MODELLING INDIVIDUAL AND HOUSEHOLD ACTIVITY-TRAVEL SCHEDULING BEHAVIOURS IN STOCHASTIC TRANSPORTATION NETWORKS

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Certificate of Originality

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_________Xiong Yiliang__________ (Name of student)

Abstract

The fundamental objective of transportation planning is to provide appropriate transportation facilities to meet future travel demand. Knowledge of travel demand can help improve the efficiency and sustainability of the existing transportation systems. Various approaches, such as network equilibrium models and simulation systems, have been proposed to predict travel demand. The activity-based network equilibrium approach for analysing travel behaviour and predicting travel demand has emerged over the last two decades. The activity-based network equilibrium approach covers a new class of models that predict where and when activities are pursued.

The main contribution of this thesis is the development of a modelling framework for the individual and household daily activity-travel scheduling behaviour. The existing activity-based network equilibrium modelling methodologies are extended in two directions: (1) developing a dynamic model for individual's daily activity-travel scheduling behaviour in congested networks, (2) taking into account the impact of intra-household interactions on daily activity-travel scheduling.

In previous activity-based network equilibrium models, travellers are assumed to care only about the utility that can be obtained immediately. Travellers would thus choose the activity that provides the highest immediate utility but would ignore the utility that could be obtained during the remainder of a day. The daily activity-travel schedule is thus composed of a sequence of repeated static choices over an entire day. In this thesis, a Markov Decision Process (MDP) model is proposed to capture two types of within-day dynamics in individual's activity-travel scheduling behaviour. Firstly, travellers take into account the expected future utility and perceive the impact of the current decision on subsequent activities and trips.

Secondly, activity-travel decisions have a strong dependency on contextual situations, such as time of the day and location.

The impact of travel time uncertainty on activity-travel scheduling is also considered in the proposed MDP model. For example, a sudden increase in travel time to the destination of a non-compulsory activity may reduce the probability of choosing this activity. The MDP model provides a framework for modelling inter-temporal activity-travel decisions in such an uncertain environment.

The development of a bridge between the activity choice behaviour and the long-standing network equilibrium models has attracted considerable research efforts. The supernetwork representation is adopted as a unified framework for modelling activity location, time of participation, duration, and route choice in congested networks. However, imposing simple constraints on the activity-travel schedules makes the supernetwork models computationally intractable. In this thesis, the supernetwork models are shown to be special cases of the MDP model. The computational burden can be alleviated by exploiting the special structure of the MDP model. Dynamic programming algorithms are developed to solve the MDP model without enumeration of all the possible activity-travel schedules.

In traditional travel demand models, travellers within a household are treated separately. Each household member makes activity-travel decisions independently. As a result, the estimation of joint activity participation can be biased. In this thesis, a household MDP model is proposed to explore the influence of intra-household interactions on individual's activity-travel scheduling behaviour. A utility function is adopted to represent joint household preference. The function consists of a weighted sum of each household member's utility, together with a term measuring the level of intra-household interactions.

The variation in intra-household interactions across activity types is thoroughly examined using the household MDP model. The intra-household interactions are considered as weak or negligible for compulsory activities, such as work and school. These activities are conducted with significant regularity and under strict spatial and temporal constraints. Joint non-compulsory activities are motivated by collaboration and companionship. The intra-household interactions are positive for these activities. For example, one household member's choice of activities, such as social visits and outdoor sports, is highly related to the other's choice. The intra-household interactions are negative for other non-compulsory activities, such as cooking and cleaning. If one household member has completed one such activity, the other does not have to undertake this activity but still benefits from its results. To minimize the effort spent on these activities, a household simply allocates each of these activities to one household member.

Publications Arising From the Thesis

The following papers were published during the author's MPhil study period:

1. Yiliang Xiong, William H.K. Lam. *An Activity-Based Approach for Estimation of Passenger O-D Matrix and Trip Chain Pattern*. World Conference on Transport Research 2010, Lisbon, Portugal.

2. Yiliang Xiong, William H.K. Lam. *Modelling Within-day Dynamics in Activity Scheduling: A Markov Decision Process Approach*. Journal of the Eastern Asia Society for Transportation Studies, Vol. 9 (2011), 452–467.

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Notations	Notes
S	The set of states
D	The set of decisions
C	The set of primitive choices
β	The discount ratio of the future utility
$p(s' \mid s,c)$	The state transition probability function in standard MDP
$p(s' \mid s,d)$	The state transition probability function in MDP with temporal abstraction
$r(s,c)$	The deterministic component of the immediate utility R in standard MDP
$r(s,d)$	The deterministic component of the immediate utility R in MDP with temporal abstraction
$\varepsilon(s,c)$	The random component of the immediate utility R in standard MDP
$\varepsilon(s,d)$	The random component of the immediate utility R in MDP with temporal abstraction
$G_{\varepsilon}(\cdot)$	The cumulative distribution function of ε
$\pi(s,c)$	A mapping from states to the probabilities of taking each choice in standard MDP
$\pi(s,d)$	A mapping from states to the probabilities of taking each decision in MDP with temporal abstraction
$V(s)$	The expected discounted utility at any state s
$Q(s,c)$	The expected utility of selecting any choice $c \in C(s)$ at state s in standard MDP
$Q(s,d)$	The expected utility of selecting any decision $d \in D(s)$ at state

	s in MDP with temporal abstraction
N	The set of activity programs, $\{1,2\ldots,N\}$
A^n	The set of activities in activity program n
T	The planning horizon, i.e., number of time episodes
t_s	The time component of state s, $t_s \in \{1,\ldots T\}$
w_s	The location of the traveler at state s
a_s	The on-going activity at state s
e_s	The remaining time of the on-going at state s
$\underline{t}_a, \bar{t}_a$	The earliest and latest time for activity a
W	The set of available activity destinations
$B(w)$	The set of available activities at location w
A_s	The set of the uncompleted activities at state s
$\overline{A}_s$	The set of the feasible activities that can be chosen at state s
a_d	The activity type component of decision d
h_d	The activity duration component of decision d
z_d	The activity location component of decision d
m_d	The travel mode component of decision d
y_r	A route in road network, denoted by $y_{d,r}$ if it is a component of decision d
y_b	A bus line in transit network, denoted by $y_{d,b}$ if it is a component of decision d
$\tau(t,w,z,m)$	The time needed to travel from w to z departing at time t
α_m	The equivalent disutility of unit travel time for travel mode m

$u(a,t)$	The discrete marginal utility of activity a at time t
$g_a(t)$	The continuous marginal utility of activity a at time t
σ_i	The weight parameter representing the relative influence of household member i
r_i	The utility of independent activity participation for household member i
r_J	The utility of joint activity participation for a household
ρ	The level of interaction between household members
G_s	The set of non-compulsory activities that can be taken by any household member at state s
$\mathcal{I}$	The set of states in which the on-going activity or travelling is completed, $\mathcal{I} \subset S$
$f(s)$	The number of traveller in state s
$f_l(t)$	The number of travellers choosing road link l
θ	The vector of model parameters to be calibrated
$l_i(\theta)$	The contribution of individual i to the log-likelihood function
s_{it}	The observed state of individual i at time t
d_{it}	The observed decision of individual i at time t

Chapter 1 Introduction

1.1 General background

The activity-based travel demand modelling systems commonly have hierarchical structures (Bowman and Ben-Akiva, 2001). These structures can be divided into two levels. The upper level involves activity generation and produces a set of activity programs. The lower level focuses on activity-travel scheduling behaviour, involving choices of activity destination, timing and duration.

The hierarchical structure of activity-based simulation models is analogous to the sequential steps in the four-step model, namely trip generation, trip distribution, travel mode choice and route choice. The simulation models generate activities, tours, and trips with flexible and complex cross-impacts on one another.

This thesis is motivated by the widely accepted concept that travel demand is derived from the need of participating in spatially separated activities. The travel demand is therefore treated as an outcome of activity choice. The goal of activity scheduling is to maximize the overall utility of engaging in activities. The activity utility varies over time of the day. For a given activity program, the sequence of activity participation is affected by the temporal profile of marginal activity utility.

Figure 1.1 illustrates the marginal utility of three activities for a normal weekday (24 hours). The optimal activity sequence is Home → Work → Shopping → Home. The optimal switching time between activities reflects the temporal profile of marginal activity utility. The departure time of each trip is then determined.

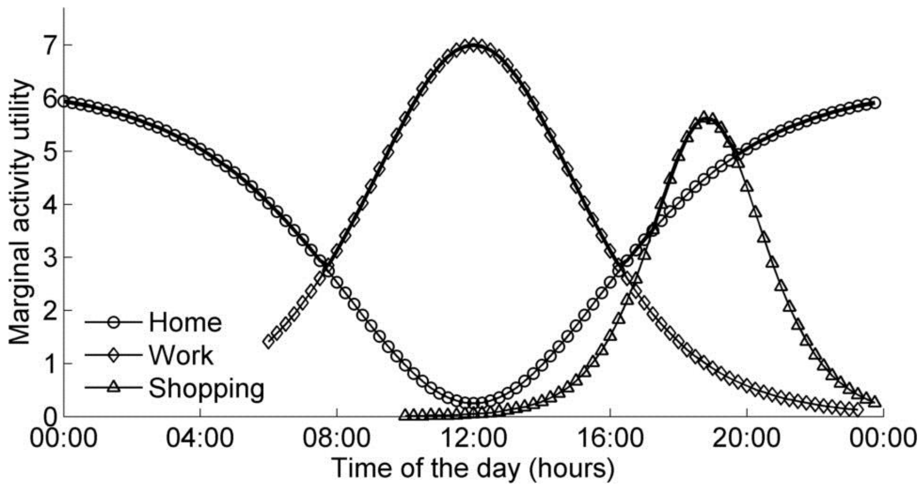

Figure 1.1 Temporal profile of the marginal activity utility

The second motivation is to develop a framework for modelling intra-household interactions associated with distinct activity types. The daily activities can be categorized into two types based on the flexibility of participation, the compulsory and non-compulsory activities. Firstly, the individuals' choices of compulsory activities, such as work and school, are independent from other household members. These activities are undertaken with significant regularity (every weekday) and under strict spatial and temporal constraints.

Secondly, non-compulsory activities, such as cooking, cleaning, grocery shopping and pick-up children from school, are usually allocated to one household member. Typically, a household is only willing to spend a minimum amount of time or the least effort to complete such activities. For example, either the husband or the wife is responsible for sending children to school.

Thirdly, collaboration and companionship motivate joint activity participation, such as social visits and having dinner outside with family. Joint participation is more efficient and satisfactory for household members than independent participation. Figure 1.2 shows that in the morning the husband escorts his wife to her workplace and then heads for his own workplace. In the evening, the husband has dinner with his wife.

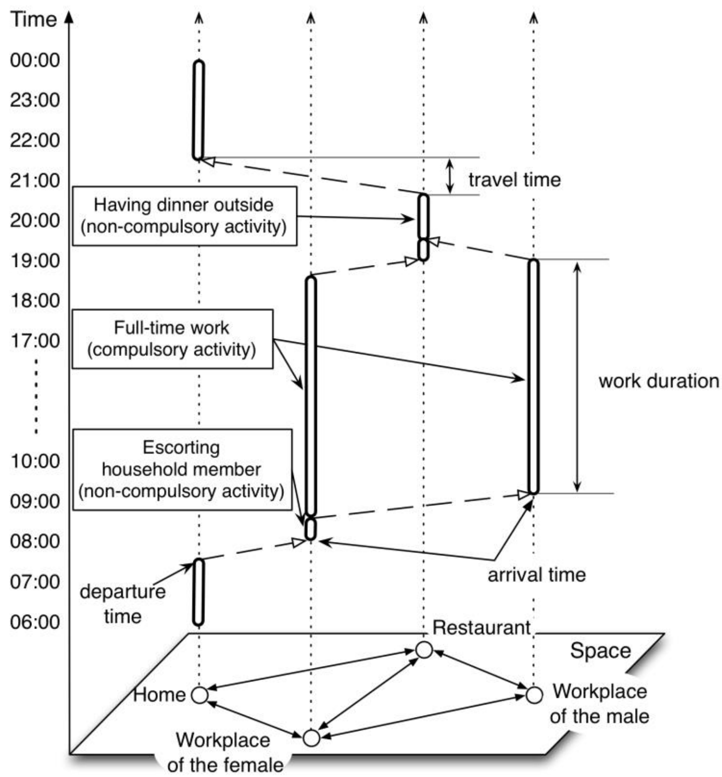

Figure 1.2 The household's daily activity-travel scheduling

1.2 Need for the study

The current development of the activity-travel scheduling models lacks a rigid and comprehensive modelling framework. Most current activity-based network equilibrium models cover only a few choice dimensions. The specification of choice dimensions is based either on the available travel survey data or on a relatively ad hoc method. Hence, this study sets out to develop a framework for modelling individual and household activity-travel scheduling behaviour in congested transportation network.

Daily activity-travel scheduling is a dynamic decision process. The dynamics in activity-travel scheduling have a two-fold meaning. First, travellers make activity-travel decisions repeatedly over time. The contextual situations at the decision epoch, such as time of the day and the traveller's location, play a crucial role in the decision-making process. Second, a rational traveller anticipates and evaluates the possible future consequences when making decision at the current moment. The traveller's present decision influences the future situations and thus has an impact on the future decisions.

It was recognised in the activity-based simulation models that the interaction between household members would influence one another's activity choices particularly in a congested transportation network. Certain types of activity can be assigned to a particular household member. Other household members would benefit from that household member's action. Household members also jointly participate in activities to obtain higher overall utility for the entire household.

The household's activity-travel scheduling behaviour is challenging for modelling framework development. A main challenge is the considerable variation in intra-household interactions across activity types. The intra-household interactions are negligible for compulsory activities, such as work and school, but play a central role in non-compulsory activities, such as social visits and outdoor sports. Another challenge is that the number of decisions available to a household with several members is greater than that of an individual. The larger decision space imposes a marked computational burden on the solution process.

1.3 Research objectives

The aim of this thesis is to incorporate within-day dynamics and intra-household interactions into the existing activity-based network equilibrium modelling methodologies. The main objectives are as follows.

1. To propose a unified and extensible activity-based network equilibrium modelling framework for activity-travel scheduling. This framework covers multi-dimensional choices and allows constraints imposed on these choices.

2. To capture the dependency of the traveller's choice on the contextual situations, and investigate how the expectation of the future affects the present decision.

3. To model and quantify the impact of intra-household interactions on individual household member's activity-travel decision.

4. To develop methods for calibrating model parameters, such as parameters of marginal activity utility function and the coefficients of intra-household interactions.

1.4 Structure of the thesis

The background and the motivations of this thesis have been described in Chapter 1. The remainder of this thesis is organized as follows. The interdependencies among chapters are illustrated in Figure 1.3.

Chapter 2 gives a review on literature on travel demand modelling. The evolution of travel demand modelling is briefly reviewed. The review then focuses on the relevant topics in activity-travel scheduling behaviour.

Chapter 3 describes the study methodology and preliminary knowledge that are required to understand the modelling methodology adopted in this thesis. An introductory example is presented to give some intuitions of the methodology.

Chapter 4 explores the with-day dynamics in individual's daily activity scheduling behaviour. Markov Decision Process (MDP) is employed as a unified framework for formulating individual's scheduling behaviour in a congested network. The supernetwork models are shown to be special cases of the MDP model.

Chapter 5 presents an MDP model with intra-household interactions, which is a direct extension of the individual's activity-travel scheduling model proposed in

Chapter 4. The impact of the intra-household interactions is analysed in a few numerical examples.

Chapter 6 introduces calibration methods for the individual and household activity-travel scheduling models. Numerical methods for solving the model calibration problem are optimized based on the mathematical property of the models. Hypothetical numerical experiments are conducted to generate time-series data for model calibration.

The final chapter summarises the findings and limitations of this thesis and gives directions for future research.

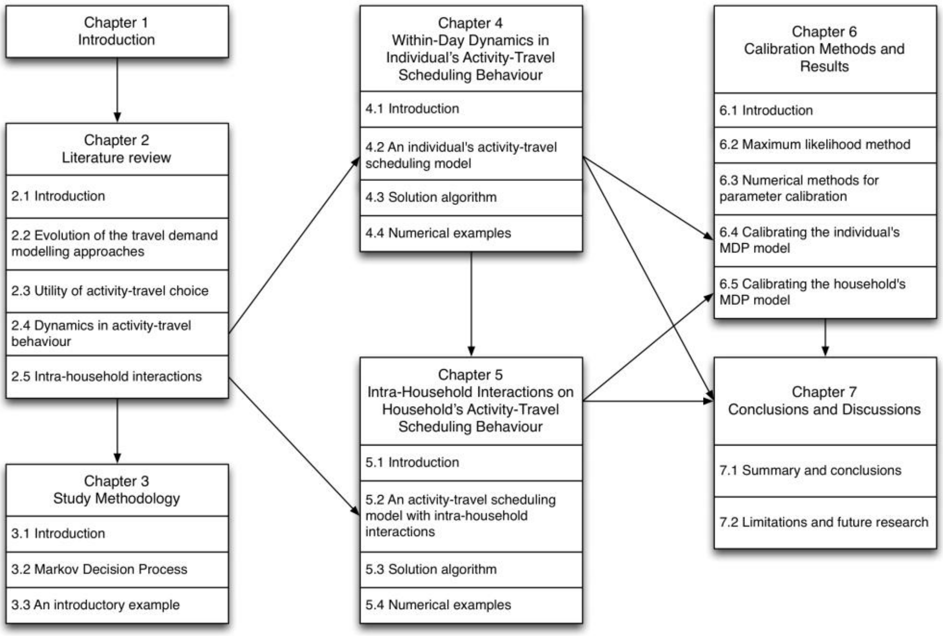

Figure 1.3 The interdependencies among chapters

Chapter 2 Literature Review

2.1 Introduction

A brief review on the evolution of the travel demand modelling approaches is presented at the beginning of this chapter. The review focuses on network equilibrium models for activity-travel choices. Since utility maximization theory is the standard framework for evaluating activity-travel choices, the concept of activity-travel utility is also reviewed.

The findings from empirical studies on dynamics in activity-travel scheduling are summarized in support of the dynamic behaviour studied in this thesis. The activity-travel scheduling behaviour can be viewed as a sequential decision process. Previous transportation studies on sequential decision process can be considered as prototypes of Markov Decision Process (MDP) models. The advantages of MDP models are explained in the context of activity-travel scheduling behaviour. Finally, previous studies on household intra-household interactions are reviewed to support the methodologies adopted in this thesis.

2.2 Evolution of the travel demand modelling approaches

The fundamental objective of transportation planning is to provide appropriate transportation facilities to meet the future travel demand (Lam et al., 2006; Li et al., 2008; Oppenheim, 1995). The knowledge of travel demand can help improve the efficiency and sustainability of the existing transportation systems (Hatzopoulou et al., 2007; Sheffi, 1985). Travel demand forecasting is a long-standing topic in transportation research. Various approaches have been proposed to modelling travel demand. Table 2.1 presents the features of three types of travel demand models. The evolution of travel demand modelling approaches is briefly reviewed in the remainder of this section.

Table 2.1 Comparison of travel demand modelling approaches

	Trip-based approach	Trip-chaining approach	Activity approach
Unit of analysis	Trip	Trip chain or Tour	Activity
Decision unit	Traffic analysis zone	Individual traveller	Household
Model structure	Sequential	Combined	Hierarchical
Solution algorithm	Numerical method	Heuristic method	Simulation*
Computational burden	Low	Intermediate	High

*: Most existing activity-travel scheduling models are based on simulation method, except for the activity-based network equilibrium models reviewed in Sections 2.2.1 and 2.2.2.

In late 1980s, the research efforts have been condensed into a complete and consistent modelling framework, the four-step method, shown as traditional approach in Table 2.1 (Oppenheim, 1995; Sheffi, 1985). The four-step method adopts *trip* as the basic unit of analysis and includes four sequential steps: trip generation, trip distribution, mode split and traffic assignment. The studied area is divided into several traffic analysis zones. In trip generation step, the production and attraction of trips in each zone are derived from the socio-economic characteristics of the zone. In trip distribution step, the number of trips between each pair of zones is computed. Trips between zones are required to be consistent with the production and attraction of each zone. In mode split step, discrete choice model is employed to determine the proportion of trips that are implemented by a specific travel mode (e.g., private car, bus). Finally, in traffic assignment step, these trips are assigned onto road links and transit routes in the transportation system. The travellers are assumed to choose the shortest or the minimum cost route.

Transportation network plays a central role in network equilibrium models. Travellers choose trip destination, departure time, travel mode and route based on the perception of travel cost in congested networks. These models are formulated as equivalent mathematical programming problems and solved with numerical methods (Florian and Hearn, 1995).

The trip-chaining model, shown as transitional approach in Table 2.1, is a direct extension of the four-step method. A trip chain is a set of ordered trips connecting consecutive stops, for example, Work → Shop → Home. The trip-chaining model takes account of the relationship between individual trips. Trips in the same trip chain influence one another in terms of total travel time and mode choice. For example, if a traveller drives to work in the morning, the traveller has to drive the car back home in the evening. The mode choice in the morning affects the choice in the evening. Since the emergence of activity-based simulation models, trip chain has been also adopted as a connection between activities and trips.

Abdelghany et al. (2001; 2003) implemented a system to simulate the traveller's choice of trip chains in a real-world transportation network. A user equilibrium solution was obtained through iterative simulation-assignment procedure. Maruyama and Harata (2005; 2007) formulated the trip-chaining behaviour as a convex nonlinear programming problem. Their trip chain choice model was built on the user equilibrium principle.

The activity-based simulation models, shown as existing approach in Table 2.1, for analysing travel behaviour and predicting travel demand emerged in the late 1980s (Jones, 1990; Kitamura, 1988; McNally, 2000). These simulation models focus on predicting where and when activities are pursued. The daily activities play a central role. Travel is considered as a demand derived from the need for participating in spatially separated activities. In the traditional four-step model, travel is a desirable activity on its own right.

The methodologies in the previous activity-based simulation models rely on a particular dataset or one perspective of travel behaviour rather than a unified modelling framework. The aspects of activity-travel behaviour modelling that have been examined are interesting but limited, such as the hierarchical structure of activity-travel choice (Bowman and Ben-Akiva, 2001), development of micro-simulation models (Abdelghany et al., 2001; Abdelghany and Mahmassani, 2003; Kitamura et al., 1996), applications of econometric theory (Bhat, 1996; Golob, 2000).

The network structure is ignored in many activity-based simulation models. Travel time between activity locations is treated as fixed and known. Travel demand is purely derived from activity participation. These models are commonly built on econometric theory (Bhat, 1996; Bowman and Ben-Akiva, 2001; Golob, 2000) or micro-simulation method (Arentze et al., 2000; Garling et al., 1994; Kitamura et al., 1996). Complicated activity-travel behaviour can be modelled in activity-based simulation models (Arentze et al., 2000; Garling et al., 1994; Kitamura et al., 1996; Roorda et al., 2008). However, most simulation models rely on ad hoc rules and lack a rigorous theoretical foundation.

2.2.1 Activity-based network equilibrium models

The activity-based simulation models and network equilibrium models have been independently developed for years. Separate groups of researchers investigate the travel demand forecasting problem from these two different perspectives (Kitamura, 1988; Mahmassani, 1988). Most activity-based simulation models lack a rigorous theoretical foundation and simulation must be repeated many times to obtain a stable solution. However, network equilibrium models have analytical forms and their solutions can be computed by numerical methods.

Considerable research efforts have been made on the development of a bridge between the activity-travel choice behaviour and the long-standing network equilibrium problem. Lam and Yin (2001) combined activity choice and the time-dependent network equilibrium model. The travellers are assumed to care only about the immediately utility obtained in a short period. The activity with the highest immediate utility is chosen in each period. Lam and Huang (2003; 2005) extended the combined model to dynamic and stochastic user equilibrium.

Lin et al. (2008) developed a conceptual framework by combining activity choice and dynamic network equilibrium. The network equilibrium model is used to generate dynamic travel time in their model. The travellers schedule daily activities with the knowledge of dynamic traffic conditions. Following this line of research, recent research efforts have been directed towards the development of a unified modelling framework.

2.2.2 Activity-based supernetwork equilibrium models

The supernetwork representation is a common framework for modelling joint travel choice among network equilibrium models, such as the joint mode split/distribution/assignment model (Sheffi, 1985). Recently, this framework is adapted to model dynamic activity-travel choices. The physical transportation networks are augmented with virtual nodes and links to represent activity-travel choices.

Ramadurai and Ukkusuri (2010, 2011) proposed a dynamic network equilibrium model to capture activity location, time of participation, duration, and route choice jointly. Virtual links are added to represent additional choice dimensions. The activity-travel schedules are represented by routes on the extended network.

