STUDY OF COMMUNICATION NETWORK PERFORMANCE FROM A COMPLEX NETWORK PERSPECTIVE

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Study of Communication Network Performance from a Complex Network Perspective

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Certificate of Originality

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Abstract

This thesis studies the performance of communication networks from a network science perspective. Our task is to establish a clear link between some structural properties of networks, such as degree distribution, average distance between nodes, and betweenness, with communication network performance, the purpose being to improve understanding of the various factors that affect the performance of communication networks and to provide design information for optimizing performance.

Communication networks are modeled, analyzed and characterized using complex network parameters. In particular, the impact of network topology, routing strategy and resource allocation on the performance of generic communication networks is studied through theoretical analysis and computer simulation. Specifically, the regular lattice, ER random, BA scale-free and Internet AS-level networks under shortest-path (SP) and minimum-degree (MD) routing strategies with various types of resource allocation schemes are considered for determination of network parameters. Performance parameters, including packet drop rate, time delay, and critical generation rate, are evaluated.

Node usage probability is proposed as a new metric for characterizing the traffic load distribution and how frequently a node is chosen to relay packets in a network. Based on the concept of node usage probability, effective network design strategies, including routing algorithms and resource allocation schemes, can be developed to maintain balanced traffic loads in the network nodes by avoiding

overuse of certain nodes. The performance of the proposed minimum-node-usage routing strategy is compared with that based on other popular routing algorithms. Moreover, the effects of different types of traffic generation sources on network performance are studied.

Finally, network design strategies for optimizing the performance of communication networks will be proposed. For efficient and reliable data transmission, the traffic load should be as uniformly distributed as possible in the network and the average distance travelled by the data should be short. This criterion has been shown to be fundamental. The key design problem is therefore to find the optimal solution that achieves this criterion. With a fixed network topology, the traffic load distribution and the transmission efficiency are determined by the specific routing algorithm and the traffic generation pattern. Specifically, a simulated annealing algorithm is employed to find the near optimal configuration of network design, which effectively balances traffic loads and improves the overall traffic performance.

Publications

Journal papers

- **J. Wu**, C. K. Tse, and F. C. M. Lau, "Concept of node usage probability from complex networks and its applications to communication network design," submitted.

- **J. Wu**, C. K. Tse, and F. C. M. Lau, "Optimizing performance of communication networks: an application of network science," *IEEE Transactions on Circuits and Systems II: Briefs*, to appear.

- F. Tan, **J. Wu**, Y. Xia, and C. K. Tse, "Traffic congestion in interconnected complex networks," *Physical Review E*, vol. 89, no. 6, p. 062813, 2014.

- **J. Wu**, C. K. Tse, F. C. M. Lau, and I. W. H. Ho, "Analysis of communication network performance from a complex network perspective," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 60, no. 12, pp. 3303–3316, Dec. 2013.

Conference papers

- **J. Wu**, C. K. Tse, and F. C. M. Lau, "Optimizing performance of communication networks: an application of network science," in *Proc. International*

Symposium on Nonlinear Theory and Its Applications, Lucern, Switzerland, Sep. 2014, pp. 264–267.

- **J. Wu**, C. K. Tse, and F. C. M. Lau, "Effective routing algorithms based on node usage probability from a complex network perspective," in *Proc. IEEE International Symposium on Circuits and Systems*, Melbourne, Australia, Jun. 2014, pp. 2209–2212.

- **J. Wu**, C. K. Tse, F. C. M. Lau, and I. W. H. Ho, "An adaptive routing algorithm for load balancing in communication networks," in *Proc. IEEE International Symposium on Circuits and Systems*, Beijing, China, May 2013, pp. 2295–2298.

- **J. Wu**, C. K. Tse, F. C. M. Lau, and I. W. H. Ho, "Concept of node usage probability for analysis and design of complex communication networks," in *Proc. International Workshop on Chaos-Fractals Theories and Applications*, Dalian, China, Oct. 2012, pp. 159–163.

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Chapter 1

Introduction

1.1 Background and Motivation

Complex systems which are composed of many interacting entities can be represented, analyzed and better understood using a *network* representation, where the entities are represented by *nodes* and the interactions by *links*. The resulting mathematical structure consisting of nodes and links is called a *graph* or a *network*. Network science is concerned with the study of the theoretical foundation of network structure and related dynamic behavior, and the application of networks to many real-world problems [1].

Network science has been applied to practical problems since 1736, when Swiss mathematician Euler solved a historically notable problem called the Seven Bridges of Königsberg using graph theory. Königsberg was a city of Prussia, which consisted of an island in the middle of river Preger and another part separated by two branches of the river. The four parts of the city were connected to each other by seven bridges. Concerning the bridges, it was asked whether it was possible to arrange a route through the city that would cross each bridge once and only once. Euler pointed out that the only important feature of a route was the sequence of bridges crossed. He therefore reformulated the problem in terms of abstract mathematical objects by replacing each landmass with a “node” and each bridge with a “link”. His work was therefore recognized as the theoretical basis for the modern graph (network) theory.

Since Euler laid the foundation in 1736, graph theory continued to be developed and became an extremely useful tool for studying interacting systems in various applied disciplines in the following 200 years.

Based on the observation from empirical study of real-world networks, researchers have proposed various theoretical network models to explain the formation and mimic the features of various network structure.

In the 1960s, two Hungarian mathematician Paul Erdös and Alfréd Rényi proposed an algorithm of constructing random graphs [2]. The Erdös-Rényi (ER) random graph is marked as a milestone in the history of graph theory. The construction of an ER random graph is quite simple. A network with n nodes is constructed by inserting m links between randomly selected pairs of nodes.

However, empirical study has demonstrated that the ER random graph cannot explain some fundamental findings from real-world systems, such as *small-world*, *high-clustering* and *scale-free* topological properties.

The small-world effect was first observed in Stanley Milgram’s “six degree of separation” experiment conducted in 1967. In his experiment, Stanley Milgram invited people randomly selected from Kansas and Nebraska to forward a folder to certain people defined by the experimenters. The experiment operated as follows. If the person who held the folder knew the target person on a personal basis, the person would forward the folder directly to the target. Otherwise, the person would forward the folder to a personal acquaintance of his/hers who was more likely to know the target. As indicated in [3], most of the folders were lost, but for the successfully arrived folders, the number of intermediaries ranged from 2 to 10, with an average of only 5.2. Hence, the social network underlying this experiment was known as a *small-world network*. Moreover, the experiment revealed that the clustering coefficient of social networks is relatively high.

In order to reproduce the small-world and high clustering features, Watts and Strogatz [4] proposed a fundamental small-world network model in 1998. It starts from a circle in which each node is connected with its k nearest neighbors, and the links are rewired with a probability p.

The small world is not the only universal topological feature of the real world. In the year 1999, researchers discovered a power-law degree distribution in the Internet and the World Wide Web [5, 6]. This kind of networks is called *scale-free networks*. A scale-free network has a small number of nodes with extremely high degree and the majority of nodes with relatively low degree. Before this time, most real networked systems were assumed to be random and follow Poisson degree distribution. Subsequently, many other important real-world infrastructural systems such as railroads, pipeline systems, power grids, and telephone networks have been shown to be scale-free [7, 8, 9]. However, both the ER random graph and the small-world network display a Poisson degree distribution. The most popular model for scale-free networks is the growth model introduced by Barabási and Albert [10] which starts from a small connected network and continuously grows using a preferential attachment mechanism. The degree distribution of the generated networks follows a power law with $\gamma = 3$, i.e., $P(k(i)) \sim k(i)^{-3}$, where $k(i)$ is the degree of node i.

In the following years, the small-world and scale-free topologies in complex networks have attracted a great deal of attention from researchers across a variety of disciplines including mathematics, physics, social science and engineering. A wealth of studies on empirical data and theoretical modeling have been conducted [11, 12, 13, 14, 15, 16, 17, 18, 19].

Moreover, the network model has been proven to be an excellent tool for understanding resiliency, synchronization, epidemic spreading, social interactions and so forth of complex systems [20, 21, 22, 23, 24, 25].

With many discoveries of the topological properties from a variety of networked systems and rigorous theoretical research in the mathematics and physics communities, the next step is to apply results from complex network science for better understanding, analyzing and designing practical systems.

The past decade has seen increasing interest in the exploitation of research results from the rapidly emerging discipline of network science for applications in engineering. Computer and communication networked systems are among the most important infrastructures of today's society. One of the most famous and widely studied examples of communication networks is the Internet, in which the computers or routers are abstracted as nodes and the physical data transmission connections as links.

Empirical studies have demonstrated that many real-life communication networks such as the Internet are heterogeneous networks and exhibit small-world and scale-free topological properties [5, 26, 21, 19, 27, 28]. Much previous work has shown that the underlying network structure is highly relevant to the traffic performance of the networks [29, 30].

1.2 Objectives of the Thesis

The principal objective of this thesis is to study the performance of communication networks from a network science perspective. We focus on a generic type of communication networks, in which packets of messages are sent from one node to another under practical operating conditions such as the use of packet buffering in communication nodes and the implementation of specific routing algorithms. Our aim is to establish a clear link between some structural properties with the intended functions of delivering information of communication networks, the purpose being to improve understanding of the various factors that affect the performance of communication networks and to provide design information for optimizing performance.

1.3 Thesis Organization

The thesis is arranged in the following order.

Chapter 2 will give an introduction of complex network theory and the communication system to be considered in this study. First, basic concepts, most commonly used models and empirical studies in complex network science will be introduced. Then, empirical studies and topological characteristics of selected communication networks will be introduced. Finally, previous study of the applications of network science on real-world networks especially communication systems will be reviewed.

In Chapter 3, the operation model of a communication network will be described and the effect of network structure, resource allocation schemes and routing algorithms on the performance of generic communication networks will be studied. The mean-field theory will be adopted to approximate the network throughput and the concept of node usage probability will be proposed as an effective metric for characterizing the traffic load distribution. The performance comparison will be presented in terms of the intended functions of delivering information through extensive simulation.

In Chapter 4, the concept of node usage probability will be discussed in detail and its implication to the choice of routing algorithms and resource allocation will be presented. Effective routing strategies will be developed to balance the traffic loads in the network nodes by avoiding overuse of some particular nodes. The performance of the proposed routing strategy will be compared with that based on other popular routing algorithms, for various network topologies and resource allocation schemes. Any effective network design is shown to necessarily involve minimization of the overall node usage for a given network topology. Moreover, the effect of different types of traffic generation sources on network performance will be discussed.

In Chapter 5, network design strategies for optimizing the performance of communication networks will be proposed. For efficient and reliable data transmission, the traffic load should be as uniformly distributed as possible in the network and the average distance travelled by the data should be short. With a fixed network topology, the traffic load distribution and the node usage probability are determined by the traffic generation pattern and the selected routing algorithm. Based on the concept of node usage probability, the simulated annealing method is adopted to find the near-optimal configuration of network design, which can effectively balance the node usage and keep the average distance relatively low.

Finally, the key conclusions of the thesis and a feasible direction for future work will be presented in Chapter 6.

Chapter 2

Overview of Complex Networks

2.1 Measures of Network Topology

To analyze network data and capture the features of network topology, a variety of measures have been developed, such as node degree, shortest path distance, clustering coefficient, and so on. In the following, we will review some of these measures and discuss about their relevance to the study of communication networks.

2.1.1 Degree

A simple but very important measure of a network is *node degree*, which is defined as the number of links connecting a node to other nodes in the network. If the network is directed, the *out-degree* of a node is the number of outward-directed links, and the *in-degree* is defined as the number of inward-directed links.

The *hub* of a network is the node with a relatively high node degree.

Although node degree is a simple measure of in a network, it can be very illuminating. For instance, in communication networks which we will discuss in detail in the rest of this thesis, node degree represents the number of physical data transmission connections of a computer/router. In this way, the node degree in communication networks gives a crude measure of the importance of this node in the network and how frequently it will be used as a relay node for data transmission.

One of the most fundamental network properties is the degree distribution. The *degree distribution* of a network, denoted by $P(k)$, is defined as the probability of finding a node with degree k. For instance, in a network with N nodes in total, if $N(k)$ of them have degree k, we have $P(k) = N(k)/N$.

Additional information is provided by the existence of degree-degree correlation, which accounts for the way in which nodes with different degrees are mixed. This correlation can be quantified by measuring the *assortativity coefficient* which is defined as follows [31]:

$$r = \frac{\frac{1}{m}\sum_{e\in M} k_i(e)k_j(e) - [\frac{1}{m}\sum_{e\in M}\frac{1}{2}[k_i(e)+k_j(e)]]^2}{\frac{1}{m}\sum_{e\in M}\frac{1}{2}[k_i(e)^2+k_j(e)^2] - [\frac{1}{m}\sum_{e\in M}\frac{1}{2}[k_i(e)+k_j(e)]]^2}, \tag{2.1}$$

where $k_i(e)$ and $k_j(e)$ are the degrees at both end of link e, M is the set of all links in the network, and m is the total number of links in the network.

If $r > 0$, the network is assortative and high-degree nodes tend to connect with high-degree nodes, whereas if $r < 0$, the network is disassortative and high-degree nodes tend to connect with low-degree nodes. The network with "$r = 0$" is referred to as "neutral assortative".

The degree-degree correlations can be used to characterize network structure and to validate the ability of theoretical network models to represent real-world networks. It has been demonstrated [32] that most networks of human connections are assortative and almost all man-made and engineering networks are disassortative. The degree-degree correlations have implications for epidemic spreading, network resilience to intentional attacks, synchronization, and many other kinds of dynamical processes occurring in the networks [33, 34, 35, 36, 37, 38, 39, 40]. For instance, in epidemic spreading, diseases targeting high degree nodes are likely to spread to other high degree nodes because most networks of human connections are assortative. On the other hand, if a fraction of nodes in the network are removed corresponding to curing or vaccinating, the network will separate into connected components, isolating the epidemic spreading. In communication networks like the Internet, intentional attacks on the high degree nodes may quickly destroy the whole communication network as in a disassortative network most low degree nodes are attached to the high degree nodes.

2.1.2 Shortest Path Distance

In an undirected network, the *distance of a path* is defined as the number of links between the source and destination nodes of the path. Then the *shortest path distance* is the number of links in the shortest path between two nodes.

In a directed network, the *directed shortest path distance* is defined as the distance of the directed shortest path from the source node to the destination node.

A characteristic of the overall network structure is the *average shortest path distance*, denoted by $\tilde{D}$, which can be expressed as follows:

$$\tilde{D} = \frac{\sum\limits_{\substack{u,w\in V,\\ u\neq w}} d(u,w)}{N(N-1)}, \tag{2.2}$$

where V is the set of all nodes in the network, N is the total number of nodes in the network, and $d(u,w)$ is the shortest path distance from node u to node w.

For communication networks to be discussed in this thesis, the average shortest path distance is a very important measure, which is closely related to the transmission efficiency and capacity. For example, a smaller $\tilde{D}$ indicates that the packets can arrive at their destinations faster on average when adopting the shortest path routing strategy, thus alleviating the traffic intensity in the network.

The *diameter* of the network is defined as the largest shortest path distance in the network.

To quantify the centrality of a node in the network, an effective measure is the *closeness* of a node i, denoted by $CL(i)$, which is given by [41, 42]:

$$CL(i) = \frac{N-1}{\sum_{j \in V} d(i,j)}, \tag{2.3}$$

The closeness of a node is essentially a measure of how long it takes to spread information to other nodes.

Another important metric related to shortest path distance is the *betweenness*. The betweenness $B(i)$ of a node or a link i is defined as:

$$B(i) = \sum_{\substack{u,w \in V, \\ u \neq w \neq i}} \frac{\rho_{uw}(i)}{\rho_{uw}}, \tag{2.4}$$

where $\rho_{uw}(i)$ is the number of shortest paths between nodes u and w that pass through the node or link i, ρ_{uw} is the total number of shortest paths between nodes u and w.

Thus, the betweenness of a node quantifies the number of times the node serves as a bridge in the paths of shortest distance between two other nodes. In communication networks, a node with a larger betweenness usually carries heavier traffic load when the shortest path routing strategy is adopted for data transmission.

2.1.3 Clustering Coefficient

Clustering coefficient is a useful metric to characterize the existence of loops of order three in the network.

There are two widely used definitions for the clustering coefficient. The first definition of clustering coefficient introduced in [43] is

$$C = \frac{3N_\Delta}{N_3}, \tag{2.5}$$

where N_Δ is the number of triangles in the network, and N_3 is the number of connected triples of nodes.

The clustering coefficient of a network varies from 0 to 1, and a higher coefficient implies more possible triangles in the network.

In the second definition of clustering coefficient, the clustering coefficient $C(i)$ of a given node i is defined as [44]

$$C(i) = \frac{N_\Delta(i)}{N_3(i)} \tag{2.6}$$

where $N_\Delta(i)$ is number of triangles containing node i, and $N_3(i)$ is the number of connected triples connected to node i.

Essentially, the clustering coefficient of a node quantifies how closely its adjacent nodes are connected.

With $C(i)$, an alternative definition of network clustering coefficient is given by

$$\tilde{C} = \frac{1}{n}\sum_{i \in V} C(i) \tag{2.7}$$

2.2 Models of Networks

Based on the topological properties observed from real-world networks, a large number of theoretical models have been derived to mimic the network formation and structure. Among them, the Erdös and Rényi (ER) random graph model, the Watts-Strogatz (WS) small-world model, and the Barabási-Albert (BA) scale-free model are most popular.

2.2.1 Erdös-Rényi Random Graph

The study of random networks can be traced back to the graph theory literature in the 1950s [45, 46]. The most well-known and widely used model of generating random networks is the one introduced by Erdös and Rényi in 1959 [2], and this model is now called the Erdös and Rényi (ER) random graph.

An ER random graph can be generated as follows. We start from N isolated nodes and connect each pair of nodes with a probability equal to p.

If N is large enough, the total number of connections in the network is a variable whose mean is $pN(N-1)/2$, and the degrees of the nodes follow a Poisson distribution, i.e.,

$$P(k(i)) = \langle k \rangle^{k(i)} \, e^{-\langle k \rangle} / k(i)! \tag{2.8}$$

where $k(i)$ is the degree of node i, and $\langle k \rangle = p(N-1) \approx pN$ is the mean value of $k(i)$.

Since each pair of nodes are connected with equal probability, the random network is a homogeneous network in which most of the nodes' degrees are around pN. In Fig. 2.1, we display some examples of ER random networks with the same number of nodes and different connecting probabilities.

The average shortest path distance of an ER network, denoted by $\tilde{D}_{ER}$, is given by [47]

$$\tilde{D}_{ER} = \frac{\ln(N) - \alpha}{\ln(pN)} + \frac{1}{2} \tag{2.9}$$

where $\alpha \approx 0.566$ is the Euler-Mascheroni constant.

Random networks are among the earliest studied networks in the history of

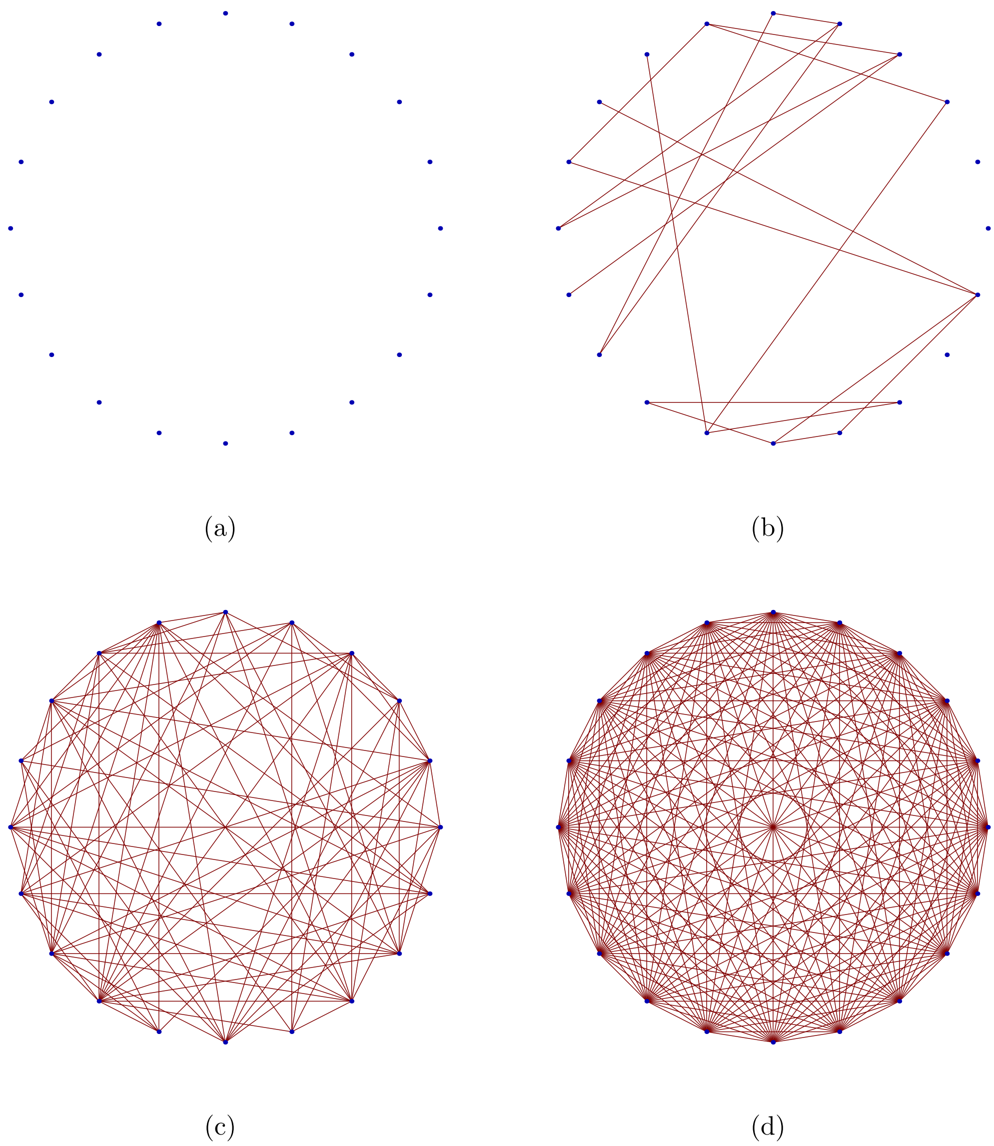


Figure 2.1: ER random networks with $N = 20$ and (a) $p = 0$, (b) $p = 0.1$, (c) $p = 0.5$, and (d) $p = 1$.

network science. However, it has been widely demonstrated that most real-world networks including natural and man-made networked systems are not random.

As indicated in (2.9), the average shortest path of an ER random network decreases rapidly with the increase of link probability p and the network displays the small-world effect. However, many other properties of an ER random network are very different from real-world networks.

Nowadays, the ER random graph is stilled widely used for theoretical modeling because of its simplicity and it also provides a baseline for comparison with non-random networks.

2.2.2 Watts-Strogatz Small-world Model

The small-world effect was firstly observed by Stanley Milgram in 1967. His famous "six degree of separation" experiment revealed that people from Kansas and Nebraska in the U.S. were separated by only about six people on average and the clustering coefficient of social networks is relatively high.

In order to reproduce the small-world and high clustering features exhibited by many real-world networks, a small-world network model was proposed by Watts and Strogatz [4]. The Watts-Strogatz (WS) model starts from a circle with N nodes in which each node is connected with its k nearest neighbours and the links are rewired with a probability p.

In Fig. 2.2, we illustrate the WS small-world networks with $N = 20$, $k = 3$, and different probabilities of rewiring.

When $p = 0$, the network is a typical regular network (see Fig. 2.2(a)) and the clustering coefficient of the network, denoted by C_{WS}, is given [44] by

$$C_{WS} = \frac{3(k-2)}{4(k-1)} \tag{2.10}$$

which indicates that the clustering coefficient of a regular network is independent of the network size, and tends to 0.75 for very large k.

In this case, the network has a relatively large average shortest path distance, which can be expressed as

$$\tilde{D}_{WS} = \frac{(N-1)(N+k-1)}{2kN} \tag{2.11}$$

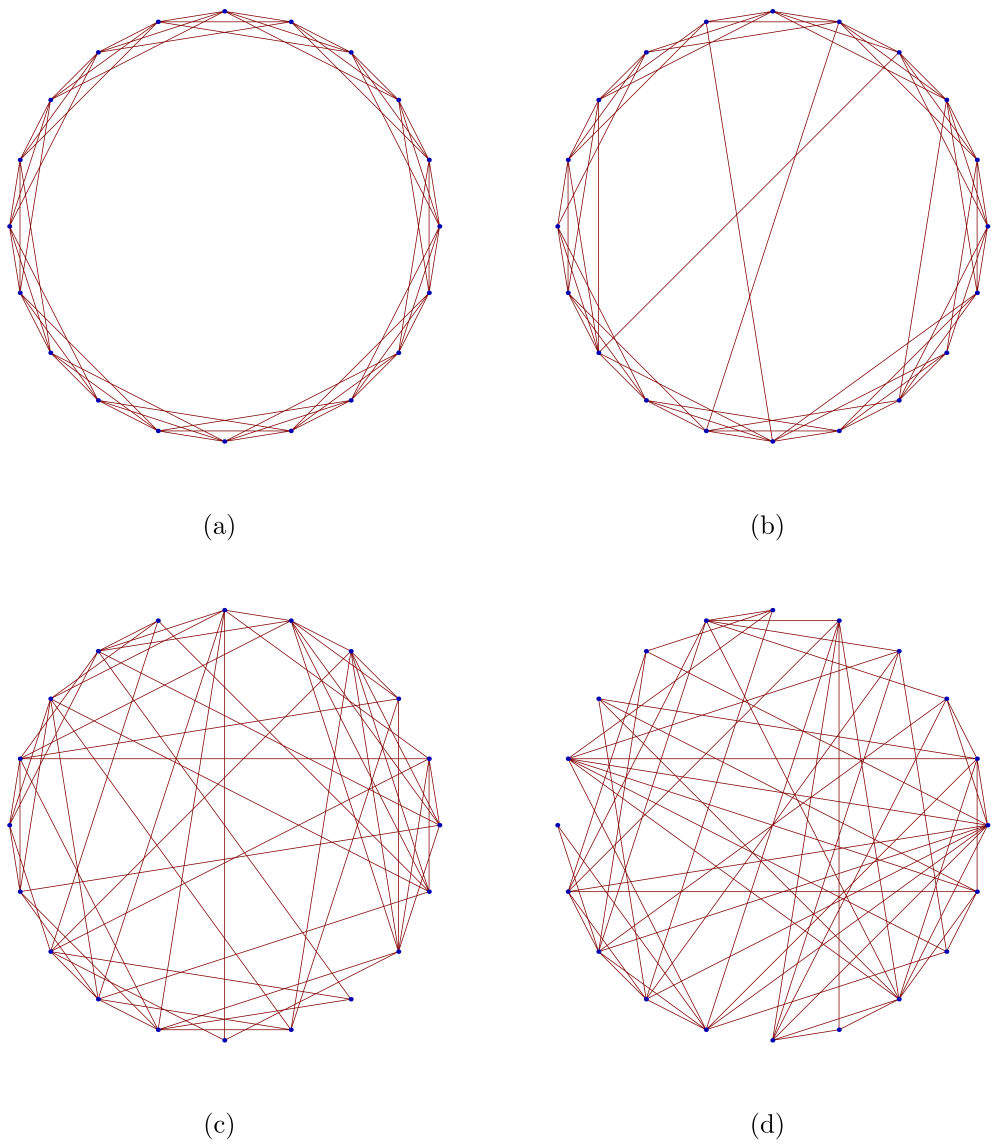
This indicates that $\tilde{D}_{WS} \propto N/2k$ when the network is sparse ($N \gg k$). As the network size grows, the average shortest path distance $\tilde{D}_{WS} \to N/2$.

As illustrated in Fig. 2.2, the randomness of WS small-world networks is scalable and depends on the rewiring probability p.

With the increase of p, the average shortest path distance decreases very rapidly and the clustering coefficient remains high. As $p \to 1$, the network tends to be an ER random network (see Fig. 2.2(d)), with $D_{WS} \propto \ln(N)/\ln(k)$ and $C_{WS} \propto k/N$.

In large-scale communication networks, packets need to find their way through a huge and complex network from their sources to destinations, and the efficiency of a communication network is closely related to the number of hops separating any two nodes in the network. Therefore, it is important to make a communication network "small" enough by limiting the growth of the average shortest path distance as the network size increases.

Compared with the ER random graph, the WS small-world network has a much larger clustering coefficient. However, the high clustering property of a WS small-world network is a consequence of the inherent clustering of the regular network used as a starting point in the generation of a WS network. On the other hand, the small-world effect of a WS network is caused by the process of random rewiring. Therefore, the formation of a WS network is very different from the evolution of many realistic small-world networks.



(a) (b) (c) (d)

Figure 2.2: WS small-world networks with $N = 20$, $k = 6$, and (a) $p = 0$, (b) $p = 0.1$, (c) $p = 0.5$, and (d) $p = 0.95$.

2.2.3 Barabási-Albert Scale-free Model

As mentioned in Section 2.1.1, degree distribution gives important clues into the structure of a network. Both the above mentioned ER random and WS small-world networks display a Poisson degree distribution. However, empirical study has revealed that many real-world networks display a power-law degree distribution.

In this distribution, the probability of a node with degree k decays as a negative power of the degree (see Fig. 2.4(b)),

$$P(k) \sim k^{-\gamma} \tag{2.12}$$

where γ is the power-law exponent.

This kind of networks is referred to as scale-free networks, in which a small number of nodes have very high node degree and the majority of nodes have relatively low degree.

Preferential attachment was first observed in evolution by Yule [48], and it has served as an explanation of why scale-free networks exist in many natural and man-made systems [5, 6].

Barabási and Albert [10] proposed a popular model of generating scale-free networks by implementation a process of preferential attachment.

The algorithm for constructing a Barabási and Albert (BA) scale-free network is as follows:

1. The starting point is a network of m_0 nodes connecting one another. At each time step, one new node is added to the network and is connected to other m existing nodes, with $m < m_0$.

2. In choosing the nodes to which a new node connects, node i will be selected to connect with the new node with probability $P_i = k(i)/\sum k(j)$.

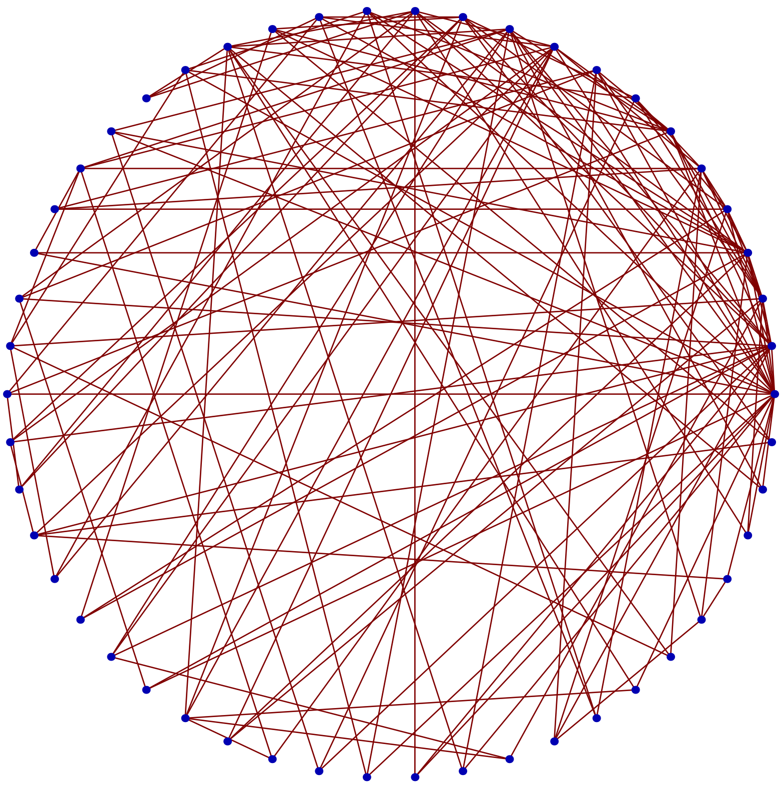


Figure 2.3: An example of BA scale-free networks with $N = 50$ and $m = 3$.

