CO-MARKETING ALLIANCE FOR INTERNATIONAL BRANDS

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M.Phil

The Hong Kong Polytechnic University

2011

The Hong Kong Polytechnic University

Department of Logistics and Maritime Studies

CO-MARKETING ALLIANCE FOR INTERNATIONAL BRANDS

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A Thesis Submitted

In Partial Fulfillment of the Requirements for

the Degree of Master of Philosophy

July 2011

CERTIFICATE OF ORIGINALITY

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ABSTRACT

Among various types of cooperation strategies, co-marketing partnership is one kind of lateral relationship between firms at the same level in the value chain. Although co-marketing partnership is growing in popularity recently, it has received little attention in the literature. This thesis aims to quantify the benefit of co-marketing partnership and to provide a game theoretical perspective on how this type of cooperation creates value for partnering firms and even international brand, their common upstream suppliers.

This thesis examines equilibrium strategies and performance of both first best situation and decentralized-decision situations. In a first-best situation, we explore the potential of a co-marketing alliance and find that its value (i.e. an additional and different marketing effort) is mainly driven by the complementing power and the reduction in cost because of the co-marketing effort. In situations where firms make their decisions separately, we compare four models with a benchmark model (before alliance) under two different business scenarios and with two different profit-sharing mechanisms. We work out the equilibrium strategies for the firms involved in all these situations, and derive market conditions under which co-marketing alliance activities can improve supplier's and distributor's performance. These conditions can provide the theoretical basis for managers to decide how to improve firms' performance during the alliance process

It is worth noting that as shown by our results, the value of a co-marketing alliance is determined by the complementing power brought by the co-marketing partner and the cost reduction effect brought by this new structure of cooperation. In a first-best situation, the alliance can benefit from an extra effort put in at a lower effort cost. In the situations where players make their decisions separately, it is intuitive that the original system can achieve better performance when a third party introduced, a co-marketing partner, can bring positive influence to demand. Unexpectedly, we find that even when the co-marketing partner might bring negative influence to the demand, the firms involved still achieve better performance under certain market conditions. It is due to the positive influence brought by this new structure of partnership which compensates the effort cost put in. In addition, as indicated in the numerical studies, the cooperation activities can help the firms with promoting products of even lower price sensitivity. It is also worth noting that the co-marketing cooperation activities are especially suitable for the products with relative low price sensitivity in the market.

The numerical studies add to the implications by comparing the performance of supplier and distributor between the models. For Scenario I where the distributor pays for the co-marketing partner's effort, the performance with revenue sharing mechanism is better than that of fixed-payment mechanism under the same settings. While in Scenario II where the supplier pays for the co-marketing partner's effort, fixed payment is favor to improve the supplier's performance compared with revenue sharing mechanism.

ACKNOWLEDGEMENTS

I would like to express my gratitude to all those who helped me during my M.Phil studies and in completing this thesis.

I would like firstly to express my deepest and sincere gratitude to my supervisor, Prof. Liming Liu, for his constant support, insightful guidance and constructive comments throughout this research and for reviewing this thesis. I would also like to thank Dr. Hengqing Ye for having been my supervisor and for having offered me so much help since this July.

I wish to express my special thanks to Dr. Xubing Zhang. He has provided many valuable suggestions on this thesis and I have benefited a great deal from his advice. I would like to thank Dr. Li Jiang and Dr. Pengfei Guo for teaching me foundation research courses, and for their encouragement.

I would like to thank Hong Kong Polytechnic University for providing good environment for research and thanks to the assistance of the wonderful staff from the Research office and Department of LMS. I also wish to thank my friends and classmates. Thank you all for your support and advice in both research work and life.

I would like to express my thanks to Mr. LAI King for providing such an interesting case, which is an important motivation of this thesis. And I also want to thank LEO Group and DCH Logistics Company for offering us opportunities for interviewing and company visits.

Finally, I would like to thank my parents and my family members, who always give me strong support and comfort.

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CHAPTER 1 INTRODUCTION

In August 2009, LEO Paper Group, a leading global printing company, and DCH Logistics Company, a global logistics company and the Asian distributor of many international brands such as Heineken, Pocari and Sunraysia, Pizza Hut, Seven-Eleven and Wal-Mart (China), announced a strategic alliance to offer their international upstream suppliers a co-marketing solution package from product designing, printing, packaging to distribution service. In this alliance, LEO provides packaging design and printing services, and DCH provides the corresponding logistics services. Both companies are headquartered in Hong Kong, and are market leaders in their respective industry segments, i.e., paper printing and logistics, especially in Hong Kong markets. More specifically, DCH client base covers fast moving consumer products, restaurants fast food chains, cosmetics, food and seasoning from all over the world. Through the alliance, LEO garners new business by serving DCH's existing customers, and DCH enhances its customer loyalty through the added value service offered by the alliance. International suppliers of DCH not only greatly benefit from sharply reduced import duties and distribution costs but also enhance the popularity of their products in the local market.

This kind of cooperation is a fascinating example of the so-called "co-marketing" partnerships. Among various types of co-operative strategies, co-marketing partnerships are becoming more and more prominent in the global environment. The activities are growing in popularity and "involve considerable sums of money" (Ebenkamp, 2007).

Unlike vertical partnership such as manufacturer-distributor partnership, co-marketing partnership are defined as “lateral relationships between firms at the same level in the value added chain … they are contractual relationships undertaken by firms whose respective products are complements in the marketplace” (Bucklin and Sengupta 1993). However, co-marketing alliance has received little attention in the literature. With the same customer base, partnering firms from different industries complement their advantages and expertise through co-marketing alliances and are able to jointly offer value-added products or service to their common customers.

Co-marketing alliances hold huge potentials. In the case of Leo and DCH’s alliance introduced above, one of DCH’s international suppliers, Almond Roca, a famous American chocolate manufacturer, has to spend a long time and incur high cost of repacking and handling in USA of its products for the Chinese market. With the value-add service offered by the alliance, Almond Roca is able to shorten the time required for delivering its products to market, and save almost 30% of material cost and 25% of logistics cost. Especially, Almond Roca estimates that the alliance might help generate 60% more sales due to the sharply reduced cost. The case of LEO and DCH demonstrates that how co-marketing partnerships create value for international brands. Another good example is in tourism service industry. It is common to combine carrier service, hotels, guided tour lines, restaurants and night clubs to offer the foreign tourists a travel package that is more convenient and modestly priced. In addition, a well-known example in telecom industry is the cooperation between Apple Inc. and China Unicom. China Unicom has developed 3G networks for iphone series in China, and has collaborated with Apple’s local distributor to jointly market smart phone such as iphone 4 in Chinese market. Therefore, it is useful to explore the process of value creation by co-marketing partnership so that managers can make decisions accordingly.

The analytical framework is built upon game-theoretical models involving three parties: the two partnering firms and an international brand, playing the role of "supplier". Prior to the formation of alliance, the supply chain consists of the international supplier and a local distributor. The local distributor places an order according to market demand, and the ordered quantities, in marketable packages, are shipped from the home country to the target market and are distributed to the market or downstream retailers. Under the co-marketing partnership, a third party joins in with the distributor to provide a value-added service that may enhance the supplier's product appeal to local market and reduce the distributor's unit import cost. The international supplier can choose to deal with either the distributor or directly with the third party, which we define as the "co-marketing partner". Both scenarios are considered to examine the impact of alliance on all the three parties and their supply chain. We also identify the necessary conditions for maintaining the alliance relationship and work out equilibrium strategies and profit of supplier, distributor and co-marketing partner under all these situations. These implications can be used for deciding whether to adopt a co-marketing alliance and how to improve firms' performance in the alliance.

We aim to explore how this type of partnership creates value for the three parties involved, i.e., international supplier, local distributor and a co-marketing partner of the distributor. We examine the business potential brought by this new mode of cooperation by first considering a first-best situation, assuming that perfect coordination between supplier, distributor/retailer and the co-marketing partner can be achieved. We compare the first-best situation with the situation before alliance. The results indicate that when the effort level from a third party (co-marketing partner) has a positive influence on market demand, the optimal system profit increases. We also find that even when the effort from the co-marketing partner has a negative influence on demand, the optimal system value may still increase due to cost savings in this kind of alliance structure under certain market conditions.

We later will examine the situations where players involved make their decisions separately, as they do in reality. We construct several three-stage games to discuss two business scenarios and two profit-sharing mechanisms. To model the alliance process, we consider two different business scenarios according to the industry practices. The scenarios are decided by the party who should pay for co-marketing partner's effort. Situations in reality vary due to different nature of services offered and different requirements from international suppliers. We also consider two profit-sharing mechanisms-- revenue sharing and unit fixed-payment between the partnering firms inspired by industrial examples presented earlier. We are principally concerned with equilibrium strategies the involved firms adopt under different situations. All these discussions shed light on how firms involved perform in co-marketing alliances.

Moreover, we compare the four models under two business scenarios and two profit-sharing mechanisms with a benchmark model that includes only an international supplier and a local distributor, and establish the conditions under which the firms will be better off than the performance of the original system. We reach the conclusion that the conditions are related with market size, price sensitivity, effort-demand sensitivity and cost-benefit ratio of the co-marketing partner. Our analysis reveals that the main influence brought by a co-marketing partner includes the following two aspects: (1) supplier's and distributor's revenue, arising because of the extra effort output from distributor and co-marketing partner under alliance; (2) the cost of effort input, influenced by distributor's and co-marketing partner's equilibrium strategies. In a first-best situation, the alliance can benefit from both extra effort output and lower effort cost. While in the situations of decentralized decisions, the costs of effort increase in the situations where players separately make their decisions. However, the benefit of effort output under these cases compensates the costs of effort input under certain conditions.

The rest of the thesis is organized as follows.

Chapter 2 provides an overview of the relevant literature. Previous work on related issues is summarized and discussed, and complementarities of this work are explained.

Chapter 3 describes formulation of the model in terms of demand and cost functions, two different business scenarios, two profit-sharing mechanisms and timing of the game. We fully discuss our model and define the relevant marketing environment. Assumptions of the model are also explained.

Chapter 4 first explores the first-best situation under co-marketing partnership and compares performance of systems before and after the cooperation. Later, we develop the models under two business scenarios where partnering firms choose to adopt profit sharing mechanisms of revenue sharing or unit fix-payments. We derive their analytical equilibrium strategies under all these situations and the relevant necessary conditions to ensure the uniqueness of equilibrium strategies. We also establish the conditions under which firms' performance is likely to be better than that of the original system.

Chapter 5 summaries the entire work and presents our conclusions. We also present the managerial implications of our results.

CHAPTER 2 LITERATURE REVIEW

In this chapter, we review the existing literature related to co-marketing alliance and profit-sharing mechanisms adopted in alliance activities under different business scenarios.

Researchers have examined several issues related to alliances in recent years, covering topics such as stability of alliances (e.g., Das and Teng, 2000), inter-partner learning (e.g., Osland and Yanrak, 1995; Anand and Khanna, 2000; Hamel, 1991), alliances processes and evolution of alliances and outcomes (e.g., Zollo, Reuer and Singh, 2002; Das and Teng, 2002; Doz, 2007). Although there is a rich mass of literature focusing on empirical examination of alliance processes and alliance outcomes evaluation, little attention has been paid to the motivation that drives the formation of alliances and the quantitative benefits brought by a certain type of alliance to the partnering firms. This thesis aims to explore how a co-marketing alliance motivates the partnering firms' cooperation and makes the firms better.

To explore motivations that drive the formation of alliances, scholars have classified alliances according to their attributes. Bleeke and Ernst (1993) developed a classification based on business relationship of allied firms. They divided the alliances into six types: collisions between competitors, alliances of the weak, disguised sales, bootstrap alliances, evolutions to a sale and alliances of complementary equals. They further pointed out that alliances of complementary equals are likely to last much longer than other types. Varadarajan and Cunningham (1995) provided a review of major contributions to the literature, describing broad categorizations such as alliances between firms with complementary and different resources, firms that access similar resources to lower cost, functional categorizations such as joint manufacturing, marketing or product development agreements, and intra- and inter-industry categorizations. Jarratt (1998) summarized the previous work and pointed out the existence of a vertical partnership between retailers and suppliers in the value chain, and lateral partnership between retailers and firms in another industry at the same level in the value chain. The two partnerships allow a company to access and configure capabilities or resources in a way not easily replicable by competitors, thus providing enhanced effectiveness in current markets or facilitating entry into new markets. This kind of classification is generally more accepted by researchers.

Co-marketing alliance, discussed in this thesis, belongs to complementary resources combination. Defined as "lateral relationships between firms at the same level in the value added chain … they are contractual relationships undertaken by firms whose respective products are complements in the marketplace" (Bucklin and Sengupta 1993), co-marketing alliances represent a form of "symbiotic marketing", which was first proposed by Adler (1966). Adler had forecast decades ago that this type of alliances "will become more and more important to business" and qualitatively analyzed the different possible modes of these alliances and the corresponding benefits. Today, the increasing number of these alliances has proven Alder's prediction. Varadarajan and Rajaratnam (1986) revisited Alder's paper and identified the differences in the nature and scope of symbiotic marketing partnerships. They explored the implementation of intensive growth and diversification strategies by citing examples of joint promotion, market development and market penetration activities.

Buklin and Sengupta (1993) defined co-marketing alliance as a form of "working partnership" that involves "contractual relationship undertaken by firms whose respective products are complementary in the marketplace". They empirically examined the characteristics of co-marketing alliances and concluded that "co-marketing alliances prosper when projects have been well selected, partners chosen carefully, and relationship structured toward balance." The authors also identified that co-marketing alliances are of significant value for research as they could provide sustainable competitive advantages for partnering firms. Wilfred did a series of research works on alliances of the same function such as alliances with similar resource and cross-functional, i.e., alliances with complementary resources. The authors (2000) first built theoretical models of three types of alliances and compared the results under the situation of same-function alliance, parallel development and cross-functional alliances. Later they (2005) established a two-stage model to explain how the number of networks and technology platforms and market sensitivity affects investments of partnering firms. Recently, they examined alliances in terms of similarity of resources and the number of partners in an alliance and found that partners in cross-functional alliances may invest more in their respective alliances than those in a same-function alliance. Our research complements their work by establishing theoretical models to discuss quantitative benefits of co-marketing alliances in the globalized business environment instead of empirical examination and qualitative analysis. We explore performances of partnering firms and their common upstream suppliers and examine how co-marketing alliances benefit all the parties involved in the alliance.

