SELECTED TOPICS IN CAPACITY AND SUSTAINABILITY INVESTMENTS FROM THE PERSPECTIVES OF PRICING AND CARBON EMISSION

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Selected Topics in Capacity and Sustainability Investments from the Perspectives of Pricing and Carbon Emission

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CERTIFICATE OF ORIGINALITY

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____________________(Signed)

Ciwei DONG (Name of student)

Abstract

As a result of global climate change and oil price volatility, energy efficiency and environmental sustainability have become top policy priorities around the world. Meanwhile, the global energy demand may grow by more than one-third after about twenty years, with the expectation that the global electricity demand will continue to grow more strongly than any other final form of energy. So it is crucial to find ways to achieve efficient capacity investments for electricity and efficient environmental sustainability investments for products. On the other hand, an efficient pricing mechanism of electricity can facilitate efficient capacity investment for electricity, and reducing carbon emission is essential to achieve environmental sustainability. Therefore, in this thesis we study capacity and sustainability investments from the perspectives of pricing and carbon emission. Three fundamental topics, which can be considered as foundations for future research, are studied in this thesis.

In the first topic, we consider the determination of the optimal capacity and pricing policies for an electricity company. The time of electricity usage consists of two periods, namely the non-peak period and the peak period. Two technologies are considered to generate electricity, where the first technology is used to generate electricity for the demands in both periods and the second technology is used to generate electricity only for the demand in the peak period. The company offers customers two tariffs, namely the flat rate (FR) tariff and time-of-use (TOU) tariff. Under the FR tariff, customers pay a flat price for electricity consumption in both periods. Under the TOU tariff, customers pay a high price and a low price for electricity consumption in the peak period and non-peak period, respectively. We first study a model with price inelasticity of total demand. Customers

who use the TOU tariff may shift some electricity consumption from the peak period to the non-peak period to take advantage of the lower price in the latter period. We apply a Stackelberg game to study this model, where the electricity company, acting as the Stackelberg leader, decides the capacity investment and prices of electricity. The customers under the TOU tariff, acting as Stackelberg followers, decide the amount of electricity consumption to shift from the peak period to the non-peak period, given the prices of electricity. We then study a model with price elasticity of demand. The optimal capacity and pricing policies for the electricity company under both models are derived. By introducing the TOU tariff to customers, the electricity company can obtain more profit while customers can save electricity cost. We also analyze the effects of the proportion of the customers using the TOU tariff and discuss the managerial implications of the findings.

In the second topic, we study the time-of-use tariff for an electricity company with stochastic shifted consumption. Similar to the first topic, the electricity company uses two technologies to generate electricity and offers both the FR tariff and TOU tariff to the customers. We consider a scenario that the amount of shifted consumption is uncertain. The optimal capacity investment decisions and the optimal pricing decisions for the electricity company are obtained. We find that shifting too much consumption from the peak period to the non-peak period may not be optimal to the electricity company. Furthermore, we study the effects of the demands, market size, proportion of customers using the TOU tariff and the cost parameters, and discuss the managerial implications of the findings.

Carbon emission abatement is a hot topic in environmental sustainability, and cap-and-trade regulation is regarded as an effective way to reduce the carbon emission. Besides, according to the real industrial practices, an environmental sustainable product usually involves decreasing carbon emission in the production process and increasing the market demand. Therefore, in the third topic, we study the environmental sustainability investment in products with emission regulation considerations. Decentralized and centralized supply chains are con-

sidered. We first examine the order quantity of the retailer and sustainability investment of the manufacturer for the decentralized supply chain with one retailer and one manufacturer. Then, we study the centralized supply chain and derive the optimal production quantity and optimal sustainability investment for the whole supply chain. In both supply chains, the sustainability investment efficiency has a significant impact on the optimal solutions. Furthermore, we conduct numerical analyses and find surprisingly that the order quantity may be increasing in the wholesale price, which is due to the effects of environmental sustainability and carbon emission. Moreover, we investigate the achievability of supply chain coordination by various contracts, and find that only revenue sharing contract can coordinate the supply chain whereas the buyback contract and two-part tariff contract cannot achieve the coordination.

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Chapter 1

Introduction

According to a report by the International Energy Agency (IEA 2010), the global energy demand may grow by more than one-third between 2008 and 2035, with the expectation that the global electricity demand will continue to grow more strongly than any other final form of energy. Electricity demand will grow by around 80% by 2035, requiring an extra 5,900 GW of total capacity, and its share of total energy consumption grows from 17% to 23% (IEA 2010). So it is crucial to find ways to achieve energy efficiency for the electricity generation and consumption (IEA 2010, ACEEE 2014). Meanwhile, there exists a peak period for the electricity usage by customers, in which the electricity demand is higher than that in the non-peak period. Reducing the electricity usage in the peak period can save huge cost for the electricity company, and may also yield energy save and improve the energy efficiency (The Electropaedia 2005, Faruqui et al. 2007, York et al. 2007, EPA 2008, WiseGEEK 2013). And it is essential to have an efficient pricing mechanism for a smart grid future (Chao 2011a). Only under an efficient pricing mechanism of electricity, can the customers have the incentive to use the electricity wisely and reduce the electricity usage in the peak period. So an efficient pricing mechanism of electricity can facilitate efficient capacity investment for electricity.

On the other hand, environmental sustainability is receiving more and more public awareness all around the globe, and increasing attention in operations management research. Meanwhile, carbon emission accelerates global warming, and reducing carbon emission is essential to achieving environmental sustainability.

So it is important to study the investment in the environmental sustainability, with the consideration of carbon emission.

Therefore, in this thesis, we study the capacity investment for an electricity company and the environmental sustainability investment in products. In particular, regarding the capacity investment for an electricity company, our study will focus on the pricing of electricity; and regarding the environmental sustainability investment in products, our study will focus on the effects of the carbon emission. The thesis is comprised of three topics. The first and second topics study the capacity investment and pricing policies for an electricity company. The third topic studies the environmental sustainability investment under a cap-and-trade regulation of carbon emission.

In Chapter 2, we determine the optimal capacity and pricing policies for an electricity company with time-of-use (TOU) tariff. Currently, electricity customers in many countries pay the same flat rate (FR) for each unit of electricity they use during a year. Under this pricing mechanism, the customers have no incentive to use the electricity wisely and reduce electricity consumption from the peak period. Motivated by this fundamental problem of in-efficient pricing mechanism for electricity, we then consider introducing another pricing mechanism, i.e., the TOU tariff, under which the electricity price varies with the time. Customers pay a high price for electricity consumption in the peak period and a low price for electricity consumption in the non-peak period. We consider that the company offers FR and TOU tariffs to the customers simultaneously. The company is regulated under the price-cap regulation, which, as its name implies, sets an upper bound on an index of the regulated company's price. Two technologies are considered to generate electricity, where the first technology is used to generate electricity for the demands in both periods and the second technology is used to generate electricity only for the demand in the peak period. We first study a model with price inelasticity of total demand, where the total demand of electricity will not be affected by prices. Customers using the TOU tariff have the incentive to shift some electricity consumption from the peak period to the non-peak period, to take advantage of the lower price in the non-peak period. Here, shifting electricity consumption means that the customers change the time to use electricity for some activities, such as doing the laundry, from the peak period to the non-peak period. We apply a Stackelberg game to model the situation, where the electricity company, acting as the Stackelberg leader, decides the capacity investment and prices of electricity. The customers under the TOU tariff, acting as Stackelberg followers, decide the amount of electricity consumption to shift from the peak period to the non-peak period, given the prices of electricity. We find that the optimal shifted consumption of customers is determined by the marginal shift cost. There are several cases for the optimal capacity, depending on the costs of the technologies and customers' shifted consumption. The price-cap regulation plays an important role in determining the optimal prices. In order to achieve a win-win outcome, where the company obtains more profit by introducing the TOU tariff and the customers save electricity cost by using the TOU tariff, the government that acts as the regulator should not set the price-cap for the electricity price in the peak period too high. We then study a model with price elasticity of demand, where demands are functions of the prices. We also derive the optimal capacity and pricing policies for the electricity company under this model.

In Chapter 3, we focus on the capacity investment and pricing of the TOU tariff with stochastic shifted consumption. Similar to the setting in Chapter 2, the electricity company uses two technologies to generate electricity and offers both the FR and TOU tariffs to the customers, and the company is also regulated under the price-cap regulation. In order to get an in-depth understanding of the TOU tariff, we further investigate the optimal capacity investment and the pricing for the electricity company, whereas we consider that the shifted consumption by customers is uncertain in this chapter. We derive the optimal capacity investment and pricing decisions for the company. The costs play critical roles in rationing the capacities to meet the demands in the non-peak and peak periods. The capacity of the second technology and the total capacity for the peak period demand both increase in the price for the non-peak period and decrease in the price for the peak period, for the TOU tariff. Regarding the optimal prices for the TOU tariff, there is a unique optimal solution for the price in the non-peak period, while there are three possible optimal solutions for the price in the peak period, depending on the price sensitivity parameters and the price-cap for the price in the peak period. We find that shifting too much consumption from the peak period to the non-peak period may not be optimal to the electricity company. Furthermore, we study the effects of the demands, market size, proportion of customers using the TOU tariff and the cost parameters, and discuss the managerial implications of the findings.

In Chapter 4, we turn to study the environmental sustainability investment with the consideration of carbon emission. Carbon emission abatement is a hot topic in environmental sustainability. Many countries have designed or adopted carbon trading mechanism, such as cap-and-trade regulation, to reduce the carbon emission (Stavins 2008, Zhang and Xu 2013). On the other hand, the investment in cleaner technologies is another way to reduce the carbon emission and achieve the environmental sustainability. Therefore, in this chapter, we study the environmental sustainability investment in sustainable products under cap-and-trade regulation of carbon emission. Here, environmental sustainable products usually involve decreasing carbon emission in the production process and increasing the market demand. For example, it will produce less carbon emissions if the company invests in cleaner technologies to produce the products in the production process, or the products will be more favourable to the customers if the company invests in more advanced technologies to promote the energy efficiency for the products. Then, from the perspective of the environment, investing in the environmental sustainability on products could reduce the carbon emission and is beneficial to the environment; and from the perspective of marketing, it could stimulate the market demand. We consider both the decentralized and centralized supply chains with one manufacturer and one retailer in this chapter. For the decentralized supply chain, we consider that the manufacturer, acting as the Stackelberg leader, determines the sustainability investment, and the retailer, acting as the Stackelberg follower, determines its order quantity. We derive the optimal sustainability investment for the manufacturer and optimal order quantity for the retailer. For the centralized supply chain, we consider that the manufacturer and the retailer are fully aligned to achieve the channel's maximal profit by determining the sustainability investment and production quantity. We derive the optimal sustainability investment and optimal production quantity for the whole supply chain. By conducting numerical studies, we find that the order quantity may be surprisingly increasing in the wholesale price, which is due to the effects of environmental sustainability and carbon emission. Furthermore, we study the coordination of the supply chain under several contracts. We find that only revenue sharing contract can coordinate the supply chain whereas the buyback contract and two-part tariff contract cannot coordinate it.

Chapter 2

Optimal Capacity and Pricing Policies for an Electricity Company with Time-of-use Tariff

2.1 Introduction

Facing growing complexity in the electricity market, it is essential to have an efficient pricing mechanism for a smart grid future (Chao 2011a). Currently, the majority of electricity customers in many countries, such as China, pay the same flat rate (FR) for each unit of electricity they use during a year. Under this pricing mechanism, they have no incentive to reduce electricity use during the peak period or to use electricity wisely. These are fundamental problems that need to be addressed in order to reduce the required capacity for electricity generation during the peak period.

The time-of-use (TOU) tariff is another pricing mechanism under which the price varies with time. In contrast to the flat rate, the TOU tariff requires customers to pay a high price for electricity consumption in the peak period and a low price in the non-peak period. With the TOU tariff, customers have the incentive to actively change the way in which they use electricity, which helps achieve the goals of reducing the peak period capacity.

The TOU tariff has been adopted in some states of the U.S. and some countries in Europe. In China, the TOU tariff has been adopted for industrial customers in some cities, such as Beijing, since the last century. Currently, the TOU tariff has been adopted for residential customers in some cities, such as Shanghai, in China (The World Bank 2005, Pepper 2010). Several papers have studied the benefits of the TOU tariff (see, e.g., Henley and Peirson 1994, Faruqui and George 2005, Spees and Lave 2007, Faruqui 2010). For example, after California's power crisis in 2000 and 2001, California's three investor-owned utilities conducted an experiment, in concert with the two regulatory commissions, to evaluate the impacts of the TOU tariff and dynamic pricing on residential, and small commercial and industrial (C&I) customers. The experiment demonstrated that, under the TOU tariff, reduction in peak-period energy use could be up to 5.9% for residential customers and 8.6% for C&I customers (Faruqui and George 2005). Earlier work has shown that even a 5% reduction in the peak electricity demand in the U.S., i.e., $757,056$ MW, is worth US$ 3 billion per year, by avoiding the installation and energy costs associated with peak-period generation, as a result of a reduced peak load, and a reduction in the transmission and distribution capacity (Faruqui et al. 2007).

Many electricity markets are considering implementing the TOU tariff for some customers, without making it mandatory for all the customers in the whole electricity market. Examples can be found in Australia, Canada, England, the U.S. etc. (Prins 2012, DEWS 2014). Consequently, there is a mixed tariff structure under which some customers use the TOU tariff while the others pay a flat rate for electricity consumption (i.e., the so-called FR tariff). In the Operations Management (OM) literature, pricing for electricity under the TOU tariff has received little research attention. One exception is Yang et al. (2013), which investigates the TOU tariff for electricity consumption with consumer behaviour consideration. However, Yang et al. (2013) do not consider the mixed tariff structure in their study.

In this chapter we study the electricity capacity and pricing policies for an electricity company that offers a mixed tariff structure under which a proportion of the customers use the TOU tariff and the rest of the customers use the FR tariff. We first examine the optimal policies for the electricity company, given the proportion of customers using the TOU tariff, and then we study the effects of the proportion of customers using the TOU tariff on the optimal policies. It is because that in some countries or cities, the TOU tariff is mandatory to the customers. For example, Toronto Hydro is the first North American utility company that mandates the TOU tariff in a major city in Canada. The results of different pricing mechanisms in the electricity market have been mixed (Tweed 2011, CEA 2009). So, we first study the capacity investment and pricing policies for a given proportion of customers using the TOU tariff. On the other hand, even the TOU tariff is mandatory to some customers, the proportion of customers may be changed in the succeeding years. Examples can be found as follows: In 2006, the Department of Public Utility Control in Connecticut in the U.S. directed all the utility companies to phase in the mandatory TOU tariff for all the customers (in each succeeding year, the mandatory TOU tariff would be applied to additional customers based on declining levels of consumption) (Friedman 2011, Jessoe and Rapson 2014); and some countries in Asia have also dabbled with the TOU tariff, e.g., China has decided to gradually move to the TOU tariff (RAP 2008). So, we further examine the impact of the proportion on the optimal capacity and pricing decisions.

In some countries, electricity companies are subject to the monitoring and control of regulators. We consider that the electricity company is regulated under the price-cap regulation. Developed in Britain in the 1980s, the price-cap regulation has been adopted globally to regulate monopolistic electricity firms, which, as its title implies, sets an upper bound on an index of the regulated firm's prices (Sappington and Sibley 1992, Braeutigam and Panzar 1993, Regulationbodyofknowledge.org 2014). We consider a vertically integrated electricity company that not only owns the generation capacity, but is also responsible for meeting the market demand for electricity. Two technologies are considered to be installed for generating electricity to meet the demands in two periods, namely the peak period and the non-peak period, respectively. The second technology will be installed only if the first technology cannot meet the demand in the peak period. We address the following key research issues in this chapter: 1) How much electricity capacity should the electricity company install? 2) What should be the electricity prices under the TOU tariff and the FR tariff? 3) What is the reaction of the customers and how much demand in the peak period will be reduced under the TOU tariff? 4) What is the benefit for the electricity company to introduce the TOU tariff?

Many prior studies have revealed that time-varying prices of electricity can reduce the peak-period demand for electricity. However, some studies show that time-varying prices may not reduce the total electricity consumption over the whole period. For example, upon analyzing the data from a British TOU pricing experiment, Henley and Peirson (1994) concluded that the widespread introduction of TOU pricing in Britain did not result in a significant reduction in electricity consumption. Faruqui and George (2005) found that there was essentially no change in the total energy use across the entire year based on the average price of the TOU tariff and dynamic pricing in the experiment of Statewide Pricing Pilot in California. Therefore, we first consider in this chapter a model with price inelasticity of total demand, i.e., the total demand for electricity is not affected by the prices of the TOU and FR tariffs. However, given that the price in the peak period is higher than that in the non-peak period under the TOU tariff, customers under the TOU tariff may save their electricity bills by shifting some electricity consumption from the peak period to the non-peak period. We apply a Stackelberg game to study this model, where the electricity company that decides the capacity investment and electricity prices acts as the Stackelberg leader, and customers under the TOU tariff who decide the amount of electricity consumption to shift from the peak period to the non-peak period act as Stackelberg followers, given the prices of electricity.

On the other hand, there are many empirical studies on the price elasticity of demand for electricity, which produce different results. For the non-TOU price elasticity, the results of price elasticity of demand from different studies are not very consistent. Under the residential sector, the numbers that come up most often are around -0.2 and -0.9 for the short run and the long run, respectively. It implies that a ten percent price increase would reduce consumption by two percent in the short run and nine percent in the long run (Bohi and Zimmerman 1984, Lafferty et al. 2001, Fan and Hyndman 2011). Under the commercial and industrial sectors, the results are even less consistent. For the TOU price elasticity, Filippini (1995) shows that the price elasticity of demand would be -1.5 for the peak period and -2.57 for the non-peak period. Besides, many researchers treat electricity demand as a function of price in their models (see, e.g., Chao 1983, Crew et al. 1995, Borenstein and Holland 2005, Chao 2011a,b, Greer 2012). It is therefore necessary to consider both models with *price inelasticity of total demand* and with *price elasticity of demand*. So we also consider a model with price elasticity of electricity demand in Section 2.5 and derive the corresponding optimal capacity and pricing policies for the electricity company.

For the model with price inelasticity of total demand, we find that the optimal shifted consumption of customers is determined by the marginal shift cost. If the cost is neither too high nor too low, the optimal shifted consumption is such that the marginal shift cost is equal to the marginal shift profit. The total consumption of electricity over the whole period is unchanged, so customers' shifted consumption under the TOU tariff amounts to the reduction in electricity usage in the peak period that is shifted to the non-peak period. The optimal capacity is divided into several cases, depending on the costs of the technologies and customers' shifted consumption under the TOU tariff. The second technology will be installed only if its cost and customers' shifted consumption are small. The price-cap regulation plays an important role in determining the optimal prices. If the upper bound on the price in the peak period is large, then the company would set the prices such that customers cannot save electricity cost by using the TOU tariff. Otherwise, customers can save some electricity cost by using the TOU tariff. By studying the effects of the proportion of the customers using the TOU tariff, we find that the company's profit can increase when the TOU tariff is offered to the customers. Therefore, in order to achieve a win-win outcome, where the company obtains more profit by introducing the TOU tariff and the customers save electricity cost by using the TOU tariff, the government that acts as the regulator should not set too high an upper bound on the electricity price in the peak period.

For the model with price elasticity of demand, we derive that the optimal capacity is similarly divided into several cases. We show that the company's profit function in each case is jointly concave in the prices, so the optimal pricing decisions are uniquely determined.

The remainder of this chapter is organized as follows: In Section 2.2 we review the related literature. In Section 2.3 we present the general model setting of this chapter. In Section 2.4 we study a model with price inelasticity of total demand. In Section 2.5 we extend the study to a model with price elasticity of demand. In Section 2.6 we conclude this chapter. We provide all the proofs in Appendix A.

2.2 Literature Review

Our work is closely related to studies on strategic technology choice and capacity investment in the Operations Management/Operations Research literature. Goyal and Netessine (2007) study the impact of competition on a firm's decisions of technology and capacity investments under demand uncertainty. Boyabatli and Toktay (2011) investigate the technology choice and capacity level for a monopolistic firm that is budget-constrained and can relax its budget constraint by borrowing money from a creditor. Yang et al. (2011) conduct a comparative analysis of five possible production strategies for flexible technology and flexible capacity investments. Kashefi (2012) examines the effect of salvage market on the strategic technology choices and capacity investment decisions of two firms in a competitive market. Recently, there is growing literature on sustainable operations that consider technology choice and capacity investment. For instance, Drake et al. (2012) analyze the impacts of emissions tax and emissions cap-and-trade regulations on a firm's technology choice and capacity portfolios. Through modelling the trade-off between renewable and nonrenewable technologies, Aflaki and Netessine (2012) explore the incentives to invest in renewable electricity generating capacity. One important feature distinguishing our work from these studies is that we consider the pricing issues associated with a time-varying electricity pricing mechanism.

On the other hand, there are papers that study time-varying electricity prices without considering technology choice and capacity investment. Garcia et al. (2005) study dynamic pricing and learning in an infinite-horizon oligopoly model, in which hydroelectric generators are engaged in dynamic price-base competition. Triki and Violi (2009) analyze a dynamic and flexible tariff structure for a distribution company that protects retail consumers against excessive fluctuations in wholesale market prices. In the Economics and Electricity literature, some papers consider the problem of time-varying prices in an electricity market, including Henley and Peirson (1994), Borenstein (2005), Borenstein and Holland (2005), and Holland and Mansur (2005).

Recently, a few papers have considered the issues of electricity generation portfolio, pricing, and investment in the electricity market, but are from perspectives different from our work. Banal-Estañol and Micola (2009) examine by simulation the impact of diversification of the electricity generation portfolio on wholesale price. Chao (2011a) presents an economic model of pricing and investment in the electricity market with intermittent resources. In the setting of intermittent resources, it is indeed a problem similar to making pricing and investment decisions under supply uncertainty. However, most of these papers neglect the customer behaviour of shifting electricity consumption from the peak period to the non-peak period under time-varying prices. Yang et al. (2013) investigate the TOU tariff in an electricity market that takes customer behaviour into consideration. Our work differs from their paper in the following fundamental ways. First, their paper adopts the TOU tariff without considering other tariffs that may co-exist, while our work considers a mixed tariff structure under which some customers use the TOU tariff and the rest of the customers use the FR tariff. Second, their paper only considers a model with price inelasticity of total demand, while our work takes both price inelasticity of total demand and elasticity of demand into consideration.

In the Economics literature, significant research attention was paid to capacity choice and peak-load pricing in electricity markets in the 1970s and 1980s, as reviewed by Crew et al. (1995). Central to the problem is to address the pricing and capacity planning issues under stochastic demand when there are diverse technologies with different cost characteristics. For instance, Crew and Kleindorfer (1976) study the problems of capacity choice and pricing when several technologies are available and demand is stochastic, which Chao (1983) extends to the case with supply uncertainty. However, their demand models are basically different from ours. Our work considers the problem from the electricity company's perspective, while the other studies consider the problems from the social welfare perspective.

2.3 The General Model Setting

For nearly a century the electricity sector has been regarded as a natural monopoly, in which all the four primary elements of electricity supply, i.e., generation, transmission, distribution, and retailing, are organized as a vertically integrated company (Aflaki and Netessine 2012). In this chapter we consider a vertically integrated electricity company that not only owns the generation capacity, but is also responsible for meeting the market demand for electricity.

The time of electricity usage spans two periods, namely the non-peak period and the peak period. We consider the scenario under which the electricity company offers customers two tariffs, namely the FR tariff and the TOU tariff. A fraction $\alpha \in (0, 1]$ of the customers use the TOU tariff while the rest of the customers use the FR tariff. We assume that α is given. A similar setting can be found in Borenstein and Holland (2005). We assume that there are N customers in the market, so αN customers use the TOU tariff. Under the FR tariff, customers pay a flat price $p_0 \in [0, \bar{p_0}]$ for electricity consumption in both the non-peak period and the peak period. Under the TOU tariff, customers pay a price $p_1 \geq 0$ for electricity consumption in the non-peak period and pay a price $p_2 \in [0, \bar{p_2}]$ for electricity consumption in the peak period. We assume that $p_1 \leq p_2$. Here, $\bar{p_0}$ and $\bar{p_2}$ are upper bounds on p_0 and p_2, respectively, which may be imposed by the regulator under the price-cap regulation. It is reasonable to assume that $\bar{p_0} \leq \bar{p_2}$. We also assume that $p_1 \leq p_0 \leq p_2$. Otherwise, no customer will be willing to use the TOU tariff if $p_1 > p_0$ and no customer will be willing to use the FR tariff if $p_2 < p_0$. Table 2.1 summarizes the tariffs for Chapter 2.

Table 2.1: Tariffs of the electricity company for Chapter 2

Tariff	**Proportion**	**Price**
FR tariff	$1-\alpha$	p_0
TOU tariff	α	p_1, p_2

We consider deterministic demand that spreads evenly throughout each period. Let $D_1 \geq 0$ and $D_2 \geq 0$ denote the demands in the non-peak and peak periods, respectively. Let T denote the total period time, e.g., one day. Without loss of generality, we let $T = 1$. Let t_1 and t_2 denote the start time and end time of the peak period, respectively. We define $\tau = t_2 - t_1$ as the percentage of time of the peak period over the whole period. Two technologies are available for generating electricity, i.e., Technology i, $i \in \{1, 2\}$. Let $k_i \geq 0$ denote the installed capacity for Technology i. Similar to the setting in Pineau and Zaccour (2007), we consider that the capacity of Technology 1 is used throughout the whole period, while the capacity of Technology 2 is installed in the peak period only when the capacity of Technology 1 cannot meet the demand. The capacity is pictorially shown in Figure 2.1.

The installed capacity is restricted by the following constraints:

$$0 \leq D_1/(1-\tau) \leq k_1; \tag{2.1}$$

$$0 \leq D_2/\tau \leq k_1 + k_2; \tag{2.2}$$

$$k_2 \begin{cases} = 0 & \text{if } D_2 \leq \tau k_1; \\ > 0 & \text{if } D_2 > \tau k_1. \end{cases} \tag{2.3}$$

Constraints (2.1) and (2.2) are the capacity constraints for the non-peak period

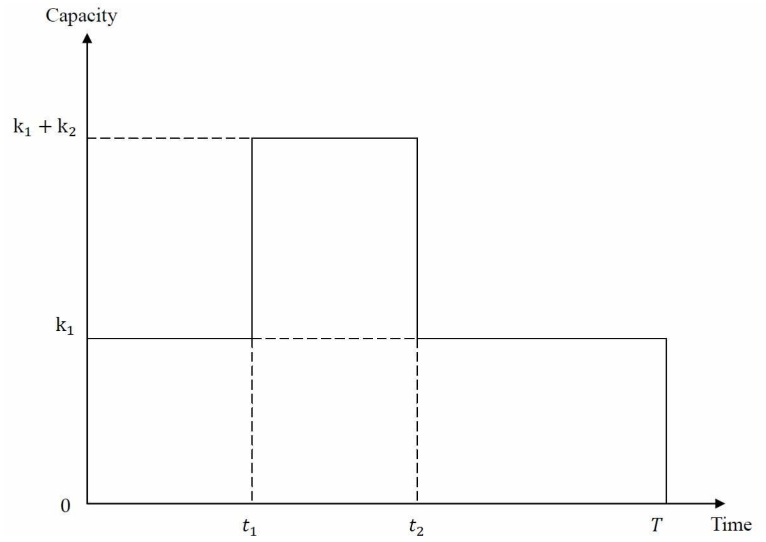


Figure 2.1: Capacity in the two periods

and the peak period, respectively; and Constraints (2.3) ensures that the capacity of Technology 2 will be installed only when Technology 1 cannot satisfy the demand in the peak period.

Let c_i and β_i denote the unit capacity cost and unit production cost of Technology i, respectively. We assume that $c_1+\beta_1 \le c_2+\beta_2$; otherwise, Technology 2 will be used in the non-peak period. Let $\gamma = \min\{(\beta_2+c_2/\tau-\beta_1-c_1))/(1-\tau), c_1/\tau\}$. If $\tau\beta_2+c_2 \ge \tau\beta_1+c_1$, then $\gamma = c_1/\tau$; otherwise, $\gamma = (\beta_2+c_2/\tau-\beta_1-c_1)/(1-\tau)$. Here, the conditions $\tau\beta_2+c_2 \ge \tau\beta_1+c_1$ and $\tau\beta_2+c_2 < \tau\beta_1+c_1$ will be used for different cases of the optimal solutions in Section 2.4.

The electricity company's cost function $C_g(k_1, k_2, D_1, D_2)$ is given by

$$\begin{aligned} C_g(k_1, k_2, D_1, D_2) \quad &= \quad c_1k_1 + c_2k_2 + \beta_1 D_1 + \beta_1 \min\{D_2, \tau k_1\} \\ &\quad +\beta_2(D_2 - \tau k_1)^+, \end{aligned} \tag{2.4}$$

where $(D_2 - \tau k_1)^+ = \max\{0, D_2 - \tau k_1\}$. In the cost function (2.4), c_1k_1 and c_2k_2 are the capacity costs of Technologies 1 and 2, respectively; $\beta_1 D_1$ is the production cost in the non-peak period; $\beta_1 \min\{D_2, \tau k_1\} + \beta_2(D_2 - \tau k_1)^+$ is the production cost in the peak period; and if the capacity of Technology 1 can meet the demand in the peak period, i.e., $D_2 \le \tau k_1$, then the production cost in the peak period is equal to $\beta_1 D_2$; otherwise, it is equal to $\beta_1\tau k_1 + \beta_2(D_2 - \tau k_1)$.

Let D_{F1} and D_{F2} denote the demands in the non-peak and peak periods, respectively, under the FR tariff. Let D_{T1} and D_{T2} denote the demands in the non-peak and peak periods, respectively, under the TOU tariff. Then $D_1 = D_{T1} + D_{F1}$ and $D_2 = D_{T2} + D_{F2}$. The company's objective is to determine the optimal capacity of the two technologies, i.e., (k_1, k_2), and the optimal prices for the two tariffs, i.e., (p_0, p_1, p_2), so as to maximize its profit:

$$\max_{\mathbf{k},\mathbf{p}} \Pi_g(\mathbf{k}, \mathbf{p}) \quad = \quad p_1 D_{T1} + p_2 D_{T2} + p_0(D_{F1} + D_{F2}) - C_g(k_1, k_2, D_1, D_2) \tag{2.5}$$

$s.t.$

$$0 \le p_1 \le p_0 \le p_2 \le \bar{p}_2; \tag{2.6}$$

$$0 \le p_0 \le \bar{p}_0; \tag{2.7}$$

$$0 \le D_1 = D_{T1} + D_{F1} \le (1-\tau)k_1;$$

$$0 \le D_2 = D_{T2} + D_{F2} \le \tau(k_1 + k_2);$$

$$k_2 \begin{cases} = 0 & \text{if } D_2 \le \tau k_1; \\ > 0 & \text{if } D_2 > \tau k_1. \end{cases}$$

Here, $C_g(k_1, k_2, D_1, D_2)$ is the company's cost function, which is determined by Equation (2.4). Constraints (2.6) and (2.7) are the price constraints. The other three constraints are the same as Constraints (2.1), (2.2), and (2.3).

In this chapter we consider two models, namely price inelasticity of total demand and price elasticity of demand. They are presented in Sections 2.4 and 2.5, respectively. Table 2.2 summarizes the major notation for the general model setting in this chapter, and we will introduce and define additional notation when needed. In Table 2.2, $i \in \{1, 2\}$ indicates Technology i.

2.4 Price Inelasticity of Total Demand

In this section we consider a model with price inelasticity of total demand. We assume that the prices do not affect the total demand for electricity. This assumption is supported by some studies in the electricity literature (see, e.g., Henley and Peirson 1994, Faruqui and George 2005). In addition, as a flat price is charged under the FR tariff, we assume that the demands in both periods are

Table 2.2: Notation of Chapter 2

k_i	installed capacity of Technology i;
τ	percentage of time of the peak period;
β_i	unit production cost of Technology i;
c_i	unit capacity cost of Technology i;
$C_g(\cdot)$	the electricity company's cost function;
α	the proportion of customers who use the TOU tariff, $\alpha \in (0, 1]$;
N	total numbers of customer in the market, so the number of customers using the TOU tariff is αN;
p_0	electricity price for the FR tariff;
p_1, p_2	electricity prices in the non-peak and peak periods, respectively, for the TOU tariff;
D_{F1}, D_{F2}	demands in the non-peak and peak periods, respectively, under the FR tariff;
D_{T1}, D_{T2}	demands in the non-peak and peak periods, respectively, under the TOU tariff;
D_1, D_2	total demands in the non-peak and peak periods, respectively.

fixed under the FR tariff. However, the prices in the non-peak and peak periods may affect the demands under the TOU tariff. Under the TOU tariff, the price in the non-peak period is lower than that in the peak period, so customers who use the TOU tariff may shift some electricity consumption from the peak period to the non-peak period.

2.4.1 Modelling

First, we let q_1 and q_2 denote the demands in the non-peak and peak periods, respectively, if the company only offers the FR tariff. When the company offers the TOU tariff, the total demand will be the same as the total demand when only the FR tariff is offered. The fraction of customers still using the FR tariff is $1 - \alpha$, so the customer demands in the non-peak and peak periods under the FR tariff are equal to $(1 - \alpha)q_1$ and $(1 - \alpha)q_2$, respectively, i.e., $D_{F1} = (1 - \alpha)q_1$ and $D_{F2} = (1-\alpha)q_2$. The customer demands in the non-peak and peak periods under the TOU tariff are equal to αq_1 and αq_2, respectively, if no consumption shifting behaviour occurs. Let $q_0 = \tau(1-\tau)(q_2/\tau - q_1/(1-\tau))$, where q_2/τ and $q_1/(1-\tau)$ are the average demands for the peak and non-peak periods, respectively, if there is no consumption shifting. It is reasonable to assume that the average demand in the peak period is not less than that in the non-peak period, so we set $q_0 \geq 0$.

A flat price is charged to customers under the FR tariff, so customers have no incentive to shift consumption from the peak period to the non-peak period. However, customers under the TOU tariff have an incentive to shift consumption if it can save their electricity bills. For example, under the TOU tariff, customers may turn off the electric water heater during the peak period, do most of the laundry and only run the dryer in the non-peak period etc. Let q_s^i be the amount of shifted consumption from the peak period to the non-peak period by an individual TOU customer, given p_1 and p_2. And we define $q_s = \alpha N q_s^i$ as the shifted consumption by all the customers under the TOU tariff. Note that the total amount of the shifted consumption will not exceed the total demand under the TOU tariff, i.e, $q_s \leq \alpha q_2$. Then, after shifting, the customer demands in the non-peak and the peak period under the TOU tariff are equal to $\alpha q_1 + q_s$ and $\alpha q_2 - q_s$, respectively, i.e., $D_{T1} = \alpha q_1 + q_s$ and $D_{T2} = \alpha q_2 - q_s$. Then the total demands in non-peak period and the peak period are determined by $D_1 = q_1 + q_s$ and $D_2 = q_2 - q_s$, respectively. Table 2.3 presents customers' electricity demands under the two tariffs after consumption shifting.

Table 2.3: Demand in the model with price inelasticity of total demand

Tariff	**Demand in the non-peak period**	**Demand in the peak period**
FR tariff	$(1-\alpha)q_1$	$(1-\alpha)q_2$
TOU tariff	$\alpha q_1 + q_s$	$\alpha q_2 - q_s$
Total	$q_1 + q_s$	$q_2 - q_s$

Meanwhile, there is inconvenience for customers to shift electricity consumption from the peak period to the non-peak period. We refer to such inconvenience as the shift cost. Let $g(q_s^i)$ denote the function of the shift cost for a TOU customer. We assume that $g(0) = 0$, which indicates that no shift cost will incur if there is no shifted consumption. It is reasonable to assume that the shift cost is a convex increasing function in the amount of shifted consumption by a customer, so we have $g'(q_s^i) > 0$ and $g''(q_s^i) > 0$. For technical convenience, we further assume $g'''(q_s^i) \geq 0$. Let $\Pi_c(q_s)$ be the function of the electricity cost for a customer under the TOU tariff, i.e.,

$$\Pi_c(q_s^i) \;=\; p_1(\frac{q_1}{N} + q_s^i) + p_2(\frac{q_2}{N} - q_s^i) + g(q_s^i).$$

Let $\Delta\Pi_c$ be the difference in the electricity cost between a customer under the TOU tariff and the customer that still uses the FR tariff, i.e.,

$$\begin{aligned}\Delta\Pi_c &= p_1(\frac{q_1}{N} + q_s^i) + p_2(\frac{q_2}{N} - q_s^i) + g(q_s^i) - p_0(\frac{q_1}{N} + \frac{q_2}{N}) \\ &= \frac{1}{N}\Big\{p_1 q_1 + p_2 q_2 - p_0(q_1 + q_2)\Big\} - (p_2 - p_1)q_s^i + g(q_s^i). \qquad (2.8)\end{aligned}$$

Let $\theta = \min\{\gamma, g'(q_0/(\alpha N))\}$. To avoid trivial outcomes, we assume $g'(0) \leq \gamma$ and $\Delta\Pi_c|_{(p_1=0,p_2=\theta)} \leq 0$[2.1]. Here, the assumption $g'(0) \leq \gamma$ ensures that the marginal shift cost is not too large; otherwise, no shifting is optimal to customers; $\Delta\Pi_c|_{(p_1=0,p_2=\theta)} \leq 0$ ensures that customers can save some electricity cost if the price in the non-peak period is equal to zero (i.e., $p_1 = 0$) and the price in the peak period is equal to the marginal shift cost or a cost that is related to the average production or capacity cost (i.e., $p_2 = \theta = \min\{g'(q_0/(\alpha N)), \gamma\}$).

We apply a Stackelberg game to study the model with price inelasticity of total demand in this chapter. The electricity company, acting as the Stackelberg leader, decides the capacity, i.e., (k_1, k_2), and the electricity prices, i.e., (p_0, p_1, p_2). The customer under the TOU tariff, acting as Stackelberg followers, decides the amount of electricity consumption to shift from the peak period to the non-peak period, i.e., q_s^i, given the prices of electricity.

2.4.2 Analysis and solution

We apply the backward sequential decision-making approach to solve our problems. First, we assume that the prices are given and known to the customers, under which we model the customer's problem and obtain the optimal response of shifted consumption, i.e., $q_s^i(p_1, p_2)$, for a customer under the TOU tariff. Note that the total shifted consumption $q_s(p_1, p_2)$ will be determined

[2.1] See Appendix A for the optimal results of these trivial cases at the same time, as we consider homogenous TOU customers and we have $q_s(p_1, p_2) = \alpha N q_s^i(p_1, p_2)$. Given the optimal response of shifted consumption, we then solve the electricity company's problem and obtain its optimal capacity and pricing decisions.

Customer's problem

We analyze and solve the customer's problem in this subsection. As a flat tariff is charged to customers under the FR tariff, the objective of the customer's problem is to minimize the customer's electricity cost under the TOU tariff by optimally setting the shifted consumption q_s^i:

$$\min_{q_s^i \in [0, q_2/N]} \Pi_c(q_s^i).$$

By minimizing the objective function $\Pi_c(q_s^i)$ over q_s^i, we obtain the following results.

Proposition 2.1 *Given p_1 and p_2, the customer's optimal response of shifted consumption is as follows: if $g'(0) \geq p_2 - p_1$, then $q_s^i(p_1, p_2) = 0$; if $g'(q_2/N) \leq p_2 - p_1$, then $q_s^i(p_1, p_2) = q_2/N$; otherwise, it is uniquely determined by*

$$g'(q_s^i) = p_2 - p_1. \tag{2.9}$$

Proposition 2.1 indicates that if the marginal shift cost is too high, i.e., $g'(0) \geq p_2 - p_1$, then there is no shifting; if it is too low, i.e., $g'(q_2/N) \leq p_2 - p_1$, then customers will shift all consumption from the peak period to the non-peak period; otherwise, it is determined by the first-order condition of the customer's cost function, where the marginal shift cost is equal to the marginal profit, i.e., $g'(q_s^i) = p_2 - p_1$. It can be shown that the optimal solutions for the other two cases lie on the boundary of the case where $g'(q_s^i) = p_2 - p_1$, implying that the global optimal solutions are obtained from the case where $g'(q_s^i) = p_2 - p_1$. For such a scenario, we say that the case where $g'(q_s^i) = p_2 - p_1$ *dominates* the other two trivial cases. So, in the sequel, we only present the analysis and results for the case $g'(0) \leq p_2 - p_1 \leq g'(q_2/N)$ such that $g'(q_s^i) = p_2 - p_1$. Next, we consider the company's problem, given the customer's optimal response of shifted consumption.

