**THE HONG KONG POLYTECHNIC UNIVERSITY**

**Department of Chinese and Bilingual Studies**

# INFERENTIAL PATTERNS OF GENERALIZED QUANTIFIERS AND THEIR APPLICATIONS TO SCALAR REASONING

**CHOW Ka Fat**

A thesis submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

March 2012

CERTIFICATE OF ORIGINALITY

I hereby declare that this thesis is my own work and that, to the best of my knowledge and belief, it reproduces no material previously published or written, nor material that has been accepted for the award of any other degree or diploma, except where due acknowledgement has been made in the text.

______________________________ (Signed)

_______CHOW Ka Fat__________ (Name of student)

## Abstract

This thesis studies the inferential patterns of generalized quantifiers (GQs) and their applications to scalar reasoning. In Chapter 1, I introduce the basic notions of Generalized Quantifier Theory (GQT) and survey the major types of right-oriented GQs traditionally studied under GQT (including both monadic and iterated GQs). I also expand the scope of this theory to the analysis of left-oriented GQs (including left conservative GQs such as "*only*" and left-iterated GQs manifested as quantified statements with relative clauses).

In Chapter 2, I introduce the major aspects of scalar reasoning to be studied in this thesis and summarize the major findings in the literature. After reviewing different notions of scales, I introduce other essential concepts and review the various theories and schools on the two main types of scalar reasoning, i.e. scalar entailments (SEs) and scalar implicatures (SIs). I then introduce four types of scalar lexical items studied under the Scalar Model Theory and Chinese grammar and discuss how their semantics / pragmatics are related to SEs and/or SIs. These include scalar operators (SOs), climax construction connectives (CCCs), subjective quantity operators (SQOs) and lexical items denoting extreme values. In the final part of this chapter, some outstanding problems in the studies on scalar reasoning are identified.

In Chapter 3, I study four main types of quantifier inferences. They are monotonicity inferences, argument structure inferences, opposition inferences and (non-classical) syllogistic inferences. The major findings are summarized in tables and theorems. Special emphasis is put on devising general principles and methods that enable us to derive valid inferential patterns of iterated GQs from the inferential properties of their constituent monadic GQs.

In Chapter 4, I apply the major findings worked out in the previous chapter

to resolve the outstanding problems identified in Chapter 2. I first develop a basic formal framework that is based on the notions of generalized fractions and I-function. This basic framework can deal with the various aspects of scalar reasoning in a uniform way. I then enrich the basic framework by adding specific ingredients to deal with the phenomena of SEs and SIs. To deal with SEs, I add a relation connecting the I-function and SEs to the basic framework, so that the derivation of SEs is reduced to comparison between the I-function values of propositions. Moreover, by capitalizing on a parallelism between SEs and monotonicity inferences, I combine findings of the two types of inferences and discover new inferential patterns, such as Proportionality Calculus and scalar syllogisms. To deal with SIs, I add the ingredients of question under discussion (QUD) foci, answer exhaustification and opposition inferences to the basic framework, so that it can account for the various types of SIs and related phenomena introduced in Chapter 2 in a uniform way. I then use the framework to conduct a cross-linguistic study on the English and Chinese scalar lexical items introduced in Chapter 2. The I-function is used to formulate the conditions of use for these lexical items. The association of SEs and SIs with different types of scalar lexical items is also explored.

Finally, Chapter 5 discusses the significance of the major findings of this thesis and possible extensions of the study.

**Publications arising from the Thesis**


Chow, K.F. (2011a) "A Semantic Model for Vague Quantifiers Combining Fuzzy Theory and Supervaluation Theory", in van Ditmarsch, H. et al (eds.) *Proceedings of the 3rd International Workshop on Logic, Rationality and Interaction*, Heidelberg: Springer, pp. 61 – 73.

Chow, K.F. (2011b) "Duidang Fangzhen Yiban Moshi ji qi Yingyong" [The General Pattern of Squares of Opposition and its Applications], in Jiang, Y. (ed.) *Approaching Formal Pragmatics*, Shanghai: Shanghai Educational Publishing House, pp. 104 – 121.

Chow, K.F. (2011c) "Dandiaoxing yu Tiji Tuili" [Monotonicity and Scalar Reasoning], in Jiang, Y. (ed.) *Approaching Formal Pragmatics*, Shanghai: Shanghai Educational Publishing House, pp. 122 – 174.

Chow, K.F. (2011d) "Yiwen Liangci de Xingshi Biaoda yu Tuili Moshi" [Formal Representation and Inferential Patterns of Interrogative Quantifiers], in Jiang, Y. (ed.) *Approaching Formal Pragmatics*, Shanghai: Shanghai Educational Publishing House, pp. 175 – 261.

Chow, K.F. (2012a) "General Patterns of Opposition Squares and 2n-gons", in Beziau, J.-Y. and Jacquette, D. (eds.) *Around and Beyond the Square of Opposition*, Basel: Birkhäuser / Springer, pp. 263 – 275.

Chow, K.F. (2012b) "Generalizing Monotonicity Inferences to Opposition Inferences", in Aloni, M. et al (eds.) *Proceedings of the 18th Amsterdam Colloquium*, Berlin: Springer, pp. 281 – 290.

Chow, K.F. (2012c) "Zhuguanliang de Tiji Moxing Jieshi" [A Scalar Model Interpretation for Subjective Quantity], *Chinese Language Learning*, 2012:4, pp. 29 – 38.

## Acknowledgements

After I obtained my first degree in 1989, I started looking for a subject area for doing research and pursuing a doctoral degree. This search lasted for nearly two decades until 2007 when I finally decided to conduct a research on quantifier inferences and scalar reasoning. The choice of this topic was of course not done randomly, but was inspired by four scholars who also provided different help for me to complete this study. I would thus like to take this opportunity to express my gratitude to them.

First and foremost, I have to extend my thanks to Dr. JIANG Yan, who is not only my PhD degree supervisor, but also the advisor of my MA dissertation (which won the Outstanding Thesis Award (MA) of the Linguistic Society of Hong Kong). In the past five years, Dr. JIANG has provided me with supportive guidance and valuable inspiration without which my thesis would not have covered certain important aspects of scalar reasoning and Chinese linguistics. He has also encouraged me to submit papers to academic conferences and journals and given me opportunity to publish parts of my works in three chapters of his edited book (*Approaching Formal Pragmatics*). These precious experiences have made the period of my doctoral study an exciting and rewarding one.

I would also like to thank the other three members of the Board of Examiners. They are Professor SHI Dingxu, Professor PAN Haihua and Professor ZOU Chongli. Professor SHI is the Chair of the Board of Examiners. He and Dr. JIANG were the instructors of the first courses that I attended for the MA in Chinese Linguistics degree in 2003. As I had no formal educational background on linguistics before I pursued the MA degree, their teaching laid a good foundation for my linguistic research later. Professor SHI's *Chomsky's Theory of Syntax – Its Evolution and Latest Development* remains one of my favourite references on Chomskyan Syntax, although I have not chosen to pursue studies in this area.

Professor PAN is the co-author (with Dr. JIANG) of *Introduction to Formal Semantics* and Professor ZOU is the author of *Studies in Natural Language Logic* and *Logic, Language and Information – Studies on Logical Grammar*. It is through these works that I got to know the fruitful ground of Formal Semantics, especially Generalized Quantifier Theory. It is indeed my honour to have the three experienced workers of Formal Semantics in China to examine my thesis and provide valuable comments so that I can improve the presentation, sharpen the arguments and correct the mistakes in my thesis.

Last but not least, I must also thank my wife Belle, who has always supported me in my studies, not only in taking care of our family, but also in sharing the ups and downs in my academic life, including accompanying me in a number of academic conferences!

**Table of Contents**

## List of Figures



## List of Tables

## List of Abbreviations

| Abbreviation | Full Name |
|---|---|
| BO | Boolean operator |
| CCC | climax construction connective |
| CP | context proposition |
| CS | context set |
| DRT | Discourse Representation Theory |
| FOPL | First Order Predicate Logic |
| GCI | generalized conversational implicature |
| GF | generalized fraction |
| GQ | generalized quantifier |
| GQT | Generalized Quantifier Theory |
| iff | if and only if |
| LHS | left-hand side |
| NP | noun phrase |
| NPI | negative polarity item |
| OP | opposition property |
| PCI | particularized conversational implicature |
| PMC | Principle of Monotonicity Calculus |
| POC | Principle of Opposition Calculus |
| PPC | Principle of Proportionality Calculus |
| PPI | positive polarity item |
| QUD | question under discussion |
| RHS | right-hand side |
| SE | scalar entailment |
| SI | scalar implicature |
| SM | scalar model |
| SMN | scalar metalinguistic negation |
| SMT | Scalar Model Theory |
| SO | scalar operator |
| SQ | subjective quantity |
| SQO | subjective quantity operator |
| TP | text proposition |
| wrt | with respect to |

# Chapter 1 Formal Properties of Generalized Quantifiers

## 1.1 General Outline of this Thesis

The main theme of this thesis is about formal reasoning in language, an important cognitive activity of the human kind. Two types of reasoning will be studied: inferences of generalized quantifiers (GQs) and scalar reasoning (including scalar entailments (SEs) and scalar implicatures (SIs)).

As quantification is a common phenomenon in natural language, a thorough study of the inferential patterns of quantifiers is of utmost importance in understanding natural language inferences. In fact, throughout the history of logic, from the ancient Aristotelian Logic to the modern First Order Predicate Logic (FOPL) as well as its offshoot Generalized Quantifier Theory (GQT), quantifiers (or quantified statements) have been a primary target of study.

Reasoning is also a primary object of study in modern pragmatics. Among the various types of pragmatic reasoning, I have chosen to study scalar reasoning because it is closely related to quantifier inferences. In fact, the classical examples of SIs are precisely about quantifiers, which can form scales. More importantly, it will be shown in this thesis that scalar reasoning shares some striking similarities with quantifier inferences, and research findings on quantifier inferences can be used to solve some problems in the studies of scalar reasoning.

In this and the next chapter, I will first introduce the basic notions of GQT and scalar reasoning. In addition to introducing and reviewing past results, I will also expand the scope of GQT study to left-oriented GQs and point out some outstanding problems in the studies of scalar reasoning. In Chapter 3, I will study four main types of quantifier inferences, namely monotonicity inferences, argument structure inferences, opposition inferences and syllogistic inferences.

In Chapter 4, I will apply the major findings worked out in the previous chapter to solve the problems identified in Chapter 2. Specifically, I will develop a formal framework for SEs and SIs and use it to account for the formal pragmatics of a number of scalar lexical items, including scalar operators, climax construction connectives, subjective quantity operators, maximizers / minimizers and Chinese idiomatic constructions with extreme numerals, etc. Chapter 5 provides some concluding remarks.

## 1.2 Overview of GQT

GQT may be roughly divided into two streams[1]. The first stream is mainly interested in the logical properties of GQs and develops into a branch of modern Mathematical Logic. It grows out of FOPL, which used to study the logical inferences of the universal and existential quantifiers. Since Mostowski (1957) and Lindström (1966), the range of quantifiers studied by logicians has been greatly expanded to generalized quantifiers. Nowadays, researches in this stream cover a wide range of topics in Model Theory, Proof Theory, Computation Theory, Game Theory, Computer Science, Artificial Intelligence and various types of modal logics, non-standard logics and theories on uncertainties (such as Fuzzy Theory and Probability Theory). Van Benthem and Westerståhl (1995) and Väänänen (1999) are good overviews of such researches.

The second stream is mainly interested in the linguistic properties of GQs. Through the works of Montague (1973), Barwise and Cooper (1981) and Keenan and Stavi (1986) and the subsequent application of GQT to linguistic studies, GQT has become an important branch of Formal Semantics. Some syntacticians

[1] Of course, such a division does not mean that every piece of works on GQT can be neatly classified into exactly one of the streams. There are in fact many studies that straddle across the two streams.

also borrow ideas from this stream to study syntactic phenomena related to quantification, such as Beghelli (1995)'s study on quantifier scope. Thus, researches in this stream also cover a wide spectrum of subject areas ranging from Formal Semantics to Formal Syntax.

As outlined by Szabolcsi (2010), the development of linguistic theories on quantification since 1970s has undergone transformation from a period of "Grand Uniformity" (1970s and 1980s), during which GQT was seen as a uniform theory for identifying the typology and general semantic properties of different types of noun phrases (NPs), to a period of "Diversity" (1980s and 1990s), during which diversified anaphoric and scopal properties of different GQs were discovered and led to diversified treatments of different types and phenomena of GQs.

The period starting from 2000 may be called a period of "In-Depth Researches"[2], during which certain subject areas not touched upon in the previous periods are now put onto the research agenda. These include, inter alia, researches on special types of quantifiers (e.g. Bernardi and Moot (2003), Glöckner (2006)), internal composition of GQs (e.g. Hackl (2000), Matthewson (2001)), polyadicity of GQs (e.g. van Eijck (2005), Robaldo (2011)), generalization or refinement of certain GQ properties (e.g. Zuber (2010b, 2011)), etc.

## 1.3 Scope of the Study

Although advances have been made in the studies of various aspects related to GQs such as vagueness, interrogative, plurality, mass terms, genericity, intensionality and dynamic semantics of anaphora and ellipsis, this thesis is

[2] Szabolcsi (2010) used the name "Internal Composition" to call this period. But since this name fails to cover some important researches during this period, I replace it by "In-Depth Researches".

targeted at the inferential patterns of GQs. Therefore, I choose to focus on the core type of GQs, i.e. sharp declarative GQs with non-collective, non-mass arguments in an episodic, extensional and static setting, to avoid distraction by considerations of issues other than quantifier inferences. Concerning embedded GQs, this thesis will only study GQs embedded within the scopes of other GQs and will not study GQs embedded under "world-creating predicates" (e.g. reporting verbs, propositional attitude verbs, etc). Despite the above restrictions, this thesis will extend the GQT framework, which has traditionally been concentrated on right-oriented GQs, to left-oriented GQs, which include left conservative monadic GQs (e.g. "*only*") and left-iterated GQs.

At present, GQT has developed to the extent that two pieces of work on GQT may differ a lot in their scopes of study, basic assumptions and notations from the classical GQT and from each other. The emergence of new theories departing from the classical GQT is often due to the need of accounting for phenomena not adequately explained by the classical GQT. This is especially so for those researches that have a strong syntactic flavor. Since one main objective of this thesis is to study the inferential patterns of GQs, this thesis will not touch on topics that are more syntax-oriented, such as scope and binding and internal composition of GQs.

## 1.4 Basic Notions of GQs and Models

A GQ can be seen as a second-order predicate with first-order predicates as arguments. Different GQs may differ in terms of the number and arities of their arguments, where "arities" refer to the number of arguments of the first-order predicates. Lindström (1966) devised a special nomenclature to denote the type of a GQ. The nomenclature takes the form of a sequence of natural numbers

<$n_1$, ... $n_k$> where k is the number of arguments of the GQ and $n_1$, ... $n_k$ are the arities of each argument. If all the numbers in the sequence are 1, the GQ is "monadic". Otherwise, it is "polyadic". For instance, the sequence <1,3> represents a polyadic GQ with two arguments, the first of which being a unary predicate and the second being a ternary predicate[3]. This sequence can be used to represent the iterated polyadic GQ "(*j* ... *some* ... *m*)[4]" in the sentence:

(1) John gave some flowers to Mary.

Note that this GQ has a unary predicate (i.e. FLOWER) and a ternary predicate (i.e. GIVE)[5] as arguments and is thus a <1,3> GQ.

In this thesis, the meaning of GQs is defined to be truth conditions. Thus, the truth of a quantified statement is not determined once and for all (unless it is a tautology or contradiction) and may vary with respect to different models. In Mathematical Logic and Formal Semantics, a model is usually represented by a pair <U, $\|\cdot\|$> where U is a set composed of all the members of the universe (or domain of discourse) and $\|\cdot\|$ is an interpretation function whose purpose is to specify the denotations of all non-logical predicates in the model. For convenience, in this thesis I will only apply this function to propositions to denote their truth values. A predicate will be denoted as a set represented by capital letters. Hence, the denotation of "boy" will be written as BOY.

In addition to the interpretation function, there is another notion – variable assignment function that assigns values to variables. Since I will use the

---

[3] Many notions in Mathematical Logic have corresponding notions in Set Theory. For example, a unary predicate corresponds to an ordinary set (composed of individual members), while an n-ary predicate corresponds to a set composed of n-tuples of individual members. In this thesis, I will switch freely between the "predicate talk" and "set talk".

[4] In this thesis, I use small letters to represent proper names in the GQ. For example, "j" and "m" represent John and Mary, respectively. See Subsection 1.6.5 for an introduction to iterated polyadic GQs.

[5] This thesis does not consider the issue of grammatical number and tense. Thus, the semantic representation of all sentences will be numberless and tenseless.

set-theoretic notations[6] to represent the argument structures of GQs (e.g. representing “Most A are B” by “*most*(A)(B)” instead of “*most* x(A(x), B(x))”) and will not consider sentences with unbound variables, all variables are hidden inside the set notation[7] and so there is no need for the variable assignment function.

## 1.5 Basic Notions of Entailments and Equivalences

Although quantifier inferences are not studied until Chapter 3, I will provide formal definitions of entailments and equivalences, the most basic notions in the study of inferences, in this section, because these notions are essential for understanding the truth conditions and certain properties of quantifiers, as well as some aspects of scalar reasoning to be introduced in the next chapter.

First, we consider entailments which are defined as follows:

(2) Let p and q be propositions, then p entails q (written “$p \Rightarrow q$”)[8], iff wrt every model, if $\|p\| = 1$, then $\|q\| = 1$.

In the above definition, p is called the premise[9] and q is called the conclusion. In the study of inferences, we also need the notion of equivalences which are in fact bilateral entailments:

(3) Let p and q be propositions, then p is equivalent to q (written “$p \Leftrightarrow q$”) iff wrt every model, $\|p\| = 1$ iff $\|q\| = 1$.

Sometimes it is useful to view the entailment and equivalence relations between quantified statements as set-theoretic relations between quantifiers. To

---

[6] By using the set notations, I am able to make use of the powerful laws and functions in Set Theory.

[7] Note that hiding an unbound variable x inside the set notation is equivalent to binding x by λ, because $\{x: A(x)\}$ is equivalent to $\lambda x(A(x))$. Also note that the symbol A for a set is in fact a short form for $\{x: A(x)\}$.

[8] In this thesis, I use “$\Rightarrow$” to denote “entailment” and “$\rightarrow$” to denote “(material) implication”.

[9] In case there is more than one premise, then p is the conjunction of these premises.

this end, we first reinterpret quantifiers as sets. Using type <1,1> GQs as an example, we can interpret any such GQ as a set of ordered pairs of sets. For example, we have

(4) $$every = \{\langle A, B\rangle : A \subseteq B\}$$

In this way, a quantified statement can be rewritten as a set-theoretic statement. For example,

(5) $$\text{Every A is B.} \Leftrightarrow \langle A, B\rangle \in every$$

Based on the above reinterpretation, we can then define two set-theoretic relations between quantifiers – "inclusion" and "equality", denoted by "$\subseteq$" and "=", respectively. Let Q, Q' be type <1,1> GQs, then

(6) Q $\subseteq$ Q' iff wrt every model and every A, B, Q(A)(B) $\Rightarrow$ Q'(A)(B).

(7) Q = Q' iff wrt every model and every A, B, Q(A)(B) $\Leftrightarrow$ Q'(A)(B).

Using the truth conditions of quantifiers (recorded in Appendix 1 and Appendix 2), one can easily derive

(8) $$no \subseteq (fewer\ than\ 2) \subseteq (fewer\ than\ 3) \subseteq \ldots$$

(9) $$most = (more\ than\ 1/2\ of)$$

In Classical Logic, we have the subalternate relation "Every A is B $\Rightarrow$ Some A is B". But under the modern interpretation of the quantifier "*every*", this relation is only conditionally valid under the condition that A is non-empty. Now, under the above reinterpretation, we can express this conditionally valid relation as "Within the domain $\{\langle A, B\rangle: A \neq \varnothing\}$, *every* $\subseteq$ *some*". In fact, we can generalize this to the following chain relation:

(10) Within the domain $\{\langle A, B\rangle: |A| \geq n\}$,

$$every \subseteq (at\ least\ n) \subseteq (at\ least\ n-1) \subseteq \ldots \subseteq (at\ least\ 2) \subseteq some$$

### 1.6 Right-Oriented GQs

In this section, I will introduce those GQs that are most studied in the literature. In this thesis, these GQs are called right-oriented GQs, which include two subtypes – right conservative GQs and right-iterated GQs. In what follows, I will first provide a typology of these GQs, and will then explain the term "right-oriented".

### 1.6.1 Determiners

Let's start from the most important GQs – type <1,1> GQs. Since these GQs have two unary predicates as arguments, we may represent the argument structure of this type of GQs in the form of a tripartite structure[10]:

(11) $$Q(A)(B)$$

where Q, A and B denote the type <1,1> GQ, its left argument (also called the "nominal argument") and right argument (also called the "predicative argument"), respectively[11]. Apart from indicating the argument structure of a GQ, the tripartite structure can also be used to represent a sentence headed by that GQ. Syntactically, Q, A and B correspond to the determiner, subject (excluding the determiner) and sentential predicate, respectively. For this reason, type <1,1> GQs are often called determiners in the literature. For example, the sentence

(12) Every boy sang.

can be represented as

(13) *every*(BOY)(SING)

Tripartite structure is a succinct means for representing the argument structure of a GQ or a quantified statement. It turns out that it is not only

---

[10] In this thesis, the argument structure of a GQ may take two forms: the flat structure, such as Q(A, B), or the tripartite structure, such as Q(A)(B). According to the Schönfinkelization Theory, the two forms are equivalent. Therefore I will switch freely between these two forms.

[11] In the literature, the left and right arguments of a tripartite structure are also called "restrictor" and "nuclear scope", respectively.

applicable to determiners, but also other kinds of GQs. Thus, this thesis will use tripartite structure as a standard notation.

The semantics of a GQ is delineated by its truth condition which is expressed by a set-theoretic proposition. For example, the truth condition of "*every*" is as follows:

(14) $$\textit{every}(\mathrm{A})(\mathrm{B}) \Leftrightarrow \mathrm{A} \subseteq \mathrm{B}$$

In this thesis, the truth conditions of right conservative GQs mainly follow those adopted in Keenan and Westerståhl (2011) (see Appendix 1).

### 1.6.2 Type <1> GQs

Type <1> GQs include those GQs corresponding syntactically to full NPs, such as "*everybody*" and "*x*" (where "x" represents an individual member of the universe expressed as a proper name in natural language). Since a type <1> GQ only requires one predicative argument, when we express such a GQ as a tripartite structure, the left argument is left empty (denoted "–"), with the right argument being the unique argument. For example, the sentence

(15) Nobody sang.

has the following argument structure and truth condition:

(16) $$\textit{nobody}(–)(\mathrm{SING}) \Leftrightarrow \mathrm{PERSON} \subseteq \neg\mathrm{SING}$$

According to Appendix 1, type <1> GQs such as "*everything*" and "*nobody*" can be reanalyzed as complex structures with type <1,1> GQs such as "*every*(THING)" and "*no*(PERSON)", respectively. Sometimes the sets THING and PERSON may be equivalent to the universe U. In this case, the truth condition can be simplified. For example, when PERSON = U, then (16) becomes

(17) $nobody(–)(\mathrm{SING}) \Leftrightarrow \mathrm{SING} = \varnothing$

In FOPL, proper names are seen as individuals of the universe. Under this view, a proper name such as "*x*" can be analyzed as a complex structure with the determiner "*every*", such as "*every*({x})". Alternatively, we may also follow Montague (1973) by analysing proper names as GQs (called "Montagovian individuals" in the literature). Under this view, the proper name "*x*" has the following truth condition:

(18) $x(–)(B) \Leftrightarrow x \in B$

One can easily check that the above truth condition is equivalent to the truth condition of "*every*({x})(B)".

Apart from the aforesaid type <1> GQs, the "determiner + common noun" structure, i.e. the "Q(A)" sub-part of the tripartite structure "Q(A)(B)", corresponds to a full NP in natural language, and so can also be seen as a type <1> GQ.

### 1.6.3 Structured GQs

Structured GQs refer to monadic GQs with more than two arguments. Beghelli (1994) has studied various types of structured GQs. This thesis will focus on structured GQs expressing quantity comparison, such as "(*more* ... *than* ...)". One characteristic of such kind of structured GQs is that the same GQ may appear in three different argument types, the most important of which being $<1^2,1>$, with 2 nominal arguments and 1 predicative argument. Syntactically, the 2 nominal arguments together with the quantifier correspond to a complex subject, while the predicative argument corresponds to the sentential predicate. To express the argument structure such kind of GQs in the form of a tripartite structure, we may write the first two arguments as an ordered pair occupying the

A position of the tripartite structure Q(A)(B). For example, the sentence

(19) More boys than girls sang.

has the following tripartite structure and truth condition[12]:

(20) $(\textit{more ... than ...})(B, G)(S) \Leftrightarrow |B \cap S| > |G \cap S|$

The other two argument types, $<1,1^2>$ and $<1^2,1^2>$, can also be represented in the same way. For example, the sentences

(21) More boys sang than danced.

(22) More boys sang than girls danced.

can be represented by the following tripartite structures and truth conditions:

(23) $(\textit{more ... than ...})(B)(S, D) \Leftrightarrow |B \cap S| > |B \cap D|$

(24) $(\textit{more ... than ...})(B, G)(S, D) \Leftrightarrow |B \cap S| > |G \cap D|$

Note that (23) and (24) can be seen as variants of (20). That is why only (20) is listed in Appendix 1. In this thesis, I will mainly study type $<1^2,1>$ structured GQs.

### 1.6.4 Existential Sentences

Existential sentences refer to the "there + be + Q(A)" structure with an existential meaning in English. Following Keenan (1987b), such kind of sentences will be seen as equivalent to "Q(A) exist(s)" where "exist(s)" is a trivially true predicate that may be represented by the universe U. For example, the sentence

(25) There are more than two boys singing.

has the following argument structure and truth condition:

(26) $(\textit{more than 2})(BOY \cap SING)(U) \Leftrightarrow |BOY \cap SING| > 2$

[12] For readability's sake, I have omitted the angled brackets (<·>) for representing ordered pairs in the tripartite structure.

In (25), "singing" is called the "coda" of the existential sentence. According to Keenan (1987b), the coda can be treated as an intersective adjective or relative clause attached to "Q(A)".

### 1.6.5 Iterated Polyadic GQs

The GQs introduced in the previous subsections are all monadic GQs, which are used to represent sentences with only unary predicates (intransitive verbs, predicative adjectives, etc). If a sentence contains n-ary predicates with n > 1 (transitive verbs, ditransitive verbs, predicative adpositions, etc), then it has to be represented by polyadic GQs. According to Keenan and Westerståhl (2011), there are various types of polyadic GQs. In this thesis I will only discuss iterated polyadic GQs.

Consider the following sentence:

(27) Every girl loves John.

This sentence may be represented as containing the following complex GQ:

(28) (*every* ... *j* ...)(GIRL, –)(LOVE)

Since LOVE is a binary predicate, the complex GQ "(*every* ... *j* ...)" above is of type <1,2> and is thus polyadic. Moreover, according to Keenan and Westerståhl (2011), (28) can also be seen as containing an iterated polyadic GQ composed of "*every*(GIRL)" and "*j*(–)". Thus, (28) may be rewritten as an iterated tripartite structure[13]:

(29) *every*(GIRL)([*j*(–)]$_2$(LOVE))

Note that (29) reflects both the scope structure and grammatical relations of

---

[13] (29) should be seen as the variable-free version of the following:

*every*({z: GIRL(z)})({x: *j*(–)({y: LOVE(x, y)})})

In the following, I will freely switch between the two versions of iterated tripartite structures.

(27)[14]. First, the inner tripartite structure "$[j(–)]_2$(LOVE)" is placed inside the outer tripartite structure "*every*(GIRL)(·)", showing that "John" and "every girl" take narrow and wide scopes in (27), respectively. Second, the subscript "2" of "$[j(–)]_2$" indicates that "John" is the second argument (i.e. object) of "love".

The truth condition of an iterated GQ may be derived from the truth conditions of its constituent monadic GQs by using the following derivation formula (adapted from Keenan (1987a) and Keenan and Westerståhl (2011)): let B be a predicate with n arguments $x_1, \ldots x_n$, then for $1 \le i \le n$, we have

(30) $[Q(A)]_i(B) = \{<x_1, \ldots x_{i-1}, x_{i+1}, \ldots x_n>: Q(A)(B_{x1, \ldots xi-1, xi+1, \ldots xn})\}$

where "A" may be "–" (if Q is a type <1> GQ), a unary predicate (if Q is a determiner) or an ordered pair of unary predicates (if Q is a type $<1^2,1>$ structured GQ), and

(31) $B_{x1, \ldots xi–1, xi+1, \ldots xn} = \{x_i: B(x_1, \ldots x_n)\}$

For illustration, let's use (30) and (31) to derive the truth condition of (29). First, we compute the following:

$[j(–)]_2$(LOVE)

= $\{x_1: j(–)(LOVE_{x1})\}$ by (30)

= $\{x_1: j(–)(\{x_2: LOVE(x_1, x_2)\})\}$ by (31)

= $\{x_1: j \in \{x_2: LOVE(x_1, x_2)\}\}$ by Appendix 1

= $\{x_1: LOVE(x_1, j)\}$

The above result shows that "$[j(–)]_2$(LOVE)" denotes the set of those who love John. Next, by using the truth condition of "*every*" and the above result, we obtain

[14] This thesis adopts a simple treatment of quantifier scopes that is compatible with Montague (1973)'s "quantifying-in" or May (1985)'s "quantifier raising". To achieve a more sophisticated treatment, one will need to borrow ideas from theories developed under other syntactic / semantic frameworks, such as Cooper (1983), Beghelli (1995), Dalrymple et al (1999), etc.

(32) $every(\mathrm{GIRL})([j(–)]_2(\mathrm{LOVE})) \Leftrightarrow \mathrm{GIRL} \subseteq \{x_1: \mathrm{LOVE}(x_1, j)\}$

An advantage of using the above notation is that we can represent the truth conditions of certain special sentence types in a format that is as close as their surface forms. These include the topicalized sentences, focused sentences and other special Chinese sentence types discussed in Jiang and Pan (2005), as well as the left and right dislocated sentence types in English and other languages studied in Cann et al (2005). Consider the following Chinese sentence:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| (33) | You | yige | nühai | meige | nanhai | dou | ai | ta. |
| | have | a | girl | every | boy | all | love | she |

*There is a girl whom every boy loves.*

This is a topicalized sentence in which the logical object of the verb “ai” is placed in front of the logical subject. Logically speaking, this sentence is equivalent to a sentence with object wide scope. Using (30) and (31), we can represent (33) in the following form and derive its truth condition as follows:

(34) $a(\mathrm{GIRL})([every(\mathrm{BOY})]_1(\mathrm{LOVE}))$

$$\Leftrightarrow |\mathrm{GIRL} \cap \{x_2: \mathrm{BOY} \subseteq \{x_1: \mathrm{LOVE}(x_1, x_2)\}\}| > 0$$

Note that in the above, the components of the iterated tripartite structure are in the same order as the components of the surface structure of (33).

### 1.6.6 Right Conservativity and Right Iteration

GQT not only tries to formulate the truth conditions of GQs, but also studies the general properties of GQs. One such property is conservativity associated with determiners. This property is defined as follows:

(35) A determiner Q is conservative iff for all A, B, $Q(A)(B) \Leftrightarrow Q(A)(A \cap B)$.

It can easily be shown that all determiners listed in Appendix 1 satisfy the above definition. In fact, early GQT scholars widely believed that conservativity was a universal property of natural language determiners.

In (35), it is the right argument of Q that undergoes a Boolean operation, namely intersection. To distinguish this property from a similar property defined on the left argument, I will henceforth rename this property "right conservativity".

The right argument is also where iteration takes place in the iterated GQs introduced in the previous subsection. In general, the iterated tripartite structures associated with these GQs have the form

(36) $$Q_1(A_1)([Q_2(A_2)]_k(\ldots))$$

where the inner tripartite structure "$[Q_2(A_2)]_k(\ldots)$" is embedded inside the right argument of the outer tripartite structure "$Q_1(A_1)(\cdot)$". To distinguish this kind from another kind of iterated GQs, this kind of iterated GQs will henceforth be called "right-iterated GQs".

The right conservative and right-iterated GQs together constitute the focus of classical GQT studies. Since both types of GQs are concerned with the right arguments, I will henceforth call them "right-oriented GQs" collectively. In the next section, I will introduce "left-oriented GQs", which include two subtypes: left conservative GQs and left-iterated GQs.

## 1.7 Left-Oriented GQs

### 1.7.1 "*only*"

The once widely-accepted view that right conservativity is a universal property of natural language determiners is not totally uncontroversial because "only" as used in the sentence

(37) Only willows weep.

with the truth condition (according to de Mey (1990))

(38) $$\textit{only}(A)(B) \Leftrightarrow A \supseteq B$$

can easily be shown to be violating the defining condition of right conservativity (35). Just choose two sets A and B such that B is not a subset of A. Then these two sets do not satisfy (38) but they do satisfy $\textit{only}(A)(A \cap B) \Leftrightarrow A \supseteq A \cap B$, and so we have $\textit{only}(A)(B)$ $\#\Leftrightarrow$ $\textit{only}(A)(A \cap B)$ [15], violating (35). But then many scholars argue that "*only*" has very different syntactic behaviour than ordinary determiners and thus should not be considered a determiner.

Challenging the traditional view, de Mey (1990) and Hobbs (1995) both maintained that "*every*" and "*only*" should be treated as converses of each other and so should not be given different statuses just because they have different syntactic behaviour. To quote de Mey's words, "syntax cannot dictate the proper analysis of 'only' "[16]. The concept of "converse" is defined below:

(39) Let Q be a determiner. Its converse, denoted $Q^{-1}$, is a determiner such that for all A, B, $Q(A)(B) \Leftrightarrow Q^{-1}(B)(A)$.

According to the above definition, we have $\textit{every}^{-1} = \textit{only}$.

De Mey (1990) also played down the special status of right conservativity by introducing similar properties of determiners:

(40) A determiner Q is left conservative iff for all A, B, $Q(A)(B) \Leftrightarrow Q(A \cap B)(B)$.

(41) A determiner Q is right progressive[17] iff for all A, B, $Q(A)(B) \Leftrightarrow$

---

[15] In this thesis, I use "$\#\Rightarrow$", "$\#\Leftrightarrow$" and "#+>" to denote "non-entailment", "non-equivalence" and "non-implicature", respectively.

[16] De Mey (1991), p. 101.

[17] "Progressivity" is a term coined by de Mey (1990) in contrast to "conservativity": while conservativity is defined in terms of set intersection, progressivity is defined in terms of set union.

$Q(A)(A \cup B)$.

(42) A determiner Q is left progressive iff for all A, B, $Q(A)(B) \Leftrightarrow Q(A \cup B)(B)$.

It can easily be shown that "*only*" is both left conservative and right progressive whereas "*every*" is both right conservative and left progressive. Thus, the logical properties of "*only*" and "*every*" are mirror images of each other and should thus be treated on a par. This thesis follows de Mey (1990) by treating "*only*" as a (left conservative) determiner.

### 1.7.2 Other Left Conservative Determiners

In addition to "*only*", "(*apart from C only*)", where C is a set of individuals, is also left conservative[18]. This determiner is the converse of "(*all … except C*)". These examples are in fact special cases of a general fact[19]:

**Theorem 1.1** A determiner Q is left conservative iff $Q^{-1}$ is right conservative.

Thus we can identify left conservative determiners by considering the converses of right conservative determiners. Based on this principle, from the following equivalence relation:

(43) More than 70% of the participants are students.

$\Leftrightarrow$ Students constitute more than 70% of the participants.

we may conclude that there should be a left conservative determiner "(*constitute more than r of*)", which is the converse of "(*more than r of*)", with the following argument structure and truth condition:

(44) (*constitute more than r of*)(A)(B) $\Leftrightarrow |B \cap A| / |B| > r$

Although syntactically speaking, this does not correspond to any natural

---

[18] According to Zuber (2004), "sami" and "oprócz C sami", the Polish equivalents of "*only*" and "(*apart from C only*)", respectively, are genuine determiners.

[19] See Appendix 3 for proofs of the theorems in this thesis.

language determiner (at least in English and Chinese), from the logico-semantic point of view, studying these "abstract" determiners can help us discover more inference patterns. Moreover, in view of the relation

(45) Within the domain $\{<A, B>: B \neq \varnothing\}$,

*only* = (*constitute exactly 100% of*)

one can see that these "abstract" determiners are a natural generalization of the left conservative determiner "*only*". Also note that the relation between "*only*" and "(*constitute exactly r of*)" is completely analogous to that between "*every*" and "(*exactly r of*)" because we have the relation

(46) Within the domain $\{<A, B>: A \neq \varnothing\}$, *every* = (*exactly 100% of*)

Furthermore, left conservative GQs can also be found among the symmetric GQs as defined below:

(47) A determiner Q is symmetric iff for all A, B, $Q(A)(B) \Leftrightarrow Q(B)(A)$.

From (39), it can easily be seen that a symmetric determiner is self-converse. Thus, according to Theorem 1.1, symmetric right conservative determiners must also be left conservative. These left-and-right conservative determiners include "*some*", "*no*", "(*more than n*)", "(*no ... except C*)", etc.

### 1.7.3 Left Conservative Structured GQs

The above discussions are concentrated on determiners. According to Keenan and Moss (1984), we may also define (right) conservativities of different types of structured GQs. Here are the definitions:

(48) A type $<1^2,1>$ structured quantifier Q is right conservative iff for all $A_1$, $A_2$, B, $Q(A_1, A_2)(B) \Leftrightarrow Q(A_1, A_2)((A_1 \cup A_2) \cap B)$.

(49) A type $<1,1^2>$ structured quantifier Q is right conservative iff for all A,

$B_1$, $B_2$, $Q(A)(B_1, B_2) \Leftrightarrow Q(A)(A \cap B_1, A \cap B_2)$.

(50) A type $<1^2,1^2>$ structured quantifier Q is right conservative iff for all $A_1$, $A_2$, $B_1$, $B_2$, $Q(A_1, A_2)(B_1, B_2) \Leftrightarrow Q(A_1, A_2)(A_1 \cap B_1, A_2 \cap B_2)$.

It can be shown that every structured GQ listed in Appendix 1 satisfies the respective definitions for the respective argument structures.

We may extend the above definitions to left conservativities:

(51) A type $<1^2,1>$ structured quantifier Q is left conservative iff for all $A_1$, $A_2$, B, $Q(A_1, A_2)(B) \Leftrightarrow Q(A_1 \cap B, A_2 \cap B)(B)$.

(52) A type $<1,1^2>$ structured quantifier Q is left conservative iff for all A, $B_1$, $B_2$, $Q(A)(B_1, B_2) \Leftrightarrow Q(A \cap (B_1 \cup B_2))(B_1, B_2)$.

(53) A type $<1^2,1^2>$ structured quantifier Q is left conservative iff for all $A_1$, $A_2$, $B_1$, $B_2$, $Q(A_1, A_2)(B_1, B_2) \Leftrightarrow Q(A_1 \cap B_1, A_2 \cap B_2)(B_1, B_2)$.

Similar to the case of determiners, the sentence

(54) Girls constitute a larger proportion of students in this class than boys.

may be seen as containing a type $<1^2,1>$ structured GQ with the following argument structure and truth condition:

(55) (*constitute a larger proportion of ... than ...*)$(A_1, A_2)(B)$

$$\Leftrightarrow |A_1 \cap B| / |B| > |A_2 \cap B| / |B|$$

It is easily shown that the GQ above satisfies (51). Moreover, the same GQ may also appear as a type $<1,1^2>$ and $<1^2,1^2>$ GQ and satisfies the respective definitions of left conservativities. A list of the left conservative GQs studied in this thesis is provided in Appendix 2.

### 1.7.4 Left-Iterated GQs

As mentioned above, we may represent a sentence with an n-ary predicate (n > 1) by a right-iterated GQ where the iteration takes place in the right argument. When it comes to a sentence with relative clause(s)[20], we need to extend the notion of iteration to the left argument and obtain left-iterated GQs. For example, the sentence

(56) Every individual who loves John is happy.

may be represented by

(57) $every([j(–)]_2(\mathrm{LOVE}))(\mathrm{HAPPY}) \Leftrightarrow \{x_1: \mathrm{LOVE}(x_1, j)\} \subseteq \mathrm{HAPPY}$

In the above, the relative clause "who loves John" is represented by the set $\{x_1: \mathrm{LOVE}(x_1, j)\}$ with the variable $x_1$ taking the place of "who". This is equivalent to Carpenter (1997) and Dalrymple (2001)'s[21] treatment of relative clauses by λ-abstraction.

In (56) the subject contains an expletive noun "individual" which functions as a place-holder to introduce the following relative clause. If we replace it by a concrete noun such as "girl", then the subject has to be represented by the intersection of two sets, following Carpenter (1997) and Dalrymple (2001), as follows:

(58)
$$every(\mathrm{GIRL} \cap [j(–)]_2(\mathrm{LOVE}))(\mathrm{HAPPY})$$
$$\Leftrightarrow \mathrm{GIRL} \cap \{x_1: \mathrm{LOVE}(x_1, j)\} \subseteq \mathrm{HAPPY}$$

In the above example, the complex subject is represented by an intersection of two sets. In some situation, they have to be represented in another way. Consider the following sentence:

(59) Only players who made some mistake received no prize.

The above sentence is ambiguous between one of the following readings:

(60) Among all the individuals, only players who made some mistake

[20] The following discussion is also applicable to non-finite clauses and verbless clauses playing the same role as relative clauses.

[21] The theories of Carpenter (1997) and Dalrymple (2001) are based on different syntactic frameworks, namely Categorial Grammar and Lexical Functional Grammar, respectively. Yet they share some commonalities in their semantic treatment of relative clauses.

received no prize. (This implies that all other individuals, including non-players, received a prize.)

(61) Among all the players, only those who made some mistake received no prize. (This says nothing about whether non-players received no prize.)

Under reading (60), the complex subject "players who made some mistake" of (59) should be represented by an intersection. Moreover, since (59) also contains an object, this sentence may be represented by a left-and-right-iterated GQ:

(62) $$only(\text{PLAYER} \cap [some(\text{MISTAKE})]_2(\text{MAKE}))([no(\text{PRIZE})]_2(\text{RECEIVE}))$$

In this representation, PLAYER only appears in the left argument of "*only*".

But under reading (61), the complex subject cannot be so represented. Here the phrase "among all the players" serves to restrict the arguments of the GQ to subsets of PLAYER. Thus, PLAYER should appear in both arguments of "*only*". For convenience, I introduce the notation of "restriction":

(63) Let Q be a monadic GQ with n arguments and S be a set. Then $(Q|S)(X_1, \ldots X_n) \Leftrightarrow Q(X_1 \cap S, \ldots X_n \cap S)$.

With this notation, we can then represent reading (61) by

(64) $$(only|\text{PLAYER})([some(\text{MISTAKE})]_2(\text{MAKE}))([no(\text{PRIZE})]_2(\text{RECEIVE}))$$

The main difference between (62) and (64) is that in the former PLAYER is part of an argument of the GQ, whereas in the latter it serves as a parameter of the GQ.

Note that the above ambiguity is mainly due to the fact that "*only*" is not right conservative, resulting in the non-equivalence of (62) and (64). If Q is a right conservative determiner, then we have $Q(S \cap A)(B) \Leftrightarrow Q(S \cap A)(S \cap A \cap B) \Leftrightarrow Q(S \cap A)(S \cap B) \Leftrightarrow (Q|S)(A)(B)$. Thus, for right conservative determiners, the two aforesaid ways of representing complex subjects are equivalent.

## 1.8 Conclusion

In this chapter, I have introduced the core type of GQs studied under GQT – right-oriented GQs and extended the study to left-oriented GQs which are less studied in the literature. Although GQs only correspond to a restricted set of syntactic categories, namely NPs and determiners, the findings of GQT researchers are in fact applicable to other syntactic categories that can be treated as quantifiers in different types of domains. For example, modals and adverbs of quantification can be treated as quantifiers in the possible worlds domain and temporal domain, respectively. This is exactly the approach taken in Chow (2006).

In this thesis, I will take another approach, i.e. extend the study on GQs to scalar reasoning. However, the association between GQT and scalar reasoning is not that they correspond to quantifiers defined on different types of domains, but that quantifier inferences and scalar reasoning share some common features. However, before talking about their commonalities, I have to provide background knowledge of scalar reasoning first, which is the topic of the next chapter.

## Chapter 2 Scalar Reasoning: Logical and Linguistic Aspects

### 2.1 Overview of Scalar Reasoning

In this thesis, the term "scalar reasoning" is used to refer to a number of phenomena related to pragmatic reasoning of scales. These phenomena arose in different periods under different subfields of linguistics.

The first phenomenon that may be categorized under scalar reasoning is the implicatures, i.e. non-literal meanings, associated with scales. Such implicatures are thus called "scalar implicatures". Scalar implicatures have close relationship with quantifier inferences because a typical instance of this kind of implicatures as exemplified below is built up on a pair of GQs: "*some*" and "*every*"[22]:

(1) Some student sang. +> Not every student sang.

But scalar implicatures have much wider applications that go beyond scales composed of GQs.

Apart from implicatures, some scholars studied another type of pragmatic inferences, called "scalar entailments" by Kay (1990), which are more like logical entailments. Such kind of inferences was first studied by Fauconnier (1975). Later, advocates of Construction Grammar systematized Fauconnier's idea and formulated the Scalar Model Theory (SMT).

Scalar reasoning is also useful for studying certain lexical items (henceforth "scalar lexical items"). For example, SMT is mainly used to study "scalar operators", which are lexical items whose meanings are built up on scalar entailments, including "let alone", "even", "even if", "at least" and a number of aspectual adverbs. Apart from scalar entailments, some scholars also studied scalar implicatures of scalar operators. Some other scholars applied SMT to lexical items denoting extreme values, such as maximizers and minimizers.

---

22 In this thesis, I use "+>" to represent "implicates".

In Chinese grammatical studies, the equivalents of the aforesaid scalar operators (e.g. “shenzhi”, “hekuang”, etc.) and extreme values are usually put under the grammatico-semantic study of climax constructions or certain idiomatic constructions. Traditionally, Chinese scholars did not use the concept of scales. But in recent years, Chinese scholars also started to use scales or even apply SMT to study individual lexical items.

Some other scholars studied Chinese scalar operators, aspectual adverbs (e.g. “cai”, “jiu”) and extreme values from a new perspective called “subjective quantity”. Although these scholars did not use the framework of SMT, it can be shown (in Chapter 4) that the phenomenon of subjective quantity is indeed a manifestation of scalar reasoning.

In the following sections, I will introduce the basic notions and previous findings obtained in the studies of the aforesaid phenomena. I will also point out some outstanding problems and debates between different schools in the studies of specific topics.

## 2.2 Different Notions of Scales

There are different notions of scales in linguistic studies. In this section, I will introduce several notions of scales that are relevant to the study of scalar reasoning. The first notion comes from scalar implicatures associated with certain scales studied by Horn (1984, 1989). These scales are thus called “Horn scales”. A Horn scale can be represented as a tuple of scalar terms $<x_1, x_2, \ldots>$ satisfying $x_j \Rightarrow_u x_i$ for all i, j such that $i < j$, where “$\Rightarrow_u$” represents a generalized notion of unilateral entailment definable between n-ary predicates of any n [23], i.e.

[23] For example, if A and B are binary predicates, then $A \Rightarrow_u B$ is defined as $A(x, y) \Rightarrow_u B(x, y)$ for any arbitrary x, y. Note that propositions can be seen as 0-ary first order predicates, while determiners can be seen as binary second order predicates.

“$x_j \Rightarrow_u x_i$” roughly means “$x_j \Rightarrow x_i$ but not vice versa” (a more rigorous definition for “$\Rightarrow_u$” will be provided in Chapter 4). For example, the following determiners which are defined within the domain {<A, B>: A ≠ ∅} form a scale:

(2) <*some*, *all*>

Note that in certain scales, the lower valued scalar terms should be understood to have an “at least” meaning, because otherwise the aforesaid unilateral entailment relation would not hold. For example, in a scale <warm, hot>, “warm” should be understood to be meaning “at least warm”, because otherwise we would not have hot $\Rightarrow_u$ warm.

After Horn, Hirschberg (1985) generalized the notion of scales to relations in any partially ordered sets (posets)[24]. These scales are thus called “Hirschberg scales”. For example, the following is a scale of qualities of a hotel:

(3) <noisy and uncomfortable, noisy but comfortable,
quiet and comfortable >

Note that the Hirschberg scale (3) differs from the Horn scale (2) in that there is no logical entailment relation among the elements of (3). For example, a hotel being quiet and comfortable does not entail its being noisy but comfortable. By generalizing the notion of scales, Hirschberg (1975) has expanded the applicability of scalar implicatures.

Studies on scalar entailments (i.e. SMT) also talk about scales. According to Israel (2011), these scales are also posets and so are essentially the same as Hirschberg scales. The only difference is that SMT scholars have generalized the notion of scales from one-dimensional to multi-dimensional ones.

Although Horn scales and Hirschberg scales differ from each other, they are

[24] A poset is a set with a partial order, i.e. a binary relation that is reflexive, antisymmetric and transitive.

not unrelated. In the following section, I will show the inherent association between the Horn scales and Hirschberg scales after introducing the basic notions of scalar entailments.

## 2.3 Scalar Entailments

### 2.3.1 Scalar Model Theory

Scalar entailments (SEs) refer to entailments associated with a scalar model (SM). In this subsection, I first introduce the basic notions of SMT. According to Kay (1990), an SM is a quadruple $<S, T, D_x, P>$, where S is a set of states of affairs, T = {0, 1} is a set of truth values, $D_x = D_1 \times \ldots \times D_n$ is the Cartesian product of scales $D_1, \ldots D_n$ and P is a propositional function which maps a member of $D_x$ to a proposition (which in turn can be seen as a function mapping S to T).

Kay (1990) illustrated the above definition with a two dimensional example in which S is about the ability of a set of jumpers who try to clear a set of obstacles. The jumpers and obstacles are represented by the following scales:

(4) $X: <x_1, x_2, \ldots>; Y: <y_1, y_2, \ldots>$

Here the elements of X (jumpers) are arranged in decreasing jumping ability (or equivalently, increasing clumsiness) while the members of Y (obstacles) are arranged in increasing difficulty. Thus S can be represented by the following propositional function:

(5) P(x, y) = "Jumper x can clear obstacle y"

where x and y are variables from X and Y, respectively and P(x, y) is a proposition that may be true or false. Moreover, P must satisfy the following property: whenever a certain jumper can clear a certain obstacle, then other things being equal, any better jumper can clear any easier obstacle, but not vice versa. Conversely, whenever a certain jumper cannot clear a certain obstacle, then other things being equal, any worse jumper cannot clear any harder obstacle, but not vice versa. The above example may be depicted by the following figure:

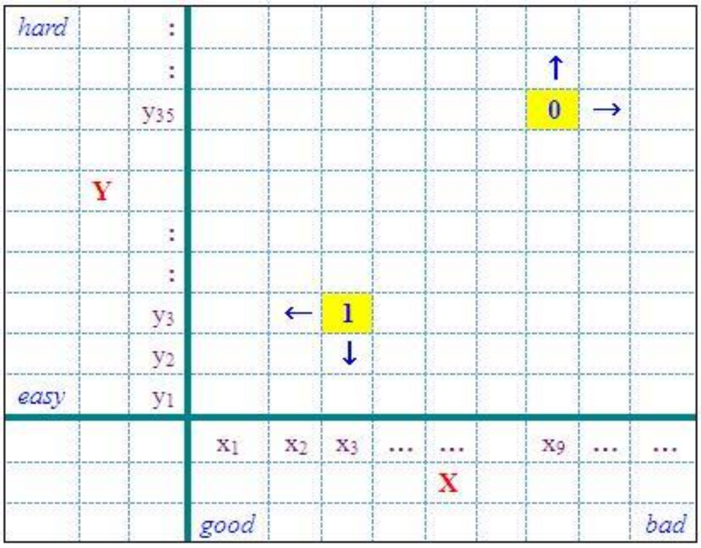


**Figure 2.1 A Scalar Model**

Each cell in the above figure represents the proposition obtained by substituting a member of X × Y into P. The number 0 or 1 is the truth value of the proposition. Whenever a cell contains "1", then all cells whose x- and y-coordinates are not greater than that cell also contain "1"; whenever a cell contains "0", then all cells whose x- and y-coordinates are not smaller than that cell also contain "0". According to the above property and figure, we can derive the following SEs:

(6) Jumper $x_3$ can clear obstacle $y_3$. $\Rightarrow_u$ Jumper $x_2$ can clear obstacle $y_2$.

(7) Jumper $x_9$ cannot clear obstacle $y_{35}$.

$\Rightarrow_u$ Jumper $x_{10}$ cannot clear obstacle $y_{36}$.

The above entailments are pragmatic in nature. Scalar entailments differ from logical entailments in that the latter are analytical in nature, i.e. the validity of logical entailments is based on the definitions of the logical operators and general laws of logic, whereas the validity of SEs is also dependent on world knowledge.

In the above example, the relevant world knowledge is the likelihood of jumpers clearing obstacles, and the SEs above can be seen as the following reasoning pattern about likelihood:

(8) Let p and q be two propositions in an SM such that p is less likely than q. Whenever p is true, then other things being equal, q is also true, but not vice versa. (Whenever q is false, then other things being equal, p is also false, but not vice versa.)

Note that the final sentence above is not necessary, because it can in fact be derived from the previous sentence by contraposition.

But "likelihood" is just one possible type of world knowledge that may feature in an SM. To make the theory more generally applicable, Kay (1990) proposed the concept of "informativeness":

(9) Let p and q be two propositions in an SM. Then p is more informative than q iff $p \Rightarrow_u q$.

Now according to (8), if p is less likely than q, then $p \Rightarrow_u q$. By (9), this is equivalent to saying that p is more informative than q. Following a similar line of reasoning, if p is more likely than q, then p is less informative than q. Thus, we have the following result:

(10) In an SM whose informativeness is reflected by the likelihood of the propositions, informativeness is inversely proportional to likelihood.

One can then interpret the SEs in (6) and (7) in terms of informativeness. For example, since "jumper $x_3$ clearing obstacle $y_3$" is less likely than "jumper $x_2$ clearing obstacle $y_2$", by (10) the former is more informative than the latter, and so by (9), the former unilaterally entails the latter, which is exactly what (6) asserts.

The validity of SEs is subject to the condition "other things being equal". Similar situations can also be found in other linguistic phenomena. For example, many scholars pointed out that the meaning of the English progressive aspect contains an "other things being equal" condition as exemplified in the following sentence[25]:

(11) Mary was baking a cake, but she didn't finish it.

i.e. had other things been equal, Mary would have finished the cake; but since other things were not equal, she didn't finish it. Borrowing ideas from Talmy (2000)'s Force Dynamic Schema, Copley and Harley (2011) proposed a framework which incorporates forces[26] in its ontology, and used it to account for the "other things being equal" condition of the progressive aspect and other associated linguistic phenomena.

It is also possible to include the treatment of SEs into this framework. Using the notion of forces, we can then interpret SEs as results of comparison between forces. However, since this will involve fundamental changes of the ontology, I will not pursue this approach. Instead, I will adopt an approach that is based on the proportionality relation between the scalar terms and the informativeness of the proposition in Chapter 4.

### 2.3.2 Relation between Horn Scales and Hirschberg Scales

Having introduced SEs, we can now develop a unified view towards Horn scales and Hirschberg scales. As mentioned above, while Horn scales are based on logical entailment relations, Hirschberg scales are based on the broader

[25] Copley and Harley (2010), (15a).

[26] According to Copley and Harley (2011), "forces" can be defined as a function mapping situations to situations, where a situation is a collection of individuals and their properties. Note that this notion of "forces" is unrelated to the "illocutionary force" in Speech Act Theory.

concept of partial order. Despite this difference, we can say that Hirschberg scales are also based on entailments, if we extend the scope of entailments to include SEs, because SEs depend on the order relation of the scalar terms, which is exactly what Hirschberg scales reflect.

For example, let's consider the following example[27]:

(12) This hotel's noisy, but at least it's comfortable.

In anticipation of the analysis to be introduced in a later section, the proper use of "at least" above is subject to the condition that

(13) "This hotel is noisy but comfortable" is more informative than "This hotel is noisy and uncomfortable".

The Hirschberg scale (3) associated with this example is based on a partial order of qualities of a hotel. Now the key point in this example is not that the first proposition in (13) is less likely than the second one, but that the first proposition is more desirable than the second one. Here we have an SM whose informativeness is reflected by the "desirability" rather than the "likelihood" of the propositions. Note that we may as well express (13) as the following SE:

(14) This hotel has attained a quality level of no less than "noisy but comfortable" in the scale (3). $\Rightarrow_u$ This hotel has attained a quality level of no less than "noisy and uncomfortable" in the scale (3).

where the scale of "quality level" is in fact a scale of desirability. Generalizing the above, we have the following relation (c.f. (10)):

(15) In an SM whose informativeness is reflected by the desirability of the propositions, informativeness is directly proportional to desirability.

This example shows that Horn scales and Hirschberg scales can be treated uniformly under SMT via the notion of informativeness. Thus, in what follows I

27 Kay (1997), (28), p. 108.

will base the order of terms of a scale on informativeness, i.e. a scale is a tuple of predicates <$x_1$, $x_2$, …> such that $x_j$ is more informative than $x_i$ for all i, j satisfying $i \leq j$.

## 2.4 Scalar Implicatures

### 2.4.1 Grice's Quantity-1 Submaxim

Scalar implicatures (SIs) are a subtype of conversational implicatures associated with pragmatic scales. In proposing the Cooperative Principle, Grice (1975) argued that our conversational exchanges involve a process of pragmatic inferences through interaction among 4 maxims of conversation (Quantity, Quality, Relation and Manner) with a number of submaxims under the Cooperative Principle. Among these, the following Quantity-1 Submaxim is most relevant to SIs[28]:

(16) Make your contribution as informative as it is required (for the current purposes of the exchange).

A classical example of SI is the following:

(17) Some student sang. +> Not all students sang.

In the above, "p +> q" means that the utterance p conveys a non-literal meaning represented by q. Using (16), we can account for the above SI in the following way: since by virtue of the scale (2), "All students sang" is more informative than "Some student sang" (according to the definition of "informativeness" given in (9)), assuming that the speaker is cooperative, when he / she utters the latter, one can infer that he / she is not in a position to utter the former (otherwise by (16) he / she should utter the former which is more informative), and thus it follows that the former is not true and the SI obtains.

[28] Grice (1975), p. 45.

One important characteristic of implicatures is that they are defeasible. According to the literature, the defeasibility of SIs is manifested by at least two properties. First, SIs are cancellable[29], i.e. a sentence with SI may be followed by a sentence which asserts the opposite of the SI without causing contradiction. For example,

(18) Some student sang yesterday. In fact / Actually, all of them did.

Second, SIs are reinforceable, i.e. a sentence with SI may be followed by a sentence which explicitly asserts the SI without causing redundancy. For example,

(19) Some student sang yesterday. But not all of them did.

The defeasibility of implicatures also means that given the literal meaning of two propositions represented by p and q, $p \wedge q$ is not necessarily false. Otherwise, p and $\neg q$ will satisfy the relation of entailments instead of implicatures. For example, if we replace "Some student sang" in (17) by "No student sang", then since "No student sang $\wedge$ all students sang" is necessarily false, what we obtain is an SE instead of an SI:

(20) No student sang. $\Rightarrow_u$ Not all students sang.