The most popular methodology recent years in modeling the alliance motivation and stability in operation management area is to adopt the cooperative game theory. The cooperative game theory can be applied in a situation where "a group of decision makers undertake a project together in order to increase the total revenue" (Imma Curiel, 1997). The cooperative game theory addresses two main concerns: (1) how to execute the project as a whole in an optimal way; and (2) how to allocate the surplus or share cost among the participants. Nagarajan and Bassok (2008) used the Nash bargaining concept to analyze the situation where a single assembler buys complementary components from multi-suppliers and assembles the components into the final products. In their model, first, a group of suppliers forms an alliance and then negotiates with the assembler to allocate profit. Yin (2010) discusses how market demand conditions influence pricing decisions among a group of perfectly complementary suppliers. She analyzed stability of the coalition under exponential demand, isoelastic demand and linear-power demand functions. Our paper, however, does not consider adopting the cooperative game theory, for two main reasons: (1) compared to the total surplus an alliance generates, we are more concerned with individual benefits and equilibrium strategies under decentralized decision making by individual partners. Especially, the participants involved in our model play different roles in the supply chain, including supplier, distributor or retailer, and a co-marketing partner, who is at the same value chain level as the distributor/retailer; (2) another aim of our paper is to discuss how different profit-sharing mechanisms between partnering firms influence firms' performances. We conduct several three-stage game theoretical models to analyze performance of the alliance under different scenarios.

Another issue involved in this thesis is profit-sharing mechanisms used in the alliance. Different contracts adopted can influence on marketing decisions and/or the alliance outcome. Chisholm (1997) examined and compared the choice between profit sharing and fixed-payment compensation using data of contracts from the motive industry. Sharing compensation constitutes an incentive to transfer a percentage of company's profits, while fixed-payment compensation implies a fixed payment without considering profit performance of the firms. Wilfred (2000) discussed alliance performance under equal profit sharing (each partner wins half of the total profit) and proportional profit sharing (partners share the gains in proportion to their individual investments). However, in real business situations, alliance managers find it difficult to precisely evaluate inputs of different partners. Nault and Tyagi (2001) explored two implementable mechanisms of horizontal alliances: a linear transfer of fees and an equal share in alliance profits generated from a royalty on each member's sales. They concluded that these two incentive mechanisms might be more useful to coordinate the situation where customers are mobile in different regions, that is to say, to coordinate demand externalities among alliance members. Pavan and Ramarao (2010) considered an adverse selection model and explored two types of contractual agreements, including outcome-based and action-based contracts, in the organizational settings with varying levels of demand externality. They focused on the impact of information asymmetry, and their results reveals that the nature of demand externality among the partner plays a critical role in the choice between the two contracts they discussed.

Our work focuses on two common profit-sharing mechanisms usually adopted in alliance: revenue-sharing mechanism and fixed-payment mechanism. We are interested in exploring how the profit-sharing mechanisms influence firms' decisions.

CHAPTER 3 THE MODEL

In this chapter, we develop our model to explore how a co-marketing alliance motivates partnering firms. A co-marketing alliance can improve the performance of the product and has potential to enhance end-customer demand, or it can help reduce the supplier's cost due to the type of service the co-marketing partner offers. We model a supply chain system with one supplier and one distributor. In the initial system, the supplier ships products to the retailer at an ex ante wholesale price, and the distributor sells the products to the market. Under a co-marketing alliance, a third party, who is at the same level in the value chain as the retailer and plays the role of the retailer's co-marketing partner, offering complementary products or services that help promote supplier's products. In general, the co-marketing partner may have an expertise in certain aspects of value-added services, technology or product development. The two firms, retailer and the co-marketing partner, make joint efforts to generate additional revenue by sharing cost or enhancing the demand.

3.1 Demand function

Prior studies have proved that in many settings, downstream distributor or retailer's sales effort has a significant influence on market demand (Jeuland,1983; Krishnan, 2004). Downstream distributor or retailers can influence demand by advertising, sales promotion or offering value added service and guiding end-customers purchases with sales personnel. Given effort-dependent demand function $D_0 = y(p, e_1)$, D is a joint function of price p and effort level e_1 without alliance. To gain insights into how the alliance operates, we consider a common specific demand curve:

$$D_0 = a + ke_1 - bp$$

where a denotes the market size, and k and b represent the effort sensitivity and price sensitivity of demand, respectively. This requirement is not restrictive and captures many other forms of effort. For example, the additive effort model can be rewritten as a multiplicative model (Krishnan, Kapuscinski and Butz, 2004).

Impact of a co-marketing alliance can be reflected in two aspects. On one hand, a co-marketing alliance can broaden the total market size, i.e., more customers may be attracted from alternative markets. On the other hand, the alliance might influence demand by influencing price elasticity, which means existing customers are willing to pay a higher price for the product. To better analyze the influence brought by co-marketing alliance, we model our demand under alliance with the multiplicative form of $D = g(e_2)(a - bp + ke_1)$, where $g(e_2)$ is a function about effort level from the co-marketing partner, representing the demand influence brought by the co-marketing partner. We model this demand influence $g(e_2)$ as a concave function, that is to say, $\frac{\partial g(e_2)}{\partial e_2} > 0$, and $\frac{\partial^2 g(e_2)}{\partial {e_2}^2} < 0$. This fits the fact that the marginal effectiveness of effort is decreasing.

Another important reason for adopting the multiplicative form of demand function is the complementary sales efforts offered by the distributor/retailer and the co-marketing partner. A co-marketing alliance depends on cooperation between distributor/retailer and the co-marketing partner in a spirit of teamwork. The complementary nature of their activities and specialized skills that these activities entail imply that the two partnering firms come together to realize the value of alliance. An additive form of demand function, such as $D = g(e_2) + (a - bp + ke_1)$, implies that the choice of one effort level is independent of the other, which does not represent the spirit of a co-marketing alliance.

In our model, we use the specific form: $g(e_2) = e_2^{\alpha}$, $0 < \alpha < 1$, where α measures the impact of under the co-marketing alliance on demand, for our analytical framework. Thus, the market demand the alliance would face would be

$$D = (a + ke_1 - bp)e_2^{\alpha}$$

In a price-sensitive market environment, we assume that when facing market demand, the distributor/retailer will sell all products by choosing an appropriate price.

3.2 Cost function

Costs involved in alliance activities mainly include supplier's production cost and effort puts in by the distributor/retailer and the co-marketing partner. We use c to denote unit production cost of supplier, $V_1(e_1)$ to denote the cost of effort of the distributor, and $V_2(e_2)$ to represent the cost of effort from the co-marketing partner.

To be consistent with the literature (Cohen and Klepper, 1992, 1996; Krishnan, 2004), we assume convex effort cost rather than linear effort costs. Zangwill (1968) pointed out that the linear cost assumption is often not realistic, and usually leads to extreme results. Convex costs are often attributed to "diminishing returns from R&D expenditures or to diseconomies of scale that, in practice, can be linked to bureaucracies in a larger firm that stifle creativity and impede innovation" (Bhaskaran and Krishnan, 2009). Therefore, we assume that both $V_1(e_1)$ and $V_2(e_2)$ are strictly increasing and convex, which means $V_i(e_i) > 0$, and $\frac{\partial V(e_i)}{\partial e_i} > 0$, $\frac{\partial^2 V(e_i)}{\partial e_i^2} > 0$, for $i = 1, 2$. Throughout this thesis, we use β_1 and β_2 to denote the convexity parameters of distributor/retailer and co-marketing partner, respectively, and $\beta_1, \beta_2 > 1$. Consequently, the effort cost of distributor/retailer $V_1(e_1)$ and the effort cost of co-marketing partner $V_2(e_2)$ would be respectively $V_1(e_1) = e_1^{\beta_1}$ and $V_2(e_2) = e_2^{\beta_2}$, respectively.

We also assume that the supplier only has a unit production cost of c during the production process, and other fixed costs such as production set up cost do not affect the decisions and will be ignored in our models.

3.3 Two different business scenarios

After introducing a co-marketing partner in the supply chain system, the distributor/retailer and its partnering firm work together to realize the value of cooperation.

In our model, the distributor/retailer contributes its effort mainly to maintain its traditional function in direct sales. By observing market demand, the distributor/retailer chooses an appropriate price to maximize its own profit.

On the other hand, we assume that the co-marketing partner contributes marketing effort by offering value-added services or providing technical innovation in products instead of involvement in the pricing decision. The service or technical innovation the co-marketing partner offers is based on the fundamental product or service provided by distributor/retailer. Therefore, to compensate co-marketing partner's effort, either supplier or distributor should establish some profit-sharing mechanisms. In accordance with these practices, we consider two common business scenarios in our models: (I) when distributor/retailer pays for co-marketing partner's effort; (II) when the supplier pays for co-marketing partner's effort.

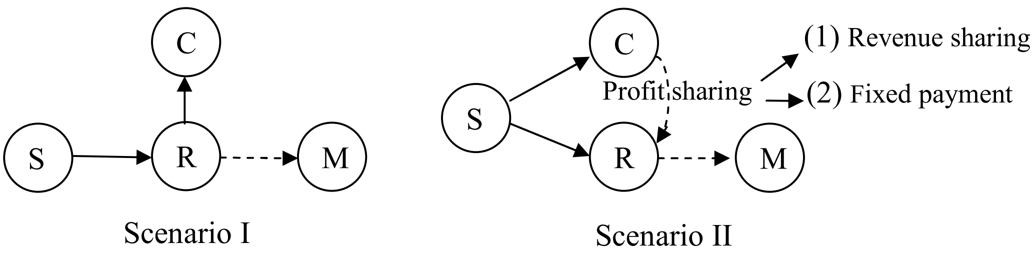

Figure 3.1 Scenarios under alliance

Note: In Figure 1, S represents supplier, R represents retailer, and C represents co-marketing partner. M refers to the market.

Figure 1 describes the transactions between the three parties under two business scenarios we summarized above.

The first scenario (Scenario I) is common in business activities, such as advertising activities. The local distributor/retailer turns to a third party for the product's advertising and promotion. In this scenario, the co-marketing partner does marketing promotion together with distributor/ retailer or adds value to the product and is paid for its effort. This case usually happens when the distributor has more market power, or is a market leader in the relevant industry.

The other scenario (Scenario II) happens when the supplier pays for co-marketing partner's effort. The supplier directly negotiates with co-marketing partner for product or service offered. This case usually happens when the supplier has a strict requirement on quality of product and wishes to exercise greater control on the alliance. The example of LEO and DCH's alliance verifies this point of view. Ferrero, a famous Italian chocolate brand, communicates directly with LEO to obtain packaging material and package design so as to ensure safety and quality.

In our model, we analyze the situations under these two business scenarios. In the later numerical analysis, we compare these two scenarios, and evaluate the effect of the two scenarios on the optimal mode of alliance.

3.4 Two types of profit-sharing mechanisms

Firms can engage in alliance activities in numerous ways. To model the alliance, we start with specific forms of revenue sharing and fixed payment between the firms.

Revenue sharing is an incentive that transfers a portion of company's profits, while fixed payment compensation is to give a fixed unit payment irrespective of the profit. Revenue sharing and fixed unit payment have been shown to be useful mechanisms for coordination in supply chain and marketing literature, and our interest focuses on how these two profit-sharing mechanisms influence firms' strategies and decisions.

With revenue sharing mechanism, the co-marketing partner gets a fraction of θ $(0 < \theta < 1)$ of the total revenue from either the supplier or the distributor/retailer (depending upon the business scenarios), whereas the supplier or distributor/retailer retains (1-θ) of the revenue. In our model, θ is a decision variable determined by the party that pays for co-marketing partner's effort.

With the fixed payment compensation mechanism, the co-marketing partner can get payment w_A on each unit of products sold. Contrary to the revenue sharing mechanism, here the sharing parameter w_A , is a decision variable of the supplier or the distributor/retailer instead of an exogenous parameter. Supplier or distributor/retailer formulate an optimal strategy on w_A after judging the market demand and its cost and finally realize its maximum equilibrium profit.

Our models analyze the situations under these two profit-sharing mechanisms and compare the performance of the three firms under optimal strategies.

3.5 Timing of the game

We focus on several three-stage games and discuss one first best situation and four models under two scenarios with two profit-sharing mechanisms. We assume risk neutrality of the three parties and assume there is no liability constraint.

The sequence of decisions under a certain business scenario is: at first, the supplier determines the unit wholesale price with the distributor/retailer; then the distributor/retailer or the supplier (depending on the scenario) decides which kind of profit-sharing mechanism will be adopted. The firm then determines how the fixed payment or revenue should be shared. Finally, the co-marketing partner decides its effort level to realize its profit according to the sharing parameter.

Table 1 summarizes the characteristics of the models. The co-marketing partner accepts the offer as long as the alliance could bring it profit, and effort levels can be chosen after the sharing parameter is settled.

Business Scenario	**Profit-sharing Mechanisms**	**Who will pay for co-marketing partner's effort**
Scenario I	Revenue Sharing	Distributor/retailer
Scenario I	Fix-payment	Distributor/retailer
Scenario II	Revenue Sharing	Supplier
Scenario II	Fix-payment	Supplier

Table 3.5 Models Characteristics

Taylor (2006) examines the sale-timing decisions when the retailer exerts sales effort prior to or during the selling season. However, our models involve three parties and two parts of effort level come from the distributor/retailer and the co-marketing partner. It is difficult for a firm, either supplier or retailer, to verify the co-marketing partner's effort level as effort is non-contractible. Thus it is reasonable to assume that distributor/retailer makes pricing decision before observing the co-marketing partner's actual effort level. Distributor/retailer might have to determine its own effort level and the selling price after estimating co-marketing partner's effort. Therefore, although some other sequences of games might also be feasible, we choose this setup to investigate channel interactions between supplier, retailer and co-marketing partner.

Without loss of generality, we take the model under scenario I with revenue-sharing mechanism as an example. The timing of the game is as follows:

(1) Supplier decides the unit wholesale price w with distributor/retailer;

(2) Distributor/retailer decides its effort level e_1 after estimating the co-marketing partner's effort level e_2 and decides a selling price p for the products;

(3) Co-marketing partner decides effort level e_2 according to revenue sharing parameter θ offered by distributor/retailer.

CHAPTER4. EQUILIBRIUM STRATEGIES UNDER ALLIANCE

In this chapter, we evaluate the equilibrium strategies and performance of firms involved the alliance process. We first examine a first-best situation, assuming the three firms can be perfectly coordinated, and then discuss the situations when players make their decisions separately under two business scenarios with two different profit-sharing mechanisms. Subsequently, we compare the decentralized situation with a benchmark model before alliance.

4.1 First-best Situation

We first present a comprehensive analysis of the first-best situation. We assume under this first-best situation, all the firms involved make centralized decisions on pricing strategy and effort output. We explore business potential of a co-marketing alliance by comparing the performance under first-best situation before and in the alliance.

The original system contains only a supplier and a distributor/retailer. Supplier ships the products to the distributor/retailer, and the latter sells the products to downstream firms or market. When a co-marketing alliance is formed, a partnering firm is introduced into the original system.

4.1.1 Uniqueness and Existence

We consider the performance of this new system, which includes a supplier, a distributor, and a third party (co-marketing partner). Recalling our notation from the earlier chapter, the new system profit under the first-best situation is given below:

$$\pi(p,e_1,e_2)=(p-c)g(e_2)y(p,e_1)-V_1(e_1)-V_2(e_2)$$

$$=(p-c)e_2^{\alpha}(a-bp+ke_1)-e_1^{\beta_1}-e_2^{\beta_2}$$

where the new system profit includes profit of the supplier, the distributor and a co-marketing partner of the distributor.

