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VEHICULAR LOAD ON BRIDGE DECK

By

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A Dissertation for the Degree of
DOCTOR OF PHILOSOPHY

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August 2001
ABSTRACT

In this thesis, two methods based on regularization technique are developed to identify the time-varying loads from vehicles moving on top of the bridge deck. The bridge deck is modeled either as a multi-span continuous beam when the bridge is narrow compared with its length, or as an orthotropic rectangular plate if otherwise. Computational simulations and laboratory tests are used to verify the feasibility and accuracy of these two methods. Two experimental setups are developed in the laboratory. One is for the beam model, and the other is for the orthotropic plate model. In the completion of the main objective to develop a moving force identification technique with a bridge-vehicle system, the following works have been performed:

Firstly, the dynamic behavior of the continuous bridge deck under moving vehicles is analyzed. The influence of different parameters such as the road surface roughness of the bridge and the surface condition of the approach, multiple vehicles and their transverse positions, braking or acceleration on the bridge are studied using computational simulations and laboratory tests.

Secondly, regularization on the ill-conditioned problem of indirect force identification is introduced to provide bounds to the identified forces. The Frequency and Time Domain Method (FTDM) (Law et al 1999) is selected in the study on the improvements due to regularization in both simulation and laboratory test results. The laboratory results from Time Domain Method(TDM) (Law et al, 1997) is also presented to compare the accuracy and the effectiveness of regularization in these two methods.

Thirdly, two new methods based on regularization are proposed to overcome the deficiencies exhibited in existing methods. A new time domain method is developed to identify moving loads on a continuous beam from the measured structural vibration responses. This method gives exact solutions to the forces with improved formulation over existing methods for a more efficient computation. Another general method based on the finite element formulation is also developed to identify moving loads on a continuous beam. A generalized orthogonal function approach is proposed to obtain the derivatives of the bridge modal responses. The moving loads are identified using least squares method with regularization on the
equation of motion in the time domain. This method is extended to identify the moving loads on non-uniform multi-span continuous Timoshenko beam. The comparative study between the results from using the Timoshenko beam theory and the Euler-Bernoulli beam theory is also included. Numerical examples on both single and multiple span bridges and the case of axle interaction forces from a four-DOFs vehicle on a triple-span bridge are used to demonstrate the feasibility and accuracy of these two methods, and factors affecting the errors in the identification are discussed. An experimental setup for the beam model is designed in the laboratory. The moving forces are identified from the measured strains using these two methods. The effect of non-uniform speed on the identified results when the forces are identified using a constant speed is also investigated.

Fourthly, the bridge deck is simplified as a rectangular orthotropic plate and the two proposed methods are extended to identify the moving loads on the three-dimensional bridge deck. Computational simulations show the effectiveness and the validity of the proposed methods in identifying loads travelling along the central line or at an eccentric path on the bridge deck. An experimental setup for the bridge deck model is designed in the laboratory. The strains of the bridge deck are measured when the model car moves across the bridge deck along different paths and at different speeds. The moving loads on the bridge deck were identified from the measured strains using the two methods, and the reconstructed responses are calculated from the identified loads to verify the performances of these two methods.
ACKNOWLEDGMENTS

I would like to express my sincere appreciation and gratitude to my chief supervisor, Dr. S.S. Law, for his valuable guidance, high responsibility and sincere personal encouragement during my research study and thesis writing. His conscientious and meticulous work attitude benefits me in exhaustibly in my research, no matter in the past or in the future. I am also grateful to my co-supervisor, Dr. T.H.T. Chan, for his supportive advice throughout my learning and research.

I gratefully acknowledge the financial support from the Hong Kong Polytechnic University in the past three years.

My thanks extend to the technicians in the department for their assistance in the laboratory work. I want to express my thanks to Mr. M.T. Ho for his assistance in the computing works.

Most of all, I want to sincerely thank my wife, Yan Wang, and my son, Hai Ming Zhu, for their support and encouragement throughout these three years. Special thanks are also due to my parents for their continuous encouragement during the course of my study.
DECLARATION

I hereby declare that the dissertation entitled "Vehicular load on bridge deck" is original and all contributions from others are duly acknowledged in the text by reference. The work has not been submitted for other degrees, either in whole or in part, previously.

Signed

Zhu Xinqun
LIST OF PUBLICATIONS IN JOURNALS DERIVED FROM THIS PROJECT


LIST OF PUBLICATIONS IN CONFERENCES DERIVED FROM THIS PROJECT


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NOTATIONS

A. $A(x)$ area of cross-section;

$A_y, B_y, C_y, D_y$ mode parameters;

$B, D$ coefficient matrix;

$C_d$ damping coefficient of the beam or plate;

$D_x, D_y$ flexural rigidities in the x- and y- directions respectively;

$D_{xy}, D_{y}$ torsional rigidity and twisting rigidity of the orthotropic plate respectively;

$E$ norm of residuals;

$E_x, E_y$ Young's moduli of orthotropic material in the x- and y-directions respectively;

$EI$ semi-norm of estimated forces;

$EI(x)$ flexural stiffness of beam;

$F_b, F_v$ generalized forces of the bridge and the vehicle system;

$F_d$ the driving force of vehicle;

$F_{d_{max}}$ amplitude of the braking force;

$G$ shear modulus;

$G_{xy}$ shear modulus of orthotropic plate;

$H_n(\omega)$ frequency response function of the $n$th mode;

$I, J$ flexural and torsional moments of inertia of I-beam;

$I_d, I_m$ impact factor from deflection and moment, respectively.

$L$ the length of the beam;

$M$ Fourier transform of bending moment vector $m$;

$M, K, C$ mass, stiffness, and damping matrices respectively of the orthotropic plate or beam;

$MM, NN$ number of vibration modes along x-direction and y-direction respectively;

$M_b, K_b, C_b$ mass, stiffness, and damping matrices of the bridge respectively;

$M_v, C_v, K_v$ mass, damping and stiffness matrices of vehicle model respectively;

$M_n$ modal mass of $n$th mode;
$N_g$ number of group of moving loads;
$N_n$ number of modes;
$N_p$ number of moving loads;
$N_{p_l}$ number of moving loads for the $l$th group of moving load;
$N_r$ number of measuring points;
$N_i$ number of data points;
$P_i(t)$ $i$th moving load;
$P_{i,j}(t)$ $j$th moving load of the $i$th group of moving load;
$P$ load vector;
$Q$ vector of modal coordinates;
$R, S$ error-weighting matrix and smoothing matrix respectively;
$R$ number of supports;
$S_1, S_2$ vehicle dimensions;
$S_d(f)$ displacement PSD of the bridge surface roughness;
$T, U$ kinetic and potential energy respectively of the bridge-vehicle system;
$U_e$ flexural energy of beam;
$U_Q$ potential energy due to point constraints;
$W$ work done by the moving loads;
$W_c$ work done due to damping in the plate;
$W_q(x, y)$ vibration mode shape of the orthotropic plate;
$W_i(x)$ mode shape function of the $i$th mode;
$W'_i(x), W''_i(x)$ first and second derivatives of $W_i(x)$;
$W_{i, j}(x)$ vibration mode of the uniform beam;
$W$ unitary matrix;
$W^*$ conjugate of unitary matrix;
$Y$ vertical displacement vector of the vehicle;
$a, b, h$ length, width and thickness of the orthotropic plate;
$a_1, a_2$ position parameters;
$b_i$ distance between two $i$-beams;
$d(x)$ the road surface roughness function;
\( d'(x) \) first derivative of \( d(x) \);
\( e \) eccentricity of moving load;
\( f \) the spatial frequency (cycles/m);
\( f_0 \) the reference spatial frequency (cycles/m);
\( h_e \) equivalent thickness of plate;
\( h_i(t) \) impulse response;
\( k_u, c_u \) stiffness and damping of the suspension system;
\( k_u, c_u \) stiffness and damping of the tires;
\( k \) stiffness of point constraint;
\( l \) span length of beam;
\( l_s \) axle spacing;
\( m \) bending moment vector;
\( m \) bending moment;
\( m_s, l_s \) mass and inertia moment of the vehicle;
\( m_i \) mass of each part of the vehicle;
\( q_i(t) \) \( i \)th generalized coordinate;
\( q_o(t) \) modal coordinate;
\( v \) speed of moving loads;
\( v_i \) moving speed of the \( l \)th group of moving load;
\( v(t) \) speed of moving load;
\( w(x, t) \) lateral deflection of the beam at time \( t \) and position \( x \);
\( \ddot{w}(x, t) \) acceleration at position \( x \) and time \( t \);
\( w(x, y, t) \) displacement of the orthotropic plate;
\( \ddot{w}(x, y, t) \) acceleration responses of the orthotropic plate;
\( \hat{x}_i(t), \hat{y}_i(t) \) location of the \( i \)th moving load;
\( \hat{x}_{i_1, i_2}(t), \hat{y}_{i_1, i_2}(t) \) location of the moving load \( P_{i_1 i_2}(t) \);
\( \hat{x}_{i_1, i_2, 0}(t), \hat{y}_{i_1, i_2, 0}(t) \) initial location of the moving load \( P_{i_1 i_2}(t) \);
\( \hat{x}_i(t) \) location of the \( i \)th axle load at time \( t \);
\( t_b \) braking rise time;
\( z_i \) distance from the neutral axis to the bottom tension surface;
\( \omega_i \)  
ith modal frequency;

\( \tilde{\omega}_i \)  
measured ith modal frequency;

\( \omega_i \)  
damped ith modal frequency

\( \omega_n \)  
circular frequency in radians per second;

\( \xi_i \)  
damping ratio of nth mode;

\( \psi(x,t) \)  
angle of rotation of cross-section;

\( \phi_i(x) \)  
shape function of the ith mode;

\( \psi_i(x), \phi_i(y) \)  
bending mode in x- and y-direction respectively;

\( \phi''''_i(y), \phi''_i(y) \)  
fourth and second derivatives of \( \phi_i(y) \);

\( \varepsilon, \dot{\varepsilon} \)  
strain and acceleration vector respectively

\( \varepsilon(x,t) \)  
strain in the beam;

\( \varepsilon_x(x,y,t) \)  
strain under the orthotropic plate in x-direction;

\( \varepsilon_y(x,y,t) \)  
strain under the orthotropic plate in y-direction;

\( \kappa \)  
shear coefficient;

\( \rho \)  
mass density of the orthotropic plate material or mass per unit length of beam;

\( \lambda' \)  
frequency parameter;

\( \lambda \)  
non-negative damping coefficient or regularization parameter;

\( \nu_{vo}, \nu_{ss} \)  
Poisson's ratio;

\( \beta, r_1 \)  
eigenvalue and eigenfunction of the continuous beam;

\( \gamma(x) \)  
radius of gyration of cross-section;

\( \delta \)  
variational symbol;

\( \delta(x), \delta(y), \delta(t) \)  
Dirac function;

\( \Delta \)  
distance interval between successive ordinates of the road surface profile;

\( \| \cdot \| \)  
norm of a vector of matrix.
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Chapter 1

INTRODUCTION

1.1 PROBLEM STATEMENT

1.1.1 Research Motivation

Bridges are civil engineering structures which are of fundamental significance to the development of a country. And the new bridges are longer and more slender than those in the past. The demand of traffic grows very rapidly, and new bridges have to be designed for very heavy vehicles and trains. Vibration limitation requirements are increasingly strict to provide bridge users with adequate safety and comfort. The use of new materials and technologies has also posed new problems in bridge engineering. All these demands bring new challenges to bridge designers and to those researchers who generate new evidences in support of the bridge design codes.

A moving truck induces dynamic response which is greater than the static response because of the interaction between the moving vehicle and the bridge. In order to evaluate the influence of a passing vehicle on a bridge deck, design codes convert the dynamic problem into a pseudo-static one and require that the weight of a specified vehicle be multiplied by a dynamic amplification factor (DAF). For example, the provisions in the Standard Specifications for Highway Bridges (AASHTO, 1996) specify dynamic load effects in terms of an impact factor that is simple in expression. It is empirically derived based on experiences from railway bridges and is solely a function of the bridge span. These AASHTO provisions seem to have served well for many years. However, modern bridge design utilizes lighter materials and longer spans, and some questions have been raised regarding the appropriateness of the AASHTO impact provisions. Research over the last 30 years has shown that the dynamic response of bridges under vehicular loading is influenced by many parameters other than bridge span, such as the characteristics of the vehicle, the velocity of the vehicle, the characteristics of the bridge, bridge surface roughness and conditions of the approach roadways. Many new bridge codes (Ministry, 1992 and AASHTO, 1998) include provisions for a dynamic load allowance (DLA) to account for all vehicular load effects, not just impact. The DLA is presented in the
form of

\[ DLA = \left( \frac{R_{\text{dyn}} - R_{\text{stat}}}{R_{\text{stat}}} \right) \times 100\% \]  \hspace{1cm} (1-1)

where \( R_{\text{dyn}} \) = maximum dynamic response; and \( R_{\text{stat}} \) = maximum static response.

There are other factors affecting greatly the dynamic responses of a modern bridge, but they are not studied too often in the past. They are: vehicle moving at non-uniform speed, i.e. the braking and acceleration of a vehicle, vehicle moving on top of a modern continuous bridge deck, and multiple vehicles and their transverse positions on multi-lane bridge deck.

The dynamic load is an important parameter in bridge design and evaluation. Traditionally, the dynamic load is considered as an equivalent static load. It is better for the bridge design if the dynamic load time history is known.

Notwithstanding the importance of vehicle-induced bridge dynamics, major bridge failures are not normally caused by dynamic wheel loads (ANON, 1992). The wheel loads cause more subtle problems and the contribute to fatigue, surface wear, and cracking of concrete which leads to corrosion. Cebon (1987) concluded that the dynamic wheel loads may increase road surface damage by a factor of 2 to 4 over that due to static wheel loads. For this reason the studies of dynamic wheel loads and ways to measure them had always been of interest. The dynamic loads continually degrade the bridge and increase the necessity of regular maintenance.

Direct measurement of the dynamic loads using instrumented vehicles is expensive and is subjected to bias (Cantineni, 1992; Heywood, 1994), while results from computation simulations are subjected to model errors. Inclusion of all the influencing parameters in the model would make it computationally expensive. Systems have been developed for weigh-in-motion of the vehicles (Peters, 1984; 1986), but they all measure only the static axle loads. All the weigh-in-motion techniques treat the bridge and the vehicle in a two-dimensional problem.

A technique to determine the vehicular loads indirectly from the vibration responses of the bridge deck is required such that the different parameters of the bridge and vehicle system are accounted for in the measured responses, and the cost involved would be much less than that by direct measurement.

Four methods have been developed to identify the moving forces on a single span beam: Interpretive Method (ITM) (O'Connor and Chan, 1988a), Time Domain
Method (TDM) (Law et al, 1997), Frequency and Time Domain Method (FTDM) (Law et al, 1999), State Estimation Optimization Method (SEOM) (Law and Fang, 2001). Comparative studies for the first three methods have been done in the laboratory by Chan et al (2000). The computational time of ITM is not long compared with, but the identification accuracy is much lower. Large errors in the identified results are induced by the direct derivation of the bridge modal responses in ITM. TDM and FTDM should take long computational time and use large computer capacity. The results obtained from the above methods are noise sensitive and they exhibit large fluctuations at beginning and the end of the time histories. But all of them could give fairly acceptable results as the total weight of the vehicle.

1.1.2 Problems to be Addressed in this Thesis

There are two distinct analytical problems to be solved in this research. The direct problem is the analysis of the dynamic behavior of the bridge under moving vehicles. The bridge can be modeled either as a multi-span continuous beam when the bridge is narrow compared with its length, or as an orthotropic rectangular plate if otherwise. An orthotropic plate possesses different mechanical properties in two mutually perpendicular directions. In practice, there are two forms of orthotropy: material orthotropy and shape orthotropy. Most bridge decks are orthotropic, such an isotropic slab, a grillage, a T-beam bridge deck, a multi-beam bridge deck, a multi-cell box-beam bridge deck, and a slab stiffened with ribs of box section. Four type of vehicle models (moving forces, quarter-truck, half-truck and full-truck) can be used in the simulations. The effects of the road surface roughness, the parameters of the bridge or the vehicle, braking and vehicle speed on the dynamic impact factor will be discussed. Several new computational methods will be developed for analyzing the bridge-vehicle interaction taking all the above factors into considerations.

The inverse problem is to develop a moving load identification technique to identify the time-varying load from the vehicle moving on the bridge deck. This is an ill-posed problem. The factors that affect the error in the identification are discussed. However, in practice, the bridge responses can only be measured at a few points. So the identification is based on the incomplete measured information. The selection of the measuring locations on the bridge is another important problem in the identification.
Experimental test facilities will be designed and studies will be conducted in the laboratory to verify the methods developed above for both the beam and the orthotropic plate model.

1.2 RESEARCH OBJECTIVES

The main objective of the research project is to develop a moving force identification technique to identify time-varying moving axle loads from vehicle on a bridge deck. This technique will be in a form such that it can be used to weigh continuously all traffic in a multi-lane highway including more than one vehicle. Provisions will be made to allow for the effect of road surface roughness, ramp or swallow hole at entry of the bridge deck and vehicle braking. Verification on the new method will be performed by simulation and laboratory studies. This research can be achieved with the completion of the following four objectives:

1) to analyze the dynamic behavior of a multi-span continuous bridge deck under moving loads. The bridge is modeled either as a multi-span continuous beam when the bridge is narrow compared with its length, or as an orthotropic rectangular plate if otherwise. The influence of different parameters such as the road-surface roughness of the bridge, the surface condition of the approach, multiple vehicles and their transverse positions, and braking or acceleration on the bridge deck is studied using computational simulations and laboratory tests.

2) to develop a moving force identification technique to identify time-varying moving axle loads on a continuous bridge deck. The bridge deck is simplified as a continuous beam. Computational simulations are used to demonstrate the feasibility and accuracy of the technique, and factors affecting the errors in the identification results are discussed.

3) to develop a moving force identification technique to identify time-varying moving axle loads on a continuous bridge deck. The bridge deck is simplified as a rectangular orthotropic plate. Computational simulations are used to demonstrate the feasibility and accuracy of the technique, and factors affecting the errors in the identification results are also discussed.

4) to design and setup two test facilities in the laboratory and to carry out experimental studies on moving load identification from measuring responses
with the beam model or plate model. The measured experimental results are used to examine the theoretical results obtained by the methods developed in this thesis.

1.3 LAYOUT OF THESIS

The contents of this thesis are divided into twelve chapters, and the layout of the thesis is shown in Figure 1.1.

- Chapter 1 introduces the motivation for this research and gives the objectives to be pursued in this Ph.D project.
- Chapter 2 is on the literature review, in which the models of the bridge-vehicle system are first reviewed. Numerical methods for vehicle-bridge interaction analysis proposed in the last few decades are summarized. The literature on load identification method, especially on moving load identification, developed by various researchers are also reviewed and summarized. The results of the literature review outline the direction and the focus of the thesis to be addressed.
- Chapter 3 studies the effect of vehicle braking on a single span and multi-span continuous bridge deck including the influence of different parameters such as initial vehicle velocity, road surface roughness (ISO 8608, 1995), braking rise time, amplitude of braking force and vehicle braking location. A method based on Hamilton principle is derived to predict the vertical motion of the vehicle-bridge system in this study. A three-axles tractor-trailer vehicle is selected for the study as the design of modern bridge deck is governed by heavy freight loading. Laboratory verification of the proposed method on both a single span beam and a two-span beam shows good matching between the measured and calculated responses.
- Chapter 4 investigates the dynamic loading on a multi-lane continuous bridge due to vehicles moving on top of the bridge deck. The bridge deck is modeled as a multi-span continuous orthotropic rectangular plate with intermediate line rigid supports. The analytical vehicle is simulated as a two-axle three-dimensional vehicle model with 7 degrees-of-freedom according to the H20-44 vehicle design loading in AASHTO(1998). The dynamic behavior of the bridge under several moving vehicles is analyzed using orthotropic plate theory and modal superposition technique. The effects of multi-lane loading from multiple vehicles
on the dynamic impact factor of the bridge are discussed. The impact factor is found varying in an opposite trend as the dynamic responses for the different loading cases under studied.

- Chapter 5 introduces a regularization method in the ill-conditioned problem to provide bounds to the identified forces in the moving force identification using existing methods. The Frequency and Time Domain method (FTDM) (Law et al, 1999) is selected in this study on the improvements due to regularization in both simulation and laboratory test results. Since Zhang (1994) has reported that identification of a stationary force by frequency domain methods would be more practical and effective than by time domain methods, laboratory results from using Time Domain method (TDM) (Law et al, 1997) is also presented to compare the accuracy and the effectiveness of regularization in these two different methods.

- In Chapter 6, two new methods are proposed to overcome the deficiencies exhibited in existing methods. A new time domain method based on regularization technique is developed to identify moving loads on a continuous beam from the measured structural vibration responses. This method gives exact solutions to the forces with improved formulation over existing methods for a more efficient computation. Regularization technique is used to provide bound to the solution. Another general method based on the finite element formulation is also developed to identify moving loads on a continuous beam. A generalized orthogonal function approach is proposed to obtain the derivatives of the bridge modal responses. The moving loads are identified using least squares method with regularization on the equation of motion in the time domain. Numerical examples of time-varying moving forces on both single and multiple span bridges and the case of axle interaction forces from a four-DOFs vehicle on a triple-span bridge are used to demonstrate the feasibility and accuracy of the two methods, and factors affecting the errors in the identification are discussed. Computational simulations show that the methods are effective for identifying moving loads on continuous bridges.

- The design and study on a bridge-vehicle system model in the laboratory are reported in Chapter 7. A uniform beam is used to model the bridge deck. The strain and acceleration responses are simultaneously measured when the model car moves across the bridge at different speeds. Since the use of accelerations requires a large number of measured modes and measuring points for a good accuracy,
only the measured strains are used in the studies. The moving forces are identified from the bridge strains using the two proposed methods. The effect of non-uniform speed on the identified results when the forces are identified using a constant speed is also investigated.

- Chapter 8 uses the finite element method presented in Chapter 6 to identify moving loads on multi-span continuous Timoshenko beam with assumed modes. The comparative study between the Timoshenko beam theory and the Euler-Bernoulli beam theory is reported. Experimental studies in laboratory are carried out to examine the theoretical results.

- Chapter 9 modeled the bridge deck as a simply supported orthotropic rectangular plate. Dynamic behavior of the bridge deck under moving loads is analyzed using the orthotropic plate theory and modal superposition method. The two moving load identification methods developed in Chapter 6 are extended to solve the three-dimensional problem of moving force identification in the time domain. Computational simulations show the effectiveness and the validity of the proposed methods in identifying forces travelling along the central line or at an eccentric path on the bridge deck.

- In Chapter 10, an experimental set-up for the bridge deck model is designed in the laboratory. The strains of the bridge deck are measured when the model car transverses the bridge deck along different paths and at different speeds. The moving loads on the bridge deck are identified from the measured strains using the two methods developed in Chapter 9, and the reconstructed responses are calculated from the identified loads to verify the performances of these two methods.

- Two methods have been developed in Chapters 6 to 10 to identify moving loads on top of a continuous bridge using measured vibration responses. Numerical studies with a single-span bridge deck are presented to illustrate the robustness and accuracy of the two approaches in Chapter 11. Parameters that may influence the accuracy of moving load identification, such as sampling frequency, number of vibration modes and measuring points in the identification are discussed. These two methods are further examined with experimental measurements obtained from a bridge-vehicle system in the laboratory.

- Chapter 12 summarizes the main conclusions obtained in this thesis with
Chapter 2

LITERATURE REVIEW

2.1 MODEL OF THE BRIDGE-VEHICLE SYSTEM

The interaction between the vehicle and the bridge is a complex phenomenon governed by a large number of different parameters. The use of simplified models is more effective to establish a clear connection between the governing parameters and the bridge response than a complex model. Normally, the bridge decks are modeled as beams (Bernoulli-Euler beam or Timoshenko beam) or plates (isotropic plates or orthotropic plates) and the vehicles are modeled as a moving force or a moving mass for simple analysis of the vehicle-bridge interaction. On the other hand, the problem of calculating the dynamic response of a continuous elastic system (string, beam and plate) subjected to a moving concentrated load is also very important in the analysis and design of highway and railway bridge, cable railways, etc. and it has been of interest to engineering designers for many years. A review of these studies is presented below.

2.1.1 A Continuous Beam Under Moving Loads

The bridge is modeled as a beam and the vehicle is modeled as a moving force, a moving mass or a moving oscillator. The response of the idealized beam structure is governed by three characterizing parameters: the speed parameter, the frequency ratio and the mass ratio. Existing studies on these parameters and on the various bridge models are presented.

1) Moving force, moving mass and moving oscillator

Dynamic response of elastic structures traversed by a moving mass or load is a very complicated function of both the mass ratio between the moving mass and the support structure and the speed of the moving load. Any analysis of the moving load problem must address fully the effects of these parameters so as to yield useful information for engineering design. For the case of low moving speed or low mass ratio, the simple moving force model can be a good approximation to the complex moving mass problem. For the case of a high mass ratio and a high moving speed, the
moving force model cannot be applied, and the complicated time-variant system analysis for the moving mass problem must be conducted to ensure an accurate investigation. For the thick beam analysis, both the effects of shear deformation and rotary inertia should be considered for an accurate analysis. The "moving-mass moving-force" problem has been discussed in detail basing on the finite element method (Lin and Trethewey, 1990). There are two extreme cases: I) The ratio of the vehicle weight and the supporting beam weight is large, so the system can be simplified as a single degree-of-freedom system having a concentrated weight moving on a massless beam; and II) The ratio of the vehicle weight and the supporting beam weight is small, so the system can be treated an elastic beam subjected to a moving concentrated force.

Moving oscillator model is more realistic in some engineering application (e.g., a softly sprung vehicle traversing a flexible structure, so that interaction effects become important). Eigen-function expansion method for calculating the dynamic response of a continuous elastic structure carrying a moving linear oscillator was developed by Pesterev and Bergman (1997). The method reduces the problem to the integration of a system of linear ordinary differential equations governing the time-dependent coefficients of the series expansion of the response in terms of the eigenfunctions of the continuous structure. An exact direct numerical procedure for the solution of moving oscillator problem was presented by Yang et al (2000). This method was extended to calculate the response of a spatially one-dimensional, non-conservative linear distributed parameter system to a moving concentrated load using complex eigen-functions of the continuous system by Pesterev and Bergman (1998). The method was also extended to the problem of multiple moving oscillators by Pesterev et al (2001).

2) Multi-span beam

For a continuous bridge, the multi-span beam model is always used in the analysis of the bridge-vehicle interaction. Modal superposition method is commonly employed to analyze the vibration of a multi-span continuous beam under moving loads. Hayashikawa and Watanabe (1981) and Wang (1997) used the eigen-stiffness matrix method; Wu and Dai (1987) studied the response of a multi-span, non-uniform beam subjected to a series of loads moving with varying speed in identical and opposite directions. The transfer matrix method is used to determine the natural
frequencies and mode shapes of a multi-span, non-uniform beam; Lee (1994) used the Euler beam theory and the assumed mode method to analyze the transverse vibration of a beam with intermediate point constraints subjected to a moving load. The vibration modes of a simply-supported beam are used as the assumed modes. The point constraints in the form of supports are assumed to be linear springs of large stiffness. Lin (1995) has mentioned that the selection of support stiffness is problem dependent, and it should be used with care if numerical stability in the solution has to be maintained. Zheng et al (1998) used the modified beam functions as the assumed modes to analyze the vibration of a multi-span non-uniform Euler beam subjected to a moving load. The modified beam functions satisfied the zero deflection conditions at all the intermediate supports as well as the boundary conditions at the two ends of the beam. Henchi et al (1997) used exact dynamic stiffness elements under the framework of finite element approximation to study the dynamic response of multi-span structures under a convoy of moving loads.

3) Timoshenko beam

The response of a beam to a high speed moving load including the shear deformation and rotatory inertia effects has been studied. Based on the Timoshenko beam theory, the dynamic response of a simply supported beam excited by moving loads has been studied by Mackertich (1990). Lee (1996) studied the dynamic response of a Timoshenko beam subjected to a concentrated mass moving with a constant speed using Lagrangian approach and the assumed mode method. The assumed functions are the vibration modes of a beam simply supported at both ends. Wang (1997) investigated the forced vibration of multi-span Timoshenko beams to a moving force using the transfer matrix method together with the modal analysis. The transfer matrices of the responses are formed by adopting the conditions of the rotatory angle continuity and the balance of the moment at the junction between two adjacent spans. The modal frequencies and their corresponding sets of mode shape functions of the multi-span Timoshenko beam are then determined by the transfer matrix method.

2.1.2 A Continuous Plate Under Moving Loads
A beam model cannot truly represent the three-dimensional behavior of the bridge, particularly when a moving vehicle has a path not along the centerline of the bridge deck. Many types of bridge decks, including those of slab bridges, hollow-core slab bridges, and deck and girder bridges can be effectively modeled by an isotropic or orthotropic plate (Bakht and Jaeger, 1985). In modeling the highway bridge as a plate, the mathematical difficulties are compounded by the complicated vehicle-bridge dynamic interaction and the difficulty in representing the behavior of the bridge subjected to traversing multi-vehicles. A review on this study is presented as follow.

1) Isotropic plate model and orthotropic plate model

The isotropic plate model and the orthotropic plate model are normally used to model the bridge deck in the analysis of the bridge-vehicle interaction. The major problems involved are in the assumptions and the implementation procedure.

The vibration analysis of an isotropic plate subjected to a moving load is based on the following assumptions (Fryba, 1972):

(1) The small elastic strains arising in the body are within the scope of Hooke’s law.
(2) There exists in the plate the so-called neutral surface. The distances between points lying on that surface do not vary with plate deflections.
(3) Mass particles lying on the normal line to the neutral surface continue to lie on it even after the plate has been deformed.

Based on this assumptions, Fryba analyzed the dynamic behavior of a simply-supported uniform isotropic plate under a force moving along a specific line by an analytical method.

When bridge decks are simplified as orthotropic plates, the orthotropic plate theory can be used in the bridge-vehicle interaction analysis. The first step is to determine the flexural rigidities and the torsional rigidity of the equivalent orthotropic plate. The equivalent rigidities have been presented for the typical slab and pseudo-slab bridge sections, such as isotropic slabs, grillages, T-beam bridge decks, multi-beam bridge decks, multi-cell box-beam bridges, slabs stiffened with ribs of box section by Bakht and Jaeger (1985).

The equation of motion and eigenfunctions for the flexural vibrations of rectangular plate of orthotropic material have been determined by Huffington and Hoppmann (1958). Leissa (1973) has presented some comprehensive and accurate
analytical results on the free vibration of rectangular plates. For a broader class of boundary-value problem, an orthogonality criterion for the eigenfunctions is established and relations on the kinetic and potential energies are derived. Free vibration of orthotropic plates with fixed-simply supported and free-free boundary conditions was investigated using orthotropic plate theory (Grace and Kennedy, 1985). Natural frequency parameter for rectangular orthotropic plates on a pair of simply supported parallel edges were studied numerically using analytical method for a variety of boundary conditions and material orthotropy (Jayaraman, et al. 1990). Transverse free vibration of beam-slab type highway bridge was analyzed by Ng and Kulkarni (1972). A modified method based on the orthotropic plate theory for computing the natural frequencies of bridge slabs was presented through a set of empirical relationships between the plate parameters.

2) Moving forces

The vibration analysis of a plate under moving loads is simply considered as the moving force problem. The moving force may be a constant force, a harmonic force and a random force. This study always assume that the plate is thin and the deformation is small, and that the load always remain in contact with the plate and is of the type of an impact force at any instant of time. Wu et al (1987) analyzed the dynamic responses of non-uniform rectangular flat plates with various boundary conditions subjected to various typical moving loads using the finite element method. The effects of eccentricity, acceleration and initial velocity of the moving load, and the effect of span length are the main aspects of the study. The dynamic behavior of a multi-span flat plate supported by the beam members of a rigid plane grid and subjected to the action of a series of moving loads is also investigated. The dynamic response of an infinite plate on elastic foundation subjected to constant amplitude or harmonic moving loads was investigated (Kim and Roesset, 1998). Larrondo et al (1998) analyzed the transverse vibrations of simply supported anisotropic rectangular plates carrying an elastically mounted concentrated mass. Laura et al (1999) analyzed the transverse dynamic response of simply supported rectangular plates of generalized anisotropy, subjected to an uniformly distributed \( P_{cos(\omega t)} \) – type excitation. Agrawall et al. (1988) have established the equation of motion on the dynamic responses of orthotropic plates under moving masses using Green's function.
When the effects of rotatory inertia and shear deformation of the plate are taken into consideration, the Mindlin plate theory can be used to analyze the bridge-vehicle interaction. Wang and Lin (1996) analyzed the vibration of a homogenous and isotropic Mindlin plate on periodic supports to a moving load basing on this theory. The component method was adopted to establish the transfer matrix and the modal frequencies, and their corresponding mode shape function for the multi-span plate were calculated by the transfer matrix method. The effect of span number, rotatory inertia and transverse shear deformation of the plate and the velocity of the load on the maximum response and the corresponding load position on the plate were investigated.

3) Quarter-truck model

The vehicle is modeled as a single degree-of-freedom (DOF) or two degrees-of-freedom sprung-mass-dashpot system. This is very useful for modeling multiple vehicles. The vehicles are modeled as a set of independent discrete units moving with the same velocity (Taheri and Ting, 1989). This eliminates the inertia effects due to roll, pitch and yaw motions of the vehicle. The masses of the vehicles are lumped on the suspension systems which are modeled as linear springs and dashpots. All movements of the suspensions, except the vertical motions, are constrained. The authors developed two methods (structural impedance method (1989) and finite element method (1990)) to analyze the dynamic response of plates subjected to moving loads basing on this model. No restriction is placed on the type of load. The algorithms accounted for the complete dynamic interactions between the moving loads and the plate. The methods can be applied to general moving mass problems and to the simplified moving force and static problems. Humar and Kashif (1995) used this model to identify the parameters governing the response of isotropic and orthotropic plate models, and to examine the nature of dynamic response of the models subjected to central and off-center moving vehicles and to multiple vehicles. The conclusions show that the characterizing parameters governing the response of a bridge structure modeled by an isotropic plate are the aspect ratio of the plate, the speed parameter, the frequency ratio between the frequency of the vehicle and the natural frequency of the bridge, and the mass ratio. The parameters governing the response of an orthotropic plate model are the speed parameter, the frequency ratio,
the mass ratio and two other factors related to the geometry and stiffness properties of the plate.

4) Half-truck model

The half-truck model is usually represented by a planar, two axle or three axle, sprung mass system with frictional device. This model was used to study the effects of the vehicle braking on the bridge response (Gupta and Traill-Nash, 1980). Three bridge models (beam, beam with torsional freedom, uniform orthotropic plate) are used in the analysis. The effect of the bridge transverse flexibility is considered firstly by including an additional freedom in the simple beam representation. The response studies are extended into the braking of vehicle on the approach as well as on the span. The effect of the bridge transverse flexibility on the bridge response is studied by obtaining the response for symmetric as well as eccentric vehicular loading on the bridge. Bridge idealization as an orthotropic plate can be used to simulate the transverse flexibility of the bridge deck. The three-axle half-truck model was also used to identify the effect of various parameters on the dynamic load (Hwang and Nowak, 1991).

2.2 DYNAMIC AMPLIFICATION FACTOR

In order to precisely represent the vehicle-bridge interaction, the bridge deck is represented with a complex model, while the vehicles will be modeled with two- or three-dimensional models (Full-truck model). Therefore the numerical method becomes important in the vehicle-bridge interaction analysis. Existing researches on numerical methods and the impact factor are discussed below.

2.2.1 Numerical Methods for Bridge-Vehicle Interaction Analysis

There are three types of algorithms to analyze the vehicle-bridge interaction, which are direct time integration method, iterative method and vehicle-bridge interaction element method.

1) Direct time integration method
When the bridge is modeled by the finite element method or one of its derivatives like the modal method or the substructure method and a full-truck model is used, the coupled equations with a large number of DOFs for both the bridge and the vehicles are constructed. The direct time integration method is used to solve the coupled equations. Henchi et al (1998) used this method to solve the coupled dynamic system using a modal superposition method for the bridge structure and the multi-body physical components of the vehicles using Lagrange's formulation. The model takes into account the road surface roughness in the form of the power spectral density function, constant speed of each vehicle, multiple vehicles at different positions in different trajectories and the linear dynamic behaviors of vehicles and the bridge.

Some advantages of this method are listed as follow: the CPU time is reduced in comparison with the uncoupled iterative method; easy and compact numerical implementation; reduced computer memory storage; no factorization of the global matrix; and no iteration in the computational process.

The principle disadvantages of the method are listed as follow: The matrices must be updated and factorized at every time step. If a new type of car or a new type of bridge is introduced, all coefficients must be changed.

2) Iterative method

The iterative method (Huang et al, 1992; Wang et al, 1992; Green and Cebon, 1994; Chatterjee et al, 1994; Chompooming and Yener, 1995; Yang and Fonder, 1996; Marchesiello et al, 1999) solved the two uncoupled sets of equations for the bridge and the vehicles separately by an iterative procedure to satisfy the geometrical compatibility conditions and equilibrium conditions for the interaction forces between the bridge and vehicles. Chompooming and Yener (1995) took into account the influence of roadway surface irregularities, vehicle-bridge interaction, the vehicle suspension system, the vehicle speed and traversing path, and the arbitrary boundary conditions and geometry of the bridge superstructures in the mathematical formulation. Green and Cebon (1994) calculated the dynamic response of the bridge by means of the convolution of modal impulse response functions and modal exciting forces, together with the modal superposition. Yang and Fonder (1996) presented in detail an iterative method with relaxation or with Aitken acceleration to solve separately the equations of motion for the bridge and vehicles and provide some
knowledge about its convergence characteristics. Marchesiello et al (1999) analyzed the dynamics of multi-span continuous straight bridges subject to the excitation from a moving multi-degree-of-freedom vehicle by applying the mode superposition principle. The modes are computed by means of the Rayleigh-Ritz method.

3) Vehicle-bridge interaction element method

The convergence rate of the iterative method is likely to be low when dealing with the more realistic case of a bridge carrying a large number of vehicles in motion. Yang et al (1995) and Yang and Yau (1997) developed a vehicle-bridge interaction element for dynamic analysis of the bridge. An interaction element is defined to consist of a bridge element and the suspension units of the vehicle resting on the element. Three types of elements can be identified for a vehicle-bridge system, which are, the bridge element, car-body element and the interaction element. By the dynamic condensation method, all the degrees-of-freedom associated with the car bodies existing within each substructure are eliminated. As all the degrees-of-freedom associated with the car bodies are condensed on the element level, conventional assembly procedure can be directly applied to form the structural equations of motion. The method was extended to the vehicle model as a rigid beam supported by two suspension units, rather than as two sprung masses by Yang et al (1999). So the pitching effect of the car body on the front and rear wheels can be taken into account.

2.2.2 Dynamic Amplification Factors

The impact factor is defined as the ratio of the maximum dynamic response to the maximum static response of the bridge minus one. Many bridge codes, including AASHTO (American Association of State Highway and Transportation Officials) specifications (1996), have adopted the same impact formula for various dynamic responses, and have related the impact factor to a single parameter of the bridge, such as the span length or frequency of vibration. This method of design is an oversimplification, and many other parameters affecting the impact factor have not been taken into account, such as the dynamic properties of the bridge, dynamic properties of the vehicle, the road-surface roughness of the bridge surface and the surface condition of the approach, multiple vehicles and their transverse positions, braking or acceleration on the bridge deck.
Cantieni (1993) studied the vehicle-bridge interaction by field tests. Hwang and Nowak (1991) developed a procedure for calculation of the dynamic load and they used it in the development of a reliability-based design code. The dynamic response of a single span multi-girder bridge under moving single vehicle and two vehicles with different speeds and road-surface roughness was studied using grillage beam theory by Wang et al (1992). Huang et al (1992) extended this method to analyze the impact factors of continuous multi-girder bridges due to moving vehicles. Chatterjee et al (1994) simplified the continuous bridge deck as a continuous Bernoulli-Euler beam with torsional vibration and the quarter truck model was used in the simulations. Yang et al (1995) discussed the effects of the speed parameter, the vehicle/bridge frequency ratio, damping of the bridge and road roughness to the impact factors. Kim and Nowak (1997) presented the results of field tests that were performed on two simply supported steel I-girder bridges to assess girder load distribution and impact factors. The factors that have an effect on the dynamic behavior of the bridge were discussed using the finite-element method for the three-dimensional bridge by Kou and Dewolf (1997). Effects of multiple vehicles on a single span bridge with two lanes were studied by Yener and Chompooming (1994). Humar and Kashif (1995) simplified the slab-type bridge as a single span orthotropic plate, and the effects of off-center vehicle and two vehicles on the bridge were also discussed with one-quarter vehicle model. Mabsout et al (1999) studied the effect of multi-lanes on wheel load distribution in steel girder bridges. Schwarz and Laman (2001) provided field-based response data for comparison to code-specified distributions of live load stress magnitudes to facilitate more accurate design and evaluation of pre-stressed concrete I-girder bridges. The methods and available data used to quantify and understand the dynamic effects of vehicle loading on bridges were presented in a synthesis by Mclean and Marsh (1998).

However most of the published works are on a two-axle vehicle moving at a constant speed on a simply supported beam. Kishan and Traill-Nash (1977) have studied the braking effect on bridge responses with the bridge represented as a simply supported beam. Gupta and Traill-Nash (1980) have presented impact factors from braking of a two-axle vehicle on a single span bridge deck using a ramped braking function. Mulcahy (1983) presented the moment amplification factors from braking of a three-axle vehicle on a single span bridge deck. Later Chompooming and Yener (1995) discussed on the effect of vehicle deceleration on bridge dynamics. But the
effect of braking of a vehicle on top of a continuous bridge deck has not been researched. A redistribution of the axle loads occurs when the vehicle is subjected to braking, and this causes large dynamic responses of the bridge. The dynamic behavior of a single span bridge deck would be different from that of a continuous bridge deck. The existing recommendation on the dynamic load allowance in AASHTO (1998) is based on 33% impact multiplier due to the dynamic components of the moving vehicle, and no differentiation is provided for single or multi-span bridge decks.

2.3 LOAD IDENTIFICATION

2.3.1 Stationary Load Identification

Accurate estimation of dynamic loads acting on a structure is very important for the structural design, control and diagnosis. The indirect load determination is of special interest when the applied loads can not be measured directly, while the responses can be measured easily (Yen and Wu, 1995a,b; Choi, 1996; Moller, 1999; Rao, 1999 and Liu et al, 2000). It is an ill-posed inverse problem because the response typically is a continuous vector function in the spatial coordinates, and it is defined at a few points of the structure only. Therefore solutions to the problem are frequently found unstable in the sense that small changes in the responses would result in large changes in the calculated load magnitudes. A number of methods have been developed to identify the loads. The methods for identifying the stationary loads can be classified into five categories, e.g., time domain method, frequency domain method, neural network method, polynomial function expansion method and regularization method.

Time domain method is based on the modal superposition principle. The load is assumed as a step function in a small time interval (Tan, 1987). One or several of the acceleration, velocity and displacement responses are measured, and the input forces can be identified from the responses. Some factors affecting the accuracy of the load identification in time domain, such as modal truncation, the relation between the number of measuring points and the number of mode shapes in the identification, the relation between the number of forces and the number of mode shapes using in the identification, sampling frequency and the noise level, were discussed (Tan, 1988). Several methods are subsequently proposed to reduce the identification errors.
Strain gauge measurements made on an plate under impact have been used to infer the contact force history in frequency domain (Doyle, 1987). With a more complex structural system, the frequency domain method becomes cumbersome. A computationally efficient de-convolution method similar to Fourier analysis and wavelet analysis was introduced (Doyle, 1996). Force reconstructions were obtained using measured acceleration responses.

On the basis of the generalized orthogonal polynomial theory, the convolution relation between the input and output of a linear system in time domain was reduced to a set of linear algebraic equations in the generalized polynomial domain (Zhang, 1996). The load identification problem also becomes a linear algebraic inverse in the generalized polynomial domain.

Artificial neural network (ANN) have attracted considerable attention and it shows good promise for modeling complex nonlinear relationships. An artificial neural network was applied to identify loads from the measuring strain responses (Cao et al, 1998). The key factors that influence the convergence of ANN learning include ANN architecture, training patterns, training algorithms and the procedures.

By investigating the characteristics of the inverse transfer function and the frequency response function (FRF) error, Lee and Park (1995) analyzed statistically the indirect force determination error and the special frequency regions where force determination error is very large. It shown that the large force determination error near the first kind of inverse pole frequencies is mainly due to the rank deficiency of the FRF error. A regularization process to reduce the error especially near the first kind of inverse pole frequencies was proposed. Since the degree of singularity of the FRF sub-matrix near the first kind of inverse pole frequencies is mainly related to system damping, a regularization procedure was suggested by adding additional damping. An optimal regularization constant was also derived.

An eigenvalue reduction technique is used to reduce the order of the system, and then dynamic programming was used to solve the inverse dynamic problem (Busby and Trujillo, 1987). The results using noisy data indicate the need to have a large regularization parameter when the noise level is high. If the data is extremely noisy, some type of filtering technique could be used before applying the dynamic programming method. Basing on dynamic programming technique, a method was developed to determine an unknown forcing function acting on a simply-supported elastic plate using a minimum number of velocity measurements at discrete points.
(Busby and Trujillo, 1993). Generalized cross validation (Golub et al, 1979) was used to determine the optimal smoothing parameter. Busby and Trujillo (1997) presented two methods (generalized cross-validation and L-curve method) to determine the optimal regularization parameter. Inoue and Kishimoto (1998) used regularization of numerical inversion of the Laplace transform for the inverse analysis of impact force. Hansen’s L-curve method (Hansen, 1992) was employed to determine the regularization parameter. A time domain method was presented for estimating the discrete input forces acting on a structure based on its measured response (Kammer, 1998). A set of inverse system Markov parameters, in which the roles of input and output are reversed, was estimated from the forward system Markov parameters using a linear predictive scheme. Inputs and acceleration outputs are assumed to be collocated to maintain minimum phase. A regularization technique was used to stabilize the computation.

2.3.2 Moving Load Identification

Information of vehicular load on a bridge deck is essential to bridge design as it constitutes the live load component in the bridge design code. Traditionally the vehicular load was either measured directly from an instrumented vehicle (Cantieni, 1992; Heywood, 1994) or computed from models of the bridge deck and the vehicle (Green and Cebon, 1997; Yang and Yau, 1997; Henchi et al, 1998). It would be very expensive and the results obtained are subjected to bias in the first approach, while the second approach is subjected to modeling errors. Systems have been developed for weigh-in-motion of the vehicles (Peters, 1984; 1986), but they all measured only the static axle loads. All the weigh-in-motion techniques treat the bridge and vehicle in a two-dimensional problem. A technique to estimate the vehicular loads from the vibration responses of the bridge deck is required such that the different parameters of the bridge and vehicle system are accounted for in the measured responses, and the cost involved would be much less than that by direct measurement.

O’conner and Chan (1988a) developed an interpretive method (IMT-I) to reconstruct the dynamic wheel loads from the bridge strains. The bridge is modeled as an assembly of lumped masses interconnected by mass-less elastic beam elements, which are not necessarily of the same length. The measured or total response equals the equivalent static response caused by the loads less the responses caused by the
inertial or D'Alembert's forces and the damping forces. Laboratory tests and field studies were used to verify the validity of the method (O'Conner and Chan, 1988b; Chan and O'Conner, 1990a). A new interpretive method (IMT-II) basing on this approach was developed to identify the moving dynamic forces (Chan et al, 1999). The method used Euler's equation for the beams to model the bridge deck in the interpretation of dynamic loads crossing the deck. A simple two-axle vehicle was used to generate the theoretical responses and the corresponding interactive forces were obtained using the method developed by Chan and O'Conner (1990b). The proposed interpretive method then used this response to identify the loads. Chan and Yung (2000) considered the effects of pre-stressing in the moving force identification. A 2-axle medium-weight lorry is modeled as a set of forces, with their magnitudes either constant or time-varying, moving across a bridge model having a span length of 28m. Results show that the identified forces are identical to actual forces only when no noise is added to the simulated bridge responses. The identified forces from responses which include noise are poor, even with a noise level as low as 1%. So a low-pass filter is used to smooth the identified results. Results show that the identified forces will be over- or under-estimated when the pre-stressing effects are neglected in the identification or in the calibration processes, respectively.

The neural network was used to determine the truck attributes (such as velocity, axle spacing and axle loads) purely from the strain response readings taken from the structure over which the truck is traveling (Gagarin et al, 1994). The radial-Gaussian incremental-learning network system is developed. The chosen approach is a two-layered modular network structure. The artificial neural network in the first layer classifies the trucks and the second-layer of the network estimates the truck's velocity, axle spacing and axle loads for each type of truck.

A methodology was developed to more accurately estimate the static response of bridges due to moving vehicles and the vehicle weights were determined by the frequency response analysis (Thater et al, 1998). This method, the equivalent dynamic filter technique, determines the dynamic and static responses using the Fast Fourier Transform, and the results are calculated based on observations that the dynamic and pseudo static responses in the frequency domain are nearly identical at DC.

On the basis of the modal superposition technique, a method was developed to identify the moving forces in time domain (DTM) (Law et al, 1997). The deck is modeled as a simply supported beam with viscous damping, and the vehicle/bridge
interaction force is modeled as one-point or two-point loads with fixed axle spacing, moving at constant speed. The correlation of the reconstructed and measured responses is a robust scoring function for evaluating the identified results. Field measurements were carried out for this method using an existing pre-stressed concrete bridge by Chan et al (2000). A two-axle heavy vehicle was used for the calibration test of the field measurements. The dynamic bending moments of the test bridge deck caused by both the calibrating vehicle and normal freight vehicles were acquired. Gross weight were obtained by summing up the equivalent axle load of each axle.

Another method based on the modal superposition technique and Fourier transform was developed to identify the interaction forces of a vehicle crossing a guide-way without knowledge of the vehicle characteristics (FTDM) (Law et al, 1999). The vehicle is modeled as a single axle and two-axle loads with fixed axle spacing moving on a simply supported beam with viscous damping. The accuracy of the identified forces is assessed by correlation of the measured responses and the responses reconstructed with the identified forces on the beam. Accurate results on the total mass of the moving vehicle can be obtained using either measured accelerations or combinations of accelerations and bending moments. Responses from more than two measurement locations are strongly recommended for the identification.

Comparative studies on the four methods (IMT-I, IMT-II, TDM, FTDM) have been carried out using theoretical study (Chan et al, 1997) and laboratory study (Chan et al, 2000). They found that IMT-I and IMT-II have a wider applicability as the locations of sensors are not fixed and they can identify more than two moving forces. TDM and FTDM are less sensitive to noise and require less number of sensors. However, the computation time for TDM or FTDM is long. From the point of view of the identification accuracy, the TDM is the best identification method with a longer computational time.

An optimal state estimation approach based on FEM (finite element method) and dynamic programming technique was developed to identify the moving load from measured responses of a simply supported beam in time domain (Law and Fang, 2001). This method provides bounds to the identified forces in solving the ill-conditioned problem, and the errors of identification are much smaller than those obtained from the existing time domain method in terms of the identified forces using
different combinations of measured responses in both simulation and laboratory studies.

2.4 CONCLUDING REMARKS

From the literature review on the three areas above, the following points are concluded illustrating the complexity of the moving load identification problem.

1) There are four methods to analyze the bridge-vehicle interaction (analytical method; iterative method; direct integration method; vehicle-bridge interaction element method). The analytical method can be used in a simplified analytical approach, specially adopted to moving loads identification. Direct integration method has good performance only for the case of a few number of vehicles on the bridge deck at the same time, and the bridge has a shape simple enough so that credible deformation modes can be estimated. Iterative method can be used to analyze the complex bridge-vehicle interaction, but the convergence characteristics of the iterative solution should be treated with care. Vehicle-bridge interaction element method is best adopted to analyze the dynamic behavior of the bridge under moving trains.

2) The dynamic amplification factors can be computed from different types of measurements (deflections, bending moments, reactions, the end shears). The parameters that effect the dynamic amplification factors can be classified into three categories. The first category consists of the parameters of the vehicle and the bridge, such as vehicle speed, axle spacing, weight and the effects of suspension system, road surface roughness, bridge span length and bridge damping. The second category consists of the parameters, such as the ratio between the weight of the vehicle and the bridge, the vehicle/bridge frequency ratio, the ratio between the axle spacing and the span length, a non-dimensional speed parameter (defined as the ratio of the driving frequency of the vehicle to the fundamental frequency of the bridge). The third category consists of the action parameters, such as braking and load distribution.

3) Four kinds of vehicle models are commonly used: a) moving forces; b) quarter-truck; c) half-truck; d) full-truck. There are four kinds of bridge models (beam, plate, finite element model and finite strip model). The "moving load" problem should be studied if the ratio of the moving mass to that of the bridge is large or
when the moving speed is large. The separation between the moving mass and the structure would be a problem in the analysis of the bridge-vehicle interaction.

4) The load identification methods are classified into five types, time domain method, frequency domain method, neural network method, regularization method and polynomial function expansion method. In practice, these methods can be combined in the identification for a better accuracy and performance.

5) There are five methods for moving load identification (interpretive method; time domain method; frequency domain method; neural network method; optimal state space method). These existing techniques to identify moving forces exhibit the common weakness of having large fluctuations in the identified results. Also TDM and FTDM require long computational time and large computer capacity.
Chapter 3

EFFECT OF VEHICLE BRAKING ON DYNAMIC RESPONSE OF CONTINUOUS BRIDGE

3.1 INTRODUCTION

The design for the dynamic effect of a moving vehicle on bridge deck is usually realized by the inclusion of an impact factor on the static load, and this approach is considered satisfactory to allow for the braking of vehicle on bridge deck. But in real life when a vehicle brakes on top of a bridge deck, its effect on the structure could be significantly different.

This chapter studies this effect on a single span and multi-span continuous bridge deck including the influence of different parameters such as initial vehicle velocity, road surface roughness (ISO 8608, 1995), braking rise time, amplitude of braking force and vehicle braking location. A method based on Hamilton principle is derived to predict the vertical motion of the vehicle-bridge system in this study. A three-axles tractor-trailer vehicle is selected for the study as the design of modern bridge deck is governed by heavy freight loading. Results from this simulation study indicate that the effect from vehicle braking is very significant in the single span bridge deck, and an impact factor larger than that allowed for in the recommendation (AASHTO, 1998) may be necessary for the bridge-vehicle combination under study. The situation is reverse in the case of the three-span bridge deck under study, in which the effect from vehicle braking is negligibly small in terms of the impact factor. The influence from other factors on the impact factor is also small. A saving in the cost of the bridge deck would result if a smaller impact factor is used. Laboratory study of the proposed method on a single and a two-span beam shows good matching between the measured and calculated responses.

3.2 DYNAMIC RESPONSE OF CONTINUOUS BRIDGE WITH VEHICLE BRAKING

The model of a seven degree-of-freedom (DOFs) tractor-trailer vehicle is shown in Figure 3.1. The seven vertical DOFs are denoted by \( y \), and those at the contact points between the wheels and the pavement are denoted by \( z \). Each vehicle
axle has a “friction” stiffness and damping from the suspensions denoted with subscripts 1, 2 and 3, and a tyre stiffness and damping denoted with subscripts 4, 5 and 6. The mass of each axle assembly is denoted by $m_3$, $m_4$ and $m_5$ and those of the tractor and trailer are $m_1$ and $m_2$ respectively.

