The Hong Kong Polytechnic University
Department of Electronic & Information Engineering

Peak-to-Average Power Ratio (PAPR) Reduction Using OFDM Null Subcarriers

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A thesis submitted in partial fulfilment of the requirements
for the degree of Master of Philosophy

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CERTIFICATE OF ORIGINALITY

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______________________________ (Signed)

WANG, Bo (Name of Student)

Abstract

The proposed schemes reduce the peak-to-average power ratio (PAPR) of multi-carrier transmission, by exploiting null subcarriers, already mandated in most OFDM wireless standards. The dissertation has two distinct but related components, which are briefly summarized as below.

(1) OFDM PAPR Reduction by Switching Null Subcarriers and Data-Subcarriers.

This new approach requires no channel-side information (CSI), imposes no rate hit, is distortionless, can complement most other PAPR reduction methods (such as tone injection, tone reservation, active constellation extension, partial transmit sequences), and can be compatible with existing standards. This work has already been published in K. T. Wong, **Bo Wang** & J. C. Chen, "OFDM PAPR Reduction by Switching Null Subcarriers and Data-Subcarriers," *Electronics Letters*, vol. 47, no. 1, pp. 62-63, January 6, 2011.

(2) OFDM PAPR Reduction by Shifting Null Subcarriers among Data-Subcarriers

This new method has low computational complexity, is distortionless, has better symbol error rate (SER) performance, requires less channel-side information (CSI), can complement most other PAPR reduction methods (such as partial transmit sequences, selective mapping, tone injection, trellis shaping, active constellation extension), and also can be compatible with existing standards.

Publications

1. K. T. Wong, **Bo WANG** & Jung-Chieh CHEN, "OFDM PAPR Reduction by Switching Null Subcarriers and Data-Subcarriers," *Electronics Letters*, vol. 47, no. 1, pp. 62-63, January 6, 2011.

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Chapter 1

Introduction

This dissertation presents much of the candidate student's research work during his past 1.6 years of study. The contents in this dissertation include two related research projects on Peak-to-Average Power Ratio Reduction for OFDM, among which one have been published in IET journals, another one has been submitted to IEEE conference for peer-review.

1.1 The Motivation & Significance of the Investigation

Orthogonal frequency division multiplexing (OFDM) as an attractive multicarrier transmission technology for wireless and wire line, provides greater bandwidth efficiently, immunity to multi-path fading and impulse noise, resistance to frequency selective fading, and also exempts the need for complex equalizers and digital signal-processor hardware implementation.

In OFDM system, by using of a discrete Fourier transform, the available bandwidth is spited into many subcarriers, which are carefully designed to keep each subcarrier orthogonal to other subcarriers to achieve bandwidth efficiency. Besides, a signal data stream is transmitted simultaneously by employing a number of low rate subcarriers, so that high data rate transmission is achieved. Since the symbol duration increases for the low data rate subcarrier, the relative amount of dispersion in time caused by multi-path delay is declined. International standards, like IEEE 802.11, IEEE 802.15, IEEE 802.16, IEEE 802.20, European Telecommunications Standards Institute (ETSI) Broadcast Radio Access Network (BRAN) committees, and 3G Long Term Revolution (LTE), adopting OFDM for high-speed wireless communications are already established or being established.

Nevertheless, some challenging issues remain unresolved in designing the OFDM systems. One of the major drawbacks is a possibly high instantaneous Peak-to-Average Power Ratio (PAPR) of transmitted OFDM signals. The OFDM signal, which superposes many individual sinusoidal subcarriers, would have a high amplitude when these sinusoids are in-phase at the IFFT input, and are thus added constructively to generate a large amplitude corresponding to a high PAPR at the IFFT output. The peak amplitude of OFDM signal could be N times that of a single-carrier system, where N denotes the number of carriers. When the peak amplitudes of OFDM signals with high PAPR reach or exceed the saturation region of Power Amplifier at the transmitter and a Low Noise Amplifier at the receiver, the OFDM signals will suffer from nonlinear distortion, spectrum spreading, in-band distortion and inter-modulation interference across the OFDM subcarriers. All these demote the bit-error-rate (BER) at the receiver. One simple solution is to use expensive power-amplifiers with large saturation region. However, as high peak amplitudes occur irregularly, these power-amplifiers would be inefficient. Besides, high peaks are also constrained by design-factors such as cost and battery power of electronics.

1.2 Literature Review

Different theoretics and hypotheses on determination of the PAPR distribution have been reported. And various schemes exist to reduce the PAPR. Various schemes could be categorized into "signal-distortion" schemes or "signal-scrambling" schemes. These techniques achieve PAPR reduction at the expense of transmit signal power increase, BER increase, data-rate loss, computational complexity increase, distortion, channel side information.

The "signal-distortion" schemes reduce high peaks by distorting the signal prior to amplification. Specific approaches include amplitude-clipping, filtering, and μ-law companding. However, "signal-distortion" schemes could cause large in-band and out-of-band noise, resulting in system performance degradation. For example, clipping [1] is computationally simple, but may cause both in-band distortion like self-interference and out-band radiation like nonlinear-distortion into OFDM signals. Besides, receiver needs to estimate two parameters of the transmitter's clipping operator: location and size, which are difficult to get.

The "signal-scrambling" techniques scramble each OFDM symbol with different scrambling sequences for the PAPR reduction. Specific approaches include partial transmit sequence (PTS) and PTS using adaptive nonlinear estimator, selective mapping, low complexity phase weighting, block coding based on Golay sequences, excess power-reduction coding, interleaving, active constellation extension, tone reservation, tone injection, and selective mapping of partial tones. In [2] [3], these various techniques are reviewed and analyzed. In [4], the authors specifically contrast "tone reservation" scheme against "adaptive constellation extension" (ACE) scheme, in terms of their "field programmable gate arrays" (FPGA) implementation. In [5], the "selected mapping" (SLM) approach and "partial transmit sequence" (PTS) scheme for PAPR reduction are specifically contrasted against each other.

Efficient schemes with explicit channel side-information (CSI) transmission have been devised in e.g. [6] [7]. Side-information embedding, i.e. side-information is not explicitly transmitted via dedicated OFDM subcarriers, has been proposed in the form of differential encoding [8] [9], marking of subcarriers [10] [6], and choosing PTS vectors from special codebooks [11]. Some of the disadvantages of these methods are peak re-growth [10] [6], increased redundancy [8] [9], inapplicability to general search algorithms [10] [6] [11], and increased detection complexity [10] [6] [11].

Meanwhile, other PAPR reduction techniques that eliminate the need of CSI are also introduced. In one of these techniques[12] [13], some of the carriers are not used for data transmission, but for PAPR reduction. However, these techniques reduce the throughput. Other technique that does not need side information uses coding technique [14]. For example, in [15], symbols switching techniques that are decoded by using an LDPC code are introduced. Nevertheless, these techniques introduce a large BER degradation. Tone reservation and Tone injection [16] are based on adding an appropriate time domain signal to the original OFDM signal to reduce PAPR without needing Channel Side Information (CSI). Tone reservation scheme is to choose the frequency-domain reserved-subcarriers as cancellation signal such that it minimizes the PAPR. At the receiver, the tone-reserved subcarriers can be discarded at the receiver without CSI. TR is computationally simple, but might violate some standards. Tone injection requires much complexity at the transmitter by substituting a point in the basic constellation for a new point in the extended constellation, thus wont be adopted on the handset uplink.

