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GENERALIZED FORM
CHARACTERIZATION OF
ULTRA-PRECISION FREEFORM
SURFACES USING AN INVARIANT
FEATURE-BASED PATTERN
ANALYSIS

REN MINGJUN

Ph.D

The Hong Kong
Polytechnic University

2012

The Hong Kong Polytechnic University
Department of Industrial and Systems Engineering

**Generalized Form Characterization of
Ultra-precision Freeform Surfaces Using an Invariant
Feature-based Pattern Analysis**

Ren Mingjun

A thesis submitted in partial fulfillment of the requirements for the degree
of Doctor of Philosophy

December 2011

CERTIFICATE OF ORIGINALITY

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Ren Mingjun_____(Name of student)

Abstract

Ultra-precision freeform surfaces are complex surfaces that possess non-rotational symmetry and are widely used in many industries, such as advanced optics and biomedical implants, due to their superior optical and mechanical properties. In view of the geometrical complexity of freeform surfaces, there is no international standard for the traceable measurement and characterization of machined ultra-precision freeform surfaces with sub-micrometre form accuracy and nanometric surface finishing.

Motivated by the need for such a standard, this thesis presents an Invariant Feature-based Pattern Analysis Method (IFPAM) for the generalized form characterization of ultra-precision freeform surfaces. The IFPAM makes use of intrinsic surface properties, such as Gaussian curvature, to map the surface into a special image to form an orientation invariant feature pattern (IFP) for the representation of the surface geometry. The digital image processing techniques are then employed to conduct the IFP registration and correspondence searching for the form characterization of the surface. Compared with traditional freeform form characterization methods, such as least squares or minimum zone methods, the IFPAM is not only independent of the type of the surface being characterized but also from the coordinate frame which brings many difficulties and uncertainties for the characterization of freeform surfaces.

The calculation of the intrinsic surface features from a machined freeform surface is susceptible to the sampling strategy and the measurement noise and outliers associated in the measured data. To address these problems, a bidirectional curve network based sampling strategy (BCNSS) combined with a robust surface fitting and

reconstruction algorithm (RSFRA) are developed for ensuring accurate extraction of the intrinsic surface features from a machined freeform surface. The BCNSS is based on scanning two sets of curves on the measured surface along two different directions to form a curve network which is used to construct a substitute surface to represent the measured surface. The RSFRA is developed to reconstruct a high fidelity surface from measured discrete points while the surface smoothness can be ensured as well. A fitting threshold, named the confidence interval of fitting error, is used to strike a balance between fitting accuracy and surface smoothness in the fitting process. Experimental study confirms that the BCNSS and RSFRA provide an effective means for the improvement of the efficiency in data sampling and in increasing the accuracy of the surface representation for the measurement of ultra-precision freeform surfaces.

To access the reliability and accuracy of the IFPAM, a task specific uncertainty analysis model is built based on a Monte Carlo method to estimate the uncertainty associated in the results of the form characterization of ultra-precision freeform surfaces. Three influential factors are identified and considered in the model, including measurement error, surface form error, and sample size. Fractional Brownian motion is used to quantify the random surface form error while the measurement error is modeled by multivariable random noise. Rather than relying on intuition, the study is more focused on mathematical modeling of the relationship between the influential factors and the resulted uncertainty so that a prediction can be made to estimate the uncertainty in the form characterization of a specific freeform surface. The developed uncertainty analysis model is helpful for control and optimization of the IFPAM so as to provide more reliable form characterization results.

The IFPAM substantially addresses the deficiencies and limitations of traditional freeform surface characterization methods, which are more susceptible to embedded

coordinated systems and possess uncertainty due to the geometrical complicity and form variety of freeform surfaces. The outcome of this study not only significantly contributes to the state-of-the-art of measurement science and technology but also provides approaches that can be used in the standardization of measurement and characterization of freeform surfaces.

Publication Arising from the Study

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Ren, M.J., Cheung, C.F. and Kong, L.B., “A Bidirectional curve network based sampling method for enhancing the performance in measuring ultra-precision freeform surfaces,” *Precision Engineering*, accepted, 2012.

Cheung, C.F., Kong, L.B. and Ren, M.J., Whitehouse, D.J. and To, S., “Generalized form characterization of ultra-precision freeform surfaces,” *Annual of CIRP*, 61, 2012.

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Cheung, C.F., Kong, L.B. and Ren, M.J., “Measurement and characterization of ultra-precision freeform surfaces using an intrinsic surface features based method,” *Measurement Science and Technology*, 21, 115109, 2010.

Conference Papers:

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Ren, M.J., Cheung, C.F., Kong, L.B., Lee, W.B. and To, S., “A study of surface reconstruction for the measurement of ultra-precision freeform surfaces,” *The 10th International Conference of European Society for Precision Engineering & Nanotechnology (EUSPEN 2010)*, May 31st – June 4^{ed}, Delft, Netherlands, vol.1, pp:254-257, 2010.

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Chapter 1

Introduction

1.1 Background of the Study

With the development of science and technology, traditional industrial components composed of simple geometries like planes, spheres and cylinders are unable to fulfill the increasing demanding functionality of products. For instance, high-added-value photo-electronic parts in the optical industry have shifted from traditional symmetrical elements, such as spherical optics, to complex optical elements with freeform shapes for improving the performance of the products, in terms of both functionality and size reduction (Lee et al, 2005b; Jiang et al, 2007b).

Freeform surfaces are classified as complex geometrical features that have no symmetry in rotation or translation and are increasingly being used in many fields, from precision optics and bio-implants to multi-functional structures (Claytor et al, 2004). To ensure the functionality, the freeform surfaces are required to possess high precision in terms of form accuracy in the micrometre to sub-micrometre range and surface finishing at the nanometric level. The rapid development of ultra-precision machining technologies, such as single-point diamond turning (SPDT), ultra-precision raster milling, computer controlled ultra-precision polishing, allows designed ultra-precision freeform surfaces to be fabricated (Lee et al, 2005b). However, a fundamental problem is how to measure and characterize such surfaces so as to examine the conformity of the machined surfaces with designer's intend.

Form accuracy plays an essential role in the characteristics of freeform elements.

Research in surface characterization started in 1933 and the focus in recent decades has shifted from simple geometries like spheres to freeform surfaces (Jiang et al, 2007a). Traditionally, the form error of freeform surfaces is characterized based on specially manufactured test gauges (Savio et al, 2007). However, the quality of the measurement heavily relies on the proficiency of the operator and the measuring accuracy is difficult to be guaranteed. Therefore, automatic form characterization techniques have been developed, in which the mechanical gauges are replaced by a computerized geometrical model. A machined freeform surface is measured and the form accuracy of the surface is characterized by comparing the measured surface with the computerized model. In this way, the human element and the mechanical gauge are no longer necessary, thereby greatly saving the time and improving measurement accuracy.

In an overview of the published literature, one of the most challenging problems in the form characterization of freeform surfaces is surface matching (also termed as surface fitting (Jiang et al, 2010) and surface localization (Li and Gu, 2004). This is due to the fact that the measured data and the computerized model are not exactly located in the same coordinate system. Hence, surface matching is required to eliminate the misalignment of the coordinate systems before the form characterization of the measured data. Researchers employ the least squares or minimum zone methods (Li and Gu, 2005; Kong et al, 2010) to perform freeform surface matching, but they encounter problems such as matching uncertainties due to the dependency of the methods on the coordinate frame or the geometry of the surface being characterized. One promising approach is the utilization of the intrinsic surface properties which are independent from the coordinate frame. The implementation of the approach for the use of intrinsic surface properties emphasizes on image processing, computer visualization, and pattern recognition (Iyer et al, 2005), while

only a limited number of applications have been found in the characterization of ultra-precision freeform surfaces with sub-micrometre form accuracy.

The measurement and form characterization of a freeform surface combines with errors which lead to uncertainty to the characterization results. Uncertainty associated in the form characterization of freeform surfaces comes from many sources, including the error of the measurement instruments (Wilhelm et al, 2001), the error of the adopted sampling strategy (Philips et al, 1998), and the error imposed by the surface matching and comparison method being used. As a result, uncertainty analysis is indispensable for form characterization which assesses the accuracy and reliability of the measurement and the characterization results for the freeform surfaces. However, current research on the analysis of the uncertainty in geometric measurement is still focused on simple geometries such as circle, sphere and cylinder (Wilhelm et al, 2001; Maihle et al, 2009), and relatively little research work has been conducted on freeform surfaces. The general model described in the Guide to the Expression of Uncertainty in Measurement (ISO, 1995; 2008) is also difficult to apply to the process of freeform surface measurement since the uncertainty varies with the nature of surfaces being measured.

The geometrical complexity and high precision of the ultra-precision freeform surfaces bring considerable challenges to the measurement and form characterization of these surfaces. Although extensive research has been conducted in recent decades, there is still a lack of international standards and definitive methodologies to characterize the form accuracy of machined ultra-precision freeform surfaces with sub-micrometre accuracy. As a result, it is necessary to develop a practical and generalized method to perform high-precision and robust form characterization of ultra-precision freeform surfaces with sub-micrometer form accuracy and nanometric surface roughness.

1.2 Research Objectives

Motivated by the demand for a standard and generalized form characterization method for the measurement of ultra-precision freeform surfaces, this research aims to develop an invariant feature pattern based form characterization method to address the deficiency and limitations of the traditional freeform surface characterization methods identified in the previous section. This research attempts to address the following key objectives:

- (i) To study the characteristics of different types of ultra-precision freeform surfaces and the orientation invariant surface features, i.e. the invariant feature patterns are identified and engaged in freeform surface representation and surface matching;
- (ii) To develop reliable sampling strategies and effective surface fitting techniques to extract invariant feature patterns from the machined freeform surfaces with high fidelity;
- (iii) To develop an Invariant Feature-based Pattern Analysis method for the generalized form characterization of ultra-precision freeform surfaces with high-precision and robustness;
- (iv) To develop an uncertainty analysis model to assess the accuracy and reliability of the form characterization results of ultra-precision freeform surfaces.

1.3 Organization of the Thesis

The thesis is divided into 6 chapters. Chapter 2 gives a literature review of the relevant topics of this research including the application of ultra-precision freeform

surfaces in different fields, the enabling ultra-precision freeform machining technologies for the fabrication of ultra-precision freeform surfaces, the state-of-the-art high precision measurement instruments for the measurement of ultra-precision freeform surfaces, and current research on the automatic form characterization techniques for ultra-precision freeform surfaces.

Chapters 3, 4 and 5 constitute the core part of this research. Chapter 3 focuses on the study of measurement strategies and surface modeling of ultra-precision freeform surfaces. This attempts to address the problem of extracting surface intrinsic features from the machined freeform surfaces. A bidirectional curve network based sampling strategy is developed to enhance the efficiency and reliability of the sampling plan in the measurement of ultra-precision freeform surfaces. A robust surface fitting algorithm is developed to reconstruct a high fidelity surface from measured discrete points while the smoothness of the surface can also be ensured. In Chapter 4, the surface intrinsic features of freeform surfaces are studied and the orientation invariant surface features are identified and engaged in freeform surface representation and surface matching. Based on this approach, an Invariant Feature-based Pattern Analysis method is developed for the generalized form characterization of the machined ultra-precision freeform surfaces. In Chapter 5, a task specific uncertainty analysis model is presented to analyze the associated uncertainty in the form characterization results with consideration of three influential factors, including the error of the measurement instruments, the error introduced by the adopted sampling plan and the surface matching and comparison method. Chapter 6 provides an overall conclusion to the thesis, and some suggestions for the future research.

Chapter 2

Literature Review

2.1 Ultra-precision Freeform Surfaces and Their Applications

Traditional industrial components mostly consist of simple geometries, such as planes, spheres and cylinders, which are fundamental for the functionality of the components. However, with the rapid development of science and technology, these simple surfaces are inadequate to fulfill the more and more particular functionality of the products. For example, in the optical industry, the performance of optical systems which employ spherical lens is limited by aberrations (Lee et al, 2005b). As a result, more and more essential components with freeform surfaces have attracted a lot of interest from academics to industry.

Differing from conventional simple surfaces, freeform surfaces are complex surfaces which usually have no symmetry in rotation or translation (Claytor et al, 2004). Structured surfaces, such as microlens arrays, V-grooves, lenticulations, echells and pyramids are sometimes also classified as freeform surfaces since they have the same aspects in regard to fabrication, alignment and measurement (Jiang et al, 2007b). Owing to the superior optical and mechanical properties of freeform surfaces, they are now widely used in many fields, ranging from the design and manufacturing of die/molds, patterns and modeling, products in plastics, automotive and aerospace industries to biomedical, entertainment and geographical data processing applications (Savio et al, 2007). To ensure the functionality of the components, these surfaces are required to have form accuracy within the micrometre to sub-micrometre range and

surface finishing at the nanometric level (Jiang et al, 2011).

In the optics industry, freeform optics breaks through the traditional concept of optical imaging. It integrates different complex surfaces in an optical system to fit differing needs of the photonics and telecommunication technologies for transmitting, receiving, converting and storage of data. As shown in Figure 2.1, the first widely used commercial product that contained freeform components was the Polaroid SX-70 folding Single Lens Reflex camera which was introduced in 1972 (Plummer, 2004). In its optical design, two freeform optical components were used for distortion correction.



Figure 2.1 Polaroid SX-70 folding Single Lens Reflex camera (Adapted from Plummer, 2004)

Along with the rapid development of photonics and telecommunication, freeform optics have been emerging and are now increasingly being used in the design and production of high-value-added products. Some freeform optics that are commonly found in the market are shown in Figure 2.2. For example, the F-theta lens (Figure 2.2 (a)) is designed to provide a flat field on the image plane for scanning and engraving applications. It is commonly used in conjunction with galvanometer scanning mirrors

for such things as laser printing, laser marking, engraving and laser machining. Micro-structured freeform surfaces are crucial components for many photoelectrical products such as microlens scanners (Daly, 2001) and microlens arrays (Figure 2.2 (b)) for flat-panel digital displays. The progressive lens (Figure 2.2 (c)) is another widely used freeform optic. It is characterized by a gradient increasing lens power, which starts at the top of the lens and reaches a maximum addition power at the bottom of the lens (Pope, 2000). Wearers can adjust the additional lens power at different viewing distances by tilting their head to look through the most appropriate part of the vertical progression.

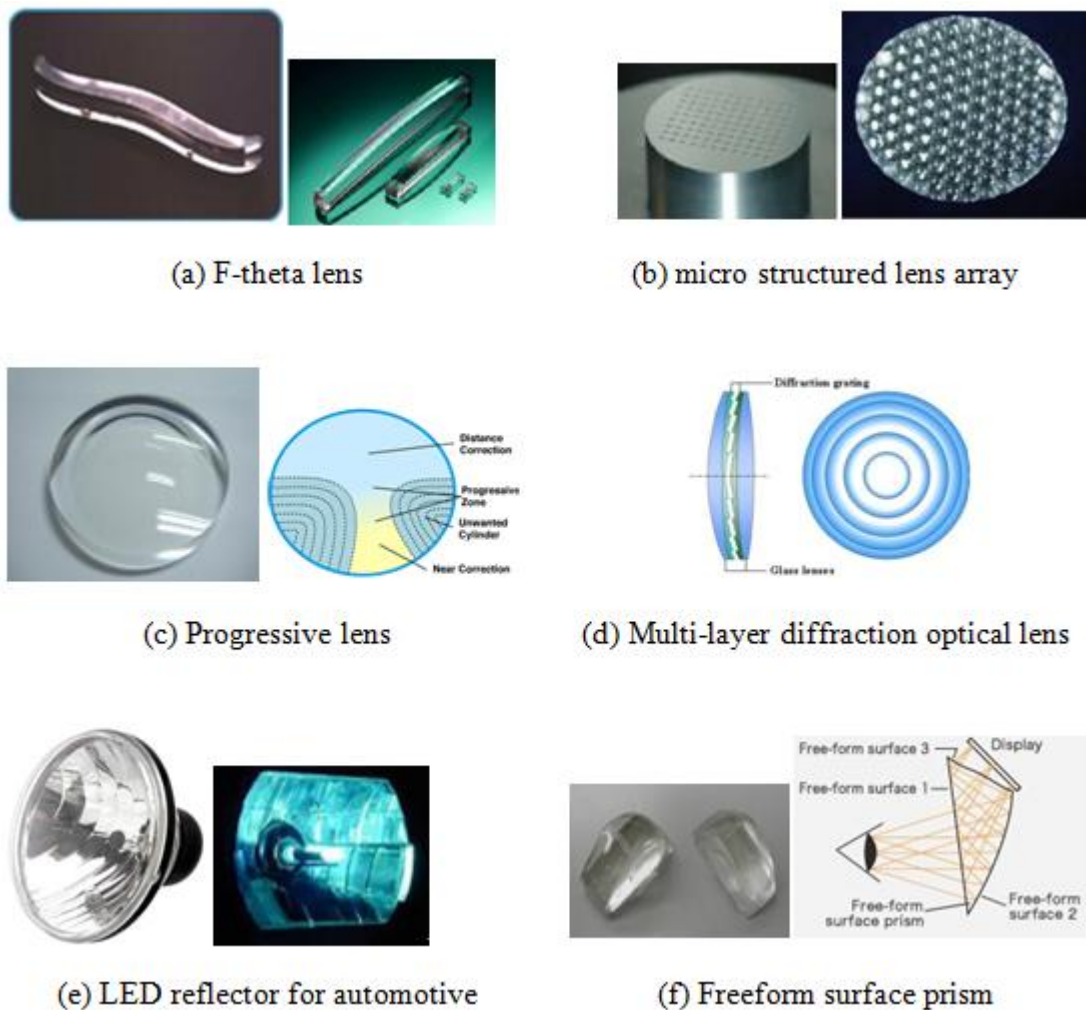


Figure 2.2 Commonly found freeform optics in the market (Adapted from Lee et al, 2005b)

Others include multi-layer diffraction optical elements (Figure 2.2 (d)) for a camera lens for improving imaging performance (Cannon, 2011), LED reflector (Figure 2.2 (e)) for automotive lighting system, and freeform surface prism (Figure 2.2 (f)) for head-mounted display systems (Lee et al, 2005b). A systematic review of freeform optics applications can be found in Lee et al (2005b) and Ledig (2010).

Over the past decades, the optical industry has grown from one based on skills and manual labour to one based on the design and manufacture of advanced products. In the United States of America alone, there are more than 5000 optical design and manufacturing companies with an estimated turnover of more than US\$50 billion (Kong, 2010c). The products are becoming more and more specialized and complicated so as to meet the increasing demands of customers. Figure 2.3 shows a roadmap of the evolution of the freeform optical components and products.

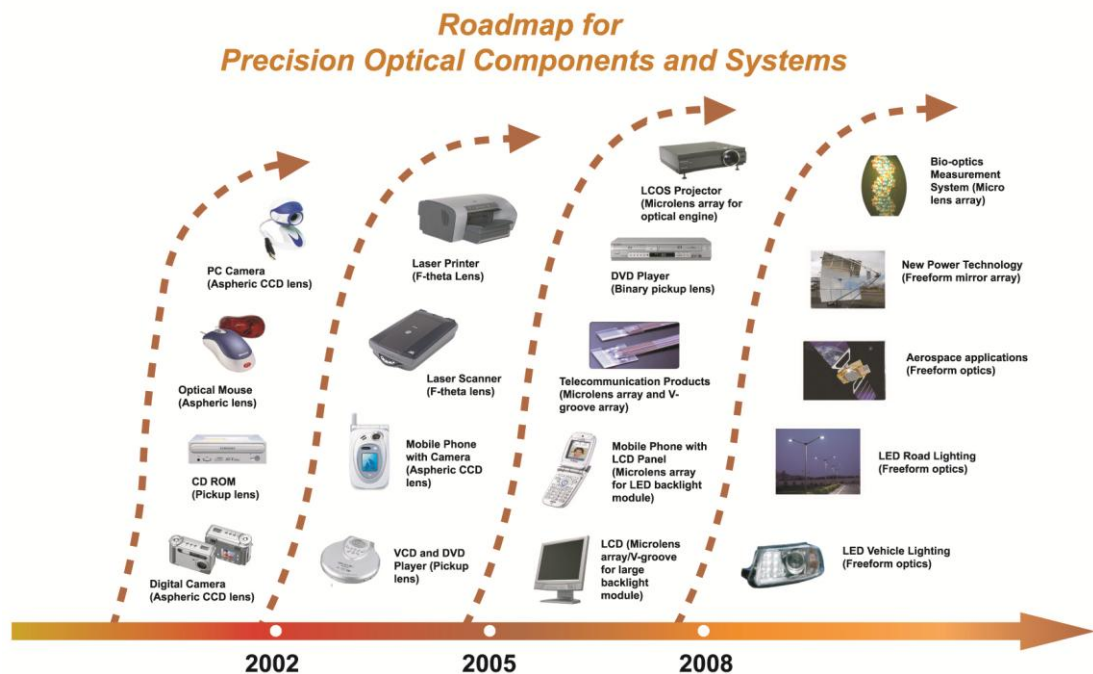


Figure 2.3 The roadmap of the application of freeform optical components

It is evident that the freeform components used in these products significantly

improve the performance in terms of both system size reduction and functionality (Lee et al, 2005b). For example, with the adoption of freeform micro-optics, biomedical measurement systems have become very small and robust; Lenticular arrays, formed from cylindrical lenses, are used in 3D displays for generating stereoscopic 3D effects; and a variety of freeform optics are used in LED vehicle lighting systems for increasing LED efficiency.

The use of the ultra-precision freeform surface is not limited to the optics field. As shown in Figure 2.4, bio-implants such as knee prostheses use freeform surfaces as bearing components. In order to prolong the implant against corrosion and wear debris, the form accuracy requirement for the implants is in the micrometre to sub-micrometre range, while that for surface roughness is less than 10 nanometres. It has been found that an increase of $0.1\mu\text{m}$ in roughness results in a 13 fold increase in wear (Charlton and Blunt, 2008).



Figure 2.4 Freeform knee joint orthopaedic implants

However, the high accuracy requirement and geometric complexity of ultra-precision freeform surfaces bring considerable challenges for the fabrication and measurement of these surfaces. Differing from traditional surfaces such as sphere and asphere, which can be produced by conventional two axis machines, more

complicated multi-axis ultra-precision machining technologies such as multi-axis milling and polishing are required in fabrication of ultra-precision freeform surfaces. The measurement and characterization of the machined ultra-precision freeform surfaces are also much more difficult and complex than conventional rotationally symmetry surfaces for which measurement may rely on the acquisition of several cross profiles.

For ultra-precision freeform surfaces, the measurement requires more advanced measurement technologies including precision 3D measuring instruments, comprehensive measurement strategies and reliable form characterization methods. Due to the geometric complexity of ultra-precision freeform surfaces, there is a lack of international standards for the form characterization of ultra-precision freeform surfaces. This brings a great deal of inconsistency in the information exchanges in industry as well as in the academic research field.

2.2 Ultra-precision Freeform Machining Technology

Ultra-precision machining represents the most advanced stage of machining and is one of the most important techniques for the manufacture of high precision components. Since its introduction in the 1970's, ultra-precision machining technology, such as single point diamond turning (SPDT), has become the most powerful solution for the manufacture of precision parts that require extremely high form accuracy and super smooth surfaces; for example, computer memory discs used in hard drives and photoreceptor components used in photocopy machines (Chapman, 2004). According to Taniguchi curves (Taniguchi, 1983), "precision" is a relative idea that varies with the development of science and technology; ultra-precision machining nowadays refers to machining technologies that can produce components with form

accuracy better than $0.1 \mu\text{m}$ and surface roughness smaller than $0.025 \mu\text{m}$.

However, ultra-precision freeform surfaces generally possess non-rotational symmetry or microstructure surfaces that have a tessellated pattern which cannot be machined by conventional two axis ultra-precision machining such as SPDT. Machining of such surfaces requires at least three numerically controlled machine axes. The increasing complexity of the surface geometry is associated with the increase of the number of controllable machine axes. Figure 2.5 shows the development of ultra-precision machining technologies towards higher accuracy and surface complexity (Riemer, 2011). This section reviews several state-of-the-art multi-axes ultra-precision machining technologies that are commonly used in the manufacturing of ultra-precision freeform surface.

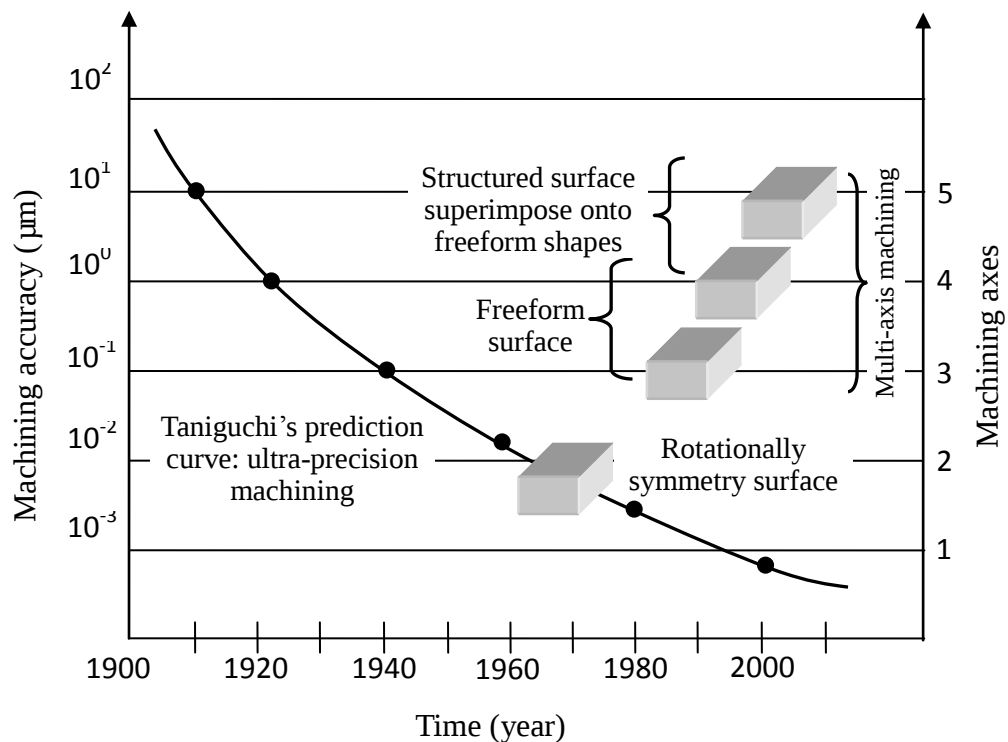


Figure 2.5 Development of ultra-precision machining technologies towards higher accuracy and surface complexity (Adapted from Riemer, 2011)

The Fast Tool Servo (FTS) is an electro-mechanical device equipped on a

diamond turning lathe. Initial development of the FTS for direct machining of freeform surfaces took place in the mid to the late 1980s (Falter and Dow, 1988). Since the tool has axial motion in coordination with the rotation of the spindle, non-rotationally symmetric parts can be made on the lathe. The axial motion of the tool is usually accomplished by a piezoelectric actuator. The high stiffness and low moving mass of the piezoelectric actuator allows high bandwidth (BW) in the axial motion. This is a significant advantage of the FTS for the fabrication of structured freeform surfaces such as microlens arrays (Luttrell, 2010).

However, most of the FTS systems have a maximum stroke less than 1 mm and therefore they are limited to certain geometries in which departure from the rotational symmetry is small. Recent innovations in FTS development include long stroke tool servos (Weck, 1995). Figure 2.6 shows a FTS actuator from Moore Nanotechnology Systems (NFTS-600) which enables displacements up to 1 mm at 100 Hz BW and 6 mm at 20 Hz BW (Moore, 2011).

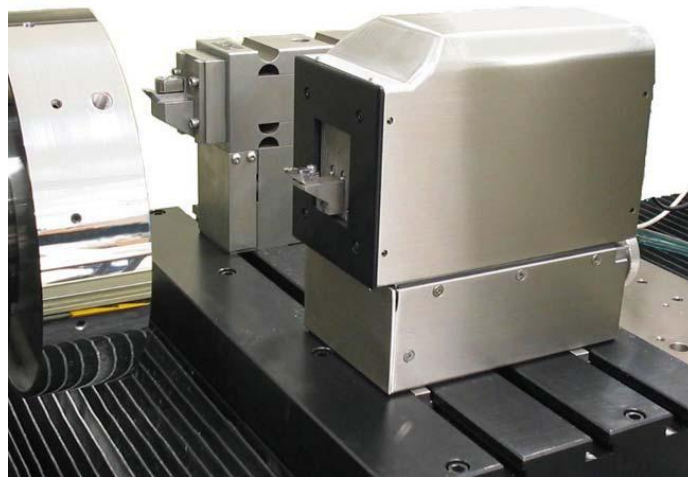


Figure 2.6 NFTS-600 Fast tool servo actuator (Adapted from Moore, 2011)

In addition, Brinksmeier et al (2010) have recently developed a nano FTS which has a 350nm stroke and is able to operate at a frequency up to 10 kHz. This system allows

the machining of complex microstructures such as diffractive optical elements.

The Slow Slide Servo (SSS) is another powerful machining technique for manufacturing ultra-precision freeform surfaces. The SSS is similar to the FTS in that it also moves the tool in coordination with the rotation of the work spindle, but uses the machine's Z slide instead of an additional motion actuator. Figure 2.7 shows the configuration of a SSS machine (Tohme and Murray, 2011). SSS is able to fabricate freeform surfaces with much larger deviations (millimeter scale) than those in the piezo-actuated FTS since the tool of the SSS can be oscillated at a range up to 25 mm (Tohme and Murray, 2011). The slow slide servo is easy to set-up, inexpensive and allows the manufacturing of high accurate freeform parts. The SSS method is commonly applied to machine torics, freeform polynomials, Zernike surfaces, and a wide variety of other freeform parts.

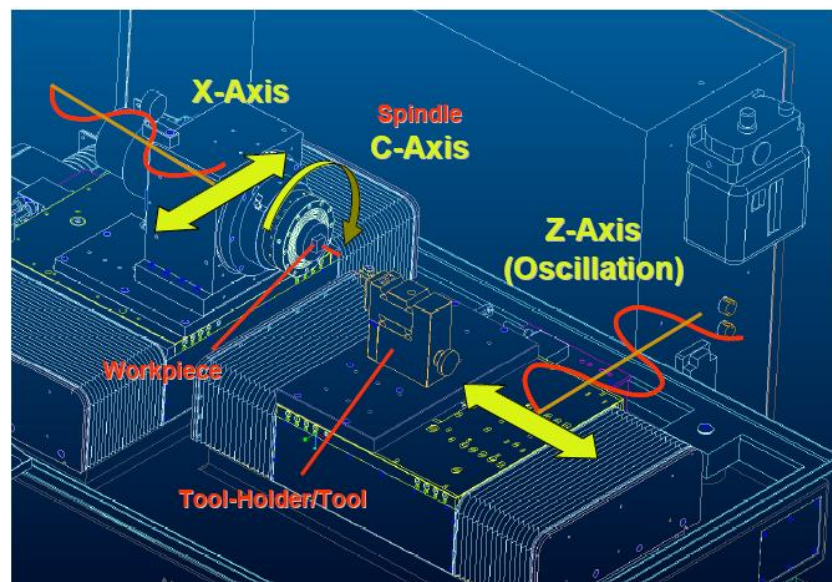


Figure 2.7 Configuration of a SSS machine (Adapted Tohme, 2011)

Ultra-precision raster milling is one of the most common techniques for manufacturing freeform surfaces with nanometric surface finishing and

sub-micrometre form accuracy without any additional post processing. Unlike the turning process, such as in a single point diamond turning machine, the generation of freeform shapes on raster milling has the workpiece relatively stationary and the cutting tool rotating on the main spindle. The motion of the machine axes is coordinated so that the cutting tool is moved across the workpiece along a series of parallel scanning lines with very close spacing. This raster type scanning allows the tool to follow very complex contours (Luttrell, 2010). Figure 2.8 shows a five axis ultra-precision freeform machine system named Freeform 705G from Precitech Inc. of USA (Precitech, 2011). There are totally five axes in the machine tool including three linear axes named X-axis, Y-axis and Z-axis, and two rotational axes named B-axis and C-axis, as shown in Figure 2.9. The machining is conducted in a raster milling style, which can machine the workpiece to a super mirror surface finish of several nanometres and form accuracy in the sub-micrometre scale using a one-pass cut (Kong, 2010c). It is frequently employed for machining ductile materials such as aluminum and copper.



Figure 2.8 Freeform 705G ultra-precision five axis raster milling machine

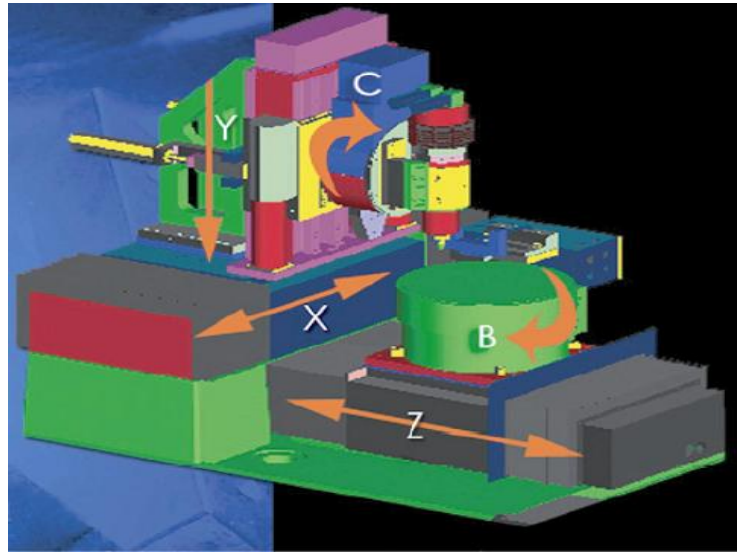


Figure 2.9 Configuration of the Freeform 705G

Ultra-precision grinding allows direct machining of a wide range of hard non-ferrous materials offering the benefits of high material removal rates with relatively low tool wear (Namba et al, 1999; Ohmori et al, 2000; Aurich et al, 2009; Xie et al, 2011). Hardened steel and brittle materials, such as glass and ceramics, are normally not amenable to using a single point diamond tool since these materials are susceptible to subsurface damage and easily cause chipping of the diamond tool. The grinding method minimizes the subsurface damage of the workpiece and provides a solution for machining brittle materials with sub-micrometre form accuracy and nanometric surface finish. (Lee et al, 2005b).

Ultra-precision freeform polishing includes bonnet based mechanical polishing, fluid jet polishing (Freeman, 2011), magnetorheological polishing (Jha and Jain, 2004), electrorheological polishing (Kuriyagawa et al, 2002), etc. It provides an important means for removing non-preferable machine signatures by diamond turning or raster milling and in machining those materials such as ferrous material that diamond machining cannot. Currently, ultra-precision polishing has been widely used in machining a variety of materials including plastics, biomedical polymeric materials,

ceramics and steels, etc. Figure 2.4 shows an ultra-precision multi-axis freeform polishing machine named the IRP200 from Zeeko Ltd. in UK (Zeeko, 2011). The machine has 7 axes of motion of which four axes control the work piece motion and the other three axes control the polishing head. It is reported as having the capability to polish freeform surfaces with sub-micrometre form accuracy and surface finishing in nanometre to sub-nanometre range (Kong, 2010).



Figure 2.10 IRP200 Ultra-precision multi-axis freeform polishing machine

Besides the ultra-precision multi-axes freeform machining technologies reviewed above, other methods such as diamond micro chiseling and micro milling are also powerful solutions in manufacturing complex structured freeform surfaces. More details of the development of ultra-precision machining over the last few decades have been summarized by Evans (1989), Chapman (Chapman, 2004) and Marsilius (2009). It is interesting to note from the literature review that the advanced development of ultra-precision freeform machining technologies constitute an enabling technology that allows the designed freeform surfaces to be fabricated with form accuracy in the sub-micrometre range and surface finishing at nanometric level.

2.3 Instrumentation for Measurement of Ultra-precision Freeform Surfaces

Two approaches are commonly used in the measurement of freeform surfaces, i.e. direct and indirect comparison (Ip and Loftus, 1996; Savio et al, 2007). With a direct comparison, the form accuracy of a machined freeform surface is characterized by checking the deviation of the surface with a corresponding mechanical gauge. The quality of the measurement heavily depends on the operator's proficiency and the manufacturing accuracy of the gauge. It is evident that the process is inefficient and the measuring accuracy is difficult to be guaranteed.

With indirect comparison, the mechanical gauge is replaced by a computerized geometrical model (e.g. computer aided design (CAD) model), and the comparison is performed between the measured surface and the computerized model. In this way, the human operation and mechanical gauge are no longer necessary, thereby greatly saving time and improving accuracy.

Savio et al (2007) classified measurement technologies for freeform surfaces into 6 categories and the typical ranges of measuring accuracy with part dimensions for different categories of measuring systems are shown in Figure 2.11. It can be seen that there are three categories that are suitable for the measurement of ultra-precision freeform surfaces with form accuracy in the micrometre to sub-micrometre range and dimensions in the several centimeters to several decimeters range, including coordinate measuring machines, profilometry and interferometric techniques.

In the following sections, the measurement principles as well as the state-of-the-art measurement instruments of these three categories are reviewed. The recently developed multi-sensor techniques as well as the typical measurement instruments are also reviewed.

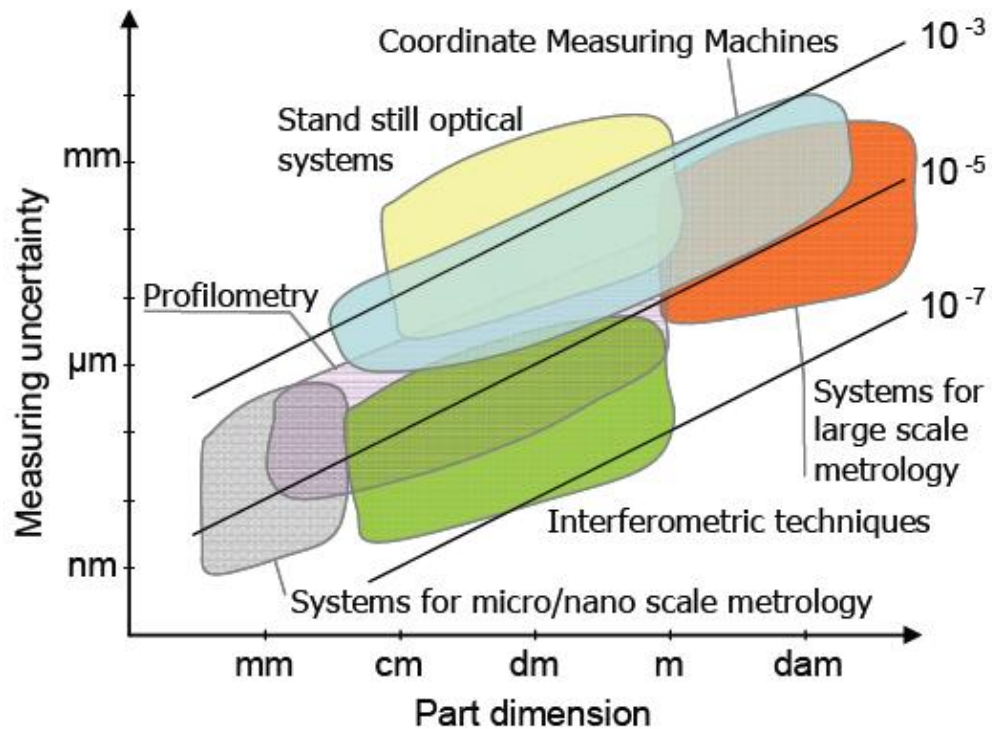


Figure 2.11 Measuring accuracy with part dimensions for different categories of measuring systems (Adapted from Savio et al, 2007)

2.3.1 Coordinate measuring machine

Coordinate measuring machines (CMMs) involve the most important measurement technology for the inspection of freeform surfaces and are widely used in industry. A CMM acquires spatial coordinates of discrete points and a points cloud is used or processed to represent the geometry of the surface shape being measured. Almost all machine designs of CMMs are based on linear axes arranged according to the Cartesian coordinate system with corresponding linear scales (Weckenmann et al, 2004). During the measurement, the measuring carriages are moved in the coordinate axes, and one of the coordinate axes, usually the Z axis, is equipped with a sensor for detecting the measured points. Hence, the accuracy of a CMM is determined by two systems: the moving system (including the positioning accuracy and perpendicularity

of the coordinate system) and the measuring sensor system (also referred to as the probing system).

Figure 2.12 shows four different types of designs of moving system for CMMs (Christoph and Neumann, 2010). The first type primarily consists of two mechanical stages (X and Y) with mechanical bearings as shown in Figure 2.12 (a). The Z axis also runs on mechanical bearings. The measuring range of this type of CMMs is approximately 200 mm to 400 mm (Christoph and Neumann, 2010).

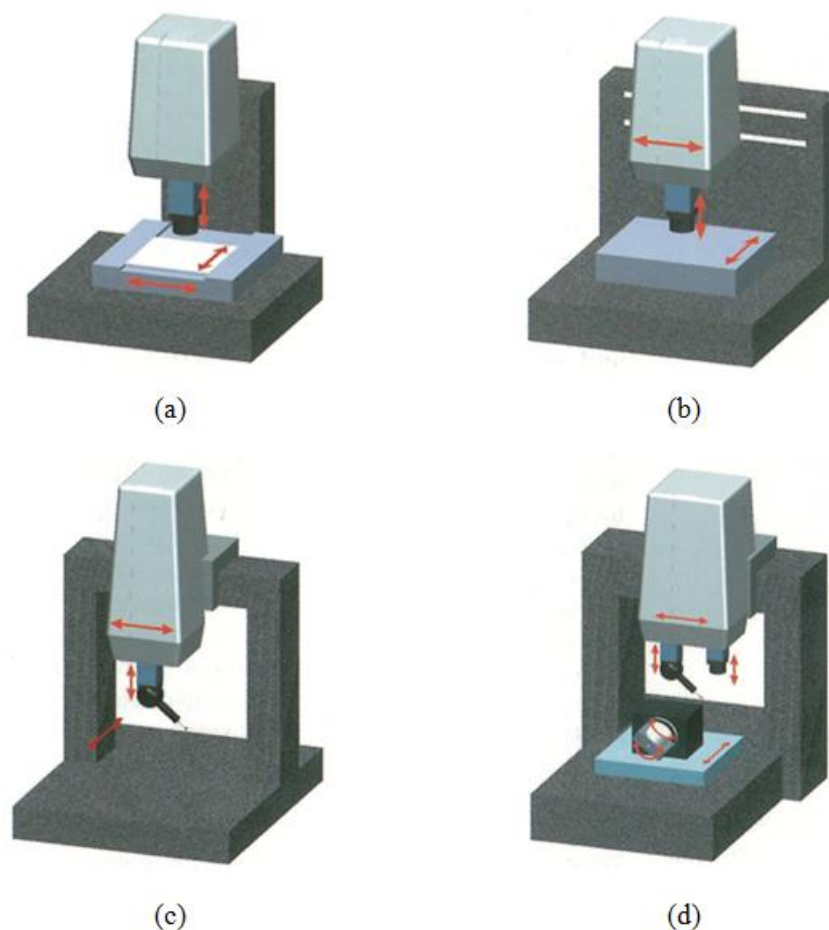


Figure 2.12 Designs of coordinate measuring machines (Adopted from Christoph and Neumann, 2010)

For higher precision requirements and larger measuring ranges, bridge-type designs are widely used as shown in Figure 2.12(b)-(d). In the design, the mechanical

guideway is replaced by moving systems with air bearings. The force required to move the carriages is relatively small and the lack of hysteresis in the positioning system results in low measuring uncertainties. For the type of design shown in Figure 2.12(b), the bridge and its columns move along the primary axis and a carriage moves laterally on the bridge along the secondary axis. The third axis is attached to this bridge. Since the workpieces are not moved during the measurement, the CMMs with this type of design are suitable for measuring extremely heavy workpieces. For fixed bridge type of designs as shown in Figure 2.12(c) and Figure 2.12(d), the workpieces are moved along the primary axis on a moving stage. The other two axes are arranged on the bridge. The chief advantage of this design is that the drive systems and scales of all three axes can be mounted centrally so as to minimize the Abbe offset effects (Christoph and Neumann, 2010).

The probing system is the most important element of a CMM which is responsible for the overall accuracy of a measurement. A probing system can be classified into two categories, i.e. contact probing systems and optical probing systems. This literature review focuses on the contact type of probing systems for micro-metrology and nano-metrology. A comprehensive review of optical probing and measurement systems can be found in Schwenke et al (2002), Estler et al (2002) and Hocken et al (2005).

Figure 2.13 shows the general principle of the contact type of probing systems. The CMMs move the probe onto the workpiece (or alternatively move the workpiece) and touch it with a sensing element (e.g. stylus tip); the coordinate of the touching point on the workpiece is determined by the relative position of the point to a reference point on the CMMs. This process is repeated and a cloud of points can be extracted from the workpiece, which are used to represent the geometry of the surface.

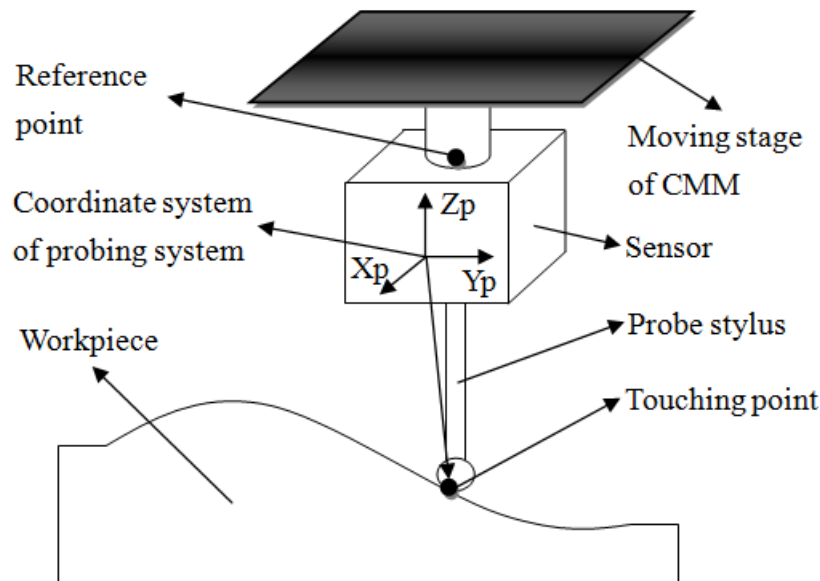


Figure 2. 13 General tactile probing systems of coordinate measuring machines

In a recent keynote speech at ISMTII 2011, Gao (2011) claimed that positioning and position measuring systems of sufficient range and resolution (several nanometres) are already available. In almost all cases, the probing system is the limiting factor of the measurement, including accessibility and the damage due to the touching force on the workpiece. To overcome these problems, micro-probing systems and nano-probing systems have been developed in recent years.

Technische Universiteit Eindhoven (TUE) developed a micro-probe and the probing system (Haitjema et al, 2001), as shown in Figure 2.14. The system adopted a three legged design, and piezo-resistive strain gauges are used to determine the position of the stylus. The resolution of this system is about 1 nm and the moving mass is limited to a mere 20 mg. The National Physics Laboratory (NPL) in UK developed a small probe (Peggs et al, 1999) that has a very light structure and relatively low probing force as shown in Figure 2.15. The probe was designed to operate with a standard probing force of 0.2 mN, corresponding to a probe tip deflection of about 10 μm . The resolution of the probe is claimed to be 3 nm. Other

recently developed micro tactile probes, such as METAS 3D probing system (Meli et al, 2003) and nonconventional probes, such as fibre opto-tactile probes developed by PTB (Schwenke et al, 2001), are also reported as having the capability to measure high aspect ratio surfaces with several nanometres resolution.

