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THE HONG KONG POLYTECHNIC UNIVERSITY

DEPARTMENT OF COMPUTING

Multicriteria Decision Analysis for Structural Decision Problems

By

Junyi Chai

A Thesis Submitted in Partial Fulfillment of

the Requirements for the Degree of

Doctor of Philosophy

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CERTIFICATE OF ORIGINALITY

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_______________________(Signed)

Junyi Chai (Name of Student)

To my family

ABSTRACT

Generally, a decision problem can be modeled as one of the three types: structural, semi-structural, and non-structural. The structural problems reveal a well organized formulation such as the high structured information tables, the measurable decision goals, and the clear boundary of problems. Both semi-structural and non-structural problems are mostly at the strategic management level. In these problems, decision information is usually non-quantitative or a lack of organization. More importantly, the human perception and judgments do play a decisive role whereas such human factor is usually intangible or unmeasurable. This thesis dedicates to the investigation of Multicriteria Decision Analysis for solving the structural decision problems.

The thesis proposes an in-depth research on Multicriteria Decision Making from three aspects: (1) *decision theories/methods* related to dominance-based rough sets; (2) *decision models* for multicriteria ranking, sorting, and choice; and (3) *decision support systems* towards group and uncertain decision-makings.

In the theory/method aspect, a new development on dominance-based rough methodology is proposed from the theoretical aspect. We developed a new Dominance-based decision Rule Induction mechanism (DRI for short). Unlike the previous rule induction methods that neglect the values of uncertain information within rough boundary regions, the proposed rule induction method can utilize both certain and uncertain decision information. This method is proved to be more sufficient and more effective than the previous methods, and applied for construction of decision models for multicriteria ranking and sorting.

In the model aspects, four new decision models are proposed towards the practical decision problems including supplier selection, warehouse evaluation, and personnel evaluation. First, we developed a novel believable rough set approach model (BRSA model for short) for multicriteria sorting problem and applied this model in *supplier selection*. Second, we developed a novel Interval Valued Intuitionistic Fuzzy Group Decision model for *warehouse evaluation* (IVIFGD model for short). This work led to a novel rule-based solution for complex warehouse evaluation problem under interval valued intuitionistic fuzzy environment. Third, we developed a novel Intuitionistic Fuzzy Superiority and Inferiority Ranking decision model (IFSIR model for short) for *supplier selection* under intuitionistic fuzzy environments. By using the proposed model, the inherent uncertainty in the form of intuitionistic information can be well propagated in the multi-step decision process. Fourth, we developed a novel dynamic tolerant skyline decision model for *personnel evaluation* (T-skyline model for short). This work additionally provided a detailed empirical study on NBA player evaluation in 2011-2012 regular seasons and consequently revealed several very interesting and valuable evaluation results with the realistic significance.

In the system aspect, two designs of decision support system (DSS) are proposed towards group decision-making and uncertain decision-making, respectively. We firstly proposed a framework of ONTOlogy-based Group Decision Support System (ONTOGDSS) for decision process which exhibits the complex structure of decision group. Secondly, we developed a framework of Uncertainty-based Group Decision Support System (UGDSS). It provides a platform for multiple processes of decision analysis in six aspects including decision environment, decision problem, decision group, decision conflict, decision schemes and group negotiation.

As a summary, this PhD research contributed a comprehensive investigation towards solving structural decision problems from multiple perspectives including the decision mathematical tool – Dominance-based rough sets (the theory aspect), the realistic problem-solving decision models (the model aspect), and the effective decision support systems (the system aspect).

Keywords: Multicriteria Decision Making, Decision Models, Dominance-based Rough Sets, Intuitionistic Fuzzy Computing, Supply Chain, Decision Support Systems.

PUBLICATIONS

The listed publications are the partial output of my PhD studies in The Hong Kong Polytechnic University.

Journal papers

1. **Junyi Chai**, James N. K. Liu. A new believable rough set approach for supplier selection, *Expert Systems with Applications*, accepted.

2. **Junyi Chai**, James N. K. Liu, Eric W. T. Ngai. Application of decision-making techniques in supplier selection: A systematic review of literature, *Expert Systems with Applications*, vol. 40, no. 10, pp. 3872-3885, 2013.

3. **Junyi Chai**, James N. K. Liu, Zeshui Xu. A rule-based group decision model for warehouse evaluation under interval valued intuitionistic fuzzy environments, *Expert Systems with Applications*, vol. 40, no. 6, pp. 1959-1970, 2013.

4. **Junyi Chai**, James N. K. Liu, Zeshui Xu. A new rule-based SIR approach to supplier selection under intuitionistic fuzzy environments, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 20, no. 3, pp. 451-471, 2012.

5. **Junyi Chai**, James N. K. Liu. Dominance-based decision rule induction for multicriteria ranking, *International Journal of Machine Learning and Cybernetics*, accepted.

6. **Junyi Chai**, James N. K. Liu. Anming Li, A novel tolerant skyline operation for decision making, *Journal of Decision Systems*, accepted.

Journal papers under review

7. **Junyi Chai**, James N. K. Liu, Man Lung Yiu. Decision-oriented skyline operation with dynamic preference, *Knowledge-Based Systems*, accepted subject to revisions.

Book chapters

1. **Junyi Chai**, James N. K. Liu. A reliable system platform for group decision support under uncertain environments, *Reliable Knowledge Discovery*, Chapter 17, Springer, pp. 291-306, 2012. (Invited Paper).

2. **Junyi Chai**, James N. K. Liu, Anming Li. A new intuitionistic fuzzy rough set approach for decision supports, *Rough Sets and Knowledge Technology (RSKT)*, LNAI 7414, Springer, pp. 71-80, 2012.

Conference papers

1. **Junyi Chai**, James N. K. Liu, Man Lung Yiu, Hongwei Wang, Anming Li. A novel dynamic skyline operation for decision support, *Hawaii International Conference on System Sciences (HICSS-46)*, Hawaii, USA, 2013.

2. **Junyi Chai**, James N. K. Liu, Dehong Gao, Jian Xu. A novel tolerant skyline operation for multicriteria decision support, *IEEE International Conference on Granular Computing (GrC)*, Hangzhou, China, 2012.

3. **Junyi Chai**, James N. K. Liu. Class-based rough approximation with dominance relations, *IEEE International Conference on Granular Computing (GrC)*, Taiwan, pp. 77-82, 2011.

4. **Junyi Chai**, James N. K. Liu. Towards a reliable framework of uncertainty-based group decision support system, *IEEE International Conference on Data Mining Workshops (ICDMW)*, Australia, pp. 851-858, 2010.

5. **Junyi Chai**, James N. K. Liu. A novel multicriteria group decision making approach with intuitionistic fuzzy SIR method, *World Automation Congress (WAC)*, Japan, pp. 1-6, 2010.

6. **Junyi Chai**, James N. K. Liu. An ontology-driven framework for supporting complex decision process, *World Automation Congress (WAC)*, Japan, pp.1-6, 2010.

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CHAPTER ONE

Introduction

1.1 MCDM: A Historical Introduction

Management is Decision-Makings.
-- Herbert A. Simon

Every science begins as philosophy and ends as art.
-- Will Durant

Multicriteria Decision Making (MCDM) can be regarded as both old and new, depending on one's perspective. We said it is the old problems since it is very natural for people to trade off multiple objectives in making decisions. The pioneer records of trade-offs in making decision by people come from the interesting discussions between two important thinkers: Ignatius of Loyola (1491-1556) and Benjamin Franklin (1706-1790). Let us firstly review a statement of St. Ignatius of Loyola from the "Spiritual Exercises" (1548).

> *To consider, reckoning up, how many advantages and utilities follow for me from holding the proposed office or benefice [...] , and, to consider likewise, on the contrary, the disadvantages and dangers which there are in having it. Doing the same in the second part, that is, looking at the advantages and utilities there are in not having it, and likewise, on the contrary, the disadvantages and dangers in not having the same. [...] After I have thus discussed and reckoned up on all sides about the thing proposed,*

to look where reason more inclines: and so, according to the greater inclination of reason, [...], deliberation should be made on the thing proposed.

We are in the letter from Benjamin Franklin to Joseph Presly, review the following discussion regarding the fountainhead of Multicriteria Decision Making.

London, Sept 19, 1772
Dear Sir,

In the affair of so much importance to you, wherein you ask my advice, I cannot, for want of sufficient premises, advise you what to determine, but if you please I will tell you how. [...], my way is to divide half a sheet of paper by a line into two columns; writing over the one Pro, and over the other Con. [...] When I have thus got them all together in one view, I endeavor to estimate their respective weights; and where I find two, one on each side, that seem equal, I strike them both out. If I find a reason pro equal to some two reasons con, I strike out the three. If I judge some two reasons con, equal to three reasons pro, I strike out the five; and thus proceeding I find at length where the balance lies; and if, after a day or two of further consideration, nothing new that is of importance occurs on either side, I come to a determination accordingly. [...] I have found great advantage from this kind of equation, and what might be called moral or prudential algebra. Wishing sincerely that you may determine for the best, I am ever, my dear friend, yours most affectionately.

B. Franklin

"What is interesting in the above two quotations is the fact that decision is strongly related to the comparison of different points of view, some in favour and some against a certain decision. This means that decision is intrinsically related to a plurality of points of view, which can roughly be defined as criteria. Contrary to this very natural observation, for many years the only way to state a decision problem was considered to be the definition of a single criterion, which amalgamates the multidimensional aspects of the decision situation into a single scale of measure (Figueira, Greco, & Ehrgott, 2005)". Therefore, decision making in consideration of multiple criteria is intuitive and also very close to the nature of human behavior. All in all, the MCDM problem may be boiled down to two segments: The evaluation problem, to which the decision maker chooses among a finite set of discrete alternatives; or The design problem, to which the set of decision alternatives is described with a mathematical model.

Since then, MCDM has gradually become the important and independent field which is very close to Management Sciences and Operation Research. In early history, many scholar had made the great achievements on this filed in a broad sense. Here, we concisely review the milestone of the early MCDM developments in Table 1-1 (Koksalan, Wallenius, & Zionts, 2011). And, it is for paying my personal respects to their great achievements.

Table 1-1 The early contributors and their works towards the modern MCDM developments

peoples	Main achievements
Marquis de Condorcet 1743-1794	French mathematician and political scientist. The pioneer in applying mathematics to the social sciences, in particular to elections. Representative Publication: "*Essay on the application of analysis to the probability of majority decisions*"
Georg Cantor 1845-1918	German mathematician born in Russia. He is known to be the creator of Set Theory.
Vilfredo Pareto 1848-1923	The economist born in Paris to Italian expatriates. Firstly introduction of the concepts of efficiency, also known as Pare-to-optimality. Representative Publication: "*Manual of political economy*"
Ragnar Frisch 1895-1973	The Nobel Prize winner in Economics His utility function elicitation technique had contributed to the MCDM field. Representative Publication: "*Numerical determination of a quadratic preference function for use in macro-economic programming*"
Paul A. Samuelson 1915-2009	The Nobel Prize winner in Economics The revealed preference theory is influential in the development of modern MCDM field. Representative Publication: "*A note on the pure theory of consumer's behavior*"
Herbert A. Simon 1916-2001	The Nobel Prize winner in Economics Aspiration Levels proposed by him pay a major role in modern MCDM techniques. Representative Publication: "*A behavioral model of rational choice*"
Gerard Debreu 1921-2004	The Nobel Prize winner in Economics Main contributor to utility and value theory in MCDM Representative Publication: "*Theory of value: An axiomatic analysis of economic equilibrium*" and "*Topological methods in cardinal utility theory*"
Kenneth Arrow 1921-	The Nobel Prize winner in Economics Arrow's Paradox: "No aggregation system can convert the ordinal preference of individual into a community-wide ranking, while also meeting certain reasonable criteria with three or more discrete options to choose from. These criteria are called unrestricted domain, non-imposition, non-dictatorship, Pare-to-efficiency, and independence of irrelevant alternatives." Representative Publication: "*Social choice and individual values*"
Lotfi A. Zadeh 1921-	The father of "Fuzzy Set Theory" and The Professor at UC Berkeley He made the original contribution to the robustness analysis by inventing fuzzy set theory which greatly influences the developments of MCDM, particularly Uncertainty MCDM subfield. Representative Publication: "*Optimality and Non-Scalar-Valued Performance Criteria*" and "*Fuzzy sets*" (Yet, the original idea still remains controversial among scholars.)

Howard Raiffa 1924-	The Bayesian decision theorist and The professor at Harvard University He worked in statistical decision theory, game theory, behavioral decision theory, risk analysis and negotiation analysis, which made the great influence on MCDM developments. Representative Publication: "*Applied Statistical Decision Theory*"; "*Decision analysis introductory lectures on choices under uncertainty*"; and "*Decisions with Multiple Objectives: Preferences and Value Tradeoffs*".
Ward Edwards 1927-2005	The father of behavior decision research. He introduced the expected utility model to psychologists and posed the interesting question: Do people actually behave as if they have a utility function? Representative Publication: "*The Theory of Decision Making*" and "*Behavioral Decision Theory*"
John F. Nash Jr. 1928-	The Nobel Prize winner in Economics Non-cooperative n-person games and to the solution of the bargaining problem Representative Publication: "*Equilibrium points in n-person games*" and "*The bargaining problem*"
Ronald A. Howard 1934-	The professor at Stanford University since 1965. The father of the morden decision analysis. He suggested using the term "Decision Analysis" for the modern MCDM research. Representative Publication: "*Sequential Decision Process*" "*Decision Analysis: Applied Decision Theory*"

The abbreviation MCDM, initially standing for Multiple Criteria Decision Making, was comprehensively accepted to the community since a paper by Stan Zionts, "MCDM – If not a Roman Numeral, then What?", which was published in 1979. After a long time of development, the current researchers more frequently make use of the term "Multicriteria" as the alternative of "Multiple Criteria". This thesis consistently employs the term "Multicriteria Decision Making" with its abbreviation "MCDM".

The current development of MCDM consists of many subfields, such as Multiobjective mathematical programming, Evolutionary multiobjective optimization, Outranking relations (also known as the French School), Fuzzy set theory, and Preference relations and modeling. In Section 2, we will make a review on the current MCDM development.

1.2 Motivation and Research Framework

This thesis made innovative works towards Multicriteria Decision Making from four aspects: 1) *Theoretical Innovation* on dominance-based rough set; 2) *Method Innovation* on multicriteria ranking, sorting, and choice; 3) *Application Developments* for the problems of supplier selection, warehouse evaluation, and personnel selection; 4) *System Innovation* for supporting the group and uncertain decision-makings. The main works can be outlined in Table 1-2.

Table 1-2 Main Contributions

Main Contributions		Abbreviations	Chapter
Theories	A new dominance-based decision rule induction	DRI	Ch.3

Models	A believable rough set approach for multicriteria sorting	BRSA Model	Ch.4
	An IVIF group decision model	IVIFGD Model	Ch.5
	An IF-SIR group decision model	IFSIR Model	Ch.6
	A dynamic tolerant skyline decision model	T-skyline Model	Ch.7
Applications	A survey of decision-making techniques on SS	SS Survey	Ch.8
Systems	A system prototype for group decision making	GDSS	Ch.9.2
	A system prototype for uncertain decision making	UDSS	Ch.9.3

1.2.1 Theory Innovations

Decision theories can be regarded as the high-modeling mathematical tools for general decision-making problems, in which some decision techniques may be involved. The current active decision theories for MCDM can be roughly summarized as (1) multiobjective/goal programming, (2) Multicriteria utility theory, (3) Outranking relations, and (4) Dominance-based rough set methodology (DRSA). The theoretical development provided by this thesis is the new strategy for dominance-based decision rule induction (Chai & Liu, 2012b; Chai, Liu, & Li, 2012). In literature, the classical strategy is induction of a minimal rule set on the basis of the lower approximations (Greco, Matarazzo, & Slowinski, 2002; 2005, Slowinski, Greco, & Matarazzo, 2009). All induced rules in the minimal rule set are certain rules. However, such mechanism neglects the values of uncertain decision information within rough boundary regions. The proposed rule induction strategy suggests induction of both certain and uncertain rules by exploring both lower approximations and rough boundary regions. This method is proved to be more sufficient and more effective for both multicriteria ranking (Chai & Liu, 2012b) and multicriteria sorting (Chai & Liu, 2013).

1.2.2 Method Innovations

Despite various decision models and problem domains, MCDM is as simple as to provide Decision Makers (DMs) with a recommendation concerning a finite set of objects (also known as actions, alternatives, candidates) evaluated from multiple viewpoints called features (also known as attributes, criteria, variables, objectives). Roy (1996) typically distinguished MCDM problems into four main issues: 1) *Criteria analysis* aiming to identify the major distinguishing features; 2) *Multicriteria choice* aiming to identify the best object or select a limited set of the best objects; 3) *Multicriteria ranking* aiming to construct an ordinal rank of the objects from the best to the worst; and 4) *Multicriteria sorting* aiming to assign objects to the predefined classes. We regard the first as the essential procedure for optimization of decision information and the latter three issues can produce specific decision outcomes. From this perspective, the innovative method in this thesis can be boiled down to the following three categories:

- **Multicriteria Sorting**

A new believable rough set approach (BRSA) is proposed for multicriteria sorting with the application of supplier selection (Chai & Liu, 2013). This approach theoretically extended the rule induction strategy presented in Chapter 3 for further handling the more complex situation of sorting: the multi-grade multicriteria sorting.

- **Multicriteria Ranking**

A new interval-valued intuitionistic fuzzy group decision (IVIFGD) model is proposed for multicriteria ranking (Chai, Liu, & Xu, 2013). This work developed a novel rule-based solution for the problem of complex warehouse evaluation. The proposed approach partly employed the rationale presented in Chapter 3 and theoretically extended it for uncertain decision makings.

A new intuitionistic fuzzy superiority and inferiority ranking (IF-SIR) decision model is proposed for ranking suppliers (Chai, Liu, & Xu, 2012). The new proposed approach extended this traditional MCDM technique for applications in uncertain environments. The proposed approach can successfully propagate the complex systematic uncertainty in whole problem-solving process.

- **Multicriteria Choice**

A dynamic tolerant skyline decision model is proposed for multicriteria choice (Chai, Liu, & Yiu, 2013). This work pioneers to application of skyline operations for the problem of personnel evaluation. Overcoming the existing weaknesses of other skyline operations that hinder the realistic decision-makings, the proposed operation possesses the merits including more controllability of hieratical skyline outputs and the effectiveness of modeling dynamic preference of DMs.

1.2.3 System Innovations

Decision Support Systems (DSS) have been proposed since the late 1960s to help decision maker improve the efficiency and correctness in decision making. Currently, two kinds of situations significantly increase the complexity of DSS: (1) multiple participants involved in decision process and (2) decision-making under uncertainty environment. This thesis provides two kinds of system prototypes for supporting group and uncertain decision-makings. They are a framework of ONTOlogy-based Group Decision Support System (ONTOGDSS) (Chai & Liu, 2010) and a framework of Uncertainty-based Group Decision Support System (UGDSS) (Chai & Liu, 2012a). These designs are the attempt for implementation of our proposed decision theories and methods in Chapter 3~7 with supports by information systems.

1.2.4 Application Developments

Decision methods (also known as decision approaches) are used for solving realistic decision problems. In order to provide a complete problem-solving approach, it generally includes problem modeling, method modeling, and mathematical justification. The developed four independent decision approaches are established for solving three key decision-making problems. They are supplier selection (Chai, Liu,

& Xu, 2012; Chai & Liu, 2013), warehouse evaluation (Chai & Liu, 2012; Chai, Liu, & Xu, 2013), and personnel evaluation (Chai, Liu, & Yiu, 2013).

Additionally, a systematic academic survey on the application of decision-making techniques on supplier selection is provided (Chai, Liu, & Ngai, 2013). In this work, we comprehensively reviewed the related literature between the periods of 2008-2012 covering 15 international journals. By using a methodological decision analysis in 4 aspects including decision problems, decision makers, decision environments, and decision approaches. This survey provides the recommendation for future research on supplier selection and facilitates knowledge accumulation and creation concerning the application of DM techniques.

1.3 Organization of the Thesis

The organization of this thesis is shown in Figure 1-1. One theoretical development on Dominance-based rough set approaches is presented in Chapter 3. Four completed decision approaches towards different problems are provided from Chapter 4 to Chapter 7. From the application perspectives, Chapter 3 and Chapter 5 present the warehouse evaluation; Chapter 4 and Chapter 6 discuss the supplier selection; Chapter 7 introduces the T-skyline model that can address typical personnel evaluation problems. Subsequently, Chapter 8 provides a detailed survey on the application of decision-making techniques based on the literature between 2008 and 2012. From the system perspective, we provide two prototyping designs of decision support systems towards uncertain decision-making and group decision-making. We conclude this thesis and outline future works in Chapter 10.

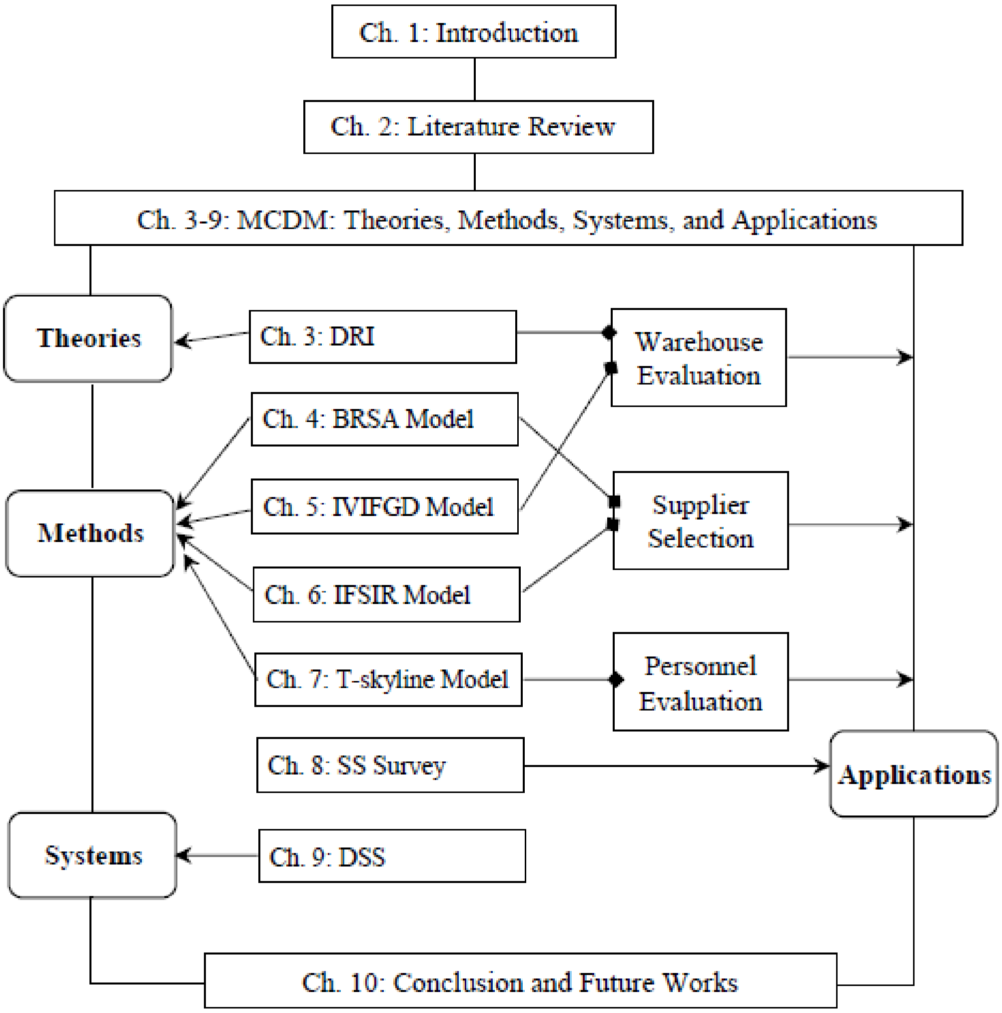


Fig 1-1 Organization of the Thesis

CHAPTER TWO

Literature Review

2.1 The MCDM Framework

MCDM can be understood as a very broad field as we introduced thereinbefore. In its narrow sense, however, MCDM can be simplified as to provide Decision Makers (DMs) with a recommendation concerning a finite set of objects (also known as actions, alternatives, candidates) evaluated from multiple viewpoints called features (also known as attributes, criteria, variables, objectives). In such narrow sense, we provide a concise literature review of the MCDM techniques from two perspectives: Classical MCDM techniques and Uncertainty MCDM techniques.

In the middle of the last century, Koopmans (1951) introduced the Efficient Point in decision area. At the same time, Kuhn and Tucker (1951) introduced the concept of Vector Optimization. Charnes and Cooper (1961) studied the model and application of Linear Programming in decision science. In 1972, the International Conference on MCDM held by Cochrane and Zeleny (1973) remarked that the normative MCDM theory had been developed as the mainstream of decision science. More recently, many applicable MCDM approaches have been used to design Decision Support System for solving specific domain problems. The MCDM with certain information or under certain decision environment is called Classical MCDM. Major methods of Classical MCDM can be roughly divided into three categories: Multicriteria Utility Theory, Outranking Relations, and Preference Disaggregation.

(1) Multicriteria Utility Theory (MAUT):

Fishburn (1974) and Huber (1974) provided very specific literature survey on Multicriteria Utility Theory. Besides, Keeney and Raiffa (1976; 1993) published a monograph which deeply influences the future development. A methodological review on MAUT has been done by Dyer (2005). The recent research achievements had been done by Abbas and his colleagues. The related litertuare includes Abbas (2004a; 2004b; 2006; 2009; 2011a; 2011b; 2013). The more studies also appeared in literature (Abbas & Howard, 2005; Abbas & Bell, 2011; 2012; Abbas & Aczel, 2010).

(2) Outranking Relations:

The outranking relations approach aims to compare every couple of alternatives and then gets overall priority ranks, which mainly includes the ELECTRE method and the PROMETHEE method. ELECTRE was firstly proposed by Roy (1968) in 1960s. Then, Roy (1977), Hugonnard and Roy (1982) extended its theory and applications. PROMETHEE method was initially established by Brans, Vincke, and Marreschal (1986). Xu (2001) extended PROMETHEE with a Superiority and Inferiority Ranking (SIR) method which integrated with the outranking approach.

(3) Preference Disaggregation:

Jacquet-Lagreze and Siskos (1982) provided a UTA method to maximize the approximation of the preference of DMs by defining a set of additive utility functions. Zopounidis and Doumpos (1999; 2000) developed the UTADIS method as a variant of UTA for sorting problems, and extended the framework of UTADIS for involving multi-participants cases called the MHDIS method.

Although Classical MCDM has already got a relatively complete theory in the past 50 years, it still cannot solve most MCDM problems in real world. One main reason is that the decision information is not usually provided completely, clearly or precisely in reality. In most cases, people have to make decisions in uncertainty environment. Therefore, many researchers pay more attention on this new research branch-Uncertainty MCDM.

Uncertainty MCDM is non-classical, and can be treated as the extension and development of Classical MCDM. We can roughly classify the uncertainty problems into three categories: (1) Stochastic type (2) Fuzzy type (3) Rough type. Accordingly, Uncertainty MCDM also has three perspectives: Stochastic MCDM, Fuzzy MCDM and Rough MCDM.

(1) Stochastic MCDM

Bayes theory is proposed for stochastic process which can improve the objectivity and veracity in stochastic decision making. Then, Bernoulli (1954) introduced the concept of Utility and Expected Utility Hypothesis Model. von Neumann and Morgenstern (1944) concluded the Expected Utility Value Theory, proposed the axiomatic of Expected Utility Model, and mathematically proved the results of maximized Expected Utility for DMs. Wald (1950) established the basis of statistical decision problem, and applied them in the selection of stochastical decision schemes. Blackwell and Girshich (1954) integrated the subjective probability with the utility theory into a clear process to solve decision problems. Savage (1954) extended the Expected Utility Model, and Howard (1966) introduced the systematical analysis approach into decision theory and developed them from theory and application aspects.

Recently, many approaches have emerged for solving Stochastic MCDM. They can be generally divided into three directions:

- Utility Theory based Approaches include Prospect theory, Cumulative Prospect theory (Baucells & Heukamp, 2006).
- Probability Aggregation based Approaches include Bayes method, Delphi method, Kaplan method (Clemem and Winkler, 1999), and so on.
- Stochastic Simulation based Approaches includes Scenario Simulation, Monte Calo method (Huaser and Tadikamalla, 1996), and so on.

(2) Fuzzy MCDM

In 1965, Zadeh (1965) proposed the Fuzzy Sets which adopted the membership functions to represent the degree of membership from elements to sets. Moreover, in 1978, Zadeh (1978) proposed a theory of possibility to represent the difference of essence in stochastic problems and fuzzy problems. Atanassov (1986) extended Zadeh's Fuzzy Sets concept into the Intuitionistic Fuzzy Sets (IFSs), and then as in the following, they extended IFSs into the Interval-Valued Intuitionistic Fuzzy Sets (IVIFSs) (Atanassov & Gargov, 1989), which are described by a membership degree and a non-membership degree whose values are intervals rather than real numbers. Based on these pioneering works, theories of IFSs and IVIFSs have received much attention from researchers. Until recently, some basic theorems such as Calculation Operators and Fuzzy Measures have just been founded for various applications (Atanassov, 1994; Xu, 2007).

(3) Rough MCDM

Pawlak (1982; 1991) systematically introduced the Rough sets theory. Then, Slowinski (1992) concluded the past achievements of Rough sets in theory and applications. Since 1992, the annual International Conference on Rough Sets has been playing a very important role in promoting the development of Rough sets in theory extension and various applications. More recently, Greco, Matarazzo and Slowinski (2001) proposed a Dominance based Rough Sets theory which produces the decision rules with stronger applicability. By now, Rough Sets theory has been applied in decision analysis, process control, knowledge discovery, machine learning, pattern recognition, and so on.

2.2 Review of Active MCDM Techniques

The classical-uncertainty partition on MCDM is actually from the viewpoint of the time of MCDM developments as we presented in section 2.1. From the perspective of MCDM problems themselves, we can consider two categorites (1) multicriteria discrete alternative problems and (2) multicriteria optimization problems.

The example of multicriteria discrete alternantive problems likes facility location problem, such as the arrangement of routers and servers in a communication network, locations of warehouses or distribution centers in a supply chain, locations of hospitials or airports in a public service system, or locations of the military bases in the country or around the world. We call it as “discrete” due to a modestly sized collection of alternatives for selection. We can typically divide this problems into two parts: (1) *Multicriteria sorting* (also known as ordinal classification): The decision target is to assign each object to one of the predefined classes, while decision values of criteria and the predefined classes are preference ordered. According to the number of predefined classes, it can be divided into two-grade sorting and multi-grade sorting. The former is regarded as the special case of the latter, and also much simpler. (2) *Multicriteria ranking*: The decision target is to order objects from the best to the worst or to select the best object. With the assistance of pairwise comparison of objects, this problem usually can be regarded as a two-grade multicriteria sorting problem.

On the other hand, the example of multicriteria optimization problems include planning such as river basin planning or energy planning, and also engineering scheduling, portfolio selection, R&D project selection, and so on. The feasible set of alternatives for such problem is usually defined by a group of equations and inequalities which identify the feasible region for the decision variables (Wallenius, Dyer, Fishburn, Steuer, Zionts, & Deb, 2008). Different from the discrete alternative problem, it may involve a very large number or even infinite number of alternatives. Therefore, such optimization problem is likely to require relatively more computational resources than discrete problems. The second difference between these two problems is that discrete problems are more likely to be constructed with uncertain variables than optimization problem. The third difference is that the consideration of utility and value functions. Wallenius et al. (2008) commented this viewpoint as “many approaches to multicrtieira discrete problems attempt to represent aspect of a decision matker’s utility or value function mathematically and then apply these results to estimate the alternatives’ (expected) utilities. In multiple criteria optimization, there is ususllay no attempt to caputure the decision maker’s utility or value function mathematically. Instead, the philosophy is to iteratively elicit and use implicit information about the decision-maker’s preferences to help steer the decision maker to her or his most preferred solustion.”

From the viewpoint of different types of MCDM problems, we can hightlight several active MCDM techniques (or called approaches) for each type. The topmost technique for solving discrete problems must be Multiattribute Utility Theory (MAUT) described by Keeney and Raiffa (1976; 1993). Also, Analytic Hierarchy Process (AHP) has been comprehensively studied and followed since its first introduction by Saaty (1999). In addition, two approaches as Elimination and Choice Expressing Reality (ELECTRE) and Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) (Figueira, Greco, & Ehrgott, 2005) can also be listed as the topmost techniques for discrete problems, both of which are taken a partial ordering of alternatives into account. MAUT and AHP are regarded as the *American school* whereas ELECTRE and PROMETHEE are regared as the *French School*, even though such partition is not encouraged (Olson 1996). For optimization problems, we list the following methodological categorties: linear/non-linear/goal/etc programmings, vector-maximum algorithems, evolutionary and interactive computations, etc. Other specific MCDM techniques have been well discussed in the literature, for example Edwards, Miles, and von Winterfeldt (2007); Ehrgott (2005); Figueira, Greco, and Ehrgott (2005); Ignizio and Romero (2003); Zopounidis and Doumpos (2002); Luenberger and Ye (2010), and Tervonen and Figueira (2008). We emphazise that the work of this thesis is majorly for solving the multicriteria discrete alternative problems, though we had made some discussions on the other type in Chapter 7.

In the past decades, the most important influence to the heart of MCDM researches is due to two aspects: (1) the development of computing power assisted by computers and (2) the development of uncertain theories/mathematical tools like fuzzy sets. In the first aspect, the topmost influence on MCDM is developments of machine leanring and knowledge discovery techniques. As a result, the concept of dominance-based rough set methodology was proposed by Greco, Matarazzo, and Slowinski (2001; 2005). This technique can be used for preference modeling which learning preference patterns in the form of “if-then” rules from a sample of past decisions, and making use these rules for decision-making types like ranking and sorting. Such Preference Learning (PL) methodology underlines and strengthens the links among Computer Sciences, Artificial Intelligence, and the MCDM field, which is as the one of most active directions to multicriteria discrete problems in current MCDM studies. This thesis is working on this direction via providng the theoretical development (see Chapter 3) and the feasible decision models (see Chapter 4, Chapter 5, and Chapter 7). For multicriteria optimization problem, the topmost direction can be the studies on Evolutionary Multiobjective Optimization (EMO). Despite prior advances, multicriteria optimization techniques failed to solve many highly nonlinear multicriteria problems, which trigger the development of EMO since its predecessor Evolutionary Algorithms (EA, Stantnikov & Matusov, 2002). More related discussions can be found in Coello and Lamont (2004) and Deb (2001).

Uncertain theories/mathematical tools are always important since being the bridge between the classical world and the real world. MCDM under uncertain environments has been comprehensively studied along with developments of uncertain theories and corresponding tools. The topmost influence to this aspect would be developments of the family of fuzzy set theories. Since its classic works (Zadeh, 1965; 1978), we have witnessed many of the related generalizations and extensions including Type-2/Type-n fuzzy sets (Dubois and Prade, 1980); Nonstationary fuzzy sets (Garibaldi, et al. 2008); Fuzzy multisets (Yager, 1986); and Intuitionistic fuzzy sets (Atanassov, 1986). Particularly, Intuitionstic fuzzy sets are characterized by the membership function and the non-membership function which can be used for preference modeling more precisely and suitably. This technique has received a great deal of attention by MCDM researchers in recent years. This thesis takes uncertain decision-making environments into account for construction of our MCDM models; particularly intuitionstic fuzzy environments (see Chapter 5 and Chapter 6).

2.3 Review of Dominance-based Rough Set Methodology

Greco, Matarazzo, and Slowinski (2001; 2005; 2009) well developed a rule-based methodology called Dominance-based Rough Set Approach (DRSA). It extends binary-relation-based indiscernibility of Classical Rough Set Approach (CRSA) to dominance relations, in order to assign objects to a set of predefined and preference-ordered decision classes. Various extended DRSA models also appeared including VC-DRSA (Greco, Matarazzo, Slowinski, & Stefanowski, 2001), VP-DRSA (Inuiguchi, Yoshioka, & Kusunoki, 2009), Stochastic DRSA (Kotlowski, Dembczynski, Greco, & Slowinski, 2008) and so on. DRSA is a promising decision methodology for MCDM which has been developed and applied in this thesis. In this section, we will review the basic principles of DRSA as our preliminaries.

Generally, an information table can be transformed into a *decision table* via distinguishing condition criteria and decision criteria. Formally, a decision table is a 4-tuple $S=\langle U,Q,V,f\rangle$, which includes (1) a finite set of objects denoted by U, $x\in U=\{x_1,...,x_m\}$; (2) a finite set of criteria denoted by $Q=C\bigcup D$, where condition criteria set $C\neq\varnothing$, decision criteria set $D\neq\varnothing$ (usually the singleton set $D=\{d\}$), and $q\in Q=\{q_1,...,q_n\}$; (3) the scale of criterion q denoted by V_q, where $V=\bigcup_{q\in Q}V_q$; (4) information function denoted by $f_q(x):U\times Q\to V$, where $f_q(x)\in V_q$ for each $q\in Q$, $x\in U$. In addition, each object x from U is described by a vector called *decision description* in terms of the decision information on the criteria, denoted by $Des_Q(x)=\left[f_{q_1}(x),...,f_{q_n}(x)\right]$. As such, information function $f_q(x)$ also can be called *decision values* in MCDM.

The objective sets of dominance-based rough approximation are upward or downward unions of predefined decision classes. Suppose the decision criterion d partitions U into a finite number of classes $CL=\{Cl_t,\ t=1,...,l\}$. We assume that Cl_{t+1} is more preferred to Cl_t. Each object x from U belongs to *one and only one* class Cl_t. The upward and downward unions of classes are represented respectively as:

$$Cl_t^{\geq}=\bigcup_{s\geq t}Cl_s\ ,\quad Cl_t^{\leq}=\bigcup_{s\leq t}Cl_s\ ,\ \text{where}\quad s,t=1,...,l\ .$$

Then, the following operational laws are valid:

$Cl_1^{\leq}=Cl_1$; $Cl_l^{\geq}=Cl_l$; $Cl_t^{\geq}=U-Cl_{t-1}^{\leq}$; $Cl_t^{\leq}=U-Cl_{t+1}^{\geq}$; $Cl_1^{\geq}=Cl_l^{\leq}=CL$; $Cl_0^{\leq}=Cl_{l+1}^{\geq}=\varnothing$.

The knowledge granules in DRSA theory are *dominance cones* regarding the value space of considered criteria. If two decision values are with dominance relation like $f_q(x)\geq f_q(y)$ for every criterion $q\in P\subseteq C$ in consideration, we say x *dominates* y, denoted by xD_Py. The dominance relation is reflexive and transitive. With this in mind, the *dominance cone* can be represented as: P-dominating set $D_P^+(x)=\{y\in U: yD_Px\}$; P-dominated set $D_P^-(x)=\{y\in U: xD_Py\}$.

The key concept in DRSA theory is the *dominance principle*: if decision value of object x is no worse than that of object y on all considered condition criteria (saying x is dominating y on $P\subseteq C$), object x should also be assigned to a decision class no worse than that of object y (saying x is dominating y on D). Objects satisfying the dominance principle are called *consistent*, and objects violating the dominance principle are called *inconsistent*. A decision table involving *inconsistent objects* is called *inconsistency table*. Founded on such dominance principle, the definitions of rough approximations are given below.

P-lower approximations of class unions $Cl_t^{\geq}$ and $Cl_t^{\leq}$, denoted by $\underline{P}(Cl_t^{\geq})$ and $\underline{P}(Cl_t^{\leq})$ respectively, are represented as: $\underline{P}(Cl_t^{\geq}) = \{x \in U : D_P^+(x) \subseteq Cl_t^{\geq}\}$; $\underline{P}(Cl_t^{\leq}) = \{x \in U : D_P^-(x) \subseteq Cl_t^{\leq}\}$.

P-upper approximations of class unions $Cl_t^{\geq}$ and $Cl_t^{\leq}$, denoted by $\overline{P}(Cl_t^{\geq})$ and $\overline{P}(Cl_t^{\leq})$ respectively, are represented as: $\overline{P}(Cl_t^{\geq}) = \{x \in U : D_P^-(x) \bigcap Cl_t^{\geq} \neq \varnothing\}$; $\overline{P}(Cl_t^{\leq}) = \{x \in U : D_P^+(x) \bigcap Cl_t^{\leq} \neq \varnothing\}$.

Rough boundary regions of class unions $Cl_t^{\geq}$ and $Cl_t^{\leq}$, denoted by $Bn_P(Cl_t^{\geq})$ and $Bn_P(Cl_t^{\leq})$ respectively, are represented as: $Bn_P(Cl_t^{\geq}) = \overline{P}(Cl_t^{\geq}) - \underline{P}(Cl_t^{\geq})$; $Bn_P(Cl_t^{\leq}) = \overline{P}(Cl_t^{\leq}) - \underline{P}(Cl_t^{\leq})$.

Obviously, we have the properties: $Bn_P(Cl_t^{\geq}) = Bn_P(Cl_{t-1}^{\leq}) = \overline{P}(Cl_t^{\geq}) \bigcap \overline{P}(Cl_{t-1}^{\leq})$
In addition, the following properties are also valid: $\underline{P}(Cl_t^{\geq}) \subseteq Cl_t^{\geq} \subseteq \overline{P}(Cl_t^{\geq})$; $\underline{P}(Cl_t^{\leq}) \subseteq Cl_t^{\leq} \subseteq \overline{P}(Cl_t^{\leq})$; $\underline{P}(Cl_t^{\geq}) = U - \overline{P}(Cl_{t-1}^{\leq})$; $\underline{P}(Cl_t^{\leq}) = U - \overline{P}(Cl_{t+1}^{\geq})$; $\overline{P}(Cl_t^{\geq}) = U - \underline{P}(Cl_{t-1}^{\leq})$; $\overline{P}(Cl_t^{\leq}) = U - \underline{P}(Cl_{t+1}^{\geq})$.

If $Q \subseteq P \subseteq C$ is satisfied, we have the following properties: $\underline{Q}(Cl_t^{\geq}) \subseteq \underline{P}(Cl_t^{\geq})$; $\overline{Q}(Cl_t^{\geq}) \supseteq \overline{P}(Cl_t^{\geq})$; $\underline{Q}(Cl_t^{\leq}) \subseteq \underline{P}(Cl_t^{\leq})$; $\overline{Q}(Cl_t^{\leq}) \supseteq \overline{P}(Cl_t^{\leq})$.

The definitions of classical DRSA are based on the strict dominance principle (as shown in above). Inspired by the Variable Precision Rough Set (Ziarko, 1993), which is the extension of Pawlak's Rough Set via relaxation of strict indiscernibility relation, Greco, Matarazzo, Slowinski, & Stefanowski (2001) provided VC-DRSA model. This approach accepts a limited number of inconsistent objects which are controlled by the predefined threshold called *consistency level*. For $P \subseteq C$, the P-lower approximations of VC-DRSA can be represented as:

$$\underline{P}^l(Cl_t^{\geq}) = \{x \in Cl_t^{\geq} : \frac{|D_P^+(x) \bigcap Cl_t^{\geq}|}{|D_P^+(x)|} \geq l\} \; ; \; \underline{P}^l(Cl_t^{\leq}) = \{x \in Cl_t^{\leq} : \frac{|D_P^-(x) \bigcap Cl_t^{\leq}|}{|D_P^-(x)|} \geq l\} \; .$$

where consistency level l means that object x from U belongs to the class union $Cl_t^{\geq}$ (or $Cl_t^{\leq}$) with no ambiguity at level $l \in (0,\ 1]$.

Then, we can obtain the P-upper approximations and the rough boundary regions as:

$$\overline{P}^l(Cl_t^{\geq}) = U - \underline{P}^l(Cl_{t-1}^{\leq}) \; ; \; Bn_P(Cl_t^{\geq}) = \overline{P}^l(Cl_t^{\geq}) - \underline{P}^l(Cl_t^{\geq}) \; ; \; \overline{P}^l(Cl_t^{\leq}) = U - \underline{P}^l(Cl_{t+1}^{\geq}) \; ; \; Bn_P(Cl_t^{\leq}) = \overline{P}^l(Cl_t^{\leq}) - \underline{P}^l(Cl_t^{\leq}) \; .$$

The VP-DRSA model is provided by Inuiguchi, Yoshioka, and Kusunoki (2009), which have the following definition. For any $P \subseteq C$, we say that $x \in U$ belongs to $Cl_t^{\geq}$ at precision level $l_2 \in (0,1]$, and

$x \in U$ belongs to $Cl_t^{\leq}$ at precision level $l_1 \in (0,1]$. The concept of lower approximations at some precision levels l_1 and l_2 are formally presented as:

$$\underline{P}^{l_2}(Cl_t^{\geq}) = \{x \in U : \frac{|D_P^-(x) \cap Cl_t^{\geq}|}{|D_P^-(x) \cap Cl_t^{\geq}| + |D_P^+(x) \cap Cl_{t-1}^{\leq}|} \geq l_2\}, \quad t = 1,...,l;$$

$$\underline{P}^{l_1}(Cl_t^{\leq}) = \{x \in U : \frac{|D_P^+(x) \cap Cl_t^{\leq}|}{|D_P^+(x) \cap Cl_t^{\leq}| + |D_P^-(x) \cap Cl_{t+1}^{\geq}|} \geq l_1\}, \quad t = 1,...,l.$$

Particularly, when $D_P^+(x) \subseteq Cl_t^{\geq}$, we have $D_P^+(x) \cap Cl_{t-1}^{\geq} = \varnothing$, and $l_2 = 1$. Accordingly, $\underline{P}^{l_2}(Cl_t^{\geq})$ becomes DRSA lower approximation $\underline{P}(Cl_t^{\geq})$. The same situation happens in $\underline{P}^{l_1}(Cl_t^{\geq})$.

Criteria reduction aims to find several subsets (called *reducts*) of original condition criteria set as alternatives, on condition that the quality of approximation of sorting (*sorting quality* for short) is not deteriorating. The intersection of all generated reducts is called the *core*. A classical measure of sorting quality is defined by Gediga and Düntsch (2002), as the ratio of the number of consistent objects from C-lower approximations and the number of all objects in the universe, denoted by $\gamma_C(CL)$:

$$\gamma_C(CL) = \frac{|U - (\bigcup_{t=2,...,l}) Bn_C(Cl_t^{\geq})|}{|U|} = \frac{|U - (\bigcup_{t=1,...,l-1}) Bn_C(Cl_t^{\leq})|}{|U|}.$$

It suggests that the reducts should be calculated by the norm of the measure $\gamma_C(CL)$. Although such measure is clear and workable in a two-grade sorting, it rather seems to be too rigorous for multi-grade sorting. Dembczyński, Greco, and Słowiński (2009) provided another two measures for calculation of reducts despite the existing drawbacks. From different viewpoints, the union-based reducts provided by Yang, Yang, Wu and Yu (2008) preserves the lower and upper approximations of downward and upward unions respectively. Such reduct offers multiple choices for DMs via tradeoffs between the reduct size and the sorting quality.

2.4 Review of Intuitionistic Fuzzy Decision Methodology

Intuitionistic Fuzzy Set (IFS), which extends the single parameter of Zadeh' fuzzy set, is characterized by three parameters: the membership function, the non-membership function and the hesitancy function. Further theoretical works were provided by Chen and Tan (Chen & Tan, 1994) who defined the score function and Hong and Choi (Hong & Choi, 2000) who defined the accuracy function. More recently, Xu (Xu & Yager, 2006; Xu, 2007d; 2007e) developed several operators (e.g. IFWA, IFWG, IFHA, IFHG) for aggregating intuitionistic fuzzy information. Compared with the Zadeh' fuzzy set, IFS can describe uncertain information (i.e. fuzzy values, symbolic values, etc) more comprehensively and in detail. This feature lets IFS be a suitable mathematical tool to preference modeling for multicriteria discrete alternative problems. In this section, we will review the basic principles of IFS as our preliminaries.

Let $X(X \neq \varnothing)$ be a finite set. IFS is defined as $A = \{< x, \mu_A(x), v_A(x) > | x \in X\}$, which contains two elements: the membership function μ_A and the non-membership function v_A with the condition $0 \le \mu_A + v_A \le 1$ for all $x \in X$. Szmidt and Kacprzyk (2000) called $\pi_A : \pi_A = 1 - \mu_A - v_A$ as the intuitionistic index of x in A, which is also the hesitancy function of x in A (Atanassov, 1986). By considering all three parameters, four kinds of distances are introduced for measuring the distance between two IFSs. Suppose A and B are two IFSs in $X = \{x_1, ..., x_n\}$, these distances can be defined as follows:

Hamming distance: $d(A,B) = \frac{1}{2}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|)$

Euclidean distance: $e(A,B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}(\mu_A(x_i) - \mu_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2}$

Normalized Hamming distance: $l(A,B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|)$

Normalized Euclidean distance: $q(A,B) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}(\mu_A(x_i) - \mu_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2}$

An intuitionistic fuzzy value (IFV) is denoted as $a = (\mu_a, v_a, \pi_a)$, where $\mu_a \in [0,1]$, $v_a \in [0,1]$, $\pi_a \in [0,1]$, and $\mu_a + v_a + \pi_a = 1$. Clearly, the maximum IFV is $a^+ = (1,0,0)$ and the minimum IFV is $a^- = (0,1,0)$. Additionally, the score function is denoted by $S(a) = \mu_a - v_a$ and the accuracy function is denoted by $H(a) = \mu_a + v_a$. In order to compare any two IFVs $a_1 = (\mu_{a_1}, v_{a_1}, \pi_{a_1})$ and $a_2 = (\mu_{a_2}, v_{a_2}, \pi_{a_2})$, a comparison law is given as follows:

(1) If $S(a_1) > S(a_2)$, then, $a_1 > a_2$;

(2) If $S(a_1) = S(a_2)$, then, a) If $H(a_1) > H(a_2)$, then $a_1 > a_2$;

b) If $H(a_1) = H(a_2)$, then $a_1 = a_2$.

Two operators IFWA and IFWG are defined for aggregating intuitionistic fuzzy information shown as follows (Xu & Cai, 2010). The aggregated value by using IFWA or IFWG is also the intuitionistic fuzzy value.

Definition Let $a_i = (\mu_{a_i}, v_{a_i})$ $(i = 1, ..., n)$ be a set of IFVs, and $IFWA: \Theta^n \to \Theta$ is defined as:

$$IFWA_\omega(a_1, a_2, ..., a_n) = \overset{n}{\underset{i=1}{\oplus}} (\omega_i a_i) = (1 - \prod_{i=1}^{n}(1 - \mu_{a_i})^{\omega_i}, \prod_{i=1}^{n}(v_{a_i})^{\omega_i})$$

where $\omega = (\omega_1, \omega_2, ... \omega_n)^T$ is the weight vector of a_i $(i = 1, ..., n)$ with $\omega_i \in [0,1]$ and $\sum_{i=1}^{n}\omega_i = 1$.

Definition Let $a_i = (\mu_{a_i}, v_{a_i})$ $(i = 1, ..., n)$ be a set of IFVs, and $IFWG: \Theta^n \to \Theta$ is defined as:

$$IFWG_\omega(a_1, a_2, ..., a_n) = \overset{n}{\underset{i=1}{\oplus}} (a_i^{\omega_i}) = (\prod_{i=1}^{n}(\mu_{a_i})^{\omega_i},\ 1 - \prod_{i=1}^{n}(1 - v_{a_i})^{\omega_i})$$

where $\omega = (\omega_1, \omega_2, ... \omega_n)^T$ is the weight vector of a_i $(i = 1, ..., n)$ with $\omega_i \in [0,1]$ and $\sum_{i=1}^{n}\omega_i = 1$.

Atanassov and Gargov (1989) introduce interval-valued intuitionistic fuzzy set (IVIFS), which consists of a membership function and a non-membership function, whose values are intervals rather than exact numbers. Compared with fuzzy set and IFS, IVIFS is more variable for depicting preference relations and consequently is a more suitable mathematical tool for expressing subjective preferences. In the following, we review the basic notions of IVIFS as one of the theortical preliminaries of this thesis (Atanassov & Gargov, 1989; Xu & Yager, 2009). Regarding a set X, an IVIFS $\tilde{A}$ over X is an object having the form:

$$\tilde{A} = \{\langle x, \tilde{\mu}_{\tilde{A}}(x), \tilde{v}_{\tilde{A}}(x)\rangle \mid x \in X\},$$

$$\text{s.t.} \quad \tilde{\mu}_{\tilde{A}}(x) = [\tilde{\mu}_{\tilde{A}}^L(x), \tilde{\mu}_{\tilde{A}}^U(x)] \subset [0,1], \quad \tilde{v}_{\tilde{A}}(x) = [\tilde{v}_{\tilde{A}}^L(x), \tilde{v}_{\tilde{A}}^U(x)] \subset [0,1], \quad \tilde{\mu}_{\tilde{A}}^U(x) + \tilde{v}_{\tilde{A}}^U(x) \le 1,$$

where $\tilde{\mu}_{\tilde{A}}^L(x) = \inf \tilde{\mu}_{\tilde{A}}(x)$, $\tilde{\mu}_{\tilde{A}}^U(x) = \sup \tilde{\mu}_{\tilde{A}}(x)$, $\tilde{v}_{\tilde{A}}^L(x) = \inf \tilde{v}_{\tilde{A}}(x)$, and $\tilde{v}_{\tilde{A}}^U(x) = \sup \tilde{v}_{\tilde{A}}(x)$.

Particularly, an IVIFS $\tilde{A}$ is reduced to an IFS if $\tilde{\mu}_{\tilde{A}}(x) = \tilde{\mu}_{\tilde{A}}^L(x) = \tilde{\mu}_{\tilde{A}}^U(x)$ and $\tilde{v}_{\tilde{A}}(x) = \tilde{v}_{\tilde{A}}^L(x) = \tilde{v}_{\tilde{A}}^U(x)$ are valid. The complement of $\tilde{A}$ is denoted as $\tilde{A}^c$, where $\tilde{A}^c = \{\langle x, \tilde{v}_{\tilde{A}}(x), \tilde{\mu}_{\tilde{A}}(x)\rangle \mid x \in X\}$.

Extracted the fundamental element from IVIFS, the interval-valued intuitionistic fuzzy value (IVIFV) (Xu, 2010) can be denoted as: $\tilde{a} = (\tilde{\mu}_{\tilde{a}}, \tilde{v}_{\tilde{a}})$, where $\tilde{\mu}_{\tilde{a}} = [\tilde{\mu}_{\tilde{a}}^L, \tilde{\mu}_{\tilde{a}}^U] \subset [0,1]$, $\tilde{v}_{\tilde{a}} = [\tilde{v}_{\tilde{a}}^L, \tilde{v}_{\tilde{a}}^U] \subset [0,1]$, $\tilde{\mu}_{\tilde{a}}^U + \tilde{v}_{\tilde{a}}^U \le 1$. Regarding two IVIFVs $\tilde{a}_i = ([\tilde{\mu}_{\tilde{a}_i}^L, \tilde{\mu}_{\tilde{a}_i}^U], [\tilde{v}_{\tilde{a}_i}^L, \tilde{v}_{\tilde{a}_i}^U])$ for $i = 1,2$, we have $\tilde{a}_1 = \tilde{a}_2$ if and only if $\tilde{\mu}_{\tilde{a}_1}^L = \tilde{\mu}_{\tilde{a}_2}^L$, $\tilde{\mu}_{\tilde{a}_1}^U = \tilde{\mu}_{\tilde{a}_2}^U$, $\tilde{v}_{\tilde{a}_1}^L = \tilde{v}_{\tilde{a}_2}^L$, $\tilde{v}_{\tilde{a}_1}^U = \tilde{v}_{\tilde{a}_2}^U$. The uniform distance between $\tilde{a}_1$ and $\tilde{a}_2$ can be calculated via:

$$d(\tilde{a}_1, \tilde{a}_2) = \left[\frac{1}{4}\left(|\tilde{\mu}_{\tilde{a}_1}^L - \tilde{\mu}_{\tilde{a}_2}^L|^\lambda + |\tilde{\mu}_{\tilde{a}_1}^U - \tilde{\mu}_{\tilde{a}_2}^U|^\lambda + |\tilde{v}_{\tilde{a}_1}^L - \tilde{v}_{\tilde{a}_2}^L|^\lambda + |\tilde{v}_{\tilde{a}_1}^U - \tilde{v}_{\tilde{a}_2}^U|^\lambda\right)\right]^{1/\lambda}, \quad \lambda \ge 1.$$

Specifically, for $\lambda = 1$, the uniform distance is reduced to the normalized Hamming distance:

$$d_H(\tilde{a}_1, \tilde{a}_2) = \frac{1}{4}\left(|\tilde{\mu}_{\tilde{a}_1}^L - \tilde{\mu}_{\tilde{a}_2}^L| + |\tilde{\mu}_{\tilde{a}_1}^U - \tilde{\mu}_{\tilde{a}_2}^U| + |\tilde{v}_{\tilde{a}_1}^L - \tilde{v}_{\tilde{a}_2}^L| + |\tilde{v}_{\tilde{a}_1}^U - \tilde{v}_{\tilde{a}_2}^U|\right).$$

And, for $\lambda = 2$, the uniform distance is reduced to the normalized Euclidean distance:

$$d_E(\tilde{a}_1, \tilde{a}_2) = \sqrt{\frac{1}{4}\left(|\tilde{\mu}_{\tilde{a}_1}^L - \tilde{\mu}_{\tilde{a}_2}^L|^2 + |\tilde{\mu}_{\tilde{a}_1}^U - \tilde{\mu}_{\tilde{a}_2}^U|^2 + |\tilde{v}_{\tilde{a}_1}^L - \tilde{v}_{\tilde{a}_2}^L|^2 + |\tilde{v}_{\tilde{a}_1}^U - \tilde{v}_{\tilde{a}_2}^U|^2\right)}.$$

According to the above provided distances, the degree of similarity between $\tilde{a}_1$ and $\tilde{a}_2$ can be defined as follows:

$$s(\tilde{a}_1, \tilde{a}_2) = d(\tilde{a}_1, \tilde{a}_2^c) \Big/ \left(d(\tilde{a}_1, \tilde{a}_2) + d(\tilde{a}_1, \tilde{a}_2^c)\right),$$

where $\tilde{a}_2^c$ be the complement of $\tilde{a}_2$. According to this definition, we can easily prove that the following assertions are valid.

(i) $0 \le s(\tilde{a}_1, \tilde{a}_2) \le 1$;

(ii) $s(\tilde{a}_1, \tilde{a}_2) = s(\tilde{a}_2, \tilde{a}_1) = s(\tilde{a}_2^c, \tilde{a}_1^c) = s(\tilde{a}_1^c, \tilde{a}_2^c)$; (iii) $s(\tilde{a}_1, \tilde{a}_2^c) = s(\tilde{a}_1^c, \tilde{a}_2)$

Especially, $s(\tilde{a}_1, \tilde{a}_2) > 0.5$ means that $\tilde{a}_1$ is more similar to $\tilde{a}_2$ rather than $\tilde{a}_2^c$, and also $s(\tilde{a}_1, \tilde{a}_2) < 0.5$ means that $\tilde{a}_1$ is more similar to $\tilde{a}_2^c$ rather than $\tilde{a}_2$.

2.5 Summary

In this chapter, we review the related works in the field of MCDM. First, we describe the MCDM framework including the three categories of MCDM techniques and the three types in uncertain MCDM studies. Second, we provide the literature review on currently active MCDM techniques from two perspectives: multicriteria discrete alternative problems and multicriteria optimization problems. Two representative influences are emphasized in current MCDM researches. Finally, Dominance-based rough set methodology and Intuitionistic fuzzy decision methodology have been reviewed in this chapter, which are as the important theoretical background of this thesis.

CHAPTER THREE

Dominance-based Decision Rule Induction

3.1 Overview

This chapter considers the issue of multicriteria ranking by decision rules induction. MCDM aims at giving people the knowledge of recommendation concerning a finite set of objects evaluated with multiple preference-ordered attributes (known as criteria). Dominance-based Rough Set Approach (DRSA) is a powerful tool for MCDM via assigning objects to several predefined and preference-ordered decision classes. Most of previous strategies are to induce a minimal set of "*if…then…*"rules. In this chapter, we provide strategies to induce a new rule set as the substitution for the classical minimal rule set. The main contributions include: (1) providing methods to induce certain rules in two situations respectively: multi-criteria and mix-attributes; (2) providing the concept of believe factor and its three measuring degrees for exploring valuable uncertain information within rough boundary regions; (3) providing the properties of believe factor. A numerical example is used for illustration of overall problem-solving procedures and for a comparison with the existing representative proposals.

3.2 Background

Despite various decision models and problem domains, MCDM is as simple as to provide DMs with a recommendation concerning a finite set of objects (also known as actions, alternatives, candidates) evaluated from multiple viewpoints called features (also known as attributes, criteria, variables, objectives). From the methodology of MCDM, we can roughly divide the decision problems into two classes as follows. (1) *Multicriteria sorting* (also known as ordinal classification): The decision target is to assign each object to one of the predefined classes, while decision values of criteria and the predefined classes are preference ordered. According to the number of predefined classes, it can be divided into two-grade sorting and multi-grade sorting. The former is regarded as the special case of the latter, and also much simpler. (2) *Multicriteria ranking*: The decision target is to order objects from the best to the worst or to select the best object. With the assistance of pairwise comparison of objects, this problem usually can be regarded as a two-grade multicriteria sorting problem. In this chapter, we mainly address the issue of multicriteria ranking by the means of dominance-based decision rules.

Figueira, Greco, and Ehrgott (2005) provided a useful and comprehensive collection of surveys on MCDM. Zopounidis and Doumpos (2002) gave a literature review of MCDM in financial decision making. Xu, Martel, and Lamond (2001) proposed a multicriteria ranking procedure taking both complete and partial orders into account. Apart from conventional MCDM methods, the unconventional MCDM has attracted more attention. Tervonen and Figueira (2008) provided a unified framework of uncertain MCDM for future researches. Actually, the uncertainties of MCDM can be divided into three types including stochastic, fuzzy and rough (Chai & Liu, 2012a). (i) For stochastic MCDM, Mareschal (1986) gave the stochastic extensions to outranking relations. Baucells and Heukamp (2006) introduced the stochastic simulation approach based on the stochastic dominance concept. (ii) For fuzzy MCDM, Pedrycz, Ekel, and Parreiras (2011) concluded new models, methods and applications in this area. Hu, Guo, Yu, and Liu (2010) generalized Shannon's entropy to handle both crisp and fuzzy ordinal classification. Chai, Liu, and Xu (2012) developed the classic SIR method (Xu X.Z., 2001) and firstly provided an Intuitionistic Fuzzy SIR (IF-SIR) method for group MCDM. (iii) Classical Rough Set Approach (CRSA) proposed by Pawlak (Pawlak & Skowron, 2007) is a fundamentally mathematical tool for uncertain decision-making. Some extensions include Variable Precision Rough Set (Ziarko, 1993), Intuitionistic Fuzzy Rough Set (Chai, Liu, & Li, 2012), Multigranulation Rough Set (Yang, Song, Chen, & Yang, 2011), Matroidal Rough Set (Ziarko, 1993) and so on. For rough MCDM, previous works (Greco, Matarazzo, & Slowinski, 2001; 2005; 2009) well developed a rule-based approach founded on the methodology of Dominance-based Rough Set Approach (DRSA). It extends binary-relation-based indiscernibility of CRSA to dominance relations, in order to assign objects to a set of predefined and preference-ordered decision classes. Various extended DRSA models also appeared including VC-DRSA (Greco, Matarazzo, Slowinski, & Stefanowski, 2001), VP-DRSA (Hu & Yu, 2004; Inuiguchi, Yoshioka, & Kusunoki, 2009), Stochastic DRSA (Kotlowski, Dembczynski, Greco, & Slowinski, 2008) and so on. In addition, the hybrid techniques were introduced with good performance in MCDM. Wang, Zhai, and Lu (2008) provided a hybrid fuzzy-rough technique to explore valuable decision information of multicriteria reducts. Hu, Yu, and Guo (2010) provided a hybrid method to extract fuzzy preference relations. Li, Shiu, and Pal (2006) attempted to combine rough set and case-based reasoning for better classification.

In our rule induction strategy, certain rules and uncertain rules are induced separated procedures. On one hand, we define superiority/inferiority matrices and functions to induce certain rules from lower rough approximations. We investigate such procedure involving two types of feature sets: homogeneous features (called multicriteria) and heterogeneous features (called mix-attributes). On the other hand, we induce the believable rule, which is a new type of uncertain rules, through the introduction of the new concept called Believe Factor. Then, we introduce three measurements of believe factor and investigate their properties from the viewpoint of class-based rough model (Chai & Liu, 2011). Finally, an illustrative example is used to indicate that our approach has a stronger capability when compared with existing representative solutions.

This chapter is organized as follows. The basic theory of DRSA is presented in Section 2.3. The next section presents the methods for induction of certain rules. In Section 3.4, we induce believable rules assisted by the concept of believe factor. In Section 3.5, a comparable example is used for illustration of the overall problem-solving procedures. Section 3.6 provides a discussion and concludes this chapter.

3.3 Certain Rule Induction

3.3.1 Motivation

For a given decision table, our proposal of forming the final rule set is to generate two types of rules: *certain rules* and *believable rules*. In this section, we induce certain rules under two circumstances: multi-criteria and mix-attributes, respectively. In literature, Slowinski, Greco, and Matarazzo (2009) provided a method to induce certain rules. It is based on constraint relations as "$\geq$" or "$\leq$" which are with the meaning of "at least as good as" or "at most as good as". This method can be used to induce certain rules and approximate rules, and hence make up a minimal rule set in one go. In contrast, An and Tong (2009) gave an idea of using the strict constraint relations as "$>$" or "$<$" to compare criterion values of two parts of objects: the objects from upward class union $Cl_t^{\geq}$ and the objects from downward class union $Cl_{t-1}^{\leq}$. Nevertheless, a deficiency of this method is that the induced certain rules actually utilize a part of uncertain information. In this chapter, we define a series of superiority/inferiority matrices and functions to extract different kinds of information from the lower approximation and its corresponding upper approximation (e.g. $\underline{P}(Cl_t^{\geq})$ and $\overline{P}(Cl_{t-1}^{\leq})$; or $\underline{P}(Cl_{t-1}^{\leq})$ and $\overline{P}(Cl_t^{\geq})$).

3.3.2 Certain rule induction in multicriteria

Let us consider a decision table $S=(U,C\bigcup\{d\})$, where a finite set of objects $x\in U$, a finite set of condition criteria $q\in C$, and the predefined classes $CL=\{Cl_t\ t=1,\dots,l\}$ known as partitions of entire U. For condition criteria set $P \subseteq C$, we can obtain the P-lower approximations $\underline{P}(Cl_t^{\geq})$ and $\underline{P}(Cl_{t-1}^{\leq})$ preserving the pairs of class unions $Cl_t^{\geq}$ and $Cl_{t-1}^{\leq}$ for $t = 2,...,l$, respectively. Then, we define discernibility matrices that include the superiority matrix over $Cl_t^{\geq}$ and the inferiority matrix over $Cl_{t-1}^{\leq}$.