The bridge superstructure is modeled as a non-uniform continuous Euler-Bernoulli beam as shown in Figure 3.2 with (R-I) intermediate vertical point supports. This model would be representative of modern bridge superstructure which is usually non-uniform and continuous. $\{P_s(t), s = 1,2,3\}$ are the axle forces from the moving vehicle. The force locations are denoted by $\hat{x}_s(t), (s = 1,2,3)$ measured from the left support. The motions of the centroids of the tractor and trailer can then be expressed in terms of the seven DOFs of the vehicle model and the coordinates at the contact points between the bridge and the vehicle.

### 3.2.1 Equation of Motion

The vertical displacements and rotations of the vehicle are relative to the static ‘at rest’ position, and the tyres are assumed to remain in contact with the bridge surface at all times. The intermediate supports are modeled as vertical linear springs with large stiffness to simulate bridge piers which are practically not perfectly rigid. The kinetic energy $T$ and the potential energy $U$ of the vehicle-bridge system can be obtained as

\[
T = T_b + T_c \\
= \frac{1}{2} \int_0^L \rho A(x) \left( \frac{\partial w(x,t)}{\partial t} \right)^2 dx + \frac{1}{2} m_1 \dot{y}_1^2 + \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} m_2 \dot{y}_2^2 \\
+ \frac{1}{2} J_2 \dot{\theta}_2^2 + \frac{1}{2} m_3 \dot{y}_3^2 + \frac{1}{2} m_4 \dot{y}_4^2 + \frac{1}{2} m_5 \dot{y}_5^2 + \frac{1}{2} m_6 \dot{y}_6^2 + \frac{1}{2} \sum_{i=3}^6 m_i \dot{x}_i(t)^2 + \frac{1}{2} m_1 \dot{x}_c^2 + \frac{1}{2} m_2 \dot{x}_c^2
\]

\[
U = U_b + U_c \\
= \frac{1}{2} \int_0^L E I(x) \left( \frac{\partial^2 w(x,t)}{\partial x^2} \right)^2 dx + \frac{1}{2} k_1 \sum_{i=1}^{6} w(x_i,t)^2 + \frac{1}{2} k_1 (y_1 - y_4)^2 + \frac{1}{2} k_2 (y_2 - y_5)^2 \\
+ \frac{1}{2} k_3 (y_3 - y_6)^2 + \frac{1}{2} k_4 (y_7 - a_1 y_2 - a_2 y_1)^2 - m_1 g z_1^c - m_2 g z_2^c - m_3 g z_1 \\
- m_4 g z_2 - m_5 g z_2,
\]

(3.1)
where subscripts \( c \) and \( b \) denote the contributions from the vehicle and the bridge respectively; \( \rho \) is the density of material of the bridge; \( A(x) \) is the cross-sectional area; \( E \) is the Young's modulus; \( I(x) \) is the moment of inertia of the beam cross-section; \( w(x,t) \) is the vertical displacement of the beam; \( x_i (i = 0,1,2,\cdots,R) \) are the coordinates of intermediate point supports and end supports; \( k_s \) is the stiffness of the linear spring at the point constraints. \( y_{cl}, y_{c2} \) are the vertical displacements and \( x_{cl}, x_{c2} \) are the horizontal locations of the centroids of the tractor and the trailer respectively, with subscripts \( 1 \) and \( 2 \) denote the tractor and the trailer respectively. \( J_l \) and \( J_2 \) are the rotational moments of inertia of the tractor and the trailer respectively, and \( g \) is the acceleration due to gravity.

By separation of variables, the vertical displacement of the beam \( w(x,t) \) can be expressed as

\[
w(x,t) = \sum_{i=1}^{n} q_i(t) W_i(x), \quad \{ \ i = 1,2,\cdots,n \ \}
\]

where \( \{ W_i(x), i=1,2,\cdots,n \ \} \) are the assumed vibration modes that satisfy the boundary conditions and \( \{ q_i(t), i=1,2,\cdots,n \ \} \) are the modal coordinates of the bridge.

The work done by the system of non-conservative forces of the bridge-vehicle system can then be written as

\[
W = W_b + W_d + W_a + W_c \\
= Q^T C_s \dot{Q} + Y^T C_s \dot{Y} + F_d \dot{x}_1(t) - P_1(t)(y_1 - y_s) - P_2(t)(y_2 - y_s) - P_3(t)(y_3 - y_s)
\]

(3.3)

where \( W_b, W_d, W_a, W_c \) are the work done by the damping force of the bridge, the driving force of the vehicle, the interaction forces, and the damping force of the vehicle respectively. \( Q = (q_1(t), q_2(t), \ldots q_n(t))^T \) is the vector of modal coordinates of the bridge; \( F_d \) is the longitudinal drive force of the vehicle; \( C_s, C_a \) are the damping coefficient matrices of the vehicle and the bridge respectively. \( \{P_1(t), P_2(t), P_3(t)\} \) are the interaction forces between the vehicle and the bridge written as

\[
P_1(t) = k_s (y_1 - z_1) + c_s (\dot{y}_1 - \dot{z}_1) \\
P_2(t) = k_s (y_2 - z_2) + c_s (\dot{y}_2 - \dot{z}_2) \\
P_3(t) = k_s (y_3 - z_3) + c_s (\dot{y}_3 - \dot{z}_3)
\]

(3.4)
Sharp changes in the road surface cause local changes in the profile acting on the base of the tyre spring. Mulcahy (1983) developed a ‘tyre enveloping’ model on the road surface roughness in which the profile is approximated by a quadratic parabola. This tends to smooth out the excitation from changes in the road profile. In the present research, the vertical displacements, \( z_i \), at the points of contact of the wheels and the bridge are given below including the road surface roughness function \( d(x) \) along the longitudinal direction:

\[
\begin{align*}
  z_i &= w(\hat{x}_i(t), t) + d(\hat{x}_i(t)), \\
  \hat{z}_i &= v \frac{\partial w(x,t)}{\partial x} |_{x=\hat{x}_i(t)} + \frac{\partial w(x,t)}{\partial t} |_{x=\hat{x}_i(t)} + v \frac{\partial d(x)}{\partial x} |_{x=\hat{x}_i(t)},
\end{align*}
\]

(3.5)

where \( v \) is the horizontal velocity of the moving vehicle. Since \( d(x) \) is independent of \( x \), the last term in \( \hat{z}_i \) can be put equal to zero. The longitudinal position of the second and the third axles are related to that of the first as

\[
\begin{align*}
  \hat{x}_2(t) &= \hat{x}_1(t) - S_1, \\
  \hat{x}_3(t) &= \hat{x}_1(t) - a_3 S_1 - a_2 S_2
\end{align*}
\]

(3.6)

Mulcahy (1983) has presented the equation of motion for a three-axle vehicle on a single span bridge using the Lagrange approach. The equation of motion of a three-axle vehicle on a multi-span bridge is presented in this chapter using the Hamilton principle:

\[
\int T - V + W dt = 0
\]

(3.7)

The equation of motion of the vertical motion of the vehicle-bridge system can be obtained in \( y \) and \( q_i \) coordinates as

\[
\begin{align*}
  M_s \ddot{y} + C_s y + K_s y &= F_s, \\
  M_b \ddot{q} + C_b \dot{q} + K_b q &= F_b
\end{align*}
\]

(3.8)

where \( y = [y_1, y_2, ..., y_7]^T \) is the vector of displacements at the seven DOFs of the vehicle; \( F_s \) and \( F_b \) are vectors of generalized forces acting on the vehicle and the bridge structure respectively. \( M_s, K_s, C_s \) and \( M_b, K_b, C_b \) are the mass, stiffness and damping matrices of the vehicle and beam respectively. All terms in the equation are referred to Appendix A.

The equation of motion of the longitudinal motion of the vehicle in \( \hat{x}_5(t) \) coordinate can also be written as
\[
\sum_{i=1}^{s} m_i \ddot{x}_i(t) - (m_1 + m_2) b_1 (\ddot{y}_1 - \ddot{y}_2) - m_2 b_2 (\ddot{y}_2 - \ddot{y}_3) \\
= F_d + \sum_{i=1}^{s} \left( \sum_{j=1}^{n} W_i(\dot{x}_i(t))q_i(t) \right) P_i(t)
\]

(3.9)

with

\[
W_i(\dot{x}_i(t)) = \left. \frac{\partial W_i(x)}{\partial x} \right|_{x=\dot{x}_i(t)} \quad (i = 1, 2, \ldots, n; \ s = 1, 2, 3)
\]

(3.10)

### 3.2.2 Mode Shape of the Bridge Deck

The mode shapes of a uniform single span simply supported beam are expressed as

\[
W_{i,0}(x) = \sin\left(\frac{i\pi x}{L}\right), \quad (i = 1, 2, \ldots, n)
\]

(3.11)

where \(n\) is the number of vibration modes. The vertical displacement of the beam can be assumed as a combination of these mode shapes satisfying the boundary conditions of the non-uniform beam as

\[
W(x) = \sum_{i=1}^{n} b_i W_{i,0}(x)
\]

(3.12)

where \(\{b_i, i = 1, 2, \ldots, n\}\) is a set of coefficients that can be found by minimizing the following integral in Ritz’s method

\[
Z = \int_a^b \left[ EI(x)(W''(x))^2 - \omega^2 \rho A(x)(W(x))^2 \right] dx
\]

(3.13)

to have

\[
(K' - \omega^2 M)b = 0
\]

(3.14)

where \(K'\) and \(M\) are \(n \times n\) matrices with their components \(k'_{ij}\) and \(m_{ij}\) given in Equation (3.15), and \(b = [b_1, b_2, \ldots, b_n]^T\) is a \(n \times 1\) vector.

\[
\begin{align*}
    k'_{ij} &= \int_a^b EI(x) W_{i\alpha} W_{j\alpha}''(x) dx \\
    m_{ij} &= \int_a^b \rho A(x) W_{i\alpha}(x) W_{j\beta}(x) dx
\end{align*}
\]

(3.15)

Rewrite Equation (3.14) into

\[
(D - \omega^2 I)b' = 0
\]

(3.16)

and solving, we have
\[ D = K' M^{-1}; \quad b' = Mb. \]  

(3.17)

where \( \omega^2, b' \) are the eigenvalues and the eigenvectors of the matrix \( D \). Coefficients \( b_i \) and hence \( \psi(x) \) can be obtained from Equation (3.12).

### 3.3 ROADSURFACE ROUGHNESS

The randomness of the road surface roughness can be represented with a periodic modulated random process. In ISO-8608 (1995) specifications, the road surface roughness is related to the vehicle speed by a formula between the velocity power spectral density (PSD) and the displacement PSD. The general form of the displacement PSD of the road surface roughness is given as

\[ S_d(f) = S_d(f_0) \cdot (f/f_0)^{-\alpha} \]  

(3.18)

where \( f_0 = 0.1 \) cycles/m is the reference spatial frequency; \( \alpha \) is an exponent of the PSD, and \( f \) is the spatial frequency (cycles/m). Equation (3.18) gives an estimate on the degree of roughness of the road from the value of \( S_d(f_0) \). This surface roughness classification is based on a constant vehicle velocity PSD and taking \( \alpha \) equals to 2.

The road surface roughness function \( d(x) \) in the time domain can be simulated by applying the Inverse Fast Fourier Transformation on \( S_d(f_i) \) to give (Henchi et al., 1998)

\[ d(x) = \sum_{i=1}^{N} \sqrt{4S_d(f_i)} \Delta f \cos(2\pi f_i x + \theta_i) \]  

(3.19)

where \( f_i = i \Delta f \) is the spatial frequency; \( \Delta f = \frac{1}{N\Delta} \); \( \Delta \) is the distance interval between successive ordinates of the surface profile; \( N \) is the number of data points, and \( \theta_i \) is a set of independent random phase angle uniformly distributed between 0 and 2\( \pi \).

### 3.4 IMPLEMENTATION AND LABORATORY VERIFICATION

#### 3.4.1 Implementation of Algorithm

The coupled equations of motion of the bridge-vehicle system presented in Equations (3.8) and (3.9) are subjected to the compatibility constraints on the interaction forces and the displacements of the two subsystems. The procedure to solve the problem is implemented as follows:
Step 1: The mode shapes $W_i(x)$ of the non-uniform multi-span continuous bridge deck are calculated from Equations (3.14) to (3.17).

Step 2: Determine the mass, stiffness and damping matrices of both the vehicle and the bridge deck.

Step 3: Calculate the road surface roughness function $d(x)$ from Equation (3.19) according to the selected road class in ISO-8608(1995).

Step 4: The responses of the bridge and vehicle are calculated by the Newmark Method. The time step, parameters of Newmark Method and the error for convergence are determined before the iteration. Set the initial values $Q_0$ and $Y_0$.

Step 5: Determine the initial vehicle position on the bridge deck.

Step 6: Calculate the excitation force on vehicle, $F_v$, and solve for the motion of the vehicle, $Y$, at time $t$ from Equations (3.8) and (3.9).

Step 7: Calculate the excitation force on the bridge, $F_b$, and solve for the motion of the bridge, $Q$, at time $t$ from Equation (3.8).

Step 8: Solve for the displacement of the bridge $w(x,t)$ from Equation (3.2).

Step 9: Repeat Steps 6 to 8 using the calculated $Q$ and $Y$. Check the convergence of the difference between the two successively calculated $w(x,t)$, and $w(x,t)_{i-1}$.

$$\|w(x,t)_{i+1} - w(x,t)_{i}\| \leq \text{tolerance error} \quad (3.20)$$

Step 10: If convergence is not achieved, repeat Steps 6 to 9. If convergence is achieved, repeat steps 5 to 10 for the next time step.

3.4.2 Laboratory Verification

The experimental setup is shown diagrammatically in Figure 3.3. The main beam, 3678mm long with a 100mm×25mm uniform cross-section, is simply supported. There is a leading beam for accelerating the vehicle and a trailing beam to accept the vehicle when it comes out of the main span. A U-shaped aluminum section is glued to the upper surface of the beams as a direction guide for the car. The model car is pulled along the guide by a string wound around the drive wheel of an electric motor. Thirteen photoelectric sensors are mounted on the beams to measure and monitor the moving speed of the car. Seven strain gauges are evenly located on the beam to measure the bending moment responses of the beam. A TEAC 14-channels magnetic tape recorder and an 8-channel dynamic testing and analysis system are used.
for data collection and analysis in the experiment. The sampling frequency is 2000Hz. The recorded length of each test lasts for six seconds. The model car has two axles at a space of 0.557m and it runs on four steel wheels with rubber band on the outside. The mass of the whole car is 16.6 Kg or 11.8Kg for the experiments described below. The braking force was applied with a set of rubber bands between two fixed anchorage points. It was placed in front of the vehicle approximately at the level of its centroid, and the braking force was adjusted by adjusting the tension in the rubber band.

Since there is no distinct suspension system in the model car, the vertical stiffness $K$, and the damping $C$, are not considered, and the car is modelled as a rigid body moving on top of the beam. The beam sub-system has very small damping, and hence the damping stiffness $C_\phi$ is set to zero in the computation. In the study with a two-span continuous beam, the same main beam is used with the intermediate support at 1.875m from the left end. $Q_\phi$ and $Y_\phi$ in the calculation are set to zero.

3.4.2.1 Experiment on a single span simply supported beam

A time step of 0.005 second is adopted in the integration of the responses in the solution of the equations-of-motion, and this covers the first six modes of the beam structure.

The first experiment is conducted with the mass of the car equals 16.6Kg. The vehicle speed is 1.22 m/second, and it brakes at 0.878m from the left support. The car gets outside the main beam at the end of braking. Figure 3.4 shows the measured and the calculated strains at $1/4L$, $1/2L$, $3/4L$. The strains for each cross-section are very close to each other indicating good estimates on the responses of the structure under the braking action of a moving vehicle using the proposed method. There is a slight underestimation in the $1/4L$ strain when the vehicle moves away from the section, and there is a slight overestimation in the $3/4L$ strain when the vehicle moves towards the section. This is due to the gentle slope in the beam under its own weight that is not included in the formulation of the road surface roughness function.

The second experiment is conducted with a larger braking force. The vehicle speed is 1.12 m/second, and the mass of the whole car is 11.8Kg. It brakes at 1.4m from the left support, and the car eventually stops on top of the beam. Figure 3.5 shows the measured and the calculated strains. There is a large difference between the
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strains at 1/2L soon after braking. This is due to the pitching motion of the vehicle at braking which is equivalent to an application of an impulsive force on the beam causing large responses.

3.4.2.2 Experiment on a two-span continuous beam:

A time step of 0.002 second is adopted in the integration of the responses in the solution of the equations-of-motion, and this covers the first five modes of the beam structure.

The third experiment is conducted with the mass of the whole car equals 16.6Kg. The vehicle speed is 1.25 m/second, and it brakes at 0.878m from the left support. The car stops outside the main beam. Figure 3.6 shows a distinct periodic response in the 1/4L strain when the vehicle is on the first span. This response corresponds to the second mode of the beam, and it is suspected to be the result of excitation by the unsteady motion of the vehicle. Similar large fluctuations in the 3/4L strain are also found when the vehicle is around the middle of the second span.

The fourth experiment is conducted with a larger breaking force. The mass of the whole car is 11.8Kg. It moves at 1.75 m/second and it brakes at 1.4m from the left support. The car eventually stops on top of the beam. Figure 3.7 shows large fluctuations in the measured strains but with the mean very close to the calculated strains. This fluctuation is again the response from the second mode of the beam caused by the motion of the vehicle.

3.5 PARAMETRIC STUDY

The three-axle tractor-trailer vehicle (Mulcahy, 1983) shown in Figure 3.1 is used in the study. The properties of the vehicle are measured and they are:

The body masses are

\[
m_1 = 3930\, \text{Kg}, \quad m_2 = 15700\, \text{Kg}, \quad m_3 = 220\, \text{Kg}, \quad m_4 = 1500\, \text{Kg}, \quad m_5 = 1000\, \text{Kg}.
\]

The vertical stiffness at each DOF is

\[
k_1 = 2.00 \times 10^6 \, \text{N/m}, \quad k_2 = 4.60 \times 10^6 \, \text{N/m}, \quad k_3 = 5.00 \times 10^6 \, \text{N/m},
\]

\[
k_4 = 1.73 \times 10^6 \, \text{N/m}, \quad k_5 = 3.74 \times 10^6 \, \text{N/m}, \quad k_6 = 4.60 \times 10^6 \, \text{N/m},
\]

\[
k_7 = 2.00 \times 10^5 \, \text{N/m}
\]

and their respective viscous damping constants are

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\[ c_1 = 5000 \text{Nm}^{-1} s, \quad c_2 = 30000 \text{Nm}^{-1} s, \quad c_3 = 40000 \text{Nm}^{-1} s, \quad c_4 = 1200 \text{Nm}^{-1} s \]
\[ c_5 = 3900 \text{Nm}^{-1} s, \quad c_6 = 4300 \text{Nm}^{-1} s, \quad c_7 = 5000 \text{Nm}^{-1} s \]

The axle spacing and the pitching moment of inertia of the tractor and the trailer are respectively
\[ S_1 = 3.66 \text{m}, \quad S_2 = 6.20 \text{m}, \quad J_1 = 1.05 \times 10^4 \text{Kgm}^2, \quad J_2 = 1.47 \times 10^5 \text{Kgm}^2 \]

The parameters on the dimensions of the vehicle are
\[ a_1 = 0.5, \quad a_2 = 0.5, \quad a_3 = 1.0, \quad a_4 = 0.0, \quad a_5 = 0.58, \quad a_6 = 0.42, \quad b_1 = 0.25, \quad b_2 = 0.40. \]

Initially both the vehicle and bridge are assumed to be at rest and the vehicle is travelling forward at a uniform velocity. A ramp function is assumed for the braking force (Gupta and Traill-Nash, 1980). This is based on the test results on highway vehicles conducted by the Transport and Road Research Laboratory, England, in 1975. The braking force increases linearly to a maximum \( F_{d, \text{max}} \) and then stays constant until the vehicle either comes to a stop or crosses the bridge span and is written as

\[
F_d = \begin{cases} 
- \frac{F_{d, \text{max}}}{t_b}, & t < t_b \\
- F_{d, \text{max}}, & t_e \geq t \geq t_b 
\end{cases} \quad (3.21)
\]

where \( t_b \) is the braking rise time. \( t_e \) is the time of the vehicle either comes to a stop or crosses the bridge span.

The braking force is the force experienced by the vehicle at the surface of the bridge due to friction in the longitudinal direction. As the braking force is not acting at the centroid of the vehicle, it induces a moment on the vehicle system causing a load re-distribution in the front axle and the rear axle. Also the road surface roughness and the deflection of the bridge have a combinational effects.

The impact factors \( I_d \) and \( I_m \) calculated from the mid-span deflections and bending moments are defined respectively as

\[
I_d = \frac{\max \| w_{\text{dynamic}} (x, t) \|}{\max \| w_{\text{static}} (x) \|} \\
I_m = \frac{\max \| B_{\text{dynamic}} (x, t) \|}{\max \| B_{\text{static}} (x) \|} \quad (3.22)
\]

where \( w_{\text{dynamic}} (x, t), w_{\text{static}} (x), B_{\text{dynamic}} (x, t), B_{\text{static}} (x) \) are the dynamic and static deflections and moments at mid-span of the beam. \( w_{\text{static}} (x), B_{\text{static}} (x) \) are obtained
from an analysis with the vehicle crossing the bridge deck at a crawling speed of 0.01 m/s.

3.5.1 A Uniform Single Span Bridge Deck

No reference can be found in the literature on the use of non-uniform beam model in simulation studies, and hence the simply supported bridge deck studied by Mulcahy (1983) is adopted for comparison. It is 32.6m long with 16m effective width, and the mass per unit area is 1240Kg/m². The flexural stiffness of the bridge superstructure is $4.592 \times 10^{10}$ Nm². The vehicle to bridge mass ratio is 0.0346. It is modeled as a simply supported uniform beam. The first ten modes are used in the solution of the equation of motion in Equations (3.8) and (3.9). The result is convergent and it also reaches a good compromise between computational time and accuracy of the result. For the identification problem, the number of modes is small because it is limited by the practical number of measuring points. A small number of modes are usually used in the computation of responses of bridge deck (Marchesiello et al, 1999; Yang et al, 2000). A time step of 0.008 second is used in the integration, and it is approximately one-tenth of the highest natural frequency of the bridge superstructure included in the analysis. This small time step is essential because braking produces an impulsive force that consists of frequency components over a wide spectrum.

The three-axle vehicle described previously is traveling at 17m/s and it brakes at one-quarter span. The impact factors from mid-span bending moments and deflections are studied for variations in the following parameters:

(a) amplitude of the braking force ($F_{d_{\max}} = 0.6m_{s}g, \ 0.4m_{s}g, \ 0.2m_{s}g$), where $m_{s}g$ is the vehicle static weight;
(b) braking rise time, $t_{b} = 0.6s, \ 0.3s$ and $0.0s$;
(c) braking position of the vehicle;
(d) vehicle velocity; and
(e) different classes of road with the road surface roughness as specified in the ISO-8608 (1995). Road Classes A to E are used in the study.

The maximum impact factors computed for different braking forces and braking locations are plotted in Figures 3.8 and 3.9 with no surface roughness included in the analysis. These factors are the maximum values for the duration when the vehicle is
on top of the bridge deck. The maximum impact factors for different classes of roads are presented in Table 3.1. The following observations are made:

(a) Figure 3.8 shows that the variation of braking force $F_{d_{\text{max}}}$ has little effect on the impact factor which is around 1.0 for all the magnitudes of braking force under study.

(b) Figure 3.9 shows that the duration of braking rise time $t_b$ has very significant effect on the maximum impact factor, and braking within the first quarter span produces large impact factor compared with braking at other locations of the span. The impact factors produced from a hard braking $t_b=0.3$ sec. is approximately 1.30 while that from a sharp braking with $t_b=0.0$ sec. is approximately 1.42.

(c) Braking on the approach is not studied. Braking on the approach produces non-zero initial displacement and velocity of the vehicle at the entry point of the bridge, but the equivalent impulsive force from braking does not act on the bridge span. The dynamic effect on bridge span depends on the characteristics of the vehicle suspension system, but it would be less severe than braking inside the bridge span in general.

(d) Table 3.1 shows the maximum impact factor for different road surface conditions when the vehicle is moving at 1.7 and 17m/s or braking with a braking force of $0.6m_{\text{g}}$ at a rise time of 0.6s at one-quarter span. There is only slight difference in the impact factor for the cases travelling at constant velocity on Classes A to D roads. The case of travelling at low velocity on Class E road has a significantly higher impact factors than that with a higher velocity. Also braking causes a distinctly higher impact factor when compared with the no braking cases on Classes C to E roads. Road Class E exhibits the worst dynamic responses with or without braking.

### 3.5.2 A Three-Span Continuous Non-Uniform Bridge Deck

A modern three-span box-section non-uniform bridge deck (Zheng et al, 1998) shown in Figure 3.10 is modeled as a 36m-48m-36m three-span non-uniform continuous beam. The vehicle to bridge mass ratio is 0.0235. The time step required for integration of the equation of motion is taken as 0.0147 second taking into account the first ten modes of the bridge superstructure in the analysis. The same three-axle vehicle is used in the study.
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The impact factors are studied with variations in the influencing parameters for comparison. The maximum impact factors are plotted in Figures 3.11 to 3.14 and in Figure 3.16, and the maximum impact factors for different classes of roads are shown in Tables 3.2 and 3.3. The following observations are made:

(a) Figure 3.11 shows that the maximum impact factors at mid-span for different constant travelling velocities as a reference. No road surface roughness is included in the analysis. The impact factors are in general small with larger values at higher velocities. The impact factor is smallest in the second span among the three spans. This may be due to its higher flexibility compared with the other two spans.

(b) Figure 3.12 shows the time histories of the impact factors resulting from braking at positions 2/7, 3/7 and 4/7 of the total bridge span, $L$. The vehicle velocity is 17 m/s, and Class B road is considered. The impact factor from the second span is largest for these braking locations among the three spans with 2/7L almost on the second support and 3/7L and 4/7L are inside the second span. These locations are selected for further studies in Figures 3.13 and 3.14. The impact factors are in general very small with the highest value of 1.04 for braking at 3/7 span.

(c) Figure 3.13 shows the time histories of impact factor from braking at 3/7L on a Class B road moving with a velocity of 17 m/s. The braking rise time $t_b$ is found to have significant effect on the impact factor with a value of 1.17 for a hard braking at $t_b=0.3$ sec. In the case of a sharp braking at $t_b=0.0$ sec., the impact factor is 1.49. It is noted that the oscillating component in the responses in Figures 3.12 to 3.14 is due to the large pitching action of the vehicle arising from braking.

(d) Figure 3.14 shows the time histories of impact factor at middle of second span from braking at 1/3L on a Class B road with a velocity of 17 m/s. The braking force $F_{d_{max}}$ has little effect on the maximum impact factor with the largest value of 1.14 for $F_{d_{max}}=0.6m_v \sigma$. The corresponding interaction axle forces are shown in Figure 3.15. The curves indicate approximately proportional increase in the interaction forces with $F_{d_{max}}$ in the first and the third axles, while that in the second axle exhibits very small change with $F_{d_{max}}$. This phenomenon is due to the pitching action of the vehicle.

(e) Figure 3.16 gives the maximum impact factors at the second span from different vehicle velocity and braking positions. Both factors have little effect on the impact factor when braking occurs at 1/7L. This is because braking occurs at the first span and the vehicle never reaches the middle of the second span. When braking
occurs at $3/7L$, all the impact factors are larger than unity, and it gradually decreases with higher velocity. The maximum value is approximately 1.15 at a velocity of 10m/s, the lower end of the velocity range. When braking occurs at $2/7L$, which is almost on top of the second support, it creates compression in the suspension system of the vehicle. And when it comes into the second span, the vehicle will bounce on top of the bridge deck causing higher impact factors than usual. However their values are not higher than those obtained when braking at the most critical location of $3/7L$ as seen in Figure 3.16.

(f) Table 3.2 shows the impact factor for different classes of roads. The impact factor increases with velocity very significantly only in the third span, and there is very small change in the other spans. Both bending moments and deflections give approximately the same impact factor. Since spans 1 and 3 are identical, this difference may be due to the different initial conditions of the vehicle at entry to the two spans (Zero initial conditions for span 1 and non-zero conditions for span 3). This observation is consistent with that observed from Figure 3.11 where the maximum impact factors are similar for both spans 1 and 3, but the impact factors here are larger due to the inclusion of road surface roughness in the analysis.

(g) Table 3.3 shows the impact factor for $F_{d_{max}}=0.6mg$, $t_b=0.6$ sec. and 17m/s velocity with braking at different locations. The maximum impact factor increases slightly with the road surface roughness in road Classes A to E, and the impact factor from deflection and moment are approximately the same for all Road Classes. Small impact factors are found when braking starts at $1/7L$ and the reason is as explained for Figure 3.16. Those factors from braking at $3/7L$ are largest, and those from braking beyond $3/7L$ are the same. This is because the braking point is beyond the midspan of the second span, and the braking effect is smaller than the effect due to travelling at constant velocity before braking. A more correct definition of impact factor for a continuous beam is therefore needed from the above observations. The impact factor from the braking effect should be a comparison of the maximum dynamic and static responses at the same span in which braking occurs.

3.5.3 Summary of Results
The effects of vehicle braking on bridge decks under study are summarized below with relative importance of the different parameters. It is noted that the vehicle to bridge mass ratio is small and is less than 0.035, and the system should produce large dynamic responses.

(a) The impact factor is largest in the same span as braking occurs in a multi-span bridge. Hence the impact factor allowed for a continuous bridge deck should be different for different span unless a conservative design is desired.

(b) When the initial braking location is in the first half of the span, the impact factor would be much higher than that from braking in the second half of the span for both the single and multi-span bridge decks under study.

(c) Initial velocity of the vehicle and the amplitude of braking force do not contribute significantly to the impact factor.

(d) The braking rise time is very significant to the impact factor. The shorter the braking rise time, the higher the impact factor, and it is less in a continuous bridge deck compared to that in a single span bridge deck.

(e) The impact factor is low at approximately 1.11 when the vehicle is travelling across the bridge deck with Road Class C roughness at a constant velocity of 1.7 m/s or at 17 m/s. When the braking effect is included while travelling at 17 m/s, the impact factor increases to approximate 1.33 for the single span bridge. This observation can also be found for other road classes. But in the case of the three-span bridge under study, there is no significant change in the parameter. This shows that the dynamic allowance recommended for design in the AASHTO (1998) for long span continuous bridge deck can be reduced leading to a reduction in the cost of construction of the structure.

3.6 CONCLUDING REMARKS

Studies from the simulation and the laboratory verification indicate that the method and the algorithm proposed in the chapter are correct and they can be used to analyze for the dynamic behavior of a multi-span continuous bridge with non-uniform cross-section under the braking action of a moving vehicle. This chapter also addresses the effect of vehicle braking on a single span and a three-span bridge decks, and the dynamic effect is compared with those from ordinary road surface roughness in terms of an impact factor. A three-axle tractor-trailer vehicle is adopted in the study
to represent freight loading which governs the design of modern bridges. The recommended 33% dynamic multiplier from AASHTO (1998) is found not sufficient for the single span bridge deck studied when vehicle braking effect is included. The recommended dynamic multiplier may also be too conservative for the three-span bridge deck under study. The cost of construction could be reduced if a smaller impact factor is recommended for multi-span bridge deck to design engineers.

The impact factor should be taken in the same span as braking occurs, and braking in the first half of the span would produce larger dynamic response in the structure. Vehicle braking causes significant dynamic responses compared with road surface roughness in single span bridge, while it has insignificant effect in the three-span bridge deck under study. The other parameters like initial vehicle velocity and amplitude of braking force have little effect on the dynamic response.
Table 3.1 Impact factors with different road surface conditions
(Single span bridge deck)

<table>
<thead>
<tr>
<th>Road Class</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7m/s no braking</td>
<td>1.09</td>
<td>1.06</td>
<td>1.14</td>
<td>1.24</td>
<td>1.79</td>
</tr>
<tr>
<td>Deflection</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moment</td>
<td>1.08</td>
<td>1.05</td>
<td>1.12</td>
<td>1.22</td>
<td>1.71</td>
</tr>
<tr>
<td>17m/s no braking</td>
<td>1.04</td>
<td>1.05</td>
<td>1.13</td>
<td>1.27</td>
<td>1.55</td>
</tr>
<tr>
<td>Deflection</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moment</td>
<td>1.04</td>
<td>1.04</td>
<td>1.10</td>
<td>1.23</td>
<td>1.49</td>
</tr>
<tr>
<td>17m/s - braking at 1/4 span</td>
<td>1.08</td>
<td>1.17</td>
<td>1.32</td>
<td>1.62</td>
<td>2.24</td>
</tr>
<tr>
<td>Deflection</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moment</td>
<td>1.07</td>
<td>1.15</td>
<td>1.29</td>
<td>1.59</td>
<td>2.20</td>
</tr>
</tbody>
</table>

Table 3.2 Impact factor from deflection and moment with different road surface conditions (Three-span bridge deck)

<table>
<thead>
<tr>
<th>Road Class</th>
<th>Span</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>17m/s no braking</td>
<td>1</td>
<td>1.01</td>
<td>1.02</td>
<td>1.08</td>
<td>1.19</td>
<td>1.40</td>
</tr>
<tr>
<td>Deflection</td>
<td>2</td>
<td>1.02</td>
<td>1.05</td>
<td>1.11</td>
<td>1.24</td>
<td>1.50</td>
</tr>
<tr>
<td>Impact Factor</td>
<td>3</td>
<td>1.07</td>
<td>1.15</td>
<td>1.32</td>
<td>1.65</td>
<td>2.29</td>
</tr>
<tr>
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<td>1.06</td>
<td>1.12</td>
<td>1.24</td>
<td>1.45</td>
</tr>
<tr>
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<td>1.02</td>
<td>1.05</td>
<td>1.11</td>
<td>1.23</td>
<td>1.49</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.11</td>
<td>1.20</td>
<td>1.37</td>
<td>1.71</td>
<td>2.33</td>
</tr>
<tr>
<td>1.7m/s no braking</td>
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<td>1.00</td>
<td>1.03</td>
<td>1.10</td>
<td>1.24</td>
<td>1.34</td>
</tr>
<tr>
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<td>2</td>
<td>1.02</td>
<td>1.08</td>
<td>1.15</td>
<td>1.30</td>
<td>1.59</td>
</tr>
<tr>
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<td>1.05</td>
<td>1.10</td>
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<td>1.46</td>
</tr>
<tr>
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<td>1.06</td>
<td>1.11</td>
<td>1.25</td>
<td>1.35</td>
</tr>
<tr>
<td>Impact Factor</td>
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<td>1.02</td>
<td>1.07</td>
<td>1.12</td>
<td>1.25</td>
<td>1.49</td>
</tr>
<tr>
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<td>3</td>
<td>1.04</td>
<td>1.08</td>
<td>1.12</td>
<td>1.17</td>
<td>1.46</td>
</tr>
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</table>
Table 3.3 Impact factor from deflection and moment in the second span with different braking position of vehicle on different classes of road 
($F_{dmax}=0.6mg$, $t_b=0.6$ sec. and 17m/s)

<table>
<thead>
<tr>
<th>Road Class</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brake at 1/7L</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deflection</td>
<td>0.43</td>
<td>0.45</td>
<td>0.51</td>
<td>0.69</td>
<td>1.05</td>
</tr>
<tr>
<td>Moment</td>
<td>0.48</td>
<td>0.49</td>
<td>0.52</td>
<td>0.67</td>
<td>0.96</td>
</tr>
<tr>
<td>Brake at 2/7L</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deflection</td>
<td>1.03</td>
<td>1.04</td>
<td>1.10</td>
<td>1.24</td>
<td>1.65</td>
</tr>
<tr>
<td>Moment</td>
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<td>1.03</td>
<td>1.09</td>
<td>1.02</td>
<td>1.55</td>
</tr>
<tr>
<td>Brake at 3/7L</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>1.06</td>
<td>1.12</td>
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<td>1.50</td>
</tr>
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<td>Moment</td>
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<td>1.06</td>
<td>1.11</td>
<td>1.23</td>
<td>1.47</td>
</tr>
<tr>
<td>Brake at 4/7L</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deflection</td>
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<td>1.05</td>
<td>1.11</td>
<td>1.24</td>
<td>1.50</td>
</tr>
<tr>
<td>Moment</td>
<td>1.02</td>
<td>1.05</td>
<td>1.11</td>
<td>1.23</td>
<td>1.47</td>
</tr>
<tr>
<td>Brake at 5/7L</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deflection</td>
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<td>1.05</td>
<td>1.11</td>
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<td>1.50</td>
</tr>
<tr>
<td>Moment</td>
<td>1.02</td>
<td>1.05</td>
<td>1.11</td>
<td>1.23</td>
<td>1.49</td>
</tr>
<tr>
<td>Brake at 6/7L</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Deflection</td>
<td>1.02</td>
<td>1.05</td>
<td>1.11</td>
<td>1.24</td>
<td>1.50</td>
</tr>
<tr>
<td>Moment</td>
<td>1.02</td>
<td>1.05</td>
<td>1.11</td>
<td>1.23</td>
<td>1.49</td>
</tr>
</tbody>
</table>
Figure 3.1 Model of a Seven Degrees-of-freedom vehicle

Figure 3.2 A continuous beam with (R-1) intermediate point supports under the moving vehicle
Figure 3.3 Diagrammatic drawing of experimental setup
Figure 3.4 Experimental measured and calculated strains (gentle braking in single span beam)
Figure 3.5 Experimental measured and calculated strains (hard braking in single span beam)
Figure 3.6 Experimental measured and calculated strains (gentle braking in two-span beam)
Figure 3.7 Experimental measured and calculated strains
(hard braking in two-span beam)
Figure 3.8 Moment ratio from different braking force
Figure 3.9 Impact factors from deflection and moment for different vehicle braking position
Figure 3.10 A three-span continuous bridge
Figure 3.11 Mid-span impact factor from deflection and moment with different vehicle speed
Figure 3.12 The ratios from deflection and moment at middle of second span with different braking position.
Figure 3.13 Ratios from deflection and moment at middle of second span with different braking rise time
Figure 3.14 Moment ratios and deflection ratios at the second span with different amplitude of braking force
Figure 3.15 Interaction force with different amplitude of braking force
Figure 3.16 Impact factor with different initial speed and initial braking position
Chapter 4

DYNAMIC LOAD
ON CONTINUOUS MULTI-LANE BRIDGE

4.1 INTRODUCTION

In the last chapter, the effect of vehicle braking on the dynamic response of continuous bridge is studied. The bridge is modeled as a non-uniform continuous Euler-Bernoulli beam. The intermediate vertical supports are modeled as linear springs with large stiffness. It is obvious that a simple beam model cannot precisely represent the three-dimensional behavior, particularly in the case of a moving vehicle with its path not along the centreline of the bridge.

This chapter investigates the dynamic loading on a multi-lane continuous bridge due to vehicles moving on top of the bridge deck. The bridge is modelled as a multi-span continuous orthotropic rectangular plate with intermediate line rigid supports. The analytical vehicle is simulated as a two-axle three-dimensional vehicle model with 7 degrees-of-freedom according to the H20-44 vehicle design loading in AASHTO(1998). The dynamic behavior of the bridge under several moving vehicles is analyzed using orthotropic plate theory and modal superposition technique. The effects of multi-lane loading from multiple vehicles on the dynamic impact factor of the bridge are discussed. The impact factor is found varying in an opposite trend as the dynamic responses for the different loading cases under study.

4.2 DYNAMIC BEHAVIOR OF BRIDGE DECK UNDER MOVING VEHICLES

4.2.1 Assumptions

The following assumptions are made for the formulation of the problem:

1) The bridge is treated as a continuous rectangular orthotropic plate with simple supports at its two ends \((x=0, x=a)\), and the other two opposite edges are free \((y=0, y=b)\) as shown in Figure 4.1. A linear elastic behavior is assumed, and the effects of shear deformation and rotary inertia are neglected.

2) The intermediate line supports of the bridge are assumed as linear rigid and they are orthogonal to the free edges of the plate.
3) The model for the H20-44 truck loading comprises three rigid masses which represent the truck body, front and rear wheel/axle set, respectively as shown in Figure 4.2.

4) Since the horizontal dimension of the bridge deck are much larger than its thickness, the thin plate assumption is made.

5) The wheel loads are assumed to be in contact with the bridge deck all the time.

4.2.2 Vibration of the Bridge Deck

From the vibrational theory of thin plate, the strain energy of the continuous orthotropic plate in Cartesian co-ordinates is

\[
U_e = \frac{1}{2} \int \int \left[ D_x \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + (D_y \nu_{xx} + D_z \nu_{yy}) \frac{\partial^2 w}{\partial x \partial y} + D_z \left( \frac{\partial^2 w}{\partial y^2} \right)^2 \right] dS
\]

where \( D_x, D_y, D_z \) are the rigidity constants of the orthotropic plate; \( \nu_{xx}, \nu_{yy} \) are Poisson’s ratio of the orthotropic material. For the bridge deck with material orthotropy and an equivalent uniform plate thickness \( h \),

\[
D_x = E_x h^3 / 12 (1 - \nu_{xx} \nu_{yy}) , \quad D_y = E_y h^3 / 12 (1 - \nu_{yy} \nu_{xx}) , \quad D_z = G_{xy} h^3 / 12 \]

in which \( E_x, E_y \) are Young’s moduli in the \( x \)- and \( y \)- directions respectively, \( G_{xy} \) is the shear modulus. These rigidities for the bridge deck with shape orthotropy can be determined by the method of Bakht and Jaeger (1985).

The kinetic energy of the system is expressed as

\[
T = \frac{1}{2} \int \int \rho \left( \frac{\partial w}{\partial t} \right)^2 dS
\]

where \( \rho \) is the mass density of plate material, and \( w \) is the vertical deflection.

The work done by the damping of the plate and moving loads are as follow,

\[
W_c = - \int \int c_k w \frac{\partial w}{\partial t} dS
\]

\[
W_e = \int \sum F_{xm}^m \delta(x - \hat{x}_m(t)) \delta(y - \hat{y}_m(t)) w dS
\]

4-2
where $c_s$ is the damping coefficient of the plate; $F_{i}^{\text{int}}$ is the $i$th interaction force between the vehicular wheel and the bridge. $(\hat{x}_i(t), \hat{y}_i(t))$ is the location of the interaction force $F_{i}^{\text{int}}$. When the vehicle is moving along one lane, $\hat{y}_i(t)$ is a constant. $\delta(x), \delta(y)$ are Dirac functions.

Based on modal superposition, the dynamic deflection $w(x, y, t)$ can be described as

$$w(x, y, t) = \sum_{i=0}^{\infty} W_i(x, y)q_i(t) \tag{4.4}$$

where $W_i(x, y)$ is the vibration mode shape of the plate and $q_i(t)$ is the corresponding modal amplitude. When Equation (4.4) is substituted into Equations (4.1) to (4.3), the equations of motion for the bridge are

$$M_s \ddot{Q} + C_s \dot{Q} + K_s Q = W_s \mathbf{F}_s^{\text{int}} \tag{4.5}$$

where $M_s, C_s, K_s$ are the mass, damping and stiffness matrices of the bridge, respectively (Appendix B); $\mathbf{F}_s^{\text{int}}$ is the vector of interaction force under the wheels of the moving vehicles. $\dot{Q}, \ddot{Q}$ are the first and second derivatives of $Q$ and $Q$ is the vector of modal amplitudes.

### 4.2.3 Modal Analysis of the Bridge Deck

For free vibration of the plate, the vertical displacement may be expressed as

$$w(x, y, t) = W(x, y)e^{\imath \omega t} \tag{4.6}$$

where $\omega$ is the natural frequency of vibration and $\imath = \sqrt{-1}$. Assuming the variables in $W(x, y)$ are separable, the mode shape function $W(x, y)$ can be expressed in terms of a series as

$$W(x, y) = \sum_{m} \sum_{n} A_{mn} \varphi_m(x) \psi_n(y) \tag{4.7}$$

where $\varphi_m(x)$ and $\psi_n(y)$ are the assumed admissible functions along the $x$- and $y$-directions respectively, while $A_{mn}$ are the unknown coefficients. A set of series consisting of a combination of beam eigenfunctions and polynomials has been selected as the admissible functions of the line-supported plates by Zhou (1994). Here we take $\varphi_m(x)$ to be the eigen-functions of the continuous multi-span Euler-Bernoulli
beam, and \( \psi_n(y) \) are the eigen-functions of the single-span Euler-Bernoulli beam satisfying the free boundary conditions. Substituting Equation (4.7) into Equations (4.6), (4.1) and (4.2), and minimizing the Rayleigh’s quotient with respect to each coefficient \( A_{mn} \) would lead to the eigenvalue equations in matrix form as follows

\[
(K - \omega^2 M_s)A = 0
\]  

(4.8)

where

\[ A = \{A_{11}, A_{12}, \ldots, A_{1N}, A_{21}, \ldots, A_{MN}\}^T; \]

\[ m_{sj} = \rho h \int_0^L \varphi_{m1}(x)\varphi_{n2}(x)dx \int_0^L \psi_{n1}(y)\psi_{n2}(y)dy \]

\[ k_{sj} = D_s \int_0^L \varphi_{m1}''(x)\varphi_{n2}''(x)dx \int_0^L \psi_{n1}(y)\psi_{n2}(y)dy \]

\[ + D_s \int_0^L \varphi_{m1}'(x)\varphi_{n2}'(x)dx \int_0^L \psi_{n1}'(y)\psi_{n2}'(y)dy \]

\[ + \nu_s D_s \int_0^L \varphi_{m1}'(x)\varphi_{n2}'(x)dx \int_0^L \psi_{n1}(y)\psi_{n2}(y)dy \]

\[ + \int_0^L \varphi_{m1}'(x)\varphi_{n2}'(x)dx \int_0^L \psi_{n1}(y)\psi_{n2}(y)dy \]

\[ + 2(1 - \nu_s)D_s \int_0^L \varphi_{m1}'(x)\varphi_{n2}'(x)dx \int_0^L \psi_{n1}'(y)\psi_{n2}'(y)dy \]

\[
(\ m1 = 1,2,\cdots,M;\ m2 = 1,2,\cdots,M;\ n1 = 1,2,\cdots,N;\ n2 = 1,2,\cdots,N;\ i = (m1 - 1)N + n1;\ j = (m2 - 1)N + n2.\)

and \( M, N \) are the number of admissible functions in x- and y- directions respectively. \( \varphi''_m(x), \varphi'_m(x) \) are the second and first derivatives of \( \varphi_m(x) \); \( \psi''_n(y), \psi'_n(y) \) are the second and first derivatives of \( \psi_n(y) \).

The natural frequencies \( \omega \) and coefficients \( A_{mn} \) can be determined from Equation (4.8). Then the mode shape functions of the continuous orthotropic plate are also determined from Equation (4.7). Since the admissible functions are eigenfunctions of the Euler-Bernoulli beam, the mode shape functions of the continuous orthotropic plate satisfy the orthogonality relationships. It should be noted that this approach is much more simple and direct than existing methods by Zhou (1994) and Marchesiello et al (1999).

**4.2.4 Vehicle Model**
Chapter 4: Dynamic Load on Continuous Multi-lane Bridges

The mathematical model for the H20-44 truck is shown in Figure 4.2. The model is similar to that employed by Marchesiello et al (1999). The vehicular body is assigned three degrees-of-freedom, corresponding to the vertical displacement (\(y\)), rotation about the transverse axis (pitch or \(\theta_p\)), and rotation about the longitudinal axis (roll or \(\theta_r\)). Each wheel/axle set is provided with two degrees-of-freedom in the vertical and roll directions (\(y_{d1}, y_{d2}, \theta_{d1}, \theta_{d2}\)). Therefore, the total number of independent degrees-of-freedom is seven. The equations of motion of the vehicle are derived using Lagrange’s formulation as follow.

\[
M \ddot{Z} + C \dot{Z} + K Z = F_{\text{int}}
\]  

(4.9)

where \(F_{\text{int}}\) is the interaction force vector applied on the vehicle; \(M, C, K\) are, respectively, the mass, damping and stiffness matrices of the vehicle system and \(Z\) is the vector of the vehicle degrees-of-freedom (Appendix B).

4.2.5 Vehicle-Bridge Interaction

The vehicle-bridge interaction forces for a single vehicle can be written as follow:

\[
F_{i1} = K_{n1} (y_{d1} - \frac{1}{2} S_{d1} \dot{\theta}_{d1} - w_1 - d_1) + C_{n1} (\dot{y}_{d1} - \frac{1}{2} S_{d1} \ddot{\theta}_{d1} - \dot{w}_1 - \dot{d}_1);
\]

\[
F_{i2} = K_{n2} (y_{d1} + \frac{1}{2} S_{d1} \dot{\theta}_{d1} - w_2 - d_2) + C_{n2} (\dot{y}_{d1} + \frac{1}{2} S_{d1} \ddot{\theta}_{d1} - \dot{w}_2 - \dot{d}_2);
\]

\[
F_{i3} = K_{n3} (y_{d2} - \frac{1}{2} S_{d2} \dot{\theta}_{d2} - w_3 - d_3) + C_{n3} (\dot{y}_{d2} - \frac{1}{2} S_{d2} \ddot{\theta}_{d2} - \dot{w}_3 - \dot{d}_3);
\]

\[
F_{i4} = K_{n4} (y_{d2} + \frac{1}{2} S_{d2} \dot{\theta}_{d2} - w_4 - d_4) + C_{n4} (\dot{y}_{d2} + \frac{1}{2} S_{d2} \ddot{\theta}_{d2} - \dot{w}_4 - \dot{d}_4);
\]

(4.10)

where \(\{K_n, i = 1,2,3,4\}\) are the stiffness of the tyres; \(\{C_n, i = 1,2,3,4\}\) are the damping coefficients of the tyres. \(S_{d1}, S_{d2}\) are the wheel spacing of the front and rear axles respectively.

\[
w_i = w(\bar{x}_i(t), \dot{\bar{y}}_i(t), t);
\]

\[
d_i = d(\bar{x}_i(t), \dot{\bar{y}}_i(t)), \quad i = 1,2,3,4
\]

(4.11)

where \(d(x,y)\) is the surface roughness of the bridge deck; \((\bar{x}_i(t), \dot{\bar{y}}_i(t))\) is the location of the \(i\)th tyre at time \(t\). As the vehicle moves along one lane,

\[
\dot{y}_i(t) = y_i + S_{d1} / 2; \quad \ddot{y}_i(t) = \dot{y}_i - S_{d1} / 2.
\]

\[
\ddot{y}_i(t) = y_i + S_{d2} / 2; \quad \dddot{y}_i(t) = \dot{y}_i - S_{d2} / 2, \quad \text{and} \quad y_0 \quad \text{is the transverse coordinate of the}
\]
centre-line of the lane. For the purpose of this study \( d(x,y) \) is taken to be one dimensional only with \( \dot{y}_i(t) \) the same for both wheels in an axle.

The dynamic responses of the bridge deck under moving vehicles can be calculated from Equations (4.5), (4.9) and (4.10) using an iterative method (such as the Newmark method) or the algorithm by Henchi et al. (1998).

4.3 IMPLEMENTATION AND SIMULATION STUDY

4.3.1 Procedure of Implementation

The coupled equations of motion of the bridge-vehicle system presented in Equations (4.5) and (4.9) are subjected to the compatibility constraints on the interaction forces and the displacements of the two subsystems. The procedure to solve the problem is implemented as follows:

Step 1: Calculate the mode shapes and natural frequencies of the multi-span bridge deck

a) Determine the assumed admissible functions \( \varphi_m(x) \) and \( \psi_n(y) \) in Equation (4.7).

b) Calculate the natural frequencies \( \omega \) and the coefficients \( A_{mn} \) from Equation (4.8). The mode shapes are determined by Equation (4.7).

Step 2: Determine the mass, stiffness and damping matrices of both the vehicle and the bridge deck.

Step 3: Calculate the road surface roughness function \( d(x) \) from Equation (3.19) according to the selected road class in ISO-8608(1995).

Step 4: The responses of the bridge and vehicle are calculated by the Newmark Method. The time step, parameters of Newmark Method and the error for convergence are determined before the iteration. Set the initial values \( Q_0 \) and \( Z_0 \).

Step 5: Determine the initial vehicle position on the bridge deck.

Step 6: Calculate the excitation force on vehicle, \( F^{\text{ext}}_v \) from Equation (4.10) and Appendix B, and solve for the motion of the vehicle, \( Z \), at time \( t \) from Equations (4.9).

Step 7: Calculate the excitation force on the bridge, \( F^{\text{ext}}_b \) from Appendix B, and solve for the motion of the bridge, \( Q \), at time \( t \) from Equation (4.5).
Step 8: Solve for the displacement of the bridge $w(x,t)$ from Equation (4.4).

Step 9: Repeat Steps 6 to 8 using the calculated $Q$ and $Z$. Check the convergence of the difference between the two successively calculated $w(x,t)$, and $w(x,t)_{m-1}$.

$$\|w(x,t)_{m-1} - w(x,t)_{m}\| \leq \text{tolerance error}$$

Step 10: If convergence is not achieved, repeat Steps 6 to 9. If convergence is achieved, repeat steps 5 to 10 for the next time step.

### 4.3.2 Verification of the Proposed Method

The bridge-vehicle system in Marchesiello et al (1999) is used to verify the theory and the algorithm developed in the chapter. No published results on an orthotropic plate can be found for comparison, and the isotropic plate from Marchesiello et al (1999) is used instead. The bridge is simplified into a continuous three-span isotropic plate with line intermediate rigid supports. The vehicle body is rigid and subject to bounce, pitch and roll motions. The parameters of the vehicle-bridge system are listed as follow:

- $l_1 = l_3 = l_5 = 26.4m$, $b = 10.7m$, $h = 0.95m$, $E = 14.54 \times 10^{10} \text{N/m}^2$.
- $\nu = 0.3$, $\rho = 2375 \text{Kg/m}^3$.
- $S_t = 4.73m$, $a_t = 0.67$, $a_z = 0.33$, $S_{x1} = S_{x2} = 2.05m$, $S_{y1} = S_{y2} = 1.41m$.
- $m_x = 17000 \text{Kg}$, $m_{x1} = 600 \text{Kg}$, $m_{x2} = 1000 \text{Kg}$, $I_x = 9 \times 10^4 \text{Kg m}^2$, $I_y = 1.3 \times 10^4 \text{Kg m}^2$.
- $I_{x1} = 550 \text{Kg m}^2$, $I_{x2} = 600 \text{Kg m}^2$.
- $K_{n_{x1}} = K_{n_{x2}} = 1.16 \times 10^4 \text{N/m}$, $K_{n_{y1}} = K_{n_{y2}} = 3.73 \times 10^4 \text{N/m}$.
- $K_{o_{x1}} = K_{o_{x2}} = 7.85 \times 10^4 \text{N/m}$, $K_{o_{y1}} = K_{o_{y2}} = 1.57 \times 10^4 \text{N/m}$.
- $C_{n_{x1}} = C_{n_{x2}} = 2.5 \times 10^4 \text{Ns/m}$, $C_{n_{y1}} = C_{n_{y2}} = 3.5 \times 10^4 \text{Ns/m}$.
- $C_{o_{x1}} = C_{o_{y1}} = 100 \text{Ns/m}$, $C_{o_{x2}} = C_{o_{y2}} = 200 \text{Ns/m}$.

$l_{x1}$ and $l_{x2}$ are the torsional moments of inertia of the two axles respectively; $S_{y1}$ and $S_{y2}$ are the spacing of the suspensions of the front and rear axle respectively; the subscript $y_i$ refers to the $i$th suspension of the vehicle. Initial $Q_0$ and $Y_0$ in the calculation are set to zero.

