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# OPTIMAL ADVERTISING AND PRICING STRATEGIES FOR LUXURY FASHION BRANDS WITH SOCIAL INFLUENCES 

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Optimal Advertising and Pricing Strategies for Luxury Fashion Brands with Social Influences

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A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

March 2011

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## Dedicated to my dear parents

# Optimal Advertising and Pricing Strategies for Luxury Fashion Brands with Social Influences ${ }^{1}$ 

## Abstract

In the fashion industry, it is well-known that social needs play an important role in the purchase of conspicuous products such as high-end fashion labels. In this thesis, motivated by various industrial cases, we analytically study the optimal advertising and pricing decisions for fashion brands in a market consisting of two groups of consumers with opposite social needs for fashion products, namely the Leader Group (LG) and the Follower Group (FG). We consider the situation when the LG consumers have a desire to distinguish themselves from the FG consumers whereas the FG consumers would like to assimilate themselves with the LG consumers. Thus, social influences exist between the two groups of consumers. Based on this market feature, we first develop an optimization model which is original and has not been proposed in the literature before for this problem and we call it the basic model. We explore the solution scheme for identifying the optimal strategy by investigating different tactics. We conduct extensive sensitivity analysis and reveal that the optimal strategies follow different scenarios and it can be optimal for a brand of conspicuous product to (1) advertise to only one group while sell to the whole market, (2) advertise and sell to FG only, or (3) advertise and sell to LG only, depending on the situation. We also derive the analytical conditions for the existence of the Veblen effect, which refers to the phenomenon that a higher selling price can lead to a higher demand for a specific consumer group. After that, we extend the model to the case when there are linear-loss penalties owing to insufficient resource allocation to each consumer group. This extension leads to a much more complicated model with a lot more possible scenarios.

[^0]Similar to the basic model, we derive the detailed mechanism to solve this extended model and conduct in-depth analysis. Important new insights are then generated. This thesis contributes to the literature not only by developing innovative optimization models for the research problem, but also deriving significant findings and managerial insights with real world relevance.

Keywords: optimal pricing, optimization, optimal advertising, social influences, fashion marketing.

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## Chapter 1 Introduction

Consumer goods marketers have long recognized a correlation between advertising and pricing strategies (Farris and Reibstein 1979). As a special category of consumer goods, luxury fashion products provide not just physical function but also symbolic function. Undoubtedly, the consumption of luxury products is strongly influenced by social reference. This thesis explores the relationship between social influence in conspicuous consumption, and optimal advertising and pricing decisions with price-dependent demands. It also analytically investigates the occurrence of the Veblen effect with luxury fashion brands. This thesis falls in the operations management and marketing interfaces and provides important academic and managerial insights. In the subsequent sections of this chapter, the industry background, motivational cases and research objectives will be discussed.

### 1.1 Industry background

The luxury industry is important in terms of both its sales volume (more than $\$ 100$ billion annually) and its influence in creating the best design, using the best materials, developing the best merchandising and packaging methods, and hence, driving both aspiration for the genuine article and the numerous mass-market imitators (Keller 2009). While enjoying opportunities from fast-growing business development and wealth, the greatest challenge faced by a luxury fashion brand nowadays is devising an optimal strategy that can cope with the extremes of the modern luxury marketplace, with a product range that may extend from $\$ 20$ socks to $\$ 20,000$ couture pieces which may be targeted to both Shanghai secretaries and Park Avenue Princesses (Bruce and Daly 2011).

The luxury goods market is made up of apparel, accessories (including handbags and shoes), perfume and cosmetics and hard luxury (including watches and jewellery). The global market of the textile-and-clothing-related luxury sector, which has grown on average by $7 \%$ per annum over the past four years, is estimated to worth
approximately $£ 145$ billion. Despite current headwinds associated with the global economic environment, several fundamental long-term drivers of growth in the luxury market still remain. These drivers include (Amaldoss and Jain 2002; Cachon and Swinney 2009):

1. Strong economic growth in the luxury market which is generally double or triple the rate of the global economy;
2. Up-rising consumption associated with the rapid expansion of emerging economies including China and Russia;
3. Significant growth in high-net-worth individuals (HNWIs) in both the core and emerging markets, where HNWIs are defined as consumers who possess financial assets (not including their primary residence) in excess of US $\$ 1$ million. HNWIs have a higher propensity to purchase luxury goods than other consumers;
4. Increasing demand for luxury brands as driven by higher consumer aspirations;
5. More international travel and tourism activities;
6. Continuous product innovation by luxury brands which creates substantial demand for new products.

It is well-agreed that a luxury fashion brand needs to reconcile the potential trade-offs between (i) exclusivity, which makes a brand out of ordinary as well as project a positive brand association of user profiles, and (ii) accessibility, which provides sufficient sales and profits (Fionda and Moore 2009; Keller 2009). In many cases, it is difficult for a luxury brand company to manage its optimal advertising and pricing strategies because there are different market segments whereas their respective consumptions interact with each other. For example, the brand Lacoste, being famous for its crocodile logo and a symbol of the modern sporting lifestyle, was adopted by many young people of African and North African origins (in the poorer outer suburbs of most French cities). This expanded market segment unfortunately has turned out to have negative impacts on the company because the brand value is lowered and many original customers no longer purchase from this brand (Chevalier and Mazzalovo 2008). To avoid such problem from occurring, many companies have imposed
restriction on the availability of their products by using exclusive distribution channels and even legal action. For example, the designer-label Christian Dior once sued supermarkets for carrying and selling its products because it fears that wide availability would hurt its exclusive high-end brand image (Week 1997). As commented in the China Daily (Chang 2010) by Denis Morisset, a director at Ecole Superieure des Sciences Economiques et Commerciales (ESSEC): "For traditional luxury goods brands, there are two directions. One is the elite path. The other seeks popularity among ordinary consumers. At present, no obvious barrier exists between two directions." As a result, how a brand should choose the optimal "path" and the subsequent optimal pricing and advertising decisions are paramount but still under-explored.

### 1.2 Motivational cases

It is a common practice in the operations management and management science (OR/MS) literature to include motivational cases in addition to the mathematical modeling analysis (Fisher and Raman 1996; Iyer and Bergen 1997). To demonstrate the motivation of our model formulation as well as the relevance of the generated research insights in advancing the practice, this thesis includes several real cases. These cases come from industrial reports.

The purpose of the case studies is to examine the current practices of the practitioners in the fashion industry regarding how they cope with socially interacted market segments. Essentially, the following details will be covered:

1. To explore the current practice of the luxury fashion brands in terms of the targeted markets, advertising and pricing strategy;
2. To collect industry's opinions on selling, advertising and pricing strategy; and
3. To obtain real world inputs to verify the findings we derived from the mathematical models and analysis.

By doing so, it is hoped that cross-reference can be obtained and more empirical insights be generated to enrich our investigation by analytical modelling.

Case 1 (Lacoste): As we have mentioned briefly earlier, Lacoste, a well-established French high-end apparel brand, was once adopted by many young people of African and North African origins in the poorer outer suburbs. The expanded market segment of Lacoste contradicted with the values that the brand promotes. (Chevalier and Mazzalovo 2008) report how the company's advertising manager Didier Calon viewed the phenomenon during an interview with Press Magazine in 2005:

It obviously had negative effects on the brand image. Certain of our customers were upset that we could pursue this target, when in fact we have no control over it. Our sales decreased for three or four years. However, as well as the negative impact on the brand image, some customers have found that it positioned the brand in a more modern trend. It is difficult to fully assess the phenomenon.

This well-reported case explains: (i) the possible negative effect on the brand caused by a certain segment of customers (such as the "lower class" consumers), and (ii) the potential "conflict" between the demands of the leader group and the follower group consumers for luxury fashion brands, which are usually related to the consumers' social classes.

Case 2 (Burberry): During the 1970s, the classic high-end British brand Burberry became popular with the British football casual cult, leading it to be associated with chavs, hooligans and members of football companies by the 1990s. The brand became something of a "national joke", particularly when actress Danniella Westbrook was photographed with her young daughter wearing matching Burberry outfits. Even Burberry admitted that "Burberry is now synonymous with Chavs and thugs" at that time (Jerath et al. 2010). To revitalize the brand, Burberry spent a huge amount of resources (e.g. in hiring advertising agency Baron\& Baron and celebrity photographer Mario Testino, using models Kate Moss and Stella Tennant) in the advertising campaign (Moore and Birtwistle 2004). Burberry also increased its market presence by launching seasonal brochures and a company web site, as well as focusing on leading lifestyle and fashion publications for its bi-annually advertising campaigns (Reed 1999). The brand also held its own fashion shows in Milan, targeting at the
fashion leaders. Besides, Burberry changed its distribution policies to reflect the company's new exclusivity, including stock withdrawal from all European stores that were deemed "unprofitable" or "non-core", and discontinuance of some of license agreements to retained overall control (Kibazo 2000). Nowadays, to strengthen the social interaction and utilize the social influence for brand development, Burberry has established a leading presence across social media platforms, creating new communities of interest. For instance, Burberry is now the leading luxury brand on Facebook with over one million fans. In fall 2009, Burberry launched "The Art of the Trench", a social site which invites users to upload photos with their wearing trench coats, and to rate and comment on the photos on the site (www.facebook.com/burberry, www.artofthetrench.com). At the same time, the brand also developed client services team to look after VIP customers globally (Source: www.burberry.com).

The sudden drop of Burberry's business and brand image in the past is an example of the negative effect brought about by social influence. In order to revitalize the brand and re-establish the proper brand positioning and equity, Burberry has spent a huge amount of effort and resources to focus on the fashion leader group.

### 1.3 Research objectives

As motivated by the above cases and based on the related literature, this thesis has several major objectives, namely:

- To develop original and innovative analytical model that captures the effects of social influence on the demand functions.
- To investigate the optimal advertising and pricing strategies in luxury fashion brands with social influence considerations;
- To examine the effects of social influence, advertising sensitivity, price sensitivity and product cost, on optimal advertising and pricing decision under the advertising and price-demand model;
- To explore the conditions that give rises to the Veblen effects.
- To examine and generate insights on the extended model that considers
more general settings with penalty on insufficiency of resource allocation to each consumer group.
- To generate significant academic and managerial insights on optimal advertising and pricing strategies for luxury fashion goods and to demonstrate the real world relevance of the findings.


### 1.4 Outline of the thesis

The organization of this thesis is as follows. Chapter 2 presents the literature review on the related fields that forms the theoretical base for this thesis research. Chapter 3 presents the basic model and the respective analysis. Chapter 4 studies the extended model and reveals the challenging structural properties of the problem. Chapter 5 discusses the managerial insights and findings from the analysis and the empirical cases. Chapter 6 concludes the research and Chapter 7 presents future research direction.

## Chapter 2 Literature Review

### 2.1 Rational expectations framework

At the heart of our model is the concept of rational expectations (REs), which assumes that all players think rationally and strategically (Muth 1961; Stokey 1981). Consequently, consumers should instantaneously be able to arrive at the equilibrium solution (Sunder and Administration 1992). Since the model developed in this thesis is partially inspired by REs, we would first review some representative papers related to the concept in this section.

The REs framework has been widely employed in the marketing and operations management literature over the past decade (Amaldoss and Jain 2005; Amaldoss and Jain 2005; Su 2007; Jerath et al. 2010; Tereyagoglu and Veeraraghavan 2010). Using the REs framework, Amaldoss and Jain (2005) analyze the pricing and production decisions of a monopolist firm that sells a product to a market with uncertain demand from conspicuous consumers. The authors show that, in equilibrium, firms may offer high availability of goods despite the presence of conspicuous consumption. They further show that scarcity strategies are harder to adopt as demand variability increases, and provide conditions under which scarcity strategies could be successfully adopted to enhance profitability. Afterwards, the authors further study the pricing decisions of a firm facing deterministic price-dependent demand in Amaldoss and Jain (2005), and show that snobs may exhibit an upward-sloping demand curve only in a heterogeneous market. They conduct laboratory experiments to confirm the existence of the equilibrium price that is theoretically derived from the model. The pricing problem in Amaldoss and Jain (2005) is also is extended and analyzed under a duopoly setting in Amaldoss and Jain (2005) and new insights are derived. In Amaldoss and Jain (2008), a market with two groups of consumers who enter the market sequentially are considered. Using a game-theoretic model, the authors explore how a firm can potentially manage the social forces between these two groups of consumers by appropriately selecting its target consumers, designing its product, setting its prices, and limiting the availability of its goods. They show that the presence of reference group effects can motivate firms to add costly features, yet
such features may only provide limited or no functional benefit to consumers. Furthermore, reference group effects can also induce product proliferation and motivate firms to offer products with "limited editions". Most recently, Amaldoss and Jain (2010) develop a discrete version of their prior works in Amaldoss and Jain (2008) that is amenable to further experimental analysis. Empirically, the authors explore the behavior of consumers in a controlled laboratory setting where they can focus on the reference group effects after controlling for the contextual and correlated effects. In Kuksov and Xie (2010), two status products are competing for demand from the "high-class" consumers who would like to signal their identity to each other through the use of a status-reflecting product. They reveal that the consumer value of a status product would increase when the proportion of the high-class consumers in the total customer base of that product increases. They also derive an interesting finding that price reduction of one product could lead to an increase in the demand for the competing product. Caulkins et al. (2010) study the pricing problem of a conspicuous product when the economy is under a recession that disrupts capital markets. The authors model the conspicuous product as a luxury good for which demand is increasing in brand image, which is built up when the good is priced high enough to make it exclusive, and is eroded if the good is discounted. In their paper, recession is modeled as having two effects: one reducing demand and the freezing capital markets such that borrowing is not possible. They reveal that at an intermediate recession level, the optimal pricing solution is history-dependent. Other related works include Stock and Balachander (2005), and Balachander and Stock (2009). The former investigate a signaling strategy to explain product shortages in order to sell 'hot' products in a market with quality uncertainty, whilst the latter provide strategic directions on the timing to offer "limited products" as a part of the product line under the REs framework.

Apart from the above reviewed works, there is also literature examining how the behaviours of customers affect branding strategies under the REs framework. In fact, empirical research has indicated that consumers tend to pay more for "brand-name products" than they do for essentially identical products that lack brand identity. Sometimes the concept of a brand as a signal of quality is an important factor that affects the decision making mechanism of rational consumers. However, brand-name markups are particularly pronounced in the fashion industry where functionality is
less important than the brand's signal of style and exclusivity. A brand's capacity to command higher prices is like a capital asset whose magnitude varies over time and thus deserves to be managed carefully. In a special Fashion Survey issue of The Economist (March 6-12, 2004, p.7), the term "brand integrity" is employed rather than "brand image". "Like everyone else in the luxury goods market, all three (Richemont, Gucci, Pinault-Printemps-Redoute) face the challenge of maintaining "brand integrity"- analyst-speak for that indefinable aura that convinces a consumer to pay a lot of money for something he, or more likely she, could buy much more cheaply elsewhere....The destroyer of brand integrity is "brand dilution", which is the perverse reward for popularity. If too many people have a supposedly exclusive Fendi handbag or Hermès scarf, it is no longer exclusive, and therefore, in the customer's view, no longer worth its vertiginous price." Obviously, brand image is highly related to consumer behaviors and so a central decision for a fashion house is sales volume: Selling too few forfeits product opportunities; selling too many dilutes brand image.

Strategic consumer behavior (under different settings) is another area that receives much attention from the operations management researchers. For instance, the operational impacts of forward-looking or strategic customers have been considered under a large variety of contexts such as seasonal goods (Aviv and Pazgal 2008), commitment in supply chain performance (Su and Zhang 2008), triggering early purchases (Liu and van Ryzin 2008), price-match guarantees (Lai et al. 2010), reservations (Cil and Lariviere 2009) and quick response strategy (Cachon and Swinney 2009). Netessine and Tang (2009) provide an excellent overview of the various strategic consumer behavior literature.

Inspired by the above works, this thesis develops economics models under the REs framework that are related to the rational consumer's behaviors and conducts further analysis.

### 2.2. Social influence in consumptionpsychology and economic studies

In addition to REs, economists point out how consumption could be beset with positive externalities that are due to: (1) social conformity and influences (Becker 1991), (2) network effects in the context of technology (Katz and Shapiro 1985), (3) market frenzies (Degraba 1995), or (4) herd behavior (Bikhchandani et al. 1992). In this section, we focus on reviewing representative works related to social influence.

In the literature, the notion of conspicuous goods dates back to Veblen (1899) who, in "The Theory of the Leisure Class", explains how individuals consumed highly conspicuous goods and services in order to advertise their wealth or social status. After that, Leibenstein (1950) emphasizes the significance of social factors in consumption, and argues that price by itself might enhance utility of consumers. Corneo and Jeanne (1997) establish that conspicuous consumption might emerge as a tool to signal wealth.

It is well-argued that consumers may purchase goods with a goal of not just satisfying their material needs but also social needs such as prestige and image (Grubb and Grathwohl 1967; Belk 1988). In particular, these social needs influence the purchase of conspicuous products, such as jewelry, perfumes, and watches. One can easily argue that the value of precious stone and metal jewelry, designer handbags (e.g., Louis Vuitton, Prada or Gucci), and fine watches (e.g., Rolex), mainly comes from the perception that using these products would elevate the person in the eyes of the onlookers.

Prior research has provided empirical evidence of social influences on consumption (Bearden and Etzel 1982; Childers and Rao 1992). For example, Grinblatt et al. (2008) analyze the purchase behavior of the residents of two Finnish provinces over several years and find that the purchases of neighbors, particularly in the recent past and by those who are geographically most proximate, influence a consumer's purchases of luxury products. Han et al. (2010) demonstrate with field experiments and market data that the preference of a market segment for conspicuously (or inconspicuously) branded luxury goods corresponds predictably
with their desire to associate (or dissociate) with members of their own and other segments. Prior studies have also identified the existence of two competing social needs among consumers, namely: a need for uniqueness, and a countervailing need for conformity (Brewer 1991; Tian et al. 2001). How these needs influence consumer choice processes are also explored (e.g. Lynn 1991; Snyder 1992; Simonson and Nowlis 2000). Research with related construct includes studies on reference groups. For example, consumers from the elite group would like to distinguish themselves from the masses in consumption, but the masses seek to emulate the choices of the elites (see Bourdieu \& Nice, 1984; Bryson, 1996 for some details).

Conspicuous consumption is also widely discussed under the economics context. From the macroeconomic perspective, Yamada (2008) presents a dynamic general equilibrium model of capital accumulation in which consumers have status preference. As claimed by the author, such an attempt is the first to investigate the macroeconomic implications of conspicuous consumption. From the microeconomic perspective, on the other hand, Becker (1991) uses conformism to show why similar retailers might eventually experience vastly different sales patterns. According to his model, it is found that at equilibrium, the demand curve for followers could be upward-sloping but the equilibrium is not stable. Pesendorfer (1995) and Bagwell and Bernheim (1996) also consider implications of such product used by consumers. In particular, Pesendorfer (1995) considers the implications of status goods of a durable-good monopoly in a dynamic model and finds that innovation cycles would endogenously occur, while Bagwell and Bernheim (1996) derive general conditions under which a market for status goods may exist.

While probing the sociological and psychological intricacies of potential consumer interdependencies, scholars may have overlooked the effects of mundane product advertising and promotions. To take into consideration of the effect of advertising, Krahmer (2006) considers a model with advertising that informs the public of brand names and creates the possibility of conspicuous consumption by rendering brands as a signaling device. In a price competition framework, the author shows that advertising increases consumers' willingness to pay and thus provides a foundation, based on optimization behavior, for persuasive approaches to advertising. However, the subjects of optimization in the model are assumed to be just the customers, while the company did not conduct optimization decisions intentionally.

As a breakthrough, McClure and Kumcu (2008) are the first to attempt to formally incorporate individual product promotions into a theory of luxury good pricing. Specifically, they formulate the relationship between the optimal price/quantity combination and the thoroughness of product promotions, in a monopoly setting. In their model, a monopolistic seller of a luxury product with imperfect information traces out a backward bending price/ quantity locus as he iterates toward the optimal combination of quantity, price and promotional thoroughness. More works on optimal decisions will be discussed in Section 2.4.

Notice that even though the above literature works that are related to social influence have provided a solid ground and empirical evidence on the importance of the topic, they have not considered the optimal strategies by taking into accounts of social influences and reference groups. This thesis research would aim at filling this important gap.

### 2.3. Network effect

Network effect (NE) (also called network externalities) refers to the market phenomenon in which the value of a product or service to consumers depends on the number of users of that product or service (for a detailed discussion, see Katz and Shapiro 1985). With rapid advances in information technology and the digital revolution, NE has become an important characteristic for an increasing number of industries and product/service categories (e.g., computers, communications, consumer electronics, software, financial exchanges, online auctions, home networking, social networking Web sites, etc). In conspicuous consumption, NE has long been regarded as an important factor in affecting purchasing decision (Leibenstein 1950). Marketing decision characterized by NE is a topic that has recently been attracting considerable amount of interests from researchers in the field of both marketing and economics for digital products and fashion products (see, e.g., Wang et al., 2010; Dube et al., 2010; Trusov et al., 2009; Friedman and Ostrov, 2008; Chien and Chu, 2008).

Literature on static pricing under NE focuses on the importance of consumer expectations and concerns the multiple equilibria problem. One commonly proposed restriction to be placed on expectations is that they will be fulfilled in the sense that consumer expectations are consistent with the actual outcome in the market (see, e.g., Leibenstein, 1950; Rohlfs, 1974; Katz and Shapiro, 1985; Economides, 1996). That is, on the induced fulfilled-expectations demand curve, each price $p$ corresponds to those quantity $q$ such that, when consumers expect quantity $q$, there will be just $q$ consumers purchasing at a price $p$. Leibenstein (1950) derives such a demand curve from fixed-expectations demand curves. He argues that, under condition of perfect knowledge (or accurate expectations), any point on the demand curve, for any given price, will be at that total quantity demanded where the marginal external consumption effect for all consumers but one, is equal to zero. Rohlfs (1974) provides an early treatment of such issues in the context of a communication network, although the fulfilled-expectations demand curve has been discussed in Leibenstein (1950). He discovers that there are typically multiple equilibria at any given price, and which equilibrium is attained depends partly on the static mode, partly on the initial disequilibrium conditions, and partly on the disequilibrium adjustment process. Some
general properties of the equilibrium user sets are derived. Sundararajan (2004) presents a static model of nonlinear pricing in a monopoly market with fulfilled expectations. He shows that the optimal pricing decision includes discounts that increase with quantity, and may also involve a two-part tariff. While NE generally raises prices, consumption may or may not rise.

As the social influence that we model is affected by and related to NE, the above reviewed works in economics will serve as the foundation for the development of our demand model.

### 2.4. Studies on optimal advertising and pricing, especially for

## conspicuous product

In marketing science and operations management, there are a considerable amount of studies that examine optimization of analytical models with pricing and/or advertising decisions under different settings. We will examine some of the related works before proceeding to explore those that are most closely related to the topic on conspicuous products.

Optimization methods such as control theories have been well-established in exploring optimal advertising problems. In particular, several authors have used optimal control theory in diffusion of innovation framework to derive normative optimal policies (Kamien and Schwartz 1991; Sethi and Thompson 2005). In a popular paper, Narasimhan et al. (1993) combine the effects of manufacturing quality with advertising and pricing, and investigate the respective optimal decisions in a control theory framework. Feichtinger et al. (1994) provide an excellent review of for the development of this well-established field, and we refer the readers to it. A stream of modeling research in interactive advertising and pricing scheme can be found in the literature regarding selling to interacted segmented markets. In this area, Buratto et al.(2006a) is the first piece of work that brings the market segmentation concepts into optimal advertising model analysis. Specifically, the authors consider that there are two scenarios for new product introduction, namely: one case when the advertising process can reach all individual selective target groups and the other case when only one advertising channel with an effectiveness segment-spectrum is available. Later on, Buratto et al. (2006b) consider a market with a finite number of segments and assume that several advertising channels, with different diffusion spectra and efficiencies, are available.

Optimal advertising/pricing models can also be classified into monopoly setting and duopoly setting. Under the monopoly setting, Mesak and Zhang (2001) formulate and solve an optimal advertising pulsation problem for a monopolistic firm using
dynamic programming (DP). In their work, the firm aims at maximizing its own profit through an optimal allocation of the advertising budget in terms of rectangular pulses over a finite planning horizon. Aggregate sales response to the advertising effort is assumed to be governed by a modified version of the Vidale-Wolfe model in continuous time. Using a numerical example in which a planning horizon of one year is divided into ten equal time periods, they develop efficient computing routines. Computational results show, among other findings, that the performance yielded by the DP policy dominates the uniform advertising policy (constant spending) for a concave advertising response function. Similarly, Amaldoss and Jain (2005) examine a continuous-time optimal advertising under an S-shaped response function. Later on, Feinberg (2005) explores extensions along three dimensions: an S-shaped response function, the value of the discount rate, and the possibility of diffusion-like response. He formulates a flexible class of S-shaped response models and derives a set of conditions on the optimal advertising response function that extend the results of a pioneering work by Sasieni (1971). Collectively, these results all suggest a set of baseline properties that reasonable analytical models should possess. In another setting, Lambertini (2005) characterizes the dynamics of optimal advertising investment in a spatial monopoly, contrasting the socially optimal behavior of a planner against that of a profit-seeking monopolist. It is found that in steady state, the monopolist always distorts both output and advertising decisions as compared to the social optimum. Later on, Grosset \& Viscolani (2009) propose a model of a firm that advertises a product in a homogeneous market, where a constant exogenous interference is present. They consider the scenario that the interference acts additively. They model the problem with a piecewise linear demand function and formulate a non-smooth optimal-control problem with an infinite horizon. By solving the respective optimal control problem, they obtain an optimal advertising policy. They also reveal that the optimal policy takes one of two forms: either a positive and constant advertising effort, or a decreasing effort starting from a positive level and eventually reaching the zero value at a finite exit time.

Under the duopoly setting, game-theoretic analysis is usually employed in the literature. For instance, Viscolani \& Zaccour (2003) consider a duopolistic market where the current sales of each firm is proportional to its goodwill stock. The evolution of the latter depends positively on a company's own advertising effort and
negatively on competitor's advertising. By relaxing the standard assumption in the literature in differential games of advertising that the players remain active throughout the whole (infinite) duration of the game, they characterize the situations under which a firm finds it optimal to remain or exit the market. They also analytically show that when both players are powerful, then the unique Nash equilibrium is the same as the one obtained in the absence of interference from the competitor's advertising effort. Ghosh \& Stock (2003) use a model of informative advertising to study the effect of penetration on competing advertisers' strategies and profits. Conditions under which an increase in penetration counter-intuitively leads firms to increase advertising levels and enjoy higher profits are identified. Bass et al (2005) examine whether, when, and how much brand advertising versus generic advertising should be done. Using differential game theory, they derive the optimal advertising decisions for a dynamic duopoly with symmetric or asymmetric competitors. They show how advertising depends on the cost and effectiveness of the type of advertising strategies for each firm, the allocation of market expansion benefits, and the profit margins determined endogenously from price competition. They find that generic advertising is proportionally more important in the short term and that there are free-riding effects leading to sub-optimal industry expenditure on generic advertising that worsen as firms become more symmetric. Afterwards, Amaldoss \& He (2009) propose and test a competitive model of advertising. They find that the brand specificity of advertising can have an inverted U-shaped relationship with profits. Most recently, Chen et al. (2009) show that depending on the nature of consumer response, combative advertising (CA) can reduce price competition to benefit competing firms by game-theoretic analysis. They also find that CA can also lead to a pro-competitive outcome where individual firms advertise to increase their own profitability, but collectively become worse off. They argue that the result is intuitive because CA can intensify price competition such that an "advertising war" leads to a "price war." Thus, similar to price competition, advertising competition can result in a prisoner's dilemma where all competing firms make less profit even when the effect of each firm's advertising is to enhance consumer preferences in its favor.

There are discussions in the literature on optimal advertising/pricing decisions from the supply chain perspective. Neslin et al. (1995) explore how retailer and consumer responses influence a manufacturer's optimal advertising and trade
promotion plans. They develop a dynamic optimization model that considers the actions of the manufacturer, retailers, and consumers. In their setting, the manufacturer attempts to maximize its profits by advertising directly to consumers and offering periodic trade deal discounts to the retailer in the hope that the retailer will in turn "pass through" a retailer promotion to the consumer. They analytically show how the manufacturer's optimal allocation depends on consumer response to advertising, consumer response to retailer promotions, retailer inventory carrying cost, and retailer's pass-through behavior. After that, Jorgensen \& Zaccour (1999) examine the conflict and channel coordination issue in a two-echelon supply chain. They propose a differential game model that includes carryover effects of advertising and identify pricing and advertising strategies for both firms under channel conflict as well as coordination. Optimal dynamic advertising policies are designed. In a symmetric case, they show that these strategies can be determined in closed form, taking into consideration explicitly the non-negativity constraints on advertising rates. Jorgensen et al. (2003) examine dynamic advertising and promotion strategies in a supply chain where the retailer promotes the manufacturer's product and the manufacturer spends on advertising to build a stock of goodwill. Assuming that sales amount depends on goodwill and promotion activities, they consider two scenarios, namely: 1. the manufacturer and retailer determine non-cooperatively their respective strategies; and 2. the game is played with the manufacturer as the leader. They find that whether or not the goodwill stock has a decreasing marginal effect on sales, the cooperative advertising program can help achieve channel coordination and Pareto improvement. Jorgensen et al. (2007) study a two-member channel in which a manufacturer and an exclusive retailer can make advertising expenditures that have both short and long term impacts on the retailer's sales. They assume that the manufacturer can support the retailer's advertising efforts through a cooperative advertising program. Different scenarios are considered and their analysis indicates that both types of retailer's advertising strategies provide more profit to both channel members than any of the two cases of partial support. Sethuraman (2009) develops an analytical model and shows that the relationship between manufacturer advertising and retail price promotion depends on the role of advertising. If advertising differentiates brands and suppresses consumer response to retail promotion, then the relationship is negative. On the contrary, if advertising is informative enough to
increase consumer response to retail promotions, then the relationship is positive. Szmerekovsky \& Zhang (2009) consider the pricing decisions and two-tier advertising levels between a manufacturer and a retailer in a two-echelon supply chain where customer demand depends on the retail price and advertisement by the manufacturer and the retailer. They solve a Stackelberg game with the manufacturer as the leader and the retailer as the follower. With price sensitive customer demand and a linear wholesale pricing contract, they derive the optimal decisions by the manufacturer and the optimal responses by the retailer. Their analytical results interestingly show that cost sharing of local advertising does not work well and it is better for the manufacturer to advertise nationally and offer the retailer a lower wholesale price. Later on, Zhang (2009) argues that convenience of home shopping and easy access to information have important implications on the retailers' channel and advertising decisions. He addresses two major research questions, namely: when a conventional bricks-and-mortar retailer should adopt a multichannel strategy, and when a multichannel retailer should use its website to advertise offline prices. His analysis shows that the answers hinge on the nature of the product, the retailer's costs, and the competitors' strategies as well as the competitiveness of the market. Recently, Buratto \& Zaccour (2009) study the cooperative and non-cooperative advertising strategies of a licensor and licensee involved in a licensing contract in the fashion business. The licensing practice that they considered is the process of leasing a legally protected entity (brand, name, logo, etc.) in conjunction with a product or product line. They analytically show that if the licensor (who is the leader) uses an incentive strategy that depends on the licensee advertising, then it can reach the jointly optimal solution in a decentralized way. Most recently, Swami and Dutta (2009) develop the optimal advertising strategies for a firm which has a new product with demand in an emerging market. They consider the case when primary channels for distribution of the firm's product do not exist (because the market has not been opened up or the firm has not entered the market). Therefore, in their model, the consumers can only employ secondary channels in other markets. They reveal insights on the optimal timing that is beneficial to advertise for the product before the market opens and how various parameters (such as the likelihood of product adoption for the primary and secondary channels, market potential and coefficient of innovation and imitation) affect the optimal advertising policies.

Targeted advertising is another important area that is related to this thesis study. In practice, most of the time, advertising is targeted (Anand and Shachar 2009). For example, advertising for handheld devices is targeted towards young professionals, and for rock climbing equipment towards outdoor enthusiasts. Targeting is ubiquitous even in product categories that cater to far broader customer segments, like apparel (e.g., Liz Claiborne, Tommy Hilfiger), and entertainment (rap versus country music). The perceived benefit of targeting is that it can reduce wasted advertising by ensuring that advertisements reach the most appropriate consumers for the firm's product. At the same time, fragmented media and new technologies make it easier to reach the individuals desired. Motivated by the significance of targeted advertising decision, Anand and Shachar (2009) study a model in which firms can target their advertisements to particular groups of consumers, and advertising is noisy. In their model, products are differentiated and consumers are heterogeneous in their tastes for product attributes. In other words, they consider the case that a particular product has a better fit with the tastes of some consumers than those of the others, and consumer-utility depends on the resulting "match" between product attributes and their tastes. They assume that the firms know the tastes of consumers whereas the consumers are uncertain about product attributes. Thus, in their model, firms can send advertisements through different media channels. However, advertising content is a noisy message on product attributes and consumer preferences over product attributes are correlated with their choice of media channel, creating a role for targeting. In this setup, they propose that advertising allows a consumer to learn about her match with the characteristics of the product. They proceed to prove the existence of a perfect Bayesian equilibrium. Most recently, Raghavan and Iyer (2010) examine advertising strategy when competing firms can target their advertising effort to different groups of consumers within a market. With targeted advertising, they find that firms advertise more to consumers who have a strong preference for their product than to comparison shoppers who can be attracted to the competition. They argue that advertising less to comparison shoppers can be seen as a way for firms to endogenously increase differentiation in the market. Interestingly, they also find that target advertising leads to higher profits, regardless of the firms' ability to set targeted prices. In addition, they prove that advertising targeting can be more valuable to firms in a competitive environment than the ability to pricing targeting.

There are some other related analytical modeling researches on exploring optimal advertising decisions and its impacts from different perspectives. For instance, Danaher and Rust (1996) adopt the point of view that advertising is an investment, and propose a simple formula for calculating the level of media spending which maximizes the return on investment. Bass et al. (2007) evaluate the dynamic effects of different themes of advertising and develop a model that jointly considers the effects of wearout as well as that of forgetting in the context of an advertising campaign that employs five different advertising themes. They quantify differential wearout effects across the different themes of advertising, and examine the interaction effects between the different themes using a Bayesian dynamic linear model (DLM). They establish a model to show how the response model parameters can be used to improve the effectiveness of advertising budget allocation across different themes. Baye and Morgan (2009) model an environment where e-retailers sell similar products and endogenously engage in both brand advertising (to create loyal customers) and price advertising (to attract "shoppers"). In contrast to models under which loyalty is exogenous, they consider the endogenizing strategy that helps create loyal customers. By game-theoretic analysis, they find several significant findings and conduct analysis based on data from a leading price comparison site to verify the analytical findings. Profits from generic advertising by the producer group often come partly at the expense of the producers of closely related commodities. The resulting tendency towards excessive advertising is exacerbated by check-off funding. Most recently, to analyze this beggar-thy-neighbor behavior, Alston et al. (2010) compare a scenario where different producer groups cooperate and choose their advertising expenditures jointly to maximize the sum of profits across the groups, and a scenario where they optimize independently.

Now, focusing on the conspicuous products, Krahmer (2006) formalizes the intuition that brands are consumed for image reasons and that advertising creates a brand's image. He argues that advertising informs the public of brand names and creates the possibility of conspicuous consumption. In a price-competition framework, he shows that advertising increases consumers' willingness to pay and thus provides a foundation for determining the optimal advertising strategy. Moreover, he finds from his analysis that an incumbent might strategically over-invest in advertising to deter entry and competition might be socially undesirable. McClure and Kumcu (2008)
formalize the relationship between the optimal price/quantity combination and the thoroughness of conspicuous product promotions. They reveal that iterating towards the profit maximizing thoroughness of product promotion will lead to a backward bending price/quantity locus. In terms of pricing of conspicuous products, Yao and Li (2005) discuss the pricing of a superior good based on its 'signalling value' and offer a different reason why in China and some other Asian countries the prices of luxury goods are extremely expensive when they are first marketed, then fall dramatically and discontinuously afterwards, when marginal costs decline to below the critical point and the goods become more popular.

As we can see from the reviewed literature above, there exists a gap in the literature to provide a decision set regarding the targeted market segment, targeted advertising segment and non-discriminant price, under the situation of two socially interacted segments with interdependent demands. Our research hence aims to fill such gap.

### 2.5. Veblen effects

The "Veblen effect" is a very important phenomenon in the marketing science literature. It is said to occur "when individuals increase their demand for a good simply because it has a higher price" or in another word, "when consumers exhibit a willingness to pay a higher price for a functionally equivalent good" (Bagwell and Bernheim 1996). This effect may actually look a bit counterintuitive because it is different from the conventional wisdom in which demand is a decreasing function of price. In the literature, Bagwell and Bernheim (1996) argue that the relationship between price and demand should emerge in equilibrium and not the simple "take-for-granted" relationship as what the classical price-demand economics model shows. They also find the conditions for such "Veblen effects" to arise in equilibrium. Despite the variety of theories in the economic literature about Veblen effects; not all explicitly admit either backward bending or upward sloping demand curves as a theoretical possibility. Although Leibenstein (1950)'s derivation of a backward bending demand curve from a Veblen effect is generally considered to be a pioneering work, a much earlier appearance of a backward bending demand explanation of conspicuous consumption is found in a textbook published by Fairchild et al. (1939). Leibenstein (p.207) distinguishes the "Veblen effect" from "bandwagon" and "snob" effects as follows:

If the Veblen effect is the predominant effect, the demand curve is less elastic than otherwise, and some portions of it may even be positively inclined; whereas if the Veblen effect is absent, the curve will be negatively inclined regardless of the importance of the snob effect in the market.

Leibenstein (1950)'s analysis of the "Veblen effect" gives rise to "demand" curves that can be backward bending. In prior research, evidence of the Veblen effects has been presented in a variety of ways. For example, empirical studies on the Veblen effect include a study of durable goods by Basmann et al. (1988), and an investigation of women's cosmetics by Chao and Schor's (1998). Although none of these articles attempt to establish the existence of a backward bending demand curve, they show the case that the prices of conspicuously consumed goods lead to Veblen effect and this effect is absent in the less conspicuous products particular to their studies. For
example, Chao and Shor explain that while the law of demand is verified for less conspicuous products, "for lipstick, price is not even a significant negative determinant of quantity demanded". Moreover, there are vivid anecdotes about Veblen effects, such as those presented by Bagwell and Bernheim (1996), who suggest that "Veblen effects may be empirically significant in marketing for luxury goods". In addition, Creedy et al. (1991), and Bagwell and Bernheim (1996) argue that: "Econometric evidence also corroborates the existence of Veblen effects". The above works have explored the existence of Veblen effects under various market settings. However, they have not explored analytically the conditions for the occurrence of the Veblen effects. As a consequence, this thesis research, via its analytical model, tries to reveal the analytical closed-form conditions for the occurrence of this important Veblen effect.

## Chapter 3 Basic model

Mathematical modeling, supplemented by real world cases, is employed to study the proposed topic stated in this thesis. In this chapter we start by introducing the background and notation for the modeling employed in multiple sections in this thesis.

### 3.1 Notation

Table 3-1 Notation list A

| $x_{L}$ and $x_{F}$ | basic demands of LG and FG |
| :--- | :--- |
| $a$ and $\alpha$ | sensitivity coefficients of advertising efforts of LG \& FG |
| $b$ and $\beta$ | sensitivity coefficients of norm of LG and FG (effects of social <br> influence). |
| g and $\gamma$ | sensitivity coefficients of price of LG and FG <br> $\lambda$ |
| $e$ | the proportion of advertising effort spent on LG (a decision variable) <br> total advertising effort (a decision variable) <br> $p$ |
| $c$ | product retail price (a decision variable) <br> production cost |

### 3.2 Model and assumption

Consider a company which sells a fashion product to the market. The unit product cost is $c$, and the unit retail price is $p$, where $p>c>0$. We consider the case when $c$ is exogenous and $p$ is decided by the company. There are two groups of customers, namely the leader-group (LG) customers and the follower-group (FG) customers. The demands of these two groups are inter-dependent. LG customers seek for a unique personal style, and they dislike a product that is owned by too many FG whereas FG customers take the purchasing decisions of LG as their references in deciding whether to buy a product or not. As a result, a higher demand of LG induces a higher demand of FG, but a higher demand of FG implies a lower demand of LG.

Therefore, mathematically, the demand of FG is increasing with the demand of LG, and the demand of LG is decreasing with the demand of FG. Both groups are price-sensitive, and demands of both groups are strictly decreasing in $p$ (Chiu et al., 2009). In order to increase the sales volume of the product, the company can implement some advertising campaigns on LG and FG. The total advertising resource and effort of the company spent on the advertising campaigns is denoted by $e$. This total advertising effort is divided into two proportions by the company: A $\lambda$ proportion of the effort is spent on LG and the rest $(1-\lambda)$ proportion is spent on FG, where $\lambda$ is bounded between 0 and 1 . The advertising and price sensitive demand model with social influence is depicted in Fig. 3.1. The advertising cost function $C(e)$ is strictly increasing in $e$ and the marginal cost of the total advertising effort is strictly increasing in $e$, i.e., $d C(e) / d e>0$ and $d^{2} C(e) / d e^{2}>0$. In order to obtain more close-form analytical insights, we consider $C(e)=e^{2}$ (P.S.: the analysis procedure will remain the same if we assume another format of this effort function). We denote the advertising and pricing strategy of the company by $\omega=(e, \lambda, p)$. We only focus on the set of finite $\omega$, i.e., $\omega \in \Omega=\{0 \leq e<+\infty, 0 \leq \lambda \leq 1, c<p<+\infty\}$. Based on the luxury fashion market we considered, we adopt the following additive demand functions of LG and FG, respectively,
$D_{L}(\omega)=\left[V_{L}(\omega)\right]^{+}$and $D_{F}(\omega)=\left[V_{F}(\omega)\right]^{+}$,
where $[Y]^{+}=\max \{0, Y\} \quad, \quad x_{L}, x_{F}, a, \alpha, b, \beta, g, \gamma \quad$ are all non-negative, and $V_{L}(\omega)=x_{L}+a \lambda e-b D_{F}(\omega)-g p$ and $V_{F}(\omega)=x_{F}+\alpha(1-\lambda) e+\beta D_{L}(\omega)-\gamma p$ are the value functions of the fashion product of LG and FG, respectively, for any given $\omega \in \Omega$.

The demand model above indicates that no consumer is willing to buy the fashion product if its value is zero or negative. However, more consumers are willing to buy the fashion product if its value to customers is higher.


Fig.3-1 Advertising and price-demand model with social influence (S.I.).

To ensure the validity of the above demand model, we have the following assumption:
Assumption 3.2.1 (a) $x_{L}>g c$ and (b) $x_{F}>\gamma c$
Notice that Assumption 3.2.1 ensures the demands of LG and FG are positive when the company sells the product to either group at cost $c$ when there is no social influence. This assumption helps avoid trivial and un-reasonable cases.

For any given feasible $\omega$, the total demand of the product and the company's profit are given as follows respectively,
$D(\omega)=D_{L}(\omega)+D_{F}(\omega)$, and
$\pi(\omega)=(p-c) D(\omega)-e^{2}$.
The profit maximization model of the luxury brand (company) is given as follows,
(P1) $\max _{\omega \in \Omega} \pi(\omega)$.
For a notational purpose, we represent the optimal advertising and pricing strategy of problem ( P 1 ) by $\omega^{*}=\left(e^{*}, \lambda^{*}, p^{*}\right)$.

### 3.3 Results

To obtain the optimal pricing and advertising strategies, notice that we need to explore the problem by investigating different cases. First, we prove the existence of feasible solutions to the optimization Problem ( $P 1$ ).
Proposition 3.3.1 There always exist multiple $\omega$ with which $\pi(\omega)>0$.
Proof: All proofs appear in the Appendix (A1).
Proposition 3.3.1 asserts that there always exist some (feasible) $\omega$ which ensures $\pi(\omega)>0$ for any given market parameters, and hence $\pi\left(\omega^{*}\right)>0$ always exists, given that Assumption 3.2.1 is valid. According to the demand models of LG and FG, there are three mutually exclusive cases, and these three cases represent three different selling tactics for the company:

- Tactic I: $D_{L}(\omega)>0$ and $D_{F}(\omega)>0$, the company sells the product to both FG and LG;
- Tactic II: $D_{L}(\omega)=0$ and $D_{F}(\omega)>0$, the company targets at FG and sells the product to FG only;
- Tactic III: $D_{L}(\omega)>0$ and $D_{F}(\omega)=0$, the company targets at LG and sells the product to LG only.
In the following, we analyze the "local" optimal solution for each tactic. The "global" optimal solution of Problem ( $P 1$ ) can finally be found by choosing the best one among the three "local" optimal solutions.


### 3.3.1 Tactic I: Selling to both LG and FG

Define $\Omega_{I}$ as the set of $\omega$ that satisfies the conditions for Tactic I: $D_{L}(\omega)>0$ and $D_{F}(\omega)>0$. The total demand of the product and the profit of the company are derived as follows,
$D(\omega)=\frac{X+e\left[\alpha(1-b)+\lambda N_{I}\right]-G p}{1+b \beta}$, and
$\pi(\omega)=\frac{p-c}{1+b \beta}\left\{X+e\left[\alpha(1-b)+\lambda N_{I}\right]-G p\right\}-e^{2}$, respectively, where
$X=(1+\beta) x_{L}+(1-b) x_{F}, G=(1+\beta) g+(1-b) \gamma$, and $N_{I}=a(1+\beta)-\alpha(1-b)$.
A closer examination of $X /(1+b \beta)$ reveals that it combines two parts, LG's basic demand with adjustment of the social influence (i.e., $x_{L}(1+\beta) /(1+b \beta)$ ) and FG 's basic demand with adjustment of the social influence (i.e., $x_{F}(1-b) /(1+b \beta)$ ). Moreover, $X /(1+b \beta)$ represents the total basic demand under Tactic I as it is not a coefficient of $e$ and $p$. Similarly, $G /(1+b \beta)$ also combines with two parts: LG's price sensitivity of demand with adjustment of the social influence (i.e., $g(1+\beta) /(1+b \beta))$ and FG's price sensitivity of demand with adjustment of the social influence to LG (i.e., $\gamma(1-b) /(1+b \beta)$ ). In addition, $-G /(1+b \beta)$ represents the price sensitivity of demand under Tactic I as it is the coefficient of $p$. Moreover, $\frac{\alpha(1-b)+\lambda N_{I}}{1+b \beta}$ is the coefficient of $e$ in $D(\omega)$. Thus, $\frac{\alpha(1-b)+\lambda N_{I}}{1+b \beta}$ represents the advertising sensitivity of total demand under Tactic I. The particular term $\lambda N_{I}$ of $\frac{\alpha(1-b)+\lambda N_{I}}{1+b \beta}$ links up the advertising sensitivity of total demand under Tactic I with $\lambda$. Under Tactic I, $D(\omega)$ is increasing (decreasing) in $\lambda$ if $N_{I}>0\left(N_{I}<0\right)$. Moreover, $D(\omega)$ is more sensitive to $\lambda$ if the absolute value of $N_{I}$ is bigger. Therefore, $N_{I}$ represents the advertising allocation sensitivity of the total demand.

Proposition 3.3.2 Suppose that Tactic $I$ is adopted with $e>0$. (a) There exists $a$ unique optimal $\lambda$ if $N_{I} \neq 0: \lambda^{*}=1$ if $N_{I}>0$ and $\lambda^{*}=0$ if $N_{I}<0$. (b) Any $\lambda \in[0,1]$ can be optimal solution of Problem (P1) if $N_{I}=0$.

Notice that in Proposition 3.3.2, we focus on the non-trivial case with $e>0$; when $e=0, \lambda$ can be ignored (as there is no advertising campaign/effort). Proposition 3.3.2 shows that under Tactic I, the company should focus only on the two extreme values of $\lambda$ if $N_{I} \neq 0$. Specifically, when $N_{I} \neq 0$, the company should either put all the advertising effort to LG, i.e., $\lambda^{*}=1$, or put all the advertising effort to FG, i.e., $\lambda^{*}=0$. On the other hand, if $N_{I}=0$, then the value of $\lambda$ is trivial, and the allocation of advertising effort between LG and FG does not affect the profit of the company. Proposition 3.3 .2 shows that the company should allocate all the advertising effort to LG (FG) if $N_{I}>0\left(N_{I}<0\right)$. Moreover, it is interesting to find
that $\lambda^{*}$ under Tactic I depends on the market parameters only, and is independent of $p$ and $e$. In addition, based on Proposition 3.3.2, the profit of the company can be further decomposed into the following mutually exclusive sub-cases:

- Tactic (IA): When $N_{I}>0$;
- Tactic (IB): When $N_{I} \leq 0$.


### 3.3.2 Tactic II: Selling to FG only

Under Tactic II, $D_{L}(\omega)=0$ and $D_{F}(\omega)>0$. Since $V_{L}(\omega)<0$ and $V_{L}(\omega)=0$ together implies $D_{L}(\omega)=0$, we further consider two sub-tactics under Tactic II:

- Tactic IIA: $V_{L}(\omega)<0$ and $D_{F}(\omega)>0$.
- Tactic IIB: $V_{L}(\omega)=0$ and $D_{F}(\omega)>0$.

For Tactic IIA, we have:
$D(\omega)=D_{F}(\omega)=x_{F}+\alpha e(1-\lambda)-\gamma p>0$, and
$\pi(\omega)=(p-c)\left(x_{F}+\alpha e(1-\lambda)-\gamma p\right)-e^{2}$.
For Tactic IIB, we have:
$x_{L}=b x_{F}-[(a+b \alpha) \lambda-b \alpha] e-(b \gamma-g) p$.
We summarize the optimal advertising allocation rule in Proposition 3.3.3.
Proposition 3.3.3 For any fixed $e>0$, it is optimal to allocate all the advertising effort to $F G$ under Tactic II, i.e., $\lambda^{*}=0$.

### 3.3.3 Tactic III: Selling to LG only

Under Tactic III, $D_{F}(\omega)=0$ and $D_{L}(\omega)>0$. Similar to Tactic II, since $V_{F}(\omega)<0$ and $\quad V_{F}(\omega)=0$ together implies $D_{F}(\omega)=0$, we further consider two sub-cases under Tactic III:

Tactic IIIA: $V_{F}(\omega)<0$ and $D_{L}(\omega)>0$.
Tactic IIIB: $V_{F}(\omega)=0$ and $D_{L}(\omega)>0$.
For Tactic IIIA, we have the following,
$D(\omega)=V_{L}(\omega)=x_{L}+a e \lambda-g p>0$, and
$\pi(\omega)=(p-c)\left(x_{L}+a e \lambda-g p\right)-e^{2}$.

For Tactic IIIB, the following holds,
$0=x_{F}+\beta x_{L}+[\alpha(1-\lambda)+a \beta \lambda] e-(g \beta+\gamma) p$.
Proposition 3.3.4 For any fixed $e>0$, it is optimal to allocate all the advertising effort to $L G$ under Tactic III, i.e., $\lambda^{*}=1$.

### 3.4 Optimal advertising and pricing

## strategies

In the above section, we have proposed the basic structure of the three different tactics. Notice that the analytical results indicate that the structures of $D(\omega)$ and $\pi(\omega)$ under different tactics are substantially different. In this section, we proceed to explore the optimal advertising and pricing strategies for each tactic one by one. In addition, we investigate the optimality conditions for each tactic. With the optimality conditions of each tactic, we derive the rules for the company to determine the optimal tactic among Tactics I, II and III. There are two steps for the company to determine the optimal tactic and the corresponding optimal advertising and pricing strategy $\omega^{*}$ :

- Step I: Find the local optimal $\omega$ of each tactic (Tactics I, II and III);
- Step II: Find the globally optimal tactic by comparing the company's profits at all local optimums of Tactics I, II and III. The tactic is globally optimal if it induces the highest company's profit.

In the following, we investigate the local optimal advertising and pricing strategy for each tactic.

### 3.4.1 Optimal advertising and pricing strategies for Tactic I

The optimal retail price and advertising effort are different between Tactic (IA) and Tactic (IB). Therefore, we consider the two sub-cases separately. Before we proceed, we define the following three advertising and pricing strategies:

$$
\begin{aligned}
& \omega_{I, A}^{*}=\left(1, \frac{a(1+\beta)(X-G c)}{4(1+b \beta) G-a^{2}(1+\beta)^{2}}, \frac{2(1+b \beta)(X+G c)-a^{2} c(1+\beta)^{2}}{4(1+b \beta) G-a^{2}(1+\beta)^{2}}\right), \\
& \omega_{I, B}^{*}=\left(0, \frac{\alpha(1-b)(X-G c)}{4(1+b \beta) G-\alpha^{2}(1-b)^{2}}, \frac{2(1+b \beta)(X+G c)-\alpha^{2} c(1-b)^{2}}{4(1+b \beta) G-\alpha^{2}(1-b)^{2}}\right), \text { and }
\end{aligned}
$$

$$
\bar{\omega}_{I, B}^{*}=\left(\text { any } 0 \leq \lambda \leq 1, \frac{\alpha(1-b)(X-G c)}{4(1+b \beta) G-\alpha^{2}(1-b)^{2}}, \frac{2(1+b \beta)(X+G c)-\alpha^{2} c(1-b)^{2}}{4(1+b \beta) G-\alpha^{2}(1-b)^{2}}\right) .
$$

We further define the following for the sake of notational simplicity:

$$
\begin{aligned}
& E_{I, A, L}=x_{L}-b x_{F}+\frac{a^{2} \beta(1+\beta)(X-G c)+(b \gamma-g)\left[2(1+b \beta)(X+G c)-a^{2} c(1+\beta)^{2}\right]}{4(1+b \beta) G-a^{2}(1+\beta)^{2}}, \\
& E_{I, A, F}=x_{F}+\beta x_{L}+\frac{a^{2} \beta(1+\beta)(X-G c)-(\gamma+\beta g)\left[2(1+b \beta)(X+G c)-a^{2} c(1+\beta)^{2}\right]}{4(1+b \beta) G-a^{2}(1+\beta)^{2}} \\
& E_{I, B, L}=x_{L}-b x_{F}-\frac{b^{2} \alpha(1-b)(X-G c)-(b \gamma-g)\left[2(1+b \beta)(X+G c)-\alpha^{2} c(1-b)^{2}\right]}{4(1+b \beta) G-\alpha^{2}(1-b)^{2}}, \\
& E_{I, B, F}=x_{F}+\beta x_{L}+\frac{\alpha^{2}(1-b)(X-G c)-(\gamma+\beta g)\left[2(1+b \beta)(X+G c)-\alpha^{2} c(1-b)^{2}\right]}{4(1+b \beta) G-\alpha^{2}(1-b)^{2}} .
\end{aligned}
$$

Proposition 3.4.1 (a) $\omega_{I, A}^{*}$ is feasible if (i) $a^{2}(1+\beta)^{2}<4 G(1+b \beta)$ and $X>G c$;
(b) $\omega_{I, B}^{*}$ and $\bar{\omega}_{I, B}^{*}$ are feasible if (ii) $b<1, \alpha^{2}(1-b)^{2}<4(1+b \beta) B$ and $A>B c$.

Case 1: $N_{I}>0$
According to Proposition 3.3.2, $\lambda^{*}=1$ if $N_{I}>0$. Let $\omega_{I, L}$ be any $\omega$ with $\lambda=1$, $\omega_{I, L} \in \Omega_{I}$ and satisfies $\pi\left(\omega_{I, L}\right)>0$. Then
$D\left(\omega_{I, L}\right)=\frac{X+e a(1+\beta)-G p}{1+b \beta}$, and
$\pi\left(\omega_{I, L}\right)=\frac{p-c}{1+b \beta}[X+e a(1+\beta)-G p]-e^{2}$.
Proposition 3.4.2 For $N_{I}>0$ and given any fixed $p>c$, the optimal advertising effort is given by $e_{I, A}^{*}(p)=\frac{a(1+\beta)}{2(1+b \beta)}(p-c)>0 \quad$ under Tactic $I$.

Proposition 3.4.2 asserts that $e_{I, A}^{*}(p)$ is positive and strictly increasing in $p$ for all $p>c$. In words, a higher advertising supports a higher selling price under Tactic (IA). Let $\omega_{A}(p)=\left(1, e_{A}^{*}(p), p\right)$. We have

$$
\pi\left(\omega_{A}(p)\right)=\frac{a^{2}(1+\beta)^{2}-4 G(1+b \beta)}{4(1+b \beta)^{2}}(p-c)^{2}+\frac{X-G c}{1+b \beta}(p-c) .
$$

Proposition 3.4.3 For $N_{I}>0$, if $\omega^{*} \in \Omega_{\mathrm{I}}$ and $\pi\left(\omega^{*}\right)<+\infty$, then (i) $E_{I, A, L}>0$, (ii)
$E_{I, A, F}>0$ and (iii) $a^{2}(1+\beta)^{2}<4 G(1+b \beta)$ (iv) $X>G c$. Moreover, $\omega^{*}$ is unique and given by $\omega_{I, A}^{*}$.

Proposition 3.4.3 shows that for the case $N_{I}>0, \omega_{I, A}^{*}$ is the unique local optimal $\omega$ of Tactic I. As shown in Proposition 3.3.2, $\lambda^{*}=1$ if $N_{I}>0$. Therefore, $\omega^{*}=\omega_{I, A}^{*}$ (which is consistent to Proposition 3.3.2). There are four necessary optimality conditions for $\omega_{I, A}^{*}$. As we mentioned in Proposition 3.4.1, Conditions (iii) and (iv) of Proposition 3.4.3 ensure that $\omega_{I, A}^{*}$ is feasible. Conditions (i) and (ii) of Proposition 3.4.3 guarantee that $\omega_{\mathrm{I}, \mathrm{A}}^{*} \in \Omega_{\mathrm{I}}$, respectively. Therefore, if any condition of Proposition 3.4.3 is not satisfied, then $\omega_{I, A}^{*}$ is either infeasible or $\omega_{\mathrm{I}, \mathrm{A}}^{*} \notin \Omega_{\mathrm{I}}$, and hence it is not optimal. Therefore, these conditions can be used for the company to check whether Tactic I should be rejected or not if $N_{I}>0$. Specifically, if $N_{I}>0$ and any condition of Proposition 3.4.3 is not fulfilled, then Tactic I is never optimal and should thus be rejected, and the company should choose from Tactics II and III which means selling the product to either LG or FG.

Case 2: $N_{I} \leq 0$
According to Proposition 3.3.2, $\lambda^{*}=0$ if $N_{I}<0$, and $\lambda^{*}$ can take any value between 0 and 1 if $N_{I}=0$ (because the resulting profit remains the same). Therefore, we take $\lambda=0$ for $N_{I}=0$ for simplicity and combine it with $N_{I}<0$ as Case 2 here. Let $\omega_{I, F}$ be any $\omega$ with $\lambda=0, \omega_{\mathrm{I}, \mathrm{F}}^{*} \in \Omega_{\mathrm{I}}$ and satisfies $\pi\left(\omega_{I, F}\right)>0$. Then we have:
$D\left(\omega_{I, F}\right)=\frac{X+e \alpha(1-b)-G p}{1+b \beta}$, and $\pi\left(\omega_{I, F}\right)=\frac{p-c}{1+b \beta}[X+e \alpha(1-b)-G p]-e^{2}$.
Proposition 3.4.4 For $N_{I} \leq 0$ and any fixed $p>c$, the optimal advertising effort is given by $e_{I, B}^{*}(p)=\frac{\alpha(1-b)}{2(1+b \beta)}(p-c)>0$ under Tactic $I$.

Proposition 3.4.4 asserts that $e_{I, B}^{*}(p)$ is positive and strictly increasing in $p$ for all $p>c$, which means that a higher selling price would lead to a higher optimal advertising effort. Let $\omega_{B}(p)=\left(1, e_{I . B}^{*}(p), p\right)$. We have

$$
\pi\left(\omega_{B}(p)\right)=\frac{\alpha^{2}(1-b)^{2}-4 G(1+b \beta)}{4(1+b \beta)^{2}}(p-c)^{2}+\frac{X-G c}{1+b \beta}(p-c) .
$$

Proposition 3.4.5 Suppose that $N_{I} \leq 0$, i.e., $\lambda^{*}=0$. If $\omega^{*} \in \Omega_{\mathrm{I}}$ and $\pi\left(\omega^{*}\right)<+\infty$, then (i) $E_{I, B, 1}>0$, (ii) $E_{I, B, 2}>0$, and (iii) $a^{2}(1+\beta)^{2}<4 G(1+b \beta)$, (iv) $X>G c$. Moreover, $\omega^{*}=\omega_{I, B}^{*}$ is unique if $N_{I}<0$, and $\quad \omega^{*}=\bar{\omega}_{I, B}^{*}$ if $N_{I}=0$.

The interpretation of Proposition 3.4.5 is similar to the interpretation of Proposition 3.4.3. Specifically, Proposition 3.4.5 shows that $\omega_{I, B}^{*}$ is the unique local optimal $\omega$ of Tactic I, for $N_{I}<0$. As shown in Proposition 3.3.2, $\lambda^{*}=0$ if $\mathrm{N}_{I}<0$, and $\lambda^{*}$ can be any value between 0 and 1 if $N_{I}=0$. Therefore, we have $\omega^{*}=\omega_{I, A}^{*}$ and $\omega^{*}=\bar{\omega}_{I, B}^{*}$, for the corresponding situations. These results are consistent with the findings of Proposition 3.3.2. There are four necessary optimality conditions of $\omega_{I, B}^{*}$ and $\bar{\omega}_{I, B}^{*}$. As we mentioned in Proposition 3.4.1, Conditions (iii) and (iv) of Proposition 3.4.5 ensure that $\omega_{I, B}^{*}$ is feasible. For $N_{I}<0$, Conditions (i) and (ii) of Proposition 3.4.5 guarantee that $\omega_{1, \mathrm{~B}}^{*} \in \Omega_{\mathrm{I}}$. Therefore, if any condition of Proposition 3.4.5 is not satisfied, then either $\omega_{I, B}^{*}$ is infeasible, or $\omega_{I, B}^{*}$ does not satisfy $\omega_{\mathrm{I}, \mathrm{B}}^{*} \in \Omega_{\mathrm{I}}$, and hence $\omega_{I, B}^{*}$ is never the optimal solution. Similarly, for $N_{I}=0$, Conditions (i) to (iv) can be used to check the feasibility of $\bar{\omega}_{I, B}^{*}$, and the conditions for $\omega_{\mathrm{I}, \mathrm{B}}^{*} \in \Omega_{\mathrm{I}}$. Therefore, these conditions can be used for the company to check whether Tactic I should be rejected or not if $N_{I} \leq 0$. Lastly, Proposition 3.4.5 shows that $\omega_{I, B}^{*}$ is the unique local optimal $\omega$ of Tactic I for $N_{I}<0$, and there exist multiple local optimal $\omega$ of Tactic I for $N_{I}=0$.

### 3.4.2 Optimal advertising and pricing strategies for Tactic II

For Tactic II, we consider the following three advertising and pricing strategies:
$\omega_{I I, 1}^{*}=\left(0, e_{I I, 1}^{*}, p_{I I, 1}^{*}\right), \omega_{I I, 2}^{*}=\left(0, e_{I I, 2}^{*}, p_{I I, 2}^{*}\right) \omega_{I I, 3}^{*}=\left(0,0, p_{I, 3}^{*}\right)$,
where

$$
\begin{aligned}
& e_{I I, 1}^{*}=\frac{\alpha\left(x_{F}-\gamma c\right)}{4 \gamma-\alpha^{2}}, e_{I, 2}^{*}=\frac{\alpha\left[2 b \gamma\left(x_{L}-g c\right)-2 b g\left(x_{F}-\gamma c\right)+(b \gamma-g) c\right]}{2\left[\alpha^{2} b g+(b \gamma-g)^{2}\right]}, \\
& p_{I I, 1}^{*}=\frac{2 x_{F}+c\left(2 \gamma-\alpha^{2}\right)}{4 \gamma-\alpha^{2}}, p_{I I, 2}^{*}=\frac{\left[\alpha^{2} b-2(b \gamma-g)\right] x_{L}+2 b(b \gamma-g) x_{F}+\alpha^{2} b g c}{2\left[\alpha^{2} b g+(b \gamma-g)^{2}\right]}, \\
& p_{I I, 3}^{*}=\frac{b x_{F}-x_{L}}{b \gamma-g} .
\end{aligned}
$$

Proposition 3.4.6 (a) $\omega_{I I, 1}^{*}$ is feasible if and only if $4 \gamma>\alpha^{2}$. (b) $\omega_{I, 2}^{*}$ is feasible if only if either of the following conditions holds:
(b.1) $\left[\alpha^{2} b-2(b \gamma-g)\right]\left(x_{L}-g c\right)+2 b(b \gamma-g)\left(x_{F}-\gamma c\right)>0$ and
(b.2) $2 b \gamma\left(x_{L}-g c\right)-2 b g\left(x_{F}-\gamma c\right)+(b \gamma-g) c \geq 0$,
(c) $\omega_{I I, 3}^{*}$ is feasible if and only if either of the following condition holds:
(c.l) $b\left(x_{F}-\gamma c\right)>x_{L}-g c$ and $b \gamma \geq g$, or
(c.2) $b\left(x_{F}-\gamma c\right)<x_{L}-g c$ and $b \gamma<g$.

Proposition 3.4.7 If $\omega^{*}$ satisfies $V_{L}\left(\omega^{*}\right)<0, D_{F}\left(\omega^{*}\right)>0$ and $\pi\left(\omega^{*}\right)<+\infty$, then
(i) $\alpha^{2}<4 \gamma$, and (ii) $E_{I I}<0$. Moreover, $\omega^{*}=\omega_{I I, 1}^{*}$ is unique, where
$E_{I I}=\left(4 \gamma-\alpha^{2}\right) x_{L}-2(\gamma b+g) x_{F}+c\left(2 b \gamma^{2}-2 g \gamma+g \alpha^{2}\right)$.
Observe that if any condition of Proposition 3.4.6 is not satisfied, then Tactic II (with $V_{L}(\omega)<0$ ) is never optimal and thus should not be adopted. As shown in Proposition 3.4.6, $\alpha^{2}<4 \gamma$ is the necessary and sufficient condition for $\omega_{I I, 1}^{*}$ to be feasible, and Condition (ii) of Proposition 3.4.7 guarantees that $V_{L}\left(\omega_{I I, 1}^{*}\right)<0$. Moreover, Proposition 3.4.7 shows that $\omega_{I I, 1}^{*}$ is the unique local optimal $\omega$ of Tactic IIA. The optimal advertising effort under Tactic IIA is strictly increasing in $p$. Therefore, a higher price will lead to a bigger optimal advertising effort under Tactic II with $D_{F}(\omega)>0$ and $V_{L}(\omega)<0$.

Proposition 3.4.8 If $\omega^{*}$ satisfies $V_{L}\left(\omega^{*}\right)=0, D_{F}\left(\omega^{*}\right)>0$ and $\pi\left(\omega^{*}\right)<+\infty$, then either (i) $\omega_{I, 2}^{*}$ is feasible, $D_{F}\left(\omega_{I I, 2}^{*}\right)>0$ and $\omega^{*}=\omega_{I, 2}^{*}$; or (ii) $\omega_{I I, 3}^{*}$ is feasible, $D_{F}\left(\omega_{I I, 3}^{*}\right)>0$ and $\omega^{*}=\omega_{I, 3}^{*}$.

Proposition 3.4.8 shows that there are two local optimums $\omega$ for Tactic IIB.

Moreover, the necessary conditions for $\omega^{*}=\omega_{I I, i}^{*}, i=2,3$, under Tactic IIB include the feasibility conditions of $\omega_{I I, i}^{*}$ and the condition $D_{F}\left(\omega_{I I, i}^{*}\right)>0, i=2,3$.

