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APPLICATION OF GPS FOR MONITORING
LONG-SPAN CABLE-SUPPORTED BRIDGES
UNDER HIGH WINDS

by

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Department of Civil and Structural Engineering

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CHAN WAI SHAN

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

August 2009
To My Family
I hereby declare that this thesis entitled "Application of GPS for Monitoring Long-Span Cable-Supported Bridges under High Winds" is my own work and that, to the best of my knowledge and belief, it reproduces no material previously published or written, nor material that has been accepted for the award of any other degree or diploma, except where due acknowledgement has been made in the text.

SIGNED

CHAN Wai Shan
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Abstract

Many long-span cable-supported bridges have been built across the world in the past two decades to meet the economic, social, and recreational needs of communities. However, due to the highly flexible nature and low damping levels of such bridges, they are vulnerable to wind-induced vibration. To protect the immense capital investments made in such bridges and ensure user comfort and safety during the service stage, the implementation of long-term structural health monitoring systems (SHMS) on long-span cable-supported bridges has become a trend in wind-prone regions. Among all measurement parameters, displacement is a paramount variable used to assess the serviceability, safety and integrity of long-span cable-supported bridges. However, it is difficult to measure the wind-induced absolute displacement of a long-span cable-supported bridge, which includes both a static component and a dynamic fluctuating component, by using traditional sensors. One alternative solution is to use global positioning systems (GPS), but the application of the GPS for bridge monitoring engenders many challenges to professionals. This thesis therefore focuses on the application and integration of global positioning systems (GPS) with structural health monitoring systems (SHMS) and computer simulation to monitor and assess the serviceability and strength of long-span cable-supported bridges under strong winds.

Although calibration works of GPS have been performed by other researchers for building structures, the performance of GPS must be thoroughly validated for application to long-span cable-supported bridges because the fundamental frequency of a long-span cable-supported bridge is often much lower than that of a building. In this connection, a motion simulation table is designed and manufactured as a test station that simulates various types of two-dimensional motions across a wider range of frequencies in either the horizontal plane or the vertical plane. A detailed calibration study is then carried out in an open area in Hong Kong using the motion simulation table to assess the dynamic displacement measurement accuracy of the GPS in the longitudinal, lateral, and vertical directions. In the calibration study, the static tests are first carried out
with stationary antennae to identify the background noise in the GPS measurements. An examination of statistical data recorded over a period of 9 hours shows that the background noise is dominated by low frequency components. A band-pass filtering scheme for sinusoidal motion and circular motion is designed and applied to the displacement data recorded by the GPS, which are then compared with those generated by the table. The comparative results show that for two-dimensional sinusoidal and circular motions in the horizontal plane and one-dimensional sinusoidal motions in the vertical direction, the GPS can be used to obtain accurate dynamic displacement measurements if the motion amplitude is no less than 5 mm in the horizontal plane or 10 mm in the vertical direction and the motion frequency is less than or equal to 1 Hz. The dynamic displacement measurement accuracy of the GPS is finally assessed using the measurement data of wind-induced two-dimensional dynamic displacement responses of the Di Wang Tower in the horizontal plane during Typhoon York and wind-induced one-dimensional dynamic displacement response of the Tsing Ma suspension bridge deck in the vertical direction during Typhoon Victor. The comparative results demonstrate that the GPS can trace complex wind-induced dynamic displacement responses of real structures satisfactorily.

However, the reduced accuracy of GPS displacement measurements due to multipath effects and the low sampling frequency of the GPS receivers is also highlighted in the motion simulation table tests described above. To enhance the measurement accuracy of the total (static plus dynamic) displacement response of civil engineering structures, the concept of integrating signals from a GPS and an accelerometer for deformation monitoring is suggested. This thesis presents two frameworks of integrated data processing techniques that use both empirical mode decomposition (EMD) and an adaptive filter. To assess the effectiveness of the proposed integrated data processing techniques, a series of motion simulation tests simulating various types of motion around a pre-defined static position are performed at a site and recorded by a GPS receiver and an accelerometer. The proposed data processing techniques are then applied to the recorded GPS and accelerometer data to find both static and dynamic displacements. These results are compared with the actual displacement motion generated by the motion simulation table. The comparative results demonstrate that the proposed technique can significantly enhance total displacement measurement accuracy.

Wind and structural health monitoring systems including GPS have been
installed on some long-span cable-supported bridges, but it is not clear how to use GPS data to assess the serviceability and strength of the bridge under strong winds. This thesis takes the Tsing Ma Bridge as an example to manifest how the GPS and anemometers installed on the bridge can be used for this purpose. The Tsing Ma Bridge in Hong Kong is a long suspension bridge, and a Wind And Structural Health Monitoring System (WASHMS) that includes 6 anemometers and 14 GPS stations has been fully operational on the Tsing Ma Bridge since 1997 (for the anemometers) and 2002 (for the GPS stations), respectively. Because Hong Kong is situated in an active typhoon region and encompassed by a complicated topography, wind characteristics around the Tsing Ma Bridge are very complicated. The wind environment surrounding the Tsing Ma Bridge is thus ascertained by analyzing long-term wind data recorded by the anemometers for both typhoons and strong monsoons. The wind measurement data taken in the field are first pre-processed in an attempt to produce a high quality database. The wind measurement data recorded by the ultrasonic anemometers in the middle of main span from 1997 to 2005 are then analyzed to obtain the mean wind speed, the mean wind direction, the mean wind inclination, the turbulence intensity, the integral scale, and the wind spectrum of both 10-minutes and 1-hour in duration.

After identifying the wind environment surrounding the Tsing Ma Bridge, the next step is to analyze wind-induced bridge displacement response. Nevertheless, the GPS monitoring displacement data for the in-service Tsing Ma suspension bridge are induced by a combination of environmental and operational loadings, which include wind, temperature, and highway and railway traffic. In this connection, the bridge displacement response data recorded by the GPS during the period from 2002 to 2005 are collected, along with the temperature and vehicle flow data from the temperature sensors and weigh-in-motion sensors, respectively. Several algorithms are developed using MATLAB as a platform to pre-process the GPS measurement data for producing high quality databases. An identification method is subsequently developed to extract wind-induced displacement response by eliminating temperature- and traffic- (highway and railway) induced displacements and GPS background noise from the measured total displacement response. The relationship between wind speed and wind-induced displacement response in the lateral and vertical directions is finally explored according to wind direction and the locations of GPS stations on the bridge deck. The results show that the magnitude of displacement response varies with wind direction, and the location of GPS stations. The relationship between wind speed and displacement response is almost quadratic in the lateral
However, the aforementioned relationships are limited to the locations where GPS receivers are installed. In addition, the maximum wind speed encountered by the bridge and measured so far is also smaller than the design wind speed. Therefore, how to perform serviceability and strength assessments of the bridge is a challenging issue. This necessitates the integration of computer simulation of the bridge under the action of wind and WASHMS-based measurements. For this purpose, a complex structural health monitoring-based finite element model (FEM) for the Tsing Ma Bridge with significant bridge deck modeling features is used. The wind forces, composed of steady-state wind loads due to mean wind, buffeting forces due to turbulent wind, and self-excited forces due to interaction between wind and bridge motion, are then generated and distributed over the bridge deck surface following a series of procedures. The displacement responses of the bridge are then computed and compared with the responses measured from the field. The comparison is found to be satisfactory in general. The statistical relationship predicted from the field at the mid-main span is thus extended to extreme wind speeds and other locations on the bridge deck through computer simulation. The results are finally compared with measurements data from wind tunnel tests and the allowable movements of the bridge under the given limit state for serviceability assessment. The outcomes demonstrate that the lateral and vertical displacements follow the designed pattern and the bridge functions properly when the bridge is subjected to strong winds.

As the span length of a cable-supported bridge increases, the bridge may suffer considerable buffeting-induced vibration across a wide range of wind speeds over almost the whole design life of the bridge. The frequent occurrence of buffeting responses of a relatively large amplitude may result in serious fatigue damage to steel structural components and connections, and may subsequently lead to catastrophic failure. Therefore, in addition to assessing its serviceability, it is also imperative that the strength of a long-span cable-supported bridge under high winds be assessed on the basis of wind-induced stress/strain analysis. However, the number of sensors available to take strain measurements on the Tsing Ma Bridge is limited. It is not possible to monitor all the stress responses of all the local components directly. In this connection, the complex structural health monitoring-based FEM which replicates the geometric details of the as-built complicated bridge deck, is used to compute wind-induced stress in all the bridge components and identify critical steel members. The wind-induced
stresses of the critical members are then linked to the wind-induced displacement response at the mid-main span through the hybrid use of the GPS measured displacements and FEM analyses. The wind-induced stresses derived at extreme wind speeds are then compared with the yield stress of the steel material to assess the strength of the bridge. The results demonstrate that the mean stresses, stress standard deviations, and total stress responses vary quadratically with mean displacement, displacement standard deviation, and total displacement, respectively. The outcomes also demonstrate that the strength of the Tsing Ma Bridge under strong winds is guaranteed.
Publications

SCI Journal Papers:


International Conference Papers:


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<tr>
<td>C/A code</td>
<td>coarse/acquisition code</td>
</tr>
<tr>
<td>CFD</td>
<td>computational fluid dynamics</td>
</tr>
<tr>
<td>CNSS</td>
<td>Compass Navigation Satellite System</td>
</tr>
<tr>
<td>DGPS</td>
<td>differencing GPS</td>
</tr>
<tr>
<td>DoD</td>
<td>Department of Defense</td>
</tr>
<tr>
<td>DOP</td>
<td>dilution of precision</td>
</tr>
<tr>
<td>EFEC</td>
<td>Earth-fixed-Earth-centred</td>
</tr>
<tr>
<td>EMD</td>
<td>empirical mode decomposition</td>
</tr>
<tr>
<td>ESD</td>
<td>extreme studentized deviate</td>
</tr>
<tr>
<td>FIR</td>
<td>finite-duration impulse response</td>
</tr>
<tr>
<td>FEM</td>
<td>finite element model</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>GLONASS</td>
<td>Global Orbiting Navigation Satellite System</td>
</tr>
<tr>
<td>GNSS</td>
<td>Global Navigation Satellite Systems</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>GPS-OSIS</td>
<td>Global Positioning System-On-Structure Instrumentation System</td>
</tr>
<tr>
<td>HDOP</td>
<td>horizontal dilution of precision</td>
</tr>
<tr>
<td>HKO</td>
<td>the Hong Kong Observatory</td>
</tr>
<tr>
<td>HKSAR</td>
<td>Hong Kong Special Administrative Region</td>
</tr>
<tr>
<td>Acronym</td>
<td>Definition</td>
</tr>
<tr>
<td>---------</td>
<td>------------</td>
</tr>
<tr>
<td>HKT</td>
<td>Hong Kong Time</td>
</tr>
<tr>
<td>HyD</td>
<td>the Highways Department</td>
</tr>
<tr>
<td>IMF</td>
<td>intrinsic mode functions</td>
</tr>
<tr>
<td>JNS</td>
<td>Javad Navigation Systems</td>
</tr>
<tr>
<td>LMS</td>
<td>least mean squares</td>
</tr>
<tr>
<td>M-code</td>
<td>military code</td>
</tr>
<tr>
<td>MCB</td>
<td>main circuit breaker</td>
</tr>
<tr>
<td>MEDELL</td>
<td>multipath estimating delay lock loop</td>
</tr>
<tr>
<td>MET</td>
<td>multipath elimination technology</td>
</tr>
<tr>
<td>MSAR</td>
<td>Macau Special Administrative Region</td>
</tr>
<tr>
<td>P-code</td>
<td>precision code</td>
</tr>
<tr>
<td>pcu/ln/hr</td>
<td>private car unit/lane/hour</td>
</tr>
<tr>
<td>PDF</td>
<td>probability density function</td>
</tr>
<tr>
<td>PDOP</td>
<td>position dilution of precision</td>
</tr>
<tr>
<td>ppm</td>
<td>parts per million</td>
</tr>
<tr>
<td>PRC</td>
<td>People’s Republic of China</td>
</tr>
<tr>
<td>PRC</td>
<td>pseudo-range correction</td>
</tr>
<tr>
<td>PSD</td>
<td>power spectral density</td>
</tr>
<tr>
<td>RD</td>
<td>relative discrepancies</td>
</tr>
<tr>
<td>RLS</td>
<td>recursive least squares</td>
</tr>
<tr>
<td>RMS</td>
<td>root-mean-square</td>
</tr>
<tr>
<td>RRC</td>
<td>range rate correction</td>
</tr>
<tr>
<td>RTK</td>
<td>real-time kinematic</td>
</tr>
<tr>
<td>SHMS</td>
<td>structural health monitoring systems</td>
</tr>
<tr>
<td>SLS</td>
<td>serviceability limit state</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>TFR</td>
<td>traffic flow rate</td>
</tr>
<tr>
<td>UTM</td>
<td>Universal Traverse Mercator</td>
</tr>
<tr>
<td>VDOP</td>
<td>vertical dilution of precision</td>
</tr>
<tr>
<td>WASHMS</td>
<td>Wind And Structural Health Monitoring System</td>
</tr>
<tr>
<td>WGS84</td>
<td>World Geodetic System 1984</td>
</tr>
<tr>
<td>WIFS</td>
<td>wavelet instantaneous frequency spectra</td>
</tr>
</tbody>
</table>
### List of Notations

<table>
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<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>$A$</td>
<td>amplitude of direct satellite signal</td>
</tr>
<tr>
<td>$A_{\text{max}}$</td>
<td>average maximum peak displacement</td>
</tr>
<tr>
<td>$A_{\text{min}}$</td>
<td>average minimum peak displacement</td>
</tr>
<tr>
<td>$A^*_{\psi}$</td>
<td>non-dimensional flutter derivatives, $\psi = 1 - 6$</td>
</tr>
<tr>
<td>$B$</td>
<td>width of the bridge deck segments</td>
</tr>
<tr>
<td>$C$</td>
<td>global structural damping matrix</td>
</tr>
<tr>
<td>$\bar{C}$</td>
<td>generalized damping matrix</td>
</tr>
<tr>
<td>$C_D$</td>
<td>static aerodynamic drag coefficient</td>
</tr>
<tr>
<td>$C_D'$</td>
<td>slope of $C_D$ at angle $\alpha$</td>
</tr>
<tr>
<td>$C_L$</td>
<td>static aerodynamic lift coefficient</td>
</tr>
<tr>
<td>$C_L'$</td>
<td>slope of $C_L$ at angle $\alpha$</td>
</tr>
<tr>
<td>$C_M$</td>
<td>static aerodynamic moment coefficient</td>
</tr>
<tr>
<td>$C_M'$</td>
<td>slope of $C_M$ at angle $\alpha$</td>
</tr>
<tr>
<td>$C_{l^\psi}$</td>
<td>dimensionless coefficients</td>
</tr>
<tr>
<td>$D$</td>
<td>elastic matrix</td>
</tr>
<tr>
<td>$D$</td>
<td>true distance between the satellite $s_{gps}$ and receiver $r_{gps}$</td>
</tr>
<tr>
<td>$\bar{D}$</td>
<td>mean displacement</td>
</tr>
<tr>
<td>$\bar{D}$</td>
<td>total displacement</td>
</tr>
<tr>
<td>$D_{\text{max}}$</td>
<td>peak displacement</td>
</tr>
</tbody>
</table>
\( D_{r_{gps}}^{s_{gps}} \)  pseudo-range between the satellite \( s_{gps} \) and receiver \( r_{gps} \)

\( d_{y}^{w} \)  dimensionless coefficients

\( E \)  elevation angle of the satellite

\( E_{o} \)  Easting of projection origin

\( E_{r_{gps}} \)  Easting of GPS receiver in UTM Grid coordinate system

\( E_{ref} \)  Easting of GPS reference receiver in UTM Grid coordinate system

\( E_{rov} \)  Easting of GPS rover receiver in UTM Grid coordinate system

\( e \)  estimation error

\( e^{2} \)  first eccentricity of reference ellipsoid

\( e_{lower} \)  lower envelope of \( x(t) \)

\( e_{upper} \)  upper envelope of \( x(t) \)

\( F^{bf} \)  modal buffeting forces

\( F^{bf}_{e} \)  equivalent buffeting forces

\( F^{se} \)  modal self-excited forces

\( F^{se}_{e} \)  equivalent self-excited forces

\( F^{sf} \)  modal static forces

\( F^{sf}_{e} \)  equivalent static wind forces

\( F^{t} \)  time-dependent part of wind force

\( f \)  flattening of the reference ellipsoid

\( f_{dom} \)  frequency generated by the motion simulation table

\( f^{bf}_{eD} \)  equivalent buffeting drag

\( f^{bf}_{eL} \)  equivalent buffeting lift

\( f^{bf}_{eM} \)  equivalent buffeting moment

\( f^{se}_{eD} \)  equivalent self-excited drag
\( f_{eL}^{se} \) equivalent self-excited lift
\( f_{eM}^{se} \) equivalent self-excited moment
\( f_{eD}^{sf} \) equivalent static drag
\( f_{el}^{sf} \) equivalent static lift
\( f_{eM}^{sf} \) equivalent static moment
\( f_m \) frequency of multipath
\( f_s \) sampling frequency

\( H_{gps} \) ellipsoidal height of geodetic position of GPS receiver
\( H_{ref} \) Height of GPS reference receiver in UTM Grid coordinate system
\( H_{rov} \) Height of GPS rover receiver in UTM Grid coordinate system
\( H_{r\psi}^* \) non-dimensional flutter derivatives, \( \psi = 1 - 6 \)
\( h \) vertical distance between the antenna and the ground
\( h_c \) vertical coordinate of the centroid
\( h_e \) equivalent vertical displacement
\( \bar{I}(\omega) \) impulse function in frequency domain
\( l_a \) turbulence intensity, \( a = u,v,w \)
\( l_{\psi} \) impulse function of self-excited force
\( I_r^{s} \) ionospheric error
\( i \) imaginary part
\( K \) stiffness matrix
\( \bar{K} \) generalized stiffness matrix
\( K \) reduced frequency
\( L \) differential operator
\( L \) span length
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$L_a^x$</td>
<td>integral scale of turbulence $a = u, v, w$</td>
</tr>
<tr>
<td>$l$</td>
<td>length of the element</td>
</tr>
<tr>
<td>$M$</td>
<td>global structural mass matrix</td>
</tr>
<tr>
<td>$\mathbf{M}$</td>
<td>generalized mass matrix</td>
</tr>
<tr>
<td>$M$</td>
<td>length of the adaptive FIR filter</td>
</tr>
<tr>
<td>$M$</td>
<td>number of data points</td>
</tr>
<tr>
<td>$m_o$</td>
<td>scale factor on central meridian</td>
</tr>
<tr>
<td>$m$</td>
<td>mean of upper and lower envelopes of $x(t)$</td>
</tr>
<tr>
<td>$N$</td>
<td>shape function</td>
</tr>
<tr>
<td>$N^{se}$</td>
<td>displacement transformation matrix</td>
</tr>
<tr>
<td>$N_{a_{gps}}$</td>
<td>integer ambiguity</td>
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<tr>
<td>$N_e$</td>
<td>number of IMF components</td>
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<td>$N_f$</td>
<td>total number of frequency interval</td>
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<tr>
<td>$N_n$</td>
<td>total number of nodes</td>
</tr>
<tr>
<td>$N_o$</td>
<td>Northing of projection origin</td>
</tr>
<tr>
<td>$N_{r_{gps}}$</td>
<td>Northing of GPS receiver in UTM Grid coordinate system</td>
</tr>
<tr>
<td>$N_{ref}$</td>
<td>Northing of GPS reference receiver in UTM Grid coordinate system</td>
</tr>
<tr>
<td>$N_{rov}$</td>
<td>Northing of GPS rover receiver in UTM Grid coordinate system</td>
</tr>
<tr>
<td>$N_s$</td>
<td>total number of sections</td>
</tr>
<tr>
<td>$N_{sgps}$</td>
<td>total number of satellites in constellation</td>
</tr>
</tbody>
</table>

xxxiii
\[ N_{\text{sat}} \quad \text{number of satellites} \]
\[ n_p \quad \text{total number of points where wind speeds are simulated} \]
\[ \Omega^z \quad \text{orbital error} \]
\[ P^m(s_i) \quad \text{mean wind pressure distribution} \]
\[ P^\psi \quad \text{non-dimensional flutter derivatives, } \psi = 1 - 6 \]
\[ p_i \quad \text{unknown variables, } i = 1, 2, 3 \]
\[ \mathbf{q} \quad \text{generalized displacement vector} \]
\[ \mathbf{Q}^{bf} \quad \text{generalized buffeting force} \]
\[ \mathbf{Q}^{se} \quad \text{generalized self-excited force} \]
\[ Q \quad \text{GPS quality indicator} \]
\[ \mathbf{q} \quad \text{generalized displacement} \]
\[ q_e \quad \text{equivalent lateral displacement} \]
\[ q_c \quad \text{lateral coordinate of the centroid} \]
\[ \mathbf{r} \quad \text{reference signal vector} \]
\[ r(n) \quad \text{reference measurement} \]
\[ r_{gps} \quad \text{GPS receiver} \]
\[ r_{Ne} \quad \text{final residue} \]
\[ r_{ref} \quad \text{GPS reference station} \]
\[ r_{rov} \quad \text{GPS rover station} \]
\[ S \quad \text{total satellite signal} \]
\[ S_{aa} \quad \text{wind spectrum, } aa = vv, ww \]
\[ S_D \quad \text{direct satellite signal} \]
\[ S_F \quad \text{abnormal displacement data set} \]
\[ S_R \quad \text{indirect satellite signal} \]
$S_T$ mean vector of normal displacement data set

$s_{gps}$ satellite

$s(n)$ primary measurement

$s_l$ local element coordinate

$T$ time duration of wind speed record

$T^s_r$ tropospheric error

$t_r$ receiver clock error

$t^s$ satellite clock error

$\bar{U}$ mean wind speed

$U_E$ horizontal eastward wind component

$\bar{U}_E$ mean wind speed of horizontal eastward wind component

$\bar{U}_\text{max}$ maximum mean wind speed

$U_S$ horizontal southward wind component

$\bar{U}_S$ mean wind speed of horizontal southward wind component

$U_U$ vertical upward wind component

$\bar{U}_U$ mean wind speed of vertical upward wind component

$U^*$ friction velocities

$u$ alongwind turbulence component

$u_p$ alongwind component at the $p$th point

$\bar{u}\bar{w}$ co-variance of the longitudinal and vertical velocity fluctuation

$V$ volume of a body formed by the intersection points of site-satellite vectors

$V^\psi_l$ convolution integrations

$v$ crosswind turbulence component

xxxv
\( \mathbf{w} \) 
- tap weight vector

\( \mathbf{w} \)
- upward wind turbulence component

\( w_p \)
- upward wind component at the \( p \)th point

\( \mathbf{X} \)
- nodal displacement vector at global coordinate system

\( \bar{\mathbf{X}} \)
- mean displacement vector

\( X_{r\text{gps}} \)
- \( X \)- component of geocentric position of GPS receiver

\( X^s_{gps} \)
- \( X \)- component of geocentric position of satellite

\( x(n) \)
- noise-free signal in primary measurement

\( x(t) \)
- displacement response time history

\( x'(t) \)
- dynamic displacement component

\( \bar{x} \)
- mean displacement component

\( \bar{x}_d \)
- contaminated dynamic displacement

\( x_g \)
- statistics of signals measured by GPS

\( \bar{x}_s \)
- dynamic background noise

\( x_{std} \)
- standard deviation of an hourly record

\( x_t \)
- statistics of signals generated by motion simulation table

\( Y_{r\text{gps}} \)
- \( Y \)- component of geocentric position of GPS receiver

\( Y^s_{gps} \)
- \( Y \)- component of geocentric position of satellite

\( Z_{r\text{gps}} \)
- \( Z \)- component of geocentric position of GPS receiver

\( Z^s_{gps} \)
- \( Z \)- component of geocentric position of satellite

\( \alpha \)
- effective wind angle of attack

\( \alpha^0 \)
- initial mean angle of attack
\( \beta \) damping factor

Aerodynamic transfer functions between fluctuating wind velocities and buffeting forces, \( i = D_{bu}, D_{bw}, L_{bu}, L_{bw}, M_{bu}, M_{bw} \)

\( \Delta s \) extra pathlength

\( \Delta t \) time interval

\( \Delta \phi \) fraction of a cycle

\( \Delta \phi_m \) phase shift of the reflected signal relative to direct signal phase

\( \Delta \omega \) frequency interval

\( \delta(t) \) Dirac delta function

\( \varepsilon \) least squares error

\( \varepsilon_r \) combination of receiver noise and multipath error

\( \Phi \) mode shape matrix

\( \phi \) latitude

\( \phi_o \) latitude of projection origin

\( \phi_{gps} \) latitude of geodetic position of GPS receiver

\( \Gamma \) modal stress

\( \varphi \) phase of the direct signal

\( \varphi_{im} \) random variable uniformly distributed between 0 and 2\( \pi \)

\( \lambda \) wavelength

\( \lambda_o \) longitude of projection origin

\( \lambda_{gps} \) longitude of geodetic position of GPS receiver

\( \mu \) step-size parameter

\( \theta \) additional attack angle
\( \theta_e \) equivalent torsional displacement

\( \rho \) pseudo-range measurement

\( \rho_a \) air density

\( \bar{\sigma} \) mean stress

\( \sigma \) total stress

\( \sigma_a \) standard deviation of fluctuating wind speed, \( a = u, v, w \)

\( \sigma_D \) displacement standard deviation

\( \bar{\sigma}_u \) mean value of standard deviation of alongwind fluctuating wind speed component

\( \bar{\sigma}_v \) mean value of standard deviation of crosswind fluctuating wind speed component

\( \bar{\sigma}_w \) mean value of standard deviation of vertical fluctuating wind speed component

\( \sigma_W \) standard deviation of wind-induced dynamic displacement

\( \sigma_{WT} \) standard deviation of total dynamic displacement

\( \sigma_\sigma \) standard deviation of stress

\( \nu \) reduced mean wind velocity

\( \nu(n) \) noise signal in primary measurement

\( \nu'(n) \) noise signal in reference measurement

\( \nu \bar{\nu} \bar{w} \) co-variance of the longitudinal and vertical velocity fluctuation

\( \omega_{up} \) upper cutoff frequency

\( \omega \) circular frequency of vibration

\( \sigma_c \) designated frequency
$\psi_s$  isometric latitude
Chapter 1

INTRODUCTION

1.1. RESEARCH MOTIVATION

Many long-span cable-supported bridges have been built across the world in the past two decades to meet the economic, social, and recreational needs of communities. Some examples include the Akashi Kaikyo suspension bridge in Japan, which has a main span of 1,991 m, and the Sutong cable-stayed bridge in China, which has a main span of 1,088 m (see Figure 1.1). Increasing numbers of long-span cable-supported bridges are expected to be constructed in the coming years. For instance, the construction of the Hong Kong-Zhuhai-Macau Bridge, which has long been promoted by the governments of the People’s Republic of China (PRC), the Macau Special Administrative Region (MSAR), and the Hong Kong Special Administrative Region (HKSAR), is expected to commence no later than 2010 (Hong Kong Special Administrative Region Government 2008), and will bestow significant socio-economic benefits in the Greater Pearl River Delta region. When completed in 2015-2016, the 29 km long bridge will be one of the longest spanned bridges in Asia.

However, the increased span length of modern cable-supported bridges significantly raises their buffeting response, causes greater fatigue damage to structural components and connections, affects the stability of vehicles traveling
on the deck, and can make crossing uncomfortable for pedestrians (Zhu 2002). This situation is likely to be amplified for long-span cable-supported bridges located in wind-prone areas. It is now well known that air temperatures are gradually increasing due to the greenhouse effect, which has resulted in stronger and more frequent typhoons (Xu 2008a). It is thus a challenge in typhoon regions to construct long-span cable-supported bridges that function properly during their long service lives and do not suffer catastrophic failure under strong winds.

Recently developed structural health monitoring technology offers good solutions to some of these problems. Structural health monitoring technology is based on comprehensive sensory systems and sophisticated data-processing systems that use advanced information technology and are supported by bespoke computer algorithms. The main objectives of structural health monitoring are to monitor the loading conditions of a structure, assess the performance of the structure under various service loads, verify or update the rules used in the design stage, detect damage or deterioration, and guide inspection and maintenance. In Hong Kong and mainland China, comprehensive structural health monitoring systems (SHMS) with a variety of sensors have been installed in more than 40 long-span bridges (Xu 2008a).

Of all of the responses of long-span cable-supported bridges measured by SHMS, displacement is the most important parameter in assessing integrity and safety, because displacements that deviate from the designed geometrical configurations redistribute the stresses and strains in the bridge components and affect the load-carrying capacity of the entire bridge (Wong et al. 2001a; 2001b). However,
before the deployment of global positioning systems (GPS), the wind-induced total displacement response of long-span cable-supported bridges, which includes a static component and a dynamic fluctuating component, was difficult to measure using traditional sensors. For instance, traditional displacement transducers can only be used to measure relative displacement (Ko & Ni 2005), and laser transducers and total stations have proved unsuitable for the long-term monitoring of long-span bridges due to line-of-sight issues. Level-sensing stations allow the real-time monitoring of displacements at typical stiffening deck sections, but can only monitor the static component (Wong et al. 2001a; 2001b). Clearly, a sensor that can effectively and accurately measure the wind-induced total displacement of a bridge is necessary and imperative to improve the efficiency and accuracy of displacement monitoring in SHMS. In recent decades, the application of GPS has been extended from its original navigational function to encompass geodesy and surveying, and due to its global coverage and continuous operation under all metrological conditions it has become a useful tool for measuring both the static and dynamic displacement responses of long-span cable-supported bridges subjected to winds. Typical examples of bridges assessed using GPS include the Humber Bridge in England (Ashkenazi & Roberts 1997), the Akashi-Kaikyo Bridge in Japan (Fujino et al. 2000; Toriumi et al. 2000; Kashima et al. 2001; Miyata et al. 2002), and the Tsing Ma Bridge in Hong Kong (Wong et al. 2001a, 2001b).

However, the accuracy of displacement measurement using GPS depends on many factors, such as the data sampling rate, satellite coverage, atmospheric effects, the multipath effect, and the GPS data-processing method used. It is
therefore questionable whether GPS can accurately measure displacement to submillimeter levels. In addition, given the significant progress in hardware technology recently, there is an urgent need to conduct a broad assessment of new types of GPS receivers and to carry out calibrations to verify whether GPS is a suitable technology for monitoring long-span cable-supported bridges. In recent years, several calibration programs have been conducted to assess the dynamic performance of GPS using motion stimulators as a test station (Tamura et al. 2002; Kijewski-Correa 2003; Nickitopoulou et al. 2006). As the technology being tested was intended for the monitoring of buildings, only horizontal uni-axial motions and bi-axial circular motions were assessed in these programs. However, in long-span bridge health monitoring it is important to measure the wind-induced dynamic displacement response of the bridge deck in the vertical plane and of the bridge towers in the horizontal plane. The fundamental frequency of the bridge in one direction, which is much lower than that of buildings, may not be the same as that in the other direction. Therefore, an advanced motion simulation table that can simulate various types of two-dimensional motions of bridge decks in the vertical plane and bridge towers in the horizontal plane at frequencies that are common in bridges is urgently required.

Although the feasibility of applying GPS in deflection monitoring has been ascertained from a number of field tests, it has also been highlighted that GPS is unable to capture the higher-mode natural frequencies of bridges due to the low sampling frequency of the receivers (Roberts et al. 2004). To overcome this problem, it has been suggested that the GPS signals be integrated with those from
another sensor for better structural deformation monitoring. As accelerometers cannot reliably measure static and low-frequency structural responses but can accurately measure high-frequency structural responses, measurement signals from accelerometers have been used to complement the GPS signals (Roberts et al. 2000, 2004; Li 2004; Li et al. 2006a, 2006b). However, the displacement values obtained by the integration algorithms proposed by Roberts et al. (2000, 2004) involve the dynamic component only, and although Li (2004) successfully isolated the static and quasi-static components from GPS data, the accuracy of GPS, particularly in the low-frequency range, is greatly reduced by the effects of multipath interference (Ray et al. 2001; Kochly & Kijewski-Correa 2006). Therefore, an advanced data-processing technique is needed to derive a noise-free total (static plus dynamic) displacement signal from the GPS and the accelerometer.

The use of GPS in bridge health monitoring clearly shows great promise, yet how to take full advantage of GPS data and SHMS to continuously monitor the serviceability and strength of long-span cable-supported bridges under strong winds remains a challenge to professionals. The Tsing Ma Bridge in Hong Kong is the longest suspension bridge in the world, carrying both highway and railway. As the bridge is located in one of the most active typhoon regions in the world, a Wind And Structural Health Monitoring System (WASHMS) that comprises 6 anemometers and 14 GPS stations has been in full operation since 2002 to ensure that the bridge functions properly during its long service life and is prevented from suffering catastrophic failure under strong winds. This full-scale and long-term monitoring campaign offers an excellent opportunity for assessing the
effect of wind on bridges by determining the statistical relationship between wind and the wind-induced bridge displacement response.

In the full-scale absolute displacement monitoring of long-span cable-supported bridges, researchers and engineers are confronted with the problem of a limited number of sensors that may not be installed in the same positions as the structural defects or degradation. In addition, because the displacement measured is limited to the wind speeds and directions measured, researchers and engineers cannot predict the risks to long-span cable-supported bridges exposed to very high wind speeds, which makes the satisfactory assessment of bridges under high winds difficult. It would thus be interesting to research how the limited GPS sensors in SHMS could be used to predict the wind-induced displacement response of an entire long-span cable-supported bridge under extreme wind speeds and at different wind skew angles. The rapid development of computer technology and computational methods has allowed the development of an analytical method for predicting the buffeting and flutter instability response in long suspension bridges in the frequency domain (Davenport 1962; Scanlan 1978) and the time domain (Bucher & Lin 1988; Xiang et al. 1995). In addition, a sophisticated finite element model (FEM) formulated using the full truss girder modeling approach has been developed in recent years that appears to be appropriate for modeling the global and local structural characteristics of a long-span cable-supported bridge. These studies indicate that it would be worthwhile to integrate computer simulation with GPS measurement to predict the wind-induced displacement response of an entire long-span cable-supported bridge under extreme wind speeds.
As the span length of cable-supported bridges increases, they suffer considerable buffeting-induced vibration that occurs over a wide range of wind speeds and lasts for almost the whole design life of the bridge. The frequent occurrence of a buffeting response of relatively large amplitude may result in serious fatigue damage to the steel structural components and connections of the bridge, which could lead to catastrophic failure. The accurate assessment of the wind-induced stresses and strains of a long-span cable-supported bridge is thus imperative. Ideally, strain measurement sensors would be installed in all of the members of a long-span cable-supported bridge, but in practice the number of sensors is always limited. To counteract this limitation, member stress evaluation using a hybrid of FEM analysis and GPS may be an appropriate method of improving the health monitoring of long-span cable-supported bridges exposed to high winds.

1.2. OBJECTIVES

This thesis focuses on the application and integration of global positioning systems (GPS) with structural health monitoring systems (SHMS) and computer simulation to continuously monitor the serviceability and strength of long-span cable-supported bridges under strong winds. The main objectives and originalities of the thesis are as follows.

1. To assess the displacement measurement accuracy of GPS in three orthogonal directions for application in long-span cable-supported bridges. For this purpose, an advanced motion simulation table will be developed as a test station to simulate various types of two-dimensional motions of towers in the horizontal plane and bridge decks in the vertical plane. The validation of the
performance of GPS will be accomplished by comparing the original motions with those generated by the motion simulation table.

2. To enhance the measurement accuracy of the total (static plus dynamic) displacement response of civil engineering structures by integrating GPS-measured signals with accelerometer-measured signals. For this purpose, integrated data-processing techniques that use both empirical mode decomposition (EMD) and an adaptive filter will be developed. A series of motion simulation tests that simulate various types of motion around a pre-defined static position will be performed on site to assess the effectiveness of the proposed integrated data-processing techniques.

3. To assess the effects of wind on in-service long-span suspension bridges by determining the statistical relationship between wind and the wind-induced displacement response. To this end, an identification method will be developed to ascertain the wind-induced displacement response of Tsing Ma suspension bridge, after first evaluating the effects of the environmental and operational loads such as traffic loads. The statistical relationships between wind speed and the wind-induced displacement response will be explored for different wind directions and different locations of the bridge deck.

4. To perform an effective and reliable serviceability assessment of in-service long-span cable-supported bridges exposed to wind by integrating computer simulation with GPS measurement. In this regard, a complex structural health monitoring finite element model for the Tsing Ma Bridge that includes the
significant features of the bridge deck will be applied. The statistical relationships predicted in the field at the mid-main span of the bridge will then be extended to extreme wind speeds and other locations of the bridge deck using FEM analysis. The results will then be compared with the responses measured during wind tunnel tests and the limit state displacements determined at the design stage of the bridge.

5. To perform a strength assessment of a long-span cable-supported bridge under high winds based on an analysis of the wind-induced stresses. For this purpose, the statistical relationship between wind and the wind-induced displacement response will be extended to the stress level to allow comparison with the load-carrying capacity of the bridge. The statistical relationship between the wind-induced stresses and the wind-induced displacement response will then be developed using a hybrid of FEM analysis and GPS.

1.3. ASSUMPTIONS AND LIMITATIONS

The application of GPS for monitoring long-span cable-supported bridges under high winds is subject to the following assumptions and limitations.

1. Currently, there are two Global Navigation Satellite Systems (GNSSs) in operation: the global positioning system (GPS) owned by the United States and the Global Navigation Satellite System (GLONASS) of the Russian Federation. As GPS has been fully used for over a decade and the receivers available on the market mainly support only GPS signals, GPS is selected for
2. Integrated data-processing techniques based on measurement data collected by GPS receivers and accelerometers using EMD and an adaptive filter have been developed to enhance the measurement accuracy of the total displacement response of civil engineering structures. However, in view as the Tsing Ma Bridge is very slender, the most majority of wind-induced displacement responses with a frequency less than or equal to 1 Hz have already been well captured by GPS. In addition, because the accelerometers and GPS receivers are positioned at different locations on the Tsing Ma Bridge, the proposed data-processing technique cannot be applied in this case.

3. Acceleration is another important variable in assessing the serviceability performance of long-span cable-supported bridges under the action of wind. The statistical relationship between wind speed and the wind-induced acceleration of the Tsing Ma Bridge has been investigated, but as the main focus of this research work is the application of GPS for the monitoring of long-span cable-supported bridges under strong winds, the information related to acceleration is not presented in this thesis.

4. The displacement responses measured by GPS for the in-service Tsing Ma Bridge are induced by a combination of environmental and operational loadings. To develop an identification method for extracting the wind-induced displacement response from the measured total displacement
response, the various displacement responses of the bridge to moving vehicles, trains, and wind are assumed to be uncorrelated.

5. The accuracy of GPS is affected by a number of factors, one of which is the multipath effect. To assess the influence of this effect on the GPS data measured on the Tsing Ma Bridge, a calibration test using the two-dimensional motion simulation table will be carried out. However, as calibration must be carried out in a stationary place that is not influenced by moving vehicles and trains, the test will be carried out on ground next to the Tsing Ma Bridge.

6. Because of time and resource limitations, wind tunnel tests cannot be performed to obtain the wind pressure distribution for the whole Tsing Ma Bridge deck. As such, the wind pressure distribution of other suspension bridges with similar sections is adopted and assumed to be the same as that experienced by the Tsing Ma Bridge deck. The fluctuating wind pressure for the whole bridge deck section is assumed to be linearly related to the fluctuating wind speeds.

7. The results from the finite element analysis are used to extend the measured statistical relationships between wind and the wind-induced displacement response. Only wind coming in at right angles to the longitudinal axis of the bridge deck is considered in the analytical process.
1.4. THESIS LAYOUT

This thesis covers a variety of research topics to achieve the aforementioned objectives. It is divided into nine chapters and is organized in the following way.

- Chapter 1 gives a brief introduction of the research motivation and objectives, the assumptions and limitations, and the layout of the thesis.

- Chapter 2 presents an extensive literature review of four relevant topics. The fundamentals of GPS are first outlined to provide an understanding of GPS concepts. The second section presents an in-depth survey of the application of GPS to civil engineering structures. The third section explains the effects of wind on long-span cable supported bridges. The final section reviews the wind and structural health monitoring of the Tsing Ma Bridge to outline the need for this research and the challenges created by the combination of computation simulation and SHMS.

- Chapter 3 presents an experimental investigation of the dynamic performance of GPS in monitoring long-span cable-supported bridges. An advanced motion simulation table that can simulate various types of two-dimensional motions of the bridge towers in the horizontal direction and the bridge deck in the vertical direction is designed and presented. Performance tests, both static and dynamic, are then conducted in an open area using a motion simulation table and GPS. A band-pass filtering scheme is presented and applied to the table motion data recorded by GPS based on an analysis of the background noise recorded in the static tests.
Finally, the dynamic measurement accuracy of GPS in the horizontal and vertical planes is assessed and described by comparing the table motion recorded by GPS with the original motion generated by the table.

- Chapter 4 focuses on the possibility of integrating GPS signals with accelerometer signals to enhance the accuracy of the measurement of the total (static plus dynamic) displacement response of civil engineering structures. Integrated data-processing techniques using both EMD and an adaptive filter are presented, and a series of motion simulation table tests performed on site using three GPS receivers, one accelerometer, and one motion simulation table are reported. The proposed data-processing techniques are then applied to the recorded GPS and accelerometer data to find the static and dynamic displacements. The effectiveness of the integrated method is assessed by comparing the integrated results with the original motion generated by the motion simulation table.

- Chapter 5 presents extreme wind studies on the Tsing Ma Bridge using wind data measured during the period 1997 to 2005. The main features of the Tsing Ma Bridge are introduced in the chapter, and the anemometers in the WASHMS installed on the Tsing Ma Bridge and the terrain surrounding the bridge are concisely described. To remove unreasonable and undesirable data, some pre-processing steps for the wind data are designed and presented. The recorded wind data, which is categorized as either typhoon or monsoon, is then analyzed to evaluate the mean wind speed, mean wind direction, turbulence components, turbulence intensity,
integral length scale, and wind spectra for time intervals of 10-minutes and 1-hour in 16 cardinal directions.

- Chapter 6 focuses on the effects of wind on the bridge by determining the statistical relationship between wind and the wind-induced bridge displacement response using wind and displacement measurement data recorded during the period 2002 to 2005. The locations of the GPS receivers used in the WASHMS installed on the Tsing Ma Bridge deck are described and a series of data pre-processing techniques are then presented and applied to the GPS data in an attempt to produce a high-quality database. Attention is then placed on assessing the influences of GPS background noise, temperature, and moving vehicles and trains on the static and dynamic displacement measurements taken by the GPS sensors. An approximate identification method is then developed to extract the wind-induced displacement response from the measured total displacement. The relationship between wind speed and wind-induced displacement response in the lateral and vertical directions is then explored for various wind directions and locations on the bridge deck using GPS.

- Chapter 7 presents an effective and reliable serviceability assessment of the Tsing Ma Bridge carried out by integrating a computer simulation of the bridge under the action of wind and the measurements taken by the WASHMS. A complex structural health monitoring finite element model (FEM) for the Tsing Ma Bridge that includes significant features of the
The wind forces, which comprise steady-state wind loads, buffeting forces, and self-excited forces, are then generated using the wind characteristics measured in the field and the aerodynamic and flutter coefficients measured in the wind tunnel tests. The displacement response of the bridge is computed and compared with the responses measured in the field. The statistical relationship predicted from the field data at the mid-main span is then extended to extreme wind speeds and other locations of the bridge deck using FEM analysis to allow a comparison of the results from the wind tunnel testing and the most tolerant movements of the bridge under given limit state.

- Chapter 8 focuses on the strength assessment of the Tsing Ma Bridge under high winds based on wind-induced stress analysis. The statistical relationship between wind and the wind-induced displacement response is first extended to the stress level to compare the load-carrying capacity of the bridge. The statistical relationship between the wind-induced stresses at the critical bridge components and the wind-induced displacement response at the mid-main span is then developed using a hybrid of FEM analysis and GPS.

- Chapter 9 summarizes the key results from the study and draws some conclusions on the application and integration of GPS with SHMS and computer simulation for monitoring the serviceability and strength of long-span cable-supported bridges under high winds. Some recommendations for future work are given at the end of the thesis.
(a) Akashi-Kaikyo Bridge (1,991 m)

(b) Sutong Bridge (1,088 m)

Figure 1.1. Examples of long-span cable-supported bridges
Chapter 2

LITERATURE REVIEW

As discussed in Chapter 1, this thesis seeks to integrate the global positioning system (GPS), a structural health monitoring system (SHMS) and computer simulation to monitor long-span cable-supported bridges under high winds. The fundamentals of GPS are first outlined to provide for an understanding of GPS concepts. An in-depth survey of the application of GPS technology to civil engineering structures including tall buildings and long-span bridges is then reviewed. Because this thesis mainly focuses on the behaviour of long-span cable-supported bridges under wind action, a review of the existing literature on the effect of wind on bridges is then presented. This chapter concludes with a review of the wind and structural health monitoring of long-span cable-supported bridges in an effort to outline the necessity for and challenges faced in a combination of computational simulation and SHMS.

2.1. THE FUNDAMENTALS OF GLOBAL POSITIONING SYSTEMS

2.1.1. Global Navigation Satellite Systems

Global Navigation Satellite Systems (GNSSs) are space-based radio positioning systems that provide 24-hour, three-dimensional position, velocity, and time information to suitably equipped users on the surface of the Earth.
There are two GNSSs currently in operation: the global positioning system (GPS) owned by the United States, and the Global Orbiting Navigation Satellite System (GLONASS) of the Russian Federation. To reduce the dependence of users on the United States and Russian systems, a third system called Galileo is being developed by the European Union. In addition, China is planning to expand its existing regional satellite system called Beidou-1 into a truly global satellite navigation system to be known as the Compass Navigation Satellite System (CNSS). Each of these systems is described in the following subsections.

**Global Positioning System**

The global positioning system (GPS) developed by U.S. Department of Defence (DoD) in 1973 was originally designed to assist soldiers, military vehicles, planes, and ships (Sahin et al. 1999). The network is composed of 32 satellites in six orbital planes. Each satellite operates in circular 20,200 km orbits at an inclination angle of 55°, and each satellite completes an orbit in approximately 11 hours and 57.96 minutes (Hofmann-Wellenhof et al. 2008). The spacing of satellites in orbits is arranged so that at least six satellites are always within line of sight from any location on the Earth’s surface at all times (Hofmann-Wellenhof et al. 2001). Nowadays, GPS is the only system that is fully operational. However, to counter competition from systems such as GLONASS and Galileo, the United States has equipped some GPS satellites with a third navigation signal on top of the two already in place. This recent modification allows civilian users to achieve point-positioning accuracy equivalent to that of the military. In addition, the power of the signals has also
been increased so they can be easily received in unfavourable conditions (Meyer 2004).

**Global Orbiting Navigation Satellite System**

The Global Orbiting Navigation Satellite System (GLONASS) operated by the government of the Russian Federation is a dual use navigation system that provides positioning, navigation and time services. The complete GLONASS network is comprised of 24 satellites in three orbital planes. Each satellite, which orbits the Earth at an altitude of approximately 19,100 km with an inclination angle of 64.8°, completes a single orbit of the Earth in approximately 11 hours and 15.73 minutes (Hofmann-Wellenhof et al. 2008). GLONASS, which has a nominal 24 satellite constellation, was first completed in 1996, but the number of available satellites declined soon afterwards due to a lack of funding. A federal GLONASS program update released in 2006 indicated that GLONASS services would be fully operational by the end of 2009 (Hofmann-Wellenhof et al. 2008).

**Galileo**

Galileo is the global navigation satellite system being developed by the European Union and the European Space Agency. Galileo consists of a constellation of 30 satellites (27 operational plus 3 active spares) distributed in 3 orbital planes at an altitude of 23,616 km (some 3,000 km higher than GPS). Each plane is inclined 56° relative to the equator. The satellites complete an orbit in approximately 14 hours and 4.75 minutes. The cost of the Galileo program as a whole is in the range of EUR 3.4 billion. Galileo is due to be fully operational by 2013 (Hofmann-Wellenhof et al. 2008).
**Compass/Beidou-2**

The Beidou navigation system is a Chinese project aimed at developing an independent satellite navigation system. While the current Beidou-1 system, which consists of 3 satellites (Chen 2006), has limited coverage and application, the new generation system known as Compass or Beidou-2 will be a constellation of 35 satellites, including 30 medium Earth orbit satellites and 5 geostationary Earth orbit satellites, offering complete coverage of the globe. The 30 medium Earth orbit satellites, with an average altitude of 21,500 km, will be distributed in 3 orbital planes. Each plane is inclined 55° relative to the equator (China Satellite Navigation Project Center 2008). The first medium Earth orbit satellite was launched in 2007 (Selding 2008).

### 2.1.2. Principles of Satellite Positioning

The previous section highlights the important system parameters of the global satellite systems: GPS, GLONASS, Galileo, and Beidou-2. However, reference is being made to GPS for satellite positioning for two reasons. The first is that GPS has been fully utilised for over a decade now, and the second is that most receivers available on the market support only GPS signals. GPS has thus been selected as the global satellite system used for this research work. The principles underlying GPS are described in the following sections.

#### 2.1.2.1. Operation of GPS

As shown in Figure 2.1, GPS operates via a process of continuous coordination among the ground/control segment, the space segment, and the user segment. The ground-control segment of GPS is composed of six master control stations
and four ground antennae located around the globe (U.S. Air Force 2008). This segment does what its name implies – it tracks the orbital configuration of the satellites and updates satellite clock correction information. A further important function of the ground-control segment is to determine the orbit of each satellite and predict its path for the following 24 hours. The space segment of GPS consists of the 32 orbiting satellites. Each satellite, which is precisely timed by an atomic clock, continuously broadcasts two L-band carrier wave signals, referred to as L1 and L2, to satellite positioning users. The L1 carrier broadcast at 1,575.42 MHz has two codes modulated upon it: a C/A code and a P-code. The C/A code, with a frequency of 1.023 MHz and a wavelength of about 300 m, is accessible to all users. This code consists of a series of 1,023 binary chips that are unique to each satellite, enabling receivers to identify the origins of received signals. The P-code, which is modulated at 10.23 MHz and has a wavelength of about 30 m, is ten times more accurate than the C/A code for positioning (Wolf & Ghilani 2002). The P-Code is transmitted on both the L1 and L2 bands. However, since September 2005, a new military signal (M-code) on both the L1 and L2 channels and a more robust civil signal (L2C) on the L2 channel have also been broadcast by GPS Block IIR-M satellites (Engineering Surveying Showcase 2007; U.S. Air Force 2008). These signals are received by GPS receivers in the user segment and converted into three-dimensional spatial coordinate information with corresponding GPS time (Leica 1999).

### 2.1.2.2. Satellite Positioning Based on Distance Measurements

To determine the position of a satellite, the range $D_{r_{gps}}^{s_{gps}}$, which is the distance between satellite $s_{gps}$ and navigation satellite receiver $r_{gps}$, is first
approximated. Conceptually speaking, there are two ways to define a satellite-receiver range: according to code range measurement and according to phase measurement. Under the code ranging method, distance is determined at the receiver by counting the time delay for a sequence of codes transmitted from the satellite and an identical sequence of codes arriving at the receiver. The propagation time is then multiplied by the velocity of light (299,792,458 m/s) to obtain the range value (Mok & Retscher 2001). However, because carrier wavelengths are short (approximately 19 cm for L1 and 24 cm for L2) in comparison with C/A and P code wavelengths, better satellite range measurement accuracy is achieved by observing a phase of GPS signals (Rizos 1999). The range measured by a sinusoidal wave signal with wavelength \( \lambda \) from the satellite to the receiver can be expressed as \( N_{a_{gps}} \) cycles of \( \lambda \) plus the fraction of cycle \( \Delta \phi \), i.e. \( D^s_{r_{gps}} = \lambda \left( N_{a_{gps}} + \Delta \phi \right) \). Based on distance observations made to multiple satellites \((X_{s_{gps}}, Y_{s_{gps}}, Z_{s_{gps}})\), the position of an unknown receiver \((X_{r_{gps}}, Y_{r_{gps}}, Z_{r_{gps}})\) can be determined using the triangulation concept and the following equation in a constellation of \( N_{s_{gps}} \) satellites:

\[
D^s_{r_{gps}} = \sqrt{\left(X^s_{gps} - X_{r_{gps}}\right)^2 + \left(Y^s_{gps} - Y_{r_{gps}}\right)^2 + \left(Z^s_{gps} - Z_{r_{gps}}\right)^2}\]  \(2.1\)

where \( s_{gps} = 1, 2, \ldots, N_{s_{gps}} \). By inspection, a minimum of three satellites are required to determine a position estimate. However, because GPS receivers do not have perfectly accurate clocks in practice, an additional satellite is used to correct the receiver’s clock error (Hofmann-Wellenhof et al. 2001).
2.1.2.3. The Geodetic Coordinate System

In practice, the unknown receiver position \( \left( X_{\text{gps}}, Y_{\text{gps}}, Z_{\text{gps}} \right) \) computed in Equation (2.1) is the geocentric coordinate on the reference frame of the Earth-fixed-Earth-centred (EFEC) World Geodetic System 1984 (WGS84). However, using the coordinate in that form is inconvenient for surveyors. The main reasons are that (1) geocentric coordinates are extremely large values; (2) the axes are unrelated to the conventional directions of north, south, east, and west on the surface of the Earth; and (3) geocentric coordinates give no indication of relative elevations between points (Wolf & Ghilani 2002). For these reasons, the geocentric coordinates of any point \( \left( X_{\text{gps}}, Y_{\text{gps}}, Z_{\text{gps}} \right) \) are usually converted into its geodetic value in latitude \( \phi_{\text{gps}} \), longitude \( \lambda_{\text{gps}} \), and ellipsoidal height \( H_{\text{gps}} \).

The longitude \( \lambda_{\text{gps}} \) is first calculated as

\[
\lambda_{\text{gps}} = \tan^{-1}\left( \frac{Y_{\text{gps}}}{X_{\text{gps}}} \right)
\]

An approximate latitude \( \phi_o \) is then computed by

\[
\phi_o = \tan^{-1}\left[ \frac{Z_{\text{gps}}}{\sqrt{\left(1-e^2\right)X_{\text{gps}}^2 + Y_{\text{gps}}^2}} \right]
\]

where \( e^2 \) is the first eccentricity of the reference ellipsoid. By inputting this approximate figure into the following equation, an improved value for the latitude \( \phi \) is obtained.
\[
\phi = \tan^{-1}\left[\frac{Z_{\text{gps}} + \frac{a e^2}{\sqrt{1 - e^2 \sin^2 \phi_o}} \sin \phi_o}{\sqrt{X_{\text{gps}}^2 + Y_{\text{gps}}^2}}\right] \tag{2.4}
\]

where \(a\) is the semi-major axis of the reference ellipsoid. The calculation in Equation (2.4) is then repeated by replacing \(\phi_o\) with \(\phi\) until the change in \(\phi\) between iterations becomes negligible. The final value of \(\phi\) will be the latitude of the GPS receiver \(\phi_{\text{gps}}\).

The following formulae are then used to compute the geodetic height \(H_{\text{gps}}\) of the GPS receiver:

\[
H_{\text{gps}} = \cos^{-1}\left(\phi_{\text{gps}}\right) \sqrt{X_{\text{gps}}^2 + Y_{\text{gps}}^2} - \frac{a}{\sqrt{1 - e^2 \sin^2 \phi_{\text{gps}}}} \text{ for } \phi_{\text{gps}} < 45^\circ \tag{2.5a}
\]

\[
H_{\text{gps}} = \sin^{-1}\left(\phi_{\text{gps}}\right) Z_{\text{gps}} - \frac{a (1-e^2)}{\sqrt{1-e^2 \sin^2 \phi_{\text{gps}}}} \text{ for } \phi_{\text{gps}} \geq 45^\circ \tag{2.5b}
\]

Although the geodetic latitude \((\phi_{\text{gps}})\) and longitude \((\lambda_{\text{gps}})\) coordinates accurately represent the position of a point on the Earth’s surface, the engineer or researcher in civil engineering is not usually interested in the coordinates of terrestrial points; rather, plane coordinates are the preferred form of results. Hence, a map projection is required to transform a point in the geodetic coordinate on the ellipsoid into a point in northing and easting \((N_{\text{gps}}, E_{\text{gps}})\) on a plane. Projection formulae are used to allow users to convert the latitude and longitude into Universal Traverse Mercator (UTM) grid coordinates (Lands Department 1995):
\[ N_{gps} = N_o + m_o \left[ (M - M_o) + \nu_s \left( \sin \phi_{gps} \right) \left( \cos \phi_{gps} \right) \left( \frac{\lambda_{gps} - \lambda_o}{2} \right)^2 \right] \quad (2.6a) \]

\[ E_{gps} = E_o + m_o \left[ \nu_s \left( \frac{\lambda_{gps} - \lambda_o}{6} \right) \left( \cos^3 \phi_{gps} \right) \left( \psi_s - \frac{\tan^2 \phi_{gps}}{\psi_s} \right) \right] \quad (2.6b) \]

where

\[ N_o, E_o, \quad \lambda_o = \text{ northing, easting of projection origin} \]
\[ \lambda_o = \text{ longitude of projection origin} \]
\[ M = \text{ meridian distance measured from the Equator to the navigation satellite receiver } r_{gps} \]
\[ M_o = \text{ meridian distance measured from the Equator to origin of projection} \]
\[ m_o = \text{ scale factor on central meridian} \]
\[ \psi_s = \text{ isometric latitude} \]
\[ \nu_s = \text{ radius of curvature in the prime vertical} \]

The meridian distance, \( M \), is calculated by:

\[ M = a \left[ A'_0 \phi_{gps} - A'_2 \sin \left( 2 \phi_{gps} \right) + A'_4 \sin \left( 4 \phi_{gps} \right) \right] \quad (2.7) \]

where

\[ A'_0 = 1 - \frac{e^2}{4} - \frac{3e^2}{64} \quad (2.8a) \]
\[ A'_2 = \frac{3}{8} \left( e^2 + \frac{e^4}{4} \right) \quad (2.8b) \]
\[ A'_4 = \frac{15}{256} e^4 \quad (2.8c) \]
Using projection formulae (2.6) to (2.8) and appropriate parameters listed in Table 2.1, the UTM grid coordinate based on WGS84 datum can be calculated.

2.1.3. **Sources of Errors in GPS Positioning**

In recent years, manufacturers have worked to develop a new generation of GPS receivers that can provide users with highly accurate real-time location determinations anywhere on Earth. However, the accuracy of GPS positioning estimates is affected by a number of factors such as the data sampling rate, the multipath effect, satellite coverage, atmospheric effects, and satellite and receiver-dependent biases. The following sections briefly review each of these factors.