Ouyang et al. (2010, 2011) defined the Activity-Time-Space (ATS) network by augmenting a physical transportation network. The nodes in the expanded network denote all the possible time and space choices for a traveller. There are two types of links in the expanded network. The activity link indicates the process of activity participation for one time episode. The road link represents travel between physical nodes. Each link is associated with a utility or disutility function, which specifies the preference of the traveller. The equilibrium flows are obtained by applying stochastic network equilibrium algorithms on the expanded network.

Each route in the ATS network is supposed to associate with an activity-travel schedule and thus, only the routes that satisfy all the conditions imposed on schedules are feasible. For example, it is reasonable to require that every daily activity-travel schedule begins with in-home activity and ends with in-home activity. There is however, no neat solution to the additional constraints in the framework of supernetwork representation.

2.3 Utility of activity-travel choice

Utility maximization theory is the standard framework for measuring the benefit and loss in most activity-based simulation models. An individual's activity-travel choice is viewed as a reflection of underlying utilities associated with each of the choice alternatives. The individual selects the choice alternative providing the highest utility.

2.3.1 Process utility and goal utility

The concepts of process utility and goal utility are proposed to express different form of benefits in activity participation (Axhausen and Garling, 1992; Winston, 1987). The process utility, which measures the utility that continuously derived during the activity participation, is related to activity timing and duration (see Figure 1.1). The goal utility, which represents the utility obtained by finishing the activity, is related to the consumption behaviour and/or the objectives achieved in the activity. The total utility of engaging in a certain activity is the sum of these two types of utilities.

Ettema and Timmermans (2003; 2004) proposed a temporal profile function for process utility of activity participation, which depends on activity timing and duration. It is suitable for activities like working and studying, where people spend time and derive utility in the process of conducting activities. However, for other activities, such as shopping and eating outside, the amount of good consumption during activity participation also affects the activity utility. Additionally, the marginal utility should diminish with the amount of consumption.

On one hand, modelling activity program formation should be based on goal utility measurement. On the other hand, the research of activity-travel scheduling focuses on the process of activity participation. Activity timing and duration plays a central role in activity-travel scheduling. Thus, the process utility is adopted in this thesis to represent the traveller's preference of activity-travel schedules. The overall utility of an activity-travel schedule is the sum of activity utility and disutility derived from travelling between activity destinations.

2.3.2 Utility of the inter-temporal choice

Instead of looking merely at the total time allocated to an activity, Winston (1982, 1987) investigated individuals' inter-temporal choice of consumption and time use. The activity timing and duration choices reflect the marginal utility of activity participation, which varies over time of the day (Supernak, 1992).

In this thesis, the utility of inter-temporal activity-travel choice is based on the discounted utility model (Charypar and Nagel, 2005). The value of a sequence of inter-temporal choices $d = (d_1, \ldots, d_T)$ is expressed by the weighted utility formula:

$$V(d) = \sum_t w_t \cdot u(d_t) \tag{2.1}$$

Equation (2.1) implies that whenever two sequences of choices differ in only two periods, the preference over them does not depend on the choices in other periods. Two additional assumptions are widely employed to evaluate inter-temporal choice:

(1) The weights, w_1, w_2 , and so forth, are declining, which indicates the individual values the utility derived in earlier periods more than that in later periods.

(2) The marginal rate of discounting between any two consecutive periods is the same $w_{t+1}/w_t = \beta$. This assumption leads to the discounted utility with constant-rate, $V(d) = \sum_t \beta^t \cdot u(d_t)$.

2.4 Dynamics in activity-travel behaviour

In recent years, the dynamics in activity-travel behaviour attract considerable research effort. Activity-travel choices are found to have strong dependency over time and history of activity participation. Two types of dynamics in activity-travel behaviour are presented in Section 2.4.1 and 2.4.2. Most of these studies are based on simulation method, but their findings can be used to support the activity-travel scheduling behaviour studied in this thesis.

Most repeated activity-travel choices over time can be viewed as a sequential decision process. The idea of sequential decisions has been found in transportation research for a long time. In this thesis, the activity-travel scheduling behaviour is modelled as a Markov decision process. The modelling methodologies for repeated activity-travel choices over time are reviewed in Section 2.4.3 and 2.4.4.

2.4.1 Within-day dynamics

One aspect of the within-day dynamics in activity-travel scheduling is that most activities are pursued once in a day, and thus, the current choice of activity depends on the history of activity participation (Kasturirangan et al., 2002). Generally, the choice of activity at present is closely related to the state of the traveller, such as the remaining time for out-of-home activities and the activities pursued before the choice.

Habib (2010) proposed an econometric modelling framework for daily activity program generation. Composite activity is introduced to integrate all the activities in the rest of the day. Travellers balance the time allocated to a specific activity and the composite activity in order to maximize the overall utility that can be obtained in the entire day.

2.4.2 Day-to-day dynamics

Cirillo and Axhausen (2010) formulated activity-travel choices in different days as a discrete choice model with correlation between inter-temporal choices. The present activity utility is assumed to depend on the past choice of activity type and the number of days passed since last activity participation. For example, the weekly grocery shopping is conducted once a week. The purchased food and other products are consumed in the next a few days. Thus, the probability of making another shopping trip increases with the number of days passed since the last grocery shopping.

A need-based model of multi-day, multi-person activity generation is proposed in Arentze and Timmermans (2009). The concept of need represents the amount of daily goods or the emotional desire for social activity. The latent attributes, such as the individual’s need and time pressure, influence the activity utility. The activity utility is assumed to vary over time since the need for the activity differs over time.

Accordingly, the utility of pursuing an activity is a function of the individual's needs. Needs grow over time and when a threshold is reached, an activity is selected to satisfy the needs. Pursuing an activity can satisfy several needs, for example, having dinner with a friend fulfils the needs of food and social gathering.

2.4.3 Sequential decision process

In transportation literature, the idea of sequential decisions was first proposed by Dial (1971). In Dial’s paper, the route choice problem was considered as a sequence of choices of links. The Dial’s network loading algorithm was proved equivalent to a logit route choice model, which was criticized for the restraints imposed by the IIA property. In addition, the algorithm only applies to a subset of the routes that are composed of the predefined efficient links.

Kitamura (1984) proposed "prospective utility" as the measurement of attractiveness of traffic zones. The attractiveness of a zone is not only determined by the accessibility and attributes of the zone but also by other opportunities that can be reached from the zone. For example, if a traveller drives to a shopping mall in the downtown, the traveller takes into account the utilities that can be obtained from adjacent activity centres. The prospective utility of a zone is thus defined as the weighted sum of the utilities that can be obtained from the zone and from potential future trips.

Baillon & Cominetti (2008) employed dynamic programming to solve the route choice problem. The traveller chooses a link to follow at every intermediate node on the trip to the destination. Route choice is the outcome of the sequential decision process. Distinct discrete choice models can be adopted at every node. User equilibrium and stochastic user equilibrium are two special cases that fit into this unified modelling framework.

2.4.4 Markov decision process

Traditionally, analytical models in transportation literature are formulated in the form of nonlinear programming and variational inequality problems (Dafermos, 1980; Sheffi, 1985). These theories provide general frameworks for formulating and solving various network equilibrium models. Since this thesis focuses on the within-day dynamics in activity-travel scheduling, Markov Decision Process (MDP) is a more expressive framework for formulating the complicated inter-temporal choices in activity-travel scheduling (Puterman, 1994). Furthermore, MDP can be integrated with network equilibrium models without of difficulty. The activity-based supernetwork equilibrium models are shown to be special cases of the MDP model for activity-travel scheduling in Section 4.2.8.

The advantage of this approach is that it does not need to consider each activity-travel schedule individually. There is thus no need to enumerate all the possible schedules. Another feature of the MDP formulation is that it takes into account the expected utility that can be obtained in the near future. The traveller makes optimal decision at present with the knowledge that he will also make optimal decisions in the future. Usually, this behaviour is hard to formulate in the other modelling frameworks.

Comprehensive reviews on MDP models can be found in (Aguirregabiria and Mira, 2010; Eckstein and Wolpin, 1989; Rust, 1994). The structural dynamic discrete choice models are built upon the framework of the discrete Markov decision processes proposed by Rust (1994). Previous applications of MDP in activity scheduling behaviour can be found in Jonsson and Karlström (2005). Charypar and Nagel (2005) applied a similar model from computer science. Recently, Cirillo and Xu (2011) presented an extensive review of MDP models from the perspective of transportation research.

2.5 Intra-household interactions

Discrete choice models are widely employed in the activity-based travel demand modelling literature. An important assumption usually adopted in discrete choice models is that each individual's choice is independent of that of other individuals (Ben-Akiva and Bierlaire, 1999; Train, 2003). This assumption is, however not satisfied in the context of household activity-travel scheduling. The interdependencies between household members influence the activity participation of each household member.

Even though the household consists of different individuals, the household is commonly assumed to act as a single decision-making unit in this thesis. Hence, the household activity choice fits into the modelling framework of discrete choice. A household utility function can be proposed to capture the joint household preference with consideration of the intra-household interactions (Bradley and Vovsha, 2004; Zhang et al., 2009). The household utility function includes the weighted sum of each household member's utility. Additional terms are incorporated to capture the intra-household interactions. A general form of the household utility function can be expressed as:

$$U(v) = v'Av + b'v \tag{2.2}$$

where $v = (v_1, v_2, \ldots)'$ is the vector of individual's activity utility. A is a symmetric matrix with entries $\rho_{ij}\,(i, j = 1, 2, \ldots)$ which measures the level of intra-household interactions. Weight parameter $b = (\sigma_1, \sigma_2, \ldots)'$ represents the relative influence of household member.

Miller and Roorda (2003) and Roorda et al. (2008) presented comprehensive simulation models for household activity-travel scheduling. A synthetic population is used to represent the households for an urban region. Detailed activity-travel schedules over twenty-four hours are generated for all individuals in a household in the simulation. Activity frequency, starting time and duration are drawn from probability distributions. The activity episodes are scheduled based on predefined rules. Intra-household interactions are captured through the generation of joint activity episodes in which more than one person in the household participate in.

To capture the conflicts and bargaining within household, game theory is widely adopted in microeconomic literature (McElroy and Horney, 1981). A household is considered as a group of individuals with different preferences, among whom a bargaining process takes place to resolve the conflicts on activity-travel choices. In the cooperative framework, household members come to an agreement on allocation of welfare within household. Although game theory can be used to capture a rich range of activity-travel behaviour, modelling the conflicts and bargaining among household members are out of the scope of this thesis.

2.6 Summary

The literature related to the methodology and basic concepts in this thesis are reviewed in this chapter. The evolution of activity-based travel demand modelling and the main approaches for analysing activity-travel behaviour are reviewed in Section 2.2. Then the concept of activity-travel utility is introduced in Section 2.3. The literature on dynamics in activity-travel behaviour and intra-household interactions are described in Sections 2.4 and 2.5. These two sections present the background of the individual's and household's activity-travel scheduling models proposed in Chapter 4 and 5.

Chapter 3 Study Methodology

3.1 Introduction

The basics of the adopted mathematical modelling framework are presented in this chapter. Markov Decision Process (MDP) for modelling dynamic behaviours is described in Section 3.2. An introductory example on departure time choice is presented in Section 3.3. This example illustrates the application of MDP in travel behaviour modelling.

3.2 Markov decision process

Nonlinear programming provides a general framework for formulating and solving optimization problems. Similarly, MDP provides a structure within which optimal control of dynamic systems can be formulated and solved (Puterman, 1994). MDP is extensively documented in economic and marketing literature (Aguirregabiria and Mira, 2010; Eckstein and Wolpin, 1989). In this thesis, the MDP model is employed to investigate the dynamic behaviour in activity-travel scheduling.

Travellers make activity-travel choices repeatedly over time. The choices depend on the contextual situations, such as time of the day and the traveller's location. The contextual situations can be represented by state in the MDP model. A rational traveller attempts to anticipate the future situations and gain more enjoyment out of the day. This behaviour is captured by the objective of the MDP model. With appropriate definition of state, state transition and objective function, the activity-travel scheduling behaviour is grounded in a rigorous mathematical framework and viewed from a broader perspective.

3.2.1 Modelling framework

MDP models can be categorized into two main groups based on how the time is modelled: the discrete-time MDP and the continuous-time MDP. In discrete-time MDP model, the planning horizon is divided into equal periods. It is reasonable to assume that the traveller takes activity-travel choices at discrete decision epochs and thus, discrete-time MDP model is adopted in this thesis.

A discrete-time MDP model consists of the following components:

(1) A time episode used to index the decision process $k = 1, 2, 3, \ldots$.
(2) A set of states $s_k \in S$.
(3) A set of primitive choices $c_k \in C(s_k)$, let $C = \bigcup_{s \in S} C(s)$ denote the union of the choice sets.
(4) A state transition probability function $p : S \times S \times C \to [0,1]$.
(5) An immediate utility function $r(s_k, c_k)$.
(6) A discount ratio for future utility β.

The MDP model describes the behaviour of a traveller who makes choices with multiple components repeatedly over time. The time episode k is used to index the choices. At any time episode k, the traveller perceives the contextual situations that is represented by state s_k and selects a primitive choice $c_k \in C(s_k)$. The choice dimensions include the activity to be pursued, the location of the activity, etc. After making a choice, the traveller implements it immediately. The traveller receives the immediate utility derived from the choice $r(s,c) = E[r_k \mid s_k = s, c_k = c]$. Then the state of the traveller advances to s_{k+1} in the next time episode $k+1$.

As the state transition depends on (s,c), the subsequent state is thus written as $s' \mid s, c$ to indicate this dependency. The state transition is modelled by the probability function $p(s'|s,c) = \Pr[s_{k+1} = s'|s_k = s, c_k = c]$, where the sum of probabilities over all states is one $\sum_{s' \in S} p(s'|s,c) = 1.0$.

A *Markov policy* is a mapping from states to probabilities of taking each primitive choice $\pi : S \times C \to [0,1]$. The traveller's objective is to find a Markov policy that maximizes the expected discounted utility at any state s:

$$\begin{aligned} V^{\pi}(s) &= E\left[r_k + \beta r_{k+1} + \beta^2 r_{k+2} + \cdots \middle| s_k = s \right] \\ &= E\left[r_k + \beta V^{\pi}(s_{k+1}) \middle| s_k = s \right] \\ &= \sum_{c \in C(s)} \pi(s,c) \left[r(s,c) + \beta \sum_{s'} p(s'|s,c) V^{\pi}(s') \right] \end{aligned} \tag{3.1}$$

where $\pi(s,c)$ is the probability of choosing $c \in C(s)$ in state s. $V^{\pi}(s)$ is the value of state under policy π and V^{π} is called the state value function for π. The optimal state value function gives the maximum expected discounted utility at each state under an optimal policy:

$$\begin{aligned} V^{*}(s) &= \max_{c \in C(s)} E\left[r_k + \beta V^{\pi}(s_{k+1}) \middle| s_k = s, a_k = a \right] \\ &= \max_{c \in C(s)} \left[r(s,c) + \beta \sum_{s'} p(s'|s,c) V^{*}(s') \right] \end{aligned} \tag{3.2}$$

Given an optimal state value function V^*, an optimal policy π^* can be constructed by selecting any choice c that yields the maximum in (3.2). That is setting $\pi^*(s,c) = 1.0$ and $\pi^*(s,c') = 0.0$ for any $c' \neq c$.

A similar value function for state-choice pair gives the expected utility of selecting any choice c in each state s under policy π:

$$\begin{aligned} Q^{\pi}(s,c) &= E\left[r_k + \beta r_{k+1} + \beta^2 r_{k+2} + \cdots \middle| s_k = s, c_k = c \right] \\ &= E\left[r_k + \beta V^{\pi}(s_{k+1}) \middle| s_k = s, c_k = c \right] \\ &= r(s,c) + \beta \sum_{s'} p(s'|s,c) \sum_{c'} \pi(s',c') Q^{\pi}(s',c') \end{aligned} \tag{3.3}$$

In addition, the optimal action value function is:

$$Q^*(s,c) = r(s,c) + \beta \sum_{s'} p(s' \mid s,c) \max_{c'} Q^*(s',c') \qquad (3.4)$$

Boundary condition has to be specified to terminate the recursion in (3.1). Thus, (3.1) applies for all state, but

$$V(s) = 0, \quad s \in S_* \qquad (3.5)$$

where S_* is a set of absorbing states in which all decisions have no effect. That is, when a traveller is in an absorbing state, all decisions immediately return to that state with a zero utility. For example, $(T, \texttt{Home})$ is a reasonable absorbing state in activity-travel scheduling.

To make sure travellers arrive home before or at *T*, $V(s)$ is set to $-\infty$ for any s such that $t_s = T$ and $a_s \neq \texttt{Home}$. That is, the utility of making decisions that lead to these states is negative infinity. The utility of choosing to return home is zero, as specified in (3.5), but greater than any other choices. This is equivalent to imposing a constraint on decisions to prohibit any out-of-home activities at midnight. Actually, the latest hour of returning home can be any reasonable time of the day, such as 3:00am.

3.2.2 Markov property

The *state* is composed of several components, such as the traveller's location (i.e., nodes in the transportation network) and the activity in which the traveller is engaged. The detailed specification of state is presented in Section 4.2.5.

The state of MDP model satisfies the Markov property. That is given the current state and choice, (s_k, c_k), the subsequent state s_{k+1} is conditionally independent of all the states and choices at time before k . In terms of conditional probability, Markov property is expressed as:

$$\Pr\left(s_k \left| s_{k-1}, c_{k-1}, \ldots, s_1, c_1\right.\right) = \Pr\left(s_k \left| s_{k-1}, c_{k-1}\right.\right) \tag{3.6}$$

Nevertheless, satisfying Markov property does not require the current decision to be irreverent to the past activity-travel choices. Two typical impacts of the past events on the successive decisions are discussed as follows.

For example, travellers are assumed to participate in any activity once and only once. Then the set of completed activities needs to be maintained and updated at each decision epoch. This dependence is handled by introducing an auxiliary set into the state. Further discussion is given in Section 4.2.5.

Another example is that, if a traveller drives to the office in the morning, the traveller must drive the car back home after work. The travel mode choice in the morning determines the choice after work. In this case, a binary variable is incorporated into the state to represent the availability of the car.

In summary, the impact of the past choices can be introduced into the state. The set of feasible choice at present depends on the current state. Thus, the state serves as an intermediate layer between the past choices and the successive choices. In other words, the past choices make an impact on the successive choices through the state.

3.2.3 Solution algorithms

The standard solution algorithms for solving the MDP model are value iteration and policy iteration (Puterman, 1994). The value iteration method is obtained by turning the Bellman optimality equation (3.1) into an update rule. Because of its simplicity, the value iteration method is adopted.

The value iteration method converges to an optimal solution for the discrete-time MDP with finite horizon. In practice, the algorithm stops once the expected utility changes by only a small amount in iteration. Figure 3.1 gives a complete value iteration algorithm with this termination criterion. The outputs of the algorithm are optimal decision $\pi(s)$ and the expected utility $V_k(s)$ for any feasible state s at iteration *k*.

for each state $s \in S$ **do**
    set $V_0(s) \leftarrow 0$
set $k \leftarrow 0$
repeat
    set $k \leftarrow k+1$
    for each state $s \in S$ **do**
        set $V_k(s) \leftarrow \max_{c \in C(s)} \left\{ r(s,c) + \beta \sum_{s'} p(s' \mid s,c) V_{k-1}(s') \right\}$
until $\left| V_k(s) - V_{k-1}(s) \right| < \epsilon$
for each state $s \in S$ **do**
    set $\pi(s) \leftarrow \arg\max_{c \in C(s)} \left\{ r(s,c) + \beta \sum_{s'} p(s' \mid s,c) V_{k-1}(s') \right\}$
return V_k and π

Figure 3.1 Value iteration method for standard MDP model

3.3 An introductory example

A departure time choice problem is presented as follows to illustrate the essential ideas of MDP. Figure 3.2 shows a 2-node transportation network: W represents workplace and H represents home of a traveller. The traveller has to arrive home at or before 00:00am. The departure time choice is made to maximize the overall utility. If the traveller returns home, he will not go back to work.

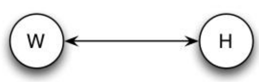

Figure 3.2 A 2-node transportation network

Let $\tau(t)$ denote the time needed to travel from W to H if the traveller departs at t o'clock. In this example, the travel time $\tau(t)$ is fixed at *1* for any $t = 1, \dots, 12$. In another word, travel time does not vary with departure time in this example. This assumption is made for simplicity and will be relaxed in Chapter 4 and 5. In addition, $r(t, W)$ and $r(t, H)$ denote the marginal utility obtained from one hour of work and in-home activity at t o'clock. Table 3.1 presents the marginal activity utility from 01:00pm to 00:00am.

Table 3.1 Marginal activity utility at each time episode

Time episode (t)	Time of the day	Marginal activity utility	
		In-home $r(t,H)$	Work $r(t,W)$
1	01:00pm	5	20
2	02:00pm	5	20
3	03:00pm	5	20
4	04:00pm	5	20
5	05:00pm	7.5	17.5
6	06:00pm	10	15
7	07:00pm	12.5	12.5
8	08:00pm	15	10
9	09:00pm	15	7.5
10	10:00pm	15	5
11	11:00pm	15	5
12	12:00am	15	5

3.3.1 Enumeration approach

Define $U(t)$ as the overall utility that the traveller can obtain from 01:00pm to 00:00am if the work ends at t o'clock. Consequently, the overall utility is expressed as the sum of the utilities derived from work and in-home activity, and the travel disutility:

$$U(t) = \sum_{i=1}^{t} r(i,W) + \alpha \cdot \tau(t) + \sum_{i=t+\tau(t)+1}^{12} r(i,H) \tag{3.7}$$

where α denotes the equivalent disutility of unit travel time. For demonstration, the value of α is set to -5.0.

The optimal solution can be obtained by enumerating all the possible departure times. Figure 3.3 shows that the overall utility reaches the maximum value at 6 o'clock. When finer time scale is of interest and more choice dimensions are included, enumerating all the choice combinations is computationally impossible.

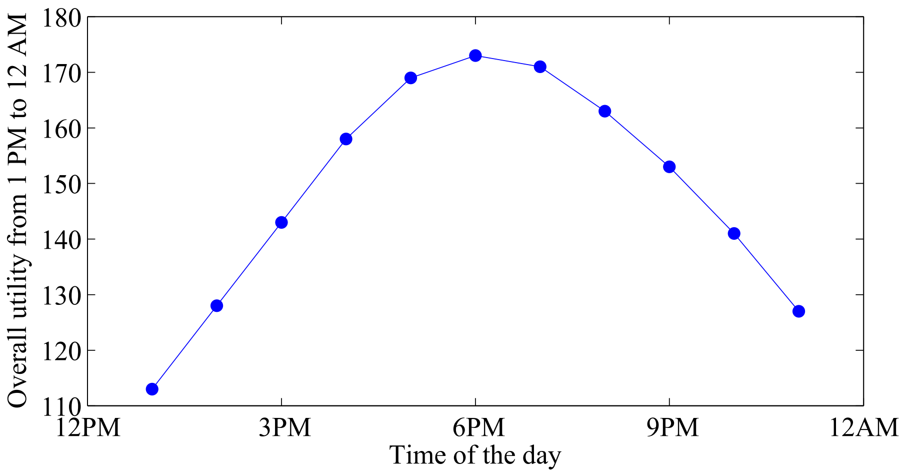

Figure 3.3 The overall utility for all possible departure times

3.3.2 Recursive method

Another approach is to formulate the departure time choice problem as a MDP model and solve it with a backward recursive method. At the beginning of each time episode, the traveller chooses the activity to be pursued in the following hour. The state has two components: the time episode and the traveller's location.

The backward recursive method starts at the last time episode. The only state that the traveller can have in the last time episode (12am) is at home. This is an absorbing state and thus, $V(12, H)$ is set to zero. At 11:01pm, there are two possibilities: the traveller is at work or at home. If the traveller is at work, the only choice is to depart for home immediately so that he can arrive home at 12:00am. In this case, the utility obtained between 11:01pm and 00:00am is $V(11, W) =$

$-\alpha \cdot \tau(11)+V(12,H)$. The other possibility is that the traveller already stays at home at 11:01pm. Then the traveller engages in in-home activity for the next hour. The utility derived from in-home activity between 11:01pm and 00:00am is

$$V(11,H)=r(11,H)+V(12,H) \tag{3.8}$$

The next solution step is to move back one time episode to 10:01pm. If the traveller has not returned home by 10:01pm, the traveller has two choices: continue at work or depart for home. The maximum utility that can be obtained in the next two hours is expressed as a maximization problem over the two choices:

$$V(10,W)=\max\begin{cases} r(10,W)+V(11,W), & \text{if continue to work} \\ \alpha \cdot \tau(10)+V(11,H), & \text{if depart for home} \end{cases} \tag{3.9}$$

Similarly, the other possibility is that the traveller already stays at home at 10:01pm. The utility derived from in-home activity from 10:01pm to 12:00am is $V(10,H)=r(10,H)+V(11,H)$.

