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Thesis entitled

Acceleration techniques for on-line transient and dynamic stability assessment

submitted by

Cheung Chi Ho

for the degree of Master of Philosophy
at The Hong Kong Polytechnic University
in August 2004

Supervisors: Dr. K.W. Chan, Prof. T.S. Chung and Dr.C.Y. Chung
(Department of Electrical Engineering)
A thesis submitted in partial fulfillment of the requirements for the Degree of Master of Philosophy
Abstract of the thesis entitled

Acceleration techniques for on-line transient and dynamic stability assessment

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Abstract:

This thesis presents several new acceleration techniques for on-line transient and dynamic stability assessment. In recent years, power systems have grown in both size and complexity owing to extensive interconnection, de-regulation, high growth rate of electric power demand. The dynamic characteristics of the power system could vary rapidly as the system conditions change. Online transient and dynamic stability assessment has now become a critical issue for secure and reliable operation of the power system.
In order to realize on-line transient and dynamic stability simulation for large-scale networks, it is necessary to use neoteric technology of hardware and software to speed up the computation. As the computation speed of serial personal computer cannot match the demand of on-line dynamic stability simulation, parallel method is an attractive low cost substitution. Furthermore, not only the parallel computer is used, but also the novel algorithm to further speed up the computation is necessary.

Early studies showed that for the transient stability, solving the linear algebraic sparse network equation is the most computational demanding process. Parallel method had been adopted included W-matrix and parallel piecewise solution method. In piecewise solution method, solve the cut-node block is required in the final step to obtain the overall solution. However, conventionally, it is solved sequentially. In this thesis, the serial cut-node block solution has been identified as the bottleneck of the parallel piecewise method. An innovative add and change scheme is proposed to solve the cut-node block in parallel. Results on the UK 811 bus power system showed that the new approach significantly improves the performance of parallel piece-wise solution method.

For dynamic stability assessment, Prony analysis has been shown as an effective method for the modal analysis of power system oscillations using measured or simulated data. However, for on-line applications, the computation speed of Prony analysis has to be improved. One of the bottlenecks of Prony analysis is as the need for solving the linear predication matrix using the single value decomposition (SVD) technique. In this thesis, a novel solution method called GEAE is proposed to accelerate the solution of the linear predication matrix. Test results showed that a significant speed-up can be achieved with an acceptable accuracy when compared to the standard SVD method.

The above two novel algorithms have been proved to be effective and could
significantly speed up the transient stability computation and dynamic stability assessment. In addition, an AI based damping classifier has been developed to automate the process of contingency screening and ranking.
Declaration

I declare that this thesis represents my own work, except where due acknowledgement is made, and that it has not been previously included in a thesis, dissertation or report submitted to this University or to any other institution for a degree, diploma or other qualification.

Signed

CHEUNG CHI HO Jason
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Jason Cheung
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Chapter 1  Introduction

This thesis describes the investigation and implementation of several acceleration techniques which could be applied for on-line transient and dynamic stability assessment. Over the last century, the power system industry has experienced a number of big and rapid changes such as from standalone to interconnected system and from regulated to de-regulated system. Undoubtedly, the changes will continue in future. However, even with the increase of size and complexity of interconnected power system, the stability of the power systems cannot be compromised. Power system is a dynamic system with states varying from time to time. It is a challenging task for maintaining a large-scale power system to operate in a secure and economic manner. Currently, industry has a strong need for fast on-line stability analysis tools which could help power system engineers to determine the dynamic security of a power system on-line. With the help on-line monitoring, system failures could be avoided and allow the system to operate less conservatively. Any breakthrough in transient stability analysis and dynamic stability assessment could be of tremendous benefit to the efficient operation of power systems. Early studies and researches have addressed and provided insights into the general framework of transient and dynamic stability analysis, and how they can be put up to establish a platform for on-line dynamic security assessment. This thesis is an attempt to explore the acceleration techniques and their practical implementation for fast on-line transient and dynamic stability assessment.

