COLLUDE OR COMPETE:

CHOICE OF P&I CLUBS AND ROLE OF

MARINE MUTUAL INSURANCE CARTEL

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CERTIFICATE OF ORIGINALITY

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______________________________ (Signed)

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FOR MY DEAR PARENTS

ABSTRACT

Protection and Indemnity (P&I) insurance covers the third-party liability of a shipowner. The mutual insurance company offering such protection is called P&I Club. The thirteen largest P&I Clubs are bound by an agreement (the Inter-Club Group Agreement, IGA). The main purpose of the IGA is to prevent one group Club from undercutting the rates charged to a shipowner who is currently entered with another holding group Club. The EU commission questions the Group's constraints on competition among the Clubs.

In order to demonstrate the impact of competition, in this research, the following tasks are accomplished to develop the competition theory of marine mutual insurance.

First, this research summarizes previous studies with regard to classic mutual insurance and stock insurance by the literature review. The research compares the marine mutual with them. It is found that marine mutual insurance, i.e., P&I insurance, has certain particular characteristics that distinguish P&I Clubs from the classic mutuals.

Secondly, this study develops the pricing model of marine mutual insurance based on the theory of Pareto efficiency. The *ex ante* and *ex post* Pareto efficient contracts of P&I insurance are proved respectively. The equilibrium contract of P&I insurance integrates the *ex post* and *ex ante* Pareto efficient contracts. This integrated P&I

insurance contract reflects not only the fundamental principle of mutuality, but also the particular ways in which P&I Clubs differ from the classic mutual.

Thirdly, given the exponential utility, the research derives the optimal Club size. This research rejects the findings of the previous studies on the size of mutual. The results show that: (a) When the per capita loss of a single Club is increasing along with the Club size, the welfare of an individual member might worsen as the Club size increases; and (b) neither freely competitive nor monopolistic markets can be formed in the P&I insurance market.

Fourthly, three competition strategies are discussed separately to verify that, in most cases, premium competition cannot benefit simultaneously the entered members and the primary members of the New Club. For each competition strategy, the research provides three criteria to (a) help the entered members to decide whether to switch membership, (b) help the primary members of the New Club to decide whether to reject the entry of the entered members, and (c) help the Holding Club to decide whether it should impose certain countermeasures.

Finally, a case study is provided to examine the proposals and criteria obtained through this research. Two P&I Clubs, North of England Club and Britannia Club, are taken as the examples for simulating the competition strategies. The results reveal that Britannia Club has a competitive advantage when compared with North of England Club, but that the three competition strategies cannot be accepted by the New Club's primary members.

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LIST OF ABBREVIATIONS

H&M	Hull and machinery
P&I	Protection and indemnity
FD&D	Freight, demurrage and defence
TDI	Trade disruption insurance
IBNR	Incurred but not reported
The Group	International Group of P&I Clubs
IGA	Inter-Club Group Agreement
PEC	Pareto efficient contract
ICR	Initial capital reserve

Collude or Compete:
Choice of P&I Clubs and Role of Marine Mutual Insurance Cartel

Chapter 1. Introduction

1.1 Background

As one of the world's most dangerous industries, shipping depends greatly on insurance coverage. During the past 150 years, marine insurance has become a very specialized field of insurance, has formed a relatively independent market, and has amassed a huge amount of capital. The coexistence of both mutual and stock underwriters is one of the distinguishing characteristics of the marine insurance market. Marine mutual underwriters (P&I Clubs) have formed a cartel, the International Group of P&I Clubs, to protect the joint interests of its members, whilst stock marine insurers carry on their business within a competitive environment.

This research focuses on the competition among P&I Clubs. Before the formal demonstration is given, an overview of the marine insurance market is provided to introduce the background to this study.

1.1.1 Marine insurance products

A. Overview of H&M and P&I

In a general sense, the marine insurance market covers four key insurance contracts available to the shipping industry [46]. They are:

(a) Hull & Machinery insurance (H&M insurance): protection for loss of or damage to the ship.

(b) Protection & Indemnity insurance (P&I insurance): protection for the ship's operating liabilities.

(c) Trade Disruption: protection for the ship's income stream.

(d) General Marine Liability: protection for the operating liabilities of other, non-ship, marine operators.

In a narrow sense, the P&I insurance sector is usually excluded from the marine insurance market. First of all, P&I insurance is used for protection against the various liabilities of shipowners to *third parties*. Stock underwriters perform poorly in this line of business, which is deemed to be market failure for commercial insurance. Secondly, P&I insurance is dominated by an oligopolistic market, whereas other types of insurance are offered in a competitive market.

H&M and P&I insurances are effectively compulsory, and occupy a significantly high proportion of a shipowner's insurance costs. Table 1-1 shows the insurance costs of operating a certain type of vessel. H&M insurance occupies about 35% - 50% of the insurance cost, while P&I takes up about 20% - 26%, except for Reefer (30%), LPG/LNG (16%) and Passenger (36%).

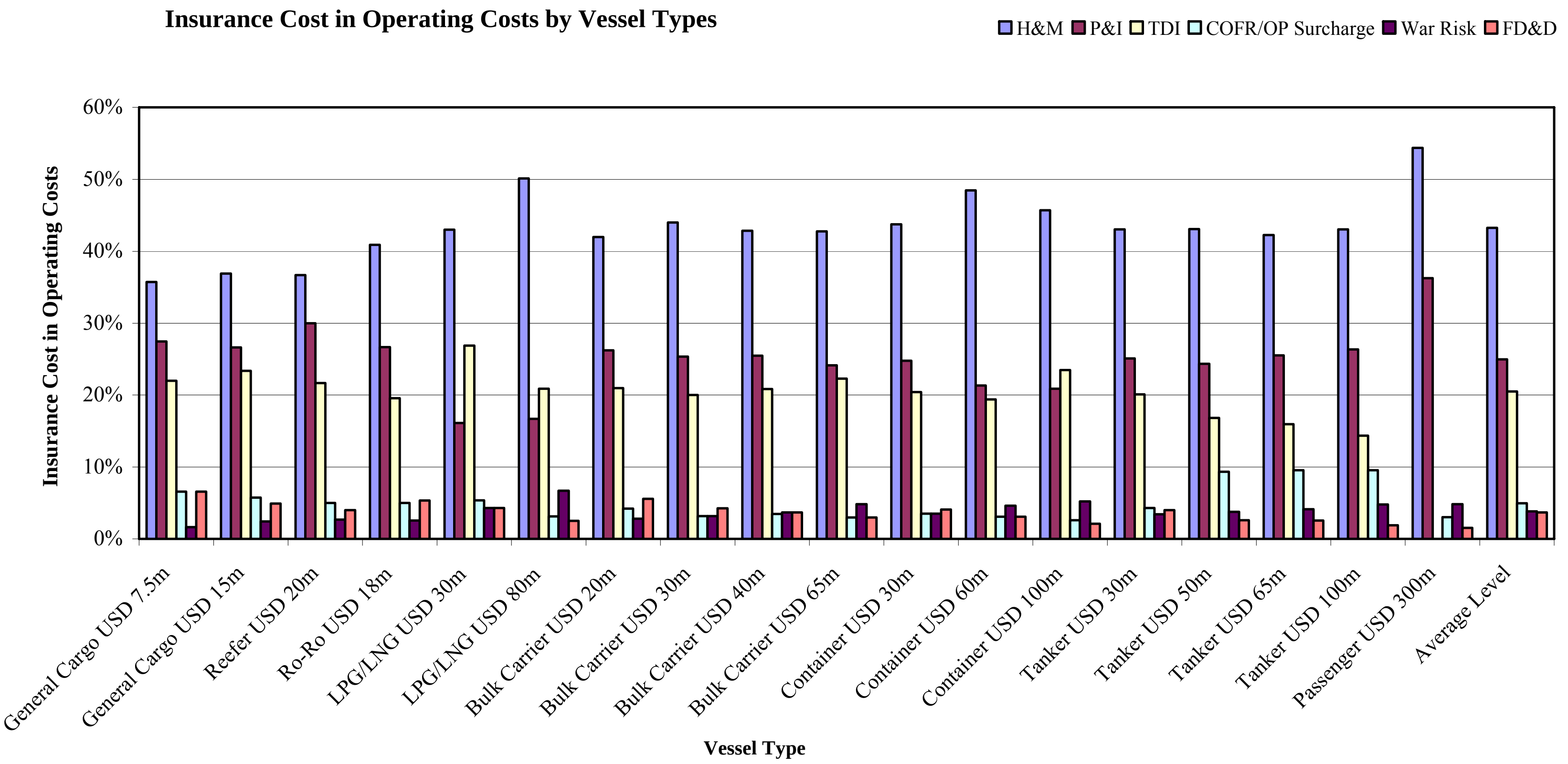
Table 1-1: Proportion of each type of insurance among overall insurance cost (2005)

Vessel Type	Declared Value (USD)	H&M	War Risk	P&I	FD&D	COFR/OP Surcharge	TDI
General Cargo	7.5m	35.71%	1.65%	27.47%	6.59%	6.59%	21.98%
	15m	36.89%	2.46%	26.64%	4.92%	5.74%	23.36%
Reefer	20m	36.67%	2.67%	30.00%	4.00%	5.00%	21.67%
Ro-Ro	18m	40.90%	2.56%	26.67%	5.33%	4.99%	19.56%
LPG/LNG	30m	43.01%	4.30%	16.13%	4.30%	5.38%	26.88%
	80m	50.10%	6.68%	16.70%	2.51%	3.13%	20.88%
Bulk Carrier	20m	41.96%	2.80%	26.22%	5.59%	4.20%	20.98%
	30m	44.00%	3.20%	25.33%	4.27%	3.20%	20.00%
	40m	42.82%	3.70%	25.46%	3.70%	3.47%	20.83%
	65m	42.75%	4.83%	24.16%	2.97%	2.97%	22.30%
Container	30m	43.73%	3.50%	24.78%	4.08%	3.50%	20.41%
	60m	48.45%	4.65%	21.32%	3.10%	3.10%	19.38%
	100m	45.69%	5.22%	20.88%	2.09%	2.61%	23.50%
Tanker	30m	43.04%	3.44%	25.11%	4.02%	4.30%	20.09%
	50m	43.07%	3.75%	24.34%	2.62%	9.36%	16.85%
	65m	42.26%	4.15%	25.52%	2.55%	9.57%	15.95%
	100m	43.06%	4.78%	26.32%	1.91%	9.57%	14.35%
Passenger	300m	54.38%	4.83%	36.25%	1.51%	3.02%	---------
Average Level		43.25%	3.84%	24.96%	3.67%	4.98%	20.53%

Data source: Drewry Shipping Consultants Ltd

Around 70% of the insurance costs are spent on H&M and P&I insurance. Figure 1-1 also displays that the share of P&I insurance costs has no obvious difference across the declared values, but that it is related closely to the vessel type. An LNG/LPG vessel's P&I cost is about 16% of the insurance cost, which is much lower than the average level of 25%. A passenger vessel, however, allocates about 36% of its insurance cost towards the P&I risks in case of injury or death of both its crew and passengers, which is much higher than the other vessel type.

Figure 1-1: Insurance Cost in Operating Costs (by Vessel Types)



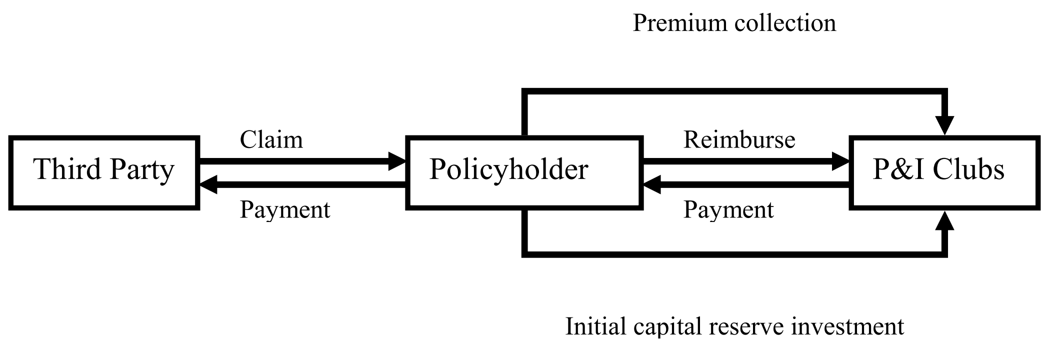
B. P&I insurance

Protection and Indemnity insurance specifically covers only the ship operator for all his/her liabilities emanating directly from the operation of the vessel. The term “ship operator” can be widely interpreted to include shipowners, ship managers, ship mortgagees and ship charterers. The Protection and Indemnity Clubs (P&I Clubs) are voluntary associations of shipowners formed on mutual understanding to insure the third party claims of marine adventure. All of these stakeholders may have an insurable interest with respect to the P&I risk, but this research does not distinguish between the different insurable interests among them.

P&I risk is of a heavy-tailed nature. P&I Clubs have flagged their concerns about escalating large claims. There have been some extreme claims for individual Clubs, for instance, two bad accidents in the USA in 2003 cost the West of England Club up to US$ 150 million, and the highest case in 2004 was Athos 1, costing US$ 125 million for the resultant oil pollution. In 2008, Hebei Spirit, insured by Skuld, caused one of the largest tanker spills in recent times. The claims processed involved more than 100,000 claimants and totaled more than US$ 550 million [2].

P&I Clubs are non-profitable insurance organizations and therefore, by definition, for each policy year, the premium and investment income is equal to the cost of claims, reinsurance and management. According to the rule of the Clubs, the member must pay his liability claim first before seeking recovery from the Club (see in Figure 1-2).

Figure 1-2: P&I insurance system



This rule reflects certain aspects of important content, and these become the assumptions in this research. Firstly, the P&I Club must have a capital reserve from which it can recover a payment already made by the member. Secondly, the actual payment made by the member against the claim must first of all be the exact amount that indemnifies the total loss of the third parties. If the Club has a sufficient capital reserve from which it can recover the claim made by the member, no further premium will be called for. Otherwise, such a prepaid claim can only be partly recovered (see Figure 1-3).

Figure 1-3: Simplified P&I insurance system

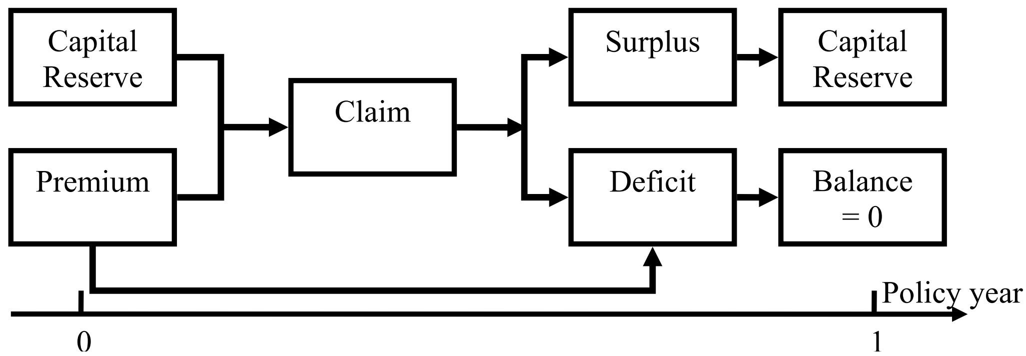


Figure 1-3 illustrates the cash flow and account status of a P&I Club in a single policy year. In this simplified model, the investment income, reinsurance and management costs are not taken into consideration. In addition, it does not distinguish between claims paid, estimated outstanding claims and incurred but not reported (IBNR) claims. If the capital reserve together with the premium exceeds the claims, the surplus will transfer to the capital reserve of the next policy year.

1.1.2 Marine insurance market

According to statistics in 2005 [46], the marine insurance market covers these six main lines of business: Marine cargo (US$ 8 billion), H&M (US$ 4 billion), P&I (US$ 2 billion), marine liabilities (US$ 1.5 billion), marine offshore (US$ 1 billion), and others (US$ 0.5 million). Albeit that marine insurance has become a global activity, it does retain concentration in its geographical spread. The maritime centers worldwide are also the key areas for marine insurance activity, such as London, Oslo, Paris, New York and Tokyo.

A. Global marine insurance market

Lloyd's of London is the oldest active marine insurance market. Lloyd's of London is a "Society and Corporation" that provides the premises, services and assistance necessary for the conducting of underwriting, and it also undertakes to assist in the regulation of the market place. In the past nine years, the capacity of

Lloyd’s of London has been growing smoothly from £10.1 billion in 2000 to £15.95 billion in 2008 (see Table 1-2).

Table 1-2: Lloyd’s overall performance

Year	Total Capacity (£ billion)	Syndicates (total)
2000	10.1	124
2001	11.3	108
2002	12.2	92
2003	14.4	75
2004	14.9	66
2005	13.7	62
2006	14.8(a)	62(b)
2007(a)	16.1	66
2008(a)	15.95	75

(a) Data source: Lloyd’s of London

(b) Data source: The Lloyd’s Market in 2006

In the meantime, the total number of syndicates dropped from 124 in the year 2000 down to as few as 62 during 2005-2006, after which the number increased back to the level of 2003. Lloyd’s remains the leader in the H&M business as a whole. With too much capacity, the underwriters may have to reduce premiums in order to solicit customers and improve competition among the syndicates.

Besides this, in the United States, hull insurance capacity remains underutilized, so it is expected that the scope for increase is limited. The Norwegian H&M market is dominated by four major players – Gard Services, Norwegian Hull Club, Bluewater and Gerling. The Norwegian H&M market has suffered several high profile claims, and the pressure on rate increases continues.

B. Marine mutual insurance market

A significant proportion of the international marine market is covered by mutuals. So far, the mutuals cover such business lines as hull, war, P&I and transport. P&I is the most important business line in the mutual market, and the thirteen largest mutuals carry on almost 90% of the world's P&I business.

Table 1-3: The 13 members of the International Group of P&I Clubs

Full Name	Annual Growth of GT	Size in GT. (m)	Annual Growth of Reserve	Free Reserve (US$)
American Steamship Owners Mutual P&I Ass., Inc. (American Club)	0%	20	5.0%	35.6
Assuranceforeningen Gard (Gard)	6%	180.2	-25.8%	430.4
Assuranceforeningen Skuld (Skuld)	13.6%	50	-29.4%	144.0
The Britannia Steam Ship Insurance Ass. Ltd (Britannia Club)	6.1%	134.8	-17.1%	191.5
The Japan Ship Owners' Mutual P&I Ass. (Japan Club)	4.5%	82.8	8.9%	117.4
The London Steam-ship Owners' Mutual Insurance Ass. (London Club)	5%	42	42.8%	115.5
The North of England Protecting & Indemnity Ass. (North of England Club)	5.5%	95	-4.0%	211.1
The Shipowners' Mutual P&I Ass. (Luxembourg) (Shipowners Club)	4%	15.9	-22.7%	95.8
The Standard Steamship Owners' P&I Ass. (Bermuda) (Standard Club)	13.7%	83	-22.1%	176.0
The Steamship Mutual Underwriting Ass. (Bermuda) Ltd (Steamship Mutual)	4.1%	75	0.9%	75.5
Sveriges Angfartygs Assurans Forening (Swedish Club)	-17.5%	101.2	7.1%	106.8
The United Kingdom Mutual Steam Ship Assurance Ass. (Bermuda) Ltd. (UK Club)	12.5%	122.7	2.9%	235.5
The West of England Ship Owners Mutual Insurance Ass. (Luxembourg), (West of England Club) *	-1.1%	53.8	-15.1%	173.6

Data source: Annual report of each club in 2009
* Data published in 2008

The policy year 2008 was a hard experience for most of the thirteen P&I Clubs. Seven of them suffered a decline in their free reserve, and except for the London Club, the reserve growth of the remaining six Clubs slowed significantly. Skuld experienced a hard policy year, when the global financial crisis had a noticeable impact on Skuld's investment performance, and the oil pollution by Hebei Spirit induced an unexpected fall in their contingency reserve of nearly 30% (see Table 1-3). Gard remains in the leading position amongst the thirteen Clubs in both Club size and free reserve.

According to statistics [46], by 2005 the collective premium of the global P&I market was approximately US$ 2 billion, and the gross free reserve of the thirteen P&I Clubs reached US$ 1.88 billion. The recent annual reports of the Clubs show that the gross free reserve has already accumulated to an amount of around US$ 2.0 billion in 2007-2008. Few underwriters undertake nearly the whole P&I business, so the marine mutual insurance market is oligopolistic.

C. International Group of P&I Clubs

The thirteen P&I Clubs mentioned above are the members of the International Group of P&I Clubs (the Group). All of them are bound by an agreement (Inter-Club Group Agreement, IGA) that governs their relationship in sharing large claims and high-level reinsurance costs, as well as the principles affecting the setting of rates for entered vessels. The main purpose of the IGA is to prevent one group Club from undercutting the rates charged to a shipowner who is currently entered with another holding group Club.

In deference to the general principle of mutuality, the Group is very much the “Club” of the thirteen P&I Clubs. The Group provides the reinsurance service to its members, which is analogous to P&I Clubs underwriting its own members. It is equivalent to that of the thirteen members pooling their risk mutually in a larger group. At present, the members of the Group each bear their own claims up to a maximum of US$ 6 million per claim. Claims in excess of this retention, and up to US$ 50 million, are shared by the pooling Clubs.

The Group bars its members from competition that undercuts the premium rates. The Group regulates the boundaries for premium competition through the Agreement 1999 [47]. A New Club wishes to underwrite a vessel that is currently insured with any one of the other Clubs. The New Club is not allowed to provide the contract at a premium that is unreasonably lower than the one offered by the Holding Club. The Holding Club has the obligation to provide the underwriter’s claim record and to state the current premium for the vessel. Using this approach, the New Club can price the contract on an identical basis with the Holding Club. At the first renewal, the Holding Club has the opportunity to verify whether or not the New Club’s rate is unreasonably low.

Release call is another method of preventing the Group’s members from such competition, because a member is forced to bear more costs on the membership switch—the so-called “switch cost” in the insurance industry. However, the EU commission questions the Group’s constraints on competition among the Clubs.

1.2 Research problems

In 1999 the European Commission renewed the exemption of full competition for ten years. As early as the year 1997, the European Commission criticized the Group, from two perspectives. Firstly, the Group obliged all the P&I Clubs to offer the same level of cover, even if a considerable number of shipowners wished to obtain substantially lower levels. By 1999, the Group lowered the common level of coverage from € 16.5 billion to € 3.9 billion, and clarified that P&I Clubs are free to provide different levels of coverage outside the Agreement.

Another critique is that the Group imposes limits on price competition between the P&I Clubs. So far, the relevant constraints are still effective in the International Group Agreement 1999. The general theory of anti-trust supposes that sufficient competition can maximize the utility of customers to the greatest extent and thus further improve social welfare. However, the insurance industry is not the typical kind of industry mentioned in economics textbooks, and P&I insurance, even more so, is poles apart from both traditional stock insurance and the classic mutual.

Bennett interviewed three P&I Clubs, several major shipping companies and various commercial underwriters [7]. Both P&I Clubs and shipping industry believe the premium competition cannot improve the P&I insurance market. His work also provided certain descriptions of P&I Clubs that correspond to an intuitive perception of marine mutual underwriters [7], as follows.

(a) All members are concerned that others minimize the risks they pose. All members are encouraged to adopt high-level safety and security standards. On balance, individuals having a similar class of risk will pool their risks together to form a mutual group.

(b) Differentiation of premium is an important principle of mutuality, so that high-risk members are not unfairly subsidized by low-risk members. If the Clubs offer an identical competitive premium to all members, it could lead to a polarization of Clubs in terms of membership quality.

(c) Each member's rate is confidential, so it will not be obvious when some are paying over the odds. In a mutual arrangement the final contribution of each member is not known until the policy year is closed; this is the final premium, and not the price rated in the policy. The rate simply determines the proportion of a Club's losses the policyholder has to bear.

(d) Although the Clubs do not make profits, they still have incentives to compete for members, because increasing the spread of risk makes losses more predictable and reduces the supplementary calls levied on a member.

(e) Restraining competition between the Clubs does appear to help maintain mutuality within the Clubs. The anti-competition agreement increases the Clubs' power over their customers, making it difficult for an individual policyholder to play off Clubs one against the other and obtain premiums that are incommensurate with the risk they pose.

Thus, in this research, there are three research problems, defined as follows:

(a) What is an optimal contract for marine mutual insurance?

An optimal contract for marine mutual insurance should maximize the joint utility of the members, in order to reach Pareto efficiency. Pareto efficiency is the basis on which to analyze the characteristics of an insurance contract. A Pareto efficient contract of stock and classic mutual insurance is examined by Fagart et al. [41, 42]. However, the P&I Club is distinct from both stock and classic mutuals. Thus, the first problem to be solved in the research is to prove the existence of a Pareto efficient contract in marine mutual insurance.

(b) How is the oligopolistic P&I market formed?

According to the Larger Number Law, an insurance company would like to underwrite as many insureds as possible. In the general sense of classic mutual, the utility of an individual insured increases with the size of risk pooling, because if the risk pooling contains more insureds, the collective risk becomes more predictable [41, 42]. However, the members of a marine mutual underwriter might have another, altogether different, story.

There is no mature approach by which to model and integrate the individuality and peculiarity of P&I insurance. Although a lot of works have been contributed on the subject of mutual insurance, these previous studies have focused merely on the classic mutuals. This research emphasizes the special characteristics of P&I insurance, and fill the gap of the oligopoly theory of the P&I market.

(c) What is the impact of competition on P&I Clubs?

The third problem involves these three aspects: (a) Which competition strategies will be conducted? (b) How can the impact of competition on the stakeholders be evaluated? (c) What countermeasures are available to the Holding Club?

Competition in the insurance market involves the competition (a) between stock underwriters, (b) between stock and mutual underwriters, and (c) between mutuals. Stock and mutual underwriters have quite dissimilar objective functions. Through price competition among stock underwriters, the insureds can obtain a much lower premium and thus reduce their insurance costs. As for marine mutuals, the premium competition might result in the members of the New Club taking on more liability in the post-incident compensation. This research will start with the peculiarities of marine mutual insurance, so as to illustrate the impact of premium competition on the P&I insurance market. The relevant conclusion will supplement the competition theory of the insurance market.

1.3 Objective of this research

This research wishes to make a contribution towards the theory of oligopoly in the marine mutual insurance industry. Traditionally, Oligopoly theory starts from the price competition and quantity competition in the production industry, and develops

Cournot equilibrium and Bertrand equilibrium. It is usually presumed that firms are maximizing their profit. By collusion, a cartel maximizes the joint profits of all members. Marine mutual insurance, though, where P&I Clubs are non-profit making entities, is quite different. The Clubs' objective function is to maximize the utility of their members through providing coverage at certain costs. Thus, P&I insurance is distinguished from both the stock and classic mutual insurance. The objective of this research revolves around the following six perspectives.

First, develop a new description of P&I insurance that systematically distinguishes the current P&I Clubs from the classic mutuals. Actually, in some aspects P&I Clubs are similar to, and have some characteristics of, stock underwriters, such as non-zero capital, pre-paid premiums, etc. However, the traditional description of mutual is no longer available for P&I Clubs [30, 41, 42].