Following the same procedure, the successive solution steps are performed by moving one time episode back in each step until the first time episode is reached. The entire solution process is described by recursive equations:

$$V(t,W)=\max\begin{cases} r(t,W)+V(t+1,W), & \text{if continue to work} \\ \alpha \cdot \tau(t)+V(t+\tau,H), & \text{if depart for home} \end{cases} \tag{3.10}$$

and

$$V(t,H)=r(t,H)+V(t+1,H) \tag{3.11}$$

3.3.3 Random errors

In the real world, each traveller can have different perceptions of utility. To capture the perception error, a random term is incorporated into the utility, denoted by

$\varepsilon(t)$. $\varepsilon(t)$ is independently and identically distributed (i.i.d.). The expected maximum utility at t is expressed as:

$$V(t,W) = E\left[\max\begin{Bmatrix} r(t,W)+\varepsilon(t,W)+V(t+1,W) \\ \alpha\cdot\tau(t)+\varepsilon(t,H)+V(t+\tau,H) \end{Bmatrix}\right] \tag{3.12}$$

If $\varepsilon(t)$ is assumed an i.i.d extreme value random variable, (3.12) has the form of a logit model:

$$V(t,W) = \frac{1}{\theta}\log\left(e^{\theta\cdot(r(t,W)+V(t+1,W))} + e^{\theta\cdot(\alpha\cdot\tau(t)+V(t+\tau,H))}\right) \tag{3.13}$$

Suppose that there are 1,000 travellers. They choose departure time based on the activity utility and travel time given in Table 3.1. The perception parameter for extreme value random variable $\varepsilon(t)$ is set to $\theta = 0.1$.

Figure 3.4 shows the expected maximum future utility at each time episode. From 1:00pm to 6:00pm, the expected maximum utility of *Work* is greater or equal to that of *Home*. From 7:00pm to midnight, travellers can obtain more utility by staying at *Home*.

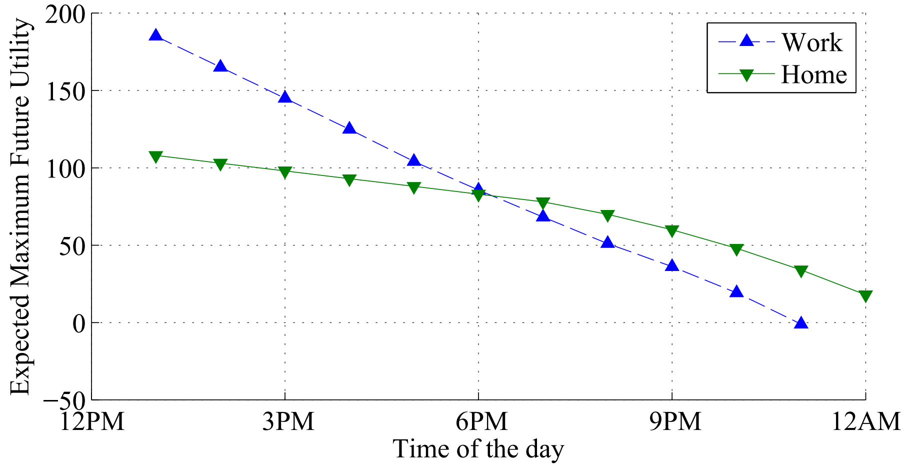

Figure 3.4 Expected maximum future utility over time of the day

Figure 3.5 presents activity participation of the entire day. The population at *Home* and *Work* reflect the travellers' activity choices. The activity choices are determined by the utility profiles shown in Figure 3.4. The expected maximum future utility of staying at *Home* surpasses that of *Work* around 6:00pm. Travellers start to depart for *Home* at that time. Indicated by the light green bars in Figure 3.5, the departure flow from *Work* to *Home* reaches the maximum value at 7:00pm.

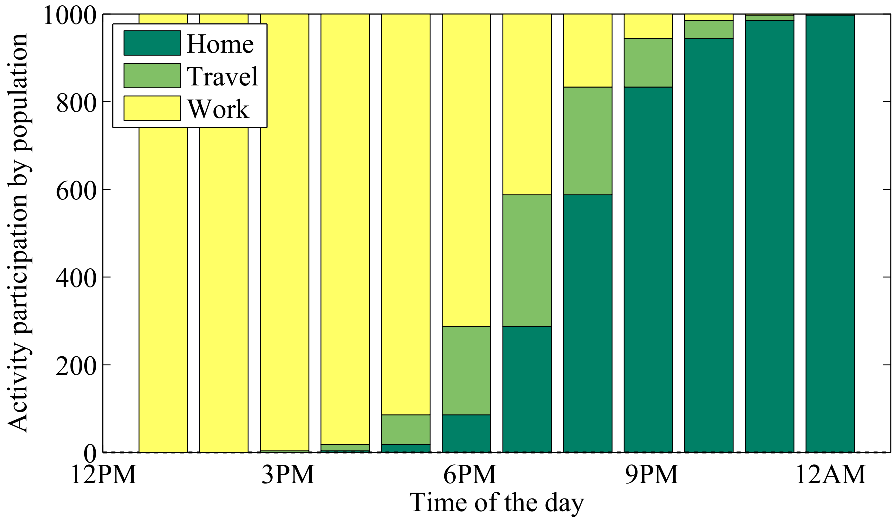

Figure 3.5 Activity participation of the entire day

3.4 Summary

The essentials of MDP modelling framework have been described in this chapter. The basic concepts of MDP, such as immediate utility, feasible state and decision, are illustrated and discussed in the introductory example.

The recursive method is more efficient than the enumeration approach, especially for larger transportation networks and multi-dimension choice (e.g. combined activity duration/timing choice and activity type/destination choice) problems. Let $|D|$ denote the number of choices and $|S|$ denote the number of states. The computational complexity of the recursive method is proportional to $|D| \times |S|$. The computational complexity of the enumeration approach is proportional to the number of all possible activity-travel schedules $\left|S\right|^{\left|D\right|}$.

Chapter 4 Within-Day Dynamics in Individual's Activity-Travel Scheduling Behaviour

4.1 Introduction

The departure time choice model presented in Section 3.3 demonstrates an elementary application of Markov Decision Process (MDP). An MDP model of activity-travel scheduling, involving activity type, destination, timing and duration choices, is formally proposed in this chapter. The purposes of the proposed model are to capture within-day dynamics in activity-travel scheduling and thus, to replicate the traveller's daily activity-travel schedules.

The within-day dynamics in activity-travel scheduling include two types of behaviour. Firstly, the traveller takes into account the expected future utility at current decision and realizes the impact of the current decision on the future utility. This is the so-called forward-looking behaviour. Secondly, the activity-travel decisions have a strong dependency on contextual situations, such as time of day and location. This behaviour exhibits state dependence in the activity-travel scheduling.

In previous time-dependent activity-based network equilibrium models, travellers are assumed to care only about the current immediate utility and not take into account of the future utility (Huang and Lam, 2005; Lam and Yin, 2001). At each period, the traveller would take the choice with the highest immediate utility.

These assumptions are not consistent with the activity-travel behaviour in reality. Firstly, the traveller is able to adjust the daily activity-travel schedule to maximise the overall utility of the entire day. Second, the current decision partly determines the subsequent state of the traveller and thus, affects the utility that can be obtained in the near future. For example, if a traveller has done grocery shopping for food, the traveller will have enough inventory of food for consumption. Thus, the traveller needs not to go for shopping in the remainder of the day.

Following this line of reasoning, it is reasonable to assume that a traveller would take into account the future utility when making a decision at the current decision epoch. The traveller cannot foresee the exact future states and does not know the future utility with certainty. However, the traveller knows that in future decisions the activity-travel schedule can be adjusted to achieve the highest level of overall utility. Hence, although the traveller does not know the exact value of that future utility, he can make decisions based on the expectation of a maximum future utility. This consideration reflects the traveller's forward-looking behaviour.

The remainder of this chapter is organized as follows. Section 4.2 presents the activity-travel scheduling behaviour in the form of an MDP model. Section 4.3 describes the solution method for the MDP model. Section 4.4 illustrates the proposed model with numerical examples. The final section summarizes the key points of this chapter.

4.2 An individual's activity-travel scheduling model

The activity-based models commonly have hierarchical structures. These complicated structures can be divided into two levels. The upper level is concerned with activity participation over a planning horizon and generates alternative activity programs. Each activity program is composed of a set of activities to be completed during the planning horizon. For example, {*Home (before work), Work, Shopping, Home (after work)*} is an activity program for a typical weekday. The common activity programs can be extracted from activity-travel diary data. Activity programs used in the following discussion and numeric examples are hypothesized and designed for illustration purpose only.

The lower level is the implementation of the activity programs in a transportation network, involving multi-dimension choices, namely activity duration, destination and route choice. For each activity program, the multi-dimension choices in the lower level constitute a complete activity-travel schedule. For example, a daily activity-travel schedule can be, depart home at 8:00am, arrive at the office at 9:00am, work in the office until 6pm, have dinner outside and head for home at 9:00pm.

This chapter focuses on the within-day dynamics in activity-travel scheduling behaviour. The set of activity programs is assumed predefined. Other long-term choices, such as workplace and residential location, are treated as fixed and known.

4.2.1 Assumptions

Three assumptions are adopted in this chapter to regulate the individual's daily activity-travel schedules.

A1. If a traveller has pursued an activity, the traveller will not re-engage in the activity within the planning horizon (Kasturirangan et al., 2002). Thus, each activity occupies a continuous period, represented by consecutive time episodes. The in-home activity is an exception, because a traveller may engage in the activity multiple times in a day. This exception can be handled by including multiple in-home activities in the activity set A. Specifically, two types of in-home activity are defined: the one occurring in the morning period $\texttt{Home-AM}$ and the other in the evening period $\texttt{Home-PM}$.

A2. If a traveller chooses to return home, the traveller must have completed all the out-of-home activities and will stay at home in the remaining time of the day. In other words, the activity-travel schedule of the traveller is a tour originating from home and terminating at home (Wen and Koppelman, 2000).

A3. A traveller terminates the current activity under two conditions: (i) voluntarily switch to another activity for higher overall utility, (ii) or exogenously be disrupted from the current activity because of some restrictions, such as supermarket opening and closing time. Thus, all out-of-home activities can only be pursued within a time interval, and let $\left[\underline{t}_a, \overline{t}_a\right]$ denote the time interval in which activity $a \in A$ can be pursued.

4.2.2 Model formulation

Suppose that for a homogenous group of travellers there is a predefined set of activity programs N. Any activity program $n \in N$ consists of a set of activities A^n. For simplicity, the superscript n is dropped. This does not cause any ambiguity, because all the formulas presented in this section apply to any activity program.

Time is discretised and a 24-hour day is evenly divided into T time episodes, denoted by $\{1,\ldots,T\}$. The state s includes three variables that describe the traveller's contextual situations, including time of the day t_s, the current location of the traveller w_s, and the set of uncompleted activities A_s.Thus, the state s can be expressed as a 3-tuple (t_s, w_s, A_s). An extensive discussion of the state transition is presented in Section 4.2.5.

At each decision epoch, a decision d is selected from a decision set $D(s)$. There are two types of decisions, activity participation and travel. If d is a decision of activity participation, d is an ordered pair (a_d, h_d), where a_d is the activity to be participated in and h_d is the chosen activity duration. Since the subsequent activity is a component of the decision, the order of activity participation is determined by the decision.

If d is a decision of travelling, d is an ordered pair $\left(z_d, m_d\right)$, where z_d is the destination of the trip and m_d is the travel mode used to get to z_d. For the simplicity of the formulation, travelling is treated as a special activity. A travel decision $\left(z_d, m_d\right)$ is also expressed in the form of $\left(a_d, h_d\right)$, where the chosen activity is $a_d = \texttt{travel}$ and the activity duration is equal to the travel time $h_d = \tau\left(t_s, w_s, z_d, m_d\right)$.

Actually, the decision defined above persists for multiple time episodes and is different from the primitive choice with unit execution time in Section 3.2.1. Appendix I presents the formal derivation of decisions as a generalization of the primitive choices in standard MDP model. In the remainder of this thesis, the choices that affect the states and utilities for multiple time episodes are termed decisions.

Let $D_{travel}\left(s\right)$ denote the set of travel decisions and $D_{activity}\left(s\right)$ denote the set of activity decisions. The decision space is the union of the two sets $D\left(s\right) = D_{activity}\left(s\right) \cup D_{travel}\left(s\right)$. The definition of the decision set can be found in Section 4.2.4. The decisions over the entire day constitute a daily activity-travel schedule.

The immediate utility function is additively separable in the deterministic and random parts:

$$R\left(s,d\right) = r\left(s,d\right) + \varepsilon\left(d\right) \tag{4.1}$$

where $\varepsilon\left(d\right)$ is a zero-mean random variable independently and identically distributed (i.i.d.) over travellers and states. The covariance structure of $\varepsilon\left(d\right)$ is examined in Section 4.2.7.

The deterministic utility of a decision is the sum of the disutility of travelling or the activity utility. The disutility of travelling is determined by the departure time, the origin and destination of the trip. The amount of activity utility depends on the activity type, the activity timing and duration. Specifically, the total discounted utility of choosing d at state s is expressed as,

$$r(s,d)=\begin{cases}\sum_{k=1}^{h_d}\beta^{k-1}u(a_d,t_s+k) & \text{if } d\in D_{activity}(s)\\ \sum_{k=1}^{h_d}\beta^{k-1}\alpha_m & \text{if } d\in D_{travel}(s)\end{cases} \tag{4.2}$$

where α_m is the equivalent disutility of unit travel time for travel mode m and $u(a,t)$ is the marginal utility of activity a at time t. h_d is the activity duration if $d\in D_{activity}(s)$ and $h_d=\tau(t_s,w_s,z_s,m_d)$ if $d\in D_{travel}(s)$.

At any decision epoch, a traveller makes a decision to maximize the weighted sum of the immediate utility and the expected future utility. The expected maximum utility is calculated by solving the recursive equation:

$$\overline{V}(s)=\mathrm{E}\left[\max_{d\in D(s)}\left\{r(s,d)+\varepsilon(d)+\beta^{h_d}\cdot\overline{V}(s')\right\}\right] \tag{4.3}$$

where $\beta\in[0,1]$ is the discount factor for future utility and is constant over time. Different values of β indicates a variety of behaviour patterns. If $\beta=0$, the traveller is only concerned with immediate utility. If $\beta=1$, the traveller places the same values on the immediate utility and the future utility of activities within the same day. As long as $\beta>0$, the current decision depends on the future utility, This dependency reveals forward-looking behaviour.

In addition, the value of β can vary across individual travellers. In another word, each traveller may have distinct preference over future utility. The preference variation over population can be captured by a probability distribution function. If the choice probability of an individual traveller follows the logit model, the choice probability of the population is modelled by the mixed logit model (Hensher and Greene, 2003; McFadden and Train, 2000).

Assume that the random component of immediate utility ε are i.i.d. over states with cumulative distribution function $G_\varepsilon(\cdot)$. The expectation of the maximum utility conditional on state s is given by the integral:

$$\overline{V}(s) = \int \max_{d \in D(s)} \left\{ r(s,d) + \varepsilon(d) + \beta^{h_d} \cdot \overline{V}(s') \right\} dG_\varepsilon(\varepsilon) \tag{4.4}$$

Given the above assumptions of independence, ε is integrated out in above integral. Thus, the expected utility $\overline{V}(s)$ is sufficient to represent the impact of ε on travellers' decisions. As the state is discrete, the expected maximum utility can be solved exactly, together with the decision-specific utility function expressed as:

$$\overline{v}(s,d) = r(s,d) + \beta^{h_d} \cdot \overline{V}(s') \tag{4.5}$$

For any $s \in S$, the conditional probability of choosing d is:

$$P(d \mid s) = \int \mathrm{I}\left[\overline{v}(s,d) + \varepsilon(d) > \overline{v}(s,d') + \varepsilon(d'), \forall d' \in D(s) \right] \cdot dG_\varepsilon(\varepsilon) \tag{4.6}$$

where $G_\varepsilon(\cdot)$ is the cumulative distribution function of ε. $\mathrm{I}[\cdot]$ is equal to one if the condition expression in the square bracket is true; otherwise, equal to zero.

4.2.3 Marginal utility

The traveller chooses the daily activity-travel schedule that provides the maximum overall utility. The traveller can schedule the start time and end time of an activity to gain as much utility as possible. The optimal starting time and the duration of activity depend on the temporal profile of activity utility. Hence, the activity-travel decision reflects the underlying temporal profile of activity utility.

The bell-shaped marginal utility function presented by Ettema and Timmermans (2003) is employed in this thesis:

$$g_a(t) = U_a^0 + \frac{\gamma_a \lambda_a U_a^{\max}}{\exp\left[\gamma_a(t-\xi_a)\right]\cdot\left\{1+\exp\left[-\gamma_a(t-\xi_a)\right]\right\}^{\lambda_a+1}} \qquad (4.7)$$

where t is time of the day, U_a^0 is the baseline utility of activity a, $U_a^{\max}$ is the maximum marginal utility of activity a, and $\gamma_a, \lambda_a, \xi_a, \eta_a$ are activity-specific parameters. The parameters can be calibrated from travel survey data. Figure 4.1 shows three examples of marginal activity utilities.

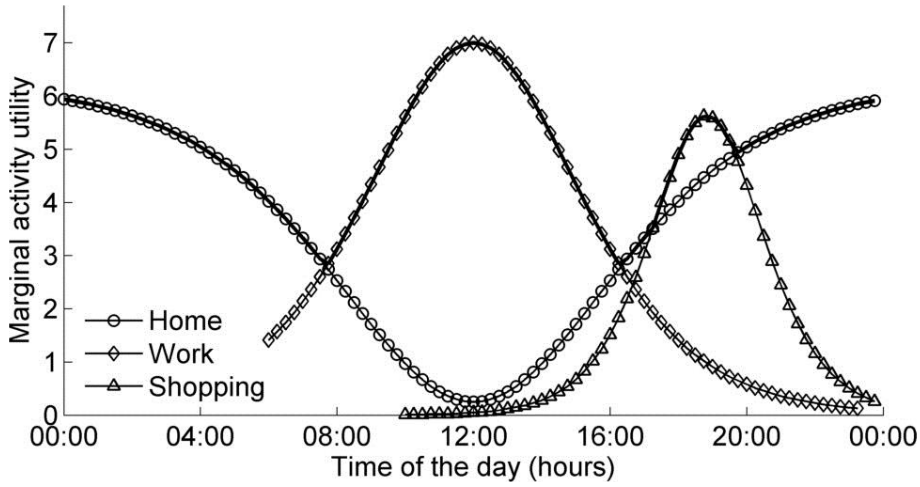

Figure 4.1 Marginal activity utilities of three activities over 24 hours

The marginal utility function is composed of two components: the baseline utility and the additional utility that defines the preference of activity timing. The baseline utility refers to the baseline preference for the activity. It is assumed to be a linear function of socio-economic and activity-specific variables and expressed as

$$U_{i,a}^0(x_i, x_a) = \mu_a x_i + \nu x_a$$

where $x_i = \left(x_i^1, x_i^2 \cdots, x_i^J\right)$ is the vector of individual-specific variables defining the socio-economic status of traveller i and $x_a = \left(x_a^1, x_a^2 \cdots, x_a^K\right)$ is the vector of activity-specific variables defining properties of activity a. μ_a is the coefficient associated with x_i and ν is the coefficient associated with x_a.

The utility of pursuing activity a in time interval $\left[t+\delta h, t+\delta(h+1)\right]$ is calculated as:

$$u_a(t,h)=\int_{\delta h}^{\delta(h+1)} g_a(t+x,x)\,dx \tag{4.8}$$

To save computational time, the values of definite integral (4.8) can be pre-computed and saved in a two-dimension array indexed by t and h.

4.2.4 Decision set

If the traveller makes a travel decision, the decision has two components, the destination and the travel mode. Let $B(z)$ denote the set of available activities at location $z \in W$. In particular, the residential location of the traveller is a location z such that $\{\texttt{Home-PM}\} \subseteq B(z)$. If $A_s \cap B(z)$, is an empty set, that is none of the uncompleted activities are available at z, location z cannot be chosen as the next activity destination. With these definitions, the set of travel decisions is expressed as:

$$D_{travel}(s)=\left\{(z,m) \mid z \in W \setminus \{w_s\}, A_s \cap B(z) \neq \varnothing, m \in M(w_s,z)\right\} \tag{4.9}$$

where $M(w_s,\mathrm{z})$ is the available travel modes to go from w_s to z.

The decision can be augmented with additional components, such as choice of route y_r or choice of bus line y_b. Decision components y_r and y_b will be passed to the traffic assignment model and aggregated into traffic flows in road network or passenger flows in transit network. Particularly, the algorithm illustrated in Figure 4.3 is adopted to compute traffic flows and passenger flows based on y_r and y_b.

If the traveller makes an activity decision, the decision also has two components, the activity type and activity duration. According to the second assumption in Section 4.2.1, the traveller returns home if and only if all the out-of-home activities have been completed. Formally, let s denote the current state and A_s denote the set of uncompleted activities in state s. If there exists any uncompleted out-of-home activity, that is $\{\texttt{Home-PM}\} \subsetneq A_s$, $\texttt{Home-PM}$ cannot be chosen and thus is excluded from the set. If $\texttt{Home-PM}$ is the only one activity left in A_s, that is $\{\texttt{Home-PM}\} = A_s$, all the out-of-home activities have been completed and thus, $\texttt{Home-PM}$ can be chosen. The set of feasible activities at state s is expressed as follows:

$$\overline{A}_s = \begin{cases} \{a \mid a \in A_s \cap B(w_s), a \neq \texttt{Home-PM}\} & \{\texttt{Home-PM}\} \subsetneq A_s \\ \{a \mid a \in A_s \cap B(w_s)\} & \{\texttt{Home-PM}\} = A_s \end{cases} \tag{4.10}$$

At the beginning of the day, A_s initially includes all the activities in the traveller's activity program. After that, at each decision epoch A_s is updated to exclude the completed activity. The formal update rule is presented in Section 4.2.5.

The third assumption in Section 4.2.1 requires that the starting time and the ending time of any activity a are within a given time interval $[\underline{t}_a, \overline{t}_a]$. Then the set of feasible activity decisions is defined as:

$$D_{activity}(s) = \{(a,h) \mid [t_a, t_a + h] \subset [\underline{t}_a, \bar{t}_a], a \in \overline{A}_s\} \tag{4.11}$$

4.2.5 Deterministic state transition

The state transition is assumed deterministic in this section. Thus, the travel time is deterministic and the subsequent state s' is uniquely determined by the current state s and the decision d. Consequently, the state transition probability function takes a reduced form: there exists $s' \in S$ such that $p(s' | s,d) = 1$ and $p(s' | s,d) = 0$ for any $s' \neq s, s' \in S$. This assumption however will be relaxed in Section 4.2.6 to incorporate travel time uncertainty into the model.

The time needed to execute decision d is equal to the activity duration h_d if $d \in D_{activity}(s)$ and equal to the travel time $\tau(t_s, w_s, z_d, m_d)$ if $d \in D_{travel}(s)$. Thus, the time of the day in the subsequent state s' is expressed by:

$$t_{s'} = \begin{cases} t_s + h_d & d \in D_{activity}(s) \\ t_s + \tau(t_s, w_s, z_d, m_d) & d \in D_{travel}(s) \end{cases} \tag{4.12}$$

If a travel decision is made, the location of the traveller is changed to the destination specified in the decision. Otherwise, the location remains unchanged:

$$w_{s'} = \begin{cases} w_s & d \in D_{activity}(s) \\ w_d & d \in D_{travel}(s) \end{cases} \tag{4.13}$$

According to the first assumption in Section 4.2.1, each activity is pursued only once within the planning horizon. Making an activity decision indicates that the current activity is completed. The completed activity is then excluded from the set of uncompleted activities $A_{s'}$:

$$A_{s'} = \begin{cases} A_s \setminus \{a_s\} & d \in D_{activity}(s) \\ A_s & d \in D_{travel}(s) \end{cases} \tag{4.14}$$

In summary, the subsequent state is defined as follows:

$$s' = \begin{cases} (t_s + h_d, w_s, A \setminus \{a_s\}) & d \in D_{activity}(s) \\ (t_s + \tau(t_s, w_s, z_d, m_d), z_d, A) & d \in D_{travel}(s) \end{cases} \tag{4.15}$$

4.2.6 Probabilistic state transition

Travel time is considered to have a stochastic nature. When travellers make activity-travel decisions, they can infer the traffic condition based on experience or using portable devices, such as smart phone and route guidance system. The travel time information obtained from these sources is not perfect. An unexpected change in travel time affects the activity-travel schedule. For example, if the travel time to a non-compulsory activity destination suddenly increases, the individual may cancel this activity and move on to another.