Definition 1. (Superiority matrix)

For object x assigned to $\underline{P}(Cl_t^{\geq})$, we have the superiority matrix as:

$$Sup^{\succ}(Cl_t^{\geq}) = [m_t(x,y)]_{|\underline{P}(Cl_t^{\geq})| \times |\overline{P}(Cl_{t-1}^{\leq})|}, \tag{3-1}$$

subject to:

$$m_t(x,y) = \{q \in P : f_q(x) > f_q(y),\ x \in \underline{P}(Cl_t^{\geq}), y \in \overline{P}(Cl_{t-1}^{\leq})\}.$$

Definition 2. (Inferiority matrix)

For object x assigned to $\underline{P}(Cl_{t-1}^{\leq})$, we have the inferiority matrix as:

$$Inf^{\prec}(Cl_{t-1}^{\leq}) = [n_{t-1}(x,y)]_{|\underline{P}(Cl_{t-1}^{\leq})| \times |\overline{P}(Cl_t^{\geq})|}, \tag{3-2}$$

subject to

$$n_{t-1}(x,y) = \{q \in P : f_q(x) < f_q(y),\ x \in \underline{P}(Cl_{t-1}^{\leq}), y \in \overline{P}(Cl_t^{\geq})\}.$$

Then, we further define discernibility functions according to the matrices acquired via Eq. (3-1) and Eq. (3-2).

Definition 3. (Superiority function)

For object x assigned to $\underline{P}(Cl_t^{\geq})$, we have the superiority function as:

$$f_P^{\geq t}(x) = \wedge(\vee a :\ a \in m_t(x,y) \neq 0, y \in \overline{P}(Cl_{t-1}^{\leq})). \tag{3-3}$$

Definition 4. (Inferiority function)

For object x assigned to $\underline{P}(Cl_{t-1}^{\leq})$, we have the inferiority function as

$$f_P^{\leq t-1}(x) = \wedge(\vee a :\ a \in n_{t-1}(x,y) \neq 0, y \in \overline{P}(Cl_t^{\geq})). \tag{3-4}$$

According to the discernibility functions Eq. (3-3) and Eq. (3-4), we extract certain rules by using the following strategies.

Strategy 3-1 (Upward certain rule)

Considering superiority function $f_C^{\geq t}(x_i) = \wedge(\vee a) = a_1 \wedge ... \wedge a_n$ preserving object $x_i \in \underline{C}(Cl_t^{\geq})$, the decision description of object x_i can be represented as: $Des_a(x_i) = [r_{a_1}^{\geq}, r_{a_2}^{\geq}, ..., r_{a_n}^{\geq}]$. Then, we can induce an upward certain rule preserving the object x:

$$If\ \ f_{a_1}(x) \geq r_{a_1}^{\geq}\ and\ f_{a_2}(x) \geq r_{a_2}^{\geq}\ and ... and\ f_{a_n}(x) \geq r_{a_n}^{\geq},\ then\ \ x \in Cl_t^{\geq}.$$

Strategy 3-2 (Downward certain rule)

Considering inferiority function $f_C^{\leq t-1}(x_i) = \wedge(\vee a) = a_1 \wedge ... \wedge a_n$ preserving object $x_i \in \underline{C}(Cl_{t-1}^{\leq})$, the decision description of object x_i can be represented as: $Des_a(x_i) = [r_{a_1}^{\leq}, r_{a_2}^{\leq}, ..., r_{a_n}^{\leq}]$. Then, we can induce a downward certain rule preserving the object x:

$$If\ \ f_{a_1}(x) \leq r_{a_1}^{\leq}\ and\ f_{a_2}(x) \leq r_{a_2}^{\leq}\ and ... and\ f_{a_n}(x) \leq r_{a_n}^{\leq},\ then\ \ x \in Cl_{t-1}^{\leq}.$$

3.3.3 Certain rule induction in mix-attributes

Heterogeneous attribute set (known as mix-attributes) involves three kinds of relations: indiscernibility I_q, similarity R_q and dominance S_q ("in-sim-dom" relations for short). Hereinto, relations I_q and R_q are defined with qualitative attributes and quantitative attributes respectively. Relations S_q are defined in criteria. In this section, we extend superiority/inferiority matrices and functions considering such mix-attributes.

Suppose information table $S=(U,Q,V,f)$, with condition criteria set C and decision criteria set D, $C \cup D = Q$. Decision criterion $D=\{d\}$ partitions U into a finite number of decision classes $CL=\{Cl_t\ t=1,...,l\}$. Each object $x \in U$ belongs to one and only one class. Three kinds of attributes are considered: criteria $C^{>}$, qualitative attributes $C^{=}$ and quantitative attributes $C^{\square}$, where $C^{>} \cup C^{=} \cup C^{\square} = C$ without any intersection. For any $P \subseteq C$, we have: 1) dominance relations on criteria, denoted as $P^{>} = P \cap C^{>}$, 2) indiscernibility relations on qualitative attributes, denoted as $P^{=} = P \cap C^{=}$ and 3) similarity relations on quantitative attributes, denoted as $P^{\square} = P \cap C^{\square}$.

Definition 5. (Superiority Matrix)

For lower approximation $\underline{P}(Cl_t^{\geq})$, $a_i \in P^{>}$ and $b_j \in P^{=}$ and $c_k \in P^{\square}$ and $P \subseteq C$, we have the superiority matrix with respect to P as:

$$Sup^{\succ}_{\underline{P}(Cl_t^{\geq})}(P) = (m_P(x_\alpha, y_\beta))_{|\underline{P}(Cl_t^{\geq})| \times |\overline{P}(Cl_{t-1}^{\leq})|}\ , \tag{3-5}$$

subject to

$$m_P(x_\alpha, y_\beta) = \left\{ \begin{array}{l} a_i, b_j, c_k \in P: \\ v(x_\alpha, a_i) Sup^{\succ} v(y_\beta, a_i), \\ v(x_\alpha, b_j) \neq v(y_\beta, b_j), \\ v(x_\alpha, c_k) \notin [v(y_\beta, c_k) \times (1 \pm \xi)], \\ x_\alpha \in \underline{P}(Cl_t^{\geq}),\ y_\beta \in \overline{P}(Cl_{t-1}^{\leq}),\ \xi \in (0,1) \end{array} \right\}$$

where ξ is a coefficient used to identify the similarity degree of quantitative attributes.

Definition 6. (Superiority Function)

For object x where $x \in \underline{P}(Cl_t^{\geq})$, we define the superiority function as:

$$f_P^{\geq}(x_\alpha) = \wedge(a_i^* \vee b_j^* \vee c_k^* : a_i, b_j, c_k \in m_P(x_\alpha, y_\beta) \neq 0, y_\beta \in \overline{P}(Cl_{t-1}^{\leq}))\ , \tag{3-6}$$

where a_i^*, b_j^*, c_k^* are corresponding to the criteria a_i, b_j, c_k respectively.

Definition 7. (Inferiority Matrix)

For lower approximation $\underline{P}(Cl_{t-1}^{\leq})$, $a_i \in P^{>}$ and $b_j \in P^{=}$ and $c_k \in P^{\square}$ and $P \subseteq C$, we have the inferiority matrix with respect to P as:

$$Inf^{\prec}_{\underline{P}(Cl_{t-1}^{\leq})}(P) = (n_P(x_\alpha, y_\beta))_{|\underline{P}(Cl_{t-1}^{\leq})| \times |\overline{P}(Cl_t^{\geq})|}\ , \tag{3-7}$$

subject to

$$n_P(x_\alpha, y_\beta) = \left\{ \begin{array}{l} a_i, b_j, c_k \in P: \\ v(x_\alpha, a_i) Inf^{\prec} v(y_\beta, a_i), \\ v(x_\alpha, b_j) \neq v(y_\beta, b_j), \\ v(x_\alpha, c_k) \notin [v(y_\beta, c_k) \times (1 \pm \xi)], \\ x_\alpha \in \underline{P}(Cl_{t-1}^{\leq}),\ y_\beta \in \overline{P}(Cl_t^{\geq}),\ \xi \in (0,1) \end{array} \right\}$$

where ξ is a coefficient used to identify the similarity degree of quantitative attributes.

Definition 8. (Inferiority Function)

For object x where $x \in \underline{P}(Cl_{t-1}^{\le})$, we define the inferiority function as:

$$f_P^{\le}(x_\alpha) = \wedge(a_i^* \vee b_j^* \vee c_k^* : a_i, b_j, c_k \in n_P(x_\alpha, y_\beta) \neq 0, y_\beta \in \overline{P}(Cl_t^{\ge})) , \quad (3\text{-}8)$$

where a_i^* , b_j^* , c_k^* are corresponding to the criteria a_i, b_j, c_k respectively.

According to acquired discernibility functions, we can generate certain rules via Strategy 3-1 and Strategy 3-2. We use an example from the literature (Greco, Matarazzo, & Slowinski, 2005) to illustrate the certain rule induction on condition of mix-attributes.

Example. Table 3-1 shows exemplary decisions of a DM concerning eight warehouses described by three condition attributes: “a” means The capacity of the sales staff; “b” means The region of the location; “c” means The inventory level, where C={a, b, c}. The decision attribute D={d} specifies the assignment into two sets of warehouses (i.e. either profit or loss in the revenue).

Table 3-1 Decision table

Warehouse	a	b	c	d
w_1	Medium	A	500	Loss
w_2	Good	A	400	Profit
w_3	Medium	A	450	Profit
w_4	Good	B	400	Loss
w_5	Good	B	475	Profit
w_6	Medium	B	425	Profit
w_7	Medium	B	350	Profit
w_8	Medium	B	350	Loss

According to the decision criterion {d}, suppose we denote “Loss” and “Profit” as class Cl_1 and class Cl_2, where $Cl_1^{\le} = Cl_1$ and $Cl_2^{\ge} = Cl_2$. We note that ‘a’ is a criterion; ‘b’ is qualitative attribute; ‘c’ is quantitative attributes. We define the coefficient ξ in similarity relations on ‘c’ as: $\xi = 10\%$.

By using DRSA, the C-lower approximations and the rough boundaries of set $Cl_1^{\le}$ and $Cl_2^{\ge}$ are: $\underline{C}(Cl_1^{\le}) = \{w_1\}$; $\underline{C}(Cl_2^{\ge}) = \{w_2, w_5\}$; $Bn_C(Cl_1^{\le}) = Bn_C(Cl_2^{\ge}) = \{w_3, w_4, w_6, w_7, w_8\}$, respectively. The superiority matrix is shown in Table 3-2. The superiority functions will be $f^{\ge}(w_2) = a \wedge b$ and $f^{\ge}(w_5) = a \wedge c$. The inferiority matrix is shown in Table 3-3. The inferiority function will be $f^{\le}(w_1) = \varnothing$.

Table 3-2 Superiority matrix

	w_1	w_3	w_4	w_6	w_7	w_8	Implication
w_2	a	a	b	ab	abc	abc	$a \wedge b$
w_5	ab	ab	c	ac	a	a	$a \wedge c$

Table 3-3 Inferiority matrix

	w_3	w_4	w_6	w_7	w_8	w_2	w_5	Implication
w_1	∅	ab	bc	∅	∅	a	ab	∅

From Table 3-2 and Table 3-3, we can induce the upward certain rules DR I and DR II. No downward certain rule can be induced in this example.

DR I: If a $\geq$ good and b $=$ A, then $w_i \in Cl_2^{\geq}$. (Support by w_2).

DR II: If a $\geq$ good and c $\approx$ 475, then $w_i \in Cl_2^{\geq}$. (Support by w_5).

3.4 Believable Rule Induction

3.4.1 Motivation

Previous works on rule-based methods have been well studied in the data engineering and database contexts. Some representative works include the fuzzy rule-refinement scheme (Wang & Dong, 2009), the measure-based association rules (Ma, Wang, & Liu, 2011), the learning automata miner (Zahiri, 2011), the fuzzy decision tree induction (Wang, Dong, & Yan, 2012) and so on. In the MCDM context, DRSA methodology provides a mathematical foundation for rule-based methods. With substitution of indiscernibility relations by dominance relations, the DRSA-rules can flexibly model DMs' semantic preference, which is significantly superior to other traditional preference models like utility functions or outranking relations. A discussion on DRSA-based decision rule approach to MCDM was given in Greco, Matarazzo, and Slowinski (2005). Most of existing proposals aim to induce a minimal set of dominance-based decision rules that are regarded as complete and non-redundant. Three types of decision rules can be considered: (a) certain rules induced from lower approximations; (b) approximate rules induced from rough boundary regions; (c) possible rules induced from upper approximations. The types (b) and (c) are regarded as uncertain rules. The minimal rule set contains certain rules and approximate rules. Let us have a discussion on these existing rules. Suppose rough approximations are with respect to the upward union $Cl_t^{\geq}$ and the downward union $Cl_s^{\leq}$. Certain rule provides an assignment described as: "at least class Cl_t" or "at most class Cl_s". The approximate rule provides an assignment described as: "some classes between Cl_s and Cl_t". And the possible rule provides an assignment like "an object possibly belongs to 'at least class Cl_t' or 'at most class Cl_s'. Obviously, to some extent uncertain rules (b) and (c) fail to provide valuable decision recommendations, because their assignments are either too wide or too ambiguous. For instance, when a decision table is established as a Pairwise Comparison Table (PCT), the Net Flow Score (NFS) (Greco, Matarazzo, & Slowinski, 1999) method can only employ induced certain rules in order to calculate the total score. All of the uncertain rules are useless for solving such ranking problem. In essence, existing uncertain rules do not effectively utilize uncertain information within rough boundary regions. Based upon the above observations, this chapter attempts to induce a new decision rule set that can effectively and sufficiently exploit both certain and uncertain information for multicriteria ranking.

3.4.2 Believe factor

For describing decision rules, several basic coefficients have been provided by Pawlak (2002), including *support, strength, confidence, coverage.* This section first revisits these principles and then gives a new concept called *believe factor*.

(1) Basic coefficients

Let $\Phi \to \Psi$ represent the decision rule "*if* Φ *then* Ψ", where condition part $\Phi \in C$ and decision part $\Psi \in D$ in decision table $S = (U, C \cup D)$. The principles of existing coefficients are summarized below.

- **Support:** An object gives *support* to a decision rule, as long as this object matches both condition part and decision part of this rule, denoted by: $\text{Support}(\Phi \wedge \Psi) = \text{Card}(\|\Phi \wedge \Psi\|_S)$.
- **Strength:** For one rule, *strength* is defined as the ratio of the number of objects supporting both condition and decision parts and the number of universal objects, denoted by: $\sigma_s(\Phi, \Psi) = \text{Support}(\Phi, \Psi) / \text{Card}(U)$.
- **Confidence:** For one rule, *confidence* is defined as the ratio of the number of objects supporting both condition and decision parts and the number of objects supporting only condition part, denoted by: $\text{Cer}(\Phi \to \Psi) = \text{Support}(\Phi \wedge \Psi)/\text{Support}(\Phi)$, where $\text{Support}(\Phi) \neq \varnothing$. It is associated with a conditional probability as $\Pr(\Psi | \Phi)$.
- **Coverage:** An object is *covered* by a rule, as long as this object matches the condition part of this rule, denoted by: $\text{Cov}(\Phi \to \Psi) = \text{Support}(\Phi \wedge \Psi)/\ \text{Support}(\Psi)$, where $\text{Support}(\Psi) \neq \varnothing$. It is associated with a conditional probability as $\Pr(\Phi | \Psi)$.

The rule would be better if it owns higher *confidence* and higher *coverage*. Apart from these basic coefficients above, we present a new concept *force*.

Definition 9. (Force of assignment of the rule, *Force* for short)

Let $\Phi \to \Psi$ represent the decision rule "*if* Φ *then* Ψ". For object x, we have multi-grade rough approximations as: $Cl_s^{\leq}, Cl_t^{\leq}, Cl_s^{\geq}, Cl_t^{\geq}$, where $s < t$.

- If two decision rules are represented as Rule I: $\Phi \to x \in Cl_s^{\leq}$ and Rule II: $\Phi \to x \in Cl_t^{\leq}$, we say Rule I has more *force*.
- If two decision rules are represented as Rule I: $\Phi \to x \in Cl_s^{\geq}$ and Rule II: $\Phi \to x \in Cl_t^{\geq}$, we say Rule II has more *force*.

(2) Believe factor and its measuring degrees

Considering the assignment of object $x \in U$, dominance cones $D_P^+(x)$ and $D_P^-(x)$ can be divided into three subsets, denoted by X_1, X_2 and X_3: (a) for $D_P^+(x)$, we have $X_1 \subseteq \underline{P}(Cl_t^{\geq})$, $X_2 \subseteq Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$, $X_3 \subseteq Cl_{t-1}^{\leq}$; (b) for $D_P^-(x)$, we have $X_1 \subseteq \underline{P}(Cl_t^{\leq})$, $X_2 \subseteq Cl_t^{\leq} - \underline{P}(Cl_t^{\leq})$, $X_3 \subseteq Cl_{t+1}^{\geq}$. With respect to the objects belonging to the predefined class unions $Cl_t^{\geq}$ and $Cl_t^{\leq}$ but failing to be assigned to the corresponding lower approximations, the following assertions are valid:

- For $t = 2,...,l$, we have $Bn_P(Cl_t^{\geq}) = Bn_P(Cl_{t-1}^{\leq}) = (Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})) \cup (Cl_{t-1}^{\leq} - \underline{P}(Cl_{t-1}^{\leq}))$.

- For $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$, $t = 2,...,l$, we have $D_P^+(x) = X_1 \cup X_2 \cup X_3$ subject to $X_1 \subseteq \underline{P}(Cl_t^{\geq})$, $X_2 \subseteq Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$, $X_3 \subseteq Cl_{t-1}^{\leq}$.
- For $x \in Cl_t^{\leq} - \underline{P}(Cl_t^{\leq})$, $t = 1,...,l-1$,we have $D_P^-(x) = X_1 \cup X_2 \cup X_3$ subject to $X_1 \subseteq \underline{P}(Cl_t^{\leq})$, $X_2 \subseteq Cl_t^{\leq} - \underline{P}(Cl_t^{\leq})$, $X_3 \subseteq Cl_{t+1}^{\geq}$.

Lemma 1.

For $x \in Bn_P(Cl_t^{\geq})$ (or $x \in Bn_P(Cl_t^{\leq})$), the following assertions are valid:

(a) $|X_1| \geq 0$; (b) $|X_2| \geq 1$; (c) $|X_3| \geq 1$

where the number of objects in one set is denoted by $|\bullet|$.

Proof: We take $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$ as example. For (a), it is given by nature. For (b), assuming $|X_2| = 0$, we get $D_P^+(x) \cap (Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})) = \varnothing$. Since we have hold $x \in D_P^+(x)$, we then infer $x \notin Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$. It is against our premises: $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$. Therefore, the assumption $|X_2| = 0$ does not hold. Finally, we obtain $|X_2| \geq 1$. For (c), assuming $|X_3| = 0$, we get $D_P^+(x) \cap Cl_{t-1}^{\leq} = \varnothing$. Since we have hold $U - Cl_{t-1}^{\leq} = Cl_t^{\geq}$, we then get $D_P^+(x) \subseteq Cl_t^{\geq}$. According to the definition of $\underline{P}(Cl_t^{\geq})$, we then hold $x \in \underline{P}(Cl_t^{\geq})$. It is against our premises : $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$. Therefore, the assumption $|X_3| = 0$ does not hold. Finally, we hold $|X_3| \geq 1$. For $x \in Cl_t^{\leq} - \underline{P}(Cl_t^{\leq})$, the proof is processed in a similar manner. □

Based on these observations, we propose a new coefficient *believe factor* to explore uncertain information of these boundary objects. The definition is given as follows:

Definition 10. (Upward believe factor)

For $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq}),\ t = 2,...,l$, we have the believe factor of upward union of decision classes (upward believe factor, for short):

$$\beta(x \to Cl_t^{\geq}) = (\mu_t^{\geq}(x),\ v_t^{\geq}(x),\ \pi_t^{\geq}(x)) , \tag{3-8}$$

subject to:

$$\begin{cases} \mu_t^{\geq}(x) = \dfrac{|D_P^+(x) \cap \underline{P}(Cl_t^{\geq})|}{|D_P^+(x)|} \\ v_t^{\geq}(x) = \dfrac{|D_P^+(x) \cap Cl_{t-1}^{\leq}|}{|D_P^+(x)|} \\ \pi_t^{\geq}(x) = \dfrac{|D_P^+(x) \cap [Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})]|}{|D_P^+(x)|} \end{cases}$$

Definition 11. (Downward believe factor)

For $x \in Cl_t^{\leq} - \underline{P}(Cl_t^{\leq}),\ t = 1,...,l-1$, we have the believe factor of downward union of decision classes (downward believe factor, for short):

$$\beta(x \to Cl_t^{\leq}) = (\mu_t^{\leq}(x),\ v_t^{\leq}(x),\ \pi_t^{\leq}(x)) , \tag{3-9}$$

subject to:

$$\begin{cases} \mu_t^{\leq}(x) = \dfrac{|D_P^-(x) \cap \underline{P}(Cl_t^{\leq})|}{|D_P^-(x)|} \\ v_t^{\leq}(x) = \dfrac{|D_P^-(x) \cap Cl_{t+1}^{\geq}|}{|D_P^-(x)|} \\ \pi_t^{\leq}(x) = \dfrac{|D_P^-(x) \cap [Cl_t^{\leq} - \underline{P}(Cl_t^{\leq})]|}{|D_P^-(x)|} \end{cases}$$

Remark that the symbol "$\rightarrow$" of believe factor $\beta(x \rightarrow Cl_t^{\geq})$ and $\beta(x \rightarrow Cl_t^{\leq})$ can be understood as "be assigned to" or "belongs to". For object $x \in U$, $\mu(x)$ (including $\mu_t^{\geq}(x)$ and $\mu_t^{\leq}(x)$) is called the *positive score*; $v(x)$ (including $v_t^{\geq}(x)$ and $v_t^{\leq}(x)$) is called the *negative score*; $\pi(x)$ (including $\pi_t^{\geq}(x)$ and $\pi_t^{\leq}(x)$) is called the *hesitancy score*. The form of believe factor can be regarded as an intuitionistic fuzzy value (Xu Z.S., 2007).

Definition 12. (Confidence degree)

For object $x \in U$, *confidence degree* of believe factor, denoted by $L(x)$, can be defined as:

$$L(x) = \mu(x) + \pi(x), \tag{3-10}$$

where $\mu(x)$ is positive score and $\pi(x)$ is hesitancy score. Specifically, we hold:

Downward confidence degree: $L(x \rightarrow Cl_t^{\leq}) = \mu_t^{\leq}(x) + \pi_t^{\leq}(x)$;

Upward confidence degree: $L(x \rightarrow Cl_t^{\geq}) = \mu_t^{\geq}(x) + \pi_t^{\geq}(x)$.

By using the confidence degree $L(x)$, the consistency level l in VC-DRSA model can be defined as:

$$L(x \rightarrow Cl_t^{\geq}) = \frac{|D_P^+(x) \cap Cl_t^{\geq}|}{|D_P^+(x)|} \geq l \ ; \ L(x \rightarrow Cl_t^{\leq}) = \frac{|D_P^-(x) \cap Cl_t^{\leq}|}{|D_P^-(x)|} \geq l,$$

where consistency level $l \in (0,1]$ controls the confidence degree of respecting objects qualified as belonging to class unions $Cl_s^{\leq}$ or $Cl_t^{\geq}$.

Definition 13. (Believe degree)

For object $x \in U$, *believe degree* of believe factor, denoted by $S(x)$, can be defined as:

$$S(x) = \mu(x) - v(x), \tag{3-11}$$

where $\mu(x)$ is positive score and $\pi(x)$ is hesitancy score. Specifically, we hold:

Downward believe degree: $S(x \rightarrow Cl_t^{\leq}) = \mu_t^{\leq}(x) - v_t^{\leq}(x)$;

Upward believe degree: $S(x \rightarrow Cl_t^{\geq}) = \mu_t^{\geq}(x) - v_t^{\geq}(x)$.

Definition 14. (Accuracy degree)

For object $x \in U$, *accuracy degree* of believe factor, denoted by $H(x)$, can be defined as:

$$H(x) = \mu(x) + v(x), \tag{3-12}$$

where $\mu(x)$ is positive score and $\pi(x)$ is hesitancy score. Specifically, we hold:

Downward accuracy degree $H(x \rightarrow Cl_t^{\leq}) = \mu_t^{\leq}(x) + v_t^{\leq}(x)$;

Upward accuracy degree: $H(x \rightarrow Cl_t^{\geq}) = \mu_t^{\geq}(x) + v_t^{\geq}(x)$.

(3) Believe factor in Class-based Rough Approximation (CRA) Model

The believe factor is defined through upper and lower approximations that are preserving the class unions (i.e. $Cl_t^{\geq}$ and $Cl_t^{\geq}$) thereinbefore. In this section, we further investigate and study the properties of believe factor via CRA model (Chai & Liu, 2011)

From the viewpoint of class-based rough model, this section investigates the properties of believe factor.

Lemma 2.

For object $x \in Cl_t,\ t = 1,\ldots,l$, the following assertions are valid:

$$\mu_t^{\geq}(x) + v_t^{\geq}(x) + \pi_t^{\geq}(x) = 1\,; \quad \mu_t^{\leq}(x) + v_t^{\leq}(x) + \pi_t^{\leq}(x) = 1\,.$$

Proof: It can be easily proved according to Eq. (4.1) and Eq. (4.2). □

Lemma 3.

For $x \in \underline{P}(Cl_t^{\geq})$, $\beta(x \to Cl_t^{\geq}) = (\mu_t^{\geq}(x),\ v_t^{\geq}(x),\ \pi_t^{\geq}(x)) = (1,0,0)$ is valid.

For $x \in \underline{P}(Cl_t^{\leq})$, $\beta(x \to Cl_t^{\leq}) = (\mu_t^{\leq}(x),\ v_t^{\leq}(x),\ \pi_t^{\leq}(x)) = (1,0,0)$ is valid.

Proof: It can be easily proved according to Eq. (4.1) and Eq. (4.2). □

Table 3-4 Value domain of believe factors in class-based rough model

Class-based	$\beta(x \to Cl_t^{\leq})$			$\beta(x \to Cl_t^{\geq})$			Assignments
approximations	$\mu^{\leq}(x)$	$v^{\leq}(x)$	$\pi^{\leq}(x)$	$\mu^{\geq}(x)$	$v^{\geq}(x)$	$\pi^{\geq}(x)$	
For $x \in P_\beta(Cl_t)$	1	0	0	[0,1)	(0,1)	(0,1)	$Cl_t^{\leq}$
For $x \in \underline{P}(Cl_t)$	1	0	0	1	0	0	$Cl_t^{\geq}$ and $Cl_t^{\leq}$
For $x \in P^\beta(Cl_t)$	[0,1)	(0,1)	(0,1)	1	0	0	$Cl_t^{\geq}$
For $x \in Cl_t$	[0,1]	(0,1)	(0,1)	[0,1]	(0,1)	(0,1)	$Cl_t^{\geq}$ or $Cl_t^{\leq}$

Lemmas 2 and 3 can be affirmed by Table 3-4 through an analysis of downward and upward believe factor from the viewpoint of class-based rough model. Taking $x \in P_\beta(Cl_t)$ as example, positive score of downward believe factor $\mu^{\leq}(x)$ equals to one, and other scores are either zero (i.e. $v^{\leq}(x)$, $\pi^{\leq}(x)$), or less than one (i.e. $\mu^{\geq}(x)$, $v^{\geq}(x)$, $\pi^{\geq}(x)$). Therefore, all objects from the low boundary region $P_\beta(Cl_t)$ are providing the assignments of class union $Cl_t^{\leq}$. Since we have $Cl_t = P_\beta(Cl_t) \cup \underline{P}(Cl_t) \cup P^\beta(Cl_t)$ in class-based rough model, for $x \in Cl_t$, we can obtain the value domains as shown in the last line of Table 3-4.

Lemma 4.

For object $x \in U$, the following assertions are valid:

$$L(x) \in (0,1]\,; \quad S(x) \in (-1,1]\,; \quad H(x) \in (0,1]\,.$$

Proof: It can be easily proved according to Eq. (4.3)-(4.5). □

Table 3-5 Value domain of measuring degrees in class-based rough model

Regions in TRM	$\beta(x \to Cl_t^{\leq})$			$\beta(x \to Cl_t^{\geq})$		
	$L^{\leq}(x)$	$S^{\leq}(x)$	$H^{\leq}(x)$	$L^{\geq}(x)$	$S^{\geq}(x)$	$H^{\geq}(x)$
For $x \in P_{\beta}(Cl_t)$	1	1	1	(0,1)	(-1,1)	(0,1)
For $x \in \underline{P}(Cl_t)$	1	1	1	1	1	1
For $x \in P^{\beta}(Cl_t)$	(0,1)	(-1,1)	(0,1)	1	1	1
For $x \in Cl_t$	(0,1]	(-1,1]	(0,1]	(0,1]	(-1,1]	(0,1]

Lemma 4 can be affirmed by Table 3-5 through an analysis of three measuring degrees from the viewpoint of class-based rough model. Taking $x \in P_{\beta}(Cl_t)$ as example, we have the domain values of six measuring degrees with respect to two believe factors $\beta(x \to Cl_t^{\leq})$ and $\beta(x \to Cl_t^{\geq})$, including $L^{\geq}(x) \in (0,1)$, $S^{\geq}(x) \in (-1,1)$, $H^{\geq}(x) \in (0,1)$, and $L^{\leq}(x) = S^{\leq}(x) = H^{\leq}(x) = 1$. Because we have $Cl_t = P_{\beta}(Cl_t) \cup \underline{P}(Cl_t) \cup P^{\beta}(Cl_t)$ in class-based rough model, for $x \in Cl_t$, we can obtain the value domains of all measuring degrees as shown in the last row of Table 3-5.

3.4.3 Believable Rule Induction

In this section, we induced believable rules with the assistance of believe factor. Given a decision table, each object x from U has a *decision description* in terms of the evaluations on the considered criteria: $Des_P(x) = [f_{q_1}(x), ..., f_{q_n}(x)]$, where information function $f_q(x) \in V_q$, for $V = \bigcup_{q \in P} V_q$, $q \in P \subseteq C$. We say each $Des_P(x)$ is able to induce an uncertain rule based on *cumulated preferences*. Considering $Des_P(x)$ of boundary object x which is coming from $Bn_P(Cl_t^{\geq})$, there are two kinds of decision descriptions in the separated rough boundary regions:

$$Des_P(x) = [r_{q_1}^{\geq}, r_{q_2}^{\geq}, ..., r_{q_n}^{\geq}] \text{, for } x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq}) \text{; } Des_P(x) = [r_{q_1}^{\leq}, r_{q_2}^{\leq}, ..., r_{q_n}^{\leq}] \text{, for } x \in Cl_{t-1}^{\leq} - \underline{P}(Cl_{t-1}^{\leq}) .$$

With this in mind, the boundary objects carry the *valuable* uncertain information for decision making on the following conditions:

(a). If believe degree $S(x \to Cl_t^{\geq}) > 0$ is satisfied, we describe the *believable* decision information carried by object x as: "Providing the assignment to class union $Cl_t^{\geq}$ in some degree".

(b). If believe factor $S(x \to Cl_{t-1}^{\leq}) > 0$ is satisfied, we describe the *believable* decision information carried by object x as: "Providing the assignment to class union $Cl_{t-1}^{\leq}$ in some degree".

The boundary objects satisfying the above conditions are called *valuable* objects. The uncertain rules induced on the basis of these *valuable* objects are called *believable rules*. In the following, the strategies are given in order to induce a set of believable rules.

Strategy 4-1 (Upward believable rule)

For object $x_i \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$, if $S(x_i \to Cl_t^{\geq}) = \mu_t^{\geq}(x_i) - v_t^{\geq}(x_i) > 0$ is satisfied, an upward believable rule $BR_t^{\geq}$ can be induced according to decision description $Des_P(x_i) = [r_{q_1}^{\geq}, r_{q_2}^{\geq}, ..., r_{q_n}^{\geq}]$ as:

$$If \ \ f_{q_1}(x) \geq r_{q_1}^{\geq} \ and \ f_{q_2}(x) \geq r_{q_2}^{\geq} \ and ... and \ f_{q_n}(x) \geq r_{q_n}^{\geq}, \ then \ \ x \in Cl_t^{\geq} ,$$

which are with confidence degree $L(x_i \to Cl_t^{\geq})$, believe degree $S(x_i \to Cl_t^{\geq})$ and accuracy degree $H(x_i \to Cl_t^{\geq})$.

Strategy 4-2 (Downward believable rule)

For object $x_i \in Cl_{t-1}^{\leq} - \underline{P}(Cl_{t-1}^{\leq})$, if $S(x_i \to Cl_{t-1}^{\leq}) = \mu_{t-1}^{\leq}(x_i) - v_{t-1}^{\leq}(x_i) > 0$ is satisfied, a downward believable rule $BR_{t-1}^{\leq}$ can be induced according to decision description $Des_P(x_i) = [r_{q_1}^{\leq}, r_{q_2}^{\leq}, ..., r_{q_n}^{\leq}]$ as:

$$If \;\; f_{q_1}(x) \leq r_{q_1}^{\leq} \;\; and \;\; f_{q_2}(x) \leq r_{q_2}^{\leq} \;\; and ... and \;\; f_{q_n}(x) \leq r_{q_n}^{\leq}, \;\; then \;\; x \in Cl_{t-1}^{\leq},$$

which are with confidence degree $L(x_i \to Cl_{t-1}^{\leq})$, believe degree $S(x_i \to Cl_{t-1}^{\leq})$ and accuracy degree $H(x_i \to Cl_{t-1}^{\leq})$.

3.5 Example Illustration

In this section, we work out an example to illustrate the problem-solving procedure using our proposed approach. In order to show the comparable results of induced rules and decision outcomes, we employ the example given by Greco, Matarazzo, and Slowinski (2005). Table 3-4 shows a DM's preference decisions concerning eight warehouses described by three criteria:

A: Capacity of the sales staff;

B: Perceived quality of goods;

C: High traffic location.

The criteria are with the preference-ordered scales in evaluation: {A}={B}={C}=[Sufficient, Medium, Good]. The decision criterion indicates the profitability of warehouses by the Return-On-Equity (ROE) ratio (%).

Table 3-6 Decision table for warehouse selection

Warehouse	A	B	C	D (ROE%)
1'	Good	Medium	Good	10.35
2'	Good	Sufficient	Good	4.58
3'	Medium	Medium	Good	5.15
4'	Sufficient	Medium	Medium	-5
5'	Sufficient	Medium	Medium	2.32
6'	Sufficient	Sufficient	Good	2.98
7'	Good	Medium	Good	15
8'	Good	Sufficient	Good	-1.55

Step 1. Establish a decision table

The value sets of criteria are described in natural language terms with multigraded preference relations. We first define the multi-grades of condition attribute by the crisp number as: {A}={B}={C}=[sufficient=1, medium=2, good=3]. Considering the warehouses x and y, we further build the outranking relations based on the decision criterion {D} by using the given laws:

(a). If $ROE(x) \geq ROE(y) - 2\%$ is satisfied, we say that x is at least as good as y in profitability, denoted by xSy.

(b). If $ROE(x) < ROE(y) - 2\%$ is satisfied, we say that x is not at least as good as y in profitability, denoted by $xS^C y$.

The Pairwise Comparison Table (PCT) with the outranking relations can be built in Table 3-7. Suppose DM does not accept those pairs that are with the same evaluations in each condition criterion. Then the PCT should not contain the pairs including (1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8); (1,7), (2,8), (4,5), (5,4), (7,1), (8,2).

Table 3-7 Pairwise Comparison Table with respect to Table 3-6

Pair	A	B	C	D	Pair	A	B	C	D	Pair	A	B	C	D
(1,2)	0	1	0	S	(3,7)	-1	0	0	S^C	(6,4)	0	-1	1	S
(1,3)	1	0	0	S	(3,8)	-1	1	0	S	(6,5)	0	-1	1	S
(1,4)	2	0	1	S	(4,1)	-2	0	-1	S^C	(6,7)	-2	-1	0	S^C
(1,5)	2	0	1	S	(4,2)	-2	1	-1	S^C	(6,8)	-2	0	0	S
(1,6)	2	1	0	S	(4,3)	-1	0	-1	S^C	(7,2)	0	1	0	S
(1,8)	0	1	0	S	(4,6)	0	1	-1	S^C	(7,3)	1	0	0	S
(2,1)	0	-1	0	S^C	(4,7)	-2	0	-1	S^C	(7,4)	2	0	1	S
(2,3)	1	-1	0	S	(4,8)	-2	1	-1	S^C	(7,5)	2	0	1	S
(2,4)	2	-1	1	S	(5,1)	-2	0	-1	S^C	(7,6)	2	1	0	S
(2,5)	2	-1	1	S	(5,2)	-2	1	-1	S^C	(7,8)	0	1	0	S
(2,6)	2	0	0	S	(5,3)	-1	0	-1	S^C	(8,1)	0	-1	0	S^C
(2,7)	0	-1	0	S^C	(5,6)	0	1	-1	S	(8,3)	1	-1	0	S^C
(3,1)	-1	0	0	S^C	(5,7)	-2	0	-1	S^C	(8,4)	2	-1	1	S
(3,2)	-1	1	0	S	(5,8)	-2	1	-1	S	(8,5)	2	-1	1	S^C
(3,4)	1	0	1	S	(6,1)	-2	-1	0	S^C	(8,6)	2	0	0	S^C
(3,5)	1	0	1	S	(6,2)	-2	0	0	S	(8,7)	0	-1	0	S^C
(3,6)	1	1	0	S	(6,3)	-1	-1	0	S^C					

Step 2. Dominance-based rough approximations

The quality of approximation of S and S^C by all criteria is equal to 0.44. The rough approximation can be calculated as shown below.

By strict dominance principles, the lower approximations of S and S^C can be obtained as:

$\underline{C}(S)$ ={(1,2),(1,4),(1,5),(1,6),(1,8),(3,2),(3,4),(3,5),(3,6),(3,8),(7,2),(7,4),(7,5),(7,6),(7,8)}

$\underline{C}(S^C)$ ={(2,1),(2,7),(4,1),(4,3),(4,7),(5,1),(5,3),(5,7),(6,1),(6,3),(6,7),(8,1),(8,7)}

The rough boundary regions preserving S and S^C can be obtained as:

$Bn_C(S) = Bn_C(S^C)$ ={(1,3),(2,3),(2,4),(2,5),(2,6),(3,1),(3,7),(4,2),(4,6),(4,8),(5,2),(5,6),(5,8),(6,2),(6,4),(6,5),(6,8),(7,3),(8,3),(8,4),(8,5),(8,6)}

Step 3. Certain rules induction

First, we induce upward certain rules from $\underline{C}(S)$. Given the superiority matrix via Eq. (3-1), we find the strict constraint relation of each criterion between two objects which are from $\underline{C}(S)$ and $\overline{C}(S^C)$, respectively. The superiority function can be obtained via Eq. (3-3): (a, b, c are respectively corresponding to criteria A, B, C)

$f_C^{\geq}(1,2) = b \wedge c$; $f_C^{\geq}(1,4) = b \wedge c$; $f_C^{\geq}(1,5) = b \wedge c$; $f_C^{\geq}(1,6) = (a \wedge b) \vee (b \wedge c)$; $f_C^{\geq}(1,8) = b \wedge c$; $f_C^{\geq}(3,2) = b \wedge c$; $f_C^{\geq}(3,4) = b \wedge c$; $f_C^{\geq}(3,5) = b \wedge c$; $f_C^{\geq}(3,6) = (a \wedge b) \vee (b \wedge c)$; $f_C^{\geq}(3,8) = b \wedge c$; $f_C^{\geq}(7,2) = b \wedge c$; $f_C^{\geq}(7,4) = b \wedge c$; $f_C^{\geq}(7,5) = b \wedge c$; $f_C^{\geq}(7,6) = (a \wedge b) \vee (b \wedge c)$; $f_C^{\geq}(7,8) = b \wedge c$.

By using Strategy 3-1, upward certain rules $DR(x|x \in S)$ can be induced from $\underline{C}(S)$ as:

From pairs (1,2) (1,6) (1,8) (3,2) (3,6) (3,8) (7,2) (7,6) (7,8), we get the rule as:

$DR(x|x \in S)$: If B $\geq$ 1 and C $\geq$ 0, then $x \in S$.

From pairs (1,4) (1,5) (3,4) (3,5) (7,4) (7,5), we get the rule as:

$DR(x|x \in S)$: If B $\geq$ 0 and C $\geq$ 1, then $x \in S$.

From pairs (1,6) (7,6), we get the rule as:

$DR(x|x \in S)$: If A $\geq$ 2 and B $\geq$ 1, then $x \in S$.

From pairs (3,6), we get the rule as:

$DR(x|x \in S)$: If A $\geq$ 1 and B $\geq$ 1, then $x \in S$.

By using Strategy 5-1**,** we obtain the optimized certain rules. We also show the objects which supports the corresponding rules.

DR [1]: If B $\geq$ 1 and C $\geq$ 0, then $x \in S$. (1,2) (1,6) (1,8) (3,2) (3,6) (3,8) (7,2) (7,6) (7,8).

DR [2]: If B $\geq$ 0 and C $\geq$ 1, then $x \in S$. (1,4) (1,5) (3,4) (3,5) (7,4) (7,5).

DR [3]: If A $\geq$ 1 and B $\geq$ 1, then $x \in S$. (1,6) (3,6) (7,6).

Analogously, we can induce downward certain rules from $\underline{C}(S^C)$. According to the inferiority matrix Eq. (3-2), we find the strict constraint relation of each criterion between two object that are from $\underline{C}(S^C)$ and $\overline{C}(S)$, respectively. The inferiority function can be obtained via Eq. (3-4): (a, b, c are respectively corresponding to criteria A, B, C)

$f_C^{\leq}(2,1) = a \wedge b \wedge c$; $f_C^{\leq}(2,7) = a \wedge b \wedge c$; $f_C^{\leq}(4,1) = b \wedge c$; $f_C^{\leq}(4,3) = b \wedge c$; $f_C^{\leq}(4,7) = b \wedge c$; $f_C^{\leq}(5,1) = b \wedge c$; $f_C^{\leq}(5,3) = b \wedge c$; $f_C^{\leq}(5,7) = b \wedge c$; $f_C^{\leq}(6,1) = a \wedge b$; $f_C^{\leq}(6,3) = a \wedge b$; $f_C^{\leq}(6,7) = a \wedge b$; $f_C^{\leq}(8,1) = a \wedge b \wedge c$; $f_C^{\leq}(8,7) = a \wedge b \wedge c$.

By using Strategy 3-2, downward certain rules $DR(x|x \in S^C)$ can be induced from $\underline{C}(S^C)$ as:

From pairs (2,1) (2,7) (8,1) (8,7), we get the rule as:

$DR(x|x \in S^C)$: If A $\leq$ 0 and B $\leq$ -1 and c $\leq$ 0, then $x \in S^C$.

From pairs (4,1) (4,3) (4,7) (5,1) (5,3) (5,7), we get the rule as:

$DR(x|x \in S^C)$: If B $\leq$ 0 and C $\leq$ -1, then $x \in S^C$.

From pairs (6,1) (6,7), we get the rule as:

$DR(x|x \in S^C)$: If A $\leq$ -2 and B $\leq$ -1, then $x \in S^C$.

From pairs (6,3), we get the rule as:

$DR(x|x \in S^C)$: If A $\leq$ -1 and B $\leq$ -1, then $x \in S^C$.

By using Strategy 5-2, we obtain the optimized certain rule as:

DR [4]: If A $\leq$ 0 and B $\leq$ -1 and C $\leq$ 0, then $x \in S^C$. (2,1) (2,7) (6,1) (6,3) (6,7) (8,1) (8,7).

DR [5]: If B $\leq$ 0 and C $\leq$ -1, then $x \in S^C$. (4,1) (4,3) (4,7) (5,1) (5,3) (5,7).

DR [6]: If A $\leq$ -1 and B $\leq$ -1, then $x \in S^C$. (6,1) (6,3) (6,7).

Remark that these induced upward and downward certain rules are with measuring degrees as [L , S , H]=[1,1,1].

Step 4. Believable rules induction

In step 3, we obtained certain rules DR[1]~DR[6] which are with believe factor (1, 0, 0) and measuring degrees: $L(x)=S(x)=H(x)=1$. In this step, we induce believable rules from regions $S-\underline{C}(S)$ and $S^C-\underline{C}(S^C)$ with measuring degrees: $S(x)\in(0,1)$; $H(x)\in(0,1)$; $L(x)\in(0,1)$. Table 3-8 gives the values of believe factor and the values of three measuring degrees. All pairs in this table are providing the assignment of class S.

Table 3-8 The value of believe factor $\beta(x\to S)$ for object $x\in S-\underline{P}(S)$

Pair	Believe Factor $\beta(x\to S)$			Confidence Degree	Believe Degree	Accuracy Degree	Assign-ment
	$\mu_s(x)$	$v_s(x)$	$\pi_s(x)$				
(1,3) (7,3)	9/13	1/13	3/13	0.923	0.615	0.769	S
(2,3)	9/19	3/19	7/19	0.842	0.316	0.632	S
(2,4)(2,5) (8,4)	4/8	1/8	3/8	0.875	0.375	0.625	S
(2,6)	6/9	2/9	1/9	0.778	0.444	0.889	S
(5,6)	7/9	1/9	1/9	0.889	0.667	0.889	S
(5,8)	9/15	4/15	2/15	0.733	0.333	0.867	S
(6,2) (6,8)	15/23	3/23	5/23	0.870	0.522	0.783	S
(6,4) (6,5)	6/12	1/12	5/12	0.917	0.417	0.583	S

By using Strategy 4-1 and Strategy 5-3, optimized upward believable rules can be induced as follows:

DR [7]: if A$\geq$1 and B$\geq$0 and C$\geq$0, then $x\in S$, subject to:

Measuring degrees as [L , S , H]=[0.923, 0.615, 0.769].

DR [8]: if A$\geq$0 and B$\geq$1 and C$\geq$-1, then $x\in S$, subject to:

Measuring degrees as [L , S , H]=[0.889, 0.667, 0.889].

DR [9]: if A$\geq$0 and B$\geq$-1 and C$\geq$1, then $x\in S$, subject to:

Measuring degrees as [L , S , H]=[0.917, 0.417, 0.583].

Table 3-9 gives the values of believe factor and the values of three measuring degrees. All pairs in this table provide the assignment of class S^C.

Table 3-9 The value of believe factor $\beta(x\to S^C)$ for object $x\in S^C-\underline{P}(S^C)$

Pair	Believe Factor $\beta(x\to S^C)$			Confidence Degree	Believe Degree	Accuracy Degree	Assign-ment
	$\mu_{s^c}(x)$	$v_{s^c}(x)$	$\pi_{s^c}(x)$				
(3,1) (3,7)	9/13	2/13	2/13	0.846	0.538	0.846	S^C
(4,6)	6/12	2/12	4/12	0.833	0.333	0.667	S^C
(4,2)(4,8) (5,2)	4/8	1/8	3/8	0.875	0.375	0.625	S^C
(8,3)	7/9	1/9	1/9	0.889	0.667	0.889	S^C
(8,5)	7/15	6/15	2/15	0.600	0.067	0.867	S^C
(8,6)	13/23	6/23	4/23	0.739	0.304	0.826	S^C

By using Strategy 4-2 and Strategy 5-4, optimized downward believable rules can be induced as follows:

DR [10]: if A $\leq$ -1 and B $\leq$ 0 and C $\leq$ 0, then $x \in S^C$, subject to:

Measuring degrees as [L, S, H]=[0.846, 0.538, 0.846].

DR [11]: if A $\leq$ -2 and B $\leq$ 1 and C $\leq$ -1, then $x \in S^C$, subject to:

Measuring degrees as [L, S, H]=[0.875, 0.375, 0.625].

DR [12]: if A $\leq$ 1 and B $\leq$ -1 and C $\leq$ 0, then $x \in S^C$, subject to:

Measuring degrees as [L, S, H]=[0.889, 0.667, 0.889].

In summary, the certain rules DR[1]~DR[6] and the believable rules DR[7]~DR[12] constitute the final optimized rule set. It provides the knowledge for decision recommendation as: assignments of class S or class S^C. Besides, the pairs that are not included in PCT also can support a rule, shown as: DR: *If* a=0 and b=0 and c=0, *then* $x \in S \cup S^C$; supported by pairs like (1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8), (1,7), (2,8), (4,5), (5,4), (7,1), (8,2). This rule is useless for ranking and of course not included in the final rule set.

3.6 Discussion

The induced certain rules (shown in Section 3.3) and the induced believable rules (shown in Section 3.4) constitute the final decision rule set. On the basis of uncertain information, new believable rules, which are also a kind of uncertain rules, are induced from the separated boundary regions. In Greco, Matarazzo, and Slowinski (2001; 2002; 2005), the induced rules were described by the concepts of *Robust*, *Base*, *Complete* and *Minimal*. The believable rule is *Robust*, as it is supported by at least one object from the given decision table. In addition, the *valuable* boundary object used to induce a believable rule is just the *Base* of this rule. Finally, we say that the subset of induced certain rules is *complete* and *minimal*, since it is able to cover all consistent objects in the decision table, and reclassify them into their original decision classes. Furthermore, the subset of induced believable rule is not *complete* or *minimal*, since there are other rules with the *included* value space of each condition criterion (using subset of elementary conditions in each considered condition criterion) and with at least the same force in the assignment.

The mathematical foundation of rule-based approach for MCDM is DRSA methodology. In order to solve multicriteria ranking problems, this chapter provides a new induction strategy of decision rule set. Such rule set consists of one certain rule subset induced from rough lower approximations and one believable rule subset induced from the separated rough boundary regions. The proposed rule set is not mutually exclusive with the classical minimal rule set. On the contrary, it provides a possible complement or alternative for DM in conducting MCDM. Firstly, both of them include the same certain rules. Then, proposed believable rules provide the assignment as "at least/most class" with quantitative measurements and approximate rules in minimal set provide the assignment like "the union of two classes”. In addition, the proposed rule set is able to provide predictive assignments with respect to PCT, in which approximate rules in minimal set are entirely disabled.

CHAPTER FOUR

The Believable Rough Set Approach for Multicriteria Sorting

4.1 Overview

In Chapter 3, we have considered multicriteria ranking from the theorical perspective. In this chapter, we further address the problem of multicriteria sorting. We consider the issue of supplier selection by using rule-based rough set methodology. Supplier Selection (SS) is an important activity in Logistics and Supply Chain Management within today’s global market. It is one of the major applications of MCDM that concerns preference-ordered decision information. The rule-based rough set methodology is proven of its effectiveness in handling preference information and also performs well in sorting or ranking alternatives. However, how to utilize them to better evaluate suppliers still remains open for more investigations. In this chapter, we propose a novel decision model for supplier selection, called Believable Rough Set Approach (BRSA). This decision model involves a complete decision-making procedures including: (i) criteria reduction; (ii) rough approximation; (iii) decision rule induction; and (iv) a scheme for rule application. Unlike our examined other representative solutions that only extract certain information, the proposed solution additionally extracts valuable uncertain information for rule induction. Due to such mechanism, BRSA performs very well in evaluation of suppliers, and outperforms other proposals. A detailed empirical study is provided for demonstration of the complete problem-solving procedure, and also for comparison with other proposals.

The rest of this chapter is organized as follows. Section 4.2 introduces the BRSA model in the first three procedures: (i) criteria reduction, (ii) rough approximation, and (iii) induction of certain rule and believable rule. Section 4.3 gives the new scheme by applying our induced rules for supplier selection. In Section 4.4, we elaborate an empirical study for illustration of the overall problem-solving procedures and demonstration of the outstanding performance. We finally make a conclusion and outline directions for future work in Section 4.5.

4.2 Background

Supplier Selection (SS) is the important issue of Logistics and Supply Chain Management (LSCM) in today's global market. Choosing and evaluating qualified suppliers depend on a wide range of factors such as value-for-money, quality-of-product, follow-up service, and so on. Despite these factors might be diverse in different business models, the essence of SS can be ascribed to the MCDM problem. MCDM aims at providing DMs a knowledge recommendation amid a finite number of objects (also known as alternatives, solutions, candidates) while being evaluated from multiple viewpoints called features (also known as attributes, criteria, objectives). Figueira, Greco, and Ehrgott (2005) distinguished MCDM into four main issues: criteria analysis, choice, ranking and sorting. We regard the first as the essential procedure for optimization of decision information, and the latter three issues can produce specific decision outcomes.

In literature, De Boer, Labro, and Morlacchi (2001) reviewed the MCDM approach for SS and suggested four stages: (i) problem definition; (ii) criteria formulation; (iii) supplier qualification; and (iv) ranking and selection. Ha and Krishnan (2008) provided a hybrid approach by merging analytic hierarchy process, data envelopment analysis and a neural network for SS. Through modeling DMs' dynamic preference, Chai, Liu, and Yiu (2013) developed a decision-oriented skyline operation that can be used for selection of potential suppliers. Currently, an active issue is to address SS under uncertain environment. Many interesting results have appeared such as fuzzy analytic hierarchy process (Haq & Kannan, 2006; Yang, Chiu, Tzeng & Yeh, 2008); fuzzy analytic network process (Onut, Kara, & Isik, 2009); fuzzy linear programming (Amin, Razmi, & Zhang, 2011; Yucel & Guneri, 2011); fuzzy TOPSIS (Wang, Cheng, & Huang, 2009). These works followed the same thinking direction that is hybridization of fuzzy logic and traditional MCDM techniques. In the meanwhile, more mathematical tools have been used to handle the uncertainties within SS including intuitionistic fuzzy set (Zhang et al., 2009; Chen, 2011; Chai, Liu, & Xu, 2012); rough set (Chang & Hung, 2010); grey systems (Li, Yamaguchi, & Nagai, 2008; Bai & Sarkis, 2010). In the aspect of Decision Support System (DSS), recent developments of intelligent context-aware DSSs by Ngai et al. (2011; 2012) can provide the great supports to LSCM. Chai and Liu (2012a) also provided a reliable DSS platform for supporting supplier selection under dynamic, uncertain, or complex environment.

In this chapter, we investigate a new problem that is different from previous common paradigms of supplier selection. Suppose a company, for example, Wal-Mart, has already got a list of registered suppliers who are graded into several predefined grades (for example, the grades can be in three levels such as Perfect suppliers, Good suppliers, and Ordinary suppliers). And also, several suppliers are considered to have the potential to join the list as registered suppliers for Wal-Mart. The circumstance is that, after establishment of the criteria system (for example three criteria as Product Quality, Supply Reliability, and Follow-up Service), Wal-Mart needs to evaluate both existing (meaning registered) suppliers as well as potential (meaning non-registered) suppliers and assign them into the predefined grades. In that case, Wal-Mart needs to infer (i) the grades of potential suppliers and (ii) the grades of existing suppliers. A intuitive and important principle is that: supposing the performance of Supplier A on all considered criteria is no worse than that of Supplier B, Supplier A should be assigned to the no worse grades when compared with the assigned grades of Supplier B. Obviously, this problem is realistic for those large-scale multi-supplier companies such as retail enterprises or assembly enterprises, and particularly important for companies to maintain a long-term and relatively stable supply chain relationship. Unfortunately, to the best of our knowledge, this problem has never been studied before.

Actually, this new paradigm of supplier selection is a typical multicriteria sorting problem that aims to assign objects (namely suppliers) to several predefined classes (namely grades), in which both criteria values (namely the performance of suppliers) and predefined classes are preference-ordered. According to the number of predefined classes, multicriteria sorting can be divided into two sub-problems: bi-grade sorting (involving two classes only) and multi-grade sorting (involving more than two classes). The former can be regarded as the special case of the latter. Greco, Matarazzo, and Slowinski (2001; 2002) extended Pawlak's rough set (Pawlak & Skowron, 2007) and proposed a Dominance-based Rough Set Approach (DRSA) by utilizing dominance relation as a substitution of binary relation. Since the employed dominance relation can properly model the preference-ordered criteria values and classes, we regard that DRSA is a suitable mathematical tool in addressing our problem. As a rule-based methodology, four procedures should be considered in using DRSA for multicriteria sorting, including (i) criteria reduction; (ii) construction of rough approximation; (iii) induction of decision rule; and (iv) the sorting scheme via rule utilization. First, criteria reduction aims to preserve all necessary relationships between condition criteria (namely, criteria system used to evaluate suppliers' performance) and decision criteria (namely, the grades of suppliers). Then, a core procedure is to calculate and obtain the rough approximations. The target is to induce decision rules and use them for affirming the assignment of rule-covered suppliers. At the end, a sorting scheme is used for utilization of induced rules for supplier evaluation. We use two measurements to evaluate the problem-solving performance, which are (i) grading existing suppliers with a higher accuracy rate and (ii) grading potential suppliers with consistency and more stability.

However, after examining the previous representative solutions, which are four "DRSA + the sorting scheme" combined decision models including Classical DRSA (C-DRSA for short, Greco, Matarazzo, & Slowinski, 2001; 2002) or Variable Consistency DRSA (VC-DRSA for short, Greco et al., 2001; Blaszczynski, Greco, & Slowinski, 2009) respectively joining the *Standard* sorting scheme (Slowinski, Greco, & Matarazzo, 2009) or the *Extended* sorting scheme (Blaszczynski, Greco, & Slowinski, 2007), we found their problem-solving performance leave more room for further improvement. In essence, their considerable shortcoming is that only certain decision information is employed for inducing decision rule and thus some *valuable* uncertain decision information is neglected. Although VC-DRSA relaxes the original strict dominance relation and thus improve the opportunity of discovering the stronger rule patterns, they are still far from satisfactory. Based upon such observations, this chapter proposes a new decision model called Believable Rough Set Approach (BRSA), in order to pursue the better problem-solving performance.

The proposed BRSA decision model is an integral problem-solving solution. Within BRSA, we first provide a unified method for both criteria reduction and certain rule induction. Subsequently, we introduce a new concept, *believe factor*, to explore uncertain information of rough boundary regions. Assisted by this concept, we induce a kind of new uncertain rule called *believable rule*, which is together with previous induced certain rule, in order to form an integral rule set. Finally, we provide a novel sorting scheme for supplier evaluation in two aspects: (i) demonstrating the grades of existing suppliers, and (ii) inferring the grades for potential suppliers. A detailed empirical study is given to illustrate the overall problem-solving procedures and the comparable results. It indicates that our solution outperforms other examined proposals, especially presenting a better quality of grading existing suppliers and a stronger ability of assessing potential suppliers.

4.3 The Believable Rough Set Approach for Multicriteria Sorting

This section introduces the first three procedures of BDRA including criteria reduction, rough approximation and induction of certain rule and believable rule for multicriteria sorting.

4.3.1 Criteria Reduction and Certain Rule Induction: A United Method

(A) Certain Rule Induction

Let us consider a decision table $S=(U,C\bigcup\{d\})$, where a finite set of objects $x\in U$; a finite set of condition criteria $q\in C$, and the predefined classes $CL=\{Cl_t\ t=1,...,l\}$ as partitions of entire U via the decision criterion $\{d\}$. By using the strict dominance principle, we can obtain the C-lower approximations $\underline{C}(Cl_t^{\geq})$ and $\underline{C}(Cl_{t-1}^{\leq})$ preserving the pairs of class unions $Cl_t^{\geq}$ and $Cl_{t-1}^{\leq}$ for $t=2,...,l$, respectively. Then, we define the discernibility matrix which includes the superiority matrix over $Cl_t^{\geq}$ and the inferiority matrix over $Cl_{t-1}^{\leq}$.

Definition 1 *(Superiority matrix)*: For object x with an assignment of $\underline{C}(Cl_t^{\geq})$, we have the superiority matrix as:

$$Sup^{\succ}(Cl_t^{\geq}) = [m_t(x,y)]_{|\underline{C}(Cl_t^{\geq})|\times|\overline{C}(Cl_{t-1}^{\leq})|}$$

s.t. $m_t(x,y) = \{q \in C : f_q(x) > f_q(y),\ x \in \underline{C}(Cl_t^{\geq}), y \in \overline{C}(Cl_{t-1}^{\leq})\}$

Definition 2 *(Inferiority matrix)*: For object x with an assignment of $\underline{C}(Cl_{t-1}^{\leq})$, we have the inferiority matrix as:

$$Inf^{\prec}(Cl_{t-1}^{\leq}) = [n_{t-1}(x,y)]_{|\underline{C}(Cl_{t-1}^{\leq})|\times|\overline{C}(Cl_t^{\geq})|}$$

s.t. $n_{t-1}(x,y) = \{q \in C : f_q(x) < f_q(y),\ x \in \underline{C}(Cl_{t-1}^{\leq}), y \in \overline{C}(Cl_t^{\geq})\}$

For discerning the objects, we use the strict preference relation $xSup^{\succ}y$ (means x is strictly superior to y) and $xInf^{\prec}y$ (means x is strictly inferior to y) to define the discernibility matrix. Then, we further define the discernibility functions according to the obtained matrices shown as follows.

Definition 3 *(Superiority function)*: For object x with an assignment of $\underline{C}(Cl_t^{\geq})$, we have the superiority function as:

$$f_C^{\geq t}(x) = \wedge(\vee a :\ a \in m_t(x,y) \neq 0, y \in \overline{C}(Cl_{t-1}^{\leq}))$$

Definition 4 *(Inferiority function)*: For object x with an assignment of $\underline{C}(Cl_{t-1}^{\leq})$, we have the inferiority function as

$$f_C^{\leq t-1}(x) = \wedge(\vee a :\ a \in n_{t-1}(x,y) \neq 0, y \in \overline{C}(Cl_t^{\geq}))$$

Based on the discernibility functions, we generate the decision rules by using the following strategies, preserving the pairs of class unions: $Cl_t^{\geq}$ and $Cl_{t-1}^{\leq}$.

Strategy 1-1 (Upward certain rule)

Considering the superiority function $f_C^{\geq t}(x_i) = \wedge(\vee a) = a_1 \wedge ... \wedge a_n$ preserving each object x_i from lower approximation $\underline{C}(Cl_t^{\geq})$, the decision description of object x_i can be represented as: $Des_a(x_i) = [r_{a_1}^{\geq}, r_{a_2}^{\geq}, ..., r_{a_n}^{\geq}]$. Then, we can induce an upward certain rule preserving the object x, denoted by $CR_t^{\geq}$: $IF\ \ f_{a_1}(x) \geq r_{a_1}^{\geq}\ and\ f_{a_2}(x) \geq r_{a_2}^{\geq}\ ...\ f_{a_n}(x) \geq r_{a_n}^{\geq},\ THEN\ \ x \in Cl_t^{\geq}$.

Strategy 1-2 (Downward certain rule)

Considering the inferiority function $f_C^{\leq t-1}(x) = \wedge(\vee a) = a_1 \wedge ... \wedge a_n$ preserving each object x_i from lower approximation $\underline{C}(Cl_{t-1}^{\leq})$, the decision description of object x_i can be represented as: $Des_a(x_i) = [r_{a_1}^{\leq}, r_{a_2}^{\leq}, ..., r_{a_n}^{\leq}]$. Then, we can induce a downward certain rule preserving the object x, denoted by $CR_{t-1}^{\leq}$: $IF\ \ f_{a_1}(x) \leq r_{a_1}^{\leq}\ and\ f_{a_2}(x) \leq r_{a_2}^{\leq}\ ...\ f_{a_n}(x) \leq r_{a_n}^{\leq},\ THEN\ \ x \in Cl_{t-1}^{\leq}$.

The condition parts of such obtained rules are implications. They are on the basis of all consistent objects which are assigned to the lower approximations. Since the assignments of these objects are based on the strict dominance principle, we express they are carrying on the certain information for decision (i.e. sorting, ranking, or choice). Appropriately, we call such acquired rules as *certain decision rules*. For example, an object y from U is with the decision description of $Des_a(y) = [f_{a_1}(y),..., f_{a_n}(y)]$. We say object y is covered by certain rule $CR_t^{\geq}$, if it satisfies the condition part of $CR_t^{\geq}$. Then the assignment of object y should follow the decision part of $CR_t^{\geq}$ (assignment of class union $Cl_t^{\geq}$).

In this process, redundant rules are induced. We provide the following strategies to optimize such obtained certain rules.

Strategy 2-1 Suppose two certain rules: $CR_s^{\geq}: \ If \ f_{q_1}(x) \geq r_{q_1}^{\geq} \ and \ f_{q_2}(x) \geq r_{q_2}^{\geq} \ ... \ f_{q_n}(x) \geq r_{q_n}^{\geq}, \ then \ x \in Cl_s^{\geq}$; and $CR_t^{\geq}: \ If \ f_{q_1}(x) \geq r_{q_1}^{\geq *} \ and \ f_{q_2}(x) \geq r_{q_2}^{\geq *} \ ... \ f_{q_n}(x) \geq r_{q_n}^{\geq *}, \ then \ x \in Cl_t^{\geq}$; rule $CR_s^{\geq}$ should be reduced if and only if $(r_{q_1}^{\geq} \geq r_{q_1}^{\geq *}) \wedge (r_{q_2}^{\geq} \geq r_{q_2}^{\geq *}) \wedge ... \wedge (r_{q_n}^{\geq} \geq r_{q_n}^{\geq *}) \wedge (s = t)$ is satisfied.

Strategy 2-2 Suppose two certain rules: $CR_t^{\leq}: \ If \ f_{q_1}(x) \leq r_{q_1}^{\leq} \ and \ f_{q_2}(x) \leq r_{q_2}^{\leq} \ ... \ f_{q_n}(x) \leq r_{q_n}^{\leq}, \ then \ x \in Cl_t^{\leq}$; and $CR_s^{\leq}: \ If \ f_{q_1}(x) \leq r_{q_1}^{\leq *} \ and \ f_{q_2}(x) \leq r_{q_2}^{\leq *} \ ... \ f_{q_n}(x) \leq r_{q_n}^{\leq *}, \ then \ x \in Cl_s^{\leq}$; rule $CR_t^{\leq}$ should be reduced if and only if $(r_{q_1}^{\leq} \leq r_{q_1}^{\leq *}) \wedge (r_{q_2}^{\leq} \leq r_{q_2}^{\leq *}) \wedge ... \wedge (r_{q_n}^{\leq} \leq r_{q_n}^{\leq *}) \wedge (s = t)$ is satisfied.

Let us carry out an analysis. To begin with, if two certain rules are with Relation I, it means they are actually the same rule that is on the basis of the same objects (also known as indiscernible objects). Secondly, if two certain rules are with Relation II; it should be optimized through Strategy 2-1 to Strategy 2-2, in order to reduce the *included* rule. Finally, if two certain rules are with Relation III and Relation IV, no rule should be reduced since they provide the different assignments.