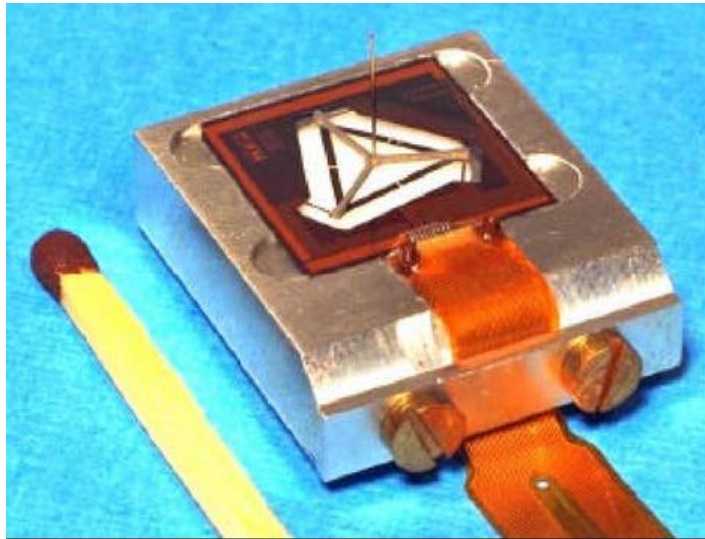


Figure 2.14 Micro-probe and the probing system developed by TUE (Adapted from Haitjema et al, 2001)



Figure 2.15 Micro probing system developed by NPL (Adapted from Peggs et al, 1999)

These micro-probing systems are now successfully integrated into the CMMs and there are several commercialized ultra-precision CMMs available in the market, such as Carl Zeiss F25 (Carl Zeiss, 2011) offered by Carl Zeiss Inc. in Germany, as shown in Figure 2.16, and the Isara series (IBS, 2011) offered by IBS Precision Engineering in Netherlands, as shown in Figure 2.17.

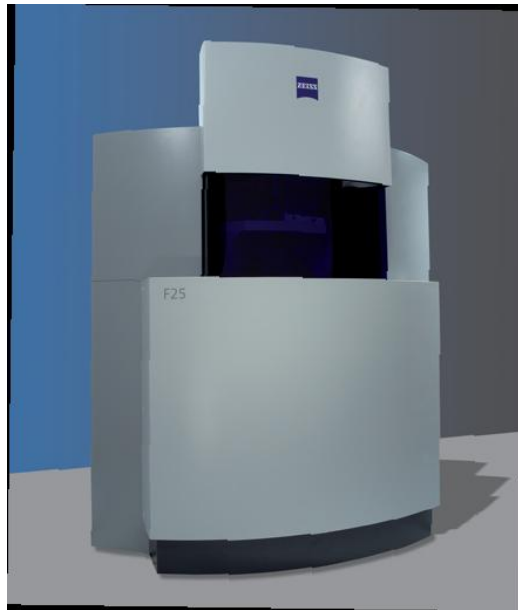


Figure 2.16 Carl Zeiss F25 ultra-precision CMM (Adapted from Carl Zeiss, 2011)

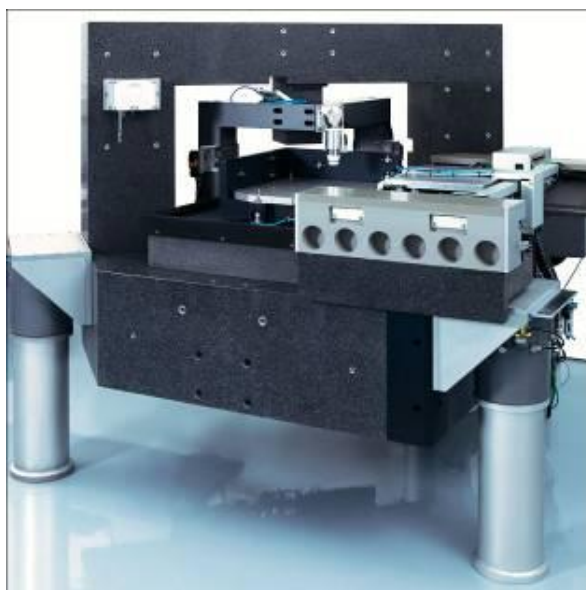


Figure 2.17 Isara 400 ultra-precision CMM (Adapted from IBS, 2011)

2.3.2 Profilometry

Profilometry is another widely used measurement technology in ultra-precision geometric measurement. In stylus profilometry, a stylus is traversed over a workpiece and a transducer measures the vertical displacement with resolutions that can be down to nanometres to sub-nanometres over a range up to 25 mm (Taylor Hobson, 2011). Conventionally, profilometry is very commonly found in industry for the testing of rotationally symmetry surfaces such as spheres and aspheres since the 1980s (Scott, 2002). Equipped with an additional moving stage, stylus profilometry is able to measure freeform surfaces in a raster scanning mode.

The transducer is the sensor responsible for the overall measuring accuracy of the profilometry. Laser interferometer is one of the most widely used methods in stylus profilometry (Jiang et al, 2007a). The measurement principle of this type of profilometer is shown in Figure 2.18.

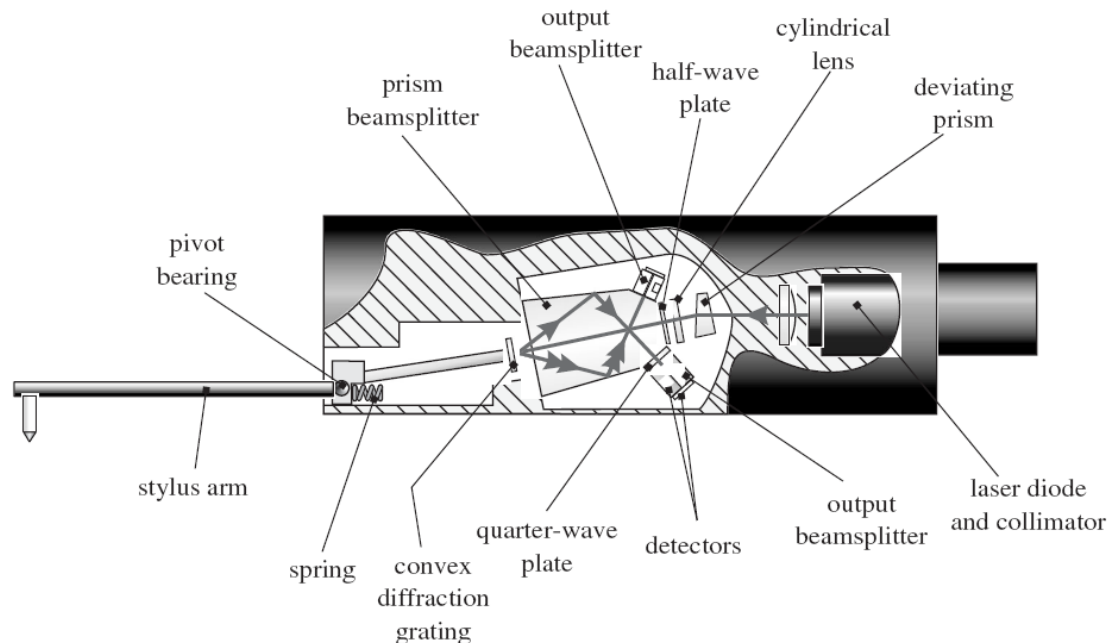


Figure 2.18 Measuring principle of laser interferometer based stylus profilometer

(Adapted from Jiang, 2007a)

A light from the laser source is split into two parts by a beam splitter, one of which goes to the reference mirror, and the other goes to the reflector fixed on the cantilever. When the stylus moves on the workpiece, the light reflected from the reflector is interfered by the light from the reference mirror and hence an interference stripe is generated. The phase of the interference stripe is relative to the displacement of the reflector, and the output data can be interpreted and the vertical displacement of the tip of the stylus is obtained. One of the typical commercialized instruments is the Surface Profiler PGI series offered by Taylor Hobson in UK (Taylor Hobson, 2011a) as shown in Figure 2.19. The recently offered PGI 2540 has a 25 mm measuring range with a resolution of only 0.4 nm.



Figure 2.19 Taylor Hobson Profiler PGI 1240 (Adapted from Taylor Hobson, 2011a)

Another typical commercialized profilometer widely used in ultra-precision freeform surfaces measurement is the Freeform profiler UA3P, developed by the Panasonic Corporation in Japan. UA3P is an ultra-high accuracy profilometer which uses an atomic force probe as the measurement probe (Panasonic, 2011). Figure 2.20 shows the measuring principle of this profiler. When the stylus moves on a workpiece,

the atomic force is generated against the measurement force that always keeps the atomic force constant, whereby the displacement is measured by the laser interferometer. UA3P has a measurement volume up to $400 \times 400 \times 90$ mm, probing force of 0.3 mN and laser interferometers for measuring the displacement in 3 axes. It has been reported that UA3P can achieve measurement accuracy down to 150 nm even when the slope of the workpiece is up to 75 degree (Panasonic, 2011).

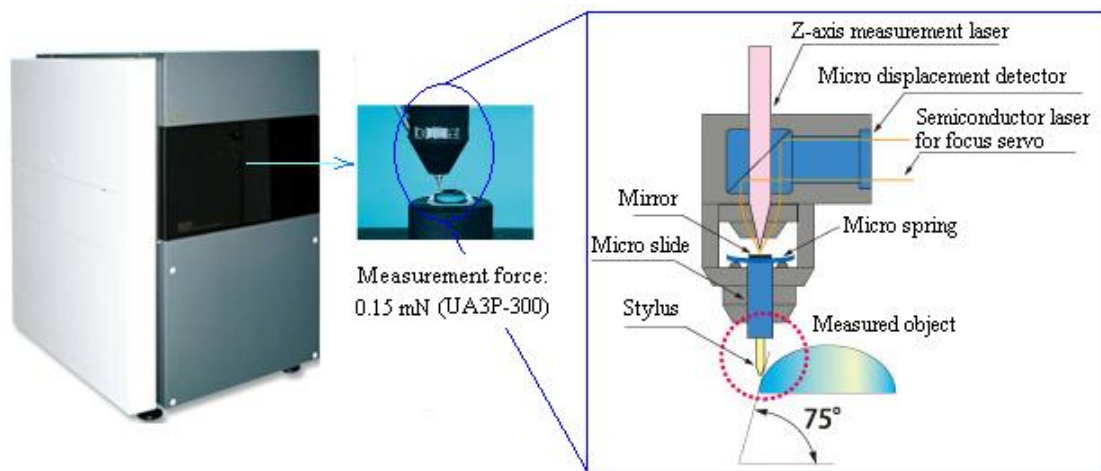


Figure 2.20 Freeform profiler UA3P and its measuring principle (Adapted from Panasonic, 2011)

2.3.3 Interferometric techniques

Interferometric techniques are well known as rapid, nondestructive and noncontact surface metrology techniques (Schwenke et al, 2002, Hocken et al, 2005). Figure 2.21 shows the general principle of interferometric methods (Blunt, 2006). The light from a light source (e.g. white light lamp) is split into two paths by a beam splitter. One path directs the light onto the workpiece under test and the other path directs the light to a reference mirror. Reflections from the two surfaces are recombined and projected onto an array detector. When the path difference between the recombined beams is of the order of a few wavelengths of light or less, the

interferometric stripes are generated and are read by the CCD detector. This interference contains information about the surface contours of the test surface. Vertical resolution can be of the order of several angstroms while the lateral resolution depends upon the objective, and is typically in the range of 0.5-5 microns (Estler et al, 2002; Blunt, 2006).

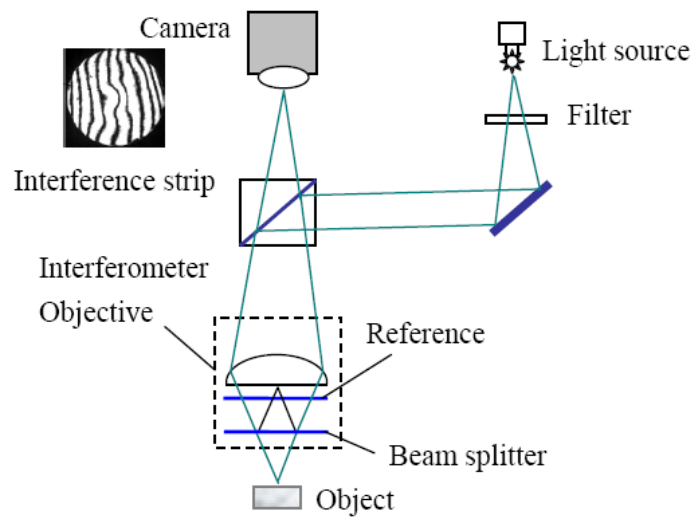
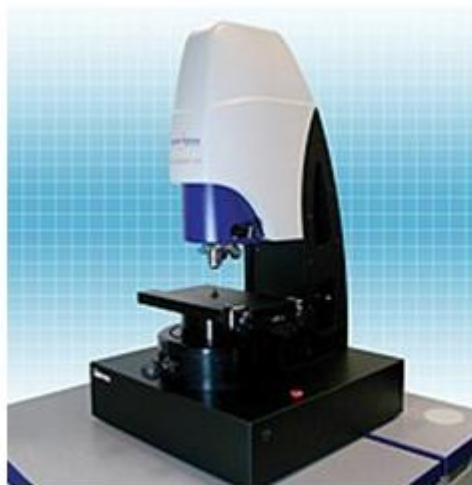


Figure 2.21 General principle of interferometric methods (Adapted from Blunt, 2006)

Typical interferometric measuring instruments in the market include Talysurf CCI series offered by Taylor Hobson Ltd in UK (Taylor Hobson, 2011b), Zygo surface profilers offered by Zygo Corp. in USA (Zygo, 2011), and Wyko optical interferometric profilers offered by Veeco Instruments Inc. in USA (Veeco, 2011). Figure 2.22 shows some noncontact optical instruments.

When measuring aspheres or freeform surfaces which have too much departure from the reference mirror, the dynamic range of the interferometers is generally insufficient. This can be overcome by using a correction element such as a null lens (Kim et al, 2004) or Computer Generated Hologram (CGH) (Pfeifer et al, 1993; Burge and Wyant, 2004). Figure 2.23 shows the schematic principle of CGH. CGH interferometry is based on the use of a surface specific diffractive element added to an

interferometer, which changes a spherical wavefront into a more complex wavefront. However, these elements have to be specially designed and manufactured for each specific surface shape. This not only adds to the cost but also increases the measurement uncertainty due to the manufacturing error and alignment error of these elements.



(a) Taylor Hobson CCI Lite Optical profiler



(b) Wyko NT9800 Optical Profiler

Figure 2.22 Noncontact optical instruments (Veeco, 2011; Taylor Hobson, 2011b)

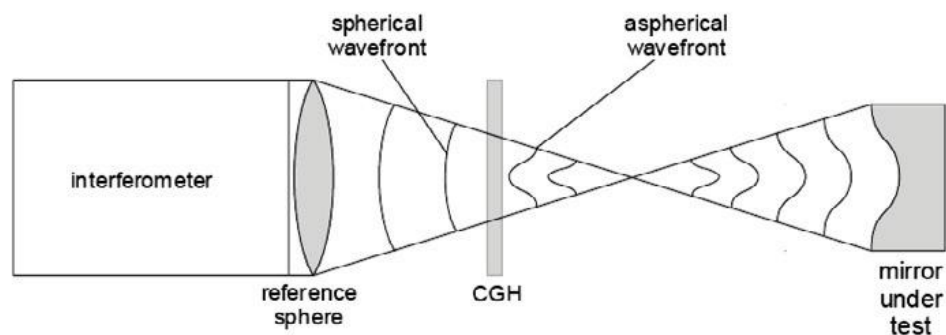


Figure 2.23 Schematic principle of a CGH (Adapted from Savio, 2007)

2.3.4 Multi-sensor techniques

There are already various measurement technologies available for the

measurement of ultra-precision freeform surfaces, as reviewed in previous sections. However all of these techniques have their own advantages and disadvantages and none of them can fulfill all the required tasks with high accuracy and efficiency (Weckenmann et al, 2006).

CMMs are powerful instruments for measuring complex surfaces with high accuracy. However, the contacting force may be unacceptable when measuring delicate parts. Further the relative slow measuring speed may take a long time for the measurement of ultra-precision freeform surfaces that require a large number of points to fully represent the surface geometry.

Stylus profilometry is good solution for measuring rotationally symmetric surfaces such as aspheres. However, the range measurement must be performed for the measurement of freeform surfaces and it will encounter the same problems as CMMs. The relatively small measuring slopes (normally no more than 30 degrees (Clayer et al, 2004)) and measurement ranges (normally no more than 20mm (Taylor Hobson, 2011)) of this method limit its application to complex surfaces with high curvature changes.

Interferometry techniques are well known rapid and noncontact surface metrology techniques that can measure thousands of points with nanometre accuracy in a second. However, the measurement range of this method is relatively small (normally no more than 2 mm) and stitching techniques are required when a large area is measured (Savio et al, 2007). Although the CGH method can be used to measure true freeform surfaces, it has to be specially designed and manufactured for each specific surface shape.

As a result, a sophisticated combination of several measuring techniques into a single system seems to be the appropriate solution for measurement quality assurance. Multi-sensor techniques have been already utilized in recent years and typical

commercialized instruments include Carl Zeiss F25 ultra-precision CMM (Carl Zeiss, 2011) by Carl Zeiss Inc. and VideoCheck UA 400 (Werth, 2011) offered by Werth Messtechnik GmbH in Germany. Taking VideoCheck UA 400 as an example as shown in Figure 2.24, it integrates an image processing video sensor, an optical distance sensor, a touch trigger, a dynamic scanning probe and a fiber probe into a single machine. With the combination of several measuring sensors, it can fully measure complex 3D geometries with accuracy down to the sub-micrometer range. However, the field of the multi-sensor measuring strategy and data fusion is emerging and has recently attracted a lot of research attention.



Figure 2.24 Werth VideoCheck UA 400 (Adapted from Werth, 2011)

2.4 Form Characterization of Ultra-precision Freeform Surfaces

A fundamental issue in the manufacturing process is to determine whether a machined workpiece meets the requirements of its original design specification. It is widely recognized that the surface form plays an essential role in the characteristics of freeform components (Iyer et al, 2005). Hence, the component must have extremely

high fidelity with the original design so as to ensure its functionality. It was explained in Section 2.3 that modern advanced measurement instruments are capable of extracting the coordinates of points on a machined freeform surface with accuracy at the nanometre level. However, the problem remains how to give an appropriate sampling strategy for a measurement so as to ensure that the extracted information is adequate for characterizing the surface form of a measured workpiece. Moreover, there are some key issues related to how to process and convert these measured data points into a useful mathematical model for representing the measured surface; and how to evaluate and characterize the form accuracy of the measured surface.

2.4.1 Sampling strategy

The sampling strategy is considered to be a major contribution to measurement uncertainty (Hocken et al, 1993; Weckenmann et al, 1997; Philips et al, 1998). This is crucial when it comes to the level of accuracy required by ultra-precision freeform surfaces. Several studies have been reported in the literature in which the sample period has been reduced, i.e. more sampled points, in order to reduce the effect of the sampling strategy on measurement uncertainty (Hocken et al, 1993; Edgeworth and Wilhelm, 1999). Most of optical sensor-based measuring instruments are able to measure huge numbers of points in a short time. However other systems, especially the contact types of measuring instruments, such as CMMs with contact probe and stylus profilometers, have limitations in measuring speed and may require a more intelligent sampling strategy.

Due to the high precision requirement and geometrical complexities, the measurement of ultra-precision freeform surfaces demands a large number of points for fully characterizing the surface geometry and reducing measurement uncertainty.

This usually takes a long measurement time. It is intuitive to perform the measurement with the lowest possible number of sample points while ensuring the accuracy as well. The traditional inspection planning software usually relies on uniformly sampling of data points on the surface, without considering surface complexity. However, a uniform sampling strategy may lead to undesirable results such as over sampling data points on low curvature regions of the surface, or under sampling data points from strong features and high curvature regions on the surface, which results in overlooking complex features of the surface.

Extensive research has been conducted on the development of various sampling methods to improve the efficiency of the sampling plan. Fiona et al (2009) have provided a thorough review of this topic for CMMs with touch-trigger probes. The main theme of these studies is the use of a feature pattern to guide the sampling process such that the complex regions of the measuring part are given more emphasis than simple regions. Cho and Kim (1995) proposed a surface curvature based sampling strategy for optimizing the distribution of sampling points with given numbers. The strategy divides the surface into a certain number of regions and ranks each region using the surface mean curvature. The regions with higher rank have more emphasis during the measurement. Edgeworth and Wilhelm (1999) presented an adaptive sampling technology for conducting measurement on CMM. Their procedure adds sample points along the workpiece profile until the errors between the nominal data and each interpolant between any two consecutive points of preliminary data set are within an acceptable tolerance range. However, this type of probe acquires data on a point-by-point basis and it takes a long time for each point as the process of approaching the surface and withdrawing has to be repeated for each probed point (Weckenmann et al, 2004).

The CMMs equipped with a continuous-contact probing system which can

acquire dense data along a curved path are becoming more widely used in industry (Neumann, 2000; Weckenmann et al, 2004). A scanning probes maintain contact with the measuring part while a set of points are sampled along a line across the part surface as it travels from one surface point to another. This type of measurement permits a much higher rate of data collection than the standard point-to-point technique. ElKott et al (2002) proposed several Non-Uniform Rational B-Spline (NURBS) surface parameters based algorithms for the measurement of freeform surfaces. NURBS parameters, surface patch size, and surface patch mean curvature are used as the criteria in the sampling process. Ainsworth et al (2000) presented a CAD-based approach for the planning of the CMM measurement path, which uses a recursive subdivision algorithm to sample the surfaces along their isoparametric lines. Chord length, minimum sampling density, and surface parameterization are the criteria used to guide the process. The work presented by ElKott and Veldhuis (2005; 2007) adopted the sampling of surface isoparametric curves, which are used to construct substitute geometry for the physical object. Both curvature change and accuracy of substitute geometry are used to determine the locations of the sampled curves.

Marinello et al (2007) proposed a bidirectional sampling strategy for three dimensional measurements. The measurement is carried out in two steps. In the first step, a set of curves is scanned along a particular direction. In the second step, another set of curves are scanned along different directions, and is used to compensate the error of the relative position of the first set of profiles caused by thermal drift.

One of the limitations of the current approaches to measuring ultra-precision freeform surfaces is that the sampling methods are based on surface features extracted from the CAD model. This makes the sampling pattern highly dependent on the coordinate system of the CAD model. Since the coordinate system of the CAD model

and the coordinate system of the measurement instruments are normally different, the relationship between the two frames has to be established before performing the measurement. However, this is a very challenging task for the measurement of ultra-precision freeform surfaces as there are uncertainties in coordinate transformation (Cheung et al, 2010). Furthermore, the form deviation of the measuring parts with the CAD model amplifies the uncertainty caused by the sampling strategy. Even in the case of circles and cylinders, the situation is complex and probably worse for ultra-precision freeform surfaces (Wilhelm et al, 2001). A compromise might be to start the measurement with an initial inspection of the surface using fast measuring devices, such as computer vision, to acquire prior knowledge about the measuring parts (Chen and Lin, 1997, Galetto and Vezzetti, 2006).

2.4.2 Surface fitting and reconstruction

Ultra-precision freeform surfaces possess non-rotationally symmetry that cannot be generalized by a universal equation used for rotational symmetric surfaces like aspheric surfaces. The representation of ultra-precision freeform surfaces is usually based either on a known surface model or a cloud of discrete measured data for an unknown surface model (Cheung et al, 2006). If the surface model is not available and only a cloud of discrete measured data of the surface is provided, the unknown surface model must first be reconstructed in order to obtain the form of the theoretical surface from the discrete data points.

In the measurement of ultra-precision freeform surfaces, a large number of discrete points are normally required for fully describing the geometry of the surface. However, performing dense measurement of freeform surfaces is very time

consuming for most of the contact-type measuring instruments such as the CMMs. Hence, in practice, a certain number of discrete points are extracted from the machined surface, with the guidance of appropriate measurement strategy, and fitted by a continuous surface to represent the measured surface, including the non-measuring area, and for further mathematical processing in the form characterization of the measured surfaces (Savio et al, 2007). This means that surface fitting and reconstruction is crucial in the measurement and form characterization so as to represent the surface shape based on a cloud of discrete points.

Due to the unlimited degree of geometrical freedom and the straightforward mathematics, B-spline surfaces are commonly adopted for constructing the surface. A B-spline surface $\mathbf{S}$ is defined as (Piegl and Tiller, 1997):

$$\mathbf{S}(u, v) \Big|_{p,q} = \sum_{i=1}^{i=n_u} \sum_{j=1}^{j=n_v} N_{i,p}(u) N_{j,q}(v) \mathbf{P}_{ij} \quad 0 \leq u \leq 1, 0 \leq v \leq 1 \quad (2.1)$$

where (p, q) are the degree of the B-spline surface; $\mathbf{P}_{ij}$ is the control point controlling the shape of the surface; n_u and n_v are the numbers of control points in u and v direction respectively; u and v are surface parameters identifying the location of point $\mathbf{S}(u, v)$ within the length of the two directions of the surface; $N_{i,p}(u)$ and $N_{j,q}(v)$ are the normalized B-spline functions uniquely defined by the degree p and a knot vector U , degree q and a knot vector V , respectively. In the present study, $(p=3, q=3)$ th degree of B-spline surface is adopted. Then the surface reconstruction can be formulated as an optimization scheme as follows (Weiss et al, 2002):

Given a set of discrete data points $\mathbf{X}_k$, $k=1, 2, \dots, m$, a B-spline surface $\mathbf{S}(u, v)$ is constructed with a set of control points $\mathbf{P}_{ij}$, $i=1, 2, \dots, n_u$, $j=1, 2, \dots, n_v$ and appropriate knots vector such that the objective function

$$F = \frac{1}{2} \sum_{k=1}^m d^2 \left(\sum_{i=1}^{n_u} \sum_{j=1}^{n_v} N_{i,3}(u) N_{j,3}(v) \mathbf{P}_{ij}, \mathbf{X}_k \right) + \lambda_s f_s \quad (2.2)$$

is minimized by satisfying the constraint $\min_{(u,v)} \|\mathbf{S}(u,v) - \mathbf{X}_k\| \leq \varepsilon_f$, where $d(\mathbf{S}(u,v), \mathbf{X}_k)$ is distance function which is used to measure the approximation error of the reconstructed surface from $\mathbf{X}_k$; f_s is a smoothing term and λ_s is the weight of f_s ; ε_f is user-defined tolerance.

This optimization scheme is a highly non-linear problem and is difficult to solve especially when the required tolerance for approximation is tight. Extensive literature addresses different problems on this topic (Franke and Nielson, 1991; Dierckx, 1993; Weiss et al, 2002). If the discrete points are given as a regular lattice (points on a grid), the problem can be solved by a tensor product B-spline surface using chord-length parametrised knot vectors. However, if the points are given as an unorganized points cloud, the surface is generally reconstructed in two steps. In the first step, an initial surface is constructed to approximate the points cloud with certain precision (Dierckx, 1993; Greiner et al, 1997; Weiss et al, 2002). An initial surface is used to estimate the minimal degree of freedom needed to characterize the real shape and to obtain a good parameter value to each data point by projecting the points cloud onto it.

There is a variety of methods to construct the initial surface, such as a plane, a bilinearly blended Coons patch (Hermann et al, 1997). Ma and Kruth (1995) used interactively defined section curves together with the four boundary curves to obtain an initial surface. Weiss et al. (2002) proposed a recursive method to construct the initial surface, starting with a simple surface with a few control points and gradually increasing the smoothness or the number of control points until the control net of the initial surface is reasonably fair and even.

The constructed initial surface is then used to assign parameters to all fitted points, i.e. locate the points on the initial surface that are close to the fitted points. Hoschek (1988) proposed an iterative method, called intrinsic parametrization, to correct a parameter using a formula that is a first order approximation to the exact foot point computation. Approximations of higher order or accurate foot point computations were discussed by Hoschek and Lasser (1993), Saux and Daniel (2003), and Hu and Wallner (2005).

Secondly, the initial surface is optimized to meet the prescribed tolerance. Once each data point is assigned corresponding parameters, the distance function from fitted data to the surface can be formulated to optimize the initial surface. Wang et al (2006) gave a thorough overview of the existing fitting method and divided them into three categories based on different distance functions (see Eq. (2.2)), i.e. point distance minimization (PDM), tangent distance minimization (TDM), and squared distance minimization (SDM). PDM minimizes the squared distance from the data points to the foot points to optimize the constructed surface. It is the most popular technique for surface fitting in computer graphics and CAD with B-spline surfaces, as well as other types of surface, due to its simplicity (Hopper et al, 1994; Farin, 1997; Hopper, 1998; Haber et al, 2001; Greiner et al, 2002;).

Blake and Isard (1999) utilized TDM from the data point to the tangent plane in the foot point as a based error term to speed up the convergence of the iterative optimization. SDM was studied by Pottmann and Hofer (2003) and was applied to solve curve and surface fitting problems (Pottmann et al, 2002). Wang et al (2006) proposed an algorithm based on a second order approximation to the distance function, named squared distance minimization (SDM), to solve the curve fitting problem. They compared the performance of the SDM with the PDM and the TDM in fitting a B-spline curve to point clouds and found that the SDM converges faster than the PDM

and is also more stable than the TDM.

Multilevel B-spline approximation of 3D discrete points was studied by Lee et al (1997; 2005a). The method is based on a coarse-to-fine hierarchy of control lattices to iteratively perform the local refinement such that a B-spline surface is reconstructed with high fidelity. Besides minimizing the approximation error of the reconstructed surfaces, various smoothness functions are usually added to improve the quality of the reconstructed surface. The most frequently used one is the simplified quadratic functional of the parametric derivatives (Dietz, 1998).

Traditional fitting technologies can efficiently reconstruct smooth surfaces from a cloud of discrete points, but few clearly demonstrate how smoothly the reconstructed surfaces should be done to represent the real form of the objects. It is important to specify an appropriate fitting threshold to balance the fitness and smoothness of reconstructed surface. This is particularly true for the reconstruction of a measurement surface from a cloud of measured data. In addition, most of the existing methods used to optimize the initial surface are based on local optimization, which makes the optimized surfaces very sensitive to the initial surface (Yang et al, 2004). It is vital to determine the appropriate number of control points and their distribution of initial surface to ensure the accuracy of the optimized surface in meeting a prescribed error threshold.

2.4.3 Form error evaluation of freeform surfaces

The form error of a measured surface is evaluated by comparing it with the design surface. A review of relevant published literature on the form characterization of freeform surfaces, reveals that one of the most important issues is surface matching (referred to by Jiang et al (2010) as surface fitting and by Li and Gu (2004) as

localization/registration) between the measured surface and the design template (Li and Gu, 2004; Savio et al, 2007; Cheung et al, 2009). This is due to the fact that the measured data points are embedded in the coordinate system of the corresponding measuring instrument, which is commonly not the same as the coordinate system of the design template. Slight misalignment between the two coordinate systems can cause serious errors in the results of the form error evaluation. This is vital for the measurement of ultra-precision freeform surfaces, which have form accuracy down to sub-micrometre level. Hence, the measured data and the designed template have to be precisely transferred to a common coordinate system before the comparison.

Form characterization of freeform surfaces is commonly formulated as an optimization problem to search for an optimal Euclidean motion (translation and rotation) based on methods such as Least Squares Method (Eq. (2.3)) or the Minimum Zone Method (Eq. (2.4)), so that the two surfaces are aligned as closely as possible (Besl and McKay, 1992; Meng et al, 1992; Yau and Meng, 1996):

$$F = \sum_{i=1}^n |\mathbf{P}_i - \mathbf{T} \mathbf{Q}_i|^2 \quad (2.3)$$

$$F = \min \left(\max |\mathbf{P}_i - \mathbf{T} \mathbf{Q}_i| - \min |\mathbf{P}_i - \mathbf{T} \mathbf{Q}_i| \right) \quad i = 1, \dots, n \quad (2.4)$$

where $\mathbf{Q}_i$ is the measured point and $\mathbf{P}_i$ is the corresponding point of $\mathbf{Q}_i$ in the template surface; $|\cdot|$ is the distance between $(\mathbf{P}_i, \mathbf{Q}_i)$ and is positive if $\mathbf{Q}_i$ is above the template surface, otherwise negative. $|\mathbf{P}_i - \mathbf{T} \mathbf{Q}_i|$ is considered as the form error of the measured surface at point $\mathbf{Q}_i$. The coordinate transformation $\mathbf{T}$ is a function of spatial rotation and translation and can be represented by following matrix

$$\mathbf{T} = \begin{bmatrix} c(r_z)c(r_y) & s(r_z)c(r_y) + c(r_z)s(r_y)s(r_x) & s(r_z)s(r_x) - c(r_z)s(r_y)c(r_x) & t_x \\ -s(r_z)c(r_y) & c(r_z)c(r_x) - s(r_z)s(r_y)s(r_x) & c(r_z)s(r_x) + s(r_z)s(r_y)c(r_x) & t_y \\ s(r_y) & -c(r_y)s(r_x) & c(r_y)c(r_x) & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.5)$$

where t_x , t_y and t_z are the translation components, and r_x , r_y and r_z are the rotation angles along the $\vec{X}$, $\vec{Y}$, and $\vec{Z}$ axis; $c()$ and $s()$ are abbreviations of the cosine and sine functions. The core of this optimization scheme is the establishment of correspondence pairs $(\mathbf{P}_i, \mathbf{Q}_i)$ between two matching objects.

In real measurement, neither the corresponding points of the measured data, nor the coordinate transformation function $\mathbf{T}$ are known knowledge. Hence, the freeform surface matching problem is generally solved in a nested approach (Besl and McKay, 1992; Ahn, 2004). Firstly, the correspondence of each measured point is established by locating the minimum distance point on the design surface in the inner iteration. In the outer iteration, $\mathbf{T}$ is estimated based on the established correspondence pairs. $\mathbf{T}$ is then used to transform the measured surface and new correspondence pairs are established. This process is iterated and is terminated when the desired accuracy is achieved.

Due to the non-convexity of the optimization problem, the matching results may be trapped at a local minimum or even become divergent if the initial relative position of the two matching surfaces is not appropriately provided (Ahn, 2004). Hence, conventional methods perform freeform surface matching in two stages, i.e. coarse matching and fine matching (Li and Gu, 2004; Zhang, 2009). Coarse matching is intended to find a rough position for the measured surface with respect to the design surface. The rough matching results are optimized at the fine matching stage.

2.4.3.1 Coarse matching of Freeform surface

If the measured surface includes some salient features like holes, slots, or pockets, these features can be used to perform coarse matching (Kyprianous, 1980; Joshi and Chang, 1988). However, most of smooth freeform surfaces do not contain any salient features. Consequently, various orientation independent mathematical and

geometrical surface features have been developed.

Wang et al (1997) defined the gravity centre of a surface as a feature point, the best fit plane as a feature plane, and the line from the gravity centre to the fastest point on the surface as a feature line. Then two surfaces are matched based on these invariant features. Cheung et al (2006) developed a method called the five-point method. The gravity centre and the four corner points of the points are used to construct a feature shape, which is used to perform surface matching. These methods are effective when the measured surface and the design template are approximately of the same size. Corney et al (2002) proposed a method to calculate the convex hull of a 3D object. Values such as hull crumpliness, hull packing and hull compactness are obtained from the convex hull and are taken as measures of the similarity between two objects.

Higuchi et al (1995) built a spherical map of curvature values called an SAI for each view of an object. The SAI are registered by rotating the spheres until the curvature values are aligned. Chua and Jarvis (1996) used principle curvatures to constrain a heuristic search for aligning matching objects. Feldmar and Ayache (1996) performed affine registration by minimizing the combined distance between position, surface normal and curvature. Thirion (1996) used crest lines to extract external points and their associated Darboux frames, which are matched in an ICP-like fashion. Soucy and Ferrie (1997) locally registered surface patches by minimizing the distance between Darboux frames over an entire neighborhood.

Yang and Allen (1998) minimized a scaled product of positional and curvature distances. Johnson and Herbert (1998) used invariants derived from the spin-image, a histogram of distances and angles to nearby surface points, to perform recognition and registration of 3D range maps. VandenWyngaerd et al (1999) matched bi-tangent curve pairs, which are pairs of curves that share the same tangent plane, between two

views for rigid and affine registration. Kase et al (1999) proposed a method to perform local matching and global matching of freeform surfaces based on both curvature change and angles formed by normal vectors. Sharp et al (2002) combined Euclidian invariants such as curvatures into the traditional ICP method, namely ICPIF, to improve the convergence to the global minimum. Osada et al (2002) presented a method to represent the surface as a signature of shape distribution. Hence the surface matching problem is converted to the comparison of probability distributions.

Ko et al (2003a; 2003b) utilized the Gaussian and the mean curvatures for surface matching, and the related iso-curvature lines were used to establish the correspondence between two objects. Other surface intrinsic properties of umbilical points and intrinsic frames were also studied and used to carry out the freeform objects matching. Li and Gu (2005) defined four types of surface shape: concave surface, convex surface, saddle surface, and flat surfaces based on different geometric information of the surfaces. The rough matching was carried out by searching four correspondences including feature similarity, inter-feature distance, curvature similarity, and frame similarity, and then the fine localization was conducted by the ICP method.

Zhu et al (2009) proposed a 3D shape matching approach based on diffusion wavelets. Jiang et al. (2010) proposed a new method for freeform surface matching whereby a new surface feature, named structured region signature, is used to conduct rough matching. Cheung et al (2006; 2009) and Kong et al. (2006) presented a series of surface matching algorithms for optical freeform surfaces. They presented several freeform surface coarse matching methods, including five-point method, couple reference data method, and a hybrid surface fitting and matching method.

Some global features like geometric moments (Funkhouser et al, 2003), Fourier descriptors (Zhang and Lu, 2002) and harmonic shape images (Groemer, 1996), which

are widely used in image retrieval, are also used in 3D surface matching. Novotni and Klein (2004) proposed a 3D Zernike descriptor for 3D shape searching. Zernike moments are considered as possessing the best performance against noise and information redundancy. Guo and Li (2009) presented a new shape descriptor named Gaussian curvature moment invariant for surface matching. It is claimed that the new descriptor is superior in accuracy and efficiency to Fourier descriptors and Zernike moments.

2.4.3.2 *Freeform surface fine matching*

The rough position obtained by coarse matching is refined by fine matching. The most commonly used method to deal with this problem is the Iterative Closest Point (ICP) method (Besl and McKay, 1992, Meng, 1992). ICP is an iterative descent procedure which seeks to minimize the sum of the squared distances between all points in measured surface and their closest points in the corresponding design surface. When the measured surface and the design surface can be represented as two sets of points with known correspondences, the coordinate transformation matrix can be determined by solving Eq. (2.3) or Eq. (2.4).

Many surface matching methods based on ICP are found in the published literature. Li and Gu (2004) provided an extensive review of this topic. Zhang (1994) extended ICP to include robust statistics and adaptive thresholds to handle outliers and occlusions. Masuda and Ykoya (1995) used ICP with random sampling and a least median squared error measurement that is robust to partial overlapping. Pottmann (2001) developed an approach similar to the ICP, which minimizes the sum of the squared distance between the measured points and the tangent plane of the nominal surface at a corresponding point. Jinkerson et al (1993) introduced a seventh parameter to the six used in the ICP minimization process, namely offset distance, for

solving the transformation; they claimed that the use of seven parameters results in smaller Root Mean Square (RMS) values after transformation than when only six are used.

Besides the development of new surface matching technologies, efforts have also been made to improve computing efficiency. Non-linear problems are usually solved by the combination of the Steepest-Descent method and the Newton-Raphson method (Kelley, 1999). However, as indicated by Menq et al (1992), solving non-linear equations takes an undesirable amount of computation time due to the complex operations involved. The problem worsens when a large number of measurement points are involved. Horn (1987) proposed a closed-form solution for providing more efficient, robust and reliable solution. Menq et al (1992) proposed a pseudo-inverse method to determine a transformation matrix that can be approximated by a rigid body transformation, claiming that the modified algorithm was 10 times faster. The method was subsequently modified by Huang and Gu (2001) to improve computing efficiency.

Although many kinds of orientation invariant surface descriptors have been developed to perform surface matching, many of them are either susceptible to the noise and outliers presented in the measured data or the features become weak when the surface being matched is relatively smooth and flat. This introduces unacceptable uncertainty to the surface matching results. Despite efforts to improve computation efficiency, surface matching under a conventional scheme still involves expensive computation. In each iteration, it requires the solution of highly non-linear equations (normally also an iterative process), and establishing correspondence pairs by searching the closest point for each measured point in the template surface. In practice, the characterization of ultra-precision freeform surfaces usually involves a large number of measured data points, which lead to an unacceptable computing time.

2.4.3.3 *Characterization of structured freeform surfaces*

As a special type of ultra-precision freeform surface, micro-structured surfaces are crucial components for many photoelectrical products such as backlight guides for display devices, microlens scanners (Daly, 2001), and microlens arrays for flat-panel digital displays. Although ultra-precision machining technologies, such as fast tool servo, enables the micro-structured surfaces to be fabricated, the measurement and characterization of such surfaces are still challenging tasks. This is due to the fact that most of micro-structured surfaces possess a high aspect ratio and high slope which makes most contact type measuring instruments unable to access the features or cause weak reflection for optical measuring instruments (Weckenmann et al, 2006).

The form characterization of the machined micro-structured surfaces is also different from that of smooth freeform surfaces. Micro-structured surfaces have tessellated pattern, for instance micro-lens arrays, micro-pyramids and micro-grooves. Hence, the characterization of a machined micro-structured surface not only needs to characterize the quality of each single feature (e.g. each lens in the surface), but also to analyze the pattern and lattice dislocation errors. Currently, optical micro-structured surfaces are usually characterized by their surface quality, such as surface roughness, as well as by their optical properties such as their modulation transfer function. Interferometric methods (Schwider and Sickinger, 2002; Ottevaere et al, 2003; Moench and Zappe, 2004; Reichelt et al, 2005) as well as wavefront measurement (Daly et al, 1994; Sickinger et al, 1999; Miyashita et al, 2004), such as Mach-Zehnder interferometers (MZI), are commonly used to test micro-optics.

Nussbaum et al (1997) characterized microlens arrays based on surface profiles, wave aberrations and surface roughness. Both geometrical features and the functional performance of the fabricated microlens arrays are considered in the characterization.

Liu et al (1998) tested micro-structured surfaces in an indirect way. The characterization is performed in terms of optical performance testing of fabricated products, such as spot uniformity, spot sizes and the positions of the focused spots at various accessible locations. Lee and Haynes (2001) proposed some parameters such as microlens pitch, fill factor, and surface quality to examine the quality of lens-let arrays for astronomical spectroscopy. Gu et al (2004) measured microlens profiles and some optical parameters, such as spot size, using laser scanning reflection/transmission confocal microscopy.

Cheung et al (2010) proposed a generic method for the characterization of freeform surfaces including micro-structured surfaces. The form deviation of each micro lens of a machined surface is evaluated by comparing with the design surface based on fine surface matching, similar to the characterization of smooth freeform surfaces. A surface matching-based method was also proposed by Yu et al (2011), but instead of quantitative pattern analysis of the lens arrays, only conventional surface height parameters, such as peak-to-valley-height and root-mean-square, are used in characterization. A pattern and feature parametric analysis method was proposed by Kong et al (2010) for characterizing optical microstructures. This involves using digital imaging technology to conduct quantitative pattern analysis and several lattice parameters to characterize the form deviation as well as the pattern and lattice distortion of the machined micro-structured surfaces.

2.4.4 Surface parameters

The manufacturing specification of freeform surfaces is commonly described by form tolerance, which is the permitted maximum value of the form deviation. According to ISO 1101 (ISO, 2004), there is a defined form tolerance zone within

which all points of the features must be contained. Within this zone, the machined surface may possess any form if it is not specified. Figure 2.25 shows the construction of a tolerance zone for a freeform surface. A set of points are sampled from design surface, then, a series of spheres are established with each sampled point as the centre point at a diameter d . Two enveloping surfaces can be constructed based on these spheres and denoted as the upper bound surface and the lower bound surface, respectively. The zone between the upper and lower surfaces is the tolerance zone of the design surface with tolerance d .

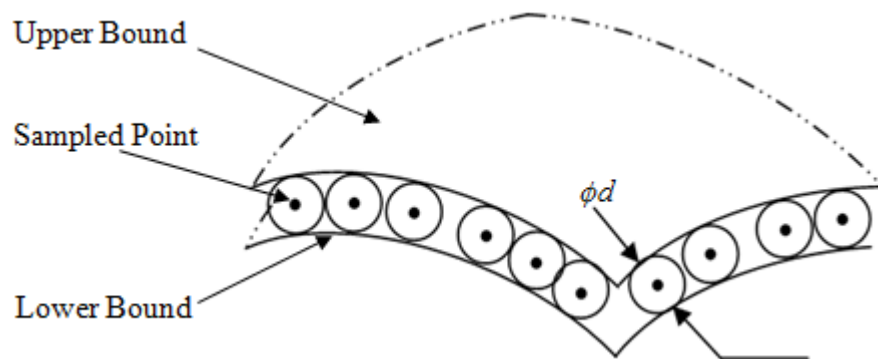


Figure 2.25 Tolerance zone of a freeform surface

For conventional simple surfaces like plane, sphere and cylinder, standard parameters such as flatness, straightness, cylindricity and circularity can be used to characterize the form accuracy of the workpiece. However, due to the geometric complexity of freeform surfaces, there are currently no standard surface parameters for the form characterization of freeform surfaces (Jiang et al, 2007b; Whitehouse, 2011). In Section 2.4.3, it is reviewed that the form error of a measured freeform surface at a measured point is determined by projecting this point onto corresponding design template after surface matching. In the present study, the form error of the measured surface is characterized by areal surface parameters (ISO, 2004) which are determined by the local form error of the measured points.

Areal surface parameters are developed for the extension of surface characterization from 2D profiles to 3D area. In 2002, a working group in ISO/TC 213 was set up to develop a new generation of areal surface texture standards (Jiang et al, 2007b). A new generation “geometric product specification and verification” (ISO/TC213-GPS) was published and some surface parameters presented for areal surface texture characterization (ISO, 2004). A similar standard is found in EUR 15178 EN (Whitehouse, 2011). The areal surface parameters can be classified into five groups, including height parameters, spatial parameters, feature parameters, functional parameters, and hybrid parameters.

Height parameters are amplitude parameters which are defined with respect to a mean plane obtained through the leveling of the mean square plane of the measured surface. Table 2.10 summarizes these height parameters. In the present study, the height parameters are used for characterizing the form errors of the measured freeform surfaces. Especially, the surface peak-to-valley height S_t and root-mean-square value S_q are usually employed for the characterization of form errors of freeform surfaces around the world, particularly Asian and European regions.

Other groups of parameters, such as spatial parameters, feature parameters, functional parameters, and hybrid parameters, have also been found to be used for characterizing freeform products for some specific applications. Spatial parameters quantify the lateral information present on the X and Y axes of the surface. Feature parameters are derived from the segmentation of a surface into motifs (hills and dales) in accordance with the watersheds algorithm. Functional parameters are calculated from the Abbott-Firestone curve obtained by the integration of the height distribution on the whole surface, while hybrid parameters quantify the information present in the X, Y and Z axes.

Table 2.1 List of surface height parameters (Adopted from EUR 15178 EN)

Parameters	Notations	Definitions	Remark
S_a	Arithmetic mean of the deviation from the mean	$S_a = \frac{1}{MN} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} z_{x,y} $	EUR 15178 EN report/ISO 25187
S_q	Root mean square height of the surface from the mean surface	$S_q = \sqrt{\frac{1}{MN} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} (z_{x,y})^2}$	EUR 15178 EN report/ISO 25187
S_p	Highest peak of the surface	Height between the highest peak and the mean plane	Conformity with 2D parameters
S_v	Deepest valley of the surface	Depth between the mean plane and the deepest valley	Conformity with 2D parameters
S_t	Total height of the surface	$S_t = S_p + S_v$	Conformity with 2D parameters
S_z	Height of the 10 points of the surface	Mean of distance between the 5 highest peaks and the 5 deepest valleys	EUR 15178 EN report/ISO 25187
S_{sk}	Skewness of the height distribution (third statistical moment, qualifying height distribution symmetry)	$S_{sk} = \frac{1}{MNS_q^3} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} (z_{x,y})^3$	EUR 15178 EN report/ISO 25187
S_{ku}	Kurtosis (fourth statistical moment, qualifying the flatness of the height distribution)	$S_{ku} = \frac{1}{MNS_q^4} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} (z_{x,y})^4$	EUR 15178 EN report/ISO 25187

2.5 Measurement Uncertainty and Traceability

Any measurement process combines with errors. Hence, when reporting a measurement result, it is obligatory that some quantitative indication of the quality of the results be given so as to assess its reliability. The indication takes the form of measurement uncertainties associated with the results (Cox and Harris, 2006). In 1995, the “Guide to the Expression of Uncertainty in Measurement” (GUM) was published by the ISO (ISO, 1995). GUM and several documents derived from it provide guidance for a wide range of uncertainty evaluation problems (Philips, 2004). A supplement to the GUM gives the guidance on the use of computer-based Monte Carlo methods, which provides more reasonable uncertainty intervals where the measurement model is highly non-linear or various input quantities have asymmetric probability distributions (JCGM, 2008).