We derive the first order conditions of system profit on p, e_1 and e_2 in (4.1.1).

$$\frac{\partial \pi}{\partial p}=e_2^{\alpha}(a+ke_1+bc-2bp)=0,\quad \frac{\partial \pi}{\partial e_1}=k(p-c)e_2^{\alpha}-\beta_1 e_1^{\beta_1-1}=0$$

$$\frac{\partial \pi}{\partial e_2}=\alpha e_2^{\alpha-1}(p-c)(a+ke_1-bp)-\beta_2 e_2^{\beta_2-1}=0 \qquad (4.1.1)$$

The following sufficient condition is given to ensure the existence and uniqueness of optimal system strategies.

Theorem 4.1.1 When $\beta_1>\frac{\beta_2+\alpha}{\beta_2-\alpha}\beta_2+1$, the system profit is jointly concave in p, e_1 and e_2, and there exist unique optimal p, e_1 and e_2 that maximize the system profit $\pi(p,e_1,e_2)$. The optimal p^*, e_1^* and e_2^* can be derived from the first order solution on p, e_1 and e_2 from (4.1.1).

Proof: The sufficient conditions to ensure that π is jointly concave in p, e_1 and e_2 are: (1) $\frac{\partial \pi}{\partial p}$, $\frac{\partial \pi}{\partial e_2}$, $\frac{\partial \pi}{\partial e_1}$=0; (2) $\begin{vmatrix} \frac{\partial^2 \pi}{\partial p^2} & \frac{\partial^2 \pi}{\partial p \partial e_1} & \frac{\partial^2 \pi}{\partial p \partial e_2} \\ \frac{\partial^2 \pi}{\partial e_1 \partial p} & \frac{\partial^2 \pi}{\partial e_1^2} & \frac{\partial^2 \pi}{\partial e_1 \partial e_2} \\ \frac{\partial^2 \pi}{\partial e_2 \partial p} & \frac{\partial^2 \pi}{\partial e_2 \partial e_1} & \frac{\partial^2 \pi}{\partial e_2^2} \end{vmatrix}$ is negative definite.

When $\frac{\partial \pi}{\partial p}$, $\frac{\partial \pi}{\partial e_2}$, $\frac{\partial \pi}{\partial e_1}$=0,

$$|H_1| = \frac{\partial^2 \pi}{\partial p^2} = -2be_2^{\alpha} < 0\,;$$

$$|H_2| = \frac{\partial^2 \pi}{\partial p^2}\frac{\partial^2 \pi}{\partial e_1^2} - \frac{\partial^2 \pi}{\partial p \partial e_1} = 2b\beta_1(\beta_1 - 1)e_2^{\alpha}e_1^{\beta-2} - k^2 e_2^{2\alpha} = 2b\beta_1 e_2^{\alpha}e_1^{\beta-2}(\beta_1 - 1 - \frac{k}{k + \frac{a-bc}{e_1}}) > 0$$

when $\beta_1 \geq 2$;

$$|H_3| = \frac{\partial^2 \pi}{\partial e_2^2}|H_2| - \frac{\partial^2 \pi}{\partial p^2}(\frac{\partial^2 \pi}{\partial e_2 \partial e_1})^2$$

$$= \beta_2 e_2^{\beta_2 - 2}(\alpha - \beta_2)e_2^{\alpha}[2b\beta_1(\beta - 1)e_1^{\beta_1 - 2} - k^2 e_2^{\alpha}] + 2bk^2\alpha^2 e_2^{3\alpha - 2}(p-c)^2$$

$$= e_2^{\alpha+\beta_2-2} 2\beta_1\beta_2 b e_1^{\beta_1-2}[-(\beta_2 - \alpha)(\beta_1 - 1) + \frac{k\beta_2(\beta_2+\alpha)}{a + ke_1 - bc}e_1] - (\beta_2 - \alpha)(\beta_1 - 1) + \frac{k\beta_2(\beta_2+\alpha)}{k + \frac{a-bc}{e_1}}$$

$$-(\beta_2 - \alpha)(\beta_1 - 1) + \frac{k\beta_2(\beta_2+\alpha)}{a + ke_1 - bc}e_1 < 0$$

Therefore, when $-(\beta_2 - \alpha)(\beta_1 - 1) + \frac{k\beta_2(\beta_2+\alpha)}{k + \frac{a-bc}{e_1}} < -(\beta_2 - \alpha)(\beta_1 - 1) + \beta_2(\beta_2 + \alpha) < 0$,

that is when $\beta_1 > \frac{\beta_2 + \alpha}{\beta_2 - \alpha}\beta_2 + 1$, $|H_3| < 0$.

So to conclude, when $\beta_1 > \frac{\beta_2 + \alpha}{\beta_2 - \alpha}\beta_2 + 1$, π is jointly concave in p, e_1 and e_2. ■

Proposition 1 characterizes a property of system profit.

Proposition 4.1.1 The system profit $\pi(p, e_1, e_2)$ is supermodular in (e_1, e_2).

Proof: It is easy to get that $\frac{\partial^2 \pi}{\partial e_1 \partial e_2} > 0$. ■

Supermodularity can be viewed as "the mathematical characterization of the notion of strategic complementarity" (Topkis, 1998). Proposition 1 tells us that given p, an increase in e_1 results in an increase in optimal e_2^*, and an increase in e_2 also results in an increase in optimal e_1^*. Complementarity between e_1 and e_2 represents that the marginal return of effort input from distributor can increase the effort input level of co-marketing partner, and the marginal return of effort input from co-marketing partner can also increase the effort input level of distributor. Therefore, the term captures the internalization of the "complementarity externality": the alliance allows partners to internalize the effect of an increase in one firm's output on its partner's profit.

4.1.2 Comparison with initial system

To better explore the performance under this alliance structure, we compare this new system profit with that of benchmark system comprising only one supplier and one distributor.

Here we use π_0^* to denote the optimal system profit under the benchmark model (before alliance), and use π_1^* to denote the optimal system profit under alliance, and e_{10}^* represents the optimal effort level from the distributor in the benchmark model, e_{11}^* represents the optimal effort level from the distributor in the alliance model. In the benchmark model, $V_0(e_{10}) = e_{10}^{\beta_0}$ indicates the cost of effort from the distributor, where β_0 impacts the cost shape.

Proposition 4.1.2 gives us some implications of the system performance comparison.

Proposition 4.1.2

(1) If $g(e_2) = e_2^{\alpha} \geq 1$, when $\frac{a-bc}{k} - \frac{\beta_2}{\beta_1(\beta_2-\alpha)} > \frac{(\beta_0-1)(a-bc)k}{\beta_0-2} > 0$, π_1^* is larger than π_0^*;

(2) Otherwise, when $\beta(\beta-2) > \beta_1\beta_2$ and $\Delta_1 > \Delta_2$, π_1^* could be larger than π_0^*,

where $\Delta_1 = \frac{[k^2(\beta-1)^2 - \beta(\beta-2)](a-bc)^2}{\beta(\beta-2)k^2}$,

$$\Delta_2 = \frac{k^2\beta_1^2(\beta_2-\alpha)^2(a-bc)^2 - \beta_1(\beta_2-\alpha)k^2[\beta_1(\beta_2-\alpha)k(a-bc)^2 - 2k^2\beta_2(a-bc]}{k\beta_1\beta_2(\beta_2-\alpha)}.$$

Proof:

In benchmark model, $\pi_0 = (p-c)(a-bp+ke_0) - e_0^{\beta_0}$

$$\frac{\partial\pi}{\partial p} = a + ke_0 + bc - 2bp,\ \frac{\partial\pi}{\partial e_0} = k(p-c) - \beta_0 e_0^{\beta_0-1}$$

Thus $\pi_0^* = \frac{\beta_0^2 b}{k^2} e_0^{2\beta_0-2} - e_0^{\beta_0}$

$$= \frac{(\beta_0-2)k^2e_0^2 + 2k(a-bc)(\beta_0-1)e_0 + \beta_0(a-bc)^2}{4b\beta_0}$$

where $e_0^{\beta_0 - 1} = \dfrac{k(a + ke_1 - bc)}{2b\beta_0}$

On the other hand,

$$\pi_1^* = (\frac{\beta_2}{\alpha} - 1)(\frac{2\beta_1 b}{k(a + ke_1 - bc)} e_1^{\beta_1 - 1})^{\frac{\beta_2}{\alpha}} - e_1^{\beta_1}$$

$$= \frac{k(\beta_1\beta_2 - \beta_1\alpha)(a + ke_1 - bc)^2 - 2k^2\beta_2(a + ke_1 - bc)}{4bk\beta_1\beta_2}$$

$$= \frac{\beta_1(\beta_2 - \alpha)k^2 e_1^2 + [2k(a - bc)\beta_1(\beta_2 - \alpha) - 2k^2\beta_2]e_1 + \beta_1(\beta_2 - \alpha)(a - bc)^2 - 2k\beta_2(a - bc)}{4b\beta_1\beta_2}$$

where $e_1^{\beta_1 - 1} = \dfrac{k(a + ke_1 - bc)}{2b\beta_1} e_2^{\alpha}$.

when $g(e_2) = e_2^{\alpha} \geq 1$, it is easy to judge: when:

$$\frac{a - bc}{k} - \frac{\beta_2}{\beta_1(\beta_2 - \alpha)} > \frac{(\beta_0 - 1)(a - bc)k}{\beta_0 - 2} > 0, \;\; \pi_1^* > \pi_0^*$$

otherwise when $\beta_0(\beta_0 - 2) > \beta_1\beta_2$ and $\Delta_1 > \Delta_2$, $\pi_1^* > \pi_0^*$.

where $\Delta_1 = \dfrac{[k^2(\beta_0 - 1)^2 - \beta_0(\beta_0 - 2)](a - bc)^2}{\beta_0(\beta_0 - 2)k^2}$,

$$\Delta_2 = \frac{k^2\beta_1^2(\beta_2 - \alpha)^2(a - bc)^2 - \beta_1(\beta_2 - \alpha)k^2[\beta_1(\beta_2 - \alpha)k(a - bc)^2 - 2k^2\beta_2(a - bc]}{k\beta_1\beta_2(\beta_2 - \alpha)}. \;\blacksquare$$

Proposition 2 gives us the insight that when the alliance output level from a third party (co-marketing partner) has a positive influence on market demand, then the optimal system profit will increase under some conditions. This is easy to understand because the optimal system value is supermodular in effort level from the distributor and the co-marketing partner. Even when the alliance output level brought by co-marketing partner has negative influence to demand, the optimal system value may increase due to the cost saving in this alliance structure. The results indicate that the value of a co-marketing alliance (i.e. an additional and different marketing effort) is driven by two basic factors: the complementing power and the cost reduction effect of the co-marketing effort.

We also notice from the above proposition that if there is no technical innovation to improve effort input efficiency of the distributor, that is to say, if $\beta = \beta_1$, then the alliance will not improve system performance when the alliance output level brought by co-marketing partner has negative influence to demand. We can interpret that under this situation, the benefit brought by co-marketing partner is not enough to cover the total effort input cost, thus making the system perform worse.

4.2 Models under Situations of Decentralized Decision Making

In this section, we are interested in discussing the situations where players make their decisions separately. Under this setting, we first build a benchmark model in the original system comprising only one supplier and one retailer, and then we explore several models where the firms perform in the alliance under two different business scenarios and with two profit-sharing mechanisms (fixed payment and revenue sharing mechanisms). We find the sufficient conditions to ensure the existence and uniqueness of equilibrium strategies and profit, and then compare the performance under these situations with that of the benchmark model. We aim to examine the conditions under which the alliance will be better off and create values for all the parties involved.

4.2.1 Benchmark model (BM)

In the benchmark model, we consider a system comprising only one supplier and one retailer, considering the retailer's sales effort. The sequence of the game is as follows:

In Stage 1, supplier will decide wholesale price w;

$$\pi_s(w) = (w-c)(a+ke^* - bp^*)$$

In Stage 2, retailer will accordingly makes decisions on its effort level e to promote the products and selling price p to sell the products in the market.

$$\pi_D(p,e) = (p-w)(a+ke-bp) - e^{\beta}$$

We can prove that when $\beta \geq 2$, $\pi_D(p,e)$ is jointly concave in p and e. Therefore, optimal p^* and e^* will fit the first-order conditions where:

$$a + ke^* + bw = 2bp^*,\ k(p^* - w) = \beta e^{\beta-1}$$

Especially, when $\beta = 2$, the effort cost becomes the form of e^2, and this quadratic form of effort cost is frequently adopted in literature (Taylor, 2006). Therefore, the optimal strategies in the second stage are:

$$e^* = \frac{ka - kbw}{4b - k^2},\ p^* = \frac{2a + 2bw - wk^2}{4b - k^2}$$

Therefore, under this situation, optimal wholesale price w^* in the first stage is:

$$w^* = \frac{a+bc}{2b}$$

Under the situation when $\beta = 2$, equilibrium strategies are:

$$p^* = \frac{6ab - k^2 a - k^2 bc + 2b^2 c}{2b(4b - k^2)},\ e^* = \frac{k(a-bc)}{2(4b-k^2)},\ w^* = \frac{a+bc}{2b}$$

and equilibrium profits of supplier and retailer are:

$$\pi_s = \frac{(a-bc)^2}{2(4b-k^2)},\ \pi_D = \frac{(a-bc)^2}{4(4b-k^2)},\ k^2 < 4b \qquad (4.2.1)$$

In the benchmark model, when $\beta = 2$, the maximum channel profit, when assuming supplier and retailer are perfectly coordinated, is $\frac{(a-bc)^2}{4b-k^2}$, which is larger than the channel profit in our model, $\frac{3(a-bc)^2}{4(4b-k^2)}$ (the sum of supplier's and retailer's profit). Thus, there exists potential for the system to improve channel performance. In the following, we explore the situations under co-marketing alliance, and find out the market conditions when the alliance would better off for all the firms.

4.2.2 Scenario I with fix-payment mechanism (M1)

Under this situation, distributor pays for the co-marketing partner's effort. With a fixed payment mechanism between distributor and the co-marketing partner, the former pays the latter a compensation price for every unit of products sold.

The sequence of a three-stage game is as follows:

(1) The supplier decides wholesale price w;

$$\pi_D(w) = (w-c)(e_2^*)^{\alpha}(a-bp^*+ke_1^*)-(e_1^*)^{\beta_1}$$

(2) The distributor decides selling price p, unit payment to co-marketing partner w_A and effort level e_1;

$$\pi_D(p, w_A, e_1) = (p-w-w_A)(e_2^*)^{\alpha}(a-bp+ke_1)-e_1^{\beta_1}$$

(3) The co-marketing partner decides effort level e_2.

$$\pi_A(e_2) = w_A e_2^{\alpha}(a-bp+ke_1)-e_2^{\beta_2}$$

The sufficient conditions for the existence and uniqueness of equilibrium strategies are given in Theorem 4.2.2.