2.1.3.1. **Data Sampling Rate**

Figure 2.2 is an extract from the GPS World Receiver Surveys of 2001 to 2007, which reveal trends in the sampling capabilities of GPS receivers available in the market which are made by some of the most common well-known manufacturers. It is noteworthy that Javad Navigation Systems (JNS) has introduced receivers with sampling rates of 100 Hz. However, its receivers can record code and carrier phase data on the L1 frequency only. Although single frequency receivers typically cost less than dual frequency receivers, dual frequency L1/L2 receivers have been in use for some time now. This is because GPS receivers capable of measuring both L1 and L2 frequencies broadcast from satellites have a comparative advantage in eliminating ionospheric interference delay, especially when the baseline between the reference and rover stations is beyond 25 km (Hofmann-Wellenhof et al. 2001). In addition, an unknown number of cycles of
ambiguity $N_{a_{gps}}$ can be computed within seconds by combining two frequencies for L1/L2 receivers. The highest sampling frequency for a modern dual-frequency GPS receiver is 25 Hz, as shown in Figure 2.2(a).

In principle, a continuous signal can be properly sampled if it does not contain frequency components of above half the sampling rate. This is called the Nyquist sampling theorem. However, the full-scale GPS monitoring of a 108 m high steel tower in Tokyo amply demonstrates the practical frequency capture ability of GPS (Li 2004; Li et al. 2006a, 2006b). In their monitoring program, the seismic and wind-induced responses of the tower were recorded by the GPS and an accelerometer at sampling frequencies of 10 and 20 Hz, respectively. By analyzing the frequency domain measurements, it was concluded that the GPS and accelerometer successfully captured the peak for both types of event at 0.57 Hz, the first mode natural frequency of the tower. While the third mode of the tower at the frequency of 4.56 Hz was successfully captured by the accelerometer in both events, the GPS failed to do so, even if the Nyquist frequency of the GPS receiver was 5 Hz. The results of this study clearly show that the Nyquist frequency (i.e. half the sampling rate) is not a good indicator of the frequency detection capability of the GPS. To further assess the performance of the GPS and the accelerometer in field monitoring of the same 108 m steel tower in Tokyo, Li (2004) and her colleagues (2006a; 2006b) found that the GPS could pick up signals at the low frequency end (0-0.2 Hz). In contrast, the accelerometer recorded high frequency signals (2 Hz and above) more easily. Based on these observations, Li (2004) and her colleagues (2006a; 2006b) concluded that the measurement performance of the accelerometer was
complementary to that of the GPS. Therefore, the possibility of integrating GPS-measured signals with accelerometer-measured signals was explored to enhance the accuracy of displacement response measurements for large civil engineering structures (Roberts et al. 2000, 2004; Li 2004; Li et al. 2006a, 2006b).

In the integration algorithms proposed by Roberts et al. (2000; 2004), the measurement signals from an accelerometer were filtered by a conventional filter to remove high-frequency noise and the measurement signals from a GPS were filtered using an adaptive filter to reduce multipath. A single integration of acceleration signals from the accelerometer was then performed to find velocity signals. The velocity signals from the accelerometer were reset using the velocity constant calculated from the GPS data. These calibrated velocity signals were integrated to obtain displacement signals, which were finally reset with the GPS coordinates to obtain the actual displacement of a structure. The results of this study revealed that the proposed integration scheme allowed millimeter-accurate positioning to be maintained within several tens of seconds. The displacement measurements obtained using the earlier method were purely dynamic displacement measurements.

Because large civil engineering structures are typically very slender, it is very difficult to measure their low-frequency responses to winds accurately with an accelerometer. Furthermore, other than wind-induced dynamic displacement, wind-induced static displacement of a structure measured by GPS is likely to be contaminated by multipath, which is described in the following subsection.
Hence, it is difficult to apply the existing integration scheme to the total
displacement response of large civil engineering structures.

2.1.3.2. The Multipath Effect

Multipath is the propagation error attributable to the fact that a satellite-emitted
signal arrives at the receiver via more than one path. Figure 2.3 shows that the
satellite signal arrives at the receiver $S$ via two different paths, one being direct
($S_D$ ) and the other indirect ($S_R$ ). In mathematical terms, these paths can be
represented by:

$$ S = S_D + S_R $$

(2.9)

in which

$$ S_D = A \cos \varphi $$

(2.10a)

$$ S_R = \beta A \cos (\varphi + \Delta \varphi_m) $$

(2.10b)

where $A$ and $\varphi$ are the amplitude and phase of the direct signal, respectively;
$\Delta \varphi_m$ is a phase shift of the reflected signal relative to the direct signal phase; $\beta$
is a damping factor of between 0 and 1, with 0 implying no reflection and 1
implying strong reflection. The indirect signal $S_R$ is mainly caused by
reflecting surfaces near the receiver, such as water and metallic surfaces and
nearby buildings (Hofmann-Wellenhof et al. 2001; Kochly & Kijewski-Correa
2006). Secondary effects are reflections at the satellite during signal transmission.
In the case of range data, the maximum multipath error that can occur is one half
the chip length of the code, which is about 150 m for C/A-code measurements
and 15 m for P-code measurements. However, carrier phase multipath does not
exceed one quarter of the wavelength (Rizos 1999). This shows why phase information is usually used for position calculations. Figure 2.3 shows that the phase shift $\Delta \phi_m$ can be expressed as a function of the extra pathlength $\Delta s$:

$$\Delta \phi_m = \frac{\Delta s}{\lambda} = \frac{2h}{\lambda} \sin E$$

(2.11)

By inspection, the multipath can be regarded as a periodic function because $E$ varies with time and its frequency $f_m$ can be represented as (Hoffmann-Wellenhof et al. 2001):

$$f_m = \frac{d(\Delta \phi_m)}{dt} = \frac{2h}{\lambda} \cos E \frac{dE}{dt}$$

(2.12)

This equation shows that the multipath frequency $f_m$ increases as the distance between the reflector and antenna $h$ increases. Today’s technology provides two techniques for mitigating multipath: better GPS instrumentation design, and data post reception processing techniques.

**Better Instrumentation:** For better GPS instrumentation, the choke ring antenna is a GPS multipath resistant antenna that can be used to mitigate signals reflected from objects below the antenna (Kijewski-Correa 2003; Zheng et al. 2005; Kijewski-Correa & Kochly 2007; Even-Tzur 2007). However, this device cannot reject multipath signals reflected from objects above the antenna plane (Kijewski-Correa & Kochly 2007; Even-Tzur 2007). Therefore, manufacturers are working to develop advanced receivers that use data processing algorithms to
eliminate code and/or carrier phase multipath effects at the signal processing level in the receiver (Zheng et al. 2005). The typical method used is to improve the receiver tracking loop design (Han & Rizos 1997) such as through narrow correlator spacing technology (van Dierendonck et al. 1992), multipath elimination technology (MET) (Townsend & Fenton 1994), or the multipath estimating delay lock loop (MEDLL) (van Nee 1992). Although these three methods can reduce the effects of multipath propagation in code and/or carrier phase locking, the residual multipath effects of these methods on position estimation can still be as large as several centimeters (Zheng et al. 2005). This represents a significant error in GPS applications with millimeter-level requirements (Zheng et al. 2005).

*Post-reception Processing Techniques:* A number of post-processing filtering techniques have been developed by end users to diagnose and remove GPS multipath effects, such as digital filters (Han & Rizos 1997; Kijewski-Correa & Kochly 2007; Zheng et al. 2005), wavelet filters (Chen et al. 2001; Xiong et al. 2004), and adaptive filters (Ge 1999; Ge et al. 2000; Roberts et al. 2002; Meng 2002). As GPS satellites repeat their orbits every sidereal day, multipath errors that are dependent on the geometry relating to the GPS satellites, reflective surfaces, and antenna will repeat in the same period if the geometry remains unchanged (Han & Rizos 1997; Ge 1999; Ge et al. 2000; Chen et al. 2001; Kijewski-Correa & Kochly 2007). Therefore, in most proposed filtering techniques, the feature of repeatability of multipath error every sidereal day (about 23 hours 56 minutes) has been applied (Han & Rizos 1997; Chen et al. 2001; Ge 1999; Ge et al. 2000; Roberts et al. 2002; Meng 2002; Zheng et al.
2005). For instance, Han and Rizo (1997) performed a spectral or frequency analysis on a discrete-time signal measured on four successive days. Based on the observations of frequency plots, they concluded that the carrier-phase double-differenced multipath effect exhibited significant correlation between successive days and was repeated at the frequency band from 1/2,400 Hz to 1/120 Hz consecutively. Therefore, a bandpass filter with a cutoff frequency between 1/2,400 and 1/120 Hz was designed to generate a multipath model that corrects the multipath effects collected on a subsequent day.

However, Ge (1999) pointed out that the GPS multipaths and the deformation signals of interest can fall in the same frequency range and that multipaths change continuously. Ge therefore suggested using an adaptive filter for multipath mitigation because it allows unknown filter parameters to be estimated in real time. Studies that have applied the adaptive filtering approach (Ge 1999; Ge et al. 2000; Roberts et al. 2002; Meng 2002) have used information from two measurement time series collected on two consecutive days. On a selected day, the first measurement input $r(n)$ (also known as the reference measurement) was the time series that contained the noise contribution from multipath $\nu'(n)$. Another measurement input $s(n)$ collected on the next day (known as the primary measurement) contained the signal free of multipath $x(n)$ and the noise contribution from multipath $\nu(n)$. By using the adaptive system to obtain a noise-free signal $x(n)$ from the primary measurement input $s(n)$, the signal $x(n)$ in the primary input $s(n)$ had to be uncorrelated not only with the multipath $\nu(n)$, but also with $\nu'(n)$ in the reference input $r(n)$ (Ge 1999; Ge et al. 2000; Roberts et al. 2002; Meng 2002). As multipath noises are repeated on
successive days in a nearly static antenna environment, the multipaths $\nu(n)$ in the primary input $s(n)$ and $\nu'(n)$ in the reference input $r(n)$ that are correlated with each other satisfy another adaptive filtering requirement.

By inputting the two measurement inputs $s(n)$ and $r(n)$ into the adaptive system as shown in Figure 2.4, the coherent component $\hat{\nu}(n)$ is calculated as:

$$\hat{\nu}(n) = \mathbf{w}^T(n)\mathbf{r}(n) = \sum_{i=0}^{M-1} w_i(n) r(n-i) \quad (2.13)$$

And will be output directly from the adaptive finite-duration impulse response (FIR) filters. In the above equation, $M$ is the length of the adaptive FIR filter, $\mathbf{r}(n) = [r(n) \ r(n-1) \cdots r(n-M+1)]^T$ is the reference signal vector, and $\mathbf{w}(n) = [w_0(n) \ w_1(n) \cdots w_{M-1}(n)]^T$ denotes a tap weight vector. In the case proposed by Ge (1999), the coherent component $\hat{\nu}(n)$ is the multipath of the primary input $s(n)$ because it is the only composition correlated with a composition of the reference signal $r(n)$. In contrast, the incoherent component, also known as the estimation error $e(n)$, is the composition of the primary signal $s(n)$ which is uncorrelated with a composition of the reference signal $r(n)$. It is given as:

$$e(n) = s(n) - \hat{\nu}(n) \quad (2.14)$$

By inspection, this estimation error $e(n)$ is actually the estimate of the signal free from multipath $\bar{x}(n)$ of the primary signal $s(n)$. 
The concept underlying the least mean squares (LMS) adaptive filter (Ge 1999; Ge et al. 2000; Roberts et al. 2002; Meng 2002) is to minimize the least squares error $\varepsilon(n)$, which is defined as:

$$\varepsilon(n) = \sum_{j=0}^{n} e^2(j)$$  \hspace{1cm} (2.15)

To minimize the least squares error function, $\varepsilon(n)$ is partially differentiated with respect to the tap weights of the filter $w(n)$ and the result set to zero:

$$\frac{\partial \varepsilon(n)}{\partial w(n)} = 2 \sum_{j=0}^{n} e(j) \frac{\partial e(j)}{\partial w(n)} = -2 \sum_{j=0}^{n} e(j)r(j) = 0$$  \hspace{1cm} (2.16)

Replacing $e(j) = s(j) - w^T(n)r(j)$, Equation (2.16) is finally equivalent to

$$R(n)w(n) = P(n)$$  \hspace{1cm} (2.17)

where

$$R(n) = \sum_{j=0}^{n} r(j)r^T(j)$$  \hspace{1cm} (2.18)

is the autocorrelation matrix for $r(j)$, while

$$P(n) = \sum_{j=0}^{n} s(j)r(j)$$  \hspace{1cm} (2.19)

is the cross-correlation between the primary input $s(j)$ and the reference input $r(j)$. By inspection, the adaptive filtering will be switched off if the compositions of the primary input $s(n)$ are completely uncorrelated with those
of the reference input $r(n)$ (Ge 1999).

Because the statistical quantities of $R(n)$ and $P(n)$ are unknown in practice, an iterative gradient descent method was used (Ge 1999; Ge et al. 2000; Roberts et al. 2002; Meng 2002) to compute the weight of an FIR filter $w(n) = R^{-1}(n)P(n)$:

$$w(n) = w(n-1) + \mu e(n)r(n)$$  \hspace{1cm} (2.20)

where $\mu$ is the step-size parameter. In a study analyzing the application of an adaptive filter for multipath mitigation, Ge (1999) found that the standard deviation of time series reduced on pseudo-range to about one quarter and on carrier phase to about one half the value of the uncorrected time series.

In addition to the repeatability of multipath, Kijewski-Correa and Kochly (2007) suggested that multipath frequency content was consistent in both displacement axes of a tall building. Their explanation was that the local coordinate system was determined by tracking the arrival time of transmissions from individual satellites. The GPS receiver would receive both the original transmission and a delayed version due to multipath effects. The delay introduced by the periodic distortion (multipaths) would infiltrate the displacement estimates along both of the local coordinate axes. Kijewski-Correa and Kochly (2007) thus performed wavelet instantaneous frequency spectra (WIFS) analysis for the 30-minute long displacements of the building along the structural axes. They identified a consistent low-frequency component at 0.02 Hz in both response directions. As it
was suspected that some additional multipath energy resided in the large-amplitude peak at 0 Hz, Kijewski-Corra and Kochly used a high-pass filter with a cut-off frequency of 0.05 Hz to eliminate the identified multipath effect at 0.02 Hz, along with any other residual effects bedded at the 0 Hz frequency for the data. Although the approach provided a new way of mitigating multipath effects, the long-span cable-supported bridge was very slender, meaning the low natural frequencies of the bridge deck may have fallen in the same frequency range.

Apart from removing multipaths based on their own characteristics, Xiong et al. (2004) suggested that the response frequency phase observation series to structure deformation should be the same for all satellites, while the multipath effects should be different for different satellites. Therefore, in the analysis conducted by Xiong et al. (2004), each double difference phase observation series was first decomposed into different frequency bands with wavelet transformation. The correlations between the phase observation series from different satellites were then computed for each frequency band. The signals with significant correlations between all the satellites were kept and reconstructed using inverse discrete wavelet transformation. The final reconstructed signal was considered to be the double difference phase observation series of structural deformation. The two tests conducted by Xiong et al. (2004) showed an improvement of more than 27% in the three directions when the method was applied. Similarly, there is always a possibility that the signals of interest will fall in the same frequency range of GPS multipaths when the proposed approach is used. In addition, the method proposed is only applicable to measurements based
on raw data, but not for real time positioning.

2.1.3.3. Satellite Coverage

Dilution of precision \((DOP)\) is a measure of the geometric strength of satellite configuration (Kaplan 1996; Hofmann-Wellenhof et al. 2001). The \(DOP\) can be geometrically viewed as the volume \(V\) of a body formed by the intersection points of site-satellite vectors with the unit sphere centered at the user on Earth (Hofmann-Wellenhof et al. 2001):

\[
DOP = \frac{1}{V}
\]  

(2.21)

By inspection, the geometry is said to be weak and the \(DOP\) value high when visible satellites are close together in the sky (i.e., the volume of geometry is small), as shown in Figure 2.5\((a)\). On the other hand, the \(DOP\) value is low when good satellite-receiver geometry has large geometric volume (Figure 2.5\((b)\)). The factors that affect the \(DOP\) are the presence of obstructions, especially in an urban environment, which make it impossible to use satellites in certain sectors of the local sky. \(DOP\) values are usually represented in horizontal, vertical and position measurements. The \(DOP\) measured in the horizontal position (i.e., latitude and longitude) is called horizontal \(DOP\) \((HDOP)\), vertical \(DOP\) \((VDOP)\) denotes the \(DOP\) measured for the vertical component (i.e., altitude), and position \(DOP\) \((PDOP)\) is related to the overall rating of precision for latitude, longitude, and altitude. The \(DOP\) value should generally be kept below 4 for reasonable accuracy (Kijewski-Correa 2003).
2.1.3.4. Atmospheric Effects

Signals transmitted from satellites are known to take the form of electromagnetic waves. When they propagate through the ionosphere and the neutral atmosphere, distance measurement errors will be induced due to changes in signal velocity. In the ionosphere region (≈ 50 km to 1000 km), free electrons are released when gas molecules are excited by solar radiation. It should be noted that the propagation of the code signal through this medium will slow down, but the carrier phase signal will propagate faster than the velocity of the modulated signal. As the ionospheric error is frequency dependent, this error can be effectively reduced by computing receiver positions using ionospheric free data derived from two frequency measurements. Tropospheric error refers to the delay in signals as they propagate through the atmosphere. When electromagnetic waves transmitted from satellites propagate through the medium, a refraction delay that depends on humidity, temperature, and pressure will be experienced. To reduce tropospheric error, mathematical formulae such as the Hopfield model, the Black model, and the Saastamoinen model have been suggested (Hofmann-Wellenhof et al. 2001).

2.1.3.5. Satellite and Receiver-dependent Biases

The position calculated by GPS is affected by a number of biases. Bias refers to measurement errors that cause measured ranges to be different from true ranges by a “systematic amount”. Satellite-dependent biases include orbital errors and satellite clock error, whilst receiver-dependent bias refers to receiver clock error. By inspection of Equation (2.1), the positions of the satellites must be known when computing the receiver position. However, due to the imperfection of the mathematical models used to describe satellite dynamics, errors in the range of 5
to 25 metres will propagate to the derived satellite position. Although satellites are equipped with high precision atomic clocks, satellite clock error is unavoidable in satellite positioning (Mok & Retscher 2001). However, in comparison with clock errors attributable to receivers, which carry inexpensive quartz crystal clocks (receiver-dependent bias), satellite clock error is much smaller. Fortunately, these satellite and receiver-dependent biases will exhibit some correlation among signals received at several receiver stations tracking the same satellites simultaneously. Different techniques can be used to take advantage of these correlations and improve the accuracy of positioning (Wells et al. 1999).

2.1.4. Positioning Techniques for Surveying

2.1.4.1. Post-processed Surveying

In conceptual terms, position can be determined by one or more receivers. If a position is described by one receiver, this type of positioning is termed ‘absolute’ or ‘single point’ positioning. On the other hand, positioning that involves at least two receivers, namely, reference and rover receivers, is called ‘relative’ positioning (Mok & Retscher 2001). In relative positioning, the reference receiver is at a location for which the WGS84 coordinate is accurately known \((N_{\text{ref}}, E_{\text{ref}}, H_{\text{ref}})\), while the roving receivers occupy stations with unknown coordinates \((N_{\text{rov}}, E_{\text{rov}}, H_{\text{rov}})\).

For post-processed surveying, simultaneous measurements are made from all stations to four or more satellites at the same epoch rate. The data collected by
the receivers are temporarily stored in the receivers until the field work is completed. The observational files from the receivers are then transferred to the computer for processing. As the simultaneous observations are collected, the most precise GPS positions can be obtained using the ‘differenced’ data.

Denoting the receivers by \( r_{\text{ref}} \) and \( r_{\text{rov}} \), and the satellites by \( s_j \) and \( s_k \), the four measured distances from satellites \( s_j \) and \( s_k \) to stations \( r_{\text{ref}} \) and \( r_{\text{rov}} \) can be expressed as their true distance plus errors, as follows:

\[
\begin{align*}
\rho_{r_{\text{ref}} s_j} &= D_{r_{\text{ref}} s_j} + t_{r_{\text{ref}} s_j} + O_{r_{\text{ref}} s_j}^{s_j} + T_{r_{\text{ref}} s_j}^{s_j} + I_{r_{\text{ref}} s_j}^{s_j} + \epsilon_{r_{\text{ref}} s_j}^{s_j} \\
\rho_{r_{\text{ref}} s_k} &= D_{r_{\text{ref}} s_k} + t_{r_{\text{ref}} s_k} + O_{r_{\text{ref}} s_k}^{s_k} + T_{r_{\text{ref}} s_k}^{s_k} + I_{r_{\text{ref}} s_k}^{s_k} + \epsilon_{r_{\text{ref}} s_k}^{s_k} \\
\rho_{r_{\text{rov}} s_j} &= D_{r_{\text{rov}} s_j} + t_{r_{\text{rov}} s_j} + O_{r_{\text{rov}} s_j}^{s_j} + T_{r_{\text{rov}} s_j}^{s_j} + I_{r_{\text{rov}} s_j}^{s_j} + \epsilon_{r_{\text{rov}} s_j}^{s_j} \\
\rho_{r_{\text{rov}} s_k} &= D_{r_{\text{rov}} s_k} + t_{r_{\text{rov}} s_k} + O_{r_{\text{rov}} s_k}^{s_k} + T_{r_{\text{rov}} s_k}^{s_k} + I_{r_{\text{rov}} s_k}^{s_k} + \epsilon_{r_{\text{rov}} s_k}^{s_k}
\end{align*}
\] (2.22a)

where \( \rho \) is the pseudo-range measurement, \( D \) is the true distance, \( t_{r_{\text{ref}}} \) is the receiver clock error, \( t_{s} \) is the satellite clock error, \( O_{r_{\text{ref}}}^{s_j} \) is the orbital error, \( T_{r_{\text{ref}}}^{s_j} \) is the tropospheric error, \( I_{r_{\text{ref}}}^{s_j} \) is the ionospheric error, and \( \epsilon_{r_{\text{ref}}}^{s_j} \) is the combination of receiver noise and multipath error. By subtracting the two pseudo-ranges from two receiver stations to the same satellite, i.e. \( \rho_{r_{\text{ref}} s_j} - \rho_{r_{\text{ref}} s_j} \) and \( \rho_{r_{\text{ref}} s_k} - \rho_{r_{\text{ref}} s_k} \), the above four equations become:

\[
\begin{align*}
\rho_{r_{\text{ref}} s_j} - \rho_{r_{\text{rov}} s_j} &= \rho_{r_{\text{ref}} s_j} - \rho_{r_{\text{rov}} s_j} = \left( D_{r_{\text{ref}} s_j} - D_{r_{\text{rov}} s_j} \right) + \left( t_{r_{\text{ref}} s_j} - t_{r_{\text{rov}} s_j} \right) + \left( \epsilon_{r_{\text{ref}} s_j}^{s_j} - \epsilon_{r_{\text{rov}} s_j}^{s_j} \right)
\end{align*}
\] (2.23a)
\[
\rho^S_{r_{ref} - r_{rov}} = \rho^S_{r_{ref}} - \rho^S_{r_{rov}} = (D^S_{r_{ref}} - D^S_{r_{rov}}) + (t^S_{r_{ref}} - t^S_{r_{rov}}) + (\varepsilon^S_{r_{ref}} - \varepsilon^S_{r_{rov}})
\]

(2.23b)

By inspection, the satellite clock error term \(t^S\) is directly cancelled in the single differencing technique. As the satellites are orbiting at approximately 20,200 km above the Earth, the signals traveling from one satellite to two different receivers pass through nearly the same atmosphere. Thus, the ionospheric \(I^S\) and tropospheric \(T^S\) effects are largely reduced by subtraction. Also, over short distances, the magnitude of orbital errors \(O^S\) can be neglected. Finally, the errors arising from the receiver clock \(t_r\) and a combination of multipath and receiver noise \(\varepsilon^S_r\) still exist, as shown in Equation (2.23). If the two single difference pseudo-ranges are further subtracted from each other, the receiver clock errors \(t_r\) are eliminated and the following double-differenced equation is obtained:

\[
\rho^S_{r_{ref} - r_{rov}} - \rho^S_{r_{ref} - r_{rov}} = \left( D^S_{r_{ref}} - D^S_{r_{rov}} - D^S_{r_{ref}} + D^S_{r_{rov}} \right) + \left( \varepsilon^S_{r_{ref}} - \varepsilon^S_{r_{rov}} - \varepsilon^S_{r_{ref}} + \varepsilon^S_{r_{rov}} \right)
\]

(2.24)

By inspection, most of the errors can either be eliminated or significantly reduced with differencing techniques. This is why the differencing GPS (DGPS) technique is most commonly used for precise surveys.

### 2.1.4.2. Real-time Kinematic Surveying

In common with post-processed surveying, real-time kinematic surveying also
requires that two or more receivers are operated simultaneously. The difference between these two forms of surveying is that in real-time kinematic surveying, radios are used to transmit the raw GPS observations from the reference receiver to the roving receiver and determine the position in real-time. Recalling Equation (2.22a), the measured pseudo-range \( \rho_{r_{\text{ref}}}^s \) is in error due to the various error sources. Because the coordinate of the reference station is known, the true geometric range \( D_{r_{\text{ref}}}^s \) can be computed using Equation (2.1). The difference in the measured pseudo-range and the true geometric range will yield pseudo-range corrections \( PRC_j^i \) for satellite \( s_j \) at reference epoch \( t_0 \):

\[
PRC_j^i \left( t_0 \right) = D_{r_{\text{ref}}}^s \left( t_0 \right) - \rho_{r_{\text{ref}}}^s \left( t_0 \right)
\] (2.25)

As the signal transmission and correction computation make it impossible to assign the \( PRC_j^i \) to the same epoch at the rover station, a range rate correction \( (RRC_j^i) \) is approximated by numerical differentiation and the \( PRC_j^i \) at any epoch \( t \) is given as (Wolf & Ghilani 2002):

\[
PRC_j^i \left( t \right) = PRC_j^i \left( t_0 \right) + RRC_j^i \left( t_0 \right) (t - t_0)
\] (2.26)

This \( PRC_j^i \left( t \right) \) information is then used to correct the range computed at the rover station. The corrected pseudo-range at the roving receiver for epoch \( t \) is \( \rho_{r_{\text{rov}}}^{s_j} \):
\[
\rho_{\text{rov}}^{s_j} (t)_{\epsilon} = \rho_{\text{rov}}^{s_j} + PRC^l (t) = D_{\text{rov}}^{s_j} (t) + t_{\text{rov}} (t) - t_{\text{ref}} (t) + \varepsilon_{\text{rov}}^{s_j} (t) - \varepsilon_{\text{ref}}^{s_j} (t)
\]

(2.27)

As noted above, the satellite clock error term \( t_{\text{rov}}^{s_j} \), the ionospheric effect \( I_r^{s_j} \), the tropospheric effect \( T_r^{s_j} \), and the orbital error \( O_r^{s_j} \) are nearly the same and thus are mathematically eliminated.

### 2.2. APPLICATION OF GPS TO CIVIL ENGINEERING STRUCTURES

Among all measurement parameters, structural displacement is one of the key parameters when assessing the safety of large civil engineering structures. Before GPS deployment, structural responses are mainly monitored by accelerometers, and dynamic displacement responses are then obtained through a double integration of the measured acceleration responses. However, the velocity and displacement integrated from the uncompensated acceleration signals will drift over time due to unknown integration constants, and a high-pass filter should be used to cope with low-frequency drift introduced during the integration process. As a result, it is recognized that an accelerometer is incapable of providing static and/or total displacement of a structure under wind load (Celebi et al. 1998; Wong et al. 2001a, 2001b; Tamura et al. 2002; Ko & Ni 2005; Kochly & Kijewski-Correa 2006). In response to this need, the use of GPS has spread from the navigation field to the civil engineering community.

#### 2.2.1. Application of GPS to Tall Structures

Lovse et al. (1995) were probably the first researchers to report on differential GPS monitoring of the 160 m Calgary Tower in Canada. The results showed that
under wind load, the Calgary Tower vibrated with a frequency of about 0.3 Hz and amplitudes of ±30 mm in both north-south and east-west directions. The success of this trial has spawned similar experiments in other countries. For instance, Chen et al. (2001) used GPS to measure the displacement and frequency signature of the 325 m Di Wang Tower in China under wind load during Typhoon York; Breuer et al. (2002) applied GPS technology to measure the wind-induced ambient vibrations of two slender high-rise structures in Germany; Ogaja et al. (2001) installed an RTK-GPS system in the 280 m Republic Plaza building in Singapore to complement existing structural monitoring instrumentation used to measure displacement during strong winds and remote earthquakes; Celebi and Sanli (2002) permanently deployed GPS units in a 34-storey San Francisco building to record data during earthquakes and strong winds; Tamura et al. (2002) erected an RTK-GPS antenna on the top of a 108 m steel tower in Japan to measure the response during typhoons; and Kareem and Kijewski-Correa (2002) deployed the GPS in an instrumented tall building in Chicago to measure the mean and background components of wind-induced responses. The literature includes many other such studies.

2.2.2. Application of GPS to Long-span Cable-supported Bridges

A field trial carried out on the Humber Bridge in England in 1996 (Ashkenazi & Roberts 1997) is probably the earliest test of the use of GPS technology to monitor a long-span suspension bridge under wind load. The success of this trial inspired similar experiments carried out in Japan over the last decade. The field measurements taken by Nakamura (2000) on a suspension bridge in Japan during the strong wind season showed that the girder displacements measured using
GPS data were in a good agreement with other measurement instrumentation, finite element values, and wind tunnel test results. GPS monitoring programs have since been introduced for the longest suspension bridge in the world, the Akashi Kaikyo Bridge (Fujino et al. 2000; Miyata et al. 2002). To improve the efficiency and accuracy of the existing Wind And Structural Health Monitoring System (WASHMS) for the Tsing Ma Bridge, the Ting Kau Bridge, and the Kap Shui Mun Bridge in Hong Kong (Wong et al. 2001a, 2001b), the Highways Department of the Hong Kong Government introduced GPS to monitor the displacement of cables, decks, and towers. Due to the success of the application of GPS to SHMS for cable-supported bridges, similar permanent and full-scale monitoring programs have been established for a number of bridges in China, such as the Shandong Binzhou Yellow River Highway Bridge (Ou 2004; Li et al. 2006), the Harbin Songhua River Bridge (Li & Ou 2005), the Humen Bridge (Guo et al. 2000; Xu et al. 2002; Guo et al. 2005), the Runyang South Bridge (Li et al. 2003), the Sutong Bridge (Ni et al. 2004), and the Zhanjiang Bay Bridge (Wang 2004).

2.2.3.   Calibration Tests

GPS has been shown to measure the displacement responses of tall structures and long-span cable-supported bridges to a satisfactory standard when the structural displacement response is large enough. However, it is questionable whether GPS can provide dynamic displacement measurement accuracy to the sub-centimeter or sub-millimeter level. Nevertheless, the significant progress made in hardware technology in recent years has resulted in new types of GPS receivers being explored to improve the dynamic measurement accuracy of GPS. According to a
survey conducted on six manufacturers who supplied the market with survey
grade receivers (Engineering Surveying Showcase 2007), the kinematic and
real-time kinematic (RTK) accuracies of GPS receivers have now attained
±10 mm + 1 ppm in the horizontal direction and ±20 mm + 1 ppm in the vertical
direction. However, as noted above, the accuracy of GPS for dynamic
displacement measurement depends on many factors, such as the data sampling
rate, the multipath effect, satellite coverage, atmospheric effects, and satellite and
receiver-dependent biases. Hence, a number of scholars have conducted a series
of calibration tests in the past few years to assess the dynamic measurement
accuracy of GPS (Celebi 2000; Tamura et al. 2002; Kijewski-Correa 2003; Park
et al. 2004; Nickitopoulou et al. 2006).

The earlier validation effort of Celebi (2000) used two steel bar specimens to
simulate 30 to 40-storey flexible buildings with fundamental frequencies of
approximately 0.25 and 0.30 Hz. By measuring the relative displacement using
GPS and performing power spectral density (PSD) analysis, it was identified that
the spectra had the same peaks corresponding to the natural frequencies of the
two bars. This research has been followed by the recent study of Park et al.
(2004), who carried out a free vibration test on a 2.44 m by 1.24 m wooden plate
supported by six vertical bars. The results of the experiment showed good
agreement between the horizontal displacement time histories produced by a
laser meter and GPS, and an identical natural frequency of 0.6 Hz was observed
after performing Fast Fourier Transform (FFT). Although both tests demonstrated
the ability of GPS to detect the first natural frequency of tall slender structures,
the amplitude and frequency sensitivities of GPS have not yet been verified.
To assess the amplitude and frequency sensitivities of *Leica* MC1000 RTK-GPS, Tamura et al. (2002) performed a one-dimensional sinusoidal calibration study using an electronic exciter. The dynamic performance of GPS in the horizontal direction was validated through a comparison of motions simultaneously measured by a wire displacement transducer. The amplitudes of the simulated motions were taken as 3, 5, 10, 20, and 50 mm, while the frequencies selected were 0.5, 1, 2, 3, and 4 Hz. A visual comparison indicated that RTK-GPS in the horizontal direction closely followed the actual displacement when the vibration frequency was lower than 2 Hz and the vibration amplitude was larger than 20 mm.

This work has been followed by a motion simulator calibration study carried out by Kijewski-Correa (2003). In testing, the physical rotation of the ball screw driving the displacement-controlled motion simulator was used to simulate one-dimensional sinusoidal waves with amplitudes ranging from 5 to 30 mm and frequencies of 0.1, 0.125, 0.15, 0.2, 1, and 2 Hz. A total of twenty-three sinusoidal signals were simulated in the horizontal direction to assess the amplitude and frequency sensitivities of *Leica* MC 500 GPS receivers. Based on the results, Kijewski-Correa (2003) noted that GPS displacement was consistent with the root-mean-square (RMS) of the actual table displacements once the signal-to-noise ratio exceeded 200%, which corresponded roughly to motions in excess of 10 mm. When the amplitude of the motion was greater than 20 mm, a consistent error of 10% or less was observed in capturing the peak values. The researcher found no appreciable dependence on frequency (Kijewski-Correa 2003).
To extend the dynamic performance validation of GPS in two-dimensional directions, the measurement accuracy of GPS for the horizontal circular movements of a structure was analyzed out by Nickitopoulou et al. (2006) based on a prototype device consisting of a horizontal rod rotating about a vertical axis powered by an electric motor. This device was used to simulate circular rotations with radii of 200, 300, 400, and 500 mm and over periods ranging from 3 to 19 sec. The outcome implied that the coordinates calculated in both axes were nearly equal in precision.

2.2.4. Insights from Previous Studies

Due to the intent of applying GPS technology to building structures, only horizontal uni-axial motions and bi-axial circular motions with frequencies common in buildings have been analyzed in previous calibration programs. However, for long-span cable-supported bridges, measuring the wind-induced dynamic displacement responses of the bridge deck in the lateral and vertical directions and of the bridge towers in the longitudinal and lateral directions is particularly important. In addition, the fundamental frequency of a bridge in one direction, which is much lower than that of a building, may not be the same as it is in another direction. Therefore, for applications in long-span cable-supported bridges, the dynamic displacement measurement accuracy of GPS in three orthogonal directions is advocated.

In addition, the displacement responses measured in previous studies on in-service long-span cable-supported bridges have been the result of environmental (i.e. wind and solar heating) and operational (i.e. traffic) loadings
(Brownjohn et al. 1994; Brownjohn et al. 1995). To define a reliable correlation of a response parameter with wind, most previous studies have been based on the response to wind over a specific period when the wind loading was predominant. To this end, the data that can be analyzed are limited; hence, the correlation of a response parameter with a single loading parameter cannot be reliably established.

2.3. WIND EFFECTS ON LONG-SPAN CABLE-SUPPORTED BRIDGES

For structural design purposes, it is important to deal with winds near the ground surface because ordinary structures are placed on the ground (Simiu & Scanlan 1996). In reality, when winds blow near the ground surface, the Earth’s surface will exert a horizontal drag force on the moving air that results in reduced wind speed and turbulence near the ground. The wind region affected by the roughness of the terrain is referred as the atmospheric boundary layer.

2.3.1. Characteristics of Natural Wind

Within the boundary layer, the wind velocity can be decomposed as a mean wind velocity in the mean wind direction and three perpendicular turbulence components. Let \( U_S(t) \), \( U_E(t) \) and \( U_U(t) \) denote three orthogonal wind components recorded by an anemometer, where \( U_S(t) \) is the horizontal southward component, \( U_E(t) \) is the horizontal eastward, and \( U_U(t) \) is the vertical component pointing upward.

The mean wind speeds of \( U_S \), \( U_E \), and \( U_U \) can be obtained by the following
formulae:

\[
\bar{U}_S = \frac{1}{T} \int_0^T U_S (t) \, dt = \frac{1}{M} \sum_{i=1}^{M} U_S (t_i) \quad (2.28a)
\]

\[
\bar{U}_E = \frac{1}{T} \int_0^T U_E (t) \, dt = \frac{1}{M} \sum_{i=1}^{M} U_E (t_i) \quad (2.28b)
\]

\[
\bar{U}_U = \frac{1}{T} \int_0^T U_U (t) \, dt = \frac{1}{M} \sum_{i=1}^{M} U_U (t_i) \quad (2.28c)
\]

where \( T \) is the time duration of wind speed record: \( T = 3,600 \) sec for hourly mean wind speed and \( T = 600 \) sec for 10-minute mean wind speed. \( M = \frac{T}{\Delta t} \) is the number of data points in the record, in which \( \Delta t \) is the time interval and \( f_s \) is the sampling frequency.

The mean wind speed \( \bar{U} \) can then be determined by the following equation:

\[
\bar{U} = \sqrt{\bar{U}_S^2 + \bar{U}_E^2 + \bar{U}_U^2} \quad (2.29)
\]

As the positive direction of the alongwind turbulence component \( u \) is normally defined in the same direction as the resultant mean wind speed, the positive direction of \( u(t) \) is determined by the cosine vector of \( (\cos \alpha_u, \cos \beta_u, \cos \gamma_u) \) for the mean wind speed (see Figure 2.6(a)). The directional cosine vector of the mean wind speed can be determined by the following equation (Zhu 2002):

\[
(\cos \alpha_u, \cos \beta_u, \cos \gamma_u) = (\bar{U}_S, \bar{U}_E, \bar{U}_U)/\bar{U} \quad (2.30)
\]

The crosswind turbulence component \( v(t) \) is normal to the mean wind speed \( \bar{U} \),
while the upward wind turbulence component $w(t)$ is perpendicular to both $u(t)$ and $v(t)$. As a result, the directional cosine vector $(\cos \alpha_v, \cos \beta_v, \cos \gamma_v)$ for the crosswind turbulence component is the normalized cross-product of vector $(0,0,1)$ and vector $(\cos \alpha_u, \cos \beta_u, \cos \gamma_u)$ (see Figure 2.6(b)), while the directional cosine vector $(\cos \alpha_w, \cos \beta_w, \cos \gamma_w)$ for the upward wind turbulence component is the normalized cross-product of vector $(\cos \alpha_u, \cos \beta_u, \cos \gamma_u)$ and $(\cos \alpha_v, \cos \beta_v, \cos \gamma_v)$ (see Figure 2.6(c)) (Zhu 2002).

$$\begin{align*}
(\cos \alpha_v, \cos \beta_v, \cos \gamma_v) &= \frac{(0,0,1) \times (\cos \alpha_u, \cos \beta_u, \cos \gamma_u)}{|(0,0,1) \times (\cos \alpha_u, \cos \beta_u, \cos \gamma_u)|} = \frac{(-\cos \beta_u \cos \alpha_u, 0)}{\sqrt{\cos^2 \alpha_u + \cos^2 \beta_u}} \quad (2.31) \\
(\cos \alpha_w, \cos \beta_w, \cos \gamma_w) &= \frac{(\cos \alpha_u, \cos \beta_u, \cos \gamma_u) \times (\cos \alpha_v, \cos \beta_v, \cos \gamma_v)}{|(\cos \alpha_u, \cos \beta_u, \cos \gamma_u) \times (\cos \alpha_v, \cos \beta_v, \cos \gamma_v)|} = \\
&= \frac{(-\cos \alpha_u \cos \gamma_u, -\cos \beta_u \cos \gamma_u, \cos^2 \alpha_u + \cos^2 \beta_u)}{\sqrt{\cos^2 \alpha_u + \cos^2 \beta_u}} \quad (2.32)
\end{align*}$$

The components of turbulent wind speed, $u(t)$, $v(t)$, and $w(t)$, in the alongwind, crosswind, and vertically upward wind directions, respectively, are computed as

$$\begin{align*}
u(t) &= U_S(t) \cos \alpha_u + U_E(t) \cos \beta_u + U_U(t) \cos \gamma_u - \bar{U} \quad (2.33a) \\
v(t) &= U_S(t) \cos \alpha_v + U_E(t) \cos \beta_v + U_U(t) \cos \gamma_v \quad (2.33b) \\
w(t) &= U_S(t) \cos \alpha_w + U_E(t) \cos \beta_w + U_U(t) \cos \gamma_w \quad (2.33c)
\end{align*}$$

To describe a turbulent flow, statistical methods must be applied. The three turbulence components described by the means of their standard deviations can
be calculated by

$$\sigma_a = \sqrt{\frac{1}{T} \int_0^T a^2(t)dt}; \ a = u, v, w$$  \hspace{1cm} (2.34)$$

and the ratio of the standard deviation of fluctuating wind speed in the definite direction to the mean wind speed is defined as the turbulence intensity:

$$I_a = \frac{\sigma_a}{\bar{U}}$$  \hspace{1cm} (2.35)$$

The turbulence intensity increases with ground roughness and decreases with height. It also varies with the duration (averaging time) used in the determination of mean velocity: a longer duration yields a smaller mean velocity and hence a larger value for turbulence intensity (Liu 1991).

Its fluctuating wind energy distribution over frequency $f$ can be described by the wind spectrum $S_{aa}(f)$. A previous study conducted by Xu et al. (2000) concluded that the measured normalized wind spectra were much better fitted by von Karman spectra than Kaimal spectra or Simiu and Scalan spectra. Therefore, the normalized von Karman spectra given by the following equations are usually used to fit the longitudinal wind power spectrum:

$$\frac{f \cdot S_{uu}}{\sigma_u^2} = \frac{4 \frac{L_x}{U} f}{1 + 70.8 \left( \frac{L_x}{U} f \right)^{\frac{5}{16}}}$$ \hspace{1cm} (2.36a)$$
and lateral and vertical wind power spectra

\[
\frac{f \cdot S_{aa}}{\sigma_a^2} = \frac{4 \frac{L_x}{U_f} \left[ 1 + 755 \left( \frac{L_x}{U_f} \right)^2 \right]}{\left[ 1 + 283 \left( \frac{L_x}{U_f} \right)^2 \right]^{11/6}}; \quad aa = v, w \quad (2.36b)
\]

where \( L_u^x, L_v^x, \) and \( L_w^x \) are the integral scales of turbulence for measuring the average longitudinal size of eddies associated with the longitudinal, lateral, and vertical velocity fluctuations, respectively.

### 2.3.2. Mean Wind Simulation and Effects

When a long-span cable-supported bridge is immersed in a given flow field, the bridge will be subjected to mean and fluctuating wind forces. To simulate such forces, a linear approximation of time-averaged static and time-varying buffeting and self-excited force components should be represented (Davenport 1962; Scanlan 1978). As described in Figure 2.7, the oncoming wind flow with a mean velocity \( \bar{U} \) attacks the bridge deck at angle \( \alpha^0 \). Owing to wind and structure interaction, the static wind action acting on the bridge deck will cause deck deformation with the torsional angle \( \theta \). The effective wind angle of attack \( \alpha = \alpha^0 + \theta \) is then formed. Therefore, the equivalent static wind force per unit span acting on the deformed deck at the \( i \)th section \( F_{ei}^{sf} \) is expressed in global bridge axes as:
where \( f_{eDi}^{sf}, f_{eLi}^{sf}, \) and \( f_{eMi}^{sf} \) = static drag, lift, and moment, respectively, on the \( i \)th section of the bridge deck; \( \rho_a \) = air density; \( B_i \) = width of the bridge deck segment; \( C_{Di}, C_{Li}, \) and \( C_{Mi} \) = the static aerodynamic drag, lift, and moment coefficients as a function of the angle of attack, respectively. The modal static forces \( F_{ei}^{sf} \) for the whole bridge can be determined by

\[
F_{ei}^{sf} = \begin{bmatrix}
0 \\
f_{eDi}^{sf} \\
f_{eLi}^{sf} \\
f_{eMi}^{sf} \\
0
\end{bmatrix}
= \frac{1}{2} \rho_a \bar{U}_i^2 B_i \begin{bmatrix}
0 \\
C_{Di}(\alpha_i) \\
C_{Li}(\alpha_i) \\
C_{Mi}(\alpha_i) B_i \\
0
\end{bmatrix}
\]

(2.37)

where \( N_s \) is the total number of sections considered in the model of the bridge.

\[
\mathbf{F}^{sf} = \begin{bmatrix}
F_{e1}^{sf} \\
F_{e2}^{sf} \\
\vdots \\
F_{eN_s}^{sf}
\end{bmatrix}
\]

(2.38)

\[
\mathbf{X}_i = [p_i, q_i, h_i, \theta_{pi}, \theta_{qi}, \theta_{hi}]^T \]

denotes the \( 6 \times 1 \) displacement vector of the \( i \)th section at the global coordinate system, as shown in Figure 2.8. The mean displacement \( \bar{\mathbf{X}} \) due to mean force is simply solved by the following static equilibrium matrix equation:

\[
\mathbf{K}\bar{\mathbf{X}} = \mathbf{F}^{sf}
\]

(2.39)

The unknown mean displacement \( \bar{\mathbf{X}} \) involves the equivalent of inverting the stiffness matrix \( \mathbf{K} \) and multiplying it by the aerostatic force vector \( \mathbf{F}^{sf} \).
However, as mentioned in Equation (2.37), the static wind force of a long suspension bridge is computed in terms of three static aerodynamic coefficients: $C_D$, $C_L$ and $C_M$. In addition, the values of these coefficients are selected on the basis of the effective wind angle of attack for the bridge $\alpha$, which is the sum of the initial mean angle of attack $\alpha^0$ and the deformation of the deck with torsional angle $\theta$. The mean angle $\alpha^0$ is simply computed using the field measurement data: the vertical wind speed component $w(t)$ is divided by $\bar{U}$. However, the additional attack angle $\theta$ caused by torsional deformation must be solved using an iteration approach because of its nonlinearity of structure and the static wind force (Zhang et al. 2002). An iteration solution scheme based on structural geometric properties has been suggested by Sun (1999); the calculation process can be described as follows:

(a) Assume $\{\theta\} = \{0\}$ where $\{\theta\} = \{\theta_1, \theta_2, \ldots, \theta_{N_n}\};$

(b) Compute $\{\alpha\} = \{\alpha^0\} + \{\theta\};$

(c) Determine the static aerodynamic coefficients $C_{Di}(\alpha_i)$, $C_{Li}(\alpha_i)$, and $C_{Mi}(\alpha_i)$ of each section $i$;

(d) Compute the nodal static forces of the bridge deck $\{F\} = \{f_D^{sf}, f_L^{sf}, f_M^{sf}\}$ and the corresponding nodal mean displacement;

(e) Compute the torsional angle $\{\theta'\};$

(f) Stop if $\|\{\theta'\} - \{\theta\}\| \leq \epsilon$, $\{\theta'\} \rightarrow \{\theta\}$; repeat steps (b)-(f) and replace $\{\theta'\} \rightarrow \{\theta\}$ if $\|\{\theta'\} - \{\theta\}\| > \epsilon$ (where $\epsilon$ should be a small number to control the accuracy of the calculation).
2.3.3. **Buffeting Force Simulations and Effects**

Buffeting action is a random vibration caused by turbulent wind that will excite certain vibration modes along the bridge as a whole, depending on the spectral distribution of the pressure vectors (Ding et al. 2000). Although a buffeting response may not lead to catastrophic failure, it can lead to structural fatigue and affect the safety of passing vehicles (Boonyapinyo et al. 1999). Hence, researchers have focused on buffeting analysis in recent years in an effort to study the structural reliability of bridges under turbulent wind load (Chen et al. 2000a, 2000b; Ding et al. 2000; Chen & Kareem 2001; Chen & Cai 2003; Xu & Guo 2003; Xu et al. 2003; Guo et al. 2007). In addition to buffeting action, flutter instability, which is caused by the self-excited forces induced by wind-structure interaction, is also important for the design and construction of long-span suspension bridges (Boonyapinyo et al. 1999), because the additional energy injected into the oscillating structure by aerodynamic forces increases the magnitude of vibrations, sometimes to catastrophic levels (Ding et al. 2000). To model the action of buffeting wind load on a long-span suspension bridge, the buffeting forces resulting from turbulent wind and the self-excited forces due to wind and bridge motion interaction should be taken into account.

Assuming no interaction between buffeting forces and self-excited forces and using quasi-steady aerodynamic force coefficients, the buffeting forces per unit span $F_{ei}^{bf}$ on the $i$th section of the bridge deck can be expressed as (Simiu & Scanlan 1996):
\[
F_{ei}^{bf} = \begin{bmatrix}
0 \\
\frac{f_{eD}^{bf}}{\gamma_{ei}} \\
\frac{f_{eL}^{bf}}{\gamma_{ei}} \\
\frac{f_{eM}^{bf}}{\gamma_{ei}} \\
0 
\end{bmatrix}
= A_i^{bf} \begin{bmatrix}
u_i(t) \\
w_i(t) 
\end{bmatrix} 
\] (2.40)

in which

\[
A_i^{bf} = \begin{bmatrix}
u_{ui} & f_{wl} \end{bmatrix} = \frac{1}{2}\rho_a \bar{U}_i^2 B_i \begin{bmatrix}
0 \\
\chi_{D_{bu}} \frac{2C_{D_i}(\alpha_i)}{\bar{U}_i} \\
\chi_{L_{bu}} \frac{2C_{L_i}(\alpha_i)}{\bar{U}_i} \\
\chi_{M_{bu}} \frac{2C_{M_i}(\alpha_i)}{\bar{U}_i} B_i \\
0 \\
0 
\end{bmatrix} \begin{bmatrix}
0 \\
\frac{c_{D_i}^{\prime\prime}}{\bar{U}_i} \\
\frac{c_{L_i}^{\prime\prime}}{\bar{U}_i} + C_{D_i}(\alpha_i) \\
\frac{c_{M_i}^{\prime\prime}}{\bar{U}_i} B_i \\
0 \\
0 
\end{bmatrix} 
\] (2.41)

and where \(f_{eD}^{bf}, f_{eL}^{bf},\) and \(f_{eM}^{bf}\) are buffeting drag, lift, and moment, respectively; \(C_{D_i}^{\prime\prime} = dC_{D_i}/d\alpha,\) \(C_{L_i}^{\prime\prime} = dC_{L_i}/d\alpha,\) and \(C_{M_i}^{\prime\prime} = dC_{M_i}/d\alpha;\) \(u_i(t)\) and \(w_i(t)\) are the horizontal and vertical components of fluctuating wind at the \(i\)th section, respectively; and \(\chi_{D_{bu}}, \chi_{D_{bw}}, \chi_{L_{bu}}, \chi_{L_{bw}}, \chi_{M_{bu}},\) and \(\chi_{M_{bw}}\) are the aerodynamic transfer functions between fluctuating wind velocities and buffeting forces. In most studies (Ding & Lee 2000; Xu et al. 2003, 2004; Guo et al. 2007), all the aerodynamic transfer functions are assumed to be unit.

From Equation (2.40), it can be seen that a series of time histories of fluctuating wind velocity \([u_i(t), w_i(t)]^T\) in the horizontal and vertical directions at various points along the bridge deck is essential to a detailed buffeting analysis. To simulate the stochastic wind velocity field, a fast spectral representation method...
proposed by Cao et al. (2000) which is based on the spectral representation method developed by Shinozuka and Jan (1972) is often adopted (Xu et al. 2003; Xu & Guo 2003; Shum 2004; Guo et al. 2007). This method depends on the assumptions that (1) the bridge deck is horizontal at the same elevation; (2) the mean wind speed and the wind spectra do not vary along the bridge deck; and (3) the distance between any two successive points where wind speeds are simulated are the same. The time histories of the alongwind component $u(t)$ and the upward wind component $w(t)$ at the $p$th point can then be generated by the following equations:

\[
\begin{align*}
u_p(t) &= \sqrt{2(\Delta \omega)} \sum_{l=1}^{N_f} \sum_{m=1}^{N_f} \sqrt{S_{uu}(\omega_{lm})} G_{pl}(\omega_{lm}) \cos(\omega_{lm} t + \varphi_{lm}) \\
w_p(t) &= \sqrt{2(\Delta \omega)} \sum_{l=1}^{N_f} \sum_{m=1}^{N_f} \sqrt{S_{ww}(\omega_{lm})} G_{pl}(\omega_{lm}) \cos(\omega_{lm} t + \varphi_{lm})
\end{align*}
\] (2.42a)

where $\Delta \omega$ is the frequency interval between the spectral lines $= \omega_{up}/N_f$; $N_f$ is the total number of frequency intervals; $\omega_{up} = \text{the upper cutoff frequency}$; $p = 1, 2, \ldots, n_p$; $n_p$ is the total number of points where wind speeds are simulated; $S_{uu}(\omega)$ and $S_{ww}(\omega)$ are the horizontal and vertical wind autospectrum, respectively; $\varphi_{lm}$ is a random variable uniformly distributed between 0 and $2\pi$; and

\[
G_{pl}(\omega) = \begin{cases} 
0, & \text{when } 1 \leq j < l \leq n_p \\
C^{\left|p-m\right|}, & \text{when } l = 1, l \leq p \leq n_p \\
C^{\left|p-m\right|}\sqrt{1 - C^2}, & \text{when } 2 \leq l \leq p \leq n_p
\end{cases}
\] (2.43)

\[
C = \exp\left(-\frac{\lambda_a \omega A}{2\pi U}\right)
\] (2.44)
\[
\omega_{lm} = (m - 1)\Delta \omega + \frac{t}{n_p} \Delta \omega, m = 1, 2, \cdots N_f
\] (2.45)

where $\Delta = \frac{L}{n_p - 1}$; $\lambda_w$ = exponential decay coefficient; and $L$ = span length.