Second, prove the existence of a Pareto efficient contract in P&I insurance, based on the actual loss (*ex post* assessment) and the expected loss (*ex ante* assessment), respectively. An integrated P&I contract is constructed through combining the *ex ante* and *ex post* Pareto efficient contracts.

Third, discuss the optimal size of a P&I Club under a concave utility function. Derive the optimal club size for a fixed total population, in order to examine the formation of the oligopolistic P&I market.

Fourth, consider three competition strategies. For each strategy, the Pareto efficient contracts are separately derived for the New and Holding Clubs. If the contracts are applicable, the various impacts of this competition are evaluated.

Fifth, clearly distinguish the risk status of the members in the competition. If the economy consists of individuals with a heterogeneous risk status, it should be ascertained as to whether or not the competition is definitely able to benefit all members.

Sixth, analyze the countermeasures of the Holding Club in the competition. The Holding Club can adopt a series of countermeasures to prevent premium competition, such as levying a release call.

1.4 Structure of this thesis

The thesis is organized as follows. Chapter 1 introduces the research problem and research motivation. Background information includes the main marine insurance products, H&M and P&I insurance, and their roles in the marine insurance market. Chapter 2 focuses on a critical review of previous studies. This chapter is divided into three sections: (1) identify the peculiarities of P&I insurance, as compared with stock insurance and classic mutuals; (2) identify the research gap in contemporary mutual insurance studies; (3) review the studies about oligopoly theory and its relevant application to the insurance market. Chapter 3 is the research framework.

Chapter 4 reveals the existence of a Pareto efficient contract in P&I insurance. Given a concave utility function, this chapter demonstrates the formation of the oligopolistic P&I insurance market. In chapter 5, premium competition is introduced into the model. The applicability and impact of the three competition strategies are discussed separately. Chapter 6 provides a simulation analysis to examine the findings of chapters 4 and 5, where the relevant parameters are generated from the empirical data (numerical approach).

Chapter 7 is the final chapter of this thesis. The main findings and conclusions are put forward in this chapter, which also points out the limitations of this program and suggests future research directions.

Chapter 2. Literature Review

2.1 P&I Insurance: an extraordinary mutual insurance

The concept of mutuality is usually defined through a comparison with stock insurance. Mutual insurance is generally depicted from four aspects: ownership, profitability, premium call and risk management. In a mutual, the insureds are also the owners of the company. Their premiums are adjusted *ex post* to balance premium revenue and indemnity expenses, which leaves the mutual with a zero profit [41, 42]. *From this definition, a P&I Club is a typical mutual.* In the International Group of P&I Clubs (Group) Agreement 1999, it is said that the parties to this Agreement are mutual, non-profit-making insurance associations of shipowners engaged in the insurance of marine risks, commonly described as “protection and indemnity” risks.

Hazelwood [48] has made a significant contribution on the systemic studies of P&I insurance with respect to the rules of the Clubs and the law cases. Kipp [56] and Reynardson [87] revealed the history and development of P&I insurance in America and Britain, respectively. During 1960s to 1990s, the scholars focused on the introduction [61, 89, 110] and legislation of P&I insurance [49, 55, 86, 111, 117]. However, in the past twenty years, the researchers turned their concerning to the new trend of marine risk, such as, terrorist attack [88], environmental damage [7, 53, 115].

The previous studies of mutual insurance, however, fail to provide a satisfactory explanation for the existence of P&I Clubs. A possible reason for this is that P&I Clubs have certain particular properties that distinguish them from the traditional mutual. In fact, P&I Clubs embody a very special type of underwriter that has most of the characteristics of a mutual, as well as some of the traits of stock underwriters. The peculiarities of a P&I Club are summarized in Table 2-1.

Table 2-1: Comparison between Classic Mutual, Stock and P&I Underwriters

Items	Classic Mutual Underwriter	P&I Club	Stock Underwriter
Compulsory or not	Not compulsory	Compulsory	Not compulsory
Ownership	Owned by insureds	Owned by member	Owned by Shareholder
Profitability	Non-profitable	Non-profitable	Profitable
Pessimism	Depends on insureds	Depends on member	Depends on Shareholder
Insured Risk	No market failure Low loss probability	Market failure Heavy tail loss	No market failure High loss probability
Initial Capital Investment	No initial capital investment	Initial capital invested by members	Initial capital invested by shareholders
Insurance Costs	*Ex post* adjusted premium	Flexible premium	Fixed premium
Insolvency	Switch to other stock or mutual	Switch to other P&I Club or form new Club	Switch to other stock or mutual

A. Ownership

There is no dispute about the ownership of a P&I Club. P&I Clubs are owned by shipowners, who are also the members. Thus, a P&I Club is an agent of its members. This coincides with the status of a general mutual.

An underwriter's strategic policy should embody the interests and risk attitudes of its principals. A mutual underwriter is the agent of its insureds. Thus, if the insured in a mutual is risk averse, the mutual underwriter is risk averse as well. By contrast, a stock underwriter is the agent of its shareholders. If the shareholder is risk neutral, the stock underwriter performs risk neutral in pricing a policy [67, 96]. Otherwise, the stock underwriter is also risk averse.

For the same reason, a P&I Club must be non-profitable. Shipowners invest an amount of capital into the Club, where the power of the Club's manager is constrained and the underwriter is unable to earn extra revenue from either the premium or the reinvestment [47]. One of the most important principles of the P&I Club is to reduce the insurance costs of its principal. Thus, the premium rate should be priced to ensure that P&I coverage is provided at all costs.

B. Insured Risk

An insured risk event can be described from two perspectives: The possibility of the event occurrence, and the losses caused by the event. Thus, there are four possible situations (as seen in Table 2-2).

Table 2-2: The choice of an insured under different risk states

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="2">Possibility of event occurrence</td></tr>
<tr><td>High</td><td>Low</td></tr>
<tr><td rowspan="2">Losses caused by the event</td><td>Unpredictable</td><td>Uninsurable</td><td>Mutual
e.g. P&I Club</td></tr>
<tr><td>Predictable</td><td>Monopoly stock/
Single mutual</td><td>Classic Mutual/
Stock</td></tr>
</table>

Generally, either a classic mutual or a stock underwriter would like to insure the risk, as it has a predictable loss size. For instance, in mutual fire insurance, the insured has to declare the insured value of his property before concluding the contract [18]. By declaring the insured value, the mutual underwriter can predict the worst possible scenario accurately.

Given a certain loss size, an insured can be considered to be a high-risk individual if he suffers the insured risk event with a high possibility [67, 96, 97]. Other insureds are deemed to be low-risk. Such classification of risk status simplifies the economic analysis of an insured's choice.

Previous studies have explained the co-existence of classic mutual and stock underwriters in certain lines of business. However, the roles of P&I Clubs in the shipping industry cannot be explained by these theories. The fundamental reasons are the *heavy tail* nature and *externalities* of P&I risks [7, 8, 50]. The heavy tail makes it difficult for an underwriter to predict the worst-case scenario of claims. Li and Cullinane [62], Li and Wonham [64], and Li et al. [65] conducted the quantitative analysis on the characteristics of the maritime risk empirically.

At the same time, P&I risk is often associated with externality, which determines the weakness of stock insurers in this business line. For instance, oil pollution can induce extreme damage to the ocean environment. When a private (stock underwriter) sector fails to provide insurance contracts, there exists market failure. The public sector (government) cannot provide P&I insurance to the private sector (shipping industry) by employing public resources. Thus, shipowners form a P&I

Club to pool the risk mutually. P&I Clubs are essentially providing the so-called "club good" [17, 22, 91, 92].

When a P&I risk event happens, the random losses usually involve different types of damage. For instance, a collision accident might induce cargo liability, injury and death of crews, and oil pollution, all at the same time. Different types of damage are not independent from each other. Thus, the heavy tail nature of P&I risk is attributed to the dependence among the random losses within different categories [6, 13, 23, 33, 54, 78, 84, 106].

C. Initial Capital Reserve

P&I claims are heavy tail distributed, which implies that an underwriter should keep a great amount of capital reserve for coverage. Owing to the expensive capital, members resort to bearing more risk through mutual companies [123]. Fagart et al [42] take into consideration the resource of the underwriter's initial capital reserve (ICR for short). For the stock insurance, the shareholders' investment forms the initial reserve. On the contrary, since there is no shareholder in a mutual, the initial capital was supposed to be zero [42, 96].

Differing from the classic mutual, P&I Clubs have ICR invested by the members. These members are called as the primary members of the risk pooling, and this initial investment is the commitment of each primary member when founding the Club.

New members do not contribute to the initial capital. Once a primary member of the Club switches to another agent, the initial capital investment cannot be refunded. Such investment becomes a part of the sunk costs of a membership switch.

D. Premium Call Policy

An insurance contract can be described in the form of two-parameter parity (α, β), where α is the premium and β is the gross indemnity [67, 83]. Premium income combined with investment income forms the main body of a Club's revenue [16, 120].

The members in a mutual retain the whole residual claim, that is, the total premium payoff of a mutual should be exactly equal to the actual losses of its members [30]. Thus, in a classic mutual, the premium is unknown before the *ex post* adjustment. If the *ex ante* premium collection is not sufficient to cover the claims, the additional premium call will balance the actual payoff coverage [96, 98]. The classic mutual's system is discontinuous. After each *ex post* adjustment, the capital of the mutual will return to zero.

In stock insurance, the shareholders of the insurance company bear the residual claim [30]. When renewing the contract, a stock underwriter will re-assess the risk level of an insured based on its previous claim record. Albeit that there might be some slight adjustments to the renewed contract, the premium rate will not generally fluctuate greatly. Thus, the premium rate is deemed to be fixed in the short term.

A P&I Club's premium call policy is essentially not different to that of a classic mutual. Nevertheless, a P&I Club provides a *flexible* call policy [63, 119]. Due to the rare-event nature of P&I risks, a Club can accumulate a huge amount of cash assets, which cause a high opportunity cost of the members. The Club runs afoul of its members' interests [51]. The objective function of a P&I Club is to minimize the insurance costs of its members [119]. If the *ex ante* premium collection fails to cover the actual claims, a high premium rate will be called in the next policy year. However, if the *ex ante* premium collection creates a surplus, the extra premium will not be refunded directly. Instead, each member will be given a premium discount [51]. This *flexible* call policy improves the utility of the members [3, 63]. Under such a call policy, P&I insurance can be modeled as a time-continuous system.

E. Insolvency

The heavy-tail nature of insured risks can result in another issue seldom discussed in previous studies, namely, the insurance strategies of an insured in the event that his stock or mutual underwriter is insolvent.

Due to the market failure in P&I risks, a member cannot buy the insurance contract from a stock underwriter. When a P&I Club suffers insolvency, the loss of members can be partially recovered [85]. Thus, what they can do is either to switch membership to another Club, or to form a new Club with other members. It is still the marine mutual underwriter that is covering the P&I claims.

Most of the previous studies avoided any discussion of the insolvency issue. Classic mutual and stock underwriters are supposed to have sufficient resources to indemnify all of the claims. What the insureds have to do is to decide whether or not to purchase an insurance contract from a mutual or a stock underwriter. Under the coexistence of both mutual and stock, the insured might switch to another insurance type or insist on the original one. The general principle of switching insurance type follows the work done by Smith and Stutzer [96, 97], and Ligon and Thistle [67].

The peculiarities of P&I Clubs are inherited from the essential characteristics of a mutual underwriter. During the long-term competition with other underwriters, numerous mutuals have been demutualized. But P&I Clubs have adopted operating strategies that differ from the classic mutuals. P&I Clubs inherit the general principles of risk sharing, but take a similar approach to that of stock underwriters on the call policy and risk management.

2.2 Previous Studies of Mutual Insurance

2.2.1 Economic analysis of mutual

The previous studies of mutual insurance were conducted at the economic analysis level and the operation research level, respectively. So far, economic analysis has been the mainstream of mutual insurance studies. Following this research clue, the researchers try to answer the following two questions:

A. Formation of mutual insurance

The four studies, Smith and Stutzer [96, 97], Cabrales et al [18], and Ligon and Thistle [67], clearly reveal the existence of mutuals and the coexistence of stock and mutual underwriters. Their models are simultaneously built up with asymmetric information. Smith and Stutzer [96] constructed a self-selection constraint to avoid the adverse selection of individuals with different risk levels. In 1995, they extended the avoidance of this adverse selection to the efforts made on accident prevention. It was found that low-risk insureds pool their risk with mutual underwriters, as this doesn't depend on whether or not their efforts are observable [97].

Following Rothschild and Stiglitz [90], Smith and Stutzer [96] classified the insureds into two types, high-risk and low-risk, and took account of two different external environments, good and bad. They found that high-risk insureds do not share the risk with the underwriter, whereas low-risk insureds would like to pool the risk with the underwriter. In 1995, Smith and Stutzer introduced the insured's effort into the model of their 1990 work [97].

Analogous to Smith and Stutzer [96], Ligon and Thistle [67] found that high-risk individuals form a single large mutual, and low-risk individuals form several small mutuals. This conclusion is based on two conditions. Firstly, a high-risk individual is rewarded with more utility by pooling risk with all high-risk individuals, rather than pooling risk with a small group of mixed individuals. Secondly, a low-risk individual obtains more utility by pooling risk with a small group of low-risk individuals rather than pooling risk with the whole population of mixed individuals.

Under the same conditions, if there is a monopolistic stock underwriter, high-risk individuals buy full-coverage policies from the monopolistic underwriter, and low-risk individuals form small mutuals. It means that for high-risk individuals the monopolistic stock insurer is preferred to a single mutual group. If competitive underwriters offer a contract from which the high-risk insured extracts more utility than joining a small risk pooling with mixed individuals, this high-risk individual prefers the fairly priced full-coverage stock insurance policies [67, 96].

Cabrales et al [18] developed a case study on a special type of mutual insurance, *La Crema*. The indemnity value in *La Crema* is determined by the announced value of the insured property. The unique Pareto-efficient risk pooling equilibrium is reachable, but only if every insured declares the insured value truthfully.

B. *Impact of switching ownership structure*

Mutual insurance is managed by professional managers. Principal-agent problem is the main reason of the demutualization of the classic mutual underwriters. Demutualization is when a mutual underwriter changes its ownership from being insured-owned to shareholder-owned. Numerous studies have provided comparisons between the two kinds of ownership [60, 74, 114].

Mayers and Smith [74] examined thirty life underwriters, who changed ownership from stock to mutual. They concluded that, for the sample companies, such ownership switch enhanced the efficiency of the underwriter. Viswanathan and

Cummins [114] focused on the reason why numerous mutual underwriters demutualized during the 1950s to 1980s. The fundamental reason for demutualization is the principal-agent problem that exists between the managers of a mutual and their insureds. Managers want to have more freedom of access to capital, and have a desire for stock-based compensation. But the insureds of a mutual prefer to constrain the behavior of its managers.

Lamm-Tennant and Starks [60] found that stock underwriters wrote more high-risk business lines than mutuals, and this finding was verified by Ligon and Thistle [67]. However, these findings cannot explain P&I Clubs, because the classification of risk status in these two studies is unavailable for marine mutual insurance (as seen in table 2-2).

2.2.2 Operational issue of mutual

On the other hand, there are only a few studies that focus on the operational issues of mutual underwriters. The studies of operational issues with respect to mutual underwriters have mainly involved the premium call policy [108, 109], investment strategies [47], risk management [5, 105], deductible levels [12, 15, 58] and reinsurance policies [5, 72, 105, 108].

Optimization and stochastic control are widely applied in the studies of insurance. Starting with different research aims, the previous studies involved different objective functions. Mutual and stock insurance companies, respectively, are owned by insureds and shareholders. A mutual underwriter minimizes the insurance cost of his agents [108, 119], or maximizes the expected utility of the insureds [47, 109]. These two objective functions are usually used in the analysis with respect to premium call policy and investment strategy.

From the perspective of risk management, both mutual and stock underwriters have the common interest of minimizing ruin probability. Intuitively, an insurer, who would like to reduce the ruin probability, naturally tends to raise the premium. However, there exists an upper limit of premium rate, beyond which the insured has no incentive to purchase this particular insurance contract. Generally, an insured pays a certain amount of premium, and claims for coverage after marine losses. The expected utility of the insured under insurance coverage should exceed the utility without any coverage.

In mutual, there is a tradeoff between ruin probability and insurance cost. Albeit that a mutual can increase the premium rate in order to reduce the ruin probability of the risk pool, the insurance costs of insureds are raised accordingly. However, very few works have been contributed on this point. Yan et al [119] made this tradeoff through supposing a very high level of bankruptcy costs. This bankruptcy cost was explained as the insureds' expenditure on finding another insurer when the mutual comes to ruin.

Cummins et al [25] applied cross-frontier analysis in a study of the efficiency of insurance organization. The objective function is a cost frontier, which is the minimum cost of concluding a certain amount of insurance contracts, given the input-oriented distance function.

The heavy-tailed nature of P&I risk is common sense for marine insurance [50]. Jaffee and Russell [51] also suggested mutual insurance as being an option to cover catastrophic losses. Generalized extreme value distribution is the general method of directly describing a heavy-tail distribution [75]. Tapiero [108] adopted the jump process to describe the claim process of a mutual. The jump process is a special case of a more general stochastic process, that is, the Lévy process [6, 19, 20, 32, 35, 36, 37, 54, 77, 81, 106, 108].

2.3 Previous Research on Oligopoly of the Insurance Market

Stiglitz [102] had already proved that in competitive markets high-risk individuals obtain complete insurance, but low-risk individuals only partial insurance. The low-risk ones avoid "subsidizing" the high-risk individuals, so the high-risk and low-risk insureds cannot pool risk together. This finding was also verified by Ligon and Thistle [67] about the mutual insurance market. Based on Ligon and Thistle's theory, the only choice of high-risk individuals is to pool risk in a single large mutual, whereas low-risk individuals form small mutuals. However, this finding is in conflict with actual observations made of the P&I insurance market. There is neither a monopoly nor numerous competitive stock underwriters who can offer full-coverage policies for the liability risk. The main share of the P&I insurance market is dominated by the thirteen largest P&I Clubs, so that the P&I insurance market is oligopolistic.

Power and Shubik [85] proved that a competitive insurance market is not always fully advantageous in the stock insurance market. Given the exponential utility function with constant absolute risk averse, there is a certain number of insurance companies at which the total quantity purchased and the insureds' payoff reach their peaks, respectively. This finding implied that a competitive market with an infinite number of insurers is not the optimal development strategy in the insurance market. Power and Shubik [85] assumed that the total investment of external investors is shared equally among all underwriters, and that the insureds can only obtain partial coverage, when the insurance company reaches insolvency. Heuristically, Power and Shubik's conclusion can be generalized to mutual, where the capital is invested by the insureds instead of by external investors.

Polborn [83] develop an oligopoly model for risk-averse underwriters without *ex ante* knowledge about the probability of a loss. The marginal cost of the i^{th} policy is defined as the premium that has to be given to the underwriter so that the underwriter's expected utility does not change in comparison to the case in which it sells only $i - 1$ policies. At the equilibrium of the oligopolistic insurance market, the number of policies sold by the underwriter corresponds exactly to the optimal cover payment that maximizes utility of the underwriter.

Aase [1] found the equilibrium of the marine mutual market with a convex operating cost function. At this equilibrium, the small clubs would like to increase pooling size to become more cost effective. The medium to larger size clubs, who have been cost effective, prefer to stay unchanged or even downsize. Aase concluded that there would be a convergence happening between both the small and the medium to large clubs. At the same time, there exists also the possibility of some extremely large clubs. Aase's work provides for a natural trend to form a monopolistic market, as well as the chance to have an oligopolistic market. Obviously, the latter situation coincides more with the actual observation of the P&I insurance market.

The Group is a cartel, through which the thirteen largest P&I Clubs collude in order to reach agreement in setting their premium, coverage, risk sharing and competition restrictions. According to general economic theory, a cartel has to maximize the joint profit of the members [101, 113]. A cartel is quite unstable, because each member of this agreement has his/her own motivation to lower the price and obtain a higher profit than the other members [113]. However, the Group is a stable organization. There should, therefore, be some self-constraints to regulate premium competition among P&I Clubs.

Chapter 3. Research Methodology

3.1 Research framework

This research is conducted based on a two-stage design. Suppose the insurance contract involved is a one-year P&I policy. Each member purchases one contract to cover the P&I risk of one vessel. In practice, a single member might insure several vessels in his/her fleets, but in this research every insured vessel is deemed to be one member corresponding to one policy.

Figure 3-1: Two stages of research

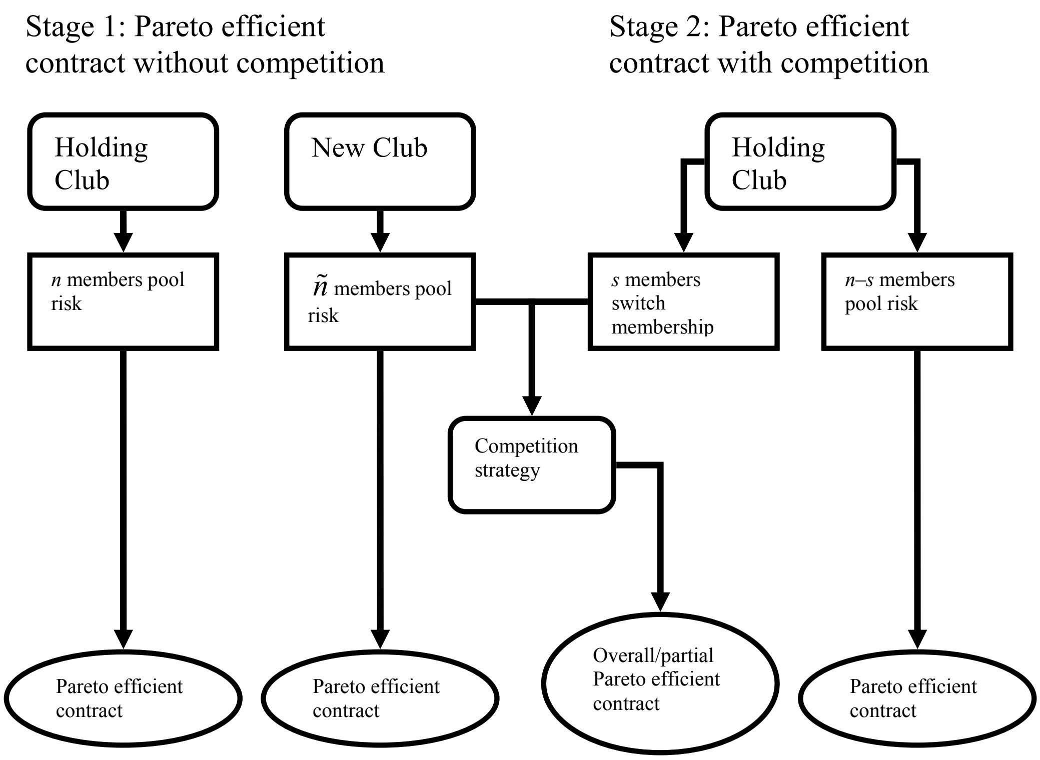
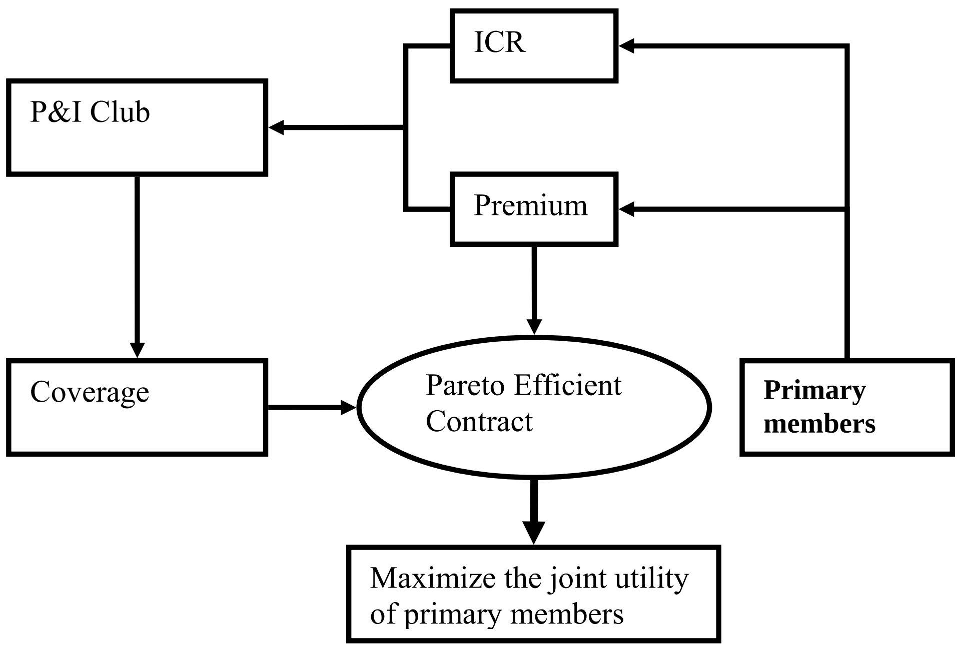


Figure 3-1 shows the framework of the two-stage design. Without the loss of generality, in a competition there must be at least one Holding Club and one New Club. In the Holding Club, there are n members, while $\tilde{n}$ members pool their risk in the New Club.

In stage 1, the members of each Club are offered Pareto efficient contracts (see figure 3-2). The primary members contribute an initial capital reserve investment, plus a certain amount of premium on the risk pooling of the Club. The P&I Club provides the coverage on the P&I risk of his agents. This coverage and premium call policy form the P&I insurance contract. When the contract maximizes the joint utility of its primary members, it is called a Pareto efficient contract. Thus, the first step of this research is to examine the existence of Pareto efficient contracts without competition [41, 42].

Figure 3-2: Pareto efficient contract of a P&I Club



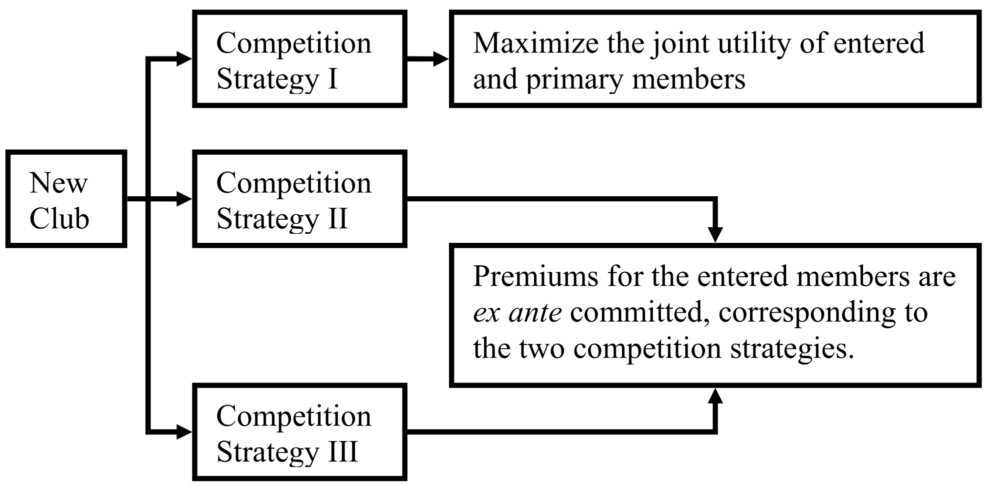
In stage 2, s of n members in the Holding Club switch their membership to the New Club. For this purpose, the New Club has to carry on a certain competition strategy (see figure 3-3), when the existence of a Pareto efficient contract can also be examined. In this research, there are only three strategies taken into consideration. In the first strategy, the joint utility of both entered and primary members of the P&I Club is maximized so that the contract concluded is Pareto efficient for all members. In the second and third strategies, respectively, the New Club provides one special offer each to the entered members. The entered members can obtain more coverage from risk pooling than the primary members of the New Club. Under these two competition strategies, except for the entered members, the contract is merely Pareto efficient for the primary members of the New Club.