The natural frequencies of the continuous three-span isotropic plate are shown in Table 4.1. Since the aspect ratio $b/L$ is small, the number of series in the assumed beam functions in $x$-direction is selected larger than that in $y$-direction as shown. The results obtained by the proposed method are very close to that obtained using Zhou's method (Zhou, 1994), and are approximately equal to the results in Marchesiello et al.
(1999). This shows that the proposed method and algorithm to obtain the natural frequencies of the continuous plate are correct.

Another check is made on the accuracy of the computed response time histories. The vehicle moves along the edge with its right tyres at a distance 1m from the right edge of the bridge. The displacements at the middle of the first span when the vehicle is moving at a speed of 32.5m/s and 37.5m/s are shown in Figure 4.3. The first thirteen modes are used in the calculation with a time step of 0.001s. Comparison between the results obtained by the proposed method and those in Marchesiello et al (1999) show that the method and algorithm proposed in the chapter are accurate to analyze for the dynamic responses of a continuous multi-lane bridge deck under a moving vehicle.

### 4.4 MULTIPLE VEHICLES ON MULTI-LANE CONTINUOUS BRIDGE DECK

Not many research has been done with multiple vehicles on top of a multi-lane bridge deck. Humar and Kashif (1995) simplified a slab-type bridge as a single span orthotropic plate, and the effects of off-center vehicle and two vehicles on the bridge are discussed with a one-quarter vehicle model. Effects of having multiple vehicles on a single span bridge deck with two lanes have been presented by Yener and Chompooming (1994). Mabsout et al (1999) has also studied the effect of multi-lanes on the wheel load distribution in a steel girder bridge.

A continuous three span multi-girder bridge as shown in Figure 4.4 is used for this study. There are four equal lanes over the total width of the bridge deck. The parameters of the bridge deck are listed as follow:

Span lengths are 24m, 30m, 24m for the first, second and third spans respectively; distance between two adjacent main girders is 2.743m; distance between two adjacent diaphragms is 6m; deck slab thickness is 0.2m, $b=13.715m$, $\rho = 3000Kg/m^3$.

$E_x=4.1682\times 10^{10} N/m^2$, $E_y=2.9733\times 10^{10} N/m^2$, $\nu_{xy} = 0.3$.

For the steel I-beam: web thickness=0.01111m, web height =1.490m, flange width=0.405m, flange thickness=0.018m.

For the diaphragms: cross-sectional area=0.001548m$^2$, $I_y = 0.707 \times 10^{-8} m^4$.

$I_z = 2 \times 10^{-8} m^4$, $J = 1.2 \times 10^{-7} m^4$. 

4-8
The rigidities of the equivalent orthotropic plate can be calculated according to Bakht and Jaeger (1985) with $D_x = 2.415 \times 10^9 \, Nm$, $D_y = 2.1807 \times 10^7 \, Nm$, $D_{xy} = 1.1424 \times 10^8 \, Nm$. The first thirteen natural frequencies of the continuous bridge are 4.13 Hz, 4.70 Hz, 6.31 Hz, 6.86 Hz, 7.76 Hz, 8.20 Hz, 15.81 Hz, 16.39 Hz, 20.84 Hz, 22.29 Hz, 22.90 Hz, 24.31 Hz, 24.86 Hz. The damping coefficients of the bridge are taken as 0.02 for all the vibration modes. Road Classes A to D according to ISO-8608 (1995) and the case without roughness are used in the simulations. The parameters of the vehicle are the same as those in Section 4.3.2.

### 4.4.1 Dynamic Loading from a Single Vehicle

The bending moment and shear force in the plate are calculated as

$$
\begin{align*}
M_x &= -(D_x \frac{\partial^2 w}{\partial x^2} + v_y D_y \frac{\partial^2 w}{\partial y^2}) \\
V_x &= -(D_x \frac{\partial^2 w}{\partial x^2} + (v_y D_y + 4 D_{xy}) \frac{\partial^2 w}{\partial x \partial y})
\end{align*}
$$

(4.12)

The impact factor and wheel-load distribution factor are defined after Huang et al. (1992) as follow.

$$
I_p = \left( \frac{R_d}{R_s} - 1 \right) \times 100\%
$$

(4.13)

$$
\eta = \frac{M_t}{M_c}
$$

where $R_d$ and $R_s$ are the absolute maximum response from the dynamic and static studies, respectively. Here $R_s$ is obtained with the vehicle moving at a very low speed of 0.1 m/s with 0.1 s for the time step in the calculation and no road surface roughness is included. $M_t$ is the maximum bending moment of one beam at the section; $M_c = M / n$; where $M$ is the sum of the bending moment of all beams at one section; $n$ is the number of wheel loads in the transverse direction.

Table 4.2 shows the static wheel load distribution, dynamic wheel load distribution and the impact factor from bending moment, displacement and strain under different loading cases. The Loading Cases from a single vehicle are shown in Figure 4.5(a). Figure 4.6 shows the time histories of the bending moments, strains, shear forces and displacements at middle of the second span of beam-1 from Load
Case 1 to 3. Figure 4.7 shows the bending moments at middle of each span and each beam under Load Case 1. The speed of the vehicle is 30m/s, and a time step of 0.001s is used in the calculation. The road surface roughness is of Class B. Figures 4.8 and 4.9 show the effects of moving speed and road surface roughness respectively on the impact factors calculated at different points of the bridge deck under different loading cases. The legends 1–3, 4–5, 6–9, 10–12 and 13–15 define the midpoints of the first, second and third spans on beams 1–5, respectively. The following observations are obtained from these figures and Table 4.2.

1) The dynamic impact factor calculated at different locations on the bridge deck varies in an opposite manner to that for the wheel load distribution factor. The former has a large value whereas the latter has a small value at the same point, and vice versa.

2) The impact factors obtained from different measurable variables, such as bending moment, strain and displacement, are similar, and hence strains can be used in further studies as they can be easily measured.

3) The bending moments in beams 1 and 2 are larger when the vehicle is close to them while those in beams 4 and 5 are smaller. This is because the motion of the vehicle in the outer lane excites the torsional modes which are significant to the responses.

4) The impact factors in the beams near to the path of the moving vehicle are smaller than those in the beams further away. But the dynamic responses behave oppositely as seen in Figure 4.6.

5) The impact factor is insensitive to the moving speed of the vehicle.

6) In Load Case 1, the impact factors on beams 4 and 5 are larger than those in the other beams. The impact factors on beam 4 are largest in all the cases studied. This may be due to the torsional modes excited in Load Case 1.

7) When the road surface roughness is increasing, the impact factors also increase especially for the case under eccentric Load Case 1. The maximum impact factor is 225% as seen in Figure 4.9.

4.4.2 Dynamic Loading from Multiple Vehicles

Two vehicles moving in different lanes are studied in the simulations. The loading cases are shown in Figure 4.5(b). Load Case 4 consists of two vehicles
moving in the same direction. Load Cases 5 to 7 consist of two vehicles moving in opposite directions. Figure 4.10 shows the bending moments, strains, shear forces and displacements at mid-span of span 2 of beam-1 under different loading cases. The speed of moving vehicles is 30m/s, and the two vehicles enter the bridge at the same time. The road surface roughness is of Class B. Time step in the computation is 0.001s. Figure 4.11 show the bending moments at middle of span 2 of beams 2, 3, 4 and 5 under loading cases 4, 5 and 7 which are more worse. Table 4.3 shows the computed static and dynamic load distribution factor and impact factor. The following observations are made from these figures and Table 4.3.

1) Tables 4.2 and 4.3 show that the impact factor generated from two vehicles is smaller than that from a single vehicle.

2) The impact factor also behaves oppositely when compared with the wheel load distribution factor.

3) The magnitude of bending moments in each beam is closely related to the transverse location of the resultant of the vehicular loads on the bridge deck. It is large when the resultant is close to the beam, and small when the force is further away from the beam.

4) Loading No. 4 gives the largest impact factors in beam-5 which are 125.19%, 120.82% and 92.60% at the middle of spans 1, 2 and 3, respectively. But the corresponding load distribution factor and the responses in Figure 4.11 are small. This is similar to the observation in point (6) made for a single vehicle in Section 4.4.1 where the contribution of torsional modes of the bridge deck is suspected. This indicates a common point that is often mixed up in design. The magnitude of the dynamic impact factor is of no significance if it is not related to the magnitude of stress or member capacity. A high impact factor, in general, corresponds to very low stress level. And only impact factors that relate to design situation are of importance.

5) The impact factors differ significantly in the three spans under different loading. This would indicate a need to have different impact factors for the three spans in the design unless a conservative design is desired.

4.5 CONCLUDING REMARKS
Chapter 4: Dynamic Load on Continuous Multi-lane Bridges

The design loading from vehicles moving on top of a three-dimensional continuous bridge deck has been investigated. The proposed method for analyzing the problem is based on a Lagrangian formulation of the vehicle-bridge system which is solved with the orthotropic plate theory and modal superposition technique. Numerical simulations have been performed to study the variation of dynamic impact factor and wheel load distribution factor on the bridge deck, and the following conclusions are obtained.

1) The transverse vehicle position has an important effect on the impact factor. The impact factors in the beam that is far away from the path of the moving vehicle are larger than those that are near. But it is the opposite with the responses and wheel load distribution factor which are larger at points close to the moving vehicle.

2) The high dynamic impact factors reported in this study correspond to low response level in the bridge deck and hence low stress level. Therefore these impact factors should be taken with care as only when they are related to the design situation that they would be of importance.

3) The impact factors associated with multiple vehicles are smaller than those for single vehicle.

4) The road surface roughness is more important to the impact factors than the moving speed of the vehicle.
Table 4.1 Natural frequencies for the three-span continuous bridge (Hz)

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<th></th>
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<td>9×5* 17×9*</td>
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<td>6.30 6.28</td>
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<td>15.05 15.04</td>
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<tr>
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<td>19.67 19.60</td>
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<td>21.49</td>
<td>22.16 22.10</td>
<td>22.59 22.40</td>
<td>22.44 22.37</td>
</tr>
</tbody>
</table>

Note: 9×5* denotes 9 number of eigen-functions in $\varphi_n(x)$ and 5 number of eigen-functions in $\psi_n(y)$, and so on.
Table 4.2 Load distribution factor (LDF) and impact factor (IMP) (single vehicle)

<table>
<thead>
<tr>
<th>Span</th>
<th>Load Case</th>
<th>Beam-1</th>
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<th>Beam-3</th>
<th>Beam-4</th>
<th>Beam-5</th>
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<td>0.486</td>
<td>0.395</td>
<td>0.309</td>
<td>0.224</td>
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<tr>
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<td>0.393</td>
<td>0.400</td>
<td>0.408</td>
<td>0.415</td>
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<td>Second</td>
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<td>0.567</td>
<td>0.319</td>
<td>0.087</td>
<td>0.153</td>
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<td>0.078</td>
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<td>0.393</td>
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Table 4.3 Load distribution factor (LDF) and impact factor (IMP) (two vehicles)

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<th>Beam-4</th>
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Static LDF

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Dynamic LDF

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IMF(%) (Displacement)

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Chapter 4: Dynamic Load on Continuous Multi-lane Bridges

Figure 4.1 Model of the continuous bridge deck

Figure 4.2 Idealization of two-axle vehicle
Figure 4.3 Displacements with different speeds

(– 32.5m/s by present method; o 32.5m/s in Marchesiello et al (1999);
--- 37.5m/s by present method; X 37.5m/s in Marchesiello et al (1999).)
Figure 4.4 Diagram of the bridge

Figure 4.5 Vehicle Loading
Figure 4.6 Responses at midpoint of beam-1 under different loading cases

(- Loading No.1; --- Loading No.2; ... Loading No.3.)
Figure 4.7 Bending moment at each beam under loading—1

(- First span; ----- Second span; ... Third span.)
Figure 4.8 Effects of moving speed on impact factor
Figure 4.9 Effects of road-surface roughness on impact factor

(a) Effects under Load Case 1

(b) Effects under Load Case 3
Figure 4.10 Responses at midpoint of beam–1 under different loading cases

(– Loading No.4; —— Loading No.5; --- Loading No.7.)
Figure 4.11 Bending moment at Beam 2~5 under different loading cases

(— Loading No.4; —— Loading No.5; ... Loading No.7.)
Chapter 5

REGULARIZATION
IN MOVING FORCE IDENTIFICATION

5.1 INTRODUCTION

Several methods have been developed recently to identify forces moving on a single span beam from measured responses. The Time Domain Approach (TDM) (Law et al, 1997) models the structure and forces with a set of second order differential equations. The forces are represented as step functions in a small time interval. These equations of motions are then expressed in the modal coordinates, and these uncoupled equations are solved by deconvolution in the time domain. The forces are then identified using the modal superposition principle. The Frequency and Time Domains Approach (FTDM) (Law et al, 1999) performs Fourier transformation on the equations of motions, which are expressed in modal coordinates. The Fourier transforms of the responses and the forces are related in the frequency domain, and the time histories of the forces are found directly by the least squares method. The Interpretive Approach (Chan et al, 1999; Yuan et al, 1998) identifies the forces completely in the modal coordinates. Measured displacements are converted into modal displacements with an assumed shape function. The modal velocities and accelerations are then obtained by differentiation. The forces are then identified solving the uncoupled equations of motions in modal coordinates.

The results obtained from all the above methods are noise sensitive and they exhibit fluctuations at the beginning and end of the time histories. These moments correspond to the switching of free vibration state of the structure to the forced vibration state, and vice versa, and the solutions are ill-conditioned. This chapter introduces a regularization method in the ill-conditioned problem to provide bounds to the identified forces. The FTDM (Law et al, 1999) is selected in this study on the improvements due to regularization in both simulation and laboratory test results. Since Zhang (1994) has reported that identification of a stationary force by frequency domain methods would be more practical and effective than by time domain methods, laboratory results from using TDM (Law et al, 1997) is also presented to compare the accuracy and the effectiveness of regularization in these two different methods. The
studies show that TDM is better than FTDM in solving the ill-posed problem, and the total weight of a vehicle can be estimated indirectly using moving force identification method with some accuracy.

5.2 THEORY OF FREQUENCY AND TIME DOMAIN METHOD

Since this chapter is dealing with the inverse problem of identification of moving forces from the measured responses, which is not familiar to most engineers, the FTDM (Law et al, 1999) is briefly described below for clarity of the formulation on the method.

5.2.1 Assumptions

The following assumptions are made on the system model;
1. The beam is simply supported.
2. The changes in the system characteristics, i.e. the stiffness, damping and mass matrices before and after the force occurrence are negligible.
3. The force is a perfect point force represented as a step function in a small time interval as shown in Equation (5.1).
4. Structural damping is included in the analysis.
5. The structure may not be at rest before an external load is applied.

5.2.2 Equation of Motion and Modal Superposition

A time-varying force is moving on a simply supported Euler-Bernoulli beam as shown in Figure 5.1. The beam is assumed to be of constant cross-section with constant mass per unit length, and having linear, viscous proportional damping and with small deflections. The effects of shear deformation and rotary inertia are neglected. The force moves from left to right at a constant speed \(v\). The equation of motion can be written as

\[
\rho \frac{\partial^2 w(x,t)}{\partial t^2} + C \frac{\partial w(x,t)}{\partial t} + EI \frac{\partial^4 w(x,t)}{\partial x^4} = \delta(x-\nu t)P(t)
\]

(5.1)
where \( w(x,t) \) is the beam deflection at point \( x \) and time \( t \); \( \rho \) is the mass per unit length; \( C \) is the viscous damping parameter; \( E \) is the Young's modulus of material; \( I \) is the second moment of inertia of the beam cross-section; \( L \) is the length of the beam; \( P(t) \) is the time-varying interaction point force; \( \delta(t) \) is the Dirac delta function.

Based on modal superposition, the dynamic deflection \( w(x,t) \) can be described as:

\[
 w(x,t) = \sum_{n=1}^{\infty} \Phi_n(x)q_n(t) 
\]

(5.2)

where \( n \) is the mode number; \( \Phi_n(x) \) is the mode shape function of the \( n \)th mode and \( q_n(t) \) is the \( n \)th modal amplitudes. Substituting Equation (5.2) into (5.1), and multiplying by \( \Phi_j(x) \), and integrating with respect to \( x \) between 0 and \( L \), and applying the orthogonality conditions, we obtain

\[
 \frac{d^2 q_n(t)}{dt^2} + 2 \zeta_n \omega_n \frac{dq_n(t)}{dt} + \omega_n^2 q_n(t) = \frac{1}{M_n} p_n(t)
\]

(5.3)

where \( \omega_n \) is the modal frequency of the \( n \)th mode; \( \zeta_n \) is the damping ratio of the \( n \)th mode; \( M_n \) is the modal mass of the \( n \)th mode; and \( p_n(t) \) is the modal force. These modal parameters are:

\[
 \omega_n = \frac{n^2 \pi^2}{L^2} \sqrt{\frac{EI}{\rho}} \\
 \Phi_n(x) = \sin(n\pi x / L) \\
 M_n = \rho L / 2 \\
 p_n(t) = P(t) \sin\left(\frac{n \pi vt}{L}\right)
\]

(5.4)

For practical structures, the modal parameters can be obtained from the finite element model and/or from the modal testing. Performing the Fast Fourier transformation (FFT) on Equation (5.3), we get

\[
 Q_n(\omega) = H_n(\omega) \frac{1}{M_n} P_n(\omega) 
\]

(5.5)

where

\[
 P_n(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} p_n(t)e^{-i\omega t} dt \\
 Q_n(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} q_n(t)e^{-i\omega t} dt
\]

(5.6)
\[
H_n(\omega) = \frac{1}{\omega_n^2 - \omega^2 + 2\xi_n \omega_n \omega} \tag{5.7}
\]

\(H_n(\omega)\) is the frequency response function of the \(n\)th mode. Performing again the Fast Fourier transformation on Equation (5.2), and substituting Equations (5.5) and (5.7) into the resultant equation, the Fourier transform of the dynamic deflection \(w(x,t)\) is obtained as

\[
W(x,\omega) = \sum_{n=1}^{\infty} \frac{1}{M_n} \Phi_n(x) H_n(\omega) P_n(\omega) \tag{5.8}
\]

### 5.2.3 Force Identification from Accelerations

The Fourier transform of the acceleration of the beam at point \(x\) is obtained from Equation (5.8) as

\[
\tilde{W}(x,\omega) = -\omega^2 \sum_{n=1}^{\infty} \frac{1}{M_n} \Phi_n(x) H_n(\omega) \Psi_n(m-k) P(k), \quad m=0,1, \ldots, N-1 \tag{5.9}
\]

Substituting the last relationship in Equation (5.4) into (5.9), and rewriting in discrete terms,

\[
\tilde{W}(m) = -\sum_{k=0}^{N-1} \sum_{n=1}^{\infty} \frac{\Delta f^2 m^2}{M_n} \Phi_n(x) H_n(m) \Psi_n(m-k) P(k), \quad m=0,1, \ldots, N-1 \tag{5.10}
\]

where \(\Psi_n\) is the Fourier transform of the \(n\)th mode shape; \(\Delta f\) is the frequency resolution and \(N\) is the number of data sample in the FFT; \(k\) and \(m\) denote the \(k\)th and the \(m\)th term in the FFT; and \(P\) is the Fourier transform of the moving force \(P(t)\). Let

\[
\tilde{H}_n(m) = -\frac{\Delta f^2 m^2}{M_n} \Phi_n(x) H_n(m) \tag{5.11}
\]

then Equation (5.10) can be rewritten in terms of the real \(P_R(k)\) and imaginary \(P_I(k)\) parts of the Fourier transforms \(P(k)\)

\[
\tilde{W}(m) = \sum_{k=0}^{N/2-1} \sum_{n=1}^{\infty} \tilde{H}_n(m) \Psi_n(m-k) [P_R(k) + iP_I(k)] + \\
+ \sum_{k=N/2}^{N-1} \sum_{n=1}^{\infty} \tilde{H}_n(m) \Psi_n(m-k) [P_R(N-k) - iP_I(N-k)] \tag{5.12}
\]

Considering the periodic property of the DFT, Equation (5.12) can be written as
\[ \tilde{W}(m) = \sum_{n=1}^{\frac{N}{2}-1} \tilde{H}_{mn}(m) \Psi_n(m) \left( P_R(0) + i P_I(0) \right) + \]
\[ + \sum_{k=1}^{\frac{N}{2}-1} \sum_{n=1}^{\frac{N}{2}-1} \tilde{H}_{mn}(m) \left[ \Psi_n(m-k) + \Psi_n(m+k-N) \right] P_R(k) + \]
\[ + i \sum_{k=1}^{\frac{N}{2}-1} \sum_{n=1}^{\frac{N}{2}-1} \tilde{H}_{mn}(m) \left[ \Psi_n(m-k) - \Psi_n(m+k-N) \right] P_I(k) + \]
\[ + \sum_{n=1}^{\frac{N}{2}} \tilde{H}_{mn}(m) \Psi_n(m-N/2) \left[ P_R(N/2) - i P_I(N/2) \right] \]
\[ m = 0, 1, \ldots, N-1 \]  
(5.13)

Rewriting Equation (5.13) in matrix form
\[ \tilde{W}_{(N-2)\times 1} = A_{(N-2)\times (N-2)} P_{(N-2)\times 1} \]  
(5.14)

where \( \tilde{W} \) and \( P \) are Fourier transforms of the acceleration vector \( \tilde{w} \) and the force vector \( p \) respectively. Writing \( P \) as its real and imaginary parts \( P_R \) and \( P_I \),
\[ \tilde{W} = (A_{rr} + i A_{ri}) P_R + i (A_{ri} + i A_{ri}) P_I \]  
(5.15)

Again, separating \( \tilde{W} \) into real and imaginary parts \( \tilde{W}_R \) and \( \tilde{W}_I \), we have
\[ \begin{bmatrix} \tilde{W}_R \\ \tilde{W}_I \end{bmatrix}_{(N-2)\times 1} = \begin{bmatrix} A_{rr} & -A_{ri} \\ A_{ri} & A_{rr} \end{bmatrix}_{(N-2)\times (N-2)} \begin{bmatrix} P_R \\ P_I \end{bmatrix}_{(N-2)\times 1} \]  
(5.16)

Since the Fourier transforms of the imaginary parts of vectors \( \tilde{w} \) and \( p \) equal to zero in the following terms, \( P_i(0) = 0, P_i(N/2) = 0, \tilde{W}_i(0) = 0, \tilde{W}_i(N/2) = 0 \), Equation (5.16) can be condensed into a set of \( N \) number of simultaneously equations as
\[ \tilde{W}_{RI} = A_{RI} P_{RI} \]  
(5.17)

Components \( P_R \) and \( P_I \) can be found from Equation (5.17) by solving the \( N \)th order linear equation. The time history of the moving force \( P(t) \) can then be obtained by performing the inverse Fourier transformation on \( P_{RI} \). The solution is obtained in the frequency domain. However the computation cost for solving Equation (5.17) is high as it involves finding the inverse of a full matrix, and therefore the following procedure in time domain is developed to overcome these difficulties.

### 5.2.4 Solution in Time Domain
If the DFTs are expressed in matrix form, the Fourier transform $P$ of the force vector $p$ can be written as follows if the terms in $p$ are real (Bendat and Piersol, 1993).

$$P = \frac{1}{N} Up$$  (5.18)

where

$$U = e^{-i2k\pi/N}$$  (5.19)

and

$$k = \begin{bmatrix}
0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 1 & 2 & \cdots & N-2 & N-1 \\
0 & 2 & 4 & \cdots & N-4 & N-2 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & N-2 & N-4 & \cdots & 4 & 2 \\
0 & N-1 & N-2 & \cdots & 2 & 1
\end{bmatrix}_{N \times N}$$

The matrix $U$ is an unitary matrix, which means

$$U^{-1} = (U^*)^t$$  (5.20)

where $U^*$ is a conjugate of $U$. Substituting Equation (5.18) into (5.14),

$$\tilde{\mathbf{W}} = \frac{1}{N} A \left[ U^B_{N\times N_B} \theta \right] \left[ p_B \right]$$  (5.21)

or

$$\tilde{\mathbf{W}} = \frac{1}{N} A U^B_{N\times N_B} p_B$$  (5.22)

linking the Fourier transform of acceleration $\tilde{w}$ with that of the force vector $p_B$ of the moving forces in the time domain. $U_B$ is the sub-matrix of $U$, $N_B = L/(\gamma \Delta t)$ is the number of data point on the beam.

Using Equation (5.22) for identification has the advantage of weighting the response data in the frequency domain. The disadvantage is that the noises of the responses during the time interval $N_B \Delta t$ to $N \Delta t$ will affect the accuracy of the identified results. Equation (5.22) can be rewritten using Equation (5.18) to relate the accelerations and force vectors in the time domain as

$$\tilde{\mathbf{W}}_{N\times 1} = (U^*)^T A U^B_{N\times N_B} p_B$$  (5.23)
If \( N = N_B \), \( p_B \) can be found by solving the \( N \)th order linear equation in Equation (5.22) or (5.23). If \( N > N_B \) or more than one acceleration are measured, the least squares method can be used to find the time history of the moving force \( P(t) \).

If only \( N_c \) (\( N_c \leq N \)) response data points of the beam are used, the equations based on these data points in Equations (5.22) and (5.23) are extracted, and described as

\[
\tilde{W} = \frac{1}{N} A U B p_B \\
\tilde{W}_c = \left(U^*_c\right)^T A U_B p_B
\]

\[\text{(5.24)}\]

In practice, more than one acceleration measurement can be used to identify a single moving force for a higher accuracy.

### 5.2.5 Identification from Bending Moments and Accelerations

Similarly, we can find the relationships between the bending moments \( m \), its Fourier transform \( M \), and the moving force \( p \) as,

\[
M = \frac{1}{N} B U p_B
\]

\[\text{(5.25)}\]

\[
m = \left(U^*_c\right)^T B U_B p_B
\]

\[\text{(5.26)}\]

\[
m_B = \left(U^*_c\right)^T B U_B p_B
\]

\[\text{(5.27)}\]

where matrix \( B \) is similar to the matrix \( A \) in Equations (5.22) and (5.23) for the accelerations. The force vector \( p_B \) can be obtained from Equations (5.25), (5.26) and (5.27) depending on the length of measured responses with respect to \( N_B \). Equations (5.25), (5.26) and (5.27) can also be combined with Equations (5.22), (5.23) and (5.24) to form an over-determined set of equations when both bending moments and accelerations are used in the identification. The equations have to be scaled as shown below to have dimensionless unit before they are used.

\[
(U^*)^T \begin{bmatrix} B/m \\ A/\tilde{w} \end{bmatrix} U_B p_B = \begin{bmatrix} m/m \\ \tilde{w}/\tilde{\tilde{w}} \end{bmatrix}
\]

\[\text{(5.28)}\]

where \( \|\cdot\| \) is the norm of the vector.
5.2.6 Two Forces Identification

The above procedure is derived for single force identification. Equation (5.26) can be modified for two forces identification using the linear superposition principle as

\[
m = (U^*)^T \begin{bmatrix}
B_a & 0 \\
B_b & B_s \\
B_c & B_s
\end{bmatrix} \begin{bmatrix}
U_a \\
U_b \\
U_c
\end{bmatrix} \begin{bmatrix}
p_1 \\
p_2
\end{bmatrix}
\]  
(5.29)

where \( B_a [N_s \times (N_b - I)] \), \( B_b [(N - I - 2N_s) \times (N_b - I)] \), and \( B_c [N_s \times (N_b - I)] \) are sub-matrices of matrix \( B \). The first row of sub-matrices in the \( B \) matrix describes the state having the first force on beam after its entry. The second and third rows of sub-matrices describe the states having two forces on beam and one force on beam after the exit of the first force respectively. The whole matrix has a dimension of \((N-1) \times (N_b - I)\). \( N_s = l_t/(v \Delta t) \) and is the number of data sample when only the first force or the second force is on the beam. \( l_t \) is the distance between the two axles. The two forces can be identified using more than one measured bending moment measurements. We can also modified Equation (5.23) in a similar way for two forces identification using more than one measured acceleration measurements.

5.3 REGULARIZATION

5.3.1 Regularization of the Solution

We take the identification using measured bending moment \( m \) as an example. Since the identified force \( p \) is not a continuous function of the measured responses at the beginning and end of the time history, solution to Equation (5.26) is ill-conditioned (Morozov, 1984). A regularization method developed by Tikhonov (Tikhonov and Arsenin, 1977) is used to provide bounds to the solution. The Tikhonov regularization method is based on the radical idea that minimizes the deviations of \( U^T B U_{sp} p \) from the measured response vector \( m \) in Equation (5.26) for a stable solution by means of an auxiliary non-negative parameter. This is equivalent to imposing certain constraints in the form of added penalty terms with adjustable weighting (regularization) parameters to the solution. The Tikhonov function is written as follow.
\[ J(p_B, \lambda) = \| R p_B - m \|^2 + \lambda \| p_B \|^2 \]  
\hspace{1cm} (5.30)

where \( \lambda \) is the non-negative regularization parameter, and \( R = U^T B U_B \). The solution of Equation (5.26) is obtained in the Tikhonov regularization with the damped least squares method as (Santantamarina and Fratta, 1998)

\[ p_{B_{N_x+1}} = (R_{N_x+1}^T R_{N_x+1} + \lambda I)^{-1} R_{N_x+1}^T m_{N_x+1} \]  
\hspace{1cm} (5.31)

where \( I \) is the identity matrix, and singular value decomposition is used in the pseudo-inverse calculation.

### 5.3.2 Regularization Parameter \( \lambda \)

The main difficulty of applying the Tikhonov regularization lies in the method to find the optimal regularization parameter \( \lambda \). Two methods to find the optimal regularization parameter are discussed in this chapter. The use of which method depends on the availability of the true force. If the true forces were known, the true force \( p_B^{(True)} \) is compared with the identified values \( p_B^{(Identify)} \), and an error curve, the S-curve (Busby and Trujillo, 1997) can be plotted for different \( \lambda \) as shown in Figure 5.2. The error of identification in the force time history is

\[ \text{error} = \frac{\| p_B^{(Identify)} - p_B^{(True)} \|}{\| p_B^{(True)} \|} \times 100\% \]  
\hspace{1cm} (5.32)

and \( \| \cdot \| \) is the norm of a matrix. It is noted from Figure 5.2 that the optimal value of \( \lambda \) corresponds to the smallest error.

In the more practical case when \( p_B^{(True)} \) is unknown as in experiment, the L-curve proposed by Hansen (1992) is used to determine the optimal \( \lambda \) value. The L-curve is a plot of the seminorm of the solution against the residual norm. The norm of residuals \( E \) of the forces is calculated as

\[ E = \| R p_B^{(Identify)} - m \| \]  
\hspace{1cm} (5.33)

and for the first-order regularization proposed by Busby and Trujillo (1997), the seminorm of the estimated forces is

\[ E_1 = \| p_{B_{N_x+1}}^{(Identify)} - p_{B_{N_x+1}} \| \]  
\hspace{1cm} (5.34)
where \( p_{B,i}^{(\text{identify})} \) and \( p_{B,j}^{(\text{identify})} \) are the identified forces with \( \lambda_i \) and \( \lambda_j + \Delta \lambda \). Typical L-curves are plotted in Figure 5.3 for different noise level in the measured data, and they all exhibit a corner in each L-curve. The value of \( \lambda \) corresponds to the point immediately to the right of the corner is the optimal value.

### 5.4 Numerical Studies

The following parameters of the forces and structure are used in the study.

**Type I Force - Single moving force from a quarter vehicle model**

The quarter vehicle model has two lump masses \( m_1 = 4,000 \text{ kg} \) and \( m_2 = 36,000 \text{ kg} \) with springs \( k_1 = 7.2 \times 10^7 \text{ kg/m} \) and \( k_2 = 1.8 \times 10^7 \text{ kg/m} \) and dampers \( c_1 = c_2 = 14.4 \times 10^4 \text{ kg/sec/m} \) as shown in Figure 5.4. The two natural frequencies of the quarter model are 3.18 Hz and 23.93 Hz which are very close to the natural frequencies of the bridge beam as given below.

**Type II Force - Two moving forces**

\[
P_1(t) = 20000 \left[ 1 - 0.1 \sin(10\pi t) + 0.05 \sin(40\pi t) \right] \text{ N}
\]

\[
P_2(t) = 20000 \left[ 1 - 0.1 \sin(10\pi t) + 0.05 \sin(50\pi t) \right] \text{ N}
\]

\( l_s = 4 \text{ m} \)

The parameters of the beam are as follows:

\( EI = 1.274916 \times 10^{11} \text{ Nm}^2 \), \( \rho = 12000 \text{ kg/m} \), \( l = 40 \text{ m} \)

\( \xi_1 = 0.02 \), \( \xi_2 = 0.02 \), \( \xi_3 = 0.04 \), \( f_1 = 3.2 \text{ Hz} \), \( f_2 = 12.8 \text{ Hz} \), \( f_3 = 28.8 \text{ Hz} \).

where \( f_i \) and \( \xi_i \) are the natural frequency and damping ratio of the beam respectively.

A moving speed of 40 m/s is studied for both force types. The analysis frequency bandwidth is from 0Hz to 40Hz to include the first three modes of the beam in the calculation. Sampling frequency is 100 samples per second with \( N_B \) equals 100 points. The record length \( N \) is 512 points, and \( N_c \) equals 110 points are used in the identification. The moving forces are identified using Equations (5.23), (5.26) and (5.28) without regularization. Please note that in the case of two moving forces, an equal but opposite component exists in each force simulating the effect of pitching motion of a vehicle.

**5.4.1 Single Force Identification**
Bending moment and/or acceleration responses at 1/2 span and/or 1/4 span are used. The effectiveness of nine combinations of the responses in the force identification are studied as follows

\[ \begin{align*}
    &1/2m & 1/2a & 1/2m \& 1/2a \\
    &1/4m & 1/4a & 1/4m \& 1/4a \\
    &1/2m \& 1/4m & 1/2a \& 1/4a & 1/2m \& 1/4a
\end{align*} \]

where 1/2 and 1/4 represent the location of span, and \( m \) and \( a \) represent the bending moment and acceleration responses respectively. Random noise is added to the calculated responses to simulate the polluted measurements as

\[ \hat{\omega} = \hat{\omega}_{\text{calculated}} + E_p \times \| \hat{\omega}_{\text{calculated}} \| \times N_{ei} \]

where \( E_p \) is a specified error level; \( N_{ei} \) is a standard normal distribution vector with zero mean and unity standard deviation.

The simulated bending moment and acceleration at 1/4 span are shown in Figures 5.5 and 5.6 for 10% noise level. The Power Spectral Density Functions (PSD)s in the figures indicate that errors due to simulated random noise exist in the higher frequency range. This error level is approximately equivalent to 90 dB dynamic range which is of the same order as those in the measurement system (65 dB). Therefore this error level could represent typical values in practical situations.

Errors in the simulated forces computed from Equation (5.32) are shown in Table 1 for 1%, 5% and 10% noise level. The errors are small for all the cases with acceleration or acceleration and bending moment responses. Those using only bending moments give larger errors in general. This is because the bending moment responses in the high frequency range is very small (referring to Figure 5.5) causing large error. Results using responses from a single measurement point only are less accurate than those using responses from multi-points as some of the modal responses may have not been used in the identification. Some of the identified results are shown in Figures 5.7 and 5.8. The PSDs of the identified forces in these figures are close to the true one although the time histories are not. There are large discrepancies between the true and the identified forces from using only the 1/2 span acceleration at the time instance of 0.3 second. Since the measured responses consist of that from the first and third modes only at 1/2 span, this large variation is due to the low sensitivity of the responses to the moving force when it passes over the nodes of the third vibration mode of the beam.
5.4.2 Two Forces Identification

Bending moment and/or acceleration responses at 1/4, 1/2 and 3/4 spans in twelve combinations described in Table 5.2 are used to identify the two forces. The error study on the identified individual forces is extended to the combined total force in an attempt to assess the accuracy of weighing-in-motion of the weight of a moving vehicle on top of the beam. The following results are obtained in a similar way as those for the quarter vehicle model.

Errors in the identification result of individual forces and the total force are shown in Table 5.2 for 1%, 5% and 10% noise levels. Sensor combination Case (d) gives the least errors among all the twelve Cases. Cases with combined bending moments and accelerations give better results than those having only bending moments or accelerations. The combinations with at least two acceleration responses give the smaller errors. Table 5.2 also shows that bending moment response at 1/2 span is more useful than that collected at 1/4 span, while it is just the opposite for acceleration responses. The accuracy in the individual force identification is lower than that in the combined force identification. One reason is that there is a force component with same amplitude and opposite phase in the two individual forces.

Regularization is also applied to improve the identified results using the damped least squares method from Equation (5.31), and the optimal regularization parameter \( \lambda \) is calculated by the S-curve method. Samples of the time histories for combinations (a) (1/2m, 1/4m) and (c) (1/2a, 1/4a) with and without regularization are shown in Figures 5.9 and 5.10. Regularization has improved the results from strains greatly but not in those from accelerations. This is explained in the following discussions. The solution to Equation (5.31) is given by (Fierro, et al, 1997)

\[
f_x = \sum_{i=1}^{c} \frac{\sigma_i^2}{\sigma_i^2 + \lambda^2} \frac{u_{i,m}}{\sigma_i} v_i
\]

(5.35)

where \( u_i \) and \( v_i \) are components of matrices \( U \) and \( V \) in the singular value decompositon of matrix \( R \); \( \sigma_i \) is the singular value of matrix \( R \). The components of the solution corresponding to the small singular values of \( R \) are suppressed through this solution. When the singular values are in general small in the case from strains, all solutions are affected greatly in this equation with the presence of parameter \( \lambda \). There is a large smoothing effect to the whole time history as shown in Figure 5.9. The
singular values are much larger in the case from accelerations. The solutions are less affected by parameter \( \lambda \), and the smoothing effect is small as seen in Figure 5.10.

There is still some noise effect in the results from strains after regularization while the curves from accelerations are relatively smooth. This is because the noise is represented in the term \( m \) in Equation (5.35). The variation from noise in the results is dependent on the magnitude of singular value \( \sigma_i \) in the denominator. Matrix \( R \) from strains has small singular values in general, and the variation in the identified forces is therefore large.

The errors in the identified forces with and without regularization are compared in Table 5.3 for the case with 5% random noise. Significant reductions are observed in all cases particularly those having large errors obtained without regularization. All combinations of sensors give errors of the same order for both the first force and the second force. Table 5.3 also shows the errors obtained from the TDM (Law et al, 1997) without using the regularization. The FTDM is shown in general better than the TDM in all the cases.

5.5 LABORATORY STUDIES

The experimental setup is shown diagrammatically in Figure 5.11. The main beam, 3376mm long with a 100mm×25mm uniform cross-section, is simply supported. A U-shaped aluminum section is glued to the upper surface of the beam as a direction guide for the car. The model car is pulled along the guide by a string wound around the drive wheel of an electric motor. Seven photoelectric sensors are mounted on the beams to measure and monitor the moving speed of the car. They are located on the beam at roughly equal spacing of 0.776m to check on the uniformity of the speed. Three strain gauges and four accelerometers are mounted at the bottom of the main beam to measure the responses. One gauge and one accelerometer are mounted at each cross-section at the 1/4, 1/2 and 3/4 span. The fourth accelerometer is mounted at the 3/8 span. A Data Translation DT2829 eight-channel dynamic A/D board is used for data collection in the experiment. The sampling frequency is 256Hz. The model car has two axles at a space of 0.203m and it runs on four rubber wheels. The mass of the whole car is 7.1Kg. The average speed for the set of responses used below is 3.102m/s.
The first three modes are used in the identification. The moving forces are identified from 1/4a, 1/2a and 1/2m with and without regularization, and the optimal regularization parameter $\lambda$ is obtained using the L-curve method. The time histories of the identified forces with and without regularization are shown in Figure 5.12 and the resultant of the two forces is shown in Figure 5.13. The results from regularization vary around the static value, and a clear pitching motion of the vehicle can be observed from the time histories. The identified resultant force varies about the total static force of the vehicle with some high frequencies due to the measurement noise. This type of error can be removed by pre-processing of the measured responses before the identification. Figure 5.13 shows that the resultant force can be used to estimate closely the total weight of the passing vehicle.

The TDM is also used to obtain another set of forces with and without regularization, and the results are shown in Figures 5.14 and 5.15. Both the resultant force and the individual forces identified are slightly poorer than those obtained from the FTDM. This is again explained by the same reason presented for the two forces identification in simulation.

Comparison between Figures 5.12 and 5.14 shows that the FTDM give less accurate results than the TDM in the first half of the time histories. Regularization improves the FTDM curves but not the TDM curves. Since regularization only provides bound to the ill-conditioned solution without any smoothing effect on the measurement noise, this observation means the TDM is better than the FTDM in solving the ill-posed problem.

5.6 CONCLUDING REMARKS

A regularization method is applied to the Time Domain Method and Frequency and Time Domain Method in the moving force identification. The results obtained are greatly improved over those without regularization with acceptable errors from using different combinations of measured responses. Time Domain Method is found better than the Frequency and Time Domain Method in solving for the ill-posed problem. Both simulation and laboratory test results indicate that the total weight of a vehicle can be estimated indirectly using moving force identification methods with some accuracy.
Table 5.1 Errors in Identified Force from Quarter Vehicle Model without Regularization

<table>
<thead>
<tr>
<th>Sensor Location</th>
<th>% error in Response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1%</td>
</tr>
<tr>
<td>1/2m</td>
<td>18.3</td>
</tr>
<tr>
<td>1/2a</td>
<td>0.5</td>
</tr>
<tr>
<td>1/2a&amp;1/2m</td>
<td>0.8</td>
</tr>
<tr>
<td>1/4m</td>
<td>11.8</td>
</tr>
<tr>
<td>1/4a</td>
<td>0.2</td>
</tr>
<tr>
<td>1/4a&amp;1/4m</td>
<td>0.2</td>
</tr>
<tr>
<td>1/4m&amp;1/2m</td>
<td>5.8</td>
</tr>
<tr>
<td>1/4a&amp;1/2a</td>
<td>0.1</td>
</tr>
<tr>
<td>1/4a&amp;1/4m&amp;1/2a&amp;1/2m</td>
<td>0.2</td>
</tr>
</tbody>
</table>

* denotes error exceeds 100%; m denotes bending moment; a denotes acceleration.

Table 5.2 Errors in Identified Forces without regularization

<table>
<thead>
<tr>
<th>Sensor Location</th>
<th>First Force</th>
<th>Second Force</th>
<th>Total Force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1%</td>
<td>5%</td>
<td>10%</td>
</tr>
<tr>
<td>Cases</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) 1/2m&amp;1/4m</td>
<td>97.6</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>(b) 1/2m&amp;1/4m&amp;3/4m</td>
<td>30.9</td>
<td>*</td>
<td>58.1</td>
</tr>
<tr>
<td>(c) 1/2a&amp;1/4a</td>
<td>43.3</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>(d) 1/2a&amp;1/4a&amp;3/4a</td>
<td>3.3</td>
<td>16.4</td>
<td>33.0</td>
</tr>
<tr>
<td>(e) 1/2m&amp;1/2a</td>
<td>63.4</td>
<td>*</td>
<td>49.1</td>
</tr>
<tr>
<td>(f) 1/2m&amp;1/4m&amp;1/2a</td>
<td>51.0</td>
<td>*</td>
<td>25.2</td>
</tr>
<tr>
<td>(g) 1/2m&amp;1/4m&amp;1/2a&amp;1/4a</td>
<td>25.7</td>
<td>*</td>
<td>9.5</td>
</tr>
<tr>
<td>(h) 1/4m&amp;1/4a</td>
<td>*</td>
<td>*</td>
<td>80.6</td>
</tr>
<tr>
<td>(i) 1/4m&amp;1/4a&amp;1/2a</td>
<td>35.2</td>
<td>*</td>
<td>10.5</td>
</tr>
<tr>
<td>(j) 1/2m&amp;1/4a</td>
<td>53.1</td>
<td>*</td>
<td>55.0</td>
</tr>
<tr>
<td>(k) 1/2m&amp;1/4a&amp;1/4m</td>
<td>58.6</td>
<td>*</td>
<td>41.0</td>
</tr>
<tr>
<td>(l) 1/4a&amp;1/2a&amp;1/2m</td>
<td>34.9</td>
<td>*</td>
<td>10.4</td>
</tr>
</tbody>
</table>

Note: * denotes error exceeds 100%.
### Table 5.3 Errors in Identified Forces with 5% noise with and without Regularization

<table>
<thead>
<tr>
<th>Sensor Location Cases</th>
<th>First Force Regularization</th>
<th>Second Force Regularization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>(a) 1/2m&amp;1/4m</td>
<td>455.2(+)</td>
<td>25.0</td>
</tr>
<tr>
<td>(b) 1/2m&amp;1/4m&amp;3/4m</td>
<td>154.5(+)</td>
<td>19.9</td>
</tr>
<tr>
<td>(c) 1/2a&amp;1/4a</td>
<td>216.7(222)</td>
<td>22.4</td>
</tr>
<tr>
<td>(d)1/2a&amp;1/4a&amp;3/4a</td>
<td>16.4(10.7)</td>
<td>8.7</td>
</tr>
<tr>
<td>(e) 1/2m&amp;1/2a</td>
<td>317.3(+)</td>
<td>24.5</td>
</tr>
<tr>
<td>(f) 1/2m&amp;1/4m&amp;1/2a</td>
<td>255.0(+)</td>
<td>22.7</td>
</tr>
<tr>
<td>(g) 1/2m&amp;1/4m&amp;1/2a&amp;1/4a</td>
<td>125.6(201)</td>
<td>21.5</td>
</tr>
<tr>
<td>(h) 1/4m&amp;1/4a</td>
<td>991.7(+)</td>
<td>22.6</td>
</tr>
<tr>
<td>(i) 1/4m&amp;1/4a&amp;1/2a</td>
<td>176.4(206)</td>
<td>21.4</td>
</tr>
<tr>
<td>(j) 1/2m&amp;1/4a</td>
<td>265.9(+)</td>
<td>24.7</td>
</tr>
<tr>
<td>(k) 1/2m&amp;1/4a&amp;1/4m</td>
<td>293.4(+)</td>
<td>22.3</td>
</tr>
<tr>
<td>(l) 1/4a&amp;1/2a&amp;1/2m</td>
<td>174.4(193)</td>
<td>23.5</td>
</tr>
</tbody>
</table>

Note: (*) is the result from Time Domain Method (Law et al. 1997)

(+) denotes error larger than 1000%

![Figure 5.1 Simply supported beam subjected to a moving force \( P(t) \)](image-url)

---

5-16
Figure 5.2 Typical S–curve (--- 1% noise; – 5% noise; ... 10% noise.)
Figure 5.3 Typical L-curves (--- 1% noise; – 5% noise; ... 10% noise.)
Figure 5.4 Quarter vehicle model
1/4-span bending moment 40m/s 10% noise

True   Simulation

PSD of the response 40m/s 10% noise

Figure 5.5 Bending moment at 1/4 span – simulation for single force identification
Figure 5.6 Acceleration at 1/4 span – Simulation for single force identification
Moving Force 40 m/s, 10% noise Id. from accelerations

PSD of the force 40 m/s, 10% noise Id. from accelerations

Figure 5.7 Identified force from acceleration

- Simulated single force identification
Moving Force 40m/s, 10% noise Id. from moments and acc.

PSD of the force 40m/s, 10% noise Id. from moments and acc.

Figure 5.8 Identified force from acceleration and bending moment
- Simulated single force identification
Figure 5.9 Identified forces in two-forces identification from 1/2m and 1/4m
(- true force; ... without regularization; —— with regularization.)
Figure 5.10 Identified forces in two-forces identification from 1/2a and 1/4a
(- true force; ... without regularization; --- with regularization.)
Figure 5.11 Diagrammatic Drawing of the Experimental Setup
Figure 5.12 Identified forces in experiment from 1/4a, 1/2a and 1/2m by FTDM
(— static force; ... without regularization; —— with regularization.)
Figure 5.13 Identified resultant force in experiment from FTDM
(– static force; ... without regularization; —— with regularization.)
Figure 5.14 Identified forces in experiment from 1/4a, 1/2a and 1/2m by TDM
(- static force; ... without regularization; ---- with regularization.)
Figure 5.15 Identified resultant force in experiment from TDM
(- static force; ... without regularization; ---- with regularization.)
Chapter 6

IDENTIFICATION OF MOVING LOADS ON MULTI-SPAN CONTINUOUS BRIDGE — EULER-BERNOULLI BEAM MODEL

6.1 INTRODUCTION

Two new methods are proposed to overcome the deficiencies exhibited in existing methods. In last chapter, the regularization technique is applied to the Time Domain Method (TDM) and Frequency and Time Domain Method (FTDM) in the moving force identification. The results obtained are greatly improved over those without regularization from using different combinations of measured responses. However, it is difficult to use these two methods to identify vehicles with multiple axles or vehicles on multi-span continuous bridge due to the long computational time and large computer capacity. Most of the computational time is spent on the computation of the system matrix on the right-hand-side of Equations (5.23) and (5.26). Therefore, a new time domain method based on regularization technique is developed to identify moving loads on a continuous beam from the measured structural vibration responses in this chapter. This method gives exact solutions to the forces with improved formulation over existing methods for a more efficient computation. Regularization technique is used to provide bound to the solution.

Compared with TDM and FTDM, the computational time of the Interpretive Method (ITM) (Chan et al, 1999) is not too long, but the identification accuracy is much lower than that by TDM and FTDM. Large errors in the identified results are induced from the direct derivatives of the bridge modal responses in ITM. A general method based on the finite element formulation is developed to identify moving loads on a continuous beam in this chapter. A generalized orthogonal function approach is proposed to obtain the derivatives of the bridge modal responses. The moving loads are identified using least squares method with regularization on the equation of motion in the time domain.

Numerical examples of moving forces on both single and multiple span bridges and the case of axle interaction forces from a four-DOFs vehicle on a triple-span bridge are used to demonstrate the feasibility and accuracy of the two methods,
and factors affecting the errors in the identification are discussed. Computational simulations show that the methods are effective for identifying moving loads on continuous bridges.

6.2 MOVING LOAD IDENTIFICATION BASED ON EXACT SOLUTION

6.2.1 Equation of Motion

A continuous beam subjected to a system of moving forces \( P_i(t) \) \( (i=1,2,...,N_p) \) is shown in Figure 6.1. The forces are assumed moving as a group at a prescribed velocity \( v(t) \), along the axial direction of the beam from left to right. The beam is assumed to be an Euler-Bernoulli beam. The equation of motion can be written as

\[
\rho A \frac{\partial^2 w(x,t)}{\partial t^2} + C \frac{\partial w(x,t)}{\partial t} + EI \frac{\partial^4 w(x,t)}{\partial x^4} = \sum_{i=1}^{N_p} P_i(t) \delta(x - \hat{x}_i(t))
\]  

(6.1)

where \( L \) is the total length of the beam; \( A \) is the cross-sectional area; \( E \) is the Young’s modulus; \( I \) is the moment of inertia of the beam cross-section; \( \rho \), \( C \) and \( w(x,t) \) are the mass per unit length, the damping and the displacement function of the beam respectively; \( \hat{x}_i(t) \) is the location of moving force \( P_i(t) \) at time \( t \); \( \delta(t) \) is the Dirac delta function and \( N_p \) is the number of forces. Express the transverse displacement \( w(x,t) \) in modal coordinates

\[
w(x,t) = \sum_{i=1}^{N} \phi_i(x) q_i(t)
\]  

(6.2)

where \( \phi_i(x) \) is the mode shape function of the \( i \)th mode, which is determined from the eigenvalue and eigenfunction analysis proposed by Hayashikawa and Watanabe (1981) as shown in Appendix C; \( q_i(t) \) is the \( i \)th modal amplitude. Substituting Equation (6.2) into Equation (6.1), and multiplying by \( \phi_i(x) \), integrating with respect to \( x \) between 0 and \( L \), and applying the orthogonality conditions, we obtain

\[
\frac{d^2 q_i(t)}{dt^2} + 2 \xi_i \omega_i \frac{dq_i(t)}{dt} + \omega_i^2 q_i(t) = \frac{1}{M_i} \sum_{i=1}^{N_p} P_i(t) \phi_i(\hat{x}_i(t))
\]  

(6.3)

where \( \omega_i, \xi_i, M_i \) are the modal frequency, the damping ratio and the modal mass of the \( i \)th mode, and

\[
M_i = \int_0^L \rho A \phi_i^2(x) dx
\]  

(6.4)
The displacement of the beam at point \( x \) and time \( t \) can be found from Equations (6.2) and (6.3).

\[
w(x,t) = \sum_{i=1}^{N_e} \frac{\phi_i(x)}{M_i} \int h_i(t-\tau) \sum_{j=1}^{N_s} P_j(\tau)\dot{\phi}_i(\tau) d\tau
\]  
(6.5)

where

\[
h_i(t) = \frac{1}{\omega'_i} e^{-\xi_i\omega'_i t} \sin \omega'_i t; \quad \omega'_i = \omega_i \sqrt{1 - \xi_i^2}
\]  
(6.6)

From the Strains

The strain in the beam at point \( x \) and time \( t \) can be written as.

\[
\varepsilon(x,t) = -z_i \frac{\partial^2 w(x,t)}{\partial x^2}
\]  
(6.7)

where \( z_i \) is the distance between the under surface and the neutral surface of the beam.

