REAL OPTIONS MODEL OF TOLL ADJUSTMENT MECHANISM IN CONCESSION CONTRACTS OF INFRASTRUCTURE PROJECTS

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Real Options Model of Toll Adjustment Mechanism in Concession Contracts of Infrastructure Projects

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CERTIFICATE OF ORGINALITY

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Chen Qing, Richard

ABSTRACT

Effective management of demand risk is of great significance for the private investor participating in a concession contract of infrastructure project, such as the traffic risk in a BOT toll road project. To attract private participation in such capital intensive projects characterized by huge sunk cost, a variety of uncertainties and risks, long-term financing agreements (usually spanning over several decades), and non-recourse/limited-recourse project financing scheme, the host government often has to provide risk mitigation mechanisms, among which traffic/revenue guarantees have been widely applied around the world in numerous projects. The contractual and managerial flexibility embedded in such arrangements can be deemed as real options; therefore real options theory has been adopted by scholars and practitioners to value the project. However, guarantees (in essence contingent liabilities for the host government) may bring heavy fiscal burdens to the host government if the actual traffic is much more pessimistic than the projected level, which is not rarely seen in economic recessions. Toll Adjustment Mechanism (TAM), a hybrid of price cap regulation mechanism and revenue sharing mechanism, is one solution to prevent the private investor from severe traffic demand risk and the government from heavy fiscal burden, at the same time to ensure the private investor a reasonable but not excessive rate of return. Nevertheless, Toll Adjustment Mechanism (TAM) has not been investigated as broadly and in-depth as guarantees; quantitative modelling and analysis of TAM is even much scarcer. Therefore this research intends to fill this gap by modelling TAM as real options, developing a framework to assess the value of flexibility of the right (but not obligation) of toll adjustments. First, a stochastic traffic demand model is developed; second, a traffic assignment (two-route choice) model is built to quantify the demand function of the toll and the traffic on the toll road, therefore the optimal pricing strategies in each period during the concession period can be obtained; then the optimal pricing strategy in multi periods, with the objective of maximizing the net present value (NPV) of the project, can be obtained through real options analysis. A hypothetical case study derived from a real life project, Western Harbour Crossing in Hong Kong, is illustrated in detail to demonstrate the application of the framework developed and to validate the effectiveness and robustness of the framework. Outcomes of the research can help the

government to design reasonable concession contracts and help the private investors to make sound investment decisions through effective management of the traffic demand risk. Therefore a win-win prospect can be achieved in public-private-partnering concession contracts for both parties.

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1. INTRODUCTION

1.1. Research Background

Facing increasing difficulties in funding public facilities and utilities, governments around the world resort to the private sector for its large amount of funds, resources and expertise. One way of achieving this objective is through Public-Private Partnership. Laws and regulations have been enacted in many countries to facilitate private finance in public infrastructure development for improved quality, efficiency, and cost effectiveness.

In Asia, transport infrastructure is the dominant sector in the PPP market, accounting for almost 50% in terms of values of concessions; the transportation sector can be categorized into toll roads with bridges, tunnels, and rest areas; railways; urban transport systems such as light rail transit (LRT), people-moving transit or subways; multipurpose terminals; fuel transport pipelines; and seaports (Kwak, 2002).

PPP projects are characterized by huge sunk cost, high level of uncertainties and risks of various sorts, long-term financing agreements (usually tens of years), and non-recourse or limited-recourse project financing scheme. UNIDO (1996) divide different types of risks, to which BOT projects are exposed, into two broad categories: general risks and specific project risks. General risks are associated with the political, economic and legal environment of the host country and over which the project sponsors generally have little or no control; while, to some extent, specific project risks are controllable by the project sponsors.

Thomas (2003) conducted a survey among the major stake holders/participants of Indian BOT road projects (Government representatives, promoters/developers, lenders and consultants) and identified 22 risks in Indian BOT road projects. Among them, the eight most critical risks were i) Traffic revenue risk, ii) Delay in land acquisition, iii) Demand risk, iv) Delay in financial closure, v) Completion risk, vi) Cost overrun risk, vii) Debt servicing risk and viii) Political risk. The conclusion applies to similar projects around the world.

Among all the aforementioned risks, demand risk plays a particularly significant role in toll road projects (Estache et al., 2000). Demand risk in transportation projects can be defined as the inability to determine the behavior of real traffic movement compared to forecasted traffic. Since revenues collected from tolls are the sole income the concessionaire could resort to for repayment of loans and recoup of initial investment, effective mitigation and management of demand risk is crucial for the private sector. Besides a sound economic environment, to attract private investors into the highly uncertain, capital intensive toll road projects, which are, worse still, vulnerable to opportunistic governmental behaviours, the host government initiated multiple policies and schemes to alleviate the risks faced by the investors, such as extensive guarantees against a variety of risks, politically and economically, in assorted forms. Minimum traffic guarantees and minimum revenue guarantees are deployed by host governments around the world to mitigate demand risk in toll road projects.

Nevertheless, cases in several developing countries have proved that poorly designed guarantee/support type of governmental schemes (in essence contingent liabilities) could induce substantial fiscal burdens to the host government and taxpayers, which can counter-intuitively diminish, even eliminate the advantage of applying PPP into infrastructure projects. Some notorious examples are: the foreign exchange guarantees provided by the Spanish government in the 1970s cost the taxpayers $2.5 billion; the failure of the Mexican toll road concessions after the 1994 Mexican crisis eventually cost $8.9 billion to the government; In 2000 alone, the amount that the Colombian government paid for the guarantees issued for 11 toll road projects was as high as 0.017% of its GDP in the same year (Irwin et al., 1999).

According to Irwin et al. (1999), to put in place sound policies that generally reduce risks and increase expected returns is better than issuing guarantees to attract private investors. Toll Adjustment Mechanism (TAM) is such a solution to the demand risk problem yet induces no fiscal burden to the host government, which is, to our knowledge, seldom studied in the academic field, although government guarantees have been extensively investigated in detail by multiple researchers, usually in the context of real options analysis. Since toll adjustment mechanism becomes more popular in toll road projects, practitioners are in need of an effective tool to analyze and manage such arrangements in the contract. This study intends to bridge this gap.

1.2. Research Aim and Objectives

The aim of this research is to conduct an extensive and in-depth investigation into whether and how real options analysis can be used in modeling the contractual / managerial flexibilities embedded within the toll adjustment mechanism in concession contracts of infrastructure projects as an effective means to mitigate and manage the demand risk in PPP projects.

Specific objectives of this research are:

1. To build and validate a traffic assignment (two-route choice, both can be congested) model of commuters with heterogeneous values-of-time to help the concessionaire optimizing the one-period revenue/profit;
2. To establish and validate a real options model of the toll adjustment mechanism in which toll adjustments are treated as

real options through active pricing strategy, to assist the concessionaire to optimize the overall net present value (NPV) of the project during a multi-period concession contract.

1.3. Research Methodology

1.3.1. General research approach

The development of a general and flexible method for valuing the toll adjustment mechanism is underpinned by real options theory. However, due to the unique and distinct characteristics of the toll adjustment mechanism which tell them different from guarantees and financial options, simple application or naive adaptation of real options analysis, such as in the case of real options analysis of guarantees in concession contracts, is no longer satisfactory or adequate within this setting. Thus, a significant portion of the research was devoted to developing a novel real options valuation framework for the toll adjustment mechanism.

The framework contributes to the existing body of knowledge of real options theory in two respects. First, it broadens the application of real options to a new family of contractual flexibilities, i.e., the toll adjustment mechanism, which will be explained in detail in Chapter 4; Second, a new model based on decision-tree analysis, branch-and-bound method and Monte-Carlo simulation is developed to value the toll adjustment mechanism as real options, because the traditional dynamic programming method is no longer applicable for this problem, due to the lack of suboptimal structure in the decision-tree. Refer to Figure 1-1 for the flow of research methodology.

1.3.2. Theoretical framework

The theoretical framework incorporates three important elements to finalize the real options value of the toll adjustment mechanism: a stochastic model of traffic demand, a traffic assignment (two-route choice) model of commuters with heterogeneous values-of-time, and, finally, the real options model of toll adjustment mechanism. The second and third elements represent the original contribution of this study. The first theoretical element is a novel method modelling risk variables characterized by little or no data history; traffic demand uncertainty is modelling in this fashion. The second theoretical element is represented by a model of traffic assignment (two-route choice) for commuters with heterogeneous values-of-time, in which the traffic demand generated in the previous model will be imported as inputs to determine the mathematical function between toll charged and corresponding traffic on the toll road in any particular period. The third theoretical element, which is the kernel of the study, is in essence an optimization problem with a maximizing function of revenues/profits as the objective, and pricing strategies in each period as the decision variables, subject to the demand functions generated in the previous model as well as relevant stipulations in the concession contract.

1.3.3. Technical analysis

Once the theoretical framework is setup, the toll adjustment mechanism is analyzed and the real options value calculated. The technical analysis was performed in three phases: first, development of a decision tree model; second, generation of prospective scenarios through Monte-Carlo simulation of uncertainties involved, foremost of which is the traffic demand; third, determination of the optimal path by branch-and-bound method

1.3.4. Case study

A hypothetical case study based on a real life project, the Western Harbour Crossing in Hong Kong, is carried out i). to demonstrate the application of the framework developed; ii). to validate and verify the theoretical framework. Reasonable and appropriate simplifications were made to modify the real life project into the hypothetical case study, in order to reduce the computation complexity and to focus on the essence of the problem. By reasonableness and appropriateness the author means that the essence of the problem is by no means altered even in the slightest manner in the process of simplifications. To get more accurate results to facilitate investment decision making of the private investors in real life, such simplifications can be safely discarded and be replaced by modelling the real world in immense details; doing so will not reduce the effectiveness of the framework, but will abate the efficiency of it. The loss of efficiency depends on to what extent the modeller wants the model to represent the real world and the size of the problem. That is to say, the modeller has to strike a balance between the optimality and computability. This dilemma can be improved by employing more accurate and faster algorithms and powerful computers.

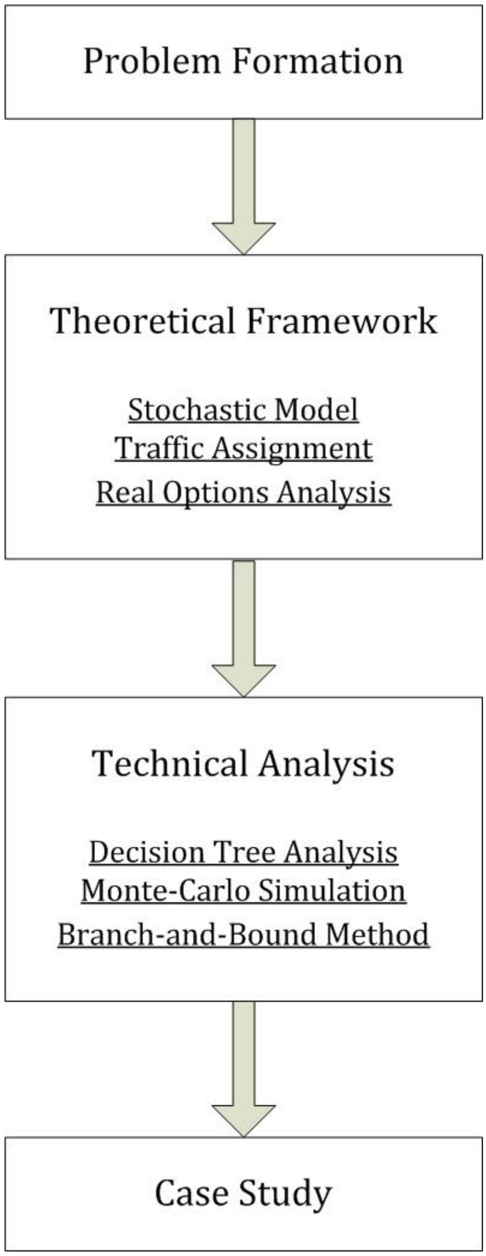

Figure 1-1 Research methodology and process

1.4. Organization of Thesis

Chapter 1 introduces the background, aim and objectives, research methodology, significance and contribution of the thesis. Chapter 2 reviews and summarizes the existing research regarding option pricing and real options theory, applications of real options theory in capital budgeting, and valuation of guarantees in infrastructure contracts as real options. Chapter 3 presents a general toll adjustment model studied in this research, elaborates the characteristics of it, and expounds the rationale of deeming the toll adjustments as real options. Chapter 4 introduces a novel stochastic model of BOT risk variables characterized by little or no historical data. Chapter 5 presents the model of traffic assignment (two-route choice) of commuters with heterogeneous values-of-time as well as solution to the model. Chapter 6 presents the real options model of the toll adjustment mechanism with the inputs from the previous two chapters and both the optimal pricing strategy for the concessionaire and the corresponding real options value of the toll adjustment are identified. Chapter 7 illustrates a hypothetical case study adapted from the Western Harbour Crossing of Hong Kong, in which both scenario analysis and Monte-Carlo simulation are applied to demonstrate and validate the framework. Chapter 8 presents conclusions and limitations of the research and recommendations for further research.

1.5. Summary of the Chapter

Chapter 1 provides an introduction to the thesis, including the research background, current research gap, research aim and objectives, research methodology, significance and contributions, and outline of the thesis. The main argument of the research is to study the toll adjustment mechanism designed in order to mitigate the demand/traffic risk in transport projects through traffic modelling and real options analysis.

2. REVIEW ON REAL OPTIONS THEORY AND ITS APPLICATIONS

2.1. Foundations of Real Options Theory

The rationale of real options is generally based on the logic of financial options. Black and Scholes (1973) and Merton (1973) were the first to provide a closed–form solution for financial options. After that, Cox et al. (1979) developed a power tool to deal with complex option settings using an equivalent discrete binomial valuation approach. Although at this early stage single options are the research focus, the following research work focused on multi-options problems or multiple assets which are regarded as special cases of a portfolio. These work includes Margrabe (1978)'s analysis on an option to exchange two assets. It was followed by Stulz (1982)'s work to study an option on the maximum or the minimum of two underlying assets. These work was later generalized and applied by Johnson (1987) and Boyle and Tse (1990) to multiple underlying assets. Laamanen (2000) extended the research to the related option on the best subset of several underlying assets. Another strand of research includes Geske (1979)'s work to develop a closed–form solution for two compounded options. Carr (1988) combined these two features, and valued a sequential exchange option.

Despite the extensive research on real option evaluation, most of them focus on options that are analyzed in isolation. Although the research on stand–alone options offered useful perspective to understand the flexibility, the usefulness is suffered due to the portfolio point of view. However, there were still some papers explicitly considering the interaction of real options and real assets as it is of great importance to value these options in combination in compound option. For example, Brennan and Schwartz (1985) considered different options to shut down, restart or abandon an energy project. Similarly, McDonald and Siegel (1985) analyzed an option of shutting down a plant temporally.

Meanwhile, some authors address the interactions of options explicitly. For example, Trigeorgis (1993a) mentioned that the existence of subsequent options is likely to increase the value of the underlying asset for earlier options. However, he also pointed out that the exercising earlier options may affect the value of the underlying asset for subsequent options. Based on this, when multiple options are written on the same underlying asset, normally option values will not add up. Following Trigeorgis (1993a), Kulatilaka (1995) confirmed his results and explicitly considered how additional options affect the critical boundaries for option exercise, i.e., the exercise schedules.

Amram and Kulatilaka (1999a) defined a real option as the right, not the obligation to take some strategic decisions depending on the uncertainty level of the conditions. Thus the real options theory helps evaluating and optimizing alternative strategies or combination to address uncertainty; In particular, real options theory helps deal with situations where future conditions are uncertain and changing strategies in later stages will incur substantial costs. Unlike making all strategic decisions at the early project stage, using flexible strategies and delaying decisions considered as real options reduce project risks and increase project value.

All in all, the real options theory help to determine what are the alternative actions for the uncertain future, when to apply these actions, and what are the prices of choosing these actions. The answers to these questions will lead to increase value of the project.

Real options theory is derived from the traditional option pricing theory which tries to value financial options (Black and Scholes 1973, Cox et al. 1979; Bookstaber 1983) A variety of methods have been developed to evaluate options (Brealey and Meyers 2000, Trigeorgis 1993b, 1995, Dixit and Pindyck 1994, Kemna and Vorst 1990). These methods have been successfully implemented in engineering and project management areas (Benaroch 2001, Baldwin and Clark 2000, Park and Herath 2000; Ford et al. 2002), and were regarded as effective tools for strategic planning (Bierman and Smidt 1992; Leslie and Michaels 1997; Kensinger 1988; Amram and Kulatilaka 1999b; Miller and Lessard 2000).

Furthermore, real options have been widely used to capture latent values in a variety of areas, which mainly include, but not limited to natural resources, research and development (R&D), technologies, property and product development (e.g. Kemna, 1993; Amram and Kulatilaka, 1999a;

Benaroch, 2001, Brennan and Trigeorgis, 2005; Dixit and Pindyck, 1994,).

2.2. Real Options in Capital Budgeting

Firms continuously make decisions whether or not to invest in risky projects. The decision becomes more challenging when a project has a great deal of uncertainty regarding its value. Firms typically learn more about the value of a project as they invest over time and as uncertainties are resolved.

A variety of research studies have been conducted to analyze and valuate investment projects. Traditional financial theory proposes a number of capital budgeting techniques, such as payback period method, accounting rate of return (ARR), internal rate of return (IRR), net present value (NPV), and decision tree analysis. However, these traditional capital budgeting methods are falling short of dealing with the operational flexibility options, especially in the situations of multiple project decision making, leading to the frequent change of original cash flows.

According to Trigeorgis (1996), the traditional capital budgeting methods cannot properly capture "management's flexibility to adapt or revise later decisions when, as uncertainty is resolved, future events turn out differently from what management expected at the outset". Academics and practitioners alike now recognize that these traditional methods such as the standard discounted cash flow techniques when applied improperly often undervalue projects with real operating options and other strategic interactions. In practice, many corporate managers overrule passive net present value (NPV) analysis and use intuition and executive judgment to value future managerial flexibility.

In the past two decades, developments in the valuation of capital investment opportunities using real option valuation methods have revolutionized capital budgeting. The theoretical foundations underpinning real options valuation methods are now well developed (Brennan and Trigeorgis, 2000; Copeland and Antikarov, 2001; Dixit and Pindyck, 1994; Trigeorgis, 1996).

As a frequently advocated approach to value projects, real option approach offers a positive and radical reassessment of the value of risk and exploration. Ford et al. (2002) proposed a real options approach for proactively using strategic flexibility to recognize and capture project values hidden in dynamic uncertainties. It is concluded that using a structured real options approach in construction management can increase returns through improved project planning and management. It is also suggested that potentially large improvements to construction management could result from the development and adoption of a structured real options approach.

Chen (2006) adopted the real option theory to develop an analytical theory of project evaluation, which enables systematic comparison of returns of different investment under different market conditions to be made. From real option theory, this refines the insights in many ways, and contributes a new foundation to investment theory by providing an analytical theory about the relation among fixed costs, variable costs and uncertainty.

According to Cheah and Garvin (2009), project evaluation, especially the economic feasibility analysis often encompasses various flexibilities and risk mitigation measures. The techniques presented in their study illustrate the limitations of valuing projects traditionally and the supremacy of deploying a real option model for valuing flexibility embedded in an infrastructure project. The real option techniques can aid in the analysis of a project's private finance potential, investment timing and thus facilitate better decision making during negotiation and procurement.

Wong (2007) investigated how uncertainty affects the investment timing in a canonical real options model. It reveals that the critical value of a project that triggers the exercise of the investment option exhibits a U-shaped pattern against the volatility of the project, which is due to the two countervailing risk and return factors in effect.

The real option approach has been widely adopted for capital budgeting within different industries. Using life-cycle analysis, Zhao et al. (2004) developed a real-options approach for optimal decision-making for highway projects in design, operation, rehabilitation, and expansion. The development of a highway system development requires huge irreversible investments, which demands for the rigorous modeling and analysis process before implementing these decisions. This decision-making process is embedded with multiple uncertainties due to changes in political, social, and environmental contexts. Zhao et al. (2004)'s model provides convincing results and dramatically improve the decision making process for highway engineering projects. Practical implications have also been provided for the optimal planning of the highway projects.

Armstrong et al. (2004) apply real option and Bayesian analysis to address the question of how to evaluate the option to acquire more information for oil projects. It helps to develop an objective procedure for evaluating the financial potential of the additional information. This is of benefit to both the operator and service companies alike as the former can properly determine ultimate economic viability. It also demonstrates that Bayesian analysis coupled with real option provides a general framework for evaluating the option to obtain additional information for project evaluation.

Smit (2003) combined real options theory and game theory to analyze the optimal and strategic features of airport investment. It is found that the combination of real option theory and game theory can help to facilitate the strategy modification in the airport industry.

Leung and Hui (2002) developed an option pricing to evaluate the real estate investment in Hong Kong. With the evidence from the Hong Kong Disneyland project, it is concluded that the real options may create added value for the project, and it is superior to the traditional NPV approach on the evaluation of real estate investment as it helps to enhance upside potential as well as reducing the downside risk.

In Taiwan, Bowe and Lee (2003) apply an accepted real options valuation methodology (log-transformed binomial numerical analysis) to the actual dataset which formed the basis for project evaluation of the Taiwan High-Speed Rail (THSR) Project, and compare results to a static cash-flow valuation benchmark, which is shown to underestimate the real value of the project. Moreover, the value placed by Taiwan High-Speed Rail Consortium (THSRC) management on the flexibility to adapt decisions in the light of future contingencies is not only significant, but appears necessary to justify the project's economic viability.

The evidence from India power project also confirms that the real options approach is superior to quantify and relocate risks and awards within the contract of large scale infrastructure project. According to Cheah and Liu (2005), the real options approach demonstrates a great promising in capturing and evaluating flexibilities.

Mattar and Cheah (2006) stated that in the early stage of the large engineering projects, project valuation is very important to stakeholders as they need to evaluate whether or not it is a wise investment. It is quite clear that it is very difficult to have an accurate valuation the level of uncertainty increases. This is particularly true if the risks cannot be measured or quantified. The review of different methods to quantify/price the value of risks for a oil and gas project revealed that different pricing methods would produce different real option values, which requires different strategies to act. These priced values for different real options are normally regarded as premium for risks.

However, it should be pointed out that although the real option approach is a frequently advocated approach to value projects and offers a positive and radical reassessment of the value of risk and exploration (Bowman, 2001), it is not always the first choice. According to Garvin and Cheah (2004)'s work, which provides the basis for examining the assumptions behind both traditional and option valuation models, the selection of a valuation model depends critically upon project characteristics, and it is an important part of decision-making process.

Mattar and Cheah (2006) compared two major different approaches: option pricing analysis, and decision analysis, and found that when both option pricing and decision analysis methods are correctly applied, they must give consistent results. Additionally, option pricing and decision analysis methods can be profitably integrated. In particular, option pricing techniques can be used to simplify decision analyses when some risks can be hedged by trading and, conversely, decision analysis techniques can be also used to extend option pricing techniques to problems with incomplete securities markets.

Furthermore, according to Bowman (2001), the assumptions incorporated in most option evaluation models can conflict with the conclusions reached by strategic analysis, and thus users of these models should understand the quantitative aspects of these models, and may need to create a customized model for each situation.

Real options can serve to capture latent value in many areas such as energy resources, research and development (R&D), different new technologies, bidding strategies, and project planning and development. Recently, the real option concept is also gaining recognition in the field of construction engineering and management. This is primarily attributed to the fact that multiple flexibilities are often embedded or intentionally structured within the various stages during the life cycle of a complex infrastructure project.

To properly value these flexibilities, some construction management researchers adopt the real option as an evaluation tool, since it is deemed to complement other conventional analytical methods such as the net present value and internal rate of return. Yiu and Tam (2006) proposed a real options model and analyze a real-life construction project tender to examine how under-pricing in tendering provides real options value. It is found that when uncertainties of cost items in a tender exist and choices are available to defer and switch modes of construction, then a valuable option is available to the bidders. The under-priced portion is the options value which the bidder is willing to pay for the flexibility and the uncertainty.

These findings enable contractors to be more competitive and to estimate construction costs more accurately in devising their bid strategies.

Liu and Cheah (2009) applied real options approach in PPP-PFI project negotiation. Modeled as real options, support and repayment features found in the case of a wastewater treatment plant in Southern China are evaluated using a combination of Monte Carlo simulation and spreadsheet-based cash flow models. It illustrates how a negotiation band incorporating these option values is constructed, which would enlarge the feasible bargaining range for both parties. The PPP-PFI case study of real option application effectively sheds new light on the design of contractual terms, project evaluation and negotiation of PPP/PFI projects.

Sing (2002) built a time to built real options model to evaluate the sequential construction process of a large scale construction project. It helps to understand the optimal payoff value, that triggers the exercise of the option to invest at a maximum rate, increases positively with the increases in cash flow volatility, input cost uncertainty, excess asset return per unit risk and maximum rate of investment.

Since more and more state departments of transportation in the U.S have used warranty provisions to protect their initial investment, Cui et al. (2004) introduces the warranty option, which gives the DOT the right to buy a warranty only if it becomes necessary at the end of construction. It is argued that the application of option mechanics in the warranty has its advantages over the conventional warranty.

Ekström and Björnsson (2005) evaluate the information technology (IT) investments in architecture, engineering and construction industry from a real option perspective. A real option model links uncertainty to the value of an underlying traded asset, providing an objective measure of this managerial flexibility. The results show that it is possible to quantify the value of managerial flexibility for IT investments in the architecture, engineering, and construction industry, but that the proper method to use is contingent on the nature of the investment project.

Zhao and Tseng (2003) viewed the expansion of a construction facility as an investment problem, in that a premium has to be paid first for an option that can be exercised later. A real option model of the foundations versus flexibility trade-off enables the competing options to be optimized by balancing the expected profits that may arise from future expansion. Use of the model is demonstrated for the construction of a public parking garage and help to determine the optimal foundation size.

2.3. Real Options in Valuation of Guarantees

The 1990s witnessed a worldwide trend towards an increase in participation of private investment in public infrastructure projects. Government guarantees, allowing the private investor to recoup part of its losses, have been used frequently in private infrastructure projects (Brandao and Saraiva, 2008). In a guarantee arrangement, if a project underperforms in a particular year, the investor has the option to demand that the government reimburse him the shortfall, up to a pre-established level of guarantee. Because of these characteristics, the valuation of these guarantees requires the use of option pricing methods known as real options analysis (Dixit and Pindyck, 1994; Trigeorgis, 1996).

The government grantees usually include three major guarantees, i.e. minimum revenue guarantee (MRG), minimum traffic guarantee (MTG), and loan guarantee in BOT road projects. Governments have long used loan guarantees that cover some or all of the risk of debt repayment to pursue a variety of policy objectives, including protecting bank depositors, promoting exports and foreign investment, and even bailing out firms in financial distress (Mody and Patro, 1996). Currently, especially in developing countries like China, governments are increasingly using guarantees to stimulate private lending for infrastructure projects. These guarantees address risks inherent in infrastructure sectors and mitigate those risks that the private sector cannot evaluate or will not bear.

As for the minimum revenue grantees, if the realized project revenue (or cash flow) is lower than initially projected, the government should provide the developer with revenue shortfall as already stipulated in the contract. This agreement helps the government to attract the private partners at early stage of financing. Conversely, if the project revenue is excessively surpassing the initially pre-stipulated level, in turn, the developer can enjoy the surplus and the government should have a way to ask claim for excessive benefit so as to quit or mitigate adverse situations of the revenue exploitation. This practice is called revenue cap agreement (RCP) that stipulates the payment from the BOT developer to the government (Cheah and Liu, 2006).

In order to assess the value of the minimum revenue guarantees, their impact on risk of the project and the expected value of the government outlays, Cheah and Liu (2006) propose a real options model to value a minimum revenue guarantee (MRG) using a Monte Carlo simulation approach and apply this to the case of the Malaysia–Singapore Second Crossing. Huang and Chou (2006) also use a real options approach to value a MRG in a project that has an option to abandon using the Taiwan High-Speed Rail project as a numerical example, and show that both the government supports and the option to abandon create value. Chiara et al. (2007) develop a model where the MRG is in the form of Bermudan and Australian options which is solved by using a least squares simulation approach and apply this to a hypothetical case. All these authors adopt the MRG model which solves for the option value by discounting project revenues at the risk free rate.

Brandão and Saraiva (2007) present a real options model that can be used to assess the value of different guarantees, which allows the government to analyze the cost/benefit of each level of support, and propose alternatives to limit the exposure of the government while still maintaining the benefits to the private investor. They conclude that a minimum traffic guarantee combined with a cap on the total government outlays for the project offers the best combination of risk reduction for the private investor and liability limits for the government.

A guarantee and an option are similar in the sense that they can provide a downside protection to their holders. The only difference is that a guarantee is often given for free. One of the major problems with guarantees is that it is difficult to value. The real options method has demonstrated to be an effective approach for value assessing, and represents an important step toward improving risk mitigation and facilitating contractual and financial negotiations.

Other applications of real options theory in assess the value of flexibility in guarantees can be found in: Ashuri (2010), Jun (2010), Charoenpornpattana (2002), Lantz (2006), Klein (1997), Alleman and Rappoport (2002), Sosin (1980), Irwin (2003), Tiong (1995), Rose (1998), Huang and Chou (2005), Cheah and Liu (2005), Wibowo (2004), Chiara and et al (2007), Kim and et al (2012), Ashuri and et al (2012), Wibowo and et al (2012), Doan and Menyah (2013), Pellegrino and et al (2011), and Iyer and Sagheer (2011).

2.4. Summary of the Chapter

Options pricing theory is first reviewed in this chapter, then is real options pricing theory derived from the former. Literature on advantages of real options valuation compared to the traditional capital budgeting method, net present value is then reviewed. Applications of real options theory can be found in a variety of fields. This study concentrates on application of real options theory to value guarantees in infrastructure projects.

3. TOLL ADJUSTMENT MECHANISM

3.1. Mechanisms to Mitigate Traffic/Demand Risk

Failures of the transport projects due to mismanagement of traffic risks have made the design and implementation of a series of risk mitigation mechanisms imminent for the governments to attract potential private investors. Among various kinds of approaches, two main purposes are consistently sought for: first, to improve the comprehensiveness and the completeness of the concession contract in order to reduce the chance of renegotiations in the operational period; second, to establish a more fair framework for sharing risks and benefits between different parties involved, such as the concessionaire (concessionaire), the government (concessioner), and the users or taxpayers (society). Four most commonly used mechanisms to mitigate traffic risk in PPP infrastructure projects are described as follows: (1) contract renegotiation for economic balance; (2) traffic/revenue guarantees; and (3) concession-duration adjusted scheme; (4) toll/tariff adjustment mechanism.

The first approach aims to address the situation in which the project IRR does not reach the minimum IRR (Gomez-Ibanez and Meyer, 1993; Vassallo and Gallego, 2005). In some cases, a maximum IRR is presented besides a minimum IRR, which prevents the concessionaire from excessive profits in case that traffic is much higher than the expected level. In order to make the project financially feasible in case that the traffic is much lower than the expected level, compensation measures, e.g. public subsides, toll adjustments, and contract length contraction/extension, etc. should be negotiated with the government. These compensation measures are often not pre-established ex ante but rather negotiated ex post when the IRR does not reach the target levels. However, in the real practice, the negotiations may be very time consuming and tedious. Additionally, the concessionaire usually lacks the motivation to reduce the cost for that a lower project IRR will allow the concessionaire to renegotiate the contract with the government. All in all, this approach has the merit of increasing the private sector's interest to enter into the contract with the government, but may not necessarily yield social benefits when re-negotiations occur.

The second mechanism, often referred to as the "guarantee" approach, is that the government guarantees the minimum level of traffic or revenues by compensation in case that actual levels of traffic or revenues fail to achieve the minimum levels stipulated in the contract ex ante; This approach has been applied in many countries such as Korea, Malaysia, Thailand, Colombia, Chile, Mexico, Spain, etc.. Along with the minimum guarantee arrangement, the host government also usually requires sharing the extra revenue which is above the higher level specified in the contract with the concessionaire. The main disadvantage of the guarantee approach lies in the link between traffic/revenue and economic situation. If an economic recession occurs or in the case that the real traffic volume is much lower than the projected level which is quite common for large scale transport projects (Skamris and Flyvbjerg, 1997; Flyvbjerg, Skamris, and Buhl, 2005.), a phenomenon termed as "optimism bias in toll road traffic forecasts" by Bain (2009), the guarantee arrangement may induce very negative consequences for the public budget, such as heavy fiscal burdens, and even social turmoil. Nevertheless, this approach has been proved to be very effective in some stable countries such as Chile where, even during an economic recession, 25 out of 29 transport concessions in operation at the end of 2004 achieved higher than the minimum income guarantee band, which meant a subsidy from the government of only 6.24 US$ million compared to the 350 US$ millions investment (Vassallo and Solino, 2006). However, in some unstable countries like Colombia, where the recession occurs quite often, this approach will put a large strain on the government's fiscal position as it has to make a huge number of payment due to the traffic/revenue lower than the minimum level (Irwin et al, 1997).

The third approach, which was first used by the UK government in 1990, sets the concession length according to the target revenue (Foice, 1998). In the UK project, namely the Second Seven Crossing, the project duration was initially predicted as long as 30 years. With the duration matching approach, which calculates the concession duration based on the target revenue and the real traffic levels during the early stage of the concession, the ultimate duration was expected to be eight years less.

According to Lemos et al (2004), a similar approach was applied in Portugal in 1990s. The purpose of this concession agreement was to ensure that the concession would expire before March 2028 or the total cumulative traffic flow reached 2,250 million vehicles. Otherwise, the concession will end before 2028.

Based on the mechanism of the third approach, the "Least Present Value of the Revenues (LPVR)" was designed by Engel, Fischer and Galetovic (1997, 2001) to address the requirement of the Ministry of Public Work of Chile. Within this scheme, those offering the LPVR will secure the concession, which will end if Least Present Value of the Revenues has been obtained. During the concession period, if the actual traffic is lower than expected, the concession will extend longer until the LPVR is reached. However if the traffic is higher, the concession will end earlier. According to this arrangement, the risk from the instability of traffic volume has been shared among different stakeholders, i.e. the concessionaire, the users, and the government.

Although the LPVR arrangement was tried in Chile, the successful awarding rate is comparatively low; only 2 concessions out of 29 were implemented under this mechanism. According to Vassallo (2006), this arrangement was opposed by the concession companies because they bore most of the contract risk if the LPVR could not be reached before the end of the concession period. Additionally, the concession companies lack the incentive to end the contract early if the traffic volume was higher than expected.

Other countries are beginning to adopt this technique. Portuguese government implemented this mechanism to the “Litoral Centro” highway project at the end of 2004. This concession will last until the net present value (NPV) of the total revenue reaches € 784 million. The concession period ranges from 22 to 30 years. Colombian government implemented the so-called “third generation of concessions” in the Caribbean Road Network and other four projects, in which the concession would be rewarded to the bidder requiring the lowest accumulated revenues. Currently, the Colombia government is considering the introduction of LPVR for the development of the next generation of concessions.

The fourth method is the toll/tariff adjustment mechanism. According to Ye and Tiong (2003a), the design of tariff is a pivotal issue in the development of privately financed infrastructure projects, which involves the determination of tariff magnitude, the choice of tariff structure, and the design of adjustment mechanisms. A well-designed tariff adjustment framework can create a 'win–win' prospect for both the public and private sectors (Ye and Tiong, 2003b). According to Athias and Saussier (2007), the contracting parties do not only try to sign complete rigid contracts in order to avoid renegotiations but also flexible contracts in order to adapt contractual framework to unanticipated contingencies and to create incentives for cooperative behavior, which gives rise to multiple toll adjustment provisions and to a tradeoff between rigid and flexible contracts. Renegotiated toll adjustment mechanism can be found in the franchise contract of Eastern Harbour Crossing; while automated toll adjustment mechanisms can be found in those of Western Harbour Crossing, Hong Kong, the Highway 407 in Canada as well as the Rizhao Power Plant in Shandong, China (Tam, 1999).

The focus of this research is develop an automated toll adjustment mechanism under which toll adjustments are following the ex ante provisions, which are stipulated in the concession contracts rather than subject to renegotiations ex post,. Price-cap regulation, in essence, is an automated toll adjustment mechanism, in which operator's prices are adjusted according to the price cap index that reflects the overall rate of inflation in the economy, the ability of the operator to gain efficiencies relative to the average firm in the economy, and the inflation in the operator's input prices relative to the average firm in the economy (Albalate, Bel and Fageda, 2009). However, price-cap regulation can only hedge the inflation risk (and possibly currency risk) for the private investors, in that the price increment is commonly indexed to the inflation rate of the entire economy or a selected subset of it; in case of severe demand risk such like lower than expected traffic volume, the price increment through the price-cap regulation may not be sufficient to prevent the concessionaire from severe losses.