Pareto efficiency is an important concept in economics, with broad applications. Pareto efficient situations are those in which any change to make any person better off is impossible without making someone else worse off. In this research, the capital reserve and premium collection are allocated among the members through recovery of the P&I claims. If there is no other allocation available to make further Pareto improvement, such allocation is defined as being Pareto efficient. Let C denote the coverage of a P&I insurance contract (a certain allocation schedule), and U_i denote the i^{th} member's utility function. Then, the Pareto efficient contract C^* should satisfy

$$C^* = \left\{ C : \max_C \left[\sum_i U_i(C) \right] \right\}$$

After the proof of a Pareto efficient contract under competition, the consequence of competition is assessed to evaluate whether such competition could improve the development of the P&I insurance industry. Ligon and Thistle [67] heuristically reveal the incentive of an individual to pool risk in a mutual. An individual wants to join a particular risk pool if doing so would increase the individual's expected utility. On the other hand, an individual is not admitted to pooling risk in a mutual unless the welfare of the other members is improved. In this research, it is proposed to prove that under premium competition, when an individual member switches his membership from one Club to another, this entered member's expected utility must be improved. But the expected utility of the members left in the Holding Club will be reduced.

Figure 3-3: Competition strategies



In this research, the model is concerned with an oligopoly of mutual underwriters who compete in premium. In order to coincide with the European Commission's comments on the Group, this research takes into account the following factors favoring oligopolies: (a) capacity constraints; (b) premium differentiation; (c) switch costs; (d) risk averse underwriters.

3.2 Data Collection

In this research, the main data is collected through analyzing the information published in the annual reports of each P&I Club. However, there are some difficulties in data collection.

(a) *Loss distribution* Since the annual reports, apart from some extreme losses, do not mention the details of every claim, the loss distribution can only be estimated through fitting the tail information to the pre-known knowledge about the distribution. Some Clubs' reports have clarified both the total number of claims and the proportion of claims exceeding a certain amount. Given the heavy tail nature of P&I risk, several heavy tail distributions are fitted, respectively. Most of the Clubs treat claims records and loss events as confidential information and thus unpublished in their annual report, but there are two P&I Clubs reporting the tail information sufficiently clearly as to enable estimation of the loss distribution. They are the

North of England Club and Britannia Club. In this research, the two Clubs are used as the examples for analysis.

(b) *Club size* Usually, the size of a P&I Club is measured through the gross tonnage of the insured fleets. However, vessels might have different tonnage, while each vessel has only one P&I insurance contract. Accordingly, the Club size is defined by the number of insured vessels. The Club size varies frequently, and is traditionally unrevealed in the annual reports, but this is not confidential information and can be collected through consulting the managers of Clubs.

(c) *Capital reserve* In this research, free reserve of a P&I Club is considered as the substitution of capital reserve. The free reserve is published annually in the financial statement of each P&I Club, and is an important financial indicator to describe the underwriting capability of the Club.

All data comes from the annual reports of two P&I Clubs, North of England and Britannia, in 2009.

Chapter 4. Pareto Efficient P&I Insurance Contract

This chapter focuses on the formation of a P&I insurance contract. Previous studies of mutual insurance reveal two different clues:

Clue A. The *ex post* adjustment of premium is carried out so as to balance the costs of coverage [41]. The member does not make any commitment on undertaking the liability. The profit of the mutual is *exactly* zero.

Clue B. A certain amount of premium is collected *ex ante* as the commitment. The *ex post* adjustment is reflected through the variation of the premiums in the next policy year. The profit of the mutual is *expectedly* zero [96, 97].

The practice of a P&I Club in premium collection integrates the characteristics of the two clues, so that neither Fagart et al. [41] nor Smith and Stutzer [96, 97] can demonstrate the full view of P&I insurance. Following the model of Fagart et al. [41], this work introduces the ICR and the *ex ante* premium commitment into the model.

In this chapter, it will be proved that there exists equilibrium for P&I Clubs, and a "**Pareto efficient contract**" (PEC), following both Clue A and Clue B, respectively. The former equilibrium is called ***ex post*** **Pareto efficient contract** (*ex post* PEC), and the latter one is called ***ex ante*** **Pareto efficient contract** (*ex ante* PEC). P&I insurance, as reviewed in Chapter 2, is essentially mutual insurance but has several peculiarities. A P&I insurance contract integrates the characteristics of both the *ex post* and *ex ante* PEC.

This chapter is organized as follow. Section 4.1 provides some basic assumptions for models. In section 4.2, the existence of *ex post* PEC and *ex ante* PEC is proved. A P&I insurance contract is formed in section 4.3, through integrating the results in section 4.2. Section 4.4 gives a brief summary.

4.1 Basic assumptions

Suppose there are N members in the economy. Each of them has an initial wealth ω and the Von Neumann Morgenstern utility function $U(\cdot)$, with $U' > 0$ and $U'' < 0$. Following the literature review in Chapter 2, it is assumed that the N members have an identical risk aversion parameter [120].

The n of N members pool their P&I risk in the same Club. An individual member indexed by i, i=1, 2, …, n, suffers a P&I risk event, in which the member faces a random pecuniary claim on a third-party liability. Let x_i denote the random losses of the third-party. The random variable x_i comes from some heavy tail distributions with finite mean and variance (e.g. Pareto distribution, log normal distribution, Weibull distribution). Denote the density function as $f_i(x_i)$, and the cumulative function as $F_i(x_i)$. The random variables x_i are independent, with a finite mean $E(x_i) = m_i$ and variance $Var(x_i) = v_i^2$.

When $f_i(x) = f_j(x)$, $\forall x \in [0, \infty[$, $i \neq j$, the members are deemed to be homogeneous. On the contrary, if $f_i(x) \neq f_j(x)$, $\exists x \in [0, \infty[$, $i \neq j$, the members are heterogeneous. The difference amongst members is determined by the mean loss and variance. If $m_i > m_j$, $i \neq j$, it is said that the i^{th} member is of higher risk than the j^{th} member; if $m_i < m_j$, $i \neq j$, the i^{th} member is of lower risk [67, 96, 97]. By contrast, given the fixed mean $m_i = m_j = m$, if $v_i > v_j$, $i \neq j$, it is said that the i^{th} member is of higher risk than the j^{th} member; if $v_i < v_j$, $i \neq j$, the i^{th} member is of lower risk [44, 69].

Briefly, there are two scenarios: (a) $m_i = m_j = m$, $i \neq j$, $i, j = 1, 2, \ldots, n$; and (b) $m_i \neq m_j$, $i \neq j$, $i, j = 1, 2, \ldots, n$. In order to simplify the second possibility, in this research it is assumed that there are only two possible values for m_i, m^H or m^L, $m^H > m^L$.

Under full indemnity and the principle of mutuality, the P&I Club concludes an agreement referred to as *treaty* β_i, which redistributes the covered losses among all of the members in this Club [11, 34]. Assume that this P&I Club has n members. Define the state vector $\boldsymbol{x} = \{x_i, i = 1, 2, \ldots, n\}$. The joint density function of $\boldsymbol{x}$ is denoted by $f_n(\boldsymbol{x})$, and the joint cumulative probability function is $F_n(\boldsymbol{x})$. Let $\beta_i(\boldsymbol{x}) \in R$ denote the treaty with respect to the i^{th} member, which is conditional on the random losses of all members in this Club. An insurance contract is then fully described by:

$$C(n) = \left\{ \beta_i(\boldsymbol{x}), \boldsymbol{x} = (x_1, x_2, \ldots, x_n) \in \Omega = [0, \infty[^n \right\}.$$

4.2 Pareto efficient contract

4.2.1 Ex post Pareto efficient contract

At the foundation of the P&I Club, each primary member of the initial wealth ω invests k into the risk pool to ensure the formation of the organization. Let α_i denote the net premium of the i^{th} member. His expected utility through the contract $C(n)$ can be written as the function of $\beta_i(\boldsymbol{x})$.

$$EU_i\left(C(n)\right) = \int_{\Omega} U\left(\omega - k - x_i + \beta_i\left(\boldsymbol{x}\right)\right) dF_n\left(\boldsymbol{x}\right)$$

Let Π denote the surplus of the capital after extracting the claim payment $\beta_i\left(\boldsymbol{x}\right)$. The constraint is that the ICR will be primarily used to cover the claim treaties $\beta_i\left(\boldsymbol{x}\right)$. If the i^{th} member's treaty cannot indemnify the actual loss x_i, the premium call α_i should fill up the gap $x_i - \beta_i\left(\boldsymbol{x}\right)$.

$$\Pi\left(C(n)\right) = nk - \sum_{i=1}^{n} \beta_i\left(\boldsymbol{x}\right)$$

The contract $C(n)$ gives the i^{th} member an expected utility $EU_i(C(n))$. For a given n, an efficient contract of P&I insurance must maximize the n members' joint expected utilities.

$$\max_{C(n)} \sum_{i=1}^{n} EU_i\left(C(n)\right) \tag{4-1}$$

$s.t.\ \Pi\left(C\left(n\right)\right)=0,\quad \forall\left(x_1,x_2,\ldots,x_n\right)$

This optimization is designed merely for the long-term contracts of P&I Clubs. It distinguishes itself from models of the classic mutual and other stock underwriters:

A. The objective function is to maximize the joint expected utility of all members in the P&I Club.

B. The model of a classic mutual does not contain capital, and the sum of the *n* members' coverage is equal to zero. In stock insurance, the total payment on coverage is no more than the capital invested by the shareholders. A P&I Club has nonnegative capital, which is invested by the primary members of the Club. The capital of a P&I Club, together with the premium income, should be equal to the expected coverage.

C. It is propounded by Smith and Stutzer [96] that the constraint is that the *ex ante* expected capital surplus of the mutual must be zero. By contrast, a non-profitable classic mutual emphasizes the *ex post* profit of the underwriter [41, 42]. As for stock insurance, there is nonnegative profit, to make sure that the stock underwriter has an incentive to provide the insurance contract. In this research, the constraint in Fagart et al's work is more suitable for studying the Club size.

Proposition 4-1. *An insurance contract C(n) offered by a P&I Club is ex post Pareto efficient if it satisfies*:

$$\beta_i\left(\boldsymbol{x}\right)=x_i-\frac{1}{n}\sum_{i=1}^{n}x_i+k \tag{4-2}$$

where x_i is the loss of the individual i in the state ($x_1, x_2, \ldots, x_n$), and the premium α and the initial investment of an individual primary member k satisfies:

$$\alpha = \frac{1}{n}\sum_{i=1}^{n} x_i - k \tag{4-3}$$

Proof. A Pareto-efficient P&I insurance contract is the solution of the following program

$$\max_{C(n)} \sum_{i=1}^{n} \int_{\Omega} U\left(\omega - k - x_i + \beta_i\left(\boldsymbol{x}\right)\right) dF_n\left(\boldsymbol{x}\right)$$

s.t. $nk - \sum_{i=1}^{n} \beta_i\left(\boldsymbol{x}\right) = 0$

Let $\lambda\left(\boldsymbol{x}\right)$ denote the Lagrangian multiplier of the constraint. The first order conditions for the i^{th} and j^{th} members' coverage $\beta_i(\boldsymbol{x})$ and $\beta_j(\boldsymbol{x})$ are given by:

$$U'\left(\omega - k - x_i + \beta_i\left(\boldsymbol{x}\right)\right) - \lambda\left(\boldsymbol{x}\right) = 0$$

$$U'\left(\omega - k - x_j + \beta_j\left(\boldsymbol{x}\right)\right) - \lambda\left(\boldsymbol{x}\right) = 0$$

From the first order conditions, it is obvious that for $i, j = 1, 2, \ldots, n$, $i \neq j$,

$$U'\left(\omega - k - x_i + \beta_i\left(\boldsymbol{x}\right)\right) = U'\left(\omega - k - x_j + \beta_j\left(\boldsymbol{x}\right)\right)$$

and thus there exists a premium call and the initial investment *k* independent of *i*, which satisfies:

$$-x_i + \beta_i\left(\boldsymbol{x}\right) = -x_j + \beta_j\left(\boldsymbol{x}\right) = -\alpha_i = -\alpha_j$$

Let $\alpha = \alpha_i = \alpha_j$, $i, j = 1, 2, \ldots, n$. Then, there is the summation that

$$-\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \beta_i(\mathbf{x}) = -n \cdot \alpha$$

Since $\sum_{i=1}^{n} \beta_i(\mathbf{x}) = n \cdot k$,

$$-\sum_{i=1}^{n} x_i + n \cdot k = -n \cdot \alpha$$

$$\Rightarrow \alpha = \frac{1}{n}\sum_{i=1}^{n} x_i - k$$

Thus, the coverage

$$\beta_i(\mathbf{x}) = x_i - \frac{1}{n}\sum_{i=1}^{n} x_i + k \qquad \square$$

In Proposition 4-1, the coverage $\beta_i(\mathbf{x})$ is constructed by three terms (see equation 4-2). The first term is when the i^{th} member suffers the loss x_i. Due to the principle of mutuality, this member is committed to bearing a share of the total loss, which is the second term of (4-2). As a primary member of the Club, the initial capital investment k will be used to cover the claim.

Corollary 4-2. *When* $k = 0$, $\alpha = \frac{1}{n}\sum_{i=1}^{n} x_i$.

Proof. This is a special case, in that the member has made no contribution to the ICR of the Club. The problem is degenerated to the mutual, as discussed by Fagart et al [41, 42]. $\square$

In *ex post* PEC, the net premium depends on the actual loss and the *ex post* adjustment. Due to $\alpha = \frac{1}{n}\sum_{i=1}^{n} x_i - k$, the net price of the mutual is unknown until all x_i are realized. The premium α is not a fixed amount but of a variance $\frac{1}{n^2}\sum_{i=1}^{n} v_i^2$, whereas the variance of the coverage $\beta_i(\boldsymbol{x})$ is $v_i^2 + \frac{1}{n^2}\sum_{j=1}^{n} v_j^2$.

4.2.2 Ex ante Pareto efficient contract

The Agreement 1999 contains the restriction that a New Club cannot write a policy at any premium unreasonably lower than the one offered by the Holding Club. The net premium of the *ex post* PEC depends on the *ex post* adjustment, that is, based on the losses that actually happened. Thus, it is hard to define the term "unreasonably lower".

The problem is what the (un)reasonably low premium is for the New Club. In order to solve this problem, the *ex ante* PEC is designed through changing the constraint in model (4-1).

Suppose there is a Club writing n members. At the foundation of the Club, each member invests capital k into the fund. Let $C^E(n)$ denote the insurance contract, where the superscript E indicates "*ex ante*". Let $\beta_i^E(\boldsymbol{x}) \in R$ denote the treaty with respect to the i^{th} member, which is conditional on the random losses of all members in this Club. Thus,

$$C^E(n) = \left\{ \beta_i^E(\boldsymbol{x}), \boldsymbol{x} = (x_1, x_2, ..., x_n) \in \Omega = [0, \infty[^n \right\}$$

The objective of the Club remains unchanged, which is to maximize the joint expected utility of the n members under contract $C^E(n)$.

$$\max_{C^E(n)} \sum_{i=1}^{n} EU_i\left(C^E(n)\right) = \max_{C^E(n)} \sum_{i=1}^{n} \int_{\Omega} U\left(\omega - k - x_i + \beta_i^E(\boldsymbol{x})\right) dF_n(\boldsymbol{x})$$

Let Π^E denote the surplus of capital after extracting the claim payment $\beta_i^E(\boldsymbol{x})$. Differing from model (4-1), the constraint in this section is that the initial capital should be sufficient to cover the claim treaties $\beta_i^E(\boldsymbol{x})$, and the surplus of the capital should be equal to zero expectedly [96, 97]. Thus, the constraint can be expressed as

$$E\Pi\left(C^E(n)\right) = nk - \sum_{i=1}^{n} \int_{\Omega} \beta_i^E(\boldsymbol{x}) dF_n(\boldsymbol{x}) = 0$$

A. $m_i = m_j = m$

Proposition 4-3. *An insurance contract* $C^E(n)$ *offered by a P&I Club is ex ante Pareto efficient if it satisfies*:

$$\beta_i^E(\boldsymbol{x}) = x_i - m + k \tag{4-4}$$

where x_i *is the loss of the individual i in the state* $(x_1, x_2, ..., x_n)$*, and the premium* α^E *and the initial investment of an individual primary member k satisfies*:

$$\alpha^E + k = m \tag{4-5}$$

Proof. The Pareto-efficient P&I insurance contract is the solution of the following program

$$\max_{C(n)} \sum_{i=1}^{n} \int_{\Omega} U\left(\omega - k - x_i + \beta_i^E(\boldsymbol{x})\right) dF_n(\boldsymbol{x})$$

s.t. $nk - \sum_{i=1}^{n} \int_{\Omega} \beta_i^E(\boldsymbol{x}) dF_n(\boldsymbol{x}) = 0$

Let $\lambda(\boldsymbol{x})$ denote the Lagrangian multiplier of the constraint; the first order conditions for the i^{th} and j^{th} members with respect to $\beta_i^E(\boldsymbol{x})$ are given by:

$$U'\left(\omega - k - x_i + \beta_i^E(\boldsymbol{x})\right) - \lambda(\boldsymbol{x}) = 0$$

$$U'\left(\omega - k - x_j + \beta_j^E(\boldsymbol{x})\right) - \lambda(\boldsymbol{x}) = 0$$

From the first order conditions, it is obvious that for $i, j = 1, 2, \ldots, n$, $i \neq j$,

$$U'\left(\omega - k - x_i + \beta_i^E(\boldsymbol{x})\right) = U'\left(\omega - k - x_j + \beta_j^E(\boldsymbol{x})\right)$$

and thus there exist a premium call and the initial investment k independent of i, which satisfies:

$$-x_i + \beta_i^E(\boldsymbol{x}) = -x_j + \beta_j^E(\boldsymbol{x}) = -\alpha_i = -\alpha_j$$

Let $\alpha^E = \alpha_i = \alpha_j$, $i, j = 1, 2, \ldots, n$. Then, there is the summation that

$$-\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \beta_i^E(\boldsymbol{x}) = -n \cdot \alpha^E$$

Take the expectation on both sides of the equation with respect to x_i and there exists

$$-\sum_{i=1}^{n} \int_0^\infty x_i f_i(x_i) dx_i + \sum_{i=1}^{n} \int_0^\infty \beta_i^E(\boldsymbol{x}) dF_n(\boldsymbol{x}) = -n \cdot \alpha^E$$

Since $\sum_{i=1}^{n} \int_{\Omega} \beta_i^E(\boldsymbol{x}) dF_n(\boldsymbol{x}) = n \cdot k$,

$$-\sum_{i=1}^{n} \int_0^{\infty} x_i f_i(x_i) dx_i + n \cdot k = -n \cdot \alpha^E$$

$$\Rightarrow \alpha^E = \frac{1}{n} \sum_{i=1}^{n} \int_0^{\infty} x_i f(x_i) dx_i - k = m - k$$

The random variables, x_i, are independent and identically distributed, so that the premium for a single member is $\alpha^E = m - k$.

Thus, the coverage

$$\beta_i^E(\boldsymbol{x}) = x_i - \frac{1}{n} \sum_{i=1}^{n} \int_0^{\infty} x_i f(x_i) dx_i + k = x_i - m + k \qquad \square$$

Differing from the coverage $\beta_i(\boldsymbol{x})$ in Proposition 4-1, the second term of $\beta_i^E(\boldsymbol{x})$ is the expected loss of the i^{th} individual member. If the expectation of random loss is less than the per capita loss that actually happened, $m < \frac{1}{n} \sum_{i=1}^{n} x_i$, the member can obtain more coverage from the *ex ante* PEC than the *ex post* one. On the contrary, if $m > \frac{1}{n} \sum_{i=1}^{n} x_i$, it is more advantageous for the member to buy the *ex post* PEC.

Proposition 4-3 shows that the premium of *ex ante* PEC, α^E, is fixed and identical across all the members. This premium can be explained as the membership fee, which is also deemed to be the commitment made by the member to undertake his liability in risk sharing.

Corollary 4-4. *For an individual member, the coverage of an ex ante Pareto efficient contract is more stable than the ex post Pareto efficient contract.*

Proof. The coverage $\beta_i^E(\boldsymbol{x})$ is the actual loss x_i minus a fixed amount, the premium α^E. Let the variance describe the volatility of the coverage. Then, there is

$$Var\left(\beta_i^E(\boldsymbol{x})\right) = v_i^2 < v_i^2 + \frac{1}{n^2}\sum_{r=1}^{n} v_r^2 = Var\left(\beta_i(\boldsymbol{x})\right)$$

It reflects that the coverage of *ex ante* PEC is more stable than the *ex post* one. □

Given the identical mean, $m_i = m_j$, $i \neq j$, $i, j = 1, 2, \ldots, n$, without the loss of generality, if we assume $v_i > v_j$, $i \neq j$, then the i^{th} member's risk is higher than the j^{th} one . Thus, there is the following corollary:

Corollary 4-5. When $m_i = m_j$, suppose there are four possible insurance contracts:

(a) high risk member buys ex post PEC

(b) low risk member buys ex post PEC

(c) high risk member buys ex ante PEC

(d) low risk member buys ex ante PEC

If $v_i^2 - v_j^2 < \frac{1}{n^2}\sum_{r=1}^{n} v_r^2$, the volatilities of the coverage in the four contracts can be ranked in this order: (d) < (c) < (b) < (a);

If $v_i^2 - v_j^2 > \frac{1}{n^2}\sum_{r=1}^{n} v_r^2$, the volatilities of the coverage in the four contracts can be ranked in this order: (d) < (b) < (c) < (a).

Proof. Without the loss of generality, suppose the i^{th} member is high-risk and the j^{th} one is low-risk. When $m_i = m_j$, there must be $v_i^2 > v_j^2$. Analogous to the proof of Corollary 4-4, there are

$$Var\left(\beta_i^E\left(\boldsymbol{x}\right)\right) = v_i^2 < v_i^2 + \frac{1}{n^2}\sum_{r=1}^{n} v_r^2 = Var\left(\beta_i\left(\boldsymbol{x}\right)\right)$$

and

$$Var\left(\beta_j^E\left(\boldsymbol{x}\right)\right) = v_j^2 < v_j^2 + \frac{1}{n^2}\sum_{r=1}^{n} v_r^2 = Var\left(\beta_j\left(\boldsymbol{x}\right)\right)$$

Due to $v_i^2 > v_j^2$, $i \neq j$, obviously, there must be $Var\left(\beta_i^E\left(\boldsymbol{x}\right)\right) > Var\left(\beta_j^E\left(\boldsymbol{x}\right)\right)$ and $Var\left(\beta_i\left(\boldsymbol{x}\right)\right) > Var\left(\beta_j\left(\boldsymbol{x}\right)\right)$. Thus, if $v_i^2 - v_j^2 < \frac{1}{n^2}\sum_{r=1}^{n} v_r^2$, then

$$Var\left(\beta_j^E\left(\boldsymbol{x}\right)\right) < Var\left(\beta_i^E\left(\boldsymbol{x}\right)\right) < Var\left(\beta_j\left(\boldsymbol{x}\right)\right) < Var\left(\beta_i\left(\boldsymbol{x}\right)\right);$$

if $v_i^2 - v_j^2 > \frac{1}{n^2}\sum_{r=1}^{n} v_r^2$, there is

$$Var\left(\beta_j^E\left(\boldsymbol{x}\right)\right) < Var\left(\beta_j\left(\boldsymbol{x}\right)\right) < Var\left(\beta_i^E\left(\boldsymbol{x}\right)\right) < Var\left(\beta_i\left(\boldsymbol{x}\right)\right).$$ □

The insurance coverage is the main payout for a P&I Club. As an underwriter, P&I Clubs dislike the unstable payout on coverage. In another words, the coverage level might fluctuate unpredictably. The unpredictable payment on the coverage complicates the estimation of the bankruptcy risk for P&I Clubs, and even influences the management strategy adopted by the managers.

B. $m_i \neq m_j$

In this research, when $m_i \neq m_j$, $i, j = 1, 2, \ldots, n$, the n members are simply divided into two groups. Assume n_1 members have the density function of random loss $f_i^H(x_i)$ with mean, m^H, and n_2 members have the density function $f_j^L(x_j)$ with mean, m^L. Since $m^H > m^L$, there are n_1 high-risk members and n_2 low-risk ones.