Thus, defeasibility can be seen as a condition that differentiates SIs from SEs and will be called "defeasibility condition" in this thesis.

Apart from the defeasibility condition, scholars have proposed various constraints on the scales associated with SIs. Matsumoto (1995) reduced these constraints into two general conditions: Conversational Condition and Scalarity Condition. In brief, the Conversational Condition states that a scale does not

[29] Not all scholars adopt the view that SIs are cancellable and reinforceable. According to these scholars, SI cancellation and SI reinforcement are interpreted as other phenomena. But for convenience of reference, I will continue to call these examples "SI cancellation" and "SI reinforcement", which should be seen as names for certain types of sentences.

license an SI "p +> ¬q" if the speaker's choice of uttering p instead of q is attributable to the avoidance of violating any maxim of conversation other than the two Quality Submaxims[30] and the Quantity-1 Submaxim. This condition precludes <Japan, Tokyo> from being an SI-licensing scale in a conversation about the countries that one has visited because the utterance of a city name "Tokyo" in such a conversation is more informative than is required and will thus violate the Quantity-2 Submaxim[31]. The Scalarity Condition states that the terms in an SI-licensing scale must be all increasing or all decreasing. This condition precludes <*some*, (*some but not all*)> from being an SI-licensing scale because "(*some but not all*)" is non-monotonic[32]. In this thesis, I assume that all scales associated with SIs satisfy the above conditions.

### 2.4.2 Epistemic Force

The studies on SIs also involve the notion of "epistemic force". This concerns whether the implicature is presented as a fact or as a description of the speaker's knowledge / belief, which may be strong, weak or nil (i.e. ignorance). In the literature there is not a uniform epistemic force applicable to all SIs. Van Rooij and Schulz (2004) have identified five possible epistemic forces. Developing a theory that fully incorporates all these possibilities will involve using modal operators.

To avoid complicating matters, unless otherwise stated, I will assume that the speaker's knowledge with respect to the subject matter is complete and so we may disregard the weak and ignorant implicatures. Moreover, I will assume that

[30] Quality-1 Submaxim: "Do not say what you believe to be false." Quality-2 Submaxim: "Do not say that for which you lack adequate evidence."

[31] Quantity-2 Submaxim: "Do not make your contribution more informative than is required."

[32] The definitions of increasing and decreasing monotonicities can be found in Subsection 3.2.1 of Chapter 3. The proof that "(*some but not all*)" is (right) non-monotonic can be found in Subsection 3.3.5 of that chapter.

the SIs generated are statements with the speaker's belief world as background. For example, the SI in (17) above should actually have the full form "The speaker believes that not all students sang". On such an understanding, we may then drop the phrase "The speaker believes that" associated with SIs, just as we may drop the phrase "other things being equal" associated with SEs on the understanding that the validity of SEs is subject to this background condition.

An advantage of adopting such an approach is that we may treat SEs and SIs uniformly using the concept of informativeness. The following figure is a schematic representation of SE and SI properties:

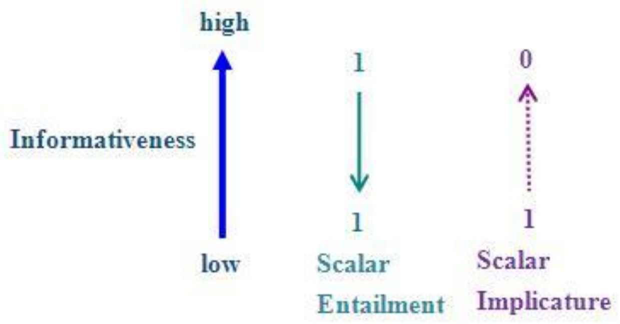


**Figure 2.2 Schematic Representation of SEs and SIs**

Just like Figure 2.1, the numbers 0 and 1 above represent truth values. The above figure shows that SEs are inferences leading from the truth of a highly informative statement to the truth of a lowly informative statement, whereas SIs are inferences leading from the truth of a lowly informative statement to the falsity of a highly informative statement.

### 2.4.3 Canonical SIs and Alternate-Value SIs

Scholars after Grice studied different types of SIs, which come in two main types: canonical SIs and alternate-value SIs. Canonical SIs are the focus of scholars studying SIs. They are based on scales consisting of higher and lower values, i.e. ordered values, of an entity. For example, (17) above exemplifies a canonical SI based on the scale (2).

Alternate-value SIs are less mentioned by scholars, apart from Hirschberg (1985), who has made in-depth study on this kind of SIs. They are based on unordered alternate values of an entity. Consider the following[33]:

(21) A: Which of Chomsky's works has John read?

B: He has read *Syntactic Structures* (SS).

+> John has not read *Aspects of the Theory of Syntax* (ATS).

The above is an example of alternate-value SIs based on the alternate values of "Chomsky's works":

(22) {SS, ATS}

Since the alternate values in an alternate-value SI are unordered, they are represented by a set, instead of a tuple as in a canonical SI.

The above example is reminiscent of the classical immediate inferences involving the contrary relations, because SS and ATS are contrary to each other[34]. Apart from this, there is another type of alternate-value SIs. Consider the following[35]:

(23) A: So did you snarf all the cakes down?

B: I didn't eat the chocolate one.

+> B ate the cheese cake.

which is based on the following set of alternate values:

---

[33] Note that the following SI satisfies the defeasibility condition introduced in Subsection 2.4.1 because "John has read SS $\wedge$ John has read ATS" under the literal meanings of the two conjuncts is not necessarily false. One can check in a similar fashion that all other SIs studied in this thesis also satisfy the defeasibility condition.

[34] SS and ATS are contrary to each other because a particular work of Chomsky cannot be SS and ATS at the same time. But it does not mean that two propositions containing these two terms are necessarily contrary propositions. For example, the propositions "John has read SS" and "John has read ATS" are not contrary to each other because they can be both true.

[35] Adapted from Hirschberg (1985), Ch. 3, (63), p. 60.

(24) {chocolate, cheese}

Note that the above SI is reminiscent of the classical immediate inferences involving the subcontrary relation, because we get a positive conclusion from a negative premise. Hirschberg (1985)'s study shows that alternate-value SIs are related to certain inferences related to the classical square of opposition.

### 2.4.4 Defaultism and Contextualism

Scholars studying SIs may be classified into camps according to their views towards a certain aspect of SIs. In this subsection, I will introduce two camps who hold opposite views towards the general nature of SIs: the Defaultists and the Contextualists[36]. These two camps differ in whether they view SIs as a subtype of generalized conversational implicatures. According to Grice (1975), conversational implicatures may be classified into two types: generalized conversational implicatures (GCIs) and particularized conversational implicatures (PCIs). The main difference between these two is that GCIs are triggered by certain lexical items (such as the less informative items in a scale in the case of SIs) and do not rely on special contexts, whereas PCIs rely heavily on special contexts.

The Defaultists include Gazdar (1979), Horn (1984, 1989), Levinson (2000) and Zhang (2008), etc. Although these scholars have proposed different frameworks to account for implicatures, they share one common feature in that they all view SIs as a subtype of GCIs. Moreover, they also contend that GCIs are generated by default, hence the name "Defaultists". Levinson (2000) even went further by proposing the use of Default Logic as the theoretical basis for GCIs, although he has not really done the formal work, which was later

[36] The names of "Defaultists" and "Contextualists" are from Zondervan (2006).

accomplished by Zhang (2008).

The Contextualists hold the opposite view towards SIs. These scholars include Sperber and Wilson (1986), van Kuppevelt (1996), van Rooij and Schulz (2004), Carston (2004, 2012), Zondervan (2006), Sevi (2009), etc. A common feature of these scholars is that they view SIs as being generated only in certain appropriate contexts instead of by default.

Despite the aforesaid commonality, the Contextualists hold different views about the key factor that determines the appropriate context. For the Relevance Theorists (including Sperber and Wilson (1986), Carston (2004, 2012)), the key factor is "relevance", which they assumed to be the underlying principle governing all types of pragmatic inferences. The Relevance Theorists also proposed the concept of "explicatures" as opposed to implicatures. Explicature is the totality of what constitutes the truth conditional meaning of a speaker's utterance, and may include, in addition to those aspects traditionally put under semantic studies (such as referent assignment, disambiguation), such aspects traditionally put under pragmatic studies, such as meaning modulation. On the other hand, implicatures come in two sorts: implicated premises and implicated conclusions, with the former being extra premises inferred implicitly from the explicatures of the speaker's utterance and the latter being conclusions entailed by implicated premises and other explicatures[37]. Since the Relevance Theorists saw SIs as the outcome of "narrowing" (to be elaborated in subsection 2.4.6), which is a subtype of meaning modulation, they classified SIs as explicatures rather than implicatures.

As for the other Contextualists, they viewed SIs as arising from answers to

[37] Note that the relation between implicated premises and their implicated conclusion is entailment relation. But since an implicated conclusion is based on its implicated premises, which are defeasible propositions, it is thus defeasible and so can be seen as an implicature.

specific (implicit or explicit) questions in the contexts. These scholars have proposed two important notions that will be useful in this thesis: question under discussion (QUD) foci and strongly exhaustive answers.

The QUD model was proposed by Roberts (1996), who viewed discourse as a process of questioning and answering about world information and modelled the (implicit or explicit) questions involved in this process by QUDs. QUDs may have a hierarchical structure consisting of a series of structured sub-QUDs requesting partial information that build up step-by-step the whole body of information requested by the main QUD.

Zondervan (2006) borrowed the notion of QUD and argued that all Contextualist approaches might be unified under the single framework of QUD. Moreover, he proposed the following QUD Focus Condition for SIs[38]:

(25) An SI will arise in a sentence iff the scalar term (with which the SI is associated) is in a constituent that answers the QUD of the context that the sentence is part of, and therefore has focus.

By adopting the above condition, one can then account for the existence and non-existence of SIs in different contexts. Compare the following question-answer pairs (in what follows, I use $[\cdot]_F$ to denote the QUD focus)[39]:

(26) Who has fourteen children?

$[\text{Nigel}]_F$ has fourteen children. #+> Nigel has at most fourteen children.

How many children does he have?

He has $[\text{twenty}]_F$. +> Nigel has at most twenty children.

It is often said that a cardinal n carries the literal meaning "at least n" and an SI "at most n"[40]. But the above example shows that this SI is not generated in all

[38] Zondervan (2006), (57'), p. 30.
[39] Adapted from van Kuppevelt (1996), (9)', p. 406.
[40] This is a controversial point that is not agreed upon by all scholars. I cite this example only for

circumstances. This difference can be accounted for by using (25). In the first answer above, since “fourteen” is not inside the QUD focus, it does not carry the SI “at most fourteen”, whereas the second answer does carry the SI “at most twenty” because “twenty” is inside the QUD focus.

Zondervan (2006)’s QUD Focus Condition states that the answer to a QUD may give rise to SI. However, not all answers but only strongly exhaustive answers will have this effect. Strong exhaustivity is an important notion in Groenendijk and Stokhof (1984)’s interrogative semantics. It requires the answer to a question to contain all and only (i.e. exactly) the true information requested by the question. This requirement is expressed as an “exh” operator with a meaning similar to “only”[41]. For example, if John and Mary are exactly the ones who sang, then “Only John and Mary” or “John and Mary and nobody else” would be a strongly exhaustive answer to the question “Who sang”.

Some Contextualists (e.g. van Rooij and Schulz (2004), Sevi (2009)) borrowed and refined the idea of strong exhaustivity and used it to account for SIs. For example, in (17) above, the sentence “Some student sang” can be seen as a strongly exhaustive answer to the QUD “What proportion of the students sang” and is equivalent to:

(27) Only some student sang.

The SI generated in (17) is then a logical consequence of (27), because “Only some student sang” is incompatible with “All students sang”.

### 2.4.5 Globalism and Localism

---

illustrative purpose.

[41] The word “only” here should be understood to carry the prejacent presupposition, i.e. it is equivalent to “all and only” and is thus different from the left conservative GQ “*only*” discussed in Chapter 1.

Apart from the debate between the Defaultists and the Contextualists, there is another debate between the Globalists and the Localists, who differ in their views towards the interpretation of embedded SIs. SIs can be classified into simple SIs and embedded SIs. In this thesis, simple SIs refer to SIs of scalar terms that do not fall under the scope of any logical operator, whereas embedded SIs are SIs of scalar terms embedded in the scope of a logical operator, which may be the negation operator or a GQ (excluding singular terms, i.e. singular proper names and singular definite descriptions)[42]. Earlier scholars paid little attention to or simply denied the existence of embedded SIs. Gazdar (1979) maintained that no SI is generated if an SI-trigger is embedded under a logical operator. But some other scholars such as Horn (1989) and Levinson (2000) pointed out that negation will give rise to SIs that are associated with a reversed scale.

Recently, some scholars began to study more general embedded SIs. Roughly speaking, these scholars fall into two camps: the Globalists and the Localists. The Globalist approach, represented by Sauerland (2004), Geurts (2010) and Russell (2012), adopts the traditional view that the SI of a sentence is generated only after the semantics of the whole sentence is computed compositionally. Thus, the negation operator arising from an SI can only be applied globally to the denotation of the whole sentence. Sauerland (2004) proposed a systematic method for generating SIs of complex sentences. Formally, let $p(x_1, \dots x_i, \dots x_n)$ be a sentence containing the scalar terms $x_1, \dots x_i, \dots x_n$, from the scales $X_1, \dots X_i, \dots X_n$, respectively and $x_i$' be a value of the scale $X_i$

[42] Since singular terms are scopeless, no scalar term will be embedded under the scope of singular terms. Moreover, scalar terms may also be embedded under propositional attitudinal predicates, such as “believe”, “know”, etc. This thesis does not consider such kind of embedded SIs.

different from $x_i$. Then we have the following:

(28) If $p(x_1, \ldots x_i', \ldots x_n) \Rightarrow_u p(x_1, \ldots x_i, \ldots x_n)$, then $p(x_1, \ldots x_i, \ldots x_n)$ +> $\neg p(x_1, \ldots x_i', \ldots x_n)$.

For example, in the sentence[43]

(29) Every student completed some of the assignments.

"some" can be seen as a scalar term from the scale (2). If we replace "some" by "all", the resultant sentence unilaterally entails the above sentence. So we have the following SI:

(30) Every student completed some of the assignments.

+> Not every student completed all of the assignments.

The Localist approach, represented by Landman (1998), Chierchia (2004) and Recanati (2010), holds the view that SIs are generated at the same time when the meaning of a sentence is computed compositionally, either by default as proposed by Landman (1998) and Chierchia (2004), or by a pragmatic process that may affect the truth condition of an utterance as proposed by Recanati (2010). Thus, the negation operator arising from an SI can be applied locally to scalar terms at any level of the sentence. Again we may express this formally. Let $p(x_1, \ldots x_i, \ldots x_n)$ and $x_i$' be defined as above. Then we have the following:

(31) If $p(x_1, \ldots x_i', \ldots x_n) \Rightarrow_u p(x_1, \ldots x_i, \ldots x_n)$, then $p(x_1, \ldots x_i, \ldots x_n)$ +> $p(x_1, \ldots \neg x_i', \ldots x_n)$.

Concerning (29), the Localists will predict the following SI:

(32) Every student completed some of the assignments.

+> Every student did not complete all of the assignments.

There is a heated debate between the two camps. One criticism raised by the Localists against the Globalists is that the Globalist approach often leads to

[43] Adapted from Chierchia (2004), (35)c.

predictions that are too weak. Comparing (30) and (32), both SIs seem to be correct. But since the SI predicted in (32) entails that in (30), the Globalists' prediction is not sharp enough.

On the other hand, the Localists also point out that the Globalist scheme (28) will lead to predictions that are too strong and thus incorrect in some cases, e.g. when the scalar term is embedded under indefinite determiners, such as the following[44]:

(33) Last year, a Dutch sailor showed some of the symptoms of delirium.

#+> Last year, no Dutch sailor showed all the symptoms of delirium.

The above incorrect SI is generated by replacing "some" by "all" and then negating the whole sentence. In comparison, the Localist approach will predict the seemingly correct SI "Last year, a Dutch sailor did not show all the symptoms of delirium".

In reply to the first criticism, Sauerland (2004) pointed out that the weaker Globalist predictions are not an undesirable feature because which prediction is preferred often depends on our background knowledge. That (32) seems to be preferable to (30) is due to our background knowledge that no students would do more than is required of them. In contrast to (29), consider the following[45]:

(34) Every student at MIT has read some of Chomsky's works.

Based on the background knowledge that MIT students study Chomsky's works very seriously, the weaker Globalist SI "Not every student at MIT has read all of Chomsky's works" is more plausible than the stronger Localist SI "Every student at MIT has not read all of Chomsky's works".

In reply to the second criticism, Geurts (2010) adopted a completely new

---

[44] Geurts (2010), Ch. 7, (29), p.144.

[45] Adapted from Sauerland (2004), (58), p. 390.

view towards SIs embedded under indefinites. Borrowing the idea of "discourse referents" from Kamp and Reyle (1993)'s Discourse Representation Theory (DRT), he argued that any SI generated from the LHS of (33) should be a statement about the specific discourse referent introduced by "a Dutch sailor", not a statement about the general "Dutch sailors". Thus, the correct SI of (33) should be "He did not show all the symptoms of delirium", where "he" refers to the specific Dutch sailor. Geurts (2010) contended that his views could be implemented by a theory that puts the study of SIs under the DRT framework, although he has not fully developed the theory.

Apart from Kamp and Reyle (1993), Fodor and Sag (1982) have also studied the referential or specific use of indefinites. They proposed treating referential indefinites like demonstratives. In this way, referential indefinites are like proper names and definite descriptions, which correspond to specific members in the universe. How can we establish the correspondence between the indefinites and specific members in the universe? Reinhart (1997) and Winter (2001)'s answer is to use the choice function. The choice function will thus be another possible way to implement Geurts (2010)'s idea. More will be said about the choice function in Chapter 4.

Geurts (2010)'s view has an advantage over the Localist approach concerning the prediction of SIs embedded under certain non-monotonic indefinite determiners. Consider the following example[46]:

(35) Thirty nine senators supported most of the bills.

Under the Localist approach, the wrong SI "Thirty nine senators did not support all of the bills" will be predicted. Using the concept of discourse referents, the SI generated should be one concerning the discourse referent denoting the 39

[46] Geurts (2010), Ch. 7, (34)b, p.146.

senators, such as “Not all of them supported all of the bills”. The problem of the Localist approach is thus avoided.

### 2.4.6 Contrastive Construals

In the literature, there is a type of negation that is closely related to SIs. Here is an example (in what follows, capitalization represents stress):

(36) Not SOME student sang yesterday. ALL of them did.

Note that (36) is similar in form to the examples of SI cancellation in (18) and SI reinforcement in (19) and so I will treat (36), (18) and (19) as different manifestations of the same phenomenon[47]. There are different explanation and terminology for this phenomenon. One view, represented by Horn (1985, 1989), contends that the negation in (36) belongs to a subtype of metalinguistic negation involving scalar terms and will henceforth be called scalar metalinguistic negation (SMN)[48]. According to this view, SMN is a metalinguistic device for registering objection to the SI generated from a previous utterance. It is different from the ordinary truth-conditional negation and is characterized by stress on the negated term. Moreover, a literal interpretation of SMN will lead to contradiction. For example, since “all” entails “some”, the above sentence apparently contains a contradiction.

Another view, held by Geurts (2010), puts the phenomenon of SMN under a broader category called “contrastive construals”. Under this view, the scalar term SOME is construed contrastively with ALL in (36), i.e. it means the same as

[47] On the surface, (18) does not involve negation. But according to Sevi (2009), the lexical item “in fact / actually” that is often used to cancel SIs should be seen as a means to mildly correct a previous utterance. Thus, SI cancellation does involve implicit negation. Moreover, Horn (1985) also pointed out that SI cancellation and scalar metalinguistic negation can achieve the same effect.

[48] According to Horn (1985, 1989), metalinguistic negation is a broad concept that may involve negation of a variety of aspects. This thesis only deals with SMN.

“some but not all”. Geurts (2010) argued that contrastive construals are achieved through the pragmatic process called “narrowing”. This strategy is to narrow down the extensions of one or all of the lexical items in contrast by enriching their intensions, thereby sharpening their meaning and avoiding semantic oddity. For example, since “some but not all” is contrary to “all”, by construing “some” as “some but not all” in (36), the apparent contradiction is eliminated.

Contrastive construals may occur in different types of contexts, of which (36), (18) and (19) are just some special examples. In fact, contrastive construals may occur in every antonymy context, such as those identified by Jones (2002)[49,50]. Geurts (2010)’s examples of contexts where contrastive construals occur correspond to Jones (2002)’s antonymy types. For example, the following correspond precisely to Jones (2002)’s negated antonymy, comparative antonymy, coordinated antonymy and ancillary antonymy, respectively[51]:

(37) Around here, we don’t LIKE coffee, we LOVE it.

(38) I’d rather have a WARM bath than a HOT one.

(39) Is a parallelogram SOMETIMES or ALWAYS a square?

(40) If it’s WARM, we’ll lie out in the sun. But if it’s VERY WARM, we’ll

---

49 Based on a corpus study, Jones (2002) identified 8 types of antonymy contexts, namely, ancillary antonymy, coordinated antonymy, comparative antonymy, distinguished antonymy, transitional antonymy, negated antonymy, extreme antonymy and idiomatic antonymy.

50 Incidentally, Levinson (2000) also identified certain “intrusive constructions” (including comparatives, negatives and conditionals) in which narrowing may occur (although he did not use this term). Most of these constructions in fact correspond to Jones (2002)’s antonymy contexts. For example, comparatives and negatives correspond to Jones (2002)’s comparative and negated antonymies, respectively. For conditionals, some of them correspond to Jones (2002)’s ancillary antonymy, such as (Levinson (2000), Ch. 3, (24)b, P. 205):

If the USA won some of the Olympic medals, other countries must have got the rest.

while some of them contain words that directly entails (without contrast) the narrowed meaning of scalar terms, such as (Levinson (2000), Ch. 3, (24)a, P. 205):

If each side in the soccer game got three goals, then the game was a draw.

The word “draw” above directly entails the “exactly three” reading of “three”. Note that this word plays a similar role as “only” in the phrase “only some” which directly entails the “some but not all” reading of “some”. More studies are required to figure out the precise relationship between antonymy contexts and intrusive constructions.

51 Geurts (2010), Ch. 8, (43)a – d, p.187.

go inside and sit in front of the air-conditioner.

Now, according to Recanati (2010) and Carston (2012), narrowing is a kind of meaning modulation that gives rise to explicatures. By recognizing contrastive construals as an outcome of narrowing, Geurts (2010) has in effect classified contrastive construals as an example of explicatures[52]. Thus, under Geurts (2010)'s view, SIs and contrastive construals are two very different phenomena which should be classified as implicatures and explicatures, respectively. This view is in sharp contrast with the Relevance Theorists who contend that both SIs and contrastive construals should be classified as explicatures. This constitutes another debate among scholars studying SIs.

## 2.5 Scalar Operators

### 2.5.1 Basic Notions

Scalar operators (SOs) refer to lexical items whose meaning and use are to be accounted for with respect to scales. A number of SOs have been studied by scholars from the perspective of SMT. These include "let alone" studied by Fillmore et al (1988), "even" studied by Kay (1990), aspectual operators studied by Israel (1997), "even if" studied by Sawada (2003), "at least" studied by Kay (1997) and Nakanishi and Rullmann (2009), etc.

We need two more notions of propositions. The first notion, called text proposition (TP), is the proposition obtained after deleting the SO from the sentence originally containing that SO (and making other necessary grammatical adjustments). The second notion, called context proposition (CP), is another proposition in the SM which differs from TP just in the scalar value and perhaps

[52] Although Geurts (2010) did not use the term "explicatures", he explicitly stated that contrastive construals "are not implicatures, but honest-to-goodness truth-conditional effects" (Geurts (2010), p. 141). This is very similar to the Relevance Theorists' view on explicatures.

polarity. It functions as an alternative statement in contrast to TP in the context and may be either implicit or explicit. For example, in the following discourse,

(41) A: Can John clear obstacle $y_2$?

B: Sure. He can even clear obstacle $y_3$.

the TP and CP are "John can clear obstacle $y_3$" and "John can clear obstacle $y_2$", respectively.

With the above definitions, one can state the conditions of use of certain SOs. Here I use the term "conditions of use" instead of "truth conditions" because according to many linguists (such as König (1991)), what SOs contribute to the meaning of sentences is not the semantic but pragmatic aspects of meaning. But which aspect of pragmatics (presupposition, conventional implicature, conversational implicature or a mixture of these) they are concerned with is still under debate. For this reason, I borrow the term "conditions of use" from Recanati (2010) to refer to the condition of proper use of these items. Moreover, these conditions of use only reflect one facet of the meaning (which may be called "scalar meaning") of these items and should be seen as necessary conditions rather than sufficient conditions.

According to Kay (1990), the meaning of "even" can be expressed as the following condition of use in the form of an SE relation between TP and CP[53]:

(42) even: TP $\Rightarrow_u$ CP

Using (41) as an example, since the following SE relation is valid:

(43) John can clear obstacle $y_3$. $\Rightarrow_u$ John can clear obstacle $y_2$.

by (42) we may conclude that "even" is properly used in (41). The conditions of

[53] Different from the classical analysis of "even" (such as Karttunen and Peters (1979)'s), the following condition of use does not require that the TP be an extreme member in a likelihood scale. Since Kay (1990) has already discussed the rationale for this difference, I will not repeat his points here. Neither will I discuss the existential presupposition of "even" (if any), which is another controversial issue.

use of other SOs can be formulated and used in an analogous way.

### 2.5.2 "even" + Negation

SOs may interact with various logical operators. In this subsection, I will introduce the interaction between "even" and the negation operator, which has drawn the most attention and led to much controversy. Consider the following sentence:

(44) John cannot even clear obstacle $y_2$. (No doubt he cannot clear obstacle $y_3$.)

The problem of the above sentence is that although on the surface "even" comes after "not", the condition of use for the above sentence cannot be expressed as the negation of (42). In other words, the above sentence does NOT mean "It is not the case that John can even clear obstacle $y_2$".

There are two opposing approaches to the analysis of sentences like (44) – the Scope Approach and the Lexical Approach. The Scope Approach, represented by Karttunen and Peters (1979), Kay (1990) and Wilkinson (1996), analyses (44) as having the following structure[54]:

(45) $even(\neg(\text{"John can clear obstacle } y_2\text{"}))$

i.e. "even" has wider scope than "not" despite the surface structure of (44).

In contrast, the Lexical approach, represented by Rooth (1985), Rullmann (1997) and Giannakidou (2007), analyses (44) as having the following structure:

(46) $\neg even_{NPI}(\text{"John can clear obstacle } y_2\text{"})$

i.e. the "even" in (44) is a negative polarity item (NPI) "$\text{even}_{NPI}$" different than the ordinary positive polarity item (PPI) "even" in sentences like (41). Being a

---

[54] Here I tentatively represent "even" as a sentential operator ignoring the focus structure of an "even"-sentence. A better representation method will be proposed in Chapter 4.

different lexical item, "even$_{NPI}$" satisfies a different condition of use than (42), which may be formulated as:

(47) $$\text{even}_{NPI}: CP \Rightarrow_u TP$$

We can then use the old SE relation (43) and this new condition of use for "even$_{NPI}$" to account for the proper use of "even" in (44).

### 2.5.3 "even" and "at least"

The studies on "even" have led to much controversy because different scholars hold different views on the condition of use for "even". According to Kay (1990)'s view, "even" is used for comparing the TP with a CP and should satisfy (42). But according to the more traditional view, "even" is used for emphatic purposes by asserting the most informative (or equivalently the least likely) proposition among all the propositions in an SM and may be called an emphatic SO. The two views are not incompatible. In fact, the emphatic use of "even" is a special case of its more general use under Kay's view because if a proposition is the most informative, then it will entail all other propositions in the SM, including the CP, and so (42) is surely satisfied. But since the emphatic use is not the unique use of "even", I maintain that (42) is preferable as it can cover more cases.

That said, the emphatic use of "even" is nonetheless the most prominent use of this SO. Thus, in this subsection I will formulate an alternative condition of use that suits the emphatic use of "even" better. Moreover, I will also contrast "even" with another SO – "at least".

First, I contend that the traditional view that "even" asserts the most informative proposition should be modified. As pointed out by Sawada (2003), "even" may trigger SIs as shown in the following example[55]:

(48) Even if you study very hard, you won't be able to get an A in that class.

+> If you study much harder than "very hard" (e.g. Study very hard without sleep, study at the risk of one's life), you can get an A in that class.

In the above sentence, "You cannot get an A if you study very hard" is very informative. But the SI brings out an even more informative proposition "You cannot get an A if you study much harder than 'very hard' " which has been negated. Thus, even under the emphatic use, "even" does not necessarily assert the most informative proposition. For this reason, an alternative condition of use for the emphatic use of "even" should be:

(49) even: TP is extremely informative, though not necessarily the most informative.

I next consider "at least"[56]. This SO differs from "even" in that the informativeness of the SM associated with an "at least"-sentence is reflected by the desirability rather than likelihood of the propositions. Consider the following:

(50) At least John can clear obstacle $y_2$. (It's better than his just being able to clear obstacle $y_1$.)

the part after "it's better than" (i.e. the CP) represents a less desirable proposition than the TP[57]. Similar to (14) above, we can express this situation as the

[55] Sawada (2003), (21), (25), pp. 429 – 430. Note that what Sawada (2003) studied is in fact "even if". But since "even if" can be seen as a special use of "even" with condtional sentences, Sawada (2003)'s findings are applicable to simple "even"-sentences.

[56] Kay (1997) identified 3 different uses of "at least" – scalar, evaluative and rhetorical, whereas Nakanishi and Rullmann (2009) identified 2 – epistemic and concessive. The "at least" studied in this thesis corresponds to Kay (1997)'s evaluative "at least" and Nakanishi and Rullmann (2009)'s concessive "at least".

[57] Kay (1997) contended that an "at least"-sentence is associated with two CPs: a less desirable and a more desirable propositions. But according to Nakanishi and Rullmann (2009), the more desirable proposition should be seen as an SI generated by the "at least"-sentence (see example below). I thus only consider the less desirable proposition as the CP of "at least". This is in accord with the above treatment that only a proposition that has an SE relation with the

following SE:

(51) John's jumping capability is up to the level of no less than obstacle $y_2$.

$\Rightarrow_u$ John's jumping capability is up to the level of no less than obstacle $y_1$.

Based on the above TP-CP relation, we can then formulate the condition of use for "at least" as follows:

(52) at least: TP $\Rightarrow_u$ CP

Note that the above condition has the same form as condition (42) for "even". Moreover, like "even", "at least" can also trigger SIs, such as

(53) At least John can clear obstacle $y_2$. +> John cannot clear obstacle $y_3$.

However, this cannot be the whole story because otherwise we would have "at least" performing the same function as "even". But we know that these two SOs are very different. While "even" is emphatic, "at least" is attenuating, i.e. conveying extremely low informativeness. Thus, just like "even", I formulate an alternative condition of use for the attenuating use of "at least" as follows:

(54) at least: TP is very uninformative, but not the most uninformative.

Note that the above condition contains the phrase "but not the most uninformative" because according to Kay (1997) and Nakanishi and Rullmann (2009), "at least" is used to settle for less, and so although the TP of an "at least"-sentence is very uninformative, it is still more informative than at least one proposition (i.e. the CP) in the same SM. In summary, "at least" shares some commonalities with "even" in certain respects, but also performs an opposite rhetorical function in contrast to "even". In this sense, "at least" can be seen as a mirror opposite of "even"[58].

---

"even"-sentence is considered as a CP of "even".

[58] A number of scholars (e.g. Sawada (2006), Giannakidou (2007), Crnič (2011)) have studied the association and contrast between "even" and "at least". Among these scholars, Sawada (2006)

## 2.6 Climax Constructions

### 2.6.1 Canonical Climax Constructions

In Chinese grammar, there is a construction called "climax construction". Traditionally, climax constructions were studied as a subtype of complex sentences. Xing (2001) classified them under the category of "generalized coordinate complex sentences"[59]. According to Xing (2001), the most typical climax construction connective (CCC) "budan … erqie …" (≈ "not only … but also …") may be used simply to coordinate two items. Under this use, p and q in "budan p, erqie q" are put on an equal footing. We may call this the additive use of "budan … erqie …". However, there is another use of "budan p, erqie q" such that q denotes a larger scope or higher degree than p. This use distinguishes an order between p and q. Since order is a characteristic of scales, we may call this the scalar use of "budan … erqie …". It is this scalar use that is related to SMT. The most commonly used CCCs in Chinese and their approximate English equivalents are set out in the following table. One will find that the English equivalents of most CCCs are in fact SOs.

**Table 2.1 Chinese CCCs and Approximate English Equivalents**

| Chinese CCC | Approximate English Equivalent |
|---|---|
| lian_dou / lian_ye[60] / shenzhi | even |
| bieshuo / hekuang | not to mention / let alone |
| faner / fandao / dao | on the contrary |
| shangqie … hekuang … | even … not to mention … |
| budan … erqie … / budan … hai … | not only … but also … |

explicitly proposed that "even" and "at least" are mirror opposites of each other.

[59] Xing (2001) classified Chinese complex sentences into three broad categories, namely "generalized coordinate", "generalized causal" and "generalized contrastive". Climax constructions were put under the "generalized coordinate" category.

[60] The discontinuous particle "lian_dou / ye" is written in this way to mean that the focus of this particle is placed between "lian" and "dou / ye". Note that "lian", "dou" and "ye" are originally three different words with their own meanings.

| | |
|---|---|
| budan bu … faner …[61] | not only not … on the contrary … |

Chinese grammarians distinguish two types of climax constructions: canonical climax constructions which are used with most of the CCCs in the above table, and anti-climax constructions which are used with "faner" and "budan bu … faner …".

The above table gives us insight for identifying SOs in English. One may suspect that "not only … but also …" (the English equivalent of "budan … erqie …") as well as the closely related form "only" may also be SOs. In fact, Horn (1969) and König (1991) have pointed out that although "only" is basically non-scalar, it may convey scalar meaning in some circumstances. Zeevat (2009) also treated "only" and "even" on a par and grouped them together under the category of "mirative particles". The same can be said of "not only … but also …". For this reason, I will thus add "only" and "not only … but also …" (under their scalar use) to the inventory of SOs.

Traditionally, Chinese scholars did not use the concept of scales to study climax constructions, even under the scalar use. For example, Zhou (2003), a monograph on the climax constructions, made no mention of "scales". In recent years, Chinese scholars (e.g. Jiang (1998), Liu (2000), Shen (2001), Jiang (2003), Shyu (2004), Gong (2006), Yuan (2008), Jiang (2011)) began to use scales or even apply SMT to study individual CCCs, but not climax constructions as a whole. In Chapter 4, I will put CCCs and SOs under the common framework of SMT.

### 2.6.2 Anti-Climax Constructions

Yuan (2008) has studied Chinese anti-climax constructions. He pointed out

[61] The word "bu" here represents the negation morpheme and may be replaced by other Chinese words conveying the negative meaning such as "meiyou" (≈ "not yet"), etc.

that the meaning of such constructions involve the two notions of scales and contrast. Thus, in the construction “budan bu p, faner q”, q is not only contrary to p, but also denotes a larger scope or higher degree than ¬p. Consider the following example[62,63]:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| (55) | Jintian | wuhou | xia | le | yi | chang | lei | zhenyu, | budan |
| | today | afternoon | fall | ASP | one | CLS | thunder | shower | budan |
| | meiyou | liang | xia | lai, | tianqi | faner | geng | menre | le. |
| | not yet | cool | down | come | weather | faner | more | stuffy | PART |

*After the thunder shower this afternoon, not only hasn't it got cooler. Quite the contrary, it gets even more stuffy.*

In this sentence, “geng menre” is not only contrary to “liang xia lai”, but denotes a higher degree of discomfort than “meiyou liang xia lai”.

Yuan (2008) also pointed out that sometimes “faner q” can be used alone without “budan bu p”, provided that there is an appropriate presupposed clause in the context, i.e. a CP, to bring out the climax effect[64]. For example, in (55) above, the clause “budan meiyou liang xia lai” can be omitted. In this case, the presupposed clause would be the following:

| | | | | | |
|---|---|---|---|---|---|
| (56) | Yuan | yiwei | keyi | liangkuai | yixie. |
| | originally | expect | possible | cool | a bit |

*Supposedly it should get a bit cooler.*

Note that the above sentence, when put in contrast with the TP “tianqi faner geng menre le”, brings out the climax effect.

According to Zhou (2003), “faner” is used only to strengthen the contrastive

62 Adpated from Yuan (2008), (42), p. 116.

63 In this thesis, I use the following abbreviations for glosses of Chinese sentences: ASP = aspect marker, CLS = classifier, PART = particle, POSS = possessive marker.

64 According to Guo (1999), “faner” (and its variants “fandao”, “dao”, etc.) can be used as an ordinary contrastive particle without climax effect. I contend that this is a non-scalar use of “faner”.

mood and is not a necessary element in the anti-climax constructions. In fact, “budan bu p, faner q” can be seen as a negative version of “budan p, erqie q”. It thus comes as no surprise that “faner” can be replaced by “erqie / hai / lian_dou / shenzhi” in this construction, according to Zhou (2003). In this thesis, I will use “budan bu p, faner q” as a representative of all these variants.

## 2.7 Subjective Quantity

### 2.7.1 Abnormal SQ

Subjective quantity (SQ) is a concept in Chinese grammar, first proposed by Chen (1994) and further developed by Li (2000) and Li (2003), etc. This concept enables us to study the meaning of certain polysemous Chinese function words from a new perspective. In this thesis, I will focus on adverbs and conjunctions that can trigger SQ (henceforth “subjective quantity operators” (SQOs)). Consider the following sentences:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (57) | Ta | 20 | jin | dou | tiao | de | qi. |
| | he | 20 | catty | dou | lift | able | up |

*He can even lift up 20 catties.*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (58) | Ta | tiao | 20 | jin | dou | juede | lei. |
| | he | lift | 20 | catty | dou | feel | tired |

*He felt tired even though he only carried 20 catties.*

Without the SQO “dou” (≈ “even”[65]), the quantity phrase “20 jin” in the above two sentences does not indicate large or small quantities. We say that it denotes “objective quantities” in this case. But with “dou”, the above sentences convey subjective evaluation of the largeness (i.e. SQ) of the quantity phrase. Intuition

[65] Historically, “dou” with the meaning “even” evolved from the CCC “lian_dou”. As the meaning of “lian” became bleached, “dou” came to denote the meaning of the whole structure, especially when it is used as an SQO.

tells us that “20 jin” denotes large and small SQ in (57) and (58), respectively.

Li (2000) has identified four sources of SQ: abnormality, infection, direct assignment and hyperbolism. This thesis will study the first three of these. Abnormal SQ is contrary to the expected quantity[66], which refers to the expected or ideal quantity in the speaker’s mind. Whenever the quantity under discussion differs from the expected quantity, abnormal SQ will arise. In a nutshell, abnormal SQ comes from “unexpectedness”.

Note that the aforesaid concepts are reminiscent of certain concepts of SMT in that the sentence containing the quantity under discussion (excluding the SQOs) and the sentence containing the expected quantity are analogous to TP and CP, respectively. For this reason, I will extend the concepts of TP and CP used in SMT to the analysis of abnormal SQ. For example, the following may serve as a CP for (57):

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| (59) | Wo | yiwei | ta | zhi | tiao | de | qi | 15 | jin. |
| | I | expect | he | only | lift | able | up | 15 | catty |

*I expected that he can only lift up 15 catties.*

The most typical abnormal SQOs are “dou” and “hai” (≈ “still”). Li (2000) has identified the conditions under which a quantity phrase in a “dou / hai”-sentence denotes large / small SQ. His findings are summarized in the following table:

**Table 2.2 SQs denoted by “dou / hai”**

| **SQO** | $\mathbf{Q_d}$ | $\mathbf{Q_i}$ |
|---|---|---|
| dou / hai | small SQ | large SQ |

In the table above, $Q_d$ and $Q_i$ represent quantity phrases that are directly and inversely proportional to the likelihood that the sentential predicate can be

[66] Li (2000) in fact used the term “normal quantity”. Li (2003) changed it to the more general term “expected quantity”.

realized, respectively. This table shows that whether a quantity phrase in a “dou / hai”-sentence denotes large / small SQ depends on the proportionality relation between the quantity phrase and the sentential predicate[67].

Let me use (57) and (58) to illustrate the idea. In (57), the quantity phrase “20 jin” is inversely proportional to the likelihood of the sentential predicate “tiao de qi”, because the larger the weight, the less likely a person can lift it up. On the other hand, in (58), “20 jin” is directly proportional to the likelihood of the sentential predicate “juede lei”, because the larger the weight, the more likely a person carrying it will feel tired. Thus, “20 jin” is a $Q_i$ in (57) and a $Q_d$ in (58). According to Table 2.2, it denotes large and small SQ in (57) and (58), respectively, in conformity with our intuition.

### 2.7.2 Infected SQ

When a quantity phrase not denoting abnormal SQ is put in contrast with another quantity phrase denoting abnormal SQ in a sentence, the former quantity phrase may acquire SQ. This kind of SQ is called infected SQ. Li (2000)’s examples for infected SQ are mainly based on the prosody of the sentence, which I do not intend to study in this thesis. But Li (2000) also pointed out that “bieshuo / hekuang”, when used in contrast with “dou”, may denote SQs. I contend that such SQs are not abnormal SQs because “bieshuo / hekuang” does not denote unexpectedness. Consider the following example[68]:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| (60) | Lian | zhe | shi | lai | fenzhong | ta | dou | shou | bu | liao, |
| | lian | this | 10 | odd | minute | she | dou | withstand | not | PART |

[67] As a matter of fact, Li (2000) contended that whether a quantity phrase in a “dou / hai”-sentence denotes large / small SQ does not only depend on the proportionality relation but also the relative position of the quantity phrase wrt the word “dou / hai”. But in fact only the proportionality relation is relevant, and so Table 2.2 has simplified Li (2000)’s findings.

[68] Adapted from Li (2000), Ch. 4, [64], p. 142.

| | | | | |
|---|---|---|---|---|
| hekuang | ban | nian | zhi | qi. |
| hekuang | half | year | POSS | period |

*She cannot even withstand 10 odd minutes, let alone a half-year period.*

In the above sentence, “hekuang” is associated with the proposition “she cannot withstand a half-year period”, which is a normal state of affairs. The quantity phrase “ban nian zhi qi” by itself does not denote SQ, but since it is put in contrast with “shi lai fenzhong” which denotes small SQ, it acquires an infected large SQ.

### 2.7.3 “jiu” and “cai”

Li (2000) has also studied the SQ associated with two Chinese aspectual operators – “jiu”[69] and “cai”. Among the SQOs, “jiu” and “cai” are special in that they have the dual nature of both abnormal and directly assigned SQOs. On the one hand, they may trigger abnormal SQs. Consider the following minimal pair:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| (61) | Liang | ge | ren | jiu | zhuan | de | 40,000 | yuan. |
| (62) | Liang | ge | ren | cai | zhuan | de | 20,000 | yuan. |
| | 2 | CLS | person | jiu / cai | earn | get | | dollar |

*The two of them earn as much as $40,000.* /

*The two of them only earn $20,000.*

Intuitively, “liang” and “40,000 yuan” denote small and large SQs in (61) , while “liang” and “20,000 yuan” denote large and small SQs in (62), respectively. The aforesaid SQs are abnormal SQs because they can be seen as being derived by contrast with a CP such as the following:

[69] According to Liu et al (2001), “jiu” is ambiguous and is synonymous with “bian” (≈ “thereupon”) and “zhi” (≈ “only”) under different stress environment. In this thesis, I only consider the sense of “jiu” that is synonymous with “bian”.

| (63) | Wo | yuqi | liang | ge | ren | zhuan | 30,000 | yuan. |
|---|---|---|---|---|---|---|---|---|
| | I | expect | 2 | CLS | person | earn | | dollar |

*I expected that 2 persons earn $30,000.*

On the other hand, according to Chen (1994) and Li (2000), unlike other abnormal SQOs, “jiu” and “cai” trigger SQs mainly by way of the relative positions of the quantity phrases wrt “jiu / cai”. In this way, “jiu” and “cai” are like SQOs such as “zuzu” (≈ “fully”) that directly assign SQs to quantity phrases located at specific neighbouring positions.

Li (2000) has identified the conditions under which a quantity phrase in a “jiu / cai”-sentence denotes large / small SQ. His findings are summarized in the following table:

**Table 2.3 SQs denoted by “jiu / cai”**

| **SQO** | **$Q_l$** | **$Q_r$** |
|---|---|---|
| jiu | small SQ | large SQ |
| cai | large SQ | small SQ |

In the table above, $Q_l$ and $Q_r$ represent quantity phrases that are located on the left and right of “jiu / cai” in a sentence, respectively. This table shows that whether a quantity phrase in a “jiu / cai”-sentence denotes large / small SQ depends on the relative location of the quantity phrase wrt “jiu / cai”.

Let me use (61) and (62) to illustrate the idea. In (61), the quantity phrases “liang” and “40,000 yuan” are located on the left and right of “jiu”; whereas in (62), “liang” and “20,000 yuan” are located on the left and right of “cai”, respectively. According to Table 2.3, “liang” and “40,000 yuan” denote small and large SQs in (61); while “liang” and “20,000 yuan” denote large and small SQs in (62) respectively, in conformity with our intuition.

## 2.8 Extreme Values

### 2.8.1 Maximizers / Minimizers and Superlatives

Israel (1996, 2011) has tried to extend SMT to a comprehensive theory on polarity items (including PPIs and NPIs). Although his theory is elegant and systematic, the types of polarity items are various and their use is extremely complicated. It is thus doubtful that a theory solely based on scalar reasoning could be an adequate theory. As a matter of fact, in the literature there is a wide variety of theories based on very different notions. In addition to scalar reasoning, these notions include domain widening (Kadmon and Landman (1993)), monotonicities (Zwarts (1997), van der Wouden (1997)), (non-)veridicality (Giannakidou (1999)), resumptive quantification (Szabolcsi (2004)), etc. Israel (1996, 2011)'s theory is just one among the many on the market.

Nevertheless, Israel (2011)'s discussion on the scalar reasoning of two subtypes of polarity items – maximizers and minimizers is insightful. Maximizers / minimizers are scalar terms with extreme (maximal or minimal) values[70]. They are typically idiomatic constructions. In this thesis, I will focus on maximizers / minimizers and will not deal with other types of polarity items.

Maximizers / minimizers may be classified into two types according to their rhetorical purposes. Emphatic maximizers / minimizers are used for achieving emphatic or hyperbolic effects, whereas attenuating maximizers / minimizers are used for achieving understating or euphemistic effects. Israel (2011) defined two scalar properties with binary values: q(uantitative)-value (high or low) tells us whether the scalar term is a maximizer or a minimizer, and i(nformative)-value (emphatic or attenuating) tells us the rhetorical purpose of the scalar term. Using

[70] Israel (1996, 2011) treated maximizers / minimizers as "scalar operators". But in fact it is more appropriate to treat these items as "scalar terms", i.e. they should be seen as values of a scale rather than functions operating on the values of a scale.

these two values together with the logic of SEs, Israel (2011) was able to account for the polarities of maximizers / minimizers.

For attenuating scalar terms, Israel (2011) observed that attenuating maximizers and minimizers are NPIs and PPIs, respectively. This conclusion is valid for any one-dimensional scale in which a proposition with a higher scalar term entails a proposition with a lower scalar term. This kind of scale is called "canonical" by Israel (2011). Since the function of attenuating maximizers / minimizers is to make weak claims, they must make the propositions where they appear highly uninformative. Now a negated proposition with a maximal scalar term or an affirmed proposition with a minimal scalar term entails few or even no other propositions in the scale, and is, by (9), very uninformative. A pair of attenuating maximizer / minimizer (underlined) is exemplified below[71]:

(64) Stella is not <u>all that</u> clever.

(65) Stella is <u>sort of</u> clever.

For emphatic scalar terms, the situation is more complicated. Israel (2011) observed that in a canonical scale, emphatic maximizers and minimizers are PPIs and NPIs, respectively. Again, one can draw this conclusion by noting that the function of emphatic maximizers / minimizers is to make strong claims. So an affirmed proposition with a maximal scalar term or a negated proposition with a minimal scalar term entails many other propositions in the scale, and is, by (9), very informative. A pair of canonical emphatic maximizer / minimizer is exemplified below[72]:

(66) Julio spent <u>a king's ransom</u> on the party.

(67) He won't spend <u>a red cent</u> on your wedding.

[71] Israel (2011), Ch. 4 (5), p. 92.
[72] Israel (2011), Ch. 4 (13)a and (12)a, p. 97.

However, Israel (2011) also observed that there are emphatic maximizers and minimizers being NPIs and PPIs, respectively. This kind of maximizers / minimizers follow reasoning patterns of "inverted" scales, i.e. scales in which a proposition with a lower scalar term entails a proposition with a higher scalar term. Since the direction of reasoning is inverted, the roles of PPIs and NPIs are interchanged. A pair of inverted emphatic maximizer / minimizer is exemplified below[73]:

(68) She wouldn't kiss him for <u>all the tea in China</u>.

(69) But he somehow got Madonna to play for <u>peanuts</u>.

Israel (2011) accounted for the difference between canonical and inverted scales in terms of the participant roles played by the scalar terms. For example, although both "a king's ransom" and "all the tea in China" represent huge amount of valuables, the former is expense while the latter is reward in the respective sentences. It is this difference that gives rise to different scales. If a person is willing to spend a certain amount of expense, then other things being equal, he / she will also be willing to spend a smaller amount. So "a king's ransom" occurs in a canonical scale in (66). On the other hand, if a person is willing to do something for a certain amount of reward, then other things being equal, he / she will also be willing to do the same thing for a larger amount. So "all the tea in China" occurs in an inverted scale in (68). The polarities of various types of maximizers / minimizers found by Israel (2011) with English examples are summarized in the following table:

**Table 2.4 Israel's Typology of Maximizers / Minimizers**

| | **Maximizer** | **Minimizer** |
|---|---|---|
| **Canonical** | PPI | NPI |

[73] Israel (2011), Ch. 4 (12)b and (13)b, p. 97.

| | | |
|---|---|---|
| **Emphatic** | "a king's ransom" | "not … a red cent" |
| **Inverted Emphatic** | NPI "not … all the tea in China" | PPI "peanuts" |
| **Attenuating** | NPI "not … all that" | PPI "sort of" |

Superlatives function like emphatic maximizers / minimizers in many cases. Fauconnier (1975) pointed out that superlatives may be used to perform a function very similar to universal quantification, as exemplified by[74]

(70) Tommy will not eat <u>the most delicious</u> food.

which in effect means "Tommy will not eat any food". In fact, "the most delicious" can be seen as an inverted emphatic maximizer. If a person will eat food that is tasty to a certain degree, then other things being equal, he / she will eat food that is more tasty. So "the most delicious" occurs in an inverted scale in (70). Now negating a maximal scalar term in an inverted scale entails negating other lower scalar terms, and so (70) has the effect of universal negation.

### 2.8.2 Chinese Idiomatic Constructions with "yi"

Parallel to the relation between SOs and maximizers / minimizers, there is also a natural association between Chinese SQOs and idiomatic constructions with extreme numerals. As a matter of fact, Li (2000) has extended his studies on SQ to Chinese idiomatic constructions with minimal numerals. In Chinese, "yi" (≈ "one") and "ban" (≈ "half") are the smallest numerals. In addition, some traditional measure words denoting very small amounts (e.g. "hao") and bare classifiers (i.e. classifiers without numerals in front, meaning "one") can also perform the function of "yi". In what follows, I use "yi" to represent all these words.

[74] Fauconnier (1975), (1), p. 353.

Li (2000) has discussed a number of idiomatic constructions in which "yi" may appear. These constructions may be grouped into two types. The first type has the main form "yi" + negation. Here are some examples of idioms in this form:

(71) bu kan yi ji

not endure one strike *cannot withstand a single blow*

(72) zhi zi wei ti

CLS word not yet mention *not utter a word*

Note that "yi" in this form can be analysed as a canonical emphatic minimizer using Israel (2011)'s framework as introduced in the previous subsection.

The second type has the main form "yi … jiu …", which is a very common construction in Chinese, as exemplified by the following sentence:

(73) Ta yi kan jiu ming.

he one see jiu understand

*He could understand by glancing through just once.*

This form is also exemplified in many idioms where "jiu" is omitted or replaced by other adverbs with similar meaning, as in the following:

(74) yi ming jing ren

one sound amaze people

*amaze the world with a single brilliant feat*

Note that the appearance of "yi" in this form conforms to Li (2000)'s theory on the SQ associated with "jiu", because according to Table 2.3, a quantity phrase appearing on the left of "jiu" denotes small SQ, which is exactly what "yi" should denote.

## 2.9 Conclusion

In this chapter, I have introduced and commented on a number of theories dealing with different aspects of scalar reasoning. In some cases, I have also presented my preliminary views on these theories. However, there are still a number of outstanding problems.

The theories introduced above were developed independently. Each is based on its own terminology and principles. Yet the various phenomena studied under these theories are all related to scalar reasoning. Can we develop a new framework on scalar reasoning that can deal with these phenomena in a uniform way?

As pointed out in Section 2.3, SEs are pragmatic reasoning different from logical entailments. However, under the assumption of "other things being equal", the robustness of SEs is indeed comparable to logical entailments. Can we discover more parallelism between the two types of entailments?

In Section 2.4, I have discussed the merits and demerits of different schools on different aspects of SIs (as well as contrastive construals). While I am sympathetic to a certain school in respect of a particular aspect, how can the theories of the various schools on different aspects be integrated to provide a complete and consistent account for the various types of SIs (including canonical and alternate-value SIs, simple and embedded SIs) and contrastive construals as well as provide answers to the debates among these schools?

As shown in Sections 2.5 – 2.8, although different types of scalar lexical items were studied independently under different theories, these theories are to a certain extent complementary to each other. First, there is a correspondence between the SOs studied under SMT and the CCCs studied under Chinese grammar. In fact, Chinese scholars have started to use SMT to study individual CCCs. Second, SMT and the theory on SQ also share some commonalities. For example, the notion of "proportionality relations" used by Li (2000) can in fact be seen as an alternative manifestation of the notion of "participant roles" used by Israel (2011). For instance, while the difference between "a king's ransom" and "all the tea in China" in (66) and (68) is seen as different participant roles under Israel (2011)'s theory (i.e. "a king's ransom" is expense for "Julio spent" while "all the tea in China" is reward for "she would kiss him"), the difference is seen as different proportionality relations under Li (2000)'s theory (i.e. "a king's ransom" is inversely proportional to "Julio spent" while "all the tea in China" is directly proportional to "she would kiss him"[75]). Given these commonalities, can we develop a unified theory for all these scalar lexical items?

Finally, although SEs and SIs are different types of reasoning, it has been shown in Subsection 2.5.3 that "even" and "at least" can both trigger SEs and SIs. What does this phenomenon tell us about the relationship between SEs and SIs? How are they manifested in the scalar lexical items?

My approach to solving the aforesaid outstanding problems is to gain insights from the studies of logical inferences, especially quantifier inferences. In fact, as shown in (17), the classical examples of SIs are about quantifiers. This shows that quantifier inferences have an inherent association with scalar reasoning. Thus, to solve the outstanding problems raised in this chapter, we first need to have a thorough understanding of the inferential patterns of GQs, which is the topic of the next chapter.

[75] The larger the sum, the less Julio is willing to spend. The larger the amount, the more she is willing to kiss him for it.

# Chapter 3 Inferential Patterns of Generalized Quantifiers

## 3.1 Introduction

In this chapter, I will study 4 main types of quantifier inferences, namely

1. monotonicity inferences;
2. argument structure inferences;
3. opposition inferences; and
4. (non-classical) syllogistic inferences.

The choice of these 4 types of inferences is not arbitrary. Right from the beginning of the linguistic stream of GQT, monotonicities of GQs have been one of the foci in GQT research. Later, some scholars (e.g. van Benthem (1986), Sanchez Valencia (1991)) studied monotonicity inferences of sentences with complex quantifier structures and called such kind of study “Natural Logic”[76]. In the 21st century, some other types of inferences were also studied as special types of “Natural Logic”. These latter types of inferences can all be seen as modern versions of classical logical inferences, among which syllogisms are the most important ones. In recent years, some scholars (van Benthem (2008), MacCartney (2009), Icard (2012), Mineshima et al (2012)) added inferences involving “exclusion relations” to the list of “Natural Logic” inferences. Exclusion relations are in fact a generalization of the opposition relations defined on the classical square of opposition. Also included in the category are inferences involving the concepts of negation and duality, which I will call “duality inferences” and inferences involving transposition of arguments or quantifiers, which I will call “transposition inferences”. These two kinds of inferences are in

[76] “Natural Logic” is a loose term without rigorous definition. Despite this, the inferences categorized under “Natural Logic” share some common features in that they are all inferences of quantified statements and can be studied using the theoretical tools of GQT. For this reason, I choose these inferences for study.

fact modern version of the classical eductive inferences[77], and will be collectively called argument structure inferences.

Although the aforesaid 4 types of inferences are studied separately, they are intimately related. Using monotonicity inferences as a reference point, the other 3 types of inferences can be seen as related to monotonicity inferences in different aspects. First, the argument structure inferences studied in Section 3.3 are inferences involving negation and / or transposition of the quantifiers or their arguments, where negation and transposition are both manipulations of the quantifier argument structures. Similarly, monotonicity inferences can be seen as involving two other types of manipulations of the quantifier argument structures, i.e. expansion (to a superset) and contraction (to a subset). Second, the opposition inferences studied in Section 3.4 are inferences involving the contradictory, contrary and subcontrary relations, which are the three core relations defined on the classical square of opposition. On the other hand, monotonicity inferences can be seen as involving the fourth relation on the square of opposition, i.e. the subalternate relation[78]. Third, as will be pointed out in Section 3.5, monotonicity inferences can be seen as extension of the classical syllogisms. Moreover, many modern scholars have also discovered various types of non-classical syllogisms. Thus, the non-classical syllogisms studied in Section 3.5 and monotonicity inferences can both be seen as extension of the classical syllogisms, albeit in different directions.

[77] Eductive inference is one of the two main types of immediate inferences in Classical Logic (the other type being the opposition inference). This type includes inferences that make use of the operations of obversion, conversion, contraposition (and sometimes inversion).

[78] According to Keenan and Faltz (1985), propositions and predicates (i.e. sets) can both form Boolean algebras in which the subalterante (i.e. unilateral entailment) relation and proper subset relation are essentially the same relation. The superset relation is just a converse of the subset relation. Note that some modern scholars (such as Smessaert (2012)) think that the subalternate relation is very different from the contradictory, contrary and subcontrary relations and so only the latter three are core opposition relations.

Before starting our study on quantifier inferences, I will first point out the scope of study in this chapter. Since the main objective of this chapter is to identify valid inferential patterns of various types of GQs, this chapter will adopt the model-theoretic framework of classical GQT, and will not consider proof-theoretic issues usually studied under modern Mathematical Logic, including formal syntax, metalogical properties, computational complexity, expressive power, proof algorithms, etc.