3.2. A General Model of Toll Adjustment Mechanism

A general model of Toll Adjustment Mechanism is built based on Zhao et al (2006). Let p_0 be the unit price of the project product (the average toll in the case of a toll road project); c_t the unit operation and maintenance cost in the operation period; n the concession duration (in years); m the granted number of toll adjustments stipulated in the concession contract, $m < n$, such adjustments should only be implemented when the revenue in the previous year is less than a pre-determined level R_t^{min} stipulated in the contact and the degree of increment should not exceed γ_i. For the simplification of demonstration without loss of generality, we suppose that the supply capacity of the project could always satisfy the demand; and the demand quantity in each year is represented in the model as a function of time and toll. Let $Q_t(p_t, t)$ denote the demand in the t-th year.

Under such toll adjustment mechanism, the concessionaire needs to determine the number of times of toll adjustments as well as when and to what extent to implement a toll adjustment. Thus we can write the theoretical model of such toll adjustment mechanism as:

$$\text{Objective function: } \max_{\substack{\{\zeta_t, k_t\} \\ t=1\ldots n}} NPV = \max_{\substack{\{\zeta_t, k_t\} \\ t=1\ldots n}} \sum_{t=1}^{n} disc^t Q_t(p_{t-} c_t)$$

$$\text{s.t.: } Q_t(p_t, t) = g(p_t) + f(t)$$

$$p_t = (1 + \zeta_t k_t) p_{t-1}$$

$$c_t = h(Q_t)$$

$$\zeta_t \in \{0, 1\} \text{ and } \zeta_t = 0, \text{if } Q_{t-1} p_{t-1} < R_{t-1}^{min}$$

$$\sum_{t=1}^{n} \zeta_t \leq m$$

$$k_t \leq \gamma_t$$

In this model, the objective function is to maximize NPV of the project; the first constraint states the mathematical demand function, where p_t is the unit price of the project product in the t-th year, $g(p_t)$ is the demand solely subject to the price, $f(t)$ is the demand solely subject to time, and there's no correlation between $g(p_t)$ and $f(t)$; the second constraint states the relationship between the unit price of the project product in the t-th year and that in the $(t-1)$th year, in which k_t represents the increment ratio of price in the t-th year; the third constraint states that the unit production cost is solely dependent on the traffic volume of the same period; in the fourth constraint, when $\zeta_t = 0$, it means that no toll adjustment is allowed in the t-th year, when $\zeta_t = 1$, it means that a toll adjustment is made in the t-th year; the fifth constraint sates that the total number of toll adjustments should not exceed the maximum number of increment times m; the sixth constraint states that the degree of increment should not exceed the maximum increment degree stipulated in the concession contract, γ.

Therefore the task faced by the concessionaire is to determine a toll adjustment strategy, that is, a series of optimum toll adjustment years strategy $\{\zeta_1^*, \zeta_2^*, \dots, \zeta_n^*\}$ and a correspondent series of optimum toll adjustment strategy $\{k_1^*, k_2^*, \dots, k_n^*\}$.

In the long duration of infrastructure concession, the dramatic change in the external economy would not be surprising to the project stakeholders. The toll adjustment mechanism proposed above can effectively resolve this conflict between such changes and fixed product prices. Under such a mechanism, through control of the maximum number of increment times m and the maximum increment degree γ, the host government can ensure, on the one hand, the social and economic benefits brought to the society, and on the other hand, a reasonable profit to the project investors, to achieve a win-win situation in the PPP project.

3.3. Toll Adjustment Mechanism as Real Options

A conventional real option is strictly correspondent to and homologous with a financial option, such as the analogue between a minimum revenue guarantee and a financial put option. In the case of a financial put option on a stock, the option holder has the right to sell the stock at price K at time t. Thus the option value is max[0, K-X], in which X is the actual price of the stock at time t; In the case of a minimum revenue guarantee, the real options value at time t is max[0, R_{min}-R], in which R_{min} is the minimum guaranteed level of revenue, if the actual revenue falls short of this level, the difference between R_{min} and R will be compensated to the option holder, that is, the concessionaire in a concession contract.

In both cases K and R_{min} are predetermined; the difference is that X is beyond the control of the option holder under the assumption that the financial market is too large to manipulate; while in fact R is in control of the concessionaire to a large extent, though facing a variety of uncertainties. However, this difference will not alter the pricing strategy of the concessionaire, be it single period or multi period investment decision making. The concessionaire will always seek to maximize the revenue or profit during each period and in adverse years when revenues fall short of the minimum levels, the real options then come to work.

Nevertheless, for a toll adjustment mechanism as aforementioned, to optimize the revenue (charging a high price) in each period is only a possible solution to the optimization problem in multi period decision making case. Charging a low price, which can induce the toll escalation for all the following periods while the cost of doing so is lower current revenue, is the other possible strategy. Therefore, in each period, the concessionaire has to weigh the two pricing strategies before making the pricing decision; a series of pricings in each period whose combination can maximize the net present value (NPV) of the project is the optimal pricing strategy for the concessionaire.

Therefore the right of toll adjustment, put it more accurately, the right to charge the toll either to initiate a toll adjustment or to maximize the revenue, is the managerial flexibility within such mechanism; that is to say, the reasoning of real options can be applied to assess the value of flexibility within toll adjustment mechanism. The real options value of the toll adjustment mechanism is then the difference between the net present value of the project under the optimal pricing strategy with the toll adjustment mechanism and the optimal net present value without it.

Due to the uniqueness and distinctness of the toll adjustment mechanism, dynamic programming method, which is usually employed to model and value real options, such as guarantees, is no longer applicable. The framework of assessing the real options value of the toll adjustment mechanism will be developed in the following chapters.

3.4. Summary of the Chapter

Chapter 3 introduces four mechanisms to mitigate the traffic/demand risk in the concession contracts and proposed a general mathematical model to elaborate the characteristics of the toll adjustment mechanism, which is the focus of this research. In Chapter 6 this model will be revisited.

4. A STOCHASTIC VARIANCE MODEL FOR TRAFFIC DEMAND

4.1. Introduction

For a Greenfield BOT project whose concession period is predetermined, usually as long as several decades, little or no historical data is available for the key risk variables, such as the traffic demand in a toll road project, which is determined by the very nature of a project, in that every project is unique in itself, as for a toll road project, it's location is unique.

Considering the uncertainty characteristics in concessions contracts, a stochastic variance model is developed to model the risk variables, based on the martingale variance model developed by Chiara and Garvin (2008). In real options practice, stochastic processes such as Geometric Brownian Motion, Ornstein–Uhlenbeck process, etc. are often employed to model the uncertainty. However, those will not apply for the risk variables in concession contracts in the sense that those stochastic processes need historical data to generate statistical parameters and they are only effective in relatively short lifespan.

4.2. Stochastic Model Features

In a concession contract with a lifespan of T years, if the expected traffic demand for each period is $\left\{\overline{Q_1}, \overline{Q_2}, \dots, \overline{Q_T}\right\}_{t=1}^{T}$. This vector can be generated through consulting traffic forecast experts. Now our task is to determine the variance of traffic demand in each period.

Markov chain

A Markov chain, named after Andrey Markov, is a mathematical system that undergoes transitions from one state to another, between a finite or countable number of possible states. It is a random process usually characterized as memorylessness: the next state depends only on the current state and not on the sequence of events that preceded it. This specific kind of "memorylessness" is called the Markov property.

For a toll road project, it is proper to model the annual traffic demand as a Markov Chain, since usually the traffic demand of two consecutive periods are dependent of each other. The traffic demand at time t depends only on that at time t-1. Formally, this process can be written in probabilistic terms as:

$$Pr\ (Q_t = q|Q_1 = q_1, Q_2 = q_2, \dots, Q_{t-1} = q_{t-1}) = Pr\ (Q_t = q|Q_{t-1} = q_{t-1})$$

and thus:

$$Pr(Q_1, Q_2, \dots, Q_t) = Pr(Q_t|Q_{t-1})P(Q_{t-1}|Q_{t-2}) \dots Pr(Q_2|Q_1)$$

Increasing uncertainty

It is straightforward to see that the traffic demand uncertainty increases monotonically with time, which is consistent with the common sense that what is to be known *m* units of time from now on is less uncertain than what is to be known *n* units of time from now on, in which *m* is less than *n.* This feature can be written in probabilistic terms as:

$$Var(Q_1) < Var(Q_2) < \cdots < Var(Q_T)$$

Increasing knowledge

Let I_t denote the information set relevant to time t, which contains all the information that the prediction of traffic demand at time t needs. New information is gained as time passes by, that is, $\{I_1, I_2, \dots, I_T | I_1 \subset I_2 \dots \subset I_T\}$. We term the information gained over time as knowledge if it can help making forecast in reducing uncertainty and therefore improving prediction, and the uncertainty of prediction in successive periods may decrease over time. For example, to forecast the traffic in year 2 based on knowledge gained only in year 1 must be more uncertain than to forecast the traffic in year 11 based on knowledge gained in year 10, which, as a matter of fact, incorporates all the information from year 1 to year 10. This feature can be written in probabilistic terms as:

$$Var(\Delta Q_1) \geq Var(\Delta Q_2) \geq \cdots \geq Var(\Delta Q_T)$$

which states that the variance of traffic increments decreases with time because of the increasing knowledge gained over time.

4.3. A Stochastic Variance Model for Traffic Demand

A discrete-time stochastic process model for traffic demand is developed as follows:

At time *t=0,* the information set I_0 includes: the duration of the operational period *T,* the expected traffic demand vector $\left\{\overline{Q_1}, \overline{Q_2}, \ldots, \overline{Q_T}\right\}_{t=1}^{T}$ and the initial status of the process, $Q_0 = \overline{Q_0}$.

If the traffic demand is represented by the discrete-time stochastic process $\{Q_t\}_{t=1}^{T}$, then we have,

$$Q_t = Q_0 + \sum_{j=1}^{t} \Delta Q_j$$

in which ΔQ_j is the annual traffic demand increment, $\Delta Q_j = Q_j - Q_{j-1}$.

We could represent $\{\Delta Q_j\}_{j=1}^{T}$ as a stochastic process:

$$\Delta Q_j = \Delta \overline{Q_j} + X_j$$

in which the random component $\{X_j\}_{j=1}^{T}$ can be modeled as a martingale process:

$$X_j = f(j)\varepsilon_j$$

in which $\{\varepsilon_j\}_{j=1}^{T}$ is an independently distributed random sequence with a mean of zero and a unit variance, and $f(j)$ is a function of time:

$$f(j) = \sigma \sqrt{\frac{1}{\sum_{i=1}^{j} \gamma^{i-1}}}$$

in which $\gamma \in [0,1]$, the coefficient of variance reduction, and $\sigma^2 = Var(Q_1) = Var(\Delta Q_1)$. Thus we have:

$$\begin{cases} X_0 = 0 \\ X_1 = f(1)\varepsilon_1 = \sigma\varepsilon_1 \\ X_2 = f(2)\varepsilon_2 = \sigma\sqrt{\dfrac{1}{1+\gamma}}\varepsilon_1 \\ \cdots \\ X_T = f(T)\varepsilon_T = \sigma\sqrt{\dfrac{1}{1+\gamma+\gamma^2+\cdots+\gamma^{T-1}}}\varepsilon_1 \end{cases}$$

From the above equations, we have

$$E[X_T] = E[X_{T-1}] = \cdots = E[X_1] = E[X_0] = 0$$

which conforms to the martingale process assumption.

Furthermore, the variance of the martingale process is given by:

$$\begin{cases} Var(X_0) = 0 \\ Var(X_1) = \sigma^2 \\ Var(X_2) = \sigma^2 \dfrac{1}{1+\gamma} \\ \cdots \\ Var(X_T) = \sigma^2 \dfrac{1}{1+\gamma+\gamma^2+\cdots+\gamma^{T-1}} \end{cases}$$

Thus, the equivalent representation of $\mathrm{Var}(X_j)$, for $\gamma \in [0,1]$, is given by:

$$Var(X_j) = \frac{1-\gamma}{1-\gamma^j}\sigma^2$$

Therefore the martingale process has an expected value and variance:

$$E[X_j] = 0 \text{ and } Var(X_j) = \tfrac{1-\gamma}{1-\gamma^j}\sigma^2$$

Therefore the expected value and variance of the stochastic process of the traffic demand increments are, respectively:

$$E[\Delta Q_j] = \Delta\overline{Q_j} \text{ and } Var(\Delta Q_j) = \tfrac{1-\gamma}{1-\gamma^j}\sigma^2$$

Observing the above equations, we have that the variance of the process $\{\Delta Q_j\}_{j=1}^{T}$ is decreasing with time, and it approaches the long-term annual variance $\mathrm{Var}(\Delta Q_T)$:

$$Var(\Delta Q_T) = (1-\gamma)\sigma^2$$

in which γ, the reduction coefficient, indicates the percentage variance reduction from the initial annual variance, $Var(\Delta Q_1) = \sigma^2$.

Thus this stochastic variance model retains the features as follows:

1. $\Delta Q_j = \Delta\overline{Q_j} + X_j$ and $X_j = f(j)\varepsilon_j$ indicates that this is a Markov Chain;

2. The variance of the process, $\mathrm{Var}(Q_j)$, is monotonically increasing with time:

$$Var(Q_j) = \sum_{i=1}^{j} Var(\Delta Q_i)$$

3. If the reduction coefficient is zero, $\gamma = 0$, then

$$Var(\Delta Q_T) = Var(\Delta Q_{T-1}) = \ldots = Var(\Delta Q_1) = \sigma^2$$

and

$$Var(Q_j) = \sigma^2 t$$

which indicates that the diffusion process characterized by $Var(Q_j) = \sigma^2 t$ is a process that shows no increasing knowledge over time.

4.4. Summary of the Chapter

A stochastic variance model is developed to reflect the uncertainty features, e.g., Markov chain, increasing uncertainty and increasing knowledge, of a Greenfield BOT project in which little or no historical data is available. The results from this model will be used as inputs to the model in Chapter 5.

5. TWO-ROUTE CHOICE MODEL WITH HETEROGENEOUS VALUES-OF-TIME

5.1. Model Set-up

A population of Q commuters traveling from city A to city B faces two options: one route is a toll road which charges each user a toll of P (unit: dpt, dollars per trip) for per usage of it and the other a free road. Q_T^{max} and Q_F^{max} are respectively the maximum traffic capacity of the toll road and that of the free road (unit: vpd, vehicles per day); Q_T and Q_F are respectively the traffic volume commuters on the toll road and that on the free road (unit: vpd); t_T^0 and t_F^0 are respectively the free-flow (no congestion) travel time per vehicle on the toll road and that on the free road (minutes); t_T and t_F are respectively the actual (with congestion) travel time per vehicle on the toll road and that on the free road (minutes). t_T is an increasing and non-concave function of Q_T, where both t_T and t_T' are continous; Similarly, t_F is an increasing and non-concave function of Q_F , where both t_F and t_F' are continous. tc_T and tc_F are respectively the travel cost per vehicle usage of the toll road and that of the free road (dollars). This situation is illustrated in Figure 5-1:

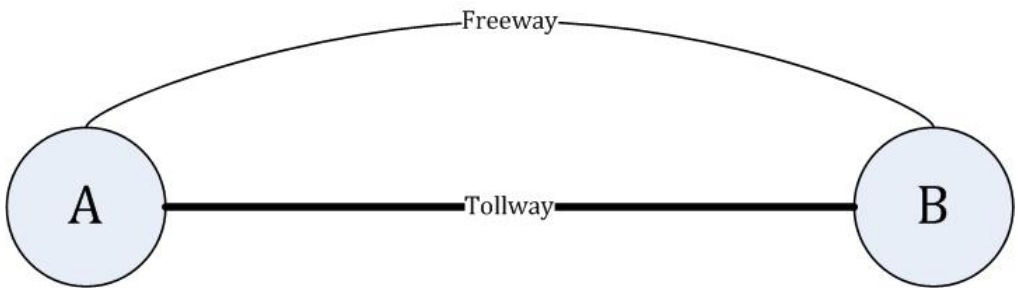

Figure 5-1 Model Set-up

Each commuter has a unique Value of Time (VOT; unit: dpm, dollars per minute). Let $f(x)$ and F(x) denote respectively the probability distribution function (PDF) and cumulative distribution function (CDF) of VOT for the whole population of commuters, and x_0 the VOT of the commuter who is indifferent in choosing the toll road or the free road, that is, travel cost for him/her is the same for the two routes. That is to say, for the commuter whose VOT is x_0, $tc_T = tc_F$, that is, $P + t_T x_0 = t_F x_0$.

5.2. Model Solution

It is assumed that:

1. All the commuters are completely rational decision makers, seeking to minimize his or her own traveling cost;
2. Each commuter knows his/her own unique VOT as well as the

distribution of VOT for the whole population of commuters;

3. The same toll is charged for all the vehicles regardless of the vehicle types.

Traffic assignment may be viewed as a non-cooperative, n-player game, where each network user endeavors to reach his destination by the best route possible which can minimize his or her travel cost. This is an n-player game in the game theory sense because the payoff (the negative of trip cost) to each player (each network user) depends on the actions of the other players.

Proposition 5-1: For any commuter whose VOT is less than x_0, he or she will definitely travel on the free road; for any commuter whose VOT is larger than x_0, he or she will definitely travel on the toll road.

Proof: Given $x_1 < x_0$, then $tc_T - tc_F = P + t_T x_1 - t_F x_1 = P + (t_T - t_F)x_1 = P - \frac{P}{x_0}x_1 = P\left(1 - \frac{x_1}{x_0}\right) > 0$, therefore $tc_T > tc_F$, thus the commuter whose VOT $x_1 < x_0$, he or she will choose the route less costly, that is, the free road in this case; Similarly, when $x_2 < x_0$, the commuter will choose the toll road.

From Proposition 5-1, we have,

$$Q_F = Q \int_0^{x_0} f(x)dx$$

that is,

$$\frac{Q_F}{Q} = F(x_0)$$

in which,

$$x_0 = \frac{P}{t_F - t_T} > 0$$

Attention should be paid to the intuitive but plain fact that only if travelling on the toll road can save time compared to on the free road, will commuters use the toll road. Otherwise, only by subsidizing those who use the toll road will attract commuters from the free road.

From the equation

$$\frac{Q_F}{Q} = F(x_0) = F\left(\frac{P}{t_F(Q_F) - t_T(Q - Q_F)}\right)$$

the number of commuters who use the free road given the distribution of VOT and toll P can be determined; Thus the number who use the toll road can therefore be determined.

It is assumed that free-flow travel time, that is, travel time with no congestion, on both routes, is solely dependent on the length of each route. If one route directly links A and B while the other indirectly, the free-flow time on the former route should be less than that of the latter. Generally speaking, as aforementioned, it takes less time to travel on the toll road than on the free road and that's the very reason commuters pay for its service. However, due to congestion, that is, when the number of actual users on the route exceeds its maximum design capacity, actual travel time can be greatly prolonged. Therefore, actual travel time on a certain route not only depends on the length of it, but also the maximum design capacity of it. Here we adopt the link (arc) congestion (or volume-delay, or link performance) function developed by The Bureau of Public Roads (BPR) of the US, which we will term as $S_a(v_a)$:

$$S_a(v_a) = t_a\left(1 + 0.15\left(\frac{v_a}{c_a}\right)^4\right)$$

in which,

t_a = free flow travel time on link a

v_a = volume of traffic on link a per unit of time (somewhat more accurately: flow attempting to use link a)

c_a = capacity of link a per unit of time

$S_a(v_a)$ = the average travel time for a vehicle on link a

Thus we have,

$$t_T = t_T^0\left(1 + 0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right) \text{ and } t_F = t_F^0\left(1 + 0.15\left(\frac{Q_F}{Q_F^{max}}\right)^4\right)$$

To obtain an analytical solution, we, herein, for the simplicity of illustration while without loss of generality, assume that VOT of the whole population conforms to a uniform distribution with lower bound a and upper bound b.

From equations as follows,

$$t_T^0\left(1+0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right)x_0+P=t_F^0\left(1+0.15\left(\frac{Q_F}{Q_F^{max}}\right)^4\right)x_0$$

$$Q_T+Q_F=Q$$

$$\frac{x_0-a}{b-a}=\frac{Q_F}{Q}$$

We could arrange P as a function of Q_T

$P=P(Q_T)$

$$=\left((t_F^0-t_T^0)+0.15\left(t_F^0\left(\frac{Q-Q_T}{Q_F^{max}}\right)^4-t_T^0\left(\frac{Q_T}{Q_T^{max}}\right)^4\right)\right)\left(\frac{Q-Q_T}{Q}b+\frac{Q_T}{Q}a\right)$$

Mapping P to Q_T, we could get

$$Q_T=P^{-1}(P)$$

Thus, we have, for the toll road owner, the daily revenue is

$$R=R(Q_T)=PQ_T$$

Let TT and TC denote the overall travel time and travel cost of the whole population of commuters, we have

$$TT = TT(Q_T) = \left(1 + 0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right) t_T^0 Q_T$$

$$+ \left(1 + 0.15\left(\frac{Q - Q_T}{Q_F^{max}}\right)^4\right) t_F^0 (Q - Q_T)$$

and

$$TC = TC(Q_T) = TC_T + TC_F$$

$$= Q_T P + \left(1 + 0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right) t_T^0 Q_T \int_{x_0}^{b} \frac{x}{b - x_0} dx$$

$$+ \left(1 + 0.15\left(\frac{Q - Q_T}{Q_F^{max}}\right)^4\right) t_F^0 (Q - Q_T) \int_{a}^{x_0} \frac{x}{x_0 - a} dx$$

$$= Q_T P + \frac{1}{2}\left(1 + 0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right) t_T^0 Q_T (b + x_0)$$

$$+ \frac{1}{2}\left(1 + 0.15\left(\frac{Q - Q_T}{Q_F^{max}}\right)^4\right) t_F^0 (Q - Q_T)(a + x_0)$$

$$= Q_T P + \frac{1}{2}\left(1 + 0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right)\left(\frac{2Q - Q_T}{Q} b + \frac{Q_T}{Q} a\right) Q_T t_T^0$$

$$+ \frac{1}{2}\left(1 + 0.15\left(\frac{Q - Q_T}{Q_F^{max}}\right)^4\right)\left(\frac{Q - Q_T}{Q} b + \frac{Q + Q_T}{Q} a\right)(Q$$

However, in more general cases, there're no analytical solutions to the problem. For instance, if VOT conforms to a normal distribution with mean μ and standard variance σ, we have

$$Q_T = Q\int_{x_0}^{+\infty} f(x)dx$$

Thus

$$\frac{Q_T}{Q} = \int_{x_0}^{+\infty} f(x)dx = \int_{\frac{P}{t_F - t_T}}^{+\infty} f(x)dx$$

$$= \int_{\frac{P}{t_F^0\left(1+0.15\left(\frac{Q-Q_T}{Q_F^{max}}\right)^4\right) - t_T^0\left(1+0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right)}}^{+\infty} f(x)dx$$

$$= \int_{\frac{P}{t_F^0\left(1+0.15\left(\frac{Q-Q_T}{Q_F^{max}}\right)^4\right) - t_T^0\left(1+0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right)}}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

that is

$$\frac{Q_T}{Q}$$

$$= \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\frac{P}{t_F^0\left(1+0.15\left(\frac{Q-Q_T}{Q_F^{max}}\right)^4\right) - t_T^0\left(1+0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right)} - \mu}{\sqrt{2\sigma^2}}\right)\right]$$

Therefore Q_T can be determined from the above equation, which is not analytically solvable; denote $P = P(Q_T)$, thus $Q_T = P^{-1}(P)$.

Thus, we have, for the toll road owner, the daily revenue is

$$R = R(Q_T)$$

$$= \frac{1}{2}\, PQ \left[1 - \operatorname{erf}\left(\frac{\dfrac{P}{t_F^0\left(1 + 0.15\left(\dfrac{Q - Q_T}{Q_F^{max}}\right)^4\right) - t_T^0\left(1 + 0.15\left(\dfrac{Q_T}{Q_T^{max}}\right)^4\right)} - \mu}{\sqrt{2\sigma^2}} \right) \right]$$

The overall travel time for the whole population of commuters is

$$TT = TT(Q_T) = \left(1 + 0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right) t_T^0 Q_T$$

$$+ \left(1 + 0.15\left(\frac{Q - Q_T}{Q_F^{max}}\right)^4\right) t_F^0 (Q - Q_T)$$

The overall travel cost for the whole population of commuters is

$$TC = TC_T + TC_F$$

$$= Q_T P + \left(1 + 0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right) t_T^0 Q_T \int_{x_0}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} x dx$$

$$+ \left(1 + 0.15\left(\frac{Q - Q_T}{Q_F^{max}}\right)^4\right) t_F^0 (Q - Q_T) \int_{-\infty}^{x_0} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} x dx$$

Arrange the above equation we have

$$TC = TC(Q_T)$$

$$= \left(\frac{1}{2}\mu\left(1+\operatorname{erf}\left(\frac{x_0-\mu}{\sqrt{2\sigma^2}}\right)\right) - \frac{\sigma}{\sqrt{2\pi}}e^{-\frac{(x_0-\mu)^2}{2\sigma^2}}\right)\left(1+0.15\left(\frac{Q-Q_T}{Q_F^{max}}\right)^4\right)Qt_F^0$$

$$+\left(\frac{1}{2}\mu\left(1-\operatorname{erf}\left(\frac{x_0-\mu}{\sqrt{2\sigma^2}}\right)\right) + \frac{\sigma}{\sqrt{2\pi}}e^{-\frac{(x_0-\mu)^2}{2\sigma^2}}\right)\left(1+0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right)Qt_T^0$$

$$+\frac{1}{2}PQ\left(1-\operatorname{erf}\left(\frac{x_0-\mu}{\sqrt{2\sigma^2}}\right)\right)$$

in which $x_0 = \frac{P}{t_F^0\left(1+0.15\left(\frac{Q-Q_T}{Q_F^{max}}\right)^4\right) - t_T^0\left(1+0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right)}$

For $\forall\, Q_T \in \{0, 1, 2, \dots, Q\}$, that is, Q_T is an integer between 0 and Q, there exists a series of corresponding $P = P(Q_T)$, $R = R(Q_T)$, $TT = TT(Q_T)$ as well as $TC = TC(Q_T)$. Exhausting the domain of Q_T from 0 to Q can generate a $(Q+1)$ dimensional array of P_k, R_k, TT_k, and TC_k, $k = 1, 2, \dots, Q+1$.

If the toll road is owned by a private investor under a concession contract, such like a BOT scheme, since the concessionaire seeks to optimize the profit, the optimal pricing strategy therefore is $\{P_j | R_j > R_i, \forall\, i \in k \; and \; i \neq j\}$; If the toll road is owned by a public agent, usually the government, then the optimal pricing strategy is either $\{P_j | TT_j < TT_i, \forall\, i \in k \; and \; i \neq j\}$ or $\{P_j | TC_j < TC_i, \forall\, i \in k \; and \; i \neq j\}$, since the government usually seeks to maximize the social benefit, either in the form of minimization of total travel time of the society, which is the former case, or of minimization of total travel cost of the society, which is the latter case.

5.3. Solution Algorithm

Step 1 Let $Q_T = 0$

Step 2 Calculate x_0 from $Q_T = Q\int_{x_0}^{+\infty} f(x)dx$

Step 3 Calculate P from $P = \frac{x_0}{t_F^0\left(1+0.15\left(\frac{Q-Q_T}{Q_F^{max}}\right)^4\right)-t_T^0\left(1+0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right)}$

Step 4 Calculate R from $R = PQ_T$

Step 5 Calculate TT from

$$TT = TT(Q_T) = \left(1 + 0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right)t_T^0 Q_T + \left(1 + 0.15\left(\frac{Q - Q_T}{Q_F^{max}}\right)^4\right)t_F^0(Q - Q_T)$$

Step 5 Calculate TC from

$$TC = TC(Q_T) = R + \left(1 + 0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right)t_T^0 Q_T \int_{x_0}^{+\infty} f(x)xdx + \left(1 + 0.15\left(\frac{Q - Q_T}{Q_F^{max}}\right)^4\right)t_F^0(Q - Q_T)\int_{-\infty}^{x_0} f(x)xdx$$

Step 6 Repeat this process for $Q_T = 1$ to Q, calculate and document the matrix $\begin{bmatrix} P_1 & R_1 & TT_1 & TC_1 \\ P_2 & R_2 & TT_2 & TC_2 \\ & & \dots & \\ P_{Q+1} & R_{Q+1} & TT_{Q+1} & TC_{Q+1} \end{bmatrix}_{(Q+1)\times 4}$ in each iteration

Step 7 for a private toll road company, find the optimal toll at P which maximize R; while for a public agency in charge of the toll road, the efficient toll is P which minimizes TT or TC

5.4. Numerical Examples

Suppose that a group of commuters need to travel from city A to city B, connected by a toll road and a free road. Travel time on the two motorways in case of free traffic flow and probability distribution of VOT of the commuters are public knowledge. In case the actual traffic flow on a certain motorway exceeds the designed capacity, congestion is induced, and thus, travel time lengthens according the degree of congestion. In real life, this relationship is often exponential than linear.

In the numerical example, to get an insightful and comprehensive understand of the model and the route-choice behavior of commuters, two scenarios are taken into consideration respectively for motorway capacity (toll road/free road has a higher capacity) and free-flow travel time (toll road/free road has a shorter free-flow travel time). Thus four scenarios will be demonstrated and analyzed as follows.

The first example is when VOT conforms to a uniform distribution; parameters are given in Table 5-1. Applying the aforementioned algorithm, the results are shown in Figure 5-2 to Figure 5-5:

Table 5-1 Two-route choice when VOT is uniformly distributed

Scenarios \ Parameters	Q	a	b	Q_F^{max}	Q_T^{max}	t_F^0	t_T^0
1	1000	2	6	400	600	4	2
2				400	200	4	2
3				400	600	4	6
4				400	200	4	6

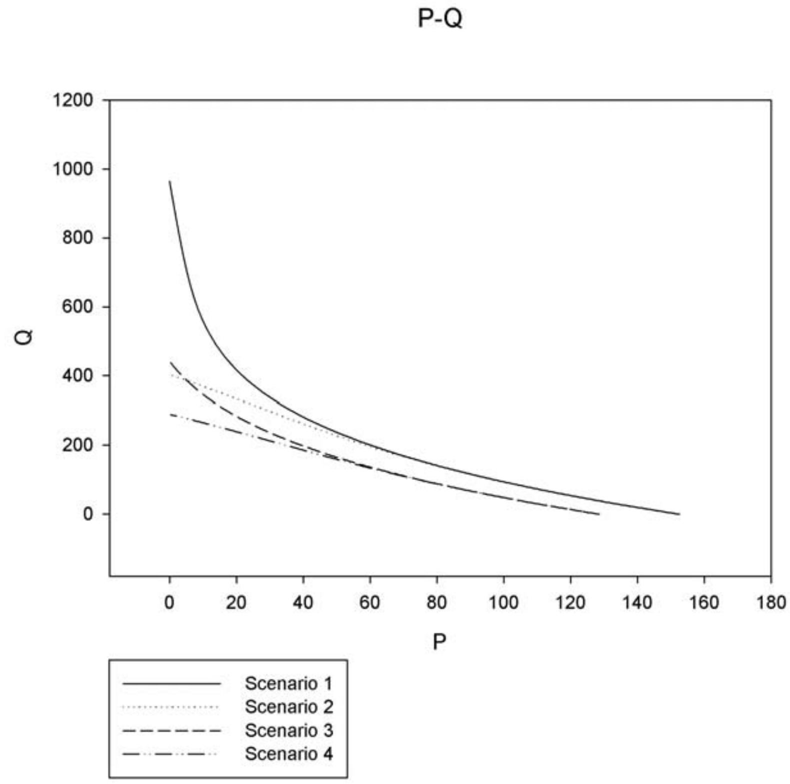

Figure 5-2 P-Q when VOT is uniformly distributed

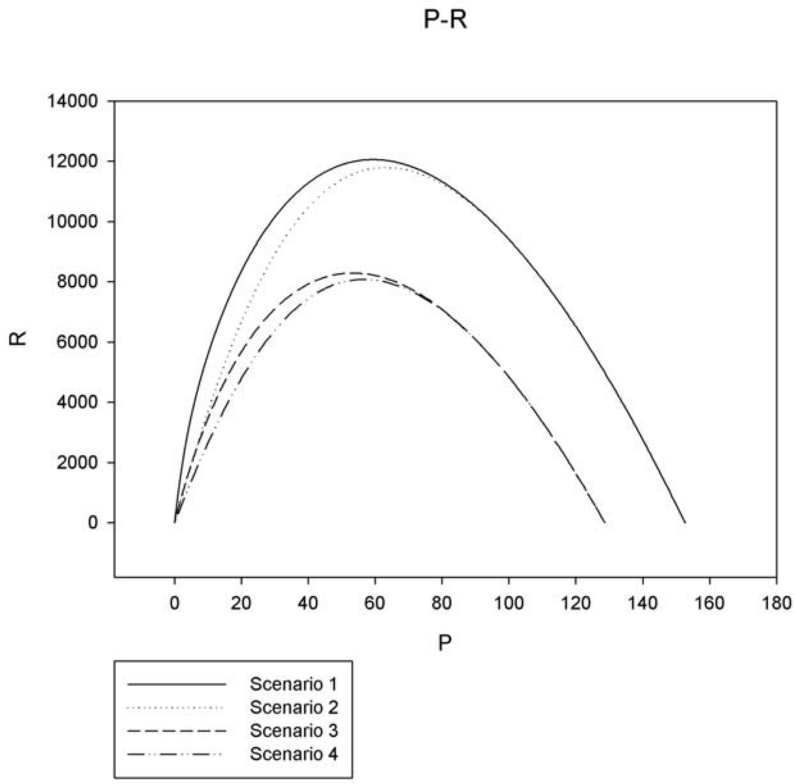

Figure 5-3 P-R when VOT is uniformly distributed

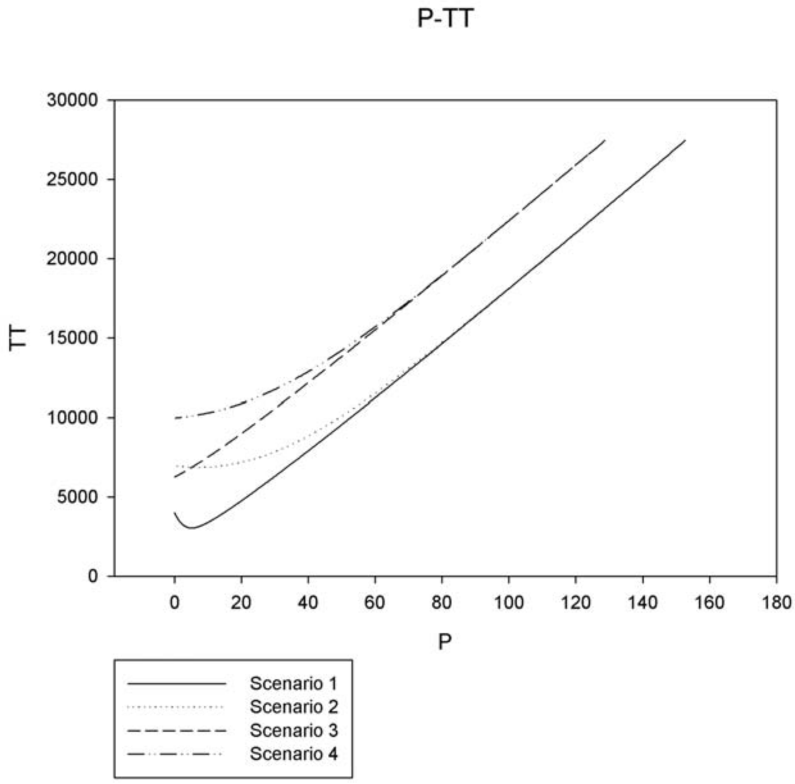

Figure 5-4 P-TT when VOT is uniformly distributed

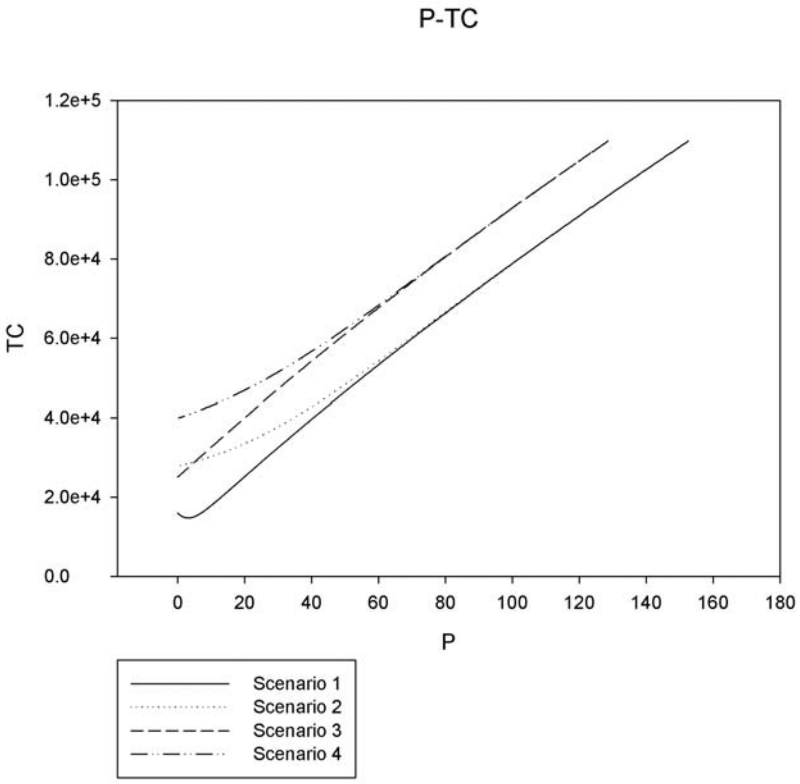

Figure 5-5 P-TC when VOT is uniformly distributed

The summary of the price corresponding to different objectives (maximization of the revenues for the toll road owner; minimization of social travel time; minimization of social travel cost) in each scenario when VOT is uniformly distributed is demonstrated in Table 5-2:

Table 5-2 Prices to different objectives with uniformaly distributed VOT

	Max R				Min TT				Min TC			
	P	R	TT	TC	P	R	TT	TC	P	R	TT	TC
S1	59.4	12062	11132	52883	5.08	3588	3045.5	15105	3.17	2516	3151	14753
S2	63.1	11795	11994	56118	7.25	2748	6872.8	29478	0.14	55.8	6964	27899
S3	53.5	8293	14408	63319	0.03	13.4	6269	25086	0.03	13.4	6269	25086
S4	56.9	8078	15216	66384	0.32	92	9971	39947	0.32	92	9971	39947

The second example is when VOT conforms to a normal distribution; parameters are given in Table 5-3. Applying the aforementioned algorithm, the results are shown in Figure 5-6 to Figure 5-9.