Proposition 4-6. *When $m_i \neq m_j$, $i, j = 1, 2, \ldots, n$, an insurance contract $C^{EM}(n)$ offered by a P&I Club is ex ante Pareto efficient if it satisfies*:

$$\beta_i^{EM}(\boldsymbol{x}) = x_i - \frac{n_1 \cdot m^H + n_2 \cdot m^L}{n_1 + n_2} + k \tag{4-6}$$

where x_i is the loss of the individual i in the state vector $(x_1, x_2, \ldots, x_n)$. The premium α^{EM} and the initial investment of an individual primary member k satisfies:

$$\alpha^{EM} + k = \frac{n_1 \cdot m^H + n_2 \cdot m^L}{n_1 + n_2} \tag{4-7}$$

Proof. The Pareto-efficient P&I insurance contract is the solution of the following program

$$\max_{C(n)} \left\{ \sum_{i=1}^{n_1} \int_{\Omega} U\left(\omega - k - x_i + \beta_i^{EM}(\boldsymbol{x})\right) dF_n(\boldsymbol{x}) + \sum_{i=1}^{n_2} \int_{\Omega} U\left(\omega - k - x_j + \beta_j^{EM}(\boldsymbol{x})\right) dF_n(\boldsymbol{x}) \right\}$$

$$\text{s.t. } (n_1 + n_2)k - \sum_{i=1}^{n_1} \int_{\Omega} \beta_i^{EM}(\boldsymbol{x}) dF_n(\boldsymbol{x}) - \sum_{j=1}^{n_2} \int_{\Omega} \beta_j^{EM}(\boldsymbol{x}) dF_n(\boldsymbol{x}) = 0$$

Let $\lambda(\boldsymbol{x})$ denote the Lagrangian multiplier of the constraint, the first order conditions for the i^{th} and j^{th} members are given by:

$$U'\left(\omega - k - x_i + \beta_i^{EM}\left(\boldsymbol{x}\right)\right) - \lambda\left(\boldsymbol{x}\right) = 0$$

$$U'\left(\omega - k - x_j + \beta_j^{EM}\left(\boldsymbol{x}\right)\right) - \lambda\left(\boldsymbol{x}\right) = 0$$

From the first order conditions, it is obvious that for $i, j = 1, 2, \ldots, n$, $i \neq j$,

$$U'\left(\omega - k - x_i + \beta_i^{EM}\left(\boldsymbol{x}\right)\right) = U'\left(\omega - k - x_j + \beta_j^{EM}\left(\boldsymbol{x}\right)\right)$$

where the i^{th} member is high risk and the j^{th} member is low risk, that is, $m_i > m_j$. Thus, there exist a premium call and the initial investment k independent of i, which satisfies

$$-x_i + \beta_i^{EM}\left(\boldsymbol{x}\right) = -x_j + \beta_j^{EM}\left(\boldsymbol{x}\right) = -\alpha_i = -\alpha_j$$

Let $\alpha^E = \alpha_i = \alpha_j$, $i, j = 1, 2, \ldots, n$. Then, there is the summation that

$$-\sum_{i=1}^{n_1} x_i - \sum_{j=1}^{n_2} x_j + \sum_{i=1}^{n_1} \beta_i^{EM}\left(\boldsymbol{x}\right) + \sum_{j=1}^{n_2} \beta_j^{EM}\left(\boldsymbol{x}\right) = -\left(n_1 + n_2\right) \cdot \alpha^E$$

Take the expectation on both sides of the equation with respect to $\boldsymbol{x}$ and there exists

$$\begin{aligned}
&-\sum_{i=1}^{n_1} \int_0^\infty x_i f_i^H\left(x_i\right) dx_i - \sum_{j=1}^{n_2} \int_0^\infty x_j f_j^L\left(x_j\right) dx_i + \sum_{i=1}^{n_1} \int_0^\infty \beta_i^{EM}\left(\boldsymbol{x}\right) dF_n\left(\boldsymbol{x}\right) \\
&+\sum_{j=1}^{n_2} \int_0^\infty \beta_j^{EM}\left(\boldsymbol{x}\right) dF_n\left(\boldsymbol{x}\right) = -\left(n_1 + n_2\right) \cdot \alpha^{EM}
\end{aligned}$$

Since $\sum_{i=1}^{n_1} \int_0^\infty \beta_i^{EM}\left(\boldsymbol{x}\right) dF_n\left(\boldsymbol{x}\right) + \sum_{j=1}^{n_2} \int_0^\infty \beta_j^{EM}\left(\boldsymbol{x}\right) dF_n\left(\boldsymbol{x}\right) = \left(n_1 + n_2\right) \cdot k$,

$$\begin{aligned}
&-n_1 \cdot m^H - n_2 \cdot m^L + \left(n_1 + n_2\right) \cdot k = -\left(n_1 + n_2\right) \cdot \alpha^{EM} \\
&\Rightarrow \alpha^{EM} = \frac{n_1 \cdot m^H + n_2 \cdot m^L}{\left(n_1 + n_2\right)} - k
\end{aligned}$$

Thus, the coverage

$$\beta_i^{EM}(\boldsymbol{x}) = x_i - \frac{n_1 \cdot m^H + n_2 \cdot m^L}{(n_1 + n_2)} + k \qquad \square$$

Recall that the premium of the *ex ante* PEC under $m_i = m_j = m$, $i \neq j$, is $\alpha^E = m - k$. There are two extreme cases: (1) If $m = m^H$, $\alpha^{EH} \triangleq \alpha^E = m^H - k$; and (2) if $m = m^L$, $\alpha^{EL} \triangleq \alpha^E = m^L - k$.

Corollary 4-7. $\alpha^{EL} < \alpha^{EM} < \alpha^{EH}$*, and* α^{EM} *increases with* n_1 *and decreases with* n_2.

Proof. Since $\alpha^{EM} = \dfrac{n_1 \cdot m^H + n_2 \cdot m^L}{n_1 + n_2} - k$, $\alpha^{EH} = m^H - k$, and $\alpha^{EL} = m^L - k$, there is

$$\alpha^{EM} = \frac{n_1 \cdot m^H + n_2 \cdot m^H + n_2 \cdot m^L - n_2 \cdot m^H}{n_1 + n_2} - k$$

$$\Rightarrow \alpha^{EM} = \frac{(n_1 + n_2) \cdot m^H - n_2 \cdot (m^H - m^L)}{n_1 + n_2} - k$$

$$\Rightarrow \alpha^{EM} = m^H - \frac{n_2}{n_1 + n_2} \cdot (m^H - m^L) - k$$

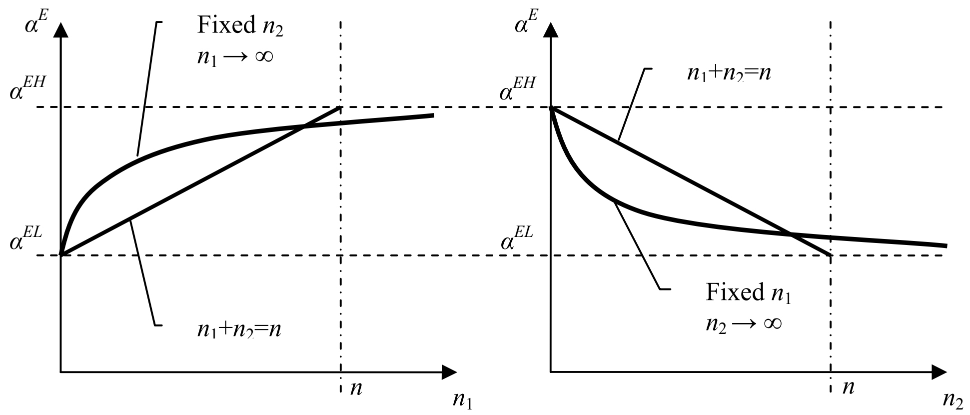
On the other hand, there is

$$\alpha^{EM} = \frac{n_1 \cdot m^H + n_2 \cdot m^L + n_1 \cdot m^L - n_1 \cdot m^L}{n_1 + n_2} - k$$

$$\Rightarrow \alpha^{EM} = \frac{\left(n_1 + n_2\right) \cdot m^L + n_1 \cdot \left(m^H - m^L\right)}{n_1 + n_2} - k$$

$$\Rightarrow \alpha^{EM} = m^L + \frac{n_1}{n_1 + n_2} \cdot \left(m^H - m^L\right) - k$$

Figure 4-1: Curve of α^{EM} with respect to n_1 and n_2



Due to $m^H > m^L$, there is $\alpha^{EL} < \alpha^{EM} < \alpha^{EH}$, and α^{EM} increases with n_1 and decreases with n_2 (as shown in Figure 4-1). □

Corollary 4-7 shows that the low-risk members are actually subsidizing the high-risk ones. Since α^{EM} increases with n_1, the low-risk members do not want to pool risk with the high-risk members, where the insurance cost will increase substantially. However, owing to the same logic, the high-risk individuals prefer to share risk with the low-risk ones. Thus, if a high-risk member wants to apply for membership of a

Club full of low-risk members, he will be rejected unless he distorts information about the mean loss and pretends to be a low-risk one [67].

4.3 P&I insurance market

4.3.1 Integrated P&I insurance contract

Neither the *ex post* nor *ex ante* PEC can fully cover the peculiarities of a P&I insurance contract. In section 4.2, it is proved that the *ex post* PEC exists — if the P&I Club follows the *ex post* adjusted premium, as demonstrated by Fagart et al [41]. Simultaneously, the existence of *ex ante* PEC is also examined, when the P&I Club offers the contract conditional on the expected non-profit. This model is based on the work of Smith and Stutzer [96, 97].

In practical P&I insurance, the premium will be collected as the commitment at the beginning of a policy year. When the premium income together with the initial capital investment cannot fully indemnify all claims, there exists a deficit in the account of the P&I Club. Then a further premium is called to balance this deficit.

On the other hand, an individual member contributes α^E and k to the risk pool and is obligated to recover a certain part of the total losses, that is, the average loss for a single member, $\frac{1}{n}\sum_{i=1}^{n} x_i$. When the actual per capita loss for a single member is less than the expectation, then the P&I Club has overestimated the risk. The Club offers the PEC at α^E. On the contrary, when the actual per capita loss exceeds the expectation, then the P&I Club has underestimated the risk. The Club offers the PEC at α.

Thus, let α^* denote the integrated premium, and $\beta_i^*(\mathbf{x})$ denote the related coverage. Then there are

$$\alpha^* = \max\left\{\alpha^E, \alpha\right\} = \max\left\{\frac{1}{n}\sum_{i=1}^{n} m_i, \frac{1}{n}\sum_{i=1}^{n} x_i\right\} - k \tag{4-8}$$

$i \neq j$, i, j=1, 2, …,n, and

$$\begin{aligned}\beta_i^*\left(\mathbf{x}\right) &= x_i - \alpha^* = x_i - \max\left\{\alpha^E, \alpha\right\} \\ &= x_i + k - \max\left\{\frac{1}{n}\sum_{i=1}^{n} m_i, \frac{1}{n}\sum_{i=1}^{n} x_i\right\}\end{aligned} \tag{4-9}$$

The P&I insurance contract is denoted by $C^*\left(n\right)$, that is,

$$C^*\left(n\right) = \left\{\beta_i^*\left(\mathbf{x}\right), \mathbf{x} = \left(x_1, x_2, ..., x_n\right) \in \Omega = \left[0, \infty\right[^n\right\}.$$

4.3.2 Club size and the P&I insurance market

Let random variable y denote the total loss of the n members, that is, $y = \sum_{i=1}^{n} x_i$. Let $g(y)$ and $G(y)$ denote the density function and cumulative probability function of random variable y. Given the density functions $f_i(x)$ for i= 1, 2, …, n, it is convenient to obtain $g(y)$ and $G(y)$ through the convolution of the n random variables x_i. Let $V(\alpha^E, n, k)$ denote the expected utility of a mutual's member, which can be written as

$$V\left(\alpha^E, n, k\right) = \int_{R^+} U\left(\min\left(\omega - k - \alpha^E, \omega - k - \frac{y}{n} + k\right)\right) dG\left(y\right) \tag{4-10}$$

Proposition 4-8. *When n is large enough, x_i are independent and identically distributed with finite mean m and finite variance v^2, and the P&I Club offers the contract at $C^*(n)$, the expected utility of an individual member, $V(\alpha^E, n, k)$, is a monotonously increasing function with respect to n.*

Proof. The independent random variable x_i, i= 1, 2,…, n, follows identical heavy-tailed distributions with finite means, m, and variances, v^2. The total loss of n members is $y = \sum_{i=1}^{n} x_i$. If n is large enough, by the Central Limit Theorem, there is a random variable z such that

$$z = \left(\frac{y}{n} - m\right) \Big/ \frac{v}{\sqrt{n}} \sim N\left(0,1\right)$$

$$\Rightarrow \frac{y}{n} = \frac{v}{\sqrt{n}} \cdot z + m$$

Thus, the random variable y and distribution $G(y)$ can be replaced by z and standard normal distribution $\Phi(z)$. Then it becomes much more convenient to talk about the property of the function (4-10).

$$V\left(\alpha^E, n, k\right) = \int_{R} U\left(\min\left(\omega - k - \alpha^E, \omega - k - \frac{v}{\sqrt{n}} \cdot z - m + k\right)\right) d\Phi\left(z\right)$$

Let A denote a value of z which is defined by

$$A = \inf\left\{ z : \omega - \alpha^E > \omega - \frac{v}{\sqrt{n}} \cdot z - m + k \right\}$$

$$\Rightarrow A = \left(\alpha^E + k - m\right) \cdot \frac{\sqrt{n}}{v}$$

Since $\alpha^E = m - k$, A = 0, then $V(\alpha^E, n, k)$ can be written as

$$V\left(\alpha^E, n, k\right) = \int_{-\infty}^{0} U\left(\omega - k - \alpha^E\right) d\Phi\left(z\right) + \int_{0}^{\infty} U\left(\omega - k - \frac{v}{\sqrt{n}} \cdot z - m + k\right) d\Phi\left(z\right)$$

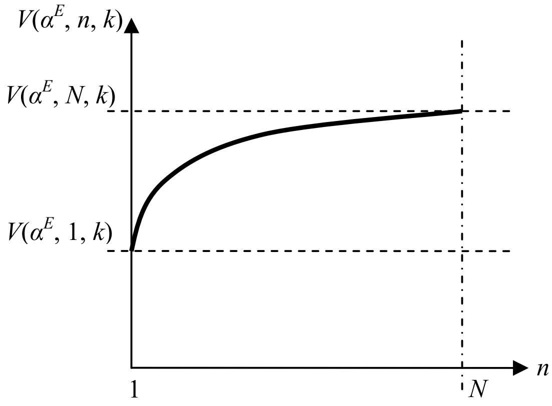
After derivation with respect to n, there is

$$\frac{\partial V\left(\alpha^E, n, k\right)}{\partial n} = \int_{0}^{\infty} U'\left(\omega - \frac{v}{\sqrt{n}} \cdot z - m\right) \frac{v}{2n^{3/2}} \cdot z d\Phi\left(z\right)$$

Due to $U' > 0$, there is $\frac{\partial V\left(\alpha^E, n, k\right)}{\partial n} > 0$. Then the proposition is proved. □

Based on the proof of Proposition 4-8, it is also easy to prove that the second order derivation is $\frac{\partial^2 V\left(\alpha^E, n, k\right)}{\partial n^2} < 0$. It demonstrates that risk sharing within a mutual improves the welfare of its members. An individual member's utility increases with the size of risk pooling (see figure 4-2).

Figure 4-2: Individual member's utility $V(\alpha^E, n, k)$



Nevertheless, there are in total N potential members in this economy. Thus, the upper limit of a P&I Club's size is N, that is, all members pool their risks in a single Club (see figure 4-2). At this moment, the utility of an individual member reaches the maximum, $V(\alpha^E, N, k)$.

However, the utility of an individual member V is not always the increasing function on n. That is because, in most of Chapter 4, it is presumed that *k is constant, and that the initial capital invested by the n members is n·k, which increases linearly with the number of new members*.

Now, consider another situation where the foundation of the P&I Clubs must be approved by the market administrative. The market administrative usually requests the organization possessing a certain amount of capital at the foundation, which is denoted by K. At this moment, the per capita investment k decreases with n, where $k = K/n$. It can also be proved that the utility of an individual member $V(\alpha^E, n, K/n)$ is increasing with the size of the risk pool.

Example 4-9. Take the exponential utility function as an example to show that, for the feasible parameters, an individual member's utility decreases with *n*.

Suppose the random variables x_i, $i = 1, 2, \ldots, n$, are independent and identically distributed with finite mean *m* and variance v^2, and the utility function *U* has the following form:

$$U(x) = u - \frac{\delta}{\gamma}\exp\{-\gamma x\}$$

where δ and γ are two positive constants [120]. This utility function plays a notable role in insurance mathematics and actuarial science.

Following the analogous approach in the proof of Proposition 4-8, $V(\alpha^E, n, K/n)$ can be written as

$$V\left(\alpha^E, n, \frac{K}{n}\right) = \int_{-\infty}^{0} U\left(\omega - \frac{K}{n} - \alpha^E\right) d\Phi(z) + \int_{0}^{\infty} U\left(\omega - \frac{K}{n} - \frac{v}{\sqrt{n}}\cdot z - m + \frac{K}{n}\right) d\Phi(z)$$

After derivation with respect to *n*, there is

$$\begin{aligned}\frac{\partial V\left(\alpha^E, n, K/n\right)}{\partial n} &= \frac{1}{2n^2}\int_0^\infty U'\left(\omega - \frac{K}{n} - \frac{v}{\sqrt{n}}\cdot z - m + \frac{K}{n}\right)\cdot\left(v\cdot z\cdot n^{1/2}\right) d\Phi(z) \\ &= \frac{\delta v}{2n^{3/2}}\exp\left\{-\gamma(\omega - m) + \frac{\gamma^2 v^2}{2n}\right\}\left[\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{\gamma^2 v^2}{2n}\right\} + \frac{\gamma v}{\sqrt{n}}\Phi\left(\frac{\gamma v}{\sqrt{n}}\right)\right]\end{aligned}$$

Hereby, the first-order derivation is always positive. The individual utility is an increasing function of pooling size, *n*, for a given exponential utility function.

Proposition 4-10. *When n is large enough,* x_i *have identical variance v, and the mean m is the function of n, denoted by m*(*n*)*. The P&I Club offers the contract at*

$C^*(n)$. *The expected utility of an individual member,* $V(\alpha^E, n, K/n)$, *is a monotonously increasing function with respect to n, when* $m'(n) < 0$.

Proof. The independent random variable x_i, i= 1, 2,…, n, follows identical heavy-tailed distributions with finite means and variances. If n is large enough, by the Central Limit Theorem, there is a random variable z that

$$z = \left(\frac{y}{n} - m(n)\right) \Big/ \frac{v}{\sqrt{n}} \sim N(0,1)$$

$$\Rightarrow \frac{y}{n} = \frac{v}{\sqrt{n}} \cdot z + m(n)$$

Thus, the random variable y and distribution $G(y)$ can be replaced by z and standard normal distribution $\Phi(z)$. Then it becomes much more convenient to talk about the property of the function (4-10).

$$V\left(\alpha^E, n, k\right) = \int_R U\left(\min\left(\omega - \frac{K}{n} - \alpha^E, \omega - \frac{K}{n} - \frac{v}{\sqrt{n}} \cdot z - m(n) + \frac{K}{n}\right)\right) d\Phi(z)$$

Let A denote a value of z which is defined by

$$A = \inf\left\{z : \omega - \alpha^E > \omega - \frac{v}{\sqrt{n}} \cdot z - m(n) + \frac{K}{n}\right\}$$

$$\Rightarrow A = \left(\alpha^E + \frac{K}{n} - m(n)\right) \cdot \frac{\sqrt{n}}{v}$$

Since $\alpha^E = m(n) - \frac{K}{n}$, $A = 0$, then $V(\alpha^E, n, K/n)$ can be written as

$$\begin{aligned}
\frac{\partial V\left(\alpha^E, n, K/n\right)}{\partial n} &= -\frac{1}{2}U'\left(\omega - m(n)\right)\cdot m'(n) \\
&\qquad + \int_0^\infty U'\left(\omega - \frac{v}{\sqrt{n}}\cdot z - m(n)\right)\cdot\left(\frac{v\cdot z}{2n^{3/2}} - m'(n)\right)d\Phi(z) \\
&= \delta\exp\left\{-\gamma\left(\omega - m(n)\right)\right\} \\
&\qquad \cdot\left(-\frac{m'(n)}{2} + \frac{v}{2\sqrt{2\pi}n^{3/2}} + \left(\frac{\gamma v^2}{2n} - m'(n)\right)\exp\left\{\frac{\gamma^2 v^2}{2n}\right\}\Phi\left(\frac{\gamma v}{\sqrt{n}}\right)\right)
\end{aligned}$$

If $m(n)$ is a decreasing function of n, $m'(n) < 0$, then $\frac{\partial V\left(\alpha^E, n, K/n\right)}{\partial n} > 0$ definitely. The utility of an individual member increases with the pooling size, and the members would like to share risk with more members. □

Conversely, when $m'(n) > 0$, $\frac{\partial V\left(\alpha^E, n, K/n\right)}{\partial n}$ is not necessarily positive. When

$$m'(n)\cdot\left[1 + 2\exp\left\{\frac{\gamma^2 v^2}{2n}\right\}\Phi\left(\frac{\gamma v}{\sqrt{n}}\right)\right] > \left[\frac{v}{\sqrt{2\pi}n^{3/2}} + \frac{\gamma v^2}{n}\exp\left\{\frac{\gamma^2 v^2}{2n}\right\}\Phi\left(\frac{\gamma v}{\sqrt{n}}\right)\right] \tag{4-11}$$

there is $\frac{\partial V\left(\alpha^E, n, K/n\right)}{\partial n} < 0$. The derivative $m'(n)$ depends on the risk structure of the members in the Club. The example below illustrates that the inequality $\frac{\partial V\left(\alpha^E, n, K/n\right)}{\partial n} < 0$ exists when the increase of n is caused by the entry of high-risk members.

Example 4-11. Suppose there are a fixed amount of low risk members, n_2. The rest $n - n_2$ members are high risk ones. The increase of n is caused by the entry of high risk members. Then, there is

$$m(n) = m^H - \frac{n_2}{n}\left(m^H - m^L\right)$$

and the first-order derivative is

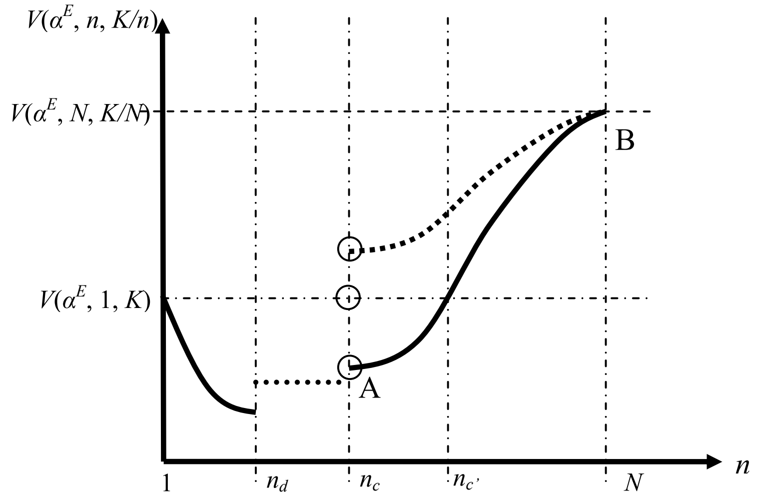
$$m'(n) = \frac{n_2}{n^2}\left(m^H - m^L\right)$$

Substitute $m'(n)$ into inequality (4-11), and there exists

$$n_2\left(m^H - m^L\right)\cdot\left[1 + 2\exp\left\{\frac{\gamma^2 v^2}{2n}\right\}\Phi\left(\frac{\gamma v}{\sqrt{n}}\right)\right] > \left[\frac{v}{\sqrt{2\pi}} n^{1/2} + n\gamma v^2 \exp\left\{\frac{\gamma^2 v^2}{2n}\right\}\Phi\left(\frac{\gamma v}{\sqrt{n}}\right)\right] \quad (4\text{-}12)$$

When $n = 0$, the left side of the inequality above is approaching positive infinite, while the right side is zero. When $n \to \infty$, the left side is approaching to $2 \cdot n_2\left(m^H - m^L\right)$, and the right side is positive infinite. For feasible γ and v, there is a certain value of n, denoted by n_c. For any pooling size greater than n_c, the inequality symbol of (4-12) is not satisfied, so that the utility of an individual member is increasing with the Club size, n. There is also another value of n, denoted by n_d. For any n less than n_d, the inequality (4-12) is satisfied, and the utility of a single member decreases with the Club size, n (see Figure 4-3).

Figure 4-3: Individual member's utility $V(\alpha^E, n, K/n)$



Strictly, the boundaries, n_c and n_d, might be different. The curve $V(\alpha^E, n, K/n)$ has complicated properties within $[n_d, n_c]$, because the right side of the inequality (4-12) is possibly a decreasing function with respect to n, when n is relatively lower. Although the solution of inequality (4-12) has no explicit form, by the numerical approach the boundary n_c can be confirmed. If the utility at n_c outperforms the self-insurance $V(\alpha^E, 1, K)$, the Club has to enter more than n_c members sufficiently so as to improve the joint utility of all members (see the increasing dotted line in figure 4-3). On the contrary, if the utility at n_c is worse than the self-insurance, the Club prefers to enter more than $n_{c'}$ members, beyond which the mutual insurance outperforms the self-insurance (see the increasing curve AB in figure 4-3).

Suppose there are P P&I Clubs, each Club being of identical size in the marine mutual insurance market. Then, each Club has n members pooling risk, where n =

N/P. Thus, n is a decreasing function of P. Substitute $n = N/P$ into $V(\alpha^E, n, K/n)$, and it is easy to find that when $V(\alpha^E, n, K/n)$ increases with n, it must decrease with P. On the other hand, when $V(\alpha^E, n, K/n)$ decreases with n, it must increase with P. Based on the boundary condition with respect to n, when $P > N/n_c$, $V(\alpha^E, P, P\cdot K/N)$ is a decreasing function of P; when $P < N/n_d$, $V(\alpha^E, P, P\cdot K/N)$ is an increasing function of P.

From the perspective of an underwriter, a single P&I Club would like to insure more than n_c members, whilst from the perspective of the whole P&I insurance market, the number of Clubs is restricted by a boundary N/n_c. Thus, neither a monopolistic nor free competitive market can form in the P&I insurance field.

4.4 Summary and conclusion

This chapter is briefly summarized as follows. In this chapter, a P&I contract $C^*(n)$ is formed through integrating the *ex post* PEC and *ex ante* PEC. At the beginning of a policy year, the P&I Club offers an *ex ante* PEC, conditional on the expected profit. By the end of the policy year, if the premium income and ICR are used up, further premium will be called for to balance the deficit through offering an *ex post* PEC.

ICR is the capital reserve of a P&I Club. Albeit Doherty and Dionne [30], and Fagart et al [41, 42] considered that having no capital reserve is one of the most important characteristics of a mutual, whereas actual observations of a P&I Club show that a mutual underwriter can have a non-zero capital reserve. The non-zero capital reserve does not influence the monotony of an individual member's utility. When every primary member of the Club has identical mean loss m, it can be proved that the utility of an individual member increases with the Club size. However, if m is the function of Club size n, the optimal Club size depends on the monotony of $m(n)$. Each Club has to try his best to solicit more low risk members to become members. As for the high risk members, there is an infinimum, n_c, and the number of high risk members has to exceed this infinimum to ensure that each individual member will benefit from the risk pooling. This explains how the oligopolistic market of P&I insurance has formed.

Chapter 5. Competition in P&I Clubs

Chapter 4 has introduced the formation of Pareto efficient contract in a single P&I Club, without the premium competition. In this chapter, the scenario is changed. Consider there are two P&I Clubs. One of them adopts a certain competition strategy based on its own ICR level, while another one conducts the countermeasure to interrupt the membership switch. Each Club has to find its Pareto efficient contract under this new scenario. Following the same methodology introduced in Chapter 4, this chapter demonstrates the welfare change of the Club members under premium competition.

In the general theory of oligopoly, a firm's objective function is to maximize the profits. By collusion, firms maximize their joint profits. Each firm has great motivation to lower their price to solicit customers [113]. However, with respect to mutual insurance, there are some peculiarities that distinguish the oligopolistic P&I Clubs from the general insurance firms. The fundamental difference is that the P&I Club is a non-profit organization. Thus, the objective function of P&I Clubs is to maximize the members' utility. The P&I Clubs collude in order to maximize the joint utility of all members in this trust.

A member may like to reduce his insurance cost through switching his membership from the **Holding Club**, who offers a high-premium contract, to a **New Club**, whose contract is offered at a low premium. The utility of this **entered**

member is definitely increased, whereas the other members in the New Club, as well as the members left in the Holding Club, probably have to undertake more risks, rather than benefit from this switching.