(B) Criteria Reduction

Criteria reduction aims to find several subsets (called *reducts*) of original condition criteria set as alternatives, on condition that the quality of approximation of sorting (*sorting quality* for short) is not deteriorating. The intersection of all generated reducts is called the *core*. A classical measure of sorting quality is defined by Gediga and Düntsch (2002), as the ratio of the number of consistent objects from C-lower approximations and the number of all objects in the universe, denoted by $\gamma_C(CL)$:

$$\gamma_C(CL) = \frac{|U - (\bigcup_{t=2,...,l})Bn_C(Cl_t^{\geq})|}{|U|} = \frac{|U - (\bigcup_{t=1,...,l-1})Bn_C(Cl_t^{\leq})|}{|U|}.$$

It suggests that the reducts should be calculated by the norm of the measure $\gamma_C(CL)$. Although such measure is clear and workable in a two-grade sorting, it rather seems to be too rigorous for multi-grade sorting. Dembczyński, Greco, and Słowiński (2009) provided another two measures for calculation of reducts despite the existing drawbacks. From different viewpoints, the union-based reducts provided by Yang, Yang, Wu and Yu (2008) preserves the lower and upper approximations of downward and upward unions respectively. Such reduct offers multiple choices for DMs via tradeoffs between the reduct size and the sorting quality. Chai and Liu (2011) investigated the class-based rough approximation and introduced the concept of class-based reducts. According to the basic principle of Chai and Liu (2011), the BRSA model is to obtain the reducts together when generating the certain rule by employing the superiority and inferiority functions. The discernibility matrix method (Yao and Zhao, 2009) is used for the establishment of implicants. We firstly define the union-based reducts.

Definition 5 *(Superiority reduct)*: If being a minimal subset P where $P \subseteq C$, which fulfills $\underline{P}(Cl_t^{\geq}) = \underline{C}(Cl_t^{\geq})$ for $t = 1,\ldots,l$, such subset is a superiority reduct, denoted by $\delta^{\geq}$-reduct.

Definition 6 *(Inferiority reduct)*: If being a minimal subset P where $P \subseteq C$, which fulfills $\underline{P}(Cl_t^{\leq}) = \underline{C}(Cl_t^{\leq})$ for $t = 1,\ldots,l$, such subset is an inferiority reduct, denoted by $\delta^{\leq}$-reduct.

Definition 7 *(Overall reduct)*: If being a minimal subset P where $P \subseteq C$, which fulfills $\underline{P}(Cl_t^{\geq}) = \underline{C}(Cl_t^{\geq})$ and $\underline{P}(Cl_t^{\leq}) = \underline{C}(Cl_t^{\leq})$ for $t = 1,\ldots,l$ simultaneously, such criteria subset is an overall reduct, denoted by P^{δ}-reduct.

The following assertions are valid:

- A $\delta^{\geq}$-reduct may/may not be the $\delta^{\leq}$-reduct; and a $\delta^{\leq}$-reduct may or may not be $\delta^{\geq}$-reduct.
- A P^{δ}-reduct may/may not be the $\delta^{\geq}$-reduct or the $\delta^{\leq}$-reduct.
- To a $\delta^{\geq}$-reduct (or a $\delta^{\leq}$-reduct), if it is also the $\delta^{\leq}$-reduct (or the $\delta^{\geq}$-reduct), such reduct must be the P^{δ}-reduct.

This reduct can be calculated by using the acquired superiority/inferiority matrices:

Superiority reduct: $\delta^{\geq} = \overset{t}{\underset{t=1,\ldots,l}{\wedge}} \overset{i,j}{\underset{x_i, x_j \in U}{\wedge}} (\vee \delta_{ij}^{\geq t})$, s.t. $\delta_{ij}^{\geq t} = m_t(x_i, y_j)$, for $x_i \in \underline{C}(Cl_t^{\geq})$ and $y_j \in \overline{C}(Cl_{t-1}^{\leq})$.

Inferiority reduct: $\delta^{\leq} = \overset{t}{\underset{t=1,\ldots,l}{\wedge}} \overset{i,j}{\underset{x_i, x_j \in U}{\wedge}} (\vee \delta_{ij}^{\leq t})$, s.t. $\delta_{ij}^{\leq t} = n_t(x_i, y_j)$, for $x_i \in \underline{C}(Cl_t^{\leq})$ and $y_j \in \overline{C}(Cl_{t+1}^{\geq})$.

According to Definition 7, the overall reduct can be obtained by:

$$P^{\delta} = \delta^{\geq} \wedge \delta^{\leq} = \overset{t}{\underset{t=1,\ldots,l}{\wedge}} \overset{i,j}{\underset{x_i, x_j \in U}{\wedge}} \vee (\delta_{ij}^{\geq t} \wedge \delta_{ij}^{\leq t}),$$

where $\delta^{\geq}$ and $\delta^{\leq}$ denote superiority reduct and inferiority reduct, respectively.

After generating certain rules, we have obtained both superiority and inferiority functions. Next, the reduct can also be easily obtained through following calculations.

Superiority reduct: $\delta^{\geq} = \overset{t}{\underset{t=2,\ldots,l}{\wedge}} \overset{i}{\underset{x_i \in \underline{C}(Cl_t^{\geq})}{\wedge}} f_P^{\geq t}(x_i)$

Inferiority reduct: $\delta^{\leq} = \overset{t}{\underset{t=1,\ldots,l-1}{\wedge}} \overset{i}{\underset{x_i \in \underline{C}(Cl_t^{\leq})}{\wedge}} f_P^{\leq t}(x_i)$

According to Definition 7, the overall reduct can be calculated by:

$$P^{\delta} = \delta^{\geq} \wedge \delta^{\leq} = (\overset{t}{\underset{t=2,\ldots,l}{\wedge}} \overset{i}{\underset{x_i \in \underline{C}(Cl_t^{\geq})}{\wedge}} f_P^{\geq t}(x_i)) \wedge (\overset{t}{\underset{t=1,\ldots,l-1}{\wedge}} \overset{i}{\underset{x_i \in \underline{C}(Cl_t^{\leq})}{\wedge}} f_P^{\leq t}(x_i)).$$

The algorithm for generation of P^{δ}-reduct is represented by the following pseudocode.

Algorithm 1: Criterion Reduction

Input: objects $x_i, y_j \in U$; original criteria set $C \neq 0$; decision class $Cl_t \in CL$; class unions $Cl_t^{\geq}$ and $Cl_t^{\leq}$; decision value $f_q(x)$.

Output: superiority reduct $\delta^{\geq}$; inferiority reduct $\delta^{\leq}$; overall reduct P^{δ}.

Description:

$P^{\delta} := \varnothing$, $\delta^{\geq} := \varnothing$, $\delta_{ij}^{\geq t} := \varnothing$, $\delta^{\leq} := \varnothing$, $\delta_{ij}^{\leq t} := \varnothing$,

For $q_k \in P$,

  If $\forall x_i \in \underline{C}(Cl_t^{\geq})$, $\forall y_j \in \overline{C}(Cl_{t-1}^{\leq})$,

    If $f_{q_k}(x_i) > f_{q_k}(y_j)$,

      $\delta_{ij}^{\geq t} := \delta_{ij}^{\geq t} \vee q_k$,

  $\delta^{\geq} := \bigwedge_{t=1,\dots,l}^{t} \bigwedge_{x_i, x_j \in U}^{i,j} \delta_{ij}^{\geq t}$, // Output: Implicants $\delta^{\geq}$ as superiority reduct //

For $q_k \in P$,

  If $\forall x_i \in \underline{C}(Cl_t^{\leq})$, $\forall y_j \in \overline{C}(Cl_{t+1}^{\geq})$,

    If $f_{q_k}(x_i) < f_{q_k}(y_j)$,

      $\delta_{ij}^{\leq t} := \delta_{ij}^{\leq t} \vee q_k$,

  $\delta^{\leq} := \bigwedge_{t=1,\dots,l}^{t} \bigwedge_{x_i, x_j \in U}^{i,j} \delta_{ij}^{\leq t}$, // Output: Implicants $\delta^{\leq}$ as inferiority reduct //

$P^{\delta} := \delta^{\geq} \wedge \delta^{\leq}$. // Output: Implicants P^{δ} as overall reduct //

End

4.3.2 Believable Rule Induction

In this section, we induce the believable rules by exploring uncertain information from rough boundary regions. Considering the assignment of object $x \in U$, dominance cones $D_P^+(x)$ and $D_P^-(x)$ can be divided into three subsets, denoted by X_1 , X_2 and X_3 : (a) for $D_P^+(x)$, we have $X_1 \subseteq \underline{P}(Cl_t^{\geq})$, $X_2 \subseteq Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$, $X_3 \subseteq Cl_{t-1}^{\leq}$; (b) for $D_P^-(x)$, we have $X_1 \subseteq \underline{P}(Cl_t^{\leq})$, $X_2 \subseteq Cl_t^{\leq} - \underline{P}(Cl_t^{\leq})$, $X_3 \subseteq Cl_{t+1}^{\geq}$. With respect to the objects belonging to the predefined class unions $Cl_t^{\geq}$ and $Cl_t^{\leq}$ but fails to be assigned to the corresponding lower approximations, the following assertions are valid:

- We have $Bn_P(Cl_t^{\geq}) = Bn_P(Cl_{t-1}^{\leq}) = (Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})) \cup (Cl_{t-1}^{\leq} - \underline{P}(Cl_{t-1}^{\leq}))$ for $t = 2,\dots,l$.
- For $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$, $t = 2,\dots,l$, we have $D_P^+(x) = X_1 \cup X_2 \cup X_3$ subject to $X_1 \subseteq \underline{P}(Cl_t^{\geq})$, $X_2 \subseteq Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$, $X_3 \subseteq Cl_{t-1}^{\leq}$.
- For $x \in Cl_t^{\leq} - \underline{P}(Cl_t^{\leq})$, $t = 1,\dots,l-1$, we have $D_P^-(x) = X_1 \cup X_2 \cup X_3$ subject to $X_1 \subseteq \underline{P}(Cl_t^{\leq})$, $X_2 \subseteq Cl_t^{\leq} - \underline{P}(Cl_t^{\leq})$, $X_3 \subseteq Cl_{t+1}^{\geq}$.

Lemma 1: For $x \in Bn_P(Cl_t^{\geq})$ (or $x \in Bn_P(Cl_t^{\leq})$), the following assertions are valid: (a) $|X_1| \geq 0$; (b) $|X_2| \geq 1$; (c) $|X_3| \geq 1$, where the number of objects in a set is denoted by $|\bullet|$.

Proof. We take $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$ as example. For (a), it is given by nature. For (b), assuming $|X_2| = 0$, we get $D_P^+(x) \cap (Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})) = \varnothing$. Since we hold $x \in D_P^+(x)$, we then infer $x \notin Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$, which is contradictory to our premises: $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$. Therefore, the assumption $|X_2| = 0$ does not hold. Finally, we obtain $|X_2| \geq 1$. For (c), assuming $|X_3| = 0$, we get $D_P^+(x) \cap Cl_{t-1}^{\leq} = \varnothing$. Since we hold $U - Cl_{t-1}^{\leq} = Cl_t^{\geq}$, we then obtain $D_P^+(x) \subseteq Cl_t^{\geq}$. According to the definition of $\underline{P}(Cl_t^{\geq})$, we then hold $x \in \underline{P}(Cl_t^{\geq})$, although contradicts our premises of $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$. Evidently, the assumption $|X_3| = 0$ does not hold. At last, we hold $|X_3| \geq 1$. The proof is similar when we take $x \in Cl_t^{\leq} - \underline{P}(Cl_t^{\leq})$. □

Based on such observations, the believe factors of upward and downward unions (simply called *believe factor*) can be defined as follows.

Upward believe factor: For $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq}),\ t = 2,\ldots,l$, we have the *believe factor of upward union* of decision classes:

$$\beta(x \to Cl_t^{\geq}) = (\mu_t^{\geq}(x),\ v_t^{\geq}(x),\ \pi_t^{\geq}(x))$$

s.t. $\mu_t^{\geq}(x) = \dfrac{|D_P^{+}(x) \cap \underline{P}(Cl_t^{\geq})|}{|D_P^{+}(x)|}$; $v_t^{\geq}(x) = \dfrac{|D_P^{+}(x) \cap Cl_{t-1}^{\leq}|}{|D_P^{+}(x)|}$; $\pi_t^{\geq}(x) = \dfrac{|D_P^{+}(x) \cap [Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})]|}{|D_P^{+}(x)|}$.

Downward believe factor: For $x \in Cl_t^{\leq} - \underline{P}(Cl_t^{\leq}),\ t = 1,\ldots,l-1$, we have the *believe factor of downward union* of decision classes:

$$\beta(x \to Cl_t^{\leq}) = (\mu_t^{\leq}(x),\ v_t^{\leq}(x),\ \pi_t^{\leq}(x))$$

s.t. $\mu_t^{\leq}(x) = \dfrac{|D_P^{-}(x) \cap \underline{P}(Cl_t^{\leq})|}{|D_P^{-}(x)|}$; $v_t^{\leq}(x) = \dfrac{|D_P^{-}(x) \cap Cl_{t+1}^{\geq}|}{|D_P^{-}(x)|}$; $\pi_t^{\leq}(x) = \dfrac{|D_P^{-}(x) \cap [Cl_t^{\leq} - \underline{P}(Cl_t^{\leq})]|}{|D_P^{-}(x)|}$.

The symbol "$\to$" can be understood as "be assigned to" or "belongs to". For object $x \in U$, $\mu(x)$ (e.g. $\mu_t^{\geq}(x)$ and $\mu_t^{\leq}(x)$) is called *positive score*; $v(x)$ (e.g. $v_t^{\geq}(x)$ and $v_t^{\leq}(x)$) is called *negative score*; $\pi(x)$ (e.g. $\pi_t^{\geq}(x)$ and $\pi_t^{\leq}(x)$) is called *hesitancy score*. The forms of upward and downward believe factors can be regarded as intuitionistic fuzzy values (Chai, Liu, & Li, 2012). Each believe factor can be measured through three different degrees, which are defined as follows.

Lemma 2: For object $x \in Cl_t,\ t = 1,\ldots,l$, the following assertions are valid:

$$\mu_t^{\geq}(x) + v_t^{\geq}(x) + \pi_t^{\geq}(x) = 1;\quad \mu_t^{\leq}(x) + v_t^{\leq}(x) + \pi_t^{\leq}(x) = 1.$$

Lemma 3: For $x \in \underline{P}(Cl_t^{\geq})$, $\beta(x \to Cl_t^{\geq}) = (\mu_t^{\geq}(x),\ v_t^{\geq}(x),\ \pi_t^{\geq}(x)) = (1,0,0)$ is valid.

For $x \in \underline{P}(Cl_t^{\leq})$, $\beta(x \to Cl_t^{\leq}) = (\mu_t^{\leq}(x),\ v_t^{\leq}(x),\ \pi_t^{\leq}(x)) = (1,0,0)$ is valid.

Proof. Lemma 2 and Lemma 3 can be easily proven according to the definitions of believe factor.

The confidence degree: For object $x \in U$, the *confidence degree* of believe factor, denoted by $L(x)$, is defined by: $L(x) = \mu(x) + \pi(x)$, where $\mu(x)$ is positive score and $\pi(x)$ is hesitancy score. Specifically, we have: $L(x \to Cl_t^{\leq}) = \mu_t^{\leq}(x) + \pi_t^{\leq}(x)$; $L(x \to Cl_t^{\geq}) = \mu_t^{\geq}(x) + \pi_t^{\geq}(x)$.

The believe degree: For object $x \in U$, the *believe degree* of believe factor, denoted by $S(x)$, is defined by: $S(x) = \mu(x) - v(x)$, where $\mu(x)$ is positive score and $v(x)$ is negative score. Specifically, we have:

$S(x \to Cl_t^{\leq}) = \mu_t^{\leq}(x) - v_t^{\leq}(x)$; $S(x \to Cl_t^{\geq}) = \mu_t^{\geq}(x) - v_t^{\geq}(x)$.

The accuracy degree: For object $x \in U$, the *accuracy degree* of believe factor, denoted by $H(x)$, is defined by $H(x) = \mu(x) + v(x)$, where $\mu(x)$ is positive score and $v(x)$ is negative score. Specifically, we have: $H(x \to Cl_t^{\leq}) = \mu_t^{\leq}(x) + v_t^{\leq}(x)$; $H(x \to Cl_t^{\geq}) = \mu_t^{\geq}(x) + v_t^{\geq}(x)$.

Lemma 4: For object $x \in U$, the following assertions are valid: $L(x) \in (0,1]$; $S(x) \in (-1,1]$; $H(x) \in (0,1]$.

Proof. It can be easily proven according to the definitions of three measuring degrees. □

Given a decision table, each object x from U has a decision description in terms of the evaluations on the considered criteria: $Des_P(x) = [f_{q_1}(x), ..., f_{q_n}(x)]$, where information function $f_q(x) \in V_q$, for $V = \bigcup_{q \in P} V_q$, $q \in P \subseteq C$. We note that each $Des_P(x)$ is able to induce an uncertain rule based on cumulated preferences. Considering $Des_P(x)$ of boundary object x which is coming from $Bn_P(Cl_t^{\geq})$, there are two kinds of decision descriptions in the separated rough boundary regions as:

$Des_P(x) = [r_{q_1}^{\geq}, r_{q_2}^{\geq}, ..., r_{q_n}^{\geq}]$, for $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$; $Des_P(x) = [r_{q_1}^{\leq}, r_{q_2}^{\leq}, ..., r_{q_n}^{\leq}]$, for $x \in Cl_{t-1}^{\leq} - \underline{P}(Cl_{t-1}^{\leq})$.

The boundary objects satisfying $S(x) > 0$ are called *valuable* objects. The induced uncertain rules on the basis of these *valuable* objects are known as *believable rules*. In the following, we provide the methods to induce a set of believable rules.

Strategy 3-1 (Upward believable rule): If $S(x_i \to Cl_t^{\geq}) = \mu_t^{\geq}(x_i) - v_t^{\geq}(x_i) > 0$ is satisfied, we then induce an upward believable rule $BR_t^{\geq}$ for $x_i \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$, which is with the three measuring degrees: $L(x_i \to Cl_t^{\geq})$, $S(x_i \to Cl_t^{\geq})$ and $H(x_i \to Cl_t^{\geq})$:

$$If\ \ f_{q_1}(x) \geq r_{q_1}^{\geq}\ and\ f_{q_2}(x) \geq r_{q_2}^{\geq}\ ...\ f_{q_n}(x) \geq r_{q_n}^{\geq},\ then\ \ x \in Cl_t^{\geq}\ .$$

Strategy 3-2 (Downward believable rule): If $S(x_i \to Cl_{t-1}^{\leq}) = \mu_{t-1}^{\leq}(x_i) - v_{t-1}^{\leq}(x_i) > 0$ is satisfied, we then induce a downward believable rule $BR_{t-1}^{\leq}$ for $x_i \in Cl_{t-1}^{\leq} - \underline{P}(Cl_{t-1}^{\leq})$, which is with the three measuring degrees: $L(x_i \to Cl_{t-1}^{\leq})$, $S(x_i \to Cl_{t-1}^{\leq})$ and $H(x_i \to Cl_{t-1}^{\leq})$:

$$If\ \ f_{q_1}(x) \leq r_{q_1}^{\leq}\ and\ f_{q_2}(x) \leq r_{q_2}^{\leq}\ ...\ f_{q_n}(x) \leq r_{q_n}^{\leq},\ then\ \ x \in Cl_{t-1}^{\leq}\ .$$

Remark that we can induce the believable rules $BR_t^{\geq}$ and $BR_{t-1}^{\leq}$, from the separated boundary regions $Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$ and $Cl_{t-1}^{\leq} - \underline{P}(Cl_{t-1}^{\leq})$, respectively. These believable rules are on the basis of the valuable boundary objects from the boundary region $Bn_P(Cl_t^{\geq})$ (or $Bn_P(Cl_{t-1}^{\leq})$). In addition, a situation should be taken note here. Taking $BR_t^{\geq}$ as an example, there may exist more than one valuable object $x_i \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$ which is with the same decision description. We regard such objects as an indiscernibility relation under considered condition criteria. By using our strategy, such objects are able to induce only one believable rule. Such rule will be supported by these indiscernible objects.

4.3.3 The Scheme for Multicriteria Sorting

For description of decision rules, several basic coefficients have been provided by Pawlak (2002) including *support*, *confidence*, *strength*, and *coverage*. We summarize these coefficients as follows: (1) *Support*: An object gives *support* to a decision rule, as long as this object affirms both condition part and decision part of this rule. (2) *Confidence*: For one rule, *confidence* is defined as the ratio of the number of objects supporting both condition and decision parts and the number of objects supporting only condition part. (3) *Strength*: For one rule, *strength* is defined as the ratio of the number of objects supporting both condition and decision parts and the number of universal objects. (4) *Coverage*: An object is *covered* by a rule, as long as this object affirms the condition part of this rule. Obviously, the decision rule would be better if it owns higher *confidence* and higher *coverage*. Apart from these basic coefficients above, we present a new concept *Force* in order to distinguish the multi-grade assignments in multicriteria sorting.

Definition 8 (Force of assignment of the rule, *Force* for short)

Suppose the decision criterion $\{d\}$ makes a partition of U into a finite number of preference-ordered classes $CL = \{Cl_t, t = 1, \ldots, l\}$. (we assume class Cl_{t+1} is superior to class Cl_t according to DM's preference). Considering the decision rule "$if\ \varphi\ then\ \psi$", the assignments given by decision part ψ can be the class unions $Cl_s^{\le},\ Cl_t^{\le},\ Cl_s^{\ge},\ Cl_t^{\ge}$ where $s < t$. Then, *force* can be represented as follows:

- If two rules denoted as $DR_{(1)}: if\ \varphi\ then\ x \in Cl_s^{\le}$ and $DR_{(2)}: if\ \varphi\ then\ x \in Cl_t^{\le}$, we say rule $DR_{(1)}$ has more *force*, due to $Cl_s^{\le} \subset Cl_t^{\le}$.
- If two rules are represented as $DR_{(1)}: if\ \varphi\ then\ x \in Cl_s^{\ge}$ and $DR_{(2)}: if\ \varphi\ then\ x \in Cl_t^{\ge}$, we say $DR_{(2)}$ has more *force*, due to $Cl_s^{\ge} \supset Cl_t^{\ge}$.

In the following, we develop a new sorting scheme for multicriteria sorting, through employing the induced final rule set, denoted as S_R. We consider U as a finite objects set of the given decision table (also known as learning objects). Our target of sorting is to provide the suitable assignment of each learning object and predict the assignment of any new objects. To this end, the whole process is to generate a univocal certain or believable rule (called the trusted rule) with respect to each object, for providing the assignment (the decision part of the trusted rule). These objects are uniformly denoted as x. We consider five situations as shown in Table 4-1, when affirming the assignment of object x by using S_R.

Table 4-1 Five situations in rule affirming process

Situations	The situations in rule matching process		The types of Affirmed rules
	Certain rules	Believable rules	
A-1	1	≥ 0	Certain rule
A-2	≥ 2	≥ 0	Certain rule
B	0	0	No rule
C-1	0	1	Believable rule
C-2	0	≥ 2	Believable rule

Situation (A): Being certain rule(s) that affirm x.

- **Situation (A-1)**: Just one certain rule affirms x.

If x is affirmed by one certain rule $CR_t^{\ge}$ (or $CR_t^{\le}$), the assignment of recommendation is "object x belongs to the class union $Cl_t^{\ge}$ (or $Cl_t^{\le}$)". Following the prudence principle (Greco, Matarazzo, &

Slowinski, 2002), the assignment can be refined to the worst case: exact class Cl_t of class union $Cl_t^{\geq}$, or exact class Cl_t of class union $Cl_t^{\leq}$.

- **Situation (A-2)**: More than one certain rule affirms x.

If x is affirmed by more than one certain rule, three sub-situations are taken into account:

(1) Being more than one $CR_t^{\geq}$ and no $CR_t^{\leq}$ affirms object x, the assignment follows this certain rule which is with the highest force, denoted by $\{x \to Cl_{t^*}^{\geq}:\ t^* = \max\{t\}\}$.

(2) Being more than one $CR_t^{\leq}$ and no $CR_t^{\geq}$ affirms object x, the assignment follows the certain rule which is with the highest force, denoted by $\{x \to Cl_{t^*}^{\leq}:\ t^* = \min\{t\}\}$.

(3) Being both $CR_t^{\geq}$ and $CR_s^{\leq}$ affirming x simultaneously, we firstly calculate $Cl_{t^*}^{\geq}$ and $Cl_{s^*}^{\leq}$ via above strategies. Then we consider the following situations:

    a) If $t^* = s^*$ is satisfied, x is exactly assigned to the class: $Cl_{t^*}(=Cl_{s^*})$.

    b) If $t^* < s^*$ is satisfied, x is assigned to the class union: $Cl_{t^*} \cup Cl_{t^*+1} \cup, ..., Cl_{s^*-1} \cup Cl_{s^*}$.

    c) If $t^* > s^*$ is satisfied, we say the induced rules cannot affirm any assignment to x, due to contradictory information.

Let us say that the above strategy is the so-called *Standard* sorting scheme. It is usually joining with C-DRSA as an integral decision model to solve sorting problems. Furthermore, Blaszczynski, Greco, and Slowinski (2007) provided an *Extended* sorting scheme in order to provide the assignment of the *exact* decision class. Considering through the strength of rules, a measurable score is calculated. A main advantage of this scheme is refining the assignment to the singleton decision class rather than the class union. Using the extended scheme joining with various DRSA as integral decision models can partially solve our problem. However, their performances are still far from satisfactory due to our experiments. In Section 4.4, we will make use of various representative DRSA (i.e. C-DRSA and VC-DRSA) respectively joining with the two types of sorting schemes (i.e. the *standard* one and the *extended* one) as the integral decision models, to compare with our proposed BRSA decision model, through the unified measurements of decision-making performance.

Situation (B): Neither certain rule nor believable rule affirms x. As such, x cannot be sorted by using the rule set S_R. It has to be assigned to the union of entire decision classes $Cl_1 \cup Cl_2 \cup, ..., Cl_{l-1} \cup Cl_l$.

Situation (C): No certain rule affirms x, as well as being believable rule(s) affirming x.

- **Situation (C-1):** Just one believable rule affirms x.

If x is affirmed by one believable rule $BR_t^{\geq}$ (or $BR_t^{\leq}$), we say the assignment is “object x belongs to the class union $Cl_t^{\geq}$ (or $Cl_t^{\leq}$) with *confidence degree* $L(x \to Cl_t^{\geq})$ *(or* $L(x \to Cl_t^{\leq})$ *)*”.

- **Situation (C-2)**: More than one believable rule affirms x. As such, three sub-situations are taken into account:

(1) Being more than one $BR_t^{\geq}$ and no $BR_t^{\leq}$, the assignment then follows the rule with the highest $L(x)$. In case of such rules being not univocal, the assignment then follows the rule with the highest $H(x)$. In case of such rules being still not univocal, the assignment then follows the rule with

the highest force. Such obtained rule $BR_{t^*}^{\geq}$ thus can be affirmed for assignment of object x.

(2) Being more than one $BR_t^{\leq}$ and no $BR_t^{\geq}$, the assignment then follows the rule with the highest $L(x)$. In case of such rules being not univocal, the assignment then follows the rule with the highest $H(x)$. In case of such rules being still not univocal, the assignment then follows the rule with the highest force. Such obtained rule $BR_{s^*}^{\leq}$ thus can be affirmed for assignment of object x.

(3) Being both $BR_t^{\geq}$ and $BR_s^{\leq}$ affirming x simultaneously, we initially calculate $BR_{t^*}^{\geq}$ and $BR_{s^*}^{\leq}$ by above strategies. In case such rules being still not univocal after comparing $L(x)$ and $H(x)$, we then consider the following three sub-situations:

   a) If $t^* = s^*$ is satisfied, x is exactly assigned to the class: $Cl_{t^*}(=Cl_{s^*})$.

   b) If $t^* < s^*$ is satisfied, x is assigned to the class union $Cl_{t^*} \cup Cl_{t^*+1} \cup, ..., Cl_{s^*-1} \cup Cl_{s^*}$.

   c) If $t^* > s^*$ is satisfied, we say the induced rule cannot affirm any assignment to object x, due to contradictory information.

For an object x, we consider the matched rule set $Mset$ that contains one matched certain rule set $Cset$ and one matched believable rule set $Bset$, where $Mset = Cset \cup Bset$. The process of finding the trusted rules from set $Mset$ can be represented through the following pseudocode.

Algorithm 2: The Scheme for Multicriteria Sorting

Input: matched rule sets $Mset$, $Cset$, $Bset$; confidence degree $L(x)$; accuracy degree $H(x)$, force.
Output: The trusted rule(s)
Description:

Begin // checking $Mset$ //
  If $Mset = \varnothing$, **then** output **<1>.**
  Else $Mset \neq \varnothing$, // checking $Cset$ //
    If $Cset = \varnothing$, // checking $Bset$ //
      If $card\{Bset\} = 1$, **then** output **<2>.**
      Else $card\{Bset\} \geq 2$
        If one rule with highest $S(x)$, **then** output **<5>.**
        Else reduce the rule(s) with the lower $L(x)$, // update $Bset$ //
          If one rule with highest $H(x)$,
            then output **<6>.**
          Else reduce the rule(s) with the lower $H(x)$, // update $Bset$ //
            If existing one $BR_t^{\geq}$, **then** output **<7>.**
            If existing one $BR_s^{\leq}$, **then** output **<8>.**
            If existing one $BR_t^{\geq}$ and one $BR_s^{\leq}$, // checking t and s //
              If $t = s$**, then** output **<9>.**
              Else if $t < s$**, then** output **<10>.**
              Else $t > s$**, then** output **<11>.**
  Else $Cset \neq \varnothing$, // checking $Cset$ //
    If $card\{Cset\} = 1$, **then** output **<3>.**
    Else $card\{Cset\} \geq 2$,// checking force of rules in $Cset$ //
      then output **<4>.**
End

A summary of outputs outlines as follows: <2> <5> <6> <7> <8> output believable rule with assignment of class union $Cl_t^{\geq}$ or else $Cl_s^{\leq}$; <9> outputs believable rule with assignment of class $Cl_t = Cl_s$; <10> outputs believable rule with assignment of class union $Cl_t \cup ... \cup Cl_s$; <3> <4> output certain rule(s) with corresponding assignments; <1> <11> output no rule, and thus it is with assignment of entire class union $\cup Cl_t, t-1,\ldots,l$. Therefore, there are a total of five kinds of assignment as sorting results of object x , including (1) $x \to \cup Cl_t, t=1,\ldots,l$; (2) $x \to Cl_t^{\geq}$; (3) $x \to Cl_s^{\leq}$; (4) $x \to Cl_t (= Cl_s)$; (5) $x \to Cl_t \cup \ldots \cup Cl_s$.

4.4 Application on Supplier Selection

4.4.1 Decision Preliminary

Suppose a Wal-Mart store owns 50 suppliers {S1, S2,…, S50} that are graded into three levels: Class I (perfect), Class II (good) and Class III (ordinary). Within this supplier evaluation, 50 existing suppliers and several potential suppliers are evaluated under three criteria: A: quality of products, B: supply reliability, C: service. Their performance is graded via scores with the value scales [1, 2, 3, 4, 5]. The higher number means the better performance. The decision table is shown as Table 4-2.

Table 4-2 Decision table

Supplier.	A	B	C	D	Supplier	A	B	C	D	Supplier	A	B	C	D	Supplier	A	B	C	D
S 1	3	4	3	I	S 14	4	3	3	I	S 27	3	4	2	II	S 40	1	2	3	III
S 2	4	3	3	I	S 15	5	3	4	I	S 28	1	4	2	II	S 41	2	3	2	III
S 3	5	3	4	I	S 16	5	3	4	I	S 29	1	4	2	II	S 42	1	3	2	III
S 4	5	3	4	I	S 17	5	4	3	I	S 30	2	1	3	II	S 43	1	3	2	III
S 5	5	4	3	I	S 18	3	4	3	I	S 31	3	2	4	II	S 44	2	3	2	III
S 6	3	4	3	I	S 19	5	2	4	II	S 32	3	2	4	II	S 45	3	2	3	III
S 7	5	3	3	I	S 20	3	4	2	II	S 33	5	2	4	II	S 46	4	2	3	III
S 8	1	3	3	I	S 21	4	2	3	II	S 34	1	3	2	III	S 47	3	2	3	III
S 9	4	3	4	I	S 22	5	2	4	II	S 35	2	3	3	III	S 48	5	3	3	III
S 10	4	3	4	I	S 23	5	2	4	II	S 36	1	2	3	III	S 49	3	2	3	III
S 11	4	4	3	I	S 24	1	4	2	II	S 37	2	3	3	III	S 50	3	2	3	III
S 12	1	3	3	I	S 25	1	4	2	II	S 38	2	2	3	III					
S 13	3	4	3	I	S 26	2	4	3	II	S 39	1	3	2	III					

4.4.2 Believable Rough Set Approach

(A) Rough approximations

Suppose 50 suppliers are denoted as the universe U . The grading levels (values of criterion D in Table 4-2) includes three decision classes: Class I = Cl_3 , Class II = Cl_2 , Class III = Cl_1 , where Cl_3 is superior to Cl_2 , and then to Cl_1 . Thus the upward and downward unions of decision classes are given as: $Cl_1^{\leq} = Cl_1$; $Cl_3^{\geq} = Cl_3$; $Cl_1^{\geq} = Cl_3^{\leq} = Cl_1 \cup Cl_2 \cup Cl_3$; $Cl_2^{\leq} = Cl_1 \cup Cl_2$; $Cl_2^{\geq} = Cl_3 \cup Cl_2$. According to the strict dominance principle, we obtain the lower approximations below. (symbol '||' is used to distinguish the different regions).

$\underline{C}(Cl_3^{\geq})$ ={ S1; S3; S4; S5; S6; S9; S10; S11; S13; S15; S16; S17; S18 }

$\underline{C}(Cl_2^{\geq})$ ={ S1; S3; S4; S5; S6; S9; S10; S11; S13; S15; S16; S17; S18; || S19; S20; S22; S23; S24; S25; S27; S28; S29; S31; S32; S33; || S26 }

$\underline{C}(Cl_1^{\leq})$ ={ S34; S36; S39; S40; S41; S42; S43; S44 }

$\underline{C}(Cl_2^{\leq})$ ={ S34; S36; S39; S40; S41; S42; S43; S44; || S19; S20; S22; S23; S24; S25; S27; S28; S29; S31; S32; S33; || S30; S21; || S38; S45; S46; S47; S49; S50 }

Figure 4-1 illustrates the partition of U and the detailed rough approximations. So we can obtain the separated rough boundary regions below.

$Cl_3^{\geq} - \underline{C}(Cl_3^{\geq})$ ={ S2; S7; S8; S12; S14 }

$Cl_2^{\geq} - \underline{C}(Cl_2^{\geq})$ ={ S2; S7; S8; S12; S14; || S30; S21 }

$Cl_1^{\leq} - \underline{C}(Cl_1^{\leq})$ ={ S35; S37; S48; || S50; S49; S47; S46; S45; S38 }

$Cl_2^{\leq} - \underline{C}(Cl_2^{\leq})$ ={ S35; S37; S48; || S26 }

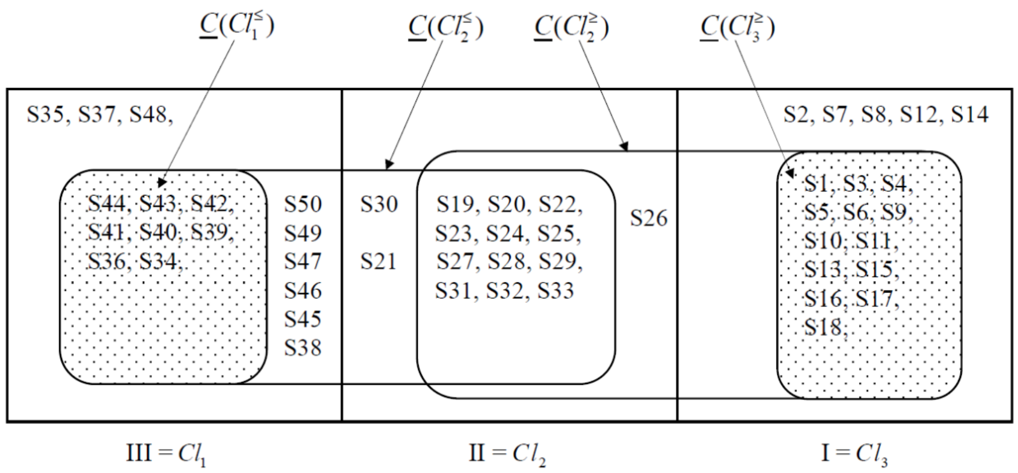


Fig 4-1 Illustration of the partition profile of the existing suppliers

(B) Certain rule induction

Based on the obtained superiority and inferiority matrices and functions, we induce the upward and downward certain rules by using Strategy 1-1 and Strategy 1-2.

(1) Induction of upward certain rules $CR_3^{\geq}$:

Step. 1. Superiority functions $f_C^{\geq 3}(x)$:

Based on the constructed superiority matrix $Sup^{\succ}(Cl_3^{\geq})$, we obtain the superiority function of S1:

$$f_C^{\geq 3}(S1) = \mathrm{B} \wedge \mathrm{B} \wedge (\mathrm{A} \vee \mathrm{B}) \wedge (\mathrm{A} \vee \mathrm{B}) \wedge \mathrm{B} \wedge \mathrm{B} \wedge \mathrm{C} \wedge \mathrm{B} \wedge \mathrm{B} \wedge \mathrm{B} \wedge (\mathrm{A} \vee \mathrm{C}) \wedge (\mathrm{A} \vee \mathrm{C}) \wedge \mathrm{A} \wedge \mathrm{C} \wedge (\mathrm{A} \vee \mathrm{C}) \wedge (\mathrm{A} \vee \mathrm{C}) \wedge (\mathrm{A} \vee \mathrm{B}) \wedge \mathrm{B} \wedge \mathrm{B}$$
$$\wedge \mathrm{B} \wedge (\mathrm{A} \vee \mathrm{B} \vee \mathrm{C}) \wedge (\mathrm{A} \vee \mathrm{B}) \wedge (\mathrm{A} \vee \mathrm{B}) \wedge (\mathrm{A} \vee \mathrm{B}) \wedge (\mathrm{A} \vee \mathrm{B}) \wedge (\mathrm{A} \vee \mathrm{B} \vee \mathrm{C}) \wedge (\mathrm{A} \vee \mathrm{B}) \wedge (\mathrm{A} \vee \mathrm{B} \vee \mathrm{C}) \wedge (\mathrm{A} \vee \mathrm{B} \vee \mathrm{C})$$
$$\wedge (\mathrm{A} \vee \mathrm{B} \vee \mathrm{C}) \wedge (\mathrm{A} \vee \mathrm{B} \vee \mathrm{C}) \wedge \mathrm{B} \wedge \mathrm{B} \wedge \mathrm{B} \wedge \mathrm{B} \wedge \mathrm{B} \wedge \mathrm{B} = \mathrm{A} \vee \mathrm{B} \vee \mathrm{C}$$

Similarly, we obtain the superiority functions as:

$f_C^{\geq 3}(S1) = f_C^{\geq 3}(S6) = f_C^{\geq 3}(S13) = f_C^{\geq 3}(S18) = \mathrm{A} \wedge \mathrm{B} \wedge \mathrm{C}$; $f_C^{\geq 3}(S3) = f_C^{\geq 3}(S4) = f_C^{\geq 3}(S15) = f_C^{\geq 3}(S16) = \mathrm{B} \wedge \mathrm{C}$;

$f_C^{\geq 3}(S9) = f_C^{\geq 3}(S10) = \mathrm{B} \wedge \mathrm{C}$; $f_C^{\geq 3}(S5) = f_C^{\geq 3}(S17) = \mathrm{A} \wedge \mathrm{B}$; $f_C^{\geq 3}(S11) = \mathrm{A} \wedge \mathrm{B}$.

Step. 2. According to Strategy 1-1, we induce the upward certain rules. We also show the suppliers who supports each corresponding rule.

$(\mathrm{A} \geq 3)$ & $(\mathrm{B} \geq 4)$ & $(\mathrm{C} \geq 3)$ => $(\mathrm{D} \geq \mathrm{I})$ (S1; S6; S13; S18);

$(\mathrm{B} \geq 3)$ & $(\mathrm{C} \geq 4)$ => $(\mathrm{D} \geq \mathrm{I})$ (S3; S4; S15; S16)

$(\mathrm{B} \geq 3)$ & $(\mathrm{C} \geq 4)$ => $(\mathrm{D} \geq \mathrm{I})$ (S9; S10)

$(\mathrm{A} \geq 5)$ & $(\mathrm{B} \geq 4)$ => $(\mathrm{D} \geq \mathrm{I})$ (S5; S17)

(A $\geq$ 4) & (B $\geq$ 4) => (D $\geq$ I) (S11)

Step. 3. According to Strategy 2-1, we obtain the optimized certain rules:

(A $\geq$ 3) & (B $\geq$ 4) & (C $\geq$ 3) => (D $\geq$ I) (S1; S6; S13; S18; || S11; || S5; S17)

(B $\geq$ 3) & (C $\geq$ 4) => (D $\geq$ I) (S3; S4; S15; S16; || S9; S10)

(A $\geq$ 4) & (B $\geq$ 4) => (D $\geq$ I) (S11; || S5; S17)

These certain rules denoted by $CR_3^{\geq}$, are induced from $\underline{C}(Cl_3^{\geq})$ with assignment of "at least Class I".

(2) Induction of upward certain rules $CR_2^{\geq}$:

Step. 1. Superiority functions $f_C^{\geq 2}(x)$:

According to the constructed superiority matrix $Sup^{\succ}(Cl_2^{\geq})$, we obtain the superiority functions as:

$f_C^{\geq 2}(S1) = f_C^{\geq 2}(S6) = f_C^{\geq 2}(S13) = f_C^{\geq 2}(S18) = \mathrm{B}$; $f_C^{\geq 2}(S5) = f_C^{\geq 2}(S17) = \mathrm{B}$; $f_C^{\geq 2}(S11) = \mathrm{B}$; $f_C^{\geq 2}(S20) = f_C^{\geq 2}(S27) = \mathrm{B}$;

$f_C^{\geq 2}(S26) = \mathrm{B}$; $f_C^{\geq 2}(S24) = f_C^{\geq 2}(S25) = f_C^{\geq 2}(S28) = f_C^{\geq 2}(S29) = \mathrm{B}$; $f_C^{\geq 2}(S9) = f_C^{\geq 2}(S10) = \mathrm{C}$; $f_C^{\geq 2}(S31) = f_C^{\geq 2}(S32) = \mathrm{C}$;

$f_C^{\geq 2}(S19) = f_C^{\geq 2}(S22) = f_C^{\geq 2}(S23) = f_C^{\geq 2}(S33) = \mathrm{C}$; $f_C^{\geq 2}(S3) = f_C^{\geq 2}(S4) = f_C^{\geq 2}(S15) = f_C^{\geq 2}(S16) = (\mathrm{A} \vee \mathrm{B}) \wedge \mathrm{C} = (\mathrm{A} \wedge \mathrm{C}) \vee (\mathrm{B} \wedge \mathrm{C})$.

Step. 2-3. According to Strategy 1-1 and Strategy 2-1, we obtain the optimized upward certain rules:

(B $\geq$ 4) => (D $\geq$ II) (S1; S6; S13; S18; S5; S17; S11; S20; S27; S24; S25; S28; S29; S26)

(C $\geq$ 4) => (D $\geq$ II) (S9; S10; S19; S22; S23; S33; S31; S32; || S3; S4; S15; S16)

These certain rules denoted by $CR_2^{\geq}$, are induced from $\underline{C}(Cl_2^{\geq})$ with assignment of "at least Class II".

(3) Induction of downward certain rules $CR_1^{\leq}$:

Step. 1. Superiority functions $f_C^{\leq 1}(x)$:

According to the constructed inferiority matrix $Inf^{\prec}(Cl_1^{\leq})$, we obtain the inferiority functions as:

$f_C^{\leq 1}(S36) = f_C^{\leq 1}(S40) = \mathrm{A} \wedge \mathrm{B}$; $f_C^{\leq 1}(S34) = f_C^{\leq 1}(S39) = f_C^{\leq 1}(S42) = f_C^{\leq 1}(S43) = \mathrm{B} \wedge \mathrm{C}$; $f_C^{\leq 1}(S41) = f_C^{\leq 1}(S44) = \mathrm{B} \wedge \mathrm{C}$.

Step. 2-3. According to Strategy 1-2 and Strategy 2-2, we obtain the optimized downward certain rules:

(A $\leq$ 1) & (B $\leq$ 2) => (D $\leq$ III) (S36; S40)

(B $\leq$ 3) & (C $\leq$ 2) => (D $\leq$ III) (S34; S39; S42; S43; || S41; S44)

These certain rules denoted by $CR_1^{\leq}$, are induced from $\underline{C}(Cl_1^{\leq})$ with assignment of "at most Class III".

(4) Induction of downward certain rules $CR_2^{\leq}$:

Step. 1. Superiority functions $f_C^{\leq 2}(x)$:

According to the constructed inferiority matrix $Inf^{\prec}(Cl_2^{\leq})$, we obtain the inferiority functions as:

$f_C^{\leq 2}(S34) = f_C^{\leq 2}(S39) = f_C^{\leq 2}(S42) = f_C^{\leq 2}(S43) = \mathrm{C}$; $f_C^{\leq 2}(S41) = f_C^{\leq 2}(S44) = \mathrm{C}$; $f_C^{\leq 2}(S20) = f_C^{\leq 2}(S27) = \mathrm{C}$;

$f_C^{\leq 2}(S24) = f_C^{\leq 2}(S25) = f_C^{\leq 2}(S28) = f_C^{\leq 2}(S29) = \mathrm{C}$; $f_C^{\leq 2}(S36) = f_C^{\leq 2}(S40) = \mathrm{B}$;

$f_C^{\leq 2}(S19) = f_C^{\leq 2}(S22) = f_C^{\leq 2}(S23) = f_C^{\leq 2}(S33) = \mathrm{B}$; $f_C^{\leq 2}(S31) = f_C^{\leq 2}(S32) = \mathrm{B}$;

$f_C^{\leq 2}(S45) = f_C^{\leq 2}(S47) = f_C^{\leq 2}(S49) = f_C^{\leq 2}(S50) = \mathrm{B}$; $f_C^{\leq 2}(S21) = f_C^{\leq 2}(S46) = \mathrm{B}$; $f_C^{\leq 2}(S30) = \mathrm{B}$; $f_C^{\leq 2}(S38) = \mathrm{B}$;

Step. 2-3. According to Strategy 1-2 and Strategy 2-2, we obtain the optimized downward certain rules:

(C $\leq$ 2) => (D $\leq$ II) (S34; S39; S42; S43; || S41; S44; || S20; S27; || S24; S25; S28; S29)

(B $\leq$ 2) => (D $\leq$ II) (S36; S40; || S19; S22; S23; S33; || S31; S32; || S45; S47; S49; S50; || S21; S46; || S38; || S30)

These certain rules denoted by $CR_2^{\leq}$, are induced from $\underline{C}(Cl_2^{\leq})$ with assignment of "at most Class II". In summary, we generated all certain rules from four C-lower approximations. The confidence degrees of these rules are equal to one. The detailed information is shown in Table 4-3.

Table 4-3 Induction of certain decision rules

Certain rules	Condition criteria A	B	C	Assignment of sorting	Strength	Coverage
[C1]	≥ 3	≥ 4	≥ 3	$\geq$ I	0.14	0.3889
[C2]		≥ 3	≥ 4	$\geq$ I	0.12	0.3333
[C3]	≥ 4	≥ 4		$\geq$ I	0.06	0.1667
[C4]		≥ 4		$\geq$ II	0.28	0.4242
[C5]			≥ 4	$\geq$ II	0.24	0.3636
[C6]	≤ 1	≤ 2		$\leq$ III	0.04	0.1176
[C7]		≤ 3	≤ 2	$\leq$ III	0.12	0.3529
[C8]		≤ 2		$\leq$ II	0.32	0.5000
[C9]			≤ 2	$\leq$ II	0.24	0.3750

(C) Criteria reduction

We calculate three union-based reducts by using the superiority and inferiority functions below.

Superiority reduct is obtained by:

$$\delta^{\geq} = \mathop{\wedge}_{t=1,\ldots,l}^{t} \mathop{\wedge}_{x_i,x_j \in U}^{i,j} (\vee \delta_{ij}^{\geq t}) = \mathop{\wedge}_{t=2,\ldots,l}^{t} \mathop{\wedge}_{x_i \in \underline{C}(Cl_t^{\geq})}^{i} f_C^{\geq t}(x_i) = (\mathrm{A} \wedge \mathrm{B} \wedge \mathrm{C}) \wedge (\mathrm{B} \wedge \mathrm{C}) \wedge (\mathrm{A} \wedge \mathrm{B}) \wedge \mathrm{B} \wedge \mathrm{C} \wedge ((\mathrm{A} \vee \mathrm{B}) \wedge \mathrm{C}) = \mathrm{A} \wedge \mathrm{B} \wedge \mathrm{C}$$

Inferiority reduct is obtained by:

$$\delta^{\leq} = \mathop{\wedge}_{t=1,\ldots,l}^{t} \mathop{\wedge}_{x_i,x_j \in U}^{i,j} (\vee \delta_{ij}^{\leq t}) = \mathop{\wedge}_{t=1,\ldots,l-1}^{t} \mathop{\wedge}_{x_i \in \underline{C}(Cl_t^{\leq})}^{i} f_C^{\leq t}(x_i) = (\mathrm{A} \wedge \mathrm{B}) \wedge (\mathrm{B} \wedge \mathrm{C}) \wedge \mathrm{C} \wedge \mathrm{B} = \mathrm{A} \wedge \mathrm{B} \wedge \mathrm{C}$$

Overall reduct is obtained by:

$$P^{\delta} = \delta^{\geq} \wedge \delta^{\leq} = (\mathrm{A} \wedge \mathrm{B} \wedge \mathrm{C}) \wedge (\mathrm{A} \wedge \mathrm{B} \wedge \mathrm{C}) = \mathrm{A} \wedge \mathrm{B} \wedge \mathrm{C}$$

Therefore, we use the overall reduct P^{δ} as the final reduct P, where $P = C = \{\mathrm{A, B, C}\}$.

(D) Believable rule induction

We calculate the believe factor of each rough boundary supplier as shown in Table 4-4. In this table, believe degree of S46 is equal to zero rather than the positive value. Therefore, S46 is not valuable for providing any assignment of sorting. Excepting S46, other suppliers are able to induce believable rules for sorting. According to Strategy 3-1 and Strategy 3-2, we generate believable rules with corresponding measuring degrees, as shown in Table 4-5.

Table 4-4 Believable factors of rough boundary suppliers

Separated boundary regions	Rough boundary suppliers	Believe factors: Positive $\mu(x)$	Negative $\nu(x)$	Hesitancy $\pi(x)$	Measuring degrees: Believe $S(x)$	Accuracy $H(x)$	Confidence $L(x)$
$Cl_3^{\geq} - \underline{P}(Cl_3^{\geq})$	S2; S14	9/13	1/13	3/13	8/13	10/13	12/13
	S7	6/8	1/8	1/8	5/8	7/8	7/8
	S8; S12	13/22	4/22	5/22	9/22	17/22	18/22
$Cl_2^{\geq} - \underline{P}(Cl_2^{\geq})$	S2; S14	9/13	1/13	3/13	8/13	10/13	12/13
	S7	6/8	1/8	1/8	5/8	7/8	7/8
	S8; S12	14/22	3/22	5/22	11/22	17/22	19/22
	S21	13/19	2/19	4/19	11/19	15/19	17/19
	S30	20/34	9/34	5/34	11/34	29/34	25/34
$Cl_1^{\leq} - \underline{P}(Cl_1^{\leq})$	S35; S37	8/14	3/14	3/14	5/14	11/14	11/14
	S38	2/4	1/4	1/4	1/4	3/4	3/4
	S45; S47; S49; S50	2/8	1/8	5/8	1/8	3/8	7/8
	S46	**2/10**	**2/10**	**6/10**	**0/10**	**4/10**	**8/10**
	S48	8/24	7/24	9/24	15/24	15/24	17/24

$Cl_2^{\leq} - \underline{P}(Cl_2^{\leq})$	S26	14/19	2/19	3/19	12/19	16/19	17/19
	S35; S37	10/14	2/14	2/14	8/14	12/14	12/14
	S48	16/24	5/24	3/24	11/24	21/24	19/24

Table 4-5 Induction of believable decision rules

Believable rules	Conditional criteria A	B	C	Assignment of sorting	Confidence degree	Accuracy degree	Base(s) of rules
[B1]	≥ 4	≥ 3	≥ 3	≥ I	0.9231	0.7692	S2; S14
[B2]	≥ 5	≥ 3	≥ 3	≥ I	0.8750	0.8750	S7
[B3]	≥ 1	≥ 3	≥ 3	≥ I	0.8182	0.7727	S8; S12
[B4]	≥ 4	≥ 3	≥ 3	≥ II	0.9231	0.7692	S2; S14
[B5]	≥ 5	≥ 3	≥ 3	≥ II	0.8750	0.8750	S7
[B6]	≥ 1	≥ 3	≥ 3	≥ II	0.8636	0.7727	S8; S12
[B7]	≥ 4	≥ 2	≥ 3	≥ II	0.8947	0.7895	S21
[B8]	≥ 2	≥ 1	≥ 3	≥ II	0.7353	0.8529	S30
[B9]	≤ 2	≤ 3	≤ 3	≤ III	0.7857	0.7857	S35; S37
[B10]	≤ 2	≤ 2	≤ 3	≤ III	0.7500	0.7500	S38
[B11]	≤ 3	≤ 2	≤ 3	≤ III	0.8750	0.3750	S45; S47; S49; S50
[B12]	≤ 5	≤ 3	≤ 3	≤ III	0.7083	0.6250	S48
[B13]	≤ 2	≤ 4	≤ 3	≤ II	0.8947	0.8421	S26
[B14]	..2	≤ 3	≤ 3	≤ II	0.8571	0.8571	S35; S37
[B15]	≤ 5	≤ 3	≤ 3	≤ II	0.7917	0.8750	S48

4.4.3 Supplier Selection via the Proposed Sorting Scheme

In this section, we use the BRSA model including the proposed sorting scheme for SS. The target is two-folds: grading existing suppliers and grading potential suppliers. The problem-solving performance is examined from two aspects: (1) the accuracy rate of grading existing suppliers (the classification ability) and (2) the consistency and stability of grading potential suppliers (the prediction ability). For comparisons, we also examine four existing rule-based approaches which are (1) C-DRSA with the standard scheme, (2) C-DRSA with the extended scheme, (3) VC-DRSA with the standard scheme, and (4) VC-DRSA with the extended scheme.

Preliminary settings: Since C-DRSA is a special case of VC-DRSA, the consistent level L=1.0 can represent C-DRSA and L<1.0 can represent VC-DRSA, for example, L=0.9, L=0.8 and L=0.7. The extended scheme is marked using the symbol "^", for example, L^=1.0, L^=0.9 and L^=0.8. The standard scheme is without this symbol. Our solution, BRSA with the proposed scheme, is shorted for BD.

(A) Classification: Grading the existing suppliers

C-DRSA aims to induce a minimal rule set with strict dominance principle that can guarantee all consistent objects classified into their original classes. VC-DRSA aims to induce the rules based on the released dominance principle that is controlled by the consistency level as threshold, in order to guarantee all consistent objects and several qualified inconsistent objects classified into their original classes. In our solution, the induced certain rules guarantee the correct classification of all consistent objects. Moreover, the induced believable rules can further guarantee that inconsistent objects are classified to the most suitable decision class, which cannot be handled by any previous solution. In this section, we illustrate this advantage and the application of supplier selection.

After rough approximations, the 50 existing suppliers can be classified into three categories according to their consistency below.

Suppliers are consistent over all related lower approximations:

{S1, S3, S4, S5, S6, S9, S10, S11, S13, S15, S16, S17, S18, || S19, S20, S22, S23, S24, S25, S27, S28, S29, S31, S32, S33, || S34, S36, S39, S40, S41, S42, S43, S44}.

Suppliers are consistent over only one lower approximation:

{S26, || S21, S30, || S38, S45, S46, S47, S49, S50}.

Suppliers are not consistent over any lower approximation:

{S2, S7, S8, S12, S14, S35, S37, S48}.

Suppliers in the first category can be correctly classified into their original classes when we employ our solution and all competitors. The appendix provides the classification results of all consistent suppliers. For suppliers in the second and third categories, Table 6 provides the classification results by respectively using (1) C-DRSA with the standard scheme (i.e. L=1.0); (2) C-DRSA with the extended scheme (i.e. L=^1.0); (3) VC-DRSA with the standard scheme (i.e. L=0.9 and L=0.8); (4) VC-DRSA with the extended scheme (i.e. L^=0.9 and L^=0.8). The original assignments are shown in the column "D". We note that some of assignments are missing (with the mark "*") and some of assignments are incorrect (in italics).

Table 4-6 Classification results of suppliers in second and third categories

Suppliers	D	L=1.0	L^=1.0	L=0.9	L^=0.9	L=0.8	L^=0.8	BD	ACR	ABR
S 2	I	*	*	I	I	I	I	I	*	**[B1]**[B3][B4][B6] [B7][B8][B12][B15]
S 7	I	*	*	I	I	I	I	I	*	**[B1]**[B2][B3][B4][B5] [B6][B7][B8] [B12][B15]
S 8	I	*	*	*	*III*	*	*II*	II+III	*	[B3][B6][B9][B12] **[B13]** [B14][B15]
S 12	I	*	*	*	*III*	*	*II*	II+III	*	[B3][B6][B9][B12] **[B13]**[B14][B15]
S 14	I	*	*	I	I	I	I	I	*	**[B1]**[B3][B4][B6] [B7][B8][B12][B15]
S 21	II	II+III	II	II+III	*III*	*	II	II+III	**[C8]**	[B7][B12][B15]
S 26	II	I+II	II	I+II	II	*	*I*	II+III	**[C4]**	[B6][B8][B13]
S 30	II	II+III	II	II+III	III	III	III	II+III	**[C8]**	[B8][B9][B10][B11] [B12][B13][B14][B15]
S 35	III	*	*	*	*	*	*II*	II+III	*	[B3][B6][B8][B9] [B12]**[B13]**[B14][B15]
S 37	III	*	*	*	*	*	*II*	II+III	*	[B3][B6][B8][B9] [B12]**[B13]**[B14][B15]
S 38	III	II+III	*II*	II+III	III	III	III	II+III	**[C8]**	[B8][B9][B10][B11] [B12][B13][B14][B15]
S 45	III	II+III	*II*	II+III	III	III	III	II+III	**[C8]**	[B8][B11][B12][B15]
S 46	III	II+III	*II*	II+III	III	*	*II*	II+III	**[C8]**	[B7] [B12] [B15]
S 47	III	II+III	*II*	II+III	III	III	III	II+III	**[C8]**	[B8][B11][B12][B15]
S 48	III	*	*	*I*	III	*I*	*I*	I	*	**[B1]**[B2][B3][B4][B5] [B6][B7][B8][B12] [B15]
S 49	III	II+III	*II*	II+III	III	III	III	II+III	**[C8]**	[B8][B11][B12][B15]
S 50	III	II+III	*II*	II+III	III	III	III	II+III	**[C8]**	[B8][B11][B12][B15]

Bold: the trusted rules; *Italics*: the incorrectly assignments; *: the missing assignments.

By using the proposed sorting scheme in section 4, the Affirmed Certain Rules (ACR) and the Affirmed Believable Rules (ABR) can be obtained as shown in Table 6. Specifically, suppliers in the second category employ Situation A of our scheme since they can be affirmed by one exact rule and several believable rules. Suppliers in the third category employ Situation C of our scheme since they only can be affirmed by believable rules. By using Algorithm 2, the trusted rule (in bold) can be generated and the corresponding assignments are shown in the column of "BD". Compared the results in the column of "D" and "BD", three suppliers (i.e. S8, S12 and S48) are with the incorrect assignments.

In this experiment, we examined our solution and competitors for our problem. As a result, all existing suppliers are graded as the classes I, II, or III. The results are the assignments in the form of a singleton class or a class union. Our target is to make assignments with the higher correct rate of sorting, which is defined as the ratio of the number of correctly classified suppliers over the total number of existing suppliers. Table 7 illustrates the statistical results. The correct rate of sorting is shown in bold. Obviously, BRSA with the proposed scheme provides the highest correct rate of 94%, which outperforms other six representative approaches.

Table 4-7 A comparison of correct classification rate

	Correctly Classified Objects		Incorrectly Classified Objects		Unknown Classified Objects	
C-DRSA (L=1.0)	42	**84%**	0	0	8	16%
C-DRSA (L^=1.0)	36	**72%**	6	12%	8	16%
VC-DRSA (L=0.9)	45	**90%**	1	2%	4	8%
VC-DRSA (L^=0.9)	43	**86%**	5	10%	2	4%
VC-DRSA (L=0.8)	41	**82%**	2	4%	7	14%
VC-DRSA (L^=0.8)	42	**84%**	8	16%	0	0
BRSA	47	**94%**	3	6%	0	0

(B) Prediction: Grading the potential suppliers

For prediction of potential suppliers using DRSA methodology, sorting results firstly should be stable to the greatest extent. It means each supplier should be affirmed by at least one decision rule. Secondly, the assignments should be consistency for satisfying the dominance principle. It means there are no suppliers with no worse evaluations on each considered criterion who are assigned to the worse class or class union. In following experiments, we evaluate the performance of our solution and competitors in predicting the assignments of potential suppliers.

We firstly examine C-DRSA with the standard scheme as the baseline approach. As a result, five decision descriptions cannot be affirmed by induced rules. They denote as N1=[1, 3, 3], N2=[2, 3, 3], N3=[3, 3, 3], N4=[4, 3, 3], and N5=[5, 3, 3]. Therefore, we need to compare sorting results over Ni for $i \in [1,2,3,4,5]$. Each Ni represents a collection of potential suppliers who are with the same decision description. Then, we examine our solution and competitors which are with different presetting consistency levels including L=1.00, L=0.95, L=0.90, L=0.85, L=0.80 L=0.75, and L=0.70. Both the standard scheme and the extended scheme (with mark "^") are taken into account. Table 8 shows the experimental results. The affirmed rules and confidence coefficients are given. The sorting results are illustrated in the last five columns. The asterisk "*" means it fails to provide any assignment of sorting.

Table 4-8 Examination of prediction ability of various decision models

	Induced decision rules	Confidence	N1	N2	N3	N4	N5
L=1.0	*	*	*	*	*	*	*
L^=1.0			*	*	*	*	*
L=0.95	*	*	*	*	*	*	*
L^=0.95			*	*	*	*	*
L=0.90	(A ≥ 4) & (B ≥ 3) => (D ≥ I)	0.9231	*	*	*	I	I
L^=0.90			*	*	*	I	I
L=0.85	(A ≥ 4) & (B ≥ 3) => (D ≥ I) (A ≥ 4) => (D ≥ II) (B ≥ 3) & (C ≥ 3) => (D ≥ II) (A ≤ 2) => (D ≤ II)	0.9231 0.8947 0.8636 0.8947	II	II	I+II	I	I
L^=0.85			II	II	II	I	I
L=0.80	(B ≥ 3) & (C ≥ 3) => (D ≥ I) (B ≥ 4) => (D ≥ II) (C ≥ 4) => (D ≥ II) (A ≥ 4) => (D ≥ II) (B ≥ 3) & (C ≥ 3) => (D ≥ II) (A ≥ 4) => (D ≥ II)	0.8182 1.0000 1.0000 0.8947 0.8636 0.8947	I	I	I	I	I
L^=0.80			II	II	I	I	I
L=0.75	(B ≥ 3) & (C ≥ 3) => (D ≥ I) (A ≤ 2) & (B ≤ 3) => (D ≤ III) …	0.8182 0.7857 …	*	*	I	I	I
L^=0.75			II	II	II	II	II
L ≤ 0.70	(B ≥ 3) & (C ≥ 3) => (D ≥ I) (B ≤ 3) & (C ≤ 3) => (D ≤ III)…	0.8182 0.7083 …	*	*	*	*	*
L^ ≤ 0.70			II	II	II	II	II
BRSA	[B1][B2][B3][B4][B5][B6][B7] [B8][B9][B12][B13][B14][B15]	$L(x)$	**II+III**	**II+III**	**I+II**	**I**	**I**

We analyze the experimental results in the following. Firstly, C-DRSA with two sorting schemes (see row of "L=1.0" and "L^=1.0") fails to provide assignments of all Ni for i ∈ [1,2,3,4,5]. It is the baseline solution with the worst problem-solving performance compared with other proposals.

Secondly, VC-DRSA with the standard scheme cannot provide an effective assignment. The comparisons of sorting results are shown in Table 9. We note VC-DRSA with the standard scheme fails to provide assignments of all Ni if the consistency level is 0.95 or lower than 0.70. Taking N1 as an example, the sorting result is Class II when L=0.85 whereas Class I when L=80. It means the results of sorting are unstable when varying L which usually needs to be preset in practice. This situation also happens over N2 and N3. Moreover, dominance relations of Ni are easily acknowledged as N5>N4>N3>N2>N1 (">" means "superior to"). Based on the dominance principle, the result of sorting should be with the preference relation: N5 ≥ N4 ≥ N3 ≥ N2 ≥ N1 (" ≥ " means "no worse than"). However, this dominance principle is violated for example when L=0.95, L=0.90, L=0.80, L=0.75, and also L ≤ 0.70. Therefore, we note that the sorting results are non-consistency when using the approach of VC-DRSA with the standard scheme.

Table 4-9 The sorting results using VC-DRSA with the *standard* scheme

	L=0.95	L=0.90	L=0.85	L=0.80	L=0.75	L ≤ 0.70	**BRSA**
N1	*	*	II	I	*	*	**II+III**
N2	*	*	II	I	*	*	**II+III**
N3	*	*	I+II	I	I	*	**I+II**
N4	*	I	I	I	I	*	**I**
N5	*	I	I	I	I	*	**I**

Thirdly, the approach of VC-DRSA with the extended scheme aims to provide a singleton class as sorting results. As experiments shown, its performance is better than other three competitors. However, this solution fails to provide assignments of all supplier Ni when L^=0.95. When L=0.75 or L $\leq$ 0.70, the assignment of Ni are all Class II which cannot make any distinction on preference. As shown in Table 10, such unsatisfactory results of sorting also appeared in N3, N4, and N5. Moreover, the sorting results by using this approach will still unstable when changing the presetting “L”. In other words, DMs will never find the most suitable “L” unless all possible “L” are examined. In this experiment, we note that only presetting L^=0.85/L=0.85 can provide a reasonable prediction.

