F \to -C_F < 0$.

Therefore, the optimal capacity k_i^* satisfies $0 < k_i^* < \dfrac{a - q_{3-i}}{2}$ and $\Pi^{(1)}(k_i^*) = 0$, i.e.,

$\int_0^{\alpha_{Li}} m_i(\alpha(a - q_{3-i} - 2m_i k_i^*) - \beta) f(\alpha) d\alpha + \int_{\alpha_{Ri}}^{\infty} (\alpha(a - q_{3-i} - 2k_i^*) - \beta) f(\alpha) d\alpha = C_F$, and

so $\Pi_i = \int_0^{\alpha_{Li}} (m_i k_i^*)^2 f(\alpha) d\alpha + \int_{\alpha_{Li}}^{\alpha_{Ri}} (q_{ib})^2 f(\alpha) d\alpha + \int_{\alpha_{Ri}}^{\infty} (k_i^*)^2 f(\alpha) d\alpha$.

This completes the proof of Proposition 7.2. □

Proposition 7.3 In a duopoly model with $0 \le m_1 \le m_2 \le 1$, given the capacities of the two firms $k_1 \ge 0$ and $k_2 \ge 0$, the production quantities of the two firms (q_1, q_2) at equilibrium are as follows.

(i) If $m_2 k_2 \le k_2 < m_1 k_1 \le k_1$, then

$$(q_1, q_2) = \begin{cases}
(m_1 k_1, m_2 k_2), & 0 \le \alpha < \dfrac{\beta}{a - m_1 k_1 - 2m_2 k_2} \\
(m_1 k_1, q_{2b-1}), & \dfrac{\beta}{a - m_1 k_1 - 2m_2 k_2} \le \alpha < \dfrac{\beta}{a - m_1 k_1 - 2k_2} \\
(m_1 k_1, k_2), & \dfrac{\beta}{a - m_1 k_1 - 2k_2} \le \alpha < \dfrac{\beta}{a - 2m_1 k_1 - k_2} \\
(q_{1b-3}, k_2), & \dfrac{\beta}{a - 2m_1 k_1 - k_2} \le \alpha < \dfrac{\beta}{a - 2k_1 - k_2} \\
(k_1, k_2), & \dfrac{\beta}{a - 2k_1 - k_2} \le \alpha
\end{cases}.$$

(ii) If $m_2k_2 < m_1k_1 \le k_2 < k_1$, then

$$(q_1, q_2) = \begin{cases} (m_1k_1, m_2k_2), & 0 \le \alpha < \dfrac{\beta}{a - m_1k_1 - 2m_2k_2} \\ (m_1k_1, q_{2b-1}), & \dfrac{\beta}{a - m_1k_1 - 2m_2k_2} \le \alpha < \dfrac{\beta}{a - 3m_1k_1} \\ (q_{1b-2}, q_{2b-2}), & \dfrac{\beta}{a - 3m_1k_1} \le \alpha < \dfrac{\beta}{a - 3k_2} \\ (q_{1b-3}, k_2), & \dfrac{\beta}{a - 3k_2} \le \alpha < \dfrac{\beta}{a - 2k_1 - k_2} \\ (k_1, k_2), & \dfrac{\beta}{a - 2k_1 - k_2} \le \alpha \end{cases}.$$

(iii) If $m_1k_1 \le m_2k_2 \le k_2 \le k_1$, then

$$(q_1, q_2) = \begin{cases} (m_1k_1, m_2k_2), & 0 \le \alpha < \dfrac{\beta}{a - 2m_1k_1 - m_2k_2} \\ (q_{1b-1}, m_2k_2), & \dfrac{\beta}{a - 2m_1k_1 - m_2k_2} \le \alpha < \dfrac{\beta}{a - 3m_2k_2} \\ (q_{1b-2}, q_{2b-2}), & \dfrac{\beta}{a - 3m_2k_2} \le \alpha < \dfrac{\beta}{a - 3k_2} \\ (q_{1b-3}, k_2), & \dfrac{\beta}{a - 3k_2} \le \alpha < \dfrac{\beta}{a - 2k_1 - k_2} \\ (k_1, k_2), & \dfrac{\beta}{a - 2k_1 - k_2} \le \alpha \end{cases}.$$

(iv) If $m_1k_1 \le m_2k_2 < k_1 < k_2$, then

$$(q_1, q_2) = \begin{cases} (m_1k_1, m_2k_2), & 0 \le \alpha < \dfrac{\beta}{a - 2m_1k_1 - m_2k_2} \\ (q_{1b-1}, m_2k_2), & \dfrac{\beta}{a - 2m_1k_1 - m_2k_2} \le \alpha < \dfrac{\beta}{a - 3m_2k_2} \\ (q_{1b-2}, q_{2b-2}), & \dfrac{\beta}{a - 3m_2k_2} \le \alpha < \dfrac{\beta}{a - 3k_1} \\ (k_1, q_{2b-3}), & \dfrac{\beta}{a - 3k_1} \le \alpha < \dfrac{\beta}{a - k_1 - 2k_2} \\ (k_1, k_2), & \dfrac{\beta}{a - k_1 - 2k_2} \le \alpha \end{cases}$$

(v) If $m_1k_1 \le k_1 \le m_2k_2 < k_2$, then

$$(q_1, q_2) = \begin{cases} (m_1k_1, m_2k_2), & 0 \le \alpha < \dfrac{\beta}{a - 2m_1k_1 - m_2k_2} \\ (q_{1b-1}, m_2k_2), & \dfrac{\beta}{a - 2m_1k_1 - m_2k_2} \le \alpha < \dfrac{\beta}{a - 2k_1 - m_2k_2} \\ (k_1, m_2k_2), & \dfrac{\beta}{a - 2k_1 - m_2k_2} \le \alpha < \dfrac{\beta}{a - k_1 - 2m_2k_2} \\ (k_1, q_{2b-3}), & \dfrac{\beta}{a - k_1 - 2m_2k_2} \le \alpha < \dfrac{\beta}{a - k_1 - 2k_2} \\ (k_1, k_2), & \dfrac{\beta}{a - k_1 - 2k_2} \le \alpha \end{cases},$$

where $q_{1b-1} = \frac{1}{2}(a - m_2k_2 - \frac{\beta}{\alpha})$, $q_{1b-2} = \frac{1}{3}(a - \frac{\beta}{\alpha})$, $q_{1b-3} = \frac{1}{2}(a - k_2 - \frac{\beta}{\alpha})$, $q_{2b-1} = \frac{1}{2}(a - m_1k_1 - \frac{\beta}{\alpha})$, $q_{2b-2} = \frac{1}{3}(a - \frac{\beta}{\alpha})$ and $q_{2b-3} = \frac{1}{2}(a - k_1 - \frac{\beta}{\alpha})$.

Proof

By Proposition 7.1, Figure 7.3 shows all possible cases of two firms' production decisions, as well as their production quantities, where $q_{ib} = \frac{1}{2}(a - \frac{\beta}{\alpha} - q_{3-i})$, $\alpha_{Li} = \dfrac{\beta}{a - 2m_ik_i - q_{3-i}}$ and $\alpha_{Ri} = \dfrac{\beta}{a - 2k_i - q_{3-i}}$, $i = 1,\ 2$. In

Firm 1 / Firm 2	$0 \le \alpha < \alpha_{L1}$	$\alpha_{L1} \le \alpha < \alpha_{R1}$	$\alpha_{R1} \le \alpha$
$0 \le \alpha < \alpha_{L2}$	I $q_1 = m_1k_1$ $q_2 = m_2k_2$	II $q_1 = q_{1b}$ $q_2 = m_2k_2$	III $q_1 = k_1$ $q_2 = m_2k_2$
$\alpha_{L2} \le \alpha < \alpha_{R2}$	IV $q_1 = m_1k_1$ $q_2 = q_{2b}$	V $q_1 = q_{1b}$ $q_2 = q_{2b}$	VI $q_1 = k_1$ $q_2 = q_{2b}$
$\alpha_{R2} \le \alpha$	VII $q_1 = m_1k_1$ $q_2 = k_2$	VIII $q_1 = q_{1b}$ $q_2 = k_2$	IX $q_1 = k_1$ $q_2 = k_2$

Figure a7.1: Possible cases of two firms' production decisions.

the following, we make an analysis on the solution and equivalent conditions of each possible case in Figure 7.3.