The time histories of modal buffeting forces $F_{ei}^{bf}$ can thus be obtained by using Equation (2.40) when the wind speed vector $[u_i(t) \ w_i(t)]^T$ at the $i$th deck section is simulated. The buffeting forces $F^{bf}(t)$ for the whole bridge can then be determined by

\[
F^{bf} = \begin{bmatrix} F_{e1}^{bf} \\ F_{e2}^{bf} \\ \vdots \\ F_{eN_f}^{bf} \end{bmatrix}
\] (2.46)

The self-excited forces on a bridge deck are attributed to the interaction between wind and bridge motion. When the energy of motion extracted from the flow exceeds the energy dissipated by the system through mechanical damping, the magnitude of vibration can attain catastrophic levels (Shum 2004). The equivalent self-excited drag $f_{eDi}^{se}$, lift $f_{eLi}^{se}$, and moment $f_{eMi}^{se}$ force per unit span at the $i$th section of the bridge deck can be described as (Scanlan 1978; Lau et al. 2000):

\[
F_{ei}^{se} = \begin{bmatrix} 0 \\ f_{eDi}^{se} \\ f_{eLi}^{se} \\ f_{eMi}^{se} \\ 0 \\ 0 \end{bmatrix}
\] (2.47)
where

\[
\begin{align*}
    f_{e_{Di}}^{se} &= \frac{1}{2} \rho_a \bar{U}_i^2 B_i \left[ KP_{i,i}^*(v) \frac{\dot{q}_{ei}}{\bar{U}_i} + KP_{2,i}^*(v) \frac{\dot{h}_{ei}}{\bar{U}_i} + K^2 P_{3,i}^*(v) \rho_{ei} \right] + K^2 P_{4,i}^*(v) \frac{q_{ei}}{B_i} + KP_{5,i}^*(v) \frac{\dot{h}_{ei}}{\bar{U}_i} + K^2 P_{6,i}^*(v) \frac{h_{ei}}{B_i} \right] \quad (2.48a) \\
    f_{e_{Li}}^{se} &= \frac{1}{2} \rho_a \bar{U}_i^2 B_i \left[ KH_{i,i}^*(v) \frac{\dot{q}_{ei}}{\bar{U}_i} + KH_{2,i}^*(v) \frac{\dot{h}_{ei}}{\bar{U}_i} + K^2 H_{3,i}^*(v) \rho_{ei} \right] + K^2 H_{4,i}^*(v) \frac{q_{ei}}{B_i} + KH_{5,i}^*(v) \frac{\dot{h}_{ei}}{\bar{U}_i} + K^2 H_{6,i}^*(v) \frac{h_{ei}}{B_i} \right] \quad (2.48b) \\
    f_{e_{Mi}}^{se} &= \frac{1}{2} \rho_a \bar{U}_i^2 B_i \left[ KA_{i,i}^*(v) \frac{\dot{q}_{ei}}{\bar{U}_i} + KA_{2,i}^*(v) \frac{\dot{h}_{ei}}{\bar{U}_i} + K^2 A_{3,i}^*(v) \rho_{ei} \right] + K^2 A_{4,i}^*(v) \frac{q_{ei}}{B_i} + KA_{5,i}^*(v) \frac{\dot{h}_{ei}}{\bar{U}_i} + K^2 A_{6,i}^*(v) \frac{h_{ei}}{B_i} \right] \quad (2.48c) \\
\end{align*}
\]

and where \( P_{\psi}^*(v) \), \( H_{\psi}^*(v) \), and \( A_{\psi}^*(v) \) (\( \psi = 1, 2, \ldots, 6 \)) are the dimensionless flutter derivatives obtained for the wind tunnel test; \( K = \omega B/\bar{U} \) is the reduced frequency; \( \nu = 2\pi/K \) is the reduced mean wind velocity; \( \omega \) is the circular frequency of vibration; and \( q_{ei} \), \( h_{ei} \), and \( \theta_{ei} \) are the equivalent lateral, vertical, and torsional displacements, respectively. Based on a linear strip theory, the self-excited forces per unit span can be expressed in terms of convolution integrals, as follows (Lin & Yang 1983):

\[
\begin{align*}
    f_{e_{Di}}^{se} &= \frac{1}{2} \rho_a \bar{U}_i^2 \int_{-\infty}^{t} \left[ I_{Dq,i}(t-\tau)q_{ei}(\tau) + I_{ Dh,i}(t-\tau)h_{ei}(\tau) \right] d\tau \quad (2.49a) \\
    f_{e_{Li}}^{se} &= \frac{1}{2} \rho_a \bar{U}_i^2 \int_{-\infty}^{t} \left[ I_{Lq,i}(t-\tau)q_{ei}(\tau) + I_{ Lh,i}(t-\tau)h_{ei}(\tau) \right] d\tau \quad (2.49b) \\
    f_{e_{Mi}}^{se} &= \frac{1}{2} \rho_a \bar{U}_i^2 \int_{-\infty}^{t} \left[ B_i I_{Mq,i}(t-\tau)q_{ei}(\tau) + B_i I_{ Mh,i}(t-\tau)h_{ei}(\tau) \right] d\tau \quad (2.49c) \\
\end{align*}
\]
where \( I_\psi (\psi = Dq, Dh, D\theta, Lq, Lh, L\theta, Mq, Mh, M\theta) \) is the impulse function of the self-excited forces in which \( \psi \) represent the corresponding force components. The relationship between the aerodynamic impulse functions and flutter derivatives can be obtained by taking the Fourier transform of Equation (2.49) and comparing it to the corresponding term in Equation (2.48) (Chen et al. 2000):

\[
\bar{I}_{Dq,i}(\omega) = K^2 \left[ P^*_{4,i}(v) + iP^*_{4,i}(v) \right] \tag{2.50a}
\]
\[
\bar{I}_{Dh,i}(\omega) = K^2 \left[ P^*_{5,i}(v) + iP^*_{5,i}(v) \right] \tag{2.50b}
\]
\[
\bar{I}_{D\theta,i}(\omega) = K^2 \left[ P^*_{3,i}(v) + iP^*_{2,i}(v) \right] \tag{2.50c}
\]
\[
\bar{I}_{Lq,i}(\omega) = K^2 \left[ H^*_{6,i}(v) + iH^*_{5,i}(v) \right] \tag{2.50d}
\]
\[
\bar{I}_{Lh,i}(\omega) = K^2 \left[ H^*_{4,i}(v) + iH^*_{3,i}(v) \right] \tag{2.50e}
\]
\[
\bar{I}_{L\theta,i}(\omega) = K^2 \left[ H^*_{3,i}(v) + iH^*_{2,i}(v) \right] \tag{2.50f}
\]
\[
\bar{I}_{Mq,i}(\omega) = K^2 \left[ A^*_{6,i}(v) + iA^*_{5,i}(v) \right] \tag{2.50g}
\]
\[
\bar{I}_{Mh,i}(\omega) = K^2 \left[ A^*_{4,i}(v) + iA^*_{3,i}(v) \right] \tag{2.50h}
\]
\[
\bar{I}_{M\theta,i}(\omega) = K^2 \left[ A^*_{3,i}(v) + iA^*_{2,i}(v) \right] \tag{2.50i}
\]

where the over-bar denotes the Fourier transform operation and terms containing \( i \) represent imaginary parts. With regard to the flutter derivatives in the frequency domain described in Equation (2.50), the following rational functions for a time domain analysis were proposed by Lin and Ariarantam (1978) to approximate the impulse functions in that domain:

\[
I(\omega) = \left[ C_i + i \frac{2\pi C_2}{\nu} + \sum_{l=1}^{m} C_{i+2} \frac{4\pi^2 + i2\pi d_{l+2}\nu}{d_{l+2}^2 \nu^2 + 4\pi^2} \right] \tag{2.51}
\]
where the value of $m$ determines the level of accuracy of the approximation and $C_1$, $C_2$, $C_{l+2}$, and $d_{l+2}$ ($l = 1, 2, \cdots, m$) are the dimensionless coefficients. The aerodynamic transfer function for each component of self-excited force can be expressed as a rational function using the following relationships:

$$\bar{I}_{D_{q,i}}(\omega) = \bar{I}(\omega); \bar{I}_{D_{h,i}}(\omega) = \bar{I}(\omega); \bar{I}_{D_{\theta,i}}(\omega) = \bar{I}(\omega) \quad (2.52a-c)$$

$$\bar{I}_{L_{q,i}}(\omega) = \bar{I}(\omega); \bar{I}_{L_{h,i}}(\omega) = \bar{I}(\omega); \bar{I}_{L_{\theta,i}}(\omega) = \bar{I}(\omega) \quad (2.52d-f)$$

$$\bar{I}_{M_{q,i}}(\omega) = \bar{I}(\omega); \bar{I}_{M_{h,i}}(\omega) = \bar{I}(\omega); \bar{I}_{M_{\theta,i}}(\omega) = \bar{I}(\omega) \quad (2.52g-i)$$

This implies that the aerodynamic transfer function $\bar{I}_{L_{h,i}}(\omega)$ for the lift induced by the upward motion can be expressed as

$$\bar{I}_{L_{h,i}}(\omega) = K^2 \left[ H_{4,i}^*(v) + iH_{1,i}^*(v) \right] = \left[ C_1 + i \frac{2\pi C_2}{\nu} + \sum_{l=1}^{m} C_{l+2} \frac{4\pi^2 + 12\pi d_{l+2}^2}{d_{l+2}^2 + 4\pi^2} \right]$$

(2.53)

By equating the real and imaginary parts of Equations (2.53), one obtains the relation between the flutter derivatives and the coefficients of $C_{1,l}^{lh}$, $C_{2,l}^{lh}$, $C_{l+2,l}^{lh}$, and $d_{l+2,l}^{lh}$ (where $l = 1, 2, \cdots, m$):  

$$H_{4,i}^* = \frac{c_{1,l}^{lh} \nu^2}{4\pi^2} + \sum_{l=1}^{m} \frac{c_{l+2,l}^{lh} \nu^2}{(d_{l+2,l}^{lh} \nu)^2 + 4\pi^2} \quad (2.54a)$$

$$H_{1,l}^* = \frac{c_{2,l}^{lh} \nu^2}{2\pi} + \sum_{l=1}^{m} \frac{c_{l+2,l}^{lh} d_{l+2,l}^{lh} \nu^3}{2\pi (d_{l+2,l}^{lh} \nu)^2 + 8\pi^3} \quad (2.54b)$$

where the coefficients of $C_{1,l}^{lh}$, $C_{2,l}^{lh}$, $C_{l+2,l}^{lh}$, and $d_{l+2,l}^{lh}$ (where $l = 1, 2, \cdots, m$)
can be determined by using the non-linear least-squares method to fit the measured flutter derivatives at different reduced frequencies.

The inverse Laplace Transform of $\tilde{I}_{L_i}(\omega)$ yields the aerodynamic impulse function

$$I_{L_i}(t) = C_{1,i}^h h(t) + C_{2,i}^h \frac{B_i}{U_i} \dot{h}(t) + C_{3,i}^h \left( \frac{B_i}{U_i} \right)^2 \ddot{h}(t)$$

$$+ \sum_{l=1}^{m} \int_{-\infty}^{t} C_{l+2,i}^h \dot{h}(t) \exp \left[ \frac{-d_{l+3,i}^h U_i}{B_i} (t - \tau) \right] d\tau$$  \hspace{1cm} (2.55)

In practice, the term $C_{3,i}^h$, which is related to the aerodynamic mass, is normally neglected and the value of $m$ is often taken as 2 (Xu et al. 2003). By applying the same procedure to $\tilde{I}_{L_q,i}(\omega)$ and $\tilde{I}_{L_\theta,i}(\omega)$ and substituting their aerodynamic impulse functions into Equation (2.49b), the self-excited lift force at the $i$th section of the bridge deck can then be determined as

$$f_{el,i}^{se} =$$

$$\frac{1}{2} \rho_a U_i^2 B_i \left\{ C_{1,i}^{L_\theta} \dot{\theta}_i(t) + C_{2,i}^{L_\theta} \left[ \frac{B_i}{U_i} \right] \ddot{\theta}_i(t) + \sum_{l=1}^{2} C_{l+1,i}^{L_\theta} \int_{-\infty}^{t} \dot{\theta}_i(t) \exp \left[ \frac{-d_{l+2,i}^{L_\theta} U_i}{B_i} (t - \tau) \right] d\tau \right\}$$

$$+ \frac{1}{2} \rho_a U_i^2 \left\{ C_{1,i}^{L_h} \dot{h}_i(t) + C_{2,i}^{L_h} \left[ \frac{B_i}{U_i} \right] \ddot{h}_i(t) + \sum_{l=1}^{2} C_{l+1,i}^{L_h} \int_{-\infty}^{t} \dot{h}_i(t) \exp \left[ \frac{-d_{l+2,i}^{L_h} U_i}{B_i} (t - \tau) \right] d\tau \right\}$$

$$+ \frac{1}{2} \rho_a U_i^2 \left\{ C_{1,i}^{L_q} \dot{q}_i(t) + C_{2,i}^{L_q} \left[ \frac{B_i}{U_i} \right] \ddot{q}_i(t) + \sum_{l=1}^{2} C_{l+1,i}^{L_q} \int_{-\infty}^{t} \dot{q}_i(t) \exp \left[ \frac{-d_{l+2,i}^{L_q} U_i}{B_i} (t - \tau) \right] d\tau \right\}$$ \hspace{1cm} (2.56)

Similar formulations for self-excited drag $f_{el,i}^{se}$ and moment $f_{eM_i}^{se}$ can be given with analogous definitions. The self-excited force at the $i$th section of the bridge deck can thus be rewritten as
\[
\mathbf{F}_{el}^{se}(t) = \mathbf{E}_{el} \mathbf{X}_{el}(t) + \mathbf{G}_{el} \dot{\mathbf{X}}_{el}(t) + \mathbf{\bar{F}}_{el}^{se}(t) \tag{2.57}
\]

where

\[
\mathbf{X}_{el}(t) = \begin{bmatrix} 0 \\
q_{el}(t) \\
h_{el}(t) \\
\theta_{el}(t) \\
0 \\
\end{bmatrix}
\]

\[
\mathbf{E}_{el} = \frac{1}{2} \rho_{a} \bar{U}_{l}^{2} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\
0 & C_{Dq}^{0} & C_{Dh}^{0} & B_{l} & C_{Dq}^{0} & 0 & 0 \\
0 & C_{Lq}^{0} & C_{Lh}^{0} & B_{l} & C_{Lq}^{0} & 0 & 0 \\
0 & B_{l} C_{Mq}^{0} & B_{l} C_{Mh}^{0} & B_{l}^{2} & C_{Mq}^{0} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\mathbf{G}_{el} = \frac{1}{2} \rho_{a} \bar{U}_{l}^{2} B_{l} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\
0 & C_{Dq}^{0} & C_{Dh}^{0} & B_{l} & C_{Dq}^{0} & 0 & 0 \\
0 & C_{Lq}^{0} & C_{Lh}^{0} & B_{l} & C_{Lq}^{0} & 0 & 0 \\
0 & B_{l} C_{Mq}^{0} & B_{l} C_{Mh}^{0} & B_{l}^{2} & C_{Mq}^{0} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\mathbf{\bar{F}}_{el}^{se} = \begin{bmatrix} 0 \\
\dot{\varphi}_{se} \\
\dot{\varphi}_{el} \\
\dot{\varphi}_{se} \\
\dot{\alpha}_{se} \\
\end{bmatrix}
\]

\[
= \frac{1}{2} \rho_{a} \bar{U}_{l}^{2} \begin{bmatrix} \Sigma_{l=1}^{2} C_{Dq}^{i+3,l} V_{Dq}^{i+3,l}(t) + \Sigma_{l=1}^{2} C_{Dh}^{i+3,l} V_{Dh}^{i+3,l}(t) + B_{l} \Sigma_{l=1}^{2} C_{Dq}^{i+3,l} V_{Dq}^{i+3,l}(t) \\
\Sigma_{l=1}^{2} C_{Lq}^{i+3,l} V_{Lq}^{i+3,l}(t) + \Sigma_{l=1}^{2} C_{Lh}^{i+3,l} V_{Lh}^{i+3,l}(t) + B_{l} \Sigma_{l=1}^{2} C_{Lq}^{i+3,l} V_{Lq}^{i+3,l}(t) \\
B_{l} \Sigma_{l=1}^{2} C_{Mq}^{i+3,l} V_{Mq}^{i+3,l}(t) + B_{l} \Sigma_{l=1}^{2} C_{Mh}^{i+3,l} V_{Mh}^{i+3,l}(t) + B_{l}^{2} \Sigma_{l=1}^{2} C_{Mq}^{i+3,l} V_{Mq}^{i+3,l}(t) \\
0 \\
0 \\
\end{bmatrix}
\]
and where $V_{i+3,l}^\psi (t)$ ($\psi = Dq, Dh, D\theta, Lq, Lh, L\theta, Mq, Mh, M\theta$) are the convolution integrations of the $i$th node and can be calculated using a recursive algorithm (Liu et al. 2004). Taking $V_{4,i}^L\theta (t)$ as an example,

$$V_{4,i}^L\theta (t) = \int_\infty^t \dot{\theta}(t) \exp \left[ \frac{-d_{4,i}^L \theta l}{B_i} (t - \tau) \right] d\tau$$

$$\approx \exp \left[ \frac{-d_{4,i}^L \theta l}{B_i} \Delta t \right] \left[ V_{4,i}^L\theta (t - \Delta t) + \Delta t \dot{\theta}(t - \Delta t) \right]$$ \hspace{1cm} (2.58)

where $\Delta t$ is the time interval. Similar formulations for $V_{i+3,l}^Lq (t)$ and $V_{i+3,l}^Lh (t)$ can be given with analogous definitions. Thus, $\hat{f}^{se}_{el_i}$ can be written as:

$$\hat{f}^{se}_{el_i} = \frac{1}{2} \rho_a \bar{U}_l^2 \sum_{l=1}^{2} C_{i+3,l} \left\{ \exp \left[ \frac{-d_{i+3,l}^L \bar{U} l}{B_i} \Delta t \right] V_{s-1}^Lq + \exp \left[ \frac{-d_{i+3,l}^L \bar{U} l}{B_i} \Delta t \right] \Delta t \dot{q}_{s-1,l} \right\}$$

$$+ \sum_{l=1}^{2} C_{i+3,l} \left\{ \exp \left[ \frac{-d_{i+3,l}^h \bar{U} l}{B_i} \Delta t \right] V_{s-1}^Lh + \exp \left[ \frac{-d_{i+3,l}^h \bar{U} l}{B_i} \Delta t \right] \Delta t \dot{h}_{s-1,l} \right\}$$

$$+ \sum_{l=1}^{2} B_i C_{i+3,l} \left\{ \exp \left[ \frac{-d_{i+3,l}^\theta \bar{U} l}{B_i} \Delta t \right] V_{s-1}^L\theta + \exp \left[ \frac{-d_{i+3,l}^\theta \bar{U} l}{B_i} \Delta t \right] \Delta t \dot{\theta}_{s-1,l} \right\}$$ \hspace{1cm} (2.59)

where the subscript ‘$s - 1$’ represents the previous time step. It is observed that Equation (2.59) is derived using a recursive algorithm for evaluating the integrals in Equation (2.56). Similar formulations exist for self-excited drag and moment.

By applying the same procedure to all deck sections, the modal self-excited force $F^{se}$ can be determined by
\[
F^{se} = \begin{bmatrix}
F_{e1}^{se} \\
F_{e2}^{se} \\
\vdots \\
F_{eN_e}^{se}
\end{bmatrix}
\] (2.60)

The governing equation for the motion of the bridge in terms of a displacement vector \( \mathbf{X} \) under wind load can thus be expressed as

\[
\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{F}^{bf}(t) + F^{se}(t)
\] (2.61)

where \( \mathbf{M} \), \( \mathbf{C} \), and \( \mathbf{K} \) are the global structural mass, damping, and stiffness matrices of the bridge, respectively, and each over-dot \( \dot{\mathbf{X}}_i(t) = [p_i(t), q_i(t), h_i(t), \theta_{pi}(t), \theta_{qi}(t), \theta_{hi}(t)]^T \) denotes one order of partial differentiation with respect to time.

Owing to the fact that the total response of a long suspension bridge can be obtained as the superposition of the solution of the independent modal equations; the modal superposition technique is adopted to solve the dynamic displacement induced by buffeting force and self-excited force in Equation (2.61). The displacement vector \( \mathbf{X}(t) \) can be expressed in terms of the modal coordinates of the bridge:

\[
\mathbf{X}(t) = \mathbf{\Phi}\mathbf{q}(t)
\] (2.62)

where \( \mathbf{q}(t) = [q_1(t), q_2(t), \ldots q_{N_m}(t)]^T \) is the generalized displacement vector and \( N_m \) is the number of modes involved in the calculation; and \( \mathbf{\Phi} = [\Phi_1, \Phi_2, \ldots, \Phi_{N_m}]^T \) is the mode shape matrix with the dimensions
The equation of motion (Equation (2.61)) can then be rewritten as

\[
\tilde{M} \ddot{q} (t) + \tilde{C} \dot{q} (t) + \tilde{K} q (t) = Q^{bf} (t) + Q^{se} (t)
\]  

(2.63)

where \( \tilde{M} \), \( \tilde{C} \), and \( \tilde{K} \) are the generalized mass, damping, and stiffness matrices, respectively; and \( Q^{bf} = \Phi^T F^{bf} \) and \( Q^{se} = \Phi^T F^{se} \) are the generalized buffeting and self-excited force vectors, respectively. Substituting Equation (2.57) into Equation (2.63) yields

\[
\tilde{M} \ddot{q} (t) + C^s \dot{q} (t) + K^s q (t) = Q^{bf} (t) + \tilde{Q}^{se} (t)
\]  

(2.64)

where

\[
\tilde{Q}^{se} = \Phi^T \tilde{F}^{se}; \quad C^s = \tilde{C} - \Phi^T \Phi; \quad K^s = \tilde{K} - \Phi^T \Phi
\]

In predicting the dynamic response of a long suspension bridge, the Newmark-\( \beta \) method is used to find the step-by-step solution for the governing equation of motion of the bridge under wind load, as shown in Equation (2.64), because of its unconditional numerical stability in comparison with other schemes (Bathe 1982). Under the Newmark-\( \beta \) method, the displacement, velocity, and acceleration at time \( t + \Delta t \) can be obtained by:

\[
q_{t+\Delta t} = \tilde{K}^{-1} \tilde{Q}_{t+\Delta t}
\]  

(2.65a)

\[
\dot{q}_{t+\Delta t} = a_0 (q_{t+\Delta t} - q_t) - a_2 \ddot{q}_t - a_3 \dot{q}_t
\]  

(2.65b)

\[
\ddot{q}_{t+\Delta t} = \ddot{q}_t + a_0 \dot{q}_t + a_2 \ddot{q}_{t+\Delta t}
\]  

(2.65c)
where
\[ \ddot{K} = K^s + a_0 \ddot{M} + a_4 C^s \]  

\[ \ddot{Q}_{t+\Delta t} = Q_{t+\Delta t}^{bf} + \ddot{Q}_{t+\Delta t}^{se} + \dot{M}(a_0 q_t + a_2 \dot{q}_t + a_3 \ddot{q}_t) + C^s(a_1 q_t + a_4 \dot{q}_t + a_5 \ddot{q}_t) \]

in which \( a_i (i = 0, 1, \ldots, 7) \) are the constant coefficients given by

\[ a_0 = \frac{1}{\beta \Delta t^2}; \quad a_1 = \frac{\gamma}{\beta \Delta t}; \quad a_2 = \frac{1}{\beta \Delta t}; \quad a_3 = \frac{1}{2\beta} - 1 \]  

\[ a_4 = \frac{\gamma}{\beta} - 1; \quad a_5 = \left[ \frac{\gamma}{\beta} - 2 \right] \frac{\Delta t}{2}; \quad a_6 = (1 - \gamma) \Delta t; \quad a_7 = \gamma \Delta t \]

where \( \gamma \) and \( \beta \) are taken as 0.5 and 0.25 (Liu et al. 2004). Hence, the dynamic responses over a time period can be obtained using a step-by-step procedure. The displacement of the bridge can be determined using Equation (2.62).

2.3.4. Insights from Previous Studies

A review of previous studies shows that all mean and buffeting forces are lumped at the centre of elasticity that neglects the spatial distribution of wind pressure on the deck surface as a whole. This neglect may not only have a considerable impact on the accuracy of wind response prediction, but may also preclude the estimation of the local behavior of the bridge deck. In this regard, Xu and his colleagues (2007b) proposed a procedure for considering the spatial distribution of both buffeting forces and self-excited forces on a bridge deck structure. Their method is based on the observation of wind pressure distribution in wind tunnel tests or computational fluid dynamics (CFD).
In the proposed procedure, Xu et al. (2007b) first defined that the forces \( \{F_k\}_{j,i} \) at the \( k \)th node and \( \{F_{k-1}\}_{j,i} \) at the \((k-1)\)th node due to the wind pressure acting on the \( j \)th element of the bridge deck at the \( i \)th section (refer to Figure 2.9) can be obtained by:

\[
\{F_{k-1}, F_k\}_{j,i} = \left\{ \int_{0}^{l_{j,i}} (1 - s_i/l_{j,i}) P_{j,i}^m(s_i) \, ds_i, \int_{0}^{l_{j,i}} (s_i/l_{j,i}) P_{j,i}^m(s_i) \, ds_i \right\}
\]

where \( l_{j,i} \) = the length of the \( j \)th element of the \( i \)th section \((j = 1,2,\ldots,N_{ei})\); \( N_{ei} \) = the total number of elements used to model the \( i \)th deck section; \( P_{j,i}^m(s_i) \) is the mean wind pressure distribution over the \( j \)th element of the \( i \)th section; and \( s_i \) is the local element coordinate. The force \( \{F_k\}_{j,i} \) at the \( k \)th node on the \( j \)th element of the \( i \)th deck section in the local coordinate can then be converted to \( \{F_{kx}, F_{kz}\}_{j,i} \) in the \( x-y-z \) global coordinate system. By adding the force vectors at the \( k \)th node contributed by all elements together, the static force vector at the \( k \)th node of the \( i \)th deck section can be represented as \( \mathbf{F}_{k,i} = [0, F_{kx}, F_{kz}, 0,0,0]^T \), in which \( k = 1,2,\ldots,N_{ni} \), and \( N_{ni} \) is the total number of nodes used to model the \( i \)th deck section.

As shown in Figure 2.10, the mean wind pressure distributions on the deck sections are described by three unknown variables: \( p_1 \), \( p_2 \), and \( p_3 \). To tackle this problem, the following relationships are applied (Xu et al. 2007b):

\[
\sum_{k=1}^{N_{ni}} f_{kx}^f = f_{eDi}
\]

\[
\sum_{k=1}^{N_{ni}} f_{kz}^f = f_{eLi}
\]

\[
\sum_{k=1}^{N_{ni}} \left( P_{kiy}^s q_{ki} + P_{kiz}^s h_{ki} \right) = M_{iz}^s + M_{iy}^s = f_{eMi}^s
\]
where $q_{ki}^c$ and $h_{ki}^c$ are the lateral and vertical coordinates, respectively, of the
$k$th node with respect to the centre of elasticity at the $i$th deck section, and $M_{iz}^{sf}$
and $M_{iy}^{sf}$ are the moments about the centre of elasticity due to the vertical and
lateral forces, respectively. In matrix notation, Equation (2.69) can be expressed
as:

$$\mathbf{A}^{sf}_i \begin{bmatrix} 0 \\ p_1 \\ p_2 \\ p_3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ f_{eDi}^{sf} \\ f_{eLi}^{sf} \\ f_{eMi}^{sf} \\ 0 \end{bmatrix} = \mathbf{F}_{ei}^{sf} \quad (2.70)$$

in which

$$\mathbf{A}^{sf}_i = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & F_y(p_1) & F_y(p_2) & F_y(p_3) & 0 & 0 \\
0 & F_z(p_1) & F_z(p_2) & F_z(p_3) & 0 & 0 \\
0 & M(p_1) & M(p_2) & M(p_3) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

where $F_y(p_\psi)$, $F_z(p_\psi)$, and $M(p_\psi)$ (for $\psi = 1,2,3$) are the coefficients of
Equation (2.69) expressed in terms of $p_1$, $p_2$, and $p_3$, as shown in Appendix A.

As it is impossible to obtain the distribution of fluctuating wind pressure for the
whole bridge deck as a function of time, an approximate approach based on $\mathbf{A}^{bf}_i$
in Equation (2.41) was suggested to tackle this problem. The suggested approach
involves assuming that the equivalent fluctuating wind force $\mathbf{F}_{ei}^{bf}(t)$ at the $i$th
deck section can be decomposed as:

$$\mathbf{F}_{ei}^{bf}(t) = \mathbf{F}_{ei}^{sf} \times \mathbf{F}_i^l(t) \quad (2.71)$$
where $\mathbf{F}_i^f(t)$ is the time-dependent part of the wind force. This part was assumed to be linearly related to the fluctuating wind speeds $u_i(t)$ and $w_i(t)$:

$$\mathbf{F}_i^f(t) = [c_1 \ c_2] \begin{bmatrix} u_i(t) \\ w_i(t) \end{bmatrix}$$  \hspace{1cm} (2.72)$$

where $c_1$ and $c_2$ are the constants. The wind buffeting force at the $i$th deck section can be rewritten as:

$$\mathbf{F}_{ei}^{bf} (t) = \mathbf{A}_i^{sf} \begin{bmatrix} 0 & 0 & p_1 c_1 & p_1 c_2 & p_2 c_1 & p_2 c_2 & p_3 c_1 & p_3 c_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_i(t) \\ w_i(t) \end{bmatrix}$$  \hspace{1cm} (2.73)$$

By comparing Equation (2.73) with Equation (2.41), the independent unknown variables $[p_1 c_1, p_2 c_1, p_3 c_1]^T$ and $[p_1 c_2, p_2 c_2, p_3 c_2]^T$ can be solved. The wind force distributions over the $i$th deck section can also be determined by replacing the result of $[p_1 c_1, p_2 c_1, p_3 c_1]^T$ with $[p_1, p_2, p_3]^T$ for $f_{ui}$ and $[p_1 c_2, p_2 c_2, p_3 c_2]^T$ with $[p_1, p_2, p_3]^T$ for $f_{wi}$ in the sections displayed in Figure 2.10. The buffeting force $\mathbf{F}_{ki}^{bf}$ of the $k$th node acting on the $i$th section of the bridge deck can then be expressed as:

$$\mathbf{F}_{ki}^{bf}(t) = \begin{bmatrix} 0 \\ f_{kiy}^{bf} \\ f_{kiz}^{bf} \\ 0 \\ 0 \\ 0 \end{bmatrix} \times u_i(t) + \begin{bmatrix} 0 \\ f_{kiy}^{bf} \\ f_{kiz}^{bf} \\ 0 \\ 0 \\ 0 \end{bmatrix} \times w_i(t)$$  \hspace{1cm} (2.74)$$
The nodal buffeting forces $\mathbf{F}^{bf}(t)$ for the whole bridge can then be determined by

$$
\mathbf{F}^{bf}(t) = \begin{bmatrix}
\mathbf{F}_1^{bf}(t) \\
\mathbf{F}_2^{bf}(t) \\
\vdots \\
\mathbf{F}_{N_s}^{bf}(t)
\end{bmatrix}
$$

(2.75)

where $\mathbf{F}_i^{bf}(t) = [\mathbf{F}_{1,i}^{bf}(t), \mathbf{F}_{2,i}^{bf}(t), \ldots, \mathbf{F}_{N_{se},i}^{bf}(t)]^T$, in which $i = 1, 2, \ldots, N_s$, and $N_s$ is the total number of sections considered in the model of the bridge.

Similarly, the self-excited force expressed by Equation (2.57) relates to the centre of elasticity of the $i$th deck section. Hence, the force model must be distributed to the nodal points of the section. The distributions based on the rigid body motion relationships between the motions at the nodal point and those at the centre of elasticity of the deck section (Lau et al. 2000) were applied by Xu et al. (2007b). The displacement relationships between the nodal points and the center of elasticity can be expressed as:

$$
\mathbf{X}_{ei} = \mathbf{N}_{i}^{se}\mathbf{X}_i
$$

(2.76)

where $\mathbf{N}_{i}^{se}$ is the displacement transformation matrix given by

$$
\mathbf{N}_{i}^{se} = \begin{bmatrix}
0 \\
N_{Di}^{se} \\
N_{Li}^{se} \\
N_{Mi}^{se} \\
0
\end{bmatrix}
$$

(2.77a)
in which

\[ N_{Bi}^{se} = \left\{ \begin{array}{c} a_{1i} h_{ci} b_{1i}, 0,0,0 \\ \vdots \\ 0, a_{Nni} h_{ci} b_{Nni}, 0,0,0 \end{array} \right\}_{\text{node } N_n} \]  

(2.77b)

\[ N_{Li}^{se} = \left\{ \begin{array}{c} 0,0, a_{1i} - q_{ci} b_{1i}, 0,0,0 \\ \vdots \\ 0,0, a_{Nni} - q_{ci} b_{Nni}, 0,0,0 \end{array} \right\}_{\text{node } N_n} \]  

(2.77c)

\[ N_{Mi}^{se} = \left\{ \begin{array}{c} 0,0, b_{1i}, 0,0,0 \\ \vdots \\ 0,0, b_{Nni}, 0,0,0 \end{array} \right\}_{\text{node } N_n} \]  

(2.77d)

where \( q_{ci} \) and \( h_{ci} \) are the lateral and vertical coordinates of the centroid of the \( i \)th deck section, respectively; \( a_{ki} = \Sigma_j l_{jki} / 2L_i \), in which \( j = 1,2,\ldots,N_{ei} \) and \( k = 1,2,\ldots,N_{ni} \), the summation \( \Sigma_j l_{jki} \) applies to all of the elements \( j \) connected at node \( k \), and \( L_i \) is the summation of the lengths of all the elements in the \( i \)th deck section; and \( b_{ki} = 1/N_{ni} q_{ki} \), in which \( q_{ki} \) is the lateral coordinate of the \( k \)th node.

To substitute Equation (2.76) into Equation (2.57), the self-excited forces for the \( i \)th deck section expressed in terms of the nodal displacement vector will become:

\[ \mathbf{F}_{ei}^{se} = \mathbf{E}_{ei} \mathbf{N}_{i}^{se} \mathbf{X}_i + \mathbf{G}_{ei} \mathbf{N}_{i}^{se} \dot{\mathbf{X}}_i + \mathbf{F}_{ei}^{se} \]  

(2.78)

By applying the virtual work principle, the self-excited forces at the centre of elasticity of the \( i \)th section can be distributed to all nodes by

\[ \mathbf{F}_{i}^{se} = (\mathbf{N}_{i}^{se})^T \mathbf{F}_{ei}^{se} = \mathbf{E}_{i} \mathbf{X}_i + \mathbf{G}_{i} \dot{\mathbf{X}}_i + (\mathbf{N}_{i}^{se})^T \dot{\mathbf{F}}_{ei}^{se} \]  

(2.79)
in which \( \mathbf{F}^{se}_i = [\mathbf{F}^{se}_{1,i}, \mathbf{F}^{se}_{2,i}, \ldots, \mathbf{F}^{se}_{N_n,i}]^T \) is the nodal self-excited force vector and
\[
\mathbf{F}^{se}_{k,i} = \left[0, F^{se}_{k,iy}, F^{se}_{k,iz}, 0, 0, 0\right]^T;
\]
and \( \mathbf{E}_i = (\mathbf{N}^{se}_i)^T \mathbf{E}_{e,i} \mathbf{N}^{se}_i \), and \( \mathbf{G}_i = (\mathbf{N}^{se}_i)^T \mathbf{G}_{e,i} \mathbf{N}^{se}_i \) are the aeroelastic stiffness and damping matrices of the \( i \)th section of the bridge deck related to the nodal self-excited forces, respectively. By applying the same procedure to all deck sections, the modal self-excited force \( \mathbf{F}^{se} \) displayed in Equation (2.61) can be determined by

\[
\mathbf{F}^{se} = \mathbf{E}\mathbf{X} + \mathbf{G}\dot{\mathbf{X}} + (\mathbf{N}^{se})^T \mathbf{F}^{se}
\]  

(2.80)

### 2.4. WIND AND STRUCTURAL HEALTH MONITORING OF LONG-SPAN CABLE-SUPPORTED BRIDGES

#### 2.4.1. Background

To secure the structural and operational safety of a bridge and issue early warnings on the occurrence of damage or deterioration before costly repairs become necessary or even catastrophic collapse, it is essential to inspect the infrastructure of the bridge from time to time. The state of in-service civil infrastructure has traditionally been assessed by way of periodic visual inspections and simple localized testing (Omenzetter & Brownjohn 2006). However, because of various shortcomings in this approach, such as significant workforce demands, insufficient frequency, the inaccessibility of critical parts of the structure, and a lack of information on actual loading, such assessments tend to result in subjective and inaccurate evaluations of structural safety and reliability (Phares et al. 2001; Omenzetter & Brownjohn 2006). The bridge engineering community has thus turned its attention to instrumented structural
health monitoring systems (SHMS) designed to complement and enhance visual inspection programs.

An analogy can be drawn between structural health monitoring and human health management, because both involve tracking a variety of performance or health measurements through the utilization of various strategies and specialized diagnostic tools. Measurement results are then interpreted in conjunction with application-specific knowledge to quantify the health condition of a structure or patient in an objective manner (Aktan et al. 2000). The significance of implementing long-term SHMS for large-scale bridges is now recognized in many parts of the world. Furthermore, the philosophy underlying the technology used in such systems has reached a certain level of maturity. A significant number of long-span bridges with spans of 100 m or longer in Europe (Brownjohn et al. 1994; Andersen & Pedersen 1994; Myrvoll et al. 2000; Casciati 2003), the United States (Barrish et al. 2000; Pines & Aktan 2002; Wang 2004), Canada (Cheung et al. 1997; Cheung & Naumoski 2002; Mufti 2002), Japan (Sumitro et al. 2001; Wu 2003; Fujino & Abe 2004), Korea (Kim et al. 2002; Koh et al. 2003; Yun et al. 2003), China (Lau et al. 1999; Xiang 2000; Ou 2004; Wong 2004), and other countries (Nigbor & Diehl 1997; Thomson et al. 2001) have been instrumented with long-term monitoring systems.

2.4.2. The Tsing Ma Bridge SHMS

Among all permanent bridge monitoring programs, the most noteworthy is the real-time bridge health monitoring system for the Tsing Ma Bridge in Hong Kong known as WASHMS (Wind And Structural Health Monitoring System).
This system is in the process of becoming the standard mechartronic system used in the design and construction of large-scale multi-disciplinary bridge projects in Hong Kong and China (Wong 2007). This makes the Tsing Ma Bridge, which had the WASHMS installed in 1997, a good choice for illustrating the functions of SHMSs.

Hong Kong’s Tsing Ma Bridge is the longest suspension bridge in the world. It carries a dual three-lane highway on the upper level of the bridge deck, in addition to two railway tracks and two protected carriageways on the lower level within the bridge deck. It spans the main shipping channel between Tsing Yi Island and Ma Wan Island, with a main span of 1,377 m and a total length of 2,132 m (see Figure 2.11). The height of the two bridge towers, the Tsing Yi tower and the Ma Wan tower, is about 206 m from the base level to the tower saddle. The two main cables, which are located 36 m apart along a north-south axis, are accommodated by the four saddles located at the top of the tower legs at the main span. On the Tsing Yi side, the main cables extend from the tower saddles to the main anchorage on the ground, forming a 300 m Tsing Yi side span. On the Ma Wan side, the main cables extended from the Ma Wan tower are first secured by pier saddles at the deck level and run a horizontal distance of about 355.5 m from the Ma Wan tower before being further secured by the main anchorage saddles on the ground. The tower legs are made of reinforced concrete, and the two towers are built on massive reinforced concrete slabs found on competent rock. The bridge deck is supported by suspenders at the main span to allow a sufficiently large navigation channel, as well as in the Ma Wan side span to minimize the number of substructures found in the sea. Due to the highway
layout requirement, the deck on the Tsing Yi side is supported by three piers rather than by suspenders. This arrangement and the difference between the spans on the Ma Wan and Tsing Yi sides introduce some asymmetry with respect to the midspan of the bridge.

Given that the Tsing Ma Bridge is located in one of the most active typhoon regions in the world, the WASHMS devised by the Highways Department of the Hong Kong Special Administrative Region (HKSAR) Government has been in place on the bridge since 1997. The system architecture of the WASHMS operated on the bridge is composed of five integrated modules: (1) the sensory system; (2) the data acquisition and transmission system; (3) the data processing and control system; (4) the bridge health evaluation system; and (5) the inspection and maintenance system (Wong et al. 2000). The WASHMS sensory system refers to the 300 sensors installed on the bridge, including anemometers, temperature sensors, corrosion cells, hygrometers, barometers, rainfall gauges, weigh-in-motion stations, digital video cameras, weldable foil-type strain gauges, servo-type accelerometers, level sensing stations, displacement transducers, tiltmeters, buffer sensors, bearing sensors, and tension magnetic gauges. The layout of these sensors on the Tsing Ma Bridge is illustrated in Figure 2.12. These sensors are grouped into four categories according to their purpose: monitoring of environmental, traffic, and global conditions, and monitoring of the local conditions of the bridge (Wong et al. 2001a, 2001b). The signals received from this sensory system are then digitalized and collected by module 2, the data acquisition and transmission system. This module delivers the data to the bridge monitoring room through a fibre optic cabling network. The data
Chapter 2

2.4.3. The Integration of GPS with SHMS

Among all measurement parameters in the WASHM system, displacement is one of the paramount variables in assessing the integrity and safety of long-span suspension bridges, because any displacement of the bridge that deviates from the geometrical design configurations will redistribute the stresses/strains in bridge components and affect the load carrying capacity of the bridge as a whole.
(Wong et al. 2001a, 2001b). Before the global positioning system (GPS) was deployed on the Tsing Ma Bridge, its displacements were usually monitored by level sensing stations and servo-type accelerometers (Wong et al. 2001a, 2001b). Although level sensing stations with a sampling rate of 2.56 Hz can provide real-time displacement information with a measurement accuracy of approximately 2 mm, they rely on the measurement of pressure changes in sensors installed along a network of flexible hoses, involving high pipeline system installation costs. In addition, they cannot measure lateral or longitudinal displacement on deck sections in the horizontal plane (Wong et al. 2001a, 2001b).

To determine the three-dimensional displacement response of the bridge, double integration of measured acceleration data using high precision servo-type accelerometers was required. However, the natural frequencies of the bridge deck are very small; the actual displacement values cannot be truly reflected by double integration. The Highways Department of the HKSAR Government thus installed a new Global Positioning System-On-Structure Instrumentation System (GPS-OSIS) in December 2000 to monitor the absolute displacements of the cables, the stiffening deck, and the bridge towers (Wong et al. 2001a, 2001b).

The real-time GPS-OSIS installed on the bridge comprises five sub-systems: a GPS sensory system, a local data acquisition system, a global data acquisition system, a computer system, and a fiber optic network system. The GPS sensory system consists of 2 base reference stations and 14 dual-frequency, 24-channel, carrier-phase rover stations. These rover stations are respectively installed on the towers, cables, and deck of the bridge where the maximum displacements are expected. To ensure the stability and reliability of data transmission in a harsh
environment, optical fiber, which is insensitive to electromagnetic waves and lightening effects and enables high-quality, rapid data transmission, is used on the Tsing Ma Bridge. Through a highly effective and stable optical fiber network system, signals from the GPS sensory system are collected and delivered to the local data acquisition station on the bridge before being transferred to the global data acquisition station located in the bridge monitoring room. Geodetic coordinates (in altitude and longitude) obtained from 14 GPS rover stations are then delivered to the GPS computer system by the global data acquisition station. This GPS computer system consists of two workstations. One of these workstations is used for the processing, archiving, storage, and graphical treatment of data to allow the real-time motions of the bridge to be displayed. This workstation is also responsible for operating and controlling the GPS sensory systems, the local and global data acquisition stations, and the optical fiber network. The other workstation is used to post-process the GPS data and to use the measured data obtained from the WASHMS in evaluating the current health condition of the bridge.

2.4.4. Use of WASHMS

The local topography surrounding the Tsing Ma Bridge is quite unique and complex, encompassing ocean, islands, and mountains of between 69 and 500 m high. This complex topography makes for complicated wind conditions at the bridge site. In addition, Hong Kong’s latitude (N22.2°) and longitude (E114.1°), as well as its location on the south eastern coast of China facing the South China Sea, mean its wind conditions are dominated by monsoons and typhoons. The WASHMS installed on the bridge makes it possible to investigate these wind
conditions and gain a better understanding of high wind loading on the bridge. On August 2, 1997, Typhoon Victor headed for Hong Kong and at one point in time was centered adjacent to the bridge. The WASHMS recorded the wind speed and bridge response time histories in a timely fashion. Wind data recorded over a seven-hour period during the typhoon were used to analyze wind characteristics at the bridge site (Xu et al. 2000). It was found that there was a sudden change of wind direction and that the wind speed fell dramatically when the eye of Typhoon Victor crossed over the bridge. The measurement results also showed that the turbulence intensities of winds during Typhoon Victor were higher than those attributable to monsoon, and that some wind samples were not stationary. These findings motivated Xu and Chen (2004) to propose a new approach for characterizing non-stationary wind speed. In the new approach they suggested, a measured wind sample of one hour in duration was decomposed into the sum of a deterministic time-varying mean wind speed plus a stationary fluctuating wind component using empirical mode decomposition (EMD). Xu (2008a, 2008b) also used a joint probability density function for monsoon for a complete population of wind speed and wind direction measured over the period January 1, 2000 - December 31, 2005. The results of these studies are essential to any assessment of the wind-induced fatigue damage to the bridge.

In practice, the real-time monitoring of structural behaviour under wind load using the WASHMS is the best way of examining the design rules currently used and of developing and verifying new analytical methods, where necessary. For instance, Xu et al. (2003) presented a framework for predicting the dynamic response of a long suspension bridge to high winds and running trains. One year
later, Xu et al. (2004) further extended their work by investigating the fully dynamic interaction of a long-span cable-stayed bridge with running trains subjected to cross winds, using the most up-to-date information in the areas of wind-bridge interaction, bridge-train interaction, and wind-train interaction. However, these two studies were based purely on numerical analysis. To verify the rationality and feasibility of the proposed framework and the accuracy of the dynamic system responses predicted by the framework, the field measurement data recorded by the WASHMS during Typhoon York were analyzed (Xu et al. 2007a; Guo et al. 2007).

Xu et al.’s earlier study (Xu et al. 2000) showed that the mean wind directions deviated from the normal of the bridge longitudinal axis. However, the analytical methods commonly used to predict the buffeting response of long suspension bridges assume that mean wind speed comes at a right angle to the longitudinal axis and that wind characteristics remain constant along the bridge deck. This leads to some difficulties in examining the accuracy of commonly used analytical methods for bridges subject to skew winds. Xu and his colleagues thus improved the commonly used methods by considering skew winds on the basis of the quasi-steady theory and the oblique strip theory, in conjunction with the finite element method and the pseudo excitation method (Xu & Zhu 2005; Zhu & Xu 2005). To verify the effectiveness of the proposed approach, the field measurement data recorded by the WASHMS during Typhoon Sam were analyzed.
2.4.5. Necessity for and Challenges Presented by a Combination of Computation Simulation and SHMS

As noted above, the purpose of installing a comprehensive WASHMS on a long-span cable-supported bridge is to facilitate bridge performance and safety assessments through the analysis and synthesis of real-time measurement data. Such real-time data include information on wind, temperature, global bridge response, local strain response, and other factors. However, many key issues remain unresolved, such as how to take full advantage of real-time data to facilitate effective and reliable bridge health assessment. Moreover, how to place sensors to measure parameters that influence the performance of the structural system is a key to success of structural health monitoring program (Ansari 2005, 2007; Liang et al. 2005b). Because civil engineering structures are large in dimension, the number of sensors available on a long suspension bridge is inevitably limited, and structural defects or degradation will not necessarily occur adjacent to the sensors. There is always a possibility that the worst structural conditions will not be directly monitored by sensors. Therefore, to enable effective assessment of the structural health of a bridge, it is imperative that the WASHMS is used to run computer simulations in harsh environments and under various types of loading.

In this regard, Xu et al. (2007b) have recently established a complex structural health monitoring-based finite element model (FEM) for the Tsing Ma Bridge. This structural health monitoring-based FEM, which uses a total of 15,904 beam elements to model the bridge deck, accurately replicates the geometric details of the as-built complicated deck, thereby enabling local structural behaviour linked
to stress and strain and which is prone to cause local damage to be estimated directly. Based on the established structural health monitoring-based FEM, Xu et al. (2007b) proposed a numerical procedure for buffeting-induced stress analysis of the Tsing Ma Bridge. The buffeting-induced acceleration at the locations of 12 accelerometers and stress responses at the locations of 9 strain gauges installed on the Tsing Ma Bridge were computed using the mode superposition method and compared with the measured results. The comparative results showed that the computed acceleration and stress time histories were similar to the measured results in both pattern and magnitude. Xu et al. (2007b) also carried out a buffeting-induced stress analysis by considering a 15 m/s mean wind perpendicular to the bridge axis. The first 80 modes of vibration of the Tsing Ma Bridge were considered in the computation. The cross section of the bridge deck at the Ma Wan tower was identified as the most critical section by comparing the maximum values and standard deviations of all stress time histories. Since the ultimate goal of the WASHMS program is to save the bridge from catastrophic collapse, the trial success of Xu and his colleagues should be further extended by carrying out a progressive collapse analysis for the bridge under extreme wind loading utilizing adequate WASHMS measurement data.

2.4.6. Insights from Previous Studies

As noted above, SHMSs installed on long-span cable-supported bridges make it possible to investigate the loading condition of a bridge and assess its performance under various service loads. Studies that have investigated the behaviour of long-span cable-supported bridges in strong winds include those conducted by Brownjohn et al. (1994), Xu et al. (2000), and Miyata et al. (2002).
However, these investigations were all based on field measurement data recorded during one or two particularly strong wind events. Wind and wind-induced bridge response statistics based on long-term SHMS measurement data have not been compiled. In particular, there has been no previous attempt to model the statistical relationships between wind and wind-induced responses, in spite of the fact that research is very important to the performance assessment of long-span cable-supported bridges.

Furthermore, given the increase in the length of cable-supported bridge spans, bridges are more likely to suffer from considerable buffeting-induced vibrations over a wide range of wind speeds. Such bridges will be subject to these types of vibrations over their entire service life. The frequent occurrence of buffeting responses of a relatively large amplitude may result in serious fatigue damage to steel structural components and connections, which may subsequently lead to catastrophic failure. Due to this risk, assessing wind-induced stresses/strains in long-span cable-supported bridges is a task that cannot be ignored. As noted above, civil engineering structures are usually large in dimension, geometrically complex with different elements and joints, and composed of different materials (Ansari 2005, 2007; Liang et al. 2005b). Localized sensing of stress/strain is only effective if the problematic areas are known in advance (Liang et al. 2005b). Given the reality that the number of sensors available for strain measurements is limited and structural defects or degradation do not necessarily occur adjacent to the sensors, Yoshida et al. (2003) and Tamura et al. (2004) have both suggested evaluating member stress in a 108 m steel tower building using a hybrid FEM analysis/GPS approach. A review of previous studies on long-span suspension
bridges shows there has been no previous investigation of the combined use of FEM and GPS sensors to quantify the risks attached to major bridge structural components. The main reason for this is that this method involves the use of a comprehensive structural health monitoring-based finite element model to replicate the geometric details of complicated bridge decks. Given the trial success of Xu and his colleagues (2007b) in investigating a structural health monitoring-based FEM for the Tsing Ma Bridge, the use of a hybrid FEM/GPS approach to quantify local stresses/strains in major bridge structural components and assess their strength can be advocated.
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Figure 2.1. GPS system elements (source: Garmin 2000)
Figure 2.2. Trends in sampling rates of GPS receivers, 2001-2007
Figure 2.3. Geometry of multipath

Figure 2.4. LMS adaptive filtering configuration
Figure 2.5. Schematic representation of satellite configurations
Figure 2.6. Definition of directional cosine vectors for alongwind, crosswind, and upward turbulent wind components
Figure 2.7. Static force components on a deck section

Figure 2.8. Displacement at global coordinate system
Figure 2.9. Force distribution at the $k$th node
Figure 2.10. Wind pressure distributions over three typical deck sections
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Figure 2.12. The layout of the sensory system and the data acquisition system (source: Wong 2007)
Chapter 3

ASSESSING THE DYNAMIC MEASUREMENT ACCURACY OF THE GPS

3.1. INTRODUCTION
The global positioning system (GPS) is an emerging tool used for measuring and monitoring both the static and dynamic displacement responses of large infrastructures to gust winds. However, the accuracy of dynamic displacement measurements taken using GPS at the sub-centimeter to millimeter level depends on many factors such as the data sampling rate, satellite coverage, atmospheric effects, the multipath effect, and the GPS data processing method used. Therefore, GPS performance must be thoroughly validated for the application to large infrastructures. In previous calibration studies conducted by Tamura et al. (2002), Kijewski-Correa (2003), and Nickitopoulou et al. (2006), only horizontal uni-axial motions or bi-axial circular motions were performed in assessing the application of the GPS to tall buildings. However, the objective of this thesis is to highlight the potential application of the GPS technology in monitoring long-span cable-supported bridges under high winds. In long-span bridge health monitoring, it is important to measure the wind-induced dynamic displacement responses of the bridge deck in the vertical plane and those of the bridge towers in the horizontal plane. In addition, the fundamental frequency of a long-span cable-supported bridge in one direction, which is much lower than that of a
building, may not be the same as that in the other direction. Therefore, a detailed calibration study assessing the dynamic displacement measurement accuracy of the GPS in the longitudinal, lateral, and vertical directions should be conducted for the application to a long-span cable-supported bridge. In this connection, an advanced motion simulation table that can simulate various types of long-span cable-supported bridge motion is desirable. Furthermore, new types of GPS receivers are released from time to time. The use of the motion simulation table to assess the measurement performance of various types of GPS for wind-induced displacement responses of long-span bridges may become routine. In Chapter 4, the motion simulation table is also used as a test station to facilitate the validation of an integrated GPS-accelerometer data processing technique.

Given the above, this study develops an advanced motion simulation table used as a test station to simulate various types of two-dimensional motions of either bridge towers in the horizontal plane or bridge decks in the vertical plane. The antenna of a GPS receiver is installed on this motion simulation table and used to measure table motion in an open area. Based on an analysis of background noise recorded in static tests, a band-pass filtering scheme is designed and applied to the table motion data recorded by the GPS. Finally, the dynamic performance validation of the GPS is accomplished through a comparison with the original motions generated by the motion simulation table. The original motions with which this chapter is concerned include one-dimensional sinusoidal motion, two-dimensional circular motion, and the wind-induced horizontal and vertical dynamic displacement responses of large civil engineering structures.
3.2. MOTION SIMULATION TABLE

3.2.1. Table Configuration

The major components of the motion simulation table (Figure 3.1) include a movable platform, two ball screws, two servomotors, an electronic control system, a power terminal box, a supporting frame, and a computer system with a 16-channel data acquisition system. The size of the movable platform (Figure 3.2) is 400 mm (length) × 400 mm (width) × 5 mm (thickness), with threaded holes for mounting the GPS antenna spaced 50 mm apart in both the X and Y directions. The movable platform is driven by two sets of ball screws controlled by two precision servomotors through the electronic control system. To maintain the stable performance of the movable platform, the power terminal box is used to provide a stable power supply. Two sets of ball screws are installed perpendicular to the supporting frame through a double-layer mechanism (Figure 3.3) to facilitate the two-dimensional motions of the platform. The supporting frame has overall dimensions of 720 mm (length) × 720 mm (width) × 800 mm (height). In addition to supporting the movable platform, it also provides space for installing the power terminal box, the data acquisition system, cables, and other accessories. The use of aluminum materials makes the table portable and facilitates onsite calibration work. The four legs of the supporting frame can be adjusted to ensure exact platform movement in the horizontal plane. The whole table can be turned over 90° on an additional steel frame, enabling two-dimensional motions in the vertical plane to be generated. The four legs of the additional steel frame can also be adjusted to ensure exact platform movement in the vertical plane.
3.2.2. Table Control System

The motion simulation table includes a sophisticated control system designed to ensure the table can accurately reproduce the targeted input motion. An ADLINK motion controller board is installed on a COMPAQ desktop, which first converts the targeted table displacement into a targeted pulse of the number of ball screw rotations in the two directions. The controller inside the motor then commands each servomotor to drive the ball screw, with feedback being given from the actuator position. Based on the feedback, 4,096 pulses are counted for one rotation of the servomotor. The rotation of the servomotor driving the platform is then converted back to the displacement of the table and the data are finally acquired by a National Instruments NI-6035E board at a sampling rate of 50 Hz. The 16-channel data acquisition system interfaces with a main computer program named the “motion creator,” which runs on the COMPAQ desktop (Figure 3.4). In addition, a power control circuit with a main circuit breaker (MCB) is installed to protect the servomotors and the computer.

3.2.3. Table Characteristics

The motion simulation table can generate sinusoidal waves, white noise random waves, circular waves, and any other wave defined by input wave time histories around a pre-defined static position in two perpendicular directions with an upper frequency of up to 2 Hz. This frequency range covers the dominant natural frequencies of most long-span cable-supported bridges. The maximum stroke (displacement) of the movable platform is ±50 mm in both directions, but the frequencies are less than 0.8 Hz. When the maximum stroke is ±20 mm, the frequencies are more than 1.6 Hz. For frequencies between 0.8 and 1.2 Hz, the
maximum displacement is ±35 mm. For frequencies between 1.2 and 1.6 Hz, the maximum stroke is ±25 mm.

To ensure the GPS antenna installed on the platform is subjected to the motion of the platform only, the supporting frame was designed to have an adequate level of stiffness. A calibration test for the platform motion was carried out using a Keyence LK501 laser displacement transducer with a displacement measurement accuracy of 0.01 mm. The laser displacement transducer was installed 350 mm from the table. Clearly, if the measurement results from both the table and the laser displacement sensor are the same, one may claim not only that the platform motion is accurate, but also that no additional motion is induced by the supporting frame. The results show that the motion of the platform, whether in the horizontal direction or in the vertical direction, is almost the same as that measured by the laser displacement transducer and the targeted one. When the standard deviations of the mean, minimum, and maximum displacements are examined (refer to Appendix B), the difference between the two sets of results is generally less than 0.7%. The accuracy of the motion simulation table is thus guaranteed.

3.3. FIELD MEASUREMENT ARRANGEMENT

Before applying the GPS technology to long-span bridges, calibrations are essential to assess GPS performance in dynamic displacement measurements. Calibration tests using the two-dimensional motion simulation table were thus carried out for a GPS consisting of two sets of a Leica GX1230 receiver and an AT504 choke ring antenna on the site of Pak Shek Kok in Hong Kong in June
2004. The calibration tests included static tests designed to find the background noise in the GPS measurements and dynamic tests aimed at assessing the dynamic displacement measurement accuracy of the GPS with the input sinusoidal motion, circular motion, and wind-induced dynamic displacement responses of large civil engineering structures measured during typhoon periods in the field.

3.3.1. Test Site

To identify the best possible performance of the GPS tested, visibility from a satellite mask angle in all directions needs to be good. This mask angle is generally taken as 15° (Hofmann-Wellenhof et al. 2001; Wolf and Ghilani 2002). With this in mind, Pak Shek Kok (Figure 3.5), which is enclosed by Tolo Harbor, the Science Park, and the Chinese University of Hong Kong, was chosen as the test site. The neighboring buildings and mountains surrounding this test site are relatively low and are located a great distance from the site, and the maximum elevation angle is estimated to be less than 12° with respect to the antenna base.

3.3.2. Hardware and Software Configurations

The field calibration work was conducted over the course of three days in June 2004, with the weather remaining fine throughout. In the field calibration work (Figure 3.6), an electricity generator was used to provide a constant power supply to all the equipment and computers. The Leica GX1230 GPS receivers and AT504 choke ring antennae were the major components of the measurement instrumentation. In the field calibration work, one of the choke ring antennae was installed on the movable platform of the motion simulation table to act as a rover.
station, while the other antenna was fixed on a tripod approximately 12 m away from the simulation table to function as a reference station.

Another GPS receiver (an *Ashtech* GG24) was used to synchronize the computer clock on the motion simulation table with the GPS time. The *Ashtech* GPS receiver first automatically reset its clock using data downloaded from satellites. Through an RS232 serial interface, the data message, including navigation and timing data from the *Ashtech* GPS receiver, was transmitted to the computer every second. An offset between the computer clock and the correct time was then calculated to steer the computer clock to the GPS time.

Raw GPS observations were logged in 32 Mb CF cards and sampled at 20 Hz, which is the highest GPS sampling rate used at both the reference and rover stations. The raw data were then regularly transferred to an onsite laptop computer for further processing. The motions of the platform were also recorded and transferred to the laptop computer for comparison with the GPS results. The raw GPS data were first converted to the RINEX format with *Leica* Geo-Office software. An in-house software application developed by the Department of Land Surveying and Geo-informatics at The Hong Kong Polytechnic University, the GPS Health Monitoring Computer Program, was then used to process the raw GPS data in kinematic mode to calculate the position of the rover antenna at each epoch. The cut-off angle for the GPS data was 15°.

To compare the results from the two measurement systems (the motion simulation table and the GPS), the dynamic trajectory in the GPS coordinate
system (WGS84) was transformed into the motion simulation table coordinate system (X, Y, Z) as follows:

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = \begin{bmatrix}
\cos \alpha & \sin \alpha & 0 \\
-\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
E_{rgps} \\
N_{rgps} \\
H_{rgps}
\end{bmatrix}
\]

(3.1)

\[
\alpha = \tan^{-1} \left[ \frac{N_{rgps (ref)} - N_{rgps (rov)}}{E_{rgps (ref)} - E_{rgps (rov)}} \right]
\]

(3.2)

The above calculation was made on the basis of the mean coordinates of both the reference (ref) and rover (rov) stations.

### 3.3.3. Test Cases

In this study, calibration work was carried out in three phases. In the first phase, background noise in GPS measurements was measured for nine hours, with the rover station remaining stationary. The GPS data were then analyzed to ascertain the basic characteristics of background noise at the site, on the basis of which an appropriate bandwidth filtering scheme was designed for processing the GPS data recorded in dynamic mode. In the second phase of calibration work, the sensitivities of the GPS were measured based on the amplitude and frequency of one-dimensional sinusoidal motions and two-dimensional circular motions. The amplitudes of the simulated motions were taken as 2 mm, 5 mm, 10 mm, 20 mm, and 40 mm, while the frequencies selected were 0.025 Hz, 0.1 Hz, 0.5 Hz, 1 Hz and 1.8 Hz, respectively. A 600-second data collection period was used for each of the simulated motions. The third phase of calibration work examined the ability of the GPS to track complex motions. The complex motions performed
included two-dimensional horizontal motions measured at the top of the Di Wang Building during Typhoon York (Xu & Zhan 2001) and one-dimensional vertical motions of the middle section of the Tsing Ma suspension bridge deck measured during Typhoon Victor (Xu et al. 2000).