Table 4-10 The sorting results using VC-DRSA with the *extended* scheme

	L^=0.95	L^=0.90	L^=0.85	L^=0.80	L^=0.75	L^ $\leq$ 0.70	**BRSA**
N1	*	*	II	II	II	II	**II+III**
N2	*	*	II	II	II	II	**II+III**
N3	*	*	II	I	II	II	**I+II**
N4	*	I	I	I	II	II	**I**
N5	*	I	I	I	II	II	**I**

Finally, we comment our solution. Using BRSA with the proposed scheme, the results of sorting are given as follows: both N1 and N2 are assigned to the class union II+III; N3 is assigned to the class union I+II; both N4 and N5 are assigned to the singleton Class I. Such result outperforms other competitors since they are stable and consistent: (1) potential suppliers Ni for i $\in$ [1,2,3,4,5] can be affirmed by one univocal believable rule which is screened by using our scheme (i.e. Algorithm 2); (2) all assignments of Ni comply with the dominance principle. Considering examined competitors in our experiments, we note that VC-DRSA-based solutions have better performance than DRSA-based solutions. The former improves the opportunity of discovering stronger rule patterns through relaxing the strict dominance relation. However, the uncertain information derived from rough boundary regions cannot be extracted insufficiently. Outperforming all these competitors, our solution can invariably provide the stable and consistent sorting results for prediction of potential suppliers. The sufficient utilization of information and the structured discovery of useful knowledge in the process of rule induction and sorting scheme make the proposed BRSA decision model be the most suitable solution for our problems.

4.5 Summary

This work makes use of dominance-based rough set methodology to address the supplier selection problem in a new paradigm. That is, to evaluate both existing suppliers and potential suppliers with a prior information of a collection of existing suppliers and their grades, and also to classify and predict the grades of pontential suppliers. We proposed a new Believable Rough Set Approach (BRSA) decision model, in order to pursue the better performance than other representative decision models which include C-DRSA and VC-DRSA respectively joining with two kinds of sorting schemes. In BRSA decision model, a unified method is initially provided to evaluate criteria and induce certain decision rules through rough approximations. Then, believable rules as a type of uncertain rule are induced with the assistance of the new concept of believe factor. Thus, both certain and uncertain information are extracted in a form of “if-then” decision rules. Lastly we provided a new sorting scheme of rule utilization for the classification of existing suppliers as well as the prediction of potential suppliers. The significance of this work includes (i) examination of various representative dominance-based rough set approaches for supplier selection, (ii) proposing a novel and complete BRSA decision model, and demonstration of its problem-solving process.

CHAPTER FIVE

The Rule-based Uncertain Group Decision Model for Warehouse Evaluation

5.1 Overview

In Chapter 3 and Chapter 4, we have developed the rule-based approaches for multicriteria ranking and sorting. This chapter and the next chapter will concern multicriteria decision analysis under uncertain environments (i.e. intuitionstic fuzzy decision information). In this chapter, we consider the issue of warehouse evaluation towards successful logistic and supply chain management. Suppose a company has managed a chain of owned warehouses, and now this company is in need of acquiring some new and profitable warehouse adding to its operation chain. A key business decisions here is how to choose the most profitable warehouses from a number of potential warehouses. In reality, the challenge is that the future profitability is unpredictable. Therefore, it is infeasible to rank potential warehouses directly for choice. To address such a problem, this chapter proposes a new rule-based decision model. This model includes the following characteristics: (i) decision information is provided via interval-valued intuitionistic fuzzy values; (ii) multiple experts as a group of DMs are involved; (iii) both *subjective* evaluations from experts and *objective* data of historical profitability are employed; (iv) both *certain* and *uncertain* information are exploited. The core decision mechanism is, making use of uncertain information of owned warehouses, to induce a collection of “if…then…”rules, and subsequently to exploit these rules for prediction of preference orders of all potential warehouses. Therein, we develop and integrate multiple techniques for the purposes of (a) aggregation of uncertain information; (b) construction of pairwise comparison; (c) induction of certain and uncertain rules; and (d) decision rules exploitation. We finally elaborate our discussion with an example illustrating the application of the proposed decision mechanism to supply-chain domain problems.

5.2 Background

Logistic and supply chain management (LSCM) plays the foundational role in today's global market. It has brought tremendous impact to organizational performance in terms of multi-dimensions such as price, quality, responsiveness, and flexibility. Broadly speaking, LSCM contains various issues like customer relations and service, demand and supply planning, inventory control, supplier selection, and warehouse management. Therein, as a typical problem regarding MCDM (Figueira, Greco, & Ehrgott, 2005), supplier selection has been comprehensively studied in literature (Chang & Hung, 2010; Amin, Rzami, & Zhang, 2011; Chai, Liu, & Xu, 2012; Yucel & Guneri, 2011; Buyukozkan & Cifci, 2012). In this chapter, in terms of MCDM methodology, we investigate a key decision-making issue towards successful LSCM, which is *warehouse evaluation*.

The problem being solved can be outlined as follows. Suppose there are several existing warehouses (we call them *owned warehouses*) in possession by a company. For business expansion, this company wants to buy some profitable warehouse from a number of potential warehouses (we call them *alternatives*). The realistic challenge is that the information about profitability of alternatives is unavailable and unpredictable. Under such a circumstance, the core difficulty is how to acquire the preference order of all alternatives in accordance with their profitability, and consequently determine which one(s) should be bought-in. Obviously this is a valuable and realistic decision problem in LSCM. Considering the retail companies such as Wal-Mart, Carrefour, and 7-Eleven, they possess a number of warehouse stores, conveniences stores, or supermarkets (we uniformly call them warehouses). In this case, they usually consider business expansion which requires decision making to acquire additional and profitable warehouses. After collection of the information about several potential warehouses, the core problem will be warehouse selection amid the lacking of related information on their profitability. The similar problems also arise in stores evaluation for fast-food-chain companies such as Mcdonald's or Kentucky Firied Chicken.

In this work, we provide a solution for solving the raised problem via knowledge discovery and utilization. The decision knowledge in the form of "if… then …" rules are generated based on known information of owned warehouses. The knowledge is then utilized for predicting the preference order of alternatives according to their profitability. We briefly outline this problem-solving process as follows. Firstly, qualified experts are invited to assess both owned warehouses and alternatives according to criteria. The preset criteria can represent the key influence factors of profitability. Secondly, we collect historical profitability records of owned warehouses, for example return-on-equity ratio. Thirdly, we extract rules based on both experts' evaluations (subjective information) and historical profitability of owned warehouses (objective information). Such obtained rules can preserve the cause-and-effect relations between the evaluation values and the profitability. Fourthly, we develop techniques to apply the induced rules for getting the preference order of all alternatives.

Preference expression is a key issue in MCDM. The invited experts need to express their evaluations with regard to each alternative under multiple criteria. Possible expression tools include real numbers (Chai, Liu, & Li, 2012), linguistic variables (Greco, Matarazzo, & Slowinski, 2005), vague values (Zhang, Zhang, Kai, & Lu, 2009), grey numbers (Bai & Sarkis, 2010), triangular fuzzy numbers (Yang, Chiu, Tzeng, & Yeh, 2008). However, these tools may face challenges since the following situations usually happen in practice: (a) experts may not possess a sufficient preparation of knowledge, or cannot acquire necessary details of alternatives; (b) experts may not explicitly discriminate the degree of preference, or to what extent one alternative is superior to others; (c) experts may not one hundred percent sure about their judgments; (d) experts may hold the opposite views regarding one alternative simultaneously. Some literature (Chen, 2011a; Chai, Liu, & Xu, 2012) has considered to employ intuitionistic fuzzy set (IFS) as preference expression tool and already got appreciable results. As the extended Zadeh's fuzzy set, IFS (Atanassov, 1986) is characterized by the membership function and the non-membership function. Atanassov and Gargov (1989) introduce interval-valued intuitionistic fuzzy set (IVIFS), which consists of a membership function and a non-membership function, whose values are intervals rather than exact numbers. Compared with fuzzy set and IFS, IVIFS is more variable for depicting preference relations and consequently is a more suitable mathematical tool for expressing subjective preferences. In this chapter, we address warehouse evaluation under uncertain environments; specifically the circumstance is that experts make use of IVIFSs as tools for their preference expression.

After necessary decision preparations including historical records of profitability; a collection of potential warehouses; construction of proper evaluation criteria; and expert evaluations in the form of IVIFSs, the next stage is to face three challenges: (a) how to determine the importance of experts and construction of expert-aggregated decision matrix; (b) how to induce decision rules based on owned warehouse; (c) how to apply acquired rules for getting preference order of all alternatives. In this work, we propose a complete decision model that has overcome these challenges remarkably. Firstly, we determine the weights of experts and aggregate intuitionistic fuzzy information via the weighted arithmetic average operator. Second, we conduct pairwise comparison with respect to owned warehouses and alternatives, respectively. Third, we adequately utilize both certain and uncertain decision information provided by owned warehouses, to generate two types of decision rules: certain rules and believable rules. Finally, we develop the method of rule application to determine the preference order of alternatives.

The rest of this chapter is organized as follows. In the next section, we revisit the IVIFS theory and the principles of the rule-based approach. They are preliminaries when constructing the proposed decision model. In Section 5.4, we model the raised decision problem and outline our proposed solution. Section 5.5 establishes the problem-solving decision model step by step. Section 5.6 presents an illustrative example with numerical calculation. We conclude the chapter in Section 5.7.

5.3 Decision Preliminaries

5.3.1 The Rule-based Rough Set Approach

Previous works on rule-based methods have been well studied in the context of data engineering and database. Some representative works include decision tree methods (Wang, Dong, & Yan, 2011) and classical rough sets (Pawlak & Skowron, 2007). However, both of them do not take human preference into account. Dominance-based rough set approach (DRSA) (Greco, Matarazzo, & Slowinski, 2001; 2005; Slowinski, Greco, & Matarazzo, 2009) utilizes preference-related dominance relation as a substitute of classical binary-relation-based indiscernibility. The induced "if… then…" rules are more suitable for decision-making towards preference-related decision table. In this section, we revisit principles of DRSA methodology as the preliminary.

The target of DRSA is to induce "if…then…" rules which can preserve the relations between two types of criteria: condition criteria and decision criteria. Generally, the antecedent (i.e. the part of "if…") refers to several condition criteria and the predecessor (i.e. the part of "then…") refers to just one singleton decision criterion. Information within both of two types of criteria, in the form of numerical data, symbol, or linguistic variables, is ordered according to human preference. Formally, a decision table is the 4-tuple $S = \langle U, Q, f \rangle$, which includes (i) a finite set U of objects x, for $x \in U$; (ii) a finite condition criteria set $C \neq \varnothing$ and one singleton decision criterion $D = \{d\}$, for $q \in Q = C \cup \{d\}$; (iii) information values $f_q(x)$ with respect to the criterion q and the object x. According to all $f_d(x)$ for $x \in U$, the universe of objects can be partitioned into a finite number of decision classes $Cl_t \in CL$ for $t = 1, ..., l$. Thus, each object x from U belongs to *one and only one* class Cl_t. Since information values under the decision criterion $f_d(x)$ are preference-ordered, we have that Cl_s is superior to Cl_t if $s > t$. The upward and downward unions of decision classes can be represented respectively as: $Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s$, $Cl_t^{\leq} = \bigcup_{s \leq t} Cl_s$, where $t = 1, ..., l$. The following equalities are valid: $Cl_1^{\leq} = Cl_1$; $Cl_l^{\geq} = Cl_l$; $Cl_t^{\geq} = U - Cl_{t-1}^{\leq}$; $Cl_t^{\leq} = U - Cl_{t+1}^{\geq}$; $Cl_1^{\geq} = Cl_l^{\leq} = CL$; $Cl_0^{\leq} = Cl_{l+1}^{\geq} = \varnothing$.

Dominance relation of DRSA can be defined according to preference-ordered values under condition criteria. Specifically, for the objects $x, y \in U$ and the criterion $q \in C$, we say $f_q(x) \geq f_q(y)$ represents $f_q(x)$ is at least as good as $f_q(y)$. If it exists $f_q(x) \geq f_q(y)$ for all $q \in P \subseteq C$, we say the object x

dominates the object y, denoted as xD_Py. We can thus define two object sets related to the object x as the dominating set $D_P^+(x)=\{y\in U: yD_Px\}$ and the dominated set $D_P^-(x)=\{y\in U: xD_Py\}$.

The core procedure of DRSA is rough approximation. As we reviewed, according to the decision criterion $\{d\}$, the universe of objects U has been partitioned into several decision classes Cl_t. The target of rough approximation is, on the basis of the partitioned universe, to refine (or called reclassify) all objects into upper or lower rough approximations. The notions are provided as follows:

(1) P-lower approximations with respect to $Cl_t^{\geq}$ and $Cl_t^{\leq}$:

$\underline{P}(Cl_t^{\geq})=\{x\in U: D_P^+(x)\subseteq Cl_t^{\geq}\}$; $\underline{P}(Cl_t^{\leq})=\{x\in U: D_P^-(x)\subseteq Cl_t^{\leq}\}$.

(2) P-upper approximations with respect to $Cl_t^{\geq}$ and $Cl_t^{\leq}$:

$\overline{P}(Cl_t^{\geq})=\{x\in U: D_P^-(x)\cap Cl_t^{\geq}\neq\varnothing\}$; $\overline{P}(Cl_t^{\leq})=\{x\in U: D_P^+(x)\cap Cl_t^{\leq}\neq\varnothing\}$.

(3) Rough boundary regions with respect to $Cl_t^{\geq}$ and $Cl_t^{\leq}$:

$Bn_P(Cl_t^{\geq})=\overline{P}(Cl_t^{\geq})-\underline{P}(Cl_t^{\geq})$; $Bn_P(Cl_t^{\leq})=\overline{P}(Cl_t^{\leq})-\underline{P}(Cl_t^{\leq})$.

In addition, the following equalities are valid: $Bn_P(Cl_t^{\geq})=Bn_P(Cl_{t-1}^{\leq})=\overline{P}(Cl_t^{\geq})\cap\overline{P}(Cl_{t-1}^{\leq})$; $\underline{P}(Cl_t^{\geq})\subseteq Cl_t^{\geq}\subseteq\overline{P}(Cl_t^{\geq})$; $\underline{P}(Cl_t^{\leq})\subseteq Cl_t^{\leq}\subseteq\overline{P}(Cl_t^{\leq})$; $\underline{P}(Cl_t^{\geq})=U-\overline{P}(Cl_{t-1}^{\leq})$; $\underline{P}(Cl_t^{\leq})=U-\overline{P}(Cl_{t+1}^{\geq})$; $\overline{P}(Cl_t^{\geq})=U-\underline{P}(Cl_{t-1}^{\leq})$; $\overline{P}(Cl_t^{\leq})=U-\underline{P}(Cl_{t+1}^{\geq})$.

5.3.2 Interval-valued Intuitionistic Fuzzy Set

In this chapter, we consider the situation that the experts' subjective evaluations are expressed by interval-valued intuitionistic fuzzy set (IVIFS). We firstly revisit the basic notions of IVIFS (Atanassov & Gargov, 1989; Xu & Yager, 2009) as preliminaries.

Regarding a set X, an IVIFS $\tilde{A}$ over X is an object having the form:

$$\tilde{A}=\{\langle x,\tilde{\mu}_{\tilde{A}}(x),\tilde{v}_{\tilde{A}}(x)\rangle \mid x\in X\},$$

s.t. $\tilde{\mu}_{\tilde{A}}(x)=[\tilde{\mu}_{\tilde{A}}^L(x),\tilde{\mu}_{\tilde{A}}^U(x)]\subset[0,1]$, $\tilde{v}_{\tilde{A}}(x)=[\tilde{v}_{\tilde{A}}^L(x),\tilde{v}_{\tilde{A}}^U(x)]\subset[0,1]$, $\tilde{\mu}_{\tilde{A}}^U(x)+\tilde{v}_{\tilde{A}}^U(x)\leq 1$,

where $\tilde{\mu}_{\tilde{A}}^L(x)=\inf\tilde{\mu}_{\tilde{A}}(x)$, $\tilde{\mu}_{\tilde{A}}^U(x)=\sup\tilde{\mu}_{\tilde{A}}(x)$, $\tilde{v}_{\tilde{A}}^L(x)=\inf\tilde{v}_{\tilde{A}}(x)$, and $\tilde{v}_{\tilde{A}}^U(x)=\sup\tilde{v}_{\tilde{A}}(x)$.

Particularly, an IVIFS $\tilde{A}$ is reduced to an IFS if $\tilde{\mu}_{\tilde{A}}(x)=\tilde{\mu}_{\tilde{A}}^L(x)=\tilde{\mu}_{\tilde{A}}^U(x)$ and $\tilde{v}_{\tilde{A}}(x)=\tilde{v}_{\tilde{A}}^L(x)=\tilde{v}_{\tilde{A}}^U(x)$ are valid. The complement of $\tilde{A}$ is denoted as $\tilde{A}^c$, where $\tilde{A}^c=\{\langle x,\tilde{v}_{\tilde{A}}(x),\tilde{\mu}_{\tilde{A}}(x)\rangle \mid x\in X\}$.

Extracted the fundamental element from IVIFS, the interval-valued intuitionistic fuzzy value (IVIFV) (Xu, 2010) can be denoted as: $\tilde{a}=(\tilde{\mu}_{\tilde{a}},\tilde{v}_{\tilde{a}})$, where $\tilde{\mu}_{\tilde{a}}=[\tilde{\mu}_{\tilde{a}}^L,\tilde{\mu}_{\tilde{a}}^U]\subset[0,1]$, $\tilde{v}_{\tilde{a}}=[\tilde{v}_{\tilde{a}}^L,\tilde{v}_{\tilde{a}}^U]\subset[0,1]$, $\tilde{\mu}_{\tilde{a}}^U+\tilde{v}_{\tilde{a}}^U\leq 1$. Regarding two IVIFVs $\tilde{a}_i=([\tilde{\mu}_{\tilde{a}_i}^L,\tilde{\mu}_{\tilde{a}_i}^U],[\tilde{v}_{\tilde{a}_i}^L,\tilde{v}_{\tilde{a}_i}^U])$ for $i=1,2$, we have $\tilde{a}_1=\tilde{a}_2$ if and only if $\tilde{\mu}_{\tilde{a}_1}^L=\tilde{\mu}_{\tilde{a}_2}^L$, $\tilde{\mu}_{\tilde{a}_1}^U=\tilde{\mu}_{\tilde{a}_2}^U$, $\tilde{v}_{\tilde{a}_1}^L=\tilde{v}_{\tilde{a}_2}^L$, $\tilde{v}_{\tilde{a}_1}^U=\tilde{v}_{\tilde{a}_2}^U$. The uniform distance between $\tilde{a}_1$ and $\tilde{a}_2$ can be calculated via:

$$d(\tilde{a}_1,\tilde{a}_2)=\left[\frac{1}{4}\left(|\tilde{\mu}_{\tilde{a}_1}^L-\tilde{\mu}_{\tilde{a}_2}^L|^\lambda+|\tilde{\mu}_{\tilde{a}_1}^U-\tilde{\mu}_{\tilde{a}_2}^U|^\lambda+|\tilde{v}_{\tilde{a}_1}^L-\tilde{v}_{\tilde{a}_2}^L|^\lambda+|\tilde{v}_{\tilde{a}_1}^U-\tilde{v}_{\tilde{a}_2}^U|^\lambda\right)\right]^{1/\lambda},\quad \lambda\geq 1. \qquad \text{Eq. (2-1)}$$

Specifically, for $\lambda=1$, the uniform distance is reduced to the normalized Hamming distance:

$$d_H(\tilde{a}_1,\tilde{a}_2)=\frac{1}{4}\left(|\tilde{\mu}^L_{\tilde{a}_1}-\tilde{\mu}^L_{\tilde{a}_2}|+|\tilde{\mu}^U_{\tilde{a}_1}-\tilde{\mu}^U_{\tilde{a}_2}|+|\tilde{v}^L_{\tilde{a}_1}-\tilde{v}^L_{\tilde{a}_2}|+|\tilde{v}^U_{\tilde{a}_1}-\tilde{v}^U_{\tilde{a}_2}|\right).$$

And, for $\lambda=2$, the uniform distance is reduced to the normalized Euclidean distance:

$$d_E(\tilde{a}_1,\tilde{a}_2)=\sqrt{\frac{1}{4}\left(|\tilde{\mu}^L_{\tilde{a}_1}-\tilde{\mu}^L_{\tilde{a}_2}|^2+|\tilde{\mu}^U_{\tilde{a}_1}-\tilde{\mu}^U_{\tilde{a}_2}|^2+|\tilde{v}^L_{\tilde{a}_1}-\tilde{v}^L_{\tilde{a}_2}|^2+|\tilde{v}^U_{\tilde{a}_1}-\tilde{v}^U_{\tilde{a}_2}|^2\right)}.$$

According to the above provided distances, the degree of similarity between $\tilde{a}_1$ and $\tilde{a}_2$ can be defined as follows:

$$s(\tilde{a}_1,\tilde{a}_2)=d(\tilde{a}_1,\tilde{a}_2^c)\Big/\left(d(\tilde{a}_1,\tilde{a}_2)+d(\tilde{a}_1,\tilde{a}_2^c)\right),$$

where $\tilde{a}_2^c$ be the complement of $\tilde{a}_2$. According to this definition, we can easily prove that the following assertions are valid.

(i) $0\le s(\tilde{a}_1,\tilde{a}_2)\le 1$; (ii) $s(\tilde{a}_1,\tilde{a}_2)=s(\tilde{a}_2,\tilde{a}_1)=s(\tilde{a}_2^c,\tilde{a}_1^c)=s(\tilde{a}_1^c,\tilde{a}_2^c)$; (iii) $s(\tilde{a}_1,\tilde{a}_2^c)=s(\tilde{a}_1^c,\tilde{a}_2)$

Especially, $s(\tilde{a}_1,\tilde{a}_2)>0.5$ means that $\tilde{a}_1$ is more similar to $\tilde{a}_2$ rather than $\tilde{a}_2^c$, and also $s(\tilde{a}_1,\tilde{a}_2)<0.5$ means that $\tilde{a}_1$ is more similar to $\tilde{a}_2^c$ rather than $\tilde{a}_2$.

5.4 An Overview of the Decision Model

Generally, there are three stages for solving decision problems: (i) problem construction, (ii) decision preliminary, and (iii) specific decision methods (Chai & Liu, 2012a). In order to cater for complex and uncertain decision environments, Chai, Liu, and Xu (2012) suggested that decision process should involve five analytic aspects on (i) decision problems, (ii) decision environment, (iv) decision group, (iii) decision scheme and, (v) group coordination and recommendation. In this section, we use a five-aspect analysis methodology to model the proposed problem and provide an overview of our problem-solving framework. In Figure 5-1, we illustrate the procedures of our solution.

(1) *Decision problem analyses*: Decision problems can be roughly divided into three categories: structured, semi-structured, and unstructured. The structured problem is well organized like structured decision tables, determinate DMs, and clear decision targets. The other two are usually in the form of text documents or interviewing dialogues. Obviously, our problem belongs to the structured decision problem.

(2) *Decision environment analyses*: Decision organizers play the key role. This role is usually played by managers of companies. The related issues should be clarified in this process.

a. Decision targets: The decision target is to obtain the preference order of all alternatives according to their profitability quantified in terms of the value of return-on-equity (RoE) ratio (in %).
b. Available resources: Company manages several similar-type warehouses (called owned warehouse) whose historical RoE ratios can be easily acquired.
c. Limitations: The profitability of alternatives is unknown.

(3) *Decision group analyses*: The experts should be selected according to their expertise, qualification, and experience. Their subjective evaluations significantly influence the reliability of final recommendations.

(4) *Decision scheme analyses*: Decision schemes are potential solutions that come from past experiences or new developed approaches. To address the present problem, the organizers invite qualified experts to assess both owned warehouses and alternatives by using IVIFS as tool for preference expression. We then develop a new rule-based decision model to extract rules and utilize them to rank alternative warehouses according to their profitability. In Figure 5-1, we outline the rule induction and utilization process in the top-right corner.

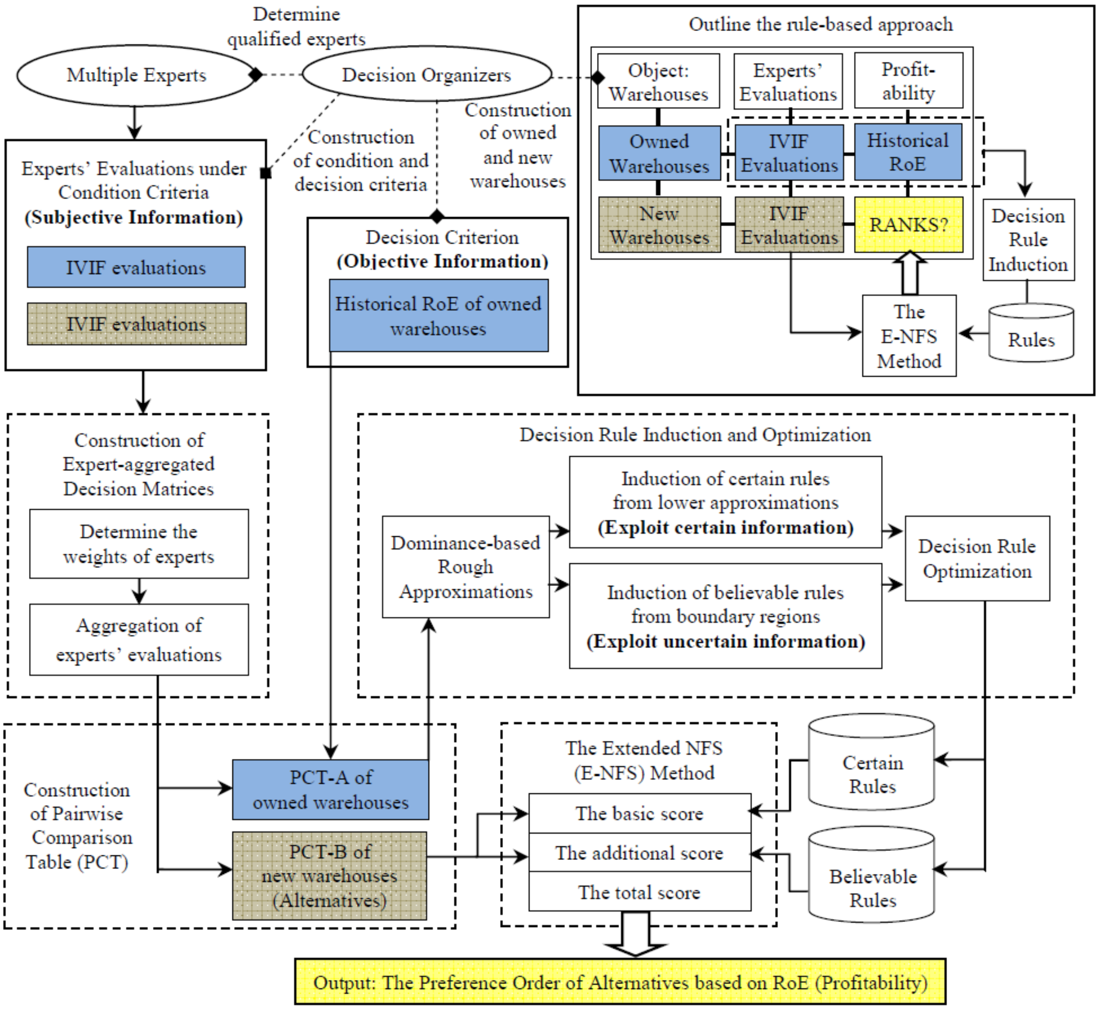


Fig 5-1 The overview of uncertain group decision model towards warehouse evaluation

(5) *Group coordination and recommendation*: This is the core procedure including group aggregation and decision recommendation. The proposed approach includes four main stages.

a. Construction of expert-aggregated decision matrices: We firstly determine the weights representing the importance of experts (Chen & Yang, 2011). Then we aggregate experts' evaluation by using interval-valued intuitionistic fuzzy aggregation operators (Xu, 2007d).

b. Construction of pairwise comparison table (PCT): We establish two kinds of PCTs (Greco, Matarazzo, & Slowinski, 1999) called PCT-A and PCT-B, which are based on the owned warehouses and alternatives, respectively. In PCT-A, the experts' evaluations are used to form the values of condition criterion and the historical RoE ratios are used to form the values of decision criterion. In PCT-B, there are just condition criteria constructed from experts' evaluations.

c. Decision rule induction and optimization: In this step, we induce decision rule and conduct optimization based on decision information of PCT-A. We firstly build the relations between subjective information and objective information via DRSA. Therein, certain rules are induced from certain information within lower approximations. Believable rules are induced from uncertain information within boundary regions. We then optimize the induced rules through reduction of redundant rules. As a result, an optimized rule set can be obtained.

d. Recommendation via rule exploitation: We introduce an extended net flow score (NFS) procedure for rule exploitation. By this means, certain rules are used to calculate the basic score and believable rules are used to calculate the additional score. After summing, the total score can be used to rank alternatives.

5.5 Group Decision Process for Warehouse Evaluation

In this decision task, we firstly set the related key inputs: (a) the owned warehouse set M and the alternative set N, where $N \cap M = \varnothing$; let warehouses $Y_i \in \mathrm{N} = N \cup M$ for $i = 1,...,m$; (b) the condition criteria set C where criteria $C_i \in \mathrm{C}$ for $j = 1,...,n$; (c) the expert group Z where experts $e_k \in Z$ for $k = 1,...,t$; (d) the decision criteria set $DC = \{d\}$.

By using IVIFVs as expression tool, experts evaluate both owned warehouses and alternatives under a number of criteria. The decision values are denoted as $r_{ij}^{(k)} = ([\mu_{ij}^{L(k)}, \mu_{ij}^{U(k)}], [v_{ij}^{L(k)}, v_{ij}^{U(k)}])$ regarding the warehouse Y_i, the condition criterion C_j, and the expert e_k, where $[\mu_{ij}^{L(k)}, \mu_{ij}^{U(k)}] \subset [0,1]$, $[v_{ij}^{L(k)}, v_{ij}^{U(k)}] \subset [0,1]$, $\mu_{ij}^{U(k)} + v_{ij}^{U(k)} \le 1$, $i = 1,...,m$, and $j = 1,...,n$. For example, the decision value $r_{23}^{(1)} = \left(\left[\mu_{23}^{L(1)}, \mu_{23}^{U(1)}\right], \left[v_{23}^{L(1)}, v_{23}^{U(1)}\right]\right)$ represents the evaluation of the warehouse Y_2 by the expert e_1 under the criterion C_3. The interval-valued intuitionistic fuzzy (IVIF) decision matrix is denoted as $R^{(k)} = (r_{ij}^{(k)})_{m \times n}$. Therein, the interval value $[\mu_{ij}^{L(k)}, \mu_{ij}^{U(k)}]$ represents the membership degree of the most preferred profile under the criterion C_j; and the interval values $[v_{ij}^{L(k)}, v_{ij}^{U(k)}]$ represents the membership degree of the most non-preferred profile under the criterion C_j.

Step 1. Construction of expert-aggregated decision matrices

Note that different experts should have a different importance according to their expertise. In MCDM, such importance is represented as the weights of experts. To determine such weights, we firstly suppose all experts are with equal importance. Thus, the mean of t-experts' decision values can be calculated via:

$$(m_{ij})_{m\times n} = ([\mu_{ij}^{L'}, \mu_{ij}^{U'}],\ [v_{ij}^{L'}, v_{ij}^{U'}])_{m\times n}, \qquad \text{Eq. (3-1)}$$

s.t. $\mu_{ij}^{L'} = \frac{1}{t}\sum_{k=1}^{t}\mu_{ij}^{L(k)}$, $\mu_{ij}^{U'} = \frac{1}{t}\sum_{k=1}^{t}\mu_{ij}^{U(k)}$, $v_{ij}^{L'} = \frac{1}{t}\sum_{k=1}^{t}v_{ij}^{L(k)}$, $v_{ij}^{U'} = \frac{1}{t}\sum_{k=1}^{t}v_{ij}^{U(k)}$, $k=1,\dots,t$, $i=1,\dots,m$, $j=1,\dots,n$.

This t-expert mean is an aggregation of the t IVIF decision values. We then define the degree of similarity between the decision value $r_{ij}^{(k)}$ and the t-expert mean m_{ij} via:

$$s(r_{ij}^{(k)}, m_{ij}) = 1 - d(r_{ij}^{(k)}, m_{ij}) \Big/ \sum_{k=1}^{t} d(r_{ij}^{(k)}, m_{ij}), \qquad \text{Eq. (3-2)}$$

where $d(r_{ij}^{(k)}, m_{ij})$ is the distance between two IVIFVs $r_{ij}^{(k)}$ and m_{ij}, which can refer to Eq. (2-1).

Finally, the weight of the expert e_k for the warehouse Y_i under the criterion G_j is determined via:

$$w_{ij}^{(k)} = s(r_{ij}^{(k)}, m_{ij}) \Big/ \sum_{k=1}^{t} s(r_{ij}^{(k)}, m_{ij}). \qquad \text{Eq. (3-3)}$$

Remark that this method to determine the weights of experts is to reduce the influence of these unduly high or low decision values caused by experts' limited expertise. The closer the expert e_k 's decision value $r_{ij}^{(k)}$ from the t-expert mean m_{ij}, indicates the lesser the consideration of $r_{ij}^{(k)}$ as the outliner, and consequently, the higher is the weight of the expert e_k.

The next step is to aggregate multiple experts' opinions into an expert-aggregated IVIF decision matrix. Here, We employ the weighted arithmetic average operator IIFWA (Xu, 2007d; 2007e) to aggregate IVIF information. Considering the expert's weight w_{ij}^{k} and the decision values $r_{ij}^{(k)}$, the expert-aggregated decision values can be obtained via:

$$\bar{r}_{ij} = IIFWA_{w_{ij}^{k}}\left(r_{ij}^{(1)}, r_{ij}^{(2)}, \dots, r_{ij}^{(t)}\right) = \left(\left[1-\prod_{k=1}^{t}\left(1-\mu_{ij}^{L(k)}\right)^{w_{ij}^{k}}, 1-\prod_{k=1}^{t}\left(1-\mu_{ij}^{U(k)}\right)^{w_{ij}^{k}}\right], \left[\prod_{k=1}^{t}\left(v_{ij}^{L(k)}\right)^{w_{ij}^{k}}, \prod_{k=1}^{t}\left(v_{ij}^{U(k)}\right)^{w_{ij}^{k}}\right]\right). \qquad \text{Eq. (3-4)}$$

Finally, all acquired IVIFVs $\bar{r}_{ij} = ([a_{ij}, b_{ij}],\ [c_{ij}, d_{ij}])$ consist of the expert-aggregated IVIF decision matrix: $\left(\bar{r}_{ij}\right)_{m\times n}$.

Step 2. Construction of Pairwise Comparison Table

Regarding the definition of IVIFSs, we can easily obtain the ideal IVIFV and the anti-ideal IVIFV, represented as $a^{+} = ([1,1],[0,0])$ and $a^{-} = ([0,0],[1,1])$, respectively. Clearly, the value a^{+} be the complement of the value a^{-}, and vise versa. Here, we make use of the concept of ideal and anti-ideal points to help constructing the PCT.

The degree of similarity between the expert-aggregated decision value $\bar{r}_{ij} = ([a_{ij}, b_{ij}],[c_{ij}, d_{ij}])$ and the ideal point $a^{+} = ([1,1],[0,0])$ can be defined as:

$$s(\bar{r}_{ij}, a^{+}) = d(\bar{r}_{ij}, a^{-}) \Big/ \left(d(\bar{r}_{ij}, a^{+}) + d(\bar{r}_{ij}, a^{-})\right), \qquad \text{Eq. (3-5)}$$

s. t. $d(\bar{r}_{ij}, a^{+}) = \left[\frac{1}{4}\left(|a_{ij}-1|^{\lambda} + |b_{ij}-1|^{\lambda} + |c_{ij}|^{\lambda} + |d_{ij}|^{\lambda}\right)\right]^{1/\lambda}$ and $d(\bar{r}_{ij}, a^{-}) = \left[\frac{1}{4}\left(|a_{ij}|^{\lambda} + |b_{ij}|^{\lambda} + |c_{ij}-1|^{\lambda} + |d_{ij}-1|^{\lambda}\right)\right]^{1/\lambda}$,

where $\lambda = 1,2$ and $s(\bar{r}_{ij}, a^{+}) \in [0,1]$.

Note that $d(\overline{r}_{ij}, a^+)$ and $d(\overline{r}_{ij}, a^-)$ are the uniform distance that can be selected as the normalized Hamming distance ($\lambda = 1$) or the normalized Euclidean distance ($\lambda = 2$). Obviously, the larger $s(\overline{r}_{ij}, a^+)$ indicates that the warehouse Y_i preserves the more preferred decision value with respect to the criterion C_j.

Let the expert-aggregated decision value be $f_j(Y_i) = s(\overline{r}_{ij}, a^+)$ with respect to the warehouse Y_i and the criterion C_j, where $f_j(Y_i) \in [0,1]$, $i = 1,...,m$, $j = 1,...,n$. The dominance relation among $f_j(Y_i)$ can be defined as:

$$\left\{P_{C_i}^h : h = h_j(\mathrm{Y}_f) - h_j(\mathrm{Y}_g), C_j \in \mathrm{C}, \ j = 1,...,n\right\}, \qquad \text{Eq. (3-6)}$$

where Y_f and Y_g denote warehouses for $f, g = 1,...,m$, and $h \in [-1,1]$. Specifically, we have:

(a) $Y_f P_{C_j}^h Y_g$ for $h > 0$ means that Y_f is preferred to Y_g by degree h with respect to criteria C_j;

(b) $Y_f P_{C_j}^h Y_g$ for $h < 0$ means that Y_f is not preferred to Y_g by degree h with respect to criteria C_j;

(c) $Y_f P_{C_j}^h Y_g$ for $h = 0$ means that Y_f is with the same/similar preference to Y_g with respect to criteria C_j.

Based on the above defined dominance relations, we can construct two types of Pairwise Comparison Table (PCT) including: PCT-A with respect to the owned warehouses and PCT-B with respect to the new warehouses. The key components of PCT-A and PCT-B are constructed as following shows:

(1) The decision objects:

The objects of both PCT-A and PCT-B are no longer the warehouse Y_i, but, the ordered pairs of warehouses $(\mathrm{Y}_f, \mathrm{Y}_g)$ for $f, g = 1,...,m$. After pairwise comparisons, the number of pairs will be: $m_1^{m_1}$ in PCT-A for m_1 owned warehouses and $m_2^{m_2}$ in PCT-B for m_2 new warehouses, where $m_1 + m_2 = m$, $m_1, \ m_2 \neq 0$. In real applications, the self-comparison pairs like $(\mathrm{Y}_f, \mathrm{Y}_f)$ or $(\mathrm{Y}_g, \mathrm{Y}_g)$ are usually deleted.

(2) The criteria of PCT:

First in both PCT-A and PCT-B, the condition criteria are the same as that of original decision table, say C_j for $j = 1,...,n$. Secondly, the values of decision criterion in PCT-A are constructed according to the pairwise comparison of owned warehouses with respect to the original decision criterion d. As we mentioned, the decision values under d are usually the historical and preference-ordered data like ROE of warehouses. According to human preference, we have:

(a) $Y_f S\, Y_g$ for $f, g = 1,...,m_1$ means that Y_f is at least as good as Y_g with respect to the pair $(\mathrm{Y}_f, \mathrm{Y}_g)$;

(b) $Y_f S^c Y_g$ for $f, g = 1,...,m_1$ means that Y_f is not at least as good as Y_g with respect to the pair $(\mathrm{Y}_f, \mathrm{Y}_g)$.

Therefore the decision criterion in PCT-A consists of two values S and S^c with respect to the pairs of owned warehouses. Notice that there is no decision criterion within PCT-B.

(3) The decision values:

The universe of warehouse pairs are denoted as $\Re$. For $x \in \Re$, the decision values in both PCT-A and PCT-B are the degree $h_j(x) = h_j(Y_f) - h_j(Y_g)$ with respect to the criterion C_j, where the pair x is (Y_f, Y_g).

Step 3. Decision rule induction

The decision rules are induced from PCT-A and applied on PCT-B. Let $\Re_A$ represent the universe of pairs in PCT-A. For $x, y \in \Re_A$, if decision values $h_j(x) \geq h_j(y)$ with respect to $C_j \in P$, we say x dominates y, denoted as $xD_P y$. According to the principles of DRSA, we can define two x-related pair sets as: the dominating set $D_P^+(x) = \{y \in U : yD_P x\}$ and the dominated set $D_P^-(x) = \{y \in U : xD_P y\}$. Then, we can define the rough approximations preserving two values of decision criterion, S and S^c, as follows:

(a) The lower approximations with respect to S and S^c:

$$\underline{P}(S) = \{x \in U : D_P^+(x) \subseteq S\}\ ;\quad \underline{P}(S^c) = \{x \in U : D_P^-(x) \subseteq S^c\}\,. \qquad \text{Eq. (3-7)}$$

(b) The upper approximations with respect to S and S^c:

$$\overline{P}(S) = \{x \in U : D_P^-(x) \cap S \neq \varnothing\}\ ;\quad \overline{P}(S^c) = \{x \in U : D_P^+(x) \cap S^c \neq \varnothing\}\,.$$

(c) The rough boundary regions with respect to S and S^c:

$$Bn_P(S) = \overline{P}(S) - \underline{P}(S)\ ;\quad Bn_P(S^c) = \overline{P}(S^c) - \underline{P}(S^c)\,. \qquad \text{Eq. (3-8)}$$

Obviously, above definitions inherit all properties of classical rough approximation of DRSA like $Bn_P(S) = Bn_P(S^c) = \overline{P}(S) \cap \overline{P}(S^c)$. We illustrate such rough approximation using Figure 5-2. As shown, the universe $\Re_A$ is partitioned into two decision classes S and S^c. The process of rough approximations refines such partitions into three regions including: two lower approximations $\underline{P}(S)$ and $\underline{P}(S^c)$, as well as the boundary region $Bn_P(S)$ (or say $Bn_P(S^c)$). In addition, the boundary region can be considered as two parts: $S - \underline{P}(S)$ and $S^c - \underline{P}(S^c)$.

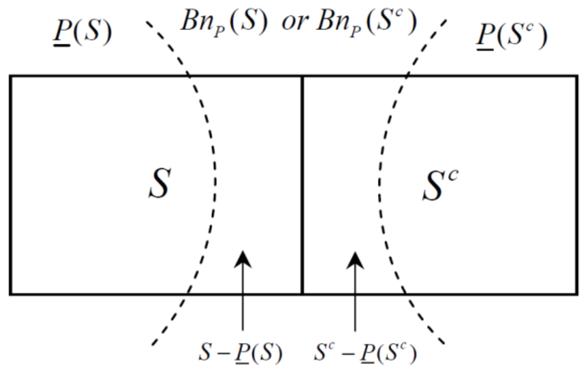


Fig 5-2 The illustration of rough approximation

The rough approximation builds the relations between two kinds of decision information coming from the decision criterion and the condition criteria, respectively. Based on above definitions, the pairs from the lower approximations are carrying *certain decision information*, because they satisfy the dominance principle. The pairs from the boundary region are carrying *uncertain decision information*, since they violate the dominance principle. In the following, we will induce two types of decision rules that preserve the relations between decision criterion and condition criteria: *certain rules* based on certain decision information within lower approximations, and *uncertain rules* based on *valuable* uncertain information within the boundary region.

(1) Certain rule induction

We firstly define discernibility matrices that include the superiority matrix with respect to S and the inferiority matrix with respect to S^c.

Superiority matrix: For $x \in \underline{P}(S)$, we have the superiority matrix as:

$Sup^{\succ}(S) = [m(x,y)]_{|\underline{P}(S)|\times|\overline{P}(S^c)|}$, s. t. $m(x,y) = \{C_j \in P : h_j(x) > h_j(y),\ x \in \underline{P}(S), y \in \overline{P}(S^c)\}$. Eq.(3-9)

Inferiority matrix: For $x \in \underline{P}(S^c)$, we have the inferiority matrix as:

$Inf^{\prec}(S^c) = [n(x,y)]_{|\underline{P}(S^c)|\times|\overline{P}(S)|}$, s. t. $n(x,y) = \{C_j \in P : h_j(x) < h_j(y),\ x \in \underline{P}(S^c), y \in \overline{P}(S)\}$. Eq.(3-10)

Superiority function: For $x \in \underline{P}(S)$, we have the superiority function as:

$f_S(x) = \wedge(\vee a:\ a \in m(x,y) \neq 0, y \in \overline{P}(S^c))$, where conjunction $\wedge$ and disjunction $\vee$. Eq.(3-11)

Inferiority function: For $x \in \underline{P}(S^c)$, we have the inferiority function as:

$f_{S^c}(x) = \wedge(\vee a:\ a \in n(x,y) \neq 0, y \in \overline{P}(S))$, where conjunction $\wedge$ and disjunction $\vee$. Eq.(3-12)

According to the superiority and inferiority functions, we can induce certain rules via the following strategies.

Strategy I: Upward certain rule:

Considering superiority function $f_S(x_i) = \wedge(\vee a) = a_1 \wedge \ldots \wedge a_n$ for $x_i \in \underline{P}(S)$, the value vector of x_i can be represented as: $V_S(x_i) = \left(h_1(x_i), h_2(x_i), \ldots, h_n(x_i)\right)$. Then, we can induce an upward certain rule preserving the object x: $IF\ h_1(x) \geq h_1(x_i)\ and\ h_2(x) \geq h_2(x_i)\ and\ \ldots\ and\ h_n(x) \geq h_n(x_i),\ THEN\ \ x \to S$.

Strategy II: Downward certain rule:

Considering inferiority function $f_{S^c}(x_i) = \wedge(\vee a) = a_1 \wedge \ldots \wedge a_n$ for $x_i \in \underline{P}(S^c)$, the value vector of x_i can be represented as: $V_{S^c}(x_i) = \left(h_1(x_i), h_2(x_i), \ldots, h_n(x_i)\right)$. Then, we can induce a downward certain rule preserving the object x: $IF\ h_1(x) \leq h_1(x_i)\ and\ h_2(x) \leq h_2(x_i)\ and\ \ldots\ and\ h_n(x) \leq h_n(x_i),\ THEN\ \ x \to S^c$.

(2) Believable rule induction

We firstly provide the definitions of believe factor with respect to all boundary pairs $x \in \left(S - \underline{P}(S)\right) \cup \left(S^c - \underline{P}(S^c)\right)$.

Upward believe factor: For $x \in S - \underline{P}(S)$, the upward believe factor can be defined as:

$$\beta(x \to S) = (\mu_S(x),\ v_S(x),\ \pi_S(x)), \quad \text{Eq. (3-13)}$$

s. t. $\mu_S(x)=\frac{|D_P^+(x)\cap \underline{P}(S)|}{|D_P^+(x)|}$, $v_S(x)=\frac{|D_P^+(x)\cap S^c|}{|D_P^+(x)|}$, $\pi_S(x)=\frac{|D_P^+(x)\cap [S-\underline{P}(S)]|}{|D_P^+(x)|}$.

Downward believe factor: For $x\in S^c-\underline{P}(S^c)$, the downward believe factor can be defined as:

$$\beta(x\to S^c)=(\mu_{S^c}(x),\ v_{S^c}(x),\ \pi_{S^c}(x)) , \qquad \text{Eq. (3-14)}$$

s. t. $\mu_{S^c}(x)=\frac{|D_P^-(x)\cap \underline{P}(S^c)|}{|D_P^-(x)|}$, $v_{S^c}(x)=\frac{|D_P^-(x)\cap S|}{|D_P^-(x)|}$, $\pi_{S^c}(x)=\frac{|D_P^-(x)\cap [S^c-\underline{P}(S^c)]|}{|D_P^-(x)|}$.

The symbol “$\to$” within believe factor $\beta(x\to S)$ and $\beta(x\to S^c)$ can be understood as “be assigned to” or “belongs to”. For $x\in\Re_A$, $\mu(x)$ (i.e. $\mu_S(x)$ and $\mu_{S^c}(x)$) is called the positive score ; $v(x)$ (i.e. $v_S(x)$ and $v_{S^c}(x)$) is called the negative score; $\pi(x)$ (i.e. $\pi_S(x)$ and $\pi_{S^c}(x)$) is called the hesitancy score. The form of believe factor can be regarded as an Intuitionistic Fuzzy Value (Chai, Liu, & Li, 2012).

According to three elements within the believe factor, we can further define three measurements.

Confidence degree: For $x\in\Re_A$, confidence degree of believe factor can be defined as:

$$L(x)=\mu(x)+\pi(x) . \qquad \text{Eq. (3-15)}$$

Specifically, we have $L(x\to S)=\mu_S(x)+\pi_S(x)$ and $L(x\to S^c)=\mu_{S^c}(x)+\pi_{S^c}(x)$.

Believe degree: For $x\in\Re_A$, believe degree of believe factor can be defined as:

$$B(x)=\mu(x)-v(x) . \qquad \text{Eq. (3-16)}$$

Specifically, we have $B(x\to S)=\mu_S(x)-v_S(x)$ and $B(x\to S^c)=\mu_{S^c}(x)-v_{S^c}(x)$.

Accuracy degree: For $x\in\Re_A$, accuracy degree of believe factor can be defined as:

$$H(x)=\mu(x)+v(x) . \qquad \text{Eq. (3-17)}$$

Specifically, we have $H(x\to S)=\mu_S(x)+v_S(x)$ and $H(x\to S^c)=\mu_{S^c}(x)+v_{S^c}(x)$.

According to Chai, Liu, and Li (2012), the following assertions are valid.

(1) For boundary objects $x\in Bn_P(S)$, we have: $L(x)\in(0,\ 1)$, $B(x)\in(-1,\ 1)$, and $H(x)\in(0,\ 1)$.

(2) For objects from lower approximations $x\in\underline{P}(S)$ or $x\in\underline{P}(S^c)$, we have $L(x)=1$, $B(x)=1$, and $H(x)=1$.

Considering $x\in Bn_P(S)$, if the positive score $\mu(x)$ is not less than the negative score $v(x)$, consequently, the believe degree $B(x\to S)>0$ or $B(x\to S^c)>0$, we say the boundary object (i.e. warehouse pair) x carries *valuable* uncertain information for decision making. The objects fulfilling above conditions are called *valuable objects*. The uncertain rules induced on the basis of valuable objects are called *believable rules*.

According to the proposed believe factor and three measurements, we can induce believable rules via the following strategies.

Strategy III: Upward believable rule:

For $x_i \in S - \underline{P}(S)$, its value vector can be represented as $V_{S-\underline{P}(S)}(x_i) = [h_1^{\geq}(x_i), h_2^{\geq}(x_i), ..., h_n^{\geq}(x_i)]$. Thus if it satisfies $B(x_i \to S) = \mu_S(x_i) - v_S(x_i) > 0$, an upward believable rule BR_S can be induced as:

$$IF\ \ h_1(x) \geq h_1^{\geq}(x_i)\ and\ h_2(x) \geq h_2^{\geq}(x_i)\ and\ ...\ and\ h_n(x) \geq h_n^{\geq}(x_i),\ THEN\ \ x \to S\ .$$

It is with the three measurements: $L(x_i \to S)$, $B(x_i \to S)$, and $H(x_i \to S)$.

Strategy IV: Downward believable rule:

For $x_i \in S^c - \underline{P}(S^c)$, its value vector can be represented as $V_{S^c-\underline{P}(S^c)}(x_i) = [h_1^{\leq}(x_i), h_2^{\leq}(x_i), ..., h_n^{\leq}(x_i)]$. Thus if it satisfies $B(x_i \to S^c) = \mu_{S^c}(x_i) - v_{S^c}(x_i) > 0$, a downward believable rule BR_{S^c} can be induced as:

$$IF\ \ h_1(x) \leq h_1^{\leq}(x_i)\ and\ h_2(x) \leq h_2^{\leq}(x_i)\ and\ ...\ and\ h_n(x) \leq h_n^{\leq}(x_i),\ THEN\ \ x \to S^c\ .$$

It is with the three measurements: $L(x_i \to S^c)$, $B(x_i \to S^c)$, and $H(x_i \to S^c)$.

(3) Decision rule optimization

During induction of decision rules, several redundant rules are also induced inevitably. In this chapter, we provide the following strategies for rule optimization.

Strategy V: Two decision rules R(A) and R(B) are given as:

R(A): $If\ \ h_1(x) \geq h_1^{\geq}(x_A)\ and\ h_2(x) \geq h_2^{\geq}(x_A)\ and\ ...\ and\ h_n(x) \geq h_n^{\geq}(x_A),\ Then\ \ x \to S$;

R(B): $If\ \ h_1(x) \geq h_1^{\geq}(x_B)\ and\ h_2(x) \geq h_2^{\geq}(x_B)\ and\ ...\ and\ h_n(x) \geq h_n^{\geq}(x_B),\ Then\ \ x \to S$;

with the condition of $h_1^{\geq}(x_A) \geq h_1^{\geq}(x_B)\ and\ h_2^{\geq}(x_A) \geq h_2^{\geq}(x_B)\ and\ ...\ and\ h_n^{\geq}(x_A) \geq h_n^{\geq}(x_B)$.

(1) If both of R(A) and R(B) are certain rules, then R(A) should be reduced.

(2) If both of R(A) and R(B) are believable rules, then the rule with smaller $L(x)$ should be reduced.

Strategy VI: Two decision rules R(A) and R(B) are given as:

R(A): $If\ \ h_1(x) \leq h_1^{\leq}(x_A)\ and\ h_2(x) \leq h_2^{\leq}(x_A)\ and\ ...\ and\ h_n(x) \leq h^{\leq}(x_A),\ Then\ \ x \to S^c$;

R(B): $If\ \ h_1(x) \leq h_1^{\leq}(x_B)\ and\ h_2(x) \leq h_2^{\leq}(x_B)\ and\ ...\ and\ h_n(x) \leq h_n^{\leq}(x_B),\ Then\ \ x \to S^c$;

with the conditions $h_1^{\leq}(x_A) \leq h_1^{\leq}(x_B)\ and\ h_2^{\leq}(x_A) \leq h_2^{\leq}(x_B)\ and\ ...\ and\ h_n^{\leq}(x_A) \leq h_n^{\leq}(x_B)$.

(1) If both of R(A) and R(B) are certain rules, then R(A) should be reduced.

(2) If both of R(A) and R(B) are believable rules, then the rule with smaller $L(x)$ should be reduced.

After optimization, we obtain the optimized decision rule set, denoted by R. It consists of one certain rule subset A and one believable rule subset B, where $\mathrm{A} \cup \mathrm{B} = \mathrm{R}$ and $\mathrm{A} \cup \mathrm{B} = \varnothing$. With respect to the object x, affirming certain rules are denoted as $CR(x) \in \mathrm{A}$, and affirming believable rules are denoted as $BR(x) \in \mathrm{B}$. We summarize the induced decision rule set in Table 5-1.

Table 5-1 The summarization of rule induction and optimization

	The induced certain rules $CR(x)$	The induced believable rules $BR(x)$
Induction regions	Induced from the lower approximation regions: $\underline{P}(S)$ and $\underline{P}(S^c)$	Induced from the separated boundary regions: $S - \underline{P}(S)$ and $S^c - \underline{P}(S^c)$

Utilized decision information	Certain information: objects fulfilling the dominance principle	Uncertain information: objects violating the dominance principle.
Induction methods	Superiority and inferiority matrices: $Sup^{\succ}(S)$ and $Inf^{\prec}(S^c)$	Induction strategies assisted by believe factors: $\beta(x \to S)$ and $\beta(x \to S^c)$
Rule optimization	The optimized certain rule set A	The optimized believable rule set B
	The optimized decision rule set R where $A \cup B = R$ and $A \cup B = \varnothing$	

Step 4. The E-NFS for warehouse evaluation

The Net Flow Score (NFS) method is an exploitation procedure firstly provided by Greco, Matarazzo, and Slowinski (1999). The key idea is to locate a suitable position for the final preference-ordered sequence. In Chai and Liu (2012b), the classical NFS method is developed as the extended Net Flow Score (E-NFS) through further considering uncertain information within boundary regions. In this section, we make use of the developed E-NFS for our decision problem. The target in using the E-NFS method is to rank the new warehouse Y_i ($i=1,...m_2$) within PCT-A by utilization of the certain and believable rule induced from PCT-B. We present the detailed definitions and decision process as follows.

Firstly, we calculate the basic score by using our induced certain rules.

With respect to the warehouse Y_i, we calculate the basic score $N(Y_i)$ via checking the rule subset A :

$$N(Y_i) = N^{++}(Y_i) - N^{+-}(Y_i) + N^{-+}(Y_i) - N^{--}(Y_i)\,, \qquad \text{Eq. (3-18)}$$

$$\text{s.t.}\begin{cases} N^{++}(\mathrm{Y}_i) = \mathrm{card}\left(\left\{\forall Y_g \in \mathrm{N} : \exists CR(\mathrm{x}) \in \mathrm{A}, \mathrm{x} = (\mathrm{Y}_i, \mathrm{Y}_g) \text{ with decision class } S\right\}\right) \\ N^{+-}(\mathrm{Y}_i) = \mathrm{card}\left(\left\{\forall Y_g \in \mathrm{N} : \exists CR(\mathrm{x}) \in \mathrm{A}, \mathrm{x} = (\mathrm{Y}_g, \mathrm{Y}_i) \text{ with decision class } S\right\}\right) \\ N^{-+}(\mathrm{Y}_i) = \mathrm{card}\left(\left\{\forall Y_g \in \mathrm{N} : \exists CR(\mathrm{x}) \in \mathrm{A}, \mathrm{x} = (\mathrm{Y}_g, \mathrm{Y}_i) \text{ with decision class } S^c\right\}\right) \\ N^{--}(\mathrm{Y}_i) = \mathrm{card}\left(\left\{\forall Y_g \in \mathrm{N} : \exists CR(\mathrm{x}) \in \mathrm{A}, \mathrm{x} = (\mathrm{Y}_i, \mathrm{Y}_g) \text{ with decision class } S^c\right\}\right) \end{cases}$$

where S and S^c are decision classes; $CR(x)$ represents certain rules.

The cardinality of the set $\{Y_g\}$ is denoted as $card(\{Y_g\})$.

Secondly, we calculate the addition score by using our induced believable rules.

With respect to the warehouse Y_i, we calculate the additional score $\bar{N}(Y_i)$ via checking the rule subset B :

$$\bar{N}(Y_i) = \bar{N}^{++}(Y_i) - \bar{N}^{+-}(Y_i) + \bar{N}^{-+}(Y_i) - \bar{N}^{--}(Y_i)\,, \qquad \text{Eq. (3-19)}$$

$$\text{s.t.}\begin{cases} \bar{N}^{++}(\mathrm{Y}_i) = \sum_{\forall Y_g \in N}\left\{B(\mathrm{x}) : \exists BR(\mathrm{x}) \in B, \mathrm{x} = (\mathrm{Y}_i, \mathrm{Y}_g) \text{ with decision class } S, Y_i, Y_g \in \mathrm{N}\right\} \\ \bar{N}^{+-}(\mathrm{Y}_i) = \sum_{\forall Y_g \in N}\left\{B(\mathrm{x}) : \exists BR(\mathrm{x}) \in B, \mathrm{x} = (\mathrm{Y}_g, \mathrm{Y}_i) \text{ with decision class } S, Y_i, Y_g \in \mathrm{N}\right\} \\ \bar{N}^{-+}(\mathrm{Y}_i) = \sum_{\forall Y_g \in N}\left\{B(\mathrm{x}) : \exists BR(\mathrm{x}) \in B, \mathrm{x} = (\mathrm{Y}_g, \mathrm{Y}_i) \text{ with decision class } S^c, Y_i, Y_g \in \mathrm{N}\right\} \\ \bar{N}^{--}(\mathrm{Y}_i) = \sum_{\forall Y_g \in N}\left\{B(\mathrm{x}) : \exists BR(\mathrm{x}) \in B, \mathrm{x} = (\mathrm{Y}_i, \mathrm{Y}_g) \text{ with decision class } S^c, Y_i, Y_g \in \mathrm{N}\right\} \end{cases}$$

where S and S^c are the decision classes; $BR(x)$ represents believable rules.

Finally, we obtain the total score as follows.

Finally, the total score is defined as sum of the basic score $N(Y_i)$ and the additional score $\bar{N}(Y_i)$ as:

$$N_F(Y_i) = N(Y_i) + \bar{N}(Y_i). \qquad \text{Eq. (3-20)}$$

This score is regarded as the final utility (real number) that is used to rank warehouses. The larger $N_F(Y_i)$ suggests the warehouse Y_i is better.

5.6 An Illustrative Example

5.6.1 Decision Preparation

This section illustrates a case of warehouse evaluation by using the proposed decision model. Suppose a company has managed five owned warehouses $Y_i \in M \subseteq \mathrm{N}$ for $i = 1,...,5$ and wants to buy some new warehouses this year. The selection criterion is their profitability that can be quantified as the return-on-equity (RoE) ratio (in %). There are eight new warehouses $Y_i \in M \subseteq \mathrm{N}$ for $i = 6,...,13$ as alternatives for selection, but their future profitability is unknown. The decision task aims to rank these alternatives and recommend the company which warehouse(s) should be bought-in.

In decision preliminary, the organizer collected historical RoE records of owned warehouses. Three qualified experts $e_k \in Z$ for $k = 1,2,3$ are invited to evaluate both own and new warehouses from three key aspects forming the condition criteria ($C_j \in \mathrm{C}$ for $j = 1,2,3$). They include:

C_1: Capacity of the sales staff;

C_2: The comprehensive quality of stable suppliers;

C_3: High traffic location.

The evaluation values are in the form of IVIFVs denoted as $r_{ij}^{(k)} = ([\mu_{ij}^{L(k)}, \mu_{ij}^{U(k)}], [\nu_{ij}^{L(k)}, \nu_{ij}^{U(k)}])$ regarding the warehouse Y_i, the condition criterion C_j, and the expert e_k. For example, e_1 assesses Y_2 under C_3 by using the IVIFV $r_{23}^{(1)} = ([0.54,\ 0.61],[0.27,\ 0.29])$. Therein, the interval value $[0.54, 0.61]$ represents the low and high membership degrees of the best performance on C_3; and the interval value $[0.27,\ 0.29]$ represents the interval membership degree of the worst performance on C_3. Table 5-2 gives the multi-expert evaluation table.

Table 5-2 The multi-expert evaluation table

	e_k	C_1	C_2	C_3	DC
The owned warehouses					
Y_1	e_1	([0.75, 0.82], [0.09, 0.14])	([0.86, 0.95], [0.01, 0.04])	([0.66, 0.68], [0.29, 0.30])	12.88
	e_2	([0.71, 0.77], [0.08, 0.10])	([0.57, 0.98], [0.00, 0.01])	([0.56, 0.59], [0.09, 0.09])	
	e_3	([0.89, 0.92], [0.05, 0.06])	([0.86, 0.99], [0.01, 0.01])	([0.77, 0.78], [0.20, 0.21])	
Y_2	e_1	([0.44, 0.52], [0.29, 0.33])	([0.51, 0.91], [0.04, 0.09])	([0.54, 0.61], [0.27, 0.29])	9.96
	e_2	([0.39, 0.45], [0.43, 0.54])	([0.59, 0.84], [0.01, 0.11])	([0.44, 0.46], [0.18, 0.19])	
	e_3	([0.61, 0.68], [0.27, 0.30])	([0.76, 0.86], [0.01, 0.09])	([0.56, 0.69], [0.25, 0.27])	
Y_3	e_1	([0.54, 0.68], [0.08, 0.09])	([0.66, 0.84], [0.06, 0.14])	([0.54, 0.59], [0.34, 0.36])	17.25
	e_2	([0.52, 0.59], [0.07, 0.17])	([0.63, 0.89], [0.02, 0.04])	([0.37, 0.39], [0.51, 0.54])	
	e_3	([0.64, 0.76], [0.10, 0.20])	([0.63, 0.93], [0.01, 0.01])	([0.59, 0.62], [0.29, 0.33])	

Y_4	e_1	([0.56, 0.81], [0.14, 0.19])	([0.50, 0.79], [0.12, 0.20])	([0.44, 0.56], [0.23, 0.26])	10.50
	e_2	([0.31, 0.88], [0.08, 0.09])	([0.52, 0.69], [0.09, 0.31])	([0.41, 0.44], [0.27, 0.29])	
	e_3	([0.68, 0.73], [0.04, 0.16])	([0.59, 0.91], [0.02, 0.03])	([0.57, 0.60], [0.31, 0.36])	
Y_5	e_1	([0.33, 0.50], [0.30, 0.31])	([0.43, 0.56], [0.02, 0.16])	([0.21, 0.33], [0.60, 0.61])	1.19
	e_2	([0.10, 0.35], [0.59, 0.61])	([0.33, 0.49], [0.13, 0.33])	([0.19, 0.24], [0.07, 0.09])	
	e_3	([0.39, 0.69], [0.10, 0.23])	([0.34, 0.69], [0.19, 0.30])	([0.29, 0.34], [0.49, 0.57])	
The new warehouses (also known as alternatives)					
Y_6	e_1	([0.68, 0.78], [0.18, 0.21])	([0.54, 0.88], [0.10, 0.12])	([0.45, 0.77], [0.13, 0.16])	
	e_2	([0.61, 0.68], [0.28, 0.30])	([0.58, 0.79], [0.03, 0.16])	([0.48, 0.49], [0.31, 0.36])	
	e_3	([0.78, 0.83], [0.06, 0.10])	([0.81, 0.96], [0.02, 0.03])	([0.73, 0.75], [0.20, 0.23])	
Y_7	e_1	([0.49, 0.77], [0.17, 0.19])	([0.56, 0.71], [0.19, 0.26])	([0.53, 0.67], [0.31, 0.32])	
	e_2	([0.49, 0.57], [0.04, 0.06])	([0.47, 0.72], [0.12, 0.26])	([0.53, 0.54], [0.25, 0.29])	
	e_3	([0.72, 0.76], [0.14, 0.22])	([0.73, 0.89], [0.06, 0.10])	([0.71, 0.75], [0.15, 0.19])	
Y_8	e_1	([0.43, 0.70], [0.22, 0.26])	([0.45, 0.66], [0.23, 0.33])	([0.76, 0.82], [0.09, 0.10])	
	e_2	([0.39, 0.56], [0.24, 0.33])	([0.41, 0.89], [0.03, 0.07])	([0.58, 0.61], [0.30, 0.31])	
	e_3	([0.57, 0.66], [0.17, 0.31])	([0.45, 0.69], [0.23, 0.26])	([0.81, 0.84], [0.06, 0.09])	
Y_9	e_1	([0.45, 0.49], [0.17, 0.20])	([0.68, 0.89], [0.06, 0.08])	([0.41, 0.56], [0.34, 0.36])	
	e_2	([0.33, 0.49], [0.03, 0.06])	([0.59, 0.67], [0.29, 0.29])	([0.39, 0.41], [0.40, 0.44])	
	e_3	([0.56, 0.78], [0.14, 0.21])	([0.66, 0.89], [0.03, 0.05])	([0.59, 0.62], [0.28, 0.36])	
Y_{10}	e_1	([0.38, 0.51], [0.41, 0.43])	([0.40, 0.45], [0.21, 0.49])	([0.21, 0.23], [0.67, 0.69])	
	e_2	([0.30, 0.31], [0.44, 0.49])	([0.28, 0.41], [0.16, 0.49])	([0.10, 0.11], [0.75, 0.78])	
	e_3	([0.53, 0.59], [0.23, 0.35])	([0.39, 0.53], [0.18, 0.40])	([0.27, 0.35], [0.57, 0.59])	
Y_{11}	e_1	([0.71, 0.76], [0.15, 0.19])	([0.65, 0.75], [0.01, 0.03])	([0.55, 0.74], [0.12, 0.17])	
	e_2	([0.60, 0.63], [0.26, 0.31])	([0.67, 0.91], [0.07, 0.08])	([0.58, 0.70], [0.21, 0.22])	
	e_3	([0.67, 0.81], [0.16, 0.17])	([0.90, 0.95], [0.02, 0.03])	([0.76, 0.79], [0.20, 0.20])	
Y_{12}	e_1	([0.91, 0.97], [0.01, 0.02])	([0.56, 0.61], [0.19, 0.20])	([0.33, 0.38], [0.21, 0.31])	
	e_2	([0.79, 0.87], [0.04, 0.07])	([0.47, 0.52], [0.12, 0.16])	([0.33, 0.35], [0.05, 0.19])	
	e_3	([0.72, 0.96], [0.03, 0.04])	([0.45, 0.56], [0.06, 0.10])	([0.51, 0.62], [0.15, 0.16])	
Y_{13}	e_1	([0.45, 0.51], [0.17, 0.20])	([0.78, 0.88], [0.06, 0.12])	([0.69, 0.79], [0.14, 0.16])	
	e_2	([0.33, 0.39], [0.13, 0.16])	([0.59, 0.61], [0.19, 0.23])	([0.79, 0.82], [0.04, 0.14])	
	e_3	([0.51, 0.58], [0.14, 0.17])	([0.54, 0.76], [0.03, 0.15])	([0.69, 0.72], [0.18, 0.26])	

5.6.2 The Numerical Illustration

Table 5-2 provides all the necessary information for decision-making. According to our proposed decision model thereinbefore, this section illustrates the problem-solving procedure step by step.