Company's problem

Knowing the customer's optimal response of shifted consumption, i.e., $q_s^i(p_1, p_2)$ (and then $q_s(p_1, p_2) = \alpha N q_s^i(p_1, p_2)$), the company's problem is to maximize its profit by optimally determining the capacity decisions, i.e., $\mathbf{k} = (k_1, k_2)$, and prices, i.e., $\mathbf{p} = (p_0, p_1, p_2)$. The objective function is derived from (2.5) and

$$\begin{aligned}\Pi_g(\mathbf{k}, \mathbf{p}) &= p_1(\alpha q_1 + q_s(p_1, p_2)) + p_2(\alpha q_2 - q_s(p_1, p_2)) + p_0(1 - \alpha)(q_1 + q_2) \\ &\quad - C_g(k_1, k_2, q_1 + q_s(p_1, p_2), q_2 - q_s(p_1, p_2)).\end{aligned}$$

Besides the Constraints (2.1), (2.2), (2.3), (2.6), and (2.7), we need to ensure that the customers will not be hurt if the TOU tariff is offered, i.e., $\Delta\Pi_c \leq 0$, where $\Delta\Pi_c$ is expressed in Equation (2.8).

We use the sequential decision-making approach to solve the company's problem. Under this approach, the company's problem can be reduced to an optimization problem over the decision variables $\mathbf{p}$ by first solving for the optimal values of $\mathbf{k}$ as functions of $\mathbf{p}$, and then substituting the results back to $\Pi_g(\mathbf{k}, \mathbf{p})$. Thus, we solve the company's problem by two steps. First, we assume that the prices are given, under which we solve the company's problem and obtain the optimal responses of capacities, i.e., $\mathbf{k}(\mathbf{p})$. In the second step, we obtain the optimal prices, i.e., $\mathbf{p}^*$, given the optimal responses of capacities. This approach can guarantee the optimality of the solution, and is widely used in the literature, such as Petruzzi and Dada (1999) and Wang et al. (2004).

(1) Capacity decisions

By analyzing the objective function of the company's problem, we find the optimal response of capacity as presented in Theorem 2.1.

Theorem 2.1 *The optimal capacity* $(k_1(\mathbf{p}), k_2(\mathbf{p}))$ *is determined as follows:*

$$(k_1(\mathbf{p}), k_2(\mathbf{p})) = \begin{cases} (\frac{q_1+q_s(p_1,p_2)}{1-\tau}, \frac{q_0-q_s(p_1,p_2)}{\tau(1-\tau)}) & \textit{if } q_s(p_1,p_2) \le q_0 \\ & \textit{and } \tau\beta_2 + c_2 < \tau\beta_1 + c_1; \\ (\frac{q_2-q_s(p_1,p_2)}{\tau}, 0) & \textit{if } q_s(p_1,p_2) \le q_0 \\ & \textit{and } \tau\beta_2 + c_2 \ge \tau\beta_1 + c_1; \\ (\frac{q_1+q_s(p_1,p_2)}{1-\tau}, 0) & \textit{if } q_s(p_1,p_2) \ge q_0. \end{cases}$$

Theorem 2.1 gives the optimal capacity for the company. Since we have $q_s = \alpha N q_s^i$, it is straightforward to see that the optimal capacity decisions will be affected by the proportion of customers using the TOU tariff. We define three cases of the optimal capacity, which are pictorially shown in Figure 2.2.

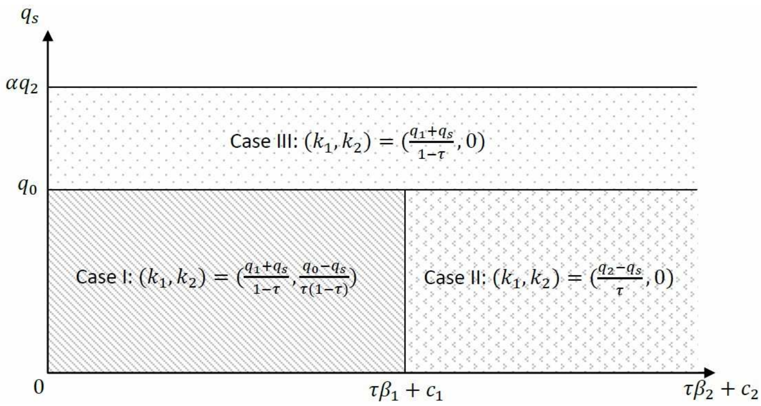


Figure 2.2: Three cases of the model with price inelasticity of total demand

Case I: $(k_1(\mathbf{p}), k_2(\mathbf{p})) = (\frac{q_1+q_s(p_1,p_2)}{1-\tau}, \frac{q_0-q_s(p_1,p_2)}{\tau(1-\tau)})$. In this case, the company installs both technologies to generate electricity for customers. This case happens when the shifted consumption and production cost of Technology 2 are small, i.e., $q_s(p_1,p_2) < q_0$ and $\tau\beta_2 + c_2 < \tau\beta_1 + c_1$.

Case II: $(k_1(\mathbf{p}), k_2(\mathbf{p})) = (\frac{q_2-q_s(p_1,p_2)}{\tau}, 0)$. In this case, the company only installs Technology 1 to generate electricity, which can meet the demands in both the non-peak period and the peak period. This case happens when the shifted consumption is small, i.e., $q_s(p_1,p_2) < q_0$, but the production cost of Technology 2 is large, i.e., $\tau\beta_2 + c_2 \ge \tau\beta_1 + c_1$.

Case III: $(k_1(\mathbf{p}), k_2(\mathbf{p})) = (\frac{q_1+q_s(p_1,p_2)}{1-\tau}, 0)$. In this case, the company also only installs Technology 1 to generate electricity. This case happens when the shifted consumption is large, i.e., $q_s(p_1, p_2) > q_0$.

The results are intuitive. When the shifted consumption is large, the remaining consumption in the peak period will be small, so the electricity generated by Technology 1 can meet the demand in the peak period. Then the company does not need to install Technology 2 (Case III happens). If the shifted consumption is small, then the installation of Technology 2 is determined by the costs of the technologies. If the cost of Technology 2 is too large, then the optimal strategy for the company is not to install this technology and only use Technology 1 to generate electricity (Case II happens); otherwise, the company will install both technologies with total capacity $(q_2 - q_s(p_1, p_2))/\tau$ (because $k_1(p_1, p_2) + k_2(p_1, p_2) = (q_2 - q_s(p_1, p_2))/\tau$) in this situation and Case I happens).

Remark 2.1 *In Figure 2.2, we assume that $q_0 \leq \alpha q_2$. If $q_0 > \alpha q_2$, then only Cases I and II are possible.*

Remark 2.2 *The optimal capacity is continuous in q_s, which means that the capacity values for the three cases are equal when $q_s(p_1, p_2) = q_0$. Regarding the horizontal axis in Figure 2.2, we have $\Pi_g(\mathbf{k}, \mathbf{p}) = \alpha(p_1 q_1 + p_2 q_2) + (1-\alpha)p_0(q_1 + q_2) - (p_2 - p_1 + \beta_1 - \beta_2)q_s(p_1, p_2) - \beta_1 q_1 - \beta_2 q_2 - c_2(k_1 + k_2)$ when $\tau\beta_2 + c_2 = \tau\beta_1 + c_1$, where the values of $c_2(k_1 + k_2)$ are equal for Cases II and III.*

(2) Price decisions

With the substitution of the optimal response of capacity into the company's profit function, our objective is to maximize the profit function by optimally setting the prices of electricity. The company's profit function can be expressed as follows:

$$\Pi_g(\mathbf{p}) = \begin{cases} \Pi_0 - \Big(p_2 - p_1 - \frac{1}{1-\tau}(\beta_2 + \frac{c_2}{\tau} - \beta_1 - c_1)\Big)q_s(p_1, p_2) - \frac{(\beta_1 + c_1)q_1 + (\beta_2 + \frac{c_2}{\tau})q_0}{1-\tau} & \\ & \text{for Case I;} \\ \Pi_0 - (p_2 - p_1 - \frac{c_1}{\tau})q_s(p_1, p_2) - \beta_1(q_1 + q_2) - c_1\frac{q_2}{\tau} & \text{for Case II;} \\ \Pi_0 - (p_2 - p_1 + \frac{c_1}{1-\tau})q_s(p_1, p_2) - \beta_1(q_1 + q_2) - c_1\frac{q_1}{1-\tau} & \text{for Case III,} \end{cases}$$

where $\Pi_0 = \alpha(p_1 q_1 + p_2 q_2) + (1-\alpha)p_0(q_1 + q_2)$.

Note that $q_s(p_1, p_2)$ is a function of p_1 and p_2, and it is not affected by changes in p_0. So it is straightforward to see that $\Pi_g(\mathbf{p})$ is increasing in p_0. Thus, the upper bound on p_0 is optimal for the company, i.e., $p_0^* = \bar{p_0}$. As Constraints (2.6) and (2.7) indicate, there is a condition for p_0 such that $p_0 \leq \min\{p_2, \bar{p_0}\}$. Then, in fact, we have two cases here: one is $p_0^* = \bar{p_0}$ and the other is $p_0^* = p_2$, but it can be shown that the latter case is dominated by the former one.

Lemma 2.1 *Case III is dominated by Case I or Case II.*

Lemma 2.1 indicates that the optimal solution for Case III lies on the boundary of Case I or Case II. Then we can solve the problem by analyzing Cases I and II, thus obtaining Theorem 2.2.

Theorem 2.2 *The optimal prices in the non-peak and peak periods, and the optimal shifted consumption by the customers are shown in Table 2.4.*

Table 2.4: Optimal prices and shifted consumption for the model with price inelasticity of total demand

Case	**Sub-case**	p_1^*	p_2^*	q_s^*	$\mathbf{\Delta\Pi_c}$
If $p_2^D \leq g'(0)$	If $\bar{p_2} \geq p_2^B$	$p_2^B - \theta$	p_2^B	$\alpha N g'^{-1}(\theta)$	$= 0$
	If $p_2^A \leq \bar{p_2} \leq p_2^B$	p_1^E	$\bar{p_2}$	$\alpha N g'^{-1}(\bar{p_2} - p_1^E)$	$= 0$
	If $\bar{p_2} \leq p_2^A$	$\bar{p_2} - g'(0)$	$\bar{p_2}$	0	< 0
If $g'(0) \leq p_2^D \leq \theta$	If $\bar{p_2} \geq p_2^B$	$p_2^B - \theta$	p_2^B	$\alpha N g'^{-1}(\theta)$	$= 0$
	If $p_2^C \leq \bar{p_2} \leq p_2^B$	p_1^E	$\bar{p_2}$	$\alpha N g'^{-1}(\bar{p_2} - p_1^E)$	$= 0$
	If $p_2^D \leq \bar{p_2} \leq p_2^C$	p_1^F	$\bar{p_2}$	$\alpha N g'^{-1}(\bar{p_2} - p_1^F)$	< 0
	If $\bar{p_2} \leq p_2^D$	0	$\bar{p_2}$	$\alpha N g'^{-1}(\bar{p_2})$	< 0
If $p_2^D \geq \theta$	If $\bar{p_2} \geq p_2^B$	$p_2^B - g'(q_0/(\alpha N))$	p_2^B	q_0	$= 0$
	If $g'(q_0/(\alpha N)) \leq \bar{p_2} \leq p_2^B$	$\bar{p_2} - g'(q_0/(\alpha N))$	$\bar{p_2}$	q_0	< 0
	If $\bar{p_2} \leq g'(q_0/(\alpha N))$	0	$\bar{p_2}$	$\alpha N g'^{-1}(\bar{p_2})$	< 0

Here, $\theta = \min\{\gamma, g'(q_0/(\alpha N))\}$; $p_2^A = \bar{p_0} + \frac{q_1}{q_1+q_2} g'(0)$; $p_2^B = \bar{p_0} + \frac{\theta(Ng'^{-1}(\theta)+q_1) - Ng(g'^{-1}(\theta))}{q_1+q_2}$; p_2^C *is the unique solution of* p_2 *for the equations:* $\Delta\Pi_c = 0$, $\frac{\partial \Pi_g(\mathbf{p})}{\partial p_1} = 0$, *and* $g'(q_s/(\alpha N)) = p_2 - p_1$; p_2^D *is the unique solution of* p_2 *for the equations:* $\frac{\partial \Pi_g(\mathbf{p})}{\partial p_1}|_{p_1=0} = 0$ *and* $g'(q_s/(\alpha N)) = p_2$; p_1^E *is the unique solution of* p_1 *for the equations:* $\Delta\Pi_c|_{p_2=\bar{p_2}} = 0$ *and* $g'(q_s/(\alpha N)) = \bar{p_2} - p_1$; p_1^F *is the unique solution of* p_1 *for the equations:* $\frac{\partial \Pi_g(\mathbf{p})}{\partial p_1}|_{p_2=\bar{p_2}} = 0$ *and* $g'(q_s/(\alpha N)) = \bar{p_2} - p_1$; *and* $\frac{\partial \Pi_g(\mathbf{p})}{\partial p_1} = \alpha q_1 + q_s + (p_2 - p_1 - \gamma)\frac{\alpha N}{g''(q_s/(\alpha N))} = 0.$

As shown in Table 2.4, there are three cases for the optimal solutions, depending on the value of p_2^D. The price-cap regulation plays an important role in determining the optimal price decisions. In any of these cases, if $\bar{p_2}$ is large, i.e., $\bar{p_2} \geq p_2^B$, then $p_2^* = p_2^B$, $p_1^* = p_2^B - \theta$, $q_s = \alpha N g'^{-1}(\theta)$, and $\Delta\Pi_c \equiv 0$. This implies that, if the price in the peak period is below its upper bound (i.e., $p_2^B < \bar{p_2}$), then the company will set the prices such that there is no difference in the electricity cost between customers under the TOU tariff and customers still using the FR tariff (i.e., $\Delta\Pi_c = 0$, and we say that customers cannot save electricity cost by using the TOU tariff). Figure 2.3 illustrates the optimal solutions for the case

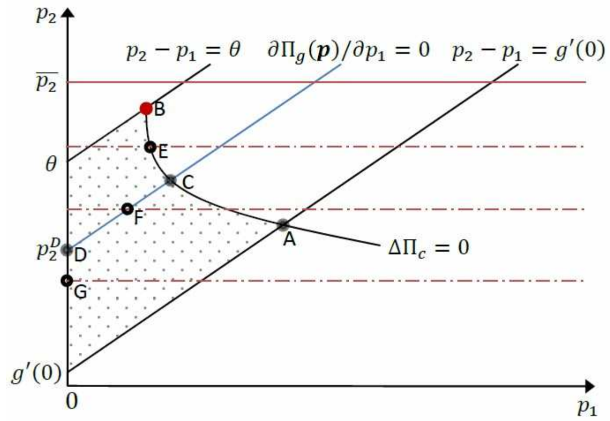


Figure 2.3: Optimal prices when $g'(0) \leq p_2^D \leq \theta$. *Note:* $A = \{(x_1, x_2)|\Delta\Pi_c = 0 \quad and \quad p_2 - p_1 = g'(0)\}$, $B = \{(x_1, x_2)|\Delta\Pi_c = 0 \quad and \quad p_2 - p_1 = \theta\}$, $C = \{(x_1, x_2)|\Delta\Pi_c = 0 \quad and \quad \frac{\partial\Pi_g(\mathbf{p})}{\partial p_1} = 0\}$, $D = \{(x_1, x_2)|\frac{\partial\Pi_g(\mathbf{p})}{\partial p_1} = 0 \quad and \quad p_1 = 0\}$, $E = \{(x_1, x_2)|\Delta\Pi_c = 0 \quad and \quad p_2 = \bar{p_2}\}$, $F = \{(x_1, x_2)|\frac{\partial\Pi_g(\mathbf{p})}{\partial p_1} = 0 \quad and \quad p_2 = \bar{p_2}\}$, $G = \{(x_1, x_2)|p_1 = 0 \quad and \quad p_2 = \bar{p_2}\}$.

where $g'(0) \leq p_2^D \leq \theta$. If $p_2 = \bar{p_2}$ is above Point C, then Point B or E is optimal for the electricity company, under which customers cannot save electricity cost by using the TOU tariff. If $p_2 = \bar{p_2}$ is below Point C, then Point F or G is optimal for the electricity company, under which customers can save some electricity cost by using the TOU tariff. Therefore, the implication of the price-cap regulation is as follows: in order to achieve a win-win situation, where the company can get more profit by introducing the TOU tariff (we will show this result in Subsection 2.4.3) and customers can save electricity cost by using the TOU tariff, the government that acts as the regulator should not set too high an upper bound on the price in the peak period.

Remark 2.3 *It is worth noting that, in our model setting, the electricity cost for a customer, i.e.,* Π_c*, consists of two parts: one is the actual amount charged in the electricity bill and the other one is the shift cost due to the inconvenience incurred for the customer to shift electricity consumption. Thus, even for the cases where* $\Delta\Pi_c = 0$*, the customers can save on their electricity bills by using the TOU tariff.*

Remark 2.4 *Instead of the price-cap regulation, if we consider an alternative regulation where the government imposes a lower bound on customers' electricity cost saving, i.e.,* $\Delta\Pi_c \leq -\delta$ *for* $\delta \geq 0$*, then the optimal prices can be obtained by solving the equations:* $\Delta\Pi_c = -\delta$ *and* $p_2 - p_1 = \theta$*. The result shows that customers will always benefit from adopting the TOU tariff. On the other hand, we can also show that the company can get more profit by introducing the TOU tariff. Therefore, introducing the TOU tariff under this alternative regulation can achieve Pareto improvement. Comparing with the results under the price-cap regulation, we find that these two regulations are similar, but this alternative regulation rules out the case where the customers cannot save electricity cost by using the TOU tariff.*

2.4.3 The effects of proportion of customers using the TOU tariff

In this subsection we consider the effects of the proportion of customers who use the TOU tariff on the company's optimal decisions, customers' shifted consumption, and the company's profit.

Note that the optimal price in the non-peak period under the TOU tariff may be implicitly determined by the first-order condition for the company's profit function. In order to keep the results neat and generate some managerial insights, we consider a quadratic shift cost for the customers here, i.e., $g(q_s^i) = c_s(q_s^i)^2$, where $c_s > 0$. Then $g'(q_s^i) = 2c_s q_s^i$, $g''(q_s^i) = 2c_s$, and $g'''(q_s^i) = 0$. We obtain that $p_2^A = \bar{p_0} + q_1 g'(0)/(q_1 + q_2) = \bar{p_0} \leq \bar{p_2}$ and $p_2^D = \gamma/2 - c_s q_1/N < \theta$, so the cases where $(p_1^*, p_2^*) = (\bar{p_2} - C_s'(0), \bar{p_2})$ and $(p_1^*, p_2^*) = (\bar{p_2} - g'(q_0/(\alpha N)), \bar{p_2})$ will not happen, and we have four cases for the optimal solutions as shown in Table 2.5.

Table 2.5: Optimal prices and shifted consumption with a quadratic shift cost for the model with price inelasticity of total demand

Cases	(p_1^*, p_2^*)	q_s^*	$\Delta\Pi_c$
$\bar{p_2} \geq p_2^B$	$(p_2^B - \theta, p_2^B)$	$\frac{\alpha N \theta}{2c_s}$	$= 0$
$(p_2^A \leq \bar{p_2} \leq p_2^B$ and $p_2^D \leq 0)$ or $(p_2^C \leq \bar{p_2} \leq p_2^B$ and $p_2^D \geq 0)$	$(p_1^E, \bar{p_2})$	$\frac{-2c_s q_1 + \sqrt{(2c_s q_1)^2 + 4c_s N w}}{2c_s}\alpha$	$= 0$
$p_2^D \leq \bar{p_2} \leq p_2^C$ and $p_2^D \geq 0$	$(p_1^F, \bar{p_2})$	$\frac{\alpha N}{2c_s}(\frac{\gamma}{2} - \frac{c_s q_1}{N})$	< 0
$\bar{p_2} \leq p_2^D$ and $p_2^D \geq 0$	$(0, \bar{p_2})$	$\frac{\alpha N}{2c_s}\bar{p_2}$	< 0

Here, $w = (\bar{p_2} - \bar{p_0})(q_1 + q_2)$, $p_2^B = \bar{p_0} + \theta(N\theta + 4c_s q_1)/(4c_s(q_1 + q_2))$, $p_1^E = \bar{p_2} - (-2c_s q_1 + \sqrt{(2c_s q_1)^2 + 4c_s N w})/N$, $p_2^C = \bar{p_0} + (N(\gamma/2 - c_s q_1/N)(\gamma/2 + 3c_s q_1/N))/(4c_s(q_1 + q_2))$, and $p_1^F = \bar{p_2} - (\gamma/2 - c_s q_1/N)$.

Proposition 2.2 *The effects of α on (p_1^*, p_2^*), q_s^*, Π_g, and $\Delta\Pi_c$ are presented in Table 2.6, where $T_1 = (-2c_s q_1 + \sqrt{(2c_s q_1)^2 + 4c_s N w})/(2c_s)$ and $T_2 = w - \bar{p_2}(2c_s q_1 + (\bar{p_2} - \gamma)N)/(2c_s)$. For all four cases, $dk_1^*/d\alpha = -(1/\tau)(dq_s^*/d\alpha) \leq 0$*

Table 2.6: The effects of α on $p_1^*, p_2^*, q_s^*, \Pi_g$, and $\Delta\Pi_c$ for the model with price inelasticity of total demand

(p_1^*, p_2^*)	$\frac{dp_1^*}{d\alpha}$	$\frac{dp_2^*}{d\alpha}$	$\frac{dq_s^*}{d\alpha}$	$\frac{d\Pi_g}{d\alpha}$	$\frac{d\Delta\Pi_c}{d\alpha}$
$(p_2^B - \theta, p_2^B)$	0	0	$\frac{N\theta}{2c_s} \geq 0$	$\frac{1}{2}N\frac{\theta}{2c_s}(2\gamma - \theta) \geq 0$	0
$(p_1^E, \bar{p_2})$	0	0	$T_1 \geq 0$	$\frac{dq_s^*}{d\alpha}(\gamma - \frac{c_s}{N}\frac{dq_s^*}{d\alpha}) \geq 0$	0
$(p_1^F, \bar{p_2})$	0	0	$\frac{N}{2c_s}(\frac{\gamma}{2} - \frac{c_s q_1}{N}) \geq 0$	$w + \frac{N}{2c_s}(\frac{\gamma}{2} - \frac{c_s q_1}{N})^2 \geq 0$	0
$(0, \bar{p_2})$	0	0	$\frac{N}{2c_s}\bar{p_2} \geq 0$	$T_2 \geq 0$	0

and $dk_2^*/d\alpha = 0$ if $\tau\beta_2 + c_2 < \tau\beta_1 + c_1$, $dk_1^*/d\alpha = (1/(1-\tau))(dq_s^*/d\alpha) \geq 0$ and $dk_2^*/d\alpha = -(1/(\tau(1-\tau)))(dq_s^*/d\alpha) \leq 0$ if $\tau\beta_2 + c_2 \geq \tau\beta_1 + c_1$, and $d(k_1^* + k_2^*)/d\alpha = -(1/\tau)(dq_s^*/d\alpha) \leq 0$.

One might expect that the company will increase the price in the non-peak period for customers under the TOU tariff if more customers use the TOU tariff, as the customers under the TOU tariff have the incentive to shift some consumption from the peak period to the non-peak period. However, our results show that the company will keep the prices unchanged when more customers use the TOU tariff, so the amount of shifted consumption by a customer is fixed. Consequently, while more consumption will be shifted from the peak period to the non-peak period as more customers use the TOU tariff, the company needs to install less capacity for the second technology, which is dedicated for the peak period demand, resulting in less total capacity for the peak period demand. From the customers' perspective, the customers who already use the TOU tariff will neither be hurt nor obtain additional benefit when more customers use the TOU tariff, as the prices for the TOU tariff are unchanged and the amount of shifted consumption by a customer is fixed if more customers use the TOU tariff. The customers who change to use the TOU tariff from the FR tariff may save electricity cost, as we have imposed the constraint $\Delta\Pi_c \leq 0$. The customers who still use the FR tariff will not be hurt either, as the price for FR tariff is also unchanged. From the company's perspective, we have the following finding:

Corollary 2.1 *The company can get more profit if more customers use the TOU tariff.*

2.5 Price Elasticity of Demand

There are some empirical studies on the price elasticity of demand for electricity (e.g., Filippini 1995, Fan and Hyndman 2011). And some researchers treat electricity demand as a function of the price in their models, such as Chao (2011a,b) and Greer (2012). In this section we consider a model with price elasticity of demand, i.e., the demand for electricity depends on prices.

2.5.1 Modelling

Recall that in the model with price inelasticity of total demand, we consider both the customer's problem and the company's problem, and we determine the shifted consumption for the customer's problem. However, the scenario is different here. In the model with price elasticity of demand, we do not consider the customer's problem. Alternatively, the electricity consumption in the non-peak and peak periods is directly reflected by the demand functions.

Let $D_{T1}(p_1, p_2)$ and $D_{T2}(p_1, p_2)$ be the demands in the non-peak period and the peak period, respectively, under the TOU tariff. Under the FR tariff, they are $D_{F1}(p_0)$ and $D_{F2}(p_0)$, respectively. We consider a linear demand function here, i.e., an additive demand function (e.g., $D(p) = a - bp$) is adopted. Examples of linear demand functions for electricity consumption can be found in Chao (2011a,b) and Greer (2012). Specifically, we let $D_{F1}(p_0) = a_{F1} - b_{F1}p_0$, $D_{F2}(p_0) = a_{F2} - b_{F2}p_0$, $D_{T1}(p_1, p_2) = a_{T1} - b_{T1}p_1 + r_1p_2$, and $D_{T2}(p_1, p_2) = a_{T2} - b_{T2}p_2 + r_2p_1$. It is reasonable to model that for the FR tariff, the electricity demands are decreasing in the price. It is also reasonable to model that for the TOU tariff, the electricity demand in the non-peak period is decreasing in the price in the non-peak period and increasing in the price in the peak period. The electricity demand in the peak period is decreasing in the price in the peak period and increasing in the price in the non-peak period. Table 2.7 presents customers' electricity demands under the two tariffs.

Table 2.7: Demand in the model with price elasticity of demand

Tariff	**Demand in the non-peak period**	**Demand in the peak period**
FR tariff	$D_{F1}(p_0) = a_{F1} - b_{F1}p_0$	$D_{F2}(p_0) = a_{F2} - b_{F2}p_0$
TOU tariff	$D_{T1}(p_1, p_2) = a_{T1} - b_{T1}p_1 + r_1p_2$	$D_{T2}(p_1, p_2) = a_{T2} - b_{T2}p_2 + r_2p_1$

Assumption 2.1 *(a) $b_{T1} \geq r_1$ and $b_{T2} \geq r_2$; (b) $b_{T1} \geq r_2$ and $b_{T2} \geq r_1$.*

Part (a) of Assumption 2.1 is the dominant assumption. It stipulates the relationships among the price and cross-price sensitivity parameters, which are treated as common constraints in the literature (e.g., Maglaras and Meissner 2006). The assumption states that the demand in each period is more sensitive to a change in its own price than it is to a simultaneous change in the prices of the other period. Part (b) indicates that the reduced demand from one period due to the price increase in this period is no less than the increased demand in the other period.

Note that the price elasticity of demand is determined by $E_d = pD'(p)/D(p)$. Here, the price elasticity of customer demands in the non-peak and peak periods under the FR tariff are determined by $-b_{F1}p_0/(a_{F1} - b_{F1}p_0)$ and $-b_{F2}p_0/(a_{F2} - b_{F2}p_0)$, respectively. If $b_{F1} = 0$ ($b_{F2} = 0$), then the customer demand under the FR tariff in the non-peak (peak) period is perfectly inelastic; otherwise, it exhibits some elasticity.

Define $D_1(p_0, p_1, p_2) = \alpha D_{T1}(p_1, p_2) + (1-\alpha)D_{F1}(p_0)$ and $D_2(p_0, p_1, p_2) = \alpha D_{T2}(p_1, p_2) + (1-\alpha)D_{F2}(p_0)$ as the aggregate demands in the non-peak period and the peak period, respectively. Similar forms of the aggregated demand function for the two tariffs can be found in Borenstein and Holland (2005). As shown in Equation (2.5), the objective function of the company's problem in this model is given by

$$\begin{aligned}\Pi_g(\mathbf{k}, \mathbf{p}) \quad = \quad & \alpha\Big(p_1 D_{T1}(p_1, p_2) + p_2 D_{T2}(p_1, p_2)\Big) + (1-\alpha)p_0\Big(D_{F1}(p_0) + D_{F2}(p_0)\Big) \\ & - C_g\Big(k_1, k_2, D_1(p_0, p_1, p_2), D_2(p_0, p_1, p_2)\Big).\end{aligned}$$

2.5.2 Analysis and solution

As before, we use the sequential decision-making approach to solve the problem under this model. First, we assume that the prices are given, under which we solve the problem and obtain the optimal responses of capacities, i.e., $(k_1(\mathbf{p}), k_2(\mathbf{p}))$. In the second step, we obtain the optimal prices, i.e., (p_0^*, p_1^*, p_2^*), given the optimal responses of capacities.

(1) Capacity decisions

By analyzing the objective function, we obtain the optimal responses of capacities, which are presented in Theorem 2.3.

Theorem 2.3 *The optimal responses of capacities $(k_1(\mathbf{p}), k_2(\mathbf{p}))$ are determined as follows:*

$$\big(k_1(\mathbf{p}), k_2(\mathbf{p})\big) = \begin{cases} \left(\frac{D_1(p_0,p_1,p_2)}{1-\tau}, \frac{D_2(p_0,p_1,p_2)}{\tau} - \frac{D_1(p_0,p_1,p_2)}{1-\tau}\right) & \text{if } \frac{D_1(p_0,p_1,p_2)}{1-\tau} \leq \frac{D_2(p_0,p_1,p_2)}{\tau} \\ & \text{and } \tau\beta_2 + c_2 < \tau\beta_1 + c_1; \\ \left(\frac{D_2(p_0,p_1,p_2)}{\tau}, 0\right) & \text{if } \frac{D_1(p_0,p_1,p_2)}{1-\tau} \leq \frac{D_2(p_0,p_1,p_2)}{\tau} \\ & \text{and } \tau\beta_2 + c_2 \geq \tau\beta_1 + c_1; \\ \left(\frac{D_1(p_0,p_1,p_2)}{1-\tau}, 0\right) & \text{if } \frac{D_1(p_0,p_1,p_2)}{1-\tau} > \frac{D_2(p_0,p_1,p_2)}{\tau}. \end{cases}$$

Theorem 2.3 gives the optimal capacities for the company. The proportion of customers using the TOU tariff will affect the capacity decisions in the model with price elasticity of demand because different types of customers have different demand functions, which affect capacity investment. If the total demand in the peak period is small, i.e., $D_2(p_0, p_1, p_2) < \tau D_1(p_0, p_1, p_2)/(1-\tau)$, then the company only needs to install Technology 1, which can meet the demands in both the non-peak period and the peak period. Even if the total demand in the peak period is large, i.e., $D_2(p_0, p_1, p_2) \geq \tau D_1(p_0, p_1, p_2)/(1-\tau)$, then the company may still not install Technology 2 because of its high cost. If the cost of Technology 2 is low, then the company will install both technologies with total capacity $D_2(p_0, p_1, p_2)/\tau$. Similar to the model with price inelasticity of total demand, in this model, we define three cases for the optimal capacities, which are pictorially shown in Figure 2.4 (where $\bar{D}_2$ is an upper bound on D_2, which can be obtained when the prices reach the lower bound).

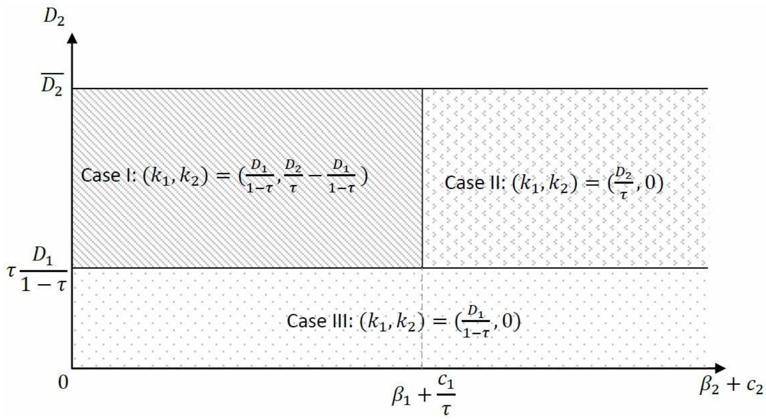


Figure 2.4: Three cases of the model with price elasticity of demand

Case I: $(k_1(\mathbf{p}), k_2(\mathbf{p})) = (\frac{D_1(p_0,p_1,p_2)}{1-\tau}, \frac{D_2(p_0,p_1,p_2)}{\tau} - \frac{D_1(p_0,p_1,p_2)}{1-\tau})$. In this case, the company installs both technologies to generate electricity. This case happens when the total demand in the peak period is large, i.e., $D_2(p_0, p_1, p_2) \geq \tau D_1(p_0, p_1, p_2)/(1-\tau)$, and the cost of Technology 2 is small, i.e., $\tau\beta_2 + c_2 < \tau\beta_1 + c_1$.

Case II: $(k_1(\mathbf{p}), k_2(\mathbf{p})) = (\frac{D_2(p_0,p_1,p_2)}{\tau}, 0)$. In this case, the company only installs Technology 1 to generate electricity, which can meet the consumption in both the non-peak period and the peak period. This case happens when the total demand in the peak period and the cost of Technology 2 are large, i.e., $D_2(p_0, p_1, p_2) \geq \tau D_1(p_0, p_1, p_2)/(1-\tau)$ and $\tau\beta_2 + c_2 \geq \tau\beta_1 + c_1$, respectively.

Case III: $(k_1(\mathbf{p}), k_2(\mathbf{p})) = (\frac{D_1(p_0,p_1,p_2)}{1-\tau}, 0)$. In this case, the company still only installs Technology 1 to generate electricity. This case happens when the total demand in the peak period is small, i.e., $D_2(p_0, p_1, p_2) < \tau D_1(p_0, p_1, p_2)/(1-\tau)$.

(2) Price decisions

After obtaining the optimal responses of capacities, we solve the problem and obtain the optimal prices for the three cases. With the substitution of the optimal responses of capacities into the company's profit function, our objective becomes maximizing the profit function by optimally setting the electricity prices. By analyzing the objective functions of three cases, we obtain the optimal prices, which are presented in Theorem 2.4.

Theorem 2.4 *The optimal values of prices* $\mathbf{p}$ *are determined by* $p_0^* = \min\{\max\{\hat{p_0}, 0\}, \bar{p_0}\}$, $p_1^* = \min\{\max\{\hat{p_1}, 0\}, p_0^*\}$, *and* $p_2^* = \min\{\max\{\hat{p_2}, p_0^*\}, \bar{p_2}\}$, *where*

$$\hat{p_0} = \begin{cases} \frac{1}{2(b_{F1}+b_{F2})}\Big(a_{F1} + a_{F2} + \frac{1}{1-\tau}(\beta_1 + c_1 - \tau\beta_2 - c_2)b_{F1} + (\beta_2 + \frac{c_2}{\tau})b_{F2}\Big) & \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \textit{for Case I;} \\ \frac{1}{2(b_{F1}+b_{F2})}\Big(a_{F1} + a_{F2} + \beta_1(b_{F1} + b_{F2}) + \frac{c_1}{\tau}b_{F2}\Big) & \textit{for Case II;} \\ \frac{1}{2(b_{F1}+b_{F2})}\Big(a_{F1} + a_{F2} + \beta_1(b_{F1} + b_{F2}) + \frac{c_1}{1-\tau}b_{F1}\Big) & \textit{for Case III.} \end{cases}$$

$$\hat{p_1} = \begin{cases} \frac{1}{4b_{T1}b_{T2}-(r_1+r_2)^2}\Big((r_1 + r_2)A_6 + 2b_{T2}A_5\Big) & \textit{for Case I;} \\ \frac{1}{4b_{T1}b_{T2}-(r_1+r_2)^2}\Big((r_1 + r_2)A_4 + 2b_{T2}A_3\Big) & \textit{for Case II;} \\ \frac{1}{4b_{T1}b_{T2}-(r_1+r_2)^2}\Big((r_1 + r_2)A_2 + 2b_{T2}A_1\Big) & \textit{for Case III.} \end{cases}$$

$$\hat{p_2} = \begin{cases} \frac{1}{4b_{T1}b_{T2}-(r_1+r_2)^2}\Big((r_1 + r_2)A_5 + 2b_{T1}A_6\Big) & \textit{for Case I;} \\ \frac{1}{4b_{T1}b_{T2}-(r_1+r_2)^2}\Big((r_1 + r_2)A_3 + 2b_{T1}A_4\Big) & \textit{for Case II;} \\ \frac{1}{4b_{T1}b_{T2}-(r_1+r_2)^2}\Big((r_1 + r_2)A_1 + 2b_{T1}A_2\Big) & \textit{for Case III.} \end{cases}$$

$$A_1 = a_{T1} + (\beta_1 + \frac{c_1}{1-\tau})b_{T1} - \beta_1 r_2, A_2 = a_{T2} + \beta_1 b_{T2} - (\beta_1 + \frac{c_1}{1-\tau})r_1,$$
$$A_3 = a_{T1} + \beta_1 b_{T1} - (\beta_1 + \frac{c_1}{\tau})r_2, A_4 = a_{T2} + (\beta_1 + \frac{c_1}{\tau})b_{T2} - \beta_1 r_1,$$

$$A_5 = a_{T1} + \frac{1}{1-\tau}\big(\beta_1 + c_1 - \tau\beta_2 - c_2\big)b_{T1} - (\beta_2 + \frac{c_2}{\tau})r_2,$$
$$A_6 = a_{T2} + (\beta_2 + \frac{c_2}{\tau})b_{T2} - \frac{1}{1-\tau}\big(\beta_1 + c_1 - \tau\beta_2 - c_2\big)r_1.$$

Theorem 2.4 shows the optimal prices for the company under the three cases. In any case, the optimal interior solutions are uniquely determined by the first-order condition of the company's profit function. These results are very different from those for the model with price inelasticity of total demand.

We can solve the problem under the model with price elasticity of demand by the following procedures: Given the parameters such as the unit production and capacity costs, we determine the optimal capacities and prices for the company according to Theorems 2.3 and 2.4. We then examine the solutions to see whether they satisfy the conditions of the cases. If the results satisfy the conditions of more than two cases, then we pick the solution that generates the most profit for the company.

2.5.3 The effects of proportion of customers using the TOU tariff

In this subsection we consider the effects of the proportion of customers who use the TOU tariff on the optimal decisions and the company's profit. For notational simplicity, we define $D_1^* = D_1(p_0^*, p_1^*, p_2^*)$, $D_2^* = D_2(p_0^*, p_1^*, p_2^*)$, $D_{F1}^* = D_{F1}(p_0^*)$, $D_{F2}^* = D_{F2}(p_0^*)$, $D_{T1}^* = D_{T1}(p_1^*, p_2^*)$, and $D_{T2}^* = D_{T2}(p_1^*, p_2^*)$.