3.4. BACKGROUND NOISE AND THE BANDWIDTH FILTERING SCHEME

3.4.1. Background Noise in GPS Measurements

When the GPS antennae at both the rover and reference stations are stationary, any displacements found in GPS measurements can be considered background noise (Kijewski-Correa 2003). Since the baseline in the measurement is very short, most of the GPS errors are cancelled when a double-differencing data processing technique is used, and any errors remaining in the background noise are mainly due to the multipath effect.

Figure 3.7 shows 30 minute-long GPS-measured time histories with three coordinates, X, Y and Z (background noise), when the antenna on the platform is stationary. The background noise fluctuates between -5.9 and 5.1 mm in the X-direction, between -4.4 and 6.3 mm in the Y-direction, and between -20.5 and 4.7 mm in the Z-direction. The standard deviations of the background noise are 1.4 mm, 1.1 mm, and 2.9 mm in the X, Y, and Z-directions, respectively. Figure 3.8 depicts the power spectral density (PSD) functions in dB/Hz corresponding to the background noise records in Figure 3.7. Figure 3.8 shows that while the background noise contains many frequency components, the dominant noise
energy is distributed over a relatively low frequency range. Since the background noise in the vertical direction is higher than that in the horizontal direction, the amplitude of the PSD function of the background noise in the vertical direction is greater than that in the horizontal direction.

Figure 3.9 shows the probability density functions (PDFs) of the background noise in the X and Z-directions recorded during the GPS static measurement process and compares them with the standard Gaussian distribution. Clearly, the background noise closely follows the Gaussian distribution. Therefore, although the background noise fluctuates, with peak values of -5.9 and 5.1 mm in the X-direction, -4.4 and 6.3 mm in the Y-direction, and -20.5 and 4.7 mm in the Z-direction, 99.7% of the data points are still within the range of the mean value $\pm 3\sigma$ ($\sigma$ is the standard deviation), i.e., between -4.2 mm and 3.9 mm in the X-direction, between -2.8 mm and 3.6 mm in the Y-direction, and between -15.5 mm and 1.9 mm in the Z-direction.

It has been recognized that GPS errors caused by the multipath effect are well correlated between sidereal days if the geometry relating to the GPS satellites, the reflective surfaces, and the antenna remain unchanged (Ge 1999). To ensure that the multipath disturbance remains closely correlated between sidereal days, even when the GPS receiver is subjected to motion, dynamically measured background noise is compared with statically measured background noise in this study. The dynamically measured background noise is obtained by subtracting the original motion generated by the simulation table from the table motion recorded by the GPS. Figures 3.10 and 3.11 illustrate the background noise...
measured during the dynamic tests and static tests on two successive days in the X-direction and Z-direction, respectively, with a duration of 1,600 seconds. It can be seen that the dynamically measured background noise is quite similar to the statically measured background noise. The effect of GPS antenna motion on the multipath disturbance is thus neglected in this study.

3.4.2. Bandwidth Filtering Scheme

Based on the understanding of background noise as discussed above, a tenth order elliptic bandpass filter is applied to dynamic displacement measurement data from the GPS to reduce background noise. For sinusoidal motion and circular motion, a bandwidth filtering scheme, depending on the frequency $f_{dom}$ of the table motion, is implemented in this study.

$$\frac{f_{dom}}{5} \leq \text{frequency range interested} \leq 5 \times f_{dom} \leq 5 \text{ Hz} \quad (3.3)$$

This bandwidth filtering scheme covers a relatively large frequency range and is easily implemented in comparison with other advanced filtering schemes. The frequencies of the sinusoidal and circular motions of the platform used in this study are 0.025 Hz, 0.1 Hz, 0.5 Hz, 1 Hz, and 1.8 Hz.

To assess the effectiveness of the band-pass filter defined above, the band-pass filter is applied to background noise data obtained from the static tests. Table 3.1 lists the statistical values of the background noises shown in Figure 3.7 in the X and Z directions before and after filtering. It is evident that after filtering, the mean values of the background noise are almost eliminated and all other statistics
are significantly lower. After filtering, the maximum standard deviation of background noise is 0.53 mm in the X-direction and 1.22 mm in the Z-direction. Since the probability density function of the filtered background noise still complies with the standard Gaussian distribution, the value of $3\sigma$ is 1.59 mm in the X-direction and 3.66 mm in the Z-direction. The dynamic displacement measurement accuracy of the GPS will be within these values.

3.5. ASSESSMENT OF DYNAMIC MEASUREMENT ACCURACY

3.5.1. Definition of Measurement Error

In this study, the dynamic displacement measurement error in GPS measurements is defined by

$$\text{Error(\%)} = \frac{x_g - x_t}{x_t} \times 100\%$$

(3.4)

where $x$ represents one of the statistics such as the minimum peak displacement, the maximum peak displacement, or the displacement standard deviation. The subscript “$g$” refers to the GPS, while “$t$” refers to the motion simulation table. As the peak in each cycle of sinusoidal motion and circular motion can demonstrate the tracking ability of the GPS, the average minimum peak displacement ($A_{\text{min}}$) and the average maximum peak displacement ($A_{\text{max}}$) are used in this study. $A_{\text{min}}$ is defined as the average value of the troughs of all cycles in the time history. $A_{\text{max}}$ is defined as the average value of the crests of all cycles in the time history.
3.5.2. 1-D Sinusoidal Motion in the Horizontal Direction

Figure 3.12 depicts sinusoidal motions of 0.025 Hz with amplitudes of 2 mm, 5 mm, 10 mm, 20 mm, and 40 mm in the Y-direction generated by the motion simulation table and those recorded by the GPS before and after filtering. It can be seen that without filtering, GPS can satisfactorily track sinusoidal motions with an amplitude of 20 mm or above; however, with filtering, GPS can satisfactorily trace sinusoidal motions with an amplitude of 5 mm or above. Furthermore, when they are filtered, GPS signals become smooth in the absence of higher frequency components, and signal shifts in GPS measurements at a very low frequency are also eliminated. The measurement errors calculated according to Equation (3.4) are listed in Table 3.2. It can be seen that except for the sinusoidal motion of 2 mm amplitude, the measurement errors are all very small for the sinusoidal motions of other amplitudes. There is an error of less than 1.5% in the average minimum peak displacement ($A_{\text{min}}$), less than 2% in the average maximum peak displacement ($A_{\text{max}}$), and less than 0.6% in the displacement standard deviation. In particular, the absolute error in the displacement standard deviation decreases with increasing motion amplitude. Therefore, one may conclude that the GPS performs satisfactorily when the amplitude of the sinusoidal motion of 0.025 Hz is no less than 5 mm.

Figure 3.13 illustrates sinusoidal motions of 20 mm amplitude with frequencies of 0.1 Hz, 0.5 Hz, 1 Hz, and 1.8 Hz in the Y-direction generated by the motion simulation table, along with those recorded by the GPS with filtering. It is evident that other than for the motion of 1.8 Hz, the GPS can satisfactorily track sinusoidal motions generated by the table. Table 3.3 lists the corresponding
measurement errors. Clearly, the tracking accuracy of the GPS is reduced with increasing frequency. For the sinusoidal motion of 1 Hz, the measurement errors of the average maximum peak displacement, the average minimum peak displacement, and the displacement standard deviation are 3.52%, 4.90%, and 3.84%, respectively. Because these measurement errors are all less than 5%, the error level considered acceptable for most large civil engineering structures, one may conclude that the GPS performs satisfactorily when the frequency of the sinusoidal motion of 20 mm amplitude is no greater than 1 Hz.

### 3.5.3. 2-D Circular Motion in the Horizontal Plane

Figure 3.14 depicts circular motions of 40 mm radius generated by the motion simulation table at 0.025 cycles per second (Hz) in the horizontal plane and compares them with those recorded by the GPS. It is evident that the GPS has an ability to track two-dimensional motions of the table in the horizontal plane. Referring to the statistical values listed in Table 3.4 for circular motions of different radii at 0.025 Hz, one can see that other than for the circular motion of 2 mm radius, errors in the displacement standard deviation, the average maximum peak displacement, and the average minimum peak displacement are all below 4% in both the X- and Y- directions for all circular motions of a radius larger than 2 mm. Hence, one may conclude that the performance of the GPS used in this study is quite satisfactory when the radius of a circular motion of 0.025 Hz is no less than 5 mm.

Table 3.5 lists the measurement errors when the GPS was used to track circular motions of 20 mm radius at different frequencies. It can be seen that the GPS can
adequately track circular motions of a frequency no greater than 1 Hz. The maximum errors in all the statistical values are less than 4%. However, as in the 1-D sinusoidal motion test, when the circular motion frequency reached 1.8 Hz, the measurement errors in the average maximum peak displacement, the average minimum peak displacement, and the displacement standard deviation are all above 14% in both the X- and Y-directions. This implies that the performance of the GPS is satisfactory only when the frequency of circular motion is no greater than 1 Hz.

3.5.4. 1-D Sinusoidal Motion in the Vertical Direction

When considering the application of GPS technology to a long-span cable-supported bridge, measuring the vertical displacement of the bridge deck is a major concern. This section thus assesses the measurement accuracy of the GPS in the vertical direction in terms of sinusoidal motion. Figure 3.15 depicts 0.5 Hz sinusoidal motions with amplitudes of 5 mm, 10 mm, 20 mm, and 40 mm generated by the table in the vertical direction, as well as those recorded by the GPS before and after filtering. Although the motion patterns recorded by the GPS before filtering are similar to those generated by the table, there is a shift in the mean value for all the sinusoidal motions. After filtering, the vertical displacement time histories recorded by the GPS are very close to those generated by the motion table. Table 3.6 lists the measurement errors of the GPS in the vertical direction after filtering. Clearly, the measurement errors are all less than 8%, even for the sinusoidal motion of only 5 mm amplitude. However, given that the $3\sigma$ value of the background noise after filtering is 3.66 mm in the Z-direction, it is safe to conclude that the GPS performs satisfactorily in vertical
displacement measurement when the amplitude of a 0.5 Hz sinusoidal motion is no less than 10 mm.

Figure 3.16 illustrates the 20 mm sinusoidal motions in the vertical direction generated by the table with frequencies of 0.025 Hz, 0.1 Hz, 1 Hz, and 1.8 Hz, respectively, together with those recorded by the GPS with filtering. The corresponding measurement errors are listed in Table 3.7. As in the case of horizontal motion, GPS performance reaches a nadir when the motion frequency is 1.8 Hz. The measurement errors in the average maximum peak displacement, the average minimum peak displacement, and the standard deviation displacement are all more than 22%. For motion frequencies of no higher than 1 Hz, the measurement errors in the displacement standard deviation are less than 12%. Clearly, the measurement accuracy of the GPS in the vertical direction is much lower than that in the horizontal direction. It is only if a measurement error of 14% is deemed acceptable that one can say that the GPS performs satisfactorily in the vertical displacement measurement when the frequency of a 20 mm sinusoidal motion is no greater than 1 Hz.

3.5.5. Wind-induced 2-D Dynamic Response in the Horizontal Plane

To emulate the dynamic responses of bridge towers, the two-dimensional wind-induced dynamic displacements of Di Wang Tower measured during Typhoon York in 1999 are used to assess the measurement accuracy of the GPS. Di Wang Tower is a 69–storey building that measures 384 m at the top of its mast and is located in the center of Shenzhen City about 2 km from the Hong Kong border. On 16 September 1999, Typhoon York, which was the strongest typhoon
to hit the region since 1983 and the one with the longest duration on record, approached Shenzhen and Hong Kong. The wind and structural monitoring system installed in Di Wang Tower recorded both the wind level and the structural responses of the building (Xu & Zhan 2001). Of the data recorded, the records on the wind-induced dynamic displacement responses of the tower at an altitude of 298 m over a period of 1,800 seconds were selected as input data for the motion simulation table. The GPS was then used to record the table motion in the field. A band-pass filter with lower and upper frequencies of 0.05 Hz and 0.5 Hz, respectively, was applied to both the input and GPS-measured dynamic displacement time histories to extract the first natural frequency of the tower in the X-direction (0.17 Hz) and Y-direction (0.20 Hz). The maximum dynamic displacement in the X-direction was 35 mm, while the maximum dynamic displacement response in the Y-direction was 9 mm. Figures 3.17(a) and 3.17(b) depict the motion trajectories extracted from 700 to 800 seconds in the displacement time histories generated by the motion simulation table and measured by the GPS after filtering, respectively. It can be seen that the trajectory measured by the GPS is similar to that generated by the motion simulation table. Referring to the measurement errors listed in Table 3.8, one can see that the tracking ability of the GPS for the wind-induced displacement responses of the tower is satisfactory. The measurement error is less than 2% in the displacement standard deviation and less than 5% in the maximum and minimum peak displacements in both directions. In addition to its displacement tracking ability, the frequency tracking ability of the GPS is also satisfactory, as depicted in Figure 3.18, where the power spectral density function of the GPS-measured displacement response closely follows that generated by the table,
other than for high frequencies above 0.3 Hz. All of these comparative results demonstrate that the GPS has the ability to track wind-induced two-dimensional dynamic displacement responses of tall buildings in the horizontal plane.

### 3.5.6. Wind-induced Vertical Dynamic Response

The measurement accuracy of the GPS is also assessed in this study using the wind-induced vertical dynamic displacement response of the Tsing Ma Bridge deck measured during Typhoon Victor in 1997. Hong Kong’s Tsing Ma Bridge is a suspension bridge with a main span of 1,377 m, with three motor lanes in each direction on the upper level of the bridge and two railway tracks and two carriageways on the lower level. On 2 August 1997, Typhoon Victor headed for Hong Kong and its center passed adjacent to the bridge. The Wind And Structural Health Monitoring System (WASHMS) installed on the Tsing Ma Bridge recorded wind level and structural response data during this typhoon event (Xu et al. 2000). The wind-induced dynamic displacement responses measured at the mid-main span over an 1,800 second period were selected as input data for the motion simulation table, after which the GPS recorded the table motion in the field. The vertical dynamic displacement time history input into the motion simulation table was extracted from the vertical acceleration response of the bridge deck measured at the mid-main span through double integrations. The 1,800 second vertical dynamic displacement time history had a maximum value of 32 mm. To extract the first (0.139 Hz) and second (0.241 Hz) modes of vibration of the bridge, a band-pass filter with lower and upper cut-off frequencies of 0.05 Hz and 0.6 Hz was applied to both the input vertical dynamic displacement time history and the GPS-recorded vertical dynamic displacement...
time history. The results are depicted in Figure 3.19(a), which shows that the wind-induced vertical dynamic displacement response of the bridge deck recorded by the GPS is similar to that generated by the table. The measurement errors listed in Table 3.9 indicate that the tracking ability of the GPS for the vertical dynamic displacement response of the bridge deck is satisfactory, with the measurement errors all being less than 7%. Figure 3.19(b) illustrates the frequency tracking ability of the GPS for the wind-induced vertical dynamic displacement response of the bridge deck. Clearly, the PSD function measured by the GPS closely resembles that generated by the table, other than for frequencies of over 0.3 Hz. All of these comparative results demonstrate that the GPS has the ability to track the dynamic displacement responses of a long-span cable-supported bridge in the vertical direction.

3.6. SUMMARY

This chapter presents an advanced two-dimensional motion simulation table designed to assess the accuracy of the GPS in measuring the dynamic displacement responses of long-span cable-supported bridges. A series of field tests were carried out in an open area in Hong Kong using the motion simulation table and the GPS, the main components used being two sets of Leica GX1230 receivers and AT504 choke ring antennae with a sampling rate of 20 Hz in either the horizontal plane or the vertical plane. The static tests were first carried out with stationary antennae to identify the background noise in the GPS measurements taken at the site. An examination of statistical data recorded over a period of 9 hours showed that the background noise was dominated by low frequency components. A band-pass filtering scheme for sinusoidal motion and
circular motion was designed and applied to the dynamically measured displacement data recorded by the GPS, which were then compared with those generated by the table. The comparative results showed that for two-dimensional sinusoidal and circular motions in the horizontal plane and one-dimensional sinusoidal motions in the vertical direction, the GPS could be used to obtain accurate dynamic displacement measurements if the motion amplitude was no less than 5 mm in the horizontal plane or 10 mm in the vertical direction and the motion frequency was less than or equal to 1 Hz. The dynamic displacement measurement accuracy of the GPS was finally assessed using data on the horizontal wind-induced two-dimensional dynamic displacement responses of Di Wang Tower during Typhoon York and the vertical wind-induced one-dimensional dynamic displacement response of the Tsing Ma suspension bridge deck during Typhoon Victor. The comparative results demonstrated that the GPS could trace complex wind-induced dynamic displacement responses of real structures satisfactorily.

Although the motion simulation table tests described above yielded positive results on the application of the GPS, the GPS is unable to capture higher-mode natural frequencies and its effectiveness is greatly reduced by multipath interference. To derive a noise-free total (static plus dynamic) displacement signal across a wider frequency range, Chapter 4 discusses the concept of integrating signals from the GPS and an accelerometer using both empirical mode decomposition (EMD) and an adaptive filter.
### Table 3.1. Statistics of GPS-measured background noise before and after filtering

<table>
<thead>
<tr>
<th>FILTERING CONDITION</th>
<th>$f_{\text{min}}$ (Hz)</th>
<th>X-DIRECTION (mm)</th>
<th>Z-DIRECTION (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_{\text{min}}$</td>
<td>MEAN</td>
<td>STD. DEVIATION</td>
</tr>
<tr>
<td>Before</td>
<td>--</td>
<td>-0.126</td>
<td>1.358</td>
</tr>
<tr>
<td>After</td>
<td>0.025</td>
<td>-0.003</td>
<td>0.405</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>-0.001</td>
<td>0.367</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>-0.000</td>
<td>0.397</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-0.000</td>
<td>0.527</td>
</tr>
<tr>
<td></td>
<td>1.8</td>
<td>-0.000</td>
<td>0.510</td>
</tr>
</tbody>
</table>

### Table 3.2. GPS measurement errors for 0.025 Hz sinusoidal motion in the horizontal direction

<table>
<thead>
<tr>
<th>ERROR (%)</th>
<th>MOTION AMPLITUDE (mm)</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\text{min}}$</td>
<td></td>
<td>22.99</td>
<td>-0.26</td>
<td>-0.48</td>
<td>-1.12</td>
<td>0.45</td>
</tr>
<tr>
<td>$A_{\text{max}}$</td>
<td></td>
<td>22.27</td>
<td>-1.65</td>
<td>0.95</td>
<td>1.95</td>
<td>0.22</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td></td>
<td>20.89</td>
<td>-0.54</td>
<td>0.28</td>
<td>0.25</td>
<td>-0.02</td>
</tr>
</tbody>
</table>

### Table 3.3. GPS measurement errors for 20 mm sinusoidal motion in the horizontal direction

<table>
<thead>
<tr>
<th>ERROR (%)</th>
<th>MOTION FREQUENCY (Hz)</th>
<th>0.1</th>
<th>0.5</th>
<th>1</th>
<th>1.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\text{min}}$</td>
<td></td>
<td>0.51</td>
<td>0.77</td>
<td>3.52</td>
<td>17.97</td>
</tr>
<tr>
<td>$A_{\text{max}}$</td>
<td></td>
<td>1.32</td>
<td>1.80</td>
<td>4.90</td>
<td>17.00</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td></td>
<td>0.77</td>
<td>1.29</td>
<td>3.84</td>
<td>18.87</td>
</tr>
</tbody>
</table>
Table 3.4. GPS measurement errors for 0.025 Hz circular motion in the horizontal direction

<table>
<thead>
<tr>
<th>MOTION RADIUS (mm)</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>ERROR (%)</td>
<td>X</td>
<td>Y</td>
<td>X</td>
<td>Y</td>
<td>X</td>
</tr>
<tr>
<td>$A_{\text{min}}$</td>
<td>8.31</td>
<td>-1.00</td>
<td>-0.33</td>
<td>-0.18</td>
<td>0.38</td>
</tr>
<tr>
<td>$A_{\text{max}}$</td>
<td>4.67</td>
<td>7.21</td>
<td>3.67</td>
<td>3.2</td>
<td>3.24</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>3.61</td>
<td>0.74</td>
<td>2.57</td>
<td>-0.44</td>
<td>1.87</td>
</tr>
</tbody>
</table>

Table 3.5. GPS measurement errors for 20 mm circular motion in the horizontal plane

<table>
<thead>
<tr>
<th>MOTION FREQUENCY (Hz)</th>
<th>0.1</th>
<th>0.5</th>
<th>1</th>
<th>1.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>ERROR (%)</td>
<td>X</td>
<td>Y</td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>$A_{\text{min}}$</td>
<td>-1.68</td>
<td>2.28</td>
<td>-0.06</td>
<td>1.85</td>
</tr>
<tr>
<td>$A_{\text{max}}$</td>
<td>-0.43</td>
<td>1.18</td>
<td>0.63</td>
<td>1.09</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>-1.11</td>
<td>1.43</td>
<td>-0.14</td>
<td>1.32</td>
</tr>
</tbody>
</table>

TABLE 3.6. GPS measurement errors for 0.5 Hz sinusoidal motion in the vertical direction

<table>
<thead>
<tr>
<th>MOTION AMPLITUDE (mm)</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>ERROR (%)</td>
<td>X</td>
<td>Y</td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>$A_{\text{min}}$</td>
<td>7.77</td>
<td>3.04</td>
<td>4.86</td>
<td>-5.12</td>
</tr>
<tr>
<td>$A_{\text{max}}$</td>
<td>7.18</td>
<td>2.21</td>
<td>6.83</td>
<td>-1.97</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>1.97</td>
<td>0.40</td>
<td>5.98</td>
<td>-4.63</td>
</tr>
</tbody>
</table>
### TABLE 3.7. GPS measurement errors for 20 mm sinusoidal motion in the vertical direction

<table>
<thead>
<tr>
<th>ERROR (%)</th>
<th>MOTION FREQUENCY (Hz)</th>
<th>0.025</th>
<th>0.1</th>
<th>1</th>
<th>1.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\text{min}}$</td>
<td></td>
<td>9.02</td>
<td>-13.40</td>
<td>11.75</td>
<td>32.94</td>
</tr>
<tr>
<td>$A_{\text{max}}$</td>
<td></td>
<td>8.77</td>
<td>-8.16</td>
<td>10.00</td>
<td>22.18</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td></td>
<td>9.14</td>
<td>-11.10</td>
<td>9.44</td>
<td>28.78</td>
</tr>
</tbody>
</table>

### Table 3.8. GPS measurement errors for wind-induced dynamic displacement responses of Di Wang Tower in the horizontal plane

<table>
<thead>
<tr>
<th>ITEMS</th>
<th>X-DIRECTION</th>
<th>Y-DIRECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MAXIMUM</td>
<td>MINIMUM</td>
</tr>
<tr>
<td>GPS(mm)</td>
<td>33.47</td>
<td>-31.48</td>
</tr>
<tr>
<td>Table(mm)</td>
<td>34.50</td>
<td>-33.09</td>
</tr>
<tr>
<td>Error(%)</td>
<td>-2.98</td>
<td>-4.87</td>
</tr>
</tbody>
</table>

### Table 3.9. GPS measurement errors for wind-induced vertical dynamic displacement response of the Tsing Ma Bridge deck

<table>
<thead>
<tr>
<th>ITEMS</th>
<th>Z-DIRECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MAXIMUM</td>
</tr>
<tr>
<td>GPS(mm)</td>
<td>39.21</td>
</tr>
<tr>
<td>Table(mm)</td>
<td>36.93</td>
</tr>
<tr>
<td>Error(%)</td>
<td>6.18</td>
</tr>
</tbody>
</table>
Chapter 3 Assessing the Dynamic Measurement Accuracy of the GPS

Figure 3.1. Two-dimensional motion simulation table

1 movable platform  2 double layer mechanism
3 power terminal box  4 supporting frame
5 16-channel data acquisition system  6 computer system

Figure 3.2. Movable platform
Figure 3.3. Double layer mechanism
Figure 3.4. Interface of “Motion Creator”
Figure 3.5. View of Pak Shek Kok
Figure 3.6. Hardware configuration for the motion simulation table tests
Figure 3.7. Time histories of background noise in GPS measurements

Figure 3.8. PSD functions of background noise in GPS measurements
Figure 3.9. PDF of background noise in the horizontal and vertical directions
Figure 3.10. Statistically measured background noise versus dynamically measured background noise in the horizontal direction.
Figure 3.11. Statistically measured background noise versus dynamically measured background noise in the vertical direction.
Figure 3.12. Comparisons of horizontal sinusoidal motions before and after filtering (frequency = 0.025 Hz)
Figure 3.13. Comparisons of horizontal sinusoidal motions after filtering (amplitude = 20 mm)
Figure 3.14. Comparison of circular motions after filtering
(frequency = 0.025 Hz; amplitude = 40 mm)
Figure 3.15. Comparisons of vertical sinusoidal motions before and after filtering (frequency = 0.5 Hz)
Figure 3.16. Comparisons of vertical sinusoidal motions after filtering (amplitude = 20 mm)
Figure 3.17. Comparison of motion trajectories of Di Wang Tower

(a) Motion simulation table

(b) GPS
Figure 3.18. Comparisons of displacement PSD function of Di Wang Tower
Chapter 3

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Figure 3.19. Comparisons of displacements and PSD functions of the Tsing Ma Bridge deck
4.1. INTRODUCTION

The feasibility of applying the global positioning system (GPS) in deflection monitoring has been ascertained from the calibration study described in Chapter 3. However, a lack of accuracy in GPS measurements of dynamic displacements with a frequency of greater than 1 Hz and of displacements susceptible to the multipath effect are both inevitable weaknesses of GPS applications. Due to the fact that accelerometers cannot measure static and low frequency displacements, but can measure high frequency displacements, this chapter explores the possibility of integrating GPS-measured signals with accelerometer-measured signals to enhance the accuracy of total (static plus dynamic) displacement response measurements for civil engineering structures across a wider range of frequencies.

In this regard, two frameworks for integrated data processing techniques using both empirical mode decomposition (EMD) and an adaptive filter are presented. The EMD developed by Huang et al. (1998) is a data-processing tool that can be used to decompose any complicated data set into a number of intrinsic mode
functions (IMFs) and a final residual. The EMD method has been successfully used to extract time-varying mean wind speed from typhoon-induced non-stationary wind records for long-span cable-supported bridges (Xu & Chen 2004) and tall buildings (Chen & Xu 2004). The adaptive filter is a signal decomposer that extracts information of interest from a contaminated signal using the cross-correlation between reference and primary time series (Ge 1999; Ge et al. 2000; Roberts et al. 2002). In recognition that the multipath is repeatable on every sidereal day, Ge (1999) successfully applied adaptive filtering to GPS data to reduce the multipath effect.

To assess the effectiveness of the integrated data processing techniques proposed, a series of motion simulation table tests are performed at a site using three GPS receivers, one accelerometer, and one motion simulation table. Static tests carried out with the GPS antenna installed on the motion simulation table in a stationary condition are first performed at the test site to estimate the effect of multipath. The motion simulation table is then used to generate various types of dynamic displacement response around a pre-defined static position. The GPS and accelerometer measurement data are recorded over the same time period as the static tests, but on the next sidereal day. The data processing techniques proposed are then applied to the recorded GPS and accelerometer data to ascertain both static and dynamic displacements. The effectiveness of the integrated methods is finally assessed through a comparison of the integrated results with the original motions generated by the motion simulation table.
4.2. **EMPIRICAL MODE DECOMPOSITION**

The EMD method used in this study involves decomposing a GPS-measured structural displacement response time history $x(t)$ into a number of IMF components and a final residue through the following sifting process (Huang et al. 1998; Xu & Chen 2004; Chen & Xu 2004; Liang et al. 2005a; Yu & Ren 2005):

(a) Construct the upper $e_{\text{upper}}(t)$ and lower $e_{\text{lower}}(t)$ envelopes of $x(t)$ by connecting the local maxima and minima of $x(t)$, respectively, using a cubic spline function;

(b) Compute the mean of the two envelopes $m_1(t)$:

$$m_1(t) = \frac{e_{\text{upper}}(t) + e_{\text{lower}}(t)}{2}$$  \hspace{1cm}(4.1)

(c) Subtract the mean envelope $m_1(t)$ from the original time history to extract the first IMF $c_1(t)$:

$$c_1(t) = x(t) - m_1(t)$$  \hspace{1cm}(4.2)

if it satisfies the following two conditions: (1) within the data range, the number of extrema and the number of zero-crossings are equal or differ at most by one; and (2) the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero;
(d) Replace $c_1(t)$ as the original signal if $c_1(t)$ does not satisfy the IMF conditions. Repeat steps (a)-(c) until the condition is satisfied;

(e) Subtract the original signal $x(t)$ from the IMF $c_1(t)$ to yield the first residue $r_1(t)$:

$$ r_1(t) = x(t) - c_1(t) \quad (4.3) $$

(f) Treat the residue $r_1(t)$ as a new time history and repeat the above steps to obtain the second IMF, $c_2(t)$. The EMD procedure continues until the residue becomes so small that it is less than a predetermined value of consequence, or the residue becomes a monotonic function. The original time history $x(t)$ is finally expressed as the sum of the IMFs plus the final residue:

$$ x(t) = \sum_{j=1}^{N_e} c_j(t) + r_{N_e}(t) \quad (4.4) $$

where $N_e$ is the number of IMF components and $r_{N_e}(t)$ is the final residue.

If the final residue $r_{N_e}(t)$ of the structural displacement response time history measured by the GPS is a monotonic function, it can be defined as the mean displacement of the structure.
As the concept underlying this decomposition process is based on the direct extraction of the energy associated with various intrinsic time scales of the time history itself, mode mixing during the sifting process is possible. Huang et al. (1999) thus suggested a criterion, termed the intermittency check, for separating the waves of different periods into different modes based on the period length. In this study, the EMD method involving an intermittency check and a cutoff frequency $\omega_c$ is also used to process the acceleration time history measured by an accelerometer to obtain the high-frequency dynamic responses of frequency components greater than the cut-off frequency $\omega_c$.

### 4.3. ADAPTIVE FILTER

#### 4.3.1. Concepts

An adaptive filter used as a signal decomposer (Figure 4.1) is operated on the basis of information from two measurement inputs: (1) a primary measurement $s(n)$ that contains the desired signal of interest $x(n)$ contaminated by noise $v(n)$; and (2) the reference measurement $r(n)$ of the noise signal $v'(n)$. Two conditions have to be satisfied to extract the desired signal $x(n)$ from the polluted primary measurement $s(n)$ using the adaptive filter: (1) the desired signal $x(n)$ and noise $v(n)$ in the primary measurement are uncorrelated with each other; and (2) the noise $v'(n)$ in the reference measurement is uncorrelated with the desired signal $x(n)$ but is correlated in some way with the noise component $v(n)$ of the primary signal. As the multipath measured by the moving receiver is similar to that measured by the stationary receiver between sidereal days, as evidenced by Figures 3.10 and 3.11, the adaptive filter can be
used to mitigate the multipath effect. The displacement measured by the GPS with a moving antenna is taken as the primary measurement $s(n)$, which includes the desired structural displacement $x(n)$ and the multipath noise $\nu(n)$. The signal measured by the GPS with a stationary antenna during the same time period as the period in which the dynamic measurement is recorded but on the next or previous sidereal day is taken as the reference measurement $r(n) = \nu'(n)$. By assuming that the desired structural displacement is uncorrelated to the multipath effect while the reference measurement is uncorrelated with the structural displacement but is correlated with the multipath in some way, the adaptive filter can be used in this study. This study not only uses the adaptive filter for multipath mitigation, but also to extract low-frequency dynamic displacement responses from the GPS-measured data by using high-frequency dynamic displacement responses from the accelerometer as reference measurements.

### 4.3.2. Recursive Adaptive Algorithm

As shown in Figure 4.1(b), the estimate of $\nu(n)$, $\hat{\nu}(n)$, is found by multiplying the reference signal vector $r(n)$ by the tap weight vector $w(n)$:

$$\hat{\nu}(n) = w^T(n) r(n) = \sum_{i=0}^{M-1} w_i(n) r(n-i) \quad (4.5)$$

where $M$ represents the filter order. The estimation error $e(n)$ for the filter is given as:

$$e(n) = s(n) - \hat{\nu}(n) \quad (4.6)$$
The idea of a recursive least squares (RLS) filter is to minimize the weighted least squares error $\varepsilon(n)$, which is defined as:

$$
\varepsilon(n) = \sum_{j=0}^{n} \lambda^{n-j} |e(j)|^2
$$

(4.7)

where $\lambda$ represents the forgetting factor with a value of between 0 and 1. The forgetting factor $\lambda$ is used to better track any statistical variations in the signal because data from the distant past are forgotten (Akay 1994; Haykin 2002). In general, the forgetting factor $\lambda$ is selected on the basis of the number of samples, $N$ (Akay 1994):

$$
\lambda = \frac{N}{N+1}
$$

(4.8)

To minimize the weighted least squares error function, $\varepsilon(n)$ is partially differentiated with respect to the tap weights of the filter $\mathbf{w}(n)$ and the result set to zero:

$$
\frac{\partial \varepsilon(n)}{\partial \mathbf{w}(n)} = 2 \sum_{j=0}^{n} \lambda^{n-j} e(j) \frac{\partial e(j)}{\partial \mathbf{w}(n)} = -2 \sum_{j=0}^{n} \lambda^{n-j} e(j) \mathbf{r}(j) = 0
$$

(4.9)

Replacing $e(j) = s(j) - \sum_{i=0}^{M-1} w_i(n) r(j - i)$, Equation (4.9) is finally equivalent to

$$
\mathbf{R}(n) \mathbf{w}(n) = \mathbf{P}(n)
$$

(4.10)

where
\[
R(n) = \sum_{j=0}^{n} \lambda^{n-j} r(j)r^T(j) \tag{4.11}
\]

is the weighted autocorrelation matrix for \( r(j) \), while

\[
P(n) = \sum_{j=0}^{n} \lambda^{n-j} s(j)r(j) \tag{4.12}
\]

is the cross-correlation between \( s(j) \) and \( r(j) \).

The optimal filter weight \( w(n) \) can be calculated by

\[
w(n) = R^{-1}(n)P(n) \tag{4.13}
\]

The main objective of employing the RLS adaptive algorithm is to update the filter weight \( w(n) \) in a recursive solution of the form

\[
w(n) = w(n - 1) + \Delta w(n - 1) \tag{4.14}
\]

where \( \Delta w(n - 1) \) is a correction factor at time \( n - 1 \). Similarly, \( R(n) \) and \( P(n) \) can be estimated recursively:

\[
R(n) = \lambda R(n - 1) + r(n)r^T(n) \tag{4.15}
\]

\[
P(n) = \lambda P(n - 1) + s(n)r(n) \tag{4.16}
\]

Using the matrix inversion lemma, the inverse of \( R(n) \), \( R^{-1}(n) \), is defined as
\[
R^{-1}(n) = \frac{1}{\lambda} \left[ R^{-1}(n-1) - \frac{R^{-1}(n-1)r(n)r^T(n)R^{-1}(n-1)}{\lambda + r^T(n)R^{-1}(n-1)r(n)} \right] \quad (4.17)
\]

Defining \( \Phi(n) = R^{-1}(n) \), Equation (4.17) will yield to

\[
\Phi(n) = \lambda^{-1} \Phi(n-1) - g(n)r^T(n)\lambda^{-1} \Phi(n-1) \quad (4.18)
\]

where

\[
g(n) = \frac{\Phi(n-1)r(n)}{\lambda + r^T(n)\Phi(n-1)r(n)} \quad (4.19)
\]

Reforming Equation (4.19), \( g(n) \) will yield to

\[
g(n) = \Phi(n)r(n) \quad (4.20)
\]

As discussed in Equation (4.13), the optimal filter weight \( w(n) \) can be obtained by

\[
w(n) = \lambda \Phi(n)P(n-1) + s(n)\Phi(n)r(n) \quad (4.21a)
\]

\[
w(n) = \lambda [\lambda^{-1} \Phi(n-1) - g(n)r^T(n)\lambda^{-1} \Phi(n-1)]P(n-1) + s(n)g(n) \quad (4.21b)
\]

\[
w(n) = \Phi(n-1)P(n-1) - g(n)r^T(n)\Phi(n-1)P(n-1) + s(n)g(n) \quad (4.21c)
\]

\[
w(n) = \Phi(n-1)P(n-1) + g(n)[s(n) - r^T(n)\Phi(n-1)P(n-1)] \quad (4.21d)
\]

\[
w(n) = w(n-1) + g(n)[s(n) - r^T(n)w(n-1)] \quad (4.21e)
\]

\[
w(n) = w(n-1) + g(n)e_{\text{prior}}(n) \quad (4.21f)
\]

where
\[ e_{\text{prior}}(n) = s(n) - r^T(n)w(n - 1) \]  

(4.22)

is the prior estimation error. This implies that the correction factor displayed in Equation (4.14) is based on the old least-squares estimate of the tap-weight vector at time \( n - 1 \):

\[ \Delta w(n - 1) = g(n)e_{\text{prior}}(n) \]  

(4.23)

By collecting Equations (4.19), (4.22), (4.21f), and (4.18) together and following them in that order, the RLS algorithm can be constituted as summarized in Table 4.1.

### 4.4. INTEGRATED DATA PROCESSING TECHNIQUE

By assuming wind excitation is a stationary random process, the displacement response of a civil engineering structure in the longitudinal direction of wind excitation can be regarded as the sum of a mean displacement component \( \bar{x} \) and a dynamic displacement component \( x'(t) \):

\[ x(t) = \bar{x} + x'(t) \]  

(4.24)

The key issue in this study is how to extract the mean displacement \( \bar{x} \), the dynamic displacement \( x'(t) \), and then the total displacement \( x(t) \) of the structure from the GPS-measured and accelerometer-measured signals for the purpose of structural deformation monitoring. Given the inherent limitations and advantages of the two measurement systems as noted above, the high-frequency
dynamic displacement response of the structure over a designated frequency $\omega_c$ will be extracted mainly from the accelerometer-measured data. Given the advantages of the GPS at low frequencies, the mean displacement response and the low-frequency dynamic displacement response below the designated frequency $\omega_c$ will be extracted mainly from the GPS-measured data. This study proposes two integrated data processing algorithms using the aforementioned EMD method and the adaptive filter.

4.4.1. Algorithm A

Figure 4.2 illustrates the integrated data processing Algorithm A, in which three sets of data are involved: (1) the primary measurement data from the GPS with a moving antenna (dynamic test); (2) the reference measurement data from the GPS with a stationary antenna (static test); and (3) the dynamic displacement measurement data from an accelerometer. In the first stage, EMD is applied to the primary measurement time history from the GPS and decomposes the time history into a number of IMFs and a final residue $\tilde{x}_d$. EMD is also applied to the reference measurement time history from the GPS to obtain another final residue $\tilde{x}_s$. The mean displacement $\bar{x}$ of the structure is then taken as the difference $\bar{x}_d - \bar{x}_s$. The difference between the primary measurement time history from the GPS and the final residue $\tilde{x}_d$ is considered the contaminated dynamic displacement time history, while the difference between the reference measurement time history from the GPS and the final residue $\tilde{x}_s$ is considered the time history of dynamic background noise. In the second stage, EMD with an intermittency frequency $\omega_c$ is applied to the accelerometer-measured dynamic displacement time history to extract the high-frequency dynamic displacement
time history. In the third stage, the adaptive filter is used twice to obtain the low frequency dynamic displacement time history. On the first occasion, the contaminated dynamic displacement time history from the GPS is taken as a primary measurement, while the high-frequency dynamic displacement time history from the accelerometer is considered as a reference measurement. The incoherent component time history resulting from the first operation of the adaptive filter is actually the contaminated low-frequency dynamic displacement. On the second occasion, the contaminated low-frequency dynamic displacement time history is taken as a primary measurement, while the dynamic background noise time history from the GPS with a stationary antenna is considered as a reference measurement. The incoherent component time history resulting from the second operation of the adaptive filter is the low-frequency dynamic displacement. Finally, the total displacement of the structure is given by the combination of the mean displacement \((\bar{x}_d - \bar{x}_s)\), the low-frequency dynamic displacement time history, and the high-frequency dynamic displacement time history.

### 4.4.2. Algorithm B

In the integrated data processing Algorithm B shown in Figure 4.3, the methods for extracting the mean displacement of a structure from the GPS and the high-frequency dynamic displacement time history from the accelerometer are the same as those used in Algorithm A. The only difference between Algorithm B and Algorithm A is in the third stage. In Algorithm B, the adaptive filter is first applied to the contaminated dynamic displacement time history from the GPS with a moving antenna by taking the dynamic background noise time history
from the GPS with a stationary antenna as a reference measurement to obtain the
dynamic displacement time history from the GPS. The adaptive filter is then
applied to this dynamic displacement time history by taking the high-frequency
dynamic displacement time history from the accelerometer as a reference
measurement to obtain the low-frequency dynamic displacement time history.
The total displacement of the structure is finally given by the combination of the
mean displacement \((\bar{x}_d - \bar{x}_s)\), the low-frequency dynamic displacement time
history, and the high-frequency dynamic displacement time history, as in
Algorithm A.

4.5. MOTION SIMULATION TABLE TESTS

To assess the effectiveness and accuracy of integrated data processing
Algorithms A and B, a series of motion simulation table tests were performed at a
site using a motion simulation table, three GPS receivers, and an accelerometer
to obtain three sets of data and record the actual displacement of the structure for
comparison purposes. One GPS receiver and the accelerometer were mounted on
the moving platform of the motion simulation table. In the first set of
measurements, the motion simulation table was stationary and the GPS receiver
thus also remained stationary. A static test carried out with a stationary antenna
was then performed continuously for two hours to obtain the reference
measurements and assess the background noise in the GPS. In the second set of
measurements, the motion simulation table generated white noise random waves
and wind-induced dynamic responses measured from a real complex civil
engineering structure around a pre-defined static position to obtain the primary
measurement data from the GPS and the dynamic displacement measurement
data from the accelerometer. Each measurement of 1,800 seconds was taken at the same time as the static test, but on the next sidereal day.

Motion simulation table tests were conducted over two days on the roof of the Pao Yue Kong Library of The Hong Kong Polytechnic University in May 2005. This five-storey rigid building is surrounded by other buildings and is susceptible to the multipath effect (Figure 4.4). As the experiments were conducted over a period in which the weather was relatively calm throughout, the measured time histories were assumed to be less affected by the motion of the building. The hardware and software configurations were almost the same as those used in the calibration study described in the previous chapter. Only one KYOWA ASQ-1BL accelerometer (Figure 4.5(a)) was used to measure the motion of the simulation table. The accelerometer was installed on the movable platform of the motion simulation table and connected to a VAQ-500A signal conditioner (Figure 4.5(b)) which was switched to displacement measurement mode. The accelerometer, with the signal conditioner switched to the displacement measurement mode, was calibrated in the laboratory against the motion simulation table prior to taking the field measurements. Figure 4.6 compares the sinusoidal motions of 20 mm amplitude generated by the table with those recorded by the accelerometer. The calibration exercise showed that the displacement time histories recorded by the accelerometer were the same as those generated by the table only when motion frequency was no less than 0.2 Hz.

### 4.6. DATA ANALYSIS AND ALGORITHM ASSESSMENT

The calibration work for both the accelerometer and the GPS showed that the
accuracy of displacement measurements taken by the accelerometer could not be
guaranteed when motion frequency was below 0.2 Hz, whereas the accuracy of
displacement measurements taken by the GPS could not be relied on when
motion frequency was greater than 1.0 Hz. Therefore, the intermittency
frequency $\omega_c$ used in EMD should be set to 0.2 Hz and 1.0 Hz, respectively, for
data analysis. Since there are two integrated data processing algorithms, a total of
four combinations of data analysis should be performed for each type of
measurement, which include white noise random waves and wind-induced
dynamic responses measured from a real complex structure during Typhoon York
around a pre-defined static position.

4.6.1. White Noise Random Waves

In the white noise random wave measurement exercise, a one-dimensional white
noise random wave was generated by the motion simulation table in the
horizontal direction around a pre-defined static displacement of 10 mm. The
white noise random wave simulated had a standard deviation of 4.5 mm, a
duration of 1,800 seconds, and a frequency range of 0.025 Hz to 1.8 Hz. Three
sets of data recorded by the GPS in a dynamic test, the GPS in a static test, and
the accelerometer in a dynamic test are analyzed according to the two integrated
data processing algorithms. Figure 4.7 depicts the IMF components of the
GPS-measured displacement time history from the dynamic test using EMD. The
first curve (Curve 1) displayed in Figure 4.7 is the original displacement time
history recorded by the GPS in the dynamic test. After applying EMD to this
time history, a total of 12 IMF components (Curves 2 to 13) and one final residue
(Curve 14) are obtained. The last curve (Curve 14) is almost a constant value of
about 4 mm. This value can be regarded as the pre-defined static displacement contaminated by the multipath effect. The first curve (Curve 1) depicted in Figure 4.8 is the corresponding GPS time history measured in the static test. A total of 12 IMF components (Curves 2 to 13) and one final residue (Curve 14) are obtained after EMD is applied to Curve 1. Curve 14 is again almost a constant value of about -6 mm. The difference between the two residues is 10 mm, which is actually the pre-defined static displacement generated by the motion simulation table, as shown in Figure 4.9(a). This result partially confirms the effectiveness of the two integrated data processing algorithms in determining the mean displacement of a structure.

The final residue of about 4 mm is then subtracted from the original displacement time history measured by the GPS in the dynamic test to obtain a contaminated dynamic displacement time history, as shown in Figure 4.9(b). The EMD with an intermittency frequency of 0.2 Hz is applied to the dynamic displacement time history measured by the accelerometer to obtain the high-frequency dynamic displacement time history, as shown in Figure 4.9(c). In the first application of the adaptive filter, the dynamic displacement time history obtained from the GPS is taken as the primary measurement, while the high-frequency displacement time history from the accelerometer is taken as the reference measurement. The resulting coherent and incoherent components are displayed in Figures 4.9(d) and (e), respectively. The incoherent component shown in Figure 4.9(e) is regarded as the low-frequency dynamic displacement time history, but is contaminated due to the multipath effect. Thus, in the second application of the adaptive filter, this low-frequency dynamic displacement time
history is taken as the primary measurement, while the corresponding dynamic background noise time history measured in the static test is taken as the reference measurement. The coherent and incoherent components are successfully separated and illustrated in Figures 4.9(f) and (g), respectively. The incoherent component shown in Figure 4.9(g) can be seen as the low-frequency dynamic displacement time history. Although the time history shown in Figure 4.9(g) should consist of the thermal noise of the receiver, which is a kind of instrumental noise, the instrumental noise of an advanced receiver is only 0.1 mm RMS after 1-second averaging for phase measurement (Byun et al. 2002). Therefore, the error due to instrumental noise is neglected in this study.

The total displacement time history is thus the combination of the mean displacement (see Figure 4.9(a)), the high-frequency dynamic displacement from the accelerometer with frequency components of more than 0.2 Hz (see Figure 4.9(c)), and the low-frequency dynamic displacement from the GPS with frequency components of less than 0.2 Hz (see Figure 4.9(g)). A portion of the total displacement time history obtained by integrated data processing Algorithm A from 300 to 400 seconds is plotted in Figure 4.10, together with the actual displacement time history generated by the motion simulation table. It can be observed that there is good agreement between the derived time history from the GPS and accelerometer measurements and the input time history from the motion simulation table. The power spectral density functions (PSD) of both the measured and targeted time histories are also computed and plotted in Figure 4.11. It can be seen that the proposed data processing algorithm can accurately capture frequency components from 0.025 Hz to 1.8 Hz of the dynamic
displacement time history generated by the motion simulation table.

Algorithm A with an intermittency frequency of 1 Hz and Algorithm B with intermittency frequencies of 0.2 Hz and 1.0 Hz are also applied to the same sets of measurement data to obtain the total displacement. Table 4.2 lists the mean, standard deviation, minimum, and maximum values of the total displacement derived from Algorithms A and B, together with those derived from the actual movement generated by the simulation table and those obtained from the displacement time histories measured directly by the GPS and the accelerometer. The results presented in the table show that the accelerometer alone is incapable of measuring the mean displacement generated by the motion simulation table. Likewise, the GPS alone cannot measure the mean displacement or dynamic displacement accurately. However, a great improvement in measurement accuracy can be achieved by using the integrated data processing algorithms. Compared with the statistical values of the actual displacement time history generated by the motion simulation table, the relative errors in the mean, standard deviation, maximum, and minimum displacements are respectively -6.2%, 12.7%, -1.4%, and 10.7% for Algorithm A with the intermittency frequency of 0.2 Hz and respectively -6.2%, 13.5%, 0.5%, and 11.7% for Algorithm B with the intermittency frequency of 0.2 Hz. It can be seen that the data processing performance of Algorithm A is statistically similar to that of Algorithm B. By comparing the statistical measurement errors from the two algorithms with different intermittency frequencies, it can be seen that the data processing performance of both algorithms with a higher intermittency frequency is similar to that achieved using a lower intermittency frequency.
4.6.2. The Wind-induced Dynamic Response of a Real Complex Structure

To further examine the data processing performance of the two proposed data processing algorithms, the wind-induced dynamic displacement response of a real complex structure with the first lateral natural frequency of 0.17 Hz, as recorded by displacement transducers during Typhoon York in 1999, was reproduced by the motion simulation table around a pre-defined static displacement of 10 mm, with the GPS and the accelerometer then being used to measure this movement.

Algorithm A with an intermittency frequency of 1 Hz is applied to the three sets of measurement data; the results are plotted in Figure 4.12. The displacement time history measured by the GPS in the dynamic test and the displacement time history measured by the accelerometer are shown in Figures 4.12(a) and (b), respectively. The mean displacement obtained from the difference in the final residues of the GPS-measured static and dynamic time histories, the high-frequency dynamic displacement response measured by the accelerometer with frequency components of more than 1 Hz, and the low-frequency dynamic displacement response measured by the GPS with frequency components of less than 1 Hz are depicted in Figures 4.12(c), (d), and (e), respectively. A portion of the total displacement time history obtained by integrated data processing Algorithm A between 800 and 1,500 seconds is plotted in Figure 4.13(a), together with the actual displacement time history generated by the motion simulation table. It can be observed that there is good agreement between the total displacement time history derived from the data processing algorithm and
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Integrating GPS and Accelerometer Data to Measure Total Displacement

the actual motion generated by the motion simulation table. The PSD functions of both the measured and targeted time histories are also computed and plotted in Figure 4.13(b). It can be seen that the proposed data processing algorithm can capture the major frequency component (0.17 Hz) of the dynamic displacement time history generated by the motion simulation table.

Algorithm A with an intermittency frequency of 0.2 Hz and Algorithm B with intermittency frequencies of 0.2 Hz and 1.0 Hz are also applied to the same sets of measurement data to obtain the total displacement. Table 4.3 lists the mean, standard deviation, minimum, and maximum values of the total displacement derived from Algorithm A and Algorithm B, together with those derived from the actual movement generated by the simulation table and those obtained from the displacement time histories measured directly by the GPS and the accelerometer. It can be seen that although the accelerometer cannot detect the mean displacement, it can detect the dynamic displacement to some extent. By comparing the results presented in Tables 4.2-4.3, one can see that the GPS is better for measuring wind-induced dynamic displacement response than for measuring white noise random waves. However, -28.7% and 25.2% measurement errors still exist in the mean and maximum values, respectively, of the wind-induced dynamic displacements. Therefore, the measurement data taken directly from the GPS need to be processed. After applying the proposed integrated approach, the statistical values of the wind-induced displacement responses are significantly improved. Compared with the statistical values of the actual displacement time history generated by the motion simulation table, the relative errors in the mean, standard deviation, maximum, and minimum
displacements are respectively -2.4%, -0.2%, 1.5%, and -1.6% for Algorithm A with the intermittency frequency of 1.0 Hz and respectively -2.4%, -0.2%, 1.1%, and -2.1% for Algorithm B with the intermittency frequency of 1.0 Hz. It can also be seen that the data processing performance of the two algorithms with two different intermittency frequencies are similar to each other.

4.7. SUMMARY

This study proposes two data processing algorithms designed to enhance the accuracy of total (static plus dynamic) displacement response measurements for large civil engineering structures through a new integration of GPS-measured data with accelerometer-measured data based on empirical mode decomposition (EMD) and adaptive filter techniques. A series of GPS and accelerometer measurement tests have been performed at a test site using a motion simulation table to generate a white noise random wave and a wind-induced dynamic displacement response around a pre-defined static position. The results obtained by applying the proposed algorithms to the recorded GPS and accelerometer data have been compared with the actual displacement motions generated by the motion simulation table.

The motion simulation table test results show that the GPS is susceptible to the multipath effect and is incapable of measuring high-frequency dynamic displacement responses, while the accelerometer is incapable of measuring mean displacement and low-frequency dynamic displacement responses. However, after adopting the two integrated data processing algorithms, the accuracy of the total displacement measurements improved significantly. The data processing
performance of Algorithm A was similar to that of Algorithm B. The data processing performance of both algorithms with a higher intermittency frequency was also similar to their performance with a lower intermittency frequency. A comparison of the integrated results for the two motions generated by the motion simulation table shows that data processing performance for the wind-induced dynamic response of a real complex structure is better than for a white noise random wave. This indicates that a more satisfactory result will be achieved for motions of a larger magnitude.
Table 4.1. Summary of the RLS algorithm

1.0 Initialization

\[ w(0) = 0 \]

\[ \Phi(0) = \delta^{-1} I \quad \text{where} \quad \delta = 0.01 \quad \text{(Akay 1994)} \]

2.0 Calculation for each instant of time, \( n = 1, 2, \ldots, N \)

2.1 \[ g(n) = \frac{\Phi(n-1)r(n)}{\lambda + r^T(n)\Phi(n-1)r(n)} \]

2.2 \[ e_{\text{prior}}(n) = s(n) - r^T(n)w(n - 1) \]

2.3 \[ w(n) = w(n - 1) + g(n)e_{\text{prior}}(n) \]

2.4 \[ \Phi(n) = \lambda^{-1}\Phi(n - 1) - g(n)r^T(n)\lambda^{-1}\Phi(n - 1) \]
### Table 4.2. Statistics from the white noise random wave measurement simulation

<table>
<thead>
<tr>
<th>STATISTICS (mm)</th>
<th>MEASUREMENT EQUIPMENT</th>
<th>INTEGRATED DATA PROCESSING METHOD</th>
<th>ALGORITHM A</th>
<th>ALGORITHM B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TABLE</td>
<td>GPS</td>
<td>ACC.</td>
<td>$\omega_c$=0.2Hz</td>
</tr>
<tr>
<td>Mean</td>
<td>10.1</td>
<td>3.5</td>
<td>0.2</td>
<td>9.5</td>
</tr>
<tr>
<td></td>
<td>(-65.3)</td>
<td>(-)</td>
<td>(-)</td>
<td>(-6.2)</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>4.5</td>
<td>10.1</td>
<td>4.6</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>(124.4)</td>
<td>(2.2)</td>
<td>(12.7)</td>
<td>(16.0)</td>
</tr>
<tr>
<td>Minimum</td>
<td>-7.9</td>
<td>-29.8</td>
<td>-17.4</td>
<td>-7.8</td>
</tr>
<tr>
<td></td>
<td>(277.2)</td>
<td>(120.3)</td>
<td>(-1.4)</td>
<td>(5.1)</td>
</tr>
<tr>
<td>Maximum</td>
<td>28.5</td>
<td>44.2</td>
<td>20.8</td>
<td>31.5</td>
</tr>
<tr>
<td></td>
<td>(55.1)</td>
<td>(-27.0)</td>
<td>(10.7)</td>
<td>(15.5)</td>
</tr>
</tbody>
</table>

Values in brackets indicate relative error percentages (%)

### Table 4.3. Statistics from the wind-induced dynamic displacement measurement simulation

<table>
<thead>
<tr>
<th>STATISTICS (mm)</th>
<th>MEASUREMENT EQUIPMENT</th>
<th>INTEGRATED DATA PROCESSING METHOD</th>
<th>ALGORITHM A</th>
<th>ALGORITHM B</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$\omega_c$=0.2Hz</td>
</tr>
<tr>
<td>Mean</td>
<td>10.8</td>
<td>7.7</td>
<td>0.2</td>
<td>10.5</td>
</tr>
<tr>
<td></td>
<td>(-28.7)</td>
<td>(-)</td>
<td>(-)</td>
<td>(-2.4)</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>11.7</td>
<td>12.1</td>
<td>9.6</td>
<td>11.4</td>
</tr>
<tr>
<td></td>
<td>(3.4)</td>
<td>(-17.9)</td>
<td>(-2.6)</td>
<td>(-0.2)</td>
</tr>
<tr>
<td>Minimum</td>
<td>-22.6</td>
<td>-28.3</td>
<td>-28.1</td>
<td>-22.5</td>
</tr>
<tr>
<td></td>
<td>(25.2)</td>
<td>(24.3)</td>
<td>(0.5)</td>
<td>(1.5)</td>
</tr>
<tr>
<td>Maximum</td>
<td>45.3</td>
<td>47.6</td>
<td>36.4</td>
<td>45.6</td>
</tr>
<tr>
<td></td>
<td>(5.1)</td>
<td>(-19.6)</td>
<td>(-5.9)</td>
<td>(-1.6)</td>
</tr>
</tbody>
</table>

Values in brackets indicate relative error percentages (%)
(a) Representation of the RLS algorithm

(b) A transversal filter with time-varying tap weights

**Figure 4.1.** The RLS adaptive filter
Figure 4.2. Algorithm A for the proposed integrated data processing approach
Figure 4.3. Algorithm B for the proposed integrated data processing approach
Figure 4.4. Views of the Pau Yue Kong Library
(a) The *KYOWA* ASQ-1BL accelerometer

(b) The *KYOWA* VAQ-500A signal conditioner

**Figure 4.5.** Accelerometer set up
Figure 4.6. Calibration results for the accelerometer for subsequent dynamic displacement measurements
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Curve 1
Curve 2
Curve 3
Curve 4
Curve 5
Curve 6
Curve 7
Curve 8
Curve 9
Curve 10
Curve 11

Time (sec)
Figure 4.7. IMF components of GPS-recorded displacement time history (dynamic test)
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Integrating GPS and Accelerometer Data to Measure Total Displacement

Curve 1

Curve 2

Curve 3

Curve 4

Curve 5

Curve 6

Curve 7

Curve 8

Curve 9

Curve 10

Curve 11

Displacement (mm)

Time (sec)
Figure 4.8. IMF components of GPS-recorded displacement time history (static test)
Figure 4.9. Displacement time histories obtained by Algorithm A (white noise random wave)
Figure 4.10. Comparative displacement time history of the white noise random wave: (a) motion simulation table; (b) integrated result
Figure 4.11. Comparative PSD function of the white noise random wave: (a) motion simulation table; (b) integrated result
Figure 4.12. Displacement time histories obtained by Algorithm A (wind-induced dynamic response)
Figure 4.13. Comparative results of wind-induced dynamic response: (a) displacement time history; (b) PSD function
Chapter 5

STRONG WIND STUDIES ON THE TSING MA BRIDGE

5.1. INTRODUCTION

Chapters 3 and 4 have discussed the dynamic measurement accuracy of the global positioning system (GPS) in three orthogonal directions and the integrated GPS-accelerometer data processing technique used for total displacement monitoring, respectively. Through a series of motion simulation table tests, the studies outlined positive results on the application of GPS in the structural health monitoring system (SHMS). However, a challenging problem for professionals is how to take full advantage of GPS and SHMS to continuously monitor the serviceability and safety of long-span cable-supported bridges under strong winds.