After t time steps, the network has $N = t+m_0$ nodes and mt links. Numerical simulations indicate that the degree distribution of the network follows a power-law with $\gamma = 3$, i.e., $P(k) \sim k^{-3}$.

Fig. 2.3 shows an example of BA scale-free networks with $N = 50$ and $m = 3$. In Figs. 2.4 and 2.5, we compare the degree distribution and degree histogram of ER random and BA scale-free networks. We observe that the BA scale-free and the ER random networks display power-law and Poisson degree distributions, respectively. Moreover, the BA scale-free network is much more heterogeneous than the ER random network.

The average shortest path distance of a BA scale-free network, denoted by $\tilde{D}_{BA}$, is given as follows [49]:

$$\tilde{D}_{BA} = \frac{\ln(N) - 1 - \alpha}{\ln\ln(N)} + \frac{3}{2} \tag{2.13}$$

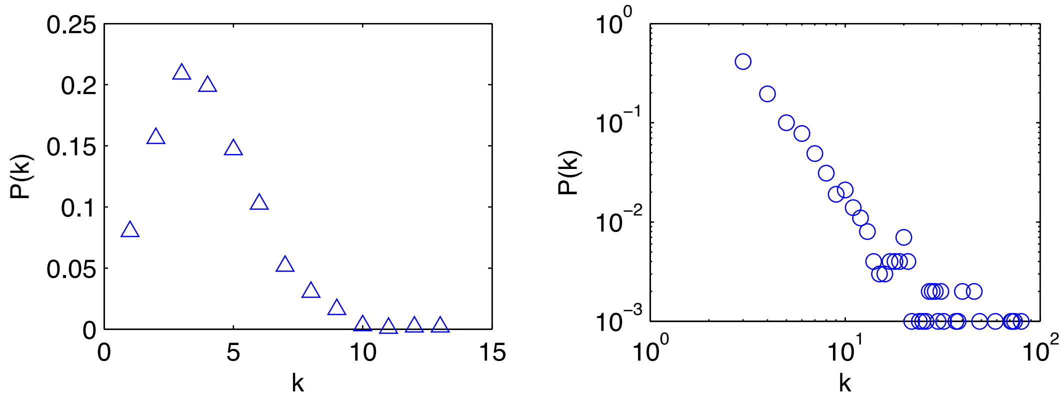
where α is the Euler-Mascheroni constant.

With the same total number of nodes and links, BA scale-free networks have a shorter average shortest path distance than ER random networks.

Cohen *et al.* [50, 51] have analytically demonstrated that scale-free networks are ultra-small worlds. Because of the existence of the hubs in the networks, the average shortest path distance becomes significantly smaller, and scales as

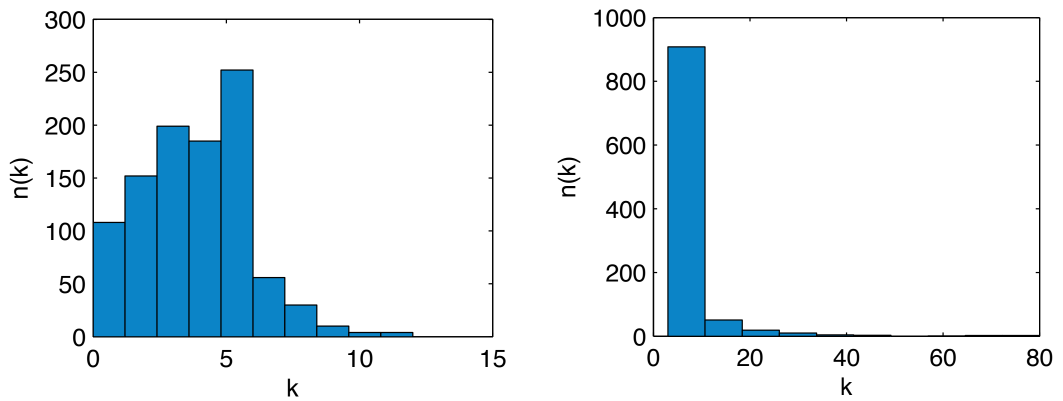
$$\tilde{D}_{BA} \propto \log\log N \tag{2.14}$$

A typical example of large scale-free networks is the Internet, which has been demonstrated to have hubs with extremely high node degree and exhibit the small-world effect. The existence of the hubs in the network can greatly shorten the average shortest path length, and the hubs usually carry much heavier traffic load than the rest of the network.



(a) (b)

Figure 2.4: Degree distribution for (a) an ER random network and (b) a BA scale-free network.



(a) (b)

Figure 2.5: Degree histogram for (a) an ER random network and (b) a BA scale-free network.

In the following years, a number of extensions and variations of the BA scale-free model have been proposed to make the model more flexible [52].

Networks generated by the BA scale-free model follow a power-law degree distribution with exponent close to 3. However, for real-world scale-free networks, the power-law exponent is typically in the range between 2 and 3.

Dorogovtsev *et al.* [17] have proposed a model which starts in a similar way as the BA scale-free model and new nodes are attached to the existing nodes with probability proportional to their degree. Then, they consider a constant χ, and add χ links between the nodes in the network with probability $p_{u,v} \propto k(u)k(v)$, if the link between nodes u and w does not yet exist. The degree distribution of this model follows:

$$P(k) \propto k^{-(2+\frac{1}{1+2\chi})} \tag{2.15}$$

As shown in (2.15), the power-law exponent of this model varies between 2 and 3.

Moreover, BA scale-free networks have a relatively low clustering coefficient, which is quite different from many real-world networks. For instance, the Internet topology at the AS level has been revealed to have the hierarchical clustering feature.

Therefore, another modification of the BA scale-free model was proposed by Holme *et al.* [53] which can construct networks with power-law degree distribution with exponent equal to 3, and different values of the clustering coefficient.

2.2.4 Other Network Models

Exponential Random Graph Models

Exponential random graph models are a family of probability models for the analysis of social and other networks, which assume that the structure of an observed network y can be explained by any statistics $s(y)$ depending on features of the observed network and its nodes [54, 55, 56, 57, 58].

For a random graph Y, the dependence between the dyadic variables can be described as

$$P(Y = y|\theta) = \frac{\exp(s(y)\theta)}{c(\theta)} \tag{2.16}$$

where θ is a vector of model parameters associated with $s(y)$ and $c(\theta)$ is a normalizing constant.

Many of the networks we observe in the real world exist only as one example of a large number of possible networks with similar topological characteristics. For example, there is only one Internet but its topology changes with time. Thus, we observe different structures of the Internet at different times and all examples of the Internet topology should have some basic properties in common.

Similar cases also exist in other kinds of real-world networks such as social networks and biological networks. Therefore, the exponential random graph model can be used for modeling an ensemble of all possible networks weighted on their similarity to the observed network.

Random Geometric Model

There are some real-world networks which grow under certain geometrical constraints, such as the road network in a city and the ant gallery network [59].

A random geometric network with N nodes can be generated as follows [60]:

1. Place N nodes randomly and independently in the region and each node i has coordinates (x_i, y_i).

2. Let r be the specified radius, and connect nodes u and v if, and only if, the distance between them is at most a threshold r, i.e., $d(u, v) = (x_u - x_v)^2 + (y_u - y_v)^2 \leq r^2$.

In a random geometric network, there exist a phase transition point, denoted by r_c, from a set of disconnected networks to a connected network, and the critical radius is given by [61]

$$r_c = \frac{\sqrt{\ln N + 1}}{\pi N} \tag{2.17}$$

Range Dependent Random Network Model

Inspired by the structure of biological networks, Grindrod has proposed a range dependent random network model [62]. In this model, the set of node index of a network with N nodes is $\{0, 1, \cdots, N-1\}$, and the nodes i and j are linked with probability $\beta\eta^{|j-i|-1}$, where $\beta > 0$ and $\eta \in (0, 1)$ are constants.

The average node degree and clustering coefficient of these networks, denoted by $\langle k \rangle_{RD}$ and C_{RD}, are given as

$$\langle k \rangle_{RD} = \frac{2\beta}{1-\eta} \tag{2.18}$$

$$C_{RD} = \frac{3\beta\eta}{(1+\eta)(1+3\eta)} \tag{2.19}$$

If $\beta = 1$, all nearest neighbors in the networks are connected and $C_{RD} \to 3/8$ as $\eta \to 1$.

With (2.18) and (2.19), the clustering coefficient can be written as

$$C_{RD} = \frac{3\langle k \rangle_{RD}(1-\eta)\eta}{2(1+\eta)(1+3\eta)} \tag{2.20}$$

This indicates that the clustering coefficient has a maximum value at $\eta = (\sqrt{8}-1)/7 \approx 0.261$, independent of the average node degree. Thus, the range dependent random network model can be used to generate large-scale sparse networks with high clustering coefficients.

2.3 Digital Communication Networks

2.3.1 Digital Transmission and the Internet

The advent of digital networked technologies in the past two decades has greatly facilitated the generation, transmission, processing and sharing of information among people in different parts of the world. The resulting highly connected community has played an important role in enhancing efficiency of many operations in commerce, business, government, education, and public services.

Digital transmission has proven to be an effective mode of communication, and one common way of transmitting digital information is to send packets from sources to destinations via specific routes in the network. Computer and communication networked systems are among the most important infrastructures of today's society.

One of the most famous and widely studied examples of communication networks is the Internet, in which computers or routers are abstracted as nodes and the physical data transmission connections as links. The history of the Internet dates back to the 1960s when packet switching networks such as ARPANET, CYCLADES, Merit Network, Tymnet and Telenet, were developed using a various communication protocols [63]. In particular, the ARPANET was the first operational packet-switching network to implement the Transmission Control Protocal/Internet Protocol (TCP/IP), leading to the development of protocols for internetworking [64].

In the following decades, in particular since the mid-1990s, the Internet continues to grow rapidly and has had a tremendous evolutionary impact on the way we live, work and study.

2.3.2 Empirical Study of the Internet

Nowadays, the Internet is a huge network of physical data connections between computers, routers and other related communication devices. In recent years, the structure of the Internet is measured and evaluated at router, subnet, domain, and autonomous system (AS) levels [65, 66, 67, 68, 69, 70, 71, 72, 73].

Router Level

At the router level, the routers are represented as nodes and the physical data transmission connections are represented as links. With a standard tool called *traceroute*, we can discover the path chosen by the packets to travel between the computers.

A pioneering work of the study of the topology of the Internet was due to Pansiot *et al.* [74], who analyzed the network database collected at the router level in 1995. Based on the collected data, it has been revealed that the degree distribution of nodes with less than 30 links is power-law [5]. However, in a larger network, the degree distribution for nodes with more than 30 links exhibits a faster cut-off than the power-law [75]. Therefore, the Weibull distribution has been used to better fit the collected data than the power-law distribution [76].

Moreover, analysis on data collected in 1999 shows that the Internet topology at router level does not have obvious hierarchical characteristics [77].

Subnet Level

A subnet is a group of IP addresses, each consisting of four numbers in the range from 0 to 255. A subnet in which the first three numbers of all the IP addresses are fixed is called a class C subnet. There are also class A and class B subnets which have all IP addresses with the first one and two numbers fixed, respectively.

As all IP addresses in a class C subnet usually belong to the same organization, it is reasonable to abstract a class C subnet as a node and place a link between two subnets if any routers in one subnet has a data transmission connection with any router in the other. Fig. 2.6 shows an example of the Internet map at the level of class C subnets [78, 79].

Domain Level

A domain is a group of computers, routers and other related devices belonging to a particular organization and identified by the same domain name. For example, the domain name for Hong Kong Polytechnic University is "polyu.edu.hk".

At the level of domain, a node in the network represents a domain, and a link is placed between two nodes if any router in one domain has a direct transmission connection with any router in the other. It has been shown that the Internet at the domain level has topological properties of small-world and scale-free [5].

Autonomous System (AS) Level

In the past decade, the Internet AS-level topology has been widely and extensively studied and widely used in a variety of research disciplines. An autonomous system (AS) is a group of computers, and routers under the control of one or more network operators that present a common and clearly defined routing policy to the Internet [80]. Within an autonomous system, packets are routed using an interior routing protocol called the Interior Gateway Protocol (IGP). While between different autonomous systems, the routing paths for the packets are calculated by an inter-domain routing protocol called the Border Gateway Protocol (BGP). To ensure efficient calculation of routing paths, BGP routers are aware of the entire paths to all destinations.

Each AS is assigned with a unique AS number by Internet Assigned Numbers Authority (IANA) [81]. The assigned AS numbers range from 0 to 65535. Most existing AS numbers have been assigned and the remaining numbers are reserved.

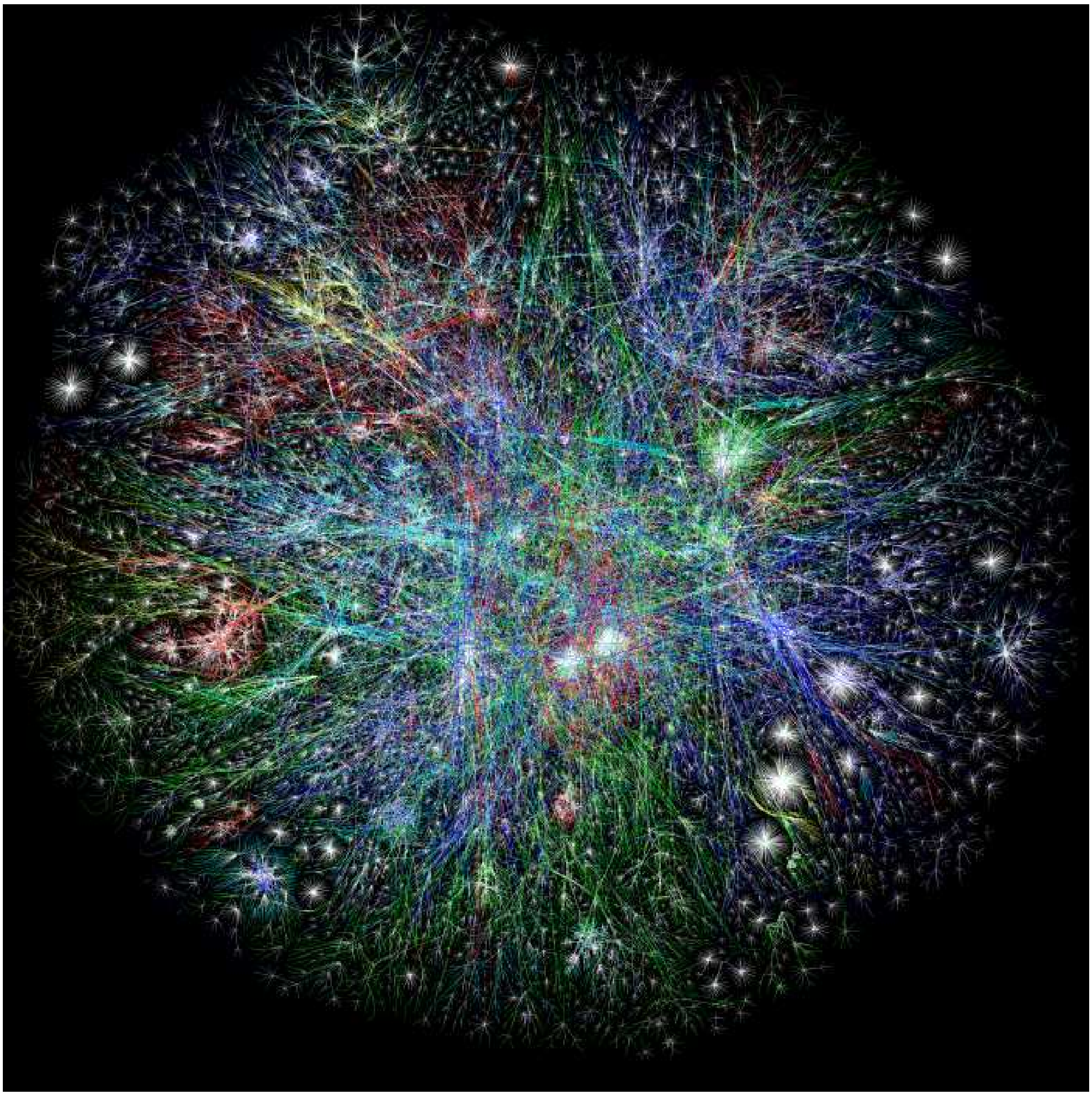


Figure 2.6: An example of the Internet map at the level of "class C subnets" on Nov. 22, 2003. The colors were based on class A allocation of IP space to different registrars in the world. Figure is obtained from the Opte Project (http://www.opte.org), under a Creative Commons License.

Date	2003-07-31	2008-07-31
Assigned AS numbers	1-30979 31810-33791 64512-65534	1-30979 30980-48127 64512-65534

Table 2.1: Assigned Autonomous System (AS) numbers.

As listed in Table 2.1, 33983 AS numbers were assigned in 2003 and in 2008 this number increased to 49149. As the BGP routers only need to know the routing path at the AS level, BGP routing tables contain a large number of routing paths which consist of sequences of the AS numbers.

Therefore, the Internet at the AS level can be constructed with the data derived from BGP routing tables. At the AS level, each AS is represented by a node and two ASs are linked if there is a BGP connection between them. The most widely used datasets of BGP routing tables are provided by Route Views and RIPE RIS [82, 83, 84]. For example, the project of Route Views uses 233 source computers all over the world to collect the BGP routing tables every two hours.

2.3.3 Topological Properties of the Internet

Using the structural data collected at router, subnet, domain, and autonomous system (AS) levels for the Internet, many intriguing topological features have been discovered.

Power-laws

Based on the structural data collected in the 1990s, power-laws have been observed in the Internet topology at router and domain levels [5, 85]. In the past decade, the analysis of the Internet topology mainly focused on the data collected at the AS level [27, 86, 87, 88]. It has been demonstrated that the structure of the Internet AS-level network can be characterized by the presence of various power-laws which consider the following relationships [5, 89, 90]:

1. $k(i) \propto r(i)^R$, where $k(i)$ is the out-degree of node i, $r(i)$ is the rank of node i, and R is the power-law exponent.

2. $p(k) \propto k^\gamma$, where $p(k)$ is the probability of finding a node with degree k, and γ is the power-law exponent.

3. $N(h) \propto h^H$, where h is a number of hops, $N(h)$ is the number of nodes within h hops, and H is the power-law exponent.

4. $\lambda(i) \propto i^\epsilon$, where $\lambda(i)$ is the eigenvalues of the network, i is the order of the eigenvalue, and ϵ is the power-law exponent.

Further analysis about power-laws has discovered that the power-law exponents have not changed substantially in spite of the growth of the network [88].

Based on a more complete AS-level network by merging more datasets, it has been observed that the network has heavy tailed degree distribution which deviates from the strict power-law [91]. It was recommended later that the power-law degree distribution should be a necessary but not sufficient condition for the Internet AS-level topology [90].

The Birth and Death of AS Nodes and Links

We define the "birth" of an AS node as an event when the AS node joins the Internet, and likewise "death" when it disappears and never appears again. When a new AS node joins the Internet, it connects to a certain number of existing nodes in the network via new links. New links also appear between the existing AS nodes. The appearance of new links is defined as the "birth" of the AS links. On the other hand, links between two AS nodes may disappear, which is defined as the "death" of the AS links.

Table 2.2: "Birth" and "death" numbers of AS nodes and links from November 1998 to November 2000

	Birth number	Death number
AS nodes	6696	1452
AS links	20253	10260

As shown in Table 2.2, the numbers of newly born AS nodes and links are much larger than that of dead AS nodes and links, which implies that the Internet grows rapidly.

In the real Internet, it has been observed that a newly born AS node usually only has one link to the existing AS nodes and the AS nodes with degree only 1 disappear with much higher probability than the others [92].

Therefore, the Internet is a rapidly evolving network with continuous "birth" and "death" of nodes.

Hierarchical Clustering

Another remarkable topological feature of the Internet is the hierarchical clustering. The Internet hierarchy at the AS level can be divided into international links, national backbones, and regional networks. Within each regional network, the AS nodes are tightly connected with a high clustering coefficient. The regional networks are then sparsely connected via national backbones and international links [93]. Based on the features of hierarchical clustering in the real Internet, Fan *et al.* have proposed a multi-local-world (MLW) model to mimic the Internet topology [92].

To capture the clustering feature of the Internet AS level network, spectral analysis of the normalized Laplacian matrix and the adjacent matrix has been employed to analyze the topological data from the Route Views and RIPE [94, 95, 96, 87]. It has been revealed that the clustering of the AS nodes and links had obvious changes from 2003 to 2008.

This shows that the Internet is a dynamical network which undergoes continuous re-organization of local connections among nodes.

Chapter 3

Analysis of Communication Network Performance

In the previous chapter, we have introduced the fundamentals of network science and empirical studies of real-world communication networks. In this chapter, we analyze the performance of communication networks from a network science perspective. Our analysis and simulation results reveal the effects of network structure, resource allocation and routing strategy on the performance of communication networks. Performance parameters, including packet drop rate, time delay, and critical generation rate, are considered. For efficient data transmission, the traffic load should be as uniformly distributed as possible in the network and the average distance between nodes should be short. We propose to use a new metric called *node usage probability*, which depends on the network topology and the routing strategy, to characterize the traffic load distribution. We show that resource allocation based on the node usage probability outperforms other uniform and degree-based allocation schemes. On the basis of the proposed analysis and routing algorithms, we compare the performances of regular networks, scale-free networks, random networks, and the Internet constructed at the autonomous system (AS) level. Results from this chapter provide important insights into the relationship between the structural properties of communication networks and their performances.

3.1 Overview

The statistical study of complex networks in the past decade has provided important insights into the way in which network topology affects the performance of communication networks [29, 97, 98, 99, 100, 101, 102, 103, 104, 105].

In this chapter we study the performance of communication networks from a network science perspective. In the physics research community, previous statistical study often assumes over-simplified communication models for the evaluation of performance without incorporating realistic network operational settings and performance parameters from the communication perspective [99, 100, 102, 103, 106, 107, 108, 109, 110]. On the other hand, researchers from the engineering community usually use very detailed and complex communication models which make it too difficult to provide useful general insights into the behavior of various realistic data networks [111, 112, 113, 114]. In our study here, we use a model that compromises simplicity and very specific modeling by incorporating realistic network operational settings in general network topologies.

Our task is to establish a clear link between some structural properties of networks such as degree distribution, average distance, and betweenness, with communication network performance, the purpose being to improve understanding of the various factors that affect the communication performance of networks and to provide design information for optimizing performance.

We focus on a generic type of communication networks, in which packets of messages are sent from one node to another under practical operational conditions such as the use of packet buffering in communication nodes and the implementation of specific routing algorithms. We analyze networks of selected topologies that are of practical relevance, including regular lattices, Erdös-Rényi random networks (ER random), and Barabási-Albert scale-free networks (BA scale-free), and the real-world Internet constructed at the autonomous system (AS) level, and investigate the performance of these networks in terms of their intended functions of delivering information. Performance parameters, including packet drop rate, time delay, and critical generation rate, are considered.

In Section 3.2, we explain the operation model of communication networks, and describe the three basic network structures of practical relevance, namely, *regular lattice*, *ER random* and *BA scale-free* networks. In Section 3.3, we adopt the mean-field theory to approximate the transition point from free-flow to congestion state for communication networks. We also introduce the node usage probability as a new essential parameter for network design. In Section 3.4, we present the performance comparison of the different network types in terms of critical generation rate, packet drop rate, and transmission time, under different routing algorithms and resource allocation schemes. In Section 3.5, we apply our analysis to the Internet constructed at the AS level, and evaluate its performance under different routing strategies and resource allocation schemes.

3.2 Communication Network Operation

In this chapter we focus on a generic type of communication networks [115], in which data or information is presented as packets and transmitted through connections in the network under specific routing algorithms.

3.2.1 Operation Model of Network Data Traffic

A communication network is assumed to have two kinds of nodes: routers and hosts. *Routers* can only store and forward packets. *Hosts* refer to nodes that can generate and receive packets, and they can also work as routers. The density of hosts ρ is the ratio of the number of hosts to the total number of nodes in the network, and in this chapter, we set $\rho = 0.2$ and randomly select hosts in the networks.

Packets are created by the hosts and sent through the links one hop at a time until they reach the destinations. Also, each node in the network has a buffer, the buffer size for node i being $B(i)$. Then, the data traffic operates as follows:

1. Packet Generation: At each time step, new packets are generated by hosts. The average number of generated packets by a host (node i) is λ_i, which is defined as the generation rate of node i.

2. Packet Transmission: The transmission capacity for node i is $R(i)$. At each time step, the first $R(i)$ packets of node i are forwarded toward their destinations by one step according to the routing algorithm.

3. Packets Dropped: If the total number of packets reaching one node is larger than its buffer $B(i)$, the outstanding packets are dropped or destroyed.

4. Packets Released: Packets already arrived at their destinations are released from the buffer.

3.2.2 Network Topology

Performance comparison is made here between three kinds of networks, namely, *regular lattice*, *ER random* and *BA scale-free* networks.

In some kinds of wireless communication networks, each node can only connect to other nodes close to it because of energy constraint. In this way, the network structure of these networks can be modeled by a regular lattice. In this chapter, a regular lattice with average degree k is generated as follows. We link each node i to k nodes with node index $i + j$ and $j \in \{-k/2, \cdots, -1, 1, \cdots, k/2\}$. For a network with N nodes, the set of node index is $\{0, 1, \cdots, N-1\}$. If $i+j > N-1$,

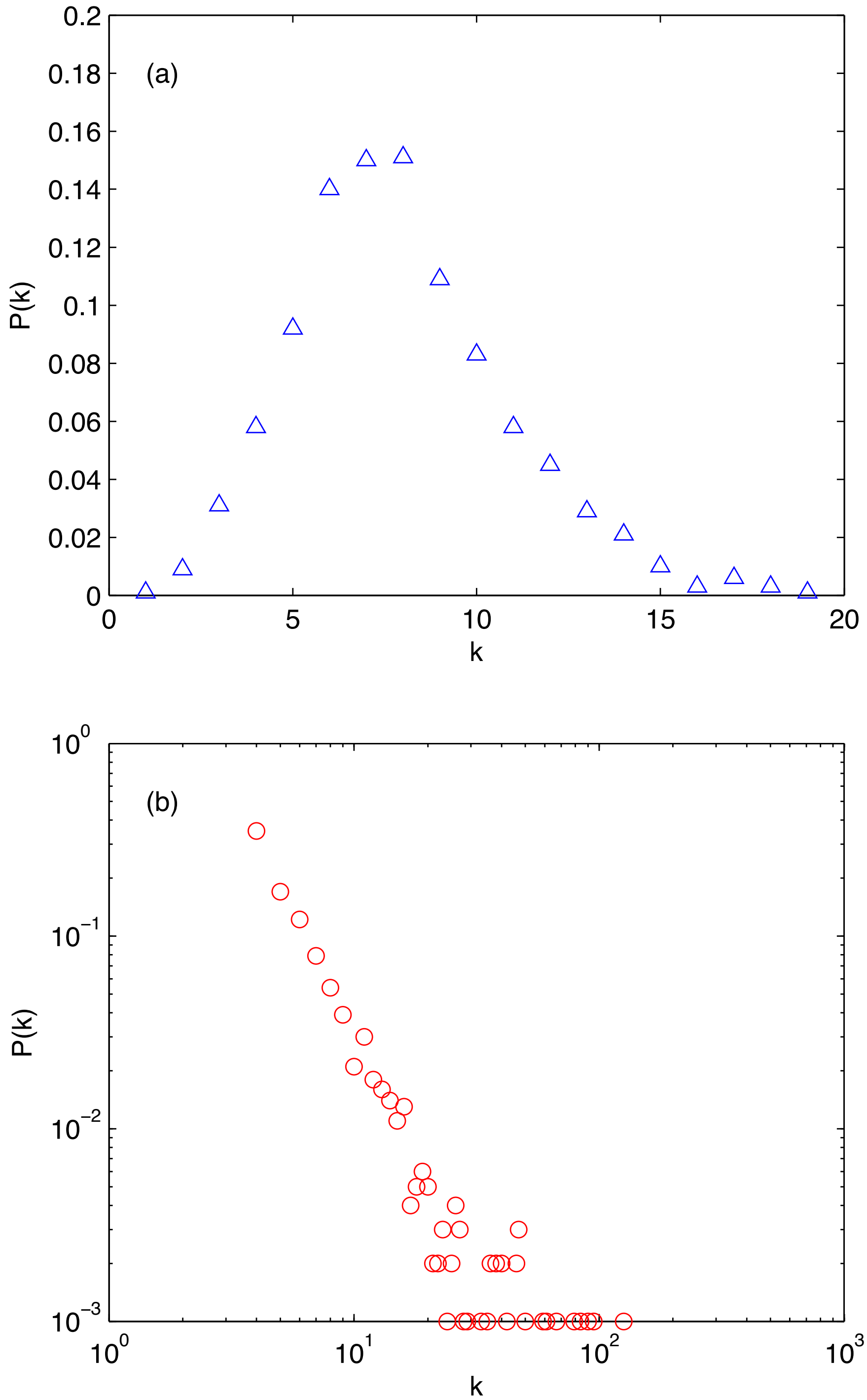


Figure 3.1: Degree distribution of (a) ER random network and (b) BA scale-free network

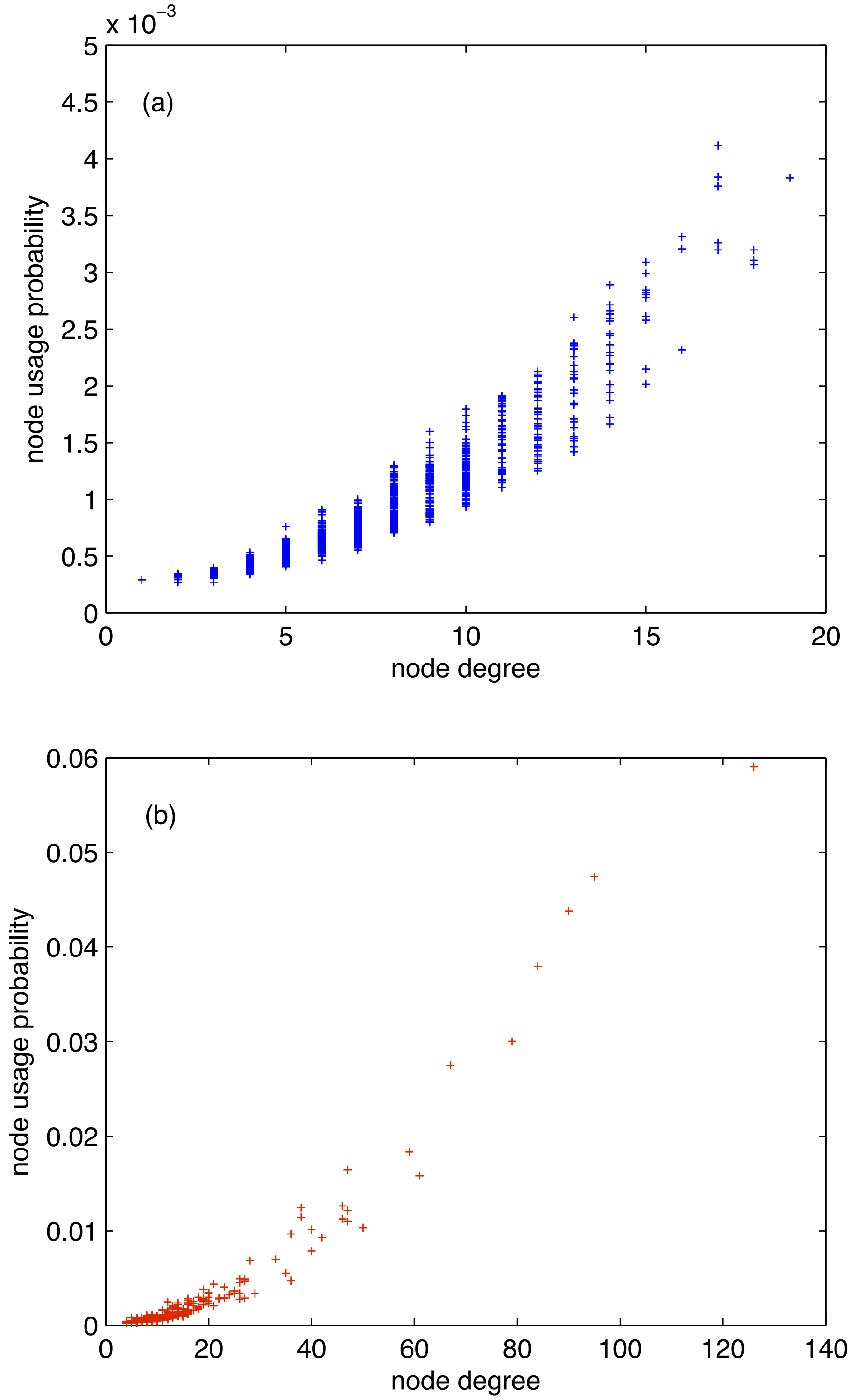


Figure 3.2: Node usage probability versus node degree of (a) ER random network and (b) BA scale-free network, under SP routing.