Proposition 2.3 *The effects of α on (k_1^*, k_2^*), (p_0^*, p_1^*, p_2^*), and Π_g are as follows:*

$$\begin{cases} \frac{dk_1^*}{d\alpha} = \frac{D_{T1}^* - D_{F1}^*}{1-\tau},\ \frac{dk_2^*}{d\alpha} = \frac{D_{T2}^* - D_{F2}^*}{\tau} - \frac{D_{T1}^* - D_{F1}^*}{1-\tau} & \text{for Case I;} \\ \frac{dk_1^*}{d\alpha} = \frac{D_{T2}^* - D_{F2}^*}{\tau},\ \frac{dk_2^*}{d\alpha} = 0 & \text{for Case II;} \\ \frac{dk_1^*}{d\alpha} = \frac{D_{T1}^* - D_{F1}^*}{1-\tau},\ \frac{dk_2^*}{d\alpha} = 0 & \text{for Case III.} \end{cases}$$

$$\frac{d\Pi_g}{d\alpha} = p_1^* D_{T1}^* + p_2^* D_{T2}^* - p_0^*(D_{F1}^* + D_{F2}^*)$$
$$- \begin{cases} \Big(\frac{\beta_1 + c_1 - \tau\beta_2 - c_2}{1-\tau}(D_{T1}^* - D_{F1}^*) + (\beta_2 + \frac{c_2}{\tau})(D_{T2}^* - D_{F2}^*)\Big) & \text{for Case I;} \\ \Big(\beta_1(D_{T1}^* - D_{F1}^* + D_{T2}^* - D_{F2}^*) + \frac{c_1}{\tau}(D_{T2}^* - D_{F2}^*)\Big) & \text{for Case II;} \\ \Big(\beta_1(D_{T1}^* - D_{F1}^* + D_{T2}^* - D_{F2}^*) + \frac{c_1}{1-\tau}(D_{T1}^* - D_{F1}^*)\Big) & \text{for Case III.} \end{cases}$$

For all the three cases, (p_0^*, p_1^*, p_2^*) *are independent of* α *and* $d^2\Pi_g/d\alpha^2 = 0$.

An interesting result is that, for all the three cases, the optimal prices are independent of the proportion of customers using the TOU tariff. This may be because the demand under one tariff is not affected by the price under the other tariff. As the company's profit under the optimal decisions is a linear function of α, it may increase or decrease with α, depending on the values of the parameters for the demand functions and the values of the costs.

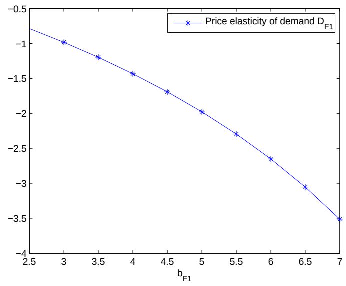
2.5.4 The effects of price elasticity of demand

We resort to numerical studies to gain an understanding of the effects of price elasticity of demand on the price and capacity decisions, and the associated profit. Recall that price elasticity of demand is determined by $E_d = pD'(p)/D(p)$. So the price elasticity of demand in the non-peak and peak periods under the FR tariff are determined by $E_d^{F1} = -b_{F1}p_0^*/(a_{F1} - b_{F1}p_0^*)$ and $E_d^{F2} = -b_{F2}p_0^*/(a_{F2} - b_{F2}p_0^*)$, respectively, and under the TOU tariff are determined by $E_d^{T1} = -b_{T1}p_1^*/(a_{T1} - b_{T1}p_1^* + r_1p_2^*)$ and $E_d^{T2} = -b_{T2}p_2^*/(a_{T2} - b_{T2}p_2^* + r_2p_1^*)$, respectively.

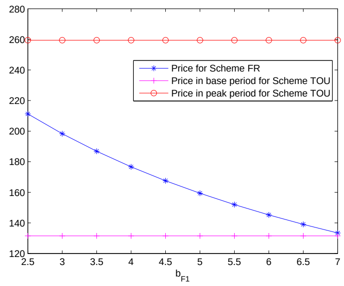
Unlike the multiplicative demand function, where elasticity is a constant independent of price, the elasticity of the additive demand function is related to price. So it is very complicated to analyze the effects of price elasticity of the additive demand function directly. On the other hand, for an additive demand function, such as $D(p) = a - bp$, demand is decreasing in b, given p. To a certain extent, the parameter of price sensitivity, i.e., b, reflects the price elasticity of demand. So, in this subsection, we study the effects of price elasticity of demand through investigating the effects of price sensitivity parameters, i.e., b_{F1}, b_{F2}, b_{T1}, and b_{T2}.

In all the numerical examples, we set $a_{F1} = 1200, a_{F2} = 1800, a_{T1} = 1300, a_{T2} = 1700, r_1 = 1, r_2 = 1, \tau = 1/3, \alpha = 1/3, \beta_1 = 5, \beta_2 = 10, c_1 = 10$, and $c_2 = 20/3$. We set $b_{T1} = 7$ and $b_{T2} = 4$; let b_{F1} change values within $\{2.5, 3, 3.5, \cdots, 6.5, 7\}$ with $b_{F2} = 5$ to assess the effects of price elasticity associated with changes in b_{F1}, and let b_{F2} change values within $\{2.5, 3, 3.5, \cdots, 6.5, 7\}$

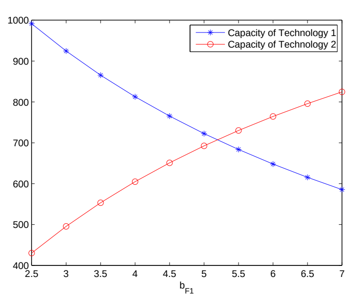
with $b_{F1} = 4$ to assess the effects of price elasticity associated with changes in b_{F2}. Regarding the effects of b_{T1} and b_{T2}, we set $b_{F1} = 3$ and $b_{F2} = 4$. We let b_{T1} change values within $\{4.5, 5, 5.5, \cdots, 8.5, 9\}$ with $b_{T2} = 4$ to assess the effects of price elasticity associated with changes in b_{T1}, and let b_{T2} change values from $\{2.2, 2.4, 2.6, \cdots, 3.8, 4\}$ with $b_{T1} = 7$ to assess the effects of price elasticity associated with changes in b_{T2}.



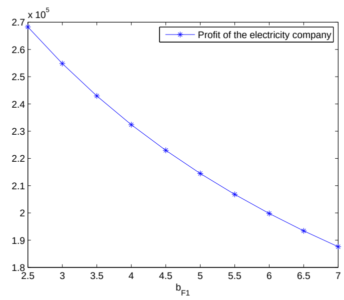
(a) Price elasticity of demand



(b) Prices for the FR and TOU tariffs



(c) Capacity investment

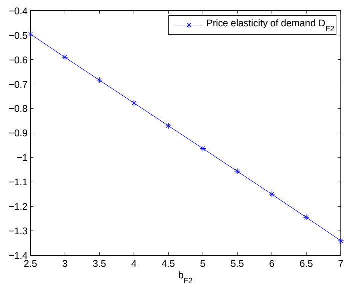


(d) Company's profit

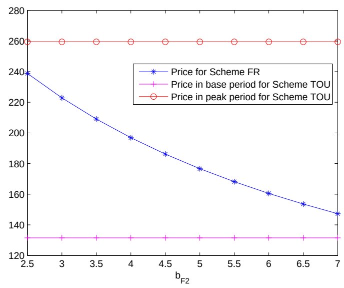
Figure 2.5: Effects of price elasticity of demand associated with changes in b_{F1}.

The effects of price elasticity of demand under the FR tariff are shown in Figures 2.5 and 2.6. The price elasticity of demand in the non-peak (peak) period under the FR tariff, i.e., E_d^{F1} (E_d^{F2}), decreases as price sensitivity b_{F1} (b_{F2}) increases. This means that the demand in the non-peak (peak) period under the FR tariff is more elastic when it is more sensitive to price. On the other hand, the price for the FR tariff and the company's profit decrease when the price sensitivity b_{F1} (b_{F2}) increases. Consequently, we conclude that the price for the FR tariff and the company's profit both decrease when the demand under the FR tariff is more elastic to price.

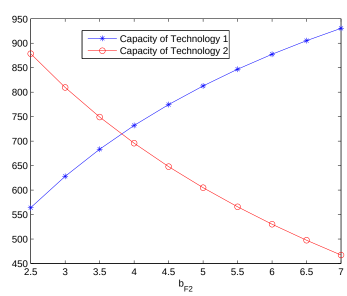
Figure 2.5(c) shows that the capacity of Technology 1 (2) decreases (increases)



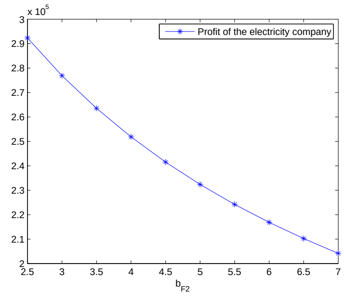
(a) Price elasticity of demand



(b) Prices for the FR and TOU tariffs



(c) Capacity investment

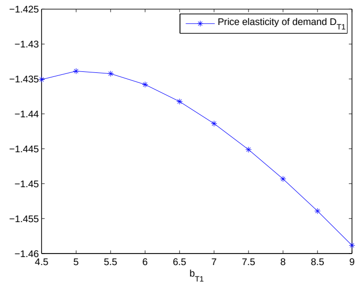


(d) Company's profit

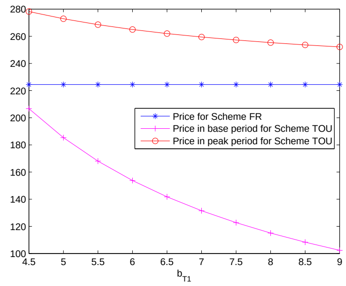
Figure 2.6: Effects of price elasticity of demand associated with changes in b_{F2}.

in b_{F1}. It is because, on the one hand, the demand in the non-peak period under the FR tariff, which directly determines the capacity of Technology 1, decreases when customers are more sensitive to price. On the other hand, the demand in the peak period under the FR tariff increases as price decreases. So the capacity of Technology 2 increases because it equals $D_2/\tau - D_1/(1-\tau)$ here. The changes in capacity as shown in Figure 2.6(c) can be explained similarly.

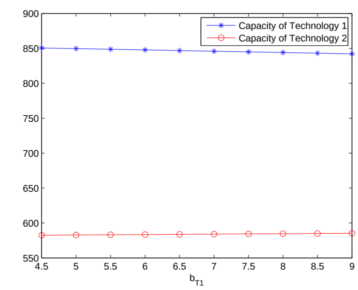
Figures 2.7 and 2.8 show the effects of price elasticity of demand under the TOU tariff. After a slight increase, the price elasticity of demand in the non-peak period under the TOU tariff, i.e., E_d^{T1}, decreases as the price sensitivity b_{T1} increases. This result is different from that for price elasticity of demand under the FR tariff, where the price elasticity of demand in the non-peak (peak) period under the FR tariff always decreases in b_{F1} (b_{F2}). Here, the price elasticity of demand in the peak period, i.e., E_d^{T2}, still decreases in the price sensitivity b_{T2}. Similar to the effects of b_{F1} and b_{F2}, both prices (i.e., the prices in the non-peak and the peak period) for the TOU tariff and the company's profit decrease as the price sensitivity b_{T1} and b_{T2} increase. In view of the non-monotonicity of



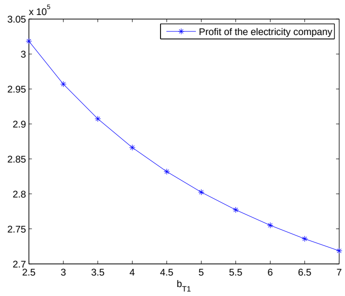
(a) Price elasticity of demand



(b) Prices for the FR and TOU tariffs

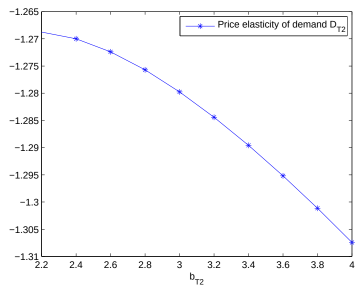


(c) Capacity investment

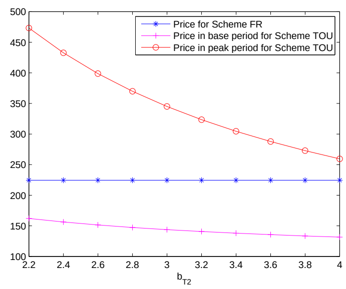


(d) Company's profit

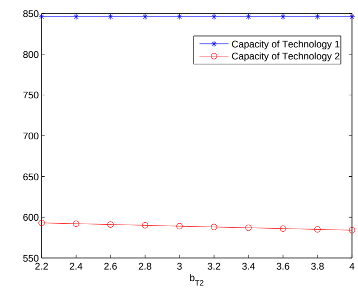
Figure 2.7: Effects of price elasticity of demand associated with changes in b_{T1}.



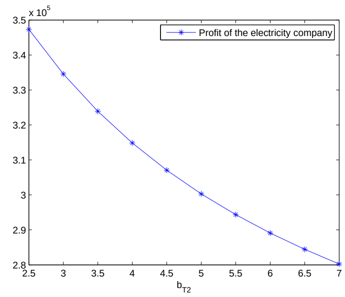
(a) Price elasticity of demand



(b) Prices for the FR and TOU tariffs



(c) Capacity investment



(d) Company's profit

Figure 2.8: Effects of price elasticity of demand associated with changes in b_{T2}.

the price elasticity of demand in the non-peak period under the TOU tariff (see Figure 2.7(a)), we cannot conclude that the prices for the TOU tariff and the company's profit decrease when the demand in the non-peak period under the TOU tariff is more elastic to price.

Figures 2.7(c) and 2.8(c) show the changes in capacity. Unlike the large changes in capacity in Figures 2.5(c) and 2.6(c), the changes in capacity are very small when b_{T1} and b_{T2} increase. The capacity of Technology 1 is even unchanged when b_{T2} increases. Note that the prices for the non-peak and the peak period under the TOU tariff, i.e., p_1 and p_2, decrease as b_{T1} (or b_{T2}) increases, so under the increasing effect of b_{T1} (or b_{T2}) and decreasing effects of p_1 and p_2, the demand in the non-peak period or the peak period under the TOU tariff may not change drastically. This leads to small changes in capacity because capacity is mainly determined by demand.

2.6 Conclusions

In this chapter we consider an electricity market in which an electricity company generates electricity for customers under a mixed tariff structure that comprises the FR and TOU tariffs. Under the FR tariff, customers pay a flat price for electricity consumption over the whole period. Under the TOU tariff, customers pay a high price for electricity consumption in the peak period and a low price for electricity consumption in the non-peak period.

We present two models in this chapter. The first model is characterized by price inelasticity of total demand, i.e., price does not affect the total consumption of electricity. Customers under the TOU tariff can save electricity cost by shifting some consumption from the peak period to the non-peak period. We find the optimal shifted consumption for the customers. The second model is characterized by price elasticity of demand, i.e., demand is affected by price. For both models, we find the optimal capacity and pricing policies for the electricity company.

By studying the effects of the proportion of customers using the TOU tariff, we find the following managerial insights, which are useful to electricity companies:

For both the model with price inelasticity of total demand and the model with price elasticity of demand, the company will not change the optimal prices when more customers use the TOU tariff. For the model with the price inelasticity of total demand, even though the amount of shifted consumption by a customer is fixed due to the unchanged optimal prices, the total consumption by all the customers will increase if more customers use the TOU tariff, leading to a decrease in capacity investment for the peak period. For the model with price elasticity of demand, the effects of the proportion of customers using the TOU tariff on capacity investment and the company's profit depend on the parameters of the demand functions. We further analyze the effects of price sensitivity on the model with price elasticity of demand. Our results show that the prices and the company's profit are non-increasing when customers are more sensitive to price.

Chapter 3

Electricity Time-of-use Tariff with Stochastic Shifted Consumption

3.1 Introduction

It is crucial to find ways to promote energy efficiency in the electricity generation and consumption (IEA 2010, ACEEE 2014). Currently, many electricity customers use the traditional flat-rate (FR) tariff, under which the customers pay the same flat price for each unit of electricity consumption. However, this tariff dampens the incentive for the customers to reduce the electricity usage in the peak period.

Reducing peak period demand may save the electricity cost, yield energy save, and improve energy efficiency (York et al. 2007, EPA 2008). First, reducing peak period demand can reduce the electricity load in the peak period, so that the electricity company can avoid additional technology installation and save its electricity costs for the peak period. Second, reducing peak period demand can reduce the transmission loss and save the electricity energy (Triki and Violi 2009, Faruqui et al. 2007). Third, in electricity generation, reducing peak period demand can improve the energy efficiency. It is because that the peak load plants for generating electricity operate on high cost and less energy-efficient fuels, such as natural gas, while the base load plants for generating electricity operate on low cost and more energy-efficient fuels, such as coal (The Electropaedia 2005, WiseGEEK 2013).

Motivated by the fundamental problem of in-efficient pricing mechanism for electricity under the FR tariff, in this chapter we further study another pricing mechanism, i.e., the time-of-use (TOU) tariff, which helps achieve the goal of energy efficiency. In Chapter 2, we have studied the optimal capacity and pricing policies for an electricity company with time-of-use tariff. Although we have shown that the customers have the incentive to shift the electricity consumption from the peak period to the non-peak period under the TOU tariff, the amount of the shifted consumption may be uncertain. So we consider the electricity time-of-use tariff with stochastic shifted consumption in this chapter.

The TOU tariff has been implemented in some countries in Europe, some states in the U.S., and some cities in Asia (RAP 2008, CEA 2009). But the fundamental questions of implementing the TOU tariff with the consideration of stochastic shifted consumption are still unanswered. First, although early work has shown that the TOU tariff can reduce the peak period demand, it is still open for the question: (1) How many capacities should be installed to meet the demands in both the peak and non-peak periods? Second, we also need to answer the question: (2) With the uncertainty of shifted consumption, what should be the optimal electricity prices for the peak and non-peak periods? Third, some electricity markets have a mixed tariff structure under which some customers use the TOU tariff while the others use the FR tariff. Examples can be found in Australia, Canada, and the U.S. (CEA 2009, Prins 2012). But, the proportion of customers using the TOU tariff may be changing. For example, the department of Public Utility Control in Connecticut in the U.S. directed all the utility companies to phase in the mandatory TOU tariff for all the customers. In other words, in each succeeding year, the mandatory TOU tariff would be applied to additional customers based on declining levels of consumption (Friedman 2011, Jessoe and Rapson 2014). On the other hand, the electricity consumption will increase in the future and the electricity market size may expand as well. So the third question is: (3) What are the effects of demands, market size, and proportion of customers using the TOU tariff on the optimal capacity and price decisions? Fourth, it is also unanswered for the question: (4) What are the effects of cost parameters, such as the production, capacity and shortage costs, on the implementation of the TOU tariff? To get an in-depth understanding of the TOU tariff, in this chapter we further investigate the optimal capacity investment and pricing decisions for an electricity company with the TOU tariff, and answer the above fundamental questions.

Similar to the setting in Chapter 2, in this chapter, we consider a vertically integrated electricity company which determines the amounts of the installed capacities and the electricity prices, and is responsible for meeting the market demand for electricity. The electricity company is regulated under the price-cap regulation, which sets an upper bound on an index of the regulated firm's price (Sappington and Sibley 1992, Braeutigam and Panzar 1993, Regulationbodyofknowledge.org 2014). The electricity company offers a mixed tariff structure to the customers. A fraction of customers use the TOU tariff and the remaining fraction of customers use the FR tariff. Two technologies are considered to be installed for generating electricity in the two periods, i.e., the peak and non-peak periods, for the customers. The first technology is used to generate electricity for demands in both periods, while the second technology is used to generate electricity only for the demand in the peak period. We refer to the first and second technologies as the base-load technology and peak-load technology, respectively. The base-load technology (e.g., using coal or nuclear to generate electricity) usually has low production cost and high capacity cost, while the peak-load technology (e.g., using natural gas to generate electricity) usually has high production cost and low capacity cost. It is a common strategy to use the base-load and peak-load technologies to generate electricity in the electricity generation industry (The Electropaedia 2005, Pineau and Zaccour 2007, WiseGEEK 2013). In Chapter 2, we study both the model with price inelasticity of total demand and the model with price elasticity of demand. However, in this chapter, we only consider the setting that the total demand will be unchanged when the TOU tariff is introduced to the customers. We first derive the optimal capacity investments and the optimal prices for the TOU tariff. After that, we analyze the effects of the demands, market size, proportion of customers using the TOU tariff, and cost parameters on the optimal solutions.

In summary, this chapter makes the following contributions:

1. We model the TOU tariff by considering the customers' uncertain behaviour of shifting the electricity consumption. Our work not only methodologically relates to the Operations Management/Operations Research (OM/OR) literature, but also contextually relates to the OM/OR, Energy, and Economics literatures. By studying the interfaces of these bodies of work, we develop novel insights for the electricity company to implement the TOU tariff.

2. We obtain the optimal capacity investments for the peak and non-peak periods. The costs play critical roles in rationing the capacities to meet the demands. The capacity of Technology 2 and total capacity for the peak period demand both increase in the price for the non-peak period and decrease in the price for the peak period, for the TOU tariff. We also obtain the optimal prices for the TOU tariff. There is a unique optimal value of the price in the non-peak period, while there are three possible optimal values for the price in the peak period, depending on the price sensitivity parameters and the upper bound of the price in the peak period (i.e., price-cap set by the regulator). It is interesting to show that it may not be optimal to the electricity company to let the customers shifting too much consumption from the peak period to the non-peak period.

3. We analytically examine the behaviour of the optimal solutions with respect to the demands, electricity company's market size, proportion of customers using the TOU tariff, and cost parameters (such as production, capacity and shortage costs). Important insights and managerial implications are discussed.

The remainder of this chapter is structured as follows: In Section 3.2, we review the related literature. In Section 3.3, we present the model setting of this chapter. In Section 3.4, we study the optimal capacity investment and pricing decisions. In Section 3.5, we do some comparative statics with respect to the demands, market size, proportion of customers using the TOU tariff, and cost parameters. Conclusions are given in Section 3.6. We provide all the proofs in Appendix B.

3.2 Literature Review

Our work is related to three streams of research. The first one is the literature on the time-varying electricity prices. In the Economics and Energy literature, some papers consider the customer or demand response to time-varying electricity prices, such as Henley and Peirson (1994), Faruqui and George (2005), Herter at al. (2007), Chao (2010), and Faruqui and Sergici (2010); some other papers study the other effects of time-varying electricity prices (e.g., the effects on capacity investments, wholesale prices), such as Holland and Mansur (2005), Borenstein and Holland (2005), Faruqui et al. (2007), Pineau and Zaccour (2007), and Chao (2011a). These studies highlight the importance of the time-varying electricity prices, but most of them do not model the customer's shifting behaviour.

In the OM/OR literature, Garcia et al. (2005) examine the dynamic pricing and learning in an infinite-horizon oligopoly model in electricity markets. With a transfer function model, Nogales and Conejo (2006) study the electricity price forecasting based on both past electricity prices and demands. Triki and Violi (2009) investigate the dynamic pricing of electricity in retail markets, through a two-stage pricing scheme where the customer is offered a first-stage TOU tariff and then a dynamic component once the real-time demand is observed. Banal-Estañol and Micola (2009) study by simulation how the diversification of electricity generation portfolios affect wholesale prices. However, most of these papers do not consider the customer behaviour of shifting electricity consumption from a period of higher price to a period of lower price. Yang et al. (2013) analyze the TOU tariff for an electricity company taking the customer behaviour into consideration. Yet unlike our model, theirs does not consider a mixed tariff structure where some customers use the TOU tariff and the rest of the customers use the FR tariff; and theirs does not consider the uncertainty in the shifted consumption.

The second stream of research related to our work is the capacity choice and peak load pricing in electricity markets in the Economics literature. This framework considers the pricing and capacity investment problems with the stochastic demand and diverse technologies with different cost characteristics. For instance, Carlton (1977) and Crew and Kleindorfer (1978) investigate some peak load pricing problems with stochastic demand, which are extended by Chao (1983) and Kleindorfer and Fernando (1993) to consider supply uncertainty. Comprehensive reviews of this subject can be found in Crew et al. (1995). This research stream is structurally similar to our setting of the TOU tariff, but our work is mainly from the electricity company's perspective and the other studies are from the social welfare perspective. Besides, our work explicitly models the shift consumption behaviour due to the price difference in the two periods, and considers the FR tariff co-existing with the TOU tariff, which allows us to study the effects of the proportion of customers using TOU tariff and other cost parameters.

In this chapter, we consider to use two technologies, as base-load and peak-load technologies, to generate the electricity, and study the capacity investments in technologies with uncertainty, so the third related stream of research includes the works on the investment in technologies. In the OM/OR literature, many papers have modified or extended the classic newsvendor model and studied the strategic capacity management (e.g., Van Mieghem 1998, Harrison and Van Mieghem 1999, Van Mieghem and Rudi 2002). Van Mieghem (2003) provides a review on the literature of strategic capacity management, which is concerned with determining the types, sizes, and timing of capacity investments and adjustments under uncertainty.

This framework has been extended to other related settings. For example, Goyal and Netessine (2007) study the technology choice and capacity investment of two firms with stochastic price-dependent demand in a competitive environment. Yang et al. (2011) examine how market uncertainty, costs, and operation timing affect a firm's strategic decisions on the flexible technology and flexible capacity. Boyabatli and Toktay (2011) consider a monopolistic firm that decides the technology choice and capacity level with demand uncertainty in imperfect capital markets, where the firm is budget constrained which can be relaxed by borrowing money from a creditor. Kashefi (2012) investigates the effect of a non-sale capacity market on the decisions of the technology choice and capacity investment of two firms with competition and uncertain demand. Recently, there has been growing literature on technology choice and capacity investment in the energy market, and on environmental issues. For instance, Sönmez et al. (2012) study the strategic technology selection, choices around technology configuration and capacity for the incumbent and emerging technologies in the liquefied natural gas industry. By modelling the trade-off between renewable and nonrenewable technologies, Aflaki and Netessine (2012) investigate the incentives for investing in renewable electricity generating capacity. Drake et al. (2012) study the technology choice and capacity investment under emission tax and emission cap-and-trade regulation, through a two-stage model where the firm determines capacities in two technologies in the first stage; demand information is realized given at certain time between two stages; and then the firm determines production quantities in the second stage. Filomena et al. (2014) analyze the technology selection and capacity investment for electricity generation in a competitive market with the consideration of uncertain marginal cost.

In the Energy Economics literature, Wickart and Madlener (2007) develop an economic model to examine the optimal technology choice and investment timing with the consideration of cost (e.g., input fuel price) uncertainty. Westner and Madlener (2012) use a spread-based real options approach to study the investment in a condensing power plant without heat utilization or a plant with combined heat-and-power generation. Tuthill (2008) and Schwerin (2013) investigate the effects of emission costs on the investments of dirty and clean technologies.

One important feature distinguishing our work from the studies in the investment in technologies is that these papers do not consider the pricing issues, while our work considers the time-varying electricity pricing mechanism to reduce the demand in the peak period. Of the exceptions, Bish and Wang (2004), Chod and Rudi (2005), Biller et al. (2006) and Bish et al. (2012) study the capacity investment with a pricing issue, but focus on responsive pricing (or price postponement) where the pricing decision is made after the demands are realized. However, in our work the electricity prices are determined before realizing the demand information.

3.3 Modelling

In this chapter we consider a vertically integrated electricity company which not only determines the capacity for generating electricity, but also sets the electricity prices for the customers. We consider a scenario that the time of electricity usage is divided into two periods: the peak period and the non-peak period. Let T denote the total period time, e.g., one day, and let τ denote the percentage of the peak period time over the total period time. Without loss of generality, we normalize $T = 1$. The fact that the peak-load capacity is only used "few" hours a day has been empirically observed, consequently, we assume that $\tau < 1/2$ (Pineau and Zaccour 2007).

Two technologies are considered to be installed for generating electricity, i.e., Technologies 1 and 2. Let k_i denote the capacity needed to be installed for Technology i, $i \in \{1, 2\}$ (e.g., in megawatt, which is a unit to measure the rate of energy conversion or transfer), and $\mathbf{k} = (k_1, k_2)$. Similar to the setting in Yang et al. (2013) and Pineau and Zaccour (2007), we assume that Technology 1 will be used all the time during the whole period, while Technology 2 will be used only during the peak period. So k_1 and k_2 are generally referred to as base-load and peak-load capacities, respectively. Then $(1-\tau)k_1$ is the installed capacity of Technology 1 for the non-peak period demand, and τk_1 is the installed capacity of Technology 1 for the peak period demand. Note that electricity generation capacity is measured in units of power, e.g., megawatt, in other words, we can say that it is measured as a maximum rate of energy per unit time, so τk_2 (rather than k_2) is the capacity of Technology 2 for the peak-period demand. We then let $k_p = \tau(k_1 + k_2)$, which is the total capacity for the peak period demand.

Let c_i and β_i denote the unit capacity cost and unit production cost of Technology i, respectively. It is well known that the peak-load technology, i.e., Technology 2, typically has lower capacity cost and higher production cost comparing with the base-load technology, i.e., Technology 1, thus we have $c_1 > c_2$ and $\beta_1 < \beta_2$ (Crew et al. 1995, Pineau and Zaccour 2007). The shortage cost will be incurred if the demands exceed the installed capacities. Let v_1 and v_2 denote the unit shortage costs for the non-peak period and peak period demands, respectively. On one hand, the shortage costs could be referred to as costs of the operating reserves. Here, *operating reserves* are often referred to as ancillary services to help ensure grid reliability for the electric power system, by keeping partially loaded spinning generators available to respond to random variation in demand and system contingencies (Hummon et al. 2013). Hummon et al. (2013) use a simulation tool to evaluate the cost of operating reserve services, and find that the total cost of providing reserves adds about 2% to the total cost of providing energy. On the other hand, the shortage costs can be referred to as the electricity prices to purchase the additional electricity from outside markets. Therefore, it is reasonable to assume that $v_1 \geq \beta_1$ and $v_2 \geq \beta_2$. Furthermore, we assume $v_1 \leq \beta_2 + c_2$, indicating that the electricity company will not use the peak-load technology to generate the electricity for the shortage in the non-peak period.

We let $x^+ = \max\{0, x\}$ for any real number x. Let $D_1 \geq 0$ and $D_2 \geq 0$ denote the demands in the non-peak and peak periods, respectively. The electricity company's expected cost function $C(\mathbf{k}, D_1, D_2)$ can be expressed as follows:

$$\begin{aligned} C(\mathbf{k}, D_1, D_2) &= \mathbb{E}[c_1 k_1 + c_2 k_2 + \beta_1 \min\{D_1, (1-\tau)k_1\} + v_1(D_1 - (1-\tau)k_1)^+ \\ &\quad + \beta_1 \min\{D_2, \tau k_1\} + \beta_2 \min\{(D_2 - \tau k_1)^+, \tau k_2\} \\ &\quad + v_2(D_2 - \tau k_1 - \tau k_2)^+]. \end{aligned} \tag{3.1}$$

In the cost function (3.1), the first and second terms are the capacity costs for Technologies 1 and 2, respectively; the third and forth terms are the production cost and shortage cost in the non-peak period, respectively; the fifth and sixth terms are the production costs associated with Technologies 1 and 2, respectively, in the peak period; and the last term is the shortage cost in the peak period.

We consider a single-period planning horizon problem. Before starting the planning horizon, the electricity company only offers the FR tariff to the customers. We define q_1 and q_2 as the original total demands in the non-peak and peak periods, respectively (i.e., the demands in the non-peak and peak periods, respectively, if the company only offers the FR tariff to the customers). It is reasonable to assume that the average demand in the peak period is not less than that in the non-peak period, i.e, $q_2/\tau \geq q_1/(1-\tau)$. At the beginning of the planning horizon, the electricity company introduces the TOU tariff to the customers, such that there is a mixed tariff structure for the customers in the planning horizon. A fraction $\alpha \in (0, 1]$ of the customers are under the TOU tariff and thus the remaining fraction $1 - \alpha$ of the customers are under the FR tariff. We first assume that α is given, and later we will investigate the effects of α on the optimal solutions. Let N denote the number of customers in the market, then the number of customers using the TOU tariff is αN. We assume that the total demand is unchanged when the TOU tariff is introduced to the customers. The fraction of customers still using the FR tariff is $1 - \alpha$. So the demands in the non-peak and peak periods under the FR tariff are $(1-\alpha)q_1$ and $(1-\alpha)q_2$, respectively. The demands in the non-peak and peak periods under the TOU tariff are αq_1 and αq_2, respectively, if there is no shifted consumption.

Under the FR tariff, the customers pay a flat price p_0 for the electricity consumption in both the non-peak period and the peak period. As the customers under the FR tariff have no incentive to shift the consumption from the peak period to the non-peak period, the electricity company will simply choose the upper bound of p_0 for FR tariff, given by the regulator, to maximize its profit. Therefore, in this chapter, we assume that p_0 is given and we focus on the TOU tariff. Under the TOU tariff, the customers pay a lower price $p_1 \geq 0$ for the consumption in the non-peak period, and pay a higher price $p_2 \in [0, \bar{p_2}]$ for the consumption in the peak period. Here, $\bar{p_2}$ is a given upper bound of p_2, which may be imposed by the regulator, e.g., government, under the price-cap regulation. We assume that $p_1 \leq p_2$, and let $\mathbf{p} = (p_1, p_2)$. Table 3.1 illustrates the tariffs in this chapter.

Table 3.1: Tariffs of the electricity company for Chapter 3

Tariffs	**Proportion**	**Prices**
FR tariff	$1 - \alpha$	p_0
TOU tariff	α	p_1, p_2

The customers under the TOU tariff will shift some consumption from the peak period to the non-peak period, due to the price difference. We consider that the customers under the TOU tariff are homogenous. Let q_s^i denote the amount of shifted consumption from the peak period to the non-peak period by a customer under the TOU tariff, for given p_1 and p_2. We consider that the amount of shifted consumption by a customer is stochastic, i.e.,

$$q_s^i(p_1, p_2) \quad = \quad y^i(p_1, p_2) + \epsilon,$$

where ϵ is a random factor with pdf $f(\cdot)$, cdf $F(\cdot)$, a mean value of $\mu = 0$, and in the range $[A, B]$, $A \leq 0$ and $B \geq 0$, and $y^i(p_1, p_2)$ is a deterministic function, namely *determined shifted consumption* by a customer (it is the expected value of the shifted consumption per customer). We let $y^i(p_1, p_2)$ take the form of

$$y^i(p_1, p_2) \quad = \quad a - b_1 p_1 + b_2 p_2, \text{ where } a > 0, b_1 > 0, b_2 > 0.$$

In order to assure that non-negative shifted consumption and positive demand in the peak period after shifting are possible for some values of p_1 and p_2, we assume

$a+A \geq 0$ and $a+B \leq q_2/N$. It is reasonable to assume that the expected value of the shifted consumption per customer is decreasing in the price p_1 and increasing in the price p_2. And examples of linear demand functions for electricity can be found in Chao (2011a) and Greer (2012). Besides, we have shown an additive form of the shifted consumption when considering the customer's problem in Chapter 2. Moreover, with the consideration of customer behaviour, Yang et al. (2013) model the customer's problem for the TOU tariff to obtain the optimal shift consumption, and their derived optimal shift consumption equation is very similar to the function of the determined shifted consumption in our setting.

Let q_s denote the total amount of shifted consumption by all customers under the TOU tariff. As we consider that the customers under the TOU tariff are homogenous customers, we have $q_s = \alpha N q_s^i$. And we let $y = \alpha N y^i$. Then after shifting the consumption, the demands in the non-peak and peak periods under the TOU tariff are $\alpha q_1 + q_s$ and $\alpha q_2 - q_s$, respectively. Table 3.2 illustrates the electricity demands of the customers under the two tariffs after introducing the TOU tariff.

Table 3.2: Electricity demand after introducing the TOU tariff

Tariff	**Demand in the non-peak period**	**Demand in the peak period**
FR tariff	$(1-\alpha)q_1$	$(1-\alpha)q_2$
TOU tariff	$\alpha q_1 + q_s$	$\alpha q_2 - q_s$
Total	$D_1 = q_1 + q_s$	$D_2 = q_2 - q_s$

Our objective is to maximize the electricity company's expected profit function $\Pi(\mathbf{k}, \mathbf{p})$ by optimally determining the installed capacities for the two technologies, i.e., $\mathbf{k} = (k_1, k_2)$, and the prices for the TOU tariff, i.e., $\mathbf{p} = (p_1, p_2)$:

$$\begin{aligned}\max_{\mathbf{k},\mathbf{p}} \Pi(\mathbf{k}, \mathbf{p}) \quad &= \quad \mathbb{E}[p_1(\alpha q_1 + q_s) + p_2(\alpha q_2 - q_s) + p_0(1-\alpha)(q_1 + q_2)] \\ &\quad\; -C(\mathbf{k}, q_1 + q_s, q_2 - q_s). \qquad (3.2)\end{aligned}$$

The objective function (3.2) is composed of two parts. The first part (i.e., the part in the squared bracket) is the revenue from customers, where the first and second terms are the revenue from the customers under the TOU tariff for the demands in the non-peak and the peak periods, respectively, and the third term is the revenue from the customers under the FR tariff. The second part (i.e., $C(\mathbf{k}, q_1 + q_s, q_2 - q_s)$) is the electricity company's cost function.

Table 3.3 summarizes the major notation used in this chapter, where $i \in \{1, 2\}$ indicates Technology i.

Table 3.3: Notation of Chapter 3

τ	percentage of time of the peak period.
k_i	installed capacity of Technology i, $\mathbf{k} = (k_1, k_2)$.
k_p	total capacity for the peak period demand.
c_i	unit capacity cost of Technology i.
β_i	unit production cost of Technology i.
v_1, v_2	unit shortage cost for the non-peak period demand and the peak period demand, respectively.
D_1, D_2	total demands in the non-peak and peak periods, respectively.
$C(\cdot)$	the electricity company's expected cost function.
α	the proportion of customers using the TOU tariff, $\alpha \in (0, 1]$.
N	total numbers of customers in the market, so the number of customers using the TOU tariff is αN.
p_0	electricity price for the FR tariff.
p_1, p_2	electricity prices in the non-peak and peak periods, respectively, for the TOU tariff, and $\bar{p_2}$ is the upper bound of p_2.
q_1, q_2	original total demands in the non-peak and peak periods, respectively, i.e., the demands in the non-peak and the peak periods, respectively, if only FR tariff is offered to the customers.
q_s^i	shifted consumption from the peak period to the non-peak period per customer under the TOU tariff, and $q_s = \alpha N q_s^i$.
y^i	determined shifted consumption, i.e., deterministic part of q_s^i, and $y = \alpha N y^i$.
$\Pi(\cdot)$	the electricity company's expected profit function.

3.4 Analysis and Solution

We use the sequential decision-making approach to find the optimal solution, denoted by $(\mathbf{k}^*, \mathbf{p}^*)$, that maximizes $\Pi(\mathbf{k}, \mathbf{p})$ in (3.2). That is, we first obtain the optimal responses of capacities, i.e., $\mathbf{k}(\mathbf{p})$, for a given $\mathbf{p}$. In the second step, we obtain the optimal prices, i.e., $\mathbf{p}^*$, by maximizing $\Pi(\mathbf{k}(\mathbf{p}), \mathbf{p})$ over $\mathbf{p}$, given the optimal responses of capacities.

The optimal response of capacities can be obtained uniquely as shown in Theorem 3.1.

Theorem 3.1 *Given* $\mathbf{p}$*, the optimal responses of capacities* $\mathbf{k}(\mathbf{p})$ *are shown as follows:*
If $c_2 > \tau(v_2 - \beta_2)$*, then* $k_2^* = 0$*; otherwise,* $\mathbf{k}(\mathbf{p})$ *is determined by the first-order conditions:*

$$\tau(\beta_2 - \beta_1)F(\frac{q_2 - y(p_1,p_2) - \tau k_1}{\alpha N}) + \tau(v_2 - \beta_2)F(\frac{q_2 - y(p_1,p_2) - \tau k_1 - \tau k_2}{\alpha N})$$
$$+(1-\tau)(v_1 - \beta_1)[1 - F(\frac{(1-\tau)k_1 - q_1 - y(p_1,p_2)}{\alpha N})] = c_1; \quad (3.3)$$
$$F(\frac{q_2 - y(p_1,p_2) - \tau k_1 - \tau k_2}{\alpha N}) = \frac{c_2}{\tau(v_2 - \beta_2)}. \quad (3.4)$$

If $c_2 > \tau(v_2 - \beta_2)$, then the smallest capacity of Technology 2 is optimal, i.e., $k_2^* = 0$. This case happens because of a too high capacity cost of Technology 2 (c_2), too small percentage of the peak period (τ), etc. Otherwise, the values of the optimal capacities can be obtained by jointly solving Equations (3.3) and (3.4), which are the first-order conditions of the objective function. The costs, such as capacity cost, production cost and shortage cost, play critical roles in rationing the capacities to meet the demands. Equation (3.4) shows that, for a given k_1, the probability of meeting the demand via Technology 2 (with capacity k_2) is proportional to the ratio of the capacity cost to the shortage cost minus the production cost of Technology 2. It follows that, the optimal capacity of Technology 2 decreases as the capacity cost (c_2) and production cost (β_2) increase, and shortage cost (v_2) decreases. The left-hand side of Equation (3.3) is a combination of the probability of rationing the capacities to meet the demand in both the non-peak and peak periods. Another observation is that, by combining Equations (3.3) and (3.4), we immediately obtain

$$\tau(\beta_2 - \beta_1)F(\frac{q_2 - y(p_1,p_2) - \tau k_1}{\alpha N}) + c_2$$
$$+(1-\tau)(v_1 - \beta_1)[1 - F(\frac{(1-\tau)k_1 - q_1 - y(p_1,p_2)}{\alpha N})] - c_1 = 0. \quad (3.5)$$

There is no k_2 in the derived Equation (3.5). It follows that the optimal capacity of Technology 1 decreases as its capacity cost (c_1) increases and the capacity cost of Technology 2 (c_2) decreases. We remark that in the following, we will not consider the trivial case where $k_2^* = 0$ and only focus on the case where $\mathbf{k}(\mathbf{p})$ is determined by the first-order conditions.