Uncertainty associated in the measurement and form characterization of freeform surface comes from many sources, and the estimation of the total uncertainty of the measurement results is non-trivial due to the fact that coordinate measuring systems are complex and the measurement uncertainties vary with the task being performed, the environment, the operator, the chosen measurement methodologies, etc. Wilhelm et al (2001) divided the sources of uncertainty in geometric measurement into five main categories: hardware, workpiece, sampling strategy, fitting and evaluation algorithm, and extrinsic factors. Figure 2.26 shows the error components that lead to uncertainty in CMM measurements.

The hardware of the coordinate measuring system contributes most to the uncertainty of the measurement result. This category refers to the sources of uncertainty caused by uncorrected systematic and apparently random probing errors (Estler et al, 1997), probe changing and probe articulation uncertainties (Weckenmann

et al, 2004; 2006), probing parameters and errors caused by dynamics of the machine structure (Pereira, 2001), and environmental and machine temperatures and vibration (ISO, 2001). The uncertainties related to the properties of the workpiece include elastic deformation due to probing forces (Weckenmann et al, 2004), coordinate transformation to the part coordinate system (Cheung et al, 2006; 2007). It is evident that sampling strategy is a major contributor to the CMM's measurement uncertainty (Hocken et al, 1993; Phillips et al, 1995). Included in this are errors due to inadequate sampling (Choi et al, 1998) and sampling distribution (Choi et al, 1998; ElKott and Veldhuis, 2007).

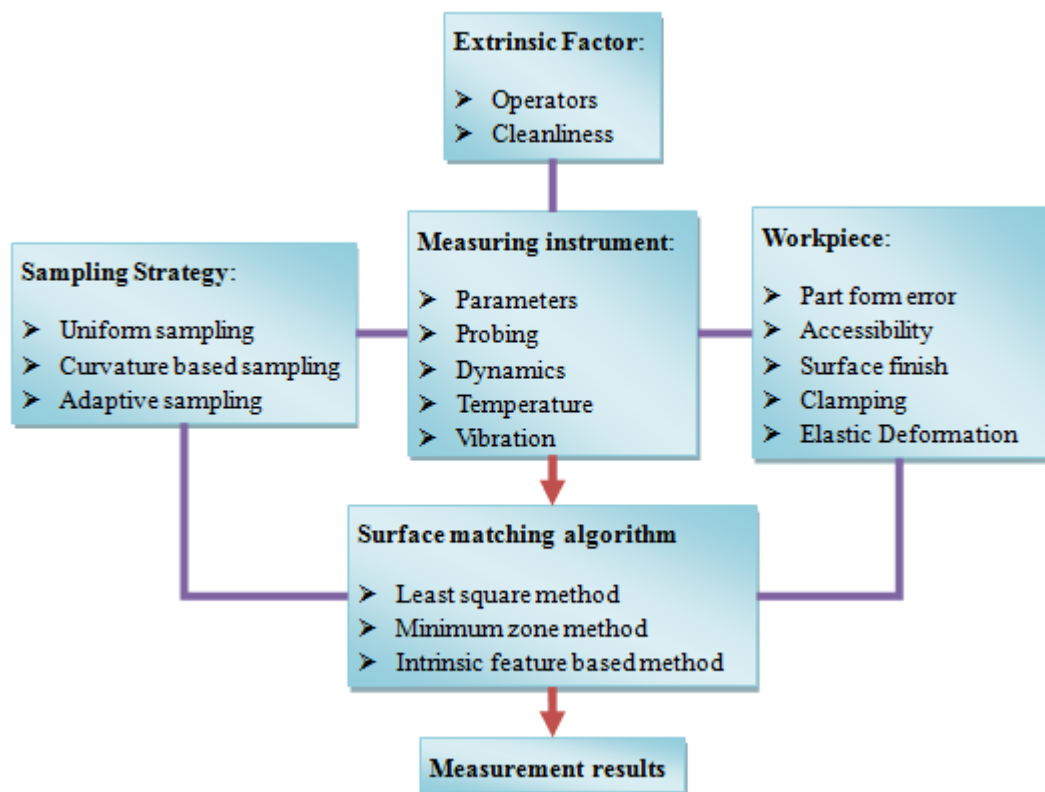


Figure 2.26 Error source in freeform surface measurement and form characterization

The uncertainties caused by the fitting and evaluation algorithms are dominated by their algorithm suitability, selection and implementation. The National Institute of Standards and Technology (NIST) in USA, Physikalisch-Technische Bundesanstalt

(PTB) in Germany, and the National Physics Laboratory (NPL) in UK have reported recent developments on software specification for uncertainty evaluation (Cox and Harris, 2008; Blunt et al, 2008). In the form characterization of freeform surfaces, the form error of measured surfaces is evaluated by comparing with the design surface based on fine surface matching. For example, the least squares based surface matching method is theoretically able to match two freeform surfaces with any desired accuracy when there is no deviation between them. Since the measured data do not perfectly match with the template, the surface matching results always contain uncertainties.

Much research has been conducted in recent decades on the uncertainty analysis of coordinate measuring instruments so as to calibrate and compensate for errors during measurement (Wilhelm et al, 2001). However, research on uncertainty analysis in data processing focuses on simple geometry, such as circles, spheres and cylinders (Wilhelm et al, 2001; Maihle et al, 2009), with relatively little research having been conducted on the measurement of freeform surfaces. In fact, uncertainty analysis is an indispensable part for form characterization, which assesses the accuracy and reliability of the characterization results and is the focus of this study. Some researchers have represented freeform surfaces by assembling simple geometry, such as spheres and cylinders, so that the problem is converted to analyzing each piece of assembled simple geometry (Savio et al, 2002). The general model described by the “Guide to the Expression of Uncertainty in Measurement” (ISO, 1995) is also difficult to apply to the freeform surface measurement process, since the uncertainty varies with the surface being measured and the sampling strategy being used.

The estimated measurement uncertainty in itself is insufficient to assess the quality of a measurement result. It is necessary to have in place a credible basis for the measurement result and associated uncertainty. The basis constitutes traceability of

the measurement results. The *International Vocabulary of Basic and General Terms in Metrology* (VIM) defines traceability as “the property of the result of a measurement or the value of a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparison all having stated uncertainties” (ISO, 1993).

This basis is strengthened by the equivalence of measures across nations by their national measurement institutes (NMI) participating in comparisons of measurement standards, known as *key comparisons*, through a mutual recognition arrangement (MRA) (BIPM, 1999).

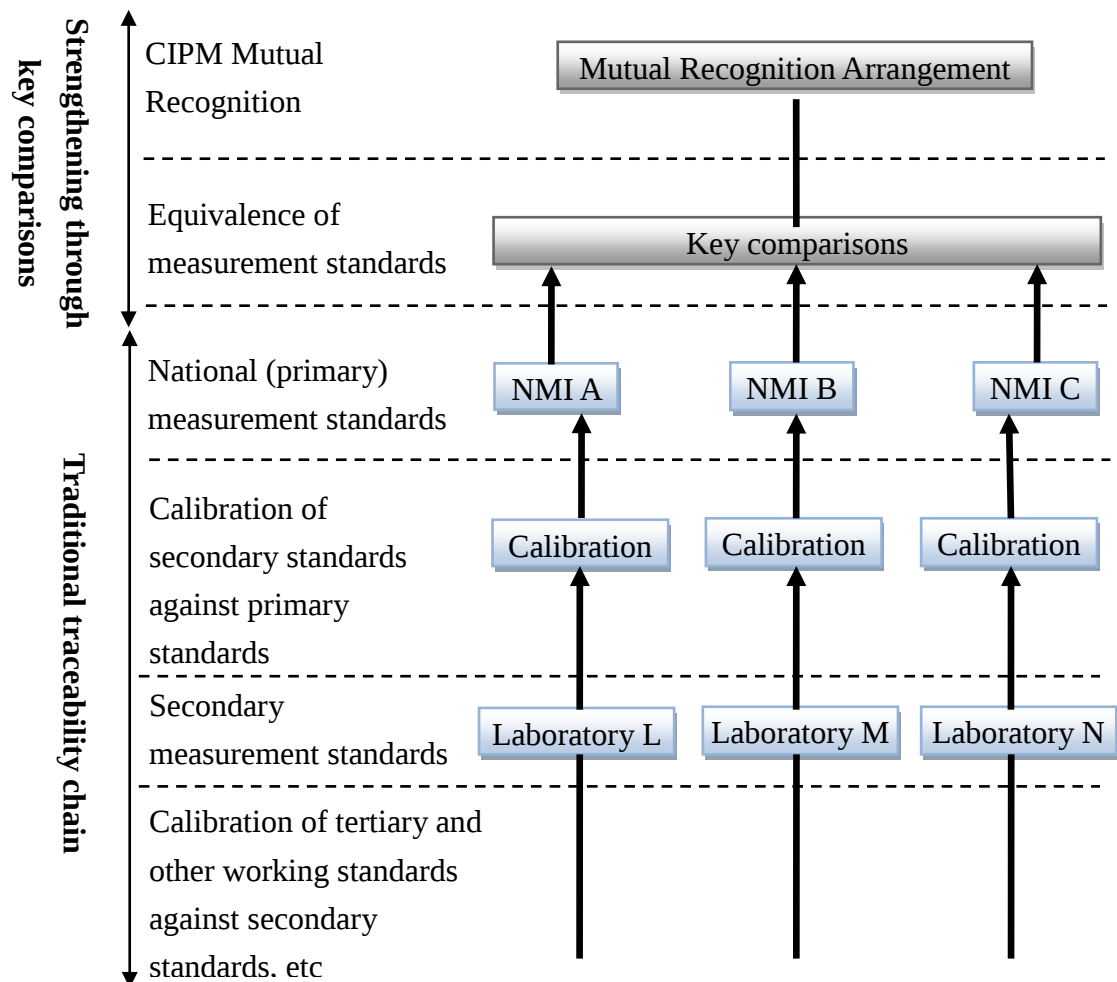


Figure 2.27 Traceability chains and their strengthening through key comparisons

(Adopted from Cox and Harris, 2006)

Figure 2.27 shows the traceability chains and their strengthening through key comparisons. Unfortunately, due to the geometric complexity of freeform surfaces, there is still no international standard for traceable measurement and form characterization of freeform surfaces. Recently, a freeform artifact was fabricated by NPL for verification of non-contact measuring systems (McCarthy, 2011). However, the form accuracy of the artifact is still at micrometre level.

2.6 Summary

Ultra-precision freeform surfaces are usually large scale surface topologies with shapes possessing no symmetry in rotation or translation. These surfaces are increasingly being used in many fields such as advanced optical systems and biomedical implants due to their superior optical and mechanical properties. To ensure the performance of the components, ultra-precision freeform surfaces are required to have form accuracy in the micrometre to sub-micrometre range and surface finishing at the nanometre level. The advanced development of modern ultra-precision freeform machining technologies constitutes an enabling technology that allows the designed freeform surfaces to be fabricated. However, there is yet a lack of international standards and well established traceable form characterization methods for the measurement of the machined ultra-precision freeform surfaces with sub-micrometre form accuracy.

Surface characterization has been studied for a considerable time and the research has shifted from simple geometry to freeform surfaces in recent decades. The geometrical complexity of freeform surfaces brings considerable challenges to the measurement and characterization of ultra-precision freeform surfaces. Although there are various enabling precision measuring techniques, e.g. ultra-precision CMMs and

stylus profilometry, which are able to measure data points from a machined freeform surface with nanometric accuracy, current research on ultra-precision freeform surfaces is hindered by generalized and accurate form characterization methods. Based on a review of relevant literature, current freeform surface characterization methods have the following deficiencies:

- (i) As freeform surfaces have non-rotational symmetry, conventional surface sampling strategies based on sampling one or several cross sections of the surface is inadequate to represent the variation of the surface characteristics at different regions. Hence, a more comprehensive sampling strategy is required for fully representing the geometry of machined freeform surfaces.
- (ii) Most of the form characterization methods for freeform surfaces depend on the embedded coordinate systems or frames which bring the barriers for surface form error evaluation by comparison between the designed model and the measured surface. There is a lack of efficient corresponding searching/surface matching methods for a freeform surfaces approach by Rigid-Body Transformation (RBT) of the coordinate system.
- (iii) The measurement and form characterization of a freeform surface combines with errors which lead to uncertainty to the characterization results. However, current research on the analysis of the uncertainty in geometric measurement is focused on simple geometries such as circles, spheres and cylinders, and relatively little research work has been conducted on freeform surfaces. In fact, uncertainty analysis is indispensable part for the form characterization which assesses the accuracy and reliability of the measurement and characterization results for freeform surfaces.

In view of the above shortcomings, there is a need for a generalized method to perform high-precision and robust form characterization of ultra-precision freeform surfaces with sub-micrometre form accuracy. One promising approach is the utilization of the intrinsic surface properties, which are independent of the coordinate frame. This research therefore focused on the development of an invariant feature pattern-based characterization method to address the stringent requirements for the measurement of ultra-precision freeform surfaces.

Chapter 3

Measurement Strategy and Surface Modeling of Ultra-precision Freeform Surfaces

The measurement and form characterization of a machined freeform surface can generally be divided into two parts: data acquisition and data processing. The former refers to the extraction of the geometric features from machined freeform surface by using precision 3D measuring instruments. To achieve the generalized form characterization of freeform surfaces, it is vital to extract the intrinsic surface features from a machined ultra-precision freeform surface with high accuracy (Cheung et al, 2010).

This is not so difficult for the design surface since it is given as a mathematical model, like a parametric surface, and the derivatives can therefore be obtained accurately. However, it is a challenging task for the machined surface. High precision measuring instruments are required to measure the surface geometry of the machined surface by extracting a large number of discrete points. Intrinsic surface features are then calculated based on the measured discrete points. Since the calculation of the intrinsic surface features, such as curvature, is very sensitive to the noise and outliers associated in the measured data, proper data processing techniques are required to eliminate the effect of the noise in the measured data.

This chapter studies the measurement strategy and the surface modeling techniques for ultra-precision freeform surfaces. A novel bidirectional curve network based sampling method is presented for enhancing the performance of the sampling

plan in the representation and the characterization of ultra-precision freeform surfaces. A robust surface fitting algorithm is developed to reconstruct a high fidelity surface from measured discrete points while the surface smoothness can also be ensured. In this way, the intrinsic surface features of the machined surface can be calculated accurately based on the reconstructed surface.

3.1 Measurement Strategy for Freeform Surfaces

In recent years, some newly developed measuring instruments (Weckenmann et al, 2006; 2011), such as IBS Isara (IBS, 2011), are reported as having the capability to extract data points from machined freeform surfaces with the nanometre level of accuracy. Due to the geometrical complexities, measuring ultra-precision freeform surfaces with the sub-micrometer form accuracy usually requires high density and intensive sampling in order to fully characterizing the surface geometry and reducing the measurement uncertainty. However, this usually imposes a lot of challenges in surface metrology and precision surface measurement, such as insufficient sampling or long measurement time, especially for contact type measuring instruments. As a result, it is desirable to perform the measurement with the lowest number of points while the accuracy can be ensured as well.

3.1.1 CAD Model based bidirectional optimal sampling strategy

Optimal sampling strategy in coordinate measurement refers to optimize the sampling plan for minimizing the number of sample points, minimizing the length of the measuring traverse path, or minimizing the sampling error. However, this can be obtained only through the knowledge of the shape and position of the measuring

object. For freeform measurement with the CMMs, the position of the measuring object can be pre-located by some reference to the object, and the computer aided design (CAD) model of the object can then be used to generate the optimal sampling strategy for the measurement.

3.1.1.1 Definition and manipulation of CAD model

(i) Nominal surface reconstruction

The CAD model of freeform surfaces is sometimes given by a cloud of discrete data points. In this case, the model should be reconstructed in order to obtain a nominal surface so as to guide the sampling process. This section only deals with the surface fitting problems when the discrete points are rectangularly arranged.

Given a grid of points $\mathbf{Q}_{ks}$ ($k=1,2,\dots,n, s=1,2,\dots,m$), a continuous surface is sought to construct by B-spline surface to fit these points. According to the definition of B-spline surface as shown in Eq. (2.1), the surface reconstruction can be formulated as follows:

$$\mathbf{Q}_{ks} = \mathbf{S}(u_k, v_s) = \sum_{i=1}^n \sum_{j=1}^m N_{i,3}(u_k) N_{j,3}(v_s) \mathbf{P}_{ij} \quad (3.1)$$

where (u_k, v_s) is the parameters of $\mathbf{Q}_{ks}$ which is obtained by Centripetal method (Piegl and Tiller, 1997). Eq. (4.1) represents $(n \times m)$ equations with $(n \times m)$ unknown $\mathbf{P}_{ij}$. Since $\mathbf{S}$ is a tensor product surface, the $\mathbf{P}_{ij}$ can be obtained efficiently as follows (Piegl and Tiller, 1997). For each fixed s , Eq. (3.1) can be written as

$$\begin{aligned}
\mathbf{Q}_{ks} &= \sum_{i=1}^n \sum_{j=1}^m N_{i,3}(u_k) N_{j,3}(v_s) \mathbf{P}_{ij} \\
&= \sum_{i=1}^n N_{i,3}(u_k) \left(\sum_{j=1}^m N_{j,3}(v_s) \mathbf{P}_{ij} \right) = \sum_{j=1}^m N_{i,3}(u_k) \mathbf{R}_{ks}
\end{aligned} \tag{3.2}$$

where

$$\mathbf{R}_{ks} = \sum_{j=1}^m N_{j,q}(v_s) \mathbf{P}_{ij} \tag{3.3}$$

Eq. (3.1) can then be solved with two steps. In the first step, a row of points $\mathbf{Q}_{ks}$ are used to determine $\mathbf{R}_{ks}$ by solving Eq. (3.2) for each fixed s . In the second step, a row of $\mathbf{R}_{ks}$ are used to determine $\mathbf{P}_{ij}$ by solving Eq. (3.3) for each fixed k .

(ii) B-spline surface and plane intersection

The cutter plane is defined by a point $\mathbf{Q}$ and the normal vector $\vec{D}$ and it is denoted as $L = L(\mathbf{Q}, \vec{D})$. The problem of solving the intersection of plane and B-spline surface can be formulated as finding the zeros of the function $f(u, v)$ as shown as follows:

$$f(u, v) = (\mathbf{S}(u, v) - \mathbf{Q}) \cdot \vec{D} = 0 \tag{3.4}$$

Recursive subdivision technology is used to determine the parametric values of the points in the intersection curve with high accuracy (Dokken, 1985). The calculated points are used to construct a number of Bezier curve segments which are the B-spline representation of a curve with knot multiplicities of $p+1$, where p is the degree of the curve. Figure 3.1 shows an example of a plane/surface intersection curve.

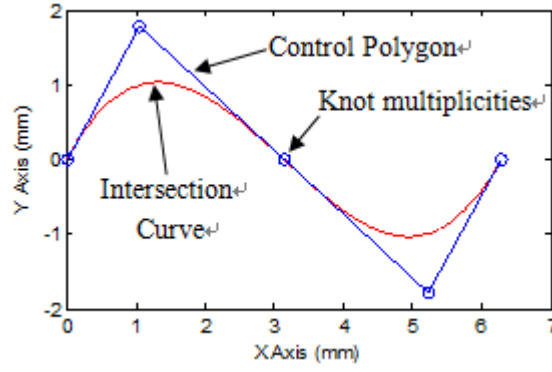


Figure 3.1 Plane/surface intersection curve

(iii) Interpolation of a bidirectional curve network

Let Eq. (3.5) and Eq. (3.6)

$$C_k(u) = \sum_{i=1}^{i=n} N_{i,3}(u) P_{ki} \quad k = 1, 2, r. \quad 0 \leq u \leq 1 \quad (3.5)$$

$$C_l(v) = \sum_{j=1}^{j=m} N_{j,3}(v) P_{lj} \quad l = 1, 2, s. \quad 0 \leq v \leq 1 \quad (3.6)$$

be two sets of B-spline curves satisfying two conditions:

- As independent sets that are compatible, i.e. all the $C_k(u)$ are defined on a common knot vector, U , and all the $C_v(l)$ are defined on a common knot vector, V .
- There exist parameters $0 = u_1 < u_2 < \dots < u_{s-1} < u_s = 1$ and $0 = v_1 < v_2 < \dots < v_{l-1} < v_l = 1$ such that

$$Q_{l,k} = C_k(u_l) = C_l(v_k) \quad k = 1, 2, r., \quad l = 1, 2, \dots, s \quad (3.7)$$

Gordon Surface technology (Piegl and Tiller, 1997) is used to construct a surface $S(u, v)$ given by Eq. (3.8) which satisfying Eq. (3.9) and Eq. (3.10) as follows

$$S(u, v) = \sum_{l=1}^s C_l(v) \phi_l(u) + \sum_{k=1}^r C_k(u) \psi_k(v) - \sum_{l=1}^s \sum_{k=1}^r Q_{l,k} \phi_l(u) \psi_k(v) \quad (3.8)$$

$$= L1(u, v) + L2(u, v) - T(u, v)$$

$$S(u_l, v) = C_l(v) \quad l = 1, 2, \dots, s. \quad (3.9)$$

$$S(u, v_k) = C_k(u) \quad k = 1, 2, \dots, r. \quad (3.10)$$

where $\{\phi_l(u)\}_{l=1}^s$ and $\{\psi_k(v)\}_{k=1}^r$ are two sets of blending functions satisfying

$$\phi_l(u_i) = \begin{cases} 0 & \text{if } i \neq l \\ 1 & \text{if } i = l \end{cases} \quad \psi_k(v_i) = \begin{cases} 0 & \text{if } i \neq k \\ 1 & \text{if } i = k \end{cases} \quad (3.11)$$

$S(u, v)$ is composed of three simpler surfaces, i.e. two scanned surfaces $L1(u, v)$, $L2(u, v)$ and a tensor product $T(u, v)$. Figure 3.2 shows an example of interpolating a bidirectional curve network.

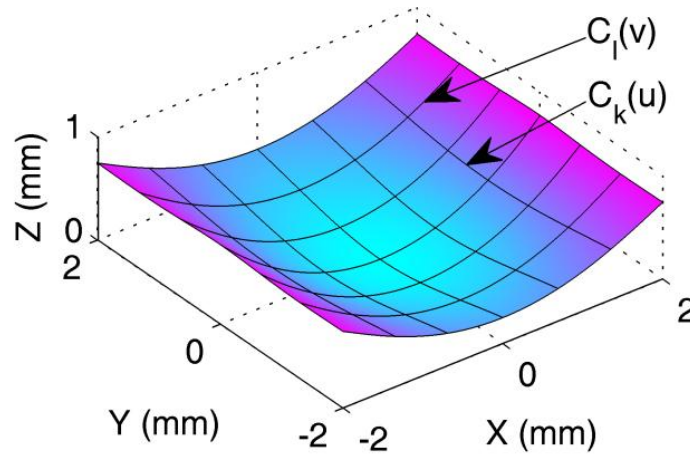


Figure 3.2 Interpolation of a bidirectional curve network

(iv) Surface complexity analysis

The curvature change is used to analyze the surface complexity in the present study. This choice is inspired by considering that both the machining error of the machined surface and the form error of the substitute surface are likely to take place at the surface regions where the curvature changes sharply. The analysis is made in the

Euclidian space. Firstly, a grid of points is evaluated on the surface. The size of the grid is user-specified. Let $Curv(i, j)$ be the surface curvature at an arbitrary point $P(x_{i,j}, y_{i,j}, z_{i,j})$ in the grid, $i=1,2,\dots,Nx$ and $j=1,2,\dots,Ny$ where (i, j) are integer indices while Nx and Ny are the size of the surface grid in the directions of $\vec{X}$ and $\vec{Y}$, respectively. A matrix C that represents the surface curvature is given by Eq. (3.12) as follows:

$$C = \begin{bmatrix} Curv(1,1) & Curv(1,2) & \dots & Curv(1,Ny) \\ Curv(2,1) & Curv(2,2) & \dots & Curv(2,Ny) \\ \vdots & \vdots & \vdots & \vdots \\ Curv(Nx,1) & Curv(Nx,2) & \dots & Curv(Nx,Ny) \end{bmatrix} \quad (3.12)$$

Secondly, the curvature changes between every two neighboring points along each of the two directions $\vec{X}$ and $\vec{Y}$ and is determined as follows:

$$CX = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & ChanX(2,1) & \dots & ChanX(2,Ny-1) & 0 \\ 0 & \vdots & \dots & \vdots & 0 \\ \vdots & ChanX(Nx-1,1) & \dots & ChanX(Nx-1,Ny-1) & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (3.13)$$

$$CY = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & ChanY(2,2) & \dots & ChanY(2,Ny-1) & 0 \\ 0 & \vdots & \dots & \vdots & 0 \\ \vdots & ChanY(Nx-1,2) & \dots & ChanY(Nx-1,Ny-1) & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (3.14)$$

where

$$ChanX(i, j) = \frac{Curv(i+1, j) - Curv(i, j)}{x_{i+1,j} - x_{i,j}} \quad (3.15)$$

$$ChanY(i, j) = \frac{Curv(i, j+1) - Curv(i, j)}{y_{i,j+1} - y_{i,j}} \quad (3.16)$$

CX is the matrix represents surface curvature change in $\bar{X}$ direction; CY is the matrix represents surface curvature change in $\bar{Y}$ direction. Hence, the above two curvature change matrices are combined to simplify the calculations and unify the representation of the surface curvature. Then, the final curvature change matrix of the surface is determined as follows:

$$CC = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & CC(2,1) & \dots & CC(2, Ny-1) & 0 \\ 0 & \vdots & \dots & \vdots & 0 \\ \vdots & CC(Nx-1,1) & \dots & CC(Nx-1, Ny-1) & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (3.17)$$

where

$$CC(i, j) = \sqrt{(ChanX(i, j))^2 + (ChanY(i, j))^2} \quad (3.18)$$

CC is the final curvature change matrix of the surface. All the points in the grid are projected to the nominal surface so as to determine the corresponding curvature. From a given surface $S(u, v)$, the Gaussian and mean curvature, K and H are calculated as follows:

$$K = \frac{LN - M^2}{EG - F^2}, \quad H = -\frac{1}{2} \left(\frac{EN + GL - 2FM}{EG - F^2} \right) \quad (3.19)$$

where $E = \bar{S}_u \cdot \bar{S}_u$, $F = \bar{S}_u \cdot \bar{S}_v$ and $G = \bar{S}_v \cdot \bar{S}_v$ are the first fundamental form coefficients, and $L = \bar{N} \cdot \bar{S}_{uu}$, $M = \bar{N} \cdot \bar{S}_{uv}$ and $N = \bar{N} \cdot \bar{S}_{vv}$ are the second fundamental form coefficients; $\bar{S}_u$ and $\bar{S}_v$ are tangent vectors of S at an point $S(u, v)$ in u , v direction respectively, and $\bar{S}_{uu}$, $\bar{S}_{uv}$ and $\bar{S}_{vv}$ are the second

derivatives. Since the Gaussian curvature may vanish for some classes of surface, the selection of the curvature measure is carried out interactively.

3.1.1.2 Automatic bidirectional optimal sampling algorithm

Traditional one directional raster fashion sampling strategies (Ainsworth et al, 2000; ElKott and Veldhuis, 2005) extract a series of curves along a direction, which is used to construct a skinned surface to represent the measured surface. However, the proposed bidirectional sampling strategy samples two sets of curves along two different directions of the surface to form a curve network, which is used to construct the substitute surface. It is emphasized that in this study all the curves are extracted through intersecting the surface with infinite planes. This is due to the fact that the freeform surfaces are measured by extracting a series of iso-planer curves along a direction for most CMMs equipped with a scanning probe, as well as stylus profilometers.

As shown in Figure 3.3, Cartesian coordinate frame is constructed based on three axes of the CAD model, i.e. $\vec{X}$, $\vec{Y}$ and $\vec{Z}$ to guide the sampling process. The sampling is carried out into two directions, $\vec{X}$ and $\vec{Y}$. A_i ($i=1,2,..4$) are obtained by projecting four corner points of the surface onto XY plane along $-\vec{Z}$ direction. Surface sectioning starts at A_1 , then A_2 is the end of the surface sectioning in the direction, $\vec{X}$. A_4 is the end of surface sectioning in the other direction, $\vec{Y}$. This ensures that the first and last surface bounding curves in two sampling directions are included in the sampling plan.

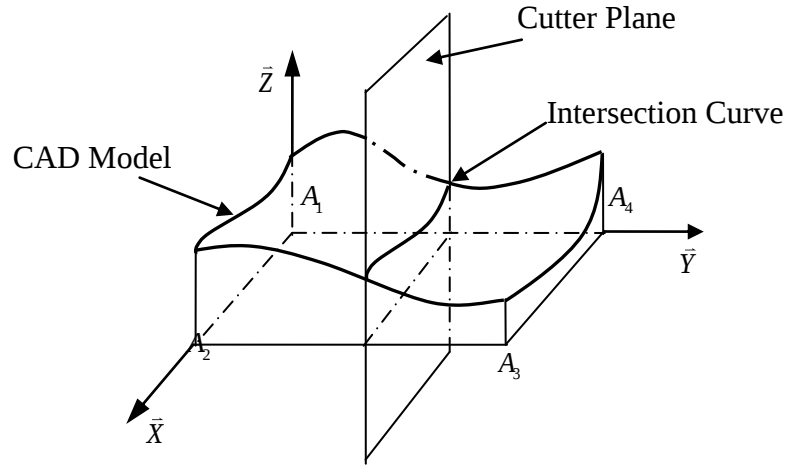


Figure 3.3 Cartesian coordinate of the CAD model

Then, four boundaries of the manufactured surface are used to construct initial bidirectional curve network to construct a substitute geometry based on the algorithm given in Section 3.1.1.1. A Grid of points is sampled on the substitute geometry and each point is projected on the nominal surface, so that the deviation of the substitute geometry from CAD model is determined. If the constructed substitute surface does not meet the required accuracy, a new curve will be extracted to subdivide the curve network.

Figure 3.4 shows the operation of the bidirectional sampling algorithm. The location of the new sampling curve is determined by considering both the surface complexity and deviation of the substitute geometry from the nominal surface. Firstly, the curvature change matrix (CC) is calculated on the sub-region where the maximum deviation of the substitute surface (MDp) from the nominal surface is located. CC represents the surface complexity of the region and is determined based on Eq. (3.12) to Eq. (3.19), given in Section 3.1.1.1.

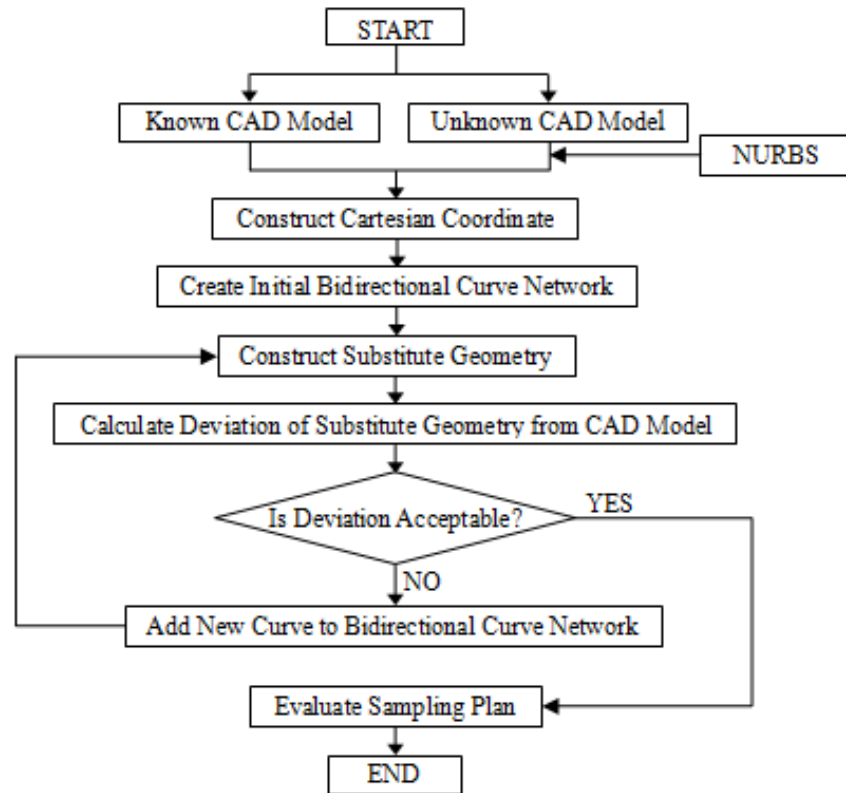


Figure 3.4 Schematic diagram of the bidirectional sampling algorithm

Then the maximum value of the curvature change matrix (MVC) is determined and compared to a curvature factor (CF) which is a threshold used to control the sensitivity of the sampling algorithm to the change of surface curvature. If MVC is smaller than CF, the curvature change is considered having little effect on sampling accuracy and a new sampling line is extracted on MDp. Otherwise, a new curve is sampled on the point (MVCp) where MVC is located. It is worthnote that when the new sampling curve is determined by MVC, the sampling location is constrained by the minimum step size (MS) between subsequent sampling curves. That is, if the new sampled curve is too close to the nearest original curves, the process is returned and another curve is sampled based on the new curvature change matrix which is obtained by setting the original MVC be zero. This is due to the consideration that the sampling plan is available only when the distance of any two adjacent sampled curves is bigger

than the minimum step size of measuring instruments.

Then, a normal vector between two sampling direction $\vec{X}$ and $\vec{Y}$ is chosen to construct cutter plan with the sampling location obtained in above step. Figure 3.5 shows the projection of the bidirectional curve network.

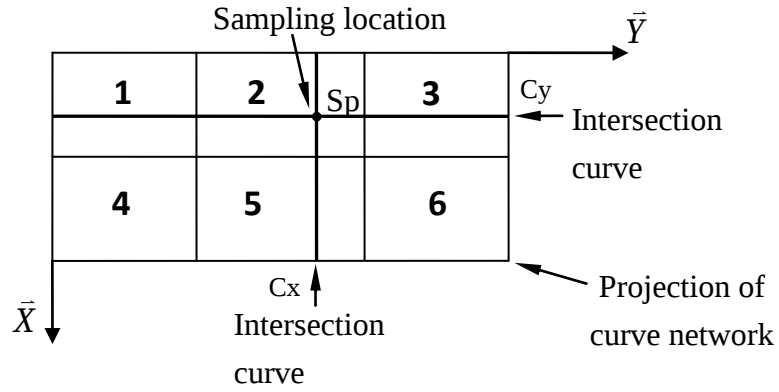


Figure 3.5 Projection of bidirectional curve network

Point Sp is the sampling location. Cx and Cy are the projection of intersection curves along $\vec{X}$ and $\vec{Y}$ respectively. If the deviation of the regions (2 and 5) that Cx passes through is bigger than the deviation of the regions (region 1, 2 and 3) that Cy passes through, Cx is selected to add to the curve network. Otherwise, Cy is selected. This ensures that the selected curve contributes more to improve the fitting accuracy of substitute geometry. Finally, the intersection curve is used to subdivide the curve network to construct a new substitute surface. Figure 3.6 shows the schematic diagram of adding a new sampling curve.

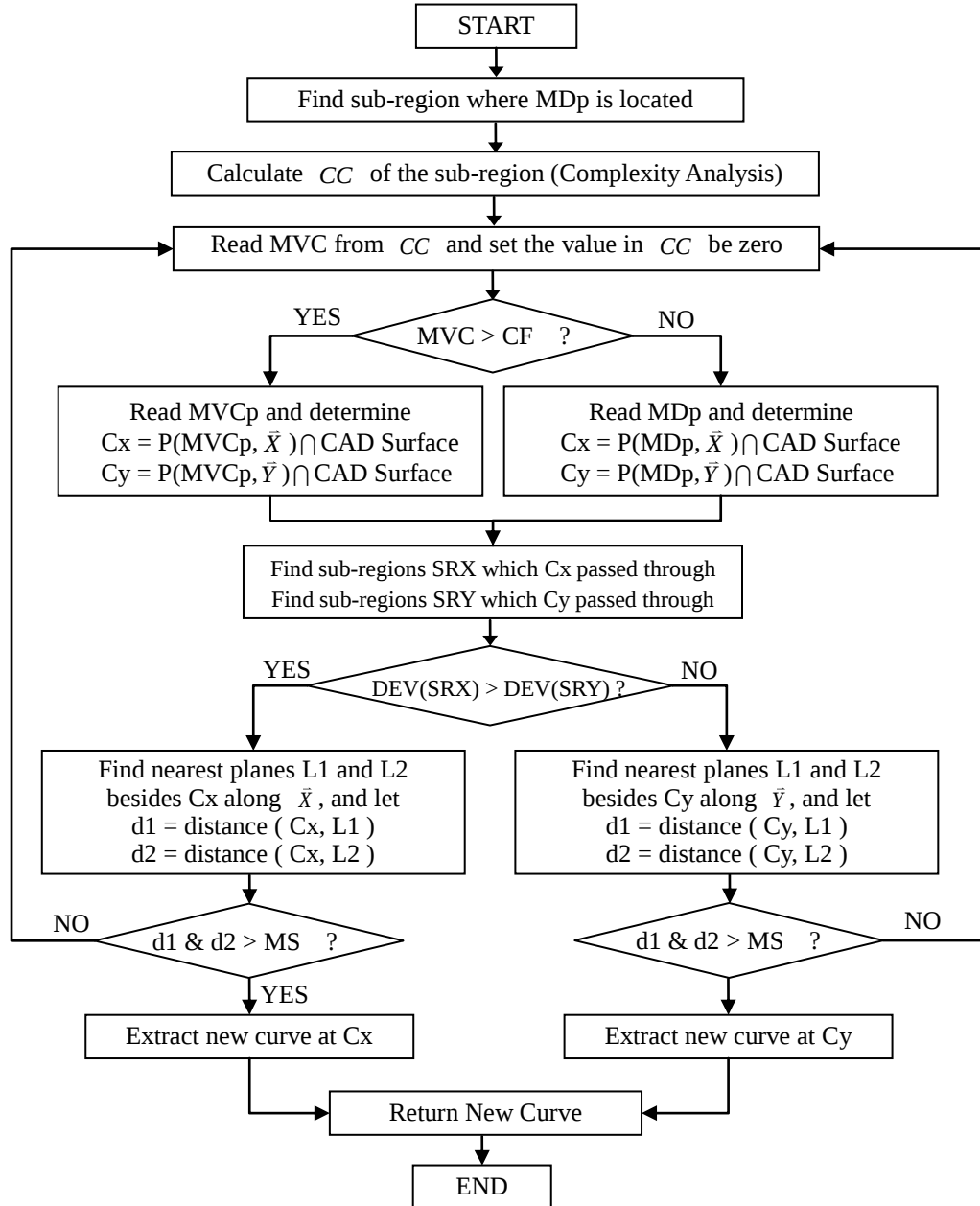


Figure 3.6 Procedure of adding a new curve

MDp refers the point where the maximum deviation between the substitute geometry and nominal surface is located; MVC is the maximum value in CC ; MVCp refers the point where MVC is located; CF is the curvature factor; $P(\cdot)$ refers to the plane which is decided by a point and a vector; SRX and SRY refers to the sub-regions that Cx and Cy pass through; DEV(SRX) and DEV(SRY) are the deviation of the SRX and the SRY from nominal surface respectively; MS is the

minimum step-over distance between two consecutive sampling curves.

3.1.1.3 Computer simulation of bidirectional optimal sampling

The bidirectional sampling method has been extensively tested for a variety of surfaces with various complexities. A case study is presented to study the efficiency and accuracy of the proposed method. The ideal design surface is given by Eq. (3.20):

$$z = 30e^{-(0.0005x^2 + 0.001y^2)} \quad (3.20)$$

with the dimensions $-50 \leq x \leq 50$ (mm) and $-50 \leq y \leq 50$ (mm). To generalize the application, a B-spline form of nominal surface is obtained by fitting a cloud of discrete points scattered on the designed surface. Bidirectional sampling method is then used to generate the sampling plan based on the reconstructed surface. The sampling is carried out along $\bar{X}$ and $\bar{Y}$ axis.

Traditional one directional sampling method is employed to compare the performance with the bidirectional sampling method. The one directional sampling method produces the sampling plan by sampling a set of iso-planar curves from given CAD model. The location of the sampled curves is obtained iteratively based on the same sampling criteria as given in Section 3.1.1.2 but only in one direction. The minimum distance between subsequent sample lines is set to be 0.1 mm. To get better sampling results, the $\bar{Y}$ direction is selected as the sampling direction of both uniform iso-planar sampling method and optimal iso-planar sampling method, since the curvature change along the $\bar{Y}$ direction is bigger than that along the $\bar{X}$ direction.

Table 3.1 shows the simulation results with the sampling accuracy at a low level, meaning that the maximum deviation of the substitute geometry is in the range of $10^{-1} \sim 10^{-3}$ mm; and Table 3.2 shows the simulation results with the sampling

accuracy at a high level, meaning that the maximum deviation of the substitute geometry is in the range of $10^{-4} \sim 10^{-6}$ mm. A summary of the sampling results are shown in Table 3.3, where N is the total number of sampled curves; N_x and N_y are the number of sampled curves in $\bar{X}$ and $\bar{Y}$ directions respectively; and E_s is the sampling accuracy denoting the maximum deviation of the substitute surface with the nominal surface.

It is interesting to note from the simulation results in Table 3.1 that there is not much difference in the efficiency between the one directional sampling method and the bidirectional sampling method when the requirement of the sampling accuracy is low, i.e. in the range of $10^{-1} \sim 10^{-3}$ mm. However, as shown in Table 3.2, along with the improvement of the sampling accuracy, the sampling plans generated by one direction sampling method possess a larger number of curves compared with that generated by the bidirectional sampling method. It can be seen from Table 3.3 that the efficiency of the sampling plan with the bidirectional sampling method is improved by almost 50% compared with the one directional sampling method when the accuracy of the sampling is in the sub-micrometer level.

The advantage of the bidirectional sampling method can be explained from the principle of the sampled curves in estimating the non-measuring area. For conventional one directional method, this is carried out by skinning a set of section curves to construct a surface so as to approximate the area between the section curves. However, for the proposed method, this is carried out by skinning two sets of section curves from two different directions to construct a curve network so as to approximate the intersection area.

Table 3.1 Sampling results with precision in the low level (10^{-1} - 10^{-3} mm)

	One directional optimal Sampling	Bidirectional optimal sampling
Sampling Error Threshold: 10^{-1} mm		
Sampling Plan		
Sampling Error		
Sampling Error Threshold: 10^{-2} mm		
Sampling Plan		
Sampling Error		
Sampling Error Threshold: 10^{-3} mm		
Sampling Plan		
Sampling Error		

Table 3.2 Sampling results with precision in the high level (10^{-4} - 10^{-6} mm)

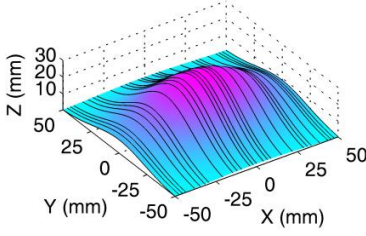
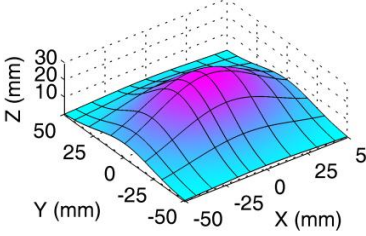
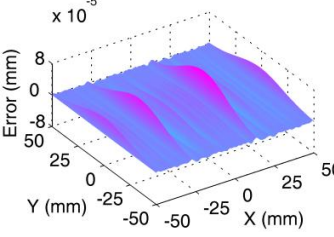
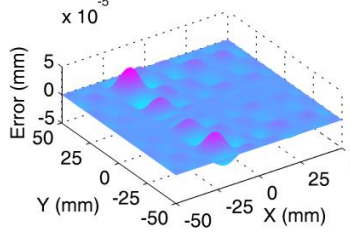
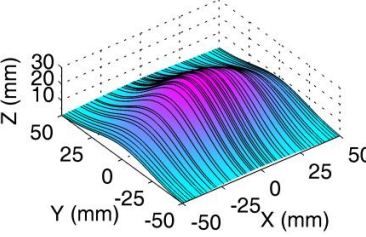
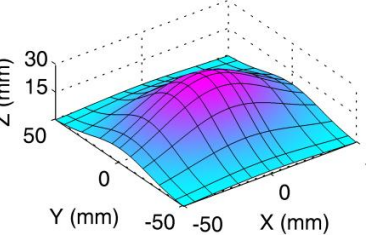
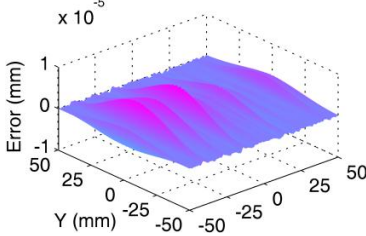
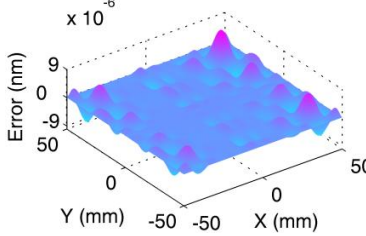
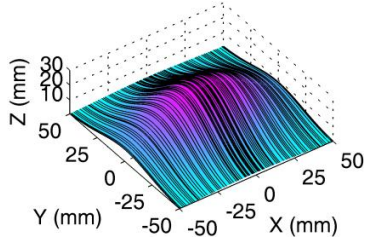
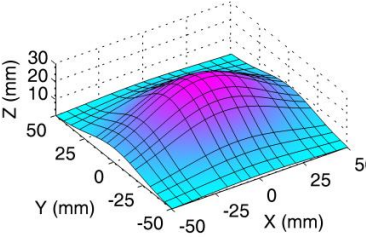
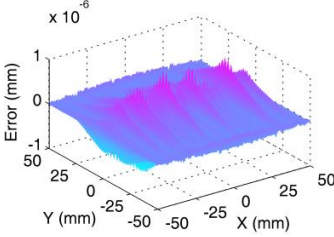
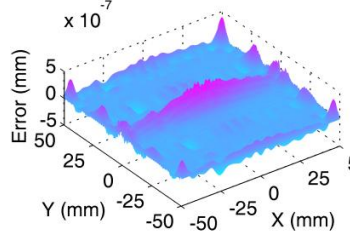
	One directional optimal Sampling	Bidirectional optimal sampling
Sampling Error Threshold: 10^{-4} mm		
Sampling Plan		
Sampling Error		
Sampling Error Threshold: 10^{-5} mm		
Sampling Plan		
Sampling Error		
Sampling Error Threshold: 10^{-6} mm		
Sampling Plan		
Sampling Error		

Table 3.3 Number of sampling curves under various sampling accuracy

Es (mm)	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}
One directional sampling						
N	8	11	18	28	49	86
Bidirectional sampling						
$N(N_x, N_y)$	(5, 3)	(5, 5)	(5, 8)	(10, 8)	(11, 11)	(15, 15)

As a result, it is proven that the curve network provides much stronger geometric constraint to the non-measuring area than a set of parallel section curves. Compared with traditional sampling methods, such as sampling curves only in one direction, the bidirectional sampling method provides a significant improvement in terms of the efficiency of freeform data sampling with high precision.