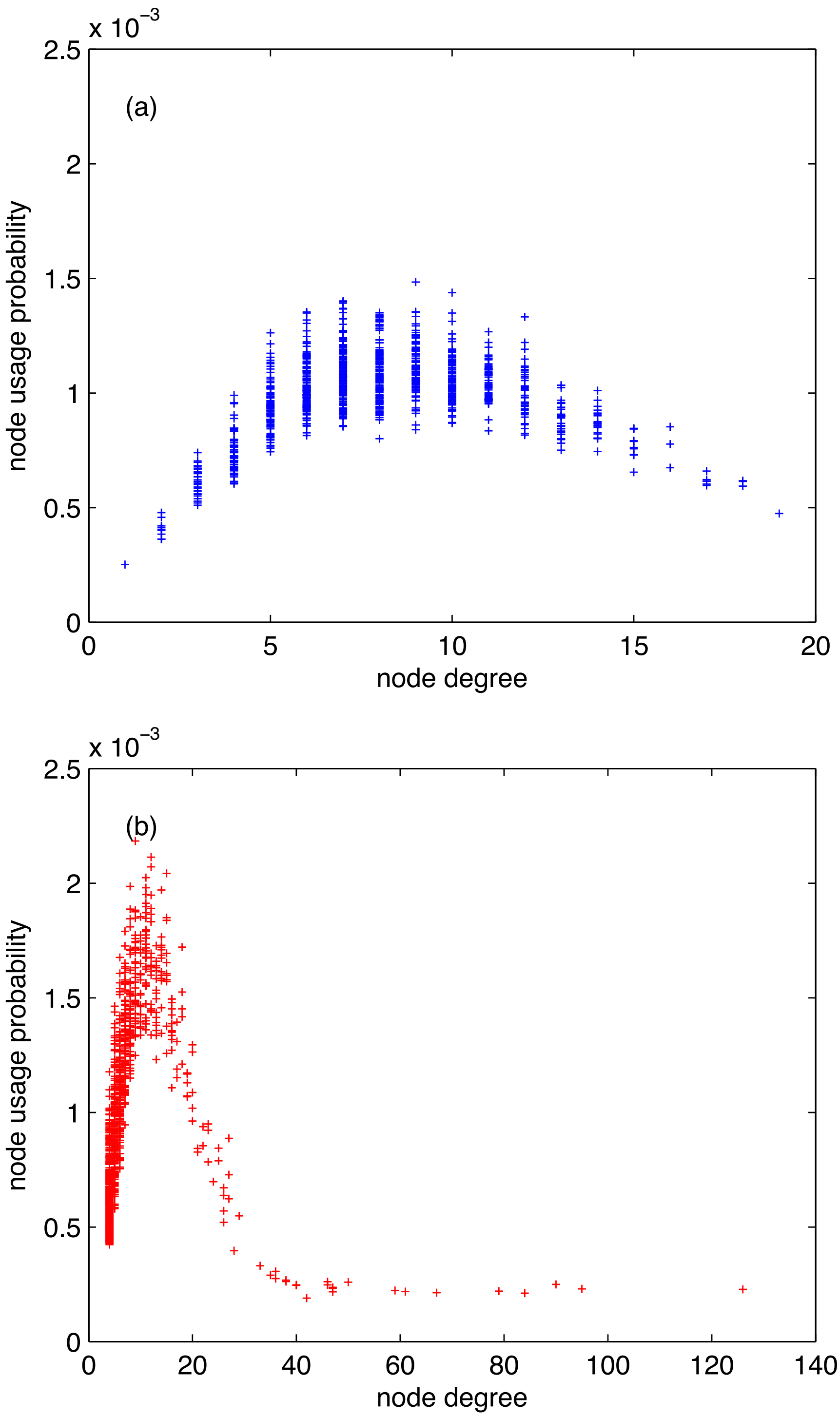


Figure 3.3: Node usage probability versus node degree of (a) ER random network and (b) BA scale-free network, under MD routing.

it is replaced by $i+j-N$, and if $i+j<0$, it is replaced by $i+j+N$. Therefore, in a regular network, all nodes have the same degree and such a network is totally deterministic.

As introduced in Chapter 2, a popular algorithm of generating random graphs was proposed by Erdös and Rényi [2] in 1959. The construction of a random network is as follows. In a network with N nodes, we connect each pair of nodes with a probability p. If N is large enough, the total number of connections in the network is a variable whose mean is $pN(N-1)/2$, and the degrees of the nodes follow a Poisson distribution, i.e., $P(k(i)) = \langle k \rangle^{k(i)} e^{-\langle k \rangle}/k(i)!$, where $k(i)$ is the degree of node i, and $\langle k \rangle = p(N-1) \approx pN$ is the mean value of $k(i)$. Since each pair of nodes are connected with equal probability, the random network is a homogeneous network in which most of the nodes' degrees are around pN.

However, prior work has shown that many real-world networks, including many communication networks, are heterogeneous networks with a power-law degree distribution [5, 6], i.e., $P(k(i)) \sim k(i)^{-\gamma}$, where γ is the characteristic exponent. Such networks are called *scale-free* networks in which a small number of nodes have a large number of connections and most other nodes have very few connections. To construct a scale-free network, we adopt the Barabási-Albert (BA) growth model here [10]. Numerical simulations indicate that the degree distribution of the network follows a power law with $\gamma = 3$, i.e., $P(k(i)) \sim k(i)^{-3}$.

3.2.3 Routing Algorithms

In this chapter, we study the performance of the network under two different routing algorithms, shortest path (SP) routing and minimized degree (MD) routing.

A shortest path refers to the path with minimum hops from the source to the destination. Here, we calculate all shortest paths for each pair of hosts. If there are more than one shortest paths, we randomly choose one.

Another routing algorithm we consider here is the minimimum degree (MD) routing algorithm introduced by Yan *et al.* [102]. Here, MD routing aims to minimize the sum of degrees of all nodes in the path.

3.3 Mean-Field Approximation

It has been shown in previous studies [116, 117, 118, 119, 120] that a data communications network may have a phase transition point from a *free-flow* state to a *congestion* state.

To ensure effective transmission, it is important to keep the network in the free-flow state, and here we use the *critical generation rate* λ_c, where the phase transition occurs from free-flow state to congestion state, as an indicator of the network *throughput*.

If $\lambda < \lambda_c$, the network reaches a steady state when the numbers of packets generated and successfully arrived are balanced. In this case, very few packets are dropped.

If $\lambda > \lambda_c$, traffic congestion occurs and packets accumulate in the nodes until those packets exceeding the buffer are dropped.

In order to estimate the critical point from free-flow state to congestion state, we adopt the mean-field approximation which has been used in previous study for theoretical estimation [107, 121, 122, 123].

3.3.1 Homogeneous Mean-Field Approximation for Regular Networks

For a regular lattice network, we adopt the homogeneous mean-field approximation which assumes that all nodes in the network are statistically equivalent, i.e.,

$$\frac{d}{dt}S(t) = \rho\lambda N - \frac{S(t)}{\tilde{\tau}(t)} \tag{3.1}$$

where $\rho\lambda N$ is the total number of newly generated packets at each time step, $S(t)$ is the total number of packets in transit at time step t, $\tilde{\tau}(t)$ is the average transmission time of successfully arrived packets, and $S(t)/\tilde{\tau}(t)$ is the number of arrived packets at time step t.

Below the critical point, Little's Law holds [124], i.e., $dS(t)/dt = 0$ and there exists a steady-state solution S^* for the total number of packets in transit. In the steady state, we have $d\tilde{\tau}(t)/dt = 0$ and steady-state solution $\tilde{\tau}^*$.

Therefore, the number of successfully arrived packets is equal to the number of newly generated packets, and the steady-state solution can be expressed as

$$\rho\lambda N = \frac{S^*}{\tilde{\tau}^*} \tag{3.2}$$

The average transmission time at steady state, $\tilde{\tau}^*$, can be approximated as the average distance $\tilde{D}$ from a packet's source to destination plus the average delay time a packet spends in the nodes' buffers,

$$\tilde{\tau}^* \approx \tilde{D}\left(1 + \frac{\tilde{S}^*}{R}\right) \tag{3.3}$$

where R is the average transmission capacity of each node, $\tilde{S}^*$ is the average packet number per node at steady state, and $\tilde{S}^* = S^*/N$. The average delay a packet spends in one node's buffer can be estimated as $\tilde{S}^*/R$, and on average a packet stays in the buffers of $\tilde{D}$ nodes.

From (3.2) and (3.3), the steady-state solution of the packet number in each node is found as

$$\tilde{S}^* = \frac{R\rho\lambda\tilde{D}}{R - \rho\lambda\tilde{D}} \tag{3.4}$$

In the regular network, each node has the same traffic load. We assume that at the critical point, each node has a full buffer. Therefore, the number of packets in each node when $\lambda = \lambda_c$, denoted by $S^*_{\lambda_c}$, can be approximated as

$$\tilde{S}^*_{\lambda_c} \approx B \tag{3.5}$$

where B is the buffer size of each node.

Therefore, the critical generation rate λ_c is given by

$$\lambda_c = \frac{BR}{\rho\tilde{D}(B+R)}. \tag{3.6}$$

3.3.2 Heterogeneous Mean-Field Approximation for Random and Scale-free Networks

The critical generation rate for the regular network can be well approximated by (3.6) because the traffic load for each node is about the same.

However, nodes in the random and scale-free networks have various degrees and varying importance. Therefore, some nodes are more likely to be chosen as routers and thus more prone to congestion. When these nodes get congested, the network gets congested too.

In previous study, researchers have used the concept of *betweenness,* which is defined as the number of shortest paths between any pair of nodes which go through a node to characterize the traffic load [43, 125, 126, 127].

In our study here, by taking both host distribution and different routing algorithms into consideration, we define a new parameter for network performance characterization, namely, **node usage probability** $U(i)$ for node i, as

$$U(i) = \frac{\sum\limits_{\substack{u,w \in H, \\ u \neq w \neq i}} \sigma_{uw}(i)}{\sum\limits_{j \in V} \sum\limits_{\substack{u,w \in H, \\ u \neq w \neq j}} \sigma_{uw}(j)}, \tag{3.7}$$

where H is the set of hosts in the network, V is the set of all nodes in the network, $\sigma_{uw}(i)$ is defined as 1 if node i lies on the path between nodes u and w under a specific routing algorithm, and as 0 otherwise.

Figs. 3.2 and 3.3 show that node usage probability $U(i)$ is related to the degree $k(i)$ in both random and scale-free networks.

Thus, we apply the heterogeneous mean-field approximation here as a basis of our theoretical analysis. The heterogeneous mean-field approach is based on the assumption that all nodes with the same node degree k are statistically equivalent [105]. By grouping the nodes in degree classes, the dynamical processes of random and scale-free networks can be analyzed by mean-field equations for each degree class.

The average number of packets in transit at nodes with degree k at time step t can be expressed as

$$\tilde{S}_k(t) = \frac{1}{N_k} \sum_{i \in V_k} S_i(t) \tag{3.8}$$

where N_k is the number of nodes with degree k, V_k is the set of nodes with degree k, $S_i(t)$ is the number of packets in node i at time step t and the summation denotes the total number of packets that exist in nodes with degree equal to k. Therefore, $\tilde{S}_k$ represents the average number of packets in transit of nodes with degree k.

The average node usage probability for a node with degree k, denoted by $\tilde{U}_k$, is

$$\tilde{U}_k = \frac{1}{N_k} \sum_{i \in V_k} U(i), \tag{3.9}$$

and the steady-state solution of the packet number in a node with degree k, denoted by $\tilde{S}_k^*$, can be approximated as

$$\tilde{S}_k^* \approx \frac{R_k \rho \lambda \tilde{D} \tilde{U}_k N}{R_k - \rho \lambda \tilde{D} \tilde{U}_k N} \tag{3.10}$$

where N is the total number of nodes in the network, and R_k is the transmission capacity of a node with degree k.

We define λ_k as the critical generation rate of the nodes with degree k, and when $\lambda = \lambda_k$, the buffers of the nodes with degree k are almost full,

$$\tilde{S}_k^* \approx B_k \tag{3.11}$$

where B_k is the buffer size allocated to a node with degree k.

Therefore,

$$\lambda_k = \frac{B_k R_k}{\rho \tilde{D} \tilde{U}_k N (B_k + R_k)} \tag{3.12}$$

As the network throughput is limited by the nodes with minimum λ_k, the critical point from free-flow to congestion, λ_c, should be equal to the minimum λ_k, i.e.,

$$\lambda_c = \min_{k \in K} \frac{B_k R_k}{\rho \tilde{D} \tilde{U}_k N (B_k + R_k)}, \tag{3.13}$$

where K is the set of all possible degree values.

If each node in the network has the same buffer size and transmission capacity, the nodes with highest node usage probability will be the first to get congested, and the critical generation rate λ_c can be simplified as

$$\lambda_c = \frac{BR}{\rho \tilde{D} \tilde{U}_{k,\max} N (B + R)} \tag{3.14}$$

where B and R are the buffer size and transmission capacity of each node, respectively, and $\tilde{U}_{k,\max}$ is the maximum value of $\tilde{U}_k$.

Equation (3.14) is also valid for the regular network, in which each node has the same node usage probability,

$$U_i = \frac{1}{N} \tag{3.15}$$

and the critical generation rate λ_c turns out to be the same as that given in (3.6).

3.4 Comparison of Network Structures, Resource Allocation, Routing and Performance

3.4.1 Network Properties

Using the network models described in Section 3.2.2, we build regular lattice, ER random and BA scale-free networks.

To do the comparison fairly, for these three networks, the number of nodes $N = 1000$ and the average degree $\langle k \rangle \approx 8$. For each network type, we build 10 networks and do 50 simulations for each network. However, as shown in Table 3.1, the maximum node degree of a scale-free network is much larger than that of regular and random networks.

For the regular lattice, each node has the same node usage probability of 0.001. For the ER random and BA scale-free networks, as shown in Figs. 3.2 and 3.3, the node usage probability is influenced by both node degree and network routing strategy. From Fig. 3.2, we observe that under SP routing, high degree nodes tend to have a higher node usage probability. In particular, for the BA scale-free network (see Fig. 3.2(b)), few high-degree nodes have much higher node usage probability than the rest of the network under shortest path routing. Fig. 3.3 shows that, under MD routing, both the highest and lowest degree nodes have relatively lower node usage probability.

Table 3.1: Node number N, average node degree $\langle k \rangle$, and maximum node degree $k_{\max}$ of regular lattice, ER random, and BA scale-free networks.

network	regular	ER random	BA scale-free
N	1000	1000	1000
$\langle k \rangle$	8	7.95	7.98
$k_{\max}$	8	19	126

Table 3.2: Maximum node usage probability $U_{\max}$ of regular lattice, ER random, and BA scale-free networks, under SP and MD routing.

$U_{\max}$	regular	ER random	BA scale-free
SP routing	0.001	0.0041	0.0596
MD routing	0.001	0.0015	0.0022

According to (3.6) and (3.14), for efficient data transmission, the maximum node usage probability should be small and the average distance should be short.

Table 3.2 indicates that under SP routing, the maximum value of node usage probability of the BA scale-free network is much larger than that of the regular and random networks. MD routing can make the traffic load more uniformly distributed and reduce the maximum node usage probability, especially for BA scale-free networks.

Table 3.3 shows that the average distance $\tilde{D}$ of a regular lattice is much longer than the ER random and BA scale-free networks. For the ER random and BA scale-free networks, SP routing brings shorter $\tilde{D}$ than MD routing.

Table 3.3: Average distance $\tilde{D}$ of regular lattice, ER random, and BA scale-free networks, under SP and MD routing.

$\tilde{D}$	regular	ER random	BA scale-free
SP routing	62.5	3.55	3.14
MD routing	62.5	3.74	4.47

3.4.2 Resource Allocation

Here we consider two kinds of resources, namely, transmission capacity and buffer size. For a fair comparison, we keep the total transmission capacity and buffer size of all nodes the same in all simulations, i.e., $R_{\text{all}} = 5000$ packets and $B_{\text{all}} = 20000$ packets.

With the same total resource, we compare the traffic performance under three resource allocation schemes, namely, uniform, degree-based, and node usage probability-based ($\tilde{U}_k$-based).

Under the degree-based resource allocation scheme, we allocate the transmission capacity $R(i)$ for node i based on its degree $k(i)$, using the following formula:

$$R(i) = \frac{(k(i))^\alpha}{\sum_{i=1}^{N}(k(i))^\alpha} R_{\text{all}}, \tag{3.16}$$

where R_{all} is the total transmission capacity of all nodes, and $k(i)$ is the degree of node i.

Similarly, the buffer size of node i, $B(i)$, is given by

$$B(i) = \frac{(k(i))^\beta}{\sum_{i=1}^{N}(k(i))^\beta} B_{\text{all}}, \tag{3.17}$$

where B_{all} is the total buffer size of all nodes, and $k(i)$ is the degree of node i.

For the $\tilde{U}_k$-based scheme, for the nodes with degree k, their transmission capacity, R_k, and buffer size, B_k, are allocated according to their respective average node usage probability $\tilde{U}_k$, i.e.,

$$R_k = \tilde{U}_k R_{\text{all}} \tag{3.18}$$

$$B_k = \tilde{U}_k B_{\text{all}} \tag{3.19}$$

It should be noted that for the $\tilde{U}_k$-based scheme, α and β no longer come into play as the optimal performance should correspond to $\alpha = \beta = 1$, which has also been verified by our simulations.

For the regular lattice, as each node has the same degree and node usage probability, the resource allocation is not influenced by the choice of the allocation schemes. While for the ER random and BA scale-free networks, under the degree-based scheme (when $\alpha, \beta > 0$) or $\tilde{U}_k$-based scheme, the nodes with higher degrees or node usage probabilities will have larger transmission capacities and buffer sizes.

3.4.3 Performance Indicators

To evaluate the communication performance, besides the *critical generation rate* λ_c described in Section 3.3, we consider two other performance parameters, namely, *average packet drop rate* and *average transmission time*.

We define the *average packet drop rate*, denoted by $\tilde{P}_d$, as

$$\tilde{P}_d = \frac{\tilde{S}_d}{\tilde{S}_g} \tag{3.20}$$

where $\tilde{S}_d$ is the average number of dropped packets per time step, and $\tilde{S}_g$ is the average number of generated packets per time step.

The *average transmission time*, denoted by $\tilde{\tau}$, is the average number of time steps a successfully arrived packet takes to arrive at the destination from the source.

In our model, each node has a limited buffer size. When $\lambda < \lambda_c$, the network works in free-flow state, and only a few packets are dropped because of the random nature of the simulation $\tilde{P}_d \approx 0$. When $\lambda > \lambda_c$, the packets accumulate in the nodes until the steady state is reached. At this state, a fraction of nodes are dropped, i.e., $\tilde{P}_d > 0$. In this model, a larger drop rate or longer transmission time means a higher congestion level in the network.

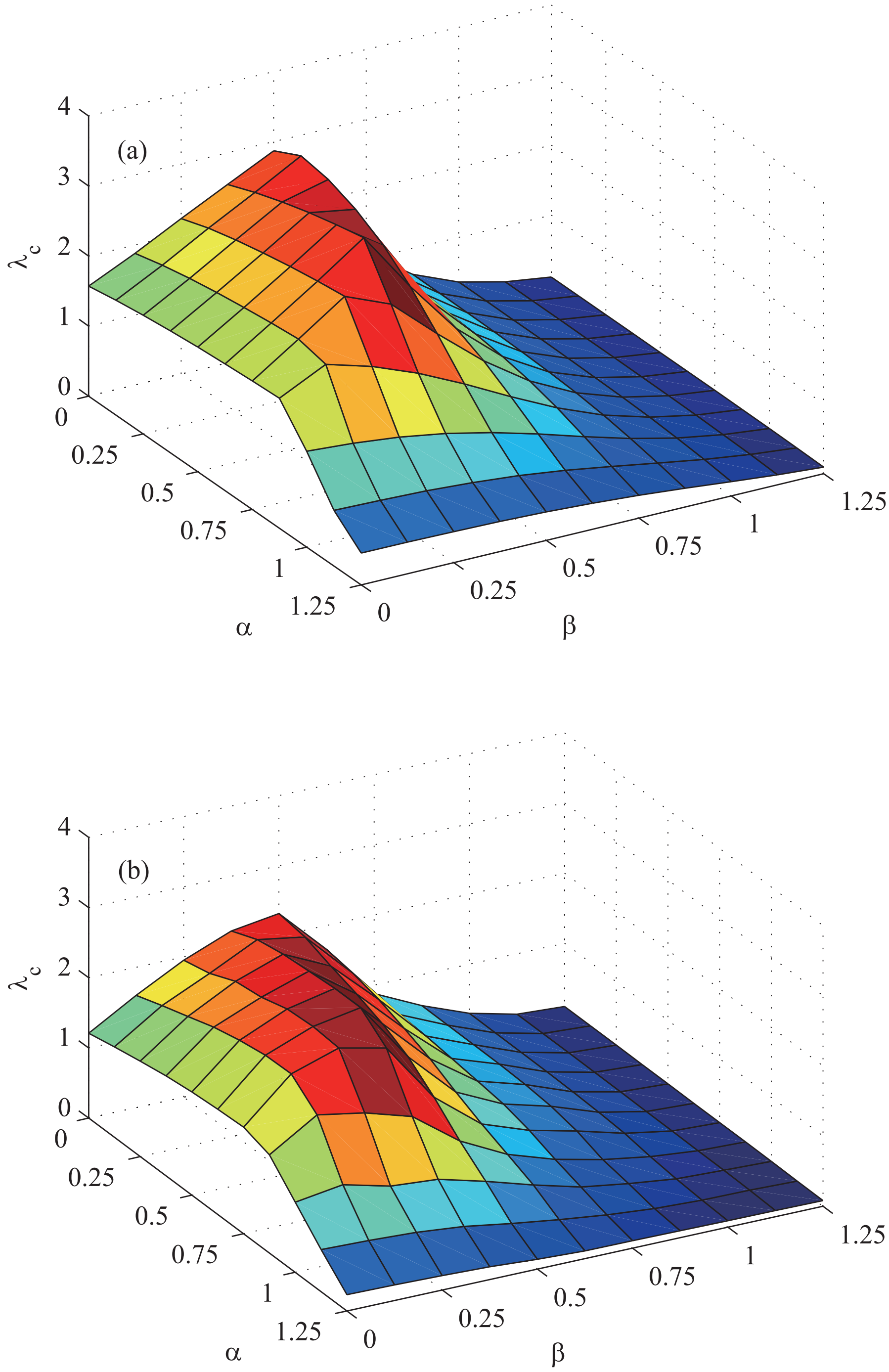


Figure 3.4: Critical generation rate λ_c of ER random network under SP routing versus α and β: (a) estimation (b) simulation.

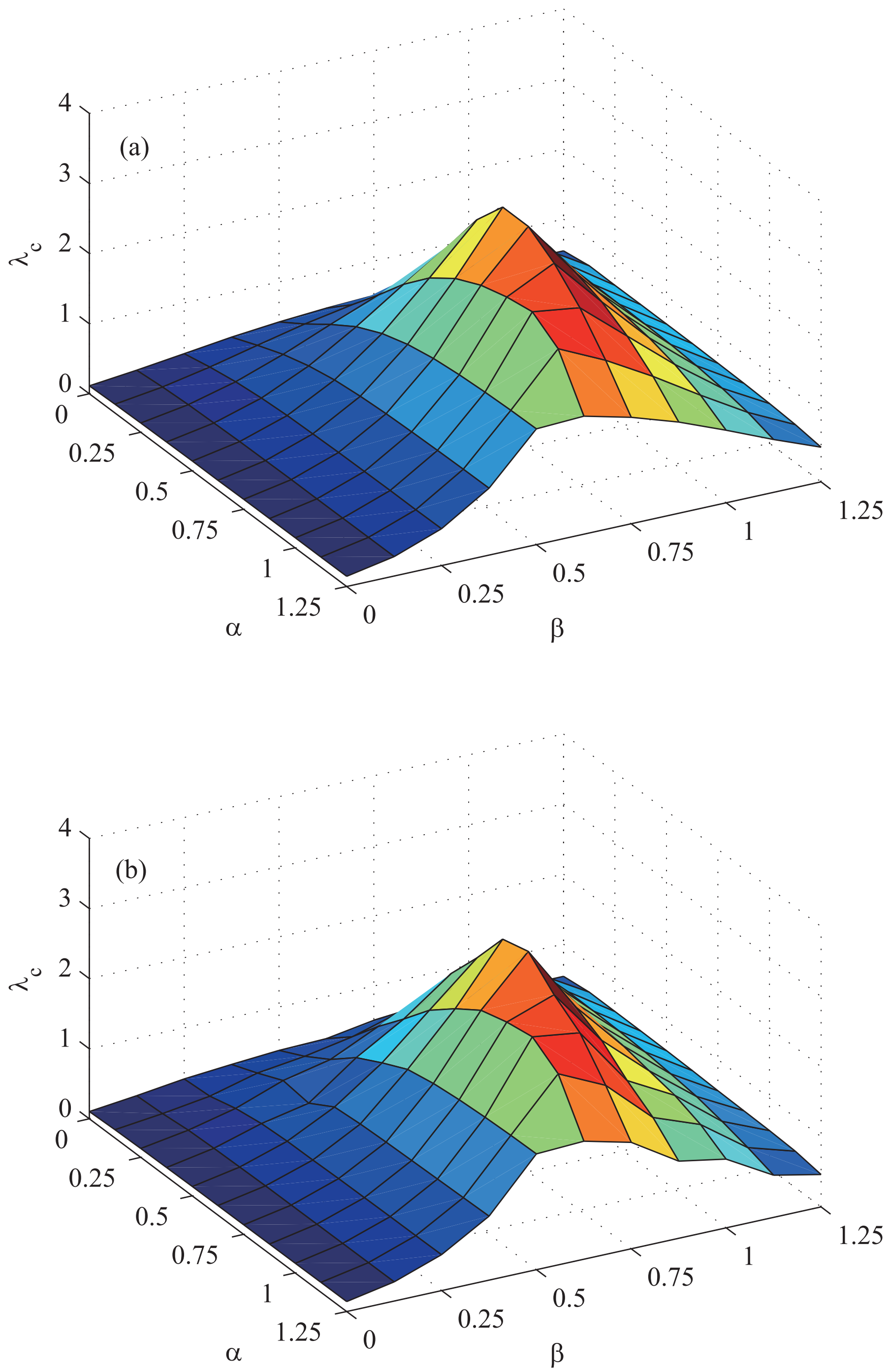


Figure 3.5: Critical generation rate λ_c of BA scale-free network under SP routing versus α and β: (a) estimation (b) simulation.

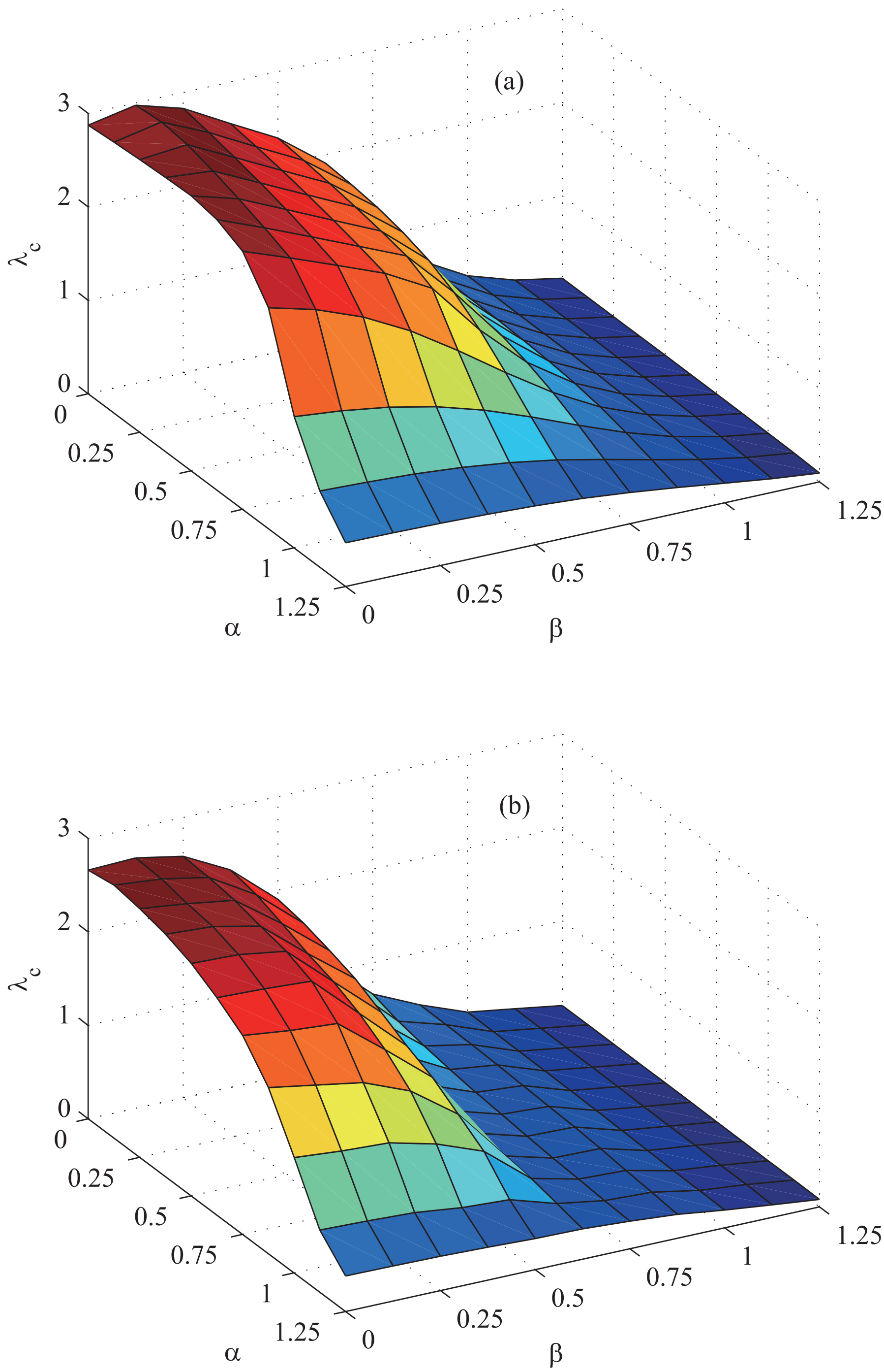


Figure 3.6: Critical generation rate λ_c of ER random network under MD routing versus α and β: (a) estimation (b) simulation.