## 3.2 Monotonicity Inferences

### 3.2.1 Basic Definitions

Monotonicity is concerned with truth preservation of a quantified statement when the arguments of the statement are replaced by their supersets / subsets. Since the concept of monotonicity can also be applied to Boolean operators (BOs)[79], here I adopt a more general definition for monotonicity:

(1) Let Q be a GQ / BO with n arguments, then Q is increasing in the $i^{th}$ argument ($1 \leq i \leq n$) iff for all $X_1, \ldots X_i, X_i', \ldots X_n$, $X_i \leq X_i' \Rightarrow Q(X_1, \ldots X_i, \ldots X_n) \leq Q(X_1, \ldots X_i', \ldots X_n)$.

(2) Let Q be as above, then Q is decreasing in the $i^{th}$ argument ($1 \leq i \leq n$) iff for all $X_1, \ldots X_i, X_i', \ldots X_n$, $X_i \geq X_i' \Rightarrow Q(X_1, \ldots X_i, \ldots X_n) \leq Q(X_1, \ldots X_i', \ldots X_n)$.

Q is called monotonic in the $i^{th}$ argument iff it is either increasing or decreasing in the $i^{th}$ argument. Otherwise, it is called non-monotonic in the $i^{th}$ argument. In the above definitions, "$\leq$" is a general partial order relation. When used between two sets, it represents "$\subseteq$" (the subset relation); when used between two

[79] Boolean operators mean the same as "propositional connectives", such as "negation" ("$\neg$"), conjunction ("$\wedge$"), etc.

propositions, it represents "$\Rightarrow$" (the entailment relation)[80].

In what follows, I introduce a notation to denote the monotonicities of GQs / BOs[81] as exemplified in:

(3) *every* $\in$ ↓MON↑; (*more ... than ...*) $\in$ ↑↓MON–

In the above, ↓MON↑ and ↑↓MON– represent sets of GQs / BOs with the specified monotonicities. The left and right sides of "MON" represent the nominal and predicative arguments of a GQ, or the first and second arguments of a binary BO, respectively. The symbols ↑, ↓ and – represent increasing, decreasing and non-monotonic, respectively. Thus, (3) tells us that "*every*" is left decreasing and right increasing, while "(*more ... than ...*)" is increasing, decreasing and non-monotonic in the 1st, 2nd and 3rd arguments, respectively.

Next I introduce the notion of "triviality":

(4) Let Q be a GQ / BO with n arguments, then Q is trivial in the $i^{th}$ argument ($1 \leq i \leq n$) iff for any particular set of $X_1, \ldots X_{i-1}, X_{i+1}, \ldots X_n$, either $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 1$ for any $X_i$, or $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 0$ for any $X_i$.

In other words, whether $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 1$ or not only depends on $X_1, \ldots X_{i-1}, X_{i+1}, \ldots X_n$ but does not depend on $X_i$. Note that no GQ / BO studied in this thesis possesses this property[82].

---

80 According to the Boolean Semantics developed by Keenan and Faltz (1985), propositions and various word classes (modeled as sets) in natural language form Boolean algebras. Under this approach, the entailment relation between propositions and the subset relation between sets are indeed the same relation, namely the domination relation (represented by "$\leq$") of a Boolean algebra.

81 Note that "monotonicity" may be manifested in two different levels – the GQ / BO level and the argument level. Using "*every*(A)(B)" as an example, on the GQ / BO level, we say that the monotonicity of the GQ "*every*" is left decreasing and right increasing; on the argument level, we say that the monotonicities of the arguments A and B under "*every*" are decreasing and increasing, respectively. Also note that some scholars use the term "polarity" to call "monotonicity on the argument level". To avoid using different terms for similar notions, I do not adopt this terminology.

82 Since $\|every(\varnothing)(B)\| = 1$ for any B, one may say that the type <1> GQ "*every*($\varnothing$)" is trivial in its only argument. But "*every*($\varnothing$)" is not a single GQ studied in this thesis.

Since increasing and decreasing are not contradictory concepts (but monotonic and non-monotonic are), it is logically possible that the same GQ / BO may have both properties in the same argument(s). But fortunately we have the following theorem:

**Theorem 3.1** If a GQ / BO is both increasing and decreasing in an argument, it is trivial in that argument.

Thanks to this theorem, once we have established that a GQ / BO studied in this thesis is increasing (decreasing) in an argument, we can be sure that it cannot also be decreasing (increasing) in that argument.

### 3.2.2 Previous Studies

Ever since the inception of GQT, many scholars have studied the monotonicities of various types of GQs, including structured GQs (Smessaert (1996)), iterated GQs (Zuber (2010a)), possessive constructions (Peters and Westerståhl (2006)) and other GQs that have not been considered in this thesis.

Some scholars proposed and studied generalized or refined concepts of monotonicity. These include continuity (Westerståhl (1989)), local monotonicities (Glöckner (2006)), (anti-)additivity / (anti-)multiplicativity as well as their combined properties, i.e. homomorphicity / antimorphicity (Zwarts (1997), van der Wouden (1997)) and "directional monotonicities", i.e. southeast increasing / southwest increasing / northeast decreasing / northwest decreasing, as well as their combined properties, i.e. antieuclidity / smoothness / intersectivity (Peters and Westerståhl (2006)). By introducing these new notions, certain GQs that are non-monotonic may turn out to have certain generalized or refined monotonicities. For example, the left non-monotonic GQ "(*at least 1/3*)" is smooth (i.e. southeast increasing and northwest decreasing).

In a complex sentence, the monotonicity of a predicate may interact with any quantifier or logical operator that has scope over that predicate. Such interaction is the subject matter of Monotonicity Calculus, whose purpose is to devise a set of rules or an algorithm for determining the monotonicities of the predicates in a complex sentence. The idea of Monotonicity Calculus was initiated by Hoeksema (1986) and van Benthem (1986), but the first practicable model for Monotonicity Calculus was developed by Sanchez Valencia (1991), who adopted the proof-theoretic approach and built up his framework using Categorial Grammar.

Following Sanchez Valencia (1991), a number of scholars extended or improved his framework using different variants of Categorial Grammar (including Dowty (1994), Kas and Zwarts (1994), Bernardi (2002), Fyodorov (2002), Zamansky (2004), van Eijck (2007), Christodoulopoulos (2008) and Moss (2012)) or other computational approaches (such as MacCartney (2009)).

### 3.2.3 Monotonicities of Monadic GQs

In Chow (2007), I have proposed some rules for determining the monotonicities of types <1> and <1,1> right conservative GQs as well as right-iterated GQs. In this thesis, I will further generalize these rules. This subsection is devoted to the rules for monadic GQs. I first state the following preliminary theorems:

**Theorem 3.2** Let X, X' and Y be sets such that $X \subseteq X'$. Then

(a) $X \cap Y \subseteq X' \cap Y$

(b) $|X \cap Y| \leq |X' \cap Y|$

**Theorem 3.3** A GQ with presupposition is monotonic only in cases where its arguments satisfy the presupposition.

Then we have the following theorems for monadic GQs:

**Theorem 3.4** Let Q's truth condition be in the form $X_1 \subseteq Y$ or $X_1 \cap X_2 \subseteq Y$, where $X_i$ ($i \in \{1, 2\}$) and Y are arguments of Q or constant sets and no $X_i$ is equal to Y. Then Q is increasing (decreasing) in all arguments Y ($X_i$). If $X_i$ or Y is replaced by its negative counterpart in the truth condition, the monotonicity of $X_i$ or Y is reversed.

**Theorem 3.5** Let Q's truth condition be in one of the following forms (after converting any division into multiplication):

(a) $|X_1 \cap X_2| \geq/>/\leq/< n$;

(b) $|X_1 \cap X_2| \geq/> |Y_1 \cap Y_2|$;

(c) $|X_1 \cap X_2| \geq/>/\leq/< r \times |X_3|$;

(d) $|X_1 \cap X_2| \times |Y_3| \geq/> |Y_1 \cap Y_2| \times |X_3|$

where n and r are constants as defined in Appendix 1, $X_i$ and $Y_j$ ($i, j \in \{1, 2\}$) are arguments of Q or constant sets and $X_3$ and $Y_3$ are equal to one of the $X_i$ and $Y_j$, respectively. Then Q is increasing (decreasing) in all arguments appearing solely on the left (right) of "≥/>" or the right (left) of "≤/<", and non-monotonic in all arguments appearing on both sides of "≥/>/≤/<". If any monotonic $X_i$ or $Y_j$ is replaced by its negative counterpart in the truth condition, then treat $\neg X_i$ or $\neg Y_j$ as if it were $X_i$ or $Y_j$ appearing on the opposite side of "≥/>/≤/<".

**Theorem 3.6** Let Q's truth condition be in the form $X_1 = Y$ or $X_1 \cap X_2 = Y$, where $X_i$ ($i, j \in \{1, 2\}$) and Y are arguments of Q or non-trivial constant sets and no $X_i$ is equal to Y. Then Q is non-monotonic in all of its arguments. This fact is unaffected if $X_i$ or Y is replaced by its negative counterpart in the truth condition.

**Theorem 3.7** Let Q's truth condition be in one of the following forms:

(a) $|X_1 \cap X_2| = n$;

(b) $m \leq |X_1 \cap X_2| \leq n$;

(c) $|X_1 \cap X_2| = |Y_1 \cap Y_2|$;

(d) $|X_1 \cap X_2| / |X_3| = r$;

(e) $q \leq |X_1 \cap X_2| / |X_3| \leq r$;

(f) $|X_1 \cap X_2| / |X_3| = |Y_1 \cap Y_2| / |Y_3|$

where m, n, q and r are constants as defined in Appendix 1, $X_i$ and $Y_j$ ($i, j \in \{1, 2\}$) are arguments of Q or constant sets and $X_3$ and $Y_3$ are equal to one of the $X_i$ and $Y_j$, respectively. Then Q is non-monotonic in all of its arguments. This fact is unaffected if $X_i$ or $Y_j$ is replaced by its negative counterpart in the truth condition.

In what follows I demonstrate how to use these theorems to determine the monotonicities of GQs. Consider the GQ "*both*" with a presupposition $|CS \cap A| = 2$. Since after replacing A by its superset or subset A', $|CS \cap A'|$ is not necessarily equal to 2, it may result in presupposition failure, by Theorem 3.3 this GQ is left non-monotonic. Since the truth condition of "*both*" is $CS \cap A \subseteq B$, by Theorem 3.4, "*both*" is right increasing. Together, we have *both* $\in$ –MON↑.

Since the truth condition of "*every*" and "*no*" can be written as $A \subseteq B$ and $A \subseteq \neg B$, respectively, by Theorem 3.4, we have *every* $\in$ ↓MON↑, *no* $\in$ ↓MON↓.

Since the truth condition of "(*constitute less than r of*)" can be written as $|B \cap A| < r \times |B|$, where A appears solely on the left and B appears on both sides of

"<", by Theorem 3.5(c), we have (*constitute more than r of*) $\in$ ↓MON–.

Since the truth condition of "(*more* ... *than* ...)" can be written as $|A_1 \cap B| > |A_2 \cap B|$, where $A_1$ appears solely on the left, $A_2$ appears solely on the right and B appears on both sides of ">", by Theorem 3.5(b), we have (*more* ... *than* ...) $\in$ ↑↓MON–.

Next consider "(*all* ... *except C*)" and "(*between m and n*)" whose truth conditions can be written as $A \cap \neg B = C$ (where C is a constant set) and $m \leq |A \cap B| \leq n$, respectively. By Theorem 3.6 and Theorem 3.7(b), we have (*all* ... *except C*), (*between m and n*) $\in$ –MON–.

The following valid inference illustrates the left decreasing and right increasing monotonicities of "*every*":

(5) Every child is jogging. $\Rightarrow$ Every boy is doing exercises.

Note that the above inference makes use of the relations BOY $\subseteq$ CHILD and JOG $\subseteq$ DO-EXERCISES. In contrast, the following invalid inference illustrates that "(*more* ... *than* ...)" is not increasing in the 3rd argument:

(6) More boys than girls are jogging.

#$\Rightarrow$ More boys than girls are doing exercises.

To prove the invalidity of the above, one can use a method similar to that introduced in the proof of Theorem 3.5 to construct a counterexample. Hence, we may let U = {a, b, c, d, e}, BOY = {a, b}, GIRL = {c, d, e}, JOG = {a, b, c}, DO-EXERCISES = {a, b, c, d, e}. Then one can check that JOG $\subseteq$ DO-EXERCISES and $\|$(*more* ... *than* ...)(BOY, GIRL)(JOG)$\|$ = 1, but $\|$(*more* ... *than* ...)(BOY, GIRL)(DO-EXERCISES)$\|$ = 0.

The following table summarizes the monotonicities of the GQs studied in this thesis[83]:

**Table 3.1 Monotonicities of GQs**

| **Monotonicity Type** | **GQ** |
|---|---|
| MON↑ | *everybody*(*-thing*), *somebody*(*-thing*), ($x_1$, $x_2$ *and* …) |
| MON↓ | *nobody*(*-thing*) |
| ↑MON↑ | *some*, (*more than n*), (*at least n*) |
| ↑MON↓ | (*not every*), *only* |
| ↑MON– | (*constitute more than r of*), (*constitute at least r of*) |
| ↓MON↑ | *every*, (*not only*) |
| ↓MON↓ | *no*, (*fewer than n*), (*at most n*) |
| ↓MON– | (*constitute less than r of*), (*constitute at most r of*) |
| –MON↑ | *most*, (*more than r of*), (*at least r of*), *the*, *C's*, *both*, *either* |
| –MON↓ | (*a minority of*), (*less than r of*), (*at most r of*), *neither* |
| ↑↓MON– | (*more … than …*), (*at least as many … as …*), (*constitute a larger proportion of … than …*), (*constitute at least the same proportion of … as …*) |
| ↓↑MON– | (*fewer … than …*), (*at most as many … as …*), (*constitute a smaller proportion of … than …*), (*constitute at most the same proportion of … as …*) |

### 3.2.4 Monotonicity Calculus

In this subsection, I will study the monotonicities of iterated GQs, which is the subject matter of Monotonicity Calculus. First consider the case in which a predicate does not fall within the argument of any GQ / BO. Let X and X' be predicates, i.e. sets. A set not falling within the argument of any GQ / BO can be seen as falling within the argument of the identity operator ι defined by ι(X) = X for any set X. Now it is obvious that if X ⊆ X', then ι(X) ⊆ ι(X'). By definition (1), ι is increasing in its argument. So we conclude that a predicate not falling

[83] Only those monotonicity types with at least one increasing / decreasing argument position are listed here. Thus, GQs studied in this thesis that are not listed below are understood to be non-monotonic in all arguments. For example, (*exactly n*) ∈ –MON–.

within the argument of any GQ / BO is increasing.

Next we consider the case in which a predicate falls within the argument of some GQ / BO. In this case, we need the following theorem:

**Theorem 3.8** Let P and P' be n-ary predicates, then $P \subseteq P' \Rightarrow \{x_i: P(x_1, \dots x_{i-1}, x_i, x_{i+1}, \dots x_n)\} \subseteq \{x_i: P'(x_1, \dots x_{i-1}, x_i, x_{i+1}, \dots x_n)\}$ for any $1 \leq i \leq n$ and any particular set of $x_1, \dots x_{i-1}, x_{i+1}, \dots x_n$.

With the above theorem, we can then conclude that a predicate is increasing (decreasing) if it falls within an even (odd) number of decreasing argument positions without at the same time falling within any non-monotonic argument position. In what follows, I will provide a proof sketch for this important result. Here I will only consider a special case which can be generalized to other cases. Suppose we have an iterated GQ with the following argument structure:

(7) $$Q_1(A_1)(\{x_1: \dots Q_n(A_n)(\{x_n: B(x_1, \dots x_n)\}) \dots \})$$

We focus on the monotonicity of B (the monotonicities of other predicates can be similarly treated). Suppose B does not fall within any non-monotonic argument position and $B \subseteq B'$. By Theorem 3.8, we know that $\{x_n: B(x_1, \dots x_n)\} \subseteq \{x_n: B'(x_1, \dots x_n)\}$ for any $x_1, \dots x_{n-1}$. According as $Q_n$ is right increasing or decreasing, we have $Q_n(A_n)(\{x_n: B(x_1, \dots x_n)\}) \Rightarrow Q_n(A_n)(\{x_n: B'(x_1, \dots x_n)\})$ or $Q_n(A_n)(\{x_n: B'(x_1, \dots x_n)\}) \Rightarrow Q_n(A_n)(\{x_n: B(x_1, \dots x_n)\})$. Thus, the increasing and decreasing monotonicities are, respectively, preserving and reversing the order relation between B and B'. The above reasoning can be seen as a kind of "upward derivation": from the set inclusion relation at the B-level, we derive an entailment relation at the $Q_n$-level.

Now $Q_n(A_n)(\{x_n: B(x_1, \dots x_n)\})$ can be seen as the argument structure of an (n – 1)-ary predicate (with $x_1, \dots x_{n-1}$ as arguments). Thus, we can carry out the aforesaid upward derivation again by making use of Theorem 3.8 and deriving an entailment relation at the $Q_{n-1}$-level. The process of determining the monotonicity of B in (7) is essentially a repetition of this upward derivation up to the $Q_1$-level. Thus, if B falls within an even (odd) number of decreasing argument positions, the order relation between B and B' will finally be reversed for an even (odd) number of times, which is equivalent to no (one) reverse and so B is increasing (decreasing).

Note that the aforesaid upward derivation breaks down when B falls within at least one non-monotonic argument position, because when we come to the $Q_k$-level ($1 \leq k \leq n$) where $Q_k$ is non-monotonic in the right argument, no entailment relation can be derived at that level. Intuitively speaking, the upward derivation is blocked by $Q_k$ and so B turns out to be non-monotonic.

With the above discussion and results, we can now formulate the following Principle of Monotonicity Calculus (PMC):

**Principle of Monotonicity Calculus (PMC)**

A singly-occurring predicate not falling within the argument of any GQ / BO is increasing. A singly-occurring predicate is increasing (decreasing) if it falls within an even (odd) number of decreasing argument positions without at the same time falling within any non-monotonic argument position. A singly-occurring predicate is non-monotonic if it falls within at least one non-monotonic argument position.

In the above principle, a "singly-occurring" predicate is a predicate that has only one occurrence in the quantified statement.

Writing a disambiguated complex quantified statement in the form of a variable-free iterated tripartite structure, we can then easily determine the monotonicity of each predicate appearing in the statement by employing PMC. For example, consider the following tripartite structure:

(8) $no(A_1)([every(A_2)]_2([(exactly\ 1)(A_3)]_3(B)))$

Since *no* $\in$ ↓MON↓, *every* $\in$ ↓MON↑, (*exactly 1*) $\in$ –MON–, one can easily see that $A_1$ is decreasing. As $A_2$ falls within the right argument of "*no*" and the left argument of "*every*", i.e. 2 decreasing argument positions, it is increasing. As $A_3$ and B fall within the left and right arguments of "(*exactly 1*)", they are both non-monotonic. Based on the above analysis, we can derive the following valid inference:

(9) No teacher recommended every romance novel to exactly 1 student.

$\Rightarrow$ No English teacher recommended every novel to exactly 1 student.

Next consider the following tripartite structure:

(10) $(exactly\ 1/2\ of)(A_1)([some(A_2)]_2(B))$

By PMC we know that B is non-monotonic in (10). Although PMC does not provide a systematic method for constructing counterexamples to prove non-monotonicities, it is not difficult to construct these counterexamples. For example, let $A_1 = \{a, b, c, d\}$, $A_2 = \{e, f\}$. We then define $B = \{<a, e>, <b, f>\}$ and $B' = \{<a, e>, <b, f>, <c, f>\}$. One can check that the above predicates satisfy $B \subseteq B'$ and that $\|(exactly\ 1/2\ of)(A_1)([some(A_2)]_2(B))\| = 1$ but $\|(exactly\ 1/2\ of)(A_1)([some(A_2)]_2(B'))\| = 0$, thus showing that B is not increasing in (10). Similarly, we can also construct counterexamples to show that B is not decreasing.

In Chapter 1, I showed that sentences with relative clauses may be represented by left-iterated GQs. We can also apply PMC to determine the monotonicities of these GQs. For example, consider the following iterated tripartite structure:

(11) $every(A \cap [some(B)]_1(C))(D)$

Since A, B and C all lie within the left argument of "*every*", these three predicates are decreasing (the decreasing monotonicity of B and C is unaffected by "*some*" because "*some*" is increasing in both arguments). This result enables us to derive the following valid inference:

(12) Every boy who is loved by some girl is happy. $\Rightarrow$

Every handsome boy who is deeply loved by some pretty girl is happy.

Of course, we can also apply PMC to left-and-right-iterated GQs. Consider the following iterated tripartite structure:

(13) $every(A \cap [no(B)]_2(C))([a(D)]_2(E))$

Since B falls within two decreasing argument positions, it is increasing. Moreover, it is obvious that A and D are decreasing and increasing, respectively. This result enables us to derive the following valid inference:

(14) Every athlete who won no track event obtained a consolation prize.

$\Rightarrow$ Every male athlete who won no event obtained a prize.

### 3.2.5 GQs as Sets and Arguments

As discussed in Chapter 1, GQs can be viewed as sets – higher order sets whose members are n-tuples of sets. Thus, GQs may also have supersets and subsets. Consider the following tripartite structure:

(15) $every(A)(B)$

Viewed as a set, since "*every*" does not fall within the argument of any GQ / BO, it is increasing. Now by virtue of the set inclusion relation given in (10) of Chapter 1, we can derive the following subalternate relation in Classical Logic:

(16) Given that $A \neq \varnothing$, $every(A)(B) \Rightarrow some(A)(B)$

Thus the classical subalternate relation can be seen as a special case of monotonicity inferences.

Moreover, GQs may also act as arguments of other GQs / BOs. In an iterated tripartite structure, a GQ in the inner tripartite structure can be seen as an argument of another GQ in the outer tripartite structure. For instance, in the following iterated tripartite structure

(17) $$every(A_1)([every(A_2)]_2(B))$$

the second "*every*" can be seen as falling within the right argument of the first "*every*" and is thus increasing. Moreover, the first "*every*", not falling within the argument of any GQ / BO, is also increasing. Thus, by making use of (10) of Chapter 1 and assuming that $A_1 \neq \varnothing$ and $A_2 \neq \varnothing$, we can derive the following valid inference schema:

(18) $$every(A_1)([every(A_2)]_2(B)) \Rightarrow some(A_1)([some(A_2)]_2(B))$$

or even the following (on condition that $|A_1| \geq 10$ and $|A_2| \geq 2$)[84]:

(19) $$every(A_1)([every(A_2)]_2(B)) \Rightarrow (at\ least\ 10)(A_1)([(at\ least\ 2)(A_2)]_2(B))$$

Schemas (18) and (19) can be seen as generalizations of the classical subalternate relation to multiply quantified sentences, which have been considered by Zou (2002). By using PMC, more generalizations can be made.

### 3.2.6 Negation Operator

As mentioned above, BOs may also affect monotonicities. In this thesis, I only consider the monotonicity of the negation operator "¬". We have the following theorem:

**Theorem 3.9** "¬" is decreasing.

Since "¬" is a polymorphic operator that may appear in different levels, the above theorem is particularly useful for exploring the interaction between monotonicity and negation. But just as I pointed out in Chow (2007), we must

[84] When deriving this inference schema, I am treating the two "*every*" as two separate GQs instead of two different occurrences of the same GQ, so that I can replace the two "*every*" by different GQs.

determine the correct scope structure of a quantified statement in order to analyse the monotonicities of its predicates correctly. Consider the following sentence:

(20) Non-professionals do not constitute less than half of the participants.

Although the above sentence appears to contain a transitive verb "constitute", this verb should be seen as part of the left conservative GQ "(*constitute less than 1/2 of*)", and the sentence should be represented as the following tripartite structure:

(21) $\neg$(*constitute less than 1/2 of*)($\neg$PROFESSIONAL)(PARTICIPANT)

Based on the above, we can now easily see that PROFESSIONAL and PARTICIPANT fall within 3 decreasing and 1 non-monotonic argument positions and are thus decreasing and non-monotonic, respectively. So we have the following valid inference (assuming that LAWYER $\subseteq$ PROFESSIONAL):

(22) Non-professionals do not constitute less than half of the participants. $\Rightarrow$

Non-lawyers do not constitute less than half of the participants.

## 3.3 Argument Structure Inferences

### 3.3.1 Basic Definitions

Argument structure inferences refer to inferences involving manipulations (such as Boolean operations and transpositions) of the arguments or quantifiers. In this section, I will focus on argument structure inferences involving negation and transposition, which can be classified into two subtypes – duality inferences and transposition inferences.

Duality inferences are argument structure inferences involving negation. Modern scholars have classified three notions of negation: outer negation, inner negation and dual. Generalizing de Mey (1990), I extend the notions of inner negation and dual to the various arguments of a multi-argument GQ. Let Q be a monadic GQ with n arguments, the outer negation (denoted $\neg Q$), inner negation in the $i^{th}$ argument ($1 \le i \le n$) (denoted $Q\neg_i$) and dual in the $i^{th}$ argument ($1 \le i \le n$) (denoted $Q^{di}$) are defined as follows[85]:

(23) $$(\neg Q)(X_1, \ldots X_n) \Leftrightarrow \neg(Q(X_1, \ldots X_n))$$

(24) $$(Q\neg_i)(X_1, \ldots X_i, \ldots X_n) \Leftrightarrow Q(X_1, \ldots \neg X_i, \ldots X_n)$$

(25) $$(Q^{di})(X_1, \ldots X_i, \ldots X_n) \Leftrightarrow \neg(Q(X_1, \ldots \neg X_i, \ldots X_n))$$

Note that dual is a composite of the outer and inner negations. Based on these definitions, we can also derive other composite relations among these notions. For example, one can easily derive $\neg(Q^{di}) = Q\neg_i$, $(Q\neg_i)^{di} = \neg Q$, etc.

Moreover, we can also combine inner negations in different arguments. For example, for determiners, we may talk about left-and-right inner negation, which is a combination of inner negations in the left and right arguments. The following tables list the outer negations, inner negations and duals of the GQs studied in this thesis[86]:

**Table 3.2 Outer Negations of GQs**

| |
|---|
| ¬*somebody*(-*thing*) = *nobody*(-*thing*); ¬*every* = (*not every*); ¬*some* = *no*; ¬(*more than n*) = (*at most n*); ¬(*fewer than n*) = (*at least n*); ¬(*more than r of*) = (*at most r of*); ¬(*less than r of*) = (*at least r of*); ¬*either* = *neither*; ¬(*more … than …*) = (*at most as many … as …*); ¬(*fewer … than …*) = (*at least as many … as …*); ¬(*proportionally more … than …*) = (*at most the same proportion of … as …*); ¬(*proportionally fewer … than …*) = (*at least the same proportion of … as …*); ¬*only* = (*not only*); ¬(*constitute more than r of*) = (*constitute at most r of*); ¬(*constitute less than r of*) = (*constitute at least r of*); ¬(*constitute a larger* |

85 For a type <1> GQ with only one argument, there is no need to specify the unique argument in which the inner negation and dual occur. For determiners, the arguments are usually called "left" and "right" arguments. Correspondingly, "l" and "r" will be used instead of numbers in the notation. Please also note that some scholars (such as Bird (1964)) used special terms (such as obversion, inversion, conversion, contraposition) to denote some of the negative notions. To avoid using too many jargons, I do not adopt these terms.

86 Since outer negation, inner negation and dual are involutive operations, each equation listed in the following tables is equivalent to one which differs from the original equation only by interchanging the positions of the GQs (and thus is not listed). For instance, ¬*somebody*(-*thing*) = *nobody*(-*thing*) is equivalent to ¬*nobody*(-*thing*) = *somebody*(-*thing*), and so the latter is not listed.

| |
|---|
| *proportion of* … *than* …) = (*constitute at most the same proportion of* … *as* …); ¬(*constitute a smaller proportion of* … *than* …) = (*constitute at least the same proportion of* … *as* …) |

**Table 3.3 Inner Negations of GQs**

| **Argument(s) Involved** | **Inner Negation** |
|---|---|
| unique | *everybody*(-*thing*)¬ = *nobody*(-*thing*); (*everybody*(-*thing*) *except C*)¬ = (*nobody*(-*thing*) *except C*) |
| right | *every*$\neg_{r}$ = *no*; (*all* … *except C*)$\neg_{r}$ = (*no* … *except C*); (*exactly n*)$\neg_{r}$ = (*all except n*); (*more than r of*)$\neg_{r}$ = (*less than 1 – r of*); (*at least r of*)$\neg_{r}$ = (*at most 1 – r of*); (*exactly r of*)$\neg_{r}$ = (*exactly 1 – r of*) = (*all except r of*); (*between q and r of*)$\neg_{r}$ = (*between 1 – r and 1 – q of*) = (*all except between q and r of*); *both*$\neg_{r}$ = *neither* |
| left | *only*$\neg_{l}$ = *no*; (*apart from C only*)$\neg_{l}$ = (*no* … *except C*); (*constitute more than r of*)$\neg_{l}$ = (*constitute less than 1 – r of*); (*constitute at least r of*)$\neg_{l}$ = (*constitute at most 1 – r of*); (*constitute exactly r of*)$\neg_{l}$ = (*constitute exactly 1 – r of*) = (*constitute all except r of*); (*constitute between q and r of*)$\neg_{l}$ = (*constitute between 1 – r and 1 – q of*) = (*constitute all except between q and r of*) |
| left and right | *every*$\neg_{l,r}$ = *only*; (*all* … *except C*)$\neg_{l,r}$ = (*apart from C only*) |
| 3rd | (*proportionally more* … *than* …)$\neg_{3}$ = (*proportionally fewer* … *than* …); (*at least the same proportion of* … *as* …)$\neg_{3}$ = (*at most the same proportion of* … *as* …) |
| 1st and 2nd | (*constitute a larger proportion of* … *than* …)$\neg_{1,2}$ = (*constitute a smaller proportion of* … *than* …); (*constitute at least the same proportion of* … *as* …)$\neg_{1,2}$ = (*constitute at most the same proportion of* … *as* …) |

**Table 3.4 Duals of GQs**

| **Argument(s) Involved** | **Dual** |
|---|---|
| unique | *everybody*(-*thing*)$^{d}$ = *somebody*(-*thing*) |
| right | *every*$^{dr}$ = *some*; (*more than r of*)$^{dr}$ = (*at least 1 – r of*); (*less than r of*)$^{dr}$ = (*at most 1 – r of*); *both*$^{dr}$ = *either* |
| left | *only*$^{dl}$ = *some*; (*constitute more than r of*)$^{dl}$ = (*constitute at least 1 – r of*); (*constitute less than r of*)$^{dl}$ = (*constitute at most 1 – r of*) |
| left and right | *every*$^{dl,r}$ = (*not only*); *only*$^{dl,r}$ = (*not every*) |
| 3rd | (*proportionally more* … *than* …)$^{d3}$ = (*at least the same* |

| | |
|---|---|
| | *proportion of ... as ...*); (*proportionally fewer ... than ...*)$^{d3}$ = (*at most the same proportion of ... as ...*) |
| 1st and 2nd | (*constitute a larger proportion of ... than ...*)$^{d1,2}$ = (*constitute at least the same proportion of ... as ...*); (*constitute a smaller proportion of ... than ...*)$^{d1,2}$ = (*constitute at most the same proportion of ... as ...*) |

The equalities in the above tables can be proved by using the truth conditions of the GQs and definitions (23) – (25). For example, we can prove (*proportionally more ... than ...*)$_{\neg 3}$ = (*proportionally fewer ... than ...*) as follows: let $A_1$, $A_2$, B be arbitrary sets, then

(*proportionally more ... than ...*)$_{\neg 3}(A_1, A_2)(B)$

$\Leftrightarrow$ (*proportionally more ... than ...*)$(A_1, A_2)(\neg B)$ by (24)

$\Leftrightarrow$ $|A_1 - B| / |A_1| > |A_2 - B| / |A_2|$ by Appendix 1

$\Leftrightarrow$ $1 - |A_1 \cap B| / |A_1| > 1 - |A_2 \cap B| / |A_2|$

$\Leftrightarrow$ $|A_1 \cap B| / |A_1| < |A_2 \cap B| / |A_2|$

$\Leftrightarrow$ (*proportionally fewer ... than ...*)$(A_1, A_2)(B)$ by Appendix 1

Note that in the above proof I have made use of the set-theoretic laws: $X \cap \neg Y = X - Y$ and $|X| = |X \cap Y| + |X - Y|$ for any sets X and Y.

I next introduce two notions that are useful in the study of duality inferences – fixed points and self-duals:

(26) Let Q be a monadic GQ with n arguments. Q is a fixed point in the $i^{th}$ argument ($1 \leq i \leq n$) iff $Q_{\neg i} = Q$.

(27) Let Q be as above. Q is a self-dual in the $i^{th}$ argument ($1 \leq i \leq n$) iff $Q^{di} = Q$.

In other words, fixed points / self-duals are GQs that are equal to their inner negations / duals.

Transposition inferences are argument structure inferences involving transpositions (i.e. interchange of positions) and, in some cases, also negations.

Transpositions may occur on two levels: arguments or quantifiers, each with its own terminology and notation. I first consider transpositions on the argument level, which involve the notions of "converse" and "symmetry". These two notions have already been defined for determiners in (39) and (47) of Chapter 1.

To generalize these two concepts to more general monadic GQs, I first define a new notion of converse:

(28) Let Q be a monadic GQ with n arguments and $1 \leq i < j \leq n$. Its converse wrt the $i^{th}$ and $j^{th}$ arguments, denoted $Q^{-1}_{i,j}$, is a monadic GQ with the same argument structure as Q such that for all $X_1, \dots X_n$, $Q(X_1, \dots X_i, \dots X_j, \dots X_n) \Leftrightarrow (Q^{-1}_{i,j})(X_1, \dots X_j, \dots X_i, \dots X_n)$.

According to this definition, the type $<1^2,1>$ structured GQs "(*more ... than ...*)" and "(*fewer ... than ...*)" are converses wrt the 1st and 2nd arguments because

(29) (*more… than …*)$(A_1, A_2)(B) \Leftrightarrow$ (*fewer… than …*)$(A_2, A_1)(B)$

A generalized notion of symmetry can now be defined[87]:

(30) Let Q be a monadic GQ with n arguments and $1 \leq i < j \leq n$. Q is symmetric wrt the $i^{th}$ and $j^{th}$ arguments iff for all $X_1, \dots X_n$, $Q(X_1, \dots X_i, \dots X_j, \dots X_n) \Leftrightarrow Q(X_1, \dots X_j, \dots X_i, \dots X_n)$.

In other words, Q is symmetric wrt the $i^{th}$ and $j^{th}$ arguments iff Q is self-converse wrt the same arguments.

I next consider transpositions on the quantifier level. For an iterated GQ composed of two type <1> GQs $Q_1(A_1)$ and $Q_2(A_2)$ and a binary predicate B, the two GQs may be transposed. The equivalence / entailment relation between the pre-transposed and post-transposed statements gives us the notion of scope independence / scope dominance as defined below:

[87] As a matter of fact, Zuber (2007) has also proposed a generalized definition of symmetry. To suit the purpose of this thesis, I adopt a different definition.

(31) $Q_1(A_1)$ and $Q_2(A_2)$ are scopally independent iff for all B, $Q_1(A_1)([Q_2(A_2)]_2(B)) \Leftrightarrow Q_2(A_2)([Q_1(A_1)]_1(B))$.

(32) $Q_1(A_1)$ is scopally dominant over $Q_2(A_2)$ iff for all B, $Q_1(A_1)([Q_2(A_2)]_2(B)) \Rightarrow Q_2(A_2)([Q_1(A_1)]_1(B))$.

In the above definitions, $Q_1(A_1)([Q_2(A_2)]_2(B))$ and $Q_2(A_2)([Q_1(A_1)]_1(B))$ represent two different readings, i.e. subject-wide-scope reading and object-wide-scope reading, of the same sentence. Thus, the notions of scope independence and scope dominance express inferential relations between different scope structures of multiply quantified statements. For example, it is well known that the sentence

(33) Some boy loves every girl.

has two scope structures that are related by the following entailment relation:

(34) Some boy is such that he loves every girl.

$\Rightarrow$ Every girl is such that some boy loves her.

On the other hand, we can define the converse of B, denoted $B^{-1}$, as follows:

(35) $$B^{-1}(x, y) \Leftrightarrow B(y, x)$$

then by (30) and (31) of Chapter 1, we have

(36) $$[Q_1(A_1)]_1(B) = [Q_1(A_1)]_2(B^{-1})$$

In natural language, pairs of converse predicates include the active and passive forms of transitive verbs such as “love” and “be loved by”, some special pairs of verbs such as “send letters to” and “receive letters from”, and others studied by Cruse (1986).

Substituting (36) into (31) – (32), we will obtain the following alternative definitions of scope independence / dominance:

(37) $Q_1(A_1)$ and $Q_2(A_2)$ are scopally independent iff for all B,

$$Q_1(A_1)([Q_2(A_2)]_2(B)) \Leftrightarrow Q_2(A_2)([Q_1(A_1)]_2(B^{-1})).$$

(38) $Q_1(A_1)$ is scopally dominant over $Q_2(A_2)$ iff for all B, $Q_1(A_1)([Q_2(A_2)]_2(B)) \Rightarrow Q_2(A_2)([Q_1(A_1)]_2(B^{-1}))$.

We can thus view scope independence / dominance as inferential relations between two different sentences (both assumed to be under subject-wide-scope reading) whose predicates are converses of each other. In this thesis, I will mainly use these two definitions. For example, (34) can be rewritten as

(39) Some boy is such that he loves every girl.

$\Rightarrow$ Every girl is such that she is loved by some boy.

By combining negation and transposition, we may come up with even more notions. In this thesis, I will study one such notion – contrapositivity, which is defined only on determiners:

(40) A determiner Q is contrapositive iff for all A, B, $Q(A)(B) \Leftrightarrow Q(\neg B)(\neg A)$.

### 3.3.2 Previous Studies

Duality inferences can be seen as a modern counterpart of the classical eductive inferences called obversion and inversion, which are equivalent to right inner negation and left-and-right inner negation, respectively. In modern times, some scholars[88] (including Piaget (1949), Gottschalk (1953), Löbner (1987, 2011), de Mey (1990), Peters and Westerståhl (2006)) studied the various notions of negation and interactions between these notions, while some scholars (including Zwarts (1996), Keenan (2003, 2008), Zuber (2005), Löbner (2011)) studied valid inference patterns involving these notions as well as "fixed points / self-duals".

[88] Some of the following scholars actually used their own terms instead of "inner negation" and "dual". But their terms are equivalent to the notions introduced here.

Transposition inferences on the argument level can be seen as a modern counterpart of the classical eductive inferences called conversion (transposition of the subject and predicate) and contraposition (transposition of the negated subject and negated predicate). In modern times, some scholars (such as Barwise and Cooper (1981), Peters and Westerståhl (2006), Zuber (2007)) studied symmetry and contrapositivity from the perspective of GQT, and shed new light on this old topic.

Individual examples of transposition inferences on the quantifier level such as (34) have been known for a long time. However, systematic studies on scope independence / dominance only have a short history, starting from Westerståhl (1986). After that, Zimmermann (1993) and Westerståhl (1996) studied special cases of scope independence called scopelessness and self-commutativity, respectively, whereas Altman et al (2001), Ben-Avi and Winter (2004) and Altman et al (2005) studied scope dominance of monotonic GQs. Altman and Winter (2005) have also devised an algorithm for computing scope dominance.

Apart from negation and transposition, scholars have also studied other types of manipulations. For example, GQT scholars such as Zwarts (1983), van Benthem (1984), Westerståhl (1984) and Zuber (2005) have studied a number of quantifier properties whose definitions involve various kinds of argument manipulations. Some examples of these quantifier properties include reflexivity, antisymmetry, transitivity, etc. These properties may also be seen as inferential patterns of GQs.

Some modern logicians have also tried to generalize the classical notion of categorical statements by manipulating the subject or predicate of the categorical statement. These manipulations include "quantification of the predicate", e.g. "Some A is every B" (according to Cavaliere (2008), this statement is equivalent to “Some A is B and every B is A”), “complement of the subject”, e.g. “Every non-A is B” (according to Richman (2004), de Morgan called this the “e-statement” in analogy to the classical “E-statement”: “Every A is non-B”) and “conjunction of quantifiers”, e.g. “Some but not all A are B” and “All or no A are B” (called “distinctive quantifiers” by Cavaliere (2008)), etc.

### 3.3.3 Double Negation Law and Duality Inferences

Based on definitions (23) – (25) and Table 3.2 – Table 3.4, we can immediately derive valid inference schemas with only one monadic GQ, such as the following:

(41) $$\textit{every}(A)(B) \Leftrightarrow \textit{no}(A)(\neg B)$$

which may be generalized to the inference schema

(42) $$Q(A)(B) \Leftrightarrow (Q\neg_r)(A)(\neg B)$$

On top of these, there are more interesting ones. Keenan (1993, 2003) and Zwarts (1996) have proposed a number of inference schemas with iterated GQs. These schemas are all based on the following Double Negation Law:

(43) $$\neg\neg X = X$$

where X may be a GQ or predicate. For example, consider the following inference schema (called “Facing Negations” by Keenan (1993)) where $Q_1$ and $Q_2$ are determiners:

(44) $$Q_1(A_1)([Q_2(A_2)]_2(B)) \Leftrightarrow (Q_1\neg_r)(A_1)([(\neg Q_2)(A_2)]_2(B))$$

To prove the validity of this inference schema, we can rewrite the RHS by using the definition of (right) inner negation given in (24):

(45) $$Q_1(A_1)([\neg\neg Q_2(A_2)]_2(B))$$

By (43), one can immediately see that the above is equivalent to the LHS of (44).

We next consider the following inference schema involving 3 determiners and all 3 notions of negation:

(46) $$Q_1(A_1)([Q_2(A_2)]_2([Q_3(A_3)]_3(B)))$$

$$\Leftrightarrow (Q_1\neg_r)(A_1)([(Q_2{}^{dr})(A_2)]_2([(\neg Q_3)(A_3)]_3(B)))$$

To prove this schema, we can rewrite the RHS by using the definitions of (right) inner negation and (right) dual given in (24) and (25):

(47) $$Q_1(A_1)([\neg\neg Q_2(A_2)]_2([\neg\neg Q_3(A_3)]_3(B)))$$

Obviously the above is equivalent to the LHS of (46).

In fact, there is no need to list all valid inference schemas. Just by considering the Double Negation Law, we can identify valid duality inferences. Consider the following sentence:

(48) Only players who made some mistake received no prize.

As discussed in Chapter 1, this sentence is ambiguous between two readings. Here I will only consider the reading that may be represented as follows:

(49) (*only*|PLAYER)([*some*(MISTAKE)]$_2$(MAKE))([*no*(PRIZE)]$_2$(RECEIVE))

Note that here PLAYER is treated as a parameter of the GQ "*only*". Under this reading, we have the following valid inference:

(50) Only players who made some mistake received no prize.

$\Leftrightarrow$ All players who made no mistake received some prize.

Since *only*$\neg_{l,r}$ = *all*, $\neg$*some* = *no* and $\neg$*no* = *some*, the above can be seen as an instance of the following inference schema[89]:

(51) $$(\textit{only}|A)([\textit{some}(B)]_2(C))([\textit{no}(D)]_2(E))$$

$$\Leftrightarrow (\textit{only}|A)\neg_{l,r}([\neg\textit{some}(B)]_2(C))([\neg\textit{no}(D)]_2(E))$$

To prove the above schema, we can rewrite the RHS by using the definition of (left-and-right) inner negation:

[89] Remember (see Subsection 1.7.4 of Chapter 1) that since "*all*" is right conservative, (*all*|A) = *all*.

(52) $(\textit{only}|\mathrm{A})([\neg\neg \textit{some}(\mathrm{B})]_2(\mathrm{C}))([\neg\neg \textit{no}(\mathrm{D})]_2(\mathrm{E}))$

One can then easily see that the above is equivalent to the LHS of (51).

Apart from occurring in quantifiers on the subject / object position, negations may also occur in predicates on various grammatical positions. For example, Zwarts (1996) talked about “verb negations” as opposed to “quantifier negations”. Thus, analogous to (46), we have the following inference schema:

(53) $Q_1(A_1)([Q_2(A_2)]_2(B)) \Leftrightarrow (Q_1\neg_r)(A_1)([(Q_2^{dr})(A_2)]_2(\neg B))$

The following is an instance of this inference schema (assuming that ¬QUIT = ATTEND):

(54) At least 1/3 of the students quit some class.

⇔ At most 2/3 of the students attend every class.

To derive the correct inference, the verbs “quit” and “attend” above must be seen as taking the narrowest scope. Note that unlike its surface structure, the tripartite structure of the LHS of (54) should in fact be

(55) $(\textit{at least 1/3 of})(\mathrm{STUDENT})([\textit{some}(\mathrm{CLASS})]_2(\mathrm{QUIT}))$

The example above shows that determining correct scope structures is essential to obtaining valid inferences. Here I demonstrate another example that is even more subtle. Consider the following Chinese example[90]:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| (56) | Ta | gan | zhe | zhong | shi | bu | keneng | cai | yi | ci. |
| | she | do | this | kind | thing | not | possible | only | one | time |

*She could not have done such kind of things only once.*

What is interesting with this sentence is that the quantified phrases (including the modal particle) appear in the latter part of the sentence. According to Shi (2006), the former part of this sentence “ta gan zhe zhong shi” (henceforth represented by TGZZS) should be analysed as a clause functioning as the subject of the

[90] Shi (2006), (26), p. 53.

whole sentence. The scope structure of (56) will then be represented as follows:

(57) *no*(W)({w: (*at most 1*)({r: TGZZS(r)})(R) in w})

The above expression has made use of notions of Possible Worlds Semantics and borrowed an idea from Nicolas (2010). First, "it could not have been the case that p" where p is a proposition is represented by *no*(W)({w: p in w}), meaning literally that no possible world w in the possible worlds domain W is such that p is true in w. Second, I treat TGZZS as a relation type[91] and use {r: TGZZS(r)} to represent the set of instances of this relation type. Moreover, I represent "only once" by the determiner "(*at most 1*)". Thus, the inner tripartite structure (*at most 1*)({r: TGZZS(r)})(R) means that there is at most one instance of TGZZS in the domain of instances of relations (represented by R). Note that this tripartite structure has the form of an existential sentence as introduced in Chapter 1. Under this interpretation, (56) means it is impossible that there is at most 1 instance of TGZZS.

Based on the above scope structure and using the schema (44) as well as the facts *no*$\neg_{r}$ = *every* and ¬(*at most 1*) = (*more than 1*), we obtain the following valid inference:

(58) *no*(W)({w: (*at most 1*)({r: TGZZS(r)})(R) in w})

⇔ *every*(W)({w: (*more than 1*)({r: TGZZS(r)})(R) in w})

According to Possible Worlds Semantics, *every*(W)({w: p in w}) represents "it must be the case that p". Thus the RHS of the above represents the following sentence:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| (59) | Ta | gan | zhe | zhong | shi | yiding | buzhi | yi | ci. |
| | she | do | this | kind | thing | necessarily | not only | one | time |

[91] According to Nicolas (2010), "relation" is a more general notion than "state", "event", "process" and may encompass the latter notions.

*She must have done such kind of things more than once.*

One can check that (56) and (59) are equivalent statements.

### 3.3.4 Fixed Points and Self-Duals

In this subsection, I study fixed points and self-duals. As a matter of fact, Keenan (2003, 2008) and Zuber (2005) have studied these notions and proposed a number of theorems. But since these theorems are focused on right conservative GQs, I will generalize them so that they can also be applied to left conservative GQs. For fixed points, we have the following two theorems (In what follows, "$\wedge$" and "$\vee$" are defined pointwise, i.e. $(Q_1 \wedge(\vee)\ Q_2)(X_1, \ldots X_n) \Leftrightarrow Q_1(X_1, \ldots X_n) \wedge(\vee)\ Q_2(X_1, \ldots X_n)$ for any $X_1, \ldots X_n$):

**Theorem 3.10** Let Q be a monadic GQ with n arguments, then $Q \wedge Q\neg_i$ and $Q \vee Q\neg_i$ are fixed points in the $i^{th}$ argument.

**Theorem 3.11** Let $Q_1$ and $Q_2$ be monadic GQs with the same argument structure. If both $Q_1$ and $Q_2$ are fixed points in the $i^{th}$ argument, then $\neg Q_1$, $Q_1 \wedge Q_2$ and $Q_1 \vee Q_2$ are also fixed points in the $i^{th}$ argument.

Based on these two theorems, we can identify a number of fixed points in natural language. First consider the classical GQs and their converses. Since *some*$\neg_r$ = (*not all*), *some*$\neg_l$ = (*not only*), *all*$\neg_{l,r}$ = *only*, by Theorem 3.10 we know that "(*some but not all*)" is a right fixed point, "(*some but not only*)" is a left fixed point, and "(*all and only*)" is a left-and-right fixed point.

Next consider the proportional quantifiers. Since (*at least r of*)$\neg_r$ = (*at most 1 – r of*), by Theorem 3.10 we know that "(*at least r and at most 1 – r of*)", where $r \leq 1/2$, is a right fixed point. Taking r = 1/3 and 1/2, we then obtain "(*between 1/3 and 2/3 of*)" and "(*exactly 1/2 of*)" as two right fixed points. The above reasoning can be extended to left conservative proportional quantifiers as well. Thus, "(*constitute between 1/3 and 2/3 of*)" and "(*constitute exactly 1/2 of*)" are two left fixed points.

Having identified some fixed points, we can use Theorem 3.11 to obtain more. For example, since ¬(*some but not all*) = (*all or no*), we know that "(*all or no*)" is a right fixed point because its negation is. Moreover, we also know that "(*between 1/3 and 2/3 of*)($A_1$) *and* (*exactly 1/2 of*)($A_2$)" is a fixed point because its two conjuncts are.

Using the above results, the definition of fixed points and the inference schemas, we can then derive valid inferences. Using "(*exactly 1/2 of*)" as an example, when we take A = {x, y} where x and y are individuals, "(*exactly 1/2 of*)(A)" is equivalent to "(*exactly one of x and y*)(–)", and so we have the following valid inference (assuming ¬WIN = LOSE, i.e. there is no tie in a presidential election):

(60) Exactly one of Obama and Romney will win the presidential election.

⇔ Exactly one of Obama and Romney will lose the presidential election.

One may wonder whether there are similar results for outer negation. But we have the following negative result:

**Theorem 3.12** There is no fixed point for outer negation.

Since dual is a combination of inner and outer negations, one may conjecture that there are relatively few self-duals in natural language. In fact, only one main type of self-duals will be discussed in this thesis. Following Keenan (2003), I first make the following definition. Let k be a non-negative integer and K be a subset of the number set {0, … k}. We define the following determiner (for A such that |A| = 2k + 1):

(61) $$Q_{k,K}(A)(B) \Leftrightarrow \begin{cases} |A \cap B| \in K, & \text{if } 0 \leq |A \cap B| \leq k \\ 2k + 1 - |A \cap B| \notin K, & \text{if } k + 1 \leq |A \cap B| \leq 2k + 1 \end{cases}$$

Then we have the following theorem:

**Theorem 3.13** The determiner $Q_{k,K}$ defined in (61) is a right self-dual.

To illustrate the above theorem, let us substitute $K = \varnothing$ into (61):

(62) $$Q_{k,\varnothing}(A)(B) \Leftrightarrow \begin{cases} |A \cap B| \in \varnothing, & \text{if } 0 \leq |A \cap B| \leq k \\ 2k + 1 - |A \cap B| \notin \varnothing, & \text{if } k + 1 \leq |A \cap B| \leq 2k + 1 \end{cases}$$

Since $|A \cap B| \in \varnothing$ must be false and $2k + 1 - |A \cap B| \notin \varnothing$ must be true for any k, A, B, (62) can be rewritten as

(63) For $|A| = 2k + 1$, $Q_{k,\varnothing}(A)(B) \Leftrightarrow |A \cap B| \geq k + 1$

This shows that $Q_{k,\varnothing}$ can be expressed as "(*more than 1/2 of*)" on condition that |A| is odd. Following a similar line of reasoning, if we substitute $K = \{0, \ldots k\}$ into (61), we will obtain

(64) For $|A| = 2k + 1$, $Q_{k,\{0, \ldots k\}}(A)(B) \Leftrightarrow |A \cap B| \leq k$

This shows that $Q_{k,\{0, \ldots k\}}$ can be expressed as "(*less than 1/2 of*)" on condition that |A| is odd. Thus, by Theorem 3.13, we may conclude that "(*more than 1/2 of*)" and "(*less than 1/2 of*)" are right self-duals on condition that |A| is odd[92]. Now in (63) and (64) above, if we exchange the roles of A and B, we will obtain two left conservative determiners: "(*constitute more than 1/2 of*)" and "(*constitute less than 1/2 of*)" on condition that |B| is odd. Thus, we may also conclude that these two determiners are left self-duals on condition that |B| is odd.

Apart from the aforesaid special proportional determiners, singular terms

[92] Note that by choosing different k and K, we can obtain even more right self-duals. For example, by choosing k = 3 and K = {0, 3}, we obtain the partitive construction "(*either none or 3 or 5 or 6 of the 7*)", which is also a right self-dual. But since such partitive constructions are quite unnatural, I do not consider them any further.

(i.e. singular proper names of the form "*x*(–)", where x is an individual, and singular definite descriptions of the form "*the*(S)" or "*C's*(S)", where S is singular) are also self-duals, because these singular terms can all be expressed as "*every*(A)", where A = {x}, CS $\cap$ S or $\text{POSSESS}_C \cap$ S. In all these cases, A is a singleton, and "*every*(A)" is equivalent to $Q_{0,\varnothing}(A)$[93]:

(65)
$$Q_{0,\varnothing}(A)(B) \Leftrightarrow \begin{cases} |A \cap B| \in \varnothing, & \text{if } |A \cap B| = 0 \\ 1 - |A \cap B| \notin \varnothing, & \text{if } |A \cap B| = 1 \end{cases}$$

Thus, "*every*(A)" is in fact a special case of $Q_{k,K}(A)$, and so any singular term is a self-dual[94].

The following tables list the fixed points and self-duals found in this thesis:

**Table 3.5 Fixed Points**

| **Argument Involved** | **Fixed Point** |
|---|---|
| right | (*exactly 1/2 of*), (*between q and r of*) (q + r = 1) |
| left | (*constitute exactly 1/2 of*), (*constitute between q and r of*) (q + r = 1) |

**Table 3.6 Self-Duals**

| **Argument Involved** | **Self-Dual** |
|---|---|
| unique | *x*, *the*(A) (A is singular), *C's*(A) (A is singular) |
| right | (*more than 1/2 of*) (\|A\| is odd), (*less than 1/2 of*) (\|A\| is odd) |
| left | (*constitute more than 1/2 of*) (\|B\| is odd), (*constitute less than 1/2 of*) (\|B\| is odd) |

Using the above results, the definition of self-duals and the inference schema (46), we can then derive valid inferences such as the following:

(66) Every policeman asked more than 3 of the 7 suspects some question.

[93] Since A is a singleton, $|A \cap B| = 1$ iff $A \subseteq B$. Thus, (65) can be rewritten as $Q_{0,\varnothing}(A)(B) \Leftrightarrow A \subseteq B$ and so $Q_{0,\varnothing}(A)$ is equivalent to "*every*(A)".

[94] Keenan (2003) also pointed out that reflexive pronouns are self-duals. But since reflexive pronouns have to be analysed as a certain type of non-iterated polyadic GQs not studied in this thesis, I do not include Keenan (2003)'s result on reflexive pronouns.

$\Leftrightarrow$ No policeman asked more than 3 of the 7 suspects no question.

(67) All except one boy gave Mary at least two roses.

$\Leftrightarrow$ Exactly one boy gave Mary fewer than two roses.

### 3.3.5 Duality and Monotonicities

In this subsection, I discuss the relation between duality and monotonicities, i.e. how monotonicities are influenced by the three notions of negation. I first state the following theorem:

**Theorem 3.14** Let Q be a monadic GQ with n arguments, then

(a) Q is increasing (decreasing) in the $i^{th}$ argument iff $\neg Q$ and $Q\neg_i$ are decreasing (increasing) in the $i^{th}$ argument iff $Q^{di}$ is increasing (decreasing) in the $i^{th}$ argument.

(b) Q is non-monotonic in the $i^{th}$ argument iff $\neg Q$, $Q\neg_i$ and $Q^{di}$ are non-monotonic in the $i^{th}$ argument.

The above theorem enables us to determine the monotonicities of GQs. For example, from the facts that "*every*" is right increasing, $\neg$*every* = (*not every*), *every*$\neg_r$ = *no* and *every*$^{dr}$ = *some,* we can deduce by Theorem 3.14(a) that both "(*not every*)" and "*no*" are right decreasing, whereas "*some*" is right increasing.

Theorem 3.14(b) enables us to identify non-monotonic GQs. In addition to this, we also have the following theorem which can be seen as a corollary of Theorem 3.14(a).

**Theorem 3.15** Let Q be a monadic GQ that is non-trivial in the $i^{th}$ argument. If Q is a fixed point in the $i^{th}$ argument, then Q is non-monotonic in that argument.

Combining the above theorem with the findings in the previous subsection, we can identify even more non-monotonic GQs. For example, by combining

Theorem 3.15 and Theorem 3.10, we know that $(Q \wedge Q_{\neg i})$ and $(Q \vee Q_{\neg i})$ where Q is any monadic GQ are non-monotonic in the $i^{th}$ argument. Thus, "(*some but not all*)" and "(*all or no*)" are both right non-monotonic.

### 3.3.6 Transposition Inferences on the Argument Level

In this subsection, I study transposition inferences on the argument level. This kind of inferences is related to three notions of monadic GQs – converses, symmetry and contrapositivity. I first propose the following theorem concerning converses:

**Theorem 3.16** Let $Q_1$ and $Q_2$ be monadic GQs with n arguments and the same argument structure and $1 \leq i < j \leq n$. Then $(\neg Q_1)^{-1}{}_{i,j} = \neg(Q_1{}^{-1}{}_{i,j})$; $(Q_1 \wedge Q_2)^{-1}{}_{i,j} = Q_1{}^{-1}{}_{i,j} \wedge Q_2{}^{-1}{}_{i,j}$; $(Q_1 \vee Q_2)^{-1}{}_{i,j} = Q_1{}^{-1}{}_{i,j} \vee Q_2{}^{-1}{}_{i,j}$.

This theorem enables us to discover more converse pairs of GQs. For example, from *every*$^{-1}$ = *only*, we may deduce that (*not every*)$^{-1}$ = (*not only*).

I next consider symmetry inferences. It is easy to identify symmetric GQs by checking their truth conditions. Whenever the truth condition of a GQ remains unchanged upon transposing a pair of arguments, that GQ is symmetric wrt that pair of arguments. For example, the truth condition of "(*no ... except C*)" is

(68) (*no ... except C*)(A)(B) $\Leftrightarrow A \cap B = C$

It is easy to see that this truth condition remains unchanged upon transposing A and B (because "$\cap$" is commutative). Thus, we know that this determiner is symmetric, as exemplified by the following valid inference:

(69) No student except John sang. $\Leftrightarrow$ No singer except John is a student.

For contrapositivity, I propose the following theorems:

**Theorem 3.17** Let Q be a determiner. Then Q is symmetric iff $Q_{\neg r}$ is contrapositive iff $Q\neg_l$ is contrapositive.

**Theorem 3.18** Let Q be a determiner. Then Q is contrapositive iff $Q^{-1}$ is contrapositive iff $\neg Q$ is contrapositive iff $Q\neg_{l,r}$ is contrapositive.