Several PAPR reduction techniques based on PTS or SLM schemes without need of side information are also proposed. Alavi [17] derives a simplified maximum likelihood (ML) decoder for SLM and PTS that operates without side information. The proposed SLM and PTS systems neither lose throughput due to side information nor degrade bit error rate (BER) due to errors in side information. However, a reduction in throughput occurs due to the pilot tones used for channel estimation (CE). Some increase in the receiver complexity is the price paid for these benefits. Giannopoulos [18] proposes a new low-complexity technique for retrieving the weighting factors in the receiver. The proposed decoder requires no additional pilot tones or explicit transmission of side information, therefore no data rate loss is implied. Furthermore this paper presents a digitally very large scale integration implementation of the proposed PTS decoder and demonstrates its low-power properties. Fujii [19] proposes 2 weighting factor estimation (WFE) methods for the receiver to estimate the phase-offsets with no CSI.

In [20], several different PAPR reduction techniques are examined specifically for implementation of multi-giga-bit-per-second 60GHz wireless CMOS Radio. Moreover, it concludes that clipping and partial transmit sequence (PTS) are more practical than other techniques for implementation of multi-giga-bit-per-second 60 GHz wireless communication systems.

1.2.1 Various PAPR-Reduction Strategies

(1) Clipping, Filtering and Peak Window

Power amplifier at transmitter with saturation level below the signal span automatically cause the signal to be clipped [1]. Receiver needs to estimate two parameters of the transmitter's clipping operator: location and size, which are difficult to get. However, clipping introduces both in-band distortion like self-interference and out-band radiation like nonlinear-distortion into OFDM signals, which degrades system's BER and spectral efficiency.

As improved clipping methods, peak windowing schemes minimize out-band radiation by using narrowband windows such as Gaussian window to attenuate peak signals. Filtering can reduce out of band radiation after clipping [21] [22]. Besides, clipping may cause some peak re-growth so that the signal after clipping will exceed the clipping level at some points. To reduce peak re-growth, a repeated clipping-and-filtering operation can be used to obtain a desirable PAPR at a cost of increasing computational complexity.

(2) Selected Mapping

In the SLM technique [23] [24], the transmitter generates a set of sufficiently different candidate data blocks by multiplying the same number of different phase sequences, all representing the same information as the original data block. And the one with the lowest PAPR is selected for transmission. Information about the selected phase sequence should be transmitted to the receiver as side information.

The SLM-OFDM transmitter is depicted as the Figure 1.1, where one of the alternative subcarrier vectors can be the unchanged original one. Differentially encoded modulation may be applied before the IDFT and right after generating the alternative OFDM symbols. At the receiver, differential demodulation has to be implemented right after the DFT.

(3) Partial Transmit Sequence

The transmitter constructs its transmit signal with low PAPR by scrambling appropriate rotation factors to subcarrier subblocks [26] [27].

The difference between SLM and PTS is that the first applies independent scrambling rotations to all subcarriers, while the latter only applies scrambling rotations to subcarrier subblocks. The PTS-OFDM transmitter is depicted in the Figure 1.2 with the hint that one PTS can always be left unrotated.

(4) Interleaving technique

In interleaving approach [28] [29] [30], a set of interleavers is used to reduce the PAPR of the multicarrier signal. An interleaver is a device that reorders data blocks. To make a set of modified data blocks, different interleavers are used to permute data blocks from the original data block. And the modified data block with the lowest

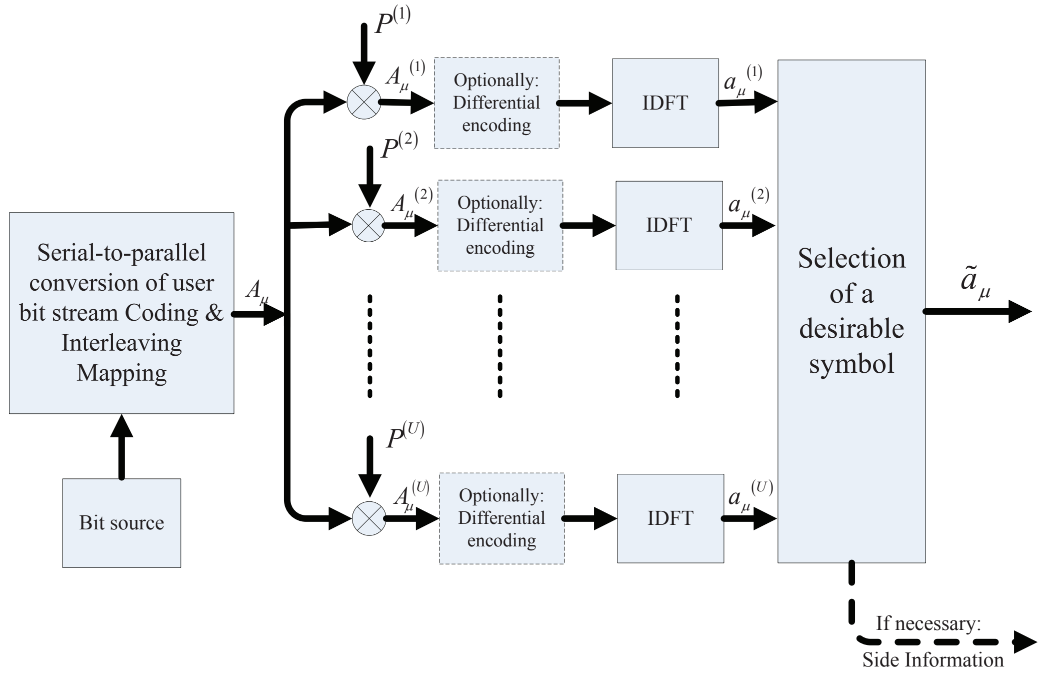

Figure 1.1: A block diagram of the SLM technique [25].

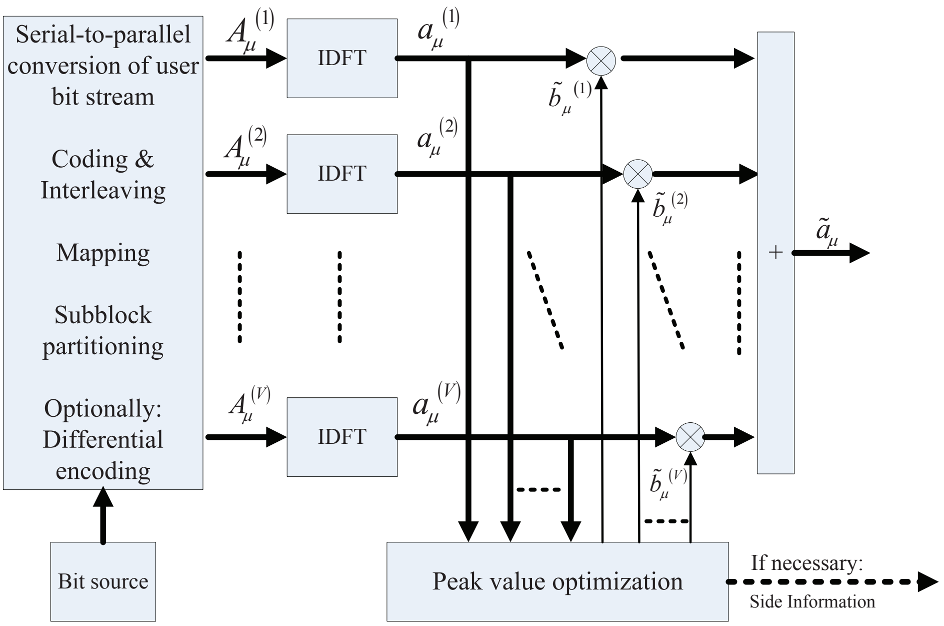

Figure 1.2: A block diagram of the PTS technique [25].