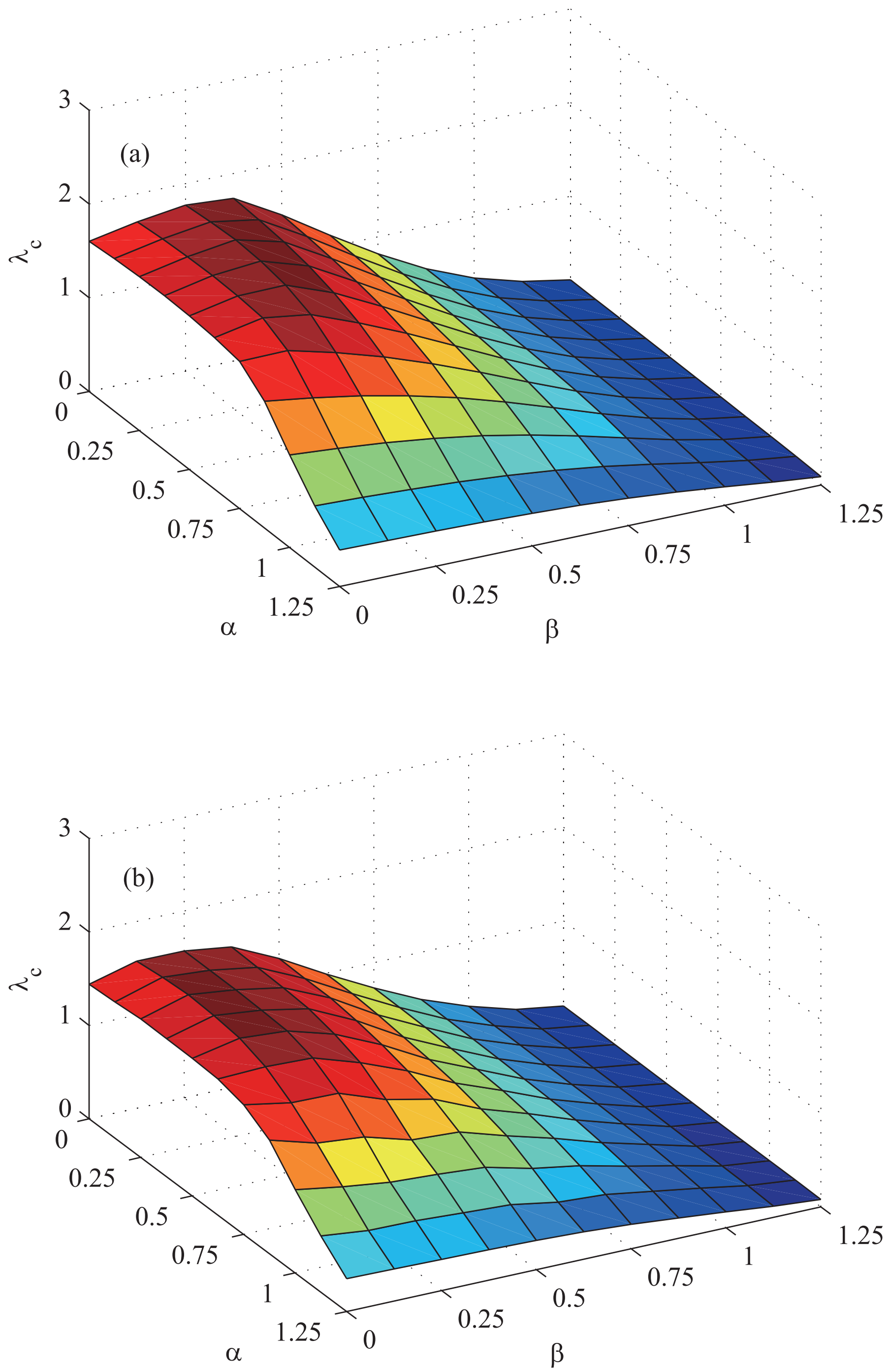


Figure 3.7: Critical generation rate λ_c of BA scale-free network under MD routing versus α and β: (a) estimation (b) simulation.

Table 3.4: Critical generation rate using uniform node resource allocation, $\lambda_{c,u}$, from theoretical estimation and from simulations (in brackets).

λ_c	regular	ER random	BA scale-free
SP routing	0.320 (0.304)	1.572 (1.211)	0.110 (0.103)
MD routing	0.320 (0.304)	2.873 (2.66)	1.602 (1.44)

Table 3.5: Peak value of critical generation rate of networks using the degree-based resource allocation, $\lambda_{c,k}$, from theoretical estimation and from simulations (in brackets).

$\lambda_{c,k}$	ER random	BA scale-free
SP routing	3.135 (2.512)	3.143 (3.110)
MD routing	3.002 (2.708)	1.819 (1.629)

Table 3.6: α, β values at optimal critical generation rate using the degree-based resource allocation, from theoretical estimation and from simulations (in brackets).

α, β	ER random	BA scale-free
SP routing	0.625, 0.375 (0.5, 0.375)	0.75, 0.75 (0.75, 0.75)
MD routing	0.125, 0.125 (0.125,0)	0.25 0.25 (0.25 0.125)

Table 3.7: Critical generation rate using $\tilde{U}_k$-based resource allocation, $\lambda_{c,\tilde{U}_k}$, from theoretical estimation and from simulations (in brackets).

$\lambda_{c,\tilde{U}_k}$	ER random	BA scale-free
SP routing	4.373 (3.936)	5.015 (4.853)
MD routing	4.052 (3.233)	3.300 (2.815)

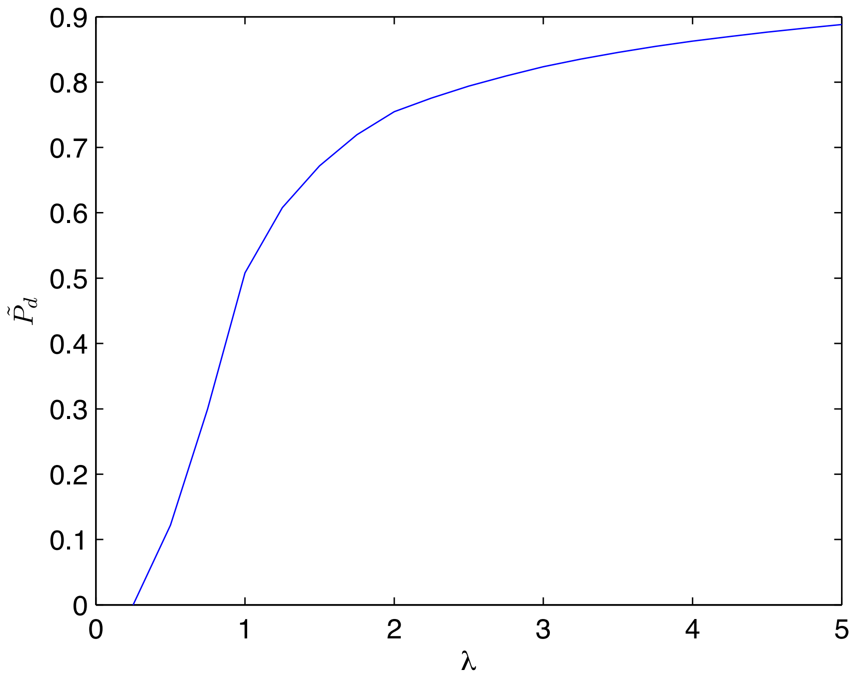


Figure 3.8: Average drop rate $\tilde{P}_d$ versus λ for regular lattice network

3.4.4 Network Performance

Table 3.4 compares the critical generation rate λ_c from theoretical estimations and from simulations for the three networks with uniformly allocated node resource, under SP and MD routings. The estimation results are obtained by (3.6) for the regular network and (3.14) for the random and scale-free networks.

Under SP routing and uniform resource allocation, the ER random network performs much better than the regular lattice and the BA scale-free network. For the BA scale-free network, the highest degree nodes have a much higher probability to be chosen as a router, making them very vulnerable to congestion and thus limiting the throughput of the whole network. While for the regular network, the reason for poor performance is the very long average distance. As shown in (3.6) and (3.14), $\lambda_c \propto 1/\tilde{D}$.

Moreover, MD routing generally gives larger λ_c for both ER random and BA scale-free networks by making the traffic load distribution more even. In

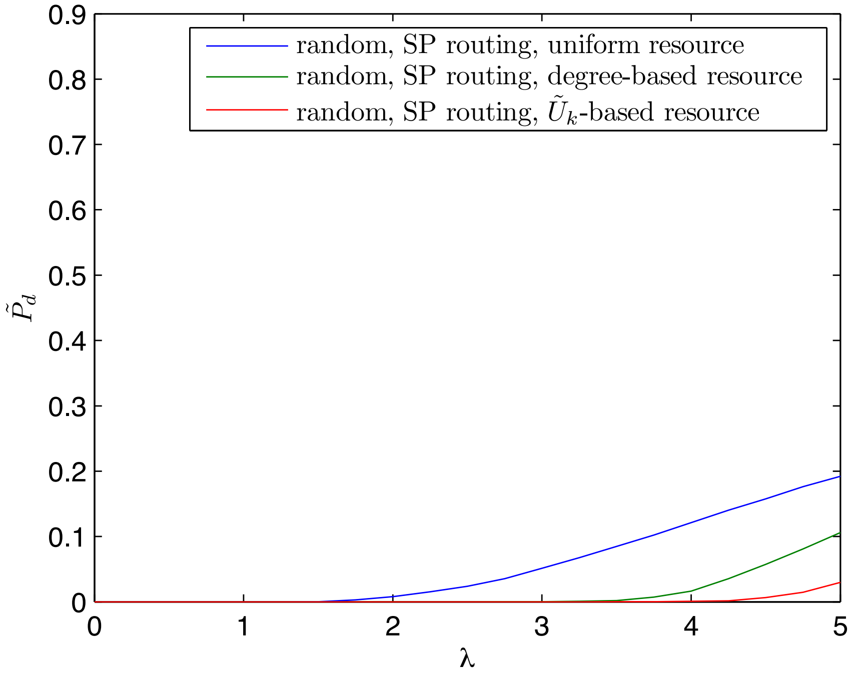


Figure 3.9: Average drop rate $\tilde{P}_d$ versus λ for ER random network, under SP routing

particular, for BA scale-free networks, λ_c is increased by more than 10 times.

Next, we study the network performance under the degree-based scheme. Figs. 3.4, 3.5, 3.6, and 3.7 show the variation of λ_c in ER random and BA scale-free networks with α and β varying from 0 to 1.25 (precision = 0.25). We observe that under SP routing, the α, β exponents for the peak value of λ_c are much larger than those of the MD routing. Moreover, under SP routing, with optimal exponent for the degree-based resource allocation scheme, the BA scale-free network can have better performance in terms of λ_c than the ER random network. As shown in Table 3.5, if the network resource is degree-based allocated using the optimal α and β (see Table 3.6), λ_c can be obviously improved.

Table 3.7 compares the critical generation rate λ_c of the ER random and BA scale-free networks using the $\tilde{U}_k$-based allocated resource, under SP and MD routings. We observe that SP routing excels over MD routing in terms of λ_c for both ER random and BA scale-free networks.

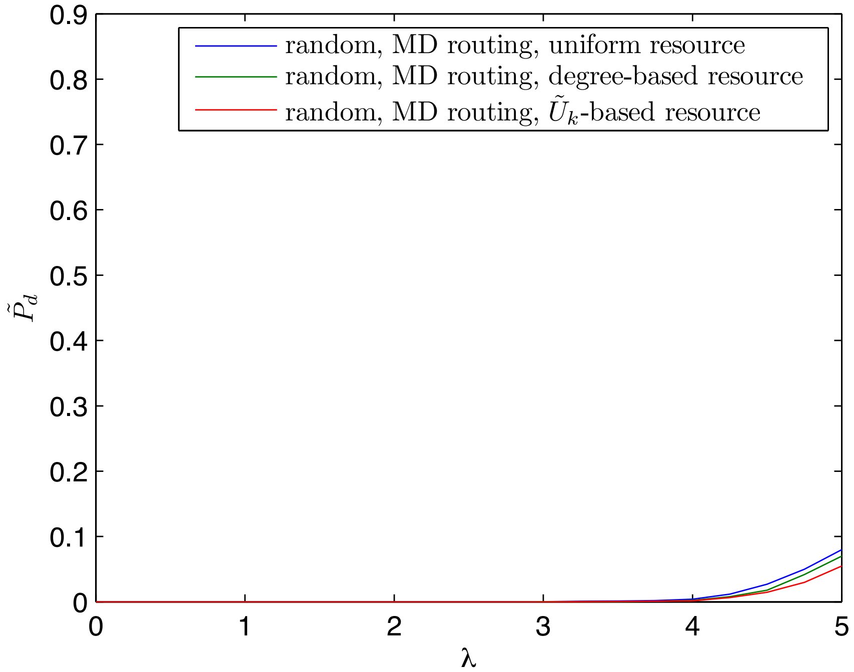


Figure 3.10: Average drop rate $\tilde{P}_d$ versus λ for ER random network, under MD routing

As shown in Tables 3.4, 3.5 and 3.7, the $\tilde{U}_k$-based resource allocation gives the best performance.

From Figs. 3.8, 3.9, and 3.11, we observe that, under SP routing and uniform resource allocation, the average packet drop rate $\tilde{P}_d$ of the regular lattice and BA scale-free networks is much larger than the ER random network.

We refer to Figs. 3.9, 3.10, 3.11 and 3.12. When each node has the same resource, for ER random and BA scale-free networks, MD routing performs better than SP routing in terms of $\tilde{P}_d$. Moreover, if the network resource is degree-based allocated using the optimal α and β observed in Figs. 3.4, 3.5, 3.6, and 3.7, $\tilde{P}_d$ can be obviously reduced, compared with the case where all nodes have the same resource. When resource is assigned based on the node usage probability, $\tilde{P}_d$ is further reduced and SP routing brings better performance than MD routing for both BA scale-free and ER random networks.

As shown in Figs. 3.13 to 3.17, the regular lattice has a much longer average

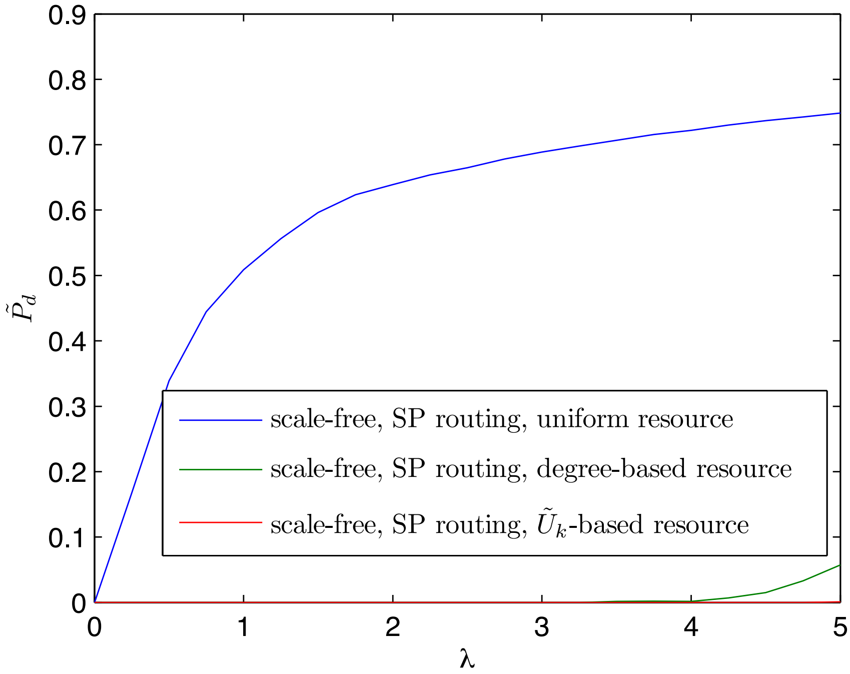


Figure 3.11: Average drop rate $\tilde{P}_d$ versus λ for BA scale-free network, under SP routing

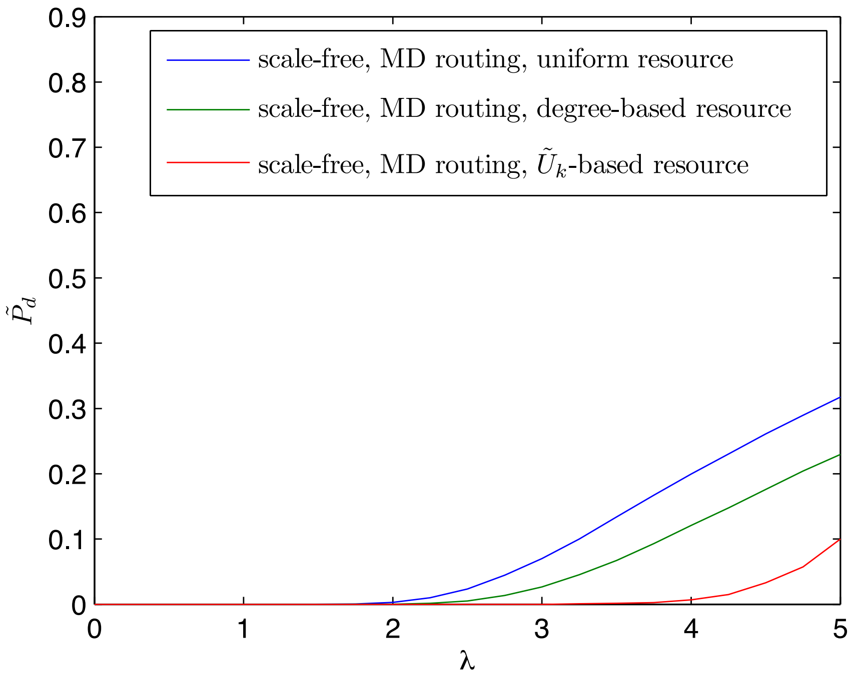


Figure 3.12: Average drop rate $\tilde{P}_d$ versus λ for BA scale-free network, under MD routing.

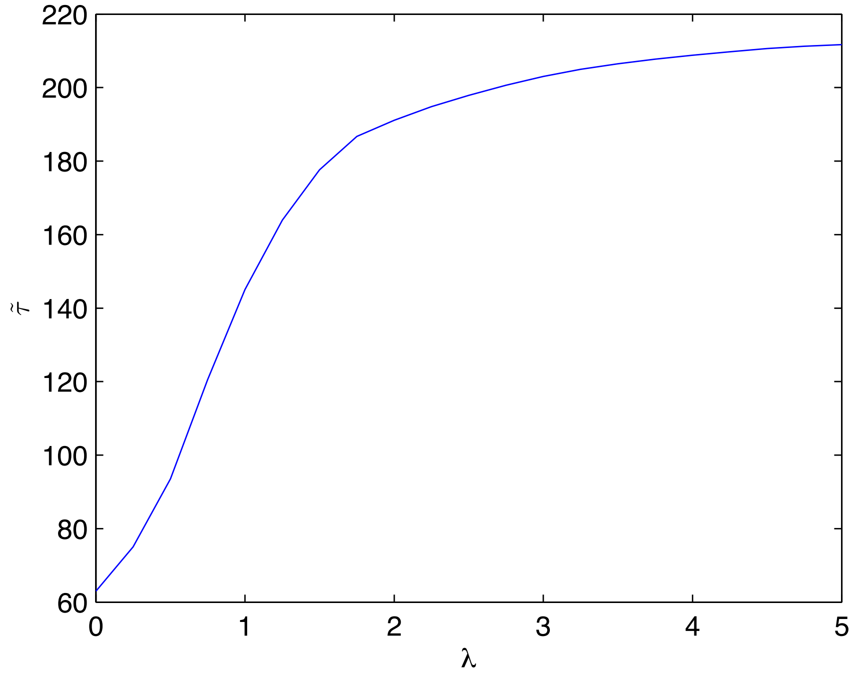


Figure 3.13: Average transmission time $\tilde{\tau}$ versus λ for regular lattice network.

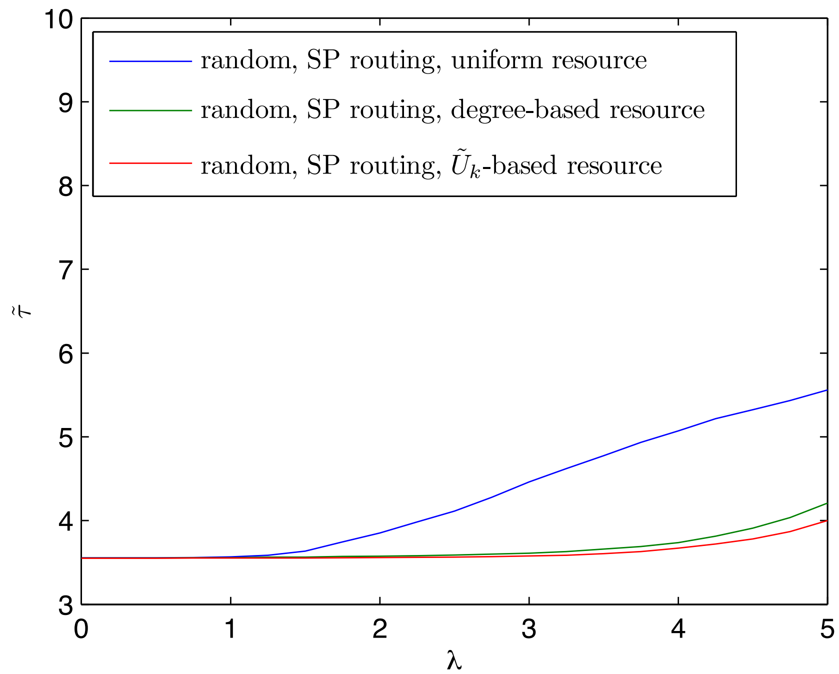


Figure 3.14: Average transmission time $\tilde{\tau}$ versus λ for ER random network, under SP routing.

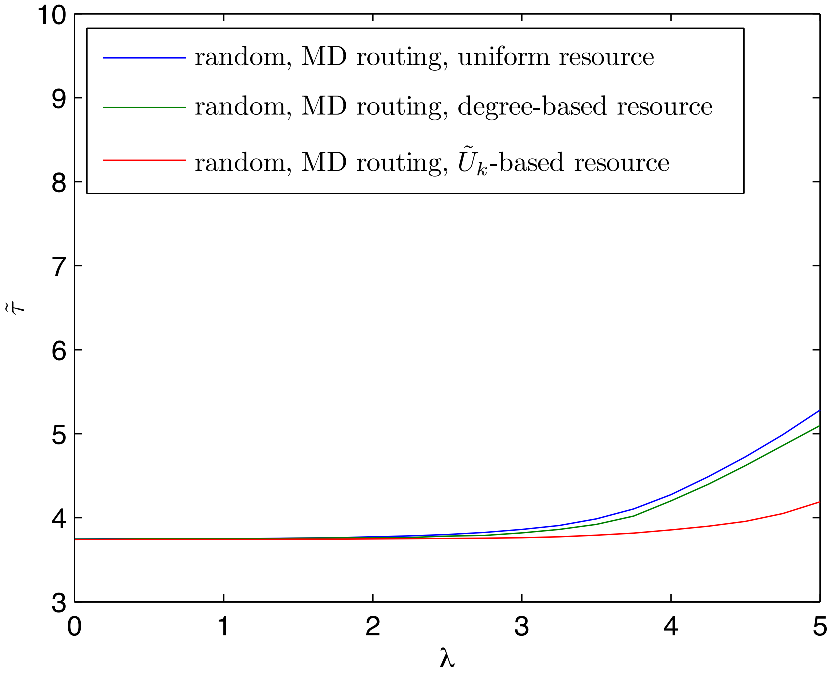


Figure 3.15: Average transmission time $\tilde{\tau}$ versus λ for ER random network, under MD routing.

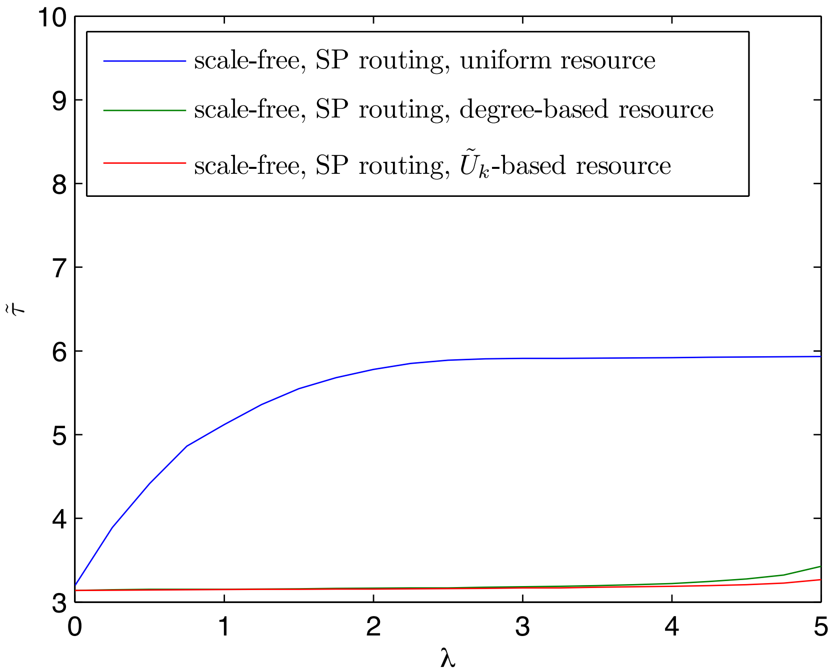


Figure 3.16: Average transmission time $\tilde{\tau}$ versus λ for BA scale-free network, under SP routing.

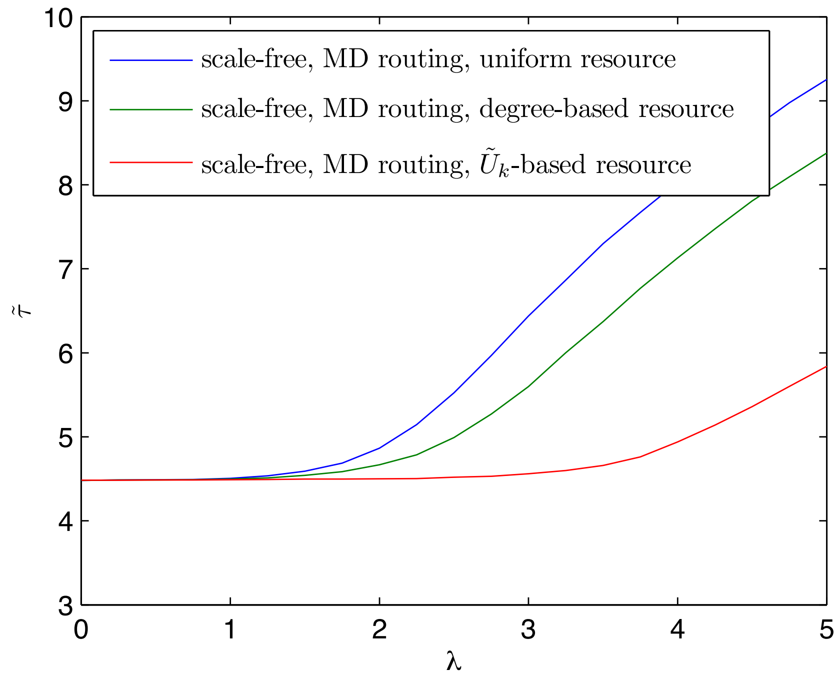


Figure 3.17: Average transmission time $\tilde{\tau}$ versus λ for BA scale-free network, under MD routing.

transmission time $\tilde{\tau}$ than the ER random and BA scale-free networks in both free-flow ($\lambda < \lambda_c$) and congestion states ($\lambda > \lambda_c$). This is because the average distance $\tilde{D}$ of the regular network is much longer.

For the ER random network, as shown in Figs. 3.14 and 3.15, when resource is uniformly distributed, SP routing performs better under a low traffic load condition, while MD routing has a shorter $\tilde{\tau}$ when the traffic becomes heavier. And if resource allocation is degree-based (using the optimal α and β) or $\tilde{U}_k$-based distributed, SP routing always performs better than the MD routing in terms of $\tilde{\tau}$.

For the BA scale-free network, as shown in Figs. 3.16 and 3.17, SP routing excels in terms of $\tilde{\tau}$ over the MD routing (except when $0.6 < \lambda < 2.6$ under uniform resource allocation), especially under high traffic intensity. Moreover, for both routing algorithms, degree-based and $\tilde{U}_k$-based resource allocation schemes can effectively shorten the $\tilde{\tau}$ compared with the uniform resource allocation.

Table 3.8: Node number N, average node degree $\langle k \rangle$, and maximum node degree $k_{\max}$ of Internet AS-level network.

N	7716
$\langle k \rangle$	5.97
$k_{\max}$	1852

Table 3.9: Maximum node usage probability $U_{\max}$ of Internet AS-level network, under SP and MD routing.

$U_{\max}$	Internet AS-level network
SP routing	0.0884
MD routing	0.0077

3.5 Application to the Internet

To compare the performance of different routing strategies and resource allocation schemes on practical communication networks, we acquire the interconnection information of the Internet at autonomous system (AS) level from online dataset (http://snap.stanford.edu/data). We incorporate the raw data collected from July 1, 2008 to July 1, 2009 to build a network containing 7716 nodes and 46065 links, which can be regarded as an example or a subnetwork of the Internet.

Comparing Tables 3.1 and 3.8, we observe that although the AS-level network is quite sparse, the maximum node degree is extremely high.

In our study, we set the average transmission capacity $\tilde{R}$ and average buffer size $\tilde{B}$ of each node as $\tilde{R} = 50$ packets and $\tilde{B} = 200$ packets.

Fig. 3.18 shows the node usage probability versus node degree of the Internet AS-level network, under SP and MD routing algorithms. Table 3.9 shows that

Table 3.10: Average distance $\tilde{D}$ of Internet AS-level network, under SP and MD routing.

$\tilde{D}$	Internet AS-level network
SP routing	6.94
MD routing	13.22

Table 3.11: α, β values at optimal critical generation rate of Internet AS-level network, using the degree-based resource allocation, from theoretical estimation.