Theorem 4.2.2 When all of the following conditions are fulfilled, the supplier, the distributor and the co-marketing partner have unique equilibrium strategies on their decisions to maximize their respective profits.

(a) $\dfrac{2\alpha}{\beta_2-\alpha}+1 \le \beta_1 \le \dfrac{\beta_2+3\alpha}{2\alpha}$;

(b) $\dfrac{\alpha}{\beta_2-\alpha} \le \dfrac{\beta_1 k}{1+\beta_1 k}$.

Proof:

In Stage 3, $\pi_A(e_2) = w_A e_2^{\alpha}(a-bp+ke_1)-e_2^{\beta_2}$, we derive the first-order condition on the effort, and get: $\dfrac{d\pi_A}{de_2} = \alpha w_A(a-bp+ke_1)e_2^{\alpha-1}-\beta_2 e_2^{\beta_2-1}=0$

So, $\dfrac{d^2\pi_A}{de_2^{\,2}} = \alpha(\alpha-1)w_A(a-bp+ke_1)e_2^{\alpha-2}-\beta_2(\beta_2-1)e_2^{\beta_2-2} < 0$ because $0<\alpha<1$.

Therefore, $\pi_A(e_2)$ is concave in e_2.

In Stage 2, we should find sufficient conditions to ensure the joint concavity of π_D on p, w_A and e_1.

$$\pi_D(p,w_A,e_1)=(p-w-w_A)(e_2^{\ *})^{\alpha}(a-bp+ke_1)-e_1^{\ \beta_1}$$

$$=(p-w-w_A)(\frac{\alpha w_A}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}}(a-bp+ke_1)^{\frac{\beta_2}{\beta_2-\alpha}}-e_1^{\ \beta_1}$$

We derive the first-order conditions on price and the efforts:

$$\frac{\partial\pi_D}{\partial p}=(\frac{\alpha w_A}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}}[(a-bp+ke_1)^{\frac{\beta_2}{\beta_2-\alpha}}-\frac{\beta_2}{\beta_2-\alpha}b(p-w-w_A)(a-bp+ke_1)^{\frac{\alpha}{\beta_2-\alpha}}]=0$$

$$\frac{\partial\pi_D}{\partial e_1}=(p-w-w_A)(\frac{\alpha w_A}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}}\frac{\beta_2}{\beta_2-\alpha}(a-bp+ke_1)^{\frac{\alpha}{\beta_2-\alpha}}k-\beta_1 e_1^{\ \beta_1-1}=0$$

$$\frac{\partial\pi_D}{\partial w_A}=(a-bp+ke_1)^{\frac{\beta_2}{\beta_2-\alpha}}[-(\frac{\alpha w_A}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}}+\frac{\alpha}{\beta_2-\alpha}\frac{\alpha}{\beta_2}(p-w-w_A)(\frac{\alpha w_A}{\beta_2})^{-\frac{\alpha}{\beta_2-\alpha}}]=0$$

The sufficient conditions to fulfill the property of concavity include:

(1) $\frac{\partial\pi_D}{\partial p^*}=\frac{\partial\pi_D}{\partial e_1^{\ *}}=\frac{\partial\pi_D}{\partial w_A^{\ *}}=0$;

(2) Hessian Matrix $\begin{vmatrix} \frac{\partial^2\pi_D}{\partial p^2} & \frac{\partial^2\pi_D}{\partial p\partial w_A} & \frac{\partial^2\pi_D}{\partial p\partial e_1} \\ \frac{\partial^2\pi_D}{\partial w_A\partial p} & \frac{\partial^2\pi_D}{\partial w_A^{\ 2}} & \frac{\partial^2\pi_D}{\partial w_A\partial e_1} \\ \frac{\partial^2\pi_D}{\partial e_1\partial p} & \frac{\partial^2\pi_D}{\partial e_1\partial w_A} & \frac{\partial^2\pi_D}{\partial e_1^{\ 2}} \end{vmatrix}$ is negative definite.

Thus, when $\frac{\partial \pi_D}{\partial p^*} = \frac{\partial \pi_D}{\partial e_1^*} = \frac{\partial \pi_D}{\partial w_A^*} = 0$, we have:

$$\frac{\partial^2 \pi_D}{\partial p^2} = (\frac{\alpha w_A}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}} (a - bp + ke_1)^{\frac{\alpha}{\beta_2-\alpha}-1} [-b\frac{\alpha}{\beta_2-\alpha} - (b + b\frac{\beta_2}{\beta_2-\alpha})(a - bp + ke_1)] < 0$$

$$\frac{\partial^2 \pi_D}{\partial e_1^2} = k^2 (p - w - w_A)(\frac{\alpha w_A}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}} \frac{\beta_2}{\beta_2-\alpha}(a - bp + ke_1)^{\frac{\alpha}{\beta_2-\alpha}-1} - \beta_1(\beta_1 - 1)e_1^{\beta_1-2} < 0$$

$$\frac{\partial^2 \pi_D}{\partial w_A^2} = (a - bp + ke_1)^{\frac{\beta_2}{\beta_2-\alpha}} (\frac{\alpha w_A}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}} w_A^{\frac{\alpha}{\beta_2-\alpha}-2} [-2\frac{\alpha}{\beta_2-\alpha} w_A + \frac{\alpha}{\beta_2-\alpha}(\frac{\alpha}{\beta_2-\alpha} - 1)(p - w - w_A)] < 0$$

$$\frac{\partial^2 \pi_D}{\partial p \partial e_1} = (\frac{\alpha w_A}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}} k (a - bp + ke_1)^{\frac{\alpha}{\beta_2-\alpha}}$$

$$\frac{\partial^2 \pi_D}{\partial p \partial w_A} = (\frac{\alpha w_A}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}-1} (a - bp + ke_1)^{\frac{\alpha}{\beta_2-\alpha}} \{\frac{\alpha}{\beta_2}\frac{\alpha}{\beta_2-\alpha}[(a - bp + ke_1) - b(p - w - w_A)\frac{\beta_2}{\beta_2-\alpha}] + \frac{\alpha w_A b}{\beta_2-\alpha}\}$$

$$\frac{\partial^2 \pi_D}{\partial e_1 \partial w_A} = 0$$

To ensure $\begin{vmatrix} \frac{\partial^2 \pi_D}{\partial p^2} & \frac{\partial^2 \pi_D}{\partial p \partial w_A} & \frac{\partial^2 \pi_D}{\partial p \partial e_1} \\ \frac{\partial^2 \pi_D}{\partial w_A \partial p} & \frac{\partial^2 \pi_D}{\partial w_A^2} & \frac{\partial^2 \pi_D}{\partial w_A \partial e_1} \\ \frac{\partial^2 \pi_D}{\partial e_1 \partial p} & \frac{\partial^2 \pi_D}{\partial e_1 \partial w_A} & \frac{\partial^2 \pi_D}{\partial e_1^2} \end{vmatrix}$ is negative definite, we should make sure that:

(i) $\frac{\partial^2 \pi_D}{\partial p^2} = (\frac{\alpha w_A}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}} (a - bp + ke_1)^{\frac{\alpha}{\beta_2-\alpha}-1} [-b\frac{\alpha}{\beta_2-\alpha} - (b + b\frac{\beta_2}{\beta_2-\alpha})(a - bp + ke_1)] < 0$

(ii) $\begin{vmatrix} \frac{\partial^2 \pi_D}{\partial p^2} & \frac{\partial^2 \pi_D}{\partial p \partial w_A} \\ \frac{\partial^2 \pi_D}{\partial w_A \partial p} & \frac{\partial^2 \pi_D}{\partial w_A^2} \end{vmatrix} = \frac{\partial^2 \pi_D}{\partial p^2} * \frac{\partial^2 \pi_D}{\partial w_A^2} - (\frac{\partial^2 \pi_D}{\partial p \partial w_A})^2$

$$=(\frac{\alpha w_A}{\beta_2})^{\frac{2\alpha}{\beta_2-\alpha}-2}(a-bp+ke_1)^{\frac{2\alpha}{\beta_2-\alpha}}(\frac{\alpha}{\beta_2})^2[b\frac{\beta_2}{\beta_2-\alpha}\frac{\alpha}{\beta_2-\alpha}w_A+b^2w_A{}^2\frac{\beta_2{}^2}{\alpha(\beta_2-\alpha)}]>0$$

thus $\begin{vmatrix} \frac{\partial^2\pi_D}{\partial e_1{}^2} & \frac{\partial^2\pi_D}{\partial e_1\partial w_A} \\ \frac{\partial^2\pi_D}{\partial w_A\partial e_1} & \frac{\partial^2\pi_D}{\partial w_A{}^2} \end{vmatrix}>0$

(iii) $\begin{vmatrix} \frac{\partial^2\pi_D}{\partial p^2} & \frac{\partial^2\pi_D}{\partial p\partial w_A} & \frac{\partial^2\pi_D}{\partial p\partial e_1} \\ \frac{\partial^2\pi_D}{\partial w_A\partial p} & \frac{\partial^2\pi_D}{\partial w_A{}^2} & \frac{\partial^2\pi_D}{\partial w_A\partial e_1} \\ \frac{\partial^2\pi_D}{\partial e_1\partial p} & \frac{\partial^2\pi_D}{\partial e_1\partial w_A} & \frac{\partial^2\pi_D}{\partial e_1{}^2} \end{vmatrix}=\frac{\partial^2\pi_D}{\partial e_1{}^2}.\begin{vmatrix} \frac{\partial^2\pi_D}{\partial p^2} & \frac{\partial^2\pi_D}{\partial p\partial w_A} \\ \frac{\partial^2\pi_D}{\partial w_A\partial p} & \frac{\partial^2\pi_D}{\partial w_A{}^2} \end{vmatrix}-\frac{\partial^2\pi_D}{\partial w_A{}^2}.(\frac{\partial^2\pi_D}{\partial e_1\partial p})^2$

$$=(\frac{\alpha w_A}{\beta_2})^{\frac{2\alpha}{\beta_2-\alpha}-2}(a-bp+ke_1)^{\frac{2\alpha}{\beta_2-\alpha}}(\frac{\alpha}{\beta_2})^2\{.\}$$

$$\{.\}=[bw_A\frac{\beta_2}{\beta_2-\alpha}\frac{\alpha}{\beta_2-\alpha}+b^2w_A{}^2\frac{\beta_2{}^2}{\alpha(\beta_2-\alpha)}]\beta_1e^{\beta_1-2}(\frac{k\alpha}{(a-bp+ke_1)(\beta_2-\alpha)}e_1-\beta_1+1)$$

$$-(\frac{\alpha w_A}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}}(a-bp+ke_1)^{\frac{\alpha}{\beta_2-\alpha}}k^2w_A\frac{\beta_2}{\beta_2-\alpha}]$$

$$=b\beta_1kw_A\frac{\beta_2}{\beta_2-\alpha}[\frac{\alpha}{(\beta_2-\alpha)\beta_1k}+\frac{bw_A\frac{\beta_2}{\beta_2-\alpha}}{a-bp+ke_1}-1]$$

As $w_A=(p-w-w_A)\frac{\alpha}{\beta_2-\alpha}=\frac{a-bp+ke_1}{b(\frac{\alpha}{\beta_2-\alpha}+1)}\frac{\alpha}{\beta_2-\alpha}$, thus $\frac{bw_A(\frac{\alpha}{\beta_2-\alpha}+1)}{a-bp+ke_1}=\frac{\alpha}{\beta_2-\alpha}$

Therefore, from the expression of (1), a group of sufficient conditions to ensure

$$\begin{vmatrix} \frac{\partial^2 \pi_D}{\partial p^2} & \frac{\partial^2 \pi_D}{\partial p \partial w_A} & \frac{\partial^2 \pi_D}{\partial p \partial e_1} \\ \frac{\partial^2 \pi_D}{\partial w_A \partial p} & \frac{\partial^2 \pi_D}{\partial w_A{}^2} & \frac{\partial^2 \pi_D}{\partial w_A \partial e_1} \\ \frac{\partial^2 \pi_D}{\partial e_1 \partial p} & \frac{\partial^2 \pi_D}{\partial e_1 \partial w_A} & \frac{\partial^2 \pi_D}{\partial e_1{}^2} \end{vmatrix}$$ to be negative definite include the following:

(1) $\frac{\alpha}{(\beta_2-\alpha)\beta_1 k} + \frac{\alpha}{(\beta_2-\alpha)} - 1 \le 0$, that is $\frac{\alpha}{\beta_2-\alpha} \le \frac{\beta_1 k}{1+\beta_1 k}$;

(2) $\beta_1 \ge \frac{2\alpha}{\beta_2-\alpha} + 1$;

(3) $2\alpha\beta_1 - \beta_2 - 3\alpha \le 0$, that is $\beta_1 \le \frac{\beta_2 + 3\alpha}{2\alpha}$

To conclude, under this group of conditions, π_s is jointly concave in p, e_1 and w_A, thus making sure that there exist unique Nash equilibrium solutions. ■

We also explore a property about the relationship between profit performance of distributor and relevant variables.

Proposition 4.2.2.1 Under this scenario of fixed-payment in co-marketing alliance, after estimation of co-marketing partner's optimal effort level e_2, for given unit payment w_A, the profit of distributor/retailer $\pi_D(e_1, p \mid w_A)$ is supermodular in (e_1, p).

Proof: From the proof of Theorem 4.2.2, we can easily get: $\frac{\partial^2 \pi_D}{\partial e_1 \partial p} > 0$. ■

Proposition 4.2.2 indicates that given an equal unit payment w_A^*, an increase in distributor's effort e_1 results in an increase in optimal selling price p^*. Therefore, when distributor makes decisions after estimating co-marketing partner's effort level, the distributor tends to sell at a higher price if its effort level increases under alliance.

In the above problem, let us recall that β_1 is the effort input parameter of distributor, and β_2 is the effort input parameter of co-marketing partner, thus these two parameters represent firms' cost input levels; while α can be interpreted as the benefit level or the alliance output level brought by co-marketing partner. Let us denote $\eta = \frac{\beta_2}{\alpha}$ to represent the cost-benefit ratio brought by the co-marketing partner. Due to the high degree equations that emerge in the middle stage of the game, it is difficult to derive close-form equilibrium strategies and equilibrium profit for the involved firms, thus making analytical comparison of the performance with benchmark model infeasible. Therefore, we consider a special case when $\beta_1 = 2$, and the cost-benefit ratio $\eta = \frac{\beta_2}{\alpha} = 3$, to discuss the properties and performance under this case.

The case of $\beta_1 = 2$, $\eta = \frac{\beta_2}{\alpha} = 3$ fits our sufficient conditions to ensure the existence and uniqueness of Nash equilibrium solutions. Therefore, in the following stages, we focus on how the equilibrium strategies come out. To ensure sub-game perfection, we solve the game backwards.