As noted in Chapter 2, the Tsing Ma Bridge in Hong Kong is the longest suspension bridge in the world carrying both highway and railway. To ensure the bridge functions properly over its long service life and to prevent it from catastrophic failure under strong winds, a Wind And Structural Health Monitoring System (WASHMS) that includes 6 anemometers and 14 GPS stations has been fully operational on the Tsing Ma Bridge since 1997 (for the anemometers) and 2002 (for the GPS stations), respectively. This full-scale,
long-term monitoring program offers an excellent opportunity for assessing the serviceability and safety of a long-span cable-supported bridge under the action of wind. As it is known that a statistical relationship between wind and wind-induced displacement response is important in assessing the serviceability of the bridge, studies of the Tsing Ma Bridge under strong winds should first be conducted to identify the mean wind speed and direction and establish the statistical relationship between displacement and wind speed discussed in Chapter 6. In addition, the statistical relationship developed is intended to be extended to extreme wind speeds on the bridge deck in Chapter 7 using finite element model (FEM) analysis. This makes it necessary to use the wind characteristics measured in the field to generate the wind forces required for such analysis. The objective of this chapter is thus to review the wind measurement data recorded in the field to define the characteristics of strong winds in the Tsing Ma Bridge area. In this regard, the WASHMS anemometers installed on the Tsing Ma Bridge and the local terrain surrounding the bridge are first described. Several conditions are then designed to pre-process the data and produce a high quality database. The pre-processed data ultimately produced are then analysed to evaluate the mean wind speed, the mean wind direction, the mean wind incidence, the turbulence intensity, the integral length scale and the wind spectra of both 10-minutes and 1-hour in duration.

5.2. THE LOCAL TOPOGRAPHY OF THE TSING MA BRIDGE

Hong Kong is situated at latitude N22.2° and longitude E114.1°. Not only does Hong Kong encompass many islands, but also features a large number of mountains that cover most of the territory. The area surrounding the Tsing Ma
Bridge in Hong Kong, with a bridge deck alignment deviating from bearing by about 73°, is no exception. The complex local topography surrounding the Tsing Ma Bridge within the dashed circle of 5 km in radius includes sea, islands, and mountains of 69 to 500 m in height. These features are shown in the Hong Kong map reproduced in Figure 5.1 (Xu et al. 2000). This complex topography is mirrored by the nature of the wind characteristics around the bridge. The following summaries describe the major topographical conditions that may affect the wind characteristics at the bridge site, taking the midpoint of the main span of the bridge as a reference and using 16 cardinal directions:

(a) Wind from N of the bridge site: The near field effect may arise due to a 300 m-plus range of mountains in Tai Lam Chung Country Park. The far field effect may be caused by Kai Kung Leng, a mountain 572 m in altitude;

(b) Wind from NNE of the bridge site: The near field may be affected by Shek Lung Kung, a 474 m mountain. The far field effect may be caused by Tai To Yan and the mountain neighboring Sheung Fa Shan, with respective heights of 566 m and 579 m;

(c) Wind from NE of the bridge site: The near field may arise due to a range of 200 m-plus mountains. The far field effect may be caused by Tai Mo Shan, a mountain of 957 m in height;

(d) Wind from ENE of the bridge site: The near field effect may be induced by mountains on Tsing Yi Island which rise to an altitude of 218 m. The far
field effect may be induced by the mountains of Grassy Hill, Needle Hill, and Ma On Shan, with respective heights of 647 m, 532 m, and 702 m;

(e) Wind from E of the bridge site: The near field effect may be induced by mountains on Tsing Yi Island with heights of around 100 m. The far field effect may be induced by Lion Rock and Kowloon Peak, mountains with respective heights of 495 m and 602 m;

(f) Wind from ESE of the bridge site: The near field effect may arise due to mountains on Tsing Yi Island with a highest level of 334 m. The far field effect may be caused by Mount Parker, which is 532 m in height;

(g) Wind from SE of the bridge site: The near field may arise due to mountains on Tsing Yi Island with a height of around 200 m. The far field effect may be caused by open sea and by Victoria Peak and Violet Hill, with respective heights of 532 m and 436 m;

(h) Wind from SSE of the bridge site: The near field and far field effects may arise due to open sea;

(i) Wind from S of the bridge site: The near field and far field effects may arise due to open sea;

(j) Wind from SSW of the bridge site: The near field and far field effects may arise due to open sea;
(k) **Wind from SW of the bridge site:** The near field may arise due to Fa Peng Teng, a mountain of 273 m in height. The far field effect may be caused by Tai Shan, Lo Fu Tau, and Sunset Peak, with respective heights of 291 m, 465 m, and 869 m;

(l) **Wind from WSW of the bridge site:** The near field may arise due to a range of mountains on Lantau Island which rise to more than 100 m in altitude, whereas the far field effect may be caused by Lantau Peak, with a height of 934 m;

(m) **Wind from W of the bridge site:** The near field may arise due to buildings on Ma Wan Island. The far field effect is caused by open sea;

(n) **Wind from WNW of the bridge site:** The near field may arise due to the mountain on Tai Lam Chung which rises to 344 m in height. The far field effect may be induced by Castle Peak, at a height of 583 m, and the mountain neighboring the Tai Lam Chung Reservoir at a height of 507 m;

(o) **Wind from NW of the bridge site:** The near field and far field effects may arise due to a range of mountains in Tai Lam Chung Country Park which rise to more than 300 m; and

(p) **Wind from NNW of the bridge site:** The near field and far field effects may arise due to a range of 200 m-plus mountains in Tai Lam Chung Country Park.
In summary, according to the near field effect on the bridge site, the terrains from the SW to the SE quarters, rotating in a clockwise direction, can be seen as an overland fetch. On the other hand, the terrains from the SE to the SW quarters, rotating in a clockwise direction, can be considered an open sea fetch.

5.3. INSTRUMENTATION AND DATA ANALYSIS

As noted above, the WASHMS for the Tsing Ma Bridge includes a total of six anemometers, of which two are located in the middle of the main span, two in the middle of the Ma Wan approach span, and one on each on the Tsing Yi and Ma Wan towers (see Figure 5.2). To prevent disturbance from the bridge deck, the anemometers at the deck level were respectively installed on the north and south sides of the bridge deck via an 8.965 m boom extended from the leading edge of the deck (see Figure 5.3). The anemometers installed on the north and south sides of the bridge deck in the middle of the main span, respectively, which are specified as WI-TJN-01 and WI-TJS-01, are digital-type Gill Wind Master ultrasonic anemometers. This type of anemometer can simultaneously measure wind speed components in three orthogonal directions. The anemometers located on the north and south sides of the bridge deck near the middle of the Ma Wan approach span, which are specified as WI-TBN-01 and WI-TBS-01, respectively, are analogue mechanical anemometers. Each analogue anemometer consists of a horizontal component (RM Young 05106) that gives the horizontal resultant wind speed and its azimuth, and a vertical component (RM Young 27106) that provides the vertical wind speed. The other two analogue mechanical anemometers, which provide a horizontal component only, are located 11 m above the top of each bridge tower on the south side. They are specified as
WI-TPT-01 for the Tsing Yi tower and WI-TET-01 for the Ma Wan tower.

Since the number of anemometers is limited, it is not possible to properly investigate the wind profiles and correlations in either the horizontal or vertical direction. Furthermore, a reference anemometer is necessary to explore the statistical relationship between wind speed and wind-induced bridge displacement response. Of all of the anemometers installed on the Tsing Ma Bridge, the winds measured by the ultrasonic anemometers in the middle of the main span are the most representative and are less influenced by the surrounding geographic conditions. The two ultrasonic anemometers in the middle of the main span are therefore targeted in this study. To avoid disturbance from the bridge deck affecting natural wind measurement, either the ultrasonic anemometer on the south side or the other ultrasonic anemometer on the north side is selected for a given wind direction. To this end, the propeller anemometer located on the Ma Wan tower is used to determine the wind direction and facilitate selection of the ultrasonic anemometer on the appropriate side of the deck. Not all available records were used to analyze the strong wind characteristics to which the bridge is subject; only two types of relatively strong wind events for the period from July 1997 to September 2005 were considered: typhoons during which signal No. 3 or above was hoisted, and events in which the Hong Kong Observatory (HKO) issued a strong monsoon signal. As a result, a total of 504 and 1,665 hourly wind records for typhoon and monsoon events, respectively, are targeted for analysis. The sampling frequency of wind measurement was set as 2.56 Hz.
5.4. DATA PRE-PROCESSING

To produce high quality databases, several algorithms are developed using MATLAB as a platform to pre-process the original wind data. The primary function of data pre-processing is to remove unreasonable and undesirable data while maintaining useful wind data for subsequent statistical data analysis. The secondary function of pre-processing is to transform directly recorded wind data into data with either physical or geometric meaning. As shown in Figure 5.4, the main steps taken in the pre-processing of original wind data include (1) eliminating unreasonable data of abnormal magnitude from the propeller anemometer on the Ma Wan tower; (2) determining which of the anemometers on the northern and southern sides of the deck should be considered for each typhoon or monsoon event; (3) eliminate unreasonable data of abnormal magnitude from the ultrasonic anemometers in the middle of the main span; and (4) obtaining 10-minute and hourly mean wind speeds and wind turbulence components based on original wind data recorded by the ultrasonic anemometers. The details of each step are discussed in the following sections.

5.4.1. Elimination of Unreasonable Data from the Propeller Anemometer (Step 1)

To obtain quality wind direction and speed readings, consideration is given to removing data of abnormal magnitude from the propeller anemometer on the Ma Wan tower. Hourly wind records with a standard deviation of zero and wind records showing extremely high mean wind speeds are the typical cases in which data are considered abnormal. A standard deviation of zero indicates a constant wind speed and direction without any fluctuation. Such abnormal data are likely
to be associated with malfunctioning anemometers in a harsh environment. To remove this type of abnormal data, the first criterion set is expressed in terms of the standard deviation of an hourly record, as follows:

\[ x_{std,p} = 0 \]  

(5.1)

in which \( x_{std,p} \) is the standard deviation of an hourly record for either wind speed or wind direction.

Wind records indicating extremely high mean wind speeds should also be eliminated, because they are not realistic. The second criterion thus eliminates wind records showing an hourly mean wind speed of greater than \( \bar{U}_{max} \).

\[ \bar{U} > \bar{U}_{max} \]  

(5.2)

where \( \bar{U} \) denotes the hourly mean wind speed recorded by the propeller anemometer at the top of the Ma Wan tower. The “Structures Design Manual for Highways and Railways” (Highways Department, 2008) specifies that the design hourly mean wind speed over a 120-year return period is 35 m/s at a reference height of 10 m. For the purpose of this study, the threshold value of \( \bar{U}_{max} \) is set at 62.8 m/s at a height of 217 m, the altitude of the anemometer on the Ma Wan tower.

### 5.4.2. Selection of the Appropriate Ultrasonic Anemometer at Deck Level (Step 2)
The wind data recorded by the two ultrasonic anemometers located at the deck level should next be selected to ensure natural winds are analyzed without disturbance from the bridge deck. Therefore, only one of the two ultrasonic anemometers located on the north and south sides of the bridge deck is selected for each wind record recorded by the propeller anemometer on the Ma Wan tower. To determine which anemometer should be considered, 1-hour and 10-minute mean wind direction information obtained from the propeller anemometer on the Ma Wan tower is used. As the alignment of the bridge deck in the main span deviates from the North by 73°, the hourly and 10-minute measurement records from the ultrasonic anemometer located on the northern side of the middle of the main span are selected when the respective hourly and 10-minute mean wind directions measured at the Ma Wan tower are between 73 and 253°, rotating in an anti-clockwise direction. Conversely, the hourly and 10-minute measurement records from the anemometer located on the southern side are selected if the corresponding hourly and 10-minute mean wind directions measured at the Ma Wan tower lie between 73 and 253°, rotating in a clockwise direction.

However, for wind records taken at the Ma Wan tower, abrupt changes in wind direction are noted when wind enters the dead zone (i.e. 0±5°) of the propeller anemometer. Figure 5.5 shows a wind direction time history from the propeller anemometer on the Ma Wan tower for 18:00 to 19:00 on December 4, 2004. The histogram of the wind direction time history is plotted in Figure 5.6. It can be seen from Figure 5.6 that there are two peaks: the higher peak is around 4.9° and is the mode of the distribution, while the smaller one is around 300°. Figure 5.6 gives the mean value of the wind direction time history, 105.7°. Clearly, the mode
of the wind direction distribution is different from the mean value of the wind direction time history. The actual wind direction should be taken as the mode of the wind direction distribution rather than as the mean value of the wind direction time history. Let us now consider another wind direction time history in which the wind blows from a direction outside the dead zone. The histogram and fitting distribution of such a wind direction time history are shown in Figure 5.7. It can be seen that there is one peak only. The mode of the wind direction distribution is almost the same as the mean value of the wind direction time history. Thus, to avoid the dead zone problem, the mode of the wind direction distribution is used as the mean wind direction rather than using the mean value of the wind direction time history.

5.4.3. Elimination of Unreasonable Data from Ultrasonic Anemometers (Step 3)

In common with abnormal wind records from the propeller anemometer on the Ma Wan tower, wind records of abnormal magnitude from the ultrasonic anemometers are also considered for removal. There are two classes of abnormal data which should be eliminated for the subsequent wind component determination analysis. Wind records with a standard deviation of zero are one type of abnormal data considered for removal. The first criterion is therefore designed to eliminate hourly and 10-minute records with an abnormal standard deviation, as follows:

$$x_{\text{std},u} = 0$$  \hspace{1cm} (5.3)
in which $x_{std, \mu}$ is the standard deviation of an hourly or 10-minute wind record for one of the three orthogonal wind components recorded directly by an ultrasonic anemometer.

The second class of abnormal data which should be eliminated is spikes in a wind speed time history. A number of methods can be used to detect such spikes, depending on the nature of the application and the number of observations in a data record. Grubbs’ test, which is normally used to identify a single spike in a sample of $n$ observations, is adopted to eliminate such abnormal data (Thode 2002). Grubbs’ test is also called the extreme studentized deviate (ESD) method. Before applying Grubbs’ test to wind records, it is assumed that most wind speed records can be approximated by the Gaussian distribution, which is indexed by the values of mean $\bar{x}$ and variance $\sigma_x^2$. Furthermore, the distribution of spikes is independent of this initial Gaussian distribution. Figure 5.8 shows the flowchart suggested for detecting multiple spikes in a wind speed record using Grubbs’ test. The original hourly or 10-minute wind component record is divided into a number of equal segments, each of which has 128 data points, or 50 seconds in this study. Statistics such as the standard deviation $\sigma_x$ and the mean $\bar{x}$ for each segment are calculated. The statistic $G$, defined as the maximum absolute deviation from the sample mean $\bar{x}$ in units of the sample standard deviation $\sigma_x$, is then calculated for each segment:

$$ G = \frac{\max |x_i - \bar{x}|}{\sigma_x} \quad (5.4) $$

In this study, the hypothesis of no spike is rejected if the statistic $G$ calculated
via Grubbs’ test is greater than the critical value at the 1% significance level. The value of $x_i$ that has the maximum absolute deviation from the sample mean is identified as a spike. Since Grubbs’ test detects only one spike at a time, the spike detected should be expunged from the data record and the test iterated until all spikes have been removed. The missing data are then simulated by a linear interpolation between the neighboring points. However, a wind speed time history will not be considered for further analysis if the percentage of spikes is greater than 10% of the total number of data points in an hourly or 10-minute wind speed time history. Figure 5.9 shows the results of three wind speed component time histories with the spikes detected by Grubbs’ test. These time histories were collected from the ultrasonic anemometer at the mid-main span of the bridge between 15:00 and 16:00 Hong Kong Time (HKT) on August 24, 2003.

5.4.4. Determination of Mean Wind and Turbulent Wind Components

(Step 4)

Before being in a position to analyze wind characteristics at the bridge site, the mean wind and three orthogonal wind turbulence components must be determined on the basis of the original hourly and 10-minute wind data recorded directly by the ultrasonic anemometers in the middle of the main bridge span.

Let $U_1 (t)$, $U_2 (t)$, and $U_3 (t)$ denote the three orthogonal wind components recorded directly by the ultrasonic anemometer in the middle of the main bridge span (see Figure 5.10). The two perpendicular horizontal wind components $U_S (t)$ in the south and $U_E (t)$ in the east, respectively, and the upward wind
component \( U_U (t) \) in the vertical direction can then be determined in terms of \( U_1 (t) \), \( U_2 (t) \), and \( U_3 (t) \):

\[
U_S (t) = -U_1 (t) \cos \theta + U_2 (t) \cos \theta \quad (5.5a)
\]
\[
U_E (t) = U_1 (t) \sin \theta + U_2 (t) \sin \theta \quad (5.5b)
\]
\[
U_U (t) = U_3 (t) \quad (5.5c)
\]

where \( \theta \) is defined as the bearing for wind component \( U_1 (t) \) as shown in Figure 5.10. Based on the horizontal southward and eastward components \( U_S (t) \) and \( U_E (t) \) of the transient wind velocity and the vertical upward component \( U_U (t) \), the mean wind speed \( \bar{U} \) and three orthogonal wind turbulence components \( u(t) \), \( v(t) \), and \( w(t) \) in the alongwind, crosswind, and vertical upward wind directions, respectively, can be determined. Despite the existence of a recently proposed non-stationary wind model for characterizing turbulent winds (Xu & Chen 2004), the mathematical formulae used to implement the aforementioned wind component transform in this study are based on the assumption of a stationary wind and can be found in Chapter 2. Table 5.1 provides the formulae for calculating the mean wind direction in the horizontal plane and the major wind characteristics for the stationary wind model.

The orientations of the two ultrasonic anemometers have been changed several times since July 1997. The bearing \( \theta \) of the two anemometers was set at 163° from July 1997 to June 1999. During the period from June 2003 to August 2005, the orientations of the northern and southern ultrasonic anemometers were set at the bearings of 343° and 273°, respectively. Because of damage to the
anemometers during strong winds, the wind measurement data they recorded during the period July 1999 to June 2003 were unreliable; as a result, the wind measurement data recorded during this period are not analyzed.

To ensure that the procedure and computer programs developed for wind component transformation are correct, the horizontal mean wind directions derived from the selected measurement data recorded at the bridge site are compared with those published by the HKO for the same wind event. Figure 5.11(a) shows variations in the 10-minute horizontal mean wind direction recorded at the bridge site during Typhoon Dujuan on September 2, 2003, while Figure 5.11(b) displays variations in the 10-minute horizontal mean wind direction for the same wind event produced by the HKO on the basis of wind measurement data taken at Sha Tin. The two sets of wind direction data are compatible, giving confidence in the procedure and computer programs developed for this study.

5.5. WIND CHARACTERISTICS

Given its sub-tropical location facing the South China Sea, Hong Kong has four distinct seasons and is subjected to extreme winds generated by both typhoon and monsoon events. In the winter months between November and February, the winter monsoon normally blows from the northerly and northeasterly directions, bringing dry conditions and occasional cold fronts to Hong Kong. The advent of spring, which generally occurs in March and April, sees occasional spells of high humidity. In the summer months between May and September, the summer monsoon blows from the southerly and southeasterly directions. During this
period, the weather is hot and humid and is interspersed with occasional showers or thunderstorms. Between July and September, Hong Kong is most likely to be affected by typhoons. Of all four seasons, the autumn, which lasts from mid-September to early November, is the shortest. The winds in this period tend towards the east (Hui 1996).

Given Hong Kong’s complex wind climate, it is helpful to gain a solid understanding of winds affecting the Tsing Ma Bridge. Therefore, statistics on mean wind direction, mean wind speed, mean wind incidence, turbulence components, turbulence intensities, wind spectra, and integral length scales during strong wind events (both typhoons and monsoons) taken from the ultrasonic anemometers located in the middle of the main span of the bridge deck are analyzed.

5.5.1. Mean Wind Direction

For the purpose of this section, mean wind direction is defined as the direction of the horizontal component of the mean wind speed, taking the N direction as the reference in the clockwise direction. The equations used to determine the mean horizontal wind direction can be found in Table 5.1. To facilitate an understanding of the mean wind speed and mean wind direction of strong winds experienced at the Tsing Ma Bridge, wind records are further split into four groups according to mean wind speed. These wind speed groups include: (1) less than 10 m/s; (2) between 10 and 18 m/s; (3) between 18 and 45.8 m/s; and (4) greater than 45.8 m/s. The latter three groups represent stages 1, 2 and 3 as specified in the high wind management system for the Tsing Ma Bridge. The
high wind management system is an operating guideline which specifies that in stage 1, wind susceptible vehicles will be banned from using the upper deck of the Tsing Ma Bridge and diverted to the lower deck when the hourly mean wind speed recorded onsite is in excess of 40 kph ($\approx 10$ m/s) but does not exceed 65 kph ($\approx 18$ m/s). In stage 2 of the high wind management system, when the hourly wind speed is in excess of 65 kph ($\approx 18$ m/s) but does not exceed 165 kph ($\approx 45.8$ m/s), the upper deck of the Tsing Ma Bridge will be completely closed. Both the upper and lower decks will be completely closed in stage 3 when the hourly mean wind speed is in excess of 165 kph ($\approx 45.8$ m/s).

Table 5.2 shows the number of hourly and 10-minute typhoon records in each 22.5° sector and each of the four distinct groups. Figure 5.12 is a polar histogram plot for the 10-minute mean wind direction during typhoon events. It can be seen that almost 25% of typhoon event records are taken from the N direction. This observation appears to contradict the received wisdom that Hong Kong typhoons follow a southeasterly or southwesterly track. However, because typhoons are huge whirlpools rotating in a counter-clockwise direction, the direction of a typhoon may not be the same as its track. In addition, there is a possibility that the flow of air is disrupted by the surrounding geographic conditions. The observation made here is consistent with that made by the HKO. During the 1997-2005 period (excluding July 1999-June 2003), the HKO issued No. 8 gale or storm signals for a total of 46 hours, of which a total of 10 and 23 hours related to northwest and northeast winds, respectively.

Figure 5.13 is the polar histogram plot of the 10-minute mean wind direction for
the monsoon events concerned. It shows that the prevailing wind during monsoons is from the E direction. To gain an understanding of the characteristics of seasonal monsoons experienced at the Tsing Ma Bridge, the monsoon data are split up by season for analysis. The four distinct seasons used are winter (from November to February), spring (from March to April), summer (from May to August), and autumn (from September to October). Table 5.3(a)-(d) present the number of monsoon wind data in each direction sector and in each of the four wind speed groups for each monsoon season, respectively. The tables show that winter is the high season for monsoons at the Tsing Ma Bridge. The winter monsoon blows mainly from the N direction at a mean wind speed of less than 10 m/s, as depicted in Figure 5.14(a). In spring and autumn, monsoon winds tend towards the east (Figure 5.14(b) and (d)). Figure 5.14(c) shows that the summer monsoon, the least frequent, blows mainly from the southwest (the S and SSW directions) at a mean wind speed in excess of 10 m/s.

5.5.2. Mean Wind Speed

This section analyzes the same wind data as those used in Section 5.5.1 to identify the mean wind speed range. Information on the maximum hourly and 10-minute mean wind speeds in each direction sector during typhoon events is given in Table 5.4. The data show that most of the maximum wind speeds are greater than 18 m/s. Amongst the 16 direction sectors, the WNW and SW directions, respectively, show the maximum hourly and 10-minute mean wind speeds experienced at the Tsing Ma Bridge. The maximum hourly and 10-minute mean speeds are 19.42 and 22.67 m/s, respectively. Figure 5.15 is a wind rose diagram for the 10-minute mean wind speed and direction during typhoon events.
Figure 5.15 also illustrates three of the wind speed stages specified in the high wind management system. The figure shows that wind speeds in the range from 10 to 18 m/s predominate during typhoon events and that a stage 1 high wind management system signal is hoisted during most typhoons experienced at the Tsing Ma Bridge.

Table 5.5 tabulates the maximum hourly and 10-minute mean wind speeds in each direction sector observed during all monsoons and in each monsoon season. It can be seen that the maximum monsoon wind speed is less than the maximum typhoon wind speed. For the winter monsoon, the maximum mean wind speeds and directions in hourly and 10-minute bases are 11.04 m/s from the E direction and 12.15 m/s from the N direction, respectively. Similar results are noted for the other monsoon seasons. For the autumn monsoon season, the maximum hourly mean wind speed of 11.43 m/s and the maximum 10-minute mean wind speed of 12.23 m/s are observed in the NNE direction. For the spring monsoon season, the maximum hourly mean wind speed of 11.27 m/s and the maximum 10-minute mean wind speed of 13.04 m/s are observed in the E direction. Among the four monsoon seasons, the summer monsoon from the open sea fetch has the maximum hourly and 10-minute mean wind speeds of 12.42 m/s and 13.60 m/s from the S direction, respectively. The wind rose diagrams for the 10-minute mean wind speeds and directions for all monsoons and the four monsoon seasons are illustrated in Figures 5.16 (a)-(e), respectively. The results imply that it is not necessary to hoist a high wind management system stage 1 signal during most strong monsoon events.
5.5.3. Mean Wind Incidence

Mean wind incidence is defined as the angle between the mean wind velocity and the horizontal plane (the sea plane). The formula used to determine the mean wind incidence is given in Table 5.1. A positive mean wind incidence indicates wind blowing in an upward direction (see Figure 5.17). According to the definition, mean wind incidences are highly unstable at very low mean wind speeds. Therefore, a threshold value of 5 m/s is applied to wind data from the ultrasonic anemometers in the middle of the main span of the bridge deck.

Figure 5.18 is a polar plot of the 10-minute mean wind incidences and directions recorded at the deck level of the bridge during typhoon events. It can be seen that all the wind incidences are within $\pm 10^\circ$ of the 95% upper and lower limits of wind incidences under a mean wind speed of 20 m/s. However, the wind incidences measured in the easterly directions (i.e. in the ENE, E, and ESE directions) are much more scattered than those measured in other directions. As the easterly wind directions are almost parallel to the longitudinal direction of the bridge deck, there is a possibility that the flow of air is disrupted by the bridge deck. Table 5.6 presents the mean values and standard deviations of the hourly and 10-minute mean wind incidences for typhoon events and the number of records in each direction. It can be seen that wind incidences tend to be approximately zero for the open sea area. The mean values recorded for 10-minute mean wind incidences are $0.19^\circ$, $0.25^\circ$, and $0.35^\circ$ in the SE, SSE, and S directions, respectively. Similar results are also noted in the NNW and N directions. The mean values of the 10-minute mean wind incidences are $0.35^\circ$ in the N direction and $-0.46^\circ$ in the NNW direction. The table also shows that
almost all the wind incidences are negative for wind blowing over the overland fetch. The most negative mean value of 10-minute mean wind incidences is $-4.99^\circ$ in the ESE direction.

Figures 5.19(a)-(e) display the polar plots of the 10-minute mean wind incidences for all monsoons and the four monsoon seasons, respectively. Similar to typhoon wind incidences, most wind incidences measured during the monsoon period are within $\pm10^\circ$ of the 95% upper and lower limits of wind incidences under a mean wind speed of 20 m/s. However, in comparison with the range of typhoon wind incidences, a relatively large range of monsoon wind incidences is noted. Tables 5.7(a)-(e) list the mean values and standard deviations of the hourly and 10-minute mean wind incidences and the number of records in each direction for all monsoons and the four monsoon seasons, respectively. Overall, the most positive mean value of 10-minute mean wind incidences is $4.25^\circ$ in the SW direction, while the most negative mean value is $-3.77^\circ$ in the NW direction. However, when seasonal characteristics are considered, the most positive mean value of 10-minute mean wind incidences for the winter monsoon is $3.01^\circ$ in the NNE direction and the most negative mean value for the same season is $-4.46^\circ$ in the ESE direction. Similar results are noted for the other monsoon seasons. For the spring monsoon, the most positive mean value of 10-minute mean wind incidences is $3.33^\circ$ in the NNE direction and the most negative mean value is $-3.07^\circ$ in the ESE direction. For the autumn monsoon, the most positive mean value of 10-minute mean wind incidences is $4.35^\circ$ in the N direction and the most negative mean value is $-4.57^\circ$ in the ENE direction. For the summer
monsoon, the most positive 10-minute mean value of wind incidences is 4.99° in the E direction and the most negative mean value is -3.94° in the ENE direction.

5.5.4. Turbulence Intensity

Turbulence intensity, which reflects the intensity of turbulent wind, is an important parameter in determining the wind-induced dynamic response of a long suspension bridge. It is defined as the ratio of the standard deviation of turbulent wind to mean wind speed for a given duration. According to the definition, turbulence intensities may not be stable at very low mean wind speeds. Therefore, a threshold value of 5 m/s is applied to the effective wind data from the ultrasonic anemometers in the middle of the main span.

Figures 5.20(a)-(c) show polar plots of the 10-minute turbulence intensities in the longitudinal, lateral, and vertical directions at the deck level of the bridge for typhoon events. It can be seen that the turbulence intensities measured in the northeasterly and easterly directions vary within a larger range than those measured in other directions. This is because the longitudinal axis of the bridge deck is nearly parallel to the winds blowing from the easterly direction, and there is a possibility that the flow of air is disrupted by the bridge deck. To gain an understanding of the terrain effect on wind fluctuation experienced at the Tsing Ma Bridge, the mean values of the 10-minute standard deviations of the three fluctuating wind speed components ($\sigma_u, \sigma_v, \sigma_w$) are plotted as functions of the friction velocities ($U^*$) for some of the wind direction sectors at an average 10-minute mean wind speed of about 10.00 m/s (see Figure 5.21). Friction velocity is computed using the following formula (Tieleman & Mullins 1979):
\[ U^* = \sqrt{\left( \bar{u}\bar{w}_u \right)^2 + \left( \bar{v}\bar{w}_v \right)^2} \]  

(5.6)

where \( \bar{u}\bar{w}_u \) and \( \bar{v}\bar{w}_v \) are the co-variances of the longitudinal and vertical velocity fluctuations, and of the lateral and vertical velocity fluctuations, respectively, and the bar means the mean value. The regression lines fitted in Figure 5.21 clearly indicate that the standard deviations of turbulent wind velocities increase with increasing friction velocity, and that the standard deviation and friction velocity are higher in the N direction than in the S direction. Since the average 10-minute mean wind speed is about 10.0 m/s for all the direction sectors represented in Figure 5.21, it can be surmised that the turbulence intensities recorded over the open sea terrain are the smallest and have the lowest friction velocities, whereas the turbulence intensities recorded over the overland terrain are the highest and have the highest friction velocities.

The variations in 10-minute turbulence intensities with 10-minute mean wind speed for each direction sector are plotted in Appendix C, resulting in a total of 48 plots. Table 5.8 summarizes the mean values and standard deviations of turbulence intensities for each direction sector. The table shows that the most turbulent winds during typhoon events come from the NE direction. The mean values of the longitudinal, lateral, and vertical turbulence intensities are 38.6%, 36.2%, and 26.1%, respectively. The least turbulent winds appear to come from the S direction, which has mean longitudinal, lateral, and vertical turbulence intensities of 8.6%, 8.4%, and 5.3%, respectively. Table 5.8 also lists the average ratio of lateral turbulence intensity to longitudinal turbulence intensity (0.903) and the ratio of vertical turbulence intensity to longitudinal turbulence intensity.
Figures 5.22-5.26 are the polar plots of the 10-minute turbulence intensities and directions recorded at the deck level of the bridge for all monsoons and the four monsoon seasons, respectively. In common with typhoon events, monsoon events produce a relatively large range of turbulence intensities in the northeasterly directions. Each monsoon season has its own prevailing wind direction. Therefore, it is better to combine the wind data measured in the four monsoon seasons to assess the terrain effect on wind fluctuations experienced at the Tsing Ma Bridge. Figure 5.27 illustrates the variations in the mean 10-minute standard deviations of the three fluctuating wind speed components ($\sigma_u, \sigma_v, \sigma_w$) and the corresponding friction velocities ($U^*$) for some of the direction sectors at an average 10-minute mean wind speed of about 8.0 m/s. The results shown in Figure 5.27 show that open sea terrain and the overland area have the lowest and highest friction velocity, respectively. The same observation applies for typhoon events.

Furthermore, the slope of the regression line for monsoon events is similar to the slope of the regression line for typhoon events. Since the friction velocity measured during typhoon events is higher than that measured during monsoon events, the standard deviations of the three fluctuating wind speeds in typhoon events are larger than those in monsoon events. Therefore, during monsoons, the turbulence intensities in the longitudinal, lateral, and vertical directions should be lower than those recorded during typhoons.
Appendix C shows plots of the variations in 10-minute turbulence intensities at the 10-minute mean wind speed for each direction sector over the four monsoon seasons; a total of 192 plots are displayed. Tables 5.9(a)-(e) summarize the mean values and standard deviations of the turbulence intensities for all monsoon events and the four monsoon seasons. As indicated in Table 5.9(a), the mean turbulence intensities for all monsoons range from 8.5% to 29.2% in the longitudinal direction, from 7.3% to 26.0% in the lateral direction, and from 4.8% to 23.3% in the vertical direction. The lowest and highest mean turbulence intensities for monsoons are smaller than those for typhoons. When seasonal characteristics are considered, during the winter monsoon season, the most turbulent wind observed is from the NE direction, with a mean longitudinal turbulence intensity of 28.7%, a mean lateral turbulence intensity of 26.4%, and a mean vertical turbulence intensity of 23.6%. The least turbulent wind during the same season is from the NW direction with mean longitudinal, lateral, and vertical turbulence intensities of 13.0%, 15.9%, and 15.9%, respectively. During the spring monsoon season, the most turbulent wind observed is from the NE direction, with mean longitudinal, lateral, and vertical turbulence intensities of 29.2%, 24.0%, and 20.8%, respectively. The least turbulent wind during the spring season is from the NNW direction, with a mean longitudinal turbulence intensity of 14.7%, a mean lateral turbulence intensity of 14.8%, and a mean vertical turbulence intensity of 12.9%. Likewise, the most turbulent wind during the autumn monsoon is from the NE direction, with a mean longitudinal turbulence intensity of 26.7%, a mean lateral turbulence intensity of 22.5%, and a mean vertical turbulence intensity of 21.1%. The least turbulent wind is from the SE direction, with mean longitudinal, lateral, and vertical turbulence intensities
of 9.4%, 9.6%, and 5.4%, respectively. Of the four seasons, the summer monsoon has the greatest extremes, bringing both the most and least turbulent winds over the course of the year. The most turbulent wind is recorded in the NE direction, with a mean longitudinal turbulence intensity of 36.8%, a mean lateral turbulence intensity of 31.4%, and a mean vertical turbulence intensity of 26.9%. The least turbulent wind blows from the SSW direction, with mean longitudinal, lateral, and vertical turbulence intensities of 8.3%, 7.2%, and 4.8%, respectively. The ratios of lateral turbulence intensity to longitudinal turbulence intensity for the four monsoon seasons range from 0.888 to 1.021, while the ratios of vertical turbulence intensity to longitudinal turbulence intensity range from 0.669 to 0.905. It is clear that due to the complexity of the terrain surrounding the Tsing Ma Bridge, the lateral and vertical turbulence intensities of winds at the site are significantly higher than for those over flat terrain.

5.5.5. Wind Spectra and Integral Length Scale

In this study, the power spectral density function (PSD) is computed for each one-hour wind turbulence component using the fast Fourier transform (FFT) technique. In the FFT, the time history of a wind turbulence component is divided into overlapping data frames, each of which has 1,536 points and overlaps the preceding and following frames by half a frame duration, i.e., by 768 points. A Hanning window is applied to all data frames. The PSD obtained by averaging all data frames is then normalized and expressed as

\[
y_a = \frac{s_{aa} \times f}{\sigma_a^2} \quad (5.7)
\]
where \( S_{aa} \) is the PSD function of wind turbulence component \( a (a = u, v, w) \), \( f \) is the frequency, and \( \sigma_a \) is the standard deviation of wind turbulence component \( a \). The frequency resolution used in this study is 0.0017 Hz.

In a previous study, Xu et al. (2000) concluded that the normalized wind spectra they measured were much better fitted by the von Karman spectra than by the Kaimal spectra or by Simiu and Scalan spectra. Therefore, the normalized von Karman spectra given in Table 5.1 are used to fit the alongwind, crosswind, and upward wind power spectra measured by the ultrasonic anemometers at the mid-main span of the bridge deck. From the equations shown in the table, the terms \( L^x_u \), \( L^x_v \), and \( L^x_w \) are the integral scales of turbulence for measuring the average longitudinal size of eddies associated with the longitudinal, transverse, and vertical velocity fluctuations, respectively. The integral scales \( L^x_a (a = u, v, w) \) are estimated by fitting the von Karman model to the normalized PSD using the least-squares method. The integral length scale \( L^x_a \) is found as the value that minimizes the square of the difference between the normalized power spectral density function \( y^*_a \) obtained from the measured data and the von Karman model \( y_a \).

\[
\varepsilon = \sum_{f = 0}^{f_{\text{limit}}} [y_a(f, \bar{U}, L^x_a) - y^*_a(f)]^2
\]

(5.8)

where \( f_{\text{limit}} \) is the upper limit of the frequency range for the curve fitting process.
In this study, relatively steady wind records measured during typhoons and monsoons are used for spectral analysis if the wind direction is perpendicular to the bridge deck and comes from either the sea or the land. Figures 5.28 and 5.29 show the alongwind, crosswind, and upward wind power spectra for the two selected samples, with winds blowing from the open sea and overland directions, respectively, during typhoon events. Another two samples shown in Figures 5.30 and 5.31 were measured during monsoon conditions in which the wind direction was from the open-sea and overland, respectively. Tables 5.10 and 5.11 summarize the statistics for all integral length scales derived using the curve fitting method that are involved in the spectral analysis for typhoons and monsoons, respectively.

Table 5.10 shows that the integral length scales of $L_{u_x}$, $L_{v_x}$, and $L_{w_x}$ measured for the overland exposure range between 64.02 m and 255.59 m (the average value being 106.56 m), 26.80 m and 65.30 m (the average value being 46.50 m), and 33.18 m and 55.72 m (the average value being 41.57 m), respectively. In contrast, the integral length scales measured for the open-sea fetch range between 110.66 m and 340.83 m (the average value being 188.73 m) for $L_{u_x}$, 41.62 m and 159.28 m (the average value being 70.54 m) for $L_{v_x}$, and 24.02 m and 33.84 m (the average value being 29.12 m) for $L_{w_x}$. It seems that the average integral length scales of $L_{u_x}$ and $L_{v_x}$ from the open-sea fetch are larger than those from the overland fetch. This observation for $L_{u_x}$ appears to be consistent with the comment in the literature that the length scale of $L_{u_x}$ is a decreasing function of terrain roughness (Simiu & Scalan 1996). However, the table also shows that the
average integral length scale of $L_w^x$ from the open-sea fetch is less than that observed from the overland fetch.

Table 5.11 also indicates that for wind recorded over the open sea terrain during monsoons, the integral length scales measured range between 212.32 m and 567.24 m (the average value being 358.25 m) for $L_u^x$, 56.42 m and 163.65 m (the average value being 87.11 m) for $L_v^x$, and 30.65 m and 60.88 m (the average value being 43.42 m) for $L_w^x$. For wind samples recorded over the overland terrain, the integral length scales vary from 65.80 m to 512.12 m (the average value being 172.61 m) for $L_u^x$, 26.76 m to 153.24 m (the average value being 58.29 m) for $L_v^x$, and 31.31 m to 99.63 m (the average value being 50.28 m) for $L_w^x$. A comparison of the length scales of wind samples recorded from the overland fetch with those from the open sea fetch shows that the integral length scales of $L_u^x$ and $L_v^x$ from the open sea fetch are larger than those from the overland fetch, and that the integral length scale of $L_w^x$ measured from the open sea fetch is less than that of the overland fetch. This observation is consistent with the observation made for typhoon events. A comparison of the typhoon and monsoon results shows that the mean values of $L_u^x$ for the length scales derived for monsoons are larger than those estimated for typhoons, while the values of $L_v^x$ and $L_w^x$ are similar for monsoons and typhoons.

5.6. SUMMARY

This chapter has analyzed wind data recorded by the WASHMS during typhoons and monsoons in an effort to evaluate the wind characteristics of the Tsing Ma suspension bridge. After describing the surroundings of the bridge and the
measurement instrumentation used, the wind measurement data recorded by WASHMS anemometers during typhoon and monsoon events were collected and pre-processed to obtain quality data enabling the wind characteristics of the bridge to be evaluated. The major results are summarized as follows:

- The terrain from the SE to the SW in an anti-clockwise direction can be regarded as the overland fetch, whereas the terrain from the SE to the SW in a clockwise direction can be considered the open sea fetch;

- The prevailing mean wind directions measured at the middle of the main span were from the N direction during typhoons and winter monsoons (from November to February), the E direction in spring (from March to April) and autumn (from September to October) monsoons, and the S direction during summer monsoons (from May to August);

- The maximum hourly and 10-minute mean wind speeds for typhoons were 19.42 m/s from the WNW direction and 22.67 m/s from the SW direction, respectively. Most of the typhoon winds recorded were within the wind management system stage 1 signal range (10-18 m/s). During the winter monsoon, the maximum mean wind speeds in hourly and 10-minute bases were 11.04 m/s from the E direction and 12.15 m/s from the N direction, respectively. During the spring monsoon, the maximum hourly mean wind speed of 11.27 m/s and the maximum 10-minute mean wind speed of 13.04 m/s were observed in the E direction. During the autumn monsoon, the maximum hourly mean wind speed of 11.43 m/s and the maximum
10-minute mean wind speed of 12.23 m/s were observed in the NNE direction. During the summer monsoon, the maximum hourly and 10-minute mean wind speeds recorded were 12.42 and 13.60 m/s from the S direction, respectively. Unlike the winds measured during typhoon events, those measured during the monsoon period were within 10 m/s;

- Mean wind incidence, defined as the angle between the mean wind velocity and the horizontal plane, has been discussed in this chapter. The results show that the hourly and 10-minute mean wind incidences were within approximately ±6° of the upper and lower mean wind incidence for typhoon and monsoon events. It was also found that the mean wind incidences measured in the easterly directions were much more scattered than those measured in other directions. This might be due to the bridge deck disrupting air flow in the easterly direction. In addition, the incidence measured for the overland fetch varied across a range larger than that indicated by open sea fetch measurements, while a mean wind incidence approaching zero was found in the open sea fetch;

- Turbulence intensity, defined as the ratio of the standard deviation of fluctuating wind to mean wind speed, has also been examined in this chapter. The evidence showed that the turbulence intensities recorded in the longitudinal, lateral, and vertical directions for the typhoon events ranged from 8.6% to 38.6%, 8.4% to 36.2%, and 5.3% to 26.1%, respectively. For all monsoon events, mean turbulence intensity ranged from 8.5% to 29.2% for the alongwind direction, 7.3% to 26.0% for the crosswind direction, and
4.8% to 23.3% for the vertical wind direction. Both the lowest and highest mean turbulence intensities recorded for monsoons were smaller than those measured for typhoons. In addition, the turbulence intensities measured for the overland fetch were higher than those measured for the open sea fetch. This is because turbulence intensity is an increasing function of friction velocity at the site;

- Using the FFT technique, the PSD functions for each wind turbulence component have been computed. The corresponding integral length scales of wind turbulence components have also been estimated by fitting the von Karman model to the normalized PSD using the least-squares method. An analysis of typhoon event data showed that the average integral length scales of $L_u^x$, $L_v^x$, and $L_w^x$ measured for the overland fetch were 106.56, 46.50, and 41.57 m, respectively, while those measured for the open sea fetch were 188.73, 70.54, and 29.12 m, respectively. For monsoon events, the average integral length scales of $L_u^x$, $L_v^x$, and $L_w^x$ measured from overland terrain were 172.61, 58.29, and 50.28 m, respectively, with the corresponding averages for open sea terrain being 358.25, 87.11, and 43.42 m, respectively. The integral length scales of the alongwind turbulence component $L_u^x$ and the crosswind turbulence component $L_v^x$ measured from the open sea fetch were larger than those measured from the overland fetch. This is because $L_u^x$ is inversely related to the terrain roughness of the site. However, this phenomenon did not occur for the integral length scale $L_w^x$. A comparison of integral length scales measured during typhoon and monsoon conditions showed that the length scales derived from the monsoon samples were larger
than those estimated from the typhoon data.

The statistical relationship between wind and wind-induced displacement response is known to be an important factor in assessing the serviceability of a bridge. In Chapter 6, this statistical relationship is established using the mean wind speed and direction for the Tsing Ma Bridge. To extend this statistical relationship to extreme wind speeds, the wind characteristic field measurements described above will be used to generate the wind forces required for the finite element model analysis presented in Chapter 7.
### Table 5.1. Definitions of wind characteristics

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>FORMULAE</th>
</tr>
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<td>(1) Mean wind direction in the</td>
<td>(a) for $\overline{U}_x \leqslant 0$ :</td>
</tr>
<tr>
<td>horizontal plane $\overline{\phi}$ (defined as the bearing angle from which the wind blows)</td>
<td>$\overline{\phi} = \cos^{-1}\left( \frac{\overline{U}_x}{\sqrt{\overline{U}_x^2 + \overline{U}_y^2}} \right)$</td>
</tr>
<tr>
<td>(b) for $\overline{U}_x &gt; 0$ :</td>
<td>$\overline{\phi} = 360^\circ - \cos^{-1}\left( \frac{\overline{U}_x}{\sqrt{\overline{U}_x^2 + \overline{U}_y^2}} \right)$</td>
</tr>
<tr>
<td>(2) Mean wind incidence $\overline{\alpha}$ (defined as the angle (from $0^\circ$ to $\pm 90^\circ$) between the mean wind velocity and the horizontal plane)</td>
<td>$\overline{\alpha} = \sin^{-1}\left( \frac{\overline{U}_y}{\sqrt{\overline{U}_x^2 + \overline{U}_y^2 + \overline{U}_z^2}} \right)$</td>
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<tr>
<td>(3) Variance $\sigma^2_a$</td>
<td>$\sigma^2_a = \frac{1}{T} \int_0^T a^2(t)dt ; \quad a = u, v, w$</td>
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<td>(4) Turbulence intensity $I_a$</td>
<td>$I_a = \frac{\sigma_a}{\overline{U}}$</td>
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<tr>
<td>(5) Wind spectrum (longitudinal, von Karman)</td>
<td>$f \cdot S_{u, a} = \frac{4 \frac{L_a}{U}}{f} \left[ \frac{1 + 70.6 \left( \frac{L_a}{U} f \right)^0.5}{\sqrt{1 + 70.6 \left( \frac{L_a}{U} f \right)^0.5}} \right]$</td>
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<tr>
<td>(6) Wind spectrum (lateral and vertical, von Karman)</td>
<td>$f \cdot S_{v, w, a} = \frac{4 \frac{L_a}{U}}{f} \left[ \frac{1 + 75.5 \left( \frac{L_a}{U} f \right)^0.5}{\sqrt{1 + 285 \left( \frac{L_a}{U} f \right)^0.5}} \right] ; \quad a = v, w$</td>
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Table 5.2. Number of typhoon records in each sector

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Table 5.3. Number of records in each sector for the four monsoon seasons

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(d) Autumn monsoon

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Table 5.4. Maximum mean wind speed in each sector for typhoon events

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Table 5.5. Maximum mean wind speed in each sector for monsoon events

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(b) 10-minute mean wind speed

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(a) All monsoons

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**AVERAGE VALUE**

0.952  0.755
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**AVERAGE VALUE**

|       | 1.021 | 0.905 |
### (c) Spring monsoons

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| AVERAGE VALUE | 0.888 | 0.669 |
### Autumn monsoons

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**AVERAGE VALUE**

0.981 0.760
Table 5.10. Statistics for integral length scales for typhoon events

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<td>Minimum</td>
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<td>Maximum</td>
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<td>Std. Deviation</td>
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Table 5.11. Statistics for integral length scales for monsoon events

<table>
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<th>OPEN SEA FETCH</th>
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<tbody>
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<td>Mean</td>
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<td>Maximum</td>
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</tr>
<tr>
<td>Std. Deviation</td>
<td>104.87</td>
<td>87.11</td>
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</table>
Figure 5.1. Location of the Tsing Ma Bridge and its surrounding topography

Figure 5.2. Distribution of anemometers on the Tsing Ma Bridge
Figure 5.3. Monitoring positions of anemometers on the bridge deck
Figure 5.4. Flowchart for wind data pre-processing
Figure 5.5. Example of time history of wind direction from propeller anemometer

Figure 5.6. Histogram distribution of wind direction time history shown in Figure 5.5
Figure 5.7. Distributions of wind direction time histories

(a) August 23, 1999: 0400-0500

mode = 236.6°
mean = 235.5°

(b) August 24, 2003: 1500-1600

mean = 87.6°
mode = 87.5°
Figure 5.8. Flowchart of spike detection and removal
Figure 5.9. Three wind speed component time histories with spikes detected

Figure 5.10. Orientation of ultrasonic anemometer for wind measurement
Figure 5.11. Comparison of mean wind direction during Typhoon Dujuan (September 2, 2003)
Figure 5.12. Histogram plot of 10-minute mean wind direction for typhoons

Figure 5.13. Histogram plot of 10-minute mean wind direction for monsoons
(a) Winter monsoons

(b) Spring monsoons
(c) Summer monsoons

(d) Autumn monsoons

Figure 5.14. Histogram plots of 10-minute mean wind direction for the four monsoon seasons
Figure 5.15. Wind rose plot of 10-minute mean wind speed for typhoon events
(a) All monsoons

(b) Winter monsoons
(c) Spring monsoons

(d) Summer monsoons
(e) Autumn monsoons

Figure 5.16. Wind rose plots of 10-minute mean wind speed for monsoon events

Figure 5.17. Sign of mean wind incidence
Figure 5.18. Wind rose plots of mean wind incidences for typhoon events
(a) All monsoons

(b) Winter monsoons
(c) Spring monsoons

(d) Summer monsoons
Figure 5.19. Wind rose plots of 10-minute mean wind incidence for monsoon events

(e) Autumn monsoons
(a) $I_u$

(b) $I_v$
Figure 5.20. Wind rose plots of turbulence intensity for typhoon events

(c) $I_w$

**Figure 5.20.** Wind rose plots of turbulence intensity for typhoon events
(a) Alongwind direction

$$\sigma_u (\text{m/s}) - U^* (\text{m/s})$$

Field data: Curve fit: $$aU^*$$

$$a = 2.0833$$

(b) Crosswind direction

$$\sigma_v (\text{m/s}) - U^* (\text{m/s})$$

Field data: Curve fit: $$aU^*$$

$$a = 1.9303$$
Figure 5.21. Variation in standard deviation with friction velocity for typhoon events

(c) Vertical upward wind direction
Chapter 5

Strong Wind Studies on the Tsing Ma Bridge

(a) $I_u$

(b) $I_v$
Figure 5.22. Wind rose plots of turbulence intensity for all monsoon events
(a) $I_u$

(b) $I_v$
Figure 5.23. Wind rose plots of turbulence intensity for winter monsoon events
(a) $I_u$

(b) $I_v$
Figure 5.24. Wind rose plots of turbulence intensity for spring monsoon events

(c) $I_w$
Chapter 5  

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(a) $I_u$

(b) $I_v$
Figure 5.25. Wind rose plots of turbulence intensity for summer monsoon events

(c) $I_w$
(a) $I_u$

(b) $I_v$
Figure 5.26. Wind rose plots of turbulence intensity for autumn monsoon events

\( I_w \)
(a) Alongwind direction

(b) Crosswind direction

Field data
Curve fit: $aU^*$

$a = 2.1495$

$a = 1.9757$
Figure 5.27. Variation in standard deviation with friction velocity for monsoon events

(c) Vertical upward wind direction
(a) Alongwind direction

(b) Crosswind direction
(c) Vertical upward wind direction

Figure 5.28. Wind spectra of typhoon from open sea fetch
(a) Alongwind direction

(b) Crosswind direction
(c) Vertical upward wind direction

Figure 5.29. Wind spectra of typhoon from overland fetch
Chapter 5

Strong Wind Studies on the Tsing Ma Bridge

(a) Alongwind direction

(b) Crosswind direction
(c) Vertical upward wind direction

Figure 5.30. Wind spectra of monsoon from open sea fetch
(a) Alongwind direction

(b) Crosswind direction
(c) Vertical upward wind direction

Figure 5.31. Wind spectra of monsoon from overland fetch
Chapter 6

STATISTICAL ANALYSIS OF THE RELATIONSHIP BETWEEN DISPLACEMENT AND WIND SPEED

6.1. INTRODUCTION

Under the assumption of stationary wind, wind velocity can be treated as a combination of a static (mean) component and three dynamic fluctuating components. These static and dynamic components will induce mean and dynamic bridge responses, respectively. To assess the effects of wind on the Tsing Ma Bridge and perform an effective serviceability assessment, this chapter analyses the statistical relationship between wind and wind-induced displacement response using the wind measurement data described in Chapter 5 and displacement measurement data recorded at the bridge. In Chapter 7, the statistical relationship thus developed is then extended to extreme wind speeds on the bridge deck using finite element model (FEM) analysis.

The bridge displacement responses for the Tsing Ma Bridge were measured by three types of sensory systems: displacement transducers, level sensing stations, and global positioning systems (GPS). Two displacement transducers are installed on the bridge: one is used to measure the relative lateral movement of the deck adjacent to the Ma Wan tower and the other is used to measure the
longitudinal movement of the bridge deck at the Tsing Yi abutment. These transducers are important for measuring thermal movement, but not for measuring the wind-induced dynamic displacement responses of the bridge. The level sensing stations mounted at six locations along the bridge deck can measure vertical displacements of the bridge deck at a sampling frequency of 2.56 Hz. They cannot measure the lateral and longitudinal displacements of the bridge deck sections in the horizontal plane. In addition, only the static vertical displacement can be measured. Therefore, only GPS stations, which can measure not only vertical, but also longitudinal and lateral static and dynamic displacement responses under all meteorological conditions, are considered in this study, which is designed to establish the statistical relationship between wind and wind-induced bridge displacement response. However, it should be noted that the GPS stations on the Tsing Ma Bridge were installed in 2000, with ongoing updates made until September 2002. Thus, the GPS data considered in this study were recorded in the October 2002-September 2005 period.

To enhance the measurement accuracy of the total displacement response of long-span cable-supported bridges, integrated data-processing techniques based on measurement data collected by GPS receivers and accelerometers using empirical mode decomposition (EMD) and an adaptive filter are developed in Chapter 4. However, in view as the Tsing Ma Bridge is very slender, the most majority of wind-induced displacement responses with a frequency less than or equal to 1 Hz have already been well captured by GPS. In addition, because the accelerometers and GPS receivers are positioned at different locations on the Tsing Ma Bridge, the proposed data-processing technique cannot be applied in this case.
This chapter first describes the locations of the GPS stations installed on the Tsing Ma Bridge and the corresponding data pre-processing techniques used. As noted above, the static and dynamic components of wind speed cause the mean and dynamic displacements of the bridge, respectively. However, at the same time, temperature variations cause mean displacements of the bridge, while moving vehicles and trains induce dynamic displacements. The wind-induced displacement response should therefore be ascertained according to total response measurements by eliminating the effects arising from environmental and operational loads. To this end, this chapter next analyzes the dynamic WIM sensors for vehicle flow and the temperature sensors for temperature. The effect of GPS background noise on displacement measurements is also evaluated by conducting calibration tests at the site of the Tsing Ma Bridge using the two-dimensional motion simulation table. This chapter concludes with an exploration of the statistical relationship between wind speed and wind-induced bridge displacement response at different locations as a function of wind direction.

6.2. GPS IN THE WASHMS

The geographic coordinates of the components of the Tsing Ma Bridge measured by GPS include bridge towers, main cables, and the bridge deck. There are a total of 14 Leica MC500 GPS rover stations installed in the three major bridge components described above; Figure 6.1 shows how they are distributed over the bridge. The Ma Wan tower and the Tsing Yi tower are each equipped with a pair of GPS receivers mounted at the top of the saddles on each tower leg (see Figure 6.2(a)). Because the position coordinates of a pair of main cables are monitored
only in the middle of the main span, a pair of GPS receivers is located at the mid-main span (see Figure 6.2(b)). The main span and the Ma Wan approach span of the bridge deck are the two other areas in which four pairs of GPS receivers are mounted. The mid-span of the Ma Wan approach span is equipped with one pair of receivers. The other three pairs of GPS receivers are located one quarter, one half and three quarters of the way along the main span of the bridge deck from the Ma Wan tower. To avoid the possibility of signal obstruction caused by high-sided vehicles, all the antennae of the GPS receivers located on the bridge deck are mounted at a height of 4 m and at a view angle of above 15° (see Figure 6.2(c)). Displacement measurements indicate positive movement in the longitudinal, lateral, or vertical direction if the bridge component moves toward Tsing Yi Island, sways to the north, or rises, respectively. The hourly real-time kinematic (RTK) records made in NMEA GPGGA format at a sampling frequency of 10 Hz are stored using the data file names listed in Table 6.1, together with the tag numbers of the GPS receivers.