Step 1. Construction of expert-aggregated decision matrices

The decision information is provided in Table 5-2 as $r_{ij}^{(k)} = ([\mu_{ij}^{L(k)}, \mu_{ij}^{U(k)}],\ [v_{ij}^{L(k)}, v_{ij}^{U(k)}])$ for $i = 1,\ldots,13$, $j = 1,2,3$, and $k = 1,2,3$. We take the value $r_{11}^{(k)}$ for $i = 1$ and $j = 1$ as an example to illustrate the procedure of expert aggregation. The inputs are $r_{11}^{(1)} = ([0.75,\ 0.82],\ [0.09,\ 0.14])$, $r_{11}^{(2)} = ([0.71,\ 0.77],\ [0.08,\ 0.10])$, and $r_{11}^{(2)} = ([0.89,\ 0.92],\ [0.05,\ 0.06])$. The mean of 3-experts' decision values can be obtained via Eq. (4-1):

$$m_{11} = ([\mu_{11}^{L'}, \mu_{11}^{U'}],\ [v_{11}^{L'}, v_{11}^{U'}]) = ([0.7833, 0.8367],\ [0.0733, 0.1000])\text{, because}$$

$$\mu_{11}^{L'} = \frac{1}{t}\sum_{k=1}^{t}\mu_{11}^{L(k)} = \frac{1}{3}\times(0.75+0.71+0.89) = 0.7833;\quad \mu_{11}^{U'} = \frac{1}{t}\sum_{k=1}^{t}\mu_{11}^{U(k)} = \frac{1}{3}\times(0.82+0.77+0.92) = 0.8367;$$

$$v_{11}^{L'} = \frac{1}{t}\sum_{k=1}^{t}v_{11}^{L(k)} = \frac{1}{3}\times(0.09+0.08+0.05) = 0.0733;\quad v_{11}^{U'} = \frac{1}{t}\sum_{k=1}^{t}v_{11}^{U(k)} = \frac{1}{3}\times(0.14+0.10+0.06) = 0.1000.$$

By using Eq. (2-1) in the case of $\lambda = 1$, we calculate the distance between m_{11} and $r_{11}^{(k)}$ for $k = 1,2,3$:

$d(r_{11}^{(1)}, m_{11}) = 0.0267$; $d(r_{11}^{(2)}, m_{11}) = 0.0367$; $d(r_{11}^{(3)}, m_{11}) = 0.0633$;

Using Eq. (4-2), we calculate the degree of similarity between m_{11} and $r_{11}^{(k)}$ for $k=1,2,3$:

$s(r_{11}^{(1)}, m_{11}) = 0.7895$; $s(r_{11}^{(2)}, m_{11}) = 0.7105$; $s(r_{11}^{(3)}, m_{11}) = 0.5000$;

Using Eq. (4-3), we calculate the weight of three experts:

$w_{11}^{(1)} = 0.3947$; $w_{11}^{(2)} = 0.3553$; $w_{11}^{(3)} = 0.2500$;

Using Eq. (4-4), we calculate the expert-aggregated decision value:

$$\bar{r}_{11} = ([a_{11}, b_{11}],\ [c_{11}, d_{11}]) = IIFWA_{w_{11}^k}\left(r_{11}^{(1)}, r_{11}^{(2)}, r_{11}^{(3)}\right) = \left(\left[1-\prod_{k=1}^{3}\left(1-\mu_{11}^{L(k)}\right)^{w_{11}^k}, 1-\prod_{k=1}^{3}\left(1-\mu_{11}^{U(k)}\right)^{w_{11}^k}\right], \left[\prod_{k=1}^{3}\left(\nu_{11}^{L(k)}\right)^{w_{11}^k}, \prod_{k=1}^{3}\left(\nu_{11}^{U(k)}\right)^{w_{11}^k}\right]\right)$$
$$= \left([0.7854, 0.8397],\ [0.0745, 0.1005]\right)$$

In the same manner, we can obtain the expert-aggregated IVIF decision table as shown in Table 5-3.

Table 5-3 The expert-integrated IVIF decision table

	C_1	C_2	C_3	DC
The owned warehouses				
Y_1	([0.7854, 0.8397], [0.0745, 0.1005])	([0.8089, 0.9786], [0.0000, 0.0163])	([0.6856, 0.7031], [0.1879, 0.1938])	12.88
Y_2	([0.4851, 0.5580], [0.3175, 0.3689])	([0.6331, 0.8702], [0.0150, 0.0976])	([0.5243, 0.6097], [0.2375, 0.2544])	9.96
Y_3	([0.5635, 0.6788], [0.0813, 0.1378])	([0.6377, 0.8953], [0.0211, 0.0352])	([0.5213, 0.5587], [0.3566, 0.3869])	17.25
Y_4	([0.5474, 0.8139], [0.0812, 0.1460])	([0.5304, 0.8074], [0.0699, 0.1433])	([0.4693, 0.5334], [0.2640, 0.2953])	10.50
Y_5	([0.2977, 0.5365], [0.2568, 0.3368])	([0.3660, 0.5869], [0.0824, 0.2553])	([0.2372, 0.3120], [0.3197, 0.3637])	1.19
The new warehouses (also known as alternatives)				
Y_6	([0.6942, 0.7728], [0.1520, 0.1902])	([0.6476, 0.8910], [0.0428, 0.0918])	([0.5846, 0.6997], [0.1973, 0.2327])	
Y_7	([0.5830, 0.7199], [0.1044, 0.1426])	([0.5833, 0.7758], [0.1199, 0.2044])	([0.5856, 0.6532], [0.2371, 0.2694])	
Y_8	([0.4706, 0.6491], [0.2072, 0.2962])	([0.4403, 0.7529], [0.1382, 0.2035])	([0.7457, 0.7905], [0.1055, 0.1279])	
Y_9	([0.4546, 0.6046], [0.0954, 0.1416])	([0.6519, 0.8552], [0.0691, 0.0930])	([0.4639, 0.5412], [0.3368, 0.3813])	
Y_{10}	([0.4041, 0.4842], [0.3579, 0.4225])	([0.3628, 0.4660], [0.1841, 0.4590])	([0.1998, 0.2354], [0.6612, 0.6836])	
Y_{11}	([0.6705, 0.7546], [0.1762, 0.2061])	([0.7669, 0.8957], [0.0255, 0.0432])	([0.6374, 0.7436], [0.1711, 0.1958])	
Y_{12}	([0.8173, 0.9439], [0.0243, 0.0401])	([0.4886, 0.5576], [0.1078, 0.1454])	([0.3910, 0.4571], [0.1144, 0.2132])	
Y_{13}	([0.4417, 0.5059], [0.1489, 0.1790])	([0.6442, 0.7741], [0.0648, 0.1598])	([0.7227, 0.7811], [0.1054, 0.1780])	

Step 2. Construction of PCT-A and PCT-B

In Table 5-3, the information of the owned warehouses is used for construction of PCT-A. And the information of the new warehouses is used to form PCT-B. At first, we construct the decision criterion of PCT-A. Considering the similarity of historical RoE, we build two laws regarding the owned warehouse pair (Y_f, Y_g) for $f, g = 1,\ldots,5$:

(a) If RoE$\left(Y_f\right) \geq$ RoE$\left(Y_g\right)$ -1%, we say that Y_f is at least as good as Y_g in profitability, denoted by $Y_f S\, Y_g$. In this case, the value under DC will be the decision class S with respect to the pair (Y_f, Y_g).

(b) If RoE$\left(Y_f\right) <$ RoE$\left(Y_g\right)$ -1%, we say that Y_f is not at least as good as Y_g in profitability, denoted by $Y_f S^c Y_g$, In this case, the value under DC will be the decision class S^c with respect to the pair (Y_f, Y_g).

By employing the above laws, we can obtain the decision criterion "S/S^c" of PCT-A as shown in Table 5-4. Taking the pair (Y_2, Y_4) as an example, its value under DC should be S since 9.96 % $\geq$ 10.50 % -1 %.

Secondly, the condition criteria of both PCT-A and PCT-B can be obtained as follows. We still take the pair (Y_2, Y_4) as an example. Regarding the ideal IVIFV a^+ and the anti-ideal IVIFV a^-, the degree of similarity between $a^+ = ([1,1],[0,0])$ and the expert-aggregated decision value $\bar{r}_{21} = ([0.4851,\ 0.5580],\ [0.3175,\ 0.3689])$ can be calculated via Eq. (4-5):

$$s(\bar{r}_{21}, a^+) = \frac{\left[\frac{1}{4}\left(|a_{21}|^\lambda + |b_{21}|^\lambda + |c_{21}-1|^\lambda + |d_{21}-1|^\lambda\right)\right]^{1/\lambda}}{\left[\frac{1}{4}\left(|a_{21}-1|^\lambda + |b_{21}-1|^\lambda + |c_{21}|^\lambda + |d_{21}|^\lambda\right)\right]^{1/\lambda} + \left[\frac{1}{4}\left(|a_{21}|^\lambda + |b_{21}|^\lambda + |c_{21}-1|^\lambda + |d_{21}-1|^\lambda\right)\right]^{1/\lambda}}$$

In this example, we employ the normalized Euclidean distance (i.e. $\lambda = 1$) to calculate the distance $d(\bar{r}_{21}, a^+)$ and $d(\bar{r}_{21}, a^-)$. We can then obtain $f_1(Y_2) = s(\bar{r}_{21}, a^+) = 0.5872$. With the same manner, we have the information vector $(f_1(Y_2),\ f_2(Y_2),\ f_3(Y_2)) = (0.5872,\ 0.8103,\ 0.6539)$ with respect to Y_2 and the information vector $(f_1(Y_4),\ f_2(Y_4),\ f_3(Y_4)) = (0.7549,\ 0.7494,\ 0.6053)$ with respect to Y_6.

According to Eq. (4-6), we can obtain the dominance relation as:

$$\left\{P_{C_1}^{h_1}:\ h_1 = h_1(Y_2) - h_1(Y_4) = 0.5872 - 0.7549 = -0.1677 \approx -0.17\right\}$$

The dominance relations $\left\{P_{C_1}^{h_1},\ P_{C_2}^{h_2},\ P_{C_3}^{h_3}\right\}$ can be calculated as $\left\{P_{C_1}^{-0.17},\ P_{C_2}^{0.06},\ P_{C_3}^{0.05}\right\}$. Thus, we can obtain the information vector of PCT-A with respect to the owned pair (Y_2, Y_4) as: $((2,4),\ -0.17,\ 0.06,\ 0.05,\ S)$. Following similar procedures, we can obtain the PCT-A of owned warehouses as shown in Table 5-4.

Table 5-4 The PCT-A of the owned warehouses

Pairs	C_1	C_2	C_3	S/S^c
(1,3)	0.12	0.08	0.16	S^c
(1,4)	0.10	0.16	0.14	S
(1,2)	0.27	0.10	0.09	S
(1,5)	0.30	0.28	0.28	S
(3,1)	-0.12	-0.08	-0.16	S
(3,4)	-0.02	0.07	-0.02	S
(3,2)	0.14	0.01	-0.07	S
(3,5)	0.17	0.19	0.12	S
(4,1)	-0.10	-0.16	-0.14	S^c
(4,3)	0.02	-0.07	0.02	S^c
(4,2)	0.17	-0.06	-0.05	S
(4,5)	0.20	0.12	0.14	S
(2,1)	-0.27	-0.10	-0.09	S^c
(2,3)	-0.14	-0.01	0.07	S^c
(2,4)	-0.17	0.06	0.05	S
(2,5)	0.03	0.18	0.19	S
(5,1)	-0.30	-0.28	-0.28	S^c
(5,3)	-0.17	-0.19	-0.12	S^c
(5,4)	-0.20	-0.12	-0.14	S^c
(5,2)	-0.03	-0.18	-0.19	S^c

In the same manner, we can obtain the PCT-B of alternatives as shown in Table 5-5.

Table 5-5 The PCT-B of the new warehouses (alternatives)

Pairs	C_1	C_2	C_3	Pairs	C_1	C_2	C_3
(6,7)	0.03	0.05	0.03	(10,6)	-0.25	-0.25	-0.43
(6,8)	0.13	0.10	-0.11	(10,7)	-0.22	-0.20	-0.40
(6,9)	0.10	-0.02	0.14	(10,8)	-0.12	-0.15	-0.54
(6,10)	0.25	0.25	0.43	(10,9)	-0.15	-0.27	-0.29

(6,11)	0.02	-0.08	-0.04	(10,11)	-0.23	-0.33	-0.47
(6,12)	-0.13	0.12	0.10	(10,12)	-0.38	-0.13	-0.33
(6,13)	0.14	0.01	-0.09	(10,13)	-0.11	-0.24	-0.52
(7,6)	-0.03	-0.05	-0.03	(11,6)	-0.02	0.08	0.04
(7,8)	0.10	0.05	-0.14	(11,7)	0.01	0.13	0.07
(7,9)	0.07	-0.07	0.11	(11,8)	0.11	0.18	-0.07
(7,10)	0.22	0.20	0.40	(11,9)	0.08	0.06	0.18
(7,11)	-0.01	-0.13	-0.07	(11,10)	0.23	0.33	0.47
(7,12)	-0.16	0.07	0.07	(11,12)	-0.15	0.20	0.14
(7,13)	0.11	-0.04	-0.19	(11,13)	0.12	0.09	-0.05
(8,6)	-0.13	-0.10	0.11	(12,6)	0.13	-0.12	-0.10
(8,7)	-0.10	-0.05	0.14	(12,7)	0.16	-0.07	-0.07
(8,9)	-0.03	-0.12	0.25	(12,8)	0.26	-0.02	-0.21
(8,10)	0.12	0.15	0.54	(12,9)	0.23	-0.14	0.04
(8,11)	-0.11	-0.18	0.07	(12,10)	0.38	0.13	0.33
(8,12)	-0.26	0.02	0.21	(12,11)	0.15	-0.20	-0.14
(8,13)	0.01	-0.09	0.02	(12,13)	0.27	-0.11	-0.19
(9,6)	-0.10	0.02	-0.14	(13,6)	-0.14	-0.01	0.09
(9,7)	-0.07	0.07	-0.11	(13,7)	-0.11	0.04	0.12
(9,8)	0.03	0.12	-0.25	(13,8)	-0.01	0.09	-0.02
(9,10)	0.15	0.27	0.29	(13,9)	-0.04	-0.03	0.23
(9,11)	-0.08	-0.06	-0.18	(13,10)	0.11	0.24	0.52
(9,12)	-0.23	0.14	-0.04	(13,11)	-0.12	-0.09	0.05
(9,13)	0.04	0.03	-0.23	(13,12)	-0.27	0.11	0.19

Note that the self-comparison pairs are removed from both PCT-A and PCT-B. They include the pairs: (1,1), (2,2), (3,3), (4,4), (5,5) from PCT-A and the pairs (6,6), (7,7), (8,8), (9,9), (10,10), (11,11), (12,12), (13,13) from PCT-B.

Step 3. Decision rule induction

Decision rules are induced according to the information of owned warehouses in PCT-A. Firstly, we need to build the relations between three condition criteria (i.e. C_1, C_2, C_3) and one decision criterion. According to Eq. (4-7) and Eq. (4-8), we can obtain the related approximation as follows.

- The lower approximations:

  $\underline{P}(S)$ = {(1,4), (1,2), (1,5), (3,2), (3,5), (4,2), (4,5), (2,5)};

  $\underline{P}(S^c)$ = {(4,1), (2,1), (2,3), (5,1), (5,3), (5,4), (5,2)}.

- The rough boundary regions:

  $Bn_P(S) = Bn_P(S^c)$ = {(3,1), (3,4), (2,4), (1,3), (4,3)}.

In addition, we can obtain the separated rough boundary regions as: $S-\underline{P}(S)$ = {(3,1), (3,4), (2,4)} and $S^c-\underline{P}(S^c)$ = {(1,3), (4,3)}.

By using Eq. (4-9) ~ Eq. (4-12) and induction strategies I and II, four certain rules can be induced as shown in Table 5-6.

Table 5-6 Induction of certain decision rules

Certain rules	Condition criteria			Assigned decision class
	C_1	C_2	C_3	
[1]	$\geq$ 0.14			S
[2]		$\geq$ 0.16		S
[3]		$\leq$ -0.10		S^c
[4]	$\leq$ -0.14	$\leq$ -0.01		S^c

According to Eq. (4-13) ~ (3-17), we calculate believe factors preserving five rough boundary pairs and their measurements. The results are given in Table 5-7.

Table 5-7 Believable factors of boundary pairs

Boundary regions	Boundary Pairs	Believe factors			Measuring degrees		
		Positive score	Hesitancy score	Negative score	Believe degree	Accuracy degree	Confidence degree
S - P(S)	(3,1)	8/12	2/12	2/12	6/12	10/12	10/12
	(3,4)	6/8	1/8	1/8	5/8	7/8	7/8
	(2,4)	6/8	1/8	1/8	5/8	7/8	7/8
S^c - P(S^c)	(1,3)	7/12	2/12	3/12	4/12	10/12	9/12
	(4,3)	6/8	1/8	1/8	5/8	7/8	7/8

All believe degrees are larger than zero in Table 5-7. Therefore, no boundary pairs should be eliminated. By using strategies III and IV, we can induce five believable rules as shown in Table 5-8.

Table 5-8 Induction of believable decision rules

Believable rules	Condition criteria			Assigned Class	Confidence degree	Accuracy degree
	C_1	C_2	C_3			
[1]	$\geq$ -0.12	$\geq$ -0.08	$\geq$ -0.16	S	0.833	0.833
[2]	$\geq$ -0.02	$\geq$ 0.07	$\geq$ -0.02	S	0.875	0.875
[3]	$\geq$ -0.17	$\geq$ 0.06	$\geq$ 0.05	S	0.875	0.875
[4]	$\leq$ 0.12	$\leq$ 0.08	$\leq$ 0.16	S^c	0.750	0.833
[5]	$\leq$ 0.02	$\leq$ -0.07	$\leq$ 0.02	S^c	0.875	0.875

In the following, we conduct the rule optimization. Based on strategies V and VI, no certain rules should be reduced. Considering believable rules, we have the following observations regarding Table 5-8.

(1) Consider the believable rules [1] and [2], we have: -0.02 $\geq$ -0.12 AND 0.07 $\geq$ -0.08 AND -0.02 $\geq$ -0.16. According to the strategy V, the believable rule [1] should be reduced because it is with the smaller confidence degree (i.e. 0.833 < 0.875).

(2) Consider the believable rules [4] and [5], we have: 0.02 $\leq$ 0.12 AND -0.07 $\leq$ 0.08 AND 0.02 $\leq$ 0.16. According to the strategy VI, the believable rule [4] should be reduced because it is with the smaller confidence degree (i.e. 0.750 < 0.875)

Therefore, we can obtain the optimized decision rule set R listed as follows. With respect to the pair x from PCT-B, we have the certain rule set A including:

1. The upward certain rule $CR(x)$:

   CR-1: If $h_1(x) \geq$ 0.14, then $x \to S$.

   CR-2: If $h_2(x) \geq$ 0.16, then $x \to S$.

2. The downward certain rule $CR(x)$:

   CR-3: If $h_2(x) \leq$ -0.10, then $x \to S^c$.

   CR-4: If $h_1(x) \leq$ -0.14 AND $h_2(x) \leq$ - 0.01, then $x \to S^c$.

We have the believable rule set B including:

1. The upward believable rule $BR(x)$:

BR-1: If $h_1(x) \geq$ -0.02 AND $h_2(x) \geq$ 0.07 AND $h_3(x) \geq$ -0.02, then $x \to S$.

BR-2: If $h_1(x) \geq$ -0.17 AND $h_2(x) \geq$ 0.06 AND $h_3(x) \geq$ 0.05, then $x \to S$.

2. The downward believable rule $BR(x)$:

BR-3: If $h_1(x) \leq$ 0.02 AND $h_2(x) \leq$ -0.07 AND $h_3(x) \leq$ 0.02, then $x \to S^c$.

Step 4. Rule exploitation for warehouse evaluation

In this step, we utilize the induced rules to predict the preference order of new warehouses. Based on PCT-B, Table 5-9 illustrates all affirmed pairs with respect to each induced rule. The believe degrees are provided in the last column.

Table 5-9 The affirmed pairs regarding the optimized decision rules

Optimized Rule sets	Decision rules		Affirmed pairs of new warehouses	Assigned class	Believe degree
Certain rule set A	Upward certain rule	CR-1	(6,10) (6,13) (7,10) (9,10) (11,10) (12,7) (12,8) (12,9) (12,10) (12,11) (12,13)	S	1.000
		CR-2	(6,10) (7,10)(9,10) (11,8) (11,10) (11,12) (13,10)	S	1.000
	Downward certain rule	CR-3	(7,11) (8,6) (8,9) (8,11) (10,6) (10,7) (10,8) (10,9) (10,11) (10,12) (10,13) (12,6) (12,9) (12,11) (12,13)	S^c	1.000
		CR-4	(10,6) (10,7) (10,9) (10,11) (10,12) (13,6)	S^c	1.000
Believable rule set B	Upward believable rule	BR-1	(6,10) (7,10) (8,10) (9,10) (11,6) (11,7) (11,10) (12,10) (13,8) (13,10)	S	0.625
		BR-2	(6,10) (6,12) (7,10) (7,12) (8,10) (9,10) (11,7) (11,9) (11,10) (11,12) (12,10) (13,10)	S	0.625
	Downward believable rule	BR-3	(6,11) (7,11) (8,13) (10,6) (10,7) (10,8) (10,9) (10,11) (10,12) (10,13)	S^c	0.625

Using the extended NFS method, we can obtain the basic score $N(Y_i)$ via Eq. (4-18), the addition score $\tilde{N}(Y_i)$ via Eq. (4-19), and the subsequent total score $N_F(Y_i)$ via Eq. (4-20). The results are given in Table 5-10.

Table 5-10 Prediction results of the preference order regarding alternatives

Warehouses	Y_6	Y_7	Y_8	Y_9	Y_{10}	Y_{11}	Y_{12}	Y_{13}
$N^{++}(x)$	2	1	0	1	0	3	6	0
$N^{+-}(x)$	0	1	2	1	6	1	1	2
$N^{-+}(x)$	4	1	1	3	0	4	1	2
$N^{--}(x)$	0	1	3	0	7	0	4	1
Basic scores	6	0	-4	3	-13	6	2	-1
$\tilde{N}^{++}(x)$	2	2	1	1	0	5	1	2
$\tilde{N}^{+-}(x)$	1	1	1	1	7	0	3	0
$\tilde{N}^{-+}(x)$	1	1	1	1	0	3	1	2
$\tilde{N}^{--}(x)$	1	1	1	0	7	0	0	0
Additional scores	1×0.625	1×0.625	0×0.625	1×0.625	-14×0.625	8×0.625	-1×0.625	4×0.625
Total score	6.625	0.625	-4	3.625	-21.7	11	1.375	1.5
Ranks	#2	#6	#7	#3	#8	#1	#5	#4

According to the total score from Table 5-10, we have:

$$(11) > (6.625) > (3.625) > (1.5) > (1.375) > (0.625) > (-4) > (-21.7).$$

Therefore, we can finally obtain the preference order of all new warehouses as

$$Y_{11} \rightarrow Y_6 \rightarrow Y_9 \rightarrow Y_{13} \rightarrow Y_{12} \rightarrow Y_7 \rightarrow Y_8 \rightarrow Y_{10}$$

As a result, a clear recommendation can be provided:

1. The new warehouse Y_{11} is with the highest rate of profitability and consequently should be recommended for buy-in.
2. Considering all alternatives, the preference order indicates the profitability from the highest rate to the lowest rate. The company could rely on such recommendation and make decision according to different business strategies.

5.6.3 Discussion

We use this example to illustrate the overall problem-solving procedures. The core mechanism of the proposed decision model is rule induction and exploitation under interval-valued intuitionistic fuzzy environments. In this section, we discuss two key points towards the application of the proposed decision model for warehouse evaluation. Firstly, we address the construction of appropriate condition criteria. The condition criteria are used for decision evaluations by experts towards good profit-making ability. Therefore, the preset condition criteria should be able to represent the most significant influence factor of the profitability. In the illustrative example, the manager considers three factors that are able to greatly influence the profitability, which are the capacity of the sales staff, the high traffic location, and the comprehensive quality of stable suppliers. In reality, considerations of setting condition criteria may not be identical. Therefore, a scientific criteria analysis is necessary before using the proposed model. Secondly, the proposed mechanism is to utilize the known information to predict the unknown information. The tool is the rules which represent the cause-and-effect linkage between own warehouses and new warehouses. Therefore, a basic precondition is that two kinds of warehouses are homogeneous and comparable.

5.7 Conclusion

In this chapter, our interest is warehouse evaluation under interval-valued intuitionistic fuzzy environment. The target is to predict the preference order of new warehouses and thus to identify the most profitable one(s) for buy-in. This prediction is not only based on the experts' evaluations, but also the historical profitability of owned warehouses. Towards this decision objective, we develop a rule-based decision model, which consists of multi-stage procedures such as uncertain information integration, pairwise comparison, decision rule induction, and rule exploitation. This hybrid decision model has three main advantages. Firstly, the importance of multiple experts is identified via objective information rather than subjective presettings. Secondly, the rule-based mechanism can well preserve the cause-and-effect linkages between subjective data (i.e. experts' evaluations under condition criteria) and objective data (i.e. historical profitability under the decision criterion). Thirdly, the proposed decision model employs both certain and uncertain information for rule induction and exploitation, which can bring the more accurate predictions. The final simulation shows the effectiveness of the proposed decision mechanism and illustrates the successful problem-solving procedures.

CHAPTER SIX

The Intuitionistic Fuzzy SIR Decision Model for Supplier Selection

6.1 Overview

In this chapter, we attempt to develop conventional MCDM techniques for dealing with uncertain decision information. MCDM aims at giving people a recommendation concerning a set of objects evaluated from multiple preference-ordered attributes. The Superiority and Inferiority Ranking (SIR) is a generation of the well-known outranking approach-PROMETHEE (Brans, Vincke, & Mareschal, 1986; Brans & Mareschal, 2005), which is an efficient approach for MCDM. As the traditional MCDM approach, however, it faces the obstacle in handling uncertainties of real world. We address the issue on how to extend the traditional MCDM approach for applications in uncertain environments. This chapter proposes a new Intuitionistic Fuzzy SIR (IF-SIR for short) approach and focuses on its application of supplier selection which is the important activity in supply chain management. Toward practical applications, two factors are considered here: (1) multiple DMs and (2) decision information in the form of linguistic terms. We firstly identify these terms via Intuitionistic Fuzzy Set (IFS) which is proven to be a powerful mathematical tool in modeling uncertain information. Then, we provide the IF-SIR approach for group aggregation and decision analysis. Hereinto, a rule-based method is developed for ranking and selection of suppliers. Finally, an illustrative example is used for illustration of the proposed approach.

6.2 Background

Supplier Selection (SS) is the important activity in supply chain management in today's global market. De Boer, Labro, and Morlacchi (2001) reviewed the MCDM approach for SS and suggested four stages in SS: problem definition; criteria formulation; supplier qualification; ranking and selection. Ha and Krishnan (2008) revisited the existing methods and provided a hybrid approach by incorporation of analytic hierarchy process, data envelopment analysis and neural network. In real world, the process of SS is often based on the uncertain information. Many people have been aware of this issue and provided several methodologies in literature including fuzzy analytic hierarchy process (Haq & Kannan, 2006; Yang, Chiu, Tzeng, & Yeh, 2008); fuzzy analytic network process (Onut, Kara, & Isik, 2009); fuzzy linear programming (Amin, Razmi, & Zhang, 2011; Yucel & Guneri, 2011); fuzzy TOPSIS (Wang, Cheng, & Huang, 2009). All of the above methods have similar direction: hybridization of traditional MCDM approaches and fuzzy logic. In addition to the fuzzy set theory, some other mathematical tools have been used to model the uncertainties of SS such as intuitionistic fuzzy set (Zhang, Zhang, Lai, & Lu, 2009; Chen, 2011a); rough set (Chang & Hung, 2010); grey systems (Li, Yamaguchi, & Nagai, 2008; Bai & Sarkis, 2010), *etc.*

Despite the diversity of cases, the basic ingredients of SS can be abstracted as the group MCDM problem. Group MCDM can be regarded as the process in which multiple DMs evaluate each alternative (also called object, action, solution, candidate, *etc*) according to multiple criteria (also called attributes, features, variables, objectives, *etc*). Figueira *et al.* (Figueira, Greco, & Ehrgott, 2005) provided a comprehensive collection of state-of-the-art surveys on MCDM problem. Many representative methods have been introduced for MCDM which can be roughly divided into three categories: (1) multi-criteria utility theory, (2) outranking relations and (3) preference disaggregation. Thereinto, outranking relations aim to compare the pairwise alternatives and then obtain overall priority ranks, which mainly contain ELECTRE methods (Roy, 1991) and PROMETHEE methods (Brans, Vincke, & Mareschal, 1986). Some authors extended these methods for further applications including ELECTRE III/TRI (Leyva-Lopez & Frenandez-Gonzalez, 2003; Lourenco & Costa, 2004), PROMETHEE with AHP (Macharis, Springael, Brucker, & Verbeke, 2004), and Superiority and Inferiority Ranking (SIR) method (Xu, 2001), *etc*. We regard these methods as the traditional MCDM methods since the factor of uncertainty is not particularly taken into account. With this in mind, our study is in the direction of developing the traditional MCDM method for their application under uncertain environments.

The classical SIR method, as a significant development of outranking relations, simultaneously employs the superiority and inferiority information, which can more comprehensively and efficiently investigate the priority among alternatives. Although this feature lets it be the very suitable tool for supplier selection, as traditional MCDM method, it is still hard to be applied in practice. That's because the precondition of the classical SIR method is that the decision information must be provided by real values, which are rarely fulfilled in real world. In order to bridge this gap, this chapter proposes a new Intuitionistic Fuzzy SIR (IF-SIR for short) approach for supplier selection. This approach considers two factors towards the practical applications: (1) DM (expert/manager/*etc*) is in the form of decision group involving multiple participants; (2) all decision information is provided by human language in the form of linguistic terms. We firstly establish the decision model of supplier selection which is an uncertainty group decision process. Following this process, we construct the decision problem and decision preliminary including three important roles (suppliers, criteria, DMs) and three required inputs (DMs' weights, criteria weights and decision values). Then, we introduce the IF-SIR approach with respect to supplier selection. An example is also provided for illustration of the proposed approach.

The rest of this chapter is organized as follows. Section 6.3 revisits principles of the IFS theory and the classical SIR method, both of which are the basis to construct our approach. Section 6.4 establishes the decision model of supplier selection and provides the details of the IF-SIR approach. Section 6.5 presents an illustrative case with numerical calculation. A discussion is given in Section 6.6. Section 6.7 provides a summary of the chapter.

6.3 The Basic Theory

6.3.1 Intuitionistic Fuzzy Set

Aiming to construct our approach, we first revisit the basic principles of IFS as follows: Let $X(X \neq \varnothing)$ be a finite set. IFS is defined as $A = \{< x, \mu_A(x), v_A(x) > | x \in X\}$, which contains two elements: the membership function μ_A and the non-membership function v_A with the condition $0 \le \mu_A + v_A \le 1$ for all $x \in X$. Szmidt and Kacprzyk (2000) called $\pi_A : \pi_A = 1 - \mu_A - v_A$ as the intuitionistic index of x in A, which is also the hesitancy function of x in A (Atanassov, 1986). By considering all three parameters, four kinds of distances are introduced for measuring the distance between two IFSs. Suppose A and B are two IFSs in $X = \{x_1, ..., x_n\}$, these distances can be defined as follows:

Hamming distance:

$$d(A,B) = \frac{1}{2}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|)$$

Euclidean distance:

$$e(A,B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}(\mu_A(x_i) - \mu_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2}$$

Normalized Hamming distance:

$$l(A,B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|)$$

Normalized Euclidean distance:

$$q(A,B) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}(\mu_A(x_i) - \mu_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2}$$

An intuitionistic fuzzy value (IFV) is denoted as $a=(\mu_a,v_a,\pi_a)$, where $\mu_a\in[0,1]$, $v_a\in[0,1]$, $\pi_a\in[0,1]$, and $\mu_a+v_a+\pi_a=1$. Clearly, the maximum IFV is $a^+=(1,0,0)$ and the minimum IFV is $a^-=(0,1,0)$. Additionally, the score function is denoted by $S(a)=\mu_a-v_a$ and the accuracy function is denoted by $H(a)=\mu_a+v_a$. In order to compare any two IFVs $a_1=(\mu_{a_1},v_{a_1},\pi_{a_1})$ and $a_2=(\mu_{a_2},v_{a_2},\pi_{a_2})$, a comparison law is given as follows:

(1) If $S(a_1)>S(a_2)$, then, $a_1>a_2$;

(2) If $S(a_1)=S(a_2)$, then, a) If $H(a_1)>H(a_2)$, then $a_1>a_2$;

b) If $H(a_1)=H(a_2)$, then $a_1=a_2$.

Two operators IFWA and IFWG are defined for aggregating intuitionistic fuzzy information shown as follows (Xu & Cai, 2010). The aggregated value by using IFWA or IFWG is also the intuitionistic fuzzy value.

Definition 1 Let $a_i=(\mu_{a_i},v_{a_i})\ (i=1,...,n)$ be a set of IFVs, and $IFWA:\Theta^n\rightarrow\Theta$ is defined as:

$$IFWA_\omega(a_1,a_2,...,a_n)=\overset{n}{\underset{i=1}{\oplus}}(\omega_i a_i)=(1-\prod_{i=1}^{n}(1-\mu_{a_i})^{\omega_i},\prod_{i=1}^{n}(v_{a_i})^{\omega_i})$$

where $\omega=(\omega_1,\omega_2,...\omega_n)^T$ is the weight vector of $a_i\ (i=1,...,n)$ with $\omega_i\in[0,1]$ and $\sum_{i=1}^{n}\omega_i=1$.

Definition 2 Let $a_i=(\mu_{a_i},v_{a_i})\ (i=1,...,n)$ be a set of IFVs, and $IFWG:\Theta^n\rightarrow\Theta$ is defined as:

$$IFWG_\omega(a_1,a_2,...,a_n)=\overset{n}{\underset{i=1}{\oplus}}(a_i^{\omega_i})=(\prod_{i=1}^{n}(\mu_{a_i})^{\omega_i},\ 1-\prod_{i=1}^{n}(1-v_{a_i})^{\omega_i})$$

where $\omega=(\omega_1,\omega_2,...\omega_n)^T$ is the weight vector of $a_i\ (i=1,...,n)$ with $\omega_i\in[0,1]$ and $\sum_{i=1}^{n}\omega_i=1$.

6.3.2 The Classic SIR Method

In this section, we concisely review the classical SIR method which is the basis for construction of our method. Suppose one DM provides the real-valued performance function $g_j(A_i)$ to m alternatives $A_i\ (i=1,...,m)$ under n criteria $g_j\ (j=1,...,n)$. Let f_j be the threshold function for the criteria g_j, which is a nondecreasing function and can be decided by DMs. For each pair $(A_i,A_k)\ i,k=1,...,m$, $P_j(A_i,A_k)=f_j(g_j(A_i)-g_j(A_k))$ is called the preference intensity which represents the superiority of A_i to A_k, and also the inferiority of A_k to A_i, with respect to the j th criterion. Then main principles of the classical SIR method are summarized as follows (Xu, 2001):

The SIR index: For alternative A_i, the superiority index $S_j(A_i)$ and the inferiority index $I_j(A_i)$ with respect to criterion j are defined by:

$$S_j(A_i)=\sum_{k=1}^{m}P_j(A_i,A_k) \quad \text{and} \quad I_j(A_i)=\sum_{k=1}^{m}P_j(A_k,A_i),$$

where P_j is the preference intensity and $j=1,...,n$, $i,k=1,...,m$.

The SIR flow: With holding the superiority matrix $S=[S_j(A_i)]_{m\times n}$ and the inferiority matrix $I=[I_j(A_i)]_{m\times n}$, the superiority flow $\varphi^{>}(A_i)$ and the inferiority flow $\varphi^{<}(A_i)$ are defined by: $\varphi^{>}(A_i)=V[S_1(A_i),...,S_n(A_i)]$ and $\varphi^{<}(A_i)=V[I_1(A_i),...,I_n(A_i)]$, where V be the aggregation function. Clearly, when the higher $\varphi^{>}(A_i)$ and the lower $\varphi^{<}(A_i)$, alternative A_i is better.

The SIR ranking: This ranking considers three relations, which are the preference relation P, the indifference relation I and the incomparability relation R. With holding the descending order of $\varphi^{>}(A_i)$, the superiority ranking $R_{>}^{*}=\{P_{>},I_{>}\}$ can be obtained by: $A_iP_{>}A_k$ iff $\varphi^{>}(A_i)>\varphi^{>}(A_k)$ and $A_iI_{>}A_k$ iff $\varphi^{>}(A_i)=\varphi^{>}(A_k)$. Similarly, with holding the ascending order of $\varphi^{<}(A_i)$, the inferiority ranking $R_{<}^{*}=\{P_{<},I_{<}\}$ can be obtained by: $A_iP_{<}A_k$ iff $\varphi^{<}(A_i)<\varphi^{<}(A_k)$ and $A_iI_{<}A_k$ iff $\varphi^{<}(A_i)=\varphi^{<}(A_k)$. The two partial ranks $R_{>}^{*}=\{P_{>},I_{>}\}$ and $R_{<}^{*}=\{P_{<},I_{<}\}$ are combined into the final ranks by means of $R^{*}=\{P,I,R\}=R_{>}^{*}\cap R_{<}^{*}$.

6.4 The Proposed Decision Model for Supplier Selection

6.4.1 Decision Problem Modeling

Despite the diversity of scenarios in supplier selection, the basic ingredients of supplier selection are abstracted as a model of Group MCDM: according to a finite set of considered criteria ($G_j\ j=1,...,m$), several DMs (experts/managers/*etc*) ($e_k\ k=1,...,l$) in decision group are required to make their own evaluations $d_{ij}^{(k)}$ to assess a finite set of alternative suppliers ($Y_i\ i=1,...,n$). The weighted linear programming is the most common approach for use, which needs another two parameters in order for measuring the importance of different DMs and different criteria: the weights of DMs ($w_k\ k=1,...,l$) and the criteria weights ($\omega_j\ j=1,...,m$). In the process of Group MCDM, three stages should be clearly considered: (1) decision problem construction; (2) decision preliminary (with inputs); and (3) various decision approaches as the third stage. Figure 6-1 illustrates the overall process of supplier selection based on the IF-SIR approach. Firstly, decision organizers should construct three important roles in the first stage, including alternative suppliers; considered criteria and multiple qualified DMs as a group. Then three inputs should be clarified in preliminary stage including: DMs' weights; criteria weights; and decision values. The input 1 is the results of assessment of qualified DMs in accompany with the construction of DM group. The inputs 2 and 3 are provided by each DM via evaluating the importance of criteria and the performance of alternative suppliers under criteria. With holding the three inputs, our proposed IF-SIR approach is responsible for the third stage. The decision target is to provide a recommendation in the form of ranking or choice.

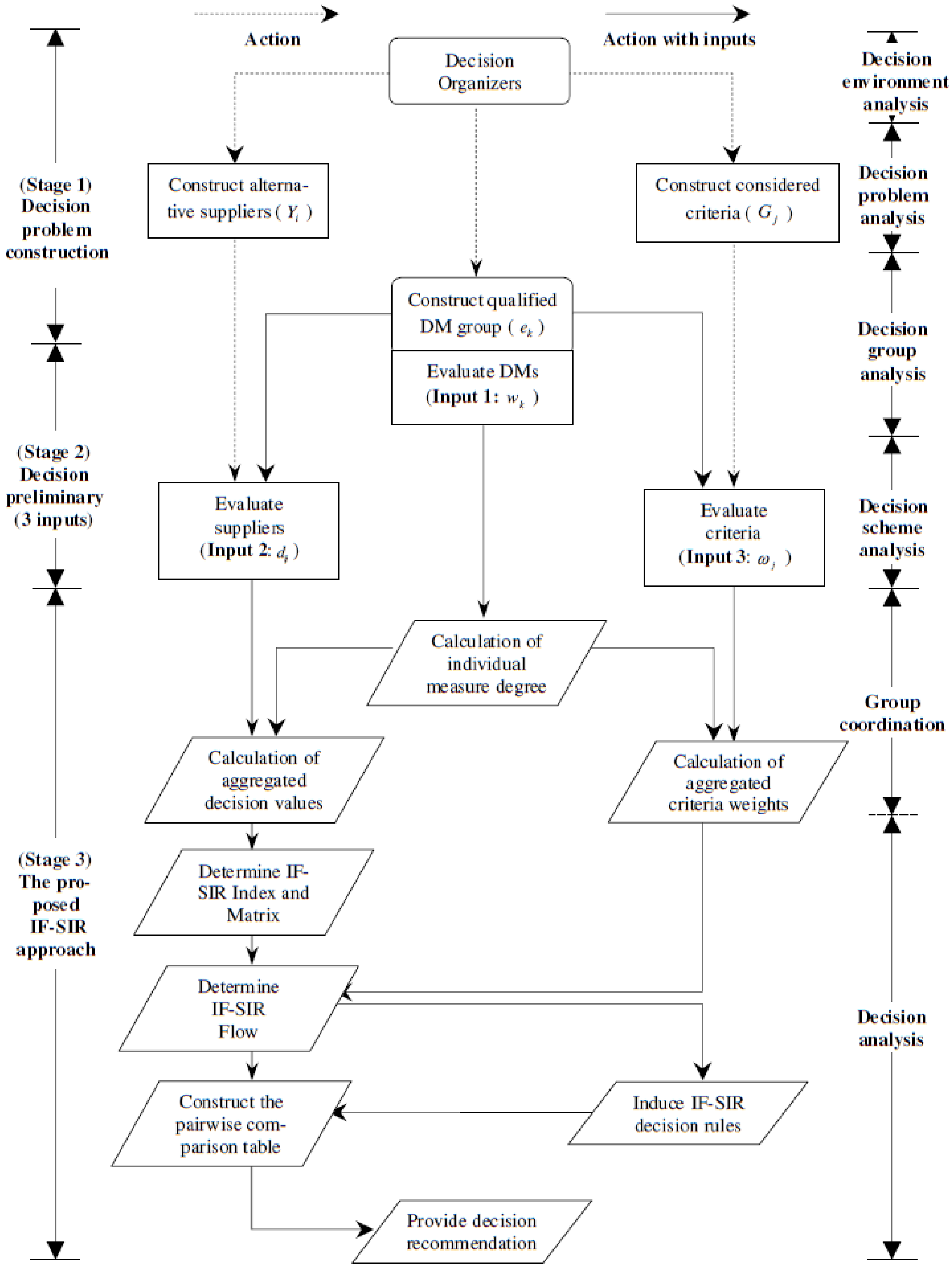


Fig 6-1 The overall process of supplier selection based on the IF-SIR approach

Chai and Liu (2012a) have provided an Uncertainty Group Decision Process by consideration of decision factors in Group MCDM under uncertain environments. This process mainly contains five analytic steps which can be aided by Decision Support System (DSS). Following this process as shown in Figure 6-1, let us remark how to establish the uncertainty group MCDM model of supplier selection and support decision by using the IF-SIR approach.

(1) Decision environment analysis

Decision organizers are the initiator of the decision process, and also are responsible for the final decision recommendation. Several important issues should be considered including decision targets, decision principles, possible limitations, available resources, possible uncertainties, *etc*. In Chai and Liu (2012a), we summarized that the uncertainty in MCDM mainly contains five situations: incompletion, unclarity, inaccuracy, dynamic and multiple uncertainties. This chapter mainly addresses the unclarity issue since all decision information is represented in fuzzy linguistic terms.

(2) Decision problem analysis

Decision problems are roughly in three categories: structured, semi-structured, and unstructured. The structured problem is well organized (e.g. information table, decision table, *etc*). And the other two kinds of problems are usually in the form of text documents or interviewing dialogues. In most cases, supplier selection belongs to structured problems since it can be abstracted into the model of group MCDM. In other words, it can be organized as a number of information tables with the suppliers in row and the criteria in column. These tables are very important for the following use.

(3) Decision group analysis

In this step, the qualified people are assessed and selected in order for construction of a DM group. Their subjective judgments make significant impact on the decision results. These DMs should be selected according to their qualification, experience, specialized field, *etc*, which may involve another decision process. In this chapter, we evaluate the importance of DMs in the form of quantified weights, which is one of the inputs in the IF-SIR approach.

(4) Decision scheme analysis

Decision schemes are the problem-solving solutions which may be derived from past experiences or new established solutions. To supplier selection, this analysis is reflected in two DMs' evaluations: 1) criteria weights: evaluate the importance of each criterion, 2) decision values: evaluate the performance of alternative suppliers with respect to each criterion. These evaluations are important inputs of our approach, since they convey the DMs' subjective judgments.

(5) Group coordination and decision analysis

This step takes the responsibility for the aggregation of group opinions and the selection of the most suitable supplier. The IF-SIR approach here is working on condition that three inputs in the form of linguistic terms have been identified by means of IFSs from above steps.

They are defined as follows:

Input 1: The k th DM's weights: $w_k = (\mu_k, \nu_k)$, $k = 1,...,l$.

Input 2: The criteria weights given by the k th DM: $\omega_j^{(k)} = (\mu_j^{(k)}, \nu_j^{(k)})$, $j = 1,...,m$.

Input 3: The decision values given by the k th DM: $d_{ij}^{(k)} = (\mu_{ij}^{(k)}, v_{ij}^{(k)})$, $i = 1,...,n$.

6.4.2 The Intuitionistic Fuzzy SIR Approach

With holding the constructed basic roles (suppliers, criteria, DMs) and three inputs in the first and second stages (see Figure 6-1), the IF-SIR approach is proposed for supplier selection. Firstly, the individual measure degree of each DM is determined according to its importance. Then, we obtain two kinds of group aggregated evaluations: decision values and criteria weights. After that, we successively calculate the IF-SIR index, matrix and flow. And finally, we induce the decision rules and introduce a simplified Net Flow Score algorithm for supplier ranking and selection. The whole process contains seven steps as follows.

Step 1. Determine individual measure degree ξ_k.

In this step, we transform the k th DM's weight $w_k = (\mu_k, v_k)$ $k = 1,...,l$ into the real-valued individual measure degree, denoted by ξ_k. In Szmidt and Kacprzyk (2000), distances between IFSs were introduced by employing all three parameters: membership degree μ_k, non-membership degree v_k, and hesitancy degree π_k. Motivated by TOPSIS (Boran, Genc, Kurt, & Akay, 2009), we can calculate the relative closeness coefficient of the IFVs $w_k = (\mu_k, v_k)$ by using these distances. So long as the acquired coefficient is a real number within the interval [0,1], we can regard this coefficient as the individual measure degree ξ_k. This process is shown as follows:

Suppose the k th DM's weight is denoted by $w_k = (\mu_k, v_k, \pi_k)$ $k = 1,...,l$ where π_k is the hesitancy value calculated by $\pi_k = 1 - \mu_k - v_k$. We define the positive-ideal IFV denoted by $w^+ = (\mu^+, v^+, \pi^+)$ and the negative-ideal IFV denoted by $w^- = (\mu^-, v^-, \pi^-)$. Clearly, there are $w^+ = (1,0,0)$ and $w^- = (0,1,0)$. People can choose anyone of the four distances (see details in Section 6.3.1) to calculate the individual measure degree ξ_k. Here, we employ Euclidean Distance for demonstration.

The distance $D_k(w_k, w^+)$ between w_k and w^+ can be obtained by:

$$D_k(w_k, w^+) = \sqrt{\frac{1}{2}\left((\mu_k - \mu^+)^2 + (v_k - v^+)^2 + (\pi_k - \pi^+)^2\right)} \tag{1}$$

The distance $D_k(w_k, w^-)$ between w_k and w^- can be obtained by:

$$D_k(w_k, w^-) = \sqrt{\frac{1}{2}\left((\mu_k - \mu^-)^2 + (v_k - v^-)^2 + (\pi_k - \pi^-)^2\right)} \tag{2}$$

Calculate the relative closeness coefficient to the positive-ideal IFV by using the acquired distances $D_k(w_k, w^+)$ and $D_k(w_k, w^-)$:

$$\xi_k(w_k, w^+) = D_k(w_k, w^-) \big/ (D_k(w_k, w^+) + D_k(w_k, w^-)) \tag{3}$$

where $0 \le \xi_k(w_k, w^+) \le 1$ and if $\xi_k(w_k, w^+) \to 1$, then $w_k \to w^+$.

Since $\xi_k(w_k, w^+)$ is the real value within the interval [0,1], this coefficient can be regarded as the individual measure degree $\xi_k = (\xi_1, \xi_2, ..., \xi_l)$.

Step 2. Calculate the group aggregated decision information.

In this step, we calculate the group aggregated decision values $\bar{d}_{ij} = (\bar{\mu}_{ij}, \bar{v}_{ij})$ and criteria weights $\bar{\omega}_j = (\bar{\mu}_j, \bar{v}_j)$ by using the intuitionistic fuzzy aggregation operators[24]. IFWA provides the normalized weights for the given IFVs and then aggregates them by addition. While IFWG provides the exponentially weights and allows for aggregation of them by multiplication. Both of them are used for aggregation of intuitionistic fuzzy information and the results are also IFVs. This process is shown as follows:

Aggregate the criteria weights by using IFWG and ξ_k:

$$\begin{aligned}\bar{\omega}_j &= IFWG_{\xi_k}(\omega_j^{(1)}, \omega_j^{(2)}, ..., \omega_j^{(k)}) = (\omega_j^{(1)})^{\xi_1} \otimes (\omega_j^{(2)})^{\xi_2} \otimes ... \otimes (\omega_j^{(l)})^{\xi_l} \\ &= (\prod_{k=1}^{l} (\mu_j^{(k)})^{\xi_k}, 1 - \prod_{k=1}^{l} (1 - v_j^{(k)})^{\xi_k}) = (\bar{\mu}_j, \bar{v}_j)\end{aligned} \tag{4}$$

Aggregate the decision values by using IFWA and ξ_k:

$$\begin{aligned}\bar{d}_{ij} &= IFWA_{\xi_k}(d_{ij}^{(1)}, d_{ij}^{(2)}, ..., d_{ij}^{(l)}) = \xi_1 d_{ij}^{(1)} \oplus \xi_2 d_{ij}^{(2)} \oplus ... \oplus \xi_l d_{ij}^{(l)} \\ &= (1 - \prod_{k=1}^{j} (1 - \mu_{ij}^{(k)})^{\xi_k}, \prod_{k=1}^{j} (v_{ij}^{(k)})^{\xi_k}) = (\bar{\mu}_{ij}, \bar{v}_{ij})\end{aligned} \tag{5}$$

From this step, we obtain the aggregated criteria weights $\bar{\omega}_j = (\bar{\mu}_j, \bar{v}_j)$ and the aggregated decision values $\bar{d}_{ij} = (\bar{\mu}_{ij}, \bar{v}_{ij})$, both of which are used in following steps.

Step 3. Determine the performance function $g_j(Y_i)$.

In this step, we transform the aggregated decision values $\bar{d}_{ij} = (\bar{\mu}_{ij}, \bar{v}_{ij})$ into the real-valued performance function $g_j(Y_i)$. The process is similar as step 1.

Suppose $D_j(\bar{d}_{ij}, d^+)$ ($D_j(\bar{d}_{ij}, d^-)$) is the distance between aggregated decision values and the positive-ideal IFV (the negative-ideal IFV). This distance can be calculated by one of the four intuitionistic fuzzy distance measures (see details in Section 6.3.1). The performance function $g_j(Y_i)$ can be obtained through calculating the relative closeness coefficient to the positive-ideal IFV $\psi(\bar{d}_{ij}, d^+)$ by:

$$\psi(\bar{d}_{ij}, d^+) = D_j(\bar{d}_{ij}, d^-) \big/ (D_j(\bar{d}_{ij}, d^+) + D_j(\bar{d}_{ij}, d^-)) \tag{6}$$

Since $\psi(\bar{d}_{ij}, d^+)$ is the real value within the interval [0,1], this coefficient can be regarded as the performance function $g_j(Y_i)$, and if $\bar{d}_{ij} \to d^+$, then $g_j(Y_i) \to 1$.

Step 4. Determine the IF-SIR Index and Matrix.

In this step, we determine the IF-SIR Index and Matrix by using the acquired performance function. Firstly, with respect to the jth criterion, we define $P_j(Y_i, Y_t)$ as the preference intensity of supplier Y_i over supplier Y_t by:

$$P_j(Y_i, Y_t) = \phi_j(g_j(Y_i) - g_j(Y_t)), \quad j = 1, 2, ..., m, \quad i, t = 1, 2, ..., n, \quad i \neq t, \tag{7}$$

where $\phi_j(\bullet)$ is the nondecreasing threshold function and its value is within the interval [0,1]. Here, $\phi_j(\bullet)$ can be defined by DMs with reference to the six generalized threshold functions (Brans, Vincke, & Mareschal, 1986).

Secondly, for supplier Y_i, we define the IF-SIR Index with respect to the jth criterion shown as follows:

The IF-superiority index can be obtained by:

$$S_j(Y_i)=\sum_{i=1}^{n}P_j(Y_i,Y_t)=\sum_{i=1}^{n}\phi_j(g(Y_i)-g(Y_t))$$

The IF-inferiority index can be obtained by:

$$I_j(Y_i)=\sum_{i=1}^{n}P_j(Y_t,Y_i)=\sum_{i=1}^{n}\phi_j(g(Y_t)-g(Y_i))$$

Then, for supplier Y_i, we obtain the IF-SIR matrices with respect to the j th criterion.

The IF-superiority matrix:

$$[S_j(Y_i)]_{n\times m}=\begin{bmatrix} S_1(Y_1) & \dots & S_j(Y_1) & \dots & S_m(Y_1) \\ \dots & \dots & \dots & \dots & \dots \\ S_1(Y_i) & \dots & S_j(Y_i) & \dots & S_m(Y_i) \\ \dots & \dots & \dots & \dots & \dots \\ S_1(Y_n) & \dots & S_j(Y_n) & \dots & S_m(Y_n) \end{bmatrix} \tag{8}$$

The IF-inferiority matrix:

$$[I_j(Y_i)]_{n\times m}=\begin{bmatrix} I_1(Y_1) & \dots & I_j(Y_1) & \dots & I_m(Y_1) \\ \dots & \dots & \dots & \dots & \dots \\ I_1(Y_i) & \dots & I_j(Y_i) & \dots & I_m(Y_i) \\ \dots & \dots & \dots & \dots & \dots \\ I_1(Y_n) & \dots & I_j(Y_n) & \dots & I_m(Y_n) \end{bmatrix} \tag{9}$$

This process is similar as the classic SIR method.

Step 5. Determine the IF-SIR flow.

With holding the IF-SIR matrices $[S_j(Y_i)]_{n\times m}$ and $[I_j(Y_i)]_{n\times m}$, the IF-SIR flow can be calculated by using intuitionistic fuzzy aggregation operators. We hold that IFWA should be used here in order to preserve the subjective evaluations derived from DMs. The process is shown as follows:

The IF-superiority flow can be obtained by:

$$\begin{aligned}\varphi^{>}(Y_i)&=\sum_{j=1}^{m}\bar{\varpi}_j S_j(Y_i)=IFWA_{S_j(Y_i)}(\bar{\varpi}_1,\bar{\varpi}_2,\dots,\bar{\varpi}_m)=(1-\prod_{j=1}^{m}(1-\bar{\mu}_j)^{S_j(Y_i)},\prod_{j=1}^{m}\bar{v}_j^{\,S_j(Y_i)})\\ &=(\mu^{>}(Y_i),v^{>}(Y_i))\end{aligned} \tag{10}$$

The IF-inferiority flow can be obtained by:

$$\begin{aligned}\varphi^{<}(Y_i)&=\sum_{j=1}^{m}\bar{\varpi}_j I_j(Y_i)=IFWA_{I_j(Y_i)}(\bar{\varpi}_1,\bar{\varpi}_2,\dots,\bar{\varpi}_m)=(1-\prod_{j=1}^{m}(1-\bar{\mu}_j)^{I_j(Y_i)},\prod_{j=1}^{m}\bar{v}_j^{\,I_j(Y_i)})\\ &=(\mu^{<}(Y_i),v^{<}(Y_i))\end{aligned} \tag{11}$$

In the IF-SIR flow, $\varphi^{>}(Y_i)$ assesses how Y_i is superior to all other suppliers and $\varphi^{<}(Y_i)$ assesses how Y_i is inferior to all other suppliers. Clearly, the higher IF-superiority flow $\varphi^{>}(Y_i)$ and the lower IF-inferiority flow $\varphi^{<}(Y_i)$, the better supplier Y_i is.

Step 6. Induce decision rules based on outranking relations.

Through pairwise comparison, we can obtain the outranking relations of pairwise suppliers. With holding the acquired IF-SIR flow, we compare the supplier $Y_i(\varphi^{>}(Y_i),\varphi^{<}(Y_i))$ with other suppliers $Y_t(\varphi^{>}(Y_t),\varphi^{<}(Y_t))$ for $i,t=1,...,n$, $t \neq i$. All the outranking relations are shown as follows:

Comparing $\varphi^{>}(Y_i)$ and $\varphi^{>}(Y_t)$, we have $\varphi^{>}(Y_i) > \varphi^{>}(Y_t)$, $\varphi^{>}(Y_i) = \varphi^{>}(Y_t)$ or $\varphi^{>}(Y_i) < \varphi^{>}(Y_t)$.

Comparing $\varphi^{<}(Y_i)$ and $\varphi^{<}(Y_t)$, we have $\varphi^{<}(Y_i) > \varphi^{<}(Y_t)$, $\varphi^{<}(Y_i) = \varphi^{<}(Y_t)$ or $\varphi^{<}(Y_i) < \varphi^{<}(Y_t)$.

When we simultaneously consider IF-superiority flow and IF-inferiority flow for construction of the decision rules, there are totally nine kinds of condition parts. But only three of them are able to affirm the outranking relations between Y_i and Y_t. For supplier Y_i, if $Y_i \succ Y_t$, we say Y_i is superior than Y_t with respect to the considered criteria, which is affirmed by the following superior rules:

[S-Rule.1] If $\varphi^{>}(Y_i) > \varphi^{>}(Y_t)$ and $\varphi^{<}(Y_i) < \varphi^{<}(Y_t)$, then $Y_i \succ Y_t$.

[S-Rule.2] If $\varphi^{>}(Y_i) > \varphi^{>}(Y_t)$ and $\varphi^{<}(Y_i) = \varphi^{<}(Y_t)$, then $Y_i \succ Y_t$.

[S-Rule.3] If $\varphi^{>}(Y_i) = \varphi^{>}(Y_t)$ and $\varphi^{<}(Y_i) < \varphi^{<}(Y_t)$, then $Y_i \succ Y_t$.

Similarly, for supplier Y_i, if $Y_i \prec Y_t$, we say Y_i is inferior than Y_t with respect to the considered criteria, which is affirmed by the following inferior rules:

[I-Rule.1] If $\varphi^{>}(Y_i) < \varphi^{>}(Y_t)$ and $\varphi^{<}(Y_i) > \varphi^{<}(Y_t)$, then $Y_i \prec Y_t$.

[I-Rule.2] If $\varphi^{>}(Y_i) < \varphi^{>}(Y_t)$ and $\varphi^{<}(Y_i) = \varphi^{<}(Y_t)$, then $Y_i \prec Y_t$.

[I-Rule.3] If $\varphi^{>}(Y_i) = \varphi^{>}(Y_t)$ and $\varphi^{<}(Y_i) > \varphi^{<}(Y_t)$, then $Y_i \prec Y_t$.

And, if the pair (Y_i,Y_t) cannot be affirmed by anyone of the above rules, we say the suppliers Y_i and Y_t are incomparable under the given decision environment. Let us remark that the acquired IF-SIR flows are IFVs shown as $\varphi^{>}(Y_i) = (\mu^{>}(Y_i),v^{>}(Y_i))$ and $\varphi^{<}(Y_i) = (\mu^{<}(Y_i),v^{<}(Y_i))$. The comparison law[23] should be used here, which is based on the score function and the accuracy function.

Step 7. Provide the decision recommendation for supplier selection

Aided by the induced decision rules, we calculate a specific score of each alternative supplier based on the established pairwise comparison table. For each supplier Y_i, the score can be obtained by:

$$Score(Y_i) = Sup(Y_i) - Inf(Y_i) \tag{12}$$

subject to

$$Sup(Y_i) = card(\{Y_t: \text{ at least one superior rule affirms } Y_i \succ Y_t\})$$

$$Inf(Y_i) = card(\{Y_t: \text{ at least one inferior rule affirms } Y_i \prec Y_t\})$$

for $i,t=1,...,n,\ i \neq t$.

In this algorithm, $Sup(Y_i)$ represents the number of suppliers and supplier Y_i is superior. Similarly, $Inf(Y_i)$ represents the number of suppliers and supplier Y_i is inferior. And $Score(Y_i)$ is the quantitative measure used for identifying the priority in the final ranking. This comparing procedure can be regarded as a simplified algorithm of Net Flow Score (Greco, Matarazzo, Slowinski, 1999).

Using the calculated scores, a clear decision recommendation can be provided:

- For ranking from better to worse, it suggests the preference-order based on the score from maximum to minimum.
- For selection, it suggests the most suitable supplier which is having the maximum score.

Let us remark that there may be more than one supplier with the same score based on Eq. (12). It means these suppliers are with the same priority for DM under the given decision environment. In information table, each column of criteria including its weights and the corresponding decision values, is regarded as one granule of knowledge for decision-making. Assuming the acquired decision recommendation cannot fulfill the decision organizer's requirement due to these suppliers who are with the same priority, the additional criteria should be taken into account. In other words, the acquired recommendation in this step can be refined by means of considering more granular knowledge in decision preliminary stage.

6.5 Application and Evaluation

A case of supplier selection is illustrated by using the IF-SIR approach. Suppose three qualified experts ($e_k, k=1,2,3$) as DMs evaluate five alternative suppliers ($Y_i, i=1,2,3,4,5$) according to the four given criteria ($G_j, j=1,2,3,4$) including:

G_1: Financial Situation

G_2: Technology Performance

G_3: Management Performance

G_4: Service Performance

The decision organizers assess these experts and identify their weights according to their importance, denoted by w_k. In addition, each DM needs to give his/her evaluations in two aspects: (1) The weights of the given criteria according to their importance, denoted by ω_j; (2) The decision values of alternative suppliers according to their performance under each criteria, denoted by d_{ij}. All these decision information are represented in linguistic terms.

6.5.1 Decision Preliminary

The preliminary gives the all the inputs in order for the proposed method to work. We firstly identify linguistic terms by using intuitionistic fuzzy sets. We call them the IFV-measures which can be set via past experiences (e.g. Zhang, Zhang, Lai, & Lu, 2009; Bai & Sarkis, 2010; Herrrera, Herrera-Viedma, & Martinez, 2000). Table 6-1 gives the IFV-measures of linguistic terms on "Importance" and "Performance", which are of nine levels.

Table 6-1 IFV-measures of linguistic terms on "Importance" and "Performance"

Levels	"Importance" terms	"Performance" terms	IFVs
L1	Extremely Important (EI)	Extremely Positive (EP)	(1.00, 0.00)

L2	Absolutely Important (AI)	Absolutely Positive (AP)	(0.90, 0.10)
L3	Very Very Important (VVI)	Very Very Positive (VVP)	(0.80, 0.10)
L4	Very Important (VI)	Very Positive (VP)	(0.70, 0.20)
L5	Important (I)	Positive (P)	(0.60, 0.30)
L6	Medium (M)	Medium (M)	(0.50, 0.40)
L7	Less Important (LI)	Negative (N)	(0.40, 0.50)
L8	Not Important (NI)	Very Negative (VN)	(0.05, 0.80)
L9	Unconsidered (UC)	Extremely Negative (EN)	(0.00, 1.00)

Based on Table 6-1, Table 6-2 presents the weights of experts which are provided by decision organizers and the weights of criteria provided by corresponding experts.

Table 6-2 The weights via using the linguistic terms on "Importance"

Experts (e_k)	Weights of experts (w_k)	Weights of criteria (ω_j)			
		ω_1	ω_2	ω_3	ω_4
e_1	EI	AI	VI	VVI	I
e_2	VVI	VVI	I	VI	VI
e_3	VI	VI	VI	I	I

Based on Table 6-1, Table 6-3 presents the decision values of alternative suppliers under each criterion which are provided by experts.

Table 6-3 The decision values via using the linguistic terms on "Performance"

Experts (e_k)	Suppliers (Y_i)	Criteria (G_j)			
		G_1	G_2	G_3	G_4
e_1	Y_1	VP	P	VVP	VP
	Y_2	P	M	VP	P
	Y_3	AP	VVP	VVP	VVP
	Y_4	P	M	VVP	VP
	Y_5	M	N	VP	M
e_2	Y_1	VVP	VP	VP	VP
	Y_2	VP	P	P	M
	Y_3	VVP	VP	VVP	VVP
	Y_4	VP	M	VP	P
	Y_5	P	M	VP	P
e_3	Y_1	VP	P	VVP	VP
	Y_2	M	VP	P	P
	Y_3	VVP	VVP	VP	VP
	Y_4	VP	P	VP	P
	Y_5	P	M	P	M

Let us remark that the distance measures used in the following experiments require three-parameter IFVs including the membership degree, the non-membership degree and the hesitancy degree. The IFV-measures presented in Table 6-1 just contain the first two parameters. The third parameter can be calculated via one minus the first two parameters, according to the definition of hesitancy degree.