When travel time is uncertain, the arrival time is also uncertain and consequently, the subsequent state is a random variable. To capture the travel time uncertainty, the subsequent state is specified by a transition probability $p\left(s' \mid s,d\right)$ rather than the deterministic transition presented in Section 4.2.5.

A discrete probability distribution is used to describe the stochastic travel time. For any $j \in \mathbb{N}$, let $p_j\left(t_s, w_s, z_d, m_d\right)$ denote the probability of travel time j being realized. The state transition probability is equal to the probability that travelling from w_s to z_d by travel mode m_d:

$$p\left(s' \mid s,d\right) = p_j\left(t_s, w_s, z_d, m_d\right) \quad \text{if } s' \in \left\{x \mid t_x - t_s = j, j \in \mathbb{N}\right\} \tag{4.16}$$

With consideration of travel time uncertainty, the expected maximum utility is expressed by:

$$\overline{V}\left(s\right) = E\left[\max_d \left\{R\left(s,d\right) + \sum_{s'} p\left(s' \mid s,d\right) \cdot \beta^{h(s,s')} \cdot \overline{V}\left(s'\right)\right\}\right] \tag{4.17}$$

where $h\left(s,s'\right) = t_{s'} - t_s$.

4.2.7 Random errors

The random term in the immediate utility $\varepsilon\left(d\right)$ is a vector of random variables with zero means. The dimension of $\varepsilon\left(d\right)$ is determined by the number of alternatives in $D(s)$. Let $R(s,d)$ be the immediate utility of choosing d at state s. ε enters the immediate utility in a separate and additive way:

$$R(s,d)=r(s,d)+\varepsilon(d) \tag{4.18}$$

where $r(s,d)$ is the deterministic component and $\varepsilon(d)$ is a zero-mean random variable.

In logit model, $\varepsilon(d)$ is an i.i.d. Type I extreme-value random variable. Then the multi-dimension integral in (4.4) has a closed form expression:

$$\bar{V}(s)=\frac{1}{\theta_1}\log\left(\sum_{d\in D(s)}\exp\left(\theta_1\cdot\bar{v}(s,d)\right)\right) \tag{4.19}$$

where θ_1 is a parameter related to the traveller's perception error and $\bar{v}(s,d)$ is the decision-specific utility in (4.5). The probability of choosing d conditional on state s is expressed by:

$$\mathrm{P}(d\mid s)=\frac{\exp\left(\theta_1\cdot\bar{v}(s,d)\right)}{\sum_{d'\in D(s)}\exp\left(\theta_1\cdot\bar{v}(s,d')\right)} \tag{4.20}$$

It is well known that logit model suffers from the property of Independence of Irrelevant Alternatives (IIA). To capture the correlation between alternatives, a probit model is introduced to relax the limits on the covariance structure. The probit model avoids the IIA property by explicitly modelling the error covariance and thus allows flexible substitution among alternatives.

Using a probit model imposes a heavy computational burden. If there are J activity types and the maximum possible activity duration is T, an unrestricted covariance matrix of size $TJ\times TJ$ is hard to estimate due to the identifiability issue and computational burden. Thus, a structured covariance matrix is more tractable and interpretable.

The random term $\varepsilon(d)$ is decomposed into two components, $\varepsilon^1(d)$ and $\varepsilon^0(d)$. $\varepsilon^1(d)$ is common for all decisions with the same activity type and $\varepsilon^0(d)$ differs over activity types and activity durations. $\varepsilon^1(d)$ and $\varepsilon^0(d)$ are assumed independent from each other. The covariance of two decisions is then expressed as:

$$\operatorname{cov}\left(\varepsilon_k(d), \varepsilon_k(d')\right) = \operatorname{cov}\left(\varepsilon^1(d), \varepsilon^1(d')\right) + \operatorname{cov}\left(\varepsilon^0(d), \varepsilon^0(d')\right) \quad (4.21)$$

where $d = (a, h)$ and $d' = (a', h')$ are decisions in $D(s)$.

The first term in (4.21) is the covariance between different activity types. This term is defined as:

$$\operatorname{cov}\left(\varepsilon^1(d), \varepsilon^1(d')\right) = \sigma^1_{aa'} \quad (4.22)$$

where $\sigma^1_{aa'}$ is constant and defined for every two activities.

It is reasonable to assume that the traveller's perception error is larger when the activity duration is longer. Thus, the second term in (4.21) increases with activity duration. The variance of $\varepsilon^0(d)$ and $\varepsilon^0(d')$ is then expressed as:

$$\operatorname{cov}\left(\varepsilon^0(d), \varepsilon^0(d')\right) = \begin{cases} \min\{h, h'\} \cdot \sigma^0_a & a = a' \\ 0 & a \neq a' \end{cases} \quad (4.23)$$

where σ^0_a is a activity-specific constant.

Since ε follows a multivariate normal distribution, the evaluation of the expected maximum utility involves a multi-dimension integral, which the conventional numerical method cannot handle. A simulation method is needed to overcome this problem. The most widely used method is the GHK simulator (Train, 2003). The maximum likelihood method for calibrating the parameters of the individual MDP model is briefly discussed in Section 6.2.

4.2.8 Relationship with activity-based supernetwork equilibrium models

As reviewed in Section 2.2.2, the supernetwork representation is used to model dynamic activity-travel choices in congested networks. The nodes in a supernetwork denote the possible time and location choices. The activity link indicates the process of activity participation for one time episode. The road link represents travelling between physical nodes.

Actually, the activity-based supernetwork equilibrium models are special cases of the proposed MDP model. Each node in a supernetwork can be represented by a state in the MDP model. Each link in the supernetwork is an activity or travel decision that connects one state to another. Each route in the supernetwork is an ordered set of states and decisions, which constitutes an activity-travel schedule. Each route begins with a state in time 0 and ends with a state in time T.

In the MDP model, the feasible schedules are defined by local rules for each state and decision. The number of states and decisions can be large, but the number of possible routes in a supernetwork is even larger. Directly imposing rules on routes makes the supernetwork models computationally intractable. The advantage of the MDP model is that it is much easier to define local rules for feasible states and decisions than to define rules for feasible routes in a supernetwork.

As discussed above, any supernetwork model can be represented by an equivalent MDP model. The supernetwork models are actually network equilibrium models with expanded networks. The MDP model can be integrated with traffic network equilibrium as well. The solution algorithms for the MDP model capturing activity-travel scheduling behaviour in a congestion network are developed in the next section.

4.3 Solution algorithms

Solution algorithms are developed to solve the MDP model and the network equilibrium problem simultaneously. The solution algorithms have a nested structure as shown in Figure 4.2.

The inner iteration is to find the optimal solution of the MDP model of the individual's activity-travel scheduling. Value iteration method presented in Section 3.2.3 is employed to solve the recursive equation (4.3). The solution includes the optimal values of $\overline{V}(s)$ and the associated choice probability $\mathrm{P}(d \mid s)$.

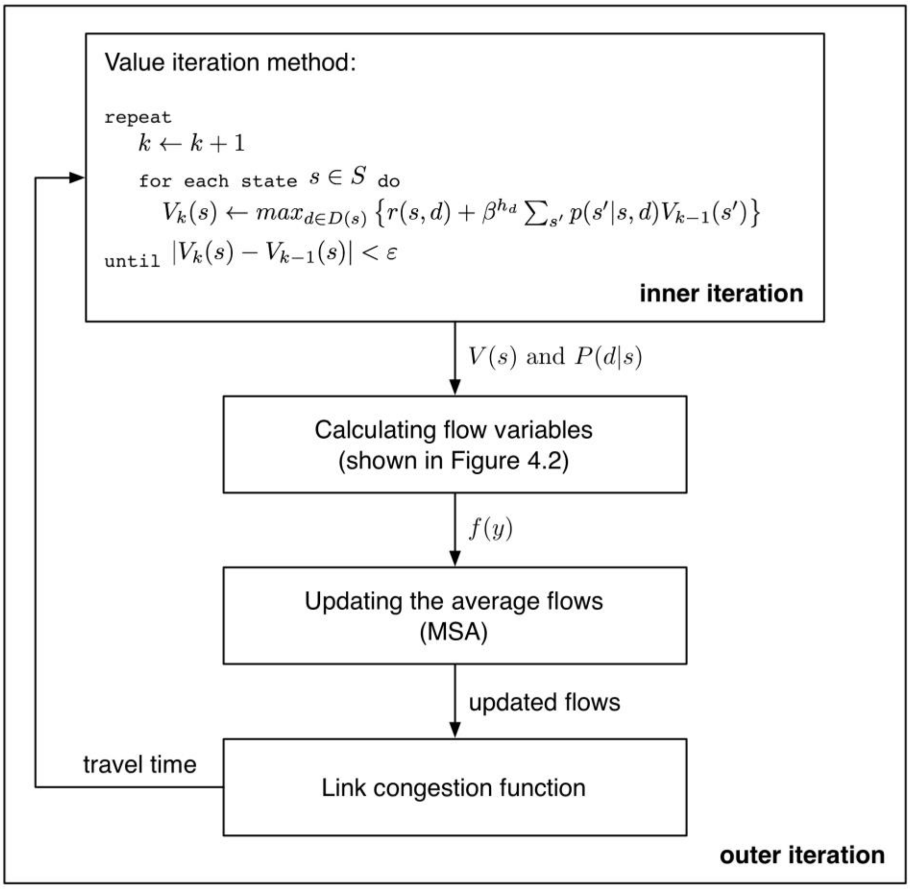

Figure 4.2 Solution algorithms for the MDP model and the network equilibrium problem

The outer iteration is to find the equilibrium traffic flows by using the Method of Successive Averages (MSA). MSA is easy to be adapted for network equilibrium models and does not require computation of derivatives. The derivative-free feature of MSA is essential since the solution procedure of the MDP model involves an inner iteration and no closed-form solution exists.

The flow variables found in network equilibrium models, such as route flows, can be derived from the optimal values $\overline{V}(s)$ and the optimal policy $\mathrm{P}(d \mid s)$. The steps of the derivation are presented as follows.

The expected maximum utility that can be obtained over an entire day by choosing activity program n is $\overline{V}(s_0^n)$, where $s_0^n = (t_0, w_0, A_0)$ is the initial state. t_0 is set to 1, the first time episode. w_0 is set to the residential location. A_0 is initialized to A^n, the set of all activities in activity program n.

If there are M travellers in the transportation system, the number of travellers choosing each activity program is assumed to follow a logit model:

$$M^n = M \cdot \frac{\exp\left(\theta_2 \cdot \overline{V}\left(s_0^n\right)\right)}{\sum_{m=1}^{N} \exp\left(\theta_2 \cdot \overline{V}\left(s_0^m\right)\right)} \tag{4.24}$$

where θ_2 is the perception error of choosing activity program.

Figure 4.3 show a breadth-first search algorithm for computing the number of travellers in state s, denoted by $f(s)$. The algorithm starts from the initial state s_0^n and searches any states that can be reached from s_0^n. The variable $f(s)$ is updated in each iteration. The algorithm terminates when all the states are visited.

for each state $s \in S$ **do**
    set $f(s) \leftarrow 0$
let Q be a first-in-first-out queue
for each activity program $n \in \{1, \ldots, N\}$ **do**

set $f\left(s_0^n\right) \leftarrow M^n$

push s_0^n into Q

while $Q \neq \varnothing$

pop the first element s from Q and remove s from Q

for each decision $d \in D(s)$ **do**

find the subsequent state s' using (4.15)

if the time component t' of state $s' > T$ **then**

break out of the for loop

else

update $f\left(s'\right) \leftarrow f\left(s'\right) + f\left(s\right) \cdot P\left(d \mid s\right)$

push s' into Q

Figure 4.3 Algorithm for calculating variables f(s)

If additional components, such as choice of route y_r or choice of bus line y_b, are incorporated into the decision, the aggregate traffic flows can be computed as follows. The traffic flow on a particular route y_r at time episode t is calculated by summing all the $f\left(s\right) \cdot P\left(d \mid s\right)$ with route y_r as a component of decision d and $t_s = t$. Then the link flow on each road can be readily derived from the route flow. The number of passenger boarding on bus line y_b at stop w is calculated in a similar manner. The pseudo code of the algorithm is presented in Figure 4.4.

initialize route flows $f\left(t, y_r\right) \leftarrow 0$ for any route y_r

initialize variables $f\left(t, y_b, w\right) \leftarrow 0$ for any bus line y_b and bus stop w

for each state $s \in S$ **do**

for each decision $d \in D(s)$ **do**

if mode choice m_d is *private car*

update $f\left(t_s, y_{d,r}\right) \leftarrow f\left(t_s, y_{d,r}\right) + f\left(s\right) \cdot P\left(d \mid s\right)$

if mode choice m_d is *public transportation*

update $f\left(t_s, y_{d,b}, w_s\right) \leftarrow f\left(t_s, y_{d,b}, w_s\right) + f\left(s\right) \cdot P\left(d \mid s\right)$

Figure 4.4 Algorithm for calculating flow variables

4.4 Numerical examples

In this section, the proposed MDP model and solution algorithm are applied to several numerical examples. The purpose of the examples is to analyse the individual's activity-travel scheduling behaviour of a homogenous group with 10,000 travellers in a period of 24 hours. Sensitivity analysis is conducted to examine how the computational results are affected by some key parameters of the model.

Figure 4.5 shows a small road network. There are three links connecting the locations of the three activities. The free travel time between any activity locations is labelled on the corresponding link. The dynamic link travel time is expressed as a simple Bureau of Public Road (BPR) function:

$$\tau_l\left(f_l\left(t\right)\right)=t_l^0\times\left(1+0.15\left(\frac{f_l\left(t\right)}{5000}\right)^4\right) \tag{4.25}$$

where $f_l\left(t\right)$ is the flow on link l at time t.

The equivalent disutility of travelling for one hour is $\alpha=60$. The discount ratio of the future utility is set to $\beta=0.99$. The random error in the immediate utility follows a i.i.d. extreme-value distribution with parameter $\theta_1=0.2$. The entire day (24 hours) is divided into 5-minute periods with 288 periods in total.

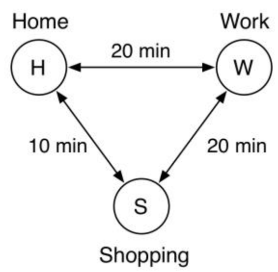

Figure 4.5 A 3-node network

There are three types of activity, namely staying at home, working and shopping. For simplicity, the choice of activity program is not explicitly considered. However, all the activity decisions effectively constitute the actual daily activity program. All the travellers are assumed to stay in home at 00:00 and have jobs with flexible working hours. Thus, travellers can fully control the duration of every activity and adjust the timing and duration to maximize the overall utility in the entire day. Table 4.1 presents the temporal constraints (e.g. opening hours) and parameters of marginal utility function for each activity.

Table 4.1 Temporal constraints and parameters of marginal utility function

Activities	Temporal constraints	Parameters of marginal utility function			
		U_m	γ	λ	ξ (min)
Home	00:00 – 23:59	1000	0.006	1.0	0
Work	06:00 – 23:59	800	0.010	1.0	720
Shopping	10:00 – 23:59	180	0.032	1.0	1110

All the utility functions have single-peaked profiles as shown in Figure 4.6. The marginal utility of staying in home reaches the maximum value at midnight. The utility of working surpasses all the other activities at near 8:00 and remains so until the evening. At shopping time, travellers can achieve a high level of utility by consumption. Therefore, shopping has the steepest profile with a peak at 18:30.

The individual MDP model is clearly defined with all the above specifications. The model is implemented in AMPL, an algebraic modelling language for linear and nonlinear optimization problems (Fourer et al., 1990). The AMPL source code with comments is included in Appendix II. The computational results are presented as follows.

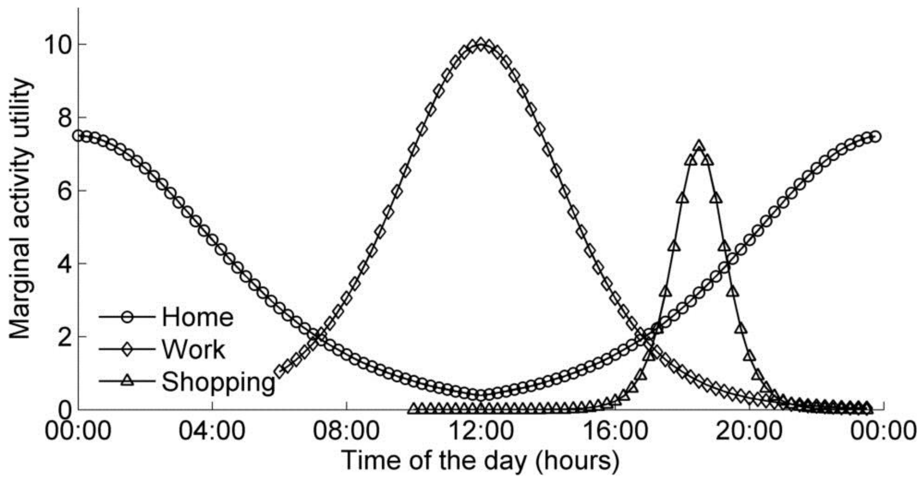

Figure 4.6 Temporal profiles of marginal utility functions

Figure 4.7 presents the number of travellers engaging in each activity over time of the day. Travellers start to leave home as early as 6:00. Most travellers arrive at office at around 8:00. The average duration of work is about 9 hours (from 8:00 to 17:00). After work, most travellers go for shopping and a small amount of traveller return home directly.

The pattern of activity participation illustrated in Figure 4.7 matches the temporal profiles of marginal utility functions depicted in Figure 4.6. This activity-travel pattern is used as the base scenario in the following sensitivity analysis. The remainder of the section presents sensitivity analysis of the key parameters, involving the discount ratio for future utility, the value of travel time and increase in travel time. The results obtained from the sensitivity analysis are compared with the base scenario.

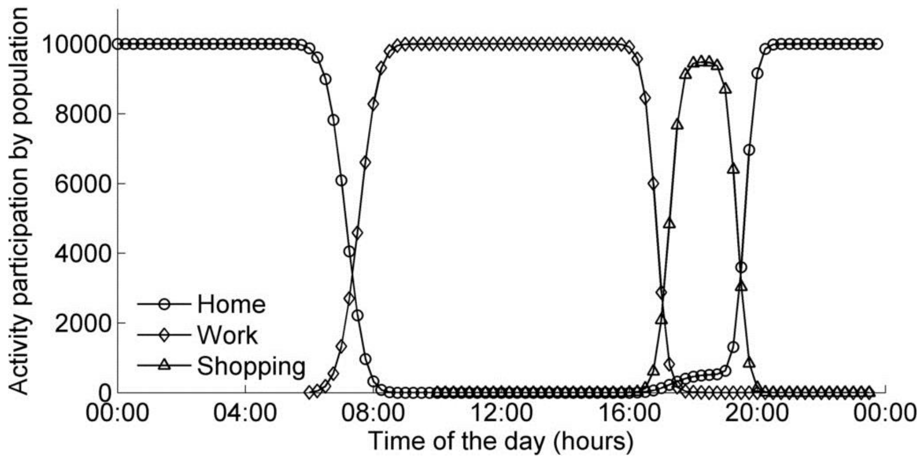

Figure 4.7 Activity participation over time of the day

4.4.1 Discount ratio for future utility

Discount ratio β reveals how the travellers value the future utility. The value of β is related to the time scale. In this example, the time scale is 5 minutes and $\beta = 0.99$. That is a unit of utility obtained in time $t+1$ is equivalent to 0.99 unit of utility obtained at time t. Generally, the utility obtained at a later time $t+n$ is equivalent to 0.99^n unit of utility at time t. For example, a unit of utility obtained one hour later is viewed as much as $0.99^{12} = 0.886$ unit of utility at present. If β is set to 0.97, the discount ratio for utility obtained one hour later is 0.694. With a 5-minute time scale, even small variation in β can lead to a significant change in travellers' preference of future utility.

Figure 4.8 shows the variation in the timing of activity. The left part illustrates the activity participation over time of the day for $\beta = 0.99$, and the right part that for $\beta = 0.97$. Each activity is postponed as the value of β is decreased from 0.99 to 0.97. That is because if the discount ratio $\beta \in [0,1]$ travellers always prefer to take a decision with greater utility as early as possible. Suppose that there are two tasks A and B that can be completed in a unit of time. The utilities that can be obtained from A and B are 5 and 10, respectively. If A is completed before B, the total discounted utility is $u_{AB} = 5 + \beta \cdot 10$. If B is completed before A, the total discounted utility is $u_{BA} = 10 + \beta \cdot 5$. Since $u_{BA} - u_{AB} = 5 - \beta \cdot 5 > 0$ for any $\beta \in [0,1]$, the individual prefers to complete B before A.

Switching from one activity to another involves travelling which causes disutility. Thus, according to the above argument, travellers try to postpone activity switching as much as possible. If the value of β decreases, travellers value future utility even less, and thus the timing of each activity is shifted to a later time.

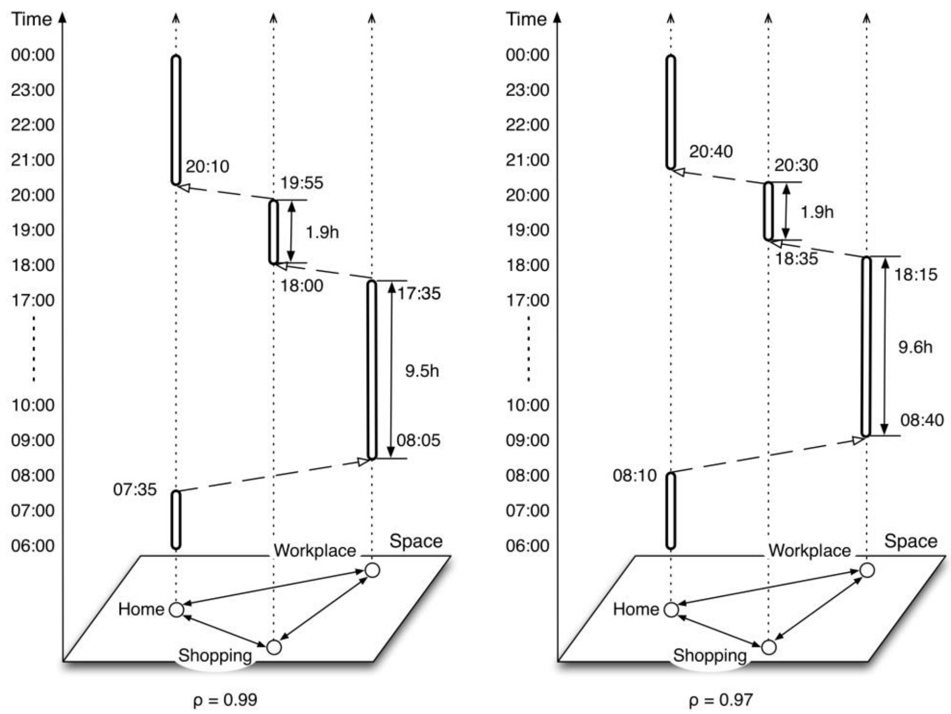

Figure 4.8 Activity participation over time of a typical traveller (β = 0.99 and β = 0.97)

4.4.2 Value of travel time

The value of travel time α is the equivalent disutility of travelling for a unit of time. As the value of α increases, the travellers tend to travel less. Travellers may even cancel their own activities to avoid the disutility of travelling. The value of travel time thus affects the activity-travel pattern. Figure 4.9 shows the activity participation for different values of α. The left part illustrates activity participation for $\alpha = 60$ and the right part illustrates that for $\alpha = 120$. Most travellers go shopping for about 2 hours after work when they have a lower value of time. If the value of travel time is doubled, 55% of the travellers return home directly from the workplace and 45% half of the traveller go shopping for about 1 hour.

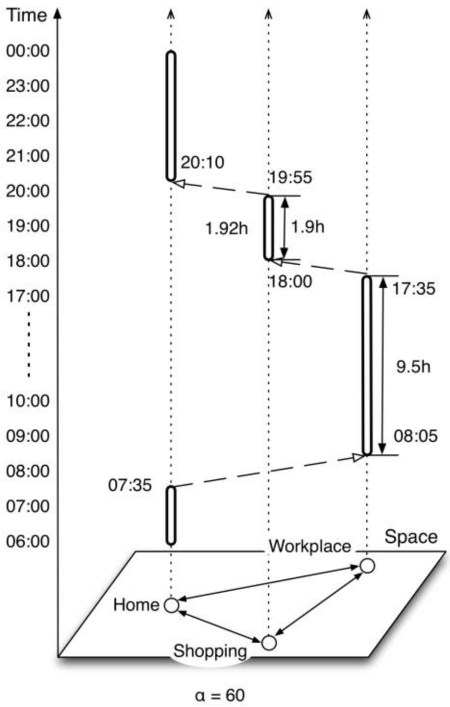

Figure 4.9 Activity participation of a typical traveller (α = 60 and α = 120)

4.4.3 Effect of increasing travel time

Travel time and the value of travel time jointly determine the travellers' perception of travel cost. However, road congestion does not only induce a longer travel time and a larger travel cost, but also reduces the time available to activity participation. This additional cost should be considered when evaluating transport policy. The individual's activity-travel scheduling model proposed in this chapter is capable of capturing the complicated interdependency between travel time and activity participation.

