COMPARISON STUDY ON TIME SERIES FORECASTING TECHNIQUES FOR APPAREL RETAILING

LI MIN

M.Phil

The Hong Kong Polytechnic University

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The Hong Kong Polytechnic University

Institute of Textiles and Clothing

Comparison Study on Time Series Forecasting Techniques for Apparel Retailing

Li Min

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____________________________________ (Signed)

Li Min (Name of student)

TO MY HUSBAND AND MY PARENTS

For their constant love, support and encouragement

Abstract

Sales forecasting is the foundation for planning various phases of a firm's business operations. It is also crucial to dynamic supply chains and greatly affects retailers and other channel members in various ways. Effective sales forecasting enables big improvement in supply chain performance. In today's apparel retailing, sales forecasting mainly rely on subjective assessment and experience of sales/marketing personnel with simple statistical analysis of historical sales data. While there exist various sales forecasting techniques, it is unknown how each technique fits different types of apparel sales data, and no research efforts have ever been made to investigate and compare the effects of different techniques on different sales data patterns for apparel retailing. The purpose of this research is to investigate and compare the forecasting performances of commonly used univariate and multivariate time series forecasting techniques for apparel retailing.

A methodology with nine procedures was presented to compare different time series forecasting techniques for apparel retailing. Four typical data patterns of apparel retailing were identified to represent various apparel sales time series. Some commonly used time series forecasting techniques, including five univariate techniques, three multivariate techniques and neural network (NN) techniques, were used and compared. Five accuracy measures were used to evaluate the forecasting results, which included the mean absolute deviation, mean absolute error, mean absolute percentage error, mean absolute scaled error and root mean square error.

Five commonly used univariate forecasting techniques were used to construct nine forecasting models. The performances of these univariate forecasting models and two

univariate NN models were compared on the basis of a large number of apparel sales time series. These apparel sales data were collected from an apparel retail company and categorized into trend, seasonal, irregular and random patterns. 10 multivariate forecasting models were constructed based on four multivariate forecasting techniques. The forecasting performances of these models and two multivariate NN (MVNN) models were compared on the basis of the same apparel sales data. Lastly, the performances generated by these univariate models were further compared with those by the multivariate models. This research also investigated the effects of different numbers of input variables and different accuracy measures on sales forecasting performances.

The comparison study showed that (1) for different data patterns, forecasting performances generated by univariate and multivariate forecasting models are mixed; for seasonal data patterns, ARX(3,2), ARMAX(3,3,2) and NN(3) models can perform better than the others; for irregular data patterns, ARMAX(3,3,2), ARMAX(3,3,1) and ARMAX(2,2,2) models can perform better than the others; for random data patterns, AR(2), ARX(3,1), ARMA(1,1) and ARMAX(3,3,1) models can outperform the other models. (2) Among the univariate techniques, the moving average technique usually generates the worse forecasting results no matter what data pattern is used. (3) NN models cannot provide better forecasting performances than the other classical models. (4) The multivariate time series forecasting models cannot always generate better results than the univariate time series forecasting models. For example, the AR(2,1) and AR(2,2) models usually cannot generate better forecasts than the AR(2) models although the ARX(3,1) and ARX(3,2) models are relatively better than the AR(3) model. (5) The MVNN models cannot outperform the other traditional multivariate models significantly. (6) Even for the same model, different parameter settings can impact forecasting results greatly. For instance, the ARX(3,2) model generates much better results than the ARX(2,2) model for seasonal patterns. (7) In addition, different accuracy measures and different numbers of input variables can impact forecasting results greatly.

These comparison results show that it is important to select appropriate forecasting models based on different data patterns, and to set appropriate model parameters, NN structures, accuracy measures and input variables based on specific forecasting tasks. The comparison presented in this thesis can provide a theoretical basis for researchers and practitioners of apparel sales forecasting, and help them select the appropriate forecasting or benchmark models for different apparel sales forecasting tasks.

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Chapter 1

Introduction

1.1 Background

1.1.1 Today's apparel retailing

Today's apparel industry is characterized by its short product life cycles, volatile customer demands, tremendous product varieties and long supply processes (Sen, 2008). Apparel retailers in this ever-changing market must have the capability of providing consumers with appropriate apparel products at the right time.

The apparel retail industry is one of the most important business sectors, which is highly dependent on consumer spending. For example, most people have to reduce their daily expenditure on apparel products in the global economic downturn. That is, apparel retailers are extremely vulnerable to economic swings. With fashion emerging as a means of self-expression, today's consumers tend to choose apparel products with brand images or logos. The branded apparel industry has benefited from demographic shifts and changes in consumer preferences due to globalization and a rising awareness of branding. The growth of emerging markets such as China is encouraging for the branded apparel industry. However, comparing with international apparel branders, China's branded apparel retailers are facing bigger market competition.

The pressure on today's apparel retailers from global competition and volatile consumer demands has been enormous, which compels apparel retailers to continuously improve their business operations and supply chain management. Due to the ever-increasing global competition, sales forecasting plays an increasingly prominent role in supply chain management. Some studies have shown that effective sales forecasting enables big improvement in supply chain performance (Zhao et al., 2002; Bayraktar et al., 2008).

Thus, apparel retailers must improve their sales forecasting performance in order to deliver appropriate apparel products at the right time. However, in today's apparel retailing, sales forecasting mainly relies on subjective assessment and experience of sales/marketing personnel with simple statistical analysis of limited historical sales data.

1.1.2 Sales forecasting

Sales forecasting uses past sales performances and analysis of expected demands and market conditions to predict the sales of a product or service. It is the foundation for planning various phases of a firm's business operations (Boulden, 1958; Lancaster & Reynolds, 2002), and is crucial to dynamic supply chains and greatly affects retailers and other channel members in various ways (Xiao & Yang, 2008). A sales forecast can provide the business with an evaluation of past and current sales performances and future changes.

Sales forecasting has been investigated by a large number of researchers, concerning sales forecasting problems in various industries, including the apparel industry (Sztander et al., 2004; Thomassey et al., 2005; Au et al., 2008), print circuit board industry (Chang et al., 2005;

Hadavandi et al., 2011), tourism and lodging industry (Andrew et al., 1990; Smith et al., 1994; Kulendran & Wong, 2011), and airline industry (Oberhausen & Koppelman, 1982; Saab & Zouein, 2001; Lin, 2006; Jing et al., 2010).

To solve sales forecasting problems, various forecasting techniques have been proposed. The current forecasting techniques can be divided into two groups: classical techniques based on mathematical and statistical models and artificial intelligence techniques. In terms of classical techniques, exponential smoothing (ES), autoregressive (AR) methods, autoregressive moving average (ARMA) methods and Kalman filter methods are categorized as linear methods which employ a linear functional form for time-series modeling (De Gooijer & Hyndman, 2006; Gardner, 2006). As these linear methods cannot capture features that commonly occur in actual time-series data like occasional outlying observations and asymmetric cycles, they may not be suitable for nonlinear real-world time series (Makridakis et al., 1998). As for artificial intelligence techniques, neural networks (NNs) are the most commonly used technique which has been studied extensively by forecasting researchers (Kuo, 2001; De Gooijer & Hyndman, 2006; Au et al., 2008). The results of most of the above studies demonstrate that the NN approach outperforms the classical models due to their capacities for nonlinearity, generalization and universal function approximation. Among the above techniques, some are extended to form multivariate models, such as AR with exogenous inputs (ARX) and ARMA with exogenous inputs (ARMAX), and consider exogenous variables as the input of a forecasting model. However, it remains unknown whether the existing multivariate models can outperform the classical univariate forecasting techniques in terms of apparel sales forecasting because no experimental comparisons have ever been made in the existing studies.

Thus, it is advisable to (1) compare the adaptabilities of different forecasting techniques based on a large amount of experimental data, and (2) investigate whether multivariate techniques perform better than univariate techniques.

1.1.3 Sales forecasting in apparel retailing

To help the apparel industry implement effective sales forecasting, some researchers have investigated apparel sales forecasting problems in different aspects. Au et al. (2008) used an evolutionary NN approach in search of an ideal network structure for a forecasting system, and compared it with traditional forecasting models. Wong and Guo (2010) developed an hybrid intelligent algorithm to solve medium-term apparel sales forecasting problems based on real-world forecasting processes in apparel retailing. However, these studies mainly focused on developing univariate models to forecast apparel sales.

Some researchers emphasized the necessity of multivariate models for forecasting (Zhang et al., 1998; Jones et al., 2009). However, multivariate apparel sales forecasting models have attracted relatively little attention so far. Sztandera et al. (2004) presented a multivariate fuzzy forecasting model by incorporating the effects of color and size as inputs to forecast the sales of women's apparel products, which exhibited a better forecasting performance than univariate NN models. Sun et al. (2008) applied a novel NN model to apparel sales forecasting and investigated the relationship between sales amounts and some significant apparel product attributes. However, these studies did not consider the effects of various exogenous factors, such as climate, economic environment and promotion activities, which can affect apparel sales, Thomassey et al. (2005) developed two complementary forecasting models for the textile and apparel industry by using a fuzzy inference system to quantify the influence of exogenous variables, including price and data calendars. However, the effectiveness of fuzzy inference rules greatly depends on expert knowledge, which is difficult to obtain in practice. In addition, it is difficult to apply these models in practice because they use multiple soft computing techniques and parameters for the setting of such techniques.

The effectiveness of forecasting techniques proposed in previous studies is usually validated by a small amount of experimental data. Due to insufficiency of experimental data, it is questionable whether the techniques can be used for other sales data. In addition, the parameter setting of each forecasting model has great influence over its forecasting performance. However, the existing studies rarely investigate the rationale behind the choice of benchmark models and their parameter setting.

On the other hand, it is generally agreed that there does not exist a forecasting technique appropriate to all sales time series. For example, linear forecasting models cannot generate ideal results for non-linear forecasting tasks. However, it is unknown how each forecasting technique fits different types of apparel sales data.

To summarize, the comparison study of time series forecasting techniques for apparel retailing is necessary and it is important to choose suitable techniques for different types of data pattern in apparel sales forecasting.

1.2 Research objectives

This research investigated and compared the performances of various commonly used forecasting techniques in forecasting different types of apparel sales pattern in apparel supply chains. The objectives of this research are as follows:

(1) To investigate and compare the performances of different types of univariate forecasting technique for apparel retailing (e.g. traditional univariate techniques and neural network techniques);

(2) To investigate and compare the performances of different types of multivariate forecasting technique for apparel retailing (e.g. traditional multivariate techniques and neural network techniques);

(3) To investigate and compare the performances of univariate and multivariate forecasting techniques investigated in (1) and (2);

(4) To identify the appropriateness and adaptability of different forecasting techniques for different types of apparel sales data pattern.

1.3 Methodology

To achieve the research objectives above, this research used typical sales data of the apparel retail industry. The data were collected from Hong Kong and China. Some commonly used univariate and multivariate time series forecasting techniques were selected and used for comparison. The comparisons were conducted on the basis of different types of data pattern and accuracy measures.

The key procedures involved in the methodology are as follows.

(1) Data collection and preprocessing

To achieve the research objectives of this project, the first step was to collect historical data on apparel product sales and various exogenous variables. Historical data spanning several years were required to establish effective forecasting models. The exogenous variables included product prices, climate factors, economic factors (i.e. gross domestic product indexes, consumer price indexes and promotion activities). Related economic indexes were singled out for further model development to analyze the correlation between the index and apparel sales.

The preprocessing process vetted the collected data by removing outliers, missing values or irregularities to reduce the effects of defective data on forecasting accuracy. As different inputs (e.g. historical sales and exogenous variables) may have different value ranges, each input needed normalizing so as to improve the accuracy of the forecasting model.

(2) Performance comparison of different univariate time series forecasting techniques for apparel sales forecasting

This research investigated and compared the forecasting performances of several commonly used univariate time series forecasting techniques, including Naïve, AR (autoregressive), MA (moving average), ARMA (autoregressive moving average) and NN (neural network), and compared their performances in apparel sales forecasting.

(3) Performance comparison of different multivariate time series forecasting techniques for apparel sales forecasting

This research investigated several commonly used multivariate forecasting methods, including ARX (AR with extra inputs), ARMAX (ARMA with extra inputs), GLM (generalized linear regression) and NN, and compared their performances in apparel sales forecasting.

(4) Performance comparison of univariate and multivariate time series forecasting techniques for apparel sales forecasting

The univariate and multivariate time series forecasting techniques used in (3) and (4) would then be compared based on the same sales time series.

To achieve a comprehensive comparison, extensive experiments needed to be conducted. Firstly, there were various sales forecasting tasks, such as mid-term aggregated sales forecasting and short-term item sales forecasting, in apparel retailing. Experiments would cover these apparel sales forecasting tasks. Secondly, numerous sales data patterns, such as seasonal, nonlinear, and even random time series, existed in apparel retailing. This research used these different types of data pattern as benchmark data to observe the performances of different techniques in each type of data pattern.

1.4 Significance of this research

The significance of this research is described as follows:

(1) The first contribution of this research is to broaden the investigation and enrich our understanding of apparel retail sales forecasting from the perspectives of both univariate and multivariate time series forecasting.

(2) The comparison study of time series forecasting techniques for apparel retailing is necessary and it is important to choose suitable techniques for different types of data pattern in apparel sales forecasting. This research provides a theoretical basis for forecasting researchers and practitioners and helps them select the appropriate forecasting or benchmark models from the commonly used time series forecasting techniques for different sales forecasting tasks.

(3) The development of this research will enrich the methodologies of sales forecasting for apparel retailing. The conclusions can also be used in dealing with sales forecasting problems in other similar retail industries.

1.5 Structure of this thesis

The aim of this research is to compare the forecasting performances of the commonly used univariate and multivariate time series forecasting techniques for apparel retailing. The subsequent chapters will detail our research work and are summarized as follows:

Chapter 2 provides a comprehensive literature review of the existing research into sales forecasting in apparel and other industries, including various sales forecasting problems, techniques for sales forecasting, and existing comparison studies of sales forecasting techniques. Various time series forecasting techniques used in this research are also introduced on the basis of analysis on research limitations of previous related studies.

Chapter 3 presents the methodology used for the comparison of sales forecasting techniques. The data patterns in apparel sales data and the commonly used accuracy measures for sales forecasting will be described therein. Besides, the details how to develop and implement these forecasting techniques are presented as well.

Chapter 4 investigate and compare the performances of univariate and multivariate time series forecasting techniques for apparel sales forecasting. Extensive comparison experiments are conducted, and the comparison results are discussed and analyzed.

Finally, Chapter 5 summarizes the findings and limitations of this research. Further research directions are also suggested.

Chapter 2

Literature Review

2.1 Introduction

As an important business process in apparel supply chain operations, apparel sales forecasting has drawn much attention in both academia and industry. This chapter is to review previous studies linked to this research project. In section 2.2, previous research in sales forecasting is reviewed. Techniques for sales forecasting are reviewed in section 2.3. Finally, previous research in apparel sales forecasting is finally reviewed in section 2.4.

2.2 Previous research in sales forecasting

Research in sales forecasting can be traced back to 1950s (Boulden 1958). Since then sales forecasting has attracted extensive attention from academia. A large number of sales forecasting papers have been published, which involves a wide variety of real-world applications in numerous industries, mainly including print circuit board industry (Chang et al. 2005, Hadavandi et al. 2011), tourism and lodging industry industry (Andrew et al. 1990, Smith et al. 1994, Kulendran and Wong 2011), airline industry (Oberhausen and Koppelman 1982, Saab and Zouein 2001, Lin 2006, Jing et al. 2010), foodservice industry (Miller et al. 1991, Chen and Ou 2009, Tsai and Kimes 2009), and apparel industry (Sztandera et al. 2004, Thomassey et al. 2005, Au et al. 2008). This section only reviews previous sales forecasting studies in several main industries except for apparel industry, which will be reviewed in section 2.4.

2.2.1 Printed circuit board industry

Chang et al. (2005) addressed the monthly sales forecasting problem to help printed circuit board companies generate effective customer demand forecasts by considering indexes from four different domains such as macroeconomic and industrial ones. Hadavandi et al. (2011) presented a novel sales forecasting approach by the integration of genetic fuzzy systems (GFS) and data clustering to construct a sales forecasting expert system. At first, all records of data are categorized into k clusters by using the K-means model. Then, all clusters will be fed into independent GFS models with the ability of rule base extraction and data base tuning. Experimental results show that the proposed approach outperforms the other previous approaches.

2.2.2 Tourism and lodging industry

Andrew et al. (1990) examined a problem of forecasting hotel occupancy rates by using two time series models, Box-Jenkins and exponential smoothing. The models are fitted and tested using actual monthly occupancy rates for a major center-city hotel. Both models show a high level of predictive accuracy. Smith et al.(1994) investigated a problem to deal with weekly sales data from a major retail chain and developed a two-stage forecasting methodology for estimating the sales responses to marketing and environmental variables when it is likely that their impacts will change unpredictably over time. Kulendran and Wong (2011) aimed at forecasting the investigated the turning points in the Hong Kong inbound tourism growth cycle in terms of various tourism demand determinants such as income, price at the destination, price at the substitute destination, and oil price.

2.2.3 Airline industry

Oberhausen and Koppelman (1982) used a multivariate time series approach to predict air travel and estimate fare elasticity. Nan and Schaefer (1995) examined the forecasting problem of international airline passenger traffic by using neural networks (NNs) in lieu of traditional statistical techniques. Saab and Zouein (2001) aimed at forecasting the number of boarding passengers on each of the next N departure dates of a particular flight leg using as input the booking levels made for the next N departure dates of that flight leg. Lin (2006) investigated how a firm forecast the future demand distribution with better precision by considering a situation where the firm does not have an accurate demand forecast, but can only roughly estimate the customer arrival rate before the sale begins in airline service industries then use the new information to dynamically adjust the product price in order to maximize the expected total revenue. Jing et al. (2010) investigated an airline demands forecasting problem based on period-decoupled booking data, by constructing an ARIMA model and forecasting for one typical period-decoupled booking data.

2.2.4 Restaurants and foodservice industry

Miller et al. (1991) investigated the forecasting of restaurant covers (dine-in guests). The demand for specific menu items could then be derived from the number of covers forecast. Chen and Ou (2009) investigated a perishable food forecasting problem with the consideration of sales data of target store and neighboring stores as well as weather data. Tsai and Kimes (2009) investigated a restaurant reservation forecasting problem based on the concept of pattern retrieval. They discussed the issues of how to retrieve booking patterns, search for influential parameters, and divide available samples for training, validating, and testing.

Gelper and Croux (2007) suggested that the multivariate regression test is the most powerful among the considered possibilities by investigating whether the consumer confidence index Granger causes retail sales in Germany, France, the Netherlands and Belgium. Danese and Kalchschmidt (2011) examined the impact of multivariate forecasting on companies' performance by analyzing the sample data from 343 manufacturing companies in six different countries. They demonstrated that companies should devote their attention to all the different forecasting variables while intending to improve cost and delivery performances

2.3 Techniques for sales forecasting

In the literature, sales forecasting problems can be classified into two categories: univariate sales forecasting and multivariate sales forecasting, no matter what industry the sales forecasting problem belongs to. To generate sales forecasts, forecasting model needs to be firstly established based on a forecasting technique, which can approximate the data generating process based on available training samples.

Forecasting models can be categorized as time-series, causal and judgmental. Time-series models utilize past data as the basis for forecasting future results. Techniques that fall into this category include decomposition, moving average, exponential smoothing, and Box-Jenkins, neural network (NN), etc. The premise of a causal model is that a particular outcome is directly influenced by some other predictable factor. Regression techniques and some intelligent techniques fall into this category. Judgmental techniques are often called subjective because they rely on intuition, opinions, and probability to generate the forecast. These techniques include expert opinion, Delphi, sales force composite, customer expectations (customer surveys), and simulation. This research only investigates time-series models. Forecasting techniques that fall into these two categories can also be divided into two groups: classical time series techniques based on mathematical and statistical models, and intelligent time series techniques.

According to different input variables of forecasting models, time series forecasting techniques can be classified into two categories: univariate and multivariate. Univariate forecasting techniques make forecasts by using input data directly from the historical sales data of time series being forecasted(Andrew et al. 1990, GarciaFerrer et al. 1997), which are usually based on a basic assumption that the underlying data-generating process of the time series is constant. This assumption is usually invalid in the real world since a variety of factors influencing product sales, called influencing factors (or exogenous variables), may cause uncertain change of data pattern, particularly in a dynamic and quick response retail industry such as apparel. As a result, the univariate forecasting model cannot handle abnormal sudden changes caused by exogenous variables such as product attributes and economic environment. Multivariate forecasting techniques are thus developed and used to handle sales forecasting in a dynamic business environment (Oberhausen and Koppelman 1982), which contain multiple inputs, including historical sales and related influencing factors (exogenous variables).

2.3.1 Classical time series forecasting techniques

Classical time series forecasting techniques include Naïve, exponential smoothing (Gardner 2006, Taylor 2007), regression (Ridley 1994), autoregressive integrated moving average (ARIMA) (Makridakis and Hibon 1997, Chu and Zhang 2003), autoregressive conditionally heteroskedastic (ARCH) methods (Engle 1982), Kalman filter (Xie et al. 1997, Jacobi et al. 2007), and so on.

Naïve model assumes that the next period will be identical to the present. The forecast is equal to the most recent observation of data. For seasonal data, all forecasts are equal to the most recent observation of the corresponding season, which is called as “Seasonal Naïve” (Athanasopoulos et al. 2011). The Naïve model is the simplest forecasting technique and usually used as benchmark technique (Lawrence et al. 2000, Thomassey et al. 2005, Athanasopoulos et al. 2011). Despite its simple form, some researchers have pointed out that the naïve model performed well for some time series when compared to more sophisticated quantitative forecasts (Makridakis et al. 1982, Makridakis et al. 1993, Makridakis and Hibon 2000, Athanasopoulos et al. 2011).

Exponential smoothing models are widely used in forecasting time series. Exponential smoothing makes an exponentially smoothed weighted average of past sales, trends, and seasonality to derive a forecast. Harrison (1967) employed a simple exponential smoothing technique to handle the short-term sales forecasting. Simple exponential smoothing does not perform well when there is a trend in the data to be forecasted. Snyder et al. (2004) investigated the applications of various exponential smoothing variants, such as Damped trend and Winters additive method, for forecasting lead-time demand (LTD) for inventory control. Gelper et al. (2010) developed a robust Holt-Winters exponential smoothing method for time series forecasting, which presented an easily implemented mechanism that automatically identifies outliers and downgrades their influence. The Theta method proposed by Assimakopoulos and Nikolopoulos (2000) is also known as the weighted method, which is equivalent to simple exponential smoothing with an added trend and a constant, where the slope of the trend is half that of a fitted trend line through the original time series (Hyndman and Billah 2003). This model performed extremely well for lots of time series in the M3 competition (Makridakis and Hibon 2000). However, this method has attracted little attention from forecasting researchers.

Regression techniques statistically relate sales to one or more explanatory (independent) variables. Explanatory variables may be economic data, competitive information, or any other variable related to sales. Regression techniques can be used for time series forecasting if the values explanatory variables are determined by time series. Chu and Zhang (2003) compared the performances of various linear and nonlinear models for forecasting aggregate retail sales. Heshmaty and Kandel (1985) presented a fuzzy linear regression technique for sales forecasting. Some researchers have used regression techniques as benchmark techniques for performance comparison (Ridley 1994, Ong and Flitman 1998, Kuo and Xue 1999).

ARIMA technique uses the auto correlative structure of sales data to develop an autoregressive moving average forecast from past sales and forecast errors, which includes a moving average process and an autoregressive process. Autoregressive (AR) techniques, moving average (MA) technique, and autoregressive moving average ARMA can all be taken as special cases of ARIMA techniques. Dalrymple (1978) developed the Box-Jenkins ARIMA models for sales forecasting. Makridakis and Hibon (1997) compared various ARIMA models and found that AR(1), AR(2) and ARMA(1,1) models can produce more accurate post-sample forecasts than those found through the application of Box–Jenkins methodology. ARIMA technique is one of the most commonly used benchmark techniques (Ansuj et al. 1996, Chu and Zhang 2003, Chen and Ou 2009, Kuo et al. 2009, Wong and Guo 2010).

Exponential smoothing, ARIMA and Kalman filter techniques are all categorized as linear methods that employ a linear functional form for time-series modeling (Chase 1993, Florance and Sawicz 1993, De Gooijer and Hyndman 2006). As these linear methods cannot capture features that commonly occur in actual time-series data like occasional outlying observations and asymmetric cycles, they may not be suitable for nonlinear real-world time series (Makridakis et al. 1998).

The Autoregressive conditional heteroskedasticity (ARCH) model was introduced by Engle (1982). ARCH models are employed usually in characterizing and modeling financial time series that exhibit time-varying volatility clustering, i.e. periods of swings followed by periods of relative calm. The ARCH family, including its various extensions such as generalized ARCH and nonlinear generalized ARCH, has been reviewed extensively (Bollerslev et al. 1992, Bera and Higgins 1993). Comparing with other classical models, the ARCH model is harder to use and attracted relatively little attention from forecasting researchers.

For many time series, the values in the time series are determined by not only its historical observations but also the historical observations of other explanatory variables. These explanatory variables need to be used as the inputs of time series forecasting models, which are so-called multivariate time series forecasting models. In classical methods, ES, regression, ARIMA, Kalman filter and ARCH techniques involve multivariate time forecasting models, in which the most commonly used multivariate models are AR with exogenous inputs (ARX) and ARMA with exogenous inputs (ARMAX) models.

Many researchers developed a variety of multivariate time series forecasting models with the consideration of exogenous variables and claimed the necessity of using multivariate models for prediction (Zhang et al. 1998, Jones et al. 2009). However, most of multivariate models in the literature are black-box models, which are developed in terms of their input, output and transfer characteristics without any knowledge required of their internal workings. Thus, these models are not helpful to analyze qualitatively the relationships between explanatory variables and product demands. More recently, fuzzy time-series and fuzzy neural network models, which are based on the fuzzy set theory, have advancedly improved the forecasting capability of various problem domains, such as enrollment (Chen 1996), inventory (Huarng and Yu 2006), and the stock index (Yu 2005). These models perform better than the conventional ones since they can handle nonlinear data directly and are able to model human knowledge (Mastorocostas et al. 2000, Van Lith et al. 2000). In addition, rigid assumptions regarding the data are not required. These fuzzy time-series models have different types, including order-1 autoregression (Chen 1996, Sah and Degtiarev 2005), order-p autoregression (Huarng and Yu 2003) and bivariate (Hsu et al. 2003) to multivariate models (Wu and Hsu 2002). However, these fuzzy neural network models are computation-intensive. It needs to be investigated further how to reduce the computation complexity of these models and apply them to extract the intrinsic relationships between different exogenous variables and apparel product demands.

Some researchers used generalized linear model (GLM) in multivariate time series forecasting (West et al. 1985, Guo et al. 2013). The generalized linear model (GLM) was developed by Nelder and Wedderburn (1972), which is a flexible generalization of various least squares regression models, including linear regression, logistic regression and Poisson regression. The GLM generalizes linear regression by allowing the linear model to be related to the response variable via a link function and by allowing the magnitude of the variance of each measurement to be a function of its predicted value. GLM can be used for time series forecasting by using historical observations of time series of sales amounts and other exogenous variables as inputs.

In recent years, these classical time series forecasting techniques, including univariate and multivariate techniques, were mainly used as benchmark techniques in the literature.

2.3.2 Intelligent forecasting techniques

Classical forecasting techniques approximate data generating process of forecasted time series based on the assumption that the forecasted series imply the same or similar mathematical relationship with the classical technique. For instance, linear regression method assumes that the forecasted data can be approximated by a linear regression equation.

However, in most forecasting cases, we cannot know the mathematical relationship of the data to be forecasted in advance. Intelligent forecasting techniques can handle these cases well because they can decide the mathematical relationship dynamically based on training data.

Intelligent forecasting techniques include neural network (NN) models (Thiesing and Vornberger 1997, Sun et al. 2008, Alekseev and Seixas 2009), expert systems (Lo 1994, Smith et al. 1996), fuzzy systems (Sakai et al. 1999, Frank et al. 2004), and hybrid intelligence techniques integrating multiple intelligent techniques (Thomassey and Fiordaliso 2006, Thomassey and Happiette 2007, Wong et al. 2010).

NN techniques have been proved to be universal approximators and can effectively model various time series and non-time series, which have the potential to generate effective forecasts due to their capacities of nonlinearity, generalization, and universal function approximation (Tang et al. 1991, Chu and Zhang 2003). Ansuj et al. (1996) used a backpropagation (BP) NN model to analyze the behavior of sales in a medium size enterprise located in Santa Maria, Brazil. The forecasts generated by the NN model were found to be more accurate than those generated by ARIMA model with interventions. Luxhoj et al. (1996) developed a hybrid econometric NN model, which integrated the structural characteristics of econometric models with the non-linear pattern recognition features of neural networks, for forecasting total monthly sales. Thiesing and Vornberger (1997) used a BP NN to forecast the future values of time series of weekly demands on items in a German supermarket. Their NN model considered the effects of the influencing indicators of prices, advertising campaigns and holidays on next week’s sales. Kotsialos et al. have used a multi-layer NN model for forecasting long-term sales (Kotsialos et al. 2005). In their study, the NN model has not show obvious superiority over Holt-Winters exponential smoothing method. Chen and Ou (2011) presented a gray extreme learning machine (GELM) NN model, integrating Gray relation analysis and extreme learning machine with Taguchi method, to forecast retail sales in Taiwan. Their experimental results demonstrate that the GELM model outperform several BPNN sales forecasting methods which are based on back-propagation neural networks such as BPNN and a multifunctional layered network model.

Expert systems use the knowledge of one or more forecasting experts to develop decision rules to arrive at a forecast. Shahabuddin (1990) has pointed out that an expert system could be easily developed to help executives in forecasting. Lo (1994) presented the application of expert systems for selecting techniques for demand forecasting. The expert system was built to capture expert knowledge and acted as an advisor for choosing suitable demand forecasting techniques for use under various general business circumstances. Simth et al (1996) developed an expert system to forecast the short-term regional gas demand. Sanchez et al. (1995) developed an expert system model to replicate the knowledge, experience, creativity, judgment, and intuition of the forecasting expert in foodservice, for forecasting various combinations of menu items. The performance of the expert system was comparable to that of the forecast expert; 81% acceptability was achieved. Nonexperts can use the expert system easily as well (Sanchez and Sanchez 1995).

Fuzzy systems use fuzzy logic theory to handle fuzzy and uncertain information in forecasting process. Sakai et al. (1999) established a fuzzy forecasting system, based on fuzzy logic and a multiple regressive model, to forecast the number of cans dispensed daily so that electricity would be used to cool only the required number of cans. Frank et al. (2004) developed a multivariate fuzzy system for forecasting women's casual sales, which generated better forecasting solutions than an NN model, a single seasonal exponential smoothing and a Winters' three parameter exponential smoothing model.

Hybrid intelligence technique is a combination of multiple intelligent techniques, such as Fuzzy logic and NN (FNN), evolutionary algorithms and NN (ENN). In recent years, more and more researchers developed hybrid intelligence models for sales forecasting (Kuo and Xue 1998, Doganis et al. 2006) , which combined the strengths of different techniques to generate better forecasting performances. Kuo and Xue (1998) constructed an FNN model to handle the sales forecasting problem under promotion. Their study reported that the FNN model provided better forecasts than single ANN and ARMA. Chang et al. (2005) developed an evolving neural network (ENN), integrating NN and genetic algorithm, to forecast the monthly sale demand in a printed circuit board company. Their experimental results showed that the ENN model can generate better forecasts than BPNN, linear regression and Winter's exponential smoothing. Doganis et al. (2006) developed a hybrid intelligence model, integrating a genetic algorithm and a radial basis function (RBF) neural network, for nonlinear time series sales forecasting. This model was applied successfully to sales data of fresh milk provided by a major manufacturing company of dairy products. Kuo et al. (2009) integrated a hybrid evolutionary algorithm with a radial basis function neural network (RBFNN) for forecasting papaya milk sales. They hybridized particle swarm optimization (PSO) and genetic algorithm (GA) to improve the learning performance of RBFNN. Their experimental results show that the proposed method outperforms PSO, GA and Box-Jenkins model in terms of forecasting accuracy. Chang et al. (2009) proposed a hybrid intelligence model, integrating K-mean cluster and fuzzy neural network, to forecast the monthly sales of a printed circuit board factory. Their experimental results showed that the proposed model generated better forecasts than Winter's exponential smoothing model and BP NN model in terms of two different performance measures, i.e., forecasted errors (RMSE) and accuracy of forecasted results (MAPE).

Hybrid intelligence models are hopeful to overcome the weakness of one intelligent technique by combining the strengths of different techniques. However, they are usually computation-intensive. It needs to be investigated further how to reduce the computation complexity of these models. In addition, hybrid intelligence models are hard to use because too many algorithm parameters need to be preset and how to set these parameters still relies on experience.

Among intelligent forecasting techniques, NN techniques were the mostly commonly used ones which have been studied extensively by forecasting researchers (Zhang et al. 1998, Au et al. 2008). NN techniques can be used to construct both univariate and multivariate forecasting models by using different input variables. For example, univariate NN models are constructed if only historical observations of sales time series are used as NN inputs; multivariate NN models are constructed if historical observations of time series of sales amounts and other exogenous variables are used as NN inputs. Moreover, by setting different NN structures and parameters, different NN forecasting models can be established.

2.3.3 Performance comparison of different forecasting techniques

In the literature, forecasting techniques can be classified into two categories: univariate and multivariate.

Multivariate time series models may be expected to generate more accurate forecasts because they have potentials to effectively model the relation between sales data and various exogenous variables. Some researchers provided evidence about this (Chiu and Shyu 2004, De Gooijer and Hyndman 2006, Danese and Kalchschmidt 2011). However, some researchers also reported contrary evidences (Du Preez and Witt 2003, De Gooijer and Hyndman 2006). Their experimental results showed that univariate forecasting models performed better than multivariate models. A possible reason is that in the multivariate sales forecasting literature, a limited number of exogenous variables are considered due to various reasons such as data inavailability, which perhaps omits important factors and thus weakens forecasting performance.

Some researchers have aimed at comparing the performances of several classical forecasting techniques based on different forecasting applications. GarciaFerrer and DelHoyo (1997) compared the forecasting performances of ARIMA approach and two univariate unobserved component models with fixed and time-varying parameters by using monthly time series of automobile sales in Spain. The accuracy of the different methods is assessed by comparing five measures of forecasting performance based on the out-of-sample predictions for various horizons, as well as different assumptions on the models' parameters. Their research showed that (1) a thorough discussion on the nature and the relationships among the forecasting criteria is necessary before any general conclusions are drawn; (2) there is no uniform dominance of one method over the others for all criteria and at all forecasting intervals.