A New Club wishes to insure a vessel that is currently insured by a Holding Club. The Agreement of the Group 1999 suggests that the New Club cannot offer the contract at a premium that is unreasonably lower than the premium of the Holding Club. The Holding Club should supply the New Club with the entered member's record, and a statement of the current premium. Through sharing these two perspectives of information, it ensures that the New Club prices the contract on an identical information basis with the Holding Club. The EU Committee questioned this agreement based on the antitrust law. The EU Committee encourages the Group to release the boundary on the price competition. However, the price competition does not necessarily lead to an increase of joint utility of the overall population.

In this chapter, certain definitions and suppositions must first be made to specify the problems involved. The framework of this chapter is built upon how to define "competition". So far, the competition in this research refers merely to price competition. However, in marine mutual insurance, competition is the behavior of a New Club to attract a member to switch membership from the Holding Club to the New Club, through offering the contract at a premium lower than the one offered by the Holding Club.

In the previous chapter, the integrated premium, α^*, depends on $\max\{\alpha^E, \alpha\}$. For a given k and a certain Club size n, the *ex ante* PEC with a fixed premium is applicable if the actual per capita loss is lower than the expectation. On the other hand, if the actual per capita loss is higher than the expectation, the *ex post* PEC is available, with a flexible premium. There then exists a practical problem with respect to the latter situation. When the actual per capita losses faced by the New Club are much lower than those of the Holding Club, it is difficult to evaluate whether or not the *ex post* contract of the New Club is unreasonably cheaper than the one of the Holding Club.

In the competition for P&I insurance, a member takes into consideration two perspectives of information when he decides on his membership: (a) Whether the per capita liability on the loss will expectedly increase; and (b) whether there are more ICR resources available that can provide more coverage.

The rest of this chapter is organized as follows: Section 5.1 provides the overall Pareto efficient contract under strategy I. Following the analogous logic, section 5.2 focuses on the insurance contract under another two special competition strategies. In section 5.3, the decision making process of the entered member, together with that of the New and Holding Clubs, will be discussed, while the countermeasures of the Holding Club in the competition are provided for in section 5.4. Section 5.5 briefly summarizes the whole chapter.

5.1 Overall Pareto efficient contract under competition

5.1.1 Basic Assumption

Consider a simplified oligopolistic market, that is, where there are only two independent P&I Clubs. One is the Holding Club, and another one is the New Club. There are *s* members switching their memberships from the Holding Club to the New Club. Some assumptions are given below.

(a) The Holding Club has n members, while the New Club has $\tilde{n}$ members.

(b) Each primary member in the New Club invests $\tilde{k}$ into the ICR $\tilde{K}$ so that $\tilde{K} = \tilde{n} \cdot \tilde{k}$.

(c) Each primary member in the Holding Club invests k into the ICR K so that $K = n{\cdot}k$.

(d) The premium of the New (Holding) Club before membership switch is called **pre-switch premium** of the New (Holding) Club, while the premium of the New (Holding) Club after membership switch is called **post-switch premium** of the New (Holding) Club.

(e) The entered member does not contribute either k or $\tilde{k}$ to the New Club.

(f) The initial investment of the entered member k in the Holding Club is not refundable.

(g) After membership switch, the entered member is indexed as the $\tilde{n}+s$ th member of the New Club. Let $\tilde{\boldsymbol{x}}_s$ denote the random vector $\left(\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_{\tilde{n}+s}\right)$, where $\tilde{x}_{\tilde{n}+1}, \tilde{x}_{\tilde{n}+2}, ..., \tilde{x}_{\tilde{n}+s}$ are indexed by $x_{n-s+1}, x_{n-s+2}, ..., x_n$ in the Holding Club, and $F_{\tilde{n}+s}\left(\tilde{\boldsymbol{x}}_s\right)$ denote the joint distribution of the random vector $\tilde{\boldsymbol{x}}_s$. The mean of $\tilde{x}_i$ is denoted by $\tilde{m}_i$ and the variance is $\tilde{v}_i^2$.

(h) The contract offered by the New Club is denoted by $\tilde{C}\left(n+s\right)$

$$\tilde{C}\left(\tilde{n}+s\right)=\left\{\beta_i\left(\tilde{\boldsymbol{x}}_s\right), \tilde{\boldsymbol{x}}_s=\left(\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_{\tilde{n}+s}\right) \in \tilde{\Omega}_s=\left[0, \infty\right[^{\tilde{n}+s}\right\}$$

5.1.2 Pareto efficient contract of the New Club

Strategy I (overall PEC strategy): An entered member becomes a new member of the New Club without any contribution to ICR. The New Club maximizes the joint utility of the primary members and his new client, where this strategy is called **overall PEC strategy**.

Proposition 5-1. *If there are s individual members switching membership, then the i^th primary member,* $i=1,2,...,\tilde{n}$, *in the New Club has the ex post PEC at:*

$$\tilde{\beta}_i^I\left(\tilde{\boldsymbol{x}}_s\right)=\tilde{x}_i-\frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{x}_i+\frac{\tilde{n}}{\tilde{n}+s}\tilde{k}-\frac{s}{\tilde{n}+s}\left(k-\tilde{k}\right) \tag{5-1}$$

and the premium is collected at:

$$\tilde{\alpha}^{I} = \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{x}_i - \frac{\tilde{n}}{\tilde{n}+s}\tilde{k} + \frac{s}{\tilde{n}+s}\left(k-\tilde{k}\right). \tag{5-2}$$

For the entered member, the coverage is

$$\beta_j^{I}\left(\tilde{\mathbf{x}}_s\right) = \tilde{x}_j - \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{x}_i + \frac{\tilde{n}}{\tilde{n}+s}k \tag{5-3}$$

$j = \tilde{n}+1,...,\tilde{n}+s$*, and the premium is*

$$\alpha^{I} = \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{x}_i - \frac{\tilde{n}}{\tilde{n}+s}k \tag{5-4}$$

Proof. The optimization problem is

$$\max_{\tilde{C}(\tilde{n}+s)}\left\{\sum_{i=\tilde{n}+1}^{\tilde{n}+s}\int_{\tilde{\Omega}_s} U\left(\omega-k-\tilde{x}_i+\beta_i^{I}\left(\tilde{\mathbf{x}}_s\right)\right)dF_{\tilde{n}+s}\left(\tilde{\mathbf{x}}_s\right)+\sum_{i=1}^{\tilde{n}}\int_{\tilde{\Omega}_s} U\left(\omega-\tilde{k}-\tilde{x}_i+\tilde{\beta}_i^{I}\left(\tilde{\mathbf{x}}_s\right)\right)dF_{\tilde{n}+s}\left(\tilde{\mathbf{x}}_s\right)\right\}$$

s.t. $\sum_{i=1}^{\tilde{n}}\tilde{\beta}_i^{I}\left(\tilde{\mathbf{x}}_s\right)+\sum_{i=\tilde{n}+1}^{\tilde{n}+s}\beta_i^{I}\left(\tilde{\mathbf{x}}_s\right) = \tilde{n}\cdot\tilde{k}$

With respect to $\beta_i^{I}\left(\tilde{\mathbf{x}}_s\right)$ and $\tilde{\beta}_i^{I}\left(\tilde{\mathbf{x}}_s\right)$, $i = 1,2,...,\tilde{n}+s$, the first order conditions are

$$U'\left(\omega-k-\tilde{x}_{i'}+\beta_{i'}^{I}\left(\tilde{\mathbf{x}}_s\right)\right)-\lambda\left(\tilde{\mathbf{x}}_s\right)=0$$

$$U'\left(\omega-k-\tilde{x}_{j'}+\beta_{j'}^{I}\left(\tilde{\mathbf{x}}_s\right)\right)-\lambda\left(\tilde{\mathbf{x}}_s\right)=0$$

$$U'\left(\omega-\tilde{k}-\tilde{x}_i+\tilde{\beta}_i^{I}\left(\tilde{\mathbf{x}}_s\right)\right)-\lambda\left(\tilde{\mathbf{x}}_s\right)=0$$

$$U'\left(\omega-\tilde{k}-\tilde{x}_j+\tilde{\beta}_j^I\left(\tilde{\mathbf{x}}_s\right)\right)-\lambda\left(\tilde{\mathbf{x}}_s\right)=0$$

where $i' \neq j'$, $i \neq j$, $i, j=1,2,...,\tilde{n}$, and $i', j'=\tilde{n}+1, \tilde{n}+2,...,\tilde{n}+s$. Then, there is

$$\begin{aligned}&-k-\tilde{x}_{j'}+\beta_{j'}^I\left(\tilde{\mathbf{x}}_s\right)=-k-\tilde{x}_{i'}+\beta_{i'}^I\left(\tilde{\mathbf{x}}_s\right)=-\tilde{k}-\tilde{x}_i+\beta_i^I\left(\tilde{\mathbf{x}}_s\right)=-\tilde{k}-\tilde{x}_j+\beta_j^I\left(\tilde{\mathbf{x}}_s\right)\\&=-\tilde{k}-\tilde{\alpha}_i^I=-\tilde{k}-\tilde{\alpha}_j^I\end{aligned}$$

Let $\tilde{\alpha}_i^I=\tilde{\alpha}_j^I=\tilde{\alpha}^I$, $i, j=1,2,...,\tilde{n}$. Summate the following equation on both sides with respect to *i*,

$$-\tilde{k}-\tilde{x}_i+\tilde{\beta}_i^I\left(\tilde{\mathbf{x}}_s\right)=-\tilde{k}-\tilde{\alpha}_i^I$$

so that

$$-\tilde{n}\cdot\tilde{k}-\sum_{i=1}^{\tilde{n}}\tilde{x}_i+\sum_{i=1}^{\tilde{n}}\tilde{\beta}_i^I\left(\tilde{\mathbf{x}}_s\right)=-\tilde{n}\cdot\tilde{k}-\tilde{n}\cdot\tilde{\alpha}^I$$

$$\Rightarrow -s\cdot k-\sum_{i=\tilde{n}+1}^{\tilde{n}+s}\tilde{x}_i-\tilde{n}\cdot\tilde{k}-\sum_{i=1}^{\tilde{n}}\tilde{x}_i+\sum_{i=1}^{\tilde{n}}\tilde{\beta}_i^I\left(\tilde{\mathbf{x}}_s\right)+\sum_{i=\tilde{n}+1}^{\tilde{n}+s}\beta_i^I\left(\tilde{\mathbf{x}}_s\right)=-\left(\tilde{n}+s\right)\cdot\tilde{k}-\left(\tilde{n}+s\right)\cdot\tilde{\alpha}^I$$

Due to $\sum_{i=1}^{\tilde{n}}\tilde{\beta}_i^I\left(\tilde{\mathbf{x}}_s\right)+\sum_{i=\tilde{n}+1}^{\tilde{n}+s}\beta_i^I\left(\tilde{\mathbf{x}}_s\right)=\tilde{n}\cdot\tilde{k}$, the equation above can be simplified so that

$$-s\cdot k-\sum_{i=\tilde{n}+1}^{\tilde{n}+s}\tilde{x}_i-\tilde{n}\cdot\tilde{k}-\sum_{i=1}^{\tilde{n}}\tilde{x}_i+\sum_{i=1}^{\tilde{n}}\tilde{\beta}_i^I\left(\tilde{\mathbf{x}}_s\right)+\sum_{i=\tilde{n}+1}^{\tilde{n}+s}\beta_i^I\left(\tilde{\mathbf{x}}_s\right)=-\left(\tilde{n}+s\right)\cdot\tilde{k}-\left(\tilde{n}+s\right)\cdot\tilde{\alpha}^I$$

$$\Rightarrow -s\cdot k-\tilde{n}\cdot\tilde{k}-\sum_{i=1}^{\tilde{n}+s}\tilde{x}_i+\tilde{n}\cdot\tilde{k}=-\left(\tilde{n}+s\right)\cdot\tilde{k}-\left(\tilde{n}+s\right)\cdot\tilde{\alpha}^I$$

$$\Rightarrow -s\cdot k-\sum_{i=1}^{\tilde{n}+s}\tilde{x}_i=-\left(\tilde{n}+s\right)\cdot\tilde{k}-\left(\tilde{n}+s\right)\cdot\tilde{\alpha}^I$$

$$\Rightarrow \tilde{\alpha}^I=\frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{x}_i-\frac{\tilde{n}}{\tilde{n}+s}\tilde{k}+\frac{s}{\tilde{n}+s}\left(k-\tilde{k}\right)$$

Then the coverage is

$$\tilde{\beta}_i^I\left(\tilde{\boldsymbol{x}}_s\right)=\tilde{x}_i-\frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{x}_i+\frac{\tilde{n}}{\tilde{n}+s}\tilde{k}-\frac{s}{\tilde{n}+s}\left(k-\tilde{k}\right)$$

Hereby, $\tilde{\alpha}^I$ and $\tilde{\beta}_i^I\left(\tilde{\boldsymbol{x}}_s\right)$ are the premium and coverage offered to the primary members of the New Club. For the $\tilde{n}+s^{\text{th}}$ member, who switches membership, the principle of premium call and coverage in the New Club means

$$\alpha^I=\frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{x}_i-\frac{\tilde{n}}{\tilde{n}+s}k$$

and

$$\beta_j^I\left(\tilde{\boldsymbol{x}}_s\right)=\tilde{x}_j-\frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{x}_i+\frac{\tilde{n}}{\tilde{n}+s}k \qquad \square$$

The *ex post* contracts, offered to the primary member and the entered member, are different with respect to the premium call policy. The primary member has to pay more premium than the entered one by $-\tilde{k}+k$, if $k>\tilde{k}$, and obtains less coverage by the same amount. Similar results can be observed in the *ex ante* PEC.

Proposition 5-2. *If there are s individual members switching membership, then the i^{th} member, $i=1,2,...,\tilde{n}$, in the New Club has the ex ante PEC at:*

$$\tilde{\beta}_i^{IE}\left(\tilde{\boldsymbol{x}}_s\right)=\tilde{x}_i-\frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{m}_i+\frac{\tilde{n}}{\tilde{n}+s}\tilde{k}-\frac{s}{\tilde{n}+s}\left(k-\tilde{k}\right) \qquad (5\text{-}5)$$

and the premium is collected at:

$$\tilde{\alpha}^{IE} = \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{m}_i - \frac{\tilde{n}}{\tilde{n}+s}\tilde{k} + \frac{s}{\tilde{n}+s}\left(k-\tilde{k}\right). \tag{5-6}$$

For the entered member, the coverage is

$$\beta_j^{IE}\left(\tilde{\mathbf{x}}_s\right) = \tilde{x}_j - \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{m}_i + \frac{\tilde{n}}{\tilde{n}+s}k \tag{5-7}$$

$j = \tilde{n}+1,\ldots,\tilde{n}+s$ *, and the premium is*

$$\alpha^{IE} = \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{m}_i - \frac{\tilde{n}}{\tilde{n}+s}k \tag{5-8}$$

Proof. The optimization problem is

$$\max_{\tilde{C}(\tilde{n}+s)}\left\{\sum_{i=\tilde{n}+1}^{\tilde{n}+s}\int_{\tilde{\Omega}_s}U\left(\omega-k-\tilde{x}_i+\beta_i^{IE}\left(\tilde{\mathbf{x}}_s\right)\right)dF_{\tilde{n}+s}\left(\tilde{\mathbf{x}}_s\right)+\sum_{i=1}^{\tilde{n}}\int_{\tilde{\Omega}_s}U\left(\omega-\tilde{k}-\tilde{x}_i+\tilde{\beta}_i^{IE}\left(\tilde{\mathbf{x}}_s\right)\right)dF_{\tilde{n}+s}\left(\tilde{\mathbf{x}}_s\right)\right\}$$

s.t. $\sum_{i=1}^{\tilde{n}}\int_{\tilde{\Omega}}\tilde{\beta}_i^{IE}\left(\tilde{\mathbf{x}}_s\right)dF_{\tilde{n}+s}\left(\tilde{\mathbf{x}}_s\right)+\sum_{i=\tilde{n}+1}^{\tilde{n}+s}\int_{\tilde{\Omega}}\beta_i^{IE}\left(\tilde{\mathbf{x}}_s\right)dF_{\tilde{n}+s}\left(\tilde{\mathbf{x}}_s\right)=\tilde{n}\cdot\tilde{k}$

With respect to $\beta_i^{IE}\left(\tilde{\mathbf{x}}_s\right)$ and $\tilde{\beta}_i^{IE}\left(\tilde{\mathbf{x}}_s\right)$, the first order conditions are

$$U'\left(\omega-k-\tilde{x}_{i'}+\beta_{i'}^{IE}\left(\tilde{\mathbf{x}}_s\right)\right)-\lambda\left(\tilde{\mathbf{x}}_s\right)=0$$

$$U'\left(\omega-k-\tilde{x}_{j'}+\beta_{j'}^{IE}\left(\tilde{\mathbf{x}}_s\right)\right)-\lambda\left(\tilde{\mathbf{x}}_s\right)=0$$

$$U'\left(\omega-\tilde{k}-\tilde{x}_i+\tilde{\beta}_i^{IE}\left(\tilde{\mathbf{x}}_s\right)\right)-\lambda\left(\tilde{\mathbf{x}}_s\right)=0$$

$$U'\left(\omega-\tilde{k}-\tilde{x}_j+\tilde{\beta}_j^{IE}\left(\tilde{\boldsymbol{x}}_s\right)\right)-\lambda\left(\tilde{\boldsymbol{x}}_s\right)=0$$

where $i' \neq j'$, $i \neq j$, $i, j = 1, 2, ..., \tilde{n},$ and $i', j' = \tilde{n}+1, \tilde{n}+2, ..., \tilde{n}+s$. Then, there is

$$-k-\tilde{x}_{j'}+\beta_{j'}^{IE}\left(\tilde{\boldsymbol{x}}_s\right)=-k-\tilde{x}_{i'}+\beta_{i'}^{IE}\left(\tilde{\boldsymbol{x}}_s\right)=-\tilde{k}-\tilde{x}_i+\tilde{\beta}_i^{IE}\left(\tilde{\boldsymbol{x}}_s\right)=-\tilde{k}-\tilde{x}_j+\tilde{\beta}_j^{IE}\left(\tilde{\boldsymbol{x}}_s\right)$$
$$=-\tilde{k}-\tilde{\alpha}_i^{IE}=-\tilde{k}-\tilde{\alpha}_j^{IE}$$

Let $\tilde{\alpha}_i^{IE}=\tilde{\alpha}_j^{IE}=\tilde{\alpha}^{IE}$, $i, j = 1, 2, ..., \tilde{n}$. Summate the following equation on both sides with respect to i

$$-\tilde{k}-\tilde{x}_i+\tilde{\beta}_i^{IE}\left(\tilde{\boldsymbol{x}}_s\right)=-\tilde{k}-\tilde{\alpha}_i^{IE}$$

so that

$$-\tilde{n}\cdot\tilde{k}-\sum_{i=1}^{\tilde{n}}\tilde{x}_i+\sum_{i=1}^{\tilde{n}}\tilde{\beta}_i^{IE}\left(\tilde{\boldsymbol{x}}_s\right)=-\tilde{n}\cdot\tilde{k}-\tilde{n}\cdot\tilde{\alpha}^{IE}$$

$$\Rightarrow -s\cdot k-\sum_{i=\tilde{n}+1}^{\tilde{n}+s}\tilde{x}_i-\tilde{n}\cdot\tilde{k}-\sum_{i=1}^{\tilde{n}}\tilde{x}_i+\sum_{i=1}^{\tilde{n}}\tilde{\beta}_i^{IE}\left(\tilde{\boldsymbol{x}}_s\right)+\sum_{i=\tilde{n}+1}^{\tilde{n}+s}\beta_i^{IE}\left(\tilde{\boldsymbol{x}}_s\right)=-\left(\tilde{n}+s\right)\cdot\tilde{k}-\left(\tilde{n}+s\right)\cdot\tilde{\alpha}^{IE}$$

Take the expectation with respect to $\tilde{x}_i$ on both sides of the equation. Due to $\sum_{i=1}^{\tilde{n}}\int_{\tilde{\Omega}}\tilde{\beta}_i^{IE}\left(\tilde{\boldsymbol{x}}_s\right)dF_{\tilde{n}+s}\left(\tilde{\boldsymbol{x}}_s\right)+\sum_{i=\tilde{n}+1}^{\tilde{n}+s}\int_{\tilde{\Omega}}\beta_i^{IE}\left(\tilde{\boldsymbol{x}}_s\right)dF_{\tilde{n}+s}\left(\tilde{\boldsymbol{x}}_s\right)=\tilde{n}\cdot\tilde{k}$, the equation above can be simplified so that

$$-s\cdot k-\sum_{i=\tilde{n}+1}^{\tilde{n}+s}\tilde{m}_i-\tilde{n}\cdot\tilde{k}-\sum_{i=1}^{\tilde{n}}\tilde{m}_i+\tilde{n}\cdot\tilde{k}=-(\tilde{n}+s)\cdot\tilde{k}-(\tilde{n}+s)\cdot\tilde{\alpha}^{IE}$$

$$\Rightarrow -s\cdot k-\sum_{i=1}^{\tilde{n}+s}\tilde{m}_i=-(\tilde{n}+s)\cdot\tilde{k}-(\tilde{n}+s)\cdot\tilde{\alpha}^{IE}$$

$$\Rightarrow \tilde{\alpha}^{IE}=\frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{m}_i-\frac{\tilde{n}}{\tilde{n}+s}\tilde{k}+\frac{s}{\tilde{n}+s}\left(k-\tilde{k}\right)$$

Then the coverage is

$$\tilde{\beta}_i^{IE}\left(\tilde{\boldsymbol{x}}_s\right)=\tilde{x}_i-\frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{m}_i+\frac{\tilde{n}}{\tilde{n}+s}\tilde{k}-\frac{s}{\tilde{n}+s}\left(k-\tilde{k}\right)$$

Hereby, $\tilde{\alpha}^{IE}$ and $\tilde{\beta}_i^{IE}\left(\tilde{\boldsymbol{x}}_s\right)$ are the premium and coverage offered to the primary members of the New Club. For the $\tilde{n}+s^{\text{th}}$ member, who switch membership, the principle of premium call and coverage in the New Club are

$$\alpha^{IE}=\frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{m}_i-\frac{\tilde{n}}{\tilde{n}+s}k$$

and

$$\beta_j^{IE}\left(\tilde{\boldsymbol{x}}_s\right)=\tilde{x}_j-\frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{m}_i+\frac{\tilde{n}}{\tilde{n}+s}k \qquad \square$$

The *ex ante* PEC is exactly the expectation of the *ex post* PEC. A P&I insurance contract is constructed through integrating the *ex post* and *ex ante* PEC. The insurance contract for a primary member of the New Club is

$$
\begin{cases}
\tilde{\alpha}^{I*} = \max\left\{ \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{x}_i, \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{m}_i \right\} - \frac{\tilde{n}}{\tilde{n}+s}\tilde{k} + \frac{s}{\tilde{n}+s}\left(k-\tilde{k}\right) \\
\tilde{\beta}_i^{I*}\left(\tilde{\mathbf{x}}_s\right) = \tilde{x}_i - \max\left\{ \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{x}_i, \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{m}_i \right\} + \frac{\tilde{n}}{\tilde{n}+s}\tilde{k} - \frac{s}{\tilde{n}+s}\left(k-\tilde{k}\right)
\end{cases} \tag{5-9}
$$

where $i = 1, 2, ..., \tilde{n}$, and the contract for the entered member is

$$
\begin{cases}
\alpha^{I*} = \max\left\{ \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{x}_i, \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{m}_i \right\} - \frac{\tilde{n}}{\tilde{n}+s}k \\
\beta_j^{I*}\left(\tilde{\mathbf{x}}_s\right) = \tilde{x}_j - \max\left\{ \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{x}_i, \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{m}_i \right\} + \frac{\tilde{n}}{\tilde{n}+s}k
\end{cases} \tag{5-10}
$$

where $j = \tilde{n}+1, \tilde{n}+2, ..., \tilde{n}+s$.

The New Club provides such a contract to solicit the primary members of the Holding Club to switch memberships. However, the conditions on which the entered member will accept this contract are then questioned, as well as whether the primary members of the New Club will support this new contract. The decision maker can only make a choice based on the expectation.

Criterion 1: For an entered member, if the expected premium under new contract $E(\alpha^{I*})$ is no more than the one in the Holding Club $E(\alpha)$, it is advantageous for the entered member to switch membership. That is

$$
\frac{1}{n}\sum_{i=1}^{n} m_i - \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{m}_i \geq \frac{s}{\tilde{n}+s}k
$$

Criterion 2: For a primary member of the New Club, if the expected premium under the new contract $E\left(\tilde{\alpha}^{I*}\right)$ is no more than the one before the entry of the entered member, it is advantageous for the primary member to accept the new members.

$$\frac{1}{\tilde{n}}\sum_{i=1}^{\tilde{n}}\tilde{m}_i - \frac{1}{s}\sum_{i=\tilde{n}+1}^{\tilde{n}+s}\tilde{m}_i \ge k$$

If this post-switch Pareto efficient contract of the New Club can benefit all members, including the *s* entered member, criteria 1 and 2 must be satisfied at the same time, that is,

$$\frac{1}{n-s}\sum_{i=1}^{n-s}m_i - \frac{1}{s}\sum_{i=n-s+1}^{n}m_i \ge \frac{n}{n-s}k \tag{5-11}$$

Inequality (5-11) is independent of the information about the New Club but depends on how many members of the Holding Club switch their membership.