Substituting Equation (6.5) into Equation (6.7), and re-write in discrete form.

\[
\varepsilon(x_i,m) = -\sum_{i=1}^{N_e} z_i \phi_i^{\prime\prime}(x_i) \frac{\Delta t}{M_i} \sum_{j=0}^{m} h_i(m-j) \sum_{j=1}^{N_s} P_j(j)\dot{\phi}_i(\tau)\\
(m = 0,1,2,\ldots,N_i; s = 1,2,\ldots,N_s)
\]  
(6.8)

where \( \Delta t \) is the time interval; \( N \) is the number of vibration modes; \( N_i \) is the number of data points; \( x_i \) is the location of the measuring point; \( N_s \) is the number of measuring points.

\[
h_i(j) = \frac{1}{\omega'_i} e^{-\xi_i\omega'_i \Delta t} \sin \omega'_i j\Delta t
\]  
(6.9)

Equation (6.8) can be re-written in matrix form

\[
BP = \varepsilon
\]  
(6.10)

where \( \varepsilon \) is \((N_i \times N_s) \times 1\) matrix; \( B \) is \((N_i \times N_s) \times (N_s \times N_p)\) matrix; \( P \) is \((N_i \times N_p) \times 1\) matrix,

\[
\varepsilon = \{\varepsilon(x_1,1),\varepsilon(x_2,1),\ldots,\varepsilon(x_{N_s},1),\varepsilon(x_1,2),\ldots,\varepsilon(x_{N_s},N_s)\}^T
\]  
(6.11)

\[
P = \{p_{1}(0), p_{2}(0), \ldots, p_{N_s}(0), p_{1}(1), \ldots, p_{N_s}(N_s-1)\}^T
\]

When the measured data is more than the number of unknown forces, Equation (6.10) can be solved using the least squares method. However the solution would involves the computation of the inverse of matrix \( B \) which would be very inefficient when the
measured data is large (Law et al. 1997,1999). Matrix $B$ can be split into smaller sub-matrices to improve the computation efficiency as follows

$$
B = \begin{bmatrix}
B_{10} & 0 & \cdots & 0 \\
B_{20} & B_{21} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
B_{N,0} & B_{N,1} & \cdots & B_{N,N-1}
\end{bmatrix}
; \quad B_{ij} = \begin{bmatrix}
b_{11} & b_{12} & \cdots & b_{1Nc} \\
b_{21} & b_{22} & \cdots & b_{2Nc} \\
\vdots & \vdots & \ddots & \vdots \\
b_{N,1} & b_{N,2} & \cdots & b_{N,Nc}
\end{bmatrix}
$$

$$
b_{ij} = z_i \Delta t \sum_{j=1}^{Nc} \frac{\phi''(x_i)}{M_j} h_i(m-j) \phi_s(\hat{x}_i(j))
$$

$$
(m = 1,2,3,\ldots, N_s; j = 0,1,2,\ldots, N_t - 1; s = 1,2,\ldots, N_s; l = 1,2,\ldots, N_p.)
$$

(6.12)

From the Accelerations

The acceleration at a point $x$ and time $t$ can be obtained from Equation (6.5) as

$$
\ddot{w}(x,t) = \sum_{i=1}^{N_s} \frac{\phi_i(x)}{M_i} \left[ \sum_{j=1}^{N_t} P_j(t) \phi_j(\hat{x}_i(t)) + \int h_i''(t-\tau) \sum_{j=1}^{N_t} P_j(\tau) \phi_j(\hat{x}_i(\tau)) d\tau \right]
$$

(6.13)

where

$$
h_i''(t) = \frac{1}{\omega_i^2} e^{-\zeta_i\omega_i t} \left\{ (\zeta_i \omega_i)^2 - \omega_i^2 \right\} \sin \omega_i t - 2 \zeta_i \omega_i \omega_i \cos \omega_i t - j \Delta t
$$

(6.14)

The acceleration at measuring point $x$, is written in discrete forms to include the $N$ modes.

$$
\ddot{w}(x_i,m) = \sum_{i=1}^{N_s} \frac{\phi_i(x)}{M_i} \left[ \sum_{j=1}^{N_t} P_j(m) \phi_j(\hat{x}_i(m)) + \sum_{j=0}^{N_t} h_i''(m-j) \sum_{j=1}^{N_t} P_j(\hat{x}_i(\hat{x}_i(m))) \Delta t \right]
$$

(6.15)

where $N_i+1$ is the total number of sampling points, and

$$
h_i''(j) = \frac{1}{\omega_i^2} e^{-\zeta_i\omega_i j \Delta t} \left\{ (\zeta_i \omega_i)^2 - \omega_i^2 \right\} \sin \omega_i j \Delta t - 2 \zeta_i \omega_i \omega_i \cos \omega_i j \Delta t
$$

(6.16)

Rewrite Equation (6.15) in matrix forms.

$$
\ddot{w} = DP
$$

(6.17)

where

\[ D = \begin{bmatrix}
\end{bmatrix}
\]

6-4
\[ \ddot{w} = \{ \ddot{w}(x_1, y_1, 1), \ddot{w}(x_2, y_2, 1), \ldots, \ddot{w}(x_{N_x}, y_{N_x}, 1), \ddot{w}(x_1, y_1, 2), \ldots, \ddot{w}(x_{N_x}, y_{N_x}, N) \}^T; \]

\[ P = \{ p_1(0), p_2(0), \ldots, p_{N_p}(0), p_1(1), \ldots, p_{N_p}(N-1) \}^T; \]

(6.18)

and again matrix \( D \) is split into smaller sub-matrices as follows

\[
D = \begin{bmatrix}
D_{11} & 0 & \cdots & 0 \\
D_{21} & D_{22} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
D_{N_1,0} & D_{N_1,1} & \cdots & D_{N_1,N_{1-1}}
\end{bmatrix}, \quad D_{ij} = \begin{bmatrix}
d_{11} & d_{12} & \cdots & d_{1N_p} \\
d_{21} & d_{22} & \cdots & d_{2N_p} \\
\vdots & \vdots & \ddots & \vdots \\
d_{N_{1},1} & d_{N_{1},2} & \cdots & d_{N_{1},N_p}
\end{bmatrix}_{N_x \times N_p}
\]

\[ d_u' = \Delta t \sum_{i=1}^{N} \frac{\phi_i(x_s)}{M_i} h_i'''(m-j)\phi_i(\hat{x}_s(j)) \]

\[ (m = 1, 2, \ldots, N_i; \quad j = 0, 1, 2, \ldots, N_i - 1; \quad s = 1, 2, \ldots, N_i; \quad l = 1, 2, \ldots, N_p) \]

When \( j < m \),

\[ d_u = d_u' \]

When \( j = m \),

\[ d_u = d_u' + \sum_{i=1}^{N} \frac{\phi_i(x_s)}{M_i} \phi_i(\hat{x}_s(m)) \]

(6.19)

The computation with smaller matrices as shown above is much more efficient than existing methods (Law et al. 1997, 1999) working with a single large matrix.

6.2.2 Statement of Problem

The natural frequencies and mode shapes obtained from modal testing and modal analysis are subject to measurement errors. Noise contamination in the test data has adverse effect on the accuracy of the identified moving loads (Law et al. 1997, 1999). If \( N_x \geq N_p \), the moving loads can be identified from Equations (6.10) and (6.17) by least squares method. But the solutions would be unstable in the sense that small perturbations in the responses would result in large deviation from their exact solutions, and this is due to the ill-conditioning of matrix \( B \) or \( D \), which increases as the dimension of the problem increases. Hence general regularization methods based on singular value decomposition, cross-validation (Golub et al. 1979) and L-Curves (Hansen, 1992; Hansen and O'Leary, 1993) are studied with an attempt to overcome the ill-conditioned problems. The problem can be formulated as

\[
\text{minimize} \quad (r - AP)^T R (r - AP)
\]
subject to \((SP)^T SP = e\) \hspace{1cm} (6.20)

where \(R\) is an error-weighting matrix depending on the measured information. Since some data are of small amplitude and therefore are highly affected by systematic errors; some data are gathered at a time when the background noise is high. \(R\) is usually taken as the inverse of the covariance of the measured data. \(S\) is a smoothing matrix, which is typically either the identity matrix or a discrete approximation to a derivative operator (Santantamarina and Fratta, 1998). \(e\) denotes a residual scalar estimation error. \(r\) is the response, which may be \(e\) or \(\bar{w}\). \(A\) is the coefficient matrix \(B\) or \(D\).

The Lagrangian expression on the problem becomes

\[
J(P, \lambda) = (r - AP, R(r - AP)) + \lambda (SP, SP)
\]

where \(\lambda\) is the regularization parameter; The first term on the right-hand-side is the Euclidian scalar product.

The moving loads can be obtained from Equation (6.21) as

\[
P = (A^T RA + \lambda S^T S)^{-1} A^T r
\]

(6.22)

Both \(S\) and \(R\) are taken as the identity matrix in this chapter, which means a minimum norm in the solution and no prior information on the measured data is available respectively. Two terms can be derived from Equation (6.21) as follows using the singular value decomposition:

\[
E_1^2 = (r - AP, r - AP) = \sum_{i=1}^{N_{P}} \left| \sigma_i (v_i^T P - (u_i^T r)) \right|^2 + \sum_{i=N_{P}+1}^{N_{S}} \left| u_i^T r \right|^2
\]

(6.23)

\[
E_2^2 = (P, P) = \sum_{i=1}^{N_{S}} \left| v_i^T P \right|^2
\]

(6.24)

where \(A = USV^T\), \(U = \{ u_1^T, u_2^T, \ldots, u_{N_p}^T \}\) and \(V = \{ v_1^T, v_2^T, \ldots, v_{N_s}^T \}\) are the right and left singular vectors respectively, and \(\Sigma = \text{diag}\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_{N_p+N_s}^2\}\) are the singular values with \((\sigma_1 \geq \sigma_2 \geq \ldots, \sigma_{N_p+N_s} \geq 0)\).

Substitute Equations (6.23) and (6.24) into Equation (6.21), the solution \(P_\lambda\) can be obtained as

\[
P_\lambda = \sum_{i=1}^{N_p} \frac{\sigma_i^2}{(\sigma_i^2 + \lambda)} \sigma_i^{-1}(u_i^T r)v_i
\]

(6.25)
When \( \lambda = 0 \), \( P_\lambda \) is the least squares solution and the noise effect will be amplified when \( \sigma_i < u_i^\top r \). When \( \lambda > 0 \), this formulation can reduce the influence of the components corresponding to those singular values \( \sigma_i \) which are smaller than \( \lambda \), so that the solution is less noise sensitive. Substitute Equation (6.25) into Equations (6.23) and (6.24), we have

\[
E_1^2 = \sum_{i=1}^{N_r \times N_s} \frac{\lambda^2}{(\sigma_i^2 + \lambda)^2} ||u_i^\top r||^2 + \sum_{i=1}^{N_r \times N_s} ||u_i^\top r||^2
\]

(6.26)

\[
E_2^2 = \sum_{i=1}^{N_r \times N_s} \frac{\sigma_i^2}{(\sigma_i^2 + \lambda)^2} ||u_i^\top r||^2
\]

(6.27)

where \( E_1^2 \) is a increasing function of \( \lambda \), whereas \( E_2^2 \) is a decreasing function of \( \lambda \). and thus an optimal regularization parameter exists with balance contribution from the two different errors.

If the true loads are known, the optimal parameter can be obtained by minimizing the error between the true loads and the estimate loads. However the true loads are not known in practice. The generalized cross validation and L-curve method are employed to determine the optimal regularization parameter.

### 6.2.3 Determination of Regularization Parameter

**Generalized Cross Validation**

The basic idea of cross-validation is as follows: One input data point \( r_k (k = 1, 2, \cdots, N_s) \) is left out at a time, and a solution of \( P_i(k) \) is determined from the remaining data points. Then the estimate of \( r_k \) computed from \( P_i(k) \) must be a good estimate. While ordinary cross-validation depends on the particular ordering of the data, generalized cross-validation is independent of the orthogonal transformation of the data vector. The generalized cross-validation (GCV) function is defined by Golub et al. (1979) as

\[
GCV(\lambda) = \frac{1}{N_s \cdot N_t} \frac{||AP_\lambda - r||^2}{(\frac{1}{N_s \cdot N_t} \text{trace}(I - A(\lambda))))^2}
\]

(6.28)

where \( A(\lambda) = A^T (A + \lambda I)^{-1} A^T \).
Since the matrix $A$ is large, it is very difficult to perform the computation directly. Equations (6.10) and (6.17) show that matrices $B$ and $D$ are lower triangular matrices, and $S$ and $R$ equal to $I$ in Equation (6.23). It is therefore possible to write

$$J(P, \lambda) = \sum_{i=1}^{N_r} \left( \left\| \sum_{j=1}^{N_i} A_{ji} P_i - r_i \right\| + \lambda \left\| P_i \right\| \right)^2$$

(6.29)

where $A = \begin{bmatrix} A_{11} & 0 & \cdots & 0 \\ A_{21} & A_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{N_r 1} & A_{N_r 2} & \cdots & A_{N_r N_r} \end{bmatrix}$

and $A_{ji} (i = 1, 2, \ldots, N_r; j = 1, 2, \ldots, N_i)$ are $N_S \times N_P$ matrices. The solution on the force $P$ can be obtained from Equation (6.29) as

$$P_{ji} = (A^T_{ji}A_{ji} + \lambda I)^{-1} A^T_{ji} r_i$$

$$P_{ji} = (A^T_{ji}A_{ji} + \lambda I)^{-1} A^T_{ji} (r_i - \sum_{k=1}^{N_r} A_{rk} P_k), \quad (j = 2, 3, \ldots, N_r)$$

(6.30)

Equation (6.28) can be re-written as follows.

$$GCV(\lambda) = \frac{1}{N_S \times N_I} \sum_{i=1}^{N_r} \sum_{j=1}^{N_i} \left\| A_{ji} P_{ji} - r_i \right\|^2$$

$$\left( \frac{1}{N_I \times N_I} \sum_{i=1}^{N_r} \text{trace}(I - A_{ii}(\lambda)) \right)^2$$

(6.31)

where $A_{ii}(\lambda) = A_{ii}(A_{ii}^T A_{ii} + \lambda I)^{-1} A_{ii}^T$.

The moving loads can then be determined from Equations (6.30) and (6.31).

**L-Curves**

The L-curve (Hansen, 1992) is a plot of $||P||$ versus $||AP - r||$ parameterized by $\lambda$. If the norm of the random noise is less than the norm of the unperturbed data vector and the discrete Picard condition is satisfied, the plot shows a sharp turn in the curve when the value of $\lambda$ is close to the optimal value $\lambda_{opt}$. Hansen and O’Leary (1993) specify the optimal regularization parameter, $\lambda_{opt}$, as the regularization parameter with maximum curvature at the corner of the log-log plot of the L-curve.

**6.2.4 Implementation and Simulation Studies**

**6.2.4.1 Verification of the proposed method**
A single span simply supported beam with two forces moving on top is studied in this section. Applications to multiple span beam will be discussed in Section 6.4.1.

\[
\begin{align*}
    p_1(t) &= 20000[1 + 0.1\sin(10\pi t) + 0.05\sin(40\pi t)] \\
    p_2(t) &= 20000[1 - 0.01\sin(10\pi t) + 0.05\sin(50\pi t)]
\end{align*}
\] (6.32)

The parameters of the beam are: \( EI = 1.274916 \times 10^{11} Nm^2 \), \( \rho A = 12000 Kg/m \), \( L = 40m \). The moving speed is \( 40m/s \). The distance between the two moving forces is \( 4.27m \). The first six natural frequencies of the beam are \( 3.2Hz, 12.8Hz, 28.8Hz, 51.2Hz, 80.0Hz \) and \( 115.2Hz \). White noise is added to the calculated responses of the beam to simulate the polluted measurements and 1, 5, 10 and 20 percent noise levels are studied with

\[
\begin{align*}
    \varepsilon &= \varepsilon_{\text{calculated}} + E_p \ast N_{\text{noise}} \ast Var(\varepsilon_{\text{calculated}}) \\
    \ddot{w} &= \ddot{w}_{\text{calculated}} + E_p \ast N_{\text{noise}} \ast Var(\varepsilon_{\text{calculated}})
\end{align*}
\] (6.33)

where \( \ddot{w} \) and \( \varepsilon \) are the acceleration and strain, respectively. \( E_p \) is the noise level. \( N_{\text{noise}} \) is a standard normal distribution vector with zero mean value and unit standard deviation. \( \ddot{w}_{\text{calculated}}, \varepsilon_{\text{calculated}} \) are the calculated acceleration and strain, and \( Var(\ddot{w}_{\text{calculated}}), Var(\varepsilon_{\text{calculated}}) \) are their standard deviations.

The errors in the identified forces are calculated as

\[
\text{Error} = \frac{\|P_{\text{identified}} - P_{\text{true}}\|}{\|P_{\text{true}}\|} \times 100\% \quad (6.34)
\]

The first six modes are used in the simulation. The time interval between adjacent data is \( 0.002s \). Six measuring points are evenly distributed on the beam at \( 1/7L \) spacing, at which both acceleration and strain are measured. Figure 6.2 shows the identified results with 5% noise level in the responses. The curves obtained from accelerations are more accurate than those from strain with less fluctuations.

The variation of errors is plotted against the regularization parameters in Figure 6.3. The error increases for a given \( \lambda \) with increases in the noise level. The optimal \( \lambda \) also increases for increasing noise level. This means that a larger regularization parameter should be used to reduce the noise effect in the solution. Since the true loads are known, the curves in Figure 6.3 can be used to determine the optimal regularization parameter. Figures 6.4 and 6.5 show the GCV function and the L-curve, respectively. A wide range of minimum is available from the GCV function.
so it is very difficult to localize the minimum value. But the optimal value can be obtained easily from the corner of the L-curve. All the above three methods yield approximately the same regularization parameter. In practice, the GCV and L-curve criteria can be used to obtain the optimal value and they are also robust against random noise.

**Comparison with Existing Methods**

Table 6.1 shows the errors in the identified forces obtained from Time Domain Method (TDM), (Law et al. 1997), Frequency and Time Domain Method (FTDM) (Law et al. 1999) and the regularization method proposed in this chapter. The parameters of the problem are the same as in last section. The first three modes are used in the calculation. The bending moments, strains and accelerations at $1/4L$, $1/2L$ and $3/4L$ are used in the identification. The errors from using TDM or FTDM are larger and more sensitive to the noise level in the responses than the proposed method. The regularization technique is seen to be an effective tool to solve the ill-posed problem in the moving load identification.

**6.2.4.2 Special Aspects in Implementation**

More studies are performed to ensure the proposed method to be a practical tool in the moving load identification. The following aspects are studied with the same load or beam system described in last section.

**Sampling Frequency**

The sampling frequency is an important parameter in the identification. Figure 6.6 shows the accelerations and strains computed from using different time intervals in Equations (6.10) and (6.17) respectively. The strain time histories remain almost the same as long as the sampling frequency is larger than or equal to twice the maximum frequency of the vibration modes used in the calculation and the frequency components in the moving loads. But the acceleration time histories are very different when the time interval is large. This is due to the large computational error in Equation (6.16). This is because the dynamic behavior of the beam under moving loads is dominated by transient responses. The impulsive response components in the strains in Equation (6.9) decreases quickly as the frequency increases and that of the
acceleration in Equation (6.14) is only slightly affected due to the presence of \( \omega \) in the first term on the right-hand-side. So there are more high frequency components in the acceleration than in the strain. This means large sampling frequency should be used with accelerations. But in practice, the signal to noise ratio is very small in the high frequency range, and it is therefore difficult to identify the moving loads from accelerations.

Effect of Different Amount of Modal Information in Identification

The effect of using different amount of modal information in the responses on the identification is studied. Table 6.2 shows the errors in the moving loads identified using different number of modes in the responses. The number of vibration modes in the responses is taken equal to the number of modes in the calculation in Equations (6.8) and (6.15). The number of measuring points is selected to be the same as the number of modes and they are evenly located on the beam. The errors in the identified moving loads decrease as the number of modes in the identification increases and no further reduction in the error can be achieved with the number of sensor greater than seven. This remaining amount of error is due to the existence of large variations at the beginning and end of the identified load histories.

Modal Truncation in Computation

In practice the number of vibration modes in the responses is much larger than the number of modes used in the calculation, and the effect of modal truncation in the computation is studied. The “measured” data contains responses from the first six modes and no noise effect is included. Six measuring points are used and they are evenly located on the beam. Table 6.3 shows the identified errors from strains and accelerations with different number of mode shapes in the computation. When six modes are used in the identification, close to zero errors can be found in the identified forces indicating the accuracy of the proposed method as an exact solution. (The small error shown is due to discretization of the mode shapes and the zero force assumption at the first and last point of the force history).

The modal truncation effect is again studied in terms of the identification of a constant force, a force varying about zero mean and an ordinary force as described in Equation (6.32) using the same beam arrangement. The identified results on the
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constant component and the varying component of the forces in Equation (6.32) and the whole forces are shown in Figure 6.7. Only the first three modes are used in the computation. Observations indicate the following intermediate conclusions:

1) When the number of the mode shapes included in the identification is not less than three, acceptable results can be obtained from both strains and accelerations. This is because the third natural frequency (28.8 Hz) is already larger than the highest frequency of the moving loads (2.5 Hz). The first three modes constitute the main components in the responses.

2) The errors in the identified loads from accelerations are more sensitive to the number of modes than that from strains. This also means that more mode shapes should be used in the identification from accelerations. In practice, more modes are excited in the responses with impulses at the beginning and end of the time histories, especially in the case of constant moving loads. Therefore more mode shapes should be used in the identification from accelerations.

3) Figure 6.7 shows that there are errors in the identified results even if the first three modes have covered the frequency range of the moving loads. This is due to the fact that the responses from moving loads consist of transient responses represented by the exponential terms in Equations (6.6) and (6.14). This is different from and more complicated when compared with general stationary load identification.

**Effect of Modelling Errors**

In practice, the modal parameters are obtained by modal testing. If they are obtained through finite element analysis, they should be updated using the measured data, but some errors should have remained in the modal parameters. Figure 6.8 shows the identified results from strains and accelerations with 1% random noise in the modal frequencies and mode shapes. The first six modes are used in the identification. The figure shows that acceptable result can still be obtained when the error in the model is small, but identification from accelerations would suffer a large reduction in the accuracy. Table 6.4 shows the errors in the identification with different error magnitudes in the model and different noise levels in the responses. The error in the model is seen to be more important than the noise level in the responses. An accurate model is therefore a requirement in the moving load identification.
6.3 MOVING LOAD IDENTIFICATION BASED ON FINITE
ELEMENT FORMULATION

6.3.1 Generalized Orthogonal Function Expansion

The same assumptions of forces moving on top of a continuous beam as
discussed in Section 6.2.1 is studied. Substitute Equation (6.2) into Equation (6.7) and
assuming there are \( N \) modes in the responses, we have

\[
\varepsilon(x,t) = \varphi \Omega
\]

(6.35)

where

\[
\varphi = [\phi_1'(x), \phi_2''(x), \ldots, \phi_N''(x)]; \quad \Omega = [q_1(t), q_2(t), \ldots, q_N(t)]^T.
\]

and \( \phi_i''(x) \) is the second derivative of \( \phi_i(x) \):

The strain can be approximated by a generalized orthogonal function \( T(t) \) as

\[
\varepsilon(x,t) = \sum_{i=1}^{N} T_i(t) C_i(x)
\]

(6.36)

where \( \{T_i(t), i=1,2,\ldots,N \} \) is the generalized orthogonal function (Appendix D);
\( \{C_i(x), i=1,2,\ldots,N \} \) is the vector of coefficients in the expansion expression. The
strains at the \( N \) measuring points can be expressed as

\[
\varepsilon = C \cdot T
\]

(6.37)

where

\[
T = [T_1(t), T_2(t), \ldots, T_N(t)]^T;
\]

\[
\varepsilon = [\varepsilon(x_1,t), \varepsilon(x_2,t), \ldots, \varepsilon(x_N,t)]^T;
\]

\[
C = \begin{bmatrix}
C_{10}(x_1) & C_{11}(x_1) & \cdots & C_{1N}(x_1) \\
C_{20}(x_2) & C_{21}(x_2) & \cdots & C_{2N}(x_2) \\
\vdots & \vdots & \ddots & \vdots \\
C_{N0}(x_N) & C_{N1}(x_N) & \cdots & C_{NN}(x_N)
\end{bmatrix}
\]

and \( \{x_1, x_2, \ldots, x_N \} \) is the vector of the location of the strain measurements. By the
least squares method, the coefficient matrix can be obtained as

\[
C = \varepsilon \cdot T^T \cdot (T \cdot T^T)^{-1}
\]

(6.38)

Substitute Equation (6.35) into Equation (6.37),

\[
Q = (\Phi^T \cdot \Phi)^{-1} \cdot \Phi^T \cdot C \cdot T
\]

(6.39)

where
\[ \Phi = \begin{bmatrix}
\phi_i''(x_1) & \phi_i''(x_2) & \cdots & \phi_i''(x_n) \\
\phi_2''(x_1) & \phi_2''(x_2) & \cdots & \phi_2''(x_n) \\
\vdots & \vdots & \ddots & \vdots \\
\phi_n''(x_1) & \phi_n''(x_2) & \cdots & \phi_n''(x_n)
\end{bmatrix} \]

and it can be obtained from Appendix C.

### 6.3.2 Regularization

The vector of generalized coordinates obtained from Equation (6.39) can be substituted into Equation (6.3), and rewrite it in matrix form to become

\[ I * \ddot{q} + C_d * \dot{q} + K * q = B * p \quad (6.40) \]

where

\[ C_d = \text{diag}(2 * \ddot{x}_i * \omega_i) \]

\[ K = \text{diag}(\omega_i^2) \]

\[ B = \begin{bmatrix}
\phi_1(\ddot{x}_1(t))/M_1 & \phi_1(\ddot{x}_2(t))/M_1 & \cdots & \phi_1(\ddot{x}_n(t))/M_1 \\
\phi_2(\ddot{x}_1(t))/M_2 & \phi_2(\ddot{x}_2(t))/M_2 & \cdots & \phi_2(\ddot{x}_n(t))/M_2 \\
\vdots & \vdots & \ddots & \vdots \\
\phi_n(\ddot{x}_1(t))/M_n & \phi_n(\ddot{x}_2(t))/M_n & \cdots & \phi_n(\ddot{x}_n(t))/M_n
\end{bmatrix} \]

The required $\ddot{q}$ and $\dot{q}$ can be obtained by directly differentiating Equation (6.39) to have

\[ \ddot{q} = (\Phi^T \Phi)^{-1} \Phi^T C \dot{\ddot{q}} \]

\[ \dot{q} = (\Phi^T \Phi)^{-1} \Phi^T C \ddot{q} \]

The moving forces obtained from Equation (6.40) using a straight forward least squares solution would be unbound. Let the left-hand-side of Equation (6.40) be represented by $U$. A regularization technique can be used to solve the ill-posed problem in the form of minimizing the function

\[ J(P, \lambda) = \|B * p - U\|^2 + \lambda \|P\|^2 \quad (6.41) \]

where $\lambda$ is the non-negative regularization parameter.

### 6.3.3 Optimal Regularization Parameter

The success of solving Equation (6.41) lies in how to determine the regularization parameter $\lambda$. Two methods are used in this chapter. If the true forces
are known, the parameter can be determined by minimizing the error between the true forces and the predicting values as

\[ S = \| \hat{P} - P \| \] (6.42)

In the real case when the true forces are not known, the method of generalized cross-validation (GCV) is used to determine the optimal regularization parameter. The GCV function to be minimized in this work is defined by (Golub et al., 1979)

\[ g(\lambda) = \frac{\| B \cdot \hat{P} - U \|_2^2}{\text{trace}[I - B \cdot (B^T \cdot B + \lambda \cdot I)^{-1} \cdot B^T \cdot B]^{-1}} \] (6.43)

where \( \hat{P} \) is the vector of estimated forces.

### 6.3.4 Implementation and Simulation Studies

#### 6.3.4.1 Moving Forces

The proposed method is illustrated in the following simulation studies. The effect of discarding some of the information contained in the measured responses on the error of identification is studied. This aspect has not been studied in previous works reported by other researchers.

#### 6.3.4.2 Single Span Beam

A single span simply supported beam is studied with two varying forces moving on top at a constant spacing of 4.27m.

\[ f_1(t) = 9.9152 \times 10^5 [1 + 0.1 \sin(10 \pi t) + 0.05 \sin(40 \pi t)] \quad N; \] \( \quad \) \( f_2(t) = 9.9152 \times 10^5 [1 - 0.1 \sin(10 \pi t) + 0.05 \sin(50 \pi t)] \quad N; \] (6.44)

The parameters of the beam are as follow: \( EI = 2.5 \times 10^{10} \) Nm², \( \rho A = 5000 \) kg/m, \( L = 30 \) m. \( h = 1 \) m. The first eight natural frequencies of the beam are 3.9, 15.61, 35.13, 62.48, 97.58, 140.51, 191.25 and 249.8 Hz, and they are used in the computation of the analytical mode shapes from Appendix C. The forces are moving at a speed of 30m/s. Random noise is added to the calculated strains to simulate the polluted measurement and 1, 5 and 10 percent noise levels are studied. The errors in the identified forces are calculated by Equation (6.34).

Table 6.5 shows the errors of identification from using different number of mode shapes in the identification. The time step is 0.001s in the calculation. The strain consists of responses from the first eight mode shapes polluted with 5% noise.
level. Ten measuring points are available in the identification and they are evenly distributed along the beam length. The different combination of number of mode shapes used in the identification and the number of measuring points are studied. Figure 6.9 shows the identified results using three and six mode shapes. The following observations are obtained.

1) Results in Table 6.5 show that the errors in the identified forces are insensitive to the noise level in the responses. This is because orthogonal functions have been used to approximate the strains in the identification, and this approximation suppresses the errors due to high frequency measurement noise.

2) When the number of mode shapes used in the identification is the same as the number of mode shapes in the responses, i.e. eight number of mode shapes, the identified errors are the smallest. The errors in identification become large when the number of mode shapes used in identification is either larger or smaller than the number of mode shapes in the responses. This indicates that the pairing of the number of mode shapes in both the responses and the identified forces has a large effect on the errors in the identification. The correct pairing can be determined from an inspection of the frequency content in the measured responses.

3) Figure 6.9 shows that there are large discrepancies in the identified forces near the beginning and the end of the moving forces when only three modes are used in the identification. These discrepancies are much less when six modes are used. This is because the sudden appearance and disappearance of the forces at these points can be represented by an equivalent impulsive force. These impulsive forces excite the beam with a broad-band vibration that covers a large number of modal frequencies. Therefore more mode shapes should be used in the identification to take advantage of the information of the forces at higher modal frequencies in the responses at the beginning and the end of the time histories.

6.3.4.3 Two-Span Continuous Beam

Table 6.6 shows the errors in the identified moving forces on a two-span continuous beam with different number of mode shapes and number of measuring points. The parameters of the beam are the same as for the single span beam except each span measures 30m long. The first eight natural frequencies of the beam are 3.9, 6.1, 15.61, 19.75, 35.12, 41.22, 62.43 and 70.48 Hz. Figure 6.10 shows the identified
forces from using strains polluted with 5% noise level at six measuring points. Inspection of the results in Table 6.6 and Figure 6.10 gives the following observations:

1) Results in Table 6.6 show that the errors increase as the noise level in the response increase. The errors are more than twice of that under similar conditions for the single-span beam. Therefore moving load identification in a multi-span beam would be less accurate than that in a single-span beam.

2) When the number of mode shapes used in the identification equals to that in the responses as shown in the first two rows and the lower part of Table 6.6, the errors in the identified forces varies only slightly with more measuring points. The number of the measuring points is best selected to be equal to the number of mode shapes.

3) Results from the upper part of Table 6.6 also show that the errors would be smallest when the number of mode shapes in the identification is the same as that in the responses. This confirms the observation made in the case with the single span beam.

4) The identified forces in Figure 6.10 have large fluctuations close to zero at the point of the intermediate support at 1.0s. This is again due to the presence of the equivalent impulsive force with the sudden appearance or disappearance of the forces at this point.

5) In Table 6.6, the error in the identified results with 14 measuring points is larger than that with 10 measuring points. This is because the measuring points are evenly distributed on the beam, and some of them are near the supports. The signal-to-noise ratio of signals from these measuring points is small. The inclusion of these signals would contaminate the useful information causing the errors in the results.

### 6.4 APPLICATIONS TO BRIDGE-VEHICLE INTERACTION FORCE IDENTIFICATION

#### 6.4.1 Identification Based on the Exact Solution

**Single Span Bridge Subject to a Moving Vehicle**

A bridge-vehicle system is represented by a simply supported beam subject to a moving vehicle. The parameters of the system are listed as follow.
Bridge: \( L = 30 \text{m}, \ EI = 2.5 \times 10^{10} \text{Nm}^2, \ \rho A = 5.0 \times 10^3 \text{Kg/m}, \ \xi = 0.02 \) for all modes.

A 4 DOFs vehicle model is presented in Figure 6.11. The characteristics of the vehicle model are adopted from Mulcahy (1983):

\[
\begin{align*}
    m_x &= 17735 \text{Kg}, \ m_1 = 1500 \text{Kg}, \ m_2 = 1000 \text{Kg}, \ S = 4.27 \text{m}, \ a_1 = 0.519, \ a_2 = 0.481, \ H = 1.80 \text{m}, \\
    k_{11} &= 2.47 \times 10^6 \text{Nm}^{-1}, \ k_{12} = 4.23 \times 10^6 \text{Nm}^{-1}, \ k_{11} = 3.74 \times 10^6 \text{Nm}^{-1}, \ k_{12} = 4.60 \times 10^6 \text{Nm}^{-1}, \\
    c_{11} &= 3.00 \times 10^4 \text{Nm}^{-1} \text{s}, \ c_{12} = 4.00 \times 10^4 \text{Nm}^{-1} \text{s}, \ c_{11} = 3.90 \times 10^3 \text{Nm}^{-1} \text{s}, \ c_{12} = 4.30 \times 10^3 \text{Nm}^{-1} \text{s}, \\
    I_x &= 1.47 \times 10^5 \text{Kgm}^2.
\end{align*}
\]

The first six natural frequencies of the bridge are 3.90Hz, 15.61Hz, 35.13Hz, 62.45Hz, 97.58Hz and 140.51Hz. The natural frequencies of the vehicle are 10.27Hz, 14.44Hz, 65.05Hz and 94.90Hz. The first six modes are used in the calculation. Six measuring points are evenly located on the bridge. The weight ratio between the vehicle and bridge is 0.135. A parameter often used to characterize the magnitude of wheel forces is the dynamic load coefficient (DLC). It is now redefined for the axle load as follow.

\[
DLC = \frac{\text{RMS dynamic axle load}}{\text{mean axle load}}
\]  

(6.45)

Table 6.7 shows the dynamic load coefficients from different speed and road surface roughness. Table 6.8 shows the dynamic load coefficients from different accelerations and braking positions. The initial speed is 30m/s. Figure 6.12 shows the identified axle loads from strains and accelerations with 5% noise when the speed of the vehicle is 30m/s. Figure 6.13 shows the identified results with Grade B road roughness and the vehicle starts braking at 1/3L. The analysis method for bridge-vehicle interaction is referred to Chapter 3. The constant acceleration from braking is -3m/s. The noise level in the responses is 5%. The following observations are made:

1) The method proposed can be used to identify the bridge-vehicle interaction forces from the responses of the bridge with or without vehicle braking on the bridge;

2) The identified results confirm that a redistribution of the axle loads occurs when the vehicle is subject to braking.

3) Tables 6.7 and 6.8 show that the dynamic load coefficients increase monotonically with the speed, roughness and acceleration. Vehicle braking on the bridge has a more significant effect than speed or road roughness, and vehicle speed has the smallest influence on the dynamic load coefficient.
4) The identified results from accelerations are much better than that from strains as shown in Figures 6.12 and 6.13. This shows that the errors in the identification from accelerations are less sensitive to the noise level in response than that from strains when the number of modes used in the identification is the same as that in the responses. This observation is compatible with the conclusion drawn for Section 6.2.4.2 that a larger number of modes should be used for an accurate result when accelerations are used.

Three-span Continuous Bridge Subject to a Moving Vehicle

A three-span continuous bridge with the following parameters is studied:

\[ L_1 = L_2 = 30\text{m}, L_3 = 60\text{m}, EI = 4.3 \times 10^{10}\ \text{Nm}^2, \ \rho A = 1.1 \times 10^4\ \text{Kg/m} \cdot \xi = 0.02 \] for all modes.

The parameters of the vehicle are the same as for the above example. The natural frequencies and mode shapes of the bridge are determined from Hayashikawa and Watanabe (1981). The first nine frequencies of 1.37Hz, 3.45Hz, 4.37Hz, 5.39Hz, 9.10Hz, 13.80Hz, 15.57Hz, 17.46Hz and 23.77Hz are used in the calculation. The weight ratio between the vehicle and the bridge is 0.0153. Two, five and two measuring points are evenly located on the first, the second and the third span of the bridge respectively. Class B road roughness is used in the simulation. Figure 6.14 shows the identified results from strains and accelerations with 1% noise level. The vehicle is moving on the bridge at 30m/s. Figure 6.15 shows the identified results with 1% and 5% noise levels in the responses and braking at 1/3L with an acceleration of −1m/s². The following observations are made.

1) The identified results from multi-span continuous bridges are more sensitive to the noise level than that from single-span bridges. The interaction forces between the multi-span continuous bridge and the vehicle are more difficult to identify accurately.

2) The identified loads at the supports are close to zero which means large errors exist near the supports. This is due to the optimization over the total duration with a single regularization parameter which is not quite appropriate near the supports. A smaller regularization parameter could be used near the supports for an improvement (Choi and Chang, 1996).
3) From results not shown in the paper, the identified results are the same as the true forces when there is no noise in the response. This shows that the proposed method is correct.

Identification with Incomplete Vehicle Speed Information

In practice, the axle spacing, the number of axles of the vehicle, and the time that the vehicle enters or exits the bridge can be measured directly by axle sensors (Chan et al. 2000). But the braking position and the acceleration are difficult to measure. The errors induced from identifying using an average speed should be studied. Figure 6.16 shows the identified results with the vehicle starts braking at the entry with an acceleration of $-1m/s^2$. The average speed of $29.39m/s$ is used. Figure 6.17 shows the identified results with the vehicle starts braking at $1/3L$ and the acceleration is $-3m/s^2$. The average speed of $29.04m/s$ is used. The initial speed is $30m/s$ and the road roughness is Class B in both cases.

It is seen that the identified results from using the average speed are acceptable when the acceleration is small. When the acceleration is larger as shown in Figure 6.17, the identified results from strains are still acceptable but those from accelerations exhibit divergence near the end of the time histories. This also shows that the accelerations of the bridge are sensitive to the vehicle speed.

6.4.2 Identification Based on Finite Element Formulation

The parameters of the vehicle and three-span beam are the same as described above. The bridge responses and the interaction forces are calculated by the method proposed in Chapter 3. The time step in the calculation is 0.005s. The road surface roughness of the bridge is not included in the simulation. The measuring points are evenly distributed along each span. The identified axle loads from the following two cases using different number of modes and measuring points are shown in Figure 6.18. The number of mode shapes in the identification is taken equal to that in the measured responses.

Case 1: The first six modes and 8 measuring points are used in the identification. There are 2 measuring points spaced at one-third span on the first span and the third span, and 4 measuring points spaced at one-fifth span on the second span.
Chapter 6: Euler-Bernoulli Continuous Beam Model

Case 2: The first twelve modes and 14 measuring points are used in the identification. There are 4 measuring points spaced at one-fifth span on the first span and the third span, and 6 measuring points spaced at one-seventh span on the second span.

1% noise is included in the responses. Figure 6.18 shows that the identified results are close to the calculated interaction forces except near the supports, and Case (2) gives more accurate results than Case (1) particularly at locations close to the supports. Therefore the proposed method is also effective for identifying the bridge-vehicle interaction forces.

6.5 CONCLUDING REMARKS

A new time domain method has been developed to identify the moving loads on a continuous beam from strains and accelerations. The large matrix relating the force and responses is splitted into smaller sub-matrices making the computation more cost effective. Regularization technique is used to stabilize the unbound solution. The GCV, L-curve and the plot of error versus the regularization parameter can locate the optimal regularization parameter effectively. If the moving loads are identified from strains, the sampling frequency can be selected to be larger or equal to twice of the maximum frequency of the moving loads and the vibration modes used in the identification. But when accelerations are used, a much higher sampling frequency should be used in the identification. In general, more vibration modes should be used in impulsive load identification. And the modelling error is found to have a very significant effect on the accuracy of the identified forces. An accurate model is therefore required. This method is put to test in environments simulating the real bridge-vehicle system in which road surface roughness and non-uniform vehicle speed are modelled. The method is found to have good performances in the identification of individual interactive forces with small variations in the vehicle speed. It can identify accurately the combined load from the axle forces even with vehicle braking on the bridge deck.

Another general method based on finite element formulation has also been developed to identify the moving loads on a continuous beam from strains in this chapter. A generalized orthogonal function approach is proposed, and the moving forces are identified with bounds in the errors using regularization in the solution.
Computational simulations show that: 1) The method can be used to identify the bridge-vehicle interaction forces from measured strains in a continuous beam, and acceptable results can be obtained; and 2) More mode shapes should be used to identify the moving forces at locations close to the supports; and 3) When the number of mode shapes in the identification is the same as that in the measured responses, the errors of identification will be the smallest.
### Table 6.1 Errors (percent) in identification from using different methods

<table>
<thead>
<tr>
<th>Noise Level</th>
<th>Loads</th>
<th>TDM</th>
<th>FTDM</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>M</td>
<td>A</td>
<td>M</td>
</tr>
<tr>
<td>1%</td>
<td>$P_1(t)$</td>
<td>230.48</td>
<td>1.76</td>
<td>41.84</td>
</tr>
<tr>
<td></td>
<td>$P_2(t)$</td>
<td>157.52</td>
<td>4.91</td>
<td>44.01</td>
</tr>
<tr>
<td>5%</td>
<td>$P_1(t)$</td>
<td>$+$</td>
<td>18.79</td>
<td>232.61</td>
</tr>
<tr>
<td></td>
<td>$P_2(t)$</td>
<td>$+$</td>
<td>16.54</td>
<td>397.20</td>
</tr>
<tr>
<td>10%</td>
<td>$P_1(t)$</td>
<td>$+$</td>
<td>40.19</td>
<td>224.76</td>
</tr>
<tr>
<td></td>
<td>$P_2(t)$</td>
<td>$+$</td>
<td>33.52</td>
<td>630.01</td>
</tr>
<tr>
<td>20%</td>
<td>$P_1(t)$</td>
<td>$+$</td>
<td>49.21</td>
<td>$+$</td>
</tr>
<tr>
<td></td>
<td>$P_2(t)$</td>
<td>$+$</td>
<td>72.68</td>
<td>882.70</td>
</tr>
</tbody>
</table>

Note: M – Bending Moment; A – Acceleration; S – Strain. $+$ means error larger than 1000%.

### Table 6.2 - Errors in identification with different number of modes

<table>
<thead>
<tr>
<th>Data Type</th>
<th>Noise Level</th>
<th>Loads</th>
<th>Number of Modes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Strain</td>
<td>1%</td>
<td>$P_1(t)$</td>
<td>14.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$P_2(t)$</td>
<td>14.41</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>$P_1(t)$</td>
<td>21.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$P_2(t)$</td>
<td>21.24</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>$P_1(t)$</td>
<td>25.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$P_2(t)$</td>
<td>25.53</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>$P_1(t)$</td>
<td>30.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$P_2(t)$</td>
<td>31.02</td>
</tr>
<tr>
<td>Acceleration</td>
<td>1%</td>
<td>$P_1(t)$</td>
<td>3.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$P_2(t)$</td>
<td>4.87</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>$P_1(t)$</td>
<td>10.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$P_2(t)$</td>
<td>11.41</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>$P_1(t)$</td>
<td>11.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$P_2(t)$</td>
<td>13.38</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>$P_1(t)$</td>
<td>15.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$P_2(t)$</td>
<td>18.38</td>
</tr>
</tbody>
</table>

6-23
Table 6.3 Identified errors (percent) with different number of mode shapes in responses

<table>
<thead>
<tr>
<th>Number Of modes</th>
<th>Data Type</th>
<th>Varying Component</th>
<th>Constant Component</th>
<th>Total Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First load</td>
<td>Second load</td>
<td>First load</td>
</tr>
<tr>
<td>6</td>
<td>S</td>
<td>0.06</td>
<td>0.12</td>
<td>0.20</td>
</tr>
<tr>
<td>5</td>
<td>S</td>
<td>0.72</td>
<td>0.84</td>
<td>0.95</td>
</tr>
<tr>
<td>4</td>
<td>S</td>
<td>1.16</td>
<td>0.97</td>
<td>1.04</td>
</tr>
<tr>
<td>3</td>
<td>S</td>
<td>1.12</td>
<td>1.49</td>
<td>1.56</td>
</tr>
<tr>
<td>2</td>
<td>S</td>
<td>106.02</td>
<td>92.92</td>
<td>2.59</td>
</tr>
<tr>
<td>6</td>
<td>A</td>
<td>0.06</td>
<td>0.07</td>
<td>0.20</td>
</tr>
<tr>
<td>5</td>
<td>A</td>
<td>5.27</td>
<td>7.30</td>
<td>3.81</td>
</tr>
<tr>
<td>4</td>
<td>A</td>
<td>13.47</td>
<td>15.98</td>
<td>13.15</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>24.19</td>
<td>25.77</td>
<td>10.60</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>18.62</td>
<td>37.44</td>
<td>69.93</td>
</tr>
</tbody>
</table>

Note: S - identified from strains; A - identified from accelerations.

Table 6.4 Identified errors with different errors in model

<table>
<thead>
<tr>
<th>Data Type</th>
<th>Model Error</th>
<th>Noise Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0%</td>
<td>1%</td>
</tr>
<tr>
<td>Strains</td>
<td>1%</td>
<td>4.88</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>15.22</td>
</tr>
<tr>
<td>Accelerations</td>
<td>1%</td>
<td>6.24</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>24.61</td>
</tr>
</tbody>
</table>

6-24
### Table 6.5 Identified errors for single span beam

<table>
<thead>
<tr>
<th>Number of mode shapes</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First force</td>
<td>Second force</td>
<td>First force</td>
</tr>
<tr>
<td>2</td>
<td>31.51</td>
<td>31.53</td>
<td>45.11</td>
</tr>
<tr>
<td>3</td>
<td>11.56</td>
<td>12.00</td>
<td>13.04</td>
</tr>
<tr>
<td>4</td>
<td>6.78</td>
<td>6.18</td>
<td>7.05</td>
</tr>
<tr>
<td>5</td>
<td>5.16</td>
<td>3.62</td>
<td>5.07</td>
</tr>
<tr>
<td>6</td>
<td>3.90</td>
<td>3.10</td>
<td>4.02</td>
</tr>
<tr>
<td>7</td>
<td>3.43</td>
<td>2.99</td>
<td>3.45</td>
</tr>
<tr>
<td>8</td>
<td>3.15</td>
<td>2.86</td>
<td>3.19</td>
</tr>
<tr>
<td>9</td>
<td>9.52</td>
<td>8.96</td>
<td>9.48</td>
</tr>
<tr>
<td>10</td>
<td>18.02</td>
<td>17.30</td>
<td>18.51</td>
</tr>
</tbody>
</table>

### Table 6.6 Identified Errors for Two-span Beam

<table>
<thead>
<tr>
<th>No. of mode shape in responses</th>
<th>No. of mode shapes in identification</th>
<th>No. of measuring points</th>
<th>Noise level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>First force</td>
</tr>
<tr>
<td>N1 10</td>
<td>N2 10</td>
<td>Ns 14</td>
<td>7.85</td>
</tr>
<tr>
<td>10 10</td>
<td>10</td>
<td>10</td>
<td>7.78</td>
</tr>
<tr>
<td>10 9</td>
<td>10</td>
<td>10</td>
<td>8.90</td>
</tr>
<tr>
<td>10 8</td>
<td>10</td>
<td>10</td>
<td>11.54</td>
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<td>10 7</td>
<td>10</td>
<td>10</td>
<td>13.81</td>
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<td>17.05</td>
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<td>15.90</td>
</tr>
<tr>
<td>6 6</td>
<td>6</td>
<td>14</td>
<td>15.85</td>
</tr>
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</table>
Table 6.7 Dynamic Load Coefficients for individual force with different grade of road surface roughness and Vehicle Speed

<table>
<thead>
<tr>
<th>velocity (m/s)</th>
<th>Smooth</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
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<tbody>
<tr>
<td>1st 2nd</td>
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<td>1st 2nd</td>
<td>1st 2nd</td>
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<td>1.069</td>
<td>1.070</td>
<td>1.071</td>
<td>1.071</td>
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<tr>
<td>20</td>
<td>1.069</td>
<td>1.07</td>
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<td>1.070</td>
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<td>30</td>
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<td>50</td>
<td>1.070</td>
<td>1.072</td>
<td>1.072</td>
<td>1.073</td>
<td>1.078</td>
</tr>
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</table>

Table 6.8 Dynamic Load Coefficients with Different road surface roughness and Braking Position

<table>
<thead>
<tr>
<th>acceleration (m/s²)</th>
<th>Braking Position</th>
<th>Road surface roughness Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Smooth</td>
<td>A</td>
</tr>
<tr>
<td>-3 1/3L</td>
<td>1.093</td>
<td>1.098</td>
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<tr>
<td></td>
<td>1.091</td>
<td>1.093</td>
</tr>
<tr>
<td>-2 1/3L</td>
<td>1.083</td>
<td>1.087</td>
</tr>
<tr>
<td></td>
<td>1.080</td>
<td>1.081</td>
</tr>
<tr>
<td>-1 1/3L</td>
<td>1.075</td>
<td>1.078</td>
</tr>
<tr>
<td></td>
<td>1.073</td>
<td>1.074</td>
</tr>
<tr>
<td>-3 0</td>
<td>1.098</td>
<td>1.095</td>
</tr>
<tr>
<td></td>
<td>1.094</td>
<td>1.093</td>
</tr>
</tbody>
</table>
Figure 6.1 A continuous beam subjected to moving loads
Figure 6.2 Identified forces from accelerations and strains with 5% noise

( - True force;   --- Identified by accelerations; ... Identified by strains.)
Figure 6.3 Errors in identification with different regularization parameters

(- 1% noise in responses; — 5% noise in responses; ... 10% noise in responses.)
Figure 6.4 GCV function

(- 1% noise in responses; — 5% noise in responses; ... 10% noise in responses.)
Figure 6.5 L–curve

(- 1% noise in responses; —— 5% noise in responses; ... 10% noise in responses.)
Figure 6.6 Acceleration and strain with different time intervals

(−0.001s; −0.002s; ... 0.005s.)
Figure 6.7 Loads identified with three modes

(— True forces; —— Identified from strains; ... Identified from accelerations.)
Figure 6.8 Forces identified with 1% error in the model

(− True forces; — identified from strains; ... identified from accelerations.)
Figure 6.9 The identified results with different number of mode shapes
( - True loads; ---- Identified results with 3 modes;
... Identified results with 6 modes.)
The first axle force

The second axle force

Figure 6.10 The identified results with different number of mode shapes
(- True loads; ---- Identified results with 3 modes;
... Identified results with 6 modes.)
Figure 6.11 Bridge-Vehicle system model
Figure 6.12 Identification from strains and accelerations with 5% noise

(- True forces; — Identified from accelerations; … Identified from strains.)
Figure 6.13 Identification with road roughness, 5% noise and braking starts at 1/3L

(- True loads; —— Identified from accelerations; ... Identified from strains.)
Figure 6.14 Identification with 1% noise and road roughness

(— True loads; —— Identified from accelerations; ... Identified from strains.)
Figure 6.15 Identification with road roughness and braking starts at 1/3L

(- True loads; --- Identified from strains with 1% noise; ... Identified from strains with 5% noise.)
(a) The first axle load

(b) The second axle load

Figure 6.16 Identification in case of small acceleration using average speed

(- True loads; — Identified from accelerations; ... Identified from strains.)
Figure 6.17 Identification in case of large acceleration using average speed

(- True loads; --- Identified from accelerations; ... Identified from strains.)
Figure 6.18 The identified results with different modes and measuring points
(- Calculated interaction forces; ---- Identified results with 6 modes
and 8 measuring points; ... Identified results with 6 modes.)
Chapter 7

LABORATORY STUDY I: UNIFORM BEAM MODEL

7.1 INTRODUCTION

Two methods based on regularization technique have been developed to identify moving loads on a continuous beam from measured structural vibration responses in last chapter. Numerical examples of moving forces on both single and multi-span bridges and the case of vehicle-bridge interaction forces from a four-DOF's vehicle on a triple-span bridge have illustrated the feasibility and accuracy of these methods. This chapter reports on the practical experimental work to further verify the performances of these methods under laboratory conditions.

A bridge-vehicle system model has been designed in the laboratory basing on the uniform beam model. The strain and acceleration responses are simultaneously measured when the model car moves across the bridge at different speeds. Since the use of accelerations requires a large number of measured modes and measuring points for a good accuracy, only the measured strains are used in the following studies. The moving forces are identified from the bridge strains using the two proposed methods. The effect of non-uniform speed on the identified results when the forces are identified using a constant speed is also investigated.

7.2 EXPERIMENTAL SETUP AND MEASUREMENTS

7.2.1 Experimental Setup

The experimental setup is shown in Photograph 7.1. The main beam located in the laboratory is 3678mm long with a 100mm×25mm uniform cross-section. There is a leading beam for accelerating the vehicle and a tailing beam to accept the vehicle when it comes out of the main span. The beams are simply supported and the ends of the beams are placed close together leaving only a very narrow gap of approximately one millimetre as shown in Photograph 7.2. This is necessary in order not to have a large impulsive force on the beam when the wheels cross the gap.
A U-shaped aluminum section is glued to the upper surface of the beams as a direction guide for the car. The model car is pulled along the guide by a string wound around the drive wheel of an electric motor. The model car, shown in Photograph 7.3(a), has two axles at a spacing of 0.557m and it runs on four steel wheels with rubber band on the outside. The mass of the whole car is 16.6 Kg with the front axle load and the rear axle load weigh 9.8kg and 6.8kg respectively. The transverse spacing between wheels is 0.08m. Thirteen photoelectric sensors are mounted on the beams at an approximately equal spacing to measure and monitor the moving speed of the car.

Seven strain gauges are evenly distributed on the beam at $1/8L$. A TEAC 14-channels magnetic tape recorder and an 8-channel dynamic testing and analysis system are used for data collection and analysis in the experiment. The layout of the experimental system is shown in photograph 7.4. The sampling frequency is 2000Hz. The recorded length of each test sample lasts for six seconds.

Braking force is applied with a set of rubber bands placed transversely in front of the vehicle approximately at the level of its centroid, and the braking force was tuned by adjusting the tension in the rubber band.

### 7.2.2 Experimental Procedure

The modal test, shown in Photograph 7.5, was performed on the model car and on the main beam. The first three natural frequencies of the model car and the main beam are shown in Table 7.1.

The signal output from the strain gauges were zero adjusted when the main beam was unloaded. This takes care of the fact that the strain gauge signals are very small and are usually with zero-shift phenomenon. Removing the zero-shift portion in the output increases the dynamic range of the measurement. The strain gauges were calibrated by adding masses at the middle of the main beam. The average sensitivities for the gauges were found.

The car was placed at the right end of the leading beam, and the data acquisition system was set in the pre-trigger state at channel 1. The power for the motor was then turned on, and the car moved on the top of the beam. The vibration signals were recorded. The zero-shift in the measured signals were removed, and the
signals were calibrated with their measured channel sensitivities. The point in the signals when the front wheel of the car just got on the main beam was identified.

7.3 MOVING LOADS IDENTIFICATION BASED ON EXACT SOLUTION

7.3.1 Identification from Strains with Uniform Speed

The first experiment is conducted with the vehicle moving approximately at 1.25 m/s, and the first three modes of the beam are used in the identification. The collected strains are re-sampled at 200Hz to include the first three vibration modes. Figure 7.1 shows the identified results from sets of strains at 1/4L, 1/2L and 3/4L and from 1/8L, 1/2L and 7/8L. The optimal regularization parameters are $7.8930 \times 10^{-16}$ and $8.4069 \times 10^{-16}$ respectively for the two sets of strains. The reconstructed strains at 5/8L are shown in Figure 7.2. The following observations are made.

1) The mean values of the identified combined loads are close to and varying around the static loads. The fluctuations in the combined load are typically from the inertia effect of the moving masses.

2) The reconstructed strains at 5/8L are close to the measured strains. The correlation coefficients between them are 0.986 and 0.968 respectively for the two sets of strains. These also indicate that the method is correct.

3) The peak of the reconstructed strain is smaller than that of the measured strain. This is because only a small number of modes are used in the identification as discussed in the simulations results presented in last chapter.

4) The identified results from strains at 1/4L, 1/2L and 3/4L are similar to that from 1/8L, 1/2L and 7/8L but with less fluctuations. This is because the signal to noise ratios at 1/8L and 7/8L are lower than that at 1/4L and 3/4L. Therefore the selection of measuring point is very important for an accurate identification.

7.3.2 Identification from Strains with Braking Starts at 0.878m

The parameters are the same as for the above experiment. The moving car starts braking at 0.878m. The actual speed between adjacent pairs of photoelectric sensors is shown in Table 7.1 with an average speed of 1.19m/s. The moving car accelerates before braking and it crosses the beam completely with deceleration.
Figure 7.3 shows the identified loads from strains at $1/4L$, $1/2L$ and $3/4L$ for the car moving at the true and the average speed. The reconstructed strains at $5/8L$ are also compared with the measured strain. The correlation coefficients between them are respectively 0.983 and 0.985, and the optimal regularization parameters are respectively $8.040 \times 10^{-16}$ and $8.267 \times 10^{-16}$ for the cases of identifying using true and average speeds. The following observations are made.