6.5.2 Numerical Illustration of Decision Process

The numerical experiments are illustrated step by step, according to the IF-SIR approach.

Step 1. Determine individual measure degree ξ_k.

The individual measure degree of three experts $\xi_k = (\xi_1, \xi_2, \xi_3)$ can be obtained by using Eqs. (1)-(3) and the inputs of Table 6-2. By employing Euclidean Distance, we take the third expert e_3 as example to illustrate this procedure.

The inputs: $w_3 = (0.70, 0.20, 0.10)$; $w^+ = (1,0,0)$; $w^- = (0,1,0)$.

The distance $D_3(w_3, w^+)$ can be obtained as:

$$D_3(w_3, w^+) = \sqrt{\frac{1}{2}\left((0.70-1)^2 + (0.20-0)^2 + (0.10-0)^2\right)} = 0.2646$$

The distance $D_3(w_3, w^-)$ can be obtained as:

$$D_3(w_3, w^-) = \sqrt{\frac{1}{2}\left((0.70-0)^2 + (0.20-1)^2 + (0.10-0)^2\right)} = 0.7550$$

The relative closeness coefficient to w^+ can be obtained as:

$$\xi_3(w_3, w^+) = D_3(w_3, w^-) \big/ (D_3(w_3, w^+) + D_3(w_3, w^-)) = 0.7550 / (0.2646 + 0.7550) = \mathbf{0.7405}$$

Following similar procedures, we can obtain the individual measure degrees as:

$$\xi_k = (\xi_1, \xi_2, \xi_3) = (1.0000, 0.8314, \mathbf{0.7405})$$

Step 2. Calculate the group aggregated decision information.

The inputs: Table 6-1; Table 6-2; the acquired ξ_k.

By using Eq. (4), the aggregated criteria weights can be obtained as:

$$\bar{\varpi}_j = (\bar{\mu}_j, \bar{v}_j) = ((0.9892,\ 0.0022), (0.9309,\ 0.0284), (0.9560\ ,\ 0.0133), (0.8900,\ 0.0532))$$

The inputs: Table 6-1; Table 6-3; the acquired ξ_k.

By using Eq. (5), the aggregated decision values can be obtained as:

$$\bar{d}_{ij} = (\bar{\mu}_{ij}, \bar{v}_{ij}) = \begin{bmatrix} (0.9677,\ 0.0090) & (0.9254,\ 0.0323) & (0.9777,\ 0.0048) & (0.9548,\ 0.0159) \\ (0.9120,\ 0.0399) & (0.9043,\ 0.0446) & (0.9289,\ 0.0301) & (0.8859,\ 0.0574) \\ (0.9920,\ 0.0027) & (0.9777,\ 0.0048) & (0.9785,\ 0.0045) & (0.9785,\ 0.0045) \\ (0.9397,\ 0.0239) & (0.8574,\ 0.0766) & (0.9699,\ 0.0080) & (0.9289,\ 0.0301) \\ (0.8816,\ 0.0603) & (0.7982,\ 0.1184) & (0.9441,\ 0.0215) & (0.8603,\ 0.0746) \end{bmatrix}$$

From this step, we obtain the group aggregated decision information $\bar{\varpi}_j$ and $\bar{d}_{ij}$, both of which are used as the inputs in following steps.

Step 3. Determine the performance function $g_j(Y_i)$.

The performance function $g_j(Y_i)$ can be obtained by using Eq. (6) and the chosen distance measure. In this case, by employing Hamming Distance, we take the aggregated decision value $\bar{d}_{11} = (\bar{\mu}_{11}, \bar{v}_{11})$ as example to calculate the performance function $g_1(Y_1)$ in the following procedure.

The inputs: $\bar{d}_{11} = (0.9677,\ 0.0090,\ 0.0233)$; $w^+ = (1,0,0)$; $w^- = (0,1,0)$.

The distance $D_1(\bar{d}_{11}, d^+)$ can be obtained as:

$$D_1(\bar{d}_{11}, d^+) = \frac{1}{2}(|\mu_1 - \mu^+| + |v_1 - v^+| + |\pi_1 - \pi^+|) = \frac{1}{2}(|0.9677 - 1| + |0.0090 - 0| + |0.0233 - 0|) = 0.0323$$

The distance $D_1(\bar{d}_{11}, d^-)$ can be obtained as:

$$D_1(\bar{d}_{11}, d^-) = \frac{1}{2}(|\mu_1 - \mu^-| + |v_1 - v^-| + |\pi_1 - \pi^-| = \frac{1}{2}(|0.9677 - 0| + |0.0090 - 1| + |0.0233 - 0|) = 0.9910$$

The relative closeness coefficient $\psi(\bar{d}_{11}, d^+)$ to w^+ can be obtained as:

$$\psi(\bar{d}_{11}, d^+) = D_1(\bar{d}_{11}, d^-) / (D_1(\bar{d}_{11}, d^+) + D_1(\bar{d}_{11}, d^-)) = 0.9910 / (0.0323 + 0.9910) = \mathbf{0.9684}$$

Following similar procedures, we can obtain the performance function as:

$$g_j(Y_i) = \psi(\bar{d}_{ij}, d^+) = \begin{bmatrix} \mathbf{0.9684} & 0.9284 & 0.9781 & 0.9561 \\ 0.9160 & 0.9090 & 0.9317 & 0.8920 \\ 0.9920 & 0.9781 & 0.9789 & 0.9789 \\ 0.9418 & 0.8662 & 0.9706 & 0.9317 \\ 0.8881 & 0.8137 & 0.9460 & 0.8688 \end{bmatrix}$$

Step 4. Determine the IF-SIR Index and Matrix.

Firstly we define the nondecreasing threshold function of Eq. (7) as:

$$\phi_j(d) = \begin{cases} 0.01 & if \quad d > 0 \\ 0.00 & if \quad d \le 0 \end{cases}$$

The form of the set function is similar to the True Criterion of the six generalized threshold functions in Brans, Vincke, and Mareschal (1986). Then, according to Eqs. (7)-(9) and the acquired performance function $g_j(Y_i)$ as inputs, the IF-SIR matrices can be obtained as:

The IF-superiority matrix

$$[S_j(Y_i)]_{n\times m} = \begin{bmatrix} 0.03 & 0.03 & 0.03 & 0.03 \\ 0.01 & 0.02 & 0 & 0.01 \\ 0.04 & 0.04 & 0.04 & 0.04 \\ 0.01 & 0.01 & 0.02 & 0.02 \\ 0 & 0 & 0.01 & 0 \end{bmatrix}$$

The IF-inferiority matrix

$$[I_j(Y_i)]_{n\times m} = \begin{bmatrix} 0.01 & 0.01 & 0.01 & 0.01 \\ 0.03 & 0.02 & 0.04 & 0.03 \\ 0 & 0 & 0 & 0 \\ 0.03 & 0.03 & 0.02 & 0.02 \\ 0.04 & 0.04 & 0.03 & 0.04 \end{bmatrix}$$

Step 5. Determine the IF-SIR flow

Based on the IF-SIR matrices $[S_j(Y_i)]_{n\times m}$ and $[I_j(Y_i)]_{n\times m}$, the IF-superiority flow $\varphi^{>}(Y_i)$ and IF-inferiority flow $\varphi^{<}(Y_i)$ can be obtained according to Eqs. (10) and (11). The IFWA operator is used to aggregate the intuitionistic fuzzy information. We take $\varphi^{>}(Y_1)$ and $\varphi^{<}(Y_1)$ as example to illustrate this procedure.

The inputs: $\bar{\omega}_j = (\bar{\mu}_j, \bar{v}_j) = ((0.9892,\ 0.0022), (0.9309,\ 0.0284), (0.9560\ ,\ 0.0133), (0.8900,\ 0.0532))$;
$S_j(Y_1) = [0.03,\ 0.03,\ 0.03,\ 0.03]$; $I_j(Y_1) = [0.01,\ 0.01,\ 0.01,\ 0.01]$.

The IF-superiority flow $\varphi^{>}(Y_1)$ can be obtained by:

$$\begin{aligned}\varphi^{>}(Y_1) &= \varpi_1 S_1(Y_1) \oplus \varpi_2 S_2(Y_1) \oplus \varpi_3 S_3(Y_1) \oplus \varpi_4 S_4(Y_1) = IFWA_{S_j(Y_1)}(\varpi_1, \varpi_2, \varpi_3, \varpi_4) \\ &= (1-(1-0.9892)^{0.03} \times (1-0.9309)^{0.03} \times (1-0.9560)^{0.03} \times (1-0.8900)^{0.03} \\ &= (0.0022^{0.03} \times 0.0284^{0.03} \times 0.0133^{0.03} \times 0.0532^{0.03}) \\ &= (\mathbf{0.3134},\ \mathbf{0.6017})\end{aligned}$$

The IF-inferiority flow $\varphi^{<}(Y_1)$ can be obtained by:

$$\begin{aligned}\varphi^{<}(Y_1) &= \varpi_1 I_1(Y_1) \times \varpi_2 I_2(Y_1) \times \varpi_3 I_3(Y_1) \times \varpi_4 I_4(Y_1) \times = IFWA_{I_j(Y_1)}(\varpi_1, \varpi_2, \varpi_3, \varpi_4) \\ &= (1-(1-0.9892)^{0.01} \times (1-0.9309)^{0.01} \times (1-0.9560)^{0.01} \times (1-0.8900)^{0.01} \\ &= (0.0022^{0.01} \times 0.0284^{0.01} \times 0.0133^{0.01} \times 0.0532^{0.01}) \\ &= (\mathbf{0.1178}, \mathbf{0.8442})\end{aligned}$$

With respect to $\varphi^{>}(Y_1)$ and $\varphi^{<}(Y_1)$, suppose $S^{>}(Y_1)$ and $S^{<}(Y_1)$ are the score functions as well as $H^{>}(Y_1)$ and $H^{<}(Y_1)$ are the accuracy functions. Based on their definitions, we can obtain their values as shown in the following:

$S^{>}(Y_1) = 0.3134 - 0.6017 = -\mathbf{0.2883}$; $H^{>}(Y_1) = 0.3134 + 0.6017 = \mathbf{0.9151}$;

$S^{<}(Y_1) = 0.1178 - 0.8442 = -\mathbf{0.7264}$; $H^{<}(Y_1) = 0.1178 + 0.8442 = \mathbf{0.9620}$.

Following by the same procedure, the IF-SIR flow of each alternative supplier Y_i can be obtained as shown in Table 6-4. Clearly, the higher $\varphi^{>}(Y_i)$ and the lower $\varphi^{<}(Y_i)$, the better supplier Y_i is.

Table 6-4 The IF-SIR flows

Suppliers	$\varphi^{>}(Y_i)$	$S^{>}(Y_i)$	$H^{>}(Y_i)$	$\varphi^{<}(Y_i)$	$S^{<}(Y_i)$	$H^{<}(Y_i)$
Y_1	$(\mathbf{0.3134},\ \mathbf{0.6017})$	-0.2883	0.9151	$(\mathbf{0.1178}, \mathbf{0.8442})$	-0.7264	0.9620
Y_2	(0.1138, 0.8506)	-0.7368	0.9644	(0.3164, 0.5971)	-0.2807	0.9135
Y_3	(0.3942, 0.5079)	-0.1137	0.9021	(0.0000, 1.0000)	-1	1.0000
Y_4	(0.1636, 0.7852)	-0.6216	0.9488	(0.2758, 0.6469)	-0.3711	0.9227
Y_5	(0.0308, 0.9577)	-0.9269	0.9885	(0.3750, 0.5304)	-0.1554	0.9054

Step 6-7. Provide the decision recommendation aided by induced decision rules.

Following the induced rules and Eq. (12), we can compare each pair of suppliers and calculate the net flow score as shown in Table 6-5.

Table 6-5 The pairwise comparison table with net flow score

	Y_1	Y_2	Y_3	Y_4	Y_5	$Sup(Y_i)$	$Inf(Y_i)$	Score
Y_1	—	$\succ$	$\prec$	$\succ$	$\succ$	3	1	2
Y_2	$\prec$	—	$\prec$	$\prec$	$\succ$	1	3	-2
Y_3	$\succ$	$\succ$	—	$\succ$	$\succ$	4	0	4
Y_4	$\prec$	$\succ$	$\prec$	—	$\succ$	2	2	0
Y_5	$\prec$	$\prec$	$\prec$	$\prec$	—	0	4	-4

Notes: "—" represents inexistent items.

Using the data in Score column, a clear decision recommendation can be provided:

- If the target is selection, it suggests the supplier Y_3 is the most suitable supplier which is with the maximum score (=4).
- If the target is ranking, it suggests the rank should be $Y_3 \succ Y_1 \succ Y_4 \succ Y_2 \succ Y_5$, from better to worse, which is according to the calculated score from larger to smaller (4>2>0>-2>-4).

6.6 Discussion

In supplier selection, many approaches have been provided in literature. Following the uncertainty group decision process (mentioned in Section 6.4.1), we carry out the analysis of several latest literature works with comparison of our IF-SIR approach in Table 6-6. In summary, the decision target of supplier selection is to rank the alternative suppliers and select the most suitable one. Three basic decision factors are necessary including alternative suppliers, considered criteria and qualified DMs. Two inputs including decision values and criteria weights derived from DMs' evaluations in the form of linguistic terms, which are identified by means of diverse mathematical tools for modeling the existing uncertainties. The commonly used tools consist of fuzzy set and its extensions (FS, IFS, etc), vague set, grey systems, rough set, etc. If considering the factor of multiple DMs, the weighted linear programming is still the frequently used means for aggregation of group opinions.

Table 6-6 A comparable analysis of latest literatures and the IF-SIR approach

No.	Steps	Yang, Chiu, Tzeng, & Yeh (2008)	Zhang, Zhang, Lai, & Lu (2009)	Bai & Sarkis (2010)	Chen T.Y. (2011a)	IF-SIR
1-1	Target	Ranking/Selection	Ranking/Selection	Ranking/Selection	Ranking/Selection	Ranking/Selection
1-2	Uncertainties	Fuzzy: TFNs	Vague: VVs	Grey: GNs	Fuzzy: IFVs	Fuzzy: IFVs
2-1	Suppliers	Assigned	Assigned	Assigned	Assigned	Assigned
2-2	Criteria	**Assigned**	Assigned	Assigned	Assigned	Assigned
3-1	DM group (weights)	Two types of DMs without weights	**Multiple DMs with weights in RNs**	Multiple DMs with weights in GNs	**Single DM**	Multiple DMs with weights in IFVs
4-1	Decision values	Assigned by TFNs	Assigned by VVs	Assigned by GNs	Assigned by IFVs	Assigned by IFVs
4-2	Criteria weights	Assigned by TFNs; Fuzzy AHP method.	Assigned by VVs	Assigned by GNs	Assigned by IFVs in three conditions	Assigned by IFVs
5-1	Group aggregation	-----	Linear weighting	Grey-based linear weighting	-----	Intuitionistic fuzzy aggregation operators
5-2	Ranking with selection	Fuzzy integral based synthetic utility	Fuzzy judgment matrix	**Grey-rough hybrid method**	An Integrated programming model	**The rule-based IF-SIR approach**
Features (marked in bold)		Consider the relationships among (sub-)criteria	Consider different preferences between individual and group	Rough set is used to refine supplies according to the historical decisions	Consider single DM's subjective attitudes (optimism or pessimism)	Simultaneously consider two types information; Agile to the dynamic criteria

Notes: TFNs: triangular fuzzy numbers; IFVs: intuitionistic fuzzy values; GNs: grey numbers; VVs: Vague values; RNs: real numbers; * represents the features of the corresponding approach; ----- represents inexistent items.

A major distinction of existing approach is how to measure the priority of suppliers via one kind of comparable values/utilities. Compared with existing approaches, the IF-SIR approach inherits the feature of classical SIR method, which is simultaneously consideration of the superiority and inferiority relations of suppliers. According to the relations, decision rules are induced for construction of the pairwise comparison table and the net flow scores are calculated as the utility of suppliers for ranking and selection. The final score merely relies on the relations of the pairwise suppliers aided by induced rules. This feature makes the proposed approach agile towards the dynamic decision criteria. In practice, it offers the opportunity to refine the achieved recommendation via consideration of more decision knowledge (i.e. additional criteria). Such mechanism of the IF-SIR method can be easily implemented in various kinds of decision support systems (e.g. Chai, 2009; Chai & Liu, 2010; 2012a).

6.7 Summary

This chapter proposes a new approach to solve the uncertainty group MCDM problem. We apply intuitionistic fuzzy sets to indentify the uncertain linguistic terms which are as DMs' evaluations in order for supplier selection. The intuitionistic fuzzy SIR approach is provided for group aggregation and decision analysis. Hereinto, we develop a rule-based method with employing net flow scores for the ranking and selection of suppliers. The proposed approach can be regarded as a new development of the classical SIR method, and also offer an easy-to-used tool serving for the real-world application.

CHAPTER SEVEN

Dynamic Tolerant Skyline Operation for Multicriteira Decision Analysis

7.1 Overview

In this chapter, we consider MCDM over large datasets in the context of database. The skyline operation is a typical MCDM activity that has been well studied in the context of database. The basic assumption of conventional skyline operation is preference-ordered values within multi-dimensional decision tables. This assumption utilizes preference-ordered values to simplify the complex human preference, which is subject to big challenges in real applications. In this chapter, we firstly investigate preference relations on skyline operation and then establish a dynamic preference model. We then introduce the concept of preference intensity and propose a new decision model called Tolerant Skyline Operation (T-skyline). We study the method for computation of T-skyline and address the issue of continuous T-skyline maintenance. Through a detailed empirical study related to the NBA player evaluation in 2010-11 regular seasons, we demonstrate the effectiveness and advantages of the proposed decision model.

7.2 Background

Considering the recent developments of MCDM, we can outline the three directions with the perspective of operational techniques: (1) Preference modeling in MCDM (Lu, Zhang, & Ruan, 2008; Ma, Lu, & Zhang, 2010; Xu, 2007a; 2007f), (2) Dominance-based rough methodology (Chai & Liu, 2012b; Chai, Liu, & Li, 2012; Greco, Matarazzo, & Slowinski, 2001), and (3) Intuitionistic fuzzy methodology (Chai, Liu, & Xu, 2012; Tan & Chen, 2010; Xu, 2011; Xu & Xia, 2011). In this chapter, we focus on the preference MCDM and attempt to develop a new skyline-based decision model taken dynamic DMs' preference into account. As far as our knowledge, it is the first time to develop skyline operation for handling the MCDM problems.

The skyline operation can be regarded as a multicriteria ranking procedure aiming to retrieve a set of qualified objects. Conventional skyline operation was firstly proposed in (Borzsonyi, Kossmann, & Stocker, 2001) and the baseline computation methods were also provided including Block-Nest-Loop and Divide-and-Conquer (Borzsonyi, Kossmann, & Stocker, 2001), Sort-filter skyline (Chomicki, Godfrey, Gryz, & Liang, 2003), Linear-estimation-sort (Godfrey, Shipley, & Gryz, 2005) and so on. Since then, several extended skyline operations were also appeared in the contexts of database. The representative works include subspace skyline operation (Pei et al. 2006), R-tree based skyline operation (Papadias, Tao, Fu, & Seeger, 2003), and the latest constrained skyline operation (Lu, Jensen, & Zhang, 2011). The current research paid more attention to the skyline-based applications for supporting related fields, for example, using skyline operation as aggregation function to build data cubes for fast OLAP (Yiu, Lo, & Yung, 2012), reviewing the skyline processing in Peer-to-Peer systems (Hose, & Vlachou, 2011), and so on.

Although literature had reported various skyline operations in the last decade, there still exist two major drawbacks that had hindered the development of skyline-based applications in real world. Firstly, an inherent weakness of skyline operations is that the outputting size of skyline is uncontrollable and lack of flexibility. Once a preference system has been confirmed, the size is accordingly settled. This problem comes from two aspects. For one thing, the predefined preference system strongly relies on DMs' subjective judgment that might be imperfect. For the other thing, nonflexible skyline results inevitably omit DMs' desirable size. Previous studies (Lin, Yuan, Zhang, & Zhang, 2007; Tao, Ding, Lin, & Pei, 2009) attempted to resolve this problem via finding a representative subset of skylines. They can surely control the size of skyline but just valid when skyline size is much larger than DMs' desires. The recent works (Lu, Jensen, & Zhang, 2011; Yiu & Mamoulis, 2009) systematically studied how to constrain the resulting size of skyline, but both of which seem still not satisfactory in practice.

Secondly, existing skyline operations rarely take human dynamic preference into account. They usually need an assumption of the fixed preference-ordered data model, but which is subject to challenge in real applications. Actually, this assumption is the simplest preference model which simplifies the realistic and complex preference of human. The most straightforward resolution is to transfer dynamic skylines to conventional skylines, thus making existing operations feasible. However, it needs full materializations of datasets and a time-consuming preprocessing. Wong, Pei, Fu, & Wang (2009) pointed out such resolution is prohibitive in real applications and hence provided a semi-materialization method via consideration of an implicit preference rather than ordered values. It has partly solved dynamic preference problems, yet is still restricted in unitary linear preference model. Jiang et al. (2008) studied preference relations in a specific problem domain. Yiu, Lu, Mamoulis, & Vaitis (2011) provided preference query techniques for spatial database. These studies partly refer to the dynamic preference issue of skyline operations. But, they were still far from satisfactory for the realistic decision-making applications.

Towards realistic decision-making applications, this chapter proposes a new decision-oriented skyline operation called Tolerant Skyline Operation (T-skyline) for overcoming the mentioned drawbacks thereinbefore. We firstly carry out an analysis on preference relations and subsequently introduce a new concept of preference intensity for dynamically modeling DMs' preference. Then, the conceptual model of T-skyline is established on the basis of preference intensity. We develop a set of frequently-updating methods for T-skyline computation and propose the algorithm for continuous T-skyline maintenance. In our empirical study, the proposed T-skyline is applied to solve a realistic personnel selection problem: the NBA player evaluation. We particularly analyze the technical statistics of total NBA players in 2010-11 regular seasons and consequently reveal several interesting evaluation results. Our experimental investigation demonstrates the effectiveness and the advantages of the proposed operation based on the multiple comparisons.

This chapter is organized as follows. In the next section, we construct dynamic preference models and establish the T-skyline operation. In Section 7.4, we develop the methods and algorithms for T-skyline computation and maintenance. Section 7.5 elaborates an empirical study related to the real NBA player evaluation. Finally, we make the conclusion in Section 7.6.

7.3 The Tolerant Skyline Operation

7.3.1 Conventional skyline operation

The conventional definition of skyline operation strongly relies on the concept of dominance relations. Considering objects A and B in multidimensional data table, A dominates B if its values are not inferior to B's values in all dimensions and at least superior to B's values in one dimension. Skyline is regarded as an elementary set in which objects cannot be dominated by any other object in the universe. Skyline operation aims to obtain such an object subset which fulfills the requirement of predefined preference. The computation methods of conventional skyline operation can refer to literature (Borzsonyi, Kossmann, & Stocker, 2001; Chomicki, Godfrey, Gryz, & Liang, 2003; Godfrey, Shipley, & Gryz, 2005). The work (Borzsonyi, Kossmann, & Stocker, 2001) provided two baseline algorithms. Block-Nest-Loop (BNL) compares every object with each other objects and identifies its skyline membership if it cannot be dominated. Divide-and-Conquer (D&C) retrieves partial skyline from several subsets of data sets and merges all obtained partial skylines into a final result. Sort-filter Skyline (SFS) (Chomicki, Godfrey, Gryz, & Liang, 2003) developed BNL through firstly sorting the objects by a monotone function. Finally, Linear-estimation-sort (LESS) (Godfrey, Shipley, & Gryz, 2005) further improves SFS through eliminating a part of objects in the sorting process.

7.3.2 Presentation of the Problem

At first, we propose the problem through a small running case related to NBA player evaluation. Table 7-1 gives technical statistic of NBA players in 2008-09 regular seasons (http://espn.go.com/nba/). Ten players are described by seven attributes, including last name (LN), team (TM), playing games (G), minutes per game (MPG), assists (A), turnovers (T), and steal (S).

Table 7-1 The running case

No.	LN	TM	G	MPG	A	T	S
P1	Nash	Sun	68	35.3	786	264	56
P2	Williams	Jazz	71	37.5	672	218	78
P3	Kidd	Nets	71	36.9	645	188	117
P4	Paul	Hornets	57	36.5	495	143	109
P5	Davis	Warriors	55	35.5	451	170	114
P6	Ford	Raptors	67	30.4	535	215	93
P7	Miller	76ers	71	36.9	562	200	99
P8	Wade	Heat	46	38.9	362	193	96
P9	Iverson	Nuggets	57	42.7	417	238	112
P10	Billups	Pistons	64	36.7	460	132	78

Conventional skyline operation will firstly predefine a preference order on the attributes. Generally speaking, there are two types of attributes: GAIN (large values are preferred) and COST (small values are preferred). In this case, attribute “LN” and “TM” generally cannot be preference-ordered. The value in attribute “G” and “MPG” can be preference-ordered in some circumstances. Normally, “A” and “S” are with GAIN type and “T” belongs to COST. If just taking “A”, “T” and “S” into account, we can easily obtain the skyline that includes P1, P2, P3, P4, P5, P10. However, many existing skyline operations will confront challenges in case like NBA team managers as DMs who would like to choose qualified players under the realistic situations as follows.

(1) The original attributes cannot be directly used to choose the qualified player. Alternatively, the manager would like to consider another three dimensions including (i) Assist per 48 min:

$(48\times A)/(G\times MPG)$; (ii) Ratio of assist over turnover: A/T ; (iii) Ratio of steal over turnover: S/T . Then, how to compute skyline under such DM-specific preference system?

(2) This DM-specified evaluation (preference) system is not one hundred percent suitable or trustable by DMs. Then, DMs would like to make such evaluation according to a relatively dynamic evaluation system that some coefficients can be adjusted.

(3) In a very large numerical database, the difference between two records (attribute values) might be so small that can be ignored. For example, records <P3, S>=117 and <P5, S>=115 can be regarded as the same in this decision process. Then conventional skyline operation needs appropriate extensions.

7.3.3 Dynamic Preference System

Skyline operation arouses great attentions because of its ability in analyzing data table. Such table is with finite objects (also called tuples, alternatives, points, items) which are described by multiple dimensions (also called attributes, criteria, features). Each cell of table is corresponding to one object in row and one dimension in column. The values of cell are called attribute values (or called information function, records, etc.). Formally, Data Table (DT) is a 4-tuple $DT=(U,\ Q,\ V,\ g)$, where a finite objects set U for $x\in U$; an attribute set Q for $q\in Q$. The scale of values of attribute q is denoted by V_q for $V=\{V_q: q\in Q\}$. And attribute values can be represented as $g_q(x)\colon U\times Q\to V$ for $g_q(x)\in V_q$. In general, attribute values can be in various forms like numbers, symbols and linguistic terms. But, they should be homogeneous with respect to a specific attribute.

We underline that only the *criteria* as dimensions are able to result skylines. In other words, the matter of skyline operations is multicriteria data table. Each criterion is defined in a predefined preference function. Formally, Preference Table (PT) can be represented as a 3-tuple $PT=(U,\ P,\ f)$. It includes a set U of objects x ; a set P of criteria p ; and a set of preference function f . Criterion values related to x and p are the values of preference function with attribute values $g_q(x)$ as independent variables. Thus, criterion values also can be denoted as $f_P[g_q(x)]$ instead. Each singleton criterion p_j is with a specific f_j . We call the set of f_j as preference system. For instance, denoting attribute values $g_{q_i}(x)=v(x,q_i)$ and criterion values $f_{p_j}[g_{q_i}(x)]=w(x,p_j)$, we understand $w(x,p_j)$ is the value of preference function f with independent variable $v(x,q_i)$, hence $w(x,p_j)=f(v(x,q_i))$.

Remark that the simplest preference system is a set of unary linear functions. If the function is monotonic increasing, we call it GAIN type like $f(q)=q$. Otherwise, we call it COST type like $f(q)=-q$. In other words, values are preference-ordered. To our running case, the criteria “A” and “S” are with $f(q)=q$, and criterion “T” is with $f(q)=-q$. Most of literatures assume such simplest preference system in skyline operation.

7.3.4 Preference Intensity

Skyline operation strongly relies on subjective judgments. Thus once the defined preference system is imprecise, the corresponding skyline is hard to be satisfied. In this work, we define the concept of preference intensity. Through specifying preference intensity of criteria, DMs have opportunities to adjust the settled preference system and thus control the resulting skylines. We define the preference intensity function as follows.

Definition 1 (preference intensity):
Considering dominance relations of two criterion values $w(x, p_j)$ and $w(y, p_j)$ on criterion p_j, preference intensity is defined as:

$$G(x, y) = \rho(d) \quad \text{s.t.} \quad d = w(x, p_j) - w(y, p_j)$$

The variable d is the difference (D-values for short) of criteria value x over y. The function value $G(x, y)$ is between 0 and 1. It requires DMs to preset certain intensity functions that reflect the DMs' preference. In this work, we recommend six generalized intensity functions (Brans, Vincke, & Mareschal, 1986; Xu, 2001) as the commonly used types for selection by DMs, as shown in Table 7-2.

Table 7-2 The six types of generalized criteria

I	Usual Criterion	$\rho(d)=\begin{cases}1 & \text{if } d \neq 0\\ 0 & \text{if } d = 0\end{cases}$	IV	Level Criterion	$\rho(d)=\begin{cases}1 & \text{if } \lvert d\rvert > \varepsilon_2\\ 1/2 & \text{if } \varepsilon_2 \geq \lvert d\rvert > \varepsilon_1\\ 0 & \text{if } \lvert d\rvert \leq \varepsilon_1\end{cases}$
II	Quasi Criterion	$\rho(d)=\begin{cases}1 & \text{if } d < -\varepsilon \text{ or } d > \varepsilon\\ 0 & \text{if } -\varepsilon \leq d \leq \varepsilon\end{cases}$	V	Linear Criterion (i)	$\rho(d)=\begin{cases}1 & \text{if } d < -\varepsilon \text{ or } d > \varepsilon\\ d/\varepsilon & \text{if } -\varepsilon \leq d \leq \varepsilon\end{cases}$
III	Gaussian criterion	$\rho(d) = 1 - \exp(-d^2 / 2\varepsilon^2)$	VI	Linear Criterion (ii)	$\rho(d)=\begin{cases}1 & \text{if } \lvert d\rvert > \varepsilon_2\\ \dfrac{\lvert d\rvert-\varepsilon_1}{\varepsilon_2-\varepsilon_1} & \text{if } \varepsilon_2 \geq \lvert d\rvert > \varepsilon_1\\ 0 & \text{if } \lvert d\rvert \leq \varepsilon_1\end{cases}$

The main value of preference intensity is to model DMs' imprecise preference. In real applications, we consider that conventional skyline operation is of type I, where $\rho(d)$ =1 meaning it is different between two criterion values. For other five types, they need DMs to define a threshold on $G(x, y)$, which offers an opportunity to measure the similarity via various preference intensity functions $\rho(d)$. For example, Quasi type can tolerate the similarity of continuous criterion values. Gaussian type is able to integrate group preference via controlling the parameter $\varepsilon\ (\varepsilon > 0)$ (e.g. ε can be obtained according to the normal distribution of preference intensities, if multiple participants are involved.). The last three types can be used in various problem domains by setting a threshold on $\rho(d)$. The preference intensity is considered in our definition of tolerant skyline.

7.3.5 Tolerant Skyline Operation

In this section, we define a new Tolerant Skyline Operation (T-skyline). It consists of three main concepts: *dominance relations*, *dominance granules*, and *(non-)preferred T-skyline*. The frequently used notations are summarized in Table 7-3.

Table 7-3 The frequently used notations

Notations	Meanings
$DT=(U,\ Q,\ V,\ g)$	Data Table DT with objects $x\in U$, attributes $q\in Q$, attribute value $g_q(x)$ and the scale of attribute values V_q
$v(x,q_i)$	Attribute value $v(x,q_i)$ of object x with respect to attribute q_i, also denoted $g_{q_i}(x)$
$w(x,p_j)$	Criterion value $w(x,p_j)$ of object x with respect to criterion p_j, also denoted $f_{P_j}[g_q(x)]$
f_j	The DM-specified preference function f_j with respect to criterion p_j; $w(x,p_j)=f_{p_j}(v(x,q_i))$
$PT=(U,\ P,\ f)$	Preference Table PT with objects $x\in U$, criteria $p\in P$ and preference function f
xD_Py	Dominance relation: for $x,y\in U$, x is superior or equal to y on criteria set P
$G(x,y)=\rho(d)$	Preference intensity of x over y; the value of function $G(x,y)$; where $d=w(x,p_j)-w(y,p_j)$
$d=w(x,p_j)-w(y,p_j)$	The D-value of x over y under criterion p_j; $d_0(x,y)$ is the expected D-value of x over y.
$D_P^+(x)$ & $D_P^-(x)$	Dominance granules: superiority set $D_P^+(x)$ and inferiority set $D_P^-(x)$ with respect to the singleton object x
$S_P^+(\lambda)$ & $S_P^-(\lambda)$	T-skyline with respect to criteria set P: preferred T-skyline $S_P^+(\lambda)$ and non-preferred T-skyline $S_P^-(\lambda)$, where the DM-specified tolerant degree $\lambda\in[1,\infty)$
$C_P(x)$ & $I_P(x)$	The comparable set $C_P(x)$ and the incomparable set $I_P(x)$ with respect to the singleton object x; properties: $D_P^+(x)\bigcup D_P^-(x)=C_P(x)$, $C_P(x)\bigcup I_P(x)=U$, $C_P(x)\bigcap I_P(x)=\varnothing$.

At first, dominance relations can be represented as: (i) For $x,y\in U$, object x is dominating object y under singleton criterion p_j if $w(x,p_j)$ is superior or equal to $w(y,p_j)$ on preference function f_j, denoted as $xD_{p_j}y$. (ii) For $x,y\in U$, object x is dominated by object y under singleton criterion p_j if $w(x,p_j)$ is inferior or equal to $w(y,p_j)$ on preference function f_j, denoted as $yD_{p_j}x$. Hereinto, three terms need to be remarked. “Equal” means the criterion values are same. “Superior” and “inferior” are typical outranking relations regarding the value of preference function.

The dominance relations are with respect to preference intensities. If adopting generalized criteria for identification, an expected D-value d_0 can be found. It is subject to threshold of $G(x,y)$ and/or parameter(s) ε. Supposing $G(x,y)=\rho(d)$ s.t. $d=w(x,p_j)-w(y,p_j)$, dominance relation can be given like: If $d\geq d_0$, then $w(x,p_j)$ is superior to $w(y,p_j)$, or $w(y,p_j)$ is inferior to $w(x,p_j)$. For example, DMs can specify $d_0=\varepsilon$ in Type III or $d_0=0.2\varepsilon_1+0.7\varepsilon_2$ in Type IV. Again, if DMs specify a group-agreed threshold $G(x,y)\geq 0.8$ on Gaussian criterion with $\varepsilon=0.5$, it is easy to obtain $|d|\geq 0.5\sqrt{2\ln 5}$. Thus, dominance relation can be defined as: If $d\geq d_0$ or $w(x,p_j)=w(y,p_j)$, then $xD_{p_j}y$.

According to dominance relations, we can define the dominance granules as: (i) the superiority set $D_P^+(x)$ where $D_P^+(x)=\{y\in U:\ \forall p_j\in P,\ yD_{p_j}x\}$, (ii) the inferiority set $D_P^-(x)$ where $D_P^-(x)=\{y\in U:\ \forall p_j\in P,\ xD_{p_j}y\}$. Then, the definition of tolerant skyline includes two parts: The preferred skyline $S_P^+(\lambda)$ and the non-preferred skyline $S_P^-(\lambda)$, where: $S_P^+(\lambda)=\{x\in U:|D_P^+(x)|\leq\lambda\}$ and $S_P^-(\lambda)=\{x\in U:|D_P^-(x)|\leq\lambda\}$. The number of objects in dominance granules is denoted by $|\bullet|$. The natural number λ, where $\lambda\in[1,\ \infty)$, is called tolerant degree. This coefficient is installed by DMs and usually alterable along with different needs and the size of data set. Let us remark this new definition below.

(1) The dominance relation of T-skyline is based on the released outranking relation. It eliminates the requirement of conventional skyline operation, which is "criterion values should strictly be superior/inferior to that of any other objects at least on one criterion".

(2) Each object x from the universe U has two dominance granules. The superiority set includes object x itself and all dominating objects which are with superior values at least on one criterion as well as not with inferior values on all criteria. Meanwhile, the inferiority set includes object x itself and all dominated object which are with inferior value at least on one criterion and not with superior values on all criteria. Thus, we can obtain a property as $D_P^+(x)\cap D_P^-(x)=\{x\}$.

(3) Tolerant degree λ is DM-specified. It is used to control the extent of skyline for meeting DMs' needs. In particular, we have $|D_P^+(x)|\leq 1$ and $|D_P^-(x)|\leq 1$ when $\lambda=1$. It implies $D_P^+(x)=\{x\}$ and $D_P^-(x)=\{x\}$.

This new operation has two features. First, it considers two boundaries according to predefined preference: preferred one and non-preferred one. The former is used to provide available alternatives. And the latter provides adverse alternatives for DMs. Secondly, it employs the DM-specified thresholds to control the level of skyline membership. It offers an opportunity to discover marginal skyline objects for consideration. Our empirical study in Section 7.5.1 will demonstrate the advantage of such mechanism.

7.4 Tolerant Skyline Computation and Maintenance

This section provides a series of algorithms to compute T-skyline and address continuous maintenance of T-skyline. Hereinto, we consider preferred T-skyline, since non-preferred skyline can be computed in the same manner.

7.4.1 The UA Method

Pairwise comparison operation is the naïve method in computing skylines. Each object in the universe compares its values of preference function with *all* other objects on *all* criteria, in order to obtain dominance granules of each object. In a table $PT:n\times k$, this operation can be done under time complexity $O(n^2)$. For $\exists x\in U$, one step of the iteration can partition the universe U into two subsets. We define them as the comparable set $C_P(x)$ and the incomparable set $I_P(x)$. The properties include: $C_P(x)\cup I_P(x)=U$ and $C_P(x)\cap I_P(x)=\varnothing$. Set $C_P(x)$ is constituted by superiority set $D_P^+(x)$ and inferiority set $D_P^-(x)$. According to their definitions, we can obtain the properties, including $D_P^+(x)\cup D_P^-(x)=C_P(x)$ and $D_P^+(x)\cap D_P^-(x)=\{x\}$. Then, the following assertions can be easily proved to be valid: (i) For objects $x,y\in U$, if $y\in D_P^+(x)$ is satisfied, then we have $D_P^+(y)\subseteq D_P^+(x)$ and $D_P^-(y)\supseteq D_P^-(x)$. (ii) For objects $x,y\in U$, if $y \in D_P^-(x)$ is satisfied, then we have $D_P^-(y) \subseteq D_P^-(x)$ and $D_P^+(y) \supseteq D_P^+(x)$. With respect to tolerant degree λ, the following assertions can be easily proved to be valid for $\exists x \in U$:

(1) If $|D_P^+(x)| \le \lambda$ is satisfied, then we have $\bigcup_{y \in D_P^+(x)} D_P^+(y) \subseteq S_P^+$.

(2) If $|D_P^-(x)| \le \lambda$ is satisfied, then we have $\bigcup_{y \in D_P^-(x)} D_P^-(y) \subseteq S_P^-$.

(3) If $|D_P^+(x)| > \lambda$ is satisfied, then we have $\bigcup_{y \in D_P^+(x)} D_P^-(y) \not\subset S_P^+$.

(4) If $|D_P^-(x)| > \lambda$ is satisfied, then we have $\bigcup_{y \in D_P^-(x)} D_P^+(y) \not\subset S_P^-$

Based on above analysis, we can compute the T-skyline through frequently updating the universe. For $\exists x \in U$ and a given λ, if $|D_P^+(x)| > \lambda$ is satisfied in an iteration, then the inferiority set $D_P^-(x)$ can be eliminated from the universe for next iteration. If $|D_P^+(x)| < \lambda$ is satisfied in an iteration, then the superiority set $D_P^+(x)$ can be eliminated from the universe for next iteration, and also $D_P^+(x)$ can be accepted to be T-skyline $S_P^+(\lambda)$. If $|D_P^+(x)| = \lambda$ is satisfied in an iteration, then the comparable set $C_P(x) = D_P^+(x) \cup D_P^-(x)$ can be eliminated from the universe for next iteration, and also $D_P^+(x)$ can be accepted to be T-skyline $S_P^+(\lambda)$.

This update-approaching (UA) method aims to minimize the number of steps of iterations by frequently updating the sets of objects. We present the UA method via pseudocode. Algorithm I calculate the dominance granule of singleton object in consideration of dynamic preference system. This operation is called by other algorithms. Algorithm II conducts the UA computation.

Algorithm I: Calculation of dominance granules

Input: The singleton object x; its criteria value $w(x, p_j)$; each criterion p_j is with preference function f_j and preference intensity ρ.

Output: dominance granules $D_P^+(x)$ and $D_P^-(x)$.

Description:

```
for ∃p_j ∈ P
    for ∃y ∈ U
        compare w(x, p_j) with w(y, p_j) on f_j and ρ
        if w(x, p_j) is superior or equal to w(y, p_j)
            D^-_{p_j}(x) ← y
        if w(x, p_j) is inferior or equal to w(y, p_j)
            D^+_{p_j}(x) ← y
        end if
    end for
    compute dominance granule
        D^+_P(x) = ⋂_{p_j∈P} D^+_{p_j}(x) and D^-_P(x) = ⋂_{p_j∈P} D^-_{p_j}(x)
end for
return D^+_P(x) and D^-_P(x).
```

Algorithm II: The UA method

Input: Database and the DM-specified tolerant degree λ.
Output: Preferred T-skyline S_P^+ with degree λ.
Description:
1: Initialization: $S_P^+ = \varnothing$ and goal set $\Delta = U$
2: **for** $\exists x \in \Delta$
3: call Algorithm I on $\exists y \in \Delta$
4: **if** $|D_P^+(x)| > \lambda$ update $\Delta = \Delta - D_P^-(x)$
5: then go to 2
6: **else if** $|D_P^+(x)| < \lambda$ update $\Delta = \Delta - D_P^+(x)$ and $S_P^+ = S_P^+ \cup D_P^+(x)$
7: then go to 2
8: **else** $|D_P^+(x)| = \lambda$ update $\Delta = \Delta - D_P^+(x) \cup D_P^-(x)$ and $S_P^+ = S_P^+ \cup D_P^+(x)$
9: then go to 2
10: **end for**

7.4.2 The EUA Method

The UA method eliminates the superiority set or the inferiority set in each step of iterations and processes the next iteration under the updated universe. It is effective for general T-skyline computation. However, for the particular situation of $\lambda = 1$, we can generate the UA method into an Extremum-Update-Approximating (EUA) method for higher efficiency.

Firstly, we provide the definitions of *extreme object* and *extreme set*, with respect to preference function f_{p_j} in criterion p_j.

Definition 2 (extreme object):

In preference table $PT: U \times P$ for $x \in U$ and $p_j \in P$, the extreme object with respect to p_j can be defined below.

$$\text{The maximum object } x_{p_j}^+ : \ x_{p_j}^+ = \{x: \max[f_{p_j}(x)],\ \forall x \in U, p_j \in P\};$$

$$\text{The minimum object } x_{p_j}^- : \ x_{p_j}^- = \{x: \min[f_{p_j}(x)],\ \forall x \in U, p_j \in P\}.$$

Definition 3 (extreme set):

In preference table $PT: U \times P$ for $x \in U$ and $p_j \in P$, the extreme set with respect to P can be defined below.

$$\text{The maximum set } X_P^+ : \ X_P^+ = \bigcup_{p_j \in P} x_{p_j}^+ ; \text{ The minimum set } X_P^- : \ X_P^- = \bigcup_{p_j \in P} x_{p_j}^- .$$

Objects with extreme values are called extreme objects. And extreme set includes all extreme objects with consideration of *all* criteria. Clearly, we have the properties: $X_P^+ \subseteq U$ and $X_P^- \subseteq U$. In addition, with respect to tolerant degree λ, the following assertions are valid for $\exists x \in U$.

(1) If $|D_P^+(x)| \neq 1$ is satisfied in an iteration, then the inferiority set $D_P^-(x)$ can be eliminated from the universe for next iteration.

(2) If $|D_P^+(x)| = 1$ is satisfied in an iteration, then the inferiority set $D_P^-(x)$ can be eliminated from the universe for next iteration, and also object x can be accepted as T-skyline $S_P^+(\lambda = 1)$.

We require that the iteration processes begin from *extreme objects* ($\exists x \in X_P^+$). In the meanwhile, the updating is processing in both X_P^+ and U. Until $X_P^+ = \varnothing$, we obtain a portion of $S_P^+(\lambda = 1)$. The next iterations will go on for the rest of the object set till it becomes empty. Remark that compared with non-extreme objects, an extreme object is more likely to be with skyline membership. Therefore, the inferiority set of extreme objects is usually with the maximum size of non-skyline objects. Once starting from the extreme set, the frequent updating can eliminate most of the non-skyline objects. Such mechanism can dramatically improve the efficiency of computation comparing with the UA method.

Algorithm III is the pseudocode of the EUA method. Hereinto, the extreme set X_P^+ is firstly calculated. Then, iterations start from $\exists x \in X_P^+$ through calling Algorithm I. Finally, iterations are conducted with initialization of goal set Δ where $\Delta \subseteq U - X_P^+$.

Algorithm III: The EUA method

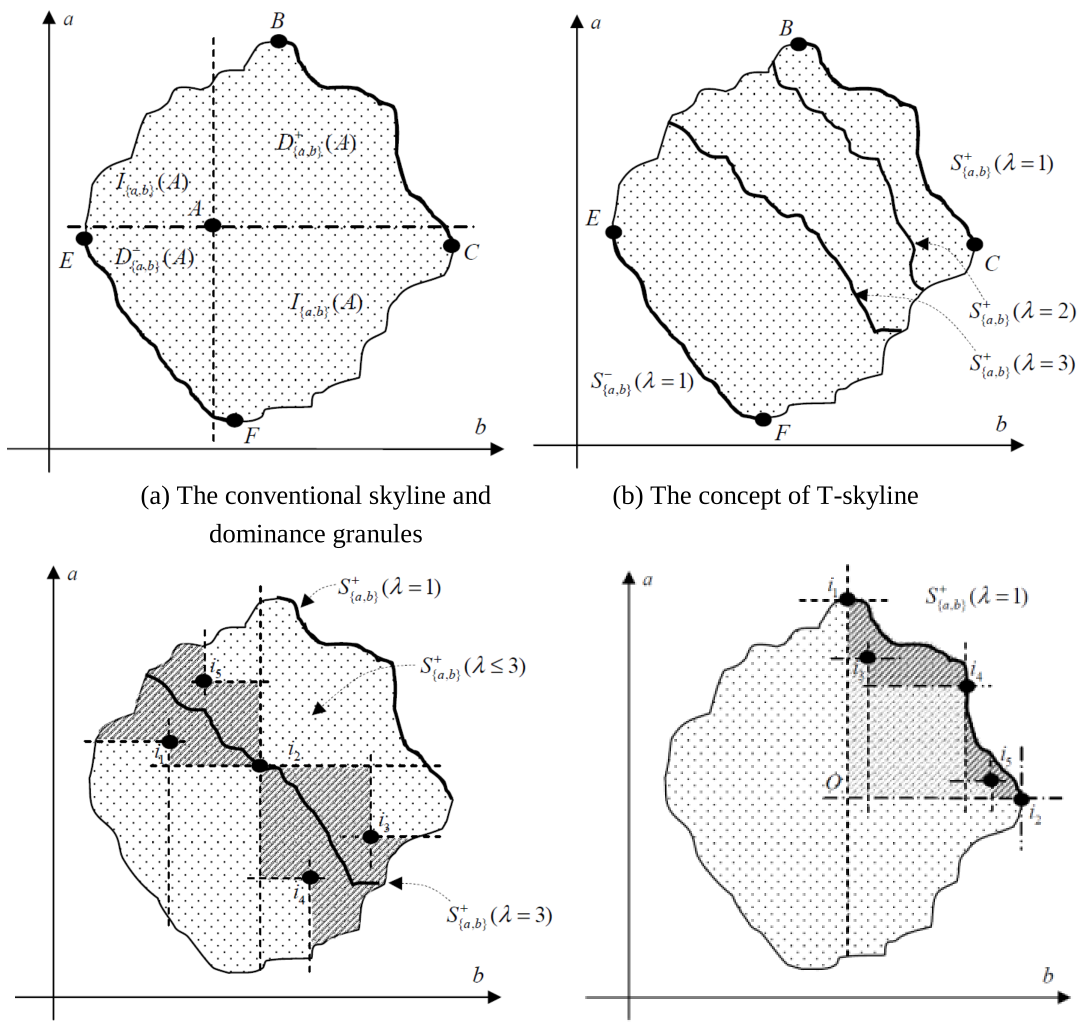
Input: Database; the tolerant degree is specified to be 1.
Output: T-skyline $S_P^+(\lambda = 1)$.
Description:

1: Initialization: $S_P^+ = \varnothing$ and goal set $\Delta = U$
2: **for** $\exists p_j \in P$
3: compute extreme set
4: $X_P^+ = \bigcup_{p_j \in P} x_{p_j}^+$ where $x_{p_j}^+ = \{x : \max[f_{p_j}(x)],\ \forall x \in U,\ p_j \in P\}$
5: **end for** %% obtain extreme set X_P^+
6: **for** $\exists x \in X_P^+$
7: $D_P^+(x)$, $D_P^-(x)$ ← Algorithm I on $\exists y \in \Delta$
8: **if** $|D_P^+(x)| \neq 1$, update $\Delta = \Delta - D_P^-(x)$
9: then go to 6
10: **else** $|D_P^+(x)| = 1$, update $\Delta = \Delta - D_P^-(x)$ and $S_P^+ \leftarrow x$
11: then go to 6
12: **end for**
13: **for** $\exists x \in \Delta$ %% initially $\Delta \subseteq U - X_P^+$
14: $D_P^+(x)$, $D_P^-(x)$ ← Algorithm I on $\exists y \in \Delta$
15: **if** $|D_P^+(x)| \neq 1$, update $\Delta = U - D_P^-(x)$
16: then go to 13
17: **else** $|D_P^+(x)| = 1$, update $\Delta = U - D_P^-(x)$ and $S_P^+ \leftarrow x$
18: then go to 13
19: **end for**

In preference table $PT : n \times k$, the naïve method needs $k \times n^2$ iterations. If the calculated object can be memorized when frequently updating, it needs $0.5 \times kn(n-1)$ iterations. Both of them are done under time complexity $O(n^2)$. In the EUA method, the runtime is subject to the route of object selection. The worst case will happen if the selected object satisfies $D_P^-(x) = \{x\}$ in each iteration. It thus just can eliminate x itself. The best case will happen if selected objects fulfill $D_P^+(x) = \{x\}$ in each iteration. Then, it needs $|S_P^+(\lambda = 1)|$ iterations.

7.4.3 An analysis of the UA and EUA Methods

In this section, we use Figure 7-1 to illustrate the T-skyline and its computation methods. For conciseness, we suppose two attributes with GAIN type constituted to a two-dimensional space. The scatterplot area represents the overall object set. Figure 7-1(a) illustrates the superiority set $D^{+}_{\{a,b\}}(A)$, the inferiority set $D^{-}_{\{a,b\}}(A)$ and the incomparable set $I_{\{a,b\}}(A)$ with respect to the point A. The curve BC roughly represents the conventional skyline. Figure 7-1(b) gives three preferred T-skyline with respect to different λ: $S^{+}_{\{a,b\}}(\lambda=1)$, $S^{+}_{\{a,b\}}(\lambda=2)$ and $S^{+}_{\{a,b\}}(\lambda=3)$. The curve EF is roughly represented as the non-preferred skyline $S^{-}_{\{a,b\}}(\lambda=1)$.



(a) The conventional skyline and dominance granules

(b) The concept of T-skyline

(c) The UA method for general T-skyline

(d) The EUA method for $\lambda=1$ T-skyline

Fig 7-1 Illustrations of T-skyline and the UA/EUA method

The UA method is illustrated in Figure 7-1(c). The conventional skyline is given as $S^{+}_{\{a,b\}}(\lambda=1)$. Supposing setting $\lambda=3$, points in area $S^{+}_{\{a,b\}}(\lambda\le 3)$ should be the resulting skyline. The middle curve, denoted as $S^{+}_{\{a,b\}}(\lambda=3)$, represents the set of points with $|D^{+}_{P}(x)|=3$. This figure represents five iterations from the point i_1 to the point i_5. For i_1 and i_4, their inferiority set $D^{-}_{P}(i_1)$ and $D^{-}_{P}(i_4)$ are eliminated since both of $|D^{+}_{P}(i_1)|$ and $|D^{+}_{P}(i_4)|$ are larger than 3. For i_3 and i_5, their superiority set $D^{+}_{P}(i_3)$ and $D^{+}_{P}(i_5)$ are accepted by $S^{+}_{\{a,b\}}(\lambda\le 3)$. Meanwhile, they are eliminated from the universe, since both of $|D^{+}_{P}(i_3)|$ and $|D^{+}_{P}(i_5)|$ are smaller than 3. For point i_2, it is accepted to be the skyline and its whole comparable set $D^{+}_{P}(i_2)\cup D^{-}_{P}(i_2)$ is eliminated. After first round of iteration, the scatterplot area shrunk to the slash area that will be the universe for the next iteration. Figure 7-1(d) illustrates the EUA method. Different from Figure 7-1(c), the skyline operation starts from the extreme points i_1 and i_2. After eliminating inferiority sets of i_1 and i_2 in the first two iterations, the universe shrunk to area i_1Oi_2. After the iterations on i_3, i_4 and i_5, the universe shrunk to the slash area. From intuitional observations on Figure 7-1(c) and 7-1(d), we can infer that the EUA method will be more efficient than the UA method.

In this work, the proposed UA/EUA method requires us to calculate dominance granules and promptly update goal sets via eliminating objects. Although it seems very similar to the classical computation methods of conventional skyline operation like SFS, LESS, BNL and D&C, they are different fundamentally. It is DM-specified tolerant degree as the threshold value to decide whether superiority set or inferiority set should be eliminated. All these operations are based on relaxed dominance relations that are established on preference functions and preference intensities. For improving computational efficiency in the particular case (i.e. $\lambda = 1$), the EUA method is developed as alternative of the UA method.

7.4.4 Continuous T-skyline Maintenance

Continuous skyline maintenance aims to keep the calculated skyline up-to-date after deleting "old" data. In general, this computation includes two aspects: (i) objects deletion when the criteria set is fixed, and (ii) criteria deletion when the objects are fixed. Many literatures have contributed to the issue of subspace skyline analysis (for example Pei et al. 2006). The concepts like skyline group and decisive subspace are provided to explore relations among original criteria sets, criteria subsets and criteria supersets. These works have partly solved the second aspect. Nevertheless, to the best of our knowledge, very rarely any literature tackles the first aspect. In this section, we provide the solutions for continuous maintenance of T-skyline in the first aspect.

Suppose the deleted object set is denoted as $V_P (\neq \varnothing)$, and $S_P^+(\lambda)$ is the calculated T-skyline. Obviously, the updated skyline should still be $S_P^+(\lambda)$ if $S_P^+(\lambda) \cap V_P = \varnothing$. However, it is a little complicated if $S_P^+(\lambda) \cap V_P \neq \varnothing$. We suppose the set $N = S_P^+(\lambda) \cap V_P$. After deletion, the rest of T-skyline will be $S_P^+(\lambda) - N$. Then, the dominance granules of objects from the set N will be correspondingly changed. More specifically, the inferiority sets of objects from N need to be considered for the updated skyline. The union of these sets is $\bigcup_{x \in N} D_P^-(x)$. Therefore, the updated skyline can be obtained in consideration of a new object set $\bigcup_{x \in N} D_P^-(x) \cup (S_P^+(\lambda) - N)$. Algorithm IV provides the pseudocode for this T-skyline maintenance.

Algorithm IV: Continuous T-skyline maintenance

Input: Known preferred skyline $S_P^+(\lambda)$; The DM-specified tolerant degree $\bar{\lambda}$ where $\bar{\lambda} \leq \lambda$; Deleted object set V_P.

Output: Updated preferred skyline $\bar{S}_P^+(\bar{\lambda})$

Description:

1: Initialization: $\bar{S}_P^+(\bar{\lambda})=\varnothing$; goal set $\Delta=\varnothing$; let set $N=S_P^+(\lambda)\cap V_P$.
2: **for** $\exists x\in N$
3: $\bar{D}_P^+(\mathrm{x})$, $\bar{D}_P^-(\mathrm{x})$ $\leftarrow$ Algorithm I on $\exists y\in U$
4: compute goal set $\Delta=\bigcup_{x\in N}\bar{D}_P^-(\mathrm{x})\cup(\mathrm{S}_P^+(\lambda)-\mathrm{N})$
5: end for %% denoted $\bar{D}_P^+(\mathrm{x})$ and $\bar{D}_P^-(\mathrm{x})$ for distinguishing.
6: **for** $\exists x\in\Delta$
7: $D_P^+(x)$, $D_P^-(x)$ $\leftarrow$ Algorithm I on $\exists y\in\Delta$
8: **if** $|\mathrm{D}_P^+(\mathrm{x})|>\bar{\lambda}$ update $\Delta=\Delta-D_P^-(x)$
9: then go to 6
10: **else if** $|\mathrm{D}_P^+(\mathrm{x})|<\bar{\lambda}$ update $\Delta=\Delta-D_P^+(x)$ and do $\bar{S}_P^+(\bar{\lambda})=\bar{S}_P^+(\bar{\lambda})\cup\mathrm{D}_P^+(\mathrm{x})$
11: then go to 6
12: **else** $|\mathrm{D}_P^+(\mathrm{x})|=\bar{\lambda}$ update $\Delta=\Delta-D_P^+(x)\cup D_P^-(x)$ and do $\bar{S}_P^+(\bar{\lambda})=\bar{S}_P^+(\bar{\lambda})\cup\mathrm{D}_P^+(\mathrm{x})$
13: then go to 6
14: **end for**

There is a prerequisite for this method. Suppose that the known T-skyline is $S_P^+(\lambda\le 3)$, the updated skyline can be obtained while $\lambda=1$, $\lambda=2$ and $\lambda=3$. In other words, the setting degree $\bar{\lambda}$ of updated skylines should not be larger than the degree λ of the known skylines, denoted as $\bar{\lambda}\le\lambda$.

7.5 An Empirical Study

In this section, we will provide a detailed empirical study related to NBA player evaluation by using the proposed T-skyline decision model. Player evaluation in NBA is the important and frequent activities. Our empirical study contains two cases. Firstly, Case (i) is used to illustrate the advantages of T-skyline through comparison with other existing skyline operations. Secondly, we use Case (ii) to illustrate the continuous maintenance of the T-skyline results. Both Case (i) and Case (ii) can demonstrate the effectiveness of the proposed decision model in problem-solving process. At last, we provide the running time analysis for verification of two aspects: (1) the efficiency of the UA/EUA methods and (2) the stability of tolerant degrees.

This study employs the real NBA dataset in 2010-11 regular seasons. The original dataset contains 468 players with 26 attributes (http://espn.go.com/nba/). In accordance with practice, we only consider the player if and only if his playing game (G) is larger or equals to 25. Then, 383 NBA players are enrolled in both Case (i) and Case (ii). All programs runs were conducted on an Intel Core2 Duo CPU (T5750 @ 2.00GHz) CPU with 4.00 GB memory.

7.5.1 T-skyline with Preference Intensities on Case (i)

In this section, we use Case (i) to demonstrate the T-skyline results with respect to various preference intensities. Suppose DMs are interested to know: *Who is/are the most efficient player(s)?* S/he firstly establish the dynamic preference system that involves three preference functions f_I , f_{II} , and f_{III} . A criteria set is defined in P ={A, B, C} as follows.

(1) Criterion A: Personal Efficiency per Game (EFF): EFF = f_I (PV,NV,G)=(PV-NV)/G,

where: Positive Values: PV=PTS+REB+STL+AST+BLK

Negative Values: NV=(FGA-FG)+(FTA-FT)+TO

(2) Criterion B: Shot Efficiency (SE): SE = f_{II} (PTS, FT, FGA)=(PTS-FT $\times$ 1)/FGA

(3) Criterion C: Score per game AVG: f_{III} (AVG)=AVG

Remark that criterion A is used to detect the overall efficiency of basketball players. It is actually the common-used criterion in real NBA scenario. Criterion B is to detect the efficiency of shot. It means the average non-Free-Throw (FT) personal total score (PTS) of each field goal attempt (FGA). These criterion values should be around one. Criterion C is the total score per game with the GAIN type.

The first advantage of the T-skyline is its hieratical skyline results. By using UA/EUA methods, we can compute the T-skyline with respect to $\lambda = 1, 2, 3...$ as shown in Table 7-4. With setting $\lambda = 1$, it can return 9 players that is identical with the results by using conventional skyline operation after materializing 3-dimensional preference table. Besides, the T-skyline can additionally provide 7 players with setting $\lambda = 2$ (e.g. R.Allen) and 6 players with setting $\lambda = 3$ (e.g. C.Anthony). Such hieratical results by using the T-skyline are benefit for decision-makings from two aspects. On the one hand, it can reveal why some favored NBA players can or cannot turn into the skyline membership. For example, DMs interest the performance of the famous player Kobe Bryant. The proposed operation can return the result as: K.Bryant is the 6-skylines*. Hereinto, in addition to the 22 players shown in Table 7-4 (from $\lambda = 1$ to $\lambda = 3$), there are 9 players as 4-skylines (e.g. S.Nash) and 4 players as 5-skylines (e.g. D.Rose), all of whom are *more efficient* than B.Kobe in 2010-11 NBA regular seasons. On the other hand, the hieratical T-skyline results will benefit for detecting the possible imperfection of preference systems. Taken Case (i) as an example, obviously obtaining one Rebound (REB)/Assist (AST)/Block shot (BLK) is much harder than getting one score from shot. In this sense, the player-evaluation system may be unfair for those good defensive players. Thus, DMs will know that two players in our empirical study, A.Bynum and P.Gasol, have been underrated, even though both of them are qualified as 3-skylines. Such merit can be very valuable for real decision-making applications, particularly for personnel evaluation.

The second advantage of the T-skyline is that the outputting size of the hierarchical T-skyline sets is more controllable. In addition to the baseline competitor as conventional skyline operation, we further exam another representative skyline operation. In Lu, Zhang, and Ruan (2008), a skyline-order (SO) based skyline operation is provided for obtaining a hierarchical skyline result. This operation is defined as a skyline sequence $S = \{S_1, S_2, ..., S_n\}$ with respect to the entire object set U, where S_j for $j \in n$

* For simplicity, the objects in each hierarchical T-skyline set with respect to the tolerant degree λ can be represented as λ -skylines. Thus, 6-skylines means the objects with the skyline membership when $\lambda = 6$ excluding the objects with the skyline membership when $\lambda = 5$.

can be understood as the conventional skyline set with respect to the object set $U - \sum_{i=1}^{j-1} S_i$. Under the same settings thereinbefore, we calculated the hierarchical SO-based skyline sets and the size of the outputting skyline sets were provided in Table 7-4. When $\lambda = 1$ and $j = 1$, both of two operations return 9 players who are also the conventional skyline objects. When $j = 2$, the skyline order S_2 contains eighteen players, among which, if using T-skyline, seven players are 2-skylines, other six players are 3-skylines, and the remaining five players are with skyline membership when $\lambda > 3$. Obviously, the skyline order S_2 actually contains more than two kinds of players (2-skylines, 3-skylines, and λ -skylines when $\lambda > 3$) which should be differentiated according to DMs' preference. Through setting the tolerant degree λ via T-skyline, DMs can control the outputting size of skyline results more precisely and flexibly than the SO-based skyline operation and the baseline conventional skyline operation. In other words, the results by using the T-skyline are more controllable.