The remainder of this chapter provides the objectives and contributions of this thesis, and an introduction to the power system stability problem, the motivation and approaches used here. Finally, the chapter concludes with an outline of the remainder of the thesis.
1.1 Power system stability

Any physical system that is designed or operated to perform certain pre-assigned tasks in a steady state mode must, in addition to performing these functions in a satisfactory manner, be stable at all times for sudden disturbances with an adequate margin of safety. Power system stability can be broadly defined as the ability of the power system to remain operating in equilibrium under normal operating conditions after response to a disturbance [1-4]. Because of the stiffness of the power system, it is difficult to have a simple classification to cover all the dynamic behaviors. Thus, the stability analysis is usually divided into three types which depend on the nature of disturbance. These are transient, dynamics and steady-state stability analysis.

The modern power system is now entering into the new century. It is in business oriented environment. The interconnections are being used for trading, which is resulting in reduced or even no system stability margins. This also put stress on the importance of system stability and control to prevent or minimize the impact of power blackouts, which have been occurring in growing numbers in recent years. Nowadays, the system stability assessment is critical for power system planning and operation. For such large and complex as a typical modern interconnected power system, the investigation of stability requires both analytical sophistication in terms of techniques employed and practical experience in interpreting the results properly.
1.1.1 Transient stability

Transient stability is defined as the ability of a power system to maintain in synchronism when it is subjected to a severe disturbance. It concerns with the behavior of the synchronous machines after they have been perturbed. For instance, if an unbalance between the supply and demand is created by a change a load, in generation, or in network conditions, a new operating state is necessary, with the subsequent adjustment of the power angles. The adjustment to the new operating condition is called the transient period. The problem of interest is whether the synchronous machines maintain in synchronism at the end of the transient period.

Since the degree of severity and probability of disturbance occurrence appeared on the power system can vary widely, the system is designed and operated to be stable for a selected set of contingencies. The contingencies usually considered are different types short-circuits or faults. They are usually assumed to occur on transmission lines, but occasionally bus or transformer faults are also considered. The fault is assumed to be cleared by the opening of appropriate breakers to isolate the faulted element. If one of these large impacts occurs, the synchronous machines may lose synchronism. This behavior is referred to in the literature as the transient stability problem. Normally, the simulation time required for the investigation of the transient stability problems is about 3-5 seconds and 10 seconds for a large-scale system.

1.1.2 Dynamic stability

Dynamic stability is associated with power oscillations between one machine against the system or between groups of machines in different areas for interconnected system or among machine groups of any combination.
In today’s large modern power system, the transient instability may not occur as first-swing instability, it could be the result of the superposition of several oscillation modes causing large shifting of rotor angle beyond the first swing which is classified as dynamic or oscillatory stability. The problem is usually due to lack of sufficient damping torque.

Figure 1.1 illustrates the behavior of a synchronous machine for stable and unstable situations. It shows the rotor angle responses after different degree of disturbance cases. In Case 1, the rotor angle continues to increase steadily until synchronism is lost. This kind of unstable or instability is usually referred to as first-swing instability which normally cased by insufficient of synchronizing torque. On the contrary, in the stable case (Case 2), the rotor angle increases to a maximum, then decreases and oscillates with decreasing amplitude until it reaches a steady state. In Case 3, it is stable in the first-swing but with sustained oscillation that may be classified as so called transient stable. However, this form of oscillatory instability phenomenon is caused by sufficient of synchronizing torque but lack of damping torque.
1.1.3 On-line stability assessment

Conventionally, transient stability assessment (TSA) [5-7] has been performed using off-line calculation which is one of the most important studies in the design and operation of an electric power system. This analysis entails the evaluation of the ability of a power system network to withstand large disturbances and survive the transient to a normal or acceptable operating condition.

It is the current practices that power utility engineers perform a huge number of off-line transient stability simulations to determine and assess the operating security limits. In
this process, the detailed stability analysis is conducted for each credible contingency under a variety of operating conditions. These limits are then used in energy management system at the control centre for on-line dynamic security monitoring. However, in the system operations, the conditions may not match well with the off-line studies. Consequently, many power utilities have increasingly faced with the threat of transient instability because of stressed operation due to economic and environmental pressure. As a result, fast direct methods based on the Extended Equal Area Criterion (EEAC) or Transient Energy Function (TEF) have been received considerable attention in the recent decade [7-12]. The drawback for the direct method is limited modeling capability. To overcome those drawbacks, the step by step time domain simulation (TDS) is usually adopted in practices. Though TDS remains the most reliable method with unlimited modeling capability, it is used to be considered as too computationally demanding for on-line use and applications. However, with the recent rapid advancement in solution algorithms and computing technologies, on-line dynamic security assessment system based on the use of fast TDS engine has now been beginning to realize, and a number of reviews and investigations on the acceleration techniques for fast time domain simulation have been triggered and initialized [13].
1.2 Current approaches for transient and dynamic stability assessment