Case-I

$$\begin{cases} q_1 = m_1 k_1 \\ q_2 = m_2 k_2 \\ 0 \le \alpha < \alpha_{L1} \\ 0 \le \alpha < \alpha_{L2} \end{cases} \Leftrightarrow \begin{cases} q_1 = m_1 k_1 \\ q_2 = m_2 k_2 \\ 0 \le \alpha < \min\left\{\dfrac{\beta}{a - 2m_1k_1 - m_2k_2}, \dfrac{\beta}{a - m_1k_1 - 2m_2k_2}\right\} \end{cases}$$

Case-II

$$\begin{cases} q_1 = q_{1b} \\ q_2 = m_2 k_2 \\ \alpha_{L1} \le \alpha < \alpha_{R1} \\ 0 \le \alpha < \alpha_{L2} \end{cases} \Leftrightarrow \begin{cases} q_1 = q_{1b} = \dfrac{1}{2}(a - \dfrac{\beta}{\alpha} - m_2k_2) \\ q_2 = m_2 k_2 \\ \dfrac{\beta}{a - 2m_1k_1 - m_2k_2} \le \alpha < \min\left\{\dfrac{\beta}{a - 2k_1 - m_2k_2}, \dfrac{\beta}{a - 3m_2k_2}\right\} \end{cases}$$

Case-III

$$\begin{cases} q_1 = k_1 \\ q_2 = m_2 k_2 \\ \alpha_{R1} \le \alpha \\ 0 \le \alpha < \alpha_{L2} \end{cases} \Leftrightarrow \begin{cases} q_1 = k_1 \\ q_2 = m_2 k_2 \\ \dfrac{\beta}{a - 2k_1 - m_2k_2} \le \alpha < \dfrac{\beta}{a - k_1 - 2m_2k_2} \end{cases}$$

Case-IV

$$\begin{cases} q_1 = m_1 k_1 \\ q_2 = q_{2b} \\ 0 \le \alpha < \alpha_{L1} \\ \alpha_{L2} \le \alpha < \alpha_{R2} \end{cases} \Leftrightarrow \begin{cases} q_1 = m_1 k_1 \\ q_2 = q_{2b} = \dfrac{1}{2}(a - \dfrac{\beta}{\alpha} - m_1k_1) \\ \dfrac{\beta}{a - m_1k_1 - 2m_2k_2} \le \alpha < \min\left\{\dfrac{\beta}{a - 3m_1k_1}, \dfrac{\beta}{a - m_1k_1 - 2k_2}\right\} \end{cases}$$

Case-V

$$\begin{cases} q_1 = q_{1b} \\ q_2 = q_{2b} \\ \alpha_{L1} \le \alpha < \alpha_{R1} \\ \alpha_{L2} \le \alpha < \alpha_{R2} \end{cases} \Leftrightarrow \begin{cases} q_1 = q_{1b} = \dfrac{1}{3}(a - \dfrac{\beta}{\alpha}) \\ q_2 = q_{2b} = \dfrac{1}{3}(a - \dfrac{\beta}{\alpha}) \\ \max\left\{\dfrac{\beta}{a - 3m_1k_1}, \dfrac{\beta}{a - 3m_2k_2}\right\} \le \alpha < \min\left\{\dfrac{\beta}{a - 3k_1}, \dfrac{\beta}{a - 3k_2}\right\} \end{cases}$$

Case-VI

$$\begin{cases} q_1 = k_1 \\ q_2 = q_{2b} \\ \alpha_{R1} \le \alpha \\ \alpha_{L2} \le \alpha < \alpha_{R2} \end{cases} \Leftrightarrow \begin{cases} q_1 = k_1 \\ q_2 = q_{2b} = \dfrac{1}{2}(a - \dfrac{\beta}{\alpha} - k_1) \\ \max\left\{ \dfrac{\beta}{a - 3k_1}, \dfrac{\beta}{a - k_1 - 2m_2k_2} \right\} \le \alpha < \dfrac{\beta}{a - k_1 - 2k_2} \end{cases}$$

Case-VII

$$\begin{cases} q_1 = m_1k_1 \\ q_2 = k_2 \\ 0 \le \alpha < \alpha_{L1} \\ \alpha_{R2} \le \alpha \end{cases} \Leftrightarrow \begin{cases} q_1 = m_1k_1 \\ q_2 = k_2 \\ \dfrac{\beta}{a - m_1k_1 - 2k_2} \le \alpha < \dfrac{\beta}{a - 2m_1k_1 - k_2} \end{cases}$$

Case-VIII

$$\begin{cases} q_1 = q_{1b} \\ q_2 = k_2 \\ \alpha_{L1} \le \alpha < \alpha_{R1} \\ \alpha_{R2} \le \alpha \end{cases} \Leftrightarrow \begin{cases} q_1 = q_{1b} = \dfrac{1}{2}(a - \dfrac{\beta}{\alpha} - k_2) \\ q_2 = k_2 \\ \max\left\{ \dfrac{\beta}{a - 2m_1k_1 - k_2}, \dfrac{\beta}{a - 3k_2} \right\} \le \alpha < \dfrac{\beta}{a - 2k_1 - k_2} \end{cases}$$

Case-IX

$$\begin{cases} q_1 = k_1 \\ q_2 = k_2 \\ \alpha_{R1} \le \alpha \\ \alpha_{R2} \le \alpha \end{cases} \Leftrightarrow \begin{cases} q_1 = k_1 \\ q_2 = k_2 \\ \max\left\{ \dfrac{\beta}{a - 2k_1 - k_2}, \dfrac{\beta}{a - k_1 - 2k_2} \right\} \le \alpha \end{cases}$$

Since $0 \le m_1 \le m_2 \le 1$, there are five possible situations.

Situation 1 $m_2k_2 \le k_2 < m_1k_1 \le k_1$

Among cases I - IX, only five possible cases hold. They can be connected as I→IV→VII→VIII→IX for increasing α in $[0, \infty)$. This results in the solution (q_1, q_2) in Case (i).

Situation 2 $m_2k_2 < m_1k_1 \le k_2 \le k_1 \Leftrightarrow m_2k_2 < m_1k_1 \le k_2 < k_1$

Among cases I - IX, only five possible cases hold. They can be connected as I→IV→V→VIII→IX for increasing α in $[0,\infty)$. This results in the solution (q_1, q_2) in Case (ii).

Situation 3 $m_1k_1 \le m_2k_2 \le k_2 \le k_1$

Among cases I - IX, only five possible cases hold. They can be connected as I→II→V→VIII→IX for increasing α in $[0,\infty)$. This results in the solution (q_1, q_2) in Case (iii).

Situation 4 $m_1k_1 \le m_2k_2 < k_1 < k_2$

Among cases I - IX, only five possible cases hold. They can be connected as I→II→V→VI→IX for increasing α in $[0,\infty)$. This results in the solution (q_1, q_2) in Case (iv).

Situation 5 $m_1k_1 \le k_1 \le m_2k_2 < k_2$

Among cases I - IX, only five possible cases hold. They can be connected as I→II→III→VI→IX for increasing α in $[0,\infty)$. This results in the solution (q_1, q_2) in Case (v).