PAPR is then chosen for transmission. To recover the original data block, the receiver need only know which interleaver is used at the transmitter.

(5) Tone Reservation

TR scheme [16] is to choose the frequency-domain reserved-subcarriers as cancellation signal such that it minimizes the PAPR of TR transmitted signal.

At the receiver, symbol is demodulated in the frequency domain on a tone-by-tone basis, so the tone-reserved subcarriers can be discarded at the receiver, thus distortionless.

(6) Tone Injection

The basic idea of tone injection is to increase the constellation size so that the same data point can be mapped into multiple possible points in the expanded constellation [16]. And substituting a point in the basic constellation for a new point in the extended constellation for PAPR reduction is equivalent to injecting a tone with appropriate frequency and phase in the original signal.

At the receiver, TI does not require the extra side information, but only needs to know how to map the redundant constellations on the original one. However, comparing with TR technique, the TI technique injected signal by occupying the same frequency band as the information bearing signal, and also increases the transmitted signal's power.

(7) Active Constellation Extension

In this technique, some of the outer signal constellation points of the data block are dynamically extended toward the outside of the original constellation so that the PAPR of the data block is reduced [31]. The ACE approach can be applied to many kinds of constellation schemes with large constellation size, such as QAM, MPSK and QPSK, in which data points that lie on the outer boundaries of the constellations have room for increased margin without increasing the error probability for other data symbols. Furthermore, there is no need for data rate hit and channel side information. However, these modifications result in a power increase in the transmitted signal.

(8) Coding

Coding methods are used to reduce the PAPR by selecting appropriate codewords for transmission. For example, Block coding [14] is based on Golay sequences [32] (with dual capabilities of error correction and peak reduction), where the data sequence is embedded in a larger sequence and only those larger sequences with low peak powers are used. The data is encoded using a block code. In order to reduce the PAPR, a different sequence is transmitted where some of the data symbols are replaced by others, instead of transmitting the data symbol sequence corresponding to the codeword. The errors that are deliberately introduced could be corrected by the error correcting code. Hence, part of the error correcting capability of the code is sacrificed to PAPR reduction.

1.2.2 Criteria to Compare Among Various Schemes

Various methods need be compared and contrasted by these criteria:

(a) the PAPR reduction achieved.

(b) computational complexity at the transmitter.

(c) computational complexity at the receiver.

(d) the amount of channel side information (CSI) required to be communicated from the transmitter to the receiver.

(e) any need by the transmitter for any prior information.

(f) required/possible synergy/combination with other PAPR-reduction techniques?

Table 1.1 compare different methods based on the criteria discussed above.

Table 1.1: Comparison of various methods

Method	PAPR reduction	Distortion	Rate hit	CSI	Complexity
Clipping	High	Yes	Low	No	Low
PTS	Moderate	No	Low-High	Yes	Low-High
SLM	Moderate	No	Low-High	Yes	Low-High
ACE	Low	No	Low	No	High
Constellation shaping	High	No	High	No	High
Trellis shaping (TS)	Moderate	No	Low-High	No	High
Tone injection (TI)	Moderate	No	High	No	Low-High
Balancing	Moderate	No	High	No	Low
Coding	High	No	High	No	High

1.3 Mathematical Formulation of the Problem

Consider OFDM transmission with L subcarriers at the frequencies $\{f_\ell, \ell = 1, \cdots, L\}$, indexed by the set $\mathcal{S}$ = Label the set $\{\ell = 1, \cdots, L\}$. Of these, N are null subcarriers, with distinct indices drawn from the ascending set $\mathcal{N} = \{g_n, n = 1, \cdots, N\} \subset \mathcal{S}$. These N null-subcarriers respectively occupy the frequencies $\{f_{g_n}, n = 1, \cdots, N\}$, ordered in ascending frequency. The remaining $L - N$ subcarriers serve as data-subcarriers, with distinct indices from the ascending set $\mathcal{D} = \{h_d, d = 1, \cdots, L - N\} \subset \mathcal{S}$. These $L - N$ data-subcarriers respectively occupy the frequencies, $\{f_{h_d}, d = 1, \cdots, L - N\}$, ordered in ascending frequency. Obviously, $\mathcal{N} \cup \mathcal{D} = \mathcal{S}$, and $f_{g_n} \neq f_{h_d}, \forall n, d$.

Assigned to the data-subcarriers at $\{f_{h_d}, 1 \leq d \leq L - N\}$ respectively are the M-ary data symbols $\{\bar{x}_d, d = 1, \cdots, L - N\}$, taken from any phase-modulated and/or amplitude-modulated constellation.

Let T be the modulation interval and LT be the duration of the OFDM symbol (excluding the guard interval). The OFDM-signal's complex envelope equals

$$x(t) = \frac{1}{\sqrt{L}} \sum_{d=1}^{D} \bar{x}_{h_d} e^{j2\pi f_{h_d} t}, \qquad 0 \leq t \leq LT.$$

Assumed above is an idealized rectangular time-domain window. The subsequent cyclic-prefix extension of $x(t)$ would not alter the peak-amplitude.

The peak-to-average power ratio (PAPR) of the continuous-time signal $x(t)$ is defined as

$$\zeta^c(x(t)) \overset{\text{def}}{=} \frac{\max\limits_{0 \leq t < LT} \{|x(t)|^2\}}{\int_{t=0}^{LT} |x(t)|^2 dt}$$

For numerical calculations, the above continuous-time definition may be approximated in discrete time as

$$x_k \triangleq x\left(\frac{kT}{K}\right) = \frac{1}{\sqrt{L}} \sum_{d=1}^{D} \bar{x}_{h_d} e^{j\frac{2\pi}{KL} f_{h_d}}, 0 \leq k \leq LT$$

where K represents the oversampling factor, which is recommended to be 4 by [36] but to be 8 by [37].

Then, we introduce the complementary cumulative density function (CCDF) of the PAPR. For large L, x_k become Gaussian distributed, each with a mean of zero and a variance 0.5. And amplitude of the OFDM signal has a Rayleigh distribution with power distribution becomes a central chi-square distribution given by

$$F(\zeta_0) = 1 - \exp(-\zeta_0) \tag{1.1}$$

Complementary Cumulative distribution function (CCDF, signal samples mutually uncorrelated when non-oversampling) is given by

$$\begin{aligned} P(\zeta > \zeta_0) &= 1 - P(\zeta \leq \zeta_0) = 1 - F(\zeta_0)^N \\ &= 1 - (1 - \exp(-\zeta_0))^N \end{aligned} \tag{1.2}$$

CCDF of the OFDM signal with K as oversampling factor, approximately given by is derived as

$$\begin{aligned} P(\zeta > \zeta_0) &= 1 - P(\zeta \leq \zeta_0) = 1 - F(\zeta_0)^{KN} \\ &= 1 - (1 - \exp(-\zeta_0))^{KN} \end{aligned} \tag{1.3}$$

where ζ_0 denotes the given threshold. This expression assumes that the N time domain signal samples are mutually independent and uncorrelated.