Table 5-3 Two-route choice when VOT is normally distributed

Scenarios \ Parameters	Q	μ	σ	Q_F^{max}	Q_T^{max}	t_F^0	t_T^0
1	1000	5	1	400	600	4	2
2				400	200	4	2
3				400	600	4	6
4				400	200	4	6

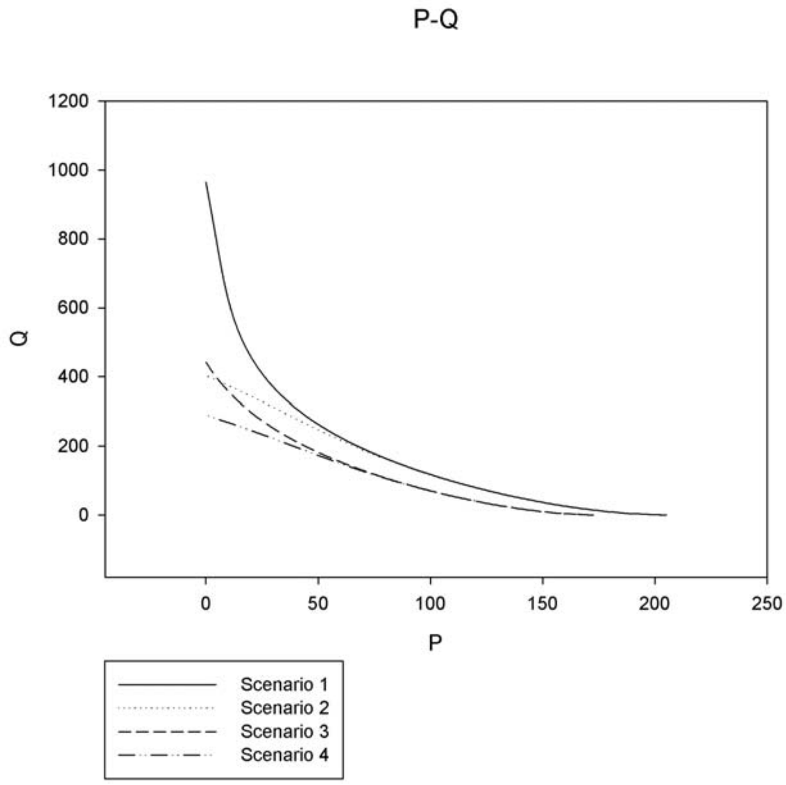

Figure 5-6 P-Q when VOT is normally distributed

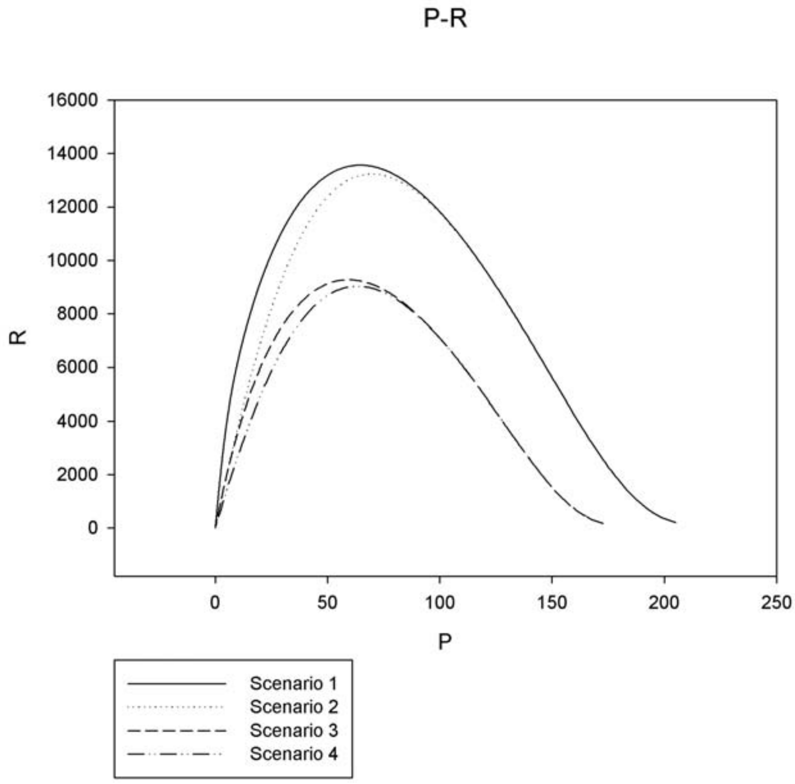

Figure 5-7 P-R when VOT is normally distributed

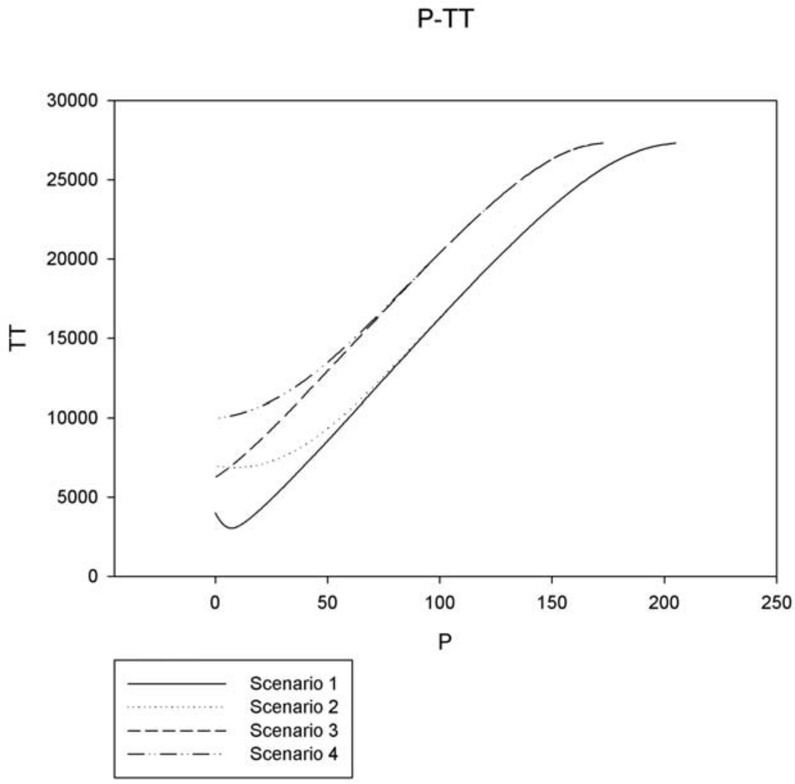

Figure 5-8 P-TT when VOT is normally distributed

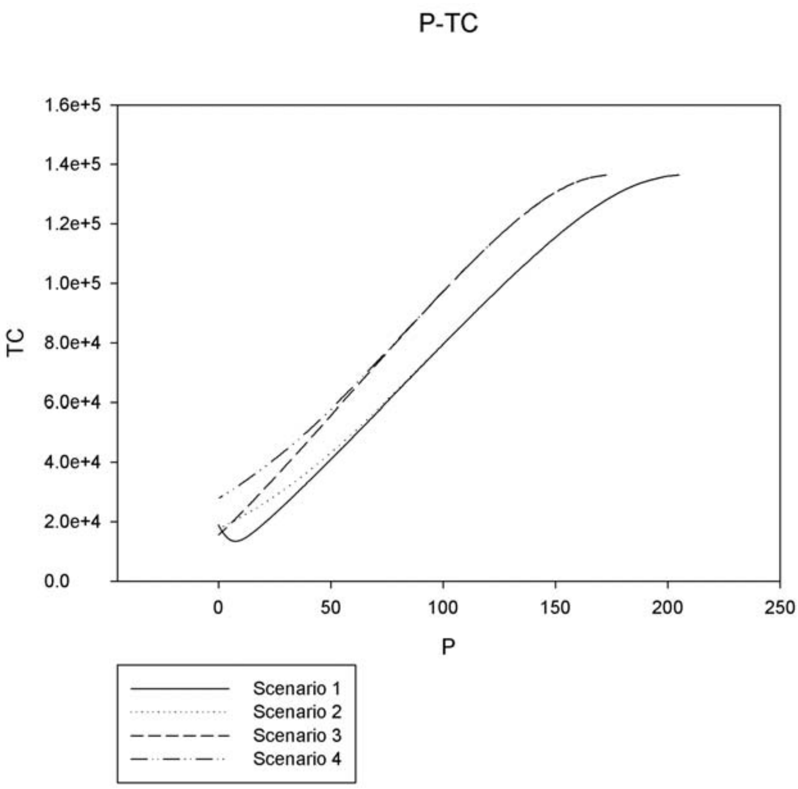

Figure 5-9 P-TC when VOT is normally distributed

Similarly, Table 5-4 can be presented as follows:

Table 5-4 Prices to different objectives with normally distributed VOT

	Max R				Min TT				Min TC			
	P	R	TT	TC	P	R	TT	TC	P	R	TT	TC
S1	64.6	13565	10793	52107	7.13	5036	3045.5	13462	7.56	5230	3048	13447
S2	69.6	13228	11839	56871	8.58	3253	6872.8	20979	0.17	66.75	6965	17612
S3	59.4	9273	14350	63622	0.04	16.28	6269	15612	0.04	16.28	6269	15612
S4	63.1	9029	15154	67763	0.36	105.4	9971	28052	0.36	105.4	9971	28052

5.5. Discussions on the Results

From Figure 5-2 and Figure 5-6, we could see that the traffic demand on the toll road is indeed a non-concave function of the toll charged; From Figure 5-3 and Figure 5-7, we could see that there is a toll which can maximize the revenue of the toll road company; From Figure 5-3 and Figure 5-7, we could see that in some cases (Scenario 1 and 2), a free charge of toll may not bring the maximization of social benefit (minimization of total travel time) for all the commuters, rather a higher toll can (5.08 dpt and 7.25 dpt in Table 5-2; 7.13 dpt and 8.58 dpt in Table 5-4); From Figure 5-4 and Figure 5-8, we could see that in some cases (Scenario 1), a free charge of toll may not bring the maximization of social benefit (minimization of total travel cost) for all the commuters, rather a higher toll can (3.17 dpt in Table 5-2; 7.56 dpt in Table 5-4). Thus for Scenario 2, if the toll road is a public property, the government has to choose between the two criteria to maximize the social benefit, which are, respectively, to minimize total travel time, and to minimize total travel cost; in the former case, the government sets the price at 7.25 dpt (Table 5-2) and 8.58 dpt (Table 5-4); while in the latter, free of charge.

For a toll beyond a certain level large enough (about 60 dpt), Scenario 1 and 2 as well as Scenario 3 and 4 will converge to each other. That is to say, the impact of shorter travel time is more significant than that of a larger capacity. For travel time on the toll road is either 2 or 6 minutes per trip, no matter the toll road capacity is either 200 or 600 vehicles per day, as long as the toll charged is beyond the level of 60 dpt, the toll road can surely attract the same traffic volume, thus the same revenue. If the toll road is owned by the government, since the government seeks to maximize the overall social benefit for all the commuters, the toll charged is far less than 60 dpt (in fact less than 10 dpt in all cases). That is to say, this phenomenon of convergence has little impact to do with the project if the owner is the government.

However, this has a huge impact on the concessionaire's investment behavior if the toll road is owned by a private investor. If in a BOT contract, it is the toll road company rather than the government to deicide the road capacity, and the toll road company will certainly choose a lower capacity to reduce its fixed investment cost, at the same time enjoy the same level of revenue if the road capacity were much larger. This will bring no efficiency loss to the commuters and the society if there are no price caps set by the government; in this situation, a lower capacity is a win-win choice for both the private and public sectors. That is to say, with no price regulations, a larger capacity may not be able to attract more commuters, because of the high travel cost on the toll road. However, if, to prevent the concessionaire from earning excessive profits and to protect the benefit of the public, the government set price caps which are lower than 60 dpt, at which the concessionaire can maximize its profit, say, at, say, 20, 30 or 40 dpt, which means that the toll charged by the concessionaire cannot exceed those levels, a lower capacity will be a lose choice for the public sector in that both the total travel time and the total travel cost are significantly larger than the case with a higher capacity; for the private sector, it may or may not be an lose choice, up to the relationship between the extent of extra revenue and the higher fixed cost brought by the higher capacity. Thus in this case, the government usually sets the desired capacity level and writes it explicitly in the documents inviting for tenders. During the negotiation period, the investor and the government discuss on the level of price caps, which should, on the one hand, make the project reasonably profitable for the investor and on the other hand, prevent the company from excessive profits.

5.6. Shadow Tolling Mechanism

First proposed by the UK Government in 1993, shadow tolls have been widely used in the UK and also in other countries, including Belgium, Canada, Finland, Netherlands, Spain and the United States, although to a more limited extent. A shadow toll is a contractual payment made by the government on behalf the commuter using the toll road to a private company that operates it in a public-private partnership concession contract such like BOT contract. Payments are based, at least in part, on the number of vehicles using the toll road, often spanning over a period of 20 to 30 years. The shadow tolls or per vehicle fees are paid directly to the company by the government without intervention or direct payment from the road users.

If shadow-tolling is introduced, for the commuters the toll road is actually free too. Then the problem is indeed a classic two-route choice problem.

The equilibrium traffic assignment will be reached when the time, therefore the cost, traveling on the two routes are the same, that is

$$t_T = t_T^0\left(1 + 0.15\left(\frac{Q_T}{Q_T^{max}}\right)^4\right) = t_F = t_F^0\left(1 + 0.15\left(\frac{Q_F}{Q_F^{max}}\right)^4\right)$$

$$Q_T + Q_F = Q$$

Then both Q_T and Q_F can be sovled.

In the Scenario 1 of the first numerical example, under a shadow tolling mechanism, there are 964 vehicles using the toll road and 36 vehicles the free road. If the government allows the concessionaire to earn a profit as high as the maximum level it could earn under a user-pay mechanism (rather than a shadow tolling, which indeed is a taxpayer-pay mechanism), which is 12062 dpd (dollars per day) when the toll is 59.4 dpt (total travel time 11132 minutes, total travel cost 52883 dpd), then the price should be set at 12.5 dpt (12062 dpd divided by 964 vehicles); in this case the total travel time (actually, regardless of the probability distribution of VOT) is 3999 minutes (which indicates that for one commuter, travelling on either the toll road or free road will cost him or her 4 minutes) and the total travel cost is 28058 dpd (total travel time, 3999 minutes, times the average VOT for the commuters, 4 dpt, plus the revenue, 12062 dpd).

For the two aforementioned numerical examples, the results of all scenarios under shadow tolling mechanisms are shown respectively in Table 5-5 and Table 5-6, with comparisons to user-pay contracts (when in both contracts the revenue is the maximum level that the concessionaire can earn under a user-pay mechanism).

Table 5-5 Taxpayer-pay v.s. user-pay with uniformly distributed VOT

	User-Pay				Taxpayer-Pay (Shadow Tolling)			
	P	Q_T	TT	TC	P	Q_T	TT	TC
S1	59.4	203	11132	52883	12.5	964	3999	28058
S2	63.1	187	11994	56118	29.3	403	6965	39655
S3	53.5	155	14408	63319	18.8	442	6269	33369
S4	56.9	142	15216	66384	27.9	290	9962	47926

Table 5-6 Taxpayer-pay v.s. user-pay with normally distributed VOT

	User-Pay				Taxpayer-Pay (Shadow Tolling)			
	P	Q_T	TT	TC	P	Q_T	TT	TC
S1	64.6	210	10793	52107	14.1	964	3999	33560
S2	69.6	109	11839	56871	32.8	403	6965	48053
S3	59.4	156	14350	63622	21.0	442	6269	40618
S4	63.1	143	15154	67763	31.1	290	9962	58839

From Table 5-5 and Table 5-6 we can see that, compared to the user-pay mechanism, the shadow tolling (taxpayer-pay mechanism) is superior in that, without even the slightest sacrifice of the concessionaire's benefit (enjoying the maximum revenue possibly in a user-pay contract), both the total travel time and total travel cost are reduced to a large extent (ranging from nearly 70% to over 30%).

Nevertheless, a taxpayer-pay mechanism such as shadow tolling may raise a very controversial issue which is absent for a user-pay contract. When most of the taxpayers are the users themselves who will use the toll road, the difference between user-pay contracts and taxpayer-pay contracts are just whether they pay directly to the concessionaire in the form of tolls or to the government in the form of taxes; However, if most of the taxpayers are not the users who will use the toll road, for instance, most of the taxpayers are not private car owners and therefore rely on public transport for commuting between city A and B, it is not fair for them since they are not users of the toll road at all but they have to pay for use of the toll road in form of taxes for others who use it. Someone may justify the legitimacy of shadow tolling even in the case that only a fraction of taxpayers are users of the toll road through citing examples such as taxation and transfer payment by the government. However, it should be noticed in the case of income tax, it is the rich who will subsidize the poor; while in the case of shadow tolling, it is the very opposite practice, the poor who cannot afford private cars will subsidize the rich who rely on their own cars for transport between city A and B. It's a dilemma of fairness and efficiency: from the perspective of maximization of social benefit, that is, efficiency, shadow tolling is superior; however, taking fairness into consideration, the principle of user-pay seems more appropriate.

5.7. Summary of the Chapter

Chapter 5 presents a two-route choice (traffic assignment) model of commuters with heterogeneous values-of-time. A theoretical model is built and both analytical solution and numerical solution are provided. Two numerical examples are presented to demonstrate the model and results are analyzed in details. Shadowing tolling as another option is discussed, with comparison to the user-pay mechanism.

6. REAL OPTIONS MODEL OF TOLL ADJUSTMENT MECHANISM

6.1. Contract Features

In a BOT toll road project, the concessionaire is granted the right to finance, build, operate and maintain the toll road; the revenues collected will be used to recoup the loans and investment costs. The concession period is N years. At the end of the concession period, the project will be transferred back to the government at no cost. The contract contains a toll adjustment mechanism, which is characterized by features as follows:

Minimum revenues

There are minimum levels of revenues specified explicitly for each year in the concession contract. If in any year, the actual revenue falls short of the the minimum level, the concessionaire will be entitled the right to adjust the toll according to the maximum toll increment level stipulated in the contract.

Revenue caps and sharing

There are maximum levels of revenues specified explicitly for each year in the concession contract. If in any year, the actual revenue exceeds the maximum level, the revenue excessive of the maximum level will go to the government; for the concessionaire, the revenue will be capped at the level of maximum revenue.

Price caps

In each year there's a price cap, specifying the maximum level of toll the concessionaire may charge. If in any year a toll adjustment is possible, the price cap will be updated accordingly.

Toll adjustment

Toll adjustment can only be made when the actual revenue is less than the minimum level stipulated, and the extent of adjustment cannot exceed the maximum level specified in the contract.

6.2. Concessionaire's Decision Making Behaviour

The two principles of behavior of the concessionaire are:

1. A profit optimizer, in this case, the concessionaire seeks to optimize the net present value (NPV) of the project;
2. Unable to foresee the future, that is, the concessionaire is subject to demand uncertainties in the operational period.

Concessionaire's decision making process is as follows:

The concessionaire starts the operation after the construction of the project at time t=0, it should decide the toll it charges for this period, which is either at the level to initiate a toll adjustment in the next period, or to maximize the revenue of current period. Repeat this process for each period until the penultimate period; in this process, the price caps should be updated correspondingly and constantly. In the last period, the concessionaire should charge the toll that can maximize the revenue.

Proposition 1: If given a general (non-concave) demand function $Q = Q(P)$ under a price cap mechanism, then $max P_1 Q(P_1) \geq max P_2 Q(P_2), s.t.: P_1 \leq P_1^c, P_2 \leq P_2^c$, and $P_2^c < P_1^c$, in which P_1^c and P_2^c are price caps.

Proof: for a function $f: X \to Y$, if $X_1 \subset X_2 \subset X$, then it is true that $Y_1 \subset Y_2 \subset Y$.

Example: for a linear demand function $Q = a + bP$, in which, $a > 0$, and $b < 0$, revenue $R = PQ = aP + bP^2$, thus when $P = P_{opt} = -\frac{a}{2b}$, $maxR = -\frac{a^2}{4b}$

- if $P_1^c \leq P_{opt}$, then $maxP_1Q(P_1) = P_1^cQ(P_1^c) > maxP_2Q(P_2) = P_2^cQ(P_2^c)$
- if $P_2^c \leq P_{opt} < P_1^c$, then

$$maxP_1Q(P_1) = P_{opt}Q\left(P_{opt}\right) \geq maxP_2Q(P_2) = P_2^cQ(P_2^c)$$

- if $P_2^c > P_{opt}$, then $maxP_1Q(P_1) = P_{opt}Q\left(P_{opt}\right) = maxP_2Q(P_2)$

Proposition 2: During any period of the contract, to optimize the overall revenues, there are only two pricing strategies can be adopted: one is to earn the minimum revenue level of that period stipulated in the contract, that is, $\underline{P_i} = P(R = R_i^{min})$; the other is to maximize the revenue during that period, $\overline{P_i} = min\ (P_i^c, P(maxR))$.

Proof: given a possible combination of prices $\{P_1, \dots, P_i, \dots, P_t\}$, in which $\forall\ i \in t,\ P_i = \underline{P_i}\ or\ \overline{P_i}$, the correspondent overall revenue is $R = \sum_{i=1}^{t} R_i$, in which $R_i = R(P_i)$, in any period k, if $P_k = \underline{P_k}$, then for any price $P_k^{\prime} < \underline{P_k}$, it is obvious that $R_k^{\prime} < R_k = R_k{}^{min}$, thus $R^{\prime} = \sum_{i=1}^{k-1} R_i + R_k^{\prime} + \sum_{i=k+1}^{t} R_i < \sum_{i=1}^{k-1} R_i + R_k + \sum_{i=k+1}^{t} R_i = R$, that is to say, pricing strategy $\{P_1, \dots, P_k^{\prime}, \dots, P_t\}$ is strictly inferior to $\{P_1, \dots, P_k, \dots, P_t\}$; in any period l, if $P_l = \overline{P_l}$, then for any price $P_l^{\prime} \neq \underline{P_l}$, it is obvious that $R_l^{\prime} < R_l = R(min\,(P_l^c, P(maxR)))$, thus we have $R^{\prime} = \sum_{i=1}^{l-1} R_i + R_l^{\prime} + \sum_{i=l+1}^{t} R_i < \sum_{i=1}^{l-1} R_i + R_l + \sum_{i=l+1}^{t} R_i = R$, that is to say, pricing strategy $\{P_1, \dots, P_l^{\prime}, \dots, P_t\}$ is strictly inferior to $\{P_1, \dots, P_l, \dots, P_t\}$.

Proposition 1 and 2 imply that, under a toll adjustment mechanism with price caps, the optimal strategy of the concessionaire for a certain period (depending on the project audit schedule, usually on quarterly, semi-annual and annual basis), may not be to optimize the current revenue; instead, may be to lower the current revenue less than the minimum level stipulated in the contract (as close to as possible), in exchange for a chance of toll adjustment, thus the price caps for the following operational periods are raised by a certain level, which is also stipulated in the contract. With higher price caps, it is possible that the revenues collected henceforth are much more than those if the price caps stay at the same level.

Considering a two-period model, for the simplicity of demonstration while without loss of generality, the demand function for both periods are $Q = Q(P)$, with P_{opt} which optimizes $R = PQ(P) = R(P)$ at the level R_{max}, which is also the revenue cap for the first period; the price cap for the first period is P^c and $P^c < P_{opt}$; at the end of the first period, the revenue collected, R, will be inspected, if $R \leq R_{min}$, in which R_{min} is the minimum revenue level stipulated in the contract and $R_{min} < R_{max}$, the concessionaire will be allowed to increase the price cap by ΔP, that is to say, the price cap for the second period is updated as $(P^c + \Delta P)$. For the simplicity of notation, let $\underline{P}$ denote the price at which R_{min} will be achieved, the discount rate of the project is r, thus the discount factor is $\beta = (1 + r)^{-1}$.

Two strategies will be considered at the beginning of the project. First, to optimize the revenue in the first period, the pricing strategy is $P_1^1 = P^c$, and the revenue is $R_1^1 = R(P^c)$ and $R_{min} < R(P^c) < R_{max}$, thus in the second period, since the price cap is still at the level of P^c, to optimize the revenue in the second period, the pricing strategy is the same as that in the first period, $P_2^1 = P_1^1 = P^c$, thus $R_2^1 = R_1^1 = R(P^c)$, and the net present value (NPV) of this scenario is $NPV_1 = R_1^1 + \beta R_2^1$; Second, to seek a toll adjustment, then the optimal pricing strategy for the first period is $P_1^2 = \underline{P}$, and the revenue is $R_1^2 = R_{min}$, thus in the second period, the price cap is increased by ΔP to $(P^c + \Delta P)$. To optimize the revenue in the second period, the pricing strategy is $P_2^2 = min\,(P^c + \Delta P, P_{opt})$, that is to say, if the price cap is higher than P_{opt}, then $P_2^2 = P_{opt}$ and $R_2^2 = R_{max}$; if not, then $P_2^2 = P^c + \Delta P$ and $R_2^2 = R(P^c + \Delta P)$, and the net present value (NPV) of this scenario is $NPV_2 = R_1^2 + \beta R_2^2 = R_{min} + \beta R_2^2$; Let $NPV_2 - NPV_1 > 0$, and it is true that $R_2^2 - R_2^1 = R\left(\min\left(P^c + \Delta P, P_{opt}\right)\right) - R(P^c) > 0$ (from Proposition 1), thus $\beta > \frac{R_1^1 - R_1^2}{R_2^2 - R_2^1} = \frac{R(P^c) - R_{min}}{R\left(\min\left(P^c + \Delta P, P_{opt}\right)\right) - R(P^c)}$, let $\Delta R_1 = R(P^c) - R_{min}$ denoting the difference between the maximum revenue in the first period and the minimum revenue stipulated in the contract, and $\Delta R_2 = R\left(min\left(P^c + \Delta P, P_{opt}\right)\right) - R(P^c)$ denoting the difference the maximum revenue in the second period and that of the first period, thus, if $\Delta R_1 < \beta \Delta R_2$, the optimal strategy is to seek a toll adjustment, otherwise, to maximize the revenue in the first period. That is to say, only when i). the optimal revenue in the second period large enough and/or ii). the optimal revenue in the first period small enough and/or iii). the minimum revenue stipulated in the contract large enough, can suffice the strategy of seeking a toll adjustment, sacrificing the revenue in the first period in favor of the revenue in the second period.

Applying the same reasoning for a general model of multi periods rather than just two periods, give any period $i \in \{1, \dots, t-1\}$, the concessionaire still faces the same two pricing strategies which can maximize the overall net present value (NPV) of the project: to earn the minimum level so as to seek a toll adjustment for the following periods or to maximize the revenue of that very period, under the according price cap; however, for $i = t$, that is, for the last period, the optimal pricing strategy is always to maximize the revenue of that period, under the according price cap, for that there's no point to seek a toll adjustment if it's the last period of operating the project, after which the project will handed back to the government at no cost.

6.3. Contract Modelling

In the BOT contract, the concessionaire is entitled to operate the project for N years, that is, the contract duration is N (excluding the construction duration), and at the end of the operational period the project will be transferred back to the government at no cost. At $t = 0$, that is, the beginning of the first operational period, the government set the price cap for this period as $P_{0_{TAM}}$, that is, the price charged by the concessionaire cannot exceed this level. The discount factor is $\beta_{TAM} = \frac{1}{1+r_{TAM}}$, in which r_{TAM} is the discount rate of the concessionaire for the project. R_t^{min} and R_t^{max} are respectively the minimum revenue and maximum revenue (revenue cap) stipulated in the toll adjustment mechanism: if in any year i the actual revenue of the concessionaire is below R_i^{min}, the price cap for the next year (and thus the following years) will be allowed to raise by ΔP_i, and let $P_i^c{}_{TAM}$ denote the price cap for period i. In each period, the demand function is modeled as $Q_t = Q_t(P_{t_{TAM}})$, in which $P_{t_{TAM}}$ is the price actually charged by the concessionaire, and the demand function is non-concave and price cannot be negative. For the convenience of illustration, let R_t^{opt} denote the maximum revenue, given $Q_t = Q_t(P_t)$, which can be attained at the price level P_t^{opt}, attention should be paid that P_t^{opt} may exceed $P_t^c{}_{TAM}$ and therefore actually neither R_t^{opt} nor P_t^{opt} is attainable for the concessionaire; in this case the project can only seek to maximize its revenue under the according price cap in the period. Q_{max} is the maximum supply capacity of the project, and in the case of a toll road, it is the maximum daily volume of vehicles it can handle. During the whole contract duration period, the maximum toll that the concessionaire may charge cannot exceed P_{max}.

As mentioned in the general model in Chapter 3, only fixed investment cost will be considered in the following analysis; variant costs in the operational period will be omitted. This assumption can simplify the calculation at no expense of sacrifice of the effectiveness and robustness of the model. If variant costs were to be taken into consideration, it is equivalent to subtract a positive number from the toll charged.

The ultimate objective of the company is to maximize the net present value of the project, so the objective function is:

$$maxNPV_{TAM} = \max_{\{P_{t_{TAM}}\}_1^N} \sum_{t=1}^{N} {\beta_{TAM}}^t R_t \left(P_{t_{TAM}}\right)$$

which indicates, to maximize the net present value (NPV) is no other than to seek a optimal series of pricing strategy for each period, $\{P_{t_{TAM}}\}_1^N = \{P_{1_{TAM}}, P_{2_{TAM}}, \dots, P_{N_{TAM}}\}$, given:

$$\beta_{TAM}$$

$$P_{0_{TAM}}$$

$$P_{max}$$

$$Q_{max}$$

$$\Delta P_t, t = 1, \dots, N-1$$

$$R_t^{min}, t = 1, \dots, N-1$$

$$R_t^{max}, t = 1, \dots, N$$

The object function is subject to the following constraints:

$$R_t(P_{t_{TAM}}) = min\{P_{t_{TAM}} min\{Q_t(P_{t_{TAM}}), Q_{max}\}, R_t^{max}\}$$

which indicates, first, the supply of the project cannot exceed the maximum capacity Q_{max}, if the traffic demand exceeds the maximum capacity, the concessionaire can only satisfy part of the demand in this case; second, the revenue collected by the concessionaire is capped at the level R_t^{max}, which is stipulated in the contract.

$$Q_{t_{TAM}} = Q_t\left(P_{t_{TAM}}\right), t = 1, \dots, N$$

which indicates, the demand is solely determined by the price (toll), and so as the revenue. For the simplicity of demonstration, we'll adopt this assumption for the time being. To be more realistic, other factors which would affect the traffic demand will be considered in the updated models, such as competition from other service providers (other roads in the case of a toll road), travel time on the road, which is of pivotal consideration for road users, and last but not least, uncertainty of traffic demand is the most significant risk factor facing a toll road company.

$$\frac{\partial Q_t}{\partial P_t} < 0, t = 1, \dots, N$$

which indicates that, the demand function is a general non-concave function. That is to say, the higher the price, the lower the demand. This applies to traffic demand as to most commodities: if the toll is high, commuters could change their mode of transportation, say, from driving private cars to using public transport, or converting from a time-saving but over-charged toll road to a time-consuming and congested free road, depending on their incomes, values of time, purposes of commuting and so forth.

$$0 \le P_{1_{TAM}} \le P_{1\ TAM}^{c} = P_0 \le P_{max}$$

which indicates that, in the first operational period, the price charged by the concessionaire cannot exceed the price cap of that period $P^c_{1\,TAM}$, that is, P_0 stipulated by the government in the contract.

$$0 \le P_{t\,TAM} \le P^c_{t\,TAM} = P_0 + \sum_{i=1}^{t-1} sgn\left(R_i^{min} - R_i\right)\Delta P_i \le P_{max}\, , t = 2, \dots , N$$

in which,

$$sgn(x) = \begin{cases} 1, if\ x > 0 \\ 0, if\ x \le 0 \end{cases}$$

which indicates that, in each period, there exists a price cap for that period, P^c_t, and P^c_t is determined by different combinations of price escalations ΔP_ts, and ultimately, determined by the revenues in each prior period. If revenue of one period is less than the minimum level stipulated in the toll adjustment mechanism, then $sgn(x) = 1$, that is, a toll adjustment is allowed; otherwise, $sgn(x) = 0$. In any period, the price cap cannot exceed the maximum level of price for the whole operational period P_{max}.

The price regulation can also take other forms rather than an automatic toll adjustment mechanism as aforementioned. For the convenience of comprehension and simplicity of analysis, a basic form of price cap regulation will be adopted here. In the previous BOT project, other conditions being the same, the concessionaire faces no minimum revenues and revenue caps but a set of price caps for each period, $\{P^c_{t\,PCM}\}_1^N = \{P^c_{1\,PCM}, P^c_{2\,PCM}, \dots, P^c_{N\,PCM}\}$, the actual toll charged during each period cannot exceed the price cap as stipulated. The discount factor is $\beta_{PCM} = \frac{1}{1+r_{PCM}}$, in which r_{PCM} is the discount rate of the concessionaire for the project under this price cap mechanism. r_{PCM} may be different from r_{TAM} in that discount factor reflects the concessionaire's perception of risks of the project. If the contract clauses change and therefore the risk profile changes too, for the same concessionaire, its discount rate (rate of return) may change accordingly. In financial markets, this is termed "high risk, high return", which explains why the yield rates of U.S. Treasury securities are much lower than some so called junk bonds. Similarly, for a BOT concessionaire, to invest the same project in a developed country with a healthy commercial environment and a mature and stable legal system and a developing country in which even property rights are not fully guaranteed and the project may even be terminated or expropriated under extreme conditions such as political turmoil, the discount rate of the latter must be higher than the former to compensate the higher level of risk.