This completes the proof of Proposition 7.3. □

Theorem 7.2 Given $m_1 = m_2 = m$, $0 \le m \le 1$, then

(i) the optimal capacity of firm 1 and firm 2 are $k_1 = k_2 = k_e$ at equilibrium;

(ii) k_e is decreasing in $m \in [0,1]$, i.e., k_e is increasing in flexibility degree η, $\eta = 1 - m$;

(iii) $k_f \le k_e \le k_0$ where k_0 satisfies $\int_{\frac{\beta}{a-3k_0}}^{\infty} (\alpha(a-3k_0)-\beta) f(\alpha) d\alpha = C_F$ and

$$k_f = \frac{1}{3}\left(a - \frac{C_F + \beta}{\mu}\right);$$

(iv) The expected profit of each firm is $\Pi_1 = \Pi_2 = \Pi_e$, where

$$\Pi_e = \int_0^{\frac{\beta}{a-3mk_e}} (mk_e)^2 \alpha f(\alpha) d\alpha + \frac{1}{9}\int_{\frac{\beta}{a-3mk_e}}^{\frac{\beta}{a-3k_e}} \left(a - \frac{\beta}{\alpha}\right)^2 \alpha f(\alpha) d\alpha + \int_{\frac{\beta}{a-3k_e}}^{\infty} k_e^2 \alpha f(\alpha) d\alpha$$

and k_e satisfies

$$m\int_0^{\frac{\beta}{a-3mk_e}} (\alpha(a-3mk_e)-\beta) f(\alpha) d\alpha + \int_{\frac{\beta}{a-3k_e}}^{\infty} (\alpha(a-3k_e)-\beta) f(\alpha) d\alpha = C_F.$$

Proof

Without loss of generality, we assume $k_1 \geq k_2$. Thus, we do not need to consider situations D and E. Since the optimal capacities satisfy $\Pi_1^{(1)}(k_1) = 0$ and $\Pi_2^{(1)}(k_2) = 0$, we have $\Pi_1^{(1)}(k_1) - \Pi_2^{(1)}(k_2) = 0$. The analysis for situations A - C are as follows.

For Situation A $mk_2 \leq k_2 < mk_1 \leq k_1$

In this case, $m \neq 0$; otherwise, $0 < k_2 < mk_1 = 0$, which is a contradiction.

By Claim 7.1, we have $\Pi_1^{(1)}(k_1) - \Pi_2^{(1)}(k_2) = \sum_{i=1}^{5} \varsigma_{Ai} = 0$, where

$$\varsigma_{A1} = m^2(k_2 - k_1)\int_0^{\frac{\beta}{a-m(k_1+2k_2)}} \alpha f(\alpha)d\alpha \leq 0;$$

$$\varsigma_{A2} = \frac{1}{2}m\int_{\frac{\beta}{a-m(k_1+2k_2)}}^{\frac{\beta}{a-mk_1-2k_2}} [\alpha(a-3mk_1)-\beta]f(\alpha)d\alpha$$

$$= \frac{1}{2}m\int_{\frac{\beta}{a-m(k_1+2k_2)}}^{\frac{\beta}{a-mk_1-2k_2}} [\alpha(a-mk_1-2k_2)-\beta]f(\alpha)d\alpha + m(k_2 - mk_1)\int_{\frac{\beta}{a-m(k_1+2k_2)}}^{\frac{\beta}{a-mk_1-2k_2}} \alpha f(\alpha)d\alpha$$

$$\leq m(k_2 - mk_1)\int_{\frac{\beta}{a-m(k_1+2k_2)}}^{\frac{\beta}{a-mk_1-2k_2}} \alpha f(\alpha)d\alpha \leq 0;$$

$$\varsigma_{A3} = \int_{\frac{\beta}{a-mk_1-2k_2}}^{\frac{\beta}{a-2mk_1-k_2}} [\alpha((m-1)a + (1-2m)mk_1 + (2-m)k_2) - (m-1)\beta]f(\alpha)d\alpha$$

$$= (m-1)\int_{\frac{\beta}{a-mk_1-2k_2}}^{\frac{\beta}{a-2mk_1-k_2}} [\alpha(a-mk_1-2k_2)-\beta]f(\alpha)d\alpha + m(k_2 - mk_1)\int_{\frac{\beta}{a-mk_1-2k_2}}^{\frac{\beta}{a-2mk_1-k_2}} \alpha f(\alpha)d\alpha$$

$$\leq m(k_2 - mk_1)\int_{\frac{\beta}{a-mk_1-2k_2}}^{\frac{\beta}{a-2mk_1-k_2}} \alpha f(\alpha)d\alpha \leq 0;$$

$$\varsigma_{A4} = -\frac{1}{2}\int_{\frac{\beta}{a-2mk_1-k_2}}^{\frac{\beta}{a-2k_1-k_2}} [\alpha(a-3k_2)-\beta]f(\alpha)d\alpha$$

$$= -\frac{1}{2}\int_{\frac{\beta}{a-2mk_1-k_2}}^{\frac{\beta}{a-2k_1-k_2}} [\alpha(a-2mk_1-k_2)-\beta)]f(\alpha)d\alpha + (k_2 - mk_1)\int_{\frac{\beta}{a-2mk_1-k_2}}^{\frac{\beta}{a-2k_1-k_2}} \alpha f(\alpha)d\alpha$$

$$\leq (k_2 - mk_1)\int_{\frac{\beta}{a-2mk_1-k_2}}^{\frac{\beta}{a-2k_1-k_2}} \alpha f(\alpha)d\alpha \leq 0;$$

$$\varsigma_{A5} = (k_2 - k_1)\int_{\frac{\beta}{a-2k_1-k_2}}^{\infty} \alpha f(\alpha)d\alpha < 0.$$

Thus, $\sum_{i=1}^{5} \varsigma_{Ai} < 0$, which is a contradiction.

For Situation B $mk_2 < mk_1 \leq k_2 < k_1$

In this case, $m \neq 0$; otherwise, $0 = mk_2 < mk_1 = 0$, which is a contradiction.

By Claim 7.1, we have $\Pi_1^{(1)}(k_1) - \Pi_2^{(1)}(k_2) = \sum_{i=1}^{4} \varsigma_{Bi} = 0$, where

$$\varsigma_{B1} = m^2(k_2 - k_1)\int_0^{\frac{\beta}{a-mk_1-2mk_2}} \alpha f(\alpha)d\alpha \le 0;$$

$$\varsigma_{B2} = \frac{1}{2}m\int_{\frac{\beta}{a-mk_1-2mk_2}}^{\frac{\beta}{a-3mk_1}} [\alpha(a-3mk_1)-\beta] f(\alpha)d\alpha \le 0;$$

$$\varsigma_{B3} = -\frac{1}{2}\int_{\frac{\beta}{a-3k_2}}^{\frac{\beta}{a-2k_1-k_2}} [\alpha(a-3k_2)-\beta] f(\alpha)d\alpha \le 0;$$

$$\varsigma_{B4} = (k_2 - k_1)\int_{\frac{\beta}{a-2k_1-k_2}}^{\infty} \alpha f(\alpha)d\alpha < 0.$$

Thus, $\sum_{i=1}^{4} \varsigma_{Bi} < 0$, which is a contradiction.