1.4 Contributions of This Work

Listed below are the main contributions of my work presented in this thesis.

(1) In Chapter 2, a new PAPR reduction method of multi-carrier transmission system, called “Null-Switching” method, is proposed. This new approach requires no channel-side information (CSI), imposes no rate hit, is distortionless, can complement most other PAPR reduction methods (such as tone injection, tone reservation, active constellation extension, partial transmit sequences), and can be compatible with existing standards.

(2) In Chapter 3, another new PAPR reduction method of OFDM multi-carrier system, called “Null-Shifting” method is presented. Comparing with the “Null-Switching” method introduced in chapter 2, this method has low computational complexity and better symbol error rate (SER) performance. Also this proposed method is distortionless, requires less channel-side information (CSI), can complement most other PAPR reduction methods (such as partial transmit sequences, selective mapping, tone injection, trellis shaping, active constellation extension), and also can be compatible with existing standards.

Chapter 2

OFDM PAPR Reduction by Switching Null Subcarriers and Data-Subcarriers

2.1 Null Subcarriers in OFDM

Inherent in many multi-carrier standards are null-subcarriers (a.k.a., virtual / unused /unmodulated subcarriers), where no energy is transmitted.

For example, in the IEEE 802.11a/g standard (i)6 null-subcarriers serve as guard-band at the low-frequency end and 5 null-subcarriers serve at the high-frequency end as guard-bands. (ii)A mid-band null subcarrier (indexed 0) for accommodating low cost RF-filters that avoids delay of DC energy.

In the IEEE 802.11a/g standard, using some of the "innermost" null subcarriers in the guard-band is sometimes tolerable because the spectral mask has its transition-band over those null subcarriers, thereby passing a good portion of the energy in these "innermost" null subcarriers as shown in Figure 2.1. See [33] for inclusion of null-subcarriers. See [34] for sub-blocking strategies.

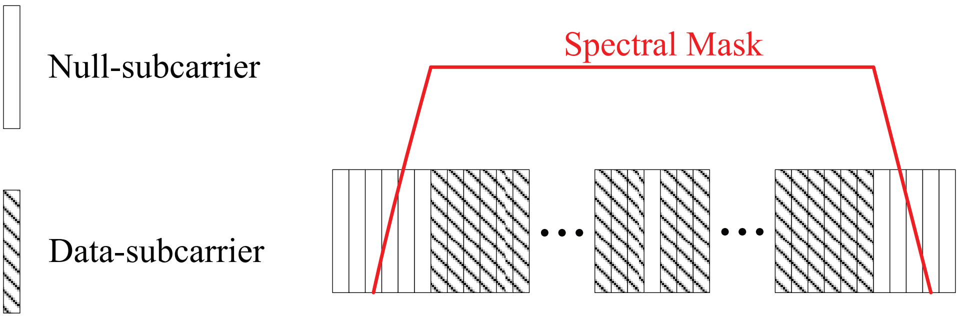

Figure 2.1: Spectral Mask

2.2 Proposed Method's Underlying Philosophy

This proposed scheme switches one or more null-subcarriers with to be identified data-subcarrier(s). This changes the input to the IFFT operator, and thus the IFFT operators output and its PAPR.

The guard-bands of many multi-carrier standards (e.g.IEEE 802.11a/g) have null-subcarriers in the transition-band of the transmit spectrum mask (i.e. the bandpass filter matched to the desired users data-subcarriers). Hence, such a null-subcarrier frequency (now switched to carry data) can pass a good portion of any energy therein onto the receiver.

For the above switching, the transmitter is to search for the data-subcarrier that (when switched with a null-subcarrier) would achieve the greatest PAPR-reduction.

2.3 Advantage of Proposed Scheme

This proposed scheme requires no channel side information (CSI) from the transmitter to the receiver, unlike some other PAPR-reduction approaches, such as selective mapping (SLM) and partial transmit sequences (PTS). Such channel side information would reduce the data-rate, and could significantly increase the bit error rate (if the channel side information is corrupted in transmission). This scheme does not distort the transmitted signal, unlike clipping. It also imposes no "rate hit", as the constellation remains unchanged at each data-subcarrier, unlike any coding-based PAPR-reduction method. The scheme is versatile, as it may be used simultaneously with any constellation-modifying PAPR-reduction scheme, such as active constellation extension, constellation shaping, partial transmit sequences (PTS), selective mapping (SLM), tone injection, trellis shaping. The proposed scheme is not a degenerate case of any constellation-modifying PAPR-reduction scheme, because the proposed scheme does not affect the constellation at the data-subcarriers. The proposed scheme may be used simultaneously with other PAPR-reduction methods that enlarge/contract/alter the group of data-subcarriers, such as tone reservation (also known as peak-reduction carrier).

2.4 The Proposed Algorithm

Without modifying L or N specified in the given OFDM-system, the proposed method switches P number of null subcarriers (i.e. P elements in $\{\tilde{g}_p, p = 1, \cdots, P\} \subset \mathcal{N} = \{g_n, n = 1, \cdots, N\}$) with P number of data-subcarriers (i.e. P members of $\{\tilde{h}_p, p = 1, \cdots, P\} \subset \mathcal{D} = \{h_d, d = 1, \cdots, L - N\}$), such that if $f_{\tilde{h}_p} < f_{\tilde{h}_{p+1}}$, then $f_{\tilde{g}_p} < f_{\tilde{g}_{p+1}}$.

With L, P, N specified, there exist altogether $\begin{pmatrix} L - N \\ P \end{pmatrix} = \frac{(L-N)!}{P!(L-N-P)!)}$ number of different "switching" possibilities. The optimization problem aims to identify which $\{\tilde{h}_p, p = 1, \cdots, P\}$ from $\{h_d, d = 1, \cdots, L - N\}$.

The above constraint (i.e. if $f_{\tilde{h}_p} < f_{\tilde{h}_{p+1}}$, then $f_{\tilde{g}_p} < f_{\tilde{g}_{p+1}}$) allows *no* channel side-information be transmitted, because: (i) The receiver has priori knowledge of $\mathcal{D}$. (ii) The received data allow the identification of $\{\tilde{h}_p, p = 1, \cdots, P\}$ on account of their low power-levels. (iii) The permutation of the P switched data-subcarriers remain unchanged after the switching. Hence, the *receiver* can "un-switch" correctly, even with *no* channel side-information.

To minimize any degradation to the guard-band, the "innermost" P null subcarriers are to be used. For the IEEE 802.11a/g standard, the number of null-subcarriers as guard-bands at low-frequency end is one more than that at high-frequency end, if P is even; the number of null-subcarriers as guard-bands at low-frequency end equals to that at high-frequency end, if P is odd.