By using these two theorems, we can easily identify contrapositive determiners. For example, since *no*$\neg_r$ = *every* and (*no ... except C*)$\neg_l$ = (*apart from C only*), and we know that "*no*" and "(*no ... except C*)" are symmetric, by Theorem 3.17 we may conclude that "*every*" and "(*apart from C only*)" are contrapositive. Moreover, since *every*$^{-1}$ = *only*, by Theorem 3.18 we may conclude that "*only*" is also contrapositive.

The following tables list the converse pairs of GQs, symmetric quantifiers and contrapositive determiners found in this thesis:

**Table 3.7 Converse Pairs of GQs**[95]

| **Arguments Involved** | **Converse Pair** |
|---|---|
| left and right | *every*$^{-1}$ = *only*; (*not every*)$^{-1}$ = (*not only*); (*more than r of*)$^{-1}$ = (*constitute more than r of*); (*less than r of*)$^{-1}$ = (*constitute less than r of*); (*at least r of*)$^{-1}$ = (*constitute at least r of*); (*at most r of*)$^{-1}$ = (*constitute at most r of*); (*exactly r of*)$^{-1}$ = (*constitute exactly r of*); (*between q and r of*)$^{-1}$ = (*constitute between q and r of*); (*all except r of*)$^{-1}$ = (*constitute all except r of*); (*all except between q and r of*)$^{-1}$ = (*constitute all except between q and r of*); (*all ... except C*)$^{-1}$ = (*apart from C only*) |
| 1st and 2nd | (*more ... than ...*)$^{-1}_{1,2}$ = (*fewer ... than ...*); (*at least as many ... as ...*)$^{-1}_{1,2}$ = (*at most as many ... as ...*); (*proportionally more... than ...*)$^{-1}_{1,2}$ = (*proportionally fewer... than ...*); (*at least the same proportion of ... as ...*)$^{-1}_{1,2}$ = (*at most the same proportion of ... as ...*); (*constitute a larger proportion of ... than ...*)$^{-1}_{1,2}$ = (*constitute a smaller proportion of ... than ...*); (*constitute at least the same proportion of ... as ...*)$^{-1}_{1,2}$ = (*constitute at most the same proportion of ... as ...*) |

[95] Since converse is an involutive operation, each equation listed below is equivalent to one with the positions of the GQs interchanged. For instance, *every*$^{-1}$ = *only* is equivalent to *only*$^{-1}$ = *every*.

**Table 3.8 Symmetric Quantifiers**

| Arguments Involved | Symmetric Quantifier |
|---|---|
| left and right | *some*, *no*, (*no … except C*), (*more than n*), (*fewer than n*), (*at least n*), (*at most n*), (*exactly n*), (*between m and n*) |
| 1st and 2nd | (*exactly as many … as …*), (*exactly the same proportion of … as …*), (*constitute exactly the same proportion of … as …*) |

**Table 3.9 Contrapositive Determiners**

| |
|---|
| *every*, (*not every*), (*all … except C*), (*all … except n*), *only*, (*not only*), (*apart from C only*) |

### 3.3.7 Transposition Inferences on the Quantifier Level

I next turn to transposition inferences on the quantifier level. This kind of inferences is related to two properties of iterated GQs – scope independence and scope dominance. Due to the technicalities of the issue, in what follows, I will mainly introduce some particular results over finite domains obtained by other scholars[96] and suggest some possible extensions. As for scope independence, the main result is that $Q_1(A_1)$ and $Q_2(A_2)$ are scopally independent where $Q_1(A_1)$ and $Q_2(A_2)$ are non-trivial and increasing iff $Q_1 = Q_2$ = "*every*" / "*some*", or either one of $Q_1(A_1)$ and $Q_2(A_2)$ is a singular term, as exemplified by the following inference schemas:

(70) $$\textit{every}(A_1)([\textit{every}(A_2)]_2(B)) \Leftrightarrow \textit{every}(A_2)([\textit{every}(A_1)]_2(B^{-1}))$$

(71) $$x(-)([\textit{most}(A_2)]_2(B)) \Leftrightarrow \textit{most}(A_2)([x(-)]_2(B^{-1}))$$

As for scope dominance, there are four main results. The first result concerns the case of two increasing non-trivial GQs. If $Q_1$ = "*some*" or $Q_2$ = "*every*", then $Q_1(A_1)$ is scopally dominant over $Q_2(A_2)$, as exemplified by the following schema:

[96] These results are from Altman et al (2005), Fact 2, Corollary 3 and Ben-Avi and Winter (2004), Corollary 7, Corollary 8, Propositon 10.

(72) $some(A_1)([(at\ least\ 2)(A_2)]_2(B)) \Rightarrow (at\ least\ 2)(A_2)([some(A_1)]_2(B^{-1}))$

The second and third results concern the case of two GQs of opposite monotonicities. If the truth conditions of $Q_1$ and $Q_2$ are of one of the following forms: (i) $|A_1 \cap B_1| \geq r_1$ and $|A_2 \cap B_2| < r_2$, respectively, such that $|A_1| / r_1 < (r_2 + 1) / r_2$; or (ii) $|A_1 \cap B_1| < r_1$ and $|A_2 \cap B_2| \geq r_2$, respectively, such that $|A_2| > (r_2 - 1)(|A_1| - r_1 + 2)$, then $Q_1(A_1)$ is scopally dominant over $Q_2(A_2)$, as exemplified by the following schemas (by taking $r_1 = |A_1|$ and $r_2 = |A_2|$ in (73), and $r_1 = |A_1|$ and $r_2 = |A_2| / 2$ in (74)):

(73) $every(A_1)([(not\ every)(A_2)]_2(B)) \Rightarrow (not\ every)(A_2)([every(A_1)]_2(B^{-1}))$

(74)
$$(not\ every)(A_1)([(at\ least\ 1/2\ of)(A_2)]_2(B))$$
$$\Rightarrow (at\ least\ 1/2\ of)(A_2)([(not\ every)(A_1)]_2(B^{-1}))$$

The fourth result concerns the case of two decreasing GQs. If the truth conditions of $Q_1$ and $Q_2$ are of the form $|A_1 \cap B_1| < r_1$ and $|A_2 \cap B_2| < r_2$, respectively, such that $2 - |A_2| / r_2 > (r_1 - 1) / (|A_1| - r_1 + 1)$, then $Q_1(A_1)$ is scopally dominant over $Q_2(A_2)$, as exemplified by the following schema (by taking $r_1 = |A_1| / 2$ and $r_2 = |A_2|$):

(75)
$$(less\ than\ 1/2\ of)(A_1)([(not\ every)(A_2)]_2(B))$$
$$\Rightarrow (not\ every)(A_2)([(less\ than\ 1/2\ of)(A_1)]_2(B^{-1}))$$

To extend the above results, I propose the following theorem:

**Theorem 3.19** $Q_1(A_1)$ is scopally dominant over $Q_2(A_2)$ iff

(a) $(Q_2^{dr})(A_2)$ is scopally dominant over $(Q_1^{dr})(A_1)$;

(b) $(Q_1\neg_1)(A_1)$ is scopally dominant over $(Q_2\neg_1)(A_2)$;

(c) $(Q_2^{dl,r})(A_2)$ is scopally dominant over $(Q_1^{dl,r})(A_1)$.

I now use the above theorem to extend results of scope dominance. By virtue of Theorem 3.19(a) and the fact that $every^{dr}$ = *some* and $(not\ every)^{dr}$ = *no*, we can deduce the following from (73):

(76) $no(A_1)([some(A_2)]_2(B)) \Rightarrow some(A_2)([no(A_1)]_2(B^{-1}))$

Next, by virtue of Theorem 3.19(b) and the fact that $no\neg_1 = only$ and $some\neg_1 =$ (*not only*), from (76) we can deduce the following schema involving left conservative GQs:

(77) $only(A_1)([(not\ only)(A_2)]_2(B)) \Rightarrow (not\ only)(A_2)([only(A_1)]_2(B^{-1}))$

Since scope independence is just a bilateral version of scope dominance, we can also use Theorem 3.19 to extend results of scope independence. For example, by using Theorem 3.19(c) on both directions and the fact that $every^{dl,r} =$ (*not only*), we can deduce the following schema from (70):

(78) $$(not\ only)(A_1)([(not\ only)(A_2)]_2(B))$$
$$\Leftrightarrow (not\ only)(A_2)([(not\ only)(A_1)]_2(B^{-1}))$$

Apart from valid inference schemas, there are also invalid ones which can be disproved by constructing counterexamples. For instance, for the following invalid schema:

(79) $$most(A_1)([(less\ than\ 1/2\ of)(A_2)]_2(B))$$
$$\#\Rightarrow (less\ than\ 1/2\ of)(A_2)([most(A_1)]_2(B^{-1}))$$

we can construct the following counterexample: $A_1 = \{a, b, c\}$, $A_2 = \{d, e, f\}$, $B = \{\langle a, d\rangle, \langle b, e\rangle, \langle c, d\rangle, \langle c, e\rangle\}$. One can check that with these predicates, the premise of (79) is true, but the conclusion is false.

## 3.4 Opposition Inferences[97]

### 3.4.1 Basic Definitions

Opposition inferences refer to inferences involving the contradictory, contrary and subcontrary relations[98] and can be defined by generalizing the

[97] Some parts of this section have been published in Chow (2012b).
[98] These three relations are the core relations defined on the classical square of opposition.

definitions of monotonicity inferences. So let us first review the definitions of the increasing and decreasing monotonicities combined below (c.f. (1) and (2)):

(80) Let Q be a GQ / BO with n arguments. Q is increasing (decreasing) in the $i^{th}$ argument ($1 \leq i \leq n$) iff for all $X_1, \ldots X_i, X_i', \ldots X_n$, $X_i \leq(\geq) X_i'$ $\Rightarrow Q(X_1, \ldots X_i, \ldots X_n) \leq Q(X_1, \ldots X_i', \ldots X_n)$.

In the definition above, "$\leq$ / $\geq$" can be seen as short form of the "subset / superset" relation between sets or "entailing / entailed by" relation between propositions. Now "$\leq$" and "$\geq$" are just two possible binary relations between sets / propositions. If we replace "$\leq$" and "$\geq$" in (80) by other binary relations (denoted by $R_1$, $R_2$), and write them in prefix form (i.e. "$R_1(X, Y)$" instead of "$X$ $R_1$ $Y$"), then we obtain the following definition:

(81) Let Q be a GQ / BO with n arguments. Q is $R_1 \rightarrow R_2$ in the $i^{th}$ argument ($1 \leq i \leq n$) iff for all $X_1, \ldots X_i, X_i', \ldots X_n$, $R_1(X_i, X_i') \Rightarrow R_2(Q(X_1, \ldots X_i, \ldots X_n), Q(X_1, \ldots X_i', \ldots X_n))$.

Under this definition, the increasing and decreasing monotonicities may be represented by "$\leq\rightarrow\leq$" (or equivalently "$\geq\rightarrow\geq$") and "$\leq\rightarrow\geq$" (or equivalently "$\geq\rightarrow\leq$"), respectively.

In addition to (81), we also need the definitions of 7 basic binary relations between sets / propositions: equivalence, subalternation, superalternation, contradiction, contrariety, subcontrariety and loose relationship. The names of these 7 relations are adapted from Brown (1984). They are defined as follows: let X and X' be sets / propositions, then

(82) (a) X is equivalent with X' iff $X = X'$;

(b) X is subalternate to X' iff $X < X'$;

(c) X is superalternate to X' iff $X > X'$;

(d) X is contradictory with X' iff X = ¬X';

(e) X is contrary to X' iff X < ¬X';

(f) X is subcontrary to X' iff ¬X < X';

(g) X is loosely related to X' iff X and X' do not satisfy (a) – (f) above.

Now "≤" and "≥" are just two possible disjunctions of these 7 binary relations, i.e. ≤ = subalternate or equivalent; ≥ = superalternate or equivalent. In this section I will study two other possible disjunctions of these relations. They are "contrary or contradictory" (denoted by "CC" for short) and "subcontrary or contradictory" (denoted by "SC" for short), which can be defined using the definitions in (82):[99]

(83) $$CC(X, X') \Leftrightarrow X \leq \neg X'; \; SC(X, X') \Leftrightarrow \neg X \leq X'$$

From the above definition and the contrapositive law[100], it is easily seen that

(84) $$CC(X, X') \Leftrightarrow CC(X', X); \; SC(X, X') \Leftrightarrow SC(X', X)$$

(85) $$CC(X, X') \Leftrightarrow SC(\neg X, \neg X')$$

When X and X' are propositions, we can also interpret the CC and SC relations alternatively as follows: two propositions satisfy the CC relation iff they cannot be both true, and they satisfy the SC relation iff they cannot be both false.

By instantiating $R_1$ and $R_2$ in definition (81) as CC and SC, we then have 4 possible properties of Q: "CC→CC", "CC→SC", "SC→CC" and "SC→SC". These 4 properties will henceforth be called "opposition properties" (OPs). We say that Q is "o(pposition)-sensitive" in the $i^{th}$ argument iff it possesses any of the aforesaid 4 OPs in that argument. Otherwise, it is o-insensitive in that argument[101]. In what follows, I will denote the sets of GQs possessing or not

---

[99] When X and Y are sets, we have CC(X, X') ⇔ *no*(X)(X'); SC(X, X') ⇔ *every*(¬X)(X').

[100] That is X ≤ X' iff ¬X' ≤ ¬X for any X and X'.

[101] Like "monotonicity", "o-sensitivity" may also be manifested on either the GQ / BO level or the argument level. Using "*every*(A)(B)" as an example, on the GQ / BO level, we say that the o-sensitivity of the GQ "*every*" is, subject to certain conditions, SC→CC in the left argument and CC→CC in the right argument; on the argument level, we say that the o-sensitivities of the arguments A and B under "*every*" are SC→CC and CC→CC, respectively.

possessing a certain OP in a certain argument by placing a "+" or "–" sign on the left and right-hand sides of the name of the OP. For example, –CC→CC+ denotes the set of those GQs that are CC→CC in the right but not left argument.

### 3.4.2 Previous Studies

Opposition inferences were originally inferences involving the opposition relations defined on the classical square of opposition and were one of the immediate inferences studied in Classical Logic. In modern times, some scholars tried to refine or generalize these relations. Reichenbach (1952) has refined the concepts of contrary, subcontrary and subalternate by introducing different presuppositions associated with these concepts. For example, he subclassified the contrary / subcontrary relation into the "proper" and "oblique" subtypes. Other scholars have generalized the classical opposition relations to more general relations. For example, by making different combinations of the 7 basic binary relations, Huang (1994) identified a series of "generalized categorical statements" that are defined on binary relations not studied by logicians before.

There are also modern scholars who studied opposition inferences. Van Benthem (2008) was the first to propose the study on this kind of inferences. After pointing out that monotonicity inferences are inferences with "inclusion premises" in the form:

(86) $$P \leq Q \text{ implies } \varphi(P) \leq \varphi(Q)$$

he proposed (but without carrying out) the study on a new type of inferences with "exclusion premises" in the form[102]:

(87) $$P \leq \neg Q \text{ implies } \varphi(P) \leq \neg\varphi(Q)$$

Note that the above is equivalent to the definition of CC→CC. After van

[102] Both formulae are from van Benthem (2008), section 6.

Benthem (2008), MacCartney (2009), Icard (2012) and Mineshima et al (2012) have started the study on opposition inferences. But their studies were based on frameworks different than that adopted in this thesis.

### 3.4.3 O-Sensitivities of Monadic GQs (Single OP)

My next task is to derive rules for determining the o-sensitivities of monadic GQs. I first propose the following general theorems:

**Theorem 3.20** A GQ with presupposition has any one of the 4 OPs only in cases where its arguments satisfy the presupposition.

**Theorem 3.21** Let Q be a GQ with n arguments. Then wrt the $i^{th}$ argument, Q possesses a certain OP iff each of $\neg Q$, $Q\neg_i$ and $Q^{di}$ possesses a different OP according to the following table:

| **Q** | **¬Q** | **$Q\neg_i$** | **$Q^{di}$** |
|---|---|---|---|
| CC→CC | CC→SC | SC→CC | SC→SC |
| CC→SC | CC→CC | SC→SC | SC→CC |
| SC→CC | SC→SC | CC→CC | CC→SC |
| SC→SC | SC→CC | CC→SC | CC→CC |

**Theorem 3.22** Let $Q_1$ and $Q_2$ be GQs of the same type with $Q_1 \leq Q_2$.

(a) If $Q_2$ is CC→CC (SC→CC) in the $i^{th}$ argument, so is $Q_1$.

(b) If $Q_1$ is CC→SC (SC→SC) in the $i^{th}$ argument, so is $Q_2$.

**Theorem 3.23** Let Q be a GQ with n arguments, $1 \leq i < j \leq n$ and Π be one of the 4 OPs.

(a) Q is Π in the $i^{th}$ argument iff $Q^{-1}_{i,j}$ is Π in the $j^{th}$ argument.

(b) If Q is symmetric wrt the $i^{th}$ and $j^{th}$ arguments, then Q is Π in both or neither of these two arguments.

**Theorem 3.24** Let Q be a contrapositive determiner. Then Q is CC→CC in an argument iff it is SC→CC in the other argument. Q is CC→SC in an argument iff it is SC→SC in the other argument.

The above are general principles. We also need the following particular result:

**Theorem 3.25** "(*at least r of*)" ($1/2 < r < 1$) is CC→CC in the right argument; "(*more than r of*)" ($1/2 \leq r < 1$) is CC→CC in the right argument; "(*between q and r of*)" ($0 < q < r < 1$) is not CC→CC in the left argument.

Based on this particular result and the general theorems above, we can then determine the properties of the proportional GQs. For example, let $1/2 < r < 1$, then since (*exactly r of*) ≤ (*at least r of*) and (*exactly r of*) = (*between r and r of*), from Theorem 3.25 and Theorem 3.22, we have (*exactly r of*), (*at least r of*) ∈ –CC→CC+ for $1/2 < r < 1$. Next let $1/2 \leq r < 1$. By Theorem 3.25, we already know that "(*more than r of*)" is CC→CC in the right argument. Moreover, since (*exactly r* + ε *of*) ≤ (*more than r of*) where ε represents an infinitesimal quantity, by Theorem 3.22, we know that "(*more than r of*)" is not CC→CC in the left argument. Thus, we have (*more than r of*) ∈ –CC→CC+ for $1/2 \leq r < 1$.

We next consider the classical determiner "*some*". First we observe that there is the relation (*at least r of*) ($0 < r \leq 1/2$) ≤ *some*, on condition that A ≠ ∅[103]. Now it can be shown that "(*at least r of*)" is SC→SC in the right argument for $0 < r \leq 1/2$[104]. So by Theorem 3.22(b), we know that "*some*" is SC→SC in the right argument on condition that A ≠ ∅. Note that this condition is essential because when A = ∅, ‖*some*(∅)(B)‖ = 0 for any B, and so we can never have SC(B, B') ⇒ SC(*some*(∅)(B), *some*(∅)(B')). As for the left argument of "*some*", by

[103] According to the truth condition of "(*at least r of*)", this GQ is undefined if A = ∅.

[104] By Theorem 3.25, "(*more than r of*)" ($1/2 \leq r < 1$) is CC→CC in the right argument. Since (*more than r of*)$^{dr}$ = (*at least 1 – r of*), by Theorem 3.21, "(*at least 1 – r of*)" ($0 < 1 - r \leq 1/2$) is SC→SC in the right argument. Replacing the arbitrary 1 – r by r, we obtain the result: "(*at least r of*)" ($0 < r \leq 1/2$) is SC→SC in the right argument.

symmetry of "*some*" and Theorem 3.23(b), we know that "*some*" is SC$\rightarrow$SC in the left argument subject to certain condition. One can easily verify that this condition is $B \neq \varnothing$. The above fact will be represented succinctly by *some* $\in$ +SC$\rightarrow$SC+ ($B \neq \varnothing$; $A \neq \varnothing$)[105].

Since *some*$^{dr}$ = *every*, by Theorem 3.21, we may conclude that "*every*" is CC$\rightarrow$CC in the right argument subject to certain condition. One can easily verify that this condition is $A \neq \varnothing$. Since "*every*" is contrapositive, by Theorem 3.24, "*every*" is SC$\rightarrow$CC in the left argument subject to $B \neq U$. Again this condition is essential because when B = U, $\|every(A)(U)\|$ = 1 for any A, and so we can never have SC(A, A') $\Rightarrow$ CC(*every*(A)(U), *every*(A')(U)). The above fact will be represented succinctly by *every* $\in$ +SC$\rightarrow$CC– $\cap$ –CC$\rightarrow$CC+ ($B \neq U$; $A \neq \varnothing$)[106]. The o-sensitivities of some other determiners can be determined in a similar way.

Concerning the absolute numerical GQs, we have the following negative result:

**Theorem 3.26** Every absolute numerical determiner and structured GQ studied in this thesis is o-insensitive in all arguments.

Based on the above results, we can derive valid inferences. For example, the following are instances exemplifying the facts that "(*at least 3/4 of*)" is CC$\rightarrow$CC in the right argument and "*some*" is SC$\rightarrow$SC in the right argument on condition that its left argument is non-empty (given that CC(TEENAGER, ELDERLY), SC(AGED-OVER-50, AGED-BELOW-51)):

(88) CC("At least 3/4 of the members are teenagers",
"At least 3/4 of the members are elderly")

---

[105] The conditions $B \neq \varnothing$; $A \neq \varnothing$ are ordered such that the first (second) condition corresponds to the left (right) argument of the determiner.

[106] The fact that "*every*" is neither SC$\rightarrow$CC in the right argument nor CC$\rightarrow$CC in the left argument can be established by constructing counterexamples.

(89) (Additional assumption: There is some member.)

SC("Some member is aged over 50",

"Some member is aged below 51")

### 3.4.4 O-Sensitivities of Monadic GQs (Multiple OPs)

In the previous subsection, I have only considered the case in which a GQ possesses a single OP in an argument. In this subsection, I will consider the possibility that a GQ may possess more than one OP in the same argument. To do this, we need to introduce some new notions[107]:

(90) Let Q be a GQ with n arguments. Q is perfectly consistent in the $i^{th}$ argument iff $Q(X_1, \ldots X_i, \ldots X_n) \Rightarrow \neg Q(X_1, \ldots Y, \ldots X_n)$ where Y is any subset or superset of $\neg X_i$.

(91) Let Q be a GQ with n arguments. Q is perfectly complete in the $i^{th}$ argument iff $\neg Q(X_1, \ldots X_i , \ldots X_n) \Rightarrow Q(X_1, \ldots Y, \ldots X_n)$ where Y is any subset or superset of $\neg X_i$.

I now propose the following theorem:

**Theorem 3.27** Let Q be a GQ with n arguments. With respect to the $i^{th}$ argument,

(a) It is impossible for Q to be CC→CC and CC→SC.

(b) It is impossible for Q to be SC→CC and SC→SC.

(c) Q is CC→CC and SC→SC iff Q is self-dual and increasing.

(d) Q is SC→CC and CC→SC iff Q is self-dual and decreasing.

(e) Q is CC→CC and SC→CC iff Q is perfectly consistent.

(f) Q is CC→SC and SC→SC iff Q is perfectly complete.

[107] The notions of "perfect consistency" and "perfect completeness" are generalization of Zwarts (1996)'s notions of "consistency" and "completeness".

From Theorem 3.27(a) and (b), we can deduce that it is impossible for any GQ to possess 3 or 4 of the OPs. Therefore we need not consider these cases.

According to Theorem 3.27(c) and (d), we can find GQs that are both CC→CC and SC→SC, or both SC→CC and CC→SC from among the self-duals identified in Subsection 3.3.4. For example, since the singular terms, i.e. monadic GQs of the form "*x*(–)", "*the*(A)" or "*C's*(A)" where A is singular, are increasing self-duals, they are both CC→CC and SC→SC. Moreover, since "(*constitute less than 1/2 of*)" (where |B| is odd) is a decreasing left self-dual, we know that this determiner is both SC→CC and CC→SC in the left argument.

According to Theorem 3.27(e), we can find GQs that are both CC→CC and SC→CC from among perfectly consistent GQs. But what GQs are these? Among the GQs studied in this thesis, the absolute numerical and proportional GQs are in general not perfectly consistent, because their truth conditions are dependent on the cardinalities or proportionalities rather than the member composition of their arguments. Consider "(*exactly 3/4 of*)". Let us construct a counterexample. Define A and B such that $\|(\textit{exactly 3/4 of})(A)(B)\| = 1$, i.e. $|A \cap B| / |A| = 0.75$. That means $|A \cap \neg B| / |A| = 0.25$. Also define a subset X of $A \cap B$ such that $|X| / |A| = 0.5$. Since ¬B and X are disjoint, we must have $|A \cap (\neg B \cup X)| / |A| = 0.25 + 0.5 = 0.75$, and so we have $\|(\textit{exactly 3/4 of})(A)(\neg B \cup X)\| = 1$. This model shows that "(*exactly 3/4 of*)" is not perfectly consistent in the right argument.

Thus, perfectly consistent GQs can only be found from among GQs that are not essentially numerical or proportional. It turns out that exceptive GQs (such as "(*all … except C*)") whose truth conditions are in the form of a set-theoretic equation are such GQs. Since the truth of a set-theoretic equation depends on the membership composition of the sets involved, changing a set X to a subset or superset of ¬X will in general make a true equation become false. Consider "(*all* ... *except C*)" as an example. Suppose $\|(all \ldots except\ C)(A)(B)\| = 1$, then we have A – B = C. That means C is disjoint from B. So provided that A – C ≠ ∅, we must have A – Y ≠ C, where Y is any subset or superset of ¬B. Thus we conclude that "(*all* ... *except C*)" is both CC→CC and SC→CC in the right argument on condition that A – C ≠ ∅.

Finally, as for GQs that are both CC→SC and SC→SC, i.e. perfectly complete GQs, by Theorem 3.27(e), (f) and Theorem 3.21, we know that this kind of GQs can be found from the outer negations of perfectly consistent GQs. But it turns out that there is no such outer negations among the GQs studied in this thesis. For example, the outer negation of "(*all* ... *except C*)" would be a determiner with the truth condition A – B ≠ C, which does not correspond to any determiner studied in this thesis[108].

The following table summarizes the OPs of the GQs studied in this thesis[109]:

**Table 3.10 OPs of GQs**

| OP Type | GQ |
|---|---|
| CC→CC+ | *everybody*(*-thing*) (S ≠ ∅), ($x_1$, $x_2$ *and* ...) |
| SC→CC+ | *nobody*(*-thing*) (S ≠ ∅) |
| SC→SC+ | *somebody*(*-thing*) (S ≠ ∅) |
| CC→CC+ ∩ SC→CC+ | (*everybody*(*-thing*) *except C*) (S – C ≠ ∅), (*nobody*(*-thing*) *except C*) (S – C ≠ ∅) |
| CC→CC+ ∩ SC→SC+ | *x*, *the*(A) (A is singular), *C's*(A) (A is singular) |

[108] Note that it can be shown that the structured GQ (studied by Beghelli (1994)) as defined by the following truth condition:

$$(the\ same\ \ldots\ as\ \ldots)(A)(B_1, B_2) \Leftrightarrow A \cap B_1 = A \cap B_2$$

is perfectly consistent in the 2nd argument subject to the condition $A - (A \cap B_2) \neq \emptyset$. (A similar statement can be made for the 3rd argument.) Therefore, its outer negation as defined by (the subscript "w" below represents a "weak" version of "different")

$$(different_w\ \ldots\ than\ \ldots)(A)(B_1, B_2) \Leftrightarrow A \cap B_1 \neq A \cap B_2$$

is perfectly complete in the 2nd argument. Therefore, there does exist in natural language GQ that is both CC→SC and SC→SC in the same argument. But these two structured GQs are not among the GQs studied in this thesis.

[109] Only those OP types with at least one o-sensitive argument position are listed here. Thus, GQs studied in this thesis that are not listed below are understood to be o-insensitive in all arguments. For example, (*exactly n*) ∈ –CC→CC– ∩ –CC→SC– ∩ –SC→CC– ∩ –SC→SC–.

| | |
|---|---|
| +CC→CC– | (*constitute more than r of*) ($1/2 \le r < 1$), (*constitute at least r of*) ($1/2 < r < 1$), (*constitute exactly r of*) ($1/2 < r < 1$), (*constitute between q and r of*) ($1/2 < q < r < 1$), (*constitute all except r of*) ($0 < r < 1/2$), (*constitute all except between q and r of*) ($0 < q < r < 1/2$) |
| –CC→CC+ | *most*, (*a majority of*), (*more than r of*) ($1/2 \le r < 1$), (*at least r of*) ($1/2 < r < 1$), (*exactly r of*) ($1/2 < r < 1$), (*between q and r of*) ($1/2 < q < r < 1$), (*all except r of*) ($0 < r < 1/2$), (*all except between q and r of*) ($0 < q < r < 1/2$), *both* |
| +CC→SC– | (*constitute less than r of*) ($1/2 < r < 1$), (*constitute at most r of*) ($1/2 \le r < 1$) |
| –CC→SC+ | (*less than r of*) ($1/2 < r < 1$), (*at most r of*) ($1/2 \le r < 1$) |
| +SC→CC+ | *no* ($B \neq \emptyset$; $A \neq \emptyset$) |
| +SC→CC– | (*constitute less than r of*) ($0 < r \le 1/2$), (*constitute at most r of*) ($0 < r < 1/2$), (*constitute exactly r of*) ($0 < r < 1/2$), (*constitute between q and r of*) ($0 < q < r < 1/2$), (*constitute all except r of*) ($1/2 < r < 1$), (*constitute all except between q and r of*) ($1/2 < q < r < 1$) |
| –SC→CC+ | (*a minority of*), (*less than r of*) ($0 < r \le 1/2$), (*at most r of*) ($0 < r < 1/2$), (*exactly r of*) ($0 < r < 1/2$), (*between q and r of*) ($0 < q < r < 1/2$), (*all except r of*) ($1/2 < r < 1$), (*all except between q and r of*) ($1/2 < q < r < 1$), *neither* |
| +SC→SC+ | *some* ($B \neq \emptyset$; $A \neq \emptyset$) |
| +SC→SC– | (*constitute more than r of*) ($0 < r < 1/2$), (*constitute at least r of*) ($0 < r \le 1/2$) |
| –SC→SC+ | (*more than r of*) ($0 < r < 1/2$), (*at least r of*) ($0 < r \le 1/2$), *either* |
| +CC→CC– ∩ +SC→SC– | (*constitute more than 1/2 of*) ($\|B\|$ is odd) |
| –CC→CC+ ∩ –SC→SC+ | (*more than 1/2 of*) ($\|A\|$ is odd) |
| +SC→CC– ∩ +CC→SC– | (*constitute less than 1/2 of*) ($\|B\|$ is odd) |
| –SC→CC+ ∩ –CC→SC+ | (*less than 1/2 of*) ($\|A\|$ is odd) |
| +CC→CC+ ∩ +SC→CC+ | (*all … except C*) ($B \cup C \neq U$; $A - C \neq \emptyset$), (*no … except C*) ($B - C \neq \emptyset$; $A - C \neq \emptyset$), (*apart from C only*) ($B - C \neq \emptyset$; $A \cup C \neq U$) |
| +CC→CC– ∩ –SC→CC+ | *only* ($B \neq \emptyset$; $A \neq U$) |

| | |
|---|---|
| $+SC \to CC-$ $\cap -CC \to CC+$ | *every* ($B \neq U$; $A \neq \varnothing$) |
| $+CC \to SC-$ $\cap -SC \to SC+$ | (*not only*) ($B \neq \varnothing$; $A \neq U$) |
| $+SC \to SC-$ $\cap -CC \to SC+$ | (*not every*) ($B \neq U$; $A \neq \varnothing$) |

### 3.4.5 Opposition Calculus

Parallel to Monotonicity Calculus, we also have Opposition Calculus, which involves opposition inferences of iterated GQs. The o-sensitivity of an iterated GQ can be determined based on those of its constituent monadic GQs. To this end, we need a principle like PMC. Before stating the principle, we first need a definition:

(92) Let X be a predicate under an iterated GQ. Suppose X is within the $i_k^{th}$ argument of $Q_k$ ($1 \leq k \leq n$), $i_{k-1}^{th}$ argument of $Q_{k-1}$, … $i_1^{th}$ argument of $Q_1$, where $Q_k$, $Q_{k-1}$, … $Q_1$ are constituent monadic GQs of the iterated GQ ordered from the innermost to the outermost layers. Then X has an OP-chain $<R_k, R_{k-1}, \dots R_0>$, where each of $R_k, R_{k-1}, \dots R_0$ is one of {CC, SC}, iff $Q_k$ is $R_k \to R_{k-1}$ in the $i_k^{th}$ argument, $Q_{k-1}$ is $R_{k-1} \to R_{k-2}$ in the $i_{k-1}^{th}$ argument, … $Q_1$ is $R_1 \to R_0$ in the $i_1^{th}$ argument.

For instance, in the following argument structure of an iterated GQ:

(93) (*at most 1/2 of*)($A_1$)({$x_1$: *no*($A_2$)({$x_2$: B($x_1$, $x_2$)})})

$A_2$ is within the left argument of "*no*" and right argument of "(*at most 1/2 of*)". Since "*no*" is SC→CC in the left argument on condition that its right argument is non-empty and "(*at most 1/2 of*)" is CC→SC in the right argument, $A_2$ has an OP-chain <SC, CC, SC> on condition that $\{x_2: B(x_1, x_2)\} \neq \varnothing$. One can also easily check that B has an OP-chain <SC, CC, SC> on condition that $A_2 \neq \varnothing$

while $A_1$ has no OP-chain.

We now consider the case in which a predicate does not fall within the argument of any GQ / BO. Let X and X' be sets. A set not falling within the argument of any GQ / BO can be seen as falling within the argument of the identity operator ι. Now it is obvious that if CC(X, X'), then CC(ι(X), ι(X')). The same is true for the case of SC(X, X'). Thus, ι is CC→CC and SC→SC in its argument. We conclude that a predicate not falling within the argument of any GQ / BO is CC→CC and SC→SC.

We next consider the case in which a predicate falls within the argument of some GQ / BO. We need the following theorems:

**Theorem 3.28** Let P be a predicate. Then $\{x: \neg P(x)\} = \neg\{x: P(x)\}$.

**Theorem 3.29** Let P and P' be n-ary predicates and R be one of {CC, SC}, then $R(P_1, P_2) \Rightarrow R(\{x_i: P(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\}, \{x_i: P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\})$ for any $1 \leq i \leq n$ and any particular set of $x_1, \ldots x_{i-1}, x_{i+1}, \ldots x_n$.

With the above theorems, we can then conclude that a predicate is $R_n \rightarrow R_0$ if it has an OP-chain $\langle R_n, \ldots R_0 \rangle$. In what follows, I will provide a proof sketch for this important result. Suppose we have an iterated GQ in the form (7) renumbered as (94) below:

(94) $$Q_1(A_1)(\{x_1: \ldots Q_n(A_n)(\{x_n: B(x_1, \ldots x_n)\}) \ldots \})$$

We focus on the o-sensitivity of B (the o-sensitivities of other predicates can be similarly treated). Let B have an OP-chain $\langle R_n, R_{n-1}, \ldots R_0 \rangle$ and $R_n(B, B')$. By Theorem 3.29, we have $R_n(\{x_n: B(x_1, \ldots x_n)\}, \{x_n: B'(x_1, \ldots x_n)\})$ for any $x_1, \ldots x_{n-1}$. Moreover, by definition (92), $Q_n$ is $R_n \rightarrow R_{n-1}$ in $\{x_n: B(x_1, \ldots x_n)\}$, and so we have $R_{n-1}(Q_n(A_n)(\{x_n: B(x_1, \ldots x_n)\}), Q_n(A_n)(\{x_n: B'(x_1, \ldots x_n)\}))$. The above reasoning is the same as the "upward derivation" introduced in Subsection 3.2.4:

from the $R_n$ relation at the B-level, we derive the $R_{n-1}$ relation at the $Q_n$-level. Now the process of determining the o-sensitivities of B is essentially a repetition of this upward derivation. After n rounds of derivation, we will finally derive the $R_0$ relation at the $Q_1$ level. The net effect is thus $R_n(B, B') \Rightarrow R_0(Q_1(A_1)(\{x_1: \ldots Q_n(A_n)(\{x_n: B(x_1, \ldots x_n)\}) \ldots \}), Q_1(A_1)(\{x_1: \ldots Q_n(A_n)(\{x_n: B'(x_1, \ldots x_n)\}) \ldots \}))$, showing that B is $R_n \rightarrow R_0$.

The above derivation relies on the condition that B has an OP-chain. This condition does not hold either when at least one of $Q_1, \ldots Q_n$ is o-insensitive, or when the OPs possessed by $Q_1, \ldots Q_n$ do not form a chain. In either case, the absence of the OP-chain blocks the upward derivation.

With the above discussion and results, we can formulate the Principle of Opposition Calculus (POC):

| **Principle of Opposition Calculus (POC)** |
|---|
| A singly-occurring predicate not falling within the argument of any GQ / BO is CC→CC and SC→SC. A singly-occurring predicate is $R_k \rightarrow R_0$ iff it has an OP-chain $\langle R_k, \ldots R_0 \rangle$. |

We can now use POC to determine the o-sensitivities of predicates in a multiply quantified statement. Consider (93) renumbered as (95) below:

(95) $(\textit{at most 1/2 of})(A_1)(\{x_1: \textit{no}(A_2)(\{x_2: B(x_1, x_2)\})\})$

In the above, it has been found that $A_1$ has no OP-chain whereas $A_2$ and B both have the OP-chain <SC, CC, SC> subject to different conditions. Thus, according to POC, we know that $A_1$ is o-insensitive, $A_2$ is SC→SC on condition that $\{x_2: B(x_1, x_2)\} \neq \varnothing$ and B is SC→SC on condition that $A_2 \neq \varnothing$. From the above result, we can derive the following valid inference (by letting $A_1$ = CLUB, $A_2$ = AGED-OVER-50, $A_2$' = AGED-BELOW-51, B = ADMIT-AS-MEMBERS):

(96) (Additional assumption: Every club admits somebody as member.)

SC("At most 1/2 of the clubs admit nobody aged over 50 as member",

"At most 1/2 of the clubs admit nobody aged below 51 as member")

Although POC does not provide a systematic method for constructing counterexamples to show that a certain argument does not possess a certain OP, it is in general not difficult to construct such counterexamples. For instance, to show that the argument $A_1$ in (95) is not CC→SC, we may define U = {a, b, c, d, e, f, g}, $A_1$ = {a, b, c}, $A_1$' = {d, e}, $A_2$ = {f}, B = {<a, g>, <c, f>, <d, g>}. Then we have CC($A_1$, $A_1$') and $\|(\textit{at most 1/2 of})(A_1)(\{x_1: \textit{no}(A_2)(\{x_2: B(x_1, x_2)\})\})\|$ = $\|(\textit{at most 1/2 of})(A_1')(\{x_1: \textit{no}(A_2)(\{x_2: B(x_1, x_2)\})\})\|$ = 0.

Opposition Calculus is also applicable to left-iterated GQs. Consider the predicate B in the argument structure of the following left-iterated GQ:

(97) $\textit{no}(A \cap \{x: \textit{some}(B)(\{y: C(x, y)\})\})(D)$

Since B falls within the left arguments of "*some*" and "*no*", which are SC→SC and SC→CC, respectively, both in its left argument on condition that its right argument is non-empty, B has an OP-chain <SC, SC, CC>. By POC, B is SC→CC subject to the condition that $\{y: C(x, y)\} \neq \varnothing \wedge D \neq \varnothing$. From the above result, we can derive the following valid inference (by letting A = COMPANY, B = AGED-OVER-50, B' = AGED-BELOW-51, C = EMPLOY, D = GO-BANKRUPT):

(98) (Additional assumption: Every company employs somebody and some company went bankrupt.)

CC("No company employing somebody aged over 50 went bankrupt",

"No company employing somebody aged below 51 went bankrupt")

Note that monotonicity inferences of iterated quantifiers are governed by the same condition as opposition inferences. If we represent increasing monotonicity as $\geq\to\geq$ or $\leq\to\leq$ and decreasing monotonicity as $\geq\to\leq$ or $\leq\to\geq$, then we can define an analogous notion of "MON-chain" by replacing {CC, SC} with {$\leq$, $\geq$} in (92) and modify POC by replacing "OP-chain" with "MON-chain". The modified condition can then be used to determine the monotonicities of iterated quantifiers in its predicates.

For illustration, consider (93) renumbered as (99) below:

(99) $(\textit{at most 1/2 of})(A_1)(\{x_1: \textit{no}(A_2)(\{x_2: B(x_1, x_2)\})\})$

Let's determine the monotonicity of $A_2$. Since $A_2$ is within the left argument of "*no*" and right argument of "(*at most 1/2 of*)", and both "*no*" and "(*at most 1/2 of*)" are decreasing in both of their arguments, $A_2$ has a MON-chain $<\leq, \geq, \leq>$ (or equivalently, $<\geq, \leq, \geq>$)[110]. According to the modified POC, we know that $A_2$ is $\leq\to\leq$ (or equivalently $\geq\to\geq$), i.e. increasing. This result is in accord with that obtained by using PMC.

### 3.4.6 GQs as Sets and Arguments

As pointed out above, GQs can be seen as higher order sets and so they may enter into the CC and / or SC relations with other GQs. For example, it is easy to see that the following holds:

(100) Within the domain $\{<A, B>: A \neq \varnothing\}$,

$\mathrm{CC}(\textit{every}, \textit{no}) \wedge \mathrm{SC}(\textit{some}, (\textit{not every}))$

(101) $\mathrm{CC}(\textit{some}, \textit{no}) \wedge \mathrm{SC}(\textit{some}, \textit{no})$

Following the same line of reasoning as in (16) and making use of the fact that a GQ not falling within the argument position of any GQ / BO is both CC$\to$CC

[110] Note that since both increasing and decreasing monotonicities have two possible representations, the determination of MON-chains is more complicated than that of OP-chains. We may need to consider all possible representations of the monotonicities involved in order to determine whether a predicate has a MON-chain.

and SC$\rightarrow$SC, we can then derive the following contrary, subcontrary and contradictory relations in Classical Logic:

(102) Given that $A \neq \emptyset$, *every*(A)(B) $\Rightarrow \neg$*no*(A)(B)

(103) Given that $A \neq \emptyset$, $\neg$*some*(A)(B) $\Rightarrow$ (*not every*)(A)(B)

(104) *some*(A)(B) $\Leftrightarrow \neg$*no*(A)(B)

Thus, the classical contrary, subcontrary and contradictory relations can be seen as special examples of the opposition inferences studied in this thesis.

Moreover, GQs as sets may also act as arguments of other GQs / BOs. For instance, consider (93) renumbered as (105):

(105) (*at most 1/2 of*)($A_1$)($\{x_1$: *no*($A_2$)($\{x_2$: $B(x_1, x_2)\}$)$\}$)

Since "*no*" falls within the right argument of "(*at most 1/2 of*)", which is CC$\rightarrow$SC in the right argument, we know that "*no*" is CC$\rightarrow$SC in (105). Using (100), we can then derive the following valid inference:

(106) SC("At most 1/2 of the clubs admit nobody aged over 50 as members",
"At most 1/2 of the clubs admit everybody aged over 50 as members")

Before closing this subsection, I will introduce and prove the following CC relation which will be useful in Chapter 4:

(107) For any A, A' such that CC(A, A'), CC(*only*(A), *some*(A'))

To proof the above, first assume that $\|$*only*(A)(B)$\|$ = 1 for an arbitrary B. According to Appendix 2, this is equivalent to $B \subseteq A$. From CC(A, A'), we have $A \subseteq \neg A'$ by (83). Combining these two subset relations, we have $B \subseteq \neg A'$, which is equivalent to *no*(A')(B). Thus, we must have $\|$*some*(A')(B)$\|$ = 0. I have shown that CC(*only*(A)(B), *some*(A')(B)) for any B. The above CC relation thus follows.

### 3.4.7 Negation Operator

Finally, we discuss the o-sensitivity of the negation operator "¬".

**Theorem 3.30** "¬" is CC→SC and SC→CC and does not possess other OPs.

With this theorem, we can determine the o-sensitivities of predicates within the scope of "¬". Consider the argument $A_2$ in the following iterated GQ:

(108) (*less than 1/2 of*)($A_1$)({$x_1$: *some*(¬$A_2$)({$x_2$: B($x_1$, $x_2$)})})

Since $A_2$ falls under the argument of "¬", the left argument of "*some*" and the right argument of "(*less than 1/2 of*)", it has an OP-chain <CC, SC, SC, CC>. Therefore, $A_2$ is CC→CC on condition that {$x_2$: B($x_1$, $x_2$)} ≠ ∅. Based on this result, we can derive the following valid inference (by letting $A_1$ = CLUB, $A_2$ = TEENAGER, $A_2$' = ELDERLY, B = ADMIT-AS-MEMBERS):

(109) (Additional assumption: Every club admits somebody as member.)

CC("Less than 1/2 of the clubs admit some non-teenager as member",

"Less than 1/2 of the clubs admit some non-elderly as member")

### 3.4.8 Comparison with Monotonicity Inferences

From the discussion above, one can see that there is a parallel relation between opposition inferences and monotonicity inferences in terms of the basic notions and principles governing the inferential patterns of these two types of inferences. More importantly, the definitions of the CC / SC relations in (83) are expressed in the form of subset relations, a characteristic relation of the monotonicity inferences. In view of this, one may doubt whether opposition inferences can be treated as a subtype of monotonicity inferences. Yet the GQs have non-parallel patterns of monotonicities and o-sensitivities. Consider the proportional determiner "(*at least r of*)" as an example. While this determiner has a uniform monotonicity throughout the whole range of 0 < r < 1 (i.e. it is –MON↑ in that range), it has two different o-sensitivities in that range (i.e. it is –CC$\rightarrow$CC+ for $1/2 < r < 1$ but –SC$\rightarrow$SC+ for $0 < r \leq 1/2$).

In fact, despite the similarity between the definitions of the CC / SC relations and that of the usual subset relation, one cannot derive results for the o-sensitivities of a GQ by simply referring to its monotonicity. Reviewing the proof of the right o-sensitivity of "(*at least r of*)" (i.e. Theorem 3.25), one can find that it contains steps using the properties of right inner negation and outer negation, as well as a step that makes use of a property of proportional determiners (i.e. deriving $\|$(*less than r of*)(A)(B')$\|$ = 1 from $\|$(*at most 1 – r of*)(A)(B')$\|$ = 1 for $1/2 < r < 1$). Note that these steps are not derivable from the right monotonicity of these determiners. Since the o-sensitivities of many other GQs depend on that of "(*at least r of*)", we may thus conclude that o-sensitivities are independent of monotonicities, and opposition inferences are not subsumable under monotonicity inferences.

The inferential relations derived from the OPs of GQs are often weaker than those derived from the monotonicities. For instance, by (83) the inferential relation in (88) can be rewritten as the following entailment:

(110) At least 3/4 of the members are teenagers.

$\Rightarrow$ Less than 3/4 of the members are elderly.

Although valid, the conclusion above seems too weak because if we make use of the relation TEENAGER $\leq \neg$ELDERLY, the right increasing monotonicity of "(*at least 3/4 of*)" and the fact that (*at least 3/4 of*)$\neg_r$ = (*at most 1/4 of*), we can obtain the following sharper inference:

(111) At least 3/4 of the members are teenagers.

$\Rightarrow$ At most 1/4 of the members are elderly.

Thus, opposition inferences seem to generate weaker conclusions.

However, entailment is not the only type of inferential relations that is of interest in logical studies. In some situations, we do need to establish some other types of inferential relations (such as the CC / SC relation) between sets / propositions. Consider the following puzzle[111]:

(112) Three persons A, B and C each made a remark about the membership of a club. Suppose the club has some member, John is a member of the club and there is only one true statement among the three remarks. Which is the only true statement?

A: Not all members of the club are teenagers.

B: Not all members of the club are elderly.

C: John is a teenager.

Based on the fact that "(*not every*)" is CC→SC in the right argument, we may conclude that A's and B's remarks satisfy the SC relation, i.e. one of them must be true. Since there is only one true statement among the three, C's remark must be false, i.e. John is not a teenager. This means that A's remark must be true, because otherwise it contradicts the fact that John is not a teenager. Thus, we conclude that A's remark is the only true statement. Apart from solving logical puzzles, opposition inferences also have linguistic uses. This will be discussed in Chapter 4.

## 3.5 Syllogistic Inferences

### 3.5.1 Basic Definitions

This section mainly studies non-classical syllogisms. In order to do so, we first need to review some basic notions of classical syllogisms. Classical syllogisms refer to inferences between quantified statements with two premises

[111] The following is adapted from a typical type of classical puzzles that make use of relations defined on the classical square of opposition.

and one conclusion each of which has specific syntactic structure (the so-called "figures" and "moods"). The following table summarizes the figures of classical syllogisms:

**Table 3.11 Figures of Classical Syllogisms**

| | **Figure 1** | **Figure 2** | **Figure 3** | **Figure 4** |
|---|---|---|---|---|
| **Major Premise** | $Q_1$(M)(P) | $Q_1$(P)(M) | $Q_1$(M)(P) | $Q_1$(P)(M) |
| **Minor Premise** | $Q_2$(S)(M) | $Q_2$(S)(M) | $Q_2$(M)(S) | $Q_2$(M)(S) |
| **Conclusion** | $Q_3$(S)(P) | $Q_3$(S)(P) | $Q_3$(S)(P) | $Q_3$(S)(P) |

In the above table, S, P and M are the minor term, major term and middle term, respectively. $Q_1$, $Q_2$ and $Q_3$ are quantifiers (restricted to the four classical quantifiers). The mood of a syllogism is denoted by the alphabets representing the classical quantifiers (A = "*every*", E = "*no*", I = "*some*", O = "(not *every*)") appearing in the premises and conclusion. In Classical Logic, we can use the format "mood-figure" to identify a particular syllogism. For example, "EIO-1" represents the following syllogism[112]:

(113) $\textit{no}(M)(P) \wedge \textit{some}(S)(M) \Rightarrow (\textit{not every})(S)(P)$

The essence of the classical syllogisms is to find out the relation between the subject (i.e. the minor term) and predicate (i.e. the major term) in the conclusion, based on the two premises which establish the relations between a middle term and each of the aforesaid two terms. The middle term, which does not appear in the conclusion, functions as a link between the minor and major terms. The following table lists the valid classical syllogisms[113]:

**Table 3.12 Valid Classical Syllogisms**

| **Additional Assumption** | **Valid Syllogism** |
|---|---|
| None | AAA-1, EAE-1, AII-1, EIO-1, EAE-2, AEE-2, EIO-2, AOO-2, IAI-3, AII-3, OAO-3, EIO-3, AEE-4, IAI-4, |

[112] In this thesis, I assume that "$\wedge$" has precedence over "$\Rightarrow$", and so no parantheses are used to contain the two premises.
[113] Adapted from Pagnan (2012).

| | |
|---|---|
| | EIO-4 |
| $S \neq \varnothing$ | AAI-1, EAO-1, AEO-2, EAO-2, AEO-4 |
| $P \neq \varnothing$ | AAI-4 |
| $M \neq \varnothing$ | AAI-3, EAO-3, EAO-4 |

In modern times, many scholars have proposed various types of non-classical syllogisms which differ from the classical ones in the syntactic structure or even the number of premises and conclusions[114]. The traditional concepts of "figure" and "mood" are no longer relevant. The distinction between the major / minor terms and the middle term has also become blurred because in some syllogisms studied by modern scholars, all three terms may appear in the conclusion.

While each of the other types of inferences studied in this chapter is associated with a specific type of operations / relations (i.e. monotonicity inferences associated with the superset and subset relations, argument structure inferences associated with negation and transposition, opposition inferences associated with the CC and SC relations), syllogistic inferences are not associated with any specific type of operations / relations, and we can only define syllogistic inferences as inferences involving quantified statements with at least 2 premises and cannot be classified under the other types of quantifier inferences. This heterogeneous nature makes it difficult to devise a method for determining the (in)validity of all syllogisms. Nevertheless, in this section I will try to formulate general principles and methods for monadic and relational syllogisms.

An important discovery in the modern study on "Natural Logic" is that classical syllogisms are subsumable under monotonicity inferences. For example, van Eijck (1984, 2007) pointed out that all valid classical syllogisms can be

[114] Strictly speaking, syllogisms with more than 2 premises and / or more than 1 conclusion should be called "polysyllogisms". For simplicity, these are also called "syllogisms" in this thesis.

accounted for by using the monotonicities of the GQs in question (plus the property of symmetry and existential presupposition in some cases). For example, the classical AAA-1 syllogism

(114) $\quad\quad\quad\quad$ *every*(M)(P) $\wedge$ *every*(S)(M) $\Rightarrow$ *every*(S)(P)

can be reinterpreted as the following inference:

(115) $\quad\quad\quad\quad$ Given M $\subseteq$ P, *every*(S)(M) $\Rightarrow$ *every*(S)(P)

which is a manifestation of the right increasing monotonicity of "*every*". Moreover, since monotonicity inferences may involve GQs, such kind of inferences can indeed be seen as extension of classical syllogisms. For example, by using the right decreasing monotonicity of "(*less than 1/2 of*)" as exemplified by

(116) $\quad\quad$ Given P $\subseteq$ M, (*less than 1/2 of*)(S)(M) $\Rightarrow$ (*less than 1/2 of*)(S)(P)

we can immediately obtain the following Figure 2 syllogism featuring the GQ "(*less than 1/2 of*)":

(117) $\quad\quad$ *every*(P)(M) $\wedge$ (*less than 1/2 of*)(S)(M) $\Rightarrow$ (*less than 1/2 of*)(S)(P)

Thus, monotonicity inferences can be seen as extension of classical syllogisms.

Apart from the above, modern scholars have also identified a large number of new syllogisms that are not subsumable under monotonicity inferences. These new syllogisms constitute another direction of extending classical syllogisms and will be called non-classical syllogisms, which is the main target of study in this section.

### 3.5.2 Previous Studies

Since the ancient times, generations of logicians had thoroughly studied classical syllogisms. Apart from identifying all valid syllogisms, they had in effect established a logical system based on the concept of "distribution".

However, with the emergence of modern mathematical logic, not only has syllogism lost its central importance in logic, its independent status is also called into question, because syllogisms may be seen as a subtype of logical inferences in FOPL. But in the latter half of the 20th century, we saw a revival of interest among some scholars in syllogisms. In what follows, I will briefly introduce the various progresses made in the modern studies on non-classical syllogisms.

Many scholars have studied syllogisms enriched with various kinds of new features. These new features include: Boolean operations (Reichenbach (1952), Nishihara and Morita (1989), Richman (2004), Moss (2010a, 2011b)[115]), numerical quantifiers (Hacker and Parry (1967), Murphree (1991, 1997), Pratt-Hartmann (2008)), vague / fuzzy quantifiers (Zadeh (1983), Dubois et al (1993), Peterson (2000)), transitive verbs (Thom (1977), Sommers and Englebretsen (2000), Moss (2010b), Pratt-Hartmann and Moss (2009), van Rooij (2012)), comparative adjectives[116] (Keene (1969), Moss (2011a)) and generalized categorical statements (Huang (1994), Cavaliere (2008)).

Apart from identifying non-classical syllogisms, modern scholars have also tried to formulate systematic theories about syllogisms. Some scholars (e.g. Reichenbach (1952), Hacker and Parry (1967), Peterson (2000)) inherited the traditional concept of distribution and tried to formulate new laws of distribution for determining the validity of non-classical syllogisms. Other scholars (e.g. Łukasiewicz (1951), Nishihara and Morita (1989), Moss (2008), Pratt-Hartmann (2008, 2011)) tried to build up formal proof systems for syllogisms and made use of various tools of modern logic to study the metalogical properties, expressive

[115] Moss (2010a) is about syllogisms with intersecting adjectives, but these can also be seen as syllogisms with the Boolean operation of intersection.
[116] Both transitive verbs and comparative adjectives are represented by binary predicates. But comparative adjectives have additional properties such as transitivity.

power and computational complexity of these systems.

### 3.5.3 Monadic Syllogisms

Given the diverse types of syllogisms, can we identify general principles to account for all these syllogisms? Traditionally, the validity of syllogisms is based on the concept of distribution. Although some modern scholars (e.g. van Eijck (1984), Hodges (1998)) have tried to provide formal interpretation for this concept, other scholars (e.g. Geach (1962), Murphree (1994)) criticized and queried the coherence and relevance of this concept. Moreover, it is not clear how to extend this concept to syllogisms with different types of GQs, and so I will not make use of this concept. Instead, I propose two general methods for constructing and proving valid monadic syllogisms.

The first method is to make use of the inferential patterns introduced in this thesis and other logical laws to transform syllogisms known to be valid to new ones. For example, by virtue of the notion of converses, we can easily derive syllogisms with left conservative GQs by replacing the GQs of a valid syllogism with their converses and transposing their arguments. For example, from the classical AEO-2 syllogism, we have the following valid syllogism with the additional assumption that P ≠ ∅ (note that the roles of S and P have been interchanged):

(118) $$\textit{only}(\mathrm{M})(\mathrm{S}) \wedge \textit{no}(\mathrm{M})(\mathrm{P}) \Rightarrow (\textit{not only})(\mathrm{S})(\mathrm{P})$$

The second method is summarized below:

(119) Given a monadic syllogistic schema, the premises may be rewritten as set-theoretic or numerical (in)equalities. Based on these (in)equalities, an appropriate (in)equality can be proved or derived and then rewritten in a

suitable form as the conclusion, if the syllogistic schema is valid; or a counterexample can be constructed, if the syllogistic schema is invalid.

In what follows, I will apply (119) to derive different types of non-classical monadic syllogisms. In this way, the various types of syllogisms can be accounted for under a unified framework. First, consider the following syllogism with Boolean operation:

(120) $(\textit{not every})(S)(M \cap P) \wedge \textit{every}(S)(M) \Rightarrow (\textit{not every})(S)(P)$

According to (119), we first rewrite the premises as (in)equalities:

(121) $|S - (M \cap P)| \geq 1 \wedge |S - M| = 0$

Using the set-theoretic formulae: $A - (B \cap C) = (A - B) \cup (A - C)$, $|A \cup B| = |A| + |B| - |A \cap B|$ and $A - B = A \cap \neg B$ for any sets A, B, C, the first conjunct of (121) can be rewritten as

(122) $|S - M| + |S - P| - |S - M - P| \geq 1$

Using the fact $|S - M| = 0$, the above can be rewritten as $|S - P| \geq 1 + |S - M - P|$, which entails $|S - P| \geq 1$. This is exactly the conclusion of (120).

In some cases, the conclusion is derived indirectly from the premises. Consider the following syllogism with numerical GQs:

(123) $(\textit{at most } n)(M)(\neg P) \wedge (\textit{at least } m + n)(S)(M) \Rightarrow (\textit{at least } m)(S)(P)$

which is in fact a generalized form of the classical AII-1 syllogism. First rewrite the premises as inequalities:

(124) $|M - P| \leq n \wedge |S \cap M| \geq m + n$

Using the set-theoretic formula: $|A| = |A \cap B| + |A - B|$ for any sets A, B, (124) can be rewritten as

(125) $|M - P \cap S| + |M - P - S| \leq n \wedge |S \cap M \cap P| + |S \cap M - P| \geq m + n$

From the first conjunct above, we have $|M - P \cap S| \leq n$, which is equivalent to $-|M - P \cap S| \geq -n$. From the second conjunct, we have $|S \cap M \cap P| \geq m + n -$

$|S \cap M - P|$. Combining the above, we have $|S \cap M \cap P| \geq m$. From this we can then derive $|S \cap P| \geq m$ and thus deduce the conclusion of (123) indirectly.