Proposition 3.1 *$\partial k_2(\mathbf{p})/\partial p_1 \geq 0$, $\partial k_2(\mathbf{p})/\partial p_2 =\leq 0$, $\partial k_p(\mathbf{p})/\partial p_1 = \alpha N b_1 \geq 0$ and $\partial k_p(\mathbf{p})/\partial p_2 = -\alpha N b_2 \leq 0$.*

Proposition 3.1 indicates that the capacity of Technology 2 (k_2) and the total capacity for the peak period demand (k_p) always increase in the price for the non-peak period (p_1) and decrease in the price for the peak period (p_2). It is because that, if the price in the non-peak period increases or the price in the peak period decreases, then the expected shifted consumption is decreased. It follows that the remaining demand in the peak period is increased, that leads to an increase effect on the installed capacities for the peak period. In addition, the total capacity for the peak period demand is linear in p_1 with increasing ratio $\alpha N b_1$ and linear in p_2 with decreasing ratio $-\alpha N b_2$. But the capacity of Technology 2 is not linear in p_1 and p_2, and the increasing or decreasing ratio depend on some parameters, such as the production and shortage costs. And it is remarked that the effects of the prices on the installed capacity of Technology 1 (k_1) are more complex and not monotone in general.

Theorem 3.2 *Given p_2, the optimal price in the non-peak period (p_1) is uniquely determined by the first-order condition:*

$$\begin{aligned} \frac{q_1}{N} + y^i(p_1, p_2) &= b_1 \big(\frac{c_1 - (v_1 - \beta_1)(1 - F(\frac{(1-\tau)k_1(\mathbf{p}) - q_1 - y(p_1, p_2)}{\alpha N}))}{\tau} \\ &\quad + p_1 - p_2 \big). \end{aligned} \tag{3.6}$$

Theorem 3.2 shows the optimal price in the non-peak period, for a given price in the peak-period for the TOU tariff. It indicates that, under the optimal solution, the expected demand in the non-peak period for a customer under the TOU tariff, i.e., the left-hand side of Equation (3.6), is equal to a value presented by the right-hand side of Equation (3.6).

Proposition 3.2 *$dp_1(p_2)/dp_2 \geq 0$; $dy(p_1(p_2), p_2)/dp_2 \geq 0$ if $b_2 \geq b_1$; and $dy(p_1(p_2), p_2)/dp_2 < 0$, otherwise.*

Proposition 3.2 shows the effects of the price in the peak period (p_2) on the optimal response of the price in the non-peak period ($p_1(p_2)$), and on the determined shifted consumption by all customers from the peak period to the non-peak period ($y(p_1(p_2), p_2)$). If the price in the peak period increases, then the company should also increase the price in the non-peak period. Consequently, the determined shifted consumption from the peak period to the non-peak period will be increased if the customers are more sensitive to the price changes in the peak period, i.e., $b_2 \geq b_1$, and it will be decreased if the customers are more sensitive to the price changes in the non-peak period, i.e., $b_2 < b_1$. As $y = \alpha N y^i$, then the effects on the determined shifted consumption by a customer (y^i) are similar with that on the determined shifted consumption by all customers.

Theorem 3.3 *The upper bound of the price in the peak period p_2^* is optimal for the TOU tariff, i.e., $p_2^* = \min\{\hat{p_2}, \bar{p_2}\}$, where $\hat{p_2}$ is the unique solution of $y(p_1(p_2), p_2) = \alpha q_2 - \alpha NB$ if $b_2 \geq b_1$, and is the unique solution of $y(p_1(p_2), p_2) = -\alpha NA$ if $b_2 < b_1$.*

It is interesting to show that, given the optimal response of the price in the non-peak period, the upper bound of the price in the peak period is optimal for the TOU tariff. Because the total demand of the electricity will not be affected by prices, the company will set the price in the peak period as high as possible, for a given optimal response of the price in the non-peak period. There are three possible values for the upper bounds of the price in the peak period. As indicated in Proposition 3.2, if the customers are more sensitive to the price changes in the peak period, then increasing in the price in the peak period will increase the determined shifted consumption. But we need to guarantee that the shifted consumption should not be larger than the demand in the peak period without any shifting, i.e., $q_s \leq \alpha q_2$, which requires that $y(p_1(p_2), p_2) \leq \alpha q_2 - \alpha NB$. So the first possible upper bound of the price in the peak period is determined by $y(p_1(p_2), p_2) = \alpha q_2 - \alpha NB$. Contrary, if the customers are more sensitive to the price changes in the non-peak period, then increasing in the price in the peak period will decrease the determined shifted consumption. We also need to guarantee that the shifted consumption should be non-negative, i.e., $q_s \geq 0$, which requires that $y(p_1(p_2), p_2) \geq -\alpha NA$. So the second possible upper bound of the price in the peak period is determined by $y(p_1(p_2), p_2) = -\alpha NA$. Meanwhile, under the price-cap regulation, we need to guarantee that the price in the peak period should not exceed the upper bound of the price set by the regulator ($\bar{p_2}$). So the last possible value of the upper bound of the price in the peak period is $\bar{p_2}$.

Corollary 3.1 *It may not be optimal for the electricity company to let the customers shifting too much consumption from the peak period to the non-peak period.*

This result is obtained directly from Theorem 3.3, which indicates that the lower bound of the determined shifted consumption may be optimal if the customers are more sensitive to the price change in the non-peak period. It is because the company can get a higher profit by setting the appropriate prices for the TOU tariff, although a high shifting consumption implies a low capacity investment in the peak period.

3.5 Comparative Statics

In this section, we evaluate the impacts of original total demand in the non-peak period (q_1), original total demand in the peak period (q_2), market size (N), proportion of customers using the TOU tariff (α), and the cost parameters ($\beta_1, \beta_2, c_1, c_2, v_1$ and v_2) on the optimal capacities ($\mathbf{k}$), prices ($\mathbf{p}$), determined shifted consumption (which includes the determined shifted consumption by a customer (y^i) and determined shifted consumption by all customers (y) under the TOU tariff), and profit (Π). These comparative statics help us to deeply understand the TOU tariff. In order to keep the presentation concisely, the terms without the word "optimal" will be used to indicate the optimal solutions. For example, capacity of Technology 1 means the optimal capacity of Technology 1.

As indicated in Theorem 3.3, there are three possible optimal values of the price in the peak period for the TOU tariff. So before we proceed further, we define three cases as follows: Case I: p_2^* is determined by $y(p_1(p_2), p_2) = -\alpha NA$, Case II: p_2^* is determined by $y(p_1(p_2), p_2) = \alpha q_2 - \alpha NB$, and Case III: $p_2^* = \bar{p}_2$. Note that for Case III we have $p_2^* = \bar{p}_2$, which is not affected by other set values (e.g., q_1, N). So it is unnecessary to present the effects on p_2^* for Case III.

3.5.1 Impact of original total demand in the non-peak period

Proposition 3.3 characterizes the behaviour of capacities, prices, and determined shifted consumption with respect to the original total demand in the non-peak period (q_1).

Proposition 3.3 *(a) For Cases I and II, capacities of Technology 1 (2) increase (decrease) in the original total demand in the non-peak period; and the total capacities for the peak period demand and the determined shifted consumption are not affected by the original total demand in the non-peak period. For Case I, prices decrease in the original total demand in the non-peak period, but for Case II, prices increase in the original total demand in the non-peak period.*
(b) For Case III, capacity of Technology 1, total capacity for the peak period demand and price in the non-peak period increase in the original total demand in the non-peak period; determined shifted consumption decreases in the original total demand in the non-peak period; and capacity of Technology 2 decreases in the original total demand in the non-peak period if $\alpha \leq 2\tau$ and increases if $a = 1$.

In any case, capacities of Technology 1 increase in the original total demand in the non-peak period. This is intuitive. However, the effects on capacities of Technology 2 vary for different cases. For Cases I and II, capacities of Technology 2 decrease in the original total demand in the non-peak period. It is because the increase of capacity of Technology 1 can help to meet the demand in the peak period (i.e., $(1-\tau)k_1$ is for peak period demand), so that the company can install less capacity for Technology 2, given the fact that determined shifted consumption is on the boundary and unchanged for these two cases. Besides, as both determined shifted consumption and the original total demand in the peak period are unchanged, the total capacities for the peak period demand are not affected by the original total demand in the non-peak period. The difference of Cases I and II is that prices decrease in the original total demand in the non-peak period for Case I but increase for Case II. For Case III, the price in the non-peak period, which is increasing in the original total demand in the non-peak period, leads to a decrease in the determined shifted consumption, given that the price in the peak period is at the boundary and unchanged. Consequently, the remaining demand in the peak period increases in the original total demand in the non-peak period, which leads to an increase in total capacity for the peak period demand. On the other hand, combining the increasing effect of the needed capacity of Technology 2 due to the decrease of determined shifted consumption with the decreasing effect due to the increase of the capacity of Technology 1, it is interesting to show that the effect on the capacity of Technology 2 for Case III is not monotone. We find that the capacity of Technology 2 decreases in the original total demand in the non-peak period if $\alpha \leq 2\tau$, while it unexpectedly increases in the original total demand in the non-peak period, if all customers use the TOU tariff.

3.5.2 Impact of original total demand in the peak period

Proposition 3.4 characterizes the behaviour of capacities, prices, and determined shifted consumption with respect to the original total demand in the peak period

(q_2).

Proposition 3.4 *(a) For Case I, capacities of Technologies* 1 *and* 2*, total capacity for the peak period demand, and prices increase in the original total demand in the peak period (and* $dk_p^*/dq_2 = 1$*); and determined shifted consumption is not affected by the original total demand in the peak period.*
(b) For Case II, capacity of Technology 1*, total capacity for the peak period demand, and determined shifted consumption increase in the original total demand in the peak period (and* $dk_p^*/dq_2 = (1-\alpha)/\tau$*,* $dy^i/dq_2 = 1/N$*, and* $dy^*/dq_2 = \alpha$*); capacity of Technology* 2 *increases in the original total demand in the peak period if* $\alpha + \tau < 1$ *and decreases if* $\alpha + \tau \geq 1$*; and prices increase in the original total demand in the peak period if* $\alpha + \tau \geq 1$*.*
(c) For Case III, capacities of Technologies 1 *and* 2*, total capacity for the peak period demand, and determined shifted consumption increase in the original total demand in the peak period; and price in the non-peak period decreases in the original total demand in the peak period.*

In any case, capacities of Technology 1 increases in the original total demand in the peak period. This is intuitive, since Technology 1 is also responsible for the peak period demand. For Case I, capacity of Technology 2 increases in the original total demand in the peak period, given that determined shifted consumption is at the boundary and unchanged. This result also holds for Case III, although the determined shifted consumption increases in the original total demand in the peak period due to a decrease of the price in the non-peak period. However, for Case II, the effect on the capacity of Technology 2 is not monotone. It is interesting to show that, for Case II, capacity of Technology 2 may decrease in the original total demand in the peak period, because of a decreasing effect on the capacity of Technology 2 due to the increase of the determined shifted consumption. But the total capacity for the peak period demand still increases in the original total demand in the peak period for any case.

3.5.3 Impact of market size

In this subsection we consider the impacts of the market size (N) on the optimal capacities, prices, and determined shifted consumption. We consider a scenario that the original total demands in the non-peak and peak periods per customer are unchanged with the market size. That is, if we define q_1^i and q_2^i as the original total demands in the non-peak and peak periods per customer, respectively, then they are not affected by N (and we have $q_1 = Nq_1^i$ and $q_2 = Nq_2^i$). Proposition 3.5 characterizes the behaviour of capacities, prices, and determined shifted consumption with respect to the market size.

Proposition 3.5 *For any case, capacities of Technologies* 1 *and* 2*, total capacities for the peak period demand, and determined shifted consumption by all customers (y) increase in the market size (and $dk_1^*/dN = k_1/N$, $dk_2^*/dN = k_2/N$, and $dk_p^*/dN = \tau(k_1 + k_2)/N$ for any case. $dy^*/dN = -\alpha A \geq 0$ for Case I, $dy^*/dN = \alpha q_2^i - \alpha B \geq 0$ for Case II, and $dy^*/dN = \alpha y^i \geq 0$ for Case III); and prices and determined shifted consumption by a customer (y^i) are not affected by the market size.*

It is intuitive that the electricity company should increase all capacities if the market size increases. Taking the second derivatives of the optimal capacities with respect to the market size, we obtain that $d^2k_1^*/dN^2 = d^2k_2^*/dN^2 = d^2k_p^*/dN^2 = 0$ for any case. We find that all capacities linearly increase in the market size. Similarly, determined shifted consumption by all customers also linearly increase in the market size. On the other hand, the electricity company should keep the prices unchanged for customers, since the original total demand in the non-peak and peak periods per customer and the determined shifted consumption by a customer are fixed for any case.

3.5.4 Impact of proportion of customers using the TOU tariff

Proposition 3.6 characterizes the behaviour of prices and determined shifted consumption, with respect to the proportion of customers using the TOU tariff (α).

Proposition 3.6 *(a) For both Cases I and II, determined shifted consumption by a customer is not affected by the proportion of customers using the TOU tariff; and determined shifted consumption by all customers increases in the proportion of customers using the TOU tariff. Prices decrease in the proportion of customers using the TOU tariff for Case I, and increase for Case II.*
(b) For Case III, price in the non-peak period increases in the proportion of customers using the TOU tariff; and determined shifted consumption by a customer decreases in the proportion of customers using the TOU tariff.

It is difficult to analyze the effects of the proportion of customers using the TOU tariff on the capacities and the company's profit. The effects of the proportion of customers using the TOU tariff on the prices for the TOU tariff vary from case to case. When more customers use the TOU tariff, the company will decrease the prices for the TOU tariff, if the customers are more sensitive to the price change in the non-peak period, i.e., $b_2 < b_1$ (Case I); and increase the prices for the TOU tariff, if the customer are more sensitive to the price change in the peak period, i.e., $b_2 \geq b_1$ (Case II). For Case III, when more customers use the TOU tariff, the electricity company will increase the price in the non-peak period. Consequently, the determined shifted consumption by a customer is decreased. For Cases I and II, the determined shifted consumption by all customers increases in the proportion, since the determined shifted consumption by a customer is at the boundary and unchanged. For Case III, combining the increase of the proportion of customers using the TOU tariff with the decrease of the determined shifted consumption by a customer, we remark that the effect on determined shifted consumption by all customers is not clear. If the determined shifted consumption by all customers decreases in the proportion of customers using the TOU tariff like the determined shifted consumption by a customer does, then the total capacity for the peak period demand will be unexpectedly increased.

3.5.5 Impact of cost parameters

In this subsection we study the effects of the costs parameters, i.e., $\beta_1, \beta_2, c_1, c_2, v_1$ and v_2, on the optimal solutions and profits.

Proposition 3.7 *The effects of the cost parameters on the optimal solutions are shown in Table 3.4, where '+', '-', and '0' mean the increase effect, decrease effect, and no effect, respectively, '?' means no clear monotone effect.*

Table 3.4: The effects of the cost parameters on the optimal solutions

	Case I							Case II							Case III						
	k_1	k_2	k_p	p_1	p_2	y	Π	k_1	k_2	k_p	p_1	p_2	y	Π	k_1	k_2	k_p	p_1	p_2	y	Π
β_1	-	+	0	?	?	0	?	-	+	0	?	?	0	?	-	+	?	?	0	?	-
β_2	+	-	-	+	+	0	?	+	-	-	-	-	0	-	+	-	-	-	0	+	-
c_1	-	+	0	?	?	0	?	-	+	0	?	?	0	?	-	+	?	?	0	?	-
c_2	+	-	-	+	+	0	?	+	-	-	-	-	0	-	+	-	-	-	0	+	-
v_1	+	-	0	-	-	0	-	+	-	0	+	+	0	?	+	-	+	+	0	-	-
v_2	0	+	+	0	0	0	-	0	+	+	0	0	0	-	0	+	+	0	0	0	-

Note that $y = \alpha N y^i$, then the effects of the cost parameters on determined shifted consumption by a customer (y^i) are the same as the effects on determined shifted consumption by all customers (y). So we do not present the effects on determined shifted consumption by a customer in Table 3.4. Recalling that, for Cases I and II, $y = -\alpha NA$ and $y = \alpha q_2 - \alpha NB$, respectively, which are independent of the costs parameters. So, for Cases I and II, determined shifted consumption is not affected by the cost parameters. Regarding the prices and profits, the effects vary from case to case. But it is interesting to show that, for any case, prices are not affected by the shortage cost for the peak period demand. It may be because the capacity investments are determined together with the pricing simultaneously, then we can keep the prices unchanged and adjust the capacity investment decisions when the shortage cost for the peak period demand changes. Regarding the effects on the capacities, for any case, capacities of Technology 1 decrease (increase) in the production and capacity costs of Technology 1 (2); and capacities of Technology 2 increase (decrease) in the production and capacity costs of Technology 1 (2). Capacities may also be affected by the shortage costs. Specifically, capacities of Technology 1 increase in the shortage cost for the non-peak period demand, while it is independent of the shortage cost for the peak period demand. Capacities of Technology 2 decrease (increase) in the shortage cost for the non-peak period demand (peak period demand). The results are intuitive. Regarding the effects on the total capacities for the peak period demand, it is worth noting that, for Cases I and II, total capacity for the peak period demand is independent of the production and capacity costs of Technology 1 and the shortage cost for the non-peak period demand; decreases in the production and capacity costs of Technology 2; and increases in the shortage cost for the peak period demand. It may be because the determined shifted consumption is unchanged in these two cases; and Technology 2 can act as capacity buffer to keep the total capacity for the peak period demand unchanged when the production and capacity costs of Technology 1 and the shortage cost for the non-peak period demand change. For Case III, total capacity for the peak period demand increases in the shortage cost for the non-peak period, which is different from that for Cases I and II. It is because, for Case III, the electricity company will increase the price for the non-peak period demand when the shortage cost for the non-peak period increases, which will lead to a decrease in the determined shifted consumption by all customers. Consequently, the remaining consumption in the peak period will increase, so that the electricity company will increase the total capacity for the peak period demand.

3.6 Conclusions

In this chapter, we study the electricity TOU tariff, under which the customers have the incentive to shift some electricity consumption from the peak period to the non-peak period, due to the price difference. We consider that the amount of shifted consumption by customers is uncertain. The electricity company offers two tariffs to the customers, a fraction of the customers use the TOU tariff, and the remaining fraction of the customers use the traditional FR tariff. Two technologies, the base-load and peak-load technologies, are considered to be installed for generating electricity for the customers. The first technology will be installed to generate electricity for the demands in both periods, while the second technology will be installed to generate electricity only for the peak period demand.

We have answered the four research questions raised in Section 3.1. For the first and second questions, we obtain the optimal capacity investment decisions of the two technologies for the electricity company and the optimal prices for the TOU tariff. For the third and fourth questions, we analyze the impacts of the demands, market size, proportion of customers using the TOU tariff, and cost parameters on the optimal solutions. Important insights and managerial implications for the electricity company are discussed and summarized:

1. Regarding the effects of the demands, we find that the effect of the original total demand in the non-peak period (i.e., the demand in the non-peak period if the company only offers the FR tariff to the customers) on the capacity of Technology 2 may not be monotone. It may unexpectedly increase in the original total demand in the non-peak period, if all customers use the TOU tariff, because of the battling between the decreasing effect due to the increase of the capacity of Technology 1 and the increasing effect due to the decrease of the expected shifted consumption. Meanwhile, the capacity of Technology 2 may surprisedly decrease in the original total demand in the peak period, due to an increase in the shifted consumption from the peak period to the non-peak period by customers.

2. Regarding the effects of the market size, the electricity company should increase all capacities if the market size increases. This is intuitive. However, we find that the company should keep the prices unchanged for the customers, as the expected demands per customer are unchanged.

3. Regarding the effects of the proportion of customers using the TOU tariff, we find that the effects on the prices vary from case to case. The prices may increase or decrease in the proportion. This result is different with that in

Chapter 2, where we show that the optimal prices are not affected by the proportion of customers using the TOU tariff. Besides, in this chapter, we find that the expected shifted consumption by a customer may be decreased when more customers use the TOU tariff, due to an increase in the price in the non-peak period. Then the effect on the expected shifted consumption by all customers becomes unclear, due to the joint effects of the decrease of the expected shifted consumption by a customer and the increase of the proportion. If the expected shifted consumption by all customers decreases in the proportion, then the total capacity for the peak period demand will be unexpectedly increased.

4. Regarding the effects of cost parameters, the total capacity for the peak period demand may be independent of the production and capacity costs of Technology 1 and the shortage cost for the non-peak period demand; decreases in the production and capacity costs of Technology 2; and increases in the shortage cost for the peak period demand. It may be mainly due to that Technology 2 can act as a capacity buffer to keep the total capacity for the peak period demand unchanged when the production and capacity costs of Technology 1 and the shortage cost for the non-peak period demand change. Besides, prices are not affected by the shortage cost for the peak period demand. It may be because that the capacity investments are determined together with the pricing simultaneously, then we can keep the prices unchanged and adjust the capacity investment decisions when the shortage cost for the peak period demand changes.

Managers of electricity companies may follow the insights when they implement the TOU tariff for the customers.

Chapter 4

Environmental Sustainability Investment under Cap-and-trade Regulation for Carbon Emission

4.1 Introduction

Carbon emission accelerates global warming. After Kyoto Protocol in 1997, many countries such as the U.S. and Australia have attempted to design carbon trading mechanism such as cap-and-trade for carbon emission reduction (Stavins 2008, Zhang and Xu 2013). Cap-and-trade policy implies that a firm is allocated a limit or cap on carbon emissions by national government. More specifically, the firm has to buy the right to emit extra carbon if it produces more than the prescribed capacity; otherwise, it can sell its surplus carbon credit (Du et al. 2011, Hua et al. 2011). Reducing carbon emission is significantly important when environmental sustainability is receiving more and more public awareness all around the globe (Nagurney and Yu 2012).

However, only implementing the carbon cap-and-trade policy is still not effective enough to reduce carbon emission (Samaras et al. 2009). In order to be more effective, the investment in the adoption of cleaner technologies is also implemented by responsible firms (Drake and Spinler 2013). For example, in the fashion apparel industry, it is well-known that the fashion supply chain produces all kinds of pollutants including carbon (de Brito et al. 2008, Lo et al. 2012). Companies such as H&M, Marks & Spencer, and Levis all promise to protect environment and reduce carbon emission. For example, H&M, the Sweden fast fashion company, has taken many approaches to minimize carbon emission in its production process by adopting new technologies and meanwhile, H&M launches the green label products which are claimed to be produced in a sustainable way (H&M conscious actions sustainability report 2010 and 2012). From the environmental perspective, producing the sustainable product could reduce the emission and is beneficial to the environment, whereas from the marketing perspective, it could stimulate the market demand. Consumers have strong willingness to purchase the sustainable products (Luchs et al. 2010, Thøgersen et al. 2012, Shen et al. 2012, Grimmer and Bingham 2013). Hence, the positive impact of sustainability on market demand should not be neglected in managing carbon emission abatement.

Motivated by the real industrial practices, in this chapter, we study a two-echelon decentralized supply chain and its centralized channel in which the channel members determine the order quantity (or production quantity) and sustainability investment with a sustainability-dependent market demand under carbon cap-and-trade regulation. For the decentralized supply chain, we consider a classical newsvendor setting in which the manufacturer, as a Stackelberg leader, determines the sustainability investment, and then the retailer, as a follower, places the decision of the order quantity. We consider that the manufacturer is operating on make-to-order basis, under which the manufacturer's production quantity is equal to the retailer's order quantity. For the centralized supply chain, we consider that the manufacturer and the retailer are fully aligned to achieve the channel's maximal profit by determining the production quantity and sustainability investment. To the best of our knowledge, this study is the first one to examine the impact of the order quantity (or production quantity) and sustainability investment in a supply chain under the carbon cap-and-trade regulation.

This chapter contributes to the literature by constructing a model in which both the order quantity (or production quantity) and the sustainability investment are considered under the carbon cap-and-trade regulation. The optimal order quantity and sustainability investment are derived for the decentralized supply chain, and the production quantity and sustainability investment are derived for the centralized supply chain as well. The effects of some emission related parameters on the optimal solutions and profits are analytically analyzed. Moreover, by comparing the optimal solutions and the profits for the decentralized and centralized supply chains, the managerial insights in the significance of carbon emission regulation in a supply chain are discussed. Finally, the coordination of the supply chain is studied under several contracts.

The remainder of this chapter is organized as follows: In Section 4.2, we review the related literature. Section 4.3 analyzes the decentralized supply chain and Section 4.4 examines the centralized supply chain. Section 4.5 compares the optimal solutions and the profits for the decentralized and centralized supply chains. Section 4.6 studies the coordination of the supply chain. The conclusions and managerial insights are given in Section 4.7. All of the proofs are relegated to Appendix C.

4.2 Literature Review

Cap-and-trade policy started to receive considerable attentions from 1970s (Montgomery 1972, Tietenberg 1985) and is regarded as an effective way to mitigate climate change (Stern 2008). Lately, cap-and-trade regulation has been extensively discussed by scholars in the field of supply chain management due to its huge impact on supply chain performance (Choi 2013). Zhao et al. (2010) study a supply chain in which the equilibrium production is affected by the allowance allocation under perfect competition and the cap-and-trade setting. Hua et al. (2011) investigate how companies optimally manage inventory under carbon cap-and-trade regulation by integrating the consideration of carbon emission into the classical economic order quantity model. They find that carbon cap and carbon price have a great impact on the retailer's order decisions. Zhang et al. (2011) derive the manufacturer's optimal production policy with a stochastic demand under the cap-and-trade regulation. Further, Song and Leng (2012) examine the optimal inventory decision in a single-period production problem under carbon cap-and-trade regulation and find that under which the firm could not only reduce carbon emission, but also enhance its business performance under some conditions. Zhang and Xu (2013) also examine a single-period but multi-item production planning supply chain under the carbon cap-and-trade regulation, and find that the firm tends to produce more carbon efficient products under the carbon cap-and-trade regulation.

Du et al. (2013) investigate a two-echelon supply chain in which the emission-dependent manufacturer trades with emission permit supplier under the cap-and-trade regulation. They prove that the manufacturer's profit increases while the supplier's profit decreases with the emission cap. More interestingly, they find that in the centralized system, there is a condition under which the supply chain can achieve coordination. Benjaafar et al. (2013) examine the impact of cap-and-trade regulation in a supply chain and find the possibility that the firms can earn additional revenue under carbon cap-and-trade regulation by leveraging differences between their emission reduction costs and the market carbon price. In addition, they explore the impact of technology adoption on carbon emission reduction and find that if the gains from alternative technologies are substantial, the carbon cap-and-trade regulation could be effective in motivating the firms to adopt the energy-efficient technologies.

Drake and Spinler (2013) indicate that the effectiveness of technology adoption should not be underestimated in a sustainable economic. To develop green supply chain such as carbon emission reduction, making investment in cleaner technologies to reduce emission, namely, sustainability investment, has been discussed and proposed in the existing literature. Krass et al. (2013) consider the case in which the environmental regulator acting as a Stackelberg leader firstly decides the tax level and the firm acting as a follower selects emission control technology, production quantity and price. They find that an initial increase in taxes may motivate a switch to a cleaner technology and if the capital cost of cleaner technologies is subsidized, the negative environmental effect would disappear and taxation becomes efficient. Drake et al. (2012) study the impact of emission tax and emissions cap-and-trade regulation on a firm's long-run technology choice and capacity decisions. They find that emissions would be reduced under cap-and-trade regulation with technology choice and by embedding the option value into the firm's production decision, and cap-and-trade could help firm to earn greater expected profits than emission tax due to the uncertainty of emissions price and the option of no production under the former. Similar with Drake et al. (2012), we also consider that the emission could be reduced by investing in the sustainable technology in production. In addition, consistent with industrial practices, we consider that the consumers will be motivated to purchase if the product is produced with lower emission, namely, the market demand is dependent on product sustainability.

Supply chain coordination represents the scenario under which the individual supply chain members will behave in a way which maximizes the total supply chain system's profitability (Xiao et al. 2005, Chopra and Meindl 2007). Some papers have discussed the supply chain coordination with carbon emission consideration. Jaber et al. (2013) investigate the problem of supply chain coordination when considering greenhouse gap emissions generated from the manufacturer's processes under the European Union Emissions Trading System. Zhang and Liu (2013) consider a supply chain in which the market demand correlates with the green degree of green product. They find that the revenue sharing contract can coordinate the supply chain and encourage positive response of the participating members to the cooperation strategy. Swami and Shah (2013) examine a two-echelon supply chain in which both supply chain members can design the greening effort. Under the deterministic demand setting, they find that a two-part tariff contract can coordinate the supply chain. In this chapter, we consider under the stochastic demand setting, whether the supply chain contracts such as revenue sharing contract, buyback contract and two-part tariff contract can achieve supply chain coordination.

As reviewed above, even though the existing literature has examined various important aspects of sustainable supply chain management with cap-and-trade regulation, how the product sustainability and cap-and-trade regulation affect the decision making in a supply chain is not yet fully known. In addition, it is important to know how such a supply chain can be coordinated. To the best of our knowledge, the above important research issues have not yet been explored in the literature. Addressing these open research questions hence outlines the contribution of this chapter. Table 4.1 shows the literature positioning of our work.

Table 4.1: The literature positioning of Chapter 4

Research	Carbon Emission Consideration	Product Sustainability	Supply Chain Coordination
Benjaafar et al. (2013)	Yes	No	No
Drake et al. (2012)	Yes	Yes	No
Hua et al. (2011)	Yes	No	No
Jaber et al. (2013)	Yes	No	Yes
Swami and Shah (2013)	No	Yes	Yes
Zhang and Xu (2013)	Yes	No	No
Zhang et al. (2011)	Yes	No	No
Zhang and Liu (2013)	No	Yes	Yes
Our work	Yes	Yes	Yes

4.3 The Decentralized Supply Chain

In this section, we consider a two-echelon decentralized supply chain, where a manufacturer (she) produces the product and trades with a retailer (he) by a wholesale price contract in a single period. The retailer is responsible for selling the product to the customer market. The decisions are made in two sequential steps. In the first step, the manufacturer decides the product's sustainability level in terms of carbon emission abatement. In the second step, given the sustainability level, the retailer decides the order quantity of the product from the manufacturer. Please note that, in this chapter, we focus on examining the optimal decisions of the sustainability level of the manufacturer and the order quantity of the retailer. So we consider that the wholesale price is exogenously given and will analyze its effects in Section 4.5.

Let p denote the market price of a product sold by the retailer, c denote the unit production cost, and w denote the wholesale price per unit product. Given the wholesale price, the retailer decides to order x units of the product from the manufacturer. Under the make-to-order setting, the manufacturer will produce the amount of product exactly as the retailer's order quantity. We assume that there are no constraints on the order quantity and production capability. The manufacturer produces the x units of the product (it is equal to the retailer's order quantity under the make-to-order setting) which results in a carbon emission level $(a - bs)x$, where $0 \leq s \leq a/b$ is the sustainability level determined by the manufacturer, a is the base emission when sustainability level is zero, and b is the coefficient of the sustainability effect on reducing the emission. Here, we assume a linear function of carbon emission reduction model, and it indicates that improving the sustainability level has diminishing return on emission. Similar models of reducing the carbon emission level by the investment can be found in Jiang and Klabjan (2012).

Consistent with the existing literature (e.g., Swami and Shah 2013), we consider a linear demand function affected by the sustainability level,

$$D(s) = d + \beta s + \epsilon,$$

where d is the base demand and irrelevant to s, coefficient $\beta > 0$ indicates that the sustainability level has a positive effect on the demand, and ϵ is a random factor with pdf $f(\cdot)$, cdf $F(\cdot)$, a mean value of μ, and in the range $[A, B]$, $A \leq 0$ and $B \geq 0$. Here, we model the demand as a function of the sustainability level, because the customers have strong willingness to purchase the sustainable products (see, e.g., Shen et al. 2012). Similar models of the positive effects on the demand function can also be found in the existing literature, such as Gurnani et al. (2007) and Gurnani and Erkoc (2008). In order to assure the non-negative demand, we further set $A \geq -d$. If the demand does not exceed the order quantity x, then the leftover $x - D$ is disposed at the unit cost c_h (it may be negative, in which it represents a per-unit salvage value). Without loss of generality, we assume that the shortage cost is equal to zero even if the demand exceeds x.

We consider the cap-and-trade regulation for the emission in this chapter. Let K denote the total permissible emission level, which is given by the regulator and assumed to be exogenous. Let c_e denote the emission price per unit emission, and we assume that emission amount can always be bought or sold at this price. Similar to Savaskan and Van Wassenhove (2006), Gurnani and Erkoc (2008), Li et al. (2013), and Swami and Shah (2013), we assume that the sustainability investment cost for the manufacturer is a quadratic function, i.e., $c_I s^2/2$, where c_I is the sustainability investment coefficient.

In current business practice, it is true that the investment cost for improving the sustainability level usually is high. So we assume that c_I is high enough such that $c_I \geq 2c_e b\beta$, though we can obtain analytical results even if without this assumption. Specifically, if $c_I < 2c_e b\beta$, we have the results that the lower bound or upper bound of the sustainability are optimal to the manufacturer, i.e., $s^* = 0$ under which the manufacture will not invest in the sustainability, or $s^* = a/b$ under which the manufacture will invest in a very high sustainability level such that no carbon emission will be produced. So in order to avoid these trivial cases and make our results more elegant, we only present our results for the case of $c_I \geq 2c_e b\beta$ hereafter[4.1].

Table 4.2 summarizes some major notations used in this chapter.

We use the backward sequential decision-making approach to analyze the problems. First, we assume that the sustainability level is given by the manufacturer, under which we solve the retailer's problem and obtain the optimal response of the order quantity, i.e., $x(s)$. In the second step, we solve the manufacturer's problem and obtain the optimal sustainability level, i.e., s^*, given the optimal response of the order quantity.

4.3.1 Retailer's problem

For a given sustainability level, the retailer maximizes his own expected profit by deciding the order quantity x. Denote $\Pi_r(x)$ as the retailer's expected profit

[4.1] See the supplement for the case of $c_I < 2c_e b\beta$ in Appendix C.

Table 4.2: Notation of Chapter 4

p	product's market price.
c	unit production cost.
w	wholesale price.
x	order quantity for decentralized supply chain (production quantity for centralized supply chain).
s	sustainability level.
a	the emission when sustainability level is zero.
b	coefficient of the sustainability effect on reducing the emission
c_I	sustainability investment coefficient
β	coefficient of the sustainability effect on increasing the demand
K	total permissible emission level.
c_e	unit emission price.
$D(\cdot)$	demand function.
c_h	unit leftover cost.
Π_m	manufacturer's profit
Π_r	retailer's profit
Π_d	supply chain's profit in the decentralized setting
Π_c	supply chain's profit in the centralized setting

function. We have

$$\max_x \Pi_r(x) \quad = \quad \mathbb{E}[p\min\{D, x\} - wx - c_h(x - D)^+]. \tag{4.1}$$

In the above profit function, the first term is the revenue from selling the product in the customer market, the second term is the cost of ordering the product from the manufacturer, and the last term is the leftover cost. After deriving Equation (4.1) with respect to order quantity x, we can have the following proposition.

Proposition 4.1 *Given s, the unique optimal response of the order quantity $x(s)$ is as follow:*

$$x(s) = F^{-1}(\frac{p - w}{p + c_h}) + d + \beta s. \tag{4.2}$$

The optimal response of the order quantity is obtained by the first-order condition of the retailer's profit function. The solution is essentially the same to the well-known newsvendor solution in the literature. Given a sustainability level s, the order quantity is increasing in the base demand d, and decreasing in the unit leftover cost c_h and wholesale price w, which are consistent with our intuitive understanding.

Corollary 4.1 $dx(s)/ds = \beta > 0$.

Corollary 4.1 indicates that the order quantity is increasing in the sustainability level. This result could be potentially explained by the fact that when the sustainability level is higher, the market demand would be also higher, which induces the retailer to order more from the manufacturer.

4.3.2 Manufacturer's problem

The manufacturer's profit function, denoted by $\Pi_m(s)$, is given by

$$\Pi_m(s) \quad = \quad wx(s) - cx(s) - c_e((a - bs)x(s) - K) - \frac{c_I}{2}s^2. \tag{4.3}$$

In the above profit function, the first term is the revenue generated from selling the product to the retailer, the second term is the production cost, the third term is the cost or revenue from buying or selling the extra allowances of the emission, and the last term is the sustainability investment cost. Knowing that the retailer orders the product x according to Equation(4.2) in response to a given sustainability level s, the manufacturer decides on s to maximize her own expected profit. By substituting $x(s)$ into Equation (4.3) and differentiating it with respect to sustainable level s, we can have the following proposition.

Proposition 4.2 *The manufacturer's optimal sustainability level is given by*

$$s^* \quad = \quad \frac{c_e b(F^{-1}(\frac{p-w}{p+c_h}) + d) + (w - c - c_e a)\beta}{c_I - 2c_e b\beta}. \tag{4.4}$$

Proposition 4.2 shows the optimal sustainability level for the manufacturer, i.e., Equation (4.4), which is solved by the first-order condition of the manufacturer's profit function. Obliviously, the optimal value is increasing in the base demand d, and decreasing in the unit production cost c and unit leftover cost c_h.

Next we analyze the effects of the parameters b, β, c_e, and c_I, which are related to the sustainability investment or the emission, on the optimal decisions x^* and s^*, the retailer's optimal profit Π_r^*, the manufacturer's optimal profit Π_m^*, and the optimal profit of the whole supply chain Π_d^* (i.e., $\Pi_d^* = \Pi_r^* + \Pi_m^*$).

Proposition 4.3 *x^* and s^* are increasing in b, and decreasing in c_I.*

Proposition 4.3 indicates that, if the coefficient of the sustainability effect on reducing the emission b is larger, then the manufacturer will invest more in the sustainability level s^* to reduce the emission. Meanwhile, a higher sustainability level will induce a larger demand, which will lead to a higher order quantity x^*. So the order quantity is increasing in the coefficient b. Intuitively, if the sustainability investment coefficient c_I is large, then the manufacturer will invest less in the sustainability level, which will lead to a lower order quantity.

Remark 4.1 *For the effects of the coefficient of the sustainability effect on increasing the demand β and unit emission price c_e, we can obtain that*

$$\begin{aligned} \frac{ds^*}{d\beta} &= \frac{w - c - c_e(a - 2bs^*)}{2c_e b\beta - c_I}; & \frac{dx^*}{d\beta} &= s^* + \frac{ds^*}{d\beta}\beta; \\ \frac{ds^*}{dc_e} &= \frac{-bx^* + \beta(a - bs^*)}{2c_e b\beta - c_I}; & \frac{dx^*}{dc_e} &= \frac{ds^*}{dc_e}\beta, \end{aligned}$$

which may be positive or negative. And the way by which Π_r^*, Π_m^*, and Π_d^* depend on b, β, c_e, and c_I are more complex and are not monotone in general also.