The link travel time depends on the traffic flows on the road. It is also affected by other external factors, such as weather condition and road construction. Suppose that such external factors cause the free travel time doubled. Table 4.2 shows that the time spent on shopping decreases from 2 hours to 0 hour and the time spent on in-home activity increases by 1.4 hours. The time that is originally intended for shopping is reallocated to in-home activity and travel.

Table 4.2 Allocation of time to activities and travel under different traffic conditions

Average duration (hours)	Home	Work	Shopping	Travel	Total
Normal traffic condition	**11.6**	9.3	**2.0**	**1.1**	24.0
Traffic congestion condition	**13.0**	9.1	**0.0**	**1.9**	24.0

Figure 4.10 shows the variation in activity participation due to the increasing travel time. The left part illustrates the activity participation for normal traffic condition. The right part illustrates that for traffic congestion condition. Compared with the normal condition, the travellers arrive at office much later due to the traffic congestion. To compensate for the late arrival at office, the travellers tend to leave office later in the evening. The working durations in both conditions are maintained at about 9 hours. The traveller gives up shopping and heads for home directly because of the postponed working hours and the longer travel time.

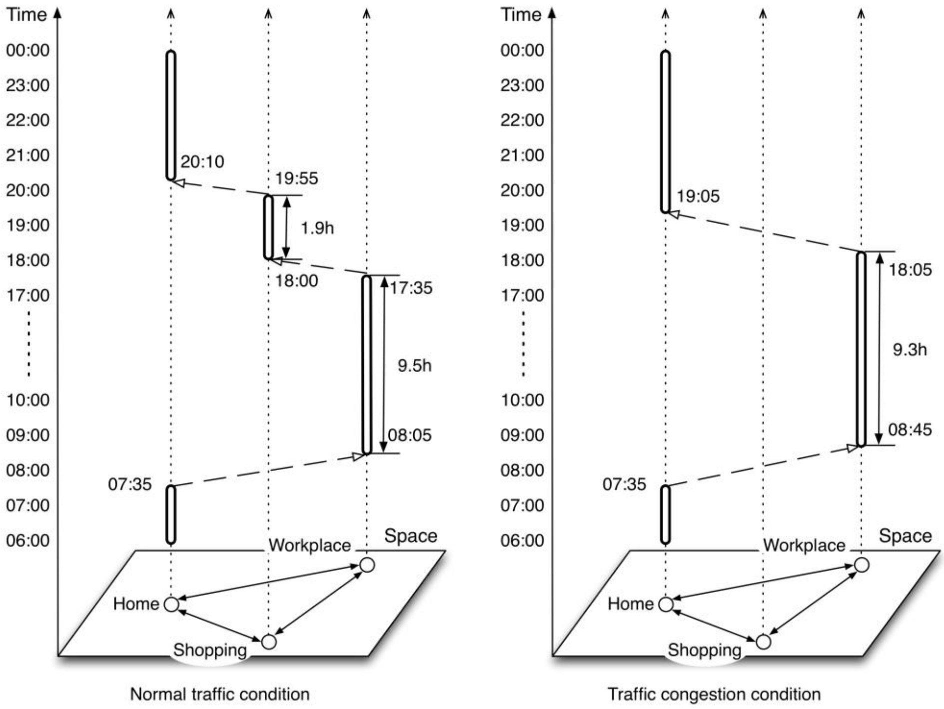

Figure 4.10 Activity participation of a typical traveller under different traffic conditions

4.5 Summary

In this chapter, a dynamic analytical model has been proposed to capture the state dependence and forward-looking behaviour in individual's daily activity-travel scheduling. The relationship between the current activity choice and the remaining uncompleted activities is explicitly considered. The model also captures travellers' choice of activity timing and duration in a congested network. Network equilibrium is incorporated into the MDP model of activity-travel scheduling. Solution algorithms are developed to solve the MDP model and the network equilibrium problem simultaneously.

The proposed model has four features. Firstly, time is treated as a scarce resource. Travellers allocate time to each activity to maximize the utility of the entire day. Second, a temporal utility function is employed in the model. The activity utility varies over time and affects the sequence of individual's activity participation.

Third, exhaustive enumeration of all possible activity schedules is avoided by defining feasible decisions and states. The computational burden is thus greatly relieved. Finally, the individual's activity-travel scheduling behaviour is formulated as an MDP model. Hence, the results are consistent with the random utility maximization framework.

Chapter 5 Intra-Household Interactions in Household's Activity-Travel Scheduling Behaviour

5.1 Introduction

Considerable evidence suggests that intra-household interactions play a crucial role in activity-travel scheduling. The individual's activity-travel scheduling model proposed in Chapter 4 ignores the interactions among household members and treats each individual separately. Further discussion is necessary to extend the proposed model to capture the intra-household interaction.

The activity-travel scheduling behaviour of a household with two members is investigated in this chapter. An analytical model is developed to explore how the interactions between household members influence the individual's activity-travel scheduling behaviour. Markov Decision Process (MDP) is employed to model household's scheduling behaviour and the intra-household interactions. MDP provides a modelling framework that allows the household's decisions to have complex interdependency over time.

The remainder of this chapter is organized as follows. Section 5.2 presents an extension of the individual's MDP model, which captures the intra-household interactions. Section 5.3 describes the solution algorithm for the extended MDP model. Section 5.4 illustrates the extended MDP model with numerical examples. The final section summarizes findings from this chapter.

5.2 An activity-travel scheduling model with intra-household interactions

An MDP model with intra-household interactions is presented in this section. The household is assumed to act as a single decision-making unit. The compulsory and non-compulsory activities of each household member are incorporated into the model by augmenting the state.

5.2.1 Assumptions

The following four assumptions are made to define the scope of the household activity-travel scheduling process.

A1. The individuals in a household jointly make activity-travel decisions (Zhang et al., 2005). Each individual voluntarily takes actions to implement the decision.

A2. Each individual has different preference over activity-travel schedules and the preference is represented by an individual utility function (Ettema et al., 2007).

A3. Household members are honest in revealing their preferences. The individual preferences are therefore public information within the household.

A4. With knowledge of individual preferences, the joint decision-making process seeks to maximize the welfare of the entire household (Bradley and Vovsha, 2004; Zhang et al., 2005).

Assumption A1 ensures that the joint decisions are effectively implemented by household members. With regard to Assumption A2, the individual utility functions can be used to derive the household utility function for activity-travel schedules. Assumptions A3 and A4 eliminate the possible strategic behaviour adopted by the individuals to gain advantages within the household. Without them, the household activity-travel scheduling process becomes a more general problem that game theory is needed to account for the individuals' strategic behaviour.

5.2.2 Model formulation

The optimal activity-travel schedule of an individual traveller is well defined and solved in Chapter 4. However, a household often have multiple members. Each household member participates in activities in parallel. The combination of optimal decisions of all household members is not always optimal in terms of the welfare of the entire household. Typically, making an activity-travel decision for one household member constrains the decisions available for the other. For example, if a household member drives the only car of the household, the other member cannot choose private car as the travel mode.

For simplicity, a household with two members, indexed by $i \in \{1, 2\}$, is considered in this chapter. The model formulation can be easily generalized to any number of household members. Let A_i be the set of daily activities for individual i in the household. Each individual can undertake a subset of the activities in A_i. The activities in A_i are categorized into two types based on the flexibility of participation in Section 5.2.4.

The activity-travel scheduling behaviour of household member i is formulated as an individual MDP model, denoted by M_i. A subscript is used to distinguish the elements of an individual MDP model. For example, S_i, D_i, p_i and R_i are the state set, decision set, transition probability function and utility function of individual MDP model M_i. The two household members share the same discount ratio for future utility, denoted by β.

Formally, the activity-travel scheduling behaviour of the entire household is defined as a household MDP model. The state set of the household MDP is a proper subset of the cross product of the individual household members' state sets, i.e., $S=\{(s_1,s_2)\mid t_{s_1}=t_{s_2}, s_1\in S_1, s_2\in S_2\}$. Notice that for any household state, the time episodes of the individual's states are synchronized.

The original individual's state defined in Chapter 4 is augmented with the on-going activity a_i and its remaining time e_i. The functionality of the additional components will be discussed later. The state of household member i is then a 5-tuple $s_i=\left(t_{s_i}, w_{s_i}, A_{s_i}, a_{s_i}, e_{s_i}\right)$. Suppose that the state of a household member is (*9AM, Office, Work,* {*Shopping, Home*}*, Work, 9 hours*). This state represents that the household member works in the office at 9AM and will keep working for 9 hour. Two additional activities, *Shopping* and *Home*, need to be undertaken in the remainder of the day.

The constraints on the activity-travel decisions imply that the decision set of the household MDP model D, is a proper subset of the cross product of the individual household members' decision sets, i.e., $D\subseteq D_1\times D_2$. Each household member's decision set is defined as follows.

A household member's activity decision consists of the choice of activity type and duration:

$$D_{activity}(s_i)=\left\{(a,h)\mid a\in A_{s_i}, [t_a, t_a+h]\subset\left[\underline{t}_a, \bar{t}_a\right]\right\} \tag{5.1}$$

Suppose that the individual decision of a household member is (*Shopping, 1 hour*). This indicates that the household member will go shopping for 1 hour. Moreover, this activity has to be conducted in the opening hours.

A household member's travel decision consists of the choice of trip destination and travel mode:

$$D_{travel}(s_i)=\left\{(z,m)\mid A_{s_i}\cap B(z)\neq\varnothing, z\in W\setminus\left\{w_{s_i}\right\}, m\in M\left(w_{s_i}, z\right)\right\} \tag{5.2}$$

The union of the individual's travel decisions and activity decisions gives all the activity-travel decisions that the traveller can make:

$$D_{new}\left(s_i\right) = D_{activity}\left(s_i\right) \bigcup D_{travel}\left(s_i\right) \tag{5.3}$$

Each household member can participate in activities in parallel. The household is at a decision epoch whenever a household member has completed an activity. The other member, however, may have not completed his/her on-going activity. Formally, if the current state $s = \left(s_1, s_2\right)$ is a decision epoch, each household member either takes a decision from $D_{new}\left(s_i\right)$ or continues the on-going activity. A special decision set is defined to account for the on-going activity of individual household member $D_{pre}\left(s_i\right) = \left\{\left(a_{s_i}, e_{s_i}\right) \mid e_{s_i} > 0\right\}$. Then the set of feasible decisions for household member i is expressed as:

$$D\left(s_i\right) = \begin{cases} D_{new}\left(s_i\right) & s_i \in \mathcal{I}_i \\ D_{pre}\left(s_i\right) & s_i \notin \mathcal{I}_i \end{cases} \tag{5.4}$$

where $\mathcal{I}_i = \left\{s_i \mid e_{s_i} = 0, s_i \in S_i\right\}$ is the set of decision epochs for individual i.

The set of feasible decisions for the entire household is the cross product of that of the two household members:

$$D\left(s\right) = D\left(s_1\right) \times D\left(s_2\right) \tag{5.5}$$

To allow the possibility of simply waiting for a household member to complete an on-going activity, the individual's decision set $D(s_i)$ is augmented with a *wait* decision. The *wait* decision has a variable duration equal to the time until the next decision epoch. Travel decision is treated as a special activity *travel* with travel time as the activity duration.

Let $Y(s,d)=\left\{\left(a_{d_i},h_{d_i}\right)\mid d_i\in D(s_i),\forall i=1,2\right\}$ denote the set of the on-going activities and their remaining times until completion. The next decision epoch is the earliest time after which any on-going activity is completed:

$$\tau_d^s=\min_{(a,h)\in Y(s,d)} h \tag{5.6}$$

When a joint household decision $d=(d_1,d_2)$ is made in state $s=(s_1,s_2)$, the subsequent state of household member i is updated as follows:

$$s_i'=\begin{cases}\left(t_{s_i}+\tau_d^s,w_{s_i},A_{s_i}\setminus\left\{a_{s_i}\right\},a_{d_i},e_{s_i}-\tau_d^s\right) & d_i\in D_{activity}(s_i)\\ \left(t_{s_i}+\tau_d^s,z_{d_i},A_{s_i},travel,e_{s_i}-\tau_d^s\right) & d_i\in D_{travel}(s_i)\\ \left(t_{s_i}+\tau_d^s,w_{s_i},A_{s_i},a_{s_i},e_{s_i}-\tau_d^s\right) & d_i\in D_{pre}(s_i)\end{cases} \tag{5.7}$$

The transition probability function of the household's state is defined as:

$$p(s'\mid s,d)=p_1(s_1'\mid s_1,d_1)\cdot p_2(s_2'\mid s_2,d_2) \tag{5.8}$$

The total discounted household utility of making joint household decision d at state s is expressed by:

$$r(s,d)=\sum_{k=1}^{\tau_d^s}\beta^{k-1}r(s,d,k) \tag{5.9}$$

According to assumption A4 in Section 5.2.1, the household's objective is to make a joint household decision to maximize the expected overall utility:

$$\overline{V}(s)=\mathrm{E}\left[\max_{d\in D(s)}\left\{r(s,d)+\varepsilon(d)+\beta^{\tau_d^s}\cdot\overline{V}(s')\right\}\right] \tag{5.10}$$

where $\varepsilon(d)$ is the random error due to unobserved characteristics as explained previously in Section 4.2.7. The joint household decisions over the entire day constitute the daily activity-travel schedule of the household.

5.2.3 Household utility function

A utility function is adopted to represent the household joint preference with consideration of intra-household interactions. The immediate utility that the household obtains at time k is decomposed as follows:

$$r(s,d,k)=\sigma_1\cdot r_1(s_1,d_1,k)+\sigma_2\cdot r_2(s_2,d_2,k)+r_J(s,d,k) \tag{5.11}$$

where σ_i is the weight parameter representing the relative influence of household member i. r_i is the individual utility that household member i can obtain when pursuing the activity independently:

$$r_i(s_i,d_i,k)=\begin{cases}\mu\left(a_{d_i},t_{s_i}+k\right) & \text{if } d_i\in D_{activity}(s_i)\\ \alpha\left(m_{d_i}\right) & \text{if } d_i\in D_{travel}(s_i)\end{cases} \tag{5.12}$$

r_J is the utility that the household can obtain for the joint activity participation:

$$r_J(s,d,k)=\rho\cdot r_1(s_1,d_1,k)\cdot r_2(s_2,d_2,k) \tag{5.13}$$

where ρ measures the level of interaction between household members' activities.

The interaction coefficient ρ takes non-zero values if the two household members jointly pursue activity a at location w, i.e., $a_{s_1}=a_{s_2}=a$ and $w_{s_1}=w_{s_2}=w$ for any state $s\in S$. Otherwise, (5.13) is equal to zero and the household utility (5.11) is reduced to the weighted sum of the individual utilities.

To model differences in intra-household interactions across activities, distinct interaction coefficient ρ_a can be specified for each activity a. Activities that require companionship and collaboration among household members have a positive intra-household interaction coefficient. Some routine activities that only need to be undertaken by any one of the individuals are specified with a negative ρ_a. That is the substitution between individuals exhibits negative intra-household interaction. In summary, there exists positive interaction between household members if $\rho > 0$, negative interaction if $\rho < 0$, and no interaction if $\rho = 0$. Figure 5.1 depicts the household utility functions with distinct interaction coefficients.

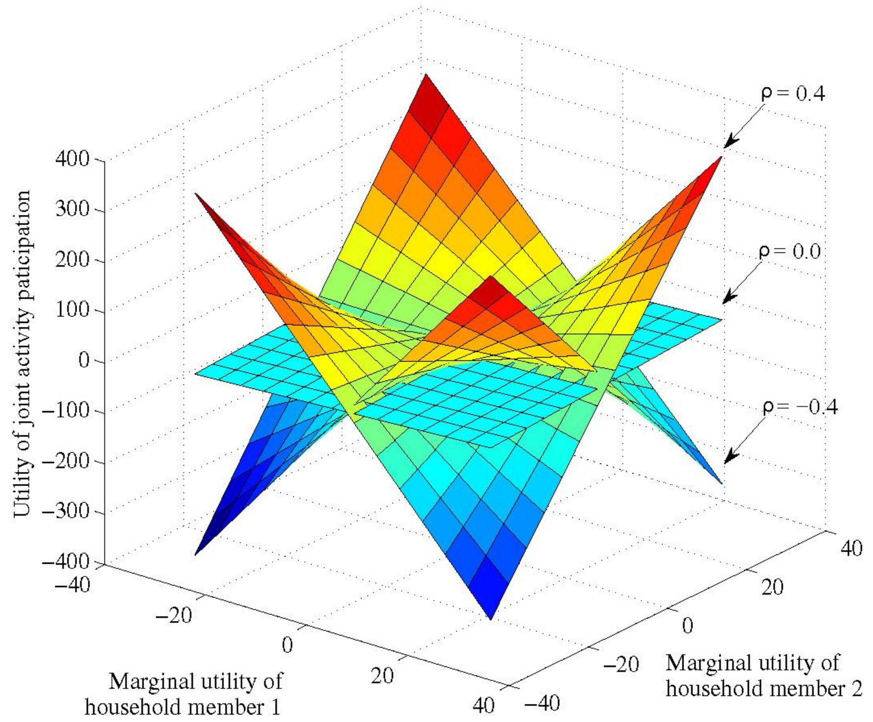

Figure 5.1 The utility of joint activity participation for the entire household

The effect of the interaction coefficient can be further illustrated in an activity choice problem with two alternatives, A and B. Suppose that the household utility function takes a simple form:

$$r(a) = r_1(a) + r_2(a) + \rho \cdot r_1(a) \cdot r_2(a) \tag{5.14}$$

The individual utilities for the two activities are given in Table 5.1. Using (5.14) with $\rho = 0$, the household must be indifferent between A and B. If the household selects A, Individual 1 would receive higher utility from his preferred activity, while Individual 2 would receive lower utility from his less preferred activity. This result seems to be unfair to Individual 2. Thus, in order to avoid inequality, the household should prefer B. When this is the case, this example indicates that ρ should take a positive value and thus, the household utility of choosing A is less than B, $r(A) < r(B)$.

Table 5.1 The utilities of independent activity participation

$r_i(a)$	Individual 1	Individual 2
Activity A	15	5
Activity B	10	10

Household members can choose the optimal individual decisions that are parts of the optimal household decision. However, there is no guarantee that the household members consider the same household decision, and thus that the actual household decision is in fact suboptimal for the entire household. Decision-making is even harder if there are multiple optimal household decisions. Coordination between household members is required to ensure that the optimal household decision is chosen.

5.2.4 Compulsory and non-compulsory activities

The daily activities can be categorized into two types based on the flexibility of participation, the compulsory and non-compulsory activities. All the compulsory activities are allocated to a specific household member and should be completed within the planning horizon. The non-compulsory activities are optional. Imposing these constraints on activity choice demonstrates the flexibility of the MDP framework.

To model the compulsory and non-compulsory activities, the original household state is augmented with an additional component G_s, the set of non-compulsory activities. Activities in G_s can be undertaken by any household member or be skipped. The original set of daily activites A_{s_i} is redefined to include compulsory activities that must be completed by individual i.

The sets of non-compulsory activities in the subsequent state s' are updated as follows:

$$G_{s'} = \begin{cases} G_s & d_i \notin D_{activity}\left(s_i\right), \forall i \\ G_s \setminus \left\{a_{s_1}, a_{s_2}\right\} & \text{otherwise} \end{cases} \tag{5.15}$$

The other components of the subsequent state are updated according to state transition equation (5.7).

At each decision epoch, individual i can select a compulsory activity or a non-compulsory one. Thus, the individual's activity decision set is expressed as:

$$D_{activity}\left(s_i\right) = \left\{\left(a,h\right) \mid a \in A_{s_i} \cup G_s, \left[t_a, t_a + h\right] \subset \left[\underline{t}_a, \bar{t}_a\right]\right\} \tag{5.16}$$

To ensure that any individual completes the compulsory activities in A_{s_i}, for any absorbing state $s \in S_*$, the set of uncompleted compulsory activities should be empty, $A_{s_1} = A_{s_2} = \varnothing, \forall \left(s_1, s_2\right) \in S_*$.

5.3 Solution algorithms

Given the household MDP model and the optimal solutions of the individual's MDP models, one heuristic solution strategy is directly combining the optimal solutions of the individual's MDP models. Due to the intra-household interactions and constraints on household decisions, this strategy is suboptimal and even results in infeasible household decisions.

The following solution algorithm narrows down the household's decision space via dynamic merging the solutions of the individual MDP models (Singh and Cohn, 1998). The dynamic merging algorithm finds the optimal solution to the household

MDP model by directly performing value iteration on the household state and decision set. The pseudo code of the algorithm is presented in Figure 5.2. The equilibrium network flows can be computed by a nested method similar to the one presented in Figure 4.2. The only difference is that the household MDP model is solved by using the dynamic merging algorithm.

For a household MDP model, $M=(M_1,M_2)$, the individual MDPs M_1 and M_2 should be solved first using the algorithm presented in Section 3.2.3. Then the optimal values of M_1 and M_2, $V(s_i), \forall s_i \in S_i, i=1,2$ are used to construct the initial lower and upper bounds in the dynamic merging algorithm.

for each state $s \in S$ **do**
  set $(s_1,s_2) \leftarrow s$
  set $V_0(s) \leftarrow 0$
  set $V_0^L(s) \leftarrow \max\{V(s_1),V(s_2)\}$ and $V_0^U(s) \leftarrow V(s_1)+V(s_2)$
set $k \leftarrow 0$
repeat
  set $k \leftarrow k+1$
  for each state $s \in S$ **do**
    update the lower and upper bounds:

$$V_k^L(s) \leftarrow \max_{d \in D_{k-1}(s)} \left\{ r(s,d) + \sum_{s' \in S} p(s'|s,d) \cdot V_{k-1}^L(s'|s,d) \right\}$$

$$V_k^U(s) \leftarrow \max_{d \in D_{k-1}(s)} \left\{ r(s,d) + \sum_{s' \in S} p(s'|s,d) \cdot V_{k-1}^U(s'|s,d) \right\}$$

    update the value of the household state:

$$V_k(s) \leftarrow \max_{d \in D_{k-1}(s)} \left\{ r(s,d) + \sum_{s' \in S} p(s'|s,d) \cdot V_{k-1}(s'|s,d) \right\}$$

    update the set of competitive decisions:

$$D_k(s) \leftarrow \left\{ d \in D_{k-1}(s) \mid Q_k^U(s,d) \geq V_t^L(s) \right\}$$

    where $Q_k^U(s,d) = r(s,d) + \sum_{s' \in S} p(s'|s,d) \cdot V_{k-1}^U(s'|s,d)$

until $|D_k(s)|=1$ for all $s \in S$ **or** $|V_k(s)-V_{k-1}(s)| < \epsilon$
for each state $s \in S$ **do**
  set $\pi(s) \leftarrow \arg\max_{d \in D_k(s)} \sum_{s' \in S} F(s'|s,d)\left[R(s,d)+V_k(s'|s,d)\right]$

return V_k and π

Figure 5.2 Dynamic merging algorithm for household MDP model

The efficiency of dynamic merging is gained by constructing lower and upper bounds on the optimal values of the household states. The bounds are initially constructed based on the optimal solution of the individual MDP models and then incrementally updated using value iteration. If the upper bound of household decision d is less than the lower bound of another household decision, the decision d is strictly dominated and can be safely excluded from the household decision set. The algorithm terminates when there is only one household decision remaining in set $D_k(s)$ for each household state s, or when the expected utility changes by a small amount in an iteration.

5.4 Numerical examples

Figure 5.3 shows a 4-node road network on which activity-travel schedules are implemented. There are 10,000 behaviourally homogeneous households and each household is composed of two adults: Individual 1 and Individual 2. Node H represents the residential location. Node W1 and W2 represent the workplaces of Individual 1 and 2. For simplicity, travel time is assumed deterministic and the congestion effect is captured by a BPR function,

$$\tau_l\left(f_l(t)\right) = t_l^0 \times \left(1 + 0.15\left(\frac{f_l(t)}{5000}\right)^4\right) \tag{5.17}$$

where $f_l(t)$ is the flow on link l at time t.

The equivalent disutility of travelling for one hour is $\alpha = 60$. The discount ratio of the future utility is set to $\beta = 0.99$. The entire day (24 hours) is divided into 5-minute periods with 288 periods in total.