5.1.3 Pareto efficient contract of the Holding Club

There are *s* members switching membership from the Holding Club, and these members leave the Holding Club with the initial contribution *k* unrefunded. Then there is the *ex post* PEC for the remaining primary members. Let $\overline{\mathbf{x}}_s$ denote the vector $(x_1, x_2, \ldots, x_{n-s})$, and $C(n-s)$ denote the contract of the Holding Club, where

$$C(n-s) = \left\{\beta_i(\overline{\mathbf{x}}_s), \overline{\mathbf{x}}_s = (x_1, x_2, \ldots, x_{n-s}) \in \Omega_{n-s} = [0,\infty[^{n-s}\right\}$$

Proposition 5-3. *If there are s members switching to the New Club, then there exists an ex post Pareto efficient contract for the rest n – s members, where the coverage is*

$$\beta_i\left(\overline{\mathbf{x}}_s\right) = x_i - \frac{1}{n-s}\sum_{i=1}^{n-s} m_i + \frac{n}{n-s}k \tag{5-12}$$

and the premium is priced at

$$\alpha = \frac{1}{n-s}\sum_{i=1}^{n-s} m_i - \frac{n}{n-s}k \tag{5-13}$$

Proof. The Pareto efficient P&I insurance contract is the solution of the following program

$$\max_{C(n-s)} \sum_{i=1}^{n-s} \int_{\Omega_{n-s}} U\left(\omega - k - x_i + \beta_i\left(\overline{\mathbf{x}}_s\right)\right) dF_{n-s}\left(\overline{\mathbf{x}}_s\right)$$

s.t. $nk - \sum_{i=1}^{n-s} \beta_i\left(\overline{\mathbf{x}}_s\right) = 0$

Let $\lambda\left(\overline{\mathbf{x}}_s\right)$ denote the Lagrangian multiplier of the constraint, the first order conditions for the i^{th} and j^{th} members are given by:

$$U'\left(\omega - k - x_i + \beta_i\left(\overline{\mathbf{x}}_s\right)\right) - \lambda\left(\overline{\mathbf{x}}_s\right) = 0$$

$$U'\left(\omega - k - x_j + \beta_j\left(\overline{\mathbf{x}}_s\right)\right) - \lambda\left(\overline{\mathbf{x}}_s\right) = 0$$

From the first order conditions, it is obvious that for $i, j = 1, 2, \ldots, n-s$, $i \neq j$,

$$U'\left(\omega - k - x_i + \beta_i\left(\overline{\mathbf{x}}_s\right)\right) = U'\left(\omega - k - x_j + \beta_j\left(\overline{\mathbf{x}}_s\right)\right)$$

and thus there exist a premium call and the initial investment k independent of i, which satisfies:

$$-x_i + \beta_i\left(\overline{\mathbf{x}}_s\right) = -x_j + \beta_j\left(\overline{\mathbf{x}}_s\right) = -\alpha_i = -\alpha_j$$

Let $\alpha = \alpha_i = \alpha_j$, $i, j = 1, 2, \ldots, n - s$. Then, there is the summation that

$$-\sum_{i=1}^{n-s} x_i + \sum_{i=1}^{n-s} \beta_i\left(\overline{\mathbf{x}}_s\right) = -\left(n-s\right)\cdot\alpha$$

Since $\sum_{i=1}^{n-s} \beta_i\left(\overline{\mathbf{x}}_s\right) = n\cdot k$,

$$-\sum_{i=1}^{n-s} x_i + n\cdot k = -\left(n-s\right)\cdot\alpha$$

$$\Rightarrow \alpha = \frac{1}{n-s}\sum_{i=1}^{n-s} x_i - \frac{n}{n-s}k$$

Thus, the coverage is

$$\beta_i\left(\overline{\mathbf{x}}_s\right) = x_i - \frac{1}{n-s}\sum_{i=1}^{n-s} x_i + \frac{n}{n-s}k \qquad \square$$

On the other hand, the *ex ante* Pareto efficient contract of the Holding Club after switch also exists.

Proposition 5-4. *If there are s members switching to the New Club, then there exists an ex ante Pareto efficient contract for the rest n – s members, where the coverage is*

$$\beta_i^E\left(\overline{\mathbf{x}}_s\right)=x_i-\frac{1}{n-s}\sum_{i=1}^{n-s}x_i+\frac{n}{n-s}k \tag{5-14}$$

and the premium is priced at

$$\alpha^E=\frac{1}{n-s}\sum_{i=1}^{n-s}x_i-\frac{n}{n-s}k \tag{5-15}$$

Proof. The Pareto-efficient P&I insurance contract is the solution of the following program

$$\max_{C(n-s)}\sum_{i=1}^{n-s}\int_{\Omega_{n-s}}U\left(\omega-k-x_i+\beta_i^E\left(\overline{\mathbf{x}}_s\right)\right)dF_{n-s}\left(\overline{\mathbf{x}}_s\right)$$

s.t. $nk-\sum_{i=1}^{n-s}\int_{\Omega_{n-s}}\beta_i^E\left(\overline{\mathbf{x}}_s\right)dF_{n-s}\left(\overline{\mathbf{x}}_s\right)=0$

Let $\lambda\left(\overline{\mathbf{x}}_s\right)$ denote the Lagrangian multiplier of the constraint, the first order conditions for the i^{th} and j^{th} members are given by:

$$U'\left(\omega-k-x_i+\beta_i^E\left(\overline{\mathbf{x}}_s\right)\right)-\lambda\left(\overline{\mathbf{x}}_s\right)=0$$

$$U'\left(\omega-k-x_j+\beta_i^E\left(\overline{\mathbf{x}}_s\right)\right)-\lambda\left(\overline{\mathbf{x}}_s\right)=0$$

From the first order conditions, it is obvious that for i, j = 1, 2, …, $n-1$, $i\neq j$,

$$U'\left(\omega-k-x_i+\beta_i^E\left(\overline{\mathbf{x}}_s\right)\right)=U'\left(\omega-k-x_j+\beta_i^E\left(\overline{\mathbf{x}}_s\right)\right)$$

and thus there exist a premium call and the initial investment k independent of i, which satisfies:

$$-x_i+\beta_i^E\left(\overline{\mathbf{x}}_s\right)=-x_j+\beta_i^E\left(\overline{\mathbf{x}}_s\right)=-\alpha_i^E=-\alpha_j^E$$

Let $\alpha_i^E=\alpha_j^E=\alpha^E$, i, j = 1, 2, …, $n-s$. Then, there is the summation that

$$-\sum_{i=1}^{n-s} x_i + \sum_{i=1}^{n-s} \beta_i^E(\overline{\mathbf{x}}_s) = -(n-s)\cdot\alpha^E$$

Since $nk - \sum_{i=1}^{n-s} \int_{\Omega_{n-s}} \beta_i^E(\overline{\mathbf{x}}_s)\, dF_{n-s}(\overline{\mathbf{x}}_s) = 0$, take the expectation on both sides of the equation above

$$-\sum_{i=1}^{n-s} m_i + n\cdot k = -(n-s)\cdot\alpha^E$$

$$\Rightarrow \alpha^E = \frac{1}{n-s}\sum_{i=1}^{n-s} m_i - \frac{n}{n-s}k$$

Thus, the coverage is

$$\beta_i^E(\overline{\mathbf{x}}_s) = x_i - \frac{1}{n-s}\sum_{i=1}^{n-s} m_i + \frac{n}{n-s}k \qquad \square$$

Before the s members switch their memberships, the rest $n-s$ members of the Holding Club have to decide whether to set some boundary to terminate the switch.

Criterion 3: For a primary member left in the Holding Club, if the expected premium under the new contract is no more than the pre–switch premium, it is advantageous for this primary member to approve the entered member's switch without any boundary.

$$\frac{1}{n-s}\sum_{i=1}^{n-s} m_i - \frac{1}{s}\sum_{i=n-s+1}^{n} m_i \le \frac{n}{n-s}k \tag{5-16}$$

The overlap of inequality (5-11) and (5-16) is exactly the point where

$$\frac{1}{n-s}\sum_{i=1}^{n-s} m_i - \frac{1}{s}\sum_{i=n-s+1}^{n} m_i = \frac{n}{n-s}k \Rightarrow \frac{1}{s}\sum_{i=n-s+1}^{n} m_i = \frac{1}{n}\sum_{i=1}^{n} m_i - k$$

5.2 Two special competition strategies

In section 5.1, the contracts offered by the New Club are Pareto efficient for all members, including the primary and the entered members. However, the treaties for them are quite different. In this section, the problem is turned towards the contract that is only Pareto efficient for the primary members of the New Club, given the entered member's coverage. It will be proved that there exists a competition strategy that allows all members, including the entered one, to obtain an identical treaty. Consider the simplest case: There is only one member switching membership. The principle of premium collection is

Premium = liability on the actual/expected loss – per capita coverage from ICR

where liability on the actual/expected loss reflects the essential idea of mutuality. This principal part of mutual insurance is unchangeable. The per capita coverage from ICR, however, will become the key component in soliciting the entered member. There are two possible competition strategies with respect to ICR:

Strategy II: the ICR of the New Club is high enough such that the per capita coverage for the entered member is larger than *k*. The entered member obtains the coverage from ICR equal to the primary members of the New Club.

Strategy III: albeit that the ICR of the New Club is not as high as strategy II, the New Club promises that the entered member can obtain the coverage, which is no less than *k*, from the ICR.

Let $\tilde{\boldsymbol{x}}$ denote the vector $(\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_{\tilde{n}+1})$ with distribution function $F_{\tilde{n}+1}(\tilde{\boldsymbol{x}})$. Let $\tilde{C}(\tilde{n}+1)$ denote the contract offered by the New Club, where

$$\tilde{C}(\tilde{n}+1) = \left\{ \tilde{\beta}_i(\tilde{\boldsymbol{x}}), \tilde{\boldsymbol{x}} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_{\tilde{n}+1}) \in \tilde{\Omega} = [0, \infty[^{\tilde{n}+1} \right\}$$

5.2.1 Pareto efficient contract for strategy II

Strategy II is carried out when there is $\tilde{n} \cdot \tilde{k} / (\tilde{n}+1) > k$. When $\tilde{k}/k \geq 2$, $\tilde{n}$ can be any positive integral; when $1 < \tilde{k}/k \leq 2$, $\tilde{n}$ has a lower limit, $\tilde{n} > k / (\tilde{k} - k)$. The coverage for the entered member is

$$\tilde{\beta}^{II}_{\tilde{n}+1}(\tilde{\boldsymbol{x}}) = \left(\tilde{x}_{\tilde{n}+1} - \frac{1}{\tilde{n}+1} \sum_{i=1}^{\tilde{n}+1} \tilde{x}_i + \frac{\tilde{n}\tilde{k}}{\tilde{n}+1} \right)$$

A. Ex post Pareto efficient contract

Proposition 5-5. *If there is an individual member switching membership, and* $\frac{\tilde{n}\cdot\tilde{k}}{\tilde{n}+1} > k$*, then the* i^{th} *member,* $i = 1, 2, ..., \tilde{n}+1$*, in the New Club has the ex post PEC at:*

$$\tilde{\beta}_i^{II}(\tilde{\boldsymbol{x}}) = \tilde{x}_i - \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i + \frac{\tilde{n}}{\tilde{n}+1}\cdot\tilde{k} \tag{5-17}$$

and the premium is collected at:

$$\tilde{\alpha}^{II} = \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i - \frac{\tilde{n}}{\tilde{n}+1}\cdot\tilde{k}\,. \tag{5-18}$$

Proof. The optimization problem is

$$\max_{\tilde{C}(\tilde{n}+1)}\left\{\int_{\tilde{\Omega}} U\left(\omega - k - \tilde{x}_{\tilde{n}+1} + \tilde{\beta}_{\tilde{n}+1}^{II}(\tilde{\boldsymbol{x}})\right)dF_{\tilde{n}+1}(\tilde{\boldsymbol{x}}) + \sum_{i=1}^{\tilde{n}}\int_{\tilde{\Omega}} U\left(\omega - \tilde{k} - \tilde{x}_i + \tilde{\beta}_i^{II}(\tilde{\boldsymbol{x}})\right)dF_{\tilde{n}+1}(\tilde{\boldsymbol{x}})\right\}$$

s.t. $\sum_{i=1}^{\tilde{n}}\tilde{\beta}_i^{II}(\tilde{\boldsymbol{x}}) + \tilde{\beta}_{\tilde{n}+1}^{II}(\tilde{\boldsymbol{x}}) = \tilde{n}\cdot\tilde{k}$

$$\tilde{\beta}_{\tilde{n}+1}^{II}(\tilde{\boldsymbol{x}}) = \left(\tilde{x}_{\tilde{n}+1} - \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i + \frac{\tilde{n}\tilde{k}}{\tilde{n}+1}\right)$$

With respect to $\tilde{\beta}_i^{II}(\tilde{\boldsymbol{x}})$, $i = 1, 2, ..., \tilde{n}$, the first order conditions are

$$U'\left(\omega - \tilde{k} - \tilde{x}_i + \tilde{\beta}_i^{II}(\tilde{\boldsymbol{x}})\right) - \lambda(\tilde{\boldsymbol{x}}) = 0$$

$$U'\left(\omega - \tilde{k} - \tilde{x}_j + \tilde{\beta}_j^{II}(\tilde{\boldsymbol{x}})\right) - \lambda(\tilde{\boldsymbol{x}}) = 0$$

where $i \neq j,\ i, j = 1, 2, ..., \tilde{n}$. Then, there is

$$-\tilde{k}-\tilde{x}_i+\tilde{\beta}_i^{II}\left(\tilde{\boldsymbol{x}}\right)=-\tilde{k}-\tilde{x}_j+\tilde{\beta}_j^{II}\left(\tilde{\boldsymbol{x}}\right)=-\tilde{k}-\tilde{\alpha}_i^{II}=-\tilde{k}-\tilde{\alpha}_j^{II}$$

Let $\tilde{\alpha}_i^{II}=\tilde{\alpha}_j^{II}=\tilde{\alpha}^{II}$, $i,j=1,2,...,\tilde{n}$. Summate the following equation on both sides with respect to i

$$-\tilde{k}-\tilde{x}_i+\tilde{\beta}_i^{II}\left(\tilde{\boldsymbol{x}}\right)=-\tilde{k}-\tilde{\alpha}^{II}$$

so that

$$-\sum_{i=1}^{\tilde{n}}\tilde{x}_i+\sum_{i=1}^{\tilde{n}}\tilde{\beta}_i^{II}\left(\tilde{\boldsymbol{x}}\right)=-\tilde{n}\cdot\tilde{\alpha}^{II}$$

$$\Rightarrow-\sum_{i=1}^{\tilde{n}}\tilde{x}_i+\sum_{i=1}^{\tilde{n}}\tilde{\beta}_i^{II}\left(\tilde{\boldsymbol{x}}\right)+\tilde{\beta}_{\tilde{n}+1}^{II}\left(\tilde{\boldsymbol{x}}\right)=-\tilde{n}\cdot\tilde{\alpha}^{II}+\tilde{\beta}_{\tilde{n}+1}^{II}\left(\tilde{\boldsymbol{x}}\right)$$

Due to $\tilde{\beta}_{\tilde{n}+1}^{II}\left(\tilde{\boldsymbol{x}}\right)=\left(\tilde{x}_{\tilde{n}+1}-\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i+\frac{\tilde{n}\tilde{k}}{\tilde{n}+1}\right)$ and $\sum_{i=1}^{\tilde{n}}\tilde{\beta}_i^{II}\left(\tilde{\boldsymbol{x}}\right)+\tilde{\beta}_{\tilde{n}+1}^{II}\left(\tilde{\boldsymbol{x}}\right)=\tilde{n}\cdot\tilde{k}$, the equation above can be simplified so that

$$-\sum_{i=1}^{\tilde{n}}\tilde{x}_i+\tilde{n}\cdot\tilde{k}=-\tilde{n}\cdot\tilde{\alpha}^{II}+\tilde{x}_{\tilde{n}+1}-\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i+\frac{\tilde{n}}{\tilde{n}+1}k$$

$$\Rightarrow-\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i+\left(\tilde{n}-\frac{\tilde{n}}{\tilde{n}+1}\right)\cdot\tilde{k}=-\tilde{n}\cdot\tilde{\alpha}^{II}-\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i$$

$$\Rightarrow\tilde{n}\cdot\tilde{\alpha}^{II}=\left(1-\frac{1}{\tilde{n}+1}\right)\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i-\left(\tilde{n}-\frac{\tilde{n}}{\tilde{n}+1}\right)\cdot k$$

$$\Rightarrow\tilde{\alpha}^{II}=\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i-\frac{\tilde{n}}{\tilde{n}+1}\cdot\tilde{k}$$

Then the coverage is

$$\tilde{\beta}_i^{II}\left(\tilde{\mathbf{x}}\right)=\tilde{x}_i-\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i+\frac{\tilde{n}}{\tilde{n}+1}\cdot\tilde{k}$$ □

Under the new *ex post* PEC, it is questionable whether the entry of the new member can benefit the primary members of the New Club. When $\tilde{n}$ is large enough, let $\frac{\tilde{y}}{\tilde{n}}=\frac{1}{\tilde{n}}\sum_{i=1}^{\tilde{n}}\tilde{x}_i$, then *α* is normally distributed with mean $\frac{1}{\tilde{n}}\sum_{i=1}^{\tilde{n}}\tilde{m}_i-\tilde{k}$ and variance $\frac{1}{\tilde{n}^2}\sum_{i=1}^{\tilde{n}}\tilde{v}_i^2$. Given the distribution of $\tilde{x}_{\tilde{n}+1}$ and a realization of $\left(\tilde{x}_1,\tilde{x}_2,...,\tilde{x}_{\tilde{n}}\right)$, the probability of suffering a utility decline can be calculated for the primary members of the New Club.

B. Ex ante Pareto efficient contract

The *ex ante* PEC under strategy II can be derived through changing the constraint of the optimization in Proposition 5-5. The constraint in the *ex ante* PEC is the expected non-profit. Let *m* denote the mean loss of the $\tilde{n}+1^{\text{st}}$ member.

Proposition 5-6. *If there is an individual member switching membership, and the New Club conducts the competition strategy II, then the i^th^ member,* $i=1,2,...,\tilde{n}+1$*, in the New Club, has the ex ante PEC at:*

$$\tilde{\beta}_i^{II,E}\left(\tilde{\mathbf{x}}\right)=\tilde{x}_i-\frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}}\tilde{m}_i+m\right)+\frac{\tilde{n}}{\tilde{n}+1}\cdot\tilde{k} \tag{5-19}$$

and the premium is collected at:

$$\tilde{\alpha}^{II,E} = \frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) - \frac{\tilde{n}}{\tilde{n}+1} \cdot \tilde{k} . \tag{5-20}$$

Proof. The optimization problem is

$$\max_{\tilde{C}(\tilde{n}+1)} \left\{ \int_{\tilde{\Omega}} U\left(\omega - k - \tilde{x}_{\tilde{n}+1} + \tilde{\beta}_{\tilde{n}+1}^{II,E}(\tilde{\boldsymbol{x}})\right) dF_{\tilde{n}+1}(\tilde{\boldsymbol{x}}) + \sum_{i=1}^{\tilde{n}} \int_{\tilde{\Omega}} U\left(\omega - \tilde{k} - \tilde{x}_i + \tilde{\beta}_i^{II,E}(\tilde{\boldsymbol{x}})\right) dF_{\tilde{n}+1}(\tilde{\boldsymbol{x}}) \right\}$$

s.t. $\sum_{i=1}^{\tilde{n}+1} \int_{\tilde{\Omega}} \tilde{\beta}_i^{II,E}(\tilde{\boldsymbol{x}}) dF_{\tilde{n}+1}(\tilde{\boldsymbol{x}}) = \tilde{n} \cdot \tilde{k}$

$$\tilde{\beta}_{\tilde{n}+1}^{II,E}(\tilde{\boldsymbol{x}}) = x_{\tilde{n}+1} - \frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) + \frac{\tilde{n}}{\tilde{n}+1}\tilde{k}$$

With respect to $\tilde{\beta}_i^{II,E}(\tilde{\boldsymbol{x}})$, the first order conditions are

$$U'\left(\omega - \tilde{k} - \tilde{x}_i + \tilde{\beta}_i^{II,E}(\tilde{\boldsymbol{x}})\right) - \lambda(\tilde{\boldsymbol{x}}) = 0$$

$$U'\left(\omega - \tilde{k} - \tilde{x}_j + \tilde{\beta}_j^{II,E}(\tilde{\boldsymbol{x}})\right) - \lambda(\tilde{\boldsymbol{x}}) = 0$$

where $i \neq j$, $i, j = 1,2,\ldots,\tilde{n}$. Then there is

$$-\tilde{x}_i + \tilde{\beta}_i^{II,E}(\tilde{\boldsymbol{x}}) = -\tilde{x}_j + \tilde{\beta}_j^{II,E}(\tilde{\boldsymbol{x}}) = -\tilde{\alpha}_i^{II,E} = -\tilde{\alpha}_j^{II,E}$$

Let $\tilde{\alpha}_i^{II,E} = \tilde{\alpha}_j^{II,E} = \tilde{\alpha}^{II,E}$, $i, j = 1, 2,\ldots, \tilde{n}$. Summate the following equation on both sides with respect to i =1, 2,…, $\tilde{n}$

$$-\tilde{x}_i + \tilde{\beta}_i^{II,E}(\tilde{\boldsymbol{x}}) = -\tilde{\alpha}_i^{II,E}$$

so that

$$-\sum_{i=1}^{\tilde{n}} \tilde{x}_i + \sum_{i=1}^{\tilde{n}} \tilde{\beta}_i^{II,E}\left(\tilde{\boldsymbol{x}}\right) = -\tilde{n}\cdot\tilde{\alpha}^{II,E}$$

$$\Rightarrow -\sum_{i=1}^{\tilde{n}} \tilde{x}_i + \sum_{i=1}^{\tilde{n}} \tilde{\beta}_i^{II,E}\left(\tilde{\boldsymbol{x}}\right) + \tilde{\beta}_{\tilde{n}+1}^{II,E}\left(\tilde{\boldsymbol{x}}\right) = -\tilde{n}\cdot\tilde{\alpha}^{II,E} + \tilde{x}_{\tilde{n}+1} - \frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) + \frac{\tilde{n}}{\tilde{n}+1}\tilde{k}$$

$$\Rightarrow -\sum_{i=1}^{\tilde{n}+1} \tilde{x}_i + \sum_{i=1}^{\tilde{n}+1} \tilde{\beta}_i^{II,E}\left(\tilde{\boldsymbol{x}}\right) = -\tilde{n}\cdot\tilde{\alpha}^{II,E} - \frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) + \frac{\tilde{n}}{\tilde{n}+1}\tilde{k}$$

Take the expectation with respect to $\tilde{x}_i$ on both sides of the equation. Then there is

$$-\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) + \tilde{n}\cdot\tilde{k} = -\tilde{n}\cdot\tilde{\alpha}^{II,E} - \frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) + \frac{\tilde{n}}{\tilde{n}+1}\tilde{k}$$

$$\Rightarrow \tilde{\alpha}^{II,E} = \frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) - \frac{\tilde{n}}{\tilde{n}+1}\cdot\tilde{k}$$

Then the coverage is

$$\tilde{\beta}_i^{II,E}\left(\tilde{\boldsymbol{x}}\right) = \tilde{x}_i - \frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) + \frac{\tilde{n}}{\tilde{n}+1}\cdot\tilde{k} \qquad \square$$

Under the new *ex ante* PEC, the $\tilde{n}$ primary members of the New Club have an identical coverage treaty to the entered one. This finding seems quite reasonable and fair for all $\tilde{n}+1$ members. In fact, the primary members have already invested $\tilde{k}$ into the ICR, which in Proposition 5-6 can be explained as the sunk cost. Then, we have

Criterion 4: Under the *ex ante* PEC and strategy II, the primary members of the New Club can benefit from a membership switch if

$$m < \frac{1}{\tilde{n}} \sum_{i=1}^{\tilde{n}} \tilde{m}_i - \tilde{k} \tag{5-21}$$

Criterion 5: Under the *ex ante* PEC and strategy II, the entered member can benefit if

$$\frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) - \frac{1}{n}\left(\sum_{i=1}^{n-1} m_i + m\right) < \frac{\tilde{n}}{\tilde{n}+1}\tilde{k} - k \tag{5-22}$$

C. Discussion

Under the symmetric information, all variables in (5-21) and (5-22) are known. Thus, the membership switch can be realized only when the two inequalities are satisfied simultaneously. These two inequalities reflect the role of the information played in the insurance decision for the New Club and the entered member:

a. When the New Club decides to accept the entered member, it only needs to know the mean loss of this new member, *m*. Thus, the Agreement 1999 regulates that the Holding Club is obligated to provide the New Club with the record of the entered member.

b. From the financial statement regularly published by the New Club, the entered member can obtain empirical information about the mean loss of the New Club, the level of ICR and the total insured tonnage of the fleets. Then, the entered member can compare the expected benefit of switching membership.

c. Consider some special cases related to the inequalities (5-21) and (5-22):

(a) The n members in the Holding Club and the $\tilde{n}$ members in the New Club have an equal mean loss. For any $\tilde{k} \geq 0$, the inequality (5-21) cannot be satisfied, which implies that the New Club's primary members disfavor the entry of this member, even though the entered member is strongly motivated to switch membership.

(b) The entered member might have a mean loss differing from both the other members of the Holding Club and the primary members of the New Club. Table 5-1 shows the possible situations.

Table 5-1: Benefit of strategy II (for entered member and New Club's members)

m	m_i $i \neq$ n	$\tilde{m}_i$	Examine Inequality (5-21)	Examine inequality (5-22)
m^H	m^H	m^L	Disadvantage for New Club's members	Always advantageous for entered member
m^H	m^L	m^L	Disadvantage for New Club's members	Conditionally advantageous for entered member: (1) $\tilde{n}+1<n$ (2) $\frac{\tilde{n}}{\tilde{n}+1}\tilde{k}-k<\frac{n-\tilde{n}-1}{n(\tilde{n}+1)}\left(m^H-m^L\right)$
m^H	m^L	m^H	Disadvantage for New Club's members	Conditionally advantageous for entered member: $\frac{\tilde{n}}{\tilde{n}+1}\tilde{k}-k>\frac{n-1}{n}\left(m^H-m^L\right)$
m^L	m^L	m^H	If $m^H-m^L>\tilde{k}$, strategy II is advantageous for New Club's member	Strategy II is not advantageous for entered member, when it is advantageous for New Club's member.
m^L	m^H	m^H	If $m^H-m^L>\tilde{k}$, strategy II is advantageous for New Club's member	If $\tilde{n}+1<n$, strategy II is advantageous for entered member.
m^L	m^H	m^L	Disadvantage for New Club's members	Advantageous for entered member

Assume that the other primary members of the Holding Club have an identical mean loss, and the primary members of the New Club also have the identical mean loss. Let vector ***m*** denote the triplet of the mean losses of the entered member, other primary members of the Holding Club, and the primary members of the New Club, that is, $\boldsymbol{m} = \left(m, m_i, \tilde{m}_j\right)$, i =1, 2, …, n -1, j = 1, 2,…, $\tilde{n}$. Table 5-1 illustrates that the primary members of the New Club cannot obtain any benefit from competition when the entered member's mean loss is larger than or equal to that of the New Club's primary members (see the 2nd, 3rd, 4th and last rows in Table 5-1, where the inequality 5-13 is not satisfied). When $\boldsymbol{m} = \left(m^H, m^L, m^L\right)$, albeit that the New Club's primary members are still disadvantaged, the entered member cannot at the same time be benefited unless $\tilde{n} + 1 < n$.

When $\boldsymbol{m} = \left(m^L, m^L, m^H\right)$ (see the 5th row of the table), the inequalities (5-21) and (5-22) cannot be satisfied at the same time. When $\boldsymbol{m} = \left(m^L, m^H, m^H\right)$ (see the 6th row of Table 5-1), however, there exists the possibility that the inequalities (5-21) and (5-22) can be satisfied at the same time, when $\tilde{n} + 1 > n$ and $\tilde{k} > nk/\left(n-1\right)$, such that

$$\tilde{k} < m^H - m^L < \frac{n\left(\tilde{n}+1\right)}{\tilde{n} - n + 1}\left(\frac{\tilde{n}}{\tilde{n}+1}\tilde{k} - k\right)$$

However, this inequality cannot be satisfied under $\tilde{n} \cdot \tilde{k} \big/ \left(\tilde{n}+1\right) > k$, in other words, $\tilde{k} > k$. If the inequality is satisfied, then there must be $\tilde{k} < n \cdot k \big/ \left(n-1\right)$, where $\tilde{k} < k$, which is in conflict with the condition of strategy II.