As shown in Table 3.12, some classical syllogisms rely on additional assumptions about the minimum cardinality of certain sets. Murphree (1997) called these "minimum presuppositions" and extended the concept to "maximum presuppositions". According to Murphree (1997), these presuppositions can be used to derive from one statement to another statement. For example, based on the minimum presupposition $|S| \geq m + n$, from "(*at most n*)(S)(¬P)" we can derive "(*at least m*)(S)(P)"[117]. On the contrary, based on the maximum presupposition $|S| \leq m + n$, from "(*at least m*)(S)(P)" we can derive "(*at most n*)(S)(¬P)".

I next consider a syllogism with maximum presupposition:

(126) $(\textit{at least } m + l)(M)(P) \wedge (\textit{at least } m + n)(S)(M) \wedge |M| \leq l + m + n$

$$\Rightarrow (\textit{at least } m)(S)(P)$$

In classical syllogisms, from two "at least"-statements one cannot derive anything. But with a maximum presupposition, we can derive an "at most"-statement which can then be used as a new premise. For example, in (126), from $|M| \leq l + m + n$ and the first premise, we can derive "(*at most n*)(M)(¬P)". Now this result and the second premise above constitute the premises of (123), and so the conclusion of (126), which is the same as the conclusion of (123), obtains immediately.

### 3.5.4 Relational Syllogisms by Direct Substitution

Relational syllogisms refer to syllogisms with iterated GQs and have posed

[117] We first rewrite $|S| \geq m + n$ as $|S \cap P| + |S - P| \geq m + n$ and "(*at most n*)(S)(¬P)" as $|S - P| \leq n$. Combining these two inequalities, we then obtain $|S \cap P| \geq m$, which gives us "(*at least m*)(S)(P)".

challenges to any theory on syllogistic inferences because of their complexities and diversity. In this and the next subsections I will propose two general methods for deriving and proving certain types of relational syllogisms.

The most straightforward way of constructing relational syllogisms is by making direct substitution for the terms of monadic syllogisms. For example, by substituting P = {x: *most*(O)({y: $R_2$(x, y)})} and S = {x: *no*(S)({y: $R_1$(x, y)})} into the monadic syllogism (123), we immediately obtain the following:

(127) (*at most n*)(M)(¬{x: *most*(O)({y: $R_2$(x, y)})}) ∧ (*at least m* + *n*)({x: *no*(S)({y: $R_1$(x, y)})})(M) ⇒ (*at least m*)({x: *no*(S)({y: $R_1$(x, y)})})({x: *most*(O)({y: $R_2$(x, y)})})

The above schema may be exemplified by the following inference (by letting n = 1, m = 2, S = EXAM, M = REPEATER, O = COURSE, $R_1$ = PASS, $R_2$ = RETAKE):

(128) At most 1 repeater did not retake most of the courses. ∧ At least 3 of those who passed no exams were repeaters. ⇒ At least 2 of those who passed no exams retook most of the courses.

Sometimes we have to make use of results of other types of inferences (such as argument structure inferences) when making substitutions. Van Rooij (2012) discussed the following inference:

(129) No man is seen by an ass. ∧ Everything that laughs sees a man.

⇒ Nothing that laughs is an ass.

which may be represented as (by letting S = LAUGH, M = MAN, O = ASS, R = SEE):

(130) *no*(M)({x: *some*(O)({y: $R^{-1}$(x, y)})}) ∧ *every*(S)({x: *some*(M)({y: R(x, y)})}) ⇒ *no*(S)(O)

To prove the validity of the above, we have to invoke the following equivalence:

(131) $no(A_1)([some(A_2)]_2(B)) \Leftrightarrow no(A_2)([some(A_1)]_2(B^{-1}))$

Note that the above can be derived from (70) by first invoking the duality inference schema (53) (using the facts $every\neg_r = no$ and $every^{dr} = some$) and then replacing the arbitrary ¬B by B on both sides. Using (131), we can then rewrite (130) as

(132) $no(O)(\{x: some(M)(\{y: R(x, y)\})\}) \wedge every(S)(\{x: some(M)(\{y: R(x, y)\})\}) \Rightarrow no(S)(O)$

The above is valid because it is just an instance of the classical EAE-2 syllogism (by substituting M = {x: *some*(M)({y: R(x, y)})}), P = O). The validity of (132) guarantees the validity of (130).

### 3.5.5 Relational Syllogisms by Syllogism Embedding

Apart from direct substitution, we may construct relational syllogisms by syllogism embedding. The idea is to make proper substitution into a monadic syllogism to obtain a syllogism with one of its premises and its conclusion containing a free variable. By binding the free variable with the set symbol, we then transform the syllogism to an immediate inference whose conclusion has the form of a subset relation $X \subseteq Y$, which can be rewritten as the proposition "*every*(X)(Y)". We then choose a suitable syllogistic scheme and make suitable substitution so that "*every*(X)(Y)" becomes one of the premises and derive the desired conclusion. The aforesaid process can be seen as embedding a monadic syllogism into another monadic syllogism.

One advantage of the aforesaid method is that we can easily construct relational syllogisms involving non-classical quantifiers. The key is to identify a valid simple syllogism with non-classical quantifiers. For illustration, consider the following inference schema involving numerical GQs and a binary relation

R:

(133) $(\textit{at least } m + n)(S)(M) \wedge \textit{every}(O)(\{x: (\textit{at most } n)(M)(\{y: \neg R(x, y)\})\})$

$$\Rightarrow \textit{every}(O)(\{x: (\textit{at least } m)(S)(\{y: R(x, y)\})\})$$

Let's see how this schema can be derived. By first substituting P = {y: R(x, y)} into the numerical syllogism (123) proved above and using Theorem 3.28, we obtain the following:

(134) $(\textit{at most } n)(M)(\{y: \neg R(x, y)\}) \wedge (\textit{at least } m + n)(S)(M)$

$$\Rightarrow (\textit{at least } m)(S)(\{y: R(x, y)\})$$

Since x is an arbitrary unbound variable, from the above we derive the following:

(135) $(\textit{at least } m + n)(S)(M) \Rightarrow$

$$\{x: (\textit{at most } n)(M)(\{y: \neg R(x, y)\})\} \subseteq \{x: (\textit{at least } m)(S)(\{y: R(x, y)\})\}$$

We next substitute S = O, P = {x: (*at least m*)(S)({y: R(x, y)})} and M = {x: (*at most n*)(M)({y: ¬R(x, y)})} into the AAA-1 syllogism and obtain

(136) $\textit{every}(\{x: (\textit{at most } n)(M)(\{y: \neg R(x, y)\})\})(\{x: (\textit{at least } m)(S)(\{y: R(x, y)\})\}) \wedge \textit{every}(O)(\{x: (\textit{at most } n)(M)(\{y: \neg R(x, y)\})\}) \Rightarrow \textit{every}(O)(\{x: (\textit{at least } m)(S)(\{y: R(x, y)\})\})$

Note that the first premise of (136) is the same as the conclusion of the immediate inference (135). It can thus be replaced by the premise of (135), i.e. (*at least m* + *n*)(S)(M) (This is equivalent to strengthening the premise of (136)). After such a replacement, we obtain (133). Note that (133) can be seen as the result of embedding the numerical syllogism (123) into the AAA-1 syllogism. Its validity is thus guaranteed by the validity of these two syllogisms. Here is an instance of (133) (by letting m = 2, n = 3, S = BOY, M = SMOKER, O = GIRL, R = LIKE and assuming that DISKLIKE = ¬LIKE):

(137) At least 5 boys are smokers. ∧ Every girl dislikes at most 3 smokers.

$\Rightarrow$ Every girl likes at least 2 boys.

In the above example, R is a general binary relation. If we now require that R possess certain specific properties, then we will obtain even more interesting results. One such property is transitivity as defined below:

(138) A binary relation R is transitive iff for all x, y, z $\in$ U, R(x, y) $\wedge$ R(y, z) $\Rightarrow$ R(x, z).

Based on the above definition, we can derive theorems involving transitive relations, such as the following:

**Theorem 3.31** Let R be a transitive relation, O a set, x an individual and Q a right increasing determiner, then *some*({z: Q(O)({w: R(z, w)})})({y: R(x, y)}) $\Rightarrow$ Q(O)({y: R(x, y)}).

Comparative adjectives, including adjectives used in equal comparative constructions (e.g. "as … as" structure) and those used in unequal comparative constructions (e.g. "more … than" structure), are typical examples of transitive relations. For example, if x is as smart as y, and y is as smart as z, then x is as smart as z. Thus, syllogisms with binary transitive relations may be manifested as syllogisms with comparative adjectives. Consider the following syllogism (where R is assumed to be transitive):

(139) *every*(S)({x: *some*(M)({y: R(x, y)})}) $\wedge$ *every*(M)({z: *most*(O)({w: R(z, w)})}) $\Rightarrow$ *every*(S)({x: *most*(O)({y: R(x, y)})})

Let's see how this schema can be derived. By substituting P = {y: R(x, y)} and S = {z: *most*(O)({w: R(z, w)})} into the classical IAI-3 syllogism, we first obtain the following:

(140) *some*(M)({y: R(x, y)})} $\wedge$ *every*(M)({z: *most*(O)({w: R(z, w)})})

$\Rightarrow$ *some*({z: *most*(O)({w: R(z, w)})})({y: R(x, y)})

Since "*most*" is a right increasing determiner, by Theorem 3.31, we have

(141) $some(\{z: most(O)(\{w: R(z, w)\})\})(\{y: R(x, y)\})$

$$\Rightarrow most(O)(\{y: R(x, y)\})$$

Combining (140) and (141), we obtain

(142) $some(M)(\{y: R(x, y)\}) \wedge every(M)(\{z: most(O)(\{w: R(z, w)\})\})$

$$\Rightarrow most(O)(\{y: R(x, y)\})$$

Since x is an arbitrary unbound variable, from the above we derive the following:

(143) $every(M)(\{z: most(O)(\{w: R(z, w)\})\})$

$$\Rightarrow \{x: some(M)(\{y: R(x, y)\})\} \subseteq \{x: most(O)(\{y: R(x, y)\})\}$$

We next substitute P = $\{x: most(O)(\{y: R(x, y)\})\}$ and M = $\{x: some(M)(\{y: R(x, y)\})\}$ into the AAA-1 syllogism and obtain

(144) $every(\{x: some(M)(\{y: R(x, y)\})\})(\{x: most(O)(\{y: R(x, y)\})\}) \wedge every(S)(\{x: some(M)(\{y: R(x, y)\})\}) \Rightarrow every(S)(\{x: most(O)(\{y: R(x, y)\})\})$

Since the first premise of (144) is the same as the conclusion of (143), it can be replaced by the premise of (143). After such a replacement, we obtain (139). Note that (139) can be seen as the result of embedding the IAI-3 syllogism into the AAA-1 syllogism. Its validity is thus guaranteed by the validity of these two classical syllogisms (as well as Theorem 3.31).

Here is an instance of (139) (by letting S = LOGICIAN, M = PHYSICIST, O = MATHEMATICIAN, R = AS-SMART-AS):

(145) Every logician is as smart as some physicist.

$\wedge$ Every physicist is as smart as most mathematicians.

$\Rightarrow$ Every logician is as smart as most mathematicians.

Note that the above syllogism is also valid if we replace “as smart as” by “smarter than” because adjectives used in unequal comparative constructions are also transitive. However, since adjectives used in unequal comparative constructions possess properties other than transitivity[118], if we are to study syllogistic inferences involving such adjectives, we need to assume more properties for R. By so doing, we can then derive more theorems related to such adjectives in addition to Theorem 3.31, and more syllogistic schemas involving such adjectives. I will leave this for future research.

### 3.5.6 Refutation of Invalid Syllogisms

The classical theory of syllogisms included rules to distinguish valid from invalid syllogisms by considering the distribution of the terms. Since I have abandoned the concept of distribution, no such rules are available. Although (119) provides the general principle of how to refute invalid syllogisms, it is not clear whether the principle can be generalized to a systematic way of constructing counterexamples for refuting invalid syllogistic schemas. Thus in this subsection, I only show some typical examples of refuting invalid syllogistic schemas. First consider an example of a purported monadic syllogistic schema:

(146) $$\textit{every}(M)(P) \wedge (\textit{at least 2})(S)(M) \# \Rightarrow (\textit{at least 3})(S)(P)$$

First rewrite the above as (in)equalities:

(147) $$|M - P| = 0 \wedge |S \cap M| \geq 2 \# \Rightarrow |S \cap P| \geq 3$$

To form a link between the premises and the conclusion, the above is then rewritten as

(148) $$|M - P \cap S| + |M - P - S| = 0 \wedge |S \cap M \cap P| + |S \cap M - P| \geq 2$$
$$\# \Rightarrow |S \cap P \cap M| + |S \cap P - M| \geq 3$$

From the above, it is clear that if we let $|M - P \cap S| = |M - P - S| = |S \cap P - M| = 0$, $|S \cap M \cap P| = 2$, then we will have true premises and false conclusion. Thus,

[118] According to Keene (1969), such adjectives also possess the properties of counter-transitivity and asymmetry.

a counterexample for (146) can be obtained by letting S = M = P = {a, b}.

Next consider an example of embedding a valid syllogism (i.e. AAA-1) into an invalid syllogistic schema (i.e. (146)):

(149) $\mathit{every}(\mathrm{O})(\mathrm{M}) \wedge (\mathit{at\ least\ 2})(\mathrm{S})(\{x: \mathit{every}(\mathrm{M})(\{y: \mathrm{R}(x, y)\})\})$

$\#\Rightarrow (\mathit{at\ least\ 3})(\mathrm{S})(\{x: \mathit{every}(\mathrm{O})(\{y: \mathrm{R}(x, y)\})\})$

Let's see how this purported schema can be derived. First, by substituting S = O and P = {y: R(x, y)} into the AAA-1 syllogistic schema and then binding the unbound variable x, we obtain:

(150) $\mathit{every}(\mathrm{O})(\mathrm{M}) \Rightarrow$

$\{x: \mathit{every}(\mathrm{M})(\{y: \mathrm{R}(x, y)\})\} \subseteq \{x: \mathit{every}(\mathrm{O})(\{y: \mathrm{R}(x, y)\})\}$

Next by substituting M = {x: *every*(M)({y: R(x, y)})} and P = {x: *every*(O)({y: R(x, y)})} into (146), we have

(151) $\mathit{every}(\{x: \mathit{every}(\mathrm{M})(\{y: \mathrm{R}(x, y)\})\})(\{x: \mathit{every}(\mathrm{O})(\{y: \mathrm{R}(x, y)\})\})$

$\wedge (\mathit{at\ least\ 2})(\mathrm{S})(\{x: \mathit{every}(\mathrm{M})(\{y: \mathrm{R}(x, y)\})\})$

$\#\Rightarrow (\mathit{at\ least\ 3})(\mathrm{S})(\{x: \mathit{every}(\mathrm{O})(\{y: \mathrm{R}(x, y)\})\})$

By combining (150) and (151), we obtain the purported inference schema (149).

To refute (149), we can make use of the counterexample for (146) to find a counterexample for (151): O = M = P = {a, b}, S = {c, d}, R = {<c, a>, <c, b>, <d, a>, <d, b>}. Since (149) is derived from (151), it turns out that the above is also a counterexample for (149).

Finally consider an example of embedding an invalid syllogism (i.e. (146)) into a valid syllogistic schema (i.e. AAA-1):

(152) $(\mathit{at\ least\ 2})(\mathrm{O})(\mathrm{M}) \wedge \mathit{every}(\mathrm{S})(\{x: \mathit{every}(\mathrm{M})(\{y: \mathrm{R}(x, y)\})\})$

$\#\Rightarrow \mathit{every}(\mathrm{S})(\{x: (\mathit{at\ least\ 3})(\mathrm{O})(\{y: \mathrm{R}(x, y)\})\})$

Let's see how this schema can be derived. First, by substituting S = O and P = {y: R(x, y)} into (146) and then binding the unbound variable x, we obtain:

(153) $(\textit{at least } 2)(O)(M) \#\Rightarrow$

$$\{x: \textit{every}(M)(\{y: R(x, y)\})\} \subseteq \{x: (\textit{at least } 3)(O)(\{y: R(x, y)\})\}$$

Next by substituting M = $\{x: \textit{every}(M)(\{y: R(x, y)\})\}$ and P = $\{x: (\textit{at least } 3)(O)(\{y: R(x, y)\})\}$ into the AAA-1 syllogistic schema, we have

(154) $\textit{every}(\{x: \textit{every}(M)(\{y: R(x, y)\})\})(\{x: (\textit{at least } 3)(O)(\{y: R(x, y)\})\})$

$$\wedge\ \textit{every}(S)(\{x: \textit{every}(M)(\{y: R(x, y)\})\})$$

$$\Rightarrow \textit{every}(S)(\{x: (\textit{at least } 3)(O)(\{y: R(x, y)\})\})$$

By combining (153) and (154), we obtain the purported inference schema (152).

To refute (152), we can make use of the counterexample for (146) to find a counterexample for (153): O = M = P = {a, b}, R = {<c, a>, <c, b>, <d, a>, <d, b>}. Since (152) is derived from (153), we can expand the above to a counterexample for (152) by just adding S = {c, d}.

### 3.5.7 Inverse Logic

In the above, I am interested in identifying inferential patterns that a given set of GQs satisfies. This kind of inquiry is called "direct logic" by van Benthem (1995). On the contrary, we may also study "inverse logic" in which we try to identify GQs that satisfy a given inferential pattern. Inverse logic has been studied by van Benthem (1984), Westerståhl (1984b), etc. Instead of repeating the results of these studies, here I will only try to generalize a result in Peters and Westerståhl (2006). This result is instructive in that it involves the concepts of conservativity, monotonicity and duality. According to Peters and Westerståhl (2006), a right conservative determiner Q is right increasing iff it satisfies the following schema:

(155) $Q(S)(P) \wedge Q^{dr}(S)(P') \Rightarrow \textit{some}(S)(P \cap P')$

I now generalize this result to the following:

**Theorem 3.32** A left conservative determiner Q is left increasing iff it satisfies the following syllogistic schema:

(156) $Q(S)(P) \wedge Q^{dl}(S')(P) \Rightarrow \textit{some}(S \cap S')(P)$

Here is an instance of (156) (by letting Q = *only,* S = MEMBER, S' = FEMALE, P = ATTEND-THE-MEETING and making use of $\textit{only}^{dl} = \textit{some}$):

(157) Only members attended the meeting. $\wedge$ Some female attended the meeting. $\Rightarrow$ Some female member attended the meeting.

The different nature of inverse logic from direct logic is revealed by the proof of Theorem 3.32 which does not invoke (119). In fact, inverse logic can be studied as a separate topic. For this reason, I will not discuss this topic any further.

## 3.6 Conclusion

Having completed the study on the 4 main types of quantifier inferences, it is now time to briefly discuss the statuses of these inferences in natural language inferences. Monotonicity inferences and opposition inferences can be seen as two main pillars of natural language inferences complementary to each other. They are defined on parallel notions and governed by parallel principles (PMC and POC). Their complementary statuses can be revealed by the fact that the relations studied under each of these two types of inferences correspond to different relations defined on the classical square of opposition: the subset and superset relations correspond to the subalternate relation, whereas the CC and SC relations correspond to the contradictory, contrary and subcontrary relations.

The argument structure inferences studied in this chapter mainly serve as useful tools for deriving the other types of inferences, in that the results of argument structure inferences are often used to transform propositions to a suitable form so that they can be manipulated by the operations of the other types of inferences. Finally, the non-classical syllogisms studied in this chapter provide additional instances of natural language inferences that are not subsumable under the other types of inferences and may be useful for constructing certain complex inferences.

Although quantifier inferences are an important subject area of modern Formal Semantics that may be studied in its own right, what is more important is that they can be used to account for certain aspects of scalar reasoning, which is the topic of the next chapter. Their different statuses mean that they will play different roles in the next chapter.

# Chapter 4 Quantifier Inferences and Scalar Reasoning

## 4.1 Introduction

In this chapter, I will apply the major findings on quantifier inferences worked out in Chapter 3 to the studies on scalar reasoning in an attempt to solve the outstanding problems raised in the end of Chapter 2. Of course, not every single result from Chapter 3 is applicable, and not all 4 types of quantifier inferences are equally important. In fact, the statuses of the 4 types of inferences in this chapter will be similar to those discussed in Section 3.6. More specifically, in the first half of this chapter, monotonicity inferences and opposition inferences will be used to establish a formal framework for scalar entailments and scalar implicatures. Non-classical syllogistic inferences will provide insights for extending the scope of scalar entailments (to "scalar syllogisms"). Argument structure inferences are mainly used to transform propositions to a suitable form so as to reveal their reasoning patterns.

In the second half of this chapter, the formal framework for scalar entailments / implicatures will be further used to study the various types of scalar lexical items introduced in Chapter 2. Although the relation between quantifier inferences and these items is indirect, one will see that some special inferential patterns (especially inferences related to the left arguments of determiners) will shed light on certain aspects of scalar lexical items that past researchers have not paid enough attention to.

## 4.2 Scalar Entailments

### 4.2.1 Generalized Fractions

In this section, I will develop a formal framework for scalar entailments (SEs). This framework is composed of two main ingredients. The first is generalized fractions. The second is the I-function. I will discuss generalized fractions first.

SEs are in essence comparison of informativeness, an attribute of propositions. Therefore, to develop a formal framework for SEs, we first need to devise a formal method for comparing attributes of entities in a domain. In our daily life, comparisons of attributes are often based on specific measurements. For example, comparison of tallness is often based on heights.

The most typical comparisons are comparisons of magnitudes of single numbers. For example, when comparing the tallness of two persons John and Mary, we measure their heights, which are expressed as numbers and compare the magnitudes of the numbers, with the greater magnitude corresponding to the taller person. Formally, I express this as the formula

(1) $$\mathrm{TALLNESS}(x) = h$$

In the above, TALLNESS is the attribute in question, x is a variable of entities (e.g. persons) whose tallness is to be compared, and h is a numerical variable representing heights. This formula says that our comparison of tallness of persons is based on their heights. That is why TALLNESS is expressed as a function. In this example, the attribute depends on only one factor – height.

Sometimes an attribute may depend on more than one factor. In this case, we have to express the attribute function in the form of fractions. Consider a case of comparing the efficiency of factories (represented by x) measured by 3 factors: number of days spent (represented by d), number of jobs completed (represented by j) and number of workers (represented by w). To make a sensible comparison, we may measure the average number of jobs completed per worker per day. This may be represented by the following formula:

(2) $$\text{EFFICIENCY}(x) \sim j / (w \times d)$$

In the above formula, "~" represents "be proportional to". Note that this formula does not provide an exact numerical formula for evaluating the efficiency of factories because efficiency may depend on many factors and it is difficult to devise a precise formula. Nevertheless, this formula does reflect the proportionality relation between efficiency and the three factors. Since it is more convenient to do comparison with an equation, we can transform the above formula to an equation:

(3) $$\text{EFFICIENCY}(x) = j / (w \times d)$$

Strictly speaking, in order for a proportionality relation to become an equation, the RHS of (3) should contain a proportionality constant k. But since every value output by this function will then contain the same constant k, we may scale down all the function values by k and thus obtain the above formula. This is equivalent to choosing a suitable unit to measure EFFICIENCY so that the measured values are equal to the calculated values of the above formula. Note that this is a standard practice adopted in natural sciences. So in what follows, I will adopt the convention that all attribute functions are expressed in the form of an equation without a proportionality constant.

In even more complicated cases, the attribute may depend on non-numerical factors and we cannot combine the factors into a numerical fraction like (3). For instance, consider a case of comparing the smartness of people (represented by x) based on two relevant factors: age (represented by a) and rank attained (represented by r). Although the ranks are ordered (e.g. in a scale like <trainee, officer, manager, CEO>), they are not numerical values. Yet it seems that the following "generalized fraction" is sensible:

(4) $$\text{SMARTNESS}(x) = r / a$$

In the above, r and a are placed at the numerator and denominator because these two factors are directly and inversely proportional to the smartness of a person, respectively, i.e. the higher rank a person attains at a younger age, the smarter he / she is.

In general, given a set of entities (represented by x) whose attribute A is to be compared based on a number of ordered (represented by a scale) but not necessarily numerical factors: $a_1, \ldots, a_m, b_1, \ldots, b_n$, where the $a_i$'s and $b_j$'s are directly and inversely proportional to A respectively, we may make comparison by evaluating a generalized fraction (GF):

(5) $$A(x) = (a_1 \times \ldots \times a_m) / (b_1 \times \ldots \times b_n)$$

For comparison between two GFs, we define the following:

(6) Let $F = (a_1 \times \ldots \times a_m) / (b_1 \times \ldots \times b_n)$ and $F' = (a_1' \times \ldots \times a_m') / (b_1' \times \ldots \times b_n')$ be two GFs of the same structure. Then we say that

(a) $F < F'$ iff for all $1 \leq i \leq m$, $a_i < a_i'$ and for all $1 \leq j \leq n$, $b_j > b_j'$;

(b) $F > F'$ iff for all $1 \leq i \leq m$, $a_i > a_i'$ and for all $1 \leq j \leq n$, $b_j < b_j'$;

(c) $F = F'$ iff for all $1 \leq i \leq m$, $a_i = a_i'$ and for all $1 \leq j \leq n$, $b_j = b_j'$;

(d) F and F' are incomparable, otherwise.

According to the above, we have the following inequality:

(7) $$\text{manager} / 25 < \text{CEO} / 24$$

Note that this inequality corresponds to the following comparison result:

(8) A person who becomes a CEO at the age of 24 is smarter than a person who becomes a manager at the age of 25.

### 4.2.2 The I-Function and SEs

As shown in Chapter 2, informativeness is a central idea in the theory on

SEs. Since the informativeness of a proposition is an attribute that may depend on one or more factors, it can be expressed as an I(nformativeness)-function in the form of a GF. The GF will be composed of variables representing members of scales associated with the proposition defined in a scalar model (SM). These scales are factors determining the informativeness of the proposition. Those factors appearing on the numerator (denominator) are directly (inversely) proportional to the informativeness of the proposition.

In many cases, the informativeness of a proposition is reflected by its likelihood or desirability. Since we have the following relation established in (10) and (15) of Chapter 2:

(9) In an SM whose informativeness is reflected by the likelihood (desirability) of the propositions, informativeness is inversely (directly) proportional to likelihood (desirability).

the GF associated with the I-function should be constructed as follows: those factors appearing on the numerator are inversely (directly) proportional to the likelihood (desirability) of the proposition, whereas those factors appearing on the denominator are directly (inversely) proportional to the likelihood (desirability) of the proposition.

For illustration, let us consider the following propositional function in an SM:

(10) "Jumper x can clear obstacle y"

where x and y are variables from the following two scales, respectively

(11) X: $<x_1, x_2, \dots>$; Y: $<y_1, y_2, \dots>$

Remember that the elements of X are arranged in increasing clumsiness while the members of Y are arranged in increasing difficulty. Now the two factors X and Y are both inversely proportional to the likelihood of (10), because the more clumsy x is and the more difficult y is, the less likely x can clear y. Thus, the I-function for (10) can be written as[119]

(12) I("Jumper x can clear obstacle y") = $x \times y$

Using the I-function and the definition of informativeness given in (9) of Chapter 2, we can now derive the following relation:

(13) Let p and q be two propositions in an SM. Then $I(p) > I(q)$ iff $p \Rightarrow_u q$.

One important point to note is that the entailment relation "$\Rightarrow_u$" above is subject to the condition "other things being equal" and is thus a type of pragmatic reasoning rather than logical inference.

By (13), we can now reduce SEs to inequalities of I-function values of propositions in an SM. An advantage of this reduction is that it provides a convenient method for calculating SEs. This is especially so if the I-function is equal to a GF with many factors and having different proportionality relations with the propositional function. Once we have determined the form of the I-function, we can then calculate easily by just comparing GFs. For illustration, consider the following SE wrt the SM set up by (10) and (11):

(14) Jumper $x_3$ can clear obstacle $y_3$. $\Rightarrow_u$ Jumper $x_2$ can clear obstacle $y_2$.

According to the two scales in (11) and the definition of comparison of GFs, we have the inequality $x_3 \times y_3 > x_2 \times y_2$, which, by (12), tells us that

(15) I("Jumper $x_3$ can clear obstacle $y_3$")
> I("Jumper $x_2$ can clear obstacle $y_2$")

By (13), (15) is just an alternative way of expressing (14). The validity of (14) can now be accounted for by the correctness of (15).

The I-function is particularly convenient for handling SEs involving negated

[119] Since both variables are at the numerator, the denominator of the following GF is 1. Following the convention of arithmetic, the denominator need not be written out in this case.

propositions. Since negation will reverse the proportionality relations, we have the following:

(16) $$I(\neg p) = 1 / I(p)$$

i.e. the I-function of a negated proposition is just the reciprocal of the I-function of the original unnegated proposition. For illustration, consider the following SE:

(17) Jumper $x_9$ cannot clear obstacle $y_{35}$.

$\Rightarrow_u$ Jumper $x_{10}$ cannot clear obstacle $y_{36}$.

By (16), we can obtain from (12) the following:

(18) $$I(\neg(\text{“Jumper x can clear obstacle y”})) = 1 / (x \times y)$$

By virtue of the inequality $1 / (x_9 \times y_{35}) > 1 / (x_{10} \times y_{36})$, we have the following:

(19) I(“Jumper $x_9$ cannot clear obstacle $y_{35}$”)

> I(“Jumper $x_{10}$ cannot clear obstacle $y_{36}$”)

The validity of (17) can now be accounted for by the correctness of (19).

### 4.2.3 Strict Monotonicity Inferences as Scalar Entailments

In this subsection I will establish the association between SEs and monotonicity inferences. First let us compare two typical examples of SEs and standard monotonicity inferences:

(20) Given the difficulty of $y_4$ < the difficulty of $y_5$,

John can clear obstacle $y_5$. $\Rightarrow_u$ John can clear obstacle $y_4$.

(21) Given BOY $\subseteq$ CHILD and JOG $\subseteq$ DO-EXERCISES,

Every child is jogging. $\Rightarrow$ Every boy is doing exercises.

An important difference between the above two examples is that while (20) is based on the strict relations “<” and “$\Rightarrow_u$”, (21) is based on the non-strict relations “$\subseteq$” and “$\Rightarrow$”. To eliminate this difference, we turn our attention from standard monotonicity inferences to strict monotonicity inferences by replacing the non-strict relations in (21) with their strict counterparts "$\subset$" and "$\Rightarrow_u$":

(22) Given BOY $\subset$ CHILD and JOG $\subset$ DO-EXERCISES,

Every child is jogging. $\Rightarrow_u$ Every boy is doing exercises.

where "$\Rightarrow_u$" is defined as follows:

(23) Let p and q be propositions. Then p $\Rightarrow_u$ q iff wrt every model, if $\|p\| = 1$, then $\|q\| = 1$, and it is not the case that wrt every model, if $\|q\| = 1$, then $\|p\| = 1$.[120]

Note that the results of standard monotonicity inferences discussed in Chapter 3 and recorded in Table 3.1 are also valid for strict monotonicity inferences except for certain extreme cases such as the following:

(24) Given $\varnothing \subset$ CHILD, JOG $\subset$ U,

*every*($\varnothing$)(JOG) #$\Rightarrow_u$ *every*(CHILD)(U)

The reason for the invalidity of the above is that the predicates $\varnothing$ and U have trivialized the right and left arguments of "*every*", respectively, by making "*every*($\varnothing$)(X)" and "*every*(X)(U)" trivially true for any set X, and so the conclusion in (24) should be an equivalence instead of a unilateral entailment. But in daily language use, we seldom use these "trivializing predicates". Therefore, by casting aside trivializing predicates, we can safely use the results of standard monotonicity inferences for strict monotonicity inferences.

Strict monotonicity inferences are unilateral entailments between two quantified statements one of whose arguments has proper superset-subset relation. Since proper supersets / subsets can form scales, strict monotonicity inferences can thus be reformulated as SEs. Consider the strict monotonicity inference given in (22) again. Let us define the following propositional function:

[120] According to this definition, p $\Rightarrow_u$ q can be true even if there is some model in which $\|p\| = \|q\| = 1$, provided not all models are such that $\|p\| = \|q\| = 1$.

(25) "Every x is y-ing"

where x and y are variables from the following two scales, respectively:

(26) X: <CHILD, BOY>; Y: <DO-EXERCISES, JOG>

The scales are ordered in a way such that each scalar term is a proper subset of any scalar term on its left. This is to accord with the convention stipulated in Chapter 2 that each scalar term is more informative than any scalar term on its left, because a set is more informative than its superset[121].

How can we determine the I-function for (25)? I will try to provide a general answer to this question instead of talking about a particular example. Let Q(a, … x, … z) be the argument structure of a monadic GQ with n arguments. Let x be an argument of Q whose possible values form a scale X: $<x_1, x_2, x_3, \ldots>$ such that $x_i \supset x_j$ for any i and j such that i < j. Now any argument of Q may be increasing, decreasing or non-monotonic. If x is increasing, then given $x_i \supset x_j$, we have $Q(a, \ldots x_j, \ldots z) \Rightarrow_u Q(a, \ldots x_i, \ldots z)$. Thus, the further right $x_i$ is located in X, the more entailments $Q(a, \ldots x_i, \ldots z)$ can generate, and the more informative $Q(a, \ldots x_i, \ldots z)$ is. Following a similar line of reasoning, if x is decreasing, then the further right $x_i$ is located in X, the less informative $Q(a, \ldots x_i, \ldots z)$ is. If x is non-monotonic, then the location of $x_i$ in X is unrelated to the informativeness of Q(a, … x, … z). Based on the above discussion, we may conclude the following:

(27) Let Q be a GQ and x be an argument of Q. If x is increasing (decreasing), it is directly (inversely) proportional to the informativeness of Q. If x is non-monotonic, it has no proportionality relation with the informativeness of Q.

[121] But since the I-function only depends on the proportionality relation between the scales and the informativeness of the proposition, whether the scalar terms are arranged from less informative to more informative ones or the other way around is in fact not very important.

Now in (25), the arguments x and y are decreasing and increasing, respectively, according to the monotonicity of "*every*". By (27), we may conclude that x and y are inversely and directly proportional to the informativeness of (25), respectively. In other words, the I-function of (25) is

(28) I("Every x is y-ing") = y / x

According to (26), we have JOG / CHILD > DO-EXERCISES / BOY. This means

(29) I("Every child is jogging") > I("Every boy is doing exercises")

We have thus reduced the strict monotonicity inference (22) to a comparison of informativeness. By using (27), we can reduce all strict monotonicity inferences to SEs. In a sense, we may thus say that SEs are a generalization of monotonicity inferences.

### 4.2.4 Proportionality Calculus

We may also consider entailments in which a scalar term is within the scope of a GQ / BO such as the following:

(30) Every jumper who cannot clear obstacle $y_6$ will receive no prize.

$\Rightarrow_u$ Every jumper who cannot clear obstacle $y_5$ will receive no prize.

To account for the validity of the above, we may represent the propositional function associated with this SE as the following tripartite structure:

(31) *every*(JUMPER $\cap$ {x: ¬(x can clear obstacle y)})(NO-PRIZE)

where y is a variable from the scale Y defined in (11)[122]. Now y is a scalar term having a particular (i.e. direct) proportionality in the embedded proposition "x can clear obstacle y". But y is also within the argument of "¬" and the left

[122] Note that in (31), the bound variable x is used as a dummy variable in the set notation and so does not contribute to the informativeness of the whole proposition.

argument of "*every*". So how can we determine the proportionality of y in (31)?

Just as we have Monotonicity Calculus for determining the monotonicity of a GQ argument located within the scope of another GQ / BO, we also have Proportionality Calculus for determining the proportionality of a scalar term located within the scope of a GQ / BO. Given the correspondence between monotonicities and proportionality relations given in (27), the principle governing Proportionality Calculus is similar to that governing Monotonicity Calculus. I now formulate the Principle of Proportionality Calculus (PPC) based on PMC given in Subsection 3.2.4:

| **Principle of Proportionality Calculus (PPC)** |
| --- |
| A singly-occurring scalar term not falling within the argument of any GQ / BO has its proportionality unaffected. A singly-occurring predicate has its proportionality retained (reversed) if it falls within an even (odd) number of inversely proportional argument positions without at the same time falling within any non-monotonic argument position. A singly-occurring predicate is non-monotonic if it falls within at least one non-monotonic argument position. |

We can now use the above principle to determine the proportionality of y in (31). As discussed above, y originally has direct proportionality within the embedded proposition "x can clear obstacle y". Since y falls within the argument of "¬" and the left argument of "*every*", which are both inversely proportional argument positions according to (16) and (27), by PPC we know that y has its proportionality retained. As a result, y is directly proportional to the informativeness of (31). This is formally recorded as

(32) $\mathrm{I}(every(\mathrm{JUMPER} \cap \{x: \neg(x \text{ can clear obstacle } y)\})(\mathrm{NO\text{-}PRIZE})) = y$

Since $y_6 > y_5$, we have

(33) I("Every jumper who cannot clear obstacle $y_6$ will receive no prize")

> I("Every jumper who cannot clear obstacle $y_5$ will receive no prize")

The validity of (30) can now be accounted for by the correctness of (33).

As pointed out in Chapter 3, GQs can be treated as sets, and so they can also act as scalar terms. We next consider a case with two scalar terms one of which being a GQ:

(34) Every jumper can clear obstacle $y_6$.

$\Rightarrow_u$ Some jumper can clear obstacle $y_5$.

We define the following propositional function:

(35) q(JUMPER)({x: x can clear obstacle y})

where y is a variable from the scale Y defined in (11) and q is a variable from[123]

(36) Q: <*some*, *every*>

In (35), q does not fall within the argument of any GQ / BO. According to PMC, such a GQ is increasing, and so has direct proportionality. By PPC, q has its proportionality unaffected. Moreover, y falls within the right argument of q, which is a directly proportional argument position because both members of Q are right increasing. Thus, by PPC, y has its proportionality retained. As a result, we have the following I-function for (35):

(37) I(q(JUMPER)({x: x can clear obstacle y})) = q × y

Since *every* × $y_6$ > *some* × $y_5$, we have

(38) I("Every jumper can clear obstacle $y_6$")

> I("Some jumper can clear obstacle $y_5$")

The validity of (34) can now be accounted for by the correctness of (38).

[123] Note that when we use the scale <*some*, *every*>, we must assume that the left arguments of these 2 GQs are non-empty, because *every*(A)(B) $\Rightarrow_u$ *some*(A)(B) only when A ≠ ∅. But in daily language use, this poses no problem because we seldom use "every" to talk about empty categories.

### 4.2.5 Scalar Syllogisms

Just as we may combine SEs with (strict) monotonicity inferences, we may also consider combining SEs with (non-classical) syllogistic inferences, which may be called scalar syllogisms.

Consider the following scalar syllogism:

(39) There are at most 3 adjudicators. $\wedge$ At least 2 teachers are adjudicators. $\wedge$ Every jumper who can clear obstacle $y_{35}$ will be praised by at least 2 adjudicators. $\Rightarrow_u$ Every jumper who can clear obstacle $y_{36}$ will be praised by at least 1 teacher.

To construct this scalar syllogism, we will use the method of syllogism embedding introduced in Subsection 3.5.5 and the non-classical syllogistic schema (126) of Chapter 3 repeated below (rewritten in the strict form):

(40) $$|M| \leq l + m + n \wedge (\textit{at least } m + n)(S)(M) \wedge (\textit{at least } m + l)(M)(P) \Rightarrow_u (\textit{at least } m)(S)(P)$$

Substituting l = m = n = 1, M = AJUDICATOR, S = TEACHER, P = {y: $\text{PRAISE}^{-1}$(x, y)} into the above, we obtain the following schema:

(41) $$|\text{AJUDICATOR}| \leq 3 \wedge (\textit{at least 2})(\text{TEACHER})(\text{AJUDICATOR}) \wedge (\textit{at least 2})(\text{AJUDICATOR})(\{y: \text{PRAISE}^{-1}(x, y)\}) \Rightarrow_u (\textit{at least 1})(\text{TEACHER})(\{y: \text{PRAISE}^{-1}(x, y)\})$$

Since the above contains an arbitrary unbound variable x, we rewrite it as

(42) $$|\text{AJUDICATOR}| \leq 3 \wedge (\textit{at least 2})(\text{TEACHER})(\text{AJUDICATOR}) \Rightarrow_u \{x: (\textit{at least 2})(\text{AJUDICATOR})(\{y: \text{PRAISE}^{-1}(x, y)\})\} \subset \{x: (\textit{at least 1})(\text{TEACHER})(\{y: \text{PRAISE}^{-1}(x, y)\})\}$$

The conclusion of (42) is a proper subset relation. It thus forms a scale:

(43) $Z: <z_1, z_2>$, where

$$z_1 = \{x: (\textit{at least 1})(\text{TEACHER})(\{y: \text{PRAISE}^{-1}(x, y)\}),$$

$$z_2 = \{x: (\textit{at least 2})(\text{AJUDICATOR})(\{y: \text{PRAISE}^{-1}(x, y)\})\}$$

We next define the following propositional function:

(44) $\textit{every}(\text{JUMPER} \cap \{x: x \text{ can clear obstacle } y\})(z)$

where y and z are variables from the scales Y and Z defined in (11) and (43), respectively. Now y has direct proportionality in the embedded proposition "x can clear obstacle y" and falls within the left argument of "*every*". By PPC, its proportionality is reversed. Moreover, since z falls within the right argument of "*every*", it has direct proportionality. As a result, the I-function of (44) is

(45) $I(\textit{every}(\text{JUMPER} \cap \{x: x \text{ can clear obstacle } y\})(z)) = z / y$

Since $z_2 / y_{35} > z_1 / y_{36}$, we have

(46) I("Every jumper who can clear obstacle $y_{35}$ will be praised by at least 2 adjudicators") > I("Every jumper who can clear obstacle $y_{36}$ will be praised by at least 1 teacher")

From (46) we obtain the following valid unilateral entailment:

(47) Every jumper who can clear obstacle $y_{35}$ will be praised by at least 2 adjudicators. $\Rightarrow_u$ Every jumper who can clear obstacle $y_{36}$ will be praised by at least 1 teacher.

Now (47) is the result of applying PPC to the I-function (45), which depends on two scales: (11) and (43), the latter being derived from the conclusion of (42). By adding the premises of (42) to the premise of (47), we then obtain (39). The validity of (39) is thus guaranteed by the validity of the non-classical syllogism (40) and the correctness of (46).

### 4.3 Scalar Implicatures

### 4.3.1 The I-function and SIs

In this section, I will turn to the topic of scalar implicatures (SIs). Certain notions developed in the previous section for SEs will be extended to the study of SIs. These include scales, GFs, informativeness and the I-function.

According to Figure 2.2, SIs are inferences leading from the truth of a lowly informative statement to the falsity of a highly informative statement in an SM. Informativeness is thus a central idea in the theory on SIs, just as it is in the theory on SEs. We may thus describe SIs in terms of comparison of I-function values.

Consider the following example of canonical SIs:

(48) Most students sang. +> Not all students sang.

It is not difficult to determine the I-function associated with (48):

(49) I(“q students sang”) = q

where q is a variable from the following scale:

(50) Q: <*most*, *all*>

Since according to (50), *most* < *all*, we have

(51) I(“Most students sang”) < I(“All students sang”)

Thus, (48) does have the characteristic of SIs, i.e. an inference from the truth of a lowly informative statement (“Most students sang”) to the falsity of a highly informative statement (“All students sang”).

In Chapter 2, I have also introduced another type of SIs – alternate-value SIs, which are based on unordered sets rather than ordered scales. How can we apply the I-function to this kind of SIs? The key is to reformulate alternate-value SIs as canonical SIs by arranging members of the set into a hierarchy, thereby assigning an order to them. For illustration, consider the following example:

(52) A: Which of Chomsky's works has John read?

B: He has read SS.

+> John has not read ATS.

which is based on the following set:

(53) Y: {SS, ATS}

Note that this set can be reformulated as the following scale of sets:

(54) Y': <{SS, ATS}, {SS}>

We then define the following propositional function associated with (52):

(55) "John has read y' "

where y' is a variable from Y'. It is not difficult to determine the I-function associated with (55):

(56) I("John has read y' ") = 1 / y'

Since according to (54), {SS, ATS} < {SS}, we have

(57) I("John has read SS") < I("John has read both SS and ATS")

As a result of this, we can then say that the SI generated in (52) is in fact a canonical one like the following:

(58) Not both SS and ATS have been read by John.

Given B's response in (52), (58) can be re-expressed more briefly as

(59) John has not read ATS.

which is precisely what appears after "+>" in (52).

In the above, I have shown that the I-function can be used to uniformly describe canonical and alternate-value SIs. However, what the I-function describes is only the symptoms of SIs as depicted in Figure 2.2. It has not explained why SIs arise. To do this, we need more tools. It turns out that these tools can be best illustrated in the case of alternate-value SIs, a topic to which we now turn.

### 4.3.2 Alternate-Value SIs

In this subsection, I will interpret alternate-value SIs as a combination of two components: exhaustivity implicatures and opposition inferences. The concept of exhaustivity implicatures comes from the Contextualist view that makes use of the notions of QUD and strongly exhaustive answers introduced in Chapter 2. This view is stated as follows (the following is a modified version of (25) in Chapter 2):

(60) An SI will arise in a sentence iff the scalar term (with which the SI is associated) is a strongly exhaustive answer[124] to the QUD and therefore has focus.

I will illustrate the idea by considering (52) renumbered as (61) below. Since (60) involves the notion "focus", the following will explicitly show the foci of the answers by putting them in $[.]_F$ (Note that WH-words are by default the foci of the questions where they appear):

(61) A: Which of Chomsky's works has John read?

B: He has read $[SS]_F$.

+> John has not read ATS.

the SI generated is based on the set Y of alternate values defined in (53).

By (60), I interpret B's answer as a strongly exhaustive answer, i.e. "John has only read SS". Note that according to a result of transposition inferences introduced in Chapter 3, a singular term like "John" is scopally independent of any GQ. In other words, "John has only read SS" and "John has read ATS" are

[124] In this thesis, I will consider only a subtype of strongly exhaustive answers. These are called "definite" answers in Groenendijk and Stokhof (1984) and may take either one of the following forms: (i) a single number or a quantifier denoting proportion, (ii) (conjoined) proper names and / or definite descriptions, (iii) universally (positive or negative) quantified terms.

respectively equivalent to “Only SS has been read by John” and “ATS has been read by John”, which may be expressed as the following tripartite structures:

(62) *only*({SS})({x: x has been read by John})

(63) *some*({ATS})({x: x has been read by John})[125]

After exhaustifying the answer, we can then bring opposition inferences into play. Since CC({SS}, {ATS}), by (107) of Chapter 3, we have

(64) CC(*only*({SS}), *some*({ATS}))

The above expression entails that for any B, CC(*only*({SS})(B), *some*({ATS})(B)). From this we may derive the following relation:

(65) CC(*only*({SS})({x: x has been read by John}),
*some*({ATS})({x: x has been read by John}))

Combining the above with (62), we can deduce that ‖*some*({ATS})({x: x has been read by John})‖ = 0, which is precisely the SI derived in (61).

The above discussion shows that there is a subtle relationship between logical entailments and SIs. According to the defeasibility condition for SIs, two propositions having the relation “p +> ¬q” must not be such that ‖p ∧ q‖ = 0 (under the literal, i.e. non-exhaustified, meaning of p). Otherwise, we will have “p ⇒ ¬q”, which is a logical entailment rather than an implicature. However, after exhaustification, we have in effect transformed p to another proposition p’ with exhaustified meaning such that ‖p’ ∧ q‖ = 0, which then gives us the logical entailment “p’ ⇒ ¬q”. For example, in (61), “John has read SS” (under its literal meaning) is compatible with “John has read ATS”. But after exhaustification, “Only SS has been read by John” is no longer compatible with “ATS has been read by John”. The SI in (61) is thus the result of the combined

[125] Since {ATS} is a singleton, *some*({ATS})({x: x has been read by John}) is equivalent to ATS ∈ {x: x has been read by John}.

effects of exhaustification and logical entailments.

In summary, I have broken down the derivation process of SIs into two steps. In the first step (i.e. the exhaustivity implicature), we exhaustify the proposition p (under the literal meaning) which is compatible with q to another proposition p' which is incompatible with q. In the second step (i.e. the opposition inference), we derive a logical entailment "p' ⇒ ¬q". The combination of these two steps gives us the SI "p +> ¬q".

One should be aware that the exhaustivity implicature is context-dependent and does not always hold. In fact, answers to questions may exhibit different types of exhaustivity. As argued by Beck and Rullman (1999), answers of different types of exhaustivity (including strong exhaustivity, weak exhaustivity and non-exhaustivity) may turn out to be ideal answers in different circumstances. For example, in the following discourse, B's response is obviously non-exhaustive but appropriate, given B's understanding of the purpose of A's question (just hoping to buy one copy for reading instead of doing a survey on the sales network):

(66) A: Where can I buy *New York Times*?

B: The newsstand at the train station.

In case of non-exhaustivity, one cannot use opposition inferences to derive SIs. That is why SIs should be seen as defeasible implicatures although their derivation process contains an element of logical entailment.

#### 4.3.3 Canonical SIs

On the surface, canonical SIs are very different from alternate-value SIs in that they are associated with ordered scales consisting of higher and lower values.

But in fact, ordered scales can be reformulated as alternate-value sets[126]. Suppose we have the following scale:

(67) $X: \langle x_1, x_2, x_3, \ldots x_{n-1}, x_n \rangle$

where the scalar terms are related by unilateral entailment relation: $\ldots x_3 \Rightarrow_u x_2 \Rightarrow_u x_1$. This unilateral entailment relation is characteristic of strict monotonicity inferences rather than opposition inferences. But we can reformulate X as

(68) $X': \{x_1 \wedge \neg x_2, x_2 \wedge \neg x_3, \ldots x_{n-1} \wedge \neg x_n, x_n\}$

The idea can be illustrated by the following figure:

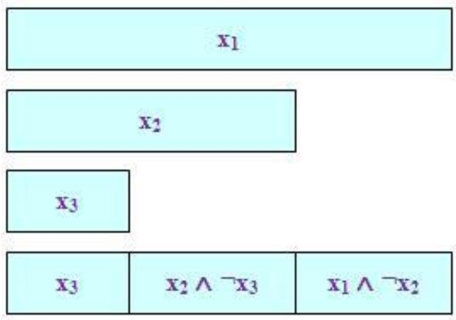


**Figure 4.1 Transforming an Ordered Scale to an Alternate-Value Set**

In the above figure, the rectangles $x_1$, $x_2$ and $x_3$ with a nested structure form a scale. However, they can be reformulated as mutually exclusive and collectively exhaustive rectangles. Essentially, this reformulation replaces the unilateral entailment relation among members of X to contrary relation among members of X'. After deriving the above set, the remaining part in the derivation of the SI is the same as in the previous subsection.

I illustrate the idea by considering the following example:

(69) [Most]$_F$ students sang. +> Not all students sang.

I assume that the QUD associated with this example is

(70) What is the proportion q such that q students sang?

whose answer is restricted to members of the scale Q defined in (50). By (60), I

---

126 In Subsection 4.3.1, I have shown that unordered sets can be reformulated as ordered scales. In this subsection, I will show that ordered scales can be reformulated as unordered sets.

interpret the LHS of (69) as a strongly exhaustive answer to this QUD. Now according to Beck and Rullmann (1999), the strongly exhaustive answer to a question asking for the quantity or degree associated with a proposition is the greatest quantity or highest degree that satisfy that proposition. Thus, the LHS of (69) is understood to be the highest member q of Q such that ‖q students sang‖ = 1, and so is equivalent to "most but not all". Moreover, since proportion serves as an object requested by the QUD (70), "most but not all" should in this case be treated as a Montagovian individual (of a domain composed of GQs) rather than an ordinary GQ[127]. To distinguish "most but not all" being a Montagovian individual from being an ordinary GQ, I will represent the former as "*MOST-BUT-NOT-ALL*". Thus, the effect of exhaustification is to reformulate the scale Q defined in (50) as the following set of Montagovian individuals:

(71) Q': {*MOST-BUT-NOT-ALL*, *ALL*}

The LHS of (69) viewed as a strongly exhaustive answer is thus equivalent to

(72) (*MOST-BUT-NOT-ALL*)(–)({q': q' students sang})

Now the two members of Q' are contrary to each other, i.e.

(73) CC(*MOST-BUT-NOT-ALL*, *ALL*)

From (73) we may derive the following relation:

(74) CC((*MOST-BUT-NOT-ALL*)(–)({q': q' students sang}),
*ALL*(–)({q': q' students sang}))

Combining this with (72), we can thus deduce ‖*ALL*(–)({q': q' students sang})‖ = 0, which is precisely the RHS of (69).

### 4.3.4 SIs Right-Embedded under "*every*"

[127] The situation is similar to a question like "What is the smallest prime number". The answer to this question "two" should be treated as an individual of a domain composed of natural numbers rather than an ordinary GQ.

The framework developed in the previous two subsections for simple SIs can be applied to embedded SIs. As mentioned in Chapter 2, there is heated debate between the Globalists and Localists, who hold different views on the particular SIs generated for embedded scalar terms. In this subsection, I consider SIs embedded under the right argument of "*every*" first.

Under the framework developed above, the difference between the two views can be accounted for by using different types of QUDs. I first consider an example of embedded alternate-value SIs (based on the set Y defined in (53)):

(75) Every student at MIT has read $[SS]_F$.

The SIs generated by this sentence under the Globalist and Localist views are, respectively:

(76) Not every student at MIT has read ATS.

(77) Every student at MIT has not read ATS.

The Globalist SI in (76) is associated with the following QUD:

(78) Which of Chomsky's works has every student at MIT read?

This question asks for a subset of Chomsky's works bearing a common relation to (i.e. having been read by) every MIT student. Viewing (75) as a strongly exhaustive answer to this question, we can express (75) as

(79) *only*({SS})({x: Every MIT student has read x})

Note that although according to Chapter 3, "*every*" and "*only*" are not scopally independent, i.e.

(80) Every student at MIT has only read SS.

#⇔ Only SS has been read by every student at MIT.

this does not affect my analysis here, because according to the Globalists, (75), with "SS" being exhaustified, does not mean the LHS of (80). In other words, the LHS of (80) is simply unrelated to the SI generated by (75) under the Globalist view.

From (79) and the CC relation (64), we may then deduce $\|$*some*({ATS})({x: Every MIT student has read x})$\|$ = 0. This means that ATS is not read by every MIT student (note that this does not rule out the possibility that ATS may be read by some MIT student), which is precisely what (76) asserts.

In contrast, the Localist SI in (77) is associated with the following QUD:

(81) Which of Chomsky's works has every student at MIT only read?

This question is stronger than the question in (78) because it requires that every student at MIT has read and only read the same set of works. The difference between (78) and (81) can be illustrated by a scenario in which the set of MIT students contains only x, y and z such that x read SS, while y and z both read SS and ATS. In this scenario, "SS" is what x, y and z have all read, whereas there is no common work that each of x, y and z has only read. Thus, under the Localist view, (75), with "SS" being exhaustified, can be expressed as

(82) *only*({SS})({x: Every MIT student has only read x})

From (82) and the CC relation (64), we may then deduce $\|$*some*({ATS})({x: Every MIT student has only read x})$\|$ = 0. This means that ATS is not something that every MIT student has only read. In other words, every MIT student has not read ATS, which is precisely what (77) asserts.

Comparing (79) and (82), we will find that while (79) contains just one "only", (82) contains two. For this reason, we may call the QUD (78) a "singly exhaustive question" and (81) a "doubly exhaustive question". Thus, while the Globalist SI is associated with a singly exhaustive QUD, the Localist SI is associated with a doubly exhaustive QUD.

Next consider an example of embedded canonical SIs (based on the scale Q defined in (50):

(83) Every student at MIT has read $[\text{most}]_F$ of Chomsky's works.

The Globalist and Localist SIs derived from this sentence are, respectively:

(84) Not every student at MIT has read all of Chomsky's works.

(85) Every student at MIT has not read all of Chomsky's works.

These two SIs are associated with the following QUDs, respectively:

(86) What is the common proportion q such that every MIT student has read q of Chomsky's works?

(87) What is the common proportion q such that every MIT student has read only q of Chomsky's works?

Similar to the previous example, (86) and (87) are a "singly exhaustive question" and a "doubly exhaustive question", respectively. Their difference is that the former does not rule out the possibility that different students may have read different proportions of Chomsky's works, while the latter requires that every MIT student has read the same proportion of Chomsky's works. The difference between (86) and (87) can be illustrated by a scenario in which the set of MIT students contains only x, y and z such that x read 70% of Chomsky's works, while y and z both read 100% of Chomsky's works. In this scenario, "*most*" is the highest common proportion from among (50) that x, y and z have all read, whereas there is no common proportion that each of x, y and z has only read. Based on the above discussion, (83) can now be expressed as two strongly exhaustive answers from the set Q' defined in (71) to the two QUDs:

(88) (*MOST-BUT-NOT-ALL*)(–)({q': q' is the common proportion such that every MIT student has read q' of Chomsky's works})

(89) (*MOST-BUT-NOT-ALL*)(–)({q': q' is the common proportion such that every MIT student has read only q' of Chomsky's works})

By using the CC relation in (73), we can then derive (84) and (85) from (88) and

(89), respectively.

The above discussion shows that the Localist view is associated with doubly exhaustive QUDs which, in my opinion, are less common and natural than the singly exhaustive QUDs associated with the Globalist view. For this reason, I conclude that the Globalist view on embedded SIs is preferable as it represents a more common and natural phenomenon, while the Localist view should be seen as representing a marked phenomenon.

#### 4.3.5 SIs Right-Embedded under Simple Indefinite Determiners

The framework introduced in the previous subsections cannot be directly applied to SIs embedded in the right argument of simple indefinite determiners such as "*some* / *a*" and bare numerals. Consider the following example:

(90) A student at MIT has read $[SS]_F$.

Note that according to Chapter 3, "*a*" and "*only*" are not scopally independent. In other words,

(91) A student at MIT has only read SS.

#$\Leftrightarrow$ Only SS has been read by a student at MIT.

Moreover, (90), with "SS" being exhaustified, does not mean the RHS of (91). Thus, the framework introduced above seems not to be applicable to this kind of sentences.

However, as mentioned in Chapter 2, Geurts (2010) claimed that such kind of SIs should be analysed as statements about the discourse referents introduced by the indefinite NPs using the DRT framework. I think Geurts (2010)'s idea is correct, but I will implement his idea by using the choice function.

A choice function is of type $(e \to t) \to e$. It is a function f mapping a non-empty set S to an individual such that $f(S) \in S$. Following Reinhart (1998), I will use choice functions to represent indefinites used referentially. Let us consider an example involving a singular indefinite determiner such as (90) above. Using choice functions, (90) can be paraphrased as

(92) $f_1$(MIT-STUDENT) has read $[SS]_F$.

In the above, $f_1$ represents a particular choice function whose output is a specific member of the set MIT-STUDENT. This means that $f_1$(MIT-STUDENT) is like a singular term such as “John”. Thus, the SI generated by (90) is just a simple alternate-value SI and may be treated in the same way as discussed in Subsection 4.3.2. Under such a view, the SI generated is

(93) $f_1$(MIT-STUDENT) has not read ATS.

which is as desired[128].

Next consider an example involving a distributive plural indefinite determiner:

(94) Thirty nine senators supported $[\text{most}]_F$ of the bills.

Following Winter (2001), I treat plural indefinites as sets of sets and so the choice function corresponding to “thirty nine senators” above takes a set of sets as its argument. The distributive meaning of the above sentence will be implemented as a universally quantified statement. Thus, the above sentence can be paraphrased as

(95) Every one of $f_2(\{X: X \subseteq \text{SENATOR} \wedge |X| = 39\})$ supported $[\text{most}]_F$ of the bills.