The concessionaire will still seek to maximize the net present value (NPV) of the project under this price cap regulation mechanism, so the objective function is similar to that under the automatic toll adjustment mechanism:

$$maxNPV_{PCM} = \max_{\{P_{t_{PCM}}\}_1^N} \sum_{t=1}^{N} \beta_{PCM}{}^t R_t\left(P_{t_{PCM}}\right)$$

which indicates, to maximize the net present value (NPV) is no other than to seek a optimal series of pricing strategy for each period, $\{P_{t_{PCM}}\}_1^N$.

The object function is subject to the following constraints:

$$R_t(P_{t_{PCM}}) = P_{t_{PCM}} min\{Q_t(P_{t_{PCM}}), Q_{max}\}$$

which indicates the supply constraint of the service provider.

$$Q_{t_{PCM}} = Q_t\left(P_{t_{PCM}}\right), t = 1, \ldots, N$$

The demand function is the same as the one in the automatic toll adjustment mechanism case, for the purpose of comparison.

$$0 \leq P_{t_{PCM}} \leq P^c_{t\ PCM}, t = 1, \ldots, N$$

which indicates that the actual price charged by the concessionaire cannot exceed the price cap in that period stipulated in the contract.

Since the pricing strategies are not interrelated, as in the case of the automatic toll adjustment mechanism, put it in another way, the prices that the concessionaire may charge are independent of each other, and only subject to the price caps which are treated as given parameters for the optimization problem. Thus to optimize the net present value (NPV) of the project is equal to optimize the individual revenues in each period:

$$maxNPV_{PCM} = \max_{\{P_{t_{PCM}}\}_1^N} \sum_{t=1}^{N} {\beta_{PCM}}^t R_t\,(P_{t_{PCM}}) \overset{equilavent\ to}{\Longleftrightarrow} \max_{\{P_{t_{PCM}}\}_1^N} \{R_t(P_{t_{PCM}})\}$$

For that R_t^{opt} is the maximum revenue, given $Q_t = Q_t(P_t)$, which can be attained at the price level P_t^{opt}, thus, if $P_{t\ PCM}^c \geq P_t^{opt}$, that is to say, the price cap is higher than the optimal price, therefore the concessionaire may charge the price at the optimal level, then $P_{t_{PCM}} = P_t^{opt}$, in this case, $R_{t_{PCM}} = R_t^{opt}$; if $P_{t\ PCM}^c < P_t^{opt}$, that is to say, the price cap is lower than the optimal price, therefore the concessionaire may not be able to charge the price at the optimal level, then to maximize the revenue, $P_{t_{PCM}} = P_{t\ PCM}^c$, in this case, $R_{t_{PCM}} = R_t(P_{t\ PCM}^c)$; Thus in any period t, the optimal pricing strategy is to charge the price at $P_{t_{PCM}} = min\,(P_t^{opt}, P_{t\ PCM}^c)$, and the corresponding revenue is $R_{t_{PCM}} = min\,(R_t^{opt}, R_t(P_{t\ PCM}^c))$.

6.4. Real Options Value of the Toll Adjustment Mechanism

Thus the real options value (ROV) of the toll adjustment mechanism is:

$$ROV = maxNPV_{TAM} - maxNPV_{PCM}$$

which indicates, the value of flexibility in the contract with the automatic toll adjustment is equal to the different between: i). the optimal net present value of the project under the optimal pricing strategy in the contract with the automatic toll adjustment mechanism and ii). the optimal net present value of the project under the optimal pricing strategy in the contract with the pure price cap regulation mechanism.

For the first optimization problem, since the pricing strategies are interrelated and interdependent (current low price – minimum revenue – toll adjustment – higher price caps for the following period – high price charged – possibly higher revenues), to maximize the revenue in each individual period is just one possible solution to the optimization problem, may or, in more cases, may not be the optimal pricing strategy. However, intuitively, this is the most likely pricing strategy that can be adopted by the concessionaire, and certainly, not the worst one. We name this strategy the passive pricing strategy, in contrast to the active strategy, which is to consider two possible pricing strategies in each period, and to find the optimal pricing strategy among 2^{N-1} possible combinations of pricing strategies. For a contract duration of N periods, the possible combinations of pricing strategies is 2^{N-1} rather than 2^{N} for that in the last period the optimal pricing strategy is always to maximize the revenue of that period:

$$\left\{P_{t_{TAM}}\right\}_1^N(pas)=\left\{\overline{P_{1_{TAM}}},\ \overline{P_{2_{TAM}}},\dots,\overline{P_{N_{TAM}}}\right\}$$

$$\overline{P_{t_{TAM}}}=min\,(P_t^c,P_t^{opt})$$

Attentions should be paid that in a passive pricing strategy, it doesn't necessarily mean that there will be no toll adjustments. If in one period, the maximum level of revenue of the concessionaire is still less than the minimum revenue stipulated in the contract, then the toll adjustment can also take place in the next period.

Thus the revenues in each individual period are:

$$\left\{R_{t_{TAM}}\right\}_1^N(pas)=\left\{R(\overline{P_{1_{TAM}}}),R(\overline{P_{2_{TAM}}}),\dots,R(\overline{P_{N_{TAM}}})\right\}$$

The net present value (NPV) of the project under this pricing strategy is:

$$NPV_{TAM}(pas) = \sum_{t=1}^{N} \beta_{TAM}{}^{t} R_t \, (\overline{P_{t\,TAM}})$$

This is akin to the case of pure price cap regulation mechanism. Actually, if the price caps are respectively the same for the two mechanisms in each individual period and the discount rates are the same too, the net present value (NPV) will be exactly the same for the toll adjustment mechanism with a passive pricing strategy and a pure price cap regulation mechanism with the optimal pricing strategy.

To solve the optimization problem, a tree structure model is built to facilitate the solving process, see figure 6-1:

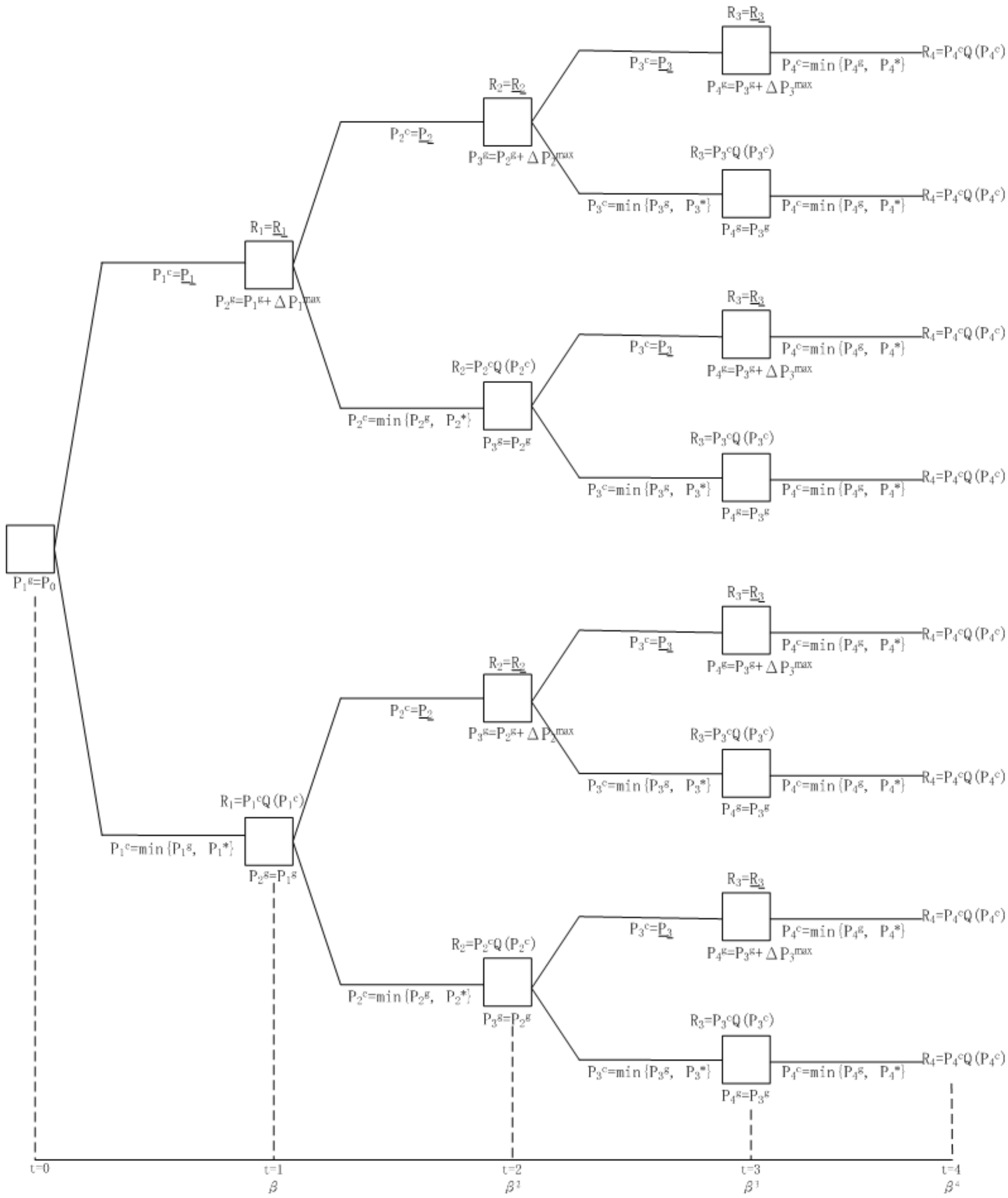

Figure 6-1 Tree structure model for real options analysis of TAM

In Figure 6-1, P_c is the actual price (concession price), it may be $\underline{P}$, corresponding to the price at which revenue is at the minimum level stipulated in the contract; or P^*, the price that can maximize the revenue; P_g is the price cap (gazetted price).

For a contract of N periods, let $t = 1, 2, \dots, N$ denote the individual periods. At the beginning of each period, $t = 0, 1, \dots, N-1$, from Proposition 2, the concessionaire will decide the pricing strategy of that period, according to the price caps; For $t = 0, 1, \dots, N-2$, $P_{t_{TAM}} = \underline{P_{t_{TAM}}}\ or\ \overline{P_{t_{TAM}}}$, correspond respectively to the minimum revenue level stipulated in the contract and the possible maximum level of revenue in that period, $R_{t_{TAM}} = R_t^{min}\ or\ min\ (R_t^{opt}, R_t^{max})$. Thus for the next period, the price caps are respectively $P_{t+1}^c = P_t^c\ or\ P_t^c + \Delta P_t$. From Proposition 1, $R_{t+1}(P_t^c) \leq R_{t+1}(P_t^c + \Delta P_t)$.

For the simplicity of notation, let $\{\tau_t = 0\ or\ 1\}_{t=1}^{N}$ denote the pricing strategy for period t, i.e., $\tau_1 = 0$ means in the first period, $P_{1_{TAM}} = \underline{P_{1_{TAM}}}$, $\tau_{N-1} = 1$ means in the penultimate period, $P_{N-1_{TAM}} = \overline{P_{N-1_{TAM}}}$. For the last period, $\tau_N \equiv 1$, that is, $P_{N_{TAM}} \equiv \overline{P_{N_{TAM}}}$. Let $k = 1 + \sum_{t=1}^{N-1} \tau_t 2^{N-t-1}$. Thus $k = 1$ corresponds to the pricing strategy $\{0, 0, \dots, 0, 0, 1\}_N$, $k = 2$ corresponds to the pricing strategy $\{0, 0, \dots, 0, 1, 1\}_N$, …, and $k = 2^{N-1}$ corresponds to the pricing strategy $\{1, 1, \dots, 1, 1, 1\}_N$.

For instance, $k = 1 + 2^{N-2}$, the pricing strategy is therefore $\{0, 1, \dots, 1, 1, 1\}_N$, which indicates that the concessionaire will only seek one opportunity of toll adjustment, which is from the second period, thus the revenue for the first period is the minimum level stipulated in the contract, and the benefit is that prices caps for the following periods are increased by ΔP_1, that is, the according price caps for each period are ${P^c_{TAM}}_1^N = \{P_{0_{TAM}}, P_{0_{TAM}} + \Delta P_1, \dots, P_{0_{TAM}} + \Delta P_1, P_{0_{TAM}} + \Delta P_1\}_N$, therefore the revenues of individual periods are ${R_{TAM}}_1^N = \{R_1^{min}, min[R_2(\overline{P_{2_{TAM}}}), R_2^{max}], \dots, min[R_N(\overline{P_{N_{TAM}}}), R_N^{max}]\}_N$, thus the net present value (NPV) of the project under this pricing strategy is:

$$NPV_{TAM}(k = 1 + 2^{N-2}) = \beta_{TAM} R_1^{min} + \sum_{t=2}^{N} {\beta_{TAM}}^t min[R_t(\overline{P_{t_{TAM}}}), R_t^{max}]$$

in which,

$$\overline{P_{t_{TAM}}} = min\ (P^c_{t\ TAM}, P_t^{opt})$$

Thus, for all the the ks, net present value (NPV) of the project under the active pricing strategy is:

$$NPV_{TAM}(act) = \max_{\{k\}_1^{2^{N-1}}} \{NPV_{TAM}(k)\}$$

which indicates, for each pricing strategy k there is a correspondent set of revenues of N periods, therefore the net present value (NPV) for each pricing strategy k can be found. Exhausting k from 1 to 2^{N-1}, thus the optimal pricing strategy for the concessionaire is the one that can maximize the net present value (NPV) of the project among the k combinations.

6.5. Numerical Solution to the Model

Theoretically the optimization problem has been solved. However, in the real world, the contract duration of such BOT projects can often be as long as 30 to 50 years, or even more. That is to say, the concessionaire has to exhaust $2^{29} = 536{,}870{,}912$ to $2^{49} = 562{,}949{,}953{,}421{,}312$ or more different combinations of pricing strategies to find the optimal one. This is a too much taunting and almost impossible task even for the most sophisticated computer in the world currently.

To make the calculation practical, one possible solution is to modify and simplify the contract itself. For instance, let $\Delta P_1 = \Delta P_2 = \cdots = \Delta P_N = \Delta P$; in this case, a binomial tree can be employed to facilitate the analysis. In the previous setting, from period $t = 1, \dots, N-1$, there are 2^t decision nodes at each period and 2^{N-1} at the last period; however, with the present setting, from period $t = 1, \dots, N-1$, there are t decision nodes at each period and $N-1$ at the last period.

However, the statement that within this setting, the size of problem can be effectively reduced from the exponential 2^n to linear n is utterly false. In fact, the size of problem is still the same 2^n rather than n. This is because the net present value (NPV) of the project is path-dependent; that is to say, for the same node at the last period, two paths may generate two net present values (NPV); unlike valuing American or European options, which are not path-dependent but only dependent on the final state, for the same node at the last period, all paths leading to this node generate the same payoff. Thus it is straightforward to see that for the n nodes in the last period, the number of possible paths leading to them is still 2^n. Therefore simplification of the contract in this fashion will not help reducing the computation complexity of the problem.

With the same conditions of the aforementioned optimization problem, branch and bound algorithm can be applied to reduce the computation complexity. Branch and bound is a general algorithm for finding optimal solutions of various kinds of optimization problems, especially in discrete and combinatorial optimization. A branch-and-bound algorithm consists of a systematic enumeration of all candidate solutions, where large subsets of fruitless candidates are discarded en masse, by using upper and lower estimated bounds of the quantity being optimized. The method was first proposed by Land and Doig (1960) for discrete programming. Application of branch and bound algorithm to the optimization problem aforementioned is as follows.

At period *t,* there are 2^{t-1} decision nodes in the tree model, each with a unique price cap, corresponding to its route of previous pricing strategies. For the jth decision node, the price cap is:

$$P^c_{t\ TAM}(j) = P_{0_{TAM}} + \sum_{i=1}^{t} (1 - \tau_t(j))\Delta P_i$$

The lower bound (LB) for this decision node is:

$$LB_t(j) = \sum_{i=t}^{N} \beta_{TAM}{}^{i} R_i(P^c_{t\ TAM}(j))$$

which indicates, from period t and on, the concessionaire will charge the toll at the level of price cap of period t, $P_{t_{TAM}} = P_{t+1_{TAM}} = \cdots = P_{N_{TAM}} = P^c_{t\ TAM}(j)$. The net present value (NPV) of this decision node under such a pricing strategy is called the lower bound for the decision node j at period t. It is straightforward to see that for any pricing strategy actually chosen by the concessionaire, $\{\tau_t = 0\ or\ 1\}_t^N$, any of the correspondent net present value (NPV) $NPV_{t_{TAM}}(j)$ is no less than the lower bound $LB_t(j)$.

Similarly, we can define the upper bound (UB) for this decision node as:

$$UB_t(j) = \sum_{i=t}^{N} \beta_{TAM}{}^{i} min\,\{R_i(min\,\{P_t^{opt}, P^c_{t\ TAM}(j) + \sum_{i=t}^{N} \Delta P_j\})\,, R_t^{max}\}$$

which indicates that from period t and on, the concessionaire will seek to maximize the revenue in each period under the condition that all price escalations are permissible regardless of the relationship between actual revenues and the minimum levels stipulated in the contract. The revenue may or may not be higher than the optimal revenue without any price caps for that it's still possible that with all the price escalations effective, the price cap can be still lower than the optimal level. The revenues must be lower than the revenue caps, that is to say, revenue caps still work. The net present value (NPV) of this decision node under such a pricing strategy is called the upper bound for the decision node j at period *t*. It is straightforward to see that for any pricing strategy actually chosen by the concessionaire, $\{\tau_t = 0 \; or \; 1\}_t^N$, any of the correspondent net present value (NPV) $NPV_{t_{TAM}}(j)$ is less than the upper bound $UB_t(j)$.

At any period, for any decision nodes, both LB and UB can be attained, in that the pricing strategies assumed in this fashion are comprised of independent prices, unlike in the case of active pricing strategy, the prices the concessionaire may charge for each period are interrelated.

At any period *t*, exhausting $\mathrm{j} = 1,2, \dots, 2^{\mathrm{t}-1}$, we will get $2^{\mathrm{t}-1}$ pairs of $(LB_t(j), UB_t(j))_{\mathrm{j}=1}^{2^{\mathrm{t}-1}}$, for any two decision nodes *m* and *n* at the same period *t*, let *k* denote one route leading to node *m* and *l* the one to node *n,* if:

$$NPV_{t_{TAM}}(k) + UB_t(m) < NPV_{t_{TAM}}(l) + LB_t(n)$$

which indicates that the pricing strategy corresponding to route *k* is strictly inferior to the pricing strategy correspondent to route *l.* Thus route *k* can be safely discarded. For example, project A will definitely generate a profit of 1 million dollars in the first year, and in the second year, the profit is no less than 5 million dollars; project B will definitely generate a profit of 2 million dollars in the first year, and in the second year, the profit is no more than 4 million dollars; other conditions such as costs and so on being the same, a rational investor will definitely choose A over B.

Applying this process for $t = 1, \dots N - 1$ and in each period for decision node $j = 1,2, \dots, 2^{t-1}$, will effectively reduce the search space, and therefore, computation complexity of the problem. Application of branch and bound algorithm will be illustrated in the next chapter.

6.6. Real Options Value of the Toll Adjustment Mechanism with Stochastic Demand

Facing the traffic uncertainty and all the other conditions being the same, the optimization problem for the concessionaire is modified as:

$$maxE(NPV)_{TAM} = \max_{\{P_{t_{TAM}}\}_1^N} \sum_{t=1}^{N} {\beta_{TAM}}^t E(R_t\,(P_{t_{TAM}}))$$

given:

$$\beta_{TAM}$$

$$P_{0_{TAM}}$$

$$P_{max}$$

$$Q_{max}$$

$$\Delta P_t, t = 1, \dots, N-1$$

$$R_t^{min}, t = 1, \dots, N-1$$

$$R_t^{max}, t = 1, \dots, N$$

s.t.:

$$R_t(P_{t_{TAM}}) = min\{P_{t_{TAM}} min\{Q_t(P_{t_{TAM}}), Q_{max}\}, R_t^{max}\}$$

$$\widetilde{Q_t}_{TAM} = Q_t\left(P_{t_{TAM}}\right)\widetilde{Q_t}, t = 1, \dots, N$$

$$0 \leq P_{1_{TAM}} \leq P_{1\ TAM}^{c} = P_0 \leq P_{max}$$

$$0 \leq P_{t_{TAM}} \leq P_{t\ TAM}^{c} = P_0 + \sum_{i=1}^{t-1} sgn\left(R_i^{min} - R_i\right)\Delta P_i \leq P_{max}, t = 2, \dots, N$$

in which,

$$sgn(x) = \begin{cases} 1, if\ x > 0 \\ 0, if\ x \leq 0 \end{cases}$$

and the probability density function of $\tilde{Q}$ is $f(x)$

Similarly, this optimization problem can also be solved by analyzing the aforementioned tree structure model. The two pricing strategies facing the concessionaire in each period other than the last one are still the same: a price low enough to induce a toll adjustment in the next period or a high price to maximize the revenue of that period. However, for the first strategy, facing the traffic uncertainty, if only one price is allowed to set at the beginning of each period and the concessionaire sets it at the level at which the expectation of revenue is equal to the minimum level stipulated in the contract, it is probable that the actual revenue is higher than the minimum level. It is straightforward that the lower the price, the lower the revenue, therefore the higher chance that the revenue is less than the minimum level. How low is enough depends on the preferences for risk of different investors, say, a concessionaire of high risk preference may choose the probability level that the revenue is less than the minimum level to be 80%, while a concessionaire of low risk preference may choose the probability level as high as 95% or even 99%; the revenue of the latter will be lower than the former while the probability of a toll adjustment is higher. For an ultra-conservative concessionaire, setting the price to be zero will definitely (probability 100%) induce the toll adjustment. Nevertheless, as a matter of fact, since the concessionaire is entitled to charge any prices under the price cap and there're no restrictions on the number of price adjustments in one period, the concessionaire can always control the revenue at the minimum level stipulated in the contract through micro adjustments of prices, e.g., charge at a very low level in the beginning, gradually increase the price, when the revenue reach the minimum level, set the price to be zero. For the time being, we suppose that the concessionaire will charge only one price for one period. Thus, to initiate a toll adjustment, the concessionaire will earn a much less revenue than the minimum level stipulated in the contract. This is the negative impact of the traffic uncertainty.

To solve the above optimization problem with demand uncertainty, the same tree structure model aforementioned will be employed. To maximize the expectation of net present value (NPV) of the project is equivalent to find an optimal pricing strategy for each period which is the same rationale in the case of deterministic traffic demand; the difference is the way to determine the price level at which a toll adjustment will be induced and the price to maximize the expected value of revenue.

In any period i, if the price cap is P_i^c, let $P_i(j) = j \times \frac{P_i^c}{M}$, in which $j = 0, 1, \dots, M$, M is a sufficiently large number decided by the concessionaire. In each period for each price $P_i(j)$ from the demand function we have $\widetilde{Q_\iota(J)} = Q_i\big(P_i(j)\big)\widetilde{Q_\iota}$. Thus, for each $P_i(j)$, there is a unique distribution of $\widetilde{R_\iota(J)}$, the probability density function of which is $g_i^j(x)$. Let α denote the risk preference of the concessionaire, i.e. given a price $\underline{P_t^\alpha}$, the probability that the revenue of that period is equal to the minimum level is $1-\alpha$.

Exhausting j, when $j = k$, $\int_0^{R_i^{min}} g_i^k(x)dx = 1-\alpha$, that is to say, at the price level $P_i(k)$, the probability that the actual revenue in period i is equal to the minimum level of revenue in that period is $1-\alpha$, therefore $\underline{P_i} = P_i(k)$; if when $j = l$, $\int_0^{R_i^{max}} xg_i^l(x)dx > \int_0^{R_i^{max}} xg_i^m(x)dx\,, m \in j = 0, 1, \dots, M, and\ m \neq l$, that is to say, at the price level $P_i(l)$, the expected revenue in period i is maximized, we have $\overline{P_i} = P_i(l)$. Thus for period i other than the last period, the two pricing strategies of the concessionaire facing traffic demand uncertainty are respectively $P_i(k)$ and $P_i(l)$. For the last period, the optimal pricing strategy is always $P_i(l)$, which is to say, to maximize the expected value of revenue of the last period.

Applying branch and bound algorithm, this optimization problem is then solved. A case study will be provided in the next chapter for the purpose of illustration.

6.7. Summary of the Chapter

Chapter 6 presents the real options model of the toll adjustment mechanism. First the contract features and concessionaire's behaviors are illustrated. Then the model is built under the assumption that traffic demand is deterministic. A tree structure model and branch and bound method are then proposed to facilitate the calculation. Numerical solutions are then provided. The case that the traffic demand is stochastic is then discussed with numerical solutions provided. Application of this model will be illustrated in the next chapter, with the results from previous two models as inputs to the current model.

7. CASE STUDY

7.1. Western Harbour Crossing

<u>Project profile</u>

Western Harbour Crossing (abbreviated as WHC), a dual three-lane immersed tube tunnel, is the third cross harbour tunnel in Hong Kong (the other two are the Cross Harbour Tunnel and the Eastern Harbour Crossing), linking West Kowloon with Sai Ying Pun on Hong Kong Island. WHC is part of the Airport Core Programme (ACP), a comprehensive set of ten infrastructure projects associated with the then newly built airport at Chek Lap Kok in the Lantau Island. Unlike the other nine infrastructure projects in ACP, WHC is a franchised build-operate-transfer (BOT) project, with the Hong Kong Government as the concessioner and Western Harbour Tunnel Company Limited (WHTCL) as the concessionaire, who will be responsible for financing, designing, building, operating and maintaining the tunnel for 30 years, and after which the tunnel will be handled to the government at no cost. That is to say, the concession duration of the BOT contract is 30 years including the construction period. The construction started in August 1993 and was finished in April 1997, which took 45 months, shorter than the proposed construction period. The total project cost is about HK$7 billion, in which HK$5.7 billion is the construction cost. The maximum capacity is 180,000 vehicles per day and the design life is 120 years. The location of WHC is shown in Figure 7-1.

Figure 7-1 Location of WHC (Source: Google Map, Hong Kong, 2012)

Toll Adjustment Mechanism in WHC Ordinance

Apart from the technical advantages, what really makes WHC distinct from the other two tunnels in Hong Kong and even the other similar projects (private financing toll roads, tunnels, etc.) is the so-called automatic Toll Adjustment Mechanism (TAM) designed in the WHC Ordinance.

There is a toll adjustment mechanism in the form of rate-of-return regulation stipulated in the Ordinance of the Eastern Harbour Crossing, which stated that when the rate of return of the project was below a certain level, the concessionaire might appeal for a toll adjustment to the government; to what extent as well as when the toll adjustment can be implemented is at the solely discretion of the government. In other transport projects such as toll roads that financed and operated by the private sector in a similar fashion (PPP, PFI, etc.), the prevalent practice in regulating tolls is in the form of price cap regulation which adjusts the operator's prices according to the price cap index that reflects the overall rate of inflation in the economy, the ability of the operator to gain efficiencies relative to the average firm in the economy, and the inflation in the operator's input prices relative to the average firm in the economy. In this price cap regulation, often referred to as CPI-X or RPI-X, toll adjustments are automatic and on a periodical basis; both the extent of adjustment and review period are specified clearly in the contract ex ante.

The Toll Adjustment Mechanism (TAM) is a hybrid of the two aforementioned: an automatic rate-of-return regulation plus revenue-cap regulation mechanism. According to the proceedings in the Legislative Council (1995), the purpose of the mechanism is to provide WHTCL with a reasonable but not excessive return to the shareholders, whilst maintaining a stable toll regime for road users. It also aims to ensure that the concessionaire will earn sufficient revenue to service its debts.

At First, the so-called Western Harbour Crossing Toll Stability Fund (TSF) is established and three levels of revenues are projected and written in the ordinance, respectively, minimum estimated net revenue, upper estimated net revenue and maximum estimated net revenue.

The revenue caps work as follows. When in any year, the net revenue of the concessionaire exceeds the maximum estimated net revenue, the concessionaire’s revenue is capped at the level of the upper estimated net revenue plus 50% of the difference between the upper estimated net revenue and the maximum estimated net revenue; the rest will be paid into the TSF; When in any year, the net revenue of the concessionaire exceeds the upper estimated net revenue but does not exceed the maximum estimated net revenue, the concessionaire’s revenue is capped at the level of the upper estimated net revenue plus 50% of of the amount in excess of the upper estimated net revenue.

Six dates are specified in the ordinance. If, in any year immediately before a specified date, the net revenue of the concessionaire is less than the upper estimated net revenue and the amount of TSF is insufficient to make the difference between the net revenue and the upper estimated net revenue, the concessionaire is allowed to initiate an anticipated toll adjustment, the amount of which is stipulated in the ordinance, too; if the amount of TSF is sufficient to make the difference between the net revenue and the minimum estimated net revenue, the concessionaire will be subsidized by the difference between the net revenue and the upper estimated net revenue from TSF and the toll adjustment will be deferred.

7.2. Hypothetical Case Study

We adapted the case of Western Harbour Crossing into the following case. The adaptation is necessary in that it leaves the kernel of the problem, the toll adjustment mechanism, intact, while at the same time, simplifies the model by ignoring trivial details which can only complicate the problem in the negative way. For instance, there are three rather than two crossing harbour tunnels in Hong Kong and none of them is free of charge. However; in the adapted case study, the number of tunnels is reduced to two and one of the two is assumed to be free of charge. The simplification of the number of tunnels will not affect the robustness of the model, because only the traffic on the tunnel studied and the total traffic demand is of concern to the modeller; to know the traffic on each of the three tunnels, a multi-route choice model should be developed, which is out of the scope of this study. The simplification that one tunnel is free of charge will not affect the robustness of the model either, because, in this case, the toll paid by the commuter using tunnel X is actually the difference between the tolls of tunnel X and Y. Using this difference to replace the actual toll of tunnel X in the model and assuming tunnel Y is free, the route choice pattern of the commuters remains the same.

The hypothetical case is as follows. A free crossing harbour tunnel connects Kowloon and Hong Kong Island. The current crossing harbour traffic is 150,000 vehicles per day (vpd), which exceeds the maximum designed capacity, 100,000 vpd. This induced severe traffic jams during peak hours of the day, which causes huge wastes of time for the commuters and therefore huge social and environmental cost for the society. The free flow travel time without congestions in the free tunnel is 6 minutes per vehicle (mpv). However, actual travel time can be as long as 10 or 20 minutes due to congestions. In light of this situation, the government of Hong Kong intends to build another crossing harbour tunnel to relieve the traffic on the existing one. The maximum designed capacity of the new tunnel is set to be 300,000 vpd and the cost of construction is estimated to be 70 billion Hong Kong dollars (HKD). Due to the better location which means a shorter travel distance and the more advanced design of the tunnel which allows for a higher speed, free travel time in the new tunnel is only half of that of the current one, just 3 mpt.

The government decided to procure this capital intensive project through a Build-Operate-Transfer (BOT) scheme, signing a concession contract with a private investor. The private investor, often in the form of a concessionaire, will be responsible for the financing and construction of the tunnel according to the technical specifications set by the government and then will be entitled the exclusive franchise to operate the tunnel for 30 years upon completion of construction; the revenues generated through toll collection will be used to recoup the huge initial investment of the project, comprising mainly of construction and financial costs. When the concession contracts terminates, the tunnel will be transferred back to the government at no cost.

To prevent the concessionaire from exploiting the commuters arbitrarily for excessive profits, which is quite politically sensitive for a public utility, the government sets both i). revenue caps for each year in the concession period, the excessive revenues beyond the caps will automatically go to the government and ii). price caps for each year; during the first operational year the maximum toll that the concessionaire may charge is 20 Hong Kong dollars per trip (dpt); during the whole concession period, 30 years, the toll cannot exceed 100 dpt. Since the project is massively capital intensive and involves a variety of risks and uncertainties, e.g., the traffic demand, to make it financially feasible and properly profitable in order to attract investors, the government also devises a toll adjustment mechanism, written in the contract, in which a set of minimum revenue levels are defined clearly ex ante. During any year, if the actual revenue is less than the minimum level, the concessionaire is entitled to raise the price cap by a certain degree for the remaining period. The extents of toll adjustment are 5 HKD each year for the first decade, 10 HKD for the middle and 15 HKD for the last, to offset inflation. Levels of maximum and minimum of revenues, can be found in Table 7-1 (in millions of Hong Kong dollars).

Table 7-1 Maximum and minimum revenues

t	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
min R	365	402	442	486	534	588	647	711	782	861
max R	1095	1205	1325	1457	1603	1764	1940	2134	2347	2582
t	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*
min R	947	1041	1146	1260	1386	1525	1677	1845	2029	2232
max R	2840	3124	3437	3780	4158	4574	5031	5535	6088	6697
t	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*
min R	2456	2701	2971	3268	3595	3955	4350	4785	5264	5790
max R	7367	8103	8914	9805	10785	11864	13050	14355	15791	17370

The concessionaire's discount rate in calculating the net present value (NPV) is 12%. The risk-free rate of return in the market is 5%.

For the simplification of analysis, we assume that i). operational costs are zero and ii). the toll is same regardless of the vehicle types. For this sort of massive infrastructure project, compared to the huge initial construction cost, the operational costs can be safely omitted; even if not, this can be easily achieved by rectifying the goal of the concessionaire from seeking maximum net present value (NPV) of revenues to maximum net present value (NPV) of profits. In the real world, price discrimination is applied to different types of vehicles. A combination of different pricing strategies for different vehicle types can be equivalent to the two pricing strategies analyzed in the previous chapter. Therefore, those two assumptions are appropriate for the hypothetical case study. Users who are discontent with the simplification can also model the real situation by minor modifications.

For the private investors, the foremost uncertainty facing them is the traffic demand, which is of crucial significance to the success of the project. Facing the price regulation, a low level of traffic may be detrimental to the investors. The other factor they concern is the willingness to pay of the users, that is, value-of-time (VOT) of the commuters, which essentially is the money they would pay to save time. A higher VOT means that more commuters tend to use the tolled tunnel than the free one to save time (in this case, 3 minutes if both tunnels are not congested) at the expense of paying tolls. We assume that the VOT of the commuters conforms to a normal distribution.

Two ways of treating uncertainties by the concessionaire are illustrated here: First, to classify i). the future of traffic in three scenarios, which are, low level of traffic, medium level of traffic and high level of traffic and ii). the future of VOT in three scenarios too, which are low level of VOT, medium level of VOT and high level of VOT; Second, to consider the probabilistic distributions of the annual traffic demand, corresponding to the three levels of VOTs.

Both the active and passive pricing strategies under the same toll adjustment mechanism will be compared. The pricing strategy under no toll adjustment mechanism will also be presented, to see the real options value of the mechanism.

Case study No. 1: Scenario analysis

As illustrated above, there are total 9 scenarios in the future, see Table 7-2.

Table 7-2 Scenarios of the future

Scenarios		Traffic Demand		
		Low	Medium	High
VOT	Low	S1	S4	S7
	Medium	S2	S5	S8
	High	S3	S6	S9

For the low VOT scenario, the expected VOT of the first year is 3 Hong Kong dollars per minute (dpm); for the medium, 5 dpm; for the high, 7 dpm; the expected VOTs in year 2 to year 30 increase by 2% annually. In all three cases the standard variance of VOT is one fifth of the expected value. For the low traffic scenario, the total crossing harbour traffic will grow at an annual rate of 2% with the first year traffic to be 100,000 vpd; for the medium, 2% with 200,000 vpd; for the high, 2% with 300,000 vpd.

If the concessionaire adopts an active pricing strategy, that is, to maximize the NPV of the project for the entire concession period, the actual tolls charged, price caps and actual revenues in each period are shown in Table 7-3 to Table 7-5; if a passive pricing strategy, that is, to seek the maximization of revenue in each period, the corresponding results are shown in Table 7-6 to Table 7-8. A contract with pure price cap regulation mechanism (with no toll adjustment mechanism) will be used as a benchmark for the real options analysis. The price caps for the first decade in the concession contract is 20 dpt; the middle 25 dpt; the last 30 dpt.