1) Large fluctuations are found in the identified loads in Figure 7.3. This is due to the pitching motion induced by the horizontal braking force. Impulsive interaction forces generated from braking would also cause these fluctuations.

2) The combined load is close to the static load after braking as shown. This shows that braking mainly induces the pitch motion of the car through a re-distribution of the axle loads.

3) The identified individual loads differs slightly when the true or average speed is used, but there is virtually no difference in the combined load. This means that the proposed method can be used to identify accurately the combined load of a moving vehicle from the bridge responses even with braking on the bridge.

4) The second axle load identified from using average speed is always smaller than that from using the true speed, while that for the first axle load is reversed (Figure 7.3). This is because the true resultant load is always in front of the location at which the combined force is computed in the case of deceleration. The identified combined force tends to match the true force to have minimum error in the identification, and this means a shift of the centroid forward causing an increase in the first axle load and a decrease in the second axle load. There is very small difference in the curves close to the ends of the beam because the difference in the locations between the true and the identified combined forces is very small. Same observations can be found in Figures 6.16 and 6.17 for the results from strains.

7.4 MOVING LOADS IDENTIFICATION BASED ON FINITE ELEMENT FORMULATION

7.4.1 Identification from Strains Using Uniform Speed

Experimental results for Section 7.3.1 are used in this study. Strains at $1/8L$, $1/4L$, $3/8L$, $1/2L$, $3/4L$, $7/8L$ are used in the identification. Table 7.2 shows the correlation coefficients between the measured and the reconstructed strains at $5/8L$. 

7-4
from several cases of identified forces using different number of mode shapes in the identification. The number of measuring points is taken equal to the number of mode shapes in the identification. Figure 7.4 shows that the identified forces from Cases (A) and (G) of the study using 3 and 6 sensors respectively. The combined force is also presented in Figure 7.4(c). The following observations are made:

1. Table 7.3 shows that the correlation coefficients are all larger than 0.9 for different combination of modes and measuring points. It shows that the proposed method is effective to identify the moving forces in practice.

2. There is a low frequency component in the identified individual forces in Figure 7.4. This is the pitching motion of the moving car.

3. The identified forces from using six modes are closer to the static forces at the beginning and the end of the time histories than that obtained from using three modes. This gives experimental evidences that more mode shapes in the computation should be used to identify the moving forces near these locations.

7.4.2 Identification from Strains with Braking Starts at 0.878m

The experimental results obtained in Section 7.3.2 are used in this study. Since there is no distinct suspension system in the model car, the vertical stiffness and the damping are not considered, and the car is modelled as a rigid body moving on top of the beam. The beam sub-system has very small damping, and hence the damping stiffness is set to zero in the computation.

The experiment is conducted with varying vehicle speeds as listed in Table 7.1, and the car brakes at 0.878m from the left support. The car gets outside the main beam at the end of braking. The measured strains at 1/4L, 1/2L, 3/4L are shown in Figure 3.4. Figures 7.5 and 7.6 show the identified results by using the average speed of the time period and the measured varying speed as shown in Table 7.1. The first three modes are used in the identification. Figure 7.7 shows the measured and calculated strain from the identified forces at 3/8L.

From the laboratory tests, the following results can be obtained:

1. The method proposed in the last chapter is effective, and acceptable results can be obtained.

2. When the speed varies slightly, the identified results from using average speed are approximately equal to the results obtained from using varying speed.
7.5 CONCLUDING REMARKS

The experimental results show that the two proposed methods are effective to identify the moving forces on a simply supported bridge in practice. They are found to have good performances in the identification of individual interactive forces with small variations in the vehicle speed when the true or average speed is used. It can identify accurately the combined load from the axle forces using the average speed even with vehicle braking on the bridge deck.
Table 7.1 Natural frequencies of the model car and main beam

<table>
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<tr>
<th>Mode</th>
<th>Model car (Hz)</th>
<th>Main beam (Hz)</th>
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<tbody>
<tr>
<td>1</td>
<td>7.82</td>
<td>3.67</td>
</tr>
<tr>
<td>2</td>
<td>9.77</td>
<td>16.83</td>
</tr>
<tr>
<td>3</td>
<td>11.72</td>
<td>37.83</td>
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Table 7.2 - Transient speed in experiment

<table>
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<tr>
<th>Range of distance (m)</th>
<th>0.0</th>
<th>0.478</th>
<th>0.878</th>
<th>1.178</th>
<th>1.478</th>
<th>1.778</th>
<th>2.078</th>
<th>2.378</th>
<th>2.678</th>
<th>3.178</th>
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<tr>
<td>Velocity (m/s)</td>
<td>1.215</td>
<td>1.305</td>
<td>1.271</td>
<td>1.250</td>
<td>1.245</td>
<td>1.139</td>
<td>1.154</td>
<td>1.149</td>
<td>1.129</td>
<td>1.119</td>
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</tbody>
</table>

Table 7.3 The correlation coefficients between measured and reconstructed responses at 5/8L

<table>
<thead>
<tr>
<th>Case</th>
<th>Number of Mode Shapes</th>
<th>Measuring locations</th>
<th>Correlation Coefficient</th>
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<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>1/4L, 1/2L, 3/4L</td>
<td>0.9809</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>1/8L, 1/4L, 1/2L, 3/4L</td>
<td>0.9470</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>1/8L, 1/4L, 1/2L, 3/4L, 7/8L</td>
<td>0.9752</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>1/4L, 1/2L, 3/4L</td>
<td>0.9853</td>
</tr>
<tr>
<td>E</td>
<td>4</td>
<td>1/8L, 1/4L, 1/2L, 3/4L</td>
<td>0.9837</td>
</tr>
<tr>
<td>F</td>
<td>5</td>
<td>1/8L, 1/4L, 1/2L, 3/4L, 7/8L</td>
<td>0.9822</td>
</tr>
<tr>
<td>G</td>
<td>6</td>
<td>1/8L, 1/4L, 3/8L, 1/2L, 3/4L, 7/8L</td>
<td>0.9716</td>
</tr>
</tbody>
</table>
Figure 7.1 Identification from strains at different measuring points

(\(-\) Static forces; \(\ldots\) By strains at \(1/4L, 1/2L, 3/4L; \ldots\) By strains at \(1/8L, 1/2L, 7/8L\).)
Figure 7.2 Reconstruct strain at 5/8L at different measuring points

(— Measured strain; —— By strains at 1/4L, 1/2L, 3/4L; ... By strains at 1/8L, 1/2L, 7/8L.)
Figure 7.3 Identification from strains at 1/4L, 1/2L, 3/4L with braking starts at 0.878m

(— Static forces; —— Identified using varying speed; ... Identified using average speed.)
Figure 7.4 Identified results from using different modes
(— Static load; —— with 3 modes (Case A); ... with 6 modes (Case G).)
Figure 7.5 The identified first force
(-- average speed; ---- varying speed)

Figure 7.6 The identified second force
(-- average speed; ---- varying speed)
Figure 7.7 Measured and calculated strains at 3/8L.

(- measured; --- average; ... varying)
Photograph 7.1 Experimental set-up
Photograph 7.2 Detail of Supports

(a) Car model  \hspace{1cm} (b) Driving motor

Photograph 7.3 Model car and driving motor
Photograph 7.4 Instrument and Data collection and analysis system

Photograph 7.5 Modal test
Chapter 8

IDENTIFICATION OF MOVING LOADS ON MULTI-SPAN CONTINUOUS BRIDGE —— TIMOSHENKO BEAM MODEL

8.1 INTRODUCTION

Two methods have been developed in the last two chapters to identify moving loads on a continuous bridge based on the Euler-Bernoulli beam theory. But there are a lot of multi-span continuous bridges of large cross-section, and the effect of variation of the cross-sectional dimensions on the dynamic properties cannot be neglected. The effect of rotatory inertia and of shear deformation must be considered.

In this chapter, the vibrational behavior of the multi-span non-uniform Timoshenko beam subjected to a set of moving loads is analyzed basing on Hamilton’s principle with the intermediate point constraints represented by very stiff linear springs. The loads can take up any initial positions on the beam. A method to identify these moving forces in time domain is then developed based on the finite element formulation with modal superposition and optimization technique. The method is then used to study the effect of different parameters on the identification results through single-force and multi-forces identification. The comparative study between the Timoshenko beam theory and the Euler-Bernoulli beam theory is included. Computation simulations and laboratory results show that the method is effective and robust, and acceptable results can be obtained from strain and displacement measurements.

8.2 DYNAMIC RESPONSES OF MULTI-SPAN CONTINUOUS TIMOSHENKO BEAM UNDER MOVING LOADS

8.2.1 Equation of Motion

Figure 8.1 shows a continuous Timoshenko beam with \((Q-1)\) intermediate point supports under \(N_f\) number of moving loads. The beam is constrained at these supports. The loads \(P_s(t)\) \((s=1,2,\ldots,N_p)\) are moving as a group at a prescribed velocity \(v\) along the axial direction of the beam from left to right. \(v\) is assumed constant in this study. The load locations are described by \(\hat{x}_1(t)\) with \(\hat{x}_1(t) = \hat{x}_1 + vt\) where \(\hat{x}_\infty\) is the
initial location of the load. The bending moment and the transverse shear force of the beam are given as

\[ M(x) = EI(x) \frac{\partial \psi(x,t)}{\partial x} \]

\[ V(x) = \kappa GA(x) \left[ \frac{\partial w(x,t)}{\partial x} - \psi(x,t) \right] \quad (8.1) \]

where \( G \) is the shear modulus of the beam material and \( A(x) \) is the cross-sectional area; \( E \) is the Young’s modulus; \( I(x) \) is the moment of the inertia of the beam cross-section; \( \kappa \) is the shear coefficient; \( w(x,t) \) is the transverse displacement function of the beam; \( \psi(x,t) \) is the angle of rotation at a cross-section.

The kinetic energy \( T \) of the beam, the strain energy \( U_s \), the potential energy due to point constraints \( U_0 \), and the work done \( W \) due to the moving loads can be written for the Timoshenko beam as follows

\[ T = \frac{1}{2} \int_0^L \rho A(x) \left( \frac{\partial \psi(x,t)}{\partial t} \right)^2 + \gamma^2(x) \left( \frac{\partial \psi(x,t)}{\partial x} \right)^2 \, dx \]

\[ U_s = \frac{1}{2} \int_0^L EI(x) \left( \frac{\partial w(x,t)}{\partial x} \right)^2 + \kappa GA(x) \left[ \frac{\partial w(x,t)}{\partial x} - \psi(x,t) \right]^2 \, dx \]

\[ U_0 = \frac{1}{2} k \sum_{i=1}^{\frac{Q}{2}} w(x_i,t)^2 \]

\[ W = \sum_{i=1}^{\frac{Q}{2}} \delta(x - \delta_i(t)) P_i(t) w(x_i,t) \, dx \quad (8.2) \]

where \( \rho \) is the mass density of material of the beam; \( x_i (i = 0,1,2,\cdots,Q) \) are coordinates of the intermediate point supports and end supports and \( \delta(t) \) is the Dirac delta function. \( \gamma(x) \) is the radius of gyration of the beam cross-section. \( k \) is the stiffness of the point constraints. Expressing the vibration responses of the beam \( w(x,t) \) and \( \psi(x,t) \) in modal coordinates

\[ w(x,t) = \sum_{i=1}^{n} q_i(t) W_i(x) \quad (i = 1,2,\cdots,n) \]

\[ \psi(x,t) = \sum_{i=1}^{n} q_i(t) \phi_i(x) \quad (8.3) \]

where \( W_i(x), \phi_i(x) \) are the assumed vibration modes that satisfy the boundary conditions and \( q_i(t) \) is the generalized co-ordinate.
Substituting Equation (8.3) into (8.2), we obtain
\[
T = \frac{1}{2} \int \rho A(x) \left( \sum_{i=1}^{n} \ddot{q}_i(t) W_i(x) \sum_{j=1}^{n} \ddot{q}_j(t) W_j(x) + \gamma^2(x) \sum_{i=1}^{n} \ddot{q}_i(t) \phi_i(x) \dot{q}_i(t) \phi_i(x) \right) dx
= \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} \ddot{q}_i(t) m_{ij} \ddot{q}_j(t)
\]

\[
U_e = \frac{1}{2} \int \left[ EI(x) \sum_{i=1}^{n} q_i(t) \phi''_i(x) + \sum_{i=1}^{n} q_i(t) \phi'_i(x) \sum_{j=1}^{n} q_j(x) \phi'_j(x) \right] dx
+ \kappa GA(x) \left( \sum_{i=1}^{n} q_i(t) W_i'(x) - \sum_{i=1}^{n} q_i(t) \phi_i(x) \sum_{j=1}^{n} q_j(x) W_j'(x) \right) dx
= \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} k''_{ij} q_i(t) q_j(t)
\]

\[
U_q = \frac{1}{2} k \sum_{i=1}^{n} \sum_{j=1}^{n} q_i(t) W_i(x_i) q_j(t) W_j(x_j) = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} q_i(t) k'_{ij} q_j(t)
\]

\[
W = \int \sum_{i=1}^{n} \delta(x - \ddot{x}_i(t)) P_i(t) \sum_{i=1}^{n} q_i(t) W_i(x_i) dx = \sum_{i=1}^{n} q_i(t) f_i(t) \tag{8.4}
\]

where
\[
m_{ij} = \int \rho A(x) [W_i(x) W_j(x) + \gamma^2(x) \phi_i(x) \phi_j(x)] dx
\]

\[
k'_{ij} = \int \left[ EI(x) \phi''_i(x) \phi''_j(x) + \kappa GA(x) [W_i'(x) - \phi_i(x)] [W_j'(x) - \phi_j(x)] \right] dx
\]

\[
k''_{ij} = k \sum_{i=1}^{n} W_i(x_i) W_j(x_j) \tag{8.5}
\]

and \( \ddot{q}_i(t) \) and \( \phi'_i(x) \) denote the first derivatives of \( q_i(t) \) and \( \phi_i(x) \), respectively: \( m_{ij} \) is the generalized mass, and \( f_i(t) \) is the generalized force. Let \( k_{ij} = k_{ij}' + k_{ij}'' \) and \( k_{ij} \) is the generalizes stiffness. The Lagrange equation may be written as follows:
\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} + \frac{\partial U}{\partial q} - \frac{\partial W_e}{\partial q} = \frac{\partial W}{\partial q} \tag{8.6}
\]

\[
W_e = -q^T C q
\]

where \( W_e \) is the work due to the viscous damping in the beam. Substituting Equation (8.4) into Equation (8.6), the equation can be written as
\[
\sum_{j=1}^{n} m_j \ddot{q}_j(t) + \sum_{j=1}^{n} c_j \dot{q}_j(t) + \sum_{j=1}^{n} k_j q_j(t) = f_j(t), \quad (i = 1, 2, \ldots, n) \quad (8.7)
\]

and in matrix form as
\[
M \ddot{q}(t) + C \dot{q}(t) + Kq(t) = F(t) \quad (8.8)
\]

where
\[
M = \begin{Bmatrix} m_i, i = 1, 2, \ldots, n; j = 1, 2, \ldots, n \end{Bmatrix},
\]
\[
K = \begin{Bmatrix} k_i, i = 1, 2, \ldots, n; j = 1, 2, \ldots, n \end{Bmatrix},
\]
\[
C = \begin{Bmatrix} c_i, i = 1, 2, \ldots, n; j = 1, 2, \ldots, n \end{Bmatrix},
\]
\[
q(t) = [q_1(t), q_2(t), \ldots, q_n(t)]^T;
\]
\[
F(t) = [f_1(t), f_2(t), \ldots, f_n(t)]^T.
\]

The dynamic responses of the structure in Equations (8.3) and (8.8) can be obtained using the precise time step integration scheme described in Appendix E.

### 8.2.2 The assumed vibration modes

The general form of the vibration mode for a uniform Timoshenko beam can be written as follow.

\[
W(x) = D_1 \cos(\alpha x) + D_2 \sin(\alpha x) + D_3 \cosh(\beta x) + D_4 \sinh(\beta x)
\]

\[
\psi(x) = D_1 \cos(\alpha x) + D_2 \sin(\alpha x) + D_3 \cosh(\beta x) + D_4 \sinh(\beta x)
\]

where \(D_1, D_2, D_3, D_4, D_5, D_6, D_7\) and \(D_8\) are constants, and \(\alpha, \beta\) are frequency parameters. The vibration modes of a Timoshenko beam with simply supported ends are obtained as follow according to Huang (1961).

\[
\begin{align*}
W_i(x) &= B_i \sin \left( \frac{i \pi x}{L} \right) \\
\phi_i(x) &= \cos \left( \frac{i \pi x}{L} \right)
\end{align*}
\]

where
\[
B_i = \frac{i \pi L}{(i \pi)^2 - b_i^2 s^2}; \quad s^2 = \frac{EI}{\kappa AG L^3}; \quad r^2 = \frac{l}{AL}
\]
\[
b_i^2 = 1 + (i \pi)^2 (r^2 + s^2) - \sqrt{(1 + (i \pi)^2 (r^2 + s^2)^2 - 4(i \pi)^2 r^2 s^2)}
\]

\[
\frac{2 r^2 s^2}{(r^2 + s^2)^2}
\]

\[
(8.12)
\]
8.3 THEORY OF MOVING LOADS IDENTIFICATION FROM BASED ON ASSUMED VIBRATION MODES

8.3.1 From Displacements

Express \( w(x, t) \) in modal coordinates,

\[
  w(x, t) = \sum_{i=1}^{n} W_i(x, t) q_i(t), \quad (s = 1, 2, \ldots, N_s)
\]  

(8.13)

where \( N_s \) is the number of measurement locations; \( \{w(x, t), s = 1, 2, \ldots, N_s\} \) are the displacements at \( x_s \). Equation (8.13) can be written as

\[
  \{w\}_{N_s \times 1} = [W]_{N_s \times n} \{q\}_{n \times 1}
\]  

(8.14)

where \( \{w\}_{N_s \times 1} \) is the vector of displacements at \( N_s \) measurement locations. The vector of generalized coordinates can then be written using the well-known least-squares pseudo inverse

\[
  \{q\}_{n \times 1} = ([W]^T_{n \times N_s} [W]_{N_s \times n})^{-1} [W]^T_{n \times N_s} \{w\}_{N_s \times 1}
\]  

(8.15)

The modal velocity and acceleration are obtained by differentiation, and they are substituted into Equation (8.8) to get

\[
  \{F\}_{n \times 1} = M_{n \times n} \{\dot{q}\}_{n \times 1} + C_{n \times n} \{\ddot{q}\}_{n \times 1} + K_{n \times n} \{q\}_{n \times 1}
\]  

(8.16)

The vector of generalized forces \( \{F\} \) can also be found from Equation (8.5) as

\[
  \{F\}_{n \times 1} = [B]_{n \times N_s} \{p\}_{N_s \times 1}
\]  

(8.17)

where \( \{p\}_{N_s \times 1} \) are the moving forces on the beam, and

\[
  [B]_{n \times N_s} = \begin{bmatrix}
    W_1(\hat{x}_1(t)) & W_1(\hat{x}_2(t)) & \cdots & W_1(\hat{x}_{N_s}(t)) \\
    W_2(\hat{x}_1(t)) & W_2(\hat{x}_2(t)) & \cdots & W_2(\hat{x}_{N_s}(t)) \\
    \vdots & \vdots & \ddots & \vdots \\
    W_n(\hat{x}_1(t)) & W_n(\hat{x}_2(t)) & \cdots & W_n(\hat{x}_{N_s}(t))
  \end{bmatrix}_{n \times N_s}
\]  

(8.18)

By the simple least squares method (LS), the moving force can be calculated directly from

\[
  \{p\}_{N_s \times 1} = ([B]^T_{N_s \times n} [B]_{n \times n})^{-1} [B]^T_{N_s \times n} \{F\}_{n \times 1}
\]  

(8.19)

But since the identified forces \( \{p\} \) are not continuous function of the generalized forces \( \{F\} \), large variations in the results would be obtained by simple least squares method. In order to have bounds on the ill-conditioned forces, the damped least squares method (DLS) (Tikhonov, 1963) is used and singular value decomposition is
used in the pseudo-inverse calculation. Equation (8.19) can be written in the following form using the DLS method:

$\{P\}_{s_1s_2} = ([B]^T \Phi_{s_1s_2} [B]_{s_1s_2} + \lambda I)^{-1} [B]^T \Phi_{s_1s_2} \{F\}_{m_1}$  \hspace{1cm} (8.20)

where $\lambda$ is the non-negative damping coefficient governing the participation of the least-squares error in the solution. The solution of Equation (8.20) is equivalent to minimizing the function

$J(P, \lambda) = \| fB(P) - \{F\} \|^2 + \lambda \| P \|^2$  \hspace{1cm} (8.21)

and the second term in Equation (8.21) provides bounds to the solution. When $\lambda$ approaches zero, the estimated vector $\{P\}$ approaches the solution obtained from the least-squares method. In practice, the expected value of $\lambda$ is not known. Methods to obtain the optimum value of $\lambda$ have been discussed in Chapter 5. In the simulation studies described below, the error between the true and the estimated forces is minimized (Santantammarina and Fratta, 1998) for a specific range of $\lambda$. In the experimental identification of the forces, the error in the identified forces between successive computation with an increment of $\lambda$ is minimized instead.

The identified forces are obtained through the following computations: The mode shapes $w_i(t)$ are computed from Equation (8.11). The generalized coordinate $q_i(t)$ is computed from Equation (8.15) with information on the measured displacements $w_i(x_s, t)$. The derivatives of $q$ and the system matrices are computed, and the generalized force vector $F$ is obtained from Equation (8.16). Matrix $B$ is obtained from Equation (8.18) with information of the normal mode shapes. The non-negative damping coefficient $\lambda$ is obtained from minimization as discussed in last paragraph, and then the moving force vector $\{P\}$ is identified from Equations (8.19) and (3.20) for the least-squares and damped least-squares method.

### 8.3.2 From Strains

The strain at the bottom of the beam can be expressed in terms of the generalized coordinate as

$\varepsilon(x_s, t) = -\frac{h}{2} \sum_{i=1}^{n} \phi_i'(x_s) q_i(t). \hspace{1cm} (s = 1, 2, \cdots, N_s)$  \hspace{1cm} (8.22)
where \( N_s \) is the number of measurement points; \( \{ \varepsilon(x_i, t), i=1,2,\ldots, N_s \} \) are the strain at \( x_i \). When written in matrix form

\[
\{ \varepsilon \}_{N_s \times 1} = [\phi']_{N_s \times n} \{ q \}_{n \times 1}
\]  

(8.23)

The vector of generalized coordinates \( \{ q \}_{n \times 1} \) can be similarly calculated as for Equation (8.14).

\[
\{ q \}_{n \times 1} = ([\phi']^T_{n \times N_s} [\phi']_{N_s \times n})^{-1} [\phi']^T_{n \times N_s} \{ \varepsilon \}_{N_s \times 1}
\]  

(8.24)

The subsequent identification process would be the same as for the displacements.

### 8.3.3 From Both Strains and Displacements

If the strains and displacements are measured at the same time, both of them can be used in the identification. But the strains and displacements should be scaled by their respective norms to have dimensionless units.

\[
\begin{bmatrix}
\varepsilon \\
\| \varepsilon \| \\
\frac{W}{\|W\|}_{N_s \times 1}
\end{bmatrix}
= 
\begin{bmatrix}
\phi' \\
\| \phi' \| \\
\frac{W}{\|W\|}_{N_s \times n}
\end{bmatrix}
\{ q \}_{n \times 1}
\]

(8.25)

where \( \| \bullet \| \) is the norm of the vector.

### 8.4 IMPLEMENTATION AND SIMULATION STUDIES

#### 8.4.1 Single Force Identification on a Single Span Beam

A single span simply supported beam with a single force moving on top is studied.

\[
f(t) = 40000 [ 1 + 0.1 \sin(10 \pi t) + 0.05 \sin(40 \pi t) ] 
\]

(8.26)

The parameters of the beam are as follow: \( EI = 1.274916 \times 10^{11} \text{ Nm}^2 \), \( \rho = 7700 \text{ kgm}^{-3} \), \( \rho A = 12000 \text{ kg/m} \), \( L = 20 \text{ metres} \), \( G = 77.6 \times 10^9 \text{Nm}^2 \). The moving speed is 20m/s. and the shear coefficient \( \kappa \) is 5/6. The first three modes of the beam are considered. White noise is added to the calculated displacements to simulate the polluted measurement as follows and 1, 5 and 10 percent noise levels are studied.

\[
w = w_{\text{calculated}} (1 + E \rho \cdot N_{\text{noise}})
\]

\[
\varepsilon = \varepsilon_{\text{calculated}} (1 + E \rho \cdot N_{\text{noise}})
\]

(8.27)
where $w$ and $\varepsilon$ are the polluted displacement and strain respectively; $E_p$ is the noise level; $N_{\text{noise}}$ is a standard normal distribution vector with zero mean value and unit standard deviation. $w_{\text{calculated}}$ and $\varepsilon_{\text{calculated}}$ are the calculated displacement and strain.

The errors in the identified forces are calculated as

$$\text{Error} = \frac{\|P_{\text{identified}} - P_{\text{true}}\|}{\|P_{\text{true}}\|} \times 100\% \quad (8.28)$$

Table 8.1 shows the errors on single force identification with different number of vibration modes. The responses at ten measurement points evenly distributed along the beam are simulated from the first ten modes. These responses are used in the identification. Table 8.2 shows the errors on single force identification with different number of measuring points, and the first three modes are used in the simulation. Figure 8.2 shows the identified results with different number of vibration modes. Displacement responses from the same number of sensors as the vibration modes are used. The sampling frequency is 2.5 times the maximum natural frequency of interest for the above studies.

The following results are obtained in the simulation studies:

1. The results show that the proposed method and algorithm for one moving force identification are correct. The identified force is very close to the true force.

2. When the number of displacement response is not less than the number of vibration modes, the error is small, and it decreases with increase in the number of responses used. This indicates that the number of displacement responses must not less than the number of the vibration modes used in the identification. The number of displacement responses can be taken equal to the number of the vibration modes in the force identification in practice.

3. The error in the identified force is not sensitive to the noise level in the response measurements used for the identification.

4. Table 8.1 shows that the error in the identified force does not change much when the number of the vibration modes is larger than 3. This may be due to the fact that the fourth mode is at a frequency of approximately 110Hz, and the corresponding modal response of the beam is small. Additional vibration modes higher than the third mode contain less information on the moving forces. This is supported by Figure 8.2 where the identified force from both 3 modes and 6
modes are very close to the true force except some variations close to the entry and exit of the force.

### 8.4.2 Two Forces Identification on Two Span Continuous Beam

Again a two-span simply supported beam is considered. The total length is 40 metres, and the intermediate support is at the middle of the beam. This support is modelled with a stiff linear spring of stiffness equals to $10^{16}$ N/m. The parameters of the beam are the same as described in Section 8.4.1. The two moving forces are

\[
\begin{align*}
    f_1(t) &= 20000[1 + 0.1 \sin(10\pi t) + 0.05 \sin(40\pi t)] \quad N; \\
    f_2(t) &= 20000[1 - 0.1 \sin(10\pi t) + 0.05 \sin(50\pi t)] \quad N; \\
\end{align*}
\]

(8.29)

and they are moving at 40 m/s. The distance $l_s$ between the two moving forces is 4 metres. Again, the first three modes are used in the calculation. The damping ratio is 0.02 for all the three modes. The sampling frequency is 100 Hz and 110 points are used in the identification. The strain and displacement measurements at 1/8, 1/4 and 5/8 span are used with 5 percent added noise. The identified forces from DLS method are shown in Figures 8.3 and 8.4, and the results from both LS and DLS methods are also shown in Tables 8.3 and 8.4. The following observations are made:

1. When the number of measured strains and displacements is not less than the number of vibration modes used in the identification, acceptable results could be obtained. This shows that the proposed method and the algorithm used for two-forces identification are correct and they can be used in practical multi-forces situation.

2. The magnitude of error remains relatively constant at different noise level. This indicates that the proposed method is not sensitive to noise.

3. There are large errors in the forces at both the entry and exit of the beam. These errors are found to decrease as the stiffness of the beam is increased in the simulation. These large errors are due to the low sensitivity of the bridge responses to the forces at the beginning and the end of the beam. The impulsive forces generated at these points also contribute greatly to these errors.

4. The moving forces on a two-span continuous beam can be identified using only the measuring data from one span of the two-span continuous structure as same in Tables 8.3 and 8.4.
(5) The forces identified from the strains are of the same accuracy as those obtained from the displacements. But since the strain measurements can be easily obtained, force identification from the strain measurements would be very convenient and useful.

(6) The error in the identified forces obtained by using LS is larger than that from using DLS. This shows the effectiveness of the damped least squares method in providing bounds on the ill-conditioned forces with an optimal damping coefficient $\lambda$.

8.5 COMPARATIVE STUDY ON USING DIFFERENT BEAM MODELS

Both Timoshenko beam model and Euler-Bernoulli beam model are used in the simulation studies for a comparison of their accuracy in the force identification problem. The effects of different influencing parameters of the dynamic system on the identification results are studied in the following two examples. The influencing parameters are the number of sensors, vibration modal truncation, sampling frequency, spacing of forces, excitation frequency, and the measurement noise level.

8.5.1 Single moving force identification on a single span beam

The parameters of the beam and the force are the same as in section 8.4.1. The first three modes are used in the calculation. Measured strains at 1/4L, 1/2L and 3/4L are used in the identification. The sampling frequency is 100 Hz.

Study 1: The optimal regularization parameter

Table 8.5 shows the optimal value of $\lambda$ and the corresponding errors for the two beam models. Since the true force is known, the optimal regularization parameter $\lambda$ is determined from both the S-curve and L-curve methods, and they are the same for both beam models. The optimal value $\lambda$ increases as the noise level increases, and yet the identified error and the norm are relatively stable. This result seems to indicate that the identified error is not sensitive to the noise level in the measurement. However this result is based on a limited study, and the noise effect will be further studied below.
Study 2: Effect of number of sensors

The number of sensors is varied, and they are evenly distributed on the beam. Table 8.6 shows that the error in the identified forces has a significant reduction when the number of sensors is equal to or larger than the number of vibration modes used (which is three in this case) in the identification. This observation is consistent with the normal practice in vibration measurement of having one sensor for each vibration mode to be detected. More sensor will not increase significantly the information collected for a particularly set of modes to be monitored. Hence the number of sensors for the identification is recommended to be at least equal to the number of vibration modes in the study.

Study 3: Effect of sampling frequency

Table 8.7 shows the relation between the sampling frequency and the error in the identification with 5% noise level. The optimal $\lambda$ is relative stable indicating consistent quality in the identified results. The error remains relatively stable as the sampling frequency increases for both beam models. Therefore the sampling frequency need not be very high in practice, and it may be taken as larger than two times the maximum frequency of interest to be consistent with the requirement in digital signal analysis. This recommendation means that the moving force identification method can use a relatively low sampling rate to be computational efficient.

Study 4: Effect of modal truncation

Table 8.8 gives the errors in the identified force with different number of vibration modes when 5% noise level is included. The number of measuring points is taken equal to the number of vibration modes, and they are evenly located on the beam. The sampling frequency is two times the highest frequency of interest as suggested above. For the beam-force combination as the one we have, the maximum natural frequency of the beam considered is larger than the highest frequency of interest or the exciting frequency of the moving force. The effect of higher modes is not included due to the low sampling rate used. As a result of this, the error in the identified force remains relatively the same and it does not depend on the number of
vibration modes used. It is therefore recommended to determine the number of vibration modes from the highest frequency of interest.

**Study 5: Effect of exciting frequency**

The major exciting frequency of the force in Equation (8.26) varies from 0 to 50 Hz. Figure 8.5 shows the error of the identified force for each of the frequencies with 5% noise level. The error in the identified force increases when the exciting frequency of the moving force approaches the natural frequency of the beam-force system. The natural frequency of the beam-force system is smaller than that for the beam alone due to the existence of the force acting on top. The error around the third natural frequency identified by Euler-Bernoulli beam model is larger than that by Timoshenko beam model, but it is the opposite around the first natural frequency. Timoshenko beam model in general gives larger errors than the Euler-Bernoulli beam model over the whole range of frequency studied.

**8.5.2 Two moving forces identification on a two-span continuous beam**

The forces are the same as described in Section 8.4.2. Note that there is an opposite component in the forces representing the pitching motion of a vehicle. The parameters of the beam are the same as in section 8.4.1 except that the two beam span lengths are each 20 metres. The moving speed is again constant at 40m/s and the initial position of the first force is at the left end of the beam. 5% noise is included in the measured responses.

The first three modes are used in the identification. The measured displacements at 1/8L, 1/4L and 3/4L are used in the calculation. The sampling frequency is 200Hz.

**Study 6: Two moving forces identification**

Figures 8.6 and 8.7 show the identified forces using Timoshenko beam theory and Euler-Bernoulli beam theory. The curves are obtained after regularization, and they exhibit large discrepancies with the true forces at the beginning and end of the curves. Improvement to the results can be made by dividing the time history into smaller time segments, and different regularization parameters can be used in each of
these segments as suggested by Choi and Chang (1996). The identified results from Timoshenko beam and Euler-Bernoulli beam models have no significant differences and they are close to each other throughout the time histories. It is also noted that the identified forces follow the main trend of the true forces except for some high frequency components which are due to the 5% noise introduced in the analysis.

Study 7: Effect of distance between two forces

When two forces are close together, their effect on the dynamic response of a beam may not be easily differentiated, and the resolution of the two forces in the force identification may be reduced. Figure 8.8 shows the plot of errors of identification against the distance between the two moving forces at 5% noise level. The figure shows that the error increases monotonically as the distance between two moving forces increases with smaller value at shorter distance. This is contrary to the usual belief of having larger error at a smaller spacing. Hence the moving force identification method can be used to identify two moving forces at a close spacing. The errors from the Timoshenko beam model and Euler-Bernoulli beam model are similar with the former slightly larger than the latter. Three small peaks in the error curves are identified at 1.25Hz, 2.5Hz and 3.75Hz which are themselves harmonics. However their existence cannot be explained in this study.

8.6 LABORATORY STUDIES

The experimental setup is shown diagrammatically in Figure 3.3. The main beam, 3678mm long with a 100mm×25mm uniform cross-section, is simply supported. A U-shaped aluminum section is glued to the upper surface of the beams as a direction guide for the car. The model car is pulled along the guide by a string wound around the drive wheel of an electric motor. Seven photoelectric sensors are mounted on the beams to measure and monitor the moving speed of the car. They are located on the beam at roughly equal spacing of 0.776m to check on the uniformity of the speed. Seven strain gauges are evenly located on the beam at one-eighth span spacing to measure the responses of the beam. A Data Translation DT2829 eight-channel dynamic A/D board is used for data collection in the experiment. The measured frequencies of the model car and the main beam are shown in Table 7.5. The sampling frequency is 2000Hz, and each data record duration lasts for 6 seconds.
The model car has two axles at a space of 0.557m and it runs on four rubber wheels. The mass of the whole car is 16.6Kg.

The first three modes are used in the identification. Correlation coefficients are calculated between the measured strain and the strain reconstructed from the identified forces for twelve combinations of measured strains, and they are shown in Table 8.9. The Euler-Bernoulli beam model gives slightly poorer correlation in all the cases than the Timoshenko beam model. There are two cases where the correlation is less than 0.3 while the latter model can still identify good results with a correlation above 0.7. However the correlation coefficients vary with different combination of measured information. Those derived from using more than three sensors are not much better than that those from using three sensors. This confirms the result from Study 2.

The optimal regularization parameter $\lambda$ is obtained from a modified plot of the $L$-curve by separately plotting the norm of the error and the seminorm of the solution against the parameter, and the intersection of the two curves gives the optimal value. The plot for the Timoshenko beam model and the Euler-Bernoulli beam model is shown in Figure 8.9, and both models give the same optimal value.

Figures 8.10 and 8.11 show the identifying forces from strains at 1/4L, 1/2L and 3/4L. Only the results from the Timoshenko beam model with and without regularization are shown. Fluctuations in the time histories are found around 0.0s, 0.5s, 3.0s and 3.6s in the curves without regularization. These moments correspond to the entry of the first and second axle on the main beam and the exit of the first and second axle from the main beam, when the forcing system switches from single force excitation to two-forces excitation or vice versa. Such large fluctuations may also be caused by the jumping of the wheels over the gaps between the beam ends at the entry and exit which induce impulsive forces on the main beam. The fluctuations in the curves disappear after regularization, and the regularization has no effect on the time histories between these moments. These observations indicate that the regularization procedure only provide bounds to the ill-conditioned solutions without any smoothing effect. Errors in the curves in the form of high frequency components has to be treated by some other means.

Figure 8.12 shows the reconstructed strains from the identified forces obtained from the Timoshenko beam model and the measured strain at 3/8 span. The
reconstructed strain varies around the curve of measured strain with some high frequency components which are due to the measurement noise.

The two identified forces are added together and the resultant is shown in Figure 8.13. The curve is higher than the static curve in the first half of the time history and it is lower than the static curve in the second half. This arises from the deflection of the beam under its own weight. The model car accelerates downwards in the first half of the beam and it decelerates upwards in the second half of the beam. The differences from the static curve are due to these acceleration forces. The resultant force obtained from regularization is close to the static weight from the car, and is a good indication of the accuracy of the moving force identification method in identifying the resultant force of a system of moving forces.

8.7 CONCLUDING REMARKS

A moving force identification method has been improved with a regularization procedure applied to the identified results. The ill-conditioned identified forces at the beginning and end of the time history are significantly improved. The regularization procedure does not have any smoothing effect on the results. The Timoshenko beam model gives better results in general than the Euler-Bernoulli beam model.

Limitations on the application of this force identification method are studied, and the following recommendations are reported: The number of modes required in the identification depends entirely on the highest frequency of interest. The sampling frequency may be taken as two times the highest frequency of interest which may be the maximum exciting frequency of the moving force or the natural frequency of the beam. The number of sensors is recommended to be at least equal to the number of vibration modes in the analysis. The error in the identified forces become large when the exciting frequency of the moving force comes close to the natural frequency of the beam-force system. The error of identification is not sensitive to the distance between the two moving forces. Therefore two forces moving at a close spacing can be resolved by this method.
### Table 8.1 Errors on single force identification (in percent)

<table>
<thead>
<tr>
<th>Number of Vibration Modes</th>
<th>1%</th>
<th>Noise level 5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.83</td>
<td>17.31</td>
<td>19.08</td>
</tr>
<tr>
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<tr>
<td>10</td>
<td>9.92</td>
<td>11.05</td>
<td>14.04</td>
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### Table 8.2 Errors on single force identification (in percent)

<table>
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<th>Number of Measuring Points</th>
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<th>Noise level 5%</th>
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<tr>
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<td>12.88</td>
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### Table 8.3 Errors on two-forces identification by LS method (in percent)

<table>
<thead>
<tr>
<th>Locations and responses</th>
<th>1% error in response</th>
<th>5% error in response</th>
<th>10% error in response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First</td>
<td>Second</td>
<td>First</td>
</tr>
<tr>
<td>1/4D,3/4D</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
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<td>112.78</td>
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<td>72.87</td>
<td>114.25</td>
<td>77.15</td>
</tr>
<tr>
<td>1/8D,1/4D,1/4s</td>
<td>72.87</td>
<td>113.35</td>
<td>73.52</td>
</tr>
<tr>
<td>1/8s,1/4s,1/8D</td>
<td>72.80</td>
<td>113.79</td>
<td>73.40</td>
</tr>
<tr>
<td>1/8s,1/4s,3/8s</td>
<td>72.58</td>
<td>111.67</td>
<td>68.83</td>
</tr>
</tbody>
</table>

Note: * represents the error is larger than 1000%.

### Table 8.4 Errors on two-forces identification by DLS method (in percent)

<table>
<thead>
<tr>
<th>Locations and responses</th>
<th>1% error in response</th>
<th>5% error in response</th>
<th>10% error in response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First</td>
<td>Second</td>
<td>First</td>
</tr>
<tr>
<td>1/4D,3/4D</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>1/8D,1/4D,3/8D</td>
<td>25.72</td>
<td>29.41</td>
<td>25.80</td>
</tr>
<tr>
<td>1/8D,1/4D,5/8D</td>
<td>25.63</td>
<td>29.31</td>
<td>26.38</td>
</tr>
<tr>
<td>1/8D,1/4D,3/4D</td>
<td>25.61</td>
<td>29.44</td>
<td>25.64</td>
</tr>
<tr>
<td>1/8D,1/8s,1/4D</td>
<td>25.60</td>
<td>29.35</td>
<td>26.04</td>
</tr>
<tr>
<td>1/8D,1/4D,1/4s</td>
<td>25.68</td>
<td>29.32</td>
<td>25.66</td>
</tr>
<tr>
<td>1/8s,1/4s,1/8D</td>
<td>25.75</td>
<td>29.38</td>
<td>25.97</td>
</tr>
<tr>
<td>1/8s,1/4s,3/8s</td>
<td>25.74</td>
<td>29.33</td>
<td>25.97</td>
</tr>
</tbody>
</table>

Note: * represents the error is larger than 1000%.
Table 8.5 The optimal regularization parameter and errors under different noise level

<table>
<thead>
<tr>
<th>Noise level(%)</th>
<th>Optimal $\lambda$</th>
<th>Error</th>
<th>Norm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.37</td>
<td>13.65</td>
<td>$2.83 \times 10^{11}$</td>
</tr>
<tr>
<td>5</td>
<td>0.54</td>
<td>13.50</td>
<td>$2.29 \times 10^{11}$</td>
</tr>
<tr>
<td>10</td>
<td>1.05</td>
<td>17.80</td>
<td>$2.16 \times 10^{11}$</td>
</tr>
</tbody>
</table>

Table 8.6 Error of identification with different number of sensors

<table>
<thead>
<tr>
<th>Beam model</th>
<th>Noise level</th>
<th>Number of sensors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1%</td>
<td>2</td>
</tr>
<tr>
<td>T</td>
<td>105.6</td>
<td>23.4</td>
</tr>
<tr>
<td></td>
<td>103.5</td>
<td>22.9</td>
</tr>
<tr>
<td></td>
<td>103.5</td>
<td>26.0</td>
</tr>
<tr>
<td>E</td>
<td>1%</td>
<td>140.5</td>
</tr>
<tr>
<td></td>
<td>140.0</td>
<td>84.1</td>
</tr>
<tr>
<td></td>
<td>139.4</td>
<td>84.1</td>
</tr>
</tbody>
</table>

Note: T—Timoshenko beam; E—Euler-Bernoulli beam.

Table 8.7 Error in identification with different sampling frequency

<table>
<thead>
<tr>
<th>Beam model</th>
<th>Sampling frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
</tr>
<tr>
<td>T</td>
<td></td>
</tr>
<tr>
<td>Error(%)</td>
<td>12.49</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.002</td>
</tr>
<tr>
<td>E</td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Note: T—Timoshenko beam; E—Euler-Bernoulli beam.
Table 8.8 Error of identification with different number of vibration modes

<table>
<thead>
<tr>
<th>Beam model</th>
<th>Number of modes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>T Error(%)</td>
<td>16.5</td>
</tr>
<tr>
<td>λ</td>
<td>0.43</td>
</tr>
<tr>
<td>E Error(%)</td>
<td>17.0</td>
</tr>
<tr>
<td>λ</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Note: T—Timoshenko beam; E—Euler-Bernoulli beam.

Table 8.9 Correlation coefficient between reconstructed and measured strains

<table>
<thead>
<tr>
<th>Sensor combinations</th>
<th>Timoshenko beam model</th>
<th>Euler-Bernoulli Beam model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/8s, 1/2s, 3/4s</td>
<td>0.939</td>
<td>0.900</td>
</tr>
<tr>
<td>1/4s, 1/2s, 3/4s</td>
<td>0.939</td>
<td>0.926</td>
</tr>
<tr>
<td>1/8s, 1/2s, 7/8s</td>
<td>0.943</td>
<td>0.897</td>
</tr>
<tr>
<td>1/8s, 1/4s, 1/2s</td>
<td>0.919</td>
<td>0.280</td>
</tr>
<tr>
<td>1/8s, 1/4s, 5/8s</td>
<td>0.901</td>
<td>0.564</td>
</tr>
<tr>
<td>1/8s, 1/4s, 7/8s</td>
<td>0.894</td>
<td>0.620</td>
</tr>
<tr>
<td>5/8s, 3/4s, 7/8s</td>
<td>0.729</td>
<td>0.278</td>
</tr>
<tr>
<td>1/8s, 1/4s, 3/4s, 5/8s</td>
<td>0.929</td>
<td>0.878</td>
</tr>
<tr>
<td>1/8s, 1/4s, 3/4s, 7/8s</td>
<td>0.893</td>
<td>0.657</td>
</tr>
<tr>
<td>1/8s, 1/4s, 1/2s, 3/4s, 7/8s</td>
<td>0.946</td>
<td>0.917</td>
</tr>
<tr>
<td>1/8s, 1/4s, 1/2s, 5/8s, 3/4s, 7/8s</td>
<td>0.941</td>
<td>0.909</td>
</tr>
<tr>
<td>1/8s, 1/4s, 5/8s, 7/8s</td>
<td>0.924</td>
<td>0.821</td>
</tr>
</tbody>
</table>

Note: 1/8s – measured strain at 1/8 span.
Figure 8.1 A continuous beam with (Q-1) intermediate point supports under $N_p$ moving forces
Figure 8.2 Time histories of identified force in single force simulation

(- true force; ---- from 3 modes; ... from 6 modes.)
Figure 8.3 Time histories of identified first force in two forces simulation
(- true force; ---- 1/8,1/4,5/8D; ... 1/8,1/4,5/8e.)
Figure 8.4 Time histories of identified second force in two forces simulation

(- True force; --- 1/8,1/4,5/8D; ... 1/8,1/4,5/8e.)
Figure 8.5 Error in identified force with different exciting frequency
(— Using Euler–Bernoulli beam model; – Using Timoshenko beam model.)
Figure 8.6 The identified first force with 5% noise

(- True force; ---- force from Timoshenko beam model; ... force from Euler–Bernoulli beam model.)
Figure 8.7 The identified second force with 5% noise
(- True force; ---- force from Timoshenko beam model;
... force from Euler–Bernoulli beam model.)
Figure 8.8 Effect of spacing of two forces on error in identification (with 5% noise)
(- from Timoshenko beam model; --- from Euler-Bernoulli beam model.)
Figure 8.9 The modified L-curve
(- Timoshenko beam model; ---- Euler-Bernoulli beam model.)
Figure 8.10 The first identified force from experiment using Timoshenko beam model
(- static force; ... without regularization; ---- with regularization.)
Figure 8.11 The second identified force from experiment using Timoshenko beam model
(— static force; ... without regularization; —— with regularization.)
Figure 8.12 The measured and reconstructed strain at 3/8 span from experiment
(--- measure; ... without regularization; —— with regularization.)
Figure 8.13 The identified resultant force from experiment
( - static force; ... without regularization; ---- with regularization.)
Chapter 9

IDENTIFICATION OF MOVING LOADS ON BRIDGE DECK

9.1 INTRODUCTION

All the identification methods described in previous chapters are based on the bridge-vehicle interaction with a simply supported beam. A beam model cannot truly represent the three-dimensional behavior of the bridge deck in practice, particularly when a vehicle travels not along the centerline of the bridge deck. Since many types of bridge decks, including those of slab bridges, hollow-core slab bridges, and deck and girder bridges can be effectively modeled by an isotropic or orthotropic plate (Bakht and Jaeger, 1985), identification of the real interaction forces in a three-dimensional problem is possible and feasible.

The bridge deck is modeled as a simply supported orthotropic rectangular plate in this chapter. Dynamic behavior of the bridge deck under moving loads is analyzed using the orthotropic plate theory and modal superposition. The two moving load identification methods developed in Chapter 6 are extended and applied to this study, and the solutions are obtained in time domain. Computational simulations show the effectiveness and the validity of the proposed methods in identifying forces travelling along the central line or at an eccentric path on the bridge deck.

9.2 DYNAMIC BEHAVIOR OF BRIDGE DECKS UNDER MOVING LOADS

9.2.1 Free Vibration of an Orthotropic plate

The problem of a plate under the action of moving forces attracted much research attention only in the last two decades. Fryba (1972) have solved analytically the dynamic responses of a uniform flat plate under a moving load along a specified path. Wu et al (1987) analyzed the dynamic responses of a flat plate subjected to various moving loads by the finite element method. Later Wang and Lin (1996) analyzed the dynamic behavior of a multi-span continuous Mindlin plate subjected to a moving load. Transfer matrix is used to determine the natural frequency and the
vibration modes of the plate. Marchesiello et al (1999) analyzed the dynamics of multi-span continuous straight bridges subjected to multi-degrees-of-freedom moving vehicle excitation by applying the mode superposition principle. The modes are computed by means of the Rayleigh-Ritz method. Chan and Chan (1999) analyzed the dynamic behavior of slab-on-girder bridges by eccentric beam elements. In this chapter, the bridge deck is modelled as an orthotropic plate, and the dynamic behavior of the bridge deck under moving loads is analyzed basing on the modal superposition principle. A brief description of the plate model used in this chapter is given below.

According to Huffington and Hoppmann (1958), the governing equations of motion of an orthotropic plate shown in Figure 9.1 can be written as follow.

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_y \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_z \frac{\partial^4 w}{\partial y^4} + C \frac{\partial w}{\partial t} + \rho h \frac{\partial^2 w}{\partial t^2} = p$$  \hspace{1cm} (9.1)

where $D_\alpha = (D_x \nu_{xx} + 2D_z)$, $D_x = \frac{E_x h^3}{12(1-\nu_{xx}^2)}$, $D_y = \frac{E_y h^3}{12(1-\nu_{yy}^2)}$, $D_z = \frac{G_{xy} h^3}{12}$.

$D_x, D_y$ are the flexural rigidities of the orthotropic plate in the $x$- and $y$- directions, respectively. $D_z$ is the twisting rigidity of the plate. $C$ is the damping coefficient. $h$ is the thickness of the plate. $E_x, E_y$ are the modulus of the plate in the $x$- and $y$- directions, respectively; $\nu_{xx}$ is the Poisson’s ratio associated with a strain in the $y$- direction for a load in the $x$- direction. $G_{xy}$ is the shear modulus. $p(x,y,t)$ is the external moving load. $w(x,y,t)$ is the displacement of plate in the $z$-direction. $\rho$ is the mass density of plate material.

The free vibration of the plate without damping is analyzed. Assuming the plate is simply supported along $x=0$ and $x=a$ with the other two sides free to vibrate, the displacement of the plate can be written as

$$w(x,y,t) = \sum_{\alpha,\mu} Y_{\alpha\mu}(y) \sin(\frac{m\pi x}{a}) \sin(\omega_{\alpha\mu} t + \theta)$$  \hspace{1cm} (9.2)

where $\omega_{\alpha\mu}$ is the natural frequency corresponds to the $m$th mode in the $x$-direction and $\alpha$th mode in the $y$-direction; $\theta$ is the initial angle. $Y_{\alpha\mu}(y) \sin(m\pi x/a)$ is the mode shape. Substituting Equation (9.2), Equation (9.1) becomes

$$D_x Y_{\alpha\mu}(y) \frac{d^4}{dx^4} - 2D_y \left(\frac{m\pi}{a}\right)^2 Y_{\alpha\mu}(y) + [D_z \left(\frac{m\pi}{a}\right)^4 - \rho h \omega_{\alpha\mu}^2] Y_{\alpha\mu}(y) = 0$$  \hspace{1cm} (9.3)
The solution on $Y_{mn}(y)$ can be obtained and classified as follows according to the following properties of the plate.

1) When $D_z < \rho h \omega_{\text{mn}}^2 \left(\frac{a}{m\pi}\right)^4$, 

$$Y_{mn}(y) = A_{mn} \sinh(r_{1mn}y) + B_{mn} \cosh(r_{1mn}y) + C_{mn} \sin(r_{1mn}y) + D_{mn} \cosh(r_{1mn}y)$$

(9.4)

where

$$r_{1mn} = \frac{m\pi}{a} \sqrt[4]{\frac{D_y + \sqrt{D_y^2 + D_z \rho h \omega_{\text{mn}}^2 \left(\frac{a}{m\pi}\right)^4 - D_z D_y}}{D_y}}$$

(9.5)

$$r_{2mn} = \frac{m\pi}{a} \sqrt[4]{\frac{-D_y + \sqrt{D_y^2 + D_z \rho h \omega_{\text{mn}}^2 \left(\frac{a}{m\pi}\right)^4 - D_z D_y}}{D_y}}$$

2) When $D_{yz}^2 + \rho h \omega_{\text{mn}}^2 \left(\frac{a}{m\pi}\right)^4 \begin{cases} > D_z > \rho h \omega_{\text{mn}}^2 \left(\frac{a}{m\pi}\right)^4, \end{cases}$

$$Y_{mn}(y) = A_{mn} \sinh(r_{1mn}y) + B_{mn} \cosh(r_{1mn}y) + C_{mn} \sinh(r_{2mn}y) + D_{mn} \cosh(r_{2mn}y)$$

(9.6)

where

$$r_{3mn} = \frac{m\pi}{a} \sqrt[4]{\frac{D_y - \sqrt{D_y^2 + D_z \rho h \omega_{\text{mn}}^2 \left(\frac{a}{m\pi}\right)^4 - D_z D_y}}{D_y}}$$

(9.7)

3) When $D_z > \frac{D_{yz}^2}{D_y} + \rho h \omega_{\text{mn}}^2 \left(\frac{a}{m\pi}\right)^4$, 

$$Y_{mn}(y) = \cosh(r_{3mn}y) \left( A_{mn} \cos(r_{3mn}y) + B_{mn} \sin(r_{3mn}y) \right) + \sinh(r_{4mn}y) \left( C_{mn} \cos(r_{3mn}y) + D_{mn} \sin(r_{3mn}y) \right)$$

(9.8)

where

$$r_{4mn} = \frac{m\pi}{a} \sqrt[4]{\frac{1}{2} \left( \frac{D_{xyz}}{D_y} + \frac{D_z - \rho h \omega_{\text{mn}}^2 \left(\frac{a}{m\pi}\right)^4}{D_y} \right)}$$

$$r_{5mn} = \frac{m\pi}{a} \sqrt[4]{\frac{1}{2} \left( \frac{D_{xyz}}{D_y} + \frac{D_z - \rho h \omega_{\text{mn}}^2 \left(\frac{a}{m\pi}\right)^4}{D_y} \right)}$$

(9.9)
Chapter 9: Identification of Moving Loads on Bridge Decks

The parameters $A_{mm}, B_{mm}, C_{mm}, D_{mm}$ and the natural frequencies are determined from the free boundary conditions at $y=0$ and $y=b$. The edge moment, the transverse shear and the torsional moment are zero at these edges, giving

$$\left\{ \begin{align*}
\frac{\partial^4 w}{\partial y^2} + \nu \frac{\partial^4 w}{\partial x^2} &= 0 \\
-D_n \frac{\partial^3 w}{\partial x \partial y} - D_r \frac{\partial^3 w}{\partial y^3} &= 0 \\
2D_4 \frac{\partial^3 w}{\partial x^2 \partial y} &= 0 \\
-D_n \frac{\partial^3 w}{\partial x^3 \partial y} - D_r \frac{\partial^3 w}{\partial x \partial y^3} - 2D_4 \frac{\partial^3 w}{\partial x^2 \partial y} &= 0
\end{align*}\right. \quad (y = 0 \text{ or } y = b) \quad (9.10)$$

Substitute Equation (9.2) and Equations (9.4)-(9.9) into Equation (9.10). The following equation can be obtained.

$$A \{ C \} = 0 \quad (9.11)$$

where $A$ is a coefficient matrix $\{a_{ij}\}$, detail formulation of each coefficient are listed in Appendix F for the different classifications presented above, and

$$\{C\} = \{A_{mm}, B_{mm}, C_{mm}, D_{mm}\}^T.$$

Since only the non-zero solution of Equation (9.11) is of interest, the determinant of the coefficient matrix is set equal to zero from which the natural frequencies $\omega_{mn} (m = 1, 2, \cdots; n = 1, 2, \cdots)$ are obtained. Vector $\{C\}$ can then be obtained for each natural frequency, and hence $Y_{mn}(y)$ can be found.