Brodie and Dekluyver (1987) have reported that Naïve model often yielded more accurate results than sophisticated models for short-term econometric market share forecasting. Chan and Hui (1999) compared the performance of different forecasting models for forecasting visitor arrival by using the Gulf War as an example of sudden environmental change. The findings showed that a Naïve model (Naïve II) generates better forecasts in dealing with unstable data than ARIMA and exponential smoothing models in terms of forecasting accuracy. Witt et al. (1994) have reported that different forecasting techniques might perform differently in handling stable vs. unstable data by using domestic and international tourism demand data.

Ong and Flitman (1998) have compared the forecasting performances of NNs with multiple regression and Holt-Winters exponential smoothing based on real-life data, and pointed out NN models showed better performance. In Ong and Chan's another work (2011), experimental results have showed that NN models could not generate superior forecasts over deseasonalized and detrended model and ARIMA model.

Alon et al. (2001) compared the performances of NN technique and three classical methods, including Winters exponential smoothing, ARIMA and multivariate regression, for forecasting US aggregate retail sales with trend and seasonal patterns. Their comparison results indicate that on average ANNs fare favorably in relation to the more traditional statistical methods, followed by the ARIMA model. Despite its simplicity, the Winters model was shown to be a viable method for multiple-step forecasting under relatively stable economic conditions.

However, no research has been conducted to compare the performances of various univariate and multivariate time series forecasting techniques for apparel sales time series so far.

2.4 Previous research in apparel sales forecasting

In apparel sales forecasting, the models tailor-made for apparel products are scarce since forecasting apparel product sales is a challenging task. Frank et al. (2003) investigated the forecasting of women's apparel sales by comparing the performance of statistical time series modeling and that of ANN. The results indicated that the ANN model performed better.

In the apparel retail industry, sales forecasting activities mainly rely on qualitative methods, including panel consensus and historical analogy. These methods are mostly based on subjective assessment and experience of sales/marketing personnel with simple statistical analysis of limited historical sales data. A sophisticated sales forecasting model designed for apparel products capable of taking account of both exogenous and endogenous factors is necessary. Forecasting models tailor-made for apparel retailing are scarce since forecasting apparel product sales is a challenging task.

Sztandera et al. (2004) addressed the sales forecasting of women's apparel by developing a multivariate fuzzy forecasting model, which used historical sales, color and size as inputs.

The model exhibits superior forecasting performance to univariate models. Thomassey et al. (2005) proposed an automatic forecasting system for apparel sales forecasting, which consists of two complementary forecasting models. The first one obtains medium-term (one-year) forecasting by using FL to quantify the influence of explanatory variables while the second one fulfills short-term (one-week) forecasting by readjusting medium-term forecasts. Their experimental results showed that the proposed model had better forecasting performance than three classical models, including Holt–Winter with multiplicative seasonality model, ARMAX model and naïve model. In a following work, Thomassey and Fiordaliso (2006) integrated clustering and classification techniques to forecast mid-term sales of new apparel items. Thomassey and Happiette (2007) handled the forecasting of sales profiles of new apparel items by developing an NN-based system. Sun et al. (2008) presented a novel NN model for apparel sales forecasting and identified the relationship between sales amount and several apparel product attributes such as color, size and price. Au et al. (2008) proposed an evolutionary NN model to forecast sales of apparel items with features of low demand uncertainty and weak seasonal trends. Their model performed better than the traditional ARIMA method does.

Most studies in apparel sales forecasting aim at forecasting sales of apparel items so far. In real-world practice, the apparel retailer usually makes sourcing budgets by forecasting the total sales of apparel items in one apparel category or in all categories. Apparel designers then determine which items need to be purchased or produced in each category based on the category forecasts. Thomassey et al. (2005) presented a forecasting support system to deal with medium-term sales forecasting at different sales aggregation levels. However, it is hard to employ the system in retailing practice because multiple soft computing techniques were integrated in the system and too many parameters for such techniques need to be pre-set. Wong and Guo (2010) developed an extreme learning machine and harmony search-based intelligent model to forecast the medium-term apparel sales in terms of apparel product category. Their model can provide better forecasts than traditional mathematical methods such as moving average and auto-regressive methods.

In the literature, the most majority of researchers employed intelligent techniques, mainly NN technique, to develop forecasting models for apparel sales forecasting. However, NN technique is prone to be over-parameterized and overfitting, which will detract from the credibility of forecasts (Wong and Guo 2010, Wong et al. 2010). Constructing a NN model needs to select appropriate NN structure and learning algorithm, which involves the setting of multiple parameters. Unfortunately, how to determine the values of these parameters is still open, which increases the complexity of selecting an appropriate NN model and improves the risk of generating poor forecasts. In addition, it is well-known that NN model can perform better if more training samples are available. Unfortunately, available historical sales data of apparel products are usually insufficient due to frequent product changes and short selling season in apparel retailing, which will probably detract from the credibility of forecasts generated by NN-based models.

Previous studies in apparel sales forecasting usually utilized several sets of data to compare the forecasting performances of proposed intelligent models and few classical techniques. No research has investigated how to select appropriate techniques for comparison and how to set the parameters of these benchmarking forecasting models. It is questionable if these comparisons are sufficient. In addition, it is still open and desirable to investigate whether the classical techniques used for performance comparison in these studies are fair and reasonable.

To a certain extent, the nature of data can determine what forecasting method can be used. For example, it is impossible to use ARIMA forecasting techniques if sufficient sample data are unavailable; it is also unnecessary to use a complicated nonlinear technique to forecast a simple linear time series. Witt et al. (1994) have reported that different forecasting techniques might perform differently in handling stable vs. unstable data. It is well accepted that no forecasting technique is appropriate to all data patterns. However, no research has investigated and compared the effects of different techniques on different sales data patterns from apparel retailing, which leaves much room for further research exploration.

2.5 Limitations of previous studies

Although various univariate forecasting techniques and NN techniques have been used widely to forecast sales, little research has been conducted to compare the performances of various univariate and multivariate time series forecasting techniques for apparel sales time series so far. Previous studies have not investigate how to select appropriate techniques for comparison and how to set the parameters of these benchmarking forecasting models in apparel sales forecasting. It is thus desirable to investigate whether the classical forecasting techniques used for performance comparison in existing literature are fair and reasonable.

On the other hand, it is well accepted that no forecasting technique is appropriate to all data patterns and different forecasting techniques should be adopted for different data patterns. However, no research has investigated and compared the effects of different techniques on different apparel sales data patterns.

The existing research gap described above leaves much room for further research exploration. This research will conduct the comparison study on different time series forecasting techniques for apparel retailing.

2.6 Forecasting techniques commonly used

To the best of the author's knowledge, the research is the first to compare different time series forecasting techniques for apparel retailing. As a start, this research compares several commonly used univariate and multivariate time series forecasting techniques.

A time-series is a collection of observations taken sequentially at specified times, usually at 'equal intervals' (e.g. sales of an apparel product in successive months, seasons or years). Suppose we have an observed time series $\left(x_1, x_2, x_3, \ldots, x_T\right)$ and need to forecast the future values such as $x_{T+1}, \ldots, x_N$. The $x_1, x_2, x_3, \ldots, x_T$ is called in-sample data or a training sample for model creation. The $x_{T+1}, \ldots, x_N$ is called out-of-sample data or a testing sample for model testing.

$$time \quad series: \quad \{x_1, x_2, ..., x_t, \ldots, x_T, x_{T+1}, \ldots, x_N\}$$

Suppose the observed data are divided into a training sample of length T and a test sample of length N. Typically T is much larger than N. Let $\hat{x}_{t+1}$ denote the forecast for period $t+1$.

2.6.1 Univariate forecasting techniques

Univariate forecasting techniques used in this research include Naïve, MA, AR, ARMA and ES, which are described as follows.

(1) Naïve: It is the most cost-effective and efficient objective forecasting approach, and provides a benchmark against which more sophisticated models can be compared. For stable time series data, this approach ensures that the forecast for any period equals the previous period's actual value. That is, the forecast $\hat{x}_{t+1}$ for period $t+1$ is

$$\hat{x}_{t+1} = x_t \,. \tag{2-1}$$

(2) MA: It is a simple and commonly used time series approach to reducing short-term fluctuations and highlighting longer-term trends or cycles. Thus, moving averages eliminate or, at least, neutralize irregular components in a time series. The forecast $\hat{x}_{t+1}$ for the $t+1$th data point in time series S is equal to a moving average of its previous n data points. The MA model using its previous n data points as the model inputs is called as MA(n) model. The forecast generated by MA(n) model is,

$$\hat{x}_{t+1} = \frac{x_t + x_{t-1} + x_{t-2} + \cdots + x_{t-n+1}}{n} \tag{2-2}$$

(3) AR: It is a multiple regression model, in which x_t is regressed on its past values. A time series $\{x_t\}$ is an AR process of order p, $AR(p)$ process, if

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \cdots + \alpha_p x_{t-p} + e_t \tag{2-3}$$

where e_t is a purely random process with mean zero and variance σ_e^2. Thus, the forecast $\hat{x}_{t+1}$ of the $AR(p)$ model for the $t+1$ th data point in time series $S = \{x_t\}$ is

$$\hat{x}_{t+1} = \alpha_1 x_t + \alpha_2 x_{t-1} + \cdots + \alpha_p x_{t-p+1} \tag{2-4}$$

(4) ARMA: It is a mixed model which consists of an AR part and an MA part. The model is usually referred to as the $ARMA(p,q)$ model where *p* is the order of the *AR* part and q is the order of the MA part. The *AR* part refers to the autoregressive model of order *p*. The MA part is written as

$$x_t = \mu + e_t + \sum_{i=1}^{q} \theta_i e_{t-i}, \tag{2-5}$$

where the θ_1,..., θ_q are the parameters of the model, μ is the expectation of x_t (often assumed to be 0), and e_t is a purely random process with mean zero and variance σ_e^2.

The notation $ARMA(p,q)$ refers to the model with *p* autoregressive terms and q moving-average terms, which is thus written as

$$\hat{x}_{t+1} = \sum_{i=1}^{p} \varphi_i x_{t-i} + \mu + e_t + \sum_{i=1}^{q} \theta_i e_{t-i} \tag{2-6}$$

The importance of the ARMA processes is that many real data sets may be approximated in a more parsimonious way (i.e. fewer parameters are needed) by a mixed ARMA model rather than a pure AR or MA process.

(5) ES: It is a technique that can be applied to time series data, either producing smoothed data for presentation or giving forecasts. The time series are a sequence of observations. The observed phenomenon may be an essentially random process or an orderly but noisy process. However, past observations are weighted equally in the simple moving average and exponential smoothing assigns exponentially decreasing weights over time.

ES is commonly applied to financial markets and economic data, and can also be used with a discrete set of repeated measurements. In this research, the Brown's double ES (DES) method and the triple ES (TES) method were used because simple exponential smoothing does not do well when there is a trend in the data. The raw data sequence is often represented by $\{x_t\}$, beginning at time t = 0. We use $\{s_t\}$ to represent the smoothed value for time t, and $\{b_t\}$ is our best estimate of the trend at time t. The forecasting output of ES is now written as $\{\hat{x}_{t+m}\}$, an estimate of the value of x at time $t+m, m>0$, based on the raw data up to time t.

The Brown's double exponential smoothing is given by the following formulas.

$$s'_0 = x_0 , \tag{2-7}$$

$$s''_0 = x_0 , \tag{2-8}$$

$$s_t' = \alpha x_t + (1-\alpha)s_{t-1}', \tag{2-9}$$

$$s_t'' = \alpha s_t' + (1-\alpha)s_{t-1}'', \tag{2-10}$$

$$\hat{x}_{t+m} = a_t + mb_t \tag{2-11}$$

where α is the *smoothing factor*, and $0 < \alpha < 1$; a_t, the estimated level at time *t* and b_t, the estimated trend at time *t* are:

$$a_t = 2s_t' - s_t'' , \tag{2-12}$$

$$b_t = \frac{\alpha}{1-\alpha}(s_t' - s_t'') . \tag{2-13}$$

The triple exponential smoothing functions as follows.

$$s_0 = x_0 , \tag{2-14}$$

$$s_0'' = x_0 , \tag{2-15}$$

$$s_t' = \alpha x_t + (1-\alpha)\cdot s_{t-1}', \tag{2-16}$$

$$s_t'' = \alpha s_t' + (1-\alpha)\cdot s_{t-1}'', \tag{2-17}$$

$$s_t''' = \alpha s_t'' + (1-\alpha)\cdot s_{t-1}' \tag{2-18}$$

$$a_t = 3s_t' - 3s_t'' + s_t''' \tag{2-19}$$

$$b_t = \frac{\alpha}{2(1-\alpha)^2}((6-5\alpha)\cdot s_t' - (10-8\alpha)\cdot s_t'' + (4-3\alpha)\cdot s_t'''), \tag{2-20}$$

$$c_t = \frac{\alpha^2}{2(1-\alpha)^2}(s_t' - 2s_t'' + s_t'''),\qquad (2\text{-}21)$$

$$\hat{x}_{t+m} = a_t + mb_t + c_t \cdot m^2 \qquad (2\text{-}22)$$

2.6.2 Multivariate forecasting techniques

It is generally agreed that exogenous variables can affect the sales of apparel products, such as life spans, climate, shop quantity and various economic factors. Exogenous variables are used as external inputs of forecasting models for constructing multivariate forecasting models. Several representative multivariate forecasting techniques, including ARX, ARMAX and generalized linear model (GLM), are used and their sales forecasting performances are compared, which are introduced below:

(1) ARX: An ARX model is denoted by ARX(p , r), which represents the model with p autoregressive terms and r exogenous input terms. This model contains the AR(p) model and a linear combination of the last r terms of a known and external time series u_t. u_t is a vector because there exist multiple exogenous inputs. The model is given by:

$$x_{t+1} = \sum_{i=1}^{p} a_i x_{t+1-i} + \sum_{i=0}^{r} \mathrm{b}_i \mathrm{u}_{t+1-i} + e_{t+1} \qquad (2\text{-}23)$$

where e_{t+1} is a purely random process with mean zero and variance σ_e^2. Thus, the forecast $\hat{x}_{t+1}$ of the ARX(p , r) model for the $t+1$ th data point in time series $S = \{x_t\}$ is

$$\hat{x}_{t+1} = \sum_{i=1}^{p} a_i x_{t+1-i} + \sum_{i=0}^{r} \mathrm{b}_i \mathrm{u}_{t+1-i} \qquad (2\text{-}24)$$

The AR model is a special case of the ARX model with no external input.

(2) ARMAX: An ARMAX model is denoted by ARMAX(p , q , r), which refers to the model with p autoregressive terms, q moving average terms and r exogenous input terms. The model is given by:

$$x_{t+1} = \sum_{i=1}^{p} a_i x_{t+1-i} + \sum_{i=1}^{q} c_i e_{t+1-i} + \sum_{i=0}^{r} \mathrm{b}_i \mathrm{u}_{t+1-i} + e_{t+1}, \tag{2-25}$$

where e_{t+1} is a purely random process with mean zero and variance σ_e^2. Thus, the forecast $\hat{x}_{t+1}$ of the ARMAX (p, q, r) model for the $t+1$ th data point in time series $S = \{x_t\}$ is

$$\hat{x}_{t+1} = \sum_{i=1}^{p} a_i x_{t+1-i} + \sum_{i=1}^{q} c_i e_{t+1-i} + \sum_{i=0}^{r} \mathrm{b}_i \mathrm{u}_{t+1-i} . \tag{2-26}$$

The ARMA model is a special case of the ARMAX model without external input.

(3) GLM: The GLM was developed by Nelder and Wedderburn (1972), which is a flexible generalization of various least squares regression models allowing for response variables having other than a normal distribution.

In a GLM, each outcome of dependent variables, Y, is assumed to be generated from a particular distribution in the exponential family, which is a large range of probability distributions (e.g. normal, binomial, Poisson and gamma). The mean, μ , of the distribution depends on independent variables, X, through:

$$\mathrm{E(Y)} = \mu = g^{-1}(\mathrm{X}\beta) \tag{2-27}$$

where $\mathrm{E(Y)}$ is the expected value of Y, $\mathrm{X}\beta$ is the linear predictor, β is a linear combination of unknown parameters and $g(\cdot)$ is the link function.

Thus, the forecast $\hat{x}_{t+1}$ of the GLM model for the $t+1$ th data point in time series $S=\{x_t\}$ is

$$\hat{x}_{t+1} = g^{-1}(\mathrm{X}\beta) . \tag{2-28}$$

In this research, we set $\mathrm{X} = (x_t, x_{t-1}, ..., x_{t-k}, \mathrm{u}_t)$, and use GLM(k) to denote a GLM model, where k denotes the latest k th historical sales observation.

2.6.3 Neural network-based forecasting techniques

An NN is a computational model inspired by research into biological neural networks, which can be used to construct a univariate or multivariate forecasting technique based on different numbers of input variables. An NN consists of a number of interconnected neurons (or nodes), which are analogous to biological neurons in the brain, according to some patterns of connectivity. In most cases, an NN is an adaptive system, which discovers the relationships between inputs and associated outputs by adjusting the network setting in terms of data patterns of training samples. Feedforward NNs (FNNs) are the most common type of NN. One of the most well-known features of NNs is that it can be used as a universal approximator. In view of this feature, FNNs have been widely applied to a variety of forecasting tasks. In this study, all NNs are FNNs.

FNNs are a type of NN where connections among units do not travel in a loop but in a single directed path. Typically, an FNN consists of an input layer of neurons (nodes), one or more hidden layers of neurons, and an output layer of neurons. The input and output layer form bookends for hidden layers of neurons. Signals are propagated from the input layer to hidden neurons and then onto output neurons, which output responses of the network to outside users. That is, signals only move in a forward direction on a layer-by-layer basis. Figure 2-1 shows a typical FNN with one hidden layer.

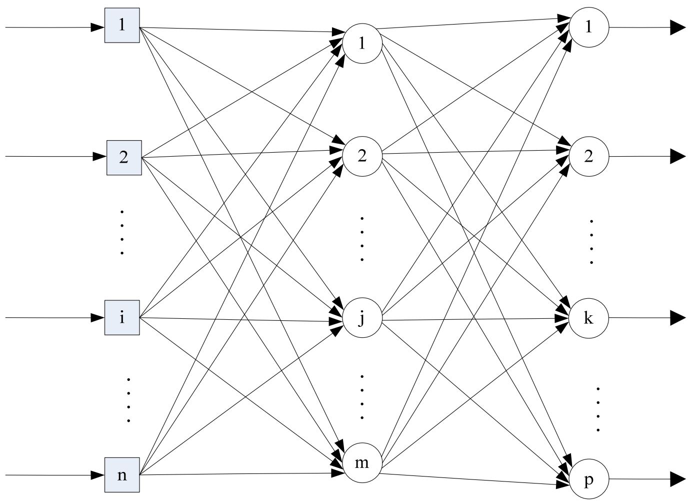

Figure 2-1: FNN with one hidden layer

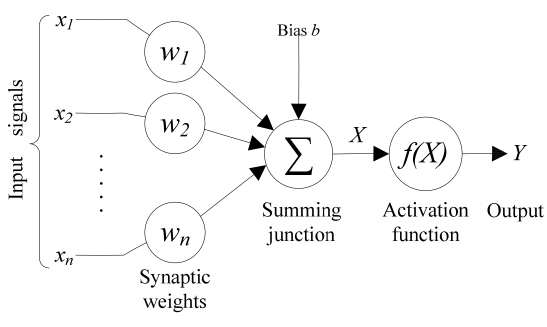

Figure 2-2: Diagram of a neuron

In an NN, a neuron is a mathematical function conceived as abstraction of biological neurons. Figure 2-2 shows a typical neuron. A neuron receives signals from its inputs x_i $(i = 1,...,n)$ (representing one or more dendrites) and an externally applied bias b. The weighted summation X $(X = \sum_{i=1}^{n} x_i w_i + b)$ of these input signals is then passed through activation function $f(X)$ to generate output signal Y (representing a biological neuron's axon). It is clear that

$$Y = f(X) = f(\sum_{i=0}^{n} x_i w_i) \tag{2-29}$$

In this equation, the effects of the bias is considered by (1) adding a new input signal fixed at +1 and (2) adding a new synaptic weight equal to bias b. That is, $x_0 = 1, w_0 = b$. The input signal $x_i (i = 1,...,n)$ can be raw data or outputs of other neurons. Output signal Y can be either a final solution to the problem or an input to other neurons. It should be noted that, for simplicity's sake, the NN shown in Figure 3-6 does not include bias signals, which is feasible in practical applications.

Various FNNs have been developed, which are constructed in terms of different settings from the following three perspectives:

(1) Network architecture, including the number of input neurons, the number of hidden layers and hidden neurons, the number of output neurons, and the interconnections among these neurons.

(2) Activation function, which determines the relationship between input and output of a neuron. Every neuron has its own activation function and generally only two activation functions are used in a particular NN. Neurons in the input layer use the identity function as the activation function. That is, the output of an input neuron equals its input.

(3) Learning algorithm, which determines the connection weights among network neurons. Traditionally, NN learning is an algorithmic procedure whereby parameters (such as weights) of an NN are estimated. The most popular learning algorithm is the backpropagation algorithm.

If p latest observations are used as the inputs of the NN forecasting model to forecast next data point $\hat{x}_{t+1}$, the forecast $\hat{x}_{t+1}$ generated by the NN is a function of the p observations. That is,

$$\hat{x}_{t+1} = f\left(x_t, x_{t-1}, \ldots, x_{t-p+1}\right) \tag{2-30}$$

where function $f(\cdot)$ is determined by training samples from time series $\{x_1, x_2, \ldots, x_T\}$, which represents the input/output relationship of the NN model.

2.7 Summary

In this chapter, previous studies on sales forecasting and existing comparison studies of sales forecasting techniques were reviewed. Some research gaps in the existing literature are pointed out, based on which we presented the time series forecasting techniques used in this research. Undoubtedly, this research will enrich greatly the study on forecasting techniques for apparel retailing. This research is also helpful to identify and select benchmark forecasting techniques for different data patterns.

Chapter 3

Methodology for Comparison on

Time Series Forecasting Techniques

3.1 Introduction

Chapter 2 presents a detailed review of the latest achievements in the field of apparel sales forecasting. However, the literature on this field is very limited and has made little impact on apparel retail practice despite the fact that effective sales forecasting is crucial to retail performance and supply chain management in the apparel industry.

This chapter presents the methodology for comparison of time series forecasting techniques for apparel retailing. Firstly, the main procedures of the comparison are introduced. Secondly, four typical data patterns in apparel retail sales data are presented. How univariate and multivariate forecasting techniques are used for comparison are then presented. Next, four accuracy measures used in this research are introduced. Finally, the development and implementation of the forecasting techniques are presented.

3.2 Procedures involved in the methodology

Figure 3-1 shows the procedures used to compare the forecasting performances of different time series forecasting techniques for apparel sales data. These procedures include :

Step 1: Collect sufficient apparel sales data from point-of-sales (POS) databases of a couple of apparel retailers headquartered in Hong Kong.

Step 2: Identify different types of sales data pattern to represent the changing trends of apparel sales data based on extensive analysis of apparel sales data.

Step 3: Select appropriate sample data on apparel sales for each data pattern, and use these data for performance comparison.

Step 4: Select appropriate accuracy measures for performance comparison.

Step 5: Select commonly used univariate and multivariate time series forecasting techniques for performance comparison.

Step 6: Compare the forecasting performances of several commonly used univariate forecasting techniques for each data pattern based on different accuracy measures, and analyze the comparison results.

Step 7: Compare the forecasting performances of several commonly used multivariate forecasting techniques for each data pattern based on different accuracy measures, and analyze the comparison results.

Step 8: Based on the comparison results in Procedures (6) and (7), compare the performances of the univariate and multivariate time series forecasting techniques.

Step 9: Identify the appropriateness and adaptability of different forecasting techniques for different types of apparel sales data pattern.

The following sections will introduce in detail the data patterns, univariate forecasting techniques, multivariate forecasting techniques and accuracy measures used in this research.

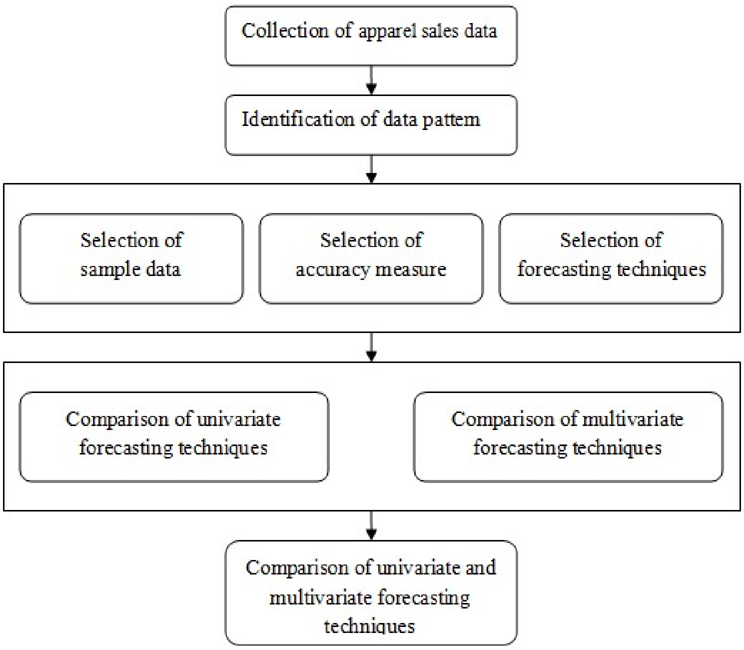

Figure 3-1: Procedures involved in the methodology

3.3 Data patterns in apparel retail sales data

In apparel retailing, there exist different sales forecasting tasks, including sales forecasting of one or more products, categories, shops or cities. In these tasks, time series of apparel sales data involve various data patterns. It is known that no forecasting technique is effective to all data patterns. Conducting a comprehensive analysis of a large amount of apparel sales data, we found these patterns can be classified into 4 types, namely trend, seasonal, irregular and random. The research thus compared the forecasting performances of different forecasting techniques based on the 4 types of data pattern.

(1) Trend pattern

In this study, a time series of apparel sales data is regarded as a trend pattern if the change of the series is characterized by an upward, downward or level direction. In apparel retailing, trend patterns usually occur in the aggregate sales data on cities. Figure 3-2 shows a time series of yearly sales data, indicating an obvious upward linear trend over time.

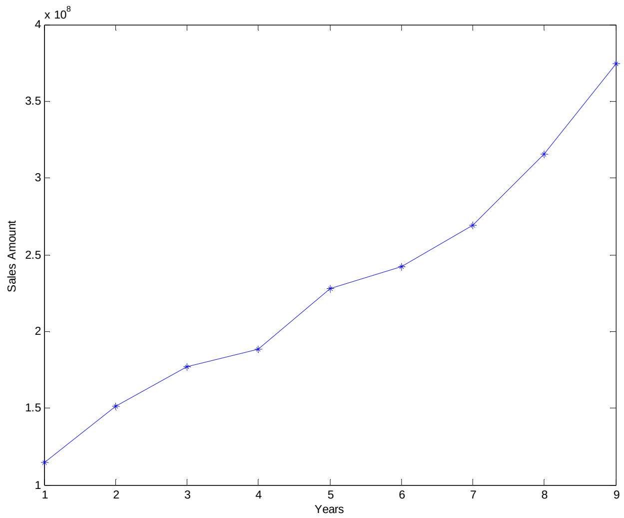

Figure 3-2: Example of a trend pattern

(2) Seasonal pattern

A seasonal pattern exists when a time series is influenced by seasonal factors, such as quarters, months and days. Seasonal series are sometimes considered “periodic” although they do not exactly repeat themselves over each period.

Seasonal patterns are popular in apparel sales data, which are usually caused by climates, selling seasons and holiday promotions. In apparel retailing, sales time series of product categories, such as coats and dresses, are usually characterized by seasonal patterns. Figure 3-3 shows a time series of sales data, indicating the monthly sales amount of a product category.

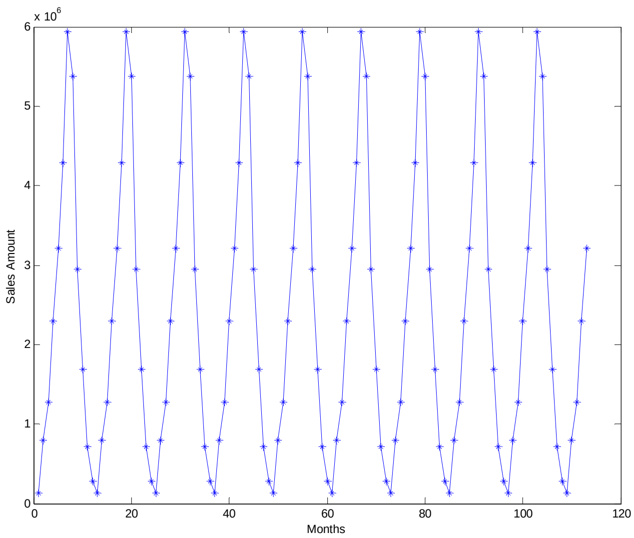

Figure 3-3: Example of a seasonal pattern

(3) Irregular pattern

If a data set features a trend or seasonal series, its time series is considered irregular. The irregular component (also known as residual) is what remains after seasonal or trend components of a time series have been estimated and removed, resulting from short-term series fluctuations which are neither systematic nor predictable. In a highly irregular series, these fluctuations can dominate movements and mask the trend or seasonality. Figure 3-4 shows an irregular time series.

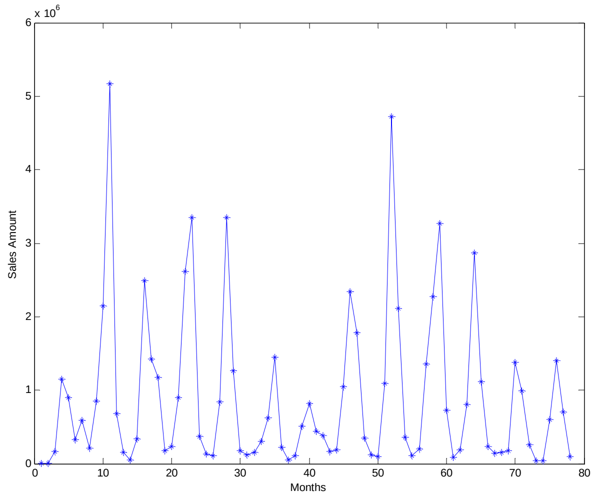

Figure 3-4: Example of an irregular pattern

(4) Random pattern

A time series, in which observations fluctuate around a constant mean, having a constant variance and being statistically independent, is regarded as random. In other words, it does not exhibit any pattern. The variance does not increase or decrease over time. The observations do not go upwards or downwards; nor do they tend to be larger in certain periods. Figure 3-5 shows a highly random time series.

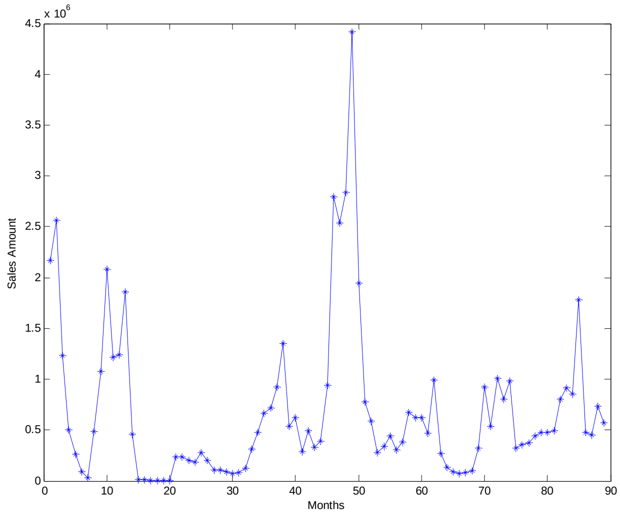

Figure 3-5: Example of a random pattern

3.4 Forecasting techniques used for comparison

For each of the four data patterns described above, this research compares the performances of several commonly used univariate forecasting techniques and multivariate forecasting techniques for apparel sales in terms of a large number of corresponding time series of apparel sales data. The NN technique is used for both univariate and multivariate forecasting tasks since it can be used as univariate and multivariate forecasting techniques. As a result, the univariate time series forecasting techniques compared include Naïve, MA, AR, ARMA, ES and NNs while the multivariate forecasting techniques compared include

ARX, ARMAX, GLM and NNs. For the details of these forecasting techniques, please see section 2.6.

The forecasting performances of these forecasting techniques are compared so as to evaluate each technique's performance on each data pattern of apparel sales data. The forecasting performances of the 6 univariate forecasting techniques are compared firstly. Then, the performances of the 4 multivariate forecasting techniques are compared. Finally, the performance of the 6 univariate techniques and 4 multivariate techniques are compared together. For each forecasting technique, different input and output relationships are established by setting different parameters based on the in-sample training data, each of which represented a forecasting model.

3.5 Accuracy measures

No accuracy measure is applicable to all forecasting problems due to various forecasting objectives and data scales (De Gooijer & Hyndman, 2006; Hyndman & Koehler, 2006). Let x_t denote the observation at time t and $\hat{x}_t$ denote the forecast of x_t. Then define the forecast error $e_t = \hat{x}_t - x_t$. In this research, five commonly used accuracy measures for sales forecasting analysis were used, namely the mean absolute deviation (MAD), mean absolute error (MAE), mean absolute percentage error (MAPE), root mean square error (RMSE) and mean absolute scaled error (MASE). The first one was used to evaluate the fitting performance of each forecasting model, and the others were used to evaluate the forecasting performance of each forecasting model:

(1) Mean absolute deviation (MAD): The MAD, also known as the mean deviation, is the data mean of absolute deviations of a data set. In other words, it is the average distance of the data set from its mean. The equation for the MAD is as follows:

$$MAD = \mathrm{mean}\left(\left|e_t - mean(e_t)\right|\right) \tag{3-1}$$

(2) Mean absolute error (MAE): The MAE is a quantity used to measure how close forecasts or predictions are to eventual outcomes. The mean absolute error is given by

$$MAE = \mathrm{mean}\left(\left|e_t\right|\right) \tag{3-2}$$

(3) Mean absolute percentage error (MAPE): This criterion is less sensitive to large errors than the RMSE and can be expressed as

$$MAPE = \mathrm{mean}\left(\frac{100e_t}{\hat{x}_t}\right) \tag{3-3}$$

(4) Mean absolute scaled error (MASE): To overcome the drawbacks of the existing measures, Hyndman and Koehler [21] proposed the MASE as the standard measure for comparing forecast accuracies across multiple time series after comparing various accuracy measures for univariate time series forecasting. The MASE is expressed as follows:

$$MASE = \mathrm{mean}\left(\left|\frac{e_t}{\frac{1}{n-1}\sum_{i=2}^{n}\left|\hat{x}_i - x_i\right|}\right|\right) \tag{3-4}$$

The MASE is less than one if it arises from a better forecast than the average one-step Naïve forecast computed in-sample. The Naïve model uses directly the last observation of a time series as a forecast. Conversely, it is more than one if the forecast is worse than the average one-step Naïve forecast computed in-sample.