### 3.4.3 Optimal advertising and pricing strategies for Tactic III

Similar to Tactic II, for Tactic III, we consider the following two advertising and pricing strategies:
$\omega_{I I I, 1}^{*}=\left(1, e_{I I, 1}^{*}, p_{I I I, 1}^{*}\right), \omega_{I I I, 2}^{*}=\left(1,0, p_{I I I, 2}^{*}\right)$, where

$$
e_{I I I, 1}^{*}=\frac{a\left(x_{L}-g c\right)}{4 g-a^{2}}, \quad p_{I I I, 1}^{*}=\frac{2 x_{L}+c\left(2 g-a^{2}\right)}{4 g-a^{2}}, \quad p_{I I, 2}^{*}=\frac{x_{F}+\beta x_{L}}{g \beta+\gamma} .
$$

Proposition 3.4.9 (a) $\omega_{I I, 1}^{*}$ is feasible if and only if $4 g>a^{2}$. (b) $\omega_{I I, 2}^{*}$ is feasible.
Proposition 3.4.9 (a) shows the necessary and sufficient conditions that $\omega_{I I, 1}^{*}$ is feasible. Moreover, Proposition 3.4.9 (b) shows that $\omega_{I I I, 2}^{*}$ is always feasible.

Proposition 3.4.10 If $\omega^{*}$ satisfies $V_{F}\left(\omega^{*}\right)<0, D_{L}\left(\omega^{*}\right)>0$ and $\pi\left(\omega^{*}\right)<+\infty$, then
(i) $4 g>a^{2}$, and (ii) $E_{I I I, A}>0$, where
$E_{I I}=\left(4 g-a^{2}\right) x_{F}+2(g \beta-\gamma) x_{F}+c\left(a^{2} \gamma-2 g \gamma-2 g^{2} \beta\right)$.
Moreover, $\omega^{*}=\omega_{I I I, 1}^{*}$ is unique.
If any condition of Proposition 3.4.10 is not satisfied, Tactic IIIA is not optimal and thus should not be adopted. As mentioned in Proposition 3.4.9, $4 g>a^{2}$ is the necessary and sufficient condition for having feasible $\omega_{I I, 1}^{*}$, and Condition (ii) of Proposition 3.4.10 guarantees that $D_{L}\left(\omega_{I I, I}^{*}\right)>0$. Moreover, Proposition 3.4.10 shows that $\omega_{I I I, 1}^{*}$ is the unique local optimal $\omega$ of Tactic IIIA. The optimal advertising effort under Tactic IIIA is strictly increasing in $p$.

Proposition 3.4.11 If $\omega^{*}$ satisfies $V_{F}\left(\omega^{*}\right)=0, D_{L}\left(\omega^{*}\right)>0$ and $\pi\left(\omega^{*}\right)<+\infty$, then $\omega_{I I, 2}^{*}$ is feasible, $D_{L}\left(\omega_{I I, 2}^{*}\right)>0$ and $\omega^{*}=\omega_{I I I, 2}^{*}$.

Proposition 3.4.11 shows that there is a unique local optimum $\omega$ of Tactic IIIB. The necessary conditions for $\omega^{*}=\omega_{I I I, 2}^{*}$ under Tactic IIIB include the feasibility conditions of $\omega_{I I I, 2}^{*}$ and the condition $D_{L}\left(\omega_{I I, 2}^{*}\right)>0$.

### 3.5 The Veblen effects

### 3.5.1 The Veblen effect for Case I.A

We denote $\omega_{A}(p)$ as any feasible $\omega$ with $\lambda=1, e=e_{I, A}^{*}(p)$ and $p>c$, and satisfies $D_{L}\left(\omega_{A}(p)\right)>0$ and $D_{F}\left(\omega_{A}(p)\right)>0$. Then
$D_{L}\left(\omega_{A}(p)\right)=\frac{1}{1+b \beta}\left\{x_{L}-b x_{F}-\frac{a^{2} c(1+\beta)}{2(1+b \beta)}+\left[\frac{a^{2}(1+\beta)}{2(1+b \beta)}+b \gamma-g\right] p\right\}$, and
$D_{F}\left(\omega_{A}(p)\right)=\frac{1}{1+b \beta}\left\{x_{F}+\beta x_{L}-\frac{a^{2} \beta c(1+\beta)}{2(1+b \beta)}+\left[\frac{a^{2} \beta(1+\beta)}{2(1+b \beta)}-\gamma-\beta g\right] p\right\}$.
Clearly, $D_{L}\left(\omega_{A}(p)\right)$ is strictly increasing in $p$ if and only if $\frac{a^{2}(1+\beta)}{2(1+b \beta)}>g-b \gamma$, and $D_{F}\left(\omega_{A}(p)\right)$ is strictly increasing in $p$ if and only if $\frac{a^{2}(1+\beta)}{2(1+b \beta)}>\frac{\gamma}{\beta}+g$. Let $p_{A, L}^{0}$ and $p_{A, F}^{0}$, respectively, satisfy $D_{L}\left(\omega_{A}\left(p_{A, L}^{0}\right)\right)=0$ and $D_{F}\left(\omega_{A}\left(p_{A, F}^{0}\right)\right)=0$.

For $N_{I} \geq 0$, the Veblen effect takes place to both $L G$ and $F G$ if $\frac{a^{2}(1+\beta)}{2(1+b \beta)}>\frac{\gamma}{\beta}+g$. Moreover, $D\left(\omega_{A}(p)\right)$ is increasing in $p$, and $\pi\left(\omega_{A}(p)\right)=(p-c) D\left(\omega_{A}(p)\right) \rightarrow \infty$ as $p \rightarrow \infty$, if $\frac{a^{2}(1+\beta)}{2(1+b \beta)}>\frac{\gamma}{\beta}+g$. Moreover, the feasible region of $p$ for $D_{L}\left(\omega_{A}(p)\right)>0$ and $D_{F}\left(\omega_{A}(p)\right)>0$ is given by $p \geq \max \left\{p_{A, F}^{0}, p_{A, L}^{0}\right\}$. Noting that $e_{I, A}^{*}(p)$ is strictly increasing in $p$ : A bigger $p$ thus implies a higher advertising effort by the company. In addition, a higher advertising effort pushes up the demand of the product. Therefore, the company can increase the demand by putting more advertising efforts. When the gain in demand due to a higher advertising effort by the company is bigger than the loss in demand due to a higher price, the Veblen effect takes place. This happens especially when the sensitivity of the advertising efforts to the demand of LG is big, i.e., $a$ is big.

$$
\text { For } \frac{\gamma}{\beta}+g \geq \frac{a^{2}(1+\beta)}{2(1+b \beta)}>g-b \gamma, \quad D_{L}\left(\omega_{A}(p)\right) \text { is strictly increasing in } p \text { and }
$$

$D_{F}\left(\omega_{A}(p)\right)$ is decreasing in $p$. Veblen effect takes place to LG only for this case. Moreover, the feasible region of $p$ for $D_{L}\left(\omega_{A}(p)\right)>0$ and $D_{F}\left(\omega_{A}(p)\right)>0$ is given by $p_{A, L}^{0} \leq p \leq p_{A, F}^{0}$.

For $\frac{a^{2}(1+\beta)}{2(1+b \beta)} \leq g-b \gamma$, both $D_{L}\left(\omega_{A}(p)\right)$ and $D_{F}\left(\omega_{A}(p)\right)$ are decreasing in $p$. Veblen effect does not take place to both LG and FG for this case because $a$ is small. Moreover, the feasible region of $p$ for $D_{L}\left(\omega_{A}(p)\right)>0$ and $D_{F}\left(\omega_{A}(p)\right)>0$ is given by $p \leq \min \left\{p_{A, F}^{0}, p_{A, L}^{0}\right\}$.

It is impossible for the occurrence of both (i) $D_{L}\left(\omega_{A}(p)\right)$ is decreasing in $p$, and (ii) $D_{F}\left(\omega_{A}(p)\right)$ is increasing in $p$ (because $g-b \gamma \geq \frac{a^{2}(1+\beta)}{2(1+b \beta)}>\frac{\gamma}{\beta}+g$ contradicts to $g-b \gamma<g+\gamma / \beta$ ). Therefore, for $N_{I} \geq 0$, the Veblen effect does not occur in both LG and FG.

Proposition 3.5.1 Under Tactic I and for $N_{I} \geq 0$, (a) if $\frac{a^{2}(1+\beta)}{2(1+b \beta)}>\frac{\gamma}{\beta}+g$, then the Veblen effect occurs in both $L G$ and $F G$, and the feasible region of $p$ is given by $p \geq \max \left\{p_{A, F}^{0}, p_{A, L}^{0}\right\}$; (b) if $\frac{\gamma}{\beta}+g \geq \frac{a^{2}(1+\beta)}{2(1+b \beta)}>g-b \gamma$, then the Veblen effect occurs in $L G$ only, and the feasible region of $p$ is given by $p_{A, L}^{0} \leq p \leq p_{A, F}^{0}$; and (c) if $\frac{a^{2}(1+\beta)}{2(1+b \beta)} \leq g-b \gamma$, then the Veblen effect does not occur in both $L G$ and $F G$, and the feasible region of $p$ is given by $p \leq \min \left\{p_{A, F}^{0}, p_{A, L}^{0}\right\}$.

Proposition 3.5.1 shows the conditions of the occurrence of the Veblen effect and associated feasible regions of $p$ under Tactic I and for $N_{I} \geq 0$ (see also Table 3-2). The feasible region of $p$ for $D_{L}\left(\omega_{A}(p)\right)>0$ and $D_{F}\left(\omega_{A}(p)\right)>0$ may not exist in some cases, and it depends on the values of market parameters.

Table 3-2 The Veblen effect and the feasible region of $p$ under Tactic I for $N_{I} \geq 0$

| Conditions | Veblen effect occurs in | Feasible region of $p$ |
| :--- | :--- | :--- |


| $\frac{a^{2}(1+\beta)}{2(1+b \beta)}>\frac{\gamma}{\beta}+g$ | Both LG and FG | $p \geq \max \left\{p_{A, F}^{0}, p_{A, L}^{0}\right\}$ |
| :---: | :---: | :---: |
| $\frac{\gamma}{\beta}+g \geq \frac{a^{2}(1+\beta)}{2(1+b \beta)}>g-b \gamma$ | LG only | $p_{A, L}^{0} \leq p \leq p_{A, F}^{0}$ |
| $\frac{a^{2}(1+\beta)}{2(1+b \beta)} \leq g-b \gamma$ | No Veblen effect | $p \leq \min \left\{p_{A, F}^{0}, p_{A, L}^{0}\right\}$ |

Next, we investigate how the social influence affects the Veblen effect. First we consider the social influence of LG to FG. For $\beta=0, N_{I} \geq 0$ implies $a \geq \alpha(1-b)$, and we have
$D_{L}\left(\omega_{A}(p)\right)=x_{L}-b x_{F}-\frac{a^{2} c}{2}+\left[\frac{a^{2}}{2}+b \gamma-g\right] p$, and
$D_{F}\left(\omega_{A}(p)\right)=x_{F}-\frac{a^{2} \beta c}{2}-\gamma p$.
Since $\gamma>0, D_{F}\left(\omega_{A}(p)\right)$ is strictly decreasing in $p$. Hence the Veblen effect never occurs in FG under Tactic I for $\beta=0$ and $N_{I} \geq 0$; essentially, if the social influence is excluded, then the Veblen effect will never occur in LG and FG simultaneously under Tactic I for $N_{I} \geq 0$.

Proposition 3.5.2 Under Tactic $I$ and for $\beta=0$ and $N_{I} \geq 0$, if $\frac{a^{2}}{2}>g-b \gamma$, then the Veblen effect occurs in $L G$.

If $b>1$, then $\frac{a^{2}}{2}>\frac{a^{2}(1+\beta)}{2(1+b \beta)}$ for any $\beta>0$. Therefore, Proposition 3.5.2 asserts that, for $b>1$, the social influence of LG to FG loosens the constraint on the occurrence of the Veblen effect in LG under Tactic I. However, for $b<1$, the social influence of LG to FG tightens the constraint on the occurrence of the Veblen effect in LG under Tactic I.

For $b=0, N_{I} \geq 0$ implies $a(1+\beta) \geq \alpha$, and we have
$D_{L}\left(\omega_{A}(p)\right)=x_{L}-\frac{a^{2} c(1+\beta)}{2}+\left[\frac{a^{2}(1+\beta)}{2}-g\right] p$, and
$D_{F}\left(\omega_{A}(p)\right)=x_{F}+\beta x_{L}-\frac{a^{2} \beta c(1+\beta)}{2}+\left[\frac{a^{2} \beta(1+\beta)}{2}-\gamma-\beta g\right] p$.
By examining the demand functions $D_{L}\left(\omega_{A}(p)\right)$ and $D_{F}\left(\omega_{A}(p)\right)$, we can find the condition for the occurrence of the Veblen effect under Tactic I for $b=0$ and $N_{I} \geq 0$.

Proposition 3.5.3 Under Tactic $I$ and for $b=0$ and $N_{I} \geq 0$, (a) if $\frac{a^{2}(1+\beta)}{2}>\frac{\gamma}{\beta}+g$, then the Veblen effect occurs in both $L G$ and $F G$; (b) if $\frac{\gamma}{\beta}+g \geq \frac{a^{2}(1+\beta)}{2}>g$, then the Veblen effect occurs in $L G$ only; and (c) if $\frac{a^{2}(1+\beta)}{2} \leq g$, then the Veblen effect does not occur in both $L G$ and $F G$.

Because $\frac{a^{2}(1+\beta)}{2}>\frac{a^{2}(1+\beta)}{2(1+b \beta)}$, Proposition 3.5.1 and Proposition 3.5.3 show that the social influence of FG to LG loosens the conditions of the occurrence of the Veblen effect in both FG and LG. Moreover, for $b<\frac{\beta g-\gamma}{\beta \gamma}$, we have $(g-b \gamma)(1+b \beta)>g$. Therefore, the social influence of FG to LG increases the chance for the Veblen not to take place in neither LG nor FG for $b<\frac{\beta g-\gamma}{\beta \gamma}$.

### 3.5.2 The Veblen effect for Case I.B

Denote $\omega_{B}(p)$ as any feasible $\omega$ with $\lambda^{*}=0, e=e_{I, B}^{*}(p)$ and $p>c$, and satisfies $D_{L}\left(\omega_{B}(p)\right)>0$ and $D_{F}\left(\omega_{B}(p)\right)>0$. Then
$D_{L}\left(\omega_{B}(p)\right)=\frac{1}{1+b \beta}\left\{x_{L}-b x_{F}+\frac{\alpha^{2} b(1-b) c}{2(1+b \beta)}-\left[\frac{\alpha^{2} b(1-b)}{2(1+b \beta)}-b \gamma+g\right] p\right\}$, and $D_{F}\left(\omega_{B}(p)\right)=\frac{1}{1+b \beta}\left\{x_{F}+\beta x_{L}-\frac{\alpha^{2}(1-b) c}{2(1+b \beta)}+\left[\frac{\alpha^{2}(1-b)}{2(1+b \beta)}-\gamma-\beta g\right] p\right\}$.

Clearly, $D_{L}\left(\omega_{B}(p)\right)$ is strictly increasing in $p$ if and only if $\gamma-\frac{g}{b}>\frac{\alpha^{2}(1-b)}{2(1+b \beta)}$,
and $D_{F}\left(\omega_{B}(p)\right)$ is strictly increasing in $p$ if and only if $\frac{\alpha^{2}(1-b)}{2(1+b \beta)}>\gamma+\beta g$. Let $p_{B, L}^{0}$ and $p_{B, F}^{0}$, respectively, satisfy $D_{L}\left(\omega_{B}\left(p_{B, L}^{0}\right)\right)=0$ and $D_{F}\left(\omega_{B}\left(p_{B, F}^{0}\right)\right)=0$.

For $N_{I} \leq 0, \quad b<1$ and it is impossible to have both $D_{L}\left(\omega_{B}(p)\right)$ and $D_{F}\left(\omega_{B}(p)\right)$ being increasing in $p$ (cf: the mutually exclusive condition for occurrence). Therefore, for $N_{I} \leq 0$, the Veblen effect does not take place to LG and FG simultaneously.

For $\frac{\alpha^{2}(1-b)}{2(1+b \beta)}<\gamma-\frac{g}{b}, D_{L}\left(\omega_{B}(p)\right)$ is strictly increasing in $p$ and $D_{F}\left(\omega_{B}(p)\right)$ is strictly decreasing in $p$. The Veblen effect occurs in LG only for this case. Moreover, the feasible region of $p$ for $D_{L}\left(\omega_{B}(p)\right)>0$ and $D_{F}\left(\omega_{B}(p)\right)>0$ is given by $p_{B, L}^{0} \leq p \leq p_{B, F}^{0}$.

For $\gamma-\frac{g}{b} \leq \frac{\alpha^{2}(1-b)}{2(1+b \beta)} \leq \gamma+\beta g, D_{L}\left(\omega_{B}(p)\right)$ and $D_{F}\left(\omega_{B}(p)\right)$ are decreasing in p. Therefore, the Veblen effect does occur in both LG and FG for this case. Moreover, the feasible region of $p$ for $D_{L}\left(\omega_{B}(p)\right)>0$ and $D_{F}\left(\omega_{B}(p)\right)>0$ is given by $p \leq \min \left\{p_{B, L}^{0}, p_{B, F}^{0}\right\}$.

For $\frac{\alpha^{2}(1-b)}{2(1+b \beta)}>\gamma+\beta g, D_{L}\left(\omega_{B}(p)\right)$ is decreasing in $p$ and $D_{F}\left(\omega_{B}(p)\right)$ is strictly increasing in $p$. The Veblen effect takes place to FG only for this case. Moreover, the feasible region of $p$ for $D_{L}\left(\omega_{B}(p)\right)>0$ and $D_{F}\left(\omega_{B}(p)\right)>0$ is given by $p_{B, F}^{0} \leq p \leq p_{B, L}^{0}$.

Proposition 3.5.4 Under Tactic I and for $N_{I}<0$, (a) the Veblen effect does not occur in both $L G$ and $F G$ simultaneously; (b) if $\frac{\alpha^{2}(1-b)}{2(1+b \beta)}<\gamma-\frac{g}{b}$, then the Veblen effect occurs in $L G$, and the feasible region of $p$ is given by $p_{B, L}^{0} \leq p \leq p_{B, F}^{0}$; (c) if $\gamma-\frac{g}{b} \leq \frac{\alpha^{2}(1-b)}{2(1+b \beta)} \leq \gamma+\beta g$, then the Veblen effect does not occur in both $L G$ and $F G$, and the feasible region of $p$ is given by $p \leq \min \left\{p_{B, L}^{0}, p_{B, F}^{0}\right\}$; and (d) if
$\frac{\alpha^{2}(1-b)}{2(1+b \beta)}>\gamma+\beta g$, then the Veblen effect occurs in $F G$, and the feasible region of $p$ is given by $p_{B, F}^{0} \leq p \leq p_{B, L}^{0}$.

Proposition 3.5.4 shows the conditions of the occurrence of the Veblen effect and associated feasible regions of $p$ under Tactic I and for $N_{I}<0$ (see also Table 4.2). As a remark, the feasible region of $p$ for $D_{L}\left(\omega_{A}(p)\right)>0$ and $D_{F}\left(\omega_{A}(p)\right)>0$ may not exist in some cases, and it depends on the values of market parameters.

Table 3-3 The Veblen effect and the feasible region of p under Tactic I for $N_{I}<0$

| Conditions | Veblen effect occurs in | Feasible region of $p$ |
| :--- | :--- | :--- |
| $\frac{\alpha^{2}(1-b)}{2(1+b \beta)}<\gamma-\frac{g}{b}$ | LG only | $p_{B, L}^{0} \leq p \leq p_{B, F}^{0}$ |
| $\gamma-\frac{g}{b} \leq \frac{\alpha^{2}(1-b)}{2(1+b \beta)} \leq \gamma+\beta g$ | No Veblen effect | $p \leq \min \left\{p_{B, L}^{0}, p_{B, F}^{0}\right\}$ |
| $\frac{\alpha^{2}(1-b)}{2(1+b \beta)}>\gamma+\beta g$ | FG only | $p_{B, F}^{0} \leq p \leq p_{B, L}^{0}$ |

Next, we investigate how the social influence affects the Veblen effect. First we consider the social influence of LG to FG. For $\beta=0, N_{I}<0$ implies $a<\alpha(1-b)$, and we have.
$D_{L}\left(\omega_{B}(p)\right)=x_{L}-b x_{F}+\frac{\alpha^{2} b(1-b) c}{2}-\left[\frac{\alpha^{2} b(1-b)}{2}-b \gamma+g\right] p$, and
$D_{F}\left(\omega_{B}(p)\right)=x_{F}-\frac{\alpha^{2}(1-b) c}{2}+\left[\frac{\alpha^{2}(1-b)}{2}-\gamma\right] p$.
By examining the demand functions $D_{L}\left(\omega_{B}(p)\right)$ and $D_{F}\left(\omega_{B}(p)\right)$, we find the condition of the occurrence of the Veblen effect under Tactic I for $\beta=0$ and $N_{I}<0$.

Proposition 3.5.5 Under Tactic $I$ and for $\beta=0$ and $N_{I}<0$, (a) if $\frac{\alpha^{2}(1-b)}{2}<\gamma-\frac{g}{b}$, then the Veblen effect occurs in $L G$; (b) if $\gamma-\frac{g}{b} \leq \frac{\alpha^{2}(1-b)}{2} \leq \gamma$, then the Veblen effect does not occur in both $L G$ and $F G$; and (c) if
$\frac{\alpha^{2}(1-b)}{2(1+b \beta)}>\gamma+\beta g$, then the Veblen effect occurs in $F G$.
Because $\frac{\alpha^{2}(1-b)}{2}>\frac{\alpha^{2}(1-b)}{2(1+b \beta)}$, Proposition 3.5.5 shows that the social influence of FG to LG loosens the constraint on the occurrence of the Veblen effect in LG under Tactic I with $N_{I}<0$. On the other hand, the social influence of FG to LG tightens the constraints on the occurrence of the Veblen effect in FG under Tactic I with $N_{I}<0$.

For $b=0, N_{I}<0$ implies $a(1+\beta)<\alpha$, and we have

$$
D_{L}\left(\omega_{B}(p)\right)=x_{L}-g p, \text { and } D_{F}\left(\omega_{B}(p)\right)=x_{F}+\beta x_{L}-\frac{\alpha^{2} c}{2}+\left[\frac{\alpha^{2}}{2}-\gamma-\beta g\right] p .
$$

Because $g>0, D_{L}\left(\omega_{B}(p)\right)$ is strictly decreasing in $p$. Hence the Veblen effect never take place in LG under Tactic I for $b=0$ and $N_{I}<0$. Therefore, if the social influence is excluded, then the Veblen effect can never take place in LG under Tactic I, for $N_{I}<0$.

Proposition 3.5.6 Under Tactic $I$ and for $b=0$ and $N_{I}<0$, if $\frac{\alpha^{2}}{2}>\beta g+\gamma$, then the Veblen effect occurs in FG only.

In summary, under Tactic I, the Veblen effect occurs in LG and FG simultaneously only if $N_{I} \geq 0$. If $N_{I}<0$, the Veblen effect only occurs in either LG or FG. Moreover, the social influence of LG to FG loosen the constraint on the occurrence of the Veblen effect for $N_{I} \geq 0$.

### 3.5.3 Veblen effect for Case IIA

Proposition 3.5.7 Under Tactic IIA, the optimal advertising effort for any fixed $p>c$ is given by $e_{I A}^{*}(p)=\alpha(p-c) / 2$.

The optimal advertising effort under Tactic IIA is strictly increasing in $p$. Therefore, a higher price needs a bigger advertising effort to support under Tactic II with $D_{F}(\omega)>0$ and $\bar{D}_{L}(\omega)<0$.

Let $\omega_{I, A}(p)$ be any feasible $\omega$ with $\lambda=0$ and $e=e_{I A}^{*}(p)$, and satisfies $D_{F}\left(\omega_{I A}(p)\right)>0$ and $\bar{D}_{L}\left(\omega_{I I A}(p)\right)<0$. Then $D\left(\omega_{I I A}(p)\right)=x_{F}+\frac{\alpha^{2}-2 \gamma}{2} p-\frac{\alpha^{2} c}{2}$.

Therefore, Veblen effect occurs in FG under Tactic IIA if $\alpha^{2}>2 \gamma$.

### 3.5.4 Veblen effect for Case IIIA

Proposition 3.5.8 Under Tactic IIIA, the optimal advertising effort for any fixed $p>c$ is given by $e_{I I I A}^{*}(p)=a(p-c) / 2$.

The optimal advertising effort under Tactic IIIA is strictly increasing in $p$. Therefore, a higher price needs a bigger advertising effort to support under Tactic IIIA.

Let $\omega_{I I I A}(p)$ be any feasible $\omega$ with $\lambda=1$ and $e=e_{I I A}^{*}(p)$, and satisfies $D_{L}\left(\omega_{I I I A}(p)\right)>0$ and $\bar{D}_{F}\left(\omega_{I I I A}(p)\right)<0$. Then $D\left(\omega_{I I I A}(p)\right)=x_{L}+\frac{a^{2}-2 g}{2} p-\frac{a^{2} c}{2}$.

Therefore, Veblen effect occurs in LG under Tactic IIIA if $a^{2}>2 g$.

### 3.6 Numerical analysis

In this section, we carry out numerical analysis to illustrate the steps to identify the local and global optimal decisions. To be specific, we consider the basic set of parameters as shown in Table 3-4 ${ }^{2}$

Table 3-4 Parameters for the numerical analysis

$$
\begin{aligned}
& X_{L}=100, X_{F}=300, a=0.1, \alpha=1, b=0.5, \beta=10, g=0.05, \gamma=0.5, \\
& c=200 .
\end{aligned}
$$

As we mentioned earlier, we need to examine each local optimal solution for each tactic before we can determine the global optimal solution for the problem. In the following, we examine the local optimal profit under each tactic. For each tactic (including the sub-tactic), we have the respective local optimal solution listed in Table N.2. By comparing the profits under all the local optimal solutions, we can obtain the global optimal solution (shown in bold, in Table 3-5). For this case, the global optimal solution is achieved under Tactic IA.

Table 3-5 Local optimal solution under each tactic

| Tactic | $\lambda$ | $e$ | $p$ | $D_{L}$ | $D_{F}$ | $\pi$ | Feasibility |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IA | $\mathbf{1}$ | $\mathbf{6 6 . 7}$ | $\mathbf{9 2 7 . 1}$ | $\mathbf{2 4}$ | $\mathbf{7 3}$ | $\mathbf{6 6 0 4 2 . 3}$ | Feasible |
| IB | 0 | 28.8 | 890.2 | 19 | 73 | 62696.6 | Feasible |
| IIA | 0 | 200.0 | 600.0 | 0 | 200 | 40000.0 | Feasible |
| IIB1 | 0 | 130.8 | 576.9 | 0 | 142 | 36538.5 | Feasible |
| IIB2 | 0 | 0 | 0 | 0 | 175 | 8750.0 | Feasible |
| IIIA | 1 | 47.4 | 1147.4 | 47 | 0 | 42631.6 | Infeasible |
| IIIB | 1 | 0 | 1300.0 | 35 | 0 | 38500.0 | Feasible |

[^1]In the following, we conduct a sensitivity analysis to explore how the change of parameters affects the optimal decision:
A. Varying $X_{L}$ and $X_{F}$

Table 3-6 shows the optimal tactic with the cases of different $X_{L}$ and $X_{F}$. Tactic IA is optimal when (i) $X_{F}$ and $X_{L}$ are both high, or (ii) $X_{F}$ is low (regardless the size of $X_{L}$ ). The result suggests that in those market situations, it is optimal for the company to sell to both market segments with the respective optimal price and allocation of advertising effort.

Table 3-6 Optimal tactic with changing $X_{L}$ and $X_{F}$

|  | Low | High |
| :--- | :--- | :--- |
| Low | IA | IA |
| High | IIA | IA |


| $X_{L}$ | (level) | $X_{F}$ | (level) | Tactic | $\lambda$ | $e$ | $p$ | $D_{L}$ | $D_{F}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 50 | (low) | 150 | (low) | IA | 1 | 28.4 | 510.2 | 13 | 28 |
| 50 | (low) | 450 | (high) | IIA | 0 | 350 | 900.0 | 0 | 350 |
| 150 | (high) | 150 | (low) | IA | 1 | 95.7 | 1243.2 |  |  |
| 150 | (high) | 450 | (high) | IA | 1 | 104.9 | 1344.0 | 34 | 84 |

For the case when $X_{L}$ is low, if $X_{F}$ increases, the optimal tactic will shift from Tactic IA to Tactic IIA which means it is optimal for the company to target only at FG. This suggests that when $X_{F}$ is large enough to compensate for the potential problem involved with losing the revenue from LG, it is globally optimal for the company to sell and advertise to FG exclusively. However, it is interesting to note that when $X_{L}$ is high: (i) The product price will go up, and (ii) the optimal tactic will still be Tactic IA no matter whether $X_{F}$ is high or low. This is a rather surprising finding and the relative effects brought by $X_{L}$ and $X_{F}$ are not "symmetric".

## B. Varying $a$ and $\alpha$

Table 3-7 shows that the optimal tactics with cases of varied $a$ and $\alpha$. The results show that the company would advertise to a segment which is more responsive to the advertising efforts. Plus, if $a$ and $\alpha$ increase, the respective optimal profits will increase because (i) the corresponding advertising efforts and (ii) the effectiveness of advertisement will both increase.

Table 3-7 Optimal tactics with changing $a$ and $\alpha$

| $\alpha$ | Low | High |
| :--- | :--- | :--- |
| Low | IB | IA |
| High | IB | IA |


| $a$ (level) | $\alpha$ (level) | Tactic | $\lambda$ | $e$ | $p$ | $D_{L}$ | $D_{F}$ | $\pi$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.01 (low) | 0.5 (low) | IB | 0 | 14.2 | 883.5 | 21 | 71 | 62082.3 |
| 0.01 (low) | $1 \quad$ (high) | IB | 0 | 28.8 | 890.2 | 19 | 73 | 62696.6 |
| 0.1 (high) | 0.5 (low) | IA | 1 | 66.6 | 927.1 | 24 | 73 | 66042.3 |
| 0.1 (high) | $1 \quad$ (high) | IA | 1 | 66.6 | 927.1 | 24 | 73 | 66042.3 |

## C. Varying $b$ and $\beta$

Table 3-8 shows the optimal tactics with cases of varied $b$ and $\beta$. From the results, we can see that as $b$ increases, the company's optimal tactic shifts from selling to both segments to only one segment and the company earns less. While as $\beta$ increases, it is optimal for the company to shift from selling to only one segment to both segments and the company can earn more only for the case with a small $b$. These results are interesting and show the asymmetry of the social influences of the two groups of consumers. Moreover, these results demonstrate the significant impacts brought upon by the sensitivities of norms (and hence the influences by social groups) on the optimal tactic choice and optimal resulting benefits.

Table 3-8 Optimal tactics and associated company's profits with changing $b$ and $\beta$

|  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Optimal Tactic |  |  |  |  |  |
| 0.4 |  |  |  |  |  |
| 0.8 | 1.2 |  |  |  |  |
| $\beta=0$ | IB | IIIA | IIIA | IIIA | IIIA |
| 4 | IB | IB | IA | IIIA | IIIA |
| 8 | IB | IA | IA | IIA | IIA |
| 12 | IA | IA | IA | IIA | IIA |
| 16 | IA | IA | IA | IIA | IIA |
| Expected Profit |  |  |  |  |  |
| $\beta=0$ | 70083 | 42632 | 42632 | 42632 | 42632 |
| 4 | 211250 | 60616 | 42647 | 42632 | 42632 |
| 8 | 364321 | 73359 | 42703 | 40000 | 40000 |
| 12 | 644983 | 81774 | 48625 | 40000 | 40000 |
| 16 | 1192390 | 87400 | 49746 | 40000 | 40000 |

## D. Varying $g$ and $\gamma$

Table 3-9 shows that the tactic with the cases of varied $g$ and $\gamma$. The results suggest that the company would like to sell to the market segment with the low price elasticity. It is also interesting to note that when the price sensitivity of $\operatorname{LG}(g)$ is low, Tactic IIIA which focuses on selling to LG becomes the dominating strategy.

Table 3-9 Optimal tactics with changing $g$ and $\gamma$

| $r$ | Low | High |
| :--- | :--- | :--- |
| Low | IIIA | IIA |
| High | IIIA | IA |


| $g$ | (level) | $\gamma \quad$ (level) | Tactic | $\lambda$ | $e$ | $p$ | $D_{L}$ | $D_{F}$ | $\pi$ |
| :--- | :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.01 | (low) | 0.5 (low) | IIIA | 1 | 326.7 | 6733.3 | 65 | 0 | 320133.3 |
| 0.01 | (low) | 1 | (high) | IIIA | 1 | 326.7 | 6733.3 | 65 | 0 |
| 320133.3 |  |  |  |  |  |  |  |  |  |
| 0.1 | (high) | 0.5 (low) | IIA | 0 | 200.0 | 600.0 | 0 | 200 | 40000.0 |
| 0.1 | (high) | $1 \quad$ (high) | IA | 1 | 27.5 | 500.1 | 25 | 55 | 23256.3 |

## E. Changing $c$

Table 3-10 shows the details of optimal tactic in each case with varied $c$. When $c$ is comparably low, the company would focus on FG as it has a big basic demand. As $c$ increases, the company has to target at the whole market to include LG as they can afford a higher price. When $c$ is comparably high, the company would focus on LG as the product with that high price is not so attractive to FG. We can observe that the company can earn more when $c$ goes down. This suggests that despite selling luxury products, a good cost control is still essential for the success of luxury goods company. Furthermore, it is interesting to observe that if $c$ increases, (i) the optimal advertising effort will decrease, and (ii) price will increase. The further implication is that the consumers in the market will also be benefited when $c$ is smaller because the retail selling price is smaller.

Table 3-10 Optimal tactics with changing $c$

| $c$ | Tactic | $\lambda$ | $e$ | $p$ | $D_{L}$ | $D_{F}$ | $\pi$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 10 | IIA | 0 | 295.0 | 600.0 | 0 | 295 | 87025.0 |
| 50 | IA | 1 | 74.0 | 857.1 | 21 | 86 | 81384.1 |
| 100 | IA | 1 | 71.5 | 880.4 | 22 | 82 | 76092.3 |
| 200 | IA | 1 | 66.6 | 927.1 | 24 | 73 | 66042.3 |
| 500 | IA | 1 | 52.0 | 1067.0 | 28 | 47 | 40161.2 |
| 1000 | IIIA | 1 | 26.3 | 1526.3 | 26 | 0 | 13157.9 |

## Chapter 4 Extended model

Advertising not only influences "immediate" purchase but also the long-term brand-equity of the luxury brand and products. Advertising, along with personal experience, is an undeniable force in creating brand equity (Aaker and Bie, 1993). One mechanism of creating brand equity via advertising is by creating and enhancing brand image. With high symbolic value, instead of solely physical vale, luxury fashion brands are considered to rely more on advertising in building brand salience.

In the basic model we explored in Chapter 3, we focus on the operational optimal decisions with respect to the effects of advertising in enhancing buying intention. It is worthwhile to consider advertising's influences to the luxury brand in the long run. Specifically, it would be worth investigating a luxury brand's optimal decision in advertising and pricing, given that a brand has concern on its budget allocation for sustaining the respective brand's role/positioning in the market with respect to both groups of consumers (P.S.: follower group (FG) and leader group (LG)).

In this chapter, we consider the scenario that a certain "basic amount" of advertising effort is taken as desirable for each market segment (FG, LG) in order to maintain the brand strength; otherwise the brand would suffer in the long run due to loss of goodwill of a group of consumers. In particular, in our proposed model below, any advertising effort lower than this basic amount for each market segment will be penalized by the respective linear loss function.

### 4.1 Model construction, assumption and notation

### 4.1.1 Model construction

Similar to Section 3.2, in this section, we consider a company which sells a fashion product to two market segments which demands are inter-dependent. Specifically, there are two groups of customer in the market: the leader group (LG) and the follower group (FG). Demands of LG and FG are
$D_{L}(\omega)=\left[V_{L}(\omega)\right]^{+}$and $D_{F}(\omega)=\left[V_{F}(\omega)\right]^{+}$,
where $V_{L}(\omega)=x_{L}+a \lambda e-b D_{F}(\omega)-g p$,
$V_{F}(\omega)=x_{F}+\alpha(1-\lambda) e+\beta D_{L}(\omega)-\gamma p$, $[Y]^{+}=\max \{0, Y\}$, and $x_{L}, x_{F}, a, \alpha, b, \beta, g, \gamma$ are all non-negative.

### 4.1.2 Assumptions

Assumption 4.1 $x_{L}>g c$ and $x_{F}>\gamma c$.
Similar to Chapter 3's basic model, we employ Assumption 4.1 to make sure that there are positive product demands in both market segments when the product is sold at production cost and the social influences are not considered.

In this chapter, we consider that certain amount of advertising is needed for each market segment in order to maintain the brand strength; otherwise the brand would suffer in the long run due to loss of goodwill of a group of consumers. We hence consider the following linear loss functions which are related to the insufficient advertising for $L G$ and $F G$,
$\Lambda_{L}(e)=m[T-e]^{+}$and $\Lambda_{F}(e)=\mu[\tau-(1-\lambda) e]^{+}$, respectively,
where $T \geq 0$ and $\tau \geq 0$ are the minimum advertising efforts/resources for LG and FG, respectively; and, $m \geq 0$ and $\mu \geq 0$ are the marginal losses due to insufficient
advertising efforts/resources that are assigned for LG and FG, respectively. Moreover, in this section, we consider a more general cost function of advertising effort which is given by

$$
C(e)=h e^{2},
$$

where $h>0$. Therefore, the profit of the company becomes

$$
\begin{align*}
\pi_{L L}(\omega) & =(p-c) D(\omega)-C(e)-\Lambda_{L}(e)-\Lambda_{F}(e), \\
& =(p-c) D(\omega)-h e^{2}-m[T-\lambda e]^{+}-\mu[\tau-(1-\lambda) e]^{+} . \tag{4.1}
\end{align*}
$$

In this section, our objective is to derive the optimal advertising and pricing strategy for the social influence model with linear loss function for insufficient advertising. Mathematically, we consider the following optimization model

$$
(\mathrm{P}-\mathrm{SILL})^{3} \max _{\omega \in \Omega} \pi_{L L}(\omega)=(p-c) D(\omega)-C(e)-\Lambda_{L}(e)-\Lambda_{F}(e),
$$

where $\Omega=\{e \geq 0,0 \leq \lambda \leq 1, p>c\}$.
Denoted by $\omega^{*}=\left\{e^{*}, \lambda^{*}, p^{*}\right\}$ the optimal solution of (P-SILL). To avoid trivial cases, we have Assumption 4.2 below which ensures that there exists at least one profitable optimal decision for (P-SILL)

Assumption 4.2 There exists at least one $\omega$ such that $\pi_{L L}(\omega)>0$ for any given market parameters.

Moreover, we ignore all the cases for which $\pi_{L L}(\omega)<0$. Notice that since $\Lambda_{L}(e)$ and $\Lambda_{F}(e)$ are non-differentiable at $\lambda e=T$ and $(1-\lambda) e=\tau$, respectively, $\pi_{L L}(\omega)$ is non-differentiable at $e=T / \lambda$ and $e=\tau /(1-\lambda)$.

### 4.1.3 Notation

To facilitate the reading, notation and the respective meanings are listed in Table 4-1.

[^2]Table4-1 Notations list B

| $X=(1-b) x_{f}+(1+\beta) x_{L}$ |
| :---: |
| $G=(1-b) \gamma+(1+\beta) g$ |
| $N_{I}=a(1+\beta)-\alpha(1-b)$ |
| $B=X-G c$ |
| $\tilde{X}=X /(1+b \beta)$ |
| $\tilde{G}=G /(1+b \beta)$ |
| $\tilde{N}=N_{I} /(1+b \beta)$ |
| $\tilde{B}=B /(1+b \beta)$ |
| $Y=4 h G(1+b \beta)-a^{2}(1+\beta)^{2}$ |
| $Z=4 h G(1+b \beta)-\alpha^{2}(1-b)^{2}$ |
| $\Theta=4 h G(1+b \beta)-a \alpha(1+\beta)(1-b)$ |

### 4.2 Extended model analysis

### 4.2.1 Cases summary

To deal with the non-differentiable property of $\pi_{L L}(\omega)$, we consider the following four exclusive cases in determining the optimal advertising and pricing strategy:

Case 1: Advertising efforts assigned to both market segments are sufficient, i.e., $\lambda e \geq T$ and $(1-\lambda) e \geq \tau$. For this case, we have $e \geq T+\tau$.

Case 2: Advertising efforts assigned to LG is sufficient but to FG is insufficient, i.e., $\lambda e \geq T$ and $(1-\lambda) e<\tau$. For this case, we have $e \geq T$.

Case 3: Advertising efforts assigned to LG is insufficient but to FG is sufficient, i.e., $\lambda e<T$ and $(1-\lambda) e \geq \tau$. For this case, we have $e \geq \tau$.

Case 4: Advertising efforts assigned to both market segments are insufficient, i.e., $\lambda e<T$ and $(1-\lambda) e<\tau$. For this case, we have $e<T+\tau$.

Moreover, similar to Section 3.3, we consider the following three marketing tactics:

- Tactic I: $D_{L}(\omega)>0$ and $D_{F}(\omega)>0$, the company sells the product to both FG and LG;
- Tactic II: $D_{L}(\omega)=0$ and $D_{F}(\omega)>0$, the company targets at FG and sells the product to FG only;
- Tactic III: $D_{L}(\omega)>0$ and $D_{F}(\omega)=0$, the company tar targets at LG and sells the product to LG only;

As there are four exclusive cases for each marketing tactic, we need to handle 12 mutually exclusive cases in the analysis for this extended model. To facilitate the presentation, we use Tactic $i . j$, for $i=\mathrm{I}$, II and III, and $j=1,2,3,4$ to represent Case $j$ of Tactic $i$. The basic conditions for each tactic are listed in Table 4-2.

Table 4-2 Basic conditions for each tactic

| Basic conditions | $\begin{aligned} & \lambda e \geq T \\ & (1-\lambda) e \geq \tau \\ & \text { and } e \geq T+\tau \end{aligned}$ | $\begin{aligned} & \lambda e \geq T \\ & (1-\lambda) e<\tau \\ & \text { and } e \geq T \end{aligned}$ | $\begin{aligned} & \lambda e<T \\ & (1-\lambda) e \geq \tau, \\ & \text { and } e \geq \tau \end{aligned}$ | $\begin{aligned} & \lambda e<T \\ & (1-\lambda) e<\tau, \\ & \text { and } e<T+\tau \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{ll} \hline D_{L}(\omega)>0 & \text { and } \\ D_{F}(\omega)>0 & \end{array}$ | Tactic I. 1 | Tactic I. 2 | Tactic I. 3 | Tactic I. 4 |
| $\begin{array}{ll} \hline D_{L}(\omega)=0 & \text { and } \\ D_{F}(\omega)>0 & \end{array}$ | Tactic II. 1 | Tactic II. 2 | Tactic II. 3 | Tactic II. 4 |
| $\begin{array}{ll} D_{L}(\omega)>0 & \text { and } \\ D_{F}(\omega)=0 \end{array}$ | Tactic III. 1 | Tactic III. 2 | Tactic III. 3 | Tactic III. 4 |

Similar to Section 3.3, in this section, we investigate the local optimal advertising and pricing strategy of each tactic individually. Moreover, we also explore the associated necessary conditions for optimality of each tactic ${ }^{4}$. The company can use the necessary conditions to screen out the tactics which are not possible to be the global optimal advertising and pricing strategy. For those tactics which satisfy the necessary conditions, the company needs to calculate the company's profit for this tactic. By comparing the company's profit of each tactic that satisfies the necessary condition, the global optimal tactic is obtained. Specifically, the local optimal advertising and pricing strategy which induces the highest company's profit is the global optimal advertising and pricing strategy. In the following, we explore the local optimal solution of each tactic individually.

[^3]
### 4.2.2 Tactic I: Selling to both LG and FG

For Tactic I, product demands of LG and FG satisfy $D_{L}(\omega)>0$ and $D_{F}(\omega)>0$, respectively. Moreover, the total demand of the product is

$$
D(\omega)=\left(B-G(p-c)+\alpha(1-b) e+\lambda N_{I} e\right) /(1+b \beta)
$$

and the associated company's profit is

$$
\begin{aligned}
\pi_{L L}(\omega)= & \left\{-G(p-c)^{2}+(p-c)\left[B+\alpha(1-b) e+\lambda N_{I} e\right]\right\} /(1+b \beta) \\
& -h e^{2}-m[T-\lambda e]^{+}-\mu[\tau-(1-\lambda) e]^{+} .
\end{aligned}
$$

## Tactic I.1:

Basic conditions: $D_{L}(\omega)>0, D_{F}(\omega)>0, \lambda e \geq T,(1-\lambda) e \geq \tau$ and $e \geq T+\tau$.
The company's profit for Tactic I. 1 is

$$
\begin{equation*}
\pi_{L L}(\omega)=\frac{-G(p-c)^{2}+(p-c)\left[B+\alpha(1-b) e+\lambda N_{I} e\right]}{1+b \beta}-h e^{2} . \tag{4.2}
\end{equation*}
$$

We first study the optimal $\lambda$ for Tactic I.1.
Proposition 4.2.1 For Tactic I.1, (a) $\lambda^{*}=T /(T+\tau)$ if $e=T+\tau$; (b) $\lambda^{*}=1-\tau / e$ if $N_{I} \geq 0$ and $e>T+\tau$; and (c) $\lambda^{*}=T / e$ if $N_{I} \leq 0$ and $e>T+\tau$.

Proposition 4.2.1 shows that the value of $\lambda^{*}$ takes different forms under various situations. Specifically, if the company wants to assign the advertising efforts such that $e=T+\tau$, then $\lambda^{*}=T /(T+\tau)$ and hence $\lambda^{*} e=T$ and $\left(1-\lambda^{*}\right) e=\tau$. In words, the optimal advertising efforts assigned to both market segments are just sufficient for $e=T+\tau$. On the other hand, if the company wants to assign the advertising efforts such that $e>T+\tau$, then the company should check the value of $N_{I}$ first. If $N_{I} \geq 0$, then $\lambda^{*}=1-\tau / e$ and $\lambda^{*} e>T$, namely, the advertising efforts assigned to LG is higher than the minimum requirement but the advertising efforts assigned to FG is just sufficient for $e>T+\tau$ and $N_{I} \geq 0$. If $N_{I} \leq 0$, then $\lambda^{*} e=T$ and $\left(1-\lambda^{*}\right) e>\tau$, namely, the advertising efforts assigned to FG is higher than the minimum requirement but the advertising effort assigned to LG is just sufficient. As a remark, $N_{I}$ represents the sensitivity of $\lambda$ to product demand. Therefore, a bigger $\lambda$ advances product demand as well as company's profit if $N_{I}$ is positive, a smaller $\lambda$ advances product demand as well as company's profit if $N_{I}$ is negative, and the
value of $\lambda$ does not affect product demand and company's profit if $N_{I}=0$.
By following Proposition 4.2.1, we further consider three sub-tactics of Tactic I.1:
(Tactic I.1.a) $\lambda^{*}=T /(T+\tau)$ and $e^{*}=T+\tau$;
(Tactic I.1.b) $\lambda^{*}=1-\tau / e^{*}, N_{I} \geq 0$ and $e^{*}>T+\tau$; and
(Tactic I.1.c) $\lambda^{*}=T / e^{*}, N_{I} \leq 0$ and $e^{*}>T+\tau$.
Notice that the above mentioned conditions are specific for the associated sub-tactics. In particular, the basic conditions of Tactic I.1, $\lambda e \geq T,(1-\lambda) e \geq \tau$ and $e \geq T+\tau$, are covered by individual's specific conditions of each sub-tactic. Therefore, we need to consider the remaining basic conditions of Tactic I.1, $D_{L}(\omega)>0$ and $D_{F}(\omega)>0$ for all sub-tactics of Tactic I.1.

Denoted by $\omega_{1,1, i}^{*}$ the local optimal advertising and pricing strategy for Tactic I.1.i, where $i=a, b, c$. Next, we explore the local optimal advertising and pricing strategies for each sub-tactic of Tactic I.1.

- Tactic I.1.a

Specific conditions: $D_{L}(\omega)>0, D_{F}(\omega)>0, \lambda^{*}=T /(T+\tau)$ and $e^{*}=T+\tau$.
By putting $e^{*}=T+\tau$ and $\lambda^{*}=T /(T+\tau)$ into (2), the company's profit for Tactic I.1.a becomes
$\pi_{L L}(\omega)=\frac{-G(p-c)^{2}+(p-c)[B+a(1+\beta) T+\alpha(1-b) \tau]}{1+b \beta}-h(T+\tau)^{2}$.
Notice that right hand side of (4.2) depends on $p$ only.
Proposition 4.2.2 For Tactic I.1.a, the local optimal advertising and pricing strategy exists only if (i) $D_{L}\left(\omega_{I .1 . a}^{*}\right)>0$ and $D_{F}\left(\omega_{I .1 . a}^{*}\right)>0$; (ii) $G>0$; and (iii) $B+a(1+\beta) T+\alpha(1-b) \tau>0$. Moreover, if the local optimal advertising and pricing strategy for Tactic I.1.a exists, then it is unique and is given by

$$
\omega_{I .1 . a}^{*}=\left\{e_{I .1 . a}^{*}=T+\tau, \lambda_{I .1 . a}^{*}=T /(T+\tau), p_{I .1 . a}^{*}=\frac{B+a(1+\beta) T+\alpha(1-b) \tau}{2 G}+c\right\}, \quad \text { and }
$$

$$
\begin{equation*}
\pi_{L L}\left(\omega_{I, 1, a}^{*}\right)=\frac{[B+a(1+\beta) T+\alpha(1-b) \tau]^{2}}{4 G(1+b \beta)}-h(T+\tau)^{2} . \tag{4.4}
\end{equation*}
$$

The necessary conditions for the existence of $\omega_{I, 1 . a}^{*}$ are shown in Proposition
4.2.2. Specifically, conditions in item (i) of Proposition 4.2.2 are the basic conditions for any sub-tactic of Tactic I.1. Conditions in item (ii) of Proposition 4.2.2 ensure that the company's profit function is concave in $p$. Item (iii) of Proposition 4.2.2 ensures that the constraint $p>c$ is satisfied. Moreover, Proposition 4.2 .2 shows the explicit formulas of the local optimal advertising and pricing strategy (if it exists), and the associated company's profit for Tactic I.1.a. Furthermore, by considering the sensitivity of $T$ to $e_{I .1, a}^{*}, \lambda_{I .1, a}^{*}$ and $p_{I .1, a}^{*}$, we find that $e_{I, 1, a}^{*}, \lambda_{I, 1, a}^{*}$ and $p_{I, 1, a}^{*}$ are all strictly increasing in $T$ (because $d e_{I .1 . a}^{*} / d T=1>0, d \lambda_{I .1 . a}^{*} / d T=\tau /(T+\tau)^{2}>0$ and
$\left.d p_{I .1 . a}^{*} / d T=a(1+\beta) /(2 G)>0\right)$. In other words, a bigger $T$ induces a higher advertising effort, a bigger portion of advertising efforts is allocated to LG, and a higher local optimal retail price of the product appears in the local optimum of Tactic I.1.a. On the other hand, $e_{I .1 . a}^{*}$ is strictly increasing in $\tau, \lambda_{I .1 . a}^{*}$ is strictly decreasing in $\tau$ and $p_{I .1 . a}^{*}$ is strictly increasing in $\tau$ for $b<1$ and $p_{I .1 . a}^{*}$ is strictly decreasing in $\tau$ for $b>1$. This shows that $T$ and $\tau$ affect $\omega_{I, 1, a}^{*}$ differently. Interestingly, the value of $m$ and $\mu$ do not affect $\omega_{I .1, a}^{*}$ and $\pi_{L L}\left(\omega_{I, 1, a}^{*}\right)$. The intuitive reason for it is that the advertising effort assigned to both market segments are sufficient, so the penalties $m$ and $\mu$ for insufficient advertisings can be ignored.

- Tactic I.1.b

Specific conditions: $D_{L}(\omega)>0, D_{F}(\omega)>0, \lambda^{*}=1-\tau / e^{*}, N_{I} \geq 0$ and $e^{*}>T+\tau$.
By putting $\lambda^{*}=1-\tau / e^{*}$ into (4.2), the profit of the company for Tactic I.1.b becomes

$$
\begin{equation*}
\pi_{L L}(\omega)=\frac{-G(p-c)^{2}+(p-c)\left[B+a(1+\beta) e-\tau N_{I}\right]}{1+b \beta}-h e^{2} . \tag{4.6}
\end{equation*}
$$

Proposition 4.2.3 For Tactic I.1.b, the local optimal advertising efforts (as a function of $p$ ) is given by
$e_{I .1 . b}^{*}(p)=\frac{a(1+\beta)}{2 h(1+b \beta)}(p-c)$,
and $e_{I .1 . b}^{*}(p)$ is strictly increasing in $p$
Proposition 4.2.3 asserts that, for Tactic I.1.b, a higher retail price induces a high optimal advertising effort. Results of Proposition 4.2.3 are not surprising especially
for luxury products. Advertising usually will provide surpluses to luxury products, and the surpluses of the luxury product are reflected by a higher retail price of the luxury product.
Proposition 4.2.4 For Tactic I.1.b, the local optimal advertising and pricing strategy exists only if (i) $N_{I} \geq 0, \quad D_{L}\left(\omega_{I .1 . b}^{*}\right)>0$ and $D_{F}\left(\omega_{I .1 . b}^{*}\right)>0$; (ii) $Y>0$; (iii) $B>Y(T+\tau) /[a(1+\beta)]+N_{I} \tau$. Moreover, if the local optimal advertising and pricing strategy for Tactic I.1.b exists, then it is unique and is given by
$\omega_{I .1 . b}^{*}=\left\{e_{I .1 . b}^{*}, \lambda_{I .1 . b}^{*}, p_{I .1 . b}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I .1 . b}^{*}\right)=h\left(B-N_{I} \tau\right)^{2} / Y$,
where $e_{I, 1 . b}^{*}=a(1+\beta)\left(B-N_{I} \tau\right) / Y, \quad \lambda_{I, 1 . b}^{*}=1-\tau / e_{I, 1 . b}^{*}$, and
$p_{\text {I.i.b }}^{*}=c+2 h\left(B-N_{I} \tau\right)(1+b \beta) / Y$.
The necessary conditions for $\omega_{I, 1 . b}^{*}$ being finite are shown in Proposition 4.2.4. Specifically, conditions in item (i) of Proposition 4.2.4 are the specific conditions of Tactic I.1.b. The condition in item (ii) of Proposition 4.2.4 ensures that the profit function of the company for Tactic I.1.b is concave in $p$. The condition in item (iii) of Proposition 4.2.4 ensures $e>T+\tau$ which is another basic condition for Tactic I.1.b. Notice that $p_{I .1 . b}^{*}>c$ if and only if $B>N_{I} \tau$. Because $B>Y(T+\tau) /[a(1+\beta)]+N_{I} \tau$ implies $B>N_{I} \tau$, the condition in item (iii) of Proposition 4.2.4 does cover the constraint $p>c$. Moreover, Proposition 4.2.4 provides the explicit formulas of the local optimal advertising and pricing strategy (if it exists), and the associated company's profit for Tactic I.1.b.

According to Proposition 4.2.4, $e_{I .1 . a}^{*}, \lambda_{I .1 . a}^{*}, p_{I, 1 . a}^{*}$ and $\pi_{L L}\left(\omega_{I .1 . b}^{*}\right)$ are decreasing in $\tau\left(d \lambda_{I .1 . a}^{*} / d T=-\frac{Y B}{a(1+\beta)\left(B-N_{I} \tau\right)^{2}}<0\right.$ and the other derivatives are trivial), but independent of $T$. In words, for Tactic I.1.b, a bigger $\tau$ induces a lower advertising effort, a smaller portion of advertising efforts is allocated to LG, and a lower optimal retail price of the product. However, $T$ does not affect $e_{I .1 . a}^{*}, \lambda_{I .1 . a}^{*}$, $p_{I .1 . a}^{*}$ and $\pi_{L L}\left(\omega_{I .1 . b}^{*}\right)$.

Finally, similar to Tactic I.1.a, the value of $m$ and $\mu$ do not affect $\omega_{I .1, b}^{*}$ and $\pi_{L L}\left(\omega_{I, 1, b}^{*}\right)$, as the advertising budgets assigned to both market segments are sufficient
and hence the penalties $m$ and $\mu$ for insufficient advertisings can be ignored for Tactic I.1.b.

- Tactic I.1.c

Specific conditions: $D_{L}(\omega)>0, D_{F}(\omega)>0, N_{I} \leq 0, e^{*}>T+\tau$ and $\lambda^{*}=T / e^{*}$.
First of all, $\quad N_{I} \leq 0$ is equivalent to $a(1+\beta) \leq \alpha(1-b)$ which implies $b<1$ as $a(1+\beta)>0$. Then, by putting $\lambda^{*}=T / e^{*}$ into (4.2), the profit of the company for Tactic I.1.c becomes

$$
\begin{equation*}
\pi_{L L}(\omega)=\frac{-G(p-c)^{2}+(p-c)\left[B+\alpha(1-b) e+T N_{I}\right]}{1+b \beta}-h e^{2} . \tag{4.10}
\end{equation*}
$$

Proposition 4.2.5 For Tactic I.1.c, the local optimal advertising effort (as a function of $p$ ) is given by

$$
\begin{equation*}
e_{I .1 . c}^{*}(p)=\frac{\alpha(1-b)}{2 h(1+b \beta)}(p-c), \tag{4.11}
\end{equation*}
$$

and $e_{I .1 . c}^{*}(p)$ is strictly increasing in $p$.
Similar to Proposition 4.2.3, Proposition 4.2 .5 shows that the optimal advertising effort is increasing in the optimal retail price for Tactic I.1.c.

Proposition 4.2.6 For Tactic I.1.c, the local optimal advertising and pricing strategy exists only if (i) $N_{I} \leq 0, \quad D_{L}\left(\omega_{I .1 . c \mid}^{*}\right)>0$ and $D_{F}\left(\omega_{I .1 . c}^{*}\right)>0$; (ii) $Z>0$; (iii) $B>Z(T+\tau) /[\alpha(1-b)]-N_{I} T$. Moreover, if the local optimal advertising and pricing strategy for Tactic I.1.c exists, then it is unique and is given by
$\omega_{I .1 . c}^{*}=\left\{e_{I, 1, c}^{*}, \lambda_{I .1 . c}^{*}, p_{I .1 . c}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I .1 . c}^{*}\right)=h\left(B+N_{I} T\right)^{2} / Z$,
where $e_{I .1 . c}^{*}=\frac{\alpha(1-b)\left(B+N_{I} T\right)}{Z}, \quad \lambda_{I .1 . c}^{*}=\frac{T Z}{\alpha(1-b)\left(B+N_{I} T\right)}$, and
$p_{I .1 . c}^{*}=\frac{2 h\left(B+N_{I} T\right)(1+b \beta)}{Z}+c$.
Similar to Proposition 4.2.4, the necessary conditions for $\omega_{I .1 . c}^{*}$ being finite are shown in Proposition 4.2.6. Specifically, conditions of item (i) of Proposition 4.2.6 are the specific conditions of Tactic I1.c. Condition in item (ii) of Proposition 4.2.6 ensures that the profit function of the company for Tactic I.1.c is concave in $p$. Condition in item (iii) of Proposition 4.2.6 ensures $e>T+\tau$ which is another basic
condition for Tactic I.1.c. Notice that $p_{I . i . c}^{*}>c$ if $B>Z(T+\tau) /[\alpha(1-b)]-N_{I} T$. Moreover, Proposition 4.2.6 provides the explicit formulas of the local optimal advertising and pricing strategy (if it exists), and the associated company's profit for Tactic I.1.c.

According to Proposition 4.2.4, $\pi_{L L}\left(\omega_{I, 1 . c}^{*}\right), e_{I .1 . c}^{*}$ and $p_{I .1 . c}^{*}$ are decreasing in $T$, while $\lambda_{I .1 . c}^{*}$ is increasing in $T$. This shows that the company will spend less on advertising, will be more focused on LG, and will set a lower retail price of the product when $T$ increases. Although there is less expenditure spent on advertising, the company still loses some profit due to a lower retail price of the product is offered to the customers. On the other hand, $e_{I .1 . c}^{*}, \lambda_{\text {I.1.c }}^{*}$ and $p_{I .1 . c}^{*}$ are independent of $\tau$. Similarly, these findings are caused by the technical issue, as $T$ does not include in any specific condition of Tactic I.1.c.

Finally, similar to Tactic I.1.a and Tactic I.1.b, the values of $m$ and $\mu$ do not affect $\omega_{I .1 . c}^{*}$ and $\pi_{L L}\left(\omega_{I .1 . c}^{*}\right)$, as the advertising efforts assigned to both market segments are sufficient and hence the penalties $m$ and $\mu$ for insufficient advertisings can be ignored for Tactic I.1.c.

## Tactic I. 2

Basic conditions: $D_{L}(\omega)>0, D_{F}(\omega)>0, \lambda e \geq T,(1-\lambda) e<\tau$ and $e \geq T$.
The company's profit for Tactic I. 2 is

$$
\begin{equation*}
\pi_{L L}(\omega)=\frac{-G(p-c)^{2}+(p-c)\left[B+\alpha(1-b) e+\lambda N_{I} e\right]}{1+b \beta}-h e^{2}-\mu[\tau-(1-\lambda) e] . \tag{4.14}
\end{equation*}
$$

Proposition 4.2.7 For Tactic I.2, (a) the local optimal advertising and pricing strategy does not satisfy $N_{I}(p-c)<\mu(1+b \beta)$ and $e \geq T+\tau$. (b) $\lambda^{*}=1$ if $e=T$;
(c) $\lambda^{*}=1$ if $N_{I}(p-c)>\mu(1+b \beta)$ and $e>T$; (d) $\lambda^{*}=T / e \quad$ if $N_{I}(p-c)<\mu(1+b \beta)$ and $T<e<T+\tau$; and (e) multiple $\lambda^{*}$ exist if $N_{I}(p-c)=\mu(1+b \beta)$ and $e>T$.

Proposition 4.2.7 asserts that we can ignore all the cases for which $N_{I}(p-c)<\mu(1+b \beta)$ and $e \geq T+\tau$, under Tactic I.2. Proposition 4.2.7 shows that the value of $\lambda^{*}$ varies under different situations. Specifically, if the company wants
to assign the advertising efforts such that $e=T$, we have $\lambda^{*}=1$. However, if the company wants to assign the advertising efforts such that $e>T$, then the company should take into account of the values of $N_{I}(p-c)$ and $\mu(1+b \beta)$. Specifically, if $N_{I}(p-c)>\mu(1+b \beta)$, then $\lambda^{*}=1$; else if $N_{I}(p-c)<\mu(1+b \beta)$, then $\lambda^{*}=T / e$; else if $N_{I}(p-c)=\mu(1+b \beta)$, then for any given $e>T$, any $\lambda$ that satisfies $(1-\lambda) e<\tau$ and $\lambda e \geq T$ is optimal for Tactic I.2. In words, if $N_{I}(p-c)>\mu(1+b \beta)$, then the company should assign the advertising efforts to LG only. However, if $N_{I}(p-c)<\mu(1+b \beta)$, then the company should assign to LG the advertising effort which just covers $T$, and the company should assign to FG the advertising effort which is less $\tau$. Lastly, if $N_{I}(p-c)=\mu(1+b \beta)$, then the company can select any $\lambda$ that satisfies $(1-\lambda) e<\tau$ and $\lambda e \geq T$. As $\lambda=1$ satisfies $(1-\lambda) e<\tau$ and $\lambda e \geq T$ for any given $e>T$, we take $\lambda^{*}=1$ for $N_{I}(p-c)=\mu(1+b \beta)$. According to the above discussions, we further consider four sub-tactics of Tactic I.2:
(Tactic I.2.a) $e=T, \lambda^{*}=1$;
(Tactic I.2.b) $N_{I}(p-c)>\mu(1+b \beta), e>T, \lambda^{*}=1$;
(Tactic I.2.c) $N_{I}(p-c)=\mu(1+b \beta), e>T, \lambda^{*}=1$; and
(Tactic I.2.d) $N_{I}(p-c)<\mu(1+b \beta)$ and $T<e<T+\tau, \lambda^{*}=T / e$.
The above mentioned conditions are specific for the associated sub-tactics. In particular, the basic conditions of Tactic I. $2, \lambda e \geq T,(1-\lambda) e<\tau$ and $e \geq T$, are covered by individual's specific conditions of each sub-tactic. Therefore, we need to consider the remaining basic conditions of Tactic I.2, $D_{L}(\omega)>0$ and $D_{F}(\omega)>0$ for all sub-tactics of Tactic I.2.

Denoted by $\omega_{I .2 . i}^{*}$, for $i=a, b, c, d$, the local optimal advertising and price strategy for Tactic I.2.i. Next, we explore the local optimal advertising and pricing strategies for each sub-tactic individually.

- Tactic I.2.a

Specific condition: $D_{L}(\omega)>0, D_{F}(\omega)>0, e=T$ and $\lambda^{*}=1$.
By putting $e=T$ and $\lambda^{*}=1$ into (4.14), the profit of the company for Tactic I.2.a becomes
$\pi_{L L}(\omega)=\frac{-G(p-c)^{2}+(p-c)[B+a(1+\beta) T]}{1+b \beta}-h T^{2}-\mu \tau$.
Proposition 4.2.8 For Tactic I.2.a, the local optimal advertising and pricing strategy exists only if (i) $D_{L}\left(\omega_{I .2 . a}^{*}\right)>0$ and $D_{F}\left(\omega_{I, 2 . a}^{*}\right)>0$; (ii) $G>0$; and (iii) $B+a(1+\beta) T>0$. Moreover, if the local optimal advertising and pricing strategy for Tactic I.2.a exists, then it is unique and is given by

$$
\begin{align*}
& \omega_{I .2 . a}^{*}=\left\{e_{I .2 . a}^{*}=T, \lambda_{I .2 . a}^{*}=1, p_{I .2 . a}^{*}=c+[B+a(1+\beta) T] /(2 G)\right\}, \text { and }  \tag{4.16}\\
& \pi_{L L}\left(\omega_{I .2 . a}^{*}\right)=\frac{B^{2}+2 B a(1+\beta) T-Y T^{2}}{4 G(1+b \beta)}-\mu \tau . \tag{4.17}
\end{align*}
$$

The necessary conditions for $\omega_{I .2 . a}^{*}$ being finite are shown in Proposition 4.2.8. Specifically, conditions in item (i) of Proposition 4.2.8 are the basic conditions of Tactic I.2. Condition in item (ii) of Proposition 4.2.8 ensures that the profit function of the company for Tactic I.2.a is concave in $p$. Condition in item (iii) of Proposition 4.2.8 ensures that $p_{1.2 . a}^{*}>c$. Moreover, Proposition 4.2 .8 provides the explicit formulas of the local optimal advertising and pricing strategy, and the associated company's profit for Tactic I.2.a. Furthermore, $e_{I .2 . a}^{*}$ and $p_{I .2 . a}^{*}$ are strictly increasing in $T$. In other words, for Tactic I.2.a, a bigger $T$ induces a higher optimal advertising effort and a higher retail price of the product. However, according to Proposition 4.2.8, we know that $e_{I .2 . a}^{*}$ and $p_{I .2 . a}^{*}$ are independent of $\tau$.

## - Tactic I.2.b

Specific conditions: $D_{L}(\omega)>0, \quad D_{F}(\omega)>0, \quad N_{I}(p-c)>\mu(1+b \beta), \quad e>T$ and $\lambda^{*}=1$.