For Situation C $mk_1 \le mk_2 \le k_2 \le k_1$, by Claim 7.1, we have

If $m_1 = m_2 = m$, Situation C is the line $k_1 = k_2 = k_e$. Therefore,

$$\Pi_1^{(1)}(k_1) = m\int_0^{\frac{\beta}{a-3mk_e}} (\alpha(a-3mk_e)-\beta) f(\alpha)d\alpha + \int_{\frac{\beta}{a-3k_e}}^{\infty} (\alpha(a-3k_e)-\beta) f(\alpha)d\alpha - C_F,$$

$$\Pi_2^{(1)}(k_2) = m\int_0^{\frac{\beta}{a-3mk_e}} (\alpha(a-3mk_e)-\beta) f(\alpha)d\alpha + \int_{\frac{\beta}{a-3k_e}}^{\infty} (\alpha(a-3k_e)-\beta) f(\alpha)d\alpha - C_F.$$

So the optimal solution satisfies $\Pi_1^{(1)}(k_1) = \Pi_2^{(1)}(k_2) = 0$, i.e.,

$$C_F = m\int_0^{\frac{\beta}{a-3mk_e}} (\alpha(a-3mk_e)-\beta) f(\alpha)d\alpha + \int_{\frac{\beta}{a-3k_e}}^{\infty} (\alpha(a-3k_e)-\beta) f(\alpha)d\alpha.$$

By Propositions 7.2 and 7.3, the expected profit of each firm is $\Pi_1 = \Pi_2 = \Pi_e$, where

$$\Pi_e = \int_0^{\frac{\beta}{a-3mk_e}} (mk_e)^2 \alpha f(\alpha)d\alpha + \frac{1}{9}\int_{\frac{\beta}{a-3mk_e}}^{\frac{\beta}{a-3k_e}} (a-\frac{\beta}{\alpha})^2 \alpha f(\alpha)d\alpha + \int_{\frac{\beta}{a-3k_e}}^{\infty} k_e^2 \alpha f(\alpha)d\alpha.$$

With respect to m, we have

$$\int_0^{\frac{\beta}{a-3mk_e}} (\alpha(a-6mk_e)-\beta) f(\alpha)d\alpha - \left(3m^2\int_0^{\frac{\beta}{a-3mk_e}} \alpha f(\alpha)d\alpha + 3\int_{\frac{\beta}{a-3k_e}}^{\infty} f(\alpha)d\alpha\right)\frac{dk_e}{dm} = 0.$$

Therefore, $\frac{dk_e}{dm} \le 0$, $m \in [0,1]$. That means k_e is decreasing in $m \in [0,1]$.

When $m=0$, set $k_0 = k_e(m=0)$, then it can be proved that there exists a unique k_0 satisfying $\int_{\frac{\beta}{a-3k_0}}^{\infty} \big(\alpha(a-3k_0)-\beta\big) f(\alpha)d\alpha = C_F$. When $m=1$, set $k_f = k_e(m=1)$, then $\int_0^{\frac{\beta}{a-3k_f}} \big(\alpha(a-3k_f)-\beta\big) f(\alpha)d\alpha + \int_{\frac{\beta}{a-3k_f}}^{\infty} \big(\alpha(a-3k_f)-\beta\big) f(\alpha)d\alpha = C_F$, i.e., $(a-3k_f)\mu - \beta = C_F$. Therefore, we have $k_f = \frac{1}{3}\left(a - \frac{C_F+\beta}{\mu}\right)$. Hence, with given $m \in [0,1]$, $k_f \le k_e \le k_0$.

This completes the proof of Theorem 7.2. □

Proposition 7.4 Given $0 \le m_1 < m_2 \le 1$, then the optimal solution $(k_1,\ k_2)$ is not in situation E.

<u>Proof</u>

For situation E $m_1k_1 \le k_1 \le m_2k_2 < k_2$, by Claim 7.1 we have

$$
\begin{aligned}
&\Pi_1^{(1)}(k_1) - \Pi_2^{(1)}(k_2) \\
&= \int_0^{\frac{\beta}{a-2m_1k_1-m_2k_2}} [m_1\alpha(a-2m_1k_1-m_2k_2-\frac{\beta}{\alpha}) - m_2\alpha(a-m_1k_1-2m_2k_2-\frac{\beta}{\alpha})] f(\alpha)d\alpha \\
&\quad - \int_{\frac{\beta}{a-2m_1k_1-m_2k_2}}^{\frac{\beta}{a-2k_1-m_2k_2}} \frac{1}{2} m_2\alpha(a-3m_2k_2-\frac{\beta}{\alpha}) f(\alpha)d\alpha \\
&\quad + \int_{\frac{\beta}{a-2k_1-m_2k_2}}^{\frac{\beta}{a-k_1-2m_2k_2}} \alpha[(a-2k_1-m_2k_2-\frac{\beta}{\alpha}) - m_2(a-k_1-2m_2k_2-\frac{\beta}{\alpha})] f(\alpha)d\alpha \\
&\quad + \int_{\frac{\beta}{a-k_1-2m_2k_2}}^{\frac{\beta}{a-k_1-2k_2}} \frac{1}{2}\alpha(a-3k_1-\frac{\beta}{\alpha}) f(\alpha)d\alpha + \int_{\frac{\beta}{a-k_1-2k_2}}^{\infty} \alpha(k_2-k_1) f(\alpha)d\alpha
\end{aligned}
$$

Set

$$\varsigma_{E1} = \int_0^{\frac{\beta}{a-2m_1k_1-m_2k_2}} \left(m_1\alpha(a-2m_1k_1-m_2k_2-\frac{\beta}{\alpha}) - m_2\alpha(a-m_1k_1-2m_2k_2-\frac{\beta}{\alpha}) \right) f(\alpha)d\alpha;$$

$$\varsigma_{E2} = -\int_{\frac{\beta}{a-2m_1k_1-m_2k_2}}^{\frac{\beta}{a-2k_1-m_2k_2}} \frac{1}{2} m_2\alpha(a-3m_2k_2-\frac{\beta}{\alpha}) f(\alpha)d\alpha;$$

$$\varsigma_{E3} = \int_{\frac{\beta}{a-2k_1-m_2k_2}}^{\frac{\beta}{a-k_1-2m_2k_2}} \alpha\left((a-2k_1-m_2k_2-\frac{\beta}{\alpha}) - m_2(a-k_1-2m_2k_2-\frac{\beta}{\alpha}) \right) f(\alpha)d\alpha;$$

$$\varsigma_{E4} = \int_{\frac{\beta}{a-k_1-2m_2k_2}}^{\frac{\beta}{a-k_1-2k_2}} \frac{1}{2}\alpha(a-3k_1-\frac{\beta}{\alpha})f(\alpha)d\alpha \text{ ; and}$$

$$\varsigma_{E5} = \int_{\frac{\beta}{a-k_1-2k_2}}^{\infty} \alpha(k_2-k_1)f(\alpha)d\alpha .$$

In the following, we analysis these 5 terms respectively,

$$\varsigma_{E1} = \int_0^{\frac{\beta}{a-2m_1k_1-m_2k_2}} \left(m_1\alpha(a-2m_1k_1-m_2k_2-\frac{\beta}{\alpha}) - m_2\alpha(a-m_1k_1-2m_2k_2-\frac{\beta}{\alpha}) \right) f(\alpha)d\alpha$$

$$= \int_0^{\frac{\beta}{a-2m_1k_1-m_2k_2}} (m_1-m_2)\alpha \left(a + \frac{2(m_2^2k_2-m_1^2k_1)}{m_1-m_2} + \frac{m_1m_2(k_1-k_2)}{m_1-m_2} - \frac{\beta}{\alpha} \right) f(\alpha)d\alpha$$

$$a + \frac{2(m_2^2k_2-m_1^2k_1)}{m_1-m_2} + \frac{m_1m_2(k_1-k_2)}{m_1-m_2} - \left(a-2m_1k_1-m_2k_2\right) = \frac{m_2(m_2k_2-m_1k_1)}{m_1-m_2} < 0$$

So that $a + \frac{2(m_2^2k_2-m_1^2k_1)}{m_1-m_2} + \frac{m_1m_2(k_1-k_2)}{m_1-m_2} < a-2m_1k_1-m_2k_2$. As $0 < \alpha < \frac{\beta}{a-2m_1k_1-m_2k_2}$, we have $\frac{\beta}{\alpha} > a-2m_1k_1-m_2k_2$, and so $a + \frac{2(m_2^2k_2-m_1^2k_1)}{m_1-m_2} + \frac{m_1m_2(k_1-k_2)}{m_1-m_2} < \frac{\beta}{\alpha}$. Therefore, $\varsigma_{E1} \geq 0$.

$$\varsigma_{E2} = -\int_{\frac{\beta}{a-2m_1k_1-m_2k_2}}^{\frac{\beta}{a-2k_1-m_2k_2}} \frac{1}{2}m_2\alpha(a-3m_2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha .$$

As $\frac{\beta}{a-2m_1k_1-m_2k_2} < \alpha < \frac{\beta}{a-2k_1-m_2k_2}$, then $\frac{\beta}{\alpha} > a-2k_1-m_2k_2 \geq a-3m_2k_2$.