(5) Root mean square error (RMSE): The RMSE is popular and often chosen by practitioners because of its user-friendliness and theoretical relevance to statistical modeling. The RMSE is expressed as follows:

$$RMSE = \sqrt{\text{mean}(e^2)} \tag{3-5}$$

3.6 Development and implementation of forecasting techniques

The forecasting techniques used in this research are developed and implemented on the MATLAB 7.0 (R14) platform. For Naïve and MA techniques, their forecasts are calculated by simply using their mathematical formulae described in section 2.6.1. For other forecasting techniques including AR, ARMA, ARMAX, GLM, NN and ES, the computation processes are much more complicated. For the first 5 techniques, their forecasts are implemented by using corresponding functions in several toolboxes of MATLAB. For the two ES (DES and TES) techniques, their forecasts are implemented by developing self-defined functions. The development and application of the functions for these techniques are described in detail as follows.

(1) AR

The AR-based forecasting models are implemented using the function ‘ar’ in the system identification toolbox of MATLAB. The AR model structure is given by the following equation:

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \cdots + \alpha_p x_{t-p} + e_t \ . \tag{3-6}$$

The ‘ar’ function is used to estimate the parameters $\{\alpha_i\}$ of AR model for univariate time series. The usage of ‘ar’ function is

m = ar(y,n),

where y represents a data object that contains the time-series data (one output channel), n represents the order p of the model one wants to estimate, and m represents the AR(p) model object containing parameters that describe the general multiple-input single-output model structure. In MATLAB, the AR model parameters are estimated using the variants of the least-squares method.

(2) ARX, ARMAX and ARMA

The ARX-based and ARMAX-based forecasting models are implemented using functions ‘arx’ and ‘armax’ respectively in the system identification toolbox. On the basis of training samples available, ‘arx’ estimates the parameters of the ARX model structure using least square while ‘armax’ estimates the parameters of the ARMAX model structure using the iterative prediction-error method.

The usage of ‘arx’ function is

m = arx(data,'na',p,'nb',r,'nc',q,'nk',nk),

where the input parameter data represents a data object of multivariate inputs and single output data pair, 'na', 'nb', and 'nc' are orders of the ARMAX model. ‘nk’ is the delay. p, r, q and nk are the corresponding integer values. For ARX model, q is set to 0. The default setting of nk is 1 in this research. The ‘arx’ function returns the ARX(p, r) model m with orders and delays specified as parameter-value pairs.

The ARMAX model is more flexible than the ARX model because the ARMAX structure contains an extra polynomial to model the additive disturbance. The usage of ‘armax’ function is

m = armax(data,'na',p,'nb',r,'nc',q,'nk',nk).

The input parameters of the ‘armax’ function are the same to those of the ‘arx’ function. The ‘armax’ function returns the ARMAX(p, q, r) model m with orders and delays specified as parameter-value pairs. The ARMA(p, q) model is a special case of ARMAX(p, q, r) model. We can thus calculate an ARMA(p, q) model by using the ‘armax’ function to armax(data, ‘na’, p ,’nb’,0,’nc’, r ,’nk’,0).

(3) GLM

The GLM-based forecasting models are implemented using functions ‘glmfit’ and ‘glmval’ in the statistics toolbox. Function ‘glmfit’ returns a vector of coefficient estimates for a generalized linear regression of the sales amount on model inputs while function ‘glmval’ computes predicted values for the generalized linear model with model inputs based on the model generated by the ‘glmfit’ function. The usage of function ‘glmfit’ is

b = glmfit(X,y,distr).

This function returns the estimates of coefficient { β } for a generalized linear regression of the responses in y (output samples) on the predictors in X (input samples), using the distribution distr. distr can be any of the following strings: 'binomial', 'gamma', 'inverse gaussian', 'normal', and 'poisson'. The default distribution ‘normal’ is used in this research.

The usage of function ‘glmfit’ is

yhat = glmval(b,X,link).

This function returns predicted values for the generalized linear model with link function link and predictors X (input samples). Distinct predictor variables should appear in different columns of X. b is a vector of coefficient estimates as returned by the ‘glmfit’ function. The link parameter is set as ‘identity’.

(4) NNs

The NN-based forecasting models are implemented on the basis of the neural networks toolbox in MATLAB. In this toolbox, function ‘newff’ is used to create feed-forward backpropagation networks, function ‘train’ is used to train neural networks, and function ‘sim’ is used to calculate predicted values for the NN-based model with model inputs. The usage of function ‘newff’ is

net = newff(P,T,[S1 S2...S(N-l)],{TF1 TF2...TFNl}, BTF),

where P represents the input sample data for neural network training, T represents the output training sample data, Si ($1 \le i \le N-1$) represents the size of ith layer (Output layer size SN is determined from T), TFi ($1 \le i \le N$) represents the transfer function of ith layer, and BTF represents the backpropagation training function. This function initializes and returns an N-layer feed-forward backpropagation network,

After a neural network is initialized, the ‘train’ function is then used to train the network based on training samples. The usage of function ‘train’ is

[net2,tr] = train(net1,P,T),

where net1 denotes the network generated by the ‘newff’ function, P denotes the network inputs (input samples), T denotes network targets (output samples). This function returns the trained network net2.

The ‘sim’ function is used to make forecasts after a neural network is trained well. The usage of function ‘sim’ is

[Y] = sim(net,P),

where net denotes the trained network, P denotes the network inputs (input samples), and Y denotes the forecast generated by the neural network model.

(5) ES

Two ES techniques (DES and TES) are used in this research, for which no corresponding functions exist in MATLAB. I thus wrote two self-defined functions ('ES_double' and 'ES_triple') to implement the two techniques respectively. The programs of two functions are placed in Appendix D.

The usage of the 'ES-double' function is

[trnErr, Fcst]=ES_double(series,L0,L1,L2,T),

where series represents the time series for model estimation, L0, L1 and L2 represent the lower limit, the step and the upper limit of α values respectively, T indicates the T-step-ahead prediction, trnErr represents the training error, and Fcst represents the forecasts.

The usage of the 'ES-triple' function is

[trnErr, Fcst]=ES_triple(series,L0,L1,L2,T),

where the input and output parameters of this function are the same to those of the 'ES-double' function.

3.7 Summary

The methodology of this research for performance comparison of the time series forecasting techniques for apparel sales was presented in detail. On the basis of the methodology, the next two chapters will present the conducted experiments to compare the univariate forecasting techniques and multivariate forecasting techniques, and present and analyze the comparison results.

Chapter 4

Comparison of Forecasting Techniques

for Apparel Retailing

4.1 Introduction

This chapter introduces experiments designed for performance comparison of the several commonly used univariate and multivariate forecasting techniques for apparel sales data. It also describes the collection of experimental data and presents the adopted forecasting models. The experimental results are shown in detail and the comparison results are analyzed and discussed.

4.2 Experimental design

To evaluate and compare the forecasting performances of different time series forecasting techniques, extensive experiments were conducted based on real-world sales data of apparel retail products. This section also presents the collection and selection of experimental data, the identification of data patterns and the construction of forecasting models.

4.2.1 Data collection

(1) Apparel sales data collection

Appropriate experimental data are the basis of reliable experimental results. A large variety of real-world apparel sales data were collected from two apparel retail companies located in Hong Kong and Mainland China. The first company is one of the most popular fashion brands in China, which sells casual wear products. Another company is an international high-ended fashion brand renowned for its stylish designs, high-quality fabrication and exquisite craftsmanship. The sales data collected from these fashion brands are typical time sales in fashion retail industry in Mainland and Hong Kong.

The point-of-sales (POS) data were collected from retail shops of different cities from January 2000 through May 2009. In apparel retailing, it is extremely difficult or even impossible to predict short-term sales of each apparel item using time series forecasting techniques due to their uncertainties and randomness. Therefore, this research used time series of medium-term aggregate sales (i.e. the aggregate sales amount of an apparel product or a product category) in retail shops (or cities) on a monthly, quarterly or yearly basis. As raw sales data are often incomplete in retailing, this research used complete sales data as experimental data for performance comparison. For each data pattern, a specified number of time series was selected for comparison.

In this research, 105 time series were used for performance comparison of univariate forecasting techniques, which include 34 yearly sales time series, 21 quarterly sales time series, and 50 monthly sales time series. The number of observations in yearly, quarterly, and monthly sales time series are 9, 37 and 113 respectively. All these sales time series are selected out from a huge number of sales data from the two retail companies' POS databases, which are complete sales data without missing points. The 105 sales time series involve 15 trend series, 30 seasonal series, 30 irregular series and 30 random series. How the 4 data patterns will be explained in detail in sub-section 4.2.2.

Due to the page limit, these time series are not presented in this thesis. For each time series, the final 15% of observations were used as out-of-samples to compare and evaluate the accuracy of the forecasting models. For each out-of-sample observation, its previous sales data were used as training samples to set up the forecasting models for making one-step-ahead forecasts.

(2) Multivariate data collection

To make multivariate sales forecasting, various influencing factors need to be used as inputs (exogenous variables) of multivariate forecasting models. These influencing factors include climate and economic indexes. Climate indexes can be represented by temperatures in a city while economic indexes can be represented by various economic indexes released by the Bureau of Statistics of each city. Some commonly used economic indexes are the Consumer Price Index (CPI), Gross Domestic Product (GDP), Producer Price Index (PPI), total retail sales index and price index for 'clothing and footwear'. However, not every Bureau of Statistics releases the same indexes or all these indexes. On the other hand, some released data are incomplete and thus not usable. As a result, it is necessary to select appropriate indexes as inputs of the multivariate forecasting model for each city. Except for the corresponding indexes described above, the input variables of the multivariate forecasting model for each city also have a certain amount of historical sales data.

Our performance comparison of the multivariate forecasting techniques was made on the basis of the 105 time series mentioned in this chapter. Most of the original time series were sales series of one or all product categories in four different cities, namely Beijing, Shanghai, Guangzhou and Hong Kong. On the basis of the actual indexes released by the Bureaus of Statistics of the four cities and the data completeness of these indexes, the following indexes were obtained and used as inputs of the multivariate forecasting models for the four different cities from January 2000 through May 2009:

Beijing: temperature, CPI, PPI, total retail sales of social consumer goods, and total retail sales of social clothing goods.

Shanghai: temperature and price index for 'clothing and footwear'.

Guangzhou: temperature, residents' disposable incomes, per capita consumption expenditure, and per capita consumption expenditure on clothing.

Hong Kong: temperature, total retail amount, and visitor arrivals.

For monthly (or quarterly) time series forecasting, the value of temperature is the monthly (or quarterly) average temperature while the values of the other indexes are the monthly (or quarterly) values released by the Bureau of Statistics of each city.

The values of these indexes are shown in two tables of Appendix B. Some of the original 105 sales time series were sales series of a product category from all cities. The above indexes were not applicable to these time series since these time series involved multiple cities. On the other hand, some of the original time series were composed of yearly sale data. The number of observations in these time series was less than 10. It was hard for these time series to establish appropriate multivariate forecasting models. Therefore, only 49 of the 105 time series were used for performance comparison of the multivariate forecasting techniques, which involved 15 seasonal time series, 27 irregular time series and 7 random time series. Due to the page limit, these time series are not presented in this thesis. The numbers of time series used in both univariate and multivariate forecasting are highlighted in yellow in the title bars of Tables A2-A4 of Appendix A.

The 15 original trend time series comprised yearly sales data due to the seasonal features of apparel retailing. However, the length of these time series was very limited, having a maximum of nine observations. It is impossible for time series with such a limited number of observations to establish effective multivariate forecasting models. This research thus did not compare the forecasting performances of the multivariate forecasting techniques for trend sales data.

For each time series, the final 15% of the observations were used as out-of-samples to compare and evaluate the accuracy of forecasting models. For each out-of-sample observation, its previous sales data were used as training samples to set up forecasting models for making one-step-ahead forecasts.

For such multivariate data as weather and economic indexes, some observations are missing. Moreover, it is possibly that an apparel sales time series includes abnormal observations due to various uncertainties such as product promotions and shop shutdown. The preprocessing process is thus utilized to vet the original time series by removing outliers, missing values or irregularities to reduce the effects of defective data on forecasting accuracy. The missing data are interpolated to keep the completeness and the change trend of time series. The missing observation is filled in by using the mean of its latest two neighboring data in its time series. The z-score normalization method is adopted to normalize the input and output variables of NN-based forecasting models so as to speed up the training time of NNs by starting the training process for each feature within the same scale.

4.2.2 Procedures to identify data patterns

Experiments were conducted based on the following four types of publicly available and widely used time series. The identification of the four types of data patterns is described below.

Trend pattern: In this research, we used a linear function to fit all observations of each time series. If the absolute error percentage between observation points and their corresponding outputs of a function was less than 5%, the time series was identified as a trend pattern. There were 15 yearly time series of product categories (or cities). Although a yearly time series with more observations was more appropriate for performance comparison, it was hard to find trend time series with more observations due to incompleteness and unavailability of raw sales data.

Seasonal pattern: In this research, we used a linear function to fit the values of the same quarters (or months) of different years for time series with periodic changes. If the absolute percentage error value between observation points and their corresponding outputs of a function was less than 5%, this time series was identified as a seasonal pattern. There were 30 time series of quarterly or monthly sales data on product categories (or cities).

Irregular pattern: If the data set of a time series consisted of features of a trend or seasonal series, the time series was identified as an irregular pattern. There were 30 time series of quarterly or monthly sales data on product categories (or cities).

Random pattern: If the data set of a time series did not include any features of the above three patterns, the time series was identified as a random pattern. There were 30 time series of quarterly or monthly sales data on product categories (or cities).

4.2.3 Univariate forecasting models used

This research adopted a wide variety of models and general comments were made on the models and their building processes. The univariate forecasting approaches introduced in Section 2.3 were adopted for performance comparison. On the basis of these approaches, different forecasting models were constructed in terms of different parameters settings, including different numbers of input variables. For some forecasting models, such as the ARMA and NN, many model parameters needed setting so that a relatively larger number of training samples were required to establish these models. Because it is usually hard to obtain sufficient training samples, three last observations at most were used as input variables of forecasting models in this research.

The adopted models were described in detail below.

(1) Naïve model:

$$\hat{x}_{t+1} = x_t \tag{4-1}$$

(2) AR(2) model: It is an AR model using the last two observations as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = \alpha_1 x_t + \alpha_2 x_{t-1} \tag{4-2}$$

(3) AR(3) model: It is an AR model using the last three observations as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = \alpha_1 x_t + \alpha_2 x_{t-1} + \alpha_3 x_{t-2} \tag{4-3}$$

(4) MA(2) model: It is an MA model using the last two observations as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = \frac{x_t + x_{t-1}}{2} \tag{4-4}$$

(5) MA(3) model: It is an MA model using the last three observations as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = \frac{x_t + x_{t-1} + x_{t-2}}{3} \tag{4-5}$$

(6) ARMA(1,1) model: It is an AR model with one autoregressive term and one moving average term, that is,

$$\hat{x}_{t+1} = c + \varepsilon_2 + \varphi_1 x_1 + \theta_1 \varepsilon_1 \tag{4-6}$$

(7) ARMA(1,2) model: It is an AR model with one autoregressive terms and two moving average terms, that is,

$$\hat{x}_{t+1} = c + \varepsilon_t + \varphi_1 x_{t-1} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} \tag{4-7}$$

(8) DES model: It is an ES model using the past observations as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = \frac{2 - \alpha}{1 - \alpha} s'_t - \frac{1}{1 - \alpha} s''_t \ . \tag{4-8}$$

(9) TES model: It is an ES model using the past observations as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = a_t + b_t + c_t \, . \tag{4-9}$$

(10) NN(2) model: It is an NN model using the last two observations as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = f\left(x_t, x_{t-1}\right) \tag{4-10}$$

(11) NN(3) model: It is an NN model using the last three observations as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = f\left(x_t, x_{t-1}, x_{t-2}\right) \tag{4-11}$$

In the two NN models, the conjugate gradient backpropagation algorithm with Fletcher-Reeves updates was used as the learning algorithm. The maximum number of training epochs was 2,000. The number of hidden neurons was 3 if the length of the training data of time series was equal to or less than 15; otherwise, it was equal to 2×the number of input variables+1. For each time series in the experiments, 30 different trials with randomly generated initial weights were run to avoid forecasting randomicity. The final forecast of each time point was the mean of forecasts generated by the 30 trials.

4.2.4 Multivariate forecasting models used

The four multivariate forecasting techniques (ARX, ARMAX, GLM and NN) introduced in Section 2.6 were adopted for performance comparison. For each forecasting technique, different parameter settings can have great effects on forecasting performance. However, it is hard to select optimal parameters for each technique. To make a fair and thorough comparison, this research used multiple parameter combinations for each technique.

Therefore, based on the four techniques, this research adopted the following 12 multivariate forecasting models with different parameters settings.

(1) ARX(2,1) model: It is an ARX model using the last two observations of sales amounts and the latest values of the influencing factors as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = a_1 x_t + a_2 x_{t-1} + \mathrm{b}_1 \mathrm{u}_t \,. \tag{4-12}$$

(2) ARX(3,1) model: It is an ARX model using the last three observations of sales amounts and the latest values of the influencing factors as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = a_1 x_t + a_2 x_{t-1} + a_3 x_{t-2} + \mathrm{b}_1 \mathrm{u}_t \,. \tag{4-13}$$

(3) ARX(2,2) model: It is an ARX model using the last two observations of sales amounts and the last two observations of the influencing factors as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = a_1 x_t + a_2 x_{t-1} + \mathrm{b}_1 \mathrm{u}_t + \mathrm{b}_2 \mathrm{u}_{t-1} . \tag{4-14}$$

(4) ARX(3,2) model: It is an ARX model using the last three observations of sales amounts and the last two observations of the influencing factors as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = a_1 x_t + a_2 x_{t-1} + a_3 x_{t-2} + \mathrm{b}_1 \mathrm{u}_t + \mathrm{b}_2 \mathrm{u}_{t-1} \,. \tag{4-15}$$

(5) ARMAX(2,2,1) model: It is an ARMAX model using the last two observations of sales amounts, the last observation of the influencing factors and the last two forecasting errors as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = a_1 x_t + a_2 x_{t-1} + c_1 e_t + c_2 e_{t-1} + \mathrm{b}_1 \mathrm{u}_t \tag{4-16}$$

(6) ARMAX(2,2,2) model: It is an ARMAX model using the last two observations of sales amounts, the last two observations of the influencing factors and the last two forecasting errors as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = a_1 x_t + a_2 x_{t-1} + c_1 e_t + c_2 e_{t-1} + \mathrm{b}_1 \mathrm{u}_t + \mathrm{b}_2 \mathrm{u}_{t-1} \tag{4-17}$$

(7) ARMAX(3,3,1) model: It is an ARMAX model using the last three observations of sales amounts, the last observation of the influencing factors and the last three forecasting errors as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = a_1 x_t + a_2 x_{t-1} + a_3 x_{t-2} + c_1 e_t + c_2 e_{t-1} + c_3 e_{t-2} + \mathrm{b}_1 \mathrm{u}_t \tag{4-18}$$

(8) ARMAX(3,3,2) model: It is an ARMAX model using the last three observations of sales amounts, the last two observations of the influencing factors and the last three forecasting errors as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = a_1 x_t + a_2 x_{t-1} + a_3 x_{t-2} + c_1 e_t + c_2 e_{t-1} + c_3 e_{t-2} + \mathrm{b}_1 \mathrm{u}_t + \mathrm{b}_2 \mathrm{u}_{t-1}. \tag{4-19}$$

(9) GLM(2) model: It is a generalized linear model using the last two observations of sales amounts and the last observation of the influencing factors as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = a_1 x_t + a_2 x_{t-1} + \mathrm{b}_1 \mathrm{u}_t + c_1. \tag{4-20}$$

(10) GLM(3) model: It is a generalized linear model using the last three observations of sales amounts and the last observation of the influencing factors as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = a_1 x_t + a_2 x_{t-1} + a_3 x_{t-2} + \mathrm{b}_1 \mathrm{u}_t + c_1 \tag{4-21}$$

(11) MVNN(2) model: It is a multivariate NN model using the last two observations of sales amounts and the last observation of the influencing factors as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = f\left(x_t, x_{t-1}, \mathrm{u}_t\right) \qquad (4\text{-}22)$$

(12) MVNN(3) model: It is a multivariate NN model using the last three observations of sales amounts and the last observation of the influencing factors as input variables to forecast the next data point, that is,

$$\hat{x}_{t+1} = f\left(x_t, x_{t-1}, x_{t-2}, \mathrm{u}_t\right) \qquad (4\text{-}23)$$

In the two NN models, the Levenberg-Marquardt backpropagation was used as the learning algorithm. The maximum number of training epochs was 2,000. The number of hidden neurons was equal to the number of input variables because the number of input variables was big and the number of training samples was small. For each time series in the experiments, 30 trials with randomly generated initial weights were run to avoid forecasting randomicity. The final forecast of each time point was the mean of forecasts generated by the 30 trials.

4.3 Comparison results of univariate forecasting techniques

The objective of this comparison was to evaluate the forecasting performances of different univariate forecasting techniques for the four typical types of apparel sales data pattern. In this research, experiments were conducted based on each type of data pattern on a desktop computer with an Intel� Core™2 Duo three GHz processor and two GB of RAM, running MATLAB version 7.0 (R14).

The comparison results for each type of data pattern are described below. Due to the page limit, the forecasts generated by each forecasting model are not shown in this chapter. Instead, this chapter presents the values of each accuracy measure generated by each forecasting technique for each time series and the comparison results of these techniques.

4.3.1 Trend pattern

1) Forecasting performances generated by different models

To evaluate the effectiveness of each forecasting model, the forecasting performances generated by 10 forecasting models for trend patterns are shown in Table A1 in Appendix A. The value in this table represents an accuracy measure generated by a forecasting model for a time series. Columns 3-17 show the forecasting performances of time series 1-15 respectively. For example, the value in the 3rd column and the 2nd row, 58744520.0, is the MAE value generated by the Naïve model for time series 1. For each model, four performance values are shown according to four different forecasting accuracy measures. The 15 trend time series are yearly sales data. The number of observations in each time series is less than 10. Such a small number of observations are insufficient to establish an ARMA(1,2) model. Thus, Table A1 does not include the results generated by this model.

Take the MAPE's performances as an example to describe the forecasting performances generated by different models. Its performances for the 15 time series are summarized in Table 4-1. In this table, the second and the third rows show the minimal and the maximal MAPE values generated by each forecasting model for the 15 time series; the fourth to sixth rows show the number of time series for which the MAPE values generated by the corresponding models are more than 10%, 15% and 20% respectively. For example, the minimal and maximal MAPEs generated by the AR(2) model are 0.1.% and 16.6% respectively. In addition, the MAPEs of two time series are greater than 15% but the MAPEs of all time series are less than 20%. In this table, the two AR models and the two ES models generate good forecasts while the two MA models produce unacceptable results.

Table 4-1: Summary of MAPE forecasting performances

	Naïve	AR(2)	AR(3)	MA(2)	MA(3)	ARMA(1,1)	DES	TES	NN(2)	NN(3)
Min.	6.9%	0.1%	0.4%	13.5%	13.7%	1.4%	0.1%	0.1%	3.3%	2.1%
Max.	62.3%	16.6%	20.0%	100.6%	135.8%	29.0%	15.9%	25.5%	46.7%	73.5%
>10%	11	3	3	15	15	5	2	3	7	8
>15%	6	2	3	12	14	2	1	2	6	6
>20%	4	0	1	8	13	1	0	1	4	5

2) Forecasting performance comparisons in terms of different accuracy measures

For the 15 trend time series investigated, there was only one out-of-sample point for performance evaluation because time series with yearly sales could not be obtained. As a result, the comparison results generated by different forecasting accuracy measures were the same as those in Table 4-2. The value in this table represents the number of time series for which the corresponding forecasting model generated the forecasting performances of corresponding ranking. For example, the value ‘3’ in the third row and the second column suggests that the AR(2) model generates the best forecasting results for three time series. The value ‘1’ in the fourth row and the second column suggests that the AR(2) model generates the second best forecasting results for one time series. The following can be deduced from Table 4-2:

Table 4-2: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking

Performance Ranking	1	2	3	4	5	6	7	8	9
Naïve	0	0	1	0	0	3	2	9	0
AR(2)	3	7	1	3	0	0	1	0	0
AR(3)	3	1	2	5	2	1	0	1	0
MA(2)	0	0	0	0	0	0	0	3	12
MA(3)	0	0	0	0	0	0	0	0	1
ARMA(1,1)	1	2	1	0	6	2	2	1	0
DES	4	2	4	3	1	1	0	0	0
TES	2	2	4	2	4	0	1	0	0
NN(2)	1	1	1	1	2	6	1	1	0
NN(3)	1	0	1	1	0	2	8	0	2

(i) No model can perform much better than the others;

(ii) The Naïve model and the two MA models are the worst three.

(iii) The two NN models perform poorly and fail to outperform the others.

4.3.2 Seasonal pattern

1) Forecasting performances generated by different models

To evaluate the effectiveness of each forecasting model, the forecasting performances generated by 11 forecasting models for seasonal patterns are shown in Table A2 in Appendix A. The structures of Tables A2-A4 are the same as that of Table A1 except for 30 seasonal time series of monthly and quarterly sales data.

Take the MASE's performances as an example to describe the forecasting performances generated by different models. The performances for the 30 time series are summarized in Table 4-3. In this table, the second and the third rows show the minimal and the maximal MASE values generated by each forecasting model for the 30 time series; the fourth to sixth rows show the number of time series for which the MASE values generated by the corresponding models are greater than 0.5, 1 and 2 respectively. Take the results generated by the AR(2) model as an example. For the 30 time series, the minimal and maximal MASEs are 0.33 and 1.64 respectively. In addition, the MASEs of eight time series are greater than 1 but the MASEs of all time series are less than 2. For the results generated by the NN(2) model, the minimal and maximal MASEs are 0.00 and 2.80 respectively while the MASEs of six time series are greater than 1 and the MASEs of two time series are greater than 2. In Table 4-3, the two MA models generate the worst forecasts. Some results generated by the two NN models are good (almost zero) while some are not because the models are prone to over-fitting.

Table 4-3: Summary of MASE forecasting performances

	Naïve	AR(2)	AR(3)	MA(2)	MA(3)	ARMA(1,1)	ARMA(1,2)	DES	TES	NN(2)	NN(3)
Min.	0.47	0.33	0.15	0.65	0.69	0.29	0.42	0.52	0.54	0.00	0.00
Max.	1.87	1.64	1.69	2.44	2.99	1.76	1.74	1.63	1.87	2.80	3.12
>0.5	29	27	18	30	30	27	29	30	30	20	20
>1	17	8	7	25	20	10	10	11	11	6	9
>2	0	0	0	1	6	0	0	0	0	2	1

2) Forecasting performance comparisons in terms of different accuracy measures

If more than one out-of-sample is forecast, different comparison results can be used when different accuracy measures are used to evaluate forecasting accuracy. Tables 4-4 to 4-7 show the performance comparison results of 11 forecasting models when using the MAE, MAPE, RMSE and MASE as forecasting accuracy measures. The structures of these tables are the same as that of Table 4-2.

Table 4-4: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MAE)

Performance Ranking	1	2	3	4	5	6	7	8	9
Naïve	0	1	2	0	2	1	5	5	7
AR(2)	2	5	5	5	4	4	1	2	1
AR(3)	8	3	6	5	6	1	1	0	0
MA(2)	0	0	0	0	0	0	0	3	9
MA(3)	0	0	0	0	0	1	1	3	2
ARMA(1,1)	0	1	3	9	3	4	6	3	1
ARMA(1,2)	0	1	4	4	2	6	6	5	2
DES	2	5	1	5	4	7	3	3	0
TES	1	1	5	1	4	4	5	3	4
NN(2)	10	6	0	1	4	1	2	1	1
NN(3)	7	7	4	0	1	1	0	2	3

Table 4-4 shows the comparison results generated by different MAE forecasting models:

(i) The NN(2) model gives better forecasts although it also generates the worst forecasts for one time series;

(ii) The Naïve model, the two MA models and the two ARMA models perform poorly and fail to generate the best forecast for even one time series.

(iii) The AR(3) model outperforms the AR(2) significantly.

Table 4-5: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MAPE)

Performance Ranking	1	2	3	4	5	6	7	8	9
Naïve	1	3	1	3	4	2	1	3	5
AR(2)	1	3	9	5	9	2	1	0	0
AR(3)	4	4	7	4	4	5	2	0	0
MA(2)	0	0	0	1	0	0	3	7	6
MA(3)	0	0	0	0	0	0	2	7	3
ARMA(1,1)	3	6	4	9	3	1	0	2	0
ARMA(1,2)	4	4	0	2	1	3	5	5	4
DES	1	0	3	1	5	6	5	4	5
TES	0	0	0	3	1	3	8	1	3
NN(2)	10	5	1	0	3	3	1	1	3
NN(3)	6	5	5	2	0	5	2	0	1

Table 4-5 shows the comparison results generated by different MAPE forecasting models:

(i) The NN(2) model is still the best and the two MA models are still the worst.

(ii) The Naïve model and the two ARMA models outperform the two ES models.

(iii) The AR(3) model performs marginally better than the AR(2).

In Table 4-6, the comparison results generated by the MASE are exactly the same as those in Table 4-4 The MASE and MAE generate the same comparison results for the 30 seasonal time series. For the other patterns, the comparison results generated by the MASE and MAE are also the same. Therefore, the results generated by the MASE are not shown in the rest of this chapter.

Table 4-6: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MASE)

Performance Ranking	1	2	3	4	5	6	7	8	9
Naïve	0	1	2	0	2	1	5	5	7
AR(2)	2	5	5	5	4	4	1	2	1
AR(3)	8	3	6	5	6	1	1	0	0
MA(2)	0	0	0	0	0	0	0	3	9
MA(3)	0	0	0	0	0	1	1	3	2
ARMA(1,1)	0	1	3	9	3	4	6	3	1
ARMA(1,2)	0	1	4	4	2	6	6	5	2
DES	2	5	1	5	4	7	3	3	0
TES	1	1	5	1	4	4	5	3	4
NN(2)	10	6	0	1	4	1	2	1	1
NN(3)	7	7	4	0	1	1	0	2	3

The comparison results of the RMSE are shown in Table 4-7, which are closer to those generated by the MAE than by the MAPE.

(i) The NN(2) model is not superior to the AR(3) model.

(ii) The Naïve model, the two MA models and the two ARMA models perform poorly and fail to generate the best forecast for even one time series.

(iii) The AR(3) model outperforms the AR(2) significantly.

Table 4-7: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (RMSE)

Performance Ranking	1	2	3	4	5	6	7	8	9
Naïve	0	0	1	1	1	2	4	7	7
AR(2)	1	6	5	7	5	1	2	2	1
AR(3)	10	4	9	5	2	0	0	0	0
MA(2)	0	0	0	0	0	0	0	3	6
MA(3)	0	0	0	0	0	2	2	2	7
ARMA(1,1)	0	2	2	5	5	9	4	2	1
ARMA(1,2)	0	1	4	4	8	5	4	3	0
DES	2	3	3	4	6	6	1	2	3
TES	1	2	2	2	2	2	10	4	2
NN(2)	9	6	1	1	0	2	1	5	0
NN(3)	7	6	3	1	1	1	2	0	3

4.3.3 Irregular pattern

1) Forecasting performances generated by different models

To evaluate the effectiveness of each forecasting model, the forecasting performances generated by 11 forecasting models for irregular patterns are shown in Table A3 in Appendix A. The 30 irregular time series include monthly and quarterly sales data.

Table 4-8: Summary of MASE forecasting performances

	Naïve	AR(2)	AR(3)	MA(2)	MA(3)	ARMA(1,1)	ARMA(1,2)	DES	TES	NN(2)	NN(3)
Min.	0.03	0.03	0.03	0.04	0.04	0.02	0.42	0.08	0.15	0.25	0.08
Max.	1.85	1.89	2.01	2.30	2.56	2.71	1.74	2.55	2.55	3.03	4.33
>0.1	24	24	24	24	24	24	24	29	30	30	29
>0.5	23	23	23	24	24	23	23	24	24	23	23
>1	14	14	15	19	20	15	15	15	16	13	18
>2	0	0	1	1	0	2	0	1	3	4	6

Take the MASE's performances as an example to describe the forecasting performances generated by different models. Its performances for the 30 time series are summarized in Table 4-8. For example, the minimal and maximal MASEs generated by the AR(2) model are 0.03 and 1.89 respectively. In addition, the MASEs for 24 time series are greater than 0.1, the MASEs for 14 time series are greater than 1 but the MASEs for all time series are less than 2. In Table 4-8, the difference between the results of the Naïve, the two AR, the two MA and the two ARMA models is not obvious but the models generate better forecasts than the two ES and the two NN models.

2) Forecasting performance comparisons in terms of different accuracy measures

Tables 4-9 to 4-11 show the performance comparison results of the 11 forecasting models when using the MAE, MAPE and RMS as forecasting accuracy measures. The structure of these tables is the same as that of Table 4-2.

Table 4-9: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MAE)

Performance Ranking	1	2	3	4	5	6	7	8	9
Naïve	5	4	4	1	8	4	3	1	0
AR(2)	3	6	3	9	4	4	0	1	0
AR(3)	1	4	9	5	3	2	3	2	0
MA(2)	0	1	1	1	1	4	4	4	10
MA(3)	0	0	0	2	1	1	3	2	2
ARMA(1,1)	0	5	6	6	1	5	3	2	2
ARMA(1,2)	5	1	4	3	7	4	3	0	1
DES	2	3	2	0	3	3	4	5	6
TES	1	4	1	0	2	1	1	9	2
NN(2)	9	2	0	2	0	1	4	1	2
NN(3)	4	0	0	1	0	1	2	3	5

When forecasting accuracy is measured by the MAE, the following can be deduced from Table 4-9:

(i) The NN(2) model gives better forecasts generally than the other models although it also generates the worst forecasts for two time series;

(ii) The two MA models and the ARMA(1,1) model perform poorly and fail to generate the best forecast for even one time series.

(iii) The NN(2) model outperforms the NN(3) significantly.

When forecasting accuracy is measured by the MAPE, the following can be deduced from Table 4-10:

(i) The two MA models are still the worst.

(ii) The AR(2) model generates most of the best forecasts.

(iii) The two ES models and the two NN models do not perform much better than the other models.