Briefly, the situations shown above explore that, in most cases, the inequalities (5-21) and (5-22) cannot be satisfied at the same time, which implies conflicting interests between the primary members of the New Club and the entered member, except when $\boldsymbol{m} = \left(m^L, m^H, m^H\right)$.

5.2.2 Pareto efficient contract for strategy III

Consider the second situation, in which the New Club promises the entered member to provide the coverage, which is no less than *k*, from the ICR. For any $\tilde{n}$, the condition is always held when $\tilde{k} \le k$. Since $\tilde{n}$ is a positive integral, $1 < \frac{\tilde{k}}{k} \le 2$. Also, there is an upper limit for $\tilde{n}$, $\tilde{n} \le k \big/ \left(\tilde{k} - k\right)$. The coverage for the entered member is

$$\tilde{\beta}_{\tilde{n}+1}^{III}\left(\tilde{\boldsymbol{x}}\right) = \left(\tilde{x}_{\tilde{n}+1} - \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i + k\right)$$

A. Ex post Pareto efficient contract

Proposition 5-7. *If there is an individual member switching membership, and competition strategy III is conducted, then the i*[th] *primary member,* $i=1,2,\ldots,\tilde{n}$*, in the New Club, has the ex post PEC at:*

$$\tilde{\beta}_i^{III}\left(\tilde{\boldsymbol{x}}\right)=\tilde{x}_i-\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i+\tilde{k}-\frac{k}{\tilde{n}} \tag{5-23}$$

and the premium is collected at:

$$\tilde{\alpha}_i^{III}=\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i-\left(\tilde{k}-\frac{k}{\tilde{n}}\right). \tag{5-24}$$

Proof. The optimization problem is

$$\max_{\tilde{C}(\tilde{n}+1)}\left\{\int_{\Omega}U\left(\omega-\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i\right)dF_{\tilde{n}+1}\left(\tilde{\boldsymbol{x}}\right)+\sum_{i=1}^{\tilde{n}}\int_{\Omega}U\left(\omega-\tilde{k}-\tilde{x}_i+\tilde{\beta}_i^{III}\left(\tilde{\boldsymbol{x}}\right)\right)dF_{\tilde{n}+1}\left(\tilde{\boldsymbol{x}}\right)\right\}$$

s.t. $\sum_{i=1}^{\tilde{n}}\tilde{\beta}_i^{III}\left(\tilde{\boldsymbol{x}}\right)+\tilde{\beta}_{\tilde{n}+1}^{III}\left(\tilde{\boldsymbol{x}}\right)=\tilde{n}\cdot\tilde{k}$

$$\tilde{\beta}_{\tilde{n}+1}^{III}\left(\tilde{\boldsymbol{x}}\right)=\left(\tilde{x}_{\tilde{n}+1}-\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i+k\right)$$

With respect to $\tilde{\beta}_i^{III}\left(\tilde{\boldsymbol{x}}\right)$, the first order conditions are

$$U'\left(\omega-\tilde{k}-\tilde{x}_i+\tilde{\beta}_i^{III}\left(\tilde{\boldsymbol{x}}\right)\right)-\lambda\left(\tilde{\boldsymbol{x}}\right)=0$$

$$U'\left(\omega-\tilde{k}-\tilde{x}_j+\tilde{\beta}_i^{III}\left(\tilde{\boldsymbol{x}}\right)\right)-\lambda\left(\tilde{\boldsymbol{x}}\right)=0$$

where $i \neq j$, $i, j = 1,2,\ldots,\tilde{n}$. Then there is

$$-\tilde{x}_i+\tilde{\beta}_i^{III}\left(\tilde{\boldsymbol{x}}\right)=-\tilde{x}_j+\tilde{\beta}_j^{III}\left(\tilde{\boldsymbol{x}}\right)=-\tilde{\alpha}_i^{III}=-\tilde{\alpha}_j^{III}$$

Let $\tilde{\alpha}_i^{III}=\tilde{\alpha}_j^{III}=\tilde{\alpha}^{III}$, $i, j = 1, 2,\ldots, \tilde{n}$. Summate the following equation on both sides with respect to i =1, 2,…, $\tilde{n}$

$$-\tilde{x}_i+\tilde{\beta}_i^{III}\left(\tilde{\boldsymbol{x}}\right)=-\tilde{\alpha}_i^{III}$$

so that

$$-\sum_{i=1}^{\tilde{n}}\tilde{x}_i+\sum_{i=1}^{\tilde{n}}\tilde{\beta}_i^{III}\left(\tilde{\boldsymbol{x}}\right)=-\tilde{n}\cdot\tilde{\alpha}^{III}$$

$$\Rightarrow-\sum_{i=1}^{\tilde{n}}\tilde{x}_i+\sum_{i=1}^{\tilde{n}}\tilde{\beta}_i^{III}\left(\tilde{\boldsymbol{x}}\right)+\tilde{\beta}_{\tilde{n}+1}^{III}\left(\tilde{\boldsymbol{x}}\right)=-\tilde{n}\cdot\tilde{\alpha}^{III}+\tilde{x}_{\tilde{n}+1}-\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i+k$$

$$\Rightarrow-\frac{\tilde{n}}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i+\left(\tilde{n}\cdot\tilde{k}-k\right)=-\tilde{n}\cdot\tilde{\alpha}^{III}$$

$$\Rightarrow\tilde{\alpha}^{III}=\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i-\left(\tilde{k}-\frac{k}{\tilde{n}}\right)$$

Thus, the new treaties for the New Club's $\tilde{n}$ primary members is

$$\tilde{\beta}_i^{III}\left(\tilde{\boldsymbol{x}}\right)=\tilde{x}_i-\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i+\tilde{k}-\frac{k}{\tilde{n}},\qquad i=1,2,\ldots,\tilde{n}\qquad\square$$

Proposition 5-7 demonstrates the *ex post* contract under the third strategy of the New Club, where the New Club can only ensure that the entered member obtains, at most, coverage k from the ICR. Thus, each primary member has to pay more premium (giving up a certain amount of coverage indirectly), $k/\tilde{n}$, to satisfy the condition of membership switch.

Corollary 5-8. *When* $\tilde{k} = k$*, the premium policy for the* i^{th} *primary member,* $i = 1, 2, ..., \tilde{n}$*, is*

$$\alpha_i = \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i - \frac{\tilde{n}-1}{\tilde{n}}k$$

and the coverage is

$$\beta_i\left(\tilde{\boldsymbol{x}}\right) = \tilde{x}_i - \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i + \frac{\tilde{n}-1}{\tilde{n}}k$$

Proof. When $\tilde{k} = k$, there is always $\tilde{n} \cdot \tilde{k} / \left(\tilde{n}+1\right) < k$. Thus, this corollary is a special case of Proposition 5-3. Substitute $\tilde{k} = k$ into the equation (5-4) and (5-5), and obtain the conclusion. □

B. Ex ante Pareto efficient contract

Proposition 5-9. *If there is an individual member switching membership, and the New Club conducts competition strategy III, then the* i^{th} *member,* $i = 1, 2, ..., \tilde{n}$*, in the New Club, has the ex ante PEC at:*

$$\tilde{\beta}_i^{III,E}\left(\tilde{\boldsymbol{x}}\right) = \tilde{x}_i - \frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}}\tilde{m}_i + m\right) + \left(\tilde{k} - \frac{k}{\tilde{n}}\right) \qquad (5\text{-}25)$$

and the premium is collected at:

$$\tilde{\alpha}_i^{III,E} = \frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) - \left(\tilde{k} - \frac{k}{\tilde{n}}\right). \tag{5-26}$$

Proof. The optimization problem is

$$\max_{\tilde{C}(\tilde{n}+1)} \left\{ \int_{\tilde{\Omega}} U\left(\omega - k - \tilde{x}_{\tilde{n}+1} + \tilde{\beta}_{\tilde{n}+1}^{III,E}(\tilde{\boldsymbol{x}})\right) dF_{\tilde{n}+1}(\tilde{\boldsymbol{x}}) + \sum_{i=1}^{\tilde{n}} \int_{\tilde{\Omega}} U\left(\omega - \tilde{k} - \tilde{x}_i + \tilde{\beta}_i^{III,E}(\tilde{\boldsymbol{x}})\right) dF_{\tilde{n}+1}(\tilde{\boldsymbol{x}}) \right\}$$

s.t. $\int_{\tilde{\Omega}} \left(\sum_{i=1}^{\tilde{n}} \tilde{\beta}_i^{III,E}(\tilde{\boldsymbol{x}}) + \tilde{\beta}_{\tilde{n}+1}^{III,E}(\tilde{\boldsymbol{x}}) \right) dF_{\tilde{n}+1}(\tilde{\boldsymbol{x}}) = \tilde{n} \cdot \tilde{k}$

$$\tilde{\beta}_{\tilde{n}+1}^{III,E}(\tilde{\boldsymbol{x}}) = \tilde{x}_{\tilde{n}+1} - \frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) + k$$

With respect to $\tilde{\beta}_i^{III,E}(\tilde{\boldsymbol{x}})$, i = 1, 2, …, $\tilde{n}$, the first order conditions are

$$U'\left(\omega - \tilde{k} - \tilde{x}_i + \tilde{\beta}_i^{III,E}(\tilde{\boldsymbol{x}})\right) - \lambda(\tilde{\boldsymbol{x}}) = 0$$

$$U'\left(\omega - \tilde{k} - \tilde{x}_j + \tilde{\beta}_j^{III,E}(\tilde{\boldsymbol{x}})\right) - \lambda(\tilde{\boldsymbol{x}}) = 0$$

where $i \neq j$, i, j = 1,2,…, $\tilde{n}$. Then, there is

$$-\tilde{k} - \tilde{x}_i + \tilde{\beta}_i^{III,E}(\tilde{\boldsymbol{x}}) = -\tilde{k} - \tilde{x}_j + \tilde{\beta}_j^{III,E}(\tilde{\boldsymbol{x}}) = -\tilde{k} - \tilde{\alpha}_i^{III,E} = -\tilde{k} - \tilde{\alpha}_j^{III,E}$$

Let $\tilde{\alpha}_i^{III,E} = \tilde{\alpha}_j^{III,E} = \tilde{\alpha}^{III,E}$, i, j = 1, 2,…, $\tilde{n}$. Summate the following equation on both sides with respect to i =1, 2,…, $\tilde{n}$

$$-\tilde{k} - \tilde{x}_i + \tilde{\beta}_i^{III,E}(\tilde{\boldsymbol{x}}) = -\tilde{k} - \tilde{\alpha}_i^{III,E}$$

so that

$$-\sum_{i=1}^{\tilde{n}} \tilde{x}_i + \sum_{i=1}^{\tilde{n}} \tilde{\beta}_i^{III,E}\left(\tilde{\boldsymbol{x}}\right) = -\tilde{n}\cdot\tilde{\alpha}^{III,E}$$

$$\Rightarrow -\sum_{i=1}^{\tilde{n}} \tilde{x}_i + \sum_{i=1}^{\tilde{n}} \tilde{\beta}_i^{III,E}\left(\tilde{\boldsymbol{x}}\right) + \tilde{\beta}_{\tilde{n}+1}^{III,E}\left(\tilde{\boldsymbol{x}}\right) = -\tilde{n}\cdot\tilde{\alpha}^{III,E} + \tilde{x}_{\tilde{n}+1} - \frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) + k$$

$$\Rightarrow -\sum_{i=1}^{\tilde{n}+1} \tilde{x}_i + \sum_{i=1}^{\tilde{n}} \tilde{\beta}_i^{III,E}\left(\tilde{\boldsymbol{x}}\right) + \tilde{\beta}_{\tilde{n}+1}^{III,E}\left(\tilde{\boldsymbol{x}}\right) = -\tilde{n}\cdot\tilde{\alpha}^{III,E} - \frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) + k$$

Take the expectation with respect to $\tilde{x}_i$ on both sides of the equation, and then there is

$$-\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) + \tilde{n}\cdot\tilde{k} = -\tilde{n}\cdot\tilde{\alpha}^{III,E} - \frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) + k$$

$$\Rightarrow \tilde{\alpha}^{III,E} = \frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) - \left(\tilde{k} - \frac{k}{\tilde{n}}\right)$$

Thus, the coverage for the i^{th} primary member of the New Club is

$$\tilde{\beta}_i^{III,E}\left(\tilde{\boldsymbol{x}}\right) = \tilde{x}_i - \frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) + \left(\tilde{k} - \frac{k}{\tilde{n}}\right) \qquad \square$$

Strategy III is conducted when the New Club has insufficient ICR to support strategy II, where each primary member of the New Club pays more premium at a certain amount, $k/\tilde{n}$, to ensure strategy III is applicable.

Criterion 6: Under the *ex ante* PEC and strategy III, the primary members can benefit from a membership switch, if

$$m < \frac{1}{\tilde{n}}\sum_{i=1}^{\tilde{n}} \tilde{m}_i - \frac{\tilde{n}+1}{\tilde{n}} k \tag{5-27}$$

Criterion 7: The entered member can benefit if

$$\frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}}\tilde{m}_i+m\right)<\frac{1}{n}\left(\sum_{i=1}^{n-1}m_i+m\right) \tag{5-28}$$

C. Discussion

Consider some special cases related to the inequalities (5-27) and (5-28):

a. The *n* members in the Holding Club and the $\tilde{n}$ members in the New Club have an equal mean loss, *m*. For any $k>0$, the inequality (5-27) cannot be satisfied, which implies that the New Club's primary members dislike the entry of this member, albeit that the entered member prefers to switch membership.

b. The entered member might have a mean loss differing from the other members of the Holding Club and from the primary members of the New Club. Similar to Table 5-1, Table 5-2 shows the possible situations under the same assumptions.

Strategy III can definitely benefit the entered member in several situations: $\boldsymbol{m}_1=\left(m^H,m^H,m^L\right)$, $\boldsymbol{m}_2=\left(m^L,m^H,m^L\right)$, $\boldsymbol{m}_3=\left(m^H,m^L,m^L\right)$ with $\tilde{n}+1>n$, and $\boldsymbol{m}_4=\left(m^L,m^H,m^H\right)$ with $\tilde{n}+1<n$. In the last situation especially, the primary members of the New Club can also obtain benefit from competition if $m^H-m^L>\frac{\tilde{n}+1}{\tilde{n}}k$. With the exception of this case, the other situations shown above explore that the inequalities (5-27) and (5-28) cannot be satisfied at the same time, which implies, in most scenarios, conflicting interests of the primary members of the New Club and the entered member.

Table 5-2: Benefit of strategy III (for entered member and New Club's members)

M	m_i $i \neq$ n	$\tilde{m}_i$	Inequality (5-17)	Inequality (5-18)	Remarks
m^H	m^H	m^L	$m^H - m^L < -\frac{\tilde{n}+1}{\tilde{n}}k$ Unsatisfied	$m^L < m^H$	Always advantageous for entered member
m^H	m^L	m^L	$m^H - m^L < -\frac{\tilde{n}+1}{\tilde{n}}k$ Unsatisfied	$\tilde{n}+1 < n$: $m^L > m^H$ $\tilde{n}+1 > n : m^L < m^H$	$\tilde{n}+1 < n$: Unsatisfied $\tilde{n}+1 > n$: Always advantageous for entered member
m^H	m^L	m^H	$0 < -\frac{\tilde{n}+1}{\tilde{n}}k$ Unsatisfied	$m^L > m^H$ Unsatisfied	Inapplicable for entered member
m^L	m^L	m^H	$m^H - m^L > \frac{\tilde{n}+1}{\tilde{n}}k$	$m^L > m^H$ Unsatisfied	Inapplicable for entered member
m^L	m^H	m^H	$m^H - m^L > \frac{\tilde{n}+1}{\tilde{n}}k$	$\tilde{n}+1 < n$: $m^L < m^H$ $\tilde{n}+1 > n : m^L > m^H$	$\tilde{n}+1 > n$: Unsatisfied $\tilde{n}+1 < n$: Always advantageous for entered member
m^L	m^H	m^L	$0 < -\frac{\tilde{n}+1}{\tilde{n}}k$ Unsatisfied	$m^L < m^H$	Always advantageous for entered member

5.2.3 Pareto efficient contract of the Holding Club

A. Ex post Pareto efficient contract of the Holding Club

The entered member switches membership, but leaves the Holding Club its initial capital investment k in the reserve. Thus, this is an insurance contract $C(n\text{-}1)$ with ICR $n\cdot k$.

Proposition 5-10. *An insurance contract* $C(n\text{-}1)$ *offered by the Holding Club for the rest of the* $n-1$ *members is ex post Pareto efficient if it satisfies*:

$$\beta_i\left(\boldsymbol{x}\right)=x_i-\frac{1}{n-1}\sum_{i=1}^{n-1}x_i+\frac{n}{n-1}k \tag{5-29}$$

where x_i *is the loss of the individual i in the state* $\overline{\mathbf{x}}=(x_1, x_2,\ldots, x_{n-1})$, *and the premium* α *is*

$$\alpha=\frac{1}{n-1}\sum_{i=1}^{n-1}x_i-\frac{n}{n-1}k \tag{5-30}$$

Proof. This is a special case of Proposition 5-3, when s=1. □

Compare the coverage in Proposition 5-10 with the one before the switch, to see that there is the following criterion.

Criterion 8: Under the *ex post* PEC, if the actual loss of the entered member is larger than the post-switch premium of the Holding Club, the membership switch is advantageous for the remaining $n-1$ members of the Holding Club.

$$x_n>\frac{1}{n-1}\sum_{i=1}^{n-1}x_i-\frac{n}{n-1}k \tag{5-31}$$

B. Ex ante Pareto efficient contract of the Holding Club

The post-switch *ex ante* PEC of the Holding Club can be obtained through the same method as Proposition 5-4.

Proposition 5-11. *An insurance contract* $C(n\text{-}1)$ *offered by the Holding Club for the remaining* $n-1$ *members is ex ante Pareto efficient if it satisfies*:

$$\beta_i\left(\bar{\mathbf{x}}\right)=x_i-\frac{1}{n-1}\sum_{i=1}^{n-1}m_i+\frac{n}{n-1}\cdot k \tag{5-32}$$

and the premium is

$$\alpha=\frac{1}{n-1}\sum_{i=1}^{n-1}m_i-\frac{n}{n-1}\cdot k \tag{5-33}$$

Proof. This is a special case of Proposition 5-4, when $s = 1$. □

Compare the post-switch coverage of the other primary members of the Holding Club with the pre-switch one, to see that there is the following criterion.

Criterion 9: Under the *ex ante* PEC, if the mean loss of the entered member is larger than the post-switch premium of the Holding Club, the membership switch is advantageous for the remaining $n-1$ members of the Holding Club.

$$m>\frac{1}{n-1}\sum_{i=1}^{n-1}m_i-\frac{n}{n-1}k \tag{5-34}$$

5.2.4 Integrated P&I insurance contract

In this section, the P&I insurance contracts of the New and Holding Clubs are developed based on certain competition strategies. The contract for the New Club integrates the *ex post* PEC revealed by Proposition 5-1, 5-3, and the *ex ante* PEC revealed by Proposition 5-7, 5-9. Also, the Holding Club's contracts after the switch are proved to be *ex post* or *ex ante* Pareto efficient in Proposition 5-5 and 5-11, respectively.

Thus, if strategy II is conducted, the integrated P&I insurance contract of the New Club is

$$\begin{cases} \tilde{\alpha}^{II*} = \max\left\{ \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{m}_i, \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i \right\} - \frac{\tilde{n}}{\tilde{n}+1}\cdot\tilde{k} \\ \tilde{\beta}_i^{II*}\left(\tilde{\boldsymbol{x}}\right) = \tilde{x}_i - \max\left\{ \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{m}_i, \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i \right\} + \frac{\tilde{n}}{\tilde{n}+1}\cdot\tilde{k} \end{cases} \tag{5-35}$$

whilst if strategy III is conducted, the integrated P&I insurance contract of the New Club is

$$\begin{cases} \tilde{\alpha}^{III*} = \max\left\{ \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{m}_i, \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i \right\} - \left(\tilde{k} - \frac{k}{\tilde{n}}\right) \\ \tilde{\beta}_i^{III*}\left(\tilde{\boldsymbol{x}}\right) = \tilde{x}_i - \max\left\{ \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{m}_i, \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i \right\} + \left(\tilde{k} - \frac{k}{\tilde{n}}\right) \end{cases} \tag{5-36}$$

This contract is only available for the primary members of the New Club. As for the entered member, under strategy III, the contract is

$$\begin{cases} \alpha^{III*} = \max\left\{ \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{m}_i, \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i \right\} - k \\ \beta_i^{III*}\left(\tilde{\boldsymbol{x}}\right) = \tilde{x}_i - \max\left\{ \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{m}_i, \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i \right\} + k \end{cases} \tag{5-37}$$

and the post-switch P&I insurance contract of the Holding Club is

$$
\begin{cases}
\alpha = \max\left\{ \frac{1}{n-1}\sum_{i=1}^{n-1} m_i, \frac{1}{n-1}\sum_{i=1}^{n-1} x_i \right\} - \frac{n}{n-1}k \\
\beta_i(\mathbf{x}) = x_i - \max\left\{ \frac{1}{n-1}\sum_{i=1}^{n-1} m_i, \frac{1}{n-1}\sum_{i=1}^{n-1} x_i \right\} + \frac{n}{n-1}k
\end{cases}
\tag{5-38}
$$

So far, the work in section 5.2 focuses on the Pareto efficient premium call policy and coverage offered to the primary members of the New Club, given the competition strategy. Under strategies II and III, the optimization problem is actually to maximize the joint utility of the $\tilde{n}$ primary members of the New Club, rather than the joint utility of the $\tilde{n}+1$ members. The equation (5-20) shows that the contracts for all $\tilde{n}+1$ members are identical when strategy II is conducted. In contrast, if it is strategy III, then the contract for the entered member differs from the one for the primary members.

5.3 Decision making under competition

In this oligopolistic P&I market, there are three kinds of stakeholders, the entered member, the primary members of the New Club, and the remaining primary members of the Holding Club. Albeit that the integrated P&I insurance contract involves both the expected and actual loss simultaneously, either the entered member or the New Club can only make a decision based on the *ex ante* expectation rather than the *ex post* knowledge about the realized loss (see Table 5-1 and 5-2).

5.3.1 Holding Club

Membership switching, in most situations, cannot benefit the Holding Club. It is easy to understand that there are two main reasons that explain why an entered member would like to switch membership, if he does so. Firstly, the entered member has a lower mean loss, but pays an equal premium and undertakes equal liability with members of a much higher mean loss. Secondly, the New Club offers a really attractive coverage and cheaper premium.

Inequality (5-11) shows that the entered member and the primary members of the New Club can all benefit from the membership switch simultaneously. However, inequality (5-16) also shows up the condition that the primary members left in the Holding Club can be benefited from the premium competition. The inequalities (5-11) and (5-16) can be satisfied simultaneously at the very point where

$$\frac{1}{n-s}\sum_{i=1}^{n-s} m_i - \frac{1}{s}\sum_{i=n-s+1}^{n} m_i = \frac{n}{n-s}k$$

This equation indicates that the n members of the Holding Club are divided into two groups. One is of the $n-s$ members who are left in the Holding Club, and the other contains the s members, who switch to the New Club. The per capita mean loss of the former group is larger than the latter group, but the distance cannot exceed the upper limit $n \cdot k/(n-s)$. If the distance is exactly the upper limit, the entered members can switch freely. If, however, the distance exceeds the upper limit, the remaining $n-s$ members in the Holding Club suffer reduced coverage and an increasing premium, so then the Holding Club will set some boundary to interrupt the membership switch.

Inequality (5-34) is a special case of (5-16), when $s = 1$. This special case is available for the Holding Club to evaluate the impact of a certain member switching his membership, because each individual member has his own risk status, especially when there are more than two Clubs competing in the market.

5.3.2 Entered member

The entered member is the accepter of the competition strategy. Criterion 1, inequalities (5-22) and (5-28), are respectively the conditions on which the entered member will accept the contract of the New Club. Inequality (5-22) is the condition for accepting strategy II, while (5-28) is the condition for accepting strategy III.

An entered member makes a decision through the following steps: (a) Collect information from the Holding Club, the total number of members, the mean loss of each member, and the ICR level; (b) collect information from the New Club about the same items as in step (a); and (c) examine the inequality (5-22) and (5-28) and Criterion 1.

5.3.3 New Club

The New Club is the designer of the competition strategies. In this research, it involves three types of competitive contracts: (a) Maximize the joint utility of both the primary members of the New Club and the entered member; (b) allow the entered member to share the ICR of the New Club equally with other $\tilde{n}$ members and thus maximize the joint utility of the primary members; and (c) allow the entered member to obtain a fixed amount, *k*, from the ICR of the New Club and thus maximize the joint utility of the primary members.

Under strategy I, there are three steps for the New Club: (a) Investigate the mean loss of each primary member of the New Club; (b) investigate the ICR level of the Holding Club; and (c) examine Criterion 2. At the same time, the New Club can also examine Criterion 1, to make sure that the contract is acceptable for the entered member.

Under strategy II, the first step is to investigate the information involved, for instance, the mean loss of the entered member, the ICR of the New and Holding Clubs, and the total number of members in the New Club. The second step is to make sure strategy II is available. Promising $\tilde{n} \cdot \tilde{k} / (\tilde{n} + 1)$ is the second competition contract, when $\tilde{k} > k$ (see Table 5-3). What should be highlighted here is that there is a lower limit for the number of members in the New Club, when $\tilde{k}/k \in]1, 2]$. This implies that there is no decisive distance between $\tilde{k}$ and *k*. Thus, there must be sufficient *primary* members to ensure that the ICR is large enough to support this strategy. Besides this, another situation for this strategy is $\tilde{k}/k \in]2,\infty[$. The distance between $\tilde{k}$ and k decisively supports this strategy. The third step is to examine inequalities (5-21).

The analogous method can be adopted to analyze the third competition strategy. After information investigation, the second step is then to make sure of the applicability of the contract. Promising k as the coverage from the ICR is the competition strategy conducted in these two situations: (a) When the per capita initial investment of the New Club is less than the counterpart of the Holding Club, i.e. $\tilde{k} \le k$; and (b) when the New Club's individual ICR investment is higher than the Holding Club's, where $1 < \frac{\tilde{k}}{k} \le 2$, but there are no more than $k/(\tilde{k}-k)$ members in the New Club.