4.4 The Centralized Supply Chain

In this section, we consider a centralized supply chain, where the manufacturer and the retailer are fully aligned to achieve the channel's maximal profit. Our objective is to maximize the expected profit of the whole supply chain by optimally choosing the production quantity and sustainability investment.

$$\begin{aligned} \max_{x,s} \Pi_c(x, s) &= \mathbb{E}[p \min\{D, x\} - cx - c_h(x - D)^+ \\ &\quad -c_e\big((a - bs)x - K\big) - \frac{c_I}{2}s^2]. \end{aligned} \tag{4.5}$$

In the above profit function, the first term is the revenue generated from selling the product in the customer market, the second term is the production cost, the third term is the leftover cost, the fourth term is the cost or revenue from buying or selling the extra allowances of the emission, and the last term is the sustainability investment cost.

We use the sequential decision-making approach to analyze the problem, and solve our problem by two steps. In the first step, we assume that the sustainable level is given, under which we solve the problem and obtain the optimal response of the production quantity, $x(s)$. In the second step, we obtain the optimal sustainability level, s^*, given the optimal response of the production quantity. This approach can guarantee the optimality of the solution, and is widely used in the literature, such as Petruzzi and Dada (1999) and Wang et al. (2004).

Proposition 4.4 shows the optimal response of the production quantity for a given sustainability level.

Proposition 4.4 *Given s, the unique optimal response of production quantity $x(s)$ is as follow:*

$$x(s) = F^{-1}\left(\frac{p - c - c_e(a - bs)}{p + c_h}\right) + d + \beta s. \tag{4.6}$$

The optimal response of the production quantity is obtained by the first-order condition of the channel's profit function, for a given s. Similar to the decentralized supply chain, the solution of the production quantity for the centralized supply chain is essentially the same to the well-known newsvendor solution in the literature. Given a sustainability level s, the order quantity is increasing in the base demand d, and decreasing in the unit leftover cost c_h, unit production cost c, and unit emission price c_e.

Corollary 4.2 $dx(s)/ds = \beta + (c_e b)/((p + c_h)f(x - d - \beta s)) > 0.$

Similar to Corollary 4.1, Corollary 4.2 indicates that the production quantity is increasing in the sustainability level.

Substituting $x = x(s)$ into Equation (4.5), the optimization problem becomes a maximization over the single variable s: $\max_s \Pi_c(x(s), s)$. By taking and rearranging the first and second derivatives of $\Pi_c(x(s), s)$ over s, we obtain

$$\begin{aligned} \frac{d\Pi_c(x(s),s)}{ds} &= (p - c - c_e(a - bs))\beta - c_I s + c_e b x(s); \\ \frac{d^2\Pi_c(x(s),s)}{ds^2} &= 2c_e b\beta - c_I + \frac{(c_e b)^2}{(p + c_h) f(x(s) - d - \beta s)}. \end{aligned} \tag{4.7}$$

As shown in Proposition 4.5, $\Pi_c(x(s), s)$ might have multiple optimal values of the sustainability level, depending on the parameters of the problem.

Proposition 4.5 *There is at most one optimal point of s that satisfies the first-order condition of the channel's profit function $\Pi_c(x(s), s)$ when $f(\cdot)$ is monotonous.*

There may be multiple points that satisfy the first-order optimality condition of the channel's profit function $\Pi_c(x(s), s)$, i.e., $d\Pi_c(x(s), s)/ds = 0$, where $d\Pi_c(x(s), s)/ds$ is represented in Equation (4.7). If $f(\cdot)$ is a non-decreasing distribution function (i.e., $f'(\cdot) \geq 0$), then we obtain $d^3\Pi_c(x(s), s)/ds^3 \leq 0$, implying that $d\Pi_c(x(s), s)/ds$ is concave in s. So $d\Pi_c(x(s), s)/ds = 0$ has at most two roots and the larger of the two makes a change of sign for $d\Pi_c(x(s), s)/ds$ from positive to negative that corresponds to a local maximum of $\Pi_c(x(s), s)$; if $f(\cdot)$ is a decreasing distribution function (i.e., $f'(\cdot) < 0$), then the smaller of the two makes a change of sign for $d\Pi_c(x(s), s)/ds$ from positive to negative that corresponds to a local maximum of $\Pi_c(x(s), s)$. We consider three general distributions of the demand: uniform, exponential, and normal distribution in Corollary 4.3.

Corollary 4.3 *For the uniform, exponential, and normal distribution of the demand, there is at most one optimal point of s that satisfies $d\Pi_c(x(s), s)/ds = 0$.*

The following proposition describes how the optimal decision x^* and s^*, and the optimal profit of the whole supply chain Π_c^* change with system parameters b, β, and c_I.

Proposition 4.6 *x^*, s^*, and Π_c^* are increasing in b and β, and are decreasing in c_I.*

Similar to the decentralized supply chain, the centralized supply chain will invest more in the sustainability level s^* and increase the production quantity x^*, if the coefficient of the sustainability effect on reducing the emission b is large and the sustainability investment coefficient c_I is small. Besides, Proposition 4.6 indicates that, if the coefficient of the sustainability effect on increasing the demand β increases, then the centralized supply chain will increase the sustainability level and the production quantity, and the channel's profit will be increased as well, and if the sustainability investment coefficient c_I increases, then the channel's profit will be decreased. Note that b, β, and c_I are the parameters related to sustainability level or emission, so Proposition 4.6 implies that, in order to increase the centralized supply chain profit, enhancing the efficiency of sustainability investment is significant.

Remark 4.2 *For the effects of the unit emission price c_e, we can obtain that*

$$\frac{ds^*}{dc_e} = \frac{-bx^* + \frac{\hat{p}\hat{f}\beta + c_e b}{\hat{p}\hat{f}}(a - bs^*)}{2c_e b\beta - c_I + \frac{(c_e b)^2}{\hat{p}\hat{f}}}; \qquad \frac{dx^*}{dc_e} = \frac{-(a - bs^*) + (\hat{p}\hat{f}\beta + c_e b)\frac{ds^*}{dc_e}}{\hat{p}\hat{f}};$$

$$\frac{d\Pi_c^*}{dc_e} = K - (a - bs^*)x^*,$$

where $\hat{p} = p + c_h$ and $\hat{f} = f(x^* - d - \beta s^*)$. Here, ds^*/dc_e, dx^*/dc_e, and $d\Pi_c^*/dc_e$ may be positive or non-positive, and the effects of the unit emission price are complicate and are not monotone in general.

4.5 The Comparison of Decentralized and Centralized Supply Chains

In this section, we numerically compare the profit of the whole supply chain and the optimal solutions under the decentralized case with those under the centralized case. Some interesting results are presented in the following subsection.

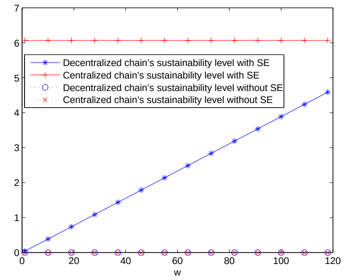
4.5.1 Numerical examples

As shown in Corollary 4.3, there is at most one optimal solution of s that satisfies the first-order condition of the profit function, for the centralized supply chain, for the uniform, normal, and exponential distributions of the demand. Figures 4.1 and 4.2 show the numerical results for the uniform and normal distributions, respectively. For the exponential distribution, we can obtain the similar numerical results.

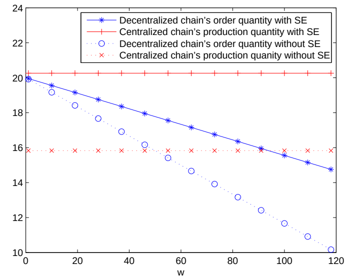
In the literature, the wholesale price is usually assumed to be larger than the unit production cost, i.e., $w > c$. However, in this chapter, after considering the cap-and-trade regulation, we could relax this assumption and set that the wholesale price can be not lager than the unit production cost. For example, if the manufacturer could obtain a higher profit by selling the quota of the allowances of the emission, rather than by selling product, then she would invest in a high sustainability level to reduce the emission in production, although the wholesale price is very small.

In all numerical examples, we set $p = 120, c = 50, d = 10, a = 5, b = 0.5, c_e = 10, \beta = 1$, and $c_I = 25$. Without loss of generality, we let the total permissible emission level equals to zero, i.e., $K = 0$. Then the manufacturer's profit would be negative if the wholesale price is lower, e.g., $w < c$. Alternatively, if we set a high total permissible emission level, e.g., $K = 500$, the manufacturer can get a positive profit even if the wholesale price is very low. For the uniform distribution, we let $\epsilon \backsim U[0, 10]$, and for the normal distribution, we let $\epsilon \backsim Normal(10, 1)$. We benchmark our results with the consideration of the sustainability and the emission, to the results without considering the sustainability and the emission (i.e., $c_e = c_I = a = b = s_d = s_c = 0$). We use 'SE' to stand for the 'Sustainability and emission', so 'with SE' means 'with the consideration of the sustainability and the emission' and 'without SE' means 'without considering the sustainability and the emission'.

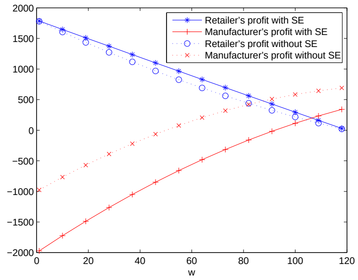
Figure 4.1 shows the effects of the wholesale price on the optimal solutions and the corresponding profits for the uniform distribution of the demand. Ob-



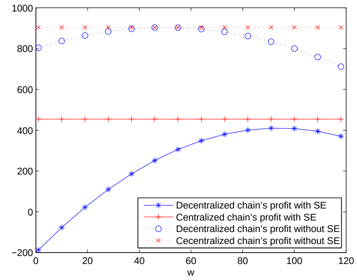
(a) The optimal sustainability levels



(b) The optimal order (production) quantities



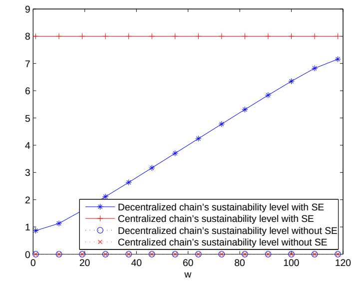
(c) The retailer's optimal profit and the manufacturer's optimal profit



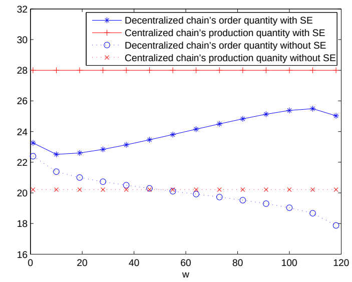
(d) The optimal profits of the whole supply chain

Figure 4.1: Effects of the wholesale price on the optimal solutions and the profits for the uniform distribution of the demand.

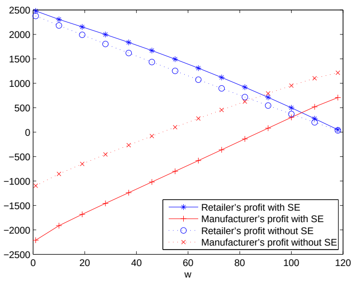
viously, the optimal solutions and the corresponding profits for the centralized case are not affected by the wholesale price. Figures 4.1(a) and 4.1(b) show that the optimal sustainability level and order quantity for the decentralized case are non-decreasing and decreasing, respectively, in the wholesale price. Figure 4.1(c) shows that the manufacturer's profit and the retailer's profit are increasing and decreasing, respectively, in the wholesale price. Besides, the manufacturer's profit with SE is smaller than that without SE, but the retailer's profit with SE is larger than that without SE. Because, with the consideration of the sustainability and the emission, the manufacturer needs to pay the emission cost and the sustainability investment cost, while the retailer can get the benefit of the sustainability effect on increasing the demand. As shown in Figure 4.1(d), the optimal profit of the whole supply chain for the decentralized case is not larger than that for the centralized case. If we do not consider the sustainability investment and the emission issues, the optimal profit of the whole supply chain is obtained when the wholesale price equals to the unit production cost (i.e., $w = c = 50$). However, as shown in Figure 4.1(d), our results indicate that the optimal profit of the whole supply chain obtains its maximum at $w = 94$ which is almost double of the unit production cost, due to the effects of the sustainability and emission consideration. Those differences are mainly due to the effects of the sustainability investment and emission consideration.



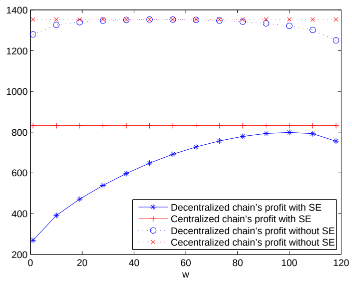
(a) The optimal sustainability levels



(b) The optimal order (production) quantities



(c) The retailer's optimal profit and the manufacturer's optimal profit



(d) The optimal profits of the whole supply chain

Figure 4.2: Effects of the wholesale price on the optimal solutions and the profits for the normal distribution of the demand.

Figure 4.2 shows the effects of the wholesale price on the optimal solutions and the corresponding profits for the normal distribution of the demand. The effects are almost the same with that for the uniform distribution, except for the effects on the sustainability level and the order quantity. As shown in Figures 4.2(a) and 4.2(b), the optimal sustainability level and the order quantity for the decentralized case are not monotonous in the wholesale price, with the consideration of the sustainability and the emission. If we do not consider the sustainability issue, then the order quantity will be decreasing in the wholesale price. However, in this chapter, we consider that the sustainability level has the direct effect on the demand which would further affect the order quantity. Besides, as shown in

Equations (4.2) and (4.4), the order quantity is increasing in the sustainability level which may be increasing or decreasing in the wholesale price, depending on the cdf of the distribution of the demand and the value of the coefficient of the sustainability effect on increasing the demand (i.e., β). For the normal distribution of the demand, we thus obtain the above result, which is different from the other distributions and the situations without the sustainability and the emission consideration.

4.6 Coordinating the Supply Chain

This section studies the coordination in a supply chain with the consideration of the sustainability and the emission. In the previous literature, several contracts have been proposed for coordinating a supply chain, including the buy-back contract, the revenue sharing contract, the two-part tariff contract, etc (Cachon 2003, Cachon and Lariviere 2005). In this chapter, we consider three contracts, i.e., buyback, revenue sharing, and two-part tariff contracts, and verify that whether they can coordinate the supply chain. Recalling that this chapter determiners the optimal order quantity (or production quantity) and sustainability level. So a key question is that whether the contracts that coordinate the retailer's order quantity and also coordinate the manufacture's sustainability level. We restrict our attention to the cases in which the sustainability levels are determined by the first-order conditions of the profit functions. Note that under some contracts, such as the buyback contract, the manufacturer needs to depose the unsold products, which may causes carbon emission. However, in this chapter we will not consider such issues, and only focus on the situation where the emission is caused when the manufacturer produces the products.

Let x_d and s_d be the optimal solutions of the order quantity and sustainability level, respectively, for the decentralized supply chain, and x_c and s_c be the optimal solutions of the production quantity and sustainability level, respectively, for the centralized chain.

4.6.1 Revenue sharing contract

We consider that, under a revenue sharing contract (w, ϕ), the retailer pays the manufacturer a unit wholesale price w for each unit ordered plus a proportion of his revenue from selling the products to the customers, where ϕ is the proportion of the revenue the retailer keeps, and thus $1-\phi$ is the proportion shared to the manufacturer. See Cachon (2003) and Cachon and Lariviere (2005) for detailed discussions of this contract. The retailer's and the manufacturer's expected profit functions are given by

$$\begin{aligned}\Pi_r &= \mathbb{E}[\phi p \min\{D, x\} - wx - c_h(x-D)^+];\\ \Pi_m &= \mathbb{E}[(1-\phi)p\min\{D, x\} + wx - cx - c_e((a-bs)x - K) - \frac{c_I}{2}s^2].\end{aligned}$$

Proposition 4.7 *For a given revenue sharing contract (w, ϕ), the optimal decision of the order quantity and sustainability investments $(x_d, s_d) = (x^*, s^*)$ are determined as follows:*

$$x - F^{-1}(\frac{\phi p - w}{\phi p + c_h}) - d - \beta s = 0; \tag{4.8}$$

$$((1-\phi)p + w - c - c_e(a-bs))\beta - c_I s + c_e bx = 0. \tag{4.9}$$

Comparing Equations (4.8) with (4.6) and (4.9) with (4.7), we find that (x_d, s_d) can be the centralized supply chain's optimal solution (x_c, s_c) if $w = \phi p$ and $\phi = (p + c_h)/(p(p - c - c_e(a - bs_c))) - c_h/p$. Therefore, a revenue sharing contract with reasonable contract parameters is sufficient to coordinate the supply chain with the sustainability and emission consideration. Besides, our result shows that there is s single coordinating revenue sharing contract such that provides only one allocation of the supply chain's profit. This result is similar to the coordination result of the revenue sharing contract with price dependent demand and non-zero lost sales penalty (Cachon 2003).

4.6.2 Buyback contract

With a buyback contract (w, b_c), the manufacturer charges the retailer a unit wholesale price w for each unit purchased, but pays the retailer b_c per unit remaining at the end of the season. See Pasternack (1985) and Cachon (2003) for detailed analysis of this contract in the context of the newsvendor problem. The retailer's and the manufacturer's expected profit functions are given by

$$\begin{aligned}\Pi_r &= \mathbb{E}[p\min\{D,x\} - wx - c_h(x-D)^+ + b_c(x-D)^+];\\ \Pi_m &= \mathbb{E}[wx - cx - c_e((a-bs)x-K) - \frac{c_I}{2}s^2 - b_c(x-D)^+].\end{aligned}$$

Proposition 4.8 *For a given buyback contract (w, b_c), the optimal decision of the order quantity and sustainability investments $(x_d, s_d) = (x^*, s^*)$ are determined as follows:*

$$x - F^{-1}(\frac{p-w}{p+c_h-b_c}) - d - \beta s = 0; \tag{4.10}$$

$$(w - c - c_e(a-bs))\beta - c_I s + c_e bx = 0. \tag{4.11}$$

Comparing Equations (4.10) with (4.6) and (4.11) with (4.7), we find that (x_d, s_d) can be the centralized supply chain's optimal solution (x_c, s_c) only if $w = p$ and $b_c = p + c_h$. Therefore, the coordination can only occur if $w = p$, which is not desirable. With $w = p$ and $b_c = p + c_h$, the retailer earns a non-positive profit, so the retailer certainly cannot be better off with buyback contract. Cachon and Lariviere (2005) prove that the revenue sharing contract is equivalent to the buyback contract with the fixed-price newsvendor setting. However, our results show that the revenue sharing contract can coordinate the supply chain with the consideration of the sustainability and the emission whereas the buyback contract cannot.

4.6.3 Two-part tariff contract

With a two-part tariff contract (w, G), the manufacturer charges the retailer a per unit wholesale price w and a fix fee G. See Cachon and Lariviere (2005) and Cachon and Kök (2010) for detailed analysis of this contract. The retailer's and the manufacturer's expected profit functions are given by

$$\begin{aligned}\Pi_r &= \mathbb{E}[p\min\{D,x\} - wx - c_h(x-D)^+ - G];\\ \Pi_m &= \mathbb{E}[wx - cx - c_e((a-bs)x-K) - \frac{c_I}{2}s^2 + G].\end{aligned}$$

Proposition 4.9 *For a given two-part tariff contract (w, G), the optimal decision of the order quantity and sustainability investment $(x_d, s_d) = (x^*, s^*)$ are determined as follows:*

$$x - F^{-1}(\frac{p-w}{p+c_h}) - d - \beta s = 0; \quad (4.12)$$

$$(w - c - c_e(a-bs))\beta - c_I s + c_e bx = 0. \quad (4.13)$$

Comparing Equations (4.12) with (4.6) and (4.13) with (4.7), we find that (x_d, s_d) can be the centralized supply chain's optimal solution (x_c, s_c) only if $w = p = c + c_e(a - bs_c)$. With $p = c + c_e(a - bs_c)$, the manufacture can get the positive profit only if the total permissible emission level K is sufficient large such that the manufacture can earn some profit by selling the extra allowances of the emission. With $w = p$, the retailer earns a non-positive profit, so the retailer cannot be better off with two-part tariff contract. Hence, the two-part tariff contract does not coordinate the supply chain with the consideration of the sustainability and the emission.

Supply chain coordination implies that under a scenario the individual supply chain members can behave in a way eliminating double marginalization and maximizing the total supply chain's profit, so as to achieve the Pareto improvement. We find that when we consider the sustainability investment and carbon emission for the supply chain, the revenue sharing contract can coordinate the supply chain while the buyback and two-part tariff contracts cannot coordinate the supply chain. It may be due to the fact that the revenue sharing contract can induce the manufacturer doing better in the sustainability investment. Therefore, we should propose the revenue sharing contract for the supply chain so as to achieve the Pareto improvement.

4.7 Conclusions

Motivated by the real industrial practices, in this chapter, we consider supply chains in which a high level of product's sustainability not only increases the market demand, but also reduces the carbon emission, and its sustainability production of carbon emission abatement requires the sustainability investment as a cost. We first investigate a two-echelon decentralized supply chain in which the manufacturer firstly decides the product's sustainability level and then the retailer places an order under the cap-and-trade regulation. We also examine the supply chain in the centralized setting and then compare their performance with those in the decentralized one.

We derive the optimal ordering quantity and sustainability investment for decentralized setting, and the optimal production quantity and sustainability investment for centralized setting as well. We find that the sustainability investment coefficient has a significant impact on the optimal order quantity (or production quantity) and sustainability investment. If the sustainability investment and the emission issues are not considered, the optimal profit of the whole supply chain will be theoretically obtained when the wholesale price equals to the unit production cost. However, by examining the effects of the wholesale price, we find that, due to the effects of the sustainability and the emission, the optimal profit of the whole supply chain obtains its maximum at a wholesale price which is almost double of the unit production cost. On the other hand, if we do not consider the sustainability and the emission issues, then the order quantity will be decreasing in the wholesale price. However, our results show that the order quantity may be unexpectedly increasing in the wholesale price, because the order quantity is increasing in the sustainability level, but which may be decreasing in the wholesale price, depending on the cdf of the distribution of the demand and the value of the coefficient of the sustainability effect on increasing the demand. Moreover, with the consideration of the sustainability and the emission, the manufacturer's profit is smaller than that without considering the sustainability and the emission, but the retailer's profit has the inverse result. It is because that, with the consideration of the sustainability and the emission, the manufacturer needs to pay the emission cost and the sustainability investment cost, while the retailer can get the benefit of the sustainability effect on increasing the demand.

Finally, we study the coordination in the supply chain by considering three contracts, i.e., buyback, revenue sharing, and two-part tariff contracts. We verify that whether the contracts that coordinate the retailer's order quantity and also coordinate the manufacturer's sustainability level. It is shown that with the consideration of the sustainability and the emission, the buyback and two-part tariff contracts cannot coordinate the supply chain but revenue sharing contract can coordinate it. The allocation of the supply chain's profit in revenue sharing contract is unique. From the coordination perspective, this finding implies that the revenue sharing contract should be suggested to be adopted in sustainable supply chain.

Chapter 5

Summary and Future Research

In this thesis, we study three topics in capacity and sustainability investments from the perspectives of pricing and carbon emission. In the first and second topics, we analyze the capacity investment and pricing policies for an electricity company with the TOU tariff. In the first topic, we consider two models, one with price inelasticity of total demand (the total demand of electricity will not be affected by prices) and the other one with price elasticity of demand (the demands of electricity are functions of prices). We derive the optimal capacity investment and pricing policies for the electricity company for both models. Besides, for the model with price inelasticity of total demand, we derive the optimal shifted consumption for the customers under the TOU tariff by solving the customer's problem. Based on the work in the first topic, we further study the capacity investment and pricing policies for the TOU tariff with uncertain shifted consumption in the second topic. By examining the behavior of the optimal solutions with respect to some parameters, we obtain some insights for the electricity company, which are practically relevant. The electricity company may follow the insights upon implementing the TOU tariff. For example, we find that the electricity company should not change the prices when the market size expands. This guideline should be helpful to the electricity company with practical relevance. The third topic studies the environmental sustainability investment under a cap-and-trade regulation of carbon emission. We consider both decentralized and centralized supply chains with one manufacturer and one retailer. The optimal sustainability investment and order quantity are derived for the decentralized supply chain, and the optimal sustainability investment and production quantity are derived for the centralized supply chain. Moreover, the coordination of the supply chain is studied under several contracts in this topic. Important insights and managerial implications are discussed in all topics.

The phenomenon of the peak and non-peak periods exists not only in the electricity industry, but also in other industries, such as the transportation industry and telecommunication industry. Thus, from the application perspective, the analysis of the capacities investment in various technologies and the pricing for the peak period and non-peak period demands in the first and second topics can be applied to extensive industries with the property of having the peak and non-peak periods. In terms of future research, an immediate extension would be to consider the competition effects of multiple companies on the capacity and pricing policies. Some industrial examples have shown that the mandatory TOU tariff may be applied to some customers in some areas (RAP 2008, Friedman 2011, Jessoe and Rapson 2014). Based on this observation, we assume that the proportion of customers who use the TOU tariff is given. However, there are some examples showing that the TOU tariff may be optional to the customers (Tweed 2011), so another future research direction would be to consider the setting in which the proportion of customers using the TOU tariff is determined endogenously by the electricity prices and customers' values, based on consumer choice behaviour.

In the third topic, we have focused on investigating the optimal decisions of order quantity (or production quantity) and sustainability investment, given the wholesale price. Although the effects of wholesale price on the optimal solutions and profits are studied in this topic, it is worth considering the setting under which the wholesale price is determined endogenously in the future research. Besides, the consideration of the joint decision of the price and sustainability investment may provide additional useful insights. Moreover, it is also interesting to study the risk issues in a supply chain under the cap-and-trade regulation.

Finally, the combination of the three topics leads to a very interesting research direction. From an environmental perspective, the generation of electricity and heat accounts for 41% of the total CO_2 emission in the world in 2010. The use of fossil fuels to generate electricity also accounts for 38% of total CO_2 emission in the U.S. in 2012 (IEA 2012, EPA 2014). So it is an important future research direction to incorporate the modelling of CO_2 emission in the third topic into the first and second topics, and examine the effects of the constraints of CO_2 emission on the capacity investment and pricing policies for the electricity company. To extend our research on the TOU tariff with consideration of cap-and-trade regulation, we can consider that the generation of electricity will emit CO_2. Each unit of electricity generated by different technologies emits different units of CO_2. The electricity company can buy or sell the emission allowances in an outside market under the cap-and-trade regulation. In this way, we relate the three topics in this thesis together. This thesis not only fills a gap in the literature, but also lays a foundation for future research.

Appendix A

Proofs and Supplement for Chapter 2

A.1 Proofs

Proof of Proposition 2.1 We can easily show that $\Pi_c(q_s^i)$ is convex in q_s^i. The first derivative of $\Pi_c(q_s^i)$ with respect to q_s^i is

$$\frac{d\Pi_c(q_s^i)}{dq_s^i} = g'(q_s^i) - (p_2 - p_1).$$

Note that $g'(q_s^i)$ is increasing in q_s^i, so by comparing the values of $g'(0)$ and $g'(q_2/N)$ with $p_2 - p_1$, we obtain the results. ■

Before presenting the proof of Theorem 2.1, we show Lemma A.1 for the model with price inelasticity of total demand.

Lemma A.1 *If $q_2 - q_s(p_1, p_2) \leq \tau k_1$, the optimal capacities $(k_1(\mathbf{p}), k_2(\mathbf{p}))$ are determined by:*

$$\big(k_1(\mathbf{p}), k_2(\mathbf{p})\big) = \begin{cases} (\frac{q_1+q_s(p_1,p_2)}{1-\tau}, 0) & \text{if } q_s(p_1,p_2) > q_0; \\ (\frac{q_2-q_s(p_1,p_2)}{\tau}, 0) & \text{if } q_s(p_1,p_2) \leq q_0. \end{cases}$$

If $q_2 - q_s(p_1, p_2) \geq \tau k_1$, the optimal capacities $(k_1(\mathbf{p}), k_2(\mathbf{p}))$ are determined by:

$$\big(k_1(\mathbf{p}), k_2(\mathbf{p})\big) = \begin{cases} (\frac{q_2-q_s(p_1,p_2)}{\tau}, 0) & \text{if } q_s(p_1,p_2) \leq q_0 \\ & \text{and } \tau\beta_2 + c_2 \geq \tau\beta_1 + c_1; \\ (\frac{q_1+q_s(p_1,p_2)}{1-\tau}, \frac{q_0-q_s(p_1,p_2)}{\tau(1-\tau)}) & \text{if } q_s(p_1,p_2) \leq q_0 \\ & \text{and } \tau\beta_2 + c_2 < \tau\beta_1 + c_1; \\ \text{no feasible solution} & \text{if } q_s(p_1,p_2) > q_0. \end{cases}$$

Proof of Lemma A.1 Note from Constraint (2.3) that k_2, determined by $q_2 - q_s(p_1, p_2)$, can be zero or positive. So we consider two cases. The first case is $q_2 - q_s(p_1, p_2) \leq \tau k_1$ such that $k_2 = 0$. In this case, the company's profit function can be expressed as

$$\begin{aligned}\Pi_g(\mathbf{k}, \mathbf{p}) &= \alpha(p_1q_1 + p_2q_2) + (1-\alpha)p_0(q_1+q_2) - (p_2 - p_1)q_s(p_1, p_2) \\ &\quad -\beta_1(q_1+q_2) - c_1k_1. \qquad \text{(A.1)}\end{aligned}$$

The second case is $q_2 - q_s(p_1, p_2) \geq \tau k_1$ such that $k_2 \geq 0$. In this case, the company's profit function can be expressed by:

$$\begin{aligned}\Pi_g(\mathbf{k}, \mathbf{p}) &= \alpha(p_1q_1 + p_2q_2) + (1-\alpha)p_0(q_1+q_2) - (p_2 - p_1 + \beta_1 - \beta_2)q_s(p_1, p_2) \\ &\quad -\beta_1q_1 - \beta_2q_2 - (c_1 + \tau\beta_1 - \tau\beta_2)k_1 - c_2k_2. \qquad \text{(A.2)}\end{aligned}$$

First we prove the results for the case $q_2 - q_s(p_1, p_2) \leq \tau k_1$ ($k_2 = 0$). From Equation (A.1), we find that $\Pi_g(\mathbf{k}, \mathbf{p})$ is decreasing in k_1. Then the optimal k_1 is equal to its lower bound, which is indicated by Constraints (2.1) and (2.2). Note that $k_2 = 0$, so we need to compare $\frac{q_1+q_s(p_1,p_2)}{1-\tau}$ and $\frac{q_2-q_s(p_1,p_2)}{\tau}$ to determine the lower bound on k_1. After that, the proof for the case $q_2 - q_s(p_1, p_2) \leq \tau k_1$ is completed.

We then show the results for the case where $q_2 - q_s(p_1, p_2) \geq \tau k_1$ ($k_2 \geq 0$). Recalling that we have two other constraints: Constraint (2.1) $q_1 + q_s(p_1, p_2) \leq (1-\tau)k_1$ and Constraint (2.2) $q_2 - q_s(p_1, p_2) \leq \tau(k_1 + k_2)$. If $\frac{q_1+q_s(p_1,p_2)}{1-\tau} > \frac{q_2-q_s(p_1,p_2)}{\tau}$ (i.e., $q_s > q_0$), combining it with $q_2 - q_s(p_1, p_2) \geq \tau k_1$ and $q_1 + q_s(p_1, p_2) \leq (1-\tau)k_1$, we get a contradiction. So there is no feasible solution if $q_s > q_0$.

Next we show the results for the case where $\frac{q_1+q_s(p_1,p_2)}{1-\tau} \leq \frac{q_2-q_s(p_1,p_2)}{\tau}$ (i.e., $q_s \leq q_0$). Note from Equation (A.2) that $\Pi_g(\mathbf{k}, \mathbf{p})$ is decreasing in k_2, and it is increasing in k_1 if $c_1 + \tau\beta_1 - \tau\beta_2 \leq 0$ and decreasing otherwise. Then we consider two cases as follows:

(1) If $c_1 + \tau\beta_1 - \tau\beta_2 \leq 0$, then $\Pi_g(\mathbf{k}, \mathbf{p})$ is increasing in k_1 and decreasing in k_2. From the conditions $q_2 - q_s(p_1, p_2) \geq \tau k_1$, $q_1 + q_s(p_1, p_2) \leq (1-\tau)k_1$, and $q_2 - q_s(p_1, p_2) \leq \tau(k_1 + k_2)$ (where $\frac{q_1+q_s(p_1,p_2)}{1-\tau} \leq \frac{q_2-q_s(p_1,p_2)}{\tau}$), we obtain that the optimal k_1 and k_2 are $k_1(p_1, p_2) = \frac{q_2-q_s(p_1,p_2)}{\tau}$ and $k_2(p_1, p_2) = 0$, respectively.

(2) If $c_1 + \tau\beta_1 - \tau\beta_2 > 0$, then $\Pi_g(\mathbf{k}, \mathbf{p})$ is decreasing in k_1 and k_2. In this case, we still have the conditions $q_2 - q_s(p_1, p_2) \geq \tau k_1$, $q_1 + q_s(p_1, p_2) \leq (1-\tau)k_1$, and $q_2 - q_s(p_1, p_2) \leq \tau(k_1 + k_2)$ (where $\frac{q_1+q_s(p_1,p_2)}{1-\tau} \leq \frac{q_2-q_s(p_1,p_2)}{\tau}$). However, in order to maximize $\Pi_g(\mathbf{k}, \mathbf{p})$, which is decreasing in k_1 and k_2, we need to compare $\frac{c_1+\tau\beta_1-\tau\beta_2}{c_2}$ and 1.

(2.1) If $\frac{c_1+\tau\beta_1-\tau\beta_2}{c_2} \geq 1$, then the optimal k_1 and k_2 are $k_1(p_1, p_2) = \frac{q_1+q_s(p_1,p_2)}{1-\tau}$ and $k_2(p_1, p_2) = \frac{q_0-q_s(p_1,p_2)}{\tau(1-\tau)}$.

(2.2) If $\frac{c_1+\tau\beta_1-\tau\beta_2}{c_2} < 1$, then the optimal k_1 and k_2 are $k_1(p_1, p_2) = \frac{q_2-q_s(p_1,p_2)}{\tau}$ and $k_2(p_1, p_2) = 0$.

Thus, by combining (1) and (2), we obtain that, for the case $q_s \leq q_0$, $(k_1(p_1, p_2), k_2(p_1, p_2)) = (\frac{q_2-q_s(p_1,p_2)}{\tau}, 0)$ if $c_1 + \tau\beta_1 - \tau\beta_2 < c_2$; $(k_1(p_1, p_2), k_2(p_1, p_2)) = (\frac{q_1+q_s(p_1,p_2)}{1-\tau}, \frac{q_0-q_s(p_1,p_2)}{\tau(1-\tau)})$ if $c_1 + \tau\beta_1 - \tau\beta_2 \geq c_2$.

By combining the results obtained above, we complete the proof. ■

Proof of Theorem 2.1 From Lemma A.1, we find that there are five cases for the capacity decision. We enumerate them in Table A.1.

Table A.1: Five cases for the capacity values

Case	$(k_1(\mathbf{p}), k_2(\mathbf{p}))$	Conditions
1	$(\frac{q_1+q_s(p_1,p_2)}{1-\tau}, 0)$	$q_s(p_1, p_2) > q_0$ and $q_2 - q_s(p_1, p_2) \leq \tau k_1$
2	$(\frac{q_2-q_s(p_1,p_2)}{\tau}, 0)$	$q_s(p_1, p_2) \leq q_0$ and $q_2 - q_s(p_1, p_2) \leq \tau k_1$
3	$(\frac{q_2-q_s(p_1,p_2)}{\tau}, 0)$	$q_s(p_1, p_2) \leq q_0$, $\tau\beta_2 + c_2 \geq \tau\beta_1 + c_1$ and $q_2 - q_s(p_1, p_2) \geq \tau k_1$
4	$(\frac{q_1+q_s(p_1,p_2)}{1-\tau}, \frac{q_0-q_s(p_1,p_2)}{\tau(1-\tau)})$	$q_s(p_1, p_2) \leq q_0$, $\tau\beta_2 + c_2 < \tau\beta_1 + c_1$ and $q_2 - q_s(p_1, p_2) \geq \tau k_1$
5	no feasible solution	$q_s(p_1, p_2) > q_0$ and $q_2 - q_s(p_1, p_2) \geq \tau k_1$

First, we divide Case 2 into two cases as shown in Table A.2:

Table A.2: Two sub-cases for the capacity values

Case	$(k_1(\mathbf{p}), k_2(\mathbf{p}))$	Conditions
2-1	$(\frac{q_2-q_s(p_1,p_2)}{\tau}, 0)$	$q_s(p_1, p_2) \leq q_0$, $\tau\beta_2 + c_2 \geq \tau\beta_1 + c_1$ and $q_2 - q_s(p_1, p_2) \leq \tau k_1$
2-2	$(\frac{q_2-q_s(p_1,p_2)}{\tau}, 0)$	$q_s(p_1, p_2) \leq q_0$, $\tau\beta_2 + c_2 < \tau\beta_1 + c_1$ and $q_2 - q_s(p_1, p_2) \leq \tau k_1$

Then, we complete the proof by considering three combinations:

(1) First combination: by combining Cases 2-1 and 3, we obtain that $(k_1(\mathbf{p}), k_2(\mathbf{p})) = (\frac{q_2 - q_s(p_1,p_2)}{\tau}, 0)$ if $q_s(p_1, p_2) \leq q_0$ and $\tau\beta_2 + c_2 \geq \tau\beta_1 + c_1$.

(2) Second combination: we combine Cases 2-2 and 4. Note that in Case 2-2, $k_1 = \frac{q_2 - q_s(p_1,p_2)}{\tau}$, so the condition $q_2 - q_s(p_1, p_2) \leq \tau k_1$ is always satisfied and the optimal value is obtained on the boundary of this condition for Case 4. This implies that Case 2-2 is dominated by Case 4. So, by combining Cases 2-2 and 4, we obtain that $(k_1(\mathbf{p}), k_2(\mathbf{p})) = (\frac{q_1 + q_s(p_1,p_2)}{1-\tau}, \frac{q_0 - q_s(p_1,p_2)}{\tau(1-\tau)})$ if $q_s(p_1, p_2) \leq q_0$ and $\tau\beta_2 + c_2 < \tau\beta_1 + c_1$.

(3) Third combination: by combining Cases 1 and 5, we obtain that $(k_1(\mathbf{p}), k_2(\mathbf{p})) = (\frac{q_1 + q_s(p_1,p_2)}{1-\tau}, 0)$ if $q_s(p_1, p_2) > q_0$. ■

Proof of Lemma 2.1 Note that $g'(q_s^i) \geq 0$ and $p_2 - p_1 = g'(q_s^i) = g'(q_s/(\alpha N))$, then the condition for Case III, i.e., $q_s(p_1, p_2) \geq q_0$, is equivalent to $p_2 - p_1 \geq g'(q_0/(\alpha N))$. We can list the conditions associated with p_1 and p_2: $g'(q_0/(\alpha N)) \leq p_2 - p_1 \leq g'(q_2/N)$; $p_1 \leq p_0^* \leq p_2 \leq \bar{p_2}$; $p_2 - p_1 = g'(q_s/(\alpha N))$, $\Delta\Pi_c \leq 0$, and $\Delta\Pi_c|_{(p_1=0, p_2=g'(q_0/(\alpha N)))} \leq 0$.

Taking the first partial derivative of $\Pi_g(\mathbf{p})$ with respect to p_1, we have

$$\begin{aligned}
\frac{\partial \Pi_g(\mathbf{p})}{\partial p_1} &= \alpha q_1 + q_s(p_1, p_2) - (p_2 - p_1 + \frac{c_1}{1-\tau})\frac{\partial q_s(p_1, p_2)}{\partial p_1} \\
&= \alpha q_1 + q_s(p_1, p_2) - (p_2 - p_1 + \frac{c_1}{1-\tau})\alpha N \frac{\partial q_s^i(p_1, p_2)}{\partial p_1} \\
&= \alpha q_1 + q_s(p_1, p_2) + (p_2 - p_1 + \frac{c_1}{1-\tau})\frac{\alpha N}{g''(q_s/(\alpha N))} > 0,
\end{aligned}$$

where $(\partial q_s^i(p_1, p_2))/\partial p_1 = -1/g''(q_s/(\alpha N))$ is derived from Equation (2.9). The inequality holds because of $p_2 > p_1$ and $g''(q_s^i) > 0$. Therefore, $\Pi_g(\mathbf{p})$ is increasing in p_1 for a given p_2. Combining with the conditions listed above, we have the result that the optimal point either lies on $p_2 - p_1 = g'(q_0/(\alpha N))$ or $\Delta\Pi_c = 0$.