Therefore, $\varsigma_{E2} \geq 0$.

$$\varsigma_{E3} = \int_{\frac{\beta}{a-2k_1-m_2k_2}}^{\frac{\beta}{a-k_1-2m_2k_2}} \alpha\left((a-2k_1-m_2k_2-\frac{\beta}{\alpha}) - m_2(a-k_1-2m_2k_2-\frac{\beta}{\alpha}) \right) f(\alpha)d\alpha$$

$$\geq \int_{\frac{\beta}{a-2k_1-m_2k_2}}^{\frac{\beta}{a-k_1-2m_2k_2}} \alpha\left((a-2k_1-m_2k_2-\frac{\beta}{\alpha}) - (a-k_1-2m_2k_2-\frac{\beta}{\alpha}) \right) f(\alpha)d\alpha$$

$$= \int_{\frac{\beta}{a-2k_1-m_2k_2}}^{\frac{\beta}{a-k_1-2m_2k_2}} \alpha\left(m_2k_2-k_1\right) f(\alpha)d\alpha \geq 0 .$$

$\varsigma_{E4} = \int_{\frac{\beta}{a-k_1-2m_2k_2}}^{\frac{\beta}{a-k_1-2k_2}} \frac{1}{2}\alpha(a-3k_1-\frac{\beta}{\alpha})f(\alpha)d\alpha$. As $\frac{\beta}{a-k_1-2m_2k_2} < \alpha < \frac{\beta}{a-k_1-2k_2}$, then $\frac{\beta}{\alpha} < a-k_1-2m_2k_2 \le a-3k_1$. Therefore, $\varsigma_{E4} \ge 0$.

$\varsigma_{E5} = \int_{\frac{\beta}{a-k_1-2k_2}}^{\infty} \alpha(k_2-k_1)f(\alpha)d\alpha > 0$.

Therefore, $\Pi_1^{(1)}(k_1) - \Pi_2^{(1)}(k_2) > 0$.

Therefore, the optimal solution $(k_1,\ k_2)$ is not in situation E.

This completes the proof of Proposition 7.4. □

Proposition 7.5 Given $0 \le m_1 \le m_2 \le 1$, then the optimal solution $(k_1,\ k_2)$ is not in situation D.

Proof

Situation D $m_1k_1 \le m_2k_2 < k_1 < k_2$, then by Claim 7.1, we have

$\Pi_1^{(1)}(k_1) - \Pi_2^{(1)}(k_2)$

$$= \int_0^{\frac{\beta}{a-2m_1k_1-m_2k_2}} \left(m_1\alpha(a-2m_1k_1-m_2k_2-\frac{\beta}{\alpha}) - m_2\alpha(a-m_1k_1-2m_2k_2-\frac{\beta}{\alpha}) \right) f(\alpha)d\alpha$$

$$-\int_{\frac{\beta}{a-2m_1k_1-m_2k_2}}^{\frac{\beta}{a-3m_2k_2}} \frac{1}{2}m_2\alpha(a-3m_2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha + \int_{\frac{\beta}{a-3k_1}}^{\frac{\beta}{a-k_1-2k_2}} \frac{1}{2}\alpha(a-3k_1-\frac{\beta}{\alpha})f(\alpha)d\alpha$$

$$+\int_{\frac{\beta}{a-k_1-2k_2}}^{\infty} \alpha(k_2-k_1)f(\alpha)d\alpha > 0.$$

Therefore, the optimal solution $(k_1,\ k_2)$ is not in situation D.

This completes the proof of Proposition 7.5. □

Theorem 7.3 Given $0 \le m_1 \le m_2 \le 1$,

(i) if $\int_{\frac{\beta}{a}}^{\infty} \alpha(a-\frac{\beta}{\alpha})f(\alpha)d\alpha \le C_F$, then $k_{1e} = k_{2e} = 0$ and $\Pi_1 = \Pi_2 = 0$;

(ii) if $\int_{\frac{\beta}{a}}^{\infty} \alpha(a-\frac{\beta}{\alpha})f(\alpha)d\alpha > C_F$, then $k_{1e} > 0$, $k_{2e} > 0$ and $\frac{m_1}{m_2}k_{1e} < k_{2e} < k_{1e}$;

(ii-1) the optimal productions $(q_1^* \quad q_2^*)$ are

$$(q_1^* \quad q_2^*) = \begin{cases} (m_1k_1 \quad m_2k_2), & \alpha \le \alpha_{L1} \\ (q_{1b-1} \quad m_2k_2), & \alpha_{L1} < \alpha < \alpha_{L2} \\ (q_{1b-2} \quad q_{2b}), & \alpha_{L2} \le \alpha \le \alpha_{R2} \\ (q_{1b-3} \quad k_2), & \alpha_{R2} < \alpha < \alpha_{R1} \\ (k_1 \quad k_2), & \alpha_{R1} \le \alpha \end{cases}.$$

(ii-2) the optimal capacity decisions $(k_1,\ k_2) = (k_{1e},\ k_{2e})$ at equilibrium satisfy

$$\int_0^{\alpha_{L1}} m_1\alpha(a-2m_1k_{1e}-m_2k_{2e}-\frac{\beta}{\alpha})f(\alpha)d\alpha + \int_{\alpha_{R1}}^{\infty} \alpha(a-2k_{1e}-k_{2e}-\frac{\beta}{\alpha})f(\alpha)d\alpha = C_F$$

and

$$\int_0^{\alpha_{L1}} m_2\alpha(a-m_1k_{1e}-2m_2k_{2e}-\frac{\beta}{\alpha})f(\alpha)d\alpha + \int_{\alpha_{L1}}^{\alpha_{L2}} \frac{1}{2}m_2\alpha(a-3m_2k_{2e}-\frac{\beta}{\alpha})f(\alpha)d\alpha$$

$$+\int_{\alpha_{R2}}^{\alpha_{R1}} \frac{1}{2}\alpha(a-3k_{2e}-\frac{\beta}{\alpha})f(\alpha)d\alpha + \int_{\alpha_{R1}}^{\infty} \alpha(a-k_{1e}-2k_{2e}-\frac{\beta}{\alpha})f(\alpha)d\alpha = C_F;$$

(ii-3) the optimal profits of firm 1 and firm 2 are

$$\Pi_1(k_1) = (m_1k_1)^2\int_0^{\alpha_{L1}} \alpha f(\alpha)d\alpha + \int_{\alpha_{L1}}^{\alpha_{L2}} q_{1b-1}^2\alpha f(\alpha)d\alpha + \int_{\alpha_{L2}}^{\alpha_{R2}} q_{1b-2}^2\alpha f(\alpha)d\alpha$$

$$+\int_{\alpha_{R2}}^{\alpha_{R1}} q_{1b-3}^2\alpha f(\alpha)d\alpha + \int_{\alpha_{R1}}^{\infty} k_1^2\alpha f(\alpha)d\alpha;$$

$$\Pi_2(k_2) = (m_2k_2)^2\int_0^{\alpha_{L2}} \alpha f(\alpha)d\alpha + \int_{\alpha_{L2}}^{\alpha_{R2}} q_{2b}^2\alpha f(\alpha)d\alpha + \int_{\alpha_{R2}}^{\infty} k_2^2\alpha f(\alpha)d\alpha;$$

where $q_{1b-1} = \frac{1}{2}(a-m_2k_2-\frac{\beta}{\alpha})$, $q_{1b-2} = \frac{1}{3}(a-\frac{\beta}{\alpha})$, $q_{1b-3} = \frac{1}{2}(a-k_2-\frac{\beta}{\alpha})$, $q_{2b} = \frac{1}{3}(a-\frac{\beta}{\alpha})$; $\alpha_{L1} = \frac{\beta}{a-2m_1k_{1e}-m_2k_{2e}}$, $\alpha_{R1} = \frac{\beta}{a-2k_{1e}-k_{2e}}$; $\alpha_{L2} = \frac{\beta}{a-3m_2k_{2e}}$, $\alpha_{R2} = \frac{\beta}{a-3k_{2e}}$.