By putting $\lambda^{*}=1$ into (4.14), the profit of the company for Tactic I.2.b becomes

$$
\begin{equation*}
\pi_{L L}(\omega)=\frac{-G(p-c)^{2}+(p-c)[B+a(1+\beta) e]}{1+b \beta}-h e^{2}-\mu \tau . \tag{4.18}
\end{equation*}
$$

As $p>c$ and $\mu(1+b \beta)>0, N_{I}(p-c)>\mu(1+b \beta)$ implies $N_{I}>0$.
Proposition 4.2.9 For Tactic I.2.b, the local optimal advertising effort is given by
$e_{I, 2 . b}^{*}(p)=e_{I, 1 . b}^{*}(p)=a(1+\beta)(p-c) /[2 h(1+b \beta)]$,
and $e_{I .2 . b}^{*}(p)$ is strictly increasing in $p$.
Proposition 4.2.9 shows that, for Tactic I.2.b, high retail price should be supported by high advertising effort of the company. Results of Proposition 4.2.9 are
similar to Proposition 4.2.3 and Proposition 4.2.5.
Proposition 4.2.10 For Tactic I.2.b, the local optimal advertising and pricing strategy exists only if (i) $N_{I}>0, \quad D_{L}\left(\omega_{I .2 . b}^{*}\right)>0$ and $D_{F}\left(\omega_{I .2 . b}^{*}\right)>0$; (ii) $Y>0$; (iii) $B>T Y /[a(1+\beta)]$; and (iv) $B \geq \mu Y /\left(2 h N_{I}\right)$. Moreover, if the local optimal advertising and pricing strategy for Tactic I.2.b exists, then it is unique and is given by
$\omega_{I .2 . b}^{*}=\left\{e_{I .2 . b}^{*}, \lambda_{I .2 . b}^{*}, p_{I .2 . b}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I .2 . b}^{*}\right)=h B^{2} / Y-\mu \tau$,
where $e_{I, 2, b}^{*}=a(1+\beta) B / Y, \quad \lambda_{I, 2, b}^{*}=1$ and $p_{I, 2, b}^{*}=2 h B(1+b \beta) / Y+c$.
The necessary conditions for $\omega_{I, 2 . b}^{*}$ being finite are shown in Proposition 4.2.10. Specifically, conditions in item (i) of Proposition 4.2.10 are the specific conditions of Tactic I.2.b. Item (ii) of Proposition 4.2.10 ensures that the profit function of the company for Tactic I.2.b is concave in $p$. Item (iii) of Proposition 4.2.10 ensures $e>T$ which is the basic condition for Tactic I.2.b. Item (iv) of Proposition 4.2.10 ensures that $N_{I}(p-c)>\mu(1+b \beta)$ which is another basic condition for Tactic I.2.b. Noting that Item (iii) of Proposition 4.2.10 already covers the constraint $p_{I .2 . b}^{*}>c$. Moreover, Proposition 4.2.10 provides the explicit formulas of the local optimal advertising and pricing strategy and the associated company's profit for Tactic I.2.b. Furthermore, $\pi_{L L}\left(\omega_{1.2, b}^{*}\right)$ is strictly decreasing in $\tau$ and $\mu$, but is independent of $T$ and $m$.

Interestingly, as shown by Proposition 4.2.10, $e_{I .2 . b}^{*}, \lambda_{I .2 . b}^{*}$ and $p_{\text {I.2.b }}^{*}$ are independent of $\tau$ and $T$. These findings are different from many previous findings in which both the optimal advertising effort and the optimal retail price depend on either $\tau$ or $T$. On the other hand $e_{I .2 . b}^{*}, \lambda_{I .2 . b}^{*}$ and $p_{I .2 . b}^{*}$ are independent of $\mu$ and $m$.

## - Tactic I.2.c

Specific conditions: $D_{L}(\omega)>0, D_{F}(\omega)>0, N_{I}(p-c)=\mu(1+b \beta), e>T$ and $\lambda^{*}=1$.
As $p>c$ and $\mu(1+b \beta)>0, N_{I}(p-c)=\mu(1+b \beta)$ implies $N_{I}>0$.
Proposition 4.2.11 For Tactic I.2.c the local optimal advertising effort as a function of $p$ is given by

$$
\begin{equation*}
e_{I, 2 . c}^{*}(p)=e_{I, 2 . b}^{*}(p)=a(1+\beta)(p-c) /[2 h(1+b \beta)], \tag{4.22}
\end{equation*}
$$

and $e_{\text {I.2.c }}^{*}(p)$ is strictly increasing in $p$.
Proposition 4.2.11 asserts that, for Tactic I.2.c, a high retail price should be supported by a high advertising effort of the brand. Results of Proposition 4.2.11 are similar to Proposition 4.2.9.

Proposition 4.2.12 For Tactic I.1.a, the local optimal advertising and pricing strategy exists only if (i) $N_{I}>0, \quad D_{L}\left(\omega_{I .2 . c}^{*}\right)>0$ and $D_{F}\left(\omega_{I .2 . c}^{*}\right)>0$; (ii) $a \mu(1+\beta)>2 h N_{I} T$. Moreover, if the local optimal advertising and pricing strategy for Tactic I.2.c exists, then it is unique and is given by
$\omega_{I .2, c}^{*}=\left\{e_{I .2 . c}^{*}, \lambda_{I .2 . c}^{*}, p_{I .2 . c}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I .2 . c}^{*}\right)=\frac{B \mu}{N_{I}}-\frac{Y \mu^{2}}{4 h N_{I}^{2}}-\mu \tau$,
where $e_{I .2 . c}^{*}=a \mu(1+\beta) /\left(2 h N_{I}\right), \quad \lambda_{I .2 . c}^{*}=1$ and $p_{I .2 . c}^{*}=c+\mu(1+b \beta) / N_{I}$.
The necessary conditions for $\omega_{I .2 . c}^{*}$ being finite are shown in Proposition 4.2.12. Specifically, conditions in item (i) of Proposition 4.2 .12 are the specific conditions of Tactic I.2.c. Item (ii) of Proposition 4.2.10 ensures $e>T$ which is the specific condition for Tactic I.2.c. Moreover, Proposition 4.2.12 provides the explicit formulas of the local optimal advertising and pricing strategy and the associated company's profit for Tactic I.2.c. Furthermore, $\pi_{L L}\left(\omega_{I .2 . c}^{*}\right)$ is strictly decreasing in $\tau$ and independent of $T$ and $m$.

As shown in Proposition 4.2.12, $e_{I .2 . c}^{*}, \lambda_{I .2 . c}^{*}$ and $p_{I .2 . c}^{*}$ are independent of $\tau$ and $T$. These findings are similar to Tactic I.2.b. On the other hand $e_{I .2 . c}^{*}$ and $p_{I .2 . c}^{*}$ are increasing in $\mu$ but are independent of $m$. Moreover, $\lambda_{I .2 . c}^{*}$ is independent of $\mu$ and $m$.

- Tactic I.2.d

Specific conditions: $D_{L}(\omega)>0, D_{F}(\omega)>0, N_{I}(p-c)<\mu(1+b \beta), T<e<T+\tau$ and $\lambda^{*}=T / e$.

By putting $\lambda^{*}=T / e$ into (4.14), the profit of the company for Tactic I.2.d becomes
$\pi_{L L}(\omega)=\frac{-G(p-c)^{2}+(p-c)\left[B+\alpha(1-b) e+T N_{I}\right]}{1+b \beta}-h e^{2}-\mu[T+\tau-e]$.

Proposition 4.2.13 For Tactic I.2.d, the local optimal advertising efforts in the function of retail price $p$ is given by
$e_{I, 2 . d}^{*}(p)=\alpha(p-c)(1-b) /[2 h(1+b \beta)]+\mu$.
Noting that $e_{I .2 . d}^{*}(p)$ is strictly increasing in $p$ only if $b<1$. If $b>1$, then $e_{I, 2 . d}^{*}(p)$ is strictly decreasing in $p$. Therefore, Proposition 4.2.13 asserts that a high advertising effort of the company does not always support a high retail price, and it does happen if $b>1$ for Tactic I.2.d.
Proposition 4.2.14 For Tactic I.2.d, the local optimal advertising and pricing strategy exists only if (i) $D_{L}\left(\omega_{I .2 . d}^{*}\right)>0$ and $D_{F}\left(\omega_{I .2 . d}^{*}\right)>0$; (ii) $Z>0$; (iii) $T<e_{I .2 . d}^{*}<T+\tau$; (iv) $2 h\left(B+N_{I} T\right)+\alpha \mu(1-b)>0$; and (v) $N_{I}\left[2 h\left(B+N_{I} T\right)+\alpha \mu(1-b)\right] \leq \mu Z$. Moreover, if the local optimal advertising and pricing strategy for Tactic I.2.d exists, then it is unique and is given by
$\omega_{I .2 . d}^{*}=\left\{e_{I .2 . d}^{*}, \lambda_{I .2 . d}^{*}, p_{I .2 . d}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I, 2 . d}^{*}\right)=\frac{\left[2 h B+2 h N_{I} T+\alpha \mu(1-b)\right]^{2}+\mu^{2} Z}{4 h Z}-\mu(T+\tau)$,
where $e_{I .2 . d}^{*}=\frac{\alpha(1-b)\left[2 h\left(B+N_{I} T\right)+\alpha \mu(1-b)\right]}{2 h Z}+\frac{\mu}{2 h}, \quad \lambda_{I .2 . d}^{*}=T / e_{I, 2 . d}^{*}$, and
$p_{I, 2, d}^{*}=\frac{(1+b \beta)\left[2 h\left(B+N_{I} T\right)+\alpha \mu(1-b)\right]}{Z}+c$.
$\pi_{L L}(\omega)=\frac{-G(p-c)^{2}+(p-c)\left[B+\alpha(1-b) e+T N_{I}\right]}{1+b \beta}-h \mu^{2}-\mu[T+\tau-\mu]$

The necessary conditions for $\omega_{I, 2 . d}^{*}$ being finite are shown in Proposition 4.2.14. Specifically, conditions in item (i) of Proposition 4.2.14 are the basic conditions of Tactic I.2. Item (ii) of Proposition 4.2.14 ensures that the profit function of the company for Tactic I.2.d is concave in $p$. Item (iii) of Proposition 4.2.14 ensures $T<e<T+\tau$ which is one of the specific conditions of Tactic I.2.d. Item (iv) of Proposition 4.2.14 ensures that $p_{I .2 . d}^{*}>c$ and Item (v) of Proposition 4.2.14 ensures that $N_{I}(p-c)<\mu(1+b \beta)$ which is another specific condition of Tactic I.2.d. Moreover, Proposition 4.2.14 provides the explicit formulas of the local optimal advertising and pricing strategy, and the associated company's profit for Tactic I.2.d.

Proposition 4.2.15 (a) If $N_{I}>0$, then $e_{I .2 . d}^{*}$ and $p_{I .2 . d}^{*}$ are increasing in $T$. (b) If $N_{I}<0$, then $e_{I, 2, d}^{*}$ and $p_{I .2, d}^{*}$ are decreasing in $T$. (c) If $N_{I}=0$, then $e_{I, 2, d}^{*}$ and $p_{I .2 . d}^{*}$ are independent of $\tau$.

Proposition 4.2.15 shows the sensitivities of $T$ with respect to $e_{I .2 . d}^{*}$ and $p_{I .2 . d}^{*}$. Specifically, the sensitivities of $T$ to $e_{I .2 . d}^{*}$ and $p_{I .2 . d}^{*}$ depend on $N_{I}$. A bigger $T$ induces bigger optimal advertising effort as well as optimal retail price for Tactic I.2.d if $N_{I}$ is positive. However, a bigger $T$ induces smaller optimal advertising effort as well as optimal retail price for Tactic I.2.d if $N_{I}$ is negative. For the sensitivities of $\tau$ to $e_{I .2 . d}^{*}, \lambda_{I .2 . d}^{*}$ and $p_{I .2 . d}^{*}$, from Proposition 4.2.14, it is obvious that $e_{I .2 . d}^{*}, \lambda_{I .2 . d}^{*}$ and $p_{I, 2 . d}^{*}$ are independent of $\tau$.

## Tactic I. 3

Basic conditions: $D_{L}(\omega)>0, D_{F}(\omega)>0, \lambda e<T,(1-\lambda) e \geq \tau$ and $e \geq \tau$.
The company's profit for Tactic I. 3 is

$$
\begin{equation*}
\pi_{L L}(\omega)=\frac{-G(p-c)^{2}+(p-c)\left[B+\alpha(1-b) e+\lambda N_{I} e\right]}{1+b \beta}-h e^{2}-m(T-\lambda e) . \tag{4.29}
\end{equation*}
$$

We first study the optimal solution of $\lambda$ for Tactic I.3.
Proposition 4.2.16 For Tactic I.3, (a) the local optimal advertising and pricing strategy does not satisfy $N_{I}(p-c)>-m(1+b \beta)$ and $e \geq T+\tau$; (b) if $e=\tau$, then $\lambda^{*}=0$; (c) if $N_{I}(p-c)>-m(1+b \beta)$ and $\tau<e<T+\tau$, then $\lambda^{*}=1-\tau / e$; (d) if $N_{I}(p-c)<-m(1+b \beta)$ and $e>\tau$, then $\lambda^{*}=0$; (e) multiple $\lambda^{*}$ exist if $N_{I}(p-c)=-m(1+b \beta)$ and $e>T$.

Proposition 4.2.16 asserts that we can ignore all the cases for which $N_{I}(p-c)>-m(1+b \beta)$ and $e \geq T+\tau$, under Tactic I.3. Moreover, Proposition 4.2.16 shows that the value of $\lambda^{*}$ varies under different situations. Specifically, if the company wants to assign the advertising efforts such that $e=\tau$, then the company should only assign advertising effort to FG, i.e., $\lambda^{*}=0$. However, if the company wants to assign advertising effort requirement such that $e>\tau$, then the company should check the value of $N_{I}(p-c)+m(1+b \beta)$. If $N_{I}(p-c)>-m(1+b \beta)$, then $\lambda^{*}=(1-\lambda) / e$. If $N_{I}(p-c)<-m(1+b \beta)$, then $\lambda^{*}=0$. If $N_{I}(p-c)=-m(1+b \beta)$,
then there exist multiple $\lambda^{*}$. In words, if $N_{I}(p-c)<-m(1+b \beta)$, then the company should assign the advertising effort to FG only. However, if $N_{I}(p-c)>-m(1+b \beta)$, then the company should assign to FG the advertising effort which just covers $\tau$, and the company should assign to LG the advertising effort which is less than $T$.

As $p>c$ and $m(1+b \beta)>0, \quad N_{I}(p-c) \leq-m(1+b \beta)$ implies that $N_{I}<0$ and hence $b<1$. Furthermore, for $N_{I}(p-c)=\mu(1+b \beta)$, the company can select any $\lambda$ that satisfies $(1-\lambda) e<\tau$ and $\lambda e \geq T$. As $\lambda=0$ satisfies $(1-\lambda) e \geq \tau$ and $\lambda e<T$ for any given $e>T$, we take $\lambda^{*}=0$ for $N_{I}(p-c)=-m(1+b \beta)$. According to the above discussions, we further consider four sub-tactics of Tactic I.3:
(Tactic I.3.a) $e=\tau$ and $\lambda^{*}=0$;
(Tactic I.3.b) $N_{I}(p-c)>-m(1+b \beta), \tau<e<T+\tau$ and $\lambda^{*}=1-\tau / e$;
(Tactic I.3.c) $b<1, N_{I}<0, N_{I}(p-c)<-m(1+b \beta), e>\tau$ and $\lambda^{*}=0$;
(Tactic I.3.d) $b<1, N_{I}<0, N_{I}(p-c)=-m(1+b \beta), e>\tau$ and $\lambda^{*}=0$.
The above mentioned conditions are specific to the associated sub-tactics. In particular, the basic conditions of Tactic I.3, $\lambda e<T,(1-\lambda) e \geq \tau$ and $e \geq \tau$, are covered by individual's specific conditions of each sub-tactic, and we still need to consider the remaining basic conditions of Tactic I.3., $D_{L}(\omega)>0$ and $D_{F}(\omega)>0$, for all sub-tactics of Tactic I.3.

We denote by $\omega_{I .3 i,}^{*}$, for $i=a, b, c, d$, the local optimal advertising and price strategy for Tactic I.3.i. We now proceed to explore the local optimal advertising and pricing strategies for each sub-tactic individually.

- Tactic I.3.a

Specific conditions: $D_{L}(\omega)>0$ and $D_{F}(\omega)>0, e=\tau$ and $\lambda^{*}=0$.
By putting $e=\tau$ and $\lambda^{*}=0$ into (4.14), the profit of the company for Tactic I.3.a becomes

$$
\begin{equation*}
\pi_{L L}(\omega)=\frac{(p-c)[B+\alpha(1-\beta) \tau]-G(p-c)^{2}}{1+b \beta}-h T^{2}-m T . \tag{4.30}
\end{equation*}
$$

Proposition 4.2.17 For Tactic I.3.a, the local optimal advertising and pricing strategy exists only if (i) $D_{L}\left(\omega_{I, 3, a}^{*}\right)>0$ and $D_{F}\left(\omega_{I, 3, a}^{*}\right)>0$; (ii) $G>0$; and (iii) $B+\alpha(1-b) \tau>0$. Moreover, if the local optimal advertising and pricing strategy for

Tactic I.3.a exists, then it is unique and is given by

$$
\begin{align*}
& \omega_{I .3, a}^{*}=\left\{e_{I .3, a}^{*}=\tau, \lambda_{I .3 . a}^{*}=0, p_{I .3 . a}^{*}=c+[B+\alpha(1-b \beta) \tau] /(2 G)\right\}, \text { and }  \tag{4.31}\\
& \pi_{L L}\left(\omega_{I, 3, a}^{*}\right)=\frac{B^{2}+2 B \alpha(1-b) \tau-Z \tau^{2}}{4 G(1+b \beta)}-m T . \tag{4.32}
\end{align*}
$$

The necessary conditions for $\omega_{I, 3, a}^{*}$ being finite are shown in Proposition 4.2.17. Specifically, conditions in item (i) of Proposition 4.2 .17 are the basic conditions of Tactic I.3. Item (ii) of Proposition 4.2.17 ensures that the profit function of the company for Tactic I.3.a is concave in $p$. Item (iii) of Proposition 4.2.17 ensures that $p_{I .3, a}^{*}>c$. Moreover, Proposition 4.2.17 shows the explicit formulas of the local optimal advertising and pricing strategy, as well as the associated company's profit for Tactic I.3.a.

For the sensitivity of $\tau$ to $e_{I .3 . a}^{*}$ and $p_{I .3, a}^{*}$, from Proposition 4.2.17, we find that $e_{I .3 . a}^{*}$ is always strictly increasing in $\tau$, while $p_{I .3 . a}^{*}$ is strictly increasing in $\tau$ only if $b<1$. If $b>1$, then $p_{I .3 . a}^{*}$ is strictly decreasing in $\tau$. In other words, for Tactic I.3.a, a bigger $\tau$, always induces a higher advertising efforts. On the other hand, a bigger $\tau$ induces a higher retail price of the product if $b>1$, but, a big $\tau$ induces a low retail price of the product if $b<1$.

- Tactic I.3.b:

Specific conditions: $D_{L}(\omega)>0, D_{F}(\omega)>0, N_{I}(p-c)>-m(1+b \beta), \tau<e<T+\tau$ and $\lambda^{*}=1-\tau / e$.

By putting $\lambda^{*}=1-\tau / e$ into (4.29), the profit of the company for Tactic I.3.b becomes

$$
\begin{equation*}
\pi_{L L}(\omega)=\frac{-G(p-c)^{2}+(p-c)\left[B+a(1+\beta) e-\tau N_{I}\right]}{1+b \beta}-h e^{2}-m(T-e+\tau) . \tag{4.33}
\end{equation*}
$$

Proposition 4.2.18 For Tactic I.3.b, the local optimal advertising effort as a function of retail price $p$ is given by

$$
\begin{equation*}
e_{I \cdot 3 . b}^{*}(p)=\frac{a(1+\beta)}{2 h(1+b \beta)}(p-c)+\frac{m}{2 h} . \tag{4.34}
\end{equation*}
$$

Moreover, $e_{I .3 . b}^{*}(p)$ is strictly increasing in $p$.
Proposition 4.2.18 implies that, for Tactic I.3.b, a high optimal retail price results a high optimal advertising efforts.

Proposition 4.2.19 For Tactic I.3.b, the local optimal advertising and pricing strategy exists only if (i) $D_{L}\left(\omega_{I .3 . b}^{*}\right)>0$ and $D_{F}\left(\omega_{I .3 . b}^{*}\right)>0$; (ii) $Y>0$; $B>N_{I} \tau-a m(1+\beta) /(2 h)$; (iv) $\frac{\Theta \tau-2 m G(1+b \beta)}{a(1+\beta)}<B<\frac{Y T+\Theta \tau-2 m G(1+b \beta)}{a(1+\beta)}$; and (v) $\left[2 h\left(B-N_{I} \tau\right)+a m(1+\beta)\right] N_{I}>-m Y$. Moreover, if the local optimal advertising and pricing strategy for Tactic I.3.b exists, then it is unique and is given by
$\omega_{I .3 . b}^{*}=\left\{e_{I .3 . b}^{*}, \lambda_{I .3 . b}^{*}, p_{I .3 . b}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I .3 . b}^{*}\right)=\frac{\left[2 h\left(B-N_{I} \tau\right)+a m(1+\beta)\right]^{2}}{4 h^{2} Y}-m(T+\tau)+\frac{m^{2}}{4 h}$,
where $e_{I, 3, b}^{*}=\frac{a(1+\beta)\left[2 h\left(B-N_{t} \tau\right)+a m(1+\beta)\right]}{2 h Y}+\frac{m}{2 h}, \quad \lambda_{I, 3, b}^{*}=1-\tau / e_{I, 3, b}^{*}$, and
$p_{I .3 . b}^{*}=c+(1+b \beta)\left[2 h\left(B-N_{I} \tau\right)+a m(1+\beta)\right] / Y$.
Proposition 4.2.19 shows the explicit formulas of the local optimal advertising and pricing strategy, and the associated company's profit for Tactic I.3.b. Moreover, conditions in item (i) of Proposition 4.2.19 are the basic conditions of Tactic I.3. Item (ii) of Proposition 4.2.19 ensures that the profit function of the company for Tactic I.3.b is concave in $p$. Item (iii) of Proposition 4.2 .19 ensures $p>c$. Item (iv) of Proposition 4.2.19 ensures that $\tau<e<T+\tau$ which is the specific condition of Tactic I.3.b. Item (v) of Proposition 4.2 .19 ensures that $N_{I}(p-c)>-m(1+b \beta)$ which is also the specific condition for Tactic I.3.b.

Proposition 4.2.20 (a) If $N_{I}>0$, then $e_{I .3 . b}^{*}, \lambda_{I .3 . b}^{*}$ and $p_{I .3 . b}^{*}$ are decreasing in $T$. (b) If $N_{I}<0$, then $e_{I .3 . b}^{*}$ and $p_{I .3 . b}^{*}$ are increasing in $T$. (c) If $N_{I}=0$, then $\lambda_{I .3 . b}^{*}$ is decreasing in $T$.

Proposition 4.2.20 shows the sensitivities of $T$ with respect to $e_{I .3 . b}^{*}, \lambda_{I .3 . b}^{*}$ and $p_{I .3, b}^{*}$. Specifically, the sensitivities of $T$ to $e_{I .3 . b}^{*}, \lambda_{I .3 . b}^{*}$ and $p_{I .3 . b}^{*}$ depend on $N_{I}$. A bigger $T$ induces smaller optimal advertising effort as well as optimal retail price for Tactic I.3.b if $N_{I}$ is positive. However, a bigger $T$ induces bigger optimal advertising effort as well as optimal retail price for Tactic I.3.b if $N_{I}$ is negative. Finally, a bigger $T$ induces a smaller $\lambda_{I .3 . b}^{*}$ if $N_{I}=0$.

- Tactic I.3.c:

Specific conditions: $D_{L}(\omega)>0, D_{F}(\omega)>0, N_{I}<0, N_{I}(p-c)<-m(1+b \beta), e>\tau$ and $\lambda^{*}=0$.

By putting $\lambda^{*}=0$ into (4.29), the profit of the company for Tactic I.3.c becomes
$\pi_{L L}(\omega)=\frac{-G(p-c)^{2}+(p-c)[B+\alpha(1-b) e]}{1+b \beta}-h e^{2}-m T$.
Proposition 4.2.21 For Tactic I.3.c, the local optimal advertising effort as a function of retail price $p$ is given by
$e_{I .3 . c}^{*}(p)=\frac{\alpha(1-b)(p-c)}{2 h(1+b \beta)}$,
and $e_{I .3 . c}^{*}(p)$ is strictly increasing in $p$.
Proposition 4.2.21 shows that, for Tactic I.3.c, higher retail price induces higher optimal advertising effort of the brand.

Proposition 4.2.22 For Tactic I.3.c, the local optimal advertising and pricing strategy exists only if (i) $N_{I}<0, D_{L}\left(\omega_{I .3 . c}^{*}\right)>0$ and $D_{F}\left(\omega_{I .3 . c}^{*}\right)>0$; (ii) $Z>0$; (iii) $B>\tau \mathrm{Z} /[\alpha(1-b)]$; and (iv) $B>-m Z /\left(2 h N_{I}\right)$. Moreover, if the local optimal advertising and pricing strategy for Tactic I.3.c exists, then it is unique and is given by $\omega_{I .3 . c}^{*}=\left\{e_{I .3 . c}^{*}=\alpha B(1-b) / \mathrm{Z}, \lambda_{I .3, c}^{*}=0, p_{I .3 . c}^{*}=c+2 h B(1+b \beta) / Z\right\}$, and $\pi_{L L}\left(\omega_{I, 3, c}^{*}\right)=h B^{2} / Z-m T$.

Proposition 4.2.22 shows the explicit formulas of the local optimal advertising and pricing strategy, and the associated company's profit for Tactic I.3.c. Moreover, it also shows the necessary conditions for having a finite $\omega_{I .3 . c}^{*}$. Plus, according to Proposition 4.2.22, $e_{I .3 . c}^{*}, \lambda_{I .3 . c}^{*}$, and $p_{I .3 . c}^{*}$ are all independent of $T \tau, m$, and $\mu$.

- Tactic I.3.d

Specific conditions: $\quad D_{L}(\omega)>0 \quad$ and $\quad D_{F}(\omega)>0, \quad b<1, \quad N_{I}<0$, $N_{I}(p-c)=-m(1+b \beta), e>\tau$ and $\lambda^{*}=0$.

Proposition 4.2.23 For Tactic I.2.c, the local optimal advertising efforts in the function of $p$ is given by

$$
\begin{equation*}
e_{I .3, d}^{*}(p)=e_{I .3 . c}^{*}(p)=\frac{\alpha(1-b)(p-c)}{2 h(1+b \beta)}, \tag{4.41}
\end{equation*}
$$

and $e_{I .2 . c}^{*}(p)$ is strictly increasing in $p$.
Proposition 4.2.23 asserts that, for Tactic I.3.d, a high retail price induces a high optimal advertising efforts of the brand. Moreover, results of Proposition 4.2.23 are similar to Proposition 4.2.21.

Proposition 4.2.24 For Tactic I.3.d, the local optimal advertising and pricing strategy exists only if (i) $N_{I}<0, \quad D_{L}\left(\omega_{I .3 . d}^{*}\right)>0$ and $D_{F}\left(\omega_{I .3 . d}^{*}\right)>0$; and (ii) $\alpha m(b-1)<2 h N_{I} \tau$. Moreover, if the local optimal advertising and pricing strategy for Tactic I.3.d exists, then it is unique and is given by

$$
\begin{equation*}
\omega_{I .3, d}^{*}=\left\{e_{I .3, d}^{*}=\alpha m(b-1) /\left(2 h N_{I}\right), \lambda_{I .3, d}^{*}=0, p_{I .3, d}^{*}=c-m(1+b \beta) / N_{I}\right\}, \text { and } \tag{4.42}
\end{equation*}
$$

$$
\begin{equation*}
\pi_{L L}\left(\omega_{I, 3, d}^{*}\right)=\frac{\alpha^{2} m^{2}(1-b)^{2}-4 h m\left[G m(1+b \beta)+B N_{I}\right]}{4 h N_{I}^{2}}-m T . \tag{4.43}
\end{equation*}
$$

Proposition 4.2.24 provides the explicit formulas of the local optimal advertising and pricing strategy and the associated company's profit for Tactic I.3.d. Moreover, the necessary conditions for having a finite $\omega_{I .3 . d}^{*}$ are shown in Proposition 4.2.24. Specifically, conditions in item (i) of Proposition 4.2.24 are the specific conditions of Tactic I.3.d. Item (ii) of Proposition 4.2.10 ensures $e>\tau$ which is also the specific condition for Tactic I.3.d. As shown in Proposition 4.2.24, $e_{I .3 . d}^{*}, \lambda_{I .3 . d}^{*}$ and $p_{I .3 . d}^{*}$ are independent of $\tau, T$, and $\mu$. However, $e_{I .3 . d}^{*}$ and $p_{I .3 . d}^{*}$ are increasing in $m$.

## Tactic I.4:

Basic conditions: $D_{L}(\omega)>0, D_{F}(\omega)>0, \lambda e<T,(1-\lambda) e<\tau$ and $e<T+\tau$.
The company's profit for Tactic I. 4 is

$$
\begin{equation*}
\pi_{L L}(\omega)=\frac{(p-c)\left[B+\alpha(1-b) e+\lambda N_{I} e\right]-G(p-c)^{2}}{1+b \beta}-h e^{2}-m T-\mu \tau+(m \lambda+\mu-\mu \lambda) e \tag{4.44}
\end{equation*}
$$

Proposition 4.2.25 For Tactic I.4, (a) if $e=0$, then $\lambda$ is ignorable. (b) if $N_{I}(p-c)>(\mu-m)(1+b \beta) \quad$ and $\quad 0<e<T$, then $\lambda^{*}=1 \quad$ (c) If $N_{I}(p-c)<(\mu-m)(1+b \beta) \quad$ and $0<e<\tau$, then $\lambda^{*}=0$; and (d) If $N_{I}(p-c)=(\mu-m)(1+b \beta)$ and $0<e<T+\tau$, then there exist multiple $\lambda^{*}$.

Proposition 4.2.25 shows that the value of $\lambda^{*}$ differs under different situations. Specifically, the value of $\lambda$ can be ignored if the company does not assign any
advertising effort, i.e., $e=0$. However, if the company wants to assign advertising effort requirement such that $e>0$, then the company should check the value of $e$ and $N_{I}(p-c)+m(1+b \beta)$. If $N_{I}(p-c)>(\mu-m)(1+b \beta)$ and $0<e<T$, then the company should assign the advertising effort to LG only. If $N_{I}(p-c)<(\mu-m)(1+b \beta)$ and $0<e<\tau$, then the company should assign the advertising effort to FG only. If $N_{I}(p-c)=(\mu-m)(1+b \beta)$ and $0<e<T+\tau$, then there exist multiple $\lambda^{*}$.

Proposition 4.2.26 For Tactic I.4, the local optimal advertising and pricing strategy does not satisfy (a) $N_{I}(p-c)>(\mu-m)(1+b \beta) \quad$ and $\quad T \leq e<T+\tau$; (b) $N_{I}(p-c)<(\mu-m)(1+b \beta)$ and $\tau \leq e<T+\tau$; or (c) $N_{I}(p-c)=(\mu-m)(1+b \beta)$.

Proposition 4.2.26 shows the two cases for which the optimal advertising and pricing strategy does not belong to Tactic I.4. Specifically, for $N_{I}(p-c)>(\mu-m)(1+b \beta)$ and $T \leq e<T+\tau$, Tactic I. 2 dominates Tactic I.4. For $N_{I}(p-c)<(\mu-m)(1+b \beta)$ and $\tau \leq e<T+\tau$, Tactic I. 3 dominates Tactic I.4. For $N_{I}(p-c)=(\mu-m)(1+b \beta)$, at least one of Tactic I. 2 and Tactic I. 3 dominates Tactic I.4.

According to Proposition 4.2.25 and Proposition 4.2.26, we further consider three sub-tactics of Tactic I.4:
(Tactic I.4.a) $e^{*}=0$;
(Tactic I.4.b) $N_{I}(p-c)>(\mu-m)(1+b \beta), 0<e<T$ and $\lambda^{*}=1$; and
(Tactic I.4.c) $N_{I}(p-c)<(\mu-m)(1+b \beta), \quad 0<e<\tau$ and $\lambda^{*}=0$.
The above mentioned conditions are specific for the associated sub-tactics. In particular, the basic conditions of Tactic I.4, $\lambda e<T,(1-\lambda) e<\tau$ and $0 \leq e<T+\tau$, are covered by individual's specific conditions of each sub-tactic, and we still need to consider the remaining basic conditions of Tactic I.4., $D_{L}(\omega)>0$ and $D_{F}(\omega)>0$, for all sub-tactics of Tactic I.4.

Denoted by $\omega_{I .4 i}^{*}$, for $i=a, b, c$, the local optimal advertising and price strategy for Tactic I.3.i. We can explore the local optimal advertising and pricing strategies for each sub-tactic individually.

## - Tactic I.4.a

Specific conditions: $e^{*}=0$.

By putting $e^{*}=0$ into (4.44), the profit of the company for Tactic I.4.a becomes $\pi_{L L}(\omega)=\frac{-G(p-c)^{2}+(p-c) B}{1+b \beta}-h e^{2}-m T-\mu \tau$.
Proposition 4.2.27 For Tactic I.4.a, the local optimal advertising and pricing strategy exists only if (i) $D_{L}\left(\omega_{I ., 4, a}^{*}\right)>0$ and $D_{F}\left(\omega_{I ., 4, a}^{*}\right)>0$; (ii) $G>0$; and (iii) $B>0$. Moreover, if the local optimal advertising and pricing strategy exists, then it is unique and is given by
$\omega_{I, 4, a}^{*}=\left\{e_{I .4, a}^{*}=0, \lambda_{I .4, a}^{*}=0, p_{I .4, a}^{*}=B /(2 G)+c\right\}$, and
$\pi_{L L}\left(\omega_{I, 4, a}^{*}\right)=\frac{B^{2}}{4 G(1+b \beta)}-m T-\mu \tau$.
Similar to the other tactics, the necessary conditions for having a finite $\omega_{\text {I.4.a }}^{*}$ are shown in Proposition 4.2.27.

## Tactic I.4.b

Specific conditions: $N_{I}(p-c)>(\mu-m)(1+b \beta), 0<e<T$ and $\lambda^{*}=1$.
By putting $\lambda^{*}=1$ into (4.44), the profit of the company for Tactic I.4.b becomes $\pi_{L L}(\omega)=\frac{-G(p-c)^{2}+(p-c)[B+a(1+\beta) e]}{1+b \beta}-h e^{2}-m T-\mu \tau+m e$.

Proposition 4.2.28 For Tactic I.4.b, the local optimal advertising efforts in the function of retail price $p$ is given by

$$
\begin{equation*}
e_{I, 4, b}^{*}(p)=\frac{a(1+\beta)}{2 h(1+b \beta)}(p-c)+\frac{m}{2 h}=e_{I .3 . b}^{*}(p) . \tag{4.49}
\end{equation*}
$$

Moreover, $e_{I .4 . b}^{*}(p)$ is strictly increasing in $p$.
Proposition 4.2.28 shows that for Tactic I.4.b, a higher retail price induces higher advertising efforts of the company.
Proposition 4.2.29 For Tactic I.4.b, the local optimal advertising and pricing strategy exists only if (i) $D_{L}\left(\omega_{I .4 . b}^{*}\right)>0$ and $\quad D_{F}\left(\omega_{I .4 . b}^{*}\right)>0$; (ii) $Y>0$;
$B>-a m(1+\beta) /(2 h) \quad$ (condition for $\quad p>c)$;
$\frac{-2 m G(1+b \beta)}{a(1+\beta)}<B<\frac{T Y-2 m G(1+b \beta)}{a(1+\beta)}$; and (iv) $2 h N_{I} B>\mu Y-m \Theta$. Moreover, if the local optimal advertising and pricing strategy for Tactic I.4.b exists, then it is unique and is given by
$\omega_{I .4 . b}^{*}=\left\{e_{I .4 . b}^{*}, \lambda_{I .4 . b}^{*}, p_{I .4 . b}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I, 4, b}^{*}\right)=\frac{h B^{2}-\operatorname{Bam}(1+\beta)+m^{2} G(1+b \beta)}{Y}-m T-\mu \tau$,
where $e_{I .4, b}^{*}=\frac{a(1+\beta) B+2 m G(1+b \beta)}{Y}, \quad \lambda_{I, 4 . b}^{*}=1$, and
$p_{I .4 . b}^{*}=\frac{[2 h B+a m(1+\beta)](1+b \beta)}{Y}+c$.
Specifically, conditions in item (i) of Proposition 4.2.29 are the basic conditions for Tactic I.4.b. Item (ii) of Proposition 4.2.29 ensures that the profit function of the company for Tactic I.4.b is concave in $p$. Item (iii) of Proposition 4.2.29 ensures $p>c$. Item (iv) and Item (v) of Proposition 4.2.29 ensure that $0<e<T$ and $N_{I}(p-c)>(\mu-m)(1+b \beta)$, respectively, which are the specific conditions for Tactic I.4.b. Moreover, Proposition 4.2.29 shows the explicit formulas of the local optimal advertising and pricing strategy, and the associated company's profit for Tactic I.4.b. The necessary conditions for having a finite $\omega_{I .4, b}^{*}$ are shown in Proposition 4.2.29. Noting that $e_{I .4 . b}^{*}, \lambda_{I .4 . b}^{*}$ and $p_{I .4 . b}^{*}$ are independent of $T$ and $\tau$.

- Tactic I.4.c

Specific conditions: $N_{I}(p-c)<(\mu-m)(1+b \beta), 0<e<\tau$ and $\lambda^{*}=0$.
By putting $\lambda^{*}=0$ into (4.44), the profit of the company for Tactic I.4.c becomes

$$
\begin{equation*}
\pi_{L L}(\omega)=\frac{-G(p-c)^{2}+(p-c)[B+\alpha(1-b) e]}{1+b \beta}-h e^{2}-m T-\mu \tau+\mu e . \tag{4.52}
\end{equation*}
$$

Proposition 4.2.30 For Tactic I.4.c, the local optimal advertising efforts in the function of retail price $p$ is given by

$$
\begin{equation*}
e_{I .4 . c}^{*}(p)=\frac{\alpha(1-b)(p-c)}{2 h(1+b \beta)}+\frac{\mu}{2 h} . \tag{4.53}
\end{equation*}
$$

Noting that $e_{I .4 . c}^{*}(p)$ is strictly increasing in $p$ only if $b<1$. If $b>1$, then $e_{I .4 . c}^{*}(p)$ is strictly decreasing in $p$. Therefore, similar to Proposition 4.2.13, Proposition 4.2.30 asserts that a high advertising effort may not support a high retail price, and it does happen if $b>1$ for Tactic I.4.c.

Proposition 4.2.31 For Tactic I.4.c, the local optimal advertising and pricing strategy
exists only if (i) $D_{L}\left(\omega_{\text {I.4.c }}^{*}\right)>0$ and $D_{F}\left(\omega_{I .4 . c}^{*}\right)>0$; (ii) $Z>0$; (iii) $2 h B+\alpha \mu(1-b)>0 \quad ; \quad$ (iv) $\quad 0<2 \mu G(1+b \beta)+\alpha(1-b) B<Z \tau \quad$; and (v) $2 h B N_{I}<\Theta \mu-Z m$. Moreover, if the local optimal advertising and pricing strategy for Tactic I.4.c exists, then it is unique and is given by
$\omega_{I .4 . c}^{*}=\left\{e_{I .4 . c}^{*}, \lambda_{I .4 . c}^{*}, p_{I .4 . c}^{*}\right\}$ and
$\pi_{L L}\left(\omega_{I .4 . c}^{*}\right)=h B^{2} / Z-m T$,
where $e_{I .4 . c}^{*}=[2 \mu G(1+b \beta)+\alpha(1-b) B] / Z, \quad \lambda_{I .4 . c}^{*}=0$, and
$p_{I .4 . c}^{*}=(1+b \beta)[2 h B+\alpha \mu(1-b)] / Z+c$.
The necessary conditions for having a finite $\omega_{I .4 . c}^{*}$ are shown in Proposition 4.2.31. Specifically, conditions in item (i) of Proposition 4.2 .31 are the basic conditions for Tactic I.4. Item (ii) of Proposition 4.2.31 ensures that the profit function of the company for Tactic I.4.c is concave in $p$. Item (iii) of Proposition 4.2.31 ensures that $p>c$. Items (iv) and (v) of Proposition 4.2.31 ensure that $0<e<\tau$ and $N_{I}(p-c)<(\mu-m)(1+b \beta)$ which are the specific conditions for Tactic I.4.c. Moreover, Proposition 4.2.31 shows the explicit formulas of the local optimal advertising and pricing strategy, and the associated company's profit for Tactic I.4.c.Notice that $e_{I .4 . c}^{*}, \lambda_{I .4, c}^{*}$, and $p_{I .4 . c}^{*}$ are all independent of $T$ and $\tau$.

This completes the derivation of all the local optimums for Tactic I. Before we go to the analysis for Tactic II and Tactic III, we summarize the major findings for Tactic I.

1. When there is no penalty for insufficient advertising, the optimal allocation of advertising effort is either i) allocates all the advertising efforts to LG or ii) allocates all the advertising efforts to FG.. When there is a penalty for insufficient advertising, the optimal allocation of advertising effort will more likely be allocated to both LG and FG. This implies that when there is penalty for insufficient advertising, the company should take a balance between allocating the advertising effort between the two groups and avoid being "polarized".
2. The optimal advertising effort is never decreasing with the optimal retail price when there is no penalty for insufficient advertising. However, this can happen (for Tactic I.2.d with $b>1$ and Tactic I.4.c with $b>1$ ) when there is penalty for insufficient advertising.

There are totally 14 sub-tactics of Tactic I. By checking carefully the necessary conditions of each sub-tactic of Tactic I, we observe that by checking the value of $N_{I}$, some sub-tactics of Tactic I can already be screened out. In specific, Tactic I.2.b, Tactic I.2.c, Tactic I.3.c, Tactic I.3.d can be screened out for $N_{I} \geq 0$; and Tactic I.1.b, Tactic I.2.b and Tactic I.2.c can be screened out for $N_{I}<0$. Therefore, in determining the global optimal advertising and pricing strategy, the company should check the value of $N_{I}$ first and then screen out the non-optimal sub-tactics. This rule can significantly reduce the computational effort required to solve the problem.

### 4.2.3 Tactic II: Selling to FG only

Under Tactic II, the demand of the product of LG and FG satisfy $D_{L}(\omega)=0$ and $D_{F}(\omega)>0$, respectively. Therefore, the total demand of the product is
$D(\omega)=D_{F}(\omega)=x_{F}+\alpha(1-\lambda) e-\gamma p$
and the associated company's profit is
$\pi_{L L}(\omega)=\left[x_{F}+\alpha(1-\lambda) e-\gamma p\right](p-c)-h e^{2}-m[T-\lambda e]^{+}-\mu[\tau-(1-\lambda) e]^{+}$.
Now, we explore the optimal advertising and pricing strategy for each sub-tactic of Tactic II.

## Tactic II.1:

Basic conditions: $D_{L}(\omega)=0, \quad D_{F}(\omega)>0, \quad \lambda e \geq T, \quad(1-\lambda) e \geq \tau$ and $e \geq T+\tau$.
The company's profit for Tactic II. 1 is
$\pi_{L L}(\omega)=\left[x_{F}+\alpha(1-\lambda) e-\gamma p\right](p-c)-h e^{2}$.
Proposition 4.2.32 If the optimal adverting and pricing strategy belongs to Tactic II.1, then $V_{L}(\omega)<0$.

The interpretation of Proposition 4.2.32 is that the company should totally ignore the LG market under Tactic II. 1 as $V_{L}\left(\omega^{*}\right)<0$, i.e., the LG's value function is negative. Next, we study the optimal solutions of $\lambda$ under Tactic II.1. As $V_{L}\left(\omega^{*}\right)<0$ is necessary for Tactic II. 1 and $V_{L}(\omega)<0$ implies $D_{L}(\omega)=0$, we replace the basic condition $D_{L}(\omega)=0$ by $V_{L}(\omega)<0$.

Proposition 4.2.33 For Tactic II.1, (a) if $e=T+\tau$, then $\lambda^{*}=T /(T+\tau)$; and (b) if $e>T+\tau$, then $\lambda^{*}=T / e$.

Proposition 4.2.33 shows that if the company wants to assign the advertising effort such that $e=T+\tau$, then we have $\lambda^{*}=T /(T+\tau)$. However, if the company wants to assign the advertising efforts such that $e>T+\tau$, then the company should set $\lambda^{*} e=T$. Moreover, according to Proposition 4.2.33, we further consider two sub-tactics for Tactic I.2:
(Tactic II.1.a) $e=T+\tau$ and $\lambda^{*}=T /(T+\tau)$; and
(Tactic II.1.b) $e>T+\tau$ and $\lambda^{*}=T / e$.

Notice that the above mentioned conditions are specific for the associated sub-tactics. In particular, the basic conditions of Tactic II.1, $\lambda e \geq T,(1-\lambda) e \geq \tau$ and $e \geq T+\tau$, are covered by individual's specific conditions of each sub-tactic. Therefore, we need to consider the remaining basic conditions of Tactic II.1, $V_{L}(\omega)<0$ and $D_{F}(\omega)>0$ for all sub-tactics of Tactic II.1. Next, we explore the local optimal advertising and pricing strategies for each sub-tactic of Tactic II.1.

Denoted by $\omega_{I I .1 . i}^{*}$, for $i=a, b$, the local optimal advertising and price strategy for Tactic II.1.i. We proceed to explore the local optimal advertising and pricing strategies for Tactic II.1.a, and Tactic II.1.b individually.

- Tactic II.1.a

Specific conditions: $V_{L}(\omega)<0, D_{F}(\omega)>0, e=T+\tau$ and $\lambda^{*}=T /(T+\tau)$.
By putting $e=T+\tau$ and $\lambda^{*}=T /(T+\tau)$ into (4.56), the profit of the company for Tactic II.1.a becomes
$\pi_{L L}(\omega)=\left(x_{F}-\gamma c+\alpha \tau\right)-\gamma(p-c)^{2}-h(T+\tau)^{2}$,
Proposition 4.2.34 For Tactic II.1.a, the local optimal advertising and pricing strategy exists only if $V_{L}\left(\omega_{\text {II. . . } ~}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .1 . a}^{*}\right)>0$. Moreover, if the local optimal advertising and pricing strategy exists, then it is unique and is given by

$$
\begin{align*}
& \omega_{I I .1 . a}^{*}=\left\{e_{I I .1 . a}^{*}=T+\tau, \lambda_{I I .1 . a}^{*}=T /(T+\tau), p_{I I .1 . a}^{*}=\left(x_{F}+\alpha \tau+\gamma c\right) /(2 \gamma)\right\} \text {, and }  \tag{4.58}\\
& \pi_{L L}\left(\omega_{I I .1 . a}^{*}\right)=\left(x_{F}-\gamma c+\alpha \tau\right)^{2} /(4 \gamma)-h(T+\tau)^{2} . \tag{4.59}
\end{align*}
$$

The necessary conditions for having a finite $\omega_{I I .1 . a}^{*}$ are shown in Proposition 4.2.34. Specifically, only the basic conditions for Tactic II.1, $V_{L}\left(\omega_{I I .1 . a}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .1 . a}^{*}\right)>0$ are required. Moreover, Proposition 4.2.34 gives the explicit formulas of the local optimal advertising and pricing strategy and the associated company's profit for Tactic II.1.a if $\omega_{I I .1 . a}^{*}$ exists. According to Proposition 4.2.34, $e_{I I .1 . a}^{*}$ is strictly increasing in $T$ and $\tau, \lambda_{I I, 1 . a}^{*}$ is strictly decreasing in $T$ and $\tau$, and $p_{I I, 1 . a}^{*}$ is strictly increasing in $\tau$ but independent of $T$.

- Tactic II.1.b

Specific conditions: $V_{L}(\omega)<0, D_{F}(\omega)>0, e>T+\tau$ and $\lambda^{*}=T / e$.
By putting $\lambda^{*}=T / e$ into (4.56), the company's profit for Tactic II.1.b becomes

$$
\begin{equation*}
\pi_{L L}(\omega)=\left[x_{F}-\gamma c+\alpha(e-T)\right]-\gamma(p-c)^{2}-h e^{2} . \tag{4.60}
\end{equation*}
$$

Proposition 4.2.35 For Tactic II.1.b, the local optimal advertising effort is given by $e_{I I .1 . b}^{*}(p)=\alpha(p-c) /(2 h)$.

Moreover, $e_{I I .1 . b}^{*}(p)$ is strictly increasing in $p$.
Proposition 4.2.35 shows that for Tactic II.1.b, a higher retail price induces higher advertising effort of the company.

Proposition 4.2.36 For Tactic II.1.b, the local optimal advertising and pricing strategy exists only if (i) $V_{L}\left(\omega_{I I .1 . b}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .1 . b}^{*}\right)>0$; (ii) $4 h \gamma>\alpha^{2}$; and (iii) $x_{F}-\gamma c>\left[4 h \gamma(T+\tau)-\alpha^{2} \tau\right] / \alpha$. Moreover, if the local optimal advertising and pricing strategy exists then it is unique and is given by
$\omega_{I I .1 . b}^{*}=\left\{e_{I I, 1 . b}^{*}, \lambda_{I I, 1 . b}^{*}, p_{I I .1 . b}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I I .1 . b}^{*}\right)=\frac{h\left(x_{F}-\gamma c-\alpha T\right)^{2}}{4 h \gamma-\alpha^{2}}$,
where $e_{I I .1 . b}^{*}=\frac{\alpha\left(x_{F}-\gamma c-\alpha T\right)}{4 h \gamma-\alpha^{2}}, \quad \lambda_{I I .1 . b}^{*}=\frac{T\left(4 h \gamma-\alpha^{2}\right)}{\alpha\left(x_{F}-\gamma c-\alpha T\right)}$, and
$p_{I I, 1 . b}^{*}=\frac{2 h\left(x_{F}-\gamma c-\alpha T\right)}{4 h \gamma-\alpha^{2}}+c$.
The necessary conditions for having a finite $\omega_{I I .1 . b}^{*}$ are shown in Proposition 4.2.36. Specifically, conditions in item (i) of Proposition 4.2 .36 are the basic conditions for Tactic II.1. Item (ii) of Proposition 4.2.36 ensures that the local optimal advertising and pricing strategy is finite. Item (iii) of Proposition 4.2.36 ensures $e>T+\tau$ which is the specific condition for Tactic II.1.b. Notice that the feasibility constraint $p>c$ is covered by item (iii) of Proposition 4.2.29. Moreover, Proposition 4.2.36 gives the explicit formulas of the local optimal advertising and pricing strategy (if it exists), and the associated company's profit for Tactic II.1.b. Furthermore, from Proposition 4.2.36, we find that $d e_{I I .1 . b}^{*} / d \tau=-\alpha /\left(4 h \gamma-\alpha^{2}\right)<0$, $d \lambda_{I I .1 . b}^{*} / d T=\frac{\left(4 h \gamma-\alpha^{2}\right)\left(x_{F}-\gamma c\right)}{\alpha\left(x_{F}-\gamma c-\alpha T\right)^{2}}>0 \quad$ and $\quad d p_{I I .1 . b}^{*} / d \tau=\frac{-2 h \alpha}{4 h \gamma-\alpha^{2}}<0 \quad, \quad$ because $4 h \gamma>\alpha^{2}$ for Tactic II.1.b. Therefore, $e_{I I .1 . b}^{*}$ and $p_{I I .1 . b}^{*}$ are decreasing in $T$, while $\lambda_{I I .1 . b}^{*}$ is increasing in $T$. On the other hand, $e_{I I .1 . b}^{*}, \lambda_{I I .1 . b}^{*}$ and $p_{I I .1 . b}^{*}$ are independent of $\tau$.

## Tactic II. 2

Basic condition $D_{L}(\omega)=0, D_{F}(\omega)>0, \lambda e \geq T,(1-\lambda) e<\tau$ and $e \geq T$.
The company's profit for Tactic II. 2 is
$\pi_{L L}(\omega)=\left[x_{F}+\alpha(1-\lambda) e-\gamma p\right](p-c)-h e^{2}-\mu[\tau-(1-\lambda) e]$.
Proposition 4.2.37 If the optimal adverting and pricing strategy belongs to Tactic II.2, then $V_{L}\left(\omega^{*}\right)<0$.

The interpretation of Proposition 4.2.37 is similar to Proposition 4.2.32. Specifically, Proposition 4.2 .37 suggests that, for Tactic II.2, the company should totally ignore the market segment of LG as $V_{L}\left(\omega^{*}\right)<0$. As $V_{L}\left(\omega^{*}\right)<0$ is necessary for Tactic II. 2 and $V_{L}(\omega)<0$ implies $D_{L}(\omega)=0$, we can replace the basic condition $D_{L}(\omega)=0$ by $V_{L}(\omega)<0$.

Proposition 4.2.38 For Tactic II.2, (a) if $e=T$, then $\lambda^{*}=1$; and (b) if $T<e<T+\tau$, then $\lambda^{*}=T / e$. Moreover, Tactic II. 1 dominates Tactic II. 2 if $e \geq T+\tau$.

Proposition 4.2.38 shows that we can ignore the case for $e \geq T+\tau$ under Tactic II.2. Moreover, Proposition 4.2 .38 shows that if the company wants to assign the advertising efforts such that $e=T$, then the company should allocate all the advertising efforts to LG. However, if the company wants to assign the advertising efforts such that $e>T$, then the company should set $\lambda^{*} e=T$. Moreover, according to Proposition 4.2.38, we further consider two sub-tactics for Tactic II.2:
(Tactic II.2.a) $e=T$ and $\lambda^{*}=1$; and
(Tactic II.2.b) $T<e<T+\tau$ and $\lambda^{*}=T / e$.
Similarly, the above mentioned conditions are specific for the associated sub-tactics. In particular, the basic conditions of Tactic II.2, $\lambda e \geq T,(1-\lambda) e<\tau$ and $e \geq T$, are covered by individual's specific conditions of each sub-tactic. Apart from the specific conditions, we still need to consider the remaining basic conditions of Tactic II.2, $V_{L}(\omega)<0$ and $D_{F}(\omega)>0$ for all sub-tactics of Tactic II.2.

Denoted by $\omega_{I I, 2 i}^{*}$, for $i=a, b$, the local optimal advertising and price strategy for Tactic II.2.i. we can investigate the local optimal advertising and pricing strategies for each sub-tactic of Tactic II.2.

- Tactic II.2.a

Specific conditions: $V_{L}(\omega)<0, D_{F}(\omega)>0, e=T$ and $\lambda^{*}=1$ :
By putting $e=T$ and $\lambda^{*}=1$ into (4.58), the profit of the company for Tactic II.2.a becomes
$\pi_{L L}(\omega)=\left(x_{F}-\gamma p\right)(p-c)-h T^{2}-\mu \tau$.
Proposition 4.2.39 For Tactic II.2.a, the local optimal advertising and pricing strategy exists only if $V_{L}\left(\omega_{I I .2 . a}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .2 . a}^{*}\right)>0$. Moreover, if $\omega_{I I, 2 . a}^{*}$ exists, then $\omega_{I I .2 . a}^{*}$ is unique and
$\omega_{I I, 2 a}^{*}=\left\{e_{I I, 2, a}^{*}=T, \lambda_{I I, 2 a}^{*}=1, p_{I I, 2 \cdot a}^{*}=c+\left(x_{F}-\gamma c\right) /(2 \gamma)\right\}$, and $\pi_{L L}\left(\omega_{I I, 2, a}^{*}\right)=\left(x_{F}-\gamma c\right)^{2} /(4 \gamma)-h T^{2}-\mu \tau$.

The necessary conditions for having a unique $\omega_{I I .2 . a}^{*}$ are shown in Proposition 4.2.39. Specifically, only the basic conditions for Tactic II.2, $V_{L}\left(\omega_{I I .2 . a}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .2 . a}^{*}\right)>0$ are required. Moreover, Proposition 4.2.39 gives the explicit formulas of the local optimal advertising and pricing strategy and the associated company's profit for Tactic II.2.a. Notice that according to Proposition 4.2.32, $e_{I I .2 . a}^{*}$ is strictly increasing in $T$, while $\lambda_{I I, 2 . a}^{*}$ and $p_{I I, 2, a}^{*}$ are independent of $T$. Furthermore, $e_{I I, 2 . a}^{*}$, $\lambda_{I I, 2 a}^{*}$ and $p_{I I, 2, a}^{*}$ are independent of $\tau$.

## - Tactic II.2.b

Specific conditions: $V_{L}(\omega)<0, D_{F}(\omega)>0, T<e<T+\tau$ and $\lambda^{*}=T / e$.
By putting $\lambda^{*}=T / e$ into (4.64), the profit of the company for Tactic II.2.b becomes
$\pi_{L L}(\omega)=\left[x_{F}-\gamma c+\alpha(e-T)\right]-\gamma(p-c)^{2}-h e^{2}-\mu(T+\tau-e)$.
Proposition 4.2.40 For Tactic II.2.b, the optimal advertising effort in the function of retail price $p$ is given by

$$
\begin{equation*}
e_{I I, 2 . b}^{*}(p)=[\alpha(p-c)+\mu] /(2 h) . \tag{4.69}
\end{equation*}
$$

Moreover, $e_{I I .2 . b}^{*}(p)$ is strictly increasing in $p$.
Proposition 4.2.40 shows that for Tactic II.2.b, the optimal advertising effort is increasing in the optimal retail price.
Proposition 4.2.41 For Tactic II.2.b, the local optimal advertising and pricing
strategy exists only if (i) $V_{L}\left(\omega_{I I .2 . b}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .2 . b}^{*}\right)>0$; (ii) $4 h \gamma>\alpha^{2}$; and (iii) $2 \gamma(2 h T-\mu) / \alpha<x_{F}-\gamma c<\left[4 h \gamma(T+\tau)-\alpha^{2} \tau-2 \gamma \mu\right] / \alpha$. Moreover, if the local optimal advertising and pricing strategy for Tactic II.2.b exists, then it is unique and is given by
$\omega_{I I, 2 . b}^{*}=\left\{e_{I I .2 . b}^{*}, \lambda_{I I .2 . b}^{*}, p_{I I .2 . b}^{*}\right\}$ and
$\pi_{L L}\left(\omega_{I I, 2, b}^{*}\right)=\frac{h\left(x_{F}-\gamma c-\alpha T\right)^{2}+\alpha \mu\left(x_{F}-\gamma c-\alpha T\right)+\gamma \mu^{2}}{4 h \gamma-\alpha^{2}}-\mu(T+\tau)$,
where $e_{I I .2 . b}^{*}=\frac{2 \gamma \mu+\alpha\left(x_{F}-\gamma c-\alpha T\right)}{4 h \gamma-\alpha^{2}}, \quad \lambda_{I I .2 . b}^{*}=\frac{T\left(4 h \gamma-\alpha^{2}\right)}{2 \gamma \mu+\alpha\left(x_{F}-\gamma c-\alpha T\right)}$, and
$p_{I I .2 b}^{*}=\frac{2 h\left(x_{F}-\gamma c-\alpha T\right)+\alpha \mu}{4 h \gamma-\alpha^{2}}+c$.
The necessary conditions for having a finite $\omega_{I I, 2 . b}^{*}$ are shown in Proposition 4.2.41. Specifically, conditions in item (i) of Proposition 4.2.41 are the basic conditions for Tactic II.2. Item (ii) of Proposition 4.2.31 ensures that the profit function of the company for Tactic II.2.b is concave in $p$. Item (iii) of Proposition 4.2.41 ensures $T<e<T+\tau$ which is the specific condition for Tactic II.2.b. Moreover, Proposition 4.2.41 gives the explicit formulas of the local optimal advertising and pricing strategy and the associated company's profit for Tactic II.2.b. Furthermore, according to Proposition 4.2.41, we find that $e_{I I, 2 . b}^{*}$ and $p_{I I, 2 b}^{*}$ are decreasing in $T$, while $\lambda_{I I .2 . b}^{*}$ is increasing in $T$. On the other hand, $e_{I I .2 . b}^{*}, \lambda_{I I .2 . b}^{*}$ and $p_{I I, 2 b}^{*}$ are independent of $\tau$.

## Tactic II. 3

Basic conditions: $D_{L}(\omega)=0, D_{F}(\omega)>0, \lambda e<T,(1-\lambda) e \geq \tau$ and $e \geq \tau$.
The company's profit for Tactic II. 2 is

$$
\begin{equation*}
\pi_{L L}(\omega)=\left[x_{F}+\alpha(1-\lambda) e-\gamma p\right](p-c)-h e^{2}-m(T-\lambda e) . \tag{4.72}
\end{equation*}
$$

Proposition 4.2.42 For Tactic II.3, (a) if $p<m / \alpha+c$ and $e=\tau$, then $\lambda^{*}=0$ and $V_{L}\left(\omega^{*}\right)=0$; (b) if $p<m / \alpha+c$ and $\tau<e<T+\tau$, then $\lambda=1-\tau / e$ and $V_{L}\left(\omega^{*}\right)=0$; (c) if $p>m / \alpha+c$ and $e=\tau$, then $\lambda^{*}=0$ and $V_{L}\left(\omega^{*}\right)<0$; (d) if $p>m / \alpha+c$ and $e>\tau$, then $\lambda^{*}=0$ and $V_{L}\left(\omega^{*}\right)<0$; (e) if $p=m / \alpha+c$ and $e>\tau$, then there are multiple $\lambda^{*}$; and (f) Tactic II. 1 dominates Tactic II. 3 if
$e \geq T+\tau$ and $p<m / \alpha+c$.
Proposition 4.2.42 shows that, for Tactic II.3, if the optimal retail price is greater than the threshold $m / \alpha+c$, then the company should totally ignore the LG market as $V_{L}\left(\omega_{L L}^{*}\right)<0$. However, if the optimal retail price is less than the threshold $m / \alpha+c$, then the company should set the optimal advertising and pricing strategy such that $V_{L}\left(\omega_{L L}^{*}\right)=0$. Moreover, according to Proposition 4.2.42, we further consider five sub-tactics for Tactic II.3:
(Tactic II.3.a) $p<m / \alpha+c, e=\tau, \lambda^{*}=0$ and $V_{L}\left(\omega^{*}\right)=0$;
(Tactic II.3.b) $p<m / \alpha+c, \tau<e<T+\tau, \lambda^{*}=1-\tau / e$ and $V_{L}\left(\omega^{*}\right)=0$;
(Tactic II.3.c) $p>m / \alpha+c, e=\tau, \lambda^{*}=0, V_{L}\left(\omega^{*}\right)<0$; and
(Tactic II.3.d) $p>m / \alpha+c, e>\tau, \lambda^{*}=0, V_{L}\left(\omega^{*}\right)<0$.
(Tactic II.3.e) $p=m / \alpha+c, e>\tau, \lambda^{*}=0$.
Notice that, according to Proposition 4.2.42, there are multiple $\lambda^{*}$ for $p=m / \alpha+c$. For $e>\tau, \lambda=0$ satisfies $(1-\lambda) e \geq \tau$ and $\lambda e<T$. Therefore, we only consider $\lambda^{*}=0$ for $p=m / \alpha+c$. Similar results can also been obtained if we consider other values of $\lambda^{*}$ for $p=m / \alpha+c$. Moreover, the above mentioned conditions are specific for the associated sub-tactics. In particular, the basic conditions of Tactic II.3, $\lambda e<T,(1-\lambda) e \geq \tau$ and $e \geq \tau$, are covered by individual's specific conditions of each sub-tactic, and $D_{F}(\omega)>0$ is covered by individual's specific conditions of each sub-tactic except Tactic II.3.e. Apart from the specific conditions, we still need to consider the remaining basic conditions of Tactic II.3, $D_{F}(\omega)>0$ for all sub-tactics of Tactic II. 3 .

We represent by $\omega_{I I, 3 i}^{*}$, for $i=a, b, c, d, e$, the local optimal advertising and price strategy for Tactic II.3.i. The local optimal advertising and pricing strategies for each sub-tactic of Tactic II. 3 are explored as follows.

## - Tactic II.3.a

Specific conditions: $D_{F}(\omega)>0, p<m / \alpha+c, e=\tau, \lambda^{*}=0$ and $V_{L}\left(\omega^{*}\right)=0$.
Proposition 4.2.43 For Tactic II.3.a, the local optimal advertising and pricing strategy exists only if (i) $D_{F}\left(\omega_{I I .3 . a}^{*}\right)>0$; and (ii) $0<\frac{x_{L}-g c-b\left(x_{F}-\gamma c\right)-\alpha b \tau}{g-b \gamma}<\frac{m}{\alpha}$.

Moreover,
if the optimal advertising and pricing strategy for Tactic II.3.a exists, then it is unique and is given by
$\omega_{I I .3 . a}^{*}=\left\{e_{I I .3 . a}^{*}=\tau, \lambda_{I I .3 . a}^{*}=0, p_{I I .3 . a}^{*}=\frac{x_{L}-b x_{F}-\alpha b \tau}{g-b \gamma}\right\}$, and
$\pi_{L L}\left(\omega_{I I, 3 . a}^{*}\right)=\left[x_{F}+\alpha \tau-\gamma c\right]\left(p_{I I, 3 . a}^{*}-c\right)-\gamma\left(p_{I I, 3 . a}^{*}-c\right)^{2}-h \tau^{2}-m T$
The necessary conditions for having a finite $\omega_{I I .3 . a}^{*}$ are shown in Proposition 4.2.43. Specifically, there are two necessary conditions for $\omega_{I I .3 . a}^{*}$ being optimal. Condition (i) of Proposition 4.2.43 is the basic condition for Tactic II.3. Item (ii) of Proposition 4.2.43 ensures that $c<p_{I I .3 . a}^{*} \leq m / \alpha+c$. Moreover, Proposition 4.2.43 provides an explicit formula of the local optimal advertising and pricing strategy for Tactic II.3.a (if it exists), and an associated company's profit. Furthermore, from Proposition 4.2.43, we find that $e_{I I .3 . a}^{*}$ is increasing in $\tau, p_{I I .3 . a}^{*}$ is decreasing in $\tau$ and $\lambda_{I I, 3, a}^{*}$ is independent of $\tau$. On the other hand, $e_{I I .3, a}^{*}, \lambda_{I I, 3, a}^{*}$ and $p_{I I, 3, a}^{*}$ are independent of $T$.

## - Tactic II.3.b

Specific conditions: $D_{F}(\omega)>0, p<m / \alpha+c, e>\tau, \lambda=1-\tau / e$ and $V_{L}\left(\omega^{*}\right)=0$.
By putting $\lambda^{*}=1-\tau / e$ into (4.72), we obtain $\pi_{L L}(\omega)=\left(x_{F}-\gamma c+\alpha \tau\right)(p-c)-\gamma(p-c)^{2}-h e^{2}-m(T+\tau-e)$, and

Proposition 4.2.44 For Tactic II.3.b, the local optimal advertising effort as a function of retail price $p$ is given by

$$
\begin{equation*}
e_{I I .3 . b}^{*}(p)=\left\{b x_{F}-x_{L}+(a+\alpha b) \tau-(b \gamma-g) p\right\} / a . \tag{4.76}
\end{equation*}
$$

Proposition 4.2.44 shows the relationship between optimal $e$ and optimal $p$ for Tactic II.3.b. Noting that $e_{I I .3 . b}^{*}(p)$ is increasing in $p$ if $g \geq b \gamma$, and $e_{I I .3 . b}^{*}(p)$ is decreasing in $p$ if $g \leq b \gamma$, as $d e_{I I .3 . b}^{*}(p) / d p=(g-b \gamma) / a$.

Proposition 4.2.45 For Tactic II.3.b, the local optimal advertising and pricing strategy exists only if (i) $D_{F}\left(\omega_{I I .3 . b}^{*}\right)>0$; (ii) $a^{2} \gamma>h(b \gamma-g)^{2}$;
(iii) $a(\gamma b+g)\left(x_{F}-\gamma c+\alpha \tau\right)>2 a \gamma\left(x_{L}-g c\right)+(2 \tau h-m)(b \gamma-g)^{2}$;
(iv) $\left[a^{2}+2 h b\right]\left(x_{F}-\gamma c+\alpha \tau\right)-2 h\left(x_{L}-g c-a \tau\right)>a m(b \gamma-g)$; and
(v) $\left[a^{2}+2 h b\right]\left(x_{F}-\gamma c+\alpha \tau\right)-2 h\left(x_{L}-g c-a \tau\right)<m\left[2 a^{2} \gamma-2 h(b \gamma-g)^{2}-a \alpha(b \gamma-g)\right] / \alpha$.