3.1.2 Bidirectional uniform sampling strategy

The CAD model based optimal sampling strategy highly depends on the embedded coordinate frame of the CAD model. For measuring instruments such as CMMs, the precise location of the machined workpiece relative to the coordinate frame of the CAD model can be captured before the measurement. However, for measuring instruments such as profilometers, this becomes a challenging task. In such cases, a uniform sampling strategy is compromisingly used in practice even though the sampling efficiency is low. In the present study, a bidirectional uniform sampling strategy is proposed to improve the sampling efficiency of freeform measurements with stylus profilometers.

3.1.2.1 Curve network extraction

Two Cartesian coordinates are constructed based on the structure of the adopted profilometer to guide the sampling process as shown in Figure 3.7. $C_o = \{\bar{X}_o, \bar{Y}_o, \bar{Z}_o\}$ represents the coordinate system of the measurement instrument, and $C_R = \{\bar{X}_R, \bar{Y}_R, \bar{Z}_R\}$ represents the embedded coordinate system of the rotating table, where $\bar{Z}_R$ is the axis of rotation. The workpiece is mounted on the rotating table and the surface curve is measured by moving the probe along $\bar{X}_o$.

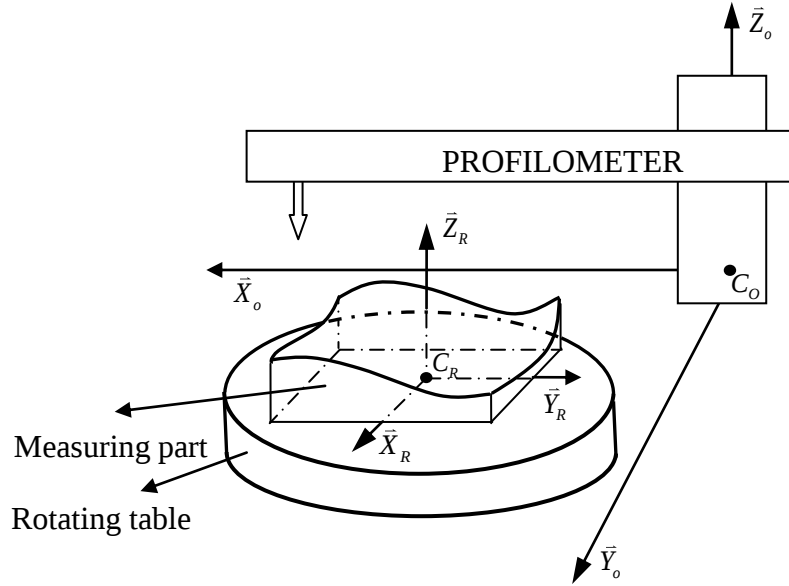


Figure 3.7 Sampling coordinate systems in the instrument

Sampling is carried out in two steps. In the first step, a set of curves are uniformly sampled along a selected direction $\bar{d}_1$ with step size d_1 , denoted as $SC1_i$, $i=1, \dots, n$. $\bar{d}_1$ is appropriately selected such that the sampled curves in this direction possess a higher curvature change. ElKott and Veldhuis (2005) found that sampled curves with high curvature change leads to a better sampling plan. The coordinate transformation from C_o to C_R is a function of spatial rotation and translation and

can be represented by the following matrix:

$$\underline{C}_{RO} = \begin{bmatrix} c\varphi c\psi & s\theta s\varphi c\psi - c\theta s\psi & c\theta s\varphi c\psi + s\theta s\psi & t_x \\ c\varphi s\psi & s\theta s\varphi s\psi + c\theta c\psi & c\theta s\varphi s\psi - s\theta c\psi & t_y \\ -s\varphi & s\theta c\varphi & c\theta c\varphi & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.21)$$

where θ , φ , ψ are the Euler angles; t_x , t_y , t_z are components of the translation vector; c and s are abbreviations of the cosine and sine functions. Then, the sampled curves under C_R can be determined as follows:

$$RSC1_i = \underline{C}_{RO} SC1_i, \quad i = 1, \dots, n \quad (3.22)$$

In the second step, the rotating table is rotated with a certain angle α . Then, another set of curves are uniformly sampled along direction $\vec{d}_2$ with step size d_2 , denoted as $SC2_j$, $j = 1, \dots, m$. There is no strict requirement for the choice of α . In this work, α is simply selected such that the sampled curve network covers as much area of the measured surface as possible. Then the new coordinate transformation from C_O to C_R can be represented as follows:

$$NC_{RO} = \underline{C}_R \underline{C}_{RO} \quad (3.23)$$

where

$$\underline{C}_R = \begin{bmatrix} c\alpha & s\alpha & 0 & 0 \\ -s\alpha & c\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The sampled curves under C_R can then be determined by

$$RSC2_j = NC_{RO} SC2_j, \quad j = 1, \dots, m \quad (3.24)$$

Hence, two sets of curves can be used to construct a curve network under the embedded coordinate system of the rotating table C_R as shown in Figure 3.7.

The sampled curves should be trimmed before surface reconstruction. Four border curves, $RSC1_1$, $RSC1_n$, $RSC2_1$ and $RSC2_m$, are used to form a quadrilateral. As shown in Figure 3.8, four planes passing through the four border curves are used to cut the part of each sampled curve that is outside the quadrilateral. This is a curve/plane intersection problem and intersection points can be determined numerically with high accuracy using recursive subdivision technology (Dokken, 1985). The trimmed surface is then used to construct the substitute surface to represent the measured surface.

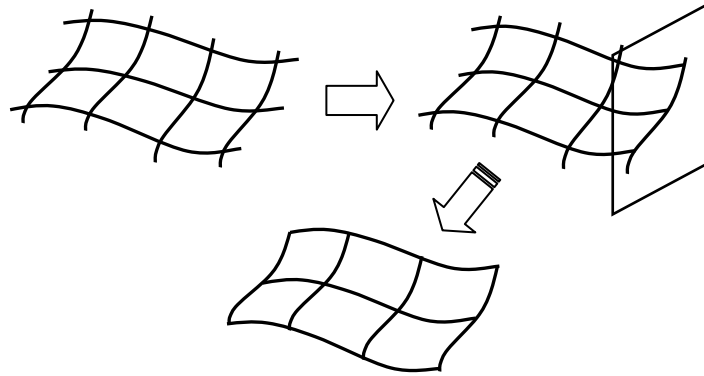


Figure 3.8 Trimming sampled curve network

3.1.2.2 Computer simulation of bidirectional uniform sampling

In the present study, the sampling efficiency of a sampling plan is characterized by the total length of sampled curves in it. It is particularly meaningful for profilometry since the measuring time is proportional to the total length of sampled curves. Sampling accuracy of a sampling plan is characterized by the deviation of the substituted surface from the nominal surface. Two case studies are presented, which

include sampling a biconic surface in B-spline format and a CAD model of a freeform streetlight lens.

An automatic sampling algorithm is implemented based on the proposed bidirectional uniform sampling method, as shown in Figure 3.9, where $Ns1$ and $Ns2$ are user-defined initial number of curves which are pre-sampled on a given surface along two different directions, $\vec{d}_1$ and $\vec{d}_2$ respectively. The sampled curves are used to construct substitute surface using the fitting algorithm as presented in Section 3.1.1.1. The sampling error is characterized by the maximum deviation of the substitute surface from the given nominal surface. If the sampling error is larger than a given error threshold, $(Ns1+1)$ and $(Ns2+1)$ curves are re-sampled on the given surface. The whole process is iterative and is terminated upon reaching the desired level of accuracy.

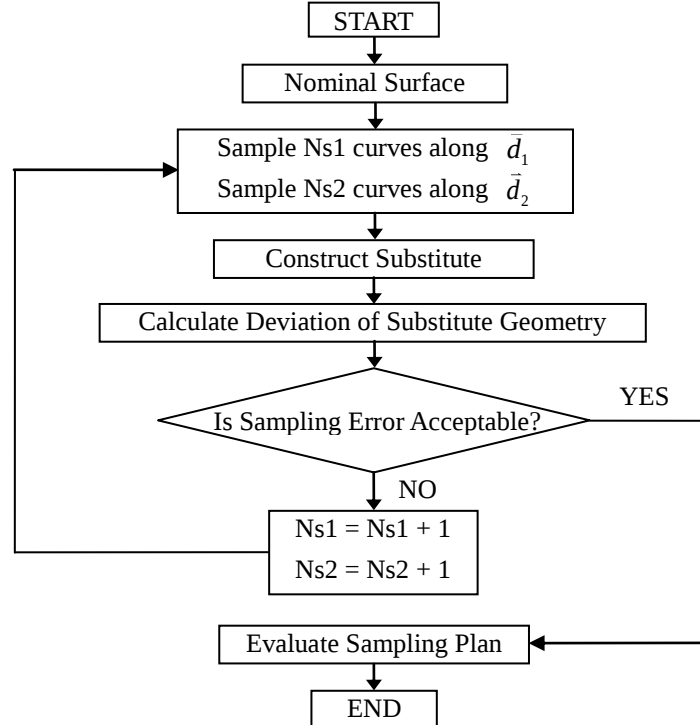


Figure 3.9 Schematic diagram of automatic bidirectional uniform sampling process

A biconic surface which is widely applied in the optical components is designed as given by Eq. (3.25):

$$z = \frac{C_x x^2 + C_y y^2}{1 + \sqrt{1 - (K_x + 1)C_x^2 x^2 - (K_y + 1)C_y^2 y^2}} + \sum_{i=1}^n A_{xi} x^i + \sum_{j=1}^m A_{yj} y^j \quad (3.25)$$

The parameters of the design surface are listed in Table 3.4. This leads to the generation of the biconic design surface as shown in Figure 3.10.

Table 3.4 Parameter of the designed biconic surface

C_x	0.3713834	C_y	0.2057346
K_x	-0.2405417	K_y	-14.23495
A_{x2}	0	A_{y2}	0
A_{x4}	$-3.4195423 \times 10^{-3}$	A_{y4}	-3.44147×10^{-3}
A_{x6}	$-2.5526374 \times 10^{-4}$	A_{y6}	2.0794629×10^{-3}
A_{x8}	$-9.4151045 \times 10^{-6}$	A_{y8}	$-7.7332897 \times 10^{-4}$
A_{x10}	$-2.2655833 \times 10^{-6}$	A_{y10}	1.8985258×10^{-4}
A_{x12}	$-5.8382646 \times 10^{-7}$	A_{y12}	$-2.9348209 \times 10^{-5}$
A_{x14}	1.3432194×10^{-7}	A_{y14}	2.4562495×10^{-6}

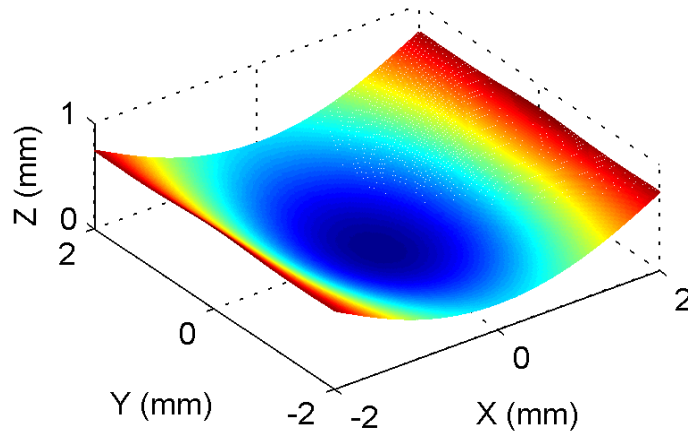
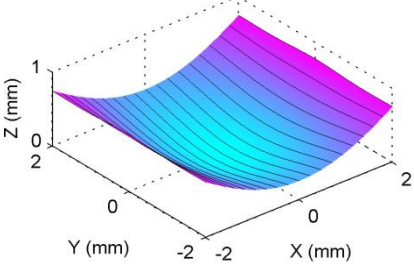
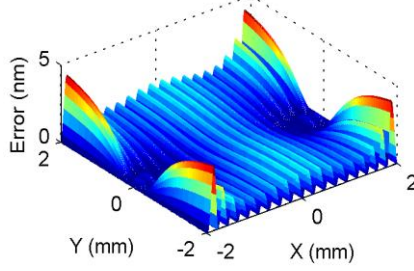
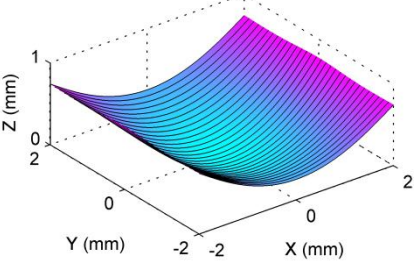
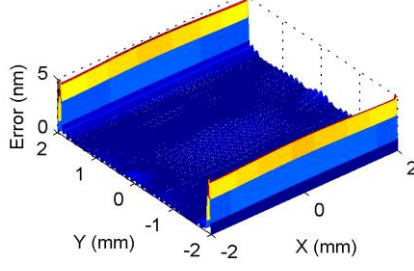
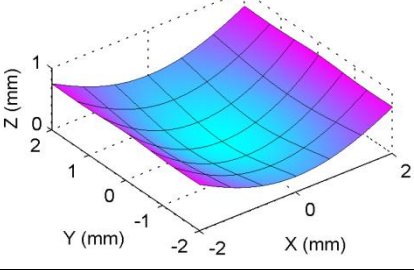
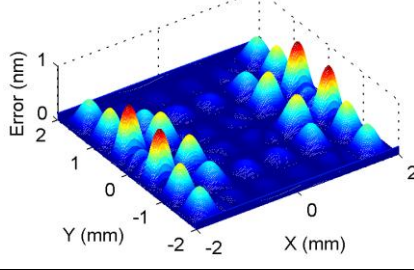


Figure 3.10 Designed biconic surface

The sampling of surface data is carried out based on three different strategies, i.e. the one directional uniform sampling (ODUS) along $\bar{X}$, $\bar{Y}$ directions respectively, and bidirectional uniform sampling (BDUS) along both $\bar{X}$ and $\bar{Y}$ directions. All sampling plans are produced with the same threshold of sampling error of 5 nm. Hence, the comparisons of the sampling efficiency among the produced sampling plans are performed by comparing the total length of the sampled curves. Table 3.5 shows the produced sampling plan and corresponding deviation of the substitute surface from the designed surface.

Table 3.5 Sampling results on biconic surface

Sampling Plan	Sampling Error	Summary
		$SD :$ $\bar{X}$ $SE :$ 4.5 nm $NsX :$ 19 $NsY :$ 0 $TLC :$ 75 mm
		$SD :$ $\bar{Y}$ $SE :$ 4.9 nm $NsX :$ 0 $NsY :$ 32 $TLC :$ 128mm
		$SD :$ $\bar{X}, \bar{Y}$ $SE :$ 0.7 nm $NsX :$ 7 $NsY :$ 7 $TLC :$ 56 mm

SD is the sampling direction; SE is the sampling error; NsX and NsY are the number of sampled curves along $\bar{X}$ and $\bar{Y}$ respectively. TLC is the total length of

sampled curves which is determined by Eq. (3.26):

$$TLC = \sum_{i=1}^{NsX} Lc_i + \sum_{j=1}^{NsY} Lc_j \quad (3.26)$$

where Lc_i / Lc_j is the length of the i th / j th sampled curves along $\bar{X} / \bar{Y}$.

The results show that the sampling plan produced by the BDUS method possesses less curves (total 14 curves) with a smaller sampling error as compared with the sampling plan produced by the ODUS method irrespective of whether it is along $\bar{X}$ (19 curves) or $\bar{Y}$ (32 curves). With the same sampling error threshold, the BDUS method produces sampling plan with shorter TLC when compared with that produced by the ODUS method. This infers that the BDUS method requires less measuring time than that for the ODUS method with the same level of accuracy.

A CAD model of a freeform surface for a streetlight lens is used to study the sampling accuracy of the proposed method (Jiang et al, 2009). Figure 3.11 shows the surface model of the freeform lens. Similar to the previous case study, three different strategies are used to sample the given CAD model, but in order to provide a comparison of the sampling accuracy of all three sampling plans which have approximately the same TLC .

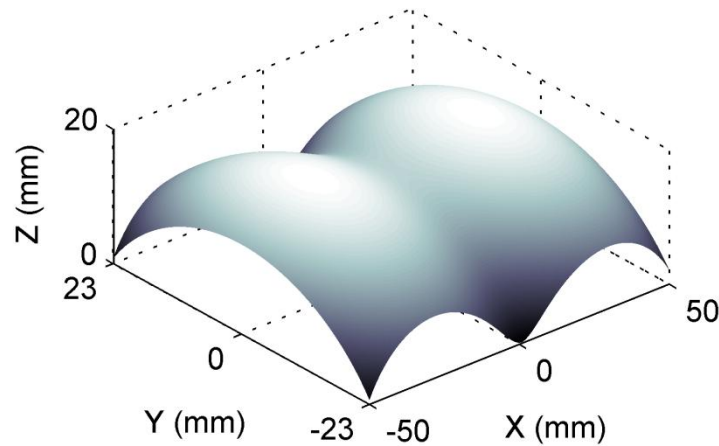
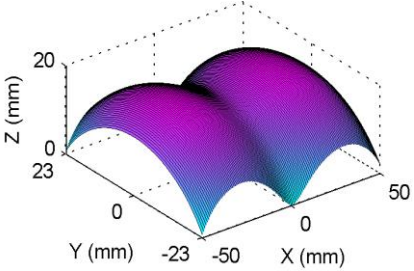
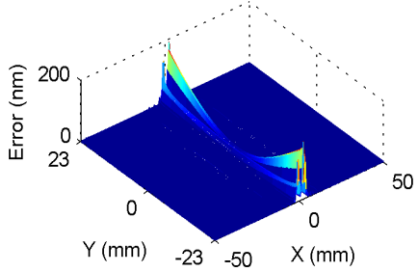
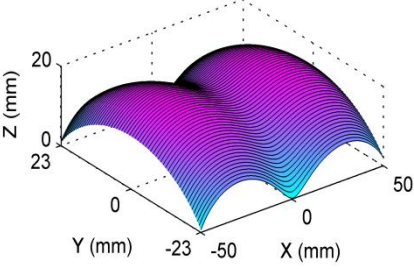
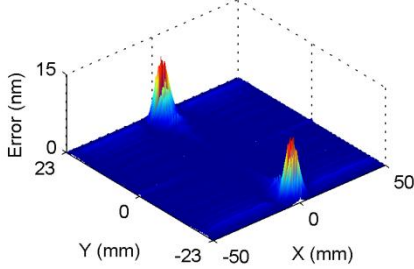
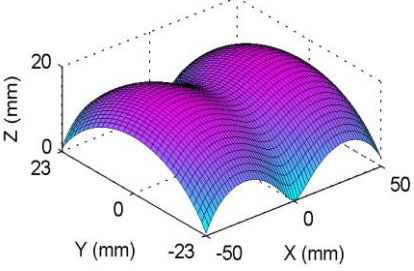
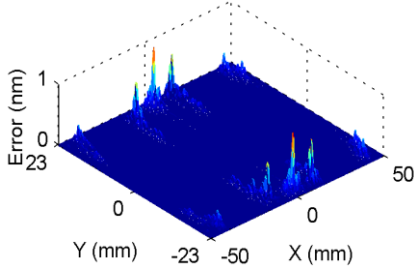


Figure 3.11 CAD model of the freeform streetlight lens

Table 3.6 shows the produced sampling plan and the corresponding deviation of the substitute surface from the designed surface. With approximately the same TLC, the results show that the sampling plan produced by the BDUS method has a much smaller sampling error (0.9 nm) than that produced by the ODUS method irrespective of whether it is in the $\bar{X}$ (187.7 nm) or $\bar{Y}$ (12.3 nm) direction. This infers that the BDUS method possesses a higher sampling accuracy than that for the ODUS method with the same measuring time.

Table 3.6 Sampling results on CAD model of a freeform surfaces for an optical lens

Sampling Plan	Sampling error	Summary	
		<i>SD</i>	$\bar{X}$
		<i>SE</i>	187.7 nm
		<i>NscX</i>	135
		<i>NscY</i>	0
		<i>TLC</i>	3105 mm
		<i>SD</i>	$\bar{Y}$
		<i>SE</i>	12.3 nm
		<i>NscX</i>	0
		<i>NscY</i>	62
		<i>TLC</i>	3100 mm
		<i>SD</i>	$\bar{X}, \bar{Y}$
		<i>SE</i>	0.9 nm
		<i>NscX</i>	67
		<i>NscY</i>	31
		<i>TLC</i>	3091 mm

It is found in the present study, as well as in the published literature, that sampled curves with a high curvature change produce a better sampling plan. As shown in Table 3.5, the ODUS method produced a more efficient sampling plan along $\bar{X}$. This

is due to the fact that the section curves sampled along $\bar{X}$ possess greater curvature change than that for the sampled section curves along $\bar{Y}$ as shown in Figure 3.12. Table 3.6 shows another example of the sampling plan along $\bar{Y}$ being more accurate than that along $\bar{X}$ (see Figure 3.13).

It is also found from the results in the Table 3.5 and Table 3.6 that all the substitute surfaces possess a much higher deviation in the regions where curvature changes sharply than in relatively flat regions. This reflects the fact that uniform sampling based strategies, irrespective of ODUS method or BDUS method, have sampled redundant points from relatively flat regions.

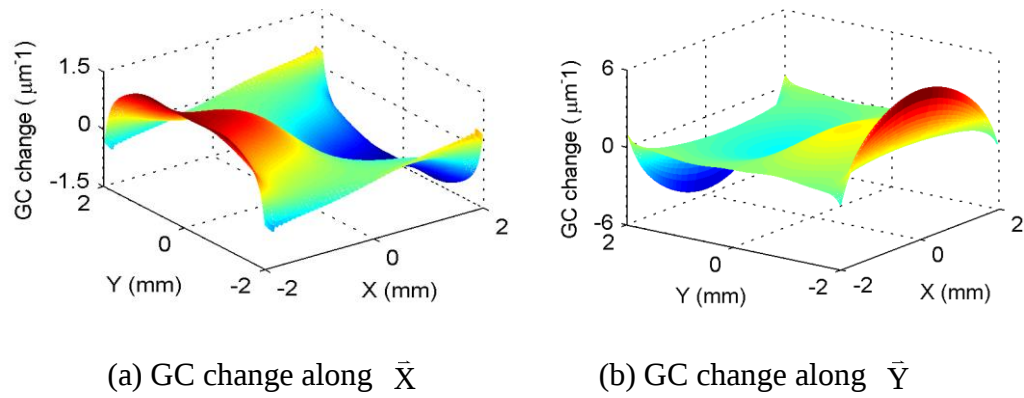


Figure 3.12 Gaussian curvature (GC) change of designed biconic surface

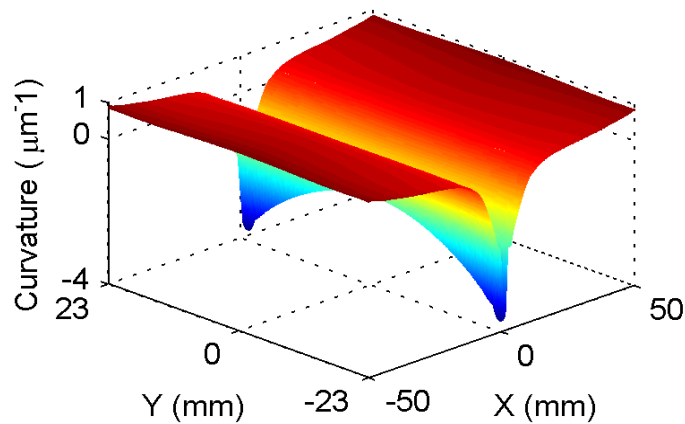


Figure 3.13 Gaussian curvature of the CAD model of the streetlight lens

3.2 Robust Surface Fitting and Reconstruction Algorithm

In the measurement of freeform surfaces, the problem of converting the measured discrete points into useful geometric models is referred to as *surface reconstruction*. Surface reconstruction plays key role in the generalized form characterization of ultra-precision freeform surfaces. This is due to the fact that the direct calculation of the intrinsic surface features from a machined surface is highly sensitive to the local surface properties, such as surface roughness, caused by tool marks. This gives rise to surface fitting as a crucial means to reconstruct a smooth surface from measured discrete points to represent the machined surface. This section presents a robust surface fitting and reconstruction algorithm to reconstruct a high fidelity surface from a cloud of unorganized discrete points, while ensuring surface smoothness.

3.2.1 Fitting criteria

If the B-spline surface is used to construct the surface, the surface reconstruction can be formulated as an optimization scheme as follows. Given a set of unorganized data points $\mathbf{X}_k$, $k=1,2,\dots,m$, a set of control points are found $\mathbf{P}_{i,j}$, $i=1,2,\dots,n_u$, $j=1,2,\dots,n_v$ and appropriate knots vector is such that the objective function

$$F = \frac{1}{2} \sum_{k=1}^m d^2 \left(\sum_{i=1}^{n_u} \sum_{j=1}^{n_v} N_i(u) N_j(v) \mathbf{P}_{i,j}, \mathbf{X}_k \right) + \lambda_s f_s \quad (3.27)$$

is minimized, where $d()$ is a measure of the fitting error of $\mathbf{X}_k$ from the reconstructed surface, f_s is a smoothing term, and λ_s is the weight of f_s . Fitting accuracy and surface smoothness usually contradict each other. If the reconstructed

surface is too close to the data points, unwanted variation may occur due to measurement noise and surface roughness. On the other hand, the reconstructed surface cannot meet the required accuracy if the surface is too smooth. Hence, it is important to strike a balance between them in an appropriate way.

To effectively demonstrate the problem, an example of curve fitting is presented. An ideal curve is designed to generate a nominal curve that is obtained by adding an artificial roughness. The ideal curve and added roughness are given by Eq. (3.28) and Eq. (3.29):

$$y = \sin(0.2x) \quad (3.28)$$

$$y = 7 \times 10^{-6} \sin(10x) + 3 \times 10^{-6} \text{rand}(0,1) \quad (3.29)$$

with the dimensions $0 \leq x \leq 10\pi$ (mm). Figure 4.14 shows the added roughness, where R_p is the maximum pick height and R_v is the maximum valley depth.

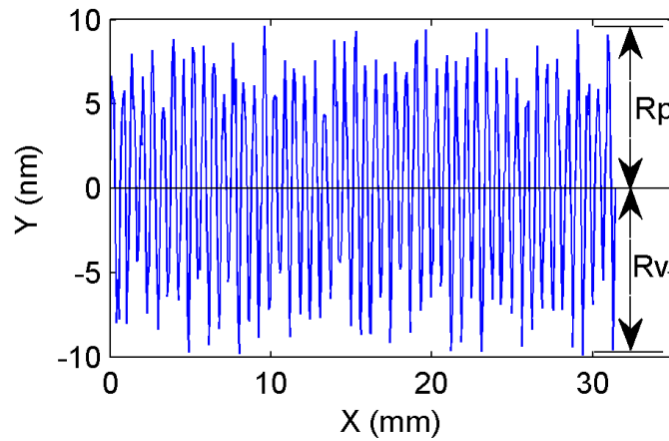


Figure 3.14 Artificial roughness added on ideal designed curve

A row of discrete data is extracted from a nominal curve and used to fit a continuous curve. Curvature is a local property that represents the ‘curvedness’ of a surface or curve. Hence, it is a good measure of local variations of the reconstructed curve which are caused by roughness or noise. To analyze the influence of added

artificial roughness to the reconstructed curve under different thresholds of fitting error, extensive simulation has been conducted to analyze the curvature accuracy of the reconstructed curve under a wide range of threshold fitting errors ranging from 0.3 nm to 480 nm. Curvature accuracy of the reconstructed curve is determined by the maximum error of the curvature of the fitted row of discrete data, which are estimated based on the reconstructed curve.

Figure 3.15 shows the curvature error of the reconstructed curve under different thresholds of the fitting error. The error of the curvature is large in both cases when the threshold of the fitting error is too tight or too loose. It is interesting to note from Figure 3.15 that the curvature error increases sharply when the fitting error decreases from $\max(R_p, R_v)$ (9.9 nm). This infers that the reconstructed curve contains unwanted variation caused by the added roughness when the fitting error is smaller than $\max(R_p, R_v)$. The smaller the fitting error is, the sharper the variations. On the other hand, the curvature error increases slowly along with the increase of fitting error from $\max(R_p, R_v)$. This infers that the reconstructed curve is no longer affected by the added roughness and the curvature error is mainly caused by low fidelity of the reconstructed curve due to the loose fitting error. Based on the above analysis, a proper threshold of fitting error for curve fitting or surface fitting is sought by minimizing the influence of surface roughness of manufactured objects to intrinsic features of the reconstructed object.

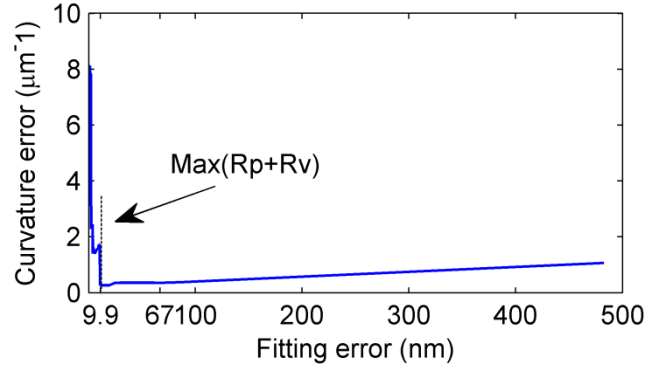


Figure 3.15 Curvature error of reconstructed curve under different fitting error thresholds

The above example shows $\max(R_p, R_v)$ to be a critical value as a threshold of fitting error, which is used to control the balance between fitness and smoothness. However, it is inevitable that the estimated $\max(R_p, R_v)$ contains error in real measurement and it is almost impossible to judge whether the value is the best fitting error or not. Hence, a new threshold of fitting error is proposed as follows:

$$MEr \in C_{Int} \quad C_{Int} = [R_t/2 \quad R_t] \quad (3.30)$$

$$\text{s.t. } Er_a \leq c_a R_a \quad Er_a = \frac{1}{m} \sum_{k=1}^m d_k$$

where $R_t = R_p + R_v$ is the maximum peak to valley height of the roughness; R_a is the arithmetic roughness; c_a is the user-defined non-negative constant; MEr and Er_a are the maximum fitting error and the average deviation of cloud points from constructed geometry, respectively.

In the criterion, an interval likely to include the best fitting error is estimated and is denoted in this study as ‘confidence interval of fitting error’ rather than estimating the best fitting error. $c_a R_a$ is used to further control the form accuracy of reconstructed objects. In the present study, c_a is set to be 1.2. It is considered that

the fitting accuracy and smoothness are well balanced if the fitting error of the reconstructed object satisfies the condition given in Eq. (3.30). It is also worth noting that when the maximum fitting error is close to the lower limit of confidence interval, the possibility of the loss of surface smoothness is increased. On the other hand, when the maximum fitting error is close to the upper limit of confidence interval, the possibility of the loss of surface fidelity is increased.

In the example, the confidence interval of fitting error can be determined by Eq. (3.30), i.e. $C_{\text{Int}} = [9.89 \ 19.78](\text{nm})$. Figure 3.16 shows a part of Figure 3.15 with a fitting error from 0.30 to 67 nm. The discrete data is reconstructed with a high curvature accuracy (less than $0.28 \mu\text{m}^{-1}$) and an acceptable fitting error in the area where the fitting error falls into the confidence interval. This infers that the fitting accuracy and curve smoothness are well balanced.

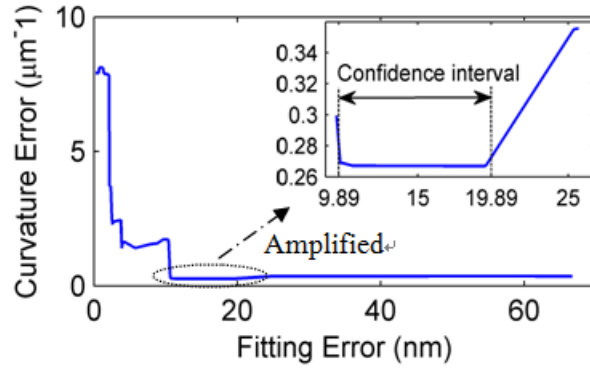


Figure 3.16 Part of Fig 3.15 with fitting error from 0.3nm to 67 nm

3.2.2 Robust surface fitting algorithm

Once the threshold of the fitting error is estimated, surface reconstruction can be carried out using the objective function of the optimization scheme as given in Eq. (3.27). This is a highly non-linear problem since the number and the distribution of

control points, the knot vector and the associated parameter values of the data points are all unknown. To simplify the process, an initial surface is constructed to approximate the points cloud with certain precision, which is used to estimate the minimal degree of freedom needed to characterize the real shape and to obtain a good value of the parameter for each data point.

3.2.2.1 Construction of an initial surface

If the measured data points are regularly distributed, e.g. the measured data points are sampled with the guidance of the bidirectional sampling strategy, then the initial surface can be easily constructed by parameterizing the measured points based on the Centripetal method (Piegl and Tiller, 1997) as given in Section 3.1.1.1. However, if the measured data points are randomly distributed, the bidirectional sampling strategy is used to extract two sets of iso-parametric rows of data from points cloud and each row of data is approximated to construct a curve network. This curve network is then fitted to obtain an initial surface with B-spline form. The details are given in the following procedures:

Step 1: Extract four boundaries from the points cloud and fit to form a curve network.

Step 2: Fit a B-spline surface to the curve network to form an initial surface.

Step 3: Project each point onto the initial surface and locate point $\mathbf{P}_d$ which possesses the maximum distance from the corresponding projection. The distance between P_d and its projection is considered as the deviation of the initial surface from points cloud.

Step 4: Update the initial surface if the deviation is within the given tolerance.

Otherwise, the parameter (u_d, v_d) of $\mathbf{P}_d$ is used to sample a new curve from points cloud. There are two sampling directions, i.e. $\bar{u}$ and $\bar{v}$. If $\bar{u}$ is selected as the sampling direction, all the points satisfying $u_d - \varepsilon < u < u_d + \varepsilon_s$ are sampled from points cloud; otherwise, if $\bar{v}$ is selected, all the points satisfying $v_d - \varepsilon < v < v_d + \varepsilon_s$ are sampled from points cloud. u and v are the parameters of an arbitrary point in points cloud; ε_s is a user specified tolerance. The criterion for the selection of the sampling direction is similar to that given in Section 3.1.1.2. The sampled points are used to construct a new curve so as to form a new curve network.

3.2.2.2 Construction of an accurate surface

Squared distance minimization technology (Wang et al., 2006) is used to optimize the constructed initial surface by minimizing the objective function as follows:

$$F_{obj} = \frac{1}{2} \sum_{k=1}^{Nn} \omega_k^2 Esd_k + \lambda_s f_s \quad (3.31)$$

where ω_k is the weight of $\mathbf{X}_k$ and Esd_k denotes the distance function from data $\mathbf{X}_k$ to the corresponding projection $\mathbf{S}(u_k, v_k)$ on the initial surface $\mathbf{S}(u, v)$. Before going into details, the formula of the B-spline surface is modified to simplify the presentation:

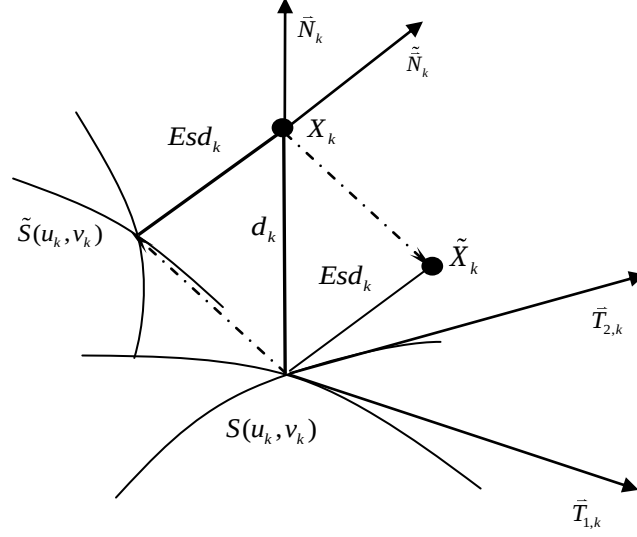
$$\mathbf{S}(u, v) = \sum_{i=0}^{n_u-1} \sum_{j=0}^{n_m-1} N_{i,p}(u) N_{j,q}(v) \mathbf{P}_{i,j} = \sum_{s=0}^{n_c} N_s(u, v) \mathbf{P}_s \quad (3.32)$$

where

$$n_c = n_u n_m, \quad \mathbf{P}_s = \mathbf{P}_{i(s), j(s)}, \quad N_s(u, v) = N_{i(s), p}(u) N_{j(s), q}(v)$$

$$i(s) = \left\lfloor \frac{s}{n_v} \right\rfloor, \quad j(s) = s \bmod n_v$$

If it is assumed that the parameter (u_k, v_k) of each point $\mathbf{X}_k$ and knot vector in the u, v directions are fixed at each iteration, the control points are the only variables in the objective function (Eq. (3.31)). Let $\tilde{\mathbf{S}}(u, v)$ denote the optimized surface with updated control points $\tilde{\mathbf{P}}_{i,j} = \mathbf{P}_{i,j} + \Delta\mathbf{P}_{i,j}$, where $\Delta\mathbf{P}_{i,j}$ is the incremental updates to $\mathbf{P}_{i,j}$. When the control points $\mathbf{P}_{i,j}$ change with the fixed parameters (u_k, v_k) , the foot point $\mathbf{S}(u_k, v_k)$ becomes a variable point $\tilde{\mathbf{S}}(u_k, v_k)$. Considering the initial surface is constructed close to the point cloud, $\tilde{\mathbf{S}}(u_k, v_k)$ is assumed to be in the neighborhood of $\mathbf{S}(u_k, v_k)$. Since the translation from $\mathbf{S}(u_k, v_k)$ to $\tilde{\mathbf{S}}(u_k, v_k)$ is relative to the data point $\mathbf{X}_k$, it can be viewed in a relative sense that $\mathbf{S}(u_k, v_k)$ is fixed and $\mathbf{X}_k$ undergoes a translation to $\tilde{\mathbf{X}}_k$ as shown in Figure 3.17. Then, the distance function $Es d_k$ can be approximated by the Squared Distance Function from $\tilde{\mathbf{X}}_k$ to $\mathbf{S}(u_k, v_k)$ under the local right-handed Cartesian system Σ_k at $\mathbf{S}(u_k, v_k)$ (see Figure 3.17), where $\bar{T}_{1,k}$, $\bar{T}_{2,k}$ and $\bar{N}_k$ are the two principle curvature directions and normal vector at $\mathbf{S}(u_k, v_k)$, respectively; $\tilde{\bar{N}}_k$ is the normal vector at $\tilde{\mathbf{S}}(u_k, v_k)$ of the optimized surface (Pottmann and Hofer, 2003).


 Figure 3.17 Relative translation under local coordinate system on $S(u_k, v_k)$

The Squared Distance Function from $\tilde{\mathbf{X}}_k$ to the original surface S in local coordinate system Σ_k is defined as follows (Pottmann and Hofer, 2003):

$$Esd_k(\tilde{\mathbf{X}}_k) = \frac{d_k}{d_k + \rho_{1,k}} x_1^2 + \frac{d_k}{d_k + \rho_{2,k}} x_2^2 + x_3^2 \quad (3.33)$$

$$\begin{aligned} x_1 &= (\mathbf{S}(u_k, v_k) - \tilde{\mathbf{X}}_k) \cdot \bar{\mathbf{T}}_{1,k} \\ x_2 &= (\mathbf{S}(u_k, v_k) - \tilde{\mathbf{X}}_k) \cdot \bar{\mathbf{T}}_{2,k} \\ x_3 &= (\mathbf{S}(u_k, v_k) - \tilde{\mathbf{X}}_k) \cdot \bar{\mathbf{N}}_k \end{aligned} \quad (3.34)$$

where (x_1, x_2, x_3) are the coordinates of the $\tilde{\mathbf{X}}_k$ (see Figure 3.17) under the local frame Σ_k ; $d_k = \|\mathbf{S}(u_k, v_k) - \mathbf{X}_k\|$ is the distance from $\mathbf{X}_k$ to $\mathbf{S}(u_k, v_k)$; $\rho_{1,k}$ and $\rho_{2,k}$ are two principle curvature radius at the point $S(u_k, v_k)$. Substituting $(\mathbf{S}(u_k, v_k) - \tilde{\mathbf{X}}_k)$ by $(\tilde{\mathbf{S}}(u_k, v_k) - \mathbf{X}_k)$, the final distance function from $\mathbf{X}_k$ to $\tilde{\mathbf{S}}(u_k, v_k)$ in the global coordinate can be expressed as:

$$Esd_k(\tilde{\mathbf{P}}) = \frac{d_k}{d_k + \rho_{1,k}} x_1^2 + \frac{d_k}{d_k + \rho_{2,k}} x_2^2 + x_3^2 \quad (3.35)$$

$$\begin{aligned}
 x_1 &= \left(\sum_{s=0}^{n_c} N_s(u_k, v_k) \tilde{\mathbf{P}}_s - \mathbf{X}_k \right) \cdot \vec{T}_{1,k} \\
 x_2 &= \left(\sum_{s=0}^{n_c} N_s(u_k, v_k) \tilde{\mathbf{P}}_s - \mathbf{X}_k \right) \cdot \vec{T}_{2,k} \\
 x_3 &= \left(\sum_{s=0}^{n_c} N_s(u_k, v_k) \tilde{\mathbf{P}}_s - \mathbf{X}_k \right) \cdot \vec{N}_k
 \end{aligned} \tag{3.36}$$

where $\tilde{\mathbf{P}}$ is the matrix of updated control points.

To further improve the quality of generated surface, the smoothness functions are usually added to smooth the reconstructed surface as much as possible. This work makes use of simplified thin plate energy (Dietz, 1998), which is a quadratic function in the second partial derivatives as given in Eq. (3.37).

$$f_s = \iint_{\Omega} (\bar{S}_{uu}^2 + 2\bar{S}_{uv}^2 + \bar{S}_{vv}^2) dudv \tag{3.37}$$

The smoothness weight λ_s is used to control the effect of the smoothing term to surface fitting. It is started with a certain big value and is adjusted in each iteration such that the resulting surface is improved, but not over dominated by the smoothness function. It is emphasized that the smoothing term is integrated explicitly without numerical approximation in the present study.

Substituting all terms into the objective function in Eq. (3.31), the final linear optimal system is arrived at as:

$$F_{opt}(\tilde{\mathbf{P}}) = \frac{1}{2} \sum_{k=1}^n \omega_k^2 Esd_k(\tilde{\mathbf{P}}) + \lambda_s \iint_{\Omega} (\bar{S}_{uu}^2 + 2\bar{S}_{uv}^2 + \bar{S}_{vv}^2) dudv \tag{3.38}$$

The objective function F_{opt} is positive and quadratic in the updated control points $\tilde{\mathbf{P}}$. Hence, these control points can be computed efficiently by quasi-Newton optimization (Kelley, 1999).

On the whole, this algorithm takes the cloud of scanned 3D data as input, while the output is a B-spline surface which satisfies the given fitting threshold. As shown

in Figure 3.18, bidirectional sampling strategy is used to extract sampling curves to construct an initial surface with certain level of accuracy. This initial surface is then optimized based on the square distance minimization technology. If the optimized surface does not reach the required accuracy, a new curve is extracted from the original data to construct a new initial surface. The whole process is iterative and is terminated upon attaining the desired level of accuracy.

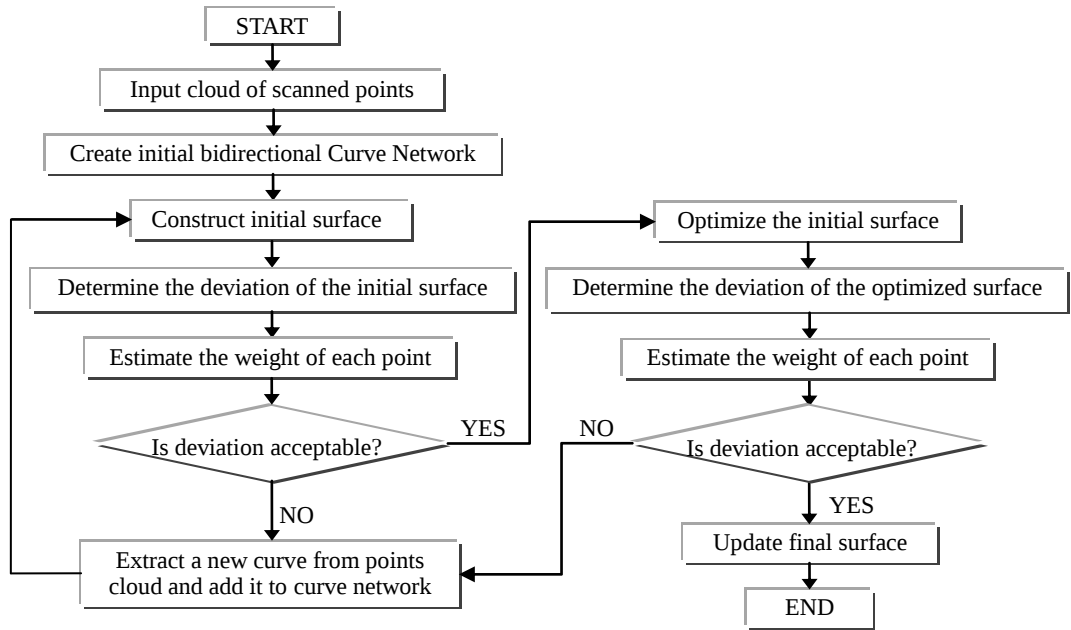


Figure 3.18 Overview of the surface fitting algorithm

3.2.3 Computer simulation of surface fitting algorithm

To evaluate the validity of the proposed fitting algorithm, an ideal freeform surface is designed to generate a measured surface that is obtained by adding the surface roughness. The ideal designed surface and the surface roughness are given by Eq. (3.39) and Eq. (3.40)

$$z = \sin(0.2x) \cos y \quad (3.39)$$

$$z = 5 \times 10^{-6} \sin(10\theta) \times 5 \times 10^{-6} \cos(10\theta) \quad (3.40)$$

with the dimensions $0 \leq x \leq 6\pi$ (mm) and $0 \leq y \leq 6\pi$ (mm). Figure 3.19 and Figure 3.20 show the ideal surface and the aided error pattern, respectively.