Table 4-10: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MAPE)

Performance Ranking	1	2	3	4	5	6	7	8	9
Naïve	2	2	4	11	3	4	3	1	0
AR(2)	14	5	6	1	2	1	1	0	0
AR(3)	2	4	4	5	9	3	2	0	1
MA(2)	0	1	0	0	3	7	5	7	7
MA(3)	0	0	0	0	1	1	4	4	4
ARMA(1,1)	1	7	10	5	4	1	0	2	0
ARMA(1,2)	4	3	4	4	4	4	3	0	0
DES	1	3	1	0	2	4	4	2	5
TES	2	2	1	0	0	2	0	3	3
NN(2)	3	1	0	2	2	0	5	4	6
NN(3)	1	2	0	2	0	3	3	7	4

When forecasting accuracy is measured by the RMSE, the following can be deduced from Table 4-11:

(i) The NN(2) model is not superior to the NN(3) model.

(ii) The DES model is not superior to the TES model.

(iii) The two MA models are still the worst.

Table 4-11: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (RMSE)

Performance Ranking	1	2	3	4	5	6	7	8	9
Naïve	2	6	2	2	6	6	3	0	1
AR(2)	5	3	7	7	2	4	0	1	1
AR(3)	3	7	6	6	3	1	3	1	0
MA(2)	1	2	0	0	2	7	7	3	6
MA(3)	0	0	1	1	2	1	3	4	4
ARMA(1,1)	2	4	6	3	6	4	3	2	0
ARMA(1,2)	5	2	5	7	5	1	1	1	2
DES	2	1	1	1	2	3	5	7	5
TES	2	2	0	1	1	1	2	6	5
NN(2)	4	3	1	2	1	1	3	2	3
NN(3)	4	0	1	0	0	1	0	3	3

4.3.4 Random pattern

1) Forecasting performances generated by different models

To evaluate the effectiveness of each forecasting model, the forecasting performances generated by 10 forecasting models for random patterns are shown in Table A4 in Appendix. The 30 random time series include monthly and quarterly sales data.

Table 4-12: Summary of MASE forecasting performances

	Naïve	AR(2)	AR(3)	MA(2)	MA(3)	ARMA(1,1)	DES	TES	NN(2)	NN(3)
Min.	0.06	0.01	0.06	0.01	0.02	0.01	0.01	0.09	0.02	0.14
Max.	3.71	4.15	4.48	4.41	4.81	3.85	3.69	3.58	5.96	5.93
>0.1	29	26	28	26	28	28	29	29	29	30
>0.5	18	18	19	18	19	15	21	22	21	24
>1	6	9	11	10	12	8	14	15	13	17
>2	5	2	6	5	6	5	3	4	11	12

Take the MASE's performances as an example to describe the forecasting performances generated by different models. Its performances for the 30 time series are summarized in Table 4-12. In the table, the two NN models perform much poorer than the other models.

2) Forecasting performance comparisons in terms of different accuracy measures

Tables 4-13 to 4-15 show the performance comparison results of 10 forecasting models when using the MAE, MAPE and RMSE as forecasting accuracy measures. The structures of these tables are the same as that of Table 4-2.

When forecasting accuracy is measured by the MAE, the following can be deduced from Table 4-13:

(i) The AR(2) model gives better forecasts although it also generates the worst forecasts for 1 time series;

(ii) The NN(2) model outperforms the NN(3) significantly.

(iii) The MA(2) model is not superior to the MA(3) model.

(iv) The DES model is not superior to the TES model.

Table 4-13: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MAE)

Performance Ranking	1	2	3	4	5	6	7	8	9
Naïve	2	6	8	3	3	1	3	4	0
AR(2)	6	6	4	3	0	6	2	2	1
AR(3)	3	4	2	2	6	2	2	5	1
MA(2)	3	1	5	7	2	3	3	4	2
MA(3)	2	0	2	3	3	7	5	4	2
ARMA(1,1)	5	5	5	5	3	4	1	1	1
DES	2	2	3	4	5	2	5	5	2
TES	3	2	0	1	5	1	3	3	8
NN(2)	4	3	0	1	2	1	4	2	5
NN(3)	0	1	1	1	1	3	2	0	8

When forecasting accuracy is measured by the MAPE, the following can be deduced from Table 4-14:

(i) The ARMA model gives better forecasts although it also generates the worst forecasts for 1 time series.

(ii) The two AR models, the two MA models and the two ES models do not perform much better than the other models.

(iii) The two NN models perform poorly and only generate the best forecasts for a few time series but the worst forecasts for most time series.

Table 4-14: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MAPE)

Performance Ranking	1	2	3	4	5	6	7	8	9
Naïve	2	6	7	5	2	0	4	4	0
AR(2)	5	7	6	2	2	3	2	2	1
AR(3)	4	3	3	4	4	2	2	4	1
MA(2)	3	3	3	6	2	6	2	3	2
MA(3)	2	0	1	3	3	7	5	5	2
ARMA(1,1)	7	5	5	3	3	5	1	0	1
DES	1	2	3	5	6	2	5	5	1
TES	3	2	0	1	4	1	3	3	6
NN(2)	3	1	1	0	3	1	4	4	6
NN(3)	0	1	1	1	1	3	2	0	10

When forecasting accuracy is measured by RMSE, the following can be deduced from Table 15:

(i) The ARMA model gives better forecasts although it also generates the worst forecasts for 1 time series.

(ii) The NN(3) model performs poorly and fails to generate the best forecast for even one time series but generates the worst forecasts for most time series.

(iii) The NN(2) model outperforms most of the models as shown in Table 4-15 and generates the best forecasts for most time series.

Table 4-15: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (RMSE)

Performance Ranking	1	2	3	4	5	6	7	8	9
Naïve	1	5	5	4	3	4	4	4	0
AR(2)	2	8	4	6	2	3	2	2	1
AR(3)	3	5	2	2	5	3	2	4	1
MA(2)	3	1	7	6	2	1	3	3	4
MA(3)	3	0	3	2	4	6	5	3	2
ARMA(1,1)	7	3	5	4	3	5	2	0	1
DES	2	3	2	4	4	2	5	7	1
TES	3	2	1	1	4	2	3	4	6
NN(2)	6	2	0	0	2	1	4	2	6
NN(3)	0	1	1	1	1	3	0	1	8

4.3.5 Discussion

From the experimental results above, the performance comparison results of different univariate forecasting techniques are summarized in Table 4-16. Each value (number) in this table represents the performance ranking of the corresponding forecasting model for a specific data pattern and accuracy measure. For example, number ‘8’ in the second row and the third column suggests that the forecasting performance generated by the Naïve model ranks eighth among all the trend time series when the MAE is used as an accuracy measure. In addition, the green cells in each row indicate that several corresponding techniques generate the best results while the olive green cells indicate that the corresponding techniques generate the worst results.

Table 4-16: Summary of performance comparison results

		Naïve	AR(2)	AR(3)	MA(2)	MA(3)	ARMA (1,1)	ARMA (1,2)	DES	TES	NN(2)	NN(3)
Trend	MAE	8	2	3	9	10	5	/	1	4	6	7
	MAPE	8	2	3	9	10	5	/	1	4	6	7
	MASE	8	2	3	9	10	5	/	1	4	6	7
	RMSE	8	2	3	9	10	5	/	1	4	6	7
Seasonal	MAE	9	4	2	11	10	8	7	5	6	1	3
	MAPE	7	6	3	10	11	5	4	8	9	1	2
	MASE	9	4	2	11	10	8	7	5	6	1	3
	RMSE	9	5	1	11	10	7	8	4	6	2	3
Irregular	MAE	2	5	7	10	11	9	3	6	8	1	4
	MAPE	5	1	4	10	11	7	2	8	6	3	9
	MASE	2	5	7	10	11	9	3	6	8	1	4
	RMSE	6	1	5	10	11	7	2	9	8	3	4
Random	MAE	7	1	4	6	9	2		8	5	3	10
	MAPE	7	2	3	4	8	1		9	5	6	10
	MASE	7	1	4	6	9	2		8	5	3	10
	RMSE	9	7	3	5	6	1		8	4	2	10

Best results (green) — Worst results (olive green)

Based on the results shown in Table 4-16, the following conclusions can be drawn:

(1) For different data patterns, the forecasting performances generated by different univariate forecasting models are mixed.

(i) For trend patterns, the forecasting results generated by the AR, ARMA, ES and NN models are acceptable in retailing practice. Among these models, the Naïve and MA models generate the worst forecasts while the AR and ES models generate the best;

(ii) For seasonal patterns, the forecasting results generated by the Naïve and MA models are unacceptable in retailing practice. The results generated by the ARMA and ES models are only adequate while the AR and NN models generate much better results;

(iii) For irregular patterns, the Naïve model generates better results than the MA. In addition, no model performs much better than the others;

(iv) For random patterns, the ARMA(1,1) and AR(2) models generate slightly better results than the others. The NN(3) model generates the worst.

It is clear that the MA model usually generates the worst forecasting results, whichever data pattern is used. In addition, the NN model cannot perform much better than the other traditional models.

(2) Even for the same model, different input sizes can impact forecasting results greatly. For instance, the AR(3) generates much better results than the AR(2) for seasonal patterns; However, the AR(2) generates better results than the AR(3) for irregular patterns.

(3) Different accuracy measures also affect forecasting results greatly. Take irregular patterns as an example, the Naïve and the AR(2) generate similar forecasts when the MAE is used as the accuracy measure; however, the AR(2) generates much better forecasts when the MAPE is used.

Table 4-17 shows the four best forecasting models for each data pattern. The results are obtained based on the comparison results shown in Table 4-16. For trend data patterns, the DES and AR(2) models generate the best forecasts. For seasonal and irregular data patterns, the NN(2) model generates the best forecasts. For random data patterns, the ARMA(1,1) model generates the best forecasts.

Table 4-17: Summary of the four best forecasting models for each data pattern

Rank	**1**	**2**	**3**	**4**
Trend	DES	AR(2)	AR(3)	TES
Seasonal	NN(2)	AR(3)	NN(3)	AR(2)
Irregular	NN(2)	AR(2)	ARMA(1,2)	Naïve
Random	ARMA(1,1)	AR(2)	NN(2)	AR(3)

4.4 Comparison results of multivariate forecasting techniques

This section presents the results of performance comparison and analysis of multivariate forecasting techniques for apparel sales time series. The objective of this comparison was to evaluate the forecasting performances of different multivariate forecasting techniques for the three typical types of apparel sales data pattern (i.e. seasonal, irregular and random). The comparison results for each type of data pattern are described below. Due to the page limit, the forecasts generated by each forecasting model are not detailed in this section. Instead, this section presents the values of each accuracy measure generated by each forecasting technique for each time series and the comparison results of these techniques.

4.4.1 Seasonal pattern

1) Forecasting performances generated by different models

To evaluate the effectiveness of each forecasting model, the forecasting performances generated by the 12 multivariate forecasting models for seasonal patterns are shown in Table C1 of Appendix C. The value in this table represents an accuracy measure generated by a forecasting model for a time series. Columns 3-17 show the forecasting performances of time series 1-15 respectively. For example, the value in the 3rd column and the 2nd row, 842769.2, is the MAE value generated by the ARX(2,1) model for time series 1. For each model, four performance values are shown according to four forecasting accuracy measures. The 15 trend time series include quarterly and monthly sales data. In Appendix C, the structures of Tables C2-C3 are the same as that of Table C1 except for the number of time series.

Take the MASE's performances as an example to describe the forecasting performances generated by different models. The performances for the 15 time series are summarized in Table 4-18. In this table, the second and the third rows show the minimal and the maximal MASE values generated by each forecasting model for the 30 time series; the fourth to sixth rows show the number of time series for which the MASE values generated by the corresponding models are greater than 0.5, 1 and 2 respectively. Take the results generated by the ARX(2,1) model as an example. For the 15 time series, the minimal and maximal MASE valuess are 0.09 and 2.24 respectively. In addition, the MASE values of five time series are greater than 1 but the MASE value of only one time series is greater than 2. For the results generated by the GLM(3) model, the minimal and maximal MASE values are 0.14 and 1.18 respectively while the MASE value of only one time series is greater than 1 and the MASE value of no time series is greater than 2. In this table, the ARX(2,1) and ARMAX(2,2,1) models generate the worst forecasts because they generate relatively large MASE values. Some MASE values generated by the two MVNN models are good (almost zero) while some are greater than 2 because the models are prone to over-fitting.

Table 4-18: Summary of MASE forecasting performances

	ARX (2,1)	ARX (3,1)	ARX (2,2)	ARX (3,2)	ARMAX (2,2,1)	ARMAX (2,2,2)	ARMAX (3,3,1)	ARMAX (3,3,2)	GLM(2)	GLM(3)	MVNN(2)	MVNN(3)
Min.	0.09	0.01	0.05	0.00	0.14	0.13	0.01	0.02	0.20	0.14	0.00	0.00
Max.	2.24	2.00	1.26	1.17	2.26	1.07	1.50	1.06	1.25	1.18	2.11	2.01
>0.1	14	12	13	13	15	15	13	12	15	15	13	13
>0.5	8	7	7	7	9	8	8	8	9	8	10	10
>1	5	4	4	4	3	2	2	1	2	1	3	3
>2	1	1	0	0	1	0	0	0	0	0	1	1

2) Forecasting performance comparisons in terms of different accuracy measures

Tables 4-19 to 4-22 show the performance comparison results of the 12 multivariate forecasting models in terms of four accuracy measures, namely the MAE, MAPE, RMSE and MASE. The value in this table represents the number of time series for which the corresponding forecasting model generates the first twelve best forecasting performances. For example, the value ‘1’ in the third row and the second column suggests that the ARX(3,1) model generates the best forecasting results for one time series. The value ‘2’ in the fourth row and the second column suggests that the ARX(2,2) model generates the second best forecasting results for one time series.

Table 4-19 shows the comparison results generated by different MAE forecasting models:

(i) The ARX(3,2) model and the ARMAX(3,3,2) model generate better forecasts than the other models;

(ii) The two GLM models perform poorly and fail to generate the best forecast for even one time series.

(iii) The MVNN(3) model and the MVNN(2) model generate similar forecasting performances.

(iv) Different ARX models and different ARMAX models generate different forecasting performances. The other models cannot provide satisfying forecasts although the ARX(3,2) model and the ARMAX(3,3,2) model perform well.

Table 4-19: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MAE)

Performance Ranking	1	2	3	4	5	6	7	8	9	10	11	12
ARX(2,1)	0	0	0	0	1	2	2	1	0	3	4	2
ARX(3,1)	1	3	1	1	1	1	2	0	1	3	1	0
ARX(2,2)	0	2	0	1	3	1	1	2	4	0	0	1
ARX(3,2)	5	2	1	0	1	2	2	0	0	1	1	0
ARMAX(2,2,1)	0	0	1	2	1	1	2	2	2	1	0	3
ARMAX(2,2,2)	0	0	2	2	3	1	3	2	0	1	1	0
ARMAX(3,3,1)	1	2	4	2	2	0	0	2	1	0	0	1
ARMAX(3,3,2)	4	2	2	5	1	0	0	1	0	0	0	0
GLM(2)	0	0	0	0	1	2	3	2	2	4	0	1
GLM(3)	0	0	2	0	0	4	0	2	4	1	1	1
MVNN(2)	1	3	2	1	1	0	0	0	1	0	3	3
MVNN(3)	3	1	0	1	0	1	0	1	0	1	4	3

Table 4-20 shows the comparison results generated by different MAPE forecasting models:

(i) The ARX(3,2) model and the MVNN(3) model generate the best forecasts among all the models.

(ii) The MVNN(2) model outperforms the two GLM models, four ARMAX models and three ARX models.

(iii) The ARX(3,2) model outperforms the other ARX models significantly while the four ARMAX models exhibit similar and poor forecasting results.

Table 4-20: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MAPE)

Performance Ranking	1	2	3	4	5	6	7	8	9	10	11	12
ARX(2,1)	0	1	0	0	0	0	0	3	1	3	3	4
ARX(3,1)	0	1	3	1	0	1	3	0	1	3	2	0
ARX(2,2)	0	2	1	2	4	2	1	0	1	0	0	2
ARX(3,2)	4	2	1	1	0	1	2	1	1	1	1	0
ARMAX(2,2,1)	1	0	2	2	0	0	0	0	1	2	4	3
ARMAX(2,2,2)	0	1	0	2	1	1	3	3	1	1	1	1
ARMAX(3,3,1)	2	1	1	1	3	3	0	2	0	0	0	2
ARMAX(3,3,2)	1	0	2	3	3	2	1	1	1	1	0	0
GLM(2)	0	0	0	2	1	1	1	3	3	3	0	1
GLM(3)	1	1	0	1	1	4	1	2	2	1	1	0
MVNN(2)	2	4	4	0	0	0	2	0	1	0	1	1
MVNN(3)	4	2	1	0	2	0	1	0	2	0	2	1

Table 4-21: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MASE)

Performance Ranking	1	2	3	4	5	6	7	8	9	10	11	12
ARX(2,1)	0	0	0	0	0	2	3	0	0	5	4	1
ARX(3,1)	1	4	0	1	1	1	1	1	1	3	0	1
ARX(2,2)	0	1	2	0	2	2	2	3	1	1	1	0
ARX(3,2)	5	2	1	1	2	1	1	0	1	1	0	0
ARMAX(2,2,1)	0	1	1	1	0	2	2	2	2	1	1	2
ARMAX(2,2,2)	0	0	1	4	1	2	2	2	1	1	1	0
ARMAX(3,3,1)	1	2	2	4	2	1	0	1	1	0	0	1
ARMAX(3,3,2)	2	2	4	2	3	1	0	1	0	0	0	0
GLM(2)	0	0	1	2	1	1	2	2	3	1	1	1
GLM(3)	3	0	0	0	1	2	0	1	5	1	2	0
MVNN(2)	1	2	2	0	1	0	1	1	0	1	2	4
MVNN(3)	2	1	1	0	1	0	1	1	0	0	3	5

The comparison results generated by different MASE forecasting models are shown in Table 4-21:

(i) The ARX(3,2) model generates the best forecasts among all 12 models.

(ii) When the last two observations of sales amounts are used as inputs of the ARX, ARMAX and GLM models, the three models fail to generate the best forecast for even one time series.

(iii) The two MVNNs models generate the worst forecasts for many time series although they also generate the best forecasts for some.

The comparison results based on the RMSE are shown in Table 4-22, which are very similar to those generated by the MAE. When taking 15 seasonal time series into account, , the models using the last three observations of sales amounts generate relatively better results than the corresponding models using the last two observations of sales amounts, whichever accuracy measure is used for the ARX, ARMAX, GLM and MVNN models.

Table 4-22: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (RMSE)

Performance Ranking	1	2	3	4	5	6	7	8	9	10	11	12
ARX(2,1)	0	0	0	0	1	2	2	0	1	3	5	1
ARX(3,1)	1	3	1	1	1	1	2	0	0	4	1	0
ARX(2,2)	0	2	0	2	2	2	0	2	4	0	0	1
ARX(3,2)	5	2	1	0	2	1	2	0	0	1	1	0
ARMAX(2,2,1)	0	0	1	2	1	1	2	2	2	1	0	3
ARMAX(2,2,2)	0	0	2	3	2	2	1	3	0	1	1	0
ARMAX(3,3,1)	1	2	4	2	2	0	0	2	1	0	0	1
ARMAX(3,3,2)	4	2	3	4	1	0	0	1	0	0	0	0
GLM(2)	0	0	0	0	1	2	3	2	4	2	0	1
GLM(3)	0	1	1	0	0	4	0	3	3	1	2	0
MVNN(2)	1	2	2	0	2	0	1	0	0	1	2	4
MVNN(3)	3	1	0	1	0	0	2	0	0	1	3	4

4.4.2 Irregular pattern

1) Forecasting performances generated by different models

To evaluate the effectiveness of each forecasting model, the forecasting performances generated by 12 forecasting models for irregular patterns are shown in Table C2 of Appendix C. The 27 irregular time series include monthly and quarterly sales data.

Take the MASE's performances as an example to describe the forecasting performances generated by different models. Its performances for the 27 time series are summarized in

Table 4-23. For example, the minimal and maximal MASE values generated by the ARX(2,1) model are 0.61 and 25.91 respectively. In addition, the MASE values for all 27 time series are greater than 0.1, the MASE values for 21 time series are greater than 1. According to the accuracy measure of the MASE, the difference in the results generated by different models is not obvious, and no forecasting model can generate much better results than the other models.

Table 4-23: Summary of MASE forecasting performances

	ARX (2,1)	ARX (3,1)	ARX (2,2)	ARX (3,2)	ARMAX (2,2,1)	ARMAX (2,2,2)	ARMAX (3,3,1)	ARMAX (3,3,2)	GLM(2)	GLM(3)	MVNN(2)	MVNN(3)
Min.	0.61	0.47	0.58	0.42	0.51	0.62	0.48	0.55	0.59	0.52	0.26	0.07
Max.	25.91	23.44	25.27	23.10	28.66	28.77	29.30	27.76	26.08	31.54	2.70	4.69
>0.1	27	27	27	27	27	27	27	27	27	27	27	26
>0.5	27	26	27	26	27	27	26	27	27	27	23	23
>1	21	18	15	10	15	11	9	8	17	14	13	18
>2	5	3	3	3	3	3	2	3	3	3	4	6

2) Forecasting performance comparison in terms of different accuracy measures

Tables 4-24 to 4-27 show the performance comparison results of the 12 forecasting models when using the MAE, MAPE, MASE and RMSE as forecasting accuracy measures. The structures of these tables are the same as that of Table 4-19.

When the forecasting accuracy is measured by the MAE, the following can be deduced from Table 4-24:

(i) The ARMAX(3,3,2) model provides the best forecasts among the 12 models although it also generates inferior forecasts for several time series;

(ii) The ARX(2,1) model generates the worst forecasts and fails to generate the best forecast for even one time series.

(iii) The performance of the MVNN(3) model for the 27 time series is unstable. This model generates superior forecasts for some time series, but also generates inferior forecasts for yet more time series. In fact, it generates the worst forecasts for 11 time series.

Table 4-24: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MAE)

Performance Ranking	1	2	3	4	5	6	7	8	9	10	11	12
ARX(2,1)	0	1	1	0	0	1	2	2	3	5	8	4
ARX(3,1)	1	2	2	0	5	3	0	2	2	3	6	1
ARX(2,2)	1	1	0	2	5	3	3	2	1	4	3	2
ARX(3,2)	2	1	4	8	2	3	4	2	1	0	0	0
ARMAX(2,2,1)	1	2	2	3	2	1	6	2	5	1	0	2
ARMAX(2,2,2)	1	5	5	3	1	2	1	1	1	4	2	1
ARMAX(3,3,1)	4	5	1	4	3	1	3	2	1	2	0	1
ARMAX(3,3,2)	8	3	2	3	2	2	1	2	2	1	1	0
GLM(2)	1	2	2	1	0	4	2	7	4	2	1	1
GLM(3)	2	2	1	1	4	3	4	2	4	2	1	1
MVNN(2)	3	1	6	1	3	2	1	2	1	1	3	3
MVNN(3)	3	2	1	1	0	2	0	1	2	2	2	11

The comparison results based on the MASE are shown in Table 4-25, which indicate:

(i) The ARMAX(2,2,1) model and the GLM(3) models cannot generate the best forecast for even one time series.

(ii) The GLM(2) model generates better results than the GLM(3) model although the ARX and ARMAX models using the last three observations of sales amounts generate relatively better results than the corresponding models using the last two observations of sales amounts .

(iii) The ARX(2,1) and MVNN(3) models generate the worst forecasts for more time series than the other models although the MVNN(3) also generates the best forecasts for five time series.

Table 4-25: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MAPE)

Performance Ranking	1	2	3	4	5	6	7	8	9	10	11	12
ARX(2,1)	1	0	1	2	1	1	2	3	2	5	3	6
ARX(3,1)	1	3	1	4	2	4	2	0	2	1	7	0
ARX(2,2)	1	3	0	0	3	1	3	4	1	2	5	4
ARX(3,2)	2	0	4	2	3	4	4	0	2	3	2	1
ARMAX(2,2,1)	0	1	1	4	0	4	3	3	4	4	2	1
ARMAX(2,2,2)	3	4	1	6	2	1	1	1	5	1	1	1
ARMAX(3,3,1)	5	2	3	3	4	3	3	1	1	0	1	1
ARMAX(3,3,2)	3	5	7	1	4	3	1	1	0	0	1	1
GLM(2)	1	0	3	1	2	0	1	7	4	3	2	3
GLM(3)	0	2	0	0	2	5	2	5	3	5	1	2
MVNN(2)	5	2	5	1	2	0	3	1	3	1	2	2
MVNN(3)	5	5	1	3	2	1	2	1	0	2	0	5

Table 4-26: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MASE)

Performance Ranking	1	2	3	4	5	6	7	8	9	10	11	12
ARX(2,1)	0	1	1	0	0	1	3	2	3	4	8	4
ARX(3,1)	1	3	0	1	6	2	0	2	2	5	3	2
ARX(2,2)	1	1	0	1	6	4	3	1	3	3	2	2
ARX(3,2)	1	2	6	6	1	4	4	2	1	0	0	0
ARMAX(2,2,1)	1	1	2	4	2	1	6	2	5	2	0	1
ARMAX(2,2,2)	2	3	5	4	1	2	1	2	0	4	2	1
ARMAX(3,3,1)	3	2	5	5	2	2	3	1	1	1	1	1
ARMAX(3,3,2)	7	2	4	3	3	2	0	1	3	0	2	0
GLM(2)	0	3	1	2	1	3	3	7	4	1	0	2
GLM(3)	1	1	3	1	4	3	4	2	4	3	0	1
MVNN(2)	6	5	0	0	1	3	0	1	0	1	8	2
MVNN(3)	4	3	0	0	0	0	0	4	1	3	1	11

Table 4-26 shows the comparison results on the basis of the MASE, from which the following can be deduced:

(i) The ARMAX(3,3,2) model and the MVNN(2) model generate relatively better forecasts than the other models.

(ii) The ARX(2,1) models and the GLM(2) model cannot generate the best forecast for even one time series.

(iii) The forecasts generated by the MVNN(3) model rank 1^{st} and 2^{nd} and also get last five places while most of the forecasting results generated by the ARX(3,2) model rank 3^{rd} to 7^{th}.

Table 4-27 shows the comparison results on the basis of the RMSE, from which the following can be deduced:

(i) The MVNN(3) model generates the worst forecasts for most of the time series although it also generates the best forecasts for three time series.

(ii) The two ARMAX models with na=3 generate the best forecasts among the 12 models.

(iii) The four ARX models and the GLM(2) model generate the best forecasts for only one time series each.

Table 4-27: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (RMSE)

Performance Ranking	1	2	3	4	5	6	7	8	9	10	11	12
ARX(2,1)	0	1	1	0	0	1	2	1	3	9	7	2
ARX(3,1)	0	3	3	1	2	2	2	2	5	2	3	2
ARX(2,2)	0	1	0	2	8	3	0	5	2	3	2	1
ARX(3,2)	1	1	5	4	5	7	1	1	1	0	1	0
ARMAX(2,2,1)	3	2	1	3	2	1	3	6	3	2	0	1
ARMAX(2,2,2)	2	2	4	6	1	2	2	2	2	1	2	1
ARMAX(3,3,1)	5	5	4	2	1	2	3	0	2	1	2	0
ARMAX(3,3,2)	8	3	2	1	2	1	3	2	3	0	1	1
GLM(2)	1	2	3	2	1	3	4	3	3	4	1	0
GLM(3)	2	2	1	2	3	2	5	4	3	1	1	1
MVNN(2)	2	2	2	4	2	1	1	1	0	3	5	4
MVNN(3)	3	3	1	0	0	2	1	0	0	1	2	14

4.4.3 Random pattern

1) Forecasting performances generated by different models

To evaluate the effectiveness of each forecasting model, the forecasting performances generated by 12 forecasting models for random patterns are shown in Table C3 of Appendix C. The 30 random time series include monthly and quarterly sales data.

Table 4-28: Summary of MASE forecasting performances

	ARX (2,1)	ARX (3,1)	ARX (2,2)	ARX (3,2)	ARMAX (2,2,1)	ARMAX (2,2,2)	ARMAX (3,3,1)	ARMAX (3,3,2)	GLM(2)	GLM(3)	MVNN(2)	MVNN(3)
Min.	0.50	0.06	0.48	0.44	0.58	0.51	0.09	0.44	0.83	0.64	0.17	0.15
Max.	5.36	3.39	3.34	5.14	5.30	3.84	5.42	6.31	11.88	15.90	3.98	5.50
>0.1	7	6	7	7	7	7	6	7	7	7	7	7
>0.5	7	4	6	6	7	7	5	6	7	7	4	5
>1	6	4	5	4	6	6	5	4	6	6	2	3
>2	4	2	3	3	3	4	3	3	3	2	1	1

Take the MASE's performances as an example to describe the forecasting performances generated by different models. Its performances for seven time series are summarized in Table 4-28. In the table, the ARX(3,1) model generates relatively better results than the other models.

2) Forecasting performance comparisons in terms of different accuracy measures

Tables 4-29 to 4-32 show the performance comparison results of the 12 forecasting models when using the MAE, MAPE, MASE and RMSE as forecasting accuracy measures. As shown in these tables, the following conclusions can be drawn:

(i) The forecasting results are very similar when the MAE, MAPE and RMSE are used as accuracy measures. The MVNN(2) model and the ARX(3,1) model generate the best forecasts for two time series, and the ARMAX(2,2,1), ARMAX(3,3,1) and ARMAX(3,3,2) models generate the best forecasts for one time series, while the other models fail to generate the best forecast for any time series.

(ii) The MVNN(3) model generates the worst forecasts for three time series when the RMSE is used as the accuracy measure, but generates the worst forecasts for only one time series when other accuracy measures are used.

(iii) When the MASE is used as the accuracy measure, the MVNN(2) model gives the best forecasting performance among the 12 models by providing the best forecasts for three time series.

Compared with other data patterns, the random time series generate relatively similar forecasting results when different accuracy measures are used, possibly because only seven random time series are used for performance comparison. Such a limited number of samples can make the comparison incomplete, and thus the corresponding comparison conclusions can be biased.

Table 4-29: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MAE)

Performance Ranking	1	2	3	4	5	6	7	8	9	10	11	12
ARX(2,1)	0	0	0	0	0	2	0	2	1	2	0	0
ARX(3,1)	2	1	0	1	0	0	1	1	1	0	0	0
ARX(2,2)	0	1	1	1	1	0	1	0	1	0	0	1
ARX(3,2)	0	1	2	0	0	1	2	0	0	1	0	0
ARMAX(2,2,1)	1	0	0	0	0	1	1	1	1	0	1	1
ARMAX(2,2,2)	0	0	0	0	1	1	1	2	0	1	1	0
ARMAX(3,3,1)	1	1	0	0	1	1	0	0	1	1	0	1
ARMAX(3,3,2)	1	1	2	2	0	0	0	0	0	1	0	0
GLM(2)	0	1	0	2	0	0	0	0	0	1	2	1
GLM(3)	0	1	0	1	0	0	0	1	1	0	1	2
MVNN(2)	2	0	1	0	2	1	0	0	0	0	1	0
MVNN(3)	0	0	1	0	2	0	1	0	1	0	1	1

Table 4-30: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MAPE)

Performance Ranking	1	2	3	4	5	6	7	8	9	10	11	12
ARX(2,1)	0	0	0	0	1	0	3	1	0	2	0	0
ARX(3,1)	2	0	1	0	1	0	1	2	0	0	0	0
ARX(2,2)	0	1	1	1	0	0	1	1	1	0	1	0
ARX(3,2)	0	1	2	0	0	1	0	0	2	0	0	1
ARMAX(2,2,1)	1	0	0	0	0	1	0	2	1	0	1	1
ARMAX(2,2,2)	0	1	0	0	1	1	1	0	1	2	0	0
ARMAX(3,3,1)	1	1	0	0	0	2	0	0	0	1	1	1
ARMAX(3,3,2)	1	1	0	2	1	1	0	0	0	1	0	0
GLM(2)	0	0	1	1	0	1	0	0	0	1	2	1
GLM(3)	0	0	1	0	1	0	0	1	1	0	1	2
MVNN(2)	2	0	0	3	0	0	1	0	0	0	1	0
MVNN(3)	0	2	1	0	2	0	0	0	1	0	0	1

Table 4-31: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (MASE)

Performance Ranking	1	2	3	4	5	6	7	8	9	10	11	12
ARX(2,1)	0	0	0	0	1	1	2	1	1	1	0	0
ARX(3,1)	2	1	0	0	3	0	0	0	1	0	0	0
ARX(2,2)	0	1	1	1	1	0	1	0	0	1	1	0
ARX(3,2)	0	2	1	0	0	1	0	2	0	1	0	0
ARMAX(2,2,1)	1	0	0	0	0	1	1	0	2	0	2	0
ARMAX(2,2,2)	0	0	0	1	0	1	1	3	0	0	1	0
ARMAX(3,3,1)	1	1	0	0	0	2	0	1	0	1	0	1
ARMAX(3,3,2)	1	0	1	2	1	1	0	0	1	0	0	0
GLM(2)	0	1	1	0	0	0	0	0	1	1	2	1
GLM(3)	0	1	0	1	0	0	1	0	1	1	0	2
MVNN(2)	3	0	1	2	0	0	1	0	0	0	0	0
MVNN(3)	0	0	2	0	1	0	0	0	0	1	0	1

Table 4-32: The number of time series for which the forecasting model generates the forecasting performances of corresponding ranking (RMSE)

Performance Ranking	1	2	3	4	5	6	7	8	9	10	11	12
ARX(2,1)	0	0	0	0	1	1	2	1	1	1	0	0
ARX(3,1)	2	1	0	0	3	0	0	0	1	0	0	0
ARX(2,2)	0	1	1	1	1	0	1	0	0	1	1	0
ARX(3,2)	0	2	1	0	0	1	0	2	0	1	0	0
ARMAX(2,2,1)	1	0	0	0	0	1	1	0	2	0	2	0
ARMAX(2,2,2)	0	0	0	1	0	1	1	3	0	0	1	0
ARMAX(3,3,1)	1	1	0	0	0	2	0	1	0	1	0	1
ARMAX(3,3,2)	1	0	1	2	1	1	0	0	1	0	0	0
GLM(2)	0	1	1	0	0	0	0	0	1	1	2	1
GLM(3)	0	1	0	1	0	0	1	0	1	1	0	2
MVNN(2)	2	0	1	2	0	0	1	0	0	0	1	0
MVNN(3)	0	0	2	0	1	0	0	0	0	1	0	3

4.4.4 Discussion

From the experimental results described above, the performance comparison results of different multivariate forecasting techniques are summarized in Table 4-33. Each value (number) in this table represents the performance ranking of the corresponding forecasting model for a specific data pattern and accuracy measure. For example, number '12' in the second row and the third column suggests that the forecasting performance generated by the ARX(2,1) model ranks 12th for all seasonal time series when the MAE is used as an accuracy measure. In addition, the green cells in each row indicate that several corresponding techniques generate the best results while the grey cells indicate that the corresponding techniques generate the worst results.