2.5 Simulation Results

The IEEE 802.11a standard with AWGN channel model is used here as an example, even though the proposed scheme could be used with any multi-carrier system with null-subcarriers. To minimize any degradation to the guard-band, in Figure 2.2, one "innnermost" null-subcarriers (indexed as $\# - 27$ in the IEEE 802.11a standard, giving $P = 1, 100000 symbols$) is used at low-frequency side of the data-subcarrier bands, with $L - N = 48$. To minimize any degradation to the guard-band, in Figure 2.3, two "innnermost" null-subcarriers (indexed as $\# \pm 27$ in the IEEE 802.11a standard, giving $P = 2, 100000 symbols$) are used at either side of the data-subcarrier bands, with $L - N = 48$.

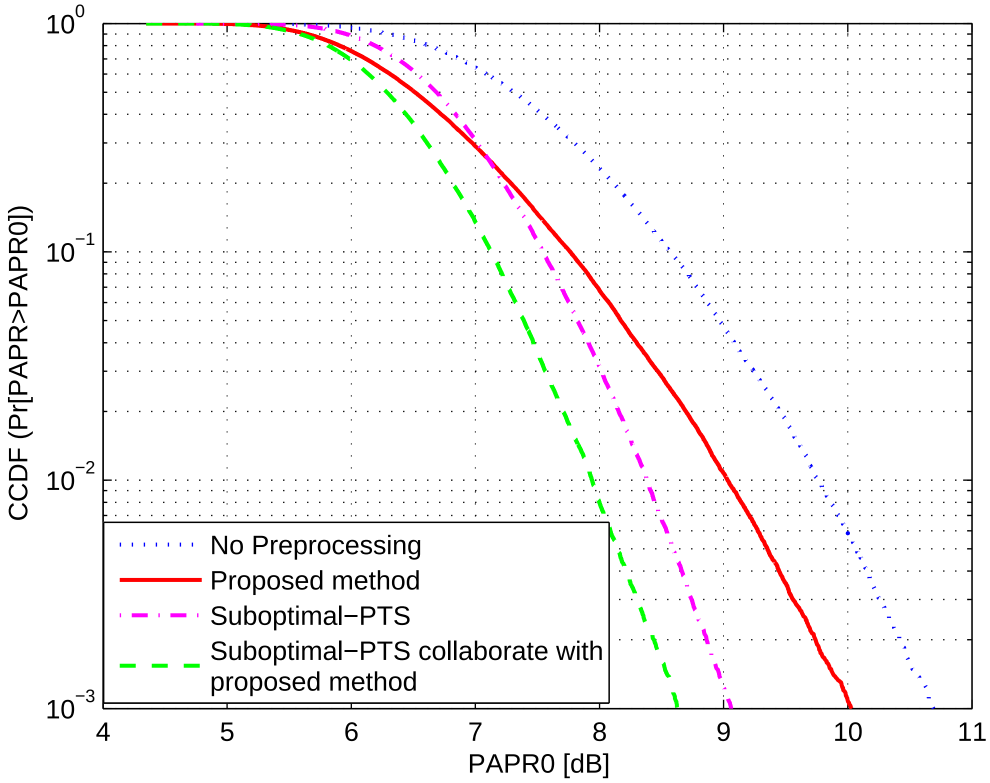

Figure 2.2: The PAPR's CCDF at $P = 1$.

A memoryless nonlinear power amplifier is used. It has a "soft limiter" input-output relationship: For a complex-value input y, the output equals $\Lambda(|y|)e^{j\angle y}$, where (see Section 2.4 on page 13 of [39])

$$\Lambda(a) = \begin{cases} a, & \text{if } a \leq A, \\ A, & \text{if } a > A. \end{cases}$$

The "clipping ratio" is defined as $\gamma = \frac{A}{\sqrt{P_{in}}}$ (see equation (2.16) in [39]), where P_{in} denotes the average power of the input signal,$\gamma = 2$ dB.

Figure 2.2 and 2.3 show the proposed schemes PAPR-reduction performance. Figure 2.4 and 2.5 show that the proposed scheme can improve the SER and Bit-Error-Rate

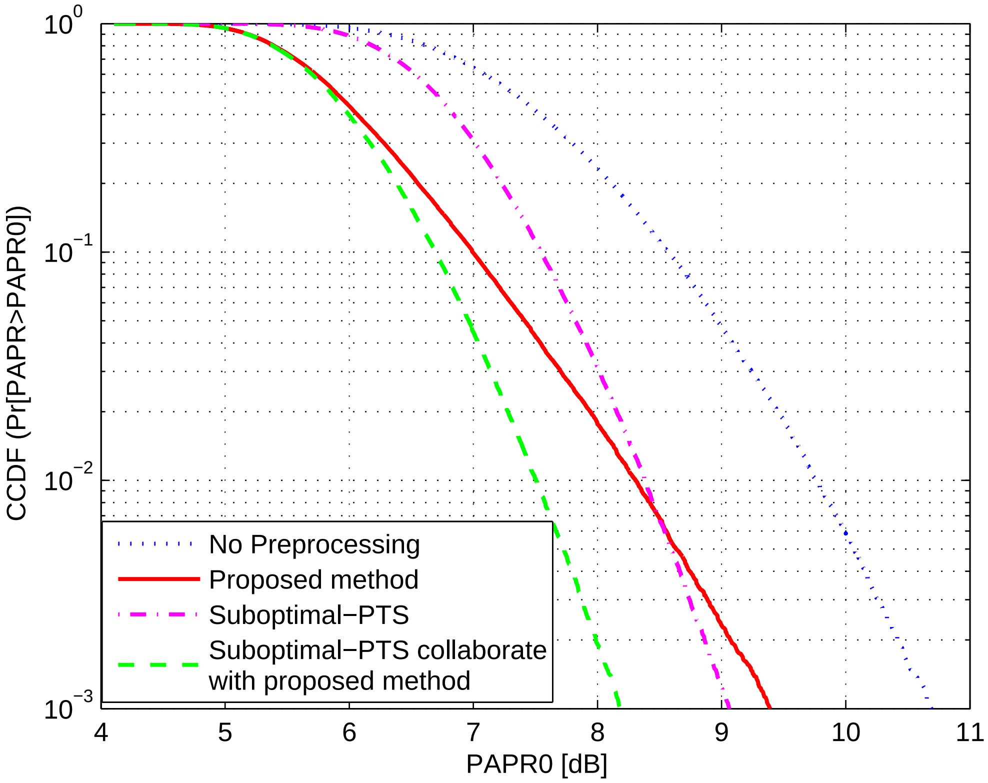

Figure 2.3: The PAPR's CCDF at $P = 2$.