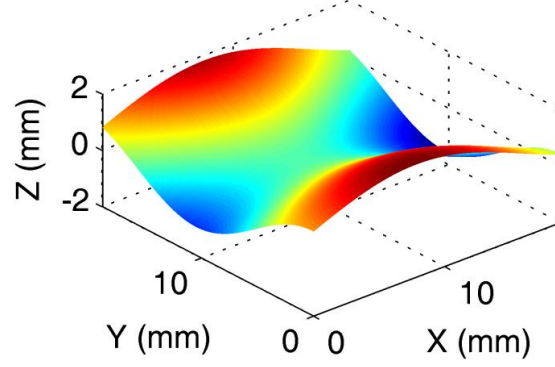


Figure 3.19 Ideal designed surface

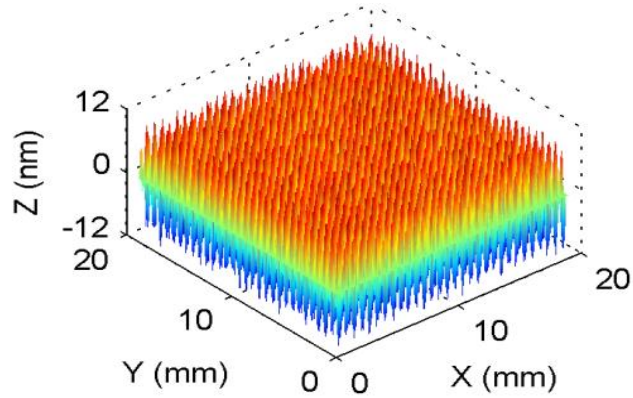
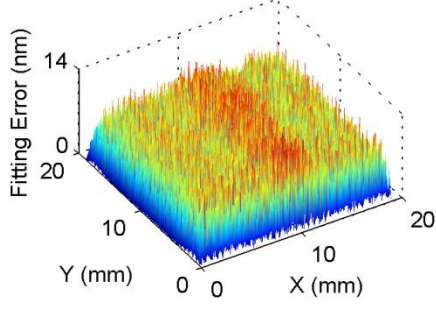
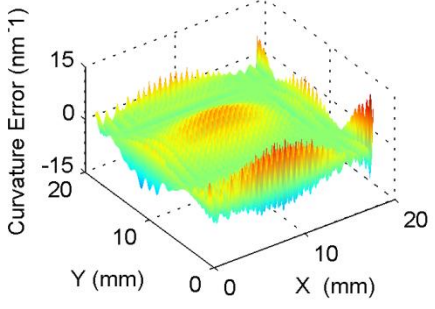
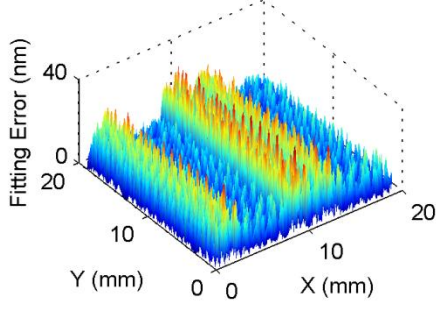
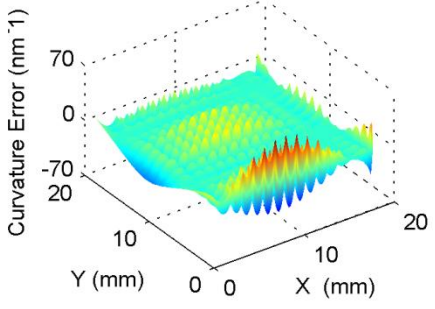
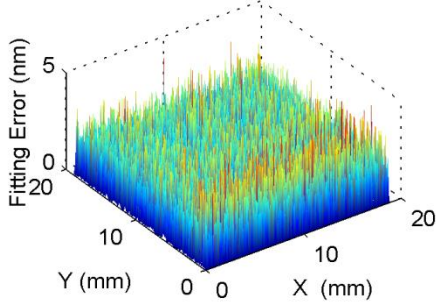
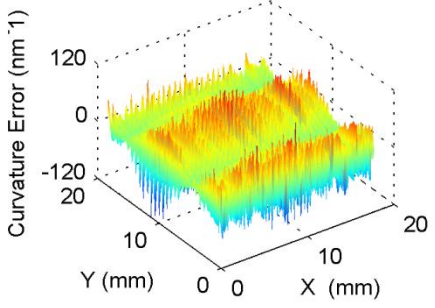


Figure 3.20 Aided error on ideal surface

The fitting error threshold can be determined by the parameters of the added surface roughness using Eq. (3.30). Based on this equation, three fitting cases are studied, i.e. well fitting, under fitting, and over fitting. In the present study, Gaussian curvature is adopted as a measure of the surface smoothness of the reconstructed surface, since Gaussian curvature is slightly more sensitive to the local variation of the surface than mean curvature. Table 3.7 shows the fitting error map and Gaussian

curvature error map of the reconstructed surface and a summary of the simulation results are shown in Table 3.8.

Table 3.7 Fitting error and curvature error of reconstructed surface

Case	Fitting error	Curvature error
Well fitting		
Under fitting		
Over fitting		

It is interesting to note from the results that an over-tight threshold of the fitting error leads to unwanted variations in the reconstructed surface when the maximum fitting error is below the confidence interval. This can be verified by the large Gaussian curvature error of the reconstructed surface. On the other hand, both poor form accuracy and Gaussian curvature accuracy arise under a tight fitting error threshold when the surface is reconstructed with the maximum fitting error above the

confidence interval. When the thresholds of the fitting error satisfy the fitting criteria, both fitting accuracy and Gaussian curvature accuracy are at a high level. This infers that the fitting accuracy and smoothness are well balanced for this surface fitting.

Table 3.8 Summary of the simulation results

	<i>Specific</i>	<i>Value</i>
Discrete data	Number of discrete points	200×200
Fitting Criteria	Confidence Interval	[11.65, 23.3] nm
	Arithmetic roughness	4.09 nm
Well fitting	Maximum fitting error	13.39 nm
	Average fitting error	3.96 nm
	Maximum Gaussian curvature error	15.8 nm ⁻¹
Over fitting	Maximum fitting error	5.22 nm
	Average fitting error	1.19 nm
	Maximum Gaussian curvature error	120.75 nm ⁻¹
Under fitting	Maximum fitting error	37.95 nm
	Average fitting error	8.23 nm
	Maximum Gaussian curvature error	63.27 nm ⁻¹

3.3 Measurement of a Freeform Mould Inserts of a Bifocal Lens

To further evaluate the capability of the proposed methods, an ultra-precision freeform mould insert of a bifocal lens made of stainless steel is produced by a 7-axis ultra-precision polishing machine (Zeeko IRP-200). The measurement is carried out using a Form Talysurf PGI 1240 freeform measurement system (Appendix I). Talysurf

PGI 1240 is an ultra-precision stylus profilometer, which has 200 mm traverse unit with $0.11 \mu\text{m}/200 \text{ mm}$ straightness and has 12.5 mm gauge range with 0.8 nm vertical resolution. Two coordinate systems are established to facilitate the measuring process as shown in Figure 3.21. C_o is the coordinate system of measurement instrument while C_R is the embedded coordinate system of the rotating table. The measurement trajectory is designed following with the sampled curves in a sequential way in each direction, respectively.

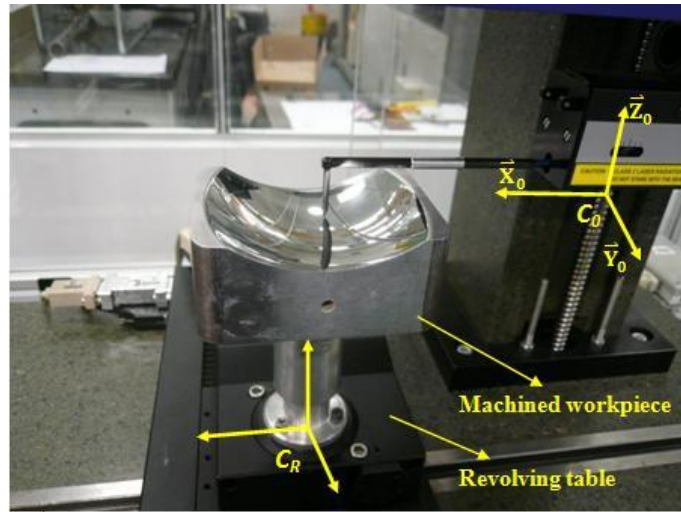


Figure 3.21 Measuring machined freeform workpiece

The measurement is carried out in two steps. Firstly, a set of uniformly distributed curves is extracted along $\bar{Y}_o$ with step size of 4.5 mm. Secondly, the rotating table is rotated 90° and another set of curves is extracted along $\bar{Y}_o$ also with the same step size of 4.5 mm. By transforming all sampled data points to the embedded coordinate system of rotating table as described in Section 3.1.2.1, a curve network is formed by the sampled curves. Figure 3.22 shows the two set of rows of data sampled along different direction and the formed curve network.

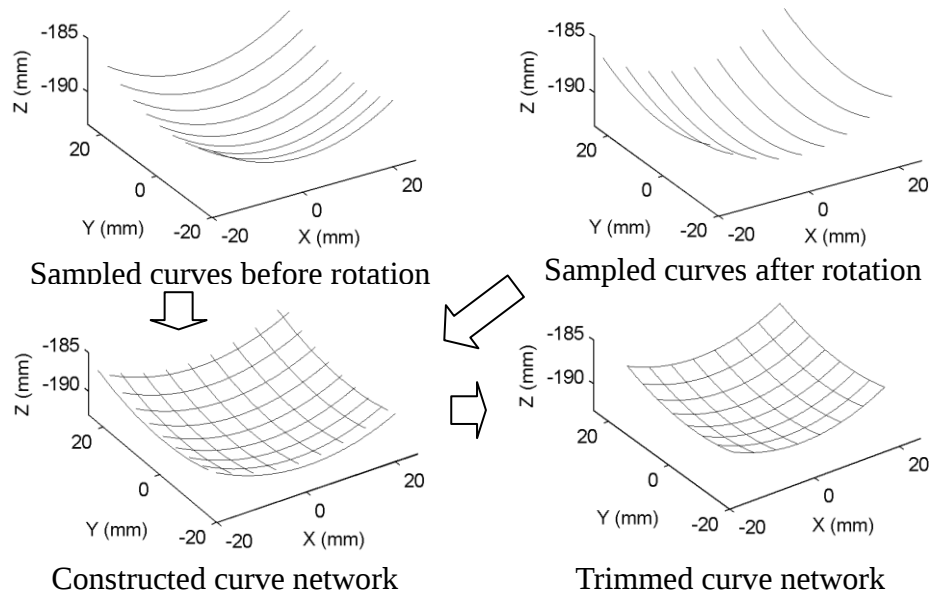


Figure 3.22 Sampled curve network based on bidirectional sampling method

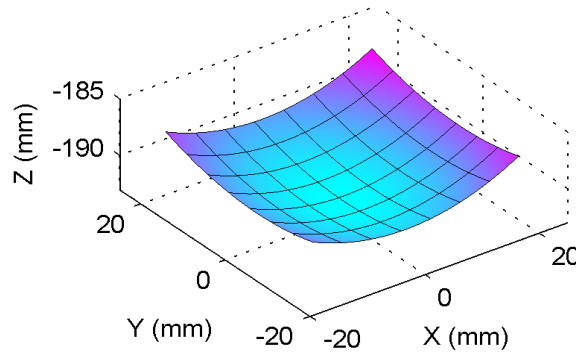


Figure 3.23 Reconstructed substitute surface based on sampled curve network

A smooth surface is then constructed by fitting the sampled curve network. To characterize the form accuracy of the reconstructed surface of the machined surface, a set of high density curves are extracted with step size of 1 mm under the coordinate system of the instrument as shown in Figure 3.24a. It is emphasized that the workpiece is mounted throughout the whole measuring process in order to avoid unnecessary misalignment of the coordinate systems. Hence, a nominal surface is constructed under the embedded coordinate system of the rotating table using the high density curve set. It is assumed that the curve set has sufficient density to fully

describe and represent the geometry of the machined surface. Figure 3.24b shows the constructed nominal model and the deviation of the sampled substitute surface from the nominal model as shown in Figure 3.25. The maximum deviation is 41.8 nm and the average deviation is 7.9 nm. This infers that the reconstructed surface represents the measured data with accuracy in nanometer range.

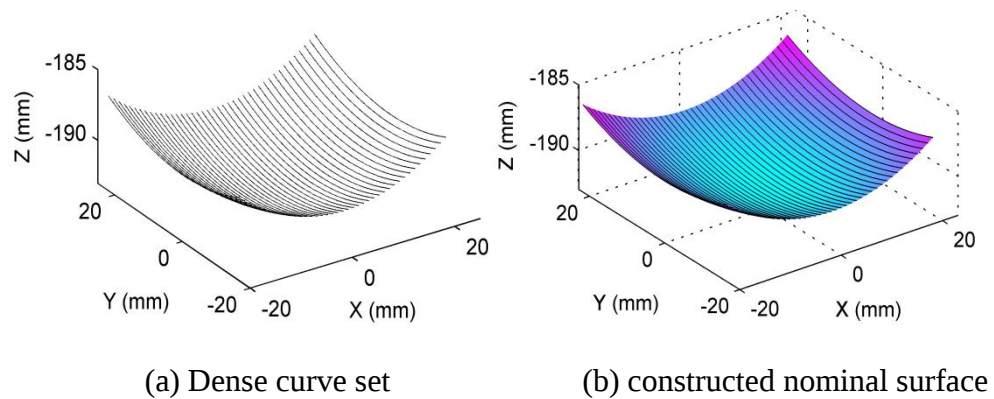


Figure 3.24 Dense curve set and constructed nominal surface

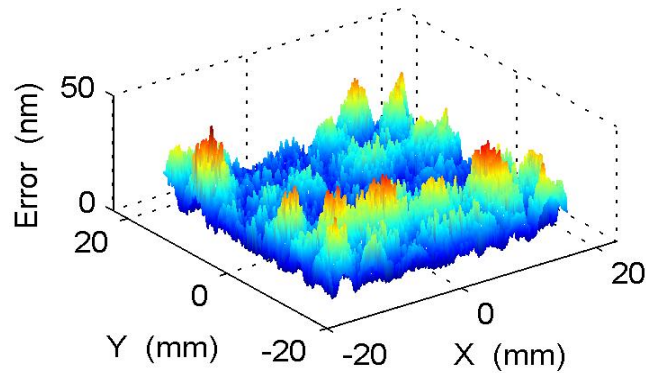


Figure 3.25 Deviation of substitute surface from nominal surface

When the bidirectional sampling method is used for profilometry as is the case of above application, a key problem is that the two sets of curves are sampled in different coordinate systems and should be transferred to a common coordinate system accurately so as to form a curve network. The accuracy of the rotating table would affect the sampling accuracy of the bidirectional sampling method. A series of

simulation experiments have been conducted to study the effect of the rotation error of the rotating table on the final sampling accuracy. Rotational error ranging from -0.5 degree to 0.5 degree are aided in the process of coordinate transformation to form a series of new curve networks. Figure 3.26 shows the relative sampling error (SE) of the rotation error aided sampling plan as comparing with the original one.

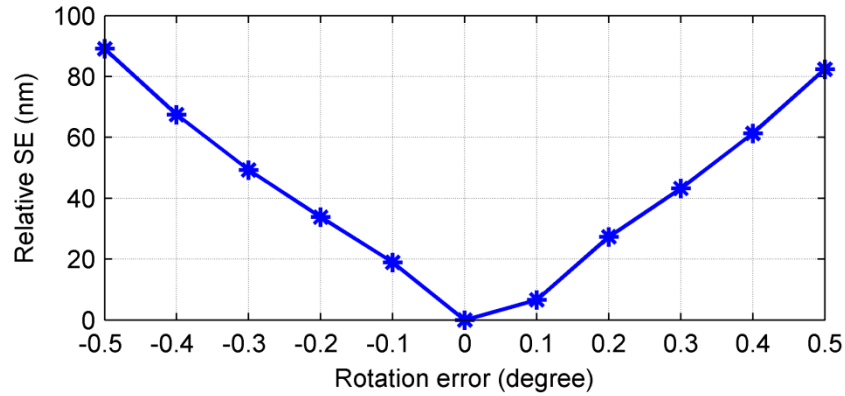


Figure 3.26 Relative sampling error of rotation error aided sampling plans

The result shows that the SE increases approximately linearly with increasing absolute value of the rotational error. In this application, the relative SE is smaller than 20 nm when the rotation error is smaller than 0.1 degree. It is also found from Figure 3.26 that there is a smaller increment of SE when the rotational error is increased from 0 to 0.1 degree as compared to others. This might show the fact that there exists rotational error in the experiment and the actual rotational angle of the rotating table is in the range of $90 \sim 90.1$ degrees.

3.4 Summary

A critical problem of the generalized form characterization of ultra-precision freeform surface is the accurate extraction of the intrinsic surface features from a machined freeform surface. To address this problem, this chapter presents a

bidirectional curve network based sampling strategy combines with a robust surface fitting and reconstruction algorithms.

Different from the conventional raster fashion sampling, the bidirectional sampling strategy attempts to enhance the efficiency and the accuracy of the sampling plan by extracting two sets of raster fashion curves along two different directions to form a curve network, which is used to generate a substitute surface. A CAD based bidirectional optimal sampling algorithm is developed to generate optimal sampling plan with the consideration of both surface complexity and the deviation of the substitute surface from the CAD model. Compared with the sampling plan that is generated by the one directional sampling strategy, the proposed method shows a significant improvement (around 30%) in terms of the efficiency of freeform data sampling with sampling accuracy at the sub-micrometre level. The method is particularly applicable to sampling for the freeform measurement with coordinate measuring machines.

The CAD based sampling strategy greatly depends on the embedded coordinate frame of the CAD model, which makes it difficult to be applied to some measuring instruments such as profilometry. To address this problem, a bidirectional uniform sampling strategy is developed. The experimental results indicate that the method requires less measuring time than that for the raster scanning method with the same level of sampling accuracy, and possesses higher sampling accuracy than that for the raster scanning method with the same measuring time. The method is particularly applicable for the measurement of freeform surfaces with continuous probe, while it can also be applied to trigger probe based measurement by further sampling discrete points on each sampled curve.

Since the direct calculation of the intrinsic features, such as curvature from the measured discrete points, is very sensitive to the noise and outliers presented in the

measured data, a robust surface fitting and reconstruction algorithm is developed to address this problem. To obtain a high quality surface, the reconstructed surface is required to be close enough to the fitted points to fully characterize the form of the measured surface, while surface smoothness should also be ensured to avoid unwanted variations caused by surface roughness and measurement noise.

A new fitting threshold named ‘confidence interval of fitting error’ has been presented to balance the fitting accuracy and surface smoothness. To simplify the fitting process and to avoid local optimization problem, an initial surface is constructed to estimate an appropriate number of control points and their distribution. The squared distance minimization method is then used to minimize the fitting error of the initial surface and experimental work conducted to verify the performance of the developed surface fitting algorithm. The results indicate that the proposed fitting criterion provides an effective means of balancing fitting accuracy and surface smoothness so as to reconstruct high fidelity surfaces with good surface smoothness.

Chapter 4

Invariant Feature Based Form Error Evaluation of Ultra-precision Freeform Surfaces

Over the considerable number of years that surface characterization has been studied, the research has shifted from profile to areal, from stochastic to structured surfaces, and from simple geometries to freeform surfaces (Jiang et al, 2007b), with more research conducted on freeform surface characterization techniques in recent decades. However, due to the geometric complexity of freeform surfaces, there is still a lack of international standards and definitive methodologies for the form characterization of ultra-precision freeform surfaces with form accuracy in the sub-micrometre range and surface finishing at the nanometre level.

The literature review conducted for this study indicates that the most conventional form characterization methods for freeform surfaces are developed based on distance errors (Li and Gu, 2004; Savio et al, 2007; Cheung et al, 2010). These methods employ the least squares method or the minimum zone method to perform the correspondence searching/surface matching and form error evaluation. However, these methods are susceptible to geometry of the surface being characterized and to the present outliers in the measured data, and there are still some problems such as the uncertainty due to the dependency of the methods on the coordinate frame or the geometry of the surface being characterized. As a result, there is a need for a generalized form characterization method for ultra-precision freeform surfaces, which is not only independent of the type of freeform surfaces being

characterized but is also free from the coordinate systems that cause the uncertainties in surface matching.

One promising approach is the utilization of the surface intrinsic properties, which are independent of the coordinate frame. This Chapter presents a generalized form characterization method, named invariant feature-based pattern analysis method (IFPAM), which provides a robust and high precision form characterization method for various types of ultra-precision freeform surfaces with sub-micrometre form accuracy and surface finish in the nanometre range. The IFPAM makes use of orientation invariant surface features, such as Gaussian curvature, to map the surface into a special 2D image pattern so that the corresponding searching or surface matching is converted into invariant feature pattern registration, which makes the method free from the embedded coordinate frame. The bidirectional curve network based sampling strategy combined with a robust surface fitting method, as presented in Chapter 3, are incorporated into the IFPAM to ensure performance of the method in the representation and characterization of the machined ultra-precision freeform surface.

4.1 Invariant Feature-based Pattern Analysis Method

Since the geometry of a machined freeform surface is measured by extracting the coordinates of a set of points on the surface under a Cartesian coordinate frame (e.g. the coordinate systems of the measurement instruments), it is intuitive that the subsequent representation and analysis of the measured surface are performed “extrinsically”, i.e. under the embedded Cartesian coordinate frame. However, this presents problems when the form error of the measured freeform surface is characterized by comparing with theoretical design surface if the two surfaces are not

embedded in the same coordinate system. Hence, surface matching must be undertaken to eliminate the misalignment between the two coordinate systems before the form error evaluation. Precise surface matching of two freeform surfaces is a challenging task since freeform surfaces have six degrees of freedom, especially when there is no strong feature in the matched surface.

Intrinsic surface feature refers to the intrinsic property of a surface, which is determined solely by the distance within the surface and is free of the embedded coordinate system of the surface (Aleksandrov, 1967). Taking Gaussian curvature as an example, it is an intrinsic property of a surface used to describe how “curve” is the surface, and is invariant under the transformation of the coordinate system. Based on the coordinate transformation invariant property of the intrinsic surface feature, an Invariant Feature-based Pattern Analysis Method (IFPAM) has been developed. Figure 4.1 shows the schematic diagram of the IFPAM divided into five parts, i.e. theoretical design surface input and processing, measured data acquisition and processing, invariant feature based surface representation, invariant feature pattern based surface matching and comparison, and parametric output of form error characterization.

Different from conventional simple surfaces, such as spheres, freeform surfaces cannot be generalized by a universal equation. The representation of ultra-precision freeform surfaces is usually based either on a known surface model or an unknown surface model (Cheung et al, 2006). Known surface model refers to the freeform surfaces generated by some specially designed equations, such as F-theta surfaces and bi-conic surfaces. Unknown surface model refers to surfaces represented by a cloud of discrete points. This kind of surface model is commonly found in freeform optics design. For example, the freeform automotive reflector is usually designed with some CAD software packages, such as Reflector CAD and ASAP, and the surface model

can be obtained as a cloud of discrete points in IGES or STEP file format (Minano et al, 2009; Kong et al, 2011).

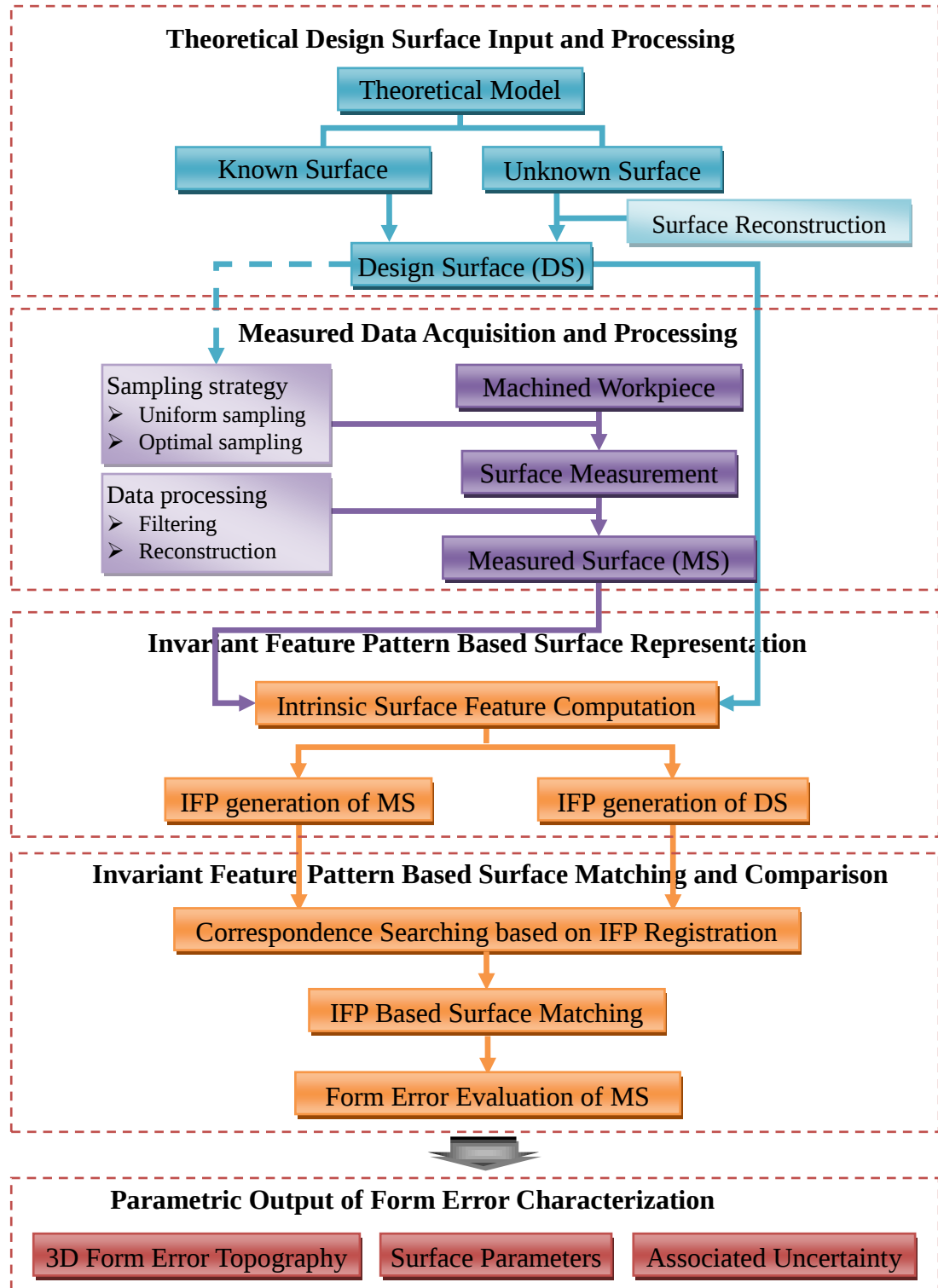


Figure 4.1 A schematic diagram of the invariant feature-based pattern analysis method

For unknown surface models, the surface is reconstructed to obtain a mathematically continuous theoretical surface. The reconstructed surface can be used to estimate the coordinates and differential geometrical features of the surface, including non-sampled area of the given surface model. A portion of the theoretical design model is considered as design surface, which is used to guide the measurement of machined workpiece and subsequent form characterization of the measured surface. The details of this part of research work have been presented in Chapter 3.

In the second part, the machined workpiece is measured by high precision measurement instruments. In the present study, Carl Zeiss PRISMO CMM from Carl Zeiss Inc. and PGI 1240 stylus profilometer from Taylor Hobson are used to measure machined ultra-precision freeform surfaces. During the measurement, an appropriate sampling strategy is generated with the guidance of the design surface so as to ensure the measured data points are adequate to fully represent the geometry of machined workpiece. The measured data points are then fitted to reconstruct a continuous surface to represent the measured surface. This is for two reasons. Firstly, the reconstructed surface can be used to estimate the non-measuring area of the machined surface. Secondly, the reconstructed surface can be easily used to calculate the differential geometrical properties of the measured surface with high accuracy. In the IFPAM, the bidirectional curve network based sampling strategy combines with a robust surface fitting and reconstruction algorithm, which has been discussed in Chapter 3.

In the third part, invariant feature patterns (IFPs) of the design surface and the measured surface are generated respectively to represent the surface geometry. IFP is generated by making use of the surface intrinsic features, such as Gaussian curvature, to map the surface into a 2D image to form a special feature pattern. Since the IFP is composed of intrinsic surface features, it is a generalized surface feature that is

independent of the type of the surface geometry and is free from the embedded coordinate system. In the fourth part, the correspondence searching between measured surface and design surface is performed in terms of image registration of IFP of the measured surface on IFP of the design surface. The form error of the measured surface can be evaluated based on the correspondence established by IFP registration results. The details of this part of work are presented in the following sections of this chapter.

Finally, surface parameters are used to characterize the evaluated form error. Any measurement process combines with errors. It is inevitable that errors are associated with the results of the form characterization, which may be due to factors such as measurement error of measuring instruments, form error of measured surface, and the utilized sampling plan. Hence, it is obligatory to analyze the uncertainty of the form characterization method so as to assess the reliability and accuracy of the characterization results. A task specific uncertainty analysis model is developed in this study to evaluate the uncertainty associated in the characterization results with the consideration of the following three factors: measurement error, surface form error, and sampling strategy. The details of uncertainty analysis are discussed in Chapter 5.

4.2 Invariant Feature Pattern Based Surface Representation

Surface representation is the first step toward the final goals of correspondence searching/surface matching and form characterization. Invariance, uniqueness and stability are the key properties for an effective and efficient surface representation, which directly affects the results of surface matching and registration. The common used freeform surface representation methods, such as parametric surfaces (e.g. NURBS) or points cloud, are highly dependent on the implicit parameterization and the embedded coordinate system, which presents difficulty in surface matching and

comparison. In the present study, an invariant feature pattern (IFP) based surface representation method is presented.

4.2.1 Invariant surface feature

Invariant surface features refer to those surface features whose values are invariant under the transformation (rotation/translation) of the embedded coordinate frame and are free to the implicit parameterization of the surface. In the form characterization of freeform surfaces, if the design surface and measured surface can be represented by invariant surface features, then the form characterization of the measured surface can be performed without the need to consider the misalignment of the coordinate frames and the parameterization of the surfaces.

For example, a sphere can be represented by a point and a radius as its invariant surface features. Hence, the form of a measured sphere can be characterized by comparing the radius of the measured surface with that of the design surface without the need to consider the embedded coordinate frame. Unfortunately, most of freeform surfaces do not have such kind of obvious and simple invariant features. As a result, intrinsic surface property based invariant features are studied in this work.

Suppose a freeform surface $\mathbf{S}$ is given in a parametric form as follows:

$$\mathbf{S} = \mathbf{S}(u, v) = \begin{bmatrix} x(u, v) \\ y(u, v) \\ z(u, v) \end{bmatrix}, \quad \mathbf{u} = (u, v) \in [\mathbf{a}, \mathbf{b}] \subset \mathbb{R}^2 \quad (4.1)$$

where u and v are parameters along $\bar{u}$ and $\bar{v}$ direction respectively; $[\mathbf{a}, \mathbf{b}]$ denotes a rectangular in the u, v -plane, as shown in Figure 4.2. It is assumed that the parametric surface $\mathbf{S}(u, v)$ possesses continuous second partial derivatives. Generally, there are two basic mathematical entities that are considered in the

differential geometry of a smooth surface, i.e. the first and second fundamental forms of a surface (Hsiung, 1981).

The first fundamental form of $\mathbf{S}(u, v)$ is given as follows:

$$\begin{aligned} I = ds^2 &= \mathbf{S}_u^2 du^2 + 2\mathbf{S}_u \mathbf{S}_v du dv + \mathbf{S}_v^2 dv^2 \\ &= Edu^2 + 2Fdudv + Gdv^2 \end{aligned} \quad (4.2)$$

where ds is the arc element of $\mathbf{S}(u, v)$;

$$E = \mathbf{S}_u^2 = \left(\frac{\partial \mathbf{S}(u, v)}{\partial u} \right)^2 \quad (4.3)$$

$$F = \mathbf{S}_u \mathbf{S}_v = \frac{\partial \mathbf{S}(u, v)}{\partial u} \frac{\partial \mathbf{S}(u, v)}{\partial v} \quad (4.4)$$

$$G = \mathbf{S}_v^2 = \left(\frac{\partial \mathbf{S}(u, v)}{\partial v} \right)^2 \quad (4.5)$$

and where $\mathbf{S}_u$ and $\mathbf{S}_v$ are referred to as the tangent vector along $\bar{u}$ and $\bar{v}$ direction respectively (see Figure 4.2).

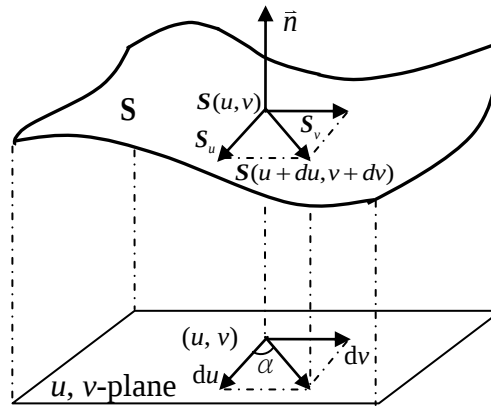


Figure 4.2 Local coordinate frame of parametric surface

As indicated by Eq. (4.2), the first fundamental form I measures the small amount of movement ds^2 on the surface at a point (u, v) along a given vector movement in the u, v -plane. It is well known that an arc element ds is invariant to

the surface parameterization changes and is free to the coordinate frame. Hence, the first fundamental form is invariant to the coordinate transformation. In other words, it depends only on the surface itself but does not depend on how the surface is embedded in the 3D space. Such properties are referred to be an intrinsic property of a surface.

The second fundamental form of $\mathbf{S}(u, v)$ is given by:

$$II = -d\mathbf{S}(u, v) \cdot d\bar{\mathbf{n}} = Ldu^2 + 2Mdudv + Ndv^2 \quad (4.6)$$

where

$$\bar{\mathbf{n}} = \frac{\mathbf{S}_u \times \mathbf{S}_v}{\|\mathbf{S}_u \times \mathbf{S}_v\|} \quad (4.7)$$

$$L = \mathbf{S}_{uu} \cdot \bar{\mathbf{n}} = \frac{\partial^2 \mathbf{S}(u, v)}{\partial u^2} \cdot \bar{\mathbf{n}} \quad (4.8)$$

$$M = \mathbf{S}_{uv} \cdot \bar{\mathbf{n}} = \frac{\partial^2 \mathbf{S}(u, v)}{\partial u \partial v} \cdot \bar{\mathbf{n}} \quad (4.9)$$

$$N = \mathbf{S}_{vv} \cdot \bar{\mathbf{n}} = \frac{\partial^2 \mathbf{S}(u, v)}{\partial v^2} \cdot \bar{\mathbf{n}} \quad (4.10)$$

As shown in Eq. (4.6), the second fundamental form II measures the correlation between the change in the surface position $d\mathbf{S}(u, v)$ and the change in the normal vector $d\bar{\mathbf{n}}$ as a function of a small movement in the u, v -plane. In contrast to first fundamental form, the second fundamental form of a surface is dependent on the embedding of the surface in 3D space.

There are two fundamental theorems, i.e. existence theorem and uniqueness theorem for 3D surfaces that are proven as a direct consequence of the fundamental theorem of ordinary differential equations. Here the uniqueness theorem is given as follows, which is extensively used in this study (Besl, 1988).

Uniqueness: If two surfaces $\mathbf{S}$ and $\mathbf{S}^*$ possess fundamental form I , II and I^* ,

II^* respectively such that the following equalities hold at every point of two surfaces

$$E = E^*, \quad F = F^*, \quad G = G^* \quad (4.11)$$

$$L = L^*, \quad M = M^*, \quad N = N^* \quad (4.12)$$

then there exists an appropriate translation and rotation such that S and S^* coincide exactly implying they have the same shape.

The theorem implies that an arbitrary smooth 3D surface shape is completely captured by six scalar functions, i.e. E, F, G, L, M , and N , i.e. an arbitrary smooth 3D surface can be uniquely represented by the six scalar functions. Although these scalar functions depend on the surface parameterization or the embedded coordinate frame, there are several combinations of these functions that yield specific features of surface shape, which are invariant surface features. Surface curvature is one of the most important invariant surface features.

Gaussian curvature K of a surface is determined by the coefficients of the first and second fundamental forms and it is given as follows:

$$K = \det \left(\begin{bmatrix} E & F \\ F & G \end{bmatrix}^{-1} \right) \det \left(\begin{bmatrix} L & M \\ M & N \end{bmatrix} \right) \quad (4.13)$$

where $\det(\cdot)$ denotes the determinant of a matrix. The mean curvature H of a surface is defined as follows:

$$H = \frac{1}{2} \text{tr} \left(\begin{bmatrix} E & F \\ F & G \end{bmatrix}^{-1} \begin{bmatrix} L & M \\ M & N \end{bmatrix} \right) \quad (4.14)$$

where $\text{tr}(\cdot)$ denotes the trace of a matrix. Both Gaussian curvature and mean curvature are obtained by mapping the two fundamental form functions into a single scalar function. The signs of the Gaussian and mean curvature determine eight basic surface types as listed in Table 4.1 (Besl, 1988).

Table 4.1 Surface Type determined by signs of curvature and mean curvature
(Adopted from Besl, 1988)

	$K<0$	$K=0$	$K>0$
$H<0$	Peak	Ridge	saddle ridge
$H=0$	Minimal	Flat	none
$H>0$	Pit	Valley	saddle valley

There are other ways of looking at surface curvature. The ratio of the first fundamental form I and the second fundamental form II is known as normal curvature function. Normal curvature at a surface point is the curvature of a curve in the surface, of which the osculating plane is perpendicular to the surface tangent plane at the surface point. It varies as a function of the direction of the differential vector (du, dv) in the parametric plane (see Figure 4.2), and is given by:

$$\kappa_{normal}(\mathbf{S}(u, v), du, dv) = \frac{Ldu^2 + 2Mdudv + Ndv^2}{Edu^2 + 2Fdudv + Gdv^2} \quad (4.15)$$

Eq. (4.15) can be rewritten as:

$$\kappa_{normal}(\mathbf{S}(u, v), \lambda) = \frac{L + 2M\lambda + N\lambda^2}{E + 2F\lambda + G\lambda^2} \quad (4.16)$$

where $\lambda = dv/du = \tan \alpha$ (see Figure 4.2). κ_{normal} has two extreme values and occur at the roots λ_1 and λ_2 of:

$$\det \begin{bmatrix} \lambda^2 & -\lambda & 1 \\ E & F & G \\ L & M & N \end{bmatrix} = 0 \quad (4.17)$$

The quantities λ_1 and λ_2 define two directions in the parametric plane and are called *principle directions*. The two extreme values of κ_{normal} at a surface point are

called *principle curvatures* and denoted as κ_1 and κ_2 . The signs of two principle curvatures determine six basic surface types as listed in Table 4.2 (Besl, 1988).

Table 4.2 Surface Type determined by signs of principle curvatures (Adopted from Besl, 1988)

	$\kappa_1 < 0$	$\kappa_1 = 0$	$\kappa_1 > 0$
$\kappa_2 < 0$	Peak	Ridge	saddle
$\kappa_2 = 0$	Ridge	Flat	valley
$\kappa_2 > 0$	Saddle	Valley	pit

The principle curvatures κ_1 and κ_2 are pair of orientation invariant surface descriptors which are similar to the Gaussian and mean curvature. In fact, the Gaussian and mean curvature can also be determined by the two principle curvatures as follows:

$$K = \kappa_1 \kappa_2, \quad H = \frac{\kappa_1 + \kappa_2}{2} \quad (4.18)$$

Gaussian and mean curvatures, as well as principle curvatures, are widely used as shape descriptors in image processing, computer visualization and pattern recognition (Iyer et al, 2005). They are all invariant to arbitrary coordinate transformations and are also free to the surface implicit parameterization. In contrast, six E , F , G , L , M , and N depend on the choice of the surface parameterizations, though they uniquely characterize the surface shape. As a result, it is more convenient to use the surface curvature as a surface shape descriptor. In practice, different kinds of surface curvature have advantages and disadvantages depending on their application. A comparison among different kinds of surface curvature is summarized.

Firstly, Gaussian and mean curvature uniquely determine the surface shape

according to the Gaussian Curvature Uniqueness Theorem (Hsiung, 1981; Horn, 1984) and the Mean Curvature Uniqueness Theorem (Hsiung, 1981; Horn, 1984). However, principle curvatures do not permit such kind of comparable theorem due to its directional dependence. Secondly, the values of the principle curvature should combine with corresponding principle directions for a richer description of the surface while the values for Gaussian and mean curvature are free to direction. Thirdly, the mean curvature and principle curvature change the sign if the orientation of the surface is reversed. However, the Gaussian curvature is entirely independent of the surface parameterization. Fourthly, surface curvatures are sensitive to the noise presented in the analyzed data in numerical computation. Since the mean curvature is the average of the principle curvatures, it is slightly less sensitive than principle curvatures. However, Gaussian curvature is more sensitive to noise. Finally, the calculation of principle computations is more complicated as compared with the mean and Gaussian curvature. Moreover, it is much simpler to compute the sign of the mean and Gaussian curvature than that for the principle curvature.

Although the Gaussian and mean curvature do not contain all the surface shape information contained in the six functions in general, the two curvature functions contain essentially all surface shape information under certain sets of constraints. The uniqueness theorem indicate that the surface corresponding searching/surface matching is able to perform without considering the embedded coordinate frame if a freeform surface is fully represented by Gaussian curvature and mean curvature.

4.2.2 Invariant surface feature pattern

Uniqueness theorem provides the basis for representing and characterizing freeform surfaces by invariant surface features. This section presents an invariant

feature pattern based freeform surface representation, which is generated based on the invariant surface features such as Gaussian curvature. The main theme of the method is shown in Figure 4.3.

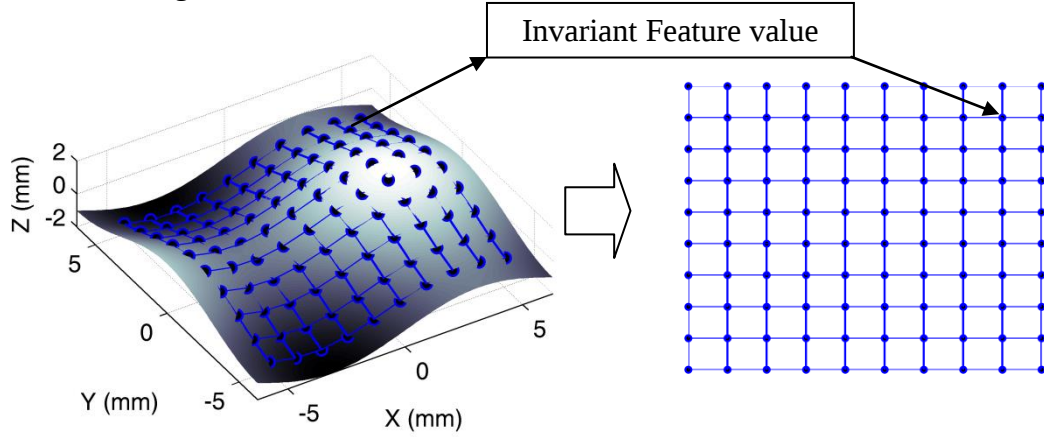


Figure 4.3 Invariant feature pattern based surface representation method

A grid of points are uniformly sampled on a freeform surface and the values of the invariant surface feature of these points are arranged on a 2D plane to form a bitmap image. This bitmap image is named invariant feature pattern of the surface since it is not only invariant to the coordinate transformation but is also free to the implicit parameterization of the surface. However, the layout of a two dimensional texture onto a general freeform surface inevitably creates distortion in all but developable surfaces, i.e. surfaces with zero Gaussian curvature, such as a cylinder (Ahlfors, 1960). Hence, the problem is how to fit a 2D pattern into a freeform surface such that the texture distortion is minimized. In the present study, a woven mesh model (Wang, 2005) is employed to address this problem.

Woven mesh model is a kind of woven fabric model, which consists of a series of vertical threads (warp) with a series of horizontal threads (weft) as shown in Figure 4.4. The directions of warp and weft are orthogonal to each other. There are three types of springs in woven mesh model, i.e. weft spring, warp spring, and diagonal spring. The three types of spring have their own initial length at which the spring has

zero energy. The initial length of the weft spring, the warp spring, and the diagonal spring are denoted as l_{weft} , l_{warp} and l_{diag} , respectively. When the mesh model is fitted into a freeform surface, the texture may be distorted and the directions and lengths of the springs are not preserved as compared with the original 2D pattern. This leads to the strain energy. The distortion can then be minimized by minimizing the strain energy in the mesh model during the fitting process.

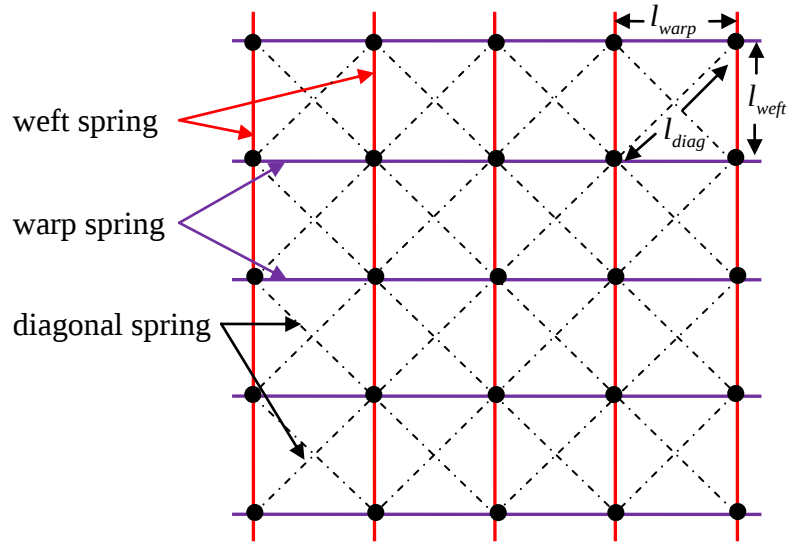


Figure 4.4 Woven mesh model

Suppose $S(u, v)$ is a parametric surface, then the mesh model is fitted into $S(u, v)$ by the following steps.

Step 1: Define a point and two perpendicular vectors on $S(u, v)$ denoted as P_c and $(\bar{A}, \bar{B})$, as shown in Figure 4.5(a). The vectors are selected in such a way that the sampled points can cover as large an area of the surface as possible. In practice, the $\bar{X}$, $\bar{Y}$ axis of the embedded coordinate system are appropriate for a measured surface.

Step 2: $N \times M$ grid of points are extracted from $S(u, v)$ with spacing l_{weft} and l_{warp} .

It starts from extracting two perpendicular curves on $S(u, v)$, as shown in Figure

4.5(a). Cutting planes A and B are determined by normal vector $\vec{N}_c$ of $S(u,v)$ at point P_c and vector $\vec{A}, \vec{B}$ respectively. Then two rows of points are sampled on the intersection curves C_A and C_B . Figure 4.5(b) shows the method for sampling the points. Suppose $P_{c,i}$ is an arbitrary sampled point on curve C_A , the subsequent point $P_{c,i+1}$ is determined in two steps. In the first step, a point $P_{c,i}^o$ is determined by Eq. (4.19) as follows:

$$P_{c,i}^o = P_{c,i} + l_{\text{weft}} \vec{T}_{c,i} / |\vec{T}_{c,i}| \quad (4.19)$$

where $\vec{T}_{c,i}$ is tangent vector of C_A in $P_{c,i}$. In the second step, $P_{c,i+1}$ is determined by projecting $P_{c,i}^o$ on C_A . The normal direction from point $P_{c,i}^o$ to C_A is determined by solving the following equation:

$$C'(u)(C(u) - P_{c,i}) = 0$$

where $C(u)$ is the B-spline form of C_A ; $C'(u)$ is the first derivative of $C(u)$. It can be solved iteratively with Newton method (Piegl and Tiller, 1997). With the same method, a row of points can be determined on C_B with sampling step l_{warp} .

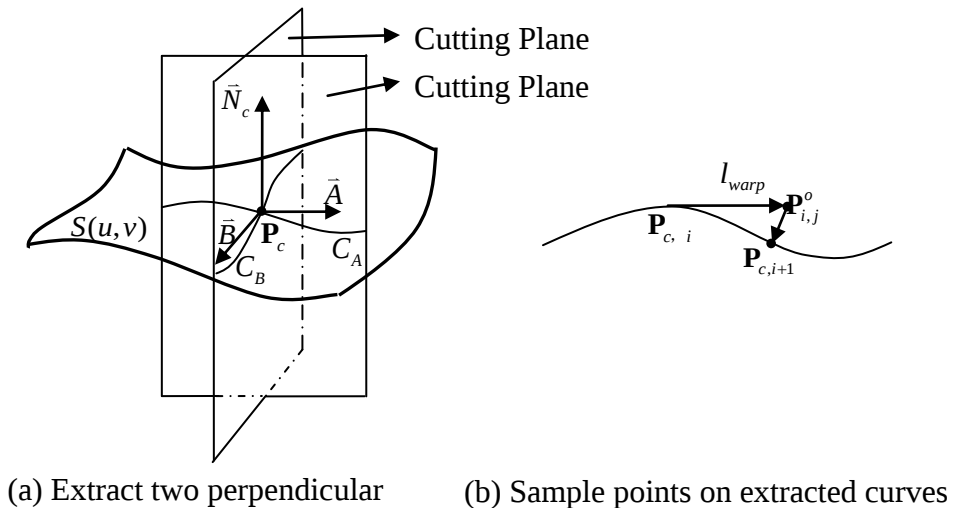


Figure 4.5 The method for selecting two perpendicular rows of points

Figure 4.6 shows the method for determining the remaining points in the grid. If points $\mathbf{P}_c$, $\mathbf{P}_{c,i+1}$ and $\mathbf{P}_{c,j+1}$ are already determined in previous steps, a point $\mathbf{P}_{i+1,j+1}^o$ is determined by:

$$\mathbf{P}_{i+1,j+1}^o = s_1 (\mathbf{P}_{c,i} - \mathbf{P}_c) / \|\mathbf{P}_{c,i} - \mathbf{P}_c\| + l_{warp} (\mathbf{P}_{c,j} - \mathbf{P}_c) / \|\mathbf{P}_{c,j} - \mathbf{P}_c\| \quad (4.20)$$

$\mathbf{P}_{i+1,j+1}$ is then determined by projecting $\mathbf{P}_{i+1,j+1}^o$ on $S(u,v)$. With the same method, $\mathbf{P}_{i+1,j+2}$ can be determined by points $\mathbf{P}_{c,j+1}$, $\mathbf{P}_{c,j+2}$ and $\mathbf{P}_{i+1,j+1}$.