### 9.2.2 Dynamic Behavior under Moving Loads

The equations of motion of a damped orthotropic plate under moving loads expressed in Equation (9.1) can be written as follows by expressing the force $p$ as a time step function.

$$D_n \frac{\partial^4 w}{\partial x^4} + 2D_n \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_r \frac{\partial^4 w}{\partial y^4} + C \frac{\partial w}{\partial t} + ph \frac{\partial^2 w}{\partial t^2} = \sum_{i=1}^{N_p} p_i(t) \delta(x - \hat{x}_i(t)) \delta(y - \hat{y}_i(t)) \quad (9.12)$$

where $\{p_i(t), l = 1, 2, \cdots, N_p\}$ are the moving loads and they are moving as a group at a fixed spacing. $(\hat{x}_i(t), \hat{y}_i(t))$ is the position of the moving load $p_i(t)$. $\delta(x), \delta(y)$ are the Dirac function. By modal superposition, the displacement of the orthotropic plate can be written as follow.
\[ w(x, y, t) = \sum_{m,n} W_{mn}(x, y) q_{mn}(t) \]  

(9.13)

where \( W_{mn}(x, y) = Y_{mn}(y) \sin\left(\frac{m \pi x}{a}\right) \) is the mode shape of the orthotropic plate, and \( q_{mn}(t) \) is the corresponding modal coordinate.

Substituting Equation (9.13) into Equation (9.12) results in

\[ \ddot{q}_{mn}(t) + 2 \zeta_{mn} \omega_{mn} \dot{q}_{mn}(t) + \omega_{mn}^2 q_{mn}(t) = \frac{2}{\rho ah} \int_{-b}^{b} \sum_{i=1}^{n_p} p_i(t) Y_{mn}(\tilde{y}_i(t)) \sinh\left(\frac{m \pi \tilde{x}_i(t)}{a}\right) dy \]

(9.14)

\[ (m, n = 1, 2, \ldots) \]

where \( \zeta_{mn} = \frac{C}{2 \rho \omega_{mn} a} \). \( a, b \) are the dimensions of the orthotropic plate in \( x \)- and \( y \)-directions respectively. Equation (9.14) can be solved in the time domain by the convolution integral with the plate initially at rest, yielding

\[ q_{mn}(t) = \frac{1}{M_{mn}} \int_{0}^{t} H_{mn}(t - \tau) f_{mn}(\tau) d\tau \]

(9.15)

where

\[ M_{mn} = \frac{\rho h a}{2} \int_{-b}^{b} Y_{mn}^2(y) dy \]

\[ H_{mn}(t) = \frac{1}{\omega_{mn}'} e^{-\omega_{mn}' t} \sin(\omega_{mn}' t), \quad t \geq 0 \]

(9.16)

\[ f_{mn}(t) = \sum_{i=1}^{n_p} p_i(t) Y_{mn}(\tilde{y}_i(t)) \sinh\left(\frac{m \pi \tilde{x}_i(t)}{a}\right) \]

\[ \omega_{mn}' = \omega_{mn} \sqrt{1 - \zeta_{mn}^2} \]

Substituting Equation (9.15) into Equation (9.13), the displacement of the orthotropic plate at point \((x, y)\) and time \( t \) can be found, as

\[ w(x, y, t) = \sum_{m,n} \sum_{i=1}^{n_p} Y_{mn}(y) \sinh\left(\frac{m \pi x}{a}\right) \frac{1}{M_{mn}} \int_{0}^{t} H_{mn}(t - \tau) f_{mn}(\tau) d\tau \]

(9.17)

### 9.3 MOVING LOAD IDENTIFICATION BASED ON EXACT SOLUTION

#### 9.3.1 Theory of Moving Load Identification

#### 9.3.1.1 From Strains
The strains under the orthotropic plate at point \((x, y)\) and time \(t\) are
\[
\varepsilon_x(x, y, t) = z \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left( \frac{m \pi}{a} \right)^2 Y_m^*(y) \sin \left( \frac{m \pi}{a} x \right) \frac{l}{M_{mm}} \int H_m^{(t-\tau)} f_m(\tau) d\tau
\]
\[
\varepsilon_y(x, y, t) = -z \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Y_m''(y) \sin \left( \frac{m \pi}{a} x \right) \frac{l}{M_{mm}} \int H_m^{(t-\tau)} f_m(\tau) d\tau
\]
(9.18)

where \(\varepsilon_x(x, y, t), \varepsilon_y(x, y, t)\) are the strains at the bottom surface of the plate along \(x\)-direction and \(y\)-direction, respectively, and \(z\) is the distance from the neutral plane to the bottom tension surface. The strains at measuring point \((x_s, y_s)\) can be written in discrete form including the \(MM \times NN\) modes along the \(x\)- and \(y\)-directions respectively,
\[
\varepsilon_x(x_s, y_s, mm) = z \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left( \frac{m \pi}{a} \right)^2 Y_m^*(y_s) \sin \left( \frac{m \pi}{a} x_s \right) \frac{l}{M_{mm}} \sum_{i=1}^{M} H_m^{(mm-k)} f_m(k) \Delta t
\]
\[
\varepsilon_y(x_s, y_s, mm) = -z \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Y_m''(y_s) \sin \left( \frac{m \pi}{a} x_s \right) \frac{l}{M_{mm}} \sum_{i=1}^{M} H_m^{(mm-k)} f_m(k) \Delta t
\]
(9.19)

\((s = 1, 2, \ldots, N_s; mm = 1, 2, \ldots, N)\)

where \(\Delta t\) is the time step; \((N+1)\) is the number of sampling points; \(N_s\) is the number of measuring points, and
\[
H_m(k) = \frac{1}{\omega_{mn}} e^{-\omega_{mn} k \Delta t} \sin(\omega_{mn} t \Delta t)
\]
(9.20)
\[
f_m(k) = \sum_{i=1}^{N_s} p_i(k \Delta t) Y_m(k \Delta t) \sin \left( \frac{m \pi}{a} \tilde{x}_i(k \Delta t) \right)
\]
\((m = 1, 2, \ldots, MM; n = 1, 2, \ldots, NN.)\)

Equation (9.19) is rewritten in matrix form (Only the \(x\)-direction strains are presented since those for the \(y\)-direction strains are similar.)
\[
\varepsilon_x = BP
\]
(9.21)

where \(\varepsilon_x\) is a \((N_s \times N)\) matrix; \(B\) is a \((N_s \times N_p)\) matrix and \(P\) is a \((N_s \times N_p)\) matrix.

\[
\varepsilon_x = [\varepsilon_x(x_1, y_1, 1), \varepsilon_x(x_2, y_2, 1), \ldots, \varepsilon_x(x_{N_s}, y_{N_s}, 1), \varepsilon_x(x_1, y_1, 2), \ldots, \varepsilon_x(x_{N_s}, y_{N_s}, N)]^T
\]
\[
P = [p_i(0), p_i(1), \ldots, p_i(N-1)]^T
\]
(9.22)
\[ B = \begin{bmatrix} B_{10} & 0 & \cdots & 0 \\ B_{20} & B_{21} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ B_{N_0} & B_{N_1} & \cdots & B_{N_0,N_{N_1}} \end{bmatrix} ; \quad B_{mn} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1N_{r}} \\ b_{21} & b_{22} & \cdots & b_{2N_{r}} \\ \vdots & \vdots & \ddots & \vdots \\ b_{N_1i} & b_{N_1i} & \cdots & b_{N_1N_{r}} \end{bmatrix} \]

\[
\begin{align*}
    b_u &= z_i \Delta t \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{l}{M_{mn} \omega_{mn}} \frac{m \pi}{a} j^j Y_{mn}(y, \xi) \sin\left(\frac{m \pi}{a} x_i\right) e^{-i \omega_{mn} \left(mm - k\right) \Delta t} \\
    &\cdot \left(\frac{m \pi}{a} \gamma_{mn}(k \Delta t) \sin\left(\frac{m \pi}{a} x_i(k \Delta t)\right) \right) \\
    \left(mm = 1,2,3,\ldots,N; k = 0,1,2,\ldots,N - 1; s = 1,2,\ldots,N_s; l = 1,2,\ldots,N_{r}\right)
\end{align*}
\] (9.23)

Since the identified force \( P \) is not a continuous function of the measured data, a regularization method developed by Tiknonov (1963) is used to solve this ill-posed problem (Morozov. 1984). The load identification problem can be formulated as the following damped least squares problem.

\[
\min J(P, \lambda) = \left(\varepsilon - BP, R(\varepsilon - BP)\right) + \lambda \langle P, P \rangle
\] (9.24)

where \( \lambda = \text{diag}(\lambda_1, \ldots, \lambda_{N_{r}}) \) is a diagonal matrix of non-negative regularization parameter, and \( \lambda_i \) is related to the moving load \( P_i(t) \) according to the "smoothing" requests in the forces. \( R \) is a weight matrix determined from the measured information.

### 9.3.1.2 From accelerations

The acceleration at a point \((x, y)\) and time \(t\) obtained from Equation (9.17) is

\[
\ddot{w}(x, y, t) = \sum_{m=1}^{N} \sum_{n=1}^{N} Y_{mn}(y) \sin\left(\frac{m \pi}{a} x\right) - \frac{1}{M_{mn}} \left[ f_{mn}(t) + \int_0^t \ddot{H}_{mn}(t - \tau) f_{mn}(\tau) d\tau \right]
\] (9.25)

where

\[
\ddot{H}_{mn}(t) = \frac{1}{\omega_{mn}} e^{-i \omega_{mn} t} \left(\left(\zeta_{mn} \omega_{mn}\right)^2 - \omega_{mn}^2 \right) \sin \omega_{mn} t - 2 \zeta_{mn} \omega_{mn} \omega_{mn} \cos \omega_{mn} t
\] (9.26)

The acceleration at measuring point \((x_i, y_i)\) can be written in discrete form including the \(MM \times NN\) modes as

\[
\ddot{w}(x_i, y_i, mm) = \sum_{m=1}^{N} \sum_{n=1}^{N} Y_{mn}(y_i) \sin\left(\frac{m \pi}{a} x_i\right) - \frac{1}{M_{mn}} \left[ f_{mn}(mm) + \sum_{k=0}^{m} \ddot{H}_{mn}(mm - k) f_{mn}(k) \Delta t \right]
\]

\[
\left(s = 1,2,\ldots,N_s; mm = 1,2,\ldots,N\right)
\] (9.27)
\[ \bar{H}_{mn}(k) = \frac{1}{\omega_{mn}} e^{-\frac{\omega_{mn}}{a} k \Delta t} \left\{ \left( \zeta_{mn} \omega_{mn} \right)^2 - \omega_{mn}^2 \right\} \sin(\omega_{mn} \cdot k \Delta t) - 2 \zeta_{mn} \omega_{mn} \omega_{mn} \cos(\omega_{mn} \cdot k \Delta t) \]

\[ f_{mn}(k) = \sum_{i=1}^{N} p_i(k \Delta t) Y_{mn}(\hat{y}_i(k \Delta t)) \sin \left( \frac{m \pi}{a} \hat{x}_i(k \Delta t) \right) \]

\[ (m = 1, 2, \ldots, MM; n = 1, 2, \ldots, NN; k = 0, 1, 2, \ldots, N-1.) \quad (9.28) \]

Equation (9.27) can also be written in matrix forms as follows.

\[ \ddot{\bar{w}} = DP \quad (9.29) \]

where

\[ \ddot{\bar{w}} = \begin{bmatrix} \ddot{w}(x_1, y_1, l), \ddot{w}(x_2, y_2, l), \ddots, \ddot{w}(x_N, y_N, l) \end{bmatrix}^T; \]

\[ P = \begin{bmatrix} p_1(0), p_2(0), \ldots, p_{N_y}(0), p_1(1), \ldots, p_{N_y}(N-1) \end{bmatrix}^T; \]

\[ D = \begin{bmatrix} D_{10} & 0 & \cdots & 0 \\ D_{20} & D_{21} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ D_{N_0} & D_{N_1} & \cdots & D_{NN-1} \end{bmatrix}_{(N_y \times N_y) \times (N_y \times N_y)}; \quad D_{mn} = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1N_y} \\ d_{21} & d_{22} & \cdots & d_{2N_y} \\ \vdots & \vdots & \ddots & \vdots \\ d_{N_1,1} & d_{N_1,2} & \cdots & d_{N_1,N_y} \end{bmatrix}_{N_y \times N_y} \]

\[ d'_u = \Delta t \sum_{m=1}^{MM} \sum_{n=1}^{NN} \frac{1}{M_{mn} \omega_{mn}} Y_{mn}(y_i) \sin \left( \frac{m \pi}{a} x_i \right) e^{-\frac{\omega_{mn}}{a} (mm-k) \Delta t} \]

\[ \left( \zeta_{mn} \omega_{mn} \right)^2 - \omega_{mn}^2 \right\} - 2 \zeta_{mn} \omega_{mn} \omega_{mn} \cos(\omega_{mn} \cdot (mm-k) \Delta t) \}

\[ Y_{mn}(\hat{y}_i(k \Delta t)) \sin \left( \frac{m \pi}{a} \hat{x}_i(k \Delta t) \right) \]

\[ (mm = 1, 2, 3, \ldots, N; k = 0, 1, 2, \ldots, N-1. \quad s = 1, 2, \ldots, N_y; l = 1, 2, \ldots, N_y.) \]

When \( k < mm, \quad d_u = d'_u; \]

When \( k = mm, \)

\[ d_u = d'_u + \sum_{m=1}^{MM} \sum_{n=1}^{NN} Y_{mn}(y_i) \sin \left( \frac{m \pi}{a} Y_{mn}(\hat{y}_i(mm \Delta t)) \sin \left( \frac{m \pi}{a} \hat{x}_i(mm \Delta t) \right) / M_{mn} \] (9.31)

Again the load identification problem is formulated as a damped least squares problem.

\[ \min J(P, \lambda) = (\ddot{\bar{w}} - DP, R(\ddot{\bar{w}} - DP)) + \lambda (P, P) \quad (9.32) \]

The moving loads are determined from Equations (9.24) and (9.32) using either strains or accelerations or both. The method to determine the optimal regularization parameter \( \lambda \) is referred to the work by Busby and Trujillo (1997).

9.3.2 Computation Algorithm
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The computational process is implemented as follows:

1) Basing on the maximum exciting frequency generated by the moving loads, the number of mode shapes $MM \times NN$, the number of the measuring points $N$, and the sampling frequency are determined;

2) The natural frequencies $\omega_{nm}$ and the mode shapes $W_{nm}(x, y)$ of the orthotropic rectangular plate are calculated according to Equation (9.11);

3) Matrix $B_{nnk}$ and $D_{nnk}$ are calculated from Equations (9.23) and (9.31);

4) Set initial $\lambda$ equals to zero.

5) Calculating $P(0)$ from.

$$P(0) = (B_{10}^T B_{10} + \lambda I)^{-1} B_{10}^T \varepsilon_x(1)$$

or

$$P(0) = (D_{10}^T D_{10} + \lambda I)^{-1} D_{10}^T \bar{w}(1)$$

where

$$\varepsilon_x(j) = \{ \varepsilon_x(x, y, j), \varepsilon_x(x, y, j), \ldots, \varepsilon_x(x, y, j) \}^T;$$

$$\bar{w}(j) = \{ \bar{w}(x, y, j), \bar{w}(x, y, j), \ldots, \bar{w}(x, y, j) \}^T; \quad (j = 1, 2, \ldots, N)$$

$$P(i) = \{ p_x(i), p_y(i), \ldots, p_{ny}(i) \}^T; \quad (i = 0, 1, 2, \ldots, N - 1)$$

6) Calculating $P(k)$ from;

$$P(k) = (B_{i(k+1)}^T B_{i(k+1)} + \lambda I)B_{i(k+1)}^T (\varepsilon_x(k + 1) - \sum_{i=0}^{k-1} B_{i+1}, P(i))$$

or

$$P(k) = (D_{i(k+1)}^T D_{i(k+1)} + \lambda I)D_{i(k+1)}^T (\bar{w}(k + 1) - \sum_{i=0}^{k-1} D_{i+1}, P(i))$$

$$(k = 1, 2, \ldots, N - 1)$$

7) Calculating one of the following parameters: error in the identification, GCV value, or the curvature of L-curve.

8) Calculating the parameter with incremental regularization parameter from steps 5 to 7 until the error or GCV value is smaller than a given value or the curvature of the L-curve is larger than a given value. The optimal regularization parameter is obtained.

9) The moving loads can then be calculated from Steps 5 and 6 with the optimal regularization parameter.

9.3.3 Verification and Discussions
9.3.3.1 Reliability of the method in experiment

The experimental results obtained for a model car moving on a simply supported beam are used for this study. The experimental setup is shown diagrammatically in Figure 3.3. The main beam located in the laboratory is 3678mm long with 100mm $\times$ 25mm uniform cross-section. The Young’s Modulus of material is $2.1 \times 10^9 N/m^2$. The mass density is $2300 Kg/m^3$, and the Poisson ratio is 0.3. A leading beam for the vehicle to pick up speed and another beam at the other end for receiving the vehicle after its exit from the main beam are also shown. The beams are simply supported and the ends of the beams are placed close together leaving only a very narrow gap of approximately 1mm. This is necessary in order not to have a large impulsive force on the beam when the wheels cross the gap.

A U-shaped aluminum section on the upper surface of the beams serves as a direction guide for the car. The model car is pulled along the guide by a string wound on a wheel mount on the axle of an electric motor where the rotating speed can be adjusted. Thirteen photoelectric sensors are mounted on the beams to measure and monitor the moving speed of the car.

Seven strain gauges are mounted at the bottom of the main beam to measure the bending moment responses of the beam. A TEAC 14-channels magnetic tape recorder and an 8-channel dynamic testing and analysis system are used for data collection and analysis in the experiment. The sampling frequency is 2000 Hz. The recorded length of each test lasts for six seconds. The model car has two axles at a spacing of 0.557m and it runs on four steel wheels with rubber band on the outside. The mass of the whole car is 16.6 Kg. The static axle weights are 8.78 kg and 7.82 kg for the front and rear axles respectively. The transverse spacing between wheels is 0.08 metre.

The car is modelled as two axle forces moving on top of the beam. The beam sub-system has very small damping, and hence the damping coefficient is taken equal to 0.02 in the computation. Figure 9.2 shows the measured strains at 1/4L, 1/2L and 3/4L. Since the transverse spacing of the wheels is very small compared with the axle spacing, the identification is in terms of the axle loads instead of the individual wheel loads in this study. Figure 9.3 shows the identified axle forces using the beam model described in Chapter 8 and using the plate model described in this chapter. The vehicle moves at an average speed of 1.1856 m/s, and the first three modes are used in
the identification. There are large fluctuations in the two sets of results especially after the entrance of the second axle and before the exit of the first axle. The two sets of curves are close to the static axle weight with large fluctuations in their time histories. The fluctuations are affected by the measurement noise, and therefore whether a beam model can replace a plate model in the analysis or not cannot be ascertained from these results. The following section gives further discussions on the conditions when a plate can be modelled as a beam without introducing significant errors in the identified forces.

9.3.3.2 The beam model versus the plate model

A beam model for the bridge deck can be derived from a simplification of Equations (9.16) and (9.17). The displacement at a point along the central line of the bridge deck with the loads moving along \( y = e \) can be obtained from Equation (9.17) as follows.

\[
 w(x, \frac{b}{2}, t) = \sum_{m, n} \frac{\xi_m}{M_b} \sin\left(\frac{m\pi x}{a}\right) \frac{l}{M_b} \int_{s}^{t} H_m(t - \tau) f_m(\tau) d\tau
\]

(9.35)

\[
 C_{bmn} = \frac{b Y_{mn}^b (e)}{2 Y_{mn}^b (y) dy} \quad M_b = \rho hab / 2:
\]

(9.36)

\[
 f_{bm} = \sum_{i=1}^{N_p} p_i(t) \sin\left(\frac{m\pi x}{a} \xi_i(t)\right)
\]

where \( e \) is the eccentricity of the moving load, and \( h \) is the thickness of the beam.

If \( n = 1 \), i.e. the torsional modes are not considered. Equation (9.35) is the same as that for the displacement of an equivalent beam. Therefore the identification can be simplified using a beam model when \( n = 1 \). Table 9.1 shows the natural frequencies of an isotropic plate with two simply supported edges and two free edges. The length of the plate is 3.678m and the thickness is 0.025m. The width of the plate varies from 0.1m to 1.8m. The lowest several modes of the plates mainly consist of longitudinal modes in the \( x \)-direction with \( n = 1 \).

Two constant forces 87.25N and 38.25N at a fixed spacing of 0.557m are moving across the plate at 1.0 m/s along the central line. The sampling rate is 100 Hz. Figure 9.4 shows the identified forces from accelerations at 1/4a, 1/2a, 3/4a and 1/2b of a 0.1m and 0.4 m wide plate using the beam and the plate models, and the lowest three modes with \( n = 1 \) are used. The resulting curves for the two plate models overlap.
and exactly match those of the true forces without error. The three sets of curves are very close to each other except near the start and end of the time histories. The plate model can exactly identify the forces using the proposed method, while results from the beam models deteriorates when a plate with a larger width is used in the model. The modal frequencies in Table 9.1 indicate that the beam model would be accurate enough for identifying the moving loads when the highest natural frequency of the plate with \( n=1 \), is larger than the frequencies used in the identification. Since the aspect ratio \( b/a \) is small, such as \( b=0.1m \), the natural frequencies from using the beam model are more accurate than those from using the plate model. This is because the mode shapes of the beam satisfy the boundary conditions but not with the plate model.

### 9.3.3.3 Two moving loads identification with plate model

A diagrammatic cross-section of a simply supported beam-slab type bridge deck is shown in Figure 9.5. The parameters of the bridge deck are listed as follow.

For the plate: \( a=20m, \; b=11m, \; E=2.1\times10^9\; N/m^2, \; \rho = 2300Kg/m^3 \).

For the I-beam: Second moment of area \( I=0.118\; m^4 \); torsional moment of inertia \( J=0.04385\; m^4 \); \( m_1=0.175m \), \( n_1=1.13m \), \( \alpha=0.3 \), \( b_1=2.25m \), and \( v=0.33 \).

The rigidities in the \( x \) - and \( y \)-directions of the equivalent orthotropic bridge deck can be calculated according to Bakht and Jaeger (1985) as

\[
D_x = \frac{Eh^4}{12(1-S\nu^2)} + \frac{EI}{b_1}
\]

\[
D_y = \frac{Eh^4}{12(1-S\nu^2)}
\]

\[
D_{w} = vD_y + \frac{Gh^4}{6} + \frac{Gm_1^2n_1\alpha}{b_1}
\]

where \( G \) is the torsional rigidity; \( \alpha \) is a constant and \( S \) is the ratio \( D_y/D_x \). The calculated rigidities are \( D_x=1.1153\times10^9 \; N\cdot m; \; D_y=1.4\times10^7 \; N\cdot m \) and \( D_{w}=2.1665\times10^7 \; N\cdot m \). Twelve natural frequencies of the orthotropic plate are shown in Table 9.2.

The two moving loads to be identified are

\[
\begin{align*}
    p_1(t) &= 150000 \cdot (1+0.1 \sin 10\pi t + 0.05 \sin 40\pi t) \quad N; \\
p_2(t) &= 150000 \cdot (1-0.1 \sin 10\pi t + 0.05 \sin 50\pi t) \quad N.
\end{align*}
\]
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Note that there is an out-of-phase component in the forces simulating the pitching motion of a vehicle. White noise is added to the calculated displacements due to the moving loads to simulate the polluted measurement as

\[
\begin{align*}
\ddot{w} &= \ddot{w}_{\text{calculated}} + \left( 1 + E_p \cdot N_{\text{mse}} \right) \\
\varepsilon &= \varepsilon_{\text{calculated}} + \left( 1 + E_p \cdot N_{\text{mse}} \right)
\end{align*}
\]

where \( \ddot{w}, \varepsilon \) are the measured accelerations and strains used for the identification; \( E_p \) is the noise level; \( N_{\text{mse}} \) is a standard normal distribution vector (with zero mean value and unit standard deviation). \( \ddot{w}_{\text{calculated}}, \varepsilon_{\text{calculated}} \) are the calculated accelerations and strains.

Calculations are made for the loads moving at a fixed spacing of 4 m along the central line and along \( y=3/8 \) b. The moving speed is 10 m/s and the sampling rate is 100 Hz. The lowest nine vibration modes are used in the simulation, and nine measurement points are located at 1/4a, 1/2a and 3/4a on the second, third and the fourth I-beams. The number of the measuring points is taken equal to the number of the vibration modes as recommended in Chapter 8.

Figure 9.6 shows the identified forces moving along the central line from the accelerations and the strains with 1% noise level. Very good results are achieved except at the start and end of the time histories. There is a large deviation between the true load and the curves from strains when the load is near the midspan of the beam. This is subjected to be the effect of noise which is large compared to the low response when the forces are at mid-span of structure where the second longitudinal mode responses are smaller. The observation contrasts with the identified results when there is no noise in the responses and the identified forces exactly match those of the true forces. The curves from acceleration exhibit no such large differences, because the acceleration responses remain relatively stable throughout the duration. Those from accelerations almost match the true curves perfectly. Table 9.3 shows the errors in the identified forces with no smoothing on data at different noise levels. Accelerations give much better results than strains at different noise levels, and the identification of eccentric load using this set of sensors gives slightly larger errors than the loads along the central line. This may be due to the smaller responses at the sensor locations caused by the eccentric loads. Further work has to be done on the best sensor locations for identifying loads moving on different paths.

9-13
Figure 9.7 shows the identified loads moving along the central line from the strains only with or without three-points smoothing on the measured data with 1% noise level. It is seen that smoothing before the identification could improve the results significantly especially on the variation in the middle of the time histories. This variation is due to the low response when the force traverses the mid-span of the structure where the second longitudinal modes responses are smallest. Table 9.4 also shows reduction in the errors over the whole time period when smoothing is used. The improvement is larger in the results from strains than from accelerations.

9.3.4 Special Aspects in Implementation

The parameters of the bridge deck are the same as for the above section. Sampling frequency is 100 Hz such that the highest measured frequency is larger than the highest frequency of interest. The number of measuring points is taken equal to the number of vibration modes in the analysis, and the sensors are placed on the five I-beams at 1/4, 1/2 and 3/4 of the span.

9.3.4.1 Effect of Sensor Locations on Identification Errors

The following two moving loads at 4 metres spacing are used in this study.

\[
\begin{align*}
    p_1(t) &= 150000 \times (1 + 0.1 \sin 10\pi t + 0.05 \sin 40\pi t) \quad N; \\
    p_2(t) &= 150000 \times (1 - 0.1 \sin 10\pi t + 0.05 \sin 50\pi t) \quad N.
\end{align*}
\]  \tag{9.37}

The group of loads is moving on the plate along \( y = \frac{3}{8} b \) at a speed of 10m/s. The following sets of sensors are used to provide data for the identification. Their locations with respect to the loads are shown in Table 9.5.

- Set I: Nine sensors on the left three I-beams looking in the direction of travel;
- Set II: Nine sensors on the middle three I-beams; and
- Set III: Nine sensors on the right three I-beams.

Table 9.5 also shows the errors in the identification from using responses with different noise levels. It is noted that the errors in the identified forces from the accelerations and the strains are very small when there is no noise in the responses, and the accelerations give more accurate results than strains. However the identified results are very noise sensitive. The errors in the identified forces are more or less the same from different sets of sensors, but slightly less errors are found from the
measurement locations that are further away from the moving loads. This may be due to the higher sensitivity of these sensors to loads along \( y = 3/8b \).

### 9.3.4.2 Identification of eccentric group of moving loads

The loads described in Equation (9.37) are moving along paths at an eccentricity of 0.0, 1/8b and 3/8b separately at a speed of 10 m/s. Set II sensors described above are used. The errors in the identified forces from using responses with different noise levels are shown in Table 9.6. Errors in the identified forces when moving at an eccentric path are slightly larger than those for the loads moving along the central line of the deck. It is noted that the torsional modes of the plate are excited significantly in the former case. Figure 9.8 shows the identified results with 1% noise in the measured accelerations. The results match the true forces very closely except at the beginning and end of the time histories. This is due to the discontinuity of the forces at these two points leading to large fluctuations in the identified results.

### 9.3.4.3 Identification of multiple groups of loads

Nowak et al (1993) have inspected three patterns of multiple vehicles on the bridge deck in his studies of transverse load distribution. They are

1. In-lane: two vehicles following each other at a distance less than 15 m;
2. Side by side in tandem: two vehicles in adjacent lanes traveling with front axles on the same line; and
3. Side by side and behind: two vehicles in adjacent lanes but one behind the other with the distance between front axles less than 15m.

Set II sensors and the lowest nine vibration modes are used for the force identification. The following loads are assumed moving on the bridge deck as a group at a spacing of 4 metres.

\[
\begin{align*}
    p_1(t) &= 75000 \times (1 + 0.1 \sin 10\pi t + 0.05 \sin 40\pi t) \ N; \\
    p_2(t) &= 75000 \times (1 - 0.1 \sin 10\pi t + 0.05 \sin 50\pi t) \ N.
\end{align*}
\]

Two groups of these forces are moving along paths at an eccentricity \( e = 1/8b \) and \( e = 3/8b \). Three patterns of vehicles on the bridge deck are studied. They are

- **Case 1**: Side by side in tandem: Two groups of forces get on the bridge at the same time with the same speed of 10m/s;
Case 2: Side by side and behind: Two groups of forces get on the bridge at the same time with different speeds of 10m/s and 12m/s along paths \( e=3/8b \) and \( e=1/8b \) respectively; and

Case 3: Side by side and behind: Two groups of forces get on the bridge with the group along \( e=3/8b \) 1.0 metre behind the group along \( e=1/8b \) and with speeds of 10m/s and 12m/s respectively.

The In-lane case of Nowak et al (1993) is considered as a special case of Case 3. Table 9.7 shows the errors in the identified forces with different noise level in the measured responses. The time histories of identified forces for all three cases are similar, and the errors increase significantly with a small increase in noise. And errors are more or less the same except those from accelerations with 5% noise. Figure 9.9 shows the results for case 3 with 1% noise. Significant pitching motions of the vehicles are found in the curves from accelerations between 1.3s to 2.0s in the first group and between 0.5s to 2.0s in the second group. These errors contribute much to the overall errors in Table 9.7. The results from strains do not exhibit this phenomenon. Most of the errors in the curves from strains come from the variations when the forces traverse the mid-span of the bridge deck. This is due to the small responses in the sensors when the force traverses this point where the vibration from the second longitudinal bending modes is smallest.

Another computation is made for Case 3 with three-point smoothing on the measured data containing 5% errors and the curves are shown in Figure 9.10. The smoothing is effective on the strains but not on the acceleration measurements since the latter contains large coupling motion in the two forces in a group which cannot be eliminated through this smoothing process. Only the forces identified from strains are shown in Figure 9.10. The smoothing could significantly improve the accuracy of the results.

The ill-conditioned solution from accelerations could be improved either with a better regularization strategy on the time histories (Choi and Chang, 1996), or use an optimal set of sensors with maximum sensitivity of response to the moving loads. A sensor optimization technique has to be developed to meet this need.

The number of moving loads in the method is not limited. The number of moving loads can be determined by installing sensors at the entry and exit of the bridge deck. Details can be referred to Chan and Yung (2000).
9.3.4.4 Identification of individual loads in a group

The group includes four moving loads at a longitudinal spacing of 4 metres and a lateral spacing of 2 metres, which are

\[
\begin{align*}
  p_1(t) &= 75000 \times (1 + 0.1 \sin 10\pi t - 0.1 \sin 20\pi t + 0.05 \sin 40\pi t); \quad N \\
  p_2(t) &= 75000 \times (1 + 0.1 \sin 10\pi t + 0.1 \sin 20\pi t + 0.05 \sin 40\pi t); \quad N \\
  p_3(t) &= 75000 \times (1 - 0.1 \sin 10\pi t - 0.1 \sin 20\pi t + 0.05 \sin 40\pi t); \quad N \\
  p_4(t) &= 75000 \times (1 - 0.1 \sin 10\pi t + 0.1 \sin 20\pi t + 0.05 \sin 40\pi t); \quad N
\end{align*}
\]

where \( p_1(t) \) and \( p_2(t) \) are the left and right loads at the front looking in the direction of the travelling path, and \( p_3(t) \) and \( p_4(t) \) are the left and right loads at the back. The loads are travelling as a group along the central line of the plate at a speed of 10m/s. Set II sensors are used in the identification. Figure 9.11 shows the identified forces with 1% noise. Both accelerations and strains can give acceptable results for all the four forces in the same group, and accelerations give better results than strains as the results match the true forces over more than 80% of the time histories except near the beginning and the end. Most of the errors in the curves from strains come from the fluctuations when the forces traverse the mid-span of the bridge deck. This is because when the forces cross this point, the components in the responses due to the second longitudinal modes become small. Since these components contribute a great part in the overall dynamic responses of the plate, the measured responses are small in the case of strains leading to large error in the identified forces.

9.4 MOVING LOAD IDENTIFICATION BASED ON FINITE ELEMENT FORMULATION

9.4.1 Identification of Moving Loads on Bridge Deck

The displacement \( w(x, y, t) \) at location \( (x, y) \) and at time \( t \) is rewritten in matrix form from Equation (9.13) as

\[
w(x, y, t) = W_iQ \quad (s=1,2,...,N_s)
\]

(9.38)

where \( N_s \) is the number of measuring points, and \( Q \) is a matrix of \( q_s(t) \) from Equation (9.13).

\[
W_s = \{W_{11}(x, y), W_{12}(x, y), \cdots, W_{n1}(x, y), W_{n2}(x, y), \cdots, W_{mn}(x, y)\}
\]

The modal strains in x-direction can be written as
\[ W_y(x_i, y_j) = -z_i \left( \frac{i \pi}{a} \right)^2 \sin \left( \frac{i \pi}{a} x_i \right) Y_j(y_j), \quad (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n). \]

where \( z_i \) is the distance from the measuring point at the outer surface to the neutral surface of bending. For \( N_t \) measuring points

\[ w_{nr} = W_{nr} Q \quad \text{(9.39)} \]

where

\[
W_{nr} = \begin{bmatrix} w(x_1, y_1, t) & w(x_2, y_2, t) & \cdots & w(x_{N_x}, y_{N_y}, t) \end{bmatrix}^T
\]

\[
W_{ns} = \begin{bmatrix} W_{11}(x_1, y_1) & W_{12}(x_1, y_1) & \cdots & W_{n1}(x_1, y_1) \\ W_{11}(x_2, y_2) & W_{12}(x_2, y_2) & \cdots & W_{n2}(x_2, y_2) \\ \vdots & \vdots & \ddots & \vdots \\ W_{11}(x_{N_x}, y_{N_y}) & W_{12}(x_{N_x}, y_{N_y}) & \cdots & W_{nN}(x_{N_x}, y_{N_y}) \end{bmatrix}_{N_t \times n}
\]

The modal displacement can be obtained from Equation (9.39) by least squares method as

\[ Q = (W_{ns}^T W_{ns})^{-1} W_{ns}^T w_{nr} \quad \text{(9.40)} \]

Since the displacements or strains are measured, the velocities and accelerations can be obtained by dynamic programming filter (Trujillo and Busby, 1983) or orthogonal polynomial method described in Chapter 6, and the modal velocities and accelerations are calculated by the least squares method from Equation (9.40). They are then substituted into Equation (9.14) to form the matrix equation

\[ B = SP \quad \text{(9.41)} \]

where

\[
S = \begin{bmatrix} \frac{2 \sin \left( \frac{m \pi}{a} x_1(t) \right) Y_{11}(y_1(t))}{\rho a} & \frac{2 \sin \left( \frac{m \pi}{a} x_2(t) \right) Y_{12}(y_1(t))}{\rho a} & \cdots & \frac{2 \sin \left( \frac{m \pi}{a} x_{N_x}(t) \right) Y_{11}(y_{N_x}(t))}{\rho a} \\ \frac{2 \sin \left( \frac{m \pi}{a} x_1(t) \right) Y_{11}(y_1(t))}{\rho a} & \frac{2 \sin \left( \frac{m \pi}{a} x_2(t) \right) Y_{12}(y_2(t))}{\rho a} & \cdots & \frac{2 \sin \left( \frac{m \pi}{a} x_{N_x}(t) \right) Y_{12}(y_{N_x}(t))}{\rho a} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{2 \sin \left( \frac{m \pi}{a} x_1(t) \right) Y_{n1}(y_1(t))}{\rho a} & \frac{2 \sin \left( \frac{m \pi}{a} x_2(t) \right) Y_{n2}(y_2(t))}{\rho a} & \cdots & \frac{2 \sin \left( \frac{m \pi}{a} x_{N_x}(t) \right) Y_{n1}(y_{N_x}(t))}{\rho a} \end{bmatrix}
\]
\[
B = \begin{bmatrix}
q_{11}(t) + 2\zeta_{11}\omega_{11}q_{11}(t) + \omega_{11}^2 q_{11}(t) \\
q_{12}(t) + 2\zeta_{12}\omega_{12}q_{12}(t) + \omega_{12}^2 q_{12}(t) \\
\vdots \\
q_{mn}(t) + 2\zeta_{mn}\omega_{mn}q_{mn}(t) + \omega_{mn}^2 q_{mn}(t)
\end{bmatrix} \quad \Rightarrow \quad P = \left[ p_1(t), p_2(t), \ldots, p_{n_p}(t) \right]^T \tag{9.42}
\]

The moving load \( P \) can be obtained by the straightforward least squares method from Equation (9.41). But the solutions are frequently unstable in the sense that small noises in the responses would result in large changes in the predicted moving force. The regularization technique can be utilized to improve the conditioning. The load identification can be formulated as a nonlinear least squares problem.

\[
\min J(P, \lambda) = (B - SP, R(B - SP)) + \lambda(P, P) \tag{9.43}
\]

where \( \lambda \) is an optimal regularization parameter or a vector. \( R \) is a weight matrix and it can be determined from the measured information (Santantamaria and Fratta, 1998). Generalized cross-validation method (Golub et al, 1979) and L-Curve method (Hansen, 1992) are then used to determine the optimal regularization parameter in this study.

### 9.4.2 Numerical Studies

A simply supported prototype bridge composes of five I-section steel girders and a concrete deck as shown in Figure 9.12. It is noted that the model is similar to the one used by Fafard et al (1993) in his study of bridge-vehicle interaction. It is wide enough to accommodate four-lanes traffic. The parameters of the bridge deck are listed as follow: \( a=24.325m, b=13.715m, h=0.2m, E_x=4.1682 \times 10^{10} N/m^2 \)

\( E_y=2.9733 \times 10^{10} N/m^2, \quad \rho = 3000 Kg/m^3, \quad v_w = 0.3 \). For the steel I-beam: web thickness\( =0.0111m \), web height\( =1.490m \), flange width\( =0.405m \), flange thickness\( =0.018m \). For the diaphragms, the distance between two diaphragms is \( 4.865m \), cross-sectional area\( =0.001548m^2 \), \( I_y = 0.707 \times 10^{-6} m^4 \), \( I_z = 2 \times 10^{-6} m^4 \), \( J = 1.2 \times 10^{-7} m^4 \). The rigidities in the x-direction of the equivalent orthotropic plate can be calculated according to Bakht and Jaeger (1985) as \( D_{xy} = 2.415 \times 10^9 Nm, D_{yy} = 2.1813 \times 10^7 Nm, D_{x} = 2.2195 \times 10^8 Nm \). The natural
frequencies of the bridge deck are listed in Table 9.8. Note that this structure is similar to but not the same as that in Chapter 4.

A two-axle vehicle model is used in the simulation. The axle spacing and wheel spacing are 4.26\( \text{m} \) and 1.829\( \text{m} \) respectively. The four wheel loads are listed as follow.

\[
\begin{align*}
P_1(t) &= 3134.*\left(1 + 0.1\sin(10\pi t) - 0.1\sin(20\pi t) + 0.05\sin(40\pi t)\right)Kg; \\
P_2(t) &= 6166.*\left(1 - 0.1\sin(10\pi t) - 0.1\sin(20\pi t) + 0.05\sin(40\pi t)\right)Kg; \\
P_3(t) &= 3134.*\left(1 + 0.1\sin(10\pi t) + 0.1\sin(20\pi t) + 0.05\sin(40\pi t)\right)Kg; \\
P_4(t) &= 6166.*\left(1 + 0.1\sin(10\pi t) + 0.1\sin(20\pi t) + 0.05\sin(40\pi t)\right)Kg;
\end{align*}
\] (9.44)

where \( P_1 \) and \( P_3 \) are the front wheels and \( P_2 \) and \( P_4 \) are the rear wheels with \( P_2 \) and \( P_4 \) after \( P_1 \) and \( P_3 \) respectively. The total vehicle load is 18.6 Tonnes and the proportion of axle loads follows the pattern of vehicle type H20-44 from AASHTO (1996). The vehicle moving speed is 20\( \text{m/s} \), and the time step of analysis is 0.001\( \text{s} \) in the simulation. White noise is added to the calculated displacements or strains to simulate the polluted measurement.

9.4.2.1 Numerical Study on the Noise Effect

The vehicle is moving along the centerline of the deck. The measured responses from 25 modes (\( n=5, r=5 \)) in Table 9.8 are used in the calculation, and the number of modes in the identification equation (9.41) is the same as that in the responses. According to Chapter 8, the number of measuring points should not be less than the number of vibration modes in the measured information. And therefore twenty-five measuring points are selected evenly distributed on the five I-beams. The identified individual loads from using displacement responses with 1% and 5% noise levels are shown in Figure 9.13. The following observations are made.

1) The beginning or end of the identified results is under-estimated when there is noise in the responses. This is due to the small responses at the beginning or the end of the time duration, and the fact that the regularization parameter has been optimized over the total time duration of the event.

2) Errors in the identified results increase with the noise level. Hence when the noise level is high, a data treatment process (such as filtering or smoothing) should be used to reduce the noise in the responses before the computation.
9.4.2.2 Identification with incomplete system information

In practice, the vibration modes selected for identification are not the same as that in the responses. And in general the lower modes of the structure are dominating the measured responses, and they are used in the identification. The moving loads are identified again with this incomplete modal information with fewer modes in the identification than those in the responses. The parameters used in the simulation are the same as those in the last study. The vehicle is moving along the centerline of the bridge deck. Table 9.9 shows the errors in the identified results, and Figure 9.14 shows the identified results using 20 modes \((m=4,n=5)\) or \((m=5,n=4)\) with 1% noise in the responses. The following observations are made from the results.

1) As \(m \geq 4, n \geq 4\), an acceptable result can be obtained with most of the errors less than 10% at 1% noise level. This is because these modes (shown in Table 9.8) have covered most of the excitation frequency range of the car as shown in Equation (9.44). In practice, the frequency range required in the identification can be obtained from the spectrum of the responses.

2) The more modes used in the identification, the lesser errors are found in the identified results (Table 9.9). However large errors still exist at the beginning and the end of the load time histories as seen in Figure 9.14. This is due to the fact that impulses are generated by the moving loads at the beginning and the end of the time duration, and a lot of higher modes of the structure are excited which are not covered by the selected vibration modes in the identification.

3) There are large errors in the identified results with \(m=5, n=3\). This shows that the torsional modes are also very important in the moving load identification on bridge decks even if the vehicle is moving along the centerline.

9.4.2.3 Effects of Eccentricity

There are four lanes on the bridge deck. Normally the vehicle is not moving exactly along the centerline. Table 9.10 shows the errors in the identified results with the car moving at different eccentricities and using different number of vibration modes in the identification. The identified results for different eccentricities with 25 modes \((m=5,n=5)\) are shown in Figure 9.15. The parameters are the same as for the
above studies, and the responses are calculated with 25 modes \((m=5, n=5)\). The following intermediate conclusions can be drawn from Table 9.10 and Figure 9.15.

1) When \(m \geq 3, n = 5\), an acceptable result can be obtained with most of the errors less than 10% at 1% noise level. This shows that the method proposed in the paper is also effective to identify the eccentric moving loads on the bridge deck.

2) As \(n<5\), there are large errors in the identified eccentric load. This shows that the torsional modes are more important in the eccentric load identification than that for the case with the car moving along the centerline.

3) As the eccentricity increases, the errors in the identified results also increase as seen in Table 9.10. Since the same measured information is used in both sets of identification, eccentric moving loads are more difficult to be identified accurately. There is a need for an optimum selection of measuring locations for different moving load configuration, and it is a subject of further research.

9.5 CONCLUDING REMARKS

Two methods to identify moving loads on top of bridge deck using measured structural responses have been developed. The bridge deck is modeled as an orthotropic plate and the vehicular load is modeled as a group of four wheel loads or two axle loads moving on top of the bridge deck at fixed spacing. Dynamic behavior of the bridge deck is analyzed by the orthotropic plate theory and mode superposition technique. Tikhonov regularization technique is used to provide bounds to the identified forces. Errors in the identified forces are noise sensitive. When the measurement noise is very large, pre-processing procedures are required to reduce the measurement noise in the responses before the regularization method can work effectively to provide bounds to the identified forces.

The proposed identification method based on exact solution can identify individual loads from the measured strains and accelerations. Acceleration measurements would provide better results than those from strain measurements. Identification of forces moving on an eccentric path is slightly less accurate than that for forces moving along the central line of the bridge deck when the sensors are around the middle of the bridge cross-section. Optimal sensor location is important to have less errors in the identified forces especially from acceleration measurements, and further study has to be made in this area. When the force traverses mid-span of
the beam, the responses from the second longitudinal modes are smallest at this moment, and this affects the strain measurement and hence the identified results greatly. Large errors are found at the beginning and the end of the time histories of the identified forces. This is due to the discontinuity of the forces at these two points leading to large fluctuations in the identified results. When the lower modes of the bridge deck are dominated by vibration modes along the longitudinal axis, a beam model instead of a plate model may be accurate enough in the identification.

Another new time domain method based on finite element formulation is also presented to identify moving loads on a bridge deck based on the measured responses. The method is also effective in the identification of eccentric moving loads. The torsional modes are also found very important in the moving load identification even when the group of loads is moving along the centerline of the bridge deck.

These two methods can be extended easily to identify moving loads on multiple span continuous bridge deck if the method developed in Chapter 4 is used to calculate the mode shapes of the structure. Certainly, it needs long computational time and large computer capacity in the calculation.
Chapter 9: Identification of Moving Loads on Bridge Decks

Table 9.1 Natural Frequencies (Hz) of beam and plate with different width

<table>
<thead>
<tr>
<th>Width of plate</th>
<th>Mode number</th>
<th>Mode type</th>
<th>n</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam model (0.1m width)</td>
<td>B</td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>0.1m</td>
<td>B</td>
<td></td>
<td>4.32</td>
<td>17.27</td>
<td>38.86</td>
<td>69.08</td>
</tr>
<tr>
<td>0.2m</td>
<td>B</td>
<td></td>
<td>4.22</td>
<td>16.89</td>
<td>38.01</td>
<td>67.59</td>
</tr>
<tr>
<td>0.3m</td>
<td>B</td>
<td>1st T</td>
<td>4.22</td>
<td>16.92</td>
<td>38.14</td>
<td>67.97</td>
</tr>
<tr>
<td>Plate model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4m</td>
<td>B</td>
<td>1st T</td>
<td>4.23</td>
<td>16.94</td>
<td>38.24</td>
<td>68.23</td>
</tr>
<tr>
<td>1.8m</td>
<td>B</td>
<td>1st T</td>
<td>4.27</td>
<td>17.35</td>
<td>39.38</td>
<td>70.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd T</td>
<td>12.72</td>
<td>29.68</td>
<td>51.57</td>
<td>85.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3rd T</td>
<td>53.46</td>
<td>97.36</td>
<td>128.93</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4th T</td>
<td>70.22</td>
<td>132.16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: * denotes the natural frequency corresponds to mode shape mainly in y-direction. (B: bending mode; 2nd T: second torsional mode, etc)

Table 9.2 Natural frequencies of the orthotropic plate (Hz)

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.22</td>
<td>5.56</td>
<td>7.89</td>
<td>14.46</td>
</tr>
<tr>
<td>2</td>
<td>20.87</td>
<td>21.21</td>
<td>22.73</td>
<td>26.89</td>
</tr>
<tr>
<td>3</td>
<td>46.96</td>
<td>47.29</td>
<td>48.57</td>
<td>51.61</td>
</tr>
</tbody>
</table>

Note: The modal frequencies used in the simulation study are bolded.
### Chapter 9: Identification of Moving Loads on Bridge Decks

Table 9.3 Errors in the identified forces with different noise levels

<table>
<thead>
<tr>
<th>Eccentricity</th>
<th>Responses</th>
<th>Noise</th>
<th>$\lambda$</th>
<th>First force (%)</th>
<th>Second force (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Acceleration</td>
<td>1%</td>
<td>$4.29 \times 10^{-13}$</td>
<td>2.74</td>
<td>2.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5%</td>
<td>$4.96 \times 10^{-11}$</td>
<td>12.30</td>
<td>11.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10%</td>
<td>$1.26 \times 10^{-10}$</td>
<td>18.97</td>
<td>17.81</td>
</tr>
<tr>
<td></td>
<td>Strain</td>
<td>1%</td>
<td>$1.23 \times 10^{-21}$</td>
<td>15.92</td>
<td>17.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5%</td>
<td>$1.31 \times 10^{-20}$</td>
<td>27.44</td>
<td>29.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10%</td>
<td>$2.44 \times 10^{-20}$</td>
<td>34.71</td>
<td>37.00</td>
</tr>
<tr>
<td>1/8b</td>
<td>Acceleration</td>
<td>1%</td>
<td>$1.60 \times 10^{-12}$</td>
<td>4.30</td>
<td>4.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5%</td>
<td>$5.51 \times 10^{-11}$</td>
<td>15.29</td>
<td>14.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10%</td>
<td>$1.17 \times 10^{-10}$</td>
<td>22.44</td>
<td>21.24</td>
</tr>
<tr>
<td></td>
<td>Strain</td>
<td>1%</td>
<td>$1.22 \times 10^{-21}$</td>
<td>16.60</td>
<td>18.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5%</td>
<td>$1.17 \times 10^{-20}$</td>
<td>28.51</td>
<td>30.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10%</td>
<td>$2.21 \times 10^{-20}$</td>
<td>35.97</td>
<td>38.13</td>
</tr>
</tbody>
</table>

Table 9.4 Errors in the identified forces with or without smoothing

<table>
<thead>
<tr>
<th>Responses</th>
<th>Noise</th>
<th>$\lambda$</th>
<th>First force (%)</th>
<th>Second force (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Smoothing</td>
<td>1%</td>
<td>$4.29 \times 10^{-13}$</td>
<td>2.74</td>
<td>2.53</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>$4.96 \times 10^{-11}$</td>
<td>12.30</td>
<td>11.25</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>$1.26 \times 10^{-10}$</td>
<td>18.97</td>
<td>17.81</td>
</tr>
<tr>
<td>Strain</td>
<td>1%</td>
<td>$1.23 \times 10^{-21}$</td>
<td>15.92</td>
<td>17.69</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>$1.31 \times 10^{-20}$</td>
<td>27.44</td>
<td>29.42</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>$2.44 \times 10^{-20}$</td>
<td>34.71</td>
<td>37.00</td>
</tr>
<tr>
<td>Smoothing</td>
<td>1%</td>
<td>$1.22 \times 10^{-20}$</td>
<td>2.72</td>
<td>3.14</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>$2.80 \times 10^{-12}$</td>
<td>9.24</td>
<td>9.43</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>$6.53 \times 10^{-11}$</td>
<td>13.59</td>
<td>13.59</td>
</tr>
<tr>
<td>Strain</td>
<td>1%</td>
<td>$7.57 \times 10^{-23}$</td>
<td>9.07</td>
<td>10.06</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>$1.84 \times 10^{-21}$</td>
<td>18.60</td>
<td>20.51</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>$6.38 \times 10^{-21}$</td>
<td>24.76</td>
<td>27.01</td>
</tr>
</tbody>
</table>
Table 9.5 Errors (percent) in the identified forces from different measuring locations

<table>
<thead>
<tr>
<th>Sensor Set</th>
<th>Noise Level</th>
<th>From strains</th>
<th>From accelerations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First force</td>
<td>Second force</td>
</tr>
<tr>
<td>I</td>
<td>0%</td>
<td>0.00</td>
<td>0.49</td>
</tr>
<tr>
<td>I</td>
<td>1%</td>
<td>16.58</td>
<td>18.55</td>
</tr>
<tr>
<td>I</td>
<td>5%</td>
<td>28.79</td>
<td>30.92</td>
</tr>
<tr>
<td>I</td>
<td>10%</td>
<td>36.53</td>
<td>38.73</td>
</tr>
<tr>
<td>II</td>
<td>0%</td>
<td>0.00</td>
<td>0.49</td>
</tr>
<tr>
<td>II</td>
<td>1%</td>
<td>16.60</td>
<td>18.48</td>
</tr>
<tr>
<td>II</td>
<td>5%</td>
<td>28.51</td>
<td>30.48</td>
</tr>
<tr>
<td>II</td>
<td>10%</td>
<td>35.97</td>
<td>38.13</td>
</tr>
<tr>
<td>III</td>
<td>0%</td>
<td>0.00</td>
<td>0.49</td>
</tr>
<tr>
<td>III</td>
<td>1%</td>
<td>16.09</td>
<td>17.56</td>
</tr>
<tr>
<td>III</td>
<td>5%</td>
<td>27.59</td>
<td>29.54</td>
</tr>
<tr>
<td>III</td>
<td>10%</td>
<td>34.80</td>
<td>37.13</td>
</tr>
</tbody>
</table>

\[ e = \frac{1}{8} b \]

- **Set I:**
- **Set II:**
- **Set III:**

\[ \text{Location of measuring point} \]
Chapter 9: Identification of Moving Loads on Bridge Decks

Table 9.6 Errors (percent) in identified forces from Set II sensor locations

<table>
<thead>
<tr>
<th>Path Eccentricity</th>
<th>Noise Level</th>
<th>From strains</th>
<th>From accelerations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First force</td>
<td>Second force</td>
</tr>
<tr>
<td>0</td>
<td>0%</td>
<td>0.00</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>1%</td>
<td>16.46</td>
<td>18.25</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>28.22</td>
<td>30.13</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>35.55</td>
<td>37.73</td>
</tr>
<tr>
<td>1/8b</td>
<td>0%</td>
<td>0.00</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>1%</td>
<td>16.60</td>
<td>18.48</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>28.51</td>
<td>30.48</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>35.97</td>
<td>38.13</td>
</tr>
<tr>
<td>3/8b</td>
<td>0%</td>
<td>0.00</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>1%</td>
<td>16.90</td>
<td>18.96</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>29.15</td>
<td>31.22</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>36.92</td>
<td>39.19</td>
</tr>
</tbody>
</table>