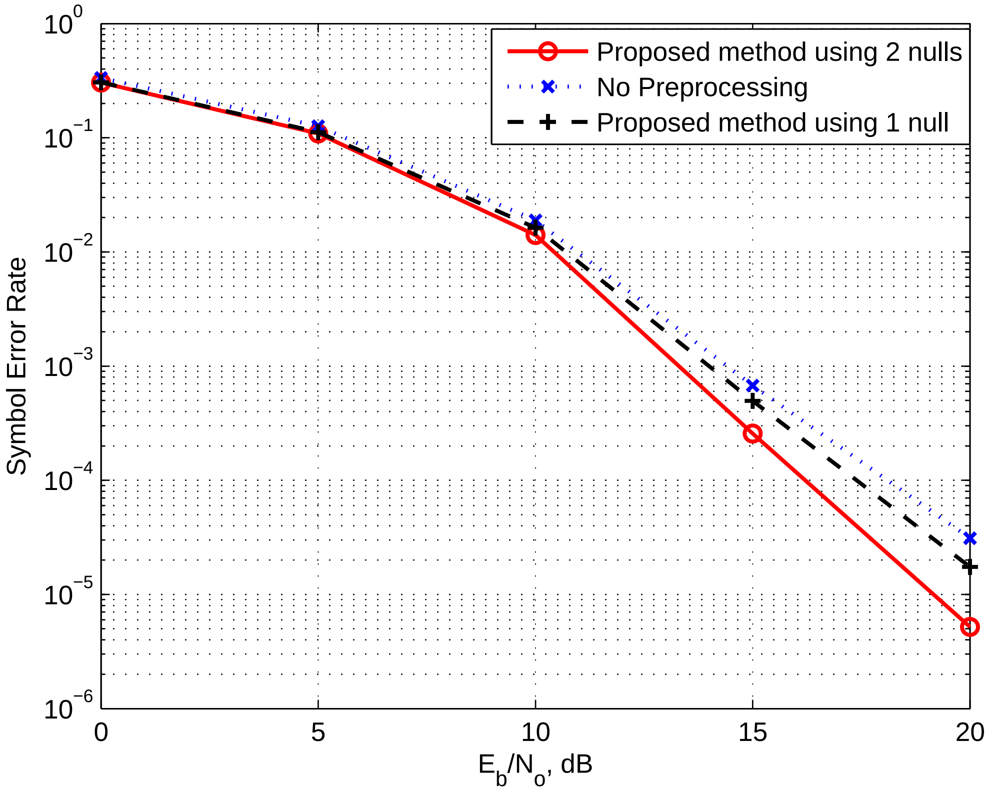

Figure 2.4: The SER versus the signal to noise ratio at $A = 1.7$.

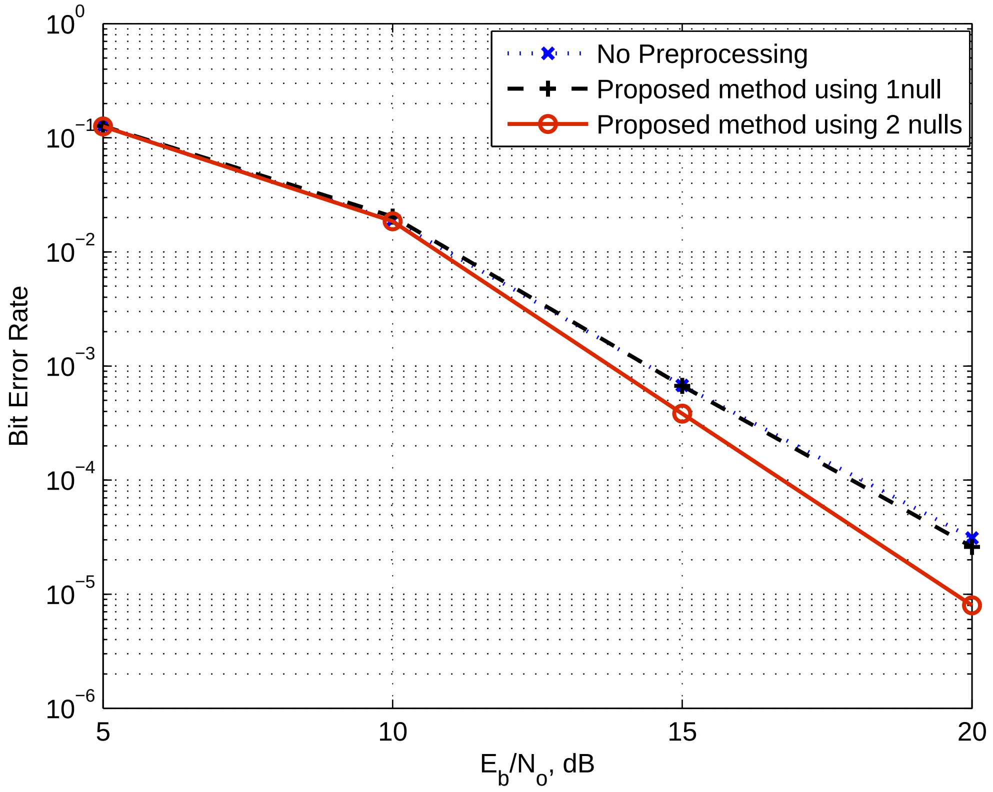

Figure 2.5: The BER versus the signal to noise ratio at $A = 1.7$.

(BER), despite needing no CSI.

2.6 Summary

Herein proposed is a new method to reduce the peak-to-average power ratio (PAPR) of multi-carrier OFDM systems by switching null-subcarriers with data-subcarriers, which is a CSI-free pre-processing algorithm, can be compatible with most existing OFDM standards (like PTS, SLM, etc), and can complement many other PAPR-reduction algorithms.

Chapter 3

OFDM PAPR Reduction by Shifting Null Subcarriers among Data-Subcarriers

3.1 The Basic Idea of Proposed Method

Using the "innermost" null subcarriers in the guard-band for PAPR reduction is originally proposed in the "Null-Switching" method [38], presented in Chapter 2. The "Null Switching" method is to switch some data-subcarrier(s) with some "innermost" null subcarrier(s) in the guard-band, in order to change the frequency(ies) of some term(s) inputted to the IFFT operator. However, this "Null-Switching" method need very high computational complexity when the number of subcarriers is large. Here, we propose a new scheme along with "reduced-complexity" version by shifting some of the "innermost" null subcarrier(s) in the guard-band among data subcarriers to minimize the PAPR at the transmitter, possessing better SER performance and low computational complexity.

3.2 Advantages of the Proposed Method

This proposed scheme possesses the following merits:

1. Unlike clipping method, this proposed method does not distort the transmitted signal.
2. Comparing with "Null-Switching" method, it has low computational complexity and better SER or BER performance.
3. The proposed scheme does not affect the constellation at the data-subcarriers.
4. This proposed method only needs to send the indices of null-subcarriers after shifting to the receiver.
5. This proposed scheme could relate with other PAPR reduction strategies , such as "partial transmit sequences", "selective mapping", "tone injection", "trellis shaping".

6. This proposed scheme can be compatible with existing standards, such as IEEE 802.11a/g.

3.3 The Proposed Algorithm

3.3.1 The Basic Version

Without modifying L or N, the proposed method shifts P elements $\{\tilde{g}_p, p = 1, \cdots, P\}$ of the null-subcarrier set $\mathcal{N} = \{g_n, n = 1, \cdots, N\}$ to the indices $\{h_{\tilde{l}_p}, p = 1, \cdots, P\}$ of the subcarrier set, such that if $f_{\tilde{l}_p} < f_{\tilde{l}_{p+1}}$, then $f_{\tilde{g}_p} < f_{\tilde{g}_{p+1}}$. Meanwhile, the permutation of the data-subcarrier set remain unchanged after the shifting. The transmitter is to search the most advantageous indices for the P "innermost" null subcarrier(s) to shift to, for the greatest PAPR reduction. The optimization problem is to identify $\{h_{\tilde{l}_p}, p = 1, \cdots, P\}$ to minimize the PAPR. There are altogether $\binom{L-N}{P} = \frac{(L-N)!}{P!(L-N-P)!)}$ different "shifting" possibilities.