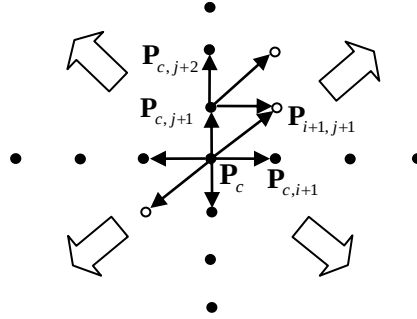


Figure 4.6 The method for determining points in the grid

Step 3: The distribution of the sampled points is optimized by minimizing the strain energy. The total strain energy of the sampled points is given by:

$$E = \sum_{i=1}^{sn} \sum_{j=1}^{sm} \left(\frac{1}{2} \sum_{adj} k_{adj} \left(\|\mathbf{P}_{i,j} - \mathbf{P}_{adj}\| - l_{adj} \right)^2 \right) \quad (4.21)$$

where $\mathbf{P}_{adj}$ is adjacent point of $\mathbf{P}_{i,j}$, k_{adj} is the spring constant of $\mathbf{P}_{i,j} - \mathbf{P}_{adj}$; l_{adj} is the initial length of the spring at zero-energy stage; sn and sm are the number of sampled points along two sampling directions. It is noted that there are 8 adjacent points for each internal point, 5 adjacent points for edge points and 3 adjacent points for corner points. To minimize the strain energy, the following should be preserved for each sampled points:

$$\frac{\partial E}{\partial \mathbf{P}_{i,j}} = \sum_{adj} k_{adj} \frac{\mathbf{P}_{i,j} - \mathbf{P}_{adj}}{\|\mathbf{P}_{i,j} - \mathbf{P}_{adj}\|} \left(\|\mathbf{P}_{i,j} - \mathbf{P}_{adj}\| - l_{adj} \right) = 0 \quad (4.22)$$

A diffusion-like process is employed to release the strain energy (Kobbelt, 2000; Wang et al, 2005). The total strain energy is released by adjusting the distribution of sampled points from those points nearby the centre point $\mathbf{P}_c$. For each point, new position is obtained iteratively by:

$$\mathbf{P}_{i,j}^{new} = \mathbf{P}_{i,j} + \lambda \frac{\partial E}{\partial \mathbf{P}_{i,j}} \quad (4.23)$$

where λ is a damping factor. The iteration is terminated when the variation of E is in the prescribed tolerance. Figure 4.7 shows an example of the generation of an invariant feature pattern for a given sinusoidal surface.

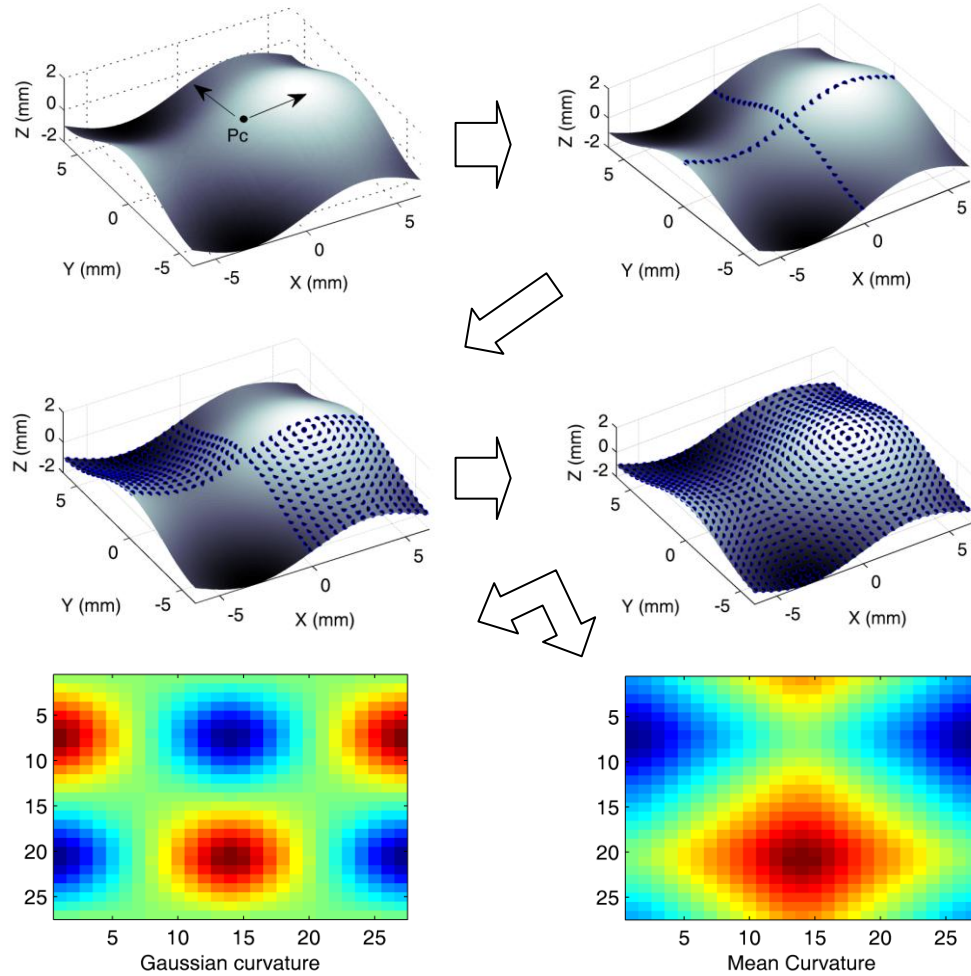


Figure 4.7 An example of generation of invariant feature pattern of a freeform surface

Step 4: The invariant surface features such as Gaussian curvature and mean curvature are calculated for each sampled point. The invariant feature values of the grid of points are then used to form a feature pattern in the 2D plane.

From the generation process, it can be seen that the IFP is composed of two kinds of geometrical features of $S(u, v)$: invariant surface feature of a range of data points, and the distance between them. This makes IFP to be invariant to the coordinate transformation and is also free from the implicit parameterization of the surface. Moreover, the uniqueness theorem implies that IFP uniquely represents the geometry of a surface when the density of the IFP is sufficient.

4.3 Invariant Feature Pattern Based Surface Matching and Comparison

Invariant feature pattern (IFP) based surface matching and comparison intends to represent the ultra-precision freeform surfaces by IFP, which is free to the embedded coordinate system. The corresponding searching/surface matching problem between the measured surface (MS) and the design surface (DS) is then converted to IFP registration. That is, the corresponding searching is performed in 2D space rather than in 3D space as shown in Figure 4.8. By registering the IFP of MS on IFP of DS, point-to-point correspondence pairs are established between MS and DS. The established correspondence pairs can then be used to estimate the coordinate transformation of the measured surface in least square scheme.

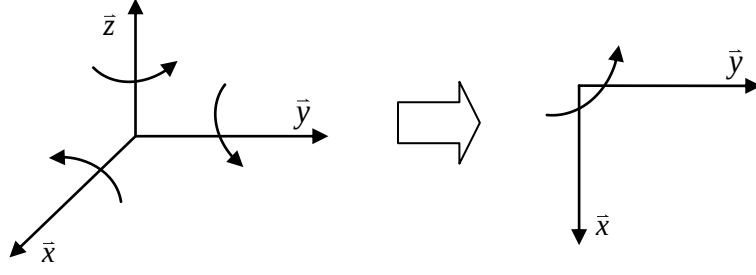


Figure 4.8 IFP based surface matching process

4.3.1 Correspondence establishment

It is noted in the literature review in Section 2.4.3 that the correspondence establishment between MS and DS is the core step of the form characterization of freeform surfaces. If the DS and MS are represented by IFP, then the correspondence between MS and DS can be established by the image registration of IFP of MS on IFP of DS. Image registration is a process of geometric alignment of two similar images so that they may be compared and analyzed in a common reference frame. Registration problems that involve translation and rotation can be recovered by applying Fourier-Mellin transform and phase correlation method (Chen, 1994; Takita, 2003). This method represents the image in frequency domain using Fourier transformation so as to recover the translation. Then polar transformation is used to the magnitude spectrum and the rotation is recovered by using phase correlation in the polar space.

Suppose $f'(x, y)$ is a translated and rotated replica of $f(x, y)$ given as follows:

$$f'(x, y) = f(x \cos(\alpha) - y \sin(\alpha) - \Delta x, x \sin(\alpha) + y \cos(\alpha) - \Delta y) \quad (4.24)$$

where $(\Delta x, \Delta y)$ is a translation distance and α is a rotation angle as shown in

Figure 4.9. According to the Fourier translation property and the Fourier rotation property, the Fourier transformation of f and f' are related by:

$$F'(\xi, \eta) = \exp(-j2\pi(\xi\Delta x + \eta\Delta y))F(\xi \cos(\alpha) - \eta \sin(\alpha), \xi \sin(\alpha) + \eta \cos(\alpha)) \quad (4.25)$$

where F and F' are Fourier transform of f and f' , respectively. Therefore, from Eq. (4.25), Eq. (4.26) preserves:

$$M'(\xi, \eta) = M(\xi \cos(\alpha) + \eta \sin(\alpha), -\xi \sin(\alpha) + \eta \cos(\alpha)) \quad (4.26)$$

where M' and M are magnitude of F and F' , respectively.

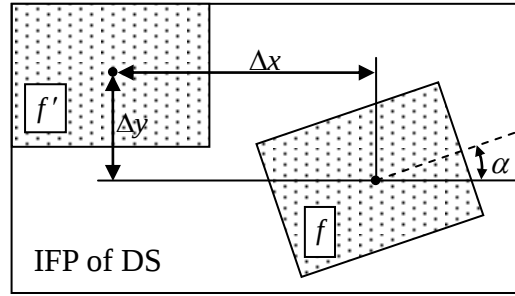


Figure 4.9 Image registration of IFP of MS on IFP of DS

According to Eq. (4.26), the spectral magnitude of the Fourier transform of an image is translational invariant, and the rotation of the image causes the spectral magnitude to be rotated with the same angle. By representing the spectral magnitude of two images in polar coordinates, the rotation angle is converted to translational offsets in polar coordinates as shown in the following equation:

$$PM'(\rho, \theta) = PM(\rho, \theta - \alpha) \quad (4.27)$$

where PM and PM' are the spectral magnitude of f and f' in polar coordinates, respectively. The translational offsets can be determined by phase correlation method (Takita et al, 2003), which is done by extracting and correlating

the phase of both PM and PM' as follows:

$$Cor(u, v) = \frac{FPM(u, v)}{|FPM(u, v)|} \cdot \frac{FPM'(u, v)}{|FPM'(u, v)|} = \exp[j\phi_{FPM}(u, v) - j\phi_{FPM'}(u, v)] \quad (4.28)$$

where FPM and FPM' are the Fourier transform of PM and PM' ; ϕ_{FPM} and $\phi_{FPM'}$ are the spectral phase of PM and PM' . In the absence of noise, Eq. (4.28) is reduced to:

$$Cor(u, v) = \exp[-2\pi(u + v\alpha)] \quad (4.29)$$

The inverse Fourier transform of Eq. (4.29) is a Dirac δ -function yielding a sharp maximum at $(0, \alpha)$. Since the spectral magnitude is a periodic function of the polar angle, the determined angle of rotation may be either α or $\alpha + 180^\circ$. Hence, the f' is rotated by α and $\alpha + 180^\circ$, and the two rotated f' are phase correlated with f . The highest maximum of the outputs is located and is considered as the translational offset $(\Delta x, \Delta y)$ of f' .

The Fourier-Mellin transform and phase correlation based image registration process is summarized as follows:

- (i) Input image f and its translated and rotated image f' ;
- (ii) Compute the Fourier transforms of f and f' , and transform the spectral magnitude of f and f' into polar coordinate;
- (iii) Determine the rotated angle α by phase correlation method;
- (iv) Rotate the f' by α and $\alpha + 180^\circ$, and the two rotated f' are matched with f by phase correlation method respectively;
- (v) Determine the translational offsets by locating the highest maximum of the outputs.

Point to point correspondence between MS and DS can be established after the IFP registration. It should be noted that IFP of MS and DS must have the same spacing in two directions, respectively. Suppose I_{MS} and I_{DS} are IFP of MS and DS, respectively; $\mathbf{Q}_{k,s}$ ($k=0,1,\dots,n$, $s=0,1,\dots,m$) are sampled points when generating I_{MS} ; $\mathbf{P}_{i,j}$ ($i=0,1,\dots,N$, $j=0,1,\dots,M$) are sampled points when generating I_{DS} . Then the corresponding index of $\mathbf{Q}_{k,s}$ with respect to the centre of I_{MS} is given as follows:

$$\text{Ind}(\mathbf{Q}_{k,s}) = \begin{bmatrix} k - n/2 \\ s - m/2 \end{bmatrix} \quad (4.30)$$

After registering I_{MS} on I_{DS} with $[\Delta x, \Delta y, \alpha]$, the new index of $\mathbf{Q}_{k,s}$ in I_{DS} is given as follows:

$$\text{NInd}(\mathbf{Q}_{k,s}) = \begin{bmatrix} (k - n/2)\cos(\alpha) - (k - n/2)\sin(\alpha) + \Delta x \\ (k - n/2)\sin(\alpha) + (s - m/2)\cos(\alpha) + \Delta y \end{bmatrix} \quad (4.31)$$

Suppose $\text{NInd}(\mathbf{Q}_{k,s})$ is located in a sub-rectangular of I_{DS} as shown in Figure 4.10, then Eq. (4.32) and Eq. (4.33) preserve

$$\begin{bmatrix} i \\ j \end{bmatrix} = \lfloor \text{NInd}(\mathbf{Q}_{k,s}) \rfloor \quad (4.32)$$

$$\begin{bmatrix} Qx_{k,s} \\ Qy_{k,s} \end{bmatrix} = \text{NInd}(\mathbf{Q}_{k,s}) - \begin{bmatrix} i \\ j \end{bmatrix} \quad (4.33)$$

where $[i, j]$ is the corresponding index of sampled point $\mathbf{P}_{i,j}$ on I_{DS} . Based on $Qx_{k,s}$ and $Qy_{k,s}$, a point $\mathbf{Q}_{k,s}^o$ can be determined as follows:

$$\mathbf{Q}_{k,s}^o = \frac{1}{4} \begin{pmatrix} \mathbf{P}_{i,j} + Qx_{k,s}(\mathbf{P}_{i+1,j} - \mathbf{P}_{i,j})/|\mathbf{P}_{i+1,j} - \mathbf{P}_{i,j}| + Qy_{k,s}(\mathbf{P}_{i,j+1} - \mathbf{P}_{i,j})/|\mathbf{P}_{i,j+1} - \mathbf{P}_{i,j}| \\ + \mathbf{P}_{i+1,j} + Qx_{k,s}(\mathbf{P}_{i,j} - \mathbf{P}_{i+1,j})/|\mathbf{P}_{i,j} - \mathbf{P}_{i+1,j}| + Qy_{k,s}(\mathbf{P}_{i+1,j+1} - \mathbf{P}_{i+1,j})/|\mathbf{P}_{i+1,j+1} - \mathbf{P}_{i+1,j}| \\ + \mathbf{P}_{i,j+1} + Qx_{k,s}(\mathbf{P}_{i+1,j+1} - \mathbf{P}_{i,j+1})/|\mathbf{P}_{i+1,j+1} - \mathbf{P}_{i,j+1}| + Qy_{k,s}(\mathbf{P}_{i,j} - \mathbf{P}_{i,j+1})/|\mathbf{P}_{i,j} - \mathbf{P}_{i,j+1}| \\ + \mathbf{P}_{i+1,j+1} + Qx_{k,s}(\mathbf{P}_{i,j+1} - \mathbf{P}_{i+1,j+1})/|\mathbf{P}_{i,j+1} - \mathbf{P}_{i+1,j+1}| + Qy_{k,s}(\mathbf{P}_{i+1,j} - \mathbf{P}_{i+1,j+1})/|\mathbf{P}_{i+1,j} - \mathbf{P}_{i+1,j+1}| \end{pmatrix} \quad (4.34)$$

$Q_{k,s}^o$ is projected on DS and the projection is considered as the corresponding point of $Q_{k,s}$ on the DS. Figure 4.11 shows an example of correspondence establishment process for given two sinusoid surfaces.

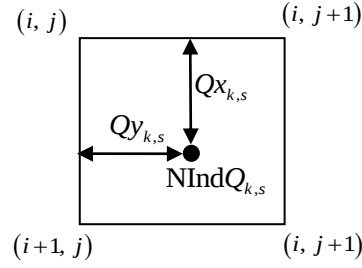


Figure 4.10 Location of index of $Q_{k,s}$ on I_{DS}

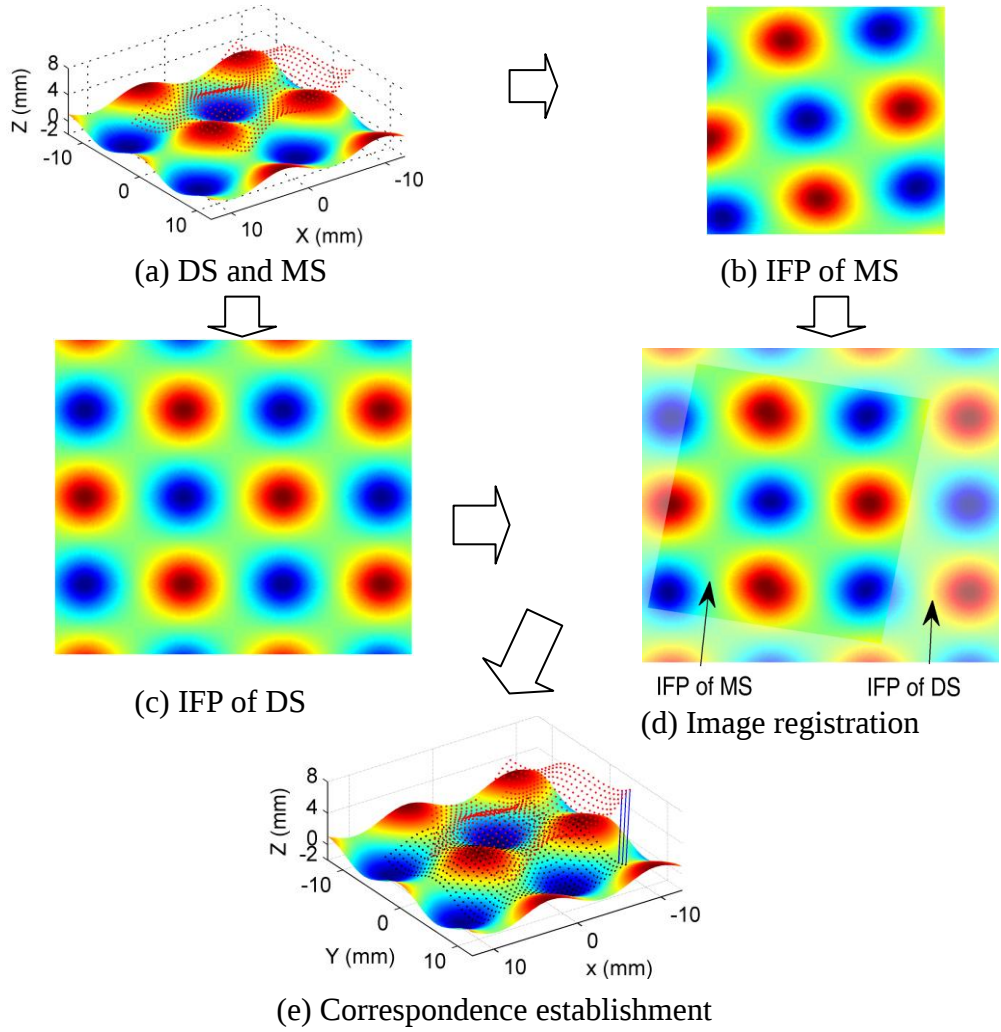


Figure 4.11 Correspondence establishment by IFP registration

4.3.2 Form error evaluation

Form error of a measured ultra-precision freeform surface is evaluated by comparing it with the theoretical design surface as follows:

$$E_i = (\mathbf{P}_i - \mathbf{T} \mathbf{Q}_i) \cdot \bar{\mathbf{n}}_i \quad (4.35)$$

where $(\mathbf{P}_i, \mathbf{Q}_i)$ is a correspondence pair in a homogeneous coordinates; $\bar{\mathbf{n}}_i$ is unit normal vector of DS at point $\mathbf{P}_i$; $\mathbf{T}$ is a coordinate transformation matrix used to remove the misalignment between MS and DS and is given as follows:

$$\mathbf{T} = \begin{bmatrix} c(r_z)c(r_y) & s(r_z)c(r_y) + c(r_z)s(r_y)s(r_x) & s(r_z)s(r_x) - c(r_z)s(r_y)c(r_x) & t_x \\ -s(r_z)c(r_y) & c(r_z)c(r_x) - s(r_z)s(r_y)s(r_x) & c(r_z)s(r_x) + s(r_z)s(r_y)c(r_x) & t_y \\ s(r_y) & -c(r_y)s(r_x) & c(r_y)c(r_x) & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.36)$$

where t_x , t_y and t_z are the translation components, and r_x , r_y and r_z are the rotation angles along $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ axis respectively; $c()$ and $s()$ are abbreviations of the cosine and sine functions. By using the correspondence established in Section 4.3.1, the six spatial parameters $\mathbf{m} = [r_x, r_y, r_z, t_x, t_y, t_z]$ can be determined by minimizing the sum of squared form error E_i of each correspondence pairs.

$$F(\mathbf{m}) = \min \sum_{i=1}^N |\mathbf{P}_i - \mathbf{T} \mathbf{Q}_i|^2 \quad (4.37)$$

where N is the number of the correspondence pairs.

A residual vector $\mathbf{R} \in \Re^{3N \times 1}$ is defined as

$$R_k = \begin{cases} px_i - qx_i & k = 3i - 2 \\ py_i - qy_i & k = 3i - 1 \\ pz_i - qz_i & k = 3i \end{cases} \quad i = 1, 2, \dots, N. \quad (4.38)$$

where $[px_i, py_i, pz_i]$ is components of $\mathbf{P}_i$; $[qx_i, qy_i, qz_i]$ is components of $\mathbf{T} \mathbf{P}_i$.

Then the Eq. (4.37) can be rewritten in matrix form as

$$F(\mathbf{m}) = \min \mathbf{R}^T \mathbf{R} \quad (4.39)$$

Then the following preserve for the local minimum

$$\frac{\partial F}{\partial \mathbf{m}} = 2 \left(\frac{\partial \mathbf{R}}{\partial \mathbf{m}} \right)^T \mathbf{R} = 0 \quad (4.40)$$

Eq. (4.40) is expanded with the Taylor series,

$$\frac{\partial F}{\partial \mathbf{m}} = 2 \left(\frac{\partial \mathbf{R}}{\partial \mathbf{m}} \right)^T \mathbf{R} + 2 \left[\left(\frac{\partial \mathbf{R}}{\partial \mathbf{m}} \right)^T \frac{\partial \mathbf{R}}{\partial \mathbf{m}} + \mathbf{R}^T \frac{\partial^2 \mathbf{R}}{\partial^2 \mathbf{m}} \right] (\mathbf{m}^* - \mathbf{m}) + \left[O(\mathbf{m}^* - \mathbf{m})^2 \right] = 0 \quad (4.41)$$

By ignoring the higher order terms in Eq. (4.42), the Newton method (Fletcher, 2000) can be used iteratively updates the solution by

$$\delta \mathbf{m} = -(\mathbf{J}^T \mathbf{J} + \mathbf{S})^{-1} \mathbf{J}^T \mathbf{R} \quad (4.42)$$

where $\mathbf{J} = \frac{\partial \mathbf{R}}{\partial \mathbf{m}}$ is $3N \times 6$ Jacobian matrix; $\mathbf{S}$ is 6×6 matrix with

$\mathbf{S}_{i,j} = \mathbf{R}^T \frac{\partial^2 \mathbf{R}}{\partial m_i \partial m_j}$. Newton method is known to exhibit a quadratic convergence rate.

Due to the computation complexity of the second order derivatives at each iteration, the term $\mathbf{S}$ is sometimes ignored and this leads to the Gauss-Newton (GN) method (Chong, 2001)

$$\delta \mathbf{m} = -(\mathbf{J}^T \mathbf{J})^{-1} \mathbf{J}^T \mathbf{R} \quad (4.43)$$

GN method has a super-linear convergence rate when the given initial guess is sufficiently close to the solution. A drawback of this method is the relatively narrow convergence domain and the results may be tracked at a local minimum or even divergent if the initial guess is not provided appropriately.

To enhance the robustness of the algorithm to the initial guess, a new method was developed by Kenneth Levenberg and Donald Marquardt, named

Levenberg-Marquardt (LM) method (Marquardt, 1963) as given by

$$\delta \mathbf{m} = -(\mathbf{J}^T \mathbf{J} + \lambda \mathbf{D})^{-1} \mathbf{J}^T \mathbf{R} \quad (4.44)$$

where λ is a damping factor and $\mathbf{D}$ is a diagonal matrix with entries equal to the diagonal elements of $\mathbf{J}^T \mathbf{J}$ while it is feasible to set $\mathbf{D}$ as an identity matrix in practice. The parameter λ is used to control the step-length to guarantee the reduction of F at each iteration (see Eq. (4.37)). To speed up the convergence rate, λ is reduced in each iteration and the LM method moves towards the GN method and allows faster convergence near the solution. A common technique for the selection of λ is given by Jiang et al (Jiang et al, 2010).

Considering the difficulty in guessing a good initial value for the iteration in real measurement, this study chooses more stable Levenberg-Marquardt method to solve Eq. (4.37). A schematic diagram for the determination of the coordinate transformation matrix is shown in Figure 4.12.

The key part of the programme is to calculate the Jacobian matrix $\mathbf{J} = \frac{\partial \mathbf{R}}{\partial \mathbf{m}}$.

Recalling that:

$$\mathbf{R} = [\mathbf{P}_1, \mathbf{P}_2, \dots, \mathbf{P}_N]^T - [\mathbf{TQ}_1, \mathbf{TQ}_2, \dots, \mathbf{TQ}_N]^T \quad (4.45)$$

Then:

$$\mathbf{J} = -[\frac{\partial \mathbf{T}}{\partial \mathbf{m}} \mathbf{Q}_1, \frac{\partial \mathbf{T}}{\partial \mathbf{m}} \mathbf{Q}_2, \dots, \frac{\partial \mathbf{T}}{\partial \mathbf{m}} \mathbf{Q}_N]^T \quad (4.46)$$

It is noted from Eq. (4.46) that the calculation of Jacobian matrix is complex and time consuming if it needs to be determined at each iteration. Hence, the six spatial parameters are estimated recursively in this study. In each iteration, $\delta \mathbf{m}$ is estimated with given $\mathbf{m}_0$ by Eq. (4.44); and it is used to transform each $\mathbf{Q}_i$ to a new position by $\delta \mathbf{T}$. In the next iteration, a new $\delta \mathbf{m}$ is estimated with given $\mathbf{m}_0$ based on the

moved $\mathbf{Q}_i$. The process is continued and is terminated upon the norm of $\delta\mathbf{m}$ is smaller than prescribed value ε_{LM} . The transformation matrix of the original correspondence pairs is obtained by cumulating the $\delta\mathbf{T}$ in each iteration. It is noted that $\mathbf{m}_0$ is always set to be $[0, 0, 0, 0, 0, 0]$ and does not change throughout the whole process.

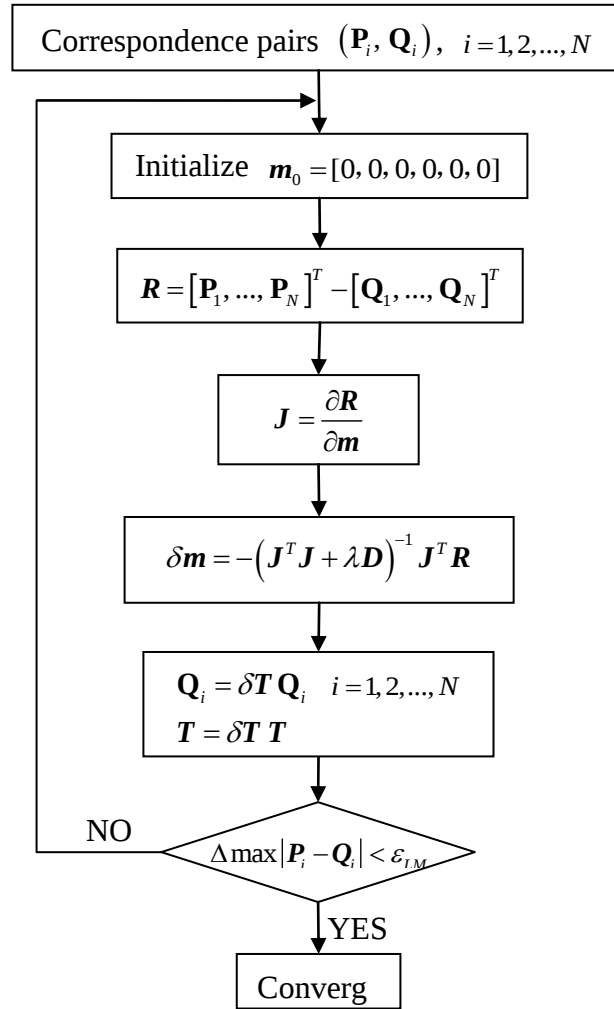


Figure 4.12 Schematic diagram of the determination of six spatial parameters

In this way, the Jacobian matrix $\mathbf{J}$ does also not change in the whole iteration and can simply be determined as follows:

$$\mathbf{J}_k = \begin{cases} 0 & -qz_i & qy_i & 1 & 0 & 0 & k = 3i - 2 \\ qz_i & 0 & qx_i & 0 & 1 & 0 & k = 3i - 1 \\ -qy_i & qx_i & 0 & 0 & 0 & 1 & k = 3i \end{cases} \quad (4.47)$$

The correspondence established by IFP registration can be further refined by orthogonal point projection if it is necessary. The refinement can be performed in a nested approach. That is, a new the correspondence is established by projecting each measured point on the design surface and the projection is considered as the correspondence pair of that point. The newly established correspondence is then used to estimate the coordinate transformation function. This process is continued upon reaching the desired accuracy. $\Delta \max |\mathbf{P}_i - \mathbf{Q}_i|$ is the variation of the maximum deviation of the measured data from the design surface in each iteration. A schematic diagram of correspondence refinement is shown in Figure 4.13.

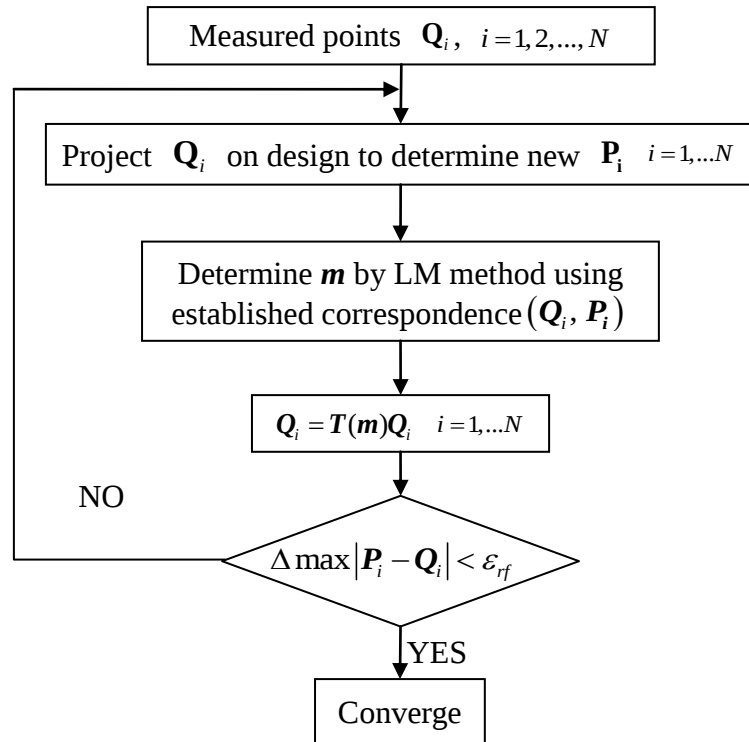


Figure 4.13 A schematic diagram of correspondence pair refinement

One of the problems needs to be addressed is the calculation of Jacobian matrix.

Recalling Eq. (4.46), the term $\frac{\partial \mathbf{P}}{\partial \mathbf{m}}$ is considered to be zero since $\mathbf{P}$ is free to $\mathbf{m}$ in LM method. However, in the correspondence refinement, $\mathbf{P}$ is the projection of $\mathbf{Q}$. When the $\mathbf{Q}$ moves, $\mathbf{P}$ moves as well, which means $\mathbf{P}$ is relevant with the motion parameters $\mathbf{m}$. $\frac{\partial \mathbf{P}_i}{\partial \mathbf{m}}$ is further expended as follows:

$$\frac{\partial \mathbf{P}_i}{\partial \mathbf{m}} = \frac{\partial \mathbf{P}_i}{\partial \mathbf{u}_i} \frac{\partial \mathbf{u}_i}{\partial \mathbf{m}} \quad (4.48)$$

where $\mathbf{u}_i = [u_i, v_i]$ is the parameters of $\mathbf{P}_i$ on the design surface. Since each correspondence pair $(\mathbf{P}_i, \mathbf{Q}_i)$ is the nearest to each other, the following relation always holds true (Ahn, 2004):

$$\frac{\partial}{\partial \mathbf{u}_i} (\mathbf{P}_i - \mathbf{Q}_i)^T (\mathbf{P}_i - \mathbf{Q}_i) = 2 \left(\frac{\partial \mathbf{P}_i}{\partial \mathbf{u}_i} \right)^T (\mathbf{P}_i - \mathbf{Q}_i) = 0 \quad (4.49)$$

Eq. (4.49) is differentiated to $\mathbf{m}$, so that:

$$\frac{\partial (\mathbf{P}_u^T (\mathbf{P} - \mathbf{Q}))}{\partial \mathbf{m}} = \mathbf{P}_u^T \mathbf{P}_u \mathbf{u}_m - \begin{bmatrix} \mathbf{P}_{uu}^T (\mathbf{P} - \mathbf{Q}) & \mathbf{P}_{uv}^T (\mathbf{P} - \mathbf{Q}) \\ \mathbf{P}_{uv}^T (\mathbf{P} - \mathbf{Q}) & \mathbf{P}_v^T (\mathbf{P} - \mathbf{Q}) \end{bmatrix} \mathbf{u}_m - \mathbf{P}_u^T \mathbf{Q}_m = 0 \quad (4.50)$$

where the subscript i is omitted and the partial derivatives $\frac{\partial \mathbf{P}}{\partial \mathbf{u}}$ is written as $\mathbf{P}_u$ for the sake of clarity. Hence:

$$\mathbf{u}_m = \left(\mathbf{P}_u^T \mathbf{P}_u - \begin{bmatrix} \mathbf{P}_{uu}^T (\mathbf{P} - \mathbf{Q}) & \mathbf{P}_{uv}^T (\mathbf{P} - \mathbf{Q}) \\ \mathbf{P}_{uv}^T (\mathbf{P} - \mathbf{Q}) & \mathbf{P}_v^T (\mathbf{P} - \mathbf{Q}) \end{bmatrix} \right)^{-1} \mathbf{P}_u^T \mathbf{Q}_m \quad (4.51)$$

Jacobian matrix can then be obtained by Eq. (4.46), Eq. (4.48), and Eq. (4.51).

4.3.3 Surface parameters for form error characterization

The form error of a measured surface is evaluated after surface matching. The

form error of the measured surface at a measured point $\mathbf{Q}_i$ is defined by:

$$E_i = \pm \sqrt{(px_i - qx_i)^2 + (py_i - qy_i)^2 + (pz_i - qz_i)^2} \quad (4.52)$$

If the measured point is above the design surface, E_i is positive otherwise is negative. Three surface parameters are defined in the present study to characterize the evaluated form error of the measured surface:

(i) Surface peak-to-valley height error S_t which is defined as:

$$S_t = |\max(E_i) - \min(E_i)| \quad (4.53)$$

(ii) Surface root-mean-square error S_q which is defined as:

$$S_q = \sqrt{\frac{1}{N} \sum_{i=1}^N E_i^2} \quad (4.54)$$

(iii) Surface average error S_a which is defined as:

$$S_a = \frac{1}{N} \sum_{i=1}^N E_i \quad (4.55)$$

4.4 Experimental Verification

To demonstrate the performance of the invariant feature-based pattern analysis method (IFPAM) for generalized form characterization of ultra-precision freeform surfaces, the method has been implemented using the MATLAB software package for the freeform surfaces in B-spline form in order to enhance the generalizability of the algorithm. The dependence of various critical factors, such as spacing of IFP, to the accuracy of the characterization results is also studied. It is worthnoting that all experiments are conducted on relatively flat freeform surfaces in which the form error is difficult to be evaluated due to the lack of strong geometric features for surface

matching.

4.4.1 Error budgeting

In the present study, the experiments for error budgeting are used to determine the systematic error of the IFPAM through a series of simulation experiments on two different kinds of surface, which include an aspherical surface and a continuous freeform surface. In the following simulation, a portion of the given design surface (DS) are sampled and transformed to a position by $\mathbf{m}=[1,1,1,1,1,1]$ to simulate the measured surface (MS). Then IFPAM is used to perform the corresponding searching/surface matching and form error evaluation. Since the MS is directly transformed from the DS, the form error of the MS should be zero. Hence, the evaluated form error of the MS is considered as the systematic error of the IFPAM.

4.4.1.1 Study on an aspherical surface

An optical aspherical surface is defined as:

$$z = \frac{Cp^2}{1 + \sqrt{1 - (K+1)C^2p^2}} + \sum_{i=1}^n A_{2i} p^{2i} \quad (4.56)$$

where, $C = 1/R$ is the radius of curvature and R is the radius of the best fit spherical surface; $p = \sqrt{x^2 + y^2}$ is the distance from the optical axis Z ; the conic constant K is a parameter for measuring the eccentricity of the conic surface; the even-numbered values of A_{2i} are aspheric deformation constants. The parameters of the aspheric surface machined in the present study are tabulated in Table 4.3.

Table 4.3 Parameters of the aspherical surface

<i>Parameters</i>	<i>Value</i>
R (mm)	-37.68811
K	0
A_2	0
A_4	2.3711917×10^{-3}
A_6	$-2.8821163 \times 10^{-5}$
A_8	9.6274405×10^{-7}
A_{10}	$-1.3718153 \times 10^{-9}$

Figure 4.14 shows the topography of the DS. From the DS, a section is chosen as the MS and is moved to an arbitrary position by adjusting the six parameters of coordinate transformation $T(\mathbf{m})$ to indicate the misalignment between the coordinate systems of MS and DS. Then the IFPAM is used to perform surface matching and form error evaluation of the MS by comparing with the DS. In the present case study, the Gaussian curvature is used as invariant surface feature to generate the IFP of both MS and DS.

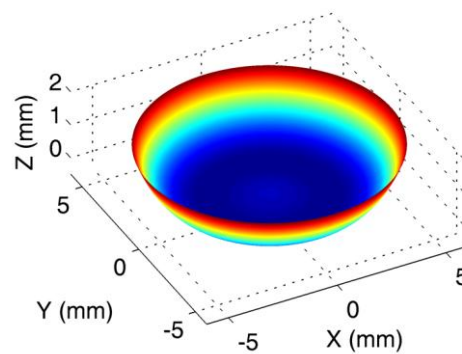


Figure 4.14 Designed aspherical surface

As shown in Figure 4.15a and Figure 4.15b, the form characterization starts from generating the IFP of the MS and the DS with a spacing of 0.1 mm in both two

sampling directions ($\bar{X}$ and $\bar{Y}$ axes), respectively. Then the image registration technique presented in Section 4.3.1 is used to register the IFP of the MS on IFP of the DS to establish the correspondence between MS and DS as shown in Figure 4.16.

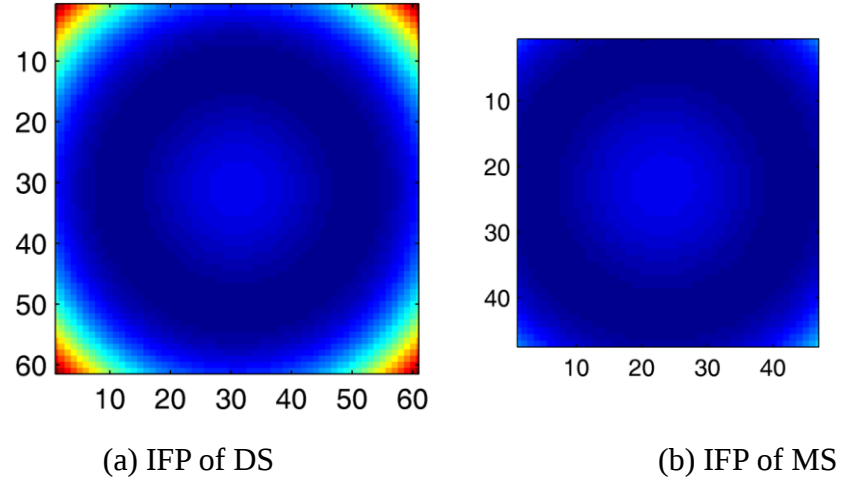


Figure 4.15 Generated IFP of DS and MS (aspherical surface)

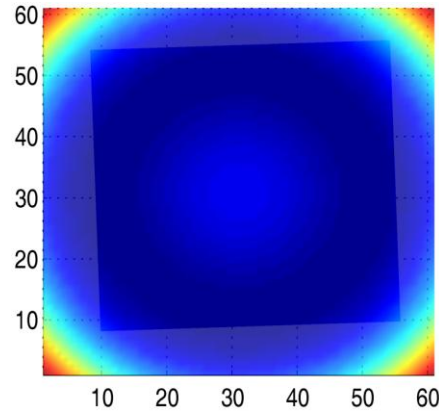


Figure 4.16 Image registration of IFP of MS on IFP of DS (aspherical surface)

The correspondence pairs are then used to estimate the six parameters of coordinate transformation. Figure 4.17 shows the evaluated form error of the MS. It is found from the results that the evaluated form error of the MS is smaller than 1 nm, which means the systematic error of IFPAM is smaller than 1 nm when the IFP is generated with spacing 0.1 mm in both sampling directions. It should be noted that IFPAM is able to characterize the form of the aspherical surfaces with accuracy at the

sub-nanometre level.

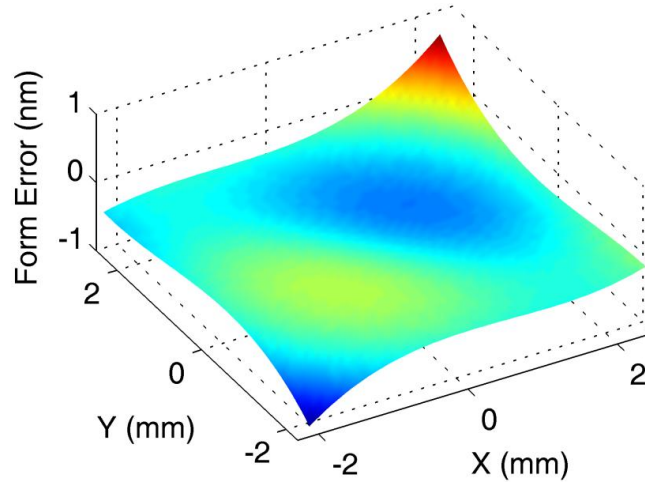


Figure 4.17 Evaluated form error of the MS (aspherical surface)

4.4.1.2 Study on continuous freeform surface

A continuous freeform surface is designed and described by the following equation:

$$\sin(0.5x) + \cos(0.5x) = 0 \quad (4.57)$$

with dimensions $-4\pi \leq x \leq 4\pi$ and $-4\pi \leq y \leq 4\pi$. From the DS, a section is chosen as the MS and is moved to an arbitrary position by adjusting the six parameters of coordinate transformation $T(\mathbf{m})$ to indicate the misalignment between the coordinate systems of the MS and the DS. To test the sensitivity of IFPAM to the initial position of the MS and DS, MS is transferred to three different positions by pure translation (PT) or translation combining with rotation (TR). It is emphasized that a rough matching process is required if least square based surface matching methods are undertaken for all the three tested initial positions.

Figure 4.18 shows the form error evaluation process. As shown in Figure 4.18a and Figure 4.18b, form characterization starts from generating the IFP of the MS and the DS with a spacing of 0.1mm in both sampling directions ($\bar{X}$ and $\bar{Y}$ axes),

respectively. Then the image registration technique is used to register the IFP of the MS on the IFP of the DS as shown in Figure 4.18c. The corresponding pairs are then used to estimate the six parameters of coordinate transformation. Figure 4.18d shows the evaluated form error of the MS.

Table 4.4 shows the results of the surface matching and the corresponding form error evaluation with different parameters of coordinate transformation. It is found that the systematic error of the IFPAM is quite small for the surface matching that only involves translation. However, when the surface matching involves translation as well as rotation, the systematic error becomes bigger along with the increase of angle of rotation. This is due to the fact that the rotation of the MS produces texture distortion to the IFP of the MS, as compared with that for the IFP of the DS, which leads to an error in the image registration process.

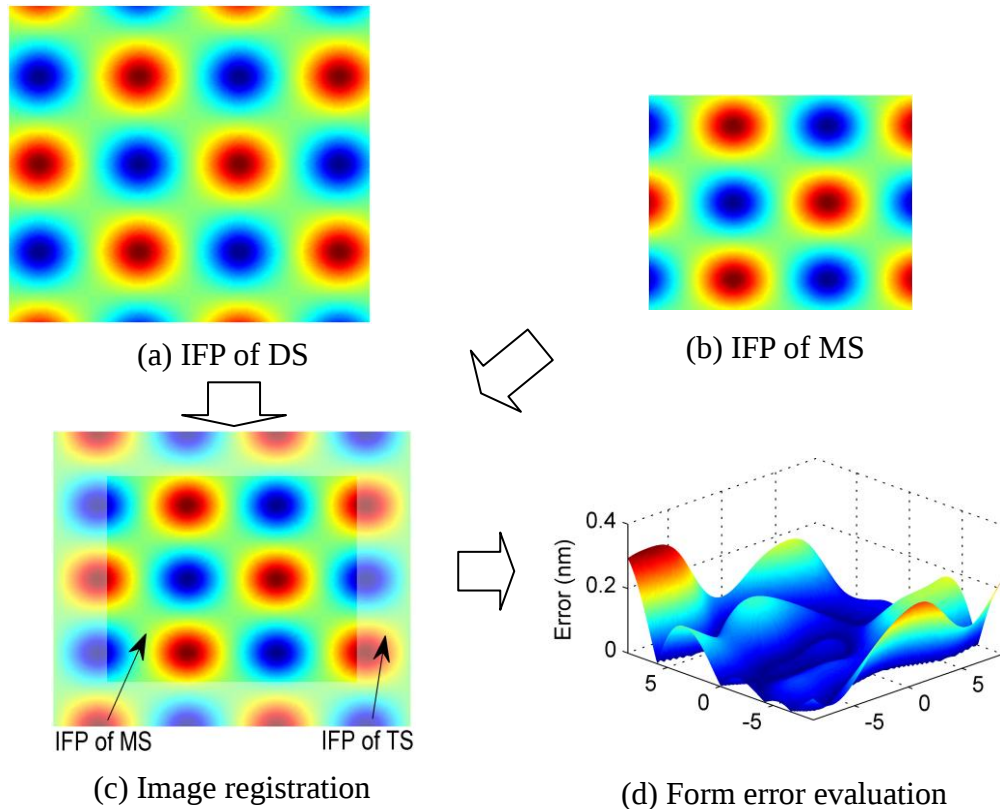


Figure 4.18 Form error evaluation process by IFPFC (smoothed freeform surface)

This problem can be easily solved by performing IFPAM several times to reduce the angle of rotation each time. Normally, the IFPAM is required to be performed no more than twice, even when the rotation angle is quite big. For the third case in Table 4.4, the IFPAM is required to be performed twice. In addition, the large angle of rotation is rare in practice since it can be simply avoided by the operators. It should be noted from the results that the IFPAM demonstrates its low sensitivity to the initial position between the MS and the DS and high accuracy (nanometre to sub-nanometre level) in the form characterization of continuous freeform surfaces.