Proof

Define $J_1 = \Pi_1^{(1)}(k_1)$ and $J_2 = \Pi_2^{(1)}(k_2)$. The optimal solution $(k_1,\ k_2)$ satisfies $J_1 = \Pi_1^{(1)}(k_1) = 0$ and $J_2 = \Pi_2^{(1)}(k_2) = 0$. Set $y = k_2 - \frac{m_1}{m_2}k_1$, i.e., $k_2 = y + \frac{m_1}{m_2}k_1$. It is noted that situation A we have $0 < k_2 < m_1k_1$; for situation B we have $m_1k_1 < k_2 < \frac{m_1}{m_2}k_1$;

for situation C we have $\frac{m_1}{m_2}k_1 < k_2 < k_1$. Therefore, (1) if $y > 0$, then the optimal solution is in situation C; (2) if $y < 0$, then the optimal situation is in situation A or situation B.

Given m_1, $m_1 \le m_2 \le 1$. Consider k_1 and y as functions of m_2. Then we have $\begin{cases} J_1\big(k_1(m_2),\ \ y(m_2),\ \ m_2\big) = 0 \\ J_2\big(k_1(m_2),\ \ y(m_2),\ \ m_2\big) = 0 \end{cases}$. With respect to m_2, we have

$$\begin{cases} \frac{\partial J_1}{\partial k_1}\frac{dk_1}{dm_2} + \frac{\partial J_1}{\partial y}\frac{dy}{dm_2} + \frac{\partial J_1}{\partial m_2} = 0 \\ \frac{\partial J_2}{\partial k_1}\frac{dk_1}{dm_2} + \frac{\partial J_2}{\partial y}\frac{dy}{dm_2} + \frac{\partial J_2}{\partial m_2} = 0 \end{cases}.$$

Therefore, if $\Delta \ne 0$, we have $\begin{cases} \frac{dk_1}{dm_2} = \frac{\Delta_1}{\Delta} \\ \frac{dy}{dm_2} = \frac{\Delta_2}{\Delta} \end{cases}$, (a7.1)

where $\Delta = \begin{vmatrix} \frac{\partial J_1}{\partial k_1} & \frac{\partial J_1}{\partial y} \\ \frac{\partial J_2}{\partial k_1} & \frac{\partial J_2}{\partial y} \end{vmatrix}$, (a7.2)

$$\Delta_1 = \begin{vmatrix} -\frac{\partial J_1}{\partial m_2} & \frac{\partial J_1}{\partial y} \\ -\frac{\partial J_2}{\partial m_2} & \frac{\partial J_2}{\partial y} \end{vmatrix}, \qquad (a7.3)$$

$$\Delta_2 = \begin{vmatrix} \frac{\partial J_1}{\partial k_1} & -\frac{\partial J_1}{\partial m_2} \\ \frac{\partial J_2}{\partial k_1} & -\frac{\partial J_2}{\partial m_2} \end{vmatrix}. \qquad (a7.4)$$

Consider situation C $\frac{m_1}{m_2}k_1 < k_2 < k_1$. Substitute $k_2 = y + \frac{m_1}{m_2}k_1$ into $J_1 = \Pi_1^{(1)}(k_1|k_2) = 0$ and $J_2 = \Pi_2^{(1)}(k_2|k_1) = 0$ we can get:

$$J_1 = \Pi_1^{(1)}(k_1|k_2) = m_1\int_0^{\alpha_{L1}} \big(\alpha(a - 3m_1k_1 - m_2 y) - \beta\big) f(\alpha)d\alpha$$

$$+\int_{\alpha_{R1}}^{\infty}\left(\alpha(a-(2+\frac{m_1}{m_2})k_1-y)-\beta\right)f(\alpha)d\alpha-C_F;$$

$$J_2=\Pi_2^{(1)}(k_1|k_2)=m_2\int_0^{\alpha_{L1}}\left(\alpha(a-3m_1k_1-2m_2y)-\beta\right)f(\alpha)d\alpha$$

$$+\frac{1}{2}m_2\int_{\alpha_{L1}}^{\alpha_{L2}}(\alpha(a-3m_1k_1-3m_2y)-\beta)f(\alpha)d\alpha$$

$$+\frac{1}{2}\int_{\alpha_{R2}}^{\alpha_{R1}}(\alpha(a-3\frac{m_1}{m_2}k_1-3y)-\beta)f(\alpha)d\alpha$$

$$+\int_{\alpha_{R1}}^{\infty}(\alpha(a-(1+2\frac{m_1}{m_2})k_1-2y)-\beta)f(\alpha)d\alpha-C_F.$$

Define $\mu_{ab}=\int_a^b\alpha f(\alpha)d\alpha$. Then we have: $\frac{\partial J_1}{\partial k_1}=-3m_1^2\mu_{0L1}-(2+\frac{m_1}{m_2})\mu_{R1\infty}$;

$\frac{\partial J_1}{\partial y}=-3m_1m_2\mu_{0L1}-\mu_{R1\infty}$; $\frac{\partial J_1}{\partial m_2}=-m_1y\mu_{0L1}+\frac{m_1}{m_2^2}k_1\mu_{R1\infty}$;

$\frac{\partial J_2}{\partial k_1}=-3m_1m_2\mu_{0L1}-\frac{3}{2}m_1m_2\mu_{L1L2}-\frac{3m_1}{2m_2}\mu_{R2R1}-(1+\frac{2m_1}{m_2})\mu_{R1\infty}$;

$\frac{\partial J_2}{\partial y}=-2m_2^2\mu_{0L1}-\frac{3}{2}m_2^2\mu_{L1L2}-\frac{3}{2}\mu_{R2R1}-2\mu_{R1\infty}$;

$$\frac{\partial J_2}{\partial m_2}=\int_0^{\alpha_{L1}}\left(\alpha(a-3m_1k_1-4m_2y)-\beta\right)f(\alpha)d\alpha$$

$$+\frac{1}{2}\int_{\alpha_{L1}}^{\alpha_{L2}}(\alpha(a-3m_1k_1-6m_2y)-\beta)f(\alpha)d\alpha-\frac{3m_1k_1}{2m_2^2}\mu_{R2R1}-\frac{2m_1k_1}{m_2^2}\mu_{R1\infty}.$$

Therefore, by (a7.2) and (a7.4) we have

$$\Delta=\frac{\partial J_1}{\partial k_1}\cdot\frac{\partial J_2}{\partial y}-\frac{\partial J_1}{\partial y}\cdot\frac{\partial J_2}{\partial k_1}$$

$$=3m_1^2m_2^2\mu_{0L1}^2+3m_1^2m_2^2\mu_{0L1}\mu_{L1L2}+3m_1^2\mu_{0L1}\mu_{R2R1}+2(m_1^2-m_1m_2+m_2^2)\mu_{0L1}\mu_{R1\infty}$$

$$+3m_2^2\mu_{L1L2}\mu_{R1\infty}+3\mu_{R2R1}\mu_{R1\infty}+3\mu_{R1\infty}^2>0$$

$$\Delta_2=-\frac{\partial J_1}{\partial k_1}\cdot\frac{\partial J_2}{\partial m_2}+\frac{\partial J_1}{\partial m_2}\cdot\frac{\partial J_2}{\partial k_1}. \quad \text{(a7.5)}$$

By (a7.1), we have $\frac{dy}{dm_2}=\frac{\Delta_2}{\Delta}$. Consider the situation of $y=0$, i.e., $m_1k_1=m_2k_2$.