Stage 3: The co-marketing partner's profit in this stage is given by:

$$\pi_A(e_2) = w_A e_2^{\alpha}(a - bp + ke_1) - e_2^{\beta_2}$$

The first-order condition on the effort of co-marketing partner is:

$$\frac{d\pi_A}{de_2} = \alpha w_A(a-bp+ke_1)e_2^{\alpha-1} - \beta_2 e_2^{\beta_2-1} = 0,$$

$$\frac{d^2\pi_A}{de_2^{\ 2}} = \alpha(\alpha-1)w_A(a-bp+ke_1)e_2^{\alpha-2} - \beta_2(\beta_2-1)e_2^{\beta_2-2} < 0 \text{ because } 0<\alpha<1.$$

Thus $\pi_A(e_2)$ is concave in e_2, so:

$$e_2^{\ *} = \left(\frac{\alpha w_A(a-bp+ke_1)}{\beta_2}\right)^{\beta_2-\alpha} = \left(\frac{w_A(a-bp+ke_1)}{3}\right)^{2\alpha}$$

Stage 2: The distributor's profit function in this stage is given by:

$$\pi_D(p, w_A, e_1) = (p-w-w_A)\left(\frac{\alpha w_A}{\beta_2}\right)^{\frac{1}{2}}(a-bp+ke_1)^{\frac{3}{2}} - e_1^{\ 2}$$

We derive the first-order conditions on p^*, e_1^* and w_A^*, and we have:

$$\frac{\partial \pi_D}{\partial p} = \left(\frac{\alpha w_A}{\beta_2}\right)^{\frac{1}{2}}\left[(a-bp+ke_1)^{\frac{3}{2}} - \frac{3}{2}b(p-w-w_A)(a-bp+ke_1)^{\frac{1}{2}}\right] = 0$$

$$\frac{\partial \pi_D}{\partial e_1} = (p-w-w_A)\left(\frac{\alpha w_A}{\beta_2}\right)^{\frac{1}{2}}\frac{3}{2}(a-bp+ke_1)^{\frac{1}{2}}k - 2e_1 = 0$$

$$\frac{\partial \pi_D}{\partial w_A} = (a-bp+ke_1)^{\frac{3}{2}}\left[-\left(\frac{\alpha w_A}{\beta_2}\right)^{\frac{1}{2}} + \frac{1}{6}(p-w-w_A)\left(\frac{\alpha w_A}{\beta_2}\right)^{-\frac{1}{2}}\right] = 0$$

Consequently, the optimal strategies involved in the second stage are given by:

$$p^* = \frac{nw+b-\sqrt{b^2+bnw-an}}{n}, \quad e_1^* = \frac{2b^2+bnw-an-2b\sqrt{b^2+bnw-an}}{n},$$

$$w_A = \frac{b-\sqrt{b^2+bnw-an}}{3n}, \text{ where } n = \frac{k^2\sqrt{b}}{6}.$$

Stage 1: The supplier's profit function in this stage is given by:

$$\pi_s = \frac{3\sqrt{b}}{k^4}(w-c)(2b^2 - an + bnw - 2b\sqrt{b^2 + bnw - an})$$

Thus, $\frac{d\pi_s}{dw} = 2b^2 - an + bnw + 2b\sqrt{b^2 + bnw - an} + (w-c)(bn - \frac{b^2 n}{\sqrt{b^2 + bnw - an}}) = 0$

Suppose $\sqrt{b^2 + bnw - an} = t$, then $2t^2 - bt + an - b^2 - bnc = 0$

When $b^2 > n(a-bc)$, $t = \frac{b + \sqrt{9b^2 - 8an + 8bnc}}{4}$

So to conclude, the equilibrium strategies are given by:

$$w^* = \frac{4an - 3b^2 + 4bnc + b\sqrt{9b^2 - 8an + 8bnc}}{8bn},\ p^* = \frac{4an + 3b^2 + 4bnc - b\sqrt{9b^2 - 8an + 8bnc}}{8bn},$$

$$e_1^* = \frac{18b^2 - 8an + 8bnc - 6b\sqrt{9b^2 - 8an + 8bnc}}{16n},\ w_A^* = \frac{3b - \sqrt{9b^2 - 8an + 8bnc}}{12n},$$

$$e_2^* = [\frac{b}{3}\frac{9b^2 - 4an + 4bnc - 3b\sqrt{9b^2 - 8an + 8bnc}}{8n^2}]^{\frac{1}{2\alpha}}$$

where $n = \frac{k^2\sqrt{b}}{6}$, and $n(a-bc) < b^2$.

Therefore, the payoffs for the retailer and the supplier are given by

$$\pi_s = \frac{9[-16n^2(a-bc)^2 + 72nb^2(a-bc) - 27b^4]}{64bk^6},$$

$$\pi_D = \frac{9n(3b - \sqrt{9b^2 - 8n(a-bc)})^3 - 2k^6(3b - \sqrt{9b^2 - 8n(a-bc)})^2}{32k^6 n} \tag{4.2.2}$$

Under this special case, we derive some properties about the equilibrium profit of supplier and distributor.

Proposition 4.2.2.2

(1) *For given* a, c, k *, there exists a fixed* b_0*, and the equilibrium profit of supplier is increasing in* b *when* $0 < b < b_0$ *and is decreasing in* b *when* $b > b_0$*;*

(2) *For given* a, c, b *, when* $k < \frac{16(a-bc)}{27b^{\frac{5}{2}}}$ *, the equilibrium profit of supplier is increasing in* k *;*

(3) *For given* a, c, b*, there exist fixed* k_1, k_2*, for* $k \in (k_1, k_2)$*, the equilibrium profit of distributor is decreasing in* k .

Proof:

(1): From the formula of (4.2.2), we differentiate with supplier's profit in b :

$$\frac{d\pi_s}{db} = \frac{8k^4c(a-bc)+162k^2a\sqrt{b}-270k^2cb^{\frac{3}{2}}-729b^2}{64k^6}$$

Thus $\frac{d^2\pi_s}{db^2} = \frac{-8k^4c^2+\frac{81k^2a}{\sqrt{b}}-405k^2c\sqrt{b}-1458b}{64k^6} < 0$, so π_s is concave in b .

In addition, when $b = 0$, $\pi_s = 0$; when $b = \frac{a}{16c}$,

$\pi_s = \frac{6480a^3c^2k^2\sqrt{\frac{a}{c}}-243a^4-1440a^3c^2k^4}{65536c^4k^6} > 0$; when $b = \frac{a}{c}$, $\pi_s = -\frac{243b^3}{64k^6} < 0$;

therefore, there exists a $b_0 \in (0, \frac{a}{c})$, where given other parameters are fixed, π_s will achieve its maximum at $b = b_0$. When $0 < b < b_0$, π_s is increasing in b; when $b > b_0$, π_s is decreasing in b.

(2) We conduct the derivative of supplier's profit function on k, and we have:

$$\frac{d\pi_s}{dk} = \frac{1}{4k^3}\left(\frac{(a-bc)^2}{2} - \frac{27\sqrt{b}(a-bc)}{k^2} + \frac{729b^3}{16k}\right)$$

Thus when $k < \frac{16(a-bc)}{27b^{\frac{5}{2}}}$, $-\frac{27\sqrt{b}(a-bc)}{k^2} + \frac{729b^3}{16k} > 0$, that is to say: $\frac{d\pi_s}{dk} > 0$.

So π_s is increasing in k when $k < \frac{16(a-bc)}{27b^{\frac{5}{2}}}$.

(3) We conduct the derivative of distributor's profit function on k, and we have:

$$\frac{d\pi_D}{dk} = \frac{\frac{6k^2\sqrt{b}(a-bc)}{\sqrt{\Delta_{11}}}(3b - \sqrt{\Delta_{11}})(-8k^4 + 9\sqrt{b}\sqrt{\Delta_{11}}) + 3(3b - \sqrt{\Delta_{11}})^2(4k^4 - 9\sqrt{b}\sqrt{\Delta_{11}})}{16\sqrt{b}k^7},$$

where $\sqrt{\Delta_{11}} = \sqrt{9b^2 - \frac{4k^2\sqrt{b}(a-bc)}{3}}$

therefore when $4k^4 < 9\sqrt{b}\sqrt{\Delta_{11}} < 8k^4$, $\frac{d\pi_D}{dk} < 0$

since $y = 9\sqrt{b}\sqrt{\Delta_{11}}$ is an increasing function in k, and it has only one point of intersection with $y = 4k^4$ and $y = 8k^4$ when $k > 0$. It is difficult to derive the point of intersection with a close form k. Suppose k_1 and k_2 $(k_1 < k_2)$ are the points. Thus when $k_1 < k < k_2$, $\frac{d\pi_D}{dk} < 0$. ■

Proposition 4.2.2.2 indicates that under this scenario, a co-marketing alliance faced with sufficiently high or sufficiently low price sensitivity affects supplier's equilibrium profit. This result is explained by the fact that when price is insensitive to demand fluctuation, or price is sufficiently sensitive to demand, the benefit of effort (due to decreasing returns of effort) does not increase at a pace faster than the cost of efforts. Analogously, low effort elasticity of demand is more favorable to the performance of alliance.

Based on the outcome in (4.2.2), Proposition 4.2.2.3 presents a comparative analysis with benchmark model (4.2.1).

Proposition 4.2.2.3

As for the supplier:

(1) if $2k^2 < b^2$, the supplier's profit under alliance is certainly larger than that in the benchmark model;

(2) if $2k^2 > b^2$, when $2k^2 + b^2 < b$ and $a - bc > \frac{53b^{2.5} - 27b^2}{32k^4 - 16b^2k^2}$, the supplier's profit under alliance will better off than in the benchmark model.

As for the distributor/retailer:

When $a-bc>m$ *, where* m *is the positive solution of:* $y=\frac{x^2}{4(4b-k^2)}$ *and*

$y=\frac{9n(3b-\sqrt{9b^2-8nx})^3-2k^6(3b-\sqrt{9b^2-8nx})^2}{32k^6 n}$*, the distributor/retailer's profit is better than the performance in the benchmark model.*

Proof: It can be easily inferred from Proposition 4.2.2.2. ■

Compared to the benchmark model, if effort sensitivity of demand is much smaller than price sensitivity, the supplier will be better off than in the benchmark model; when effort sensitivity of demand is larger than price sensitivity, the supplier may have better performance than in the benchmark model under some strict conditions. We can interpret this conclusion to imply that the supplier benefits in the case when the influence of price on demand is relatively larger than the influence of effort on demand. An analogous conclusion exists for the distributor.

To deeply explore the managerial implications under this scenario, we conduct numerical studies to illustrate the influence of co-marketing alliance to supplier's and distributor's profit and the system effort input level. We consider the special case where $a=30$, $c=10$ and $k=1.25$, and we vary b from 0.01 to 3.

Given other parameters, Figure 4.1 demonstrates the range of price sensitivity where both supplier and distributor get better performance compared with the benchmark model. To ensure the positive of the equilibrium profit of the distributor and the supplier, the bound of dashed line above requires the price sensitivity should be larger than 1.6. Figure 4.1 demonstrates that when price sensitivity is in the medium range, the distributor and the supplier can achieve a better performance.

Figure 4.2 indicates that distributor's and co-marketing partner's effort input levels are higher than the effort level in the benchmark model, while the benefit of effort (e_2^{α}) is sufficiently large to compensate the effort cost and, therefore, both supplier and distributor can achieve better performance under this scenario. Although the effort cost greatly increase under cooperation in this case, the revenue generated by the cooperation compensates the effort put in.

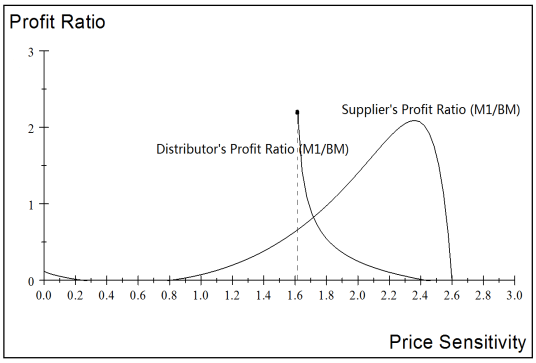

Figure 4.1 Profit Ratio (M1/BM Model)

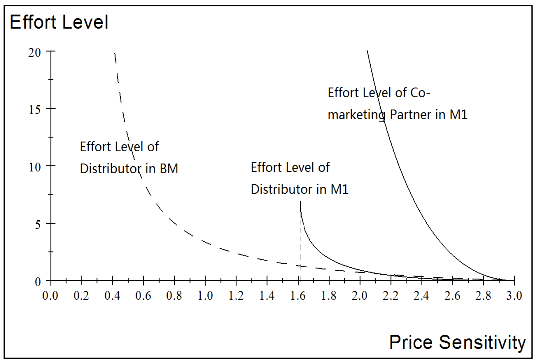

Figure 4.2 Effort Level (M1/BM Model)

4.2.3 Scenario I with revenue sharing mechanism (M2)

In this model, distributor/retailer pays for co-marketing partner's effort through a revenue sharing mechanism. The distributor/retailer transfers part of its revenue to the co-marketing partner as compensation.

The sequence of the game is as follows:

(1) The supplier decides wholesale price w;

$$\pi_s(w) = (w-c)e_2^{*\alpha}(a - bp^* + ke_1^*)$$

(2) The distributor decides selling price p, effort level e_1 and percentage θ of revenue, which is transferred to the co-marketing partner;

$$\pi_D(p,e_1,\theta)=[(1-\theta)p-w]e_2^{*\alpha}(a-bp+ke_1)-e_1^{\beta_1}$$

(3) The co-marketing partner decides effort level e_2.

$$\pi_A(e_2)=\theta pe_2^{\alpha}(a-bp+ke_1)-e_2^{\beta_2}$$

Theorem 4.2.3 gives the sufficient conditions for uniqueness of Nash equilibrium strategies under this scenario.

Theorem 4.2.3 When all the following conditions are fulfilled, the supplier, the distributor and the co-marketing partner have unique equilibrium strategies on their decisions to maximize their respective profits.

(a) $\beta_2 \geq 3\alpha$;

(b) $\beta_1 \geq \dfrac{2\alpha}{\beta_2-\alpha}+1$;

(c) $k\theta\beta_2+\dfrac{\Delta'}{2\Delta_1}>b\alpha\Delta_2$, where

$\Delta_1=b^2(\beta_2+\alpha)^2+[(a-bc)(1-\theta)\beta_2]^2+2b\beta_2(1-\theta)(\beta_2-3\alpha)$,

$\Delta_2=(1-\theta)^2\beta_2^{\,2}k+2b\beta_2k(1-\theta)(\beta_2-3\alpha)$.

Proof: Similar to Proof of Theorem 4.2.2. ■

To better explore the properties and compare performance, we use a case with $\eta=\dfrac{\beta_2}{\alpha}=3$ and $\beta_1=2$ as example. When $\eta=\dfrac{\beta_2}{\alpha}=3$ and $\beta_1=2$, we solve the game backwards to ensure sub-game perfection.