Table 9.7 Errors in the identified forces from multiple groups of moving loads

<table>
<thead>
<tr>
<th>Vehicle pattern Noise level</th>
<th>From strains</th>
<th>From accelerations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First Group</td>
<td>Second Group</td>
</tr>
<tr>
<td></td>
<td>1st axle</td>
<td>2nd axle</td>
</tr>
<tr>
<td></td>
<td>2nd axle</td>
<td>1st axle</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td>0.995</td>
<td>0.991</td>
</tr>
<tr>
<td>1%</td>
<td>19.0</td>
<td>19.3</td>
</tr>
<tr>
<td>5%</td>
<td>30.8</td>
<td>32.2</td>
</tr>
<tr>
<td>Case 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td>0.995</td>
<td>0.991</td>
</tr>
<tr>
<td>1%</td>
<td>19.3</td>
<td>20.1</td>
</tr>
<tr>
<td>5%</td>
<td>30.6</td>
<td>33.4</td>
</tr>
<tr>
<td>Case 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td>0.995</td>
<td>0.991</td>
</tr>
<tr>
<td>1%</td>
<td>19.4</td>
<td>19.9</td>
</tr>
<tr>
<td>5%</td>
<td>30.9</td>
<td>33.3</td>
</tr>
<tr>
<td>Case 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td>0.995</td>
<td>0.991</td>
</tr>
<tr>
<td>1%</td>
<td>19.9</td>
<td>19.4</td>
</tr>
<tr>
<td>5%</td>
<td>30.9</td>
<td>33.3</td>
</tr>
</tbody>
</table>

9-27
Table 9.8 Natural Frequencies of the equivalent orthotropic plate (Hz)

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.96*</td>
<td>6.31</td>
<td>10.01</td>
<td>16.07</td>
<td>24.81</td>
<td>36.46</td>
<td>51.08</td>
<td>68.70</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>19.84*</td>
<td>21.26</td>
<td>25.41</td>
<td>32.17</td>
<td>41.51</td>
<td>53.49</td>
<td>68.22</td>
<td>85.82</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>44.65*</td>
<td>46.07</td>
<td>50.32</td>
<td>57.33</td>
<td>67.06</td>
<td>79.48</td>
<td>94.60</td>
<td>112.49</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>79.37*</td>
<td>80.80</td>
<td>85.07</td>
<td>92.17</td>
<td>102.06</td>
<td>102.50</td>
<td>114.74</td>
<td>115.58</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>124.01*</td>
<td>124.02</td>
<td>125.42</td>
<td>125.44</td>
<td>125.46</td>
<td>129.40</td>
<td>129.43</td>
<td>129.51</td>
<td></td>
</tr>
</tbody>
</table>

* longitudinal bending modes

Table 9.9 Errors (percent) in the identified loads from different measured information

<table>
<thead>
<tr>
<th>Noise Level</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axle-1</td>
<td>Axle-2</td>
<td>Axle-1</td>
<td>Axle-2</td>
</tr>
<tr>
<td>5 5 25</td>
<td>6.97</td>
<td>6.00</td>
<td>17.90</td>
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<tr>
<td></td>
<td>6.51</td>
<td>5.98</td>
<td>15.58</td>
</tr>
<tr>
<td>5 4 20</td>
<td>9.05</td>
<td>7.74</td>
<td>17.95</td>
</tr>
<tr>
<td></td>
<td>9.23</td>
<td>6.60</td>
<td>16.50</td>
</tr>
<tr>
<td>5 3 15</td>
<td>30.56</td>
<td>18.62</td>
<td>32.25</td>
</tr>
<tr>
<td></td>
<td>31.27</td>
<td>18.29</td>
<td>32.64</td>
</tr>
<tr>
<td>4 5 20</td>
<td>7.06</td>
<td>6.10</td>
<td>18.01</td>
</tr>
<tr>
<td></td>
<td>6.59</td>
<td>6.06</td>
<td>15.69</td>
</tr>
<tr>
<td>4 4 16</td>
<td>9.13</td>
<td>7.82</td>
<td>18.05</td>
</tr>
<tr>
<td></td>
<td>9.34</td>
<td>6.68</td>
<td>16.63</td>
</tr>
<tr>
<td>4 3 12</td>
<td>30.86</td>
<td>18.82</td>
<td>32.53</td>
</tr>
<tr>
<td></td>
<td>31.57</td>
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<td>10.82</td>
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<td></td>
<td>9.90</td>
<td>9.37</td>
<td>20.68</td>
</tr>
<tr>
<td>3 4 12</td>
<td>12.57</td>
<td>12.03</td>
<td>23.23</td>
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<tr>
<td></td>
<td>14.67</td>
<td>10.29</td>
<td>23.12</td>
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<tr>
<td>3 3 9</td>
<td>34.69</td>
<td>23.48</td>
<td>36.19</td>
</tr>
<tr>
<td></td>
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<td>38.86</td>
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<tr>
<td>2 5 10</td>
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<td>18.95</td>
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</tr>
<tr>
<td></td>
<td>19.71</td>
<td>17.13</td>
<td>32.53</td>
</tr>
</tbody>
</table>

Note: The errors in table correspond to each wheel load as

wheel 1 | wheel 2 |
wheel 3 | wheel 4

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## Chapter 9: Identification of Moving Loads on Bridge Decks

### Table 9.10 Errors (percent) in the identified load moving at different eccentricities

<table>
<thead>
<tr>
<th>Eccentricity</th>
<th>m</th>
<th>n</th>
<th>1% Axle-1</th>
<th>1% Axle-2</th>
<th>5% Axle-1</th>
<th>5% Axle-2</th>
<th>10% Axle-1</th>
<th>10% Axle-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/8b</td>
<td>5</td>
<td>5</td>
<td>7.20</td>
<td>6.35</td>
<td>14.33</td>
<td>11.33</td>
<td>23.16</td>
<td>17.93</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5</td>
<td>6.72</td>
<td>6.41</td>
<td>15.25</td>
<td>13.26</td>
<td>24.41</td>
<td>20.99</td>
</tr>
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<td></td>
<td>3</td>
<td>5</td>
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<td>11.56</td>
<td>21.06</td>
<td>22.25</td>
<td>29.91</td>
<td>33.50</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>4</td>
<td>13.56</td>
<td>13.25</td>
<td>29.46</td>
<td>27.37</td>
<td>45.00</td>
<td>40.97</td>
</tr>
<tr>
<td>1/8b</td>
<td>5</td>
<td>5</td>
<td>6.46</td>
<td>5.56</td>
<td>15.88</td>
<td>12.86</td>
<td>25.22</td>
<td>19.60</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5</td>
<td>6.54</td>
<td>5.64</td>
<td>15.96</td>
<td>12.97</td>
<td>25.31</td>
<td>19.72</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5</td>
<td>7.80</td>
<td>7.16</td>
<td>15.51</td>
<td>14.48</td>
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<td>9.17</td>
<td>21.23</td>
<td>19.25</td>
<td>30.81</td>
<td>27.89</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>11.22</td>
<td>10.93</td>
<td>20.08</td>
<td>21.19</td>
<td>27.42</td>
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<td>23.83</td>
<td>22.02</td>
<td>30.45</td>
<td>26.05</td>
<td>22.03</td>
<td>18.21</td>
</tr>
</tbody>
</table>

Note: The errors in table correspond to each wheel load as `wheel 1 | wheel 2 | wheel 3 | wheel 4`
Chapter 9: Identification of Moving Loads on Bridge Decks

Figure 9.1 An orthotropic plate subject to action of moving loads
Figure 9.2 Strains at 1/4L, 1/2L and 3/4L
( - 1/4L; --- 1/2L; ... 3/4L.)
Figure 9.3 Identified forces by beam model and plate model
(—Beam Model; ... Plate Model.)
Figure 9.4 Identified forces by beam model and plate model

( — Plate models; —— Beam model(0.1m wide); ... Beam model(0.4m wide))
Figure 9.5 Cross-section of the orthotropic plate
Figure 9.6 Identified forces from accelerations and strains
(— True force; —— from accelerations; ... from strains.)
Figure 9.7 Identified forces from strains with or without smoothing

( — True force; —— with smoothing; ... without smoothing.)
Figure 9.8 Identified results for groups of loads moving at different eccentricity
( - True force; ---- e=1/8b; ... e=3/8b.)
Figure 9.9 Identified results of two groups of moving loads (pattern Case 3, 1% noise)

(— True force; --- from strains; ... from accelerations.)
Figure 9.9 Identified results of two groups of moving loads (pattern Case 3, 1% noise)
( - True force; ---- from strains; ... from accelerations.)
Figure 9.10 Identified results from strains of two groups of moving loads
(pattern Case 3, 1% noise)

(a) First group moving load (e=3/8b)
Figure 9.10 Identified results from strains of two groups of moving loads
(pattern Case 3, 1% noise)

( — True force; ---- without smoothing; ... with smoothing.)
Figure 9.11 Identified results of a group of four wheel loads

(- True force; --- from acceleration; ... from strain.)
Figure 9.12 A typical single span bridge
Figure 9.13 Identification with different noise levels

(- True loads; — 1% noise; ... 5% noise.)
Figure 9.14 Identification with different mode combination (1% noise)

(- True loads; — m=4,n=5; ... m=5,n=4.)
Figure 9.15 Identified results for different eccentricities (1% noise)

(— True loads; — e=0; —— e=1/8b; ... e=3/8b.)
Chapter 10

LABORATORY STUDY II:
BRIDGE DECK MODEL

10.1 INTRODUCTION

Two methods have been developed to identify moving loads on bridge deck in Chapter 9. Computational simulations show the feasibility and accuracy of the two methods to identify the moving loads travelling along the central line or at an eccentric path on the bridge deck. In order to further verify the performances of the two methods, an experimental set-up for the bridge-vehicle system was designed in the laboratory. The strains of the bridge deck are measured when the model car moves across the bridge deck along different trails at different speeds. The moving loads on the bridge deck were identified from the measured strains using these two methods. The reconstructed responses are calculated from the identified loads for comparison with measured responses to verify the performances of these two methods.

10.2 EXPERIMENTAL SETUP AND MEASUREMENTS

10.2.1 Experimental Set-up

The experimental system design includes the model design and measuring system design. The experimental model is used to simulate the vehicle-bridge system which includes the interaction between the vehicle and the bridge. The one-tenth model bridge deck simulates the single span bridge presented by Fafard et al(1993), which is similar to the bridge deck shown in Figure 9.12. According to the similarity theory, the length and width of the model are selected as 8' and 4' respectively. The beam-slab type bridge deck is used, and the model is composed of five rectangular-section steel ribs and a steel deck. According to AASHTO specifications (H20-44 or H15-44), the ratio of the wheel spacing and the axle spacing and the ratio between the front axle weight and the back axle weight are selected as 3:7.

The final model vehicle-bridge system fabricated in the laboratory is shown in Figure 10.1 and Photograph 10.1. The bridge deck is a uniform steel plate (8'×4'×0.25") stiffened with five rectangular ribs (1"×0.5") welded underneath the
plate and simply supported at the two ends. The point constraints are used to simulate the simple supports in the laboratory because the line supports are difficult to construct in practice. Also the point constraints are close to the real case for multi-girder bridge. On the other hand, it is complicated to analyze the dynamic behavior of the bridge deck on point supports under moving loads or to identify moving loads from measuring responses. So the simple supports are used in the analysis. The experimental set-up is shown in Photograph 10.1. The bridge deck is supported on two steel I-beams, and the I-beams are fixed to the ground through bolts as shown in Photograph 10.2. Moreover, spherical metal balls are placed in between the I-beams and the ribs of the bridge deck to simulate the point supports. At the entrance end of the deck, metal balls are welded and connected to both the I-beams and the ribs along the axis of rotation. However, at the exit end, the metal balls are welded on the ribs only.

Three U-shaped aluminum sections are glued to the upper surface of the deck as direction guides for the car. It is located at the 1/8b, 3/8b and 1/2b as shown in Figure 10.2(a) where b is the width of the deck. The model car is pulled along the guide by a string wound around the drive wheel of an electric motor in Photograph 10.3. During the test, a leading beam and a tailing beam are provided for the acceleration and deceleration of the model car as shown in Figure 10.1 and Photograph 10.1. The leading beam and tailing beam are independently supported on the ground. So there is no excitation to the bridge deck as the model car moves on the top of the leading beam and the tailing beam.

The model car, shown in Photograph 10.4, has four rubber wheels with an axle spacing of 0.457m and wheel spacing of 0.2m. The front and the rear axles weigh 5.2Kg and 14.7Kg respectively. The mass ratio between the model car and the bridge deck is 0.12. The ratio between the wheel spacing and the axle spacing is 0.44. At the bottom of the car, there are two studs to guide the car moving along the rails during the tests. Moreover a trigger arm is extended from the car to detect the car location and the moving speed as shown in Figure 10.1.

Nine photoelectric sensors are mounted evenly in a line on the deck to monitor the speed of the car. They are located on the plate at roughly equal spacing of one foot to check on the uniformly of the speed as shown in Figure 10.2(a) and Photograph 10.5. Twenty-five strain gauges are located at the bottom of the beam ribs to measure the responses of the plate. Their locations are shown in Figure 10.2(b). A 16-channel
data acquisition system and a KYOWA data recorder model RTP800A shown in Photograph 10.6 are used for data collection in the experiments.

10.2.2 Testing Procedure

The testing procedure consists of four main steps. The first step is the calibration of the measuring system by a static test. Loads from 0 to 30Kg are added to the middle points of the three rails. The corresponding strain values at the measuring points are recorded. At the same time, the strains at the measuring points under these loads are calculated using the finite element method by SAP2000 software package. Comparison between the measured values and the calculated strains gives the sensitivity coefficients for each of these measuring point.

In the second step, the modes of the bridge deck and the model car are measured using modal test. Table 10.1 shows the identified frequencies and the damping ratios of the bridge deck, and Figures 10.3, 10.4 and 10.5 show the first twelve mode shapes of the bridge deck. Figure 10.6 shows the first four modes of the model car. By Bakht’s simplified method, the rigidities of the equivalent orthotropic plate are $D_x = 7.3677 \times 10^4 \text{Nm}$, $D_y = 4.2696 \times 10^3 \text{Nm}$, $D_{xy} = 8.6018 \times 10^3 \text{Nm}$. The natural frequencies of the plate are calculated by the proposed method given in Chapter 9. The measured natural frequencies of the bridge deck are compared with the results from finite element method with point constraints or simple supports and the proposed method and they are shown in Table 10.2. The frequencies for the proposed method and the finite element methods using simple supports are consistently close to each other. This set of frequencies is closed to the experimental results of the plate than those from the finite element method using point supports. Therefore it is considered opportune to represent the bridge deck approximately by an orthotropic plate on simple supports.

In step three, the responses of the bridge deck when the model car moves along different rails with different speeds are measured. The sampling frequency is 1000Hz, and the number of data in each recorded segment is 7680.

Finally, the moving loads on the bridge deck are identified from the measured responses, and they are compared with the static loads to verify the performances of the two proposed methods.
10.3 IMPACT STUDY IN LABORATORY

The dynamic loading on a multi-lane continuous bridge deck under single or multiple vehicles moving in different lanes has been investigated using the orthotropic plate theory and modal superposition technique in Chapter 4. Numerical simulations have been performed to study the variation of the dynamic impact factor and wheel load distribution factor on the bridge deck. This section is to present a laboratory study on the dynamic load of the bridge deck under moving vehicle.

When the model car is moving along the centerline (Rail 3), Rail 1 or Rail 2 in turn, the strains at 1/2a of each beam are used to calculate the impact factors. The paths along Rail 1 and Rail 2 correspond to an eccentricity of 3/8b and 1/8b respectively. The moving speed is adjusted by changing the setting of the electric motor to have speeds of 0.5m/s, 0.75m/s, 1m/s, 1.5m/s. Figure 10.7 shows the strains on different beams as the model car is moving along the central line with different speeds. Figure 10.8 shows the strains on the different beams as the model car is moving along Rail 1 with different speeds. Table 10.3 shows the impact factors at the midpoints of each beam as the model car is moving along Rail 1, 2 and 3 with different speeds. The impact factor has been defined in Equation (4.13). The maximum static strains at the midpoints of each beam are determined by low pass filtering the strains when the model car is moving with a low speed (Thater et al, 1998) of 0.5m/s. From the figures and the table, the following results can be obtained.

1) The impact factors on each beam are different, especially from the eccentric moving loads. The impact factors on the beam that is further away from the moving model car are larger, but the responses are opposite.

2) As the moving speed increases, the impact factors increased prominently. This is due to an impulse induced when the wheels pass over the gap between the leading beam and the bridge deck at the entry of the bridge deck, and large responses in the model car are induced. This could lead to some related study in the assessment of "Bridge-friendliness" of the vehicle’s suspension system in analyzing the bridge-vehicle interaction.

10.4 MOVING LOAD IDENTIFICATION BASED ON EXACT SOLUTION
With the limitation of the computer capacity and computation time in matrix manipulation using this method, only the axle loads are identified to limit the size of the matrix to an acceptable size. The 9 modes \( m=3, \ n=3 \) and the strains at nine measuring points are used in the identification. The measured signals are re-sampled to have a time interval of 0.005s to reduce the computation time at the expense of accuracy. As the model car is moving along the centerline (Rail 3), Rail 1 or Rail 2 in turn, the strains at 1/4a, 1/2a and 3/4a of each beam are used to identify the moving axle loads. Three sensor sets are used for a comparative study on the sensor location selection. The sensor sets are the same as in Section 9.3.4.1 and they are shown in Table 9.5. Sensor set I consists of the strains at 1/4a, 1/2a and 3/4a of the three beams on the left. Sensor set II consists of the nine measuring points on the middle three beams, and sensor set III consists of the nine measured strains on the three beams on the right.

10.4.1 Identification of Axle Loads at Different Eccentricities

Figure 10.9 shows the identified axle loads from the strains at 1/4a, 1/2a and 3/4a of the three beams in the middle(Sensor Set II). Figure 10.10 shows the identified combined load. Table 10.4 shows the correlation coefficients between the reconstructed strains and the measured strains at 3/8a of the beams. The following observations are made from the figures and Table 10.4:

1) The proposed method based on exact solution in Chapter 9 is effective to identify the axle loads from measuring strains, and acceptable results can be obtained.

2) When the strains at 1/4a, 1/2a and 3/4a of the middle three beam are used in the identification, the correlation coefficients between the reconstructed and measured strains are all over 0.9 even when the model car is moving on Rail 1 or Rail 2 at an eccentricity of 1/8b and 3/8b, respectively.

3) From Figures 10.9 and 10.10, the identified eccentric forces are smaller than that with no eccentricity with the same sensors on the middle three beams.

4) The identified forces are in general smaller than static loads. One possible reason may be due to a calibration error as only a static calibration has been used. Another reason is due to a different number of modes used in the identification compared to the number of modes in the measured responses. Detail discussions are given in Section 10.5.2.
10.4.2 Identification of Axle Loads with strains at Different Measuring Points

The sensor sets and the parameters are the same as the above. Figures 10.11 and 10.12 show the identified forces from the strains at different measuring points. The correlation coefficients between the reconstructed and measured strains at 3/8a of the beams are shown in Table 10.4. The following observations can be obtained from Figures 10.11 and 10.12 and Table 10.4:

1) The correlation coefficients are all above 0.9 as the model car is moving along the central line (Rail 3) or on Rail 2. But the correlation coefficients are very small as sensor set I is used and the model car is moving along Rail 1. This is due to the measuring points are further away from the moving model car. It further supports the findings obtained in Section 9.3.4.1.

2) From Figures 10.11 and 10.12, the identified loads from Set II are intermediate between those from Set I and Set III. Set I sensors give a large identified forces in the front half of the time histories while Set III sensors give large identified forces in the latter half of the time histories. Set I sensors are on the left-most three beams and Set III are on the right-most three beams. The reason of this observation can not be explained.

10.5 MOVING LOAD IDENTIFICATION BASED ON FINITE ELEMENT FORMULATION

10.5.1 Wheel Load Identification

The measured strains are re-sampled to have a time interval of 0.003s. When the model car is moving along the centerline (Rail 3), Rail 1 or Rail 2 in turn, the strains at 1/4a, 1/2a and 3/4a of each beam are measured to identify the moving wheel loads. The average speed of the model car is 0.54m/s. Figures 10.13, 10.14 and 10.15 show the identified wheel loads, axle loads and the combined load from different vibration modes used in the identification with different number of measuring points when the model car moves along the centerline. Table 10.5 shows the correlation coefficients between the reconstructed and measured strains at 3/8a of each beam for different moving paths of the car. The following observations are made from the Figures and Table 10.5.
1) The method proposed in Chapter 9 is effective to identify individual moving wheel loads, and acceptable results can be obtained.

2) The correlation coefficients between the reconstructed and measured strains on the beams adjacent to the moving path of the car are larger than 0.8. This shows that the method is effective to identify the wheel loads moving with or without an eccentricity.

3) When the distance between the measuring point and the path of moving car is large, the correlation coefficient is small as seen from Beam #1 for e=3/8b. This is because of the small responses at the measuring points, and the reconstructed response is very sensitive to error in the identified loads.

4) The identified loads from the case with (m=3, n=3(15)) is always smaller in all the results shown in Figures 10.13 to 10.15. The reason is due to an unequal number of modes used in the responses and in the identification, and it will be discussed in next section.

10.5.2 Effect of Unequal Number of Modes in the Response and in the Identification

Figures 10.13 to 10.15 show that the identified loads from (m=3 and n=3) is less than the loads identified from (m=3 and n=2). This difference cannot be the result of any calibration error. An inspection of Equations (9.39) to (9.42) gives the following reasons for the existence of this error.

Equation (9.39) is valid for both the measured responses and for the identification. Let \( N_r = m_r \times n_r \) and \( N_f = m_f \times n_f \) be the number of the modes in the responses and in identification respectively. Equation (9.39) can be rewritten as follow:

\[
q_{n_r} = W_{N_r, N_r} \cdot \mathcal{Q}_{N_r} 
\]

where

\[
W_{N_r, N_r} = \begin{bmatrix}
W_1(x_1, y_1) & W_2(x_1, y_1) & \cdots & W_{N_r}(x_1, y_1) \\
W_1(x_2, y_2) & W_2(x_2, y_2) & \cdots & W_{N_r}(x_2, y_2) \\
\vdots & \vdots & \ddots & \vdots \\
W_1(x_{N_r}, y_{N_r}) & W_2(x_{N_r}, y_{N_r}) & \cdots & W_{N_r}(x_{N_r}, y_{N_r})
\end{bmatrix}_{N_r \times N_r}
\]

\[
\mathcal{Q}_{N_r} = \begin{bmatrix}
q_1(t) & q_2(t) & \cdots & q_{N_r}(t)
\end{bmatrix}^T
\]

We have two possible cases:
Case (a): When \( N_I < N_R \), \( Q_{N_I} \) can be obtained from partitions of Equation (10.1) as

\[
Q_{N_I} = (W_{N_I,N_I}^t W_{N_I,N_I})^{-1} W_{N_I,N_I}^t (w_m - W_{N_I,N_I} W_{N_I,N_R} Q_{N_R-N_I})
\]  
(10.2)

with

\[
W_{N_I,N_R-N_I} = \begin{bmatrix}
W_{N_I,N_I}(x_1, y_1) & W_{N_I,N_R-N_I}(x_1, y_1) & \cdots & W_{N_R}(x_1, y_1) \\
W_{N_I,N_R-N_I}(x_2, y_2) & W_{N_I,N_R-N_I}(x_2, y_2) & \cdots & W_{N_R}(x_2, y_2) \\
\vdots & \vdots & \ddots & \vdots \\
W_{N_I,N_R-N_I}(x_N, y_N) & W_{N_I,N_R-N_I}(x_N, y_N) & \cdots & W_{N_R}(x_N, y_N)
\end{bmatrix}
\]  
(10.3)

\[
Q_{N_R-N_I} = [q_{N_I,1}(t), q_{N_I,2}(t), \ldots, q_{N_I,N}(t)]^T
\]

The terms in last bracket in Equation (10.2) represents the responses from the lower sets of \( N_I \) modes in identification. But in practice, the total measured responses are used instead leading to an over-estimation of the forces when substituting \( Q_{N_I} \) and its derivatives into Equation (9.14).

Case (b): When \( N_R < N_I \), \( Q_{N_I} \) can be obtained in a similar way as

\[
Q_{N_I} = (W_{N_I,N_I}^t W_{N_I,N_I})^{-1} W_{N_I,N_I}^t (w_m + W_{N_I,N_R} Q_{N_R-N_I})
\]  
(10.4)

where \( W_{N_I,N_R-N_I} \) and \( Q_{N_R-N_I} \) are similarly defined as in Equation (10.3). The last term in bracket in Equation (10.4) represents the total responses corresponding to the modes used in identification. But in practice, only the measured responses \( w_m \) is used in the equation leading to an under-estimation in the forces.

In the simulation studies in Chapter 9, the measured responses are computed from 25 modes \( (m=n=5) \). In the cases with \( N_I < N_R \), the modes using in the identification covered the excitation frequency range of the moving loads. 200 terms in the orthogonal polynomial have been used in obtaining the derivatives of \( Q_{N_I} \). The small magnitudes of modes higher than \( N_I \) are further reduced by the low pass filtering effect. The term \( Q_{N_R-N_I} \) becomes insignificant small and the over-estimation from Equation (10.2) is therefore not noticeable.

In the experiment, the excitation or natural frequencies of the car are not small. But only 50 terms in the orthogonal polynomial have been used because of a very small signal to noise ratio in the higher mode responses. The number of vibration modes left after filtering is greatly reduced leading to \( N_R < N_I \). The term \( W_{N_R-N_I} Q_{N_R-N_I} \) does not exist in the measured responses, and hence the modal
coordinates $Q_x$, and its derivatives are under-estimated giving smaller than true loads when substituting into Equation (9.14). There is a difference of three modes between the cases of $(m=3,n=3)$ and $(m=3,n=2)$, and yet the final results differs by a great percentage. This is because of the large responses in these three modes which should contribute greatly to the final identified results if they are included in the identification.

10.6 CONCLUDING REMARKS

An experimental setup for bridge deck model has been designed to verify the moving force identification technique in the laboratory. A laboratory study on dynamic load of the bridge deck under moving vehicle is also presented. Jumping of the model car at the gap between the leading beam and the bridge deck at the entry of the bridge deck has a large effect on the impact factor. This is due to the initial vibration of the model car excited by this impulse. The moving forces are identified from the measured bridge strains using the two proposed methods. The experimental results show that the two proposed methods are effective to identify the eccentric moving forces on a simply supported bridge deck in practice. Optimal sensor location is important to have less errors in the identified forces. The range of vibration modes used in the identification should cover all the frequency components in the responses. When the number of vibration mode used for identification is larger than that in the responses, an under-estimation of the results would follow, and vice versa.
Table 10.1 The Identified frequencies and damping ratios of the bridge deck

<table>
<thead>
<tr>
<th>MODE No.</th>
<th>Frequency (Hz)</th>
<th>Damping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.416</td>
<td>0.6875</td>
</tr>
<tr>
<td>2</td>
<td>12.763</td>
<td>0.2196</td>
</tr>
<tr>
<td>3</td>
<td>27.009</td>
<td>0.2544</td>
</tr>
<tr>
<td>4</td>
<td>34.901</td>
<td>0.2606</td>
</tr>
<tr>
<td>5</td>
<td>38.455</td>
<td>0.1719</td>
</tr>
<tr>
<td>6</td>
<td>49.769</td>
<td>0.1285</td>
</tr>
<tr>
<td>7</td>
<td>61.648</td>
<td>0.3518</td>
</tr>
<tr>
<td>8</td>
<td>67.206</td>
<td>0.2711</td>
</tr>
<tr>
<td>9</td>
<td>70.456</td>
<td>0.2662</td>
</tr>
<tr>
<td>10</td>
<td>73.199</td>
<td>0.3356</td>
</tr>
<tr>
<td>11</td>
<td>76.601</td>
<td>0.3339</td>
</tr>
<tr>
<td>12</td>
<td>79.942</td>
<td>0.4993</td>
</tr>
<tr>
<td>13</td>
<td>86.096</td>
<td>0.1949</td>
</tr>
</tbody>
</table>

Note: Response Type: Acceleration.

Table 10.2 Natural frequencies of the bridge deck and model car (Hz)

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
<td>2</td>
<td>3</td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Deck Test</td>
<td>9.42*</td>
<td>12.76</td>
<td>27.01</td>
<td>70.46</td>
<td>34.90*</td>
<td>38.46</td>
<td>49.77</td>
<td>73.20*</td>
<td>76.71</td>
<td>86.12</td>
<td></td>
</tr>
<tr>
<td>Deck FEM(P)</td>
<td>9.12</td>
<td>15.10</td>
<td>30.10</td>
<td>64.60</td>
<td>36.10</td>
<td>43.20</td>
<td>57.70</td>
<td>79.50</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Deck FEM(S)</td>
<td>9.13*</td>
<td>12.10</td>
<td>28.10</td>
<td>63.70</td>
<td>36.30*</td>
<td>39.30</td>
<td>52.80</td>
<td>80.40*</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Deck Proposed method</td>
<td>9.27*</td>
<td>12.89</td>
<td>29.65</td>
<td>65.35</td>
<td>37.14*</td>
<td>41.15</td>
<td>56.41</td>
<td>83.58*</td>
<td>87.65</td>
<td>102.07</td>
<td></td>
</tr>
</tbody>
</table>

Mode no. of Car

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq. (Hz)</td>
<td>27.04</td>
<td>44.22</td>
<td>58.74</td>
<td>87.38</td>
</tr>
</tbody>
</table>

Note: * the longitudinal bending mode.

- FEM(P) gives results with point constraints from finite element method.
- FEM(S) gives results with simple supports from finite element method.
Table 10.3 Impact factors with vehicle on different lanes and different moving speeds

<table>
<thead>
<tr>
<th>Rail No.</th>
<th>Speed (m/s)</th>
<th>Beam #1</th>
<th>Beam #2</th>
<th>Beam #3</th>
<th>Beam #4</th>
<th>Beam #5</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.54</td>
<td>13.34</td>
<td>5.60</td>
<td>2.11</td>
<td>1.91</td>
<td>10.15</td>
</tr>
<tr>
<td></td>
<td>0.76</td>
<td>15.11</td>
<td>6.69</td>
<td>7.56</td>
<td>6.19</td>
<td>16.76</td>
</tr>
<tr>
<td></td>
<td>1.13</td>
<td>25.28</td>
<td>15.75</td>
<td>12.80</td>
<td>10.57</td>
<td>23.22</td>
</tr>
<tr>
<td></td>
<td>1.48</td>
<td>93.65</td>
<td>59.17</td>
<td>51.47</td>
<td>55.42</td>
<td>95.60</td>
</tr>
<tr>
<td>2</td>
<td>0.54</td>
<td>16.52</td>
<td>12.20</td>
<td>3.40</td>
<td>2.93</td>
<td>5.39</td>
</tr>
<tr>
<td></td>
<td>0.77</td>
<td>13.51</td>
<td>-3.22</td>
<td>10.24</td>
<td>-1.30</td>
<td>-4.02</td>
</tr>
<tr>
<td></td>
<td>1.09</td>
<td>51.87</td>
<td>25.99</td>
<td>22.29</td>
<td>10.89</td>
<td>24.90</td>
</tr>
<tr>
<td></td>
<td>1.43</td>
<td>95.98</td>
<td>54.57</td>
<td>53.62</td>
<td>37.76</td>
<td>50.38</td>
</tr>
<tr>
<td>1</td>
<td>0.53</td>
<td>137.79</td>
<td>25.41</td>
<td>7.14</td>
<td>2.21</td>
<td>3.78</td>
</tr>
<tr>
<td></td>
<td>0.70</td>
<td>205.77</td>
<td>31.90</td>
<td>51.63</td>
<td>16.51</td>
<td>7.64</td>
</tr>
<tr>
<td></td>
<td>1.09</td>
<td>157.50</td>
<td>20.20</td>
<td>56.49</td>
<td>14.02</td>
<td>5.37</td>
</tr>
<tr>
<td></td>
<td>1.43</td>
<td>857.51</td>
<td>330.04</td>
<td>305.29</td>
<td>132.18</td>
<td>86.51</td>
</tr>
</tbody>
</table>

Table 10.4 Correlation coefficient between reconstructed and measured strains at 3/8a

<table>
<thead>
<tr>
<th>Sensor Set</th>
<th>Rail Number</th>
<th>Moving Speed (m/s)</th>
<th>Beam #1</th>
<th>Beam #2</th>
<th>Beam #3</th>
<th>Beam #4</th>
<th>Beam #5</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>1.0919</td>
<td>0.0126</td>
<td>0.1095</td>
<td>0.0862</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.0896</td>
<td>0.8974</td>
<td>0.9689</td>
<td>0.9685</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.1327</td>
<td>0.9194</td>
<td>0.9163</td>
<td>0.8837</td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>1</td>
<td>1.0919</td>
<td>0.8772</td>
<td>0.8481</td>
<td>0.9204</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.0896</td>
<td>0.9884</td>
<td>0.9688</td>
<td>0.9635</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.1327</td>
<td>0.9421</td>
<td>0.9110</td>
<td>0.9443</td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>1</td>
<td>1.0919</td>
<td></td>
<td>0.9634</td>
<td>0.9697</td>
<td>0.9529</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.0896</td>
<td>0.9587</td>
<td>0.9554</td>
<td>0.9815</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.1327</td>
<td>0.9544</td>
<td>0.9794</td>
<td>0.9829</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 10.5 Correlation coefficient between reconstructed and measured strains at 3/8a

<table>
<thead>
<tr>
<th>Eccentricity</th>
<th>Modes</th>
<th>Beam1</th>
<th>Beam2</th>
<th>Beam3</th>
<th>Beam4</th>
<th>Beam5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (Rail 3)</td>
<td>m=3;n=4(15)</td>
<td>0.783</td>
<td>0.897</td>
<td>0.931</td>
<td>0.909</td>
<td>0.799</td>
</tr>
<tr>
<td></td>
<td>m=3;n=3(15)</td>
<td>0.935</td>
<td>0.944</td>
<td>0.931</td>
<td>0.951</td>
<td>0.941</td>
</tr>
<tr>
<td></td>
<td>m=3;n=2(15)</td>
<td>0.901</td>
<td>0.935</td>
<td>0.929</td>
<td>0.944</td>
<td>0.922</td>
</tr>
<tr>
<td></td>
<td>m=3;n=2(9)</td>
<td>0.932</td>
<td>0.949</td>
<td>0.933</td>
<td>0.953</td>
<td>0.947</td>
</tr>
<tr>
<td>3/8b (Rail 1)</td>
<td>m=3;n=4(15)</td>
<td>0.112</td>
<td>0.772</td>
<td>0.897</td>
<td>0.939</td>
<td>0.936</td>
</tr>
<tr>
<td></td>
<td>m=3;n=3(15)</td>
<td>0.166</td>
<td>0.793</td>
<td>0.915</td>
<td>0.951</td>
<td>0.948</td>
</tr>
<tr>
<td></td>
<td>m=3;n=2(15)</td>
<td>0.039</td>
<td>0.813</td>
<td>0.914</td>
<td>0.948</td>
<td>0.947</td>
</tr>
<tr>
<td></td>
<td>m=3;n=2(9)</td>
<td>0.044</td>
<td>0.692</td>
<td>0.839</td>
<td>0.849</td>
<td>0.837</td>
</tr>
<tr>
<td>1/8b (Rail 2)</td>
<td>m=3;n=4(15)</td>
<td>0.550</td>
<td>0.794</td>
<td>0.848</td>
<td>0.790</td>
<td>0.758</td>
</tr>
<tr>
<td></td>
<td>m=3;n=3(15)</td>
<td>0.859</td>
<td>0.937</td>
<td>0.945</td>
<td>0.949</td>
<td>0.974</td>
</tr>
<tr>
<td></td>
<td>m=3;n=2(15)</td>
<td>0.896</td>
<td>0.947</td>
<td>0.953</td>
<td>0.948</td>
<td>0.966</td>
</tr>
<tr>
<td></td>
<td>m=3;n=2(9)</td>
<td>0.880</td>
<td>0.922</td>
<td>0.920</td>
<td>0.929</td>
<td>0.945</td>
</tr>
</tbody>
</table>

Note: (15) denotes 15 measuring points located evenly on the five beams; (9) denotes 9 measuring points located evenly on the three beams near the moving path of the car.
Figure 10.1 Vehicle moving on the deck
Chapter 10: Laboratory Study II: Bridge Deck Model

(a) Top face of the bridge deck

(b) Bottom face of the bridge deck

(c) Section A-A

Figure 10.2 Layout of the bridge deck
Figure 10.3 Mode 1, 2, 3 and 4 of the bridge deck
Figure 10.4 Mode 5, 6, 7 and 8 of the bridge deck
Figure 10.5 Mode 9, 10, 11 and 12 of the bridge deck
Figure 10.6 Mode 1, 2, 3 and 4 of the model car
Figure 10.7 Strains with different speeds with car along rail 3

(− 0.54m/s; −−− 0.76m/s; ... 1.13m/s; −−−− 1.48m/s.)
Figure 10.8 Strains with different speeds with car along rail 1

(- 0.53m/s; --- 0.70m/s; . . . . 1.09m/s; . . . . 1.43m/s.)
Figure 10.9 Identified axle loads from strains with the car moving at different eccentricities

(— Static loads; — e=0; — — e=1/8b; ... e=3/8b)
Figure 10.10 Identified combined loads from strains with the car moving at different eccentricities
(— Static loads; — e=0; --- e=1/8b; ... e=3/8b)
Figure 10.11 Identified axle loads using different sensor sets

(— Static loads; — Set I; —— Set II; … Set III)
Figure 10.12 Identified combined loads using different sensor sets

(— Static loads; — Set I; —— Set II; ... Set III)
Figure 10.13 Identified wheel loads for different combinations of measured modal information

(- static loads; -- m=3, n=2(9); ---- m=3, n=3(15); ... m=3, n=2(15).)
Figure 10.14 Identified axle loads for different combinations of measured modal information
(− static loads; − m=3, n=2(9); ---- m=3, n=3(15); ... m=3, n=2(15).)
Figure 10.15 Identified total load from using different modes

(— static loads; — m=3,n=2(9); —— m=3,n=3(15); ... m=3,n=2(15).)
Photograph 10.1 Experimental Set-up

Photograph 10.2 Supports of the bridge deck model
Photograph 10.3 Electric motor

Photograph 10.4 Model Car
Photograph 10.5 Model car along the central rail

Photograph 10.6 Appearance of the measuring system
Chapter 11
COMPARATIVE STUDIES

11.1 INTRODUCTION

Two methods have been developed in Chapters 6 to 10 to identify moving loads on top of a continuous bridge using measured vibration responses. One is based on the exact solution (ESM) and the other is based on the finite element formulation (FEM). Numerical studies with a single-span bridge deck are presented to illustrate the robustness and accuracy of the two approaches in this chapter. Parameters that may influence the accuracy of moving load identification, such as sampling frequency, number of vibration modes and measuring points in the identification are discussed. Practical aspects in the identification of bridge-vehicle interaction forces, such as road surface roughness and incomplete velocity information of vehicle are also studied. Comparison of the errors in the identified forces show that the finite element approach with the orthogonal function approximation of the responses, gives more accurate results in general than the exact solution approach for all the studies presented in this thesis. The road surface roughness and the large variation in the moving speed are identified as the two main factors affecting the accuracy of identification in both methods.

These two methods are further examined with experimental measurements obtained from a bridge-vehicle system in the laboratory. The strains in the bridge deck are measured when a model car moves across the bridge deck along different paths. The moving loads on the bridge deck are identified from the measured strains using these two methods, and the responses are reconstructed from the identified loads for comparison with the measured responses to verify the performances of these methods. Studies on the identification accuracy due to the effect of the number of vibration mode used, the number of measuring points and eccentricities of travelling paths are conducted. Results show that both methods could identify the moving loads individually or as axle loads when they are travelling at a small eccentricity, and the accuracy of the FEM is dependent on the amount of measured information used in the identification.
11.2 NUMERICAL STUDIES

Two examples are used to study the effect of different parameters on the accuracy of the two methods.

11.2.1 Example 1: Identification of Two Moving Loads on a Single Span Beam

A single span simply supported beam with two forces $p_1(t)$ and $p_2(t)$ moving on top is studied.

\[
\begin{align*}
  p_1(t) &= 20000[1 + 0.1\sin(10\pi t) + 0.05\sin(40\pi t)] N \\
  p_2(t) &= 20000[1 - 0.01\sin(10\pi t) + 0.05\sin(50\pi t)] N
\end{align*}
\]  

(11.1)

The parameters of the beam are: $EI = 2.5 \times 10^{10} Nm^2$, $\rho A = 5000 Kg/m$, $L = 30m$. The distance between the two moving forces is $4.27m$. The first six natural frequencies of the beam are 3.90Hz, 15.61Hz, 35.13Hz, 62.45Hz, 97.58Hz and 140.51Hz. White noise is added to the calculated responses of the beam to simulate the polluted measurements with

\[
\varepsilon = \varepsilon_{\text{calculated}} + E_{\nu} \cdot N_{\text{noise}} \cdot \sigma(\varepsilon_{\text{calculated}})
\]

(11.2)

where $\varepsilon$ and $\varepsilon_{\text{calculated}}$ are the polluted and the original strains respectively. $E_{\nu}$ is the noise level. $N_{\text{noise}}$ is a standard normal distribution vector with zero mean value and unit standard deviation and $\sigma(\varepsilon_{\text{calculated}})$ is the standard deviation of the original strains.

Study 1: Effect of Noise Level

The first six modes are used in the simulation. The time interval between adjacent data points is 0.002s. Six measuring points are evenly distributed on the beam at $1/7L$ spacing. The moving speed is $30m/s$, and twenty terms are used in the orthogonal function. Monte Carlo method is used to simulate the noise in the responses for twenty times, and noise levels are from 1% to 10%. Figure 11.1 shows the mean and standard deviation of the errors in the identified moving loads using the method based on the exact solution method, and Figure 11.2 shows those from using the method based on finite element formulation.

The errors from using ESM vary approximately linearly with the noise levels in the responses. The standard deviation in the errors is largest with 6% noise in the responses. The errors from using FEM exhibit little change with the noise level in the
responses. This is because the orthogonal function approach in the identification reduces the effect of noise by its own filtering effect. When the noise level in the responses increases, the standard deviation in the errors also increases. This indicates the ESM could give very accurate results at low noise level, but it is greatly influenced by the noise effect. While the orthogonal function approximation in the FEM reduces consistently the noise effect to give accuracy results in all cases studied.

Study 2: Effect of Mode Truncation

The first six modes are included in the responses, and six measuring points are evenly distributed on the beam at $1/L$ spacing. The first 2, 3, 4, 5 and 6 modes are used in the identification in turn. Other parameters are the same as described in last section. Figures 11.3 and 11.4 show the errors in the identified results with different number of modes using ESM and FEM, respectively. Figure 11.5 shows the errors in the identified results with different number of terms in the orthogonal function in FEM when six modes are included in the responses.

The errors derived from ESM increase roughly proportional to the noise level in the responses and with similar rate of change for different number of modes. The errors from using FEM exhibit little change with noise. This shows that the errors in the identified results using FEM are mainly governed by the efficiency of the filtering effect in the orthogonal function approach. FEM is in general much better than ESM in the identification.

The errors shown in Figures 11.3 and 11.4 increase by a large extent when the number of the modes in the identification is less than three. This is because the first three natural frequencies of the beam cover the frequency range of the moving loads, and there is a loss of measured information in the identification when only two methods are used. The errors in the identified forces in Figure 11.5 remain relatively constant for different noise levels when the number of terms in the orthogonal function in FEM is less than 20. And the noise level would have a negative effect on the errors when there are more terms in the orthogonal function. This is because the frequency range in the orthogonal function increases with increasing number of terms, and the high frequency components in the noise would be retained in the calculation and thus affecting the final results.

Study 3: Effect of Number of Measuring Points
Again the first six modes are used in the simulation. The number of measuring points is selected as 6, 7, 8, 9, 10 in turn. The measuring points are evenly distributed on the beam. The other parameters are the same as in last section. Figures 11.6 and 11.7 show the errors in the identified results with different number of measuring points as the noise level in the responses is increased. The number of measuring points is shown insignificant to the identified results. It should be noted that the number of measuring points used are all larger than the number of the modes in the identification.

**Study 4: Effect of Sampling Frequency**

The first six modes are used in the simulation. The responses are calculated with 0.001s time interval between data points, and they are re-sampled with a time interval of 0.002s and 0.003s in turn. The moving speed is 30m/s. Figures 11.8 and 11.9 show the errors in the identified results with different sampling frequencies and noise levels using these two methods.

The errors in the identified results for ESM are largest when the noise level is above 2% and the sampling time interval is 0.001s. This is again due to the inclusion of the high frequency components of noise in the calculation with a higher sampling frequency. The errors from using FEM are smaller than those from using SEM and with smaller variations.

**11.2.2 Example 2: Single Span Bridge Subject to a Moving Vehicle**

The bridge-vehicle system is shown in Figure 6.11 and is represented by a simply supported beam subject to a moving vehicle with two axles and four degrees-of-freedom. The parameters of the system are the same as those in Section 6.4.1.

The first six natural frequencies of the bridge deck are 3.90Hz, 15.61Hz, 35.13Hz, 62.45Hz, 97.58Hz and 140.51Hz. The natural frequencies of the vehicle are 10.27Hz, 14.44Hz, 65.05Hz and 94.90Hz. The first six bridge modes are used in the calculation of the interaction forces by the method in Chapter 3. Six measuring points are evenly located on the bridge. The weight ratio between the vehicle and bridge is 0.135. The time interval between two adjacent data points is 0.002s.

**Study 5: Effect of Road Surface Roughness and Moving Speed**
Based on ISO-8606 (1995) specification, the road surface roughness in the time domain is simulated by applying the Inverse Fast Fourier Transformation (Henchi et al, 1998). Tables 11.1 and 11.2 show the errors in the identified moving loads with different moving speeds and road surface roughness using these two methods. Figure 11.10 shows the identified moving loads with Class B road surface roughness and 5% noise level in the responses. Only the strain responses are used.

The identified time histories are shown varying about the true time histories in Figure 11.10. These two methods can be used to identify the bridge-vehicle interaction forces from the bridge responses, and acceptable results can be obtained with different road surface roughness and moving speeds in the identification. The moving speed has little effect on the accuracy in the identified moving loads from both methods.

In the FEM, the errors in the identified results increase as the road surface roughness increases but they change slightly for different noise levels. This is because the high frequency components caused by the road surface roughness are reduced by the fitting of the orthogonal function. In the ESM, the errors in the identified results change slightly as the road surface condition deteriorates. However, they are sensitive to the noise level in the responses. It may be concluded that the ESM is good for low noise level and FEM is good for high noise level in the responses.

Study 6: Identification of Moving Loads on Bridge Deck with Varying Speeds

Normally a vehicle is moving on top of the bridge deck with varying speed, and we shall discuss the moving load identification when the varying speed is known in this section. The first six modes are used in the simulation. Six measuring points are evenly distributed on the bridge, and the time interval between data points is 0.002s. Other parameters are the same as in previous discussions. The responses are calculated by the method in Chapter 3. Tables 11.3 and 11.4 show the errors in the identified results with different noise levels in the responses using both methods. Figure 11.11 shows the identified moving loads with the vehicle starts braking at the entry of the bridge deck with an acceleration of $-1m/s^2$ and 5% noise in the responses using these two methods. Figure 11.12 shows the identified results with the vehicle starts braking at $1/3L$ and the acceleration is $-3m/s^2$ and with 5% noise in
the responses. The initial speed is 30\( \text{m/s} \) and the road roughness is Class B in both cases. The results are shown under the heading "instantaneous" in the Tables.

Since the instantaneous speed of the forces is known, both methods can be used to identify the moving loads from bridge strains, and acceptable results can be obtained from both methods at low noise level. Those from using FEM are consistently much better than those from using ESM for different noise levels in the study.

**Study 7: Identification with Incomplete Vehicle Speed Information**

In practice, the axle spacing, the number of axles of the vehicle, and the time that the vehicle enters or exits the bridge can be measured directly by axle sensors (Chan et al. 2000a). But the braking position and the acceleration are difficult to measure. The errors induced from identifying using an average speed should be studied. Figure 11.13 shows the identified results with the vehicle starts braking at the entry with an acceleration of \( -1\text{m/s}^2 \) using both methods. The average speed of 29.39\( \text{m/s} \) is used. Figure 11.14 shows the identified results with the vehicle starts braking at \( 1/3L \) and the acceleration is \( -3\text{m/s}^2 \). The average speed of 29.04\( \text{m/s} \) is used. The initial speed is 30\( \text{m/s} \) and the road roughness is Class B in both cases. Tables 11.3 and 11.4 show the errors in the identified results with different noise levels in the responses. The results are shown under the heading "average" in the Tables.

It is seen that the identified results from both methods using the average speed are acceptable when the acceleration is \( -1\text{m/s}^2 \). But for the case with \( -3\text{m/s}^2 \) acceleration, a large increase in the error for the second axle load is observed. This shows that the moving loads can be identified from strains using the average speed when the acceleration is not very large.

In the figures, the first moving load is seen over-estimated and the second moving load is under-estimated in both methods. This is because the moving loads are estimated by minimizing the error between the measured and reconstructed responses from the identified moving loads. The location of the resultant load using average speed lags behind that of the true resultant load and this difference is largest approximately at mid-span of the bridge deck in these two cases. The optimization however, yields a location of the resultant load which is close and behind the true load. This leads to an over-estimated first axle load and an under-estimated second axle load. This behaviour is opposite in the case of having acceleration of the vehicle.
11.3 LABORATORY STUDIES

The laboratory test results mentioned in Chapter 10 are used in these studies.

11.3.1 Axle Load Identification from Bridge Strains

Nine of the vibration modes \((m=3, n=3)\) shown in Table 10.2 and the strains at nine and fifteen measuring points are used in the identification. The measured signals are re-sampled to have a time interval of 0.005s to reduce the computation time at the expense of accuracy. As the model car is moving along the central line (Rail 3), Rail 1 or Rail 2 in turn, the strains at 1/4a, 1/2a and 3/4a of each beam are used to identify the moving loads. Five sensor sets shown in Figure 11.15 are used for a comparative study on the effect of sensor location selection. Sensor set I consists of the strains from the three beams on the left. Set II consists of the nine measuring points on the middle three beams, and set III consists of the nine measured strains from the three beams on the right. Sensor set IV consists of the measured strains in Beams 1, 3 and 5, and sensor set V consists of the measured strains of all the five beams.

Study 1: Effect of Number of Modes

Figures 11.16 and 11.17 show the identified results from different number of modes using the ESM and FEM respectively. The model car moves along Rail 3 at a speed of 1.1079 m/s. Sensor set V is used in the identification. Table 11.5 shows the correlation coefficients between the reconstructed and measured strains at 3/8a on the five beams. Figure 11.18 shows the identified axle loads from nine modes \((m=3; n=3)\) using both methods for comparison. In the FEM, fifty terms in the orthogonal functions are used to approximate the measured strains.

The identified axle loads from using the two methods are close to the static force and the correlation coefficients between the reconstructed and measured strains at 3/8a on each beams are all over 0.9. This shows that both methods are effective to identify the moving vehicular axle loads from bridge strains and acceptable results can be obtained. Figures 11.16 and 11.17 show that the identified results from three longitudinal bending modes \((m=3; n=1)\) is close to that from using nine modes \((m=3; n=3)\). This shows that the effect of torsional vibration is small when the model car moves along Rail 3, and there is little difference in the results when the torsional
modes are included in the calculation. The axle loads can be identified approximately using an equivalent beam model when the vehicle is moving along the central line of the bridge deck. Figure 11.18 also indicates the identified results using ESM are close to that using FEM when sensor set V is used in the identification with FEM giving larger fluctuations than ESM. This indicates that both methods are effective and accurate to identify the dynamic axle loads in practice using different number of vibration modes with fifteen measuring points.

Study 2: Effect of Measuring Location

The sensor sets and the parameters are the same as above. Figures 11.19 and 11.20 show the identified loads from strains at different measuring points using the ESM and FEM respectively as the model car moves along Rail 3. The correlation coefficients between the reconstructed and measured strains at 3/8a of the beams are shown in Tables 11.6 and 11.7. Figures 11.21 and 11.22 show the identified results with different sensor sets using ESM as the model car moves along Rails 2 and 1 respectively.

The ESM gives correlation coefficients above 0.9 as the model car is moving along the central line (Rail 3) or Rail 2. But the correlation coefficients are very small as sensor set I or II is used with the model car moving along Rail 1. This is because the measuring points are far away from the moving model car, and they cannot pick up the dominating bending modes whereas the torsional modal responses are small as found in Study 1. Figure 11.22 shows the extreme case from sensor set I as the model car moves along Rail 1. It is concluded that the identified force time histories from ESM are satisfactory from different sensor sets when the car moves along Rails 2 or 3.

The identified results are different from sensor sets I, II and III using FEM as shown in Figure 11.20. They are also different with the identified results from sensor set V shown in Figure 11.17. This shows that the performance of FEM is very dependent on the sensor locations and on the amount of measured information, and more sensors should be used to identify the moving loads using FEM. This observation is also supported by the correlation coefficients in Table 11.7 which are in general poorer than those from ESM in Table 11.6. The results from FEM also exhibit unsatisfactory performance in identifying loads moving along Rails 1 and 2 from sensor sets I to IV.
Study 3: Effect of Eccentricities

The following discussions refer to the results from ESM shown in Figures 11.19, 11.21 and 11.22 and in Table 11.6. The ESM is effective to identify the axle loads from measured strains, and acceptable results can be obtained. When either one of sensor set II, III or IV is used in the identification, almost all the correlation coefficients between the reconstructed and measured strains are all over 0.9 even when the model car is moving on Rail 1 or Rail 2 at an eccentricity of 1/8b and 3/8b, respectively. The identified eccentric loads are close to that with no eccentricity as shown in Figure 11.23.

The identified results from sensor set III in Figure 11.22 is the largest in the group and that from sensor set II is larger than that from sensor I as the model car moves along Rail 1. This is because sensor set III is close to the moving loads and is more subject to the bending modes of the bridge deck, and the signal to noise ratios of the measured strains are larger than that from sensor sets I or II. Therefore the identified results from sensor sets I and II are over-smoothed and are smaller than those from sensor set III.

11.3.2 Wheel Load Identification from Bridge Strains

Study 4: Effect of Measuring Locations

The sensor sets and the parameters are the same as for the axle load identification. The first and third wheels of the vehicle are on the front axle and the second and fourth wheels are on the rear axle. The second and fourth wheels follow the first and third wheels respectively. Figures 11.24 and 11.25 show the identified wheel loads with different sensor sets using the ESM and FEM respectively as the model car moves along Rail 3. Table 11.8 shows the correlation coefficients between the reconstructed and measured strains at 3/8a. The sampling frequency is 200Hz, and fifty terms of the orthogonal functions are used in the FEM. Figure 11.26 shows the comparative results from both methods from sensor set V as the model car moves along the central line (Rail 3).

Both methods are effective to identify dynamic wheel loads along the central line from bridge strains, and acceptable results can be obtained as indicated by the correlation coefficients in Table 11.8. The identified force time histories from sensor set II are nearly the same as from sensor sets I or III using ESM, but they are different
from each other when using FEM. Table 11.8 also shows that more measured information is needed in FEM than in ESM in the wheel load identification.