α, β	Internet AS-level network
SP routing	0.375, 0.375
MD routing	0.375, 0.125

Table 3.12: Critical generation rate of Internet AS-level network using uniform, degree-based, and $\tilde{U}_k$-based node resource allocation schemes, from theoretical estimation and from simulations (in bracket).

λ_c	uniform	degree-based	$\tilde{U}_k$-based
SP routing	0.042(0.038)	3.121(3.041)	28.828(27.960)
MD routing	0.265(0.24)	0.662(0.625)	15.028(14.678)

MD routing can greatly reduce the maximum node usage probability.

From Table 3.10, we observe that the average distance $\tilde{D}$ of MD routing is much longer than that of SP routing.

Table 3.12 compares the critical generation rate λ_c from theoretical estimations and from simulations for the AS-level network using three different resource allocation schemes, under SP and MD routings.

When resources are uniformly distributed, the MD routing performs better in terms of λ_c and $\tilde{P}_d$ (see Table 3.12 and Fig. 3.21) and SP routing excels in terms of $\tilde{\tau}$ (see Fig. 3.24).

Under the degree-based resource allocation scheme, Figs. 3.19 and 3.20 show the variation of λ_c with different α and β values from theoretical estimations. Table 3.11 shows the optimal α and β values which can achieve the peak value of λ_c using the degree-based resource allocation. As shown in Table 3.12, Figs. 3.22 and 3.25, if the network resources are degree-based allocated using the optimal α and β, SP routing excels over the MD routing in terms of λ_c, $\tilde{P}_d$ and $\tilde{\tau}$.

Table 3.12, Figs. 3.23 and 3.26 show that, under $\tilde{U}_k$-based resource allocation, SP routing performs better than the MD routing in terms of λ_c, $\tilde{P}_d$ and $\tilde{\tau}$.

From Tables 3.12, and Figs. 3.21 to 3.26, we can see that the $\tilde{U}_k$-based resource allocation offers much better performance than both uniform and degree-based resource allocations.

However, for some networks with large size, the calculation of $U(i)$ for each node in the network involves rather intensive computation. Therefore, we estimate the node usage probability $U(i)$ for node i as

$$U(i) \approx \frac{1}{\rho\lambda N\tilde{\tau}} \frac{\sum_{t=P+1}^{P+Q} S_i(t)}{Q} \tag{3.21}$$

where $\lambda < \lambda_c$ for the network to work in free-flow state, $S_i(t)$ is the number of packets in node i at time step t and $\sum_{t=P+1}^{P+Q} S_i(t)/Q$ is the average number of packets existing in nodes at each time step from time step $P+1$ to time step $P+Q$. Here P is set to be large enough to ensure that the network reaches the steady state. $\rho\lambda N\tilde{\tau}$ is the average packet number of each node in steady state.

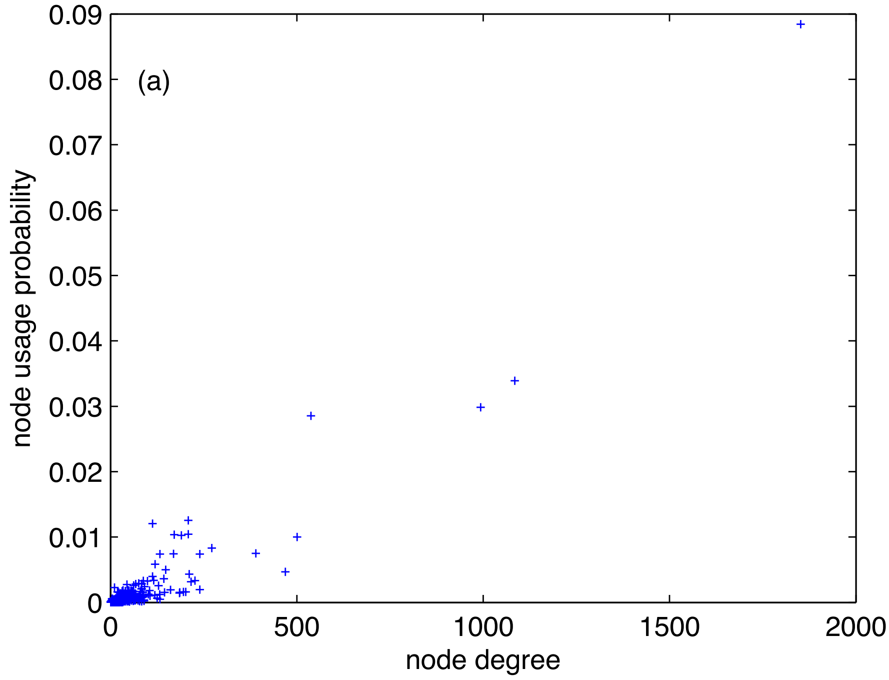
Using (3.21), we calculate the node usage probability of the M highest degree nodes and assign their transmission capacity $R(i)$ and buffer size $B(i)$ as

$$R(i) = U(i)\tilde{R}N, B(i) = U(i)\tilde{B}N, \tag{3.22}$$

where N is the total node number, $\tilde{R}$ and $\tilde{B}$ are the average transmission capacity and average buffer size of each node in the network.

Then, we allocate resource uniformly to the remaining $(N - M)$ nodes and ensure that the total transmission capacity and buffer size are unchanged.

Fig. 3.27 shows the value of λ_c versus the number of nodes with node usage probability-based allocated resources (M). We observe that under SP routing, for the network with 7716 nodes, with only about 500 nodes' resources U-based allocated, i.e., $M = 500$, the value of λ_c is higher than that of both uniform or degree-based schemes.



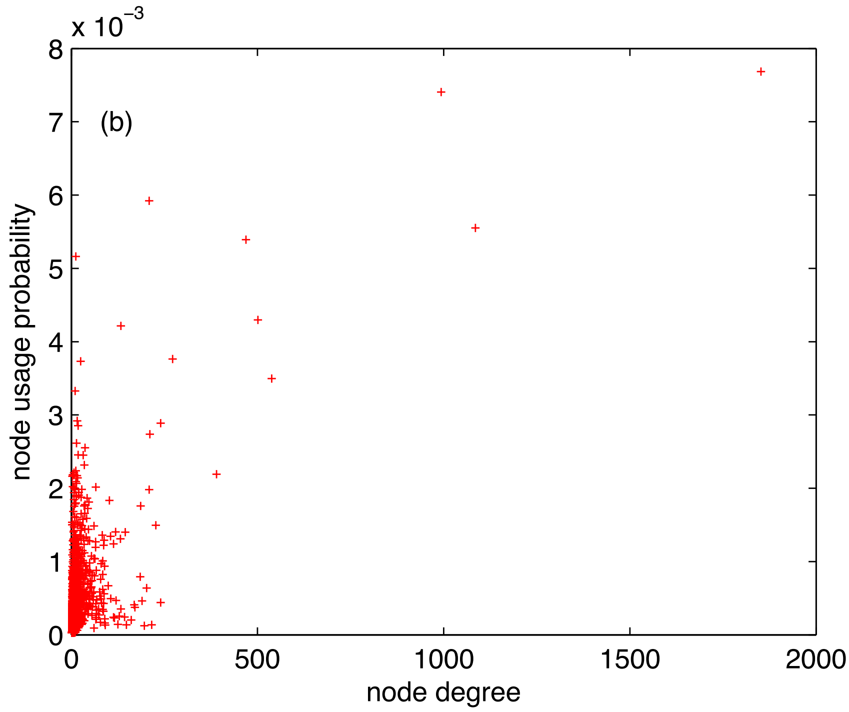


Figure 3.18: Node usage probability versus node degree of Internet AS-level network under (a) SP and (b) MD routing algorithms.

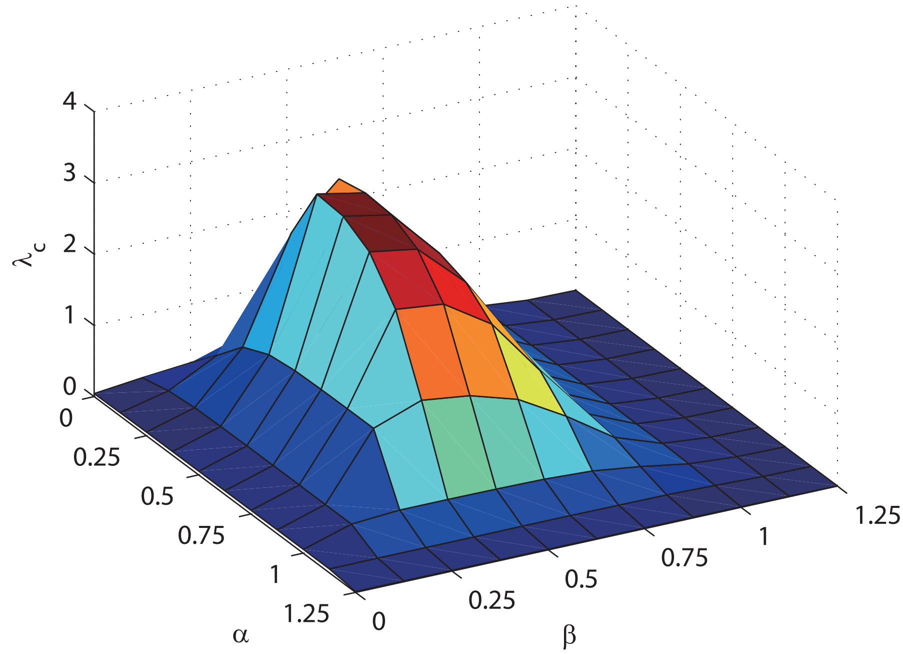


Figure 3.19: Critical generation rate λ_c of Internet AS-level network, under SP routing versus α and β.

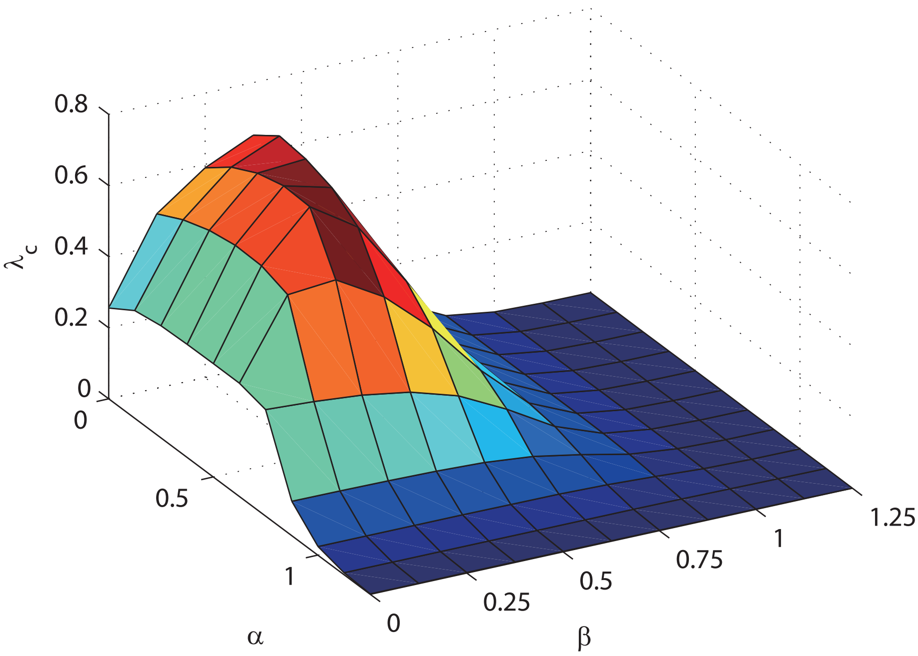


Figure 3.20: Critical generation rate λ_c of Internet AS-level network, under MD routing versus α and β.

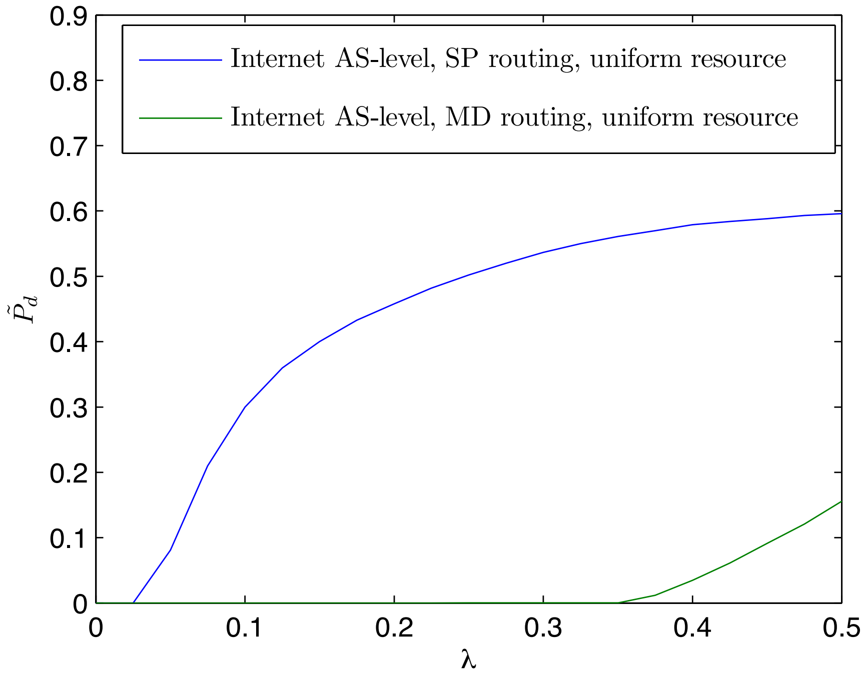


Figure 3.21: Average drop rate $\tilde{P}_d$ versus λ for Internet AS-level network, using uniform node resource allocation.

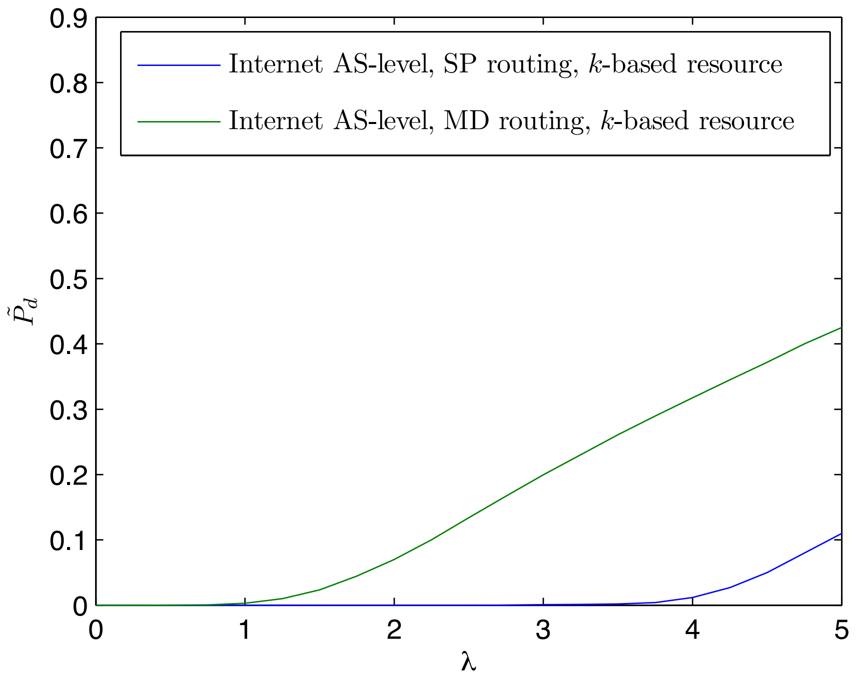


Figure 3.22: Average drop rate $\tilde{P}_d$ versus λ for Internet AS-level network, using degree-based node resource allocation.

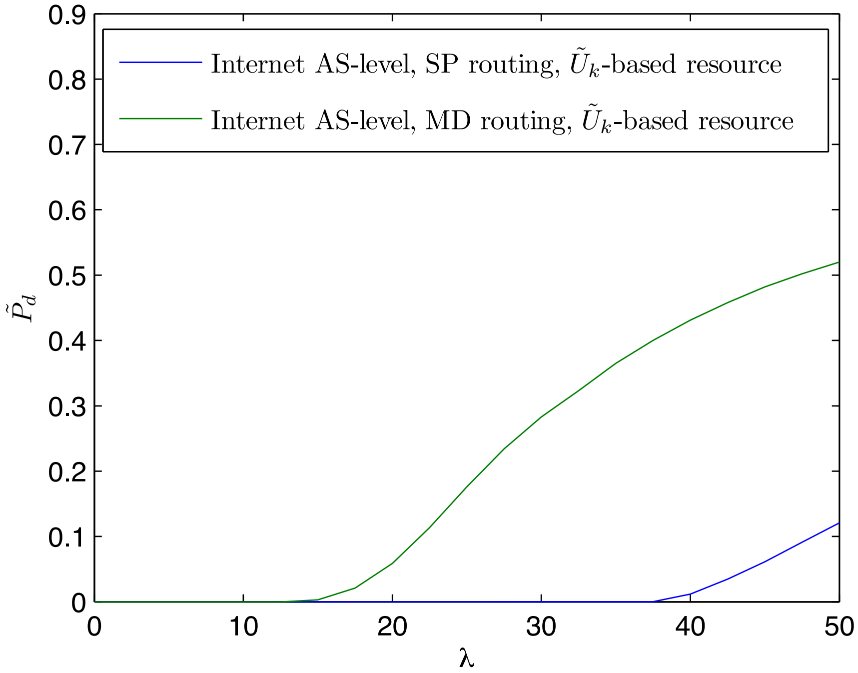


Figure 3.23: Average drop rate $\tilde{P}_d$ versus λ for Internet AS-level network, using $\tilde{U}_k$-based node resource allocation.

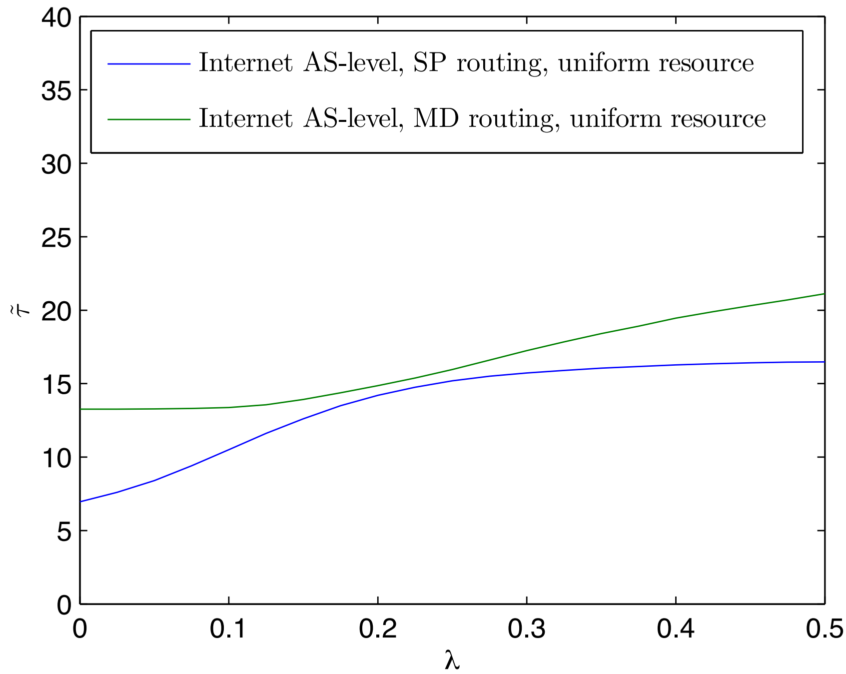


Figure 3.24: Average transmission time $\tilde{\tau}$ versus λ for Internet AS-level network, using uniform node resource allocation.

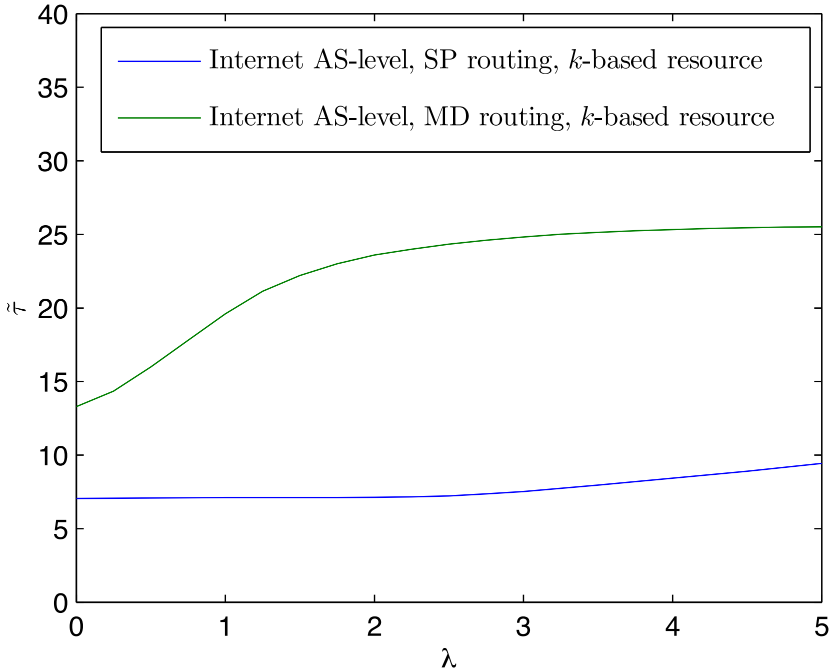


Figure 3.25: Average transmission time $\tilde{\tau}$ versus λ for Internet AS-level network, using degree-based node resource allocation.

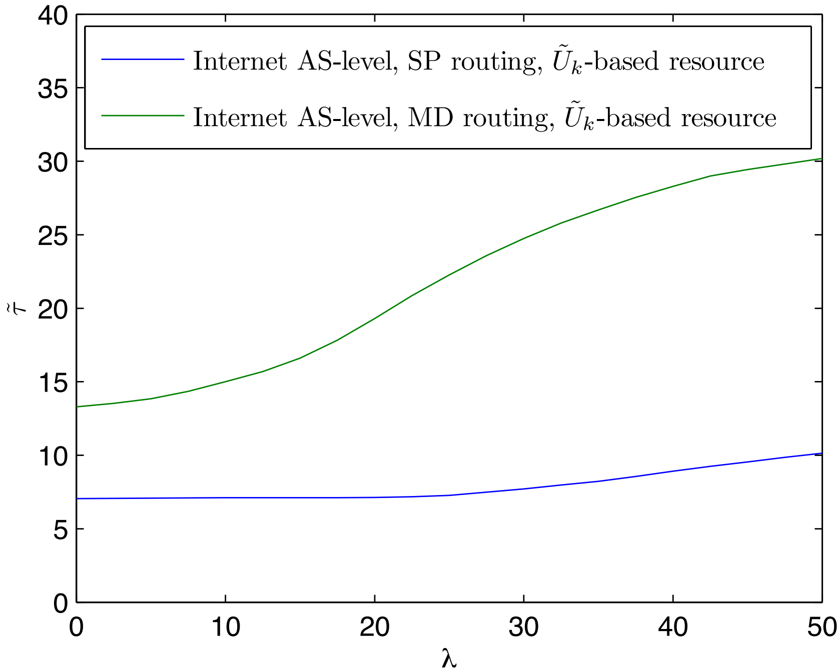


Figure 3.26: Average transmission time $\tilde{\tau}$ versus λ for Internet AS-level network, using $\tilde{U}_k$-based node resource allocation.

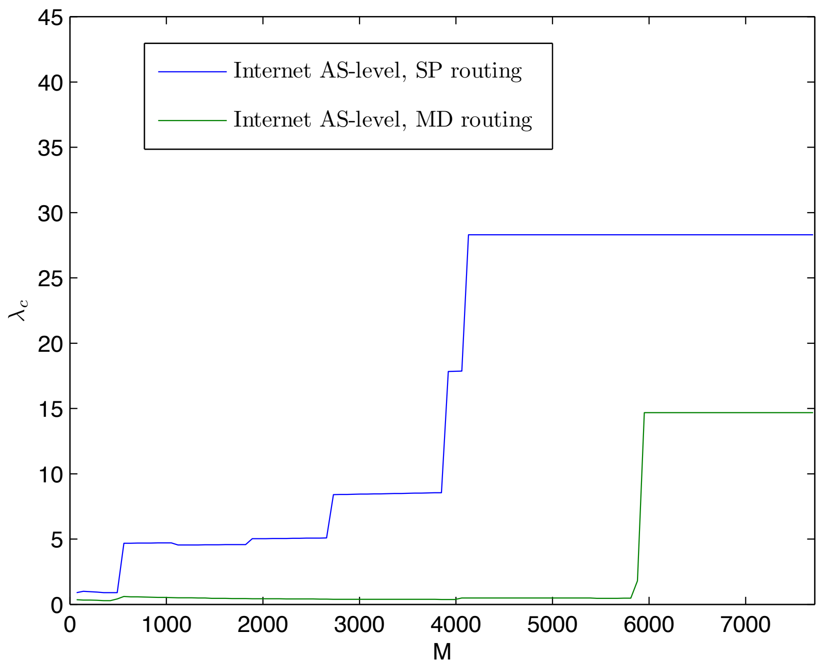


Figure 3.27: Critical generation rate λ_c versus the number of U-based resource allocated nodes.

3.6 Main Findings and Discussions

As demonstrated in the foregoing, the network topology and routing method have significant influence on the performance of a communication network. The key results and observations can be summarized as follows.

1. The node usage probability, as defined in this chapter, is an effective metric for characterizing the traffic load distribution and how frequently a node is chosen to relay packets in a network. This metric, which depends on the network topology and routing strategy, can be used for effective resource allocation in communication networks. Our simulation results compare the performance of the three networks under both SP and MD routings, and show that resource allocation based on the node usage probability outperforms the uniform and degree-based allocation schemes.

2. We have derived analytically the critical generation rate, which represents

the maximum offered load that can be supported by the network without congestion, in terms of the node usage probability, average distance of the communication paths and allocated resources. This analytical result suggests that, for efficient data transmission, the traffic load should be distributed as uniformly as possible in the network and the average path distance should be as short as possible to maximize the critical generation rate when resources are assigned uniformly. It also allows us to identify the optimal operating point in resource allocation.

3. We have evaluated the performance of generic communication networks under different network topologies, routing algorithms and resource allocation schemes through extensive simulation. Performance parameters, including packet drop rate, time delay, and critical generation rate, have been considered. Overall, the regular lattice network has the worst performance among the three due to its long average path distance. The ER random network has the best performance with SP routing in most scenarios, and it is especially prominent under uniform resource allocation. The degree-based and node usage probability-based resource allocation can further enhance the network performance in all cases, and the node usage probability-based allocation always gives the best performance. A scenario with significant improvement is the BA scale-free network with SP routing. This is because the highest-degree nodes in BA scale-free networks are extremely busy and vulnerable to congestion under SP routing, and degree-based or node usage probability-based resource allocation assigns more resources to these busy nodes hence prevents bottlenecks from forming in the network and boosts the overall network performance. Moreover, our simulation results show that when resources are uniformly assigned, MD routing excels SP routing because it causes the traffic to be distributed more evenly. While under

degree and node usage probability resource allocation schemes, SP routing results in better performance due to shorter average distance.

4. We have evaluated the relative advantages and disadvantages of the two routing algorithms and three resource allocation schemes in terms of critical generation rate, packet drop rate, and average transmission time. Our results show that the performance of Internet network (AS-level) is similar to that of the BA scale-free network. When resources are node usage probability-based allocated and under SP routing, the Internet can achieve the best performance. Moreover, for some large practical networks, the calculation of node usage probability of all nodes in the network becomes excessively intensive. We therefore propose a method to reduce the computational burden. Our simulation results indicate that under SP routing, the network can perform better than uniform and degree-based allocation schemes with only about 6.5% nodes' resources U-based allocated.

3.7 Summary

We demonstrate in this chapter the use of complex network concepts in analyzing communication networks and their performance. We show that communication networks can be effectively characterized using complex network parameters which contain information about the structure of networks. In particular, we study the impact of network topology, routing strategy and resource allocation on the performance of generic communication networks. Specifically, we have considered the regular lattice, ER random, BA scale-free and Internet AS-level networks under shortest-path (SP) and minimum-degree (MD) routings with uniform, degree-based and node usage probability-based resource allocation schemes for determination of network parameters. Our results provide important insights into how network management algorithms should be designed and developed for achieving optimum network performance.

Chapter 4

Design of Communication Networks Based on Node Usage Probability

In the previous chapter, we have analyzed the performance of communication networks from a network science perspective. The node usage probability has been proposed as an effective metric for characterizing the traffic load distribution and how frequently a node is chosen to relay packets in a network. In this chapter, we discuss the concept of node usage probability in detail. Based on the concept of node usage probability, effective network design strategies, including routing algorithms and resource allocation schemes, can be developed to improve the overall traffic performance. We compare the performance of a minimum-node-usage routing algorithm with that based on other popular routing algorithms, such as shortest path (SP) and minimum degree (MD) routing algorithms, for various network topologies and resource allocation schemes. Simulation results show that routing algorithms based on minimizing node usage can effectively balance traffic loads and resource allocation based on the node usage probability outperforms the uniform and degree-based allocation schemes. Our analysis and simulation results provide insights into how networks should be designed, including the choice of topology, the routing method, and the resource allocation scheme, for achieving optimal network performance.

4.1 Digital Communication Networks

Digital communication networks play an essential role in connecting the modern world, one prominent example being the Internet. The rapid development of society has inevitably escalated traffic congestion in many communication networks. In the past decades, the issue of traffic congestion has attracted much attention in the physics and engineering communities.

In early studies, the network structure of communication networks was ignored or simply assumed to be completely regular or random [121, 122, 128, 129, 130]. In the past decade, it has been widely revealed that many real-world communication networked systems exhibit power-law degree distribution and small-world properties and traffic congestion is closely related to the underlying network structure [27, 28, 131, 132, 133].

In practice, transmitting digital information from a source to a destination involves sending “packets” through a set of intermediate nodes in the network, commonly called a *path*, which is determined by the specific choice of routing algorithm.

The function of routing is to find a path to transmit a packet from its source to destination. Thus, the routing method plays a deciding role in relating the structure of a network with its ultimate traffic performance.

Intuitively, effective routing in a network can be formulated on the basis of the strategy of using shortest paths. However, in heterogeneous networks like the Internet, the widely used shortest-path (SP) routing strategy leads to high traffic loads at some hubs in the network, causing congestion of the whole network.

To avoid high traffic congestion in hubs and to improve the efficiency and reliability of information flow, a number of routing algorithms were proposed, such as the traffic awareness algorithm [134], the degree-based routing algorithm [102], the local routing algorithm [135], the next nearest neighbour strategy [136], the local routing strategy [137], the global dynamic routing strategy [138], and so on [139, 140, 141, 142, 143, 144, 145, 146, 147].

Among the various kinds of routing strategies, the degree-based routing algorithm [102] is known for its simplicity and efficiency. This routing strategy aims to find the path for each pair of nodes with the minimum sum of nodes' degrees, and this routing algorithm is referred to as minimum degree (MD) routing here. Based on the static topological information only, same as the traditional SP routing, the MD routing can systematically avoid the high degree nodes in the network and effectively improve the overall network performance.