Moreover, if the local optimal advertising and pricing strategy for Tactic II.3.b exists, then it is unique and is given by
$\omega_{I I, 3 . b}^{*}=\left\{e_{I I .3 . b}^{*}, \lambda_{I I, 3 . b}^{*}, p_{I I .3 . b}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I I, . b}^{*}\right)=\left(x_{F}-\gamma p_{I I, 3, b}^{*}+\alpha \tau\right)\left(p_{I I, 3 . b}^{*}-c\right)-h\left(e_{I I, 3 . b}^{*}\right)^{2}-m\left(T+\tau-e_{I I, 3, b}^{*}\right)$,
where $\quad e_{I I .3 . b}^{*}=\frac{a(\gamma b+g)\left(x_{F}-\gamma c\right)+m(b \gamma-g)^{2}+a \tau(\alpha g+2 a \gamma+\alpha b \gamma)-2 a \gamma\left(x_{L}-g c\right)}{2\left[a^{2} \gamma+h(b \gamma-g)^{2}\right]}$
$\lambda_{I I .3 . b}^{*}=1-\tau / e_{I I .3 . b}^{*}$, and
$p_{I I .3 . b}^{*}=\frac{\left[a^{2}+2 h b\right]\left(x_{F}-\gamma c+\alpha \tau\right)-2 h\left(x_{L}-g c-a \tau\right)-a m(b \gamma-g)}{2\left[a^{2} \gamma-h(b \gamma-g)^{2}\right]}+c$.
The necessary conditions for having a finite $\omega_{I I .3 . b}^{*}$ are shown in Proposition 4.2.38. Specifically, item (i) of Proposition 4.2.45 is the basic condition for Tactic II.3. Item (ii) of Proposition 4.2.45 ensures that the company's profit for Tactic II.3.b is strictly concave in $p$. Items (iii), (iv) and (v) of Proposition 4.2.45 ensure that $e_{I I, 3, b}^{*}>\tau$ and $c<p_{I I, 3, b}^{*} \leq m / \alpha+c$ respectively, which are the specific conditions for Tactic II.3.b. Moreover, Proposition 4.2.45 provides the explicit formulas of the local optimal advertising and pricing strategy for Tactic II.3.b (if it exists), and an associated company's profit.

- Tactic II.3.c

Specific conditions: $D_{F}(\omega)>0, p>m / \alpha+c, e=\tau, \lambda=0$ and $V_{L}\left(\omega^{*}\right)<0$.
By putting $\lambda^{*}=0$ into (4.66), we obtain
$\pi_{L L}(\omega)=\left[x_{F}+\alpha e-\gamma p\right](p-c)-h e^{2}-m T$.
Proposition 4.2.46 For Tactic II.3.c, the local optimal advertising and pricing strategy exists only if (i) $V_{L}\left(\omega_{I I .3 .}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .3 . c}^{*}\right)>0$; and (ii) $x_{F}-\gamma c>2 \gamma m / \alpha-\alpha \tau$. Moreover, if the optimal advertising and pricing strategy for Tactic II.3.c exists, then it is unique and is given by
$\omega_{I I .3 . c}^{*}=\left\{e_{I I .3 . c}^{*}=\tau, \lambda_{I I .3 . c}^{*}=0, p_{I I .3 . c}^{*}=c+\left(x_{F}-\gamma c+\alpha \tau\right) /(2 \gamma)\right\}$, and
$\pi_{L L}\left(\omega_{I I .3 . C}^{*}\right)=\left(x_{F}-\gamma c+\alpha \tau\right)^{2} /(4 \gamma)-h \tau^{2}-m T$,
The necessary conditions for the existence of local optimal advertising and pricing strategy for Tactic II.3.c are shown in Proposition 4.2.46. Specifically, item (i)
of Proposition 4.2.46 is the basic condition for Tactic II.3. Item (ii) of Proposition 4.2.46 ensures that $p_{I I .3 . c}^{*} \geq m / \alpha+c$, which is the specific conditions for Tactic II.3.c. Moreover, Proposition 4.2.46 shows the explicit formula of the local optimal advertising and pricing strategies for Tactic II.3.c, and the associated company's profit. Noting that $e_{I I .3 . c}^{*}$ and $p_{I I .3 . c}^{*}$ are increasing in $\tau$. In words, a bigger $\tau$ induces a higher total advertising effort assigned by the company and a higher retail price of the product for Tactic II.3.c. On the other hand, $e_{I I .3 . c}^{*}, \lambda_{I I .3 . c}^{*}$ and $p_{I I .3 . c}^{*}$ are independent of $T$.

## - Tactic II.3.d

Specific conditions: $D_{F}(\omega)>0, p>m / \alpha+c, e>\tau, \quad \lambda^{*}=0$ and $V_{L}\left(\omega^{*}\right)<0$.
Proposition 4.2.47 For Tactic II.3.d, the local optimal advertising effort as a function of retail price $p$ is given by

$$
\begin{equation*}
e_{I I .3 . d}^{*}(p)=\alpha(p-c) /(2 h) . \tag{4.82}
\end{equation*}
$$

Moreover, $e_{I I .3 . d}^{*}(p)$ is strictly increasing in $p$.
Proposition 4.2.47 shows that the optimal advertising effort is increasing in the optimal retail price for Tactic II.3.d, namely, a higher advertising effort induces a higher retail price of the product for Tactic II.3.d.

Proposition 4.2.48 For Tactic II.3.d, the local optimal advertising and pricing strategy exists only if (i) $V_{L}\left(\omega_{I I .3 . d}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .3 . d}^{*}\right)>0$; (ii) $4 h \gamma>\alpha^{2}$; (iii) $x_{F}-\gamma c>\tau\left(4 h \gamma-\alpha^{2}\right) / \alpha$; and (iv) $x_{F}-\gamma c>m\left(4 h \gamma-\alpha^{2}\right) /(2 h \alpha)$. Moreover, if the optimal advertising and pricing strategy for Tactic II.3.d exists, then it is unique and is given by
$\omega_{I I .3 . d}^{*}=\left\{e_{I I .3 . d}^{*}, \lambda_{I I .3 . d}^{*}, p_{I I .3 . d}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I I, 3 . d}^{*}\right)=h\left(x_{F}-\gamma c\right)^{2} /\left(4 h \gamma-\alpha^{2}\right)-m T$,
where $e_{I I, 3 . d}^{*}=\alpha\left(x_{F}-\gamma c\right) /\left(4 h \gamma-\alpha^{2}\right), \quad \lambda_{I I, 3 . d}^{*}=0$, and
$p_{I I .3 . d}^{*}=2 h\left(x_{F}-\gamma c\right) /\left(4 h \gamma-\alpha^{2}\right)+c$.
The necessary conditions for the existence of local optimal advertising and pricing strategy for Tactic II.3.d are shown in Proposition 4.2.48. Specifically, item (i) of Proposition 4.2.48 is the basic condition for Tactic II.3. Item (ii) of Proposition 4.2.48 ensures that the company's profit for Tactic II.3.d is strictly concave in $p$. Items
(iii) and (iv) of Proposition 4.2.48 ensure that $e_{I I, . d}^{*}>\tau$ and $p_{I I .3 . d}^{*} \geq m / \alpha+c$, respectively, which are the specific conditions for Tactic II.3.d. Moreover, Proposition 4.2.48 shows the explicit formula of the local optimal advertising and pricing strategies for Tactic II.3.d, and the associated company's profit. Noting that $e_{I I .3 . d}^{*}$, $\lambda_{I I .3 . d}^{*}$ and $p_{I I .3 . d}^{*}$ are independent of $\tau$ and $T$.

## - Tactic II.3.e

Specific conditions: $D_{F}(\omega)>0 \quad D_{F}(\omega)>0, \quad p=m / \alpha+c, e>\tau$ and $\lambda^{*}=0$.
Proposition 4.2.49 For Tactic II.3.e, the local optimal advertising and pricing strategy exists only if (i) $D_{F}\left(\omega_{I I .3 . e}^{*}\right)>0$; (ii) $2 h \tau<m$; and
(iii) $\alpha\left(x_{L}-g c\right)-b \alpha\left(x_{F}-\gamma c\right) \leq m\left[\alpha^{2} b+2 h(1-b)\right] /(2 h)$. Moreover, if the optimal advertising and pricing strategy for Tactic II.3.e exists, then it is unique and is given by
$\omega_{I I .3 . e}^{*}=\left\{e_{I I .3 . e}^{*}=m /(2 h), \lambda_{I I .3 . e}^{*}=0, p_{I I .3 . e}^{*}=m / \alpha+c\right\}$, and
$\pi_{L L}\left(\omega_{I I .3 . e}^{*}\right)=h\left(x_{F}-\gamma c\right)^{2} /\left(4 h \gamma-\alpha^{2}\right)-m T$.
The necessary conditions for the existence of local optimal advertising and pricing strategy for Tactic II.3.e are shown in Proposition 4.2.49. Specifically, item (i) of Proposition 4.2.49 is the basic condition for Tactic II.3. Items (ii) and (iii) of Proposition 4.2.49 ensure that $e_{I I .3 . e}^{*}>\tau$ and $V_{L}\left(\omega_{I I .3 . e}^{*}\right) \leq 0$, respectively which are also the basic conditions for Tactic II.3. Moreover, Proposition 4.2.49 shows the explicit formula of the local optimal advertising and pricing strategies for Tactic II.3.e, and the associated company's profit.

## Tactic II. 4

Basic conditions: $D_{L}(\omega)=0, D_{F}(\omega)>0, \lambda e<T,(1-\lambda) e<\tau$ and $0 \leq e<T+\tau$.
The company's profit for Tactic II. 4 is
$\pi_{L L}(\omega)=\left[x_{F}+\alpha(1-\lambda) e-\gamma p\right](p-c)-h e^{2}-m(T-\lambda e)-\mu(\tau-(1-\lambda) e)$.
Proposition 4.2.50 For Tactic II.4, (a) if $p<(m-\mu) / \alpha+c$ and $e=0$, then $V_{L}\left(\omega^{*}\right)=0$; (b) if $p<(m-\mu) / \alpha+c$ and $0<e<T$, then $\lambda^{*}=1$ and $V_{L}\left(\omega^{*}\right)=0$;
(c) if $p>(m-\mu) / \alpha+c$ and $e=0$, then $V_{L}\left(\omega^{*}\right)<0$; (d) if $p>(m-\mu) / \alpha+c$ and $0<e<\tau$, then $\lambda^{*}=0$ and $V_{L}\left(\omega^{*}\right)<0$; (e) if $p=(m-\mu) / \alpha+c$, then there are
multiple $\lambda^{*}$; (f) if $p<(m-\mu) / \alpha+c$ and $T \leq e<T+\tau$, then Tactic II. 2 dominates Tactic II.4; and (g) if $p>(m-\mu) / \alpha+c$ and $\tau \leq e<T+\tau$, Tactic II. 3 dominates Tactic II. 4 .

Proposition 4.2.50 shows that, for Tactic II.4, if the optimal retail price is bigger than the threshold $(m-\mu) / \alpha+c$, then the company should set the optimal advertising and pricing strategy such that $V_{L}\left(\omega^{*}\right)=0$. If the optimal retail price is greater than the threshold $(m-\mu) / \alpha+c$, then the company should totally ignore the LG market as $V_{L}\left(\omega^{*}\right)<0$. Moreover, if $p=(m-\mu) / \alpha+c$, then there are multiple $\lambda^{*}$. Furthermore, Proposition 4.2 .50 shows that Tactic II. 4 is dominated by Tactic II. 3 for $p<(m-\mu) / \alpha+c$ and $T \leq e<T+\tau$, and is dominated by Tactic II. 3 for $p>(m-\mu) / \alpha+c$ and $\tau \leq e<T+\tau$. Therefore, we can ignore these cases when we investigate the local optimal advertising and pricing strategies for Tactic II. 4.

According to Proposition 4.2.50, we further consider five sub-tactics for Tactic II.4:
(Tactic II.4.a) $p<(m-\mu) / \alpha+c, e=0$ and $V_{L}\left(\omega^{*}\right)=0$;
(Tactic II.4.b) $p<(m-\mu) / \alpha+c, 0<e<T, \lambda^{*}=1$ and $V_{L}\left(\omega^{*}\right)=0$;
(Tactic II.4.c) $p>(m-\mu) / \alpha+c, e=0$ and $V_{L}\left(\omega^{*}\right)<0$;
(Tactic II.4.d) $p>(m-\mu) / \alpha+c, 0<e<\tau, \lambda^{*}=0$ and $V_{L}\left(\omega^{*}\right)<0$; and
(Tactic II.4.e) $p=(m-\mu) / \alpha+c, \quad 0<e<\tau, \lambda^{*}=0$.
Notice that, according to Proposition 4.2.50, there are multiple $\lambda^{*}$ for $p=(m-\mu) / \alpha+c$. As $0<e<\tau$ and $\lambda=0$ satisfy $(1-\lambda) e \geq \tau$ and $\lambda e<T$, we consider $\lambda^{*}=0$ for $p=(m-\mu) / \alpha+c$. Similar results can also been obtained if we consider other values of $\lambda^{*}$ and $0<e<T+\tau$, for $p=(m-\mu) / \alpha+c$. Notice that we still need to take care of the basic conditions for Tactic II. 4 which are not covered by the above mentioned specific conditions for each sub-tactic.

Denoted by $\omega_{I I .4 i}^{*}$, for $i=a, b, c, d, e$, the local optimal advertising and price strategy for Tactic II.4.i. We proceed to explore the local optimal advertising and pricing strategies for each sub-tactic of Tactic II. 4 in the following.

## - Tactic II.4.a

Specific conditions: $D_{F}\left(\omega^{*}\right)>0, p<(m-\mu) / \alpha+c, e^{*}=0$ and $V_{L}\left(\omega^{*}\right)=0$.
Proposition 4.2.51 For Tactic II.4.a, the local optimal advertising and pricing strategy exists only if (i) $D_{F}\left(\omega_{I I .4 . a}^{*}\right)>0$; and (ii) $c<\frac{b x_{F}-x_{L}}{b \gamma-g}<\frac{m-\mu}{\alpha}+c$. Moreover, if the optimal advertising and pricing strategy for Tactic II.4.a exists, then it is unique and is given by
$\omega_{I I, 4, a}^{*}=\left\{e_{I I .4, a}^{*}=0, \lambda_{I I .4, a}^{*}=0, p_{I I .4, a}^{*}=\left(b x_{F}-x_{L}\right) /(b \gamma-g)\right\}$, and
$\pi_{L L}\left(\omega_{I I, 4, a}^{*}\right)=\left(x_{F}-\gamma p_{I I, 4, a}^{*}\right)\left(p_{I I .4, a}^{*}-c\right)-m T-\mu \tau$.
The necessary conditions for having a finite local optimal advertising and pricing strategy for Tactic II.4.a are shown in Proposition 4.2.51. Moreover, Proposition 4.2.51 shows the explicit formula of the optimal advertising and pricing strategies and the associated company's profit for Tactic II.4.a. Furthermore, the optimal advertising and pricing strategy for Tactic II.4.a is independent of $T$ and $\tau$.

## - Tactic II.4.b

Specific conditions: $D_{F}\left(\omega^{*}\right)>0, \quad p<(m-\mu) / \alpha+c, \quad 0<e<T, \quad \lambda^{*}=1$ and $V_{L}\left(\omega^{*}\right)=0:$

By putting $\lambda^{*}=1$ into (4.87), we obtain
$\pi_{L L}(\omega)=\left(x_{F}-\gamma p\right)(p-c)-h e^{2}-m(T-e)-\mu \tau$.
Proposition 4.2.52 For Tactic II.4.b, the local optimal advertising effort as a function of retail price $p$ is given by

$$
\begin{equation*}
e_{I I, 4, b}^{*}(p)=\left\{b x_{F}-x_{L}+(g-b \gamma) p\right\} / a . \tag{4.91}
\end{equation*}
$$

Proposition 4.2.52 shows the relationship between optimal $e$ and optimal $p$ for Tactic II.4.b. Noting that $e_{I I .4 . b}^{*}(p)$ is increasing in $p$ if $g \geq b \gamma$, and $e_{I I .4 . b}^{*}(p)$ is decreasing in $p$ if $g \leq b \gamma$, which is similar as $e_{I I .3 . b}^{*}(p)$ for Tactic II.3.b.

Proposition 4.2.53 For Tactic II.4.b, the local optimal advertising and pricing strategy exists only if (i) $D_{F}\left(\omega_{\text {II.4.b }}^{*}\right)>0$;
(ii) $0<a(b \gamma-g)\left(x_{F}-\gamma c\right)+m(b \gamma-g)^{2}-2 a \gamma\left(x_{L}-g c\right)<T$; and
(iii) $0<\frac{\left[a^{2}+2 h b(b \gamma-g)\right]\left(x_{F}-\gamma c\right)-\left[2 h\left(x_{L}-g c\right)+a m\right](b \gamma-g)}{2\left[a^{2} \gamma+h(b \gamma-g)^{2}\right]}<\frac{m-\mu}{\alpha}$. Moreover, if the local optimal advertising and pricing strategy for Tactic II.4.b, then it is unique
and is given by

$$
\begin{align*}
& \omega_{I I, 4, b}^{*}=\left\{e_{I I, 4, b}^{*}, \lambda_{I I, 4, b}^{*}, p_{I I, 4, b}^{*}\right\}, \text { and }  \tag{4.92}\\
& \pi_{L L}\left(\omega_{I I, 4, b}^{*}\right)=\left(x_{F}-\gamma p_{I I, 4, b}^{*}\right)\left(p_{I I, 4, b}^{*}-c\right)-h\left(e_{I I, 4, b}^{*}\right)^{2}-m\left(T+\tau-e_{I I, 4, b}^{*}\right),  \tag{4.93}\\
& \text { where } e_{I I, 4, b}^{*}=\frac{a(b \gamma-g)\left(x_{F}-\gamma c\right)+m(b \gamma-g)^{2}-2 a \gamma\left(x_{L}-g c\right)}{2\left[a^{2} \gamma+h(b \gamma-g)^{2}\right]}, \lambda_{I I, 4, b}^{*}=1, \text { and } \\
& p_{I I, 4, b}^{*}=c+\frac{\left[a^{2}+2 h b(b \gamma-g)\right]\left(x_{F}-\gamma c\right)-\left[2 h\left(x_{L}-g c\right)+a m\right](b \gamma-g)}{2\left[a^{2} \gamma+h(b \gamma-g)^{2}\right]} .
\end{align*}
$$

The necessary conditions for having a finite $\omega_{I I .4 . b}^{*}$ are shown in Proposition 4.2.53. Specifically, condition in item (i) of Proposition 4.2.53 is the basic condition for Tactic II.4. Items (ii) and (iii) of Proposition 4.2.53 ensure that $0<e_{I I .4 . b}^{*}<T$ and $c<p_{I I .4 . b}^{*} \leq(m-\mu) / \alpha+c$ respectively, which are the specific conditions for Tactic II.4.b. Moreover, Proposition 4.2 .53 shows the explicit formula of the optimal advertising and pricing strategies, and the associated company's profit for Tactic II.4.b.

## - Tactic II.4.c

Specific conditions: $D_{F}\left(\omega^{*}\right)>0, p^{*}>(m-\mu) / \alpha+c, e^{*}=0$ and $V_{L}\left(\omega^{*}\right)<0$
Proposition 4.2.54 For Tactic II.4.c, the local optimal advertising and pricing strategy exists only if (i) $V_{L}\left(\omega_{I I .4 . c}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .4 . c}^{*}\right)>0$; and (ii) $x_{F}-\gamma c>2 \gamma(m-\mu) / \alpha$. Moreover, if the local optimal advertising and pricing strategy for Tactic II.4.c, then it is unique and is given by

$$
\begin{align*}
& \omega_{I I .4 . c}^{*}=\left\{e_{I I .4 . c}^{*}=0, \lambda_{I I .4 . c}^{*}=0, p_{I I .4 . c}^{*}=c+\left(x_{F}-\gamma c\right) /(2 \gamma)\right\} \text {, and }  \tag{4.94}\\
& \pi_{L L}\left(\omega_{I I .4 . c}^{*}\right)=\left(x_{F}-\gamma c\right)^{2} /(4 \gamma)-\mu \tau-m T . \tag{4.95}
\end{align*}
$$

The necessary conditions for having a finite local optimal advertising and pricing strategy for Tactic II.4.c are shown in Proposition 4.2.54. Moreover, Proposition 4.2.54 shows the explicit formula of the optimal advertising and pricing strategy, and the associated company's profit for Tactic II.4.c. Furthermore, $e_{I I .4 . c}^{*}, e_{I I .4 . c}^{*}$ and $p_{I I .4 . c}^{*}$ are independent of $T$ and $\tau$.

## - Tactic II.4.d

Specific conditions: $D_{F}\left(\omega^{*}\right)>0, \quad p>(m-\mu) / \alpha+c, \quad 0<e<\tau, \quad \lambda^{*}=0$ and $V_{L}\left(\omega^{*}\right)<0:$

By putting $\lambda^{*}=0$ into (4.79), we obtain
$\pi_{L L}(\omega)=\left[x_{F}+\alpha e-\gamma p\right](p-c)-h e^{2}-m T-\mu(\tau-e)$.
Proposition 4.2.55 For Tactic II.4.d, the local optimal advertising efforts in the function of retail price $p$ is given by

$$
\begin{equation*}
e_{I I, 4, d}^{*}(p)=[\alpha(p-c)+\mu] /(2 h) . \tag{4.97}
\end{equation*}
$$

Moreover, $e_{I I .4 . d}^{*}(p)$ is strictly increasing in $p$.
Proposition 4.2.55 implies that, for Tactic II.4.d, a higher advertising effort induces a higher retail price of the product.

Proposition 4.2.56 For Tactic II.4.d, the local optimal advertising and pricing strategy exists only if (i) $V_{L}\left(\omega_{I I .4 . d}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .4 . d}^{*}\right)>0$; (ii) $4 h \gamma>\alpha^{2}$; (iii) $x_{F}-\gamma c>\tau\left(4 h \gamma-\alpha^{2}\right) / \alpha$; and (iv) $x_{F}-\gamma c>\left[m\left(4 h \gamma-\alpha^{2}\right)-4 h \gamma \mu\right] /(2 h \alpha)$. Moreover, if the local optimal advertising and pricing strategy for Tactic II.4.d exists, then it is unique and is given by

$$
\begin{align*}
& \omega_{I I .4 . d}^{*}=\left\{e_{I I .4 . d}^{*}, \lambda_{I I .4 . d}^{*}, p_{I I .4 . d}^{*}\right\} \text {, and }  \tag{4.98}\\
& \pi_{L L}\left(\omega_{I I, 4, d}^{*}\right)=\frac{\left[2 h\left(x_{F}-\gamma c\right)+\alpha \mu\right]^{2}}{4 h\left(4 h \gamma-\alpha^{2}\right)}+\frac{\mu^{2}}{4 h}-m T-\mu \tau,  \tag{4.99}\\
& \text { where } e_{\text {II.4.d }}^{*}=\frac{\alpha\left(x_{F}-\gamma c\right)+2 \gamma \mu}{4 h \gamma-\alpha^{2}}, \lambda_{I I .4 . d}^{*}=0 \text {, and } p_{I I .4, d}^{*}=c+\frac{2 h\left(x_{F}-\gamma c\right)+\alpha \mu}{4 h \gamma-\alpha^{2}} \text {. }
\end{align*}
$$

The necessary conditions for having a finite local optimal advertising and pricing strategy for Tactic II.4.d are shown in Proposition 4.2.56. Moreover, Proposition 4.2.56 shows the explicit formula of the local optimal advertising and pricing strategies, and the associated company's profit for Tactic II.4.d. Furthermore, $e_{I I .4 . d}^{*}$, $\lambda_{I I .4 . d}^{*}$ and $p_{I I .4 . d}^{*}$ are independent of $\tau$ and $T$.

## - Tactic II.4.e

Specific conditions: $D_{F}\left(\omega^{*}\right)>0, D_{L}\left(\omega^{*}\right)=0, p=(m-\mu) / \alpha+c, 0 \leq e<\tau$, and $\lambda^{*}=0$.

Proposition 4.2.56b For Tactic II.4.e, the local optimal advertising and pricing strategy exists only if (i) $D_{F}\left(\omega_{\text {II.4.e }}^{*}\right)>0$; (ii) $\mu<m<2 h \tau$; and
(iii) $\left(x_{L}-g c\right)-b\left(x_{F}-\gamma c\right) \leq[\alpha b m+2 h(\gamma b-g)(\mu-m)] /(2 h \alpha)$. Moreover, if the optimal advertising and pricing strategy for Tactic II.4.e exists, then it is unique and
is given by

$$
\begin{align*}
& \omega_{I I .4 . e}^{*}=\left\{e_{I I .4 . e}^{*}=m /(2 h), \lambda_{I I .4 . e}^{*}=0, p_{I I .4 . e}^{*}=(m-\mu) / \alpha+c\right\}, \text { and }  \tag{4.100}\\
& \pi_{L L}\left(\omega_{I I .4 . e}^{*}\right)=\frac{(m-\mu)\left[2 h\left(x_{F}-\gamma c\right)+\alpha \mu\right]}{2 h \alpha}-\frac{(m-\mu)^{2}\left(4 h \gamma-\alpha^{2}\right)}{4 h \alpha^{2}}-m T-\mu \tau+\frac{\mu^{2}}{4 h} . \tag{4.101}
\end{align*}
$$

The necessary conditions for the existence of local optimal advertising and pricing strategy for Tactic II.4.e are shown in Proposition 4.2.56b. Specifically, item (i) of Proposition 4.2.56b is the basic condition for Tactic II.4. Item (ii) of Proposition 4.2 .56 b ensures that $p_{I I .4 . e}^{*}>c$ and $e_{I I, 4, e}^{*}>\tau$. Item (iii) of Proposition 4.2.56b ensures that $V_{L}\left(\omega_{I I .4 . e}^{*}\right) \leq 0$ which is the basic condition for Tactic II.4. Moreover, Proposition 4.2.56b shows the explicit formula of the local optimal advertising and pricing strategies for Tactic II.4.e, and the associated company's profit.

This completes the derivation of all the local optimums for Tactic II. Similarly, we summarize the major findings for Tactic II.

1. When there is no penalty for insufficient advertising, it is always optimal to allocate all the advertising effort to FG under Tactic II. When there is penalty for insufficient advertising, the optimal allocation of advertising effort could also be allocated to both LG and FG., and even only be allocated to LG (for Tactic II.2.a and Tactic II.4.b). This shows that when there is penalty for insufficient advertising, the company should strike a balance between allocating the advertising efforts to LG and FG. If the penalty for insufficient advertising to LG is very heavy, then it is optimal to advertise to LG even the company targets of the market segment of FG.
2. For Tactic II.3.b and Tactic II.4.b, the optimal advertising effort could be decreasing with the optimal retail price.

### 4.2.4 Tactic III: Selling to LG only

Under Tactic III, the demand of the product of LG and FG satisfy $D_{L}(\omega)>0$ and $D_{F}(\omega)=0$, respectively. Therefore, the total demand of the product is
$D(\omega)=D_{L}(\omega)=x_{L}+a \lambda e-g p$,
and the associated company's profit is
$\pi_{L L}(\omega)=\left(x_{L}+a \lambda e-g p\right)(p-c)-h e^{2}-m(T-\lambda e, 0)^{+}-\mu(\tau-(1-\lambda) e, 0)^{+}$.
Next, we explore the optimal advertising and pricing strategy for each sub-tactic of Tactic II.

## Tactic III. 1

Basic conditions: $D_{L}(\omega)>0, \quad D_{F}(\omega)=0, \quad \lambda e \geq T,(1-\lambda) e \geq \tau$ and $e \geq T+\tau$.
The company's profit for Tactic III. 1 is

$$
\begin{equation*}
\pi_{L L}(\omega)=\left(x_{F}+a \lambda e-g p\right)(p-c)-h e^{2} . \tag{4.102}
\end{equation*}
$$

Proposition 4.2.57 For Tactic III.1, (a) if $a \beta<\alpha$, then $V_{F}\left(\omega^{*}\right)<0$; (b) if $a \beta=\alpha$, then $V_{F}\left(\omega^{*}\right)$ is independent of $\lambda$; and (c) if a $\beta>\alpha$, then $V_{F}\left(\omega^{*}\right)=0$.

Proposition 4.2.57 shows that, for Tactic III.1, if $a \beta \neq \alpha$, then we can immediately know whether $V_{F}\left(\omega^{*}\right)$ is negative or not. Specifically, if $a \beta<\alpha$, then $V_{F}\left(\omega^{*}\right)<0$, namely, the company should totally ignore the LG market. On the other hand, if $a \beta>\alpha$, then the company should set the optimal advertising and pricing strategy such that $V_{F}\left(\omega^{*}\right)=0$. However, if $a \beta=\alpha$, it is not sure that which one of $V_{F}\left(\omega^{*}\right)<0$ and $V_{F}\left(\omega^{*}\right)=0$ is true. Therefore, for $a \beta=\alpha$, we need to consider both $V_{F}\left(\omega^{*}\right)<0$ and $V_{F}\left(\omega^{*}\right)=0$ in the later analysis. Next, we investigate the rules for determining the optimal allocation of the advertising efforts for Tactic III.1.

Proposition 4.2.58 For Tactic III.1, (a) If $e=T+\tau$, then $\lambda^{*}=T /(T+\tau)$; and (b) if $e>T+\tau$, then $\lambda^{*}=1-\tau / e$.

According to Proposition 4.2 .57 and Proposition 4.2.58, we consider four sub-tactics for Tactic III.1:
(Tactic III.1.a) $a \beta \leq \alpha, e^{*}=T+\tau, \lambda^{*}=T /(T+\tau)$ and $V_{F}\left(\omega^{*}\right)<0$;
(Tactic III.1.b) $a \beta \leq \alpha, e^{*}>T+\tau, \lambda^{*}=1-\tau / e$ and $V_{F}\left(\omega^{*}\right)<0$;
(Tactic III.1.c) $a \beta \geq \alpha, e^{*}=T+\tau, \lambda^{*}=T /(T+\tau)$ and $V_{F}\left(\omega^{*}\right)=0$; and
(Tactic III.1.d) $a \beta \geq \alpha, e^{*}>T+\tau, \lambda^{*}=1-\tau / e$ and $V_{F}\left(\omega^{*}\right)=0$.
Similarly, we need to take care of the basic conditions for Tactic III. 1 which are not covered by the specific conditions for each sub-tactic. Denoted by $\omega_{I I I .1 . i}^{*}$, for $i=a$, $b, c, d$, the local optimal advertising and price strategy for Tactic III.1.i. We explore the local optimal advertising and pricing strategies for each sub-tactic of Tactic III. 1 in the following.

- Tactic III.1.a

Specific conditions: $D_{L}(\omega)>0, \quad a \beta \leq \alpha, \quad e^{*}=T+\tau, \quad \lambda^{*}=T /(T+\tau)$ and $V_{F}\left(\omega^{*}\right)<0:$

By putting $\lambda^{*}=T /(T+\tau)$ into (4.102), we obtain
$\pi_{L L}(\omega)=\left(x_{L}-g c+a T\right)(p-c)-g(p-c)^{2}-h(T+\tau)^{2}$.
Proposition 4.2.59 For Tactic III.1.a, the local optimal advertising and pricing strategy exists only if $a \beta \leq \alpha, V_{F}\left(\omega_{I I I .1 . a}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .1 . a}^{*}\right)>0$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.1.a exists, then it is unique and is given by
$\omega_{I I I, 3, a}^{*}=\left\{e_{I I I, 3, a}^{*}=T+\tau, \lambda_{I I I, . a}^{*}=T /(T+\tau), p_{I I I, 3, a}^{*}=\left(x_{L}-g c+a T\right) /(2 g)+c\right\}, \quad$ and
$\pi_{L L}\left(\omega_{I I I .1 . a}^{*}\right)=\left(x_{L}-g c+a T\right)^{2} /(4 g)-h(T+\tau)^{2}$,
The necessary conditions for having a finite local optimal advertising and pricing strategy for Tactic III.1.a are shown in Proposition 4.2.59. Moreover, Proposition 4.2.59 shows the explicit formula of the optimal advertising and pricing strategies and the associated company's profit for Tactic III.1.a. Furthermore, $e_{I I I .1 . a}^{*}, \lambda_{I I I .1 . a}^{*}$ and $p_{I I I .1 . a}^{*}$ are increasing in $T$. In words, for Tactic III.1.a, a bigger $T$ induces a higher total advertising effort assigned by the company, a bigger proportion of advertising effort is allocated to LG, and a higher retail price of the product. On the other hand, $e_{I I I .1 . a}^{*}$ is increasing in $\tau, \lambda_{I I I .1 . a}^{*}$ is decreasing in $\tau$, and $p_{I I I .1 . a}^{*}$ is independent of $\tau$. In words, for Tactic III.1.a, a bigger $\tau$ induces a higher total advertising efforts
assigned by the company and a smaller proportion of advertising effort is allocated to LG.

- Tactic III.1.b:

Specific conditions: $D_{L}(\omega)>0, a \beta \leq \alpha, e>T+\tau, \quad \lambda^{*}=1-\tau / e$ and $V_{F}\left(\omega^{*}\right)<0$ :
By putting $\lambda^{*}=1-\tau / e$ into (4.102), we obtain
$\pi_{L L}(\omega)=\left(x_{L}-g c+a e-a \tau\right)(p-c)-g(p-c)^{2}-h e^{2}$.
Proposition 4.2.60 For Tactic III.1.b, the local optimal advertising effort as a function of retail price $p$ is given by

$$
\begin{equation*}
e_{I I I .1 . b}^{*}(p)=a(p-c) /(2 h) . \tag{4.107}
\end{equation*}
$$

Moreover, $e_{I I I .1 . b}^{*}(p)$ is strictly increasing in $p$.
Proposition 4.2.60 asserts that, for Tactic III.1.b, a higher retail price induces a higher local optimal advertising effort.
Proposition 4.2.61 For Tactic III.1.b, the local optimal advertising and pricing strategy exists only if (i) $a \beta \leq \alpha, V_{F}\left(\omega_{I I .1 . b}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .1 .1}^{*}\right)>0$; (ii) $4 h g>a^{2}$; (iii) $x_{L}-g c \geq T\left(4 h g-a^{2}\right)+4 h g \tau$; and (iv) $x_{L}-g c>a \tau$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.1.b exists, it is unique and $\omega_{L L}^{*}$ is given by
$\omega_{I I I .1 . b}^{*}=\left\{e_{I I I .1 . b}^{*}, \lambda_{I I I .1 . b}^{*}, p_{I I I .1 . b}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I I I .1 . b}^{*}\right)=h\left(x_{L}-g c-a \tau\right)^{2} /\left(4 h g-a^{2}\right)$,
where $e_{I I I .1 . b}^{*}=a\left(x_{L}-g c-a \tau\right) /\left(4 h g-a^{2}\right), \quad \lambda_{I I I .1 . b}^{*}=\frac{a\left(x_{L}-g c\right)-4 h g \tau}{a\left(x_{L}-g c-a \tau\right)}$, and
$p_{\text {III... }}^{*}=c+2 h\left(x_{L}-g c-a \tau\right) /\left(4 h g-a^{2}\right)$.
The necessary conditions for having a finite local optimal advertising and pricing strategy for Tactic III.1.b are shown in Proposition 4.2.61. Specifically, conditions in item (ii) of Proposition 4.2 .61 ensure that concavity of $\pi_{L L}(\omega)$. Items (iii) and (iv) of Proposition 4.2.61 ensure that constraints of $e_{I I I .1 . b}^{*} \geq T+\tau$ and $p_{I I I .1 . b}^{*} \geq c$ are satisified, respectively. Moreover, Proposition 4.2 .61 shows the explicit formula of the local optimal advertising and pricing strategies, and the associated company's profit for Tactic III.1.b. Furthermore, for Tactic III.1.b, $e_{I I I .1 . b}^{*}, \lambda_{I I I .1 . b}^{*}$ and $p_{I I I .1 b}^{*}$ are decreasing in $\tau$, namely, a bigger $\tau$ induces a lower total advertising efforts
assigned by the company, a lower proportion of advertising effort is allocated to LG, and a lower retail price of the product. On the other hand, $e_{I I I .1 . b}^{*}, \lambda_{I I I .1 . b}^{*}$ and $p_{I I I .1 . b}^{*}$ are independent of $T$.

- Tactic III.1.c:

Specific conditions: $D_{L}(\omega)>0, \quad a \beta \geq \alpha, \quad e^{*}=T+\tau, \quad \lambda^{*}=T /(T+\tau) \quad$ and $V_{F}\left(\omega^{*}\right)=0$.

Proposition 4.2.62 For Tactic III.1.c, the local optimal advertising and pricing strategy exists only if $a \beta \geq \alpha$ and $D_{L}\left(\omega_{\text {III.... }}^{*}\right)>0$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.1.c exists, then it is unique and is given by
$\omega_{I I I .1 . c}^{*}=\left\{e_{I I .1 . c}^{*}=T+\tau, \lambda_{I I I .1 . c}^{*}=T /(T+\tau), p_{I I I .1 . c}^{*}=\frac{x_{F}+\beta x_{L}+\alpha \tau+a \beta T}{\gamma+\beta g}\right\}$, and
$\pi_{L L}\left(\omega_{I I I .1 . c}^{*}\right)=\left(x_{L}-g p_{I I .1 . c}^{*}+a T\right)\left(p_{I I I .1 . c}^{*}-c\right)-h(T+\tau)^{2}$
The necessary conditions for having a finite local optimal advertising and pricing strategy for Tactic III.1.c are shown in Proposition 4.2.62. Moreover, Proposition 4.2.61 shows the explicit formula of the local optimal advertising and pricing strategies, and the associated company's profit for Tactic III.1.c. Furthermore, from Proposition 4.2.62, we find that $e_{I I I .1 . c}^{*}$ is increasing in $T$ and $\tau, \lambda_{I I I .1 . c}^{*}$ is increasing in $T$ but decreasing in $\tau$, and $p_{I I I .1 . c}^{*}$ is increasing in $T$ but independent of $\tau$. In words, for Tactic III.1.c, a bigger $T$ induces a higher local optimal advertising effort, a higher proportion of advertising effort to be allocated to LG, and a higher local optimal retail price of the product. On the other hand, for Tactic III.1.c, a bigger $\tau$ induces a higher total advertising effort, and a higher proportion of advertising effort to be allocated to FG.

- Tactic III.1.d

Specific conditions: $D_{L}(\omega)>0, a \beta \geq \alpha, e>T+\tau, \lambda^{*}=1-\tau / e$ and $V_{F}\left(\omega^{*}\right)=0$.
Proposition 4.2.63 For Tactic III.1.d, the local optimal advertising effort as a function of retail price $p$ is given by
$e_{I I I .1 . d}^{*}(p)=\left[(a \beta-\alpha) \tau-\beta x_{L}-x_{F}+(\gamma+\beta g) p\right] /(a \beta)$.
Moreover, $e_{\text {III. } 1 . d}^{*}(p)$ is strictly increasing in $p$.

Proposition 4.2.63 shows that, for Tactic III.1.d, a higher retail price of the product induces a higher local optimal of advertising effort.

Proposition 4.2.64 For Tactic III.1.d, the local optimal advertising and pricing strategy exists only if (i) $a \beta \geq \alpha$ and $D_{L}\left(\omega_{\text {III. } 1 . d}^{*}\right)>0$; (ii) $h(\gamma+\beta g)^{2}>a^{2} \beta \gamma$;
(iii) $2 a \beta \gamma\left(x_{L}-g c-a \tau\right)-a(\gamma+\beta g)\left(x_{F}-\gamma c+\alpha \tau\right)>2(T+\tau)\left[h(\gamma+\beta g)^{2}-a^{2} \beta \gamma\right]$;
(iv) $2 h \beta(\gamma+\beta g)\left(x_{L}-g c-a \tau\right)>\left[a^{2} \beta-2 h(\gamma+\beta g)\right]\left(x_{F}-\gamma c+\alpha \tau\right)$. Moreover, if the optimal advertising and pricing strategy for Tactic III.1.d exists, then it is unique and is given by
$\omega_{I I I .1 . d}^{*}=\left\{e_{I I I .1 . d}^{*}, \lambda_{I I I .1 . d}^{*}, p_{I I I .1 . d}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I I I .1 . d}^{*}\right)=\left(x_{L}-g p_{I I I .1 . d}^{*}+a e-a \tau\right)\left(p_{I I I .1 . d}^{*}-c\right)-h\left(e_{I I I .1 . d}^{*}\right)^{2}$,
where $e_{I I I .1 . d}^{*}=\frac{2 a \beta \gamma\left(x_{L}-g c-a \tau\right)-a(\gamma+\beta g)\left(x_{F}-\gamma c+\alpha \tau\right)}{2\left[h(\gamma+\beta g)^{2}-a^{2} \beta \gamma\right]}, \lambda_{I I I .1 . d}^{*}=1-\tau / e_{I I I .1 . d}^{*}$, and
$p_{\text {III. } 1 . d}^{*}=c+\frac{2 h \beta(\gamma+\beta g)\left(x_{L}-g c-a \tau\right)-\left[a^{2} \beta-2 h(\gamma+\beta g)\right]\left(x_{F}-\gamma c+\alpha \tau\right)}{2\left[h(\gamma+\beta g)^{2}-a^{2} \beta \gamma\right]}$.
The necessary conditions for having a finite local optimal advertising and pricing strategy for Tactic III.1.d are shown in Proposition 4.2.64. Moreover, Proposition 4.2.64 shows the explicit formula of the optimal advertising and pricing strategies and the associated company's profit for Tactic III.1.d. Specifically, conditions in item (i) of Proposition 4.2.64 are the specific conditions for Tactic III.1.d. Item (ii) of Proposition 4.2.64 ensures that, for Tactic III.1.d, $\pi_{L L}(\omega)$ is strictly concave in $p$. Items (iii) and (iv) of Proposition 4.2.64 ensure that $e_{I I I .1 d}^{*}>T+\tau$ and $p_{I I I .1 d}^{*}>c$, respectively, which are also specific conditions for Tactic III.1.d.

## Tactic III. 2

Basic conditions: $D_{L}(\omega)>0, D_{F}(\omega)=0, \lambda e \geq T,(1-\lambda) e<\tau$ and $e \geq T$.
The profit of the company for Tactic III. 2 is

$$
\begin{equation*}
\pi_{L L}(\omega)=\left(x_{F}+a \lambda e-g p\right)(p-c)-h e^{2}-\mu[\tau-(1-\lambda) e] . \tag{4.115}
\end{equation*}
$$

Proposition 4.2.65 Suppose that $\omega^{*}$ belongs to Tactic III.2: (a) If $p^{*}>\mu / a+c$, then $\lambda^{*}=1$. If $p^{*}=\mu / a+c$, then there are multiple $\lambda^{*}$. If $p^{*}<\mu / a+c$, then $\lambda^{*}=T / e^{*}$.

Proposition 4.2.65 shows that, for Tactic III.2, the local optimal allocations of the
advertising effort are different for different cases.
Proposition 4.2.66 Suppose that $\omega^{*}$ belongs to Tactic III.2, (a) $V_{F}\left(\omega^{*}\right)<0$ if $\left[a\left(p^{*}-c\right)-\mu\right](a \beta-\alpha)<0 ; V_{F}\left(\omega^{*}\right)=0$ if $\left[a\left(p^{*}-c\right)-\mu\right](a \beta-\alpha)>0$.

Proposition 4.2.66 shows the rules for company to determine the value of $V_{F}\left(\omega^{*}\right)$ for Tactic III.2. According Proposition 4.2.65 and Proposition 4.2.66, we further consider nine sub-tactics for Tactic III.2, which cover all possible cases of Tactic III. 2.
(Tactic III.2.a) $a \beta \leq \alpha, p>\mu / a+c, e^{*}=T, \lambda^{*}=1$ and $V_{F}(\omega)<0$;
(Tactic III.2.b) $a \beta \leq \alpha, p>\mu / a+c, e>T, \lambda^{*}=1$ and $V_{F}(\omega)<0$;
(Tactic III.2.c) $a \beta \leq \alpha, p<\mu / a+c, e^{*}=T, \lambda^{*}=1$ and $V_{F}(\omega)=0$;
(Tactic III.2.d) $a \beta \leq \alpha, p<\mu / a+c, e>T, \lambda^{*}=T / e$ and $V_{F}(\omega)=0$.
(Tactic III.2.e) $a \beta \geq \alpha, p>\mu / a+c, e^{*}=T, \lambda^{*}=1$ and $V_{F}(\omega)=0$;
(Tactic III.2.f) $a \beta \geq \alpha, p>\mu / a+c, e>T, \lambda^{*}=1$ and $V_{F}(\omega)=0$;
(Tactic III.2.g) $a \beta \geq \alpha, p<\mu / a+c, e^{*}=T, \lambda^{*}=1$ and $V_{F}(\omega)<0$;
(Tactic III.2.h) $a \beta \geq \alpha, \quad p<\mu / a+c, e>T, \lambda^{*}=T / e$ and $V_{F}(\omega)<0$; and
(Tactic III.2.k) $p^{*}=\mu / a+c, \quad \lambda^{*}=1, e^{*} \geq T, V_{F}(\omega) \leq 0$.
Notice that, for $\left[a\left(p^{*}-c\right)-\mu\right](a \beta-\alpha)=0$, we either have $a \beta=\alpha$ or $p^{*}=\mu / a+c$. For $a \beta=\alpha$, the value of $V_{F}\left(\omega^{*}\right)$ is still undetermined. Therefore, we study both cases of $V_{F}\left(\omega^{*}\right)=0$ and $V_{F}\left(\omega^{*}\right)<0$ for $a \beta=\alpha$ in the paper. For $p^{*}=\mu / a+c$, by Proposition 4.2.65, there are multiple $\lambda^{*}$ for Tactic III.2. For simplicity, we only consider $\lambda^{*}$ in Tactic III.2.k. Similar results can be obtained if we consider other values of $\lambda^{*}$ for Tactic III.2.k.

Similarly, we need to consider the basic conditions for Tactic III. 2 which are not covered by the specific conditions for each sub-tactic. Denoted by $\omega_{I I I .2 i}^{*}$, for $i=a, b, c$, $d, e, f, g, h, k$, the local optimal advertising and price strategy for Tactic III.2.i. Next, we explore the local optimal advertising and pricing strategies for each sub-tactic of Tactic III.2.

## - Tactic III.2.a

Specific conditions: $D_{L}(\omega)>0, \quad a \beta \leq \alpha, \quad p^{*} \geq \mu / a+c, \quad e^{*}=T, \quad \lambda^{*}=1$ and
$V_{F}(\omega)<0$ :
Proposition 4.2.67 For Tactic III.2.a, the local optimal advertising and pricing strategy exists only if (i) a $\beta \leq \alpha, V_{F}\left(\omega_{\text {III. } 2 . a}^{*}\right)<0$ and $D_{L}\left(\omega_{\text {III..a. }}^{*}\right)>0$; and
(ii) $x_{L}-g c>2 g \mu / a-a T$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.2.a exists, then it is unique and is given by

$$
\begin{align*}
& \omega_{I I I .2 . a}^{*}=\left\{e_{I I I .2 . a}^{*}=T, \lambda_{I I I .2 a}^{*}=1, p_{I I I .2 . a}^{*}=\left(x_{L}+g c+a T\right) /(2 g)\right\} \text {, and }  \tag{4.116}\\
& \pi_{L L}\left(\omega_{I I I .2 a}^{*}\right)=\left(x_{L}-g c+a T\right) /(4 g)-h T^{2}-\mu \tau \tag{4.117}
\end{align*}
$$

The necessary conditions for having a finite local optimal advertising and pricing strategy for Tactic III.2.a are shown in Proposition 4.2.67. Moreover, Proposition 4.2.67 shows the explicit formula of the optimal advertising and pricing strategies and the associated company's profit for Tactic III.2.a. Specifically, conditions in item (i) of Proposition 4.2.67 are the specific conditions for Tactic III.2.a. Item (ii) of Proposition 4.2.67 ensures that $p_{I I I .2 a}^{*}>\mu / a+c$ which is the specific condition for Tactic III.2.a. Furthermore, from Proposition 4.2.67, we find that a bigger $T$ induces a higher total advertising effort assigned by the company and a higher retail price of the product. On the other hand, $e_{I I I .2 a}^{*}$ and $p_{I I I .2 a}^{*}$ are independent of $\tau$, and $\lambda_{I I I .2 a}^{*}$ is independent of $T$ and $\tau$.

- Tactic III.2.b

Specific conditions: $D_{L}(\omega)>0, a \beta \leq \alpha, \quad p>\mu / a+c, \quad e>T, \quad \lambda^{*}=1$ and $V_{F}(\omega)<0$.

By putting $\lambda^{*}=1$ into (4.115), we obtain
$\pi_{L L}(\omega)=\left(x_{F}+a e-g p\right)(p-c)-h e^{2}-\mu \tau$.
Proposition 4.2.68 For Tactic III.2.b, the local optimal advertising effort as a function of retail price $p$ is given by $e_{I I .3 . b}^{*}(p)=a(p-c) /(2 h)$.

As $a>0, e_{I I .3 . b}^{*}(p)$ is increasing in $p$, namely, a higher retail price of the product induces a higher advertising effort for Tactic III.2.b.

Proposition 4.2.69 For Tactic III.2.b, the local optimal advertising and pricing strategy exists only if (i) $a \beta \leq \alpha, V_{F}\left(\omega_{I I I .2 . b}^{*}\right)<0$ and $D_{L}\left(\omega_{I I .2 . b}^{*}\right)>0$; (ii) $4 h g>a^{2}$; (iii) $x_{L}-g c>T\left(4 h g-a^{2}\right) / a$; and (iv) $x_{L}-g c>\mu\left(4 h g-a^{2}\right) /(2 a h)$. Moreover, if the
local optimal advertising and pricing strategy for Tactic III.2.b exists, then it is and is given by
$\omega_{\text {III } 2 . b}^{*}=\left\{e_{\text {III. } 2 . b}^{*}, \lambda_{\text {III. } 2 . b}^{*}, p_{\text {III.2.b }}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I I I .2 . b}^{*}\right)=h\left(x_{L}-g c\right)^{2} /\left(4 h g-a^{2}\right)-\mu \tau$,
where $e_{\text {III.2.b }}^{*}=a\left(x_{L}-g c\right) /\left(4 h g-a^{2}\right), \quad \lambda_{I I I .2 . b}^{*}=1$, and
$p_{\text {III. } 2 . b}^{*}=c+2 h\left(x_{L}-g c\right) /\left(4 h g-a^{2}\right)$.
The necessary conditions for having a finite $\omega_{I I L .2 . b}^{*}$ are shown in Proposition 4.2.69.

## Tactic III.2.c

Specific conditions: $D_{L}(\omega)>0, a \beta \leq \alpha, \quad p<\mu / a+c, \quad e^{*}=T, \quad \lambda^{*}=1$ and $V_{F}(\omega)=0$.

Proposition 4.2.70 For Tactic III.2.c, the local optimal advertising and pricing strategy exists only if (i) $a \beta \leq \alpha$ and $D_{L}\left(\omega_{I I I .2 . c}^{*}\right)>0$; and
(ii) $x_{F}-\gamma c+\beta\left(x_{L}-g c\right)<\mu(\gamma+\beta g) / a-a \beta T$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.2.c exists, then it is unique and is given by

$$
\begin{align*}
& \omega_{I I I .2 . c}^{*}=\left\{e_{I I I .2 . c}^{*}=T, \lambda_{I I I .2 . c}^{*}=1, p_{I I I .2 . c}^{*}=\frac{x_{F}+\beta x_{L}+a \beta T}{\gamma+\beta g}\right\}, \text { and }  \tag{4.122}\\
& \pi_{L L}\left(\omega_{I I L .2 . c}^{*}\right)=\left(x_{L}-g p_{I I I .2 . c}^{*}+a T\right)\left(p_{I I I .2 . c}^{*}-c\right)-h T^{2}-\mu \tau . \tag{4.123}
\end{align*}
$$

Proposition 4.2.70 shows the explicit formula of the local optimal advertising and pricing strategy (if it is finite), and the associated company's profit Tactic III.2.c. Moreover, the necessary conditions for having a finite $\omega_{\text {III...c }}^{*}$ are shown in Proposition 4.2.70.

- Tactic III.2.d

Specific conditions: $D_{L}(\omega)>0, \quad a \beta \leq \alpha, \quad p<\mu / a+c, \quad e>T, \quad \lambda^{*}=T / e$ and $V_{F}(\omega)=0$.

By putting $\lambda^{*}=T / e$ into (4.115) we obtain
$\pi_{L L}(\omega)=\left(x_{F}+a T-g p\right)(p-c)-h e^{2}-\mu(T+\tau-e)$.
Moreover, as $\left(1-\lambda^{*}\right) e^{*}<\tau$ and $\lambda^{*} e^{*}=T$ for Tactic III.2.d, we have the condition
$T<e^{*}<T+\tau$ for Tactic III.2.d.
Proposition 4.2.71 For Tactic III.2.d, then the local optimal advertising efforts in the function of retail price $p$ is given by $e_{I I I, . d}^{*}(p)=\left[(\alpha-a \beta) T-\beta x_{L}-x_{F}+(\gamma+\beta g) p\right] / \alpha$.

Proposition 4.2.71 shows the relationship of optimal $e$ and optimal $p$ for Tactic III.2.d. As $\gamma+\beta g>0$ and $a>0$, the $e_{I I I .2 . d}^{*}(p)$ is increasing in $p$, namely, a higher advertising effort is induced by a higher retail price of the product for Tactic III.2.d.

Proposition 4.2.72 For Tactic III.2.d, the local optimal advertising and pricing strategy exists only if (i) $a \beta \leq \alpha$ and $D_{L}\left(\omega_{I I I .2 . d}^{*}\right)>0$; (ii) $T<e_{\text {III. . . }}^{*}<T+\tau$; and (iii) $p_{\text {III.2.d }}^{*}>\mu / a+c$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.2.d exists, then it is unique and is given by
$\omega_{I I I .2 . d}^{*}=\left\{e_{\text {III.2.d }}^{*}, \lambda_{I I I .2 . d}^{*}, p_{I I I .2 . d}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I I I .2 . d}^{*}\right)=\left(x_{F}+a T-g p_{I I I .2 . d}^{*}\right)\left(p_{I I I .2 . d}^{*}-c\right)-h e^{2}-\mu\left(T+\tau-e_{I I I .2 . d}^{*}\right)$,
where $e_{\text {III. } 2 . d}^{*}=\frac{\alpha(\gamma-\beta g)\left(x_{L}-g c+a T\right)+\mu(\gamma+\beta g)^{2}-2 \alpha g\left(x_{F}-\gamma c-\alpha T\right)}{2\left[h(\gamma+\beta g)^{2}+\alpha^{2} g\right]}$,
$\lambda_{\text {III.2.d }}^{*}=T / e_{\text {III.2.d }}^{*}$, and
$p_{\text {III. } 2 . d}^{*}=\frac{\left[\alpha^{2}+2 h \beta(\gamma+\beta g)\right]\left(x_{L}-g c+a T\right)+2 h(\gamma+\beta g)\left(x_{F}-\gamma c-\alpha T\right)+\alpha \mu(\gamma+\beta g)}{2\left[h(\gamma+\beta g)^{2}+\alpha^{2} g\right]}+c$.
Proposition 4.2.72 provides the explicit formula of the local optimal advertising and pricing strategy for Tactic III.2.d, and an associated company's profit. Moreover, the necessary conditions for having a finite local optimum for Tactic III.2.d are shown in Proposition 4.2.72.

## - Tactic III.2.e

Specific conditions: $D_{L}(\omega)>0, \quad a \beta \geq \alpha, \quad p>\mu / a+c, \quad e^{*}=T, \quad \lambda^{*}=1$ and $V_{F}(\omega)=0$.

Proposition 4.2.73 For Tactic III.2.e, the local optimal advertising and pricing strategy exists only if (i) $a \beta \geq \alpha$ and $D_{L}\left(\omega_{\text {III.2.e }}^{*}\right)>0$; and
(ii) $x_{F}-\gamma c+\beta\left(x_{L}-g c\right)>\mu(\gamma+\beta g) / a-a \beta T$. Moreover, if the optimal advertising and pricing strategy for Tactic III.2.e, then it is unique and is given by
$\omega_{I I I .2 . e}^{*}=\left\{e_{I I I .2 e}^{*}=T, \lambda_{I I I .2 . e}^{*}=1, p_{I I I .2 . e}^{*}=\frac{x_{F}+\beta x_{L}+a \beta T}{\gamma+\beta g}\right\}$, and
$\pi_{L L}\left(\omega_{I I I .2 . e}^{*}\right)=\left(x_{L}-g p_{I I I .2 .}^{*}+a T\right)\left(p_{\text {III.2.e }}^{*}-c\right)-h T^{2}-\mu \tau$
Proposition 4.2.71 provides the explicit formula of the local optimal solution of for Tactic III.2.e (if it exists), and the associated company's profit. Moreover, the necessary conditions for having a finite $\omega_{I I I .2 . e}^{*}$ are shown in Proposition 4.2.73. Specifically, conditions in item (i) of Proposition 4.2.73 are the specific conditions for Tactic III.2.e. Item (ii) of Proposition 4.2.73 ensures that $p_{I I I .2 e}^{*} \geq \mu / a+c$, which is also the specific condition for Tactic III.2.e. Furthermore, from Proposition 4.2.73, we observe that $e_{I I I .2 . e}^{*}$ and $p_{\text {III.2.e }}^{*}$ are increasing in $T$ but independent of $\tau$, and $\lambda_{I I I .2 . e}^{*}$ is independent of $T$ and $\tau$. In words, for Tactic III.2.e, a bigger $T$ induces a higher total advertising effort and a higher retail price of the product.

By noting that, $\omega_{I I I .2 . e}^{*}=\omega_{I I I .2 . c}^{*}$ and hence $\pi_{L L}\left(\omega_{I I I .2 . e}^{*}\right)=\pi_{L L}\left(\omega_{I I I .2 . c}^{*}\right)$. In words the optimal advertising and pricing strategies for Tactic III.2.e and Tactic III.2.c, the associated company's profits, are the same. However, according to Proposition 4.2.69 and Proposition 4.2.73, Tactic III.2.e and Tactic III.2.c have different necessary conditions for the existence of the local optimum.

- Tactic III.2.f.

Specific conditions: $D_{L}(\omega)>0, a \beta \geq \alpha, \quad p>\mu / a+c, \quad e>T, \quad \lambda^{*}=1$ and $V_{F}(\omega)=0$.

By putting $\lambda^{*}=1$ into (4.115), we obtain
$\pi_{L L}(\omega)=\left(x_{F}+a e-g p\right)(p-c)-h e^{2}-\mu \tau$.
Proposition 4.2.74 For Tactic III.2.f, the local optimal advertising effort as a function of retail price $p$ is given by

$$
\begin{equation*}
e_{I I I \cdot 2 \cdot f}^{*}(p)=\left[(\gamma+\beta g) p-\beta x_{L}-x_{F}\right] /(a \beta) . \tag{4.131}
\end{equation*}
$$

Proposition 4.2.74 shows the relationship of optimal $e$ and optimal $p$ for Tactic III.2.f. As $\gamma+\beta g>0$ and $a \beta>0, e_{\text {III. } 2 . f}^{*}(p)$ is increasing in $p$, namely, a higher advertising effort is induced by a higher retail price of the product for Tactic III.2.f.
Proposition 4.2.75 For Tactic III.2.f, the local optimal advertising and pricing strategy exists only if (i) $a \beta \geq \alpha$ and $D_{L}\left(\omega_{I I I .2 . f}^{*}\right)>0$; (ii) $h(\gamma+\beta g)>a^{2} \beta$;
(ii) $x_{F}-\gamma c+\beta\left(x_{L}-g c\right)>2 T\left[a^{2} \beta-h(\gamma+\beta g)\right] / a$;
(iii)

$$
\left[a^{2} \beta-2 h(\gamma+\beta g)\right]\left(x_{F}-\gamma c\right)-2 h \beta(\gamma+\beta g)\left(x_{L}-g c\right)
$$

$$
>2 \mu(\gamma+\beta g)\left[a^{2} \beta-h(\gamma+\beta g)\right] / a
$$

Moreover, if the local optimal advertising and pricing strategy for Tactic III.2.f exists, then it is unique and is given by
$\omega_{I I I .2 . f}^{*}=\left\{e_{I I I .2 . f}^{*}, \lambda_{I I I .2 f}^{*}, p_{I I I .2 . f}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I I I .2 . f}^{*}\right)=\left(x_{F}+a e_{I I I .2 . f}^{*}-g p_{I I I .2 . f}^{*}\right)\left(p_{I I I .2 . f}^{*}-c\right)-h\left(e_{I I I .2 . f}^{*}\right)^{2}-\mu \tau$,
where $e_{I I I .2 . f}^{*}=\frac{a\left[x_{F}-\gamma c+\beta\left(x_{L}-g c\right)\right]}{2\left[a^{2} \beta-h(\gamma+\beta g)\right]}, \lambda_{\text {III..f }}^{*}=1$, and
$p_{\text {III. } 2 . f}^{*}=\frac{\left[a^{2} \beta-2 h(\gamma+\beta g)\right]\left(x_{F}-\gamma c\right)-2 h \beta(\gamma+\beta g)\left(x_{L}-g c\right)}{2(\gamma+\beta g)\left[a^{2} \beta-h(\gamma+\beta g)\right]}+c$.
Proposition 4.2.75 provides the formula of the local optimal advertising and pricing strategy for Tactic III.2.f (if it exists), and the associated company's profit. Moreover, the necessary conditions for a finite $\omega_{\text {III. } 2 . f}^{*}>T$ are shown in Proposition 4.2.75. Specifically, conditions in item (i) of Proposition 4.2 .75 are the specific conditions for Tactic III.2.f. Condition in item (ii) of Proposition 4.2.75 ensures that the profit function of the company for Tactic III.2.f is concave in $p$. Conditions in items (iii) and (iv) of Proposition 4.2.75 ensure that $e_{I I I .2 . f}^{*}>T$ and $p_{I I I .2 . f}^{*}>\mu / a+c$, respectively, which are also the specific conditions for Tactic III.2.f.

## - Tactic III.2.g

Specific conditions: $D_{L}(\omega)>0, a \beta \geq \alpha, p<\mu / a+c, \quad e^{*}=T, \quad \lambda^{*}=1$ and $V_{F}(\omega)<0$.

Proposition 4.2.76 For Tactic III.2.g, the local optimal advertising and pricing strategy exists only if (i) a $\beta \geq \alpha, V_{F}\left(\omega_{I I I .2 . g}^{*}\right)<0$ and $D_{L}\left(\omega_{\text {III.2.g }}^{*}\right)>0$; and
(ii) $g c-a T<x_{L}-g c<2 g \mu / a-a T$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.2.g exists, then it is unique and is given by
$\omega_{I I I .2 . g}^{*}=\left\{e_{I I I .2 \cdot g}^{*}=T, \lambda_{I I I .2 . g}^{*}=1, p_{I I I .2 \cdot g}^{*}=\left(x_{L}+g c+a T\right) /(2 g)\right\}$, and
$\pi_{L L}\left(\omega_{I I I .2 . g}^{*}\right)=\left(x_{L}-g c+a T\right) /(4 g)-h T^{2}-\mu \tau$
Proposition 4.2.76 provides the explicit formula of the local optimal advertising and pricing strategy for Tactic III.2.f (if it exists), and the associated company's profit.

Moreover, the necessary conditions for a finite $\omega_{\text {III.2.g }}^{*}$ are shown in Proposition 4.2.76. Specifically, conditions in item (i) of Proposition 4.2.76 are directly obtained from the specific conditions for Tactic III.2.f. Item (ii) of Proposition 4.2.76 ensures that $c<p_{\text {III. } 2 . g}^{*}<\mu / a+c$, which is also the special condition for Tactic III.2.f. Furthermore, from Proposition 4.2.76, we find that $e_{I I I .2 . g}^{*}$ and $p_{I I I .2 . g}^{*}$ are increasing in $T$, but independent of $\tau$. On the other hand $\lambda_{\text {III. } 2 . g}^{*}$ is independent of $T$ and $\tau$. In other words, for Tactic III.2.g, a bigger $T$ induces a higher local optimal advertising effort and a higher local optimal retail price of the product.

Notice that $\omega_{I I I, 2 . g}^{*}=\omega_{I I I, 2, a}^{*}$ and hence $\pi_{L L}\left(\omega_{I I I, 2, g}^{*}\right)=\pi_{L L}\left(\omega_{I I I, 2, a}^{*}\right)$. In words the local optimal advertising and pricing strategies for Tactic III.2.f and Tactic III.2.a, and the associated company's profits, are the same. However, according to Proposition 4.2.76 and Proposition 4.2.67, Tactic III.2.g and Tactic III.2.a have different necessary conditions for the finite local optimum.

- Tactic III.2.h

Specific conditions: $D_{L}(\omega)>0, \quad a \beta \geq \alpha, \quad p<\mu / a+c, \quad e>T, \quad \lambda^{*}=T / e$ and $V_{F}(\omega)<0$.

By putting $\lambda^{*}=T / e^{*}$ into (4.105), we obtain
$\pi_{L L}(\omega)=\left(x_{F}+a T-g p\right)(p-c)-h e^{2}-\mu(T+\tau-e)$.
Moreover, as $\left(1-\lambda^{*}\right) e^{*}<\tau$ and $\lambda^{*} e^{*}=T$ for Tactic III.2.h, we have the condition $T<e^{*}<T+\tau$ for Tactic III.2.h.

Proposition 4.2.77 For Tactic III.2.h, the local optimal advertising effort is given by

$$
\begin{equation*}
e_{I I I, 2 h}^{*}=\mu /(2 h) . \tag{4.137}
\end{equation*}
$$

Proposition 4.2.77 shows that, for Tactic III.2.h, the optimal advertising effort assigned by the company is independent of $p$. Noting that, for all the sub-tactics that we considered previously, the optimal advertising effort assigned by the company does not depend on $p$. Proposition 4.2.77 implies that the positive correlation between the optimal $e$ and the optimal $p$ is not always true.
Proposition 4.2.78 For Tactic III.2.h, the local optimal advertising and pricing strategy exists only if (i) $a \beta \geq \alpha, \quad V_{F}\left(\omega_{I I .2 . h}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .2 . h}^{*}\right)>0$; (ii) $2 h T<\mu<2 h(T+\tau)$; and (iii) $x_{L}-g c<2 g \mu-a T$. Moreover, if the local optimal
advertising and pricing strategy for Tactic III.2.h exists, then it is unique and is given by
$\omega_{I I .2 . h}^{*}=\left\{e_{\text {III.2.h }}^{*}=\mu(2 h), \lambda_{I I I .2 . h}^{*}=2 h T / \mu, p_{I I I .2 . h}^{*}=\left(x_{L}+g c+a T\right) /(2 g)\right\}$, and
$\pi_{L L}\left(\omega_{I I I, 2, h}^{*}\right)=\frac{\left(x_{L}-g c+a T\right)^{2}}{4 g}+\frac{\mu^{2}}{4 h}-\mu(T+\tau)$.
Proposition 4.2.78 shows the explicit formula of the local optimal advertising and pricing strategies for Tactic III.2.h (if it exists), and the associated company's profit. Moreover, the necessary conditions for a finite $\omega_{I I I, 2 . h}^{*}$ are shown in Proposition 4.2.78. All the necessary conditions are derived from the specific conditions for Tactic III.2.h. Furthermore, for Tactic III.2.h, $e_{\text {III.2.h }}^{*}, \lambda_{\text {III.2.h }}^{*}$ and $p_{I I I .2 . h}^{*}$ are independent $\tau$. On the other hand, $\lambda_{I I I .2, h}^{*}$ and $p_{I I I .2, h}^{*}$ are increasing in $T$ but $e_{\text {III. } 2 . h}^{*}$ is independent of $T$. In other words, for Tactic III.2.h, the local optimal advertising effort is independent of $T$ and $\tau$. However, a bigger $T$ induces a bigger proportion of advertising effort to be allocated to LG and a higher local optimal retail price of the product.