In the above, $f_2(\{X: X \subseteq \text{SENATOR} \wedge |X| = 39\})$ is a specific set of 39 senators. Now the above paraphrase shows that the SI generated by (94) is just a canonical

[128] Since $f_1$ is a variable, we need to bind it by means of “existential closure”. However, since $f_1$(MIT-STUDENT) is also used in the SI of (90), existential closure should be applied over the whole discourse (including SIs that are not overtly uttered) instead of just one sentence. This will involve theory on discourses and thus will not be further discussed in this thesis.

SI right-embedded under "*every*" and may be treated in the same way as discussed in Subsection 4.3.4. Under this view, the SI generated is

(96) Not every one of $f_2(\{X: X \subseteq \text{SENATOR} \wedge |X| = 39\})$ supported all of the bills.

which is as desired.

### 4.3.6 Left-Embedded SIs

Scalar terms may also be embedded in the left argument of a GQ within a relative clause. In this subsection, I will only consider the case of SIs left-embedded under "*every*". For left-embedded alternate-value SIs, the analysis is the same as that for right-embedded alternate-value SIs, as shown by the following example:

(97) Every student who has read $[\text{SS}]_F$ admires Chomsky.

+> Not every student who has read ATS admires Chomsky.

The QUD associated with the above is

(98) Which of Chomsky's works y is such that every student who has read y admires Chomsky?

The LHS of (97), with "SS" being exhaustified, can be expressed as

(99) *only*({SS})({y: Every student who has read y admires Chomsky})

By using the CC relation (64), one can then derive the RHS of (97).

As for left-embedded canonical SIs, the situation is more complicated, because left-embedded and right-embedded canonical SIs are based on scales with different structures. So far in this section, I have only considered the following "right-implicating scales" ((50) is an example of this kind of scales):

(100) A scale <$x_1$, $x_2$, $x_3$, …> is right-implicating[129] iff $x_1$ is the scalar term in focus and $x_1$, $x_2$, $x_3$, … satisfy … $\Rightarrow_u x_3 \Rightarrow_u x_2 \Rightarrow_u x_1$.

This kind of scales is suitable for the previous examples of canonical SIs because according to Figure 2.2, SIs are inferences from the affirmation of lowly informative propositions to the negation of highly informative propositions. Now in the scale (100), the proposition with $x_1$ is the lowest informative proposition in the SM, and so it will implicate the negation of the propositions with $x_2$, $x_3$, …

The above conclusion is valid only when the focused scalar term is in an increasing position. If it is in a decreasing position, then we will need "left-implicating scales":

(101) A scale <… $x_3$, $x_2$, $x_1$> is left-implicating iff $x_1$ is the scalar term in focus and $x_1$, $x_2$, $x_3$, … satisfy $x_1 \Rightarrow_u x_2 \Rightarrow_u x_3 \Rightarrow_u$ …

because the direction of inference of a term in decreasing position is opposite that of a term in increasing position. Thus, we see monotonicity inferences also have a role to play in the theory on SIs. To summarize, we have the following:

(102) A scalar term in an increasing (decreasing) position is based on a right- (left-) implicating scale.

For illustration, consider the following example:

(103) Every student who has read [most]$_F$ of Chomsky's works admires Chomsky. +> Not every student who has read some of Chomsky's works admires Chomsky.

Note that the scalar term in focus, "*most*" falls within the left argument of "*every*", which is a decreasing position. So by (102), (103) is based on a left-implicating scale, such as

[129] This kind of scales is called "right-implicating" because it is associated with SIs in which a proposition with the scalar term $x_1$ implicates the negation of all propositions with scalar terms that are on the right of $x_1$.

(104) Q: <*some*, *most*>

The QUD associated with (103) is

(105) What is the common proportion q such that every student who has read q of Chomsky’s works admires Chomsky?

Since q falls within a decreasing position, a strongly exhaustive answer to this QUD would be the lowest member q of Q satisfying the proposition after “such that” above. The effect of this exhaustification is to transform (104) into the following set of Montagovian individuals:

(106) Q’: <*SOME-BUT-NOT-MOST*, *MOST*>

whose members satisfy the following relation:

(107) CC(*SOME-BUT-NOT-MOST*, *MOST*)

The LHS of (103) can thus be expressed as

(108) *MOST*(–)({q’: q’ is the common proportion such that every student who has read q’ of Chomsky’s works admires Chomsky})

From (107), we can derive ‖*SOME-BUT-NOT-MOST*(–)({q’: q’ is the common proportion such that every student who has read q’ of Chomsky’s works admires Chomsky})‖ = 0, which is precisely the RHS in (103).

### 4.3.7 SIs with Negative Scalar Terms

Scalar terms may also be embedded under the negation operator. As the negation operator may appear in different sentence levels, there are different types of negative SIs. In this subsection, I will study SIs with negative scalar terms. First consider an example of alternate-value SI with a negative scalar term:

(109) A: So did you snarf all the cakes down?

B: I didn't eat the $[\text{chocolate}]_F$ one.

+> B ate the cheese cake.

which is based on the following set of alternate values:

(110) Y: {chocolate, cheese}

In Chapter 2, I have pointed out that the above SI is reminiscent of inferences involving the subcontrary relation. But for uniform analysis, I will interpret the SI as involving contrary inferences. After all, contrary and subcontrary relations are interdefinable thanks to (85) of Chapter 3. Thus, I will make use of the following relation:

(111) CC(*only*({chocolate}), *some*({cheese}))

The key point is to note that B's response is not a direct answer to A's question. According to the QUD model, B's response can be seen as an answer to a sub-QUD that incorporates the negation operator:

(112) Which cake didn't you eat?

With respect to this sub-QUD, B's response under exhaustive interpretation can be represented by

(113) *only*({chocolate})({y: ¬(B ate the y cake)})

By (111) and (113), we can deduce ‖*some*({cheese})({y: ¬(B ate the y cake)})‖ = 0. This means that the cheese cake is not among the cakes that B didn't eat, which is equivalent to the SI derived in (109).

Next consider an example of canonical SI with a negative scalar term:

(114) Not $[\text{most}]_F$ students sang. +> Some student sang.

Note that the scalar term in focus, "most", is in a decreasing position. So by (102), (114) is based on a left-implicating scale such as (104). Similar to the previous example, we may assume that the above SI is associated with a QUD that incorporates the negation operator:

(115) What is the proportion q such that not q student(s) sang?

Under exhaustive interpretation, the scale (104) is transformed to the set Q' defined in (106) and the LHS in (114) can thus be expressed as

(116) $MOST(-)(\{q': \neg(q' \text{ student(s) sang})\})$

From (107) and (116) we can deduce $\|SOME\text{-}BUT\text{-}NOT\text{-}MOST(-)(\{q': \neg(q' \text{ student(s) sang})\})\| = 0$. This is equivalent to $\|$Some but not most student sang$\|$ = 1, which is precisely the RHS of (114).

### 4.3.8 SIs Right-Embedded under "*no*"

In this subsection, I will study SIs embedded in the right argument of "*no*". Consider the following example:

(117) No student at MIT has read $[\text{most}]_F$ of Chomsky's works. +> Some student at MIT has read some of Chomsky's works.

Note that "*most*" is in a decreasing position. So by (102), (117) is based on a left-implicating scale such as (104). The key point for analyzing (117) is to transform the LHS of (117) to a quantified statement headed by "*every*". By using the following duality inference schema ((42) of Chapter 3):

(118) $Q(A)(B) \Leftrightarrow (Q\neg_r)(A)(\neg B)$

the LHS of (117) can be transformed to

(119) Every student at MIT has not read $[\text{most}]_F$ of Chomsky's works.

We may assume that the above is associated with the following QUD:

(120) What is the common proportion q such that every student at MIT has not read q of Chomsky's works?

Under exhaustive interpretation, the scale (104) is transformed to the set Q' defined in (106) and (119) can thus be expressed as

(121) *MOST*(–)({q’: q’ is the common proportion such that every MIT student has not read q’ of Chomsky’s works}))

From (107) we can deduce ‖*SOME-BUT-NOT-MOST*(–)({q’: q’ is the common proportion such that every MIT student has not read q’ of Chomsky’s works)})‖ = 0, which is equivalent to[130]

(122) Not every student at MIT has not read any of Chomsky’s works.

By using (118) again, the above can be transformed to

(123) Some student at MIT has read some of Chomsky’s works.

which is precisely the RHS of (117).

### 4.3.9 Contrastive Construals and SIs

In Chapter 2, I have introduced the notion of contrastive construals which can account for the phenomena of SI cancellation, SI reinforcement and scalar metalinguistic negation (SMN), all involving scalar terms in antonymy contexts, as exemplified in the following examples (in what follows, I adopt Iwata (1998)’s idea by treating the contrasted terms as focused terms):

(124) $[\text{Some}]_F$ student sang yesterday. In fact / Actually, $[\text{all}]_F$ of them did.

(125) $[\text{Some}]_F$ student sang yesterday. But not $[\text{all}]_F$ of them did.

(126) Not $[\text{some}]_F$ student sang yesterday. $[\text{All}]_F$ of them did.

All these examples can be accounted for by adopting Geurts (2010)’s view that contrastive construals are explicatures resulted from narrowing, a subtype of meaning modulation. Under this view, the meaning of “some” above has been narrowed to “some but not all”, so that these sentences all involve ordinary negation corresponding to Jones (2002)’s negated antonymy context and there is

[130] In what follows I have used the NPI “any” instead of “some” because it is within the scope of a negation operator.

no semantic oddity.

Now that we have identified two related aspects involving scalar terms, i.e. SIs and contrastive construals, should we view them as belonging to the same kind of phenomena (i.e. explicatures) as suggested by the Relevance Theorists, or different kinds of phenomena (i.e. implicatures and explicatures, respectively) as suggested by Geurts (2010)? I think Geurts (2010)'s view is preferable for the following reason.

One important difference between explicature and implicature is that the former is part of the truth-conditional content albeit obtained via a pragmatic process, whereas the latter is not part of the truth-conditional content. In an example of contrastive construal such as (126), "some but not all" must be seen as the truth-conditional meaning of the term "some" because otherwise the two clauses will lead to contradiction, a truth-conditional oddity. On the other hand, in an example of SI such as (69) above, the "most but not all" meaning of the term "most" is a result of the interaction of exhaustivity implicature and opposition inferences. As argued in Subsection 4.3.2, the exhaustivity implicature is subject to certain conditions including the QUD focus and relevance of strongly exhaustive reading of the answer to the QUD. Failing to provide a strongly exhaustive answer will only lead to violation of Grice's cooperative principle, which is by no means a truth-conditional oddity, and may in some contexts even be desirable (e.g. (66)). Thus, we see that SIs and contrastive construals have very different nature and it is preferable to classify them as belonging to two different classes of phenomena.

Moreover, the reinterpretation of SIs proposed in this thesis also shares a striking similarity with the Relevance Theorists' notion of implicatures. Compare the following example of conversational implicature from Carston (2004)[131]:

(127) X: Have you read Susan's book?

Y: I don't read autobiographical books. (i)

Implicated premise: Susan's book is autobiographical. (ii)

Implicated conclusion: Y hasn't read Susan's book. (iii)

(i) $\wedge$ (ii) $\Rightarrow$ (iii)

with the following example of alternate-value SI adapted from (61) above:

(128) A: Which of Chomsky's works has John read?

B: He has read $[\text{SS}]_F$. (i)

Exhaustivity implicature: B's answer is strongly exhaustive. (ii)

Opposition inference: John has not read ATS. (iii)

(i) $\wedge$ (ii) $\Rightarrow$ (iii)

The similarity of these two examples shows that the SIs reinterpreted under the framework of this section are compatible with the Relevance-Theoretic notion of implicatures.

### 4.3.10 Other Applications of Opposition Inferences

So far in this section, I have only shown how the results of opposition inferences can be applied to account for the process of SI generation. In fact, these results have wider applications. One such application is to determine the incompatibility between two predicates. For instance, from the fact that *every* $\in$ –CC$\rightarrow$CC+, we know that "clubs all members of which are teenagers" and "clubs all members of which are elderly" are incompatible, whereas "clubs of which all teenagers are members" and "clubs of which all elderly are members" are not (because it is logically possible to have a club that includes all teenagers

[131] Adapted from Carston (2004), (18).

and elderly as members).

As incompatibility is an essential element of antonyms that feature in certain linguistic structures, such as those identified by Jones (2002), the determination of incompatibility can help us determine the well-formedness of certain linguistic structures. For example, “X rather than Y” is a structure where X and Y should be antonyms. Thus, based on the above discussion, we know that the following sentence is well-formed:

(129) I would rather work for [a club all members of which are teenagers]$_F$ than [a club all members of which are elderly]$_F$.

Of course, this does not mean that (129) will necessarily become not well-formed if it becomes

(130) I would rather work for [a club of which all teenagers are members]$_F$ than [a club of which all elderly are members]$_F$.

because when appearing in an antonymy context like “X rather than Y”, the meanings of X and Y will often be construed contrastively so as to become mutually incompatible. This is precisely the process of narrowing. For example, in (130) the meanings of “club of which all teenagers / elderly are members” may be narrowed down to say “club that includes all and only teenagers / elderly as members”, so as to make the two types of clubs contrary to each other. Thus, the results of opposition inferences can help us determine in what occasion narrowing is needed.

## 4.4 Scalar Operators and Climax Construction Connectives[132]

### 4.4.1 Focus Structure

As introduced in Chapter 2, scalar operators (SOs) and climax construction

[132] Some parts of this section have been published in Chow (2011c) (in Chinese).

connectives (CCCs) are studied independently by different scholars based on very different frameworks. Yet there is a certain degree of overlapping between the two types of lexical items. Table 2.1 shows a rough correspondence between Chinese CCCs and English particles that have been counted as SOs in the literature. Thus, it is instructive to treat SOs and CCCs on a par. More specifically, I claim that CCCs can be treated as SOs. In this section, I will reformulate some of the findings on SOs and CCCs introduced in Chapter 2 using the I-function. But before doing so, I have to introduce some basic definitions.

According to König (1991), SOs are a subtype of focus particles. Thus, a sentence with an SO has a focus structure. For analysis we only need to consider the portion of the sentence constituting the scope of the SO. The focus structure of this portion will be represented as follows:

(131) $O(f)(\lambda x(p(x)))$

where O represents the SO, f represents the focus value, p represents the scope of O (not including O itself) and $\lambda x(p(x))$ is the result of λ-abstracting the focus from p. If p contains more than one focus, then f will be in the form of an ordered tuple. In the following example (with reference to Figure 2.1),[133]

(132) A: Can jumper $[x_1]_{F1}$ clear obstacle $[y_6]_{F2}$?

B: Sure. Jumper $[x_2]_{F1}$ can even clear obstacle $[y_7]_{F2}$.

B's response contains two foci and may be represented by

(133) *even*($\langle x_2, y_7 \rangle$)($\lambda\langle x, y\rangle$("Jumper x can clear obstacle y"))

The notions TP and CP introduced in Chapter 2 can also be expressed using the aforesaid notation. TP may be represented by $\lambda x(p(x))(f)$. CP may be

[133] The numbers attached with the foci show the correspondence between the foci in the two sentences.

represented by $\lambda x(p(x))(f')$ or $\neg\lambda x(p(x))(f')$, where f' represents an alternative focus value. Using this notation, the TP and CP of B's response in (132) are, respectively,

(134) $\lambda\langle x, y\rangle$("Jumper x can clear obstacle y")($\langle x_2, y_7\rangle$)

= "Jumper $x_2$ can clear obstacle $y_7$"

(135) $\lambda\langle x, y\rangle$("Jumper x can clear obstacle y")($\langle x_1, y_6\rangle$)

= "Jumper $x_1$ can clear obstacle $y_6$"

### 4.4.2 Standalone SOs

The SOs in Table 2.1 can be classified into two types: "standalone" SOs (e.g. "even") and correlative SOs (e.g. "not only … but also …"). In this subsection, I will study the three most basic standalone SOs first. These include "even", "not to mention"[134] and "only" (under the scalar meaning). The objective of the study is to identify the conditions of use for these SOs.

Kay (1990) has proposed the condition of use for "even", i.e. (42) of Chapter 2, which is based on the relative informativeness of the TP and CP associated with an "even"-sentence. I assume that this condition is also valid for the Chinese equivalents of "even". By (42) of Chapter 2 and (13), this condition can easily be reformulated in terms of the I-function as follows:

(136) even: I(TP) > I(CP)

Using (132) as an example, we first determine the following I-function (which is identical to (12)):

(137) I("Jumper x can clear obstacle y") = $x \times y$

Since with reference to Figure 2.1, $x_2 \times y_7 > x_1 \times y_6$, we have

[134] Apart from "not to mention", "let alone" and "much less" also convey similar meaning. But since the latter two are NPIs, I choose the more general item "not to mention" as representative.

(138) $I(\text{“Jumper } x_2 \text{ can clear obstacle } y_7\text{”})$

$> I(\text{“Jumper } x_1 \text{ can clear obstacle } y_6\text{”})$

By (134) – (136), we may thus conclude that “even” is properly used in (132) because the “even”-sentence satisfies the condition of use for “even”.

In a similar fashion, the conditions of use for “not to mention” and “only” (as well as their Chinese equivalents) may be formulated respectively as follows:

(139) not to mention: $I(TP) < I(CP)$

(140) only: $I(TP) < I(\neg CP)$

The following are sentences containing “not to mention” and “only” (the CPs are given in parentheses):

(141) (John can clear obstacle $[y_7]_F$), not to mention obstacle $[y_6]_F$.

(142) John can only clear obstacle $[y_6]_F$. (He cannot clear obstacle $[y_7]_F$.)

These two sentences are associated with the following common I-function:

(143) $I(\text{“John can clear obstacle } y\text{”}) = y$

The proper use of SOs in (141) and (142) can both be accounted for by the inequality $y_6 < y_7$. For example, for (142), $\neg CP$ = “John can clear obstacle $y_7$”. Since $y_6 < y_7$, we have

(144) $I(\text{“John can clear obstacle } y_6\text{”}) < I(\text{“John can clear obstacle } y_7\text{”})$

By (140), we may then conclude that “only” is properly used in (142).

The correct formulation of the I-function is crucial to the correct analysis of SOs. In different contexts, an SO may be associated with the same scale in opposite ways. Compare the following sentences which König (1991) classified as expressing sufficient and necessary conditions, respectively[135]:

(145) Only a $[B]_F$ grade is required. (An $[A]_F$ grade is not required.)

---

[135] Adapted from König (1991), Ch. 5, (15)a, b, p. 103.

(146) Only a $[B]_F$ grade is satisfactory. (A $[C]_F$ grade is not satisfactory.)

Despite the fact that the two sentences may be seen as associated with the same scale:

(147) X: <… C, B, A>

the I-functions for the two sentences are very different because of the different SE patterns satisfied by them:

(148) An A grade is required. $\Rightarrow_u$ A B grade is required. $\Rightarrow_u$ …

(149) … $\Rightarrow_u$ A B grade is satisfactory. $\Rightarrow_u$ An A grade is satisfactory.

Based on the above entailment patterns, we can determine the I-functions for (145) and (146), respectively, as follows:

(150) I("A x grade is required") = x

(151) I("A x grade is satisfactory") = 1 / x

where x is a variable from (147). By using these I-functions, one can check that (145) and (146) satisfy condition (140) for "only".

Incidentally, in Chinese the same morpheme "zhi" can appear as part of the conjunction for sufficient conditions "zhiyao" (≈ "provided that") as well as the conjunction for necessary conditions "zhiyou" (≈ "only if"). This lends support to the assertion that "only" can enter into opposite reasoning directions in different contexts.

### 4.4.3 Correlative SOs

An advantage of pursuing cross-linguistic study of SOs and CCCs is that we can gain some insight that may otherwise be overlooked. In Chinese grammar, correlatives[136] are important constructions, and there are a number of correlative

[136] Correlatives do not constitute a distinctive part of speech in Chinese grammar. They refer to constructions composed of two (sometimes even three) conjunctions, adverbs or particles that are often used together.

CCCs. This suggests that we may also study “correlative SOs”, which has not been studied under SMT before. In this subsection, I will consider the correlative SOs shown in Table 2.1.

Comparing (136) and (139), one can find that the conditions of use for “even” and “not to mention” are opposite inequalities. This implies that the TP of the one can be the CP of the other, and so “even” and “not to mention” can readily form correlative SOs. In fact, there is a well-established Chinese correlative CCC – “shangqie … hekuang …” that corresponds to “even … not to mention …”. The condition of use for “even … not to mention …” is given below (in what follows, p and q represent propositions):

(152) even p, not to mention q: I(p) > I(q)

Here is a Chinese example of this correlative SO:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| (153) | Ta | $[gao]_F$ | lan | shangqie | tiao | de | guo, | hekuang | $[di]_F$ | lan. |
| | he | high | hurdle | shangqie | jump | able | over | hekuang | low | hurdle |

*He can even jump over high hurdles, not to mention low hurdles.*

The I-function associated with this sentence can be determined as follows:

(154) I(“Ta x lan tiao de guo”) = x

where x is a variable from the following scale:

(155) X: <di, gao>

Since gao > di, we have

(156) I(“Ta gao lan tiao de guo”) > I(“Ta di lan tiao de guo”)

Since condition (152) is satisfied, we may conclude that “shangqie … hekuang …” is properly used in (153).

Next consider “not only … but also …”. Here we have a case of an SO embedded under the scope of the negation operator. Since the framework adopted in this thesis considers just one facet of the meaning of the SOs, I am not going to derive the meaning of "not only" compositionally. Instead, I will treat "not only" holistically (just like the GQ "fewer than n") by contrasting it with "only". While "only" implies that an alternative proposition with a higher informativeness, i.e. the CP, is not true, "not only" implies that the CP is true. Thus, the condition of use for "not only" should be

(157) not only: I(TP) < I(CP)

i.e. identical to condition (139) for "not to mention".

But "not only" is seldom used in isolation. Its condition of use shows that it is readily paired with "even". In English, "not only … but also …" is a commonly used correlative which uses the non-scalar particle "also". But just like "only", "also" may have scalar use, especially when it appears in the aforesaid correlative. In such a context, the function of "also" is in fact very similar to that of "even", because both are additive focus particles according to König (1991). Therefore, I claim that "not only …, but even …" and "not only …, but also …" are near-variants of each other. Interestingly, in Chinese we also have the two variants "budan … erqie …" and "budan … lian_dou / shenzhi …" corresponding to these two English variants. Here is the condition of use for this correlative:

(158) not only p, but also q: I(p) < I(q)

Given the scalar use of the originally non-scalar additive particle "also", one may conjecture that other additive particles may also have scalar use. This conjecture is borne out as Zhou (2007) pointed out that in Chinese the inclusive construction "chule … hai / you / zai / qie / bingqie / erqie …" (corresponding to the English construction "besides … also …") can also be used as a CCC, as exemplified by

(159) Ta chule hui zuo $[\text{jiashu}]_F$, hai hui zuo $[\text{chengshu}]_F$.

he chule know do addition hai know do multiplication

*Besides knowing how to add, he also knows how to multiply.*

In the above sentence, "chule p, hai q" conveys the same meaning as "budan p, erqie q". The I-function associated with this sentence can be determined as follows:

(160) I("Ta hui zuo x") = x

where x is a variable from the following scale:

(161) X: <jiashu, chengshu>

Since jiashu < chengshu, we have

(162) I("Ta hui zuo jiashu") < I("Ta hui zuo chengshu")

Since condition (158) is satisfied, we may conclude that "chule … hai …" is properly used in (159).

Based on the discussion above, we find that "even" appears in two correlatives: "even … not to mention …" and "not only … but even …" (being a near-variant of "not only … but also …"). One may thus conjecture that the two correlatives may be combined in one sentence with "even" acting as a bridge. This conjecture is borne out as Zhou (2007) pointed out that there does exist a construction "budan … lian_dou … hekuang …" in Chinese.

But in fact "also" and "even" are a bit different in that the latter is more emphatic than the former. This can be illustrated by the fact that in Chinese, "budan", "erqie" and "shenzhi" can form a three-part correlative "budan … erqie … shenzhi …". Interestingly, in English we can also find sentences with the structure "not only p, but also q, and even r", such as the following (through google search)[137]:

(163) The study of astrology, he argues, offers a practical method of <u>not only</u> becoming more conscious of these subtle connections <u>but also</u> of testing <u>and even</u> predicting their occurrence throughout our lives.

The condition of use of this three-part correlative is given below:

(164) not only p, but also q, and even r: I(p) < I(q) < I(r)

Next consider the Chinese correlative "budan bu … faner …". As introduced in Chapter 2, in the construction "budan bu p, faner q", q is not only contrary to p, but also denotes a larger scope or higher degree than ¬p. Thus, the condition of use for "budan bu … faner …" can be formulated as:

(165) budan bu p, faner q: I(¬p) < I(q)

Here is an example of "budan bu … faner …":

(166)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Jintian | wuhou | xia | le | yi | chang | lei | zhenyu, | budan |
| today | afternoon | fall | ASP | one | CLS | thunder | shower | budan |
| meiyou | liang | xia | lai, | tianqi | faner | geng | menre | le. |
| not yet | cool | down | come | weather | faner | more | stuffy | PART |

*After the thunder shower this afternoon, not only hasn't it got cooler. Quite the contrary, it gets even more stuffy.*

The I-function associated with this sentence can be determined as follows:

(167) I("Jintian wuhou xia le yi chang lei zhenyu, tianqi x") = x

where x is a variable from the following scale:

(168) X: <¬liang xia lai, geng menre>

Since ¬liang xia lai < geng menre, we have

(169) I(¬("Jintian wuhou xia le yi chang lei zhenyu, tianqi liang xia lai")) <

[137] http://en.wikipedia.org/wiki/Synchronicity

I("Jintian wuhou xia le yi chang lei zhenyu, tianqi geng menre")

Since condition (165) is satisfied, we may conclude that "budan bu … faner …" is properly used in (166).

It was also pointed out in Chapter 2 that sometimes "faner q" may be used alone, provided that there is an appropriate presupposed clause in the context playing the same role as p in "budan bu p, faner q". Using the terminology of this thesis, this p is the CP of the "faner"-sentence. Based on (165), we may then formulate the condition of use for a standalone "faner" as follows:

(170) faner: $I(TP) > I(\neg CP)$

#### 4.4.4 SO + Conjunction

Some SOs may combine with conjunctions to form composite conjunctions such as "even if", "even though", "not to mention if", "only if", "only when", etc. In Chinese, we have the conjunctions "jishi", "jiusuan", etc. which perform the same function as "even if", although they do not have the transparent form of "CCC + Conjunction".

I will analyse these composite conjunctions as an SO acting on a complex sentence. In this case, the focus of the SO will be a proposition. For example, consider the following sentence[138]:

(171) Even if [it snows]$_F$, the match will not be cancelled. (If [it rains]$_F$, the match of course will not be cancelled.)

The TP of this sentence may be represented as

(172) *even*("it snows")(λx("The match will not be cancelled if x"))

We may adopt the following I-function for (171):

[138] Adapted from Sawada (2003) (5)a.

(173) I("The match will not be cancelled if x") = x

where x is a variable from the scale <"it is windy", "it rains", "it snows">, one can easily verify that (171) satisfies condition (136) for "even".

### 4.4.5 SO + Negation

In Chapter 2, I have introduced two different approaches for analyzing "even" + negation – the Scope Approach and the Lexical Approach. Each of the two approaches has its own merits and demerits. In this thesis, I adopt the Scope Approach, which maintains that a sentence like

(174) John cannot even clear obstacle $[y_3]_F$. (No doubt he cannot clear obstacle $[y_4]_F$.)

should be analysed as "even" taking wider scope than "not", i.e.

(175) $even(y_3)(\lambda y(\neg(\text{"John can clear obstacle y"})))$

Based on (143) and (16), we can then obtain the following I-function:

(176) $I(\neg(\text{"John can clear obstacle y"})) = 1 / y$

Since $1 / y_3 > 1 / y_4$, we have

(177) I("John cannot clear obstacle $y_3$") > I("John cannot clear obstacle $y_4$")

By condition (136) for "even", we may then conclude that "even" is properly used in (174).

One reason why I prefer the Scope Approach is that in English, negative "even"-sentences can be reorganized as having "even" before "not", as exemplified by the following reorganized version of (174):

(178) John even cannot clear obstacle $[y_3]_F$. (No doubt he cannot clear obstacle $[y_4]_F$.)

More interestingly, in Chinese, "lian / shenzhi" must appear before the negation operator, as exemplified in the following sentence:

(179) Zhangsan lian $[di]_F$ lan dou tiao bu guo.

Zhangsan lian low hurdle dou jump not over

*Zhangsan cannot even jump over low hurdles.*

When we consider correlatives like the "even … not to mention …" + negation structure, we will find another advantage of the Scope Approach. Consider the following sentence:

(180) He cannot even clear obstacle $[y_3]_F$, not to mention obstacle $[y_4]_F$.

Under the Scope Approach, both "even" and "not to mention" are seen to be taking wider scope than "not", and so both clauses have the same I-function (176). The proper use of "even … not to mention …" in the above sentence can thus be accounted for by the condition of use (152) and the correctness of the inequality $1 / y_3 > 1 / y_4$.

In contrast, under the Lexical Approach, the "even" in (180) is seen to be an NPI "$\text{even}_{\text{NPI}}$" taking narrower scope than "not" with the following condition of use (c.f. (136)):

(181) $\text{even}_{\text{NPI}}$: I(TP) < I(CP)

In order to account for the proper use of the correlative "even … not to mention …" in (180), we then have to postulate an NPI "$\text{not-to-mention}_{\text{NPI}}$" with the following condition of use (c.f. (139)):

(182) $\text{not-to-mention}_{\text{NPI}}$: I(TP) > I(CP)

as well as the following condition of use for the correlative (c.f. (152)):

(183) $\text{even}_{\text{NPI}}$ p, $\text{not-to-mention}_{\text{NPI}}$ q: I(p) < I(q)

This analysis is equivalent to saying that "not to mention" also takes narrower scope than "not" in (180), just like "even". But this is untenable because the second clause of (180) can be expanded to[139]

(184) … not to mention the fact that he cannot clear obstacle $[y_4]_F$.

in which “not to mention” takes wider scope than “not” rather than the other way around. The above argument shows that the Scope Approach is more plausible.

### 4.4.6 Certain Complex Sentence Types

As shown by (152), “even” and “not to mention” play contrastive roles in the correlative they form. Apart from pairs of lexical items, certain pairs of complex sentence types may also enter into such a relationship. According to Talmy (2000)’s Force Dynamics Schema, causal and concessive sentences represent different consequences of interaction between an external force and a hindrance. Causal sentences represent the consequence of the external force successfully overcoming the hindrance and yielding the normal result, whereas concessive sentences represent the consequence of the external force failing to overcome the hindrance and yield the normal result. Talmy (2000)’s analysis can be extended to two more sentence types that are closely related to causal sentences and concessive sentences. They are hypothetical conditional sentences and hypothetical concessive sentences, respectively. Note that according to Xing (2001)’s classification scheme, causal and hypothetical conditional sentences both are subtypes of “generalized causal complex sentences”, whereas concessive and hypothetical concessive sentences both are subtypes of “generalized contrastive complex sentences”.

The above results can be reinterpreted in terms of informativeness. The external force successfully overcoming the hindrance and yielding the normal result represents an expected scenario and so have relatively low informativeness.

[139] A lot of examples of “not to mention the fact that” can be found on the Internet.

In contrast, the external force failing to overcome the hindrance and yielding the normal result represents an unexpected scenario and so have relatively high informativeness. Based on (152), we can thus establish the correspondence between "not to mention / "even" and the four types of complex sentences mentioned above as summarized in the following table:

**Table 4.1 Correspondence between SOs and Complex Sentence Types**

| **Informativeness** | **SO** | **Complex Sentence Type** |
|---|---|---|
| relatively high | even | concessive sentence / hypothetical concessive sentence |
| relatively low | not to mention | causal sentence / hypothetical conditional sentence |

Based on the above table, I predict that causal / hypothetical conditional sentences and concessive / hypothetical concessive sentences may form multiple complex sentences[140] such that the two parts denote contrastive informativeness just as "not to mention" and "even" do in (152). The above prediction is borne out by the following Chinese multiple complex sentence[141]:

(185) Jishi wo $[\text{tiantian}]_F$ dagong, ye zhuan bu dao 20,000.
even if I everyday work also earn not able 20,000
Ruguo wo $[\text{getian}]_F$ dagong, jiu geng zhuan bu dao 20,000.
if every other day then more
*Even if I work everyday, I cannot earn $20,000, much less if I work every other day.*

I assume that the concessive conditional and hypothetical conditional sentences above are both associated with the following I-function:

(186) I("Wo x dagong, zhuan bu dao 20,000") = x

where x is a variable from the following scale of frequency:

[140] Multiple complex sentence is a notion in Chinese grammar. It refers to a complex sentence that is made up of multiple levels of constituent complex sentences.
[141] Adapted from Li (2000), Ch. 4, [30], p. 138.

(187) X: <getian, tiantian>

Since according to this scale, tiantian > getian, we have

(188) I("Wo tiantian dagong, zhuan bu dao 20,000")

> I("Wo getian dagong, zhuan bu dao 20,000")

One can thus see that the concessive conditional and hypothetical conditional sentences in (185) play the same roles as "even p" and "not to mention q" in (152), respectively. This is consistent with the result in Table 4.1.

### 4.4.7 Comparative Constructions

As pointed out in the previous sections, scalar reasoning can be seen as comparison of informativeness, an attribute of propositions. Thus, scalar reasoning is closely related to comparison of attributes. We thus expect that lexical items used in comparative constructions may be used as SOs. In fact, in Chinese there does exist one such item – "geng / gengjia". Roughly equivalent to "more", "geng" is often used in comparative constructions, and may also be used in climax constructions. However, since "geng" only denotes higher order of a scalar term in a scale regardless of the nature of the scale, it may play very different roles in different types of climax constructions.

According to Xing (2001), "geng" may appear as part of the complex sentence schemas "budan p, (erqie) geng q" (≈ "not only p, but even q") and "shangqie p, geng (hekuang) q" (≈ "even p, not to mention q"). Note that according to conditions (158) and (152), "geng" plays opposite roles in these two schemas. In "budan p, (erqie) geng q", q has a higher informativeness than p, whereas in "shangqie p, geng (hekuang) q", q has a lower informativeness than p. But in the latter case, we may also say that q has a higher likelihood than p. Thus, despite the opposite roles, "geng" in fact expresses the same core meaning (i.e.

"more") in the two schemas.

There is another word displaying similar features – "guran / ziran". Roughly equivalent to "of course / naturally", "guran" is usually used for assertion rather than comparison. But it can also be used in contrast with "geng" to denote relative lower order in a scale, just like the case of a positive degree adjective used in contrast with a comparative degree adjective. Interestingly, when used in the schemas "guran p, geng q", "guran" may also play opposite roles in different contexts.

According to Xing (2001), "guran p, geng q" may be rewritten as "budan p, geng q" or "shangqie p, geng q" in different contexts. As discussed in the above, clauses p and q have opposite comparative relations in terms of informativeness in these two schemas. Despite this, "guran p, geng q" expresses the same core meaning, i.e. q has a higher order than p in a scale (be it an informativeness scale or a likelihood scale).

### 4.4.8 "even" and "at least"

In Chapter 2, I have categorized "even" and "at least" as an emphatic SO and an attenuating SO, respectively. I have also formulated two alternative conditions of use for each of them. With the notion of I-function, I will now reformulate these conditions of use as follows (c.f. (42), (49), (52) and (54) of Chapter 2):

(189) even: I(TP) > I(CP) or I(TP) is extremely high, though not necessarily the highest

(190) at least: I(TP) > I(CP) or I(TP) is very low, but not the lowest

An advantage of using the I-function is that the I-function, in conjunction with the Proportionality Calculus introduced in Subsection 4.2.4, can clearly show the interaction between the direction of scalar reasoning and the argument structure of a sentence, and thus reveal certain features that would otherwise be overlooked.

Since “even” and “at least” are in a sense opposite to each other, they are mainly associated with different scalar reasoning. Based on Figure 2.2, “even”, being a highly informative SO, is mainly associated with SEs; whereas “at least”, being a lowly informative SO, is mainly associated with SIs. I thus predict that these two SOs will give rise to inferences in opposite directions. This prediction is borne out by the following examples:

(191) Every jumper can even clear obstacle $[y_6]_F$, (not to mention obstacle $[y_5]_F$.)

(192) At least every jumper can clear obstacle $[y_5]_F$. (But it’s hard to say if they can clear obstacle $[y_6]_F$.)

In the above, the “not to mention”-sentence is a lowly informative but certain statement entailed by the “even”-sentence, while the “hard to say”-sentence is a highly informative but uncertain statement implicated by the “at least”-sentence[142]. The felicity of these two examples can be accounted for by using the following I-function[143]:

(193) $I(\textit{every}(\text{JUMPER})(\{x\text{: x can clear obstacle y}\})) = y$

where y is a variable from the scale Y defined in (11). An important point to note

[142] Note that the “hard to say”-sentence represents the “ignorant” epistemic force of an implicature, which is different from the “strong” epistemic force assumed in Subsection 2.4.2 of Chapter 2. Since the main theme of this section is not to discuss the pragmatics of SIs, there is no harm to assume a different epistemic force here.

[143] Note that although the informativeness of the SM associated with an “at least”-sentence is reflected by the desirability rather than likelihood of the propositions, the I-function for the “at least”-sentence has the same form as that for the “even”-sentence in this particular example. This is because the harder the obstacle, the more desirable (and thus more informative) that every jumper can clear that obstacle. On the other hand, the harder the obstacle, the less likely (and thus also more informative) that every jumper can clear that obstacle. Hence the I-function has the same form in both cases.

is that in the above two examples, the positions where "$y_6$" and "$y_5$" appear in the main propositions and the entailed / implicated propositions are exactly opposite.

In (191) and (192), "$y_6$" and "$y_5$" fall within the right argument of "*every*". In case they fall within the left argument of "*every*", I predict that the positions where they appear will be opposite to those in (191) and (192), because the left and right arguments of "*every*" have opposite monotonicities. Again, this prediction is borne out by the following examples:

(194) Even all those who can clear obstacle $[y_5]_F$ will get a medal, (not to mention those who can clear obstacle $[y_6]_F$.)

(195) At least all those who can clear obstacle $[y_6]_F$ will get a medal. (But it's hard to say if those who can clear obstacle $[y_5]_F$ will get a medal.)

Note that these two examples are associated with the following I-function:

(196) I(*every*({x: x can clear obstacle y})(GET-MEDAL)) = 1 / y

Comparing (196) and (193), one can see that the variable y has opposite proportionalities in (194) – (195) and (191) – (192). This explains why the differences between the two sets of examples arise.

Despite the opposite rhetorical functions performed by "even" and "at least", these two SOs in fact share some commonalities, which are best illustrated by the fact that each of them has an alternative condition of use in the same form (i.e. I(TP) > I(CP), see (189) and (190)). An interesting consequence of this is that in some languages, the same word (used with different intonation and / or co-occurring particles) may perform the dual functions of "even" and "at least". One such example is Slovenian "magari", as proposed by Crnič (2011). I propose that Cantonese "dou" is another example[144].

[144] In this thesis, the Cantonese words are transcribed using Jyutping, a Romanization scheme devised by the Linguistic Society of Hong Kong.

Cantonese "dou" can be used to convey a variety of different meanings, including scalar meanings. Among its scalar meanings, the emphatic scalar meaning is similar to that of Mandarin Chinese "dou", which may be seen as a short form of the CCC "lian_dou", a counterpart of English "even". Thus, Cantonese "dou" can be used like "even", as in the following example:

(197) Keoi [singsou]$_F$ dou sik laa, [gaasou]$_F$ ganggaa m sai gong.

he multiplication dou know PART addition more not need say

*He even knows how to multiply, not to mention add.*

In the above example, the use of "dou" is accompanied by the sentence-final particle "laa" (first tone), which expresses an emphatic / hyperbolic mood. But "dou" can also be used as an attenuating SO similar to "at least". This use of "dou" is often accompanied by a sentence-final particle that expresses a concessive mood, such as "ge" (second tone). This concessive mood conveys the "settle for less" meaning associated with "at least". Here is an example of this use of "dou":

(198) Keoi [gaasou]$_F$ dou sik ge, [singsou]$_F$ zau naan gong laa.

he addition dou know PART multiplication then hard say PART

*It's hard to say whether he knows how to multiply. But at least he knows how to add.*

Note that the two scalar terms "singsou" and "gaasou" have exchanged positions, showing that "dou" is playing opposite roles in these two examples.

### 4.4.9 Chinese Rhetorical Questions and "ba"-questions

In Chinese, there are two types of non-canonical questions – rhetorical questions and "ba"-questions (i.e. questions formed by adding the sentence-final particle "ba"). Chinese rhetorical questions may appear in various forms. In this thesis, I will use rhetorical questions in the form of "ma"-questions (i.e.

questions formed by adding the sentence-final particle “ma”) as representative. According to Shao (1996), Chinese questions may be classified according to their “degrees of interrogation”, and rhetorical questions and “ba”-questions have extremely low and very low degrees of interrogation, respectively[145]. This thesis will not study the semantics of interrogatives. Suffice it to say that Chinese rhetorical questions and “ba”-questions are used not to request for information but to assert the high certainty of their associated propositions. For a “ba”-question, the associated proposition is the declarative obtained after deleting “ba”. For a rhetorical “ma”-question, the associated proposition is the negation of the declarative obtained after deleting “ma”.

The certainty of a proposition is often manifested as the likelihood that the proposition is realized. So by virtue of (9), rhetorical questions and “ba”-questions denote propositions with extremely low and very low informativeness, respectively. A natural corollary of this fact is that these two types of questions may interact with SOs, especially the Chinese counterparts of “even” (i.e. “lian_dou / shenzhi”) and “at least” (i.e. “zhishao / qima”), whose conditions of use involve opposite ends of the informativeness scale.

The previous subsection has shown that “even” and “at least” satisfy the following patterns of scalar reasoning:

| | | | |
|---|---|---|---|
| (199) | “even”-sentence | $\Rightarrow_u$ | “not to mention”-sentence |
| | (highly informative) | | (lowly informative and certain) |
| (200) | “at least”-sentence | +> | “hard to say”-sentence |
| | (lowly informative) | | (highly informative and uncertain) |

Based on the above observation, I predict that rhetorical questions and

[145] Within the range of [0, 1], Shao (1996) assigned 0 and 1/4 as the degrees of interrogation of rhetorical questions and “ba”-questions, respectively.

“ba”-questions, which are lowly informative and certain, can take the place of “not to mention”-sentences and “at least”-sentences in the above patterns. This prediction is borne out by the following examples (c.f. (191) and (192)):

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| (201) | Ta | lian | $[gao]_F$ | lan | dou | tiao | de | guo, |
| | he | lian | high | hurdle | dou | jump | able | over |
| | hui | tiao | bu | guo | $[di]_F$ | lan | ma? | |
| | will | jump | not | over | low | hurdle | ma | |

*He can even jump over high hurdles. Can't he jump over low hurdles?*

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| (202) | Hen | nan | shuo | ta | shifou | tiao | de | guo | $[gao]_F$ | lan, |
| | very | hard | say | he | whether | jump | able | over | high | hurdle |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | dan | yinggai | tiao | de | guo | $[di]_F$ | lan | ba? |
| | but | should | jump | able | over | low | hurdle | ba |

*It's hard to say if he can jump over high hurdles. But he should be able to jump over low hurdles, right?*

In (201), the rhetorical question actually conveys the meaning “Needless to say he can jump over low hurdles”. It thus functions like a “not to mention”-sentence in (199). In (202), the “ba”-question actually conveys the meaning “At least he should be able to jump over low hurdles”. It thus functions like an “at least”-sentence in (200). An interesting point to note here is that “hekuang”, which is a Chinese equivalent of “not to mention”, was historically an ancient Chinese WH-phrase often used to form rhetorical questions. This fact lends further support to my analysis[146].

The relation between “lian_dou / zhishao” and rhetorical / “ba”-questions are so closed that the former can even form part of the latter. As mentioned above,

[146] In this thesis, I follow some scholars (such as Xing (2001)) by treating “hekuang” as an unanalysed SO instead of an interrogative structure.

a rhetorical “ma”-question asserts extremely high likelihood of the negation of the declarative obtained after deleting “ma”. But if ¬p is extremely likely, then p is extremely unlikely, and is thus extremely informative. By (189), I predict that a “lian_dou”-sentence can form part of a rhetorical question with the “lian_dou”-subpart and the whole rhetorical question denoting opposite ends of the informativeness scale. On the other hand, as a “ba”-question asserts very high likelihood of its associated proposition, by (190), I predict that a “zhishao”-sentence can form part of a “ba”-question with the “zhishao”-subpart and the whole “ba”-question both denoting very low informativeness. The above predictions are borne out by the following example:

(203)

| | | | | | | |
|---|---|---|---|---|---|---|
| Ta | shi | zui | hao | de | xuanshou, | hui |
| he | is | most | good | POSS | athlete | will |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| lian | $[di]_F$ | lan | dou | tiao | bu | guo | ma? |
| lian | low | hurdle | dou | jump | not | over | ma |

*Being the best athlete, couldn't he jump over even the low hurdle?*

(204)

| | | | | | | |
|---|---|---|---|---|---|---|
| Ta | bu | shi | zui | cha | de | xuanshou, |
| he | not | is | most | bad | POSS | athlete |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| yinggai | zhishao | tiao | de | guo | $[di]_F$ | lan | ba? |
| should | zhishao | jump | able | over | low | hurdle | ba |

*He is not the worst athlete. At least he should be able to jump over the low hurdle, right?*

In (203), the “lian_dou”-subpart (i.e. “lian di lan dou tiao bu guo”) denotes high informativeness, while the whole rhetorical question denotes low informativeness. In (204), both the “zhishao”-subpart (i.e. “zhishao tiao de guo di lan”) and the whole “ba”-question denote low informativeness.

To conclude this section, I now summarize the conditions of use of the standalone and correlative SOs studied in this thesis in the following table:[147]:

**Table 4.2 Conditions of Use of SOs**

| SO | Condition of Use |
|---|---|
| even | I(TP) > I(CP) or I(TP) is extremely high, though not necessarily the highest |
| at least | I(TP) > I(CP) or I(TP) is very low, but not the lowest |
| not to mention | I(TP) < I(CP) |
| only | I(TP) < I(¬CP) |
| not only | I(TP) < I(CP) |
| on the contrary | I(TP) > I(¬CP) |
| even p, not to mention q | I(p) > I(q) |
| not only p, but also / even q | I(p) < I(q) |
| not only p, but also q, and even r | I(p) < I(q) < I(r) |
| not only not p, on the contrary q | I(¬p) < I(q) |

## 4.5 Subjective Quantity[148]

### 4.5.1 SQOs Based on Informativeness

In Chapter 2, I have introduced the notion of subjective quantity (SQ) and distinguished two types of subjective quantity operators (SQOs): abnormal SQOs represented by "dou" and infected SQOs represented by "hekuang". Now these two SQOs are also SOs[149] denoting different informativeness. In this subsection, I will show that the contrast between some abnormal and infected SQOs is in fact manifestation of the contrast between high and low informativeness. Thus, I will use the more general concept of high / low informativeness instead of

[147] For convenience, the scalar lexical items listed here are only given in English. See Table 2.1 for the rough Chinese equivalents of some of these items.

[148] Some parts of this section have been published in Chow (2011c) (in Chinese) and will be published in Chow (2012c) (in Chinese).

[149] Both Liu (2000) and Shen (2001) contended that "hai" is also an SO. But as pointed out by Zhang (2003), the scalar meaning of "hai" is derived from its meaning as an aspectual operator (i.e. "continue" or "remain"). Therefore, the formal pragmatics of "hai" is complicated and I will not consider it in this chapter.

abnormality / infection to distinguish these SQOs.

In Chapter 2, I have summarized Li (2000)'s findings on the SQs triggered by "dou" in Table 2.2. I now provide an account for these findings by considering two specific examples:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (205) | Ta | [20 | jin]$_F$ | dou | tiao | de | qi. |
| | he | 20 | catty | dou | lift | able | up |

*He can even lift up 20 catties.*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (206) | Ta | tiao | [20 | jin]$_F$ | dou | juede | lei. |
| | he | lift | 20 | catty | dou | feel | tired |

*He felt tired even though he only carried 20 catties.*

The I-functions of these two sentences can be determined as follows:

(207) I("Ta x tiao de qi") = x

(208) I("Ta tiao x juede lei") = 1 / x

where x is a variable of weights. The rationale of these functions is based on the informativeness of the proposition, which is inversely proportional to the likelihood of the sentential predicate.

Assuming "dou" is subject to the same condition of use for "even", i.e. (136), the felicitous use of "dou" in (205) and (206) requires that I(TP) > I(CP). In (205), I(TP) = 20 jin. If the quantity in the CP (which in the case of "dou" is also the expected value) is $x_1$ say, then in order to satisfy (136), we must have 20 jin > $x_1$. Therefore, the focused scalar term[150] "20 jin" in (205) must denote an SQ larger than expected, i.e. a large SQ. Similarly, in (206), if the quantity in the CP is $x_2$ say, then in order to satisfy (136), we must have 1 / 20 jin > 1 / $x_2$, or equivalently, 20 jin < $x_2$. Therefore, "20 jin" in (206) must denote an SQ smaller

[150] In Chapter 2, these terms are called "quantity phrases". Since I now treat SQ as a scalar phenomenon, I will adopt the terminology of SMT and call these "scalar terms".

than expected, i.e. a small SQ.

The above analyses can be generalized as follows: if x is directly (inversely) proportional to the informativeness of the sentence, or equivalently inversely (directly) proportional to the likelihood of the sentential predicate, then x must denote large (small) SQ. This conclusion is in accord with Table 2.2. The above discussion also shows that the SQ triggered by “dou” is in fact a by-product of the meaning of this particle as an SO.

Apart from “dou”, I have also studied other SOs denoting relatively high informativeness in the previous section. These SOs may also trigger SQ. For example, consider the following sentence with the SO “faner”:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| (209) | (Ta | tiao | bu | qi | na | ge | zhong | $[10]_F$ | jin | de), |
| | he | lift | not | up | that | CLS | weigh | 10 | catty | POSS |
| | faner | tiao | de | qi | zhe | ge | zhong | $[20]_F$ | jin | de. |
| | faner | lift | able | up | this | CLS | weigh | 20 | catty | POSS |

*(He could not lift up that 10-catty weight.) Yet he could lift up this 20-catty weight.*

I assume that this sentence is associated with the I-function given in (207). According to (170), the felicitous use of “faner” requires that I(TP) > I(¬CP). In (209), I(TP) = 20 jin and I(¬CP) = 10 jin. Thus, condition (170) for “faner” is satisfied. Moreover, since the focused scalar term “20 jin” is larger than the expected quantity “10 jin”, “20 jin” denotes large SQ in (209), which is in accord with our intuition about this sentence.

SQOs denoting relatively low informativeness such as “bieshuo / hekuang” can also be analysed in a similar fashion. Consider the following example:

(210) Bieshuo [10 $jin]_F$, ta [20 $jin]_F$ dou tiao de qi.

bieshuo 10 catty he 20 catty dou lift able up

*He can even lift up 20 catties, not to mention 10 catties.*

Here I focus on the "bieshuo"-clause above. According to condition (139), the felicitous use of "bieshuo" above requires that I(TP) < I(CP). Using (207) as the I-function associated with (210), I(TP) = 10 jin. If the quantity in the CP is $x_1$ say, then in order to satisfy (139), we must have 10 jin < $x_1$. Therefore, "10 jin" in (210) must denote a small SQ.

In Section 4.4, I have also shown that certain complex sentence types, comparative constructions and non-canonical questions may function like constructions with SOs. It turns out that these can also trigger SQs. Consider the following examples:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (211) | Ta | [10 | jin]$_F$ | dou | tiao | bu | qi, |
| | he | 10 | catty | dou | lift | not | up |
| | neng | tiao | de | qi | [20 | jin]$_F$ | ma? |
| | can | lift | able | up | 20 | catty | ma |

*He cannot even lift up 10 catties. Can he lift up 20 catties?*

(212) Jishi [10 jin]$_F$ de danzi, ta dou tiao bu qi.

even if POSS load

Ruguo huan le [20 jin]$_F$ de danzi, ta gengjia tiao bu qi.

if change ASP more

*Even if the load weighs only 10 catties, he cannot lift it up, much less if the load weighs 20 catties.*

In (211), the second clause is a rhetorical question. In (212), the second complex sentence is a hypothetical conditional sentence with the comparative particle "gengjia". Both these constructions function like a "hekuang"-clause with relatively low informativeness and the scalar term "20 jin" in these two sentences denotes large SQ.

### 4.5.2 "jiu", "cai" and "zhi"

In Chapter 2, I pointed out that abnormal SQ comes from unexpectedness, whereas in the previous subsection I correlated abnormal SQ with high informativeness. In many situations, unexpectedness does coincide with high informativeness, because many unexpected events are unlikely and so highly informative. However, unexpectedness and informativeness are independent concepts and may not coincide in some situations, especially when it involves personal expectation which may not coincide with the normal state of affairs.

I contend that some SQOs are based on unexpectedness rather than informativeness. These include "jiu" and "cai". Moreover, as shown in Table 2.3, the use of "jiu" and "cai" are peculiar in that they trigger SQs according to the relative locations of the focused scalar terms. Therefore, we need to provide a separate treatment for these two SQOs. It turns out that this treatment is also based on proportionality relation albeit of a different kind than the SOs.

In what follows I will only consider the case that "jiu / cai" appears as an adverbial in front of the sentential predicate. Let $L_1$, … $L_m$ be scalar terms located on the left of "jiu / cai", and $R_1$, … $R_n$ be scalar terms located on the right of "jiu / cai" in a sentence. Then I define the following function:

(213) $$\text{LARGENESS} = (R_1 \times \ldots \times R_n) / (L_1 \times \ldots \times L_m)$$

which is a combined measure of the largeness of the scalar terms. The rationale of this formula is as follows: scalar terms on the right of "jiu / cai" are in the object / complement positions that are directly related to the sentential predicate and thus reflect (i.e. directly proportional to) the largeness of the quantity expressed by it; whereas scalar terms on the left are in contrast to (i.e. inversely proportional to) those on the right. Thus, the scalar terms on the right (left) appear in the numerator (denominator) of the above function.

For example, consider the following sentences:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| (214) | [Liang]$_F$ | ge | ren | jiu | zhuan | de | [40,000 | yuan]$_F$. |
| (215) | [Liang]$_F$ | ge | ren | cai | zhuan | de | [20,000 | yuan]$_F$. |
| | 2 | CLS | person | jiu / cai | earn | get | | dollar |

*The two of them earn as much as $40,000.* /

*The two of them only earn $20,000.*

Let n and m be variables of natural number and monetary value, respectively. Then by (213), we have the following LARGENESS function for (214) and (215):

(216) LARGENESS = m / n

Note that the above is in fact a ratio measuring the amount of money earned per person. Thus, we may say that (213) is a ratio measuring the relative largeness of the quantity expressed by the sentential predicate, and “jiu” and “cai” denote that this ratio is large and small, respectively. Based on the above observation, I now formulate the conditions of use for “jiu” and “cai” as follows:

(217) jiu: LARGENESS(TP) > LARGENESS(CP)

(218) cai: LARGENESS(TP) < LARGENESS(CP)

We can now account for the SQs triggered by “jiu / cai” in (214) and (215). For (214), by (216), we have LARGENESS(TP) = 40,000 / 2. If the expected quantity in the CP is m / n say, then in order to satisfy condition (217), we must have 40,000 yuan ≥ m and 2 ≤ n. Therefore, “40,000 yuan” and “liang” must denote large and small SQs, respectively. A similar analysis on (215) will show that “20,000 yuan” and “liang” denote small and large SQs, respectively.

The above analyses can be generalized as follows: if x is located on the left (right) of “jiu”, then x must denote small (large) SQ; if x is located on the left (right) of “cai”, then x must denote large (small) SQ. This conclusion is in accord with Table 2.3.

To summarize, conditions (217) and (218) show that the proper use of “jiu” and “cai” depends on a comparison between the quantities under discussion and the expected quantities. So “jiu” and “cai” are abnormal SQOs. On the other hand, since the LARGENESS function as defined in (213) is determined by the relative positions of the scalar terms wrt “jiu / cai”, this shows that “jiu” and “cai” are also directly assigned SQOs. This explains the dual nature of “jiu” and “cai”.

Finally, we come to the problematic “zhi” (≈ “only”). Li (2000) classified “zhi” as a directly assigned SQO. In fact, “zhi” may take the place of “cai” in some sentences. For example, (215) above may be rewritten as

(219) [Liang]$_F$ ge ren zhi zhuan de [20,000 yuan]$_F$.

Moreover, as pointed out by Zeevat (2009), “only” is a mirative particle denoting unexpectedness. It is thus plausible to analyse “zhi” in a fashion similar to “cai”.

However, according to condition (140), “zhi” is also an SO denoting relatively low informativeness. Therefore, I predict that “zhi”, just like other SOs based on informativeness, should be able to trigger both large and small SQs in different contexts. This prediction is borne out by the following examples[151]:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| (220) | Zhongguo | sheng | Yilang | zhi | xu | chu | [ban]$_F$ | li. |
| | China | win | Iran | zhi | need | use | half | strength |

*China only needs to use half of their strength to beat Iran.*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| (221) | Yu | qu | sheng | zhi | neng | [quan]$_F$ | li | yi | fu. |

[151] Adapted from headlines of sports news found on the Internet: http://sports.iyaxin.com/content/2010-07/30/content_2019091.htm and http://sports.sina.com.cn/cba/2012-01-06/09115896193.shtml.

wish gain victory zhi able full strength PART go

*Only with all-out efforts can you win a victory.*

In the above examples, “zhi” denotes the same meanings as “zhiyao” and “zhiyou”, respectively. As pointed out in Subsection 4.4.2, “zhiyao” and “zhiyou” are conjunctions for sufficient and necessary conditions, respectively, and enter into opposite reasoning directions. Thus, the I-functions associated with (220) and (221) should have a form similar to (150) and (151):

(222) I(“China needs to use x of their strength to beat Iran”) = x

(223) I(“You can win a victory with x of your efforts”) = 1 / x

where x is a variable from the following scale:

(224) X: <half, all>

Now according to (140), the felicitous use of “zhi” requires that I(TP) < I(¬CP)[152]. In (220), I(TP) = half. If the expected quantity in ¬CP is $x_1$ say, then in order to satisfy (140), we must have half < $x_1$. Therefore, “ban” in (220) must denote small SQ. Similarly, in (221), I(TP) = 1 / full. If the quantity in ¬CP is $x_2$ say, then in order to satisfy (140), we must have 1 / full < 1 / $x_2$, or equivalently, full > $x_2$. Therefore, “quan” in (221) must denote large SQ.

The above analyses show that “zhi” has the dual nature of being an SQO denoting unexpectedness like “cai” as well as an SO denoting relatively low informativeness like “hekuang”. This duality should be seen as a phenomenon of polysemy. In fact, the polysemy of “zhi” is multi-faceted. As pointed out above, “zhi” has both scalar and non-scalar uses. The aforesaid duality is only one facet of this polysemy. It requires more researches to clarify the various facets in the meaning of “zhi”.

[152] The TP and CP of a “zhi”-sentence has opposite polarities. For example, a possible CP of (220) might be “Bu xu chu quan li” (≈ “There is no need to use full strength”). That is why the condition of use for “zhi” involves the negation of CP.