Table 7-4 The results of T-skyline with the comparisons under the general assumption

The T-skyline operation			The SO-based skyline operation	
λ	The T-skyline objects	Size	S_j	Size
$\lambda = 1$	T.Chandler, K.Durant, N.Hilario, D.Howard, L.James, L.Jordan, S.Novak, D.Nowitzki, S.O'Neal	9	$j = 1$	9
$\lambda = 2$	R.Allen, S.Curry, A.Horford, K.Love, L.Odom, P.Pierce, D.Wade	7	$j = 2$	18
$\lambda = 3$	A.Afflalo, C.Anthony, A.Bynum, J.Evans, P.Gasol, B.Griffin,	6	$j = 3$	23

Table 7-5 Three assemblies of preference intensities

PI	Criterion A	Criterion B	Criterion C
PI-1	Type I and DM-specified $G(x, y) = 1$	Type I and DM-specified $G(x, y) = 1$	Type I and DM-specified $G(x, y) = 1$
PI-2	Type III with $\varepsilon = 0.5$, and DM-specified $G(x, y) \geq 0.8$	Type II with $\varepsilon = 0.05$, and DM-specified $G(x, y) = 1$	Type V with $\varepsilon = 0.5$, and DM-specified $G(x, y) \geq 0.8$
PI-3	Type III with $\varepsilon = 0.5$, and DM-specified $G(x, y) \geq 0.9$	Type II with $\varepsilon = 0.08$, and DM-specified $G(x, y) = 1$	Type V with $\varepsilon = 0.5$, and DM-specified $G(x, y) \geq 0.9$

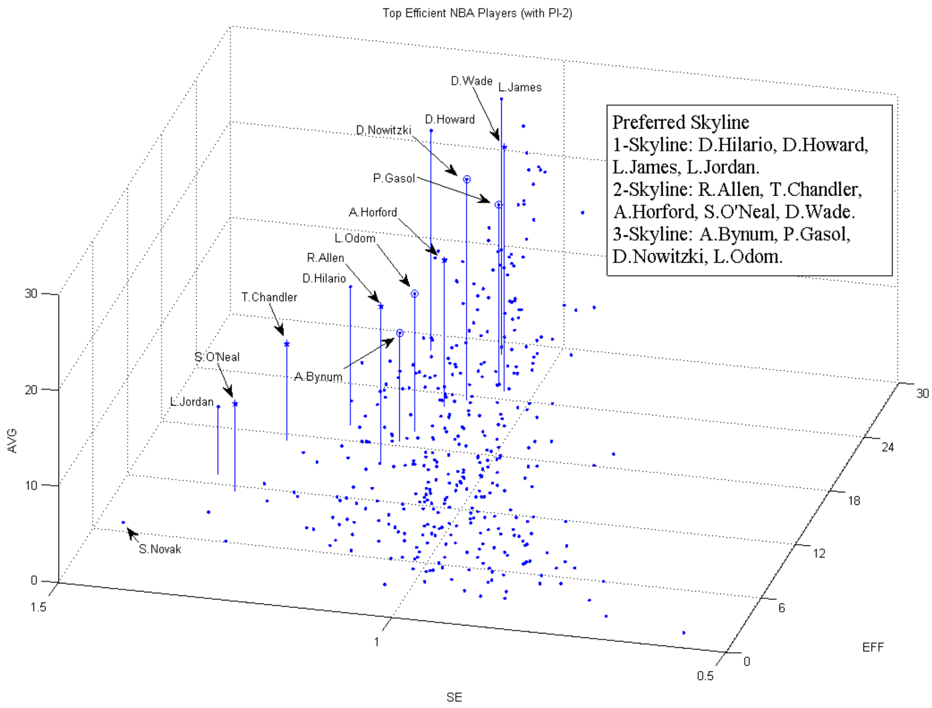
Table 7-6 The results of T-skyline with the preference intensities PI-2 and PI-3

PI	1-skylines	2-skylines	3-skylines	n-skylines
PI-2	N.Hilario, D.Howard, L.James, L.Jordan	R.Allen, T.Chandler, A.Horford, S.O'Neal, D.Wade	A.Bynum, P.Gasol, **D.Nowitzki**, L.Odom	…
PI-3	T.Chandler, N.Hilario, D.Howard, L.James, D.Jordan	R.Allen, **D.Nowitzki**, S.O'Neal,	A.Bynum, S.Curry, L.Odom, P.Pierce	…

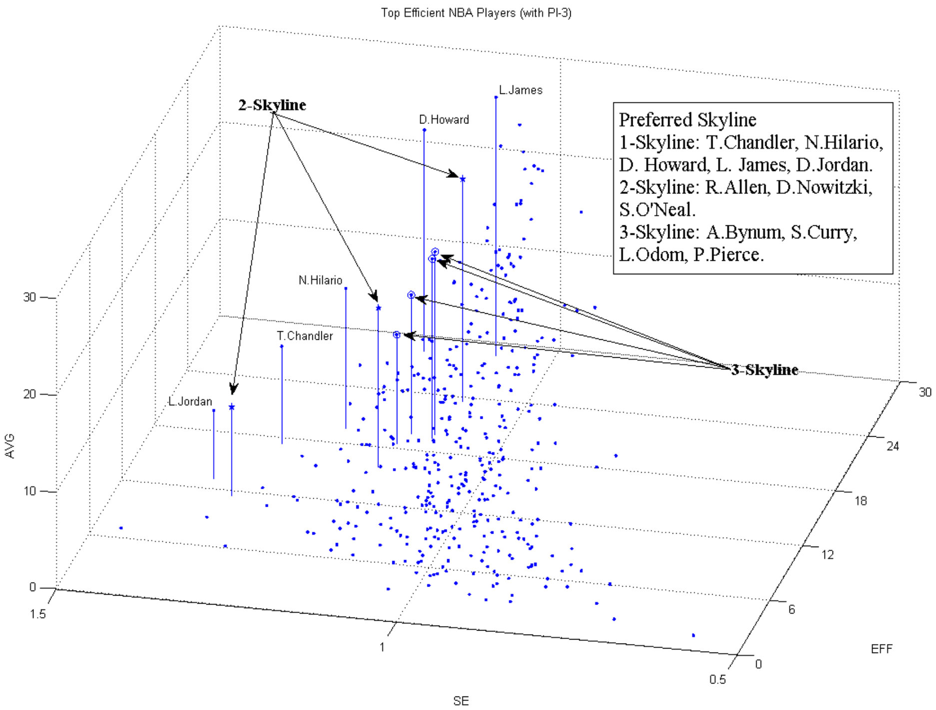
The third advantage of the proposed operation is its particular setting of preference intensity that can further handle dynamic preference of DMs. This characteristic is a breakthrough since the T-skyline computation is able to no longer under the general assumption which can be represented as the simplest assemblies of preference intensities PI-1 as shown in Table 7-5. For comparisons, the above experiments are actually with the setting of PI-1. In the following, we will conduct T-skyline with the dynamic preference settings that are specified as two assemblies of preference intensities PI-2 and PI-3 as shown in Table 7-5, for which the competitors are invalid.

In Table 7-6, we provide the detailed computation results by using the T-skyline with PI-2 and PI-3. The preferred T-skyline with PI-2 is illustrated in Figure 7-2(a). We show the first three hierarchical T-skyline as 1-skylines (using the symbol of *point*), 2-skylines (using the symbol of *star*), and 3-skylines (using the symbol of *circle*). The name of the corresponding player is also marked in this figure. Figure 7-2(b) illustrates the preferred T-skyline with PI-3. In this figure, 5 players (T.Chandler, N.Hilario, D.Howard, L.James, and D.Jordan) are marked as the 1-skylines. It further includes 3 players as 2-skylines and 4 players as 3-skylines. By contrast, we can clearly view the changes with respect to the different PIs. As illustrated in Figure 7-2, the same setting of preference system (i.e. Criteria A, B, and C) can lead to the different skyline results due to the changes of intensity function $\rho(d)$. Compared with the T-skyline objects with PI-1 in Table 7-4, the obtained results under both PI-2 and PI-3 narrowed down the number of qualified skyline objects in each hierarchy. Specifically, 4 players (i.e. K.Durant, S.Novak, D.Nowitzki, S.O'Neal) are no longer as 1-skylines. Some players also vary their skyline membership with respect to different PIs. For example, the player D.Nowitzki is as 1-skylines in PI-1, 2-skylines in PI-3, and 3-skylines in PI-2.

With respect to the same preference system, the different settings of PIs can generate the multiple skyline results, which bring the benefits for better decision performances. Generally speaking, skyline operations contain an inherent property: an object, which is with the most preferred value in at least one criterion, tends to be the 1-skylines. Such property is adverse in some decision-making situations. For instance, DMs may feel hard to accept the experimental result that S.Novak is as good as D.Nowitzki (being 1-skylines of PI-1). The reason is that S.Novak is with the most preferred values in both Criterion B and Criterion C. Such matter make the DM-specified evaluation system be questionable. In T-skyline, via setting preference intensity functions, the expected D-value d_0 can control the similarity in dominance relations. Thus, various kinds of human factors (e.g. group opinions, statistical results, and so on) can be further taken into account in the process of player evaluation, and thus eliminate the strong influence of extreme values to the final resulting skyline. In our experiments of PI-2 and PI-3, S.Novak is no longer with any skyline membership when setting $\lambda=1$, $\lambda=2$ or $\lambda=3$ (see Figure 7-2). This mechanism gives benefits to many real-world applications.



(a) The preferred T-skyline with PI-2



(b) The preferred T-skyline with PI-3

Fig 7-2 Tolerant skyline with different preference intensities

7.5.2 Continuous T-skyline Maintenance on Case (ii)

In this section, we use Case (ii) to demonstrate continuous T-skyline maintenance. Suppose DMs are interested to know: *Who is/are the most efficient ball-stealer(s)?* We firstly establish the dynamic preference system that includes three preference functions { f_I , f_{II} , f_{III} }. A criteria set is defined in $P=\{A,\ B,\ C\}$ below.

(1) Criterion A: f_I (STL, TO) =STL/TO

(2) Criterion B: f_{II} (STL, PF) =STL/PF

(3) Criterion C: f_{III} (STL, MIN) =(STL/MIN) × 48

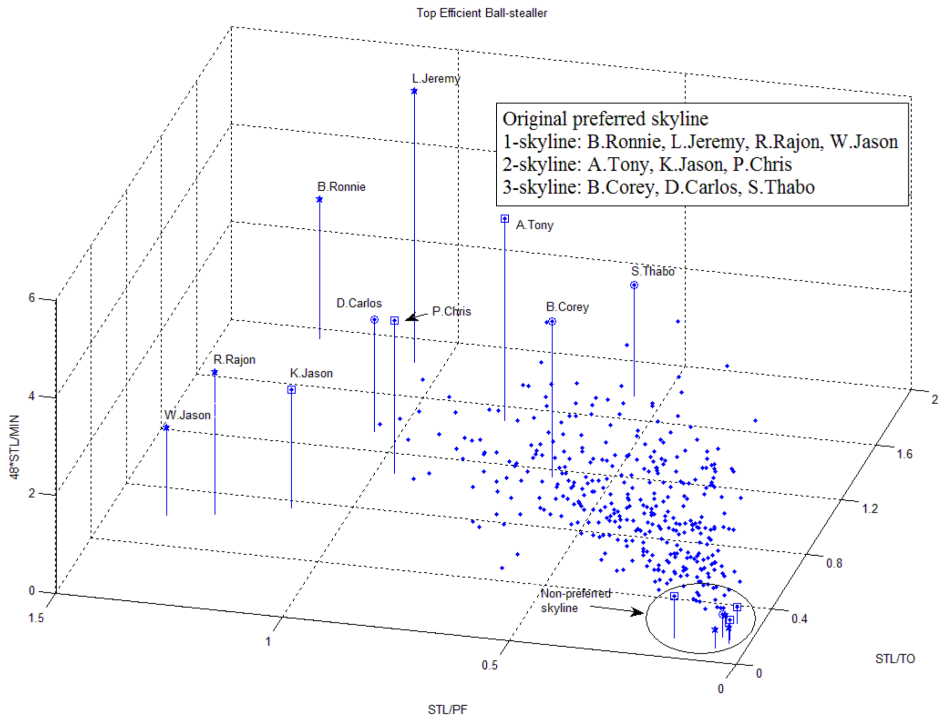
Remark that criterion A is the ratio of the number of steal (STL) and the number of turnover (TO). Criterion B is the ratio of the number of steal and the number of personal foul (PF). Criterion C is the number of steal per 48 minutes. In practice, this preference system is commonly used for evaluation of the GUARD player's ability in ball-stealing.

In this experiment, we consider to update T-skyline after a portion of objects is deleted from the original dataset (n=383). Suppose DMs want to find the T-skyline when just considering *the game starters*. Then, the objects (n=51) are deleted because their values on the attribute "game starting (GS)" are equals to zero. Using algorithm IV, the resulting T-skyline are shown in Table 7-7.

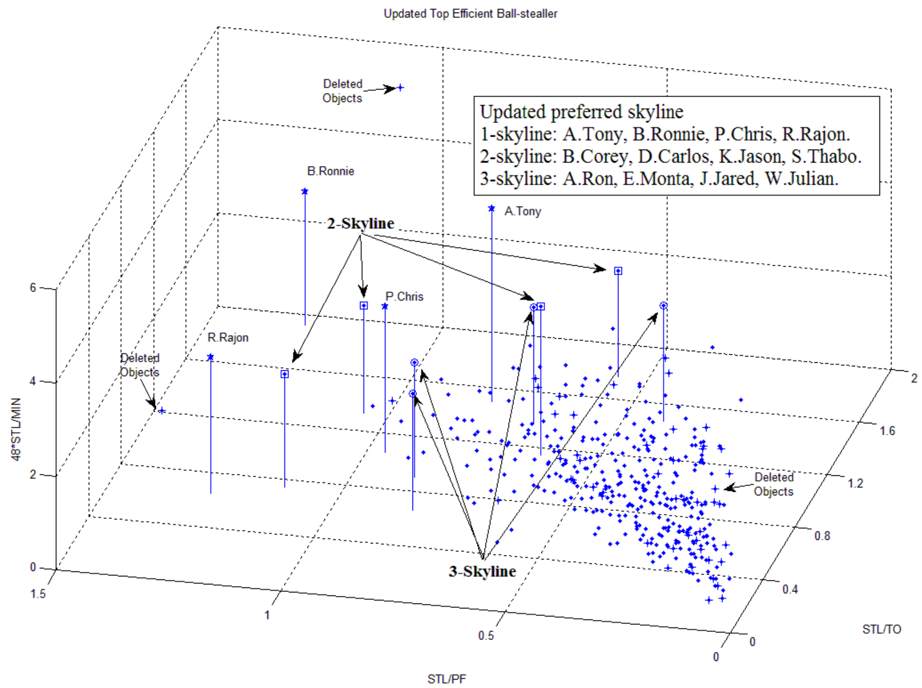
Figure 7-3 illustrates the comparable T-skyline results. The preferred T-skyline without object deletion is illustrated in Figure 7-3(a). We show the first three hierarchical T-skyline as 1-skylines (using the symbol of *star*), 2-skylines (using the symbol of *box*), and 3-skylines (using the symbol of *circle*). The names of the corresponding players are also marked in this figure. Several players as non-preferred T-skyline are also marked. Figure 7-3(b) illustrates the preferred T-skyline after object deletion. The deleted non-game-starters (totally 51 players) are represented by the symbols "+". We also mark the players' names of 1-skylines including A.Tony, B.Ronnie, P.Chris, R.Rajon, and highlight 4 players as 2-skylines and another 4 players as 3-skylines. Clearly *Jeremy Lin* and *Jason Williams* as non-game-starters are no longer with any skyline membership.

Table 7-7 The comparison of T-skyline after object deletion

T-skyline	1-skylines	2-skylines	3-skylines	n-skylines
n=383	B.Ronnie, **L.Jeremy**, R.Rajon, **W.Jason**	A.Tony, K.Jason, P.Chris	B.Corey, D.Carlos, S.Thabo	…
n=332	A.Tony, B.Ronnie, P.Chris, R.Rajon	B.Corey, D.Carlos, K.Jason, S.Thabo	A.Ron, E.Monta, J.Jared, W.Julian	…



(a) The T-skyline without object deletion



(b) The updated T-skyline with object deletion

Fig 7-3 Continuous T-skyline maintenance on case (ii).

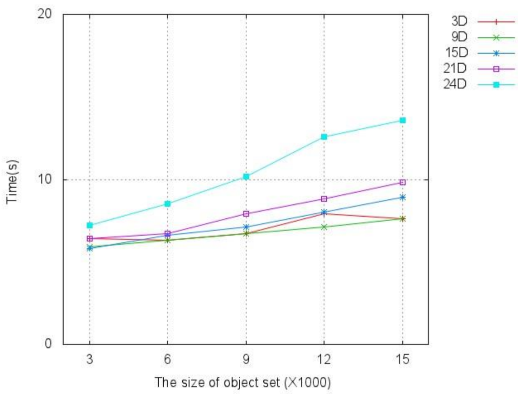
7.5.3 Running Time Analysis

This section provides the running time analysis in two aspects: (i) the efficiency of EUA and UA methods, (ii) the stability of tolerant degree, particularly examining the influence of tolerant degree in different nature of datasets (i.e. various dimensions, various sizes of objects). We employ the Case (i) to conduct this experiment.

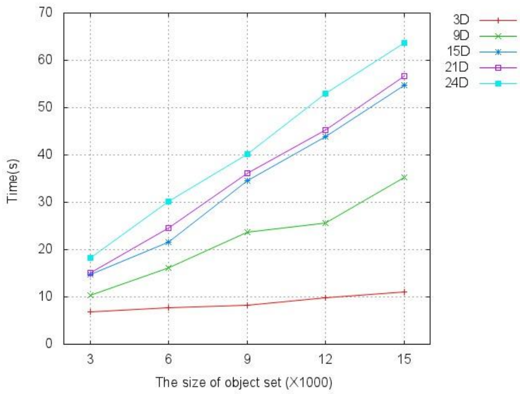
(1) The efficiency of EUA and UA methods

We examine the running time of EUA and UA, respectively, with respect to five kinds of object sets (i.e. 3k, 6k, 9k, 12k, 15k) in consideration of five kinds of dimensions (i.e. 3D, 9D, 15D, 21D, 24D). Generally, the curves in Figure 7-4(a) are smoother and the curves in Figure 7-4(b) are steeper. It illustrates that the running time of both two algorithms increases linearly along with the increment of the number of objects or dimensions. Nevertheless, EUA is more sensitive to the increment of dimensions, and UA is more sensitive to the increment of objects.

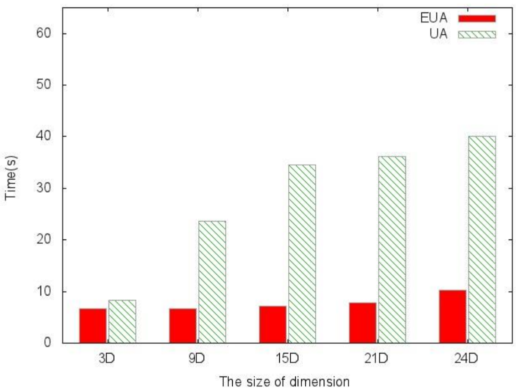
From Figures 7-4(c) and 7-4(d), the running time of EUA is very stable (around 7s-10s) although dimensions and sizes of objects are various. And the running time of UA is increasing distinctly. Combining above results, we remark that EUA is more efficient than UA in computing of 1-skylines while the object set is large and of low dimensions. But, it cannot be used for all T-skyline (i.e. $\lambda \geq 2$). UA can effectively conduct all T-skyline computations, although its efficiency leaves room for future improvement.



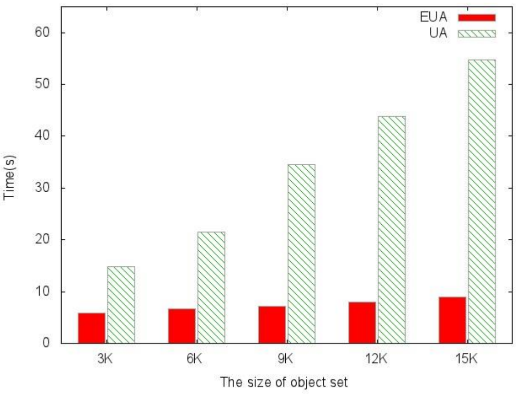
(a) Testing of EUA in diverse sizes of dimension and object set



(b) Testing of UA ($\lambda = 1$) in diverse sizes of dimension and object set



(c) Comparison in diverse sizes of dimensions ($n = 9000$)



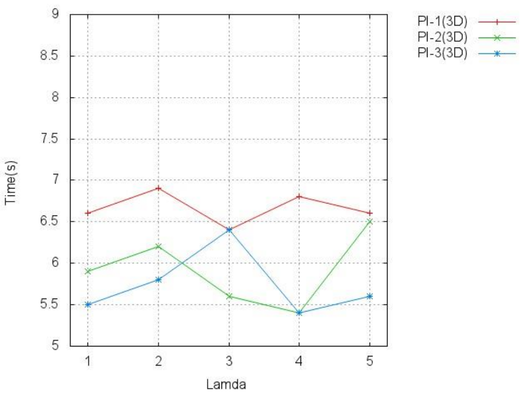
(d) Comparison in diverse size of object sets ($d = 15$D)

Fig 7-4 Running time comparison of EUA and UA (λ=1)

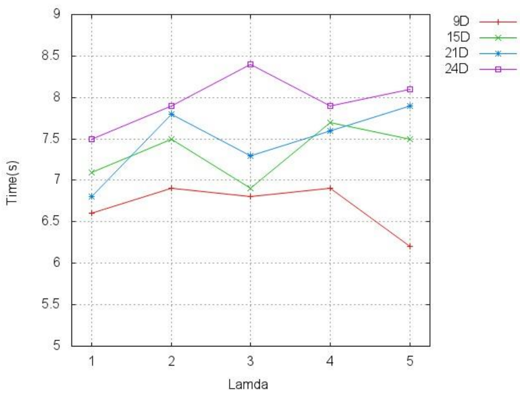
(2) The stability of tolerant degree

Tolerant degree is the important coefficient that controls the outputting size of skyline. In this section, we examine its stability in various natures of datasets. We firstly employ the preprocessed dataset (n=383) in Figures 7-5(a) and 7-5(b). Figure 7-5(a) shows the running time under three preference intensities (fixed in 3D), and Figure 7-5(b) shows another four dimensions (i.e. 9D, 15D, 21D, 24D) with the same PI. It shows that, in the small dataset (n=3), tolerant degree (λ , Lamda) is very stable when varying PIs (6 ± 0.8s in Figure 7-5(a)) or vary Dimensions (7.3 ± 1.1s in Figure 7-5(b)).

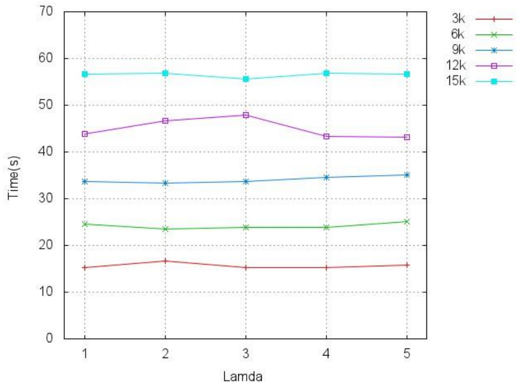
Figures 7-5(c) and 7-5(d) vary the size of objects from 3k to 15k and the size of dimensions from 3D to 24D, in order to further examine the stability of tolerant degree under larger dimensions and larger object sets. From this experiment, it also shows that the tolerant degree is very stable in (i) various sizes of objects (± 2 for n ≥ 3k), (ii) various dimensions (± 2 for n ≥ 3D). All in all, we can draw the conclusion that the tolerant degree will not impact the computational efficiency in T-skyline computation.



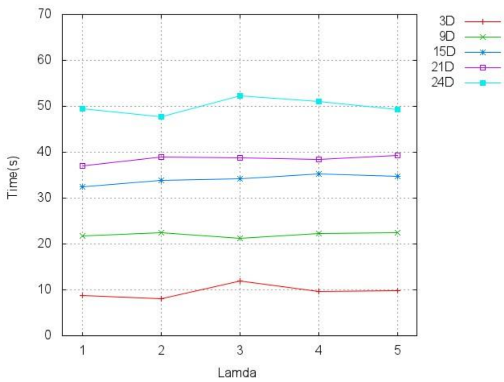
(a) Testing in diverse preference intensities (n=383, 3D)



(b) Testing in diverse dimensions (n=383, PI-1)



(c) Testing in diverse sizes of object set (15D, PI-1)



(d) Testing in diverse dimensions (n=9k, PI-1)

Fig 7-5 Running time with various tolerant degrees

7.6 Summary

This chapter proposes a novel skyline operation for decision supports. We firstly investigate preference relations on conventional skyline operation and thus establish a dynamic preference model through introduction of the concept of preference intensity. Based on this model, we define a new decision-oriented skyline operation called T-skyline. The result of this operation is a set of hieratical skylines with respect to the coefficients called tolerant degree. This new operation provides multiple parameters which make the outputting size of skyline adjustable and partly controllable. In the meanwhile, such flexible mechanism offers the opportunity for decision-makers to correct the possibly imperfect preference system. For computation, we provide two polynomial-time algorithms. Hereinto, the UA algorithm is used for the general T-skyline computation and the EUA algorithm is used for the special T-skyline to achieve the higher efficiency. In addition, we study the method for continuous maintenance of T-skyline when object deletion. Finally, an empirical study related to the NBA player evaluation is elaborated. We use it for demonstrating the effectiveness and advantages of this new operation, as well as illustrating its continuous maintenance. We also provide the running time analyses on the efficiency and the stability. As the conclusion, the proposed operation outperforms other existing skyline operations in solving the presented decision-making problems and the developed computation methods are effective and practical. This proposed operation posseses the potential for such employee evaluations as other similar applications of personnal selections.

CHAPTER EIGHT

Investigation of Integrated Decision-Making Techniques for Structured Preference Evaluation and Selection

8.1 Overview

Since Chapter 3, we have considered the different decision scenarios including supplier selection, warehouse evaluation, and personnel selection. All these applications comply with the basic paradigm of MCDM: providing decision recommendations for DMs concerning a finite set of objects evaluated with multiple preference-related criteria. In this chapter, we made a comprehensive literature investigation of integrated decision-making (DM) techniques. We review the articles published from 2008 to 2012 on the application of DM techniques related to supplier selection. By using a methodological decision analysis in four aspects including decision problems, decision makers, decision environments, and decision approaches, we finally selected and reviewed 123 journal articles for applying DM techniques to supplier selection. To cater for the research trend on uncertain supplier selection, these articles are roughly classified into seven categories according to the different types of uncertainties. Under such classification framework, 26 DM techniques are identified from three perspectives: (1) Multicriteria decision making (MCDM) techniques, (2) Mathematical programming (MP) techniques, and (3) Artificial intelligence (AI) techniques. We categorically reviewed each of the 26 techniques and analyzed the means of integrating these techniques for supplier selection. Our survey provides the recommendation for future research and facilitates knowledge accumulation and creation concerning the application of DM techniques in supplier selection.

8.2 Background

Supplier selection (SS) has received considerable attention for its significant effect toward successful Logistic and supply chain management (LSCM). At least two valuable academic surveys had well reviewed the literature on SS. Jain, Wadhwa, and Deshmukh (2009) reviewed the main approaches to supplier-related issues including SS, supplier-buyer relationships, and supplier-buyer flexibility in relationships based on a summary of existing research before 2007. Ho, Xu, and Dey (2010) analyzed MCDM approaches for SS based on journal articles from 2000 to 2008. However, great developments on SS have emerged over the last five years. A large number of new ideas, techniques, and approaches have been contributing to this promising area. Existing surveys are not keeping pace. Therefore, we believe that a new and systematic survey is useful for consolidating the most recent research efforts on this area.

In this chapter, we comprehensively collected the literature associated with the descriptors “supplier selection,” “vendor selection,” and “decision making” from academic databases including Science Direct, Emerald, Springer-Link Journals, IEEE Xplore, Academic Search Premier, and World Scientific Net. After a methodological decision analysis of all collected articles, we reviewed 123 international journal articles published from 2008 to 2012. We attempt to answer the following four questions: (1) Which decision-making (DM) techniques have frequently been applied? (2) What are the relationships and categories among these DM techniques? (3) How can the DM techniques discussed in literature be effectively integrated to achieve complex decision goals? (4) What are the development status and research trends for uncertain SS?

The emerging trend in current research is the integration of DM techniques in constructing an effective decision model to address realistic and complex SS problems, particularly for the consideration of multitudinous uncertainty factors. Given the diversity and the complexity of SS research, we particularly use a methodological decision analysis framework for the selection of the collected articles. This framework provides a guide for the analysis of the literature based on four aspects: (1) decision problems, (2) decision makers, (3) decision environments, and (4) decision approaches. First, we confine our survey on structural SS and thus eliminate the literature that discusses semi-structural or non-structural decision problems. Consequently, a total of 123 articles are selected for detailed review. Second, the literature that involves multiple decision makers as a group is specifically indicated as reference for readers. Third, we classify the selected articles into seven categories after a decision environment analysis. Fourth, the emerging decision approaches are investigated in detail. Specifically, 26 DM techniques are independently reviewed from three perspectives: MCDM techniques, mathematical programming (MP) techniques, and artificial intelligence (AI) techniques. Major integrated approaches are separately reviewed. These approaches include the integrated analytic hierarchy process (AHP), integrated analytic network process (ANP), integrated data envelopment analysis (DEA), and integrated uncertain approaches, among others.

The remainder of this chapter is organized as follows: This section describes the methods for selecting the literature. Section 8.3 presents the research methodology. In Section 8.4, we use a methodological decision analysis model to sort the selected articles and then subsequently form a summary table. Section 8.5 provides a detailed literature review on the DM techniques. Section 8.6 gives suggestions for future works. We conclude this chapter in Section 8.7.

8.3 Research Methodology

The research methodology of this survey is depicted in Figure 8-1. Our initial objective is to investigate the applications of DM techniques in current research on SS. Thus, we define the following conditions to limit our collection of the articles:

1. Only articles that had been published on decision sciences, computer sciences, or business management-related fields were selected because such articles are most possibly in accordance with the focus of this survey. The articles were searched from academic databases including Science Direct, Emerald, Springer-Link Journals, IEEE Xplore, Academic Search Premier, and World Scientific Net.
2. The keywords for our search were “supplier selection,” “vendor selection,” “decision making,” and so on. Only the literature that had been published between 2008 and 2012 was adopted.
3. To achieve the highest level of relevance, only international journal articles were selected to serve the related research communities better. Thus, conference articles, master and doctoral dissertations, textbooks, unpublished articles, and notes are not included in this review.

Based on these considerations, more than 300 articles were collected. These articles refer to (1) structural, semi-structural, and non-structural DM problems; (2) individual and group-involved DM problems; and (3) certain and uncertain DM problems. Each article was carefully reviewed and selected strictly according to our scope by using a methodological decision analysis model (see Section 3). Consequently, a total of 123 articles were identified as suitable for our survey. In the next section, we aim to provide a summarization of the selected articles. The details of the reviews are presented in Section 8.5.

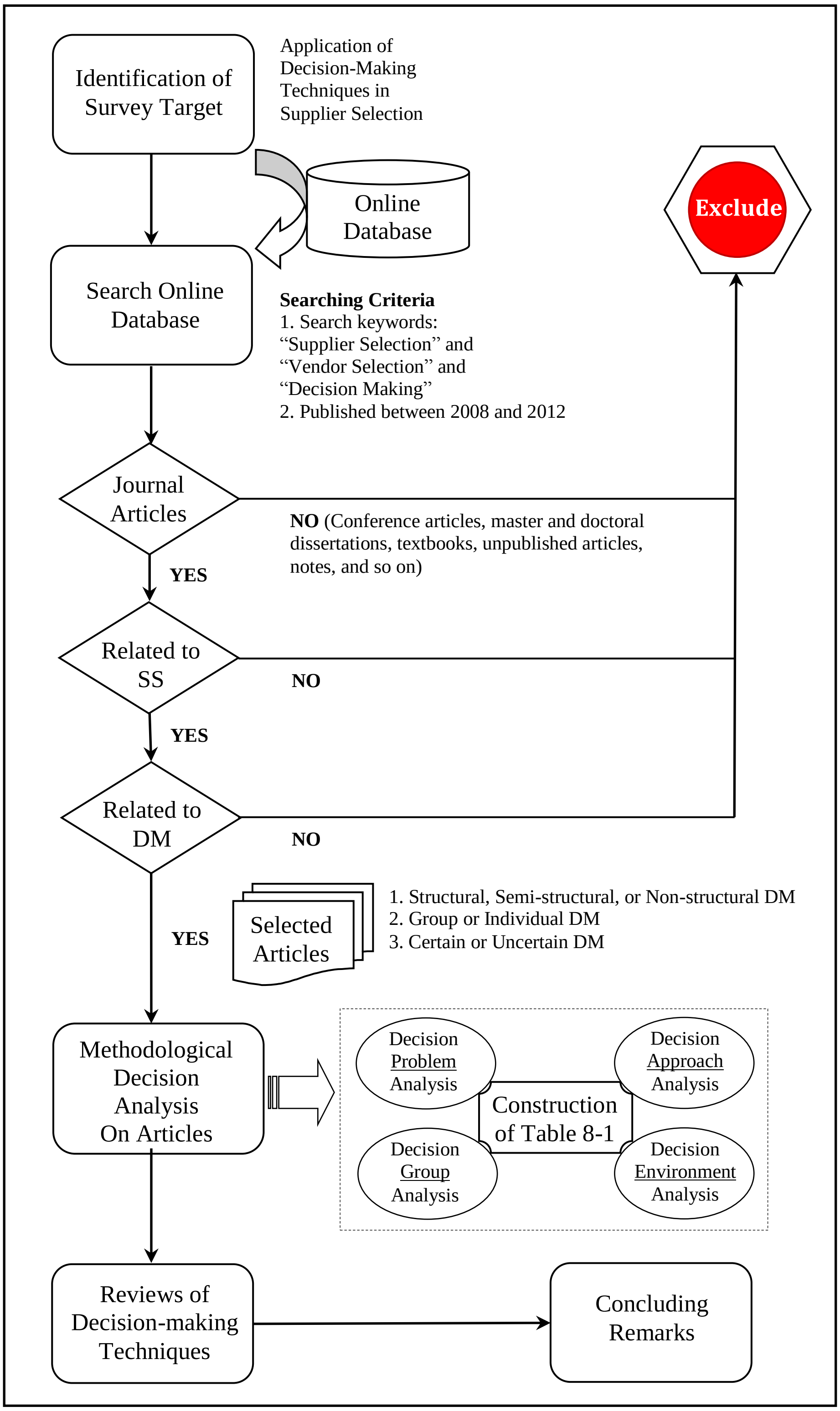


Fig 8-1 Research methodology of this survey.

8.4 Methodological Decision Analysis on Selected Articles

SS is a typical DM activity. Considering its diversity and complexity, we establish a methodological decision analysis model for a standardized analysis of all collected articles. This model contains four analytic aspects: (1) decision problems, (2) decision makers, (3) decision environments, and (4) decision approaches.

1. Decision problem analysis

The first step in scientific decision analysis is to model the realistic decision problem with a clear formulation. In this sense, problems can be modeled as any of the following three types: structural, non-structural, and semi-structural. Structural problems reveal a well-organized formulation, such as highly structured information tables, measurable decision goals, and clear problem boundaries. Both non-structural and semi-structural problems are mostly at the strategic management level. In these problems, decision information usually lacks organization or is non-quantitative. More importantly, human perception and judgment play decisive roles, although such human factors are usually intangible or unmeasurable. These three types of modeled decision problems are often found in the literature.

In this survey, we mainly focus on the application of DM techniques for structural SS. Thus, the literature that discusses non-structural and semi-structural problems are not considered in this survey. The interested reader can refer to the related literature for details. For example, Shen and Yu (2009; 2012) formulated SS as a semi-structural problem. Ordoobadi and Wang (2011) as well as Wu (2009c) formulated SS as a non-structural problem.

2. Decision maker analysis

Decisions can be made by individuals. For example, an investor can decide on which stock to buy by considering the rate of return. Large complex decisions, particularly at high managerial levels, usually involve multiple decision makers who need to work effectively in groups. Therefore, dealing with often conflicting objectives, inconsistent judgments, and incompatible opinions is a challenge in group DM settings.

For structural SS, multiple experts are usually involved as decision makers. The common scenario is that qualified experts need to provide their professional evaluations on alternative suppliers according to given criteria. Different weights are set for each expert according to their profession, expertise, qualification, or experience. In any case, the key step in DM processes is information fusion, for example, the weighted average or the ordered weighted average. In this survey, we specify the literature that involves group DMs as reference for readers (see Table 8-1).

3. Decision environment analysis

In a broad sense, decision environments contain decision goals, decision principles, available resources, and possible uncertainties. For SS, a number of studies formulated the addressed DM problems in terms of deterministic conditions without considering any involved uncertainties. Nevertheless, recent research tends to cater to more realistic SS problems via uncertainty hybrid approaches. According to our reviews on the selected articles from 2008 to 2012, the dominant method is multitudinous fuzzy hybridization. We can roughly group these methods into the following five categories: (1) basic concept of fuzzy logic, (2) triangular fuzzy sets, (3) trapezoidal fuzzy sets, (4) intuitionistic fuzzy sets, and (5) interval-valued intuitionistic fuzzy sets. Moreover, a number of non-fuzzy uncertain formulations also emerged, including stochastic and probabilistic formulations as well as incomplete and imprecise decision information.

4. Decision approaches analysis

A decision approach is understood as a complete problem-solving model (also called scheme or solution) that is capable of effectively achieving the stated decision goals. In today's global market, SS has become a very important activity in LSCM. Given the complexity of the process in the real world, current research tends to integrate multiple DM techniques in establishing a decision model. Different techniques can separately deal with the corresponding sub-problems, thus improving the performance of the whole decision approach significantly.

Along with the rapid growth of realistic demand and the development of information techniques in past decades, current studies have directed more attention to addressing uncertain SS via unconventional means under non-classical assumptions or non-deterministic conditions. For this reason, we roughly classify all selected articles into seven categories based on different decision environments. These categories are defined as follows:

(1) Certain DM

This category includes classical assumptions, deterministic conditions, certain decision environments, and conventional means.

(2) Basic fuzzy logic hybridization

This category includes Zadeh's fuzzy sets, fuzzy logic, basic fuzzy values, and classical fuzzy preference relations.

(3) Triangular fuzzy hybridization

This category includes triangular fuzzy values and triangular fuzzy preference relations.

(4) Trapezoidal Fuzzy Hybridization:

This category includes trapezoidal fuzzy values and trapezoidal fuzzy preference relations.

(5) Intuitionistic fuzzy hybridization

This category includes intuitionistic fuzzy values, intuitionistic fuzzy preference relations, and vague values [vague sets are intuitionistic fuzzy sets that had been proven by Bustince and Burillo (1996)].

(6) Interval-valued intuitionistic fuzzy hybridization

This category includes interval-valued intuitionistic fuzzy values and interval-valued intuitionistic fuzzy preference relations.

(7) Non-fuzzy hybrid uncertain DM

This category includes stochastic, probabilistic and grey-valued formulations, as well as incomplete and imprecise decision information.

Based on these classifications, we provide the overall summary of 26 DM techniques discussed in the 123 selected journal articles in Table 8-1. We highlight 26 DM techniques, which are listed in the third column of Table 8-1, in terms of core DM techniques. In the last column of Table 8-1, we note the additional remarks on the proposed decision approaches for reference by readers. In addition, we indicate the literature that involved group DMs via the term "Group." The detailed reviews are presented in the next section.

Table 8-1 Summarization of the decision approaches with respect to DM techniques

Approaches	Literature	Core DM Techniques	Additional Features of Decision Approaches
Certain Decision Approaches	Levary (2008)	AHP	Reliability chain
	Chan & Chan (2010)	AHP	AHP model for apparel industry
	Mafakheri, Breton, & Ghoniem (2011)	AHP	Two-stage dynamic programming
	Ishizaka, Pearman, & Nemery (2012).	AHP	AHP-based sorting approach
	Kull & Talluri (2008)	AHP, GP	Product life cycle consideration
	Bhattacharya et al. (2010)	AHP	Cost factor measure; QFD technique
	Ordoobadi (2010)	AHP	Taguchi loss function (TLF)
	Ustun & Demirtas (2008)	ANP, LP	Benefit, Opportunity, Cost, and Risk model
	Demirtas & Ustün (2008)	ANP, LP	Benefit, Opportunity, Cost, and Risk model
	Demirtas & Ustun (2009)	ANP, GP, LP	Benefit, Opportunity, Cost, and Risk model
	Lin, Lin, Yu, & Tzeng (2010)	ANP	Interpretive structural modeling for wafer testing
	Lin, Chen, & Ting (2011)	ANP, TOPSIS, LP	Case study related to manufacturing enterprise
	Tseng, Chiang, & Lan (2009)	ANP	Choquet integral
	Razmi & Rafiei (2010)	ANP, NLP	Mixed integer NLP
	Ho, Dey, & Lockstrom (2011)	ANP	Integrated approach; QFD technique
	Wu (2009a)	DEA, DT, NN	A hybrid model for classification and prediction
	Wu & Blackhurst (2009)	DEA	An augmented DEA approach
	Toloo & Nalchigar (2011)	DEA, LP	Regarding both cardinal and ordinal data
	Falagario et al. (2012)	DEA	Regarding the case of public procurement tenders
	Ng (2008)	LP	A simple weighted LP
	Feng, Fan, & Li (2011)	MOP	Collaborative utility; Tabu search based algorithm
	Che (2010a)	GA	Guided-Pareto GA; Multi-period SS
	Yeh & Chuang (2011)	GA, MOP, NLP	Multiobjective mixed integer NLP
	Rezaei & Davoodi (2012)	GA, MOP, NLP	Multiobjective mixed integer NLP
	Vahdani et al. (2010)	ELECTRE	Extend ELECTRE for interval values
	Liu & Zhang (2011)	ELECTRE	Combine entropy weight and ELECTRE-III
	Lee & Ouyang (2009)	NN	NN-based predictive model; Negotiation process
	Guo, Yuan, & Tian (2009)	SVM, DT	Hierarchical potential SVM method
	Chang & Hung (2010)	RST	Rule-based approach
	Zhao & Yu (2011)	CBR	(Group) Information entropy; Petroleum enterprises
	Lin, Chuang, Liou, & Wu (2009)	AR	Combination of association rule and set theory
	Tsai, Yang, & Lin (2010)	ACA	Attribute-based ant colony system
Basic Fuzzy Hybrid Approaches	Sevkli et al. (2008)	AHP, LP	AHP weighted fuzzy logic hybridization
	Wang & Yang (2009)	AHP, MOP, LP,	Fuzzy compromise programming
	Tsai & Hung (2009)	AHP, GP	Fuzzy GP; Green SCM
	Labib (2011)	AHP	Fuzzy linguistic expression; Fuzzy logic
	Amid, Ghodsypour, & O'Brien (2011)	AHP	Weighted max-min fuzzy decision model

	Chen & Chao (2012)	AHP	Consistent fuzzy preference relations
	Chamodrakas et al. (2010).	AHP	Fuzzy AHP; Interval valued pairwise comparison
	Lin (2012)	ANP, MOP, LP	Fuzzy multi-objective LP
	Amid, Ghodsypour, & O'Brien (2009)	MOP	A weighted additive fuzzy MOP
	Ozkok & Tiryaki (2011)	MOP	Compensatory fuzzy approach
	Keskin, llhan, & Ozkan (2010)	NN	Fuzzy adaptive resonance theory, clustering
	Güneri, Ertay, Yücel (2011)	NN	Utilization of Adaptive neuro-fuzzy inference system
	Wang (2008)	GA	Configuration change assessment
	Wu (2009b)	GST, DST	(Group) Grey relational analysis
	Crispim & De Sousa (2010)	TOPSIS	Fuzzy values; Regarding virtual enterprises
	Xu & Yan (2011)	PSO	Level-2 fuzzy values
Triangular Fuzzy Hybrid Approaches	Chan et al. (2008)	AHP	Global supplier selection
	Bottani & Rizzi (2008)	AHP	AHP-based clustering technique
	Yang, Chiu, Tzeng, & Yeh (2008)	AHP	Non-additive fuzzy integral
	Lee (2009a)	AHP	Benefit, Opportunity, Cost, and Risk model
	Lee (2009b)	AHP	Benefit, Opportunity, Cost, and Risk model
	Lee, Kang, & Chang (2009)	AHP, GP	Multiple goal programming
	Wang, Cheng, & Huang (2009)	AHP, TOPSIS	A fuzzy hierarchical TOPSIS
	Che (2010b)	AHP, PSO	Green SCM
	Şen, Şen, Başlgil (2010)	AHP	Max-min method
	Kilincci & Onal (2011)	AHP	Fuzzy AHP; Regarding washing machine companies
	Punniyamoorthy et al. (2011)	AHP	Structural equation modeling (SEM)
	Yucenur, Vayvay, & Demirel (2011)	AHP, ANP	Linguistic variables
	Zeydan, Colpan, & Cobanoglu (2011)	AHP, TOPSIS, DEA	(Group) Integration of multiple techniques
	Shaw et al. (2012)	AHP, MOP	Low carbon SCM
	Yu, Goh, & Lin (2012)	AHP, MOP	Soft time window
	Razmi, Rafiei, & Hashemi (2009)	ANP, NLP	Network formation and pairwise comparisons
	Amin & Razmi (2009)	ANP	Case study; QFD technique
	Onut, Kara, & Isik (2009)	ANP, TOPSIS	Regarding telecommunication industries
	Vinodh, Ramiya, & Gautham (2011)	ANP	Regarding manufacturing industries
	Buyukozkan & Cifci (2012)	ANP, TOPSIS, DE-MATEL	Green SCM
	Azadeh & Alem (2010)	DEA, TOPSIS	Regarding environmental performance of suppliers
	Chen (2011b)	DEA, TOPSIS	Strengths, Weakness, Opportunities, Threats model
	Razmi, Songhori, & Khakbaz (2009)	TOPSIS, LP	(Group) fuzzy TOPSIS integrated with fuzzy LP
	Hsu, Chiang, & Shu (2010)	NLP	Fuzzy preference relations
	Singh, Kumar, & Gupta (2010)	LP	Fuzzy statistical method
	Haleh & Hamidi (2011)	MOP, LP	Regarding multi-period time horizon
	Amin & Zhang (2012)	MOP, LP	(Group) Multiobjective mixed integer LP
	Kara (2011)	SP, TOPSIS	Fuzzy TOPSIS
	Amin, Razmi, & Zhang (2011)	SWOT, LP	(Group) Fuzzy SWOT model
	Awasthi, Chauhan, & Goyal (2010)	TOPSIS	Environmental performance
	Deng & Chan (2011)	TOPSIS	(Group) Dempster Shafer Theory of evidence
	Dalalah, Hayajneh, & Batieha (2011)	TOPSIS, DEMATEL	(Group) Fuzzy DEMATEL; Causal diagram
	Montazer, Saremi, & Ramezani (2009)	ELECTRE	Fuzzy ELECTRE-III
	Sevkli (2010)	ELECTRE	Linguistic variable
	Chen, Wang, & Wu (2011)	PROMETHEE	(Group) Fuzzy PROMETHEE; Case study
	Chang, Chang, & Wu (2011)	DEMATEL	Fuzzy DEMATEL; Evaluate performance
	Chen & Wang (2009)	VIKOR	Fuzzy VIKOR
	Chou & Chang (2008)	SMART	(Group) Fuzzy SMART
	Tseng (2011)	GST	Integrating grey degrees and fuzzy system; Green SCM
	Golmohammadi & Mellat-Parast (2012)	GST	(Group) Grey relational analysis
	Vahdani, & Zandieh (2010)	***	A fuzzy balancing and ranking method
Trapezoidal Fuzzy Hybrid Approaches	Guneri, Yucel, & Ayyildiz (2009)	TOPSIS, LP	LP model under fuzzy environments
	Faez, Ghodsypour, & O'Brien (2009)	CBR	Fuzzy CBR approach
	Yucel & Guneri (2011)	TOPSIS, LP	(Group) Weighted additive fuzzy programming
	Liao & Kao (2011)	TOPSIS, GP	(Group) Multi-choice goal programming
	Sanayei et al. (2010)	VIKOR	(Group) Linguistic variables expression
	Shemshadi et al. (2011)	VIKOR	(Group) Linguistic terms; Entropy
	Ferreira & Borenstein (2012)	BN	(Group) Fuzzy Bayesian model; Influence diagrams
	Wu, Zhang, Wu, & Olson (2010)	MOP	Possibility fuzzy hybridization; Risk analysis
	Ordoobadi (2009)	***	Fuzzy arithmetic operators
	Amindoust et al. (2012)	***	(Group) Fuzzy inference system
Intuitionistic Fuzzy Hybrid Approaches	Boran, Genc, Kurt, & Akay (2009)	TOPSIS	(Group) Intuitionistic fuzzy information aggregation
	Chai, Liu, & Xu (2012)	PROMETHEE	Extended superiority and inferiority ranking approach
	Chen (2011a)	LP, GP	Optimism-pessimism model; Net predisposition; An integrated programming model
	Khaleie et al. (2012)	***	(Group) Intuitionistic fuzzy clustering; Intuitionistic fuzzy information fusion
	Zhang, Zhang, Lai, & Lu (2009)	***	(Group) Linguistic variables converted to vague sets
	Khaleie & Fasanghari (2012)	***	(Group) Intuitionistic fuzzy information entropy;

			Association coefficient
Interval valued Intuitionistic Fuzzy Hybrid Approaches	Wang, Li, & Xu (2011)	TOPSIS, FP	Quadratic and fractional programming
	Chen, Wang, & Lu (2011)	LP	(Group) Interval-valued intuitionistic fuzzy preference relations
Non-Fuzzy Uncertain Hybrid Approaches	Pitchipoo et al. (2012)	AHP, GST	Regarding chemical processing industries
	Wu (2010)	DEA	Stochastic DEA
	Saen (2008)	DEA	Assurance region-imprecise DEA
	Saen (2010)	DEA	Regarding undesirable outputs and imprecise data
	Wu & Olson (2008a)	DEA	Stochastic DEA; Stochastic uncertainty environments
	Wu & Olson (2008b)	DEA, MOP	Chance-constrained programming; Monte Carlo simulation
	Celebi & Bayraktar (2008)	DEA, NN	Regarding incomplete information of criteria
	Xu & Ding (2011)	GA, MOP, LP	Chance-constrained MOLP model
	Li & Zabinsky (2011)	SP	Chance-constrained programming model
	Zhang & Ma (2009)	NLP	General algebraic; Regarding demand uncertainty
	Li, Yamaguchi, & Nagai (2008)	RST, GST	(Group) Grey relational analysis
	Bai & Sarkis (2010)	RST, GST	(Group) Grey values; Sustainability
	Sadeghieh et al. (2012)	GA, GST, GP	(Group) A GA-based grey goal programming approach
	Lin & Yeh (2010)	GA	Stochastic logistic network
	Yang, Wee, Pai, & Tseng (2011)	GA	Stochastic model
	Dogan & Aydin (2011)	BN	Total cost of ownership

1. The mark "***" means that no particular Core DM Techniques need to be emphasized.

2. Details of the abbreviations shown in the third column can be obtained in Table 8-2.

8.5 Categorical Reviews of Decision-Making Techniques

Considering the realistic complexity of SS, current research tends to integrate multiple DM techniques into a hybrid decision approach. In the beginning of this section, we systematically summarized the 26 DM techniques that had been integrated into the decision approaches discussed in our reviewed literature. We then separately reviewed six kinds of major integrated approaches. These approaches include the integrated AHP approaches in Section 8.5.2, the integrated ANP approaches in Section 8.5.3, the integrated DEA approaches in Section 8.5.4, the integrated uncertain decision approaches in Section 8.5.5, and other integrated approaches in Section 8.5.6.

8.5.1 Overview of Independent DM Techniques

Based on our investigation, we summarize 26 DM techniques that had been used for supplier evaluation and selection. We classify these techniques into three categories, namely: Multicriteria decision making (MCDM) techniques (Section 4.1.1), Mathematical programming (MP) techniques (Section 4.1.2), and Artificial intelligence (AI) techniques (Section 4.1.3). In Table 8-2, we provide the names of the techniques and their abbreviations. We provide one representative article for each independent DM technique. Readers can find the typical usage of these techniques in the corresponding articles. In addition, Table 8-2 shows the distribution of articles based on DM technique.

Table 8-2 The summarization of the used DM techniques

The Used DM Techniques	Abbreviation	Representative Literature	Amount	Percentage (%)
Multiattribute Decision Making (MCDM) Techniques				
1. Analytic Hierarchy Process	AHP	Levary (2008)	30	24.39
2. Analytic Network Process	ANP	Lin, Lin, Yu, & Tzeng (2010)	15	12.20
3. Elimination and Choice Expressing Reality	ELECTRE	Sevkli (2010)	4	3.25
4. Preference Ranking Organization Method for Enrichment Evaluation	PROMETHEE	Chen, Wang, & Wu (2011)	2	1.63
5. Technique for Order Preference by Similarity to Ideal Solution	TOPSIS	Saen (2010)	18	14.63

6. VlseKri-terijumska Optimizacija I Kompromisno Resenje	VIKOR	Chen & Wang (2009)	3	2.44
7. Decision Making Trial and Evaluation Laboratory	DEMATEL	Chang, Chang, & Wu (2011)	3	2.44
8. Simple Multiattribute Rating Technique	SMART	Chou & Chang (2008)	1	0.81
Mathematical Programming (MP) Techniques				
1. Data Envelopment Analysis	DEA	Wu & Blackhurst (2009)	13	10.57
2. Linear Programming	LP	Lin, Chen, & Ting 2011	19	15.44
3. Nonlinear Programming	NLP	Hsu, Chiang, & Shu (2010)	6	4.88
4. Multiobjective Programming	MOP	Yu, Goh, & Lin (2012)	13	10.57
5. Goal Programming	GP	Kull & Talluri (2008)	7	5.69
6. Stochastic Programming	SP	Li & Zabinsky (2011)	2	1.63
Artificial Intelligence (AI) Techniques				
1. Genetic Algorithm	GA	Güneri, Ertay, & Yücel (2011)	8	6.50
2. Grey System Theory	GST	Tseng (2011); Wu (2009b)	6	4.88
3. Neural Networks	NN	Lee & Ouyang (2009)	5	4.07
4. Rough Set Theory	RST	Chang & Hung (2010)	4	3.25
5. Bayesian Networks	BN	Ferreira & Borenstein (2012)	2	1.63
6. Decision Tree	DT	Guo, Yuan, & Tian (2009)	2	1.63
7. Case-Based Reasoning	CBR	Faez, Ghodsypour, & O'Brien (2009)	2	1.63
8. Particle Swarm Optimization	PSO	Xu & Yan (2011)	2	1.63
9. Support Vector Machine	SVM	Guo, Yuan, & Tian (2009)	1	0.81
10. Association rule	AR	Lin, Chuang, Liou, & Wu (2009)	1	0.81
11. Ant Colony Algorithm	ACA	Tsai, Yang, & Lin (2010)	1	0.81
12. Dempster Shafer Theory of evidence	DST	Wu (2009b)	1	0.81

(A) MCDM techniques

MCDM is a methodological framework that aims to provide decision makers a knowledgeable recommendation amid a finite set of alternatives (also known as actions, objects, solutions, or candidates), while being evaluated from multiple viewpoints, called criteria (also known as attributes, features, or objectives). In the literature, the problem of structural SS is usually regarded as MCDM. Therefore, a number of classical MCDM techniques have been employed in problem-solving processes. Based on the principle behind these MCDM techniques, we can classify them into four categories: (1) multiattribute utility methods such as AHP and ANP, (2) outranking methods such as Elimination and Choice Expressing Reality (ELECTRE) and Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE); (3) compromise methods such as Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and VlseKri-terijumska Optimizacija I Kompromisno Resenje (VIKOR, means Multicriteria Optimization and Compromise Solution), and (4) other MCDM techniques such as Simple Multiattribute Rating Technique (SMART) and Decision-Making Trial and Evaluation Laboratory (DEMATEL).

1. Multiattribute utility methods: AHP and ANP

Multiattribute utility methods essentially attempt to assign a utility value to each alternative. The utility value represents the degree preference that can be the basis for ranking or choice. Both AHP and ANP are well-known multiattribute utility methodologies. AHP uses pairwise comparisons along with expert judgments to handle the measurement of qualitative or intangible attributes. As an extension of AHP, ANP is a general theory of relative intangible attribute measurement. AHP and ANP remain the most important and commonly used components that constitute up-to-date decision approaches for SS. AHP and ANP are reviewed in Sections 8.5.2 and 8.5.3, respectively.

2. Outranking methods: ELECTRE and PROMETHEE

On the premise of known decision-makers' preference and evaluation values of alternatives (e.g. suppliers), an outranking relation is a binary relation S defined on the set of potential alternatives such that aSb if sufficient justification exists to decide that alternative a is at least as good as alternative b with no essential justification to disprove such statement (Figueira, Greco, & Ehrgott, 2005). The ELECTRE methods strictly follow this definition. The well-known PROMETHEE methods are further based on the situation of a pairwise comparison of alternatives. Six articles that use these two techniques are reviewed in Section 8.5.6.

3. Compromise methods: TOPSIS and VIKOR

The foundation of compromise methods was established by Yu (1973). A compromise solution is the closest to the ideal solution, and a compromise denotes an agreement on the basis of mutual concessions. As typical compromise programming methods, both TOPSIS and VIKOR are based on an aggregating function that represents closeness to the ideal. The difference is that TOPSIS uses linear normalization to eliminate the units of criteria function, whereas VIKOR uses vector normalization (Opricovic & Tzeng, 2004). In our review, 22 articles employ these two techniques as part of their decision approaches.

4. Other MCDM methods: SMART and DEMATEL

SMART is a basic ranking technique that uses the simple additive weight method to obtain total values as the ranking index. This technique can deal with both quantitative and qualitative criteria, but cannot effectively handle uncertain decision information such as linguistic terms, interval values, and various fuzzy values. Chou and Chang (2008) developed the modified SMART approach for SS. In this work, a fuzzy integrated SMART decision model was proposed for a strategy-aligned SS.

DEMATEL is a structural model for analyzing the influential relation among complex evaluation criteria. Three articles use this model as part of the whole decision approach. Buyukozkan and Cifci (2012) used DEMATEL as well as the strength of the interdependence to generate the mutual relationships of interdependencies among criteria. Dalalah, Hayajneh, and Batieha (2011) modified DEMATEL to deal with fuzzy rating and evaluations by converting the relationship between the causes and effects of the criteria into an intelligible structural model. Finally, Chang, Chang, and Wu (2011) designed a fuzzy DEMATEL questionnaire for determining the direct and indirect influence among criteria.

(B) MP techniques

MP is a general term in DM research. For selections applications, we specify the following six MP techniques for detailed reviews.

1. Data Envelopment Analysis (DEA)

DEA is a nonparametric MP technique for evaluating the relative efficiency of comparable entities in terms of decision-making units (DMUs). A basic DEA model is a performance measurement that can be used to evaluate the relative efficiency of DMUs according to multiple inputs and outputs (Adler, Friedman, & Sinuany-Stern, 2002). Given its effectiveness, DEA can be a valuable complement to various SS decision models. A total of 13 articles refer to this technique in our reviewed literature. Section 8.5.4 provides the detailed reviews.

2. Linear programming (LP)

LP is a mathematical optimization method for determining a way to achieve the best outcome in a given mathematical model under a number of requirements represented as linear relationships. In our review, the use of LP for SS can be classified into four categories: (1) The simple LP employment (Ng, 2008; Guneri, Yucel, & Ayyildiz, 2009; Chen, 2011; Lin, Chen, & Ting, 2011; Chen, Wang, & Lu, 2011), (2) The fuzzy LP (Sevkli, 2008; Yucel & Guneri, 2011; Amin, Razmi, & Zhang, 2011; Lin, 2012), (3) The multiobjective LP (MOLP) (Xu & Ding, 2011; Ozkok & Tiryaki, 2011; Yucel & Guneri, 2011), and (4) The mixed integer LP (Ustun & Demirtas, 2008; Demirtas & Ustun, 2008; Demirtas & Ustun, 2009; Razmi, Songhori, & Khakbaz, 2009; Wang & Yang, 2009; Toloo & Nalchigar, 2011; Amin & Zhang, 2012).

3. Nonlinear programming (NLP)

A number of studies modeled realistic SS processes into NLP problems and then designed various objective functions and constraints for resolution. In contrast to LP, NLP allows for some of the constraints or objective functions to be nonlinear. Based on our survey, two directions can be identified. First is the simple utilization of NLP as a decision tool. Related literature includes Hsu, Chiang, and Shu (2010) and Razmi, Rafiei, and Hashemi (2009). The second direction is to model problems by using the mixed integer NLP formulations. Related literature includes Zhang and Ma (2009), Razmi and Rafiei (2010), Yeh and Chuang (2011), as well as Rezaei and Davoodi (2012).

4. Multiobjective programming (MOP)

MOP is a kind of MP for decision problems characterized by multiple and conflicting objective functions that can be optimized over a set of feasible solutions. From 2008 to 2012, research on fuzzy MOLP for SS was the mainstream direction. Related literature includes Haleh and Hamidi (2011), Ozkok and Tiryaki (2011), Lin (2012), Yu, Goh, and Lin (2012), Shaw, Shankar, Yadav, and Thakur (2012), as well as Amin and Zhang (2012). Nevertheless, Yeh and Chuang (2011) formulated a mixed integer multiobjective NLP (MONLP) model for partner selection. In addition, Wu, Zhang, Wu, and Olson (2010) studied other possible MOP models. Feng, Fan, and Li (2011) introduced a multiobjective 0–1 programming model. Xu and Ding (2011) developed a chance constrained MOLP model with birandom coefficients.

5. Goal programming (GP)

GP is a branch of optimization method. This technique can be regarded as an extension or generalization of MOLP that can be used to deal with multiple and conflicting objective measures. Each of these measures is given a goal value to be achieved. In our literature review, seven constructed GP models for selections were found. The most direct employment of GP as a decision tool is in Kull and Talluri (2008). Tsai and Hung (2009) provided a fuzzy GP approach. Demirtas and Ustun (2009) provided a GP and ANP hybrid decision model considering a multi-period planning horizon. Chen (2011a) integrated multiple MP techniques, among which GP is an important component. Sadeghieh, Dehghanbaghi, Dabbaghi, and Barak (2012) developed a genetic algorithm (GA)-based grey GP approach. Finally, Lee, Kang, and Chang (2009) and Liao and Kao (2011) reduced real-world SS problems to a formulation of multi-choice GP.

6. Stochastic programming (SP)

SP is a framework for modeling uncertainty optimization problems in which probability distributions governing the data are known or can be estimated despite the involvement of a number of unknown parameters. This technique is a suitable mathematical tool for dealing with several real-world SS problems. Based on our survey, two articles refer to SP, including Kara (2011) as well as Li and Zabinsky (2011). Both of these articles developed two-stage SP decision models. The former consolidated SP and fuzzy TOPSIS methods, whereas the latter integrated SP with chance-constrained LP.

(C) AI techniques

In this review, 12 techniques can be regarded as AI techniques. Four of these techniques are major ones: genetic algorithm (GA), neural network (NN), rough set theory (RST), and grey system theory (GST). This section also summarizes the eight other AI techniques, including case-based reasoning (CBR), Bayesian networks (BN), particle swarm optimization (PSO), ant colony algorithm (ACA), Dempster–Shafer theory (DST), association rule (AR), support vector machine (SVM), and decision tree (DT).

1. Major AI techniques: GA, NN, GST, and RST

GA is a kind of global search technique used to identify approximate solutions for complex optimization problems. Conceptually following the steps of the biological process of evolution, GA is considered a heuristic method considering that it cannot guarantee a truly optimal solution. Eight articles refer to this technique. The literature that considered typical GA for SS includes Yang, Wee, Pai, and Tseng

(2011) as well as Yeh and Chuang (2011). Moreover, Xu and Ding (2011) designed a bi-random simulation-based GA. Che (2010a) provided a heuristic algorithm combining guided GA and Pareto GA. Rezaei and Davoodi (2012) formulated an MONLP, applying a non-dominated sorting GA. Three articles, including Wang (2008), Lin and Yeh (2010), as well as Sadeghieh, Dehghanbaghi, Dabbaghi, and Barak (2012), utilized GA as an element to construct their decision model.

An NN is generally a set of connected input or output units, in which each connection has an associated weight. The weights are adjusted during the learning phase to help the network predict the correct class label for the input objects (Han, Kamber, & Pei, 2012). We found five articles that refer to this technique. The typical NN use was found in the studies of Celebi and Bayraktar (2008) and Wu (2009). The former study employed NN to refine the general evaluation criteria set into a set of common performance measures, whereas the latter adopted the backpropagation NN for feature extraction and classification. Güneri, Ertay, and Yücel (2011) improved the performance of the adaptive neuro-fuzzy inference system for SS. Keskin, llhan, and Ozkan (2010) developed a decision approach by using adaptive resonance theory NNs. Both works employed the basic fuzzy logic for hybridization. Lee and Ouyang (2009) provided an NN-based predictive model to forecast the supplier's bid prices.

GST is a mathematical method that is applied to imprecise information in the form of interval values (Deng, 1989). The reviewed literature introduced GST for SS from two perspectives: (1) decision information in the form of grey values (Bai & Sarkis, 2010; Tseng, 2011; Sadeghieh, Dehghanbaghi, Dabbaghi, & Barak, 2012) and (2) grey relational analysis (GRA) (Li, Yamaguchi, & Nagai, 2008; Wu, 2009b; Golmohammadi & Mellat–Parast, 2012; Pitchipoo, Venkumar, & Rajakarunakaran,, 2012).

RST can be used to identify structural relationships within imprecise or noisy data. Classical RST is based on binary indiscernibility relations, which result in the establishment of equivalence classes. In our review, three articles refer to the classical RST, including Bai and Sarkis (2010), Li, Yamaguchi, and Nagai (2008), and Chang and Hung (2010).

2. Minor AI techniques: CBR, BN, PSO, ACA, DST, AR, SVM, and DT

CBR is also referred to as instance-based learners. This approach uses a collection of solutions to solve new problems. The premise is that new problems are often similar to those that were previously encountered. Thus, past successful solutions may be useful in the new situation. Two articles typically utilized this technique for SS, namely, Zhao and Yu (2011) and Faez, Ghodsypour, and O'Brien (2009).

BNs, also known as belief networks and probabilistic networks, are probabilistic graphical models. The premise is that future states of nature can be characterized probabilistically. This technique is effective for dealing with SS problems under uncertainty via probability distributions. Dogan and Aydin (2011) as well as Ferreira and Borenstein (2012) introduced BN for handling existing uncertainty. The latter study was combined with fuzzy logic, which selects suppliers under triangular fuzzy information.

PSO is an evolutionary algorithm (Kennedy & Eberhart, 1995; Kennedy, Eberhart, & Shi, 2001) that simulates the animal social behavior of birds that flock to a desired location in a multi-dimensional space for certain objectives. Che (2010b) and Xu and Yan (2011) integrated PSO as an element into their decision models for solving balanced and defective supply chain problems as well as material supply problems in large-scale water conservancy and hydropower construction projects, respectively.

ACA is a typical AI optimization method (Dorigo, Maniezzo, & Colorni, 1996; Dorigo & Gambardella, 1997) that simulates a colony of artificial ants that aid one another to obtain effective solutions in complex optimization problems. Tsai, Yang, and Lin (2010) aimed to utilize an attribute-based ant colony system for supplier evaluation.

DST is an uncertainty reasoning tool (Thierry, 1997)that can be used to combine unexpected empirical evidence regarding an individual’s opinion and consequently organize a coherent picture of reality. For SS, Wu (2009b) extended the DST to aggregate individual preferences into a collective preference.

AR is a frequent pattern mining technique that utilizes rules in the form of implications to discover the associations among data entities. The study of Lin, Chuang, Liou, and Wu (2009) presents a typical application of AR for SS.

SVM is a classification and prediction tool for both linear and nonlinear data. DT is a widely used technique for classification and prediction. Guo, Yuan, and Tian (2009) developed a potential SVM technique combined with DT for SS. Wu (2009a) attempted to integrate DT with other two techniques, such as NN and DEA, for assessing supplier performance.

8.5.2 Integrated AHP Approaches

A total of 30 articles (24.39%) that refer to the AHP technique for SS are listed in Table 8-1. Four articles independently utilize AHP for decision making. For example, Mafakheri, Breton, and Ghoniem (2011) provided an AHP-based two-stage dynamic programming approach. Towards different application fields, Levary (2008) introduced AHP to rank potential suppliers in manufacturing industries; Chan and Chan (2010) applied AHP in the fast-changing apparel industry; Ishizaka, Pearman, and Nemery (2012) developed a new variant of AHP for the sorting of suppliers into predefined ordered categories.

Three articles built the integrated AHP approaches under deterministic conditions. Kull and Talluri (2008) provided an evaluation model that used AHP for calculating a risk index based on each alternative supplier. Such indexes were then incorporated into a GP model for selecting suppliers. This model was applied to a case on product life cycle. Ordoobadi (2010) provided an integrated decision model using AHP and the Taguchi loss function. In this model, AHP was used to determine the weights that represent the importance of tangible and intangible decision factors. The weighted Taguchi loss scores were calculated for ranking suppliers. Bhattacharya, Geraghty, and Young (2010) provided an integrated AHP and quality function deployment (QFD) for ranking alternative suppliers.