Table 7-3 P-t under active pricing (dpt)

t	Scenario								
	1	2	3	4	5	6	7	8	9
1	7.0	11.7	16.4	19.9	11.8	16.6	20.0	20.0	20.0
2	7.2	12.0	16.8	5.5	12.1	16.9	20.0	19.9	20.0
3	7.3	12.2	17.1	25.0	5.8	17.3	3.9	19.9	20.0
4	7.5	12.5	17.4	8.4	25.0	17.7	4.2	19.9	20.0
5	7.6	12.7	17.8	13.0	6.6	18.1	20.8	20.0	20.0
6	7.8	13.0	18.1	34.9	29.9	18.6	30.0	4.9	20.0
7	7.9	13.2	18.5	17.0	7.8	19.1	29.9	24.3	20.0
8	8.1	13.5	18.9	39.8	35.0	19.7	29.8	25.0	20.0
9	8.3	13.8	19.3	39.9	35.0	20.0	29.9	25.0	20.0
10	8.4	14.0	19.6	40.0	34.9	20.0	29.9	25.0	20.0
11	8.6	14.3	20.0	25.6	11.0	10.2	9.0	7.2	6.6
12	8.8	14.6	20.4	27.9	12.2	11.4	9.9	7.8	7.6
13	8.9	14.9	20.9	30.4	14.5	12.3	49.4	39.2	37.8
14	9.1	15.2	21.3	33.0	19.4	13.4	49.9	44.8	40.0
15	9.3	15.5	21.7	36.0	22.3	14.6	13.1	10.7	9.8
16	9.5	15.8	22.1	38.9	85.0	69.9	59.6	53.3	48.6
17	9.7	16.1	22.6	99.7	84.7	69.8	15.6	54.7	49.6
18	9.9	16.4	23.0	99.8	84.8	69.9	69.5	14.7	13.2
19	10.1	16.8	23.5	99.8	84.7	70.0	69.5	64.7	60.0
20	10.3	17.1	23.9	99.7	84.9	69.9	69.9	64.8	59.8
21	10.5	17.5	24.4	99.9	40.5	32.4	21.5	19.4	17.9
22	10.7	17.8	24.9	100.0	99.9	36.1	23.4	80.0	74.9
23	10.9	18.2	25.4	99.9	100.0	99.9	99.8	23.0	21.6
24	11.1	18.5	25.9	99.6	99.9	99.8	99.6	94.5	89.3
25	11.3	18.9	26.5	99.7	99.6	100.0	99.9	94.4	89.8
26	11.6	19.3	27.0	99.6	99.7	99.5	99.2	94.8	89.3
27	11.8	19.7	27.6	99.9	99.6	100.0	99.1	94.2	89.3
28	12.1	20.1	28.1	99.4	100.0	99.7	99.7	94.2	89.1
29	12.3	20.5	28.7	100.0	99.6	99.8	99.1	94.9	89.6
30	12.6	20.9	29.3	99.8	99.6	99.8	99.2	94.4	89.8

Table 7-4 P_c-t under active pricing (dpt)

t	Scenario								
	1	2	3	4	5	6	7	8	9
1	20	20	20	20	20	20	20	20	20
2	25	20	20	20	20	20	20	20	20
3	30	25	20	25	20	20	20	20	20
4	35	30	20	25	25	20	25	20	20
5	40	35	20	30	25	20	30	20	20
6	45	40	20	35	30	20	30	20	20
7	50	45	20	35	30	20	30	25	20
8	55	50	20	40	35	20	30	25	20
9	60	55	25	40	35	20	30	25	20
10	65	60	30	40	35	20	30	25	20
11	70	65	35	40	35	20	30	25	20
12	80	75	45	50	45	30	40	35	30
13	90	85	55	60	55	40	50	45	40
14	100	95	65	70	65	50	50	45	40
15	100	100	75	80	75	60	50	45	40
16	100	100	85	90	85	70	60	55	50
17	100	100	95	100	85	70	60	55	50
18	100	100	100	100	85	70	70	55	50
19	100	100	100	100	85	70	70	65	60
20	100	100	100	100	85	70	70	65	60
21	100	100	100	100	85	70	70	65	60
22	100	100	100	100	100	85	85	80	75
23	100	100	100	100	100	100	100	80	75
24	100	100	100	100	100	100	100	95	90
25	100	100	100	100	100	100	100	95	90
26	100	100	100	100	100	100	100	95	90
27	100	100	100	100	100	100	100	95	90
28	100	100	100	100	100	100	100	95	90
29	100	100	100	100	100	100	100	95	90
30	100	100	100	100	100	100	100	95	90

Table 7-5 R-t under active pricing (millions of HKD)

t	Scenario								
	1	2	3	4	5	6	7	8	9
1	221	369	516	466	728	1020	1145	1380	1598
2	230	384	537	400	757	1060	1194	1429	1653
3	239	399	559	556	437	1101	437	1481	1705
4	249	415	581	486	881	1145	480	1533	1761
5	259	432	605	534	521	1190	1388	1588	1816
6	270	449	629	737	1039	1236	1854	588	1869
7	280	467	654	646	639	1285	1934	1943	1925
8	292	486	681	889	1229	1335	2012	2049	1978
9	304	506	708	965	1299	1388	2098	2119	2033
10	316	526	737	1043	1368	1441	2182	2188	2089
11	328	547	766	946	944	902	946	946	874
12	342	569	797	1040	1041	1031	1039	1037	1031
13	355	592	829	1143	1145	1128	3552	3394	3577
14	370	616	863	1258	1260	1250	3730	3897	3851
15	385	641	897	1386	1385	1384	1381	1383	1377
16	400	667	933	1523	2880	3106	4622	4785	4780
17	416	693	971	2287	3064	3260	1674	5054	5014
18	433	721	1010	2510	3257	3424	5625	1842	1841
19	450	750	1050	2737	3451	3588	5858	6196	6219
20	468	780	1092	2968	3656	3752	6123	6415	6403
21	487	811	1136	3208	2454	2453	2449	2452	2450
22	506	844	1182	3451	4467	2697	2700	8150	8200
23	527	878	1229	3696	4717	5379	9071	2966	2961
24	548	913	1278	3943	4967	5627	9420	10004	10116
25	570	949	1329	4201	5215	5894	9820	10337	10489
26	592	987	1382	4460	5480	6135	10141	10735	10759
27	616	1027	1437	4734	5744	6424	10515	11030	11095
28	640	1067	1494	4992	6031	6683	10955	11392	11406
29	666	1110	1554	5287	6289	6967	11297	11836	11799
30	692	1154	1616	5561	6570	7247	11708	12154	12171

Table 7-6 P-t under passive pricing (dpt)

t	Scenario								
	1	2	3	4	5	6	7	8	9
1	7.0	11.7	16.4	19.9	11.8	16.6	20.0	20.0	20.0
2	7.2	12.0	16.8	19.9	12.1	16.9	20.0	19.9	20.0
3	7.3	12.2	17.1	20.0	20.0	17.3	20.0	19.9	20.0
4	7.5	12.5	17.4	19.9	20.0	17.7	19.9	19.9	20.0
5	7.6	12.7	17.8	19.9	20.0	18.1	19.9	20.0	20.0
6	7.8	13.0	18.1	19.9	20.0	18.6	19.9	20.0	20.0
7	7.9	13.2	18.5	20.0	20.0	19.1	20.0	20.0	20.0
8	8.1	13.5	18.9	20.0	20.0	19.7	20.0	19.9	20.0
9	8.3	13.8	19.3	20.0	20.0	20.0	20.0	19.9	20.0
10	8.4	14.0	19.6	25.0	20.0	20.0	20.0	19.9	20.0
11	8.6	14.3	20.0	25.0	20.0	20.0	19.9	19.9	20.0
12	8.8	14.6	20.4	34.9	20.0	20.0	19.9	19.9	20.0
13	8.9	14.9	20.9	35.0	20.0	20.0	19.9	19.9	19.9
14	9.1	15.2	21.3	35.0	20.0	20.0	19.9	19.9	19.9
15	9.3	15.5	21.7	34.9	20.0	20.0	20.0	19.9	20.0
16	9.5	15.8	22.1	44.9	30.0	20.0	20.0	20.0	20.0
17	9.7	16.1	22.6	44.9	29.9	20.0	19.9	20.0	19.9
18	9.9	16.4	23.0	44.9	30.0	20.0	20.0	19.9	19.9
19	10.1	16.8	23.5	54.7	39.9	20.0	20.0	19.9	20.0
20	10.3	17.1	23.9	54.9	39.9	30.0	19.9	19.9	20.0
21	10.5	17.5	24.4	54.8	39.9	30.0	30.0	20.0	20.0
22	10.7	17.8	24.9	69.8	54.8	45.0	29.9	20.0	19.9
23	10.9	18.2	25.4	69.6	54.9	44.9	30.0	34.8	20.0
24	11.1	18.5	25.9	69.7	54.8	44.9	29.7	35.0	34.8
25	11.3	18.9	26.5	84.7	54.7	44.9	29.9	34.8	34.9
26	11.6	19.3	27.0	84.8	69.8	59.9	29.8	34.8	34.7
27	11.8	19.7	27.6	84.8	69.7	59.7	44.6	34.7	34.7
28	12.1	20.1	28.1	99.4	69.9	60.0	44.7	34.8	34.9
29	12.3	20.5	28.7	100.0	84.8	74.9	44.7	34.9	34.7
30	12.6	20.9	29.3	99.8	84.5	74.9	44.6	49.6	50.0

Table 7-7 P_c-t under passive pricing (dpt)

t	Scenario								
	1	2	3	4	5	6	7	8	9
1	20	20	20	20	20	20	20	20	20
2	25	20	20	20	20	20	20	20	20
3	30	25	20	20	20	20	20	20	20
4	35	30	20	20	20	20	20	20	20
5	40	35	20	20	20	20	20	20	20
6	45	40	20	20	20	20	20	20	20
7	50	45	20	20	20	20	20	20	20
8	55	50	20	20	20	20	20	20	20
9	60	55	25	20	20	20	20	20	20
10	65	60	30	25	20	20	20	20	20
11	70	65	35	25	20	20	20	20	20
12	80	75	45	35	20	20	20	20	20
13	90	85	55	35	20	20	20	20	20
14	100	95	65	35	20	20	20	20	20
15	100	100	75	35	20	20	20	20	20
16	100	100	85	45	30	20	20	20	20
17	100	100	95	45	30	20	20	20	20
18	100	100	100	45	30	20	20	20	20
19	100	100	100	55	40	20	20	20	20
20	100	100	100	55	40	30	20	20	20
21	100	100	100	55	40	30	30	20	20
22	100	100	100	70	55	45	30	20	20
23	100	100	100	70	55	45	30	35	20
24	100	100	100	70	55	45	30	35	35
25	100	100	100	85	55	45	30	35	35
26	100	100	100	85	70	60	30	35	35
27	100	100	100	85	70	60	45	35	35
28	100	100	100	100	70	60	45	35	35
29	100	100	100	100	85	75	45	35	35
30	100	100	100	100	85	75	45	50	50

Table 7-8 R-t under passive pricing (millions of HKD)

t	Scenario								
	1	2	3	4	5	6	7	8	9
1	221	369	516	466	728	1020	1145	1380	1598
2	230	384	537	500	757	1060	1194	1429	1653
3	239	399	559	536	790	1101	1244	1481	1705
4	249	415	581	572	830	1145	1292	1533	1761
5	259	432	605	608	871	1190	1345	1588	1816
6	270	449	629	646	912	1236	1397	1643	1869
7	280	467	654	684	954	1285	1455	1696	1925
8	292	486	681	724	997	1335	1511	1749	1978
9	304	506	708	763	1041	1388	1567	1805	2033
10	316	526	737	885	1086	1441	1622	1860	2089
11	328	547	766	936	1132	1494	1675	1920	2142
12	342	569	797	1144	1178	1546	1734	1977	2197
13	355	592	829	1217	1225	1598	1791	2032	2249
14	370	616	863	1292	1274	1651	1856	2092	2302
15	385	641	897	1367	1322	1702	1919	2149	2358
16	400	667	933	1634	1674	1752	1980	2211	2410
17	416	693	971	1734	1740	1803	2038	2271	2458
18	433	721	1010	1834	1815	1853	2106	2326	2510
19	450	750	1050	2151	2239	1903	2170	2388	2567
20	468	780	1092	2285	2335	2281	2230	2445	2621
21	487	811	1136	2412	2431	2361	3221	2511	2672
22	506	844	1182	2910	3106	3085	3315	2573	2717
23	527	878	1229	3080	3245	3196	3427	4209	2774
24	548	913	1278	3259	3383	3314	3506	4350	4542
25	570	949	1329	3848	3519	3432	3634	4445	4677
26	592	987	1382	4074	4329	4328	3725	4566	4760
27	616	1027	1437	4301	4511	4476	5414	4675	4871
28	640	1067	1494	4992	4711	4656	5596	4816	5018
29	666	1110	1554	5287	5624	5665	5768	4945	5100
30	692	1154	1616	5561	5849	5878	5927	6941	7246

Comparing Table 7-3 and 7-6, we could see the difference between active and passive pricing strategies. The latter seeks to optimize the revenues in the individual periods, and because of the fact that traffic volume is monotonically increasing over time, the tolls maximizing the revenues in each period will increase monotonically too; however, in the case of the former, since this strategy seeks to maximize the overall NPV, there're several tolls in certain years which are notably low and several notably high, in other words, the volatility of tolls between consecutive periods are much higher. For example, the toll can be increased from 6 dpt to 30 dpt, and then be reduced to 7.8 dpt, and then jumped to a much higher level of 35 dpt. When the traffic is low, the tolls are the same for the two pricing strategies. This is because that even the maximum revenues in each period are less than the minimum levels accordingly stipulated in the contract. In this situation, the active and passive pricing strategies are essentially the same. However, when the traffic is on a medium or high level, since in some years revenues higher than the minimum revenues are achievable through certain patterns of pricing, the concessionaire may seek a higher revenue in one period by artificially reducing the revenue in the prior period, resulting in a series of toll escalations. Therefore the actual tolls when the traffic is high are much higher than those when the traffic is low.

Comparing Table 7-4 and 7-7, we could also see the implications by the two pricing strategies. As aforementioned, the active pricing strategy may allow for toll escalations while the passive cannot (except for the cases in which the maximum revenues are less than the minimum levels), therefore the price caps for the active pricing strategy are no less than those for the passive in all scenarios; vice versa, high price caps allow higher actual tolls, and therefore higher revenues.

Comparing Table 7-5 and 7-8, we could see the stark difference in patterns of revenues brought by the two pricing strategies. When the traffic is low, revenues are the same as aforementioned. However, with higher traffic levels, revenues in the active pricing strategy setting is much more volatile than those in the passive setting, that is, in some years, as low as the minimum levels, while in others, even as high as twice of the latter. Especially when the traffic is high, and in the last several periods, the revenues under the active pricing setting are usually far higher, which can offset the prior lower revenues. The net present value of the former strategy is often higher than that of the latter, since the traffic is increasing monotonically over time, which can be seen through the analysis as follows.

Table 7-9 shows the maximum NPV under an active pricing strategy in a concession contract with a toll adjustment mechanism, the maximum NPV under a passive pricing strategy in a concession contract with a toll adjustment mechanism, and the maximum NPV under a pure price regulation mechanism:

Table 7-9 NPV under three different pricings (millions of HKD)

Scenario	1	2	3	4	5	6	7	8	9
NPV(active)	2310	3849	5389	7931	9970	12114	15813	16816	17298
NPV(passive)	2310	3849	5389	7436	8971	11070	12389	14094	15660
NPV(pure PCs)	2310	3849	5389	6086	8223	10661	12761	14646	16330

From Table 7-9 we can see that in Scenario 1 to 3, the two pricing strategies are actually the same. That is, however, not to say that the active pricing strategy is inferior to the passive one. On the one hand, the former performs the same in the first three scenarios as the latter; and better in the other six scenarios. Considering this fact, we can safely say that the active pricing strategy is superior, or in a more conservative way, no inferior to the passive pricing strategy. On the other hand, the active pricing strategy doesn't perform better than the passive one in the first three cases is not because the strategy itself is defective, but that the traffic level in the future is too low to generate enough profits to make it financially acceptable for the project, though it still can be economically feasible to the government taking into the externalities it brought to the society. In this case, the government should devise more incentives other than the toll adjustment mechanism in the contract, such as subsidies or guarantees, etc., to make the project attractive for private investors, otherwise no tenders will be proposed at all. In fact this is the very situation facing the Western Harbour Crossing Company now. Due to a variety of reasons, the actual traffic in the tunnel is far less than the projected level, only about one quarter of the designed capacity after nearly 20 years of operations in a 30-year concession contract. In face of this, even the toll adjustment mechanism cannot protect the concessionaire from the far-less-than-expected or even negative NPV, which may not be sufficient to recoup the huge initial capital investment

(2310 million HKD, 3849 million HKD, and 5389 million HKD are all less than the 7 billion HKD cost).

In Scenario 4 to 9, all the NPVs of both pricing strategies are greater than 7 billion HKD, and in each scenario, NPV of the active pricing strategy is higher than that of the passive, which proves the superiority of the former to the latter. Especially in Scenario 7 to 9, if the passive pricing strategy is adopted, the NPV in each scenario is even less than that in a pure price caps regulation contract; however, an active pricing strategy still performs with flying colors. Details of comparison are as follows.

The Real Options Values of the toll adjustment mechanism under both the active pricing strategy and the passive pricing strategy are shown in Table 7-10:

Table 7-10 Real Options Values (ROV) under two pricings (millions of HKD)

Scenario	1	2	3	4	5	6	7	8	9
ROV(active)	0	0	0	1845	1747	1452	3052	2170	969
ROV(passive)	0	0	0	1350	748	409	-371	-552	-670

As analyzed above, the active pricing strategy is strictly superior to the passive pricing strategy. When the traffic level is medium, both are feasible but the former is obviously better; when the traffic is high, only the former is feasible; and only through the active pricing can the intrinsic potentiality of the toll adjustment mechanism be exploited to the maximum extent. From Table 7-10 we can also see that the toll adjustment mechanism is of significance to the concessionaire if the traffic is medium or not very high. On the one hand, if the traffic is so low that even the maximum revenues are less than the minimum levels stipulated in the contract, the toll adjustment mechanism is pointless, in other words, it will work, toll escalations will be allowed, but the revenues are still low; On the other hand, if the traffic is so high that even a pure price caps regulations mechanism can guarantee the revenues to be beyond the maximum levels stipulated in the contract, there's no point to seek for multiple toll escalations through the toll adjustment mechanism, in that higher price caps won't help to increase revenues, which is, exactly, the same with the case of low traffic.

Impact of VOTs and levels of traffic demand on the concessionaire's pricing strategies (Ps), price caps (P_cs) and revenues (Rs) will be discussed.

Impacts of VOTs to Ps, P_cs and Rs under the active pricing strategy

To see the impacts of different values-of-time (VOTs) on the concessionaire's pricing strategies, price caps and revenues, differentiating the traffic demand is necessary. When traffic demand is low (Scenario 1 to 3), impacts of different VOTs to concessionaire's pricing strategies, price caps and revenues are shown in Figure 7-2 to Figure 7-4:

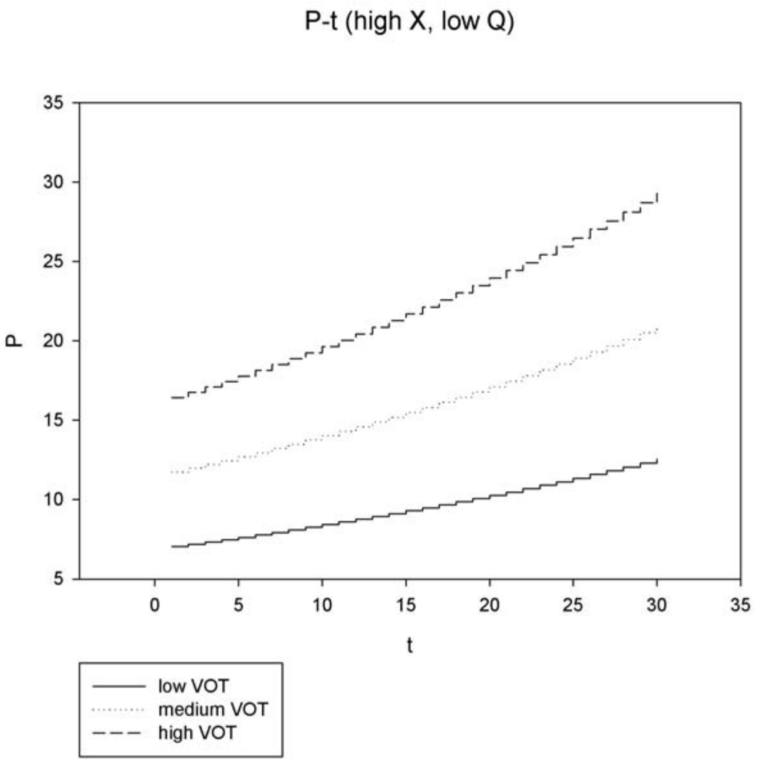

Figure 7-2 Active pricing with low traffic

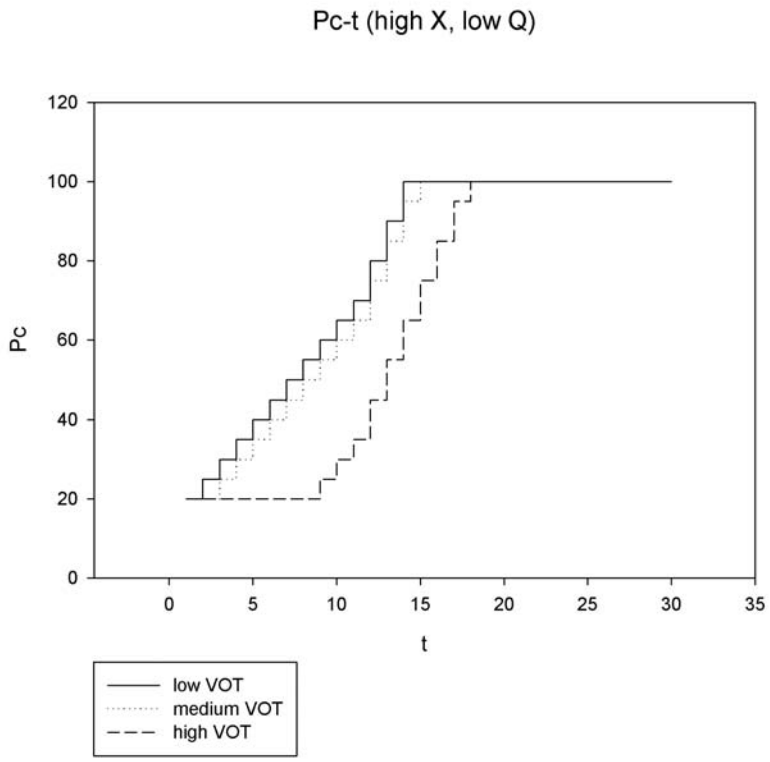

Figure 7-3 Price caps under active pricing with low traffic

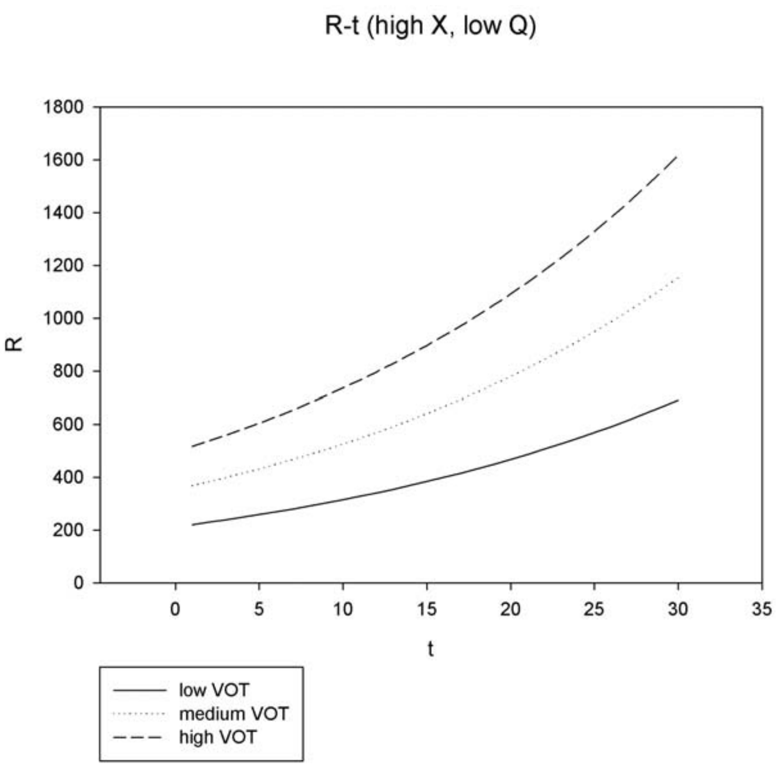

Figure 7-4 Revenues under active pricing with low traffic

With a low or medium level of VOT and a low level of traffic, the concessionaire will maximize its revenues in each individual period, and even though, they are still less than the minimum levels stipulated in the contract, thus the price caps increase from 20 dpt to the maximum level 100 dpt in 15 years; However, with a high level of VOT, in the first ten years, the maximum revenues are higher than the minimum levels, therefore the price caps remain at 20 dpt for those years. However, hence after, the revenues are less than the minimum revenues, like the situations with low and medium VOTs. Revenues of high VOT is superior to those with medium VOT, which, again, is superior to those with low VOT.

When traffic demand is medium (Scenario 4 to 6), impacts of different VOTs to concessionaire's pricing strategies, price caps and revenues are shown in Figure 7-5 to Figure 7-7:

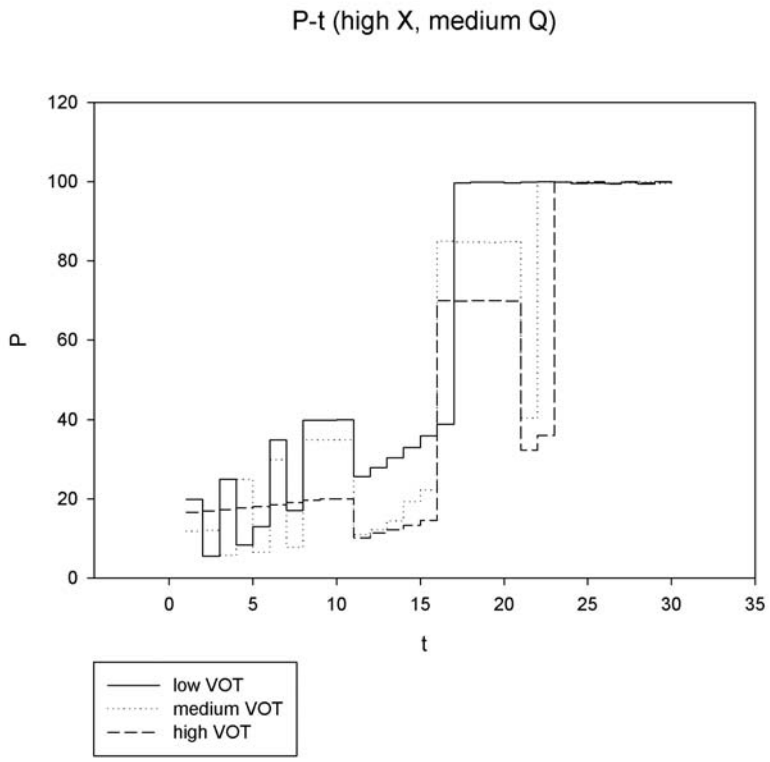

Figure 7-5 Active pricing with medium traffic

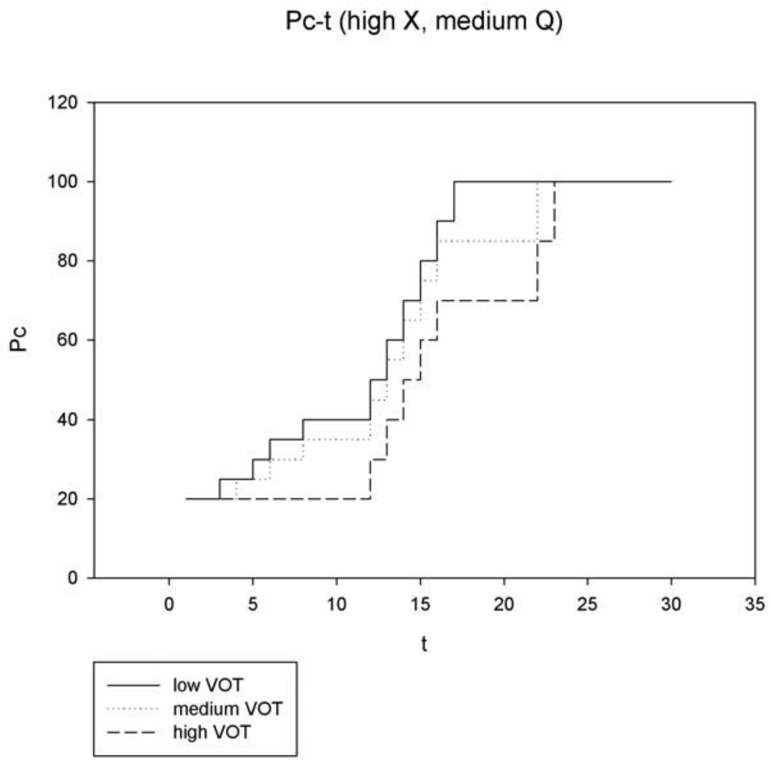

Figure 7-6 Price caps under active pricing with medium traffic

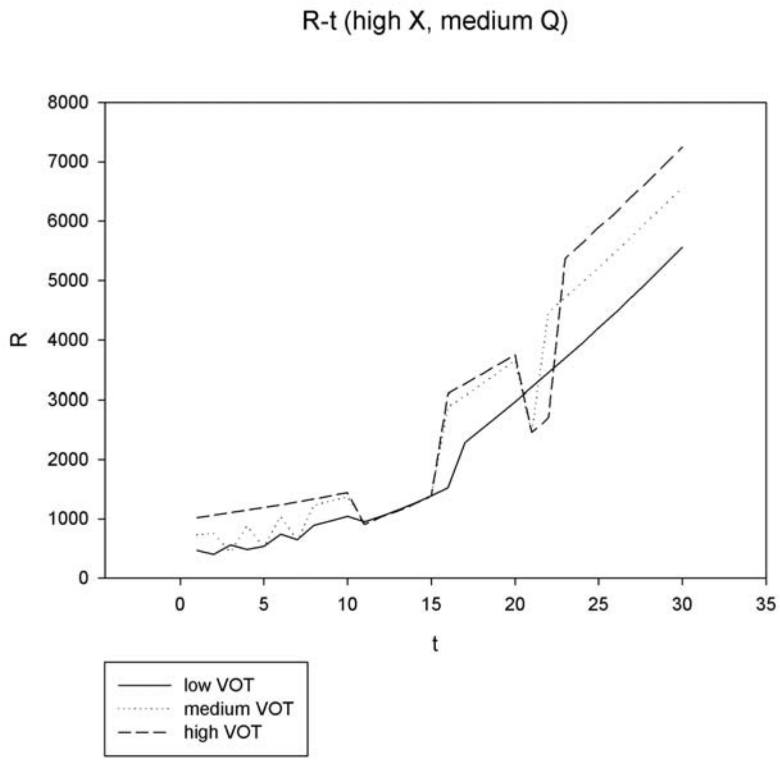

Figure 7-7 Revenues under active pricing with medium traffic

From Figure 7-5 we could see that a dynamic pricing strategy is adopted by the concessionaire and the tolls in the final stage reach the maximum level stipulated in the contract. From Figure 7-6 we could see that the higher the VOT is, the slower the price cap reaches the maximum level. From Figure 7-7 we could see that through the dynamic pricing, revenues are arranged to maximize the NPV in a zigzag form. The higher the VOT is, the higher the revenues are.

When traffic demand is high (Scenario 7 to 9), impacts of different VOTs to concessionaire's pricing strategies, price caps and revenues are shown in Figure 7-8 to Figure 7-10:

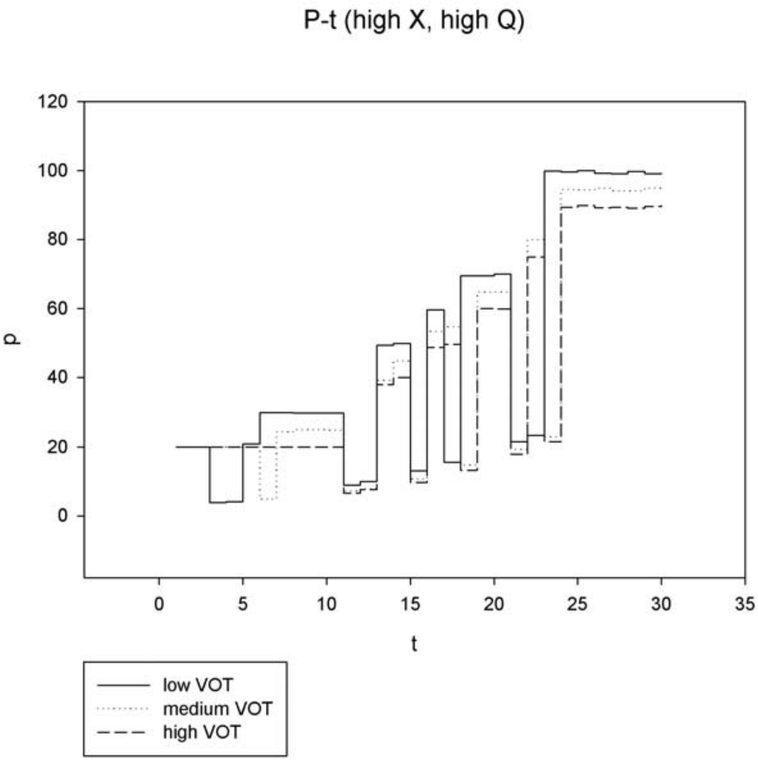

Figure 7-8 Active pricing with high traffic

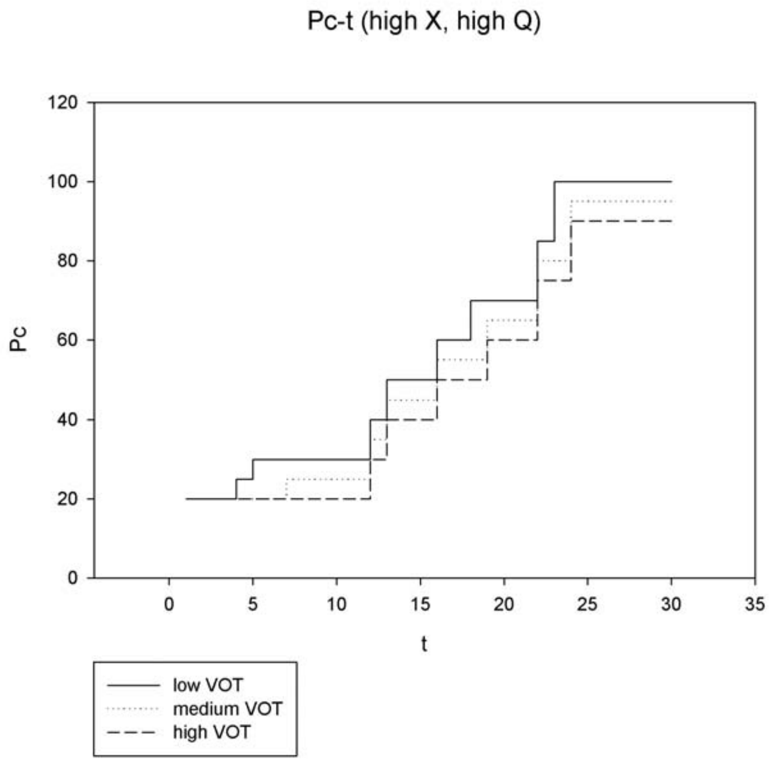

Figure 7-9 Price caps under active pricing with high traffic

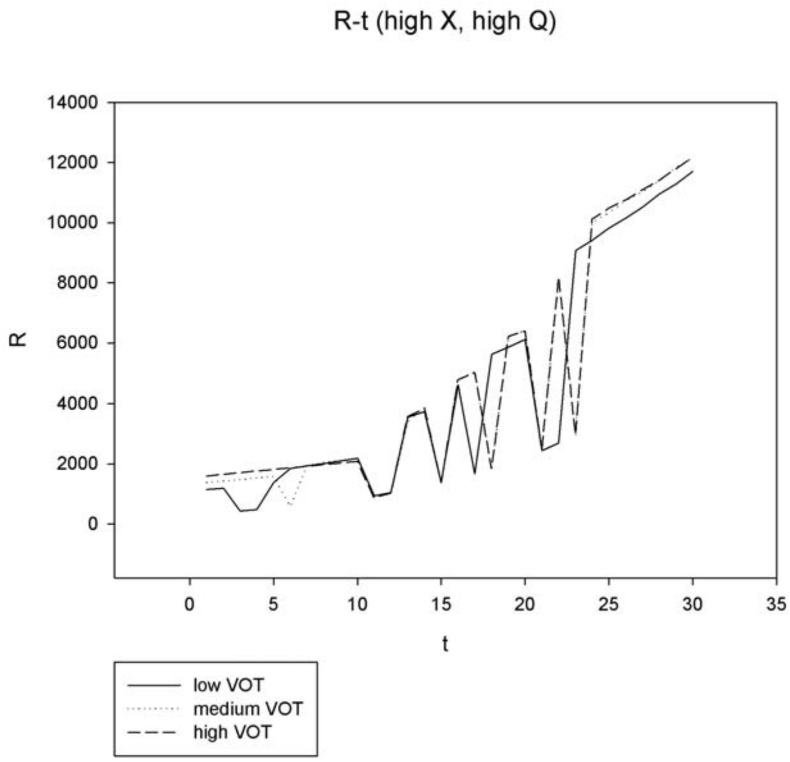

Figure 7-10 Revenues under active pricing with high traffic

From Figure 7-8 we could see that the pricing strategy adopted by the concessionaire is even more dynamic and only the tolls with low VOT reach the maximum level stipulated in the contract in the final stage. From Figure 7-9 we could still see that the higher the VOT is, the slower the price cap reaches its maximum level. From Figure 7-10 we could see that through the more dynamic pricing, revenues are arranged to maximize the NPV in a zigzag form which is more dynamic too. The revenues in consecutive years are more volatile than those in the previous case. In the final stage, the revenues with medium and high VOTs are almost the same, because of the existence and function of the revenue caps, which can limit the revenues even though the traffic is huge.