## 4.6 Extreme Values[153]

### 4.6.1 Maximizers / Minimizers

In Chapter 2, I have introduced Israel (2011)'s theory and typology for maximizers / minimizers. In this section, I will reformulate his theory using the I-function and extend the applicability of the theory to other linguistic phenomena involving extreme values.

Consider emphatic maximizers / minimizers first. As pointed out in Chapter 2, these items are very informative. I thus propose the following condition of use for these items:

(225) Emphatic Maximizers / Minimizers: I(TP) is maximal

According to Israel (2011), maximizers / minimizers may play different participant roles in different sentences. To account for this difference, he classified two types of emphatic maximizers / minimizers: "canonical" and "inverted" as recorded in Table 2.4 and explained their difference in terms of different participant roles. In this thesis, I interpret this difference in terms of different proportionality relations between the scalar terms and the I-function values. In this way, I am able to account for Israel's two types of emphatic maximizers / minimizers by using just one condition (225).

The idea can be illustrated by the following examples:

(226) Julio spent [a king's ransom]$_F$ on the party.

(227) She wouldn't kiss him for [all the tea in China]$_F$.

(226) shows an example of Israel (2011)'s canonical emphatic maximizers which play the participant role of "expenses". Since the larger the expense, the less likely a person is willing to spend and so the more informative the sentence is,

153 Some parts of this section have been published in Chow (2011c) (in Chinese).

the I-function associated with (226) is

(228) I(“Julio spent x on the party”) = x

where x is a variable of monetary value. Since “a king’s ransom” is a maximizer, substituting a maximal value into x above will yield a maximal function value. Thus condition (225) is satisfied, and we may conclude that the maximizer is properly used in (226).

In contrast, (227) shows an example of Israel (2011)’s inverted emphatic maximizers which play the participant role of “rewards”. Since the larger the reward, the less likely a person is not willing to do some things for it and so the more informative the sentence is, the I-function associated with (227) is

(229) I(¬(“She would kiss him for x”)) = x

where x is a variable of amounts of reward. Since “all the tea in China” is a maximizer, following the same line of reasoning as above, we may conclude that the maximizer is also properly used in (227).

Comparing (225) with (189), one can see that the two are compatible. This implies that we can rewrite any sentences containing emphatic maximizers / minimizers as sentences with the emphatic SO “even”. For example, we can easily add “even” to each of (226) and (227) without altering its meaning.

Next consider attenuating maximizers / minimizers. Since these items are very uninformative, I propose the following condition of use for these items:

(230) Attenuating Maximizers / Minimizers: I(TP) is minimal

I will use the following examples to illustrate the idea:

(231) Stella is [sort of]$_F$ clever.

(232) Stella is not [all that]$_F$ clever.

The following I-function will be used to analyse these two sentences:

(233) I(“Stella is x clever”) = x

where x is a variable of degree. The rationale for this I-function is that the higher the degree is, the less likely a person is clever to that degree, and so the more informative the sentence is. Note that according to (16), the I-function associated with (232) is just the reciprocal of (233). Substituting a minimal value into x in (233) and a maximal value into 1 / x, which is the reciprocal of the RHS of (233), will both yield minimal function values. We may thus conclude that the attenuating minimizer “sort of” and the attenuating maximizer “all that” are both properly used in (231) and (232).

A point which can easily be shown by Figure 2.2 is that attenuating maximizers / minimizers may generate SIs because of their low informativeness. For example, (231) and (232) may generate the SIs “Stella is not very clever” and “Stella is at least a bit clever”, respectively. These two SIs show that attenuating maximizers / minimizers can be used to avoid overpraising or overcriticizing, which is precisely what attenuation is supposed to achieve.

Comparing (230) with (190), one can see that the two are compatible. This implies that attenuating maximizers / minimizers share the same core meaning with the attenuating SO “at least”. However, unlike “even”, it is not always the case that we can readily add “at least” to sentences containing attenuating maximizers / minimizers. Sometimes we need to construct a suitable context to do this. For example, for (232), we may construct the following context:

(234) A: I heard that Stella is extremely cheerful and clever.

B: Not so. At least she’s not all that clever.

A final point to note is that by using conditions (225) and (230), we can easily account for the polarities shown in Table 2.4. For example, if we change (232) to a positive sentence, the associated I-function will become identical to

(233). Substituting a maximal value into x in (233) will yield a maximal I-function value. But this is in conflict with condition (230). Thus, if we want “all that” to be an attenuating maximizer, it can only appear in a negative context, as recorded in Table 2.4.

I now summarize the correspondence between SOs, maximizers / minimizers and typical types of scalar reasoning in the following table:

**Table 4.3 Correspondence between SOs, Maximizers / Minimizers and Scalar Reasoning**

| **SO** | **Type of Maximizers / Minimizers** | **Typical Type of Scalar Reasoning** |
|---|---|---|
| even | emphatic | SE |
| at least | attenuating | SI |

A striking point of the above table is that it is consistent with the diagnostics for emphasis (attenuation) proposed by Israel (2011). One diagnostic is co-occurrence with “even” (“at least”). The other diagnostics are derivable from the meaning and use of “even” (“at least”). In what follows, I will reformulate some of Israel (2011)’s diagnostics in terms of the notions developed in this thesis.

According to Israel (2011), two diagnostics for emphatic lexical items are that (i) they can co-occur with constructions conveying an exclamative / mirative meaning such as “you’ll never believe it” and (ii) their TP must be stronger (i.e. more informative) than their CP. Note that (i) is consistent with the fact that “even” is often used to denote low likelihood, while (ii) is consistent with the condition of use (136) for “even”.

Regarding the attenuating lexical items, Israel (2011) proposed that they can be used in two types of constructions – “hedged concessions” and “anti-concessives”, as exemplified by the following[154]:

(235) Well, I guess he’s not here [yet]$_F$, (but I still think he will come).

(236) She may not be brilliant, but she is [fairly]$_F$ clever.

In the hedged concession in (235), the CP (i.e. the clause in parenthesis) represents the negation of a proposition with higher informativeness than the TP[155]. Thus the TP and CP satisfy the condition of SI as depicted in Figure 2.2, i.e. TP +> ¬CP. In the anti-concessive in (236), the attenuator “fairly” expresses a weak claim and “settle for less” meaning, which is the core meaning of “at least”. In conclusion, Israel (2011)’s diagnostics for emphasis / attenuation lend further support to the correlation shown in Table 4.3.

### 4.6.2 Superlatives and Extreme Degree Modifiers

Maximizers / minimizers usually refer to idiomatic lexical items like those mentioned in the previous subsection. However, in natural language there are some other lexical items that can perform certain functions of maximizers / minimizers. I will discuss two types of these – superlatives and extreme degree modifiers such as “extremely”, “unusually”, “amazingly”, etc. These lexical items can be seen as extreme scalar terms in their respective scales of degrees. But unlike emphatic / attenuating maximizers / minimizers, superlatives and extreme degree modifiers are non-polar (i.e. they can appear in both positive and negative contexts), and so they should be seen as “neutral” maximizers / minimizers according to Israel (1996). Moreover, they can perform both the emphatic and attenuating functions[156].

[154] Israel (2011), Ch. 5, (24)a, (27), p. 119.
[155] The negation of the CP – “he won’t come” unilaterally entails, and is thus more informative than, the TP – “he’s not here yet”.
[156] As mentioned in Chapter 2, Fauconnier (1975) pointed out that superlatives can perform the function of emphasis. I contend that superlatives can also perform the function of attenuation, not

For illustration, consider the following I-functions:

(237) I("Tommy will eat the x delicious food") = 1 / x

(238) I("Stella is x clever") = x

where x is a variable of degree in both I-functions. The rationale of (238) has already been explained above, while the rationale of (237) is that the more delicious the food, the more likely a person will eat it, and so the less informative the sentence is. From these two I-functions, one can predict that the superlative "most" can perform the emphatic and attenuating functions in the negative and positive forms of the propositional function in (237), respectively, whereas the extreme degree modifier "unusually" can perform the emphatic and attenuating functions in the positive and negative forms of the propositional function in (238), respectively. The above predictions are borne out by the following examples, which also illustrate the correlation between emphasis / attenuation and "even" / "at least" as shown in Table 4.3:

(239) Tommy will not even eat the $[\text{most}]_F$ delicious food, let alone the less delicious one. (emphatic)

(240) Tommy is not that choosy. At least he will eat the $[\text{most}]_F$ delicious food. (attenuating)

(241) Stella is not just bright, but even $[\text{unusually}]_F$ clever. (emphatic)

(242) Stella is not a genius. At least she is not $[\text{unusually}]_F$ clever. (attenuating)

### 4.6.3 Chinese Idiomatic Constructions with Extreme Numerals

In Chapter 2, I have introduced two Chinese idiomatic schemas containing the numeral "yi" studied by Li (2000) based on his SQ theory. These two

---

just emphasis.

schemas are “yi … jiu …” and “yi” + negation. As I have put the study on SQ under the SMT framework in this chapter, I will now reinterpret and extend these results.

First consider the “yi … jiu …” schema exemplified by the following:

| (243) | Ta | $[\text{yi}]_F$ | kan | jiu | ming. |
|---|---|---|---|---|---|
| | he | one | see | jiu | understand |

*He could understand by glancing through just once.*

In Subsection 4.5.2, I have shown that Li (2000)’s findings about the SQ triggered by “jiu” and “cai” as recorded in Table 2.3 can be accounted for by using the I-function. Therefore, the following discussion will be based on Table 2.3. According to Table 2.3, scalar terms located on the left of “jiu” denote small SQ. Since “yi” represents the smallest natural number, “yi … jiu …” is a legitimate construction.

But there are more schemas that can be predicted. According to Table 2.3, scalar terms located on the left of “cai” denote large SQ. In Chinese, “bai” (≈ “hundred”), “qian” (≈ “thousand”) and “wan” (≈ “ten thousand”) are often used to denote large quantities. Therefore we expect that there is also a “bai / qian / wan … cai …” schema in Chinese, although “cai”, just like “jiu”, may be omitted or replaced by words with similar meaning (such as “shi”). The following is an instance of this schema:

| (244) | $[\text{qian}]_F$ | hu | $[\text{wan}]_F$ | huan | shi | chu | lai |
|---|---|---|---|---|---|---|---|
| | 1,000 | call | 10,000 | call | shi | out | come |

*appear only after repeated calls*

Moreover, “yi” and “bai / qian / wan” may also co-occur to form the “yi … jiu … bai / qian / wan” and “bai / qian / wan … cai … yi” schemas, where “jiu / cai” may be omitted or replaced by words with similar meanings. Note that the "yi" and "bai / qian / wan" are located on the correct positions wrt "jiu / cai" in these two schemas. The following are two instances of these two schemas:

| | | | | | |
|---|---|---|---|---|---|
| (245) | [yi | ri]$_F$ | [qian | li]$_F$ | |
| | 1 | day | 1,000 | mile | *make progress with giant strides* |
| (246) | [bai | nian]$_F$ | [yi]$_F$ | yu | |
| | 100 | year | 1 | meet | *happen only once in a century* |

Note that these two idioms contain an implicit "jiu" and "cai", respectively.

Next consider the "yi" + negation schema exemplified by the following:

| | | | | | |
|---|---|---|---|---|---|
| (247) | [zhi]$_F$ | zi | wei | ti | |
| | CLS | word | not yet | mention | *not utter a word* |

Obviously, the phrase "zhi" (being a variant of "yi") above serves as an emphatic minimizer NPI performing the same function as "a red cent" in (67) of Chapter 2.

But according to Table 2.4, there are in fact 4 types of emphatic maximizers / minimizers. Thus, apart from emphatic minimizer NPIs, there are also emphatic minimizer PPIs, emphatic maximizer NPIs and emphatic maximizer PPIs. I predict that these should also be found in Chinese idioms. This prediction is borne out by the following instances:

| | | | | | |
|---|---|---|---|---|---|
| (248) | [fen | miao]$_F$ | bi | zheng | |
| | minute | second | must | contest | *count every minute and second* |
| (249) | [qian | zai]$_F$ | bu | bian | |
| | 1,000 | year | not | change | *unchanged for a thousand years* |
| (250) | [wan | gu]$_F$ | chang | cun | |
| | 10,000 | ancient | long | exist | *last forever* |

In the above, "fen miao" (being a variant of "yi") serves as an emphatic minimizer PPI performing the same function as "peanut" in (69) of Chapter 2; "qian zai" serves as an emphatic maximizer NPI performing the same function as "all the tea in China" in (68) of Chapter 2; "wan gu" serves as an emphatic maximizer PPI performing the same function as "a king's ransom" in (66) of Chapter 2.

Furthermore, extreme numerals should also be able to serve as attenuating maximizers / minimizers. Based on Table 2.4, I predict that there should be attenuating maximizer NPIs and attenuating minimizer PPIs in Chinese idioms. Again, this prediction is borne out by the following instances:

(251) lüe zhi [yi er]$_F$

brief know 1 2 *know something about*

(252) bu [jin]$_F$[157] ru yi

not total accord wish *not totally in accord with one's wishes*

In the above, "yi er" serves as an attenuating minimizer PPI performing the same function as "sort of" in (65) of Chapter 2; "jin" serves as an attenuating maximizer NPI performing the same function as "all that" in (64) of Chapter 2.

There is a final question. Just as we have 4 possible combinations of emphatic maximizers / minimizers and PPIs / NPIs, can we also have 4 combinations of attenuating maximizers / minimizers and PPIs / NPIs? More specifically, apart from the attenuating maximizer NPIs and attenuating minimizer PPIs introduced above, are there attenuating maximizer PPIs and attenuating minimizer NPIs? A positive answer will mean expansion of Israel (2011)'s typology as recorded in Table 2.4. It turns out that it is not straightforward to find these examples, because the canonical use of attenuation is to assert a small quantity or to deny a large quantity, whereas attenuating maximizer PPIs and attenuating minimizer NPIs represent non-canonical use of

[157] The adverb "jin" is not a numeral. But since its meaning is similar to "100%", we can view it as a "quasi-numeral".

attenuation. Yet I do find two possible candidates of attenuating maximizer PPIs and attenuating minimizer NPIs:

(253) bu wei [wu dou]$_F$ mi zhe yao
not for 5 bushel rice bend waist
*not to bend one's back just for five bushels of rice*

(254) [zhong]$_F$[158] shang zhixia you yong fu
great reward under have brave man
*generous rewards rouse one to heroism*

To show that (253) and (254) are attenuators, we first note that both idioms denote highly likely, or equivalently lowly informative, propositions and are thus potential candidates of attenuators. Moreover, both idioms satisfy the diagnostics introduced at the end of Subsection 4.6.1, as exemplified by the following:

(255) Ta bu shi hen you qijie,
he not is very have moral integrity
dan zhishao bu hui wei [wu dou]$_F$ mi zhe yao.
but at least not will for 5 bushel rice bend waist
*He is not a person with high moral integrity. But at least he won't bend his back for just five bushels.*

(256) Zhe jian renwu [zhong]$_F$ shang zhixia shi you yong fu.
this CLS task great reward under is have brave man
(Dan ruo meiyou zhong shang jiu mei ren yuan zuo.)
but if no great reward then no person willing do
*Well, you can find some brave fellows to do the task by offering generous rewards. (But nobody is willing to do it without generous rewards.)*

[158] "Zhong" is not a numeral, but is a maximizer as it denotes large amount. Moreover, note that (254) is a variant of the standard Chinese idiom "zhong shang zhixia bi you yong fu" with the modal particle "bi" expressing necessity.

Note that (255) is an anti-concessive like (236) because the sentence expresses a weak claim and conveys a “settle for less” meaning[159], whereas (256) is a hedged concession like (235) because the negation of its CP, i.e. somebody is willing to do the task (even) without generous rewards, is more informative than its TP, i.e. you can find somebody to do the task by offering generous rewards, and so satisfy the relation TP +> ¬CP. These two examples show that both (253) and (254) can be used as attenuators.

Based on the above discussion, I now propose an expanded typology of maximizers / minimizers with examples of Chinese idiomatic constructions:

**Table 4.4 A New Typology of Maximizers / Minimizers**

<table>
<tr><th></th><th>Maximizer</th><th>Minimizer</th></tr>
<tr><td rowspan="2">Emphatic</td><td>PPI<br>“<u>wan gu</u> chang cun”</td><td>PPI<br>“<u>fen miao</u> bi zheng”</td></tr>
<tr><td>NPI<br>“<u>qian zai</u> bu bian”</td><td>NPI<br>“<u>zhi</u> zi wei ti”</td></tr>
<tr><td rowspan="2">Attenuating</td><td>PPI<br>“<u>zhong</u> shang zhixia you yong fu”</td><td>PPI<br>“lüe zhi <u>yi er</u>”</td></tr>
<tr><td>NPI<br>“bu <u>jin</u> ru yi”</td><td>NPI<br>“bu wei <u>wu dou</u> mi zhe yao”</td></tr>
</table>

Note that in this typology I do not differentiate “canonical / inverted” emphatic maximizers / minimizers. Moreover, this table is more symmetric than Table 2.4.

## 4.7 Conclusion

In this chapter, I have studied a number of linguistic phenomena related to scalar reasoning and addressed to the outstanding problems identified in the end of Chapter 2. Based on the ingredients of GFs and I-function, I have developed a

[159] Traditionally, “bu wei wu dou mi zhe yao” (≈ “not to bend one’s back just for five bushels of rice”) is used to commend somebody who does not succumb to the authority just for a scanty pay. But if we compare it with a possible alternative “bu wei wan dou mi zhe yao” (≈ “not to bend one’s back (even) for ten thousand bushels of rice”), then we will see that it in fact expresses a weak claim.

basic framework that can deal with the various aspects of scalar reasoning in a uniform way. Of course, since each of the 3 major aspects, namely SEs, SIs and scalar lexical items, has its own peculiarities, I have to add specific assumptions or ingredients to deal with these aspects. But one can still see the uniformity of the overall framework.

By adding relation (13) to the framework, I have formulated a formalized theory of SEs that enables one to calculate SEs by comparing I-function values of propositions in an SM. I have also shown the parallel relationship between SEs and monotonicity inferences. By capitalizing on this parallelism, I have combined findings of the two types of inferences and discovered new inferential patterns, such as Proportionality Calculus and scalar syllogisms, thus greatly expanding the scope of logical inferences and scalar reasoning.

I have enriched the basic framework by adding the ingredients of QUD-foci, answer exhaustification and opposition inferences, so that it can account for the various types of SIs (alternate-value and canonical, simple and embedded) in a uniform way. Moreover, I have also provided my own solution to the Defaultism-Contextualism debate, the Globalism-Localism debate and the implicature-explicature debate on the status of SIs.

I have linked up lexical items that were traditionally studied under different frameworks. This approach has shed new light on the studies of these lexical items. For example, by borrowing the notion of proportionality relations from the SQ theory, I have reformulated the research findings of SMT for different types of lexical items (including SOs, maximizers / minimizers, superlatives) using the same notion – the I-function. More importantly, it turns out that this reformulated SMT can also account for the formal pragmatics of lexical items that were not traditionally studied under SMT (including CCCs, SQOs and Chinese idiomatic constructions with extreme numerals), thereby expanding the applicability of SMT.

Finally, I have explored the association of SEs and SIs with different types of scalar lexical items (summarized in Table 4.3). The study on the relation between SIs and attenuating maximizers / minimizers is particularly fruitful as it has led to deeper understanding of the rhetoric of attenuation as well as a possible expansion of Israel's typology of maximizers / minimizers by using Chinese data. Moreover, the fact that "even" and "at least" can trigger both SEs and SIs has given me insight to explore the dual functions of Cantonese "dou", a phenomenon that has not been taken notice of by researchers before.

# Chapter 5 Concluding Remarks

## 5.1 Significance of the Present Study

In this section, I will discuss the theoretical and applicational significance of this study. I have chosen a modest set of GQs to study in this thesis. Despite this, I have made contribution to GQT. The main novelty of this thesis is that it has paid special attention to the left argument of the tripartite structure by studying left-oriented GQs. This has enabled a more symmetrical treatment of certain notions used in GQT. For example, traditionally the notions of conservativity, inner negation and dual are only defined on the right argument. Yet the notion of monotonicity is defined on both the left and right arguments. Thus, extending the definitions of conservativity, inner negation and dual to the left argument is a natural move. If monotonicity can be defined on the left argument, why can't the other notions?

More importantly, paying attention to the left argument enables me to extend the applicability of certain findings to left conservative GQs and sentences with relative clauses. For example, PMC, POC and PPC are powerful principles for determining the monotonicities, o-sensitivies and proportionalities of the various predicates of iterated GQs. If we restrict our attention on right-iterated GQs only, we will not bring the power of these principles into full play and will fail to discover inferential patterns of sentences with relative clauses that we can discover by employing these principles.

Moreover, paying attention to the left argument also enables me to explore new areas of scalar reasoning. These areas include left-embedded SIs and the contrast between "even / at least" in increasing and decreasing argument positions. For example, in Chapter 4, I have studied how monotonicities interact with the formal pragmatics of "even / at least", as illustrated in (191), (192), (194) and (195) of that chapter. Had I not studied sentences with relative clauses, I would not have considered (194) and (195) and would not have got a full picture of this linguistic phenomenon.

In Chapter 3, I have studied quantifier inferences by proving theorems and proposing general principles and methods that enable us to discover the inferential patterns of GQs. The most important principles and methods include PMC, POC, the Double Negation Law for deducing valid patterns of duality inferences and the two methods for constructing relational syllogisms. These have resulted in systematic methods for deriving valid inferential patterns of iterated GQs from the inferential properties of their constituent monadic GQs.

The findings of Chapter 3 have enriched the content of GQT and “Natural Logic”, because quantifier inferences are a major object of study in these theories. For example, some earlier studies on GQT (e.g. Zwarts (1983), van Benthem (1984), Westerståhl (1984)) were about the inferential patterns of GQs. The 4 types of quantifier inferences are each studied by some scholars working on “Natural Logic”. This thesis is a continuation of these scholars’ work.

In Chapter 4, I have formulated a uniform framework based on the theories and findings of predecessors of scalar reasoning research, and studied various aspects of SEs, SIs and scalar lexical items, thereby making contribution to Formal Pragmatics. For the first time, research findings obtained separately under different theories or even different branches of linguistics, i.e. SMT (studying SEs and SOs, maximizers / minimizers, superlatives), pragmatic theories on conversational implicatures (studying SIs) and Chinese grammar (studying CCCs, SQOs, idiomatic constructions with extreme numerals), are integrated and reformulated under the framework proposed in this chapter.

I have also shown the close relation between quantifier inferences and scalar reasoning. To be sure, it is generally agreed that reasoning is among the primary objects of study in pragmatics. Topics like conversational implicatures, explicatures, presupposition accommodation, Illocutionary Logic (an offshoot of Speech Act Theory), etc. all involve reasoning. However, it is not clear how concrete research findings of semantic / logical inferences can be applied to pragmatic reasoning.

To a certain extent, this thesis has achieved this. In Chapter 4, I have shown how monotonicity inferences permeate into various aspects of scalar reasoning. First, by viewing strict monotonicity inferences as special cases of SEs, I am able to extend the notions and methods of Monotonicity Calculus and syllogism embedding to scalar reasoning, resulting in two brand new topics in the study of SEs, namely Proportionality Calculus and scalar syllogisms. Second, the notion of monotonicity is also useful for determining whether an SI is associated with a left-implicating or right-implicating scale, and the reasoning direction of certain SOs such as “even” and “at least”.

Opposition inferences play an important role in the theory of SIs, because SIs are essentially negative inferences, which are precisely what opposition inferences are about. In Chapter 4, I have shown that opposition inferences constitute one of the two components of SIs. I have also discussed how the results of opposition inferences may be useful in the studies of antonymy and narrowing. Moreover, results of argument structure inferences are also useful for transforming quantified statements to a suitable form for analysis.

The findings of this study will also have important applications in various areas. Specifically, the findings on quantifier inferences will be useful to subfields of Computational Semantics and Artificial Intelligence that attach great importance to inferences. In fact, the study on opposition inferences in this thesis may be seen as implementing van Benthem (2008)'s proposal and extending MacCartney (2009)'s exclusion inferences as a subtask for tackling the Recognising Textual Entailment (RTE) problem, which is being studied by researchers in Computational Semantics and Artificial Intelligence.

Traditionally, Computational Semantics and Artificial Intelligence are mainly interested in logical items, such as GQs. The discovery of the parallelism between monotonicity inferences and SEs as well as mixed inferences involving both GQs and scalar terms has opened new possibilities in the research of these two fields.

## 5.2 Possible Extensions of the Present Study

In this final section I will point out some possible extensions of the present study. Since this thesis has chosen a rather restricted set of GQs for study, one possible direction is to extend the study to other types of GQs. A whole class of GQs that have not been studied in this thesis is the non-iterated polyadic GQs, which include resumptive GQs, branching GQs, cumulative GQs, reciprocal GQs, generalized determiners, etc. (according to Peters and Westerståhl (2006), Keenan and Westerståhl (2011), Zuber (2010b, 2011)). Even within the class of monadic GQs, there are some types of quantifiers that I have not considered. These include vague quantifiers, interrogative quantifiers, plural quantifiers, quantifiers in a generic, opaque or dynamic setting, etc. There is no doubt that these GQs have rich inferential properties. A study on these GQs will thus expand our inventory of valid inferential patterns of GQs. Some scholars have studied inferences of these GQs. For example, I have carried out study on vague quantifiers and interrogative quantifiers[160] and discovered certain inferential patterns of these quantifiers, such as the following syllogistic schema involving vague quantifiers:

(1) $\mathit{no}(\mathrm{M})(\mathrm{P}) \wedge (\mathit{almost\ every})(\mathrm{S})(\mathrm{M}) \Rightarrow (\mathit{a\ large\ proportion\ of})(\mathrm{S})(\neg\mathrm{P})$

A direction for future work is to integrate the findings of different scholars with the framework established in this thesis.

In this thesis, quantification is mainly defined on domains composed of individuals[161]. But quantification can also be defined on other domains composed of say possible worlds, time, events, locations, etc. A study on quantifier inferences defined on these domains will surely reveal interesting inferential patterns in lexical items other than quantifiers, such as modals, adverbs of quantification, locative adpositions, etc. For example, the following is a valid inference in the event domain:

(2) John prays (each time) before he has meals. $\Rightarrow$

John performs a religious ritual (each time) before he has breakfast.

Note that this inference is a manifestation of the left decreasing and right increasing monotonicities of "*every*" in the event domain. Since these domains have different structures than individual domains, we have to make substantial modification to the ontology before we can apply the results of GQT to these domains. This constitutes another direction for future studies.

As mentioned in the previous section, this thesis has contributed to the study of "Natural Logic" and will have applications in Computational Semantics and Artificial Intelligence. However, modern studies in these fields are more than just

[160] Some of the results have been published in Chow (2011a, d). But to keep this thesis better focused, I have not included these results in this thesis.

[161] Note that in the study of SIs, I have viewed GQs as Montagovian individuals in some occasions. This is equivalent to placing GQs in a higher-order domain composed of quantifiers.

identifying valid inferential patterns. Thus, the findings in Chapter 3 are only raw materials for building a logical proof system or computation algorithm that can deal with natural language inferences. More work need be done in this direction.

Even if we restrict our attention to identifying valid inferential patterns of the 4 main types of quantifier inferences studied in this thesis, there are still many areas that this thesis has not touched upon, such as the following "opposition syllogism" (studied by MacCartney (2009) and Icard (2012)):

(3) Fish and humans are contraries. $\wedge$ Humans and non-humans are contradictories. $\Rightarrow$ Fish is subalternate to non-humans.

Researches in these areas will surely yield fruitful results.

Concerning scalar lexical items, this thesis has only considered the most basic types. According to Israel (2011), there are other types of scalar lexical items. These include certain aspectual operators (such as "yet"), modals (such as "need"), connectives (such as "either") and indefinites (such as "any"). Since the meaning of these lexical items contain other elements than scalarity, the semantics / pragmatics of these items is more complicated and demands more work. Moreover, as many of these items are polarity sensitive, it requires further study to have a clear understanding of the subtle relation between scalar reasoning and polarity sensitivity.

Even for those items that have been studied in this thesis, further work is still needed. This thesis has dealt with the scalar meaning of these items. However, scalarity is only one aspect of meaning. There are other aspects of meaning. Moreover, the use of these function words each has its own subtlety. How we should interpret the interaction between these aspects / subtleties and the scalar meaning of these words in order to get a fuller picture of the meaning and use of these words requires more work.

Finally, scalarity is a widespread phenomenon in natural language. This thesis has only touched on a particular aspect – scalar reasoning. What are the other aspects? How do they interact with scalar reasoning? Are there other types of scalar reasoning apart from SEs and SIs? These remain open questions for future studies.

Thus, while this thesis has addressed to a number of problems in the study of scalar reasoning raised in the final section of Chapter 2, new problems are also discovered. But I believe that this thesis has laid a good foundation for tackling these outstanding problems in future.

## Appendix 1 Truth Conditions of Right Conservative GQs

In what follows, m, n are natural numbers with $0 < m < n$; q, r are rational numbers with $0 < q < r < 1$; $x_1$, $x_2$, … are individual members of the universe manifested as proper names; C is a non-empty set of individuals manifested as conjoined proper names and / or definite descriptions; S is equal to PERSON or THING, according as the GQ ends with "-body" or "-thing".

| Tripartite Structure | Truth Condition |
|---|---|
| *everybody*(*-thing*)(–)(B) | $S \subseteq B$ or<br>$S - B = \varnothing$ |
| *somebody*(*-thing*)(–)(B) | $\lvert S \cap B\rvert > 0$ or<br>$S \cap B \neq \varnothing$ |
| *nobody*(*-thing*)(–)(B) | $S \subseteq \neg B$ or<br>$S \cap B = \varnothing$ |
| (*everybody*(*-thing*) *except C*)(–)(B) | $S - B = C$ [162] |
| (*nobody*(*-thing*) *except C*)(–)(B) | $S \cap B = C$ |
| ($x_1$, $x_2$ *and …*)(–)(B) | $\{x_1, x_2, \ldots\} \subseteq B$ [163] |
| *every*(A)(B)<br>*all*(A)(B) | $A \subseteq B$ or<br>$A - B = \varnothing$ |
| (*not every*)(A)(B)<br>(*not all*)(A)(B) | $\lvert A - B\rvert > 0$ or<br>$A - B \neq \varnothing$ |
| *some*(A)(B)<br>*a*(A)(B) | $\lvert A \cap B\rvert > 0$ or<br>$A \cap B \neq \varnothing$ |
| *no*(A)(B) | $A \subseteq \neg B$ or<br>$A \cap B = \varnothing$ |
| (*all … except C*)(A)(B) | $A - B = C$ |
| (*no … except C*)(A)(B) | $A \cap B = C$ |
| (*more* (*fewer*) *than n*)(A)(B) | $\lvert A \cap B\rvert >(<) n$ |
| (*at least* (*most*) *n*)(A)(B) | $\lvert A \cap B\rvert \geq(\leq) n$ |
| (*exactly n*)(A)(B) | $\lvert A \cap B\rvert = n$ |
| (*between m and n*)(A)(B) | $m \leq \lvert A \cap B\rvert \leq n$ |
| (*all except n*)(A)(B) | $\lvert A - B\rvert = n$ |

[162] In this thesis, I only consider exceptive constructions in the form "all / no … except C" where C is manifested as conjoined proper names and / or definite descriptions and is represented set-theoretically by the union of the sets representing its components. For example, when C is manifested as "John, Mary and the teacher", we have $C = \{j\} \cup \{m\} \cup (CS \cap TEACHER)$.

[163] For a single individual x, the truth condition can alternatively be written as $x \in B$.

| | |
|---|---|
| (*all except between m and n*)(A)(B) | $m \leq \|A - B\| \leq n$ |
| *most*(A)(B)<br>(*a majority of*)(A)(B) | $\|A \cap B\| / \|A\| > 0.5$ |
| (*a minority of*)(A)(B) | $\|A \cap B\| / \|A\| < 0.5$ |
| (*more* (*less*) *than r of*)(A)(B) | $\|A \cap B\| / \|A\| >(<) r$ |
| (*at least* (*most*) *r of*)(A)(B) | $\|A \cap B\| / \|A\| \geq(\leq) r$ |
| (*exactly r of*)(A)(B) | $\|A \cap B\| / \|A\| = r$ |
| (*between q and r of*)(A)(B) | $q \leq \|A \cap B\| / \|A\| \leq r$ |
| (*all except r of*)(A)(B) | $\|A - B\| / \|A\| = r$ |
| (*all except between q and r of*)(A)(B) | $q \leq \|A - B\| / \|A\| \leq r$ |
| *the*(A)(B), where A is singular | $CS \cap A \subseteq B$, if $\|CS \cap A\| = 1$ [164] |
| *C's*(A)(B), where A is singular | $POSSESS_C \cap A \subseteq B$,<br>if $\|POSSESS_C \cap A\| = 1$ [165] |
| *both*(A)(B) | $CS \cap A \subseteq B$, if $\|CS \cap A\| = 2$ |
| *either*(A)(B) | $\|CS \cap A \cap B\| > 0$, if $\|CS \cap A\| = 2$ |
| *neither*(A)(B) | $CS \cap A \subseteq \neg B$, if $\|CS \cap A\| = 2$ |
| (*more* (*fewer*) *… than …*)($A_1$, $A_2$)(B) | $\|A_1 \cap B\| >(<) \|A_2 \cap B\|$ [166] |
| (*at least* (*most*) *as many … as …*)($A_1$, $A_2$)(B) | $\|A_1 \cap B\| \geq(\leq) \|A_2 \cap B\|$ |
| (*exactly as many … as …*)($A_1$, $A_2$)(B) | $\|A_1 \cap B\| = \|A_2 \cap B\|$ |
| (*proportionally more* (*fewer*) *… than …*)($A_1$, $A_2$)(B) | $\|A_1 \cap B\| / \|A_1\| >(<) \|A_2 \cap B\| / \|A_2\|$ |
| (*at least* (*most*) *the same proportion of … as …*)($A_1$, $A_2$)(B) | $\|A_1 \cap B\| / \|A_1\| \geq(\leq) \|A_2 \cap B\| / \|A_2\|$ |
| (*exactly the same proportion of … as …*)($A_1$, $A_2$)(B) | $\|A_1 \cap B\| / \|A_1\| = \|A_2 \cap B\| / \|A_2\|$ |

[164] In this thesis I adopt Westerståhl (1984a)'s semantic analysis of definite determiners which uses a context set (CS) that serves to restrict the domain by intersection. Moreover, I also assume that definite determiners carry presuppositions which, following Heim and Kratzer (1998), are implemented by partial functions that are only defined in cases where certain conditions are satisfied.

[165] $POSSESS_C = \{x: \forall y \in C\ (POSSESS(y, x))\}$. Here POSSESS represents the possessive relation and has a broad meaning, as proposed in Langacker (1991). Moreover, I do not consider such relational nouns as "father", "friend", etc which may appear in possessive constructions, as these relational nouns involve more complicated semantics.

[166] Each type $<1^2,1>$ structured GQ listed here also has type $<1,1^2>$ and $<1^2,1^2>$ variants. For simplicity, I have only included the truth condition of the $<1^2,1>$ variant, because the truth conditions of the other two variants can be derived from that of the $<1^2,1>$ variant.

## Appendix 2 Truth Conditions of Left Conservative GQs

In what follows, m, n are natural numbers with $0 < m < n$; q and r are rational numbers with $0 < q < r < 1$; C is a non-empty set of individuals manifested as conjoined proper names and / or definite descriptions.

| **Tripartite Structure** | **Truth Condition** |
|---|---|
| *only*(A)(B) | $A \supseteq B$ or<br>$B - A = \varnothing$ |
| (*not only*)(A)(B) | $\|B - A\| > 0$ or<br>$B - A \neq \varnothing$ |
| (*apart from C only*)(A)(B) | $B - A = C$ |
| (*constitute more* (*less*) *than r of*)(A)(B) | $\|B \cap A\| / \|B\|$ >(<) r |
| (*constitute at least* (*most*) *r of*)(A)(B) | $\|B \cap A\| / \|B\|$ ≥(≤) r |
| (*constitute exactly r of*)(A)(B) | $\|B \cap A\| / \|B\| = r$ |
| (*constitute between q and r of*)(A)(B) | $q \leq \|B \cap A\| / \|B\| \leq r$ |
| (*constitute all except r of*)(A)(B) | $\|B - A\| / \|B\| = r$ |
| (*constitute all except between q and r of*)(A)(B) | $q \leq \|B - A\| / \|B\| \leq r$ |
| (*constitute a larger* (*smaller*) *proportion of ... than ...*)($A_1$, $A_2$)(B) | $\|A_1 \cap B\| / \|B\|$ >(<) $\|A_2 \cap B\| / \|B\|$ |
| (*constitute at least* (*most*) *the same proportion of ... as ...*)($A_1$, $A_2$)(B) | $\|A_1 \cap B\| / \|B\|$ ≥(≤) $\|A_2 \cap B\| / \|B\|$ |
| (*constitute exactly the same proportion of ... as ...*)($A_1$, $A_2$)(B) | $\|A_1 \cap B\| / \|B\| = \|A_2 \cap B\| / \|B\|$ |

## Appendix 3 Proofs of Theorems

**Theorem 1.1** A determiner Q is left conservative iff $Q^{-1}$ is right conservative.

**Proof:** Here I only prove one direction of the theorem. The other direction is similar. Let Q be a left conservative determiner and A, B be arbitrary sets. Then by (40), we have $Q(A)(B) \Leftrightarrow Q(A \cap B)(B)$. Let $Q^{-1}$ be the converse of Q. Then by (39), we have $Q(A)(B) \Leftrightarrow Q^{-1}(B)(A)$ and $Q(A \cap B)(B) \Leftrightarrow Q^{-1}(B)(A \cap B)$. Combining the above, we have $Q^{-1}(B)(A) \Leftrightarrow Q^{-1}(B)(A \cap B)$. Since A, B are arbitrary, by (35) we conclude that $Q^{-1}$ is right conservative. □

**Theorem 3.1** If a GQ / BO is both increasing and decreasing in an argument, it is trivial in that argument.

**Proof:** Suppose Q is a GQ / BO with n arguments that is both increasing and decreasing in the $i^{th}$ argument. For any particular set of $X_1, \dots X_{i-1}, X_{i+1}, \dots X_n$, either $\| Q(X_1, \dots X_i, \dots X_n) \| = 0$ for all $X_i$, or $\| Q(X_1, \dots X_i', \dots X_n) \| = 1$ for at least an $X_i'$. In the latter case, take an arbitrary $X_i$. Since Q is increasing in the $i^{th}$ argument, we can deduce $\| Q(X_1, \dots X_i \cup X_i', \dots X_n) \| = 1$. Since Q is also decreasing in the $i^{th}$ argument, we can next deduce $\| Q(X_1, \dots X_i, \dots X_n) \| = 1$. I have thus proved that for any particular set of $X_1, \dots X_{i-1}, X_{i+1}, \dots X_n$, either $\| Q(X_1, \dots X_i, \dots X_n) \| = 1$ for any $X_i$, or $\| Q(X_1, \dots X_i, \dots X_n) \| = 0$ for any $X_i$, i.e. Q is trivial in the $i^{th}$ argument. □

**Theorem 3.2** Let X, X' and Y be sets such that $X \subseteq X'$. Then

(a) $X \cap Y \subseteq X' \cap Y$

(b) $|X \cap Y| \leq |X' \cap Y|$

**Proof:** If $X \cap Y = \varnothing$, then (a) and (b) are satisfied automatically. So let x be an arbitrary element of $X \cap Y$. By the assumption, x is also an element of X'. So x is also an element of $X' \cap Y$. Both (a) and (b) are thus satisfied. □

**Theorem 3.3** A GQ with presupposition is monotonic only in cases where its arguments satisfy the presupposition.

**Proof:** In cases where the arguments of a GQ do not satisfy its presupposition, the quantified statement is undefined and has no truth value, and so does not satisfy the definitions of the increasing and decreasing monotonicities. □

**Theorem 3.4** Let Q's truth condition be in the form $X_1 \subseteq Y$ or $X_1 \cap X_2 \subseteq Y$, where $X_i$ ($i \in \{1, 2\}$) and Y are arguments of Q or constant sets and no $X_i$ is equal to Y. Then Q is increasing (decreasing) in all arguments Y ($X_i$). If $X_i$ or Y is replaced by its negative counterpart in the truth condition, the monotonicity of $X_i$ or Y is reversed.

**Proof:** Here I only prove the monotonicity of $X_2$ under Q with the truth condition $X_1 \cap X_2 \subseteq Y$. The proofs for other cases are similar. Suppose the truth condition is satisfied and let $X_2 \supseteq X_2'$. Then by Theorem 3.2(a), $X_1 \cap X_2' \subseteq X_1 \cap X_2$, and so $X_1 \cap X_2' \subseteq Y$, i.e. the truth condition of Q is satisfied with $X_2'$ replacing $X_2$. We have thus shown that $X_2$ is decreasing.

Suppose the truth condition becomes $X_1 \cap \neg X_2 \subseteq Y$ and let $X_2 \subseteq X_2'$. Then we have $\neg X_2' \subseteq \neg X_2$. By Theorem 3.2(a) again, $X_1 \cap \neg X_2' \subseteq X_1 \cap \neg X_2$, and so $X_1 \cap \neg X_2' \subseteq Y$, i.e. the truth condition of Q is satisfied with $X_2'$ replacing $X_2$. We have thus shown that $X_2$ is increasing. □

**Theorem 3.5** Let Q's truth condition be in one of the following forms (after converting any division into multiplication):

(a) $|X_1 \cap X_2| \geq/>/\leq/< n$;

(b) $|X_1 \cap X_2| \geq/> |Y_1 \cap Y_2|$;

(c) $|X_1 \cap X_2| \geq/>/\leq/< r \times |X_3|$;

(d) $|X_1 \cap X_2| \times |Y_3| \geq/> |Y_1 \cap Y_2| \times |X_3|$

where n and r are constants as defined in Appendix 1, $X_i$ and $Y_j$ ($i, j \in \{1, 2\}$) are arguments of Q or constant sets and $X_3$ and $Y_3$ are equal to one of the $X_i$ and $Y_j$, respectively. Then Q is increasing (decreasing) in all arguments appearing solely on the left (right) of "≥/>" or the right (left) of "≤/<", and non-monotonic in all arguments appearing on both sides of "≥/>/≤/<". If any monotonic $X_i$ or $Y_j$ is replaced by its negative counterpart in the truth condition, then treat $\neg X_i$ or $\neg Y_j$ as if it were $X_i$ or $Y_j$ appearing on the opposite side of "≥/>/≤/<".

**Proof:** For truth conditions (a) and (b), I only prove the monotonicity of $Y_2$ under Q with the truth condition $|X_1 \cap X_2| \geq |Y_1 \cap Y_2|$. The proofs for other cases are similar. I first consider the case that $Y_2$ is not the same as any $X_i$. Suppose the truth condition is satisfied and let $Y_2 \supseteq Y_2'$. Then by Theorem 3.2(b), $|Y_1 \cap Y_2'| \leq |Y_1 \cap Y_2|$, and so $|X_1 \cap X_2| \geq |Y_1 \cap Y_2'|$, i.e. the truth condition of Q is satisfied with $Y_2'$ replacing $Y_2$. We have thus shown that $Y_2$, which appears solely on the right of "≥/>", is decreasing.

I next consider the case that $Y_2$ is the same as some $X_i$. Without loss of generality, let the truth condition be $|X_1 \cap Y_2| \geq |Y_1 \cap Y_2|$. I will prove the non-monotonicity of $Y_2$ by showing how to construct a counterexample. First choose three non-trivial sets $X_1$, $Y_1$ and $Y_2$ satisfying the following three conditions: (i) $|X_1 \cap Y_2| \geq |Y_1 \cap Y_2|$; (ii) $|X_1| < |Y_1|$; (iii) $(Y_1 - X_1) \cap Y_2 \neq \emptyset$. Define $Y_2' = (Y_1 - X_1) \cap Y_2$. Then we have $Y_2' \subseteq Y_2$, $X_1 \cap Y_2' = \emptyset$ and $Y_1 \cap Y_2' = Y_2'$, and so $|X_1 \cap Y_2'| < |Y_1 \cap Y_2'|$. Next define $Y_2'' = U$. Then we have $Y_2 \subseteq Y_2''$, $X_1 \cap Y_2'' = X_1$ and $Y_1 \cap Y_2'' = Y_1$, and so $|X_1 \cap Y_2''| < |Y_1 \cap Y_2''|$ by (ii) above. I have thus shown that $Y_2$ is neither decreasing nor increasing, i.e. it is non-monotonic.

In case the truth condition has the form (c), say $|X_1 \cap Y_2| \geq r \times |Y_2|$, we can prove the non-monotonicity of $Y_2$ in the same way as in the previous paragraph by writing U instead of $Y_1$ and modifying the first two conditions in the previous paragraph as (i) $|X_1 \cap Y_2| \geq r \times |Y_2|$; (ii) $|X_1| < r \times |U|$. One can check that the method described above will provide the required counterexample. Note that by interchanging the roles of $X_1$ and $Y_1$, one can then prove the non-monotonicity of $Y_2$ in $|X_1 \cap Y_2| \leq |Y_1 \cap Y_2|$ and $|X_1 \cap Y_2| \leq r \times |Y_2|$.

In case the truth condition has the form (d), say $|X_1 \cap X_2| \times |Y_2| \geq |Y_1 \cap Y_2| \times |X_2|$, we can prove the non-monotonicity of $X_2$ in the same way as in the previous paragraph by moving $|Y_2|$ to the RHS of "≥" and treating $|Y_1 \cap Y_2| / |Y_2|$ as a constant.

If any monotonic $X_i$ or $Y_j$ is replaced by its negative counterpart in the truth condition, then we can follow a similar line of reasoning as in the proof of Theorem 3.4 to show that the monotonicity of $\neg X_i$ or $\neg Y_j$ is opposite that of $X_i$ or $Y_j$. Thus, $\neg X_i$ or $\neg Y_j$ can be treated as if it were $X_i$ or $Y_j$ appearing on the opposite side of "≥/>/≤/<". □

**Theorem 3.6** Let Q's truth condition be in the form $X_1 = Y$ or $X_1 \cap X_2 = Y$, where $X_i$ (i, j $\in$ {1, 2}) and Y are arguments of Q or non-trivial constant sets and no $X_i$ is equal to Y. Then Q is non-monotonic in all of its arguments. This fact is unaffected if $X_i$ or Y is replaced by its negative counterpart in the truth condition.

**Proof:** Here I only prove the non-monotonicity of $X_2$ under Q with the truth condition $X_1 \cap X_2 = Y$. The proofs for other cases are similar. I will show how to construct a counterexample. First choose three non-trivial sets $X_1$, $X_2$ and Y such that (i) $X_1 \cap X_2 = Y$ and (ii) $X_1 \neq Y$. Then choose an element x from Y. Define $X_2' = X_2 - \{x\}$. Then we have $X_2' \subseteq X_2$ and $X_1 \cap X_2' \neq Y$. Next define $X_2'' = X_1$

$\cup X_2$. Then we have $X_2 \subseteq X_2''$ and $X_1 \cap X_2'' = X_1 \neq Y$ by (ii) above. I have thus shown that $X_2$ is neither decreasing nor increasing, i.e. it is non-monotonic.

In case the truth condition contains a negative set, say $X_1 \cap \neg Z = Y$, we can define $X_1$ and Y as in the previous paragraph and define Z, Z' and Z'' as $\neg X_2$, $\neg X_2'$ and $\neg X_2''$, respectively, where $X_2$, $X_2'$ and $X_2''$ are as defined in the previous paragraph. Then we will have $Z \subseteq Z'$, $Z'' \subseteq Z$ and $X_1 \cap \neg Z = Y$, $X_1 \cap \neg Z' \neq Y$, $X_1 \cap \neg Z'' \neq Y$, and so Z is non-monotonic. ☐

**Theorem 3.7** Let Q's truth condition be in one of the following forms:

(a) $|X_1 \cap X_2| = n$;

(b) $m \leq |X_1 \cap X_2| \leq n$;

(c) $|X_1 \cap X_2| = |Y_1 \cap Y_2|$;

(d) $|X_1 \cap X_2| / |X_3| = r$;

(e) $q \leq |X_1 \cap X_2| / |X_3| \leq r$;

(f) $|X_1 \cap X_2| / |X_3| = |Y_1 \cap Y_2| / |Y_3|$

where m, n, q and r are constants as defined in Appendix 1, $X_i$ and $Y_j$ ($i, j \in \{1, 2\}$) are arguments of Q or constant sets and $X_3$ and $Y_3$ are equal to one of the $X_i$ and $Y_j$, respectively. Then Q is non-monotonic in all of its arguments. This fact is unaffected if $X_i$ or $Y_j$ is replaced by its negative counterpart in the truth condition.

**Proof:** I first prove the non-monotonicity of $X_2$ under Q with truth condition in the form (b). First choose two non-trivial sets $X_1$ and $X_2$ such that (i) $m \leq |X_1 \cap X_2| \leq n$ and (ii) $|X_1| > n$. Then choose a subset Z consisting of $|X_1 \cap X_2| - m + 1$ elements from the intersection $X_1 \cap X_2$. Define $X_2' = X_2 - Z$. Then we have $X_2' \subseteq X_2$ and $|X_1 \cap X_2'| = |X_1 \cap X_2| - (|X_1 \cap X_2| - m + 1) = m - 1 < m$. Next define $X_2'' = X_1 \cup X_2$. Then we have $X_2 \subseteq X_2''$ and $|X_1 \cap X_2''| = |X_1| > n$ by (ii) above.

I have thus shown that $X_2$ is neither decreasing nor increasing, i.e. it is non-monotonic. The proof method described above can be applied to truth conditions in the form (a) because (a) is equivalent to $n \leq |X_1 \cap X_2| \leq n$. Then the proof method can be further applied to truth conditions in the form (c). For instance, when proving the non-monotonicity of $X_2$ in (c), one can treat $|Y_1 \cap Y_2|$ as a constant.

Next consider truth conditions in the form (e), say $q \leq |X_1 \cap X_2| / |X_2| \leq r$. To prove the non-monotonicity of $X_1$, we can rewrite this truth condition as $q \times |X_2| \leq |X_1 \cap X_2| \leq r \times |X_2|$, and then treat $q \times |X_2|$ and $r \times |X_2|$ as constants. The proof method is then similar to that for (b) above. To prove the non-monotonicity of $X_2$, we can rewrite this truth condition as the conjunction $|X_1 \cap X_2| / |X_2| \geq q \wedge |X_1 \cap X_2| / |X_2| \leq r$. Then we can employ the method in the proof of Theorem 3.5 to construct counterexamples for either conjunct. The proof method described above can be applied to truth conditions in the form (d) or (f).

In case the truth condition contains a negative set, we can follow a similar line of reasoning as in the proof of Theorem 3.6 to prove the non-monotonicity of the arguments in question. □

**Theorem 3.8** Let P and P' be n-ary predicates, then $P \subseteq P' \Rightarrow \{x_i: P(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\} \subseteq \{x_i: P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\}$ for any $1 \leq i \leq n$ and any particular set of $x_1, \ldots x_{i-1}, x_{i+1}, \ldots x_n$.

**Proof:** Suppose $P \subseteq P'$. Then for any particular set of $x_1, \ldots x_{i-1}, x_{i+1}, \ldots x_n$ and any arbitrary $x_i$, we have $P(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n) \Rightarrow P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)$, which is equivalent to saying that if $x_i \in \{x_i: P(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\}$, then $x_i \in \{x_i: P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\}$, thus showing that $\{x_i: P(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\} \subseteq \{x_i: P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\}$. □

**Theorem 3.9** "¬" is decreasing.

**Proof:** Let X and X' be sets or propositions such that $X \geq X'$. Then according to Set Theory and Propositional Logic, we have $\neg X \leq \neg X'$. Thus, by definition (2), "¬" is decreasing. ☐

**Theorem 3.10** Let Q be a monadic GQ with n arguments, then $Q \wedge Q\neg_i$ and $Q \vee Q\neg_i$ are fixed points in the $i^{th}$ argument.

**Proof:** For any $X_1, \dots X_n$, $((Q \wedge Q\neg_i)\neg_i)(X_1, \dots X_i, \dots X_n) \Leftrightarrow (Q \wedge Q\neg_i)(X_1, \dots \neg X_i, \dots X_n) \Leftrightarrow Q(X_1, \dots \neg X_i, \dots X_n) \wedge (Q\neg_i)(X_1, \dots \neg X_i, \dots X_n) \Leftrightarrow (Q\neg_i)(X_1, \dots X_i, \dots X_n) \wedge Q(X_1, \dots X_i, \dots X_n) \Leftrightarrow (Q \wedge Q\neg_i)(X_1, \dots X_i, \dots X_n)$. So by definition (26), $Q \wedge Q\neg_i$ is a fixed point in the $i^{th}$ argument. The proof for $Q \vee Q\neg_i$ is similar. ☐

**Theorem 3.11** Let $Q_1$ and $Q_2$ be monadic GQs with the same argument structure. If both $Q_1$ and $Q_2$ are fixed points in the $i^{th}$ argument, then $\neg Q_1$, $Q_1 \wedge Q_2$ and $Q_1 \vee Q_2$ are also fixed points in the $i^{th}$ argument.

**Proof:** Let $Q_1$ be a fixed point in the $i^{th}$ argument, then $((\neg Q_1)\neg_i)(X_1, \dots X_i, \dots X_n) \Leftrightarrow (\neg Q_1)(X_1, \dots \neg X_i, \dots X_n) \Leftrightarrow \neg(Q_1(X_1, \dots \neg X_i, \dots X_n)) \Leftrightarrow \neg((Q_1\neg_i)(X_1, \dots X_i, \dots X_n)) \Leftrightarrow \neg(Q_1(X_1, \dots X_i, \dots X_n)) \Leftrightarrow (\neg Q_1)(X_1, \dots X_i, \dots X_n)$. So by definition (26), $\neg Q_1$ is also a fixed point in the $i^{th}$ argument.

Let $Q_1$ and $Q_2$ be fixed points in the $i^{th}$ argument, then $((Q_1 \wedge Q_2)\neg_i)(X_1, \dots X_i, \dots X_n) \Leftrightarrow (Q_1 \wedge Q_2)(X_1, \dots \neg X_i, \dots X_n) \Leftrightarrow Q_1(X_1, \dots \neg X_i, \dots X_n) \wedge Q_2(X_1, \dots \neg X_i, \dots X_n) \Leftrightarrow (Q_1\neg_i)(X_1, \dots X_i, \dots X_n) \wedge (Q_2\neg_i)(X_1, \dots X_i, \dots X_n) \Leftrightarrow Q_1(X_1, \dots X_i, \dots X_n) \wedge Q_2(X_1, \dots X_i, \dots X_n) \Leftrightarrow (Q_1 \wedge Q_2)(X_1, \dots X_i, \dots X_n)$. So by definition (26), $Q_1 \wedge Q_2$ is also a fixed point in the $i^{th}$ argument. The proof for $Q_1 \vee Q_2$ is similar. ☐

**Theorem 3.12** There is no fixed point for outer negation.

**Proof:** This can be proved by contradiction. Assume that Q is a fixed point for outer negation. Then, by the definition of fixed points, we must have $Q(X_1, \ldots X_n) \Leftrightarrow \neg Q(X_1, \ldots X_n)$ for any $X_1, \ldots X_n$. But this is impossible because any proposition cannot be equivalent to its negation. So there cannot be a fixed point for outer negation. □

**Theorem 3.13** The determiner $Q_{k,K}$ defined in (61) is a right self-dual.

**Proof:** On the one hand, we have

(A1) $$((Q_{k,K})\neg_r)(A)(B) \Leftrightarrow \begin{cases} |A - B| \in K, & \text{if } 0 \leq |A - B| \leq k \\ 2k + 1 - |A - B| \notin K, & \text{if } k + 1 \leq |A - B| \leq 2k + 1 \end{cases}$$

On the other hand, since $2k + 1 = |A| = |A \cap B| + |A - B|$, we have

(A2) $$0 \leq |A - B| \leq k \Leftrightarrow k + 1 \leq |A \cap B| \leq 2k + 1$$

$$k + 1 \leq |A - B| \leq 2k + 1 \Leftrightarrow 0 \leq |A \cap B| \leq k$$

Therefore we can rewrite (A1) as

(A3) $$((Q_{k,K})\neg_r)(A)(B) \Leftrightarrow \begin{cases} 2k + 1 - |A \cap B| \in K, & \text{if } k + 1 \leq |A \cap B| \leq 2k + 1 \\ |A \cap B| \notin K, & \text{if } 0 \leq |A \cap B| \leq k \end{cases}$$

The outer negation of (A3), i.e. $(Q_{k,K})^{dr}$, is

(A4) $$((Q_{k,K})^{dr})(A)(B) \Leftrightarrow \begin{cases} 2k + 1 - |A \cap B| \notin K, & \text{if } k + 1 \leq |A \cap B| \leq 2k + 1 \\ |A \cap B| \in K, & \text{if } 0 \leq |A \cap B| \leq k \end{cases}$$

Since (A4) is equivalent to (61), we have thus proved that $(Q_{k,K})^{dr} = Q_{k,K}$. By (27), $Q_{k,K}$ is a right self-dual. □

**Theorem 3.14** Let Q be a monadic GQ with n arguments, then

(a) Q is increasing (decreasing) in the $i^{th}$ argument iff $\neg Q$ and $Q\neg_i$ are decreasing (increasing) in the $i^{th}$ argument iff $Q^{di}$ is increasing (decreasing) in the $i^{th}$ argument.

(b) Q is non-monotonic in the $i^{th}$ argument iff $\neg Q$, $Q\neg_i$ and $Q^{di}$ are non-monotonic in the $i^{th}$ argument.

**Proof:** Since outer negation, inner negation and dual are involutive operations, we only need to prove one direction of both parts of the theorem.

(a) Let Q be increasing in the $i^{th}$ argument. We consider the monotonicities of $\neg Q$, $Q\neg_i$ and $Q^{di}$ in the $i^{th}$ argument in turn. First, by Theorem 3.9 and PMC, "$\neg$" reverses the monotonicity of any predicate within its scope, and so $\neg Q$ is decreasing in the $i^{th}$ argument. Second, by definition (1), we have the entailment $X_i \leq X_i' \Rightarrow Q(X_1, \dots X_i, \dots X_n) \leq Q(X_1, \dots X_i', \dots X_n)$. Replacing the arbitrary $X_i$ and $X_i'$ by their negations, we obtain the equivalent entailment $\neg X_i \leq \neg X_i' \Rightarrow Q(X_1, \dots \neg X_i, \dots X_n) \leq Q(X_1, \dots \neg X_i', \dots X_n)$, which is in turn equivalent to $X_i' \leq X_i \Rightarrow Q\neg_i(X_1, \dots X_i, \dots X_n) \leq Q\neg_i(X_1, \dots X_i', \dots X_n)$. By definition (2), $Q\neg_i$ is decreasing in the $i^{th}$ argument. Third, since $Q^{di}$ is the combination of $\neg Q$ and $Q\neg_i$, by PMC, $Q^{di}$ will preserve the monotonicity of the $i^{th}$ argument, which is thus increasing. The proof for the case when Q is decreasing is similar.

(b) Suppose Q is non-monotonic in the $i^{th}$ argument. Then Q is neither increasing nor decreasing in the $i^{th}$ argument. So by (a), $\neg Q$ and $Q\neg_i$ are neither decreasing nor increasing in the $i^{th}$ argument, and $Q^{di}$ is neither increasing nor decreasing in the $i^{th}$ argument. The conclusion thus obtains. □

**Theorem 3.15** Let Q be a monadic GQ that is non-trivial in the $i^{th}$ argument. If Q is a fixed point in the $i^{th}$ argument, then Q is non-monotonic in that argument.