Stage 3: The co-marketing partner's profit function in this stage is given by:

$$\pi_A(e_2) = \theta p e_2{}^{\alpha}(a - bp + ke_1) - e_2{}^{\beta_2}$$

We derive the first-order condition and we have:

$$\frac{d\pi_A}{de_2} = \theta p(a - bp + ke_1)\alpha e_2{}^{\alpha-1} - \beta_2 e_2{}^{\beta_2-1} = 0\,,\ \ \frac{d^2\pi_A}{de_2{}^2} < 0$$

Therefore, $e_2{}^* = (\dfrac{\alpha\theta p(a - bp + ke_1)}{\beta_2})^{\frac{1}{\beta_2-\alpha}}$

Stage 2: The distributor's profit function in this stage is given by:

$$\pi_D(p, e_1, \theta) = [(1-\theta)p - w]e_2{}^{*\alpha}(a - bp + ke_1) - e_1{}^{\beta_1}$$

$$= [(1-\theta)p - w](\frac{\alpha\theta p}{p})^{\frac{\alpha}{\beta_2-\alpha}}(a - bp + ke_1)^{\frac{\beta_2}{\beta_2-\alpha}} - e_1{}^{\beta_1}$$

We derive the first-order conditions on price, effort level of distributor and transfer percentage, and we have:

$$\frac{\partial\pi_D}{\partial p} = (\frac{\alpha\theta p}{p})^{\frac{\alpha}{\beta_2-\alpha}-1}(a - bp + ke_1)^{\frac{\alpha}{\beta_2-\alpha}}\{(1-\theta)\frac{\alpha\theta p(a - bp + ke_1)}{\beta_2} +$$

$$[(1-\theta)p - w]\frac{\alpha^2\theta(a - bp + ke_1)}{\beta_2(\beta_2 - \alpha)} - b[(1-\theta)p - w]\frac{\alpha\theta p}{\beta_2 - \alpha}\} = 0$$

$$\frac{\partial\pi_D}{\partial e_1} = [(1-\theta)p - w](\frac{\alpha\theta p}{p})^{\frac{\alpha}{\beta_2-\alpha}}(a - bp + ke_1)^{\frac{\alpha}{\beta_2-\alpha}}\frac{\beta_2}{\beta_2 - \alpha} - \beta_1 e_1{}^{\beta_1-1} = 0$$

$$\frac{\partial\pi_D}{\partial\theta} = (a - bp + ke_1)^{\frac{\beta_2}{\beta_2-\alpha}}\{-p(\frac{\alpha\theta p}{p})^{\frac{\alpha}{\beta_2-\alpha}} + [(1-\theta)p - w]\frac{\alpha^2}{\beta_2 - \alpha}(\frac{\alpha\theta p}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}-1} = 0$$

When $\beta_2 = 3\alpha$ and $\beta_1 = 2$,

$$\theta^* = \frac{-4bw + \sqrt{16b^2w^2 - 4kt(bw-a)}}{2kt - 8bw + 2\sqrt{16b^2w^2 - 4kt(bw-a)}},\ e_1^* = \frac{a}{k} - \frac{bw(1+2\theta^*)}{k(1-2\theta^*)},\ p^* = \frac{w}{1-2\theta^*}$$

where $t = \sqrt{\frac{2b}{3}\frac{3kw^2}{4}}$

Stage 1: The supplier's profit function in this stage is given by:

$$\pi_s = \frac{4\sqrt{2b}}{3\sqrt{3}k^4}(w-c)(-4b + \sqrt{16b^2 - k^2\sqrt{6b}(bw-a)})^2$$

From $\frac{d\pi_s}{dw} = 0$, we can get the equation:

$$-4b\sqrt{16b^2 - k^2\sqrt{6b}(bw-a)} + 16b^2 - k^2\sqrt{6b}(bw-a) = (w-c)\sqrt{6b}k^2b$$

Let $m = \sqrt{16b^2 - k^2\sqrt{6b}(bw-a)}$, then $bw = a + \frac{16b^2 - m^2}{\sqrt{6b}k^2}$

The equation turns out to be: $2m^2 - 4bm - 16b^2 - \sqrt{6b}k^2(a-bc) = 0$

Thus, the unique solution is: $m = \frac{2b + \sqrt{36b^2 + 2\sqrt{6b}k^2(a-bc)}}{2}$

Therefore, the equilibrium strategies involved and equilibrium profit of supplier and distributor are given by:

$$w^* = \frac{8b^2 - 2b\sqrt{36b^2 + 2\sqrt{6b}k^2(a-bc)} + \sqrt{6b}k^2b(a+bc)}{4bk^2\sqrt{6b}};$$

$$\theta^* = \frac{-64b^2 + 4m^2 - 4\sqrt{6b}ak^2 + \sqrt{6b}mk^2}{3bk^4w^2 - 8\sqrt{6b}ak^2 + 128b^2 - 8m^2 + 2m},\ p^* = \frac{w^*}{1-2\theta^*},\ e_1^* = \frac{a}{k} - \frac{bw(1+2\theta^*)}{k(1-2\theta^*)}$$

$$\pi_s = \frac{3024b^3 + 192bk^2(a-bc)\sqrt{6b} + 6k^4(a-bc)^2 - [504b^2 + 38k^2(a-bc)\sqrt{6b}]\sqrt{36b^2 + 2\sqrt{6b}k^2(a-bc)}}{9k^4}$$

$$\pi_D = \frac{8(m-4b)^3}{9k^6} - \frac{(m-4b)^4}{6bk^6}, \text{ where } m = \frac{2b + \sqrt{36b^2 + 2\sqrt{6b}k^2(a-bc)}}{2} \qquad (4.2.3)$$

To conclude, from (4.2.3), we can generalize some properties of supplier and distributor's equilibrium profit.

Proposition 4.2.3.1

(1) *For given* a,c,b *, when* $0 < k < \sqrt{\frac{128+288b}{9\sqrt{6b}(a-bc)}}$ *, equilibrium profit of the distributor is increasing in* k *, when* $k > \sqrt{\frac{128+288b}{9\sqrt{6b}(a-bc)}}$ *, equilibrium profit of the distributor is decreasing in* k

(2) *For given* a,c,k *, when* $\frac{a}{2c} < b < \frac{a}{c}$ *, equilibrium profit of the distributor is decreasing in* b *;*

Proof:

(1) $$\frac{d\pi_D}{dk} = \frac{24(m-4b)^2\frac{\partial m}{\partial k}}{81k^{12}} - \frac{4(m-4b)^3\frac{\partial m}{\partial k}}{36b^2k^{12}} = \frac{12(m-4b)^2\frac{\partial m}{\partial k}[8-3(m-4b)]}{324b^2k^{12}}$$

It is obvious that $\frac{\partial m}{\partial k} > 0$,

Therefore, when $0 < k < \sqrt{\frac{128+288b}{9\sqrt{6b}(a-bc)}}$, $8-3(m-4b) > 0$, so $\frac{d\pi_D}{dk} > 0$;

When $k > \sqrt{\dfrac{128+288b}{9\sqrt{6b}(a-bc)}}$, $8-3(m-4b)<0$, so $\dfrac{d\pi_D}{dk}<0$.

(2)

$$\frac{d\pi_D}{db}=\frac{(m-4b)^2}{6b^2k^6}(34b^2-13b\sqrt{36b^2+2\sqrt{6b}k^2(a-bc)}+\sqrt{6b}k^2(a-2bc)+\frac{b\sqrt{b}(12b\sqrt{6b}+k^2(a-3bc))}{\sqrt{36b^2+2\sqrt{6b}k^2(a-bc)}})$$

$$\frac{b\sqrt{b}(12b\sqrt{6b}+k^2(a-3bc))}{\sqrt{36b^2+2\sqrt{6b}k^2(a-bc)}}=2b\sqrt{36b^2+2\sqrt{6b}k^2(a-bc)}-\frac{3\sqrt{6b}k^2(a-bc)}{\sqrt{36b^2+2\sqrt{6b}k^2(a-bc)}}$$

So:

$$\frac{d\pi_D}{db}=\frac{(m-4b)^2}{6b^2k^6}(34b^2-11b\sqrt{36b^2+2\sqrt{6b}k^2(a-bc)}+\sqrt{6b}k^2(a-2bc)-\frac{3\sqrt{6b}k^2(a-bc)}{\sqrt{36b^2+2\sqrt{6b}k^2(a-bc)}})$$

while $34b^2-11b\sqrt{36b^2+2\sqrt{6b}k^2(a-bc)}<34b^2-66b^2=-32b^2<0$

$$\sqrt{6b}k^2(a-2bc)-\frac{3\sqrt{6b}k^2(a-bc)}{\sqrt{36b^2+2\sqrt{6b}k^2(a-bc)}}=\sqrt{6b}k^2[(a-2bc)-\frac{3(a-bc)}{\sqrt{36b^2+2\sqrt{6b}k^2(a-bc)}}]$$

When $\dfrac{a}{2c}<b<\dfrac{a}{c}$, $\sqrt{6b}k^2(a-2bc)-\dfrac{3\sqrt{6b}k^2(a-bc)}{\sqrt{36b^2+2\sqrt{6b}k^2(a-bc)}}<0$， therefore, $\dfrac{d\pi_D}{db}<0$. ■

Proposition 4.2.3.1 indicates that under this scenario, a co-marketing faced with sufficiently high price sensitivity will affect distributor's performance; and as effort sensitivity to demand grows, the profit of distributor first increases and then decrease.

Proposition 4.2.3.2 compares the performance under this scenario and benchmark model in Section 4.2.1.

Proposition 4.2.3.2

(1) When the following two conditions are fulfilled, equilibrium profit of the supplier is larger than in the benchmark model;

(a) $3024b^3+192bk^2(a-bc)\sqrt{6b}>[504b^2+38k^2(a-bc)\sqrt{6b}]\sqrt{36b^2+2\sqrt{6b}k^2(a-bc)}$;

(b) $4k^2+9<24b$.

(2) When: $16b(\frac{9b+16}{6})^3-3(\frac{9b+16}{6})^4>(a-bc)^2\frac{18b(\frac{128+288b}{9\sqrt{6b}(a-bc)})^3}{4(4b-\frac{128+288b}{9\sqrt{6b}(a-bc)})}$, equilibrium profit of the distributor is larger than in the benchmark model.

Proof:

(1) It can be easily achieved from the expression of $\pi_s-\pi_0$.

(2) From (3) in Proposition 4.2.3.1, we know that given other parameters, when $k=\sqrt{\frac{128+288b}{9\sqrt{6b}(a-bc)}}$, π_D reaches its maximum value.

Therefore, when $\pi_D|_{k=\sqrt{\frac{128+288b}{9\sqrt{6b}(a-bc)}}}>\pi_{D_0}|_{k=\sqrt{\frac{128+288b}{9\sqrt{6b}(a-bc)}}}$, there exists a range of k_1,k_2. When $k\in(k_1,k_2)$, π_D is larger than π_{D_0} in the benchmark model. ■

Proposition 4.2.3.2 lists the sufficient conditions when the performance in this scenario is better than in the benchmark model. To gain some implications from the strict sufficient conditions, numerical examples in this case are presented, We consider our special case where $a=30$, $c=10$ and $k=1.25$, and we vary b from 0.01 to 3.

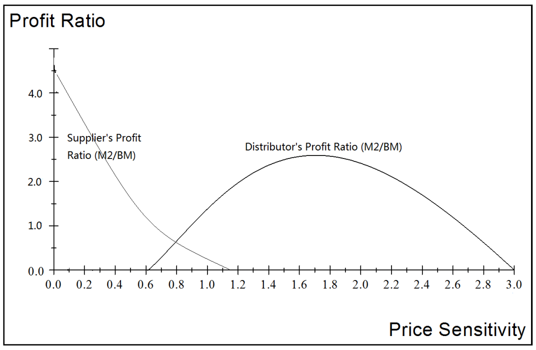

Figure 4.3 Profit Ratio (M2/BM Model)

Figure 4.3 indicates that when price sensitivity is at a relative low level, the distributor and the supplier achieve better performance.

4.2.4 Scenario II with fixed payment mechanism (M3)

In Scenario II, supplier pays for co-marketing partner's effort. Under the fixed payment mechanism, supplier pays co-marketing partner some compensation per unit of products sold.

The sequence of the game is as follows:

(1) The supplier decides wholesale price w and unit payment to co-marketing partner w_A;

$$\pi_s(w, w_A) = (w - w_A - c)(e_2^*)^\alpha (a - bp^* + ke_1^*)$$

(2) The distributor decides selling price p and effort level e_1;

$$\pi_D(p, e_1) = (p - w)(e_2^*)^\alpha (a - bp + ke_1) - e_1^{\beta_1}$$

(3) The co-marketing partner decides effort level e_2.

$$\pi_A(e_2) = w_A e_2^{\alpha}(a - bp + ke_1) - e_2^{\beta_2}$$

Theorem 4.2.4 gives sufficient conditions to ensure the existence and uniqueness of Nash Equilibrium solutions in this game.

Theorem 4.2.4 When all the following conditions are fulfilled, the supplier, the distributor and the co-marketing partner have unique equilibrium strategies on their decisions to maximize their respective profits.

(a) $\beta_1 > \dfrac{\alpha}{\beta_2 - \alpha} + 2$;

(b) $\beta_1 > \dfrac{\beta_2 + 2\alpha}{\beta_2 + \alpha} + \dfrac{2\beta_2 - \alpha}{\beta_2 k}$.

Proof: Similar to Theorem 4.2.2. ■

We also derive a property similar to Proposition 4.2.2.1 under this scenario.

Proposition 4.2.4.1 Under this scenario of unit-payment in co-marketing alliance, after estimation of co-marketing partner's optimal effort level e_2, for given unit payment w_A, the profit of distributor/retailer $\pi_D(p, w_A, e_1 | e_2^*)$ is supermodular in (e_1, p).

Proof: Similar to Proposition 4.2.2.1. ■

Proposition 4.2.4.1 indicates that given an equal unit payment w_A^*, an increase in distributor's effort e_1 results in a rise in optimal selling price p^*.

To better explore the properties and compare performance, we use a case with $\eta = \frac{\beta_2}{\alpha} = 2$ and $\beta_1 = 2$ as example.

When $\beta_2 = 2\alpha$ and $\beta_1 = 2$, we solve the game backwards to ensure sub-game perfection.