Next we consider the point along with the curve $\Delta\Pi_c = 0$, and we treat p_2 as a function of p_1. Then, we have

$$\begin{aligned}
\frac{dp_2(p_1)}{dp_1} &= -\frac{\partial \Delta\Pi_c/\partial p_1}{\partial \Delta\Pi_c/\partial p_2} \\
&= -\frac{\alpha q_1 + q_s(p_1, p_2) - \frac{\partial q_s(p_1,p_2)}{\partial p_1}(p_2 - p_1 - g'(q_s/(\alpha N)))}{\alpha q_2 - q_s(p_1, p_2) - \frac{\partial q_s(p_1,p_2)}{\partial p_2}(p_2 - p_1 - g'(q_s/(\alpha N)))}
\end{aligned}$$

$$= -\frac{\alpha q_1 + q_s(p_1,p_2)}{\alpha q_2 - q_s(p_1,p_2)} < 0. \tag{A.3}$$

The last equality holds because of $p_2 - p_1 = g'(q_s/(\alpha N))$, which is derived by the customer's optimal response of shifted consumption. Furthermore, we have

$$\Pi_g(\mathbf{p}|_{\Delta\Pi_c=0}) = \bar{p_0}(q_1+q_2) - \alpha N g(q_s/(\alpha N)) - \frac{c_1}{1-\tau}q_s - \beta_1(q_1+q_2) - \frac{c_1}{1-\tau}q_1.$$

By taking the first derivative of $\Pi_g(\mathbf{p})|_{\Delta\Pi_c=0}$ with respect to p_1, we have

$$\begin{aligned}
\frac{d\Pi_g(\mathbf{p}|_{\Delta\Pi_c=0})}{dp_1} &= \frac{\partial\Pi_g(\mathbf{p}|_{\Delta\Pi_c=0})}{\partial p_2}\frac{dp_2(p_1)}{dp_1} + \frac{\partial\Pi_g(\mathbf{p}|_{\Delta\Pi_c=0})}{\partial p_1} \\
&= (g'(q_s/(\alpha N)) + \frac{c_1}{1-\tau})\frac{\alpha N}{g''(q_s/(\alpha N))}(-\frac{dp_2(p_1)}{dp_1}+1) \\
&= (g'(q_s/(\alpha N)) + \frac{c_1}{1-\tau})\frac{\alpha N}{g''(q_s/(\alpha N))}\frac{\alpha(q_1+q_2)}{\alpha q_2 - q_s(p_1,p_2)} > 0.
\end{aligned}$$

This implies that along with $\Delta\Pi_c = 0$, $\Pi_g(\mathbf{p})$ is increasing in p_1. Note that $\frac{dp_2(p_1)}{dp_1}$ indicates that Equation (A.3) is negative. So the optimal point is the intersection of $p_2 - p_1 = g'(q_0/(\alpha N))$ and $\Delta\Pi_c = 0$ regardless of the value of $\bar{p_2}$. Although the value of $\bar{p_2}$ is taken into consideration, the optimal point is on $p_2 - p_1 = g'(q_0/(\alpha N))$, which is the boundary of Cases I and II, if $\bar{p_2} \geq g'(q_0/(\alpha N))$; otherwise, there is no feasible solution for Case III. Therefore, we say that Case III is dominated by Cases I and II. ■

Proof of Theorem 2.2 Please note that $\gamma = \frac{c_1}{\tau}$ if $\tau\beta_2 + c_2 \geq \tau\beta_1 + c_1$, and $\gamma = \frac{\beta_2 + c_2/\tau - \beta_1 - c_1}{1-\tau}$ if $\tau\beta_2 + c_2 \leq \tau\beta_1 + c_1$. Then combining Cases I and II, we express the profit function as follows:

$$\begin{aligned}
\Pi_g(\mathbf{p}) &= \alpha(p_1q_1 + p_2q_2) + (1-\alpha)p_0(q_1+q_2) - (p_2 - p_1 - \gamma)q_s(p_1,p_2) \\
&\quad - \begin{cases} \frac{(\beta_1+c_1)q_1 + (\beta_2 + c_2/\tau)q_0}{1-\tau} & \text{if } \tau\beta_2 + c_2 \leq \tau\beta_1 + c_1; \\ \beta_1(q_1+q_2) + c_1\frac{q_2}{\tau} & \text{otherwise.} \end{cases}
\end{aligned} \tag{A.4}$$

We next list the conditions associated with p_1 and p_2: $g'(0) \leq p_2 - p_1 \leq g'(q_0/(\alpha N))$; $p_1 \leq \bar{p_0} \leq p_2 \leq \bar{p_2}$; $p_2 - p_1 = g'(q_s/(\alpha N))$, $\Delta\Pi_c \leq 0$ and $\Delta\Pi_c|_{(p_1=0,p_2=\theta)} \leq 0$. Let ω be the feasible region of p_1 and p_2 such that $g'(0) \leq p_2 - p_1 \leq g'(q_0/(\alpha N))$ and $\Delta\Pi_c \leq 0$.

Taking the first partial derivative of $\Pi_g(\mathbf{p})$ with respect to p_1, we have

$$\begin{aligned} \frac{\partial \Pi_g(\mathbf{p})}{\partial p_1} &= \alpha q_1 + q_s(p_1, p_2) - (p_2 - p_1 - \gamma)\frac{\partial q_s(p_1, p_2)}{\partial p_1} \\ &= \alpha q_1 + q_s(p_1, p_2) + (p_2 - p_1 - \gamma)\frac{\alpha N}{g''(q_s/(\alpha N))}, \end{aligned} \tag{A.5}$$

where $(\partial q_s(p_1, p_2))/\partial p_1 = -(\alpha N)/g''(q_s/(\alpha N))$ is derived from Equation (2.9).

Then we consider that, if $p_2 - p_1 - \gamma \geq 0$, then $\frac{\partial \Pi_g(\mathbf{p})}{\partial p_1} > 0$ and $\Pi_g(\mathbf{p})$ is increasing in p_1; if $p_2 - p_1 - \gamma < 0$, then by taking the second partial derivative of $\Pi_g(\mathbf{p})$ with respect to p_1, we have

$$\frac{\partial^2 \Pi_g(\mathbf{p})}{\partial p_1^2} = -\frac{\alpha N}{g''(q_s/(\alpha N))}\Big(2 - (p_2 - p_1 - \gamma)\frac{g'''(q_s/(\alpha N))}{(g''(q_s/(\alpha N)))^2}\Big) \leq 0.$$

Moreover, note from Equation (A.5) that p_1 and p_2 only explicitly appear in the term of $p_2 - p_1$. Combining it with $q_s = \alpha N g'^{-1}(p_2 - p_1)$, we find that the solution of $\frac{\partial \Pi_g(\mathbf{p})}{\partial p_1} = 0$ is $p_2 - p_1 = p_2^D$, where p_2^D is the solution of p_2 from the equation: $\frac{\partial \Pi_g(\mathbf{p})}{\partial p_1}|_{p_1=0} = \alpha q_1 + \alpha N g'^{-1}(p_2) + (p_2 - \gamma)\frac{\alpha N}{g''(g'^{-1}(p_2))} = 0$. This implies that the solutions obtained from the first-order condition of $\Pi_g(\mathbf{p})$ are a straight line, e.g., $p_2 - p_1 = p_2^D$.

On the other hand, we have

$$\begin{aligned} \Pi_g(\mathbf{p}|_{\Delta\Pi_c=0}) &= \bar{p_0}(q_1 + q_2) - \alpha N g(q_s/(\alpha N)) + \gamma q_s \\ &\quad - \begin{cases} \frac{(\beta_1+c_1)q_1+(\beta_2+c_2/\tau)q_0}{1-\tau} & \text{if } \tau\beta_2 + c_2 \leq \tau\beta_1 + c_1; \\ \beta_1(q_1 + q_2) + c_1\frac{q_2}{\tau} & \text{otherwise.} \end{cases} \end{aligned}$$

By taking the first derivative of $\Pi_g(\mathbf{p}|_{\Delta\Pi_c=0})$ with respect to p_1, we have

$$\begin{aligned} \frac{d\Pi_g(\mathbf{p}|_{\Delta\Pi_c=0})}{dp_1} &= \frac{\partial \Pi_g(\mathbf{p}|_{\Delta\Pi_c=0})}{\partial p_2}\frac{dp_2(p_1)}{dp_1} + \frac{\partial \Pi_g(\mathbf{p}|_{\Delta\Pi_c=0})}{\partial p_1} \\ &= (g'(q_s/(\alpha N)) - \gamma)\frac{\alpha N}{g''(q_s/(\alpha N))}\frac{\alpha(q_1 + q_2)}{\alpha q_2 - q_s(p_1, p_2)} \\ &= (p_2 - p_1 - \gamma)\frac{\alpha N}{g''(q_s/(\alpha N))}\frac{\alpha(q_1 + q_2)}{\alpha q_2 - q_s(p_1, p_2)}, \end{aligned}$$

where the value of $\frac{dp_2(p_1)}{dp_1}$ is indicated in Equation (A.3). This implies that along with $\Delta\Pi_c = 0$, $\Pi_g(\mathbf{p})$ increases in p_1 when $p_2 - p_1 - \gamma \geq 0$ and decreases otherwise. Here, we consider two cases: Case (1) $\gamma \leq g'(q_0/(\alpha N))$ and Case (2) $\gamma \geq g'(q_0/(\alpha N))$.

Case (1) $\gamma \leq g'(q_0/(\alpha N))$. Then we consider the value of p_2^D. Since

$$\frac{\partial \Pi_g(\mathbf{p})}{\partial p_1}|_{p_2-p_1=\gamma} = \alpha q_1 + q_s(p_1, p_2) \geq 0,$$

we have the result that $p_2^D \leq \gamma$ for Case (1), so we consider two subcases: Subcase (1.1) $p_2^D \leq g'(0)$ and Subcase (1.2) $p_2^D \geq g'(0)$.

Subcase (1.1) $p_2^D \leq g'(0)$. Then in the feasible region ω, we have $\frac{\partial \Pi_g(\mathbf{p})}{\partial p_1} \geq 0$. This implies that the optimal point is either on the line $p_2 - p_1 = g'(0)$ or on the curve $\Delta\Pi_c = 0$. Along with line $p_2 - p_1 = g'(0)$, we have

$$\frac{d\Pi_g(\mathbf{p}|_{p_2-p_1=C_s'(0)})}{dp_1} = \alpha(q_1 + q_2) \geq 0.$$

This means that the intersection of $p_2 - p_1 = g'(0)$ and $\Delta\Pi_c = 0$ is optimal on the line $p_2 - p_1 = g'(0)$. So we only need to consider the curve $\Delta\Pi_c = 0$. Recalling that along with $\Delta\Pi_c = 0$, $\Pi_g(\mathbf{p})$ increases in p_1 when $p_2 - p_1 - \gamma \geq 0$, and decreases otherwise. Moreover, along with the curve $\Delta\Pi_c = 0$, we have $\frac{dp_2(p_1)}{dp_1} < 0$, which is indicated in Equation (A.3). So the intersection of $\Delta\Pi_c = 0$ and $p_2 - p_1 = \gamma$ is optimal on the curve $\Delta\Pi_c = 0$. Therefore, the intersection of $\Delta\Pi_c = 0$ and $p_2 - p_1 = \gamma$ is an optimal solution for Subcase (1.1) regardless of the value of $\bar{p_2}$.

Next we consider the effect of the value of $\bar{p_2}$: the intersection of $\Delta\Pi_c = 0$ and $p_2 - p_1 = \gamma$ is still an optimal point if the line segment $p_2 = \bar{p_2}$ is above it (i.e., $p_1^* = p_2^B - \gamma$ and $p_2^* = p_2^B$ if $\bar{p_2} \geq p_2^B$); the intersection of $p_2 = \bar{p_2}$ and $\Delta\Pi_c = 0$ is an optimal point if the line segment $p_2 = \bar{p_2}$ is between the intersection of $p_2 - p_1 = \gamma$ and $\Delta\Pi_c = 0$, and the intersection of $p_2 - p_1 = g'(0)$ and $\Delta\Pi_c = 0$, since $\Pi_g(\mathbf{p})$ increases in p_2 along with the curve $\Delta\Pi_c = 0$ when $p_2 - p_1 - \gamma \leq 0$ (i.e., $p_1^* = p_1^E$ and $p_2^* = \bar{p_2}$ if $p_2^A \leq \bar{p_2} \leq p_2^B$); the intersection of $p_2 = \bar{p_2}$ and $p_2 - p_1 = g'(0)$ is an optimal point if the line segment $p_2 = \bar{p_2}$ is between the intersection of $p_2 - p_1 = g'(0)$ and $\Delta\Pi_c = 0$, and $p_2 = g'(0)$, since $\Pi_g(\mathbf{p})$ increases in p_1 along with the line segment $p_2 - p_1 = g'(0)$ (i.e., $p_1^* = \bar{p_2} - g'(0)$ and $p_2^* = \bar{p_2}$ if $g'(0) \leq \bar{p_2} \leq p_2^A$); and there is no feasible solution if $\bar{p_2} \leq g'(0)$, we omit this extremely special case. The optimal shifted consumption is determined immediately as long as the optimal prices are obtained.

Subcase (1.2) $p_2^D \geq g'(0)$. Then there is the line segment $p_2 - p_1 = p_2^D$ in the feasible region ω. The optimal point is either on the line segment $p_2 - p_1 = p_2^D$ as $\frac{\partial^2 \Pi_g(\mathbf{p})}{\partial p_1^2} \leq 0$ when $p_2 - p_1 - \gamma \leq 0$, or on the curve $\Delta\Pi_c = 0$. Along with line $p_2 - p_1 = p_2^D$, we have

$$\frac{d\Pi_g(\mathbf{p}|_{p_2-p_1=p_2^D})}{dp_2} = \alpha(q_1 + q_2) \geq 0.$$

This means that the intersection of $p_2 - p_1 = p_2^D$ and $\Delta\Pi_c = 0$ is optimal on the line $p_2 - p_1 = p_2^D$. So we only need to consider the curve $\Delta\Pi_c = 0$. Then the rest of the analysis for Subcase (1.2) is the same as that in Subcase (1.1). Therefore, we conclude that the intersection of $\Delta\Pi_c = 0$ and $p_2 - p_1 = \gamma$ is an optimal solution for Subcase (1.2) regardless of the value of $\bar{p_2}$. Similarly, the optimal solutions can be easily obtained when the value of $\bar{p_2}$ is taken into consideration.

Case (2) $\gamma \geq g'(q_0/(\alpha N))$. Then we have

$$\frac{\partial\Pi_g(\mathbf{p})}{\partial p_1}|_{p_2-p_1=g'(q_0/(\alpha N))} = \alpha q_1 + q_s(p_1, p_2) + (g'(q_0/(\alpha N)) - \gamma)\frac{\alpha N}{g''(q_s/(\alpha N))},$$

which may be positive or negative. So we need to consider three subcases: Subcase (2.1) $p_2^D \leq g'(0)$, Subcase (2.2) $g'(0) \leq p_2^D \leq g'(q_0/(\alpha N))$, and Subcase (2.3) $p_2^D \geq g'(q_0/(\alpha N))$. The analysis and results for these subcases are very similar to those presented above, so we omit the details here. Combining the results of Cases (1) and (2), we complete the proof. ■

Proof of Proposition 2.2 The results associated with k_1^*, k_2^*, $k_1^* + k_2^*$, p_1^*, p_2^*, and q_s^* can be obtained directly by taking the derivatives of these optimal values with respect to α. Note that for $(p_1^*, p_2^*) = (p_1^F, \bar{p_2})$, $\frac{dq_s^*}{d\alpha} = \frac{N}{2c_s}(\frac{\gamma}{2} - \frac{c_s q_1}{N}) \geq 0$ as $\frac{\gamma}{2} - \frac{c_s q_1}{N} = p_2^D \geq 0$ for this case. Next we show the results for $\frac{d\Pi_g}{d\alpha}$ and $\frac{\Delta\Pi_c}{d\alpha}$ as follows:

(1) $(p_1^*, p_2^*) = (p_2^B - \theta, p_2^B)$. By taking the derivative of Π_g with respect to α, we can obtain that $\frac{d\Pi_g}{d\alpha} = \frac{1}{2}\frac{N\theta}{2c_s}(2\gamma - \theta) \geq 0$, where the inequality holds because $\theta \leq \gamma$. Since $\Delta\Pi_c = 0$ in this case, we have $\frac{d\Pi_c}{d\alpha} = 0$.

(2) $(p_1^*, p_2^*) = (p_1^E, \bar{p_2})$. By taking the derivative of Π_g with respect to α, we can obtain that $\frac{d\Pi_g}{d\alpha} = \frac{dq_s^*}{d\alpha}(\gamma - \frac{c_s}{N}\frac{dq_s^*}{d\alpha}) = -Nq_s^{i*}(c_s q_s^{i*} - \gamma) = -\frac{1}{2}Nq_s^{i*}(p_2^* - p_1^* - \gamma - \gamma) \geq 0,$

where the inequality holds because $p_2 - p_1 - \gamma \leq 0$ in this case. Similarly, $\Delta\Pi_c = 0$ in this case, so we have $\frac{d\Pi_c}{d\alpha} = 0$.

(3) $(p_1^*, p_2^*) = (p_1^F, \bar{p_2})$. By taking the derivative of Π_g with respect to α, we can obtain that $\frac{d\Pi_g}{d\alpha} = w + \frac{N}{2c_s}(\frac{\gamma}{2} - \frac{c_s q_1}{N})^2 \geq 0$. By rearranging $\Delta\Pi_c$, we obtain that $\Delta\Pi_c = \frac{1}{N}(p_1^* q_1 + p_2^* q_2 - \bar{p_0}(q_1 + q_2)) - c_s ({q_s^i}^*)^2$ which is independent of α, so we have $\frac{d\Pi_c}{d\alpha} = 0$.

(4) $(p_1^*, p_2^*) = (0, \bar{p_2})$. First we note that in this case $\frac{\partial\Pi_g}{\partial p_1} = \alpha q_s + (p_2 - p_1 - \gamma)\frac{\alpha N}{2c_s} \leq 0$, implying that $-(2c_s q_1 + (p_2^* - p_1^* - \gamma)N) = -(2c_s q_1 + (\bar{p_2} - \gamma)N) \geq 2c_s N {q_s^i}^*$. Then by taking the derivative of Π_g with respect to α, we can obtain that $\frac{d\Pi_g}{d\alpha} = w - \frac{\bar{p_2}}{2c_s}(2c_s q_1 + (\bar{p_2} - \gamma)N) \geq w + \frac{\bar{p_2}}{2c_s} 2c_s N {q_s^i}^* = w + \bar{p_2} N {q_s^i}^* \geq 0$. Similarly, by rearranging $\Delta\Pi_c$, we obtain that $\Delta\Pi_c = \frac{1}{N}(p_1^* q_1 + p_2^* q_2 - \bar{p_0}(q_1 + q_2)) - c_s({q_s^i}^*)^2$ which is independent of α, so we have $\frac{d\Pi_c}{d\alpha} = 0$. ■

Proof of Theorem 2.3 Similar to the model with price inelasticity of total demand, from Constraint (2.3), we find that two cases need to be considered, according to the values of $D_2(p_0, p_1, p_2)$ and τk_1. The first case is $D_2(p_0, p_1, p_2) \leq \tau k_1$ such that $k_2 = 0$. In this case, the company's profit function can be expressed by

$$\begin{aligned}
\Pi_g(\mathbf{k}, \mathbf{p}) &= \alpha\Big(p_1 D_{T1}(p_1, p_2) + p_2 D_{T2}(p_1, p_2)\Big) + (1-\alpha)p_0\Big(D_{F1}(p_0) + D_{F2}(p_0)\Big) \\
&\quad -\beta_1 D_1(p_0, p_1, p_2) - \beta_1 D_2(p_0, p_1, p_2) - c_1 k_1.
\end{aligned}$$

The second case is $D_2(p_0, p_1, p_2) \geq \tau k_1$ such that $k_2 \geq 0$. In this case, the company's profit function can be expressed by

$$\begin{aligned}
\Pi_g(\mathbf{k}, \mathbf{p}) &= \alpha\Big(p_1 D_{T1}(p_1, p_2) + p_2 D_{T2}(p_1, p_2)\Big) + (1-\alpha)p_0\Big(D_{F1}(p_0) + D_{F2}(p_0)\Big) \\
&\quad -\beta_1 D_1(p_0, p_1, p_2) - \beta_2 D_2(p_0, p_1, p_2) - \Big(c_1 + \tau\beta_1 - \tau\beta_2\Big)k_1 - c_2 k_2.
\end{aligned}$$

The rest of the analysis and proof are almost the same as those for the model with inelastic total demand, so we omit the proof here. ■

Proof of Theorem 2.4 By substituting the optimal response of capacity into the company's profit function, our objective becomes maximizing the profit function by optimally setting the prices for electricity. The company's profit function can be expressed as follows:

$$\Pi_g(\mathbf{p}) = \begin{cases} \Pi_0 - \beta_1 D_1(p_0,p_1,p_2) - \beta_2 D_2(p_0,p_1,p_2) - \big(c_1+\tau\beta_1-\tau\beta_2\big)\frac{D_1(p_0,p_1,p_2)}{1-\tau} \\ \qquad -c_2\big(\frac{D_2(p_0,p_1,p_2)}{\tau} - \frac{D_1(p_0,p_1,p_2)}{1-\tau}\big) & \text{for Case I;} \\ \Pi_0 - \beta_1 D_1(p_0,p_1,p_2) - \beta_1 D_2(p_0,p_1,p_2) - c_1\frac{D_2(p_0,p_1,p_2)}{\tau} & \text{for Case II;} \\ \Pi_0 - \beta_1 D_1(p_0,p_1,p_2) - \beta_1 D_2(p_0,p_1,p_2) - c_1\frac{D_1(p_0,p_1,p_2)}{1-\tau} & \text{for Case III,} \end{cases}$$

where $\Pi_0 = \alpha\Big(p_1 D_{T1}(p_1,p_2) + p_2 D_{T2}(p_1,p_2)\Big) + (1-\alpha)p_0\Big(D_{F1}(p_0) + D_{F2}(p_0)\Big)$.

The proofs for Cases I, II, and III are almost the same. So we just present the proof for Case III here.

By considering the Hessian matrix of the profit function, we have

$$\begin{aligned} H(\Pi_g) &= \begin{pmatrix} \frac{\partial^2\Pi_g}{\partial p_0^2} & \frac{\partial^2\Pi_g}{\partial p_0 p_1} & \frac{\partial^2\Pi_g}{\partial p_0 p_2} \\ \frac{\partial^2\Pi_g}{\partial p_1 p_0} & \frac{\partial^2\Pi_g}{\partial p_1^2} & \frac{\partial^2\Pi_g}{\partial p_1 p_2} \\ \frac{\partial^2\Pi_g}{\partial p_2 p_0} & \frac{\partial^2\Pi_g}{\partial p_2 p_1} & \frac{\partial^2\Pi_g}{\partial p_2^2} \end{pmatrix} \\ &= \begin{pmatrix} -2(1-\alpha)(b_{F1}+b_{F2}) & 0 & 0 \\ 0 & -2\alpha b_{T1} & \alpha(r_1+r_2) \\ 0 & \alpha(r_1+r_2) & -2\alpha b_{T2} \end{pmatrix}, \end{aligned}$$

and

$$\begin{aligned} |H_1^1| = -2(1-\alpha)(b_{F1}+b_{F2}) \le 0; |H_2^1| &= -2\alpha b_{T1} \le 0; |H_3^1| = -2\alpha b_{T2} \le 0; \\ |H_{12}^2| &= \Big(-2(1-\alpha)(b_{F1}+b_{F2})\Big)\Big(-2\alpha b_{T1}\Big) \ge 0; \\ |H_{13}^2| &= \Big(-2(1-\alpha)(b_{F1}+b_{F2})\Big)\Big(-2\alpha b_{T2}\Big) \ge 0; \\ |H_{23}^2| &= \Big(-2\alpha b_{T1}\Big)\Big(-2\alpha b_{T2}\Big) - \Big(\alpha(r_1+r_2)\Big)^2 \ge 0, \end{aligned}$$

where the last inequality holds by Assumption 2.1. Furthermore, we have

$$|H_{123}^3| = \Big(-2(1-\alpha)(b_{F1}+b_{F2})\Big)|H_{23}^2| + 0 + 0 \le 0.$$

Thus, the Hessian matrix is negative semi-definite, implying that Π_g is joint concave in p_0 and p_1, and p_2. $\hat{p_0}, \hat{p_1}$ and $\hat{p_2}$ can be solved by the first-order condition, i.e.,

$$\begin{aligned} \frac{\partial\Pi_g}{\partial p_0} &= (1-\alpha)\Big(a_{F1}+a_{F2}-2(b_{F1}+b_{F2})p_0 + (b_{F1}+b_{F2})\beta_1 + b_{F1}\frac{c_1}{1-\tau}\Big) = 0; \\ \frac{\partial\Pi_g}{\partial p_1} &= \alpha\Big(a_{T1} - 2b_{T1}p_1 + (r_1+r_2)p_2 + (b_{T1}-r_2)\beta_1 + b_{T1}\frac{c_1}{1-\tau}\Big) = 0; \\ \frac{\partial\Pi_g}{\partial p_2} &= \alpha\Big(a_{T2} - 2b_{T2}p_2 + (r_1+r_2)p_1 + (b_{T2}-r_1)\beta_1 - r_1\frac{c_1}{1-\tau}\Big) = 0. \end{aligned}$$

The optimal prices can be obtained by comparing the prices obtained from the first-order condition for the company's profit function and their boundary values.

■

Proof of Proposition 2.3 Note that for all the cases, (p_0^*, p_1^*, p_2^*) is independent of α. After taking the derivative of the optimal capacity and the company's optimal profit with respect to α, the proof is completed. ■

A.2 Supplement for Some Trivial Cases

Here, we present the optimal results when $\Delta\Pi_c|_{(p_1=0,p_2=\theta)} > 0$ or $g'(0) \geq \gamma$.

If $\Delta\Pi_c|_{(p_1=0,p_2=g'(0))} > 0$, then there is no solution for the electricity company.

If $\Delta\Pi_c|_{(p_1=0,p_2=g'(0))} \leq 0$ and $g'(0) \geq \gamma$, then the optimal solutions are shown in Table A.3. Here, $q_s^* \equiv 0$.

Table A.3: Optimal prices and shifted consumption for the case of $\Delta\Pi_c|_{(p_1=0,p_2=g'(0))} \leq 0$ and $g'(0) \geq \gamma$

Case	p_1^*	p_2^*	q_s^*	$\mathbf{\Delta\Pi_c}$
If $\bar{p_2} \geq p_2^A$	$p_2^A - g'(0)$	p_2^A	0	$=0$
If $\bar{p_2} \leq p_2^A$	$\bar{p_2} - g'(0)$	$\bar{p_2}$	0	<0

If $g'(0) \leq \gamma$ and $\Delta\Pi_c|_{(p_1=0,p_2=g'(0))} \leq 0 < \Delta\Pi_c|_{(p_1=0,p_2=\theta)}$, then the optimal solutions are shown in Table A.4. Here, p_2^A, p_2^C, p_2^D, p_1^E, and p_1^F are indicated in

Table A.4: Optimal prices and shifted consumption for the case of $g'(0) \leq \gamma$ and $\Delta\Pi_c|_{(p_1=0,p_2=g'(0))} \leq 0 < \Delta\Pi_c|_{(p_1=0,p_2=\theta)}$

Case	**Sub-case**	p_1^*	p_2^*	q_s^*	$\mathbf{\Delta\Pi_c}$
If $p_2^D \leq g'(0)$	If $\bar{p_2} \geq p_2^H$	0	p_2^H	$\alpha N g'^{-1}(p_2^H)$	$=0$
	If $p_2^A \leq \bar{p_2} \leq p_2^H$	p_1^E	$\bar{p_2}$	$\alpha N g'^{-1}(\bar{p_2} - p_1^E)$	$=0$
	If $\bar{p_2} \leq p_2^A$	$\bar{p_2} - g'(0)$	$\bar{p_2}$	0	<0
If $g'(0) \leq p_2^D \leq p_2^H$	If $\bar{p_2} \geq p_2^H$	0	p_2^H	$\alpha N g'^{-1}(p_2^H)$	$=0$
	If $p_2^C \leq \bar{p_2} \leq p_2^H$	p_1^E	$\bar{p_2}$	$\alpha N g'^{-1}(\bar{p_2} - p_1^E)$	$=0$
	If $p_2^D \leq \bar{p_2} \leq p_2^C$	p_1^F	$\bar{p_2}$	$\alpha N g'^{-1}(\bar{p_2} - p_1^F)$	<0
	If $\bar{p_2} \leq p_2^D$	0	$\bar{p_2}$	$\alpha N g'^{-1}(\bar{p_2})$	<0
If $p_2^D \geq p_2^H$	If $\bar{p_2} \geq p_2^H$	0	p_2^H	$\alpha N g'^{-1}(p_2^H)$	$=0$
	If $\bar{p_2} \leq p_2^H$	0	$\bar{p_2}$	$\alpha N g'^{-1}(\bar{p_2})$	<0

Theorem 2.2. p_2^H is the unique solution of p_2 for the equations:

$$\begin{cases} \Delta\Pi_c|_{p_1=0} = \frac{1}{N}(p_2 q_2 - \bar{p_0}(q_1 + q_2)) - p_2 q_s^i + g(q_s^i) = 0; \\ g'(q_s^i) = p_2. \end{cases}$$

Similarly, if $\bar{p_2}$ is large, e.g., $\bar{p_2} \geq p_2^H$, then we have $p_2^* = p_2^H$, $p_1^* \equiv 0$, $q_s = \alpha N g'^{-1}(p_2^H)$, and $\Delta\Pi_c \equiv 0$. The proof of the above results is very similar to that of Theorem 2.2, so we omit it here.

Appendix B

Proofs for Chapter 3

Proof of Theorem 3.1 Note that there are no $\mathbf{k}$ in the first part of $\Pi(\mathbf{k}, \mathbf{p})$ in Equation (3.2) and $\mathbf{k}$ only exist in the expected cost function $C(\mathbf{k}, \mathbf{p})$, so we prove the results for $C(\mathbf{k}, \mathbf{p})$ instead (As the decision variables are $\mathbf{k}$ and $\mathbf{p}$, here we replace $C(\mathbf{k}, D_1, D_2)$ by $C(\mathbf{k}, \mathbf{p})$ to have a consistent presentation with $\Pi(\mathbf{k}, \mathbf{p})$). For the notation simplicity, we replace $y(p_1, p_2)$ by y.

$$
\begin{aligned}
C(\mathbf{k}, \mathbf{p}) &= \mathbb{E}[c_1 k_1 + c_2 k_2 + \beta_1 \min\{D_1, (1-\tau)k_1\} + v_1(D_1 - (1-\tau)k_1)^+ \\
&\quad + \beta_1 \min\{D_2, \tau k_1\} + \beta_2 \min\{(D_2 - \tau k_1)^+, \tau k_2\} + v_2(D_2 - \tau k_1 - \tau k_2)^+] \\
&= c_1 k_1 + c_2 k_2 + \beta_1 \Big(\int_A^{\frac{(1-\tau)k_1 - q_1 - y}{\alpha N}} (q_1 + y + \alpha N u) f(u) du \\
&\quad + \int_{\frac{(1-\tau)k_1 - q_1 - y}{\alpha N}}^B (1-\tau) k_1 f(u) du \Big) \\
&\quad + v_1 \int_{\frac{(1-\tau)k_1 - q_1 - y}{\alpha N}}^B (q_1 + y + \alpha N u - (1-\tau)k_1) f(u) du \\
&\quad + \beta_1 \Big(\int_{\frac{q_2 - \tau k_1 - y}{\alpha N}}^B (q_2 - y - \alpha N u) f(u) du + \int_A^{\frac{q_2 - y - \tau k_1}{\alpha N}} \tau k_1 f(u) du \Big) \\
&\quad + \beta_2 \Big(\int_{\frac{q_2 - y - \tau k_1 - \tau k_2}{\alpha N}}^{\frac{q_2 - y - \tau k_1}{\alpha N}} (q_2 - y - \alpha N u - \tau k_1) f(u) du \\
&\quad + \int_A^{\frac{q_2 - y - \tau k_1 - \tau k_2}{\alpha N}} \tau k_2 f(u) du \Big) \\
&\quad + v_2 \int_A^{\frac{q_2 - y - \tau k_1 - \tau k_2}{\alpha N}} (q_2 - y - \alpha N u - \tau k_1 - \tau k_2) f(u) du.
\end{aligned}
$$

Consider the first and second partial derivatives of $C(\mathbf{k}, \mathbf{p})$ taken with respect to k_1 and k_2:

$$\begin{aligned}
\frac{\partial C(\mathbf{k},\mathbf{p})}{\partial k_1} &= c_1 + (1-\tau)(\beta_1 - v_1)[1 - F(\frac{(1-\tau)k_1 - q_1 - y}{\alpha N})] \\
&\quad + \tau(\beta_1 - \beta_2)F(\frac{q_2 - y - \tau k_1}{\alpha N}) + \tau(\beta_2 - v_2)F(\frac{q_2 - y - \tau k_1 - \tau k_2}{\alpha N}), \\
\frac{\partial^2 C(\mathbf{k},\mathbf{p})}{\partial k_1^2} &= \frac{1}{\alpha N}\Big\{(1-\tau)^2(v_1 - \beta_1)f(\frac{(1-\tau)k_1 - q_1 - y}{\alpha N}) \\
&\quad + \tau^2(\beta_2 - \beta_1)f(\frac{q_2 - y - \tau k_1}{\alpha N}) \\
&\quad + \tau^2(v_2 - \beta_2)f(\frac{q_2 - y - \tau k_1 - \tau k_2}{\alpha N})\Big\} \geq 0, \\
\frac{\partial C(\mathbf{k},\mathbf{p})}{\partial k_2} &= c_2 + \tau(\beta_2 - v_2)F(\frac{q_2 - y - \tau k_1 - \tau k_2}{\alpha N}), \\
\frac{\partial^2 C(\mathbf{k},\mathbf{p})}{\partial k_2^2} &= \frac{1}{\alpha N}\tau^2(v_2 - \beta_2)f(\frac{q_2 - y - \tau k_1 - \tau k_2}{\alpha N}) \geq 0.
\end{aligned}$$

If $c_2 > \tau(v_2 - \beta_2)$, then $\frac{\partial C(\mathbf{k},\mathbf{p})}{\partial k_2}$ is positive. It means that, under this setting, smallest capacity of Technology 2 is optimal, i.e., $k_2^* = 0$. Otherwise, we consider that

$$\frac{\partial^2 C(\mathbf{k},\mathbf{p})}{\partial k_1 \partial k_2} = \frac{1}{\alpha N}\tau^2(v_2 - \beta_2)f(\frac{q_2 - y - \tau k_1 - \tau k_2}{\alpha N}) \geq 0.$$

Then,

$$\begin{aligned}
&\frac{\partial^2 C(\mathbf{k},\mathbf{p})}{\partial k_1^2}\frac{\partial^2 C(\mathbf{k},\mathbf{p})}{\partial k_2^2} - \Big(\frac{\partial^2 C(\mathbf{k},\mathbf{p})}{\partial k_1 \partial k_2}\Big)^2 = \\
&[(1-\tau)^2(v_1 - \beta_1)f(\frac{(1-\tau)k_1 - q_1 - y}{\alpha N}) \\
&\quad + \tau^2(\beta_2 - \beta_1)f(\frac{q_2 - y - \tau k_1}{\alpha N})]\frac{1}{(\alpha N)^2}\tau^2(v_2 - \beta_2)f(\frac{q_2 - y - \tau k_1 - \tau k_2}{\alpha N}) \geq 0,
\end{aligned}$$

together with $\frac{\partial^2 C(\mathbf{k},\mathbf{p})}{\partial k_1^2} \geq 0$ and $\frac{\partial^2 C(\mathbf{k},\mathbf{p})}{\partial k_2^2} \geq 0$, we conclude that $C(\mathbf{k},\mathbf{p})$ is jointly convex in k_1 and k_2. Then, the optimal capacities $\mathbf{k}(\mathbf{p})$ can be obtained by $\frac{\partial C(\mathbf{k},\mathbf{p})}{\partial k_1} = 0$ and $\frac{\partial C(\mathbf{k},\mathbf{p})}{\partial k_2} = 0$. ■

Proof of Proposition 3.1 Let $f_1 = f(((1-\tau)k_1 - q_1 - y)/(\alpha N))$, $f_2 = f((q_2 - y - \tau k_1)/(\alpha N))$, $f_3 = f((q_2 - y - \tau k_1 - \tau k_2)/(\alpha N))$. First we prove the results for the effects of p_1.

By Equation (3.4), we can obtain that $\tau k_1 + \tau k_2 = q_2 - y - \alpha N F^{-1}(c_2/(\tau(v_2 - \beta_2)))$, from which we obtain that

$$\frac{\partial k_p(\mathbf{p})}{\partial p_1} = \frac{\partial(\tau k_1(\mathbf{p}) + \tau k_2(\mathbf{p}))}{\partial p_1} = -\frac{\partial y(\mathbf{p})}{\partial p_1} = \alpha N b_1 \geq 0.$$

Note that the optimal value of k_1 can be obtained by solving Equation (3.5). Then by taking the derivatives of both sides of Equation (3.5) with respect to p_1, we obtain that

$$\tau(\beta_2-\beta_1)f_2(-\frac{\partial y(\mathbf{p})}{\partial p_1}-\tau\frac{\partial k_1(\mathbf{p})}{\partial p_1})=(1-\tau)(v_1-\beta_1)f_1((1-\tau)\frac{\partial k_1(\mathbf{p})}{\partial p_1}-\frac{\partial y(\mathbf{p})}{\partial p_1}).$$

By solving this equation, we obtain that

$$\frac{\partial k_1(\mathbf{p})}{\partial p_1}=\frac{\tau(\beta_2-\beta_1)f_2-(1-\tau)(v_1-\beta_1)f_1}{\tau^2(\beta_2-\beta_1)f_2+(1-\tau)^2(v_1-\beta_1)f_1}\alpha N b_1,$$

which may be positive or negative. Now, we can obtain that

$$\begin{aligned}\frac{\partial k_2(\mathbf{p})}{\partial p_1} &= \frac{1}{\tau}(\frac{\partial k_p(\mathbf{p})}{\partial p_1}-\tau\frac{\partial k_1(\mathbf{p})}{\partial p_1})\\ &= \frac{(1-\tau)(v_1-\beta_1)f_1}{\tau^2(\beta_2-\beta_1)f_2+(1-\tau)^2(v_1-\beta_1)f_1}\frac{\alpha N b_1}{\tau}\geq 0.\end{aligned}$$

Similarly, for the effects of p_2, we can obtain that

$$\begin{aligned}\frac{\partial k_p(\mathbf{p})}{\partial p_2} &= -\alpha N b_2\leq 0;\\ \frac{\partial k_1(\mathbf{p})}{\partial p_2} &= -\frac{\tau(\beta_2-\beta_1)f_2-(1-\tau)(v_1-\beta_1)f_1}{\tau^2(\beta_2-\beta_1)f_2+(1-\tau)^2(v_1-\beta_1)f_1}\alpha N b_2;\\ \frac{\partial k_2(\mathbf{p})}{\partial p_2} &= -\frac{(1-\tau)(v_1-\beta_1)f_1}{\tau^2(\beta_2-\beta_1)f_2+(1-\tau)^2(v_1-\beta_1)f_1}\frac{\alpha N b_2}{\tau}\leq 0,\end{aligned}$$

where $\frac{\partial k_1(\mathbf{p})}{\partial p_2}$ may be positive or negative. ■

Proof of Theorem 3.2 For the notation simplicity, we first let $F_1 = F(\frac{(1-\tau)k_1(\mathbf{p})-q_1-y}{\alpha N})$, $F_2 = F(\frac{q_2-y-\tau k_1(\mathbf{p})}{\alpha N})$, and $F_3 = F(\frac{q_2-y-\tau k_1(\mathbf{p})-\tau k_2(\mathbf{p})}{\alpha N})$. After substituting $(k_1(\mathbf{p}), k_2(\mathbf{p}))$ back into $\Pi(\mathbf{k},\mathbf{p})$ and taking first derivative of $\Pi(\mathbf{k},\mathbf{p})$ with respect to p_1, we have

$$\begin{aligned}\frac{\partial \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1} &= \frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial k_1}\frac{\partial k_1(\mathbf{p})}{\partial p_1}+\frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial k_2}\frac{\partial k_2(\mathbf{p})}{\partial p_1}+\frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial p_1}\\ &= \frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial p_1}|_{\mathbf{k}=\mathbf{k}(\mathbf{p})}\\ &= \alpha q_1+y+\alpha N b_1\Big\{p_2-p_1+(v_1-\beta_1)(1-F_1)+(\beta_1-\beta_2)F_2\\ &\quad +(\beta_2-v_2)F_3\Big\}\\ &= \alpha q_1+y+\alpha N b_1\Big\{p_2-p_1+\frac{(v_1-\beta_1)(1-F_1)-c_1}{\tau}\Big\}.\end{aligned}$$

The second equality holds because $\frac{\partial \Pi(\mathbf{k},\mathbf{p})}{\partial k_1} = 0$ and $\frac{\partial \Pi(\mathbf{k},\mathbf{p})}{\partial k_2} = 0$ when $\mathbf{k} = \mathbf{k}(\mathbf{p})$. The last equality holds because of Equation (3.3), where optimal k_1 is obtained.