**Proof:** Let Q be a fixed point in the $i^{th}$ argument. Then by definition (26), we have $Q\neg_i = Q$. Suppose Q is increasing (decreasing) in the $i^{th}$ argument, then by Theorem 3.14(a), $Q\neg_i$ is decreasing (increasing) in the $i^{th}$ argument. So Q is both increasing and decreasing in the $i^{th}$ argument. But by Theorem 3.1, this entails that Q is trivial in the $i^{th}$ argument. This contradiction shows that Q is non-monotonic in the $i^{th}$ argument. □

**Theorem 3.16** Let $Q_1$ and $Q_2$ be monadic GQs with n arguments and the same argument structure and $1 \leq i < j \leq n$. Then $(\neg Q_1)^{-1}{}_{i,j} = \neg(Q_1{}^{-1}{}_{i,j})$; $(Q_1 \wedge Q_2)^{-1}{}_{i,j} = Q_1{}^{-1}{}_{i,j} \wedge Q_2{}^{-1}{}_{i,j}$; $(Q_1 \vee Q_2)^{-1}{}_{i,j} = Q_1{}^{-1}{}_{i,j} \vee Q_2{}^{-1}{}_{i,j}$.

**Proof:** Let $X_1, \dots X_i, \dots X_j, \dots X_n$ be arbitrary sets. Then $(\neg Q_1)^{-1}{}_{i,j}(X_1, \dots X_i, \dots X_j, \dots X_n) \Leftrightarrow (\neg Q_1)(X_1, \dots X_j, \dots X_i, \dots X_n) \Leftrightarrow \neg(Q_1(X_1, \dots X_j, \dots X_i, \dots X_n)) \Leftrightarrow \neg((Q_1{}^{-1}{}_{i,j})(X_1, \dots X_i, \dots X_j, \dots X_n))$. So we have $(\neg Q_1)^{-1}{}_{i,j} = \neg(Q_1{}^{-1}{}_{i,j})$.

Let $X_1, \dots X_i, \dots X_j, \dots X_n$ be arbitrary sets. Then $(Q_1 \wedge Q_2)^{-1}{}_{i,j}(X_1, \dots X_i, \dots X_j, \dots X_n) \Leftrightarrow (Q_1 \wedge Q_2)(X_1, \dots X_j, \dots X_i, \dots X_n) \Leftrightarrow Q_1(X_1, \dots X_j, \dots X_i, \dots X_n) \wedge Q_2(X_1, \dots X_j, \dots X_i, \dots X_n) \Leftrightarrow Q_1{}^{-1}{}_{i,j}(X_1, \dots X_i, \dots X_j, \dots X_n) \wedge Q_2{}^{-1}{}_{i,j}(X_1, \dots X_i, \dots X_j, \dots X_n)$. So we have $(Q_1 \wedge Q_2)^{-1}{}_{i,j} = Q_1{}^{-1}{}_{i,j} \wedge Q_2{}^{-1}{}_{i,j}$. The proof for $(Q_1 \vee Q_2)^{-1}{}_{i,j} = Q_1{}^{-1}{}_{i,j} \vee Q_2{}^{-1}{}_{i,j}$ is similar. □

**Theorem 3.17** Let Q be a determiner. Then Q is symmetric iff $Q\neg_r$ is contrapositive iff $Q\neg_l$ is contrapositive.

**Proof:** Here I only prove the case for "$Q\neg_r$". The proof for the other case is similar. Let Q be symmetric. By definition of symmetry, we have $Q(A)(B) \Leftrightarrow Q(B)(A)$. By definition of "$\neg_r$", this is equivalent to $Q\neg_r(A)(\neg B) \Leftrightarrow Q\neg_r(B)(\neg A)$, which is in turn equivalent to $Q\neg_r(A)(B) \Leftrightarrow Q\neg_r(\neg B)(\neg A)$ because we can replace the arbitrary B by its negation. By definition (40), $Q\neg_r$ is contrapositive. □

**Theorem 3.18** Let Q be a determiner. Then Q is contrapositive iff $Q^{-1}$ is contrapositive iff $\neg Q$ is contrapositive iff $Q\neg_{l,r}$ is contrapositive.

**Proof**: Let Q be contrapositive. Then by (40), we have $Q(A)(B) \Leftrightarrow Q(\neg B)(\neg A)$. By definition of converse, this is equivalent to $Q^{-1}(B)(A) \Leftrightarrow Q^{-1}(\neg A)(\neg B)$. Thus, $Q^{-1}$ is contrapositive.

Since $Q(A)(B) \Leftrightarrow Q(\neg B)(\neg A)$ can be equivalently rewritten as $\neg Q(A)(B) \Leftrightarrow \neg Q(\neg B)(\neg A)$, we may conclude that Q is contrapositive iff $\neg Q$ is contrapositive. Finally, by definition of "$\neg_{l,r}$", $Q(A)(B) \Leftrightarrow Q(\neg B)(\neg A)$ is equivalent to $Q\neg_{l,r}(\neg A)(\neg B) \Leftrightarrow Q\neg_{l,r}(B)(A)$, which is in turn equivalent to $Q\neg_{l,r}(A)(B) \Leftrightarrow Q\neg_{l,r}(\neg B)(\neg A)$ because we can replace the arbitrary A and B by their negations. Thus, Q is contrapositive iff $Q\neg_{l,r}$ is contrapositive. ☐

**Theorem 3.19** $Q_1(A_1)$ is scopally dominant over $Q_2(A_2)$ iff

(a) $(Q_2^{dr})(A_2)$ is scopally dominant over $(Q_1^{dr})(A_1)$;

(b) $(Q_1\neg_l)(A_1)$ is scopally dominant over $(Q_2\neg_l)(A_2)$;

(c) $(Q_2^{dl,r})(A_2)$ is scopally dominant over $(Q_1^{dl,r})(A_1)$.

**Proof:**

(a) By definition (38), $Q_1(A_1)$ is scopally dominant over $Q_2(A_2)$ iff $Q_1(A_1)([Q_2(A_2)]_2(B)) \Rightarrow Q_2(A_2)([Q_1(A_1)]_2(B^{-1}))$, which is equivalent to $\neg Q_2(A_2)([Q_1(A_1)]_2(B^{-1})) \Rightarrow \neg Q_1(A_1)([Q_2(A_2)]_2(B))$. This last entailment can be rewritten as $(Q_2^{dr})(A_2)([(Q_1^{dr})(A_1)]_2(\neg B^{-1})) \Rightarrow (Q_1^{dr})(A_1)([(Q_2^{dr})(A_2)]_2(\neg B))$. Replacing the arbitrary B by $\neg B^{-1}$, we obtain $(Q_2^{dr})(A_2)([(Q_1^{dr})(A_1)]_2(B)) \Rightarrow (Q_1^{dr})(A_1)([(Q_2^{dr})(A_2)]_2(B^{-1}))$, i.e. $(Q_2^{dr})(A_2)$ is scopally dominant over $(Q_1^{dr})(A_1)$[167].

(b) Similar as above, $Q_1(A_1)$ is scopally dominant over $Q_2(A_2)$ iff $Q_1(A_1)([Q_2(A_2)]_2(B)) \Rightarrow Q_2(A_2)([Q_1(A_1)]_2(B^{-1}))$. This last entailment can be rewritten as $(Q_1\neg_l)(\neg A_1)([(Q_2\neg_l)(\neg A_2)]_2(B)) \Rightarrow (Q_2\neg_l)(\neg A_2)([(Q_1\neg_l)(\neg A_1)]_2(B^{-1}))$. Replacing the arbitrary $\neg A_1$ and $\neg A_2$ by their negations, we obtain $(Q_1\neg_l)(A_1)([(Q_2\neg_l)(A_2)]_2(B)) \Rightarrow (Q_2\neg_l)(A_2)([(Q_1\neg_l)(A_1)]_2(B^{-1}))$, i.e. $(Q_1\neg_l)(A_1)$ is scopally dominant over $(Q_2\neg_l)(A_2)$.

[167] Here I have made use of the fact that $(\neg B)^{-1} = \neg(B^{-1})$. This fact can be proved by modifying the proof of Theorem 3.16 to make it applicable to predicates.

(c) Since left-and-right dual is the combination of right dual and left inner negation, the result of (c) follows immediately from (a) and (b). ☐

**Theorem 3.20** A GQ with presupposition has any one of the 4 OPs only in cases where its arguments satisfy the presupposition.

**Proof:** In cases where the arguments of a GQ do not satisfy its presupposition, the quantified statement is undefined and has no truth value and so does not satisfy the definitions of the OPs. ☐

**Theorem 3.21** Let Q be a GQ with n arguments. Then wrt the $i^{th}$ argument, Q possesses a certain OP iff each of ¬Q, $Q\neg_i$ and $Q^{di}$ possesses a different OP according to the following table:

| **Q** | **¬Q** | **$Q\neg_i$** | **$Q^{di}$** |
|---|---|---|---|
| CC→CC | CC→SC | SC→CC | SC→SC |
| CC→SC | CC→CC | SC→SC | SC→CC |
| SC→CC | SC→SC | CC→CC | CC→SC |
| SC→SC | SC→CC | CC→SC | CC→CC |

**Proof:** Here I only prove the first row of the table. The remaining rows can be derived from the first row by using the composite relations among Q, ¬Q, $Q\neg_i$ and $Q^{di}$.

By definitions (81) and (83), Q is CC→CC in the $i^{th}$ argument iff

(A5) $$CC(X_i, X_i') \Rightarrow Q(X_1, \dots X_i, \dots X_n) \leq \neg Q(X_1, \dots X_i', \dots X_n)$$

Now (A5) is equivalent to

(A6) $$CC(X_i, X_i') \Rightarrow \neg(\neg Q)(X_1, \dots X_i, \dots X_n) \leq (\neg Q)(X_1, \dots X_i', \dots X_n)$$

Substituting the arbitrary $X_i$ and $X_i'$ by their negations and using (85) and the definitions of inner negation and dual, (A5) and (A6) can be rewritten as

(A7) $$SC(X_i, X_i') \Rightarrow (Q\neg_i)(X_1, \dots X_i, \dots X_n) \leq \neg(Q\neg_i)(X_1, \dots X_i', \dots X_n)$$

(A8) $$SC(X_i, X_i') \Rightarrow \neg(Q^{di})(X_1, \dots X_i, \dots X_n) \leq (Q^{di})(X_1, \dots X_i', \dots X_n)$$

From (A6) – (A8), we may conclude that ¬Q is CC→SC, $Q\neg_i$ is SC→CC and $Q^{di}$

is SC→SC in the $i^{th}$ argument. ☐

**Theorem 3.22** Let $Q_1$ and $Q_2$ be GQs of the same type with $Q_1 \leq Q_2$.

(a) If $Q_2$ is CC→CC (SC→CC) in the $i^{th}$ argument, so is $Q_1$.

(b) If $Q_1$ is CC→SC (SC→SC) in the $i^{th}$ argument, so is $Q_2$.

**Proof:**

(a) Suppose CC($X_i$, $X_i$') and $\|Q_1(X_1, \ldots X_i, \ldots X_n)\| = 1$, then since $Q_1 \leq Q_2$, we have $\|Q_2(X_1, \ldots X_i, \ldots X_n)\| = 1$. But since $Q_2$ is CC→CC in the $i^{th}$ argument, we have $\|Q_2(X_1, \ldots X_i', \ldots X_n)\| = 0$. By $Q_1 \leq Q_2$ again, we have $\|Q_1(X_1, \ldots X_i', \ldots X_n)\| = 0$. We have thus proved that CC($Q_1(X_1, \ldots X_i, \ldots X_n)$, $Q_1(X_1, \ldots X_i', \ldots X_n)$), i.e. $Q_1$ is CC→CC in the $i^{th}$ argument. The proof for the case SC→CC is exactly the same.

(b) Suppose CC($X_i$, $X_i$') and $\|Q_2(X_1, \ldots X_i, \ldots X_n)\| = 0$, then since $Q_1 \leq Q_2$, we have $\|Q_1(X_1, \ldots X_i, \ldots X_n)\| = 0$. But since $Q_1$ is CC→SC in the $i^{th}$ argument, we have $\|Q_1(X_1, \ldots X_i', \ldots X_n)\| = 1$. By $Q_1 \leq Q_2$ again, we have $\|Q_2(X_1, \ldots X_i', \ldots X_n)\| = 1$. We have thus proved that SC($Q_2(X_1, \ldots X_i, \ldots X_n)$, $Q_2(X_1, \ldots X_i', \ldots X_n)$), i.e. $Q_2$ is CC→SC in the $i^{th}$ argument. The proof for the case SC→SC is exactly the same. ☐

**Theorem 3.23** Let Q be a GQ with n arguments, $1 \leq i < j \leq n$ and Π be one of the 4 OPs.

(a) Q is Π in the $i^{th}$ argument iff $Q^{-1}{}_{i,j}$ is Π in the $j^{th}$ argument.

(b) If Q is symmetric wrt the $i^{th}$ and $j^{th}$ arguments, then Q is Π in both or neither of these two arguments.

**Proof:**

(a) Here I only prove the case when Π = CC→CC. The proofs of the other cases are similar. Suppose CC($X_i$, $X_i$') and Q is CC→CC in the $i^{th}$ argument. Then we have $Q(X_1, \ldots X_i, \ldots X_j, \ldots X_n) \leq \neg Q(X_1, \ldots X_i', \ldots X_j, \ldots X_n)$, which by (28) may be rewritten as $(Q^{-1}_{i,j})(X_1, \dots X_j, \dots X_i, \dots X_n) \le \neg(Q^{-1}_{i,j})(X_1, \dots X_j, \dots X_i', \dots X_n)$. This shows that $Q^{-1}_{i,j}$ is CC→CC in the $j^{th}$ argument.

(b) Let Q be symmetric wrt the $i^{th}$ and $j^{th}$ arguments, then Q is self-converse wrt the same arguments. So by (a), Q is Π in the $i^{th}$ argument iff it is Π in the $j^{th}$ argument, i.e. Q is Π in both or neither of these two arguments. □

**Theorem 3.24** Let Q be a contrapositive determiner. Then Q is CC→CC in an argument iff it is SC→CC in the other argument. Q is CC→SC in an argument iff it is SC→SC in the other argument.

**Proof**: Suppose Q is CC→CC in the right argument and SC(A, A'), which by (85) is equivalent to CC(¬A, ¬A'). Let $\|Q(A)(B)\| = 1$. By contrapositivity of Q, this is equivalent to $\|Q(\neg B)(\neg A)\| = 1$. But then we must have $\|Q(\neg B)(\neg A')\| = 0$. By contrapositivity of Q again, this is in turn equivalent to $\|Q(A')(B)\| = 0$. We have thus proved that SC(A, A') ⇒ CC(Q(A)(B), Q(A')(B)), i.e. Q is SC→CC in the left argument. Similarly, we can prove that if Q is SC→CC in the left argument, then Q is CC→CC in the right argument. The proofs for the cases Q is CC→CC in the left argument and CC→SC in either argument follow the same line. □

**Theorem 3.25** "(*at least r of*)" ($1/2 < r < 1$) is CC→CC in the right argument; "(*more than r of*)" ($1/2 \le r < 1$) is CC→CC in the right argument; "(*between q and r of*)" ($0 < q < r < 1$) is not CC→CC in the left argument.

**Proof:** I first prove "(*at least r of*)" ($1/2 < r < 1$) is CC→CC in the right argument. Let $\|$(*at least r of*)(A)(B)$\| = 1$ and CC(B, B'). Then by (83), B ⊆ ¬B'. Since (*at least r of*) ∈ –MON↑, we have $\|$(*at least r of*)(A)(¬B')$\| = 1$, which is equivalent to $\|$(*at most 1 – r of*)(A)(B')$\| = 1$. Since $1/2 < r < 1$, this entails $\|$(*less than r of*)(A)(B')$\| = 1$, which is equivalent to $\|\neg$(*at least r of*)(A)(B')$\| =$

1. I have thus shown that CC((*at least r of*)(A)(B), (*at least r of*)(A)(B')). Thus, "(*at least r of*)" is CC→CC in the right argument. The fact that "(*more than r of*)" ($1/2 \leq r < 1$) is CC→CC in the right argument can be proved similarly.

Next I show that "(*between q and r of*)" ($0 < q < r < 1$) is not CC→CC in the left argument by devising a method for constructing counterexamples for any $0 < q < r < 1$. Choose any rational number x/y such that $q \leq x/y \leq r$. Construct two sets A and A' such that $|A| = |A'| = y$ and $A \cap A' = \emptyset$. Choose a subset X of A and a subset X' of A' such that $|X| = |X'| = x$. Then set $B = X \cup X'$. It is easy to check that with these predicates, we have CC(A, A') and ‖(*between q and r of*)(A)(B)‖ = ‖(*between q and r of*)(A')(B)‖ = 1. In other words, we do not have CC((*between q and r of*)(A)(B), (*between q and r of*)(A')(B)), thus completing the proof. ☐

**Theorem 3.26** Every absolute numerical determiner and structured GQ studied in this thesis is o-insensitive in all arguments.

**Proof:** According to Table 3.10, a proportional determiner possesses a certain OP only within a certain range. Now, an absolute numerical determiner can be made equivalent to any proportional determiner by setting an appropriate cardinality of its left or right argument. Thus, given an absolute numerical determiner Q, a certain OP and a certain argument, we can construct a model in which Q is equivalent to a proportional determiner which does not possess that OP in that argument. Thus, every absolute numerical determiner is both left and right o-insensitive. For example, to show that "(*at least 5*)" is not SC→SC in the right argument, we first observe that "(*at least 5*)(A)(B)" is equivalent to "(*constitute at least 1/2 of*)(A)(B)" in a model where $|B| = 10$. Since "(*constitute at least 1/2 of*)" is not SC→SC in the right argument, we can then use a method similar to that shown in the proof of Theorem 3.25 to construct a model in which $|B| = 10$

and "(*constitute at least 1/2 of*)" is not SC→SC in the right argument. This model is thus a counterexample showing that "(*at least 5*)" is not SC→SC in the right argument.

Next consider the structured GQs. Since these GQs do not denote fixed quantities, their arguments can be made equivalent to the arguments of different proportional determiners under different models. Thus, based on the same argument as above, we may conclude that the structured GQs are o-insensitive in all arguments. ☐

**Theorem 3.27** Let Q be a GQ with n arguments. With respect to the $i^{th}$ argument,

(a) It is impossible for Q to be CC→CC and CC→SC.

(b) It is impossible for Q to be SC→CC and SC→SC.

(c) Q is CC→CC and SC→SC iff Q is self-dual and increasing.

(d) Q is SC→CC and CC→SC iff Q is self-dual and decreasing.

(e) Q is CC→CC and SC→CC iff Q is perfectly consistent.

(f) Q is CC→SC and SC→SC iff Q is perfectly complete.

**Proof:**

(a) Suppose Q is CC→CC and CC→SC. Take an arbitrary $X_i$. For any particular set of $X_1, \ldots X_{i-1}, X_{i+1}, \ldots X_n$, $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 1$ or 0. Let $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 1$. Since $CC(X_i, \neg X_i)$, we have $\|Q(X_1, \ldots \neg X_i, \ldots X_n)\| = 0$. Since $CC(\neg X_i, \varnothing)$, we then have $\|Q(X_1, \ldots \varnothing, \ldots X_n)\| = 1$. But since $CC(\varnothing, X_i)$, we then have $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 0$. Thus, starting from $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 1$, I can derive $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 0$. Similarly, starting from $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 0$, I can derive $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 1$. This contradiction shows that it is impossible for Q to be CC→CC and CC→SC. The proof of (b) follows a similar line of reasoning.

(c) First let Q be CC→CC and SC→SC. Then since $CC(X_i, \neg X_i)$ and $SC(\neg X_i, X_i)$, we have $Q(X_1, \ldots X_i, \ldots X_n) \Rightarrow \neg Q(X_1, \ldots \neg X_i, \ldots X_n)$ and $\neg Q(X_1, \ldots \neg X_i, \ldots X_n) \Rightarrow Q(X_1, \ldots X_i, \ldots X_n)$, respectively. Combining the above, we have $Q(X_1, \ldots X_i, \ldots X_n) \Leftrightarrow \neg Q(X_1, \ldots \neg X_i, \ldots X_n)$. So by (27), Q is self-dual. Next, let $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 1$ and $X_i \subseteq X_i'$. Then since $CC(X_i, \neg X_i')$, we have $\|Q(X_1, \ldots \neg X_i', \ldots X_n)\| = 0$. But since $SC(\neg X_i', X_i')$, we have $\|Q(X_1, \ldots X_i', \ldots X_n)\| = 1$. Thus, Q is increasing.

Next let Q be self-dual and increasing. Suppose $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 1$ and $CC(X_i, X_i')$. Since Q is self-dual, we have $\|\neg Q(X_1, \ldots \neg X_i, \ldots X_n)\| = 1$. From $CC(X_i, X_i')$ we have $X_i' \subseteq \neg X_i$. Since Q is increasing, by Theorem 3.14(a) ¬Q is decreasing and so we have $\|\neg Q(X_1, \ldots X_i', \ldots X_n)\| = 1$, i.e. $\|Q(X_1, \ldots X_i', \ldots X_n)\| = 0$. So Q is CC→CC. Similarly, one can prove that Q is also SC→SC, thus completing the proof of (c). The proof of (d) follows a similar line of reasoning.

(e) First let Q be CC→CC and SC→CC. When Y is a subset of $\neg X_i$, we have $CC(X_i, Y)$. From this we have $Q(X_1, \ldots X_i, \ldots X_n) \Rightarrow \neg Q(X_1, \ldots Y, \ldots X_n)$. When Y is a superset of $\neg X_i$, we have $SC(X_i, Y)$. From this we also have $Q(X_1, \ldots X_i, \ldots X_n) \Rightarrow \neg Q(X_1, \ldots Y, \ldots X_n)$. So by definition (90), Q is perfectly consistent.

Next let Q be perfectly consistent and $CC(X_i, X_i')$. By (83), $X_i' \subseteq \neg X_i$, i.e. $X_i'$ is a subset of $\neg X_i$. So by (90) we must have $Q(X_1, \ldots X_i, \ldots X_n) \Rightarrow \neg Q(X_1, \ldots X_i', \ldots X_n)$. Thus, Q is CC→CC. Similarly, one can prove that Q is also SC→CC. The proof of (f) follows a similar line of reasoning. □

**Theorem 3.28** Let P be a predicate. Then $\{x: \neg P(x)\} = \neg\{x: P(x)\}$.

**Proof:** For any member x of U, we have $\|x \in \{x: \neg P(x)\}\| = \|\neg P(x)\| = \|x \in \neg\{x: P(x)\}\|$. Thus $\{x: \neg P(x)\} = \neg\{x: P(x)\}$. □

**Theorem 3.29** Let P and P' be n-ary predicates and R be one of {CC, SC}, then $R(P_1, P_2) \Rightarrow R(\{x_i: P(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\}, \{x_i: P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\})$ for any $1 \leq i \leq n$ and any particular set of $x_1, \ldots x_{i-1}, x_{i+1}, \ldots x_n$.

**Proof:** Here I only prove the case in which R = CC. The case in which R = SC is similar. Suppose CC(P, P'). By (83), this is equivalent to $P \subseteq \neg P'$. By Theorem 3.8, we have $\{x_i: P(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\} \subseteq \{x_i: \neg P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\}$ for any $1 \leq i \leq n$ and any set of $x_1, \ldots x_{i-1}, x_{i+1}, \ldots x_n$. Now by Theorem 3.28, $\{x_i: \neg P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\} = \neg\{x_i: P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\}$. Thus, by (83) again, we have $CC(\{x_i: P(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\}, \{x_i: P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\})$. ☐

**Theorem 3.30** "¬" is CC→SC and SC→CC and does not possess other OPs.

**Proof:** Suppose CC(X, X'). Then by (85), we have SC(¬X, ¬X'), thus showing that "¬" is CC→SC. I next show that "¬" is not CC→CC by constructing a counterexample. Let X and 0 be a non-trivial member and the zero member of a Boolean algebra, respectively. Then we have CC(X, 0) (because $X \leq \neg 0$ for any X) but not CC(¬X, ¬0) (because $\neg X > 0$ for any non-trivial X). So "¬" cannot be CC→CC. The proofs that "¬" is SC→CC but not SC→SC are similar. ☐

**Theorem 3.31** Let R be a transitive relation, O a set, x an individual and Q a right increasing determiner, then *some*({z: Q(O)({w: R(z, w)})})({y: R(x, y)}) ⇒ Q(O)({y: R(x, y)}).

**Proof:** Let *some*({z: Q(O)({w: R(z, w)})})({y: R(x, y)}) be true, then there exists a z such that Q(O)({w: R(z, w)}) and R(x, z) are both true. By transitivity of R, we have for every w, $R(x, z) \wedge R(z, w) \Rightarrow R(x, w)$. This means, on condition that R(x, z), $\{w: R(z, w)\} \subseteq \{w: R(x, w)\}$, and so we have Q(O)({w: R(z, w)}) ⇒ Q(O)({w: R(x, w)}) by the right increasing monotonicity of Q. The above argument shows that *some*($\{z: Q(O)(\{w: R(z, w)\})\})(\{y: R(x, y)\}) \Rightarrow Q(O)(\{w: R(x, w)\})$. Since "w" in the conclusion is just a dummy variable, we can replace it by "y", thus completing the proof. ☐

**Theorem 3.32** A left conservative determiner Q is left increasing iff it satisfies the following syllogistic schema:

(156) $Q(S)(P) \wedge Q^{dl}(S')(P) \Rightarrow$ *some*$(S \cap S')(P)$

**Proof:** First let Q be left increasing and $\|Q(S)(P)\| = \|Q^{dl}(S')(P)\| = 1$. I will prove that $\|$*some*$(S \cap S')(P)\| = 1$ by contradiction. So let the conclusion be false, i.e. $S \cap S' \cap P = \emptyset$. Then we have $S \cap P \subseteq \neg S'$. By the left conservativity of Q and $\|Q(S)(P)\| = 1$, we can deduce $\|Q(S \cap P)(P)\| = 1$. Then since Q is left increasing, we have $\|Q(\neg S')(P)\| = 1$, i.e. $\|\neg(Q(\neg S')(P))\| = 0$. By the definition of left dual, this is equivalent to $\|Q^{dl}(S')(P)\| = 0$, which contradicts the assumption that $\|Q^{dl}(S')(P)\| = 1$. This contradiction shows that Q must satisfy (156).

Next suppose Q satisfies (156), $S \subseteq S'$ and $\|Q(S)(P)\| = 1$. I will show that Q is left increasing by proving $\|Q(S')(P)\| = 1$ by contradiction. So let the conclusion be false, i.e. $\|Q(S')(P)\| = 0$, which can be rewritten as $\|\neg(Q(\neg\neg S')(P))\| = 1$. By the definition of left dual, this is equivalent to $\|Q^{dl}(\neg S')(P)\| = 1$. Since Q satisfies (156), we then have $\|$*some*$(S \cap \neg S')(P)\| = 1$, i.e. $S \cap \neg S' \cap P \neq \emptyset$. From this we can deduce that $S \cap \neg S' \neq \emptyset$, but this contradicts the assumption that $S \subseteq S'$. This contradiction shows that Q must be left increasing. ☐

## References


Altman, A., Keenan, E.L. and Winter, Y. (2001) "Monotonicity and Relative Scope Entailments", in van Rooij, R. and Stokhof, M. (eds.) *Proceedings of the 13th Amsterdam Colloquium*, pp. 25 – 30.

Altman, A., Peterzil, Y. and Winter, Y. (2005) "Scope Dominance with Upward Monotone Quantifiers", *Journal of Logic, Language and Information*, 14.4, pp. 445 – 455.

Altman, A. and Winter, Y. (2005) "Computing Dominant Readings with Upward Monotone Quantifiers", *Research on Language and Computation*, 3, pp. 1 – 43.

Barwise, J. and Cooper, R. (1981) "Generalized Quantifiers and Natural Language", *Linguistics and Philosophy*, 4, pp. 159 – 219.

Beck, S. and Rullmann, H. (1999) "A Flexible Approach to Exhaustivity in Questions", *Natural Language Semantics* 7, pp. 249 – 298.

Beghelli, F. (1994) "Structured Quantifiers", in Kanazawa, M. and Pinon, C. (eds.) *Dynamics, Polarity and Quantification*, Stanford: CSLI Publications, pp. 119 – 143.

Beghelli, F. (1995) *The Phrase Structure of Quantifier Scope*, PhD thesis, University of California.

Ben-Avi, G. and Winter, Y. (2004) "Scope Dominance with Monotone Quantifiers over Finite Domains", *Journal of Logic, Language, and Information*, 13.4, pp. 385 – 402.

van Benthem, J. (1984) "Questions about Quantifiers", *The Journal of Symbolic Logic*, Vol. 49, No. 2, pp. 443 – 466.

van Benthem, J. (1986) *Essays in Logical Semantics*, Dordrecht: Reidel Publishing Company.

van Benthem, J. (1995) “Quantifiers and Inference”, in Krynicki, M. et al (eds.) *Quantifiers: Logic, Models and Computation*, Vol. II, pp. 1 – 20.

van Benthem, J. (2008) *A Brief History of Natural Logic*, Technical Report PP-2008-05, Institute for Logic, Language and Computation, University of Amsterdam.

van Benthem, J. and Westerståhl, D. (1995) “Directions in Generalized Quantifier Theory”, *Studia Logica*, 55, pp. 389 – 419.

Bernardi, R. (2002) *Reasoning with Polarity in Categorial Type Logic*, PhD thesis, University of Utrecht.

Bernardi, R. and Moot R. (2003) “Generalized Quantifiers in Declarative and Interrogative Sentences”, *Logic Journal of the IGPL*, Vol. 11, No. 4, pp. 419 – 434.

Bird, O. (1964) *Syllogistic and its Extensions*, Englewood Cliffs: Prentice-Hall, Inc.

Brown, M. (1984) “Generalized Quantifiers and the Square of Opposition”, *Notre Dame Journal of Formal Logic*, Vol. 25, No. 4, pp. 303 – 322.

Cann, R., Kempson, R. and Marten, L. (2005) *The Dynamics of Language: An Introduction*, Amsterdam: Elsevier.

Carpenter, B. (1997) *Type-Logical Semantics*, Cambridge: MIT Press.

Carston, R.A. (2004) “Relevance Theory and the Saying/Implicating Distinction”, in Horn, L. and Ward, G. (eds.) *The Handbook of Pragmatics*, Oxford: Blackwells, pp.633 – 656.

Carston, R. A. (2012) “Implicature and Explicature”, in Schmid, H.-J. and Geeraerts, D. (eds.) *Cognitive Pragmatics, Handbooks of Pragmatics, Vol. 4*, Berlin: Mouton de Gruyter.

Cavaliere, F. (2008) *The Numerical Square: a Bridge between Classical and Fuzzy Logics*, manuscript.

Chen, X.H. (1994) "Zhuguan Liang Wenti Chutan – Jian Tan Fuci 'Jiu', 'Cai', 'Dou' " ["A Preliminary Study on the Issue of Subjective Quantity – On the Adverbs 'Jiu', 'Cai', 'Dou' "], *Chinese Teaching in the World*, 1994:4.

Chierchia, G. (2004) "Scalar Implicatures, Polarity Phenomena, and the Syntax / Pragmatics Interface", in Belletti, A. (ed.) *Structures and Beyond*, Oxford: Oxford University Press.

Chow, K.F. (2007) *Lun Ziran Yuyan Lianghua Jiegou de Dandiao Tuili Guanxi [On Monotonic Inferential Relationships of Quantified Structures in Natural Language]*, M.A. dissertation, Hong Kong Polytechnic University.

Christodoulopoulos, C. (2008) *Creating a Natural Logic Inference System with Combinatory Categorial Grammar*, MSc thesis, University of Edinburgh.

Cooper, R. (1983) *Quantification and Syntactic Theory*, Dordrecht: D. Reidel Publishing Company.

Copley, B. and Harley, H. (2011) *Force dynamics for event semantics: Reifying causation in event structures*, manuscript.

Crnič, L. (2011) "On the meaning and distribution of concessive scalar particles", in LaCara, N. et al (eds.) *Proceedings of NELS 41*.

Cruse, D.A. (1986) *Lexical Semantics*, Cambridge: Cambridge University Press.

Dalrymple, M. (2001) *Lexical Functional Grammar, Syntax and Semantics, Vol. 34*, San Diego: Academic Press.

Dalrymple, M., Lamping, J., Pereira, F.C.N. and Saraswat, V. (1999) "Quantification, Anaphora, and Intensionality", in Dalrymple, M. (ed.) *Semantics and Syntax in Lexical Functional Grammar*, Cambridge: MIT Press, pp. 39 – 89.

Dowty, D.R. (1994) “The Role of Negative Polarity and Concord Marking in Natural Language Reasoning”, *Semantics and Linguistic Theory*, 4, pp. 114 – 144.

Dubois, D., Godo, L., de Mantaras, R.L., and Prade, H. (1993) “Qualitative Reasoning with Imprecise Probabilities”, *Journal of Intelligent Information Systems*, 2, pp. 319 – 363.

van Eijck, J. (1984) “Generalized Quantifiers and Traditional Logic”, in van Benthem, J. and ter Meulen, A. (eds.) *Generalized Quantifiers and Natural Language*, Foris, Dordrecht, pp. 1 – 19.

van Eijck, J. (2005) “Normal Forms for Characteristic Functions on n-ary Relations”, *Journal of Logic and Computation*, Vol. 15 No. 2, pp. 85 – 98.

van Eijck, J. (2007) “Natural Logic for Natural Language”, in ten Cate, B.D. and Zeevat, H.W. (eds.) *Logic, Language, and Computation; 6th International Tbilisi Symposium on Logic, Language, and Computation*, Berlin: Springer-Verlag, pp. 216 – 230.

Fauconnier, G. (1975) “Pragmatic Scales and Logical Structure”, *Linguistic Inquiry*, Vol. VI, No. 3, pp. 353 – 375.

Fillmore, C.J., Kay, P. and O’Connor, M.C. (1988) “Regularity and Idiomaticity in Grammatical Constructions: The Case of Let Alone”, *Language*, 64, pp. 501 – 538.

Fodor, J. and Sag, I. (1982) “Referential and Quantificational Indefinites”, *Linguistics and Philosophy*, 5, pp. 355 – 398.

Fyodorov, Y., (2002) *Implementing and Extending Natural Logic*, MSc thesis, Israel Institute of Technology.

Gazdar, G. (1979) *Pragmatics: Implicature, Presupposition, and Logical Form*, New York: Academic Press.

Geach, P. (1962) *Reference and Generality: An Examination of Some Medieval and Modern Theories*, New York: Cornell University Press.

Geurts, B. (2010) *Quantity Implicatures*, Cambridge: Cambridge University Press.

Giannakidou, A. (1999) "Affective Dependencies", *Linguistics and Philosophy*, 22, pp. 367 – 421.

Giannakidou, A. (2007) "The Landscape of EVEN", *Natural Language and Linguistic Theory*, 25, pp. 39 – 81.

Glöckner, I. (2006) *Fuzzy Quantifiers: A Computational Theory*, Berlin: Springer-Verlag.

Gong, W.D. (2006) *The Extended Theory of Scalar Implicatures and its Applications*, PhD thesis, Shanghai International Studies University.

Gottschalk, W.H. (1953) "The Theory of Quaternality", *The Journal of Symbolic Logic*, Vol. 18, No. 3, pp. 193 – 196.

Grice, H.P. (1975) "Logic and Conversation", in Cole, P. and Morgan J.L. (eds.) *Speech Acts*, *Syntax and Semantics*, Vol. 3, New York: Academic Press.

Groenendijk, J. and Stokhof, M. (1984) *Studies on the Semantics of Questions and the Pragmatics of Answers*, PhD. thesis, Universiteit van Amsterdam.

Guo, Z.L. (1999) *Xiandai Hanyu Zhuanzhe Ciyu Yanjiu [A Study on Contrastive Words in Modern Chinese]*, Beijing: Beijing Language and Culture University Press.

Hacker, E.A. and Parry, W.T. (1967) "Pure Numerical Boolean Syllogisms", *Notre Dame Journal of Formal Logic*, Vol. VIII, No. 4, pp. 321 – 324.

Hackl, M. (2000) *Comparative Quantifiers*, PhD thesis, Massachusetts Institute of Technology.

Heim, I.R. and Kratzer, A. (1998) *Semantics in Generative Grammar*, Malden:

Blackwell Publishers Inc.

Hirschberg, J.B. (1985) *A Theory of Scalar Implicature*, PhD thesis, University of Pennsylvania.

Hobbs, J.R. (1995) “Monotone Decreasing Quantifiers in a Scope-Free Logical Form”, in van Deemter, K. and Peters, S. (eds.) *Semantic Ambiguity and Underspecification*, Stanford: CSLI Publications, pp. 55 – 76.

Hodges, W. (1998) “The Laws of Distribution for Syllogisms”, *Notre Dame Journal of Formal Logic*, Vol. 39, No. 2, pp. 221 – 230.

Hoeksema, J. (1986) “Monotonicity Phenomena in Natural Language”, *Linguistic Analysis*, 16, pp. 25 – 40.

Horn, L.R. (1969) “A Presuppositional Analysis of Only and Even”, in Binnick, R.I. (ed.) *Papers from the Fifth Regional Meeting of the Chicago Linguistics Society*, Chicago: University of Chicago, pp. 97 – 108.

Horn, L. R. (1984) “Toward a New Taxonomy for Pragmatic Inference: Q-based and R-based Implicature”, in Schiffrin, D. (ed.) *Meaning, Form, and Use in Context: Linguistic Applications (GURT ’84)*, Washington: Georgetown University Press, pp. 11 – 42.

Horn, L.R. (1985) “Metalinguistic Negation and Pragmatic Ambiguity”, *Language*, Vol. 61, No. 1, pp. 121 – 174.

Horn, L.R. (1989) *A natural history of negation*, Chicago: University of Chicago Press.

Huang, W.X. (1994) “Guangyi Zhiyan Panduan he Guangyi Sanduanlun Xintan” [“A Novel Study on Generalized Categorical Statements and Generalized Syllogisms”], in *Traditional Logic and Modern Logic*, Beijing: Kaiming Press, pp. 40 – 49.

Icard, T. (2012) “Inclusion and Exclusion in Natural Language”, *Studia Logica*.

Israel, M. (1996) “Polarity Sensitivity as Lexical Semantics”, *Linguistics and Philosophy*, 19, pp. 619 – 666.

Israel, M. (1997) “The Scalar Model of Polarity Sensitivity: the case of the aspectual operators”, in Forget, D. et al (eds.) *Negation and Polarity: Syntax and Semantics*, Amsterdam: John Benjamins, pp. 209 – 230.

Israel, M. (2011) *The Grammar of Polarity: Pragmatics, Sensitivity and the Logic of Scales*, Cambridge: Cambridge University Press.

Iwata, S. (1998), “Some extensions of the echoic analysis of metalinguistic negation”, *Lingua*, 105, pp. 49 – 65.

Jiang, Yan (1998) “Yuyong Tuili yu ‘Dou’ de Jufa / Yuyi Tezheng” [“Pragmatic Reasoning and Syntactic / Semantic Characterization of ‘Dou’ ”], *Modern Foreign Languages*, 1998:1.

Jiang, Yan (2011) “ ‘Dou’ de Xingshi Yuyongxue” [“The Formal Pragmatics of ‘dou’ ”], in Jiang, Y. (ed.) *Approaching Formal Pragmatics*, Shanghai: Shanghai Educational Publishing House.

Jiang, Yong (2003) “Kuazhang, Xugou yu Tiji Hanyi” [“Counterfactuals, Hyperboles and Scalar Implicatures”], *Research in Foreign Language and Literature*, Vol. 3, No. 4, pp. 20 – 27.

Jiang, Y. and Pan, H.H. (2005) *Xingshi Yuyixue Yinlun [Introduction to Formal Semantics]* (Second edition), Beijing: China Social Sciences Press.

Jones, S. (2002) *Antonymy: A corpus-based perspective*, London: Routledge.

Kadmon, N. and Landman, F. (1993) “Any”, *Linguistics and Philosophy*, 16, pp. 353 – 422.

Kamp, H. and Reyle, U. (1993) *From Discourse to Logic: Introduction to Modeltheoretic Semantics of Natural Language, Formal Logic and Discourse Representation Theory*, Dordrecht: Kluwer Academic Publishers.

Karttunen, L. and Peters, S. (1979) “Conventional Implicature”, in Oh, C.K. and Dinneen, D.A. (eds.) *Presupposition, Syntax and Semantics, Vol. 11*, New York: Academic Press.

Kas, M. and Zwarts, F. (1994) “Intervention Phenomena: Towards an Extended Monotonicity Calculus”, *Studies in the Linguistic Sciences*, Vol. 24, No. 1/2, pp. 279 – 292.

Kay, P. (1990) “Even”, *Linguistics and Philosophy*, 13, pp. 59 – 111.

Kay, P. (1997) “At least”, in Kay, P. (ed.) *Words and the Grammar of Context*, Stanford: CSLI Publications, pp. 99 – 121.

Keenan, E.L. (1987a) “Semantic Case Theory”, in Groenendijk, J. et al (eds.) *Proceedings of the Sixth Amsterdam Colloquium*, ITLI.

Keenan, E.L. (1987b) “A Semantic Definition of ‘Indefinite NP’ ”, in Reuland, E. and ter Meulen, A. (eds.) *The Representation of (In)definiteness*, Cambridge: MIT Press, pp. 286 – 317.

Keenan, E.L. (1993) “Natural Language, Sortal Reducibility and Generalized Quantifiers”, *The Journal of Symbolic Logic*, Vol. 58, No. 1, pp. 314 – 325.

Keenan, E.L. (2003) “Excursions in Natural Logic”, in Casadio, C. et al (eds.) *Language and Grammar: Studies in Mathematical Linguistics and Natural Language*, Stanford: CSLI, pp. 31 – 52.

Keenan, E.L. (2008) “Further Excursions in Natural Logic: The Mid-Point Theorems”, in Hamm, F. and Kepser, S. (eds.) *Logics for Linguistic Structures*, Berlin: Mouton de Gruyter, pp. 87 – 104.

Keenan, E.L. and Faltz, L.M. (1985) *Boolean Semantics for Natural Language*, Dordrecht: Reidel Publishing Company.

Keenan, E.L. and Moss, L.S. (1984) “Generalized Quantifiers and the Expressive Power of Natural Language”, in van Benthem, J. and ter Meulen, A. (eds.)

*Generalized Quantifiers in Natural Language*, Dordrecht: Foris, pp. 73 – 124.

Keenan, E.L. and Stavi, J. (1986) “A Semantic Characterization of Natural Language Determiners”, *Linguistics and Philosophy*, 9, pp. 253 – 326.

Keenan, E.L. and Westerståhl, D. (2011) “Generalized Quantifiers in Linguistics and Logic”, in van Benthem, J. and ter Meulen, A. (eds.) *Handbook of Logic and Language* (Second edition), Amsterdam: Elsevier Science, pp. 859 – 910.

Keene, G.B. (1969) *The Relational Syllogism: A Systematic Approach to Relational Logic*, Exeter: University of Exeter Press.

König, E. (1991) *The Meaning of Focus Particles – A Comparative Perspective*, London: Routledge.

van Kuppevelt, J. (1996) “Inferring From Topics: Scalar Implicatures as Topic-Dependent Inferences”, *Linguistics and Philosophy*, 19, pp. 393 – 443.

Landman, F. (1998) “Plurals and Maximalization”, in Rothstein, S. (ed.) *Events and Grammar*, Dordrecht: Kluwer, pp. 237 – 271.

Langacker, R.W. (1991) *Foundations of Cognitive Grammar – Volume II: Descriptive Application*, Stanford: Stanford University Press.

Levinson, S.C. (2000) *Presumptive Meanings: The Theory of Generalized Conversational Implicature*, Massachusetts: MIT Press.

Li, S.X. (2003) *Hanyu “Zuguanliang” de Biaoda Yanjiu [On the Expression of Subjective Quantity in Chinese]*, PhD thesis, Graduate School of the Chinese Academy of Social Sciences.

Li, Y.M. (2000) *Hanyu Liang Fanchou Yanjiu [Studies on the Quantity Category in Chinese Language]*, Wuhan: Huazhong Normal University Press.

Lindström, P. (1966) “First Order Predicate Logic with Generalized Quantifiers”, *Theoria*, 32, pp. 186 – 195.

Liu, F.H. (2000) “The scalar particle *hai* in Chinese”, *Cahiers de linguistique – Asie orientale*, Vol. 29, No. 1, pp. 41 – 84.

Liu, Y.H., Pan, W.Y. and Gu W. (2001) *Shiyong Xiandai Hanyu Yufa [A Practical Grammar of Modern Chinese]* (Revised edition), Beijing: The Commercial Press.

Löbner, S. (1987) “Quantification as a Major Module of Natural Language Semantics”, in Groenendijk, J. et al (eds.), *Studies in Discourse Representation Theory and the Theory of Generalized Quantifiers*, Berlin: Foris, pp. 53 – 85.

Löbner, S. (2011) “Dual oppositions in lexical meaning”, in Maienborn et al (eds.), *Semantics: An International Handbook of Natural Language Meaning*, Vol. 1, Berlin: de Gruyter, pp. 479 – 506.

Łukasiewicz, J. (1951) *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, Oxford: Clarendon Press.

MacCartney, B. (2009) *Natural Language Inference*, PhD thesis, Stanford University.

Matthewson, L. (2001) “Quantification and the Nature of Crosslinguistic Variation”, *Natural Language Semantics*, 9, pp. 145 – 189.

Matsumoto, Y. (1995) “The Conversational Condition on Horn Scales”, *Linguistics and Philosophy*, 18, pp. 21 – 60.

May, R. (1985) *Logical Form*, Cambridge: MIT Press.

de Mey, S. (1990) *Determiner Logic or the Grammar of the NP*, PhD thesis, University of Groningen.

Mineshima, K., Okada, M. and Takemura, R. (2012), “A Generalized Syllogistic Inference System based on Inclusion and Exclusion Relations”, *Studia*

*Logica*.

Montague, R. (1973) "The Proper Treatment of Quantification in Ordinary English", in Hintikka, K.J.J. et al (eds.) *Approaches to Natural Language: Proceedings of the 1970 Stanford Workshop on Grammar and Semantics*, Dordrecht: Reidel Publishing Company.

Moss, L.S. (2008) "Completeness Theorems for Syllogistic Fragments", in Hamm, F. and Kepser, S. (eds.) *Logics for Linguistic Structures*, Berlin: Mouton de Gruyter, pp. 143 – 174.

Moss, L.S. (2010a) "Intersecting Adjectives in Syllogistic Logic", in Ebert, C. et al (eds.) *The Mathematics of Language*, Berlin: Springer, pp. 223 – 237.

Moss, L.S. (2010b) "Syllogistic Logics with Verbs", *Journal of Logic and Computation*, Vol. 20, Issue 4, pp. 947 – 967.

Moss, L.S. (2011a) "Syllogistic Logic with Comparative Adjectives", *Journal of Logic, Language and Information*, Vol. 20, No. 3, pp. 397 – 417.

Moss, L.S. (2011b) "Syllogistic Logic with Complements", in van Benthem, J. et al (eds.) *Games, Norms and Reasons – Logic at the Crossroads*, Berlin: Springer, pp. 179 – 197.

Moss, L.S. (2012) "The Soundness of Internalized Polarity Marking", *Studia Logica*.

Mostowski, A. (1957) "On a generalization of quantifiers", *Fundamenta Mathematicae*, Vol. 44, pp. 12 – 36.

Murphree, W.A. (1991) *Numerically Exceptive Logic: A Reduction of the Classical Syllogism*, New York: Peter Lang Publishing Inc.

Murphree, W.A. (1994) "The Irrelevance of Distribution for the Syllogism", *Notre Dame Journal of Formal Logic*, Vol. 35, No. 3, pp. 433 – 449.

Murphree, W.A. (1997) "The Numerical Syllogism and Existential

Presupposition", *Notre Dame Journal of Formal Logic*, Vol. 38, No. 1, pp. 49 – 64.

Nakanishi, K. and Rullmann, H. (2009) *Epistemic and Concessive Interpretations of At Least*, slides.

Nicolas, D. (2010) "Towards a semantics for mass expressions derived from gradable expressions", *Recherches Linguistiques de Vincennes*, 39.

Nishihara, N. and Morita, K. (1989) "An Extended Syllogistic System with Conjunctive and Complementary Terms, and Its Completeness Proof", *Systems and Computers in Japan*, Vol. 20, No. 5, pp. 80 – 95.

Pagnan, R. (2012) "A Diagrammatic Calculus of Syllogisms", *Journal of Logic, Language and Information*.

Peters, S. and Westerståhl, D. (2006) *Quantifiers in Language and Logic*, Oxford: Clarendon Press.

Peterson, P.L. (2000) *Intermediate Quantifiers – Logic, linguistics, and Aristotelian semantics*, Aldershot: Ashgate Publishing Limited.

Piaget, J. (1949) *Traité de Logique: Essai de Logistique Opératoire*, Paris: Armand Colin.

Pratt-Hartmann, I. (2008) "On the Computational Complexity of the Numerically Definite Syllogistic and Related Logics", *Bulletin of Symbolic Logic*, 14(1), pp. 1 – 28.

Pratt-Hartmann, I. and Moss, L.S. (2009) "Logics for the Relational Syllogism", *Review of Symbolic Logic*, 2(4), pp. 647 – 683.

Recanati, F. (2010) *Truth-Conditional Pragmatics*, Oxford: Oxford University Press.

Reichenbach, H. (1952) "The Syllogism Revised", *Philosophy of Science*, Vol. 19, No. 1, pp. 1 – 16.

Reinhart, T. (1997) "Quantifier Scope – How labor is divided between QR and choice functions", *Linguistics and Philosophy*, 20, pp. 335 – 397.

Richman, F. (2004) "Equivalence of Syllogisms", *Notre Dame Journal of Formal Logic*, 45(4), pp. 215 – 233.

Robaldo, L. (2011) "Distributivity, Collectivity, and Cumulativity in terms of (In)dependence and Maximality", *Journal of Logic, Language, and Information*, Vol. 20, No. 2, pp. 233 – 271.

Roberts, C. (1996) "Information Structure in Discourse: Towards an Integrated Formal Theory of Pragmatics", in Yoon, J.H. and Kathol, A. (eds.) *Ohio State University Working Papers in Linguistics*, Vol. 49.

van Rooij, R. (2012) "The propositional and relational syllogistic", *Logique et Analyse*, Vol. 55, No. 217.

van Rooij, R. and Schulz, K. (2004) "Exhaustive interpretation of complex sentences", *Journal of Logic, Language and Information*, 13, pp. 491 – 519.

Rooth, M. (1985) *Association with Focus*, PhD thesis, University of Massachusetts Amherst.

Rullmann, H. (1997) "Even, Polarity and Scope", in Gibson, M. et al (eds.) *Papers in Experimental and Theoretical Linguistics*, Vol. 4, Edmonton: University of Alberta, pp. 40 – 64.

Russell, B. (2012) *Probabilistic reasoning and the computation of scalar implicatures*, PhD thesis, Brown University.

Sanchez Valencia, V. (1991) *Studies on Natural Logic and Categorial Grammar*, PhD thesis, University of Amsterdam.

Sauerland, U. (2004) "Scalar Implicatures in Complex Sentences", *Linguistics and Philosophy*, 27, pp. 367 – 391.

Sawada, O. (2003) "The Two-dimensional Scale of the Concessive Conditional

Construction: The Case of English Even if Construction”, in *Proceedings of the 8th Conference of Pan-Pacific Association of Applied Linguistics*.

Sawada, O. (2006) “The Japanese Contrastive Wa: A Mirror Image of EVEN”, in *Proceedings of the 33rd Annual Meeting of the Berkeley Linguistics Society*.

Sevi, A. (2009) *Exhaustivity: A Semantic Account of ‘Quantity’ Implicatures*, Saarbrücken: VDM Verlag Dr. Müller.

Shao, J.M. (1996) *Xiandai Hanyu Yiwenju Yanjiu [Studies on Interrogative Sentences in Modern Chinese]*, Shanghai: East China Normal University Press.

Shen, J.X. (2001) “Gen Fuci ‘Hai’ Youguan de Liangge Jushi” [“Two constructions with ‘Hai’ in Mandarin Chinese”], *Studies of the Chinese Language*, 2001:6.

Shi, D.X. (2006) “Dongci hou Sulian Duanyu de Jufa Diwei” [“On the Syntactic Status of the Post-Verbal Numeral Phrases”], *Chinese Linguistics*, Vol. 13, pp. 51 – 58.

Shyu, S.I. (2004) “(A)symmetries between Mandarin Chinese Lian…dou and Shenzhi”, *Journal of Chinese Linguistics*, 32: 1, pp.71 – 128.

Smessaert, H. (1996) “Monotonicity Properties of Comparative Determiners”, *Linguistics and Philosophy*, 19, pp. 295 – 336.

Smessaert, H. (2012), “The Classical Aristotelian Hexagon Versus the Modern Duality Hexagon”, *Logica Universalis*, 6, pp. 171 – 199.

Sommers, F. and Englebretsen, G. (2000) *An Invitation to Formal Reasoning – The logic of terms*, Aldershot: Ashgate Publishing Ltd.

Sperber, D. and Wilson, D. (1986) *Relevance: Communication and Cognition* (Second edition), Oxford: Blackwell.

Szabolcsi, A. (2004) “Positive Polarity – Negative Polarity”, *Natural Language* *and Linguistic Theory*, 22, pp. 409 – 452.

Szabolcsi, A. (2010) *Quantification*, Cambridge: Cambridge University Press.

Talmy, L. (2000) *Towards a Cognitive Semantics*, Cambridge: MIT Press.

Thom, P. (1977) “Termini Obliqui and the Logic of Relations”, *Archive für Geschichte der Philosophie*, Vol. 59, pp. 143 – 155.

Väänänen, J. (1999) “Generalized Quantifiers, an Introduction”, in Väänänen, J. (ed.) *Generalized Quantifiers and Computation*, Berlin: Springer Verlag.

Westerståhl, D. (1984a) “Determiners and Context Sets”, in van Benthem, J. and ter Meulen, A. (eds.) *Generalized Quantifiers in Natural Language*, Foris: Dordrecht, pp. 45 – 71.

Westerståhl, D. (1984b) “Some Results on Quantifiers”, *Notre Dame Journal of Formal Logic*, Vol. 25, No. 2, pp. 152 – 170.

Westerståhl, D. (1986) “On the order between quantifiers”, in Furberg, M. et al (eds.) *Logic and Abstraction: Essays Dedicated to Per Lindström on his Fiftieth Birthday*, Acta Universitatis Gothoburgensis, 1, Göteborg University, pp. 273 – 285.

Westerståhl, D. (1989) “Quantifiers in Formal and Natural Language”, in Gabbay, D. and Guenthner, F. (eds.) *Handbook of Philosophical Logic*, Vol. IV, Dordrecht: Reidel Publishing Company, pp. 1 – 131.

Westerståhl, D. (1996) “Self-Commuting Quantifiers”, *The Journal of Symbolic Logic*, Vol. 61, No. 1, pp. 212 – 224.

Wilkinson, K. (1996) “The scope of even”, *Natural Language Semantics*, 4, pp. 193 – 215.

Winter, Y. (2001) *Flexibility Principles in Boolean Semantics: The interpretation of coordination, plurality and scope in natural language*, Cambridge: MIT Press.

van der Wouden, T. (1997) *Negative contexts: collocation, polarity and multiple negation*, London: Routledge.

Xing, F.Y. (2001) *Hanyu Fuju Yanjiu [Studies on Chinese Complex Sentences]*, Beijing: The Commercial Press.

Yuan, Y.L. (2008) "Fanyuqi, Dijin Guanxi he Yuyong Chidu de Leixing – 'Shenzhi' he 'Faner' de Yuyi Gongneng Bijiao" ["Counter-expectation, additive relation and the types of pragmatic scales: The comparative analyses of the semantic function of 'shenzhi' and 'faner' "], *Contemporary Linguistics*, 2008:2, pp. 109 – 121.

Zadeh, L.A. (1983) "A Computational Approach to Fuzzy Quantifiers in Natural Languages", *Computers and Mathematics with Applications*, Vol. 9, No. 1, pp. 149 – 184.

Zamansky, A. (2004) *A 'Natural Logic' inference system based on the Lambek calculus*, MSc Thesis, Israel Institute of Technology.

Zeevat, H.W. (2009) " 'Only' as a Mirative Particle", in Riester, A. and Onea, E. (eds.) *Focus at the Syntax-Semantics Interface, Working Papers of the SFB 732*, Vol. 3, pp. 121 – 141.

Zhang, B.S. (2003) "Fuci 'hai' de Zhuguanxing" ["Subjectivity of Adverb 'hai' "], *Linguistic Sciences*, Vol. 2, No. 5, pp. 71 – 76.

Zhang, R.X. (2008) *Xingshi Yuyongxue Daolun [Introduction to Formal Pragmatics]*, Shanghai: Fudan Press.

Zhou, J. (2007) *Xiandai Hanyu Dijin Fanchou Yanjiu [A Study on the Climax Category in Modern Chinese]*, Beijing: Communication University of China Press.

Zimmermann, T.E. (1993) "Scopeless quantifiers and operators", *Journal of Philosophical Logic*, Vol. 22, pp. 545 – 561.

Zondervan, A. (2006) *The Question Under Discussion Focus Condition for Scalar Implicatures*, M.A. thesis, University of Utrecht.

Zou, C.L. (2002) *Luoji, Yuyan he Xinxi – Luoji Yufa Yanjiu [Logic, Language and Information – Studies on Logical Grammar]*, Beijing: People's Publishing House.

Zuber, R. (2004) "A class of non-conservative determiners in Polish", *Linguisticae Investigationes*, 27:1, pp. 147 – 165.

Zuber, R. (2005) "More Algebras for Determiners", in Blache, P. and Stabler, E. (eds.) *Logical Aspects of Computational Linguistics, Lecture Notes in Artificial Intelligence, Vol. 3492,* Springer-Verlag, pp. 363 – 378.

Zuber, R. (2007) "Symmetric and contrapositional quantifiers", *Journal of Logic, Language and Information*, Vol. 16, No. 1, pp. 1 – 13.

Zuber, R. (2010a) "A Note on the Monotonicity of Reducible Quantifiers", *Journal of Logic, Language and Information*, Vol. 19, No. 1, pp. 123 – 128.

Zuber, R. (2010b) "Semantic Constraints on Anaphoric Determiners", *Research on Language and Computation*, 8, pp. 255 – 271.

Zuber, R. (2011) "Some Generalised Comparative Determiners", in Pogodalla, S. and Prost, J.- P. (eds.) *Logical Aspects of Computational Linguistics*, Berlin: Springer, pp. 267 – 281.

Zwarts, F. (1983) "Determiners: A Relational Perspective", in ter Meulen, A. (eds.) *Studies in Model-Theoretic Semantics*, Dordrecht: Foris, pp. 37 – 62.

Zwarts, F. (1996) "Facets of Negation", in van der Does, J. and van Eijck, J. (eds.) *Quantifiers, logic, and language*, Stanford: CSLI Publications, pp. 385 – 422.

Zwarts, F. (1997) "Three Types of Polarity", in Hamm, F. and Hinrichs, E. (eds.) *Plurality and Quantification*, Dordrecht: Kluwer Academic Publishers, pp. 177 – 237.