## - Tactic III.2.k

Specific conditions: $D_{L}(\omega)>0, p^{*}=\mu / a+c, \lambda^{*}=1, e^{*} \geq T, V_{F}(\omega) \leq 0$.
Proposition 4.2.79 For Tactic III.2.k, the local optimal advertising and pricing strategy exists only if (i) $D_{L}\left(\omega_{I I I, 2, k}^{*}\right)>0$; (ii) $2 h T \leq \mu$; and
(iii) $x_{F}-\gamma c+\beta\left(x_{L}-g c\right) \leq \mu\left[2 h(\gamma+\beta g)-a^{2} \beta\right] /(2 a h)$. Moreover, if the optimal advertising and pricing strategy for Tactic III.2.k exists, then it is unique and is given by

$$
\begin{align*}
& \omega_{I I I, 2, k}^{*}=\left\{e_{I I I, 2, k}^{*}=\mu /(2 h), \lambda_{I I I, 2, k}^{*}=0, p_{I I I, 2, k}^{*}=\mu / a+c\right\}, \text { and }  \tag{4.140}\\
& \pi_{L L}\left(\omega_{I I \cdot 2, k}^{*}\right)=\frac{\mu\left(x_{F}-g c\right)}{a}-\frac{\mu^{2}\left(4 h g-a^{2}\right)}{4 h a^{2}}-\mu \tau . \tag{4.141}
\end{align*}
$$

The necessary conditions for the existence of local optimal advertising and pricing strategy for Tactic III.2.k are shown in Proposition 4.2.79. Moreover, Proposition 4.2.79 shows the explicit formula of the local optimal advertising and pricing strategies for Tactic III.2.k, and the associated company's profit.

## Tactic III. 3

Basic conditions: $D_{L}(\omega)>0, D_{F}(\omega)=0, \lambda e<T,(1-\lambda) e \geq \tau$ and $e \geq \tau$.
The company's profit for Tactic III. 3 is
$\pi_{L L}(\omega)=\left(x_{L}+a \lambda e-g p\right)(p-c)-h e^{2}-m(T-\lambda e)$.
Proposition 4.2.80 Suppose that the global optimal advertising and pricing strategy belongs to Tactic III.3, $V_{F}\left(\omega^{*}\right)<0$ if a $\beta \leq \alpha$, and $V_{F}\left(\omega^{*}\right)=0$ if a $\beta \geq \alpha$. Moreover, $\lambda^{*}=0$ if $e^{*}=\tau$, and $\lambda^{*}=1-\tau / e^{*}$ if $e^{*}>\tau$.

Proposition 4.2.80 shows that, for Tactic III.3, the company should set $V_{F}\left(\omega^{*}\right)<0$ if $a \beta \leq \alpha$, and the company should set $V_{F}\left(\omega^{*}\right)=0$ if $a \beta \geq \alpha$. Notice that both $V_{F}\left(\omega^{*}\right)<0$ and $V_{F}\left(\omega^{*}\right)=0$ are possible for $a \beta=\alpha$. According to Proposition 4.2.80, we further consider four sub-tactics of Tactic III.3.
(Tactic III.3.a) $a \beta \leq \alpha, e^{*}=\tau, \lambda^{*}=0$ and $V_{F}\left(\omega^{*}\right)<0$;
(Tactic III.3.b) $a \beta \leq \alpha, e^{*}>\tau, \lambda^{*}=1-\tau / e^{*}$ and $V_{F}\left(\omega^{*}\right)<0$;
(Tactic III.3.c) $a \beta \geq \alpha, e^{*}=\tau, \lambda^{*}=0$ and $V_{F}\left(\omega^{*}\right)=0$; and
(Tactic III.3.d) $a \beta \geq \alpha, e^{*}>\tau, \lambda^{*}=1-\tau / e^{*}$ and $V_{F}\left(\omega^{*}\right)=0$.
Similarly, we still need to consider the basic conditions for Tactic III. 3 which are not covered by the specific conditions for the sub-tactics. Denoted by $\omega_{\text {III. } 3 . i}^{*}$, for $i=a$, $b, c, d$, the local optimal advertising and price strategy for Tactic III.3.i. We now examine the local optimal advertising and pricing strategies for each sub-tactic of Tactic III.3.

- Tactic III.3.a

Specific conditions: $D_{L}\left(\omega^{*}\right)>0, a \beta \leq \alpha, e^{*}=\tau, \lambda^{*}=0$ and $V_{F}\left(\omega^{*}\right)<0$.
By putting $\lambda^{*}=0$ and $e^{*}=\tau$ into (4.130), we obtain
$\pi_{L L}(\omega)=\left(x_{L}-g p\right)(p-c)-h \tau^{2}-m T$.
Proposition 4.2.81 For Tactic III.3.a, the local optimal advertising and pricing strategy exists only if $a \beta \leq \alpha, V_{F}\left(\omega_{I I I .3 . a}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .3 . a}^{*}\right)>0$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.3.a exists, then it is unique and is given by

$$
\begin{align*}
& \omega_{I I I .3 . a}^{*}=\left\{e_{I I I .3 . a}^{*}=\tau, \lambda_{I I I .3 . a}^{*}=0, p_{I I I .3 . a}^{*}=\left(x_{L}-g c\right) /(2 g)+c\right\}, \text { and }  \tag{4.144}\\
& \pi_{L L}\left(\omega_{I I I .3 . a}^{*}\right)=\left(x_{L}-g c\right)^{2} /(4 g)-h \tau^{2}-m T, \tag{4.145}
\end{align*}
$$

Proposition 4.2.81 shows the explicit formula of the optimal advertising and pricing strategies (if it exists), and the associated company's profit for Tactic III.3.a. Moreover, the necessary conditions for a finite $\omega_{I I I .3 . a}^{*}$ are shown in Proposition 4.2.81. Furthermore, $e_{I I .3 . a}^{*}$ is increasing in $\tau$ and independent of $T$. Moreover, $\lambda_{I I I .3 a}^{*}$ and $p_{I I I .3 a}^{*}$ are independent of $T$ and $\tau$. In words, for Tactic III.3.a., a bigger $\tau$ only induces a higher local optimal advertising effort.

- Tactic III.3.b

Specific conditions: $D_{L}\left(\omega^{*}\right)>0,, e^{*}>\tau, \lambda^{*}=1-\tau / e^{*}$ and $V_{F}\left(\omega^{*}\right)<0$ :
By putting $\lambda^{*}=1-\tau / e^{*}$ into (4.130), we obtain
$\pi_{L L}(\omega)=\left[x_{L}+a(e-\tau)-g p\right](p-c)-h e^{2}-m(T+\tau-e)$.
Moreover, as $\left(1-\lambda^{*}\right) e^{*}<\tau$ and $\lambda^{*} e^{*}=T$ for Tactic III.3.b, we have the condition $\tau<e^{*}<T+\tau$ for Tactic III.3.b.

Proposition 4.2.82 ForTacticIII.3.b, the optimal advertising effort in a function of $p$ is given by
$e_{\text {III. } 3 . b}^{*}(p)=[a(p-c)+m] /(2 h)$.
Moreover, $e_{\text {III. } 3 . b}^{*}(p)$ is increasing in $p$.
Proposition 4.2.82 shows that, for Tactic III.3.b, the local optimal advertising effort is increasing in $p$.

Proposition 4.2.83 For Tactic III.3.b, the local optimal advertising and pricing strategy exists only if (i) $a \beta \leq \alpha, V_{F}\left(\omega_{I I I .3 . b}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .3 . b}^{*}\right)>0$; (ii) $4 h g>a^{2}$;
(iii) $(4 h g \tau-2 g m) / a<x_{L}-g c<\left[\left(4 h g-a^{2}\right) T+4 h g \tau-2 g m\right] / a$; and
(iv) $x_{L}-g c>a \tau-a m /(2 h)$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.3.b exists, then it is unique and is given by
$\omega_{I I I .3 . b}^{*}=\left\{e_{I I I .3 . b}^{*}, \lambda_{I I I .3 . b}^{*}, p_{I I I .3 . b}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I I, 3, b}^{*}\right)=\frac{\left[2 h\left(x_{L}-g c-a \tau\right)+a m\right]^{2}}{4 h\left(4 h g-a^{2}\right)}+\frac{m^{2}}{4 h}-m(T+\tau)$,
where $e_{I I I .3 . b}^{*}=\frac{2 g m+a\left(x_{L}-g c-a \tau\right)}{4 h g-a^{2}}, \lambda_{I I I .3 . b}^{*}=1-\tau / e_{I I I .3 . b}^{*}$, and
$p_{I I I .3 . b}^{*}=c+\frac{2 h\left(x_{L}-g c-a \tau\right)+a m}{4 h g-a^{2}}$.

Proposition 4.2.83 shows the explicit formula of the optimal advertising and pricing strategies (if it exists), and the associated company's profit for Tactic III.3.b. Moreover, the corresponding necessary conditions for $\omega_{I I I .3 . b}^{*}$ being global optimal are shown in Proposition 4.2.83. In particular, item (ii) of Proposition 4.2.83 ensures that the profit function is strictly concave in $p$ and hence $\omega_{\text {III..b }}^{*}$ is unique. For other conditions, they are all directly derived from the specific conditions of Tactic III.3.b. Furthermore, for Tactic III.3.b, $e_{I I I .3 . b}^{*}, \lambda_{I I I .3 . b}^{*}$ and $p_{I I I .3 . b}^{*}$ are decreasing in $\tau$ and independent of $T$. In words, for Tactic III.3.b, a bigger $\tau$ induces a bigger optimal advertising effort, a higher proportion of advertising effort which is allocated to LG, and a higher retailer price of the product.

- Tactic III.3.c

Specific conditions: $D_{L}\left(\omega^{*}\right)>0, a \beta \geq \alpha, e^{*}=\tau, \lambda^{*}=0$ and $V_{F}\left(\omega^{*}\right)=0$ :
Proposition 4.2.84 For Tactic III.3.c, the local optimal advertising and pricing strategy exists only if $a \beta \geq \alpha$ and $D_{L}\left(\omega_{\text {III...c }}^{*}\right)>0$. If the local optimal advertising and pricing strategy for Tactic III.3.c exists, then it is unique and is given by

$$
\begin{align*}
& \omega_{I I I .3 . c}^{*}=\left\{e_{I I I .3 . c}^{*}=\tau, \lambda_{I I I .3 . c}^{*}=0, p_{I I I .3 . c}^{*}=\frac{x_{F}+\beta x_{L}+\alpha \tau}{\gamma+\beta g}\right\} \text {, and }  \tag{4.150}\\
& \pi_{L L}\left(\omega_{I I .3 . c}^{*}\right)=\left(x_{L}-g p_{I I I .3 . c}^{*}\right)\left(p_{I I I .3 . c}^{*}-c\right)-h \tau^{2}-m T \tag{4.151}
\end{align*}
$$

Proposition 4.2.84 shows the explicit formula of the local optimal advertising and pricing strategy for Tactic II.3.c (if it exists), and the associated company's profit. Moreover, the necessary conditions for having a finite $\omega_{\text {III... }}^{*}$ are shown in Proposition 4.2.84. Moreover, from Proposition 4.2.84, we find that $e_{I I I .3 . c}^{*}$ and $p_{I I I .3 . c}^{*}$ are increasing in $\tau$ but independent of $T$, and $\lambda_{I I I .3 . c}^{*}$ is independent of $T$ and $\tau$. In words, for Tactic III.3.c, a bigger $\tau$ induces a higher local optimal advertising efforts and a higher local optimal retail price of the product.

- Tactic III.3.d

Specific conditions: $D_{L}\left(\omega^{*}\right)>0, a \beta \geq \alpha$ and $e^{*}>\tau$, then $\lambda^{*}=1-\tau / e^{*}$ and $V_{F}\left(\omega^{*}\right)=0:$

By putting $\lambda=1-\tau / e$ into (4.142), we obtain

$$
\begin{equation*}
\pi_{L L}(\omega)=\left[x_{L}+a(e-\tau)-g p\right](p-c)-h e^{2}-m(T+\tau-e) . \tag{4.152}
\end{equation*}
$$

Proposition 4.2.85 For Tactic III.3.d, the local optimal advertising effort as a function of retail price $p$ is given by
$e_{\text {III. } 3 . d}^{*}(p)=\left[(\gamma+\beta g) p-\beta x_{L}-x_{F}+(a \beta-\alpha) \tau\right] /(a \beta)$.
Proposition 4.2.85 shows the relationship of optimal $e$ and optimal $p$ for Tactic III.3.d. As $\gamma+\beta g>0$ and $a \beta>0, e_{I I I . d . d}^{*}(p)$ is increasing in $p$, namely, for Tactic III.3.d, a higher local optimal advertising effort is induced by a higher local optimal retail price of the product.

Proposition 4.2.86 For Tactic III.3.d, the local optimal advertising and pricing strategy exists only if (i) $a \beta>0$ and $D_{L}\left(\omega_{I I I .3 . d}^{*}\right)>0$; (ii) $h(\gamma+\beta g)^{2}>a^{2} \beta \gamma$; (iii) $\tau<e_{\text {III. } 3 . d}^{*}<T+\tau$; and (iv) $p_{\text {III..d }}^{*}>c$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.3.d exists, then it is unique and is given by

$$
\begin{align*}
& \omega_{I I .3 . d}^{*}=\left\{e_{I I I .3 . d}^{*}, \lambda_{I I I .3 . d}^{*}, p_{I I .3 . d}^{*}\right\} \text {, and }  \tag{4.154}\\
& \pi_{L L}\left(\omega_{I I I .3 . d}^{*}\right)=\left[x_{F}+a\left(e_{I I I .3 . d}^{*}-\tau\right)-g p_{I I I .3 . d}^{*}\right]\left(p_{I I I .3 . d}^{*}-c\right)-h\left(e_{I I I .3 . d}^{*}\right)^{2}-m\left(T+\tau-e_{I I I .3, d}^{*}\right), \tag{4.1.15}
\end{align*}
$$

where $e_{I I I .3 . d}^{*}=\frac{a(\gamma-\beta g)\left(x_{F}-\gamma c+\alpha \tau\right)+a \gamma\left(x_{L}-g c-a \tau\right)+m(\gamma+\beta g)}{2\left[h(\gamma+\beta g)^{2}-a^{2} \beta \gamma\right]}$, $\lambda_{\text {III...d }}^{*}=1-\tau / e_{\text {III.3.d }}^{*}$, and $p_{\text {III... }}^{*}=\frac{\left[2 h(\gamma+\beta g)-a^{2} \beta\right]\left(x_{F}-\gamma c+\alpha \tau\right)+2 h \beta(\gamma+\beta g)\left(x_{L}-g c-a \tau\right)}{2\left[h(\gamma+\beta g)^{2}-a^{2} \beta \gamma\right]}+c$.

Proposition 4.2.86 provides the explicit formula of the local optimal advertising and pricing strategy for Tactic III.3.d (if it exists), and the associated company's profit. Moreover, the necessary conditions for a finite local optimum for Tactic III.3.d are shown in Proposition 4.2.86. In particular, item (ii) of Proposition 4.2.86 ensures that the profit functions is strictly concave in $p$ and hence $\omega_{I I I ., d}^{*}$ is unique. For other conditions, they are all directly derived from the specific conditions of Tactic III.3.d.

## Tactic III.4:

Basic conditions: $D_{L}(\omega)>0, D_{F}(\omega)>0 \quad \lambda e<T,(1-\lambda) e<\tau$ and $0 \leq e<T+\tau$.
The company's profit for Tactic III. 4 is

$$
\begin{equation*}
\pi_{L L}(\omega)=\left(x_{L}+a \lambda e-g p\right)(p-c)-h e^{2}-\mu[\tau-(1-\lambda) e]-m(T-\lambda e) . \tag{4.156}
\end{equation*}
$$

Proposition 4.2.87 Suppose that $\omega^{*}$ belongs to Tactic III.4, (a) $\lambda^{*}=1$ and
$0 \leq e^{*}<T$ if $p^{*}>(\mu-m) / a+c$ and $0 \leq e^{*}<T$, (b) $\lambda^{*}=0$ and $0 \leq e^{*}<\tau$ if $p^{*}<(\mu-m) / a+c$; and (c) there exist multiple $\lambda^{*}$ and $0 \leq e^{*}<T+\tau$ if $p^{*}=(\mu-m) / a+c$.

Proposition 4.2.87 shows that if $\omega^{*}$ belongs to Tactic III.4. In particular, if the local optimal retail price is higher than the threshold $(\mu-m) / a+c$ then $\lambda^{*}=1$ and $0 \leq e^{*}<T$. while $\lambda^{*}=0$ and $0 \leq e^{*}<\tau$ if the local optimal retail price is less than the threshold $(\mu-m) / a+c$. Next, we have the following result for the value of $V_{F}\left(\omega^{*}\right)$ if $\omega^{*}$ belongs to Tactic III.4.

Proposition 4.2.88 Suppose that $\omega^{*}$ belongs to Tactic III.4, (a) if $V_{F}\left(\omega^{*}\right)<0$, then $\left[a\left(p^{*}-c\right)-\mu+m\right](a \beta-\alpha) \leq 0$, (b) if $V_{F}\left(\omega^{*}\right)=0$ then $\left[a\left(p^{*}-c\right)-\mu+m\right](a \beta-\alpha) \geq 0$.

According to Proposition 4.2 .87 and Proposition 4.2.88, we consider nine sub-tactics for Tactic III.4:
(Tactic III.4.a) $a \beta \leq \alpha, p^{*}>(\mu-m) / a+c, e^{*}=0$ and $V_{F}\left(\omega^{*}\right)<0$;
(Tactic III.4.b) $a \beta \leq \alpha, p^{*}>(\mu-m) / a+c, 0<e^{*}<T, \lambda^{*}=1$ and $V_{F}\left(\omega^{*}\right)<0$;
(Tactic III.4.c) $a \beta \leq \alpha, p^{*}<(\mu-m) / a+c, e^{*}=0, \lambda^{*}=0$ and $V_{F}\left(\omega^{*}\right)=0$;
(Tactic III.4.d) $a \beta \leq \alpha, p^{*}<(\mu-m) / a+c, 0<e^{*}<\tau, \quad \lambda^{*}=0$ and $V_{F}\left(\omega^{*}\right)=0$.
(Tactic III.4.e) $a \beta \geq \alpha, p^{*}>(\mu-m) / a+c, e^{*}=0, \lambda^{*}=1$ and $V_{F}\left(\omega^{*}\right)=0$;
(Tactic III.4.f) $a \beta \geq \alpha, p^{*}>(\mu-m) / a+c, 0<e^{*}<T, \lambda^{*}=1$ and $V_{F}\left(\omega^{*}\right)=0$;
(Tactic III.4.g) $a \beta \geq \alpha, p^{*}<(\mu-m) / a+c, e^{*}=0, \lambda^{*}=0$ and $V_{F}\left(\omega^{*}\right)<0 ;$
(Tactic III.4.h) $a \beta \geq \alpha, p^{*}<(\mu-m) / a+c, 0<e^{*}<\tau, \lambda^{*}=0$, and $V_{F}\left(\omega^{*}\right)<0$;
(Tactic III.4.k) $p^{*}=(\mu-m) / a+c, 0 \leq e^{*}<\tau$ and $\lambda^{*}=0$.
Notice that, for $p^{*}=(\mu-m) / a+c$, there exists multiple $\lambda^{*}$. For simplicity, we only consider the pair of $\lambda^{*}=0$ and $0 \leq e^{*}<\tau \quad\left(\lambda^{*}=0\right.$ and $0 \leq e^{*}<\tau$ satisfy $\lambda e<T$ and $(1-\lambda) e<\tau)$, for $p^{*}=(\mu-m) / a+c$. Similar results can be obtained if we consider other pairs of $\lambda^{*}$ and $e^{*}$ which satisfy the basic condition for Tactic III.4.

Similarly, we still need to consider of the basic conditions for Tactic III. 4 that are not covered by the specific conditions for the sub-tactics. Denoted by $\omega_{I I .4 . i}^{*}$, for $i=a$, $b, c, d, e, f, g, h$ and $k$, the local optimal advertising and price strategy for Tactic III.4.i.

We now study the local optimal advertising and pricing strategies for each sub-tactic of Tactic III.4.

## - Tactic III.4.a

Specific conditions: $D_{L}\left(\omega^{*}\right)>0, a \beta \leq \alpha, \quad p^{*}>(\mu-m) / a+c, \quad e^{*}=0$ and $V_{F}\left(\omega^{*}\right)<0$.

Proposition 4.2.89 For Tactic III.4.a, the local optimal advertising and pricing strategy exists only if (i) $a \beta \leq \alpha, V_{F}\left(\omega_{\text {III. } 4 . a}^{*}\right)<0$ and $D_{L}\left(\omega_{\text {III. } 4 . a}^{*}\right)>0$; and (ii) $x_{L}-g c>2 g(\mu-m) / a$. Moreover, If the local optimal advertising and pricing strategy for Tactic III.4.a exists, then it is unique and is given by
$\omega_{I I I .4 . a}^{*}=\left\{e_{I I I .4, a}^{*}=0, \lambda_{I I .4 . a}^{*}=0, p_{I I I .4 a}^{*}=\left(x_{L}+g c\right)(2 \mathrm{~g})\right\}$, and
$\pi_{L L}\left(\omega_{I I I .4, a}^{*}\right)=\left(x_{L}-g c\right)^{2} /(4 g)-m T-\mu \tau$.
The local optimal advertising and pricing strategy for Tactic III.4.a (if it exists), and the associated company's profit are shown in Proposition 4.2.89. Moreover, the necessary conditions for a finite $\omega_{\text {III. } 4 . a}^{*}$ are shown in Proposition 4.2.89. Moreover, from Proposition 4.2.89, we find that $e_{I I I .4 a}^{*}, \lambda_{I I I .4 a}^{*}$ and $p_{I I I .4 a}^{*}$ are independent of $T$ and $\tau$. In other words, for Tactic III.4.a, the values of $T$ and $\tau$ do not affect the values of the optimal advertising effort and the optimal retail price of the product.

## - Tactic III.4.b

Specific conditions: $D_{L}\left(\omega^{*}\right)>0, a \beta \leq \alpha, p^{*}>(\mu-m) / a+c, \quad 0<e^{*}<T, \quad \lambda^{*}=1$ and $V_{F}\left(\omega^{*}\right)<0$.

By putting $\lambda^{*}=1$ into (4.156), we obtain
$\pi_{L L}(\omega)=\left(x_{F}+a e-g p\right)(p-c)-h e^{2}-\mu \tau$.
Proposition 4.2.90 For Tactic III.4.b, then the optimal advertising effort in the function of retail price $p$ is given by

$$
\begin{equation*}
e_{I I I .4, b}^{*}(p)=[a(p-c)+m] /(2 h) . \tag{4.160}
\end{equation*}
$$

Moreover, $e_{\text {III. } 4 . b}^{*}(p)$ is increasing in $p$.
Proposition 4.2.90 shows that $e_{\text {III. . . }}^{*}(p)$ is increasing in $p$.
Proposition 4.2.91 For Tactic III.4.b, the local optimal advertising and pricing strategy exists only if (i) $a \beta \leq \alpha, V_{F}\left(\omega_{I I .4 . b}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .4 . b}^{*}\right)>0$; (ii) $4 h g>a^{2}$;
(iii) $x_{L}-g c<\left[T\left(4 h g-a^{2}\right)-2 g m\right] / a$; and (iv)
$x_{L}-g c>\left[(\mu-m)\left(4 h g-a^{2}\right)-a^{2} m\right] /(2 a h)$.
Moreover, if the optimal advertising and pricing strategy for Tactic III.4.b exists, then it is unique and is given by
$\omega_{I I I .4 . b}^{*}=\left\{e_{I I I .4 . b}^{*}, \lambda_{I I I .4 . b}^{*}, p_{I I I .4 . b}^{*}\right\}$, and
$\pi_{L L}\left(\omega_{I I I .4 . b}^{*}\right)=\frac{\left[2 h\left(x_{L}-g c\right)+a m\right]^{2}}{4 h\left(4 h g-a^{2}\right)}+\frac{m^{2}}{4 h}-\mu \tau-m T$,
where $e_{\text {III.4.b }}^{*}=\left[2 g m+a\left(x_{L}-g c\right)\right] /\left(4 h g-a^{2}\right), \quad \lambda_{I I I .4 . b}^{*}=1$, and
$p_{I I I .4 . b}^{*}=\left[2 h\left(x_{L}-g c\right)+a m\right] /\left(4 h g-a^{2}\right)+c$.
The local optimal advertising and pricing strategy (if it exists), and the associated company's profit for Tactic III.4.b are shown in Proposition 4.2.91. Moreover, the necessary conditions for $\omega_{I I I .4 . b}^{*}$ being global optimal are provided in Proposition 4.2.91. In particular, item (ii) of Proposition 4.2.91 ensures that the profit function of the company is a strictly concave function of $\omega$ and hence there exists a unique local optimum for Tactic III.4.b. Other conditions in Proposition 4.2.91 are directly derived from the specific conditions for Tactic III.4.b. Furthermore, for Tactic III.4.b, $e_{I I I .4 . b}^{*}$, $\lambda_{I I .4 . b}^{*}$ and $p_{I I .4 . b}^{*}$ are independent of $T$ and $\tau$. In other words, the values of $T$ and $\tau$ do not affect the values of the optimal advertising and pricing strategy for Tactic III.4.b.

- Tactic III.4.c

Specific conditions: $D_{L}\left(\omega^{*}\right)>0, a \beta \leq \alpha, \quad p^{*}<(\mu-m) / a+c, \quad e^{*}=0, \quad \lambda^{*}=0$ and $V_{F}\left(\omega^{*}\right)=0 ;$

Proposition 4.2.92 For Tactic III.4.c, the local optimal advertising and pricing strategy exists only if (i) $a \beta \leq \alpha$ and $D_{L}\left(\omega_{I I I .4 . c}^{*}\right)>0$;
(ii) $x_{F}-\gamma c+\beta\left(x_{L}-g c\right)<(\gamma+\beta g)(\mu-m) / a$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.4.c exists, then it is unique and is given by
$\omega_{I I .4 . c}^{*}=\left\{e_{I I I .4 . c}^{*}=0, \lambda_{I I I .4 . c}^{*}=0, p_{I I I .4 . c}^{*} \frac{x_{F}+\beta x_{L}}{\gamma+\beta g}\right\}$, and
$\pi_{L L}\left(\omega_{I I I .4 . c}^{*}\right)=\frac{\left[x_{F}-\gamma c+\beta\left(x_{L}-g c\right)\right]\left(x_{L} \gamma-x_{F} g\right)}{(\gamma+\beta g)^{2}}-\mu \tau-m T$
The local optimal advertising and pricing strategy (if it exists), and the associated company's profit for Tactic III.4.c is shown in Proposition 4.2.91. Moreover, the necessary conditions for a finite $\omega_{\text {III.4.c }}^{*}$ are also shown in Proposition 4.2.91. All the conditions are derived from the specific conditions for Tactic III.4.c. Furthermore, from Proposition 4.2.92, we can observe that $e_{I I I .4 . c}^{*}, \lambda_{I I .4 . c}^{*}$ and $p_{I I I .4 . c}^{*}$ are independent of $T$ and $\tau$.

- Tactic III.4.d

Specific conditions: $D_{L}\left(\omega^{*}\right)>0, a \beta \leq \alpha, p^{*}<(\mu-m) / a+c, 0<e^{*}<\tau, \quad \lambda^{*}=0$ and $V_{F}\left(\omega^{*}\right)=0$.

By putting $\lambda^{*}=0$ into (4.144) we obtain

$$
\begin{equation*}
\pi_{L L}(\omega)=\left(x_{L}-g p\right)(p-c)-h e^{2}-\mu(\tau-e)-m T . \tag{4.165}
\end{equation*}
$$

Proposition 4.2.93 For Tactic III.4.d, the local optimal advertising effort as a function of retail price $p$ is given by
$e_{\text {III. . . }}^{*}(p)=\left[(\gamma+\beta g) p-\beta x_{L}-x_{F}\right] / \alpha$,
and $e_{\text {III. } 4 . d}^{*}(p)$ is increasing in $p$.
Proposition 4.2.93 shows that, for Tactic III.4.d, a higher local optimal advertising effort is induced by a higher local optimal retail price of the product.
Proposition 4.2.94 For Tactic III.4.d, the local optimal advertising and pricing strategy exists only if (i) $a \beta \leq \alpha$ and $D_{L}\left(\omega_{I I I 4 . d}^{*}\right)>0$; (ii) $0<e_{I I I .4 . d}^{*}<\tau$; and (iii) $p_{\text {III. } 4 . d}^{*}<(\mu-m) / a+c$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.4.d exists, then it is unique and is given by

$$
\begin{align*}
& \omega_{I I .4 . d}^{*}=\left\{e_{I I I .4, d}^{*}, \lambda_{I I I .4 . d}^{*}, p_{I I I .4 . d}^{*}\right\} \text {, and }  \tag{4.167}\\
& \pi_{L L}\left(\omega_{I I I .4 . d}^{*}\right)=\left(x_{L}-g p_{I I I .4 . d}^{*}\right)\left(p_{I I I .4 . d}^{*}-c\right)-h\left(e_{I I I .4 . d}^{*}\right)^{2}-\mu\left(\tau-e_{I I I 4 . d}^{*}\right)-m T, \tag{4.168}
\end{align*}
$$

where $e_{\text {III. } 4 . d}^{*}=\frac{\alpha(\gamma-\beta g)\left(x_{L}-g c\right)+\mu(\gamma+\beta g)^{2}-2 \alpha g\left(x_{F}-\gamma c\right)}{2\left[h(\gamma+\beta g)^{2}+\alpha^{2} g\right]}, \lambda_{\text {III. } 4, d}^{*}=0$, and

$$
p_{\text {III. } 4 . d}^{*}=\frac{\left[\alpha^{2}+2 h \beta(\gamma+\beta g)\right]\left(x_{L}-g c\right)+2 h(\gamma+\beta g)\left(x_{F}-\gamma c\right)+\alpha \mu(\gamma+\beta g)}{2\left[h(\gamma+\beta g)^{2}+\alpha^{2} g\right]}+c .
$$

Proposition 4.2.94 shows the explicit formula of the local optimal advertising and
pricing strategy for Tactic III.4.d (if it exists), and the associated company's profit. Moreover, the necessary conditions for a finite and unique $\omega_{\text {III. .d }}^{*}$ are shown in Proposition 4.2.94. In particular, item (ii) of Proposition 4.2.94 ensures that the profit functions is strictly concave in $p$ and hence $\omega_{\text {III.4.d }}^{*}$ is unique. For other necessary conditions, they are all directly derived from the specific conditions of Tactic III.4.d.

## - Tactic III.4.e

Specific conditions: $D_{L}\left(\omega^{*}\right)>0, a \beta \geq \alpha, p^{*}>(\mu-m) / a+c, e^{*}=0, \lambda^{*}=1$ and $V_{F}\left(\omega^{*}\right)=0$.

Proposition 4.2.95 For Tactic III.4.e, the local optimal advertising and pricing strategy exists only if (i) $a \beta \geq \alpha$ and $D_{L}\left(\omega_{\text {III. . . }}^{*}\right)>0$; and
(ii) $x_{F}-\gamma c+\beta\left(x_{L}-g c\right)>(\mu-m)(\gamma+\beta g) / a$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.4.e exists, then it is unique and is given by
$\omega_{\text {III.4.e }}^{*}=\left\{e_{\text {III.4.e }}^{*}=0, \lambda_{\text {III.4.e }}^{*}=0, p_{\text {III.4.e }}^{*}=\frac{x_{F}+\beta x_{L}}{\gamma+\beta g}\right\}$, and
$\pi_{L L}\left(\omega_{I I I .4 . e}^{*}\right)=\frac{\left[x_{F}-\gamma c+\beta\left(x_{L}-g c\right)\right]\left(x_{L} \gamma-x_{F} g\right)}{(\gamma+\beta g)^{2}}-\mu \tau-m T$
Proposition 4.2.95 shows the explicit formula of the local optimal advertising and pricing strategy for Tactic III.4.e (if it exists), and the associated company's profit. Moreover, the necessary conditions for a finite and unique $\omega_{\text {III.4.e }}^{*}$ are shown in Proposition 4.2.95. All the necessary conditions are directly derived from the specific conditions of Tactic III.3.b. Furthermore, from Proposition 4.2.95, we find that $e_{I I I .4 . e}^{*}$, $\lambda_{\text {III.4.e }}^{*}$ and $p_{I I I .4 . e}^{*}$ independent of $T$ and $\tau$.

## - Tactic III.4.f

Specific conditions: $D_{L}\left(\omega^{*}\right)>0, a \beta \geq \alpha, p^{*}>(\mu-m) / a+c, \quad 0<e^{*}<T, \quad \lambda^{*}=1$ and $V_{F}\left(\omega^{*}\right)=0$.

By putting $\lambda^{*}=1$ into (4.156), we obtain
$\pi_{L L}(\omega)=\left(x_{L}+a e-g p\right)(p-c)-h e^{2}-\mu \tau-m(T-e)$.
Proposition 4.2.96 For Tactic III.4.f, the local optimal advertising effort is given by

$$
\begin{equation*}
e_{I I I .4, f}^{*}(p)=\left[(\gamma+\beta g) p-\beta x_{L}-x_{F}\right] /(a \beta), \tag{4.172}
\end{equation*}
$$

and $e_{\text {III. } 4 . f}^{*}(p)$ is strictly increasing in $p$.
Proposition 4.2.96 shows that, for Tactic III.4.f, a high local optimal advertising effort is induced by a high local optimal retail price of the product.

Proposition 4.2.97 For Tactic III.4.f, the local optimal advertising and pricing strategy exists only if (i) $a \beta \geq \alpha$ and $D_{L}\left(\omega_{\text {III. .f }}^{*}\right)>0$; (ii) $h(\gamma+\beta g)^{2}>a^{2} \beta \gamma$; (iii) $0<e_{\text {III. } 4 . f}^{*}<T$; (iv) $p_{\text {III. } 4 . f}^{*}>(\mu-m) / a+c$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.4.f, then it is unique and is given by

$$
\begin{align*}
& \omega_{I I I .4 . f}^{*}=\left\{e_{I I I .4 . f}^{*}, \lambda_{I I I .4 . f}^{*}, p_{I I I .4 . f}^{*}\right\}, \text { and }  \tag{4.173}\\
& \pi_{L L}\left(\omega_{I I I 4 . f}^{*}\right)=\left(x_{L}+a e_{I I I .4 . f}^{*}-g p_{I I I 4 . f}^{*}\right)\left(p_{I I I .4 . f}^{*}-c\right)-h\left(e_{I I I .4 . f}^{*}\right)^{2}-\mu \tau-m\left(T-e_{I I I .4 . f}^{*}\right) \tag{4.174}
\end{align*}
$$

where $e_{\text {III.4.f }}^{*}=\frac{2 a \beta \gamma\left(x_{L}-g c\right)-(\gamma+\beta g)\left[a\left(x_{F}-\gamma c\right)+m\right]}{2\left[h(\gamma+\beta g)^{2}+\alpha^{2} \beta \gamma\right]}, \lambda_{\text {III.4.f }}^{*}=1$, and

$$
p_{\text {III. } 4 . f}^{*}=\frac{\beta(\gamma+\beta g)\left[2 h\left(x_{L}-g c\right)+a m\right]+\left[2 h(\gamma+\beta g)-a^{2} \beta\right]\left(x_{F}-\gamma c\right)}{2\left[h(\gamma+\beta g)^{2}-a^{2} \beta \gamma\right]}+c .
$$

Proposition 4.2.97 shows the explicit formula of the local optimal advertising and pricing strategy for Tactic III.4.f (if it exists), and the associated company's profit. Moreover, the necessary conditions for a finite and unique $\omega_{I I I .4 . f}^{*}$ are shown in Proposition 4.2.97. In particular, item (ii) of Proposition 4.2.97 ensures that the profit function is strictly concave in $p$ and hence $\omega_{\text {III.4.f }}^{*}$ is unique. For other conditions, they are all directly derived from the specific conditions of Tactic III.4.f. Furthermore, from Proposition 4.2.97, we can easily find that $e_{I I I .4 . f}^{*}, \lambda_{I I .4 . f}^{*}$ and $p_{I I I .4 . f}^{*}$ are independent of $T$ and $\tau$.

## - Tactic III.4.g

Specific conditions: $D_{L}\left(\omega^{*}\right)>0, a \beta \geq \alpha, p^{*}<(\mu-m) / a+c, e^{*}=0, \lambda^{*}=0$ and $V_{F}\left(\omega^{*}\right)<0$.

Proposition 4.2.98 For Tactic III.4.g, the local optimal advertising and pricing strategy exists only if (i) $a \beta \geq \alpha, \quad V_{F}\left(\omega_{I I I .4 . g}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .4 . g}^{*}\right)>0$; (ii) $x_{L}-g c<2 g(\mu-m) / a$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.4.g exists, then it is unique and is given by
$\omega_{\text {III.4.g }}^{*}=\left\{e_{\text {III. } 4 . g}^{*}=0, \lambda_{\text {III.4.g }}^{*}=0, p_{\text {III.4.g }}^{*}=\left(x_{L}+g c\right) /(2 g)\right\}$, and
$\pi_{L L}\left(\omega_{\text {III.4.g }}^{*}\right)=\left(x_{L}-g c\right)^{2} /(4 g)-m T-\mu \tau$.
The local optimal advertising and pricing strategy for Tactic III.4.g (if it exists), and the associated company's profit are shown in Proposition 4.2.98. Moreover, the necessary conditions for a finite and unique $\omega_{I I I .4 . g}^{*}$ are also shown in Proposition 4.2.98. All the necessary conditions are directly derived from the specific conditions of Tactic III.4.g. Furthermore, for Tactic III.4.g, $e_{I I I .4 . g}^{*}, \lambda_{\text {III.4.g }}^{*}$ and $p_{I I I .4 . g}^{*}$ are independent of $T$ and $\tau$.

Notice that $\omega_{\text {III. } 4 . g}^{*}=\omega_{\text {III. } 4, a}^{*}$. In other words, the local optimal advertising and pricing strategy for Tactic III.4.g is the same as Tactic III.4.a. However, according to Proposition 4.2.89 and Proposition 4.2.98, the necessary conditions for a finite $\omega_{\text {III.4.g }}^{*}$ and a finite $\omega_{I I I .4, a}^{*}$ are different.

- Tactic III.4.h

Specific conditions: $D_{L}\left(\omega^{*}\right)>0, a \beta \geq \alpha, p^{*}<(\mu-m) / a+c, 0<e^{*}<\tau \quad \lambda^{*}=0$, and $V_{F}\left(\omega^{*}\right)<0$.

By putting $\lambda^{*}=0$ into (4.144), we obtain
$\pi_{L L}(\omega)=\left(x_{F}-g p\right)(p-c)-\mu(\tau-e)-m T-h e^{2}$.
Proposition 4.2.99 For Tactic III.4.h, the local optimal advertising effort as a function of retail price $p$ is given by

$$
\begin{equation*}
e_{I I I .4, h}^{*}=\mu /(2 h)>0 . \tag{4.178}
\end{equation*}
$$

Proposition 4.2.9 shows that the local optimal advertising effort for Tactic III.4.h is a positive constant.
Proposition 4.2.100 For Tactic III.4.h, the local optimal advertising and pricing strategy exists only if (i) $a \beta \geq \alpha, V_{F}\left(\omega_{I I I .4 . h}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .4 . h}^{*}\right)>0$; (ii) $\mu<2 g \tau$; and
(iii) $x_{L}-g c>2 g(\mu-m) / a$. Moreover, if the local optimal advertising and pricing strategy for Tactic III.4.h exists, then it is unique and is given by

$$
\begin{equation*}
\omega_{I I I .4, h}^{*}=\left\{e_{I I I .4 . h}^{*}=\mu /(2 h), \lambda_{I I I .4, h}^{*}=0, p_{I I I .4, h}^{*}=\left(x_{L}+g c\right) /(2 g)\right\} \text {, and } \tag{4.179}
\end{equation*}
$$

$\pi_{L L}\left(\omega_{I I I .4 . h}^{*}\right)=\frac{\left(x_{L}-g c\right)^{2}}{4 g}+\frac{\mu^{2}}{4 h}-\mu \tau-m T$,
The local optimal advertising and pricing strategy for Tactic III.4.h (if it exists), and the associated company's profit are shown in Proposition 4.2.100. Moreover, the necessary conditions for a finite and unique $\omega_{\text {III.4.h }}^{*}$ are provided in Proposition 4.2.99. All the necessary conditions are directly derived from the specific conditions of Tactic III.4.h. Furthermore, for Tactic III.4.h, $e_{\text {III.4.h }}^{*}, \lambda_{\text {III.4.h }}^{*}$ and $p_{\text {III.4.h }}^{*}$ are independent of $T$ and $\tau$.

## - Tactic III.4.k

Specific conditions: $D_{L}\left(\omega^{*}\right)>0, D_{F}\left(\omega^{*}\right)=0 \quad p^{*}=(\mu-m) / a+c, 0 \leq e^{*}<\tau$ and $\lambda^{*}=0$.

Proposition 4.2.101 For Tactic III.4.k, the local optimal advertising and pricing strategy exists only if (i) $D_{L}\left(\omega_{I I I .4 . k}^{*}\right)>0$; (ii) $2 h \tau>\mu$; (iii) $\mu>m$; and
(iv) $x_{F}-\gamma c+\beta\left(x_{L}-g c\right) \leq(\gamma+\beta g)(\mu-m) / a-\alpha \mu /(2 h)$. Moreover, if the optimal advertising and pricing strategy for Tactic III.4.k exists, then it is unique and is given by
$\omega_{\text {III. } 4, k}^{*}=\left\{e_{\text {III. } 4, k}^{*}=\mu /(2 h), \lambda_{\text {III. } 4, k}^{*}=0, p_{I I I .4, k}^{*}=(\mu-m) / a+c\right\}$, and
$\pi_{L L}\left(\omega_{I I .4 . k}^{*}\right)=\frac{(\mu-m)\left[a\left(x_{F}-g c\right)-g(\mu-m)\right]}{a^{2}}+\frac{\mu^{2}}{4 h}-m T-\mu \tau$.
The necessary conditions for the existence of local optimal advertising and pricing strategy for Tactic III.4.k are shown in Proposition 4.2.101. Moreover, Proposition 4.2.101 shows the explicit formula of the local optimal advertising and pricing strategies for Tactic III.4.k, and the associated company's profit.

This completes the derivation of all the local optimums for Tactic III. Similarly, we summarize the major findings for Tactic III.

1. When there is no penalty for insufficient advertising, it is always optimal to allocate all the advertising effort to LG. When there is penalty for insufficient advertising, the optimal allocation of advertising effort could also be allocated to both LG and FG., and even be solely allocated to FG (for Tactic III.2.k, Tactic III.3.a, Tactic III.c, Tactic III.4.h and Tactic III.4.k). This shows that when there is penalty for insufficient advertising, it is optimal for the company to achieve a
balance between allocating the advertising efforts to the two groups. If the penalty for insufficient advertising in FG is very heavy, then it is still optimal to advertise to FG even the company mainly targets at the market segment of LG.
2. Tactic III includes the largest number of sub-tactics (totally 26 sub-tactics, and nearly half number of sub-tactics) among Tactic I, Tactic II and Tactic III. This shows that although both Tactic II and Tactic III target only at one market segment, as the two customers groups react differently under the mutual social influences, the complexities of Tactic II and Tactic III are very different. When there is penalty for insufficient advertising, Tactic III is indeed much more complicated than Tactic II.

### 4.3 Numerical Analysis

In this section, we carry out a numerical analysis to illustrate the steps to identify the local and global optimal decisions with penalties for insufficient advertising. We consider the same set of parameters which are considered the numerical analysis without penalties for insufficient advertising in the basic model (Section 3.6), i.e., $h=1, X_{L}=100, X_{F}=300, a=0.1, \alpha=1, b=0.5, \quad \beta=10, g=0.05$, $\gamma=0.5$, and $c=200$. In addition, for the extended model we explored in this chapter, we need to consider the following additional set of parameters ${ }^{5}$ which are specific for the penalties for insufficient advertising, $m=100, \mu=100, T=50, \tau=5$. All these parameters are the base parameters we set for the numerical analysis. As we mentioned in Section 3.6, we need to examine each local optimal solution for each tactic before we can determine the global optimal solution for the problem. In the following, we will examine the local optimal profit under each tactic. For each tactic (including the sub-tactic), we have the respective local optimal solution listed in Table 4-3. As shown in Table 4-3, not all the sub-tactics satisfy all the associated necessary conditions for a finite local optimum. By comparing the profits under all the local optimal solutions which satisfy all the necessary conditions for a finite local optimum, we can obtain the global optimal solution (shown in bold, in Table 4-3(a)). For this case, the global optimal solution is achieved under Tactic I.1.b.

[^4]Table 4-3(a) Local optimal solutions for Tactic I

| Tactic | $e$ | $\lambda$ | $p$ | $D_{L}(\omega)$ | $D_{F}(\omega)$ | $\pi_{\Lambda}(\omega)$ | $e$ for LG | $e$ for FG | Necessary <br> conditions <br> fulfilled? |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I.1.a | 55.0 | 0.9 | 917.2 | 23 | 73 | 65556.1 | 50.0 | 5.0 | Yes |
| I.1.b | $\mathbf{6 6 . 5}$ | $\mathbf{0 . 9}$ | $\mathbf{9 2 5 . 1}$ | $\mathbf{2 3}$ | $\mathbf{7 4}$ | $\mathbf{6 5 6 7 9 . 2}$ | $\mathbf{6 1 . 5}$ | $\mathbf{5 . 0}$ | Yes |
| I.1.c | 29.6 | 1.7 | 909.2 | 25 | 70 | 0.0 | 50.0 | -20.4 | No |
| I.2.a | 50.0 | 1.0 | 915.6 | 23 | 72 | 65282.6 | 50.0 | 0.0 | Yes |
| I.2.b | 66.6 | 1.0 | 927.1 | 24 | 73 | 0.0 | 66.6 | 0.0 | No |
| I.2.c | 91.7 | 1.0 | 1200.0 | 33 | 32 | 56236.1 | 91.7 | 0.0 | Yes |
| I.2.d | 80.2 | 0.6 | 925.1 | 21 | 76 | 0.0 | 50.0 | 30.2 | No |
| I.3.a | 5.0 | 0.0 | 868.8 | 20 | 73 | 57113.0 | 0.0 | 5.0 | Yes |
| I.3.b | 119.8 | 1.0 | 961.8 | 25 | 76 | 0.0 | 114.8 | 5.0 | 119.8 |
| I.3.c | 28.8 | 0.0 | 890.2 | 19 | 73 | 0.0 | 0.0 | 28.8 | No |
| I.3.d | -41.7 | 0.0 | -800.0 | -32 | 343 | 0.0 | 0.0 | -41.7 | No |
| I.4.a | 0.0 | 0.0 | 881.3 | 21 | 70 | 56380.2 | 0.0 | 0.0 | Yes |
| I.4.b | 120.0 | 1.0 | 963.8 | 26 | 76 | 0.0 | 120.0 | 0.0 | No |
| I.4.c | 79.4 | 0.0 | 216.1 | -8 | 194 | 0.0 | 0.0 | 79.4 | No |

Table 4-3(b) Local optimal solutions for Tactic II

| Tactic | $e$ |  | $\lambda$ | $p$ | $D_{L}(\omega)$ | $D_{F}(\omega)$ | $\pi_{\Lambda}(\omega)$ | $e$ for LG | $e$ for FG |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| II.1.a | 55.0 | 0.9 | 405.0 | 0 | 103 | 0.0 | 50.0 | 5.0 | Necessary |
| conditions |  |  |  |  |  |  |  |  |  |
| fulfilled? |  |  |  |  |  |  |  |  |  |$|$

Table 4-3(c) Local optimal solutions for Tactic III

| Tactic | $e$ | $\lambda$ | $p$ | $D_{L}(\omega)$ | $D_{F}(\omega)$ | $\pi_{\Lambda}(\omega)$ | $e$ for LG | $e$ for FG | conditions <br> fulfilled? |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| III.1.a | 55.0 | 0.9 | 1150.0 | 48 | 0 | 0.0 | 50.0 | 5.0 | No |
| III.1.b | 47.1 | 0.9 | 1142.1 | 47 | 0 | 0.0 | 42.1 | 5.0 | No |
| III.1.c | 55.0 | 0.9 | 1355.0 | 37 | 0 | 39998.8 | 50.0 | 5.0 | Yes |
| III.1.d | 36.3 | 0.9 | 499.2 | 78 | 0 | 0.0 | 31.3 | 5.0 | No |
| III.2.a | 50.0 | 1.0 | 1401.0 | 35 | 0 | 0.0 | 50.0 | 0.0 | No |
| III.2.b | 47.4 | 1.0 | 1147.4 | 47 | 0 | 0.0 | 47.4 | 0.0 | No |
| III.2.c | 50.0 | 1.0 | 1350.0 | 38 | 0 | 0.0 | 50.0 | 0.0 | No |
| III.2.d | 40.5 | 1.2 | 1340.5 | 38 | 0 | 0.0 | 50.0 | -9.5 | No |
| III.2.e | 50.0 | 1.0 | 1350.0 | 38 | 0 | 40125.0 | 50.0 | 0.0 | Yes |
| III.2.f | -61.1 | 1.0 | 2433.3 | -28 | 0 | 0.0 | -61.1 | 0.0 | No |
| III.2.g | 50.0 | 1.0 | 950.0 | 58 | 0 | 0.0 | 50.0 | 0.0 | No |
| III.2.h | 50.0 | 1.0 | 1150.0 | 48 | 0 | 0.0 | 50.0 | 0.0 | No |
| III.2.k | 50.0 | 0.0 | 1200.0 | 40 | 0 | 0.0 | 0.0 | 50.0 | No |
| III.3.a | 5.0 | 0.0 | 1100.0 | 45 | 0 | 0.0 | 0.0 | 5.0 | No |
| III.3.b | 99.7 | 0.9 | 1194.7 | 50 | 0 | 0.0 | 94.7 | 5.0 | No |
| III.3.c | 5.0 | 0.0 | 1305.0 | 35 | 0 | 33373.8 | 0.0 | 5.0 | Yes |
| III.3.d | 55.0 | 0.9 | 1347.1 | 38 | 0 | 0.0 | 50.0 | 5.0 | No |
| III.4.a | 0.0 | 0.0 | 1100.0 | 45 | 0 | 0.0 | 0.0 | 0.0 | No |
| III.4.b | 100.0 | 1.0 | 1200.0 | 50 | 0 | 0.0 | 100.0 | 0.0 | No |
| III.4.c | 0.0 | 0.0 | 1300.0 | 35 | 0 | 0.0 | 0.0 | 0.0 | No |
| III.4.d | 38.1 | 0.0 | 1338.1 | 33 | 0 | 0.0 | 0.0 | 38.1 | No |
| IIII.4.e | 0.0 | 0.0 | 1300.0 | 35 | 0 | 33000.0 | 0.0 | 0.0 | Yes |
| IIII.4.4.4. | -15.8 | 1.0 | 1400.0 | 28 | 0 | 0.0 | -15.8 | 0.0 | No |
| 0.0 | 0.0 | 1100.0 | 45 | 0 | 0.0 | 0.0 | 0.0 | No |  |
| 50.0 | 0.0 | 200.0 | 90 | 0 | 0.0 | 0.0 | 50.0 | No |  |
|  | 1100.0 | 45 | 0 | 0.0 | 0.0 | 50.0 | No |  |  |
|  |  |  |  |  |  |  |  |  |  |

Comparing the global optimums between the situations with and without penalties for insufficient advertising (shown in Table 3-5, and please refer to Section 3.6 for the details of the situation without penalties for insufficient advertising), we observe that the global optimal advertising and pricing strategies with penalties for insufficient advertising has lower $e$ and $p$, and the advertising efforts are allocated to both LG and FG instead of allocated only to LG or FG. The results suggest that the optimal advertising and pricing strategy can be very different between the situations with and without penalties for insufficient advertising, especially for the optimal value of $\lambda$. In particular, the case without penalty for insufficient advertising favors polarized decisions whereas the case with penalty naturally allows for more variety of optimal allocations.

Table 4-4(a) Global optimum without penalties for insufficient advertising

| Tactic | $e$ | $\lambda$ | $p$ | $D_{L}$ | $D_{F}$ | $\pi(\omega)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IA | 66.7 | 1 | 927.1 | 24 | 73 | 66042.3 |

Table 4-4(b) Global optimum with penalties for insufficient advertising

| Tactic | $e$ | $\lambda$ | $p$ | $D_{L}(\omega)$ | $D_{F}(\omega)$ | $\pi_{\Lambda}(\omega)$ | $e$ for LG | $e$ for FG |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I.1.b | 66.5 | 0.9 | 925.1 | 23 | 74 | 65679.2 | 61.5 | 5.0 |

In the following, we conduct a sensitivity analysis to explore how a change of the penalties for insufficient advertising related parameters affects the optimal decision.

## F. Varying $T$ and $\tau$

Table 4-5 shows the optimal tactics with cases of different ${ }_{T}$ and $\tau$. Tactic I.1.b (sufficient advertising on both LG and FG) is optimal when $T$ is low regardless of the value of $\tau$. The optimal tactic switches from Tactic I.1.b to Tactic I.4.b (insufficient advertising to both LG and FG ) when $T$ becomes high regardless of the value of $\tau$. The result suggests that in such market situations, there is a tradeoff for the company between the cost of advertising and the penalties for insufficient advertising. To be specific, the company should try to avoid the penalties when $T$ is low. However, when $T$ becomes high, it is better for the company to pay some penalties for insufficient advertising instead of paying a high advertising cost to avoid the penalties.

Table 4-5(a) Optimal tactic with changing $T$ and $\tau$

| $\tau \backslash T$ | Low | High |
| :--- | :--- | :--- |
| Low | I.1.b | I.4.b |
| High | I.1.b | I.4.b |

Table 4-5(b) Optimal tactic (in details) with changing $T$ and $\tau$

| $T$ | $\tau$ | Tactic | $e$ | $\lambda$ | $p$ | $D_{L}(\omega)$ | $D_{F}(\omega)$ | $\pi_{\Lambda}(\omega)$ | $e$ for LG | $e$ for FG |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 25(low) | 2.5(low) | I.1.b | 66.6 | 0.96 | 926.1 | 23 | 73 | 65860.6 | 64.1 | 2.5 |
| 25(low) | 15(high) | I.1.b | 66.1 | 0.8 | 921.1 | 22 | 74 | 64956.1 | 51.1 | 15.0 |
| 150(high) | 2.5(low) | I.4.b | 120.0 | 1.0 | 963.8 | 26 | 76 | 60125.2 | 120.0 | 0.0 |
| 150(high) | 15(high) | I.4.b | 120.0 | 1.0 | 963.8 | 26 | 76 | 58875.2 | 120.0 | 0.0 |

## G. Varying $m$ and $\mu$

Table 4-6 shows the optimal tactics with cases of different $m$ and $\mu$. The optimal tactic is Tactic I.2.b (sufficient advertising to LG but insufficient to FG) when $\mu$ is low. The optimal tactic switches from Tactic I.2.b to Tactic I.1.b (sufficient advertising to both LG and FG) when $\mu$ becomes high. The result suggests that in such market situations, the decision of advertising target, LG or FG, depends on $m$. In particular, the company should try to avoid the penalties for having insufficient advertising to FG when $\mu$ is low. However, when $\mu$ becomes high, it is optimal for the company to avoid the penalties for insufficient advertising to LG.

Table 4-6(a) Optimal tactic with changing $m$ and $\mu$

| $\mu \backslash m$ | Low | High |
| :--- | :--- | :--- |
| Low | I.2.b | I.2.b |
| High | I.1.b | I.1.b |

Table 4-6(b) Optimal tactic (in details) with changing $m$ and $\mu$

| $m$ | $\mu$ | Tactic | $e$ | $\lambda$ | $p$ | $D_{L}(\omega)$ | $D_{F}(\omega)$ | $\pi_{\Lambda}(\omega)$ | $e$ for LG | $e$ for FG |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50(low) | 50 (low) | I.2.b | 66.6 | 1.0 | 927.1 | 24 | 73 | 65792.2 | 66.6 | 0.0 |
| 50(low) | 150 (high) | I.1.b | 66.5 | 0.92 | 925.1 | 23 | 74 | 65679.2 | 61.5 | 5.0 |
| 150 (high) | 50 (low) | I.2.b | 66.6 | 1.0 | 927.1 | 24 | 73 | 65792.2 | 66.6 | 0.0 |
| 150 (high) | 150 (high) | I.1.b | 66.5 | 0.92 | 925.1 | 23 | 74 | 65679.2 | 61.5 | 5.0 |

H. Varying $T$ and $m$ for $\tau=0$

Table 4-7 shows the optimal tactics with cases of different $T$ and $m$ with $\tau=0$. Tactic I.1.b is optimal when $T$ is low regardless of the value of $m$. The optimal tactic switches from Tactic I.1.b to Tactic I.3.b when $T$ becomes high regardless of the value of $m$. The result suggests that in such market situations, the value of $T$ affects more than the value of $m$ on the optimal choice of tactic.

Table 4-7(a) Optimal tactic with changing $m$ and $T$ for $\tau=0$

| $T \backslash m$ | Low | High |
| :--- | :--- | :--- |
| Low | I.1.b | I.1.b |
| High | I.3.b | I.3.b |

Table 4-7(b) Optimal tactic (in details) with changing $m$ and $T$ for $\tau=0$

| $m$ | $T$ | Tactic | $e$ | $\lambda$ | $p$ | $D_{L}(\omega)$ | $D_{F}(\omega)$ | $\pi_{\Lambda}(\omega)$ | $e$ for LG | $e$ for FG |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50(low) | 25(low) | I.1.b | 66.6 | 1.00 | 927.1 | 24 | 73 | 66042.2 | 66.6 | 0.0 |
| 50(low) | 150(high) | I.3.b | 93.3 | 1.0 | 945.4 | 25 | 75 | 62541.7 | 93.3 | 0.0 |
| 150(high) | 25(low) | I.1.b | 66.6 | 1.00 | 927.1 | 24 | 73 | 66042.2 | 66.6 | 0.0 |
| 150(high) | 150 (high) | I.3.b | 146.7 | 1.0 | 982.1 | 27 | 77 | 59542.8 | 146.7 | 0.0 |

I. Varying $\tau$ and $\mu$ for $T=0$

Table 4-8 shows the optimal tactics with cases of different $\tau$ and $\mu$ with $T=0$. Tactic I.2.b is optimal when $\mu$ is low regardless of the value of $\tau$. The optimal tactic switches from Tactic I.2.b to Tactic I.1.b when $\mu$ becomes high regardless of the value of $\tau$. The result suggests that in such market situations, the value of $\mu$ affects more than the value of $\tau$ on the choice of the optimal tactic. Notice that this case is different from the situation when $\tau=0$.

Table 4-8(a) Optimal tactic with changing $\tau$ and $\mu$ for $T=0$

| $\tau \backslash \mu$ | Low | High |
| :--- | :--- | :--- |
| Low | I.2.b | II.1.b |
| High | I.2.b | II.1.b |

Table 4-8(b) Optimal tactic (in details) with changing $\tau$ and $\mu$ for $T=0$

| $\mu$ | $\tau$ | Tactic | $e$ | $\lambda$ | $p$ | $D_{L}(\omega)$ | $D_{F}(\omega)$ | $\pi_{\Lambda}(\omega)$ | $e$ for LG | $e$ for FG |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50(low) | 2.5 (low) | I.2.b | 66.6 | 1.0 | 927.1 | 24 | 73 | 65917.2 | 66.6 | 0.0 |
| 50(low) | 15 (high) | I.2.b | 66.6 | 1.0 | 927.1 | 24 | 73 | 65292.2 | 66.6 | 0.0 |
| 150(high) | 2.5 (low) | II.1.b | 200.0 | 0.0 | 600.0 | 0 | 200 | 69625.0 | 0.0 | 200.0 |
| 150(high) | 15 (high) | II.1.b | 200.0 | 0.0 | 600.0 | 0 | 200 | 67750.0 | 0.0 | 200.0 |

## Chapter 5 Discussion

### 5.1 Findings and managerial insights in basic model

We have found that the optimal strategies follow different scenarios under different settings. Interestingly, we have analytically shown that it can be optimal to (1) advertise to only one group (either follower group (FG) or leader group (LG)) while sell to the whole market, (2) advertise and sell to the FG only, and (3) advertise and sell to the LG only. The specific choice depends on the given parameters which include the sensitivity coefficients on price, advertising efforts, and social influences. In particular, the impacts brought upon by the sensitivity coefficients on social influences are very significant in affecting the optimal tactic choice and decisions. This suggests that the impacts brought upon by social influences are so significant that they should not be neglected in making a scientifically sound optimal decision.

For the social influence, both mathematical and numerical analyses show that the values of $b$ and $\beta$ significantly shape the optimal strategy. We find that as $b$ increases, the company shifts from selling to both segments to only one segment and earns less. While as $\beta$ increases, the company shifts from selling to only one segment to both segments and earns more only for the case when $b$ has a small value. These findings show the interesting asymmetry of the social influences of the two groups of consumers which call for careful considerations when the luxury brands make their optimal tactic and decisions. These results also demonstrate the significant impacts brought upon by the sensitivities of norms (and hence the influences by social groups). Furthermore, the analysis of $b$ and $\beta$ answers a question that has long been discussed in international business and associated with transferring image-based values developed in the country of origin to foreign markets: "does a retail brand mean the same to a group of customers in one country as it does to customers in another?" Brown and Burt (1992) wrote in their conclusion to a special issue in the European Journal of Marketing on "Retail Marketing: International Perspectives" that
"one view of internationalisation is that based on the transfer of a retail brand, with its associated image for consumers, across national borders" (p. 81). Moore et al.(2000) considered that the marketing of designer fashion ensures that this shared international understanding of brand identity and meaning is developed and preserved through the standardization of communications strategies, and by the exercising of tight controls over merchandising, distribution and pricing strategies.

However, our analysis offers alternative answer here. As shown in our model, for a specific designer label or luxury brand product, because different social influence leads to varied optimal advertising and pricing strategy in different countries/societies, the buyer group combination could be very different, and thus the user-image might be very different. Otherwise, to convey the same image and sell for same type of consumers, the internationalized brand owner might not be able to maximize its own profit. Thus, our analysis gives deeper insights to the existing proposals in the literature.

### 5.2 Findings and managerial insights in extended model

For the extended model with linear loss functions, we find that under Tactic I:
3. When there is no penalty for insufficient advertising, the optimal allocation of advertising effort is either to i) allocate all the advertising effort to LG or ii) allocate all the advertising effort to FG.. When there is a penalty for insufficient advertising, the optimal allocation of advertising effort will more likely be allocated to both LG and FG. This implies that when there is penalty for insufficient advertising, the company should take a balance between allocating the advertising effort between the two groups and avoid being "polarized".
4. The optimal advertising effort is never decreasing with the optimal retail price when there is no penalty for insufficient advertising. However, this can happen (for Tactic I.2.d with $b>1$ and Tactic I.4.c with $b>1$ ) when there is penalty for insufficient advertising.

For Tactic II of the extended model, we find that,
3. When there is no penalty for insufficient advertising, it is always optimal to allocate all the advertising effort to FG under Tactic II. When there is penalty for insufficient advertising, the optimal allocation of advertising effort could also be allocated to both LG and FG, and even be allocated only to LG (for Tactic II.2.a and Tactic II.4.b). This shows that when there is penalty for insufficient advertising, the company should strike a balance between allocating the advertising effort to LG and FG. If the penalty for insufficient advertising to LG is very heavy, then it is optimal to advertise to LG even when the company targets of the market segment of FG.
4. For Tactic II.3.b and Tactic II.4.b, the optimal advertising effort could be decreasing with the optimal retail price.

For Tactic III of the extended model, we find that,
3. When there is no penalty for insufficient advertising, it is always optimal to allocate all the advertising effort to LG. When there is penalty for insufficient advertising, the optimal allocation of advertising effort could also be allocated to both LG and FG., and even be solely allocated to FG (for Tactic III.2.k, Tactic III.3.a, Tactic III.3.c, Tactic III.4.h and Tactic III.4.k). This shows that when there is penalty for insufficient advertising, it is optimal for the company to achieve a balance between allocating the advertising effort to the two groups. If the penalty for insufficient advertising in FG is very heavy, then it is still optimal to advertise to FG even the company mainly targets at the market segment of LG.
4. Tactic III includes the largest number of sub-tactics (totally 26 sub-tactics, and nearly half number of sub-tactics) among Tactic I, Tactic II and Tactic III. This shows that although both Tactic II and Tactic III target only at one market segment, as the two customers groups react differently under the mutual social influences, the complexities of Tactic II and Tactic III are very different. When there is penalty for insufficient advertising, Tactic III is indeed much more complicated than Tactic II in terms of the analysis.

Comparing the global optimums between the situations with and without penalties for insufficient advertising (shown in Table 3-5, and please refer to Section 3.6 for the details of the situation without penalties for insufficient advertising), we observe that the global optimal advertising and pricing strategies with penalties for insufficient advertising has lower $e$ and $p$, and the advertising efforts are allocated to both LG and FG instead of allocated only to LG or FG.

The results suggest that the optimal advertising and pricing strategy can be very different between the situations with and without penalties for insufficient advertising, especially for the optimal value of $\lambda$. In particular, the case without penalty for insufficient advertising favors polarized decisions whereas the case with penalty naturally allows for more variety of optimal allocations.

### 5.3 Findings and managerial insights about Veblen effects

Just as Adam Smith says that "with the greater part of rich people, the chief enjoyment of riches consists in the parade of riches, which in their eyes in never so complete as when they appear to possess those decisive marks of opulence which nobody can possess but themselves (c.f. Heilbroner 1986, p. 190). One potential sociological explanation for the upward slope in the demand curve for prestige goods is that consumers could use these goods to impress others of their relative wealth (Coleman 1990). However, Corneo and Jeanne (1997a) show that under a signaling framework the desire for exclusivity always leads to a downward-sloping curve. Interestingly, their analysis reveals that an upward-sloping demand curve can be observed only if the consumers are followers. Followers are consumers who prefer to conform with the society and derive more utility from a product if it is purchased by more people (Ross, Bierbrauer and Hoffman 1976, Jones 1984). Conformism is seen in products such as garments. Besides, in the literature, there are two other explanations which could explain the presence of an upward-sloping demand curve. The first reason is that consumers use price to infer quality, especially when it is difficult to determine quality by inspection (Zikmund and d'Amico 2000, p.624). The second reason is that certain goods (called "Giffen Goods") are so inferior that the income effect is larger than the substitution effect. In reality, the likelihood of such goods is viewed with skepticism as the share of expenditure on a good in comparison to total spending is likely to be so small that income effect would be negligible (Hicks 1946, Heiner 1974). The above mentioned results are popular findings related to the Veblen effect as developed in the literature. In contrast to these approaches, we also explore the Veblen effect while our focus is on social influences.

Under the model we developed in Chapter 3, we have explored the conditions for the Veblen effect to happen under each tactic.

| Tactic | Conditions | Veblen effect occurs in | Feasible region of $p$ |
| :---: | :---: | :---: | :---: |
| IA | $\frac{a^{2}(1+\beta)}{2(1+b \beta)}>\frac{\gamma}{\beta}+g$ | Both LG and FG | $p \geq \max \left\{p_{A, F}^{0}, p_{A, L}^{0}\right\}$ |
|  | $\frac{\gamma}{\beta}+g \geq \frac{a^{2}(1+\beta)}{2(1+b \beta)}>g-b \gamma$ | LG only | $p_{A, L}^{0} \leq p \leq p_{A, F}^{0}$ |
|  | $\frac{a^{2}(1+\beta)}{2(1+b \beta)} \leq g-b \gamma$ | No Veblen effect | $p \leq \min \left\{p_{A, F}^{0}, p_{A, L}^{0}\right\}$ |
| IB | $\frac{\alpha^{2}(1-b)}{2(1+b \beta)}<\gamma-\frac{g}{b}$ | LG only | $p_{B, L}^{0} \leq p \leq p_{B, F}^{0}$ |
|  | $\gamma-\frac{g}{b} \leq \frac{\alpha^{2}(1-b)}{2(1+b \beta)} \leq \gamma+\beta g$ | No Veblen effect | $p \leq \min \left\{p_{B, L}^{0}, p_{B, F}^{0}\right\}$ |
|  | $\frac{\alpha^{2}(1-b)}{2(1+b \beta)}>\gamma+\beta g$ | FG only | $p_{B, F}^{0} \leq p \leq p_{B, L}^{0}$ |
| IIA | $\alpha^{2}>2 \gamma$ | FG only | $p>c$ |
| IIIA | $a^{2}>2 g$ | LG only | $p>c$ |

We now try to generate some new insights by comparing our results with those in Amaldoss and Jain (2002) which uses similar setting with utility model but does not include advertising effect. Amaldoss and Jain (2002) suggests that if the market is comprised of only snobs (similar to Tactic III) then the demand curve is downward slopping, and firm's profits decline as snobbish behavior increases. If the market is comprised of only followers (similar to Tactic II), again the stable demand curve is downward sloping, but firm's profits increase as the degree of conformity increases. They find that if the market includes both snobs and followers (similar to Tactic I), then it is possible for the snobs to have an upward-sloping demand curve. One could see that, under our model with advertising effects, Veblen effect happens if $\alpha^{2}>2 \gamma$ in Tactic II or $a^{2}>2 g$ in Tactic III. This result is implied by the setting of advertising response function, and advertising cost and effort relationship. What we want to emphasize is that, under Tactic I, no matter which group is the advertising
target, there is a chance for the Veblen effect to happen in the other group. What's more, in some situation, the Veblen effect only happens in the group that is not advertised to. Here, our explanations for the existence Veblen effect are (1) high price supports the advertising effort for LG and thus produces stronger social effect for FG to buy; or (2) high price reduces buyers in FG, and thus produces stronger social effect for LG to buy.