The following conclusions can be drawn:

(1) For different data patterns, the forecasting performances generated by different multivariate forecasting models are mixed.

(i) For seasonal patterns, the ARX(3,2) and ARMAX(3,3,2) models give better forecasting results than the other models while the ARX(2,1) and GLM(2) models generate the worst results;

(ii) For irregular patterns, the ARMAX(3,3,2) and ARMAX(3,3,1) models generate better results than the other models while the ARX(2, 1) and ARX(2,2) models generate the worst results;

(iii) For random patterns, the ARX(3,1) and MVNN(2) models generate better results than the others while the ARX(2,1) and ARMAX(2,2,2) models generate the worst performances.

Table 4-33: Summary of performance comparison results of multivariate forecasting models

		ARX (2, 1)	ARX (3, 1)	ARX (2, 2)	ARX (3, 2)	ARMAX (2, 2, 1)	ARMAX (2, 2, 2)	ARMAX (3, 3, 1)	ARMAX (3, 3, 2)	GLM(2)	GLM(3)	MVNN (2)	MVNN (3)
Seasonal	MAE	12	6	7	1	10	8	4	2	11	9	5	3
	MAPE	11	9	7	1	8	10	4	5	12	6	3	2
	MASE	12	5	7	1	10	9	4	2	11	8	6	3
	RMSE	12	4	8	1	9	10	5	2	11	3	7	6
Irregular	MAE	12	10	11	5	8	4	2	1	9	7	3	6
	MAPE	10	7	8	6	12	5	4	2	9	11	3	1
	MASE	12	7	11	6	9	5	3	1	10	8	2	4
	RMSE	12	10	11	8	3	4	2	1	9	6	7	5
Random	MAE	12	1	7	5	6	11	4	3	8	9	2	10
	MAPE	12	1	8	7	6	9	4	3	10	11	2	5
	MASE	12	5	10	6	9	11	4	3	8	7	1	2
	RMSE	11	1	7	5	6	12	3	4	8	9	2	10

Best results Worst results

It is clear that the ARX(2,1) model usually generates the worst forecasting results, whichever data pattern is used. In addition, the MVNN models cannot perform much better than the other traditional multivariate models.

(2) Even for the same forecasting technique, different parameter settings can impact forecasting results greatly. For instance, the ARX(3,2) generates much better results than the ARX(2,2) for seasonal patterns. The ARMAX(3,3,1) and ARMAX(3,3,2) models can generate better forecasts than the ARMAX(2,2,1) and ARMAX(2,2,2) models.

(3) Different accuracy measures also affect forecasting results greatly. For example, the GLM(3) model generates different forecasting results when different accuracy measures are used.

Table 4-34 shows the four best multivariate forecasting models for each data pattern. The results are obtained based on the comparison results shown in Table 4-33. For seasonal data patterns, the ARX(3,2) model generates the best forecasts. For irregular and random data patterns, the ARMAX(3,3,2) model and the ARX(3,1) model generate the best forecasts respectively. The multivariate forecasting models shown in this table should be used more often in future forecasting practice.

Table 4-34: Summary of the four best forecasting models for each data pattern

Rank	1	2	3	4
Seasonal	ARX(3,2)	MVNN(3)	ARMAX(3,3,2)	MVNN(2)
Irregular	ARMAX(3,3,2)	MVNN(3)	ARMAX(3,3,1)	MVNN(2)
Random	ARX(3,1)	MVNN(2)	ARMAX(3,3,1)	ARMAX(3,3,2)

4.5 Comparison of univariate and multivariate forecasting techniques

This section further compares the forecasting performance of all univariate and multivariate time series forecasting models, used in this research, on the basis of the same sales time series. The comparison results of different univariate and multivariate forecasting techniques are summarized in Table 4-35. The table has the same structure with table 4-33. As shown in table 4-35, 23 time series forecasting models are used in performance comparison, which consists of:

(1) Eleven univariate time series forecasting models: Naïve, AR(2), AR(3), MA(2), MA(3), ARMA(1,1), ARMA(1,2), DES, TES, NN(2) and NN(3) models; and

(2) Twelve multivariate time series forecasting models: ARX(2,1), ARX(3,1), ARX(2,2), ARX(3,2), ARMAX(2,2,1), ARMAX(2,2,2), ARMAX(3,3,1), ARMAX(3,3,2), GLM(2), GLM(3), NN(2) and NN(3) models.

From the results shown in table 4-35, the following conclusions can be drawn:

(1) For different data patterns, the forecasting performances generated by different multivariate forecasting models are also mixed.

(i) For the seasonal pattern, the forecasting results generated by the ARX(3,2), ARMAX(3,3,2) and NN(3) models are relatively better than other models while MA(2), MA(3) and TES models generate the worst results;

(ii) For the irregular pattern, the ARMAX(3,3,2) and ARMAX(3,3,1) models generate better results than other models while two MA models and two ES models generate the worst results;

(iii) For the random pattern, the ARX(3,1) and AR(2) models generate better results than the others while ARX(2,1) and GLM(3) models generate the worst performance.

Table 4-35: Summary of performance comparison results of all forecasting models used

		Naïve	AR(2)	AR(3)	MA(2)	MA(3)	ARMA (1,1)	ARMA (1,2)	DES	TES	NN(2)	NN(3)	ARX (2, 1)	ARX (3, 1)	ARX (2, 2)	ARX (3, 2)	ARMAX (2, 2, 1)	ARMAX (2, 2, 2)	ARMAX (3, 3, 1)	ARMAX (3, 3, 2)	GLM(2)	GLM(3)	MVNN (2)	MVNN (3)
Seasonal	MAE	17	15	12	22	23	18	20	10	21	7	3	19	4	9	1	14	11	5	2	16	13	8	6
	MAPE	8	10	7	21	23	11	9	14	22	4	2	18	13	17	1	15	19	6	16	20	12	3	5
	MASE	21	15	10	23	22	19	17	18	20	12	6	16	4	9	1	14	11	5	2	13	3	7	8
	RMSE	16	19	12	22	23	17	20	10	21	8	6	18	4	9	1	14	11	5	2	15	13	7	3
Irregular	MAE	11	5	12	21	23	13	6	20	22	15	17	19	8	14	4	10	3	2	1	18	7	9	16
	MAPE	7	1	4	20	23	2	3	15	22	18	10	17	11	12	8	16	9	6	5	13	19	14	21
	MASE	15	8	11	20	23	10	6	21	22	5	16	18	9	14	4	12	3	2	1	17	13	7	19
	RMSE	18	15	13	17	23	14	7	21	22	12	11	20	9	19	6	3	4	2	1	8	5	10	16
Random	MAE	6	1	10	13	11	8	/	12	15	4	21	22	2	9	7	17	18	5	3	16	19	14	20
	MAPE	8	3	5	10	11	1	/	12	18	13	16	19	2	9	7	20	17	6	4	21	22	14	15
	MASE	3	1	11	10	12	8	/	6	14	5	15	22	7	17	13	20	19	4	9	18	21	2	16
	RMSE	15	8	12	10	13	7	/	11	16	2	21	17	1	9	6	20	18	4	5	19	14	3	22

Best results   Worst results

For seasonal and irregular data patterns, MA(2), MA(3) and TES models should not be used as benchmark models in sales forecasting research since they generate the worst forecasts.

(2) In general, ARMAX(3,3,1) and ARMAX(3,3,2) models can provide relatively better forecasts than other models, although ARMAX(2,2,1) and ARMAX(2,2,2) cannot perform well. This indicates it is very important to select appropriate model parameters.

(3) Comparing with classical time series forecasting techniques, NN models cannot provide superior forecasting performances, which is consistent with Ong and Chan's results (Ong and Chan 2011), but inconsistent with Au et al. (2008)'s and Frank et al. (2003)'s

results. Although NN models have the potentials of outperforming the classical models due to their capacities of nonlinearity, generalization, and universal function approximation, the forecasting performances of NN models are influenced by lots of factors, such as training sample size, NN network structure and parameters. In general, larger sample size can generate more accurate forecasts. Due to time limit, this research has not considered these factors' effects.

(4) Comparing with univariate time series forecasting models, multivariate time series forecasting models cannot exhibit obvious advantages. It is consistent with some experimental results in the literature (Du Preez and Witt 2003, De Gooijer and Hyndman 2006). A possible reason is that a limited number of exogenous variables are considered due to various reasons such as data inavailability, which perhaps omits important factors and thus weakens forecasting performance. In addition, the relationships between various exogenous variables and retail sales have not been identified clearly. For example, exogenous variables with low correlation can lower the forecasting performance and some exogenous variables can be intercorrelated.

Table 4-36: Summary of the four best forecasting models for each data pattern

Rank	**1**	**2**	**3**	**4**
Seasonal	ARX(3,2)	ARMAX(3,3,2)	NN(3)	MVNN(3)
Irregular	ARMAX(3,3,2)	ARMAX(3,3,1)	ARMAX(2,2,2)	ARMA(1,2)
Random	AR(2)	ARX(3,1)	ARMA(1,1)	ARMAX(3,3,1)

Table 4-36 shows the four best forecasting models for each data patterns by comparing the forecasting performances generated by univariate and multivariate forecasting models. For the seasonal data pattern, the ARX(3,2) model generates the best forecasts. For the irregular and random data patterns, the ARMAX(3,3,2) model and the AR(2) model generate the best forecasts respectively. These forecasting models shown in this table should be used in a higher priority for each corresponding data pattern in forecasting practice.

4.6 Summary

This chapter aims at addressing the performance comparison of several commonly used time series forecasting techniques for apparel sales forecasting. A large number of apparel sales time series were used and categorized into trend, seasonal, irregular and random patterns. Forecasting performances of different forecasting techniques were compared in terms of different data patterns and different accuracy measures.

11 univariate forecasting models, based on 6 univariate forecasting techniques, were compared firstly. Next, 12 multivariate models were constructed respecitvely on the basis of 4 multivariate techniques (ARX, ARMAX, GLM and MVNN), and their forecasting performances were then compared. For these multivariate models, exogenous variables considered possible influencing factors, including city temperature and various available economic indices. Due to these incompleteness and unavailability of various official data, we cannot consider more exogenous variables' effects on forecasting performance. Lastly, the comparison between univariate time series forecasting models and multivariate models were also conducted. Based on the comparison results, we further analyzed the performances of various time series forecasting models in terms of different data patterns and accuracy measures.

In addition, the chapter also investigates the effects of different input sizes and accuracy measures on sales forecasting performances. The comparison presented in this chapter can provide a theoretical basis for forecasting researchers and practitioners, and help them select appropriate forecasting or benchmark models for different apparel sales forecasting tasks.

Chapter 5

Conclusion and Future Work

This chapter starts with the conclusion of this research and presents the contributions and limitations of this research as well as suggestions for future work.

5.1 Conclusion

The purpose of this research was to investigate and compare the forecasting performances of different time series forecasting techniques for apparel retailing.

To compare the forecasting performances of different forecasting techniques, a large number of apparel sales time series were collected and analyzed and then categorized into trend, seasonal, irregular and random data patterns.

The performances of several commonly used univariate time series forecasting techniques were compared. The forecasting techniques adopted in this research were the Naïve, AR, MA, ARMA, DES, TES and NNs. On the basis of these forecasting techniques, various forecasting models with different input sizes were constructed. The sales forecasting performances of these models were compared using four commonly used accuracy measures, namely the MAE, MAPE, MASE and RMSE.

Then, the performances of several commonly used multivariate time series forecasting techniques for apparel retailing were compared. The multivariate forecasting techniques adopted in this research were the ARX, ARMAX, GLM and MVNNs. Twelve multivariate forecasting models were established on the basis of these techniques and different parameter settings. The inputs of these models included historical sales amounts and exogenous variables. The exogenous variables were regarded as influencing factors, including city temperature and economic indexes such as CPI and GDP.

Finally, we also analyzed and compared the performances of different univariate and multivariate forecasting techniques. According to the comparison results presented in Chapter 4, the following conclusions can be drawn:

(1) For different data patterns, the forecasting performances generated by different forecasting models are mixed. For seasonal data patterns, the ARX(3,2), ARMAX(3,3,2) and NN(3) models perform better than the others. For irregular data patterns, the ARMAX(3,3,2), ARMAX(3,3,1) and ARMAX(2,2,2) models perform better than the others. The AR(2), ARX(3,1), ARMA(1,1) and ARMAX(3,3,1) models outperform the other models for random data patterns. Clearly, it is important to select appropriate forecasting models based on different data patterns.

(2) Among the univariate forecasting techniques, the MA usually generates the worst forecasting results, whichever data pattern is used. In addition, the NN models cannot perform much better than the other traditional models.

(3) The NN models cannot give better forecasting performances than the other classical models. For example, the forecasts generated by the NN models are not superior to those by the ARMAX models possibly because of overfitting. It is important to set an appropriate network structure and parameters based on specific forecasting tasks.

(4) In multivariate sales forecasting, the ARX(2,1) model usually generates the worst forecasting results, whichever data pattern is used. For different data patterns, the forecasting performances generated by different multivariate forecasting models are mixed. For example, the forecasting results generated by the ARX(3,2) and ARMAX(3,3,2) models are better than those by the other models for seasonal patterns while the ARX(3,1) and MVNN(2) models generate better results than the other models for random patterns. In addition, the MVNN models cannot perform much better than the other traditional multivariate models.

(5) The multivariate time series forecasting models do not always generate better results than the univariate time series forecasting models. For example, the AR(2,1) and AR(2,2) models usually do not generate better forecasts than the AR(2) model although the ARX(3,1) and ARX(3,2) models perform relatively better than the AR(3) model.

(6) Even for the same model, different parameter settings can impact forecasting results greatly. For univariate sales forecasting, different numbers of input variables can have large effects on forecasting results. For instance, the AR(3) generates much better results than the AR(2) for seasonal patterns while the AR(2) generates better results than the AR(3) for irregular patterns. For multivariate sales forecasting, different parameter settings can have great effects too. For instance, the ARX(3,2) model generates much better results than the ARX(2,2) for seasonal patterns.

(7) Different accuracy measures can affect forecasting results greatly. Take irregular patterns as an example. The Naïve and the AR(2) generate similar forecasts when the MAE is used as the accuracy measure.; however, the AR(2) generates much better forecasts when the MAPE is used. If a seasonal pattern is used as an example, the GLM(2) and GLM(3) models generate similar forecasts when the MAE is used as the accuracy measure; however, the GLM(3) model generates much better forecasts when the MASE is used.

5.2 Contributions of this research

It is generally agreed that no forecasting model is suitable to all sales data or patterns. To be best of the author's knowledge, this research is the first to compare the forecasting performances of commonly used time series forecasting techniques for apparel sales data. This research helps enrich our understanding of retail sales forecasting from the perspectives of both univariate and multivariate time series forecasting.

The comparisons presented in this research showed that it is important to select appropriate forecasting models based on different data patterns. The comparison results and conclusions presented in this research can provide a theoretical basis for forecasting researchers and practitioners, and help them choose the most appropriate forecasting or benchmark models for different sales forecasting tasks in apparel retailing. For example, on the basis of the comparison results presented in this research, it is clear that the MA should not be used as a forecasting or benchmark model in either apparel sales research or practice.

The development of this research helps enrich the methodologies of sales forecasting for apparel retailing. Different apparel sales data patterns are identified and defined in this research, which provides us an alternative method to choose forecasting models according to specified data patterns. In addition, the conclusions drawn can also be used in dealing with sales forecasting problems in other similar retail industries.

5.3 Limitations of this research and suggestions for future work

While this research compared the performances of various time series forecasting techniques for apparel sales based on different data patterns and accuracy measures, the limitations of this research leave a great deal of work to be done.

Due to the incompleteness and unavailability of official statistical data, we could not assess the full effect of a large number of exogenous variables on forecasting performance in multivariate sales forecasting. In this research, exogenous variables were selected according to retail forecaster’s experience, the availability of official statistical data, and simple correlation analysis. However, it is questionable if there exist other exogenous variables influencing apparel sales. Future research should identify other potential influencing factors and investigate the effects of these exogenous variables on forecasting performance.

The number of apparel sales time series used for the multivariate techniques was relatively less than that used for the univariate techniques because not much data could be obtained from the sponsoring apparel retail company. Furthermore, the company had not conducted performance comparison of trend data because the trend time series of apparel sales are usually annual sales data and it is hard to collect so much consecutive annual data. For future research, it is advisable to compare and evaluate the multivariate forecasting techniques on the basis of more sample data. More sales time series, especially with noisy data, should be collected from the two retail companies' POS databases for comparison and evaluation of the multivariate forecasting techniques.

As some exogenous variables correlate, the relationships between these variables and apparel sales have great effects on forecasting performance. We could not use all these variables as the inputs of forecasting models because some variables are possibly redundant, which will increase the complexity while decrease the accuracy of forecasting model. However, this research did not investigate the relationships among these exogenous variables and their effects on apparel sales amounts due to time limit. Future research should investigate how to select the most appropriate exogenous variables and establish their relationships with apparel sales by eliminating the side effects of redundant variables.

The experimental results showed that forecasting accuracy greatly depends on the choice of forecasting techniques and their parameters settings. However, it is yet far from certain of selecting appropriate techniques and setting model parameters. Future research should examine how to select the most appropriate forecasting technique and how to set the parameters of the forecasting technique on the basis of the data nature of time series to be forecasted.

5.4 Related publications

The author demonstrated the originality of this research through the following publications.

Refereed Journal Paper:

Li Min, Wong W.K., and Leung S.Y.S. (2013). “Comparison study of univariate forecasting techniques for apparel sales”. submitted to International Journal of Clothing Science and Technology (Under review)

Li Min, Wong W.K., and Leung S.Y.S. (2013). “Comparison study of univariate and multivariate forecasting techniques for apparel sales”. To be submitted

Conference Paper:

Li, Min, Wong, W.K. and Leung, S.Y.S. 2013, ‘Comparison on univariate time series forecasting techniques for apparel sales’, Proceedings of International Symposium on Forecasting (ISF) 2013, Seoul, Korea.

Appendix A

Table A1: Forecasting performances generated by different univariate forecasting models for trend pattern

Data Series No.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Naïve	MAE	58744520.0	3663342.0	4935292.0	3663342.0	4935292.0	2309077.0	1625899.0	4386948.0	2269446.0	4755973.0	1199101.0	1720320.5	1887062.0	3541757.0	2184167.0
	MAPE	15.69%	6.91%	17.53%	6.91%	24.49%	8.26%	13.73%	14.18%	10.32%	12.22%	8.43%	30.56%	13.56%	62.34%	38.40%
	RMSE	58744520.0	3663342.0	4935292.0	3663342.0	4935292.0	2309077.0	1625899.0	4386948.0	2269446.0	4755973.0	1199101.0	1720320.5	1887062.0	3541757.0	2184167.0
	MASE	2.04	0.61	1.52	1.01	2.45	0.60	0.96	1.30	0.83	1.58	0.68	0.99	1.57	1.10	0.80
AR(2)	MAE	18599581.4	738995.3	131152.5	291750.3	3350831.9	4389316.0	232590.0	405179.2	28597.3	187550.5	204732.9	179614.7	267885.8	746432.1	443691.5
	MAPE	4.97%	1.39%	0.47%	0.55%	16.63%	15.70%	1.96%	1.31%	0.13%	0.48%	1.44%	3.19%	1.93%	13.14%	7.80%
	RMSE	18599581.4	738995.3	131152.5	291750.3	3350831.9	4389316.0	232590.0	405179.2	28597.3	187550.5	204732.9	179614.7	267885.8	746432.1	443691.5
	MASE	0.65	0.12	0.04	0.08	1.66	1.13	0.14	0.12	0.01	0.06	0.12	0.10	0.22	0.23	0.16
AR(3)	MAE	21269509.8	574353.5	606465.8	598005.1	4037010.7	4383283.7	678672.2	110834.3	1652585.3	329740.8	1374326.6	482416.3	226383.1	1108975.8	158655.7
	MAPE	5.7%	1.1%	2.2%	1.1%	20.0%	15.7%	5.7%	0.4%	7.5%	0.8%	9.7%	8.6%	1.6%	19.5%	2.8%
	RMSE	21269509.8	574353.5	606465.8	598005.1	4037010.7	4383283.7	678672.2	110834.3	1652585.3	329740.8	1374326.6	482416.3	226383.1	1108975.8	158655.7
	MASE	0.74	0.10	0.19	0.16	2.00	1.13	0.40	0.03	0.61	0.11	0.78	0.28	0.19	0.34	0.06
MA(2)	MAE	82026280.0	7118123.0	7348170.0	5618123.0	5848170.0	5117207.5	2447088.0	6470592.5	3615830.5	7008687.5	1924778.5	2722385.5	2632421.0	5716087.5	3464534.0
	MAPE	21.9%	13.4%	26.1%	10.6%	29.0%	18.3%	20.7%	20.9%	16.4%	18.0%	13.5%	48.4%	18.9%	100.6%	60.9%
	RMSE	82026280.0	7118123.0	7348170.0	5618123.0	5848170.0	5117207.5	2447088.0	6470592.5	3615830.5	7008687.5	1924778.5	2722385.5	2632421.0	5716087.5	3464534.0
	MASE	2.86	1.19	2.27	1.55	2.90	1.32	1.44	1.91	1.33	2.33	1.10	1.57	2.19	1.77	1.27
MA(3)	MAE	98717243.3	9622468.3	8906554.3	7289135.0	6573221.0	7572737.7	3629202.7	8624100.7	4479560.3	8966995.0	2990598.7	3591271.8	3340512.3	7714725.7	4961822.0
	MAPE	26.4%	18.2%	31.6%	13.7%	32.6%	27.1%	30.6%	27.9%	20.4%	23.0%	21.0%	63.8%	24.0%	135.8%	87.2%
	RMSE	98717243.3	9622468.3	8906554.3	7289135.0	6573221.0	7572737.7	3629202.7	8624100.7	4479560.3	8966995.0	2990598.7	3591271.8	3340512.3	7714725.7	4961822.0
	MASE	3.44	1.60	2.75	2.01	3.26	1.95	2.14	2.55	1.65	2.98	1.70	2.07	2.78	2.39	1.81
ARMA(1,1)	MAE	19729847.1	4254387.4	1129959.1	729107.0	2455084.9	4677616.6	1684850.5	1531652.4	1664872.0	1835894.8	302153.9	700406.5	198248.2	1645200.1	559911.2
	MAPE	5.3%	8.0%	4.0%	1.4%	12.2%	16.7%	14.2%	5.0%	7.6%	4.7%	2.1%	12.4%	1.4%	29.0%	9.8%
	RMSE	19729847.1	4254387.4	1129959.1	729107.0	2455084.9	4677616.6	1684850.5	1531652.4	1664872.0	1835894.8	302153.9	700406.5	198248.2	1645200.1	559911.2
	MASE	0.69	0.71	0.35	0.20	1.22	1.21	0.99	0.45	0.61	0.61	0.17	0.40	0.16	0.51	0.20
DES	MAE	32984334.0	1506974.3	1092859.7	48289.2	3208276.9	3305063.0	120941.8	219242.1	420433.0	252316.4	579115.7	4522.0	482354.8	521038.7	377868.5
	MAPE	8.8%	2.8%	3.9%	0.1%	15.9%	11.8%	1.0%	0.7%	1.9%	0.6%	4.1%	0.1%	3.5%	9.2%	6.6%
	RMSE	32984334.0	1506974.3	1092859.7	48289.2	3208276.9	3305063.0	120941.8	219242.1	420433.0	252316.4	579115.7	4522.0	482354.8	521038.7	377868.5
	MASE	1.15	0.25	0.34	0.01	1.59	0.85	0.07	0.06	0.15	0.08	0.33	0.00	0.40	0.16	0.14
TES	MAE	26392613.9	2730553.8	715292.7	583303.6	3176627.8	3852632.9	752118.1	266697.0	282923.7	31333.6	1286479.8	254960.3	135676.0	1446347.3	558352.2
	MAPE	7.0%	5.2%	2.5%	1.1%	15.8%	13.8%	6.3%	0.9%	1.3%	0.1%	9.0%	4.5%	1.0%	25.5%	9.8%
	RMSE	26392613.9	2730553.8	715292.7	583303.6	3176627.8	3852632.9	752118.1	266697.0	282923.7	31333.6	1286479.8	254960.3	135676.0	1446347.3	558352.2
	MASE	0.92	0.46	0.22	0.16	1.57	0.99	0.44	0.08	0.10	0.01	0.73	0.15	0.11	0.45	0.20
NN(2)	MAE	16130667.0	11041838.9	1743059.3	1474885.7	4455107.3	1971830.9	391854.8	4745834.1	1532319.2	1711162.0	1127255.0	843282.3	1681397.8	2651960.3	1675609.0
	MAPE	4.3%	20.8%	6.2%	2.8%	22.1%	7.1%	3.3%	15.3%	7.0%	4.4%	7.9%	15.0%	12.1%	46.7%	29.5%
	RMSE	16130667.0	11041838.9	1743059.3	1474885.7	4455107.3	1971830.9	391854.8	4745834.1	1532319.2	1711162.0	1127255.0	843282.3	1681397.8	2651960.3	1675609.0
	MASE	0.56	1.84	0.54	0.41	2.21	0.51	0.23	1.40	0.56	0.57	0.64	0.49	1.40	0.82	0.61
NN(3)	MAE	58119923.0	2376634.3	2459482.2	1169095.4	4510844.1	1753548.3	3121646.2	3053572.4	2250198.4	3539368.0	305337.1	1700046.1	1818051.5	3538278.9	4181989.2
	MAPE	15.5%	4.5%	8.7%	2.2%	22.4%	6.3%	26.4%	9.9%	10.2%	9.1%	2.1%	30.2%	13.1%	62.3%	73.5%
	RMSE	58119923.0	2376634.3	2459482.2	1169095.4	4510844.1	1753548.3	3121646.2	3053572.4	2250198.4	3539368.0	305337.1	1700046.1	1818051.5	3538278.9	4181989.2
	MASE	2.02	0.40	0.76	0.32	2.23	0.45	1.84	0.90	0.83	1.18	0.17	0.98	1.51	1.10	1.53

Table A2: Forecasting performances generated by different univariate forecasting models for seasonal pattern

Data Series No.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Naïve	MAE	872006.3	438275.4	4352019.2	1554198.0	2399583.9	901069.0	2065219.6	802122.9	2253046.3	703145.7	2513682.4	608483.1	2073935.3	558669.4	4973485.2
	MAPE	65.28%	138.65%	68.35%	95.81%	99.98%	153.52%	181.07%	114.71%	103.18%	165.81%	296.93%	62.03%	197.81%	330.41%	269.42%
	RMSE	1027407.1	586534.0	4799408.6	2050587.4	2980104.2	1161447.9	2744158.7	1080928.2	2852865.6	1013142.3	3926089.8	889731.3	2836502.4	718342.3	9347738.8
	MASE	0.89	0.47	1.03	1.35	1.36	1.87	0.94	1.11	1.31	1.37	1.20	0.95	0.97	0.91	0.90
AR(2)	MAE	610829.3	372192.7	3082376.7	1224900.6	1920891.2	792213.9	1871813.8	779286.7	1774441.2	575856.5	1696074.7	632283.2	1903968.5	541734.2	4988173.2
	MAPE	41.28%	102.44%	40.30%	83.89%	85.34%	113.87%	204.41%	80.22%	74.19%	152.22%	346.04%	47.33%	229.40%	286.09%	277.60%
	RMSE	747560.1	525810.4	3769573.4	1785862.6	2379352.2	1051946.2	2448602.2	1024255.2	2295147.7	823697.6	2731671.9	872969.3	2546075.0	692974.2	8177670.8
	MASE	0.63	0.40	0.73	1.07	1.09	1.64	0.85	1.08	1.04	1.12	0.81	0.99	0.89	0.88	0.90
AR(3)	MAE	583820.9	374757.7	3076570.6	1205818.4	1942226.2	811895.9	1892037.0	781748.8	1825254.9	588124.3	1750364.0	633632.3	1919953.8	554250.3	5140740.3
	MAPE	32.12%	75.76%	46.55%	101.63%	102.03%	302.99%	288.06%	144.67%	135.48%	314.42%	472.03%	47.76%	253.85%	337.05%	387.75%
	RMSE	726978.7	524211.7	3709128.6	1778887.4	2384311.8	1028984.3	2454609.9	1017217.5	2317271.3	820051.9	2762975.9	874664.1	2555576.9	697322.5	8272872.9
	MASE	0.60	0.40	0.73	1.05	1.10	1.69	0.86	1.08	1.07	1.14	0.83	0.99	0.90	0.90	0.93
MA(2)	MAE	1210934.6	606872.6	5756736.7	2107993.7	3205650.8	1175794.2	2617140.9	960343.5	3039968.6	1003828.4	3708093.9	859743.5	2640277.7	711943.1	7242540.9
	MAPE	110.03%	394.76%	120.45%	204.46%	218.44%	410.88%	463.39%	231.14%	229.65%	461.60%	848.66%	102.45%	521.65%	522.65%	731.62%
	RMSE	1405786.3	765849.8	6527991.8	2680820.8	4023831.1	1502364.0	3453202.7	1295163.0	3902194.5	1365070.2	5266451.5	1085520.0	3569226.8	785346.5	11105676.0
	MASE	1.24	0.65	1.36	1.83	1.81	2.44	1.20	1.33	1.77	1.95	1.77	1.34	1.24	1.16	1.31
MA(3)	MAE	1516016.4	755443.1	7108021.1	2606726.1	3958373.6	1438824.7	3146781.4	1145104.5	3805841.9	1267044.1	4549331.5	1104245.4	3223133.5	771313.0	9412701.5
	MAPE	168.59%	915.56%	178.64%	367.29%	428.83%	840.89%	1391.45%	471.73%	455.62%	956.38%	1907.72%	152.27%	1591.76%	866.59%	2494.22%
	RMSE	1710073.8	905533.5	8082264.4	3225030.4	4791700.0	1787162.5	4011509.9	1475284.3	4691617.5	1645093.7	6235610.6	1273100.2	4167050.9	860079.9	12555151.7
	MASE	1.55	0.81	1.68	2.27	2.24	2.99	1.44	1.59	2.22	2.47	2.17	1.72	1.51	1.25	1.71
ARMA(1,1)	MAE	615315.0	390655.2	3718568.7	1353256.9	2053239.5	850048.2	2041192.7	787608.9	2096397.4	598624.9	2042142.7	631257.0	2372090.5	543609.9	6257765.6
	MAPE	50.43%	55.33%	40.18%	51.10%	73.56%	63.97%	63.56%	145.12%	71.53%	63.43%	437.53%	48.37%	229.31%	286.96%	1072.17%
	RMSE	766733.4	533509.4	4408683.6	1883065.2	2575610.3	1120544.6	2655174.6	1041046.0	2710451.0	900208.8	3148435.0	876466.9	3197350.1	697958.1	9104752.9
	MASE	0.63	0.42	0.88	1.18	1.16	1.76	0.93	1.09	1.22	1.17	0.97	0.99	1.11	0.88	1.13
ARMA(1,2)	MAE	614073.6	390950.4	3416171.7	1233786.9	2076448.5	838053.9	2026077.9	808597.5	1963233.6	601993.6	1726528.2	871030.4	2368141.5	561911.2	6189949.8
	MAPE	50.42%	55.24%	39.59%	44.99%	56.77%	233.50%	127.76%	158.50%	63.01%	134.31%	487.80%	92.50%	115.85%	970.56%	1028.77%
	RMSE	744181.6	533837.6	4031662.5	1815257.6	2634423.5	1104331.5	2624550.5	1065733.3	2722013.2	897774.8	2798322.3	1012322.6	3197058.3	681432.7	8968521.4
	MASE	0.63	0.42	0.81	1.07	1.17	1.74	0.93	1.12	1.15	1.17	0.82	1.36	1.11	0.91	1.12
DES	MAE	562516.5	489990.8	2915572.7	1657799.5	1829287.9	786877.9	1874112.5	799263.5	1805955.1	576437.5	2405660.7	603080.9	2010509.6	474956.1	6404638.1
	MAPE	31.84%	899.50%	61.31%	225.62%	194.57%	548.53%	428.56%	278.30%	207.38%	547.91%	822.94%	44.95%	710.76%	341.07%	745.42%
	RMSE	843533.0	594841.1	3827830.3	1981677.9	2351137.0	1024844.8	2617308.5	1120051.0	2355248.3	730580.2	3344149.1	965260.9	2780420.8	788052.6	11605101.9
	MASE	0.58	0.52	0.69	1.44	1.03	1.63	0.86	1.11	1.05	1.12	1.15	0.94	0.94	0.77	1.16
TES	MAE	537132.7	607374.3	3732410.7	1670577.8	2016896.2	773214.1	1705835.5	824891.9	2053435.6	624242.6	2893319.5	761266.5	1828825.2	458592.2	10334786.0
	MAPE	44.93%	1323.03%	84.20%	302.72%	336.01%	843.24%	1019.01%	397.05%	374.48%	850.38%	2198.09%	75.65%	1245.25%	562.61%	8406.25%
	RMSE	787262.3	691424.4	4308769.0	2105826.4	2674827.6	1081050.8	2715995.5	1206058.3	2736877.5	805020.5	4282187.4	1064755.9	2868638.7	867358.0	14443722.2
	MASE	0.55	0.65	0.88	1.45	1.14	1.61	0.78	1.14	1.20	1.22	1.38	1.19	0.86	0.75	1.87
NN(2)	MAE	207.0	563024.0	3160780.6	2190136.7	1165778.1	1350239.9	1164877.9	500445.5	1518759.1	861667.7	1741574.8	998964.9	1247018.8	223.6	2573516.1
	MAPE	0.05%	481.70%	57.22%	131.93%	44.43%	242.45%	516.93%	127.15%	189.16%	258.37%	457.67%	160.93%	850.62%	0.08%	1222.79%
	RMSE	308.9	734626.6	4245219.8	2666984.4	1566078.4	2101833.5	2026877.4	690682.1	1930319.4	1107235.5	2735145.1	1496423.0	1620220.6	477.9	4089453.7
	MASE	0.00	0.60	0.75	1.91	0.66	2.80	0.53	0.69	0.89	1.68	0.83	1.56	0.59	0.00	0.47
NN(3)	MAE	81.6	735090.7	2702981.6	2204459.5	2232774.7	822653.7	966258.2	512891.9	2964587.4	871617.3	1430239.4	643312.4	1761616.1	21.4	2260503.3
	MAPE	0.01%	161.28%	33.93%	80.88%	63.85%	214.86%	155.50%	177.62%	1276.69%	708.97%	161.07%	47.98%	372.93%	0.02%	1017.68%
	RMSE	101.9	1297441.6	3176577.3	2943313.7	3308335.6	1147806.9	1360165.0	838314.1	5165603.3	1288728.9	2665320.6	946189.7	2656961.1	30.1	5373825.1
	MASE	0.00	0.78	0.64	1.92	1.26	1.71	0.44	0.71	1.73	1.70	0.68	1.00	0.83	0.00	0.41