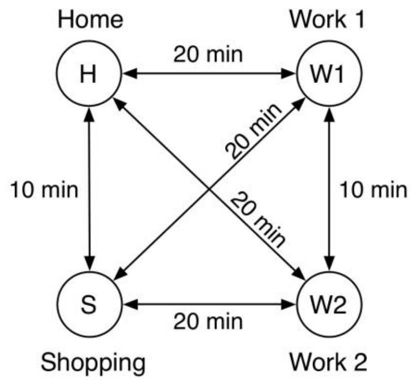

Figure 5.3 A 4-node road network

The utility of pursuing an activity varies over the course of the day. The optimal starting time and the duration of activity depend on the temporal profile of activity utility. The bell-shaped marginal utility function proposed in (Ettema and Timmermans, 2003) is adopted in this example.

Three types of activity are considered in the example: *Home*, *Work*, and *Shopping*. The parameters of utility function for each activity are presented in Table 5.2. Figure 5.4 depicts the temporal profiles of the individual's marginal activity utility functions.

Table 5.2 Parameters of utility function for each activity

Activity	Parameters of utility function			
	U_m	γ	λ	ξ (min)
Home	1000	0.006	1.0	0
Work	800	0.010	1.0	720
Shopping by Individual 1	**180**	0.032	1.0	1110
Shopping by Individual 2	**60**	0.032	1.0	1110

The two household members have distinct preferences for shopping activity. Individual 1 is more willing to go shopping than Individual 2. The distinct preferences of Individual 1 and 2 for shopping are represented by the bolded values of U_m in Table 5.2.

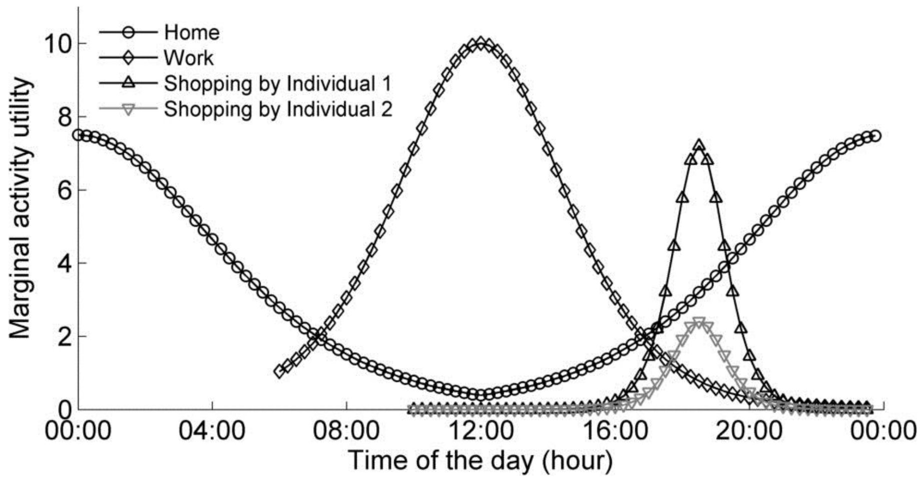

Figure 5.4 The individual's utility for independent activity participation with heterogeneous preferences

The welfare of the individuals in a household is treated equally important. The weight parameter, σ_i, representing the relative influence of household member i is thus assigned the same value: $\sigma_1 = \sigma_2 = 1.0$. The interaction coefficients of *Home* and *Work* are set to zero. The interaction coefficient of *Shopping* is denoted by ρ.

The household MDP model is defined by the above specifications. The solution algorithm in Section 5.3 is implemented in AMPL and used to solve the examples in this section. The source code is included in Appendix II. The model is examined for different types of intra-household interactions. The key findings are discussed as follows.

Initially, the interaction coefficient of *Shopping* is set to 0. Each individual makes activity and travel decisions independently. The activity-travel pattern of each individual reflects the underlying individual preference. Figure 5.5 illustrates the activity participation of Individual 1 and 2 over time of the day. Since Individual 1 gains a high level of utility from shopping, this individual goes shopping after work. Individual 2 prefers to return home directly after work.

The patterns of activity participation for Individual 1 and 2 depicted in Figure 5.5 are used as the base scenario for further analysis. The results of the positive and negative intra-household interactions are discussed and compared with the base scenario in the following sections.

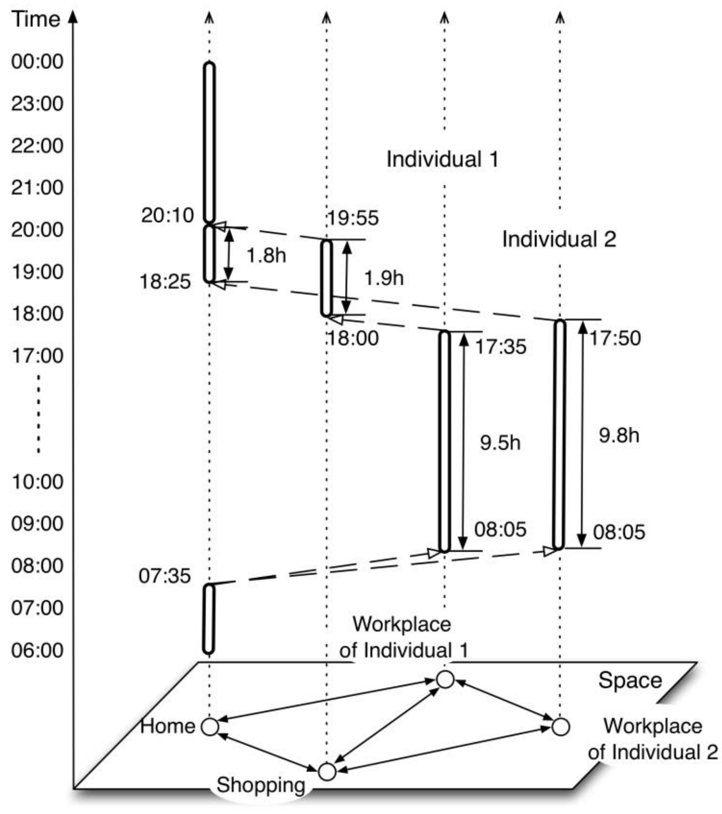

Figure 5.5 Activity participation of a two-person household (ρ = 0)

5.4.1 Positive intra-household interaction for shopping

If a household considers shopping as a non-compulsory activity with positive intra-household interaction, the two household members will prefer to participate in the activity together to interact more with each other. Thus, the interaction coefficient should take a positive value, $\rho = 0.2$ for example. The overall household's utility should be higher than the sum of individuals' utilities from independent activity participation. The extra utility received by the household is measured by (5.13).

Figure 5.6 shows the extra utility for different combinations of shopping timings and durations. For a given duration of shopping, an optimal timing gives the maximum utility. For any timing of shopping before 18:00, the utility increases rapidly with the duration of shopping. However, after 18:00 the gain of utility for spending an extra unit of time on shopping approaches zero. This tendency is illustrated by the contour lines parallel with y-axis between 18:00 and 19:00. This observation demonstrates that joint activity has an optimal timing and duration.

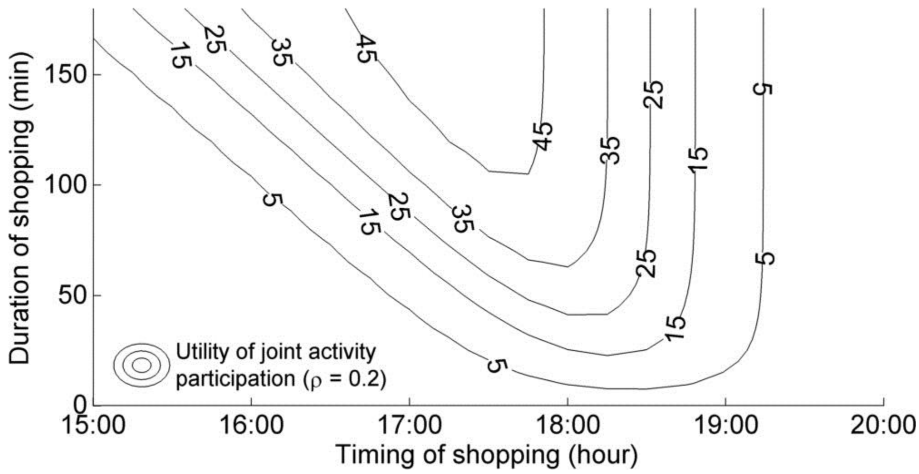

Figure 5.6 The utility of joint activity participation

Since joint participation provides a higher overall household utility than independent participation, the activity-travel pattern of Individual 2 is changed for the welfare of the household. Figure 5.7 shows that Individual 2 joins Individual 1 for shopping activity. On the other hand, the activity-travel pattern of Individual 1 does not show significant variation for the increase in ρ.

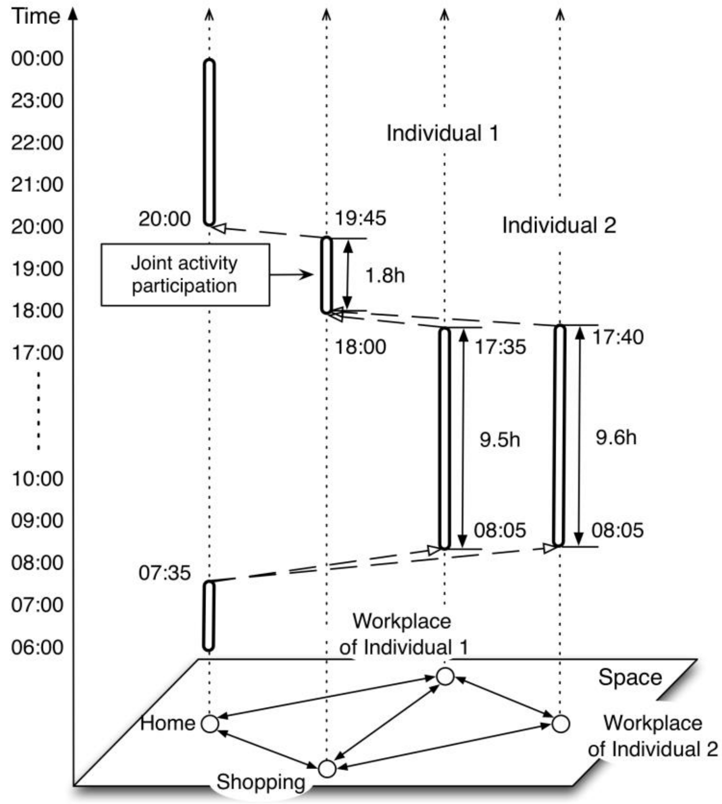

Figure 5.7 Activity participation of a two-person household (ρ = 0.2)

Table 5.3 presents the allocation of time to activities and travel for a large range of interaction coefficients. The duration of shopping for Individual 2 increases rapidly when ρ is increased from 0.0 to 0.2. However, this trend slows down when ρ approaches 0.5. Individual 2 spends less time on in-home activity to compensate the increased time in shopping. The working duration of Individual 2 is always maintained at 8 hours to 9 hours.

Table 5.3 Allocation of time to activities and travel for different values of ρ (Individual 2)

Values of ρ	Home	Work	Shopping	Travel	Total
0.0	13.8	9.4	0.0	0.8	24.0
0.2	11.9	9.2	1.8	1.1	24.0
0.5	11.7	8.9	2.3	1.1	24.0
1.0	11.6	8.4	2.9	1.1	24.0

5.4.2 Negative intra-household interaction for shopping

If the household considers shopping as a non-compulsory activity with negative intra-household interaction, only one household member will take action to complete the shopping activity and the entire household benefits from that action. If a household member has done the shopping task, the benefit of another shopping trip is negligible and the cost of the trip is significant, particularly in a congested transportation network.

The action of any household member is thus substitutable within the household. The interaction coefficient takes a negative value in this case. Figure 5.8 depicts the utility of independent activity participation with homogenous individual preferences over shopping.

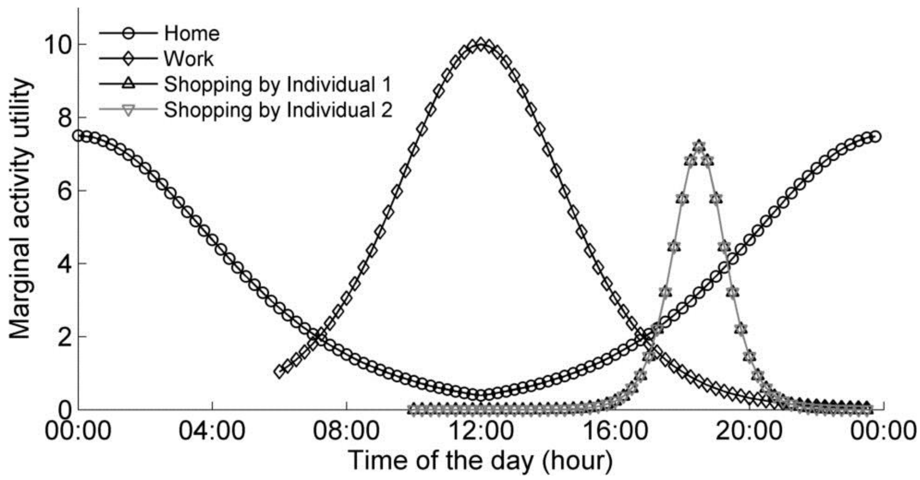

Figure 5.8 The utility of independent activity participation with homogenous preferences

The simulated time-space path in Figure 5.9 shows the possible activity-travel patterns. The two individuals share the same temporal profile of marginal utility for shopping. The household can assign the shopping task to either individual and get the same overall utility. The probability of going shopping for any individual is 50%.

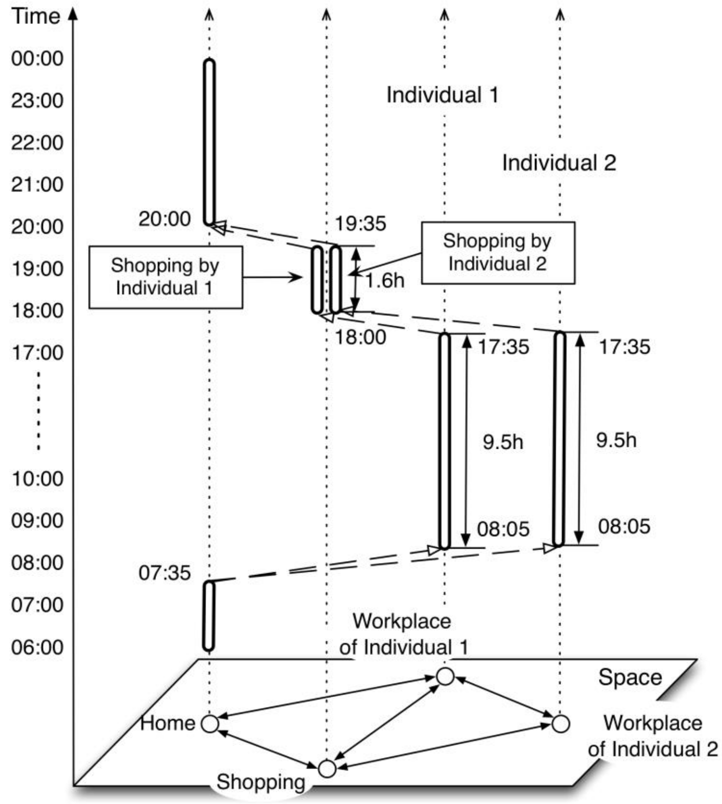

Figure 5.9 Activity participation over time of the day (ρ = -0.2)

5.5 Summary

A household MDP model with consideration of the intra-household interactions has been proposed to describe household members' daily activity-travel scheduling behaviour. The proposed model allows decomposing the household's utility into two components: the utility of engaging in an activity independently and the utility derived from the joint activity participation with other household members. The model also enables complicated activity-travel decisions over time.

The results drawn from the numerical analysis are summarized as follows. Firstly, household members may have distinct preferences for the same activity. If the intra-household interaction for an activity is zero, each household member chooses activity-travel schedule based on the individual utility. Secondly, the strength of the intra-household interaction varies across activity types. If the intra-household interaction is positive for an activity, household members participate in the activity jointly to obtain higher household utility. If the intra-household interaction is negative for an activity, this activity can be assigned to either household member. The household utility is maintained at the same level.

Chapter 6 Calibration Methods and Results

6.1 Introduction

This chapter aims to introduce statistical methods for calibrating the parameters of the MDP models proposed in Chapter 4 and Chapter 5. Time-series data are required for calibration of model parameters. The dataset should include travellers' activity choices and geographic locations over time. Due to the cost of collecting time-series activity-travel data, hypothetical numerical experiments are conducted to generate the necessary dataset.

This chapter is organized as follows. Section 6.2 presents the formulation of maximum likelihood method. Section 6.3 reviews two numerical methods for solving the calibration problem. Section 6.4 presents the data generated from numerical experiments and the calibration results.

6.2 Maximum likelihood method

Rust (1994) proposed a unified framework for calibration of MDP model. The contribution of individual i to the log-likelihood function is expressed as follows:

$$l_i(\eta)=\sum_{t=1}^{T_i}\log P\left(d_{it}\mid s_{it},\eta\right)+\sum_{t=1}^{T_i-1}\log p\left(s_{i,t+1}\mid d_{it},x_{it},\eta\right)+\log\Pr\left(s_{i1}\mid\eta\right) \quad (6.1)$$

where η represents the vector of model parameters and $p\left(s_{i,t+1}\mid d_{it},x_{it},\eta\right)$ is the state transition probability function conditional on d_{it},x_{it} and η.

Assume that ε is normally distributed with $N\times M$-variate probability density function $G_\varepsilon(\cdot)$. The probability of observing choice d_i can be calculated by integrating over all possible values of ε,

$$P\left(d_{it} \mid \eta\right)=\int\cdots\int G_{\varepsilon}\left(\varepsilon\right)d\varepsilon \tag{6.2}$$

If the choice is independent over individuals, the likelihood of all individuals' activity choices can be expressed as the product of each individual's activity choice probability:

$$L\left(d_1,\ldots,d_I \mid \eta\right)=\prod_{i=1}^{I} l_i\left(\eta\right) \tag{6.3}$$

As the choice probability involves multi-dimensional integral, (6.3) is evaluated using the GHK simulator. Consistent results can be obtained by the simulated likelihood method.

6.3 Numerical methods for parameter calibration

The MDP model is traditionally calibrated by using Nested Fixed-Point (NFXP) algorithm (Rust, 1988, 1987). However, this method is computational demanding and is thought to be impractical in many contexts. Formulating the calibration problem as Mathematical Programming with Equilibrium Constraints (MPEC) greatly reduce the computational burden (Su and Judd, 2008). This study thus adopts the MPEC approach for solving the maximum likelihood problem.

6.3.1 Nested fixed-point algorithm

The NFXP algorithm finds solutions in a nested manner. An inner fixed-point algorithm computes the unknown endogenous variables for each value of model parameter. In the activity-travel scheduling models, the endogenous variables are the travellers' decisions over time. An outer hill climbing algorithm searches for the model parameter that maximizes the likelihood function. The NFXP algorithm is an intuitional and natural method of implementing the maximum likelihood method. The drawback is the computational burden of solving the dynamic programming problem thousands of times in the inner loop. Even though the original author implemented the algorithm in GAUSS language and the program is in the public domain, the code has not been updated for years. The researcher has to implement the algorithm from scratch if it is adopted to solve the calibration problem.

6.3.2 Mathematical programming with equilibrium constraints

The idea behind the Mathematical Programming with Equilibrium Constraints (MPEC) approach is simple. It aims to search the model parameters and endogenous variables to maximize the likelihood function subject to the equilibrium constraint. The endogenous variables fulfil the equilibrium condition defined by the model parameters. The researcher can simply write down the likelihood function and the equilibrium constraints in algebraic modelling languages. Then the model parameters are calibrated with the state-of-the-art constrained nonlinear optimization solvers. The calibration of individual and household activity-travel scheduling models is implemented in AMPL and solved with KNITRO. The AMPL source code can be found in Appendix II.

The NFXP algorithm solves the dynamic programming problem with high accuracy for each guess of the model parameters. In contrast, most modern solvers of MPEC only need to solve the dynamic programming problem at the final iteration for calculating the results. The computational burden is greatly reduced by this strategy. Su and Judd (2008) showed that if MPEC and NFXP are used to solve the same calibration problem, the two methods yield the same calibration results.

6.4 Calibrating the individual's MDP model

The calibration of parameters in individual's activity-travel scheduling model is presented in this section. For a given set of parameters, Monte Carlo experiments were conducted to generate time-series data. Based on the generated data, the

MPEC approach was then employed to calibrate the model parameters. The difference of the actual parameters and the calibrated parameters is used to evaluate the accuracy of the calibration method.

6.4.1 Marginal utility functions

The first marginal utility function is a bell-shaped function (Ettema and Timmermans, 2003):

$$g_a(t) = \frac{\gamma_a \lambda_a U_a^{\max}}{\exp\left[\gamma_a (t-\xi_a)\right] \cdot \left\{1 + \exp\left[-\gamma_a (t-\xi_a)\right]\right\}^{\lambda_a+1}} \tag{6.4}$$

The second marginal utility function is based on a scaled probability density function of the scaled Cauchy distribution (Ettema et al., 2004):

$$g_a(t) = \frac{U_a^{\max}}{\pi c_a \left[1 + \left(\frac{t-b_a}{c_a}\right)^2\right]} \tag{6.5}$$

where $U_a^{\max}$ is the maximum marginal utility, b_a is the time at which the marginal utility reaches the maximum value and c_a determines the period in which a satisfactory marginal utility can be obtained.

Figure 6.1 depicts the temporal profiles of these two marginal utility functions. Both functions are unimodal (having a single local maximum) and ensure that the marginal activity utility increases in the warm-up period and decreases in the saturated period. Although the shapes of the two curves are similar, the scaled Cauchy distribution has a sharp peak and a long tail. Essentially, either marginal utility function can be adopted for empirical analysis. However, the scaled Cauchy distribution has fewer parameters and a simpler functional form. Thus, the scaled Cauchy distribution has fewer identifiability problems and it is employed in the following numerical experiment.

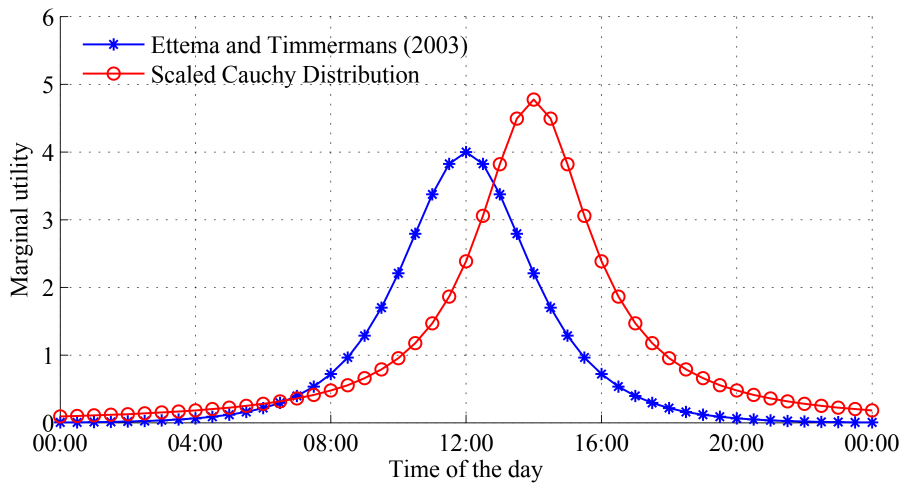

Figure 6.1 Temporal profiles of two types of marginal utility function

6.4.2 Activity-travel data generation

Figure 6.2 shows a transportation network with 3 nodes. Each node represents an activity destination. The free flow travel time of each link is given in the figure. The travel time in congestion is captured by a BPR function:

$$t_l\left(f_l\left(t\right)\right) = t_l^0 \times \left(1 + 0.15\left(\frac{f_l\left(t\right)}{5000}\right)^4\right) \tag{6.6}$$

where t_l^0 is the free flow travel time and $f_l\left(t\right)$ is the flow on link l at time t.

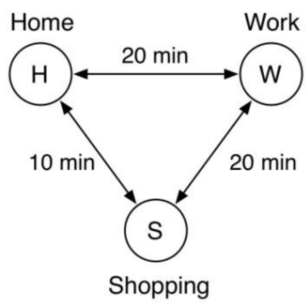

Figure 6.2 A 3-node transportation network

The time is evenly divided into 5-minute intervals. The value of time is set to 60. The discount ratio of the future utility is set to 0.95. These two parameters are treated as fixed and known to maintain the identifiability of the problem.