A total of 23 articles refer to the AHP techniques when considering uncertain decision environments. These techniques can be divided into three categories: (1) AHP-based fuzzy logic hybridization approaches, (2) AHP and triangular fuzzy set integrated approaches, and (3) AHP-based non-fuzzy hybridization approaches. In the first category, Labib (2011) introduced a simple decision model that integrated AHP with the basic fuzzy logic. Sevkli, et al. (2008) provided a hybrid decision model that used AHP to determine the weights of criteria and weighted fuzzy LP to rank suppliers. Amid, Ghodsypour, and O'Brien (2011) provided a weighted max–min fuzzy model that used an AHP technique to determine the weights of criteria. Tsai and Hung (2009) provided a fuzzy goal programming approach that used AHP to determine the objective structure. Chamodrakas, Batis, and Martakos (2010) as well as Chen and Chao (2012) integrated AHP with fuzzy preference relations to construct decision matrices. Finally, Wang and Yang (2009) integrated AHP with fuzzy compromise programming. The proposed decision model is alleged to outperform the conventional mixed integer programming by further considering scaling and subjective weighting issues.

In the second category, we found 15 articles that referred to AHP and constructed the decision approaches under triangular fuzzy environments. Two articles (Chan, et al., 2008; Bottani & Rizzi, 2009) utilized the AHP technique and linguistic pairwise comparisons for ranking suppliers under triangular fuzzy environments. Kilincci and Onal (2011) introduced a simple AHP decision model converting the linguistic variables into decision information by using triangular fuzzy values. Yucenur Vayvay, and Demirel (2011) integrated AHP with ANP, wherein triangular fuzzy values are employed to form pairwise comparison matrices. Zeydan, Colpan, and Cobanoglu (2011) integrated multiple techniques, including AHP, TOPSIS, and DEA. In this previous work, the authors considered multiple persons as a decision group. Two similar articles by Lee (2009a, 2009b) constructed an uncertain AHP decision model that utilizes the concept of benefits, opportunities, costs, and risks (BOCR). Apart from AHP, Wang, Cheng, and Huang (2009) employed hierarchical TOPSIS; Lee, Kang, and Chang (2009) employed multiple goal programming; and Che (2010b) employed the PSO for green SS. Sen, Sen, and Baslgil (2010) integrated AHP with the max-min method. Punniyamoorthy, Mathiyalagan, and Parthiban (2011) integrated AHP with the structural equation modeling model. Yang, Chiu, Tzeng, and Yeh (2008) provided a multi-stage hybrid approach that involves preference expression, interpretive structural modeling, AHP, and non-additive fuzzy integral for the selection of the best supplier. Yu, Goh, and Lin (2012) and Shaw, Shankar, Yadav, and Thakur (2012) attempted to integrate AHP with MOLP. The former study considered the time factor via a soft time-window mechanism, whereas the latter focused on the issue of carbon emission during supplier evaluation.

In the third category, Pitchipoo, Venkumar, and Rajakarunakaran (2012) developed a hybrid decision model via the integration of the AHP technique with GRA. In this model, AHP was used to determine the weights of the evaluation criteria, whereas GRA was used to identify the best supplier.

8.5.3 Integrated ANP Approaches

A total of 15 articles (12.20%) that refer to the ANP technique for SS are shown in Table 8-1. Eight articles utilized ANP under a certain decision environment. Lin, Lin, Yu, and Tzeng (2010) introduced a simple hybrid approach, in which ANP is used to determine the weights of criteria. Razmi and Rafiei (2010) provided an ANP-integrated mixed-integer non-linear decision model. Ho, Dey, and Lockstrom (2011) integrated ANP and QFD. In the integrated model of Tseng, Chiang, and Lan (2009), ANP is used for criteria analysis, whereas the Choquet integral is used for the optimization of decision makers' subjective judgments. Lin, Chen, and Ting (2011) integrated ANP, TOPSIS, and LP for the enterprise resource planning system used for the applications of a manufacturing enterprise. Three articles from the same authors (Demirtas & Üstün, 2008; 2009; Üstün & Demirtas, 2008) constructed integrated decision models that involve the ANP technique, the mixed integer LP, and the concept of BOCR.

Considering ANP and fuzzy logic hybrid approaches, Lin (2012) integrated the ANP techniques with fuzzy LP for selecting the best suppliers and handling the inherent uncertainty. Six articles constructed ANP-related models under triangular fuzzy environments. Onut, Kara, and Isik (2009) provided a case study regarding telecommunication companies, in which ANP and TOPSIS were utilized. Razmi Rafiei, and Hashemi (2009) independently utilized ANP and NLP for SS. Amin and Razmi (2009) investigated a specific case of Internet service provider selection and evaluation, in which ANP and QFD were employed independently. Vinodh, Ramiya, and Gautham (2011) introduced a simple fuzzy ANP approach regarding manufacturing companies. Yucenur, Vayvay, and Demirel (2011) integrated AHP with ANP. Büyüközkan and Cifci (2012) integrated three techniques: DEMATEL, TOPSIS, and ANP, for green SS.

8.5.4 Integrated DEA Approaches

DEA is among the most used techniques for SS. Of our reviewed articles, 14 applied DEA as an element for the construction of decision approaches. According to different decision environments, these works can be classified into three categories. In the first category, four articles employed DEA with certain decision information. Wu and Blackhurst (2009) provided an augmented DEA approach for supplier ranking. Wu (2009a) introduced a hybrid model using DEA, DT, and NN for supplier classification and prediction. Toloo and Nalchigar (2011) considered decision environments involving both cardinal and ordinal data. Falagario, Sciancalepore, Costantino, and Pietroforte (2012) integrated DEA with the cross efficiency evaluation for a specific application of public procurement tenders.

In the second category, three articles (Azadeh & Alem, 2010; Chen, 2011b; Zeydan, Colpan, & Cobanoglu, 2011) constructed DEA-related models considering a triangular fuzzy environment. These works simultaneously employed DEA and TOPSIS in the SS process. Apart from fuzzy decision environments, six articles under the third category discussed the utilization of DEA in handling other types of uncertainty. Wu and Olson (2008a; 2008b) discussed the stochastic DEA simulation models for conflicting criteria analysis, risk evaluation, and SS. Wu (2010) provided a DEA-based stochastic analysis model for dealing with imbedded uncertainty. Saen (2008; 2010) proposed effective DEA-based decision models to handle imprecise data in the SS process. Saen (2010) further considered such undesirable outputs as the uncertainty factor. Celebi and Bayraktar (2008) proposed a novel integration of DEA with NN under the condition of incomplete information within evaluation criteria.

8.5.5 Integrated Uncertain Decision Approaches

Apart from the formulated SS problem under deterministic conditions, current studies address realistic problems under different types of uncertainties. According to our reviews, fuzzy formulations dominated other types of uncertainties in recent studies. Table 8-3 presents the summary of the uncertain decision approaches based on fuzzy formulations and highlights a corresponding representative article.

Table 8-3 Representatives of the integrated fuzzy decision approaches

The Integrated Fuzzy Decision Approaches		**Representative Articles**
Integrated Fuzzy MCDM Approaches	Fuzzy AHP	Amid, Ghodsypour, & O'Brien (2011)
	Fuzzy ANP	Vinodh, Ramiya, & Gautham (2011)
	Fuzzy ELECTRE	Montazer, Saremi, & Ramezani (2009)
	Fuzzy PROMETHEE	Chen, Wang, & Wu (2011)
	Fuzzy TOPSIS	Wang, Cheng, & Huang (2009)
	Fuzzy VIKOR	Chen & Wang (2009)
	Fuzzy DEMATEL	Chang, Chang, & Wu (2011)
	Fuzzy SMART	Chou & Chang (2008)
Integrated Fuzzy MP Approaches	Fuzzy DEA	Azadeh & Alem (2010)
	Fuzzy LP	Lin (2012)
	Fuzzy NLP	Razmi, Rafiei, & Hashemi (2009)
	Fuzzy GP	Tsai & Hung (2009)
	Fuzzy MOP	Amid, Ghodsypour, & O'Brien (2009)
	Fuzzy MOLP	Yu, Goh, & Lin (2012)
	Fuzzy MONLP	Yeh & Chuang (2011)
Integrated Fuzzy AI Approaches	Fuzzy GA	Wang (2008)
	Fuzzy NN	Güneri, Ertay, & Yücel (2011)
	Fuzzy GST	Tseng (2011), Wu (2009b)
	Fuzzy BN	Ferreira & Borenstein (2012)
	Fuzzy CBR	Faez, Ghodsypour, & O'Brien (2009)
	Fuzzy PSO	Xu & Yan (2011)
	Fuzzy DST	Deng & Chan (2011)
Other Fuzzy Hybridization Approaches	Fuzzy preference relations	Chen & Chao (2012)
	Fuzzy inference system	Amindoust et al. (2012)
	Fuzzy adaptive resonance theory	Keskin, llhan, & Ozkan 2010)
	Fuzzy integral	Yang, Chiu, Tzeng, & Yeh (2008)
	Fuzzy SWOT	Amin, Razmi, & Zhang (2011)
	Fuzzy BOCR	Lee (2009a; 2009b)

Apart from multitudinous fuzzy formulations, we can roughly group the emerging types of other uncertainties into five categories. We summarize the categories based on the relatedness of the study as follows: (1) Stochastic formulations: Wu and Olson (2008a; 2008b), Wu (2010), Lin and Yeh (2010), Li and Zabinsky (2011), and Yang, Wee, Pai, and Tseng (2011); (2) Probabilistic formulations: Zhang and Ma (2009), Dogan and Aydin (2011), and Xu and Ding (2011); (3) Formulations with incomplete data: Celebi and Bayraktar (2008); (4) Formulations with imprecise data: Saen (2008; 2010); and (5) Formulations with grey values: Li, Yamaguchi, and Nagai (2008), Bai and Sarkis (2010), Pitchipoo Venkumar, and Rajakarunakaran (2012), and Sadeghieh, Dehghanbaghi, Dabbaghi, and Barak (2012).

8.5.6 Other Integrated Decision Approaches

Apart from the widely used integrated approaches regarding AHP, ANP, and DEA, we provide other integrated approaches regarding (1) MCDM techniques: ELECTRE, PROMETHEE, and VIKOR as well as (2) AI techniques: RST and GST.

1. Integrated approaches regarding ELECTRE, PROMETHEE, and VIKOR

Four articles provided the ELECTRE integrated decision models. Liu and Zhang (2011) integrated the extension of ELECTRE, called ELECTRE-III, with entropy weights. Vahdani, Jabbari, Roshanaei, and Zandieh (2010) considered interval values as decision information in the application of ELECTRE. Montazer, Saremi, and Ramezani (2009) and Sevkli (2010) extended ELECTRE for SS when triangular fuzzy values provided the decision information. The former study employed ELECTRE III, whereas the latter integrated the conventional ELECTRE with fuzzy concepts.

Considering other outranking methods, Chen, Wang, and Wu (2011) integrated PROMETHEE with the extended fuzzy concept and studied a case of information system (IS) outsourcing under triangular fuzzy environments. Chai, Liu, and Xu (2012) developed a novel superiority and inferiority ranking (SIR) group decision approach for SS under intuitionistic fuzzy environments, in which the SIR method can be regarded as an extension of the conventional PROMETHEE method.

We found three articles referring to the VIKOR method. Chen and Wang (2009) provided a fuzzy VIKOR for the application of IS/information technology (IT) outsourcing projects. This study simulated linguistic variables as decision information and then converted such variables into triangular fuzzy values. Sanayei, Mousavi, and Yazdankhah (2010) and Shemshadi, Shirazi, Toreihi, and Tarokh (2011) also integrated the VIKOR method with fuzzy concepts. The former study converted linguistic variables by using the tools of triangular and trapezoidal fuzzy values, whereas the latter considered the possible application of the Shannon entropy concept in the proposed fuzzy VIKOR decision model.

2. Integrated approaches regarding RST and GST

GST and RST are recently introduced techniques for SS. In GST, Wu (2009b) and Golmohannadi, Mellat–Parast (2012) employed GRA to deal with interval-valued decision information. Tseng (2011) employed triangular fuzzy numbers to express linguistic preferences while utilizing a grey degree to calculate the incomplete information in the green SS process. Bai and Sarkis (2010) and Li, Yamaguchi, and Nagai (2008) used both GRT and RST methods for SS. For the typical application of RST, Chang and Hung (2010) provided a rule-based decision model.

8.6. Some Observations Remarks

8.6.1 Distribution of Articles by Journal

Table 8-4 provides the distribution of the articles based on the journal in which they appeared. The articles related to the application of DM techniques for SS are distributed across 15 journals that cover a wide array of disciplines, including IS, operation research, soft computing, and production management. The journal *Expert Systems with Applications* contains the most relevant articles, comprising 55 out of the 123 articles reviewed (44.7%). Two journals with similar scopes, *International Journal of Production Research* and *International Journal of Production Economics*, contributed a combined 33 articles (26.8 %) to this research field. Since the current research tends towards uncertain SS, seven articles (5.7%) have been reported in the related journals, including *Applied Soft Computing Journal*, *Soft Computing*, and *International Journal of Uncertainty, Fuzziness and Knowledge-based Systems*.

Table 8-4 Distribution of the selected articles by journal

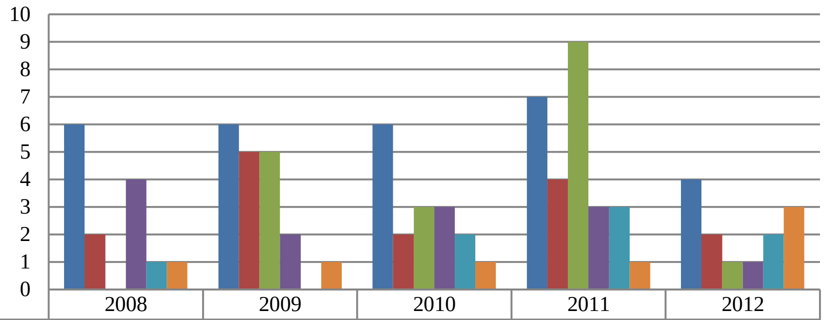
Journal title	Amount	Percentage (%)
1. Expert systems with applications	56	45.5
2. International Journal of Production Research	17	13.8
3. International Journal of Production Economics	15	12.2
4. International Journal of Advanced Manufacturing Technology	8	6.6
5. Applied Soft Computing Journal	5	4.1
6. Computers and Industrial Engineering	5	4.1
7. European Journal of Operational Research	4	3.3
8. Information Sciences	3	2.4
9. Industrial Management and Data Systems	2	1.6
10. Supply Chain Management	2	1.6
11. Omega	2	1.6
12. International Journal of Logistics Systems and Management	1	0.8
13. IEEE Transactions on Engineering Management	1	0.8
14. International Journal of Uncertainty, Fuzziness and Knowledge-based Systems	1	0.8
15. Soft Computing	1	0.8
Total	123	100

8.6.2 Statistical Analysis on Popular DM Techniques

A generally accepted methodological framework for operating an effective SS is not been to be determined because of the complexity and diversity of the real world. According to our survey, the overwhelming majority of reviewed articles attempted to integrate multiple techniques into an effective decision model for dealing with different SS issues such as group aggregation, uncertain information fusion, classification, prediction, and clustering. Therefore, we highlight the first current study trend: *Multiple techniques integrated decision approaches*.

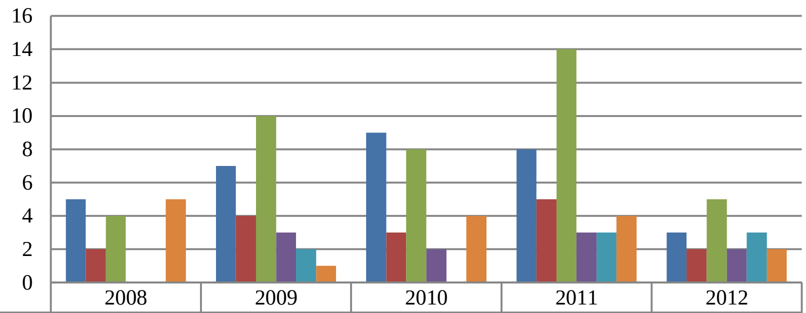
Table 8-2 indicates that the most frequently used technique is AHP (24.39%), followed by LP (15.44%), TOPSIS (14.63%), ANP (12.20%), DEA (10.57%), and multiobjective optimization (10.57%). Figure 8-2 provides the distribution of a number of major techniques that appeared during the period. The multiattribute utility methods, including AHP and ANP, dominate other techniques because of their effectiveness in ranking and task choices. TOPSIS and DEA remain significant in the construction of decision models. AI techniques, including GA and GST, are receiving considerable research attention. Several emerging AI techniques, including SVM, AR, ACA, and DST, necessitate more transfer learning on future works.

Table 8-1 presents the 75 articles (60.98%) that provide multitudinous fuzzy hybrid approaches, followed by certain decision approaches (26.01%) and non-fuzzy uncertain decision approaches (13.01%). Therefore, we highlight the second current study trend: *Multitudinous uncertain decision approaches.* Figure 8-3 provides the distribution of each uncertainty category based on the year in which they appeared. Based on this analysis, SS under triangular fuzzy environments is the main stream between 2008 and 2012. Certain decision approaches and basic fuzzy hybrid approaches were regularly reported. Two branches of basic fuzzy theory, such as trapezoidal fuzzy sets and (interval-valued) intuitionistic fuzzy sets, became the new directions toward flexible and realistic SS processes.



	2008	2009	2010	2011	2012
AHP	6	6	6	7	4
ANP	2	5	2	4	2
TOPSIS	0	5	3	9	1
DEA	4	2	3	3	1
GA	1	0	2	3	2
GST	1	1	1	1	3

Fig 8-2 Chronological distribution of some major DM techniques



	2008	2009	2010	2011	2012
Certain DM	5	7	9	8	3
Basic FS	2	4	3	5	2
Tri-FS	4	10	8	14	5
Tra-FS	0	3	2	3	2
(IV)IFS	0	2	0	3	3
Non-Fuzzy	5	1	4	4	2

Fig 8-3 Chronological distribution of the uncertainty categories

8.6.3 Other Remarks

1. Preference relations are the tools used by the decision makers to provide their preference information in the decision-making processes. Based on our reviewed articles, *the first important trend* is that from the certain preference relation to the uncertain preference relation. In this case, we summarize four kinds of preference relations that had been used by previous studies as follows: (1) multiplicative preference relation (also known as pairwise comparison relation): Levary (2008); (2) linguistic reference relation: Tan, Wu, and Ma (2011); (3) fuzzy preference relation: Chen and Chao (2012); and (4) incomplete preference relation: Tseng (2011).

   The second important trend is that from the singleton preference relation to the hybrid preference relations, which is summarized as follows: (1) interval valued multiplicative preference relation: Chamodrakas, Batis, and Martakos (2010); (2) triangular fuzzy multiplicative preference relation: Golmohammadi and Mellat–Parast (2012); (3) fuzzy linguistic preference relation: Bottani and Rizzi (2008); (4) triangular fuzzy preference relation: Chan, et al. (2008); (5) intuitionistic fuzzy preference relation: Chai, Liu, and Xu (2012); and (6) interval valued intuitionistic fuzzy preference relation: Chen, Wang, and Lu (2011). Considering that current studies mainly consider static preference relations, future works should focus on SS with dynamic preference relations when considering the time factor.

2. The methodologies of the hybridization of fuzzy sets and rough sets have been comprehensively studied in the data mining context. Several mature methodologies include rough fuzzy sets (Dubois & Prade, 1990), fuzzy rough sets (Yeung, et al. 2005), and intuitionistic fuzzy rough sets (Chai, Liu, & Li, 2012). Such methods had been successfully applied for attribute reduction (Tsang, et al. 2008), feature selection (Jensen & Shen, 2009), classification and prediction (Chai & Liu, 2012). The application of fuzzy-rough hybrid methodologies for solving real-world SS problems must be investigated.

3. We have witnessed the rapid development of the generalization of Zadeh's fuzzy set over the past several decades. Several developments including interval-valued, triangular, trapezoidal, intuitionistic, and interval-valued intuitionistic fuzzy sets were employed in the current study for SS. A new generalization of fuzzy sets called hesitant fuzzy sets has recently been proposed by Torra (2010), it's the theoretical basis of which has received considerable attention (e.g. Xu & Xia, 2011, 2012; Rodriguez, Martinez, & Herrera, 2012). Therefore, evaluating suppliers under hesitation fuzzy environments along with the trend of uncertain SS can be a very promising direction in the future studies.

8.7 Summary

This chapter provides a systematic literature review on articles published from 2008 to 2012 on the application of DM techniques for SS. We aim to analyze the collected articles from four analytical aspects: decision problems, decision makers, decision environments, and decision approaches. A total of 123 journal articles were carefully selected and reviewed in detail. We systematically summarized 26 applied DM techniques from three perspectives: MDCM, MP, and AI. The techniques that integrated decision models in the literature were particularly reviewed in terms of AHP, ANP, DEA, and so on. This chapter provides valuable knowledge accumulation on current studies and recommendations for future works.

This study has two major limitations. First, our review focuses on the application of DM techniques for SS. Other important aspects such as criteria analysis and evaluation in SS processes, were not involved in this survey because of our limited research scope. Second, the reviewed articles were published from 2008 to 2012 and searched based on the keywords “supplier selection,” “vendor selection,” and “decision making.” A number of articles published in late 2012, if any, may not be included in this survey because of the limitation of reporting time. A future review could be expanded in scope.

CHAPTER NINE

Decision Support Systems

9.1 Overview

This chapter addresses MCDM from the perspective of Information Systems. Decision Support Systems (DSS) have been proposed since the late 1960s in order to help DMs improve the efficiency and correctness in decision making. Nowadays, companies are usually working in an uncertain and rapidly changing business environment. Therefore, more timely and accurate information are required for decision-making, in order to improve customer satisfaction, support profitable business analysis, and increase their competitive advantages. In addition to the use of data and mathematical models, some managerial decisions are qualitative in nature and need judgmental knowledge that resides in human experts. Thus, it is necessary to incorporate such knowledge in developing Decision Support System (DSS). A system that integrates knowledge from experts is called a Knowledge-based Decision Support System (KBDSS) or an Intelligent Decision Support System (IDSS) (Turban, Aronson, & Liang, 2005). Moreover, two kinds of situations significantly increase the complexity of decision problem: (1) multiple participants involved in decision process; (2) decision-making under uncertainty environment.

In this chapter, we provide two designs of decision support system towards group decision-making and uncertain decision-making. We firstly proposed a framework of ONTOlogy-based Group Decision Support System (ONTOGDSS) for decision process which exhibits the complex structure of decision group. It is capable of reducing the complexity of problem structure and group relations. The system allows decision makers to participate in group decision-making through the web environment, via the ontology relation. It facilitates the management of decision process as a whole, from criteria generation, alternative evaluation, and opinion interaction to decision aggregation. The embedded ontology structure in ONTOGDSS provides the important formal description features to facilitate decision analysis and verification. It examines the software architecture, the selection methods, the decision path, etc. Finally, the ontology application of this system is illustrated with specific real case to demonstrate its potentials towards decision-making development.

Secondly, we developed a framework of Uncertainty-based Group Decision Support System (UGDSS). It provides a platform for multiple processes of decision analysis in six aspects including decision environment, decision problem, decision group, decision conflict, decision schemes and group negotiation. Based on knowledge engineering and multiple artificial intelligent technologies, this framework provides reliable support for the comprehensive manipulation of real applications and advanced uncertainty decision approaches through the design of an integrated multi-agents architecture.

9.2 Complex Large Group Decision Support System

9.2.1 Motivation

Along the development of DSS, researchers notice that the decision making in reality is not just individual decision but often involving multiple peoples. As a matter of fact, many decision problems (such as great strategic decision of government or industry, the managing decision of large company), have the complex internal structure and in need of making decision by a large decision group with complex relationship among people. To address these problems, we provide a group decision process structure and system framework coupled with relatively complex decision groups and tasks.

9.2.2 Group Argumentation Model

(A) Task Decomposition

Ontology is defined as “a set of knowledge terms, including the vocabulary, the semantic interconnections and some simple rules of inference and logic, for some particular topic” (Hendler, 2001). That is to say, ontology captures the model of knowledge for a particular domain. They allow us to describe resources on the web and the relationships between those resources. Accordingly, ontology can be regarded as metadata which play an important role in decision process. System provides the methods for generating a series of alternatives for comparison and evaluation of different decision-makers. Thus, ONTOGDSS relies on metadata to describe the attributes, objectives, context, constraints, types, criteria of the complex decision problem in real world, and therefore will be ontology-driven. So, it is necessary to develop ontologies which can encode the semantic representation of the structural complex decision problem, in order to form a specific, clear decision path.

Based on Herhert A Simon’s (Herbert, 1962) dichotomy of decision problem, we develop the idea of dividing decision problems into three categories: structured problems, semi-structured problems and unstructured problems. For structured problems, we can load decision models, methods, data and other information as reference. For other two problems, since semi-structured and unstructured problems mean that they have never been shown up before and usually presented as qualitative textual form/document with complex semantic structure, therefore, besides loading necessary data in database, it is important to make the reference via ontology-approached knowledge management system in various decision domains.

The ONTOGDSS is designed as an ontology-based intelligent information system platform. It highlights the needs for considering contextual aspects in system perspective. Besides, ontology in specific decision-problem domains would include basic concepts such as decision targets, principles, limitations, and additional concepts of problem style, characteristics, evaluation criteria and etc. Therefore, problem representative and description in ontology approach are not only important to those structure-problems for better searching and matching in previous models or methods, but also used especially for those semi-structured / unstructured problems for group decision process.

(B) Group Selection

DSS ontology can be defined as formal descriptions of decision concepts by basic terms and relationships as well as the rules for combining these terms in a certain problem domain. While abstraction of an ontology development is similar to definition of a conceptual model, the focus is on extended definitions of relationships and concepts, and having the explicit goal of reuse and sharing knowledge by using a common framework. In GDSSs, the concept of decision-group usually is presented in contextual form with complicated relationship and structure. However, the concept of group is usually defined in literature as a kind of individual-aggregated entity which does not depend on individual properties with conceptualization. This section analyzes and establishes the decision-group through ontology-based conceptual extraction in contextual decision-group domain. This approach can eliminate the confusions associated with the term “Group”. Once various structures are established, the unique characteristics of each would be emerged. Thus, researches can be focused on the various interactions among participants as well.

Based on literature review of “Group” concept, and previous group selection methods (Malakooti & Yang, 2004), we provide a Double Selection Model to process group selection. It requires decision group to be selected in two aspects at least. For example, we need to evaluate the work performance of four peoples (alternatives) $Y_i = \{Y_1, Y_2, Y_3, Y_4\}$ ($i = 1,2,3,4$) by five suitable evaluators (decision maker) d_{ij}^*, who are respectively from five different parts: higher authorities G_1; peer authorities G_2; lower authorities G_3; independent people outside of the company G_4; alternatives themselves G_5; where $G_j = \{G_1, G_2, G_3, G_4, G_5\}$ ($j = 1,2,3,4,5$). For alternative Y_i, the suitable evaluators d_{ij}^* are respectively selected from five different parts G_j through the Double Selection Model. First selection can base on the decision task types, and second selection can base on decision maker’s characters. After these processes, evaluator candidates d_{ij} ($d_{ij} \in D_j$) of alternative Y_i can be selected to be the suitable evaluators d_{ij}^*. The key issue of this approach is to establish a proper assessment criteria system. Once it is established, many classic multi-criteria decision-making approaches can be adopted to solve this problem, such as outranking relations approaches including ELECTRE (Roy, 1991) and PROMETHEE (Brans, Mareschal, & Vincke, 1985), or preference disaggregation approaches including UTA (Jacquet-Lagreze & Siskos, 1982). In this example, the criteria of first selection can be set as the different professional fields: computing, economic, management. And the other one can be set on individual characteristics of decision makers: age, sex, nationality, education background, etc.

(C) Argumentation based on Ontology Approach and Metasynthesis Methodology

Argumentation has become a keyword of Artificial Intelligence, especially in sub-fields such as multiple-source information system with natural language processing. One of the abstract frameworks of Argumentation system is Dung's one (Dung, 1995) which shows that several formalisms for non-monotonic reasoning can be expressed in terms of this argumentation system. Ontology technique can be used to model natural language for data integration, data interoperability and data visualization. By using this, humans and computers (software agents) can have a consensus on the resource structure (Wang, Liu, & Wang, 2009). In the past, ontology approaches have been a universal technique to build explicit understanding of the structure of complex problem such as those in World Wide Web design, medical informatics, bioinformatics and geospatial informatics (Liu, Kwong, & Chan, 2012). In these cases, ontology was not only used for data integration and interoperability, but also for outlining system metadata. In this study, based on semantic ontology, we try to establish a workshop system framework for argumentation processes.

Workshop system is, for specific complicated problem, a kind of Meta-synthetic process from qualitative to quantitative, which integrated the knowledge and intelligence of expert group, data, and useful equipments. In this chapter, argumentation process originates from the complex decision-problem and group structure. Therefore, it is necessary to apply the metasynthesis methodology to design the workshop system for more efficient decision processes. In the design of decision processes, experts establish some qualitative and non-precise thinking or ideas based on the availability of synthetic knowledge. Through ontology representation process, such information can be clearly described or defined, and form the quantitative expression. By this express process from qualitative to quantitative, most of the knowledge which is used in group decision process can be rationally represented and verified. In fact, the problem-solve process is also from qualitative to quantitative. Therefore, this qualitative knowledge, useful information or other knowledge in expert's mind are raised to the quantitative reorganization as whole by organization, synthesis, model establishment, iterative evaluation and modification.

(D) Group Argumentation Model

In workshop system, the participants in argumentation process are constructed as a group. From the view of Metasynthesis methodology (Gu & Tang, 2005), the integration of human's qualitative intelligence and computer's quantitative intelligence is one feasible processing method to solve complex problem in reality. We notice that, previous argumentation models did not include the properties of decision task and did not consider the particularities of complex decision task. However, in reality, these factors are very important for complex-task oriented decision making. Therefore, the chapter proposes a multi-layer structural group argumentation model as shown in Figure 9-1.

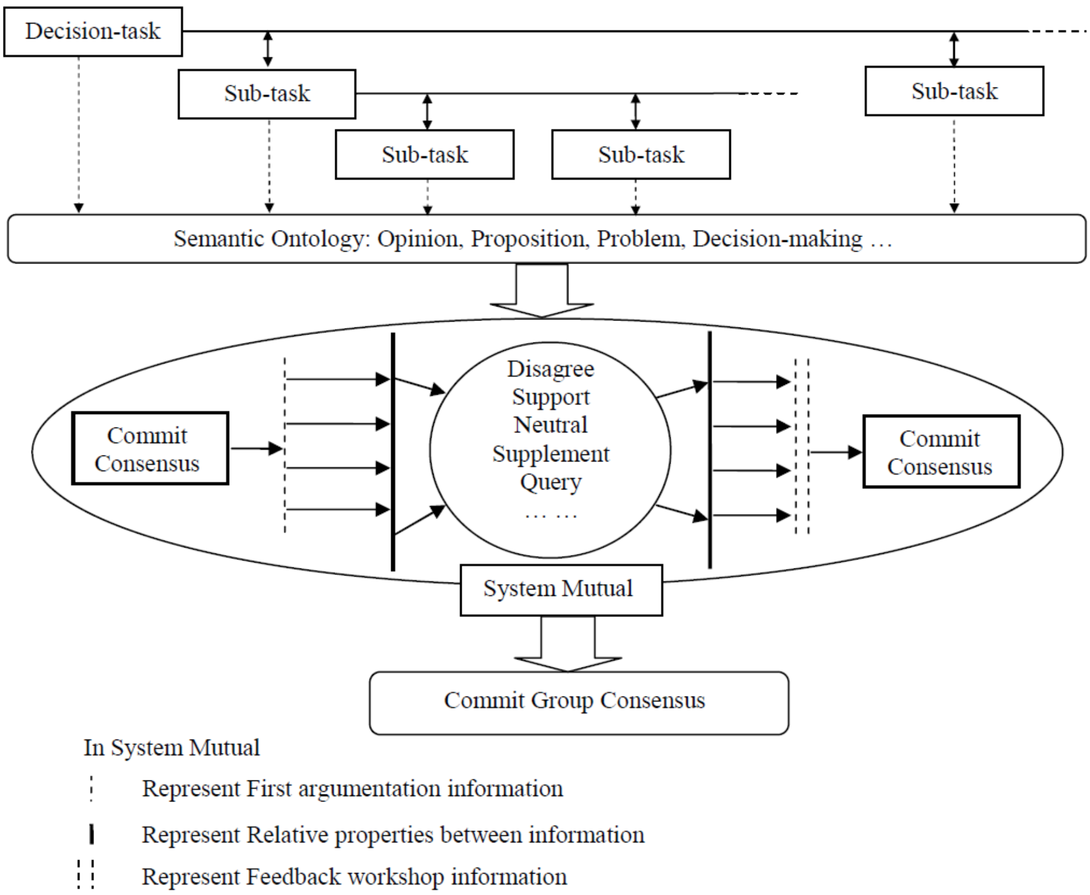


Fig 9-1 Multi-layer structural group argumentation model

Through ontology analysis, system extracts semantic objects as opinions, proposition, problems and etc, to form basic elements for argumentation. Then, these basic information elements input into Workshop system and interact with others. In this group argumentation model, we define five kinds of basic relations between information elements to model the interactions. They are Disagree, Support, Neutral, Supplement, and Query. Finally, system commits the consensus and feedback these results for following decision making section.

9.2.3 Group Decision Process and System Prototype

(A) Group Decision Process

In this section, the ONTOlogy-based Group Decision Support System (ONTOGDSS) is presented. This system framework consists of two aspects: group decision process, system hierarchical structure. All of them are based on ontology driven representative and description of decision-problem.

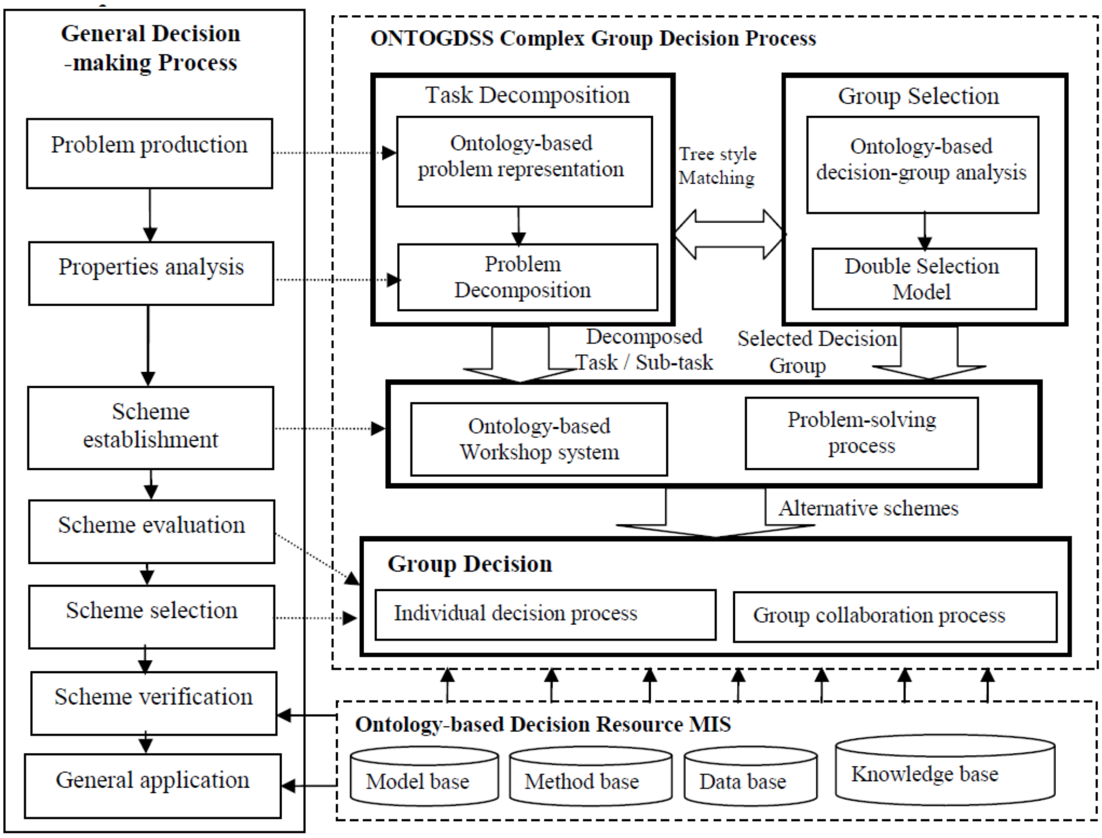


Fig 9-2 Ontology-driven complex group decision process

Figure 9-2 shows the complex group decision process in ONTOGDSS. At first, we summarize the general process of decision-making as seven stages including (1) problem production (2) properties analysis (3) scheme establishment (4) Scheme evaluation (5) Scheme Selection (6) Scheme verification (7) General application. Following this general process, our proposed decision process considers two important situations. First, this process is used to figure out the complex decision task. Second, it is used for the complex large decision group. Oriented by these two situations, we design the Group argumentation process and problem-solving process to establish alternative schemes. And through group decision algorithm, the system selected the alternative schemes. Besides, the ontology-based decision resource MIS provide the support in data accessing and information storage.

(B) System Hierarchical Structure

For processing the complex group decision, the structure of ONTOGDSS includes four layers.

1. Task decomposition layer

Based on ontology-approached representation and description of decision-problem, we can clarify its properties and limitations. Then, a tree-like decision-task structure is formed after confirming the decomposition direction. In general, task decomposition process in this layer provides the important basis and targets, and also provides some alternative decision paths.

2. Decision problem-solving layer

The system needs to organize all useful experts (or selected decision-people) to solve the task and finally form a set of problem-solving schemes, and storage into the corresponding scheme base. In this process, the workshop system which is a sub-system of ONTOGDSS and with a useful problem-solving method provides systematical supports to ontology-based group argumentation process.

3. Group decision layer

This layer includes individual decision process and group collaboration process. The main responsibility of this layer is to appoint task via mathematical algorithms, allow decision-makers to rank alternative schemes and commit a consensus at last. Through the summarization of the whole decision process and final results, we can obtain the most satisfactory scheme for this appointed task/node.

4. Ontology based Decision-resource layer

Note that this layer is based on ontology approach. As we mentioned above, for structured decision-problem, Model MIS and Method MIS can provide the model and method of previous decision experience, case or theory. Based on ontology-approach such as semantic extraction (Liu, He, Lim, & Wang, 2012; Liu, 2007), we can represent and describe these decision problems which are stored in corresponding bases.

9.3 Uncertainty Group Decision Support System

9.3.1 Background

In this section, we propose a framework of Uncertainty-based Group Decision Support System (UGBSS). Unlike existing DSS designs, this framework is based on multiagent technology and standalone knowledge management process. Through the adoption of agent technologies, this design provides an integrated system platform to support the uncertainty problem in group MCDM. Firstly, based on the literature review on MCDM researches, we analyze and provide a general model of group MCDM. Then, we carry out an analysis on uncertainty-based group MCDM, and present our designing basis of UGDSS. Thirdly, we propose the architectures and structures of UGDSS, including other two kinds of knowledge-related system components: (1) Decision Resource MIS and (2) Knowledge Base Management System (KBMS).

9.3.2 Uncertain Group Decision Process

In this section, we propose the framework of Uncertainty Group Decision Support System (UGDSS). Here, the term “Knowledge” is a comprehensive concept, which includes data, model, human knowledge and other forms of information, so long as it can be used in uncertainty group decision making.

In uncertainty group decision process, we mainly consider three factors which increase the complexity of decision-making in reality: (1) Uncertain decision environment, (2) Unstructured decision problem, (3) Complex decision group, and one issue in group decision making: Group unification of decision conflict. This process provides a mechanism to address the three kinds of complexities and group conflict, which consist of six analysis stages:

- Decision Environment Analysis
- Decision Problem Analysis
- Decision Group Analysis
- Decision Scheme Analysis
- Decision Conflict Analysis
- Group Coordination and Decision Analysis

From decision-makers' view, these stages have the basic sequence. Suppose there is a MCDM problem with complex internal structure involving multiple participants. We firstly need to analyze the existing internal and external environments, and figure out what are the decision conditions; whether the decision information is complete, certain and quantizable; what kinds of uncertainty type it belongs to. Secondly, the specific decision problems need to be analyzed, including ontological investigation, problem representation and decomposition, etc. Thirdly, an ontological group analysis is required in order to reduce the complexity of human organizational structure. Fourthly, people need to establish problem-solving solutions which may derive from various resources including previous problem-solving schemes in knowledge bases, decision schemes from domain experts, or results of group discussion, etc. Fifthly, we need to integrate those dispersive, multipurpose, individual or incomplete decision opinions into one or a set of applicable final decision results. Besides, the five stages mentioned above can momentarily call Negotiation Support System in conflict analysis stage for possible decision conflicts.

From the view of system process, each stage consists of several subsystems with different functions. For example, we adopt the ontological problem analysis tools to represent, scrutinize and decompose the complex decision problem. These subsystems as the middleware are integrated in UGDSS platform with supports of interface technologies and intelligent agent technologies. In this design, parallel computation in subsystems and middleware is quite important, which can lead to better system efficiency.

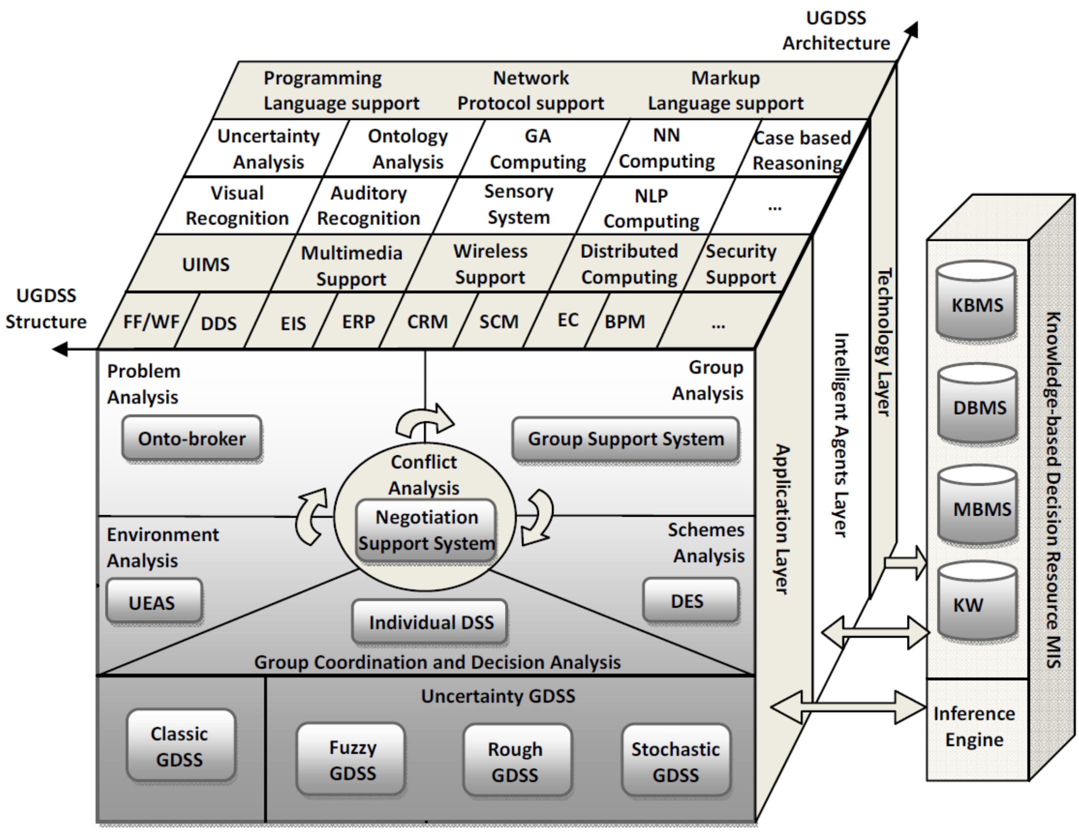


Fig 9-3 UGDSS Architecture

Decision environment analysis

Decision environment is an important factor which significantly influent other decision stages. It may contain different aspects such as decision targets, decision principles, possible limitations, available resources, etc. More importantly, people need to analyze whether there are any uncertainty information. In this chapter, we define that the uncertainty decision information which consists of the following situations:

- Information deficiency
- Information incompletion
- Dynamic information
- Unclarity information
- Inaccuracy information
- Multiple uncertainties

Although several MCDM approaches have been developed, it is not enough for solving complex uncertainty decision problem in reality. Therefore, one of our works in future aims to establish an Uncertainty Environment Analysis Sub-system (UEAS) to handle uncertainty information, and then extend its capability to solve other uncertainty MCDM problems.

1) Decision problem analysis

We can generally divide decision problems into three categories: (1) Structured, (2) Semi-structured, (3) Unstructured. To the first one, problems are well organized and represented for ontological analysis and decomposition. To another two, problems are usually represented in the form of text or interviewing dialogues. Therefore, these problems need be ontological represented and described at this stage.

2) Decision group analysis

Many decision problems in reality (such as great strategic decision of government or industry, the organizational decision of large corporation, etc.), involve multiple participants with complex human relationship or organizational structure. A good group analysis can result in much efficient decision process and impartial decision results. Group Support System (GSS) is used for group analysis including decomposition, reorganization, character analysis, integration, etc. Some methods such as Double Selection Model (Chai & Liu, 2010) are a feasible approach to realize group analysis in GSS.

3) Decision scheme analysis

Decision schemes are the problem-solving solutions to specific decision problem. These schemes may be derived from previous decision schemes reorganized in Scheme Base; new problem-solving schemes established by domain experts; solutions produced in group discussion and negotiation; all kinds of information on Web or somewhere, etc. This stage is supported by Domain Expert System and corresponding Decision Resource MIS.

4) Decision Conflict Analysis

Decision conflict analysis is the core process in UGDSS. The conflicts may be derived at each stage of decision-making process. Therefore, the subsystem in each stage may call the programs of Negotiation Support System (NSS) for conflict analysis. Chai and Liu (2010) provided a Group Argumentation Model in order to solve complex decision conflicts. This model can be used to design and develop Negotiation Support System.

5) Group Coordination and Decision Analysis

This stage takes the responsibility for the integration of group opinion. Many methods can be used to solve this problem including Vector Space Clustering, Entropy Weight Clustering, Intuitionistic Fuzzy Weight Average (IFWA) method (Xu, 2007e), etc. Besides, Individual Decision Support System is a helper of decision maker to develop their own opinion, and corresponded with Domain Expert Systems to form high quality individual decision schemes.

9.3.3 UGDSS Prototype

Unlike existing designs of DSS which mainly focus on specific problem domains, the UGDSS architecture provides an integrated system platform for complete decision analyses and comprehensive applications. The system architecture is shown in Figure 9-3, which consists of three layers:

1. The Application Layer
2. The Intelligent Agents Layer
3. The Technology Layer

1) Application layer

a) Basic function modules

- User Interface Management System (UIMS)

  UIMS, as a subsystem of UGDSS, is composed of several programs and functional interface components in intelligent agent layer such as natural language process, uncertainty analysis process, visual reorganization function, etc.
- Multimedia support

  Multimedia technologies are comprehensive used in UGDSS. The interfaces in application layer are related to many intelligent agents including Visual recognition, Audio recognition, etc.
- Wireless support

  Many mobile application devices such as PDA, mobile phone, wireless facilities are used to support group decision-making
- Security support

  In order to guarantee the security of system and data transmission, security support is indispensible in system establishment. Some main technologies include internal control mechanism, firewall, ID authentication, encryption techniques, digital signature, etc.

b) Application domain modules

The application domain modules aim to solve specific problems in different domains. For example, the Fuzzy MCDM method (Chai, Liu, & Xu, 2012) is used to solve the problem of supply chain partner selection. This application requires general domain knowledge of Supply Chain Management. These application modules as middleware of UGDSS can provide the necessary supports to various specific application domains. Several domains are shown in the following.

- Financial/Weather Forecasting (FF/WF)
- Director Decision Support (DDS)
- Enterprise Information System (EIS)
- Enterprise Resource Planning (ERP)
- Customer Relationship Management (CRM)
- Supply Chain Management (SCM)
- E-Commerce (EC)
- Business Process Management (BPM)

2) Intelligent agent layer

- Sensory system

  Sensory systems, such as vision systems, tactile system, and signal-processing systems, provide a tool to interpret and analyze the collected knowledge and to respond and adapt to changes when facing different environment.

- Genetic Algorithm (GA) computing agent

  Genetic Algorithms are sets of computational procedures, which learnt by producing offsprings that are better and better as measured by a fitness function. Algorithms of this type have been used in decision-making process such as Web search, financial forecasting, vehicle routing, etc.

- Neural Network (NN) computing agent

  A Neural Network is a set of mathematical models that simulate the way a human brain functions. A typical intelligent agent based on NN technology can be used in stock forecasting for decision making (Liu & You, 2003).

- Uncertainty analysis agent

  This agent is used to analyze the environment and conditions of decision problem.

- Case Based Reasoning (CBR) agent

  Case Based Reasoning is a means for solving new problems by using or adapting solutions of old problems. It provides a foundation for reasoning, remembering, and learning. Besides, it simulates natural language expressions, and provides access to organizational memory (Liu & Kwong, 2007).

- Natural Language Process (NLP) computing

  Natural Language Process (NLP) technology provides people the ability to communicate with a computer in their native language. The goal of NLP is to capture the meaning of sentences, which involves finding a representation for the sentences that can be connected to more general knowledge for decision making.

Besides, all of these intelligent agents with various group decision functions may consist of different kinds of Knowledge/Information bases which are united embodied in Decision Resource Management Information System (MIS). This design can improve the efficiency of information processing and the robustness of system.

3) Technology layer

This layer provides the necessary system supports to other two layers and Decision Resource MIS. It mainly includes (1) programming language support (VS.Net, C#, Java, etc.) (2) network protocol support (HTTP, HTTPS, ATP, etc.) (3) markup language support (HTML, XML, WML, etc). Besides, the technology layer also provides various technology supports for construct the four Bases and Inference engine, etc.

C. Knowledge-related system designs

1) Knowledge-based Decision Resource MIS framework

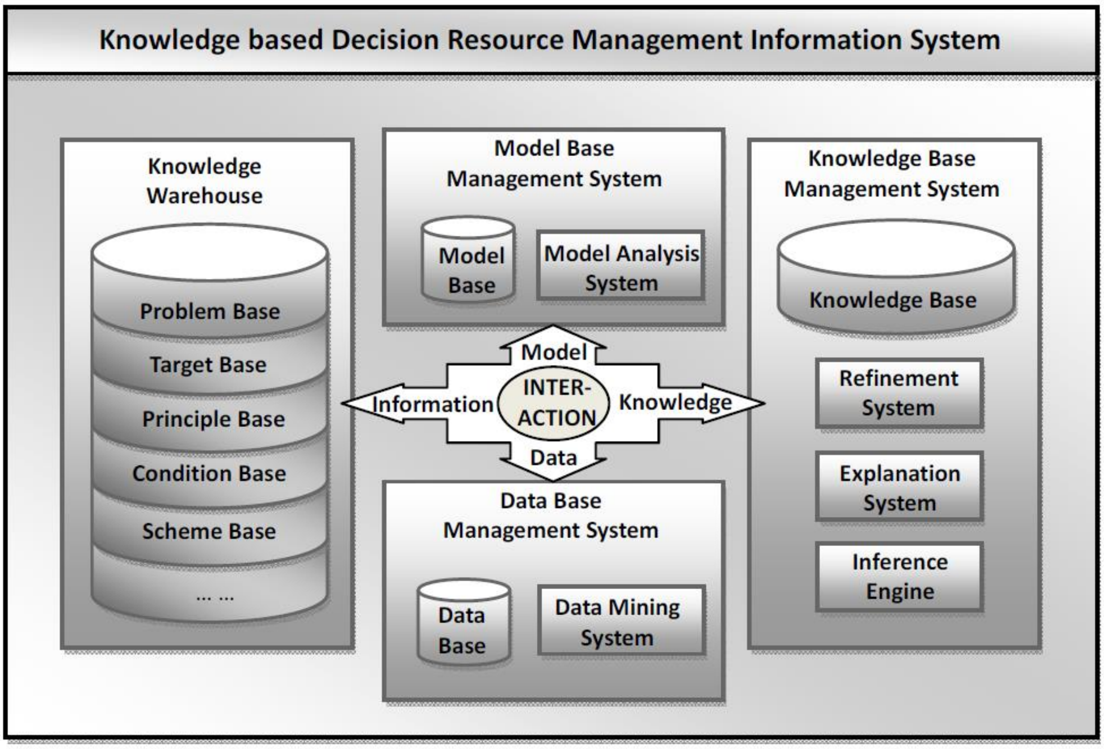


Fig 9-4 Knowledge-based Decision Resource MIS Framework

Figure 9-4 shows the framework of Knowledge-based Decision Resource MIS. It mainly consists of four kinds of subsystem: KBMS, DBMS, MBMS, KW. In this system, different kinds of information, knowledge, models and data interact together and provide the supports to the whole UGDSS.

- *Data Base Management System (DBMS)*

  Generally, DSS needs a standalone database. Especially, this DSS is required to solve the uncertainty complex group decision problems. Therefore, DBMS is a necessary component in UGDSS, which consists of a DSS database and a Data Mining System. A database is created, accessed, and updated by a DBMS. And Data Mining System is used to discover knowledge from data resources. Many technologies are applicable to mining data, such as statistical approaches (Bayes's theorem, cluster analysis, etc), case-based reasoning, neural computing, genetic algorithms, etc. These technologies are developed as intelligent agents located in the second layer in Figure 9-3.

- *Model Base Management System (MBMS)*

  MBMS mainly includes Model Base and Model Analysis System. Model base contains routine and special statistical, financial forecasting (Liu & Hu, 2012), management science, and other quantitative models which provide the resources for Model Analysis System. Turban, Aronson, and Liang (2005) divided the models into four major categories: Strategic, Tactical, Operational, and Analytical. In addition, there are model building blocks and routines. Based on these model resources, Model Analysis System is used to build blocks; generate the new routines and reports; update and change model; and manipulate model data, etc.

- *Knowledge Base Management System (KBMS)*

  There are three kinds of knowledge which will be used in decision-making: (1) structured (2) semi-structured (3) unstructured. The structured knowledge is usually will reorganize in available models and stored in model base. Much semi-structured and unstructured knowledge are so complex that they cannot be easily represented and reorganized. Therefore, more professional knowledge processing system called KBMS is required to enhance the capability of knowledge management. In next section, we present a detailed knowledge management process in KBMS.

- *Knowledge Warehouse (KW)*

  In UGDSS, KW mainly contains multiple bases for classified storage of decision knowledge including decision problems, targets, principles, conditions, schemes, etc. It is responsible for storage, extraction, maintenance, interaction and other knowledge manipulations.

2) Knowledge Management Process in KBMS

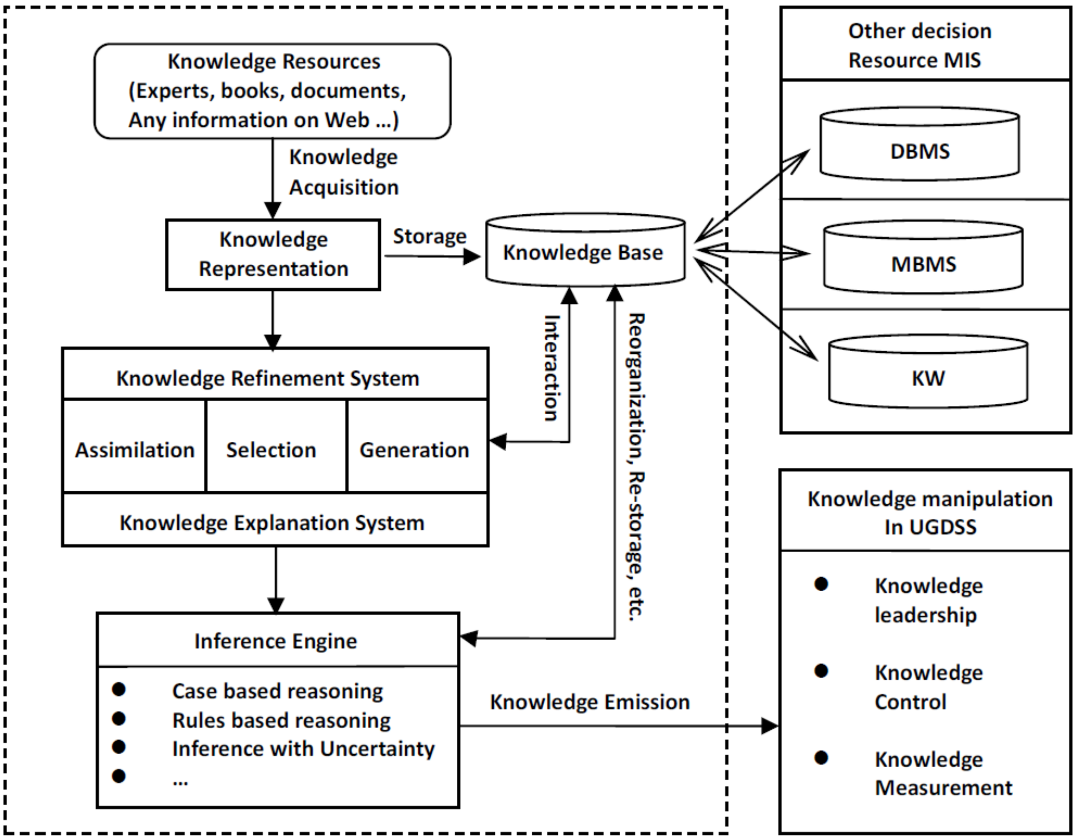


Fig 9-5 Knowledge Management Process in KBMS

In this process, there are five basic classes of knowledge manipulation activities including: acquisition, selection, generation, assimilation, and emission (Holsapple & Jones, 2004). These activities are the basis for problem founding and solving, as well as involved in each stage in decision making process. In this chapter, we provide a knowledge management process in KBMS (see Figure 9-5).

- *Knowledge Resource:*

  Some possible knowledge sources include domain experts, books, documents, computer files, research reports, database, sensors, and any information available on the Web.

- *Knowledge Acquisition:*

  This activity is the accumulation, transmission, and transformation of documented knowledge resources or problem-solving scheme of experts.

- *Knowledge Representation:*

  The acquired knowledge is organized in this activity, which involves preparation of a knowledge map and encoding the knowledge in the knowledge base.

- *Knowledge Selection:*

  In knowledge refinement and explanation system, the knowledge is validated and verified until its quality is acceptable. There are three activities to refine and explain acquired knowledge: (1) selection, (2) generation, (3) assimilation. In selection activity, systems select knowledge from information resources and making it suitable for subsequent use.

- *Knowledge Generation:*

  In this activity, knowledge is produced based on the decision incident by either discovery or derivation from existing knowledge.

- *Knowledge Assimilation:*

  In assimilation activity, this knowledge refinement and explanation system alter the state of the decision makers' knowledge resources by distributing and storing acquired, selected, or generated knowledge (Holsapple & Jones, 2004).

- *Inference Engine:*

  In knowledge base, knowledge has been organized properly and represented in a machine-understandable format. The inference engine can then use the knowledge to infer new conclusions from existing facts and rules. There are many different ways of representing human knowledge, including Production rules, Semantic networks, Logic statement, and Uncertainty information representation, etc. Here, knowledge is recognized and restored in knowledge base which also conducts the communication with other decision resource MISs.

- *Knowledge Emission:*

  This activity embeds knowledge into the outputs of KBMS, and input the useful knowledge of specific decisional episode into UGDSS for further decisional knowledge manipulation activities including knowledge leadership, control and measurement.

9.4 Summary

In this chapter, we provided two new system prototypes for complex decision supports. We firstly proposes an ontology-driven complex group decision process and corresponding decision support system ONTOGDSS in section 9.2. In section 9.3, we provided the framework of Uncertainty Group Decision Support System (UGDSS) and other two kinds of knowledge-related system components: Decision Resource MIS and Knowledge Base Management System (KBMS). The future works to this point are from two aspects. In system aspect, we need to develop multiple intelligent agents, middleware or subsystems which are integrated in UGDSS. In decision theory aspect, we will consider how to develop more applicable uncertainty group decision-making approaches based on Fuzzy sets, Rough sets, Grey system theory and other uncertainty theories in addition to what have already been discussed thereinbefore.

CHAPTER TEN

Contributions and Future Works

One theoretical development in Dominance-based Rough Set Methodology is provided. Several effective and advanced solutions are proposed for dealing with the MCDM problems. The system prototyping and the application survey are also studied. The Main contributions of the thesis and suggestions for future research are summarized in the next two subsections.

10.1 Main Contributions of the Thesis

Multicriteria decision analysis aims to provide a better recommendation for DMs. This thesis studies this issue from the perspectives of theories, methods, systems, and applications. The main contributions of this thesis can be outlined as follows:

1. In Chapter 3, a new method for induction of dominance-based decision rule set is proposed. Such rule set consists of two kinds of rules: certain rules from lower approximations and uncertain rules from rough boundary regions. The methods of certain rule induction are provided considering two kinds of nature of decision tables: multi-criteria and mix attributes. And the new defined uncertain rules called believable rules are induced on the basis of the concept of believe factor. We demonstrated the effectiveness and the advantages of the induced rule set after the comparisons with the classical minimal rule set in dealing with the issue of multicriteria ranking.

2. In Chapter 4, a complete problem-solving approach is proposed for supplier evaluation. This approach theoretically extended the techniques presented in Chapter 3 in order to further handle the more complex issue of multi-grade multicriteria sorting. Towards the realistic needs in Logistic and Supplier Chain Management, we are successful to model the supplier evaluation problem as the multicriteria sorting and pioneer to use Dominance-based Rough Set Methodology for solving it. Four representative “DRSA + Sorting Schemes” solutions are established and also as competitors for evaluation of the proposed solution: Believable Rough Set Decision Model.

3. In Chapter 5, a new rule-based decision model is proposed for warehouse evaluation under interval-valued intuitionistic fuzzy environments. The first step is to mathematically model the problem of warehouse evaluation towards logistic and supplier chain management. We elaborate the

construction of uncertain group decision process and present the multi-stage decision model by means of decision rules. The proposed approach partly employed the rationale presented in Chapter 3 and theoretically extended it for uncertain decision makings.

4. In Chapter 6, an Intuitionistic fuzzy SIR decision model is proposed for supplier selection. The Superiority and Inferiority Ranking (SIR) is a generation of the well-known PROMETHE which is an efficient approach for MCDM. The new proposed approach extended this traditional MCDM technique for applications in intuitionistic fuzzy environment. The significance of the proposed approach lies on its successfully propagating the complex systematic uncertainty in the whole problem-solving process.

5. In Chapter 7, a new tolerant skyline decision model is proposed for the NBA player evaluation. This work pioneers the application of skyline operations to the problem of personnel evaluation. Overcoming the existing weaknesses of other skyline operations that hinder the realistic decision-makings, the proposed operation possesses the merits including more controllability of hieratical skyline outputs and the effectiveness of modeling dynamic preference of DMs. The detailed empirical study on NBA player evaluation in 2010-11 regular seasons demonstrates the effectiveness and the advantages of the proposed decision model and consequently several interesting results with the realistic significance.

6. In Chapter 8, a systematic academic survey on the application of decision making techniques on supplier selection is proposed. In this work, we comprehensively reviewed the related literature between the periods of 2008-2012 covering 15 international journals. By using a methodological decision analysis in 4 aspects including decision problems, decision makers, decision environments, and decision approaches. We identified 26 used DM techniques from 3 perspectives: Multicriteria decision making (MCDM) techniques, Mathematical programming (MP) techniques, and Artificial intelligence (AI) techniques. This survey provides the recommendation for future research on supplier selection and facilitates knowledge accumulation and creation concerning the application of DM techniques.

7. In Chapter 9, two designs of decision support systems towards group and uncertain decision-making are provided. Firstly, the framework of ONTOGDSS is provided. It is capable of reducing the complexity of problem structure and group relations. The system allows decision makers to participate in group decision-making through the web environment, via the ontology relation. Secondly, the framework of UGDSS is developed for multiple processes of decision analysis in six aspects including decision environment, decision problem, decision group, decision conflict, decision schemes and group negotiation. These designs are the attempt for implementation of our proposed decision methods/models in Chapter 3~7 with supports by information systems.

As a result of both the theoretical developments and the real world applications, it is believed that this thesis has contributed efforts in laying down the foundation of MCDM for the future.

10.2 Limitations and Future Works

In this thesis, we have made some progress in the field of multicriteria decision analysis. Even so, there are still some limitations. From the theory perspective, our developments of the DRSA methodology comply with the precondition: each decision table is just with a unique decision attribute. We do not consider the situation of MCDM over multi-decision-attribute decision tables. From the method perspective, our proposed BRSA model just refers to preference-ordered values for evaluation. In many real-world applications, these values could be similar or identical. Hence, apart from dominance relations, another two relations should be considered: similarity and indiscernibility. It is essential to investigate the application of BRSA in handling such heterogeneous attribute set for supplier selections. From the system perspective, our proposed two DSSs are actually the conceptual model. They need to be verified and tested through application-oriented implementations in the future.

The development of this thesis not only provides several approaches to modeling and analyzing the structured decision problems of the real world, but it also opens up new avenues to further research in related fields. Here, we would like to identify some possible directions for future works.

1. In *theory* perspective, although the foundation of DRSA is nearly mature, it is still lack of the investigations on the effective hybridization of DRSA and other uncertain tools such as intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set, and the new appeared hesitation fuzzy sets (Torra, 2010; Xu & Xia, 2011; 2012; Rodriguez, Martinez, & Herrera, 2012).

2. In *methods* perspective, we have provided the effective solutions for the three of four key issues in MCDM, which are multicriteria sorting, ranking, and choice. For another issue, criteria analysis, this thesis discusses this issue in Chapter 4 (i.e. A united method for criteria reduction and certain rule induction). A more specialized investigation on criteria analysis would be valuable for future work.

3. In *system* perspective, a software-based decision support system (DSS) is always important in current information world. The more developments on DSS could help decision makers implement the proposed decision approaches easily and expeditiously.

4. In *application* perspective, this thesis has provided the solutions for some problems regarding Logistic and Supplier Chain Management, e.g. supplier selection and warehouse evaluation. Our

work mainly plays attention to the static decision analysis. The future work can focus on dynamic decision analysis when time factor is further taken into account. In addition, personnel evaluation, as an independent and important issue regarding decision making, is worth more studies in future.

This thesis contributed a comprehensive investigation on MCDM for structured decision problems from multiple perspectives including the decision mathematical tools (the theory aspect), the realistic problem-solving decision models (the method aspect), the effective decision support systems (the system aspect), and the real applications. As Figueira, Greco, and Ehrgott (2005) emphasized, however, the MCDM field is so large and developments so heterogeneous. I believe many promosing researches need to be done as the extension of this thesis. Particularly, I put forward two fields as our future researches. Following by the decision-making framework of Dyer (2005), this thesis just deals with the ordinal and measurable theories, namely *preference value functions*. The other aspect, *preference utility functions* which are related to risky choice such as lotteries or gambles, can be the first direction of future works. Following by the decision-making framework of Wallenius, et al. (2008), this thesis just concerns *multicriteria discrete alternative problems*. The other side, *multicriteria optimization problems* such as energy planning, portfolio selection, and R&D project selection, thus need to be investigated as our second direction of future works.

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