6.3. GPS DATA PRE-PROCESSING

The main steps taken in the pre-processing of original GPS data are: (1) convert the GPS data from the HK80 geographic coordinate to the Universal Transverse Mercator (UTM) grid coordinate; (2) compute the bridge displacement coordinates with respect to the reference coordinates measured on November 28, 2000; (3) eliminate unreasonable data with abnormal magnitude caused by an abrupt change in the number of satellites or unsatisfactory geometry configurations; (4) obtain bridge displacement responses in three orthogonal directions; (5) filter displacement response time histories using a low-pass filter
with an upper frequency limit of 1 Hz to maintain high quality data for subsequent analysis; and (6) decompose the mean and dynamic displacements based on the total response measurements recorded by GPS.

6.3.1. Conversion of the Coordinate System (Step 1)

Given that original GPS data are recorded by the HK80 geographic coordinate system, the first step in the data pre-processing process is to transform the original data into the UTM grid coordinate system using the parameters listed in Table 6.2. The transformation equations defined by the Survey & Mapping Office of the Lands Department of Hong Kong are given in Chapter 2.

6.3.2. Computation of Displacement Responses (Step 2)

The next step is to compute the bridge displacement responses with respect to the reference coordinates measured between 04:00 and 05:00 HKT on November 28, 2000. The reference coordinates given by the Bridges and Structures Division of the Highways Department (HyD) are tabulated in Table 6.3. A positive movement in the corresponding longitudinal, lateral, or vertical direction is indicated if the bridge deck moves toward Tsing Yi Island, sways to the north, or rises, respectively.

6.3.3. Elimination of Unreasonable Data from GPS (Step 3)

Three parameters included in the GPS data are designed to reflect the quality of position estimates. These are the GPS quality indicator, the number of satellites, and the dilution of precision. The GPS quality indicator, which is expressed as an integer between 0 and 3, refers to the condition of the GPS receiver in solving
any ambiguity in RTK measurements. A measurement is reliable when the integer is 3. Apart from the GPS quality indicator, the dilution of precision parameter used to evaluate the geometry of the satellite constellation is another indicator of the quality of GPS position estimates. Positions tagged with a higher dilution of precision generally represent poorer measurement results than those tagged with a lower dilution of precision. The number of satellites available at any time is also important in determining the quality of GPS position information. To deliver the maximum level of precision, it is necessary that the GPS receives signals from a large number of satellites. In summary, the first three criteria designed to eliminate poor quality GPS data are set and expressed as:

\[
\begin{align*}
Q & \neq 3 \\
N_{\text{sat}} & < 5 \\
HDOP & > 4
\end{align*}
\]

where \( Q \), \( N_{\text{sat}} \) and \( HDOP \) denote the GPS quality indicator, the number of satellites, and the horizontal dilution of precision, respectively. If a GPS data point meets one of these criteria, it is expunged from the database. The missing data point is then added by a linear interpolation between the neighboring points. However, a displacement response time history will be eliminated without further analysis if the poor data points account for more than 10% of the total data points recorded in an hourly time history.

Some displacement time histories recorded by GPS undergo an abrupt change when the number of satellites in view suddenly falls. Figure 6.3 shows the displacement time history measured by the northern GPS station on the main
cable from 2:11 to 2:15 HKT on September 24, 2005, in which a sudden decrease in the number of satellites and an abrupt increase in the displacement time history can be observed. Since this phenomenon occurs from time to time, it is necessary to find a general method of correcting it. The objective of correcting such data is to move the displacement data set subject to an abrupt change back to the normal displacement time history. Figure 6.4 presents a flow chart of the method used to make such corrections. The most frequent number of satellites in view $Y$ is first calculated on the basis of probability theory. The displacement time history $X(t)$ with $N_{sat}(t)$ less than $Y$ is considered to be corrected. Using the criterion expressed by Equation (6.1b), the correction procedure starts when $N_{sat}(t)$ is equal to 5. The $i$th set of abnormal displacement data $\{X\}_i$ with respect to the given number $N$ is defined as:

$$\{X\}_i = X\{t\}_i = \{X_k X_{k+1} \cdots X_{k+m}\}_i$$  \hspace{1cm} (6.2)

where $\{t\}_i = \{t_k t_{k+1} \cdots t_{k+m}\}_i$ and $i = 1, 2, \ldots, p$; and $m$ is the number of data points in the $i$th data set. To ensure the $i$th set of abnormal displacement data can reasonably be moved back to the normal displacement time history, the mean values of the $i$th abnormal and normal displacement data sets should be computed. The mean vector $\{S_F\}_i (m \times 1)$ of the $i$th abnormal displacement data set is calculated using the following equation:

$$\{S_F\}_i = X_{k_i} + \frac{X_{(k+m)i} - X_{k_i}}{m} (\{t\}_i - t_{k_i})$$  \hspace{1cm} (6.3)

The mean vector $\{S_T\}_i (m \times 1)$ of the $i$th normal displacement data set is
computed by

\[ \{S_T\}_i = X_{(k-1)} + \frac{X_{(k+m+1)} - X_{(k-1)}}{m+2} (\{t\}_i - t_{(k-1)}) \] (6.4)

The \( i \)th set of the corrected (new) displacement data, \( \{X\}_i \), can be then obtained by

\[ \text{new} \{X\}_i = \{X\}_i - \{S_F\}_i + \{S_T\}_i \] (6.5)

This procedure is repeated for \( i = i + 1 \) until \( p \) to complete one cycle for the given number \( N \). The procedure is finally repeated for \( N_{sat}(t) = 6 \) until \( Y \) to complete the correction procedure as a whole. To demonstrate the effectiveness of the proposed correction procedure, the GPS-measured displacement time history shown in Figure 6.3 is corrected; the results are shown in Figure 6.5.

6.3.4. Determination of Bridge Displacement Components (Step 4)

To facilitate the establishment of statistical relationship between wind and bridge displacement, the displacement time histories for a given section recorded by a pair of GPS stations (north and south) are averaged to obtain the average displacement time history in the longitudinal, lateral, and vertical directions, respectively. The tag numbers used thereafter are TM-01, TM-02, and TM-04 for one quarter, one half and three quarters of the way along the main span of the bridge deck from the Ma Wan tower, and TM-05 for halfway along the Ma Wan approach span, respectively. The tag numbers of TM-03 for the main cable, TM-TY for the Tsing Yi tower, and TM-MW for the Ma Wan tower, are also used.
thereafter.

**6.3.5. Elimination of High-frequency Components (Step 5)**

In Chapter 3, a series of calibration tests in which a motion simulation table was used were carried out in an open area in Hong Kong to assess the accuracy of GPS in measuring dynamic displacements in three orthogonal directions. The test results showed that GPS could measure dynamic displacements accurately if the motion frequency was less than or equal to 1 Hz. Based on these results, all the displacement time histories measured by GPS in this study are filtered using a low-pass filter with a cut-off frequency of 1 Hz.

**6.3.6. Decomposition of Total Displacement Responses Measured Using GPS (Step 6)**

As noted above, GPS response measurements are a combination of static and dynamic components. To extract a time-varying mean displacement response from a total displacement time history, a wavelet transform method is employed. Wavelet transform is a mathematical tool that can be used to decompose a temporal signal into a summation of time-domain functions of various resolutions. In this step, the wavelet transform is used to decompose GPS displacement signals into a series of wavelet component signals. Following a 14-level decomposition process, the signal within the frequency range of 0-0.0003 Hz is then treated as the time-varying mean displacement response, because such a low frequency range will not involve any dynamic effect. The residual, i.e., the difference between the total displacement time history and the time-varying mean displacement response, is then treated as the dynamic
displacement response of the bridge.

6.4. IDENTIFICATION OF WIND-INDUCED BRIDGE DISPLACEMENT

Because GPS is capable of measuring both static and dynamic displacement components, this chapter focuses on evaluating the predominant effects of winds on not only dynamic displacements, but also on mean bridge displacements. However, mean bridge displacement measurements are affected by temperature variation and GPS background noise, while dynamic bridge displacement measurements are affected by GPS background noise and moving trains and vehicles. This section therefore assesses the effects of GPS background noise and temperature on mean displacement measurements and the effects of GPS background noise and traffic on dynamic displacement measurements.

6.4.1. GPS Background Noise

To quantify the effects of GPS background noise, calibration tests using a two-dimensional motion simulation table were conducted at the site of the Tsing Ma Bridge in September 2005, the weather remaining fine throughout the testing period. In field measurement testing (see Figure 6.6), two sets of *Leica* GX1230 dual frequency GPS receivers were used. One set was connected to a *Leica* AT504 choke ring antenna mounted on a tripod, which acted as a reference station. Another set was connected to a *Leica* AX1202 light right antenna installed on the movable platform of the motion simulation table, which acted as a rover station. As the RTK mode was used in testing, the corrections computed by the reference receiver were regularly transmitted to the rover receiver via a Pacific Crest PDL radio modem. The Hong Kong Observatory (HKO) time
service was used to synchronize the motion simulation table computer clock with GPS time. Internet access to the HK Observatory network time server at stdtime.gov.hk enabled the computer clock to be synchronized with the HKO atomic clock once every 16 seconds using Automachron software. The GPRS service was used to assess Internet service during testing, with a NOKIA 6680 mobile phone being used as a modem.

RTK GPS observations in NMEA GPPLK format was selected as the output mode for local position and satellite information from the rover station at a sampling rate of 10 Hz. The output, with a cut-off angle of 15°, was then transferred to a Dell desktop via an RS232 serial port. The dynamic trajectory measured by GPS, which was defined according to the motion simulation table coordinate system, was then computed and displayed in real time using in-house software. Platform motions were also recorded and regularly transferred to the Dell desktop to enable a comparison with GPS results.

In the first phase of testing, the background noises in GPS measurements taken at Ma Wan Island and Tsing Yi Island were measured for four hours (Tests 1-4), with the rover and reference stations remaining stationary. Tables 6.4 and 6.5 present the 6 segments of 10-minute mean and RMS (root-mean squares) for each measurement test carried out at Ma Wan Island and Tsing Yi Island, respectively. These tables show that the maximum absolute mean values in the horizontal and vertical directions were 4.09 and 7.78 mm, respectively, at Ma Wan Island, and 1.28 and 9.89 mm, respectively, at Tsing Yi Island. In addition to the mean displacements, the background noises at Ma Wan Island were also
recorded, with maximum RMSs of 2.22 mm in the horizontal direction and 5.30 mm in the vertical direction. At Tsing Yi Island, the maximum RMSs observed were 3.34 mm in the horizontal direction and 5.54 mm in the vertical direction. Nevertheless, the average values of the 10-minute means measured on both islands were 0.07 mm and 0.11 mm in the horizontal and vertical directions, respectively. The average values of the 10-minute RMSs were 1.62 mm in the horizontal direction and 3.67 mm in the vertical direction. These results demonstrated that GPS background noise had a small effect on the mean displacement of Tsing Ma Bridge and thus could be neglected. However, the threshold values for the standard deviations of the dynamic displacement response should be applied.

To find the threshold values for dynamic displacements without any significant effect from the GPS measurement noise, the two-dimensional motion simulation table was used as a test station to simulate wind-induced bridge deck motions on the Tsing Ma Bridge. The motions input into the table were based on measurement data in which the standard deviations were adjusted. Table 6.6 shows the GPS measurement errors expressed as standard deviations for different levels of input motion. It can be seen that there are measurement errors of more than 35% in both the horizontal and vertical directions, for which the standard deviations of the motions are 1.44 and 3.14 mm, respectively. However, when the standard deviations of the motions reach 4.5 mm in the horizontal direction and 9.7 mm in the vertical direction, the measurement errors in the displacement standard deviations are less than 5% in both directions. This implies that the dynamic performance of GPS is satisfactory only when the standard deviations
are larger than 4.5 mm in the horizontal direction and 9.7 mm in the vertical direction.

### 6.4.2. Temperature-induced Static Displacement Response

To assess the effects of temperature on the mean displacement response of the bridge deck, the measurement data from both the temperature sensors and GPS stations were selected for the 40 days in 2005 on which the mean wind speed was less than 5 m/s. The hourly mean displacements of the Ma Wan tower in the longitudinal and lateral directions (see Figure 6.7), the mid-main span in the longitudinal, lateral, and vertical directions (see Figure 6.8), and the cable in the longitudinal, lateral, and vertical directions (see Figure 6.9) are plotted as functions of hourly mean temperature. The coefficients of what is either a linear or parabolic relationship between temperature and displacement are summarized in the respective figures. The figures show that as the temperature rises, the main cables expand and move downward, the deck moves downward, and the two towers move inward. Therefore, the temperature affects the mean displacement of the towers in the longitudinal direction only, and the mean displacements of the deck and cable in the vertical and longitudinal directions only.

In addition, the hourly mean displacement responses of the Ma Wan tower in the longitudinal direction (see Figure 6.10), the mid-main span in the longitudinal and vertical directions (see Figure 6.11), and the cable in the longitudinal and vertical directions (see Figure 6.12) were obtained in strong winds, including typhoon and monsoon conditions, and plotted against the effective temperature of the deck. The temperature-mean displacement models, including the
corresponding 95% confidence intervals depicted in Figures 6.7-6.9, are also plotted in Figures 6.10-6.12, respectively. These figures show that the temperature-mean displacement relationships measured during strong winds are well-predicted by the temperature-mean displacement models during weak winds. This implies that the mean displacements measured on the tower in the longitudinal direction, and on the deck and cable in the longitudinal and vertical directions, during strong winds are all attributable to temperature effects. This leads to the conclusion that the mean wind does not affect the longitudinal mean displacement of the towers or the longitudinal or vertical mean displacement of the bridge deck and cable. On the contrary, examining variations in the lateral mean displacements of the bridge deck and cable at the mean wind speed (see Figure 6.13) indicates that the mean wind affects the lateral mean displacements of the bridge deck and cable. These observations provide a way of distinguishing temperature effects from wind effects on mean bridge displacements.

6.4.3. Traffic-induced Dynamic Displacement Response

In addition to the possible influence of wind, there is a possibility that the dynamic displacement responses of the bridge recorded by GPS are affected by moving road vehicles and moving trains. To assess the dynamic effect of trains running on the bridge, the traffic-induced displacement time histories of the Ma Wan tower, the mid-main span of the bridge deck, and the main cable, all of which were measured in November 2005, are examined and shown in Figures 6.14-6.16. It can be observed that the longitudinal displacement of the towers, the vertical displacement of the deck, and the longitudinal and vertical displacements of the cable, suddenly increase when a train is running on the bridge. To extract
the effect of railway loading on the GPS displacement data, a comparison has been made between cases in which there was a train on the bridge and those in which there was not, using wavelet packet transform (Chen et al. 2007). It was found that a railway loading had a predominating effect at frequencies of less than 0.08 Hz. The time histories with a frequency of less than 0.08 Hz are plotted in Figures 6.14-6.16.

In addition to train-induced deformation, another concern in this study is the dynamic displacement response of the bridge induced by moving road vehicles. To examine the effect of road vehicles on the dynamic displacement response of the bridge, one-week measurement data from both the WIM sensors and GPS stations which were recorded from November 7-14, 2005, during which period the mean wind speed was less than 5 m/s, are selected. Figure 6.17 depicts the standard deviations of displacement responses measured at the Ma Wan tower in the longitudinal and lateral directions, the mid-main span in the lateral and vertical directions, and the main cable in the lateral and vertical directions, together with traffic flow rate (TFR). It can be seen that the longitudinal and lateral displacement standard deviations of the tower are less than the threshold value derived in the calibration tests. Thus, the road vehicle-induced displacement standard deviation on the towers can be neglected. For other bridge components, the displacement standard deviations are larger than the threshold value when TFR is higher. Displacement standard deviations larger than the threshold value should be taken into account. In this study, the average value of these displacement standard deviations is taken as a constant value ($c$) and regarded as the highway-induced displacement standard deviation. Furthermore,
by assuming the wind- and highway-induced displacements are uncorrelated with each other, the standard deviation of the wind-induced dynamic displacement response ($\sigma_W$) can be determined by:

$$\sigma_W = \sqrt{\sigma_{WT}^2 - c^2}$$  \hspace{1cm} (6.6)

where $\sigma_{WT}$ is the standard deviation of the total dynamic displacement response induced by both road vehicles and wind. The constant values of the bridge deck and main cable displacements in the presence of road vehicles are listed in Table 6.7. As the highway-induced displacement standard deviation is assumed to be constant, the highway-induced standard deviation is significantly small or can even be neglected when the total displacement standard deviation is large enough. According to Equation (6.6), an error of less than 5% error is present in the wind-induced displacement standard deviation if the total displacement standard deviation is greater than or equal to $3.28 \times c$.

6.5. THE STATISTICAL RELATIONSHIP BETWEEN WIND AND MEAN WIND-INDUCED DISPLACEMENT

Once mean wind-induced displacement responses have been estimated, the statistical relationships between mean wind speed and mean wind-induced bridge displacement for strong wind events (both typhoon and monsoon) can be explored. All valid 10-minute mean wind speeds higher than 5 m/s and the corresponding 10-minute lateral mean displacement of the bridge deck and main cable included in the database from October 2002 to September 2005 are considered. The power law function represented by Equation (6.7) is adopted to
fit the measurement data.

\[ \bar{D} = a\bar{U}^b \]  \hspace{1cm} (6.7)

where \( \bar{D} \) is the mean displacement of the bridge deck at a given location for a given wind direction, \( \bar{U} \) denotes the mean wind speed, and \( a \) and \( b \) are the two parameters to be determined using the regression software package SPSS 13.0. It is only where \( b \) has a positive, non-zero value that the larger the mean wind speed, the larger the absolute mean wind-induced displacement. Hence, the one-tailed testing hypothesis concerning the value of parameter \( b \) in the regression model is performed:

\[ H_0: b = 0 \]  \hspace{1cm} (6.8a)
\[ H_a: b > 0 \]  \hspace{1cm} (6.8b)

Given the nature of the problem, the sign of the parameter \( a \) indicates the direction of deck movement. Hence, the parameter \( a \) can be either positive or negative. The two-tailed testing hypothesis concerning the value of parameter \( a \) in the regression model is thus set as:

\[ H_0: a = 0 \]  \hspace{1cm} (6.9a)
\[ H_a: a \neq 0 \]  \hspace{1cm} (6.9b)

The above null hypotheses \( H_0 \) can be rejected if the \( p \)-value (probability-value) is less than the chosen value of the level of significance, \( \alpha \) (=0.05 in this study).
Therefore, curve-fitting is said to fail when the \( p \)-values of the parameters \( a \) and \( b \) are greater than 0.05. Because SPSS only gives the result with \( p \)-value based upon two-tailed statistical tests, the reported \( p \)-value should be modified to fit a one-tailed test by dividing it by 2.

### 6.5.1. Statistical Relationships at the Mid-main Span in the N Direction Sector

The relationships between the 10-minute mean wind speed and the 10-minute lateral mean displacement recorded at the four positions on the bridge deck and main cable are explored using the regression model given in Equation (6.7). The relationship to be explored is based on the measurement data for both typhoon and monsoon events from 2002 to 2005. The results of the regressions for the mid-main span and the mid-main cable in the lateral direction in the N direction sector are displayed in Table 6.8 and plotted in Figure 6.18. The regression results show that the \( p \)-value for the coefficients \( a \) and \( b \) in the lateral direction are approximately zero. Therefore, the null hypotheses that the value of parameter \( b \) is not greater than zero and the value of parameter \( a \) is equal to zero can be rejected. The table shows that the estimated exponents in the power law functions obtained for the lateral direction of the mid-main span and the main cable are 1.824 and 1.722, respectively. This means that the mean wind-induced displacement of the bridge in the lateral direction is approximately proportional to the squares of the wind speed. When the mean wind speed is taken as 10 m/s, the lateral mean displacements of the mid-main span and the main cable estimated using the power law functions are 129.76 and 123.53 mm, respectively. Clearly, the mean displacement response of the bridge deck at the
middle of the main span in the N direction sector is similar to that of the main cable.

6.5.2. Statistical Relationships Along the Bridge Deck in the NNW Direction Sector

This section analyzes the statistical relationships between the mean wind speed and mean wind-induced displacement responses of the bridge deck at the four locations and the main cable when the wind direction is from the NNW direction sector. The regression factors determined using the least-squares method with a \( p \)-value of less than 0.05 and \( R^2 \) (coefficient of determination) of greater than 0.6 are accepted and presented in Table 6.9. Curve fitting fails for the measurement data from the deck section in the Ma Wan side span because \( R^2 \) is less than 0.6. It can be seen from Table 6.9 that the relationship between wind and mean wind-induced displacement varies with the deck section. Figure 6.19 illustrates the statistical relationships obtained by Equation (6.7) with the regression parameters listed in Table 6.9 for the three locations on the bridge deck and the main cable in the lateral direction. The figure shows that the lateral mean displacements are almost symmetric with respect to the middle of the main span. It is also observed that the mean displacements of the main cable are similar to those of the bridge deck.

6.5.3. Statistical Relationships in Different Direction Sectors at the Mid-main Span

This section investigates the statistical relationships between wind and the mean wind-induced displacement response of the bridge deck at the middle of the main
span for different wind directions. The regression factors determined using the least-squares method with a $p$-value of less than 0.05 and the whole regression with $R^2$ of greater than 0.6 are accepted for seven wind directions and listed in Table 6.10. The table shows that the relationship between wind and mean wind-induced displacement response varies with wind direction. Using Equation (6.7) and the regression factors listed in Table 6.10, the mean displacement response of the bridge at the middle of the main span in the lateral direction for the seven direction sectors are estimated and illustrated in Figure 6.20. The figure shows that the most negative mean displacement responses occur in response to wind from the N and NNW sectors, whereas the most positive mean displacement occurs when the wind is coming from the SW direction. By comparing their absolute magnitudes, it can be seen that the absolute mean displacement responses for wind blowing from overland terrain are slightly larger than those from the open-sea fetch. This is consistent with wind tunnel test results (King & Davenport 1997).

6.6. THE STATISTICAL RELATIONSHIP BETWEEN WIND AND WIND-INDUCED DYNAMIC DISPLACEMENT

This section explores the statistical relationship between wind speed and the wind-induced dynamic displacement response to strong wind events (both typhoon and monsoon). All valid 10-minute mean wind speeds of higher than 5 m/s and the corresponding 10-minute displacement standard deviations of the bridge towers, main cable, and deck are considered for the period from October 2002 to September 2005. The power law function expressed by Equation (6.10) is adopted to fit the measurement data.
where $\sigma_D$ is the displacement standard deviation of the bridge components at a given section in a given direction, $\bar{U}$ denotes the mean wind speed, and $a$ and $b$ are the two parameters to be determined. It is only where $a$ and $b$ have positive, non-zero values that the larger the mean wind speed, the larger the wind-induced displacement standard deviation. Hence, the null hypotheses $H_0: a = 0$ and $H_0: b = 0$ can be rejected and the alternative hypotheses $H_a: a > 0$ and $H_a: b > 0$ accepted if the $p$-value is less than 0.05.

### 6.6.1. Statistical Relationships at the Mid-main Span in the N Direction Sector

Since the number of displacement records after processing is very small at the tower level, only the relationships between the 10-minute mean wind speed and the 10-minute displacement standard deviations for the bridge deck and the main cable are explored in the lateral and vertical directions using the regression model given in Equation (6.10). The results from the regressions for the relationships in the lateral and vertical directions of the mid-main span and the main cable in the N direction sector are displayed in Table 6.11 and plotted in Figures 6.21 and 6.22. The table shows that the $p$-value for the coefficients $a$ and $b$ for the lateral and vertical directions are approximately zero in these regressions. Therefore, the alternative hypotheses $H_a$ that the values of the parameters $a$ and $b$ are greater than zero are accepted. The table shows that the estimated exponents in the power law functions obtained for the lateral and vertical directions of the mid-main span are 2.143 and 2.654, respectively, while
those for the main cable are 2.013 and 2.479, respectively. This means that wind-induced displacement standard deviations in the lateral and vertical directions are approximately proportional to the squares of the mean wind speed. When the mean wind speed is taken as 10 m/s, the lateral displacement standard deviations estimated using the power law functions are 20.57 mm at the middle of the main span and 19.58 mm at the main cable, whilst the vertical displacement standard deviations are 28.40 mm at the middle of the main span and 27.72 mm at the main cable. Clearly, the displacement responses of the bridge deck at the middle of the main span in the lateral and vertical directions when the wind is blowing from the N direction sector are similar to those measured at the main cable.

### 6.6.2. Statistical Relationships Along the Bridge Deck in the NNW Direction Sector

This section explores the statistical relationships between wind and the wind-induced dynamic displacement response of the bridge deck at the four locations and the main cable for the NNW direction sector. The regression factors determined using the least-squares method, with a $p$-value of less than 0.05 and $R^2$ of greater than 0.6, are accepted and presented in Table 6.12. Curve fitting fails for the measurement data from the deck section in the Ma Wan side span because $R^2$ is less than 0.6. It can be seen from Table 6.12 that the relationship between wind and wind-induced displacement standard deviation varies with the deck section and type of response. Figure 6.23 illustrates the statistical relationships obtained by Equation (6.10) with the regression parameters listed in Table 6.12 for the three locations on the bridge deck and the
main cable in the lateral and vertical directions. The figure shows that the
dynamic displacement responses are almost symmetric with respect to the middle
of the main span in the lateral direction. However, the vertical displacement
responses at the ¼ span and ¾ span of the main span appear to have higher
standard deviations than those measured at the mid-main span and the main cable
when the mean wind speed is less than 13 m/s. This observation may be related
to the dynamic characteristics of the bridge: the first lateral mode shape is almost
symmetric with respect to the middle of the main span, but the first vertical mode
shape is almost anti-symmetric with respect to the middle of the main span (Xu
et al. 1997).

6.6.3. Statistical Relationships in Different Direction Sectors at the
Mid-main Span
This section investigates statistical relationships between wind and the
wind-induced displacement standard deviations of the bridge deck at the middle
of the main span for different wind directions. The regression factors determined
using the least-squares method with a p-value of less than 0.05 and the whole
regression with $R^2$ of greater than 0.6 are accepted for four wind directions only
and listed in Table 6.13. The table indicates that the relationship between wind
and wind-induced displacement response varies with the wind direction and type
of response. By using Equation (6.10) and the regression factors in Table 6.13,
the standard deviations of displacement responses at the middle of the main span
in the lateral and vertical directions for the four wind direction sectors are
illustrated in Figure 6.24. It is noted that the maximum standard deviation of
displacement response occurs when the wind is from the NNW direction and the
minimum standard deviation of displacement response occurs when the wind is from the SSE direction. As explained previously, the dynamic response of the bridge is an increasing function of the friction velocity. As the friction velocity for the open sea area is smaller, this demonstrates that the standard deviations of displacement responses from the overland fetch are larger than those from the open sea fetch.

### 6.7. THE STATISTICAL RELATIONSHIP BETWEEN WIND AND WIND-INDUCED TOTAL DISPLACEMENT

This section explores the statistical relationship between wind and wind-induced total displacement, defined as the mean displacement plus or minus the peak displacement, for strong wind events (including both typhoon and monsoon). The peak displacement ($D_{\text{max}}$) is defined as the standard deviation ($\sigma_D$) multiplied by a statistical peak factor ($m$):

$$D_{\text{max}} = m\sigma_D \quad (6.11)$$

The value of $m$ in Equation (6.11) is estimated on the basis of a number of measurement data recorded during typhoon events in which TFR was less than 100 pcu/in/hr and $\sigma_D$ was larger than $3.28 \times c$. These criteria ensure that the peak displacement and standard deviation displacement derived directly from the measured displacement time histories are less affected by traffic. The statistical relationship between the 10-minute wind-induced peak displacement and the 10-minute standard deviation displacement of the bridge deck at the middle of the main span is explored and shown in Figure 6.25. It can be seen that the lateral
and vertical peak displacements are almost proportional to the corresponding standard deviation with a slope of 3, which deviates slightly from the value of 3.5 applied in the wind tunnel test conducted by King & Davenport (1997). This may be due the fact that this study considers only displacements with a frequency of less than 1 Hz.

All valid 10-minute mean wind speeds of higher than 5 m/s and the corresponding 10-minute total displacements of the bridge deck and the main cable are considered for the time period from October 2002 to September 2005. The power law function expressed by Equation (6.12) is adopted to fit the measurement data:

\[
\hat{D} = \bar{D} \pm 3\sigma_D = a\bar{U}^b
\]  

(6.12)

where \(\hat{D}\) is the wind-induced total displacement, \(\bar{D}\) represents the mean wind-induced displacement, \(\bar{U}\) denotes the mean wind speed, and \(a\) and \(b\) are the two parameters to be determined. As the mean wind-induced displacement \(\bar{D}\) in the vertical direction is almost zero, the total displacements in the vertical direction for the bridge deck and the main cable are simply three times their standard deviations. Correspondingly, the statistical relationships for the vertical direction represented by Equation (6.12) are the same as those derived for wind-induced standard deviation only if the value of parameter \(a\) in Equation (6.10) is multiplied by 3. Therefore, the regression equation to be estimated in this section is based on the measurement data for the lateral direction of the bridge deck and the main cable. Because of the nature of the problem, it is only
where the value of $b$ in Equation (6.12) is positive and non-zero that the larger the mean wind speed, the larger the wind-induced total displacement. Hence, the null hypothesis $H_0: b = 0$ can be rejected and the alternative hypothesis $H_a: b > 0$ can be accepted if the $p$-value is less than 0.05. The sign of parameter $a$ representing the movement of the bridge components can be either positive or negative. Hence, the null hypothesis $H_0: a = 0$ can be rejected and the alternative hypothesis $H_a: a \neq 0$ can be accepted if the $p$-value is less than 0.05.

6.7.1. Statistical Relationships at the Mid-main Span in the N Direction Sector

The relationships between the 10-minute mean wind speeds and the total lateral displacements recorded at the four positions on the bridge deck and the main cable are explored using the regression model given in Equation (6.12). The relationship to be explored is based on the measurement data for both typhoon and monsoon events from 2002 to 2005. The results of the regressions for the mid-main span and the mid-main cable in the lateral direction in the N direction sector are displayed in Table 6.14 and plotted in Figure 6.26. The results show that the $p$-value for the coefficients $a$ and $b$ for the lateral direction are approximately zero in these regressions. Therefore, the alternative hypotheses $H_a$ that the value of the parameter $b$ is greater than zero and the value of the parameter $a$ is not equal to zero can be accepted. It is noted from the table that the estimated exponents in the power law functions obtained for the lateral direction of the mid-main span and the main cable are 1.822 and 1.741, respectively. This means that the wind-induced total displacement in the lateral direction is approximately proportional to the squares of the wind speed. When
the mean wind speed is taken as 10 m/s, the total lateral displacements of the bridge deck at the mid-main span and the main cable estimated using the power law functions are -191.29 and -179.73 mm, respectively. These results are similar to the sum of the lateral mean displacement derived from Equation (6.7) minus three times the lateral standard deviation $\sigma_D$ predicted by Equation (6.10).

### 6.7.2. Statistical Relationships Along the Bridge Deck in the NNW Direction Sector

This section explores the statistical relationships between wind and the total wind-induced displacement responses at four locations on the bridge deck and main cable for the NNW wind direction sector. The regression factors determined using the least-squares method with a $p$-value of less than 0.05 and $R^2$ of greater than 0.6 are accepted and presented in Table 6.15. Curve fitting fails for the measurement data from the deck section in the Ma Wan side span because $R^2$ is less than 0.6. Table 6.15 shows that the relationship between wind and total wind-induced displacement varies with the deck section. Figure 6.27 illustrates the statistical relationships obtained by Equation (6.12) with the regression parameters listed in Table 6.15 for the three locations of the bridge deck and the main cable in the lateral direction. The figure indicates that the total displacements are almost symmetric with respect to the middle of the main span. It is also observed that the total displacement of the main cable is similar to that of the mid-main span.

### 6.7.3. Statistical Relationships in Different Direction Sectors at the Mid-main Span
This section investigates the statistical relationships between wind and the total wind-induced displacement response of the bridge deck at the middle of the main span for different wind directions. The regression factors determined using the least-squares method with a $p$-value of less than 0.05 and $R^2$ of greater than 0.6 for the whole regression are accepted for six wind directions only and listed in Table 6.16. It can be observed from the table that the relationship between wind and total wind-induced displacement response varies with the wind direction. By using Equation (6.12) and the regression factors in Table 6.16, the total displacement responses at the middle of the main span in the lateral direction for the six wind direction sectors are plotted in Figure 6.28. The figure shows that the total displacement responses reach their most negative values in the N direction sector, whereas the most positive total displacements occur when the wind is coming from the SSE direction. As explained previously, the response of the bridge is an increasing function of the friction velocity. As the friction velocity for the overland area is larger than for the open sea area, the total displacement responses from the overland fetch are larger than those from the open sea fetch.

6.8. SUMMARY

This chapter describes a statistical analysis of wind and the wind-induced displacement responses of the Tsing Ma Bridge carried out using field measurement data recorded by the WASHMS from 2002 to 2005. Data pre-processing for GPS was first conducted to obtain high quality data. The effects of GPS background noise, temperature, and moving trains and vehicles on the mean and dynamic displacements of the bridge were then assessed. This
allowed for the statistical relationships between wind speed and mean wind-induced displacement, wind speed and dynamic displacement, and wind speed and total displacement to be subsequently established. The major results are summarized as follows:

- Calibration tests using a two-dimensional motion simulation table were carried out to assess the possible effects of GPS background noise on bridge displacement responses. The calibration results showed that the effect of GPS background noise on the mean displacement response was small and could be neglected. However, the threshold values of 4.5 for the horizontal direction and 9.7 for the vertical direction should be applied to the dynamic displacement response;

- In analyzing the effects of temperature on mean displacement, it was found that during strong winds, the mean displacement of the towers in the longitudinal direction and the mean displacements of the deck and cable in the vertical and longitudinal directions are all attributable to temperature effects;

- An analysis of the effects of moving trains showed train-induced deformation with a frequency of less than 0.08 Hz for the deck in the vertical direction, the main cable in the longitudinal and vertical directions, and the towers in the longitudinal direction. This train-induced deformation can be eliminated by wavelet decomposition;
In exploring the effects of moving vehicles, it was found that the highway-induced standard deviations were close to a constant value \( c \) when the TFR was high. Therefore, the standard deviation of wind-induced displacement response can be calculated by assuming the wind- and highway-induced displacements are uncorrelated with each other. It was also found that less than 5% of the error in wind-induced displacement standard deviation will be predicted if the displacement standard deviation is greater than or equal to \( 3.28 \times c \).

A statistical analysis of bridge displacement responses to strong winds including typhoons and monsoons was carried out using field measurement data recorded by WASHMS from 2002 to 2005. The power function was used to fit the measurement data on the means and standard deviations of the displacement responses and the total displacement responses using the least-squares method. Because this study considered only displacement responses with a frequency of less than 1 Hz, total displacement was calculated as the mean plus or minus the standard deviation multiplied by 3. The results showed that in most cases, the relationships between wind speed and the mean and standard deviation of wind-induced displacement and total wind-induced displacement were almost quadratic in the lateral and vertical directions of the bridge deck and the main cable. The displacement responses of the bridge deck in the lateral direction were almost symmetric with respect to the middle of the main span, but the displacement response of the bridge deck in the vertical direction did not conform to such a trend. The displacement response of the main cable was similar to that measured at the middle of the main span. At the middle of the main span, the displacement
response was an increasing function of the terrain roughness of the site.

As noted above, the statistical relationship between wind and wind-induced displacement response is an important factor in assessing the serviceability of the bridge. However, the degree to which the statistical relationship explored in this chapter could be developed was limited by the number of sensors available and the number of wind directions analyzed, as well as by the wind speeds measured. Hence, Chapter 7 discussed the adoption of a finite element-based approach to assess the performance of the entire Tsing Ma Bridge with respect to the possible effects of extreme wind speeds. The statistical relationship developed in this chapter will be used to verify the results of computer simulations and extended to extreme wind speeds on the bridge deck.
### Table 6.1. Details of GPS receivers on the Tsing Ma Bridge

<table>
<thead>
<tr>
<th>TAG NUMBER</th>
<th>LOCATION</th>
<th>MEASUREMENT COMPONENTS*</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM-TYN</td>
<td>Tsing Yi tower – North</td>
<td>Horizontal and vertical geographic coordinates $(\phi, \lambda, H)$</td>
</tr>
<tr>
<td>TM-TYS</td>
<td>Tsing Yi tower – South</td>
<td>Horizontal and vertical geographic coordinates $(\phi, \lambda, H)$</td>
</tr>
<tr>
<td>TM-01N</td>
<td>3/4 span of the main span from Ma Wan tower – North</td>
<td>Horizontal and vertical geographic coordinates $(\phi, \lambda, H)$</td>
</tr>
<tr>
<td>TM-01S</td>
<td>3/4 span of the main span from Ma Wan tower – South</td>
<td>Horizontal and vertical geographic coordinates $(\phi, \lambda, H)$</td>
</tr>
<tr>
<td>TM-02N</td>
<td>1/2 span of the main span from Ma Wan tower – North</td>
<td>Horizontal and vertical geographic coordinates $(\phi, \lambda, H)$</td>
</tr>
<tr>
<td>TM-02S</td>
<td>1/2 span of the main span from Ma Wan tower – South</td>
<td>Horizontal and vertical geographic coordinates $(\phi, \lambda, H)$</td>
</tr>
<tr>
<td>TM-03N</td>
<td>Main cable – North</td>
<td>Horizontal and vertical geographic coordinates $(\phi, \lambda, H)$</td>
</tr>
<tr>
<td>TM-03S</td>
<td>Main cable – South</td>
<td>Horizontal and vertical geographic coordinates $(\phi, \lambda, H)$</td>
</tr>
<tr>
<td>TM-04N</td>
<td>1/4 span of the main span from Ma Wan tower – North</td>
<td>Horizontal and vertical geographic coordinates $(\phi, \lambda, H)$</td>
</tr>
<tr>
<td>TM-04S</td>
<td>1/4 span of the main span from Ma Wan tower – South</td>
<td>Horizontal and vertical geographic coordinates $(\phi, \lambda, H)$</td>
</tr>
<tr>
<td>TM-05N</td>
<td>Mid-span of Ma Wan approach span – North</td>
<td>Horizontal and vertical geographic coordinates $(\phi, \lambda, H)$</td>
</tr>
<tr>
<td>TM-05S</td>
<td>Mid-span of Ma Wan approach span – South</td>
<td>Horizontal and vertical geographic coordinates $(\phi, \lambda, H)$</td>
</tr>
<tr>
<td>TM-MWN</td>
<td>Ma Wan tower – North</td>
<td>Horizontal and vertical geographic coordinates $(\phi, \lambda, H)$</td>
</tr>
<tr>
<td>TM-MWS</td>
<td>Ma Wan tower - South</td>
<td>Horizontal and vertical geographic coordinates $(\phi, \lambda, H)$</td>
</tr>
</tbody>
</table>

*Note: $\phi$ = latitude (°); $\lambda$ = longitude (°); $H$ = height (m)
Table 6.2. HK80 datum parameters for the projection formulae

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>HK80 DATUM (UTM ↔ φ, λ)</th>
</tr>
</thead>
<tbody>
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<td>( N_o )</td>
<td>0 m N</td>
</tr>
<tr>
<td>( E_o )</td>
<td>500,000 m E</td>
</tr>
<tr>
<td>( λ_o )</td>
<td>Zone 49Q: 111° E</td>
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<tr>
<td></td>
<td>Zone 50Q: 117° E</td>
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<tr>
<td>( m_o )</td>
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<tr>
<td>( M_o )</td>
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</tr>
<tr>
<td>( ν_s )</td>
<td>6,381,480.500 m</td>
</tr>
<tr>
<td>( ψ_s )</td>
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<tr>
<td>( a )</td>
<td>6,378,388 m</td>
</tr>
<tr>
<td>( e^2 )</td>
<td>6.722670022×10⁻³</td>
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Table 6.3. Reference coordinates measured on November 28, 2000 at 0400-0500

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<td>TM-03S</td>
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<td>78.064</td>
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<td>11403.945723</td>
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</tr>
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<td>TM-05S</td>
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</tr>
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<td>TM-MWS</td>
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Table 6.4. Statistics on background noise at Ma Wan Island

(a) Horizontal direction

<table>
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<tr>
<th>SEGMENT</th>
<th>TEST 1</th>
<th>TEST 2</th>
<th>TEST 3</th>
<th>TEST 4</th>
<th>MEAN (mm)</th>
<th>STD. DEVIATION (mm)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>1.63 1.18 1.39</td>
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<td>1.61 1.56 1.42</td>
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<td>1.41 1.40 1.25</td>
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(b) Vertical direction

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<th>TEST 3</th>
<th>TEST 4</th>
<th>MEAN (mm)</th>
<th>STD. DEVIATION (mm)</th>
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<td>4.50 2.67 3.42</td>
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<td>0.12</td>
<td>2.78</td>
<td>1.64</td>
<td>3.60</td>
<td>4.23 3.75 2.99</td>
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<td>-4.91</td>
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<td>2.16 4.45 3.95</td>
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Table 6.5. Statistics on background noise at Tsing Yi Island

(a) Horizontal direction

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<th>TEST 4</th>
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<th>TEST 2</th>
<th>TEST 3</th>
<th>TEST 4</th>
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<td>1.85</td>
<td>1.54</td>
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<td>1.63</td>
<td>1.70</td>
<td>1.56</td>
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(b) Vertical direction

<table>
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<th>TEST 3</th>
<th>TEST 4</th>
<th>TEST 1</th>
<th>TEST 2</th>
<th>TEST 3</th>
<th>TEST 4</th>
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<td>4.19</td>
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<td>4.69</td>
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<td>4.00</td>
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<td>4.58</td>
<td>2.84</td>
<td>3.47</td>
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Table 6.6. GPS measurement errors for wind-induced dynamic displacement response

<table>
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<tr>
<th>DIRECTION</th>
<th>INPUT MOTION</th>
<th>TEST</th>
<th>LEVEL (mm)</th>
<th>GPS (mm)</th>
<th>ERROR (%)</th>
</tr>
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<tbody>
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<td>35.42</td>
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<td>4.62</td>
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<td>5.42</td>
<td>2.65</td>
</tr>
<tr>
<td>Vertical</td>
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<td>1</td>
<td>3.14</td>
<td>4.99</td>
<td>58.92</td>
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<td></td>
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<td>9.68</td>
<td>9.25</td>
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Table 6.7. Constants for each section of the bridge deck and the main cable

<table>
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<tr>
<th>GPS STATION</th>
<th>DIRECTION</th>
<th>LATERAL (mm)</th>
<th>VERTICAL (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM-01</td>
<td>LATERAL</td>
<td>4.92</td>
<td>12.62</td>
</tr>
<tr>
<td>TM-02</td>
<td>LATERAL</td>
<td>5.05</td>
<td>10.66</td>
</tr>
<tr>
<td>TM-03</td>
<td>LATERAL</td>
<td>4.78</td>
<td>10.65</td>
</tr>
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<td>TM-04</td>
<td>LATERAL</td>
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<td>13.47</td>
</tr>
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<td>TM-05</td>
<td>LATERAL</td>
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**Table 6.8.** Regression factors for relationships between wind and mean displacement response at the mid-main span and the main cable in direction sector N

<table>
<thead>
<tr>
<th>GPS STATION</th>
<th>VARIABLE</th>
<th>PARAMETER</th>
<th>ESTIMATE</th>
<th>t - VALUE</th>
<th>p - VALUE</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS-02</td>
<td>a</td>
<td>1.946</td>
<td>6.389</td>
<td>&lt;0.000</td>
<td>0.765</td>
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</tr>
<tr>
<td></td>
<td>b</td>
<td>1.824</td>
<td>23.404</td>
<td>&lt;0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GPS-03</td>
<td>a</td>
<td>2.343</td>
<td>6.964</td>
<td>&lt;0.000</td>
<td>0.788</td>
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</table>

**Table 6.9.** Regression factors for relationships between wind and mean displacement response for different deck sections in direction sector NNW

<table>
<thead>
<tr>
<th>GPS STATION</th>
<th>VARIABLE</th>
<th>PARAMETER</th>
<th>p - VALUE</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS-01</td>
<td>a</td>
<td>2.337</td>
<td>0.043</td>
<td>0.651</td>
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<td>b</td>
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<tr>
<td>GPS-02</td>
<td>a</td>
<td>2.704</td>
<td>0.005</td>
<td>0.809</td>
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<tr>
<td></td>
<td>b</td>
<td>1.706</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>GPS-03</td>
<td>a</td>
<td>3.418</td>
<td>0.003</td>
<td>0.823</td>
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<td></td>
<td>b</td>
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<td>a</td>
<td>2.551</td>
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<td>b</td>
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### Table 6.10. Regression factors for relationships between wind and mean displacement response for different wind directions at the mid-main span

<table>
<thead>
<tr>
<th>WIND DIRECTION</th>
<th>VARIABLE</th>
<th>PARAMETER</th>
<th>p - VALUE</th>
<th>R²</th>
</tr>
</thead>
<tbody>
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<td>&lt;0.000</td>
<td>0.765</td>
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<tr>
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<td>b</td>
<td>1.824</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>NNE</td>
<td>a</td>
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<td>&lt;0.000</td>
<td>0.707</td>
</tr>
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<td>b</td>
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<td>&lt;0.000</td>
<td></td>
</tr>
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<td>0.354</td>
<td>0.045</td>
<td>0.979</td>
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<td>0.012</td>
<td>0.948</td>
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<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
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<td>0.019</td>
<td>0.837</td>
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<td>b</td>
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</tr>
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<td>NW</td>
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</tr>
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<td>b</td>
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<td>-2.704</td>
<td>0.001</td>
<td>0.724</td>
</tr>
<tr>
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<td>b</td>
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</table>
Table 6.11. Regression factors for relationships between wind and dynamic displacement response for the mid-main span and the main cable in direction sector N

<table>
<thead>
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<th>GPS STATION (DIRECTION)</th>
<th>VARIABLE</th>
<th>PARAMETER</th>
<th>t - VALUE</th>
<th>p - VALUE</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS-02 (Lateral)</td>
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<td>b</td>
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</tr>
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<td></td>
</tr>
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<td>GPS-03 (Vertical)</td>
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<td>2.815</td>
<td>0.003</td>
<td>0.609</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>2.479</td>
<td>14.112</td>
<td>&lt;0.000</td>
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</tr>
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</table>

Table 6.12. Regression factors for relationships between wind and dynamic displacement response for different deck sections in direction sector NNW

<table>
<thead>
<tr>
<th>GPS STATION (DIRECTION)</th>
<th>VARIABLE</th>
<th>PARAMETER</th>
<th>p - VALUE</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS-01 (Lateral)</td>
<td>a</td>
<td>0.069</td>
<td>0.044</td>
<td>0.746</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>2.342</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>GPS-01 (Vertical)</td>
<td>a</td>
<td>0.495</td>
<td>0.038</td>
<td>0.635</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>1.872</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>GPS-02 (Lateral)</td>
<td>a</td>
<td>0.142</td>
<td>0.039</td>
<td>0.645</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>2.185</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>GPS-02 (Vertical)</td>
<td>a</td>
<td>0.087</td>
<td>0.036</td>
<td>0.748</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>2.540</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>GPS-03 (Lateral)</td>
<td>a</td>
<td>0.167</td>
<td>0.033</td>
<td>0.678</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>2.096</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>GPS-03 (Vertical)</td>
<td>a</td>
<td>0.076</td>
<td>0.028</td>
<td>0.813</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>2.605</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>GPS-04 (Lateral)</td>
<td>a</td>
<td>0.066</td>
<td>0.041</td>
<td>0.678</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>2.385</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>GPS-04 (Vertical)</td>
<td>a</td>
<td>0.509</td>
<td>0.032</td>
<td>0.636</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>1.853</td>
<td>&lt;0.000</td>
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</tr>
</tbody>
</table>
**Table 6.13.** Regression factors for relationships between wind and dynamic displacement response for different wind directions at the mid-main span

<table>
<thead>
<tr>
<th>WIND DIRECTION (BRIDGE DIRECTION)</th>
<th>VARIABLE</th>
<th>PARAMETER ESTIMATE</th>
<th>p - VALUE</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>N (Lateral)</td>
<td>a</td>
<td>0.148</td>
<td>&lt;0.000</td>
<td>0.623</td>
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<td>b</td>
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<td></td>
</tr>
<tr>
<td>N (Vertical)</td>
<td>a</td>
<td>0.063</td>
<td>0.004</td>
<td>0.604</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>2.654</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>SSE (Lateral)</td>
<td>a</td>
<td>0.041</td>
<td>0.049</td>
<td>0.927</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>2.047</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>SSE (Vertical)</td>
<td>a</td>
<td>0.043</td>
<td>0.045</td>
<td>0.939</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>2.210</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>NW (Lateral)</td>
<td>a</td>
<td>0.428</td>
<td>0.015</td>
<td>0.686</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>1.580</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>NW (Vertical)</td>
<td>a</td>
<td>0.190</td>
<td>0.031</td>
<td>0.722</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>2.011</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>NNW (Lateral)</td>
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<td>0.039</td>
<td>0.645</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>2.185</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>NNW (Vertical)</td>
<td>a</td>
<td>0.087</td>
<td>0.036</td>
<td>0.748</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>2.540</td>
<td>&lt;0.000</td>
<td></td>
</tr>
</tbody>
</table>

**Table 6.14.** Regression factors for relationships between wind and total displacement response for the mid-main span and the main cable in direction sector N

<table>
<thead>
<tr>
<th>GPS STATION</th>
<th>VARIABLE</th>
<th>PARAMETER ESTIMATE</th>
<th>t - VALUE</th>
<th>p - VALUE</th>
<th>R²</th>
</tr>
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<tbody>
<tr>
<td>GPS-02</td>
<td>a</td>
<td>-2.882</td>
<td>6.120</td>
<td>&lt;0.000</td>
<td>0.753</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>1.822</td>
<td>22.450</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>GPS-03</td>
<td>a</td>
<td>-3.263</td>
<td>6.389</td>
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<td>0.758</td>
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<td></td>
<td>b</td>
<td>1.741</td>
<td>22.390</td>
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Table 6.15. Regression factors for relationships between wind and total displacement response for different deck sections in direction sector NNW

<table>
<thead>
<tr>
<th>GPS STATION</th>
<th>VARIABLE</th>
<th>PARAMETER</th>
<th>p - VALUE</th>
<th>R²</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>ESTIMATE</td>
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<td></td>
</tr>
<tr>
<td>GPS-01</td>
<td></td>
<td>a</td>
<td>3.517</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b</td>
<td>1.603</td>
<td>&lt;0.000</td>
</tr>
<tr>
<td>GPS-02</td>
<td></td>
<td>a</td>
<td>5.528</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b</td>
<td>1.545</td>
<td>&lt;0.000</td>
</tr>
<tr>
<td>GPS-03</td>
<td></td>
<td>a</td>
<td>5.251</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b</td>
<td>1.554</td>
<td>&lt;0.000</td>
</tr>
<tr>
<td>GPS-04</td>
<td></td>
<td>a</td>
<td>4.147</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b</td>
<td>1.542</td>
<td>&lt;0.000</td>
</tr>
</tbody>
</table>

Table 6.16. Regression factors for relationships between wind and total displacement response for different wind directions at the mid-main span

<table>
<thead>
<tr>
<th>WIND DIRECTION</th>
<th>VARIABLE</th>
<th>PARAMETER</th>
<th>p - VALUE</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ESTIMATE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>a</td>
<td>-2.882</td>
<td>&lt;0.000</td>
<td>0.753</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>1.822</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>NNE</td>
<td>a</td>
<td>-5.103</td>
<td>&lt;0.000</td>
<td>0.634</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>1.478</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>SSE</td>
<td>a</td>
<td>1.058</td>
<td>0.010</td>
<td>0.921</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>1.909</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>a</td>
<td>0.519</td>
<td>0.008</td>
<td>0.873</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>2.088</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>NW</td>
<td>a</td>
<td>-6.998</td>
<td>&lt;0.000</td>
<td>0.874</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>1.341</td>
<td>&lt;0.000</td>
<td></td>
</tr>
<tr>
<td>NNW</td>
<td>a</td>
<td>-5.528</td>
<td>0.001</td>
<td>0.797</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>1.545</td>
<td>&lt;0.000</td>
<td></td>
</tr>
</tbody>
</table>
Figure 6.1. Distribution of GPS receivers on the Tsing Ma Bridge

Figure 6.2. Installation of GPS receivers on different bridge components

(a) Tower top  (b) Main cable  (c) Deck
Figure 6.3. Displacement time history and number of satellites with abrupt changes
Figure 6.4. Procedure for correcting GPS displacement data subject to an abrupt change
Figure 6.5. Comparison of GPS displacement data before and after correction

Figure 6.6. Hardware configuration for GPS assessment tests on the Tsing Ma Bridge
Figure 6.7. Mean displacement responses to temperature at the Ma Wan tower
(weak wind)
Chapter 6  Statistical Analysis of the Relationship between Displacement and Wind Speed

(a) Longitudinal direction

(b) Lateral direction
(c) Vertical direction

Figure 6.8. Mean displacement responses to temperature at the mid-main span (weak wind)
(a) Longitudinal direction

(b) Lateral direction
Figure 6.9. Mean displacement responses to temperature at the main cable (weak wind)
Figure 6.10. Mean displacement responses to temperature at the Ma Wan tower (strong wind)
Figure 6.11. Mean displacement responses to temperature at the mid-main span (strong wind)
Figure 6.12. Mean displacement responses to temperature at the main cable (strong wind)
Figure 6.13. Lateral mean displacement responses to wind
Figure 6.14. Traffic-induced dynamic displacements of the Ma Wan tower

Figure 6.15. Traffic-induced dynamic displacements of the mid-main span
Figure 6.16. Traffic-induced dynamic displacements of the main cable
Figure 6.17. Highway-induced displacement standard deviations with TFR
Figure 6.18. Statistical relationships between wind and mean displacement in direction sector N
Figure 6.19. Statistical relationships between wind and mean displacement for three deck sections and the main cable in direction sector NNW.

Figure 6.20. Statistical relationships between wind and mean displacement for different wind directions at the mid-main span.
Figure 6.21. Statistical relationships of wind and dynamic displacement for mid-main span in direction sector N.

(a) Lateral direction

(b) Vertical direction
Figure 6.22. Statistical relationships between wind and dynamic displacement for the main cable in direction sector N.
Figure 6.23. Statistical relationships of wind and dynamic displacement for three deck sections and main cable in direction sector NNW
Figure 6.24. Statistical relationships between wind and dynamic displacement for different wind directions at the mid-main span
Figure 6.25. Statistical relationships between peak and standard deviation displacements at the mid-main span
Figure 6.26. Statistical relationships between wind and total displacement in direction sector N

(a) Mid-main span

(b) Main cable
Figure 6.27. Statistical relationships between wind and total displacement for three deck sections in direction sector NNW

Figure 6.28. Statistical relationships between wind and total displacement for different wind directions at the mid-main span
Chapter 7

STRUCTURAL HEALTH MONITORING-BASED PERFORMANCE ASSESSMENT: DISPLACEMENT

7.1. INTRODUCTION

Chapters 5 and 6 have presented the field measurement results from the anemometers and the global positioning systems (GPS) on Hong Kong’s Tsing Ma Bridge during strong wind periods (including typhoons and monsoons). The results clearly demonstrate that variations in the statistical relationship between wind speed and wind-induced displacement response depend on wind direction and sensor location. Nevertheless, the investigations carried out were limited by the number of sensors available and the wind speeds during the measurement period. A good solution to this problem is to employ finite element method, a method which can help with the analysis of extreme wind speeds along a long-span suspension bridge as a whole. However, in the modeling process, the deck of the bridge is normally based on a simplified spine beam finite element model (FEM) of equivalent sectional properties (Xu et al. 2000; Xu et al. 2003). Such a simplified model is not able to capture the local stress and strain behavior of the bridge, an issue to be discussed in Chapter 8. This study thus requires that a complex structural health monitoring-based three-dimensional FEM that closely replicates the geometric details of the as-built complicated bridge deck be
This chapter first introduces the significant features of the Tsing Ma Bridge deck modeling process. A formulation of distributing wind forces over the cross section of the bridge deck, including steady-state wind forces, buffeting forces, and self-excited forces, is then presented. Using the formulation described, wind forces are generated on each node of the bridge deck based on wind characteristics measured in the field and aerodynamic coefficients and flutter derivatives calculated in wind tunnel tests. The next step is to predict the displacements of the bridge deck in the time domain using the mode superposition method. The displacement responses calculated are then compared with those measured in the field, as described in Chapter 6, to verify the validity of the model presented. The statistical relationships developed in the preceding chapter are then extended through FEM analysis to allow for the possible effects of extreme wind speeds on the Tsing Ma Bridge as a whole to be estimated. In the final step, the results are compared with the responses measured during wind tunnel tests and the limit state displacements determined at the design stage of the bridge.

### 7.2. STRUCTURAL HEALTH MONITORING-BASED FEM

A three-dimensional FEM for the Tsing Ma Bridge (Figure 7.1) was established in earlier years (Xu et al. 1997). However, the model of the bridge deck as a simplified spine beam of equivalent sectional properties was not able to capture the local stress and strain behavior of the bridge. Xu et al. (2007b) recently sought to address this weakness by establishing a complex structural health monitoring-based FEM for the bridge. This structural health monitoring-based
Chapter 7  Structural Health Monitoring-Based Performance Assessment: Displacement

FEM, which uses a total of 15,904 beam elements to model the bridge deck, closely replicates the geometric details of the as-built complicated deck. Hence, this model is used in this study.

7.2.1. The Main Features of the Tsing Ma Bridge Deck

The bridge deck of the Tsing Ma Bridge is a hybrid steel structure consisting of Vierendeel cross-frames supported on two longitudinal diagonally braced trusses acting compositely with stiffened steel plates that carry the upper and lower highways. The stiffened plates, which act with two 7.2 m-deep longitudinal braced trusses at 26 m centre, provide the vertical bending stiffness of the bridge deck. Transverse shear forces are carried by the steel plates, together with the plane bracing systems that join the plates at both the upper and lower flanges and the span vent openings. The mixed plane bracing-plate systems enable the longitudinal trusses to provide lateral bending stiffness. At the main span and the Ma Wan side span, the deck is suspended from the main cables at 18 m intervals. Near the Ma Wan and Tsing Yi Bridge towers, the bridge deck changes to incorporate two additional inner longitudinal trusses that share forces with the main trusses, with the deck plates extending over the centre to cover the full width of the bridge without vent openings. The cross-section of the bridge deck also changes at the Tsing Yi side span and in the area near the Ma Wan abutment, where the deck is supported by piers rather than suspenders.