We argue that this phenomenon manifests indirect effect of advertising through social influence which is an essential practice in luxury fashion brand marketing. Numerous investment is spent on getting the attention of a very limited number of "leading" people, e.g., bloggers, fashion editors, status persons, etc, while the general public generate the major sales. This is actually efficient and has been practiced since long time ago when luxury brands were born, while our model suggests that it can be the optimal tactic for the company, which is basically to use the social effects wisely and generate the maximized profit.

## Chapter 6 Conclusion

In many situations, the outcomes of one's choices depend on the choices made by others. This strategic interdependence raises many fundamental research questions that are absent in individual choice literature (Cachon and Swinney 2009; Jerath et al. 2010). Moreover, results on individual decision making may not hold in the contexts with strategic interactions among a group of decision makers (such as consumers).

Motivated by the importance of social influences in affecting consumer demand of luxury fashion goods and based on a monopoly setting in the context of luxury fashion brand market, we have first developed an original analytical optimization model for analysis (the basic model). To be specific, we have explored the optimal advertising and pricing decisions for a luxury fashion brand in a market consisting of two groups of consumers with opposite social needs for fashion products, namely the leader group (LG) and the follower group (FG). There are three tactics (that determine the respective optimal advertising and pricing decisions) for the luxury fashion brand to choose from, namely: Tactic I is to sell the product to both groups; Tactic II is to sell the product to FG only; and Tactic III is to sell to the LG only.

From this basic model, we have found that the optimal strategies follow different scenarios. Interestingly, we have analytically shown that it can be optimal to advertise to only one group while selling to the whole market (e.g., Tactic I is adopted by the brand) under certain situations. Specifically, under Tactic I, the brand should advertise only to LG when the advertising allocation sensitivity of the total demand, $N_{I}$, is positive, while the brand should advertise to only FG when $N_{I}$ is negative. Moreover, it can also be optimal for the brand to advertise and sell to FG only, or advertise and sell to LG only.

To extend the model by considering the double function of advertising on (i) buying intention enhancement and (ii) long-term brand equity building, we develop an extended model in which a company will suffer a loss when the advertising effort to each group is not up to a certain level. This setting is supported by empirical evidence that advertising creates brand image, builds brand equity and prevents brand dilution.

For this extended model, similar to the basic model, we have developed the mechanism to identify the optimal tactic and decisions.

For both models, we have analytically found that the specific choice of target consumer and advertising strategy a company should adopt depend on the following given parameters: (i) the sensitivity coefficients on price, (ii) advertising effort, (iii) social influences, and (iv) loss due to insufficient advertising (Remark: (iv) is related to the extended model only). In particular, the impacts brought about by the sensitivity coefficients on social influences are very significant in affecting the optimal tactic choice and the corresponding optimal advertising and pricing decisions. This suggests that: (i) It can be optimal for a luxury company to focus on only one group of consumers, and (ii) the impacts brought by social influences are so significant that they should never be neglected.

Besides, we have also investigated the conditions of the Veblen effect, which is considered as a special phenomenon in luxury brand consumption, to take place under the first model setting. We have found that the Veblen effect takes place in both LG and FG only if (1) Tactic I is adopted; (2) the advertising allocation sensitivity of the total demand $N_{I} \geq 0$; and (3) the social influence from LG to FG exists, i.e., $\beta>0$. For other cases, the Veblen effect may only take place in either group or simply do not exist.

In this thesis research, we contribute by filling the gap of literature in investigating simultaneously the strategic consumer behavior under the rational expectation framework and the strategic marketing behavior (including segmented advertising, non-discriminant pricing and) in luxury brand industry. We also establish the original analytical optimization models and derive the solution schemes in solving them. We believe that the results attained from our analytical findings and numerical analyses provide important academic and managerial insights for the management of luxury fashion brands as well as explanations to many real-world phenomena. Future research directions will be proposed and discussed in the following chapter.

## Chapter 7 Limitations and Future Research

Similar to other mathematical modeling research in economics and business studies, this piece of research has a few limitations. First, the model assumes that the demand function is known for sure and is explicitly given by the linear structure. Second, the quadratic advertising cost function and the linear loss penalty functions are also assumed to be valid in order to get more findings and insights. For all these assumptions, despite the fact that they are well-supported by many prior literature studies, it is important to notice that the corresponding analytical results do depend on the specific functional forms. For future research, there are two important directions:

1) In the real world, branding decisions and consumer purchasing happen in more than one single period. In fact, there is ample literature studying the dynamics of consumer action in this setting. An obvious extension is to combine a dynamical version of consumer behavior with the optimization framework in this model. A simple two-period set-up should help to capture the leader/follow dynamics and help further explore the associated pricing and advertising implications.
2) In the real world, demand is uncertain, and even the key parameters are unobservable. Thus, another reasonable extension is to introduce stochastic elements into the two-stage model. In this case, one important issue which deserves deep investigation would be to explore how a firm can learn in the first stage, and exploits the learning in the second stage. This line of thinking would further enrich the tactical space the firm considers, from an optimization paradigm to an adaptive learning one. In addition, by considering the stochastic version of the problem, important analysis on the associated level of risk can be conducted. This will be a challenging and promising direction for future research. Last but not least, it will be interesting to explore the optimal pricing and inventory decisions of the extended problem with respect to some risk related objectives such as VaR (Chiu et al. 2011) and CVaR.

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## Appendix 1: Proofs in Chapter 3

Proof of Proposition 3.3.1 Let $y=x_{F} / \gamma$. Since $x_{F}-\gamma c>0$, we have $y>c$. Therefore, there exist multiple feasible $\omega$ with $p \in(c, y)$ such that $D_{F}(\omega) \geq x_{F}-\gamma p>0$ and $\pi(\omega) \geq(p-c)\left(x_{F}-\gamma p\right)>0$.

Proof of Proposition 3.3.2 When Tactic I is adopted, $D_{L}(\omega)>0$ and $D_{F}(\omega)>0$, and
$D(\omega)=\frac{X+e\left[\alpha(1-b)+\lambda N_{I}\right]-G p}{1+b \beta}$. Consider the first-order partial derivative of $D(\omega)$ with respect to $\lambda$, we obtain $\frac{\partial D(\omega)}{\partial \lambda}=\frac{e N_{I}}{1+b \beta}$. Observe that $\frac{\partial D(\omega)}{\partial \lambda}$ is independent of $p$. For any given $e>0, D(\omega)$ is strictly increasing in $\lambda$ if $N_{I}>0$, is strictly decreasing in $\lambda$ if $N_{I}<0$ and remains unchanged if $N_{I}=0$. Therefore, for any given $p>c$ and $e>0, \lambda^{*}=1$ if $N_{I}>0, \lambda^{*}=0$ if $N_{I}<0$, and any feasible $\lambda \in[0,1]$ is an optimal solution of problem (P1) if $N_{I}=0$.

Proof of Proposition 3.3.3 Since $\partial \pi(\omega) / \partial \lambda=-a e(p-c)<0$ (because $p>c$ ), $\lambda^{*}=0$ under Tactic IIA. For Tactic IIB, consider $\omega^{\prime}=\left\{e^{\prime}, \lambda^{\prime}, p^{\prime}\right)$, where $\omega^{\prime} \in \Omega$, $\lambda^{\prime}>0, V_{L}\left(\omega^{\prime}\right)=0$ and $D_{F}\left(\omega^{\prime}\right)>0$. There always exists $\omega^{\prime \prime}=\left\{e^{\prime}, 0, p^{\prime}\right)$, where $\omega^{\prime \prime} \in \Omega, \quad V_{L}\left(\omega^{\prime \prime}\right)<0$ and $D_{F}\left(\omega^{\prime \prime}\right)>0$ such that $\pi\left(\omega^{\prime}\right)<\pi\left(\omega^{\prime \prime}\right)$. Therefore, $\lambda^{*}=0$ under Tactic II.

Proof of Proposition 3.3.4 Since $\partial \pi(\omega) / \partial \lambda=a e(p-c)>0, \lambda^{*}=0$ under Tactic IIIA. For Tactic IIIB, consider $\omega^{\prime}=\left\{e^{\prime}, \lambda^{\prime}, p^{\prime}\right)$, where $\omega^{\prime} \in \Omega, \quad \lambda^{\prime}>0, V_{F}\left(\omega^{\prime}\right)=0$ and $D_{L}\left(\omega^{\prime}\right)>0$. There always exists $\omega^{\prime \prime}=\left\{e^{\prime \prime}, 1, p^{\prime}\right)$, where $\omega^{\prime \prime} \in \Omega, \quad e^{\prime \prime}<e^{\prime}$, $V_{F}\left(\omega^{\prime \prime}\right)<0$ and $D_{L}\left(\omega^{\prime \prime}\right)=D_{L}\left(\omega^{\prime}\right)$ such that $\pi\left(\omega^{\prime}\right)<\pi\left(\omega^{\prime \prime}\right)$. Therefore, $\lambda^{*}=1$ under Tactic III.
(Q.E.D.)

Proof of Proposition 3.4.1 First of all, $\lambda$ of $\omega_{I, A}^{*}, \omega_{I, B}^{*}$ and $\bar{\omega}_{I, B}^{*}$, are always feasible, so we focus only on the feasibility of $p$ and $e$.

Part (a): If $\alpha^{2}(1-b)^{2}<4(1+b \beta) B$ and $X>G c$, then $\omega_{I, A}^{*}$ is finite, $\frac{\alpha(1-b)(X-G c)}{4(1+b \beta) G-\alpha^{2}(1-b)^{2}}>0$, and $\frac{2(1+b \beta)(X+G c)-a^{2} c(1+\beta)^{2}}{4(1+b \beta) G-a^{2}(1+\beta)^{2}}-c=\frac{2(1+b \beta)(X-G c)}{4(1+b \beta) G-a^{2}(1+\beta)^{2}}>0$.

Therefore, $\omega_{I, A}^{*}$ is feasible if $a^{2}(1+\beta)^{2} \leq 4 G(1+b \beta)$ and $X>G c$.
Part (b): If $b<1, \alpha^{2}(1-b)^{2}<4(1+b \beta) G$ and $X>G c$, then $\omega_{I, B}^{*}$ and $\bar{\omega}_{I, B}^{*}$ are finite, $\frac{\alpha(1-b)(X-G c)}{4(1+b \beta) G-\alpha^{2}(1-b)^{2}}>0$, and $\frac{2(1+b \beta)(X+G c)-\alpha^{2} c(1-b)^{2}}{4(1+b \beta) G-\alpha^{2}(1-b)^{2}}-c=\frac{2(1+b \beta)(X-G c)}{4(1+b \beta) G-\alpha^{2}(1-b)^{2}}>0$.

Therefore, $\omega_{I, B}^{*}$ and $\bar{\omega}_{I, B}^{*}$ are feasible if $b<1, a^{2}(1+\beta)^{2} \leq 4 G(1+b \beta)$ and $X>G c$.

Proof of Proposition 3.4.2 The first- and second-order partial derivatives of $\pi\left(\omega_{I, L}\right)$ with respect to $e$ are $\frac{\partial \pi\left(\omega_{I, L}\right)}{\partial e}=\frac{a(1+\beta)}{1+b \beta}(p-c)-2 e$, and $\frac{\partial^{2} \pi\left(\omega_{I, L}\right)}{\partial e^{2}}=-2$, respectively. Therefore, for any fixed $p, \pi\left(\omega_{I, L}\right)$ is concave in $e$. By considering $\partial \pi\left(\omega_{I, L}\right) / \partial e=0$, the unique optimal advertising effort for any fixed $p$ is given by $e_{I, A}^{*}(p)=\frac{a(1+\beta)}{2(1+b \beta)}(p-c)$.

Proof of Proposition 3.4.3 In this proof, at the boundary means that $D_{L}(\omega)=0$ or/and $D_{F}(\omega)=0$ for any given $\omega$. According to Proposition 3.4.1(a), $\omega_{I, A}^{*}$ is feasible if Conditions (iii) and (iv) of Proposition 3.4.3 hold. The first- and second-order derivatives of $\pi\left(\omega_{A}(p)\right)$ with respect to $p$ are
$\frac{d \pi\left(\omega_{A}(p)\right)}{d p}=\frac{a^{2}(1+\beta)^{2}-4 G(1+b \beta)}{2(1+b \beta)^{2}}(p-c)+\frac{X-G c}{1+b \beta}$, and
$\frac{d^{2} \pi\left(\omega_{A}(p)\right)}{d p^{2}}=\frac{a^{2}(1+\beta)^{2}-4 G(1+b \beta)}{2(1+b \beta)^{2}}$, respectively, If $a^{2}(1+\beta)^{2}>4 G(1+b \beta)$, then $\pi\left(\omega_{A}(p)\right)$ is convex in $p$, and $\pi\left(\omega^{\prime}(p)\right)$ is maximized at the boundary or is maximized at $p \rightarrow \infty$. Noting that $d \pi\left(\omega_{A}(p)\right) /\left.d p\right|_{p=c}=\frac{X-G c}{1+b \beta}$ and $\omega_{A}(c)=0$, if $a^{2}(1+\beta)^{2}=4 G(1+b \beta)$, then 1) $\pi\left(\omega_{A}(p)\right)$ is strictly increasing in $p$, and hence $\pi\left(\omega_{A}(p)\right)$ is maximized at the boundary or is maximized at $p \rightarrow \infty$, if $\left.A>B c ; 2\right)$ $\pi\left(\omega_{A}(p)\right)$ is strictly decreasing in $p$ and $\pi\left(\omega_{A}(p)\right) \leq 0$ for all $p>c$ if $X<G c$; and 3) $\pi\left(\omega_{A}(p)\right)=0$ for all $p>c$ if $X=G c$. Therefore, $\omega^{*}$ satisfies $D_{L}\left(\omega^{*}\right) \leq 0$ and/or $D_{F}\left(\omega^{*}\right) \leq 0$, if Condition (iii) of Proposition 3.4.3 does not hold. If Condition (iii) of Proposition 3.4.3 holds, then $\pi\left(\omega_{A}(p)\right)$ is concave in $p$. Considering $d \pi\left(\omega_{A}(p)\right) / d p=0$, the unique optimal $p$, for the case $a^{2}(1+\beta)^{2}<4 G(1+b \beta)$, is given by $p_{I, A}^{*}=\frac{2(1+b \beta)(X+G c)-a^{2} c(1+\beta)^{2}}{4(1+b \beta) G-a^{2}(1+\beta)^{2}}>c$, and $e_{I, A}^{*}\left(p_{I, A}^{*}\right)=\frac{a(1+\beta)(X-G c)}{4(1+b \beta) G-a^{2}(1+\beta)^{2}}$. Therefore, $\omega_{I, A}^{*}$ is the unique optimal $\omega$ for the case considered in Proposition 3.4.3. It remains to find the conditions such that $D_{L}\left(\omega_{I, A}^{*}\right)>0$ and $D_{F}\left(\omega_{I, A}^{*}\right)>0$ to complete the proof of Proposition 3.4.3. Putting $\omega_{I, A}^{*}$ into $D_{L}(\omega)$ and $D_{F}(\omega)$, and performing some calculations, we obtain $D_{L}\left(\omega_{I, A}^{*}\right)=E_{I, A, L} /(1+b \beta)$ and $D_{F}\left(\omega_{I, A}^{*}\right)=E_{I, A, F} /(1+b \beta)$. Since $1+b \beta>0$, we have $D_{L}\left(\omega_{I, A}^{*}\right)>0$ and $D_{F}\left(\omega_{I, A}^{*}\right)>0$ if and only if $E_{I, A, L}>0$ and $E_{I, A, F}>0$.

Proof of Proposition 3.4.4 The first- and second- order partial derivatives of $\pi\left(\omega_{I, F}\right) \quad$ with respect to $e \quad$ are $\quad \frac{\partial \pi\left(\omega_{I, F}\right)}{\partial e}=\frac{\alpha(1-b)}{1+b \beta}(p-c)-2 e, \quad$ and $\frac{\partial^{2} \pi\left(\omega_{I, F}\right)}{\partial e^{2}}=-2$, respectively. Therefore, for any fixed $p, \pi\left(\omega_{I, F}\right)$ is concave in $e$.

By considering $\partial \pi\left(\omega_{I, F}\right) / \partial e=0$, the optimal advertising effort for any fixed $p$ is given by $e_{I, B}^{*}(p)=\frac{\alpha(1-b)}{2(1+b \beta)}(p-c)$.

Proof of Proposition 3.4.5 In this proof, at the boundary means that $D_{L}(\omega)=0$ or/and $D_{F}(\omega)=0$ for any considered $\omega$. Next, $a(1+\beta) \leq \alpha(1-b)$ implies $b<1$. Then according to Proposition 3.3.2(b), $\omega_{I, B}^{*}$ and $\bar{\omega}_{I, B}^{*}$ are feasible if Conditions (iii) and (iv) of Proposition 3.4.5 hold. The first and second order derivatives of $\pi\left(\omega_{B}(p)\right)$ with $\quad$ respect $\quad$ to $\quad p \quad$ are $\frac{d \pi\left(\omega_{B}(p)\right)}{d p}=\frac{\alpha^{2}(1-b)^{2}-4 G(1+b \beta)}{2(1+b \beta)^{2}}(p-c)+\frac{X-G c}{1+b \beta}$, and $\frac{d^{2} \pi\left(\omega_{B}(p)\right)}{d p^{2}}=\frac{\alpha^{2}(1-b)^{2}-4 G(1+b \beta)}{2(1+b \beta)^{2}}$, respectively. If $\alpha^{2}(1-b)^{2}>4 G(1+b \beta)$, then $\pi\left(\omega_{B}(p)\right)$ is convex in $p$, and $\pi\left(\omega_{B}(p)\right)$ is maximized at the boundary or $p \rightarrow \infty$. Noting that $d \pi\left(\omega_{B}(p)\right) /\left.d p\right|_{p=c}=\frac{X-G c}{1+b \beta} \quad$ and $\quad \omega_{B}(c)=0 \quad$. If $\alpha^{2}(1-b)^{2}=4 G(1+b \beta)$, then 1) $\pi\left(\omega_{B}(p)\right)$ is strictly increasing in $p$ and $\pi\left(\omega_{B}(p)\right)$ is maximized at the boundary or $p \rightarrow \infty$ if $X>G c$; 2) $\pi\left(\omega_{B}(p)\right)$ is strictly decreasing in $p$ and $\pi\left(\omega_{B}(p)\right) \leq 0$ for all $p>c$ if $X<G c$; and 3) $\pi\left(\omega_{B}(p)\right)=0$ for all $p>c$ if $X=G c$. Therefore, $\omega^{*}$ satisfies $D_{L}\left(\omega^{*}\right) \leq 0$ and/or $D_{F}\left(\omega^{*}\right) \leq 0$, if Condition (iii) of Proposition 3.4.5 does not hold. If Condition (iii) of Proposition 3.4.5 holds, then $\pi\left(\omega_{B}(p)\right)$ is concave in $p$. Considering $d \pi\left(\omega_{B}(p)\right) / d p=0$, the unique optimal $p$, for the case $\alpha^{2}(1-b)^{2}<4 G(1+b \beta)$, is given by $p_{I, B}^{*}=\frac{2(1+b \beta)(X+G c)-\alpha^{2} c(1-b)^{2}}{4(1+b \beta) G-\alpha^{2}(1-b)^{2}}>c$, and $e_{I, A}^{*}\left(p_{I, A}^{*}\right)=\frac{\alpha(1-b)(X-G c)}{4(1+b \beta) G-\alpha^{2}(1-b)^{2}}$. Therefore, $\omega_{I, B}^{*}$ is the unique optimal $\omega$ for $a(1+\beta)<\alpha(1-b)$. Next, putting $\omega_{I, B}^{*}$ into $D_{L}(\omega)$ and $D_{F}(\omega)$, and perform some calculations, we obtain $D_{L}\left(\omega_{I, B}^{*}\right)=E_{I, B, L} /(1+b \beta)$ and $D_{F}\left(\omega_{I, B}^{*}\right)=E_{I, B, F} /(1+b \beta)$. Since $1+b \beta>0$, we have $D_{L}\left(\omega_{I, B}^{*}\right)>0$ and $D_{S}\left(\omega_{I, B}^{*}\right)>0$ if and only if
$E_{I, A, L}>0$ and $E_{I, A, F}>0$. This completes the proof for $a(1+\beta)<\alpha(1-b)$. The proof for $a(1+\beta)<\alpha(1-b)$ and $a(1+\beta)=\alpha(1-b)$ are similar. For $a(1+\beta)<\alpha(1-b)$, we consider $\lambda=0$ and $\omega_{I, B}^{*}$. For $a(1+\beta)=\alpha(1-b)$, we consider any $0 \leq \lambda \leq 1$ and $\bar{\omega}_{I, B}^{*}$. So, we omit the detail of the proof for $a(1+\beta)=\alpha(1-b)$ here. (Q.E.D.)

Proof of Proposition 3.4.6 Part (a): Clearly, $\lambda=0$ is feasible. Because $x_{F}>\gamma$, $e_{I I, 1}^{*}=\frac{\alpha\left(x_{F}-\gamma c\right)}{4 \gamma-\alpha^{2}}>0$ if and only if $4 \gamma>\alpha^{2} . \frac{2 x_{F}+c\left(2 \gamma-\alpha^{2}\right)}{4 \gamma-\alpha^{2}}-c=\frac{2\left(x_{F}-\gamma c\right)}{4 \gamma-\alpha^{2}}$, and hence $\frac{2 x_{F}+c\left(2 \gamma-\alpha^{2}\right)}{4 \gamma-\alpha^{2}}>c$ if and only if $4 \gamma>\alpha^{2}$.

Part (b): Clearly, $\lambda=0$ is feasible. Because $\alpha^{2} b g+(b \gamma-g)^{2}>0, e_{I I, 2}^{*} \geq 0$ if and only if $\alpha\left[2 b \gamma\left(x_{L}-g c\right)-2 b g\left(x_{F}-\gamma c\right)+(b \gamma-g) c \geq 0\right.$, and $p_{I I, 2}^{*}>c$ if and only if $\left[\alpha^{2} b-2(b \gamma-g)\right]\left(x_{L}-g c\right)+2 b(b \gamma-g)\left(x_{F}-\gamma c\right)>0$.

Part (c): Clearly $\lambda=0$ and $e=0$ are feasible. Next, $p_{I I, 3}^{*}-c=\frac{g c-x_{L}+b\left(x_{F}-\gamma c\right)}{b \gamma-g}$. Therefore, $p_{I I, 3}^{*}>c$ if and only if condition (c.1) or (c.2) of Proposition 3.4.6 holds.

Proof of Proposition 3.4.7 In this proof, at the boundary means that $\bar{D}_{L}(\omega)=0$ or/and $D_{F}(\omega)=0$ for any considered $\omega$. First of all, $D_{F}\left(\omega_{I, 1}^{*}\right)=\frac{2 \gamma\left(x_{F}-\gamma c\right)}{4 \gamma-\alpha^{2}}>0$ and $\bar{D}_{L}\left(\omega_{I, 1}^{*}\right)<0$ if $\alpha^{2}<4 \gamma$ and $E_{I I}>0$. According to Proposition 3.4.6(a), $\omega_{I I, 1}^{*}$ is feasible if and only if $\alpha^{2}<4 \gamma$. The first- and second-order derivatives of $\pi\left(\omega_{I I, A}(p)\right) \quad$ with respect to $\quad p$ are $d \pi\left(\omega_{I I, A}(p)\right) / d p=\left(\alpha^{2}-4 \gamma\right)(p-c) / 2$ $+\left(x_{F}-\gamma c\right)$ and $d^{2} \pi\left(\omega_{I, A}(p)\right) / d p^{2}=\left(\alpha^{2}-4 \gamma\right) / 2 \quad$, respectively. $d \pi\left(\omega_{I I, A}(p)\right) /\left.d p\right|_{p=c}=x_{F}-\gamma c>0, \pi\left(\omega_{I I, A}(p)\right)$ is convex in $p$, and $\pi\left(\omega_{I I, A}(p)\right)$ is maximized at the boundary or $p \rightarrow \infty$ if $\alpha^{2} \geq \gamma$. Therefore, $\omega^{*}$ satisfies $\bar{D}_{L}\left(\omega^{*}\right) \leq 0$ and/or $D_{F}\left(\omega^{*}\right) \leq 0$, if $\alpha^{2} \geq 4 \gamma$. If $\alpha^{2}<4 \gamma$, then $\pi(\hat{\omega}(p))$ is
uniquely maximized at $p=\frac{2\left(x_{F}-\gamma c\right)}{4 \gamma-\alpha^{2}}+c>c$, i.e. $\hat{\omega}\left(p^{*}\right)=\omega_{I, 1}^{*}$.

Proof of Proposition 3.4.8 Because $\lambda^{*}=0$ for Tactic II, we only consider $\lambda=0$ in this proof. Let $\omega_{I I, B}=\left(0, e_{I I, B}, p_{I I, B}\right)$ be any feasible $\omega$ that satisfies $\bar{D}_{L}\left(\omega_{I I, B}\right)=0$ and $D_{F}\left(\omega_{I I, B}\right)>0$. Then $\pi\left(\omega_{I I, B}\right)=(p-c)\left(x_{F}+\alpha e_{I I, B}-\gamma p_{I I, B}\right)-e_{I I, B}^{2}$. and $x_{L}=b x_{F}+b \alpha e_{I I, B}+(b \gamma-g) p_{I I, B}$.

By taking the derivative on the both sides of (A.1), we obtain
$d e_{I I, B} / d p_{I I, B}=(b \gamma-g) / \alpha b$. The first- and second- derivatives of $\pi\left(\omega_{I I, B}\right)$ with respect to $p_{I I, B}$ are

$$
\begin{gathered}
\frac{d \pi\left(\omega_{I I, B}\right)}{d p_{I, B}}=\frac{1}{\alpha^{2} b^{2}}\left\{\left[\alpha^{2} b-2(b \gamma-g)\right] x_{L}+2 b(b \gamma-g) x_{F}\right. \\
\left.+\alpha^{2} b g c-2\left[\alpha^{2} b g+(b \gamma-g)^{2}\right] p_{I I, B}\right\},
\end{gathered}
$$

and $\frac{d 2 \pi\left(\omega_{I I, B}\right)}{d p_{I I, B}{ }^{2}}=\frac{-2}{\alpha^{2} b^{2}}\left[\alpha^{2} b g+(b \gamma-g)^{2}\right]<0$, respectively. Therefore, $\pi\left(\omega_{I, B}\right)$ is concave in $p_{I I, B}$, and $\pi\left(\omega_{I, B}\right)$ is maximized at $p_{I I, B}=p_{I I, 2}^{*}$. By putting $p_{I I, B}=p_{I I, 2}^{*}$ into (A.1), we obtain $e_{I I, B}=e_{I I, 2}^{*}$. Hence, $\pi\left(\omega_{I I, B}\right)$ is maximized at $\omega_{I I, B}=\omega_{I I, 2}^{*}$, if $\omega_{I I, 2}^{*}$ is feasible and satisfies $D_{F}\left(\omega_{I, 2}^{*}\right)>0\left(\omega_{I, B}\right.$ is bounded by $D_{F}(\omega)>0, p_{I I, B}>c$ and $\left.e_{I I, B} \geq 0\right)$. If $\omega_{I I, 2}^{*}$ is infeasible and/or $D_{F}\left(\omega_{I, 2}^{*}\right)>0$ does not hold, then $\pi\left(\omega_{I I, B}\right)$ is maximized at one of the boundary $D_{F}\left(\omega_{I I, B}\right)=0$, $p_{I I, B}=c$, and $e_{I I, B}=0$. Because $\pi\left(\omega_{I I, B}\right) \leq 0$ if $D_{F}\left(\omega_{I I, B}\right)=0$ and/or $p_{I I, B}=c$, the two boundaries $D_{F}\left(\omega_{I I, B}\right)=0$ and $p_{I I, B}=c$ can be ignored. This remains the boundary $e_{I I, B}=0$. Putting $e_{I I, B}=0$, we obtain $p_{I I, B}=p_{I I, B}^{*}$. Therefore, $\omega_{I I, B}=\omega_{I, 3}^{*}$ is the boundary that maximizes $\pi\left(\omega_{I, B}\right)$. Similarly, we require $\omega_{I, 3}^{*}$ is feasible and satisfies $D_{F}\left(\omega_{I, 3}^{*}\right)>0$.

Proof of Proposition 3.4.9 Part (a): Clearly, $\lambda=1$ is feasible. Because $x_{L}>g c$,
$e_{I I I, 1}^{*}=\frac{a\left(x_{L}-g c\right)}{4 g-a^{2}}>c$ if and only if $4 g>a^{2}$. Lastly, $p_{I I I, 1}^{*}-c=\frac{2\left(x_{L}-g c\right)}{4 g-a^{2}}>0$ if and only if $4 g>a^{2}$.

Part (b): Clearly, $\lambda=1$ and $e=0$ are feasible. Moreover,
$p_{I I t, 2}^{*}-c=\frac{x_{F}-\gamma c+\beta\left(x_{L}-g c\right)}{g \beta+\gamma}>0$. Therefore, $\omega_{I I I, 2}^{*}$ is feasible.

Proof of Proposition 3.4.10 In this proof, at the boundary means that $\bar{D}_{F}(\omega)=0$ or/and $D_{L}(\omega)=0$ for any considered $\omega$. First of all, $D_{L}\left(\omega_{I I, 1}^{*}\right)=\frac{2 g\left(x_{L}-g c\right)}{4 g-a^{2}}>0$ and $\bar{D}_{F}\left(\omega_{I I I, 1}^{*}\right)<0$ if $a^{2}<4 g$ and $E_{I I I, A}>0$. According to Proposition 3.3.10, $\omega_{I I, 1}^{*}$ if and only if $a^{2}<4 g$. The first- and second-order derivatives of $\pi\left(\omega_{I I, A}(p)\right)$ with respect to $p$ are $d \pi\left(\omega_{I I I, A}(p)\right) / d p=\frac{a^{2}-4 g}{2}(p-c)+x_{L}-g c$, and $d^{2} \pi\left(\omega_{I I, A}(p)\right) / d p^{2}=\frac{a^{2}-4 g}{2}$, respectively. $\quad d \pi\left(\omega_{I I I, A}(p)\right) /\left.d p\right|_{p=c}=x_{L}-g c>0$, $\pi\left(\omega_{I I, A}(p)\right)$ is convex in $p$, and $\pi\left(\omega_{I I, A}(p)\right)$ is maximized at the boundary or $p \rightarrow \infty$ if $a^{2} \geq g$. Therefore, $\omega^{*}$ satisfies $\bar{D}_{F}\left(\omega^{*}\right) \leq 0$ and/or $D_{L}\left(\omega^{*}\right) \leq 0$, if $a^{2} \geq 4 g$. If $a^{2}<4 g$, then $\pi\left(\omega_{I I I, A}(p)\right)$ is uniquely maximized at $p=\frac{2\left(x_{L}-g c\right)}{4 g-a^{2}}+c>c$, i.e. $\omega_{I I I, A}\left(p^{*}\right)=\omega_{I I, 1}^{*}$.

Proof of Proposition 3.4.11 Because $\lambda^{*}=1$ for Tactic II, we only consider $\lambda=1$ in this proof. Let $\omega_{I I I, B}=\left(1, e_{I I, B}, p_{I I I, B}\right)$ be any feasible $\omega$ that satisfies $\bar{D}_{F}\left(\omega_{I I, B}\right)=0$ and $D_{L}\left(\omega_{I I, B}\right)>0$. Then
$\pi\left(\omega_{I I, B}\right)=(p-c)\left(x_{L}+a e_{I I I, B}-g p_{I I, B}\right)-e_{I I I, B}^{2}$. and
$0=\beta x_{L}+x_{F}+a \beta e_{I I I, B}-(g \beta+\gamma) p_{I I, B}$.
By taking the derivative on the both sides of (A.2), we obtain
$d e_{I I I, B} / d p_{I I, B}=(g \beta+\gamma) / a \beta$
The first- and second- derivatives of $\pi\left(\omega_{I I I, B}\right)$ with respect to $p_{I I, B}$ are

$$
\frac{d \pi\left(\omega_{I I, B}\right)}{d p_{I I, B}}=\frac{-1}{\beta}\left[2 g \beta p_{I I I, B}+x_{L}+\gamma c\right]-\frac{2(g \beta+\gamma)}{\alpha^{2} \beta^{2}}\left[(g \beta+\gamma) p_{I I, B}-x_{F}-\beta x_{L}\right]
$$

and $\frac{d 2 \pi\left(\omega_{I I, B}\right)}{d p_{I I I, B}{ }^{2}}=\frac{-2}{\alpha^{2} \beta^{2}}\left[\alpha^{2} \beta^{2} g+(g \beta+\gamma)^{2}\right]<0$, respectively. Therefore, $\pi\left(\omega_{I I I, B}\right)$ is concave in $p_{I I I, B}$, and $\pi\left(\omega_{I I, B}\right)$ is maximized at

$$
p_{I I I, B}=\frac{\left[2(b+g \beta)-\alpha^{2} \beta\right] x_{F}+2 \beta(b+g \beta) x_{L}-\alpha^{2} \beta \gamma c}{2\left[\alpha^{2} \beta^{2} g+(b+g \beta)^{2}\right]} .
$$

However, by putting $p_{I I I, B}$ into (A.4), we obtain
$e_{I I, B}=\frac{-\alpha\left[2 x_{F}(1+g \beta)+2 \beta^{2} g x_{L}+2 \gamma c(\gamma+\beta g)\right]}{2\left[\alpha^{2} \beta^{2} g+(\gamma+\beta g)^{2}\right]}<0$.
Therefore, $\pi\left(\omega_{I I I, B}\right)$ is maximized at one of the boundary $D_{L}\left(\omega_{I I, B}\right)=0, p_{I I I, B}=c$, and $e_{I I I, B}=0$. Because $\pi\left(\omega_{I I, B}\right) \leq 0$ if $D_{L}\left(\omega_{I I I, B}\right)=0$ and/or $p_{I I I, B}=c$, the two boundaries $D_{L}\left(\omega_{I I, B}\right)=0$ and $p_{I I, B}=c$ can be ignored. This remains the boundary $e_{I I I, B}=0$. Putting $e_{I I I, B}=0$, we obtain $p_{I I I, B}=p_{I I I, 3}^{*}$. Therefore, $\omega_{I I I, B}=\omega_{I I I, 3}^{*}$ is the boundary that maximizes $\pi\left(\omega_{I I, B}\right)$. Finally, we require $\omega_{I I I, 3}^{*}$ is feasible and satisfies $D_{L}\left(\omega_{I I, 3}^{*}\right)>0$.

Proof of Proposition 3.4.12 As we mentioned before, $\lambda^{*}=0$ for Tactic II. Let $\widetilde{\omega}_{I I}$ be a feasible $\omega$ with $\lambda=0$ and satisfies $D_{F}\left(\widetilde{\omega}_{I I}\right)>0$ and $\bar{D}_{L}\left(\widetilde{\omega}_{I I}\right)<0$. Then $\pi\left(\widetilde{\omega}_{I I}\right)=(p-c)\left(x_{F}+\alpha e-\not p\right)-e^{2}$. Taking the first- and second-order partial derivative of $\pi\left(\widetilde{\omega}_{I I}\right)$ with respect to $e$, we obtain $\partial \pi\left(\widetilde{\omega}_{I I}\right) / \partial e=\alpha(p-c)-2 e$ and $\partial^{2} \pi\left(\widetilde{\omega}_{I I}\right) / \partial e^{2}=-2$. Therefore, for any given $p>c, \pi\left(\widetilde{\omega}_{I I}\right)$ is strictly concave in $e$ and maximized at $e=\alpha(p-c) / 2$.

Proof of Proposition 3.4.13 As we mentioned before, $\lambda^{*}=1$ for Tactic III. Let $\widetilde{a}_{I I I}$ be a feasible $\omega$ with $\lambda=1$ and satisfies $D_{L}\left(\widetilde{\omega}_{I I}\right)>0$ and $\bar{D}_{F}\left(\widetilde{\omega}_{I I}\right)<0$. Then $\pi\left(\widetilde{\omega}_{I I}\right)=(p-c)\left(x_{L}+a e-g p\right)-e^{2}$. Taking the first- and second-order partial derivative of $\pi\left(\widetilde{\omega}_{I I}\right)$ with respect to $e$, we obtain $\partial \pi\left(\widetilde{\omega}_{I I}\right) / \partial e=a(p-c)-2 e$ and
$\partial^{2} \pi\left(\widetilde{\omega}_{I I I}\right) / \partial e^{2}=-2$. Therefore, for any given $p>c, \pi\left(\widetilde{\omega}_{I I I}\right)$ is strictly concave in $e$ and maximized at $e=a(p-c) / 2$.

Proof of Proposition 3.4.14 to Proposition 3.4.19: For Proposition 3.4.14 to Proposition 3.4.19, the results are directly obtained by following discussions which are reported before the respective propositions in the main content.
(Q.E.D.)

## Appendix 2: Proofs in Chapter 4

Proof of Proposition 4.2.1: First of all, $e \geq T+\tau$ holds under Tactic I.1. For $e=T+\tau, \lambda=T /(T+\tau)$ is the only feasible solution for Tactic I.1. Therefore, $\lambda^{*}=T /(T+\tau)$ for $e=T+\tau$ under Tactic I.1. For $e>T+\tau$, by taking the first order partial derivative of (2) with respect to $\lambda$, we have $\partial \pi_{L L}(\omega) / \partial \lambda=e N_{I}(p-c) /(1+b \beta)$. Therefore, for any given $p>c$ and $e>T+\tau$, if $N_{I} \geq 0$, then $\pi_{L L}(\omega)$ is increasing in $\lambda$. Because $\lambda e \geq T$ and $(1-\lambda) e \geq \tau$ hold under Tactic I.1, we have $\left(1-\lambda^{*}\right) e=\tau$ for $N_{I} \geq 0$. On the other hand, if $N_{I} \leq 0$, then $\pi_{L L}(\omega)$ is decreasing in $\lambda$. Hence, $\lambda^{*} e=T$ for $N_{I} \leq 0$. Noting that $\lambda^{*}$ can take any value which satisfies $\left(1-\lambda^{*}\right) e \geq \tau$ and $\lambda^{*} e \geq T$ if $N_{I}=0$. However, for simplicity, we only demonstrate here the two special cases: $\lambda^{*}$ satisfies $\left(1-\lambda^{*}\right) e=\tau$, and $\lambda^{*}$ satisfies $\lambda^{*} e=T$, for $N_{I}=0$. Similar results can also be obtained if $\lambda^{*}$ takes any value that satisfy $\left(1-\lambda^{*}\right) e \geq \tau$ and $\lambda^{*} e \geq T$.

Proof of Proposition 4.2.2: As the feasible set of $\omega$ for Tactic I.1.a is an open set, the local optimum for Tactic I.1.a is an interior point, if it exists. Clearly, conditions $D_{L}\left(\omega_{I .1 . a}^{*}\right)>0$ and $D_{F}\left(\omega_{I .1 . a}^{*}\right)>0$ are basic for Tactic I.1. By taking the first and second order partial derivatives of (3) with respect to $p$, we have $\frac{\partial \pi_{L L}(\omega)}{\partial p}=\frac{-2 G(p-c)+[B+a(1+\beta) T+\alpha(1-b) \tau]}{1+b \beta}$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2 \tilde{G}$. If $G \leq 0$, then $\pi_{L L}(\omega)$ is convex in $p$, and hence the local optimal solution is infinite or $\pi_{L L}(\omega)=-h(T+\tau)^{2}$ which is always negative and we ignore this case. Therefore, $G>0$ is the necessary condition for having a finite local optimum for Tactic I.1.a. If $G>0$, then $\pi_{L L}(\omega)$ is strictly concave in $p$, and then by the first optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I, 1, a}^{*}$. By considering $p_{I, 1, a}^{*}>c$, we obtain $B+a(1+\beta) T+\alpha(1-b) \tau>0$. Finally, by putting $\omega_{I, 1 . a}^{*}$ into (3), we obtain (5). (Q.E.D.)

Proof of Proposition 4.2.3: By taking the first order and second order partial derivatives of (6) with respect to $e$, we obtain
$\partial \pi_{L L}(\omega) / \partial e=a(p-c)(1+\beta) /(1+b \beta)-2 h e$, and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$, for Tactic
I.1.b. Therefore, for Tactic I.1.b, $\partial \pi_{L L}(\omega)$ is a concave function of $e$. Then by considering $\partial \pi_{L L}(\omega) / \partial e=0$, we obtain (7). As $a(1+\beta) /[2 h(1+b \beta)]>0, e_{I, 1 . b}^{*}(p)$ is strictly increasing in $p$.

Proof of Proposition 4.2.4: As the feasible set of $\omega$ for Tactic I.1.b is an open set, the local optimum for Tactic I.1.b is an interior point solution, if it exists. Condition $N_{I} \geq 0, D_{L}\left(\omega_{I . i . b}^{*}\right)>0$ and $D_{F}\left(\omega_{I . i . b}^{*}\right)>0$ are the basic conditions for Tactic I.1.b. By putting (7) into (6), we obtain $\pi_{L L}(\omega)=\frac{-Y(p-c)^{2}}{4 h(1+b \beta)^{2}}+\frac{\left(B-\tau N_{t}\right)(p-c)}{1+b \beta}$, and, for Tactic I.1.b, we have $\frac{\partial \pi_{L L}(\omega)}{\partial p}=\frac{-Y(p-c)}{2 h(1+b \beta)^{2}}+\frac{\left(B-\tau N_{I}\right)}{1+b \beta} \quad$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=\frac{-Y}{2 h(1+b \beta)^{2}}$. If $Y \leq 0$, then $\pi_{L L}(\omega)$ is convex in $p$, and hence the local optimal solution for Tactic I.1.b is infinite, or $\pi_{L L}(\omega)$ is always negative and we ignore this case. Therefore, $Y>0$ is the necessary condition for having a finite local optimum of Tactic I.1.b. For $Y>0, \pi_{L L}(\omega)$ is strictly concave in $p$, and then by the first-order optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I .1 . b}^{*}$. By considering $p_{I .1 . b}^{*}>c$, we obtain $B>N_{I} \tau$. Then by putting $p_{I .1 . b}^{*}$ into (7), we obtain $e_{I, 1, b}^{*}$. By considering $e_{I, 1 . b}^{*}>T+\tau$ and by $Y>0$, we obtain item (iii) of Proposition 4.2.4. Notice that item (iii) of Proposition 4.2 .4 covers the case for $B>N_{I} \tau$. Lastly, by putting $\omega_{I, 1 . b}^{*}$ into (6), we obtain (9).

Proof of Proposition 4.2.5: By taking the first order and second order partial derivatives of (10) with respect to $e$, we obtain $\partial \pi_{L L}(\omega) / \partial e=\alpha(p-c)(1-b) /(1+b \beta)-2 h e$, and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, for Tactic I.1.c, $\pi_{L L}(\omega)$ is a concave function of $e$, and hence the optimal advertising
efforts in the function of retail price $p$ is given by (11). As $\alpha(1-b) /[2 h(1+b \beta)]>0$, $e_{I .1 . c}^{*}(p)$ is strictly increasing in $p$.

Proof of Proposition 4.2.6: As the feasible set of $\omega$ for Tactic I.1.c is an open set, the local optimum for Tactic I.1.c is an interior point solution, if it exists. Condition $N_{I} \leq 0, D_{L}\left(\omega_{\text {I...c. }}^{*}\right)>0$ and $D_{F}\left(\omega_{\text {I...c }}^{*}\right)>0$ are basic conditions for Tactic I.1.c. By putting (11) into (10), we obtain
$\pi_{L L}(\omega)=\frac{-Z(p-c)^{2}}{4 h(1+b \beta)^{2}}+\frac{\left(B+T N_{I}\right)(p-c)}{1+b \beta}$, and we have
$\frac{\partial \pi_{L L}(\omega)}{\partial p}=\frac{-Z(p-c)}{2 h(1+b \beta)^{2}}+\frac{\left(B+T N_{I}\right)}{1+b \beta}$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=\frac{-Z}{2 h(1+b \beta)^{2}}$.
If $Z \leq 0$, then $\pi_{L L}(\omega)$ is convex in $p$, and hence the local optimum is infinite, or $\pi_{L L}(\omega)$ is always non-positive and we ignore this case. Therefore, $Z>0$ is the necessary condition for having a finite local optimum of Tactic I.1.c. For $Z>0$, $\pi_{L L}(\omega)$ is strictly concave in $p$, and then by the first optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I .1 . c}^{*}$. By considering $p_{I .1 . c}^{*}>c$, as $Z>0$, we have $B>-N_{I} T$. Then by putting $p_{I .1 . c}^{*}$ into (11), we obtain $e_{I, 1 . c}^{*}$. By considering $e_{I .1 . c}^{*}>T+\tau$, we obtain item (iii) of Proposition 4.2.6, and the condition in item (iii) covers $B>-N_{I} T$. Finally, by putting $\omega_{I .1 . c}^{*}$ into (10), we obtain (13). (Q.E.D.)

Proof of Proposition 4.2.7: First of all, $e \geq T$ for tactic I.2. For $e=T, \lambda=1$ is the only feasible solution for Tactic I.2. Therefore, the optimal $\lambda^{*}=1$ for Tactic I. 2 with $e=T$. For $e>T$, by taking the first order partial derivative of (14) with respect to $\lambda$, we have $\partial \pi_{L L}(\omega) / \partial \lambda=N_{I} e(p-c) /(1+b \beta)-\mu e$ for Tactic I.2. Therefore, for any given $p>c$ and $e>T$, if $N_{I}(p-c)>\mu(1+b \beta)$, then $\pi_{L L}(\omega)$ is increasing in $\lambda$. As $\lambda=1$ satisfies $\lambda e \geq T$ and $(1-\lambda) e<\tau, \lambda^{*}=1$ for $N_{I}(p-c)>\mu(1+b \beta)$. If $N_{I}(p-c)<\mu(1+b \beta)$, then $\pi_{L L}(\omega)$ is decreasing in $\lambda$. Therefore, for $T<e<T+\tau$ and $N_{I}(p-c)<\mu(1+b \beta)$, the optimal $\lambda^{*}$ for Tactic I. 2 satisfies $\lambda^{*} e=T$. Next, consider $\quad \omega^{\prime}=\left(e, \lambda^{\prime}, p\right)$ and $\omega^{\prime \prime}=\left(e, \lambda^{\prime \prime}, p\right)$, where $e \geq T+\tau, \quad p>\mathrm{c}$, $N_{I}(p-c)<\mu(1+b \beta), \quad 0 \leq \lambda^{\prime \prime}<\lambda^{\prime} \leq 1, \quad D_{L}\left(\omega^{\prime}\right)>0, \quad D_{F}\left(\omega^{\prime}\right)>0, \quad D_{L}\left(\omega^{\prime \prime}\right)>0$,
$D_{F}\left(\omega^{\prime \prime}\right)>0, \quad \lambda^{\prime} e \geq T, \quad \lambda^{\prime \prime} e \geq T, \quad\left(1-\lambda^{\prime}\right) e<\tau,\left(1-\lambda^{\prime \prime}\right)=\tau$. We have $\pi_{L L}\left(\omega^{\prime}\right)<\pi_{L L}\left(\omega^{\prime \prime}\right)$. Therefore, Tactic I. 1 dominates Tactic I. 2 for $e \geq T+\tau$ and $N_{I}(p-c)<\mu(1+b \beta)$. If $N_{I}(p-c)=\mu(1+b \beta)$, then $\pi_{L L}(\omega)$ keeps constant for varied $\lambda$. Therefore any $\lambda$ that satisfy $(1-\lambda) e<\tau$ and $\lambda e \geq T$ is optimal solution of $\lambda$. As there exist multiple $\lambda$ which satisfy $(1-\lambda) e<\tau$ and $\lambda e \geq T$, there exist multiple $\lambda^{*}$ if $N_{I}(p-c)=\mu(1+b \beta)$.

Proof of Proposition 4.2.8: Similarly, the local optimum for Tactic I.2.a is an interior point solution if it exists. Conditions $D_{L}\left(\omega_{I, 2, a}^{*}\right)>0$ and $D_{F}\left(\omega_{I, 2, a}^{*}\right)>0$ are the basic conditions for Tactic I.2, so it is necessary for Tactic I.2.a too. By taking the first and second order partial derivatives of (15) with respect to $p$, we have $\frac{\partial \pi_{L L}(\omega)}{\partial p}=\frac{-2 G(p-c)+[B+a(1+\beta) T]}{1+b \beta}$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2 \tilde{G}$. If $G \leq 0$, then $\pi_{L L}(\omega)$ is convex in $p$ or strictly increasing in $p$, and hence the local optimum for Tactic I.2.a is infinite, i.e., $p \rightarrow \infty$. Therefore, $G>0$ is the necessary condition for Tactic I.2.a such that the local optimum is finite. For $G>0, \pi_{L L}(\omega)$ is strictly concave in $p$, and then by the first order optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I 2 . a}^{*}$. By considering $p_{I .2, a}^{*}>c$, we obtain item (iii) of Proposition 4.2.8. Lastly, by putting $\omega_{I .2 . a}^{*}$ into (15), we obtain (17).

Proof of Proposition 4.2.9: By taking the first order and second order partial derivatives of (18) with respect to $e$, we obtain
$\partial \pi_{L L}(\omega) / \partial e=\frac{a(p-c)(1+\beta)}{1+b \beta}-2 h e$, and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$.
Therefore, $\partial \pi_{L L}(\omega)$ is a concave function of $e$ for Tactic I.2.b. Hence the optimal advertising efforts in the function of retail price $p$ are given by (19). $e_{\text {I.2.b }}^{*}(p)$ is strictly increasing in $p$ as $e_{I .2 . b}^{*}(p)$ is strictly increasing in $p$.

Proof of Proposition 4.2.10: Similarly, the local optimum for Tactic I.2.b is an interior point, if it exists. Clearly, conditions $N_{I}>0, \quad D_{L}\left(\omega_{I .2 . b}^{*}\right)>0$ and
$D_{F}\left(\omega_{I .2 . b}^{*}\right)>0$ are necessary for Tactic I.2.b. By putting (19) into (18), we obtain $\pi_{L L}(\omega)=\frac{-Y(p-c)^{2}}{4 h(1+b \beta)^{2}}+\frac{B(p-c)}{1+b \beta}-\mu \tau$, and we have $\frac{\partial \pi_{L L}(\omega)}{\partial p}=\frac{-Y(p-c)}{2 h(1+b \beta)^{2}}+\frac{B}{1+b \beta}$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=\frac{-Y}{2 h(1+b \beta)^{2}}$.

If $Y \leq 0$, then $\pi_{L L}(\omega)$ is convex in $p$, or is strictly increasing in $p$, and hence the local optimum of Tactic I.2.b is infinite Therefore, $Y>0$ is the necessary for Tactic I.2.b. For $Y>0, \pi_{L L}(\omega)$ is strictly concave in $p$, and then by the first optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I .2 . b}^{*}$. As $p>c$, we obtain $B>0$. By putting $p_{I .2 . b}^{*}$ into (19), we obtain $e_{I .2 . b}^{*}$. As $e>T$ for Tactic I.2.b, we need $B>Y T /[a(1+\beta)]>0$, which covers the condition for $p>c$, By considering $N_{I}\left(p_{I .2 . b}^{*}-c\right)>\mu(1+b \beta)$, we obtain item (iv) of Proposition 10. Lastly, by putting $\omega_{I, 2 . b}^{*}$ into (18), we obtain (21).
(Q.E.D.)

Proof of Proposition 4.2.11: Similar to Proposition 4.2.9.

Proof of Proposition 4.2.12: Similarly, the local optimum for Tactic I.1.c is an interior point, if it exists. Condition $N_{I}>0, D_{L}\left(\omega_{I .2 . c}^{*}\right)>0$ and $D_{F}\left(\omega_{I .2 . c}^{*}\right)>0$ are necessary for Tactic I.2.c. As $N_{I}(p-c)=\mu(1+b \beta)$ for Tactic I.2.c, we obtain $p_{I .2 . c}^{*}=c+\mu(1+b \beta) / N_{I}>c$. According to Proposition 4.2.11, we obtain $e_{I .2 . c}^{*}=e_{I 2 . c}^{*}\left(p_{I, 2 . c}^{*}\right)=a \mu(1+\beta) /\left(2 h N_{I}\right)$. By putting $\omega_{I, 2 . c}^{*}$ to $\pi_{L L}(\omega)$, we obtain (24). Finally, by considering $e_{I .2 . c}^{*}>T$, we obtain item (ii) of Proposition 4.2.12. (Q.E.D.)

Proof of Proposition 4.2.13: By taking the first order and second order partial derivatives of (25) with respect to $e$, we obtain $\partial \pi_{L L}(\omega) / \partial e=\frac{\alpha(p-c)(1-b)}{1+b \beta}-2 h e+\mu$, and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, $\partial \pi_{L L}(\omega)$ is a concave function of $e$ for Tactic I.2.d and for any given $p>c$, and the optimal advertising efforts in the function of retail price $p$ is given by (22).

Proof of Proposition 4.2.14: Similarly, the local optimum for Tactic I.1.d is an interior point, if it exists. $D_{L}\left(\omega_{I .2 . d}^{*}\right)>0$ and $D_{F}\left(\omega_{I .2 . d}^{*}\right)>0$ are basic conditions of Tactic I.2. By putting (26) into (25), we obtain $\pi_{L L}(\omega)=\frac{-Z(p-c)^{2}}{4 h(1+b \beta)^{2}}+\frac{\left[2 h\left(B+T N_{I}\right)+\alpha \mu(1-b)\right](p-c)}{2 h(1+b \beta)}-h \mu^{2}-\mu[T+\tau-\mu]$, and we have $\frac{\partial \pi_{L L}(\omega)}{\partial p}=\frac{-Z(p-c)}{2 h(1+b \beta)^{2}}+\frac{2 h\left(B+T N_{I}\right)+\alpha \mu(1-b)}{2 h(1+b \beta)}$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=\frac{-Z}{2 h(1+b \beta)^{2}}$.

If $Z \leq 0$, then $\pi_{L L}(\omega)$ is convex in $p$, and hence the local optimum is infinite, or $\pi_{L L}(\omega)$ is always non-positive (and we ignore this case). Therefore, $Z>0$ is the necessary condition for the local optimum of Tactic I.2.d being finite. If $Z>0$, then $\pi_{L L}(\omega)$ is strictly concave in $p$, and then by the first optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I 2 . d}^{*}$. By considering $T<e_{I 2 . d}^{*}<T+\tau$, we obtain item (iii) of Proposition 4.2.14. By considering $p_{I 2 . d}^{*}>c$, we obtain item (iv) of Proposition 4.2.14. Then by considering $N_{I}\left(p_{I .2 . d}^{*}-c\right)<\mu(1+b \beta)$, we obtain item (v) of Proposition 4.2.14. Then by putting $p_{I .2 . d}^{*}$ into (26), we obtain $e_{I .2 . d}^{*}$. Finally, by putting $\omega_{I .2 . d}^{*}$ into (25), we obtain (28).

Proof of Proposition 4.2.15: By considering $d e_{I .2 . d}^{*} / d T=\alpha(1-b) N_{I} / Z$ and $d p_{I .2 . d}^{*} / d \tau=2 h N_{I}(1+b \beta) / Z$. We obtain the results of Proposition 4.2.15. (Q.E.D.)

Proof of Proposition 4.2.16: First of all, $e \geq \tau$ for tactic I.3. For $e=\tau, \lambda=0$ is the only feasible solution for Tactic I.3. Therefore, the optimal $\lambda^{*}=0$ for Tactic I. 3 when $e=\tau$. For $e>\tau$, by taking the first order partial derivative of (29) with respect to $\lambda$, we have $\partial \pi_{L L}(\omega) / \partial \lambda=N_{I} e(p-c) /(1+b \beta)+m e$.

Therefore, for any given $p>c$ and $e>\tau$, if $N_{I}(p-c)>-m(1+b \beta)$, then $\pi_{L L}(\omega)$ is strictly increasing in $\lambda$. For $\tau<e<T+\tau, \lambda=1-\tau / e$ is the biggest that satisfies $\lambda e<T$ and $(1-\lambda) e \geq \tau$. Therefore, $\lambda^{*}=1-\tau / e$ for Tactic I. 3 if $N_{I}(p-c)>-m(1+b \beta)$ and $\tau<e<T+\tau$. Next, consider $\omega^{\prime}=\left(e, \lambda^{\prime}, p\right)$ and $\omega^{\prime \prime}=\left(e, \lambda^{\prime \prime}, p\right)$, where $e \geq T+\tau, p>\mathrm{c}, N_{I}(p-c)>-m(1+b \beta), \quad 0 \leq \lambda^{\prime}<\lambda^{\prime \prime} \leq 1$, $D_{L}\left(\omega^{\prime}\right)>0, \quad D_{F}\left(\omega^{\prime}\right)>0, \quad D_{L}\left(\omega^{\prime \prime}\right)>0, \quad D_{F}\left(\omega^{\prime \prime}\right)>0, \quad \lambda^{\prime} e<T, \quad \lambda^{\prime \prime} e=T$,
$\left(1-\lambda^{\prime}\right) e \geq \tau, \quad\left(1-\lambda^{\prime \prime}\right) \geq \tau$. We have $\pi_{L L}\left(\omega^{\prime}\right)<\pi_{L L}\left(\omega^{\prime \prime}\right)$. Therefore, Tactic I. 1 dominates Tactic I. 3 for $e \geq T+\tau$ and $N_{I}(p-c)>-m(1+b \beta)$. If $N_{I}(p-c)<-m(1+b \beta)$, then $\pi_{L L}(\omega)$ is strictly decreasing in $\lambda$. As $\lambda=0$ satisfies $\lambda e<T$ and $(1-\lambda) e \geq \tau, \quad \lambda^{*}=0$ for Tactic I. 3 if $N_{I}(p-c)<-m(1+b \beta)$. If $N_{I}(p-c)=-m(1+b \beta)$, then $\pi_{L L}(\omega)$ is independent of $\lambda$ for Tactic I.3. As there are multiple $\lambda$ which satisfy $\left(1-\lambda^{*}\right) e<\tau$ and $\lambda^{*} e \geq T$, there exist multiple $\lambda^{*}$ if $N_{I}(p-c)=-m(1+b \beta)$.

Proof of Proposition 4.2.17: Similarly, the local optimum for Tactic I.3.a is an interior point, if it exists. $D_{L}\left(\omega_{I .3 . a}^{*}\right)>0$ and $D_{F}\left(\omega_{I .3, a}^{*}\right)>0$ are necessary for any sub-tactics of Tactic I.3. By taking the first and second order partial derivatives of (30) with respect to $p$, we have $\frac{\partial \pi_{L L}(\omega)}{\partial p}=\frac{-2 G(p-c)+[B+\alpha(1-b) \tau]}{1+b \beta}$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2 \tilde{G}$. If $G \leq 0$, then $\pi_{L L}(\omega)$ is convex in $p$, and hence the local optimum for Tactic I.3.c is infinite, or $\pi_{L L}(\omega)$ is always non-positive (and we ignore this case). Therefore, $G>0$ is necessary for having a finite $\omega_{I, 3 . a}^{*}$. If $G>0$, then $\pi_{L L}(\omega)$ is strictly concave in $p$. By the first optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I .3, a}^{*}$. As $p>c$, we obtain item (iii) of Proposition 17. Finally, by putting $\omega_{I, 3, a}^{*}$ into (30), we obtain (31).
(Q.E.D.)

Proof of Proposition 4.2.18: By taking the first order and second order partial derivatives of (33) with respect to $e$, we obtain $\partial \pi_{L L}(\omega) / \partial e=\frac{a(p-c)(1+\beta)}{1+b \beta}-2 h e+m$, and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, $\partial \pi_{L L}(\omega)$ is a concave function of $e$ for Tactic I.3.b and for any given $p>c$. Hence, the optimal advertising efforts in the function of retail price $p$ is given by (34). Next, as $a(1+\beta) /[2 h(1+b \beta)]>0$. Therefore, $e_{I .3 . b}^{*}(p)$ is strictly increasing in $p$.
(Q.E.D.)

Proof of Proposition 4.2.19: Similarly, the local optimum for Tactic I.1.a is an
interior point, if it exists. $D_{L}\left(\omega_{I .3, b}^{*}\right)>0$ and $D_{F}\left(\omega_{I .3, b}^{*}\right)>0$ are the basic conditions of Tactic I.3. By putting (34) into (33), we obtain

$$
\begin{aligned}
& \pi_{L L}(\omega)=\frac{-Y(p-c)^{2}}{4 h(1+b \beta)^{2}}+\frac{\left[2 h\left(B-N_{I} \tau\right)+a m(1+\beta)\right](p-c)}{2 h(1+b \beta)}-m(T+\tau)+\frac{m^{2}}{4 h}, \text { and we have } \\
& \frac{\partial \pi_{L L}(\omega)}{\partial p}=\frac{-Y(p-c)}{2 h(1+b \beta)^{2}}+\frac{\left[2 h\left(B-N_{I} \tau\right)+a m(1+\beta)\right]}{2 h(1+b \beta)} \text { and } \partial^{2} \pi_{L L}(\omega) / \partial p^{2}=\frac{-Y}{2 h(1+b \beta)^{2}} .
\end{aligned}
$$

If $Y \leq 0$, then $\pi_{L L}(\omega)$ is convex in $p$, and hence the local optimum for Tactic I.3.b is infinite, or $\pi_{L L}(\omega)$ is always non-positive (and we ignore this case). Therefore, $Y>0$ is the necessary condition for having a finite $\omega_{I .3 . b}^{*}$. If $Y>0$, then $\pi_{L L}(\omega)$ is strictly concave in $p$, and then by the first optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I .3 . b}^{*}$. By considering $p_{I .3 . b}^{*}>c$, we obtain item (iii) of Proposition 4.2.18. Similarly, by considering $\tau<e_{I .3 . b}^{*}<T+\tau$ and $N_{I}\left(p_{I .3 . b}^{*}-c\right)>-m(1+b \beta)$, we obtain items (iv) and (v) of Proposition 18, respectively. By putting $p_{I .3, b}^{*}$ into (34), we obtain $e_{I, 3, b}^{*}$. Finally, by putting (35) into (33). we obtain (36).
(Q.E.D.)

Proof of Proposition 4.2.20: By considering $d e_{I .3, b}^{*} / d \tau=-a N_{I}(1+\beta) / Y$, $d \lambda_{I . i i i . b}^{*} / d \tau=-\left[e_{I, 3 . b}^{*}+\tau a N_{I}(1+\beta)\right] /\left[Y\left(e_{I, 3 . b}^{*}\right)^{2}\right]$ and $d p_{I ., . b}^{*} / d \tau=-2 N_{I} h(1+b \beta) / Y$. We obtain the results of Proposition 4.2.20.

Proof of Proposition 4.2.21: By taking the first order and second order partial derivatives of (37) with respect to $e$, we obtain $\partial \pi_{L L}(\omega) / \partial e=\frac{\alpha(1-b)(p-c)}{1+b \beta}-2 h e$, and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, for any given $p>c, \pi_{L L}(\omega)$ is a concave function of $p$ under Tactic I.3.c. Therefore, the optimal advertising efforts in the function of retail price $p$ are given by (38). As $\alpha(1-b) /[2 h(1+b \beta)]>0, e_{I .3 . c}^{*}(p)$ is strictly increasing in $p$.

Proof of Proposition 4.2.22: Similarly, the local optimum for Tactic I.3.c is an interior point, if it exists. $N_{I}<0, D_{L}\left(\omega_{I .3 . c}^{*}\right)>0$ and $D_{F}\left(\omega_{I .3 . c}^{*}\right)>0$ are basic conditions for Tactic I.3. By putting (38) into (37), we obtain
$\pi_{L L}(\omega)=\frac{-Z(p-c)^{2}}{4 h(1+b \beta)^{2}}+\frac{B(p-c)}{(1+b \beta)}-m T$, and we have $\frac{\partial \pi_{L L}(\omega)}{\partial p}=\frac{-Z(p-c)}{2 h(1+b \beta)^{2}}+\frac{B}{(1+b \beta)}$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=\frac{-Z}{2 h(1+b \beta)^{2}}$. If $Z \leq 0$, then $\pi_{L L}(\omega)$ is convex in $p$, and hence the local optimum for Tactic I.3.c is infinite, or $\pi_{L L}(\omega)$ is always non-positive (and we ignore this case). Therefore, $Z>0$ is the necessary condition for having a finite $\omega_{I .3, c}^{*}$. If $Z>0$, then $\pi_{L L}(\omega)$ is strictly concave in $p$, and then by the first optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I .3 . c}^{*}$. By considering $p_{I .3 . c}^{*}>c$, we obtain $B>0$. By considering $e_{I .3 . c}^{*}>\tau$, we obtain item (iii) of Proposition 4.2.22. By considering $N_{I}\left(p_{I .3 . c}^{*}-c\right)<-m(1+b \beta)$, we obtain $B \geq-m Z /\left(2 h N_{I}\right)>0$. Finally, by putting $\omega_{I .3 . c}^{*}$ into (37), we obtain (40).

Proof of Proposition 4.2.23: Similar to Proposition 4.2.21.

Proof of Proposition 4.2.24: Similarly, the local optimum for Tactic I.3.da is an interior point, if it exists. Condition $N_{I}<0, D_{L}\left(\omega_{I .3 . d}^{*}\right)>0$ and $D_{F}\left(\omega_{I .3 . d}^{*}\right)>0$ are specific conditions of Tactic I.3.d. As $N_{I}(p-c)=-m(1+b \beta)$ for Tactic I.3.d, we obtain $p_{I .3 . d}^{*}$, and $p_{I .3 . d}^{*}>0$ as $N_{I}<0$ for Tactic I.3.d. According to Proposition 4.2.23, we obtain $e_{I .3 . d}^{*}=e_{I .3 . d}^{*}\left(p_{I .3 . d}^{*}\right)$. By putting $\omega_{I .3 . d}^{*}$ to (14), we obtain (43). Finally, by considering $e_{I .3 . d}^{*}>\tau$, we obtain item (ii) of Proposition 4.2.24. (Q.E.D.)

Proof of Proposition 4.2.25: First of all, $e<T+\tau$ for tactic I.4. If $e=0$, then $\lambda$ can be ignored. For $0<e<T+\tau$, by taking the first order partial derivative of (44) with respect to $\lambda$, we have $\partial \pi_{L L}(\omega) / \partial \lambda=N_{I} e(p-c) /(1+b \beta)+(m-\mu) e$. Therefore, for any fixed $p>c$ and $0<e<T$, if $N_{I}(p-c)>(\mu-m)(1+b \beta)$, then $\pi_{L L}(\omega)$ is increasing in $\lambda$. Hence $\lambda^{*}=1 \quad\left(\lambda^{*}=1\right.$ satisfies $\lambda e<T$ and $(1-\lambda) e<\tau$ for $0<e<T$ ). Moreover, for any fixed $p>c$ and $0<e<\tau$, If $N_{I}(p-c)<(\mu-m)(1+b \beta)$, then $\pi_{L L}(\omega)$ is decreasing in $\lambda$. Hence $\lambda^{*}=0\left(\lambda^{*}=0\right.$ satisfies $\lambda e<T$ and $(1-\lambda) e<\tau$ for $0<e<\tau)$. If $N_{I}(p-c)=(\mu-m)(1+b \beta)$, then
$\pi_{L L}(\omega)$ is independent of $\lambda$. As there are multiple $\lambda$ which satisfy $\left(1-\lambda^{*}\right) e<\tau$ and $\lambda^{*} e<T$, for $0<e<T+\tau$, there exists multiple $\lambda^{*}$ for $N_{I}(p-c)=(\mu-m)(1+b \beta)$.