By taking the derivative of $\frac{\partial \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1}$ with respect to p_1, we have

$$\frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2} = -\alpha N b_1\{2 + \frac{(v_1-\beta_1)}{\tau}\frac{\partial F_1}{\partial p_1}\}.$$

Here,

$$\begin{aligned}\frac{\partial F_1}{\partial p_1} &= \frac{\partial \frac{(1-\tau)k_1(\mathbf{p})-q_1-y}{\alpha N}}{\partial p_1} f_1 = \frac{1}{\alpha N}((1-\tau)\frac{\partial k_1(\mathbf{p})}{\partial p_1} - \frac{\partial y}{\partial p_1}) f_1 \\ &= b_1 \frac{\tau(\beta_2-\beta_1)f_2}{(1-\tau)^2(v_1-\beta_1)f_1 + \tau^2(\beta_2-\beta_1)f_2} f_1 \geq 0. \qquad \text{(B.1)}\end{aligned}$$

Then we obtain that

$$\frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2} = -2\alpha N b_1 - \alpha N b_1 \frac{(v_1-\beta_1)}{\tau}\frac{\partial F_1}{\partial p_1} \leq 0. \qquad \text{(B.2)}$$

Although it may be possible that $\frac{\partial \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1}$ is always positive as p_1 increase, we do not consider the extreme case and only consider the interesting case that p_1^* can be obtained at the first-order condition of $\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})$, i.e., $\frac{\partial \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1} = 0$, which leads to Equation (3.6). ■

Proof of Proposition 3.2 We first prove that $\frac{dp_1(p_2)}{dp_2} > 0$. Note that, from Proof of Theorem 3.2, we have

$$\frac{\partial \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1} = \alpha q_1 + y + \alpha N b_1 \Big\{ p_2 - p_1 + \frac{(v_1-\beta_1)(1-F_1)-c_1}{\tau} \Big\}.$$

Then we obtain that

$$\frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1 \partial p_2} = \alpha N \Big\{ b_2 + b_1 - b_1 \frac{v_1-\beta_1}{\tau}\frac{\partial F_1}{\partial p_2} \Big\}.$$

Here,

$$\begin{aligned}\frac{\partial F_1}{\partial p_2} &= \frac{\partial \frac{(1-\tau)k_1(\mathbf{p})-q_1-y}{\alpha N}}{\partial p_2} f_1 = \frac{1}{\alpha N}((1-\tau)\frac{\partial k_1(\mathbf{p})}{\partial p_2} - \frac{\partial y}{\partial p_2}) f_1 \\ &= -b_2 \frac{\tau(\beta_2-\beta_1)f_2}{(1-\tau)^2(v_1-\beta_1)f_1 + \tau^2(\beta_2-\beta_1)f_2} f_1 \leq 0.\end{aligned}$$

Then we have that $\frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1 \partial p_2} \geq 0$, so

$$\frac{dp_1(p_2)}{dp_2} = -\frac{\frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1 \partial p_2}}{\frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}} \geq 0.$$

The inequality holds because $\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1 \partial p_2} \geq 0$, and $\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2} \leq 0$ which is indicated in Equation (B.2).

Next, we prove the results for $\frac{dy(p_1(p_2),p_2)}{dp_2}$.

$$\begin{aligned}
\frac{dy(p_1(p_2),p_2)}{dp_2} &= \alpha N\Big\{-b_1\frac{dp_1(p_2)}{dp_2}+b_2\Big\} \\
&= \alpha N\Big\{-\frac{b_2+b_1-b_1\frac{v_1-\beta_1}{\tau}\frac{\partial F_1}{\partial p_2}}{2+\frac{v_1-\beta_1}{\tau}\frac{\partial F_1}{\partial p_1}}+b_2\Big\} \\
&= \frac{\alpha N(b_2-b_1)}{2+\frac{v_1-\beta_1}{\tau}\frac{\partial F_1}{\partial p_1}}.
\end{aligned}$$

Note that $\frac{\partial F_1}{\partial p_1} \geq 0$, as indicated in Equation (B.1). Thus, we have

$$\frac{dy(p_1(p_2),p_2)}{dp_2}\begin{cases}\geq 0 & \text{if } b_2 \geq b_1;\\ < 0 & \text{otherwise.}\end{cases}$$

■

Proof of Theorem 3.3 Consider that

$$\begin{aligned}
\frac{d\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{dp_2} &= \frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial p_1}\frac{dp_1(p_2)}{dp_2}+\frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial p_2} \\
&= \frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial p_2}|_{p_1=p_1(p_2)} \\
&= \alpha q_2 - y - \alpha N b_2\Big\{p_2-p_1+(v_1-\beta_1)(1-F_1)+(\beta_1-\beta_2)F_2 \\
&\quad +(\beta_2-v_2)F_3\Big\} \\
&= \alpha q_2 - y - \alpha N b_2\Big\{p_2-p_1+\frac{(v_1-\beta_1)(1-F_1)-c_1}{\tau}\Big\} \\
&= \frac{1}{b_1}\Big\{b_1(\alpha q_2 - y)+b_2(\alpha q_1 + y)\Big\}.
\end{aligned}$$

The second equality holds because that $\frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial p_1} = 0$ when p_1 reaches its optimal point. The fourth equality holds because of Equation (3.3), where optimal k_1 is obtained. The last equality holds because of Equation (3.6), where optimal p_1 is obtained.

Then we consider the second derivative of $\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})$ with respect to p_2, and we have

$$\begin{aligned}\frac{d^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{dp_2^2} &= \frac{1}{b_1}(b_2-b_1)\frac{dy(p_1(p_2),p_2)}{dp_2}\\ &= \frac{1}{b_1}\frac{\alpha N(b_2-b_1)^2}{2+\frac{v_1-\beta_1}{\tau}\frac{\partial F_1}{\partial p_1}}\\ &\geq 0.\end{aligned}$$

The inequality holds because $\frac{\partial F_1}{\partial p_1} \geq 0$, which is indicated in Equation (B.1). Thus, $\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})$ is convex in p_2.

Note that the price in the peak period is not less than that in the non-peak period, it means that we have the condition $p_2 \geq p_1$. Then we consider

$$\begin{aligned}\frac{d\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{dp_2}|_{p_2=p_1} &= \frac{1}{b_1}\Big\{b_1(\alpha q_2 - y(p_1,p_1)) + b_2(\alpha q_1 + y(p_1,p_1))\Big\}\\ &= \frac{1}{b_1}\Big\{b_1(\alpha q_2 - \alpha Na) + b_2(\alpha q_1 + \alpha Na) + \alpha N(b_1-b_2)^2 p_1\Big\}\\ &\geq 0.\end{aligned}$$

The inequality holds because of $a \leq a+B \leq q_2/N$ which implies that $\alpha q_2 - \alpha Na \geq 0$. Combining it with the result that $\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})$ is convex in p_2, we then obtain that $\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})$ increases in p_2 for $p_2 \geq p_1$. Thus, we have the result that the upper bound of p_2 is optimal.

Recalling that the shifted consumption and the remanning consumption in the peak period after the shift both should be non-negative, i.e., $q_s \geq 0$ and $\alpha q_2 - q_s \geq 0$, which require that $-\alpha NA \leq y(p_1,p_2) \leq \alpha q_2 - \alpha NB$. On the other hand, from Proposition 3.2, we have the result that $\frac{dy(p_1(p_2),p_2)}{dp_2} \geq 0$ if $b_2 \geq b_1$, and $\frac{dy(p_1(p_2),p_2)}{dp_2} < 0$ otherwise. So the upper bound value of p_2 is the minimum of two values between $\bar{p_2}$ and the value of p_2 such that $y(p_1(p_2),p_2) = \alpha q_2 - \alpha NB$ if $b_2 \geq b_1$, and such that $y(p_1(p_2),p_2) = -\alpha NA$ if $b_2 < b_1$. Therefore, $p_2^* = \min\{\bar{p_2},\hat{p_2}\}$, where $\hat{p_2}$ is the unique solution of $y(p_1(p_2),p_2) = \alpha N(a - b_1 p_1(p_2) + b_2 p_2) = \alpha q_2 - \alpha NB$ if $b_2 \geq b_1$, and is the unique solution of $y(p_1(p_2),p_2) = \alpha N(a - b_1 p_1(p_2) + b_2 p_2) = -\alpha NA$ if $b_2 < b_1$. ∎

Proof of Proposition 3.3 Recalling that the optimal capacities are determined by Equations (3.3) and (3.4), and the optimal price in the non-peak period is determined by Equation (3.6). We rearrange these three equations, and let

$$
\begin{aligned}
G_1 &= \tau(\beta_2-\beta_1)F_2+\tau(v_2-\beta_2)F_3+(1-\tau)(v_1-\beta_1)(1-F_1)-c_1=0,\\
G_2 &= \tau(v_2-\beta_2)F_3-c_2=0,\\
G_3 &= q_1+Ny^i+Nb_1(p_2-p_1+\frac{(v_1-\beta_1)(1-F_1)-c_1}{\tau})=0.
\end{aligned}
$$

And we have that $\frac{\partial G_2}{\partial k_1}=\frac{\partial G_2}{\partial k_2}=\frac{\partial G_1}{\partial k_2}=-\frac{1}{\alpha N}\tau^2(v_2-\beta_2)f_3$, $\frac{\partial G_1}{\partial k_1}=-\frac{1}{\alpha N}\{(1-\tau)^2(v_1-\beta_1)f_1+\tau^2(\beta_2-\beta_1)f_2+\tau^2(v_2-\beta_2)f_3\}$, $\frac{\partial G_3}{\partial k_1}=-\frac{1}{\alpha N}\frac{Nb_1(1-\tau)(v_1-\beta_1)f_1}{\tau}$, and $\frac{\partial G_3}{\partial p_1}=\frac{Nb_1(b_1-b_2)}{b_2}$. Before considering the three cases, we define $L=(1-\tau)^2(v_1-\beta_1)f_1+\tau^2(\beta_2-\beta_1)f_2\geq 0$, $H=(v_1-\beta_1)f_1(\beta_2-\beta_1)f_2\geq 0$ and $M=2L+b_1H\geq 0$.

(1) For Case I, i.e., p_2^* is determined by $y=-\alpha NA$, then we have condition $b_2<b_1$ and we have $y^i=-A$. So $F_1=F(\frac{(1-\tau)k_1-q_1+\alpha NA}{\alpha N})$, $F_2=F(\frac{q_2+\alpha NA-\tau k_1}{\alpha N})$, $F_3=F(\frac{q_2+\alpha NA-\tau k_1-\tau k_2}{\alpha N})$, $f_1=f(\frac{(1-\tau)k_1-q_1+\alpha NA}{\alpha N})$, $f_2=f(\frac{q_2+\alpha NA-\tau k_1}{\alpha N})$, and $f_3=f(\frac{q_2+\alpha NA-\tau k_1-\tau k_2}{\alpha N})$. Note that $y^i=-A$, by which we obtain that $p_2=\frac{b_1p_1-a-A}{b_2}$. Then G_3 can be rewritten as $G_3=q_1-NA+Nb_1(\frac{b_1p_1-a-A}{b_2}-p_1+\frac{(v_1-\beta_1)(1-F_1)-c_1}{\tau})$. And we directly obtain that $\frac{dy^i}{dq_1}=\frac{dy}{dq_1}=0$.

By taking the first derivatives of G_1,G_2 and G_3 with respect to q_1, we have

$$
\begin{aligned}
\frac{dG_1}{dq_1} &= \frac{\partial G_1}{\partial q_1}+\frac{\partial G_1}{\partial k_1}\frac{dk_1^*}{dq_1}+\frac{\partial G_1}{\partial k_2}\frac{dk_2^*}{dq_1}=0,\\
\frac{dG_2}{dq_1} &= \frac{\partial G_2}{\partial q_1}+\frac{\partial G_2}{\partial k_1}\frac{dk_1^*}{dq_1}+\frac{\partial G_2}{\partial k_2}\frac{dk_2^*}{dq_1}=0,\\
\frac{dG_3}{dq_1} &= \frac{\partial G_3}{\partial q_1}+\frac{\partial G_3}{\partial k_1}\frac{dk_1^*}{dq_1}+\frac{\partial G_3}{\partial p_1}\frac{dp_1^*}{dq_1}=0.
\end{aligned}
$$

Here, $\frac{\partial G_2}{\partial q_1}=0$, $\frac{\partial G_1}{\partial q_1}=\frac{1}{\alpha N}(1-\tau)(v_1-\beta_1)f_1$, and $\frac{\partial G_3}{\partial q_1}=1+\frac{1}{\alpha N}\frac{Nb_1(v_1-\beta_1)f_1}{\tau}$. By solving the above equations, we obtain that

$$
\begin{aligned}
&\frac{dk_1^*}{dq_1}=\frac{(1-\tau)(v_1-\beta_1)f_1}{L}\geq 0;\frac{dk_2^*}{dq_1}=-\frac{(1-\tau)(v_1-\beta_1)f_1}{L}\leq 0;\\
&\frac{dk_p^*}{dq_1}=0;\frac{dp_1^*}{dq_1}=-\frac{b_2}{Nb_1(b_1-b_2)}(1+\frac{\tau Nb_1H}{\alpha NL})\leq 0;\\
&\frac{dp_2^*}{dq_1}=\frac{b_1}{b_2}\frac{dp_1^*}{dq_1}\leq 0.
\end{aligned}
$$

(2) For Case II, i.e., p_2^* is determined by $y=\alpha q_2-\alpha NB$, then we have condition $b_2\geq b_1$ and we have $y^i=q_2/N-B$. So $F_1=F(\frac{(1-\tau)k_1-q_1-\alpha(q_2-NB)}{\alpha N})$, $F_2=F(\frac{q_2-\alpha(q_2-NB)-\tau k_1}{\alpha N})$, $F_3=F(\frac{q_2-\alpha(q_2-NB)-\tau k_1-\tau k_2}{\alpha N})$, $f_1=f(\frac{(1-\tau)k_1-q_1-\alpha(q_2-NB)}{\alpha N})$,

$f_2 = f(\frac{q_2-\alpha(q_2-NB)-\tau k_1}{\alpha N})$, and $f_3 = f(\frac{q_2-\alpha(q_2-NB)-\tau k_1-\tau k_2}{\alpha N})$. Note that $y^i = q_2/N - B$, from which we obtain that $p_2 = \frac{b_1p_1-a+q_2/N-B}{b_2}$. Then G_3 can be rewritten as $G_3 = q_1 + q_2 - NB + Nb_1(\frac{b_1p_1-a+q_2/N-B}{b_2} - p_1 + \frac{(v_1-\beta_1)(1-F_1)-c_1}{\tau})$. We directly obtain that $\frac{dy^i}{dq_1} = \frac{dy}{dq_1} = 0$.

By taking the similar method with that for case I, we obtain that

$$\begin{aligned}
&\frac{dk_1^*}{dq_1} = \frac{(1-\tau)(v_1-\beta_1)f_1}{L} \geq 0; \frac{dk_2^*}{dq_1} = -\frac{(1-\tau)(v_1-\beta_1)f_1}{L} \leq 0;\\
&\frac{dk_p^*}{dq_1} = 0; \frac{dp_1^*}{dq_1} = \frac{b_2}{Nb_1(b_2-b_1)}(1+\frac{\tau Nb_1H}{\alpha NL}) \geq 0;\\
&\frac{dp_2^*}{dq_1} = \frac{b_1}{b_2}\frac{dp_1^*}{dq_1} \geq 0.
\end{aligned}$$

(3) For Case III, i.e., $p_2^* = \bar{p_2}$, then we have $F_1 = F(\frac{(1-\tau)k_1-q_1-y}{\alpha N}), F_2 = F(\frac{q_2-y-\tau k_1}{\alpha N}), F_3 = F(\frac{q_2-y-\tau k_1-\tau k_2}{\alpha N}), f_1 = f(\frac{(1-\tau)k_1-q_1-y}{\alpha N}), f_2 = f(\frac{q_2-y-\tau k_1}{\alpha N})$, and $f_3 = f(\frac{q_2-y-\tau k_1-\tau k_2}{\alpha N})$. We immediately obtain that $\frac{dp_2^*}{dq_1} = \frac{b_1}{b_2} = 0$.

By taking the similar method with that for case I, we obtain that

$$\begin{aligned}
\frac{dp_1^*}{dq_1} &= \frac{\alpha L+\tau b_1H}{\alpha Nb_1M} \geq 0; \frac{dk_p^*}{dq_1} = \frac{\alpha L+\tau b_1H}{\tau M} \geq 0;\\
\frac{dk_1^*}{dq_1} &= \frac{1}{LM}\Big\{(1-\tau)(v_1-\beta_1)f_1\alpha N[(2-\alpha)L+b_1(1-\tau)H]\\
&\quad +\tau(\beta_2-\beta_1)f_2[\alpha NL+\tau Nb_1H]\Big\} \geq 0;\\
\frac{dk_2^*}{dq_1} &= \frac{1}{LM\tau}\Big\{(1-\tau)(v_1-\beta_1)f_1\alpha N(\alpha-2\tau)L\\
&\quad -\tau^2(\beta_2-\beta_1)f_2(1-\alpha)[\alpha NL+\tau Nb_1H]\Big\}.
\end{aligned}$$

So if $\alpha < 2\tau$ then $\frac{dk_2^*}{dq_1} < 0$, and if $\alpha = 1$ and $\tau < \frac{1}{2}$ (and we assume that $\tau < \frac{1}{2}$) then $\frac{dk_2^*}{dq_1} > 0$. For y^i and y, we have $\frac{dy^i}{dq_1} = -b_1\frac{dp_1^*}{dq_1} \leq 0$ and $\frac{dy}{dq_1} = -\alpha Nb_1\frac{dp_1^*}{dq_1} \leq 0$. ■

Proof of Proposition 3.4 The results can be obtained by following the similar proof approach of Proposition 3.3. So we omit the details of the proof and only show the results here.

(1) For Case I, we have

$$\frac{dk_1^*}{dq_2}=\frac{\tau(\beta_2-\beta_1)f_2}{L}\geq 0;\frac{dk_2^*}{dq_2}=\frac{(1-\tau)^2(v_1-\beta_1)f_1}{\tau L}\geq 0;$$
$$\frac{dk_p^*}{dq_2}=1>0;\frac{dp_1^*}{dq_2}=\frac{b_2}{b_1-b_2}\frac{(1-\tau)H}{\alpha NL}\geq 0;$$
$$\frac{dp_2^*}{dq_2}=\frac{b_1}{b_2}\frac{dp_1^*}{dq_2}\geq 0;\frac{dy^i}{dq_2}=\frac{dy}{dq_2}=0.$$

(2) For Case II, we have

$$\frac{dk_1^*}{dq_2}=\frac{\tau(\beta_2-\beta_1)f_2(1-\alpha)+(1-\tau)(v_1-\beta_1)f_1\alpha}{L}\geq 0;$$
$$\frac{dk_2^*}{dq_2}=\frac{(1-\tau)(v_1-\beta_1)f_1(1-\tau-\alpha)}{\tau L};$$
$$\frac{dk_p^*}{dq_2}=1-\alpha>0;\frac{dp_1^*}{dq_2}=\frac{b_2}{Nb_1(b_2-b_1)}(1+\frac{b_1}{b_2}+\frac{Nb_1H(\alpha+\tau-1)}{\alpha NL});$$
$$\frac{dp_2^*}{dq_2}=\frac{b_1}{b_2}\frac{dp_1^*}{dq_2}+\frac{1}{Nb_2};\frac{dy^i}{dq_2}=\frac{1}{N}>0;\frac{dy}{dq_2}=\alpha>0.$$

So we have $\frac{dk_2^*}{dq_2}\geq 0$ if $1-\tau-\alpha\geq 0$; $\frac{dk_2^*}{dq_2}\leq 0$ otherwise, and if $\alpha+\tau\geq 1$ then $\frac{dp_1^*}{dq_2}\geq 0$ and $\frac{dp_2^*}{dq_2}\geq 0$.

(3) For Case III, we have

$$\frac{dk_1^*}{dq_2}=\frac{2\tau+b_1(v_1-\beta_1)f_1}{(v_1-\beta_1)f_1}\frac{H}{M}\geq 0;\frac{dk_2^*}{dq_2}=\frac{2(1-\tau)^2(v_1-\beta_1)f_1}{\tau M}\geq 0;$$
$$\frac{dk_p^*}{dq_2}=\tau(\frac{dk_1^*}{dq_2}+\frac{dk_2^*}{dq_2})\geq 0;\frac{dp_1^*}{dq_2}=-\frac{(1-\tau)H}{\alpha NM}\leq 0;$$
$$\frac{dp_2^*}{dq_2}=0;\frac{dy^i}{dq_2}=-b_1\frac{dp_1^*}{dq_2}\geq 0;\frac{dy}{dq_2}=-\alpha Nb_1\frac{dp_1^*}{dq_2}\geq 0.$$

■

Proof of Proposition 3.5 Note that for the impact of the market size (N), we consider the scenario that the original total demand in the non-peak period per customer (q_1^i) and the original total demand in the peak period per customer (q_2^i) are unchanged with the market size, and we have $q_1=Nq_1^i$ and $q_2=Nq_2^i$. Then by following the similar proof approach of Proposition 3.3, we obtain the results which are shown as follows:

For any case, we have

$$\frac{dk_1^*}{dN}=\frac{k_1}{N}\geq 0;\frac{dk_2^*}{dN}=\frac{k_2}{N}\geq 0;\frac{dk_p^*}{dN}=\frac{\tau(k_1+k_2)}{N}>0;\frac{dp_1^*}{dN}=0;\frac{dp_2^*}{dN}=0;\frac{dy^i}{dN}=0.$$

For Case I, we have $\frac{dy}{dN} = -\alpha A \geq 0$; for Case II, we have $\frac{dy}{dN} = \alpha q_2^i - \alpha B \geq 0$; and for Case III, we have $\frac{dy}{dN} = \alpha y^i \geq 0$. ■

Proof of Proposition 3.6 Similarly, the results can be obtained by following the proof approach of Proposition 3.3. So we omit the details of the proof and only show the results here.

(1) For Case I, we have

$$
\begin{aligned}
\frac{dk_1^*}{d\alpha} &= \frac{-(q_2 - \tau k_1)\tau(\beta_2 - \beta_1)f_2 + ((1-\tau)k_1 - q_1)(1-\tau)(v_1 - \beta_1)f_1}{\alpha L};\\
\frac{dk_2^*}{d\alpha} &= -\frac{-k_2\tau^2(\beta_2 - \beta_1)f_2 + (1-\tau)^2(v_1 - \beta_1)f_1[\frac{q_2}{\tau} - \frac{q_1}{1-\tau} - k_2]}{\alpha L};\\
\frac{dk_p^*}{d\alpha} &= -\frac{q_2 - \tau k_1 - \tau k_2}{\alpha}; \frac{dp_1^*}{d\alpha} = -\frac{b_2\tau(1-\tau)[\frac{q_2}{\tau} - \frac{q_1}{1-\tau}]H}{\alpha^2 N(b_1 - b_2)L} \leq 0;\\
\frac{dp_2^*}{d\alpha} &= \frac{b_1}{b_2}\frac{dp_1^*}{d\alpha} \leq 0; \frac{dy^i}{d\alpha} = 0; \frac{dy}{d\alpha} = -NA \geq 0.
\end{aligned}
$$

(2) For Case II, we have

$$
\begin{aligned}
\frac{dk_1^*}{d\alpha} &= \frac{-(q_2 - \tau k_1)\tau(\beta_2 - \beta_1)f_2 + ((1-\tau)k_1 - q_1)(1-\tau)(v_1 - \beta_1)f_1}{\alpha L};\\
\frac{dk_2^*}{d\alpha} &= -\frac{-k_2\tau^2(\beta_2 - \beta_1)f_2 + (1-\tau)^2(v_1 - \beta_1)f_1[\frac{q_2}{\tau} - \frac{q_1}{1-\tau} - k_2]}{\alpha L};\\
\frac{dk_p^*}{d\alpha} &= -\frac{q_2 - \tau k_1 - \tau k_2}{\alpha}; \frac{dp_1^*}{d\alpha} = \frac{b_2\tau(1-\tau)[\frac{q_2}{\tau} - \frac{q_1}{1-\tau}]H}{\alpha^2 N(b_2 - b_1)L} \geq 0;\\
\frac{dp_2^*}{d\alpha} &= \frac{b_1}{b_2}\frac{dp_1^*}{d\alpha} \geq 0; \frac{dy^i}{d\alpha} = 0; \frac{dy}{d\alpha} = q_2 - NB \geq 0.
\end{aligned}
$$

(3) For Case III, we have

$$
\begin{aligned}
\frac{dk_1^*}{d\alpha} &= \frac{1}{\alpha M}\Big\{[k_1 - q_1 - q_2]b_1 H - 2(q_2 - \tau k_1)\tau(\beta_2 - \beta_1)f_2\\
&\quad +2[(1-\tau)k_1 - q_1](1-\tau)(v_1 - \beta_1)f_1\Big\};\\
\frac{dk_2^*}{d\alpha} &= \frac{2k_2\tau^2(\beta_2 - \beta_1)f_2 + 2(1-\tau)^2(v_1 - \beta_1)f_1[k_2 - \frac{q_2}{\tau} + \frac{q_1}{1-\tau}] + k_2 b_1 H}{\alpha M};\\
\frac{dk_p^*}{d\alpha} &= -\frac{q_2 - \tau k_1 - \tau k_2}{\alpha} + \frac{\tau(1-\tau)[\frac{q_2}{\tau} - \frac{q_1}{1-\tau}]b_1 H}{\alpha M};\\
\frac{dp_1^*}{d\alpha} &= \frac{\tau(1-\tau)[\frac{q_2}{\tau} - \frac{q_1}{1-\tau}]H}{\alpha^2 NM} \geq 0;\\
\frac{dp_2^*}{d\alpha} &= 0; \frac{dy^i}{d\alpha} = -b_1\frac{dp_1^*}{d\alpha} \leq 0; \frac{dy}{d\alpha} = N(y^i - \alpha b_1\frac{dp_1^*}{d\alpha}).
\end{aligned}
$$

The inequalities hold for $\frac{dp_1^*}{d\alpha}$ because that $\frac{q_2}{\tau} \geq \frac{q_1}{1-\tau}$ and $H \geq 0$. Although the effects of α on the optimal capacities may not be monotone, here, we present the values of the derivatives of the capacities with respect to α for completeness. ■

Proof of Proposition 3.7 The results can be obtained by following the similar proof approach of Proposition 3.3. So we omit the details of the proof and only show the results here.

(1) First we consider the impacts for Case I.

1.1) For c_2, we have

$$\frac{dk_1^*}{dc_2} = \frac{\alpha N}{L} \geq 0; \frac{dk_2^*}{dc_2} = -\frac{\alpha N}{\tau^2(v_2-\beta_2)f_3} - \frac{\alpha N}{L} \leq 0;$$
$$\frac{dk_p^*}{dc_2} = -\frac{\alpha N}{\tau(v_2-\beta_2)f_3} \leq 0; \frac{dp_1^*}{dc_2} = -\frac{b_2(1-\tau)(v_1-\beta_1)f_1}{\tau(b_2-b_1)L} \geq 0;$$
$$\frac{dp_2^*}{dc_2} = -\frac{b_1(1-\tau)(v_1-\beta_1)f_1}{\tau(b_2-b_1)L} \geq 0; \frac{dy^i}{dc_2} = \frac{dy}{dc_2} = 0.$$

$$\begin{aligned}
\frac{d\Pi^*}{dc_2} &= \frac{\partial \Pi}{\partial c_2} + \frac{\partial \Pi}{\partial k_1}\frac{dk_1^*}{dc_2} + \frac{\partial \Pi}{\partial k_2}\frac{dk_2^*}{dc_2} + \frac{\partial \Pi}{\partial p_1}\frac{dp_1^*}{dc_2} + \frac{\partial \Pi}{\partial p_2}\frac{dp_2^*}{dc_2} \\
&= \frac{\partial \Pi}{\partial c_2} + \frac{\partial \Pi}{\partial p_2}\frac{dp_2^*}{dc_2} \\
&= -k_2^* + \frac{\partial \Pi}{\partial p_2}\frac{dp_2^*}{dc_2}.
\end{aligned}$$

For $\frac{d\Pi^*}{dc_2}$, the second equality holds because $\frac{\partial \Pi}{\partial k_1} = \frac{\partial \Pi}{\partial k_2} = \frac{\partial \Pi}{\partial p_1} = 0$ when the optimal solutions are obtained. However, $\frac{d\Pi^*}{dc_2}$ may be positive or negative, as $\frac{dp_2^*}{dc_2} \geq 0$ and $\frac{\partial \Pi}{\partial p_2} \geq 0$ (as indicated in the proof of Theorem 3.3, $\frac{\partial \Pi}{\partial p_2} \geq 0$ when the optimal solutions of k_1, k_2 and p_1 are obtained).

1.2) For c_1, we have

$$\frac{dk_1^*}{dc_1} = -\frac{\alpha N}{L} \leq 0; \frac{dk_2^*}{dc_1} = \frac{\alpha N}{L} \geq 0;$$
$$\frac{dk_p^*}{dc_1} = 0; \frac{dp_1^*}{dc_1} = b_2\frac{(1-\tau)(v_1-\beta_1)f_1 - \tau(\beta_2-\beta_1)f_2}{(b_2-b_1)L};$$
$$\frac{dp_2^*}{dc_1} = \frac{b_1}{b_2}\frac{dp_1^*}{dc_1}; \frac{dy^i}{dc_1} = \frac{dy}{dc_1} = 0.$$

Here, $\frac{dp_1^*}{dc_1}$ and $\frac{dp_2^*}{dc_1}$ may be positive or negative. Consequently, $\frac{d\Pi^*}{dc_1} = \frac{\partial \Pi}{\partial c_1} + \frac{\partial \Pi}{\partial p_2}\frac{dp_2^*}{dc_1}$ may be positive or negative, though $\frac{\partial \Pi}{\partial c_1} \leq 0$ and $\frac{\partial \Pi}{\partial p_2} \geq 0$.

1.3) For β_2, we have

$$\frac{dk_1^*}{d\beta_2} = \frac{\alpha N \tau F_2}{L} \geq 0; \frac{dk_2^*}{d\beta_2} = -\frac{\alpha N F_3}{\tau(v_2-\beta_2)f_3} - \frac{\alpha N \tau F_2}{L} \leq 0;$$
$$\frac{dk_p^*}{d\beta_2} = -\frac{\alpha N F_3}{(v_2-\beta_2)f_3} \leq 0; \frac{dp_1^*}{d\beta_2} = -b_2\frac{(1-\tau)(v_1-\beta_1)f_1F_2}{(b_2-b_1)L} \geq 0;$$
$$\frac{dp_2^*}{d\beta_2} = -b_1\frac{(1-\tau)(v_1-\beta_1)f_1F_2}{(b_2-b_1)L} \geq 0; \frac{dy^i}{d\beta_2} = \frac{dy}{d\beta_2} = 0.$$

Note that $\frac{dp_2^*}{d\beta_2} \geq 0$, so $\frac{d\Pi^*}{d\beta_2} = \frac{\partial \Pi}{\partial \beta_2} + \frac{\partial \Pi}{\partial p_2}\frac{dp_2^*}{d\beta_2}$ may be positive or negative, though $\frac{\partial \Pi}{\partial \beta_2} \leq 0$ and $\frac{\partial \Pi}{\partial p_2} \geq 0$.

1.4) For β_1, we have

$$\frac{dk_1^*}{d\beta_1} = -\alpha N\frac{\tau F_2+(1-\tau)(1-F_1)}{L} \leq 0; \frac{dk_2^*}{d\beta_1} = \alpha N\frac{\tau F_2+(1-\tau)(1-F_1)}{L} \geq 0;$$
$$\frac{dp_1^*}{d\beta_1} = b_2\frac{(1-\tau)(v_1-\beta_1)f_1F_2-\tau(\beta_2-\beta_1)f_2(1-F_1)}{(b_2-b_1)L}; \frac{dk_p^*}{d\beta_1} = 0;$$
$$\frac{dp_2^*}{d\beta_1} = \frac{b_1}{b_2}\frac{dp_1^*}{d\beta_1}; \frac{dy^i}{d\beta_1} = \frac{dy}{d\beta_1} = 0.$$

Here, $\frac{dp_1^*}{d\beta_1}$ and $\frac{dp_2^*}{d\beta_1}$ may be positive or negative. Consequently, $\frac{d\Pi^*}{d\beta_1} = \frac{\partial \Pi}{\partial \beta_1} + \frac{\partial \Pi}{\partial p_2}\frac{dp_2^*}{d\beta_1}$ may be positive or negative.

1.5) For v_2, we have

$$\frac{dk_1^*}{dv_2} = 0; \frac{dk_2^*}{dv_2} = \frac{\alpha N F_3}{\tau(v_2-\beta_2)f_3} \geq 0; \frac{dk_p^*}{dv_2} = \frac{\alpha N F_3}{(v_2-\beta_2)f_3} \geq 0;$$
$$\frac{dp_1^*}{dv_2} = 0; \frac{dp_2^*}{dv_2} = 0; \frac{dy^i}{dv_2} = \frac{dy}{dv_2} = 0.$$

And, $\frac{d\Pi^*}{dv_2} = \frac{\partial \Pi}{\partial v_2} + \frac{\partial \Pi}{\partial p_2}\frac{dp_2^*}{dv_2} \leq 0$ as $\frac{\partial \Pi}{\partial v_2} \leq 0$ and $\frac{dp_2^*}{dv_2} = 0$.

1.6) For v_1, we have

$$\frac{dk_1^*}{dv_1} = \frac{\alpha N(1-\tau)(1-F_1)}{L} \geq 0; \frac{dk_2^*}{dv_1} = -\frac{\alpha N(1-\tau)(1-F_1)}{L} \leq 0; \frac{dk_p^*}{dv_1} = 0;$$
$$\frac{dp_1^*}{dv_1} = b_2\frac{\tau(\beta_2-\beta_1)f_2(1-F_1)}{(b_2-b_1)L} \leq 0; \frac{dp_2^*}{dv_1} = \frac{b_1}{b_2}\frac{dp_1^*}{dv_1} \leq 0; \frac{dy^i}{dv_1} = \frac{dy}{dv_1} = 0.$$

Note that $\frac{dp_2^*}{dv_1} \leq 0$, then $\frac{d\Pi^*}{dv_1} = \frac{\partial \Pi}{\partial v_1} + \frac{\partial \Pi}{\partial p_2}\frac{dp_2^*}{dv_1} \leq 0$, as $\frac{\partial \Pi}{\partial v_1} \leq 0$ and $\frac{\partial \Pi}{\partial p_2} \geq 0$.

(2) Then we consider the impacts for Case II.

2.1) For c_2, we have

$$\frac{dk_1^*}{dc_2} = \frac{\alpha N}{L} \geq 0; \frac{dk_2^*}{dc_2} = -\frac{\alpha N}{\tau^2(v_2-\beta_2)f_3} - \frac{\alpha N}{L} \leq 0;$$
$$\frac{dk_p^*}{dc_2} = -\frac{\alpha N}{\tau(v_2-\beta_2)f_3} \leq 0; \frac{dp_1^*}{dc_2} = -\frac{b_2(1-\tau)(v_1-\beta_1)f_1}{\tau(b_2-b_1)L} \leq 0;$$
$$\frac{dp_2^*}{dc_2} = -\frac{b_1(1-\tau)(v_1-\beta_1)f_1}{\tau(b_2-b_1)L} \leq 0; \frac{dy^i}{dc_2} = \frac{dy}{dc_2} = 0.$$

Note that $\frac{dp_2^*}{dc_2} \leq 0$, then $\frac{d\Pi^*}{dc_2} = \frac{\partial \Pi}{\partial c_2} + \frac{\partial \Pi}{\partial p_2}\frac{dp_2^*}{dc_2} \leq 0$, as $\frac{\partial \Pi}{\partial c_2} \leq 0$ and $\frac{\partial \Pi}{\partial p_2} \geq 0$.

2.2) For c_1, we have

$$\frac{dk_1^*}{dc_1} = -\frac{\alpha N}{L} \leq 0; \frac{dk_2^*}{dc_1} = \frac{\alpha N}{L} \geq 0; \frac{dk_p^*}{dc_1} = 0;$$
$$\frac{dp_1^*}{dc_1} = b_2\frac{(1-\tau)(v_1-\beta_1)f_1 - \tau(\beta_2-\beta_1)f_2}{(b_2-b_1)L}; \frac{dp_2^*}{dc_1} = \frac{b_1}{b_2}\frac{dp_1^*}{dc_1}; \frac{dy^i}{dc_1} = \frac{dy}{dc_1} = 0.$$

Here, $\frac{dp_1^*}{dc_1}$ and $\frac{dp_2^*}{dc_1}$ may be positive or negative. Consequently, $\frac{d\Pi^*}{dc_1} = \frac{\partial \Pi}{\partial c_1} + \frac{\partial \Pi}{\partial p_2}\frac{dp_2^*}{dc_1}$ may be positive or negative, though $\frac{\partial \Pi}{\partial c_1} \leq 0$ and $\frac{\partial \Pi}{\partial p_2} \geq 0$.

2.3) For β_2, we have

$$\frac{dk_1^*}{d\beta_2} = \frac{\alpha N\tau F_2}{L} \geq 0; \frac{dk_2^*}{d\beta_2} = -\frac{\alpha N F_3}{\tau(v_2-\beta_2)f_3} - \frac{\alpha N\tau F_2}{L} \leq 0;$$
$$\frac{dk_p^*}{d\beta_2} = -\frac{\alpha N F_3}{(v_2-\beta_2)f_3} \leq 0; \frac{dp_1^*}{d\beta_2} = -b_2\frac{(1-\tau)(v_1-\beta_1)f_1F_2}{(b_2-b_1)L} \leq 0;$$
$$\frac{dp_2^*}{d\beta_2} = -b_1\frac{(1-\tau)(v_1-\beta_1)f_1F_2}{(b_2-b_1)L} \leq 0; \frac{dy^i}{d\beta_2} = \frac{dy}{d\beta_2} = 0.$$

Note that $\frac{dp_2^*}{d\beta_2} \leq 0$, then $\frac{d\Pi^*}{d\beta_2} = \frac{\partial \Pi}{\partial \beta_2} + \frac{\partial \Pi}{\partial p_2}\frac{dp_2^*}{d\beta_2} \leq 0$, as $\frac{\partial \Pi}{\partial \beta_2} \leq 0$ and $\frac{\partial \Pi}{\partial p_2} \geq 0$.

2.4) For β_1, we have

$$\frac{dk_1^*}{d\beta_1} = -\alpha N\frac{\tau F_2 + (1-\tau)(1-F_1)}{L} \leq 0; \frac{dk_2^*}{d\beta_1} = \alpha N\frac{\tau F_2 + (1-\tau)(1-F_1)}{L} \geq 0;$$
$$\frac{dk_p^*}{d\beta_1} = 0; \frac{dp_1^*}{d\beta_1} = b_2\frac{(1-\tau)(v_1-\beta_1)f_1F_2 - \tau(\beta_2-\beta_1)f_2(1-F_1)}{(b_2-b_1)L};$$
$$\frac{dp_2^*}{d\beta_1} = \frac{b_1}{b_2}\frac{dp_1^*}{d\beta_1}; \frac{dy^i}{d\beta_1} = \frac{dy}{d\beta_1} = 0.$$

Here, $\frac{dp_1^*}{d\beta_1}$ and $\frac{dp_2^*}{d\beta_1}$ may be positive or negative. Consequently, $\frac{d\Pi^*}{d\beta_1} = \frac{\partial \Pi}{\partial \beta_1} + \frac{\partial \Pi}{\partial p_2}\frac{dp_2^*}{d\beta_1}$ may be positive or negative.