The marginal activity utility varies over time and is defined by marginal utility function (6.5). Table 6.1 presents the actual values of the parameters of the marginal utility function. These are the parameters to be calibrated. The calibration results will be compared with the actual values in the following.

Table 6.1 Parameters of marginal utility function

No	Activity types	Parameters					
1	Home-AM/PM	U_1^m	3600	b_1	0	c_1	320
2	Work	U_2^m	2500	b_2	840	c_2	180
3	Shopping	U_3^m	2000	b_3	1140	c_3	210

Figure 6.3 depicts the shape of the marginal utility function. There are two in-home activities: *Home-AM* and *Home-PM*. The parameters of the marginal utility functions of the two activities are the same as shown in Table 6.1. From 00:00 to 08:00, *Home-AM* is the activity with the maximum marginal utility. From 08:00 to 17:00, *Work* dominates the other activities in terms of marginal utility. From 17:00 to 21:00, *Shopping* is the dominate activity. From 21:00 to midnight, *Home-PM* is the dominate activity.

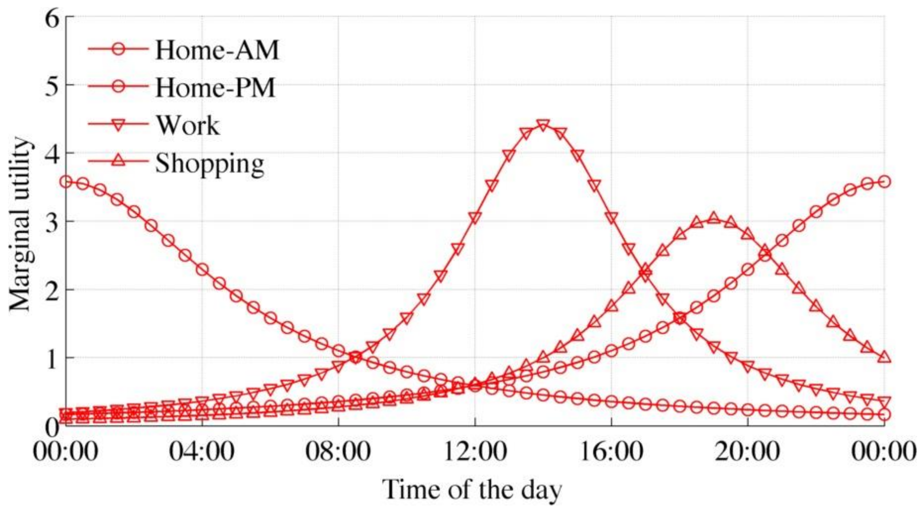

Figure 6.3 Temporal profile of the marginal utility function

The travellers are assumed to choose the daily activity program, activity duration and departure time to maximize the overall utility of the entire day. Their utility maximization behaviours are described by the MDP model. Under these assumptions, time-series data for 288 time intervals (24 hours) and 10000 travellers were generated in the numerical experiment. The choice probability is assumed to follow equation (4.20) and the parameter θ is fixed at 0.2.

The Monte Carlo method was conducted as follows: (1) fix the model parameters at actual values and solve the Bellman equation (4.4) to obtain the optimal value of $\bar{V}(s), s \in S$; (2) use the actual values of the model parameters and $\bar{V}(s)$ to compute the conditional choice probability (4.6); (3) generate choices and state transitions for 10000 travellers in 288 periods based on the choice probability and the travel time.

6.4.3 Calibration results

Before reporting the calibration results, the profile of log-likelihood function and the maximum values are illustrated and discussed. Given the activity-travel data, the log-likelihood function depends on a vector of parameters. Visualizing a multidimensional function is hard. This section thus seeks to illustrate the impact of one or two parameters of interest on the log-likelihood function.

The parameters of marginal utility function can be represented as a vector, $\eta = \left(U^{\max}, b, c\right)$, where $U^{\max} = \left(U_1^{\max}, U_2^{\max}, U_3^{\max}\right)$, $b = \left(b_1, b_2, b_3\right)$, $c = \left(c_1, c_2, c_3\right)$. Let η be the overall maximum likelihood estimate of η and $\eta\left(b_2\right)$ be the vector of parameters with all the parameters except b_2 fixed at the maximum likelihood estimate of η. Then the log-likelihood function is defined by:

$$l_i\left(b_2\right) = l_i\left(\eta\left(b_2\right)\right) \tag{6.7}$$

Figure 6.4 illustrates the log likelihood $l_i\left(b_2\right)$ as a function of b_2 with other parameters fixed at the optimal values. The log-likelihood function $l_i\left(b_2\right)$ is non-convex and has a unique maximum value at $b_2 = 841.4$, which is very close the true value 840. If b_2 is shifted a little from the optimal value, the value of the log likelihood changes dramatically. Vector b determines the time at which the marginal utility function reaches the maximum value and thus, has a strong influence on the activity timing choice.

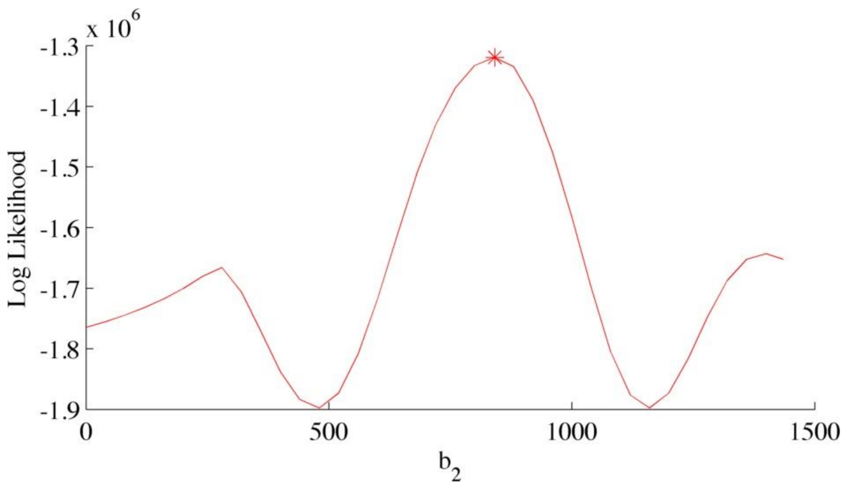

Figure 6.4 The log-likelihood function $l_i(b_2)$

Let $\eta(b_2, b_3)$ be the vector of parameters with all the parameters except b_2 and b_3 fixed at the maximum likelihood estimate of η. Then the log-likelihood function is defined by:

$$l_i(b_2, b_3) = l_i\left(\eta(b_2, b_3)\right) \tag{6.8}$$

Figure 6.5 depicts the log likelihood as a function of b_2 and b_3. The overall appearance of the log-likelihood function reveals a rather complicated relationship between the log likelihood and the model parameters b_2 and b_3. Multiple local optimal solutions can be found in the figure.

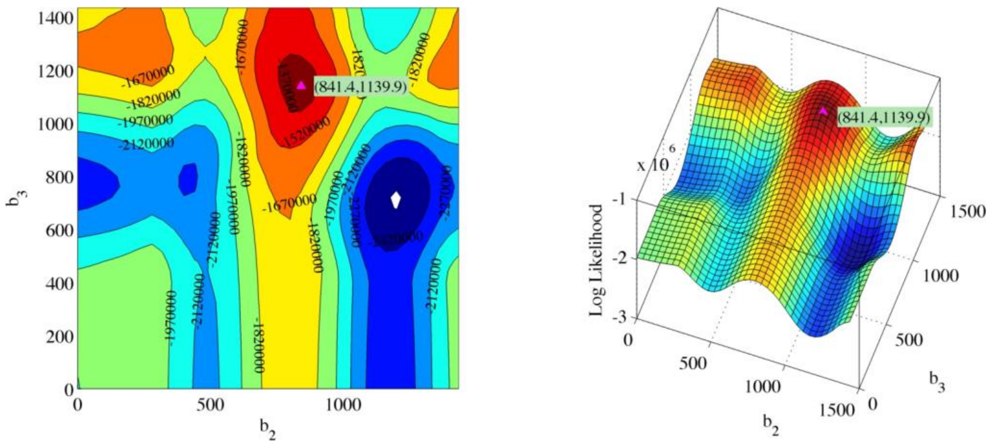

Figure 6.5 Contour and 3-D plot of the log-likelihood function $l_i(b_2, b_3)$

Table 6.2 presents the calibration results of the model parameters. In general, the relative errors of the calibrated values are within 10%. The calibrated maximum marginal utility $U^{\max}$ is smaller than the actual value. The calibrated location parameter b is very close to the actual values. b has a greater impact on the dominate period of each activity than $U^{\max}$ and thus to a great extent determines the activity choice probability and the log likelihood. This is the reason that the calibration of b is more accurate than that of $U^{\max}$. Similarly, the same argument applies to the calibration of c, which determines the width of the marginal utility curve.

Table 6.2 Calibration results of parameters in the marginal utility function

Parameters	$U^{\max}$			$\hat{b}$			$\hat{c}$		
Activity types	H	W	S	H	W	S	H	W	S
Actual values	3600	2500	2000	0	840	1140	320	180	210
Calibrated values	3517	2481	2051	0	841	1140	305	168	219

6.5 Calibrating the household's MDP model

This section presents the calibration of the intra-household interaction coefficient. The household's MDP models include two types of parameters, the intra-household interaction coefficients and the parameters of marginal utility function. Household members have distinct preference over the timing of activities and thus, the parameters of marginal utility function for each member are defined and calibrated.

6.5.1 Household activity-travel data generation

Figure 6.6 shows a 4-node road network on which activity-travel decisions are made. There are 10,000 behaviourally homogeneous households and each household is composed of two adults: Individual 1 and Individual 2. Node H represents the residential location. Node W1 and W2 are the workplaces of the household members respectively. For simplicity, travel time is assumed deterministic and the congestion effect is captured by a BPR function,

$$\tau_l\left(f_l(t)\right) = t_l^0 \times \left(1 + 0.15\left(\frac{f_l(t)}{5000}\right)^4\right) \tag{6.9}$$

where $f_l(t)$ is the flow on link l at time t.

The equivalent disutility of travelling for one hour is $\alpha = 60$. The discount ratio of the future utility is set to $\beta = 0.95$. The entire day (24 hours) is divided into 5-minute periods and there are 288 periods in total.

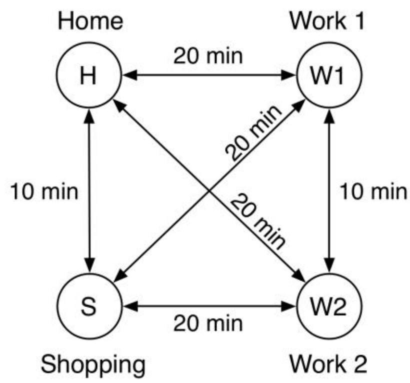

Figure 6.6 A 4-node road network

The scaled Cauchy distribution proposed in (Ettema and Timmermans, 2003) is adopted as the marginal utility function in this example. Three types of activity are considered in the example: *Home*, *Work*, and *Shopping*. The parameters of utility function for each activity are presented in Table 6.3. The two household members have distinct preferences for work and shopping activity. Individual 1 is more willing to go shopping than Individual 2, but receives less utility from work (represented by the bold values in Table 6.3). Figure 6.7 depicts the temporal profiles of the individual's marginal activity utility functions.

Table 6.3 Parameters of marginal utility functions for the household

No	Activity	Parameters of utility function					
		Individual 1			Individual 2		
		U^m	b (min)	c	U^m	b	c
1	Home-AM/PM	3600	0	320	3600	0	320
2	Work	**2500**	840	180	**3000**	840	180
3	Shopping	**2000**	1140	210	**1500**	1140	210

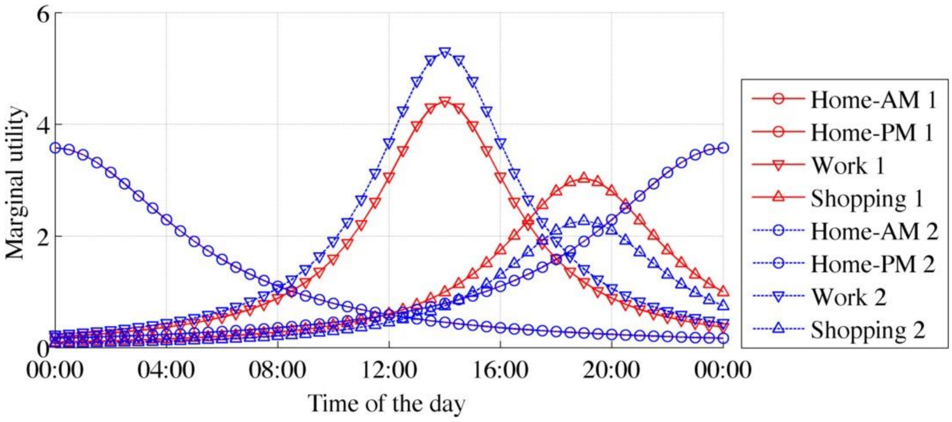

Figure 6.7 Temporal profile of the marginal utility function

The marginal utility for a household defined by equations (5.11) and (5.13) are replicated as follows.

$$r_a = \sigma_1 \cdot r_{1,a} + \sigma_2 \cdot r_{2,a} + \rho_a \cdot r_{1,a} \cdot r_{2,a} \tag{6.10}$$

where $r_{i,a}$ is the individual utility that household member i can obtain when pursuing the activity a independently. The welfare of the individuals in a household is treated equally important. The weight parameter σ_i, representing the relative influence of household member i, is thus fixed at the same value, $\sigma_1 = \sigma_2 = 1.0$. The interaction coefficient *Work* ρ_2 is set to zero. The interaction coefficient of *Home* and *Shopping* is set to $\rho_1 = 0.3$ and $\rho = 0.2$.

6.5.2 Calibration results

The main focus of this section is to show how to calibrate the intra-household interaction coefficient ρ. The log-likelihood function is a multidimensional function of a vector of parameters and hard to be visualized. Following the approach presented in Section 6.4.3, the impact of one or two parameters of interest, i.e., the interaction coefficients, on log-likelihood function is visualized and discussed.

The parameters of the household's MDP model can be represented as a vector, $\eta = \left(U_1^{\max}, b_1, c_1, U_2^{\max}, b_2, c_2, \rho\right)$, where $U_i^{\max}$, b_i and c_i are the vectors of parameters defined for household member i's marginal utility function, and ρ is the vector of intra-household interaction coefficients defined for *Home*, *Work*, and *Shopping*, $\rho = (\rho_1, \rho_2, \rho_3)$.

Denote by η the overall maximum likelihood estimate of η and let $\eta(\rho_3)$ be the vector of parameters with all the parameters except ρ_3 fixed at the maximum likelihood estimate of η. Then the log-likelihood function is defined by:

$$l_i(\rho_3) = l_i\left(\eta(\rho_3)\right) \tag{6.11}$$

Figure 6.8 illustrates the log likelihood as a function of ρ_3 with other parameters fixed at the calibrated values. The only local maximum of log-likelihood function $l_i(\rho_3)$ is also a global maximum. $l_i(\rho_3)$ has the global maximum at $\rho_3 = 0.188$. The figure shows that log-likelihood function $l_i(\rho_3)$ is concave in interval $[0,1]$. However, no formal proof is obtained to confirm this observation.

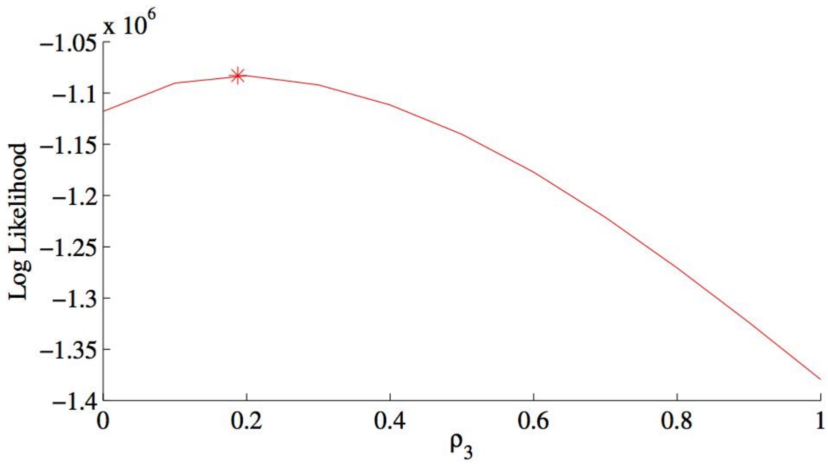

Figure 6.8 The log-likelihood function $l_i(\rho_3)$

Similarly, given the activity-travel data, the log likelihood can be defined as a function of ρ_1 and ρ_3, i.e., $l_i(\rho_1,\rho_3)$. Figure 6.9 shows that the log-likelihood function $l_i(\rho_1,\rho_3)$ is concave in the unit square $[0,1]^2$ and has a global maximum at point $(\rho_1,\rho_3)=(0.282,0.188)$.

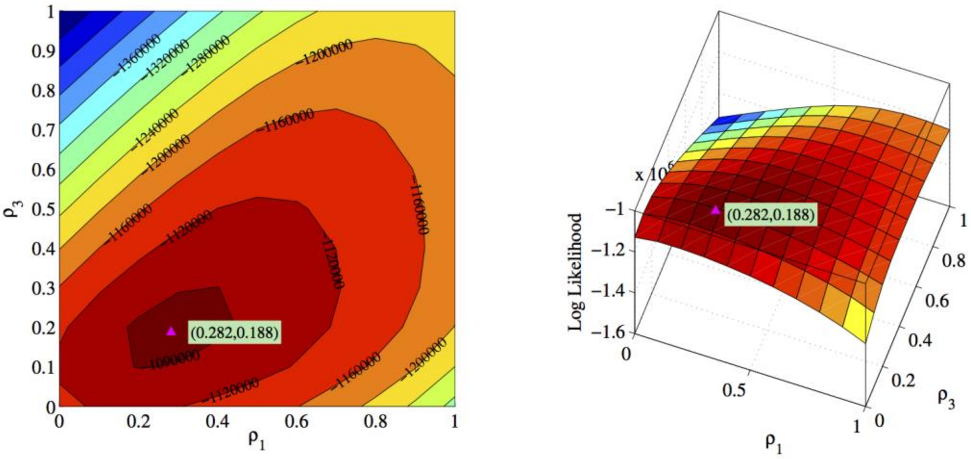

Figure 6.9 Contour and 3-D plot of the log-likelihood function $l_i(\rho_1, \rho_3)$

The true value of intra-household interaction coefficient ρ is $(0.3,0.0,0.2)$ and the calibrated value of ρ is $(0.282,0.011,0.188)$. The accurate calibration of ρ can be contributed to the concaveness of the log-likelihood function over the unit cube $[0,1]^3$. Table 6.4 presents the calibration results of the parameters in marginal utility functions.

The household utility function (6.10) is symmetric with respect to individual's utilities $r_{1,3}$ and $r_{2,3}$. Therefore, the respective calibrated values of $U_{1,3}^{\max}$ and $U_{2,3}^{\max}$ have little effect on the household utility as long as their product is comparable to that of the true values. As shown in Table 6.4, the calibrated values of $U_{1,3}^{\max}$ and $U_{2,3}^{\max}$ are 1791 and 1757, and their product is 3146787. The true values of $U_{1,3}^{\max}$ and $U_{2,3}^{\max}$ are 2000 and 1500 and their product is 3000000. This explains why the calibration results of these two parameters have less accuracy. The relative errors of the other calibrated values are within 10%.

Table 6.4 Calibration results of parameters in the marginal utility function for each household member

Household members	Parameters	$U^{\max}$			$\hat{b}$			$\hat{c}$		
	Activity types	H	W	S	H	W	S	H	W	S
1	Actual values	3600	2500	2000	0	840	1140	320	180	210
	Calibrated values	3654	2389	1791	1	828	1149	338	178	196
2	Actual values	3600	3000	1500	0	840	1140	320	180	210
	Calibrated values	3667	3114	1757	0	849	1128	305	182	226

6.6 Summary

This chapter presents the maximum likelihood method for calibrating the MDP model of individual and household activity-travel scheduling. The activity-travel data required for calibration were generated from numerical experiments. The calibration method was tested and evaluated with these hypothetical data. The calibration results were found satisfactory and the relative errors of most results are within 10%. Notably, numerical experiments reveal that the log-likelihood function is concave over the domain of intra-household interaction coefficient. This property enables efficient and accurate calibration of the coefficient.

Chapter 7 Conclusions and Discussion

7.1 Summary and conclusions

In this thesis, an activity-based network equilibrium modelling framework has been proposed to investigate within-day dynamics and intra-household interactions in activity-travel scheduling behaviour. The activity-travel scheduling behaviour is formulated as Markov Decision Process (MDP). The advantage of the MDP model is that constraints can be imposed on the activity-travel decisions without increasing the computational burden. The MDP model is extended to take into account intra-household interactions for compulsory and non-compulsory activities. Complicated interdependency between household member's activity-travel decisions over time is enabled because of the flexibility of the MDP model.

The individual MDP model has been used to assess the effect of road congestion on activity-travel scheduling behaviour. Road congestion not only induces a larger travel cost, but also reduces the time available for activity participation. Hence, the full cost of road congestion is widely greater than increased financial travel cost. The traditional trip-based models cannot be used to determine how the durations of activities, such as work and shopping, are affected by travel time. The financial cost benefit analysis based on trip-based models is thus biased.

The household MDP model has been used to examine the effect of intra-household interactions on the activity-travel schedule choice. Joint participation of certain non-compulsory activities, such as social visits and outdoor sports, provides a higher overall household utility because of the positive intra-household interaction. Thus, household members tend to participate in the same activity and at the same location. If the intra-household interaction is ignored, the travel demand at the locations of these activities will be underestimated. The intra-household interaction for other non-compulsory activities, such as cooking and cleaning, can be negative. Household utility is maximized by allocating this type of non-compulsory activity to one household member. As a result, the travel demand estimated by traditional trip-based models without consideration of negative intra-household interactions will be overestimated.

The maximum likelihood method is employed to calibrate the intra-household interaction coefficient and marginal utility function in the household's MDP model. The calibration method requires observations of household members' activity-travel decisions over time episodes. The activity-travel data required for calibration were generated from numerical experiments. The calibration method was tested and evaluated with these hypothetical data.

The achievements of this thesis on the development of a modelling framework are summarized and classified into the following five aspects.

(1) In previous activity-based network equilibrium models, travellers are assumed to care only about the utility that can be obtained immediately and choose the activity that has the highest immediate utility. In the proposed MDP model, two types of within-day dynamics in activity-travel scheduling have been captured (Chapter 4). Firstly, to obtain the maximum overall utility in a day, the traveller not only cares about the utility derived from the current activity immediately but also is concerned about the utility that could be obtained during the remainder of the day. Secondly, the activity-travel decisions depend on contextual situations, such as time of day and location.

(2) Each activity-travel schedule is individually treated and enumerated in the previous activity-travel scheduling models. The multi-dimension choices of activity types, destinations, timing and duration constitute many activity-travel schedules. These schedules impose a computational burden on the

implementation of the models. The proposed MDP model avoids activity-travel schedule enumeration by introducing a new structure (i.e., state) to represent activity-travel schedules (Chapter 4 and Chapter 5).

(3) Intra-household interactions affect the individual household member's activity-travel scheduling. In the proposed household MDP model, the interactions between household member's activities are incorporated into a household utility function (Chapter 5) and examined for different activity types. As indicated above, the calibration of travel demand for locations of activities with nonzero interactions is biased in traditional models.

(4) The allocation of activities within a household is also considered in the proposed household MDP model. Certain activities are compulsory for a specific household member, such as work and school, but the non-compulsory activities can be allocated to either household member (Chapter 5). Imposing these constraints on activity choice demonstrates the flexibility of the MDP framework.

(5) Numerical methods are formulated and implemented to calibrate the MDP models. Notably, numerical experiments show that the log-likelihood function is *concave* over the domain of intra-household interaction coefficient ρ (Chapter 6). This property enables efficient and accurate calibration of coefficient ρ.

7.2 Limitations of the study and future research

Although this thesis covers many aspects of individual and household activity-travel scheduling, some questions deserve further investigations. The direction for future research is outlined as follows.

(1) A potential extension of the individual and household MDP models is to consider day-to-day dynamics in activity-travel scheduling. The effect of certain activities can persist for multiple days and thus the activities participated in one particular day can influence the later activity-travel schedules (Arentze and Timmermans, 2009; Habib and Miller, 2008). For example, the goods purchased during a shopping trip can be adequate for a few days' usage. The probability of making another shopping trip on the next day will be very low. Another point to note is that activity-travel schedules on weekdays and weekends differ significantly. Compulsory activities, such as work and school are regular occurrences on weekdays, while some non-compulsory activities, such as physical exercise, are usually performed at the weekend.