Impacts of Qs to Ps, P_cs and Rs under the active pricing strategy

To see the impacts of different traffic demands (Qs) on the concessionaire's pricing strategies, price caps and revenues, differentiating values-of-time (VOTs) is necessary. When traffic VOT is low (Scenario 1, 4, and 7), impacts of different Qs to concessionaire's pricing strategies, price caps and revenues are shown in Figure 7-11 to Figure 7-13:

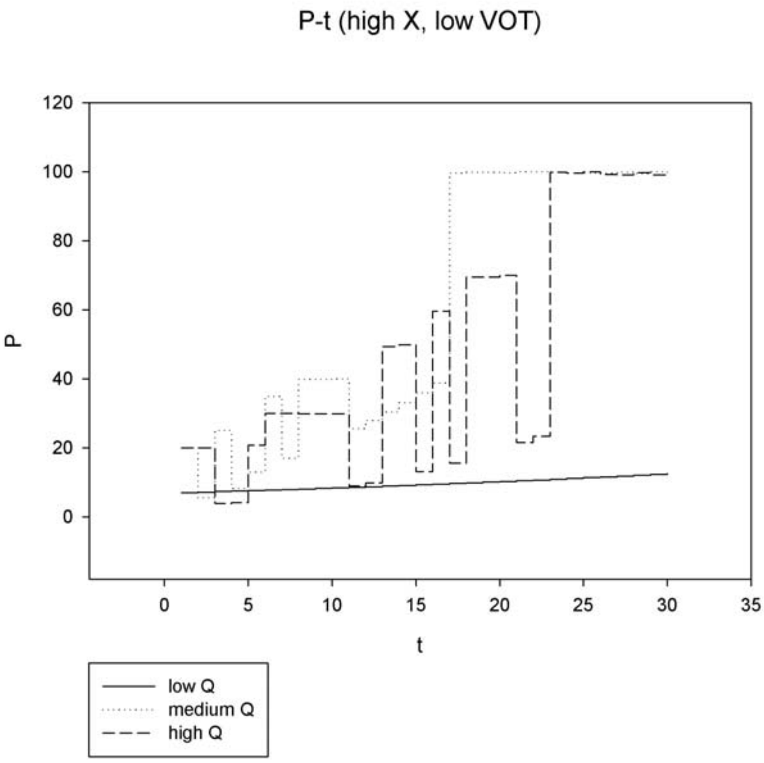

Figure 7-11 Active pricing with low VOT

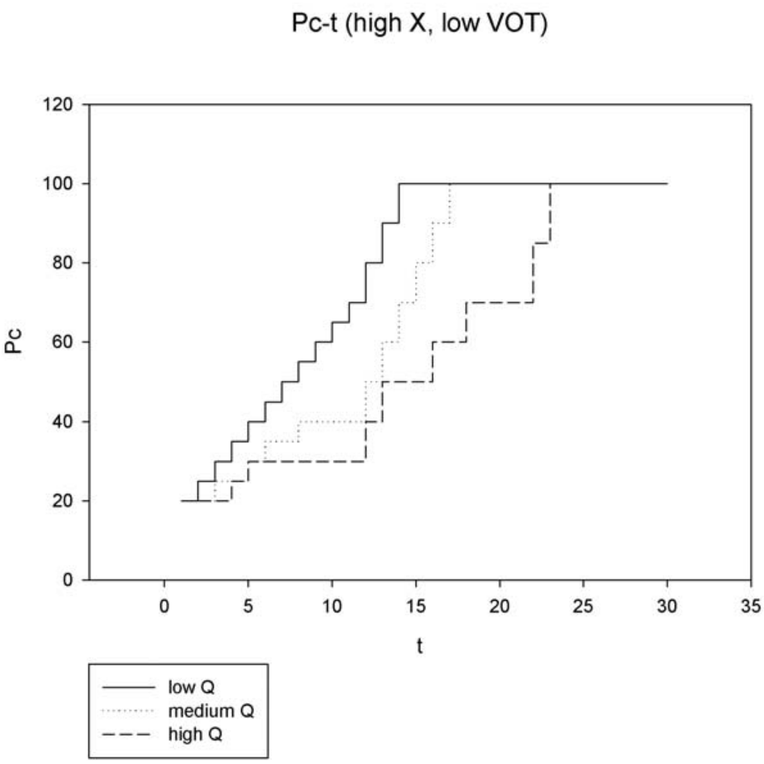

Figure 7-12 Price caps under active pricing with low VOT

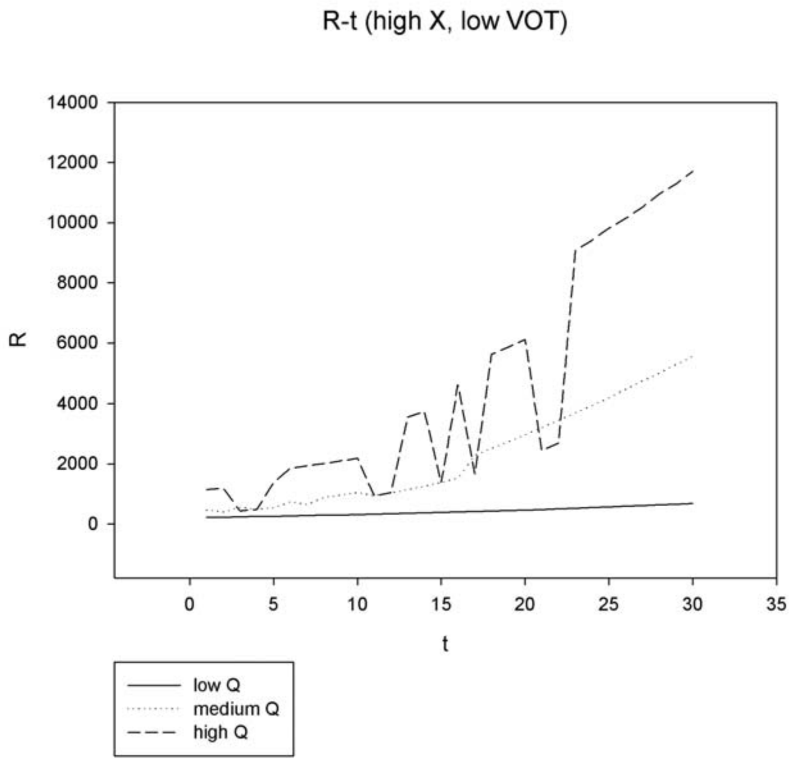

Figure 7-13 Revenues under active pricing with low VOT

From Figure 11 to 13 it is straightforward to see that the impact of traffic demand on the performance of the project is far more significant than that of the VOTs, that is to say, traffic demand risk, which can determine the success of the project, is of the foremost concern to the concessionaire. In Figure 7-11, when the traffic demand is low, the optimal prices are far less than the price caps, as analyzed before; when the traffic demand is medium or high, the actual tolls will reach the maximum level. From Figure 7-12 we could see that the higher the traffic demand is, the slower the price cap reaches the maximum level. From Figure 7-13 we could see that the differences between revenues in three levels of traffic demand are much larger than those between revenues in three levels of VOTs. When the traffic demand is high, the revenues are much more dynamic than those when the traffic demand is low or medium, in which cases the annual revenue fluctuates in a more smooth way.

When traffic VOT is medium (Scenario 2, 5, and 8), impacts of different Qs to concessionaire's pricing strategies, price caps and revenues are show in Figure 7-14 to Figure 7-16:

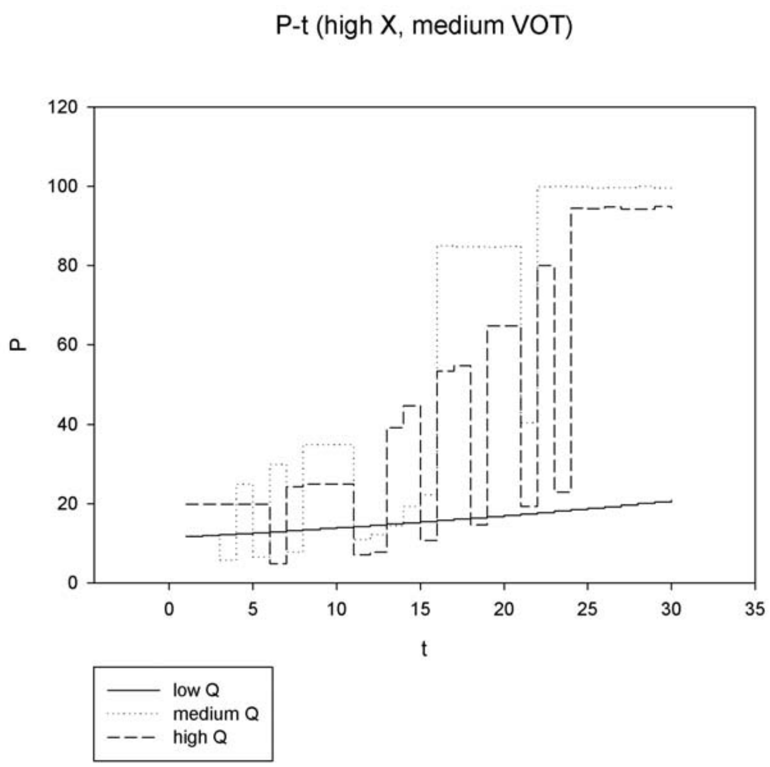

Figure 7-14 Active pricing with medium VOT

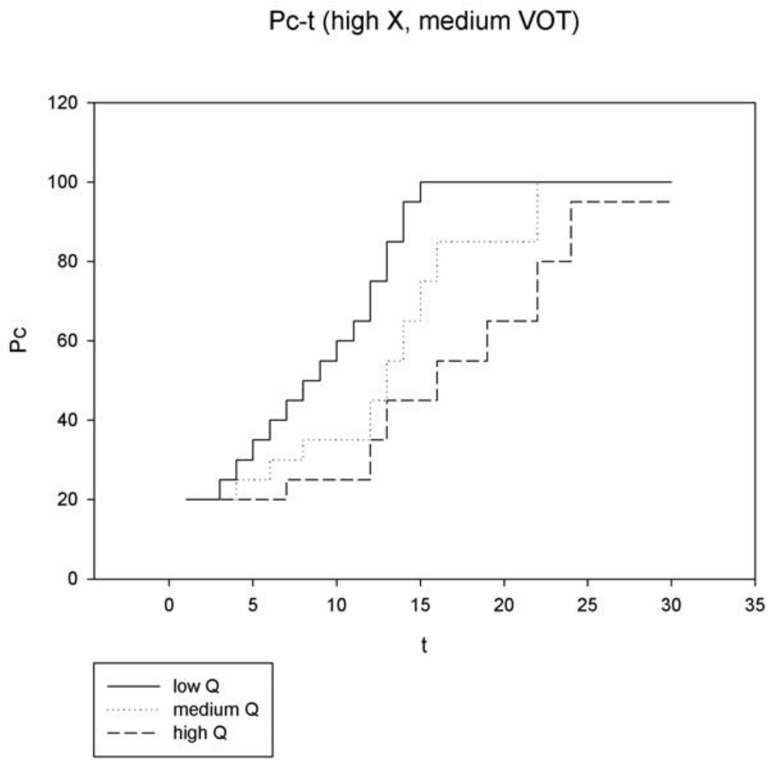

Figure 7-15 Price caps under active pricing with medium VOT

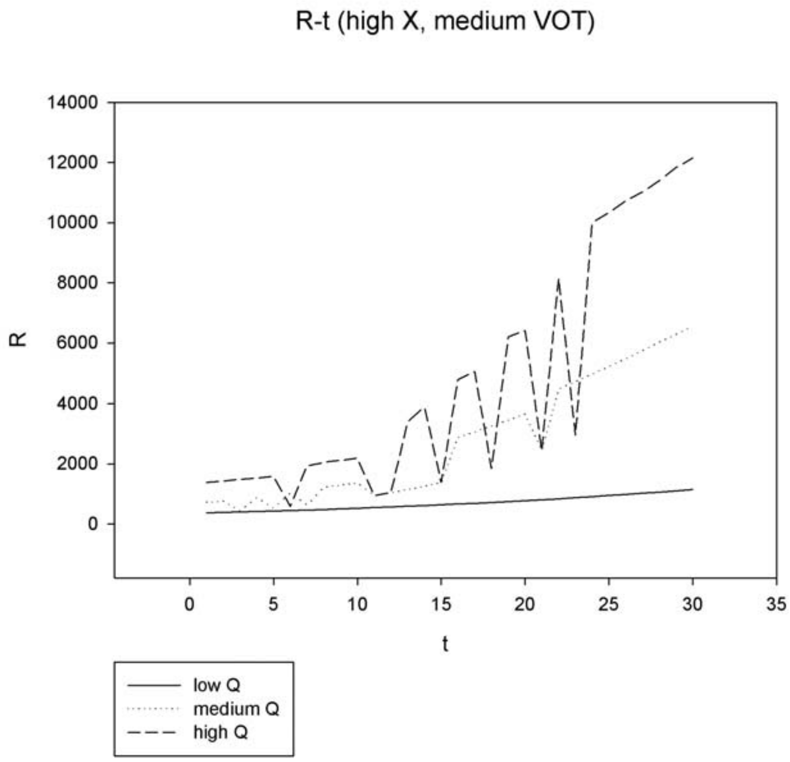

Figure 7-16 Revenues under active pricing with medium VOT

Figure 7-14 shows that only with medium traffic demand that the actual final toll can reach the maximum level; the final actual toll for high traffic demand is slightly smaller while those for the low traffic demand is much less than the price caps. Figure 7-15 shows that in each period, the price caps for the low traffic demand scenario is higher than either that for medium or high traffic demand scenario. Figure 7-16 shows that with a higher level of VOT, the dynamic feature of the revenues magnifies and the adjustment tend to be in a periodical fashion. Attention should be paid that in the final stages, the revenues under high traffic demand are as high as twice and even ten times of the revenues under medium and low traffic demand respectively.

When traffic VOT is high (Scenario 3, 6, and 9), impacts of different Qs to concessionaire's pricing strategies, price caps and revenues are shown in Figure 7-17 to Figure 7-19:

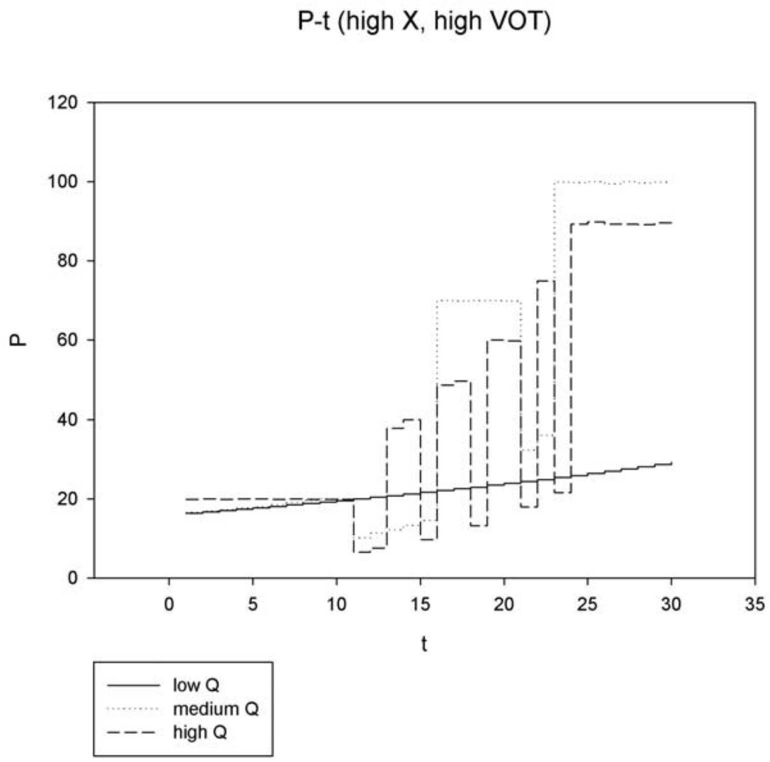

Figure 7-17 Active pricing with high VOT

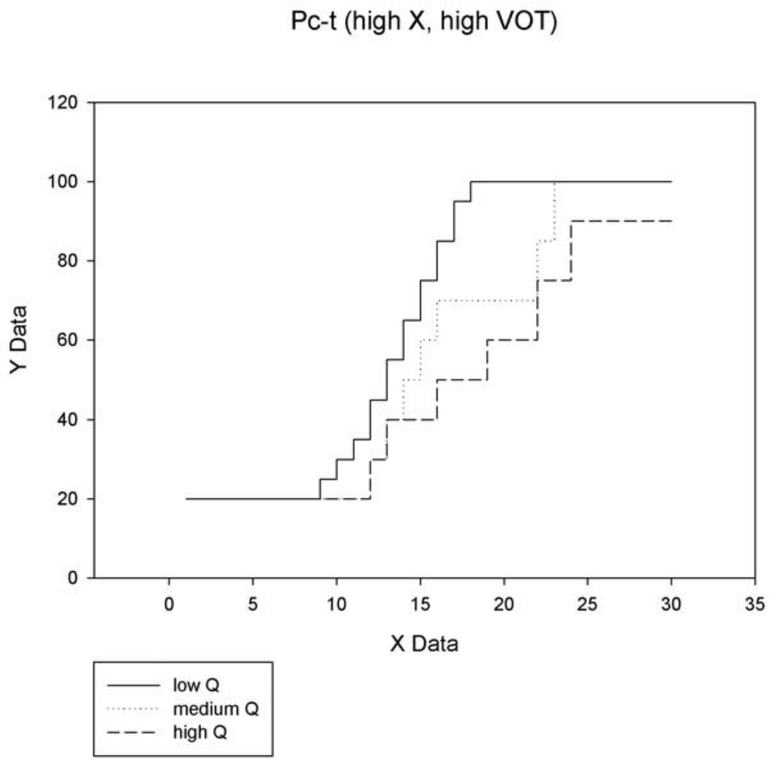

Figure 7-18 Price caps under active pricing with high VOT

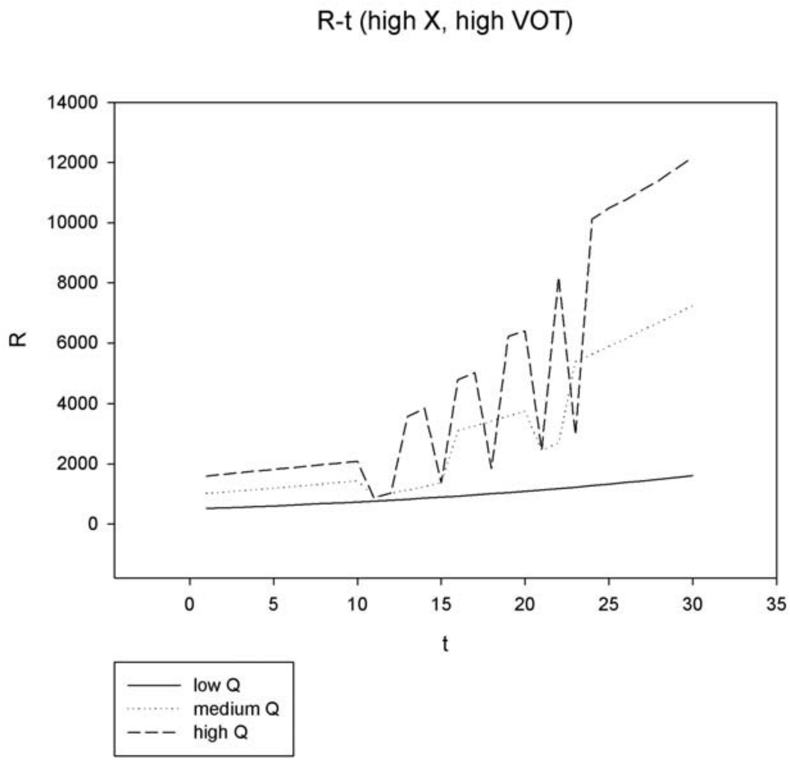

Figure 7-19 Revenues under active pricing with high VOT

Figure 7-17 shows that only with medium traffic demand that the actual final toll can reach the maximum level and adjustments of tolls under high traffic demand are more frequent. Figure 7-18 shows that in each period, the price caps for the low traffic demand scenario are higher than either that for medium or high traffic demand scenario. Figure 7-19 shows that with a higher level of VOT, the dynamic feature of the revenues magnifies and the adjustment tends to be in a periodical fashion. Attention should also be paid that in the final stages, the revenues under high traffic demand are as high as twice and even ten times of the revenues under medium and low traffic demand respectively.

Toll Adjustment Restriction Factor X

The original consideration for the government's provision of the toll adjustment mechanism is to prevent the private investor from possibly adverse traffic conditions, in case of which, the concessionaire may adjust the toll to increase the revenues to compensate the negative impacts by a too low level of traffic demand. To the government, the proper behavior of the private investor should be that in any year it charges the toll at the level which can maximize the revenue for that particular year, within the price cap for that year, that is, the passive pricing strategy aforementioned. In the active pricing strategy, the fluctuation of the tolls from year to year can be so volatile that may irritate the commuters who have been accustomed to periodical toll adjustment usually based on inflation and production improvement, therefore the extent of which is often predictable and monotonic.

Thus the government may deem the active pricing strategy as opportunistic behaviors and restrict the frequent toll adjustments and speculative pricing strategy (charging far less than the level permitted to earn the revenue at the minimum level stipulated in the contract, artificially creating a chance for toll adjustment in the following period) by introducing X, the toll adjustment restriction factor, which means that the rate of the toll in any year to that in the previous year shall not exceed X. For example, if X=2, in year t the price corresponding to the minimum level of revenue is 5 dpt, by doing so the price cap for the next year t+1 can be increased from 20 dpt to 25 dpt, suppose that 25 dpt can maximize the revenue of year t+1; in the previous analysis without X, it is possible for the concessionaire to charge 5 dpt at year t and 25 dpt at year t+1; however, under the new setting, if the toll for year t is 5 dpt, the maximum toll for year t+1 is 10 dpt rather than 20 or 25 dpt.

In theory, this will effectively suppress the speculative practice through the abusing of the toll adjustment mechanism by the private investor, for that artificially charging a low toll may not be able to induce the expected toll escalation. The new price cap should be the minimum of the previous price cap plus the toll adjustment extent stipulated, and the previous actual toll times X.

We will see the effect of introduction of X into the concession contract with toll adjustment mechanism, other conditions and parameters being the same. The results are listed in Table 7-11 to Table 7-16.

Table 7-11 P-t under active pricing with X=2 (dpt)

t	Scenario								
	1	2	3	4	5	6	7	8	9
1	7.0	11.7	16.4	19.9	11.8	16.6	20.0	20.0	20.0
2	7.2	12.0	16.8	19.9	12.1	16.9	20.0	19.9	20.0
3	7.3	12.2	17.1	20.0	20.0	17.3	20.0	19.9	20.0
4	7.5	12.5	17.4	8.4	20.0	17.7	19.9	19.9	20.0
5	7.6	12.7	17.8	13.0	20.0	18.1	19.9	20.0	20.0
6	7.8	13.0	18.1	15.1	20.0	18.6	19.9	20.0	20.0
7	7.9	13.2	18.5	30.0	20.0	19.1	20.0	20.0	20.0
8	8.1	13.5	18.9	34.9	20.0	19.7	20.0	19.9	20.0
9	8.3	13.8	19.3	34.9	20.0	20.0	20.0	19.9	20.0
10	8.4	14.0	19.6	34.9	10.0	20.0	20.0	19.9	20.0
11	8.6	14.3	20.0	25.6	11.0	10.2	9.0	7.2	6.6
12	8.8	14.6	20.4	27.9	12.2	11.4	9.9	7.8	7.6
13	8.9	14.9	20.9	30.4	14.5	12.3	10.9	8.6	8.2
14	9.1	15.2	21.3	33.0	19.4	13.4	12.0	9.6	16.4
15	9.3	15.5	21.7	36.0	22.3	14.6	13.1	10.7	32.8
16	9.5	15.8	22.1	71.8	25.1	29.2	14.3	12.0	49.7
17	9.7	16.1	22.6	84.8	50.1	58.3	28.3	23.9	49.6
18	9.9	16.4	23.0	84.7	84.8	69.9	56.5	47.8	49.9
19	10.1	16.8	23.5	84.9	84.7	70.0	79.8	79.9	50.0
20	10.3	17.1	23.9	84.6	84.9	69.9	79.9	79.6	50.0
21	10.5	17.5	24.4	56.3	85.0	32.4	79.9	79.9	17.9
22	10.7	17.8	24.9	100.0	84.9	36.1	79.6	80.0	19.6
23	10.9	18.2	25.4	99.9	47.9	71.9	79.8	79.9	21.6
24	11.1	18.5	25.9	99.6	95.3	99.8	79.9	79.6	43.0
25	11.3	18.9	26.5	99.7	99.6	100.0	79.8	79.8	85.6
26	11.6	19.3	27.0	99.6	99.7	99.5	79.4	79.8	94.7
27	11.8	19.7	27.6	99.9	99.6	100.0	79.6	79.7	94.9
28	12.1	20.1	28.1	99.4	100.0	99.7	79.6	79.2	94.9
29	12.3	20.5	28.7	100.0	99.6	99.8	79.4	79.4	94.6
30	12.6	20.9	29.3	99.8	99.6	99.8	79.8	79.4	94.0

Table 7-12 P_c-t under active pricing with X=2 (dpt)

t	Scenario								
	1	2	3	4	5	6	7	8	9
1	20	20	20	20	20	20	20	20	20
2	25	20	20	20	20	20	20	20	20
3	30	25	20	25	20	20	20	20	20
4	35	30	20	25	25	20	25	20	20
5	40	35	20	30	25	20	30	20	20
6	45	40	20	35	30	20	30	20	20
7	50	45	20	35	30	20	30	25	20
8	55	50	20	40	35	20	30	25	20
9	60	55	25	40	35	20	30	25	20
10	65	60	30	40	35	20	30	25	20
11	70	65	35	40	35	20	30	25	20
12	80	75	45	50	45	30	40	35	30
13	90	85	55	60	55	40	50	45	40
14	100	95	65	70	65	50	50	45	40
15	100	100	75	80	75	60	50	45	40
16	100	100	85	90	85	70	60	55	50
17	100	100	95	100	85	70	60	55	50
18	100	100	100	100	85	70	70	55	50
19	100	100	100	100	85	70	70	65	60
20	100	100	100	100	85	70	70	65	60
21	100	100	100	100	85	70	70	65	60
22	100	100	100	100	100	85	85	80	75
23	100	100	100	100	100	100	100	80	75
24	100	100	100	100	100	100	100	95	90
25	100	100	100	100	100	100	100	95	90
26	100	100	100	100	100	100	100	95	90
27	100	100	100	100	100	100	100	95	90
28	100	100	100	100	100	100	100	95	90
29	100	100	100	100	100	100	100	95	90
30	100	100	100	100	100	100	100	95	90

Table 7-13 R-t under active pricing with X=2 (millions of HKD)

t	Scenario								
	1	2	3	4	5	6	7	8	9
1	221	369	516	466	728	1020	1145	1380	1598
2	230	384	537	500	757	1060	1194	1429	1653
3	239	399	559	536	790	1101	1244	1481	1705
4	249	415	581	486	830	1145	1292	1533	1761
5	259	432	605	534	871	1190	1345	1588	1816
6	270	449	629	587	912	1236	1397	1643	1869
7	280	467	654	777	954	1285	1455	1696	1925
8	292	486	681	867	997	1335	1511	1749	1978
9	304	506	708	934	1041	1388	1567	1805	2033
10	316	526	737	1003	858	1441	1622	1860	2089
11	328	547	766	946	944	902	946	946	874
12	342	569	797	1040	1041	1031	1039	1037	1031
13	355	592	829	1143	1145	1128	1143	1144	1136
14	370	616	863	1258	1260	1250	1260	1258	2024
15	385	641	897	1386	1385	1384	1381	1383	3404
16	400	667	933	1955	1522	1955	1519	1523	4862
17	416	693	971	2218	2335	2943	2701	2619	5014
18	433	721	1010	2407	3257	3424	4805	4700	5190
19	450	750	1050	2603	3451	3588	6505	7319	5361
20	468	780	1092	2797	3656	3752	6784	7571	5523
21	487	811	1136	2449	3862	2453	7057	7865	2450
22	506	844	1182	3451	4068	2697	7320	8150	2686
23	527	878	1229	3696	2970	4352	7623	8425	2961
24	548	913	1278	3943	4828	5627	7919	8684	5446
25	570	949	1329	4201	5215	5894	8204	8996	10073
26	592	987	1382	4460	5480	6135	8476	9294	11313
27	616	1027	1437	4734	5744	6424	8798	9575	11690
28	640	1067	1494	4992	6031	6683	9108	9835	12045
29	666	1110	1554	5287	6289	6967	9403	10159	12372
30	692	1154	1616	5561	6570	7247	9761	10464	12664

Table 7-14 P-t under passive pricing with X=2 (dpt)

t	Scenario								
	1	2	3	4	5	6	7	8	9
1	7.0	11.7	16.4	19.9	11.8	16.6	20.0	20.0	20.0
2	7.2	12.0	16.8	19.9	12.1	16.9	20.0	19.9	20.0
3	7.3	12.2	17.1	20.0	20.0	17.3	20.0	19.9	20.0
4	7.5	12.5	17.4	19.9	20.0	17.7	19.9	19.9	20.0
5	7.6	12.7	17.8	19.9	20.0	18.1	19.9	20.0	20.0
6	7.8	13.0	18.1	19.9	20.0	18.6	19.9	20.0	20.0
7	7.9	13.2	18.5	20.0	20.0	19.1	20.0	20.0	20.0
8	8.1	13.5	18.9	20.0	20.0	19.7	20.0	19.9	20.0
9	8.3	13.8	19.3	20.0	20.0	20.0	20.0	19.9	20.0
10	8.4	14.0	19.6	25.0	20.0	20.0	20.0	19.9	20.0
11	8.6	14.3	20.0	25.0	20.0	20.0	19.9	19.9	20.0
12	8.8	14.6	20.4	34.9	20.0	20.0	19.9	19.9	20.0
13	8.9	14.9	20.9	35.0	20.0	20.0	19.9	19.9	19.9
14	9.1	15.2	21.3	35.0	20.0	20.0	19.9	19.9	19.9
15	9.3	15.5	21.7	34.9	20.0	20.0	20.0	19.9	20.0
16	9.5	15.8	22.1	44.9	30.0	20.0	20.0	20.0	20.0
17	9.7	16.1	22.6	44.9	29.9	20.0	19.9	20.0	19.9
18	9.9	16.4	23.0	44.9	30.0	20.0	20.0	19.9	19.9
19	10.1	16.8	23.5	54.7	39.9	20.0	20.0	19.9	20.0
20	10.3	17.1	23.9	54.9	39.9	30.0	19.9	19.9	20.0
21	10.5	17.5	24.4	54.8	39.9	30.0	30.0	20.0	20.0
22	10.7	17.8	24.9	69.8	54.8	45.0	29.9	20.0	19.9
23	10.9	18.2	25.4	69.6	54.9	44.9	30.0	34.8	20.0
24	11.1	18.5	25.9	69.7	54.8	44.9	29.7	35.0	34.8
25	11.3	18.9	26.5	84.7	54.7	44.9	29.9	34.8	34.9
26	11.6	19.3	27.0	84.8	69.8	59.9	29.8	34.8	34.7
27	11.8	19.7	27.6	84.8	69.7	59.7	44.6	34.7	34.7
28	12.1	20.1	28.1	99.4	69.9	60.0	44.7	34.8	34.9
29	12.3	20.5	28.7	100.0	84.8	74.9	44.7	34.9	34.7
30	12.6	20.9	29.3	99.8	84.5	74.9	44.6	49.6	50.0

Table 7-15 P_c-t under passive pricing with X=2 (dpt)

t	Scenario								
	1	2	3	4	5	6	7	8	9
1	20	20	20	20	20	20	20	20	20
2	25	20	20	20	20	20	20	20	20
3	30	25	20	20	20	20	20	20	20
4	35	30	20	20	20	20	20	20	20
5	40	35	20	20	20	20	20	20	20
6	45	40	20	20	20	20	20	20	20
7	50	45	20	20	20	20	20	20	20
8	55	50	20	20	20	20	20	20	20
9	60	55	25	20	20	20	20	20	20
10	65	60	30	25	20	20	20	20	20
11	70	65	35	25	20	20	20	20	20
12	80	75	45	35	20	20	20	20	20
13	90	85	55	35	20	20	20	20	20
14	100	95	65	35	20	20	20	20	20
15	100	100	75	35	20	20	20	20	20
16	100	100	85	45	30	20	20	20	20
17	100	100	95	45	30	20	20	20	20
18	100	100	100	45	30	20	20	20	20
19	100	100	100	55	40	20	20	20	20
20	100	100	100	55	40	30	20	20	20
21	100	100	100	55	40	30	30	20	20
22	100	100	100	70	55	45	30	20	20
23	100	100	100	70	55	45	30	35	20
24	100	100	100	70	55	45	30	35	35
25	100	100	100	85	55	45	30	35	35
26	100	100	100	85	70	60	30	35	35
27	100	100	100	85	70	60	45	35	35
28	100	100	100	100	70	60	45	35	35
29	100	100	100	100	85	75	45	35	35
30	100	100	100	100	85	75	45	50	50

Table 7-16 R-t under passive pricing with X=2 (millions of HKD)

t	Scenario								
	1	2	3	4	5	6	7	8	9
1	221	369	516	466	728	1020	1145	1380	1598
2	230	384	537	500	757	1060	1194	1429	1653
3	239	399	559	536	790	1101	1244	1481	1705
4	249	415	581	572	830	1145	1292	1533	1761
5	259	432	605	608	871	1190	1345	1588	1816
6	270	449	629	646	912	1236	1397	1643	1869
7	280	467	654	684	954	1285	1455	1696	1925
8	292	486	681	724	997	1335	1511	1749	1978
9	304	506	708	763	1041	1388	1567	1805	2033
10	316	526	737	885	1086	1441	1622	1860	2089
11	328	547	766	936	1132	1494	1675	1920	2142
12	342	569	797	1144	1178	1546	1734	1977	2197
13	355	592	829	1217	1225	1598	1791	2032	2249
14	370	616	863	1292	1274	1651	1856	2092	2302
15	385	641	897	1367	1322	1702	1919	2149	2358
16	400	667	933	1634	1674	1752	1980	2211	2410
17	416	693	971	1734	1740	1803	2038	2271	2458
18	433	721	1010	1834	1815	1853	2106	2326	2510
19	450	750	1050	2151	2239	1903	2170	2388	2567
20	468	780	1092	2285	2335	2281	2230	2445	2621
21	487	811	1136	2412	2431	2361	3221	2511	2672
22	506	844	1182	2910	3106	3085	3315	2573	2717
23	527	878	1229	3080	3245	3196	3427	4209	2774
24	548	913	1278	3259	3383	3314	3506	4350	4542
25	570	949	1329	3848	3519	3432	3634	4445	4677
26	592	987	1382	4074	4329	4328	3725	4566	4760
27	616	1027	1437	4301	4511	4476	5414	4675	4871
28	640	1067	1494	4992	4711	4656	5596	4816	5018
29	666	1110	1554	5287	5624	5665	5768	4945	5100
30	692	1154	1616	5561	5849	5878	5927	6941	7246

When with X=2, the maximum NPV under an active pricing strategy in a concession contract with a toll adjustment mechanism, the maximum NPV under a passive pricing strategy in a concession contract with a toll adjustment mechanism, and the maximum NPV under a pure price regulation mechanism are shown in Table 7-17:

Table 7-17 NPV under three different pricings (X=2) (millions of HKD)

Scenario	1	2	3	4	5	6	7	8	9
NPV(active)	2310	3849	5389	7901	9794	11838	14802	15894	16504
NPV(passive)	2310	3849	5389	7436	8971	11070	12389	14094	15660
NPV(pure PCs)	2310	3849	5389	6086	8223	10661	12761	14646	16330

From Table 7-17 we can see that only if the traffic demand is medium or high, is the project financially feasible for the private investor and the active pricing strategy is superior to the passive pricing strategy. When the traffic demand level is high, the performance of the passive pricing strategy is even inferior to that under a pure price caps regulation mechanism, which is to say, to utilize the benefit of the toll adjustment mechanism to the maximum extent, an active pricing strategy should be adopted rather than the passive one. If, in any case, this strategy is banned by the government, the private investor should negotiate with the government to replace the toll adjustment mechanism with a pure price caps regulation mechanism.