In Chapter 3, we have shown that for efficient and reliable data transmission, the traffic load should be as uniformly distributed as possible in the network and the average distance traveled by the data should be short. We have introduced the *node usage probability* as an effective metric for characterizing the traffic load distribution and how frequently a node is chosen to relay packets in a network. Based on the concept of node usage probability, we infer in this chapter effective design strategies to balance the traffic loads in the network nodes by avoiding overuse of some particular nodes. Such effective network design is shown to necessarily involve minimization of the overall node usage for a given network topology.

To evaluate the performance of the different routing strategies, we build a network using the interconnection information at autonomous system (AS) level from online database (http://snap.stanford.edu/data), which contains 3015 nodes and 5348 links. For comparison of various topologies, we adopt the Barabási-Albert (BA) growth model [6] to build a BA scale-free network and the random graph model proposed by Erdös and Rényi [2] to build a random network. Performance parameters, including critical generation rate, packet drop rate and time delay, are considered. Simulation results show that algorithms based on maintaining uniform node usage can effectively balance traffic loads and improve the overall traffic performance. Furthermore, to put our work in a practical context, we explore the relative advantages and disadvantages of various routing strategies for the three networks under different resource allocation schemes and show that resource allocation based on the node usage probability outperforms the uniform and degree-based allocation schemes. It also allows us to identify the optimal operating point in resource allocation.

4.2 Packet Generation Pattern

In much of the previous work, packet traffic generation was simulated using the Poisson model. In this case, a packet is generated if a random number with a uniform random distribution between 0 to 1 is below λ.

However, studies in the 1990's [148, 149, 150] have demonstrated that the Poisson model cannot capture all kinds of statistical features of Internet-like traffic such as *long range dependence* (LRD). The LRD manifests as bursts in the packet generation over a large number of time scales. The traffic behavior in a network has profound influence on the overall performance, and the bursty feature might make the network more vulnerable to traffic congestion. There exist many kinds of models to simulate the bursty traffic, and here we use the family of Erramilli maps [151] to model the LRD feature of real packet traffic.

We adopt both the Poisson and LRD packet generators in this chapter to simulate the network traffic and compare the network performance under different packet generators.

4.3 Node Usage Probability

4.3.1 Critical Point and Concept of Node Usage Probability

In irregular networks, especially some heterogeneous networks like the scale-free and Internet-like networks, nodes have various degrees and varying importance. Therefore, some nodes in the networks are chosen as routers with a higher probability, and the traffic intensity of them is higher.

In some previous study, researchers have used the concept of *betweenness*, which is defined as the number of shortest paths between any pair of nodes which go through a node to characterize the traffic load. In the last chapter, by taking different routing algorithms into consideration, we have defined *node usage probability* $U(i)$ for node i as

$$U(i) = \frac{\sum\limits_{\substack{u,w \in V, \\ u \neq w \neq i}} \sigma_{uw}(i)}{\sum\limits_{j \in V} \sum\limits_{\substack{u,w \in V, \\ u \neq w \neq j}} \sigma_{uw}(j)}, \tag{4.1}$$

where V is the set of all nodes in the network, $\sigma_{uw}(i)$ is defined as 1 if node i lies on the path between nodes u and w under a specific routing algorithm, and as 0 otherwise.

The total number of paths that pass through node i, denoted by $C(i)$, can be expressed as

$$C(i) = \sum_{\substack{u,w \in V, \\ u \neq w \neq i}} \sigma_{uw}(i) \tag{4.2}$$

Therefore, we have

$$U(i) = \frac{C(i)}{\sum\limits_{j \in V} C(j)} \tag{4.3}$$

The average transmission distance $\tilde{D}$ can be approximated as

$$\tilde{D} \approx \frac{\sum_{j \in V} C(j)}{N(N-1)} \tag{4.4}$$

where N is the total node number in the network.

It has been demonstrated that there exists a phase transition point from a free-flow state to a congestion state. To ensure reliable data transmission, it is necessary to keep the network in the free-flow state. Same as in Chapter 3, here we adopt the critical generation rate λ_c, where the phase transition occurs, as an indicator of the network *throughput*.

In Chapter 3, we have derived analytically an expression for λ_c in terms of the node usage probability, average distance of the communication paths and allocated resources as follows:

$$\lambda_c = \min_{i \in V} \frac{B(i)R(i)}{\tilde{D}U(i)N(B(i)+R(i))}, \tag{4.5}$$

where $B(i)$ and $R(i)$ are the buffer size and transmission capacity of node i, respectively.

For the special case where each node in the network has the same buffer size and transmission capacity, the nodes with highest node usage probability will be the first to get congested, and the critical generation rate λ_c can be simplified as

$$\lambda_c = \frac{BR}{\tilde{D}U_{\max}N(B+R)}, \tag{4.6}$$

where B and R are the buffer size and transmission capacity of each node, respectively, and $U_{\max}$ is the maximum value of $U(i)$.

As shown in (4.6), with fixed network topology and uniformly allocated network resource, we have

$$\lambda_c \propto \frac{1}{\tilde{D}U_{\max}}, \tag{4.7}$$

And using (4.3) and (4.4), we get

$$\tilde{D}U(i) \approx \frac{C(i)}{N(N-1)} \tag{4.8}$$

Therefore, a larger $C_{\max}$, which is defined as the maximum value of $C(i)$, implies a larger $\tilde{D}U_{\max}$ and a smaller λ_c.

4.3.2 Implication to Routing Strategy

In this chapter, we use the same operation model as introduced in Section 3.2 to mimic the traffic transport in communication systems. However, we assume here each node can work as either a host or router to generate or forward packets.

From (4.1), we can see that, with the fixed network topology, the traffic load distribution and the node usage probability are determined by the selected routing algorithm.

Figs. 4.2, 4.3 and 4.4 show that the node usage probability $U(i)$ is related to the node degree and selected routing algorithms in the three networks.

Besides, the routing algorithm also influences the average path distance from the source to the destination, which is the $\tilde{D}$ in (4.5) and (4.6) (see Table 4.2).

According to (4.7) and (4.8), the maximum node usage probability should be small and the average distance should be short to improve the network throughput λ_c.

Shortest path (SP) routing is a widely used algorithm in communication networks. A shortest path refers to the path with minimum hops from the source to the destination. The shortest path routing strategy is widely used in many kinds of real world communication networks because of its simplicity and efficiency.

However, in heterogeneous networks like the Internet, packets are more likely to pass through the high degree nodes under SP routing, thus causing congestion of the whole network. This problem consequently motivates the exploration for new routing strategies to balance the load distribution in the networks.

Yan *et al.* [102] proposed a routing strategy that aims to minimize the sum of the degrees of all nodes in the path, and this routing algorithm is referred to as *minimum degree* (MD) routing here. This algorithm can systematically avoid the high degree nodes in the network and effectively improve the overall network performance.

However, MD routing will increase the average distance for the packets to arrive at the destinations from the sources, as shown in Table 4.2. Intuitively, the average transmission distance is closely related to the transmission efficiency of the network. Moreover, as indicated in (4.7), longer average distance leads to smaller network throughput.

The concept of node usage probability has clearly highlighted the crucial factors for effective network design that optimize performances, namely, routing algorithms that minimize the maximum value of node usage, denoted by $C_{\max}$, for given topologies. Here, we illustrate the basic strategy with the following simple procedure, referred to as minimum-node-usage (MNU) routing here, and we stress that further optimization is possible if specific performance cost function is defined for a particular application.

1. At the starting point, we assign the weight of each node i at step 0, denoted as $w(i, 0)$, to be 1.

2. We begin with a particular pair of source and destination and compute the shortest path between them. When the path is decided, the weight of each node along the path (including the source and the destination) is increased by 1. The weight thus serves as a counter of node usage.

3. At each step t, we calculate the path of a new pair of source and destination with the minimum sum of $w(i,t)$ of all nodes in the path at time step t and update the $w(i,t+1)$ of each selected node by $w(i,t)$+1, until the paths of all pairs of nodes are calculated.

4.3.3 Implication to Resource Allocation

As we can see from (4.5), besides routing strategy, resource allocation is also an important factor that affects the overall network throughput.

For the heterogeneous network, it is unfair and not efficient to assign each node in the network with the same resource. Take the Internet AS-level network as an example. With the average node degree close to 4, the biggest hub has as many as 591 neighbors.

The hubs in the network will have much heavier traffic load than the rest of the network. It is thus reasonable to assign them more network resource to improve the network performance. Therefore, given the same total resource, we consider three resource allocation schemes, namely, uniform, degree-based (k-based), and node usage probability-based (U-based).

For a fair comparison, we keep the average transmission capacity and buffer size of all nodes the same in all simulations, i.e., $R_{\text{avr}} = 5$ packets and $B_{\text{avr}} = 10$ packets.

Under the k-based resource allocation scheme, we allocate the transmission capacity $R(i)$ for node i based on its degree $k(i)$, using the following formula:

$$R(i) = \frac{k(i)}{\sum_{i=1}^{N} k(i)} R_{\text{avr}} N, \tag{4.9}$$

where R_{avr} is the average transmission capacity of all nodes, and $k(i)$ is the degree of node i.

Similarly, the buffer size of node i, $B(i)$, is given by

$$B(i) = \frac{k(i)}{\sum_{i=1}^{N} k(i)} B_{\text{avr}} N \tag{4.10}$$

where B_{avr} is the average buffer size of all nodes, and $k(i)$ is the degree of node i.

For the U-based scheme, the transmission capacity $R(i)$, and buffer size $B(i)$ of the node i, are allocated according to its respective node usage probability $U(i)$, i.e.,

$$R(i) = U(i) R_{\text{avr}} N \tag{4.11}$$

$$B(i) = U(i) B_{\text{avr}} N \tag{4.12}$$

4.4 Network Performance

4.4.1 Network Topology

To compare the performance of different routing strategies on practical communication networks, we acquire the Internet interconnection information at autonomous system (AS) level collected from October 1, 2011 to October 31, 2011 from online dataset (http://snap.stanford.edu/data), which has 3015 nodes and 5348 links.

Moreover, for comparison, we adopt the Barabási-Albert (BA) growth model [6] to build a BA scale-free network and the random graph model proposed by Erdös and Rényi [2] to build a random network.

For each network type, we build 10 networks and run 50 simulations for each network.

For a fair comparison, the total numbers of nodes and of links in the random and BA scale-free networks are set to be similar to those of the Internet AS-level

Table 4.1: Number of nodes N, average node degree $\langle k \rangle$, and maximum node degree $k_{\max}$ of ER random, BA scale-free and Internet AS-level networks

Parameter	ER Random	BA Scale-free	Internet AS-level
N	3015	3015	3015
$\langle k \rangle$	3.95	3.98	3.97
$k_{\max}$	13	150	591

Table 4.2: Average distance $\tilde{D}$ of ER random, BA scale-free and Internet AS-level networks, under SP, MD and MNU routing algorithms

$\tilde{D}$	ER Random	BA Scale-free	Internet AS-level
SP routing	5.88	4.43	3.76
MD routing	6.18	7.23	5.98
MNU routing	5.92	4.84	4.63

network (see Table 4.1). Fig.4.1 shows that both Internet AS-level data and the BA scale-free networks follow the power-law degree distribution, and the random network has a Poisson degree distribution.

As shown in Table 4.1, the Internet AS-level network has the highest maximum node degree and the ER random network is the most homogeneous among the three networks.

Under SP routing, hubs tend to have a higher node usage probability as they are chosen as routers with a higher probability, and thus more vulnerable to congestion. In particular, for the BA scale-free and Internet AS-level network (see Fig. 4.2, and 4.4), a few high-degree nodes have much higher node usage probability than the rest of the network under shortest path routing. If these nodes get congested, the whole network gets congested. SP routing always has

Table 4.3: Maximum node usage probability $U_{\max}$ of ER random, BA scale-free and Internet AS-level networks, under SP, MD and MNU routing algorithms

$U_{\max}$	ER Random	BA Scale-free	Internet AS-level
SP routing	0.0018	0.0505	0.0751
MD routing	0.0007	0.0015	0.0176
MNU routing	0.0005	0.0017	0.0122

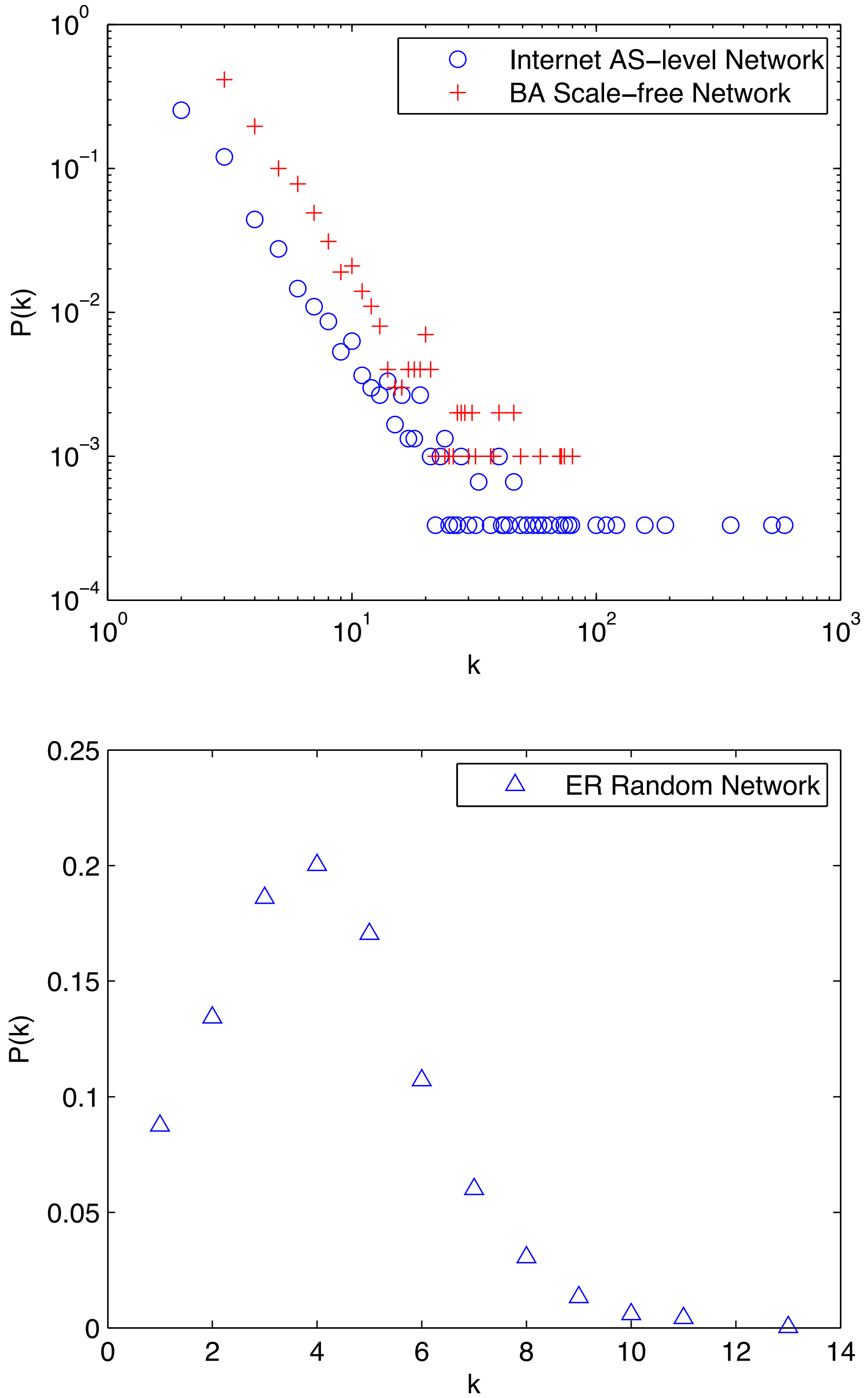


Figure 4.1: Degree distribution of BA scale-free, ER random and Internet AS-level networks.

Table 4.4: Critical generation rate λ_c of ER random, BA scale-free and Internet AS-level networks under SP, MD and MNU routing algorithms, with uniformly distributed network resource and Poisson traffic.

λ_c	ER Random	BA Scale-free	Internet AS-level
SP routing	0.142	0.007	0.005
MD routing	0.402	0.128	0.015
MNU routing	0.558	0.191	0.028

the shortest average distance for the three networks as indicated in Table 4.2. According to (4.5) and (4.6), for efficient data transmission, short average distance will benefit the network throughput.

When MD routing is adopted, the packets will automatically avoid the hubs as they move toward the destinations (see Figs. 4.2, 4.3 and 4.4). As shown in Table 4.3, the maximum values of node usage probability under MD are much lower than that under SP routing, especially for the BA scale-free and Internet AS-level networks. However, under MD routing, the high degree nodes are rarely used (see Figs. 4.2, 4.3 and 4.4), causing much longer average distance (see Table 4.2) than SP routing.

The minimum-node-usage (MNU) routing introduced in this chapter can also effectively balance the traffic loads in the network by avoiding overuse of some particular nodes as the MD routing (see Figs. 4.2, 4.3 and 4.4 and Table 4.3). Moreover, as indicated in Table 4.2, MNU routing algorithm can achieve shorter $\tilde{D}$ than MD routing.

It should be noted that for the Internet AS-level network, even when MD or MNU routing algorithm is adopted to balance the traffic distribution, the nodes with extremely high degree still have a high node usage probability. This result means that the high degree nodes are inevitably used for the traffic transmitted between many other nodes.

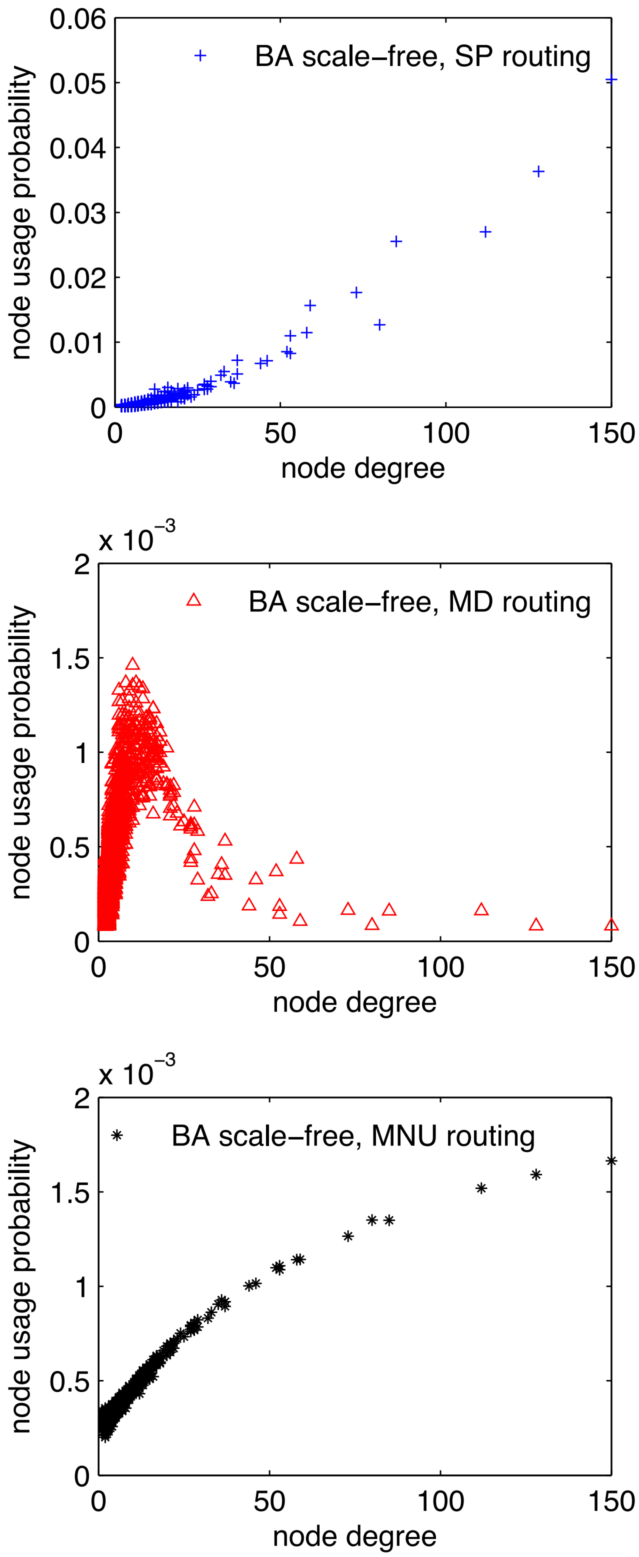


Figure 4.2: Node usage probability versus node degree of BA scale-free network under SP, MD and MNU routing algorithms.

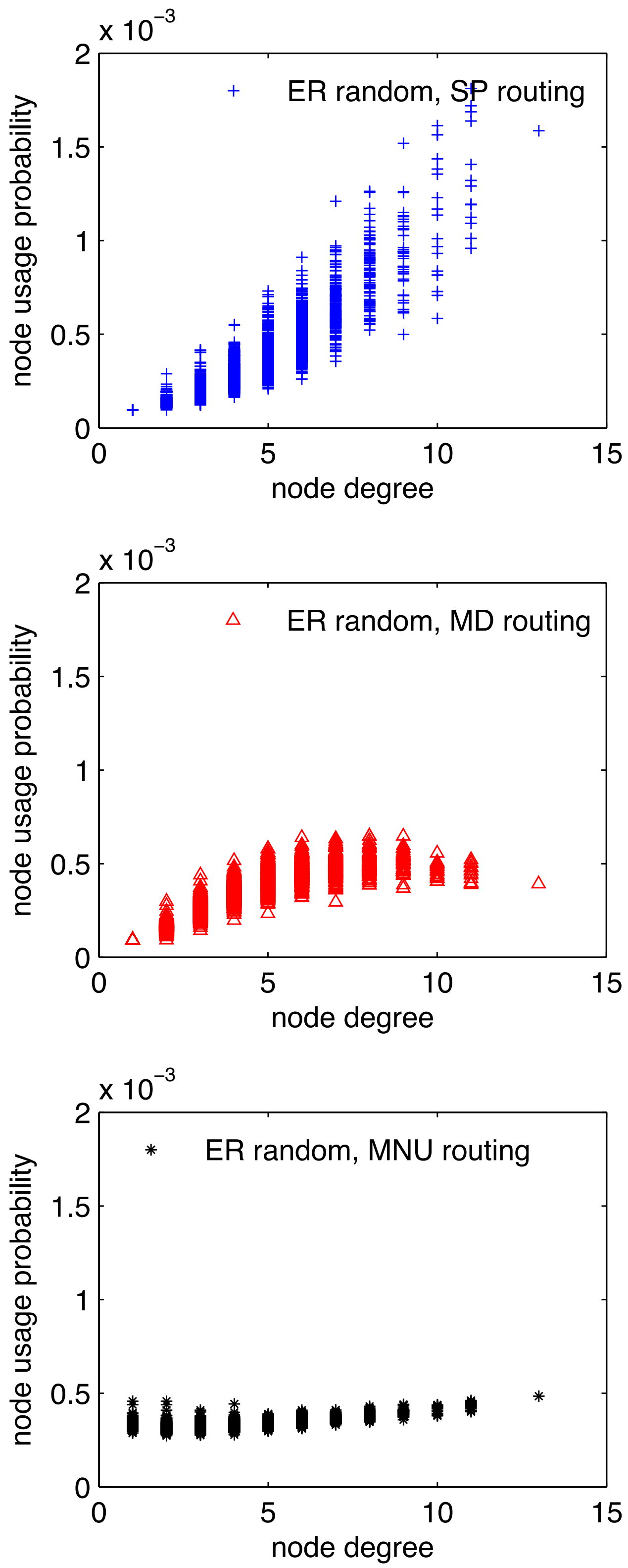


Figure 4.3: Node usage probability versus node degree of ER random network under SP, MD and MNU routing algorithms.

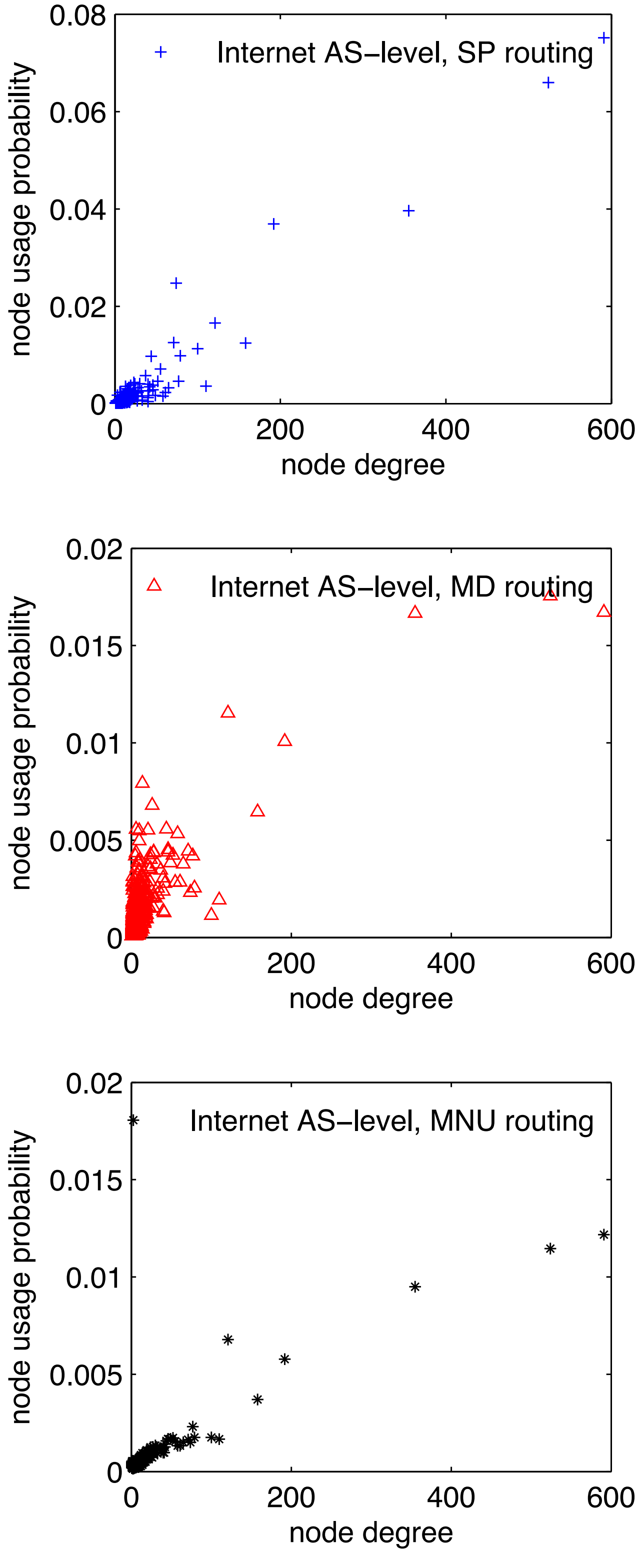


Figure 4.4: Node usage probability versus node degree of Internet AS-level network under SP, MD and MNU routing algorithms.

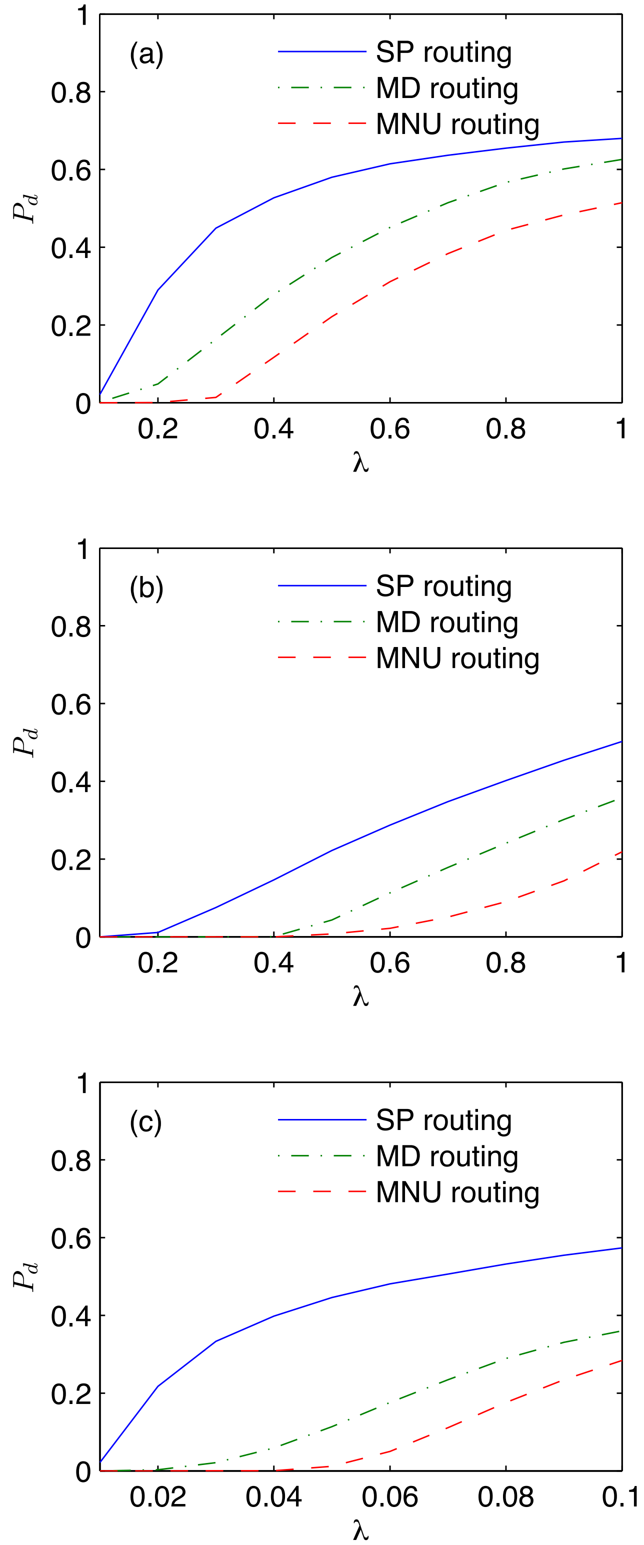


Figure 4.5: Average drop rate $\tilde{P}_d$ versus λ for (a) BA scale-free, (b) ER random, and (c) Internet AS-level networks, under Poisson traffic.