(2) In this thesis, all activities are categorized into two types: compulsory and non-compulsory. Based on the values of intra-household interactions, the non-compulsory activities are further categorized into activities with positive interactions and activities with negative interactions. It would be of interest to further categorize activities into even smaller groups based on their socio-economic characteristics. In line with the contentions of Bradley and Vovsha (2004), the variation in intra-household interactions across activity types can then be examined at a finer level of detail.

(3) This thesis only captures the intra-household interactions of a two-person household. In reality, there are different types of households, such as two full-time workers with children, non-worker or part-time worker, and two retired persons (Vovsha et al., 2004). The difference between household members affects the household's activity-travel schedules. For example, young children cannot undertake grocery shopping by themselves. In addition, the number of feasible household states increases exponentially with the number of household

members. The computational burden is the major difficulty for modelling households with three or more individuals. However, approximate dynamic programming with interpolation can be employed to alleviate the computational burden as indicated by Keane and Wolpin (1994).

7.2.1 Challenges in data collection

The limitations presented above are concerned with the modelling methodology. Another limitation is how to apply the proposed models in practice, especially, collecting the data required for model calibration. Travellers' activity-travel choices and geographic locations in a period are required for calibrating parameters of the MDP models. These fine-grained data cannot be readily collected by traditional travel survey methods. For example, travellers may not accurately recall the start time and duration of every activity in a household interview.

To extract more comprehensive information from traditional travel survey data, resampling techniques are adopted to generate data for model calibration (Miller and Roorda, 2003; Roorda et al., 2008). The activity generation is based on random draws of activities from observed probability distribution functions of activity frequency. Activity start time is then randomly drawn from a joint frequency-start time probability distribution function, conditional upon activity frequency. Finally, activity duration is randomly drawn from a joint start time-duration probability distribution function.

In addition to utilization of traditional travel survey data, applying the MDP models in real world also relies on advances in data collection technologies. For example, data mining technology can be used to extract activity locations and travel sequences from travellers' GPS trajectories (Zheng et al., 2010, 2009). As depicted in the left part of Figure 7.1, a GPS log is a collection of GPS points

$P = \{p_1, p_2, ..., p_n\}$. Each GPS point p_i contains latitude lat_i, longitude $long_i$ and timestamp T_i.

	Latitude	Longitude	Time
p_1	Lat_1	$Long_1$	T_1
p_2	Lat_2	$Long_2$	T_2
...	...	...	...
p_n	Lat_n	$Long_n$	T_n

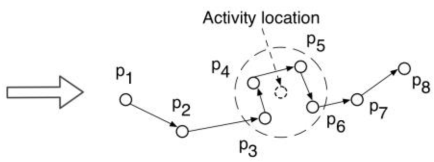

Figure 7.1 GPS points, trajectories and an activity location

The method for identifying activity location replies on the observation that an activity location is a geographic area where a traveller stays for some time to pursue activity. Thus, an activity location can be represented by a cluster of GPS points, $L = \{p_a, p_{a+1}, p_{a+2}, ..., p_b\}$ satisfying the following conditions:

$$\begin{gathered} \text{Distance}\left(p_i, p_j\right) \leq d_{thres} \quad \forall a \leq i, j \leq b \\ T_b - T_a \geq t_{thres} \end{gathered}$$

where d_{thres} and t_{thres} are two parameters defining the space and time thresholds. d_{thres} and t_{thres} can be assumed to be fixed or adaptively change with characteristics of GPS points. Further research is needed to determine the right values for d_{thres} and t_{thres} such that all the activity locations are accurately identified.

The start time and end time of the activity is simply T_a and T_b. The cluster of points is classified as a location of certain activity, such as shopping and work. The classification of cluster of points can be reported by the traveller in travel survey interview. However, if no travel survey interview is conducted, how to infer the corresponding activities for clusters of points is a major challenge in data processing.

Appendix I: MDP with temporal abstraction

In the standard MDP model, the time episode is used to index the decision process. If the current choice is taken at t, the previous choice was made at $t-1$ and the next one at $t+1$. This formulation is convenient when the travel time between activity destinations is much shorter than the period represented by a time episode. In this case, travellers can make an activity-travel choice at every time episode.

If the travel time is longer than a time episode, travelling exclusively takes these time episodes and no activity can be scheduled during this period. For example, a traveller departs from home at 8:30am and arrives at the office at 9:00am. If each time episode represents a 10-minute period, the travel time between home and workplace equals to three time episodes. During this period, the traveller cannot make or implement any decisions. Thus, the time interval between consecutive choices should not be fixed at a constant value.

MDP models are conventionally based on a discrete time episode: the choice made at time t affects the state and utility at time $t+1$. There is no notion of a course of choice that persists over a period of time. Thus, a higher level of temporal abstraction is necessary to handle the activity-travel choice that lasts for a number of time episodes.

The choices that affect the states and utilities for multiple time episodes are termed *decisions*. They are also termed *options* in artificial intelligence literature (Sutton et al., 1999). A decision is viewed as a generalization of the primitive activity-travel choice made at each time episode. A decision d consists of three components: a policy for the primitive choices, $\pi : S \times C \rightarrow [0,1]$, a termination condition, $\gamma : S \rightarrow [0,1]$, and an initiation set $\mathcal{I} \subseteq S$.

For any state s if a decision is taken, activity-travel choices are selected according to π until the decision terminates according to γ. A decision satisfies the *Markov* property if its rule, termination condition and the initiation set depend only on the current state. In particular, a *Markov* decision executes as follows. First, the next choice c_k is selected according to the choice probability function $\pi\left(s_k,\cdot\right)$. The state then transitions from s_k to s_{k+1}. The decision either terminates with probability $\gamma\left(s_{k+1}\right)$, or continues. If the decision continues, a new choice c_{k+1} is selected according to $\pi\left(s_{k+1},\cdot\right)$. This process continues until the termination condition is reached. When the decision terminates, the traveller is able to make another decision. For example, a traveller makes a decision to go for shopping by taking the subway. The primitive choices may involve choosing a subway line, getting off the train, walking to the shopping mall and starting to shop. The decision terminates when the items on the shopping list are purchased or the traveller has been shopping long enough. The initiation set restricts engagement of shopping to states in which the previous activity has been completed and the shopping mall is open.

Given a set of decisions D, let $D\left(s\right)$ denote the set of decisions in D that are available in state s according to the initiation set. $D\left(s\right)$ resembles $C\left(s\right)$ in the standard MDP model, in which $C\left(s\right)$ denotes the set of primitive choices. Similarly, policies over decisions are defined. When initiated in state s_k, the Markov policy over decisions $\mu: S\times D\rightarrow\left[0,1\right]$ selects a decision $d_k\in D\left(s_k\right)$ according to the probability distribution $\mu\left(s_k,\cdot\right)$. The decision d_k determines primitive choices until it terminates in $s_{k+\kappa}$, where $k+\kappa$ is a random time episode. A new decision $d_{k+\kappa}$ is then selected in state $s_{k+\kappa}$ according to $\mu\left(s_{k+\kappa},\cdot\right)$ and so on.

For any decision d and state s, let $\mathcal{E}\left(d,s,k\right)$ denote the event of d being initiated in state s at time k. The total discounted utility of choosing d in state s is defined as:

$$r\left(s,d\right)=\mathrm{E}\left[r_{k+1}+\beta r_{k+2}+\cdots+\beta^{\kappa-1}r_{k+\kappa}\mid\mathcal{E}\left(d,s,k\right)\right]$$

where $k+\kappa$ is the random time at which d terminates. The state transition probability for choosing d in state s is then:

$$p\left(s'\mid s,d\right)=\sum_{\kappa=1}^{\infty}p\left(s',\kappa\mid s,d\right)\cdot\beta^{\kappa}$$

where $p\left(s',\kappa\mid s,d\right)$ is the probability that the decision is initiated in s and terminates in s' after κ steps.

Given the utility and state transition probability of decision d, the Bellman equation for any Markov policy μ in state s can be expressed as:

$$V^{\mu}\left(s\right)=\sum_{d\in D(s)}\mu\left(s,d\right)\cdot\left[r\left(s,d\right)+\sum_{s'}p\left(s'\mid s,d\right)V^{\mu}\left(s'\right)\right]$$

which is analogous to (3.1). The corresponding Bellman equation for the value of a decision d in state $s\in\mathcal{I}$ is:

$$Q^{\mu}\left(s,d\right)=r\left(s,d\right)+\sum_{s'}p\left(s'\mid s,d\right)\sum_{d'\in D(s')}\mu\left(s',d'\right)\cdot Q^{\mu}\left(s',d'\right)$$

Finally, the optimal Bellman equations are as follows:

$$V^{*}\left(s\right)=\max_{d\in D(s)}\left\{r\left(s,d\right)+\sum_{s'}p\left(s'\mid s,d\right)V^{*}\left(s'\right)\right\}$$

$$Q^*(s,d) = r(s,d) + \sum_{s'} p(s'|s,d) \cdot \max_{d' \in D(s')} \{Q^*(s',d')\}$$

The policy over the primitive choices is defined as follows. If $a_s \in A$ and $e_s > 0$, that is the current activity is not completed, the traveller chooses to continue the current activity in the next period $t_s + 1$. If $a_s = \texttt{travel}$ and $e_s > 0$, that is the traveller has not arrived at the destination, the traveller must continue the trip in the next period $t_s + 1$. The initiation set includes the states that the current on-going activity or travelling is completed $\mathcal{I} = \{s | e_s = 0\}$.

If a decision is made, the associated policy over primitive choices is followed until the decision terminates. The corresponding termination rule is defined as: $\gamma(s) = 0$ for any $s \notin \mathcal{I}$ and $\gamma(s) = 1$ for any $s \in \mathcal{I}$. This assumption is restrictive. Suppose that a traveller is heading for an activity destination. The traveller may cancel the planned activity and go to another location. The model formulation can be adapted to capture this type of interruption. Instead of following the decision until termination, the traveller can re-evaluate the decision at each step. That is comparing the utility of continuing with d, which is $Q^\mu(s,d)$, with the utility of making a new decision according to the policy μ, which is $V^\mu(s) = \sum_{d'} \mu(s,d') Q^\mu(s,d')$. The MDP models in Chapter 4 and 5 do not capture this type of interruption. However, the MDP models can be extended in this direction without difficulty.

Appendix II: AMPL code for solving MDP models and calibrating model parameters

```
# Title: A Dynamic Markov Activity-Travel Scheduler
# Author: Xiong Yiliang <wlxiong@gmail.com> 2013

# Go to the NEOS Server (google "NEOS Server for Optimization").
# Click on "NEOS Solvers" and then go to "Nonlinearly Constrained
Optimization"
# You can use any of the constrained optimization solvers that take AMPL
input.

# AMPL Model File:   MarkovActv.mod
# AMPL Data File:    MarkovActv.dat
# AMPL Command File: MDPNonlinearEqn.run, JointMDPNonlinearEqn.run
#                    MDPStateAction.run, JointMDPStateAction.run
#                    MLEMathProgEC.run, JointMLEMathProgEC.run

# control of debug logging
param debug_log;

# SET UP THE MODEL and DATA #

#  Define and process the data
param T;        # the equivalent minutes of a time episode
param H;        # number of time episode in the data
param DH;       # the longest duration for a decision
set TIME := 0..(H-1);   # TIME is the vector of time episodes
param N := 2;           # number of individuals in the household
set PERS := 1..N;       # PERS is the index set of individuals
param M;                # number of activities, including HOME
set ACTV := 1..M;       # ACTV is the index set of activities
param HOME;             # define HOME activity
set WORK {n in PERS};   # define the work activity for each household member

# generated time serise data
param n1;                # a specific household member
param I;             # sample size
set SAMPLE := 1..I;       # sample IDs
param xt {SAMPLE, TIME};  # state: current activity
param dx {SAMPLE, TIME};  # decision: activity type
param dh {SAMPLE, TIME};  # decision: activity duration
param xt1 {SAMPLE, TIME}; # state: current activity of person 1
param xt2 {SAMPLE, TIME}; # state: current activity of person 2
param dx1 {SAMPLE, TIME}; # decision: activity type of person 1
param dx2 {SAMPLE, TIME}; # decision: activity type of person 2

# shortcuts for set union and set product
set AUW {n in PERS} := ACTV union WORK[n];
set AW1xAW2 := AUW[1] cross AUW[2];
set ALLACTV := ACTV union WORK[1] union WORK[2];

# Travel time varies over time of the day
param travelTime {TIME cross AW1xAW2};

param opening {ALLACTV}; # activity opening time
param closing {ALLACTV}; # activity closing time

# Declare the feasible states
param isFeasibleState {n in PERS, t in 0..H, j in AUW[n]} default 0;
param isFeasibleCoState {t in 0..H, (j1,j2) in AW1xAW2} default 0;
# Declare the feasible choices
param isFeasibleChoice {n in PERS, t in 0..H, j in AUW[n], k in AUW[n], h in
1..DH} default 0;

# Define the state space used in the dynamic programming part
# X is the index set of states
set X {n in PERS}:= {t in TIME, j in AUW[n]: isFeasibleState[n,t,j] == 1};
# XX is the set of composite states
set XX := {t in TIME, (j1,j2) in AW1xAW2:
    isFeasibleState[1,t,j1] == 1 and
```

```
    isFeasibleState[2,t,j2] == 1 and
    isFeasibleCoState[t,j1,j2] == 1};
# DTRAVEL is the index set of travel decisions
set DT {n in PERS, (t,j) in X[n]} := {k in AUW[n], h in 1..DH:
    k != j and h == travelTime[t,j,k] and isFeasibleChoice[n,t,j,k,h] == 1};
# DA is the index set of activity decisions
set DA {n in PERS, (t,j) in X[n]} := {k in AUW[n], h in 1..DH:
    k == j and isFeasibleChoice[n,t,j,k,h] == 1};
# D is the union of sets of travel and activity decisions
set D {n in PERS, (t,j) in X[n]} := DT[n,t,j] union DA[n,t,j];
# DD is the set of composite decisions. To simplify the state transition,
# the activity durations of the component decisions should be the same.
set DD {(t,j1,j2) in XX} := {a1 in AUW[1], a2 in AUW[2], h in 1..DH:
    (a1,h) in D[1,t,j1] and (a2,h) in D[2,t,j2]};

# Parameters and definition of transition process

# Define discount factor. We fix beta since it can't be identified.
param beta;           # discount factor

# END OF MODEL and DATA SETUP #


# DEFINING STRUCTURAL PARAMETERS and ENDOGENOUS VARIABLES TO BE SOLVED #

# value of time
param VoT >= 0;

# theta: parameter of the logit choice model
param theta >= 0;

# intra-household interaction coeffcient for each activity
var rho {ALLACTV} >= -1.0, <= 1.0;
# true value
param rho0 {ALLACTV} >= -1.0, <= 1.0;
# calibrated value
param rho_ {ALLACTV} >= -1.0, <= 1.0;

# PARAMETERS OF CAUCHY DISTRIBUTION
# Is Cauchy distribution used ?
var IS_CAUCHY;
# Activity parameters
var Um {PERS cross ALLACTV} >= 0, <= 5000;
var b  {PERS cross ALLACTV} >= 0, <= 1440;
var c  {PERS cross ALLACTV} >= 0, <= 600;
# True parameters
param Um0 {PERS cross ALLACTV} >= 0, <= 5000;
param b0  {PERS cross ALLACTV} >= 0, <= 1440;
param c0  {PERS cross ALLACTV} >= 0, <= 600;
# Calibrated parameters
param Um_ {PERS cross ALLACTV} >= 0, <= 5000;
param b_  {PERS cross ALLACTV} >= 0, <= 1440;
param c_  {PERS cross ALLACTV} >= 0, <= 600;


# PARAMETERS OF BELL-SHAPED FUNCTION
# Activity parameters
param U0 {PERS cross ALLACTV};
param U1 {PERS cross ALLACTV};
param xi {PERS cross ALLACTV} >= 0, <= 1440;
param gamma {PERS cross ALLACTV};
param lambda {PERS cross ALLACTV};

# Marginal activity utility
param PI := 3.141592653;
var actvUtil {n in PERS, j in AUW[n], t in 0..H} =
    if IS_CAUCHY == 1 then
        # Scaled Cauchy distribution
        if j == HOME and t >= H/2 then
            Um[n,j]/PI*( atan( ( t*T+T-(b[n,j]+1440) )/c[n,j] ) - atan( (
t*T-(b[n,j]+1440) )/c[n,j]) )
        else
            Um[n,j]/PI*( atan( ( t*T+T-b[n,j])/c[n,j] ) - atan( ( t*T-
b[n,j])/c[n,j]) )
    else
        # Bell-shaped marginal utility function
        if j == HOME and t >= H/2 then
```

```
            T * ( U0[n,j] +
                  gamma[n,j]*lambda[n,j]*U1[n,j] /
                     ( exp( gamma[n,j] * (1440.0 - t*T - xi[n,j]) ) *
                       ( 1 + exp( -gamma[n,j]*(1440.0 - t*T - xi[n,j]) )
)**(lambda[n,j]+1) ) )
        else
            T * ( U0[n,j] +
                  gamma[n,j]*lambda[n,j]*U1[n,j] /
                     ( exp( gamma[n,j] * (t*T - xi[n,j]) ) *
                       ( 1 + exp( -gamma[n,j]*(t*T - xi[n,j]) )
)**(lambda[n,j]+1) ) );


# DECLARE EQUILIBRIUM CONSTRAINT VARIABLES
# The NLP approach requires us to solve equilibrium constraint variables

# Define initial values for EV
param initEV;

# Declare expected value of each component state
var EV {n in PERS, t in 0..H, j in AUW[n]} default initEV;

# Declare expected value of each composite state
var EW {t in 0..H, (j1,j2) in AW1xAW2} default initEV;
# Declare lower bound of EW
var lower {t in 0..H, (j1,j2) in AW1xAW2};
# Declare upper bound of EW
var upper {t in 0..H, (j1,j2) in AW1xAW2};

# END OF DEFINING STRUCTURAL PARAMETERS AND ENDOGENOUS VARIABLES #


#  DECLARE AUXILIARY VARIABLES  #
#  Define auxiliary variables to economize on expressions

#  Define the total discounted utility of pursuing activity j in time (t,
t+h-1)
var sumActvUtil {n in PERS, (t,j) in X[n], (k,h) in DA[n,t,j]} =
    sum {s in 1..h} beta**(s-1) * actvUtil[n,k,t+s];
#  Define the total discounted utility of traveling from j to k departing at
t
param sumTravelCost {n in PERS, (t,j) in X[n], (k,h) in DT[n,t,j]} =
    sum {s in 1..h} beta**(s-1) * T*VoT/60;
# Define the joint utility
var jointActvUtil {(t,j1,j2) in XX, (a1, a2, h) in DD[t,j1,j2]} =
    if a1 == a2 then
        sum {s in 1..h} beta**(s-1) *
sqrt(actvUtil[1,a1,t+s]*actvUtil[2,a2,t+s])
    else
        0.0;

# Define the utility of selecting decision (k,h)
var choiceUtil {n in PERS, (t,j) in X[n], (k,h) in D[n,t,j]} =
    if k == j then
          sumActvUtil[n,t,j,k,h]
    else
        - sumTravelCost[n,t,j,k,h];

# Declare the choice probability
var choiceProb {n in PERS, (t,j) in X[n], (k,h) in D[n,t,j]} =
    exp( theta * (choiceUtil[n,t,j,k,h] +
                  beta**h * EV[n,(t+h),k]) -
         theta * EV[n,t,j] );

# Define the joint decision utility
var jointChoiceUtil {(t,j1,j2) in XX, (a1, a2, h) in DD[t,j1,j2]} =
    if a1 == j1 and a2 == j2 and a1 == a2 then
        sumActvUtil[1,t,j1,a1,h] + sumActvUtil[2,t,j2,a2,h] +
        rho[a1] * jointActvUtil[t,j1,j2,a1,a2,h]
    else if a1 == j1 and a2 == j2 then
        sumActvUtil[1,t,j1,a1,h] + sumActvUtil[2,t,j2,a2,h]
    else if a1 == j1 then
        sumActvUtil[1,t,j1,a1,h] - sumTravelCost[2,t,j2,a2,h]
    else if a2 == j2 then
        - sumTravelCost[1,t,j1,a1,h] + sumActvUtil[2,t,j2,a2,h]
    else
        - sumTravelCost[1,t,j1,a1,h] - sumTravelCost[2,t,j2,a2,h];
```

```
# Declare the joint choice probability
var jointChoiceProb {(t,j1,j2) in XX, (a1, a2, h) in DD[t,j1,j2]} =
    exp( theta * (jointChoiceUtil[t,j1,j2,a1,a2,h] +
                  beta**h * EW[t,a1,a2]) -
         theta * EW[t,j1,j2] );


#  END OF DECLARING AUXILIARY VARIABLES #

# DEFINE OBJECTIVE FUNCTION AND CONSTRAINTS #

# TODO calculate profile of likelihood and draw 3D diagrams

# Define the objective: Likelihood function
#   The likelihood function contains two parts
#   First is the likelihood that the engine is replaced given time t state
in the data.
#   Second is the likelihood that the observed transition between t-1 and t
would have occurred.
maximize likelihood0: 0;

maximize likelihood:
    sum {i in SAMPLE, t in TIME}
        if (t, xt1[i,t]) in X[n1] and (dx1[i,t], dh[i,t]) in D[n1, t,
xt1[i,t]] then
            log( choiceProb[ n1, t, xt1[i,t], dx1[i,t], dh[i,t] ] )
        else
            0.0;

maximize likelihood_joint:
    sum {i in SAMPLE, t in TIME}
        if (t, xt1[i,t], xt2[i,t]) in XX and (dx1[i,t], dx2[i,t], dh[i,t])
in DD[ t, xt1[i,t], xt2[i,t] ] then
            log( jointChoiceProb[ t, xt1[i,t], xt2[i,t], dx1[i,t], dx2[i,t],
dh[i,t] ] )
        else
            0.0;


#  Define the Bellman equation of the component MDP model
subject to Bellman_Eqn {n in PERS, (t,j) in X[n]}:
    EV[n,t,j] = if card(D[n,t,j]) > 1 then
                    log( sum {(k,h) in D[n,t,j]}
                            exp( theta * (choiceUtil[n,t,j,k,h] +
                                          beta**h * EV[n,(t+h),k]) ) ) /
theta
                else sum {(k,h) in D[n,t,j]}
                            (choiceUtil[n,t,j,k,h] + beta**h *
EV[n,(t+h),k]);
subject to Bellman_EqnH {n in PERS}:
    EV[n,H,HOME] = EV[n,0,HOME];


# Define the Bellman equation of the composite MDP model
subject to Bellman_Joint {(t,j1,j2) in XX}:
    EW[t,j1,j2] = if card(DD[t,j1,j2]) > 1 then
                    log( sum {(a1,a2,h) in DD[t,j1,j2]}
                            exp( theta * (jointChoiceUtil[t,j1,j2,a1,a2,h]
+
                                          beta**h * EW[t,a1,a2]) ) ) /
theta
                  else   sum {(a1,a2,h) in DD[t,j1,j2]}
                            (jointChoiceUtil[t,j1,j2,a1,a2,h] + beta**h *
EW[t,a1,a2]);
subject to Bellman_JointH:
    EW[H,HOME,HOME] = EW[0,HOME,HOME];

# Define the Bellman equation for updating the lower and upper bounds
subject to Bellman_Lower {(t,j1,j2) in XX}:
    lower[t,j1,j2] = log( sum {(a1,a2,h) in DD[t,j1,j2]}
                            exp( theta *
(jointChoiceUtil[t,j1,j2,a1,a2,h] +
                                          beta**h * lower[t,a1,a2]) ) )
/ theta;
subject to Bellman_LowerH:
    lower[H,HOME,HOME] = lower[0,HOME,HOME];
```

```
subject to Bellman_Upper {(t,j1,j2) in XX}:
    upper[t,j1,j2] = log( sum {(a1,a2,h) in DD[t,j1,j2]}
                                 exp( theta *
(jointChoiceUtil[t,j1,j2,a1,a2,h] +
                                              beta**h * upper[t,a1,a2]) ) )
/ theta;
subject to Bellman_UpperH:
    upper[H,HOME,HOME] = upper[0,HOME,HOME];

# Define the lower and upper bounds for EW
subject to LowerBound {(t,j1,j2) in XX}:
    EW[t,j1,j2] >= lower[t,j1,j2];
subject to UpperBound {(t,j1,j2) in XX}:
    EW[t,j1,j2] <= upper[t,j1,j2];

# Symmetric parameters
subject to Symmetric_Um {j in ACTV}: Um[1,j] = Um[2,j];
subject to Symmetric_b  {j in ACTV}:  b[1,j] =  b[2,j];
subject to Symmetric_c  {j in ACTV}:  c[1,j] =  c[2,j];

# END OF DEFINING OBJECTIVE FUNCTION AND CONSTRAINTS
```

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