The Real Options Values of the toll adjustment mechanism under both the active and passive pricing strategies with X=2 are shown in Table 7-18:

Table 7-18 Real Options Values (ROV) under two pricings (X=2) (millions of HKD)

Scenario	1	2	3	4	5	6	7	8	9
RO(active)	0	0	0	1815	1570	1177	2041	1248	174
RO(passive)	0	0	0	1350	748	409	-371	-552	-670

By comparing Table 7-3 with Table 7-11, we can conclude that the toll volatility and dynamics among consecutive periods with X=2 is much less than that without X=2. By comparing Table 7-4 with Table 7-12 we can see that price caps are the same when traffic demand is low or medium; while if the traffic demand is high, price caps with X=2 are higher than those without X=2. The implication is that even though the presence of X will constrain the toll adjustment patterns, the concessionaire can still reach a higher level of price caps through more frequent actions. Comparing Table 7-5 with Table 7-13, we can see that generally, the revenues with X=2 and without X=2 are the same when the traffic demand is low; while the traffic demand is medium or high, the revenues with X=2 are less than those without X=2, which means that introduction of X indeed can effectively reduce the power of the concessionaire's active pricing strategies. However, by comparing Table 7-6 with Table 7-14, Table 7-7 with Table 7-15, and Table 7-8 with Table 7-16, we can see that the pricing strategies with or without the Toll Adjustment Restriction Factor X are the same. This is because that under a passive pricing strategy, the concessionaire seeks the prices that can maximize the individual revenues, therefore the fluctuations of consecutive are much less volatile and dynamic than those under the active pricing strategies. Even in the case of toll adjustment, the maximum level of escalation is still less than 100%, which is equivalent to X=2. By comparing Table 7-9 and Table 7-17, Table 7-10 and Table 7-18, we could see the effect of X more intuitively.

Impact of different values-of-time (VOTs) as well as different levels of traffic demand on the concessionaire's pricing strategies (Ps), price caps (P_cs) and revenues (Rs) can also be seen accordingly with X=2. Analysis of the results is similar to that without X=2, therefore for the purpose of briefness, only results will be presented as figures while no analysis will be provided.

The results are shown from Figure 7-20 to Figure 7-28 (Impacts of VOTs to Ps, P_cs and Rs under the active pricing strategy with X=2) and from Figure 7-29 to Figure 7-37 (Impacts of Qs to Ps, P_cs and Rs under the active pricing strategy with X=2):

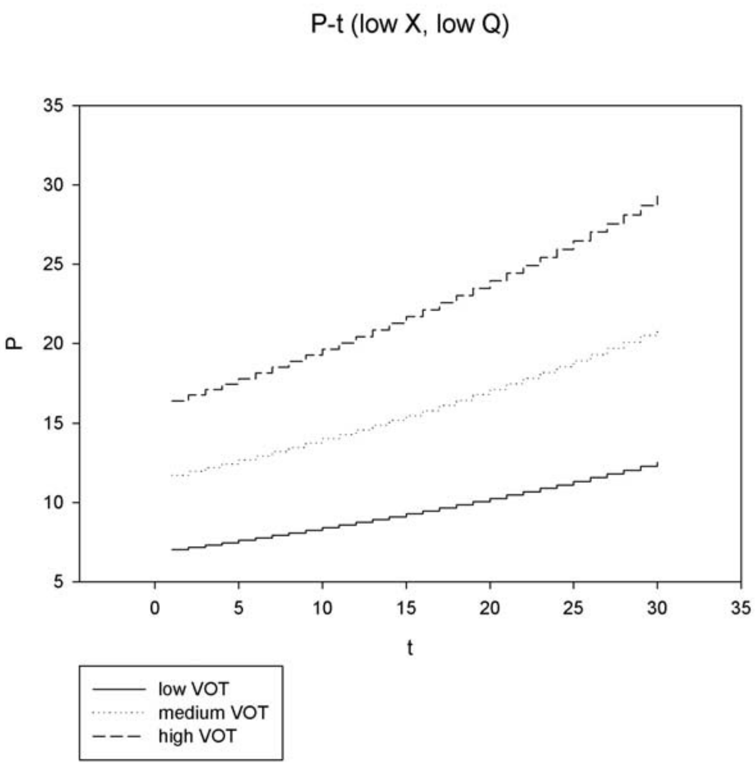

Figure 7-20 Active pricing with low traffic and X=2

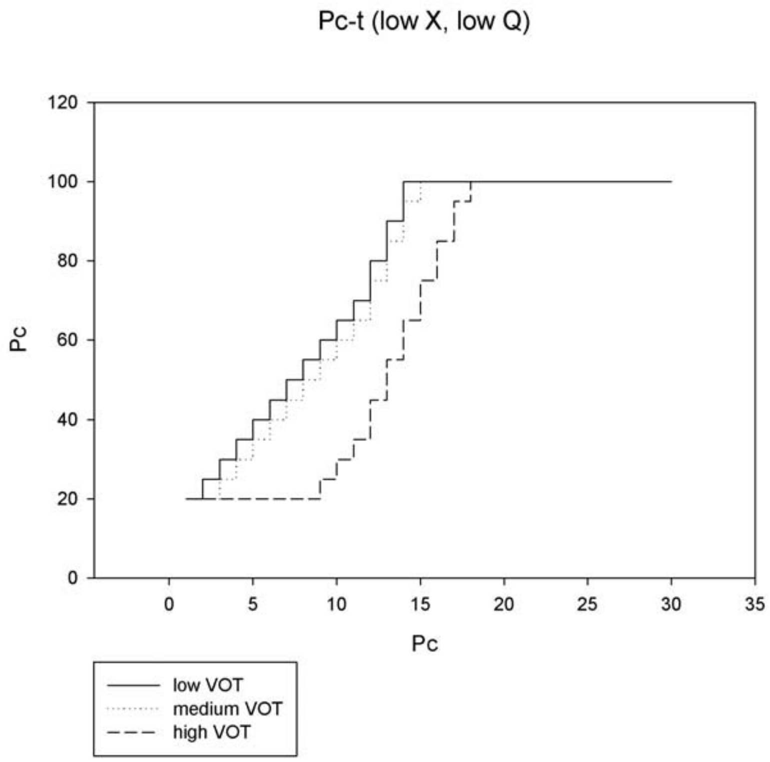

Figure 7-21 Price caps under active pricing with low traffic and X=2

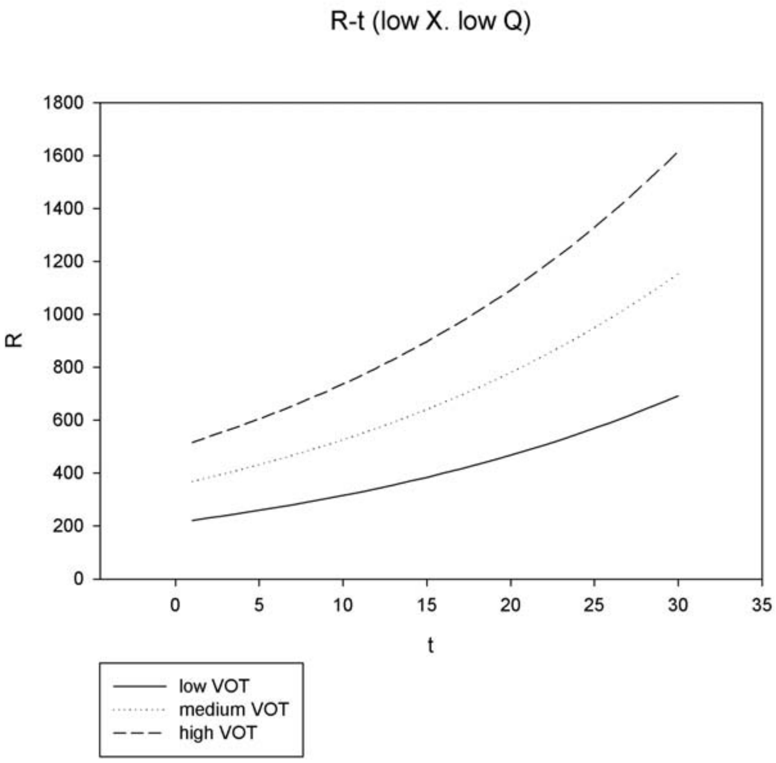

Figure 7-22 Revenues under active pricing with low traffic and X=2

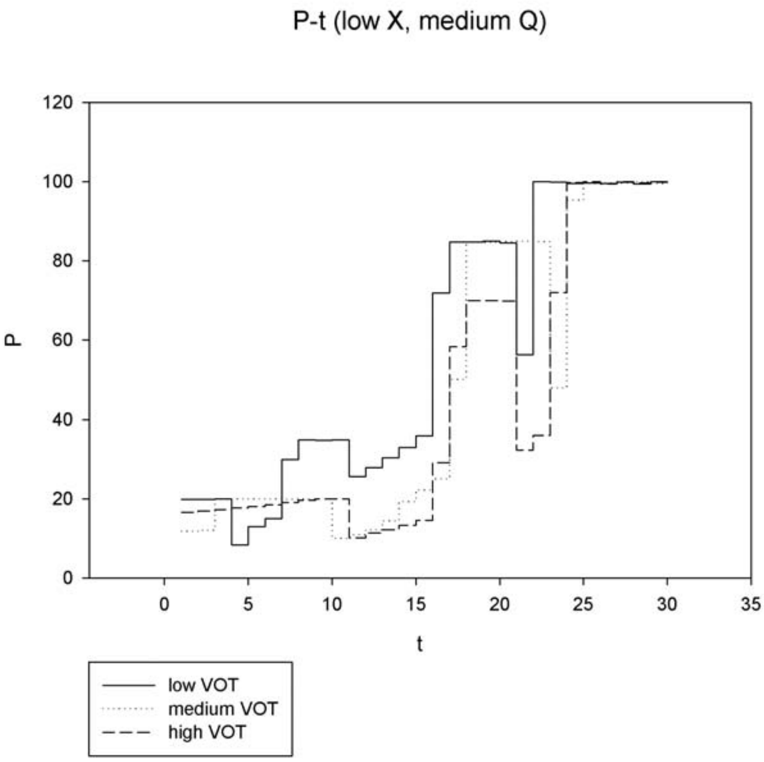

Figure 7-23 Active pricing with medium traffic and X=2

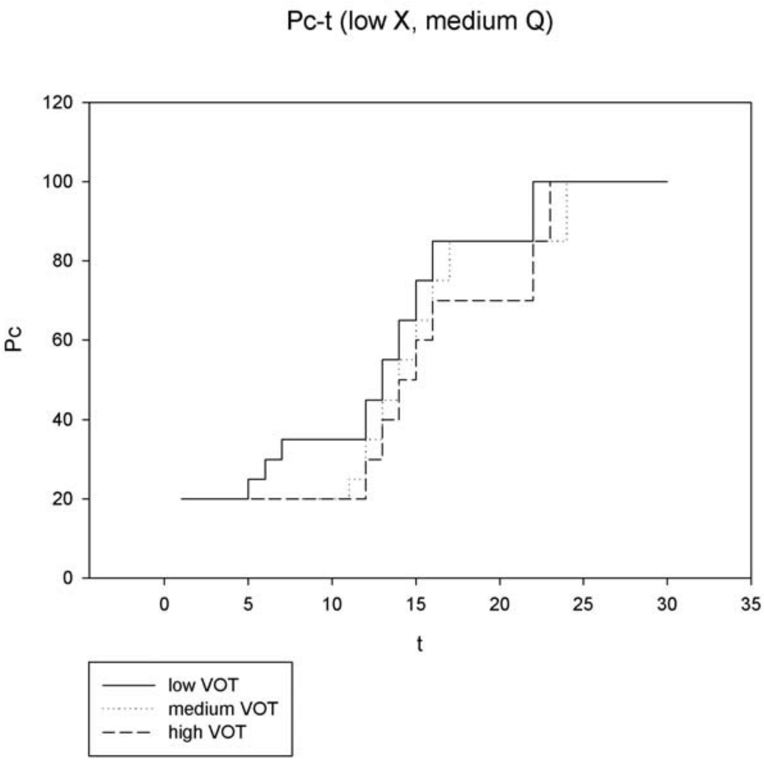

Figure 7-24 Price caps under active pricing with medium traffic and X=2

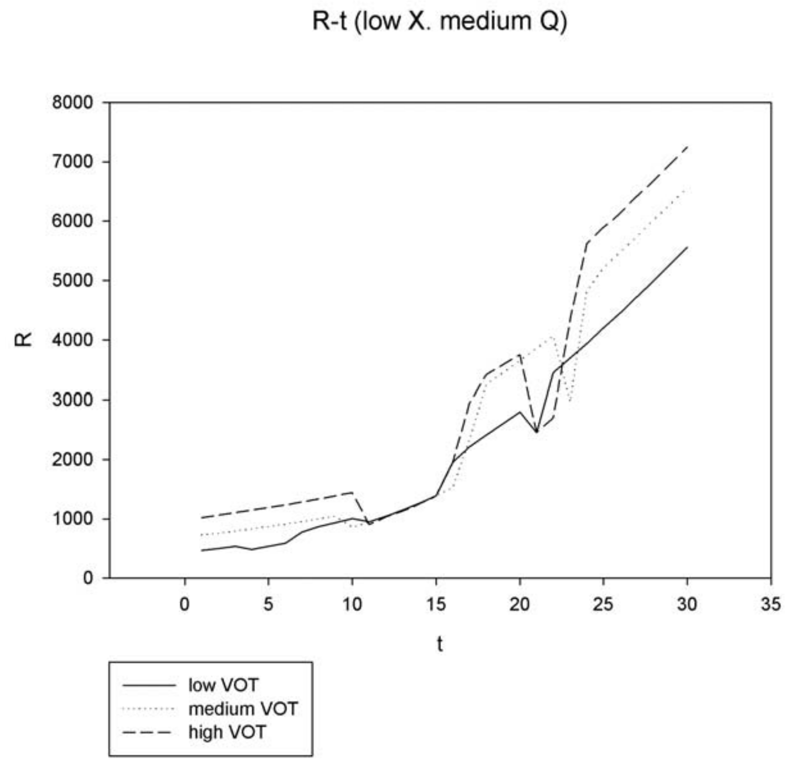

Figure 7-25 Revenues under active pricing with medium traffic and X=2

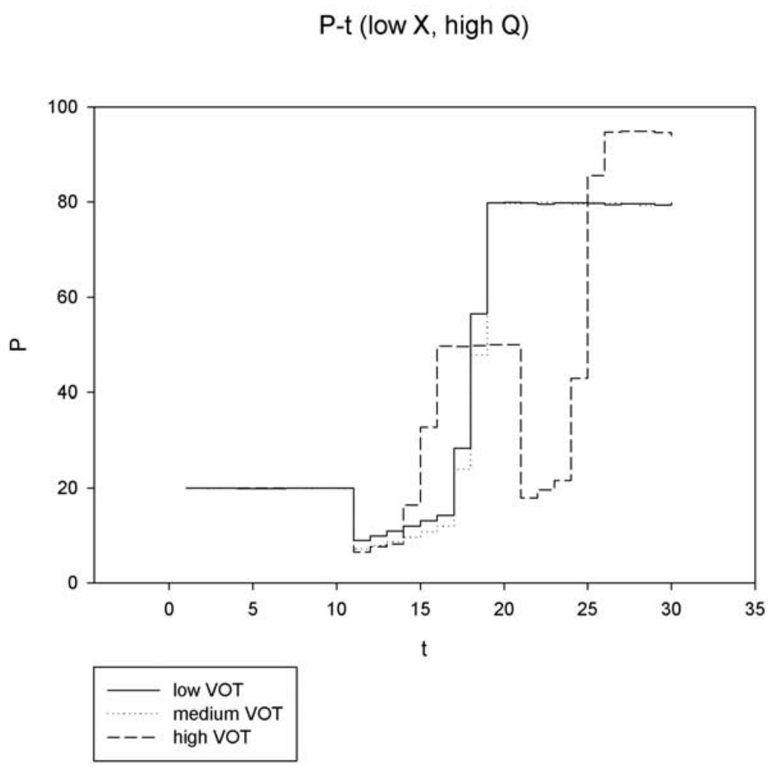

Figure 7-26 Active pricing with high traffic and X=2

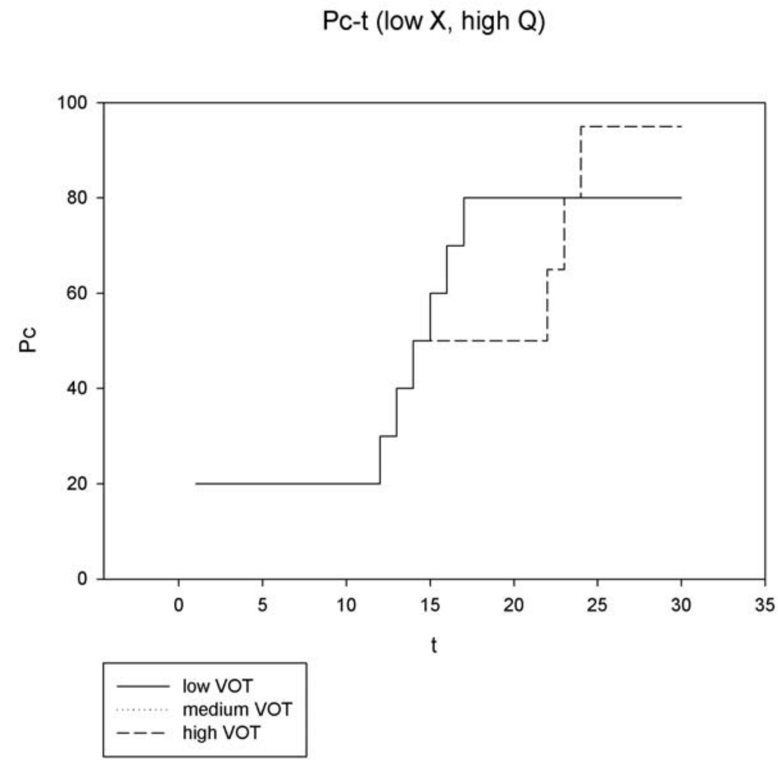

Figure 7-27 Price caps under active pricing with high traffic and X=2

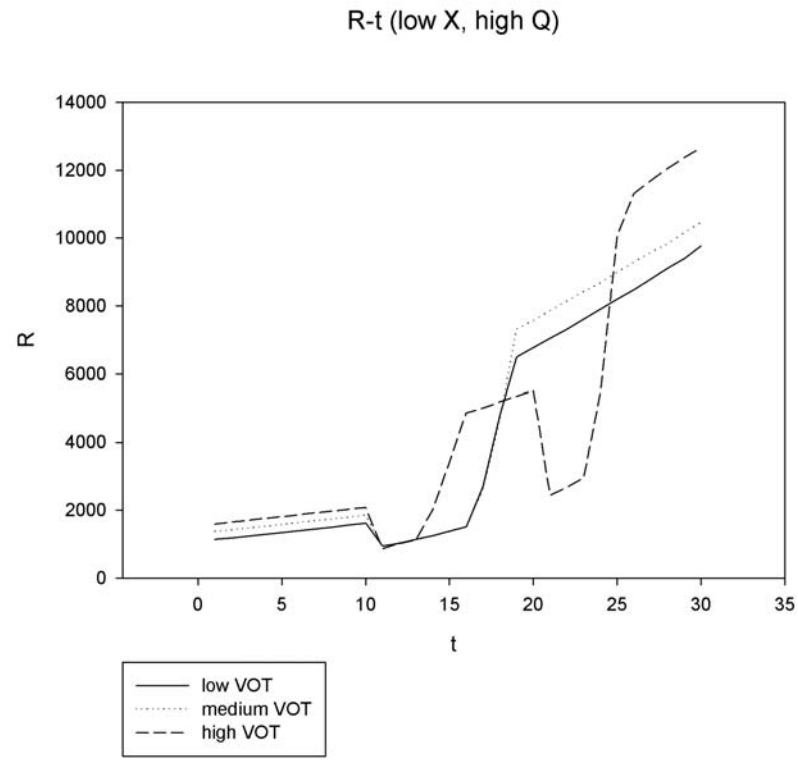

Figure 7-28 Revenues under active pricing with high traffic and X=2

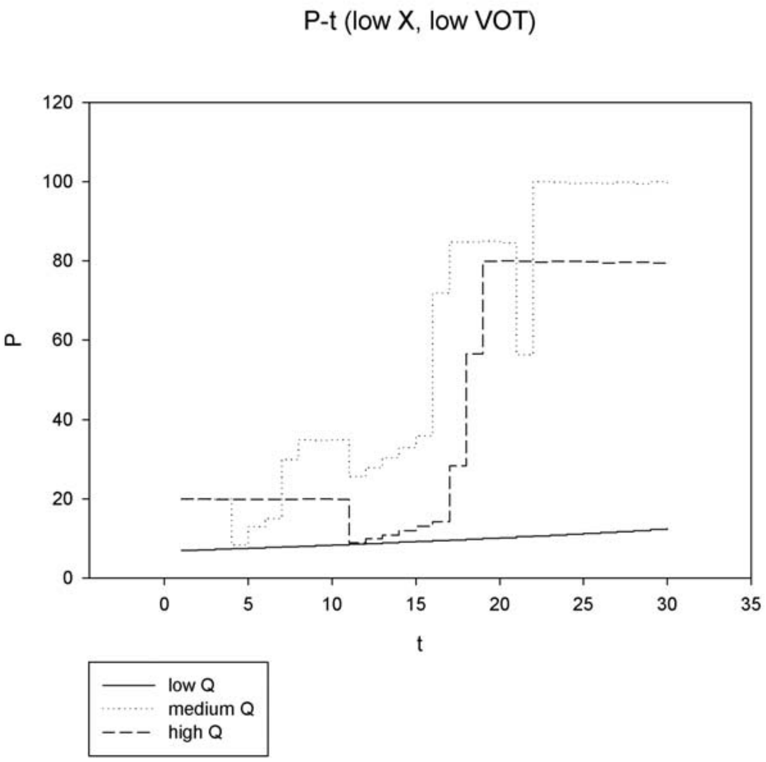

Figure 7-29 Active pricing with low VOT and X=2

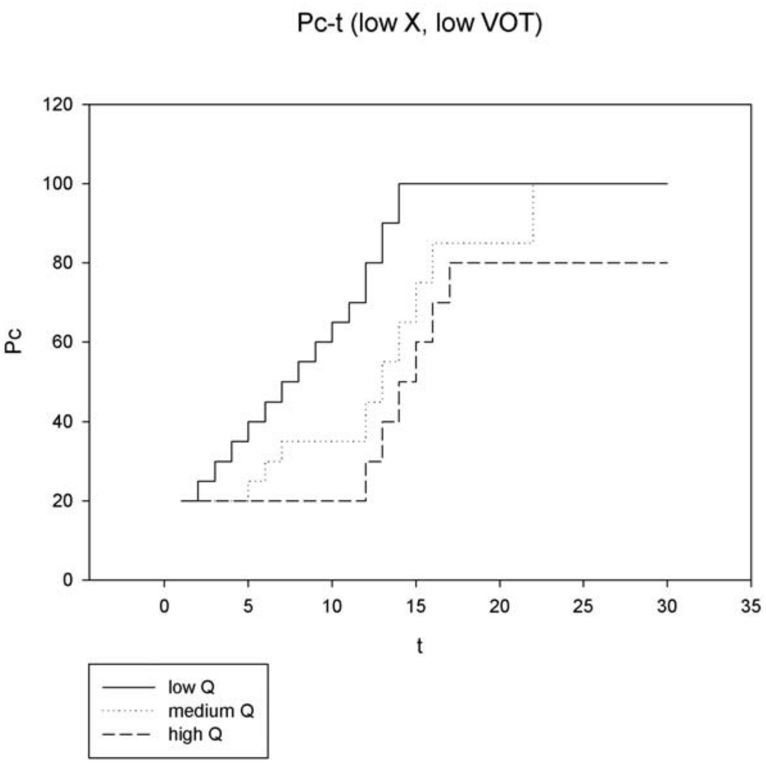

Figure 7-30 Price caps under active pricing with low VOT and X=2

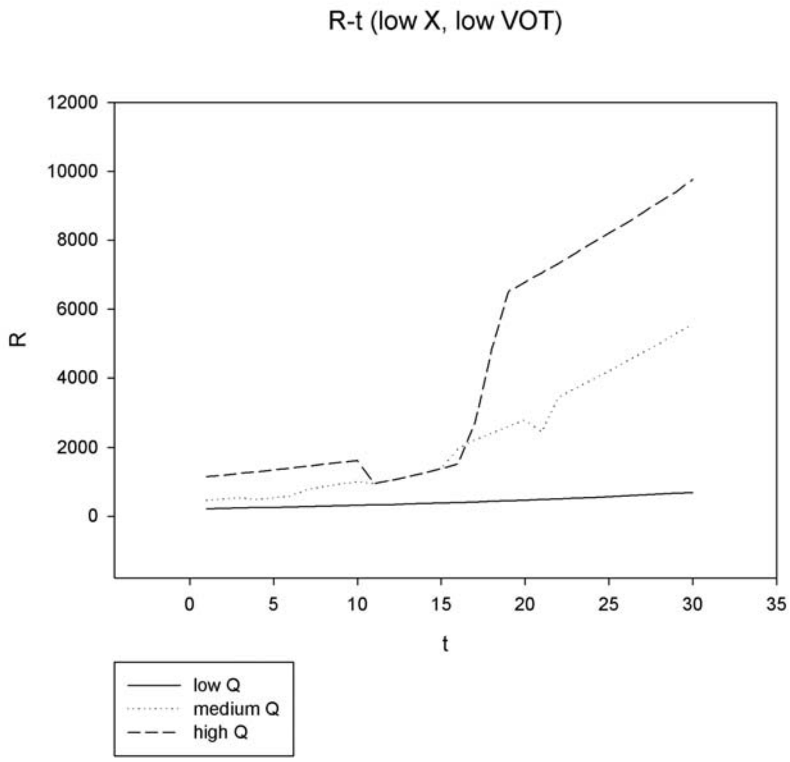

Figure 7-31 Revenues under active pricing with low VOT and X=2

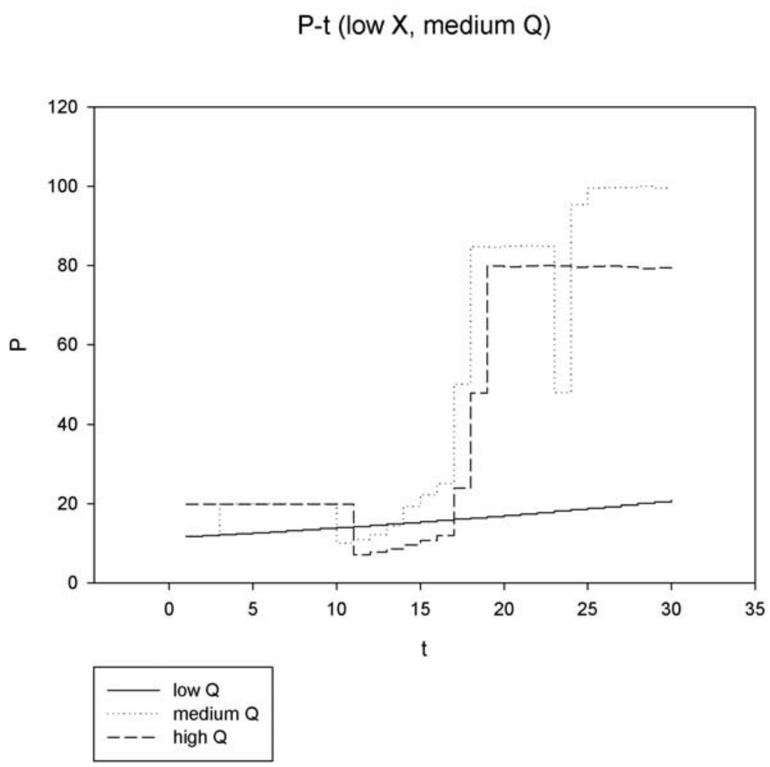

Figure 7-32 Active pricing with medium VOT and X=2

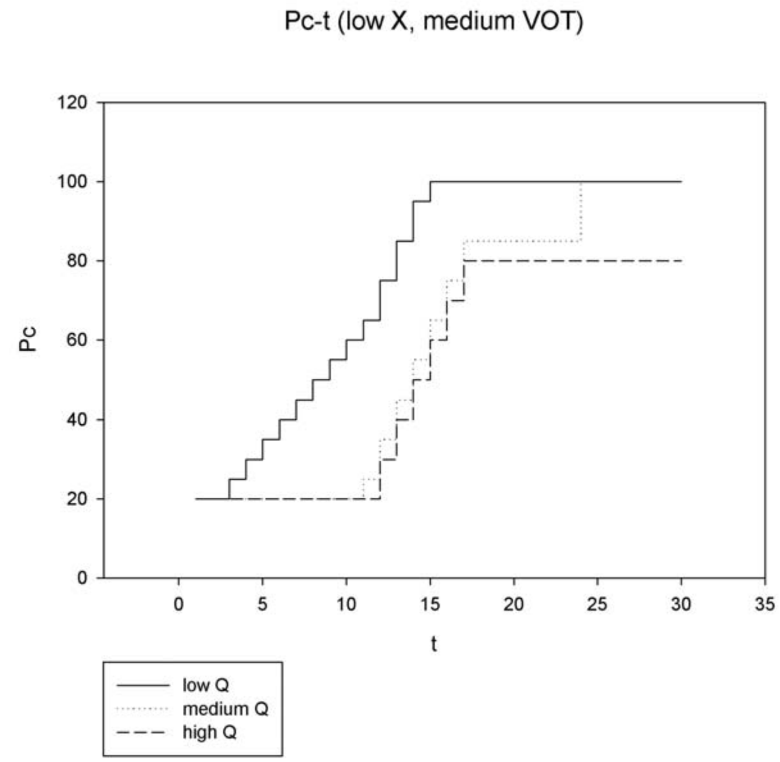

Figure 7-33 Price caps under active pricing with medium VOT and X=2

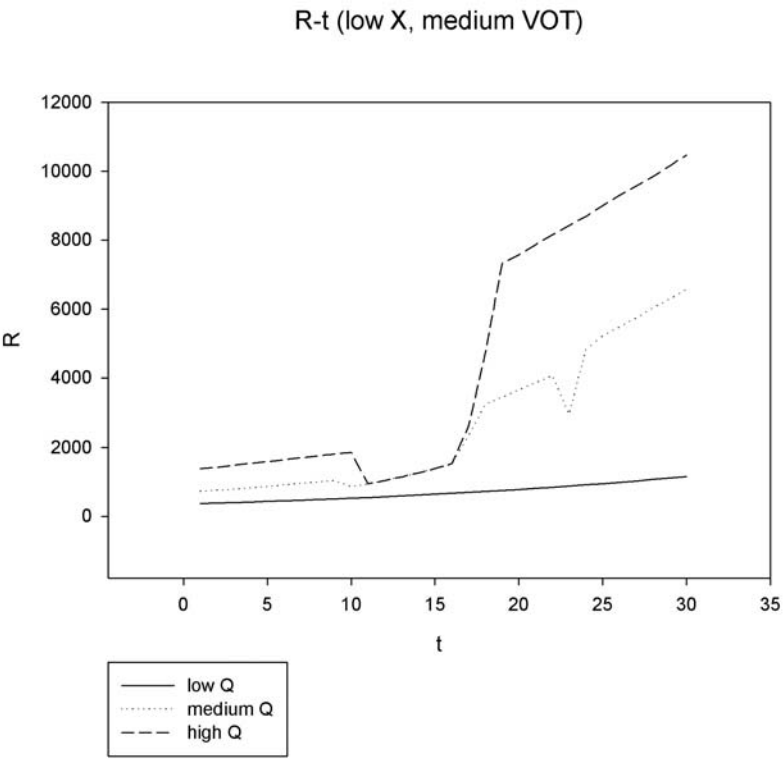

Figure 7-34 Revenues under active pricing with medium VOT and X=2

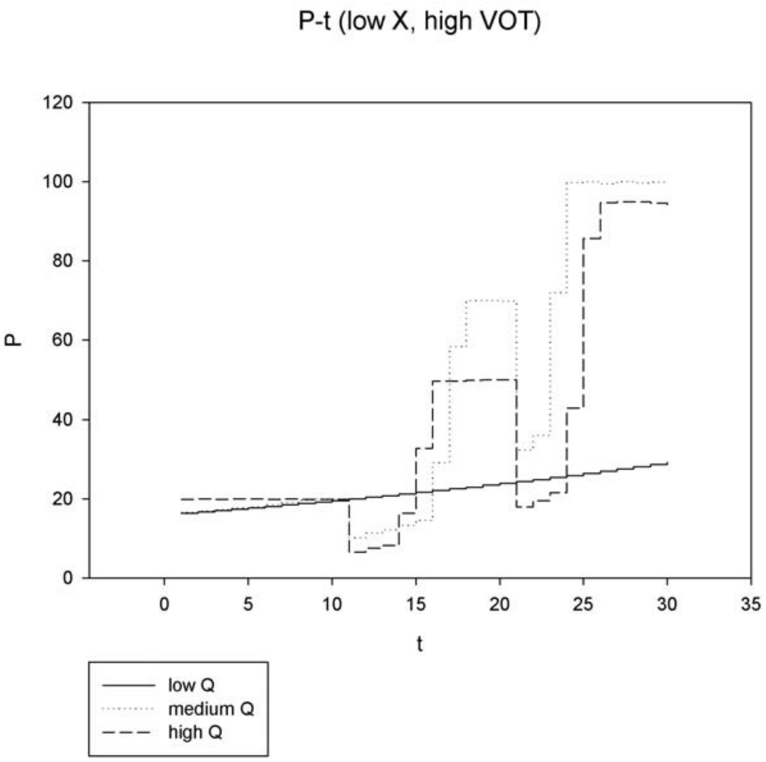

Figure 7-35 Active pricing with high VOT and X=2

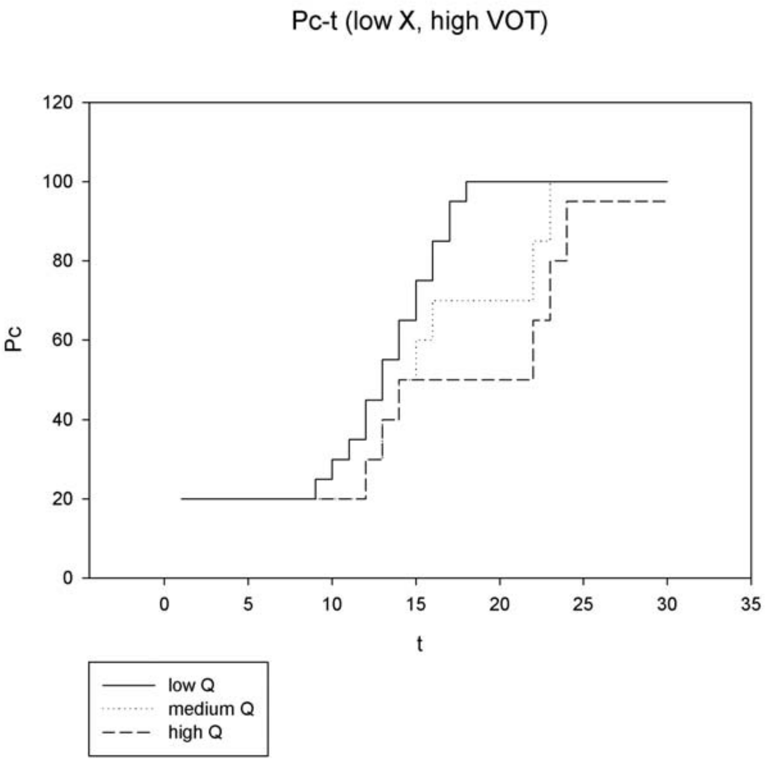

Figure 7-36 Price caps under active pricing with high VOT and X=2

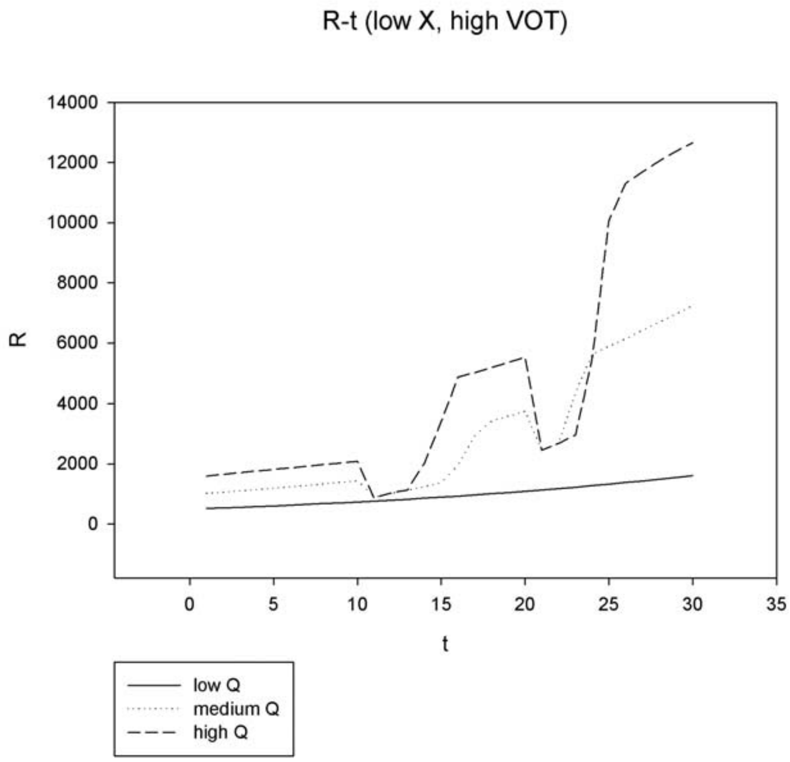

Figure 7-37 Revenues under active pricing with high VOT and X=2

Case Study No.2: When the traffic demand is uncertain

The second way to treat the traffic demand uncertainties is to model each annual traffic demand as stochastic variables conforming to certain probabilistic distributions, such as normal distributions, log-normal distributions, etc. The variance model developed in Chapter 4 will be used hereinafter to generate the stochastic traffic demand vector, i.e., in each year the traffic demand conforms to a normal distribution, the expectation and standard variance of which can be generated from the martingale variance model.