Proof of Proposition 4.2.26: Consider $\omega^{\prime}=\left(e^{\prime}, \lambda^{\prime}, p^{\prime}\right)$ and $\omega^{\prime \prime}=\left(e^{\prime}, \lambda^{\prime \prime}, p^{\prime}\right)$, where $0<\lambda^{\prime}<\lambda^{\prime \prime}<1, \quad T \leq e^{\prime}<T+\tau, N_{I}\left(p^{\prime}-c\right)>(\mu-m)(1+b \beta), \quad D_{L}\left(\omega^{\prime}\right)>0, \quad D_{L}\left(\omega^{\prime \prime}\right)>0$ $D_{F}\left(\omega^{\prime}\right)>0$ and $D_{F}\left(\omega^{\prime \prime}\right)>0$. By following the proof of Proposition 4.2.25, for any fixed $p>c$ and $T \leq e<T+\tau, \pi_{L L}(\omega)$ is strictly increasing in $\lambda$. Therefore, $\pi_{L L}\left(\omega^{\prime \prime}\right)>\pi_{L L}\left(\omega^{\prime}\right)$. However, for $T \leq e^{\prime}<T+\tau, \quad \lambda^{\prime \prime} \rightarrow 1$ does not belong to Tactic I. 4 (which belongs to Tactic I.2), as $\lambda^{\prime \prime} e>T$ when $\lambda^{\prime \prime} \rightarrow 1$. Hence, the optimal $\lambda^{*}$ for Tactic I. 4 does not exist (and Tactic I. 2 dominates Tactic I.4). This completes the proof of part (a). The proof of part (b) and part (c) are similar to the proof of part (a).

Proof of Proposition 4.2.27: Similarly, the local optimum for Tactic I.4.a is an interior point, if it exists. Conditions $D_{L}\left(\omega_{I .4, a}^{*}\right)>0$ and $D_{F}\left(\omega_{I .4 . a}^{*}\right)>0$ are necessary for any sub-tactics of Tactic I.4. By taking the first and second order partial derivatives of (45) with respect to $p$, we have $\frac{\partial \pi_{L L}(\omega)}{\partial p}=\frac{-2 G(p-c)+B}{1+b \beta}$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2 \tilde{G}$. If $G \leq 0$, then $\pi_{L L}(\omega)$ is convex in $p$, and hence the local optimum for Tactic I.4.a is infinite, or $\pi_{L L}(\omega)$ is always non-positive. Therefore, $G>0$ is necessary for having a finite $\omega_{I .4 . a}^{*}$. If $G>0$, then $\pi_{L L}(\omega)$ is strictly concave in $p$, and then by the first order optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I .4, a}^{*}$. By considering $p_{I .4, a}^{*}>c$, we obtain $B>0$, By putting $\omega_{I .4, a}^{*}$ into (45), we obtain (47).

Proof of Proposition 4.2.28: By taking the first order and second order partial derivatives of (48) with respect to $e$, we obtain $\partial \pi_{L L}(\omega) / \partial e=\frac{a(p-c)(1+\beta)}{1+b \beta}-2 h e+m$, and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, $\partial \pi_{L L}(\omega)$ is a concave function of $e$ for

Tactic I.4.b and for any given $p>c$, and the optimal advertising efforts in the function of retail price $p$ is given by (49).According to Proposition 15, $e_{I, 3 . b}^{*}(p)$ is strictly increasing in $p$. Therefore $e_{\text {I.4.b }}^{*}(p)$ is strictly increasing in $p$

Proof of Proposition 4.2.29: Similarly, the local optimum for Tactic I.4.b is an interior point, if it exists. Conditions $D_{L}\left(\omega_{I .4 . b}^{*}\right)>0$ and $D_{F}\left(\omega_{I .4 . b}^{*}\right)>0$ are necessary for Tactic I.4.b. By putting (49) into (48), we obtain $\pi_{L L}(\omega)=\frac{-Y(p-c)^{2}}{4 h(1+b \beta)^{2}}+\frac{[2 h B+a m(1+\beta)](p-c)}{2 h(1+b \beta)}-m T-\mu \tau+\frac{m^{2}}{4 h}$, and we have $\frac{\partial \pi_{L L}(\omega)}{\partial p}=\frac{-Y(p-c)}{2 h(1+b \beta)^{2}}+\frac{2 h B+a m(1+\beta)}{2 h(1+b \beta)}$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=\frac{-Y}{2 h(1+b \beta)^{2}}$.

If $Y \leq 0$, then $\pi_{L L}(\omega)$ is convex in $p$, and hence the local optimum for Tactic I.4.b is infinite, or $\pi_{L L}(\omega)$ is always non-positive. Therefore, $Y>0$ is necessary for having a finite $\omega_{I, 4, b}^{*}$. If $Y>0$, then $\pi_{L L}(\omega)$ is strictly concave in $p$, and then by the first order optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I, 4, b}^{*}$. By considering $p_{I .4 . b}^{*}>c$, we obtain $2 h B+a m(1+\beta)>0$. Then by putting $p_{I .4 . b}^{*}$ into (49), we obtain $e_{I .4, b}^{*}$. By considering $N_{I}\left(p_{I .4, b}^{*}-c\right)>(\mu-m)(1+b \beta)$ and $0<e_{I .4, b}^{*}<T$, we obtain item (iv) and item (v) of Proposition 4.2.29 respectively. Finally, by putting $\omega_{I .4 . b}^{*}$ into (48), we obtain (51)

Proof of Proposition 4.2.30: By taking the first order and second order partial derivatives of (52) with respect to $e$, we obtain $\partial \pi_{L L}(\omega) / \partial e=\frac{\alpha(1-b)(p-c)}{1+b \beta}-2 h e+\mu$, and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, $\partial \pi_{L L}(\omega)$ is a concave function of $e$ for Tactic I.4.c and for any given $p>c$, and the optimal advertising efforts in the function of retail price $p$ is given by (47).
(Q.E.D.)

Proof of Proposition 4.2.31: Similarly, the local optimum for Tactic I.1.a is an interior point, if it exists. Conditions $D_{L}\left(\omega_{I .4 . c}^{*}\right)>0$ and $D_{F}\left(\omega_{I .4 . c}^{*}\right)>0$ are necessary for Tactic I.4.c. By putting (53) into (52), we obtain
$\pi_{L L}(\omega)=\frac{-Z(p-c)^{2}}{4 h(1+b \beta)^{2}}+\frac{[2 h B+\alpha \mu(1-b)](p-c)}{2 h(1+b \beta)}-m T-\mu \tau+\frac{\mu^{2}}{4 h}$, and we have $\frac{\partial \pi_{L L}(\omega)}{\partial p}=\frac{-Z(p-c)}{2 h(1+b \beta)^{2}}+\frac{2 h B+\alpha \mu(1-b)}{2 h(1+b \beta)}$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=\frac{-Z}{2 h(1+b \beta)^{2}}$.

If $Z \leq 0$, then $\pi_{L L}(\omega)$ is convex in $p$, and hence $p^{*} \rightarrow \infty$ and the local optimum for Tactic I.4.c is infinite, or $\pi_{L L}(\omega)$ is always non-positive. Therefore, $Z>0$ is necessary for having a finite $\omega_{I .4 . c}^{*}$. If $Z>0$, then $\pi_{L L}(\omega)$ is strictly concave in $p$, and then by the first order optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I .4, c}^{*}$. By considering $p_{I .4 . c}^{*}>c$, we obtain item (ii) of Proposition 4.2.31. By considering $0<e_{I .4 . c}^{*}<\tau$ and $N_{I}\left(p_{I .4 . c}^{*}-c\right)<(\mu-m)(1+b \beta)$, we obtain Item (iv) and Item (v) of Proposition 4.2.31, respectively. Finally, by putting $\omega_{\text {I.4.c }}^{*}$ into (52), we obtain (53).
(Q.E.D.)

Proof of Proposition 4.2.32: Consider two specific $\omega, \omega^{\prime}=\left(e, \lambda^{\prime}, p\right)$ and $\omega^{\prime \prime}=\left(e, \lambda^{\prime \prime}, p\right)$, where, $p>c, \quad e>0,0 \leq \lambda^{\prime}, \lambda^{\prime \prime} \leq 1 \quad D_{F}\left(\omega^{\prime}\right)>0, \quad D_{F}\left(\omega^{\prime \prime}\right)>0$, $V_{L}\left(\omega^{\prime}\right)=0$ and $V_{L}\left(\omega^{\prime \prime}\right)<0$. By taking the first order partial derivative of (56) with respect to $\lambda$, we have $\partial \pi_{L L}(\omega) / \partial \lambda=-\alpha e(p-c)$. Therefore, for any given $p>c$ and $e>0, \pi_{L L}(\omega)$ is decreasing in $\lambda$. Moreover, $V_{L}(\omega)=x_{L}-b x_{F}-\alpha b e+(a+\alpha b) \lambda e+(\gamma b-g) p$. Therefore, for any given $e>0$, $V_{L}(\omega)$ is increasing in $\lambda$ as $a+\alpha b>0$. Hence, $0 \leq \lambda^{\prime \prime}<\lambda^{\prime} \leq 1$ and $\pi_{L L}\left(\omega^{\prime \prime}\right)>\pi_{L L}\left(\omega^{\prime}\right)$. Therefore, $\omega^{\prime}$ is not optimal for Tactic II.1.

Proof of Proposition 4.2.33: First of all, $e \geq T+\tau$ for Tactic II.1. For $e=T+\tau$, $\lambda=T /(T+\tau)$ is the only feasible solution which satisfies $\lambda e \geq T,(1-\lambda) e \geq \tau$ and $e \geq T+\tau$. Therefore, the optimal $\lambda^{*}=T /(T+\tau)$ for Tactic II. 1 with $e=T+\tau$. For $e>T+\tau$, by taking the first order partial derivative of (56) with respect to $\lambda$, we have $\partial \pi_{L L}(\omega) / \partial \lambda=-\alpha e(p-c)$. For any given $p>c$ and $e>T+\tau>0, \pi_{L L}(\omega)$ is decreasing in $\lambda$. Moreover, as $\lambda e \geq T$ for Tactic II.1, the optimal $\lambda^{*}$ satisfies $\lambda^{*} e=T$.
(Q.E.D.)

Proof of Proposition 4.2.34: Similarly, the local optimum for Tactic II.1.a is an interior point solution, if it exists. Conditions $V_{L}\left(\omega_{I I .1 . a}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .1 . a}^{*}\right)>0$ are necessary for Tactic II.1.a. By taking the first and second order partial derivatives of (57) with respect to $p$, we have $\partial \pi_{L L}(\omega) / \partial p=x_{F}-\gamma c-2 \gamma(p-c)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2 \gamma<0$. Therefore $\pi_{L L}(\omega)$ is strictly concave in $p$ for Tactic II.1.a, and hence $\pi_{L L}(\omega)$ is maximized at $p_{I I, . a \mid a}^{*}=\left(x_{F}+\alpha \tau+\gamma c\right) /(2 \gamma)>c \quad$ (because $x_{F}>\gamma c$ ). Finally, by putting $\omega_{I I, 1 . a}^{*}$ into (57), we obtain (59).

Proof of Proposition 4.2.35: By taking the first order and second order partial derivatives of (60) with respect to $e$, we obtain $\partial \pi_{L L}(\omega) / \partial e=\alpha(p-c)-2 h e$, and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, $\partial \pi_{L L}(\omega)$ is a strictly concave function of $e$ for Tactic II.1.b and for any given $p>c$, and we obtain (61). As $\alpha>0 . e_{I I .1 . b}^{*}(p)$ is strictly increasing in $p$.

Proof of Proposition 4.2.36: Similarly, the local optimum for Tactic II.1.b is an interior point solution, if it exists. Conditions $V_{L}\left(\omega_{I I .1 . b}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .1 . b}^{*}\right)>0$ are necessary for Tactic II.1.b. By putting (61) into (60), we obtain $\pi_{L L}(\omega)=\left(x_{F}-\gamma c-\alpha T\right)(p-c)-\frac{4 h \gamma-\alpha^{2}}{4 h}(p-c)^{2}$, and we have $\frac{\partial \pi_{L L}(\omega)}{\partial p}=x_{F}-\gamma c-\alpha T-\frac{4 h \gamma-\alpha^{2}}{2 h}(p-c)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-\frac{4 h \gamma-\alpha^{2}}{2 h}$.

If $4 h \gamma \leq \alpha^{2}$, then $\pi_{L L}(\omega)$ is convex in $p$, and hence the local optimum for Tactic II.1.b is infinite, or $\pi_{L L}(\omega)$ is always non-positive. Therefore, $4 h \gamma>\alpha^{2}$ is the necessary condition for $\omega^{*}$ belongs to Tactic II.1.b. If $4 h \gamma>\alpha^{2}$, then $\pi_{L L}(\omega)$ is strictly concave in $p$. By the first order optimality condition, $\pi_{L L}(\omega)$ is maximized at $p_{I I, 1 . b}^{*}$. By considering $p_{I I . \mid b}^{*}>c$, we obtain $x_{F}-\gamma c>\alpha T$. Then by putting $p_{I I .1 . b}^{*}$ into (61), we obtain $e_{I I .1 . b}^{*}$. By considering $e_{I I .1 . b}^{*}>T+\tau$, we obtain item (iii) of Proposition 4.2.36. As $x_{F}-\gamma c>\alpha T, x_{F}-\gamma c>\left[4 h \gamma(T+\tau)-\alpha^{2} \tau\right] / \alpha$ implies $x_{F}-\gamma c>\alpha T$. Hence, $p_{I I .1 . b}^{*}>c$. Finally, by putting $\omega_{I I .1 . b}^{*}$ into (60), we obtain (63).

Proof of Proposition 4.2.37: Consider two specific $\omega, \omega^{\prime}=\left(e, \lambda^{\prime}, p\right)$ and $\omega^{\prime \prime}=\left(e, \lambda^{\prime \prime}, p\right)$, where, $p>c, \quad e>0,0 \leq \lambda^{\prime}, \lambda^{\prime \prime} \leq 1 \quad D_{F}\left(\omega^{\prime}\right)>0, \quad D_{F}\left(\omega^{\prime \prime}\right)>0$, $V_{L}\left(\omega^{\prime}\right)=0$ and $V_{L}\left(\omega^{\prime \prime}\right)<0$. By taking the first order partial derivative of (64) with respect to $\lambda$, we have $\partial \pi_{L L}(\omega) / \partial \lambda=-[\alpha(p-c)+\mu] e$. Therefore, for any given $p>c$ and $e>0, \pi_{L L}(\omega)$ is strictly decreasing in $\lambda$. Moreover, as shown in the proof of Proposition 4.2.32, $0 \leq \lambda^{\prime \prime}<\lambda^{\prime} \leq 1$ and hence $\pi_{L L}\left(\omega^{\prime \prime}\right)>\pi_{L L}\left(\omega^{\prime}\right)$. Therefore, $\omega^{\prime}$ is not optimal for Tactic II.2.

Proof of Proposition 4.2.38: First of all, $e \geq T$ for tactic II.2. For $e=T, \lambda=1$ is the only possible solution for Tactic II.2. For $T<e<T+\tau$, we have $\partial \pi_{L L}(\omega) / \partial \lambda=-[\alpha(p-c)+\mu] e$ under Tactic II.2. For any given $p>c$ and $e>T$, $\pi_{L L}(\omega)$ under Tactic II. 2 is decreasing in $\lambda$. Moreover, as $\lambda e \geq T$ for Tactic II.2, the optimal $\lambda^{*}$ satisfies $\lambda^{*} e=T$. For any fixed $e \geq T+\tau$ and $p>\mathrm{c}$, consider $\omega^{\prime}=\left(e, \lambda^{\prime}, p\right)$ and $\omega^{\prime \prime}=\left(e, \lambda^{\prime \prime}, p\right)$, where $0 \leq \lambda^{\prime \prime}<\lambda^{\prime} \leq 1, D_{L}\left(\omega^{\prime}\right)=0, D_{F}\left(\omega^{\prime}\right)>0$, $D_{L}\left(\omega^{\prime \prime}\right)=0, D_{F}\left(\omega^{\prime \prime}\right)>0, \lambda^{\prime} e \geq T, \lambda^{\prime \prime} e \geq T,\left(1-\lambda^{\prime}\right) e<\tau$ and $(1-\lambda ") e=\tau$. $\pi_{L L}\left(\omega^{\prime}\right)<\pi_{L L}\left(\omega^{\prime \prime}\right)$. Therefore, Tactic II. 1 dominates Tactic II. 2 for $e \geq T+\tau$.

Proof of Proposition 4.2.39: Similarly, the local optimum for Tactic I.1.a is an interior point solution, if it exists. Conditions $V_{L}\left(\omega_{I I .2 . a}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .2 . a}^{*}\right)>0$ are necessary for Tactic II.2.a. By taking the first and second order partial derivatives of (65) with respect to $p$, we have $\partial \pi_{L L}(\omega) / \partial p=x_{F}-\gamma c-2 \gamma(p-c)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2 \gamma<0$. Therefore $\pi_{L L}(\omega)$ is concave in $p$ for Tactic II.2.a, and hence $\pi_{L L}(\omega)$ is maximized at $p_{I I, 2 a}^{*}$. As $x_{F}>\gamma c, p_{I I, 2 a}^{*}>c$. Finally, by putting $\omega_{I I, 2 . a}^{*}$ into (65), we obtain (67).

Proof of Proposition 4.2.40: By taking the first order and second order partial derivatives of (68) with respect to $e$, we obtain $\partial \pi_{L L}(\omega) / \partial e=\alpha(p-c)+\mu-2 h e$ and
$\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, $\partial \pi_{L L}(\omega)$ is a concave function of $e$ for Tactic II.2.b, and the optimal advertising efforts in the function of retail price $p$ is given by (69). As $\alpha /(2 h), e_{I I \cdot 2 . b}^{*}(p)$ is strictly increasing in $p$.

Proof of Proposition 4.2.41: Similarly, the local optimum for Tactic I.1.a is an interior point solution, if it exists. Conditions $V_{L}\left(\omega_{I I .2 . b}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .2 . b}^{*}\right)>0$ are necessary for Tactic II.2.b. By putting (69) into (68), we obtain $\pi_{L L}(\omega)=\left[x_{F}-\gamma c-\alpha T+\frac{\alpha \mu}{2 h}\right](p-c)-\frac{4 h \gamma-\alpha^{2}}{4 h}(p-c)^{2}+\frac{\mu^{2}}{4 h}-\mu(T+\tau)$, and we have $\frac{\partial \pi_{L L}(\omega)}{\partial p}=x_{F}-\gamma c-\alpha T+\frac{\alpha \mu}{2 h}-\frac{4 h \gamma-\alpha^{2}}{2 h}(p-c)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-\frac{4 h \gamma-\alpha^{2}}{2 h}$.
If $4 h \gamma \leq \alpha^{2}$, then $\pi_{L L}(\omega)$ is convex in $p$, and hence the local optimum for Tactic II.2.b is infinite or $\pi_{L L}(\omega)$ is always non-positive. Therefore, $4 h \gamma>\alpha^{2}$ is the necessary for $\omega_{L L}^{*}$ belongs to Tactic II.2.b. If $4 h \gamma>\alpha^{2}$, then $\pi_{L L}(\omega)$ is concave in $p$, and then by the first order optimality condition, $\pi_{L L}(\omega)$ is maximized at $p_{I I, 2 . b}^{*}$. By considering $p_{I I, 2 . b}^{*}>c$, we obtain $x_{F}-\gamma c>\alpha T-\alpha \mu /(2 h)$. Then by putting $p_{I I, 2, b}^{*}$ into (69), we obtain $e_{I I, 2, b}^{*}$. By considering $T<e_{I I, 2 b}^{*}<T+\tau$, we obtain item (iii) of Proposition 4.2.41. As $4 h \gamma>\alpha^{2}, \alpha T-\alpha \mu /(2 h)<2 \gamma(2 h T-\mu) / \alpha$. Hence, the first inequality of item (iii) of Proposition 4.2 .41 implies $p_{I I .2 . b}^{*}>c$. Finally, by putting $\omega_{I I .2 . b}^{*}$ into (68), we obtain (71).
(Q.E.D.)

Proof of Proposition 4.2.42: Consider two specific $\omega, \omega^{\prime}=\left(e, \lambda^{\prime}, p\right)$ and $\omega^{\prime \prime}=\left(e, \lambda^{\prime \prime}, p\right)$, where, $p>c, \quad e>0,0 \leq \lambda^{\prime}, \lambda^{\prime \prime} \leq 1 \quad D_{F}\left(\omega^{\prime}\right)>0, \quad D_{F}\left(\omega^{\prime \prime}\right)>0$, $V_{L}\left(\omega^{\prime}\right)=0$ and $V_{L}\left(\omega^{\prime \prime}\right)<0$. By taking the first order partial derivative of (72) with respect to $\lambda$, we have $\partial \pi_{L L}(\omega) / \partial \lambda=[m-\alpha(p-c)] e$. Therefore, for any given $p>c$ and $e>0, \pi_{L L}(\omega)$ is strictly increasing in $\lambda$ if $p<m / \alpha+c, \pi_{L L}(\omega)$ is strictly decreasing in $\lambda$ if $p>m / \alpha+c$, and $\pi_{L L}(\omega)$ is independent of $\lambda$ if $p=m / \alpha+c$. Moreover, according to the proof of Proposition 4.2.32, $V_{L}(\omega)$ is increasing in $\lambda$. Hence, $0 \leq \lambda^{\prime \prime}<\lambda^{\prime} \leq 1$. Therefore, if $p<m / \alpha+c$, then $\pi_{L L}\left(\omega^{\prime}\right)>\pi_{L L}\left(\omega^{\prime \prime}\right)$ and $\omega^{\prime}$
is not optimal for Tactic II.3. if $p>m / \alpha+c$, then $\pi_{L L}\left(\omega^{\prime \prime}\right)>\pi_{L L}\left(\omega^{\prime}\right)$ and $\omega^{\prime \prime}$ is not optimal for Tactic II.3, if $p=m / \alpha+c$, then $\pi_{L L}\left(\omega^{\prime \prime}\right)=\pi_{L L}\left(\omega^{\prime}\right)$. Next, we investigate the optimal solution of $\lambda$. If $e=\tau$, then the only solution of $\lambda$ which satisfies $(1-\lambda) e \geq \tau$ and $\lambda e<T$, is $\lambda=0$. For $p<m / \alpha+c$ and $\tau<e<T+\tau$, the biggest $\lambda$ that satisfies $(1-\lambda) e \geq \tau$ and $\lambda e<T$ is $\lambda=1-\tau / e$. For $p<m / \alpha+c$ and $e \geq T+\tau$, re-consider $\omega^{\prime}=\left(e, \lambda^{\prime}, p\right)$ and $\omega^{\prime \prime}=\left(e, \lambda^{\prime \prime}, p\right)$, where $0 \leq \lambda^{\prime}<\lambda^{\prime \prime} \leq 1, \quad D_{L}\left(\omega^{\prime}\right)=0, \quad D_{F}\left(\omega^{\prime}\right)>0, \quad D_{L}\left(\omega^{\prime \prime}\right)=0, \quad D_{F}\left(\omega^{\prime \prime}\right)>0, \quad \lambda^{\prime} e<T$, $\lambda^{\prime \prime} e=T,\left(1-\lambda^{\prime}\right) e \geq \tau,\left(1-\lambda^{\prime \prime}\right) \geq \tau$. We have $\pi_{L L}\left(\omega^{\prime}\right)<\pi_{L L}\left(\omega^{\prime \prime}\right)$. Therefore, Tactic II. 1 dominates Tactic II. 3 for $e \geq T+\tau$ and $p<m / \alpha+c$. For $p>m / \alpha+c$, a smaller $\lambda$ is better, so $\lambda^{*}=0$. For $p=m / \alpha+c$, there exists multiple $\lambda^{*}$ as there are multiple $\lambda$ which satisfies $(1-\lambda) e \geq \tau$ and $\lambda e<T$.

Proof of Proposition 4.2.43: By putting $e^{*}=\tau$ and $\lambda^{*}=0$ into $V_{L}\left(\omega_{L L}^{*}\right)=0$, we obtain (73). Then by putting (73) into (72), we obtain (74). Condition (i) of Proposition 4.2.43 is the basic condition for Tactic II.3.a. By considering $c<p_{I I, 3 . a}^{*} \leq m / \alpha+c$, we obtain item (ii) of Proposition 4.2.43.

Proof of Proposition 4.2.44: By putting $\lambda^{*}=1-\tau / e$ into $V_{L}(\omega)=0$, we obtain (76).

Proof of Proposition 4.2.45: Consider the first order optimality condition of $\pi_{L L}(\omega)$, we obtain $p_{I I .3 . b}^{*}$. For the second order optimality condition, we have $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2\left[a^{2} \gamma-h(b \gamma-g)^{2}\right]$. Therefore, we require item (ii) of Proposition 4.2.45 to ensure that $\pi_{L L}(\omega)$ is strictly concave in $p$. Otherwise, $\pi_{L L}(\omega)$ is convex in $p$ or $\pi_{L L}(\omega)$ is strictly increasing in $p$, and hence $\omega_{I I .3 . b}^{*}$ is infinite. If $a^{2} \gamma>h(b \gamma-g)^{2}$, then the first order optimality condition is necessary and sufficient. Condition (i) of Proposition 4.2.45 is the basic condition for Tactic II.3. By considering $e>\tau$ and $c<p \leq m / \alpha+c$, we obtain items (iii), (iv) and (v) of

Proof of Proposition 4.2.46: Similarly, the local optimum for Tactic I.1.a is an interior point solution, if it exists. Conditions $V_{L}\left(\omega_{I I .3 . c}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .3 . c}^{*}\right)>0$ are necessary for Tactic II.3.c. For Tactic II.3.c, the company's profit is $\pi_{L L}(\omega)=\left(x_{F}-\gamma c+\alpha \tau\right)(p-c)-\gamma(p-c)^{2}-h \tau^{2}-m T$, and we have $\partial \pi_{L L}(\omega) / \partial p=x_{F}-\gamma c+\alpha \tau-2 \gamma(p-c)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2 \gamma<0$.

Therefore, $\pi_{L L}(\omega)$ is strictly concave in $p$. Then by the first optimality condition, $\pi_{L L}(\omega)$ is maximized at $p_{I I .3 . c}^{*}$. As $x_{F}>\gamma c, p_{I I . i i i . c}^{*}>c$ which satisfies the constraint $p>c$. By considering $p_{I I, ., c}^{*} \geq m / \alpha+c$, we obtain item (ii) of Proposition 4.2.46. Lastly, by putting $\omega_{I I .3 . c}^{*}$ into (79), we obtain (81).

Proof of Proposition 4.2.47: By taking the first order and second order partial derivatives of (79) with respect to $e$, we obtain
$\partial \pi_{L L}(\omega) / \partial e=\alpha(p-c)-2 h e$ and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, $\partial \pi_{L L}(\omega)$ is a concave function of $e$ for Tactic II.3.d and for any given $p>c$. Then by considering $\partial \pi_{L L}(\omega) / \partial e=0$, we obtain $e=\alpha(p-c) /(2 h)$. As $\alpha /(2 h)>0, e_{I I, 3 . d}^{*}(p)$ is strictly increasing in $p$.
(Q.E.D.)

Proof of Proposition 4.2.48: Similarly, the local optimum for Tactic II.3.d is an interior point solution, if it exists. Conditions $V_{L}\left(\omega_{I I .3 . d}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .3 . d}^{*}\right)>0$ are necessary for Tactic II.3.d. By putting (82) into (79), we obtain $\pi_{L L}(\omega)=\left(x_{F}-\gamma c\right)(p-c)-\frac{4 h \gamma-\alpha^{2}}{4 h}(p-c)^{2}-m T$, and we have $\frac{\partial \pi_{L L}(\omega)}{\partial p}=x_{F}-\gamma c-\frac{4 h \gamma-\alpha^{2}}{2 h}(p-c)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-\frac{4 h \gamma-\alpha^{2}}{2 h}$.
If $4 h \gamma<\alpha^{2}$, then $\pi_{L L}(\omega)$ is strictly convex in $p$. If $4 h \gamma=\alpha^{2}$, then $\pi_{L L}(\omega)$ is strictly increasing in $p$. Hence, the local optimum for Tactic II.3.d does not exist if $4 h \gamma \leq \alpha^{2}$. Therefore, $4 h \gamma>\alpha^{2}$ is necessary for having a finite local optimum for Tactic II.3.d. If $4 h \gamma>\alpha^{2}$, then $\pi_{L L}(\omega)$ is strictly concave in $p$, and then by the first
optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I I .3 . d}^{*}$ if all the specific conditions for Tactic II.3.d are met. As $x_{F}>\gamma c, p_{I I .3 . d}^{*}>c$. By putting $p_{I I ., d}^{*}$ into (82), we obtain $e_{I I, . d}^{*}$. By considering $e_{I I, . d}^{*}>\tau$ and $p_{I I, . d}^{*}>m / \alpha+c$, we obtain items (iii) and (iv) of Proposition 4.2.48. Finally, by putting $\omega_{I I .3 . d}^{*}$ into (79), we obtain (84).

Proof of Proposition 4.2.49: Clearly, $\lambda_{I I .3 . e}^{*}=0$ and $p_{I I .3 . e}^{*}=m / \alpha+c$. As $\lambda^{*}=0$ for Tactic II.3.e, which is the same as Tactic II.3.d, Proposition 4.2.47 is valid for Tactic II.3.e too. By putting $p_{I I .3 . e}^{*}$ into (82), we obtain $e_{I I .3 . e}^{*}=m /(2 h)$. By considering $e_{I I .3 . e}^{*}>\tau$, we obtain item (ii) of Proposition 4.2.49. Then by considering $D_{L}(\omega)=0$, or equivalently $V_{L}(\omega) \leq 0$, we obtain item (iii) of Proposition 4.2.49. Finally, by putting (85) into (72), we obtain (86).
(Q.E.D.)

Proof of Proposition 4.2.50: Consider two specific $\omega, \omega^{\prime}=\left(e, \lambda^{\prime}, p\right)$ and $\omega^{\prime \prime}=\left(e, \lambda^{\prime \prime}, p\right)$, where, $p>c, \quad e>0,0 \leq \lambda^{\prime}, \lambda^{\prime \prime} \leq 1 \quad D_{F}\left(\omega^{\prime}\right)>0, \quad D_{F}\left(\omega^{\prime \prime}\right)>0$, $V_{L}\left(\omega^{\prime}\right)=0$ and $V_{L}\left(\omega^{\prime \prime}\right)<0$. By taking the first order partial derivative of (79) with respect to $\lambda$, we have $\partial \pi_{L L}(\omega) / \partial \lambda=[m-\mu-\alpha(p-c)] e$. Therefore, for any given $p>c$ and $e>0, \pi_{L L}(\omega)$ is strictly increasing in $\lambda$ if $p<(m-\mu) / \alpha+c$, is strictly decreasing in $\lambda$ if $p>(m-\mu) / \alpha+c$, and is independent of $\lambda$ if $p=(m-\mu) / \alpha+c$. Moreover, according to the proof of Proposition 4.2.32, $V_{L}(\omega)$ is increasing in $\lambda$. Hence, $0 \leq \lambda^{\prime \prime}<\lambda^{\prime} \leq 1$. Therefore, if $p<(m-\mu) / \alpha+c$, then $\pi_{L L}\left(\omega^{\prime}\right)>\pi_{L L}\left(\omega^{\prime \prime}\right)$ and $\omega^{\prime}$ is not optimal for Tactic II.4. If $p>(m-\mu) / \alpha+c$, then $\pi_{L L}\left(\omega^{\prime \prime}\right)>\pi_{L L}\left(\omega^{\prime}\right)$ and $\omega^{\prime \prime}$ is not optimal for Tactic II.4. Next, we investigate the optimal solution of $\lambda$. If $e^{*}=0$, then the value of $\lambda$ can be ignored. For $p<(m-\mu) / \alpha+c$, a bigger $\lambda$ is better, and the biggest $\lambda$ that satisfies $\lambda e<T$ $(1-\lambda) e<\tau$ for $0 \leq e<T$ is $\lambda=1$. For $p>(m-\mu) / \alpha+c$, a smaller $\lambda$ is better, so $\lambda^{*}=0$, for $0 \leq e<\tau$. As $\pi_{L L}(\omega)$ is independent of $\lambda$ if $p=(m-\mu) / \alpha+c$, and there exist multiple $\lambda$ which satisfy $\lambda e<T(1-\lambda) e<\tau$ for $0 \leq e<T+\tau$, the exist multiple $\lambda^{*}$ if $p=(m-\mu) / \alpha+c$. These complete the proofs of parts (a) to (e)

For the proofs part (f) and (g), they are similar to the proof of part (f) of Proposition 4.2.42.
(Q.E.D.)

Proof of Proposition 4.2.51: Similarly, the local optimum for Tactic II.4.a is an interior point solution, if it exists. Condition $D_{F}\left(\omega_{\text {II.4.a }}^{*}\right)>0$ are the basic condition for Tactic II.4. By putting $e=0$ into $V_{L}(\omega)=0$, we obtain $p_{\text {II.4.a }}^{*}$. By considering $c<p_{\text {II. } 4 . a}^{*}<(m-\mu) / \alpha+c$, we obtain item (ii) of Proposition 4.2.51. Finally, by putting $\omega_{I I, 4, a}^{*}$ into (87), we obtain (89).

Proof of Proposition 4.2.52: By putting $\lambda=1$ into $V_{L}(\omega)=0$, we obtain (91).

Proof of Proposition 4.2.53: Similarly, the local optimum for Tactic I.1.a is an interior point solution, if it exists. Consider the first order optimality condition of $\pi_{L L}(\omega)$, we obtain $p_{I I, 4, b}^{*}$. For the second order optimality condition, we have $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2\left[a^{2} \gamma+h(b \gamma-g)^{2}\right]<0$. Therefore, the first order optimality condition is necessary and sufficient. Item (i) of Proposition 4.2 .53 is the basic condition for Tactic II.4, and by considering $0<e_{I I .4 . b}^{*}<T$ and $c<p_{I I .4 . b}^{*}<(m-\mu) / \alpha+c$, we obtain items (ii) and (iii) of Proposition 4.2.53, respectively.

Proof of Proposition 4.2.54: Similarly, the local optimum for Tactic II.4.c is an interior point solution, if it exists. Conditions $V_{L}\left(\omega_{\text {II.4.c }}^{*}\right)<0$ and $D_{F}\left(\omega_{\text {II.4.c }}^{*}\right)>0$ are the specific conditions for Tactic II.4.c. For Tactic II.4.c, the company's profit is $\pi_{L L}(\omega)=\left(x_{F}-\gamma c\right)(p-c)-\gamma(p-c)^{2}-\mu \tau-m T$, and we have $\partial \pi_{L L}(\omega) / \partial p=x_{F}-\gamma c-2 \gamma(p-c)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2 \gamma<0$.

Therefore, $\pi_{L L}(\omega)$ is strictly concave in $p$. Then by the first optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I I .4 . c}^{*}$. As $x_{F}>\gamma c, p_{I I .4, c}^{*}>c$. By considering $p_{I I .4 . c}^{*}>(m-\mu) / \alpha+c$, we obtain item (ii) of Proposition 4.2.54. Finally, by putting $\omega_{I I .4 . c}^{*}$ into (87), we obtain (95).

Proof of Proposition 4.2.55: By taking the first order and second order partial derivatives of (96) with respect to $e$, we obtain $\partial \pi_{L L}(\omega) / \partial e=\alpha(p-c)-2 h e+\mu$ and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, $\partial \pi_{L L}(\omega)$ is a concave function of $e$ for Tactic II.4.d and for any given $p>c$. Then by considering $\partial \pi_{L L}(\omega) / \partial e=0$, we obtain (97). As $\alpha /(2 h)>0, e_{I I .4 . c}^{*}(p)$ is increasing in $p$.

Proof of Proposition 4.2.56: Similarly, the local optimum for Tactic II.4.d is an interior point solution, if it exists. Conditions $V_{L}\left(\omega_{I I .4 . d}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .4 . d}^{*}\right)>0$ are the basic conditions for Tactic II.4. By putting (97) into (96), we obtain $\pi_{L L}(\omega)=\left[x_{F}-\gamma c+\frac{\alpha \mu}{2 h}\right](p-c)-\frac{4 h \gamma-\alpha^{2}}{4 h}(p-c)^{2}-m T-\mu \tau+\frac{\mu^{2}}{4 h}$, and we have
$\frac{\partial \pi_{L L}(\omega)}{\partial p}=x_{F}-\gamma c+\frac{\alpha \mu}{2 h}-\frac{4 h \gamma-\alpha^{2}}{2 h}(p-c)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-\frac{4 h \gamma-\alpha^{2}}{2 h}$.
If $4 h \gamma \leq \alpha^{2}$, then $\pi_{L L}(\omega)$ is convex in $p$ and hence $p_{I I, 4, d}^{*}$ is infinite, or $\pi_{L L}(\omega)$ is always non-positive. Therefore, $4 h \gamma>\alpha^{2}$ is the necessary condition for Tactic II.4.d. If $4 h \gamma>\alpha^{2}$, then $\pi_{L L}(\omega)$ is strictly concave in $p$, and then by the first optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I I .4 . d}^{*}$. By putting $p_{I I .4 . d}^{*}$ into (97), we obtain $e_{I I .4 . d}^{*}$. As $x_{F}>\gamma c, e_{I I .4 . d}^{*}>0$ and $p_{I I ., d}^{*}>c$. By considering $e_{I I .4 . d}^{*}<\tau$ and $p_{I I .4 . d}^{*}>(m-\mu) / \alpha+c$, we obtain items (iii) and (iv) of Proposition 4.2.56, respectively. Finally, by putting $\omega_{I I, 4 . d}^{*}$ into (97), we obtain (99).

Proof of Proposition 4.2.56b: Clearly, $\lambda_{I I .4 . e}^{*}=0$ and $p_{I I .4 . e}^{*}=(m-\mu) / \alpha+c$ by the specific conditions of Tactic II.4.e. As $\lambda^{*}=0$ for Tactic II.4.e, which is the same as Tactic II.4.d, Proposition 4.2 .56 is valid for Tactic II.4.e too. By putting $p_{I I .4, e}^{*}$ into (97), we obtain $e_{I I .4 . e}^{*}=m /(2 h)$. By considering $e_{I I .4 . e}^{*}>\tau$ and $p_{I I .4 . e}^{*}>c$, we obtain item (ii) of Proposition 4.2.56. Then by considering $D_{L}\left(\omega_{I I, 4, e}^{*}\right)=0$, or equivalently $V_{L}\left(\omega_{I I, . e}^{*}\right) \leq 0$, we obtain item (iii) of Proposition 4.2.56. Finally, by putting (100) into (87), we obtain (101).

Proof of Proposition 4.2.57: Consider two specific $\omega, \omega^{\prime}=\left(e, \lambda^{\prime}, p\right)$ and $\omega^{\prime \prime}=\left(e, \lambda^{\prime \prime}, p\right)$, where, $p>c, e>0,0 \leq \lambda^{\prime}, \lambda^{\prime \prime} \leq 1 \quad D_{L}\left(\omega^{\prime}\right)>0, \quad D_{L}\left(\omega^{\prime \prime}\right)>0$, $V_{F}\left(\omega^{\prime}\right)=0$ and $V_{F}\left(\omega^{\prime \prime}\right)<0$. By taking the first order partial derivative of (92) with respect to $\lambda$, we have $\partial \pi_{L L}(\omega) / \partial \lambda=a e(p-c)>0$, for $e>0$ and $p>c$. Therefore, for any given $p>c$ and $e>0, \pi_{L L}(\omega)$ is increasing in $\lambda$. Next, $V_{F}(\omega)=x_{F}+\beta x_{L}+[\alpha+(a \beta-\alpha) \lambda] e-(\gamma+\beta g) p$. If $a \beta=\alpha$, then $V_{F}(\omega)$ is independent of $\lambda$. If $a \beta<\alpha$, then $V_{F}(\omega)$ is decreasing in $\lambda$. If $a \beta>\alpha$, then $V_{F}(\omega)$ is increasing in $\lambda$. Therefore, if $a \beta<\alpha$, then $0 \leq \lambda^{\prime}<\lambda^{\prime \prime} \leq 1$, and hence $\pi_{L L}\left(\omega^{\prime \prime}\right)>\pi_{L L}\left(\omega^{\prime}\right)$. As $V_{F}\left(\omega^{*}\right) \leq 0$ for Tactic III, we have $V_{F}\left(\omega^{*}\right)<0$ for $a \beta<\alpha$. If $a \beta>\alpha$, then $0 \leq \lambda^{\prime \prime}<\lambda^{\prime} \leq 1$, and hence $\pi_{L L}\left(\omega^{\prime}\right)>\pi_{L L}\left(\omega^{\prime \prime}\right)$. As $V_{F}\left(\omega^{*}\right) \leq 0$ for Tactic III, we have $V_{F}\left(\omega^{*}\right)=0$ for $a \beta>\alpha$.

Proof of Proposition 4.2.58: If $e^{*}=T+\tau$, then the only solution of $\lambda$ which satisfies $(1-\lambda) e \geq \tau$ and $\lambda e \geq T$, is $\lambda=T /(T+\tau)$. For $e^{*}>T+\tau$, as $\pi_{L L}(\omega)$ is increasing in $\lambda$ (according to the proof of Proposition 4.2.57), and the biggest $\lambda$ that satisfies $(1-\lambda) e \geq \tau$ and $\lambda e \geq T$ is $\lambda=1-\tau / e$, we obtain $\lambda^{*}=T /(T+\tau)$.

Proof of Proposition 4.2.59: Conditions $a \beta \leq \alpha, V_{F}\left(\omega_{L L}^{*}\right)<0$ and $D_{L}\left(\omega_{L L}^{*}\right)>0$ are the specific condition for Tactic III.1. Therefore, they are necessary. For Tactic III.1.a, the company's profit is $\pi_{L L}(\omega)=\left(x_{L}-g c+a T\right)(p-c)-g(p-c)^{2}-h(T+\tau)^{2}$, and we have $\partial \pi_{L L}(\omega) / \partial p=x_{L}-g c+a T-2 g(p-c)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2 g<0$. Therefore, $\pi_{L L}(\omega)$ is strictly concave in $p$. Then by the first optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I I I .1 . a}^{*}$. As $x_{L}>g c, p_{I I I .1 . a}^{*}>c$ which satisfies the constraint $p>c$. Finally, by putting $\omega_{I I I .1 . a}^{*}$ into (103), we obtain (105).

Proof of Proposition 4.2.60: By taking the first order and second order partial derivatives of (106) with respect to $e$, we obtain $\partial \pi_{L L}(\omega) / \partial e=a(p-c)-2 h e$ and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, $\pi_{L L}(\omega)$ is a strictly concave function of $e$ for

Tactic III.1.b. Then by considering $\partial \pi_{L L}(\omega) / \partial e=0$, we obtain (107). As $a>0$, $e_{I I I, 1, b}^{*}(p)$ is increasing in $p$.
(Q.E.D.)

Proof of Proposition 4.2.61: Similarly, the local optimum for Tactic III.1.b is an interior point solution, if it exists. Conditions $a \beta \leq \alpha, \quad V_{F}\left(\omega_{I I I .1 . b}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .1 . b}^{*}\right)>0$ are necessary for Tactic III.1.b. By putting (107) into (106), we obtain $\pi_{L L}(\omega)=\left(x_{L}-g c-a \tau\right)(p-c)-\frac{4 h g-a^{2}}{4 h}(p-c)^{2}$, and we have
$\frac{\partial \pi_{L L}(\omega)}{\partial p}=x_{L}-g c-a \tau-\left(4 h g-a^{2}\right)(p-c) /(2 h)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-\left(4 h g-a^{2}\right) /(2 h)$. If $4 h g \leq a^{2}$, then $\pi_{L L}(\omega)$ is either 1) convex/linear in $p$ and hence $\omega_{I I .1 . b}^{*}$ is infinite, or 2) always non-positive Therefore, $4 h g>a^{2}$ is the necessary condition for Tactic III.1.b. If $4 h g>a^{2}$, then $\pi_{L L}(\omega)$ is strictly concave in $p$, and then by the first optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I I I .1, b}^{*}$. By putting $p_{I I I .1, b}^{*}$ into (107), we obtain $e_{I I I .1 . b}^{*}$. By considering $e_{I I I . \mid b}^{*} \geq T+\tau$ and $p_{I I I . \mid b}^{*} \geq c$, we obtain items (iii) and (iv) of Proposition 4.2.61, respectively. Finally, by putting $\omega_{\text {III. } 1 . b}^{*}$ into (106), we obtain (109).

Proof of Proposition 4.2.62: Conditions $a \beta \geq \alpha$ and $D_{L}\left(\omega_{I I I . c}^{*}\right)>0$ are the specific conditions for Tactic III.1.c. By putting $e_{\text {III.1.c }}^{*}=T+\tau \quad$ and $\lambda_{I I I .1 . c}^{*}=T /(T+\tau)$ into $V_{F}(\omega)=0$, we obtain $p_{\text {III.1.c }}^{*}$. Finally, by putting (110) into (102), we obtain (111).

Proof of Proposition 4.2.63: By putting $\lambda^{*}=1-\tau / e$ into $V_{F}(\omega)=0$, we obtain (112). As $\gamma+\beta g>0$ and $a \beta>0$, the $e_{I I I .1 . d}^{*}(p)$ is strictly increasing in $p$. (Q.E.D.)

Proof of Proposition 4.2.64: Similarly, the local optimum for Tactic III.1.d is an interior point solution, if it exists. Consider the first order optimality condition of $\pi_{L L}(\omega)$, we obtain $p_{I I I .1 . d}^{*}$. For the second order optimality condition, we have $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2\left[h(\gamma+\beta g)^{2}-a^{2} \beta \gamma\right]$. Therefore, we require item (ii) of Proposition
4.2.64 to ensure that $\pi_{L L}(\omega)$ is strictly concave in $p$. Otherwise, $\omega_{I I I .1 . d}^{*}$ does not exist. If $h(\gamma+\beta g)^{2}>a^{2} \beta \gamma$, then the first order optimality condition is necessary and sufficient. Item (i) of Proposition 4.2.64 are the specific conditions for Tactic III.1.d. By considering $e_{I I I .1, d}^{*}>T+\tau$ and $p_{I I I .1 . d}^{*}>c$, we obtain items (iii) and (iv) of Proposition 4.2.64, respectively.

Proof of Proposition 4.2.65: Consider two specific $\omega, \omega^{\prime}=\left(e, \lambda^{\prime}, p\right)$ and $\omega^{\prime \prime}=\left(e, \lambda^{\prime \prime}, p\right)$, where, $p>c, \quad e>0,0 \leq \lambda^{\prime}, \lambda^{\prime \prime} \leq 1 \quad D_{F}\left(\omega^{\prime}\right)>0, \quad D_{F}\left(\omega^{\prime \prime}\right)>0$, $V_{L}\left(\omega^{\prime}\right)=0$ and $V_{L}\left(\omega^{\prime \prime}\right)<0$. By taking the first order partial derivative of (115) with respect to $\lambda$, we have $\partial \pi_{L L}(\omega) / \partial \lambda=[a(p-c)-\mu] e$ for Tactic III.2. For Tactic III. 2 and any given $e \geq T, 1$ ) if $p>\mu / a+c$, then $\pi_{L L}(\omega)$ is strictly increasing in $\lambda ; 2$ ) if $p<\mu / a+c$, then $\pi_{L L}(\omega)$ is strictly decreasing in $\lambda$; and 3) if $p=\mu / a+c$, then $\pi_{L L}(\omega)$ is independent of $\lambda$. The biggest $\lambda$ that satisfies $\lambda e \geq T$ and $(1-\lambda) e<\tau$ is $\lambda=1$, and the smallest $\lambda$ that satisfies $\lambda e \geq T$ and $(1-\lambda) e<\tau$ is $\lambda=T / e$, Therefore, $\lambda^{*}=1$ if $p^{*}>\mu / a+c, \lambda^{*}=T / e^{*}$ if $p^{*}<\mu / a+c$, and there are multiple $\lambda^{*}$ if $p^{*}=\mu / a+c$.

Proof of Proposition 4.2.66: For Tactic III. 2
$V_{F}(\omega)=x_{F}+\beta x_{L}+[\alpha+(a \beta-\alpha) \lambda] e-(\gamma+\beta g) p$. If $a \beta<\alpha$, then $V_{F}(\omega)$ is strictly decreasing in $\lambda$ and $0 \leq \lambda^{\prime}<\lambda^{\prime \prime} \leq 1$. On the other hand, if $a \beta>\alpha$, then $V_{F}(\omega)$ is strictly increasing in $\lambda$ and hence $0 \leq \lambda^{\prime \prime}<\lambda^{\prime} \leq 1$. Moreover, according to the proof of Proposition 4.2.65, for Tactic III. 2 and any given $e \geq T, 1$ ) if $p>\mu / a+c$, then $\pi_{L L}(\omega)$ is strictly increasing in $\lambda ; 2$ ) if $p<\mu / a+c$, then $\pi_{L L}(\omega)$ is strictly decreasing in $\lambda$; and 3) if $p=\mu / a+c$, then $\pi_{L L}(\omega)$ is independent of $\lambda$. Therefore, $\pi_{L L}\left(\omega^{\prime \prime}\right)>\pi_{L L}\left(\omega^{\prime}\right)$ if $\left[a\left(p^{*}-c\right)-\mu\right](a \beta-\alpha)<0$, and $\pi_{L L}\left(\omega^{\prime}\right)>\pi_{L L}\left(\omega^{\prime \prime}\right)$ if $\left[a\left(p^{*}-c\right)-\mu\right](a \beta-\alpha)>0$. As $V_{F}\left(\omega_{L L}^{*}\right) \leq 0$ for Tactic III.2, we have $V_{F}\left(\omega_{L L}^{*}\right)<0$ if $\left[a\left(p^{*}-c\right)-\mu\right](a \beta-\alpha)<0$, and $V_{F}\left(\omega_{L L}^{*}\right)=0$ if $\left[a\left(p^{*}-c\right)-\mu\right](a \beta-\alpha)>0$.(Q.E.D.)

Proof of Proposition 4.2.67: Similarly, the local optimum for Tactic III.2.a is an
interior point solution, if it exists. Conditions, $a \beta \leq \alpha, \quad V_{F}\left(\omega_{I I I . a . a}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .2 a}^{*}\right)>0$ is the specific conditions for Tactic III.2.a. By putting $e^{*}=T$ and $\lambda^{*}=1$ into (115), we obtain $\pi_{L L}(\omega)=\left(x_{F}+a T-g p\right)(p-c)-h T^{2}-\mu \tau$. By considering the first and second order derivatives of $\pi_{L L}(\omega)$ with respect to $p$, we have
$d \pi_{L L}(\omega) / d p=x_{F}-g c+a T-2 g(p-c) \quad$ and $\quad d^{2} \pi_{L L}(\omega) / d p^{2}=-2 g \quad$. Therefore $\pi_{L L}(\omega)$ is strictly concave in $p$, and the optimal $p$ for Tactic III.2.a is $p_{I I I, 2 . a}^{*}$. Next, by considering $p_{I I I .2 . a}^{*}>\mu / a+c$, we obtain item (ii) of Proposition 4.2.67. Finally, by putting (116) into $\pi_{L L}(\omega)$, we obtain (117)

Proof of Proposition 4.2.68: By taking the first order and second order partial derivatives of (118) with respect to $e$, we obtain $\partial \pi_{L L}(\omega) / \partial e=a(p-c)-2 h e$ and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, $\partial \pi_{L L}(\omega)$ is a strictly concave function of $e$ for Tactic III.2.b. Then by considering $\partial \pi_{L L}(\omega) / \partial e=0$, we obtain (119).
(Q.E.D.)

Proof of Proposition 4.2.69: Conditions $a \beta \leq \alpha, \quad V_{F}\left(\omega_{I I I .2 . b}^{*}\right)<0 \quad$ and $D_{L}\left(\omega_{I I I .2 . b}^{*}\right)>0$ are the specific conditions for Tactic III.2.b, and hence they are necessary. By putting (119) into (118), we obtain $\pi_{L L}(\omega)=\left(x_{L}-g c\right)(p-c)-\frac{4 h g-a^{2}}{4 h}(p-c)^{2}-\mu \tau \quad, \quad$ and $\quad$ we have $\partial \pi_{L L}(\omega) / \partial p=x_{L}-g c-\left(4 h g-a^{2}\right)(p-c) /(2 h)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-\left(4 h g-a^{2}\right) /(2 h)$. If $4 h g \leq a^{2}$, then $\pi_{L L}(\omega)$ is either convex in $p$ and hence $p \rightarrow \infty$, or is always negative. Therefore, $4 h g>a^{2}$ is necessary for having a finite $\omega_{I I I .2 b}^{*}$. If $4 h g>a^{2}$, then $\pi_{L L}(\omega)$ is strictly concave in $p$, and then by the first optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I I I, 2 b}^{*}$. By putting $p_{\text {III. . } b}^{*}$ into (119), we obtain $e_{\text {III.2.b }}^{*}$. By considering $e_{\text {III.2.b }}^{*}>T$ and $p_{\text {III.2. }}^{*} \geq \mu / a+c$, we obtain items (iii) and (iv) of Proposition 4.2.69. Finally, by putting $\omega_{\text {III.2.b }}^{*}$ into (118), we obtain (121).
(Q.E.D.)

Proof of Proposition 4.2.70: By putting $e^{*}=T$ and $\lambda^{*}=1$ into $V_{F}\left(\omega_{L L}^{*}\right)=0$, we obtain $p_{I I I .2 . c}^{*}$. Then by putting (122) into (115), we obtain (123). Conditions $a \beta \leq \alpha$ and $D_{L}\left(\omega_{I I I .2 . c}^{*}\right)>0$ are the specific conditions for Tactic III.2.c. Then, by considering $p_{I I I .2 . c}^{*} \leq \mu / a+c$, we obtain Condition (ii) of Proposition 4.2.70.

Proof of Proposition 4.2.71: By putting $\lambda^{*}=T / e^{*}$ into $V_{F}(\omega)=0$, we obtain (125).
(Q.E.D.)

Proof of Proposition 4.2.72: Similarly, the local optimum for Tactic III.2.d is an interior point solution, if it exists. Consider the first order optimality condition of (127), we obtain $p_{\text {III.,d }}^{*}$. For the second order optimality condition, we have $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2\left[h(\gamma+\beta g)^{2}+\alpha^{2} g\right]$. Therefore, $\pi_{L L}(\omega)$ is strictly concave in $p$ for Tactic III.2.d. Items (i), (ii) and (iii) of Proposition 4.2 .72 are the specific conditions for Tactic III.2.d. Therefore, they are necessary.
(Q.E.D.)

Proof of Proposition 4.2.73: By putting $e^{*}=T$ and $\lambda^{*}=1$ into $V_{F}\left(\omega_{L L}^{*}\right)=0$, we obtain $p_{\text {III.2.e }}^{*}$. Then by putting (128) into (115), we obtain (129). Conditions $a \beta \geq \alpha$ and $D_{L}\left(\omega_{I I I .2 . e}^{*}\right)>0$ are the specific conditions for Tactic III.2.e. Finally, by considering $p_{I I I .2 . e}^{*} \geq \mu / a+c$, we obtain item (ii) of Proposition 4.2.73. (Q.E.D.)

Proof of Proposition 4.2.74: By putting $\lambda^{*}=1$ into $V_{F}(\omega)=0$, we obtain (130).

Proof of Proposition 4.2.75: Consider the first order optimality condition of (130), we obtain $p_{I I I, 2 . f}^{*}$. For the second order optimality condition, we have $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2(\gamma+\beta g)\left[h(\gamma+\beta g)-a^{2} \beta\right] /(a \beta)^{2}$. Therefore, for Tactic III.2.d, $\pi_{L L}(\omega)$ is strictly concave in $p$ if $h(\gamma+\beta g)>a^{2} \beta$. If $h(\gamma+\beta g) \leq a^{2} \beta$, then $\pi_{L L}(\omega)$ is either 1) convex in $p$, and the optimal $p$ is infinite, or 2) $\pi_{L L}(\omega)$ is always non-positive. Therefore, $h(\gamma+\beta g)>a^{2} \beta$ is necessary for the existence of the local
optimal solution of Tactic.2.f. Conditions in item (i) of Proposition 4.2.75 are the specific conditions for Tactic III.2.f. By considering $e_{I I I .2 . f}^{*}>T$ and $p_{I I I .2 . f}^{*}>\mu / a+c$, we obtain items (iii) and item (iv) of Proposition 4.2.75, respectively.
(Q.E.D.)

Proof of Proposition 4.2.76: By putting $e^{*}=T$ and $\lambda^{*}=1$ into (115), we obtain $\pi_{L L}(\omega)=\left(x_{F}+a T-g p\right)(p-c)-h T^{2}-\mu \tau$. By considering the first and second order derivatives of $\pi_{L L}(\omega)$ with respect to $p$, we have $d \pi_{L L}(\omega) / d p=x_{F}-g c+a T-2 g(p-c) \quad$ and $\quad d^{2} \pi_{L L}(\omega) / d p^{2}=-2 g \quad$. Therefore $\pi_{L L}(\omega)$ is strictly concave in $p$, and the optimal $p$ for Tactic III.2.f is $p_{I I I .2 . f}^{*}$. Conditions $a \beta \geq \alpha, V_{F}\left(\omega_{I I I .2 . g}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .2 . g}^{*}\right)>0$ are the specific conditions for Tactic III.2.g. Next, by considering $c<p_{I I I .2 . g}^{*}<\mu / a+c$, we obtain item (ii) of Proposition 4.2.76. Finally, by putting (134) into (115), we obtain (135) (Q.E.D.)

Proof of Proposition 4.2.77: By taking the first order and second order partial derivatives of (137) with respect to $e$, we obtain $\partial \pi_{L L}(\omega) / \partial e=-2 h e+\mu$ and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, for Tactic III.2.h, $\partial \pi_{L L}(\omega)$ is a concave function of $e$. Then by considering $\partial \pi_{L L}(\omega) / \partial e=0$, we obtain (137).

Proof of Proposition 4.2.78: By putting (137) into (136), we obtain $\pi_{L L}(\omega)=\left(x_{L}-g c+a T\right)(p-c)-g(p-c)^{2}+\mu^{2} /(4 h)-\mu(T+\tau)$, and we have $\partial \pi_{L L}(\omega) / \partial p=x_{L}-g c+a T-2 g(p-c)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2 g$. Therefore, $\pi_{L L}(\omega)$ is strictly concave in $p$ for Tactic III.2.h. Then by the first optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{\text {IIL.2.h }}^{*}$. Conditions $a \beta \geq \alpha, V_{F}\left(\omega_{I I I .2 . h}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .2 . h}^{*}\right)>0$ are the specific condition for Tactic III.2.h. Therefore, they are necessary for $\omega_{I I I .2 . h}^{*}$. By considering $T<e_{I I I .2 . h}^{*}<T+\tau$ and $c<p_{\text {III } 2 . h}^{*}<\mu / a+c$, we obtain items (ii) and (iii) of Proposition 4.2.78. Finally, by putting $\omega_{I I I .2 h}^{*}$ into (136), we obtain (139).

Proof of Proposition 4.2.79: Clearly, $\lambda_{\text {III. } 2, k}^{*}=1$ and $p_{I I I .2, k}^{*}=\mu / a+c$. As $\lambda^{*}=1$
for Tactic III.2.k, which is the same as Tactic III.2.b, the local optimal advertising effort in the function of $p$ is given by $e_{I I I \cdot 2, k}^{*}(p)=a(p-c) /(2 h)$. By putting $p_{I I I, 2, k}^{*}$ into $e_{I I I .2 . k}^{*}(p)$, we obtain $e_{I I I .2, k}^{*}$. By considering $e_{I I I .2, k}^{*} \geq T$, we obtain item (ii) of Proposition 4.2.79. Then by considering $D_{F}\left(\omega_{I I I .2 . k}^{*}\right)=0$, or equivalently $V_{F}\left(\omega_{I I I .2, k}^{*}\right) \leq 0$, we obtain item (iii) of Proposition 4.2.79. Finally, by putting (140) into (115), we obtain (141).

Proof of Proposition 4.2.80: Consider two specific $\omega, \omega^{\prime}=\left(e, \lambda^{\prime}, p\right)$ and $\omega^{\prime \prime}=\left(e, \lambda^{\prime \prime}, p\right)$, where, $p>c, \quad e \geq 0,0 \leq \lambda^{\prime}, \lambda^{\prime \prime} \leq 1 \quad D_{L}\left(\omega^{\prime}\right)>0, \quad D_{L}\left(\omega^{\prime \prime}\right)>0$, $V_{F}\left(\omega^{\prime}\right)=0$ and $V_{F}\left(\omega^{\prime \prime}\right)<0$. By taking the first order partial derivative of (142) with respect to $\lambda$, we have $\partial \pi_{L L}(\omega) / \partial \lambda=[a(p-c)+m] e \geq 0$. Therefore, for any given $p>c$ and $e \geq 0, \pi_{L L}(\omega)$ is increasing in $\lambda$ for Tactic III.3. Moreover, according to the proof of Proposition 4.2.57, $V_{F}(\omega)$ is decreasing in $\lambda$ if $a \beta \leq \alpha$, but $V_{F}(\omega)$ is increasing in $\lambda$ if $a \beta \geq \alpha$. Hence, if $a \beta \leq \alpha$ then $0 \leq \lambda^{\prime}<\lambda^{\prime \prime} \leq 1$, and $\pi_{L L}\left(\omega^{\prime \prime}\right)>\pi_{L L}\left(\omega^{\prime}\right)$ and $\omega^{\prime}$ is not optimal for Tactic III.3. On the other hand, if $a \beta \geq \alpha$ then $0 \leq \lambda^{\prime \prime}<\lambda^{\prime} \leq 1$, and $\pi_{L L}\left(\omega^{\prime}\right)>\pi_{L L}\left(\omega^{\prime \prime}\right)$ and $\omega^{\prime \prime}$ is not the local optimum for Tactic III.3. Next, we investigate the optimal solution of $\lambda$. If $e^{*}=\tau$, then the only solution of $\lambda$ which satisfies $(1-\lambda) e \geq \tau$, is $\lambda=0 . \pi_{L L}(\omega)$ is increasing in $\lambda$ for Tactic III.3, a bigger $\lambda$ is better. The biggest $\lambda$ that satisfies $(1-\lambda) e \geq \tau$ is $\lambda=1-\tau / e$. Therefore $\lambda^{*}=1-\tau / e^{*}$ for $e^{*}>\tau$.

Proof of Proposition 4.2.81: Similarly, the local optimum for Tactic III.1.a is an interior point solution, if it exists. Conditions $a \beta \leq \alpha, \quad V_{F}\left(\omega_{I I I .3 . a}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .3 a}^{*}\right)>0$ are directly derived from the specific conditions for Tactic III.3.a. By taking the first and second order derivatives of (143) with respect to $p$, we obtain $\partial \pi_{L L}(\omega) / \partial p=x_{L}-g c-2 g(p-c)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2 g<0$. Therefore, $\pi_{L L}(\omega)$ is strictly concave in $p$. Then by the first optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I I I .3 . a}^{*}$. As $x_{L}>g c, p_{I I I, a, a}^{*}>c$. Finally, by putting $\omega_{I I I, . a}^{*}$ into (143), we obtain (145).

Proof of Proposition 4.2.82: By taking the first order and second order partial derivatives of (146) with respect to $e$, we obtain $\partial \pi_{L L}(\omega) / \partial e=a(p-c)-2 h e+m$ and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, for Tactic III.3.b, $\partial \pi_{L L}(\omega)$ is a strictly concave function of $e$. Then by considering $\partial \pi_{L L}(\omega) / \partial e=0$, we obtain (147). As $a>0$, $e_{I I I .3 . b}^{*}(p)$ is increasing in p .

Proof of Proposition 4.2.83: Similarly, the local optimum for Tactic III.3.b is an interior point solution, if it exists. By putting (147) into (146), and then consider the first and second order derivatives of we obtain $\pi_{L L}(\omega)$ with respect to $p$, we have $\partial \pi_{L L}(\omega) / \partial p=\left\{2 h\left(x_{L}-g c-a \tau\right)+a m-\left(4 h g-a^{2}\right)(p-c)\right\} /(2 h)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-\left(4 h g-a^{2}\right)$. Therefore, if $4 h g \leq a^{2}$, then $\pi_{L L}(\omega)$ is either 1$)$ convex in $p$, and hence there does not exist a finite local optimal solution for Tactic III.3.b., or $2)$ is always non-positive. If $4 h g>a^{2}$, then $\pi_{L L}(\omega)$ is strictly concave in $p$, and by the first order optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I I I ., b}^{*}$. Conditions $a \beta \leq \alpha, \quad V_{F}\left(\omega_{I I I .3 . b}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .3 . b}^{*}\right)>0$ are necessary for Tactic III.3.b. By considering $T<e_{I I I .3 . b}^{*}<T+\tau$, we obtain item (iii) of Proposition 4.2.83. By considering $p_{\text {III... }}^{*}>c$, we obtain item (iv) of Proposition 4.2.83. Lastly, by putting $\omega_{\text {III.ii.b }}^{*}$ into (146), we obtain (149).
(Q.E.D.)

Proof of Proposition 4.2.84: First of all, conditions $a \beta \geq \alpha$ and $D_{L}\left(\omega_{I I I .3 . c}^{*}\right)>0$ are essential for Tactic III.3.c. By putting $e^{*}=\tau$ and $\lambda^{*}=0$ into $V_{F}\left(\omega^{*}\right)=0$, we obtain $p_{I I I .3 . c}^{*}$. As $x_{F}>\gamma c$ and $x_{L}>g c, p_{I I I .3 . c}^{*}>c$. Then by putting (150) into (142), we obtain (151).
(Q.E.D.)

Proof of Proposition 4.2.85: By putting $\lambda=1-\tau / e$ into $V_{F}(\omega)=0$, we obtain (153).
(Q.E.D.)

Proof of Proposition 4.2.86: By putting (153) into (152) and then by considering the
first order optimality condition of (152), we obtain $p_{I I I .3 . d}^{*}$. For the second order optimality condition, we have $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2\left[h(\gamma+\beta g)^{2}-a^{2} \beta \gamma\right] /(a \beta)^{2}$. Therefore, for Tactic III.3.d, $\pi_{L L}(\omega)$ is strictly concave in $p$ if $h(\gamma+\beta g)^{2}>a^{2} \beta \gamma$. Otherwise, if $h(\gamma+\beta g)^{2} \leq a^{2} \beta \gamma$, then $\pi_{L L}(\omega)$ is either 1) convex in $p$, and hence Tactic III.3.d does not have a finite optimum, or always non-positive. Therefore, $h(\gamma+\beta g)^{2}>a^{2} \beta \gamma$ is necessary for the existence of the local optimal solution for Tactic.3.d. Conditions in item (i) of Proposition 4.2.63 are the specific conditions for Tactic III.3.d. Then by considering $\tau<e_{I I I . d}^{*}<T+\tau$ and $p_{I I I .3 . d}^{*} \geq c$, we obtain the necessary conditions in items (iii) and (iv) of Proposition 4.2.86, respectively.(Q.E.D.)