For the IEEE 802.11a/g standard, the number of null-subcarriers as guard-bands at low-frequency end is one more than that at high-frequency end, if P is even; the number of null-subcarriers as guard-bands at low-frequency end equals to that at high-frequency end, if P is odd.

This proposed scheme would require low CSI be communicated from the transmitter to the receiver. This advantage is due to these three properties: (a) the receiver has priori knowledge of the indices of data-subcarrier set; (b) only the indices of $\{h_{\tilde{l}_p}, p = 1, \cdots, P\}$ is to be sent to the receiver, and $6P$ bits as CSI is needed to transmit for IEEE 802.11a/g standard. (c) the permutation of the data-subcarrier set remains unchanged after the shifting.

3.3.2 A Reduced-Complexity Version

A sub-optimal method:

The OFDM symbols $\bar{x}$ are partitioned into V disjoint subblocks $\bar{x}^{(v)} \triangleq [\bar{x}_0^{(v)} ... \bar{x}_{N-1}^{(v)}]$ with $\bar{x}_k^{(v)} = \bar{x}_k$ or 0, $0 \leq v \leq V-1$, such that

$$\bar{x} = \sum_{v=0}^{V-1} \bar{x}^{(v)} \tag{3.1}$$

Only the subblock with the largest PAPR will undergo the basic scheme in section 3.3.1. Thus, the number of "shifting" possibilities is reduced significantly. There are altogether $\binom{(L-N)/V+P}{P} = \frac{((L-N)/V+P)!}{P!((L-N)/V)!}$ different "shifting" possibilities. With the increasing of V, the computational complexity is reduced dramatically. For example, when $P = 2$, $V = 8$, there are only 28 different "shifting" possibilities.

3.4 Simulation Results

To verify the proposed PAPR reduction technique, herein the IEEE 802.11a standard with AWGN channel model is used as an example. We consider the number of subcarriers in an OFDM system equals 64 with QPSK data symbols, where $L - N = 48$ is the number of data-subcarriers, and P is the number of null-subcarriers used for PAPR reduction. To minimize possible degradation to the guard-bands, we use the "innermost" null-subcarrier(s) at low-frequency edge or high-frequency edge. In order to generate the CCDF of the PAPR, 10^5 OFDM data-blocks are generated randomly. For the "Reduced-Complexity" version, the signal block is partitioned into $V = 4$ subblocks, and only the subblock with the largest PAPR undergoes the proposed switching scheme. In figure 3.1 and figure 3.2, the indices of the two "innermost" null-subcarriers are -27 at the low-frequency edge and 27 at the high-frequency edge.

A memoryless nonlinear power amplifier is used. It has a "soft limiter" input-output relationship: For a complex-value input y, the output equals $\Lambda(|y|)e^{j\angle y}$, where (see Section 2.4 on page 13 of [39])

$$\Lambda(a) = \begin{cases} a, & \text{if } a \leq A, \\ A, & \text{if } a > A. \end{cases}$$

The "clipping ratio" is defined as $\gamma = \frac{A}{\sqrt{P_{in}}}$ (see equation (2.16) in [39]), where P_{in} denotes the average power of the input signal,$\gamma = 3$ dB.

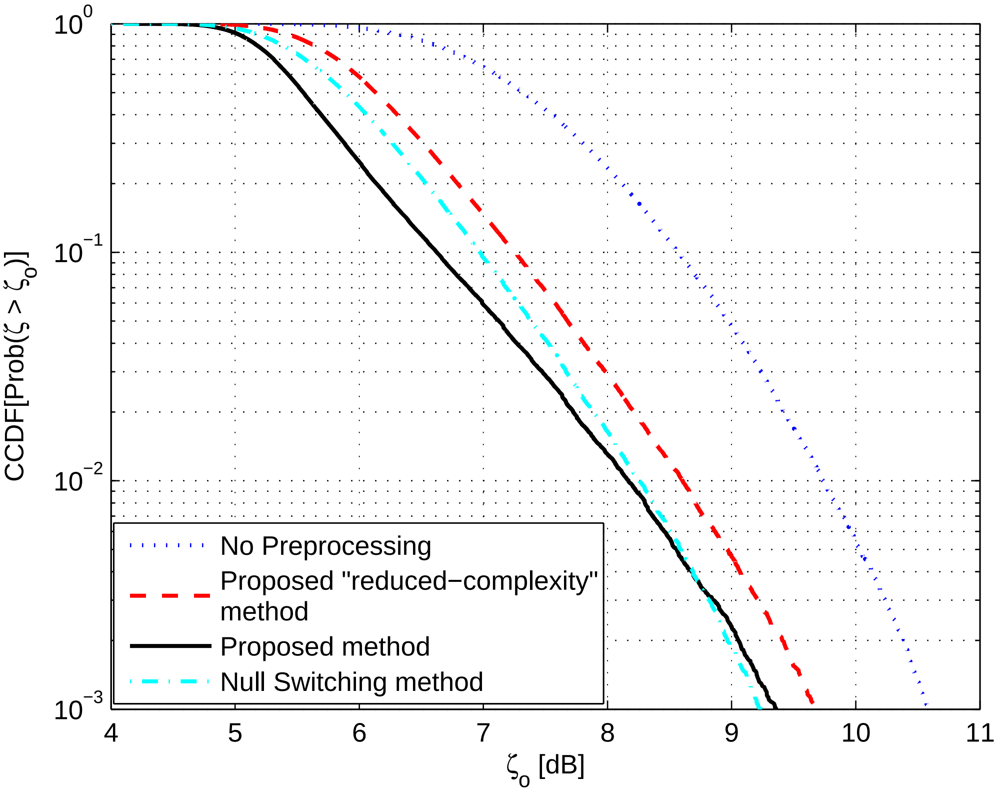

Figure 3.1: The PAPR's CCDF at $P = 2$.