Table A2: Forecasting performances generated by different univariate forecasting models for seasonal pattern (cont'd)

Data Series No.		16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Naïve	MAE	2463630.7	3641942.3	29836557.2	9470442.8	13403464.3	3779001.7	10191960.7	3729681.7	14742018.0	3766062.8	3185185.5	19154472.0	4327835.2	2635841.8	6139764.3
	MAPE	74.38%	345.83%	251.15%	366.93%	1675.25%	2860.94%	1126.89%	777.57%	773.17%	1970.17%	139.91%	1182.74%	168.39%	84.55%	330.15%
	RMSE	3801417.8	4349046.7	33149942.9	10666493.0	15371208.8	4414136.6	12272399.3	4601920.0	16568274.8	4604774.8	3267337.3	22987288.2	4618336.3	3094165.0	6413388.5
	MASE	1.02	0.74	0.95	1.07	1.26	0.98	0.63	0.76	1.31	1.01	1.10	1.36	1.18	0.92	1.43
AR(2)	MAE	2797614.9	3163536.3	22795047.0	8036986.8	6258524.4	2648926.7	5437068.3	2103329.7	8369436.4	3125646.4	2689890.3	10688792.8	3159717.5	2677333.9	4851722.3
	MAPE	70.13%	201.28%	140.24%	206.58%	410.30%	994.52%	344.66%	243.81%	209.67%	816.44%	76.26%	188.77%	93.19%	78.63%	182.30%
	RMSE	3881505.6	3474607.1	26426112.6	8543533.7	11222039.9	3405472.7	8743132.5	3503416.8	13603189.6	3906966.1	3057202.4	19087291.8	3861850.2	2942177.8	5611232.7
	MASE	1.16	0.64	0.72	0.91	0.59	0.69	0.33	0.43	0.74	0.84	0.93	0.76	0.86	0.94	1.13
AR(3)	MAE	2684999.0	2156906.7	4663342.7	6532558.7	5179760.4	1759973.9	3269835.2	1504684.2	5582197.1	1796894.0	582289.2	12043633.4	695175.5	1978868.7	1967302.2
	MAPE	61.35%	96.35%	23.59%	141.35%	307.75%	536.42%	171.94%	121.31%	126.44%	528.78%	19.58%	361.00%	27.19%	81.04%	42.38%
	RMSE	3883934.0	2702325.1	5136875.3	8018624.1	6099988.9	2097683.3	3888984.8	2169505.6	6826280.5	2024864.2	767458.9	15943166.9	834713.7	2333614.5	2435445.9
	MASE	1.12	0.44	0.15	0.74	0.49	0.46	0.20	0.31	0.50	0.48	0.20	0.86	0.19	0.69	0.46
MA(2)	MAE	3088418.5	4541201.1	28856363.8	9078135.0	13927529.1	4221209.6	11877195.3	4522577.4	14088035.6	4685336.0	3628451.9	17757260.6	3663801.1	3193317.5	5193767.4
	MAPE	132.14%	423.11%	271.04%	355.91%	2146.51%	3442.55%	1405.94%	1036.43%	971.28%	2183.42%	198.40%	1809.74%	161.16%	109.16%	371.47%
	RMSE	3986888.1	4903070.4	36718855.1	10541903.0	17485043.4	5156464.7	13949815.6	5311280.2	18058321.6	5396089.5	3786582.7	22410693.8	4854777.5	3263372.5	6440648.9
	MASE	1.28	0.92	0.92	1.02	1.31	1.10	0.73	0.92	1.25	1.26	1.26	1.26	1.00	1.12	1.21
MA(3)	MAE	3674525.1	4136723.3	29095045.5	8587717.0	13976842.6	4333198.4	11300366.1	4503307.2	14189606.7	4794447.5	2625789.7	16137947.9	3461293.3	2532041.5	3994334.1
	MAPE	233.68%	328.85%	229.68%	287.09%	1595.97%	2693.09%	1066.41%	857.95%	742.28%	1751.94%	165.79%	1360.83%	137.25%	102.14%	284.21%
	RMSE	4388271.8	4306746.5	31030203.5	9390358.5	14798301.3	4563617.9	11633072.0	4666295.1	15277048.6	4915509.5	3252428.1	19601954.9	3852143.0	2904391.4	5381085.0
	MASE	1.53	0.84	0.93	0.97	1.31	1.13	0.69	0.92	1.26	1.29	0.91	1.15	0.94	0.89	0.93
ARMA(1,1)	MAE	2765071.7	2777601.2	16704657.7	7173078.4	6759505.7	3750115.3	4794523.9	1803025.6	9860951.8	2977608.6	2452410.1	11236547.1	2406728.8	2379645.3	4318695.2
	MAPE	69.08%	170.63%	104.09%	194.96%	350.49%	2657.06%	259.78%	184.21%	114.20%	775.68%	175.76%	157.83%	68.73%	110.36%	131.86%
	RMSE	3890279.9	3301115.4	19474511.8	8655976.1	10625549.3	3978558.9	8070155.9	3285270.6	14658985.0	4176208.4	2836183.4	18998385.7	2690566.1	3052723.4	4846597.7
	MASE	1.15	0.56	0.53	0.81	0.63	0.97	0.29	0.37	0.87	0.80	0.85	0.80	0.66	0.83	1.00
ARMA(1,2)	MAE	2840355.8	2635937.7	20743117.9	7953159.9	10444191.1	2696333.2	9264601.0	3539390.5	8768366.5	2711110.1	2155974.4	13606206.6	2421314.9	2371002.2	4209894.4
	MAPE	73.14%	315.02%	175.17%	348.21%	1603.73%	2689.29%	1126.35%	737.72%	288.60%	1101.05%	104.33%	1284.27%	140.66%	121.03%	316.10%
	RMSE	3936452.2	3737158.6	25801205.6	9900374.4	11751598.4	3532883.8	10175290.7	4094694.0	13101177.9	3539162.0	2455454.8	16128599.8	3341933.3	3375456.3	4321713.2
	MASE	1.18	0.53	0.66	0.90	0.98	0.70	0.57	0.72	0.78	0.73	0.75	0.97	0.66	0.83	0.98
DES	MAE	3007849.4	3024147.0	19469695.9	5712230.3	9991272.8	3075145.1	8595206.2	3749893.7	12384308.0	3997622.2	2119394.5	13098307.0	2654883.4	2050700.8	3541167.5
	MAPE	175.00%	263.31%	131.41%	170.60%	947.16%	1529.51%	813.64%	655.99%	501.43%	1219.41%	107.60%	931.59%	82.64%	86.39%	210.33%
	RMSE	4274972.6	3230557.2	23447976.6	7094186.3	12667696.4	3510960.9	9608955.9	4063283.9	14596559.8	4216657.5	2534568.3	15790592.4	3236851.5	2362388.5	4684706.2
	MASE	1.25	0.61	0.62	0.64	0.94	0.80	0.53	0.76	1.10	1.08	0.73	0.93	0.72	0.72	0.82
TES	MAE	3557204.2	3024839.4	19596073.0	5762508.5	10328131.3	3112210.1	8742768.1	3818532.4	12570841.4	4041375.5	2130020.8	13098307.0	2723358.3	2037883.4	3541167.5
	MAPE	330.15%	262.81%	133.28%	173.33%	1023.45%	1585.16%	845.32%	690.59%	538.68%	1280.68%	112.59%	931.59%	87.49%	88.55%	210.33%
	RMSE	4627695.7	3222152.7	23570014.1	7093950.2	12886680.7	3550034.6	9771471.2	4117621.0	14669417.4	4253003.2	2521158.8	15790592.4	3306200.9	2399236.1	4684706.2
	MASE	1.48	0.61	0.62	0.65	0.97	0.81	0.54	0.78	1.11	1.09	0.74	0.93	0.74	0.71	0.82
NN(2)	MAE	41280.2	5865887.7	2973585.5	7362250.7	9424102.6	2236674.0	3217002.3	421850.2	9116352.3	8317082.4	2468750.3	3321342.2	3199448.2	946945.8	1041081.0
	MAPE	9.27%	591.63%	16.86%	128.91%	113.22%	155.51%	51.52%	42.71%	70.06%	6280.24%	59.32%	43.42%	89.96%	25.26%	14.03%
	RMSE	74118.3	7098444.1	4005669.0	8310345.6	12849730.4	3623500.5	4540421.4	582356.9	14850986.8	11678570.7	3725845.9	4621824.7	4587498.7	968494.1	1886896.2
	MASE	0.02	1.19	0.09	0.83	0.88	0.58	0.20	0.09	0.81	2.24	0.86	0.24	0.87	0.33	0.24
NN(3)	MAE	632.0	3795878.9	5942563.6	12231261.3	6723021.6	12000764.0	3447183.8	1054331.0	6530351.2	2267011.9	2751654.5	11102971.3	7290532.7	1278159.8	1319927.8
	MAPE	0.08%	312.85%	26.81%	431.23%	170.86%	8936.04%	177.36%	98.56%	53.84%	1636.89%	111.02%	74.51%	264.28%	34.94%	29.46%
	RMSE	829.9	4297990.3	7436943.9	22824805.8	10748657.4	19841610.0	5786036.0	1635717.3	11586388.5	3117321.4	3720205.2	17618813.9	7962349.9	1451058.7	1830512.3
	MASE	0.00	0.77	0.19	1.38	0.63	3.12	0.21	0.21	0.58	0.61	0.95	0.79	1.99	0.45	0.31

Table A3: Forecasting performances generated by different univariate forecasting models for irregular pattern

Data Series No.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Naïve	MAE	5357178.3	22508527.8	7852333.5	6821698.2	62389.4	13505.8	22895.8	722277.8	531401.2	14712.4	214656.0	493276.9	1042040.6	788354.5	508365.3
	MAPE	28.62%	31.08%	27.25%	28.31%	48.46%	146.53%	54.96%	31.54%	45.12%	178.38%	170.11%	205.06%	74.23%	121.15%	56.97%
	RMSE	8548222.2	32808384.6	11319788.2	10683189.5	107948.2	17827.8	36351.3	1027349.3	620489.4	19587.7	294020.1	661513.3	1790648.4	1106156.3	684926.7
	MASE	1.10	1.54	1.85	1.03	0.06	0.04	0.03	1.55	1.28	0.04	0.67	1.75	1.43	1.09	0.73
AR(2)	MAE	5549034.2	22579484.7	7989846.6	6907476.8	82606.9	11210.4	23902.2	734429.6	489383.2	11424.2	207950.2	470512.5	1210252.6	785491.4	560879.7
	MAPE	29.33%	31.97%	26.97%	27.53%	53.72%	60.79%	46.65%	29.31%	36.54%	67.04%	89.30%	90.28%	71.86%	86.29%	46.88%
	RMSE	8470331.4	30633908.5	10330814.8	9628536.7	119051.3	17024.6	35783.7	1038642.8	625140.6	17844.5	277906.6	665545.7	1786518.0	1058081.1	737525.0
	MASE	1.14	1.54	1.89	1.04	0.07	0.04	0.03	1.58	1.17	0.03	0.65	1.67	1.66	1.09	0.80
AR(3)	MAE	5564830.0	23416756.1	7460628.7	7280977.0	88893.9	11486.3	22054.1	752780.0	458569.1	10072.1	197113.3	565703.1	1274049.3	785250.9	561870.4
	MAPE	29.71%	33.06%	25.41%	29.09%	61.92%	114.83%	42.01%	29.98%	31.67%	122.05%	202.50%	347.00%	80.82%	148.01%	51.35%
	RMSE	8359996.5	30546865.0	9840056.8	9621853.5	131092.5	17368.9	34909.1	1058629.6	605077.8	16639.3	262341.7	698918.0	1815054.8	1048397.1	709713.2
	MASE	1.14	1.60	1.76	1.10	0.08	0.04	0.03	1.62	1.10	0.03	0.62	2.01	1.75	1.09	0.81
MA(2)	MAE	6604031.5	23332914.3	8123037.2	7096635.8	75498.3	13853.6	26390.5	787988.6	640553.5	15930.9	279464.9	648952.4	1400582.1	938649.0	565226.5
	MAPE	35.82%	33.87%	27.89%	28.74%	61.82%	184.04%	72.41%	36.51%	61.20%	231.65%	348.10%	457.80%	106.82%	253.58%	84.38%
	RMSE	8960334.6	30936637.1	10241561.1	9623877.3	110692.8	20801.4	39029.5	1066873.7	721151.3	23123.7	338916.6	759982.7	1909255.7	1317325.1	716147.7
	MASE	1.35	1.59	1.92	1.07	0.07	0.04	0.04	1.70	1.54	0.05	0.88	2.30	1.92	1.30	0.81
MA(3)	MAE	7419175.2	24197720.4	7634577.6	7469034.3	79125.3	16497.4	28183.3	965767.7	786457.5	16921.3	295794.9	720998.3	1753431.1	1139441.6	684036.4
	MAPE	39.43%	34.99%	26.33%	30.03%	68.47%	245.39%	77.88%	47.78%	85.85%	293.77%	541.97%	783.09%	152.82%	530.48%	126.02%
	RMSE	9194353.1	31046784.4	9789441.7	9655385.1	116713.8	22516.2	39327.3	1109356.3	852534.1	23302.0	349189.3	841195.8	2104490.2	1508510.3	802865.0
	MASE	1.52	1.65	1.80	1.13	0.07	0.05	0.04	2.08	1.89	0.05	0.93	2.56	2.40	1.58	0.98
ARMA(1,1)	MAE	6013884.9	22423305.0	6915741.5	7022434.9	86856.0	9992.4	23820.0	739922.5	519585.4	10170.2	195191.1	545033.5	1246741.0	839931.2	565825.0
	MAPE	32.69%	31.83%	24.21%	28.18%	58.62%	63.10%	46.99%	29.81%	39.48%	71.44%	169.17%	154.81%	76.55%	188.72%	48.92%
	RMSE	8820444.6	26979442.8	8958521.2	9273795.5	124276.6	15545.6	35639.8	1045433.8	647197.7	16511.2	251958.1	724567.3	1857268.8	1146888.6	740906.7
	MASE	1.23	1.53	1.63	1.06	0.08	0.03	0.03	1.59	1.25	0.03	0.61	1.93	1.71	1.17	0.81
ARMA(1,2)	MAE	5458580.0	21929387.1	7063418.1	7000201.2	72930.6	8580.0	17675.6	921050.1	504568.9	10042.9	197424.1	583279.9	1979921.9	829967.9	499106.6
	MAPE	30.08%	31.97%	25.05%	29.45%	59.72%	168.16%	39.79%	34.56%	37.85%	248.37%	342.86%	374.47%	186.20%	134.64%	44.99%
	RMSE	7461654.6	27373911.3	9162064.4	9059357.7	112314.4	13528.0	30421.3	1221472.4	628891.2	15071.7	236599.4	693602.6	2565898.0	1182423.2	663334.5
	MASE	1.12	1.50	1.67	1.06	0.07	0.03	0.02	1.98	1.21	0.03	0.62	2.07	2.71	1.15	0.72
DES	MAE	5964029.1	24058510.1	6160662.4	6261530.3	294698.2	154741.2	273771.1	844465.7	439337.9	112649.1	247298.1	719324.5	1157408.6	843089.8	519401.7
	MAPE	30.59%	33.84%	21.34%	23.79%	187.36%	2580.32%	921.94%	35.07%	39.50%	2699.15%	812.90%	1360.38%	90.54%	309.35%	51.37%
	RMSE	7329938.5	31815998.5	8390998.0	8103278.4	515850.5	164570.0	287347.3	1161422.1	670801.0	120464.9	305884.3	834153.6	2004225.1	1202095.2	769558.9
	MASE	1.22	1.64	1.45	0.95	0.26	0.50	0.39	1.82	1.05	0.33	0.78	2.55	1.59	1.17	0.75
TES	MAE	5945405.9	21125388.0	6262131.6	6284003.6	333925.4	150881.7	273793.5	956359.9	436095.7	103098.4	247298.1	718331.2	1336932.6	895945.7	559569.1
	MAPE	30.47%	27.09%	21.85%	24.05%	266.35%	2611.78%	882.16%	40.13%	39.78%	2743.61%	812.90%	1378.40%	129.73%	448.48%	76.83%
	RMSE	7327632.2	27990359.4	8428293.3	8117296.2	544401.2	162536.0	295406.8	1280131.6	704386.8	113917.0	305884.3	830392.9	2148322.6	1286199.6	813348.5
	MASE	1.22	1.44	1.48	0.95	0.30	0.49	0.39	2.06	1.05	0.30	0.78	2.55	1.83	1.24	0.80
NN(2)	MAE	5155391.5	19663678.6	9958719.3	5999242.9	381938.4	116143.3	328275.4	1410161.8	641508.3	123328.6	194236.2	436058.0	1734507.1	631420.8	546907.4
	MAPE	27.96%	24.73%	32.75%	20.63%	398.65%	1106.65%	1100.61%	60.00%	67.83%	1079.04%	175.98%	124.28%	120.90%	224.55%	75.04%
	RMSE	7498758.9	27057894.8	12378773.5	8278953.0	450643.9	143174.1	346993.1	1954332.8	819446.1	198528.2	292120.7	552080.8	2456094.0	854773.3	695897.5
	MASE	1.06	1.34	2.35	0.91	0.34	0.38	0.46	3.03	1.54	0.36	0.61	1.55	2.38	0.88	0.78
NN(3)	MAE	6566909.3	41898597.1	12064288.9	6820405.9	184928.9	126480.7	123974.8	1300647.1	701653.0	139531.1	569978.2	1220328.5	943536.9	581110.9	653694.5
	MAPE	36.63%	57.15%	41.96%	25.68%	178.51%	597.68%	380.13%	53.07%	67.44%	857.39%	151.60%	786.83%	72.10%	192.00%	90.49%
	RMSE	8885667.9	68580421.5	13732200.4	8229182.9	214145.2	183578.2	163463.6	1485048.9	922524.6	216373.0	1128394.9	1729515.9	1165052.1	817550.3	799602.7
	MASE	1.35	2.86	2.85	1.03	0.17	0.41	0.17	2.80	1.68	0.41	1.79	4.33	1.29	0.81	0.94

Table A3: Forecasting performances generated by different univariate forecasting models for irregular pattern (cont'd)

Data Series No.		16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Naïve	MAE	25292.1	211292.0	675643.0	3844433.9	761355.4	672763.7	4208699.1	1352686.5	1373091.0	1141081.8	75017.0	701329.6	1432789.7	2216483.6	660704.0
	MAPE	64.96%	233.72%	326.91%	367.84%	113.02%	80.76%	74.38%	104.79%	46.39%	42.66%	59.69%	185.75%	208.10%	96.37%	861.94%
	RMSE	39562.0	282949.8	875718.1	7037593.4	1401083.6	1017809.0	4468250.1	1765750.3	1767976.0	1368517.3	122559.4	938954.4	1919314.6	3764369.1	824091.4
	MASE	0.04	0.39	1.01	0.97	1.02	0.85	0.94	1.17	1.07	0.71	0.07	0.83	1.57	1.10	0.71
AR(2)	MAE	21627.5	166741.1	568122.5	3953676.6	811862.2	588092.9	2841016.8	1137533.8	1372654.3	1222086.0	85462.2	628864.8	1306716.0	2247903.0	538710.6
	MAPE	46.98%	184.57%	460.26%	150.78%	83.39%	55.41%	37.78%	85.74%	41.68%	44.22%	56.29%	71.49%	78.42%	83.18%	219.88%
	RMSE	35438.9	263520.2	761391.4	6637127.5	1312718.5	977202.6	3319394.0	1486959.5	1775938.6	1436568.1	127621.2	891956.9	1903853.5	3676075.6	662518.0
	MASE	0.03	0.31	0.85	1.00	1.09	0.74	0.64	0.98	1.07	0.76	0.08	0.74	1.43	1.11	0.58
AR(3)	MAE	20390.9	185877.8	524370.7	4011031.8	817259.0	593076.4	2816117.1	1161268.1	1389474.3	1226275.2	100640.7	607510.8	1448028.7	2242113.2	560959.1
	MAPE	44.78%	406.16%	1032.87%	180.19%	86.55%	56.63%	45.81%	104.47%	41.55%	42.69%	71.23%	213.01%	388.01%	84.02%	315.81%
	RMSE	34813.6	270581.8	698580.7	6695807.9	1314232.7	981452.0	3214776.7	1499818.7	1780875.5	1427967.8	146056.2	837326.3	1993518.5	3658408.8	671321.6
	MASE	0.03	0.34	0.79	1.01	1.09	0.75	0.63	1.00	1.08	0.76	0.09	0.72	1.59	1.11	0.60
MA(2)	MAE	27509.0	279051.0	988317.9	5139116.9	1112092.1	848975.7	5578454.6	1870880.3	1474979.7	1148979.1	81719.3	847805.9	1764530.5	2731361.3	906162.7
	MAPE	84.77%	631.46%	1263.10%	804.10%	182.81%	123.16%	138.60%	244.19%	51.42%	39.98%	70.01%	481.86%	605.74%	143.02%	2142.99%
	RMSE	41717.4	361285.0	1159559.2	7644153.2	1641590.1	1103319.2	6222530.0	2350653.2	1726452.7	1456917.5	118702.9	1107716.2	2183324.6	3896009.2	1122661.3
	MASE	0.04	0.52	1.48	1.30	1.49	1.07	1.25	1.62	1.15	0.72	0.07	1.00	1.93	1.35	0.98
MA(3)	MAE	25731.7	356132.2	1077502.3	6327777.3	1467454.3	943222.2	7183470.6	2448363.4	1679656.9	1232059.0	85714.9	1043245.8	2107904.4	3535507.4	1148937.2
	MAPE	80.85%	1412.62%	3311.28%	2014.81%	306.49%	190.71%	215.06%	459.82%	64.14%	46.96%	78.19%	1007.63%	1350.14%	221.39%	3367.62%
	RMSE	39478.3	420624.2	1251466.7	8432190.9	1886333.1	1193990.8	7984234.1	2921322.7	1935896.3	1578063.6	124948.0	1349589.0	2506061.5	4433237.6	1376357.6
	MASE	0.04	0.66	1.62	1.59	1.97	1.19	1.61	2.11	1.31	0.77	0.08	1.23	2.31	1.75	1.24
ARMA(1,1)	MAE	22057.7	168413.2	545460.8	4560209.6	870436.4	596678.4	3947490.9	1198597.0	1364224.1	1273655.1	98741.7	662157.2	1332468.0	2287489.5	532227.5
	MAPE	48.41%	94.57%	391.40%	709.98%	111.56%	57.57%	41.24%	48.71%	41.77%	46.39%	67.70%	101.47%	134.70%	87.20%	116.88%
	RMSE	35431.5	258225.5	720707.9	7095431.5	1331734.2	981907.2	4589541.5	1551637.0	1770566.6	1485145.8	138838.7	922811.6	1977240.7	3767661.5	685674.6
	MASE	0.03	0.31	0.82	1.15	1.17	0.76	0.88	1.04	1.06	0.79	0.09	0.78	1.46	1.13	0.57
ARMA(1,2)	MAE	29675.5	169105.9	613409.5	4421625.4	902399.2	602537.1	3778266.8	1191831.3	2239830.7	1288532.2	87276.7	613240.5	1525999.0	2273650.0	519800.8
	MAPE	97.23%	104.76%	670.95%	339.61%	139.69%	62.90%	41.09%	43.06%	94.51%	47.37%	73.63%	81.05%	208.80%	83.34%	92.95%
	RMSE	36565.0	260355.8	856031.8	7045812.4	1348970.1	983155.3	4333843.6	1547077.1	2618916.7	1515676.1	127518.3	853513.3	2069368.8	3788431.9	669908.4
	MASE	0.04	0.31	0.92	1.11	1.21	0.76	0.85	1.03	1.75	0.80	0.08	0.73	1.67	1.12	0.56
DES	MAE	176135.4	248243.3	747437.7	5552054.1	887172.0	811202.5	2637978.5	1440585.9	1267381.4	1197891.3	90489.6	731760.5	1753214.6	2475353.4	651930.7
	MAPE	775.84%	988.07%	2616.65%	2101.10%	236.47%	94.74%	67.47%	257.01%	35.10%	41.99%	56.94%	766.98%	1160.23%	154.04%	1587.28%
	RMSE	186420.4	329626.0	941938.9	10022812.7	1515065.3	1167244.7	3539889.3	1643509.2	1751900.2	1486651.4	133295.7	1043356.5	2204545.4	4307476.2	771235.1
	MASE	0.25	0.46	1.12	1.40	1.19	1.03	0.59	1.24	0.99	0.75	0.08	0.87	1.92	1.22	0.70
TES	MAE	181512.5	404316.4	750577.8	5133680.6	1262495.1	1007697.9	3704970.6	1668326.8	1307008.1	1124144.8	165737.6	837021.4	2299752.8	2832909.1	812621.0
	MAPE	735.37%	5139.60%	2592.65%	12271.06%	377.45%	186.87%	109.32%	428.32%	30.84%	38.17%	209.16%	1276.62%	2830.04%	247.30%	2566.29%
	RMSE	188965.1	444010.4	940660.9	9274032.1	1687548.7	1263259.4	4351467.3	2006639.2	1680469.5	1442076.7	271846.4	1183136.6	2714530.8	4651946.6	983372.9
	MASE	0.26	0.75	1.13	1.29	1.69	1.28	0.83	1.44	1.02	0.70	0.15	0.99	2.52	1.40	0.88
NN(2)	MAE	179032.4	237455.1	333400.8	4083311.9	1859753.1	541839.4	2616284.7	1546915.0	2294227.3	1123405.2	397742.7	596643.7	1706832.2	2438628.1	724351.1
	MAPE	721.19%	1661.22%	625.86%	1128.69%	294.89%	113.61%	43.06%	102.78%	72.69%	45.95%	393.15%	397.32%	814.26%	174.64%	1707.79%
	RMSE	204531.9	361775.6	561417.4	8173432.9	3653672.5	713809.3	3277761.7	1735458.6	2697602.3	1329561.0	706479.5	966930.0	2383728.7	4950452.1	839698.3
	MASE	0.25	0.44	0.50	1.03	2.49	0.69	0.59	1.34	1.79	0.70	0.35	0.71	1.87	1.21	0.78
NN(3)	MAE	59029.8	221172.0	1197951.4	4695528.8	1843661.9	759717.9	4823203.2	1985098.9	3405856.2	2013576.3	138766.4	479178.8	1795601.9	1465363.7	1294401.9
	MAPE	220.91%	1022.76%	566.51%	2054.61%	267.68%	111.34%	320.46%	189.03%	158.77%	69.77%	188.91%	236.57%	694.37%	79.46%	1574.85%
	RMSE	66829.2	321404.0	2441224.3	10482280.4	3415756.4	1271745.9	6609607.7	2586387.7	4971199.3	2708207.5	166801.2	782779.5	2850865.9	1875150.2	2318418.8
	MASE	0.08	0.41	1.80	1.18	2.47	0.96	1.08	1.71	2.65	1.25	0.12	0.57	1.97	0.72	1.39

Table A4: Forecasting performances generated by different univariate forecasting models for random pattern

Data Series No.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Naïve	MAE	246974.9	64945.0	300881.4	254533.0	17342660.5	804185.3	285966.3	94544.2	64486.5	279982.3	256454.5	8244240.0	123160450.0	20711030.0	1603172.0
	MAPE	77.40%	40.51%	86.03%	37.24%	19.22%	161.69%	55.77%	134.74%	54.47%	58.66%	38.71%	3.40%	13.12%	6.38%	9.79%
	RMSE	370941.2	93712.0	422098.2	460790.9	18646294.9	939464.9	378618.2	142008.2	75297.9	405880.9	291913.9	8244240.0	123160450.0	20711030.0	1603172.0
	MASE	0.70	0.33	0.78	0.74	2.11	0.69	0.11	0.11	0.06	0.11	0.31	0.44	1.72	0.88	0.18
AR(2)	MAE	216321.6	69146.8	261219.5	291875.1	14620573.3	743739.2	263098.8	235795.0	59186.6	270284.7	197182.7	24569479.8	124649356.3	28124728.2	1457927.0
	MAPE	57.93%	38.81%	60.74%	42.00%	16.02%	152.09%	46.35%	255.87%	56.85%	51.68%	26.88%	10.13%	13.27%	8.67%	8.90%
	RMSE	344096.3	91345.4	390427.2	452995.0	16323777.2	913638.6	359850.9	407205.8	69582.9	403074.2	280107.2	24569479.8	124649356.3	28124728.2	1457927.0
	MASE	0.61	0.36	0.68	0.85	1.78	0.64	0.10	0.28	0.06	0.11	0.24	1.31	1.74	1.20	0.16
AR(3)	MAE	219007.6	69304.1	267543.3	289781.4	14934904.6	757497.7	531588.1	314487.0	64527.3	308451.0	306428.3	5307622.0	130607770.8	28117387.2	6841108.3
	MAPE	71.34%	38.70%	81.51%	41.87%	16.27%	154.40%	92.85%	323.54%	57.57%	70.43%	54.35%	2.19%	13.91%	8.66%	41.78%
	RMSE	338293.5	92053.6	384852.3	454906.8	16838530.3	922822.6	673282.0	562634.5	73032.1	388728.2	336880.9	5307622.0	130607770.8	28117387.2	6841108.3
	MASE	0.62	0.36	0.69	0.85	1.82	0.65	0.20	0.37	0.06	0.13	0.37	0.28	1.82	1.20	0.75
MA(2)	MAE	276767.6	68313.5	341149.9	268766.1	12014454.3	829181.8	315386.5	196267.2	64559.5	342071.3	282855.8	9906645.0	177506620.0	11906500.0	3005069.0
	MAPE	106.08%	40.34%	124.73%	40.99%	13.25%	217.93%	69.64%	226.80%	63.97%	83.46%	46.49%	4.09%	18.90%	3.67%	18.35%
	RMSE	405695.0	92470.4	461881.3	413168.4	13004966.6	970917.9	362716.0	330353.0	72982.7	402205.5	329522.1	9906645.0	177506620.0	11906500.0	3005069.0
	MASE	0.78	0.35	0.89	0.79	1.46	0.71	0.12	0.23	0.06	0.14	0.35	0.53	2.48	0.51	0.33
MA(3)	MAE	268345.4	67394.7	327508.8	277333.3	14031088.4	843345.0	570635.9	348844.3	76631.3	362444.8	321337.3	25254496.7	176447473.3	10604590.0	2776404.7
	MAPE	133.14%	39.28%	159.69%	42.37%	15.33%	231.50%	109.22%	391.57%	74.34%	98.30%	57.19%	10.42%	18.79%	3.27%	16.96%
	RMSE	399646.7	92827.8	454554.4	391595.5	14962260.6	907258.6	701705.5	553830.8	77117.4	393834.8	393308.0	25254496.7	176447473.3	10604590.0	2776404.7
	MASE	0.76	0.35	0.85	0.81	1.71	0.72	0.22	0.42	0.08	0.15	0.39	1.35	2.46	0.45	0.31
ARMA(1,1)	MAE	226532.6	69264.1	277410.4	308636.9	14326175.4	689318.2	282500.4	181243.0	59467.4	293907.9	261723.9	22425812.2	142871049.6	25375963.0	2713364.5
	MAPE	65.58%	38.94%	72.09%	44.92%	16.10%	140.56%	61.46%	331.65%	53.07%	51.12%	37.55%	9.25%	15.21%	7.82%	16.57%
	RMSE	344961.8	91472.2	391619.6	461725.0	16558213.2	834926.6	341325.8	207128.4	64347.0	414529.1	314118.7	22425812.2	142871049.6	25375963.0	2713364.5
	MASE	0.64	0.36	0.72	0.90	1.75	0.59	0.11	0.22	0.06	0.12	0.32	1.20	2.00	1.08	0.30
DES	MAE	289061.8	68372.9	352110.6	300510.8	12867214.8	901799.0	1473334.5	712103.5	146448.1	267325.1	589388.4	24211572.0	114501285.7	26790891.9	79576.1
	MAPE	86.79%	42.46%	99.25%	48.41%	14.48%	294.55%	251.73%	866.99%	162.92%	59.38%	103.54%	9.99%	12.19%	8.26%	0.49%
	RMSE	405261.8	102344.8	460264.9	485854.8	14697265.7	1011372.0	2246823.4	979184.1	159909.7	333580.5	676313.2	24211572.0	114501285.7	26790891.9	79576.1
	MASE	0.82	0.35	0.91	0.88	1.57	0.77	0.56	0.85	0.14	0.11	0.72	1.29	1.60	1.14	0.01
TES	MAE	312984.5	72359.0	286922.2	381505.0	12744574.0	906897.8	2227083.7	837255.3	383067.6	396078.7	724299.3	24598810.6	114141914.8	28735813.7	8614644.0
	MAPE	377.11%	46.31%	394.57%	62.57%	14.42%	296.44%	442.21%	1231.91%	387.01%	170.01%	128.39%	10.15%	12.15%	8.86%	52.61%
	RMSE	378221.5	107968.9	376070.2	635531.2	14808237.6	1016790.1	2809251.5	1066577.2	390166.1	474367.1	754959.7	24598810.6	114141914.8	28735813.7	8614644.0
	MASE	0.89	0.37	0.74	1.12	1.55	0.78	0.84	1.00	0.38	0.16	0.88	1.31	1.59	1.22	0.95
NN(2)	MAE	239167.5	64703.9	266230.4	268387.7	31912175.4	584433.7	800714.4	611010.4	181330.8	429508.3	873007.2	111692570.2	333805809.4	36425446.2	6315231.9
	MAPE	162.00%	39.44%	158.44%	41.65%	35.92%	106.71%	181.66%	994.29%	175.53%	120.10%	140.17%	46.06%	35.55%	11.23%	38.57%
	RMSE	286243.9	89681.6	315204.2	397776.2	41973158.6	663719.7	1032097.4	846759.3	215043.0	459932.6	1134921.9	111692570.2	333805809.4	36425446.2	6315231.9
	MASE	0.68	0.33	0.69	0.78	3.89	0.50	0.30	0.73	0.18	0.17	1.07	5.96	4.66	1.55	0.69
NN(3)	MAE	315241.3	141692.1	406386.4	413974.3	43035059.0	1421233.8	1779690.9	2067507.2	328367.2	352482.5	554108.6	62792301.3	210471763.9	39543137.6	7069129.1
	MAPE	259.35%	78.11%	160.54%	65.43%	47.49%	304.82%	315.87%	2861.33%	205.31%	86.16%	85.52%	25.90%	22.41%	12.19%	43.18%
	RMSE	432152.8	156518.9	688209.5	675451.6	49474351.4	1676018.6	2324106.6	2822104.8	565474.5	428987.8	898658.5	62792301.3	210471763.9	39543137.6	7069129.1
	MASE	0.89	0.73	1.05	1.21	5.25	1.22	0.67	2.46	0.32	0.14	0.68	3.35	2.94	1.69	0.78