Stage 3: The co-marketing partner's profit function is given by:

$$\pi_A(e_2) = w_A e_2^{\alpha}(a - bp + ke_1) - e_2^{\beta_2}$$

We derive the first-order condition and we have:

$$\frac{d\pi_A}{de_2} = w_A(a - bp + ke_1)\alpha e_2^{\alpha-1} - \beta_2 e_2^{\beta_2-1},$$

$$\frac{d^2\pi_A}{de_2^{\ 2}} = w_A(a - bp + ke_1)\alpha(\alpha-1)e_2^{\alpha-2} - \beta_2(\beta_2 - 1)e_2^{\beta_2-2} < 0$$

Therefore, $e_2^{\ *} = (\frac{\alpha w_A(a - bp + ke_1)}{\beta_2})^{\frac{1}{\beta_2-\alpha}}$

Stage 2: The distributor's profit function is given by:

$$\pi_D(p, e_1) = (p - w)(e_2^{\ *})^{\alpha}(a - bp + ke_1) - e_1^{\beta_1}$$

$$= (p - w)(\frac{\alpha w_A}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}}(a - bp + ke_1)^{\frac{\beta_2}{\beta_2-\alpha}} - e_1^{\beta_1}$$

Hence, the first-order conditions on price, effort level are:

$$\frac{\partial\pi_D}{\partial p} = (\frac{\alpha w_A}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}}[(a - bp + ke_1)^{\frac{\beta_2}{\beta_2-\alpha}} - b(p - w)\frac{\beta_2}{\beta_2 - \alpha}(a - bp + ke_1)^{\frac{\alpha}{\beta_2-\alpha}}] = 0$$

$$\frac{\partial \pi_D}{\partial e_1} = (p-w)(\frac{\alpha w_A}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}} \frac{\beta_2}{\beta_2-\alpha}(a-bp+ke_1)^{\frac{\alpha}{\beta_2-\alpha}} k - \beta_1 e_1^{\beta_1} = 0$$

When $\beta_2 = 2\alpha$ and $\beta_1 = 2$, the above first-order conditions become:

$$a-bp+ke_1 = 2b(p-w), \quad bw_A(p-w)^2 k = e_1$$

Thus $p^* = \dfrac{2bww_A k^2 + 3b - \sqrt{9b^2 - 4k^2 bw_A(a-bw)}}{2bw_A k^2}$,

$$e_1^* = \frac{18b^2 + 4k^2 bw_A(a-bw) - 6b\sqrt{9b^2 - 4k^2 bw_A(a-bw)}}{4bw_A k^3}$$

Stage 1: The supplier's profit function is given by:

$$\pi_s(w, w_A) = (w - w_A - c)(e_2^*)^\alpha (a - bp^* + ke_1^*)$$

$$= (w - w_A - c)\frac{18b^2 + 4k^2 bw_A(a-bw) - 6b\sqrt{9b^2 - 4k^2 bw_A(a-bw)}}{2k^4 w_A}$$

$$\frac{\partial \pi_s}{\partial w} = \frac{(3b - \sqrt{9b^2 - 4k^2 bw_A(a-bw)})^2}{2k^4 w_A} + (w - w_A - c)\frac{2b^2}{k^2}(1 - \frac{3b}{\sqrt{9b^2 - 4k^2 bw_A(a-bw)}}) = 0$$

$$\frac{\partial \pi_s}{\partial w_A} = -\frac{(w-c)}{2}(\frac{3b - \sqrt{9b^2 - 4k^2 bw_A(a-bw)}}{k^2 w_A})^2 + (w - w_A - c)(\frac{3b - \sqrt{9b^2 - 4k^2 bw_A(a-bw)}}{k^2 w_A})$$
$$*(-\frac{2b(bw-a)}{\sqrt{9b^2 - 4k^2 bw_A(a-bw)}} - \frac{3b - \sqrt{9b^2 - 4k^2 bw_A(a-bw)}}{w_A}) = 0$$

On solving the above equations, we get:

$w^* = \dfrac{a+bc}{2b} + w_A$, $8bk^2 w_A{}^3 - 36bw_A + 9(a-bc) = 0$

According to Cardano's formula, we can calculate the solution of cubic equation of w_A as:

$$w_A = \sqrt[3]{\frac{9(a-bc)}{16bk^2} + \frac{3}{16bk^3}\sqrt{9k^2(a-bc)^2 - 96b^2}} + \sqrt[3]{\frac{9(a-bc)}{16bk^2} - \frac{3}{16bk^3}\sqrt{9k^2(a-bc)^2 - 96b^2}}$$

Therefore, equilibrium strategies and equilibrium profit of supplier and distributor are:

$$p^* = \frac{k^2 w_A(a+bc+2bw_A) + 3b - \sqrt{9b^2 - 2bk^2 w_A(a-bc) + 4b^2k^2 w_A{}^2}}{2bk^2 w_A},$$

$$e_1^* = \frac{9b^2 + k^2 w_A(a-bc) - 2bk^2 w_A{}^2 - \sqrt{9b^2 - 2bk^2 w_A(a-bc) + 4b^2k^2 w_A{}^2}}{2k^3 w_A}$$

$$\pi_s = \frac{a-bc-2w_A}{4k^4 w_A}(3b - \sqrt{9b^2 - 2bk^2 w_A(a-bc)})^2,$$

$$\pi_s = \frac{(3b - \sqrt{9b^2 - 2bk^2 w_A(a-bc)})^3}{4bk^6 w_A{}^2} - \left(\frac{3b - \sqrt{9b^2 - 2bk^2 w_A(a-bc)}}{2k^3 w_A}\right)^2$$

Proposition 4.2.4.2 w_A^* is decreasing in b.

Proof:

let $m = \frac{a-bc}{b}$, it is obvious that m is decreasing in b.

$$w_A(m,k) = \sqrt[3]{\frac{9m}{16k^2} + \frac{3}{16k^3}\sqrt{9k^2m^2 - 96}} + \sqrt[3]{\frac{9m}{16k^2} - \frac{3}{16k^3}\sqrt{9k^2m^2 - 96}}$$

For given k, $\frac{dw_A}{dm} = \frac{1}{3}\left(\frac{9m}{16k^2} + \frac{3}{16k^3}\sqrt{9k^2m^2 - 96}\right)^{-\frac{2}{3}}\left(\frac{9}{16k^2} + \frac{27m}{16k\sqrt{9k^2m^2 - 96}}\right) +$

$$+\frac{1}{3}(\frac{9m}{16k^2}-\frac{3}{16k^3}\sqrt{9k^2m^2-96})^{-\frac{2}{3}}(\frac{9}{16k^2}-\frac{27m}{16k\sqrt{9k^2m^2-96}})>0$$

Therefore, $\frac{dw_A}{db}=\frac{dw_A}{dm}.\frac{dm}{db}<0$, so w_A^* is decreasing in b. ■

Proposition 4.2.4.2 indicates the relationship between unit payment with price sensitivity and reaches the conclusion that low price sensitivity results in low unit payment. We can also interpret the conclusion to mean that low price sensitivity makes co-marketing partner less willing to spend marketing effort for alliance activities by reducing the unit payment price by the supplier.

Proposition 4.2.4.3

(1) When $k<\sqrt{4b-\frac{288}{(a-bc)^2}}$, equilibrium profit of the supplier is larger than that in the benchmark model;

(2) when $\frac{(a-bc)}{27b}(\frac{9(a-bc)}{16bk^2})^{\frac{2}{3}}>\frac{1}{4b-k^2}+\frac{(\frac{9(a-bc)}{16bk^2})^{\frac{1}{3}}}{9k^2}$, the equilibrium profit of the distributor is larger than in the benchmark model.

Proof:

(1) $\pi_s=\frac{a-bc-2w_A}{4k^4w_A}(3b-\sqrt{9b^2-2bk^2w_A(a-bc)})^2$

$$=\frac{a-bc-2w_A}{4k^4w_A}(\frac{2bk^2w_A(a-bc)}{3b+\sqrt{9b^2-2bk^2w_A(a-bc)}})^2=\frac{b^2(a-bc)^2w_A(a-bc-2w_A)}{(3b+\sqrt{9b^2-2bk^2w_A(a-bc)})^2}$$

$$>\frac{(a-bc)^2w_A(a-bc-2w_A)}{36}$$

$$w_A(a-bc-2w_A)=\frac{1}{2}2w_A(a-bc-2w_A)\le\frac{1}{2}(\frac{a-bc-2w_A+2w_A}{2})^2=\frac{(a-bc)^2}{8}$$

When $\pi_s>\frac{(a-bc)^4}{288}>\pi_{s_0}=\frac{(a-bc)^2}{4b-k^2}$, that is $k<\sqrt{4b-\frac{288}{(a-bc)^2}}$, $\pi_s>\pi_{s_0}$.

(2) Similar to (1). ■

Corollary 4.2.4.3

If $k<k_0<2\sqrt{b}$, then the equilibrium profit of the distributor is larger than that of benchmark model, where k_0 fit the equation:

$$\frac{(a-bc)}{27b}(\frac{9(a-bc)}{16bk_0^{\ 2}})^{\frac{2}{3}}=\frac{1}{4b-k_0^{\ 2}}+\frac{(\frac{9(a-bc)}{16bk_0^{\ 2}})^{\frac{1}{3}}}{9k_0^{\ 2}}$$

Proof:

The sufficient condition in Proposition 4.2.4.3 is

$$\frac{(a-bc)}{27b}(\frac{9(a-bc)}{16bk^2})^{\frac{2}{3}}>\frac{1}{(4b-k^2)}+\frac{(\frac{9(a-bc)}{16bk^2})^{\frac{1}{3}}}{9k^2}$$

which equals to

$$\frac{(a-bc)}{27b}(\frac{9(a-bc)}{16b})^{\frac{2}{3}}>\frac{k^{\frac{4}{3}}}{(4b-k^2)}+\frac{(\frac{9(a-bc)}{16b})^{\frac{1}{3}}}{9}$$

While $\dfrac{d\dfrac{k^{\frac{4}{3}}}{(4b-k^2)}}{dk}=\dfrac{\frac{4}{3}k^{\frac{1}{3}}(4b-k^2)+2kk^{\frac{4}{3}}}{(4b-k^2)^2}>0$

Therefore, when $k < k_0 < 2\sqrt{b}$,

$$\frac{k^{\frac{4}{3}}}{(4b-k^2)} + \frac{(\frac{9(a-bc)}{16b})^{\frac{1}{3}}}{9} < \frac{k_0^{\frac{4}{3}}}{(4b-k_0^2)} + \frac{(\frac{9(a-bc)}{16b})^{\frac{1}{3}}}{9} = \frac{(a-bc)}{27b}(\frac{9(a-bc)}{16b})^{\frac{2}{3}}$$, which fits

the sufficient condition in Proposition 4.2.4.3. ■

Proposition 4.2.4.3 gives the sufficient condition under which performance of the supplier and the distributor under alliance will be better than that in the benchmark model. The results together with Corollary 4.2.4.3 indicate that when effort sensitivity to demand is sufficiently small, distributor and supplier can be better off in alliance.

We also consider numerical examples in our special case where $a = 30$, $c = 10$ and $k = 1.25$, and we vary b from 0.01 to 3.

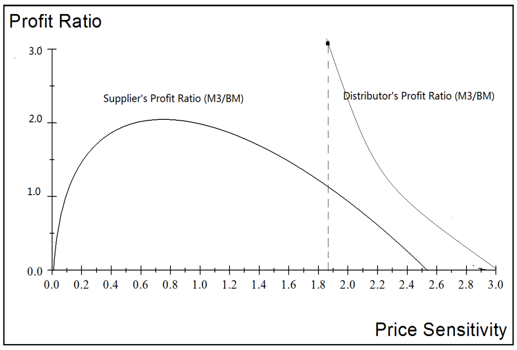

Figure 4.4 Profit Ratio (M3/BM Model)

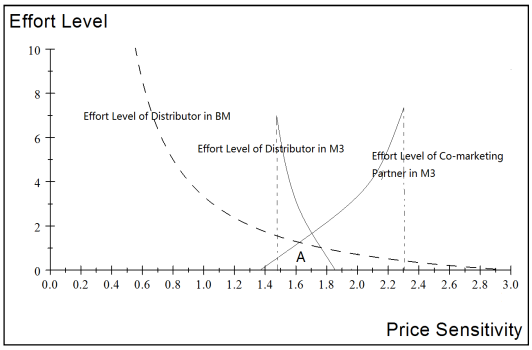

Figure 4.5 Effort Level (M3/BM Model)

Figure 4.4 indicates that low price sensitivity does favor to the supplier's performance. Especially, when price sensitivity is in a certain range, the partnership help both the supplier and the distributor achieve better performance. Figure 4.5 illustrates the effort level of benchmark model and the situation under alliance. The results can be concluded from the figure that in most cases, this partnership benefit the firms by its complementing power; however unexpectedly, even the benefit from the co-marketing partner brings negative influence to the partnering firms, the cooperative activities may still benefit the firms under certain conditions. (See Region A in Figure 4.5).

4.2.5 Scenario II with revenue sharing mechanism (M4)

In the revenue sharing mechanism, the supplier transfers part of its revenue to co-marketing partner as compensation.

The sequence of the game is as follows:

(1) The supplier decides wholesale price w and transfers θ percentage of revenue to the co-marketing partner;

$$\pi_s(w,\theta) = ((1-\theta)w - c)e_2^{*\alpha}(a - bp^* + ke_1^*)$$

(2)The distributor decides selling price p and effort level e_1;

$$\pi_D(p,e_1) = (p-w)e_2^{*\alpha}(a - bp + ke_1) - e_1^{\beta_1}$$

(3) The co-marketing partner decides effort level e_2.

$$\pi_A(e_2) = \theta w e_2^{\alpha}(a - bp + ke_1) - e_2^{\beta_2}$$

Theorem 4.2.5 gives the sufficient conditions to ensure the uniqueness of Nash equilibrium solutions for the game under this situation.

Theorem 4.2.5 When all the following conditions are fulfilled, the supplier, the distributor and the co-marketing partner have unique equilibrium strategies on their decisions to maximize their respective profits.

(a) $\beta_1 > \dfrac{\beta_2}{\beta_2 - \alpha}$;

(b) $\beta_1 > \dfrac{\beta_2 + 2\alpha}{\beta_2 + \alpha} + \dfrac{2\beta_2 - \alpha}{\beta_2 k}$;

(c) $(1-\theta)(\dfrac{\beta_2}{(\beta_2 - \alpha)(\beta_1 - 1) - \alpha} + 2) < \dfrac{\beta_2 c}{(\beta_2 - \alpha)(\beta_1 - 1) - \alpha}$.

Proof: Similar to Proof of Theorem 4.2.2.

To better explore the properties and compare performance, we use a case where $\eta = \dfrac{\beta_2}{\alpha} = 2$ and $\beta_1 = 2$ as an example. When $\beta_2 = 2\alpha$ and $\beta_1 = 2$, we solve the game backwards to ensure sub-game perfection.