2.5) For v_2, we have

$$\frac{dk_1^*}{dv_2} = 0; \frac{dk_2^*}{dv_2} = \frac{\alpha N F_3}{\tau(v_2-\beta_2)f_3} \geq 0; \frac{dk_p^*}{dv_2} = \frac{\alpha N F_3}{(v_2-\beta_2)f_3} \geq 0;$$
$$\frac{dp_1^*}{dv_2} = 0; \frac{dp_2^*}{dv_2} = 0; \frac{dy^i}{dv_2} = \frac{dy}{dv_2} = 0.$$

And, $\frac{d\Pi^*}{dv_2} = \frac{\partial \Pi}{\partial v_2} + \frac{\partial \Pi}{\partial p_2}\frac{dp_2^*}{dv_2} \leq 0$ as $\frac{\partial \Pi}{\partial v_2} \leq 0$ and $\frac{dp_2^*}{dv_2} = 0$.

2.6) For v_1, we have

$$\frac{dk_1^*}{dv_1} = \frac{\alpha N(1-\tau)(1-F_1)}{L} \geq 0; \frac{dk_2^*}{dv_1} = -\frac{\alpha N(1-\tau)(1-F_1)}{L} \leq 0; \frac{dk_p^*}{dv_1} = 0;$$
$$\frac{dp_1^*}{dv_1} = b_2\frac{\tau(\beta_2-\beta_1)f_2(1-F_1)}{(b_2-b_1)L} \geq 0; \frac{dp_2^*}{dv_1} = \frac{b_1}{b_2}\frac{dp_1^*}{dv_1} \geq 0; \frac{dy^i}{dv_1} = \frac{dy}{dv_1} = 0.$$

Note that $\frac{dp_2^*}{dv_1} \geq 0$, so $\frac{d\Pi^*}{dv_1} = \frac{\partial \Pi}{\partial v_1} + \frac{\partial \Pi}{\partial p_2}\frac{dp_2^*}{dv_1}$ may be positive or negative, though $\frac{\partial \Pi}{\partial v_1} \leq 0$, and $\frac{\partial \Pi}{\partial p_2} \geq 0$.

(3) Last we consider the impacts for Case III.

3.1) For c_2, we have

$$\begin{aligned}
\frac{dp_1^*}{dc_2} &= -\frac{(1-\tau)(v_1-\beta_1)f_1}{\tau M} \leq 0; \frac{dp_2^*}{dc_2} = 0;\\
\frac{dk_1^*}{dc_2} &= -(\frac{2\alpha N\tau}{(1-\tau)(v_1-\beta_1)f_1} + \frac{\alpha N b_1}{1-\tau})\frac{dp_1^*}{dc_2} \geq 0;\\
\frac{dk_2^*}{dc_2} &= -\frac{\alpha N}{\tau^2(v_2-\beta_2)f_3} + \frac{\alpha N b_1}{\tau}\frac{dp_1^*}{dc_2} - \frac{dk_1^*}{dc_2} \leq 0;\\
\frac{dk_p^*}{dc_2} &= -\frac{\alpha N}{\tau(v_2-\beta_2)f_3} + \alpha N b_1\frac{dp_1^*}{dc_2} \leq 0;\\
\frac{dy^i}{dc_2} &= -b_1\frac{dp_1^*}{dc_2} \geq 0; \frac{dy}{dc_2} = -\alpha N b_1\frac{dp_1^*}{dc_2} \geq 0.
\end{aligned}$$

$$\begin{aligned}
\frac{d\Pi^*}{dc_2} &= \frac{\partial \Pi}{\partial c_2} + \frac{\partial \Pi}{\partial k_1}\frac{dk_1^*}{dc_2} + \frac{\partial \Pi}{\partial k_2}\frac{dk_2^*}{dc_2} + \frac{\partial \Pi}{\partial p_1}\frac{dp_1^*}{dc_2} + \frac{\partial \Pi}{\partial p_2}\frac{dp_2^*}{dc_2}\\
&= \frac{\partial \Pi}{\partial c_2}\\
&= -k_2^* \leq 0
\end{aligned}$$

For $\frac{d\Pi^*}{dc_2}$, the second equality holds because $\frac{\partial \Pi}{\partial k_1} = \frac{\partial \Pi}{\partial k_2} = \frac{\partial \Pi}{\partial p_1} = 0$ when the optimal solutions are obtained, and $p_2^* = \bar{p_2}$ which follows that $\frac{dp_2^*}{dc_2} = 0$.

3.2) For c_1, we have

$$\begin{aligned}
\frac{dp_1^*}{dc_1} &= \frac{(1-\tau)(v_1-\beta_1)f_1 - \tau(\beta_2-\beta_1)f_2}{M}; \frac{dp_2^*}{dc_1} = 0;\\
\frac{dk_1^*}{dc_1} &= -\alpha N\frac{2+b_1((v_1-\beta_1)f_1+(\beta_2-\beta_1)f_2)}{M} \leq 0;\\
\frac{dk_2^*}{dc_1} &= \alpha N\frac{2\tau+b_1(v_1-\beta_1)f_1}{\tau M} \geq 0;\\
\frac{dk_p^*}{dc_1} &= \alpha N b_1\frac{dp_1^*}{dc_1}; \frac{dy^i}{dc_1} = -b_1\frac{dp_1^*}{dc_1}; \frac{dy}{dc_1} = -\alpha N b_1\frac{dp_1^*}{dc_1}.
\end{aligned}$$

Here, $\frac{dp_1^*}{dc_1}$, $\frac{dk_p^*}{dc_1}$, $\frac{dy^i}{dc_1}$ and $\frac{dy}{dc_1}$ may be positive or negative. Similarly, $\frac{d\Pi^*}{dc_1} = \frac{\partial \Pi}{\partial c_1} \leq 0$.

3.3) For β_2, we have

$$\begin{aligned}
\frac{dp_1^*}{d\beta_2} &= -\frac{(1-\tau)(v_1-\beta_1)f_1F_2}{M} \leq 0; \frac{dp_2^*}{d\beta_2} = 0;\\
\frac{dk_1^*}{d\beta_2} &= -(\frac{\alpha N b_1}{1-\tau} + \frac{2\alpha N\tau}{(1-\tau)(v_1-\beta_1)f_1})\frac{dp_1^*}{d\beta_2} \geq 0;\\
\frac{dk_2^*}{d\beta_2} &= -\frac{\alpha N F_3}{\tau(v_2-\beta_2)f_3} + \frac{\alpha N b_1}{\tau}\frac{dp_1^*}{d\beta_2} - \frac{dk_1^*}{d\beta_2} \leq 0;\\
\frac{dk_p^*}{d\beta_2} &= -\frac{\alpha N F_3}{(v_2-\beta_2)f_3} + \alpha N b_1\frac{dp_1^*}{d\beta_2} \leq 0;\\
\frac{dy^i}{d\beta_2} &= -b_1\frac{dp_1^*}{d\beta_2} \geq 0; \frac{dy}{d\beta_2} = -\alpha N b_1\frac{dp_1^*}{d\beta_2} \geq 0; \frac{d\Pi^*}{d\beta_2} = \frac{\partial \Pi}{\partial \beta_2} \leq 0.
\end{aligned}$$

3.4) For β_1, we have

$$\begin{aligned}
\frac{dp_1^*}{d\beta_1} &= \frac{(1-\tau)(v_1-\beta-1)f_1F_2 - \tau(\beta_2-\beta_1)f_2(1-F_1)}{M}; \frac{dp_2^*}{d\beta_1} = 0;\\
\frac{dk_1^*}{d\beta_1} &= -\alpha N\frac{[2\tau + b_1(v_1-\beta_1)f_1]F_2 + (1-F_1)[2(1-\tau) + b_1(\beta_2-\beta_1)f_2]}{M} \leq 0;\\
\frac{dk_2^*}{d\beta_1} &= \alpha N\frac{F_2[2\tau^2 + b_1(v_1-\beta_1)f_1] + 2\tau(1-\tau)(1-F_1)}{\tau M} \geq 0;\\
\frac{dk_p^*}{d\beta_1} &= \alpha N b_1\frac{dp_1^*}{d\beta_1}; \frac{dy^i}{d\beta_1} = -b_1\frac{dp_1^*}{d\beta_1}; \frac{dy}{d\beta_1} = -\alpha N b_1\frac{dp_1^*}{d\beta_1}.
\end{aligned}$$

Here, $\frac{dp_1^*}{d\beta_1}$, $\frac{dk_p^*}{d\beta_1}$, $\frac{dy^i}{d\beta_1}$, and $\frac{dy}{d\beta_1}$ may be positive or negative, and $\frac{d\Pi^*}{d\beta_1} = \frac{\partial \Pi}{\partial \beta_1} \leq 0$.

3.5) For V_2, we have

$$\begin{aligned}
&\frac{dp_1^*}{dv_2} = 0; \frac{dp_2^*}{dv_2} = 0; \frac{dk_1^*}{dv_2} = 0; \frac{dk_2^*}{dv_2} = \frac{\alpha N F_3}{\tau(v_2-\beta_2)f_3} \geq 0;\\
&\frac{dk_p^*}{dv_2} = \frac{\alpha N F_3}{(v_2-\beta_2)f_3} \geq 0; \frac{dy^i}{dv_2} = \frac{dy}{dv_2} = 0; \frac{d\Pi^*}{dv_2} = \frac{\partial \Pi}{\partial v_2} \leq 0.
\end{aligned}$$

3.6) For v_1, we have

$$\begin{aligned}
\frac{dp_1^*}{dv_1} &= \frac{\tau(\beta_2-\beta_1)f_2(1-F_1)}{M} \geq 0; \frac{dp_2^*}{dv_1} = 0;\\
\frac{dk_1^*}{dv_1} &= \alpha N(1-F_1)\frac{2(1-\tau) + b_1(\beta_2-\beta_1)f_2}{M} \geq 0;\\
\frac{dk_2^*}{dv_1} &= -\frac{2\alpha N(1-\tau)(1-F_1)}{M} \leq 0; \frac{dk_p^*}{dv_1} = \alpha N b_1\frac{dp_1^*}{dv_1} \geq 0;\\
\frac{dy^i}{dv_1} &= -b_1\frac{dp_1^*}{dv_1} \leq 0; \frac{dy}{dv_1} = -\alpha N b_1\frac{dp_1^*}{dv_1} \leq 0; \frac{d\Pi^*}{dv_1} = \frac{\partial \Pi}{\partial v_1} \leq 0.
\end{aligned}$$

■

Appendix C

Proofs and Supplement for Chapter 4

C.1 Proofs

Proof of Proposition 4.1 Note that Equation (4.1) is a newsvendor model, so we can obtain the results immediately. ∎

Proof of Proposition 4.2 By taking the first and second derivatives of the profit function $\Pi_m(s)$ with respect to s, we have

$$\begin{aligned}\frac{d\Pi_m(s)}{ds} &= c_e b(F^{-1}(\frac{p-w}{p+c_h})+d)+(w-c-c_e a)\beta+(2c_e b\beta-c_I)s;\\ \frac{d^2\Pi_m(s)}{ds^2} &= 2c_e b\beta-c_I.\end{aligned}$$

Then $\Pi_m(s)$ is concave in s, given that $c_I \geq 2c_e b\beta$. By solving the first-order condition, i.e., $\frac{d\Pi_m(s)}{ds}=0$, we obtain that

$$s^* = \frac{c_e b(F^{-1}(\frac{p-w}{p+c_h})+d)+(w-c-c_e a)\beta}{c_I-2c_e b\beta}.$$

∎

Proof of Proposition 4.3 The effects of b and c_I can be obtained by just looking at the formula of x^* and s^*, i.e., Equations(4.2) and (4.4), respectively.

However, the effects of β and c_e are more complex and not monotones in general. For completeness, we show the value of $\frac{ds^*}{d\beta}$, $\frac{dx^*}{d\beta}$, $\frac{ds^*}{dc_e}$, and $\frac{dx^*}{dc_e}$ in this proof as follows:

Recalling that $x(s)$ is determined by the first-order condition of the retailer's profit function:

$$\frac{\partial \Pi_r(x)}{\partial x} = (p + c_h)F(x - d - \beta s) - p + w = 0,$$

and s^* is determined by the first-order condition of the manufacturer's profit function:

$$\frac{d\Pi_m(s)}{ds} = (w - c)\beta + c_e bx(s) - c_e\beta(a - bs) - c_I s = 0.$$

Let $G_1 = \frac{\partial \Pi_r(x)}{\partial x} = (p + c_h)F(x - d - \beta s) - p + w$ and $G_2 = \frac{d\Pi_m(s)}{ds} = (w - c)\beta + c_e bx(s) - c_e\beta(a - bs) - c_I s$.

By taking the first derivatives of G_1 and G_2 with respect to β, we have

$$\begin{aligned} \frac{dG_1}{d\beta} &= \hat{p}\hat{f}(\frac{dx^*}{d\beta} - s^* - \beta\frac{ds^*}{d\beta}) = 0; \\ \frac{dG_2}{d\beta} &= w - c + c_e b\frac{dx^*}{d\beta} - c_e(a - bs^*) + c_e b\beta\frac{ds^*}{d\beta} - c_I\frac{ds^*}{d\beta} = 0, \end{aligned}$$

where $\hat{p} = p + c_h$ and $\hat{f} = f(x^* - d - \beta s^*)$. Solving the above two equations obtains that $\frac{ds^*}{d\beta} = \frac{w-c-c_e(a-2bs^*)}{2c_e b\beta - c_I}$, $\frac{dx^*}{d\beta} = s^* + \frac{ds^*}{d\beta}\beta$, but they may be positive or non-positive. By taking the above approach to consider the effects of β and c_e, we can obtain that , $\frac{ds^*}{dc_e} = \frac{-bx^*+\beta(a-bs^*)}{2c_e b\beta - c_I}$, and $\frac{dx^*}{dc_e} = \frac{ds^*}{dc_e}\beta$, but they may be positive or non-positive too.

For the effects on the profits, by taking the first derivatives of Π_r^*, Π_m^*, and Π_d^* with respect to b, we have

$$\begin{aligned} \frac{d\Pi_r^*}{db} &= \frac{\partial \Pi_r}{\partial b} + \frac{\partial \Pi_r}{\partial x}\frac{dx^*}{db} + \frac{\partial \Pi_r}{\partial s}\frac{ds^*}{db} = \frac{\partial \Pi_r}{\partial b}|_{(x=x^*,s=s^*)} + \frac{\partial \Pi_r}{\partial s}\frac{ds^*}{db}|_{(x=x^*,s=s^*)} \\ &= \beta((p + c_h)F(x^* - d - \beta s^*))\frac{c_e x^* + c_e\beta s^*}{c_I - 2c_e b\beta}; \\ \frac{d\Pi_m^*}{db} &= \frac{\partial \Pi_m}{\partial b} + \frac{\partial \Pi_m}{\partial x}\frac{dx^*}{db} + \frac{\partial \Pi_m}{\partial s}\frac{ds^*}{db} \\ &= \frac{\partial \Pi_m}{\partial b}|_{(x=x^*,s=s^*)} + \frac{\partial \Pi_m}{\partial x}\frac{dx^*}{db}|_{(x=x^*,s=s^*)} \\ &= c_e s^* x^* + (w - c - c_e(a - bs^*))\frac{c_e x^* + c_e\beta s^*}{c_I - 2c_e b\beta}\beta; \\ \frac{d\Pi_d^*}{db} &= \frac{\partial \Pi_d}{\partial b} + \frac{\partial \Pi_d}{\partial x}\frac{dx^*}{db} + \frac{\partial \Pi_d}{\partial s}\frac{ds^*}{db} \\ &= c_e x^* + ((p - c - c_e(a - bs^*)\beta - c_I s^* + c_e bx^*)\frac{c_e x^* + c_e\beta s^*}{c_I - 2c_e b\beta}. \end{aligned}$$

In the first equation, the second equality holds because $\frac{\partial \Pi_r}{\partial x} = 0$ when $(x = x^*, s = s^*)$, and in the second equation, the second equality holds because $\frac{\partial \Pi_m}{\partial s} = 0$ when $(x = x^*, s = s^*)$. However, $\frac{d\Pi_r^*}{db}$, $\frac{d\Pi_m^*}{db}$, and $\frac{d\Pi_d^*}{db}$ may be positive or non-positive. Similarly, we can obtain the values of the first derivative of Π_r^*, Π_m^*, and Π_d^* with respect to β, c_e, and c_I. Unfortunately, they are complex and are not monotone in general. ■

Proof of Proposition 4.4 By taking the first and second partial derivatives of the profit function $\Pi_c(x, s)$ with respect to x, we have

$$\begin{aligned} \frac{\partial \Pi_c(x,s)}{\partial x} &= (p + c_h)(1 - F(x - d - \beta s)) - c - c_h - c_e(a - bs); \\ \frac{\partial^2 \Pi_c(x,s)}{\partial x^2} &= -(p + c_h) f(x - d - \beta s) \leq 0. \end{aligned}$$

As the second partial derivative is non-positive, $\Pi_c(x, s)$ is concave in x, and the optimal response of the production quantity is uniquely determined by the first order condition of the profit function, i.e., $\frac{\partial \Pi_c(x,s)}{\partial x} = 0$. ■

Proof of Corollary 4.2 By taking the derivative of $\frac{\partial \Pi_c(x,s)}{\partial x}$ with respect to s, we have

$$\frac{\partial^2 \Pi_c(x,s)}{\partial x \partial s} = (p + c_h) f(x - d - \beta s)\beta + c_e b.$$

Then, by the Implicit Function Theorem, i.e., $\frac{dx(s)}{ds} = -\frac{\frac{\partial^2 \Pi_c(x,s)}{\partial x \partial s}}{\frac{\partial^2 \Pi_c(x,s)}{\partial x^2}}$, we have

$$\begin{aligned} \frac{dx(s)}{ds} &= -\frac{(p + c_h) f(x - d - \beta s)\beta + c_e b}{-(p + c_h) f(x - d - \beta s)} \\ &= \beta + \frac{c_e b}{(p + c_h) f(x - d - \beta s)} > 0. \end{aligned}$$

■

Proof of Proposition 4.5 Given that $c_I \geq 2c_e b\beta$, it is difficult to determine the sign of $\frac{d^2 \Pi_c(x(s),s)}{dx^2}$ directly. So we take the third derivative of $\Pi_c(x(s), s)$ over s, and we have

$$\frac{d^3 \Pi_c(x(s), s)}{ds^3} = -\frac{(c_e b)^3 f'(x(s) - d - \beta s)}{(p + c_h)^2 (f(x(s) - d - \beta s))^3}.$$

When $f'(\cdot) \geq 0$, we have $\frac{d^3\Pi_c(x(s),s)}{ds^3} \leq 0$, it implies that $\frac{d\Pi_c(x(s),s)}{ds}$ is concave in s. So $\frac{d\Pi_c(x(s),s)}{ds} = 0$ has at most two roots and the larger of the two makes a change of sign for $\frac{d\Pi_c(x(s),s)}{ds}$ from positive to negative that corresponds to a local maximum of $\Pi_c(x(s), s)$.

When $f'(\cdot) < 0$, we have $\frac{d^3\Pi_c(x(s),s)}{ds^3} > 0$, it implies that $\frac{d\Pi_c(x(s),s)}{ds}$ is convex in s. So $\frac{d\Pi_c(x(s),s)}{ds} = 0$ has at most two roots and the smaller of the two makes a change of sign for $\frac{d\Pi_c(x(s),s)}{ds}$ from positive to negative that corresponds to a local maximum of $\Pi_c(x(s), s)$. ■

Proof of Corollary 4.3 For the uniform distribution of the demand, $\epsilon \backsim U[A, B]$, then $f(z) = \frac{1}{B-A}$ and $f'(z) = 0$. We have

$$\frac{d^2\Pi_c(x(s), s)}{ds^2} = 2c_e b\beta - c_I + \frac{(c_e b)^2(B-A)}{(p + c_h)} \begin{cases} \geq 0 & \text{if } c_I \leq 2c_e b\beta + \frac{(c_e b)^2(B-A)}{(p+c_h)}; \\ < 0 & \text{otherwise,} \end{cases}$$

which means that $\Pi_c(x(s), s)$ is a convex function if $c_I \leq 2c_e b\beta + \frac{(c_e b)^2(B-A)}{(p+c_h)}$, and concave otherwise. So there is at most one optimal point of s that satisfies $d\Pi_c(x(s), s)/ds = 0$ for the uniform distribution.

For the exponential distribution, $\epsilon \backsim Exp(1/\theta)$, then $f(z) = \frac{1}{\theta}e^{-\frac{z}{\theta}}$ and $f'(z) = -\frac{1}{\theta}f(s) = -\frac{1}{\theta^2}e^{-\frac{z}{\theta}}$. We have

$$\frac{d^3\Pi_c(x(s), s)}{ds^3} = \frac{(c_e b)^3}{(p + c_h)^2(f(x(s) - d - \beta s))^2\theta} \geq 0.$$

So $d\Pi_c(x(s), s)/ds = 0$ has at most two roots, and the smaller of the two makes a change of sign for $d\Pi_c(x(s), s)/ds$ from positive to negative that corresponds to a local maximum of $\Pi_c(x(s), s)$.

For the normal distribution, $\epsilon \backsim Normal(\mu, \sigma)$, then $f(z) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(z-\mu)^2}{2\sigma^2}}$ and $f'(z) = -\frac{z-\mu}{\sigma^2}f(z)$. We have

$$\begin{aligned} \frac{d^3\Pi_c(x(s), s)}{ds^3} &= \frac{(c_e b)^3}{(p + c_h)^2(f(x(s) - d - \beta s))^2} \frac{x(s) - d - \beta s - \mu}{\sigma^2} \\ &\begin{cases} \geq 0 & \text{if } x(s) - d - \beta s \geq \mu; \\ < 0 & \text{otherwise.} \end{cases} \end{aligned}$$

By Corollary 4.2, we obtain that

$$\frac{d(x(s) - d - \beta s)}{ds} = \frac{c_e b}{(p + c_h)f(x - d - \beta s)} \geq 0,$$

which means that $x(s) - d - \beta s$ increases in s. Let s_t be the solution of $x(s) - d - \beta s = \mu$. Then we have that, if $s < s_t$, then $d\Pi_c(x(s), s)/ds$ is concave in s; if $s \geq s_t$, then $d\Pi_c(x(s), s)/ds$ is convex in s. In other words, $d\Pi_c(x(s), s)/ds$ changes from a concave function to a convex function as s increases. Therefore, $d\Pi_c(x(s), s)/ds$ has at most three roots, and the one (and has at most one) makes a changes of sign for $d\Pi_c(x(s), s)/ds$ from positive to negative that corresponds to a local maximum of $\Pi_c(x(s), s)$. ∎

Proof of Proposition 4.6 (a) Recalling that $x(s)$ is determined by

$$\frac{\partial \Pi_c(x, s)}{\partial x} = (p + c_h)(1 - F(x - d - \beta s)) - c - c_h - c_e(a - bs) = 0,$$

and s^* is determined by

$$\frac{d\Pi_c(x(s), s)}{ds} = (p + c_h)F(x - d - \beta s)\beta - c_I s + c_e bx(s) = 0.$$

Let $G_1 = -\frac{\partial \Pi_c(x,s)}{\partial x} = (p + c_h)F(x - d - \beta s) - p + c + c_e(a - bs)$ and $G_2 = \frac{d\Pi_c(x(s),s)}{ds} = (p + c_h)F(x - d - \beta s)\beta - c_I s + c_e bx(s)$.

By taking the first derivatives of G_1 and G_2 with respect to b, we have

$$\frac{dG_1}{db} = -c_e s^* + \hat{p}\hat{f}\frac{dx^*}{db} - (\hat{p}\hat{f}\beta + c_e b)\frac{ds^*}{db} = 0;$$
$$\frac{dG_2}{db} = c_e x^* + (\hat{p}\hat{f}\beta + c_e b)\frac{dx^*}{db} - (\hat{p}\hat{f}\beta^2 + c_I)\frac{ds^*}{db} = 0,$$

where $\hat{p} = p + c_h$ and $\hat{f} = f(x^* - d - \beta s^*)$.

Solving the above two equations obtains that

$$\frac{ds^*}{db} = -\frac{c_e x^* + c_e \beta s^* + \frac{c_e b c_e s^*}{\hat{p}\hat{f}}}{2c_e b\beta - c_I + \frac{(c_e b)^2}{\hat{p}\hat{f}}} \geq 0;$$
$$\frac{dx^*}{db} = \frac{c_e s^* + (\hat{p}\hat{f}\beta + c_e b)\frac{ds^*}{db}}{\hat{p}\hat{f}} \geq 0.$$

The inequalities hold because that, when s is obtained at the optimal point, $2c_e b\beta - c_I + \frac{(c_e b)^2}{\hat{p}\hat{f}} = \frac{d^2\Pi_c(x(s),s)}{ds^2} \leq 0$.

Similarly, by taking the first derivatives of G_1 and G_2 with respect to c_I, we have

$$\frac{dG_1}{dc_I} = \hat{p}\hat{f}\frac{dx^*}{dc_I} - (\hat{p}\hat{f}\beta + c_e b)\frac{ds^*}{dc_I} = 0;$$
$$\frac{dG_2}{dc_I} = -s^* + (\hat{p}\hat{f}\beta + c_e b)\frac{dx^*}{dc_I} - (\hat{p}\hat{f}\beta^2 + c_I)\frac{ds^*}{dc_I} = 0,$$

Solving the above two equations obtains that

$$\frac{ds^*}{dc_I} = \frac{s^*}{2c_e b\beta - c_I + \frac{(c_e b)^2}{\hat{p}\hat{f}}} \leq 0;$$
$$\frac{dx^*}{dc_I} = \frac{(\hat{p}\hat{f}\beta + c_e b)\frac{ds^*}{dc_I}}{\hat{p}\hat{f}} \leq 0.$$

The inequalities hold because we have that, here, $2c_e b\beta - c_I + \frac{(c_e b)^2}{\hat{p}\hat{f}} \leq 0$ when $s = s^*$.

By taking the above approach to consider the effects of β and c_e, we can obtain that, for β, $\frac{ds^*}{d\beta} = \frac{-c_e b s^*}{2c_e b\beta - c_I + \frac{(c_e b)^2}{\hat{p}\hat{f}}} \geq 0$ and $\frac{dx^*}{d\beta} = s + \frac{\hat{p}\hat{f}\beta + c_e b}{\hat{p}\hat{f}}\frac{ds^*}{d\beta} \geq 0$; for c_e, $\frac{ds^*}{dc_e} = \frac{-bx^* + \frac{\hat{p}\hat{f}\beta + c_e b}{\hat{p}\hat{f}}(a - bs^*)}{2c_e b\beta - c_I + \frac{(c_e b)^2}{\hat{p}\hat{f}}}$ and $\frac{dx^*}{dc_e} = \frac{-(a - bs^*) + (\hat{p}\hat{f}\beta + c_e b)\frac{ds^*}{dc_e}}{\hat{p}\hat{f}}$, but which may be positive or non-positive.

By taking the first derivative of Π_c^* with respect to b, we have

$$\begin{aligned}
\frac{d\Pi_c^*}{db} &= \frac{\partial \Pi_c}{\partial b} + \frac{\partial \Pi_c}{\partial x}\frac{dx^*}{db} + \frac{\partial \Pi_c}{\partial s}\frac{ds^*}{db} \\
&= \frac{\partial \Pi_c}{\partial b}|_{(x=x^*, s=s^*)} \\
&= c_e s^* x^* \geq 0
\end{aligned}$$

The second equality holds because $\frac{\partial \Pi_c}{\partial x} = \frac{\partial \Pi_c}{\partial s} = 0$ when $(x = x^*, s = s^*)$. Similarly, we can obtain that $\frac{d\Pi_c^*}{dc_I} = -\frac{(s^*)^2}{2} \leq 0$, $\frac{d\Pi_c^*}{d\beta} = s^*\hat{p}F(x^* - d - \beta s^*) \geq 0$, and $\frac{d\Pi_c^*}{dc_e} = K - (a - bs^*)x^*$ but which may be positive or non-positive. ■

Proof of Proposition 4.7 The retailer's problem is a newsvendor problem, so we can easily obtain that

$$x(s) = F^{-1}(\frac{\phi p - w}{\phi p + c_h}) - d - \beta s;$$

By substituting the x into the manufacturer's profit function, and taking the derivatives with respect to s, we have

$$\frac{d\Pi_m}{ds} = ((1-\phi)p + w - c - c_e(a - bs))\beta - c_I s + c_e b x(s);$$
$$\frac{d^2\Pi_m}{ds^2} = 2c_e b\beta - c_I \leq 0.$$

The inequality holds because in this section we restrict our attention to the case in which the sustainability level are determined by the first-order condition of the profit function for the centralized supply chain, i.e., $c_I \geq 2c_e b\beta$. Thus, the optimal s is uniquely determined by $\frac{d\Pi_m}{ds} = 0$. ■

Proof of Proposition 4.8 The proof is similar to the proof for Proposition 4.7 and omitted. ■

Proof of Proposition 4.9 The proof is similar to the proof for Proposition 4.7 and omitted. ■

C.2 Supplement for the Case of $c_I < 2c_e b\beta$

The optimal response of the order quantity (or production quantity) $x(s)$ in this case is the same as that in the case of $c_I \geq 2c_e b\beta$. So we only present the results of the optimal sustainability level here.

C.2.1 The decentralized supply chain

Proposition C.1 *The manufacturer's optimal sustainability level is as follows:*

$$s^* = \begin{cases} 0 & \text{if } c_I > \big(c_e b(F^{-1}(\frac{p-w}{p+c_h}) + d) + (w-c)\beta\big)2b/a; \\ \frac{a}{b} & \text{if } c_I \leq \big(c_e b(F^{-1}(\frac{p-w}{p+c_h}) + d) + (w-c)\beta\big)2b/a. \end{cases}$$

Proposition C.1 shows the optimal sustainability level for the manufacturer. We find that the manufacturer's profit function is a convex function in s. So either the lower bound $s = 0$ or the upper bound $s = a/b$ is optimal for the manufacturer. By comparing the manufacturer's profit when $s = 0$ and the profit when $s = a/b$, we find that, if the sustainability investment coefficient is very small, e.g., $c_I \leq (c_e b(F^{-1}((p-w)/(p+c_h)) + d) + (w-c)\beta)2b/a$, then $s^* = a/b$ such that no emission is produced, i.e., $a - bs^* = 0$; otherwise, $s^* = 0$.

Proposition C.2 *(a) If $s^* = 0$, then x^* is not affected by b, β, c_e, and c_I; Π_r^* is not affected by b, β, c_e, and c_I; Π_m^* and Π_d^* are not affected by b, β, and c_I.*

(b) If $s^* = a/b$, then x^* is decreasing in b, and is increasing in β, but is not affected by c_e and c_I; Π_r^* is not affected by b, c_e, and c_I, and increasing in β; and Π_m^* and Π_d^* are increasing in c_e, and are decreasing in c_I.

Clearly, if $s^* = 0$ or $s^* = a/b$, then s^* will not be affected by b, β, c_e, and c_I. Part (a) of Proposition C.2 indicates that, if $s^* = 0$, then the order quantity and retailer's expected profit will not be affected by parameters related to the sustainability level and the emission, and the manufacturer's profit and the whole supply chain's profit will only be affected by the emission price. Part (b) of Proposition C.2 indicates that Π_m^* and Π_d^* are increasing in c_e, that is because, in this case, the manufacturer would invest in a very high sustainability level and no emission would be produced, then the manufacturer could get extra profit by selling the quota of the permissible emission level. Here, the extra profit is increasing in the emission price. In other words, under this scenario, the manufacturer is encouraged to produce a higher sustainable product without any carbon emission when the emission price is higher. In addition, when $s^* = a/b$, namely, the upper bound of stainability level, the optimal order quantity would not be affected by both c_e and c_I.

Remark C.1 *If $s^* = 0$, then $d\Pi_m^*/dc_e = d\Pi_d^*/dc_e = K - ax^*$; if $s^* = a/b$, then $d\Pi_m^*/db = c_e x^* - (w-c)\beta a/(b^2)$, $d\Pi_m^*/d\beta = (w-c)a/b$, $d\Pi_d^*/db = c_e x^* a/b - (\hat{p}(1-\hat{F}(a/b)) - c - c_h)\beta a/(b^2)$, and $d\Pi_d^*/d\beta = (p-c)a/b$, where $\hat{p} = p + c_h$ and $\hat{F}(s^*) = F(x^* - d - \beta s^*)$.*

C.2.2 The centralized supply chain

Proposition C.3 *$(d^2\Pi_c(x(s), s))/(ds^2) > 0$, and $s^* = 0$ or $s^* = a/b$.*

Proposition C.3 shows that, if $2c_e b\beta > c_I$, then $\Pi_c(x(s), s)$ is a convex function in s. So the lower bound $s = 0$ or upper bound $s = a/b$ is optimal for $\Pi_c(x(s), s)$. By comparing the profit when $s = 0$ and the profit when $s = a/b$, we can obtain the optimal solution: If $\Pi_c(x(0), 0) \geq \Pi_c(x(a/b), a/b)$, then $s^* = 0$; otherwise $s^* = a/b$.

Proposition C.4 *(a) If $s^* = 0$, then x^* is decreasing in c_e, and x^* and Π_c^* are not affected by b, β, and c_I.*

(b) If $s^* = a/b$, then x^* is decreasing in b, and is increasing in β, but is not affected by c_e and c_I; Π_c^* is increasing in b, β and c_e, and is decreasing in c_I.

From Proposition C.4, we can see that c_I has an impact to the optimal profit of the whole supply chain Π_c^*. More specifically, when s^* is larger than zero, a higher sustainability investment coefficient would lead to a lower centralized supply chain profit. Therefore, in order to increase the centralized supply chain profit, enhancing the efficiency of sustainability investment is significant.

Remark C.2 *If $s^* = 0$, then $d\Pi_c^*/dc_e = K - ax^*$.*

C.2.3 The comparison of decentralized and centralized supply chains

In this section, we compare the whole supply chain's profit and the optimal solutions under the decentralized case with those under the centralized case. Let x_d and s_d be the optimal solutions of the order quantity and sustainability level, respectively, for the decentralized supply chain, and x_c and s_c be the optimal solutions of the production quantity and sustainability level, respectively, for the centralized chain.

Case I: $s_d = s_c = 0$. Then, we have

$$\begin{aligned} x_d &= F^{-1}(\frac{p-w}{p+c_h}) + d; \\ x_c &= F^{-1}(\frac{p-c-c_e a}{p+c_h}) + d. \end{aligned}$$

From the above equations, we easily find that, if $w \leq c + c_e a$, then $x_d \geq x_c$; otherwise, $x_d < x_c$. By comparing the profit of the whole supply chain under the decentralized case, i.e., Π_d^* ($\Pi_d^* = \Pi_r^* + \Pi_m^*$), with that under the centralized case, i.e., Π_c^*, we have

$$\Pi_d^* - \Pi_c^* = (w - c - c_e a)(x_d - d) + (p + c_h)\int_{x_c-d}^{x_d-d} \epsilon f(\epsilon) d\epsilon. \quad \text{(C.1)}$$

Corollary C.1 *For the uniform distribution of the demand, i.e., $\epsilon \backsim U[A, B]$, Equation (C.1) can be expressed as*

$$\Pi_d^* - \Pi_c^* = -\frac{(w - c - c_e a)^2(B - A)}{2(p + c_h)} \leq 0.$$

Corollary C.1 indicates that, when the demand is followed the uniform distribution and no sustainability level is invested in, i.e., $s_d = s_c = 0$, the optimal profit of the whole supply chain under the decentralized case is lower than that under the centralized case.

Case II: $s_d = s_c = a/b$. Then, we have

$$\begin{aligned} x_d &= F^{-1}\left(\frac{p - w}{p + c_h}\right) + d + \beta\frac{a}{b}; \\ x_c &= F^{-1}\left(\frac{p - c}{p + c_h}\right) + d + \beta\frac{a}{b}. \end{aligned}$$

From the above equation, we easily find that, if $w \leq c$, then $x_d \geq x_c$; otherwise, $x_d < x_c$.

By comparing the profit of the whole supply chain under the decentralized case, with that under the centralized case, we have

$$\Pi_d^* - \Pi_c^* = (w - c)(x_d - d - \beta\frac{a}{b}) + (p + c_h)\int_{x_c-d-\beta\frac{a}{b}}^{x_d-d-\beta\frac{a}{b}} \epsilon f(\epsilon) d\epsilon. \quad \text{(C.2)}$$

Corollary C.2 *For the uniform distribution of the demand, i.e., $\epsilon \backsim U[A, B]$, Equation (C.2) can be expressed as*

$$\Pi_d^* - \Pi_c^* = -\frac{(w - c)^2(B - A)}{2(p + c_h)} \leq 0.$$

Similarly, Corollary C.2 indicates that, when the demand follows the uniform distribution and a very high sustainability level is invested in such that no emission is produced, i.e., $s_d = s_c = a/b$, the optimal profit of the whole supply chain under the decentralized case is lower than that under the centralized case.

Proofs for the case of $c_I < 2c_e b\beta$

Proof of Proposition C.1 By taking the first and second derivatives of the profit function $\Pi_m(s)$ with respect to s, we have

$$\begin{aligned}\frac{d\Pi_m(s)}{ds} &= c_e b(F^{-1}(\frac{p-w}{p+c_h})+d)+(w-c-c_e a)\beta+(2c_e b\beta-c_I)s;\\ \frac{d^2\Pi_m(s)}{ds^2} &= 2c_e b\beta-c_I.\end{aligned}$$

Given $2c_e b\beta - c_I > 0$, $\Pi_m(s)$ is convex in s; either the lower bound $s = 0$ or the upper bound $s = \frac{a}{b}$ is optimum. By comparing $\Pi_m(s = 0)$ and $\Pi_m(s = \frac{a}{b})$, we obtain that

$$s^* = \begin{cases} 0 & \text{if } c_I > \big(c_e b(F^{-1}(\frac{p-w}{p+c_h})+d)+(w-c)\beta\big)2b/a; \\ \frac{a}{b} & \text{if } c_I \leq \big(c_e b(F^{-1}(\frac{p-w}{p+c_h})+d)+(w-c)\beta\big)2b/a. \end{cases}$$

■

Proof of Proposition C.2 Given $s^* = 0$ or $s^* = \frac{a}{b}$, the proof is trivial, so we omit it. ■

Proof of Proposition C.3 Clearly, given $2c_e b\beta > c_I$, we have $\frac{d^2\Pi_c(x(s),s)}{dx^2} \geq 0$, it implies that $\Pi_c(x(s), s)$ is convex in s. So either the lower bound of s or the upper bound of s is optimal. By comparing the profit under this two values, we can obtain the optimal solution. If $\Pi_c(x(0), 0) \geq \Pi_c(x(a/b), a/b)$, then $s^* = 0$; otherwise $s^* = a/b$. ■

Proof of Proposition C.4 Given $s^* = 0$ or $s^* = \frac{a}{b}$, the proof is trivial, so we omit it. ■

Proof of Corollary C.1 For the uniform distribution of the demand, $\epsilon \backsim U[A, B]$, then $f(z) = \frac{1}{B-A}$. We have

$$\begin{aligned}x_d &= \frac{p-w}{p+c_h}(B-A)+A+d;\\ x_c &= \frac{p-c-c_e a}{p+c_h}(B-A)+A+d,\end{aligned}$$

and Equation (C.1) can be expressed as

$$\begin{aligned}\Pi_d^* - \Pi_c^* &= (w-c-c_e a)(x_d - d) + (p + c_h)\int_{x_c-d}^{x_d-d} \epsilon f(\epsilon) d\epsilon \\ &= -\frac{(w-c-c_e a)^2 (B-A)}{2(p+c_h)} \leq 0.\end{aligned}$$

■

Proof of Corollary C.2 The proof is very similar with that for Corollary C.1, so we omit the details here. ■

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