Table A4: Forecasting performances generated by different univariate forecasting models for random pattern (Cont'd)

Data Series No.		16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Naïve	MAE	1729493.0	8938840.0	42147988.0	6470883.0	3973670.0	4755973.0	922822.0	3002769.0	2465189.0	3973670.0	1346526.1	11969342.0	922822.0	1119729.0	3002769.0
	MAPE	55.91%	5.99%	56.46%	436.37%	1717.06%	12.22%	5.76%	600.31%	15.48%	1717.06%	28.68%	21.11%	5.76%	5.55%	600.31%
	RMSE	1729493.0	8938840.0	42147988.0	6470883.0	3973670.0	4755973.0	922822.0	3002769.0	2465189.0	3973670.0	1346526.1	11969342.0	922822.0	1119729.0	3002769.0
	MASE	0.67	0.37	3.71	0.74	3.33	0.52	0.22	0.59	0.61	2.83	0.88	2.57	0.43	0.26	0.58
AR(2)	MAE	1576881.4	14951457.2	47215100.3	5321633.6	1806767.6	151295.7	1457749.1	2213773.9	1178662.2	1452270.7	912154.4	17705574.3	2973501.3	54186.1	3225137.4
	MAPE	50.97%	10.02%	63.25%	358.87%	780.72%	0.39%	9.10%	442.58%	7.40%	627.54%	19.43%	31.23%	18.55%	0.27%	644.77%
	RMSE	1576881.4	14951457.2	47215100.3	5321633.6	1806767.6	151295.7	1457749.1	2213773.9	1178662.2	1452270.7	912154.4	17705574.3	2973501.3	54186.1	3225137.4
	MASE	0.61	0.62	4.15	0.61	1.52	0.02	0.34	0.43	0.29	1.04	0.60	3.80	1.38	0.01	0.62
AR(3)	MAE	1342582.4	7964026.9	50991699.4	20639567.4	1230271.6	654166.0	2561495.3	11421519.6	762874.8	1083170.6	1841981.9	15267146.9	5462883.5	841802.8	13179606.8
	MAPE	43.40%	5.34%	68.31%	1391.84%	531.61%	1.68%	15.98%	2283.38%	4.79%	468.05%	39.23%	26.93%	34.08%	4.17%	2634.85%
	RMSE	1342582.4	7964026.9	50991699.4	20639567.4	1230271.6	654166.0	2561495.3	11421519.6	762874.8	1083170.6	1841981.9	15267146.9	5462883.5	841802.8	13179606.8
	MASE	0.52	0.33	4.48	2.36	1.03	0.07	0.61	2.23	0.19	0.77	1.21	3.27	2.54	0.20	2.53
MA(2)	MAE	3106408.0	21013950.0	45000456.5	11947620.5	5260748.5	7508687.5	29664.0	5424354.5	1435516.5	5260748.5	342204.1	15300362.5	29664.0	350248.5	5424354.5
	MAPE	100.42%	14.08%	60.28%	805.69%	2273.22%	19.30%	0.19%	1084.43%	9.01%	2273.22%	7.29%	26.99%	0.19%	1.74%	1084.43%
	RMSE	3106408.0	21013950.0	45000456.5	11947620.5	5260748.5	7508687.5	29664.0	5424354.5	1435516.5	5260748.5	342204.1	15300362.5	29664.0	350248.5	5424354.5
	MASE	1.21	0.87	3.96	1.37	4.41	0.82	0.01	1.06	0.35	3.75	0.22	3.28	0.01	0.08	1.04
MA(3)	MAE	4001207.3	21332933.3	46796653.7	18094068.3	5738791.3	11633661.7	1243465.3	8919850.0	2558899.0	5738791.3	24366.2	13450635.0	1243465.3	428282.0	8919850.0
	MAPE	129.34%	14.29%	62.69%	1220.18%	2479.78%	29.90%	7.76%	1783.25%	16.06%	2479.78%	0.52%	23.72%	7.76%	2.12%	1783.25%
	RMSE	4001207.3	21332933.3	46796653.7	18094068.3	5738791.3	11633661.7	1243465.3	8919850.0	2558899.0	5738791.3	24366.2	13450635.0	1243465.3	428282.0	8919850.0
	MASE	1.56	0.89	4.12	2.07	4.81	1.27	0.29	1.74	0.63	4.09	0.02	2.88	0.58	0.10	1.71
ARMA(1,1)	MAE	1489923.7	14296379.0	43800824.2	4202180.9	3570891.3	1817724.9	926905.9	2045918.7	1793287.1	3444579.5	1433089.7	11208405.5	921273.1	23759.7	2153893.5
	MAPE	48.16%	9.58%	58.68%	283.38%	1543.01%	4.67%	5.78%	409.02%	11.26%	1488.43%	30.52%	19.77%	5.75%	0.12%	430.60%
	RMSE	1489923.7	14296379.0	43800824.2	4202180.9	3570891.3	1817724.9	926905.9	2045918.7	1793287.1	3444579.5	1433089.7	11208405.5	921273.1	23759.7	2153893.5
	MASE	0.58	0.59	3.85	0.48	2.99	0.20	0.22	0.40	0.44	2.46	0.94	2.40	0.43	0.01	0.41
DES	MAE	4445962.3	7056484.5	41916895.5	16374224.5	2463415.1	4065392.5	1524674.1	8665982.7	4991153.0	1404372.1	493824.6	15148347.0	2787218.4	740759.2	7109441.2
	MAPE	143.72%	4.73%	56.15%	1104.20%	1064.46%	10.45%	9.51%	1732.49%	31.33%	606.84%	10.52%	26.72%	17.39%	3.67%	1421.31%
	RMSE	4445962.3	7056484.5	41916895.5	16374224.5	2463415.1	4065392.5	1524674.1	8665982.7	4991153.0	1404372.1	493824.6	15148347.0	2787218.4	740759.2	7109441.2
	MASE	1.73	0.29	3.69	1.88	2.07	0.44	0.36	1.69	1.23	1.00	0.32	3.25	1.29	0.17	1.36
TES	MAE	5178017.5	2209859.9	40667890.4	16505920.1	2890366.2	4544943.7	2501279.7	9038120.5	4991153.0	562362.0	493824.6	15148347.0	3889406.3	609291.9	6616102.7
	MAPE	167.38%	1.48%	54.48%	1113.08%	1248.95%	11.68%	15.61%	1806.89%	31.33%	243.00%	10.52%	26.72%	24.27%	3.02%	1322.68%
	RMSE	5178017.5	2209859.9	40667890.4	16505920.1	2890366.2	4544943.7	2501279.7	9038120.5	4991153.0	562362.0	493824.6	15148347.0	3889406.3	609291.9	6616102.7
	MASE	2.01	0.09	3.58	1.89	2.42	0.49	0.59	1.76	1.23	0.40	0.32	3.25	1.81	0.14	1.27
NN(2)	MAE	8908126.1	16906151.7	33878585.7	23401664.6	5224060.2	4134953.7	2726190.9	14934466.2	89379.4	2803008.9	813474.1	18706000.4	524153.1	1770159.8	16341218.4
	MAPE	287.96%	11.33%	45.38%	1578.10%	2257.36%	10.63%	17.01%	2985.68%	0.56%	1211.21%	17.33%	32.99%	3.27%	8.77%	3266.92%
	RMSE	8908126.1	16906151.7	33878585.7	23401664.6	5224060.2	4134953.7	2726190.9	14934466.2	89379.4	2803008.9	813474.1	18706000.4	524153.1	1770159.8	16341218.4
	MASE	3.46	0.70	2.98	2.68	4.38	0.45	0.64	2.91	0.02	2.00	0.53	4.01	0.24	0.42	3.14
NN(3)	MAE	7104200.2	22833767.5	41716924.2	36451923.3	2996942.5	4237401.2	958408.2	11996249.2	1912770.6	8316141.0	259178.9	18518642.7	2530213.5	2190308.0	18189257.0
	MAPE	229.65%	15.30%	55.88%	2458.15%	1295.01%	10.89%	5.98%	2398.28%	12.01%	3593.48%	5.52%	32.66%	15.79%	10.86%	3636.38%
	RMSE	7104200.2	22833767.5	41716924.2	36451923.3	2996942.5	4237401.2	958408.2	11996249.2	1912770.6	8316141.0	259178.9	18518642.7	2530213.5	2190308.0	18189257.0
	MASE	2.76	0.95	3.67	4.18	2.51	0.46	0.23	2.34	0.47	5.93	0.17	3.97	1.17	0.52	3.49

Appendix B

Table B1. Values of related influencing factors for different cities (Monthly)

		Beijing					Shanghai		Guangzhou				Hong Kong		
Year	Month	Temp.	CPI	PPI	Total retail sales of social consumer goods	Total retail sales of social chothing goods	Temp	price index for 'clothing and footwear'	Temp	city residents' disposable incomes	per capita consumption expenditure	per capita consumption expenditure of clothing	Temp	Total Retail Amount	Visitor Arrivals
2000	1	-6.4	103.5	101.1	125.4	12.1	5.1	94.6	14.6	1247.2	970	65.8	5.1	10193	1159871
2000	2	-1.5	101.6	101.6	155.5	11.1	4.3	92.9	14	1642.9	1158.8	102	4.3	10728	1094495
2000	3	8	103.3	102	147.2	12.2	10.7	93.3	19	1112.6	883.3	36	10.7	9386.9	1253498
2000	4	14.6	101.6	104.9	126.9	11.2	16.1	94.8	22.7	1052.9	841.2	36.4	16.1	8182.4	1126217
2000	5	20.4	103	100.1	154	11.6	21.3	93.1	26.1	1073.5	930	48.1	21.3	10802	1225581
2000	6	26.7	105.8	102.6	127.2	10.9	24.8	92.4	28.4	1040.1	810.2	39.1	24.8	8322.1	997919.6
2000	7	29.6	104.8	101.4	152.1	13.6	29.1	92.2	28.9	1093.3	888.6	40.6	29.1	8364.7	968697
2000	8	25.7	104.8	102.3	146.1	12	28.3	93.9	28.2	1077.7	988.4	44.9	28.3	9297.8	966507.3
2000	9	21.8	102.1	101.7	155.4	12.4	24.3	93.7	27.1	1202.7	1102.6	38.4	24.3	9842.8	1012495
2000	10	12.6	100.4	102.3	129.2	13.5	19.9	95.4	24.9	1140.7	973.5	38.7	19.9	9285.8	1092120
2000	11	3	101.6	102.6	145.7	12.8	13.2	92.4	19	1128.3	918	51.7	13.2	8040.7	1191879
2000	12	-0.6	101	101.4	125.6	14	9.1	93.6	16.5	1154.6	885	45.2	9.1	8513.9	1004062
2001	1	-5.4	103.1	101.7	174.3	14.6	5.9	99.8	15.4	1269.6	1000.6	63.6	5.9	10081	1128576
2001	2	-1.5	102.7	101.2	185.3	14.4	6.8	99.8	15.1	1659.5	1198.4	81	6.8	10130	1302410
2001	3	7.3	102.8	101.5	168.9	11.1	11	96.8	19.4	1083.3	829.7	36.4	11	11984	1165103
2001	4	14.4	105.8	102.5	168.3	14.2	15.2	96.1	21.7	1078.4	850.4	37.8	15.2	12678	1047820
2001	5	23.1	102.9	103.1	185.1	12.4	20.8	99.9	26.5	1139.9	931.2	45.6	20.8	12239	1203840
2001	6	25.7	103.4	102.8	152.7	14.5	24.2	99.4	27.2	1093.3	829	39.5	24.2	9009.9	985481.9
2001	7	27.3	102.8	103.2	167.8	13.2	29.7	97.9	28.2	1102.4	908.6	34.9	29.7	12118	1239565
2001	8	25.8	105.9	101.6	165.5	14.4	27	97.2	29.1	1152.8	975.3	44.9	27	11580	1019772
2001	9	21.2	102.2	100.7	183.9	12.1	24.9	97.7	27.8	1168.3	1059.3	33.3	24.9	12971	1068482
2001	10	13.8	100.4	103.3	156.2	12.9	20.2	97.2	25.4	1285.7	981.4	40.3	20.2	10568	1208787
2001	11	5.3	101.4	103	150.7	15	13.7	98	19.7	1181	808.3	46	13.7	12501	988993.8
2001	12	-2.4	102.7	102.1	175.5	14.1	7.1	96.8	14.9	1211.9	789.5	53.5	7.1	12038	1201484
2002	1	0	97.9	97.6	169.7	13.6	7	96.7	14.6	1308.2	1003	68.8	7	12093	1210803
2002	2	3.3	96.2	95.8	167.2	15.4	8.5	98.8	17.4	1779.2	1039.8	85.3	8.5	11686	1427420
2002	3	9.7	98.7	96.6	187.2	15.3	13	97.6	20.6	1140.2	854.9	38.4	13	12381	1471667
2002	4	14	99.5	94.6	186.5	13.8	17	96.2	24.4	1093.2	890.9	43.4	17	12967	1343743
2002	5	21.8	99.3	95.4	195.6	16.2	19.4	97.7	27	1051.1	906.6	45.2	19.4	12760	1437397
2002	6	23.5	99.6	95.2	202.9	16.9	25.2	95.5	28.8	1013.6	840.9	31.3	25.2	10071	1528371
2002	7	27.4	99	96.8	167.5	15	27.6	96	28.3	1003.2	904.4	36.8	27.6	13763	1356402
2002	8	25.6	98.3	96.8	200.4	15.8	27.3	98.2	28.4	1009.7	947.2	47.1	27.3	11513	1232998
2002	9	20.4	99	94.1	189.4	16.2	24.8	96.2	26.3	1054.9	1118.3	37.3	24.8	13524	1218537
2002	10	10.6	96.9	97.7	199.8	15.9	20	97.9	23.8	1156.9	998.8	47.9	20	13112	1197413
2002	11	3.3	97.1	94	182.1	14.2	12.9	95.1	19.3	1031.4	834.7	47	12.9	11992	1369970
2002	12	-3	98.2	94.2	182.2	16.7	7.7	97.9	15.7	1132.3	782.9	48.8	7.7	11112	1228663
2003	1	-3.2	100.7	97.3	210.3	18.9	3.6	98.7	13.8	1418.4	1006.1	96.6	3.6	14544	1124292
2003	2	0.8	100.7	95.4	216.2	18.5	6.8	96	17.8	1774.5	1125.1	111	6.8	11874	1359271
2003	3	6.2	98.2	97.5	198.8	18.4	9.8	98.5	18.5	1183.2	849.3	43.4	9.8	12735	1280556
2003	4	15.2	99.1	98	182.7	16.8	15.4	98.6	23.9	1130.8	813.9	39.1	15.4	12840	1402186
2003	5	20.9	101.8	94	200.4	16.1	19.8	98.9	27.3	1144.6	857.8	43.9	19.8	12299	1187399
2003	6	24.6	98.1	95.2	211.8	18.9	24.5	99	27.5	1129.8	872.3	51.2	24.5	14213	1203184
2003	7	26	101	94.9	181.8	15.6	29.5	95.4	30.3	1175.7	936.3	48.7	29.5	12755	1286706
2003	8	26.1	100.9	97.9	202.8	16.5	29.2	95.1	29.2	1187	1065.4	42.5	29.2	12771	1125784
2003	9	20.5	101.3	96.2	211.6	15.2	26.1	98.9	27.4	1236.1	1210	40.8	26.1	14181	1126470
2003	10	13.1	100.5	99	195.8	18.3	18.7	95.2	24.2	1177	997.8	59.2	18.7	13843	1346971
2003	11	3.4	102	98.9	216.9	17	13.9	95.2	20.3	1144.2	853.6	49.3	13.9	13383	1486340
2003	12	0.2	101.3	98.1	217.4	18.8	6.4	95.2	14.6	1301.5	982.5	68.2	6.4	13578	1470324

		Beijing					Shanghai		Guangzhou				Hong Kong		
Year	Month	Temp.	CPI	PPI	Total retail sales of social consumer goods	Total retail sales of social chothing goods	Temp	price index for 'clothing and footwear'	Temp	city residents' disposable incomes	per capita consumption expenditure	per capita consumption expenditure of clothing	Temp	Total Retail Amount	Visitor Arrivals
2004	1	-2.3	102.6	99.4	204.6	19.6	4.1	95.9	13.4	1684.8	1072.1	110.5	4.1	14909	1786378
2004	2	2.9	99.8	95.2	220.4	21.7	8.6	95.3	16.4	1938.1	1398.3	106.5	8.6	13741	1857513
2004	3	7.8	99.2	100.8	227	19.5	9.9	92.3	18.1	1337.5	943	62.3	9.9	16176	1757711
2004	4	16.3	99.4	97.8	237.3	21.4	16.2	92.6	23.7	1247.7	967.8	57.3	16.2	14807	1683464
2004	5	20.5	100.6	98.9	229.6	20.2	20.9	94.6	25.9	1327.9	1042.4	75.2	20.9	14187	1882516
2004	6	24.9	102.2	95.5	210.8	20	24.4	95.6	28.9	1300.9	967.9	57.4	24.4	16352	1724333
2004	7	26	101.8	97.7	237.9	18.1	29.8	92.5	28.7	1285.7	954.5	52.2	29.8	15584	1770081
2004	8	24.9	100.8	100	218.1	18.3	28.9	95.1	29.4	1316.7	1185.5	62.1	28.9	16235	2017906
2004	9	21.2	101.8	100.1	213.4	19.1	24.3	93.2	27.8	1306.4	1137.4	60.8	24.3	16503	1688775
2004	10	14	101.7	98.9	228.6	20.1	19.2	92.2	23.9	1447.8	1321	76.4	19.2	16594	1795097
2004	11	6.4	100.1	99.7	222.7	18.7	14.6	94.3	21.2	1312.8	1013	73.1	14.6	15851	1752331
2004	12	-0.5	102.9	99.8	239.5	18.7	9.1	94.8	16.5	1368.8	1138	60.6	9.1	14771	1772677
2005	1	-2.7	100.8	100.6	241	27.3	3.8	93.5	13.7	1652.9	1104.3	114.1	3.4	18909	2026889
2005	2	-2.8	102.8	98.6	226.7	26.1	4.8	91.9	14.4	2107.5	1476.1	114.4	4.3	17018	1937347
2005	3	6.3	101.7	100.6	223.5	22.3	9.4	92.8	17.2	1498.5	1050.7	62.9	9	18293	2027882
2005	4	16.4	101.7	98.8	232.3	23.2	18.2	90.8	22.6	1482.6	1237	68.2	17.8	16159	1825759
2005	5	19.8	101.8	98.8	243.6	25	21.1	91.6	27.3	1445.1	1458	73.4	20.7	17621	1821923
2005	6	25.6	101.5	102.8	240.9	20.5	26.8	92.6	27.9	1460.2	1058.9	71.6	26.5	16534	1742745
2005	7	27.9	101.6	101.2	247	20.1	29.9	90.7	29.9	1409.2	1069.3	60	29.3	17392	2067240
2005	8	26	101.4	99.2	242.6	19.7	28.7	92.5	28.7	1451.4	1282.1	68.1	28.3	16497	2120635
2005	9	22	101	101.4	246.3	21.7	26.8	92	28.5	1492.2	1283.3	61	26.3	15676	1787728
2005	10	14.9	101.2	104	253.1	24.9	19.6	92.1	25.8	1480.4	1317.7	59.4	19.2	16588	2145335
2005	11	7.5	101	102.9	248.6	25.4	15.8	90.2	22.2	1468.7	1051.8	66.2	15.2	15760	2061493
2005	12	-2.5	101.1	100.7	257.2	25.3	5.6	90.1	15.1	1463.5	1118.8	83.3	4.9	19251	2198938
2006	1	-1.9	101.4	100.1	281.7	33	6.5	105.6	15.8	1825.1	1092.6	115.4	5.7	21112	2188816
2006	2	-0.9	100.5	96.1	238.1	23.8	5.8	107.8	17.3	2494.1	1692.2	138.2	5.6	16132	1934666
2006	3	8	101.1	99.1	260.5	25	11.6	106.8	17.9	1626.2	1228.9	64.2	11.1	17910	2102768
2006	4	13.5	100.9	101.1	260.2	24.1	17	104.1	23.6	1528.3	1086	64.6	16.6	18561	2127254
2006	5	20.4	101.2	97	272.2	26.9	21.3	105.1	25.3	1589.3	1168	68.9	20.8	18536	1955615
2006	6	25.9	101.5	96.9	270.1	23.7	25.9	107.1	27.8	1430.5	1238.3	67	25.6	17371	1888126
2006	7	25.9	100.7	96.3	274.4	23.2	29.8	106.5	29.8	1527	1208.7	79.3	29.4	18604	2185610
2006	8	26.4	100.5	98.1	276.1	22.2	30.4	107	29.4	1537.4	1386.6	78.2	30.2	17855	2358932
2006	9	21.8	100.6	100	278.9	25	24.2	107	27	1564.5	1374.9	74	23.9	16847	1830189
2006	10	16.1	100.6	100.3	286	28	22.3	105.1	26.4	1597.4	1390.8	66	22.1	17724	2194130
2006	11	6.7	100.7	101.3	282.7	28.5	15.7	107.9	21.9	1542.1	1107.7	69.6	15.7	16906	2081772
2006	12	-1	101.1	101	294.3	29.4	8.6	105.7	16	1591.1	1215.1	78	8.2	21446	2403246
2007	1	-1.5	100.5	102.1	315.6	30.6	5.9	102.8	13.9	1880.5	1285.5	111	5.5	20782	2166553
2007	2	3.7	100.8	100.8	286.9	32.3	9.8	101.2	18.6	2733.8	1573.3	147.2	9.4	20730	2286100
2007	3	6.2	101.4	103.3	296.8	27.7	12.1	99.7	18.9	1839.2	1418.4	71.4	12	18780	2163164
2007	4	15.2	100.8	98.1	297.8	26.6	16	99.6	21.2	1693.9	1289.3	65.1	15.6	19132	2176854
2007	5	22.6	100.7	103.8	311.8	30.2	22.9	101.5	26.9	1757.3	1544.5	107.6	22.8	20421	2202343
2007	6	26.2	101	98.3	309.5	26	25	100.3	28.5	1656.9	1411.3	91	24.7	19852	2034354
2007	7	26.9	102.1	102.4	315.2	26.6	30.4	100.6	30.8	1682.5	1475	94.5	29.7	21257	2455224
2007	8	26.6	103.5	102.8	318.8	26.4	29.7	102.1	29	1720.7	1515.2	82.4	29.5	20568	2758337
2007	9	22.4	103.7	103.9	323.4	28.3	25.4	102.9	27.9	1803.9	1600.6	71.1	25.1	19506	2120835
2007	10	13.6	104.4	98	333.2	33.4	20.6	99.3	25.4	1776.6	1642.1	81.5	20.4	20704	2568040
2007	11	5.6	105.3	99	328	33.6	14.1	99.1	19.8	1928.2	1719.5	105.1	13.9	20214	2448135
2007	12	0.5	105	100.8	363.2	34.3	9.6	99.9	17.5	1781.4	1928.1	118	9.2	25053	2789354
2008	1	-4.7	105.3	103.4	390.9	37.3	4.5	99.8	12.8	2569.4	1815.2	155.5	4.2	25634	2518566
2008	2	-2.3	106.6	103.3	327.6	35.4	4.2	99.4	11.6	2268.9	1772.8	144.5	4	22718	2352456
2008	3	4.4	106	104.4	365.5	33.5	11.6	100.8	20.1	2530.8	1901.6	130.7	11.4	22541	2404499
2008	4	13.2	106.5	103.2	366.9	32.1	16.1	100.2	23.2	1888.8	1629.2	100.3	15.9	22680	2419911
2008	5	20.2	106.3	104.1	380.4	35.1	21.8	101.3	25.6	1758.5	1716.6	82.5	21.6	23070	2342146
2008	6	24.2	106.1	104.3	379.3	31.2	24.2	101	27	2018	1801.3	72.4	24	22147	2147926
2008	7	26	106.3	105.1	380.8	30.8	30.4	101.4	29.1	1956.4	1856.4	100.2	30.2	24150	2709375
2008	8	24.6	105.5	104.8	372.1	32.4	28.6	101.4	29.3	1859.5	1515.4	95.8	28.4	22695	2679092
2008	9	19.5	105.2	104.3	392.4	35.2	26	99.8	28.6	2143.5	1796.3	90.7	25.6	20899	2194840
2008	10	12.5	104.2	103.4	406.2	41.7	21	99.2	26.1	1938.8	1672.4	105	20.7	21386	2309628
2008	11	4	102.1	100.8	398.1	43.6	13.3	102.2	19.9	2081.9	1860.6	101.8	13.1	20990	2532782
2008	12	-2.8	101	98.6	428.6	42.3	7.9	101.1	15.6	2014.3	1645.6	84.6	7.5	23194	2579770
2009	1	-3.5	100.7	96.1	348.6	36.5	6.1	100.1	12.4	1988.3	1543.6	88.2	3.8	20861	2435154
2009	2	-1.8	98.9	95.3	366.5	34.5	9.3	103.1	21	1985.1	1523.3	93.3	8.9	23997	2604395
2009	3	4.2	99	94.6	411.1	36.9	12.1	99.4	18.2	1913.9	1600.5	109.9	10.6	22748	2270115
2009	4	11.1	98.6	94	415.8	36.5	17.2	98.3	22.2	2019.2	1797.6	86.2	16.3	23038	2512091
2009	5	18.9	98.3	93.2	436.3	42	22.5	98.8	25.9	2153.8	1797.1	81	21.9	23855	2319674

Table B2. Values of related influencing factors for different cities (Quarterly)

		Beijing					Shanghai		Guangzhou				Hong Kong		
Year	Month	Temp.	CPI	PPI	Total retail sales of social consumer goods	Total retail sales of social chothing goods	Temp	price index for 'clothing and footwear'	Temp	city residents' disposable incomes	per capita consumption expenditure	per capita consumption expenditure of clothing	Temp	Total Retail Amount	Visitor Arrivals
2000	1	0.0	101.8	103.1	407.2	34.9	6.7	93.9	15.9	4002.7	3012.0	203.8	6.7	31993.2	3007979.5
2000	2	20.6	104.7	104.4	418.7	33.0	20.7	94.2	25.7	3166.5	2581.4	123.5	20.7	34208.4	3551073.0
2000	3	25.7	102.8	104.2	425.7	33.9	27.2	94.0	28.1	3373.6	2979.6	123.8	27.2	28167.6	3492269.1
2000	4	5.0	102.9	102.5	449.7	37.6	14.1	94.1	20.1	3423.6	2776.5	135.5	14.1	28786.6	3172493.6
2001	1	0.1	102.1	101.9	510.3	39.7	7.9	97.1	16.6	4012.4	3028.7	181.1	7.9	32982.3	3399044.7
2001	2	21.1	105.5	101.8	505.5	36.5	20.1	98.3	25.1	3311.6	2610.6	122.9	20.1	32512.8	3206114.1
2001	3	24.8	103.7	103.0	519.2	39.2	27.2	97.4	28.4	3423.5	2943.2	113.1	27.2	34017.8	3494530.3
2001	4	5.6	104.2	100.7	511.9	35.4	13.7	98.3	20.0	3678.6	2579.2	139.8	13.7	33941.6	3404942.0
2002	1	4.3	97.7	95.0	574.9	43.3	9.5	95.9	17.5	4227.6	2897.7	192.6	9.5	33811.0	3876337.6
2002	2	19.8	98.7	95.8	550.5	47.1	20.5	96.7	26.7	3157.8	2638.3	119.9	20.5	34383.6	4014854.5
2002	3	24.5	97.0	96.7	543.7	47.1	26.6	96.2	27.7	3067.8	2969.9	121.2	26.6	33788.0	4154299.3
2002	4	3.6	98.2	96.0	549.9	44.5	13.5	96.5	19.6	3320.5	2616.4	143.7	13.5	37440.2	4253034.9
2003	1	1.3	100.1	96.0	590.7	52.4	6.7	96.6	16.7	4376.1	2980.5	251.0	6.7	41225.3	3700235.4
2003	2	20.2	101.0	97.2	574.9	49.8	19.9	96.9	26.2	3405.3	2543.9	134.2	19.9	40392.7	4053570.7
2003	3	24.2	100.8	98.2	595.0	51.0	28.3	97.5	29.0	3598.9	3211.7	132.0	28.3	42867.8	4062912.0
2003	4	5.6	99.4	96.9	636.5	49.4	13.0	97.7	19.7	3622.7	2833.9	176.7	13.0	40574.2	3728474.7
2004	1	2.8	101.2	98.8	667.3	59.7	7.5	93.4	16.0	4960.4	3413.4	279.3	7.5	43216.0	5130670.6
2004	2	20.6	101.1	99.7	643.2	64.4	20.5	93.0	26.2	3876.5	2978.0	189.9	20.5	48040.6	5569647.8
2004	3	24.0	100.0	98.8	660.2	58.9	27.7	94.9	28.6	3908.8	3277.4	175.1	27.7	42875.3	5381095.0
2004	4	6.6	100.0	97.5	649.3	60.6	14.3	94.0	20.5	4129.4	3472.0	210.2	14.3	46135.0	6011880.5
2005	1	0.3	101.8	102.4	691.2	75.7	5.6	92.6	15.1	5258.8	3631.1	291.3	5.6	47031.9	5727552.9
2005	2	20.6	101.7	100.8	716.8	68.7	21.7	92.3	25.9	4387.9	3753.9	213.2	21.7	49910.6	5592769.4
2005	3	25.3	101.3	100.5	735.9	61.5	28.0	92.2	29.0	4352.8	3634.7	189.1	28.0	49565.0	5975603.0
2005	4	6.6	101.1	101.1	758.9	75.6	13.1	92.2	21.0	4412.5	3488.2	209.0	13.1	51599.0	6405766.0
2006	1	1.7	101.0	99.2	780.3	81.8	7.5	105.8	17.0	5945.5	4013.8	317.7	7.5	55154.0	6226250.0
2006	2	19.9	101.2	99.3	802.5	74.7	21.0	105.4	25.6	4548.1	3492.2	200.4	21.0	54468.0	5970995.0
2006	3	24.7	100.6	99.4	829.4	70.4	27.8	106.2	28.7	4628.9	3970.2	231.5	27.8	53306.0	6374731.0
2006	4	7.3	100.8	97.2	863.0	85.9	15.3	106.7	21.4	4730.6	3713.6	213.6	15.3	56076.0	6679148.0
2007	1	2.8	100.9	101.8	899.3	90.6	9.0	100.7	17.1	6453.5	4277.2	329.6	9.0	60292.0	6615817.0
2007	2	21.3	100.8	102.0	919.1	82.8	21.0	101.0	25.5	5108.0	4245.0	263.8	21.0	59405.0	6413551.0
2007	3	25.3	103.1	99.5	957.4	81.3	28.1	101.5	29.2	5207.1	4590.8	248.0	28.1	61331.0	7334396.0
2007	4	6.6	104.9	101.3	1024.4	101.3	14.5	100.7	20.9	5486.2	5289.7	304.6	14.5	65971.0	7805529.0
2008	1	2.2	106.0	103.7	1084.1	106.1	6.5	101.3	14.8	7387.1	4963.8	438.3	6.5	70893.0	7275521.0
2008	2	19.8	106.3	103.9	1126.6	98.3	20.5	101.8	25.3	5859.5	4866.0	249.4	20.5	67897.0	6909983.0
2008	3	24.7	105.7	104.7	1145.4	98.4	28.1	100.3	29.0	6411.2	5249.6	295.6	28.1	67744.0	7583307.0
2008	4	6.6	102.4	100.9	1232.9	127.6	13.8	102.3	20.5	5634.1	5128.3	266.1	13.8	65232.4	7782453.0
2009	1	1.7	99.5	95.4	1126.1	107.9	7.8	100.9	17.2	6067.6	5287.1	242.9	7.8	63279.4	7348929.7

Appendix C

Table C1: Forecasting performances generated by different multivariate forecasting models for seasonal pattern