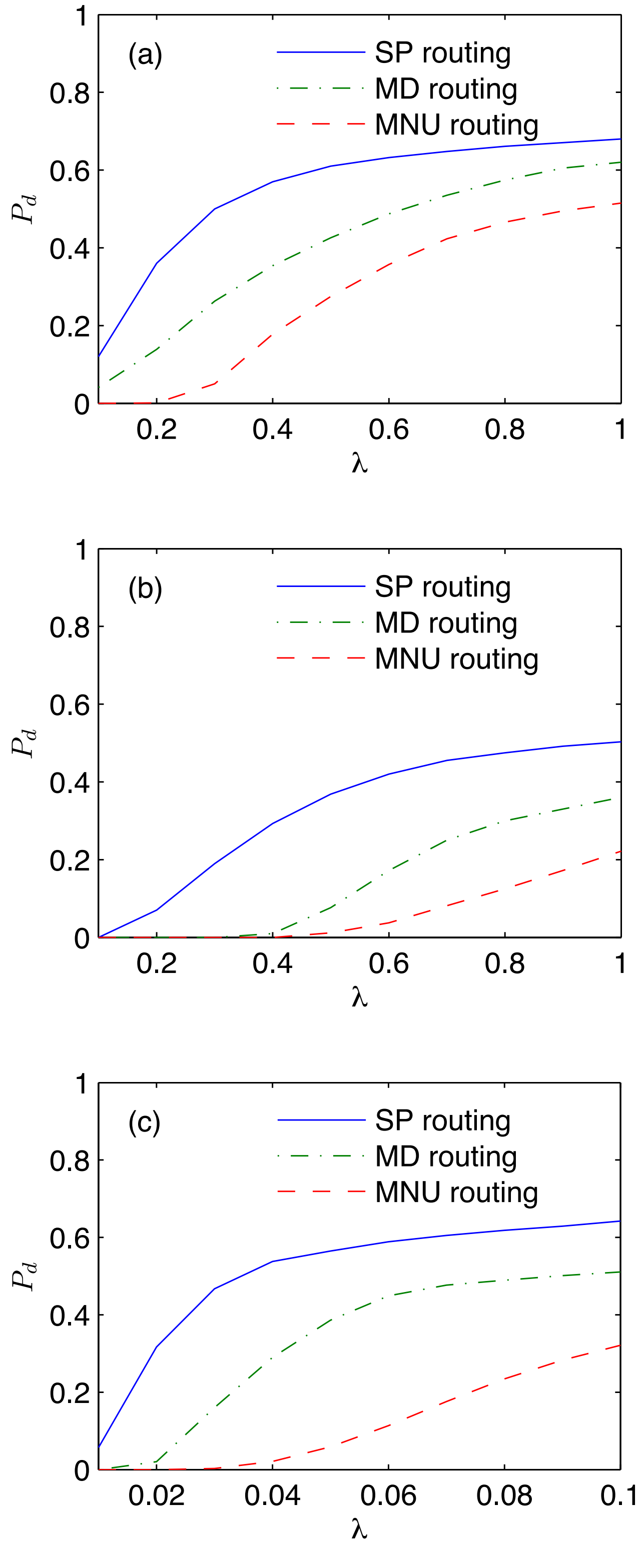


Figure 4.6: Average drop rate $\tilde{P}_d$ versus λ for (a) BA scale-free, (b) ER random, and (c) Internet AS-level networks, under LRD traffic.

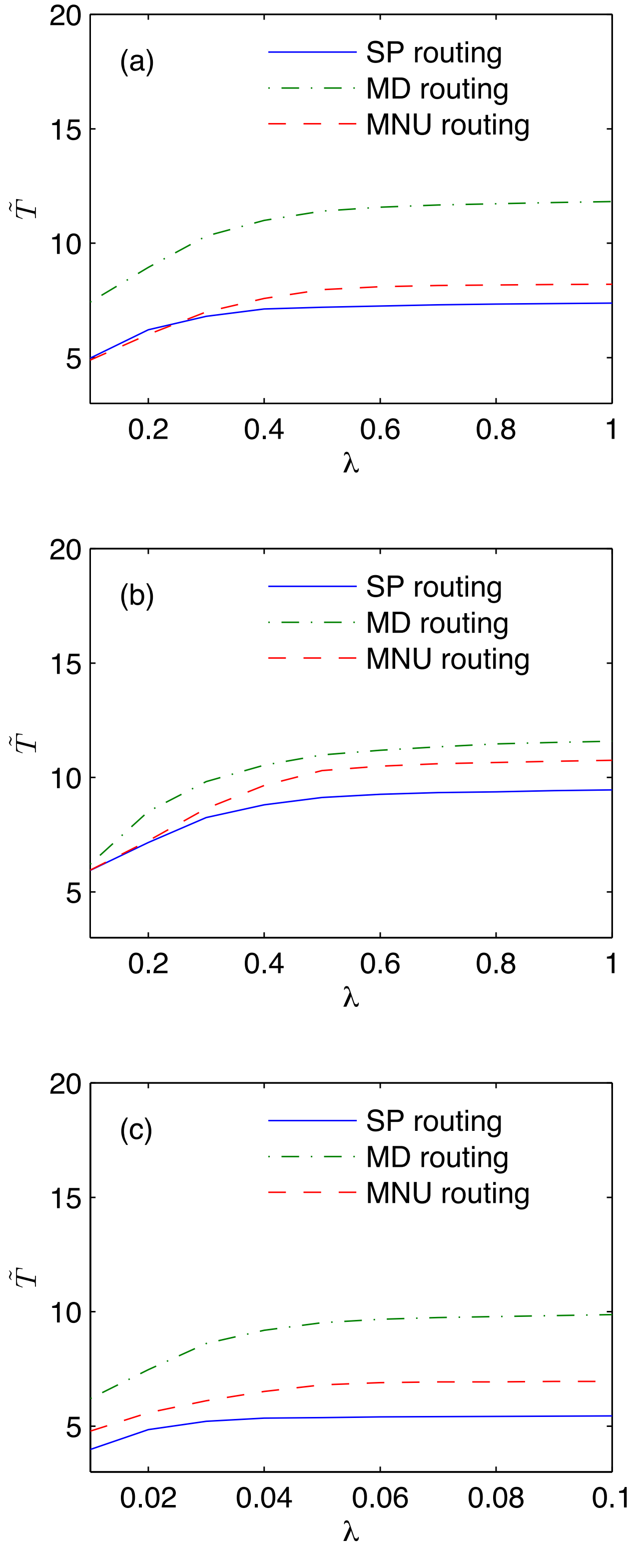


Figure 4.7: Average transmission time $\tilde{T}$ versus λ for (a) BA scale-free, (b) ER random, and (c) Internet AS-level networks, under Poisson traffic.

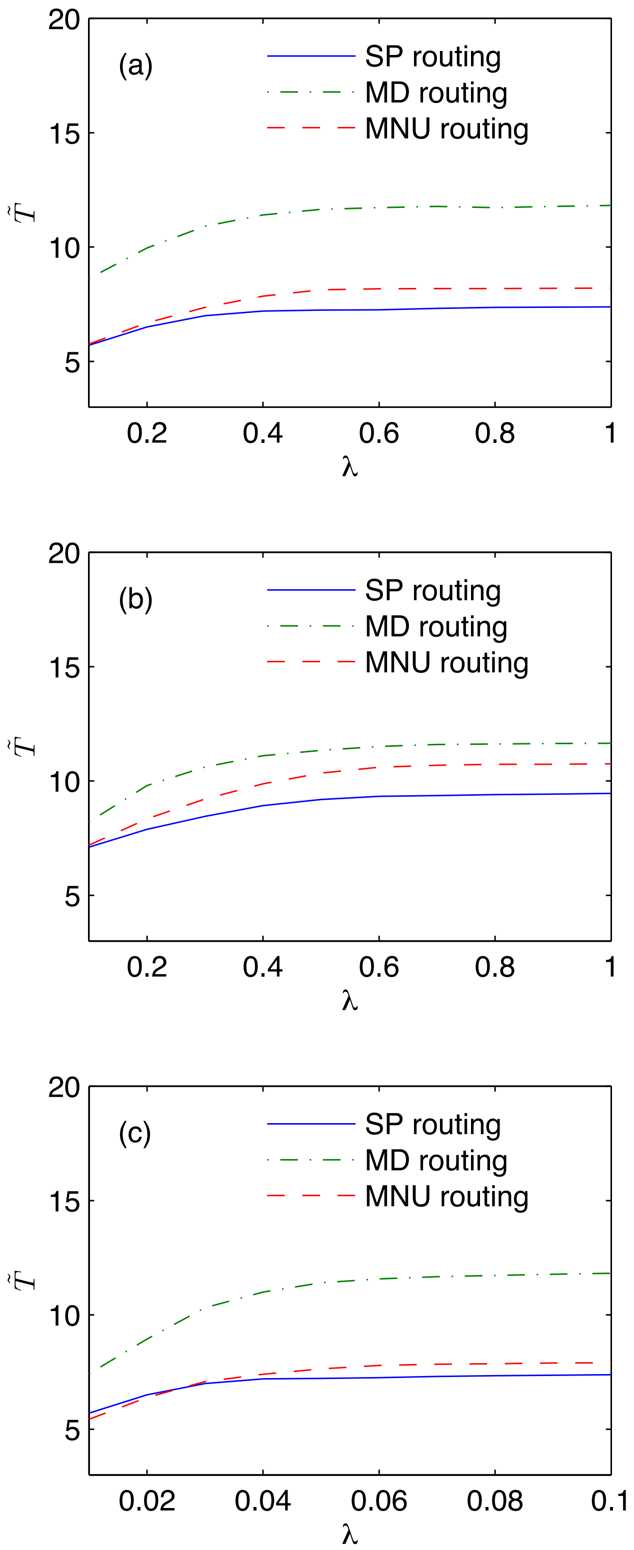


Figure 4.8: Average transmission time $\tilde{T}$ versus λ for (a) BA scale-free, (b) ER random, and (c) Internet AS-level networks, under LRD traffic.

Table 4.5: Critical generation rate λ_c of ER random, BA scale-free and Internet AS-level networks under SP, MD and MNU routing algorithms, with uniformly distributed network resource and LRD traffic.

λ_c	ER Random	BA Scale-free	Internet AS-level
SP routing	0.122	0.005	0.003
MD routing	0.353	0.095	0.009
MNU routing	0.462	0.162	0.022

4.4.2 Uniform Resource Allocation

First, we study the effect of SP, MD, and MNU routing algorithms on the three networks with uniformly distributed resources. Here we consider two kinds of resources, namely, transmission capacity and buffer size. We set the transmission capacity of each node R as 5 packets and the buffer size of each node B as 10 packets.

In order to evaluate the performance of algorithms on different networks in terms of their intended functions of delivering information, performance parameters, including packet drop rate, time delay, and critical generation rate, are considered. The definitions of these parameters have been given in Section 3.4.3.

Tables 4.4 and 4.5 summarize the critical generation rate λ_c of each scenario with Poisson and LRD traffic sources, respectively. Because of the extremely unbalanced traffic as discussed in Section 4.4.1, we can observe that the SP routing has the worst network throughput for all the three networks. By effectively reducing the maximum value of the node usage probability, MD routing can achieve much higher λ_c than SP routing. And MNU routing algorithm can further improve λ_c for all the three networks, especially for the BA scale-free and Internet AS-level networks. This is because MNU routing can effectively balance the node usage and keep the average distance relatively low. Taking the BA scale-free network as an example, although the MNU routing algorithm has a slightly higher $U_{\max}$ than MD routing (see Table 4.3), the λ_c of MNU routing algorithm is still the highest thanks to the much shorter $\tilde{D}$ compared to MD routing (see Table 4.2). This result is in perfect agreement with our analysis in Section 4.3.2 ($\lambda_c \propto 1/(\tilde{D}U_{\max})$).

Figs. 4.5 and 4.6 compare the performance of the three networks in terms of $\tilde{P}_d$ under SP, MD and MNU routing algorithms and we can see SP routing perform the worst and MNU routing perform the best.

As shown in Figs. 4.7 and 4.8, in terms of $\tilde{T}$, SP routing has the best performance, especially when the traffic intensity is relatively low. However, this is at the expense of a much smaller network throughput λ_c and higher $\tilde{P}_d$. Moreover, MNU algorithm has obvious shorter $\tilde{T}$ than the MD routing. Under a low traffic intensity, MNU routing algorithm has similar transmission time as SP routing.

By comparison between Tables 4.4 and 4.5, we observe that under the same scenario, the LRD traffic always results in smaller λ_c than the Poisson traffic. Figs. 4.5 to 4.8 indicate that LRD traffic cause higher $\tilde{P}_d$ and larger $\tilde{T}$ than the Poisson traffic at the same λ. The reason for this might be the unstable network traffic intensity caused by the bursty feature of the LRD sources. With the Poisson traffic source, the network traffic intensity is quite stable over a large number of time scales. However, with LRD traffic, the real time traffic load of the nodes in the network might change from time to time, and therefore make the network more vulnerable to traffic congestion.

4.4.3 Non-uniform Resource Allocation

Next, we evaluate different routing algorithms under nonuniform resource allocation schemes and study the effect of resource allocation on the overall traffic performance.

As LRD traffic can better capture the statistical feature of real Internet traffic, we will only use the LRD source to make the comparison in this section.

Table 4.6: Critical generation rate λ_c of ER random, under various routing algorithms and resource allocation schemes.

λ_c	uniform	k-based	U-based
SP routing	0.122	0.332	0.756
MD routing	0.353	0.383	0.667
MNU routing	0.462	0.101	0.688

Table 4.7: Critical generation rate λ_c of BA scale-free network, under various routing algorithms and resource allocation schemes.

λ_c	uniform	k-based	U-based
SP routing	0.005	0.252	0.916
MD routing	0.095	0.213	0.517
MNU routing	0.162	0.398	0.796

Table 4.8: Critical generation rate λ_c of Internet AS-level network, under various routing algorithms and resource allocation schemes.

λ_c	uniform	k-based	U-based
SP routing	0.003	0.171	0.911
MD routing	0.009	0.048	0.614
MNU routing	0.022	0.168	0.815

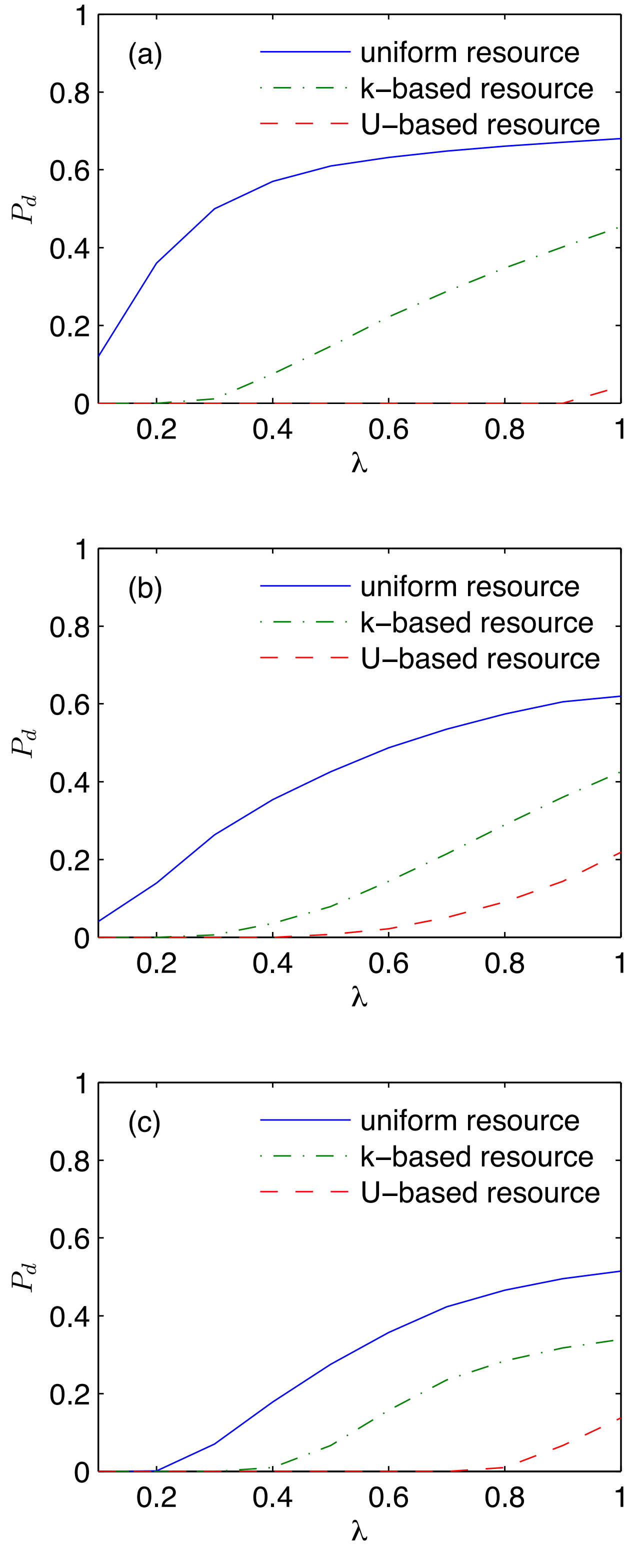


Figure 4.9: Average drop rate $\tilde{P}_d$ versus λ for BA scale-free network, under (a) SP, (b) MD, and (c) MNU routing algorithms, with different resource allocation schemes.

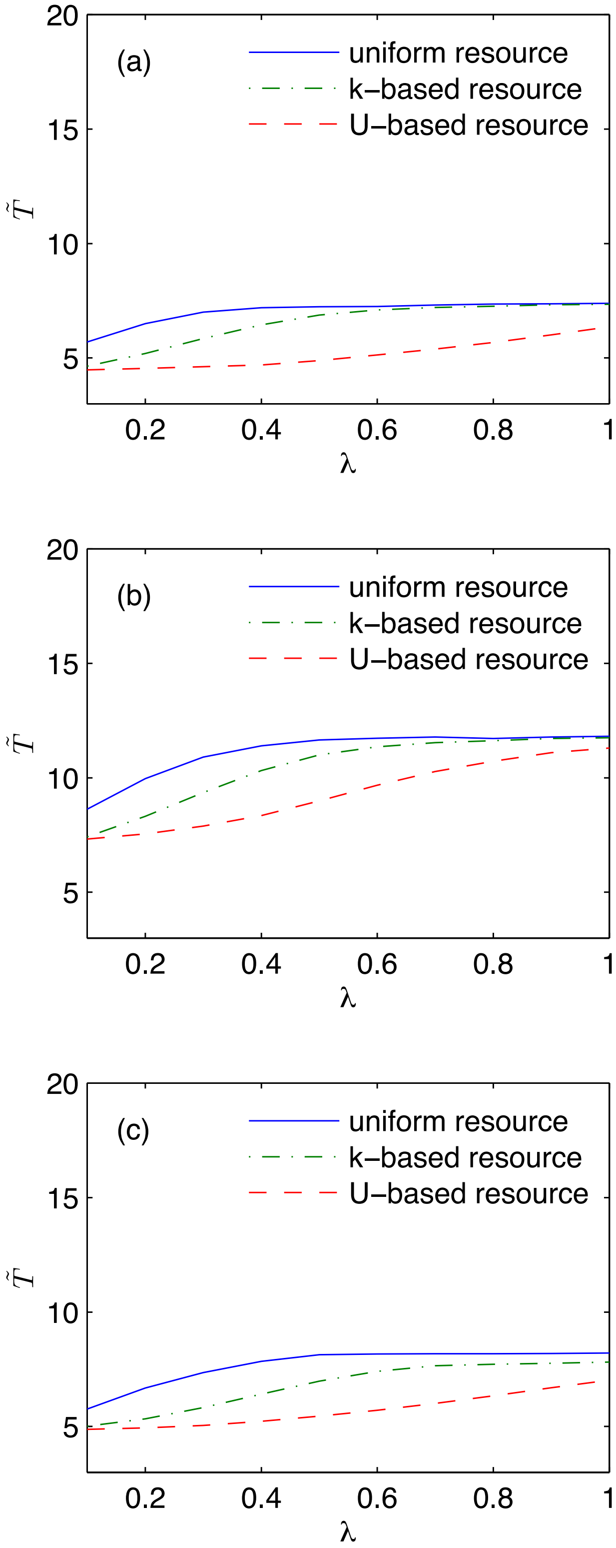


Figure 4.10: Average transmission time $\tilde{T}$ versus λ for BA scale-free network, under (a) SP, (b) MD, and (c) MNU routing algorithms, with different resource allocation schemes.

In Tables 4.6, 4.7 and 4.8, we summarize the values of λ_c under different routing algorithms and resource allocation schemes, for ER random, BA scale-free, and Internet AS-level networks, respectively. We can see that under all scenarios, the U-based scheme gives the best performance. The node usage probability depends on both the underlying network topology as well as the selected routing algorithm. The U-based scheme assigns more resource to the busier nodes hence prevents bottlenecks from forming and thus boosts the overall network performance.

However, the effect of the k-based scheme is related to the selected routing algorithm. When SP routing is adopted, the k-based scheme can make λ_c much better than that under the uniform resource allocation scheme for all the three networks (see Tables 4.6, 4.7 and 4.8). This is because under SP routing, high degree nodes are more likely to be chosen as routers for data transmission and have much higher traffic intensity than the rest of the network.

While under MD and MNU routing algorithms, nodes of higher degree do not necessarily have higher node usage probability (see Fig. 4.3). Therefore, the k-based scheme sometimes does not benefit the performance compared with the uniform scheme. In particular, for the cases when the minimum-node-usage routing algorithm is adopted in the ER random network, the k-based scheme will make the λ_c much lower than the case when all nodes have the same resource.

Moreover, under the U-based scheme, we observe that SP routing performs the best and the MD routing performs the worst in terms of λ_c. Referring to equation (4.5), under the U-based scheme, as $B(i), R(i) \propto U(i)$, we have $\lambda_c \propto 1/\tilde{D}$.

Therefore, SP routing performs the best as it has the shortest $\tilde{D}$ (see Table 4.2). This result further verifies our analytical result that $\tilde{D}$ is a very important parameter for the design of effective routing algorithms.

When the network source is uniformly allocated, the traffic performance of BA scale-free and Internet AS-level networks is much poorer than ER random networks. For heterogeneous networks, although MD and MNU routing algorithms are aimed to balance the network traffic, the traffic load still accumulates in some particular nodes which are inevitable for the traffic between other nodes. Especially for the Internet AS-level network, some nodes have extremely high node degrees and play a crucial role for data transmission. In these scenarios, if the network resource is allocated based on the U-based scheme, the network performance can have a significant improvement.

Next, we evaluate the routing algorithms in terms of $\tilde{P}_d$ and $\tilde{T}$ under different resource allocation schemes. In our simulations, we observe that the resource allocation schemes have similar effect on the ER random, BA scale-free, and Internet AS-level networks. Therefore, here in this section, we only use the results of BA scale-free networks to illustrate the effect of both resource allocation schemes and routing algorithms on network performance in terms of $\tilde{P}_d$ and $\tilde{T}$.

As shown in Fig. 4.9, when the network resource is k-based and U-based allocated, $\tilde{P}_d$ can be obviously reduced compared with the case when all nodes have the same resource. As mentioned in Section 4.4.2, when each node has the same resource, SP routing performs the worst and MNU strategy performs the best in terms of $\tilde{P}_d$. However, if the network resource is U-based allocated, SP routing has a lower $\tilde{P}_d$ than MD and MNU routing algorithms.

From Fig. 4.10, we observe that for all the three routing algorithms, k-based and U-based resource allocation schemes can effectively shorten the $\tilde{T}$ compared with the uniform resource allocation, especially when the traffic intensity is relatively low. Because of the much shorter $\tilde{D}$, SP routing always gives the best performance in terms of $\tilde{T}$.

4.5 Summary

It should be clear that node usage probability should be the key consideration in network design. Specifically, for efficient data transmission, we need to make the traffic load distributed as evenly as possible in the network and the average distance for network as short as possible. The node usage probability is an effective metric for characterizing the traffic load distribution and how frequently a node is chosen to relay packets in a network. Based on this concept, we can infer that routing based on minimizing node usage would lead to balanced traffic loads in the network nodes. However, *we should stress that our aim is not to beat the previous strategies in terms of any particular performance indicator, but to illustrate why and how a routing algorithm can improve the network performance.*

We compare the performance of minimum-node-usage (MNU) routing algorithm with that of shortest path (SP) and minimum degree (MD) routing algorithms for different network topologies and resource allocation schemes through extensive simulations. Performance parameters, including packet drop rate, time delay, and critical generation rate, are considered. Our simulation results show that when the network resources are uniformly assigned, MNU routing algorithm gives the best performance because it can effectively balance the node usage and keep the average distance relatively low. Moreover, if the network resource is node usage probability-based allocated, the routing algorithm can achieve the best performance. While under the node usage probability resource allocation scheme, the SP routing results in the best performance due to the shortest average transmission distance. Simulation results show that in both real Internet AS-level network and the networks built by typical models, MNU algorithm can effectively balance the traffic load and improve the overall traffic performance. These results perfectly agree with our analytical claim of the importance of uniform traffic distribution and short transmission distance.

In summary, network design boils down to consideration of the type of network topology and routing strategy under given resources. In this chapter we demonstrate that all network designs must ultimately comply with balance of traffic load. Node usage probability is therefore the only crucial parameter when the network topology and resource are already defined and allocated.

Chapter 5

A Methodology for Optimizing Network Performance

As demonstrated by the analytical and simulation results in the last two chapters, the traffic load should be as uniformly distributed as possible in the network and the average distance travelled by the data should be short to ensure efficient and reliable data transmission. This criterion has been shown to be fundamental. The key design problem is therefore to find the optimal solution that achieves this criterion. With a fixed network topology, the traffic load distribution and the transmission efficiency are determined by the specific routing algorithm. Based on the concept of node usage probability, it is theoretically possible to develop optimal routing strategies. In this chapter we apply a simulated annealing algorithm to find a near optimal configuration of routing paths, which effectively balances traffic loads and improves the overall traffic performance.

5.1 Communication Network Operation

5.1.1 Operation Model

In this chapter, we assume that all nodes can work as either hosts or routers to generate or forward packets. Packets are generated by the nodes and sent through the links one hop at a time until they reach the destinations. Different from previous chapters, each node in the network has an infinite buffer to store the packets which are waiting to be processed. Thus, the data traffic operates as follows:

1. Packet generation: At each time step, new packets are generated. The average number of generated packets by each node is λ, which is defined as the generation rate of each node. When a packet is generated, its destination is randomly chosen from the rest of the network. The newly generated packets are put at the end of the buffer of the source.

2. Packet transmission: The transmission capacity for node i is $R(i)$. Packets already arrived at their destinations are released from the buffer. At each time step, the first $R(i)$ packets of node i are forwarded to their destinations by one step according to the routing algorithms which we will describe in detail in Section 5.2.

5.1.2 Critical Point and Concept of Node Usage Probability

In Chapter 3, we have defined the *node usage probability* as an effective metric for characterizing the traffic load distribution and how frequently a node is chosen to relay packets in a network. For the sake of convenience, we repeat the definition of node usage probability here, i.e.,

$$U(i) = \frac{\sum\limits_{\substack{u,w \in V, \\ u \neq w \neq i}} \sigma_{uw}(i)}{\sum\limits_{j \in V} \sum\limits_{\substack{u,w \in V, \\ u \neq w \neq j}} \sigma_{uw}(j)}, \tag{5.1}$$

where V is the set of all nodes in the network, $\sigma_{uw}(i)$ is defined as 1 if node i lies on the path between nodes u and w under a specific routing algorithm, and as 0 otherwise.

The total number of paths that pass through node i, denoted by $C(i)$, can be expressed as

$$C(i) = \sum_{\substack{u,w \in V, \\ u \neq w \neq i}} \sigma_{uw}(i) \tag{5.2}$$

Therefore, we have

$$U(i) = \frac{C(i)}{\sum\limits_{j \in V} C(j)} \tag{5.3}$$

And the average transmission distance $\tilde{D}$ can be approximated as

$$\tilde{D} \approx \frac{\sum\limits_{j \in V} C(j)}{N(N-1)} \tag{5.4}$$

where N is the total node number in the network.

Previous studies have shown that there exists a phase transition point from a *free-flow* state to a *congestion* state. To ensure reliable data transmission, it is necessary to keep the network in the free-flow state. The *critical generation rate* λ_c, where the phase transition occurs, is regarded as an indicator of the network capacity. A larger λ_c implies that the network can transmit more traffic without congestion.

If $\lambda < \lambda_c$, the network reaches a steady state when the numbers of packets generated and successfully arrived are balanced. If $\lambda > \lambda_c$, packets accumulate in some nodes and traffic congestion occurs.

In Chapter 3, we have derived analytically an expression for λ_c in terms of node usage probability, average distance of the communication paths and allocated resources as follows:

$$\lambda_c = \min_{i \in N} \frac{R(i)}{\tilde{D}U(i)N}, \tag{5.5}$$

where $R(i)$ is the transmission capacity of node i.

If each node in the network has the same transmission capacity, the nodes with highest node usage probability will be the first to get congested, and the critical generation rate λ_c can be simplified as

$$\lambda_c = \frac{R}{\tilde{D}U_{\max}N}, \tag{5.6}$$

where R is the transmission capacity of each node and $U_{\max}$ is the maximum value of $U(i)$.

As shown in (5.6), with fixed network topology and uniformly allocated network resource,

$$\lambda_c \propto \frac{1}{\tilde{D}U_{\max}}, \tag{5.7}$$

With (5.3) and (5.4), we have

$$\tilde{D}U(i) \approx \frac{C(i)}{N(N-1)} \tag{5.8}$$

Therefore, a larger $C_{\max}$, which is defined as the maximum value of $C(i)$, implies a larger $\tilde{D}U_{\max}$ and a smaller λ_c.

5.2 Optimized Routing Strategy

From (5.1), we can see that, with the fixed network topology, the traffic load distribution and the node usage probability are determined by the selected routing algorithm. Therefore, we aim to find the optimal configuration of routing paths to make $C_{\max}$ as small as possible. However, in a large and irregular network like the scale-free network we consider here, finding the optimal configuration of routing paths by evaluating all possible paths between each pair of nodes in the network is infeasible. The problem of finding all possible paths between two nodes in the network was proven to be non-deterministic polynomial-time (NP) hard [152, 153].

In Chapter 4, it has been shown that performance can be optimized by minimizing the node usage probability and shortening the average distance. A minimum-node-usage (MNU) routing algorithm has been proposed and illustrated for its effectiveness. However, a strategy to find the actual optimal set of routing paths has not been developed. Therefore, in this chapter, we propose to use a nature inspired algorithm, namely, simulated annealing (SA) [154, 155], to find a near-optimal solution of this problem. Thus, the SA algorithm presented here is essentially an optimized MNU algorithm. The procedure of the algorithm is as follows:

1. Start from an initial solution, S_0. Calculate the $C^0_{\max}$ and set the best solution as $S_{\text{best}} = S_0$ and $C^{\text{best}}_{\max} = C^0_{\max}$. For fast convergence, we start from the shortest path routing. Set the system time $t = 1$ and epoch count $k = 1$;

2. Randomly pick a pair of source and destination and change the routing path between them randomly [156]. Denote this new configuration as S_t, and calculate $C^t_{\max}$.

3. If $C^t_{\max} < C^{\text{best}}_{\max}$, then we accept the new routing path and set $S_{\text{best}} = S_t$ and $C^{\text{best}}_{\max} = C^t_{\max}$. If $C^t_{\max} \geq C^{\text{best}}_{\max}$, we accept the new configuration with the probability $e^{-\Delta/T}$, where T is a control parameter called *temperature*

and $\Delta = C^t_{\max} - C^{\text{best}}_{\max}$. If the new configuration is accepted, set $k = k + 1$. Otherwise, keep k unchanged.

4. For high-quality solution, the iteration time should be long enough. If the $C^{\text{best}}_{\max}$ is unchanged in the latest 10000 steps, we stop. If not, set $t = t + 1$ and go to step 2.

The parameter T must be carefully selected since the values of parameters may have a significant influence on the performance of the algorithm [157, 158]. At the start of the algorithm, the value of T should be set large enough to make the initial probability of accepting new solution be close to 1 [154]. Then T is gradually decreased by a *cooling function* during the optimizing process. Here we adopt a simple cooling function which changes T to αT after every L epoch count, where α and L are control parameters called cooling ratio and epoch length, respectively. In our simulations, the values of α and L are given by the method proposed in [159].

For comparison and more in-depth discussion of the effects of routing on performance, we implement three other algorithms.

1. Shortest path (SP) routing is a widely used algorithm in many real communication networks because of its simplicity and efficiency. However, in heterogeneous networks, packets are more likely to pass through the high degree nodes under SP routing, thus causing congestion of the whole network.