Therefore, (1) if $\Delta_2>0$, then $\frac{dy}{dm_2}=\frac{\Delta_2}{\Delta}>0$, which implies only situation C occurs; (2)

if $\Delta_2 < 0$, then $\frac{dy}{dm_2} = \frac{\Delta_2}{\Delta} < 0$ which implies that situation B or situation A may occur.

For situation of $m_1k_1 = m_2k_2$, there are two cases (1) $m_1k_1 = m_2k_2$ and $m_1 = m_2$; (2) $m_1k_1 = m_2k_2$ and $m_1 < m_2$.

Case-i if $m_1k_1 = m_2k_2$ and $m_1 = m_2$.

By Theorem 7.2, we have $k_1 = k_2 = k \in [k_f,\ k_0]$, and $\mu_{R2R1} = 0$. By (a7.5) we have

$$\Delta_2 = 3(m_1\mu_{0b1} + \frac{1}{m_1}\mu_{R1\infty})\left(m_1\int_0^{\alpha_{L1}}(\alpha(a-3m_1k_1)-\beta)f(\alpha)d\alpha + k_1\mu_{R1\infty}\right). \tag{a7.6}$$

By the optimal solution's necessary condition $J_1 = \Pi_1^{(1)}(k_1|k_2) = 0$, we have

Since $J_1 = m_1\int_0^{\alpha_{L1}}\left(\alpha(a-3m_1k_1)-\beta\right)f(\alpha)d\alpha + \int_{\alpha_{R1}}^{\infty}\left(\alpha(a-3k_1)-\beta\right)f(\alpha)d\alpha - C_F = 0$.

Together with (a7.6) we have

$$\Delta_2 = 3(m_1\mu_{0b1} + \frac{1}{m_1}\mu_{R1\infty})\left(C_F - \int_{\alpha_{R1}}^{\infty}(\alpha(a-3k_1)-\beta)f(\alpha)d\alpha + k_1\mu_{R1\infty}\right)$$

$= 3(m_1\mu_{0b1} + \frac{1}{m_1}\mu_{R1\infty})\left(C_F - \int_{\alpha_{R1}}^{\infty}\left(\alpha(a-4k_1)-\beta\right)f(\alpha)d\alpha\right)$. By Proposition 7.5, we have $k_1 \ge k_f = \frac{1}{3}(a - \frac{C_F+\beta}{\mu})$, i.e., $a - 4k_1 \le -\frac{1}{3}(a - \frac{4(C_F+\beta)}{\mu})$. Assume that $a > \frac{4(C_F+\beta)}{\mu}$, we have $a - 4k_1 < 0$.

Then $C_F - \int_{\alpha_{R1}}^{\infty}\left(\alpha(a-4k_1)-\beta\right)f(\alpha)d\alpha > 0$. Therefore, $\Delta_2 > 0$, i.e., $\frac{dy}{dm_2} > 0$.

Therefore, when $m_1 = m_2$, the optimal situation is in Situation C.

Case-ii if $m_1k_1 = m_2k_2$ and $m_1 < m_2$, then we can get

$$m_2J_1 - m_1J_2 = (m_2-m_1)\int_{\alpha_{R1}}^{\infty}\left(\alpha(a - \frac{2(m_1+m_2)}{m_2}k_1) - \beta\right)f(\alpha)d\alpha$$

$$-\frac{m_1}{2}\int_{\alpha_{R2}}^{\alpha_{R1}}\left(\alpha(a-\frac{3m_1}{m_2}k_1)-\beta\right)f(\alpha)d\alpha - (m_2-m_1)C_F.$$

By the optimal solution's necessary condition $\begin{cases} J_1 = 0 \\ J_2 = 0 \end{cases}$, we can have $m_2 J_1 - m_1 J_2 = 0$.

To hold this result, it must have $\int_{\alpha_{R1}}^{\infty}\left(\alpha(a - \frac{2(m_1+m_2)}{m_2}k_1) - \beta\right) f(\alpha)d\alpha > 0$, so that $a - \frac{2(m_1+m_2)}{m_2}k_1 > 0$. Therefore, we have $k_1 < \frac{am_2}{2(m_1+m_2)}$. (a7.7)

Consider situation C

By $J_1 = 0$, we have

$$m_1\int_0^{\alpha_{L1}}(\alpha(a-2m_1k_1-m_2k_2)-\beta)f(\alpha)d\alpha + \int_{\alpha_{R1}}^{\infty}(\alpha(a-2k_1-k_2)-\beta)f(\alpha)d\alpha - C_F = 0 \quad .$$

By $\alpha_{L1} = \frac{\beta}{a-2m_1k_1-m_2k_2}$, we have $m_1\int_0^{\alpha_{L1}}\left(\alpha(a-2m_1k_1-m_2k_2)-\beta\right)f(\alpha)d\alpha < 0$.

Therefore, by $J_1 = 0$, we must have $\int_{\alpha_{R1}}^{\infty}\left(\alpha(a-2k_1-k_2)-\beta\right)f(\alpha)d\alpha > 0$, which requires $a - 2k_1 - k_2 > 0$.

Therefore, we have $2k_1 + k_2 < a$. (a7.8)

By $J_2 = 0$, we have

$$J_2 = \int_0^{\alpha_{L1}} m_2\alpha(a - m_1k_1 - 2m_2k_2 - \frac{\beta}{\alpha})f(\alpha)d\alpha + \int_{\alpha_{L1}}^{\alpha_{L2}}\frac{1}{2}m_2\alpha(a-3m_2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha$$
$$+ \int_{\alpha_{R2}}^{\alpha_{R1}}\frac{1}{2}\alpha(a-3k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha + \int_{\alpha_{R1}}^{\infty}\alpha(a-k_1-2k_2-\frac{\beta}{\alpha})f(\alpha)d\alpha - C_F = 0.$$

Then $\frac{\partial J_2}{\partial m_1} = -m_2k_1\mu_{0L1} < 0$. Therefore, J_2 is decreasing in m_1, $0 \le m_1 < m_2 \le 1$.

Therefore, we have $J_2(m_1 = m_2) \le J_2(m_1, m_2) = 0$. Set $I_2 = J_2(m_1 = m_2)$. Therefore,

$$I_2 = m_2\int_0^{\alpha_{L1}}\left(\alpha(a - m_2(k_1+2k_2)) - \beta\right)f(\alpha)d\alpha + \frac{1}{2}m_2\int_{\alpha_{L1}}^{\alpha_{L2}}\left(\alpha(a-3m_2k_2)-\beta\right)f(\alpha)d\alpha$$
$$+\frac{1}{2}\int_{\alpha_{R2}}^{\alpha_{R1}}\left(\alpha(a-3k_2)-\beta\right)f(\alpha)d\alpha + \int_{\alpha_{R1}}^{\infty}\left(\alpha(a-k_1-2k_2)-\beta\right)f(\alpha)d\alpha - C_F < 0.$$

With respect to m_2, we have $\frac{\partial I_2}{\partial m_2} = \int_0^{\frac{\beta}{a-2m_2(k_1+2k_2)}}\left(\alpha(a-2m_2(k_1+2k_2))-\beta\right)f(\alpha)d\alpha$

$$+\frac{1}{2}\int_{\frac{\beta}{a-2m_2(k_1+2k_2)}}^{\frac{\beta}{a-3m_2k_2}}(\alpha(a-6m_2k_2)-\beta)f(\alpha)d\alpha \le 0.$$

Therefore, $I_2(m_2 = 1) \le I_2 \le 0$. $I_2(m_2 = 1) = \mu(a - 2k_2 - k_1) - \beta - C_F \le 0$, i.e., $2k_2 + k_1 \ge a - \frac{\beta + C_F}{\mu} = 3k_f$. (a7.9)

By (a7.8) and (a7.9), we have $\begin{cases} 2k_1 + k_2 < a \\ 2k_1 + k_2 \ge 3k_f \end{cases}$, i.e., $\begin{cases} k_1 \le \frac{1}{3}(a + \frac{\beta + C_F}{\mu}) \\ k_2 \ge \frac{1}{3}(a - \frac{2(\beta + C_F)}{\mu}) \end{cases}$.