Data Series No.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
ARX(2, 1)	MAE	842769.2	5701911.2	1045885.5	1573162.1	396279.5	7300569.3	3084663.1	2785138.2	1316181.6	685504.6	1670981.9	10842675.1	1403412.2	1365400.2	1800852.8
	MAPE	1296.02%	5647.25%	183.84%	1453.40%	1600.76%	3198.19%	809.27%	246.20%	89.46%	14.47%	111.84%	416.88%	40.67%	58.30%	43.12%
	RMSE	966502.3	7144103.4	1251000.3	2070554.4	563926.2	11822101.2	3892023.1	3834859.4	1505498.6	961428.0	1904839.9	11966411.1	1470155.4	1831199.8	2243888.6
	MASE	1.37	2.24	1.62	0.86	0.70	1.44	1.31	0.20	0.09	0.17	0.49	0.62	0.36	0.48	0.35
ARX(3, 1)	MAE	566370.8	5092112.9	857361.4	1576865.4	363333.9	6595968.4	2954019.1	2304138.3	346496.8	374090.0	1354733.6	4611599.6	45313.4	1282011.8	578560.5
	MAPE	547.15%	7000.66%	118.61%	1820.31%	1200.66%	3439.70%	779.96%	124.46%	34.59%	208.11%	102.84%	458.64%	1.65%	51.25%	18.09%
	RMSE	648867.2	6472994.8	1140870.0	2092816.2	554359.8	11797806.8	3910556.3	3100922.2	483071.1	498509.1	1606812.9	6165009.8	46258.6	1470355.7	676599.1
	MASE	0.92	2.00	1.33	0.86	0.64	1.30	1.25	0.16	0.02	0.09	0.40	0.26	0.01	0.45	0.11
ARX(2, 2)	MAE	536891.0	3187187.6	812244.2	2079685.8	381517.5	2854634.5	2680319.3	1873811.2	721160.5	621982.2	723429.9	5493513.9	354476.4	1392282.8	1173803.0
	MAPE	327.50%	1040.99%	94.13%	4355.88%	1640.56%	1158.61%	484.72%	148.23%	46.43%	319.71%	36.16%	216.10%	6.76%	30.31%	28.90%
	RMSE	664255.8	4611255.3	1098051.7	2356594.3	502364.5	3897183.1	3209211.0	3128377.8	803043.7	756731.6	883950.3	6346232.2	546879.3	1588481.5	1869479.3
	MASE	0.87	1.25	1.26	1.13	0.67	0.56	1.14	0.13	0.05	0.15	0.21	0.31	0.09	0.49	0.23
ARX(3, 2)	MAE	402782.5	2981071.9	739101.6	1875692.3	350148.2	2849727.2	2712314.7	1845421.5	321644.4	510305.3	390401.1	1927730.4	1282.7	706268.9	1203172.7
	MAPE	222.94%	732.86%	80.88%	3294.42%	1317.12%	2383.96%	494.92%	27.08%	34.42%	337.35%	34.49%	226.38%	0.03%	29.52%	30.50%
	RMSE	514806.6	4346383.0	984794.9	2227021.9	482214.3	3805373.7	3154331.5	2891593.6	449363.6	670758.9	517722.4	2322904.9	1634.3	873132.1	1669689.6
	MASE	0.65	1.17	1.15	1.02	0.62	0.56	1.15	0.13	0.02	0.12	0.11	0.11	0.00	0.25	0.23
ARMAX(2, 2, 1)	MAE	573038.8	5759598.5	779280.1	1375473.3	319435.2	5959906.7	1722319.4	2769977.6	2081963.7	619561.8	1718109.0	7622584.9	1175084.9	2230178.3	2431788.6
	MAPE	267.25%	7531.41%	196.74%	431.22%	737.64%	6128.64%	153.84%	234.95%	133.76%	9.61%	119.94%	1135.36%	39.51%	76.55%	35.60%
	RMSE	689063.2	7108964.4	1040068.2	1883369.0	432370.8	8984425.0	2201842.6	3537526.6	2423352.9	882175.8	1982572.6	9797932.5	1216793.3	2340161.9	3266785.4
	MASE	0.93	2.26	1.21	0.75	0.56	1.18	0.73	0.19	0.14	0.15	0.50	0.44	0.30	0.79	0.47
ARMAX(2, 2, 2)	MAE	467375.7	2719810.6	664511.4	1532794.6	372707.3	2511884.0	2013894.8	2688396.7	3548982.6	549422.3	1676488.2	3366263.0	1033428.2	1701455.2	1233521.7
	MAPE	343.02%	1452.30%	48.92%	1356.80%	1392.68%	1502.83%	439.78%	220.35%	255.22%	331.21%	62.38%	309.54%	30.60%	44.71%	39.61%
	RMSE	573154.1	3972753.7	1035719.8	2101450.7	491012.7	3500083.0	2495916.2	3256966.4	3939031.4	742705.7	2073045.3	3860816.8	1195184.6	1837033.3	1848312.8
	MASE	0.76	1.07	1.03	0.83	0.66	0.50	0.85	0.19	0.24	0.13	0.49	0.19	0.27	0.60	0.24
ARMAX(3, 3, 1)	MAE	520559.6	3832187.4	641251.7	1367725.6	285269.5	8155489.4	1818635.8	3165620.3	927630.4	409055.4	1059110.6	2764186.5	48580.4	1438394.1	498525.0
	MAPE	195.13%	12509.10%	115.83%	721.15%	841.58%	7651.41%	122.81%	183.86%	66.61%	217.95%	81.12%	187.13%	1.56%	52.80%	10.15%
	RMSE	635469.3	4959378.6	869399.5	1917646.1	452391.4	12160257.5	2297709.8	4164838.9	1073949.5	548989.0	1246870.4	3107357.1	48698.8	1543137.2	585045.3
	MASE	0.85	1.50	0.99	0.74	0.50	1.61	0.77	0.22	0.06	0.10	0.31	0.16	0.01	0.51	0.10
ARMAX(3, 3, 2)	MAE	353504.8	2705544.0	640954.6	1355744.2	338913.4	1905232.9	1765771.7	2475580.4	694707.2	881480.0	899148.1	911981.3	84417.6	834546.2	1172406.9
	MAPE	498.01%	5228.41%	46.18%	870.74%	776.49%	2095.75%	177.10%	224.75%	53.02%	334.00%	63.47%	129.72%	1.90%	30.13%	25.07%
	RMSE	466934.0	3700416.4	1002069.0	1892427.7	462907.4	3349462.1	2195593.9	3260015.8	851248.8	1034065.9	1051399.1	1138389.7	105752.0	901146.8	1641664.2
	MASE	0.57	1.06	0.99	0.74	0.60	0.38	0.75	0.17	0.05	0.21	0.26	0.05	0.02	0.29	0.23
GLM(2)	MAE	597063.3	2763792.9	807776.7	1384582.6	519616.3	2716864.4	2153193.9	3874444.3	2893127.1	1202066.4	3023492.9	4617501.2	943031.8	1450489.9	1616911.6
	MAPE	484.84%	5724.60%	172.78%	552.81%	1334.32%	3457.35%	236.28%	149.82%	74.80%	352.86%	176.40%	152.37%	38.42%	53.34%	23.31%
	RMSE	695350.7	3258210.7	972602.8	1988926.0	618423.5	3431220.1	3009477.8	4343076.8	3145060.8	1389200.4	3292594.1	6047814.5	1007487.1	1684188.7	2379954.6
	MASE	0.97	1.09	1.25	0.75	0.91	0.54	0.91	0.27	0.20	0.29	0.89	0.26	0.24	0.51	0.31
GLM(3)	MAE	540881.1	1972996.6	761634.3	1402100.4	516728.6	2351724.2	2338084.9	3837587.2	3350103.1	1161551.9	3212517.3	5222378.4	562571.1	1496032.6	1475074.5
	MAPE	340.24%	3518.05%	148.97%	140.90%	1323.59%	2902.85%	228.24%	74.10%	87.06%	318.52%	176.05%	313.06%	23.93%	56.64%	20.73%
	RMSE	637262.7	2528156.7	866189.4	2010027.4	616857.2	2687548.2	3131906.9	4768009.9	3582625.6	1319067.8	3461994.4	6891115.5	711839.1	1741032.9	2099879.2
	MASE	0.88	0.77	1.18	0.76	0.91	0.46	0.99	0.27	0.23	0.28	0.94	0.30	0.14	0.53	0.28
NN(2)	MAE	940502	1925651	1024323	1273042	210	2545263.029	45585	9953631	9934095	8860467	2552181	3201822	3405281	951272	1009433
	MAPE	250.23%	478.67%	168.37%	949.84%	0.07%	1308.21%	10.31%	105.72%	70.99%	6408.18%	57.42%	43.62%	89.50%	23.20%	13.39%
	RMSE	1084335	2768299	1534157	1553977	477	4262404.55	70686	13125478	14771382	11507386	3441571	4873562	4833282	950980	2041034
	MASE	1.57	0.87	1.66	0.55	0.00	0.50	0.02	0.90	0.85	2.11	0.91	0.23	0.92	0.33	0.24
NN(3)	MAE	954520	1543234	691769	1768354	23	2337784.705	674	6879128	6873903	2141576	3036774	12139253	7576146	1377278	1361468
	MAPE	704.84%	172.05%	53.14%	404.29%	0.02%	962.85%	0.09%	159.40%	54.66%	1515.50%	117.27%	83.13%	294.91%	33.58%	32.01%
	RMSE	1380223	2642333	928446	2845453	31	5133561.866	892	10075483	11044453	3390340	3843432	16117204	8136900	1474143	1949132
	MASE	1.69	0.65	1.06	0.79	0.00	0.41	0.00	0.59	0.59	0.63	1.03	0.86	2.01	0.49	0.32

Table C2: Forecasting performances generated by different multivariate forecasting models for irregular pattern

Data Series No.		1	2	3	4	5	6	7	8	9	10	11	12	13	14
ARX(2, 1)	MAE	12476259.5	19684485.3	5296732.1	15084893.9	917837.7	601959.8	272599.7	354864.7	576196.0	1451320.6	436282.8	800778.2	587623.8	312682.9
	MAPE	67.77%	25.96%	16.52%	62.16%	79.15%	175.24%	4571.88%	1382.40%	1180.00%	152.33%	314.26%	274.39%	2044.16%	4563.73%
	RMSE	13809527.4	24257917.6	7727734.5	16992536.7	1328546.5	883415.2	339843.9	469945.6	734022.5	1784237.3	509560.8	973327.9	670792.5	390150.0
	MASE	2.22	0.91	0.70	2.29	1.17	1.17	22.34	1.56	1.08	1.36	0.61	1.61	25.91	1.24
ARX(3, 1)	MAE	9720462.4	19730421.4	5186054.9	10595471.1	867018.6	630060.4	257652.0	280146.2	450863.7	1208665.1	410835.4	905045.3	531534.1	260538.3
	MAPE	52.35%	26.52%	16.73%	43.75%	75.42%	128.83%	4675.20%	523.59%	609.05%	104.26%	83.46%	184.11%	1859.74%	1016.72%
	RMSE	11218805.7	23812403.9	7552655.5	12745531.5	1263465.7	749861.9	326338.7	328352.2	655447.3	1524543.1	578407.4	993497.2	629795.6	294437.5
	MASE	1.73	0.91	0.69	1.61	1.11	1.23	21.12	1.23	0.85	1.13	0.57	1.82	23.44	1.04
ARX(2, 2)	MAE	6850038.4	12516258.2	5976237.0	9987140.6	778355.6	491233.6	303216.9	363721.1	595410.0	1152872.6	620554.5	584746.7	573088.6	359766.9
	MAPE	35.44%	16.48%	20.25%	40.70%	55.85%	102.45%	6993.03%	1580.89%	1344.85%	82.99%	900.30%	90.77%	2142.49%	3989.88%
	RMSE	9207492.0	19468475.5	8654907.2	11478757.6	915427.0	595766.3	342647.4	481188.6	763532.8	1695162.1	719890.8	768710.7	676198.9	486091.5
	MASE	1.22	0.58	0.79	1.52	0.99	0.96	24.85	1.60	1.12	1.08	0.87	1.17	25.27	1.43
ARX(3, 2)	MAE	4911461.1	13815998.5	5597725.7	6999069.7	665069.5	412010.3	266591.9	305250.7	474021.9	1035345.2	553317.6	603986.5	523971.2	300996.3
	MAPE	24.56%	18.65%	18.81%	27.87%	51.39%	96.95%	6766.66%	779.67%	675.44%	70.39%	458.87%	109.36%	1956.41%	569.53%
	RMSE	7680555.0	19964823.8	8304088.7	8575962.5	839107.8	540539.4	324171.1	409404.8	664117.5	1622520.7	697592.7	794290.3	645224.7	419528.1
	MASE	0.87	0.64	0.74	1.06	0.85	0.80	21.85	1.34	0.89	0.97	0.77	1.21	23.10	1.20
ARMAX(2, 2, 1)	MAE	8857347.1	17615773.4	5771117.5	4603700.2	908892.3	599562.3	240536.2	211541.9	479324.4	1384049.7	407016.5	731089.8	650063.0	185993.8
	MAPE	45.73%	22.23%	19.79%	16.74%	65.63%	115.50%	7092.95%	579.39%	780.80%	142.17%	252.22%	254.42%	2004.67%	1547.62%
	RMSE	10254680.8	23040696.5	7763855.8	6880094.2	1178065.9	697311.4	294181.6	266513.6	668829.3	1778907.0	487471.3	907154.5	699726.8	204091.4
	MASE	1.58	0.81	0.77	0.70	1.16	1.17	19.72	0.93	0.90	1.30	0.57	1.47	28.66	0.74
ARMAX(2, 2, 2)	MAE	9136597.3	14766316.3	5509910.0	4323764.3	666159.5	456413.9	260856.0	331452.7	426041.6	899558.3	546859.9	793995.4	652505.5	325557.0
	MAPE	44.93%	20.28%	18.67%	15.38%	43.40%	85.72%	8033.09%	1331.86%	494.94%	58.02%	146.90%	92.42%	2126.76%	2340.90%
	RMSE	12262196.4	20067878.3	8321565.5	7336381.8	782685.5	504902.0	339036.5	433114.3	573140.0	1559711.9	708329.3	911505.6	701831.8	419291.1
	MASE	1.62	0.68	0.73	0.66	0.85	0.89	21.38	1.46	0.80	0.84	0.76	1.60	28.77	1.29
ARMAX(3, 3, 1)	MAE	5463753.0	17047373.3	5975983.0	6721519.0	563762.7	478116.8	357498.5	171858.9	538808.1	1431594.0	469227.5	747465.1	18290.3	174069.3
	MAPE	26.57%	22.07%	19.80%	25.59%	33.00%	89.88%	8852.95%	313.89%	1361.74%	136.36%	134.49%	71.55%	66.24%	1785.87%
	RMSE	6664734.8	23107252.3	7275876.6	8458023.6	646640.7	565543.6	402976.4	223579.7	693186.9	1876714.7	630985.2	914839.3	23367.4	202885.3
	MASE	0.97	0.79	0.79	1.02	0.72	0.93	29.30	0.76	1.01	1.34	0.65	1.50	0.81	0.69
ARMAX(3, 3, 2)	MAE	6247420.6	13524705.1	5548708.8	3981857.4	522717.7	354083.7	338665.9	153928.7	501628.1	869203.1	468784.1	738600.8	622974.0	270445.7
	MAPE	30.84%	18.61%	18.59%	14.57%	37.52%	81.32%	10820.17%	270.66%	909.34%	54.96%	95.95%	75.53%	1932.66%	1594.83%
	RMSE	8327892.3	19447675.7	8387974.0	7047995.3	703414.2	495707.0	416048.2	205621.3	660867.5	1530792.5	666546.7	896293.5	680018.8	373550.1
	MASE	1.11	0.62	0.74	0.60	0.67	0.69	27.76	0.68	0.94	0.82	0.65	1.48	27.47	1.07
GLM(2)	MAE	4150704.7	17587709.3	7095852.5	6448977.2	877015.4	613148.6	241141.5	322241.6	590241.0	1368724.3	563132.4	566016.8	591399.5	286039.6
	MAPE	22.73%	21.25%	23.46%	27.25%	53.80%	110.13%	5465.53%	1227.04%	1204.52%	153.80%	223.20%	103.55%	2178.73%	3313.02%
	RMSE	5999989.8	24374153.8	9091421.0	8470263.2	1054997.6	677830.7	256943.7	361035.4	678103.2	1773999.2	808369.6	691431.6	593447.6	325575.1
	MASE	0.74	0.81	0.94	0.98	1.12	1.19	19.77	1.42	1.11	1.28	0.79	1.14	26.08	1.14
GLM(3)	MAE	4121687.0	18200794.7	6690424.5	6311246.8	874667.0	546194.6	260738.3	332989.0	600746.9	1276195.3	569997.0	488378.6	715235.7	277231.9
	MAPE	22.83%	22.43%	22.25%	26.53%	53.06%	102.93%	6190.22%	1335.34%	1264.19%	135.46%	206.32%	97.11%	2630.27%	3601.81%
	RMSE	5902256.8	24768628.0	8698351.1	8294256.1	1058500.5	629532.3	273868.0	378413.2	688134.6	1698780.4	735092.7	644152.4	717272.8	319089.8
	MASE	0.73	0.84	0.89	0.96	1.12	1.06	21.37	1.47	1.13	1.20	0.80	0.98	31.54	1.10
NN(2)	MAE	5456484	18361008	9306251	6441028	1432374	648837.4049	124361	189019	395632	1544072	572874	557993	185639	214922
	MAPE	27.46%	22.63%	34.93%	19.24%	63.93%	63.52%	1083.97%	156.72%	111.04%	126.97%	215.56%	76.23%	674.99%	1794.49%
	RMSE	7399700	24399496	12737357	7999262	1922391	813366.2618	193371	294647	587094	2250241	859567	677810	207982	323666
	MASE	1.04	1.43	2.49	0.98	2.70	1.59	0.34	0.66	1.45	2.53	0.88	0.71	0.26	0.46
NN(3)	MAE	6098427	38674324	12051580	7351450	1170872	689232.5602	129841	543911	1182514	935887	627435	629761	54271	225234
	MAPE	37.74%	52.82%	45.15%	27.08%	49.90%	68.53%	826.19%	164.33%	771.00%	77.12%	201.84%	88.43%	216.92%	1088.57%
	RMSE	8671733	69276484	14137409	8298580	1608680	895554.0207	193503	1126263	1818738	1205077	881242	854363	66539	292507
	MASE	1.31	2.69	2.91	1.11	2.91	1.62	0.40	1.89	4.69	1.29	0.82	0.84	0.07	0.40

Table C2: Forecasting performances generated by different multivariate forecasting models for irregular pattern (cont'd)

Data Series No.		15	16	17	18	19	20	21	22	23	24	25	26	27
ARX(2, 1)	MAE	630198.0	4838218.2	1308472.7	761487.0	3787856.0	2043683.4	1305919.6	1132913.2	1304943.5	884915.2	1769077.8	2860825.6	1032452.6
	MAPE	1816.56%	8147.11%	235.61%	195.00%	67.30%	168.10%	44.55%	31.72%	1502.00%	607.83%	1925.47%	89.05%	1279.05%
	RMSE	803395.6	7178044.8	1771443.4	946753.3	4416168.5	2562881.9	1640220.4	1521654.9	1521015.7	1178523.2	2146270.5	4447898.0	1277573.9
	MASE	0.79	1.26	1.71	1.12	0.86	1.59	0.95	1.03	18.22	1.18	1.13	1.27	1.52
ARX(3, 1)	MAE	555699.0	4431713.0	1312544.5	741742.1	2042789.5	1526336.5	1364238.8	996144.9	1288717.8	844221.0	1529406.8	2803111.4	1146149.0
	MAPE	1249.34%	8709.46%	262.95%	175.58%	34.34%	111.74%	40.62%	34.87%	1590.10%	590.39%	1278.86%	85.59%	1094.85%
	RMSE	701255.7	6991666.4	1753694.3	938781.1	2733935.3	1896536.9	1755830.9	1221871.8	1528067.9	1181173.0	1902918.7	4383809.1	1323297.7
	MASE	0.70	1.15	1.71	1.10	0.47	1.19	0.99	0.91	17.99	1.12	0.98	1.24	1.68
ARX(2, 2)	MAE	601924.1	2831444.0	694730.6	549845.5	2551529.1	1883321.9	1246283.2	1005808.3	780884.5	909802.6	1780385.0	2265566.9	790792.8
	MAPE	1182.09%	12387.73%	94.92%	63.98%	100.92%	241.69%	29.49%	31.82%	1074.69%	1151.84%	2834.11%	170.84%	1792.12%
	RMSE	739317.9	4228584.0	970306.9	730812.4	3010152.2	2333197.5	1568557.4	1226982.1	1004906.3	1006182.0	2066919.6	2779888.9	957351.8
	MASE	0.75	0.74	0.91	0.81	0.58	1.46	0.91	0.91	10.90	1.21	1.14	1.00	1.16
ARX(3, 2)	MAE	528869.0	2744295.5	765497.4	586270.6	1826334.7	1438559.9	1137673.0	716544.3	737646.8	716547.4	1391414.3	1805429.6	711486.2
	MAPE	1180.62%	15040.60%	158.06%	84.62%	92.00%	252.09%	29.69%	24.41%	1119.50%	983.70%	1427.77%	135.55%	1817.26%
	RMSE	657348.5	4145601.3	989134.1	737867.9	2241006.4	1736541.7	1473162.9	1041336.3	1005931.8	890555.2	1618946.9	2400421.2	801924.7
	MASE	0.66	0.71	1.00	0.87	0.42	1.12	0.83	0.65	10.30	0.95	0.89	0.80	1.04
ARMAX(2, 2, 1)	MAE	587431.2	4277159.6	750885.2	525240.8	2221295.5	1591151.5	1064898.4	1140048.5	1769132.5	784581.6	1793527.1	2329398.2	882199.1
	MAPE	1190.31%	5672.09%	155.10%	119.49%	30.38%	239.16%	42.84%	33.80%	1999.24%	821.20%	2416.98%	158.15%	1007.77%
	RMSE	705557.1	6834859.9	967887.8	600497.6	2912480.5	1932367.8	1238357.3	1526499.2	1946898.0	915636.4	2219673.9	2894456.6	1132843.9
	MASE	0.73	1.11	0.98	0.78	0.51	1.24	0.78	1.04	24.70	1.04	1.15	1.03	1.30
ARMAX(2, 2, 2)	MAE	494685.7	2737537.5	691219.4	522943.6	2969984.1	2298168.5	1116119.2	796571.1	645133.8	858398.1	1294540.4	2343873.0	792169.2
	MAPE	891.89%	13711.93%	94.12%	61.63%	50.57%	255.86%	31.13%	24.47%	730.86%	1039.68%	1046.55%	160.45%	788.87%
	RMSE	602381.5	4005612.4	943730.8	699688.5	3474284.7	2894159.4	1484817.4	1009419.4	831916.2	932601.7	1594106.3	2760045.2	1041793.0
	MASE	0.62	0.71	0.90	0.77	0.68	1.79	0.81	0.72	9.01	1.14	0.83	1.04	1.16
ARMAX(3, 3, 1)	MAE	502123.9	3958099.3	725018.4	461518.8	2089776.9	1246083.4	1024191.6	1152462.7	220407.8	581433.6	1437626.0	1633004.1	1038653.2
	MAPE	931.48%	11812.88%	186.56%	97.93%	21.75%	36.59%	39.39%	31.15%	275.69%	328.94%	976.13%	87.16%	1310.63%
	RMSE	591127.8	5985627.1	922193.0	553631.8	2777812.2	1735125.2	1238585.0	1481589.4	247685.7	816631.9	1904817.4	2331437.0	1232299.0
	MASE	0.63	1.03	0.95	0.68	0.48	0.97	0.75	1.05	3.08	0.77	0.92	0.72	1.52
ARMAX(3, 3, 2)	MAE	488172.9	2389601.4	695935.7	499837.8	2874153.2	1030640.2	1185444.8	1399525.3	726981.6	557657.9	856667.1	1753397.0	934666.0
	MAPE	813.78%	3486.53%	135.40%	77.58%	49.83%	156.29%	28.35%	61.22%	460.73%	608.13%	732.73%	96.83%	1232.47%
	RMSE	602655.2	3638530.8	892412.4	673312.2	3522173.8	1366634.2	1696985.2	1710235.2	1103096.7	752695.8	1162608.1	2275838.9	1138658.5
	MASE	0.61	0.62	0.91	0.74	0.66	0.80	0.86	1.27	10.15	0.74	0.55	0.78	1.37
GLM(2)	MAE	470130.8	3345722.1	948148.6	777314.1	2817168.3	1298763.2	1503207.8	1448783.1	1032999.3	700199.3	1562713.9	2413217.0	556804.7
	MAPE	1817.33%	17458.97%	213.95%	226.08%	30.65%	97.60%	42.05%	46.38%	1188.34%	420.46%	2214.68%	170.53%	866.23%
	RMSE	593146.9	4555647.7	1222385.2	977948.9	3678500.8	1684945.6	1833631.6	1690299.3	1044102.1	814763.5	1834804.2	3099028.3	626106.9
	MASE	0.59	0.87	1.24	1.15	0.64	1.01	1.10	1.32	14.42	0.93	1.00	1.07	0.82
GLM(3)	MAE	483397.6	2391460.3	912819.2	758119.0	2271863.9	1555518.5	1654157.6	1307417.7	1022718.9	661983.7	1509858.2	2097055.9	571483.7
	MAPE	1611.77%	17258.99%	183.60%	226.92%	52.70%	249.33%	38.16%	38.08%	1193.07%	484.25%	2106.22%	136.18%	794.87%
	RMSE	613071.8	3646012.0	1174048.7	937405.1	2967566.0	1933977.5	2013004.6	1651772.1	1044110.9	781512.6	1788779.4	2794906.6	641269.3
	MASE	0.60	0.62	1.19	1.12	0.52	1.21	1.21	1.19	14.28	0.88	0.97	0.93	0.84
NN(2)	MAE	351219	3886974	1945735	533778	2478859	1623565	2445579	1037521	412885	570492	1610396	2178162	684642
	MAPE	562.33%	1182.82%	264.98%	123.18%	40.38%	93.51%	77.83%	47.28%	423.42%	369.68%	816.80%	172.45%	1575.10%
	RMSE	559297	8232148	3450384	739667	3485849	1836392	2675815	1399477	675494	1009155	2315955	4670191	812220
	MASE	0.54	1.08	2.53	0.69	0.58	1.20	1.63	0.63	0.36	0.73	1.70	1.16	0.80
NN(3)	MAE	1130596	4294673	1941336	693119	4358468	1925823	3611257	1812059	134279	432288	1744179	1561859	1339944
	MAPE	614.04%	2046.88%	248.88%	117.38%	314.26%	186.68%	160.19%	68.52%	187.58%	223.61%	649.45%	84.77%	1679.08%
	RMSE	2462308	11026137	3401867	1355800	6961963	2361925	4677039	2792259	177643	754089	2869238	1968768	2089656
	MASE	1.69	1.24	2.62	0.89	1.06	1.56	2.81	1.19	0.13	0.60	1.94	0.74	1.33

Table C3: Forecasting performances generated by different multivariate forecasting models for random pattern

Data Series No.		1	2	3	4	5	6	7
ARX(2, 1)	MAE	252399.0	419962.4	357429.3	8057902.8	264070.0	485381.4	875682.5
	MAPE	143.27%	415.50%	62.53%	8.77%	231.67%	192.21%	155.18%
	RMSE	299089.7	508746.1	431139.2	10518142.0	298703.9	551524.4	1209530.6
	MASE	3.77	1.44	1.46	0.50	5.36	2.57	3.16
ARX(3, 1)	MAE	248487.9	413038.9	297176.4	7424995.5	167015.8	92190.9	17916.7
	MAPE	133.92%	442.90%	49.18%	8.11%	174.90%	29.97%	2.66%
	RMSE	290299.1	481646.3	402753.5	10163283.2	191230.2	123535.6	18861.1
	MASE	3.71	1.42	1.21	0.46	3.39	0.49	0.06
ARX(2, 2)	MAE	210828.0	457944.6	256323.8	7685841.9	187391.2	119056.8	925897.9
	MAPE	121.75%	475.68%	39.76%	8.08%	231.77%	39.65%	168.90%
	RMSE	259631.5	545052.9	323182.3	10045385.6	227411.2	151207.8	1356297.3
	MASE	3.15	1.57	1.05	0.48	3.80	0.63	3.34
ARX(3, 2)	MAE	189634.7	437609.3	234179.8	6968851.6	253171.7	657570.4	174198.5
	MAPE	109.44%	492.16%	34.89%	7.62%	323.05%	306.04%	26.20%
	RMSE	232181.8	510589.7	309300.9	8635220.0	336429.1	882104.3	180595.1
	MASE	2.83	1.50	0.96	0.44	5.14	3.48	0.63
ARMAX(2, 2, 1)	MAE	354997.3	405840.9	399494.0	9333839.2	95002.0	798493.6	809546.1
	MAPE	204.12%	444.15%	71.33%	10.71%	81.26%	296.52%	146.54%
	RMSE	398867.7	501555.0	469816.8	11627905.7	101893.9	966600.0	1055675.7
	MASE	5.30	1.40	1.63	0.58	1.93	4.22	2.92
ARMAX(2, 2, 2)	MAE	245109.0	447255.9	295354.8	8223759.2	189252.8	716109.1	940695.3
	MAPE	142.63%	452.15%	42.21%	8.77%	159.88%	282.53%	177.02%
	RMSE	272516.1	508579.6	428220.6	10722495.3	224542.6	765409.0	1419356.6
	MASE	3.66	1.54	1.21	0.51	3.84	3.79	3.40
ARMAX(3, 3, 1)	MAE	363149.7	360189.8	352276.4	6439220.4	199399.0	1124393.8	24962.5
	MAPE	217.28%	352.00%	63.32%	7.09%	199.14%	455.02%	3.63%
	RMSE	445524.0	483923.2	445186.3	8585030.4	231850.3	1352204.9	26928.2
	MASE	5.42	1.24	1.44	0.40	4.05	5.95	0.09
ARMAX(3, 3, 2)	MAE	142395.8	299797.1	195892.4	6967585.6	310880.2	387903.0	182923.0
	MAPE	83.63%	321.46%	28.98%	7.56%	369.73%	193.03%	27.38%
	RMSE	183072.0	390746.2	288520.1	9321902.1	359646.6	566172.7	190909.8
	MASE	2.13	1.03	0.80	0.44	6.31	2.05	0.66
GLM(2)	MAE	121212.3	301855.4	281937.4	13250477.8	585329.5	2056017.4	1463079.4
	MAPE	76.91%	269.54%	47.55%	15.39%	544.85%	869.02%	246.74%
	RMSE	144473.4	344830.8	456260.8	13912106.7	768632.4	2647216.7	1559621.8
	MASE	1.81	1.04	1.15	0.83	11.88	10.88	5.28
GLM(3)	MAE	144547.3	290882.4	305311.6	10300461.3	643647.7	3005049.1	1182927.7
	MAPE	90.33%	254.08%	52.89%	11.67%	613.23%	1194.39%	202.63%
	RMSE	167662.3	338672.2	457420.1	10586814.1	828844.9	3509133.3	1308274.7
	MASE	2.16	1.00	1.25	0.64	13.07	15.90	4.27
NN(2)	MAE	68235	281602	286497	28849434	182673	412188.1825	798190
	MAPE	40.56%	171.65%	40.39%	32.75%	183.99%	125.00%	148.35%
	RMSE	82505	318022	382214	43476948	224443	466155.3611	1100440
	MASE	0.31	0.64	0.83	3.98	0.19	0.17	1.09
NN(3)	MAE	148851	406065	377645	46949754	303435	364114.0662	525861
	MAPE	71.99%	173.95%	59.58%	48.83%	189.73%	90.48%	90.24%
	RMSE	145185	730942	621212	45933377	582651	437072.0759	985374
	MASE	0.74	1.10	1.28	5.50	0.31	0.15	0.73

Appendix D

```
function [trnErr, Fcst]=ES_double(series,L0,L1,L2,T)
% % %    Brown's double exponential smoothing or called Broun's linear ES
% % % series, time series for model estimation
% % % L0, lower limit of alpha values
% % % L1, step of alpha values
% % % L2, upper limit of alpha values
% % % T, T-step-ahead prediction
% % % trnRMSE,      RMSE of errors of traning outputs
% % % Forecast, 1-step-ahead forecast

obsQty=length(series);
s1=zeros(round((L2-L0)/L1),obsQty);    %%% st'
s2=zeros(round((L2-L0)/L1),obsQty);    %%% st''
a2=zeros(round((L2-L0)/L1),obsQty);    %%% at
b2=zeros(round((L2-L0)/L1),obsQty);    %%% bt
F=zeros(round((L2-L0)/L1),obsQty+T); %%% forecast F(t+T)
e2=zeros(round((L2-L0)/L1),obsQty);
MAD2=zeros(1,round((L2-L0)/L1));k=0;
for alpha=L0:L1:L2
    k=k+1;      %%% indicates the order/sequence of alpha
    s1(k,1)=series(1);
    s2(k,1)=series(1);
    for t=2:obsQty
        s1(k,t)=alpha*series(t)+(1-alpha)*s1(k,t-1);
        s2(k,t)=alpha*s1(k,t)+(1-alpha)*s2(k,t-1);
        a2(k,t)=2*s1(k,t)-s2(k,t);
        b2(k,t)=(s1(k,t)-s2(k,t))*(alpha/(1-alpha));
        F(k,t+T)=a2(k,t)+b2(k,t)*T;
        if   t+T<=obsQty
            e2(k,t+T)=series(t+T)-F(k,t+T);
        end
    end

    MAD2(k)=mean(abs(e2(k,:)));
end
[MAD2,k]=min(MAD2);
alpha=L0+L1*(k-1);
Forecast=abs(F(k,obsQty+T));

ind=find(F(k,:)~=0);
startPoint=ind(1);
NNOut_tr = abs(F(k,startPoint:obsQty));
ActualOut_tr=series(startPoint:obsQty);
e=ActualOut_tr-NNOut_tr;
trnRMSE=sqrt(sum(sum(e.^2))/length(e));
```

```
function [trnErr, Fcst]=ES_triple(series,L0,L1,L2,T)
% % % series, time series for model estimation
% % % L0, lower limit of alpha values
% % % L1, step of alpha values
% % % L2, upper limit of alpha values
% % % T, T-step-ahead prediction
% % % trnRMSE,      RMSE of errors of traning outputs
% % % Forecast, 1-step-ahead forecast

obsQty=length(series);
s1=zeros(round((L2-L0)/L1),obsQty);   %%% st'
s2=zeros(round((L2-L0)/L1),obsQty);   %%% st''
s3=zeros(round((L2-L0)/L1),obsQty);   %%% st'''
a3=zeros(round((L2-L0)/L1),obsQty);   %%% at
b3=zeros(round((L2-L0)/L1),obsQty);   %%% bt
c3=zeros(round((L2-L0)/L1),obsQty);   %%% ct
F=zeros(round((L2-L0)/L1),obsQty+T);      %%% forecast F(t+T)
e3=zeros(round((L2-L0)/L1),obsQty);
MAD3=zeros(1,round((L2-L0)/L1));k=0;
for alpha=L0:L1:L2
      k=k+1;          %%% indicates the order/sequence of alpha
      s1(k,1)=series(1);
      s2(k,1)=series(1);
      s3(k,1)=series(1);
      for i=2:obsQty
              s1(k,i)=alpha*series(i)+(1-alpha)*s1(k,i-1);
              s2(k,i)=alpha*s1(k,i)+(1-alpha)*s2(k,i-1);
              s3(k,i)=alpha*s2(k,i)+(1-alpha)*s3(k,i-1);
              a3(k,i)=3*s1(k,i)-3*s2(k,i)+s3(k,i);
              b3(k,i)=(alpha/(2*(1-alpha)^2))*((6-5*alpha)*s1(k,i)-(10-8*alpha)*s2(k,i)+ …
                (4-3*alpha)*s3(k,i));
              c3(k,i)=(alpha^2/(2*(1-alpha)^2))*(s1(k,i)-2*s2(k,i)+s3(k,i));
              F(k,i+T)=a3(k,i)+b3(k,i)*T+c3(k,i)*T^2;
              if    i+T<=obsQty
                    e3(k,i+T)=series(i+T)-F(k,i+T);
              end
      end
      MAD3(k)=mean(abs(e3(k,:)));
end

[MAD3,k]=min(MAD3);
a=L0+L1*(k-1);
Forecast=abs(F(k,obsQty+T));

ind=find(F(k,:)~=0);
startPoint=ind(1);
NNOut_tr = abs(F(k,startPoint:obsQty));
ActualOut_tr=series(startPoint:obsQty);
e=ActualOut_tr-NNOut_tr;
trnRMSE=sqrt(sum(sum(e.^2))/length(e));
```

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