For the ESM results in Figure 11.24, the left wheel loads (the first and second loads) from sensor set I are larger than those from sensor set III and the right wheel loads (the third and fourth loads) from sensor set I are smaller than those from sensor set III. No rolling motion of the vehicle is observed in the identified forces. Since there is no spring component in each wheel in the model car and the wheel spacing is very small, the four wheel loads behave similar to a single moving mass with some pitching effects. Therefore this difference in the identified forces can only be due to the proximity of the sensors to the loads, i.e. sensor set I is close to the left wheel loads and sensor set III is close to the right wheel loads.

For the FEM results in Figure 11.25, the front wheel loads (the first and third loads) vary greatly from using different sensor sets while the rear wheel loads (the second and fourth loads) are relatively the same from using the sensor sets I, II and III. The identified force time histories are not consistent.

When all fifteen sensors are used, the identified force time histories from both methods are similar with the FEM results showing larger fluctuations then the ESM results as shown in Figure 11.26. This observation is similar to that in the axle load identification shown in Figure 11.18.

It may be concluded that the ESM is effective to identify accurately loads moving along the central line of the bridge deck, while FEM does not give consistent results from using different sets of sensors.

**Study 5: Effect of Eccentricities**

The sensor sets and the parameters are the same as above. Figure 11.27 shows the identified results from sensor set II using ESM as the model car moves along different rails. Figure 11.28 shows the identified results from sensor set V using FEM as the model car moves along different rails. All measured information is used as the accuracy of FEM is found from previous studies to be dependent on the amount of measured information. The related correlation coefficients between the reconstructed and measured strain at 3/8a are also shown in Table 11.8. These two methods are found effective to identify the dynamic wheel loads along Rails 2 and 3 based on sensor set II for the ESM and sensor set V for the FEM. Both methods fail to identify forces along Rail 1 and the reason is given in Study 3 above.
Study 6: Effect of Number of Modes

The sensor sets and parameters are the same as above. Figure 11.29 shows the identified results from different number of modes using ESM as the model car moves along the central line (Rail 3). Sensor set II is used in the identification. Figure 11.30 shows the identified results using FEM and sensor set V is used in the identification. The correlation coefficients between the reconstructed and measured strain at 3/8a are listed in Table 11.9. The correlation coefficients are all above 0.89 for both sets of results. The identified force time histories in Figures 11.29 and 11.30 are approximately the same when different number of the modes is used in the identification. This shows that the effect of torsional vibration is small when the model car moves along Rail 3, and there is little difference when the torsional modes are included in the calculation. This also supports the findings in Studies 1, 2 and 4 that the sensors should be close to the moving loads to pick up the dominating bending modes to have an accurate identified result.

11.4 CONCLUSIONS

Both the FEM and the ESM can be used to identify the moving loads or the bridge-vehicle interaction forces from strains with road roughness and vehicle braking on the bridge. The FEM gives consistently smaller error in the results for all noise levels while the accuracy of ESM is greatly affected by noise. This indicates the importance of having pre-processing of the measured data to remove the measurement noise before the identification. The orthogonal function approximation of the measured strains is also shown to be effective in filtering the high frequency noise components in the responses.

From the parametric studies conducted, the speed of forces and the number of sensors have no effect on the accuracy of results. The sampling frequency, the number of measuring points and the number of modes included in the identification greatly influence the accuracy. The sampling frequency should be higher than the frequency range in the responses while the number of modes should cover the frequency range of the responses. The number of measuring points should be larger than the number of modes used in the identification. The road surface roughness Class A, B and C have little effect on the accuracy while Class C and D would cause large error in the identified results from both methods. A small acceleration of \(-1m/s^2\) on an average of
30 m/s$^2$ does not have large effect on the accuracy of identification but a large acceleration of -3m/s$^2$ would cause large error in the second axle force. The accuracy of both methods and the effect of acceleration on the results are further confirmed in the laboratory work.

Results obtained from a comprehensive laboratory experiment indicate that a group of forces moving on top of the model bridge deck can be identified individually or in terms of axle loads with accuracy. The identified results for individual loads are poorer than those for axle loads. Both the ESM and FEM can identify moving loads with a small eccentricity, but FEM requires a lot more of measured information to have the same accuracy as ESM. Both methods fail to identify loads with a large eccentricity.

Since the longitudinal bending modes in the experiment are dominating in the responses, the dynamic loads from the model car can be identified with the bridge deck simplified as a beam model.
Table 11.1 Errors in the identified moving loads using FEM with different moving speeds and road surface roughness (in percent)

<table>
<thead>
<tr>
<th>Speed (m/s)</th>
<th>Roughness</th>
<th>1% noise</th>
<th>5% noise</th>
<th>10% noise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Error1</td>
<td>Error2</td>
<td>Error1</td>
</tr>
<tr>
<td>No</td>
<td></td>
<td>2.988</td>
<td>3.909</td>
<td>3.128</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>3.412</td>
<td>3.416</td>
<td>3.562</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>4.062</td>
<td>4.884</td>
<td>4.158</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>11.312</td>
<td>15.747</td>
<td>11.316</td>
</tr>
<tr>
<td>No</td>
<td>2.290</td>
<td>2.457</td>
<td>2.404</td>
<td>2.523</td>
</tr>
<tr>
<td>B</td>
<td>4.043</td>
<td>4.056</td>
<td>4.046</td>
<td>4.118</td>
</tr>
<tr>
<td>C</td>
<td>11.691</td>
<td>13.279</td>
<td>11.654</td>
<td>13.344</td>
</tr>
<tr>
<td>No</td>
<td>2.827</td>
<td>3.279</td>
<td>3.264</td>
<td>3.352</td>
</tr>
<tr>
<td>B</td>
<td>3.879</td>
<td>4.971</td>
<td>4.111</td>
<td>4.908</td>
</tr>
<tr>
<td>D</td>
<td>18.798</td>
<td>24.385</td>
<td>18.756</td>
<td>24.319</td>
</tr>
</tbody>
</table>

Error 1 – error in the first axle load; Error 2 – error in the second axle load.
Table 11.2 Errors in the identified moving loads using ESM with different moving speeds and road surface roughness (in percent)

<table>
<thead>
<tr>
<th>Speed (m/s)</th>
<th>Roughness</th>
<th>1% noise</th>
<th>5% noise</th>
<th>10% noise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Error1</td>
<td>Error2</td>
<td>Error1</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>2.858</td>
<td>2.648</td>
<td>12.247</td>
</tr>
<tr>
<td>40</td>
<td>No</td>
<td>2.008</td>
<td>2.349</td>
<td>8.843</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>2.622</td>
<td>2.979</td>
<td>11.190</td>
</tr>
</tbody>
</table>

Error1 – error in the first axle load; Error 2 – error in the second axle load.
### Table 11.3 Errors (in percent) in the moving loads identified with varying speeds using FEM

<table>
<thead>
<tr>
<th>Noise Level (%)</th>
<th>(-1 \text{m/s}^2) (braking at entry)</th>
<th>(-3 \text{m/s}^2) (braking at 1/3L)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Instantaneous Error1 Error2 Average Error1 Error2 Instantaneous Error1 Error2 Average Error1 Error2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4.051 5.203 5.038 6.985 3.516 5.588 7.000 14.776</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4.045 5.214 5.029 7.096 3.518 5.609 7.005 14.756</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>4.060 5.705 5.042 7.468 3.552 5.703 7.027 14.704</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>4.076 5.894 5.055 7.604 3.574 5.743 7.037 14.696</td>
<td></td>
</tr>
</tbody>
</table>

Error1 – error in the first axle load; Error 2 – error in the second axle load.
Table 11.4 Errors (in percent) in the moving loads identified with varying speeds using ESM

<table>
<thead>
<tr>
<th>Noise Level (km/s²)</th>
<th>Instantaneous</th>
<th>Average</th>
<th>Instantaneous</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.930</td>
<td>2.630</td>
<td>4.907</td>
<td>6.895</td>
</tr>
<tr>
<td>2</td>
<td>3.644</td>
<td>4.655</td>
<td>5.659</td>
<td>7.825</td>
</tr>
<tr>
<td>3</td>
<td>5.297</td>
<td>6.674</td>
<td>6.730</td>
<td>9.158</td>
</tr>
<tr>
<td>4</td>
<td>6.968</td>
<td>8.663</td>
<td>7.955</td>
<td>10.689</td>
</tr>
<tr>
<td>5</td>
<td>8.622</td>
<td>10.647</td>
<td>9.322</td>
<td>12.309</td>
</tr>
<tr>
<td>6</td>
<td>10.825</td>
<td>12.583</td>
<td>10.715</td>
<td>14.004</td>
</tr>
<tr>
<td>7</td>
<td>11.825</td>
<td>14.440</td>
<td>11.137</td>
<td>14.773</td>
</tr>
<tr>
<td>8</td>
<td>12.291</td>
<td>13.724</td>
<td>11.778</td>
<td>15.660</td>
</tr>
<tr>
<td>10</td>
<td>13.858</td>
<td>15.244</td>
<td>12.989</td>
<td>17.321</td>
</tr>
</tbody>
</table>

Error1 - error in the first axle load; Error2 - error in the second axle load.

Table 11.5 Correlation coefficients at 3/8a on the five beams

<table>
<thead>
<tr>
<th>Method</th>
<th>Modes</th>
<th>Correlation Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Beam1</td>
</tr>
<tr>
<td>ESM</td>
<td>3;3;3</td>
<td>0.979</td>
</tr>
<tr>
<td></td>
<td>4;3;2</td>
<td>0.982</td>
</tr>
<tr>
<td></td>
<td>1;1;1</td>
<td>0.967</td>
</tr>
<tr>
<td>FEM</td>
<td>3;3;3</td>
<td>0.905</td>
</tr>
<tr>
<td></td>
<td>2;2;2</td>
<td>0.923</td>
</tr>
<tr>
<td></td>
<td>1;1;1</td>
<td>0.923</td>
</tr>
</tbody>
</table>

Note: 4;3;2 indicates four modes with $m=1$, three modes with $m=2$ and two modes with $m=3$. 

11-16
### Table 11.6 Correlation coefficient between reconstructed and measured strains at 3/8a using ESM

<table>
<thead>
<tr>
<th>Sensor Set</th>
<th>Rail Number</th>
<th>Average Speed (m/s)</th>
<th>Correlation Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Beam1</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
<td>1.09</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.09</td>
<td>0.938</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.11</td>
<td>0.976</td>
</tr>
<tr>
<td>II</td>
<td>1</td>
<td>1.09</td>
<td>0.072</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.09</td>
<td>0.955</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.11</td>
<td>0.971</td>
</tr>
<tr>
<td>III</td>
<td>1</td>
<td>1.09</td>
<td>0.154</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.09</td>
<td>0.958</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.11</td>
<td>0.965</td>
</tr>
<tr>
<td>IV</td>
<td>1</td>
<td>1.09</td>
<td>0.071</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.09</td>
<td>0.953</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.11</td>
<td>0.966</td>
</tr>
<tr>
<td>V</td>
<td>1</td>
<td>1.09</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.09</td>
<td>0.911</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.11</td>
<td>0.979</td>
</tr>
</tbody>
</table>
Table 11.7 Correlation coefficient between reconstructed and measured strains at 3/8a using FEM

<table>
<thead>
<tr>
<th>Sensor Set</th>
<th>Rail Number</th>
<th>Average Speed (m/s)</th>
<th>Correlation Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Beam1</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
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<td>0.067</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.09</td>
<td>0.409</td>
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<td></td>
<td>3</td>
<td>1.11</td>
<td>0.887</td>
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<tr>
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<td>0.036</td>
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<td></td>
<td>2</td>
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<tr>
<td>III</td>
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<td>1.09</td>
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<tr>
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<td>3</td>
<td>1.11</td>
<td>0.621</td>
</tr>
<tr>
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<td>1.09</td>
<td>0.700</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.11</td>
<td>0.880</td>
</tr>
<tr>
<td>V</td>
<td>1</td>
<td>1.09</td>
<td>0.072</td>
</tr>
<tr>
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<td>0.741</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.11</td>
<td>0.913</td>
</tr>
</tbody>
</table>
Table 11.8 Correlation coefficients between reconstructed and measured strain at 3/8a for wheel load identification

<table>
<thead>
<tr>
<th>Method</th>
<th>Sensor Set</th>
<th>Average Speed (m/s)</th>
<th>Rail</th>
<th>Correlation Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Beam1</td>
</tr>
<tr>
<td>I</td>
<td></td>
<td></td>
<td></td>
<td>0.977</td>
</tr>
<tr>
<td>II</td>
<td></td>
<td></td>
<td></td>
<td>0.976</td>
</tr>
<tr>
<td>III</td>
<td>1.11</td>
<td>3</td>
<td></td>
<td>0.970</td>
</tr>
<tr>
<td>ESM</td>
<td>IV</td>
<td></td>
<td></td>
<td>0.969</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td></td>
<td></td>
<td>0.977</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>1.09</td>
<td>2</td>
<td>0.959</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>1.09</td>
<td>1</td>
<td>0.153</td>
</tr>
<tr>
<td>I</td>
<td></td>
<td></td>
<td></td>
<td>0.894</td>
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<td>0.818</td>
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<td>III</td>
<td>1.11</td>
<td>3</td>
<td></td>
<td>0.820</td>
</tr>
<tr>
<td>FEM</td>
<td>IV</td>
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<td>0.894</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td></td>
<td></td>
<td>0.911</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>1.09</td>
<td>2</td>
<td>0.734</td>
</tr>
<tr>
<td></td>
<td>II</td>
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<td>1</td>
<td>0.029</td>
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</table>

Table 11.9 Correlation coefficients for wheel load identification with different number of modes

<table>
<thead>
<tr>
<th>Method</th>
<th>Sensor Set</th>
<th>Modes</th>
<th>Correlation Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Beam1</td>
</tr>
<tr>
<td>ESM</td>
<td>II</td>
<td>m=3; n=3</td>
<td>0.976</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m=2; n=2</td>
<td>0.959</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m=1; n=1</td>
<td>0.958</td>
</tr>
<tr>
<td>FEM</td>
<td>V</td>
<td>m=3; n=3</td>
<td>0.911</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m=2; n=2</td>
<td>0.924</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m=1; n=1</td>
<td>0.923</td>
</tr>
</tbody>
</table>
Figure 11.1 The mean (circles) and standard deviation (error bars) of the errors in the identified moving loads using ESM.
Figure 11.2 The mean (circles) and standard deviation (error bars) of the errors in the identified moving loads using FEM
Figure 11.3 Errors in the identified moving loads using ESM
(a) Error in the identified first moving load

(b) Error in the identified second moving load

Figure 11.4 Errors in the identified moving loads using FEM
Figure 11.5 Error in the identified moving loads using different number of terms in the orthogonal function
(a) Error in the identified first moving load

(b) Error in the identified second moving load

Figure 11.6 Errors in the identified moving loads using ESM with different number of measuring points
Figure 11.7 Errors in the identified moving loads using FEM with different number of measuring points
Figure 11.8 Errors in the identified results with different sampling frequencies using FEM
Figure 11.9 Errors in the identified moving loads with different sampling frequencies using ESM
Figure 11.10 Identified forces on Class B road with 5% noise
(— True forces; —— Using FEM; ... Using ESM.)
Figure 11.11 Identified forces with instantaneous speed and braking at entry

(— True forces; ——— Using FEM; ... Using ESM.)
Figure 11.12 Identified forces with instantaneous speed and braking at 1/3L
(- True forces; --- Using FEM; ... Using ESM.)
Figure 11.13 Identified forces using average speed with braking at entry
(— True forces; —— Using FEM; ... Using ESM.)
Figure 11.14 Identified forces using average speed with braking at 1/3L
(— True forces; --- Using FEM; ... Using ESM.)
Figure 11.15 Sensor Sets for moving load identification
Figure 11.16 Identification of axle loads along Rail 3 with different modes using ESM

(— Static forces; — Identified with [3;3;3]; —— Identified with [4;3;2]; ... Identified with [1;1;1].)
Figure 11.17 Identification of axle loads along Rail 3 with different modes using FEM

(– Static forces; – Identified with [3;3;3]; — Identified with [2;2;2]; ... Identified with [1;1;1].)
Figure 11.18 Identification of axle loads along Rail 3 using different methods

(- Static forces; — Using ESM; ... Using FEM.)
Figure 11.19 Identification of axle loads along Rail 3 using ESM

(— Static forces; — Identified with set I; —— Identified with set II; ... Identified with set III.)
Figure 11.20 Identification of axle loads along Rail 3 using FEM

(— Static forces; – Identified with set I; —— Identified with set II; ... Identified with set III.)
Figure 11.21 Identification of axle loads along Rail 2 using ESM

(- Static forces; - Identified with set I; --- Identified with set II; ... Identified with set III.)
Figure 11.22 Identification of axle loads along Rail 1 using ESM

( - Static forces; - Identified with set I; --- Identified with set II; ... Identified with set III.)
Figure 11.23 Identification of axle loads along different rails from sensor set II using ESM

(- Static forces; - Rail 3; -- Rail 2; ... Rail 1.)
Figure 11.24 Identification of wheel loads along Rail 3 using ESM

(– Static forces; – Sensor Set I; — Sensor Set II; ... Sensor Set III.)
Figure 11.25 Identification of wheel loads along Rail 3 using FEM

(— Static forces; — Sensor Set I; —— Sensor Set II; ... Sensor Set III.)
Figure 11.26 Identification of wheel loads along Rail 3 using different methods

(— Static forces; —— Using ESM; ... Using FEM.)
Figure 11.27 Wheel load identification along different rails with sensor set II using ESM

(- Static forces; -- Rail 3; --- Rail 2; ... Rail 1.)
Figure 11.28 Wheel load identification along different rails with sensor set V using FEM

(- Static forces; – Rail 3; --- Rail 2; ... Rail 1.)
Figure 11.29 Identification of wheel loads along Rail 3 using different modes

(— Static forces; − [3;3;3]; ——— [2;2;2]; ... [1;1;1].)
Figure 11.30 Identification of wheel loads along Rail 3 with different modes using FEM

(— Static forces; — [3;3;3]; —— [2;2;2]; ... [1;1;1].)
Chapter 12

CONCLUSIONS AND RECOMMENDATIONS

12.1 CONCLUSIONS

Load identification techniques are not original. The two new methods developed in this thesis is to identify moving loads from measured responses. These methods achieve good identification accuracy as well as maintaining the computational efficiency, and they are also more robust than existing methods. These methods can be used for application to any types of structures. The achievement of the above objective is supported by the following works in the thesis:

1) A method has been developed to analyze the dynamic behavior of a multi-span continuous bridge with non-uniform cross-section under the braking action of a moving vehicle. A three-axle tractor vehicle is adopted in the study to represent freight loading which governs the design of modern bridges. The recommended 33% dynamic multiplier from AASHTO (1998) is found not sufficient for the single span bridge deck studied when vehicle braking effect is included. The recommended dynamic multiplier may also be too conservative for the three-span bridge deck under studied. The cost of construction could be reduced if a smaller impact factor is recommended for multi-span bridge to design engineers. The impact factor should be taken in the same span as braking occurs, and braking in the first half of the span would produce larger dynamic response in the structure. Vehicle braking causes very significant dynamic responses compared with road surface roughness in single span bridge, while it has insignificant effect in the three-span bridge deck under studied. The other parameters like initial vehicle velocity and amplitude of braking force have little effect on the dynamic response.

2) A method based on the orthotropic plate theory and modal superposition technique has been developed to investigate the loading on a multi-lane continuous bridge due to vehicles moving on top of the bridge deck. The transverse vehicle position has an important effect on the impact factor. The impact factors in the bridge deck that is far away from the path of the moving vehicle are larger than those that are near. But it is the opposite with the responses and wheel load distribution factor which are larger at points closer to the moving
vehicle. The high dynamic impact factors reported in this study correspond to low response level in the bridge deck and hence low stress level. Therefore these impact factors should be taken with care as only when they are related to the design situation that they would be of importance. The impact factors associated with multiple vehicles are smaller than those for single vehicle.

3) A regularization methods is applied to the Time Domain method and Frequency and Time Domain Method in the moving force identification. The results obtained are greatly improved over those without regularization with acceptable errors from using different combinations of measured responses. Time Domain Method is found better than the Frequency and Time Domain Method in solving for the ill-posed problem. Both simulation and laboratory test results indicate that the total weight of a vehicle can be estimated indirectly using moving force identification methods with some accuracy.

4) Two new methods based on regularization technique have been developed to overcome the deficiencies exhibited in existing methods. A new time domain method has been developed to identify the moving loads on a continuous beam from strains and accelerations. The large matrix relating the force and responses is split into smaller sub-matrices making the computation more cost effective. Regularization technique is used to stabilize the unbound solution. The Generalized Cross-Validation L-curve and the plot of error versus the regularization parameter can be locate the optimal regularization parameter effectively. If the moving loads are identified from strains, the sampling frequency can be selected to be larger or equal to twice of the maximum frequency of the moving loads and the vibration modes used in the identification. But when accelerations are used, a much higher sampling frequency should be used in the identification. In general, more vibration modes should be used in impulsive load identification. And the modelling error is found to have a very significant effect on the accuracy of the identified forces. An accurate model is therefore required. This method is put to test in environments simulating the real bridge-vehicle system in which road surface roughness and non-uniform vehicle speed are modelled. The method is found to have good performances in the identification of individual interactive forces with small variations in the vehicle speed. It can identify accuracy the combined load from the axle forces even with vehicle braking on the bridge deck.
Another general method based on finite element formulation has also been developed to identify the moving loads on a continuous beam from strains. A generalized orthogonal function approach is proposed, and the moving forces are identified with bounds in the errors using regularization in the solution. More supports, and when the number of mode shapes in the identification is more than or equal to that in the measured responses, the errors of identification will be the smallest.

5) An experimental setup with an uniform beam model has been designed in the laboratory. The moving forces are identified from the measured bridge strains using the two proposed methods. The experimental results show that the two methods are effective to identify the moving forces on a simply supported bridge in practice. They are found to have good performances in the identification of individual interactive forces with small variations in the vehicle speed when the true or average speed is used. It can identify accurately the combined load from the axle forces using the average speed even with vehicle braking on the bridge deck.

6) The general method based on finite element formulation has been extended to identify moving loads on a multi-span continuous Timoshenko beam with non-uniform cross-section. Limitations on the application of this force identification method are studied, and the following recommendations are reported: The number of modes required in the identification depends entirely on the highest frequency of interest. The sampling frequency may be taken as two times the highest frequency of interest which may be the maximum exciting frequency of the moving force or the natural frequency of the beam. The number of sensors is recommended to be at least equal to the number of vibration modes in the analysis. The error in the identified forces become large when the exciting frequency of the moving force comes close to the natural frequency of the beam-force system. The error of identification is not sensitive to the distance between the two moving forces. Therefore two forces moving at a close spacing can be resolved by this method.

7) The two proposed methods have been modified and applied to identify moving loads on top of bridge deck using measured structural responses. The bridge deck is modeled as an orthotropic plate and the vehicular load is modeled as a group of four wheel loads or two axle loads moving on top of the bridge deck at fixed
spacing. The proposed identification method based on exact solution can identify individual loads from the measured strains and accelerations. Acceleration measurements would provide better results than those from strain measurements. Identification of forces moving at an eccentric path is slightly less accurate than that for forces moving along the central line of the bridge deck when the sensors are around the middle of the bridge cross-section. Optimal sensor location is important to have less errors in the identified forces especially from acceleration measurements, and further study has to be made in this area. When the force traverses mid-span of the beam, the responses from the second longitudinal modes are smallest at this moment, and this affects the strain measurement and hence the identified results greatly. Large errors are found at the beginning and the end of the time histories of the identified forces. This is due to the discontinuity of the forces at these two points leading to large fluctuations in the identified results. When the lower modes of the bridge deck are dominated by vibration modes along the longitudinal axis, a beam model instead of a plate mode may be accurate enough in the identification.

The other proposed time domain method based on finite element formulation is also modified to identify moving loads on the three-dimensional bridge deck based on the measured responses. The method is also effective in the identification of eccentric moving loads. The torsional modes are also very important in the moving load identification even when the group of loads is moving along the centerline of the bridge deck.

8) An experimental setup for the bridge deck model has been designed to verify the moving force identification techniques in the laboratory. Results from the laboratory study on the dynamic load from moving vehicle are presented. The gap between the leading beam and the bridge deck at the entry of the bridge deck has a large effect on the impact factor as there is no obvious suspension system in the vehicle system. The moving forces are identified from the measured bridge strains using the two proposed methods. The experimental results show that the two methods are effective to identify the eccentric moving forces on a simply supported bridge deck in practice.

9) Comparative studies for the FEM and the ESM have been done by numerical and laboratory studies. The FEM gives consistently smaller error in the results for all noise levels while the accuracy of ESM is greatly affected by noise. This indicates
the importance of having pre-processing of the measured data to remove the measurement noise before the identification. The orthogonal function approximation of the measured strains is also shown to be effective in filtering the high frequency noise components in the responses.

Results obtained from a comprehensive laboratory experiment indicate that a group of forces moving on top of the model bridge deck can be identified individually or in terms of axle loads with accuracy. The identified results for individual loads are poorer than those for axle loads. Both the ESM and FEM can identify moving loads with a small eccentricity, but FEM requires a lot more of measured information to have the same accuracy as ESM. Both methods fail to identify loads with a large eccentricity. Since the longitudinal bending modes in the experiment are dominating in the responses, the dynamic loads from the model car can be identified with the bridge deck simplified as a beam model.

12.2 RECOMMENDATIONS FOR FUTURE WORK

Future studies on the moving load identification are recommended. The present work in this thesis provides some exploratory experiences and suggestions for the following additional studies:

1. The method based on the finite element formulation has been developed to identify the moving loads on the continuous bridge. Simulations for the simplified models have been presented in the thesis. The equations can be constructed for a more complex bridge structure using finite element analysis, and the moving loads can be identified using this method in further studies.

2. In Chapter 6, the effect of modeling error in the moving force identification has been studied. It is found to have a very significant effect on the accuracy of the identified forces. In practice, the modal parameters are obtained by modal testing. If they are obtained through finite element analysis, they should be updated using the measured data, but some of the modeling errors should have been retained in the modal parameters. The moving force identification with model error is an important problem, and the identification methods should be modified to consider the effect of model errors in further studies.

3. In Chapter 9, the errors in the identified forces are more or less the same from different sets of sensors, but slightly less errors are found from the measurement
locations that are further away from the moving loads. In practice, the bridge responses can only be measured at a few points, and the identification is based on incomplete measured information. Optimal sensor locations and sensor validation should be further studied.

4. As a modern research approach, the target and design values for traffic effects on bridge deck are obtained through suitable stochastic traffic models to process the recorded traffic data. The dynamic amplification factors and the load distribution of the bridge deck under random traffic flow are still not clear. The dynamic behavior of the bridge deck under random traffic flow is an area for further study, and its inverse problem is also very important to modern bridge design, assessment and vibration control.
REFERENCES


Appendix A

VEHICLE AND BRIDGE MODEL IN CHAPTER 3

The equations-of-motion of the vertical motion of the vehicle-bridge system are

\[
\begin{align*}
M_v \ddot{Y} + C_v \dot{Y} + K_v Y &= F_v \\
M_b \ddot{Q} + C_b \dot{Q} + K_b Q &= F_b
\end{align*}
\]

where

\[
F_v = \left\{ \left( m_1 + m_2 \right) b_1 \ddot{x}_1(t), \left( m_1 + m_2 \right) b_1 \dot{x}_1(t), m_2 b_2 \ddot{x}_1(t), -P_1(t), -P_2(t), -P_3(t), -m_2 b_2 \ddot{x}_1(t) \right\}^T
\]

\[
F_b = \left\{ \sum_{i=1}^{n} W_i \left( \ddot{x}_i(t) \right) P_i(t), \quad i = 1, 2, \ldots, n \right\}
\]

\[
P_i'(t) = P_i(t) + \left( m_i a_1 + m_2 a_2 + m_2 a_3 a_6 + m_3 \right) g
\]

\[
P_i'(t) = P_i(t) + \left( m_i a_1 + m_2 a_2 + m_3 \right) g
\]

\[
P_i'(t) = P_i(t) + \left( m_2 a_5 + m_3 \right) g
\]

\[
Y = \left\{ y_1, y_2, \ldots, y_7 \right\}^T;
\]

\[
Q = \left\{ q_1(t), q_2(t), \ldots, q_7(t) \right\}^T.
\]

\[
M_v = \left\{ \int \rho A(x) W_i(x) W_j(x) \, dx, (i, j = 1, 2, \ldots, n) \right\}
\]

\[
K_v = \left\{ \int E I(x) W_i(x) W_j(x) \, dx + \sum_{i=1}^{n} W_i(x_i) W_j(x_i), \quad (i, j = 1, 2, \ldots, n) \right\}
\]

\[
\begin{bmatrix}
K_{v1} & -K_{v2} & K_{v3} \\
-K_{v2} & K_{v2} & 0 \\
K_{v3} & 0 & K_{v4}
\end{bmatrix}
\]

\[
M_v = \begin{bmatrix}
M_{v1} & 0 & M_{v1}^T \\
0 & M_{v2} & 0 \\
M_{v3} & 0 & M_{v4}
\end{bmatrix}
\]

\[
M_{v1} = \begin{bmatrix}
m_1 a_1^2 + \frac{J_1}{S_1} + \left( m_1 + m_2 \right) b_1^2 & m_1 a_1 a_2 - \frac{J_1}{S_1} - \left( m_1 + m_2 \right) b_1^2 & -m_2 b_1 b_2 \\
m_1 a_1 a_2 - \frac{J_1}{S_1} - \left( m_1 + m_2 \right) b_1^2 & m_1 a_1^2 + \frac{J_1}{S_1} + \left( m_1 + m_2 \right) b_1^2 & m_2 b_1 b_2 \\
-m_2 b_1 b_2 & m_2 b_1 b_2 & \frac{J_2}{S_2} + \left( a_3^2 + b_3^2 \right)
\end{bmatrix}
\]

A-1
\[ M_{r2} = \text{diag}(m_3, m_4, m_5) \]

\[ M_{r3} = \begin{bmatrix} m_2 b_1 b_2 & -m_2 b_1 b_2 & m_2 a_3 a_6 - \frac{J_z}{S_z} - m_1 b_2^2 \end{bmatrix} \]

\[ M_{r4} = \begin{bmatrix} m_2 (a_6^2 + b_2^2) + \frac{J_z}{S_z} \end{bmatrix} \]

\[ K_{s1} = \begin{bmatrix} k_1 + k_2 a_i^2 & k_1 a_3 a_4 & 0 \\ k_1 a_3 a_4 & k_2 + k_3 a_4^2 & 0 \\ 0 & 0 & k_3 \end{bmatrix} \]

\[ K_{s2} = \text{diag}(k_1, k_2, k_3, k_4) \]

\[ K_{s3} = \{-k, a_4, -k, a_4, 0\} \]

\[ K_{s4} = \{k_7\} \]

(A.2)
Appendix B

VEHICLE AND BRIDGE MODEL IN CHAPTER 4

B.1 Vehicle Model:

\[ Z = \{y_c, \theta_r, \theta_r, y_{a1}, \theta_{a1}, y_{a2}, \theta_{a2}\}^T; \]

\[ F^{*}_{e} = \left\{0, 0, 0, -F_{t1} - F_{t2}, \frac{S_{y1}}{2} (F_{t1} - F_{t2}), -F_{t3} - F_{t4}, \frac{S_{y2}}{2} (F_{t3} - F_{t4})\right\}^T; \]

\[ M_{e} = \text{diag}\{m_c, I_c, I_r, m_{a1}, I_{a1}, m_{a2}, I_{a2}\}; \]

\[ K_{e} = [K_{y1}, K_{y2}, K_{y3}, K_{y4}, K_{y5}, K_{y6}, K_{y7}, \ldots, K_{yj}]; \]

\[ K_{v11} = \sum_{i=1}^{4} K_{y_i}; \quad K_{v12} = (K_{y1} + K_{y2})a_{1}S_{x} - (K_{y3} + K_{y4})a_{2}S_{x}; \]

\[ K_{v13} = \frac{S_{y1}}{2} (K_{y1} - K_{y2}); \quad K_{v14} = \frac{S_{y2}}{2} (K_{y3} - K_{y4}); \]

\[ K_{v15} = \frac{S_{y1}}{2} (K_{y1} + K_{y2}); \quad K_{v16} = \frac{S_{y2}}{2} (K_{y3} + K_{y4}); \]

\[ K_{v22} = (K_{y1} + K_{y2})a_{1}S_{x} + (K_{y3} + K_{y4})a_{2}S_{x}; \]

\[ K_{v23} = \frac{1}{2} (K_{y1} + K_{y2})a_{1}S_{x}S_{y_1} + \frac{1}{2} (K_{y3} + K_{y4})a_{2}S_{x}S_{y_2}; \]

\[ K_{v24} = -\frac{1}{2} (K_{y1} - K_{y2})a_{1}S_{x}S_{y_1}; \quad K_{v25} = \frac{1}{2} (K_{y1} - K_{y2})a_{1}S_{x}S_{y_1}; \]

\[ K_{v26} = (K_{y3} + K_{y4})a_{2}S_{x}; \quad K_{v27} = -\frac{1}{2} (K_{y3} - K_{y4})a_{2}S_{x}S_{y_1}; \]

\[ K_{v33} = \frac{S_{y1}^2}{4} (K_{y1} + K_{y2}) + \frac{S_{y2}^2}{4} (K_{y3} + K_{y4}); \]

\[ K_{v34} = \frac{S_{y1}}{2} (K_{y1} - K_{y2}); \quad K_{v35} = -\frac{S_{y1}}{4} (K_{y1} + K_{y2}); \quad K_{v36} = \frac{S_{y2}}{2} (K_{y3} - K_{y4}); \]

\[ K_{v37} = -\frac{S_{y2}^2}{4} (K_{y3} + K_{y4}); \]

\[ K_{v44} = (K_{y1} + K_{y2}); \quad K_{v45} = \frac{S_{y1}}{2} (K_{y1} + K_{y2}); \quad K_{v46} = K_{v47} = 0; \]

\[ K_{v55} = \frac{S_{y1}^2}{4} (K_{y1} + K_{y2}); \quad K_{v56} = K_{v57} = 0; \]

\[ K_{v66} = K_{y3} + K_{y4}; \quad K_{v67} = \frac{S_{y2}}{2} (K_{y3} - K_{y4}); \quad K_{v77} = \frac{S_{y2}^2}{4} (K_{y3} + K_{y4}); \]

where \( S_{y1}, S_{y2} \) are the spacing of suspensions in the front and rear axles respectively; \( S_{x} \) is the axle spacing.
B.2 Bridge Model:

\[ Q = \{ q_1(t), q_2(t), \ldots, q_{M_s}(t) \}^T; \quad W = \{ W_1(x, y), W_2(x, y), \ldots, W_{M_s}(x, y) \} \]

\( M_s \) is the number of mode shapes for the continuous orthotropic plate.

\[ M_s = \iint \rho h W^T W dS \quad C_s = \iint c_s W^T W dS \]

\[ W_h = \begin{bmatrix}
W_1(\hat{x}_1(t), \hat{y}_1(t)) & W_1(\hat{x}_2(t), \hat{y}_2(t)) & \cdots & W_1(\hat{x}_{N_p}(t), \hat{y}_{N_p}(t)) \\
W_2(\hat{x}_1(t), \hat{y}_1(t)) & W_2(\hat{x}_2(t), \hat{y}_2(t)) & \cdots & W_2(\hat{x}_{N_p}(t), \hat{y}_{N_p}(t)) \\
\vdots & \vdots & \ddots & \vdots \\
W_{M_s}(\hat{x}_1(t), \hat{y}_1(t)) & W_{M_s}(\hat{x}_2(t), \hat{y}_2(t)) & \cdots & W_{M_s}(\hat{x}_{N_p}(t), \hat{y}_{N_p}(t))
\end{bmatrix} \]

\[ K_h = \iint D_x W_x n^T W_x n + \frac{1}{2} (D_y V_{xx} + D_y V_{yy}) (W_x n^T W_x n + W_y n^T W_y n) + D_y W_y n^T W_y n + 4 D_y W_y n^T W_y n \]

\[ W_{x,n} = \sum_m \sum_n A_{mn} \varphi_m(x) \psi_n(y); \quad W_{y,n} = \sum_m \sum_n A_{mn} \varphi_m(x) \psi_n(y); \quad W_{x,y,n} = \sum_m \sum_n A_{mn} \varphi_m(x) \psi_n(y) \]

\( i = 1, 2, \ldots, M_s \)

\[ F_{h}^{int} = F_{h} + F_{b}; \quad F_{b} = \{ F_{11}, F_{12}, F_{13}, F_{14} \}^T \]

\[ F_{g} = \left( m \cdot a_1 + m_{sl} \right) g / 2, (m \cdot a_1 + m_{sl}) g / 2, (m \cdot a_2 + m_{sl}) g / 2, (m \cdot a_2 + m_{sl}) g / 2 \]^T \]

where \( F_{g} \) is the force vector caused by the effect of gravitation.
Appendix C

EIGENVALUE AND EIGENFUNCTION OF A CONTINUOUS BEAM

The eigenfunction of a $Q$ span Euler-Bernoulli continuous beam as shown in Figure C.1 can be written in the following form.

$$r_i(x_i) = A_i \sin \beta x_i + B_i \cos \beta x_i + C_i \sinh \beta x_i + D_i \cosh \beta x_i, \quad (i = 1, 2, \ldots, Q)$$

(C.1)

where $r_i(x_i)$ is the eigenfunction for the $i$th span, and $\beta$ is the eigenvalue. Hayashikawa and Watanabe (1981) have presented the formulation of the eigenfunction with arbitrary boundary conditions. The same problem is solved for a simply supported beam by the authors, and the boundary conditions are listed as follows

$$r_i(x_i)|_{x_i=0} = r_i(x_i)|_{x_i=l_i} = 0; \quad (i = 1, 2, \ldots, Q)$$

$$\left. \frac{\partial^3 r_i(x_i)}{\partial x_i^3} \right|_{x_i=0} = \left. \frac{\partial^3 r_0(x_0)}{\partial x_0^3} \right|_{x_0=0} = 0$$

$$\left\{ \frac{\partial r_i(x_i)}{\partial x_i} |_{x_i=l_i} = \frac{\partial r_{i+1}(x_{i+1})}{\partial x_{i+1}} |_{x_{i+1}=0}; \right. \left. \frac{\partial^2 r_i(x_i)}{\partial x_i^2} |_{x_i=l_i} = \frac{\partial^2 r_{i+1}(x_{i+1})}{\partial x_{i+1}^2} |_{x_{i+1}=0}; \quad (i = 1, 2, \ldots, Q - 2) \right. \left. \frac{\partial r_{Q-1}(x_{Q-1})}{\partial x_{Q-1}} |_{x_{Q-1}=0} = \frac{\partial r_0(x_0)}{\partial x_0} |_{x_0=0} \right. \left. \frac{\partial^2 r_{Q-1}(x_{Q-1})}{\partial x_{Q-1}^2} |_{x_{Q-1}=0} = \frac{\partial^2 r_0(x_0)}{\partial x_0^2} |_{x_0=0} \right.$$

(C.2)

Substituting the boundary conditions into Equation (C.1), the mode shape of the continuous beam can be written as
Appendix C: Eigenvalue and Eigenfunction of a Continuous Beam

\[
\phi(x) = \begin{cases} 
A_1 \left( \sin(\beta x) - \frac{\sin(\beta l_1)}{\sinh(\beta l_1)} \sinh(\beta x) \right), & 0 \leq x \leq l_1 \\
A_2 \left( \sin(\beta x) - \sum_{j=1}^{i-1} l_j \right) - \frac{\sin(\beta l_i)}{\sinh(\beta l_i)} \sinh(\beta x - \sum_{j=1}^{i-1} l_j)) \right) + B_i \left( \cos(\beta (x - \sum_{j=1}^{i-1} l_j)) \right) \\
- \cosh(\beta (x - \sum_{j=1}^{i-1} l_j)) \right) + \frac{\cosh(\beta l_i) - \cos(\beta l_i)}{\sinh(\beta l_i)} \sinh(\beta x - \sum_{j=1}^{i-1} l_j))), & \sum_{j=1}^{i-1} l_j \leq x \leq \sum_{j=1}^{i-1} l_j, \\
A_Q \left( \sin(\beta (L - x)) - \frac{\sin(\beta l_Q)}{\sinh(\beta l_Q)} \sinh(\beta (L - x)) \right), & L - l_Q \leq x \leq L 
\end{cases} 
\]

(C.3)

where parameters \(\beta, A_1, A_2, B_i (i = 2,3,\cdots, Q - 1), A_Q\) are determined from Equation (C.4) by solving the following set of equations (Gorman, 1975)

\[
[F][A] = 0 
\]

(C.4)

where

\[
A = \{A_1, A_2, B_2, \cdots, A_{Q-1}, B_{Q-1}, A_Q\}^T 
\]

The elements in matrix \(F\) are given by

\[
\begin{align*}
&f_{11} = \cos(\beta l_i) - \theta_1 \cdot \cosh(\beta l_i); f_{12} = \phi_1 - 1; f_{13} = -\phi_2; \\
&f_{21} = \sin(\beta l_i); f_{22} = -l; \\
&f_{2i-1,2i-1} = \cos(\beta l_i) - \theta_1 \cdot \cosh(\beta l_i); \\
&f_{2i-1,2i} = -\sin(\beta l_i) - \sinh(\beta l_i) + \phi_1 \cdot \cosh(\beta l_i); \\
&f_{2i-1,2i+1} = \theta_i - l; \\
&f_{2i,2i+1} = -\phi_i; \\
&f_{2i,2i+1} = -\sin(\beta l_i) - \theta_1 \cdot \sinh(\beta l_i); \\
&f_{2i,2i+2} = -\cos(\beta l_i) - \cosh(\beta l_i) + \phi_1 \cdot \sinh(\beta l_i); \\
&f_{2i,2i+2} = 2. \\
&\quad \quad (i = 2,3,\cdots, Q - 2)
\end{align*}
\]

(C.5)

where

\[
\theta_i = \frac{\sin(\beta l_i)}{\sinh(\beta l_i)}, \phi_i = \frac{\cosh(\beta l_i) - \cos(\beta l_i)}{\sinh(\beta l_i)}; \quad (i = 1,2,\cdots, Q)
\]

and the other coefficients \(f_{ij}\) equal to zero.
Figure C.1 A Q-span continuous beam
Appendix D

ORTHOGONAL POLYNOMIAL FUNCTION

\[ T_1 = \frac{1}{\sqrt{\pi}} \]

\[ T_2 = \sqrt{\frac{2}{\pi}} (\frac{2}{T} t - 1) \]

\[ T_3 = \sqrt{\frac{2}{\pi}} (2(\frac{2}{T} t - 1)^2 - 1) \]

.............

\[ T_{j+1} = 2(\frac{2}{T} t - 1)T_j - T_{j-1} \]
Appendix E

PRECISE TIME STEP INTEGRATION FOR THE DYNAMIC RESPONSE OF A CONTINUOUS BEAM UNDER MOVING LOADS

E.1 THE HIGH PRECISION INTEGRATION SCHEME

According to the precise time step integration method (Zhong and Williams, 1994), the equation of motion of the beam in Equation (8.8) can be written as

\[ \dot{u} = Hu + f \]  
(E.1)

where \( u \) is the response vector of size \( 2n \times 1 \); \( H \) is \( 2n \times 2n \) matrix; and \( f \) is the force vector of size \( 2n \times 1 \), with

\[
H = \begin{bmatrix}
-\frac{M^{-1}C}{2} & M^{-1} \\
-(K - \frac{CM^{-1}C}{4}) & \frac{CM^{-1}}{2}
\end{bmatrix},
\]

\[
f = \begin{bmatrix}
0 \\
F(t)
\end{bmatrix} = \begin{bmatrix}
0 \\
A(t)
\end{bmatrix}P(t);
\]

\[
A(t) = \begin{bmatrix}
W_1(\hat{x}_1(t)) & W_1(\hat{x}_2(t)) & \cdots & W_1(\hat{x}_{n_p}(t)) \\
W_2(\hat{x}_1(t)) & W_2(\hat{x}_2(t)) & \cdots & W_2(\hat{x}_{n_p}(t)) \\
\vdots & \vdots & \ddots & \vdots \\
W_{n_p}(\hat{x}_1(t)) & W_{n_p}(\hat{x}_2(t)) & \cdots & W_{n_p}(\hat{x}_{n_p}(t))
\end{bmatrix}
\]

\[ p(t) = M^* q(t) + \frac{Cq(t)}{2} \]

Matrix \( A(t) \) is obtained from Equation (8.10). Equation (E.1) can be written into discrete equations using the exponential matrix representation. Integrating Equation (E.1), we can have

\[ u(t) = e^{H(t-t_0)}u(t_0) + \int_{t_0}^{t} e^{H(t-\tau)f(\tau)}d\tau \]  
(E.3)

Expressing Equation (E.3) in discrete form

\[ u((j+1)h) = e^{Hh}u(jh) + \sum_{i=j}^{j+m} e^{iHh}f(\tau)d\tau \]  
(E.4)
where \( h \) is the time step of integration. The force \( f(\tau) \) is assumed constant within the time interval from \( jh \) to \((j+1)h\),

\[
u((j+1)h) = e^{iHh}u(jh) + \left[ \int_{jH}^{(j+1)H} e^{iH\tau'} d\tau' \right] f(jh) \tag{E.5}
\]

\[
= e^{iHh}u(jh) + H^{-1} \left[ e^{iHh} - I \right] f(jh)
\]

And the final discrete model for the \((j+1)\)th step is rewritten as

\[
u_{j+1} = \exp(H^* h)u_j + H^{-1}(\exp(H^* h) - I) f_j, \quad (j=0, 1, 2, \ldots) \tag{E.6}
\]

The precision of integration depends on the accuracy of \( \exp(H^* h) \). The 2\(N \) algorithm presented by Zhong and Yang (1991) is used and \( \exp(H^* h) \) has the form

\[
\exp(H^* h) = [\exp\left(\frac{H^* h}{N_i}\right)]^{N_i} = [\exp(H^* \Delta t)]^{N_i} \tag{E.7}
\]

where \( \Delta t = h / N_i \), \( N_i = 2^n \) and \( N \) can be any positive integer. Since \( h \) is not large and \( \Delta t \) would be extremely small. A truncated Taylor expansion of \( \exp(H^* \Delta t) \) may be used.

\[
\exp(H^* \Delta t) \approx I + H\Delta t + \frac{(H\Delta t)^2}{2!} + \frac{(H\Delta t)^3}{3!} + \frac{(H\Delta t)^4}{4!}
\]

\[
= I + R_0 \tag{E.8}
\]

where

\[
R_0 = H\Delta t + \frac{(H\Delta t)^2}{2!} + \frac{(H\Delta t)^3}{3!} + \frac{(H\Delta t)^4}{4!}.
\]

And \( R_i \), at the \( i \)th step of computation can be proved to take up the form of

\[
R_i = 2R_{i-1} + R_{i-1}R_{i-1}, \quad (i=1, 2, \ldots, N),
\]

Then

\[
\exp(H^* h) = [\exp(H^* \Delta t)]^{N_i} \approx I + R_N \tag{E.9}
\]

The term \( \exp(H^* h) \) can be computed from Equations (E.8) and (E.9), and the vibration response can be calculated from Equation (E.6). The computed results are compared with those from Newmark method using the same time step \( h \) in Equation (E.7), and the number of data points in the computation should be a multiple of two. It is noted that the accuracy of \( \exp(H^* h) \) and the vibration response \( u \) depends on the size of time step \( \Delta t = h / N_i \), adopted, and there is no convergence error involved in the final results.
E.2 RELIABILITY AND ACCURACY OF THE PROPOSED METHOD

A simply support uniform beam subjected to the excitation of a moving load is considered. The cross-sectional area and the material density of the beam are, respectively, $1.146 \times 10^{-3}$ m$^2$ and 7700 Kg/m$^3$. The overall length is one metre and the Young’s modulus is $2.07 \times 10^5$ Mpa. The speed of the moving load is 17.3 m/s. Computation of the responses was done using the first twelve vibration modes with the integration time step $h$ equals $9.0315 \times 10^{-4}$ s. The number of data used is 64 and $N=9$. $\Delta t$ equals $h/2^9=1.76396 \times 10^{-6}$ s, and the Taylor series expansion for $\exp(H\Delta t)$ contains infinitesimal approximation errors. Figure E.1 shows the results obtained from using the proposed method, the Newmark method and the exact solution (Fryba, 1972). The deflection under the moving load has been normalized with the static deflection when the load is at midspan. Comparison on the computation error and computer time using PII-300 personal computer from the precise method and the Newmark method is also presented in Table E.1. The computation error is defined as

$$\text{Error} = \frac{\|x - x_{\text{exact}}\|}{\|x_{\text{exact}}\|} \cdot 100\%$$

where $x$ and $x_{\text{exact}}$ are the computed result and the exact solution respectively.

Both the curves from the precise integration method and the Newmark method match with the exact solution closely. The computation errors are almost the same for both methods as seen in Table E.1. Table E.1 also shows that the computation errors from both methods are comparable for different time step of integration, but the computer time required in the precise method is only one-quarter of that in the Newmark method. The precise method also gives a larger error than the Newmark method when a very large time step is used as seen in the last row of Table E.1. This would indicate a large time step should go together with a larger $N$ value in the computation.
Table E.1 Comparison between two Methods

<table>
<thead>
<tr>
<th>No. of data</th>
<th>N</th>
<th>Time Step(s)</th>
<th>Precise method</th>
<th>Newmark</th>
</tr>
</thead>
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<tr>
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<td></td>
<td></td>
<td>Error(%)</td>
<td>Time(s)</td>
</tr>
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<td>1.4910</td>
<td>2.25</td>
</tr>
<tr>
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<td>0.99</td>
</tr>
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<td>1.6078</td>
<td>0.60</td>
</tr>
<tr>
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<td>9.0318x10^{-4}</td>
<td>2.5074</td>
<td>0.25</td>
</tr>
<tr>
<td>32</td>
<td>9</td>
<td>0.0018</td>
<td>7.2589</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Figure E.1 Deflection under the moving force at $v=17.3\text{m/s}$.
- exact solution; -- precise integration; … Newmark method.
Appendix F

FORMULATION OF COEFFICIENTS $a_{ij}$

The coefficients $a_{ij}$ stated in Equation (9.11) are listed as follows.

When $D_x < \rho \omega_\infty^2 \left( \frac{a}{m \pi} \right)^4$

$$a_{11} = 0; a_{12} = -r_{2\infty}^2 - v\left( \frac{m \pi}{a} \right)^2; a_{13} = 0; a_{14} = -r_{2\infty}^2 - v\left( \frac{m \pi}{a} \right)^2;$$

$$a_{21} = a_{12} \sinh(r_{2\infty} b); a_{22} = a_{14} \cosh(r_{1\infty} b); a_{23} = a_{12} \cos(r_{2\infty} b); a_{24} = a_{14} \cosh(r_{1\infty} b);$$

$$a_{31} = D_y r_{3\infty}^3 + (D_y + 2D_k)\left( \frac{m \pi}{a} \right)^2 r_{3\infty} a_{32} = 0;$$

$$a_{33} = -D_x r_{1\infty}^3 + (D_y + 2D_k)\left( \frac{m \pi}{a} \right)^2 r_{1\infty} a_{34} = 0;$$

$$a_{41} = a_{31} \cos(r_{2\infty} b); a_{42} = a_{32} \sinh(r_{2\infty} b); a_{43} = \cos(r_{1\infty} b); a_{44} = \sinh(r_{1\infty} b);$$

(F.1)

When \( \frac{D_y^2}{D_x} + \rho \omega_\infty^2 \left( \frac{a}{m \pi} \right)^4 > D_x > \rho \omega_\infty^2 \left( \frac{a}{m \pi} \right)^4 \)

$$a_{11} = 0; a_{12} = r_{1\infty}^2 - v\left( \frac{m \pi}{a} \right)^2; a_{13} = 0; a_{14} = r_{3\infty}^2 - v\left( \frac{m \pi}{a} \right)^2;$$

$$a_{21} = a_{12} \sinh(r_{1\infty} b); a_{22} = a_{14} \cosh(r_{1\infty} b); a_{23} = a_{12} \sinh(r_{3\infty} b); a_{24} = a_{14} \cosh(r_{3\infty} b);$$

$$a_{31} = -D_y r_{1\infty}^3 + (D_y + 2D_k)\left( \frac{m \pi}{a} \right)^2 r_{1\infty} a_{32} = 0;$$

$$a_{33} = -D_y r_{3\infty}^3 + (D_y + 2D_k)\left( \frac{m \pi}{a} \right)^2 r_{3\infty} a_{34} = 0;$$

$$a_{41} = a_{31} \cosh(r_{1\infty} b); a_{42} = a_{32} \sinh(r_{1\infty} b); a_{43} = a_{33} \cosh(r_{3\infty} b); a_{44} = a_{33} \sinh(r_{3\infty} b);$$

(F.2)
When $D_x > \frac{D_y^2}{D_y} + \rho \sigma_x^2 \left( \frac{a}{m \pi} \right)^4$

\[ a_{11} = r_{4nn}^2 - r_{5nn}^2 - \nu \left( \frac{m \pi}{a} \right)^2; a_{12} = 0; a_{13} = 0; a_{14} = 2r_{4nn}r_{5nn}; \]

\[ a_{21} = a_{11} \cosh(r_{4nn}b) \cos(r_{5nn}b) - 2r_{4nn}r_{5nn} \sinh(r_{4nn}b) \sin(r_{5nn}b); \]

\[ a_{22} = a_{11} \cosh(r_{4nn}b) \sin(r_{5nn}b) + 2r_{4nn}r_{5nn} \sinh(r_{4nn}b) \cos(r_{5nn}b); \]

\[ a_{23} = a_{11} \sinh(r_{4nn}b) \cos(r_{5nn}b) - 2r_{4nn}r_{5nn} \cosh(r_{4nn}b) \sin(r_{5nn}b); \]

\[ a_{24} = a_{11} \sinh(r_{4nn}b) \sin(r_{5nn}b) + 2r_{4nn}r_{5nn} \cosh(r_{4nn}b) \cos(r_{5nn}b); \]

\[ a_{31} = 0; a_{32} = (D_{xy} + 2D_k) \left( \frac{m \pi}{a} \right)^2 r_{4nn} - 3D_y r_{4nn}^2 r_{5nn} + D_y r_{5nn}^2; \]

\[ a_{33} = -D_y r_{4nn}^3 + (D_{xy} + 2D_k) \left( \frac{m \pi}{a} \right)^3 r_{4nn} + 3D_y r_{4nn}^2 r_{5nn}; a_{34} = 0; \]

\[ a_{41} = a_{33} \sinh(r_{4nn}b) \cos(r_{5nn}b) - a_{32} \cosh(r_{4nn}b) \sin(r_{5nn}b); \]

\[ a_{42} = a_{33} \sinh(r_{4nn}b) \sin(r_{5nn}b) + a_{32} \cosh(r_{4nn}b) \cos(r_{5nn}b); \]

\[ a_{43} = a_{33} \cosh(r_{4nn}b) \cos(r_{5nn}b) - a_{32} \sinh(r_{4nn}b) \sin(r_{5nn}b); \]

\[ a_{44} = a_{33} \cosh(r_{4nn}b) \sin(r_{5nn}b) + a_{32} \sinh(r_{4nn}b) \cos(r_{5nn}b); \]

\[(F.3)\]