Therefore, we get $\frac{k_2}{k_1} \ge \frac{a - \frac{2(\beta + C_F)}{\mu}}{a + \frac{\beta + C_F}{\mu}}$. Assume $a \ge 5\frac{(\beta + C_F)}{\mu}$,

we have $\frac{k_2}{k_1} \ge \frac{1}{2}$. (a7.10)

Since $m_1 k_1 = m_2 k_2$ and $0 \le m_1 < m_2 \le 1$, we have $k_1 > k_2$. $J_1 = 0$ can be expressed as

$$m_1 \int_0^{\alpha_{L1}} \left(\alpha(a - 3m_1 k_1) - \beta\right) f(\alpha) d\alpha + \int_{\alpha_{R1}}^{\infty} \left(\alpha(a - (2 + \frac{m_1}{m_2})k_1) - \beta\right) f(\alpha) d\alpha - C_F = 0 ,$$

where $\alpha_{L1} = \frac{\beta}{a - 3m_1 k_1}$, $\alpha_{R1} = \frac{\beta}{a - 2k_1 - k_2}$.

Therefore, $\int_0^{\alpha_{L1}} \left(\alpha(a - 3m_1 k_1) - \beta\right) f(\alpha) d\alpha + \int_{\alpha_{R1}}^{\infty} \left(\alpha(a - (2 + \frac{m_1}{m_2})k_1) - \beta\right) f(\alpha) d\alpha \le C_F$,

i.e., $\int_0^{\alpha_{L1}} \left(\alpha(a - (2 + \frac{m_1}{m_2})k_1) - \beta\right) f(\alpha) d\alpha + \int_0^{\alpha_{L1}} \left(\alpha(2 + \frac{m_1}{m_2} - 3m_1)k_1\right) f(\alpha) d\alpha$

$+ \int_{\alpha_{R1}}^{\infty} \left(\alpha(a - (2 + \frac{m_1}{m_2})k_1) - \beta\right) f(\alpha) d\alpha \le C_F$.

Since $\int_{\alpha_{L1}}^{\alpha_{R1}} \left(\alpha(a - (2 + \frac{m_1}{m_2})k_1) - \beta\right) f(\alpha) d\alpha < 0$, we have

$$\int_0^{\infty} \left(\alpha(a - (2 + \frac{m_1}{m_2})k_1) - \beta\right) f(\alpha) d\alpha + \int_0^{\alpha_{L1}} \left(\alpha(2 + \frac{m_1}{m_2} - 3m_1)k_1\right) f(\alpha) d\alpha \le C_F ,$$

i.e., $\mu(a - (2 + \frac{m_1}{m_2})k_1) - \beta + \mu(2 + \frac{m_1}{m_2} - 3m_1)k_1 \le C_F$. Therefore, we can get

$$k_1 \ge \frac{m_2}{2m_2 + m_1}(a - \frac{\beta + C_F}{\mu}) \cdot \frac{1}{1 - \frac{2m_2 + m_1 - 3m_1 m_2}{\mu(2m_2 + m_1)}} > \frac{m_2}{2m_2 + m_1}(a - \frac{\beta + C_F}{\mu}) . \quad \text{(a7.11)}$$

The following discussion is to determine the feasible range of k_1 and k_2.

By (a7.7), (a7.10) and (a7.11), we have $k_1 < \frac{am_2}{2(m_1+m_2)}$, $\frac{k_2}{k_1} \ge \frac{1}{2}$ and $k_1 > \frac{m_2}{2m_2+m_1}(a - \frac{\beta + C_F}{\mu})$. Therefore, we have

$$k_1 - \frac{am_2}{2(m_1+m_2)} > \frac{m_2}{2m_2+m_1}(a - \frac{\beta + C_F}{\mu}) - \frac{am_2}{2(m_1+m_2)}$$

$= \frac{m_2}{2m_2+m_1}\left(\frac{am_1}{2(m_1+m_2)} - \frac{\beta + C_F}{\mu}\right)$. Consider $\frac{am_1}{2(m_1+m_2)} - \frac{\beta + C_F}{\mu}$, we have two cases:

(1) if $\frac{am_1}{2(m_1+m_2)} - \frac{\beta + C_F}{\mu} \ge 0$, then we have $\frac{m_2}{m_1} \le \frac{a\mu}{2(\beta + C_F)} - 1$ and so

$$k_1 > \frac{am_2}{2(m_1+m_2)}. \tag{a7.12}$$

It is noted that (a7.11) and (a7.12) are contradictions to each other. Therefore, the feasible solutions are not in this case.

(2) if $\frac{am_1}{2(m_1+m_2)} - \frac{\beta + C_F}{\mu} < 0$, then we have $\frac{m_2}{m_1} > \frac{a\mu}{2(\beta + C_F)} - 1$. Hence, $\frac{m_1}{m_2} < \frac{\frac{2(\beta + C_F)}{\mu}}{a - \frac{2(\beta + C_F)}{\mu}}$. We assume $a \ge 6\frac{(\beta + C_F)}{\mu}$. Therefore $\frac{m_1}{m_2} < \frac{1}{2}$. As

$$m_1 k_1 = m_2 k_2 \text{, we have } \frac{k_2}{k_1} \ge \frac{1}{2}. \tag{a7.13}$$

(a7.10) and (a7.13) contradict to each other. Therefore, the feasible solutions are not in this case. Therefore, there is no feasible optimal solution in case $m_1 k_1 = m_2 k_2$ and $m_1 < m_2$. Therefore, the only Situation C occurs.

This completes the proof of Theorem 7.3. □

Appendix-II

Papers written and presentations made during my PhD study

Journal Papers

- Yang, L., C.T. Ng and T.C.E. Cheng, 2010. Evaluating the effects of distribution centres on the performance of vendor-managed inventory systems. *European Journal of Operational Research,* 201(1), 112-122.

- Yang, L. and C.T. Ng. Flexible capacity strategy in an asymmetric oligopoly market with competition and demand uncertainty. Submitted to *Management Science*.

- Yang, L., C.T. Ng and T.C.E. Cheng. Optimal production strategy under fluctuating demands: technology versus capacity. Submitted to *Operations Research*.

- Yang, L. and C.T. Ng. Endogenous flexibility of flexible capacity strategy in an n-firm competition under demand uncertainty. *Working Paper*.

- Yang, L. and C.T. Ng. Modeling capacity strategies with different flexibility degrees in a competitive market under fluctuating demands. *Working Paper*.

Conference Presentations

- Yang, L. and C.T. Ng, 2009. Optimal production strategy under fluctuating demands: technology versus capacity. *Proceedings of The 23th European Conference on Operational Research*. 5-8 July 2009, Bonn, German. (CD-ROM).

- Yang, L. and C.T. Ng, 2008. Investments in flexibility and productivity with demand uncertainty. *Presented in International Forum on Shipping, Ports and Airports 2008 Conference*. 25-28 May 2008, HK.

- Yang, L., C.T. Ng and T.C.E. Cheng, 2009. Effects of distribution centre on vendor-managed inventory system with multiple retailers. *Presented in International Forum on Shipping, Ports and Airports 2009 Conference.* 24-27 May 2009, HK.