2. Minimum degree (MD) routing strategy proposed by Yan *et al.* [102] aims to minimize the sum of the degrees of all nodes in the path. This algorithm can systematically avoid the high degree nodes in the network and effectively improve the overall network performance.

3. An illustrative minimum-node-usage (MNU) routing algorithm has been

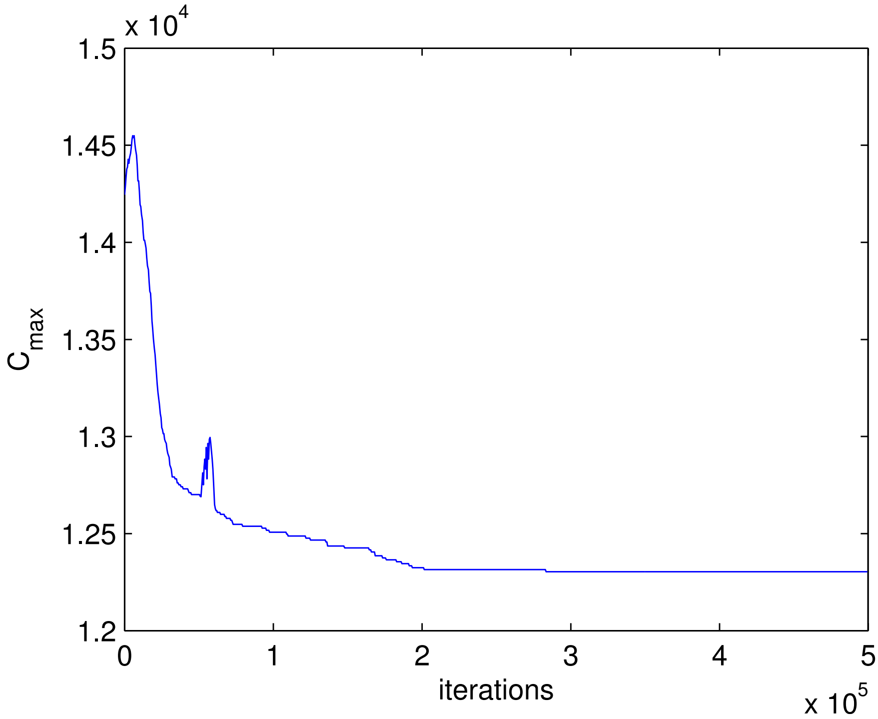


Figure 5.1: Objective value $C_{\max}$ versus number of iterations in the optimization process for BA scale-free network with network size $N = 1000$.

Table 5.1: Maximum node usage probability $U_{\max}$, average transmission distance $\tilde{D}$, and critical generation rate λ_c under SP, MD, MNU and SA routing algorithms for BA scale-free network with network size $N = 1000$

Parameter	SP	MD	MNU	SA
$U_{\max}$	0.0511	0.0033	0.0036	0.0034
$\tilde{D}$	3.477	5.090	3.9147	3.610
λ_c	0.028	0.298	0.351	0.407

proposed in Chapter 4 for balancing the traffic loads in the network nodes by avoiding overuse of some particular nodes.

5.3 Network Performance

Empirical studies have revealed that many kinds of real-world communication networks are scale-free [5, 26, 21, 19, 27, 28]. In this chapter, we adopt the widely used BA scale-free model [6] to construct the scale-free networks.

In our simulations, the number of nodes N varies from 100 to 1000, the mean

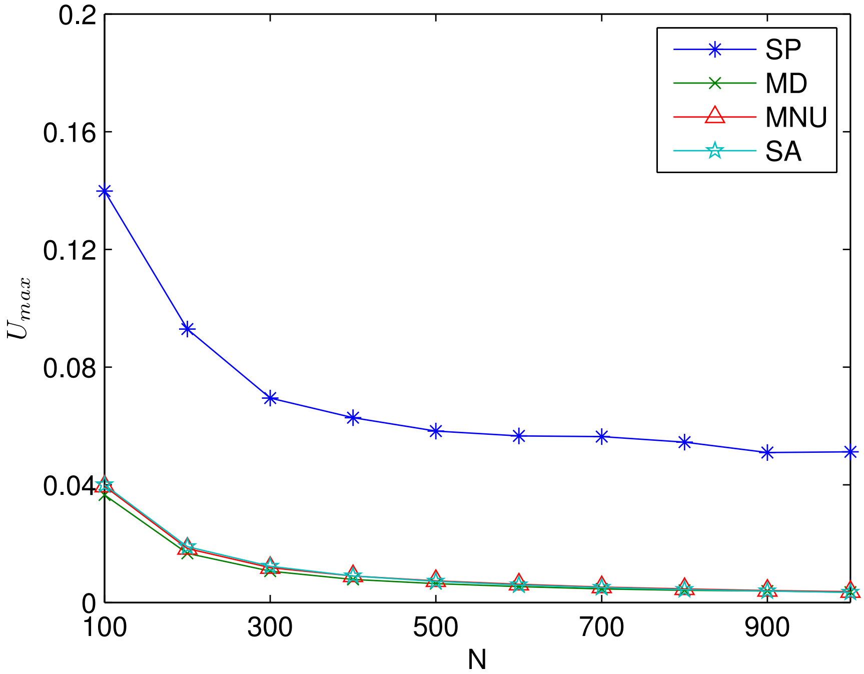


Figure 5.2: Maximum node usage probability $U_{\max}$ versus network size N for BA scale-free network under SP, MD, MNU, and SA routing algorithms.

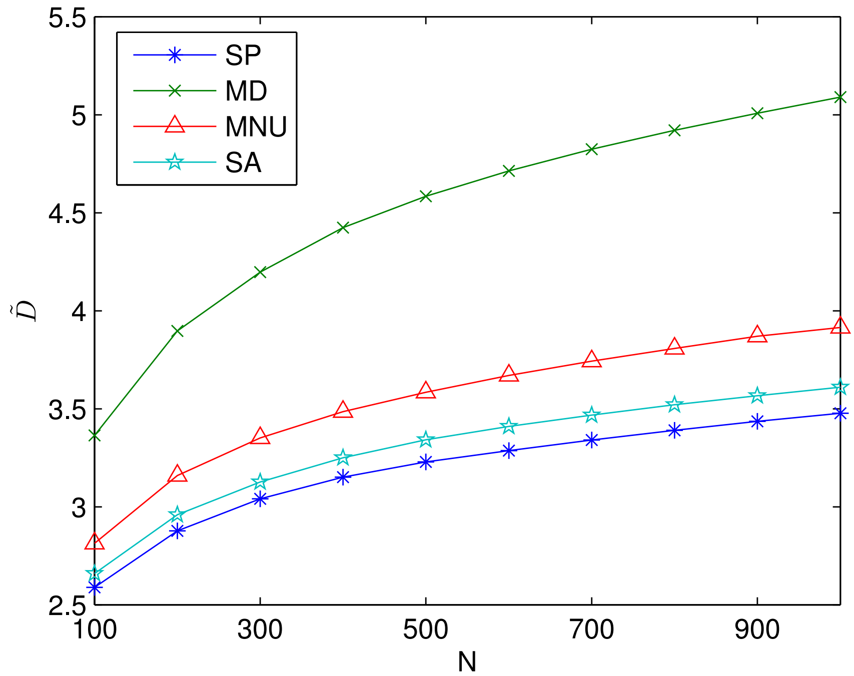


Figure 5.3: Average transmission distance $\tilde{D}$ versus network size N for BA scale-free network under SP, MD, MNU, and SA routing algorithms.

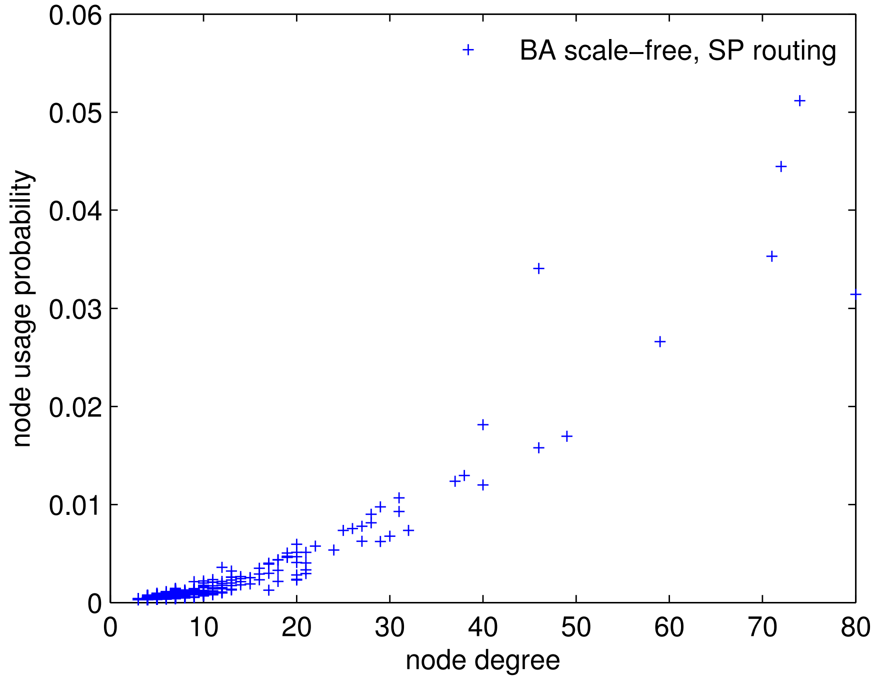


Figure 5.4: Node usage probability versus node degree under SP routing algorithm for BA scale-free network with network size $N = 1000$.

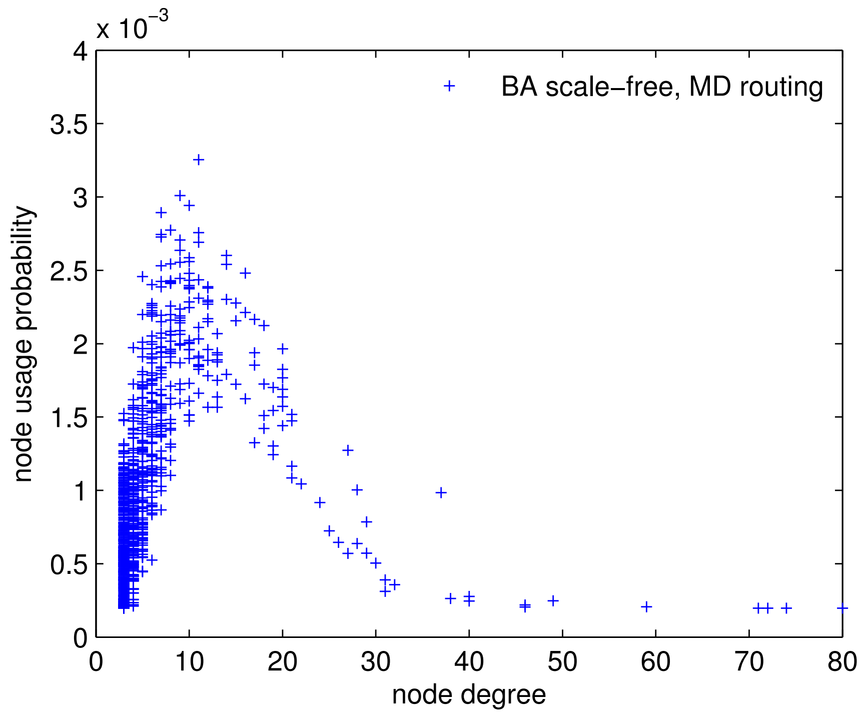


Figure 5.5: Node usage probability versus node degree under MD routing algorithm for BA scale-free network with network size $N = 1000$.

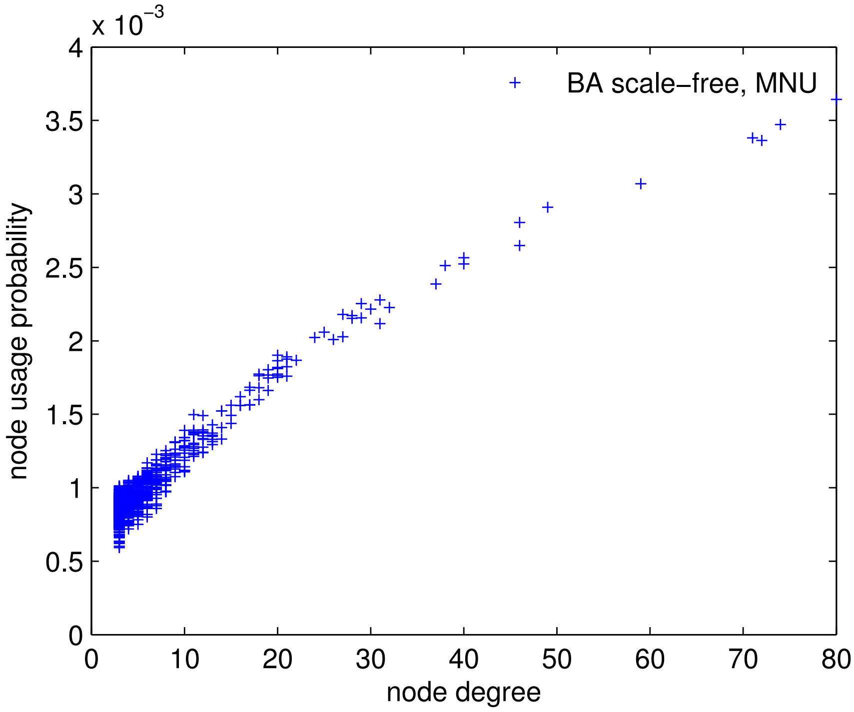


Figure 5.6: Node usage probability versus node degree under MNU routing algorithm for BA scale-free network with network size $N = 1000$.

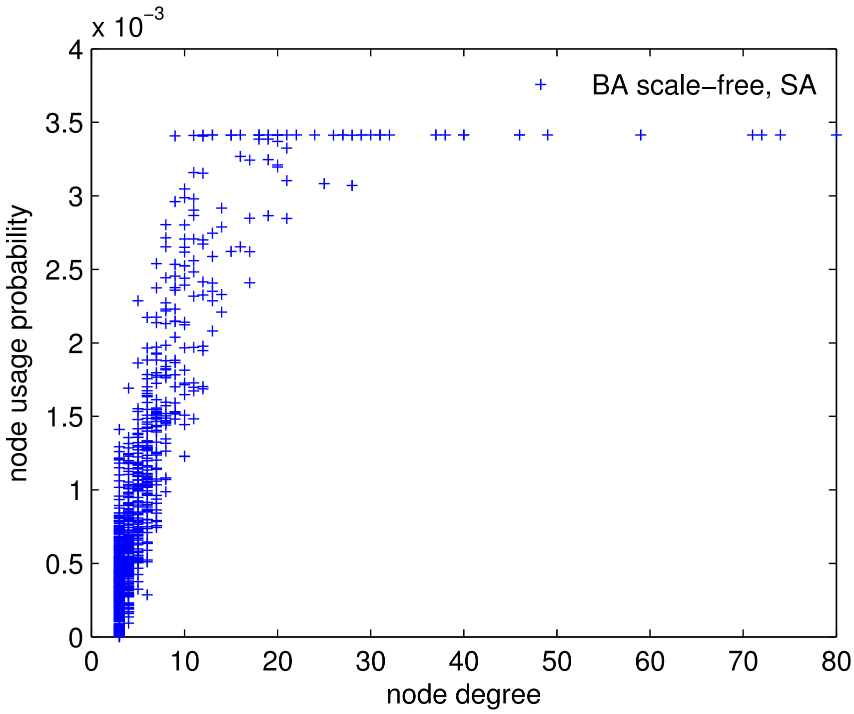


Figure 5.7: Node usage probability versus node degree under SA routing algorithm for BA scale-free network with network size $N = 1000$.

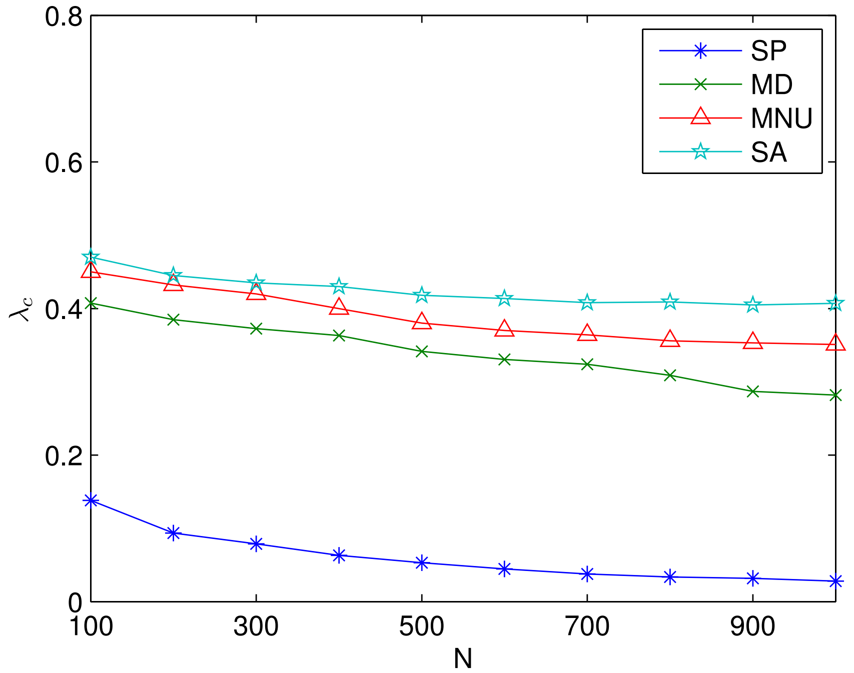


Figure 5.8: Critical generation rate λ_c versus network size N for BA scale-free network under SP, MD, MNU, and SA routing algorithms.

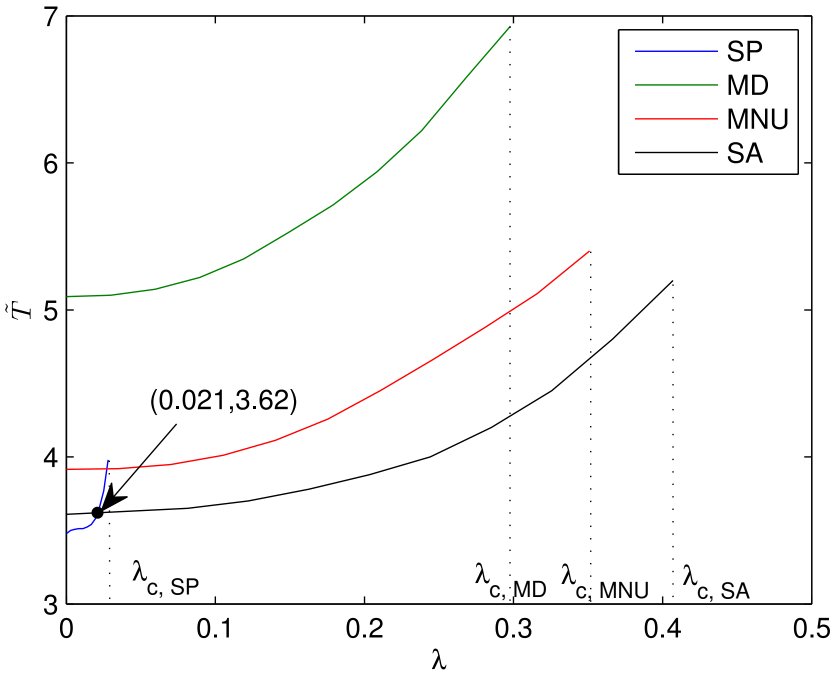


Figure 5.9: Average transmission time $\tilde{T}$ versus packet generation rate λ for BA scale-free network with network size $N = 1000$.

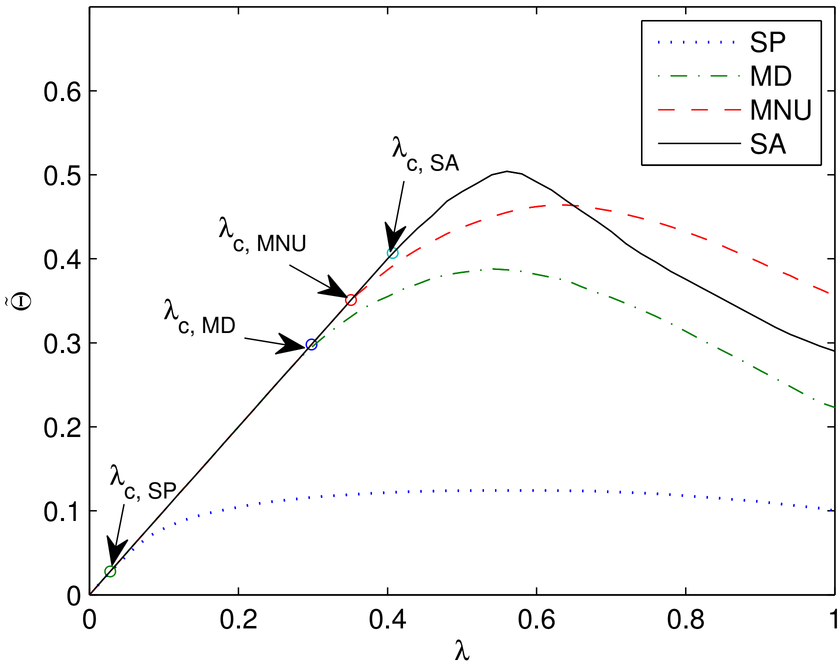


Figure 5.10: Average throughput $\tilde{\Theta}$ versus packet generation rate λ for BA scale-free network with network size $N = 1000$.

value of node degree is kept at $\langle k \rangle \approx 6$, the transmission capacity of each node $R = 5$ packets.

5.3.1 Performance Indicators

As described in Section 5.1.2, critical generation rate λ_c characterizing the phase transition point from free-flow to congestion state can be viewed as an indicator of the network capacity. Besides, we consider two other performance indicators, namely, average transmission time, and average throughput.

In order to improve the transmission efficiency of the network, with the same value of λ_c, we want the packets to arrive at their destinations as fast as possible. The average transmission time, denoted by $\tilde{T}$, is the average number of time steps for a successfully arrived packet takes to arrive at the destination from the source.

Another performance indicator we consider in this chapter is the network average throughput, denoted by $\tilde{\Theta}$, which is defined as the average number of packets successfully arriving at their destinations per node per time step.

5.3.2 Simulation Results

Fig. 5.1 plots the optimizing process for a BA scale-free network with 1000 nodes. Fig. 5.2 shows that for each kind of routing algorithm, the maximum value of node usage probability $U_{\max}$ decreases with the increase of network size N. Moreover, it is apparent that the value of $U_{\max}$ under SP routing is much higher than those of MD, MNU, and SA routings.

In Fig. 5.3, we observe that the average transmission distance $\tilde{D}$ increases with network size N slowly. SP routing has the shortest transmission distance while MD routing has the longest among the four routing strategies.

When SP routing is adopted, high degree nodes tend to have a high node usage probability as they are chosen as routers more frequently (see Fig. 5.4). Under MD routing, the packets will systematically avoid the hubs in the network (see Fig. 5.5) and $U_{\max}$ under MD is much lower than that under SP routing (see Fig. 5.2 and Table 5.1). However, under MD routing, the high degree nodes are rarely used (see Fig. 5.5), thus increasing the average transmission distance in a large scale (see Fig. 5.3 and Table 5.1) .

MNU and SA routings can also effectively reduce the maximum value of the node usage probability by avoiding overuse of some particular nodes and keeping the average distance relatively low (see Figs. 5.2, 5.3, 5.6, 5.7 and Table 3.1). Note that SA is effectively an optimized form of MNU.

Next, we evaluate the routing algorithms in terms of critical generation rate λ_c, average transmission time $\tilde{T}$, and average throughput $\tilde{\Theta}$.

Fig. 5.8 summarizes the values of λ_c under the four routing algorithms. We observe that λ_c declines with the increase of network size. Moreover, SA routing performs best and SP routing performs worst in terms of λ_c. Results in Table 5.1

indicate that in order to achieve a higher λ_c, the routing algorithm need to effectively balance the node usage and keep the average distance relatively low. This result is in perfect agreement with our analysis in Section 5.1.2 ($\lambda_c \propto 1/(\tilde{D}U_{\max})$).

When $\lambda < \lambda_c$, the network operates in free-flow state and $\tilde{T}$ is finite. When $\lambda > \lambda_c$, the number of packets congested in the network accumulates with time and $\tilde{T} \to \infty$ when time step $t \to \infty$. Therefore, it is not meaningful to calculate $\tilde{T}$ after the transition point. As shown in Fig. 5.9, MD routing displays the highest $\tilde{T}$. When the traffic load is very low ($\lambda < 0.021$), the SP routing has the lowest transmission time. However, when $\lambda > 0.021$, SA routing performs best.

From Fig. 5.10, we observe that when $\lambda < \lambda_c$, the network throughput $\tilde{\Theta}$ increases proportionally with traffic load λ. When λ exceeds the critical point λ_c, the increase of $\tilde{\Theta}$ becomes slower as the packets start to accumulate in the network. This period can be regarded as a moderate congestion sate [160], during which only part of the network is congested. When λ continues to increase, $\tilde{\Theta}$ eventually decreases. We say that the network is in a heavy congestion state.

Fig. 5.10 shows that SP routing leads to the earliest transition into a moderate congestion state because of its small value of λ_c. SA routing is the last that enters moderate congestion but it has the shortest duration of moderate congestion state before entering into heavy congestion. This is because when SP routing is adopted, at the start of network congestion, only a few nodes with a high node usage probability are congested and a small number of packets can still be successfully transmitted via other nodes which are not congested. As shown in Fig. 5.7, under SA algorithm, the upper edge of the node usage probability is quite well defined and the nodes with a relatively high node usage probability are spread over a very wide degree range, compared with other algorithms. Therefore, as the traffic load is more uniformly distributed in the network, most nodes become congested after the network enters a congestion state, thus causing a rapid decrease of the overall throughput.

5.4 Summary

For efficient data transmission, the traffic load should be as uniformly distributed as possible in the network and the average distance traveled by the data should be short. This fundamental criterion remains universal, and the key problem is therefore to find the optimal routing solution that can achieve a most balanced node usage in the network and the shortest average distance. While methods are non-exhaustive, we present one particular algorithm based on simulated annealing in this chapter. Simulation results show that the proposed SA algorithm outperforms the other three algorithms in terms of critical generation rate, average transmission time (unless under very low traffic intensity) and network throughput until the point of heavy congestion.

Chapter 6

Conclusions

In this chapter, we summarize the main contributions achieved in this project and give some suggestions for future research extensions.

6.1 Main Contributions of the Thesis

In the past decade, research on network science and its applications in engineering have received increasing interest. Computer and communication networked systems are among the most important infrastructures of today's society. Empirical studies have demonstrated that many real-life communication networks such as the Internet are heterogeneous networks and exhibit small-world and scale-free topological properties. Much previous work has shown that the underlying network structure is highly relevant to the traffic performance of a network.

In this thesis, we analyze the communication network performance from a complex network perspective. The key contributions can be summarized as follows.

1. Communication networks have been modeled, analyzed and characterized using complex network parameters, and the effects of network topology, routing strategy, and resource allocation on the communication performance have been

studied through theoretical analysis and extensive simulations.

In this thesis, we focus on a generic type of communication networks, in which packets of messages are sent from one node to another under practical operational conditions such as the use of packet buffering in communication nodes and the implementation of specific routing algorithms. In particular, we analyze networks of selected topologies that are of practical relevance, including regular lattices, Erdös-Rényi random networks (ER random), and Barabási-Albert scale-free networks (BA scale-free), and the real-world Internet constructed at the autonomous system (AS) level, and investigate the performance of these networks in terms of their intended functions of delivering information. Performance parameters, including packet drop rate, time delay, and critical generation rate, are considered.

2. The critical generation rate, which represents the maximum offered load that can be supported by the communication network without traffic congestion, has been derived analytically in terms of the node usage probability, average distance of the communication paths and allocated resources.

This analytical result suggests that the traffic load should be distributed as uniformly as possible in the network and the average path distance should be as short as possible to maximize the critical generation rate when resources are assigned uniformly. It also allows us to identify the optimal operating point in resource allocation.

3. The node usage probability has been proposed as a new metric for characterizing the traffic load distribution and how frequently a node is chosen to relay packets in a network.

Based on the concept of node usage probability, effective design strategies, including routing algorithms and resource allocation schemes, have been developed. The performance of the proposed minimum-node-usage routing algorithm has been compared with that of other popular routing algorithms. Node usage probability has been demonstrated to be the key consideration in network design, and any effective network design has been shown to necessarily involve minimization of the overall node usage for a given network topology. Our analysis and simulation results provide insights into how networks should be designed, including the choice of topology and the routing method, for achieving optimal network performance.

4. Network design strategies for optimizing the performance of communication networks has been proposed.

The performance of the communication networks can be optimized by minimizing the node usage probability and shortening the average transmission distance. With a fixed network topology, the traffic load distribution and the transmission efficiency are determined by the specific routing algorithm. However, the problem of finding all possible paths between two nodes in the network has been proven to be NP hard, and it is infeasible to find the optimal configuration of routing paths by evaluating all possible paths between each pair of nodes in a large and irregular network. Therefore, a simulated annealing algorithm has been adopted to find a near optimal configuration of routing paths, which effectively balances traffic loads and improves the overall traffic performance.

6.2 Suggestions for Future Research

Based on the research achievements presented in this thesis, we propose some possible directions of future research.

6.2.1 Analysis of a Flow Model

In this thesis, we model and analyze the communication network using a packet model in which the information or data has been presented as packets. In our future work, the traffic performance can be studied using a flow model in which the traffic is considered as continuous flows.

This macroscopic model might be more convenient for analytical study on the traffic performance with different network settings and theoretical optimization of network design.

Moreover, the transmission control protocols (TCP) could be taken into consideration to adjust the rate of data generation and transmission as a function of time delay and drop rate.

6.2.2 Traffic Generation Pattern

In most existing work, each node in a network is equally defined as either host or router to generate or deliver packets and the traffic generation rate of each node is identical.

However, many realistic communication networked systems are far from this assumption. For example, the Internet AS-level network consists of two kinds of nodes, namely, stubs and transits. Stub nodes can generate and receive packets, while transit nodes connected to stub nodes can only store and forward packets. In this way, the traffic load distribution is very sensitive to the locations of stub ASes. Moreover, in many real-world communication networks, the traffic generation rate of each node should be different. One attempt to model the uneven nature of traffic generation assumes the generation rate of each node to be proportional to its node degree, and thus high degree nodes in the network can generate more packets. However, this assumption is still too simple to reflect the real situation in real-life communication networks.

Therefore, a more realistic modeling of the traffic generation pattern is a problem worth of future exploration.

As demonstrated in this thesis, the efficiency and reliability of data transmission are closely related to the traffic load distribution in the network. For a given network topology, the traffic load distribution is determined by the selected routing algorithm as well as the traffic generation pattern. It is thus of interest to investigate the optimal traffic generation pattern for maximizing the network capacity.

6.2.3 Applications to Other Communication Networks

This thesis focuses on a wired and fixed packet switching network. A possible research direction in the future is the application of network science on other kinds of communication networks. The underlying network structure of different kinds of communication networks varies a lot. For example, the wireless Ad Hoc communication network has been demonstrated to be much more homogeneous than the wired communication network like the Internet. This is because each node in the wireless Ad Hoc communication networks can only communicate with the nodes close to it due to the power constraint. Moreover, new traffic models need to be proposed by incorporating practical operational conditions and protocols of the specific communication network.

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