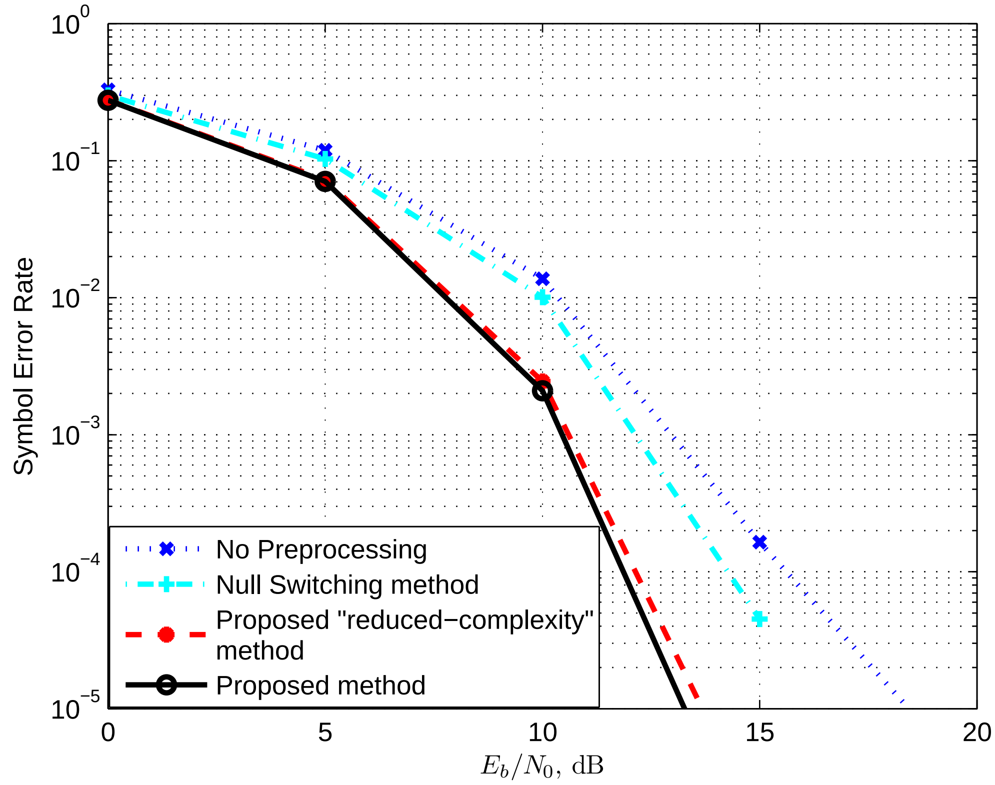

Figure 3.2: The SER versus the signal to noise ratio at $P = 2$.

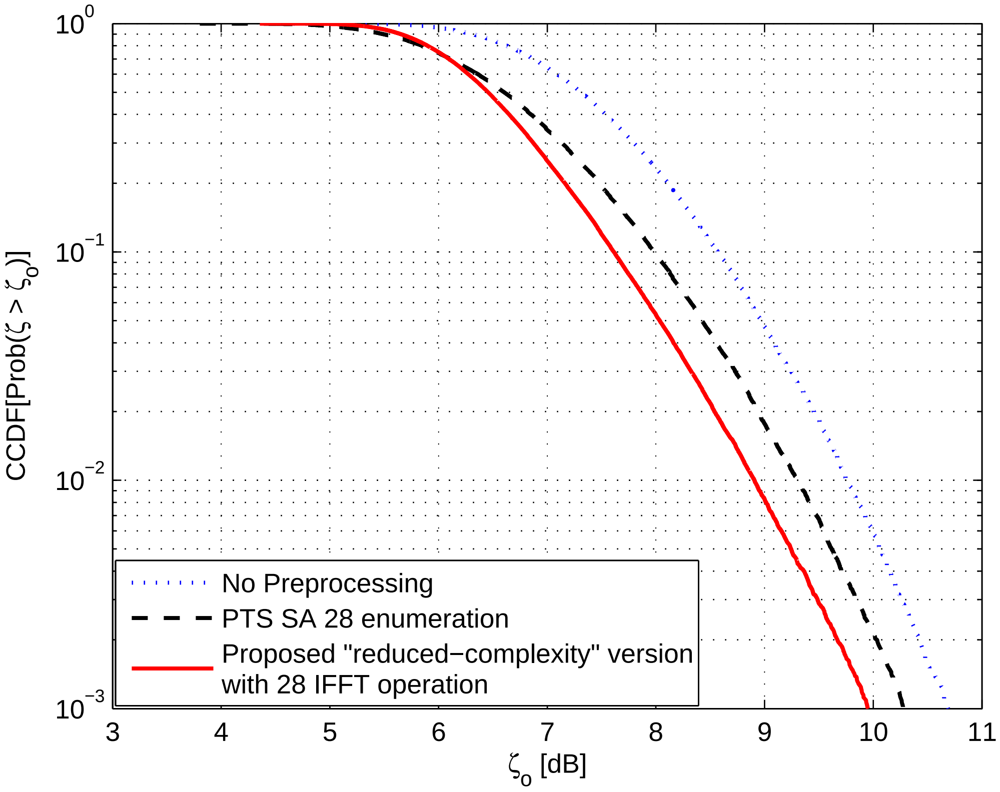

Figure 3.3: The PAPR's CCDF at $P = 2, V = 8$, QPSK modulation, 4 phase factors.

Table 3.1: Comparison of Null Switching, Null Shifting and “Reduced-Complexity” Null Shifting (P=2,V=8,IEEE 802.11a standard)

Criteria	Null Switching	Null Shifting	“Reduced-Complexity” version
PAPR reduction	High	High	Moderate - High
Distortion	No	No	No
Rate hit	No	Low	Low
CSI	No	Low	Low
SER reduction	Moderate	High	High
Computation Complexity	1128 IFFT operation	1128 IFFT operation	28 IFFT operation

Figure 3.1 illustrates that the PAPR-reduction performance of proposed method is better than that of “Null Switching” method in most cases, and that of “Reduced-Complexity” version is close to that of the basic version.

Figure 3.2 shows that the SER performance of proposed method is much superior than that of “Null Switching” method, and that of “Reduced-Complexity” version is almost same as that of the basic version.

In Figure 3.3, “Reduced-Complexity” version with 28 IFFT operations, only using 12 bits CSI, which is less than the 16 bits CSI of PTS SA method, still has better PAPR reduction performance than PTS SA method under same computational complexity.

The comparison of these three methods based on different criteria is presented in Table 3.1. And the computational complexity of proposed methods has been compared by specified values, based on same condition.

3.5 Summary

In this chapter, we propose a new scheme by reordering the null-subcarriers and data-subcarriers. This new method shifts the “innermost” null-subcarriers among different data-subcarriers at different indices to minimize the PAPR. The Reduced-Complexity version degrades the computational load by applying this idea only to the subblock with the highest PAPR. This proposed method is distortionless, does not affect the constellation at the data-subcarriers, has low computational complexity and better SER performance, can be related with most other PAPR-reduction methods, and can be compatible with existing standards.

Chapter 4

Conclusion

In Chapter 2, we propose a new method to reduce the peak-to-average power ratio (PAPR) of multi-carrier OFDM systems by switching null-subcarriers with data-subcarriers, which is a CSI-free preprocessing algorithm in frequency domain, can be compatible with most existing OFDM standards, and can complement many other PAPR-reduction algorithms.

In Chapter 3, the "Null Switching" method need very high computational complexity when the number of subcarriers is large. Although this method remove the need of CSI, the BER performance is not good enough when the signal-to-noise ratio is low. Here, we proposed a new scheme along with "Reduced-complexity" version by shifting some of the "innermost" null subcarrier(s) in the guard-band among data subcarriers to minimize the PAPR at the transmitter, possessing better SER performance and low computational complexity.

Some future work may be conducted, such as

(1) Investigate the trade-off between the computation load, BER performance, and other criteria.

(2) Test and modify the methods proposed in chapter 2 and chapter 3 based on other international standards, like IEEE 802.16e, IEEE 802.15, IEEE 802.20, etc.

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