7.2.2. Advances in Modeling of the Bridge Deck

In the complex structural health monitoring-based FEM, the bridge deck is modeled and assembled using a number of bridge deck modules. These bridge
deck modules include (1) the deck module at the main span; (2) the deck module at the Ma Wan tower; (3) the deck module at the Ma Wan approach span; (4) the deck module at the Tsing Yi tower; and (5) the deck module at the Tsing Yi approach span. The bridge deck at the main span is a suspended deck and the structural configuration is typical for every 18 m segment. Figure 7.2(a) illustrates a typical 18 m suspended deck module consisting mainly of longitudinal trusses, cross-frames, highway decks, railway tracks, and bracings. Two longitudinal trusses link up the cross-frames along the longitudinal axis of the bridge, acting as its main girder. Each longitudinal truss is comprised of upper and lower chords and vertical and diagonal members. Each 18 m deck module includes one main cross-frame and four neighboring intermediate cross-frames, with two on each side of the main cross-frame. The five cross-frames spaced 4.5 m apart are connected by two outer longitudinal trusses. Each cross-frame is comprised of upper and lower chords, inner struts, outer struts, and upper and lower inclined edge members. This deck module is suspended to the main cable through suspender units connected to the intersections of the edge members of the main cross-frame. To improve structural stability, two pairs of sway bracings connect the suspension points at the main cross-frame to the outer ends of the upper chords of the two adjacent intermediate cross-frames. Two symmetrical bays on the top highway deck plates are supported by the upper chords of the cross-frames and longitudinal trusses. A row of central cross bracing stretching from neighboring cross-frames runs between these symmetrical bays. On the bottom deck, there are two railway tracks laid on the central bay, with one row of central cross bracing and two rows of outer cross bracing added to brace the bottom chords of the cross-frames. Two bays of deck plates on the two outer sides of the bottom deck
are supported on the bottom chords of the cross-frames and longitudinal trusses. This deck module is symmetric to the middle vertical plane, with a width of $2 \times 20.5$ m, a lateral distance between the two suspension points at the main cross-frame of $2 \times 18$ m, a height of 8.0 m, and an inner clearance of 5.35 m in the middle.

The upper and lower chords of the longitudinal trusses are of the box section type, while the vertical and diagonal members of the longitudinal trusses are of the I-section type. They are all modeled as 12-DOF beam elements (CBAR) based on the principle of one element for one member. The actual section properties are computed by the program automatically. The upper and lower chords of the cross-frames are predominantly of the T-section type, other than for some segments with an I-section for the cross-bracing systems. The inner struts, outer struts, and upper and lower inclined edge members of the cross-frames are all of the I-section type. With the exception of the edge members, which are assigned large elastic modulus and significantly small density to reflect the actual situation where the joint is heavily stiffened for the connection with the suspender, all the members in the cross-frames are modeled as 12-DOF beam elements (CBAR) with actual section properties. All the members in the cross bracings are of the box–section type, while all the members in the sway bracings are circular hollow sections. These members all are modeled as 12-DOF beam elements (CBAR) with actual section properties. Each railway track is modeled as an equivalent beam modeled by special 14-DOF beam elements (CBEAM), which are similar to the elements (CBAR) but have additional properties such as a variable cross-section, a shear centre offset from the neutral axis, a wrap coefficient, and other features.
This simplified railway track model is feasible for this study, which is not concerned with the serviceability or safety of running trains. The railway tracks are meshed every 4.5 m according to the intervals between the adjacent cross-frames. The modulus of elasticity, density, and Poisson’s ratio for all members other than the edge members are taken as $2.05 \times 10^{11} \text{ N/m}^2$, 8,500 kg/m$^3$, and 0.3, respectively. The material properties of the edge members are determined through the model updating process.

The deck plates and deck troughs comprise orthotropic decks, and the accurate modelling of the stiffened deck plates is complicated. To keep the problem manageable, two-dimensional anisotropic quadrilateral plate-bending elements (CQUAD4) are employed to model the stiffened deck plates. The equivalent section properties of the elements are estimated roughly by static analysis, with the material properties of steel first being used, although these properties are subsequently updated. The deck plates are meshed in the longitudinal direction by each pair of adjacent cross-frames. Along the cross section of the bridge deck, the top highway deck plates in each bay are further divided into two elements at the positions of the longitudinal trusses. The connections between the deck plates and the chords of the cross-frames and longitudinal trusses involve the use of MPC (multi-point connection). Proper offsets of neutral axes for the connections between the components are considered to maintain the original configuration. In modeling the typical 18-m deck module, a total of 130 nodes with 172 CBAR elements, 16 CBEAM elements, 24 CQUAD4 elements, and 50 MPCs are used. Figure 7.2(b) shows the skeleton view of the 3-D finite element model of the 18-m deck module.
Although the deck modules at the Ma Wan tower, the Ma Wan approach span, the Tsing Yi tower and the Tsing Yi approach span are all constructed using the same principle as the deck module at the main span, they each take into consideration differences in the shapes and sizes of their respective cross-frames, longitudinal trusses, and other members. For convenience in integrating these bridge deck modules to form a complete bridge deck model, a global coordinate system for the whole bridge and a profile of the bridge deck were set up before the deck modules were built. In the global coordinate system (X-Y-Z), the X-axis is along the longitudinal bridge axis (from west to east), originating from the location of the Ma Wan abutment bearings and ending at the location of the Tsing Yi abutment bearings with a total length of 2,160 m; the Y-axis is along the lateral direction (perpendicular to the bridge axis) with a positive direction from the Hong Kong side (south) to the New Territories side (north); and because the Z-axis is along the vertical direction commencing from Principal Datum Hong Kong, the Z-ordinates are the same as the elevation levels used in the construction drawings. Since the bridge deck is structurally formed by 481 cross-frames interconnected by the longitudinal trusses, the route profile datum line of the deck can be geometrically illustrated by the locations of these cross-frames in terms of the upper freeway level.

7.2.3. Computer Program and Modal Analysis

The final complex structural health monitoring-based FEM for the Tsing Ma suspension bridge shown in Figure 7.3 involving 12,898 nodes, 21,946 elements (2,906 plate elements and 19,040 beam elements), and 4,788 MPCs was established using the MSC/PATRAN commercial software package. Another
sophisticated computer program called MSC/NASTRAN is used to carry out the modal analysis of the Tsing Ma Bridge. The analysis results show that the natural frequencies of the bridge basically agree with the first 18 measured natural frequencies reported by Xu et al. (1997), as shown in Table 7.1. Figure 7.4 shows the first two modes of vibration in the lateral, vertical, and torsional directions, respectively.

7.3. FORMULATION AND METHOD OF SOLUTION

7.3.1. Aerostatic Wind Force Model

Static wind action on a long-span suspension bridge usually leads to large deformations in the bridge structure (Zhang et al. 2002). As described in Figure 7.5, the oncoming wind flow with a mean velocity $\bar{U}$ attacks the bridge deck at angle $\alpha^0$. Owing to wind-structure interaction, the static wind action acting on the bridge deck will cause deck deformation with torsional angle $\theta$. The effective wind angle of attack $\alpha = \alpha^0 + \theta$ is then formed. Therefore, the equivalent static wind force per unit span acting on the deformed deck at the $i$th section $\mathbf{F}_{ei}$ is expressed in global bridge axes as:

$$
\mathbf{F}_{ei}^{sf} = \begin{bmatrix}
0 \\
\frac{1}{2} \rho_d \bar{U}_i^2 B_i \\
0
\end{bmatrix}
= \begin{bmatrix}
0 \\
C_{Di_i}(\alpha_i) \\
C_{Li_i}(\alpha_i)
\end{bmatrix}
\begin{bmatrix}
0 \\
C_{Mi_i}(\alpha_i) B_i \\
0
\end{bmatrix}
$$

(7.1)

where $f_{ed_i}^{sf}$, $f_{el_i}^{sf}$, and $f_{em_i}^{sf}$ = static drag, lift, and moment, respectively, on the $i$th
section of the bridge deck; \( \rho_a \) = air density; \( B_i \) = the width of the bridge deck segment; and \( C_{Di}, C_{Li}, \) and \( C_{Mi} \) = static aerodynamic drag, lift, and moment coefficients as a function of the angle of attack, respectively.

However, the force displayed in Equation (7.1) is the equivalent force acting at the centre of elasticity of the deck section, which neglects the spatial distribution of wind pressures on the whole surface of deck, as shown in Figure 7.6. This ignorance may not only have a considerable impact on the accuracy of static response prediction, but may also preclude estimation of the local behavior of the bridge deck. Therefore, it is necessary to redistribute the aerostatic wind force acting at the elasticity centre to each node of the element based on the wind pressure distribution derived from wind tunnel tests or computational fluid dynamics (CFD).

The static forces \( \{F_k\}_{j,i} \) at the \( k \)th node and \( \{F_{k-1}\}_{j,i} \) at the \((k-1)\)th node due to the wind pressure acting on the \( j \)th element of the bridge deck at the \( i \)th section (refer to Figure 7.7) can be obtained by:

\[
\{F_{k-1}, F_k\}_{j,i} = \left\{ \int_{0}^{s_{j,i}} (1 - s_i/b_{j,i}) P^m_{j,i}(s_i) \, ds_i, \int_{0}^{s_{j,i}} (s_i/b_{j,i}) P^m_{j,i}(s_i) \, ds_i \right\} \quad (7.2)
\]

where \( b_{j,i} \) = the length of the \( j \)th element of the \( i \)th section \( (j = 1, 2, \ldots, N_{ei}) \); \( N_{ei} \) = the total number of elements used to model the \( i \)th deck section; \( P^m_{j,i}(s_i) \) is mean wind pressure distribution over the \( j \)th element of the \( i \)th section; and \( s_i \) is the local element coordinate. The static force \( \{F_k\}_{j,i} \) at the \( k \)th node on the \( j \)th element of the \( i \)th deck section in the local coordinate can then be converted to \( \{F_{ky}, F_{kz}\}_{j,i} \)
in the $x$-$y$-$z$ global coordinate system. By adding the static force vectors at the $k$th node contributed by all elements together, the static force vector at the $k$th node of the $i$th deck section can be represented as $\mathbf{F}_{k,i} = [0, F_{k,iy}, F_{k,iz}, 0, 0, 0]^T$, in which $k = 1, 2, \ldots, N_{nl}$, and $N_{nl}$ is the total number of nodes used to model the $i$th deck section.

As shown in Figure 7.8, the mean wind pressure distributions on the deck sections are described by three unknown variables: $p_1$, $p_2$, and $p_3$. To tackle this problem, the following relationships should be applied:

$$\sum_{k=1}^{N_{nl}} p_{kiy} = f_{eDi}$$  \hspace{1cm} (7.3a)
$$\sum_{k=1}^{N_{nl}} p_{kiz} = f_{eLi}$$  \hspace{1cm} (7.3b)
$$\sum_{k=1}^{N_{nl}} (p_{kiy} q_{ki} + p_{kiz} h_{ki}) = M_{izi}^{sf} + M_{ily}^{sf} = f_{eMi}$$  \hspace{1cm} (7.3c)

where $q_{ki}$ and $h_{ki}$ are the lateral and vertical coordinates, respectively, of the $k$th node with respect to the centre of elasticity at the $i$th deck section; and $M_{izi}^{sf}$ and $M_{ily}^{sf}$ are the moments about the centre of elasticity due to the vertical and lateral forces, respectively. In matrix notation, Equation (7.3) can be expressed as:

$$\begin{bmatrix} 0 \\ p_1 \\ p_2 \\ p_3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ f_{eDi}^{sf} \\ f_{eLi}^{sf} \\ f_{eMi}^{sf} \\ 0 \\ 0 \end{bmatrix} = \mathbf{F}_{ei}^{se}$$  \hspace{1cm} (7.4)
in which \( \mathbf{A}_i^{sf} \) =

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & F_y(p_1) & F_y(p_2) & F_y(p_3) & 0 & 0 \\
0 & F_z(p_1) & F_z(p_2) & F_z(p_3) & 0 & 0 \\
0 & M(p_1) & M(p_2) & M(p_3) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

where \( F_y(p) \), \( F_z(p) \), and \( M(p) \) (for \( \Psi = 1,2,3 \)) are the coefficients of Equation (7.3) expressed in terms of \( p_1, p_2, \) and \( p_3 \), as shown in Appendix A.

After solving \( p_1, p_2, \) and \( p_3 \) in Equation (7.4), the static force \( \mathbf{F}_i^{sf} \) of the \( k \)th node of the \( i \)th deck section can be obtained by using Equation (7.2). The modal static forces \( \mathbf{F}^{sf} \) for the whole bridge can finally be determined by:

\[
\mathbf{F}^{sf} = 
\begin{bmatrix}
\mathbf{F}_1^{sf} \\
\mathbf{F}_2^{sf} \\
\vdots \\
\mathbf{F}_{N_s}^{sf}
\end{bmatrix}
\]

where \( \mathbf{F}_i^{sf} = [\mathbf{F}_{1,i}^{sf}, \mathbf{F}_{2,i}^{sf}, \ldots, \mathbf{F}_{N_s,i}^{sf}]^T \), in which \( i = 1,2,\ldots, N_s \) and \( N_s \) is the total number of sections considered in the model of the bridge.

### 7.3.2. Buffeting Force Model

Buffeting forces are caused by fluctuating wind components, \( u \) (alongwind) and \( w \) (upward). Using quasi-steady aerodynamic force coefficients, the equivalent buffeting forces per unit span \( \mathbf{F}_{ei}^{bf} \) on the \( i \)th section of the bridge deck can be expressed as (Simiu & Scanlan 1996):
$$\mathbf{F}_{ei}^{bf} = \begin{bmatrix} 0 \\ f_{eDti}^{bf} \\ f_{eLi}^{bf} \\ f_{eMi}^{bf} \\ 0 \\ 0 \end{bmatrix} = \mathbf{A}_{i}^{bf} \begin{bmatrix} u_i(t) \\ w_i(t) \end{bmatrix}$$  \hspace{1cm} (7.6)$$

in which

$$\mathbf{A}_{i}^{bf} = \begin{bmatrix} f_{ui} & f_{wi} \end{bmatrix} = \frac{1}{2} \rho_d \overline{U_i}^2 B_i \begin{bmatrix} 0 \\ \chi_{Dwu} \frac{2C_{Di}(\alpha_i)}{\bar{U}_i} \\ \chi_{Lwu} \frac{2C_{Li}(\alpha_i)}{\bar{\sigma}_i} \\ \chi_{Mwu} \frac{2C_{Mi}(\alpha_i)}{\bar{U}_i} B_i \\ 0 \\ 0 \end{bmatrix}$$ \hspace{1cm} (7.7)$$

where $f_{eDti}^{bf}$, $f_{eLi}^{bf}$, and $f_{eMi}^{bf}$ are buffeting drag, lift, and moment, respectively; $C'_{Di} = dC_{Di}/d\alpha$, $C'_{Li} = dC_{Li}/d\alpha$, and $C'_{Mi} = dC_{Mi}/d\alpha$; $u_i(t)$ and $w_i(t)$ are the horizontal and vertical components of fluctuating wind at the $i$th section, respectively; and $\chi_{Dwu}$, $\chi_{Dbw}$, $\chi_{Lwu}$, $\chi_{Lbw}$, $\chi_{Mwu}$, and $\chi_{Mbw}$ are the aerodynamic transfer functions between fluctuating wind velocities and buffeting forces.

Similarly, the spatial distribution of buffeting forces over the surface of the $i$th section is not reflected in Equation (7.6). For the purpose of this research, the buffeting force displayed in the above equation with respect to the centre of elasticity should be distributed to the nodal points on each element of the deck section. However, it is impossible to obtain the distribution of fluctuating wind pressure for the whole bridge deck as a function of time. To tackle this problem, an approximate approach is suggested on the basis of $\mathbf{A}_{i}^{bf}$ in Equation (7.7) by assuming the equivalent fluctuating wind force $\mathbf{F}_{ei}^{bf}(t)$ at the $i$th deck section can
be decomposed as

$$F_{ei}^{bf}(t) = F_{ei}^{sf} \times F_i^t(t) \quad (7.8)$$

where $F_i^t(t)$ is the time-dependent part of the wind force. This part is assumed to be linearly related to the fluctuating wind speeds $u_i(t)$ and $w_i(t)$:

$$F_i^t(t) = [c_1 \ c_2] \begin{bmatrix} u_i(t) \\ w_i(t) \end{bmatrix} \quad (7.9)$$

where $c_1$ and $c_2$ are the constants. The wind buffeting force at the $i$th deck section can be rewritten as

$$F_{ei}^{bf}(t) = A_i^{sf} \begin{bmatrix} 0 & 0 \\ p_1c_1 & p_1c_2 \\ p_2c_1 & p_2c_2 \\ p_3c_1 & p_3c_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_i(t) \\ w_i(t) \end{bmatrix} \quad (7.10)$$

By comparing Equation (7.10) with Equation (7.7), the independent unknown variables $[p_1c_1, p_2c_1, p_3c_1]^T$ and $[p_1c_2, p_2c_2, p_3c_2]^T$ can be solved. The wind force distributions over the $i$th deck section can also be determined by replacing the result of $[p_1c_1, p_2c_1, p_3c_1]^T$ with $[p_1, p_2, p_3]^T$ for $f_{ui}$ and $[p_1c_2, p_2c_2, p_3c_2]^T$ with $[p_1, p_2, p_3]^T$ for $f_{wi}$ in the figures displayed in Figure 7.8. The buffeting force $F_{ki}^{bf}$ of the $k$th node acting on the $i$th section of the bridge deck can then be expressed as
and the nodal buffeting forces $F^{bf}(t)$ for the whole bridge can finally be
determined by

$$
F^{bf}(t) = \left[ \begin{array}{c}
0 \\
F_{kly}^{bf} \\
F_{kiz}^{bf} \\
0 \\
0 \\
0 \\
\end{array} \right] \times u_i(t) + \left[ \begin{array}{c}
0 \\
F_{kly}^{bf} \\
F_{kiz}^{bf} \\
0 \\
0 \\
0 \\
\end{array} \right] \times w_i(t)
$$

(7.11)

where $F^{bf}_{i}(t) = \left[ F^{bf}_{1,i}(t), F^{bf}_{2,i}(t), \ldots, F^{bf}_{N_{i},i}(t) \right]^T$ and $i = 1, 2, \ldots, N_s$.

### 7.3.3. Self-excited Force Model

The self-excited forces on a bridge deck are due to the interaction between wind
and bridge motion. When the energy of motion extracted from the flow exceeds
the energy dissipated by the system through mechanical damping, the magnitude
of vibration can attain catastrophic levels (Shum 2004). The equivalent
self-excited drag $f^{se}_{eDi}$, lift $f^{se}_{eLi}$, and moment $f^{se}_{eMi}$ force per unit span at the $i$th
section of the bridge deck can be expressed in matrix form, as follows (Liu et al.
2004)

$$
F_{ei}^{se}(t) = E_{ei}X_{ei}(t) + G_{ei}\dot{X}_{ei}(t) + \ddot{F}_{ei}^{se}(t)
$$

(7.13)
where \( \mathbf{F}_{ei}^{se}(t) = [0, f_{eiD}^{se}(t), f_{eiL}^{se}(t), f_{eiM}^{se}(t), 0, 0]^T \) is the equivalent self-excited force acting at the centre of elasticity of the \( i \)th deck section; \( \mathbf{X}_{ei}(t) \) and \( \dot{\mathbf{X}}_{ei}(t) \) are the displacement and velocity vectors, respectively, with respect to the centre of elasticity of the \( i \)th section in the \( p-h-q \) coordinate system; \( \mathbf{F}_{ei}^{se}(t) \) is the part of the self-excited forces that reflect aerodynamic phase lag; and \( \mathbf{E}_{ei} \) and \( \mathbf{G}_{ei} \) are the aeroelastic stiffness and damping, respectively, of the \( i \)th deck section with respect to the centre of elasticity. Equation (2.57) gives the details of these matrices.

Similarly, the self-excited force expressed by Equation (7.12) relates to the centre of elasticity of the \( i \)th deck section. Hence, the force model must be distributed to the nodal points of this section. The relevant distributions are based on the rigid body motion relationships between the motions at the nodal point and those at the centre of elasticity of the deck section (Lau et al. 2000). Based on the finite element model of the given \( i \)th deck section, the position of the centroid of the \( i \)th deck section can be determined in terms of the geometry of the section. The displacement relationship between the nodal lines and the centre of elasticity of the \( i \)th section can then be given as follows:

\[
\mathbf{X}_{ei} = \mathbf{N}_{i}^{se} \mathbf{X}_i
\]

(7.14)

where \( \mathbf{X}_i = [\mathbf{X}_{1i}, \mathbf{X}_{2i}, \ldots, \mathbf{X}_{N_{di}}]^T \) is the displacement vector of all the nodes in the \( i \)th deck section in the \( p-h-q \) coordinate system and \( \mathbf{N}_{i}^{se} \) is the displacement transformation matrix which can be expressed as
\[
\mathbf{N}^{se}_i = \begin{bmatrix}
0 \\
N^{se}_{Di} \\
N^{se}_{Li} \\
N^{se}_{Mi} \\
0
\end{bmatrix}
\] (7.15a)

in which

\[
N^{se}_{Di} = \begin{bmatrix}
a_{1i} h_{cl} b_{1i}, 0,0,0,0,0 \\
\vdots \\
0,0,a_{N_{ni}}, h_{cl} b_{N_{ni}}, 0,0,0
\end{bmatrix}_{\text{node } N_n}
\] (7.15b)

\[
N^{se}_{Li} = \begin{bmatrix}
a_{1i} - q_{ci} b_{1i}, 0,0,0,0,0 \\
\vdots \\
0,0,a_{N_{ni}} - q_{ci} b_{N_{ni}}, 0,0,0
\end{bmatrix}_{\text{node } N_n}
\] (7.15c)

\[
N^{se}_{Mi} = \begin{bmatrix}
b_{1i}, 0,0,0,0,0,0,0,0 \\
\vdots \\
0,0,b_{N_{ni}}, 0,0,0,0,0,0,0
\end{bmatrix}_{\text{node } N_n}
\] (7.15d)

where \(q_{ci}\) and \(h_{cl}\) are the lateral and vertical coordinates of the centroid of the \(i\)th deck section, respectively; \(a_{ki} = \sum_j l_{jki} / 2L_i\), in which \(j = 1,2,\ldots,N_{ei}\) and \(k = 1,2,\ldots,N_{ni}\), the summation \(\sum_j l_{jki}\) applies to all of the elements \(j\) connected at node \(k\) and \(L_i\) is the summation of the lengths of all the elements in the \(i\)th deck section; and \(b_{ki} = 1/N_{ni} q_{ki}\), in which \(q_{ki}\) is the lateral coordinate of the \(k\)th node.

To substitute Equation (7.14) into Equation (7.13), the self-excited forces for the \(i\)th deck section expressed in terms of the nodal displacement vector become:

\[
\mathbf{F}^{se}_{ei} = \mathbf{E}_{ei} \mathbf{N}^{se}_i \mathbf{X}_i + \mathbf{G}_{ei} \mathbf{N}^{se}_i \dot{\mathbf{X}}_i + \mathbf{F}^{se}_{ei}
\] (7.16)

By applying the virtual work principle, the self-excited forces at the centre of elasticity of the \(i\)th section can be distributed to all nodes by
\[
\mathbf{F}_{se}^i = \left( \mathbf{N}_{se}^i \right)^T \mathbf{F}_{sei}^i = \mathbf{E}_i \mathbf{X}_i + \mathbf{G}_i \mathbf{\dot{X}}_i + \left( \mathbf{N}_{se}^i \right)^T \mathbf{\ddot{F}}_{sei}^i \tag{7.17}
\]

in which \( \mathbf{F}_{se}^i = \left[ \mathbf{F}_{1,se}^i, \mathbf{F}_{2,se}^i, \ldots, \mathbf{F}_{N_{se},se}^i \right]^T \) is the nodal self-excited force vector and \( \mathbf{F}_{k,se}^i = \left[ 0, \mathbf{F}_{k,se}^{kly}, \mathbf{F}_{k,se}^{klz}, 0,0,0 \right]^T \), \( \mathbf{E}_i = \left( \mathbf{N}_{se}^i \right)^T \mathbf{E}_{ei} \mathbf{N}_{se}^i \), and \( \mathbf{G}_i = \left( \mathbf{N}_{se}^i \right)^T \mathbf{G}_{ei} \mathbf{N}_{se}^i \) are the aeroelastic stiffness and damping matrices of the \( i \)-th section of the bridge deck related to the nodal self-excited forces, respectively. By applying the same procedure to all deck sections, the modal self-excited force \( \mathbf{F}^{se} \) can be determined by

\[
\mathbf{F}^{se} = \mathbf{E}\mathbf{X} + \mathbf{G}\mathbf{\dot{X}} + \left( \mathbf{N}^{se} \right)^T \mathbf{\ddot{F}}^{se} \tag{7.18}
\]

where

\[
\mathbf{F}^{se} (t) = \left[ \mathbf{F}_{1}^{se} (t), \mathbf{F}_{2}^{se} (t), \ldots, \mathbf{F}_{N_{se}}^{se} (t) \right]^T ;
\]

\[
\mathbf{E} = \left[ \mathbf{E}_1 , \mathbf{E}_2 , \ldots, \mathbf{E}_{N_{se}} \right]^T
\]

\[
\mathbf{G} = \left[ \mathbf{G}_1 , \mathbf{G}_2 , \ldots, \mathbf{G}_{N_{se}} \right]^T
\]

\[
\mathbf{N}^{se} = \left[ \mathbf{N}_{se}^1 , \mathbf{N}_{se}^2 , \ldots, \mathbf{N}_{se}^{N_{se}} \right]^T
\]

\[
\mathbf{\ddot{F}}^{se} (t) = \left[ \mathbf{\ddot{F}}_{1}^{se} (t), \mathbf{\ddot{F}}_{2}^{se} (t), \ldots, \mathbf{\ddot{F}}_{N_{se}}^{se} (t) \right]^T
\]

### 7.3.4. Equation of Motion and Method of Solution

The governing equation of motion for the bridge in terms of a nodal displacement vector \( \mathbf{X} \) under wind loads can be expressed as

\[
\mathbf{M}\mathbf{\ddot{X}}(t) + \mathbf{C}\mathbf{\dot{X}}(t) + \mathbf{K}\mathbf{X}(t) + \mathbf{K}\mathbf{\ddot{X}} = \mathbf{F}^{sf} + \mathbf{\ddot{F}}^{hf} (t) + \mathbf{F}^{se} (t) \tag{7.19}
\]

where \( \mathbf{M}, \mathbf{C}, \) and \( \mathbf{K} \) are the global structural mass, damping, and stiffness matrices of the bridge with dimensions of \( 6N \times 6N \), in which \( N \) is the total number of nodes.
in the FEM; \( \mathbf{X}(t) = [\mathbf{X}(t), \mathbf{X}(t), \cdots, \mathbf{X}_N(t)]^T \) is the nodal displacement vector of the bridge and \( \mathbf{X}_k = [p_k, q_k, h_k, \theta_{pk}, \theta_{qk}, \theta_{hk}]^T \) is the \( 6 \times 1 \) displacement vector of the \( k \)th node at the global coordinate system as shown in Figure 7.9; each over-dot denotes one order of partial differentiation with respect to time; and \( \bar{\mathbf{X}} \) is the mean displacement.

Referring to Equation (7.19), the excitation of a long suspension bridge can be decomposed into a static force component \( \mathbf{F}^{sf} \) which will induce mean displacement of the bridge \( \bar{\mathbf{X}} \) and dynamic components \( \mathbf{F}^{bf} \) and \( \mathbf{F}^{se} \) which will both produce fluctuating displacement \( \mathbf{X}(t) \), velocity \( \mathbf{X}(t) \) and acceleration \( \mathbf{X}(t) \).

In the aerostatic analysis, the mean displacement \( \bar{\mathbf{X}} \) is simply solved by the static equilibrium matrix equation given as

\[
\mathbf{K}\bar{\mathbf{X}} = \mathbf{F}^{sf}
\]

(7.20)

The unknown nodal mean displacement \( \bar{\mathbf{X}} \) involves the equivalent of inverting the stiffness matrix \( \mathbf{K} \) and multiplying it by the aerostatic force vector \( \mathbf{F}^{sf} \). However, as mentioned in Equation (7.1), static wind force for a long suspension bridge is calculated in terms of three static aerodynamic coefficients: \( C_D, C_L \) and \( C_M \). The values of these coefficients are selected on the basis of the effective wind angle of attack of the bridge \( \alpha \), which is the summation of the initial mean angle of attack \( \alpha^0 \) and the deformation of the deck with torsional angle \( \theta \). The mean angle \( \alpha^0 \) is computed on the basis of field measurement data by simply dividing the vertical wind speed component \( w(t) \) by \( \bar{U} \). However, because of its structural nonlinearity and the static wind force, the additional attack angle \( \theta \) caused by the torsional
deformation must be solved using an iteration approach (Zhang et al. 2002). The iteration approach proposed by Sun (1999) is applied in this study:

(a) Assume \( \{ \theta \} = \{ 0 \} \) where \( \{ \theta \} = \{ \theta_1, \theta_2, \cdots \theta_{N_n} \} \);
(b) Compute \( \{ \alpha \} = \{ \alpha^0 \} + \{ \theta \} \);
(c) Determine the static aerodynamic coefficients \( C_{Di}(\alpha_i), C_{Li}(\alpha_i) \), and \( C_{Mi}(\alpha_i) \) of each section \( i \);
(d) Compute the nodal static forces of the bridge deck \( \{ \mathbf{F} \} = \{ f_{D}^{sf}, f_{L}^{sf}, f_{M}^{sf} \} \) and the corresponding nodal mean displacement;
(e) Compute the torsional angle \( \{ \theta' \} \);
(f) Stop if \( \| \{ \theta' \} - \{ \theta \} \| \leq \epsilon, \{ \theta' \} \rightarrow \{ \theta \} \); repeat steps (b)-(f) and replace \( \{ \theta' \} \rightarrow \{ \theta \} \) if \( \| \{ \theta' \} - \{ \theta \} \| > \epsilon \) (where \( \epsilon \) should be a small number to control the accuracy of the calculation).

In the dynamic part, as the total response of a long suspension bridge can be obtained as the superposition of the solutions of the independent modal equations (Paz 1997), in Equation (7.19), the modal superposition technique is adopted to solve the dynamic displacement induced by buffeting force and self-excited force. The nodal displacement vector \( \mathbf{X}(t) \) can be expressed in terms of the modal coordinates of the bridge:

\[
\mathbf{X}(t) = \Phi \mathbf{q}(t) \quad (7.21)
\]

where \( \mathbf{q}(t) = [q_1(t), q_2(t), \cdots q_{N_m}(t)]^T \) is the generalized displacement vector and \( N_m \) is the number of modes of interest involved in the computation; and
\[ \Phi = \left[ \Phi_1, \Phi_2, \ldots, \Phi_{N_m} \right]^T \] is the mode shape matrix. The equation of motion displayed in Equation (7.19) can then be rewritten as

\[ \ddot{\mathbf{M}} \dot{\mathbf{q}}(t) + \mathbf{C} \dot{\mathbf{q}}(t) + \mathbf{K} \mathbf{q}(t) = \mathbf{Q}^{bf}\,(t) + \mathbf{Q}^{se}\,(t) \quad (7.22) \]

where \( \mathbf{M}, \mathbf{C}, \) and \( \mathbf{K} \) are the generalized mass, damping, and stiffness matrices, respectively; and \( \mathbf{Q}^{bf} = \Phi^T \mathbf{F}^{bf} \) and \( \mathbf{Q}^{se} = \Phi^T \mathbf{F}^{se} \) are the generalized buffeting and self-excited force vectors, respectively. Substituting Equation (7.18) into Equation (7.22) yields

\[ \ddot{\mathbf{M}} \dot{\mathbf{q}}(t) + \mathbf{C}^{s} \dot{\mathbf{q}}(t) + \mathbf{K}^{s} \mathbf{q}(t) = \mathbf{Q}^{bf}\,(t) + \tilde{\mathbf{Q}}^{se}\,(t) \quad (7.23) \]

where

\[ \tilde{\mathbf{Q}}^{se} = \Phi^T (\mathbf{N}^{se})^T \mathbf{F}^{se}; \quad \mathbf{C}^{s} = \mathbf{C} - \Phi^T \mathbf{G} \Phi; \quad \mathbf{K}^{s} = \mathbf{K} - \Phi^T \mathbf{E} \Phi \]

The generalized displacement vector \( \mathbf{q}(t) \) in Equation (7.23) can be solved using the Newmark implicit integral algorithm. The nodal displacement, velocity, and acceleration vectors can then be determined by Equation (7.21).

### 7.4. COMPARISONS BETWEEN NUMERICAL AND MEASUREMENT RESULTS

The WASHMS-based statistical analysis of wind speed and displacement response for the Tsing Ma Bridge described in Chapter 6 is used to verify the accuracy of displacement measures taken using the new structural health monitoring-based FEM. As the winds from the open-sea fetch measured at the Tsing Ma Bridge in
the typhoon period cover a wider range of speeds, it is essential to use the wind characteristics derived from typhoon events at the open-sea exposure, as described in Chapter 5, in determining the major parameters for the wind force simulation.

### 7.4.1. Simulation of Wind Forces

Because of time and resource limitations, wind tunnel tests cannot be performed to obtain the wind pressure distribution for the whole of the Tsing Ma Bridge deck. As such, the wind pressure distributions of other suspension bridges with similar deck sections, as shown in Figure 7.8, are adopted and assumed to be the same as those experienced by the Tsing Ma Bridge deck. The three typical deck sections shown in Figure 7.8 have 12, 10, and 8 nodes, respectively. It can be seen that the wind pressure over each deck section contains three independent variables: \( p_1 \), \( p_2 \), and \( p_3 \). Values for these three variables can be determined using the equations described in Section 7.3. However, in determining the values of the three variables, drag \( C_D \), lift \( C_L \), and moment \( C_M \) coefficients should be used. Figure 7.10 depicts the aerodynamic force coefficient curves for \( C_D \), \( C_L \), and \( C_M \) for a 12-node deck section of the Tsing Ma Bridge measured during the wind tunnel tests (Lau & Wong 1997). Since the wind incidences measured in the typhoon period were approximately equal to zero for winds from the open-sea fetch, the analyses are performed with the initial wind attack angle of 0°. At the zero wind angle of attack, the static coefficients are shown as \( C_D = 0.135 \), \( C_L = 0.090 \), and \( C_M = 0.063 \). Their first derivatives are given as \( dC_D / d\alpha = -0.253 \), \( dC_L / d\alpha = 1.324 \), and \( dC_M / d\alpha = 0.278 \) (Xu et al. 2003). Due to the lack of wind tunnel test results for the other two deck sections, the aforementioned coefficients for the 12-node section are applied to the other two deck sections.
From Equation (7.7), all the aerodynamic transfer functions are assumed to be unit. This assumption may lead to overestimation of the bridge buffeting response. In this regard, a series of time histories of fluctuating wind velocity \([u(t) \ w(t)]^T\) in the horizontal and vertical directions at various points along the bridge deck is essential to perform a detail buffeting analysis. However, because only two anemometers are installed on the bridge deck, one on each side, in this study the horizontal and vertical wind auto-spectra, the mean wind speed, the mean wind incidence, and the turbulent intensities obtained from the wind velocities measured at the mid-main span of the bridge deck are assumed to be constant along the bridge deck. A fast spectral representation approach proposed by Cao et al. (2000) which is based on the spectral representation method is adopted here for the digital simulation of the stochastic wind velocity field. To simulate the horizontal \(u\) and vertical \(w\) fluctuating wind speed time histories, the von Karman longitudinal \(S_{uu}\) and vertical \(S_{ww}\) wind auto-spectra are used. The major wind parameters at the deck level for the open-sea exposure are given as: \(L_{u}^x = 192.53\ m,\ L_{w}^x = 29.03\ m,\ I_{u} = 8.6\%\) and \(I_{w} = 5.3\%\). The wind velocity fields in both the alongwind \(u\) and vertical wind \(w\) directions along the bridge girder are simulated by 120 fluctuating wind velocity time histories at 120 \(n_p\) evenly distributed points along the bridge deck with an interval distance of 18.0 m \(\Delta\). The parameter \(\lambda_w\) in Equation (2.44) is taken as a constant value of 16. The upper cutoff frequency \(\omega_{up}\) is taken as \(50\pi\ rad/s\) and the frequency dividing number \(N_f\) is \(2^{14}\). The corresponding frequency interval \(\Delta\omega\) thus becomes \(0.009587\ rad/s\). The sampling frequency and duration used in the simulation of wind speeds are \(50\ Hz\) and \(10\ minutes\), respectively. The use of a 10-minute duration reflects the fact that the regression models developed in the preceding chapters are all
based on the same duration. Figure 7.11 illustrates the simulated turbulent wind velocity time histories of 10 minutes’ duration with a time interval (\(\Delta t\)) of 0.02 sec in the alongwind and upward wind directions at the mid-main span of the bridge deck for wind approaching from the sea with a mean wind speed of 50 m/s.

To determine the self-excited forces of the Tsing Ma Bridge, the aeroelastic stiffness matrix \(E_{el}\) and the aeroelastic damping matrix \(G_{el}\) of the deck section with respect to the centre of elasticity as shown in Equation (7.13) should first be determined. Due to the lack of wind tunnel test results on lateral flutter derivatives and the negligible effect of the coupled terms on the self-excited forces, a total of 12 frequency-independent coefficients \((C_{1e}^{h}, C_{2e}^{h}, C_{4e}^{h}, C_{5e}^{h}, d_{4e}^{h}, d_{5e}^{h}, C_{1e}^{\theta}, C_{2e}^{\theta}, C_{4e}^{\theta}, C_{5e}^{\theta}, d_{4e}^{\theta}, d_{5e}^{\theta})\) listed in Table 7.2 are determined by using the measured flutter derivatives \((H_{1e}^{*}, H_{4e}^{*}, A_{2e}^{*}, A_{3e}^{*})\) and the least squares fitting method (Ding and Lee 2000). They are used to determine the matrices \(E_{el}\) and \(G_{el}\) and the coefficients in the vector \(\hat{F}_{se}(t)\) for the \(i\)th section of the bridge deck. Because of the geometrical symmetry with respect to the mid-vertical axis of the bridge deck section, the centre of elasticity and the centroid of the deck cross-section are both in the vertical axis. By taking this geometric feature into account, the displacement transformation matrix \(N_{se}^{i}\) in Equation (7.14) for the \(i\)th deck section can be easily determined. The aeroelastic stiffness matrix \(E\), the aeroelastic damping matrix \(G\), the coefficients in the vector \(\hat{F}_{se}\), and the self-excited forces at the nodes of the FEM of the Tsing Ma Bridge in the global coordinate system can then be determined on the basis of Equations (7.17) and (7.18). Due to the lack of flutter derivatives, the whole aerodynamic analysis can be performed only at the zero effective wind angle of attack. As a result, the effective wind angle of attack
\[ \alpha = \alpha^0 + \theta \] in Equation (7.7) is assumed to be zero for the purpose of aerodynamic analysis.

### 7.4.2. Comparison of Mean Displacement Responses

To evaluate the accuracy of mean displacement estimates from the FEM, the aerostatic lateral displacements derived by nonlinear analysis with a wind speed of between 5 and 15 m/s are computed and compared with those measured at the site in the SSE direction sector. Table 7.3 illustrates the mean lateral displacement results computed and measured at the mid-main span section, together with the relative discrepancies (RDs). The relative discrepancy is defined as \((\text{computed result} - \text{measured result})/\text{measured result}\). From the table, it can be seen that the computed mean lateral displacement responses at the mid-main span are all close to the measured responses. The absolute relative discrepancies are less than 13% when the mean wind speed is 10 m/s or more. These comparable results illustrate the validity of the FEM for nonlinear aerostatic computation.

### 7.4.3. Comparison of Dynamic Displacement Responses

The buffeting-induced displacement responses of the Tsing Ma Bridge are computed using the mode superposition technique. The vibration of the bridge at a frequency of up to 1.1 Hz is taken into account in the computation process. This frequency range adequately covers the first 80 modes of vibration of the bridge. The structural damping ratio for each mode of vibration is taken as 0.2%. To evaluate the accuracy of computed dynamic displacements, the aerodynamic displacements at the mid-main span section in the lateral and vertical directions at a wind speed of between 5 and 15 m/s are compared with those measured at the
site in the SSE direction sector. Table 7.4 displays the computed and measured standard deviations of displacement. The table also lists the relative discrepancies between the two sets of results, which show that the displacements computed at the mid-main span in the lateral and vertical directions for the open-sea fetch are close to the measured results. The absolute relative discrepancies are all less than 15% in the both lateral and vertical directions. These comparable results confirm the validity of the FEM for aerodynamic computation for the open-sea exposure.

### 7.4.4. Comparison of Total Displacement Responses

In wind engineering, the expected peak value of displacement $\bar{X}$ during an interval of time is usually expressed as

$$\bar{X} = \bar{x} + m\sigma_X$$  \hspace{1cm} (7.24)

in which $\bar{X}$ is the mean displacement; $\sigma_X$ is the displacement standard deviation; and $m$ is a peak factor. In most cases, $m$ will have a value of between 3 and 4; a value of 3.5 is a fair approximation. However, to take into account displacements with a frequency of less than 1 Hz in this study, a lower peak factor should be obtained. As noted in Chapter 6, the lateral and vertical peak displacements are almost proportional to the corresponding standard deviation with a slope of 3. To verify the peak factor estimated by the FEM, the statistical relationship between the computed dynamic peak displacement ($\bar{X} - \bar{x}$) and the standard deviation displacement $\sigma_X$ of the bridge deck at the middle of the main span for wind speeds of between 5 and 15 m/s is explored and illustrated in Figure 7.12. It can be seen that the lateral and vertical peak displacements are almost proportional to the
corresponding standard deviation, with respective slopes of 2.80 and 3.18. These values are close to the slope of 3 obtained from the field measurement exercise.

To evaluate the accuracy of the FEM, the total displacement responses of the mid-main span section computed in the lateral and vertical directions at a wind speed of between 5 and 15 m/s are compared with those measured at the site in the SSE direction sector. The total displacement estimated by the FEM is calculated by Equation (7.24). Table 7.5 displays the computed and measured total displacement responses. The table also lists the relative discrepancies between the two sets of results, which show that the total displacements computed at the mid-main span in the lateral direction for the open-sea fetch are close to the measured results. While the absolute relative discrepancies in the lateral direction are all less than 17%, those in the vertical direction are more than 30%. As noted in Chapter 6, because the mean vertical displacement of the bridge is mostly affected by the temperature, the wind-induced mean vertical displacement is assumed to be zero. In fact, the mean vertical displacements computed for the bridge are 1.74, 6.99, and 15.85 mm, respectively, for mean wind speeds of 5, 10, and 15 m/s. As shown in Table 7.3, these values are much lower than those computed in the lateral direction. If the mean vertical displacement of the bridge is neglected in the total vertical displacement computation, the absolute relative discrepancies of the total displacement response in the vertical direction will all be less than 9%. These comparable results confirm the validity of the FEM.
7.5. DISPLACEMENT RESPONSE OF THE FULL BRIDGE TO HIGH WINDS

As noted in Chapter 6, the results of the field investigations were limited by the number of sensors available and the wind speeds measured. This necessitated designing a method that could be used to help analyze the response of the entire bridge to extreme wind speeds. The preceding section confirms the validity of the complex structural health monitoring-based FEM and indicates that this model can be used to produce accurate estimates of the possible effects of extreme wind speeds on the Tsing Ma Bridge as a whole.

7.5.1. Mean Displacement Response

This section considers the statistical relationship between mean lateral displacement and wind speed in the SSE direction sector. The statistical relationship for mean wind speeds of up to 15 m/s developed on the basis of measurement data in Chapter 6 is extended to extreme wind speeds through FEM analysis. Figure 7.13(a) illustrates the resultant curve of the mean lateral displacement at the mid-main span. As shown in the figure, the mean lateral displacement response at the mid-main span in the SSE direction sector is 1,524 mm when the deck-height wind speed is 50 m/s. Table 7.6 compares the computed results with the wind tunnel tests and shows relative discrepancies (RDs) of around 28% of the absolute between the two set of results. The lower magnitude of the computed results may be explained by the fact that the computation process does not take account of aerostatic forces exerted on the towers and cables.

Although the mean vertical displacement of the bridge could not be estimated in
the field, this does not imply that the wind does not induce any vertical displacement. The wind-induced mean vertical displacement of the bridge in the SSE direction sector can now be clearly identified by FEM analysis with a mean wind speed of 0 to 50 m/s, as illustrated in Figure 7.13(b). As shown in the figure, when the deck-height wind speed is 50 m/s, the mean vertical displacement at the mid-main span in the SSE direction sector is 204 mm. This result clearly shows that the wind-induced mean displacement in the vertical direction is much smaller than that in the lateral direction. To further illustrate the mean displacement response of the entire Tsing Ma Bridge, Figure 7.14 depicts the variations in the lateral and vertical mean displacement responses along the bridge at different mean wind speeds. The figure shows that the mean displacement responses are almost symmetric with respect to the middle of the main span.

7.5.2. Dynamic Displacement Response

This section extends the lateral and vertical statistical relationships for dynamic displacement in the SSE direction sector developed in the basis of the measurement data in Chapter 6 to extreme wind speeds through FEM analysis. Figures 7.15(a) and (b), respectively, illustrate the resultant curves of the lateral and vertical displacement standard deviations at the mid-main span. The figures show that when the deck-height wind speed is 50 m/s, the displacement standard deviations at the mid-main span are 278.59 and 206.27 mm in the lateral and vertical directions, respectively. These results show that the vertical standard deviation increases more slowly than the lateral standard deviation as the mean wind speed increases. Table 7.7 reports similar results from the wind tunnel tests, and indicates absolute relative discrepancies (RDs) of 26% and 13% between the
two set of results in the lateral and vertical directions, respectively. As noted above, this may be because the computation process does not take account of aerodynamic forces exerted on the towers and cables. An additional factor may be that due to the limited aerodynamic parameters available, only a zero wind angle of attack was considered in the aerodynamic computation process. To further illustrate the dynamic displacement response of the entire Tsing Ma Bridge, Figure 7.16 depicts the variations in the lateral and vertical displacement standard deviation responses along the bridge deck at different mean wind speeds. In common with the mean displacement responses, the lateral and vertical displacement standard deviation responses are almost symmetric with respect to the middle of the main span.

### 7.5.3. Total Displacement Response

This section extends the lateral and vertical statistical relationships developed in Chapter 6 for total displacement in the SSE direction sector to extreme wind speeds through FEM analysis. Figures 7.17(a) and (b), respectively, illustrate the resultant curves of the lateral and vertical total displacements at the mid-main span in which displacements under winds from 20 to 50 m/s are calculated using Equation (7.24). As noted above, the mean vertical displacement is not considered in computing the vertical total displacement response in the field. However, as shown in Figure 7.15(b), the mean displacement in the vertical direction is only 16 mm at a mean wind speed of 15 m/s. It is acceptable to neglect the mean vertical displacement response in computing the total displacement at such low mean wind speeds. As shown in the figures, when the deck-height wind speed is 50 m/s, the total displacement responses at the mid-main span are 2,304.84 and 859.78 mm in
the lateral and vertical directions, respectively. Table 7.8 presents a comparison of the computed results with those from the wind tunnel test and shows absolute relative discrepancies (RD) of about 22% between the two set of results in the lateral direction. This difference may be explained by the fact the aerostatic and aerodynamic forces exerted on the towers and cables are not considered in the computation process. An additional factor may be that only a zero wind angle of attack was considered in the aerodynamic computation process. Due to the limited set of wind tunnel test results on mean vertical displacement at wind speeds of below 50 m/s, no comparison is made between field and wind tunnel test results in the vertical direction. To further illustrate the dynamic displacement response of the entire Tsing Ma Bridge, Figure 7.18 depicts variations in the lateral and vertical total displacement responses along the bridge deck at different mean wind speeds. In common with the mean displacement responses, the lateral and vertical total displacement responses are almost symmetric with respect to the middle of the main span.

7.6. SERVICEABILITY ASSESSMENT OF THE TSING MA BRIDGE

The previous section uses the FEM to extend the statistical relationships between wind and displacement response to extreme wind speeds. However, because the long-term WASHMS of the Tsing Ma Bridge will produce a huge amount of wind and displacement data in the future, the iteration task is performed to establish the final statistical relationship through an updating process. This section compares the maximum tolerance movements of the bridge under the serviceability limit state with the maximum total displacement to assess the serviceability of the bridge.
In civil engineering, structures should be designed by considering the serviceability limit state (SLS) at which they would become unfit for their intended use. Under serviceability loads, the deflection of a structure should not impair the strength or efficiency of the structure or its components. For the design of the Tsing Ma Bridge, the guidelines used in the SLS analysis were mainly set in accordance with BS5400 and the Structural Design Manual for Highways and Railways. To ensure the bridge was suitable for Hong Kong conditions, the wind speed criteria used in designing the Tsing Ma Bridge were based on data obtained from Waglan Island, where the open-sea exposure closely matches that of the bridge site. The hourly design wind speed criteria at the deck level were then established as 50 m/s for a 120 year return period in the SLS. The resultant lateral deflection of the bridge due to wind is therefore given as 2.9 m in the SLS. Owing to the fact that the wind-induced vertical deflection is minimal in comparison with other loadings, vertical deflection of the bridge due to wind is ignored. Figure 7.18(a) shows that at a deck-height wind speed of 50 m/s, the total displacement response at the mid-main span is 2,304.84 mm in the lateral direction. In comparison with the SLS, this total lateral displacement movement at the wind speed under consideration falls within the maximum tolerance movements specified in the design criteria. This indicates the serviceability of the Tsing Ma Bridge and its components are guaranteed.

7.7. SUMMARY

This chapter uses FEM analysis to extend the statistical relationships between wind and displacement responses by investigating the static and dynamic behavior of the Tsing Ma Bridge under high winds. The analysis presented involves using
the complex structural health monitoring-based three-dimensional FEM, which closely replicates the geometric details of the as-built complicated bridge deck. The displacement responses of the bridge deck in the time domain are predicted by distributing wind forces over the cross-section of the bridge deck. The major results are summarized as follows:

- The aerostatic lateral displacement, aerodynamic lateral and vertical displacements, and total lateral and vertical displacements of the mid-main span at wind speeds of between 5 and 15 m/s were computed and compared with those measured at the site. The absolute relative discrepancies were all less than 17% when the wind speed was greater than 10 m/s. Therefore, the validity of the FEM was confirmed;

- The statistical relationships between wind and mean displacement response in the SSE direction sector at mean wind speeds of up to 15 m/s were extended to extreme wind speeds through FEM analysis. At a deck-height wind speed of 50 m/s, the lateral and mean vertical displacement responses at the mid-main span were 1,524 mm and 204 mm, respectively. At the same wind speed, the lateral and vertical dynamic standard deviations at the mid-main span were 279 mm and 206 mm, respectively, while the total displacement responses at the mid-main span were 2,305 and 860 mm in the lateral and vertical directions, respectively. The results computed for the bridge deck were then compared with the wind tunnel test results, showing absolute relative discrepancies of less than 28% between the two set of results;
The new statistical relationships between wind and displacement responses were also extended to other sections of the bridge deck. The relationship between wind and the total lateral displacement response of the bridge deck at the mid-main span were also compared to the serviceability limit state (2.9 m). It was found that the total lateral displacement at a wind speed of 50 m/s fell within the design criterion, thereby guaranteeing the serviceability of the Tsing Ma Bridge and its components.
Table 7.1. The first eighteen vibration frequencies of the Tsing Ma Bridge

<table>
<thead>
<tr>
<th>MODE</th>
<th>FREQUENCY (Hz) MEASURED</th>
<th>FREQUENCY (Hz) COMPUTED</th>
<th>MODE TYPE</th>
<th>DIFFERENCE (%)</th>
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<tbody>
<tr>
<td>1</td>
<td>0.069</td>
<td>0.069</td>
<td>Lateral motion of deck and cables</td>
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<tr>
<td>2</td>
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<td>0.122</td>
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<td>0.160</td>
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<tr>
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<td>0.184</td>
<td>0.198</td>
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</tr>
<tr>
<td>6</td>
<td>0.214</td>
<td>0.222</td>
<td>Lateral motion of main span cables</td>
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<tr>
<td>7</td>
<td>0.226</td>
<td>0.231</td>
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<tr>
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<td>0.233</td>
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<tr>
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<tr>
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<tr>
<td>11</td>
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<tr>
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<td>0.283</td>
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<tr>
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<tr>
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Table 7.2. Frequency independent coefficients

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<thead>
<tr>
<th>α</th>
<th>C־</th>
<th>C־</th>
<th>C־</th>
<th>C־</th>
<th>d־</th>
<th>d־</th>
</tr>
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<tr>
<td>Lh</td>
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<td>1.06341</td>
<td>13.09312</td>
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<td>13.43210</td>
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Table 7.3. Computed mean lateral displacements and relative discrepancies in comparison with field measurement data (mid-main span)

<table>
<thead>
<tr>
<th>WIND SPEED (m/s)</th>
<th>COMPUTED (mm)</th>
<th>MEASURED (mm)</th>
<th>RD (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>15.47</td>
<td>12.00</td>
<td>28.96</td>
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<tr>
<td>10</td>
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<td>12.54</td>
</tr>
<tr>
<td>15</td>
<td>139.08</td>
<td>133.90</td>
<td>3.87</td>
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Table 7.4. Computed standard deviations and relative discrepancies in comparison with field measurement data (mid-main span)

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<th>VERTICAL DIRECTION</th>
</tr>
</thead>
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<td>COMPUTED (mm)</td>
<td>MEASURED (mm)</td>
</tr>
<tr>
<td></td>
<td>COMPUTED (mm)</td>
<td>MEASURED (mm)</td>
</tr>
<tr>
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<td>1.26</td>
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</tr>
<tr>
<td></td>
<td>1.54</td>
<td>1.51</td>
</tr>
<tr>
<td>10</td>
<td>4.59</td>
<td>4.57</td>
</tr>
<tr>
<td></td>
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<td>6.97</td>
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<tr>
<td>15</td>
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<td>17.09</td>
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</table>

Table 7.5. Computed total displacements and relative discrepancies in comparison with field measurement data (mid-main span)

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<th>LATERAL DIRECTION</th>
<th>VERTICAL DIRECTION</th>
</tr>
</thead>
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<td>COMPUTED (mm)</td>
<td>MEASURED (mm)</td>
</tr>
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<td></td>
<td>COMPUTED (mm)</td>
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</tr>
<tr>
<td>15</td>
<td>168.84</td>
<td>186.06</td>
</tr>
<tr>
<td></td>
<td>69.94</td>
<td>51.26</td>
</tr>
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</table>
Table 7.6. Computed mean lateral displacement responses and relative discrepancies in comparison with wind tunnel test results (mid-main span)

<table>
<thead>
<tr>
<th>WIND SPEED (m/s)</th>
<th>COMPUTED (mm)</th>
<th>WIND TUNNEL (mm)</th>
<th>RD (%)</th>
</tr>
</thead>
<tbody>
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<td>20</td>
<td>247.02</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>30</td>
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</tr>
<tr>
<td>40</td>
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<td>-23.92</td>
</tr>
<tr>
<td>50</td>
<td>1524.79</td>
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<td>-27.91</td>
</tr>
</tbody>
</table>

Table 7.7. Computed dynamic displacement responses and relative discrepancies in comparison with wind tunnel test results (mid-main span)

<table>
<thead>
<tr>
<th>WIND SPEED (m/s)</th>
<th>LATERAL DIRECTION</th>
<th>VERTICAL DIRECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>COMPUTED (mm)</td>
<td>WIND TUNNEL (mm)</td>
</tr>
<tr>
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<td>--</td>
</tr>
<tr>
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<td>62.74</td>
<td>--</td>
</tr>
<tr>
<td>40</td>
<td>144.96</td>
<td>145.14</td>
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<tr>
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<td>278.59</td>
<td>220</td>
</tr>
</tbody>
</table>

Table 7.8. Computed total displacement responses and relative discrepancies in comparison with wind tunnel test results (mid-main span)

<table>
<thead>
<tr>
<th>WIND SPEED (m/s)</th>
<th>LATERAL DIRECTION</th>
<th>VERTICAL DIRECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>COMPUTED (mm)</td>
<td>WIND TUNNEL (mm)</td>
</tr>
<tr>
<td>20</td>
<td>299.88</td>
<td>--</td>
</tr>
<tr>
<td>30</td>
<td>729.94</td>
<td>--</td>
</tr>
<tr>
<td>40</td>
<td>1387.27</td>
<td>1797.99</td>
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<tr>
<td>50</td>
<td>2304.84</td>
<td>2950.77</td>
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</table>
Figure 7.1. The Tsing Ma Bridge
Figure 7.2. A typical 18 m deck section at the main span
Figure 7.3. Finite element model of the Tsing Ma suspension bridge

(a) First lateral mode (0.069 Hz)

(b) Second lateral mode (0.164 Hz)
Figure 7.4. The first two modes of vibration of the Tsing Ma Bridge in the lateral, vertical, and torsional directions
Figure 7.5. Static force components at the deck section

Figure 7.6. Static force distribution at the $i$th deck section

Figure 7.7. Static force distribution at the $k$th node
Figure 7.8. Wind pressure distributions over three typical decks sections
Figure 7.9. Nodal point displacement in the global coordinate system

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Figure 7.12. Statistical relationships between computed peak and standard deviation displacements at the mid-main span.
Figure 7.13. Statistical relationships between wind and mean displacement response at the mid-main span in the SSE direction sector.
Figure 7.14. Mean displacement response of the bridge deck
Figure 7.15. Statistical relationships between wind and dynamic displacement response at the mid-main span in the SSE direction sector.
Figure 7.16. Dynamic displacement response of the bridge deck.
Figure 7.17. Statistical relationships between wind and total displacement response at the mid-main span in the SSE direction sector.
Figure 7.18. Total displacement response of the bridge deck
Chapter 8

STRUCTURAL HEALTH MONITORING-BASED PERFORMANCE ASSESSMENT: STRESS

8.1. INTRODUCTION

Chapter 7 has assessed the serviceability of the Tsing Ma Bridge in terms of displacement through an extension of the statistical relationship developed on the basis of wind speed measurement data using finite element model (FEM) analysis. The main focus of this chapter is to assess the performance of the Tsing Ma Bridge in terms of stress. As noted above, due to the trend of constructing bridges with longer spans, modern suspension bridges are more sensitive to strong winds than ever before. The significant increase in the aerodynamic instability of modern bridges may result in serious fatigue damage to structural components and connections (Zhu 2002; Shum 2004). Assessing the wind-induced stresses of a long-span suspension bridge is thus an indispensable task.