Table 4.4 Form characterization results with various transformation parameters

$T(t_x, t_y, t_z, r_x, r_y, r_z)$	Translation Error (nm)			Rotation Error (μ rad)			RMS (nm)
	Δt_x	Δt_y	Δt_z	Δr_x	Δr_y	Δr_z	S_q
1 $PT(3, 3, 3, 0, 0, 0)$	-1.9	8.4	0.1	0.3	0.7	4.0	0.2
2 $TR(3, 3, 3, 2, 2, 2)$	17.1	9.8	1.8	4.3	2.5	9.8	13.3
3 $TR(3, 3, 3, 10, 10, 10)$	1337	1155	329	161	223	582	297.6

Note: RMS: Root-mean-square

4.4.2 Factors influencing accuracy of form characterization

In the IFPAM, there are several factors that influence the reliability of the characterization results. The form error of a measured ultra-precision freeform surface is evaluated by Eq. (4.37). This is carried out by transforming the measured surface (MS) into the coordinate of design surface (DS) using the coordinate transformation matrix presented in Eq. (4.36), which is obtained based on the correspondence pairs $(\mathbf{P}_i, \mathbf{Q}_i)$ $i = 1, \dots, N$. Hence, the accuracy of the form characterization results mainly

depends on the quality of the correspondence pairs. Theoretically, if there is no deviation between the MS and the DS, there should be a unique $\mathbf{P}_i$ in the DS for each $\mathbf{Q}_i$ such that a coordinate transformation matrix exists to perfectly match $\mathbf{P}_i$ and $T\mathbf{Q}_i$ for $i=1, \dots, N$.

In the IFPAM, the correspondence pairs are determined by IFP registration. As a result, the accuracy of the IFPAM mainly depends on the accuracy of IFP registration of the MS on the DS. In IFP registration, there are three factors that may affect the accuracy of the registration results, including the spacing of the IFP, i.e. the sampling density of the IFP; the texture distortion of the IFP; and the error generated in 2D-3D projection. The second and third sources of error are closely related to the complexity of the surface being characterized and the initial relative position between the MS and the DS. In this section, a study of the effect of these factors on the accuracy of the IFPAM is presented.

4.4.2.1 *The effect of the spacing of invariant feature pattern*

The spacing of the IFP is the most critical parameter that will seriously affect the accuracy of the IFP registration. In Section 4.2.2, the spacing in two sampling directions are given by l_{weft} and l_{warp} . The effect of the spacing of IFP on the accuracy of the IFP based form characterization method is analyzed. Suppose the error of the estimated is $[\Delta x, \Delta y, \alpha]$ (see Eq. (4.31)), while that after IFP registration is $[\Delta ex, \Delta ey, \Delta \alpha]$, then the error of the registered index of a measured point $\text{NInd}(\mathbf{Q}_{k,s})$ in IFP of DS can be determined by:

$$\begin{bmatrix} \text{EIndX}(\mathbf{Q}_i) \\ \text{EIndY}(\mathbf{Q}_i) \end{bmatrix} \cong \begin{bmatrix} \Delta\alpha(\text{IndX}(\mathbf{Q}_i)\sin(\alpha) + \text{IndY}(\mathbf{Q}_i)\cos(\alpha)) + \Delta ex \\ \Delta\alpha(-\text{IndX}(\mathbf{Q}_i)\cos(\alpha) + \text{IndY}(\mathbf{Q}_i)\sin(\alpha)) + \Delta ey \end{bmatrix} \quad (4.58)$$

According to Eq. (4.34), the displacement of the correspondence of $\mathbf{Q}_{k,s}$ in design surface can be approximated as:

$$\Delta\mathbf{P}_i \cong \text{EIndX}(\mathbf{Q}_i)l_{\text{weft}}\bar{\mathbf{A}} + \text{EIndY}(\mathbf{Q}_i)l_{\text{warp}}\bar{\mathbf{B}} \quad (4.59)$$

$\Delta\mathbf{P}_i$ leads to error to the estimation of six spatial parameters and the error can be approximated by:

$$\Delta\mathbf{m} \cong -(\mathbf{J}^T \mathbf{J})^{-1} \mathbf{J}^T \Delta\mathbf{P} \quad (4.60)$$

where $\Delta\mathbf{P} = [\Delta\mathbf{P}_1, \dots, \Delta\mathbf{P}_N]^T$ is 3×4 residual vector; $\mathbf{J}$ is the Jacobian matrix as given by Eq. (4.47); $\Delta\mathbf{m} = [\Delta r_x, \Delta r_y, \Delta r_z, \Delta t_x, \Delta t_y, \Delta t_z]$.

With a small transformation perturbation, the resulting form error can be approximated as:

$$fe_i \cong (\mathbf{Q}_i - \Delta\mathbf{T}\mathbf{Q}_i) \cdot \bar{\mathbf{n}}_i \quad (4.61)$$

where $\Delta\mathbf{T}$ is the perturbed transformation matrix and can be approximated as follows

$$\Delta\mathbf{T} \cong \begin{bmatrix} 1 & \Delta r_z & -\Delta r_y & \Delta t_x \\ -\Delta r_z & 1 & \Delta r_x & \Delta t_y \\ \Delta r_y & -\Delta r_x & 1 & \Delta t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.62)$$

Eqs. (4.58-4.62) demonstrates the relationship between the IFP registration accuracy and the evaluated form error. It is seen from the deduction that the propagation of the error of IFP registration to the evaluated form error is a complex process that may vary with the change of distribution of the measured points and the

complexity of the surface being matched. In practice, $\mathbf{Q}_i$ can be replaced by the $\mathbf{P}_i$ since $\mathbf{Q}_i$ and $\mathbf{P}_i$ are close to each other after surface matching. In the present study, IFP registration is carried out by the phase correlation method presented in Section 4.3.1 which is able to achieve the registration with translation accuracy down to 0.1-0.01 pixel (spacing) accuracy and rotation down to 1/40 degree. Hence, the spacing of the IFP can be estimated with respect to the required characterization accuracy for a specific freeform surface.

To more effectively demonstrate the effect of the IFP spacing to the form characterization results, an analysis is carried out on a sinusoidal surface given by Eq. (4.57) based on Eqs. (4.58) to (4.62). Figure 4.19 shows the maximum error of the evaluated peak-to-valley height S_t with different IFP spacing ranging from 0.1mm to 1mm. It should be noted from the results that the error of S_t shows exponential growth along with the increase of the spacing of IFP. When the spacing is smaller than 0.2mm, the IFPAM is able to characterize the form of the given freeform surface with accuracy at the nanometre level (smaller than 10 nm).

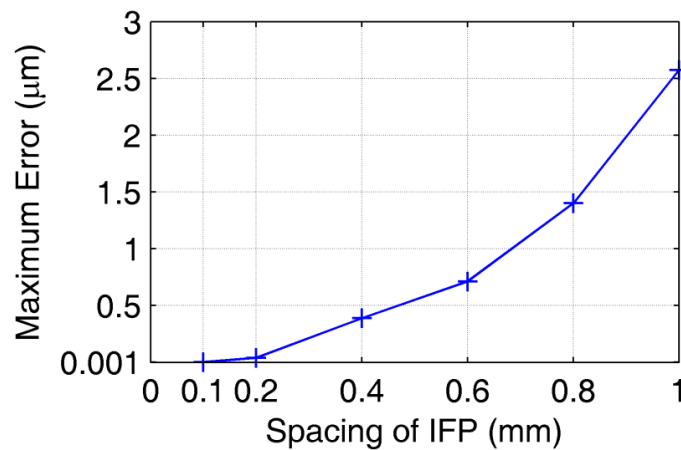


Figure 4.19 Maximum error of the evaluated S_t with different IFP spacings

4.4.2.2 *Effect of the initial relative position between measured surface and design surface*

It is noted in Section 4.2.2 that the layout of a two dimensional texture map onto a general freeform surface inevitably creates distortion in all but developable surfaces. The distortion of IFP is closely related to the Gaussian curvature of the surface, which is an indicator of how far a surface is developable (Wang et al, 2005). Surface with zero Gaussian curvature corresponds to a developable surface which can be mapped to a plane with zero distortion. In fact, the distortion is a monotone increasing function of the Gaussian curvatures magnitude. Intuitively, the texture distortion of IFP affects the accuracy of IFP registration. This implies that the accuracy of IFP based method is affected by the surface complexity.

However, IFP of the MS and the DS should have similar distortion since DS and MS are the same surface in different positions. That is, they possess the same surface complexity and the same curvature distribution. As presented in Section 4.2.2, the IFP is generated with a given centre point and two sample directions. Suppose the IFP of the MS and the DS are generated with the same centre point and the same sample directions, theoretically, the IFP of the MS should be exactly the same as the IFP of the DS, even though texture distortion occurred during the generation. Intuitively, the difference between the IFP of the MS and the DS is related to the difference between their centre point and sampling directions. This implies that the accuracy of the IFPAM may be affected by the initial relative position between the MS and the DS.

As indicated in Section 4.2.2, in practice, the $\bar{X}$, $\bar{Y}$ axis of the embedded coordinate system are selected as sampling directions to generate the IFP of the MS and the DS. Hence, the sampling directions of MS are different from that of DS with the change of the relative position of MS with respect to DS. A series of experiments

have been conducted to study the effect of different sampling directions of MS and DS to the accuracy of the form characterization results. As shown in Figure 4.5, the IFP of the DS is generated along the $(\bar{A}, \bar{B})$. Then a series of IFP of the MS are generated by rotating the $(\bar{A}, \bar{B})$ with given angle θ . The generated set of IFP of the MS is then registered on the IFP of the DS and the results are used to evaluate the form error of the MS.

Figure 4.20 shows the maximum error of evaluated peak-to-valley height S_t with different θ ranging from 0 to 20 degree. The results that the error of S_t shows exponential growth along with the increase of the spacing of IFP. The systematic error of IFP based method increases dramatically when the θ is larger than 10 degrees. This is due to the fact that the rotation of the MS produces texture distortion to the IFP of the MS as compared with the IFP of the DS, which lead to an error in the image registration process. This problem can be easily solved by performing IFPAM several times to reduce the angle of rotation each time. Normally, the IFPAM is required to be performed no more than two times even when the angle of rotation is quite big. Indeed, the angle of rotation between the the MS and the DS can be easily controlled in the measuring process by operators. When the angle between the sample directions of MS and DS is smaller than 5 degrees, the accuracy of the IFPAM can be maintained smaller than 100 nm.

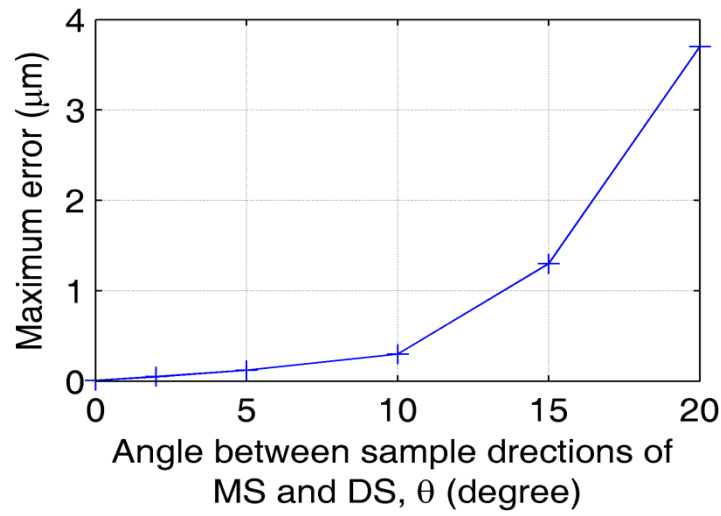
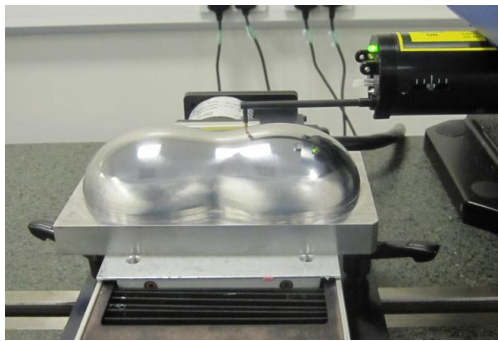


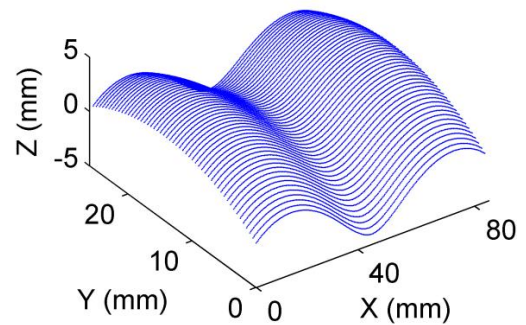
Figure 4.20 Maximum error of evaluated S_t with different θ

4.4.3 Measurement of a freeform mould insert of a streetlight lens

To further evaluate the capability of the developed IFPAM in real measurement of ultraprecision freeform surfaces, a freeform mould insert of a streetlight lens was machined by a 7-axis freeform polishing machine as shown in Figure 4.21a. The machined workpiece was measured by a Form Talysurf PGI 1240 freeform measurement system with a sampled area of $85 \times 26 \text{ mm}^2$. Figure 4.21b shows the measured discrete points.



(a) Machined freeform mould insert



(b) Measured discrete points

Figure 4.21 Measurement of a machined freeform mould insert of streetlight lens

Robust surface fitting and reconstruction algorithm presented in Chapter 3 are used to reconstruct a smooth surface from the measured data points for the extraction of the invariant feature pattern. For freeform surfaces, the confidence interval of fitting error is not easily determined from a pure geometric standpoint. In order to estimate the surface roughness of the machined workpiece, 10 different regions over the workpiece are sampled and measured by the Wyko NT8000 optical measuring system (Appendix II). Wyko NT8000 is an ultra-precision optical profiler which has 8mm vertical scan range with sub-nanometer resolution. The average of S_t and S_q of 10 sampled regions are considered as roughness parameters of the workpiece, i.e. $S_t = 3.07 \mu\text{m}$, $S_q = 0.29 \mu\text{m}$. Then, the confidence interval can be determined according to Eq. 4.63 as follows:

$$MEr \in [1.53 \quad 3.07] \quad \text{s.t.} \quad Er_a \leq 0.35 \quad (\mu\text{m}) \quad (4.63)$$

A smooth surface is fitted on the measured data with the maximum fitting error $1.97 \mu\text{m}$, which is a value in the confidence interval. Figure 4.22a shows the fitting error map. Based on the reconstructed surface, Gaussian curvature of the measured surface can be calculated. It is seen from Figure 4.22b that there is no sharp variation caused by the surface roughness. An over fitting case is also studied and is shown in Figure 4.23. The surface is reconstructed with maximum fitting error $0.58 \mu\text{m}$ (see Figure 4.23a), which is a value below the confidence interval. It is clearly seen from Figure 4.23b that the Gaussian curvature of the reconstructed surface varies sharply, which indicates that the tight fitting threshold makes the surface roughness significantly affecting the smoothness of the reconstructed surface.

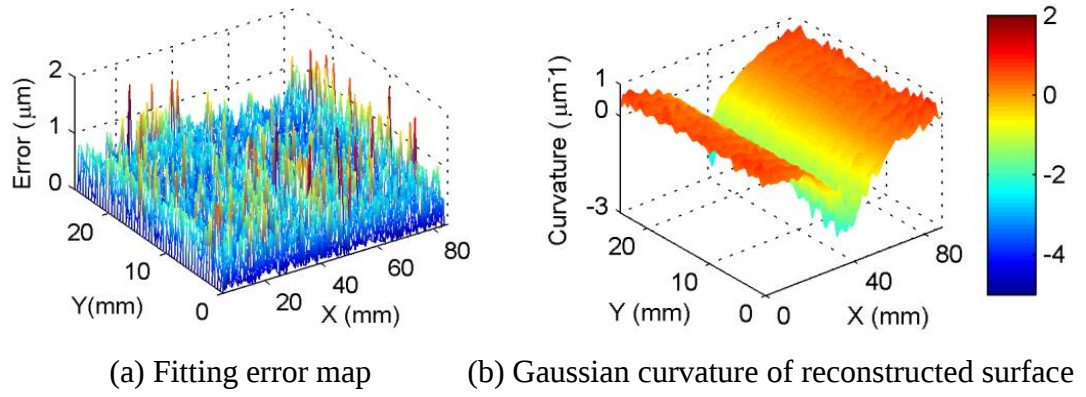


Figure 4.22 Experimental results of well surface fitting

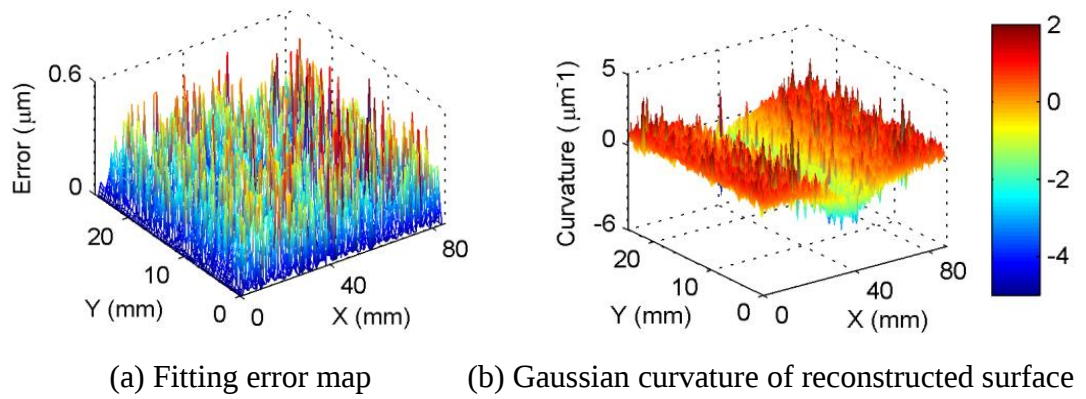


Figure 4.23 Experimental results of over surface fitting

The form characterization of the machined surface is performed based on the reconstructed MS. Figure 4.24 shows the IFP registration result, and the evaluated form error of the MS is shown in Figure 4.25. To verify the accuracy of the IFPAM, the results are compared with that is obtained by a Robust Form Characterization Method (RFCM) (Cheung et al, 2006). RFCM is developed based on the idea of the traditional iterative closest point (ICP) method. A comparison of the form characterization results is summarized in Table 4.5. It is noteworthy that the characterization results based on the reconstructed surface agree with those obtained by the traditional method.

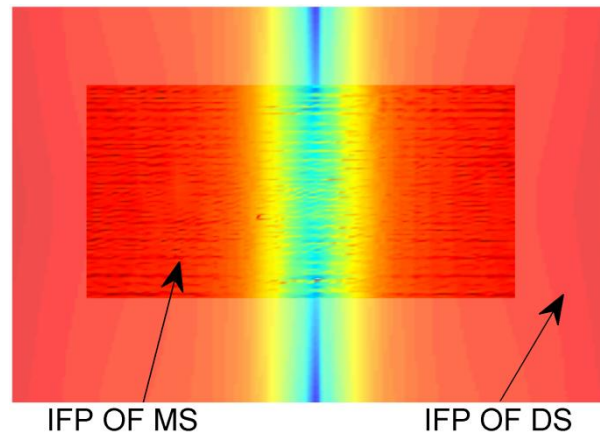


Figure 4.24 IFP registration of the MS on the DS of the streetlight lens

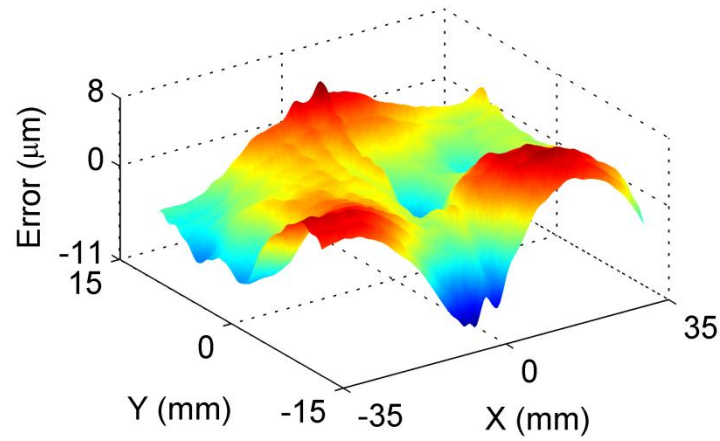


Figure 4.25 Evaluated form error of the MS of the streetlight lens

Table 4.5 Form Characterization results comparison

<i>Surface parameters (μm)</i>	Reconstructed surface	Measured data
Surface Profile error, S_t	18.8	19.7
Root mean square error, S_q	6.39	6.42

4.5 Summary

Traditional form characterization methods for freeform surfaces depend on

embedded coordinate systems, which present barriers for surface form error evaluation by the comparison between the designed model and the measured surface. Although extensive research work has been conducted in recent years on the development of more effective and accurate freeform surface matching techniques, there are still some problems, such as the uncertainty due to the dependency of the methods on the coordinate frame and the geometry of the surface being characterized.

This chapter presents an invariant feature-based pattern analysis method (IFPAM) for generalized characterization of ultra-precision freeform surfaces with sub-micrometer form accuracy. The IFPAM makes use of orientation invariant surface features to represent the geometry of the freeform surface with an invariant feature pattern (IFP). Surface corresponding searching is performed by IFP registration based on the Fourier-Mellin transforms and phase correlation. The technological advantages and merits of IFPFAM are summarized as follows:

- (i) It is a generalized method for form characterization of different types of surfaces. The IFP is generated by mapping the intrinsic surface feature of a freeform surface, such as Gaussian curvature, into a 2D pattern. Hence, the IFP is a generalized surface feature that is independent of the type of surface being characterized.
- (ii) It is robust to the initial position and the proportion of the measured surface relative to the design surface. The IFPAM represents the surface geometry by an orientation independent IFP, which makes the IFPAM free from the coordinate system. This addresses the deficiencies of traditional distance error based approaches, such as least squares or minimum zoom methods.
- (iii) The IFPAM is computationally efficient. The method does not involve iteration and high quality corresponding pairs can be established in one step in terms of

IFP registration. Hence, fast surface matching can be performed even if a large number of measured points are involved.

- (iv) The results of computer simulation demonstrate the systematic error of the IFPAM in freeform surface matching and form error evaluation in a noise free ideal case. The results show that systematic error of the IFPAM is found at sub-nanometre level, which implies that IFPAM is capable of realizing precise matching between the measured and the designed surfaces for characterizing the form error of ultra-precision freeform surfaces with sub-micrometre form accuracy.

A critical challenge of IFPAM is that the calculation of the intrinsic surface features from a machined freeform surface is susceptible to the sampling strategy and the measurement noise and outliers associated in the measured data. This has been successfully addressed by incorporating the bidirectional curve network based sampling strategy combined with a robust surface fitting and reconstruction algorithm in the IFPAM, as presented in Chapter 3. This not only provides an important means for determining the appropriate number of measured data points for fully representing the geometry of the machined surface, but also for ensuring the accuracy of the extraction of the invariant feature pattern from the measured data points.

Chapter 5

Uncertainty Analysis in Form Characterization of Ultra-precision Freeform Surfaces

Although the invariant feature-based pattern analysis method (IFPAM), presented in Chapter 4, provides an important means for the generalized form characterization of ultra-precision freeform surfaces, the process of the measurement and form characterization inevitably combines with errors that lead to uncertainty of the measurement results. Uncertainty associated in the measurement and form characterization of freeform surfaces comes from many sources, such as uncorrected systematic and random error of the measurement instrument, inadequate sampling, and errors imposed during surface matching and comparison. As a result, uncertainty analysis is an indispensable part of form characterization, which assesses the accuracy and reliability of the characterization results.

In this Chapter, a task specific uncertainty analysis model is presented, in which the associated uncertainty in the form characterization results is estimated when the measured data is extracted from a specific surface using a specific sampling strategy. Three factors are identified and considered in the uncertainty analysis model: measurement error, surface form error, and sample size. Rather than relying on intuition, the present study is more focused on the mathematically modeling of the relationship among the influential factors and the resulted uncertainty, so that a prediction can be made to estimate the uncertainty associated in the result of the form characterization for a specific freeform surface measurement.

5.1 Establishment of Uncertainty Analysis Model

Modern uncertainty analysis models for geometric measurement can be classified into three groups. The first group is theoretical estimation (statistical approach). One potential benefit of theoretical estimation of the uncertainty is that it avoids the need for a large number of measurements and allows the operator to express it as an exact mathematical expression. Any theoretical model contains two basic components: uncertainty of the coordinates of each measurement points, and a method to propagate the point coordinate uncertainty into uncertainty of the substitute geometry. The second group is computer-based simulation of measurement process. Monte Carlo simulation can be used to estimate uncertainty by repeatedly calculating the parameter in question with different random errors in the input data and examining the frequency distribution of the results (JCGM, 2008). The third group is experimental method. Different artifacts, user-defined parts or geometrical gauges, can be used to calibrate and determine uncertainty in the measurement results. In the present study, a Monte Carlo method based uncertainty analysis model is developed.

5.1.1 Error source identification and quantification

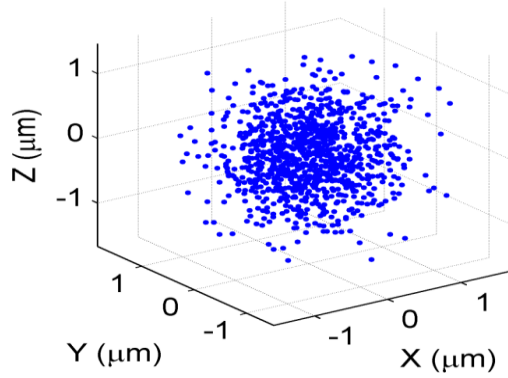
The uncertainty in the form characterization of freeform surface can be analyzed from three aspects. Firstly, the measurement errors associated in the measured data points propagate to the characterization results. Normally, the uncertainty caused by the random measurement error can generally be averaged by taking a large number of data points. Secondly, insufficient sampling makes the measured surface failing to fully describe the geometry of the workpiece and thus leads to uncertainty to the results of the form characterization. Intuitively, the uncertainty caused by sampling

can also be reduced by taking a large number of data points from the workpiece. However, the trade-off is the cost of more measurement time, especially for contact type measurement instruments such as coordinate measurement machines (CMMs). Hence, the key issue is how to determine the minimum sample size for a measurement so that the uncertainty of the sampling strategy is in a prescribed tolerance.

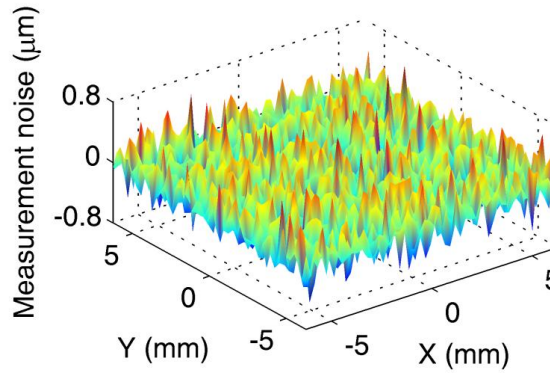
The third part is the uncertainty due to inaccurate surface matching. The measured surface always contains measurement error and form error of the workpiece so that it does not perfectly match with the design surface. Hence, the estimated coordinate transformation matrix would contain errors that cause the coordinate systems misalignment between the measured surface and design surface. Typically, the larger the deviation between the measured surface and the design surface is, the larger is the uncertainty of the surface matching results. However, rather than intuition, uncertainty analysis is required to mathematically model such relationships in order to estimate the uncertainty associated in a surface matching result with respect to the magnitude of the form deviation of measured surface.

The error of a measurement instrument is inherent and it is contained in each measured data. Since much effort has been applied to calibrate and compensate the systematic errors of the measurement instruments in recent years (Wilhelm et al, 2001), the error of a well calibrated measurement instrument can be modeled as multivariable random noise, so that the measurement error in an arbitrary measured point can be regarded as a sample from the distribution. In this study, the random error of the measuring instrument is described using a simple model, known as the single parameter model, proposed by Philips et al. (1995). The single parameter model considers the random error of the measured points within the workzone of the measuring instrument has the same uncertainty and these points are independent of one another. A typical measurement noise generated by multivariable Gaussian

distribution (MGD) is shown in Figure 5.1. Figure 5.1a shows some random points drawn from MGD, and a 3D error map of a measured surface due to the MGD noise is shown in Figure 5.1b.



(a) Points drawn from multivariable Gaussian noise



(b) 3D error map due to measurement noise

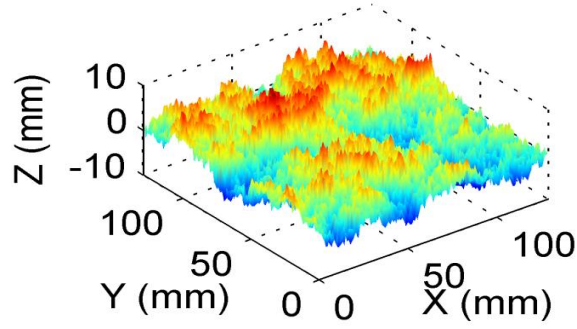
Figure 5.1 Typical measurement noise generated by multivariable Gaussian noise

The form error of a machined surface is surface variation caused by imperfect manufacturing and is dominated by the error of the relative motion between cutting tool and machining workpiece. Suppose the relative positioning error of cutting tool and machining workpiece is random noise with Gaussian distribution, the form error can be considered to be a signal generated by random walks which increments has Gaussian distribution. Hence, fractional Brownian motion (fBm) is used in this study to generate fractal surface to simulate the form error of machined surface. fBm is a

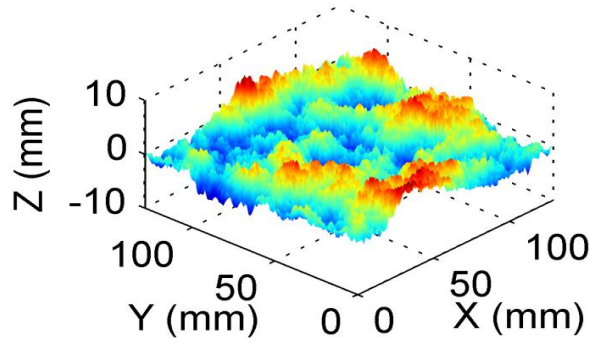
continuous-time Gaussian process $B^H(t)$ on $[0, T]$, which has expectation zero for all t in $[0, T]$ as the following covariance function (Mandelbrot, 1968):

$$E[B^H(x)B^H(y)] = \frac{1}{2}(|x|^{2H} + |y|^{2H} - |x-y|^{2H}) \quad (5.1)$$

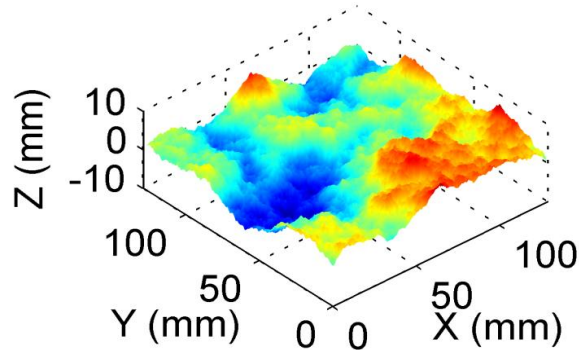
where $x, y \in [0, T]$; H is a real number in $[0, 1]$, called the Hurst index which can be used to control the ‘roughness’ of generated signal. Figure 5.2 shows the surface generated with the Hurst index 0.2, 0.5 and 0.8.



(a) $H=0.2$



(b) $H=0.5$



(c) $H=0.8$

Figure 5.2 Surface generated by fBm with different Hurst index

Figure 5.2 shows that the generated surface becomes smooth with the increase of the Hurst index and it exhibits strong low-frequency component and has irregular behavior when the H is 0.8. It is well match with the properties of the form error of a machined surface. Hence, fractional Brownian motion with Hurst index 0.8 is suitable to simulate the random form error for the machined surface. The magnitude of the generated random form error is controlled by the standard deviation (Std) of the Gaussian noise. Figure 5.3 shows the peak-to-valley height (PV) of three sets of random form errors which are generated by fBm with Std $0.5\ \mu\text{m}$, $1.5\ \mu\text{m}$ and $2.5\ \mu\text{m}$ respectively.

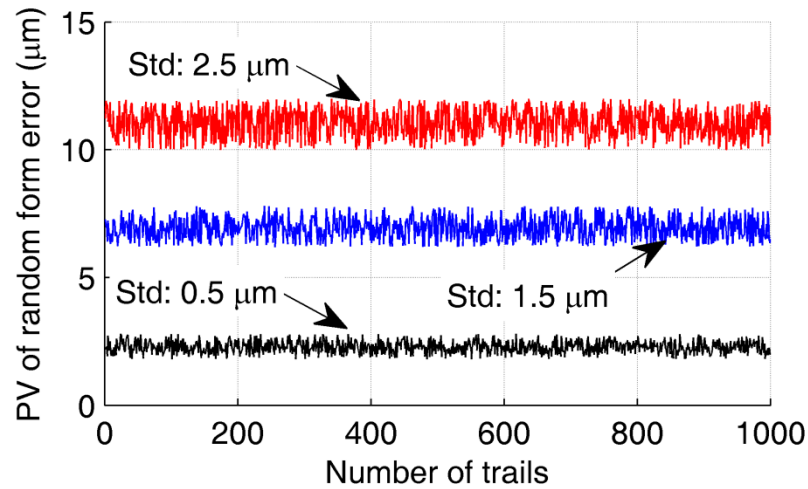


Figure 5.3 PV of random form errors by fBm with different standard deviations

It can be seen from Figure 5.3 that the PV of the generated form errors fluctuates in a certain range. For instance, the PV of generated form errors fluctuates in a range of $[6.2, 7.8]\ \mu\text{m}$ when Std of the fBm is $1.5\ \mu\text{m}$. That is, if fBm with Std $1.5\ \mu\text{m}$ is used as random variable to describe the form error, the generated random form errors may cover all possible form errors of a machined surface within the range of $[6.2, 7.8]\ \mu\text{m}$ if the number of the trials are sufficiently large. Hence, the uncertainty of the surface matching due to form error within the range of $[6.2, 7.8]\ \mu\text{m}$ can be estimated

based on Monte Carlo method by using fBm with Std 1.5 μm as form error variable.

5.1.2 Uncertainty propagation and evaluation

Based on knowledge of the measurement and characterization of ultra-precision freeform surfaces, a mathematical uncertainty evaluation model is established. In the measurement of freeform surfaces, the form of a machined workpiece is characterized by comparing the measured surface with the corresponding design surface and can be given by:

$$Ef_{eva} = f(DS, Q) \quad (5.2)$$

where $Q = [Q_1 \dots Q_N]$ represents N measured points; Ef_{eva} represents evaluated surface form error; DS represents the design surface; $f(\cdot)$ is a function to determine the deviation of Q from DS . The measured points Q can be represented by:

$$Q = T_{(m)}(P + \bar{n}Ef_{real}) + E_{meas} \quad (5.3)$$

where P are the corresponding points of Q on DS ; $\bar{n}$ is normal vectors of DS at P ; Ef_{real} is the form error of measured surface at Q ; E_{meas} is the measurement error at Q ; $T_{(m)}$ is coordinate transformation matrix determined by spatial parameters m to indicate the misalignment of the coordinate system between the measured surface and the design surface. Eq. (5.2) can be solved by the IFPAM for generalized form characterization of ultra-precision freeform surfaces as discussed in Chapter 4.

In accordance with the “Guide to the Expression of Uncertainty in Measurement” (GUM) concept (ISO, 1995), the measurement process is expressed as a model

equation while the affecting factors are expressed by means of appropriate probability distributions functions (PDF). Based on Eq. (5.2) and Eq. (5.3), the model equation for uncertainty analysis can be expressed by:

$$Y = f \left(\underset{6}{\mathbf{DS}}, \underset{N}{\mathbf{m}}, \underset{N}{\mathbf{E}f_{real}}, \underset{3N}{\mathbf{E}_{meas}} \right) \quad (5.4)$$

Eq. (5.4) demonstrates that there are total $4N+6$ factors affecting the form characterization results for a given $\mathbf{DS}$. If an affecting factor is denoted as x_i , the expectation X_i and standard deviation of this factor is given by (ISO, 1995):

$$X_i = \int_{-\infty}^{+\infty} g_{x_i}(\xi_i) \xi_i d\xi_i \quad (5.5)$$

$$u_{x_i} = \left(\int_{-\infty}^{+\infty} g_{x_i}(\xi_i) (\xi_i - X_i)^2 d\xi_i \right)^{0.5} \quad (5.6)$$

where g_{x_i} is the PDF of x_i and ξ_i is the possible value of x_i . Then the PDF of the Y is given by:

$$g_y(\eta) = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} g_{x_1} \dots g_{x_{4N+6}}(\xi_1 \dots \xi_{4N+6}) (\eta - f(\xi_1 \dots \xi_{4N+6})) d\xi_1 \dots \xi_{4N+6} \quad (5.7)$$

where η is an estimated value of Y . Hence, the expectation $E(Y)$ and the standard deviation $u(Y)$ can be given by:

$$E(Y) = \int_{-\infty}^{+\infty} g_y(\eta) \eta d\eta \quad (5.8)$$

$$u(Y) = \left(\int_{-\infty}^{+\infty} g_y(\eta) (\eta - E(Y))^2 d\eta \right)^{0.5} \quad (5.9)$$

Due to the high non-linearity of the form characterization of freeform surfaces, a Monte Carlo method is used to evaluate the uncertainty based on the latest GUM

(JCGM, 2008). In Eq. (5.4), E_{meas} is modeled by multivariable Gaussian noise and $E_{f_{real}}$ is generated by fractional Brownian motion as given in Section 5.1.1.

5.1.3 Task specific uncertainty analysis model

The uncertainty in the form characterization of freeform surfaces may be caused by three factors, including the form error of the workpiece, the measurement error associated in the measured data, and the sample size. However, due to the geometric variety of freeform surfaces, the propagation of the errors during form characterization may also be different from the geometry of the surface being characterized and the sampling strategy being utilized. This makes it difficult to establish a universal model to analyze the uncertainty for all types of freeform surfaces. As a result, a task specific uncertainty analysis method is proposed in the present study. That is, the associated uncertainty in characterization results is estimated when the measured data is extracted from a specific surface with specific sampling strategy.

Figure 5.4 shows a schematic diagram of the task specific uncertainty analysis model which is basically composed of two processes, i.e. form characterization and uncertainty evaluation. The form characterization process starts from inputting a design surface (DS), and a random form error is added on the DS to simulate the machined surface. A certain number of points are then sampled from the machined surface, with the guidance of a utilized sampling strategy, and added by a random measurement error to simulate the measurement instrument error. The measured surface (MS) is generated by moving the sampled data (SD) to an arbitrary position to indicate the misalignment of the embedded coordinate system of MS from that of DS.

Least square based method is then used to perform form characterization of the machined surface based on the generated MS. In this process, the uncertainty due to the adopted sampling strategy is evaluated by comparing the deviation of the SD from DS with the added form error. The uncertainty due to the least square based surface matching is evaluated by comparing the deviation of the MS from the DS with the deviation of the SD from the DS. Hence, the whole uncertainty in the form characterization process can be evaluated by both considering the part of uncertainty caused by insufficient sampling, and the part caused by misalignment of coordinate systems of the MS and the DS due to inaccurate surface matching.

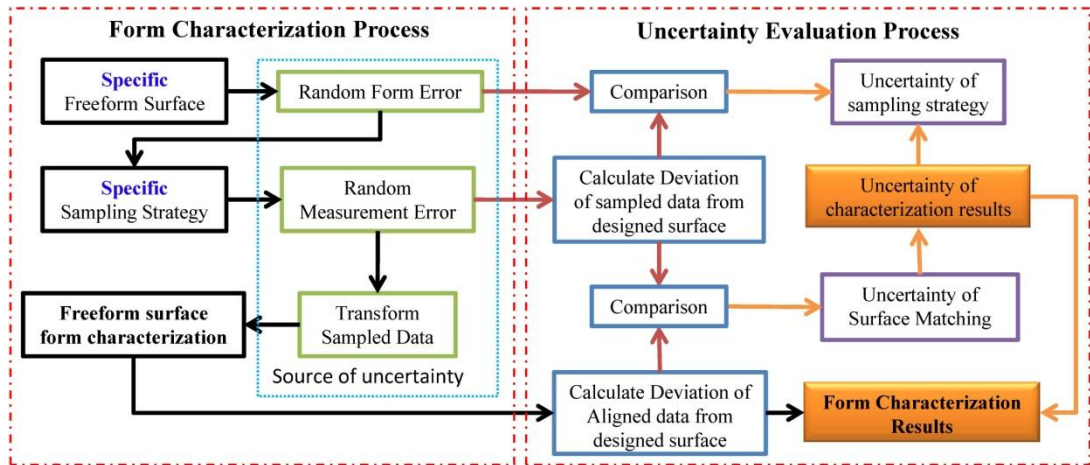


Figure 5.4 Schematic diagram of task specific uncertainty analysis method

5.2 Computer Simulation

An F-theta surface for a laser scanner lens is designed and given as follows:

$$\begin{cases} z = a x^2 + b^4 x^2 + c^2 y^2 \\ a = -1 / 250, b = 1 / 920, c = 0 \\ -20 \leq x \leq 20 \text{ mm}, -25 \leq y \leq 7.5 \end{cases} \quad (5.10)$$

The uncertainty of the IFPAM presented in Chapter 4 is studied based on the designed F-theta surface with a uniform sampling strategy. Although the uniform sampling

strategy may not be the best sampling method, it is widely used in practice.

In the following simulation, a certainty number of points are uniformly sampled from the design surface, with dimensions $-12 \leq x \leq 8$ mm and $-4.5 \leq y \leq 3$ mm, and are used to simulate the measured data points based on the transformation given by Eq. (5.3). The six spatial parameters $\mathbf{m} = [rx, ry, rz, tx, ty, tz]$ in coordinate transformation matrix $T_{(m)}$ are randomly selected to indicate the coordinate frame misalignment between the measured surface and the design surface. The IFPAM is then used to characterize the measured surface. To further improve the accuracy of the results of the form characterization, the correspondence pairs established by invariant feature pattern registration is refined by orthogonal projection as given in Section 4.3.2.

The uncertainty is analyzed as follows:

- (i) The systematic error of the implemented algorithm is studied in a noise free ideal case;
- (ii) The uncertainty due to inaccurate surface matching and the uncertainty due to inadequate sampling strategy are studied with respect to different magnitudes of form error, measurement noise and sample size;
- (iii) The confidence interval of the results of the form characterization is evaluated and tested.

Two surface parameters, i.e. peak-to-valley height S_t and root mean square error S_q , are used to characterize the form error of machined surface and are determined as follows:

$$S_t = |\max(|\mathbf{P}_i - \mathbf{TQ}_i|) - \min(|\mathbf{P}_i - \mathbf{TQ}_i|)| \quad (5.11)$$

$$S_q = \sqrt{\frac{1}{N} \sum_{i=1}^N |\mathbf{P}_i - \mathbf{TQ}|_i^2} \quad (5.12)$$

where N is total number of sampled points.

5.2.1 Systematic error in error free ideal case

Since the IFPAM is an optimization process, the implemented algorithm always contains systematic error which may affect the accuracy of the subsequent uncertainty analysis. To clarify this effect, the systematic error of the implemented algorithm is evaluated in an error free ideal case, i.e. with setting of random form error and measurement noise to be zero in Eq. (5.3). Since the measured surface is directly sampled from the design surface, the form error should be zero. Hence, the evaluated form error is considered to be the systematic error of the implemented algorithm.

The magnitude of the systematic error is determined by the terminate threshold of the surface matching optimization process. In the present study, the terminate threshold ε_{rf} (see Figure 4.13) is set to be 10^{-6} mm. Figure 5.5 shows the evaluated 3D form error of the measured surface. It is worthy of note that the peak-to-valley height S_t of the evaluated form error is at the level of 10^{-8} mm, which is small enough to ignore the effect of this error on the subsequent uncertainty analysis.

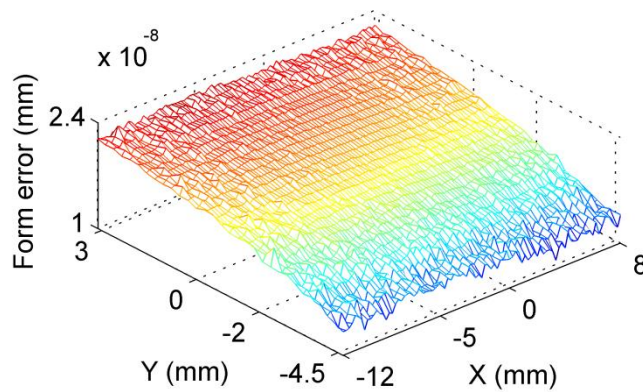


Figure 5.5 Uncertainty of form characterization result in ideal case

5.2.2 Uncertainty in freeform surface matching

The uncertainty of the IFP based surface matching is analyzed with the consideration of three factors including the measurement error associated in the measured data, the form error of the workpiece, and the number and the distribution of measured data points.

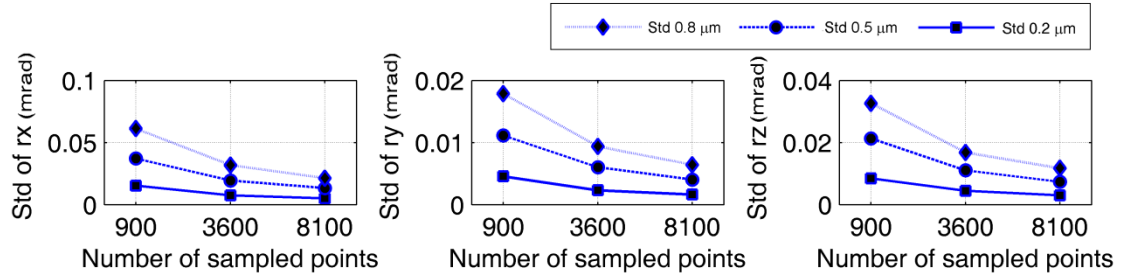
5.2.2.1 Effect of measurement noise to freeform surface matching

The uncertainty analysis is conducted with different magnitudes of measurement noise and different number of sampled points. A total 9 cases are studied as given in Table 5.1. In each case study, 1500 Monte Carlo trials are made based on 3-dimensional Gaussian noise with given standard deviation in order to evaluate the uncertainty of the surface matching results due to measurement noise.

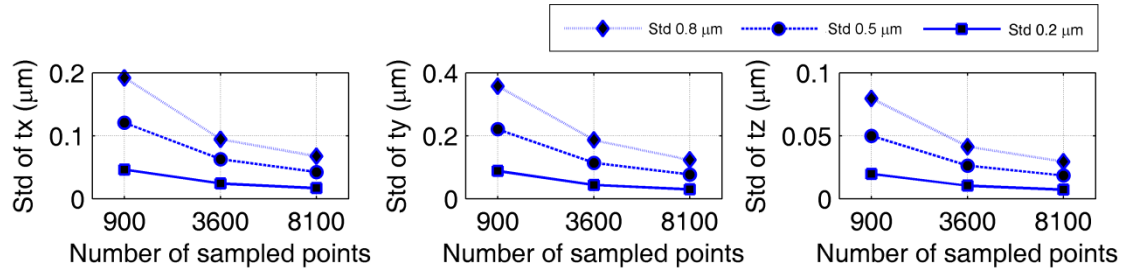
Table 5.1 Magnitudes of added measurement error and number of sampled points

<i>Measurement error</i>	<i>Number of sample points</i>		
Std: 0.2 μm	30×30	60×60	90×90
Std: 0.5 μm	30×30	60×60	90×90
Std: 0.8 μm	30×30	60×60	90×90

Figure 5.6 shows the standard deviation (Std) of evaluated six spatial parameters with different magnitudes of measurement noise and different number of sample points. It is clearly found from the results that the uncertainties of all six parameters decrease along with increasing number of sample points. This is a good match to the intuition that the effect of the random measurement error to the surface matching can generally be averaged by taking a large number of data points.