Table 5-3: Competition strategy and the P&I insurance contract

$\tilde{k}/k$	$\tilde{n}$	Competition Strategy conducted	Premium of P&I insurance contract
$]0,1[$	Any positive integrals	Strategy III	$\max\left\{\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i, \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{m}_i\right\} - \left(\tilde{k}-\frac{k}{\tilde{n}}\right)$
$[1]$	Any positive integrals	Strategy III	$\max\left\{\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i, \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{m}_i\right\} - \frac{\tilde{n}-1}{\tilde{n}}k$
$]1,2]$	$\tilde{n} > k/(\tilde{k}-k)$	Strategy II	$\max\left\{\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i, \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{m}_i\right\} - \frac{\tilde{n}}{\tilde{n}+1}\cdot\tilde{k}$
	$\tilde{n} \le k/(\tilde{k}-k)$	Strategy III	$\max\left\{\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i, \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{m}_i\right\} - \left(\tilde{k}-\frac{k}{\tilde{n}}\right)$
$]2,\infty[$	Any positive integrals	Strategy II	$\max\left\{\frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i, \frac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{m}_i\right\} - \frac{\tilde{n}}{\tilde{n}+1}\cdot\tilde{k}$

In strategy II, the entered member shares the initial capital equally with the primary members of the New Club. Since $\frac{\tilde{n} \cdot \tilde{k}}{\tilde{n}+1} > k$, the entered member benefits from this contract. However, this is only one facet of the contract. It cannot be the only reason for the entered member to switch membership. Detailed explanations and discussion have been provided in Table 5-1 and 5-2.

5.4 Holding Club's Countermeasures

Facing up to competition from the New Club, the Holding Club adopts certain countermeasures to interrupt the membership switch or to reduce the competitiveness in the game. In the previous sections of this chapter, the three competition approaches are not always applicable. They must be conducted under particular conditions.

Inequality (5-11), and the discussion in Table 5-1 and 5-2, show that if a competition strategy is applicable, it must be of benefit to both the entered member and the primary members of the New Club. It is still assumed that the mean loss of an individual member is either m^H or m^L in this economy. In agreement with the discussion in Table 5-1 and Table 5-2, it is supposed that only the entered member may have a mean loss differing from the other members of the Holding Club.

Table 5-4 below shows that there is only one situation listed in Table 5-1 and 5-2 that can benefit both the entered member and the primary members of the New Club. That is when an entered member switches his membership from a Holding Club full of high-mean-loss members to another Club of the same characteristics.

Table 5-4: Applicable competition scenario

<table>
<tr><th rowspan="2">Triplet of the mean loss
$\boldsymbol{m}$</th><th colspan="3">Competition strategy</th></tr>
<tr><th>Strategy I</th><th>Strategy II</th><th>Strategy III</th></tr>
<tr><td rowspan="2">$\boldsymbol{m} = \left(m^L, m^H, m^H\right)$</td><td rowspan="2">$m^H - m^L \geq \frac{n}{n-1}k$</td><td>$1 < \tilde{k}/k \leq 2$;
$\tilde{n} > k/\left(\tilde{k} - k\right)$;
$n > \tilde{n} + 1$;
$m^H - m^L \geq k$</td><td>$0 < \tilde{k}/k \leq 1$;
Any $\tilde{n}$;
$n > \tilde{n} + 1$;
$m^H - m^L \geq \frac{\tilde{n}+1}{\tilde{n}}k$</td></tr>
<tr><td>$\tilde{k}/k \geq 2$;
Any $\tilde{n}$;
$n > \tilde{n} + 1$;
$m^H - m^L \geq k$</td><td>$1 < \tilde{k}/k \leq 2$;
$\tilde{n} \leq k/\left(\tilde{k} - k\right)$;
$n > \tilde{n} + 1$;
$m^H - m^L \geq \frac{\tilde{n}+1}{\tilde{n}}k$</td></tr>
</table>

It is not difficult to understand this one possible scenario. In the oligopolistic market of P&I insurance, each P&I Club occupies a share of P&I insurance business and insures a proportion of the global fleets. If we let the amount m^H represent this average level, any Club would always like to convince another Club's member who has a lower mean loss, for instance, m^L, to switch membership.

Through analyzing Table 5-4, certain principles can be found to identify whether a certain strategy is available.

(a) The entered member should have a mean loss sufficiently lower than the others'. For strategy I, this distance is at least larger than $n \cdot k/(n-1)$; for strategy II, it is at least larger than k; and for strategy III, the distance should be larger than $(\tilde{n}+1) \cdot k/\tilde{n}$.

(b) The New Club should have fewer members than the Holding Club, apart from the entered member. "Fewer members" implies that, for an individual member, more ICR can be allocated to him. Thus, if possible, the entered member will pool risk within a small-size group rather than a large-size one.

(c) The per capita ICR investment of the Holding Club k is generally smaller than that of the New Club $\tilde{k}$, except for that in strategy III, where k could be larger. As known, strategy III is conducted when the New Club does not have sufficient ICR to support strategy II. Thus, when $0 < \tilde{k}/k \le 1$, there is no constraint on the New Club's size. However, if $1 < \tilde{k}/k \le 2$, there is an upper limit to the Club size.

Strategy II always requires that $\tilde{k} > k$. When $\tilde{k}$ is more than twice k, there is no constraint on the New Club's size. But if $1 < \tilde{k}/k \le 2$, there is a lower limit to the Club size. The constraint on the Club size demonstrates that it requires sufficient ICR to support strategy II.

The countermeasures of the Holding Club in the competition have to purposely make the competition conditions dissatisfied. In each competition strategy, given the high mean loss, m^H, and the low mean loss, m^L, there are only two variables controlled by the Holding Club, these being the per capita ICR, k, and the primary member number, n. Let Δm denote $m^H - m^L$.

(a) Seek external resources. It is easy to prove the following results: (a) If k increases to and exceeds $\tilde{n} \cdot \Delta m / \tilde{n} + 1$, strategy III is inapplicable; (b) if k increases to and exceeds $(n-1) \cdot \Delta m / n$, strategy I and strategy III are inapplicable; and (c) if k increases to and exceeds Δm, strategy I, as well as strategies II and III, are inapplicable.

(b) Compress the Club size. In this research, the entered member is the only one who has a low mean loss that distinguishes himself from the other members of the Holding Club. The Pareto efficient contract of the Holding Club is based on this fact. When the entered member switches his membership, the original Pareto efficient equilibrium will change, and consequently the optimal size of the Holding Club will also vary. The Club size will decrease to a level below $\tilde{n}+1$, which results in the failure of strategies II and III.

For strategy I, when $n \to 1$, the left of inequality $m^H - m^L \geq \frac{n}{n-1} k$ will approach to infinite. Since each member in this economy has a finite mean loss, Δm must be finite. With the decrease of n, there should be a threshold n_o, such that $m^H - m^L < \frac{n_o}{n_o - 1} k$. However, if $\Delta m \geq 2k$, compressing the Club size does not work as a countermeasure against strategy I.

(c) Increase the release. Let c denote the release call of the entered member paid to the Holding Club, which is deemed to be a kind of switch cost levied as a compensation to the other members. Let subscript (I), (II) and (III) indicate the strategy. The penalty should be larger than or equal to the benefit of the entered member obtained in the New Club. The value of the penalty is based on the expectation of the benefit. For strategy I, the contract for the entered member becomes

$$\begin{cases} \alpha_{\tilde{n}+1}^{I*} = \max\left\{ \dfrac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i, \dfrac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{m}_i \right\} - \dfrac{\tilde{n}}{\tilde{n}+1}\left(k + c_{(I)}\right) \\ \beta_{\tilde{n}+1}^{I*}\left(\tilde{\boldsymbol{x}}\right) = \tilde{x}_{\tilde{n}+1} - \max\left\{ \dfrac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{x}_i, \dfrac{1}{\tilde{n}+1}\sum_{i=1}^{\tilde{n}+1}\tilde{m}_i \right\} + \dfrac{\tilde{n}}{\tilde{n}+1}\left(k + c_{(I)}\right) \end{cases}$$

where the entered member transfers this switch cost to the primary members of the New Club. The Holding Club offers the remaining n – 1 members the PEC that

$$\begin{cases} \alpha = \max\left\{ \dfrac{1}{n-1}\sum_{i=1}^{n-1}x_i, \dfrac{1}{n-1}\sum_{i=1}^{n-1}m_i \right\} - \dfrac{n}{n-1}k - \dfrac{1}{n-1}c_{(I)} \\ \beta_i\left(\overline{\boldsymbol{x}}\right) = x_i - \max\left\{ \dfrac{1}{n-1}\sum_{i=1}^{n-1}x_i, \dfrac{1}{n-1}\sum_{i=1}^{n-1}m_i \right\} + \dfrac{n}{n-1}k + \dfrac{1}{n-1}c_{(I)} \end{cases}$$

When $\boldsymbol{m} = \left(m^L, m^H, m^H\right)$, $c_{(I)} = \dfrac{n-1}{n}\left(m^H - m^L\right) - k$.

For strategies II and III, premium call policies are not Pareto efficient for all of the $\tilde{n}+1$ members of the New Club. Based on $\boldsymbol{m} = \left(m^L, m^H, m^H\right)$, in strategy II, the penalty $c_{(II)}$ should be equal to $E\left[\alpha^* - \tilde{\alpha}^{II*}\right]$ so that

$$c_{(II)} = \frac{n-\left(\tilde{n}+1\right)}{n\left(\tilde{n}+1\right)}\left(m^H - m^L\right) - \left(\frac{\tilde{n}}{\tilde{n}+1}\tilde{k} - k\right)$$

For strategy III, the penalty $c_{(\mathrm{III})}$ is equal to $E\left[\alpha^{*}-\alpha^{III*}\right]$ so that

$$c_{(III)}=\frac{n-\left(\tilde{n}+1\right)}{n\left(\tilde{n}+1\right)}\left(m^{H}-m^{L}\right)$$

5.5 Conclusion

This chapter contributes to the competition strategies conducted by the New Club, analyzes the impact of these strategies on the stakeholders involved, and provides possible countermeasures for the Holding Club.

Three competition strategies are discussed respectively. Strategy I is a Pareto efficient contract priced on maximizing the joint utility of the overall members of the New Club, including the entered member. Strategy II and III are Pareto efficient contracts merely for the primary members of the New Club. Here, the premium call policy for the entered member is *ex ante* committed.

These three strategies, however, are not always applicable. In most situations, the entered and primary members of the New Club cannot both benefit from the membership switch simultaneously. If the strategy is advantageous for the entered member (or the primary members), it will be disadvantageous for the primary members (or the entered member). There are several components that must be taken into consideration in the competition decision: (a) The triplet ***m***; (b) the ICR of both the Holding and New Clubs; and (c) the size of both the Holding and New Clubs.

It is found that there is only one situation under which all of the three strategies are applicable. An individual member with a lower mean loss switches membership from the Holding Club to the New Club. Apart from the entered member, the average levels of the other members' mean loss are quite approximate, and this average level is higher than the entered member's mean loss.

So, at the end of this chapter, three countermeasures are provided for the Holding Club to deal with competition from the New Club: (a) Seek for an external resource to increase k; (b) compress the Holding Club size to increase the subsidy on the coverage from ICR; and (c) levy a penalty to increase the switch fee. The first countermeasure might lead to demutualization of the P&I Club, if an external investment comes from the private sector or public investors. Compressing the Club size is inapplicable when $m^H - m^L \geq 2k$. As a final point, the last countermeasure is quite practical for the Holding Club in the competition. The question, then, is how the Holding Club can know which strategy to conduct when all three are applicable. Thus, what the Holding Club can do is to choose the largest one among $c_{(\mathrm{I})}$, $c_{(\mathrm{II})}$ and $c_{(\mathrm{III})}$.

Chapter 6. Simulation and Discussion

6.1 Basic design of simulation

Currently, price competition in P&I Clubs is restricted by the Group Agreement 1999. It means that there are no competition records with a large-scale database to support the empirical study. Thus, in order to assess the impact of price competition, the information from multiple P&I Clubs is collected, including the loss distribution, number of members, capital reserve, etc. However, not all P&I Clubs have the relevant data published in their annual reports, especially the loss distribution used in their actuarial models. In this section, some technical measures are conducted to solve these practical difficulties.

(a) Claims can be described from two perspectives, that is, the number of claims and the pecuniary value of claims.

The annual reports of some Clubs provide merely the tail information of the individual claims, for example, the proportion of claims exceeding US$ 5 million (in the rest of this chapter, the fiscal items are measured in the scale of millions of US dollars). Suppose the random losses of members are independent and identically distributed variables. Then, the claim records can be used as samples to estimate the loss distribution of an individual member. Given the tail information, a distribution with unknown parameters can be derived through solving the cumulative function of random loss.

There are two P&I Clubs providing sufficient tail information to estimate the parameters, that is, North of England Club and Britannia Club. In this chapter, these two Clubs will be considered as an example to analyze the competition between Clubs.

(b) Albeit that the number of members is also not shown in most of the P&I Clubs' annual reports, this information can be obtained through making enquiry to the Club directly. The gross entered tonnage of North of England Club was 95 million GT, and the number of insured vessels was around 3750. Britannia Club's was 134.8 million GT, and by 20th February 2009 Britannia Club underwrote 3887 vessels in total.

Britannia Club writes bulk carrier (28%), tanker (42%), container (25%), general cargo (4%), and other vessel type (1%). North of England Club insures bulk carrier (36%), tanker (30%), container (23%), general cargo (3%) and other vessel type (8%).

(c) Initial capital reserve is the free reserve of a P&I Club at the beginning of a policy year. Free reserve is funds available to the P&I Club for investment and coverage. The free reserves of the two Clubs are quite close to each other, and higher than the average level of the thirteen members of the Group.

The rest of this chapter is based on the above assumptions and expanded from the three heavy-tail distributions, respectively.

6.2 Loss distribution

The heavy-tail nature of P&I risk is of common understanding in the shipping industry [7, 8, 50]. In order to satisfy the Central Limit Theorem, the heavy-tail distribution of a random loss must have finite mean and variance. Thus, Pareto distribution, lognormal distribution and Weibull distribution are undertaken as the distribution options to simulate the claim process.

Each of these three distributions has two parameters. The reports of Britannia Club and North of England Club provide the tail distributions of the claim sizes, i.e., the proportions of the claims exceeding some certain levels. Since each of the distribution functions has an explicit form, the two parameters can be obtained through solving the distribution function as an equation. Due to there are two parameters, the solution of the parameters can be calculated for, at least, two different loss levels and the percentages of claims larger than these two levels.

6.2.1 Pareto distribution

The random losses in each P&I Club are independent and identically distributed with Pareto distribution. Let x_m denote the positive minimum possible value of random variable x_i, and let θ be a positive parameter. Then, the density function of Pareto distribution is

$$f_i(x)=\begin{cases}\theta x_m^{\theta}/x^{\theta+1}, & x>x_m\\ 0, & x<x_m\end{cases}$$

and the cumulative function is

$$F_i(x)=\begin{cases}1-(x_m/x)^{\theta}, & x\ge x_m\\ 0, & x<x_m\end{cases}$$

Mean is $E(x)=\dfrac{\theta x_m}{\theta-1}$, and variance is $Var(x)=\dfrac{\theta x_m^2}{(\theta-1)^2(\theta-2)}$, for $\theta>2$.

According to the 2009 report from North of England Club, there were in total 6355 P&I claims. The average value of claims was 0.037174. About 1.27% of the total number of claims exceeded 0.5. It is found that θ is a complex number. Thus, Pareto distribution is not applicable to model North of England Club.

By 20 February 2009, in Britannia Club, a total of 6897 claims had been reported. There were 27 claims expected to cost 1 or more, and only eight claims exceeded 2. Thus, θ^B = 1.755 and $x_m{}^B$ = 0.0425. The mean loss of Britannia Club is 0.0988 under Pareto distribution.

6.2.2 Lognormal distribution

The random losses in each P&I Club are independent and identically distributed with lognormal distribution. The density function is

$$f_i(x) = \frac{1}{\sqrt{2\pi}\sigma x}\exp\left\{-\frac{(\ln x-\mu)^2}{2\sigma^2}\right\} \qquad \text{for } x > 0$$

with mean $E(x) = \exp\{\mu + \sigma^2/2\}$ and variance $Var(x) = \left(e^{\sigma^2} - 1\right)\cdot e^{2\mu+\sigma^2}$. Based on the data from North of England Club, $\mu^N = -3.764$, $\sigma^N = 0.971$. Lognormal distribution is not applicable for Britannia Club, because the claim records in 2009 do not support the simultaneous existence of the parameter μ and σ. The mean loss of North of England Club is 0.0372 under lognormal distribution.

6.2.3 Weibull distribution

The random losses in each P&I Club are independent and identically distributed with Weibull distribution. The density function is

$$f_i(x) = \begin{cases} \frac{\eta}{\theta}\cdot\left(\frac{x}{\theta}\right)^{\eta-1}\cdot\exp\left\{-\left(\frac{x}{\theta}\right)^{\eta}\right\}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

with mean $E(x) = \theta\cdot\Gamma(1+1/\eta)$. The cumulative distribution function is

$$F_i(x) = 1 - e^{-(x/\theta)^{\eta}}$$

Britannia Club had the parameter $\eta^B = 0.286$ and $\theta^B = 0.00252$. The claim process of North of England Club is still unable to be modeled by Weibull distribution. The mean loss of Britannia Club under Weibull distribution is 0.0292.

Thus, there are two possible distributions for Britannia Club, Pareto distribution and Weibull distribution. The annual report reveals that there were in total 6897 claims in 2009, and the net claims incurred were 177.022. For a single claim, on average, the claim payment was a mere 0.0257, which was quite close to the result under Weibull distribution rather than the Pareto distribution calculated previously.

Thus, Britannia Club underwrote the random losses following Weibull distribution, while North of England Club's members had lognormal distributed claims. Britannia Club was of a larger size than North of England Club.

Table 6-1: Summary of the two P&I Clubs

P&I Club Name	Loss distribution			Club size		Free Reserve
	Type	Mean	Variance	Tonnage	Number	
Britannia Club	Weibull (0.286, 0.00252)	0.0292	0.0324	134.8 million GT	3887	US$ 191.5 million
North of England	Lognormal (-3.764, 0.971)	0.0372	0.00216	95 million GT	3750	US$ 211.1 million

6.3 Competition assessment

6.3.1 North of England Club: the New Club

Without the loss of generality, in this section suppose that North of England Club is the New Club, and Britannia Club is the Holding Club. Based on the information shown in table 6-1, n = 3887, $\tilde{n}$ = 3750, K = 191.5, $\tilde{K}$ = 211.1, m^H = 0.0372, m^L = 0.0292. Thus, k = 0.0493, $\tilde{k}$ = 0.0563, so that $\tilde{k}/k = 1.142 \in (1,2)$ and $\tilde{n} > k/(\tilde{k}-k)$.

North of England Club can reduce the whole Club's risk level by inducing members of the Holding Club to switch membership. Table 6-2 displays the constraints on the Holding Club's size n. By examining whether the criteria are satisfied under the condition mentioned above, the New and Holding P&I Clubs, as well as the entered member, can make a decision on whether or not to accept the competition strategy.

Table 6-2: Criteria for competition strategy

Strategy	Criterion		n & $\tilde{n}$
Strategy I	Criterion 1	$\frac{1}{n}\sum_{i=1}^{n} m_i - \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s}\tilde{m}_i \geq \frac{s}{\tilde{n}+s}k$	n does not exist
	Criterion 2	$\frac{1}{\tilde{n}}\sum_{i=1}^{\tilde{n}}\tilde{m}_i - \frac{1}{s}\sum_{i=\tilde{n}+1}^{\tilde{n}+s}\tilde{m}_i \geq k$	n unsatisfied
	Criterion 3	$\frac{1}{n-s}\sum_{i=1}^{n-s} m_i - \frac{1}{s}\sum_{i=n-s+1}^{n} m_i \leq \frac{n}{n-s}k$	Any n
Strategy II	Criterion 4	$m < \frac{1}{\tilde{n}}\sum_{i=1}^{\tilde{n}}\tilde{m}_i - \tilde{k}$	$\tilde{n}$ unsatisfied
	Criterion 5	$\frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}}\tilde{m}_i + m\right) - \frac{1}{n}\left(\sum_{i=1}^{n-1} m_i + m\right) < \frac{\tilde{n}}{\tilde{n}+1}\tilde{k} - k$	n unsatisfied
Strategy III	Criterion 6	$m < \frac{1}{\tilde{n}}\sum_{i=1}^{\tilde{n}}\tilde{m}_i - \frac{\tilde{n}+1}{\tilde{n}}k$	n unsatisfied
	Criterion 7	$\frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}}\tilde{m}_i + m\right) < \frac{1}{n}\left(\sum_{i=1}^{n-1} m_i + m\right)$	n does not exist
	Criterion 9	$m > \frac{1}{n-1}\sum_{i=1}^{n-1} m_i - \frac{n}{n-1}k$	Any n

Strategy I and III are not applicable. The Holding Club size n does not exist in order to satisfy criterion 1 and 7. Criterion 1 and 7 are the standards to assess if strategy I and III are advantageous for the entered member, respectively. The non-existence of n means that these two strategies fail to benefit the entered member sufficiently. It is intuitive to understand these findings. The North of England Club has fewer members but a higher free reserve, which indicates the stronger capability of this Club in bearing more risk and underwriting more members. Especially when an entered member receives an attractive contract from a New Club like the North of England Club, he or she would prefer to obtain an equal right, like the other members, so as to employ the free reserve mutually.

Strategy II is also unacceptable. Criterion 4 is the condition under which the primary members of the New Club will accept the entry of the entered member. Criterion 4 shows that the New Club should underwrite at least 26,387 members. Since $n > \tilde{n}+1$, n should be at least 26,388. However, the actual member number of the New Club was only around 3,750. Thus, the primary members of the New Club suffer absolute utility reduction in this competition. Criterion 5 is the constraint to ensure that the entered member can benefit from the membership switch. Given $\tilde{n}=3{,}750$ and due to $n > \tilde{n}+1$, criterion 5 requests that the Holding Club should have at least 3,967 members, including the entered member. However, the actual member number of Britannia is 3,887, less than the minimal requirement. Thus, strategy II cannot be conducted.

6.3.2 Britannia Club: the New Club

Conversely to section 6.3.1, Britannia Club is now considered as the New Club, whereas North of England Club becomes the Holding Club. Thus, the parameters will be changed accordingly, where $\tilde{n}$ = 3887, n = 3750, $\tilde{K}$ = 191.5, K = 211.1, m^H = 0.0372, m^L = 0.0292. Thus, $\tilde{k}$ = 0.0493, k = 0.0563, so that $\tilde{k}/k = 0.876 \in (0,1)$. Similar to table 6-2, the constraints with respect to n can be calculated by examining the identical criteria. The relevant results are shown in table 6-3.

Table 6-3: Criteria for competition strategy

Strategy	Criterion		n, s & $\tilde{n}$
Strategy I	Criterion 1	$\frac{1}{n}\sum_{i=1}^{n} m_i - \frac{1}{\tilde{n}+s}\sum_{i=1}^{\tilde{n}+s} \tilde{m}_i \geq \frac{s}{\tilde{n}+s}k$	$s \leq 552$
	Criterion 2	$\frac{1}{\tilde{n}}\sum_{i=1}^{\tilde{n}} \tilde{m}_i - \frac{1}{s}\sum_{i=\tilde{n}+1}^{\tilde{n}+s} \tilde{m}_i \geq k$	n does not exist
	Criterion 3	$\frac{1}{n-s}\sum_{i=1}^{n-s} m_i - \frac{1}{s}\sum_{i=n-s+1}^{n} m_i \leq \frac{n}{n-s}k$	n and $\tilde{n}$ satisfied
Strategy II	Criterion 4	$m < \frac{1}{\tilde{n}}\sum_{i=1}^{\tilde{n}} \tilde{m}_i - \tilde{k}$	n does not exist
	Criterion 5	$\frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) - \frac{1}{n}\left(\sum_{i=1}^{n-1} m_i + m\right) < \frac{\tilde{n}}{\tilde{n}+1}\tilde{k} - k$	n and $\tilde{n}$ satisfied
Strategy III	Criterion 6	$m < \frac{1}{\tilde{n}}\sum_{i=1}^{\tilde{n}} \tilde{m}_i - \frac{\tilde{n}+1}{\tilde{n}}k$	n does not exist
	Criterion 7	$\frac{1}{\tilde{n}+1}\left(\sum_{i=1}^{\tilde{n}} \tilde{m}_i + m\right) < \frac{1}{n}\left(\sum_{i=1}^{n-1} m_i + m\right)$	n and $\tilde{n}$ satisfied
	Criterion 9	$m > \frac{1}{n-1}\sum_{i=1}^{n-1} m_i - \frac{n}{n-1}k$	n and $\tilde{n}$ satisfied

None of the three strategies is advantageous for the primary members of the New Club. The managers of Britannia Club have no motivation to conduct any competition strategy, mainly because the mean loss of the Holding Club is much higher. The New Club cannot reduce the overall risk level through attracting members of the Holding Club.

Chapter 7. Conclusion

In 1999, the International Group Agreement, which imposes a degree of restriction on price competition, was granted an exemption by the European Commission for 10 years from 20 February 1999. That exemption has therefore technically now expired. However, EU rules now no longer require or allow a renewal of the specific exemption, and the Group in effect has to self-certify that the operation of the market remains similar to that which prevailed in 1999, and that the Agreement is essential to support the Pooling Agreement, which in turn allows shipowners to be provided with the very high limits of coverage under the Group system.

- **Pareto Efficient P&I Insurance Contract**

In this research, the first contribution is to develop the premium call policies demonstrated by Fagart et al [41, 42] and Smith and Stutzer [96, 97]. Through examining the existence of both ex post and ex ante Pareto efficient contracts, an integrated Pareto efficient contract can be formed without competition.

- **Optimal P&I Club Size**

Given the exponential utility function of an individual member, it is proved that the utility of a member does not necessarily increase with Club size. In agreement with Powers and Shubik [85], the number of P&I Clubs in the market is not infinite, which explains the formation of the oligopolistic P&I market.

- **P&I Insurance Contract under Competition**

In the second stage, three competition strategies are discussed separately. Strategy I maximizes the joint utility of the entered and primary members of the New Club. In strategies II and III, the contract for the entered member is known, and the strategies have to maximize the joint utility of the primary members of the New Club. The analytic results reveal that competition among the P&I Clubs might not improve the utility of both the entered and primary members in the New Club simultaneously.

The entered and primary members of the New Club can both benefit at the same time when a low-risk individual switches membership from a Holding Club full of high-risk members to a New Club that is also full of high-risk members. On the other hand, the primary members left in the Holding Club are usually disadvantaged by premium competition, except in the one situation where a high-risk individual switches his/her membership out of a Holding Club that is full of low-risk members.

- **Case Study**

In the case study section, the loss distribution is obtained from the actual data of two P&I Clubs, North of England and Britannia Club. Based on the Clubs' annual and management reports, it is seen that Britannia Club has a competitive advantage relative to North of England Club. When North of England Club conducts a competition strategy, the welfare of the entered member from Britannia Club fails to be improved. On the contrary, when Britannia Club is the New Club, the competition strategies are not welcome for the primary members of the New Club.

- **Future Works**

The assumptions of this study should be generalized in the future work, with respect to the heterogeneous risk aversion of P&I members and the loss distributions. The theories of reinsurance policy and deductible of P&I insurance should be revisited under the background of Pareto efficient contract. China P&I Club can purchase the reinsurance contract from the Group through other formal members. Thus, how to improve the cooperation between China P&I Club and the members of the Group becomes one of the most practical problems facing to the global P&I market.

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