Stochastic modelling can be incorporated in the previous scenario analysis by replacing the low, medium and high levels of traffic demand with three distributions of traffic demand, in which those levels are the expectations of the probabilistic distributions. Therefore there will also be 9 scenarios. For the briefness of illustration, only the 3 scenarios of medium traffic demand will be discussed here. The other 6 scenarios with low and high traffic demand can be concluded easily by the same reasoning.

Given $Q_0 = \overline{Q_0} = 200{,}000$ vpd , $\gamma = 0.8$, $\sigma = 0.02$, $\Delta\overline{Q_1} = 10{,}000$ vpd , $\Delta\overline{Q_j} = 0.95\Delta\overline{Q_{j-1}}$, for $j = 2, \dots, 30$, if in each year the traffic demand conforms to a normal distribution, the corresponding expectation and standard variance are shown in Table 7-19.

Table 7-19 Expectation and standard variance of traffic demand

t	1	2	3	4	5	6	7	8	9	10
$E(Q_t)$	209568	219495	228335	237078	245051	252909	260365	267137	273820	280101
$stdQ_t$	26199	32856	36964	39927	42473	44785	46429	48134	49991	51618
t	11	12	13	14	15	16	17	18	19	20
$E(Q_t)$	286238	291914	297427	302415	307547	312465	316650	320787	324821	328600
$stdQ_t$	53056	54494	55805	57153	58517	59855	61283	62420	63485	64708
t	21	22	23	24	25	26	27	28	29	30
$E(Q_t)$	332233	335801	339099	342189	345358	348120	350492	352979	355467	357833
$stdQ_t$	65950	67039	67932	68871	69929	70872	71662	72610	73468	74446

The analysis results are shown in Table 7-20 and Table 7-21, with $\alpha = 0.2$.

Table 7-20 Analysis results under active pricing with VOTs (X=2)

Scenarios / t	Low VOT				Medium VOT				High VOT			
	P_c	P	E[R]	StdR	P_c	P	E[R]	StdR	P_c	P	E[R]	StdR
1	20	4.3	326	41	20	20	774	169	20	16.5	1025	85
2	25	4.4	348	53	20	20	842	209	20	17	1091	116
3	30	8.8	513	101	20	20	911	237	20	17.4	1164	148
4	30	17.6	691	242	20	20	978	258	20	18.1	1238	176
5	30	30	927	403	20	20	1043	278	20	20	1314	219
6	30	30	1018	430	20	20	1108	296	20	20	1391	242
7	30	30	1097	461	20	20	1165	313	20	20	1459	262
8	30	30	1182	482	20	20	1223	321	20	20	1525	274
9	30	30	1264	511	20	20	1281	335	20	20	1588	286
10	30	30	1341	533	20	20	1334	343	20	20	1645	291
11	30	10.8	790	184	20	8	815	148	20	7.9	813	155
12	40	11.9	859	213	30	8.6	890	158	30	8.5	893	168
13	50	13	931	242	40	9.4	980	169	40	9.1	972	182
14	60	26	1490	518	50	10.4	1089	179	50	18.2	1774	279
15	60	52	2362	1005	60	11.7	1208	193	50	36.4	2581	719
16	60	60	2659	1153	70	13.3	1336	210	50	50	3216	947
17	60	60	2778	1200	80	26.6	1974	544	50	50	3332	1000
18	60	60	2909	1243	80	53.2	3223	1124	50	50	3447	1043
19	60	60	3032	1274	80	80	4234	1491	50	50	3553	1071
20	60	60	3135	1299	80	80	4423	1595	50	50	3640	1090
21	60	28	1927	625	80	80	4607	1681	50	20.7	2180	342
22	75	30.8	2103	702	80	80	4747	1769	65	23.3	2366	401
23	90	61.6	3482	1401	80	80	4885	1832	80	26.1	2559	483
24	90	90	4595	2029	80	80	5021	1877	95	52.2	4093	1202
25	90	90	4737	2060	80	80	5154	1894	95	95	6403	2217
26	90	90	4829	2106	80	80	5239	1930	95	95	6518	2290
27	90	90	4964	2133	80	80	5343	1969	95	95	6651	2336
28	90	90	5055	2183	80	80	5443	1989	95	95	6763	2368
29	90	90	5189	2203	80	80	5557	2010	95	95	6897	2392
30	90	90	5278	2243	80	80	5655	2024	95	95	7007	2417

Table 7-21 Analysis results under passive pricing with VOTs (X=2)

Scenarios / t	Low VOT				Medium VOT				High VOT			
	P_c	P	E[R]	StdR	P_c	P	E[R]	StdR	P_c	P	E[R]	StdR
1	20	20.0	528	172	20	20.0	774	169	20	16.5	1025	85
2	20	20.0	597	221	20	20.0	842	209	20	17.0	1091	116
3	20	20.0	665	254	20	20.0	911	237	20	17.4	1164	148
4	20	20.0	729	274	20	20.0	978	258	20	18.1	1238	176
5	20	20.0	789	292	20	20.0	1043	278	20	20.0	1314	219
6	20	20.0	850	308	20	20.0	1108	296	20	20.0	1391	242
7	20	20.0	904	325	20	20.0	1165	313	20	20.0	1459	262
8	20	20.0	959	334	20	20.0	1223	321	20	20.0	1525	274
9	20	20.0	1014	350	20	20.0	1281	335	20	20.0	1588	286
10	20	20.0	1064	360	20	20.0	1334	343	20	20.0	1645	291
11	20	20.0	1112	372	20	20.0	1385	353	20	20.0	1699	298
12	20	20.0	1164	380	20	20.0	1438	358	20	20.0	1755	300
13	20	20.0	1208	389	20	20.0	1486	365	20	20.0	1805	305
14	20	20.0	1259	395	20	20.0	1539	369	20	20.0	1858	306
15	20	20.0	1298	406	20	20.0	1581	376	20	20.0	1901	313
16	20	20.0	1337	411	20	20.0	1623	378	20	20.0	1943	314
17	20	20.0	1373	417	20	20.0	1664	381	20	20.0	1983	318
18	20	20.0	1414	423	20	20.0	1707	383	20	20.0	2024	321
19	30	30.0	1918	652	20	20.0	1750	384	20	20.0	2064	324
20	30	30.0	1969	662	20	20.0	1788	386	20	20.0	2099	327
21	30	30.0	2022	671	30	30.0	2363	652	20	20.0	2135	331
22	30	30.0	2064	684	30	30.0	2406	662	35	35.0	2982	748
23	45	45.0	2808	1039	30	30.0	2456	668	35	35.0	3042	752
24	45	45.0	2882	1057	30	30.0	2509	675	35	35.0	3106	758
25	45	45.0	2956	1065	45	45.0	3416	1058	35	35.0	3166	761
26	45	45.0	3001	1089	45	45.0	3465	1076	35	35.0	3209	770
27	60	60.0	3763	1457	45	45.0	3527	1093	50	50.0	4176	1193
28	60	60.0	3829	1482	45	45.0	3581	1107	50	50.0	4237	1207
29	60	60.0	3915	1498	60	60.0	4503	1507	50	50.0	4310	1216
30	75	75.0	4663	1879	60	60.0	4573	1523	50	50.0	4370	1227

The net present values (NPVs) of the project under low, medium and high VOTs, by employing active and passive pricing strategies, are shown in Table 7-22:

Table 7-22 NPV under different VOTs with stochastic traffic demand (X=2) (millions of HKD)

Scenario	Low VOT	Medium VOT	High VOT
E[NPV(active)]	8977	10716	12620
Std NPV(active)	684	608	504
E[NPV(passive)]	7768	9579	11693
Std NPV(passive)	552	514	389
E[NPV(pure PCs)]	7626	9678	11797
Std NPV(pure PCs)	558	535	416

From Table 7-22 we can see that the active pricing strategy is superior to the passive pricing strategy. If the VOT is medium or high, the passive pricing strategy is even inferior to the strategy under a pure price caps regulation mechanism. These results are consistent to the conclusions aforementioned.

If the VOT is low, then under an active pricing strategy, the net present value (NPV) of the project conforms to a normal distribution with mean of 8,977 millions of HKD and standard variance of 684 millions of HKD, since the investment cost is 7,000 millions of HKD, then the probability that the net present value of the project (net present value of the operational revenues minus the initial investment cost) is greater than zero is

$$\frac{1}{\sqrt{2\pi}}\int_{x}^{+\infty} e^{-\frac{u^2}{2}}du = \frac{1}{\sqrt{2\pi}}\int_{\frac{7000-8977}{684}}^{+\infty} e^{-\frac{u^2}{2}}du = 99.81\%$$

Applying this calculation for each pricing strategy with each level of VOT, we can get Table 7-23:

Table 7-23 Financial feasibility of the project with stochastic traffic demand

Scenario	Low VOT	Medium VOT	High VOT
Pr(net NPV > 0|active pricing)	99.81%	100%	100%
Pr(net NPV > 0|passive pricing)	91.79%	100%	100%
Pr(net NPV > 0|pure price caps)	86.90%	100%	100%

From Table 7-23 we can see that, under the projected level of medium traffic demand, if the level of VOT for the commuters is medium or high, the probability that the project is financially feasible for the private investor is 100%; however, if the level of values-of-time (VOT) is low, the probability that the project is financially feasible for the private investor is 99.81% under an active pricing strategy while 91.79% under an passive pricing strategy. With a pure price caps regulation mechanism, this probability is even lower, that is, 86.90%.

The Real Options Values of the toll adjustment mechanism under both the active pricing strategy and the passive pricing strategy are shown in Table 7-24:

Table 7-24 Real Options Values under two pricings with stochastic traffic demand (X=2) (millions of HKD)

Scenario	Low VOT	Medium VOT	High VOT
E[RO(active)]	1351	1038	823
E[RO(passive)]	142	-99	-104

Figure 7-38 shows the active pricing strategy under three levels of VOTs. Figure 7-39 shows the corresponding price caps for each year. Figure 7-40 shows the expected revenues over time.

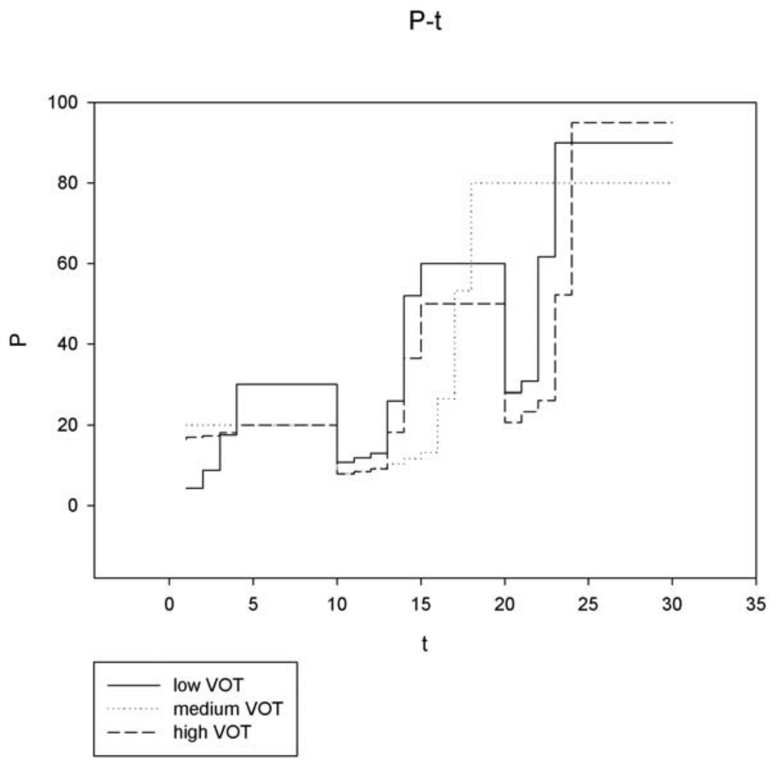

Figure 7-38 Active pricing with stochastic traffic demand (X=2)

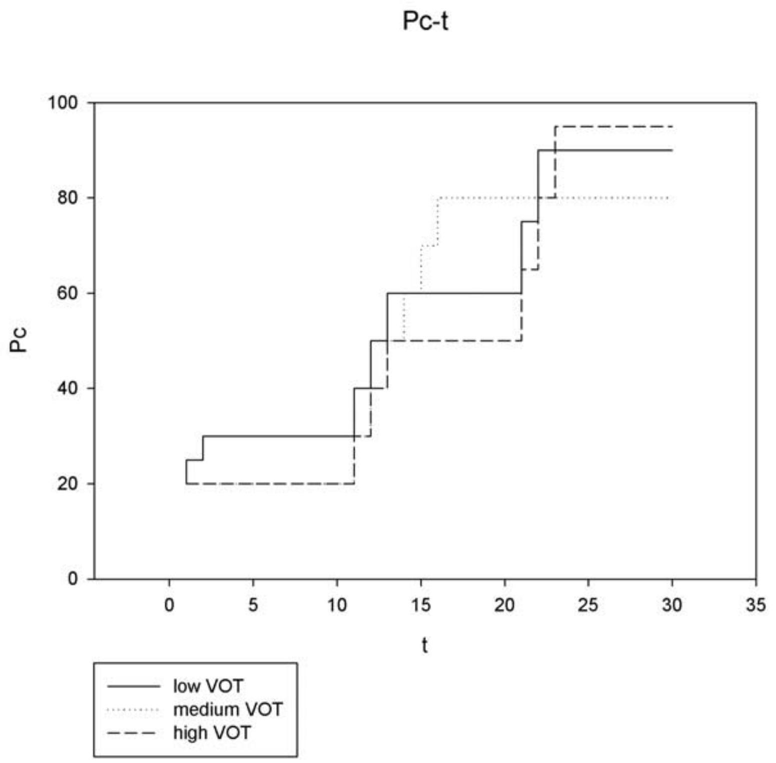

Figure 7-39 Price caps under active pricing with stochastic traffic demand (X=2)

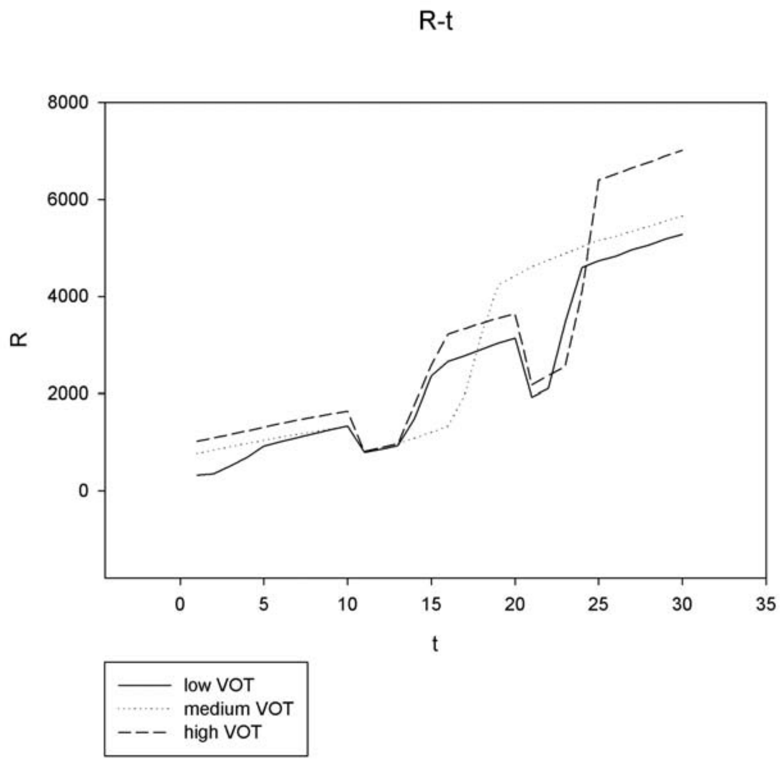

Figure 7-40 Revenues under active pricing with stochastic traffic demand (X=2)

Figure 7-41 to Figure 7-46 show the distributions of revenues under three levels of VOTs; in each level of VOT, both the price that can maximize the revenue and the price that can induce a toll adjustment are presented.

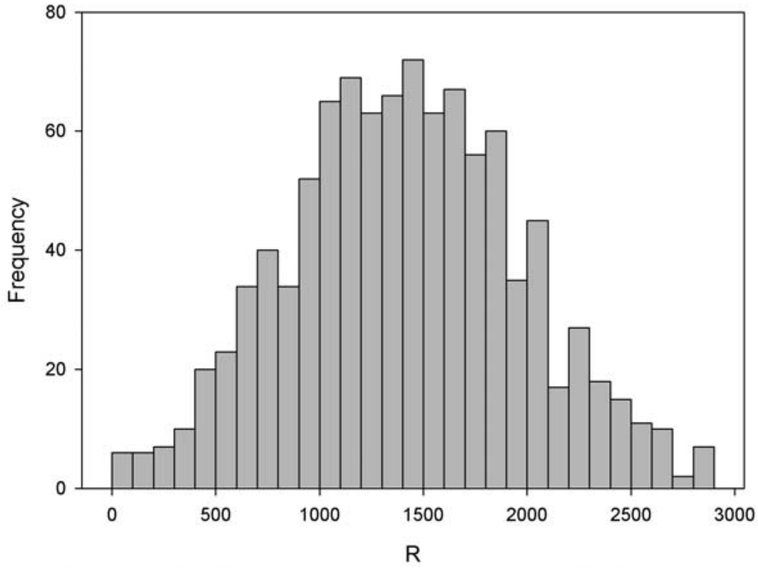

Figure 7-41 Distribution of R when the VOT is low, t=10, and P=30

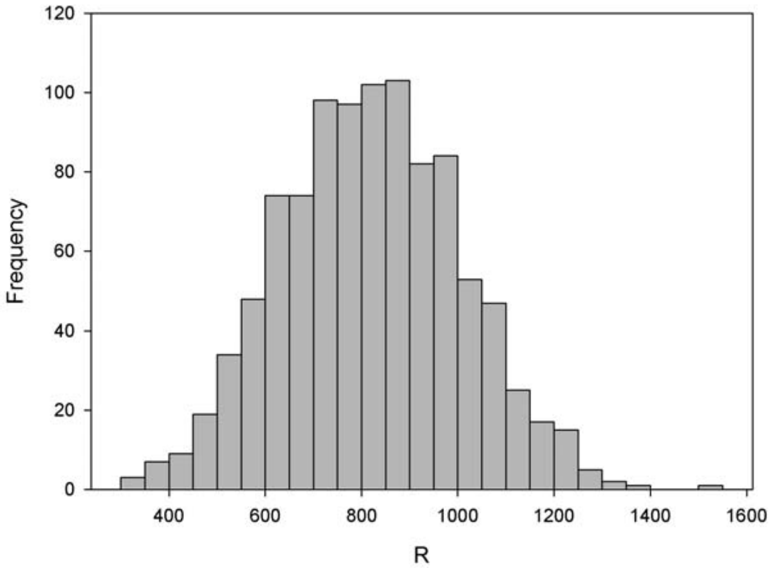

Figure 7-42 Distribution of R when the VOT is low, t=11, and P=10.8

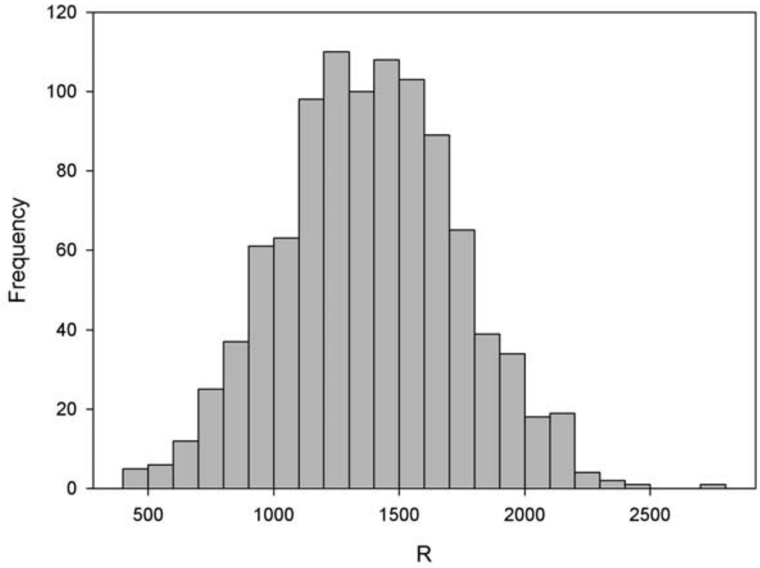

Figure 7-43 Distribution of R when the VOT is medium, t=10, and P=20

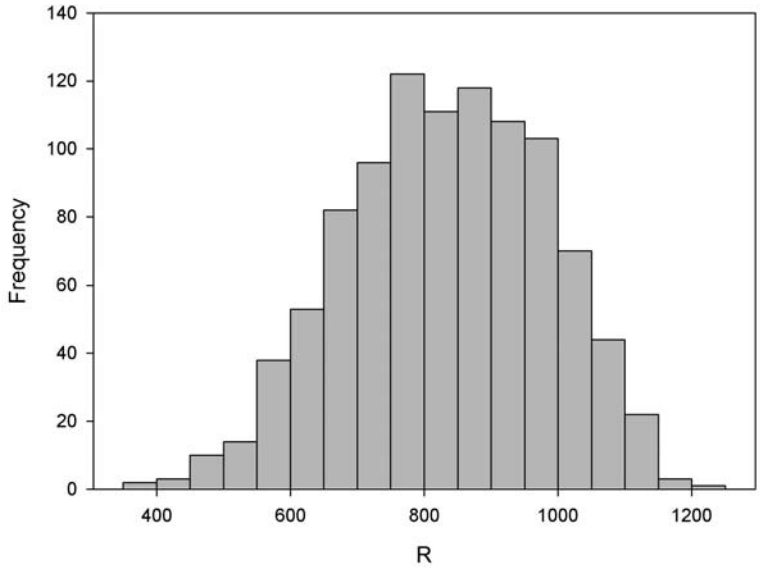

Figure 7-44 Distribution of R when the VOT is medium, t=11, and P=8

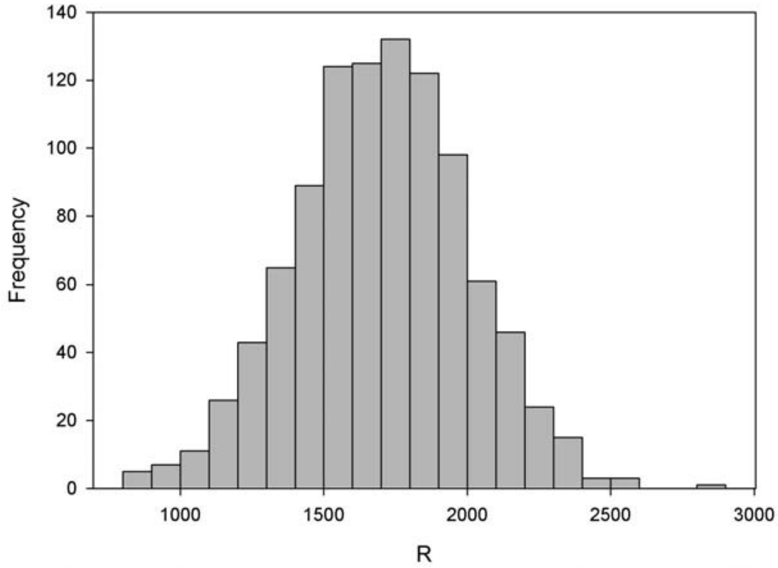

Figure 7-45 Distribution of R when the VOT is high, t=10, and P=20

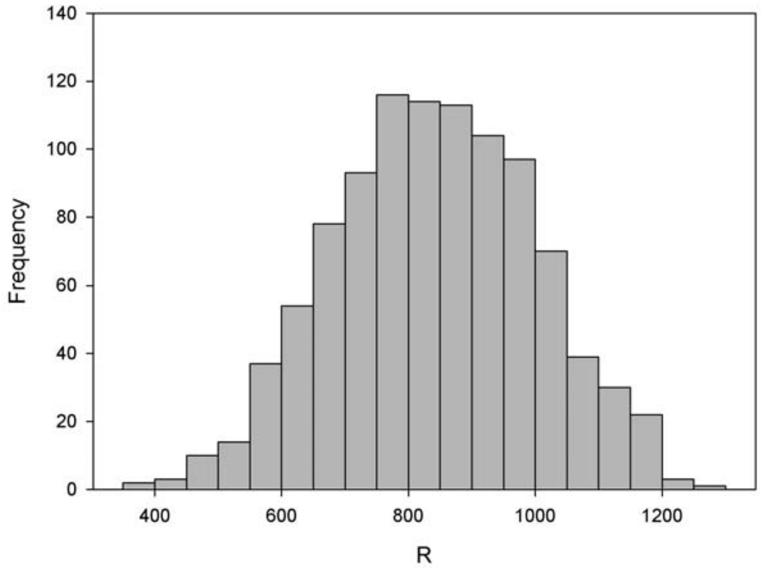

Figure 7-46 Distribution of R when the VOT is high, t=11, and P=8

7.3. Summary of the Chapter

Chapter 7 analyzes a hypothetical case study derived from a real life project in detail to demonstrate the application of the real options valuation framework developed. Both scenario analysis and stochastic modelling are illustrated, assisting the private investor to be better informed of the traffic uncertainty and to make sound investment decisions.

8. CONCLUSION AND RECOMMENDATIONS FOR FURTHER RESEARCH

8.1. Conclusion of the Research

This research presented a comprehensive and in-depth framework consisting of three models for real options valuation of the toll adjustment mechanism in concession contracts of infrastructure projects (toll road).

First, a stochastic variance model was proposed for modelling the risk variables in Greenfield projects characterized by little or no historical data, as well as features such as Markov chain, uncertainty and learning property.

Second, a two-route choice (traffic assignment) model was built to simulate the behaviors of commuters with heterogeneous values-of-time. From numerical simulations in different settings of economy (in terms of values-of-time, which reflect the wealth of the commuters, strongly and positively correlated to the economy) and traffic demand (with the outcomes of the previous model as inputs), the demand function of toll and the traffic on the toll road can be derived. Therefore the optimal pricing strategy in a single period for the concessionaire can be determined with the objective to maximize the revenue; however, if the toll road were owned by the government, the objective should be maximization of social benefit, in forms of minimization of total travel cost or time, which can also be deduced from the model. We have also proved that in the case that most of the taxpayers were users of the road, shadow tolling might be a better option for the public, in that both social travel time and cost could be mitigated to a large extent, while the benefit of the concessionaire was intact.

At last, the real options model of the toll adjustment mechanism was constructed. Two pricing strategies were proposed and compared to illustrate the managerial flexibility within such contracts. The passive pricing strategy was intuitive (to optimize the revenue in each individual period) but far less effective and profitable than the active price strategy, in which in each period, both pricing strategies to maximize the current revenue and to make it at the minimum guaranteed level through micro toll adjustments within single period, were considered. Therefore the optimal decision comprised of a series of tolls charged in each period can be determined, which was the one maximizing the net present value of the project. The real options value of managerial flexibility embedded in the toll adjustment mechanism can then be determined. A tree structure model combined with branch and bound method was employed to solve the optimization problem numerically. Monte-Carlo simulation method was used when the traffic demand uncertainty was taken into consideration.

The toll adjustment mechanism was an ideal alternative for guarantee arrangement in concession contracts of infrastructure projects such as toll roads. The framework developed to assess the real options value of such mechanism can benefit both the public and private sectors; for the public sector, ensuring a reasonable but not excessive profit margin to attract private investors and, at the same time, with no compromise of the benefit of the public, and preventing possible opportunistic behaviors through clauses amended to the toll adjustment mechanism; for the private, making investment decisions and mitigating the demand risk in a more sound way and beating competitors who failed to recognize the value of flexibility and therefore undervalued the project. A win-win prospect was then achieved for both parties.

8.2. Contributions of the Research

The outcomes of this thesis contribute to:

1. Fostering a win-win prospect for PPP infrastructure projects, by, i). assisting host government to establish effective toll adjustment mechanisms (combined with other regulation mechanisms) to mitigate the demand risk. Due to the nature of Public-Private-Partnership infrastructure projects, the kernel of the concession contract is to strike a balance between to maintain a constant and reasonable rate of return for the private investor(s), and to protect the public, i.e., users, taxpayers, etc., from over exploration by the concessionaire in a monopolistic or semi-monopolistic status. A sound toll adjustment mechanism (combined with other regulation mechanisms) can help to achieve this goal while bring no fiscal burdens to the host government, unlike guarantees and other mechanisms involving governmental payment based on contingencies; and ii). assisting private

investors to find both the optimal price which can maximize the revenue in one particular period and the optimal pricing strategy which can maximize the overall net present value (NPV) of the project. In face of the toll adjustment mechanism in the concession contract, a rational investor, that is to say, a profit-optimizer, should not only obey the rules, but also make use of them to achieve a higher level of rate of return. The models developed in this study can help the project investors to forecast the traffic under different prices and therefore to maximize the overall net present value (NPV) of the project through the optimal pricing strategy through deliberate arrangement of prices in each year rather than the normal practice, to maximize the revenue / profit in each year;

2. Broadening real options theory by providing a new family of stochastic variables and models, and decision tree analysis through branch-and-bound method and Monte-Carlo simulation. Due to the uniqueness and distinctness of the toll adjustment mechanism (in the terms of options, the option holder's behavior may influence the market and therefore the strike price of the

option), dynamic programming method, which is usually employed to model and value real options, such as guarantees, is no longer applicable in the sense of simplifying the computation, because, to apply the dynamic programming method, the revenues corresponding to each node in the decision tree have to be calculated before its application, and ironically, that is where the complexity of this problem lies (considering a shortest-path problem in which the lengths of all the paths are given, dynamic programming can solve the problem perfectly and elegantly through backward induction; however, in our problem it's none other than the lengths of the paths that are needed to be calculated beforehand; on such basis, the superiority of dynamic programming to exhaustive enumeration is quite oblivious). By forward induction rather than backward, the branch-and-bound method can effectively simplify the computation by cutting off the inferior branches instantly. The framework of assessing the real options value of the toll adjustment mechanism will be developed by decision tree analysis through branch-and-bound method and

Monte-Carlo simulation, on the basis of introducing a new family of stochastic variables.

8.3. Limitations of the Research

Limitations of the research are mainly in the two-route choice model for commuters with heterogeneous values-of-time, in that:

1. Only two routes are considered. In real life, connections between two cities can be more than two. For example, there are three tunnels linking Kowloon and Hong Kong Island;
2. Only one transport mode, that is, travelling by driving private car, is considered. As a matter of fact, there are other choices such as, buses, taxis, etc. Besides those vehicles travelling on the roads, competing routes such as subway (in Hong Kong, Massive Transport Railway) can also be chosen by the commuters;
3. The same toll is charged for all types of vehicles. In practice, different tolls are charged for different types of vehicles based on their sizes, potential damage made to the road, and other factors;
4. The fact of traffic peak hours is neglected. In this research, congestion is considered by observing the actual daily traffic

divided by the daily road capacity. However, in real life, traffic flow on a road is not even at all; during peak hours of the workdays, the traffic can be several times of that during the rest of day.

8.4. Recommendations for Further Research

Four main research threads are recommended for future research:

First, the traffic assignment model can be expanded. In the real world, the route choice can be much more complicated than the neat two-route choice model depicted in the research. Besides the route choices, transport mode choices are various too. Public transport can be incorporated into consideration with passengers with distinct values-of-time from drivers.

Second, to be more realistic, differentiations of tolls among different vehicle types as well as operational costs should be taken into consideration. In this case the objective of the private investor is to optimize the net present value of profit rather than that of the revenue, therefore pricing strategies may change accordingly; with different tolls for different vehicle types, a combination of prices rather than a single price can be charged to achieve a certain revenue/profit goal, which gives the concessionaire even more managerial flexibilities.

Third, regulations on opportunistic pricing behavior, which is exactly the active pricing strategy described in the research, should be introduced, because in practice commuters would dislike the constant fluctuations of tolls, especially in some cases adjustment extent of consecutive tolls can be as high as twice, three times or even more. However, such regulations should also ensure the private investor with a reasonable rate or return too.

Last, mechanisms to mitigate demand risk in a more fundamental way should be investigated. For the private investors, the toll adjustment mechanism described in the research is not as effective as minimum revenue guarantees when the traffic demand is so low that optimal revenues in each period are still less than the minimum levels respectively, which is exactly the case with Western Harbour Crossing in Hong Kong.

APPENDIX

List of Abbreviations

act	Active pricing strategy
BOT	Build-operate-transfer
dpd	Dollars per day, unit for daily revenue
dpm	Dollars per minute, unit for value-of-time
dpt	Dollars per trip, unit for toll
HPO	High pricing option
IRR	Internal rate of return
LB	Lower bound
LPO	Low pricing option
MRG	Minimum revenue guarantee
NPV	Net present value
pas	Passive pricing strategy
PC	Price cap
PCM	Pure price cap regulation mechanism
PFI	Private finance initiative
PPP	Public private partnership
ROV	Real options value
TAM	Toll adjustment mechanism
UB	Upper bound
VOT	Value of time
vpd	Vehicles per day, unit for daily traffic
WHC	Western harbour crossing

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