Stage 3: The co-marketing partner's profit function is given by:

$$\pi_A(e_2) = \theta w e_2^{\alpha}(a - bp + ke_1) - e_2^{\beta_2}$$

We derive the first-order condition and we have:

$$\frac{d\pi_A}{de_2} = \theta w(a - bp + ke_1)\alpha e_2^{\alpha-1} - \beta_2 e_2^{\beta_2 - 1},$$

$$\frac{d^2\pi_A}{de_2^2} = \theta w(a - bp + ke_1)\alpha(\alpha - 1)e_2^{\alpha-2} - \beta_2(\beta_2 - 1)e_2^{\beta_2 - 2} < 0$$

Therefore, $e_2^* = (\dfrac{\alpha\theta w(a - bp + ke_1)}{\beta_2})^{\frac{1}{\beta_2 - \alpha}}$

Stage 2: The distributor's profit function is given by:

$$\pi_D(p, e_1) = (p - w)e_2^{*\alpha}(a - bp + ke_1) - e_1^{\beta_1}$$

$$= (p - w)(\frac{\alpha\theta w}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}}(a - bp + ke_1)^{\frac{\beta_2}{\beta_2-\alpha}} - e_1^{\beta_1}$$

We derive the first-order conditions on price and effort level and we have:

$$\frac{\partial\pi_D}{\partial p} = (\frac{\alpha\theta w}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}}[(a - bp + ke_1)^{\frac{\beta_2}{\beta_2-\alpha}} - b(p - w)\frac{\beta_2}{\beta_2 - \alpha}(a - bp + ke_1)^{\frac{\alpha}{\beta_2-\alpha}}] = 0$$

$$\frac{\partial\pi_D}{\partial e_1} = (p - w)(\frac{\alpha\theta w}{\beta_2})^{\frac{\alpha}{\beta_2-\alpha}}\frac{\beta_2}{\beta_2 - \alpha}(a - bp + ke_1)^{\frac{\alpha}{\beta_2-\alpha}}k - \beta_1 e_1^{\beta_1} = 0$$

When $\beta_2 = 2\alpha$ and $\beta_1 = 2$, the above first-order conditions become:

$$a - bp + ke_1 = 2b(p - w), \quad b\theta w(p - w)^2 k = e_1$$

Thus $p^* = \dfrac{2b\theta w^2k^2 + 3b - \sqrt{9b^2 - 4k^2b\theta w(a - bw)}}{2b\theta wk^2}$,

$$e_1^* = \frac{18b^2 + 4k^2b\theta w(a - bw) - 6b\sqrt{9b^2 - 4k^2b\theta w(a - bw)}}{4b\theta wk^3}$$

Stage 1: The supplier's profit function is given by:

$$\pi_s(w, \theta) = [(1 - \theta)w - c]e_2^{*\alpha}(a - bp^* + ke_1^*)$$

$$= [(1 - \theta)w - c]\frac{\theta w}{2}(\frac{3b - \sqrt{9b^2 - 4k^2b\theta w(a - bw)}}{k^2\theta w})^2$$

We derive the first-order conditions on wholesale price and transfer percentage, and we have:

$$\frac{\partial \pi_s}{\partial w} = \frac{c\theta}{2} \frac{(3b-\sqrt{9b^2-4k^2b\theta w(a-bw)})^2}{2k^4\theta w} + [(1-\theta)w-c]\theta w(3b-\sqrt{9b^2-4k^2b\theta w(a-bw)})$$
$$(-\frac{3b-\sqrt{9b^2-4k^2b\theta w(a-bw)}}{k^2\theta w^2} - \frac{2ab+4b^2w}{w^2\sqrt{9b^2-4k^2b\theta w(a-bw)}}) = 0$$

$$\frac{\partial \pi_s}{\partial \theta} = -\frac{w(w-c)}{2}(\frac{3b-\sqrt{9b^2-4k^2bw_A(a-bw)}}{k^2w_A})^2 + [(1-\theta)w-c]\theta w(3b-\sqrt{9b^2-4k^2bw_A(a-bw)})$$
$$(-\frac{3b-\sqrt{9b^2-4k^2b\theta w(a-bw)}}{k^2\theta^2 w} + \frac{2b(a-bw)}{\theta\sqrt{9b^2-4k^2b\theta w(a-bw)}}) = 0$$

Solving the above equations, we get:

$$w^* = \frac{a+bc}{2b},\ k^2(a+bc)^3 w_A{}^3 - 18b^2(a+bc)\theta - 9b^2(a-bc) = 0$$

According to Cardano's formula, we can calculate the solution of cubic equation of θ as:

$$\theta = \sqrt[3]{\frac{9b^2(a-bc)}{2k^2(a+bc)} + \sqrt{\frac{81b^4(a-bc)^2}{k^4(a+bc)^6} - \frac{216b^6}{k^6(a+bc)^6}}} + \sqrt[3]{\frac{9b^2(a-bc)}{2k^2(a+bc)} - \sqrt{\frac{81b^4(a-bc)^2}{k^4(a+bc)^6} - \frac{216b^6}{k^6(a+bc)^6}}}$$

Therefore, equilibrium strategies and equilibrium profit of supplier and distributor are:

$$p^* = \frac{\frac{k^2\theta(a+bc)^2}{2b} + 3b - \sqrt{9b^2-k^2\theta(a^2-b^2c^2)}}{k^2\theta(a+bc)},\quad e_1^* = \frac{(3b-\sqrt{9b^2-k^2\theta(a^2-b^2c^2)})^2}{2k^3\theta(a+bc)}$$

$$\pi_s = \frac{(1-\theta)(a+bc)-2bc}{2k^4\theta(a+bc)}(3b-\sqrt{9b^2-k^2\theta(a^2-b^2c^2)})^2,$$

$$\pi_D = \frac{4b(3b-\sqrt{9b^2-k^2\theta(a^2-b^2c^2)})^3 - (3b-\sqrt{9b^2-k^2\theta(a^2-b^2c^2)})^4}{4k^6\theta^2(a+bc)^2} \qquad (4.2.5)$$

Proposition 4.2.5 gives the sufficient condition when performance of supplier and distributor will be better than that in the benchmark model.

Proposition 4.2.5

(1) When the following conditions are fulfilled, equilibrium profit of the supplier is larger than that in the benchmark model;

(i) $a > 3bc$;

(ii) $(\frac{9b^2(a-bc)}{k^2(a+bc)})^{\frac{1}{3}}(a+bc)(a-3bc) > \frac{72b}{4b-k^2}$

(2) when $(12(\frac{9b^2(a-bc)}{k^2(a+bc)})^{\frac{1}{3}} - k^2)(a^2-b^2c^2) > \frac{81b^4(\frac{9b^2(a-bc)}{k^2(a+bc)})^{\frac{1}{3}}}{4b-k^2}$, equilibrium profit of the distributor is larger than in the benchmark model.

Proof:

(1) $\pi_s = \frac{(1-\theta)(a+bc)-2bc}{2k^4\theta(a+bc)}(3b-\sqrt{9b^2-k^2\theta(a^2-b^2c^2)})^2$

$= \frac{(1-\theta)(a+bc)-2bc}{2k^4\theta(a+bc)}(\frac{k^2\theta(a^2-b^2c^2)}{3b+\sqrt{9b^2-k^2\theta(a^2-b^2c^2)}})^2$

$= \frac{[(1-\theta)(a+bc)-2bc]\theta(a-bc)^2(a+bc)}{2(3b+\sqrt{9b^2-k^2\theta(a^2-b^2c^2)})^2} > \frac{[(1-\theta)(a+bc)-2bc]\theta(a-bc)^2(a+bc)}{72b}$

$\theta > (\frac{9b^2(a-bc)}{k^2(a+bc)})^{\frac{1}{3}}$, $1-\theta > 1-2(\frac{9b^2(a-bc)}{k^2(a+bc)})^{\frac{1}{3}}$

thus when: $\pi_s > \dfrac{(\dfrac{9b^2(a-bc)}{k^2(a+bc)})^{\frac{1}{3}}(a-bc)^2(a+bc)(a-3bc)}{72b} > \pi_{s_0} = \dfrac{(a-bc)^2}{4b-k^2}$, $\pi_s > \pi_{s_0}$.

(2) Similar to (1). ■

Corollary 4.2.5

When the following conditions are fulfilled, equilibrium profit of the supplier is larger than in the benchmark model;

(i) $a > 3bc$;

(ii) $k < k_0 < 2\sqrt{b}$, where k_0 is the solution of equation:

$$(\frac{9b^2(a-bc)}{{k_0}^2(a+bc)})^{\frac{1}{3}}(a+bc)(a-3bc) > \frac{72b}{4b-{k_0}^2}$$

Proof:

(1) The sufficient condition in Proposition 4.2.5 is

$$(\frac{9b^2(a-bc)}{k^2(a+bc)})^{\frac{1}{3}}(a+bc)(a-3bc) > \frac{72b}{4b-k^2}$$

which equals to

$$(\frac{9b^2(a-bc)}{a+bc})^{\frac{1}{3}}(a+bc)(a-3bc) > \frac{72bk^{\frac{2}{3}}}{4b-k^2}$$

While $\dfrac{d\dfrac{k^{\frac{2}{3}}}{(4b-k^2)}}{dk} = \dfrac{\frac{2}{3}k^{-\frac{1}{3}}(4b-k^2)+2kk^{\frac{2}{3}}}{(4b-k^2)^2} > 0$

Therefore, when $k < k_0 < 2\sqrt{b}$,

$\frac{72bk^{\frac{2}{3}}}{4b-k^2} < \frac{72bk_0^{\frac{2}{3}}}{4b-k_0^2} = (\frac{9b^2(a-bc)}{a+bc})^{\frac{1}{3}}(a+bc)(a-3bc)$, which fits the sufficient condition in Proposition 4.2.5. ■

Proposition 4.2.5 and its corollary shed light on the implication that supplier benefits from the low effort-demand sensitivity. The situation for the distributor is more complicated. Only when a condition involving marking size, price sensitivity, effort sensitivity and cost is fulfilled, could the distributor have better performance than in the benchmark model.

To deeply explore the implications of performance of co-marketing alliance under this scenario, we conduct numerical analysis in our special case. Without loss of generality, we suppose $a = 30$, $c = 10$ and $k = 1.25$, and we vary b from 0.01 to 3.

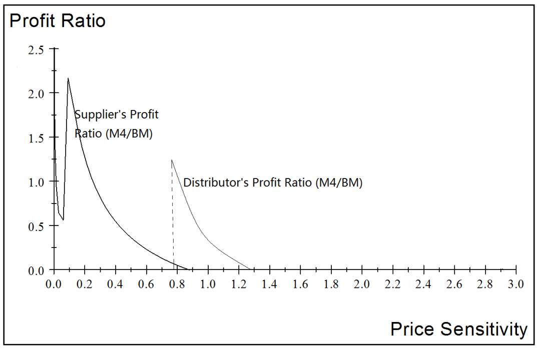

Figure 4.6 Profit Ratio (M4/BM Model)

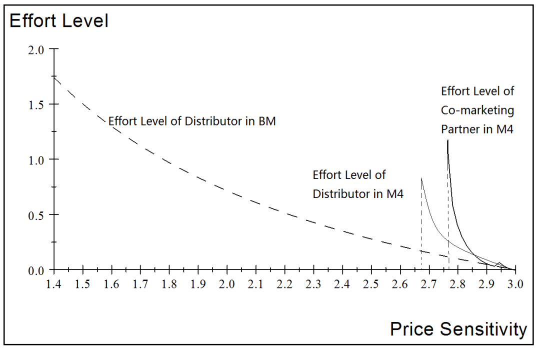

Figure 4.7 Effort Level (M4/BM Model)

Figure 4.6 demonstrates that when the price sensitivity is relative small, the partnering firms benefit from the cooperative activities. Figure 4.7 illustrates the effort levels of the distributor and the co-marketing partner under alliance. Compared with benchmark effort, the effort input greatly increases under alliance, but the effort output compensates the cost of effort, thus making the partnering firms better off.

CHAPTER 5 CONCLUSIONS

5.1 Summary

Although co-marketing alliance is growing in popularity and spans many types of businesses, it has received little attention in the literature. Our research complements previous work by quantifying the benefits of co-marketing alliance and by exploring how a co-marketing alliance creates value for partnering firms, especially their upstream suppliers, under different business scenarios. We develop game theoretical models to examine the influence of co-marketing alliance.

We first consider a first-best situation, i.e. when partnering firms and the supplier in a supply system are perfectly coordinated, and explore the potential of this alliance. The results indicate that when the effort level of the third party (co-marketing partner) has a positive influence on market demand, the optimal system profit may increase. We also find that even when the effort from the co-marketing partner has a negative influence on demand, the optimal system value may still increase due to the cost saving in this kind of alliance structure. Hence, our analysis shows that co-marketing alliance holds huge potential, and the value of a co-marketing alliance (i.e. an additional and different marketing effort) is driven by the complementing power and the effect of the cost reduction facilitated by the co-marketing effort.

Later, we examine how the co-marketing alliance may work when decisions of the partners are decentralized. We contrast the models under two different business scenarios (when the supplier pays for co-marketing partner's effort and when distributor pays for co-marketing partner's effort) and two profit-sharing mechanisms (fixed payment and revenue sharing) with a benchmark model before alliance. Our results indicate that equilibrium strategies and profits of supplier and distributor are mainly influenced by market size, price sensitivity and effort sensitivity of demand and cost-benefit ratio related with effort put in by the co-marketing partner. We, therefore, explore the cases with specific cost-benefit ratios, and derive relevant market conditions for cases where co-marketing alliance activities can improve supplier's and distributor's performance. These conditions can provide the theoretical basis for managers to decide how to improve firms' performance during the alliance process

It is worth noting that as shown by our results, the value of a co-marketing alliance is determined by the benefits and the cost of effort. In a first-best situation, the alliance can benefit from an extra effort put in at a lower effort cost. In the situations where players make their decisions separately, when supplier and distributor achieve better performance in the alliance, the costs of effort increase compared to the benchmark model. However, the benefit of effort under these cases can compensate the costs of effort input. Unexpectedly, even when the co-marketing partner might bring negative influence to the demand, the firms involved still achieve better performance under certain market conditions.

Our analysis also adds to the intuition by highlighting that price sensitivity and effort sensitivity of demand may influence the alliance's performance. In our model setting, when price sensitivity and effort sensitivity of demand are relatively low, both supplier and distributor may be better off under alliance. Products with low price sensitivity are usually non-homogenous products, such as chocolate and wine, because each well-known brand has its own customer base and the customers will not easily change their taste when price fluctuates in the market.

The numerical studies also add to the implications by comparing the performance of supplier and distributor between the models. For Scenario I where the distributor pays for the co-marketing partner's effort, compared the performance with two different profit sharing mechanisms, the performance with revenue sharing mechanism is better than that of fixed-payment mechanism under the same settings. In Scenario II where the supplier pays for the co-marketing partner's effort, fixed payment is favor to improve the supplier's performance compared with revenue sharing mechanism.

So to conclude, this thesis investigates strategic implications of co-marketing alliance, and produces several findings that provide significant managerial insights into value creation in the co-marketing alliance process.

5.2 Limitations and Future Research

The models presented in this thesis examine the cases under different scenarios with fixed cost-benefit ratio of co-marketing partner. Actually, however, the co-marketing partner, as a third party, can provide various types of services, which may result in different degrees of benefit or cost. Future studies may try to relax this restriction and explore situations with flexible cost-benefit ratios.

We also believe there are many other research issues related with co-marketing alliance that remain to be explored in future work. One interesting direction is to investigate the effect of information asymmetry in firms' decisions and its influence on firms' performance. Some interesting implications might be generated by examining the choice issues in this situation.

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