Proof of Proposition 4.2.87: Consider two specific $\omega, \omega^{\prime}=\left(e, \lambda^{\prime}, p\right)$ and $\omega^{\prime \prime}=\left(e, \lambda^{\prime \prime}, p\right)$, where, $p>c, e>0,0 \leq \lambda^{\prime}, \lambda^{\prime \prime} \leq 1 \quad D_{L}\left(\omega^{\prime}\right)>0, \quad D_{L}\left(\omega^{\prime \prime}\right)>0$, $V_{F}\left(\omega^{\prime}\right)=0$ and $V_{F}\left(\omega^{\prime \prime}\right)<0$. By taking the first order partial derivative of (144) with respect to $\lambda$, we have $\partial \pi_{L L}(\omega) / \partial \lambda=[a(p-c)-\mu+m] e$. For any given $e>0$ and $p>$ $c$, if $p^{*}>(\mu-m) / a+c$, then $\pi_{L L}(\omega)$ is increasing in $\lambda$. If $p^{*}<(\mu-m) / a+c$, $\pi_{L L}(\omega)$ is decreasing in $\lambda$. If $p^{*}=(\mu-m) / a+c \pi_{L L}(\omega)$ is independent of $\lambda$. Therefore, $\lambda^{*}=1$ if $p^{*}>(\mu-m) / a+c$ and $\lambda^{*}=0$ if $p^{*}>(\mu-m) / a+c$. As there are multiple $\lambda$ satisfy $\lambda e<T$ and $(1-\lambda) e<\tau$, there exist multiple $\lambda^{*}$ and $0 \leq e^{*}<T+\tau$. As $\lambda^{*} e^{*}<T$ and $\left(1-\lambda^{*}\right) e^{*}<\tau$ for Tactic III.4, $0 \leq e^{*}<T$ for $p^{*}<(\mu-m) / a+c$. Similarly, we have $0 \leq e^{*}<\tau$ for $p^{*}<(\mu-m) / a+c$.

Proof of Proposition 4.2.88: $V_{F}(\omega)=x_{F}+\beta x_{L}+[\alpha+(a \beta-\alpha) \lambda] e-(\gamma+\beta g) p$. If $a \beta \leq \alpha$, then $V_{F}(\omega)$ is decreasing in $\lambda$ and $0 \leq \lambda^{\prime}<\lambda^{\prime \prime} \leq 1$. On the other hand, if $a \beta \geq \alpha$, then $V_{F}(\omega)$ is increasing in $\lambda$ and $0 \leq \lambda^{\prime \prime}<\lambda^{\prime} \leq 1$. Therefore, if $\left[a\left(p^{*}-c\right)-\mu+m\right](a \beta-\alpha) \leq 0$, then $\pi_{L L}\left(\omega^{\prime \prime}\right)>\pi_{L L}\left(\omega^{\prime}\right)$. Else $\pi_{L L}\left(\omega^{\prime}\right)>\pi_{L L}\left(\omega^{\prime \prime}\right)$. As $V_{F}\left(\omega^{*}\right) \leq 0$ for Tactic III.4, we have $V_{F}\left(\omega^{*}\right)<0$ for $a \beta \leq \alpha$. On the other hand, if $a \beta \geq \alpha$, then $0 \leq \lambda^{\prime \prime}<\lambda^{\prime} \leq 1$, and hence $\pi_{L L}\left(\omega^{\prime}\right)>\pi_{L L}\left(\omega^{\prime \prime}\right)$. As $V_{F}\left(\omega^{*}\right) \leq 0$ for Tactic III.4, if $V_{F}\left(\omega^{*}\right)<0$, then $\left[a\left(p^{*}-c\right)-\mu+m\right](a \beta-\alpha) \leq 0$, and if $V_{F}\left(\omega^{*}\right)=0$
then $\left[a\left(p^{*}-c\right)-\mu+m\right](a \beta-\alpha) \geq 0$.

Proof of Proposition 4.2.89: By putting $e^{*}=0$ into (144), we obtain $\pi_{L L}(\omega)=\left(x_{L}-g p\right)(p-c)-\mu \tau-m T$. By considering the first and second order derivatives of $\pi_{L L}(\omega)$ with respect to $p$, we have $d \pi_{L L}(\omega) / d p=x_{L}-g c-2 g(p-c)$ and $d^{2} \pi_{L L}(\omega) / d p^{2}=-2 g$. Therefore $\pi_{L L}(\omega)$ is strictly concave in $p$, and the optimal $p$ for Tactic III.4.a is $p_{I I I .4 . a}^{*}$. Conditions $a \beta \leq \alpha$, $V_{F}\left(\omega_{I I I .4 a}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .4 . a}^{*}\right)>0$ are necessary for Tactic III.4.a. As $x_{L}>g c$, $p_{I I I .4, a}^{*}>c$. Then by considering $p^{*}>(\mu-m) / a+c$, we obtain item (ii) of Proposition 4.2.89. Finally, by putting (157) into (156), we obtain (158)
(Q.E.D.)

Proof of Proposition 4.2.90: By taking the first order and second order partial derivatives of (159) with respect to $e$, we obtain $\partial \pi_{L L}(\omega) / \partial e=a(p-c)+m-2 h e$ and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, for Tactic III.4.b, $\partial \pi_{L L}(\omega)$ is a strictly concave function of $e$. Then by considering $\partial \pi_{L L}(\omega) / \partial e=0$, we obtain (160).

Proof of Proposition 4.2.91: Similarly, the local optimum for Tactic III.4.b is an interior point solution, if it exists. Conditions $a \beta \leq \alpha, \quad V_{F}\left(\omega_{I I I .4 . b}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .4 . b}^{*}\right)>0$ are the specific conditions for Tactic III.4.b. By putting (160) into (159), and then by considering the first and second order derivatives of $\pi_{L L}(\omega)$ with respect to $p$, we obtain $\partial \pi_{L L}(\omega) / \partial p=\left\{2 h\left(x_{L}-g c\right\}+a m-\left(4 h g-a^{2}\right)(p-c)\right\} /(2 h)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-\left(4 h g-a^{2}\right) /(2 h)$. If $4 h g \leq a^{2}$, then $\pi_{L L}(\omega)$ is either 1) convex in $p$ and hence the local optimum for Tactic III.4.b is infinite, or 2) always non-positive. If $4 h g>a^{2}, \pi_{L L}(\omega)$ is strictly concave in $p$. Therefore, $4 h g>a^{2}$ is necessary for having a local optimum for Tactic III.4.b being finite. By the first order optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{\text {III.4.b }}^{*}$. By putting $p_{I I I .4 . b}^{*}$ into (159), we obtain $e_{\text {III. } 4 . b}^{*}$. By considering $e_{\text {III. } 4 . b}^{*}<T$ and $p_{I I I .4 . b}^{*}>(\mu-m) / a+c$, we obtain items (iii) and (iv) of Proposition 4.2.91, respectively. Finally, by putting $\omega_{I I I .4 b}^{*}$ into (156), we obtain (162).

Proof of Proposition 4.2.92: By putting $e^{*}=0$ into $V_{F}(\omega)=0$, we obtain $p_{I I I .4 . c}^{*}$. Then by putting (163) into (156), we obtain (164). Conditions $a \beta \leq \alpha$ and $D_{L}\left(\omega_{I I I .4 . c}^{*}\right)>0$ are the specific conditions for Tactic III.4.c. Finally, by considering $p_{I I I .4 . c}^{*}<(\mu-m) / a+c$, we obtain item (ii) of Proposition 4.2.92.

Proof of Proposition 4.2.93: By putting $\lambda^{*}=0$ into $V_{F}(\omega)=0$, we obtain (166). As $\gamma+\beta g>0$ and $\alpha>c, e_{\text {III. . . }}^{*}(p)$ is increasing in $p$.

Proof of Proposition 4.2.94: Consider the first order optimality condition of (165), we obtain $p_{\text {III. } 4 . d}^{*}$. For the second order optimality condition, we have $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2\left[h(\gamma+\beta g)^{2}+\alpha^{2} g\right] / \alpha^{2}$. Therefore, $\pi_{L L}(\omega)$ is strictly concave in $p$ for Tactic III.4.d, and $p_{\text {III.4.d }}^{*}$ is the unique local optimum for Tactic III.4.d. Conditions in item (i) of Proposition 94and item (iii) of Proposition 94 are the specific conditions for Tactic III.4.d. As $x_{L}>g c$ and $x_{F}>\gamma c, p_{I I I .4 . d}^{*}>c$. By considering $0<e_{\text {III. } 4 . d}^{*}<\tau$, we obtain item (ii) of Proposition 80.

Proof of Proposition 4.2.95: By putting $e_{I I I .4 . e}^{*}=0$ into $V_{F}(\omega)=0$, we obtain $p_{\text {III. } 4 . e}^{*}$. Then by putting (157) into (144), we obtain (158). Conditions $a \beta \geq \alpha$ and $D_{L}\left(\omega_{I I I .4 . e}^{*}\right)>0$ are the basic conditions for Tactic III.4.e. As $x_{L}>g c$ and $x_{F}>\gamma c$, $p_{\text {III. } 4 . e}^{*}>c$. Finally, by considering $p^{*}>(\mu-m) / a+c$, we obtain item (ii) of Proposition 95.
(Q.E.D.)

Proof of Proposition 4.2.96: By putting $\lambda^{*}=1$ into $V_{F}(\omega)=0$, we obtain (172). As $\gamma+\beta g>0$ and $a \beta>0, e_{\text {III. } 4 . f}^{*}(p)$ is strictly increasing in $p$.

Proof of Proposition 4.2.97: Consider the first order optimality condition of (171), we obtain $p_{\text {III. } 4 . f}^{*}$. For the second order optimality condition, we have $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2\left[h(\gamma+\beta g)-a^{2} \beta \gamma\right] /(a \beta)^{2}$. Therefore, for Tactic III.4.f, $\pi_{L L}(\omega)$ is
strictly concave in $p$ if $h(\gamma+\beta g)>a^{2} \beta$. Otherwise, if $h(\gamma+\beta g) \leq a^{2} \beta$, then $\pi_{L L}(\omega)$ is either 1) convex in $p$ and Tactic III.4.f does not have a finite optimum, or 2) is always non-positive. Therefore, $h(\gamma+\beta g)>a^{2} \beta$ is necessary for a finite $\omega_{I I I .4 . f}^{*}$ which satisfies $\pi_{L L}\left(\omega_{I I I .4 . f}^{*}\right)>0$. Conditions in item (i) and item (iv) of Proposition 97 are the specific conditions for Tactic III.4.f. Then by considering $0<e_{\text {III. } 4 . f}^{*}<T$, we obtain items (iii) of Proposition 97. Finally, by putting (173) into (171), we obtain (174).
(Q.E.D.)

Proof of Proposition 4.2.98: By putting $e_{\text {III.4.g }}^{*}=0$ into (156), we obtain $\pi_{L L}(\omega)=\left(x_{L}-g p\right)(p-c)-\mu \tau-m T$. By considering the first and second order derivatives of $\pi_{L L}(\omega)$ with respect to $p$, we have $d \pi_{L L}(\omega) / d p=x_{L}-g c-2 g(p-c)$ and $d^{2} \pi_{L L}(\omega) / d p^{2}=-2 g$. Therefore $\pi_{L L}(\omega)$ is strictly concave in $p$, and the optimal $p$ for Tactic III.4.g is $p_{\text {III.4.g }}^{*}$. Conditions $a \beta \geq \alpha, \quad V_{F}\left(\omega_{I I I .4 . g}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .4 . g}^{*}\right)>0$ are basic condition for Tactic III.4.g. As $x_{L}>g c, p_{I I I .4 . g}^{*}>c$. Then by considering $p^{*}<(\mu-m) / a+c$, we obtain item (ii) of Proposition 98. Finally, by putting (175) into (156), we obtain (176).

Proof of Proposition 4.2.99: By taking the first order and second order partial derivatives of (159) with respect to $e$, we obtain $\partial \pi_{L L}(\omega) / \partial e=\mu-2 h e$ and $\partial^{2} \pi_{L L}(\omega) / \partial e^{2}=-2 h<0$. Therefore, $\pi_{L L}(\omega)$ is a strictly concave function of $e$ for Tactic III.4.h. Then by considering $\partial \pi_{L L}(\omega) / \partial e=0$, we obtain (178).

Proof of Proposition 4.2.100: By putting (178) into (177), and then by considering the first and second order derivatives of $\pi_{L L}(\omega)$ with respect to $p$, we obtain $\partial \pi_{L L}(\omega) / \partial p=x_{L}-g c-2 g(p-c)$ and $\partial^{2} \pi_{L L}(\omega) / \partial p^{2}=-2 g<0$. Therefore, for Tactic III.4.h, $\pi_{L L}(\omega)$ is a strictly concave function of $p$. By the first order optimality condition, $\pi_{L L}(\omega)$ is uniquely maximized at $p_{I I I .4 h}^{*}$. Conditions $a \beta \geq \alpha$, $V_{F}\left(\omega_{I I I .4 . h}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .4 . h}^{*}\right)>0$ are the specific conditions for Tactic III.4.h. Then
by considering $e_{I I I .4 . h}^{*}<T$ and $p_{I I I .4 h}^{*}>(\mu-m) / a+c$, we obtain items (ii) and (iii) of Proposition 4.2.99, respectively. Finally, by putting $\omega_{I I .4 . h}^{*}$ into (156), we obtain (180).
(Q.E.D.)

Proof of Proposition 4.2.101: Clearly, $\lambda_{\text {III. } 4 . k}^{*}=0$ and $p_{I I I .4 . k}^{*}=(\mu-m) / a+c$. As $\lambda_{\text {III.4.k }}^{*}=0$, which is the same as Tactic III.4.h, the local optimal advertising effort is given by $e_{\text {III. } 4 . k}^{*}=\mu /(2 h)>0$. By considering $e_{\text {III. } 4 . k}^{*}<\tau$, we obtain item (ii) of Proposition 4.2.101. By considering $p_{I I I 4 . k}^{*}>c$, we obtain item (iii) of Proposition 4.2.101. By considering $D_{F}\left(\omega_{I I I .4 . k}^{*}\right)=0$, or equivalently $V_{F}\left(\omega_{I I I .4 . k}^{*}\right) \leq 0$, we obtain item (iv) of Proposition 4.2.101. Finally, by putting (181) into (156), we obtain (182).
(Q.E.D.)

## Appendix 3: Extended model's results (summary)

Table A-1 Summary of the local optimal advertising and pricing strategies with double linear loss for Tactic I

| Tactic | Local optimal advertising and pricing strategy | Necessary conditions for the existence of the local optimum | Profit of the company |
| :---: | :---: | :---: | :---: |
| I.1.a | $\begin{aligned} & e_{I, 1, a}^{*}=T+\tau \\ & \lambda_{I, 1, a}^{*}=T /(T+\tau) \\ & p_{I, 1, a}^{*}=\frac{B+a(1+\beta) T+\alpha(1-b) \tau}{2 G}+c \end{aligned}$ | (i) $D_{L}\left(\omega_{I, 1 . a}^{*}\right)>0$ and $D_{F}\left(\omega_{I .1 . a}^{*}\right)>0$; <br> (ii) $G>0$; and <br> (iii) $B+a(1+\beta) T+\alpha(1-b) \tau>0$. | $\frac{[B+a(1+\beta) T+\alpha(1-b) \tau]^{2}}{4 G(1+b \beta)}-h(T+\tau)^{2}$ |
| I.1.b | $\begin{aligned} & e_{I .1 . b}^{*}=a(1+\beta)\left(B-N_{I} \tau\right) / Y \\ & \lambda_{I, 1 . b}^{*}=1-\tau / e_{I, 1 . b}^{*} \\ & p_{I ., . b}^{*}=c+2 h\left(B-N_{I} \tau\right)(1+b \beta) / Y \end{aligned}$ | (i) $N_{I} \geq 0, D_{L}\left(\omega_{I, 1 . b}^{*}\right)>0$ and $D_{F}\left(\omega_{I, 1 . b}^{*}\right)>0$; <br> (ii) $Y>0$; and <br> (iii) $B>Y(T+\tau) /[a(1+\beta)]+N_{I} \tau$. | $h\left(B-N_{I} \tau\right)^{2} / Y$ |


| I.1.c | $\begin{aligned} & e_{I .1 . c}^{*}=\frac{\alpha(1-b)\left(B+N_{I} T\right)}{Z} \\ & \lambda_{I .1 . c}^{*}=\frac{T Z}{\alpha(1-b)\left(B+N_{I} T\right)} \\ & p_{I .1 . c}^{*}=\frac{2 h\left(B+N_{I} T\right)(1+b \beta)}{Z}+c \end{aligned}$ | (i) $N_{I} \leq 0, D_{L}\left(\omega_{I .1 . c}^{*}\right)>0$ and $D_{F}\left(\omega_{I .1 . c}^{*}\right)>0$; <br> (ii) $Z>0$; and <br> (iii) $B>Z(T+\tau) /[\alpha(1-b)]-N_{I} T$. | $h\left(B+N_{I} T\right)^{2} / Z$ |
| :---: | :---: | :---: | :---: |
| I.2.a | $\begin{aligned} & e_{I, 2, a}^{*}=T \\ & \lambda_{I, 2, a}^{*}=1 \\ & p_{I, 2, a}^{*}=c+[B+a(1+\beta) T] /(2 G) \end{aligned}$ | (i) $D_{L}\left(\omega_{I, 2, a}^{*}\right)>0$ and $D_{F}\left(\omega_{I, 2, a}^{*}\right)>0$; <br> (ii) $G>0$; and <br> (iii) $B+a(1+\beta) T>0$. | $\frac{B^{2}+2 B a(1+\beta) T-Y T^{2}}{4 G(1+b \beta)}-\mu \tau$ |
| I.2.b | $\begin{aligned} & e_{I .2 . b}^{*}=a(1+\beta) B / Y \\ & \lambda_{I .2 . b}^{*}=1 \\ & p_{I .2 . b}^{*}=2 h B(1+b \beta) / Y+c \end{aligned}$ | (i) $N_{I}>0, D_{L}\left(\omega_{I .2, b}^{*}\right)>0$ and $D_{F}\left(\omega_{I .2, b}^{*}\right)>0$; <br> (ii) $Y>0$; (iii) $B>T Y /[a(1+\beta)]$; and <br> (iv) $B \geq \mu Y /\left(2 h N_{I}\right)$. | $h B^{2} / Y-\mu \tau$ |
| I.2.c | $\begin{aligned} & e_{I, 2 . c}^{*}=a \mu(1+\beta) /\left(2 h N_{I}\right) \\ & \lambda_{I .2 . c}^{*}=1 \\ & p_{I .2 . c}^{*}=c+\mu(1+b \beta) / N_{I} \end{aligned}$ | (i) $N_{I}>0, D_{L}\left(\omega_{I, 2 . c}^{*}\right)>0$ and $D_{F}\left(\omega_{I, 2 . c}^{*}\right)>0$; <br> (ii) $a \mu(1+\beta)>2 h N_{I} T$. | $\frac{B \mu}{N_{I}}-\frac{Y \mu^{2}}{4 h N_{I}^{2}}-\mu \tau$ |


| I.2.d | $\begin{align*} & e_{I .2 . d}^{*}=\frac{\alpha(1-b)\left[2 h\left(B+N_{I} T\right)+\alpha \mu(1-b)\right]}{2 h Z}+\frac{\mu}{2 h} \\ & \lambda_{I .2 . d}^{*}=T / e_{I .2 . d}^{*} \\ & p_{I .2 . d}^{*}=\frac{(1+b \beta)\left[2 h\left(B+N_{I} T\right)+\alpha \mu(1-b)\right]}{Z}+c \tag{v} \end{align*}$ | (i) $D_{L}\left(\omega_{I .2 . d}^{*}\right)>0$ and $D_{F}\left(\omega_{I .2 . d}^{*}\right)>0$; (ii) $Z>0$; <br> (iii) $T<e_{I .2 . d}^{*}<T+\tau$ <br> ;(iv) <br> $2 h\left(B+N_{I} T\right)+\alpha \mu(1-b)>0 \quad ; \quad$ and $N_{I}\left[2 h\left(B+N_{I} T\right)+\alpha \mu(1-b)\right] \leq \mu Z$ | $\frac{-G(p-c)^{2}+(p-c)\left[B+\alpha(1-b) e+T N_{I}\right]}{1+b \beta}-h \mu^{2}-\mu[T+\tau-\mu]$ |
| :---: | :---: | :---: | :---: |
| I.3.a | $\begin{aligned} & e_{I .3 . a}^{*}=\tau \\ & \lambda_{I .3 . a}^{*}=0 \\ & p_{I .3 . a}^{*}=c+[B+\alpha(1-b \beta) \tau] /(2 G) \end{aligned}$ | (i) $D_{L}\left(\omega_{I .3 . a}^{*}\right)>0$ and $D_{F}\left(\omega_{I .3 . a}^{*}\right)>0$; (ii) $G>0$; and (iii) $B+\alpha(1-b) \tau>0$. | $\frac{B^{2}+2 B \alpha(1-b) \tau-Z \tau^{2}}{4 G(1+b \beta)}-m T$ |
| I.3.b | $\begin{aligned} & e_{I .3 . b}^{*}=\frac{a(1+\beta)\left[2 h\left(B-N_{I} \tau\right)+a m(1+\beta)\right]}{2 h Y}+\frac{m}{4 h} \\ & \lambda_{I .3, b}^{*}=1-\tau / e_{I .3 . b}^{*} \\ & p_{I .3 . b}^{*}=c+(1+b \beta)\left[2 h\left(B-N_{I} \tau\right)+a m(1+\beta)\right] / Y \end{aligned}$ | (i) $D_{L}\left(\omega_{I .3 . b}^{*}\right)>0$ and $D_{F}\left(\omega_{I .3 . b}^{*}\right)>0$; (ii) $Y>0$; <br> (iii) $\quad B>N_{I} \tau-a m(1+\beta) /(2 h) \quad$; <br> $\frac{\Theta \tau-2 m G(1+b \beta)}{a(1+\beta)}<B<\frac{Y T+\Theta \tau-2 m G(1+b \beta)}{a(1+\beta)} ;$ and (v) $\left[2 h\left(B-N_{I} \tau\right)+a m(1+\beta)\right] N_{I}>-m Y$ | $\frac{\left[2 h\left(B-N_{I} \tau\right)+a m(1+\beta)\right]^{2}}{4 h^{2} Y}-m(T+\tau)+\frac{m^{2}}{4 h}$ |
| I.3.c | $\begin{aligned} e_{I .3 . c c}^{*} & =\alpha B(1-b) / \mathrm{Z} \\ \lambda_{I .3 . c}^{*} & =0 \\ p_{I .3 . c}^{*} & =c+2 h B(1+b \beta) / Z \end{aligned}$ | (i) $N_{I}<0, D_{L}\left(\omega_{I .3 . c}^{*}\right)>0$ and $D_{F}\left(\omega_{I .3 . c}^{*}\right)>0$; <br> $Z>0 \quad$; (iii) $\quad B>\tau \mathrm{Z} /[\alpha(1-b)] \quad$; and (iv) $B>-m Z /\left(2 h N_{I}\right)$. | $h B^{2} / Z-m T$ |


| I.3.d | $\begin{aligned} & e_{I .3 . d}^{*}=\alpha m(b-1) /\left(2 h N_{I}\right) \\ & \lambda_{I .3 . d}^{*}=0 \\ & p_{I .3 . d}^{*}=c-m(1+b \beta) / N_{I} \end{aligned}$ | (i) $N_{I}<0, D_{L}\left(\omega_{I .3 . d}^{*}\right)>0$ and $D_{F}\left(\omega_{I .3 . d}^{*}\right)>0$; and <br> (ii) $\alpha m(b-1)<2 h N_{I} \tau$. | $\frac{\alpha^{2} m^{2}(1-b)^{2}-4 h m\left[G m(1+b \beta)+B N_{I}\right]}{4 h N_{I}^{2}}-m T$ |
| :---: | :---: | :---: | :---: |
| I.4.a | $\begin{aligned} & e_{I .4 . a}^{*}=0 \\ & \lambda_{I .4 . a}^{*}=0 \\ & p_{I .4 . a}^{*}=B /(2 G)+c \end{aligned}$ | (i) $D_{L}\left(\omega_{I .4 . a}^{*}\right)>0$ and $D_{F}\left(\omega_{I .4 . a}^{*}\right)>0$; (ii) $G>0$; and (iii) $B>0$. | $\frac{B^{2}}{4 G(1+b \beta)}-m T-\mu \tau$ |
| I.4.b | $\begin{aligned} & e_{I .4 . b}^{*}=\frac{a(1+\beta) B+2 m G(1+b \beta)}{Y} \\ & \lambda_{I .4 . b}^{*}=1 \\ & p_{I .4 . b}^{*}=\frac{[2 h B+a m(1+\beta)](1+b \beta)}{Y}+c \end{aligned}$ | (i) $D_{L}\left(\omega_{I .4 . b}^{*}\right)>0$ and $D_{F}\left(\omega_{I .4 . b}^{*}\right)>0$; (ii) $Y>0$; <br> (iii) $B>-a m(1+\beta) /(2 h) \quad$ (condition for $p>c)$; <br> (iii) $\frac{-2 m G(1+b \beta)}{a(1+\beta)}<B<\frac{T Y-2 m G(1+b \beta)}{a(1+\beta)}$; and (iv) $2 h N_{I} B>\mu Y-m \Theta$. | $\frac{h B^{2}-\operatorname{Bam}(1+\beta)+m^{2} G(1+b \beta)}{Y}-m T-\mu \tau$ |
| I.4.c | $\begin{align*} & e_{I .4, c}^{*}=[2 \mu G(1+b \beta)+\alpha(1-b) B] / Z \\ & \lambda_{I .4, c}^{*}=0  \tag{iv}\\ & p_{I .4 . c c}^{*}=(1+b \beta)[2 h B+\alpha \mu(1-b)] / Z+c \tag{v} \end{align*}$ | (i) $D_{L}\left(\omega_{I .4 . c}^{*}\right)>0$ and $D_{F}\left(\omega_{I .4 . c}^{*}\right)>0$; (ii) $Z>0$; <br> (iii) $\quad 2 h B+\alpha \mu(1-b)>0 \quad$; <br> $0<2 \mu G(1+b \beta)+\alpha(1-b) B<Z \tau \quad ; \quad$ and $2 h B N_{I}<\Theta \mu-Z m$. | $h B^{2} / Z-m T$ |

Table A-2 Summary of the local optimal advertising and pricing strategies for Tactic II

| Tactic | Local optimal advertising and pricing strategy | Necessary conditions for the existence of the local optimum |
| :---: | :---: | :---: |
| II.1.a | $\begin{aligned} & e_{I I .1 . a}^{*}=T+\tau, \lambda_{I I .1 . a}^{*}=T /(T+\tau), \text { and } p_{I I .1 . a}^{*}=\left(x_{F}+\alpha \tau+\gamma c\right) /(2 \gamma) . \\ & \pi_{L L}\left(\omega_{I I .1 . a}^{*}\right)=\left(x_{F}-\gamma c+\alpha \tau\right)^{2} /(4 \gamma)-h(T+\tau)^{2} \end{aligned}$ | $V_{L}\left(\omega_{I I, 1 . a}^{*}\right)<0$ and $D_{F}\left(\omega_{I I, . a}^{*}\right)>0$. |
| II.1.b | $\begin{aligned} & e_{I I, 1 . b}^{*}=\frac{\alpha\left(x_{F}-\gamma c-\alpha T\right)}{4 h \gamma-\alpha^{2}}, \quad \lambda_{I I .1 . b}^{*}=\frac{T\left(4 h \gamma-\alpha^{2}\right)}{\alpha\left(x_{F}-\gamma c-\alpha T\right)} \\ & \text { and } p_{I I, 1 . b}^{*}=\frac{2 h\left(x_{F}-\gamma c-\alpha T\right)}{4 h \gamma-\alpha^{2}}+c . \\ & \pi_{L L}\left(\omega_{I I, 1 . b}^{*}\right)=\frac{h\left(x_{F}-\gamma c-\alpha T\right)^{2}}{4 h \gamma-\alpha^{2}} \end{aligned}$ | $\begin{aligned} & \text { (i) } V_{L}\left(\omega_{I I .1 . b}^{*}\right)<0 \text { and } D_{F}\left(\omega_{I I .1 . b}^{*}\right)>0 \text {; (ii) } 4 h \gamma>\alpha^{2} \text {; and (iii) } \\ & x_{F}-\gamma c>\left[4 h \gamma(T+\tau)-\alpha^{2} \tau\right] / \alpha . \end{aligned}$ |
| II.2.a | $\begin{aligned} & e_{I I, . a}^{*}=T, \lambda_{I I .2 a a}^{*}=1, \text { and } p_{I I .2, a}^{*}=c+\left(x_{F}-\gamma c\right) /(2 \gamma) . \\ & \pi_{L L}\left(\omega_{I I .2 a a}^{*}\right)=\left(x_{F}-\gamma c\right)^{2} /(4 \gamma)-h T^{2}-\mu \tau \end{aligned}$ | $V_{L}\left(\omega_{I I, 2, a}^{*}\right)<0$ and $D_{F}\left(\omega_{I I, 2, a}^{*}\right)>0$. |
| II.2.b | $\begin{aligned} & e_{I I, 2 . b}^{*}=\frac{2 \gamma \mu+\alpha\left(x_{F}-\gamma c-\alpha T\right)}{4 h \gamma-\alpha^{2}}, \lambda_{I I, 2 . b}^{*}=\frac{T\left(4 h \gamma-\alpha^{2}\right)}{2 \gamma \mu+\alpha\left(x_{F}-\gamma c-\alpha T\right)}, \text { and } \\ & p_{I I, 2 b}^{*}=\frac{2 h\left(x_{F}-\gamma c-\alpha T\right)+\alpha \mu}{4 h \gamma-\alpha^{2}}+c . \\ & \pi_{L L}\left(\omega_{I I, 2 b}^{*}\right)=\frac{h\left(x_{F}-\gamma c-\alpha T\right)^{2}+\alpha \mu\left(x_{F}-\gamma c-\alpha T\right)+\gamma \mu^{2}}{4 h \gamma-\alpha^{2}}-\mu(T+\tau) \end{aligned}$ | (i) $V_{L}\left(\omega_{I I .2 . b}^{*}\right)<0$ and $D_{F}\left(\omega_{I I .2 . b}^{*}\right)>0$; (ii) $4 h \gamma>\alpha^{2}$; and (iii) $\frac{2 \gamma(2 h T-\mu)}{\alpha}<x_{F}-\gamma c<\frac{4 h \gamma(T+\tau)-\alpha^{2} \tau-2 \gamma \mu}{\alpha}$. |


| II.3.a | $\begin{aligned} & e_{I I, 3 a}^{*}=\tau, \quad \lambda_{I I, 3, a}^{*}=0, \text { and } p_{I I, 3, a}^{*}=\frac{x_{L}-b x_{F}-\alpha b \tau}{g-b \gamma} . \\ & \pi_{L L}\left(\omega_{I I, 3 a}^{*}\right)=\left[x_{F}+\alpha \tau-\gamma c\right]\left(p_{I I, 3 a a}^{*}-c\right)-\gamma\left(p_{I I, 3 a}^{*}-c\right)^{2}-h \tau^{2}-m T \end{aligned}$ | (i) $D_{F}\left(\omega_{\text {II.ii.a. }}^{*}\right)>0$; and <br> (ii) $0<\frac{x_{L}-g c-b\left(x_{F}-\gamma c\right)-\alpha b \tau}{g-b \gamma}<\frac{m}{\alpha}$. |
| :---: | :---: | :---: |
| II.3.b |  | (i) $D_{F}\left(\omega_{I I .3 . b}^{*}\right)>0$; (ii) $a^{2} \gamma>h(b \gamma-g)^{2}$; <br> (iii) $a(\gamma b+g)\left(x_{F}-\gamma c+\alpha \tau\right)>2 a \gamma\left(x_{L}-g c\right)+(2 \tau h-m)(b \gamma-g)^{2}$; <br> (iv) $\left[a^{2}+2 h b\right]\left(x_{F}-\gamma c+\alpha \tau\right)-2 h\left(x_{L}-g c-a \tau\right)>a m(b \gamma-g)$; and <br> (v) $\left[a^{2}+2 h b\right]\left(x_{F}-\gamma c+\alpha \tau\right)-2 h\left(x_{L}-g c-a \tau\right)<\frac{m\left[2 a^{2} \gamma-2 h(b \gamma-g)^{2}-a \alpha(b \gamma-g)\right]}{\alpha} .$ |
| II.3.c | $\begin{aligned} & e_{I I .3 . c}^{*}=\tau, \lambda_{I I .3 . c}^{*}=0, \text { and } p_{I I .3 . c}^{*}=c+\left(x_{F}-\gamma c+\alpha \tau\right) /(2 \gamma) . \\ & \pi_{L L}\left(\omega_{I I .3 . c)}^{*}\right)=\left(x_{F}-\gamma c+\alpha \tau\right)^{2} /(4 \gamma)-h \tau^{2}-m T \end{aligned}$ | $\begin{aligned} & \text { (i) } \quad V_{L}\left(\omega_{I I .3 . c}^{*}\right)<0 \quad \text { and } \quad D_{F}\left(\omega_{I I .3 . c}^{*}\right)>0 \quad ; \quad \text { and } \quad \text { (ii) } \\ & x_{F}-\gamma c>2 \gamma m / \alpha-\alpha \tau . \end{aligned}$ |
| II.3.d | $\begin{aligned} & e_{I I .3 . d}^{*}=\alpha\left(x_{F}-\gamma c\right) /\left(4 h \gamma-\alpha^{2}\right), \quad \lambda_{I I .3 . d}^{*}=0, \text { and } \\ & p_{I I .3 . d}^{*}=2 h\left(x_{F}-\gamma c\right) /\left(4 h \gamma-\alpha^{2}\right)+c . \\ & \pi_{L L}\left(\omega_{I I, . d}^{*}\right)=h\left(x_{F}-\gamma c\right)^{2} /\left(4 h \gamma-\alpha^{2}\right)-m T \end{aligned}$ | $\begin{aligned} & \text { (i) } V_{L}\left(\omega_{I I ., d}^{*}\right)<0 \quad \text { and } \quad D_{F}\left(\omega_{I I .3 . d}^{*}\right)>0 \text {; (ii) } 4 h \gamma>\alpha^{2} \text {; (iii) } \\ & x_{F}-\gamma c>\tau\left(4 h \gamma-\alpha^{2}\right) / \alpha \text {; and (iv) } x_{F}-\gamma c>m\left(4 h \gamma-\alpha^{2}\right) /(2 h \alpha) . \end{aligned}$ |
| II.3.e | $\begin{aligned} & e_{I I .3 . e}^{*}=m /(2 h), \quad \lambda_{I I .3 . e}^{*}=0, \text { and } p_{I I .3 . e}^{*}=m / \alpha+c . \\ & \pi_{L L}\left(\omega_{I I .3 . e}^{*}\right)=h\left(x_{F}-\gamma c\right)^{2} /\left(4 h \gamma-\alpha^{2}\right)-m T \end{aligned}$ | (i) $D_{F}\left(\omega_{I I .3 . e}^{*}\right)>0$; (ii) $2 h \tau<m$; and <br> (iii) $\alpha\left(x_{L}-g c\right)-b \alpha\left(x_{F}-\gamma c\right) \leq m\left[\alpha^{2} b+2 h(1-b)\right] /(2 h)$. |


| II.4.a | $\begin{aligned} & e_{I I .4, a}^{*}=0, \quad \lambda_{I I .4 . a}^{*}=0, \text { and } p_{I I .4, a}^{*}=\left(b x_{F}-x_{L}\right) /(b \gamma-g) . \\ & \pi_{L L}\left(\omega_{I I .4, a}^{*}\right)=\left(x_{F}-\gamma p_{I I .4, a}^{*}\right)\left(p_{I I .4, a}^{*}-c\right)-m T-\mu \tau \end{aligned}$ | (i) $D_{F}\left(\omega_{I I, 4 a}^{*}\right)>0$; and <br> (ii) $c<\frac{b x_{F}-x_{L}}{b \gamma-g}<\frac{m-\mu}{\alpha}+c$. |
| :---: | :---: | :---: |
| II.4.b | $\begin{aligned} & e_{I I .4 . b}^{*}=\frac{a(b \gamma-g)\left(x_{F}-\gamma c\right)+m(b \gamma-g)^{2}-2 a \gamma\left(x_{L}-g c\right)}{2\left[a^{2} \gamma+h(b \gamma-g)^{2}\right]}, \quad \lambda_{I I .4 . b}^{*}=1, \text { and } \\ & p_{I I .4 . b}^{*}=c+\frac{\left[a^{2}+2 h b(b \gamma-g)\right]\left(x_{F}-\gamma c\right)-\left[2 h\left(x_{L}-g c\right)+a m\right](b \gamma-g)}{2\left[a^{2} \gamma+h(b \gamma-g)^{2}\right]} . \\ & \pi_{L L}\left(\omega_{I I 4 . b}^{*}\right)=\left(x_{F}-\gamma p_{I I .4 . b}^{*}\right)\left(p_{I I .4 . b}^{*}-c\right)-h\left(e_{I I .4 . b}^{*}\right)^{2}-m\left(T+\tau-e_{I I .4 . b}^{*}\right) \end{aligned}$ | (i) $D_{F}\left(\omega_{I I .4 . b}^{*}\right)>0$; <br> (ii) $0<a(b \gamma-g)\left(x_{F}-\gamma c\right)+m(b \gamma-g)^{2}-2 a \gamma\left(x_{L}-g c\right)<T$; and <br> (iii) $0<\frac{\left[a^{2}+2 h b(b \gamma-g)\right]\left(x_{F}-\gamma c\right)-\left[2 h\left(x_{L}-g c\right)+a m\right](b \gamma-g)}{2\left[a^{2} \gamma+h(b \gamma-g)^{2}\right]}<\frac{m-\mu}{\alpha} .$ |
| II.4.c | $\begin{aligned} & e_{I I .4 . c}^{*}=0, \quad \lambda_{I I .4 . c}^{*}=0, \text { and } p_{I I .4 . c}^{*}=c+\left(x_{F}-\gamma c\right) /(2 \gamma) . \\ & \pi_{L L}\left(\omega_{I I .4 . c}^{*}\right)=\left(x_{F}-\gamma c\right)^{2} /(4 \gamma)-\mu \tau-m T \end{aligned}$ | $\begin{aligned} & \text { (i) } \quad V_{L}\left(\omega_{I I .4 . c}^{*}\right)<0 \quad \text { and } \quad D_{F}\left(\omega_{I I .4 . c}^{*}\right)>0 \quad ; \quad \text { and } \\ & x_{F}-\gamma c>2 \gamma(m-\mu) / \alpha \text {. } \end{aligned}$ |
| II.4.d | $\begin{align*} & e_{I I \cdot 4 . d}^{*}=\frac{\alpha\left(x_{F}-\gamma c\right)+2 \gamma \mu}{4 h \gamma-\alpha^{2}}, \lambda_{I I \cdot 4 . d}^{*}=0, \text { and } p_{I I .4 . d}^{*}=c+\frac{2 h\left(x_{F}-\gamma c\right)+\alpha \mu}{4 h \gamma-\alpha^{2}} .  \tag{iii}\\ & \pi_{L L}\left(\omega_{I I .4 . d}^{*}\right)=\frac{\left[2 h\left(x_{F}-\gamma c\right)+\alpha \mu\right]^{2}}{4 h\left(4 h \gamma-\alpha^{2}\right)}+\frac{\mu^{2}}{4 h}-m T-\mu \tau . \tag{iv} \end{align*}$ |  |
| II.4.e | $\begin{aligned} & e_{I I .4 . e}^{*}=m /(2 h), \lambda_{I I .4 . e}^{*}=0, \text { and } p_{I I .4 . e}^{*}=(m-\mu) / \alpha+c . \\ & \pi_{L L}\left(\omega_{I I .4 . e}^{*}\right)=\frac{(m-\mu)\left[2 h\left(x_{F}-\gamma c\right)+\alpha \mu\right]}{2 h \alpha}-\frac{(m-\mu)^{2}\left(4 h \gamma-\alpha^{2}\right)}{4 h \alpha^{2}}-m T-\mu \tau+\frac{\mu^{2}}{4 h} \end{aligned}$ | (i) $D_{F}\left(\omega_{I I .4 . e}^{*}\right)>0$; <br> (ii) $\mu<m<2 h \tau$; and <br> (iii) $\left(x_{L}-g c\right)-b\left(x_{F}-\gamma c\right) \leq[\alpha b m+2 h(\gamma b-g)(\mu-m)] /(2 h \alpha)$. |

Table A-3 Summary of the local optimal advertising and pricing strategies for Tactic III

| Tactic | Local optimal advertising and pricing strategy | Necessary conditions for the existence of the local optimum |
| :---: | :---: | :---: |
| III.1.a | $\begin{aligned} & e_{\text {III..a. }}^{*}=T+\tau \quad \lambda_{\text {III.3.a }}^{*}=T /(T+\tau) \quad, \quad \text { and } \\ & p_{\text {III.3.a }}^{*}=\left(x_{L}-g c+a T\right) /(2 g)+c . \\ & \pi_{L L}\left(\omega_{I I I .1 . a}^{*}\right)=\left(x_{L}-g c+a T\right)^{2} /(4 g)-h(T+\tau)^{2} \end{aligned}$ | $a \beta \leq \alpha, \quad V_{F}\left(\omega_{I I I .1 . a}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .1 . a}^{*}\right)>0$. |
| III.1.b | $e_{I I I .1 . b}^{*}=a\left(x_{L}-g c-a \tau\right) /\left(4 h g-a^{2}\right), \quad \lambda_{I I I .1 . b}^{*}=\frac{a\left(x_{L}-g c\right)-4 h g \tau}{a\left(x_{L}-g c-a \tau\right)}$, and $\begin{aligned} & p_{I I I .1 . b}^{*}=c+2 h\left(x_{L}-g c-a \tau\right) /\left(4 h g-a^{2}\right) . \\ & \pi_{L L}\left(\omega_{I I I .1 . b}^{*}\right)=h\left(x_{L}-g c-a \tau\right)^{2} /\left(4 h g-a^{2}\right) \end{aligned}$ | $\begin{aligned} & \text { (i) } a \beta \leq \alpha, V_{F}\left(\omega_{I I I .1 . b}^{*}\right)<0 \text { and } D_{L}\left(\omega_{I I I .1 . b}^{*}\right)>0 \text {; (ii) } 4 h g>a^{2} \text {; (iii) } \\ & x_{L}-g c \geq T\left(4 h g-a^{2}\right)+4 h g \tau ; \text { and (iv) } x_{L}-g c>a \tau \text {. } \end{aligned}$ |
| III.1.c | $\begin{aligned} & e_{I I I .1 . c}^{*}=T+\tau \quad \lambda_{I I I .1 . c}^{*}=T /(T+\tau), \quad \text { and } \\ & p_{I I I .1 . c}^{*}=\frac{x_{F}+\beta x_{L}+\alpha \tau+a \beta T}{\gamma+\beta g} . \\ & \pi_{L L}\left(\omega_{I I I .1 . c}^{*}\right)=\left(x_{L}-g p_{I I I .1 . c}^{*}+a T\right)\left(p_{I I I .1 . c}^{*}-c\right)-h(T+\tau)^{2} \end{aligned}$ | $a \beta \geq \alpha$ and $D_{L}\left(\omega_{I I I .1 . c}^{*}\right)>0$. |


| III.1.d | $\begin{aligned} & e_{I I I .1 . d}^{*}=\frac{2 a \beta \gamma\left(x_{L}-g c-a \tau\right)-a(\gamma+\beta g)\left(x_{F}-\gamma c+\alpha \tau\right)}{2\left[h(\gamma+\beta g)^{2}-a^{2} \beta \gamma\right]} \\ & \lambda_{I I I .1 . d}^{*}=1-\tau / e_{I I I .1 . d}^{*}, \text { and } \\ & p_{I I I .1 . d}^{*}=c+\frac{2 h \beta(\gamma+\beta g)\left(x_{L}-g c-a \tau\right)-\left[a^{2} \beta-2 h(\gamma+\beta g)\right]\left(x_{F}-\gamma c+\alpha \tau\right)}{2\left[h(\gamma+\beta g)^{2}-a^{2} \beta \gamma\right]} . \\ & \pi_{L L}\left(\omega_{I I I .1 . d}^{*}\right)=\left(x_{L}-g p_{I I I .1 . d}^{*}+a e-a \tau\right)\left(p_{I I I .1 . d}^{*}-c\right)-h\left(e_{I I I .1 . d}^{*}\right)^{2} \end{aligned}$ | (i) $a \beta \geq \alpha$ and $D_{L}\left(\omega_{I I .1 . d}^{*}\right)>0$; (ii) $h(\gamma+\beta g)^{2}>a^{2} \beta \gamma$; <br> (iii) $2 a \beta \gamma\left(x_{L}-g c-a \tau\right)-a(\gamma+\beta g)\left(x_{F}-\gamma c+\alpha \tau\right)>2(T+\tau)\left[h(\gamma+\beta g)^{2}-a^{2} \beta \gamma\right] ;$ <br> (iv) $2 h \beta(\gamma+\beta g)\left(x_{L}-g c-a \tau\right)>\left[a^{2} \beta-2 h(\gamma+\beta g)\right]\left(x_{F}-\gamma c+\alpha \tau\right)$. |
| :---: | :---: | :---: |
| III.2.a | $\begin{aligned} & e_{I I I .2 . a}^{*}=T, \quad \lambda_{I I I .2, a}^{*}=1, \text { and } p_{I I I .2 . a}^{*}=\left(x_{L}+g c+a T\right) /(2 g) . \\ & \pi_{L L}\left(\omega_{I I I .2 a}^{*}\right)=\left(x_{L}-g c+a T\right) /(4 g)-h T^{2}-\mu \tau \end{aligned}$ | (i) $a \beta \leq \alpha, \quad V_{F}\left(\omega_{I I I .2 . a}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .2 . a}^{*}\right)>0$; and <br> (ii) $x_{L}-g c>2 g \mu / a-a T$. |
| III.2.b | $\begin{aligned} & e_{I I I .2 . b}^{*}=a\left(x_{L}-g c\right) /\left(4 h g-a^{2}\right), \quad \lambda_{I I I .2 . b}^{*}=1, \text { and } \\ & p_{I I I .2 . b}^{*}=c+2 h\left(x_{L}-g c\right) /\left(4 h g-a^{2}\right) . \\ & \pi_{L L}\left(\omega_{I I I .2 . b}^{*}\right)=h\left(x_{L}-g c\right)^{2} /\left(4 h g-a^{2}\right)-\mu \tau \end{aligned}$ | (i) $a \beta \leq \alpha, \quad V_{F}\left(\omega_{I I I .2 . b}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .2 . b}^{*}\right)>0$; (ii) $4 h g>a^{2}$; <br> (iii) $x_{L}-g c>T\left(4 h g-a^{2}\right) / a$; and (iv) $x_{L}-g c>\mu\left(4 h g-a^{2}\right) /(2 a h)$. |
| III.2.c | $\begin{aligned} & e_{I I I .2 . c}^{*}=T, \quad \lambda_{I I I .2 . c}^{*}=1, \text { and } p_{I I I .2 . c}^{*}=\frac{x_{F}+\beta x_{L}+a \beta T}{\gamma+\beta g} \\ & \pi_{L L}\left(\omega_{I I I .2 . c}^{*}\right)=\left(x_{L}-g p_{I I I .2 . c}^{*}+a T\right)\left(p_{I I I .2 . c}^{*}-c\right)-h T^{2}-\mu \tau \end{aligned}$ | (i) $a \beta \leq \alpha$ and $D_{L}\left(\omega_{I I I .2 . c}^{*}\right)>0$; and <br> (ii) $x_{F}-\gamma c+\beta\left(x_{L}-g c\right)<\mu(\gamma+\beta g) / a-a \beta T$. |


| III.2.d | $\begin{aligned} & e_{I I I .2 . d}^{*}=\frac{\alpha(\gamma-\beta g)\left(x_{L}-g c+a T\right)+\mu(\gamma+\beta g)^{2}-2 \alpha g\left(x_{F}-\gamma c-\alpha T\right)}{2\left[h(\gamma+\beta g)^{2}+\alpha^{2} g\right]}, \\ & \lambda_{I I I .2 . d}^{*}=T / e_{I I I .2 . d}^{*}, \text { and } \\ & p_{I I I .2 . d}^{*}=\frac{\left[\alpha^{2}+2 h \beta(\gamma+\beta g)\right]\left(x_{L}-g c+a T\right)+2 h(\gamma+\beta g)\left(x_{F}-\gamma c-\alpha T\right)+\alpha \mu(\gamma+\beta g)}{2\left[h(\gamma+\beta g)^{2}+\alpha^{2} g\right]}+c . \\ & \pi_{L L}\left(\omega_{I I I .2 . d}^{*}\right)=\left(x_{F}+a T-g p_{I I I .2 . d}^{*}\right)\left(p_{I I I .2 . d}^{*}-c\right)-h e^{2}-\mu\left(T+\tau-e_{I I I .2 . d}^{*}\right) \end{aligned}$ | (i) $a \beta \leq \alpha$ and $D_{L}\left(\omega_{I I I .2 . d}^{*}\right)>0$; (ii) $T<e_{I I I .2 . d}^{*}<T+\tau$; and (iii) $p_{I I I, 2, d}^{*}>\mu / a+c$. |
| :---: | :---: | :---: |
| III.2.e | $\begin{aligned} & e_{I I I .2 . e}^{*}=T, \quad \lambda_{\text {III.2.e }}^{*}=1, \text { and } p_{I I I .2 . e}^{*}=\frac{x_{F}+\beta x_{L}+a \beta T}{\gamma+\beta g} . \\ & \pi_{L L}\left(\omega_{I I I .2 . e}^{*}\right)=\left(x_{L}-g p_{I I I .2 . e}^{*}+a T\right)\left(p_{I I I .2 . e}^{*}-c\right)-h T^{2}-\mu \tau \end{aligned}$ | (i) $a \beta \geq \alpha$ and $D_{L}\left(\omega_{I I I . i . e}^{*}\right)>0$; and <br> (ii) $x_{F}-\gamma c+\beta\left(x_{L}-g c\right)>\mu(\gamma+\beta g) / a-a \beta T$. |
| III.2.f | $\begin{aligned} & e_{I I I .2 . f}^{*}=\frac{a\left[x_{F}-\gamma c+\beta\left(x_{L}-g c\right)\right]}{2\left[a^{2} \beta-h(\gamma+\beta g)\right]}, \lambda_{I I I .2 . f}^{*}=1, \text { and } \\ & p_{I I I .2 . f}^{*}=\frac{\left[a^{2} \beta-2 h(\gamma+\beta g)\right]\left(x_{F}-\gamma c\right)-2 h \beta(\gamma+\beta g)\left(x_{L}-g c\right)}{2(\gamma+\beta g)\left[a^{2} \beta-h(\gamma+\beta g)\right]}+c . \\ & \pi_{L L}\left(\omega_{I I I .2 . f}^{*}\right)=\left(x_{F}+a e_{I I I .2 . f}^{*}-g p_{I I I .2 . f}^{*}\right)\left(p_{I I I .2 . f}^{*}-c\right)-h\left(e_{I I I .2 . f}^{*}\right)^{2}-\mu \tau \end{aligned}$ | (i) $a \beta \geq \alpha$ and $D_{L}\left(\omega_{I I I .2 . f}^{*}\right)>0$; (ii) $h(\gamma+\beta g)>a^{2} \beta$; <br> (ii) $x_{F}-\gamma c+\beta\left(x_{L}-g c\right)>2 T\left[a^{2} \beta-h(\gamma+\beta g)\right] / a$; <br> (iii) $\begin{aligned} & {\left[a^{2} \beta-2 h(\gamma+\beta g)\right]\left(x_{F}-\gamma c\right)-2 h \beta(\gamma+\beta g)\left(x_{L}-g c\right)} \\ & >2 \mu(\gamma+\beta g)\left[a^{2} \beta-h(\gamma+\beta g)\right] / a . \end{aligned}$ |
| III.2.g | $\begin{aligned} & e_{\text {III. } 2 . g}^{*}=T, \quad \lambda_{\text {III. } 2 . g}^{*}=1, \text { and } p_{I I I .2 . g}^{*}=\left(x_{L}+g c+a T\right) /(2 g) . \\ & \pi_{L L}\left(\omega_{I I .2 . g}^{*}\right)=\left(x_{L}-g c+a T\right) /(4 g)-h T^{2}-\mu \tau \end{aligned}$ | (i) $a \beta \geq \alpha, \quad V_{F}\left(\omega_{I I I .2 . g}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .2 . g}^{*}\right)>0$; and <br> (ii) $g c-a T<x_{L}-g c<2 g \mu / a-a T$. |


| III.2.h | $\begin{align*} & e_{I I I .2 . h}^{*}=\mu(2 h), \quad \lambda_{I I I .2 . h}^{*}=2 h T / \mu, \quad p_{I I I \cdot 2 \cdot h}^{*}=\left(x_{L}+g c+a T\right) /(2 g) .  \tag{ii}\\ & \pi_{L L}\left(\omega_{I I I \cdot 2 . h}^{*}\right)=\frac{\left(x_{L}-g c+a T\right)^{2}}{4 g}+\frac{\mu^{2}}{4 h}-\mu(T+\tau) \end{align*}$ | (i) $\quad a \beta \geq \alpha, \quad V_{F}\left(\omega_{I I I .2 . h}^{*}\right)<0 \quad$ and $\quad D_{L}\left(\omega_{I I I .2 . h}^{*}\right)>0$; $2 h T<\mu<2 h(T+\tau) \text {; and (iii) } x_{L}-g c<2 g \mu-a T \text {. }$ |
| :---: | :---: | :---: |
| III.2.k | $\begin{aligned} & e_{I I I .2, k}^{*}=\mu /(2 h), \quad \lambda_{I I I \cdot 2 \cdot k}^{*}=0, \quad p_{I I I .2 . k}^{*}=\mu / a+c . \\ & \pi_{L L}\left(\omega_{I I I, 2, k}^{*}\right)=\frac{\mu\left(x_{F}-g c\right)}{a}-\frac{\mu^{2}\left(4 h g-a^{2}\right)}{4 h a^{2}}-\mu \tau \end{aligned}$ | (i) $D_{L}\left(\omega_{I I I, 2, k}^{*}\right)>0$; <br> (ii) $2 h T \leq \mu$; and <br> (iii) $x_{F}-\gamma c+\beta\left(x_{L}-g c\right) \leq \mu\left[2 h(\gamma+\beta g)-a^{2} \beta\right] /(2 a h)$. |
| III.3.a | $\begin{aligned} & e_{I I I .3 . a}^{*}=\tau, \quad \lambda_{I I I .3 . a}^{*}=0, \text { and } p_{I I I .3 . a}^{*}=\left(x_{L}-g c\right) /(2 g)+c . \\ & \pi_{L L}\left(\omega_{I I I .3 . a}^{*}\right)=\left(x_{L}-g c\right)^{2} /(4 g)-h \tau^{2}-m T \end{aligned}$ | $a \beta \leq \alpha, \quad V_{F}\left(\omega_{I I I .3 . a}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .3 . a}^{*}\right)>0$. |
| III.3.b | $\begin{aligned} & e_{I I I .3 . b}^{*}=\frac{2 g m+a\left(x_{L}-g c-a \tau\right)}{4 h g-a^{2}}, \lambda_{I I I .3 . b}^{*}=1-\tau / e_{I I I .3 . b}^{*}, \text { and } \\ & p_{I I I .3 . b}^{*}=c+\frac{2 h\left(x_{L}-g c-a \tau\right)+a m}{4 h g-a^{2}} . \\ & \pi_{L L}\left(\omega_{I I I .3 . b}^{*}\right)=\frac{\left[2 h\left(x_{L}-g c-a \tau\right)+a m\right]^{2}}{4 h\left(4 h g-a^{2}\right)}+\frac{m^{2}}{4 h}-m(T+\tau) \end{aligned}$ | (i) $a \beta \leq \alpha, V_{F}\left(\omega_{I I I .3 . b}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .3 . b}^{*}\right)>0$; (ii) $4 h g>a^{2}$; <br> (iii) $(4 h g \tau-2 g m) / a<x_{L}-g c<\left[\left(4 h g-a^{2}\right) T+4 h g \tau-2 g m\right] / a$; <br> and (iv) $x_{L}-g c>a \tau-a m /(2 h)$. |
| III.3.c | $\begin{aligned} & e_{I I I .3 . c}^{*}=\tau, \quad \lambda_{I I I .3 . c}^{*}=0 \text {, and } p_{I I I .3 . c}^{*}=\frac{x_{F}+\beta x_{L}+\alpha \tau}{\gamma+\beta g} . \\ & \pi_{L L}\left(\omega_{I I I .3 . c}^{*}\right)=\left(x_{L}-g p_{I I I .3 . c}^{*}\right)\left(p_{I I I .3 . c}^{*}-c\right)-h \tau^{2}-m T \end{aligned}$ | $a \beta \geq \alpha$ and $D_{L}\left(\omega_{I I I .3 . c}^{*}\right)>0$. |


| III.3.d | $\begin{align*} & e_{I I I .3 . d}^{*}=\frac{a(\gamma-\beta g)\left(x_{F}-\gamma c+\alpha \tau\right)+a \gamma\left(x_{L}-g c-a \tau\right)+m(\gamma+\beta g)}{2\left[h(\gamma+\beta g)^{2}-a^{2} \beta \gamma\right]},  \tag{iii}\\ & \lambda_{I I I .3 . d}^{*}=1-\tau / e_{I I I .3 . d}^{*}, \text { and } \\ & p_{I I I .3 . d}^{*}=\frac{\left[2 h(\gamma+\beta g)-a^{2} \beta\right]\left(x_{F}-\gamma c+\alpha \tau\right)+2 h \beta(\gamma+\beta g)\left(x_{L}-g c-a \tau\right)}{2\left[h(\gamma+\beta g)^{2}-a^{2} \beta \gamma\right]}+c . \\ & \pi_{L L}\left(\omega_{I I I, 3 . d}^{*}\right)=\left[x_{F}+a\left(e_{I I I .3 . d}^{*}-\tau\right)-g p_{I I I .3 . d}^{*}\right]\left(p_{I I I .3 . d}^{*}-c\right)-h\left(e_{I I I .3 . d}^{*}\right)^{2}-m\left(T+\tau-e_{I I I .3 . d}^{*}\right) \end{align*}$ | (i) $a \beta>0$ and $D_{L}\left(\omega_{I I I .3 . d}^{*}\right)>0$; <br> (ii) $h(\gamma+\beta g)^{2}>a^{2} \beta \gamma$; $\tau<e_{I I I .3 d}^{*}<T+\tau ; \text { and (iv) } p_{I I I ., d}^{*}>c \text {. }$ |
| :---: | :---: | :---: |
| III.4.a | $\begin{align*} & e_{I I I .4 a}^{*}=0, \quad \lambda_{I I I .4 a}^{*}=0, \text { and } p_{I I I .4, a}^{*}=\left(x_{L}+g c\right)(2 \mathrm{~g}) .  \tag{ii}\\ & \pi_{L L}\left(\omega_{I I I .4, a}^{*}\right)=\left(x_{L}-g c\right)^{2} /(4 g)-m T-\mu \tau \end{align*}$ | (i) $a \beta \leq \alpha, \quad V_{F}\left(\omega_{I I I .4 . a}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .4 . a}^{*}\right)>0$; and $x_{L}-g c>2 g(\mu-m) / a$. |
| III.4.b | $\begin{aligned} & e_{I I I .4 . b}^{*}=\left[2 g m+a\left(x_{L}-g c\right)\right] /\left(4 h g-a^{2}\right), \quad \lambda_{I I I .4 . b}^{*}=1, \text { and } \\ & p_{I I I .4 . b}^{*}=\left[2 h\left(x_{L}-g c\right)+a m\right] /\left(4 h g-a^{2}\right)+c . \\ & \pi_{L L}\left(\omega_{I I I .4 . b}^{*}\right)=\frac{\left[2 h\left(x_{L}-g c\right)+a m\right]^{2}}{4 h\left(4 h g-a^{2}\right)}+\frac{m^{2}}{4 h}-\mu \tau-m T \end{aligned}$ | (i) $a \beta \leq \alpha, \quad V_{F}\left(\omega_{I I .4 . b}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .4 . b}^{*}\right)>0$; (ii) $4 h g>a^{2}$; <br> (iii) $x_{L}-g c<\left[T\left(4 h g-a^{2}\right)-2 g m\right] / a \quad$ and <br> (iv) $x_{L}-g c>\left[(\mu-m)\left(4 h g-a^{2}\right)-a^{2} m\right] /(2 a h)$ |
| III.4.c | $\begin{aligned} & e_{I I I .4 . c}^{*}=0, \quad \lambda_{I I I .4 . c}^{*}=0, \text { and } p_{I I I .4 . c}^{*} \frac{x_{F}+\beta x_{L}}{\gamma+\beta g} \\ & \pi_{L L}\left(\omega_{I I I .4 . c}^{*}\right)=\frac{\left[x_{F}-\gamma c+\beta\left(x_{L}-g c\right)\right]\left(x_{L} \gamma-x_{F} g\right)}{(\gamma+\beta g)^{2}}-\mu \tau-m T \end{aligned}$ | (i) $a \beta \leq \alpha$ and $D_{L}\left(\omega_{I I I .4 . c}^{*}\right)>0$; <br> (ii) $x_{F}-\gamma c+\beta\left(x_{L}-g c\right)<(\gamma+\beta g)(\mu-m) / a$. |


| III.4.d | $e_{I I I .4 . d}^{*}=\frac{\alpha(\gamma-\beta g)\left(x_{L}-g c\right)+\mu(\gamma+\beta g)^{2}-2 \alpha g\left(x_{F}-\gamma c\right)}{2\left[h(\gamma+\beta g)^{2}+\alpha^{2} g\right]}, \quad \lambda_{I I I .4 . d}^{*}=0,$ <br> and $\begin{aligned} & p_{I I I .4 . d}^{*}=\frac{\left[\alpha^{2}+2 h \beta(\gamma+\beta g)\right]\left(x_{L}-g c\right)+2 h(\gamma+\beta g)\left(x_{F}-\gamma c\right)+\alpha \mu(\gamma+\beta g)}{2\left[h(\gamma+\beta g)^{2}+\alpha^{2} g\right]}+c . \\ & \pi_{L L}\left(\omega_{I I I .4 . d}^{*}\right)=\left(x_{L}-g p_{I I I .4 . d}^{*}\right)\left(p_{I I I .4 . d}^{*}-c\right)-h\left(e_{I I I .4 . d}^{*}\right)^{2}-\mu\left(\tau-e_{I I I .4 . d}^{*}\right)-m T \end{aligned}$ | (i) $a \beta \leq \alpha$ and $D_{L}\left(\omega_{I I .4 . d}^{*}\right)>0$; (ii) $0<e_{\text {III. } 4 . d}^{*}<\tau$; and <br> (iii) $p_{I I I .4 . d}^{*}<(\mu-m) / a+c$. |
| :---: | :---: | :---: |
| III.4.e | $\begin{aligned} & e_{I I I .4 . e}^{*}=0, \lambda_{I I I .4 . e}^{*}=0, \text { and } p_{I I I .4 . e}^{*}=\frac{x_{F}+\beta x_{L}}{\gamma+\beta g} . \\ & \pi_{L L}\left(\omega_{I I I .4 . e}^{*}\right)=\frac{\left[x_{F}-\gamma c+\beta\left(x_{L}-g c\right)\right]\left(x_{L} \gamma-x_{F} g\right)}{(\gamma+\beta g)^{2}}-\mu \tau-m T \end{aligned}$ | (i) $a \beta \geq \alpha$ and $D_{L}\left(\omega_{I I .4 . e}^{*}\right)>0$; and <br> (ii) $x_{F}-\gamma c+\beta\left(x_{L}-g c\right)>(\mu-m)(\gamma+\beta g) / a$. |
| III.4.f | $\begin{align*} & e_{I I I .4 . f}^{*}=\frac{2 a \beta \gamma\left(x_{L}-g c\right)-(\gamma+\beta g)\left[a\left(x_{F}-\gamma c\right)+m\right]}{2\left[h(\gamma+\beta g)^{2}+\alpha^{2} \beta \gamma\right]}, \quad \lambda_{I I I .4 . f}^{*}=1, \text { and }  \tag{iii}\\ & p_{I I I .4 . f}^{*}=\frac{\beta(\gamma+\beta g)\left[2 h\left(x_{L}-g c\right)+a m\right]+\left[2 h(\gamma+\beta g)-a^{2} \beta\right]\left(x_{F}-\gamma c\right)}{2\left[h(\gamma+\beta g)^{2}-a^{2} \beta \gamma\right]}+c . \\ & \pi_{L L}\left(\omega_{I I I .4 . f}^{*}\right)=\left(x_{L}+a e_{I I I .4 . f}^{*}-g p_{I I I .4 . f}^{*}\right)\left(p_{I I I .4 . f}^{*}-c\right)-h\left(e_{I I .4 . f}^{*}\right)^{2}-\mu \tau-m\left(T-e_{I I I .4 . f}^{*}\right) \end{align*}$ | $\begin{aligned} & \text { (i) } a \beta \geq \alpha \text { and } D_{L}\left(\omega_{I I I .4 . f}^{*}\right)>0 \text {; (ii) } h(\gamma+\beta g)^{2}>a^{2} \beta \gamma \text {; } \\ & 0<e_{\text {III.4.f }}^{*}<T \text {; (iv) } p_{\text {III.4.f }}^{*}>(\mu-m) / a+c \text {. } \end{aligned}$ |
| III.4.g | $\begin{align*} & e_{I I I .4 . g}^{*}=0 \quad \lambda_{I I I .4 . g}^{*}=0, \text { and } \quad p_{I I I .4 . g}^{*}=\left(x_{L}+g c\right) /(2 g) .  \tag{ii}\\ & \pi_{L L}\left(\omega_{I I I .4 . g}^{*}\right)=\left(x_{L}-g c\right)^{2} /(4 g)-m T-\mu \tau \end{align*}$ | $\begin{aligned} & \text { (i) } \quad a \beta \geq \alpha, \quad V_{F}\left(\omega_{I I I .4 . g}^{*}\right)<0 \quad \text { and } \quad D_{L}\left(\omega_{I I I .4 . g}^{*}\right)>0 ; \\ & x_{L}-g c<2 g(\mu-m) / a . \end{aligned}$ |


| III.4.h | $\begin{aligned} & e_{I I I .4 . h}^{*}=\mu /(2 h), \quad \lambda_{I I I .4 . h}^{*}=0, \text { and } p_{I I I .4 . h}^{*}=\left(x_{L}+g c\right) /(2 g) . \\ & \pi_{L L}\left(\omega_{I I I .4 . h}^{*}\right)=\frac{\left(x_{L}-g c\right)^{2}}{4 g}+\frac{\mu^{2}}{4 h}-\mu \tau-m T \end{aligned}$ | (i) $a \beta \geq \alpha, \quad V_{F}\left(\omega_{I I .4 . h}^{*}\right)<0$ and $D_{L}\left(\omega_{I I I .4 . h}^{*}\right)>0$; (ii) $\mu<2 g \tau$; and (iii) $x_{L}-g c>2 g(\mu-m) / a$. |
| :---: | :---: | :---: |
| III.4.k | $\begin{aligned} & e_{I I I .4 . k}^{*}=\mu /(2 h), \quad \lambda_{I I I .4 . k}^{*}=0, \text { and } p_{I I I .4 . k}^{*}=(\mu-m) / a+c . \\ & \pi_{L L}\left(\omega_{I I I .4 . k}^{*}\right)=\frac{(\mu-m)\left[a\left(x_{F}-g c\right)-g(\mu-m)\right]}{a^{2}}+\frac{\mu^{2}}{4 h}-m T-\mu \tau \end{aligned}$ | (I) $D_{L}\left(\omega_{I I I .4 . k}^{*}\right)>0$; <br> (ii) $2 h \tau>\mu$; <br> (iii) $\mu>m$; and <br> (iv) $x_{F}-\gamma c+\beta\left(x_{L}-g c\right) \leq(\gamma+\beta g)(\mu-m) / a-\alpha \mu /(2 h)$. |


[^0]:    ${ }^{1}$ A part of this thesis has been accepted for publication in Zheng et al. (2011).

[^1]:    ${ }^{2}$ Notice that these parameters satisfy the model requirements and assumptions. The qualitative results of the analysis do not depend on the specific choice of the parameters, as shown in our analytical results in the preceding sections.

[^2]:    ${ }^{3}$ SILL stands for (S)ocial (I)nfluence with (L)inear (L)oss penalty for insufficient advertising.

[^3]:    ${ }^{4}$ Due to the complexity of the problem, it is very difficult to obtain the necessary and sufficient conditions for optimality. Therefore, we explore only the necessary conditions for optimality.

[^4]:    ${ }^{5}$ Notice that these parameters satisfy the model requirements and assumptions. The qualitative results of the analysis do not depend on the specific choice of the parameters, as shown in our analytical results in the previous sections.