For full-scale monitoring of a long-span suspension bridge, strain-gauges are usually installed on the assumed critical components for damage detection. However, there remains a possibility that structural defects or degradation will occur away from the positions in which such strain-gauges are installed. In this
connection, Yoshida et al. (2003) proposed evaluating member stress through the hybrid use of the global positioning system (GPS) and FEM analysis. A review of previous studies on long-span suspension bridges shows there has been no previous investigation of the combined use of FEM and GPS sensors to quantify the stresses of major bridge structural components. The main reason for this is that this method involves the use of a comprehensive structural health monitoring-based finite element model to replicate the geometric details of complicated bridge decks. Hence, the complex structural health monitoring-based three-dimensional FEM that was described in Chapter 7 is utilized in this study. Through a comprehensive structural health monitoring-based FEM, the stresses of the critical bridge components under different wind speeds are computed. The wind-induced stresses of these critical members are then linked to the wind-induced displacement response at the mid-main span through the hybrid use of the GPS and FEM analysis. The wind-induced stresses derived at extreme wind speeds are then compared with the yield stress of the material to assess the strength of the bridge.

8.2. FORMULATION

The formulation used to compute the nodal displacement has been described in Chapter 7. In this section, the relationship between stress response and the nodal displacement of a finite element is derived. Without considering initial strains and stresses, the element stress induced by the elastic deformation of element \( j \) can be computed by

\[
\sigma_j = D_j L_j N_j X_j
\]  

(8.1)
where $\mathbf{\sigma}_j = [\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{xz}]_j^T$ is the stress vector of the element $j$; $\mathbf{X}_j$ is the nodal displacement vector of the element $j$; $\mathbf{N}_j$ is the shape function of the element $j$; $\mathbf{L}_j$ is the differential operator that can transfer the displacement field to the strain field, and $\mathbf{D}_j$ is the elastic matrix which establishes the relationship between the stress and the strain of the element $j$.

The modal stresses of all the elements at their end sections in the global coordinate system can be derived from the following relationship

$$\Gamma = \mathbf{D} \mathbf{L} \mathbf{N} \Phi \quad (8.2)$$

where $\mathbf{D}$, $\mathbf{L}$, and $\mathbf{N}$ are the elastic matrix, the differential operator, and the shape function, respectively, of the bridge structure in the global coordinate system. With the introduction of the modal stresses, the stress time histories at 5 points of the end section of each element $\mathbf{\sigma}_j(t)$ can be obtained from the superposition of the modal stresses $\Gamma$ with the generalized displacements $\mathbf{q}(t)$:

$$\mathbf{\sigma}_j(t) = \Gamma \mathbf{q}(t) \quad (8.3)$$

Among the 5 points, 4 points are located at each corner of the end section and 1 point is situated at the centroid of the element. The maximum stress at the middle section of each element is then computed based on the stresses at the two ends of the element and its axial stress. A positive stress indicates that the element is under compression, whereas a negative stress indicates that the element is under tension.
8.3. **WIND-INDUCED STRESS ANALYSIS**

Buffeting-induced stress has been computed using the structural health monitoring-based FEM established and compared with measured results by Xu et al. (2007b). The comparative results showed that the computed stress time histories are similar in both pattern and magnitude to the measured stresses. Xu (2008b) also carried out a buffeting-induced stress analysis to identify the most critical beam elements and the most critical corresponding stresses on the bridge deck under a mean wind speed of 15 m/s. Of the 15,904 beam elements, the cross section of the bridge deck at the Ma Wan tower was identified as the most critical section (CH23623 in Figure 8.1), in which the four elements numbered 34111, 38111, 40881, and 48611 were identified as the most critical. Elements 34111 and 38111 are the bottom chords of the outer north and south longitudinal trusses, respectively, on the main span side. Elements 40881 and 48611 are the bottom chords of the inner north and south longitudinal trusses, respectively, on the main span side. The analysis presented in this study is thus based on the aforementioned critical members.

**8.3.1. Relationship between Wind Speed and Wind-induced Mean Stress**

Using the wind force simulation parameters described in Chapter 7, the statistical relationship between wind speed and mean stress is explored and illustrated in Figure 8.2. The computed stresses shown in the figure correspond to mean winds of 5 to 50 m/s blowing perpendicular to the bridge axis from the south. As the figure shows, the stresses on the bottom chords of the south longitudinal trusses (elements 38111 and 48611) are in an increasing function with mean wind speed,
while those on the north longitudinal trusses (elements 34111 and 40881) are in a decreasing function when the wind is blowing from the south. A positive stress value indicates that the bottom chords of the south longitudinal trusses are experiencing compression, while a negative stress value indicates that the bottom chords of the north longitudinal trusses are experiencing tension. A comparison of their absolute magnitudes shows that the absolute mean stresses on the bottom chords of the south longitudinal trusses (elements 38111 and 48611) are slightly larger than those of the north longitudinal trusses (elements 34111 and 40881), and that the mean stress on the outer truss (elements 34111 and 38111) are larger than those on the inner truss (elements 40881 and 48611). When the deck height wind speed is 50 m/s, the mean stress for elements 34111, 38111, 40881, and 48611 are -75.71, 86.27, -43.69, and 52.83 MPa, respectively.

### 8.3.2. Relationship between Wind Speed and Wind-induced Dynamic Stress

In addition to the mean stress, this section also explores the statistical relationship between wind speed and the wind-induced dynamic stress responses of the critical elements. By considering the first 80 modes of vibration of the bridge, the dynamic stress response time history which involves the highest frequency of 1.1 Hz is computed using the mode superposition technique. Figure 8.3 depicts the dynamic stress response time history for element 38111 corresponding to the 10-minute wind time history (wind speed = 50 m/s) simulated by the parameters described in Chapter 7. The same procedures are applied to the other elements with wind speeds ranging from 5 to 50 m/s. Figure 8.4 illustrates the relationship between wind speed and the standard deviation of stress response for each critical
element. The figure graphically illustrates that the standard deviation of stress response increases with the wind speed. Similar to the mean stress, the standard deviations of stress responses for the bottom chords of the outer longitudinal trusses (elements 34111 and 38111) are larger than those for the inner longitudinal trusses (elements 40881 and 48611). In addition, the stress responses for the bottom chords of the south longitudinal trusses (elements 38111 and 48611) are slightly larger than those for the north longitudinal trusses (elements 34111 and 40881). When the deck height wind speed is 50 m/s, the standard deviations of stress responses for elements 34111, 38111, 40881, and 48611 are 14.45, 14.94, 10.11, and 10.29 MPa, respectively.

8.3.3. Relationship between Wind Speed and Wind-induced Total Stress

This section explores the statistical relationship between wind speed and the wind-induced total stress responses of the critical elements. Figure 8.5 illustrates the relationship between wind speed and total stress response for each critical element. As shown in the figure, when the wind is blowing from the south, the stresses on the bottom chords of the south longitudinal trusses (elements 38111 and 48611) are in an increasing function with mean wind speed, while those on the north longitudinal trusses (elements 34111 and 40881) are in a decreasing function. In common with the mean and dynamic stress responses, it can be seen that the absolute value of the total stresses on the bottom chords of the south longitudinal trusses (elements 38111 and 48611) are slightly larger than those of the north longitudinal trusses (elements 34111 and 40881), and that the total stresses on the outer truss (elements 34111 and 38111) are larger than those on
the inner truss (elements 40881 and 48611). When the deck height wind speed is 50 m/s, the mean stresses for elements 34111, 38111, 40881, and 48611 are -114.62, 133.24, -80.97, and 83.77 MPa, respectively.

8.4. HYBRID USE OF THE GPS AND FEM FOR DISPLACEMENT -STRESS ANALYSIS

Yoshida et al. (2003) proposed evaluating member stresses through the hybrid use of FEM analysis and the GPS. In their study, a 108 m-high steel tower was taken as a field measurement structure where the tip displacement obtained by the GPS was converted to member stresses based on FEM analysis:

\[
\sigma = \sigma_{X=1} \times X_{GPS} + \sigma_{Y=1} \times Y_{GPS}
\]

(8.4)

where \(\sigma\) is the member stress obtained through the hybrid use of FEM analysis and the GPS (for axial forces only), \(\sigma_{X=1}\) and \(\sigma_{Y=1}\) are the member stresses calculated by FEM analysis when tip displacement is 1 cm in the X and Y directions, respectively; and \(X_{GPS}\) and \(Y_{GPS}\) are the tip displacements in the X and Y directions, respectively, measured by the GPS. As such, all member stresses calculated in the FEM analysis can be converted from the tip displacement obtained by the GPS.

By inspection, Equation (8.4) implies the assumption that only the first linear mode of vibration is experienced in both the X and Y directions. Admittedly, this is a normal assumption made for some building structures. However, for the Tsing Ma suspension bridge, the natural frequencies are much more closely
spaced; the first 80 natural frequencies range from 0.068 to 1.1 Hz. As such, the form of equation will become more complicated if these 80 modes of vibration are considered. In practice, the relationship between stress and displacement can be solved statistically. In this connection, the stresses derived in the FEM and the displacements measured by the GPS in the field and computed by FEM analysis with natural frequencies of less than 1 Hz are used.

The element stress of the bridge is affected by motions in the lateral, vertical, and longitudinal directions and rotations around these three axes. The multiple regression formula should, therefore, take account of the interplay between stress and displacements. In the multiple regression proposed, displacements in the lateral, vertical, and longitudinal directions, and rotations around these three axes, are regarded as independent variables which together predict the stress response of the bridge. However, the common feature of all these independent variables in the regression formula is that they are each related to wind speed. The concept of multicollinearity in multiple regression analysis describes a condition in which two or more of the independent variables in a multiple regression are highly correlated with each other. This condition distorts the standard error of estimate and the coefficient standard errors, leading to problems when conducting $t$-tests to assess the statistical significance of the parameters (Bowerman & O’Connell 1990). The most common method used to correct for multicollinearity is to omit one or more of the correlated independent variables. In this connection, the statistical relationships derived in the following sections are based on a simple regression formula in which the lateral displacement at the mid-main span is the sole independent variable.
8.4.1. Relationship between Mean Stress and Mean Displacement

This section explores the statistical relationships between mean lateral displacement at the mid-main span and the wind-induced bridge mean stress responses of the critical elements. An assessment of various fitting functions (linear, power, and quadratic) shows that the quadratic function in Equation (8.5) is the best fit to the data, and it is thus adopted. This conclusion is reached via the FEM, in which a cubic function is used as a shape function of a beam element. Because the strain is the first order difference of the displacement, a square relationship between the strain and the displacement exists.

$$\bar{\sigma} = a\bar{D}^2 + b\bar{D}$$  \hspace{1cm} (8.5)

where $\bar{D}$ is the mean lateral displacement at the mid-main span for the SSE direction (i.e. perpendicular to the bridge axis from the south); $\bar{\sigma}$ denotes the mean stress of each critical element; and $a$ and $b$ are the two parameters to be determined. The two-tailed testing hypotheses concerning the values of parameters $a$ and $b$ in the regression model are set to determine the variables which are statistically significantly different from zero. Hence, the null hypotheses $H_0: a = 0$ and $H_0: b = 0$ can be rejected and the alternative hypotheses $H_a: a \neq 0$ and $H_a: b \neq 0$ accepted if the $p$-value is less than 0.05.

The relationships between the mean lateral displacement at the mid-main span and the mean stress responses of the four critical elements are explored by using the regression model given in Equation (8.5). The results of the regressions for each of the critical elements are displayed in Table 8.1 and graphically illustrated.
in Figure 8.6. It can be seen that the $p$-values for the coefficients $a$ and $b$ for all critical elements are approximately zero in these regressions. Therefore, the null hypotheses that the values of the parameters $a$ and $b$ are equal to zero can be rejected. It is noted from the table that the coefficients of determination $R^2$ in the quadratic functions are all 0.999 or above. Such high values indicate that the wind-induced mean stress responses of the critical members vary quadratically with the mean lateral displacement at the mid-main span.

### 8.4.2. Relationship between Dynamic Stress and Dynamic Displacement

This section explores the statistical relationships between the lateral displacement standard deviation at the mid-main span and the wind-induced dynamic stress responses of the critical members. The quadratic function expressed by Equation (8.6) is adopted to fit the data.

$$\sigma_\sigma = a\sigma_D^2 + b\sigma_D$$  \hspace{1cm} (8.6)

where $\sigma_D$ is the displacement standard deviation at the mid-main span for the SSE direction (i.e. perpendicular to the bridge axis from the south); $\sigma_\sigma$ denotes the standard deviation of the stress response; and $a$ and $b$ are the two parameters to be determined. The two-tailed testing hypotheses concerning the values of parameters $a$ and $b$ in the regression model are set to determine the variables which are statistically significantly different from zero. Hence, the null hypotheses $H_0: a = 0$ and $H_0: b = 0$ can be rejected and the alternative hypotheses $H_a: a \neq 0$ and $H_a: b \neq 0$ can be accepted if the $p$-value is less than
The relationships between the lateral displacement standard deviation at the mid-main span and the standard deviations of the stress responses of the four critical elements are explored by using the regression model given in Equation (8.6). The regression results for these relationships are displayed in Table 8.2 and graphically illustrated in Figure 8.7. The table shows that the $p$-values for the coefficients $a$ and $b$ for all the critical elements are approximately zero in these regressions. Therefore, the null hypotheses that the values of the parameters $a$ and $b$ equal zero can be rejected. It is noted from the table that the $R^2$ values are all equal to or greater than 0.998. This means that the wind-induced stress standard deviations vary quadratically with the displacement standard deviation of the bridge at the mid-main span.

### 8.4.3. Relationship between Total Stress and Total Displacement

This section investigates the statistical relationship between the total lateral displacement of the bridge at the mid-main span and the wind-induced total stress response of the critical members. The quadratic function expressed by Equation (8.7) is adopted to fit the data.

$$\tilde{\sigma} = a\tilde{D}^2 + b\tilde{D}$$  

(8.7)

where $\tilde{D}$ is the wind-induced total displacement of the bridge at the mid-main span; $\tilde{\sigma}$ denotes the total stress of a critical element; and $a$ and $b$ are the two parameters to be determined. The two-tailed testing hypotheses concerning the
values of parameters $a$ and $b$ in the regression model are set to determine the variables which are statistically significantly different from zero. Hence, the null hypotheses $H_0: a = 0$ and $H_0: b = 0$ can be rejected and the alternative hypotheses $H_a: a \neq 0$ and $H_a: b \neq 0$ accepted if the $p$-value is less than 0.05.

The relationships between the lateral total displacement at the mid-main span and the total stress responses of the four critical elements are explored by using the regression model given in Equation (8.7). The regression results for these relationships are displayed in Table 8.3 and graphically illustrated in Figure 8.8. The table shows that the $p$-values for the coefficient $a$ for all the critical elements are approximately zero in these regressions. The null hypotheses that values of the parameters $a$ and $b$ equal zero can thus be rejected. In addition, Table 8.3 reports that the $R^2$ values are all 1.000. This indicates that the wind-induced total stresses are best fitted quadratically with the total lateral displacement.

8.5. STRENGTH ASSESSMENT OF THE TSING MA BRIDGE

This section compares the yield strength of the bridge material with the maximum total stress to assess the strength of the bridge exposed to high winds. On the Tsing Ma Bridge, EN10025 Grade Fe510C structural steel was used to build the longitudinal truss of the bridge deck. The yield strength of this type of material is 355 MPa. As shown in Figure 8.5, at a deck-height wind speed of 50 m/s, the maximum total stress is found to be 133.24 MPa for element 38111 (the bottom chord of the outer south longitudinal truss). This total stress at the wind speed under consideration falls within the maximum yield strength of the
material. This indicates that the strength of the Tsing Ma Bridge exposed to high wind speed speeds is guaranteed.

8.6. SUMMARY

This chapter has assessed the stress characteristics of the four critical elements at the cross section of the bridge deck at the Ma Wan tower. These elements are the bottom chords of the outer north, outer south, inner north, and inner south longitudinal trusses on the main span side. The major results are summarized as follows:

- The relationships between wind speed and the stress responses of the four critical elements at the cross section of the bridge deck at the Ma Wan tower were explored in the SSE direction through FEM analysis. Through a comparison of the absolute magnitudes of the means, standard deviations and total stresses of the four critical elements, it was seen that the stress responses on the bottom chords of the outer longitudinal trusses are larger than those of the inner longitudinal trusses. In addition, the stress responses on the bottom chords of the south longitudinal trusses are slightly larger than those on the north longitudinal trusses.

- The statistical relationships between lateral displacement at the mid-main span of the bridge and the stress responses of the four critical elements at the cross section of the bridge deck at the Ma Wan tower have also been explored. It was found that the mean stresses, stress standard deviations and total stress...
responses varied quadratically with mean displacement, displacement standard deviation, and total displacement, respectively.

- The total stress responses of the four critical elements were compared to the yield strength of the bridge material (355 MPa). It was found that the total stress response at a wind speed of 50 m/s fell within the design criterion, thereby guaranteeing the strength of the Tsing Ma Bridge exposed to high wind speeds.
### Table 8.1. Regression factors for relationships between mean lateral displacement and the mean stresses of different critical elements

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>VARIABLE</th>
<th>PARAMETER</th>
<th>( p )-VALUE</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>34111</td>
<td>( a )</td>
<td>-6.30e-6</td>
<td>0.004</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>( b )</td>
<td>-0.040</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>38111</td>
<td>( a )</td>
<td>2.95e-6</td>
<td>0.004</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>( b )</td>
<td>0.052</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>40881</td>
<td>( a )</td>
<td>-2.26e-6</td>
<td>0.003</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>( b )</td>
<td>-0.025</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>48611</td>
<td>( a )</td>
<td>-5.38e-7</td>
<td>0.011</td>
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</tr>
<tr>
<td></td>
<td>( b )</td>
<td>0.036</td>
<td>0.000</td>
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</tbody>
</table>

### Table 8.2. Regression factors for relationships between lateral displacement standard deviation and the stress standard deviations of different critical elements

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>VARIABLE</th>
<th>PARAMETER</th>
<th>( p )-VALUE</th>
<th>( R^2 )</th>
</tr>
</thead>
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<td>-4.91e-5</td>
<td>0.008</td>
<td>0.999</td>
</tr>
<tr>
<td></td>
<td>( b )</td>
<td>0.065</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>38111</td>
<td>( a )</td>
<td>-4.68e-5</td>
<td>0.008</td>
<td>0.999</td>
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<td></td>
<td>( b )</td>
<td>0.066</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>40881</td>
<td>( a )</td>
<td>-5.16e-5</td>
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<td>0.998</td>
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<tr>
<td></td>
<td>( b )</td>
<td>0.050</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>48611</td>
<td>( a )</td>
<td>-5.37e-5</td>
<td>0.004</td>
<td>0.998</td>
</tr>
<tr>
<td></td>
<td>( b )</td>
<td>0.052</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>
Table 8.3. Regression factors for relationships between lateral total displacement and the total stress responses of different critical elements

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>VARIABLE</th>
<th>PARAMETER</th>
<th>p - VALUE</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ESTIMATE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>34111</td>
<td>a</td>
<td>1.62e-6</td>
<td>0.014</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>-0.053</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>38111</td>
<td>a</td>
<td>-2.82e-6</td>
<td>0.000</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>0.064</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>40881</td>
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<td>0.026</td>
<td>1.00</td>
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<td></td>
<td>b</td>
<td>-0.037</td>
<td>0.000</td>
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</tr>
<tr>
<td>48611</td>
<td>a</td>
<td>-2.52e-6</td>
<td>0.000</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>0.042</td>
<td>0.000</td>
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</table>
Figure 8.1. The critical deck section and the critical elements identified

Figure 8.2. Relationships between wind and the mean stress responses of the critical elements
Figure 8.3. Stress response time history for critical element 38111 corresponding to a 50 m/s mean wind.

Figure 8.4. Relationships between wind and the stress standard deviation responses of the critical elements.
Figure 8.5. Relationships between wind and the peak stresses of the critical elements

Figure 8.6. Statistical relationships between mean lateral displacement and the mean stresses of the critical elements
**Chapter 8**  
**Structural Health Monitoring Based-Performance Assessment: Stress**

![Figure 8.7](image1.png)

**Figure 8.7.** Statistical relationships between lateral displacement standard deviation and the stress standard deviations of the critical elements

![Figure 8.8](image2.png)

**Figure 8.8.** Statistical relationships between total lateral displacement and the total stresses of the critical elements
Chapter 9

CONCLUSIONS AND RECOMMENDATIONS

9.1. CONCLUSIONS

This dissertation has analyzed the application and integration of global positioning systems (GPS) with structural health monitoring systems (SHMS) and computer simulation to continuously monitor the serviceability and strength of long-span cable-supported bridges under strong winds. The dissertation first assessed the performance of the GPS in the SHMS. Thus, a series of calibration tests were carried out using a two-dimensional motion simulation table to assess the dynamic measurement accuracy of the GPS in three orthogonal directions. Data processing algorithms have also been developed to enhance the measurement accuracy of the SHMS for total (static and dynamic) displacement response through the integration of GPS-measured data with accelerometer-measured data based on empirical mode decomposition (EMD) and an adaptive filtering technique. The dissertation then evaluated the performance of Hong Kong’s Tsing Ma Bridge under wind excitation using long-term field measurement data measured by anemometers and the GPS. In this connection, wind structures and structural responses measured during typhoons and strong monsoons by the Wind And Structural Health Monitoring System (WASHMS) installed on the Tsing Ma Bridge have been investigated in detail to explore the relationships between wind speed and wind-induced
displacement response according to wind direction and the locations of GPS stations on the bridge deck. The dissertation then comprehensively investigated bridge responses using a finite element modeling approach. The finite element model (FEM) used for the Tsing Ma Bridge, which features significant modeling geometric details of the as-built complicated bridge deck, has been used to predict the aerostatic and buffeting responses of the bridge under extreme wind speeds. The largest bridge movements tolerated under the serviceability limit state have been compared with regression models used to estimate the functionality of the bridge. This dissertation has also estimated the strength of the Tsing Ma Bridge in terms of stress. To enable direct computation of stresses in the critical bridge components, the statistical relationship between stress and displacement has been explored through the hybrid use of FEM analysis and the GPS. The wind-induced stresses derived at extreme wind speeds have been compared with the yield stress of the material to assess the strength of the bridge.

The main contributions of this thesis and the conclusions reached are summarized as follows:

1. **Assessment of Dynamic Measurement Accuracy of the GPS**

An advanced two-dimensional motion simulation table has been designed and manufactured to assess the measurement performance of various types of GPS receivers and used as a test station to validate an integrated GPS-accelerometer data processing technique. The table was designed to generate sinusoidal waves, white noise random waves, circular waves, and any other wave defined by input wave time histories around a pre-defined static position in two perpendicular
directions with an upper frequency of up to 2 Hz. Through a sophisticated control system, the targeted input motion was guaranteed to be accurately reproduced.

A detailed calibration study carried out in an open area in three phases used the motion simulation table to assess the dynamic displacement measurement accuracy of the GPS in the longitudinal, lateral, and vertical directions. In the first phase, the background noise in GPS measurements was measured for nine hours to ascertain the basic characteristics of background noise at the site. The test results showed that the background noise was dominated mainly by low frequency components and that the background noise in the vertical direction was greater than that in the horizontal direction. In addition, a comparison of the dynamically and statistically measured background noises showed that they follow similar patterns. Based on the understanding of background noise, a bandpass filtering scheme was designed to reduce background noise in dynamic displacement measurement data from the GPS. In the second phase of calibration work, the sensitivities of the GPS were measured based on the amplitude and frequency of one-dimensional sinusoidal motions and two-dimensional circular motions. The amplitudes of the simulated motions were taken as 2 mm, 5 mm, 10 mm, 20 mm, and 40 mm, while the frequencies selected were 0.025 Hz, 0.1 Hz, 0.5 Hz, 1 Hz, and 1.8 Hz, respectively. The test results showed that the GPS could measure dynamic displacements accurately if the motion amplitude was not less than 5 mm in the horizontal plane or 10 mm in the vertical direction, provided that the motion frequency was less than or equal to 1 Hz. The third phase of calibration work examined the ability of the GPS to track complex two-dimensional horizontal motions measured at the top of the Di Wang
Building during Typhoon York and one-dimensional vertical motion of the middle section of the Tsing Ma suspension bridge deck measured during Typhoon Victor. The test results demonstrated that the GPS could trace wind-induced dynamic displacement responses measured from real complex structures in the horizontal and vertical planes, with satisfactory results.

2. **Integrating GPS and Accelerometer Data to Measure Total Displacement**

This dissertation has explored the possibility of integrating GPS-measured signals with accelerometer-measured signals to enhance the accuracy of total (static plus dynamic) displacement response measurements for civil engineering structures across a wider range of frequencies. Two frameworks for integrated data processing techniques using both empirical mode decomposition (EMD) and an adaptive filter were presented. The EMD method used in this study involves decomposing a GPS-measured structural displacement response time history into a number of IMF components and a final residue. This final residue presented in a monotonic function is defined as the mean displacement. The EMD method involving an intermittency check and a cutoff frequency was also used to process the acceleration time history measured by an accelerometer and obtain the high-frequency dynamic responses. The adaptive filter used in this study involved not only mitigating the multipath effect from the GPS-measured data, but also extracting low-frequency dynamic displacement responses.

A series of motion simulation table tests were performed at a site to assess the effectiveness of two integrated data processing techniques proposed with intermittency frequencies of 0.2 Hz and 1.0 Hz. A white noise random wave and
Chapter 9  

Conclusions and Recommendations

the wind-induced dynamic displacement response of a real complex structure were simulated by the motion simulation table in the pre-defined static position. The motion simulation table test results showed that the GPS is susceptible to the multipath effect and is incapable of measuring high-frequency dynamic displacement responses, while the accelerometer is incapable of measuring mean displacement and low-frequency dynamic displacement responses. However, after adopting the two integrated data processing algorithms, the accuracy of the total displacement measurements improved significantly. The data processing performance of the two frameworks were similar to each other. In addition, the data processing performance of both frameworks with a higher intermittency frequency was similar to their performance with a lower intermittency frequency.

A comparison of the integrated results for the two motions generated by the motion simulation table showed that data processing performance for the wind-induced dynamic response of a real complex structure was better than for a white noise random wave. This indicated that a more satisfactory result would be achieved for motions of a larger magnitude.

3. **Strong Wind Studies on the Tsing Ma Bridge**

Two types of relatively strong wind events for the period from July 1997 to September 2005 have been considered: typhoons during which signal No. 3 or above was hoisted, and events in which the Hong Kong Observatory (HKO) issued a strong monsoon signal. As a result, a total of 504 and 1,665 hourly wind records measured by the ultrasonic anemometers in the middle of the main span for typhoon and monsoon events, respectively, were targeted for analysis. To produce high quality databases, several algorithms have been developed using
MATLAB as a platform to pre-process the original wind data. The main steps taken in the pre-processing of original wind data included: (1) eliminating unreasonable data of abnormal magnitude from the propeller anemometer on the Ma Wan tower; (2) determining which of the anemometers on the northern and southern sides of the deck should be considered for each typhoon or monsoon event; (3) eliminate unreasonable data of abnormal magnitude from the ultrasonic anemometers in the middle of the main span; and (4) obtaining 10-minute and hourly mean wind speeds and wind turbulence components based on original wind data recorded by the ultrasonic anemometers. To gain an understanding of the characteristics of seasonal monsoons experienced at the Tsing Ma Bridge, the monsoon data were split up by season for analysis. The four distinct seasons were winter (from November to February), spring (from March to April), summer (from May to August), and autumn (from September to October).

The results showed that both the mean and turbulent wind characteristics varied considerably due changes of wind direction and the nature of the wind type. The mean wind on the northern part of the bridge was dominated by typhoons, while monsoons dominated the eastern part of the bridge. With regard to the mean wind speed, most of the typhoons were within the wind management system stage 1 signal range (10-18 m/s); whereas the monsoons were within the lower mean wind speed for stage 1 of the high wind management system (10 m/s). In terms of mean wind incidence, the results showed that the hourly and 10-minute mean wind incidences for typhoon and monsoon events were within ±10° of the 95% upper and lower limits of wind incidences under a mean wind speed of 20 m/s. In
addition, the evidence showed that the turbulence intensities recorded for monsoons were smaller than those measured for typhoons. Because turbulence intensity is an increasing function of friction velocity at the site, the turbulence intensities measured for the overland fetch were higher than those measured for the open sea fetch. In addition, the results showed that the integral length scales of the alongwind turbulence component and the crosswind turbulence component measured from the open sea fetch were larger than those measured from the overland fetch. This is because the integral length scale of the alongwind turbulence component was inversely related to the terrain roughness of the site. However, this phenomenon did not occur for the integral length scale of the vertical turbulence component. A comparison of integral length scales measured during typhoon and monsoon conditions showed that the length scales derived from the monsoon samples were larger than those estimated from the typhoon data.

4. **Statistical Analysis of the Relationship between Displacement and Wind Speed**

The statistical relationship between wind speed and the mean and standard deviation of wind-induced displacement and total wind-induced displacement has been evaluated using the wind measurement data and displacement measurement data recorded in the 2002-2005 period at the Tsing Ma Bridge. To produce high quality databases, several algorithms have been developed to pre-process the original GPS data. The main steps taken in the pre-processing of original GPS data were: (1) convert the GPS data from the HK80 geographic coordinate to the Universal Transverse Mercator (UTM) grid coordinate; (2) compute the bridge
displacement coordinates with respect to the reference coordinates measured on November 28, 2000; (3) eliminate unreasonable data with abnormal magnitude caused by an abrupt change in the number of satellites or unsatisfactory geometry configurations; (4) obtain bridge displacement responses in three orthogonal directions; (5) filter displacement response time histories using a low-pass filter with an upper frequency limit of 1 Hz to maintain high quality data for subsequent analysis; and (6) decompose the mean and dynamic displacements based on the total response measurements recorded by GPS. It is known that the static and dynamic components of wind speed cause the mean and dynamic displacements of the bridge, respectively. However, at the same time, temperature variations cause mean displacements of the bridge, while moving vehicles and trains induce dynamic displacements. The effects of GPS background noise and temperature on mean displacement measurements and the effects of GPS background noise and traffic on dynamic displacement measurements were assessed. The wind-induced displacement response was then ascertained according to total response measurements by eliminating the effects arising from environmental and operational loads. Bridge displacement responses to strong winds, including typhoons and monsoons, were statistically analyzed as a function of wind direction at different locations on the bridge.

The results showed that in most cases, the relationships between wind speed and the mean and standard deviation of wind-induced displacement and total wind-induced displacement were almost quadratic in the lateral and vertical directions of the bridge deck and the main cable. The displacement responses of the bridge deck in the lateral direction were almost symmetric with respect to the
middle of the main span, but the displacement response of the bridge deck in the vertical direction did not conform to such a trend. The displacement response of the main cable was similar to that measured at the middle of the main span. At the middle of the main span, the displacement response was an increasing function of the terrain roughness of the site.

5. **Structural Health Monitoring-based Performance Assessment: Displacement**

Finite element method was employed to help with the analysis of extreme wind speeds along the Tsing Ma Bridge as a whole. The analysis involved using the complex structural health monitoring-based three-dimensional FEM, which closely replicates the geometric details of the as-built complicated bridge deck. By distributing wind forces over the cross-section of the bridge deck (including non-linear steady-state wind forces, buffeting forces, and self-excited forces), the displacement responses of the bridge deck in the time domain were predicted. The validity of the FEM was confirmed by comparing the displacement responses calculated with those measured in the field.

The statistical relationships between wind speed and the mean and standard deviation of wind-induced displacement and total wind-induced displacement in the SSE direction sector at mean wind speeds of up to 15 m/s were extended to extreme wind speeds and other sections of the bridge deck through FEM analysis. The results showed that at a deck-height wind speed of 50 m/s, the lateral and mean vertical displacement responses at the mid-main span were 1,524 mm and 204 mm, respectively. At the same wind speed, the lateral and vertical dynamic
standard deviations at the mid-main span were 279 mm and 206 mm, respectively, while the total displacement responses at the mid-main span were 2,305 and 860 mm in the lateral and vertical directions, respectively. The results computed for the bridge deck have been compared with the wind tunnel test results, showing absolute relative discrepancies of less than 28% between the two set of results. The relationship between wind and the total lateral displacement response of the bridge deck at the mid-main span was also compared to the serviceability limit state (2.9 m). It was found that the total lateral displacement at a wind speed of 50 m/s fell within the design criterion, thereby guaranteeing the serviceability of the Tsing Ma Bridge and its components.

6. **Structural Health Monitoring-based Performance Assessment: Stress**

The relationships between wind speed and the stress responses of the four critical elements of the bridge deck at the Ma Wan tower were explored in the SSE direction through FEM analysis. These elements were the bottom chords of the outer north, outer south, inner north, and inner south longitudinal trusses on the main span side. Through a comparison of the absolute magnitudes of the means, standard deviations, and total stresses of the four critical elements, it was seen that the stress responses on the bottom chords of the outer longitudinal trusses were larger than those on the inner longitudinal trusses. In addition, the stress responses on the bottom chords of the south longitudinal trusses were slightly larger than those on the north longitudinal trusses.

The statistical relationships between lateral displacement at the mid-main span of the bridge and the stress responses of the four critical elements at the cross...
section of the bridge deck at the Ma Wan tower have also been explored. It was found that the mean stresses, stress standard deviations, and total stress responses varied quadratically with mean displacement, displacement standard deviation, and total displacement, respectively. The total stress responses of the four critical elements were compared to the yield strength of the bridge material (355 MPa). It was found that the total stress responses at a wind speed of 50 m/s fell within the design criterion, thereby guaranteeing the strength of the Tsing Ma Bridge.

9.2. RECOMMENDATIONS

Although this dissertation has made some progress in developing field monitoring and numerical analysis techniques for long-span suspension bridges under wind excitation, to enhance our understanding, there remain some important issues deserving of further attention in future studies.

1. To enhance the accuracy of total displacement response measurements, integrated data-processing techniques based on measurement data collected by GPS receivers and accelerometers using empirical mode decomposition (EMD) and an adaptive filter were developed in Chapter 4. However, given that most long-span cable-supported bridges are very slender, the majority of wind-induced displacement responses with a frequency of less than or equal to 1 Hz have already been captured by GPS. It is therefore desirable that the effectiveness of the integrated data processing techniques proposed in this dissertation be assessed for real complex tall building structures with natural frequencies much higher than those of bridges.
Chapter 7 used the FEM to extend the statistical relationship developed in Chapter 6 on the basis of wind speed measurement data to extreme wind speeds. However, because the long-term WASHMS of the Tsing Ma Bridge will produce a large amount of wind and displacement data in the future, it is desirable that an iterative approach be adopted to establish the final statistical relationship through an updating process.

3. The statistical relationships between wind speed and the mean and standard deviation of wind-induced displacement and total wind-induced displacement were extended to extreme wind speeds and other sections of the bridge deck through FEM analysis. However, the computed results correspond to mean winds blowing perpendicular to the bridge axis from the south. A useful exercise for the future would be to integrate the computer simulation technique described in this dissertation with GPS measurements to predict the wind-induced displacement response of a long-span cable-supported bridge as a whole under skew winds.

4. It is understood that the strength and integrity of the Tsing Ma Bridge will decrease over its service life due to the effects of wind. However, other factors such as traffic, temperature, stress corrosion, and environmental deterioration also affect the integrity, durability, and reliability of the bridge. It is therefore desirable that statistical models of measured data including wind, temperature, and highway and railway traffic be combined in the future to estimate the strength and integrity of the Tsing Ma Bridge from an overall perspective.
Appendix A

WIND PRESSURE DISTRIBUTIONS OVER THREE TYPICAL DECK SECTIONS
A1. 12-NODE SECTION

Figure A1. Wind pressure distribution over 12-node section

Table A1. Forces in horizontal direction $F_y$ at 12-node section

<table>
<thead>
<tr>
<th>ITEM</th>
<th>DIAGRAM</th>
<th>EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_y (p_1)$</td>
<td>0.92$p_1\left(l_{z1} + l_{z5}\right)$</td>
<td></td>
</tr>
<tr>
<td>$F_y (p_2)$</td>
<td>$p_2l_{z1}$</td>
<td></td>
</tr>
<tr>
<td>$F_y (p_3)$</td>
<td>$\frac{1}{2}p_3l_{z5}$</td>
<td></td>
</tr>
</tbody>
</table>
### Table A2. Forces in vertical direction $F_z$ at 12-node section

<table>
<thead>
<tr>
<th>ITEM</th>
<th>DIAGRAM</th>
<th>EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_z (p_1)$</td>
<td><img src="image1.png" alt="Diagram 1" /></td>
<td>$p_1 \left[ 3l_{y2} - l_{y3} + 0.92(l_{y1} - l_{y5}) \right]$</td>
</tr>
<tr>
<td>$F_z (p_2)$</td>
<td><img src="image2.png" alt="Diagram 2" /></td>
<td>$p_2 l_{y1}$</td>
</tr>
<tr>
<td>$F_z (p_3)$</td>
<td><img src="image3.png" alt="Diagram 3" /></td>
<td>$\frac{1}{2} p_3 l_{y5}$</td>
</tr>
</tbody>
</table>
### Table A3. Moment in y-z direction $M$ at 12-node section

<table>
<thead>
<tr>
<th>ITEM</th>
<th>DIAGRAM</th>
<th>EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1(p_1)$</td>
<td><img src="#" alt="Diagram 1" /></td>
<td>$0.46p_1\left(-l_{z1}^2 + l_{z5}^2\right)$</td>
</tr>
<tr>
<td>$t_2(p_1)$</td>
<td><img src="#" alt="Diagram 2" /></td>
<td>$0.46p_1l_{y1}\left(l_{y1} + 2l_{y2} + 2l_{y3} + l_{y4}\right)$</td>
</tr>
<tr>
<td>$t_3(p_1)$</td>
<td><img src="#" alt="Diagram 3" /></td>
<td>$-0.46p_1l_{y5}\left(l_{y1} + l_{y2} + 2l_{y3} + l_{y4}\right)$</td>
</tr>
<tr>
<td>$t_4(p_1)$</td>
<td><img src="#" alt="Diagram 4" /></td>
<td>$0.33p_1l_{y3}\left(2l_{y3} + \frac{3}{2}l_{y4}\right)$</td>
</tr>
</tbody>
</table>
Appendix A

Wind Pressure Distributions over Three Typical Deck Sections

\[ t_5(p_1) \]
\[ M(p_1) \]
\[ M(p_2) \]
\[ M(p_3) \]

\[ -0.33p_1 l_{y2} \left( 3l_{y3} + \frac{3}{2} l_{y4} + 2l_{y2} \right) \]

\[ t_1(p_1) + t_2(p_1) + t_3(p_1) + t_4(p_1) + t_5(p_1) \]

\[ -p_1 l_{y1} \left( 0.3 l_{y1} + 0.5 l_{y4} + l_{y3} + l_{y2} \right) + 0.67p_2 l_{z2}^2 \]

\[ -0.17p_3 l_{z3} \left( 4l_{y1} + 4l_{y2} + 3l_{y3} \right) + 1.5l_{y4} - 0.17p_5 l_{z5}^2 \]
A2. 10-NODE SECTION

![Wind pressure distribution over 10-node section](image)

**Figure A2.** Wind pressure distribution over 10-node section

**Table A4.** Forces in horizontal direction $F_y$ at 10-node section

<table>
<thead>
<tr>
<th>ITEM</th>
<th>DIAGRAM</th>
<th>EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_y (p_1)$</td>
<td><img src="image" alt="Diagram of $F_y (p_1)$" /></td>
<td>$0$</td>
</tr>
<tr>
<td>$F_y (p_2)$</td>
<td><img src="image" alt="Diagram of $F_y (p_2)$" /></td>
<td>$p_2 l_{z1}$</td>
</tr>
<tr>
<td>$F_y (p_3)$</td>
<td><img src="image" alt="Diagram of $F_y (p_3)$" /></td>
<td>$\frac{1}{2} p_3 l_{z1}$</td>
</tr>
</tbody>
</table>
Table A5. Forces in vertical direction $F_z$ at 10-node section

<table>
<thead>
<tr>
<th>ITEM</th>
<th>DIAGRAM</th>
<th>EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_z (p_1)$</td>
<td><img src="image1" alt="Diagram 1" /></td>
<td>$p_1 \left[ 4l_{y2} + l_{y3} \right]$</td>
</tr>
<tr>
<td>$F_z (p_2)$</td>
<td><img src="image2" alt="Diagram 2" /></td>
<td>$-p_2 l_{y1}$</td>
</tr>
<tr>
<td>$F_z (p_3)$</td>
<td><img src="image3" alt="Diagram 3" /></td>
<td>$\frac{1}{2} p_3 l_{y1}$</td>
</tr>
</tbody>
</table>
Table A6. Moment in y-z direction $M$ at 10-node section

<table>
<thead>
<tr>
<th>ITEM</th>
<th>DIAGRAM</th>
<th>EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M(p_1)$</td>
<td><img src="image1.png" alt="Diagram 1" /></td>
<td>$-p_1 l_{y2} \left( \frac{4}{3} (l_{y2} + l_{y3}) + l_{y4} \right) + p_1 l_{y2} \left( \frac{2}{3} l_{y3} + \frac{1}{2} l_{y4} \right)$</td>
</tr>
<tr>
<td>$M(p_2)$</td>
<td><img src="image2.png" alt="Diagram 2" /></td>
<td>$-p_2 l_{y1} \left( 0.3 l_{y1} + 0.5 l_{y4} + l_{y3} + l_{y2} \right) + 0.67 p_2 l_{z1}^2$</td>
</tr>
<tr>
<td>$M(p_3)$</td>
<td><img src="image3.png" alt="Diagram 3" /></td>
<td>$-0.17 p_3 l_{y5} \left( 4 l_{y1} + 4 l_{y2} + 3 l_{y3} \right) + 1.5 l_{y4}$ $-0.17 p_3 l_{y5}$</td>
</tr>
</tbody>
</table>
A3. 8-NODE SECTION

Figure A3. Wind pressure distribution over 8-node section

Table A7. Forces in horizontal direction $F_y$ at 8-node section

<table>
<thead>
<tr>
<th>ITEM</th>
<th>DIAGRAM</th>
<th>EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_y(p_1)$</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$F_y(p_2)$</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$F_y(p_3)$</td>
<td>$p_3$</td>
<td>$2p_3z_1$</td>
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</tbody>
</table>
### Table A8. Forces in vertical direction \( F_z \) at 8-node section

<table>
<thead>
<tr>
<th>ITEM</th>
<th>DIAGRAM</th>
<th>EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_z (p_1) )</td>
<td><img src="image1.png" alt="Diagram 1" /></td>
<td>( p_1 \left{ \frac{5}{2} l_{y2} + l_{y3} \right} )</td>
</tr>
<tr>
<td>( F_z (p_2) )</td>
<td><img src="image2.png" alt="Diagram 2" /></td>
<td>( -p_2 \left{ \frac{9}{4} l_{y2} + l_{y3} \right} )</td>
</tr>
<tr>
<td>( F_z (p_3) )</td>
<td><img src="image3.png" alt="Diagram 3" /></td>
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</table>

### Table A9. Moment in y-z direction \( M \) at 8-node section

<table>
<thead>
<tr>
<th>ITEM</th>
<th>DIAGRAM</th>
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</tr>
</thead>
<tbody>
<tr>
<td>( M(p_1) )</td>
<td><img src="image4.png" alt="Diagram 4" /></td>
<td>( -p_1 l_{y2} \left{ \frac{4}{3} l_{y2} + l_{y3} + l_{y4} \right} + p_1 l_{y2} \left{ \frac{2}{3} l_{y3} + \frac{1}{2} l_{y4} \right} )</td>
</tr>
<tr>
<td>( M(p_2) )</td>
<td><img src="image5.png" alt="Diagram 5" /></td>
<td>( -p_2 l_{y1} \left{ 0.3 l_{y1} + 0.5 l_{y4} + l_{y3} + l_{y2} \right} + 0.67 p_2 l_{z1}^2 )</td>
</tr>
<tr>
<td>( M(p_3) )</td>
<td><img src="image6.png" alt="Diagram 6" /></td>
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</table>
Appendix B

PERFORMANCE ASSESSMENT OF MOTION SIMULATION TABLE
Table B1. Statistical measures for 1-D sinusoidal motion in horizontal direction. Movable platform loaded with 5 kg mass.

<table>
<thead>
<tr>
<th>AMPLITUDE (mm)</th>
<th>ITEMS</th>
<th>FREQUENCY (Hz)</th>
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<th></th>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.025</td>
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<td>1</td>
<td>1.8</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A_{\min}$</td>
<td>$A_{\max}$</td>
<td>Std. Dev.</td>
<td>$A_{\min}$</td>
<td>$A_{\max}$</td>
<td>Std. Dev.</td>
<td>$A_{\min}$</td>
<td>$A_{\max}$</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>2</td>
<td>Table (mm)</td>
<td>-1.94</td>
<td>1.94</td>
<td>1.35</td>
<td>-1.94</td>
<td>1.94</td>
<td>1.35</td>
<td>-1.94</td>
<td>1.94</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td>Laser (mm)</td>
<td>-1.96</td>
<td>1.95</td>
<td>1.36</td>
<td>-1.96</td>
<td>1.97</td>
<td>1.36</td>
<td>-1.94</td>
<td>1.95</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td>Error (%)</td>
<td>-0.95</td>
<td>-0.57</td>
<td>-0.78</td>
<td>-0.65</td>
<td>-1.66</td>
<td>-0.95</td>
<td>0.23</td>
<td>-0.43</td>
<td>-0.23</td>
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<tr>
<td>5</td>
<td>Table (mm)</td>
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<td>4.87</td>
<td>3.46</td>
<td>-4.93</td>
<td>4.93</td>
<td>3.46</td>
<td>-4.93</td>
<td>4.92</td>
<td>3.46</td>
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<td>Laser (mm)</td>
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<td>3.45</td>
</tr>
<tr>
<td></td>
<td>Error (%)</td>
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<td>-0.20</td>
<td>-0.33</td>
<td>-0.35</td>
<td>-0.06</td>
<td>-0.17</td>
<td>-0.22</td>
<td>-0.04</td>
<td>-0.26</td>
</tr>
<tr>
<td>10</td>
<td>Table (mm)</td>
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<td>9.93</td>
<td>7.00</td>
<td>-9.93</td>
<td>9.93</td>
<td>7.00</td>
<td>-9.91</td>
<td>9.91</td>
<td>6.98</td>
</tr>
<tr>
<td></td>
<td>Error (%)</td>
<td>-0.36</td>
<td>0.06</td>
<td>-0.14</td>
<td>-0.49</td>
<td>0.23</td>
<td>-0.08</td>
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Table B2. Statistical measures for 2-D circular motion in horizontal direction. Movable platform loaded with 5 kg mass.

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FREQUENCY (Hz)
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Error (%) calculated using Equation (2).
Table B3. Statistical measures for 1-D sinusoidal motion in vertical direction. Movable platform loaded with 5 kg mass.

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Notes:

\[ A_{\text{min}} = \frac{1}{n} \sum_{i=1}^{n} (\text{min. peak disp})_i \quad A_{\text{max}} = \frac{1}{n} \sum_{i=1}^{n} (\text{max. peak disp})_i \quad \text{Error} (%) = \left( \frac{x_{\text{Table}} - x_{\text{Laser}}}{x_{\text{Laser}}} \right) \times 100 \]
Appendix C

VARIATION OF TURBULENCE INTENSITIES WITH MEAN WIND SPEED
Appendix C  
Variation of Turbulence Intensities with Mean Wind Speed

(a) N  
Sector: N  
No. of data = 175  
Mean = 0.183  
Std = 0.038

(b) NNE  
Sector: NNE  
No. of data = 2  
Mean = 0.232  
Std = 0.034

(c) NE  
Sector: NE  
No. of data = 9  
Mean = 0.386  
Std = 0.064

(d) ENE  
Sector: ENE  
No. of data = 114  
Mean = 0.296  
Std = 0.074

(e) E  
Sector: E  
No. of data = 128  
Mean = 0.246  
Std = 0.089

(f) ESE  
Sector: ESE  
No. of data = 47  
Mean = 0.204  
Std = 0.033
Appendix C  Variation of Turbulence Intensities with Mean Wind Speed

(g) SE

Sector: SE
No. of data = 18
Mean = 0.102
Std = 0.02

(h) SSE

Sector: SSE
No. of data = 66
Mean = 0.108
Std = 0.029

(i) S

Sector: S
No. of data = 56
Mean = 0.086
Std = 0.016

(j) SSW

Sector: SSW
No. of data = 5
Mean = 0.147
Std = 0.046

(k) SW

Sector: SW
No. of data = 4
Mean = 0.214
Std = 0.035

(l) WSW

Sector: WSW
No. of data = 3
Mean = 0.276
Std = 0.036
Figure C1. Variation of longitudinal turbulence intensity ($I_u$) with mean wind speed (typhoon, 10-minute duration, deck level)
Appendix C

Variation of Turbulence Intensities with Mean Wind Speed

(a) N

(b) NNE

(c) NE

(d) ENE

(e) E

(f) ESE
Appendix C

Variation of Turbulence Intensities with Mean Wind Speed

(g) SE

(h) SSE

(i) S

(j) SSW

(k) SW

(l) WSW
Figure C2. Variation of lateral turbulence intensity ($I_v$) with mean wind speed (typhoon, 10-minute duration, deck level)
Appendix C  
Variation of Turbulence Intensities with Mean Wind Speed

(a) N

(b) NNE

(c) NE

(d) ENE

(e) E

(f) ESE
Appendix C

Variation of Turbulence Intensities with Mean Wind Speed

(g) SE

(h) SSE

(i) S

(j) SSW

(k) SW

(l) WSW

No. of data = 18
Mean = 0.064
Std = 0.019

No. of data = 66
Mean = 0.067
Std = 0.018

No. of data = 56
Mean = 0.053
Std = 0.008

No. of data = 5
Mean = 0.072
Std = 0.012

No. of data = 4
Mean = 0.124
Std = 0.017

No. of data = 3
Mean = 0.161
Std = 0.008
Figure C3. Variation of vertical turbulence intensity ($I_w$) with mean wind speed (typhoon, 10-minute duration, deck level)
Appendix C

Variation of Turbulence Intensities with Mean Wind Speed

(a) N

Sector: N
No. of data = 240
Mean = 0.19
Std = 0.055

(b) NNE

Sector: NNE
No. of data = 85
Mean = 0.207
Std = 0.06

(c) NE

Sector: NE
No. of data = 117
Mean = 0.287
Std = 0.084

(d) ENE

Sector: ENE
No. of data = 121
Mean = 0.275
Std = 0.067

(e) E

Sector: E
No. of data = 122
Mean = 0.248
Std = 0.055

(f) ESE

Sector: ESE
No. of data = 13
Mean = 0.216
Std = 0.044
Appendix C

Variation of Turbulence Intensities with Mean Wind Speed

(g) SE

(h) SSE

(i) S

(j) SSW

(k) SW

(l) WSW
Figure C4. Variation of longitudinal turbulence intensity ($I_u$) with mean wind speed (winter monsoon, 10-minute duration, deck level)
Appendix C  
Variation of Turbulence Intensities with Mean Wind Speed

(a) N

(b) NNE

(c) NE

(d) ENE

(e) E

(f) ESE
**Figure C5.** Variation of lateral turbulence intensity ($I_v$) with mean wind speed (winter monsoon, 10-minute duration, deck level)
Appendix C  
Variation of Turbulence Intensities with Mean Wind Speed

(a) N

(b) NNE

(c) NE

(d) ENE

(e) E

(f) ESE
Appendix C  

Variation of Turbulence Intensities with Mean Wind Speed

(g) SE  

(h) SSE  

(i) S  

(j) SSW  

(k) SW  

(l) WSW
Figure C6. Variation of vertical turbulence intensity ($I_w$) with mean wind speed (winter monsoon, 10-minute duration, deck level)
Appendix C

Variation of Turbulence Intensities with Mean Wind Speed

(a) N

Turbulence intensity (I_u) vs. 10-minute mean wind speed (m/s)

No. of data = 78
Mean = 0.192
Std = 0.048

(b) NNE

Turbulence intensity (I_u) vs. 10-minute mean wind speed (m/s)

No. of data = 18
Mean = 0.231
Std = 0.057

(c) NE

Turbulence intensity (I_u) vs. 10-minute mean wind speed (m/s)

No. of data = 13
Mean = 0.292
Std = 0.037

(d) ENE

Turbulence intensity (I_u) vs. 10-minute mean wind speed (m/s)

No. of data = 69
Mean = 0.249
Std = 0.066

(e) E

Turbulence intensity (I_u) vs. 10-minute mean wind speed (m/s)

No. of data = 419
Mean = 0.218
Std = 0.058

(f) ESE

Turbulence intensity (I_u) vs. 10-minute mean wind speed (m/s)

No. of data = 90
Mean = 0.214
Std = 0.044
Appendix C  
Variation of Turbulence Intensities with Mean Wind Speed

(g) SE

(h) SSE

(i) S

(j) SSW

(k) SW

(l) WSW
Figure C7. Variation of longitudinal turbulence intensity ($I_u$) with mean wind speed (spring monsoon, 10-minute duration, deck level)
Appendix C  
Variation of Turbulence Intensities with Mean Wind Speed

(a) N

(b) NNE

(c) NE

(d) ENE

(e) E

(f) ESE
Appendix C

Variation of Turbulence Intensities with Mean Wind Speed

(g) SE

(h) SSE

(i) S

(j) SSW

(k) SW

(l) WSW
Figure C8. Variation of lateral turbulence intensity \( (I_v) \) with mean wind speed (spring monsoon, 10-minute duration, deck level)
Appendix C

Variation of Turbulence Intensities with Mean Wind Speed

![Graphs showing turbulence intensities for different wind sectors](image-url)

(a) N

(b) NNE

(c) NE

(d) ENE

(e) E

(f) ESE

- **Sector: N**
  - No. of data = 78
  - Mean = 0.159
  - Std = 0.033

- **Sector: NNE**
  - No. of data = 18
  - Mean = 0.197
  - Std = 0.054

- **Sector: NE**
  - No. of data = 13
  - Mean = 0.208
  - Std = 0.044

- **Sector: ENE**
  - No. of data = 69
  - Mean = 0.189
  - Std = 0.049

- **Sector: E**
  - No. of data = 419
  - Mean = 0.188
  - Std = 0.051

- **Sector: ESE**
  - No. of data = 90
  - Mean = 0.181
  - Std = 0.042


Appendix C  
Variation of Turbulence Intensities with Mean Wind Speed

(g) SE

(h) SSE

(i) S

(j) SSW

(k) SW

(l) WSW
Figure C9. Variation of vertical turbulence intensity ($I_w$) with mean wind speed (spring monsoon, 10-minute duration, deck level)
Appendix C

Variation of Turbulence Intensities with Mean Wind Speed

(a) N

Sector: N
No. of data = 4
Mean = 0.193
Std = 0.027

(b) NNE

Sector: NNE
No. of data = 0

(c) NE

Sector: ENE
No. of data = 3
Mean = 0.358
Std = 0.054

(d) ENE

Sector: NE
No. of data = 4
Mean = 0.368
Std = 0.095

(e) E

Sector: E
No. of data = 2
Mean = 0.344
Std = 0.063

(f) ESE

Sector: ESE
No. of data = 0
Appendix C  
Variation of Turbulence Intensities with Mean Wind Speed

(g) SE

(h) SSE

(i) S

(j) SSW

(k) SW

(l) WSW
Figure C10. Variation of longitudinal turbulence intensity ($I_u$) with mean wind speed (summer monsoon, 10-minute duration, deck level)
Appendix C

Variation of Turbulence Intensities with Mean Wind Speed

(a) N

Sector: N
No. of data = 4
Mean = 0.166
Std = 0.027

(b) NNE

Sector: NNE
No. of data = 0

(c) NE

Sector: NE
No. of data = 4
Mean = 0.314
Std = 0.071

(d) ENE

Sector: ENE
No. of data = 3
Mean = 0.333
Std = 0.04

(e) E

Sector: E
No. of data = 2
Mean = 0.304
Std = 0.053

(f) ESE

Sector: ESE
No. of data = 0
Appendix C
Variation of Turbulence Intensities with Mean Wind Speed

(g) SE

(h) SSE

(i) S

(j) SSW

(k) SW

(l) WSW
Figure C11. Variation of lateral turbulence intensity ($I_v$) with mean wind speed (summer monsoon, 10-minute duration, deck level)
Appendix C  
Variation of Turbulence Intensities with Mean Wind Speed

(a) N

(b) NNE

(c) NE

(d) ENE

(e) E

(f) ESE
Appendix C  

**Variation of Turbulence Intensities with Mean Wind Speed**

(g) SE

(h) SSE

(i) S

(j) SSW

(k) SW

(l) WSW
Figure C12. Variation of vertical turbulence intensity ($I_w$) with mean wind speed (summer monsoon, 10-minute duration, deck level)
Appendix C

Variation of Turbulence Intensities with Mean Wind Speed

(a) N

Sector: N
No. of data = 19
Mean = 0.211
Std = 0.035

(b) NNE

Sector: NNE
No. of data = 26
Mean = 0.186
Std = 0.052

(c) NE

Sector: NE
No. of data = 17
Mean = 0.267
Std = 0.061

(d) ENE

Sector: ENE
No. of data = 92
Mean = 0.215
Std = 0.049

(e) E

Sector: E
No. of data = 113
Mean = 0.192
Std = 0.041

(f) ESE

Sector: ESE
No. of data = 32
Mean = 0.152
Std = 0.042
Appendix C

Variation of Turbulence Intensities with Mean Wind Speed

(g) SE

(h) SSE

(i) S

(j) SSW

(k) SW

(l) WSW
Figure C13. Variation of longitudinal turbulence intensity ($I_u$) with mean wind speed (autumn monsoon, 10-minute duration, deck level)
Appendix C

Variation of Turbulence Intensities with Mean Wind Speed

(a) N

(b) NNE

(c) NE

(d) ENE

(e) E

(f) ESE
Appendix C  Variation of Turbulence Intensities with Mean Wind Speed

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(g) SE

(h) SSE

(i) S

(j) SSW

(k) SW

(l) WSW
Figure C14. Variation of lateral turbulence intensity ($I_v$) with mean wind speed (autumn monsoon, 10-minute duration, deck level)
Appendix C

Variation of Turbulence Intensities with Mean Wind Speed

(a) N

(b) NNE

(c) NE

(d) ENE

(e) E

(f) ESE
Appendix C  
Variation of Turbulence Intensities with Mean Wind Speed

(g) SE

Sector: SE  
No. of data = 3  
Mean = 0.054  
Std = 0.014

(h) SSE

Sector: SSE  
No. of data = 0

(i) S

Sector: S  
No. of data = 0

(j) SSW

Sector: SSW  
No. of data = 0

(k) SW

Sector: SW  
No. of data = 0

(l) WSW

Sector: WSW  
No. of data = 0
Figure C15. Variation of vertical turbulence intensity ($I_w$) with mean wind speed (autumn monsoon, 10-minute duration, deck level)
References


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