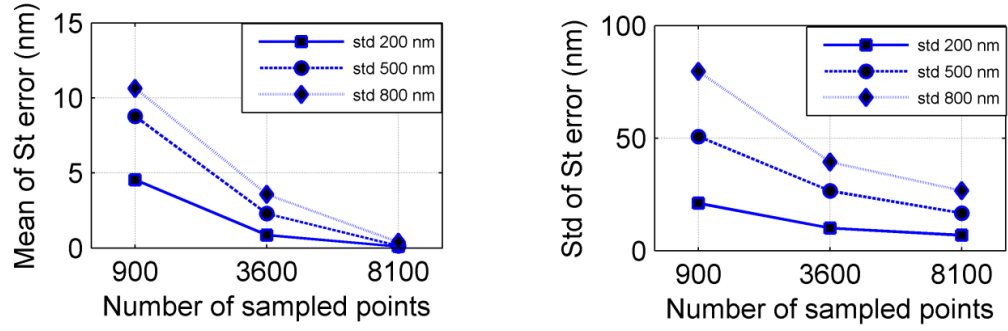
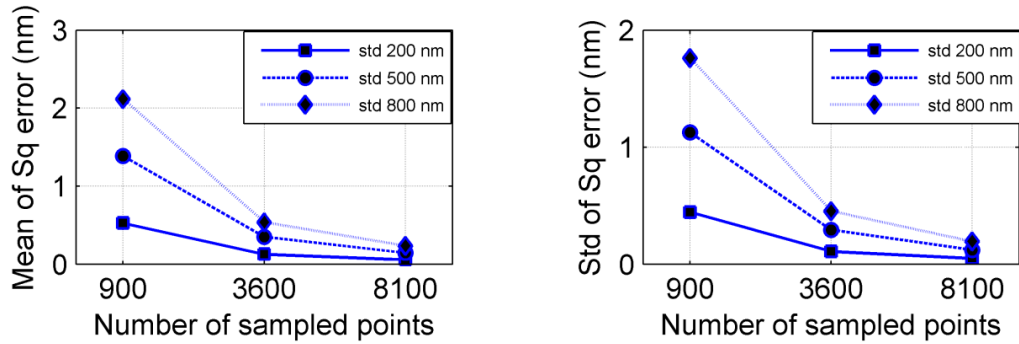
(a) Std of estimated rotation angles



(b) Std of estimated translation offsets

Figure 5.6 Uncertainty of evaluated spatial parameters due to measurement noise

Figures 5.7 and Figure 5.8 show the bias and Std of the estimated surface parameters due to inaccurate surface matching. It is found that the bias and the uncertainty of the evaluated surface parameters (both S_t and S_q) decreases dramatically with an increasing number of sampled points. Although a large magnitude of noise causes a large error to the surface parameters, the difference also decreases with an increasing number of sampled points. It is interesting to note from the simulation results that for the designed F-theta surface, the uncertainty of the surface matching results (S_t and S_q) due to measurement noise, of which Std is in the sub-micrometre range and is smaller than 50 nm if more than 3600 points are uniformly sampled. The proposed form characterization method shows its low sensitivity to the random measurement noise.

Figure 5.7 Bias and uncertainty of evaluated S_t Figure 5.8 Bias and uncertainty of evaluated S_q

5.2.2.2 Effect of surface form error to freeform surface matching

In the present study, fBm is used as a random variable to model the form errors. It was noted in Section 5.1.1 that the magnitude of the random form errors is generated by fBm with a given standard deviation fluctuating in a certain range. If the number of trials is sufficiently large, the uncertainty of the surface matching due to form errors with magnitude in that range can be estimated by the Monte Carlo method. To establish the relationship between the uncertainty of the surface matching and the magnitude of the form errors for the given F-theta surface, fBm is used to generate 5 sets of random form errors with different magnitudes. Since the sample size may also affect the surface matching results, surface matching with different sample sizes are also studied. Table 5.2 shows 25 case studies with different magnitudes of form error

and different sample sizes. The magnitude of the form error is represented by S_t .

The number of Monte Carlo trials in each case study is determined adaptively such that the variance of Std of the evaluated surface parameters are in a prescribed threshold. Figure 5.9 shows an example of a case study (30×10 points, $St\ 10\mu m$). As shown in Figure 5.9, it is found that the variation of the Std of S_t and S_q are smaller than 10 nm and 5nm when the number of trials increases to 1500. Hence, for the case study, 1500 trials are enough for accurately estimating the uncertainty of evaluated surface parameters.

Table 5.2 Magnitude of random form error and number of sampled points

$Std\ of\ fBm$	$S_t\ of\ form\ error$	<i>Number of sample points</i>				
0.4 μm	$1.5 \leq St \leq 2.5\ \mu m$	30 $\times$ 10	60 $\times$ 20	90 $\times$ 30	120 $\times$ 40	150 $\times$ 50
0.9 μm	$3.5 \leq St \leq 4.5\ \mu m$	30 $\times$ 10	60 $\times$ 20	90 $\times$ 30	120 $\times$ 40	150 $\times$ 50
1.3 μm	$5.5 \leq St \leq 6.5\ \mu m$	30 $\times$ 10	60 $\times$ 20	90 $\times$ 30	120 $\times$ 40	150 $\times$ 50
1.8 μm	$7.5 \leq St \leq 8.5\ \mu m$	30 $\times$ 10	60 $\times$ 20	90 $\times$ 30	120 $\times$ 40	150 $\times$ 50
2.2 μm	$9.5 \leq St \leq 10.5\ \mu m$	30 $\times$ 10	60 $\times$ 20	90 $\times$ 30	120 $\times$ 40	150 $\times$ 50

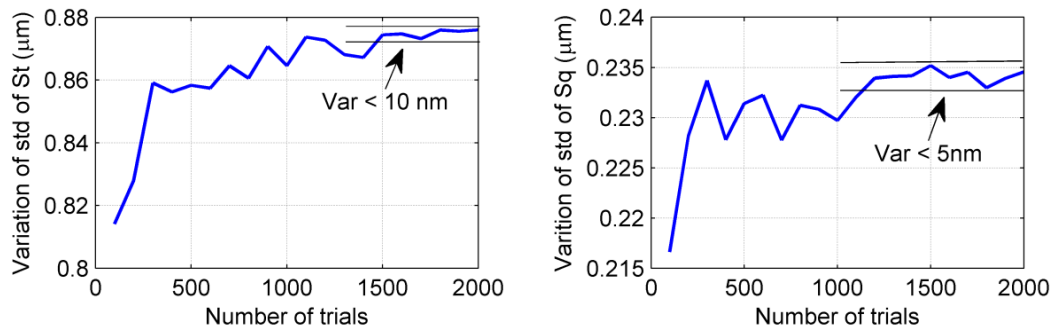


Figure 5.9 Variation in Std of surface parameters related to number of trials

First of all, the robustness and accuracy of the invariant feature pattern (IFP)

registration against different noise levels is studied. During the simulation, all of the IFP of the measured surface and the design surface are generated with spacing 0.1 mm. Peak signal-to-noise ratio (PSNR) and signal-to-noise ratio (SNR) are used to characterize the quality of the generated IFP of measured surface and are determined as follows (Netravali, 1995):

$$PSNR = 20 \log_{10} \left(\frac{\max(I)}{RMSE} \right), \quad SNR = 20 \log_{10} \left(\frac{RMS}{RMSE} \right) \quad (5.13)$$

where

$$RMSE = \sqrt{\sum_{i=1}^n \sum_{j=1}^m (I_n(i, j) - I(i, j))^2 / mn}, \quad RMS = \sqrt{\sum_{i=1}^n \sum_{j=1}^m (I(i, j))^2 / mn}$$

I is the ideal IFP of the MS generated without error; I_n is the IFP of the MS generated with error. The mean of the results of each group are shown in Table 5.3.

Intuitively, both PSNR and SNR decrease with the increase of the scale of the error. However, it is interesting to note from the results that the generated IFP has high quality (SNR>36dB) when the PV of the error is smaller than 5 μm , and are at an acceptable level (SNR>32dB) when the PV of the error is at a level up to 10 μm . This is due to the fact that the high frequency part of the added error is well smoothed in the surface reconstruction process, and the error of the generated IFP is mostly contributed by the low frequency part of the added error, which does not seriously affect the calculation of the curvature. The mean of the IFP registration results are also shown in Table 5.3. By comparing the results with those in an ideal case, it is interesting to note that the accuracy of IFP registration is smaller than 0.1 pixel in translation and 0.05 degree in rotation.

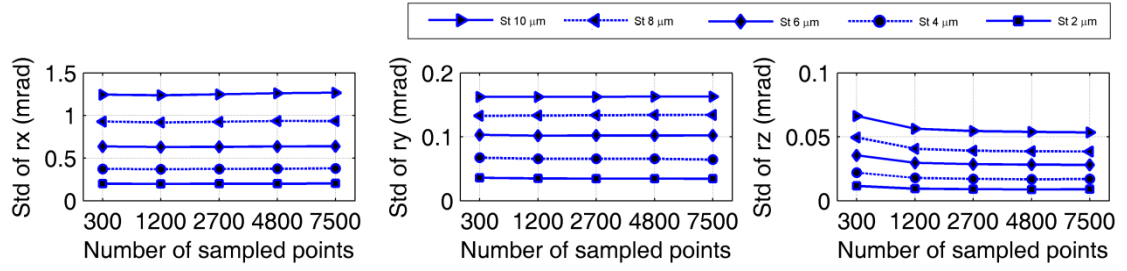
Table 5.3 IFP registration results with different scales of added error

Added Error (μm)	PSNR (dB)	SNR (dB)	Ind_X (pixel)	Ind_Y (pixel)	Angle (degree)
0	Infinite	Infinite	43.55	43.55	0.5
1.5<PV<2.5	41.2	39.1	43.54	43.54	0.46
3.5<PV<4.5	39.7	37.3	43.54	43.54	0.45
5.5<PV<6.5	36.4	35.4	43.53	43.54	0.45
7.5<PV<8.5	35.0	34.6	43.53	43.53	0.46
9.5<PV<10.5	34.8	33.7	43.53	43.53	0.45

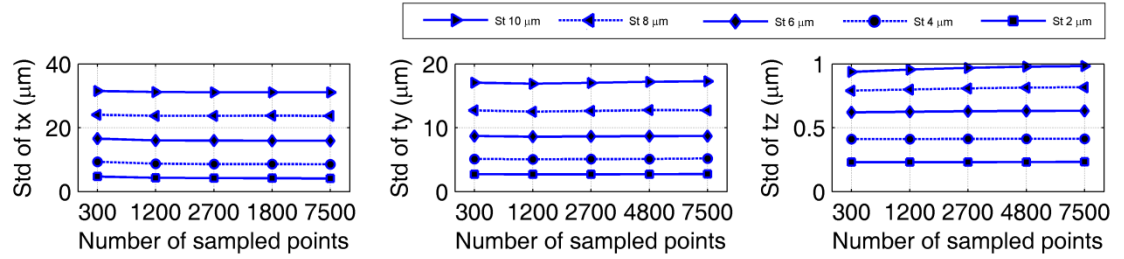
Note: Ind_X , Ind_Y are the index of the centre of the IFP of MS on the IFP of DS after registration.

After IFP registration, the correspondence pairs are established and refined so that the coordinate transformation matrix is evaluated and hence the form error of the measured surface. Figure 5.10 shows the Std of the estimated six spatial parameters due to different magnitudes of form errors and different sample sizes. The bias and standard deviation of the estimated surface parameters due to inaccurate surface matching are shown in Figures 5.11 and Figure 5.12.

The results show that the uncertainty of the surface matching due to the form error is insensitive to the sample size. This may be explained by the utilization of the uniform sampling strategy. With the uniform sampling strategy, points are sampled uniformly over the surface and the global geometry of the surface can be captured with relatively few points. The increase of the sample points may contribute little to further improve the quality of the captured global geometry of the surface.



(a) Uncertainty of the estimated rotation angles



(b) Uncertainty of the estimated translation offsets

Figure 5.10 The uncertainty of the six spatial parameters due to surface form error

It is found from Figure 5.10 that the uncertainty of the estimated translation offsets along X and Y directions are much larger than that for the other parameters. This may be explained by the criteria of the surface matching and the flatness of the given F-theta surface. As discussed in Section 4.3.2, the correspondence refinement in the surface matching is performed by minimizing the square sum of the orthogonal distance between the measured points and the design surface. When the surface is relatively flat, similar to the designed F-theta surface in the present study, the offsets of the coordinate system of the measured surface along X and Y directions contribute relatively little to the orthogonal distance between the points and the design surface.

It is also found from Figure 5.11 and Figure 5.12 that both bias and uncertainty of the estimated surface parameters increases with increasing magnitude of the form error. Although the increment becomes smaller, the results show that the accuracy of the surface matching results is still adversely affected by the form error. This is due to the fact that the established correspondence between the measured surface and the

design surface becomes poor with the increasing magnitude of the form error. Based on the simulation results, it can be estimated for the given F-theta surface that the uncertainty of the estimated S_t and S_q are around 10% and 3% of the S_t of the form error, respectively.

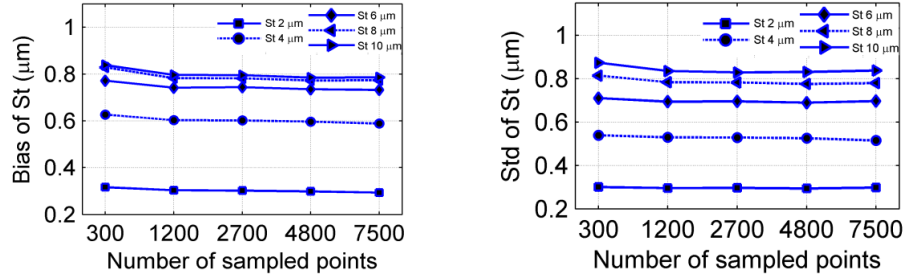


Figure 5.11 Bias and uncertainty of S_t due to inaccurate surface matching

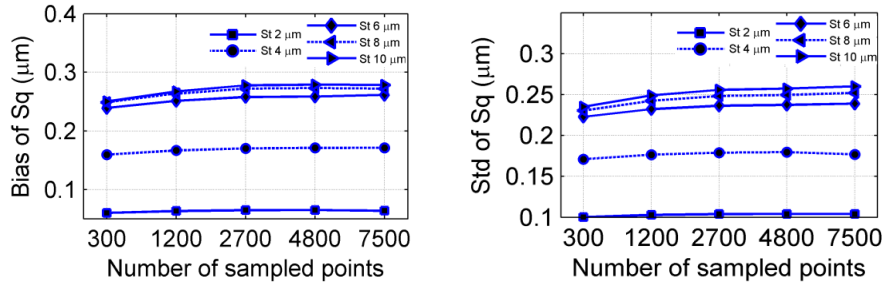


Figure 5.12 Bias and uncertainty of S_q due to inaccurate surface matching

5.2.3 Uncertainty in freeform surface sampling

Sampling strategy is considered as one of the major contributors to the measurement of uncertainty. This is due to the fact that the form error of the machined surface is characterized by a finite number of points on the surface. Based on the same idea as described in Section 5.2.1, the uncertainty of the utilized sampling strategy due to the form errors of which magnitude in a certain range can be estimated by the Monte Carlo method. The same 5 sets of random form errors given in Table 5.2 are used in the present study. Figure 5.13 shows the Std of the surface parameters due to

the utilized sampling strategy.

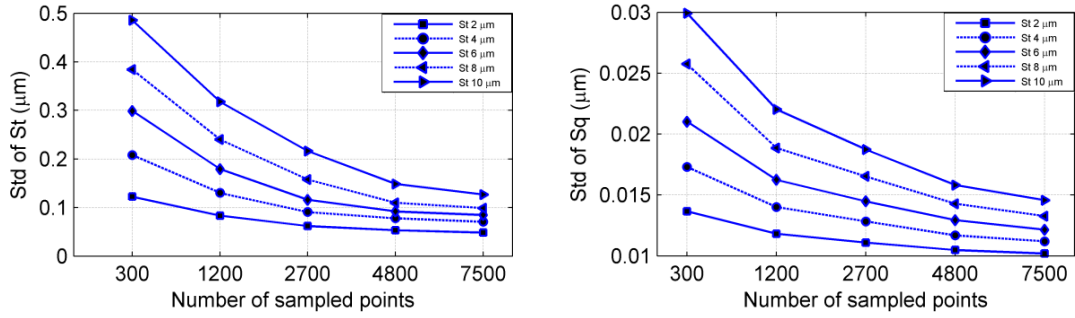


Figure 5.13 Uncertainty of the surface parameters with respect to sampling size

It is seen from the results that the uncertainty of the evaluated surface parameters decreases with increasing number of the sample points, and decreases with increasing magnitude of the surface form error. Although the results are intuition, rather than the trend, it is important to evaluate the extract uncertainty of a surface parameter for a freeform surface measurement. It is found from the results that the uncertainty of S_t , for the designed F-theta surface due to the sampling, is around 1% of the form error when the number of sample points is 7500. However, the trade-off is the cost of more measurement time. As shown in Figure 5.13, 2700 points are required if the requirement of the sampling accuracy is 2%.

5.2.4 Evaluation of confidence interval

Based on the results obtained in the previous sections, the confidence interval of the results of the form characterization can be given for the designed F-theta surface. Figure 5.14 shows regions covering a 95% confidence interval of S_t and S_q when 120×40 points are uniformly sampled from a machined F-theta surface, which may have form errors ranging from 0 to 10 μm . This confidence region can be used to estimate the uncertainty of the results of the form characterization with respect to the

magnitude of the surface form error being characterized

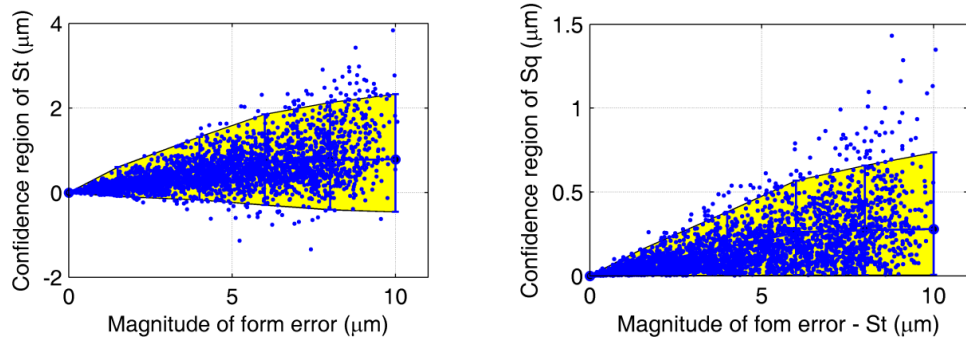


Figure 5.14 95% Confidence interval of evaluated surface parameters

To verify the validity of the estimated confidence regions, a total of 2922 trials are generated based on Eq. (5.2). E_{form} is randomly generated by fBm with Std in the range of $[0, 1.3] \mu\text{m}$; E_{meas} is randomly generated by MGN with Std in the range of $[0.2, 0.8] \mu\text{m}$; $\mathbf{m}$ is set to be $[1, 1, 1, 1, 1, 1]$. For each trial, the sampled points are characterized by the form characterization method presented in Chapter 4. Then the error of the results of the form characterization is pointed on corresponding region with respect to the magnitude of the surface form error. For the evaluated S_t , a total of 151 trials are located outside of the confidence region and the validity of the confidence region is 94.8%. For the evaluated S_q , a total of 137 trials are located outside of the confidence region and the validity of the confidence region is 95.3%. It should be noted that the results match well with the given confidence level of the region.

5.3 Experimental Study

The developed task specific uncertainty analysis model has been incorporated

into the invariant feature-based pattern analysis method (IFPAM), and hence into the generalized form characterization system presented in Chapter 4. To further evaluate the capability of this system, a series of experiments have been conducted on different types of freeform surfaces. A case study on a machined bifocal surface is presented, in which Gaussian curvature is used as the intrinsic surface feature to generate the invariant feature pattern.

An ultra-precision freeform mould insert of a bifocal optical lens is produced by a 7-axis ultra-precision polishing machine (Zeeko IRP-200). The produced workpiece is measured by Talysurf PGI 1240. The measured surface is characterized by the generalized form characterization system built based on the IFPAM. First of all, the measured surface is sampled with the guidance of bidirectional sampling strategy, and the measured discrete data points are reconstructed by the developed robust surface fitting algorithm presented in Section 3.2. The machined surface is then characterized by IFPAM based on the measured surface, i.e. the reconstructed surface.

Figure 5.15 and Figure 5.16 show the initial positions of the design surface (DS) and the measured surface (MS). It is emphasized that the initial positions of the DS and the MS are quite far away from each other (which can be inferred from the coordinates of two surfaces from the figures), such that a rough matching process is required if the conventional least square based form characterization methods are undertaken.

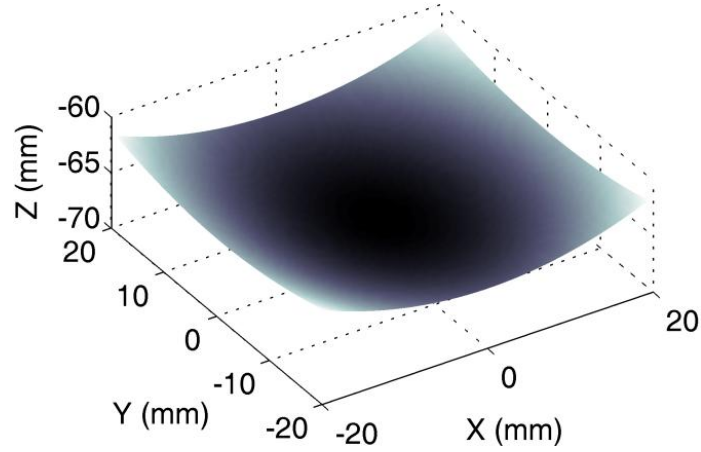


Figure 5.15 Designed bifocal surface

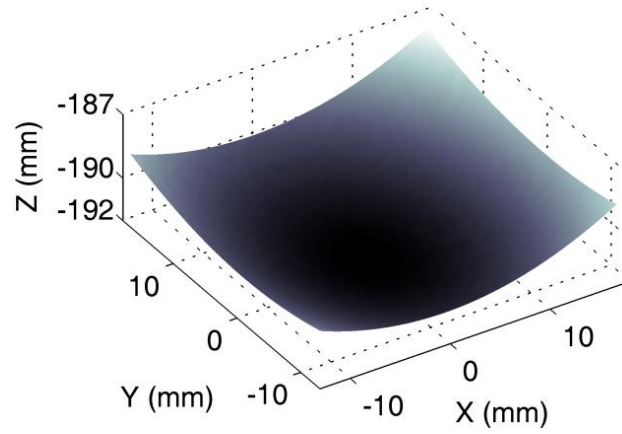


Figure 5.16 Measured bifocal surface

The invariant feature pattern (IFP) based surface matching and form error evaluation process is shown in Figure 5.17. It starts from generating the IFP of the MS and the DS with a spacing of 0.5 mm along $\bar{X}$ and $\bar{Y}$ axes of the corresponding coordinate frame, as shown in Figure 5.17a and Figure 5.17b, respectively. Then the phase correlation method is used to register the IFP of the MS on the IFP of the DS as shown in Figure 5.17c. Corresponding pairs are then established and are used to evaluate the form error of the MS as shown in Figure 5.17d. From the results, S_t of the form error is found to be 2.02 μm and S_q of the form error is found to be 0.44

μm .

The reliability of the results of the form characterization is tested by the developed task specific uncertainty analysis model. Two sources of errors are considered in the present study including the measurement error and the form error of the machined surface. Since the magnitude of the form error of the machined bifocal surface is found to be $2.02 \mu\text{m}$, fractional Brownian motion is used to generate a large number of random form errors, the magnitude of which varies around $2.02 \mu\text{m}$.

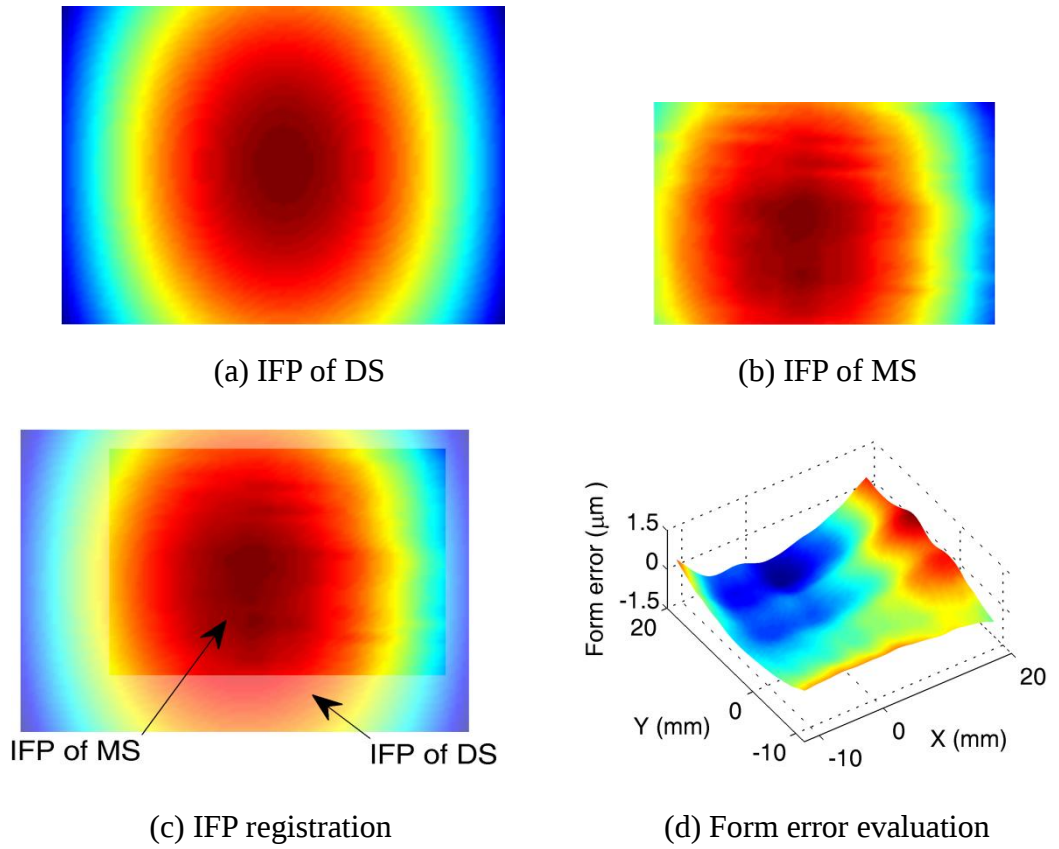


Figure 5.17 Form characterization of measured bifocal surface

Figure 5.18 shows the S_t of the 1500 random form errors, which are generated by the fractional Brownian motion with a standard deviation of $0.4 \mu\text{m}$. It can be seen from Figure 5.18 that the magnitude of the random form error varies in the range of $[1.5, 2.5] \mu\text{m}$. The measurement noise associated with the measured points is simulated by the three dimensional Gaussian noise with a standard deviation of $0.2 \mu\text{m}$

in all three axes. Based on the uncertainty analysis by the Monte Carlo method with 1500 trials, the uncertainty of the estimated S_t obtained in this case study is found to be $0.15\ \mu\text{m}$, and the uncertainty of the estimated S_q is found to be $0.09\ \mu\text{m}$.

To further evaluate the accuracy of the IFPAM for generalized form characterization of ultra-precision freeform surfaces, the results are also compared with that are obtained by conventional least squares based method (LSM) (Cheung et al, 2006). Figure 5.19 shows the uncertainty of the form characterization results when the magnitude of the surface form error is in the range of $[1, 30]\ \mu\text{m}$ and standard deviation of the measurement noise is $0.3\ \mu\text{m}$.

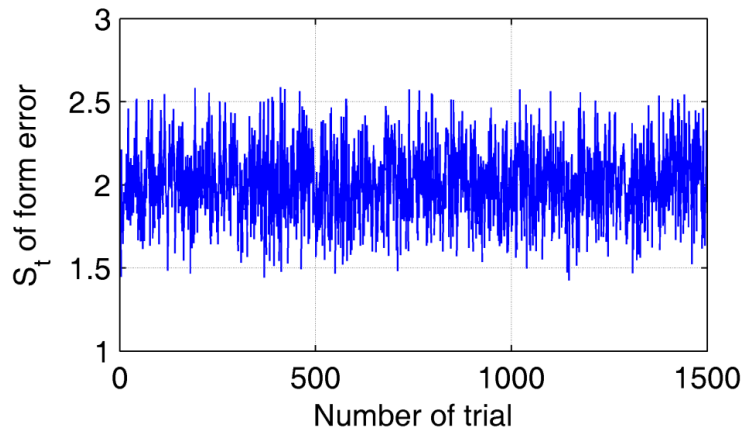


Figure 5.18 S_t of the 1500 random form errors for bifocal surface

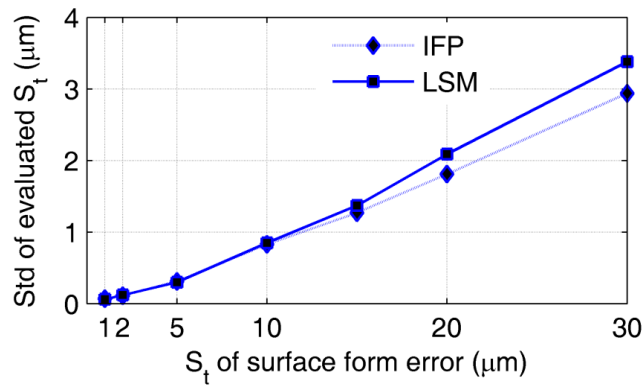


Figure 5.19 Uncertainty of evaluated S_t related to magnitude of surface form error

The results show that the standard deviation (std) of the evaluated peak-to-valley (PV) by the IFPAM and the LSM both increase along with increasing magnitude of the surface form error. However, when the S_t of the surface form error is larger than $15\mu\text{m}$, the IFPAM possesses a lower uncertainty than that for the LSM. This is due to the fact that the quality of the corresponding pairs by LSM becomes poor with increasing form error of the MS, since it is obtained by the closest point principle (Li, 2004). On the other hand, the IFPAM establishes the corresponding pairs by the IFP registration. The high frequency part of the surface form error is well smoothed in the surface reconstruction process, and the error of the generated IFP of MS is mostly contributed to by the low frequency part of the added error, which does not have a serious effect on the calculation of the curvature. Indeed, the advanced image registration technique ensures that the IFPAM is highly accurate and robust.

5.4 Summary

This chapter presents a task specific uncertainty analysis model to analyze uncertainties associated with the results of the form characterization of ultra-precision freeform surfaces. Monte Carlo method is used to evaluate the uncertainty with the consideration of three factors, including measurement error, sample size, and form error of the workpiece. Fractional Brownian motion is used as a random variable to simulate the possible form error of a measured surface, and multivariable Gaussian noise is used to generate the random measurement error. Since the propagation of the uncertainty in form characterization varies with the surface geometry being characterized, and the sampling strategy being adopted, a task specific uncertainty evaluation method is suggested in the present study. That is, the associated uncertainty in the results of the form characterization is estimated when the measured data is

extracted from a specific surface with a specific sampling strategy.

The developed task specific uncertainty analysis model has been incorporated into the generalized form characterization system, based on the IFPAM presented in Chapter 4. Experimental results show that the proposed uncertainty analysis model is able to estimate the uncertainty associated with the results of the form characterization in respect of the magnitude of the form error of the surface. Based on extensive simulation, the proposed model is also able to establish a relationship between the uncertainty of the results of the form characterization and the magnitude of contributed sources of error (e.g. the adopted sampling plan, and the magnitude of the measurement error), so that a prediction can be made for a specific form characterization of freeform surface. This provides an important means for the control and the optimization of the measurement and characterization process, so as to improve the reliability and accuracy of the results of the form characterization of ultra-precision freeform surfaces.

Chapter 6

Overall Conclusion and Suggestions for Further Research

6.1 Overall Conclusion

Ultra-precision freeform surfaces possessing non-rotationally symmetry are increasingly being used in various industries such as advanced optical systems (e.g. F-theta lens in scanner, automotive lighting systems, telecommunications, and photonics), with the functional aims of improving the performance of the products in terms of size reduction and functionality. To ensure the performance of the components, these surfaces are fabricated by modern ultra-precision freeform machining technologies, such as ultra-precision raster milling, ultra-precision polishing and ultra-precision grinding, with sub-micrometre form accuracy and surface finish in the nanometer range.

However, the geometric complexity and high precision requirements of ultra-precision freeform surfaces bring considerable challenges to the measurement and characterization of these surfaces. Although the advanced development of the measurement instruments constitutes enabling technologies to extracts data points from the machined freeform surfaces with a nanometric level of accuracy, there is still a lack of international standards and definitive methodologies to characterize the form of the machined ultra-precision freeform surfaces with sub-micrometre form accuracy. Currently, the least-squares-based or minimum-zone-based form characterization

methods are widely used in the measurement and form characterization of ultra-precision freeform surfaces. However, these methods are susceptible to the dependent coordinate frames and to the geometry of the surface being characterized. There is a stringent requirement to develop a practical and generalized method to perform high-precision and robust form characterization of ultra-precision freeform surfaces with sub-micrometer form accuracy.

Motivated by the demand for a standard and generalized form characterization method for ultra-precision freeform surfaces, this thesis presents an invariant feature-based pattern analysis method (IFPAM) for generalized form characterization of ultra-precision freeform surfaces, so as to address the deficiency and limitations of traditional form characterization methods. The IFPAM makes use of intrinsic surface properties such as Gaussian curvature to map the surface into a special image to form a generalized orientation invariant feature pattern (IFP) for the representation of the surface geometry. Digital image processing techniques are then employed to conduct the IFP registration and correspondence searching for the form characterization of the surface. In this way, the IFPAM is not only free of the type of the freeform surfaces being characterized but is also independent of the embedded coordinate frame, which brings many difficulties and uncertainties to the form characterization of ultra-precision freeform surfaces.

The calculation of the intrinsic surface features from a machined freeform surface is susceptible to the sampling strategy, the measurement noise and outliers associated with the measured data. To address these problems, a bidirectional curve network based sampling strategy (BCNSS) combined with a robust surface fitting and reconstruction algorithm (RSFRA) are developed for ensuring the accurate extraction of the intrinsic surface features from machined freeform surfaces. Different from the traditional one directional raster sampling strategy, the BCNSS samples two sets of

curves from two different directions to form a curve network, which is used to construct a substitute surface to represent the measured surface. Experimental work shows that the sampling plan produced by the BCNSS has significant improvement in terms of the efficiency of sampling of the freeform data, with the sampling accuracy at a sub-micrometre level.

The RSFRA is developed to reconstruct a high fidelity surface from the discrete measured points, while the surface smoothness is also ensured to avoid unwanted variation caused by surface roughness and measurement noise. A new fitting threshold named the confidence interval of fitting error has been presented to balance the fitting accuracy and surface smoothness. To simplify the fitting process and to avoid local optimization problems, an initial surface is constructed to estimate an appropriate number of control points and their distribution. The squared distance minimization method is then used to minimize the fitting error of the initial surface. The results of the experimental work indicate that the RSFRA provides an effective means in balancing fitting accuracy and surface smoothness, so as to reconstruct high fidelity surfaces with well surface smoothness. Both the BCNSS and the RSFRA have been incorporated into the developed IFPAM to enhance its measurement performance.

To assess the reliability and accuracy of the IFPAM, a Monte Carlo method based task specific uncertainty analysis model is built to estimate the associated uncertainty in the results of the form characterization of ultra-precision freeform surfaces. Three factors are identified and considered in the model: measurement error, surface form error, and sample size. Fractional Brownian motion is used as a random variable to simulate the possible form error of a measured surface, and multivariable Gaussian noise is used to generate the random measurement error. Since the propagation of the uncertainty in form characterization may vary with the surface

geometry being characterized, a task specific uncertainty evaluation method is developed. That is, the associated uncertainty in the characterization results is estimated when the measured data is extracted from a specific surface with a specific sampling strategy. The developed task specific uncertainty analysis model has been incorporated into the IFPAM to assess the reliability of the results of form characterization in real freeform surface measurement. Experimental results show that the proposed uncertainty analysis model is able to estimate the uncertainty associated with the results of the form characterization, with respect to the magnitude of the form error of the surface being characterized.

The major contributions of the research are summarized as follows:

- (i) Form quality plays an essential role in the characterization of ultra-precision freeform surfaces. At present, there are no international standards and definitive methods for the form characterization of ultra-precision freeform surfaces with sub-micrometre form accuracy. In this study, an IFPAM has been developed, which can perform a robust and accurate form characterization of ultra-precision freeform surfaces. The IFPAM is robust to the embedded coordinate frames of the freeform surface being characterized. It makes use of the orientation invariant surface features as surface matching and optimization criteria, which makes the method free from the coordinate frames. The IFPAM is a generalized form characterization method for ultra-precision freeform surfaces. It represents the surface geometry by the invariant feature pattern (IFP), which is generated by mapping the intrinsic surface features of a freeform surface, such as Gaussian curvature, into a 2D pattern. Hence, the IFP is a generalized surface feature that makes the IFPAM independent of the type of the freeform surfaces being characterized. They provide an important means for the advancement and standardization of freeform surface measurement.

- (ii) The effectiveness of the IFPAM depends on the accurate extraction of the intrinsic features from the machined freeform surfaces. In this study, a bidirectional curve network based sampling strategy (BCNSS) combined with a robust surface fitting and reconstruction algorithm (RSFRA) has been developed for ensuring the accurate extraction of the intrinsic surface features from a machined freeform surfaces. The BCNSS is able to produce a more efficient sampling plan than the traditional one directional method in the sampling of the data from the ultra-precision freeform surfaces with sampling accuracy in sub-micrometre to nanometre range. The RSFRA is able to reconstruct a mathematically continuous surface model from discrete measured points, which attempts to strike a good balance between the surface fitness and the surface smoothness. The BCNSS and the RSFRA provides an important means to ensure the efficiency and accuracy of the IFPAM in the measurement and generalized form characterization of machined ultra-precision freeform surfaces.
- (iii) Uncertainty analysis is indispensable part of the form characterization of ultra-precision freeform surfaces. In this study, a task specific uncertainty analysis model has been built to assess the accuracy and reliability of the IFPAM. Based on the results of extensive simulation experiments, the uncertainty analysis model is verified as being able to establish a relationship between the uncertainty of the results of the IFPAM and the magnitude of contributed sources of errors (e.g. the adopted sampling plan, the magnitude of the measurement error), so that a prediction can be made for the form characterization of a specific freeform surface. This provides an important means for the control and optimization of the measurement and form characterization process so as to improve the reliability and accuracy of the results of the generalized form characterization of ultra-precision freeform surfaces.

The present study provides a practical and generalized methodology for performing high-precision and robust form characterization of ultra-precision freeform surfaces with sub-micrometer form accuracy. The advanced technical merits of the IFPAM not only provide a breakthrough for the advancement of state-of-the-art generalized form characterization of ultra-precision freeform surfaces, but also contribute significantly to the advancement of measurement science and technology, as well as to international efforts to standardize freeform measurement.

6.2 Suggestions for Further Research

Due to the geometric complexity and factors affecting the measurement uncertainty, there is currently still a lack of international standards and well established measurement technologies for the form characterization of ultra-precision freeform surfaces with sub-micrometre form accuracy. The present study provides a practical and generalized solution to perform high-precision and robust form characterization of ultra-precision freeform surfaces. However, the theory and the practice are far from performing highly efficient and traceable measurement and form characterization on all kinds of ultra-precision freeform surfaces. Some topics for further research are suggested in the following sections:

- (i) *Research of a multi-intrinsic-surface-feature based generic method for the representation and form characterization of all kinds of freeform surfaces.*

The present study is focused on the form characterization of continuous freeform surfaces. However, the structured surfaces and conjunct surfaces with steps or edges are also classified as special kinds of freeform surfaces, since they have the same aspects in regard to the fabrication, alignment and measurement (Jiang, 2007b). Some

structured surfaces which contain ruled or revolute surfaces such as cylindrical surfaces along with their conjunct surfaces (e.g. cylindrical lens arrays), have zero Gaussian curvatures (GC). As a result, the registration of these surfaces cannot be realized by only using GC. For freeform surfaces wholly described by flat and cylindrical surfaces which have zero GC, the principal curvatures (minimum and maximum curvatures) can be separately used to identify the two types of surface. All the salient features like holes, slots, pockets, and transition or intersection curves between two adjacent surfaces can also be used as important intrinsic surface features to form a multi-intrinsic surface feature pattern for enhancing the performance of the representation and characterization of the complex surface geometry. In this way, the multi-intrinsic surface feature based form characterization method is able to be applied to all kinds of freeform surfaces, including continuous surface and structured surface. This provides an important means for supporting the standardization of freeform surface measurement and form characterization.

(ii) Development of the measurement strategy and the data fusion technologies for the multi-sensor based freeform surface measurement.

For the future development of ultra-precision freeform surfaces, there is increasing geometrical complexity and more difficulty in terms of form characterization with sub-micrometer form accuracy. Although substantial strides have been made in the development of high precision coordinate measurement instruments, none of the techniques is capable of fulfilling all the required measurement tasks with a high degree of accuracy and efficiency.

As a result, a sophisticated combination of several measuring techniques into one system appears to be a solution for assuring the quality of surface measurements. Multi-sensor based measurement technologies are used in order to get holistic, more

accurate and reliable measurements, which are not possible with a single sensor system (Weckenmann, 2009). However, since it is relatively new, very little research has been conducted into multi-sensor measuring strategies and data fusion.

Measurement strategy contributes significantly to the efficiency of the measurement and the reliability of the measurement results. Differing from single sensor based measurement, the multi-sensor based measurement requires more comprehensive measurement strategies and sampling plans to guide the measurement process. For example, tactile and optical sensors are commonly integrated in a coordinate measurement machine, such as the VideoCheck UA 400 (Werth, 2011), to perform complex measuring tasks. To further enhance the efficiency of freeform measurement, an integrated measurement strategy and sampling criteria should be developed to produce a sampling plan for both optical and tactile sensors. For instance, optical sensors, such as video-cameras are used to capture information of the global shape of the measurand, which can provide a more intelligent sampling plan for the tactile sensor to perform precision and dense measurements of the area of interest. Proper data fusion methods are also required to combine the data from different sensors into a common representation format, so that measurement results can benefit from all available sensors and data. This provides an important means for the realization of the multi-sensor based measurement technique to perform a holistic, accurate and reliable measurement of freeform surfaces.

(iii) Development of a generalized uncertainty analysis model for the evaluation of the uncertainty in the form characterization of freeform surfaces.

Due to the geometric variety of freeform surfaces, this study suggested a task specific uncertainty analysis method for the form characterization of freeform surfaces. However, this method currently can only be used to analyze the uncertainty

for a specific freeform surface each time, and the whole process needs to be carried out again if another kind of freeform surface is characterized. In further work, surface geometrical feature parameters should be developed to describe the geometry of the surface so that it can be incorporated into the uncertainty analysis model. For the sampling strategy, commonly used methods should be summarized and standardized as several modules so that appropriate module can be selected for different situation. The effect of the error sources affecting uncertainty in the measurement, characterization and evaluation also needs an in-depth study in order to provide a better understanding of the control and optimization of the measurement process for improving measuring accuracy. The study should provide an important means to generalize the capability of the uncertainty evaluation in measurement and form characterization of more types of ultra-precision freeform surfaces.

(iv) Study of functional and surface parameters for the form characterization of freeform surfaces.

Currently, there is a lack of international standards for surface parameters used for the form characterization of ultra-precision freeform surfaces. Although some areal parameters have been published by the ISO project for evaluating the surface quality of 3D surfaces, most of them are limited to use in surfaces possessing rotational symmetry such as aspheric surfaces. As a result, a series of surface parameters including global parameters and local parameters should be studied to characterize the magnitude and variation of the form errors of ultra-precision freeform surfaces. The surface parameters also need to be correlated with the specific functional characteristics of the surface (e.g. optical properties, self cleaning, and wettability) so that, based on the parameters, a decision can be made to examine if the surface meets the designer's requirements for its functional uses.

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Appendices

Appendix I Form Talysurf PGI 1240 Freeform Profiler

The Form Talysurf PGI 1240 (Figure 2.19) from Taylor Hobson is a premium specification measurement system for the small to medium sized optics market. It is designed for applications where optimum component quality and consistency cannot be compromised. The PGI 1240 Measurement System represents a new level of performance for applications such as:

- Diamond-turned or Ultra-precision ground molds for plastic or glass optics
- Aspherics for laser applications
- Digital cameras, projectors, etc
- IR & Diffractive optics up to 200mm diameter

The Form Talysurf PGI 1240 has been developed specifically for the optics industry. Every aspect of manufacture is to the highest possible level which results in unparalleled system performance.

- 200 mm Traverse unit with 0.11 μm /200 mm straightness
- 12.5 mm Gauge range, 0.8 nm vertical resolution
- 50 mm Gauge range possible with special styli
- 1 nm RMS noise floor provides market-leading optics surface finish measurement capability
- Extensive Range of Optional Accessories, including styli for demanding applications

Appendix II WYKO NT 8000 Optical Profiling System

The Wyko NT8000 is the most capable optical profiler available for the non-contact measurement of step heights, roughness and surface topography of MEMS, metal materials, semiconductors, medical devices, precision lenses and more.

Major features include:

- Gauge-capable measurement with sub-nanometer vertical resolution
- 8mm vertical scan range with sub - nanometer resolution.
- Internal reference enabling self-calibrating accuracy over the entire range

The Wyko NT 8000 system consists of several key components which work together to provide information on your sample, see Figure A-1. The system includes:

- A Wyko Profiler head mounted on a Z-axis and automated tip/tilt cradle
- Various magnification objectives mounted on a turret
- A vibration-isolation table
- A motorized x/y sample stage

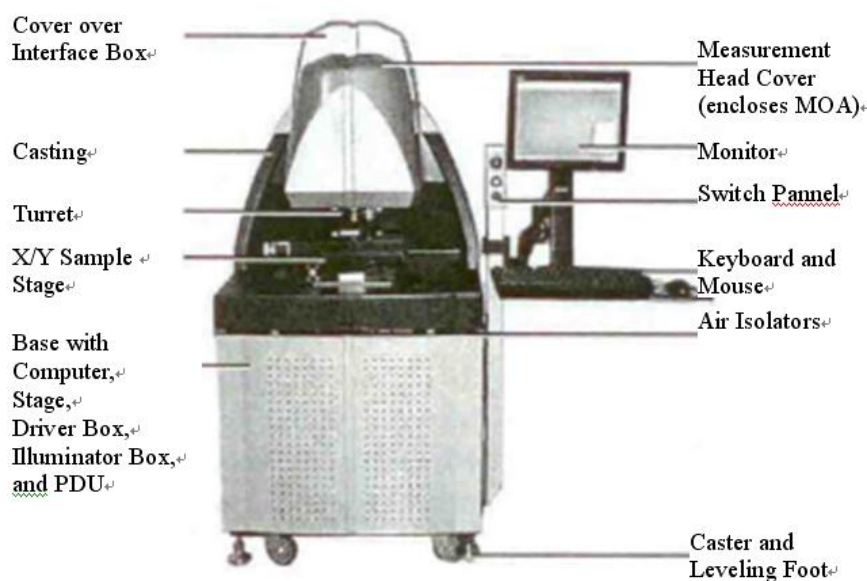


Figure A- 1 Wyko NT 8000 (Reduced Footprint Configuration)