Parallel processing is one of the commonly used techniques for speeding up computational intensive tasks. The idea of parallel processing is that, by dividing a task into a number of smaller sub-tasks, multiple processing units could be used to solve the sub-tasks in parallel with less overall execution time. The speedup and efficiency obtained are generally application dependent. For the applications of power system analysis, parallel processing techniques have been applied since the early Seventies, and a fair number of parallel algorithms have been proposed in various journals over the years [13].

For transient stability analysis, power system models can be divided into two parts - generators and transmission network, and the transient stability problem can be considered as solving a set of non-linear differential algebraic equations. Early studies showed that the Newton method could be to solve the non-linear differential algebraic equations for each period at one time, then solve them in parallel [14,15]. Alternatively, parallelism can be exploited by examining the general structure of a power system. The power system itself has inherent parallelism in that each generator is only affected by other generators via the transmission network. This can be exploited by a partitioned solution method such that each set of equations can be solved concurrently on a different processor. Fig 1.2 shows the basic algorithm for the transient stability simulation with parallel machine and parallel network solutions using 4 processors. Exploiting concurrency in the network solution is less obvious. Though a number of parallel algorithms have been proposed by various researchers [13], the two most promising methods are the W-matrix [14-16] and parallel piece-wise solution method [17-19]. In this thesis, both of those two methods will be fully considered and a recommendation will be given for their suitability in the applications of transient stability analysis.
For the dynamic stability assessment, the common approach is to use eigenvalue analysis to determine the modes of oscillations and hence the damping performance of the system. However, this approach is not well suited for on-line applications, especially for large-scale power systems, due to its high computational demand and the difficulties in obtaining the system parameters needed for the eigenvalue analysis. An alternative approach, which could be more suitable for on-line dynamic stability assessment, is based on the Prony analysis. Hauer firstly proposed and reported the use of Prony analysis for power system response signal [20]. The application of Prony analysis to power system oscillations are also reported in previous studies [21-24]. The advantage of Prony analysis is that the size of the model is not limited and only the output response of the system is required. In addition, the effects of non-linearity could be included implicitly in the output responses of the system. However, the drawback of Prony analysis is that the observable modes are limited by the types of system disturbances. Some of unstable modes may not be excited and hence cannot be
observed as a result. In case time-domain simulation based transient stability assessment is adopted, the simulation results obtained could be used as the input for further dynamic stability assessment using the Prony analysis technique. As a result, the application of Prony analysis has been proposed as a viable approach for on-line oscillatory stability assessment [25, 26].

For the application of online oscillatory stability assessment, an automated approach for ranking the severity of the power oscillations in each contingency case is highly desirable. Algorithmic techniques include curve fitting [27], filtering [28] and Prony analysis [20] of machine rotor swing curves are not totally reliable when dealing with multi-mode oscillations as masking effects due to, say, slow governing actions can lead to mis-ranking and hence wrong conclusions. While algorithmic techniques of damping evaluation can produce erroneous conclusion when faced realistic data, engineers running power system stability studies will also be skeptical of results evaluated by computer. It was because the requirements corresponding to the opinions of the engineers are difficult to describe algorithmically. It was suggested that alternative approach using AI techniques [29,30] could overcome the difficulties in incorporating “human judgments” into the classification algorithm.

1.3 Proposed enhancements

For the transient stability analysis, the previous section has been discussed that the major focus to accelerate the transient stability analysis is solving the sparse network equations for large scale power system. Based on the existing parallel piece-wise solution method [17-19], the major overhead identified is that the cut-node composing and the solution of cut-node block are performed sequentially. Speeding-up the cut-node is in consequence to increase the efficiency. To overcome overhead, a new data add and exchange scheme for composing cut-node block and parallelize the
cut-node solution method is investigated and tested. The UK National Grid 811 bus-bars power system network [31,32] were tested in 8 processors MIMD SGI Origin 2100 IRIX platform. Since the proposed method deals with the network admittance matrix and is independent of simulation time-step and system models, it can be applied in large-scale ac-dc interconnected power system.

Based on the rotor swing curves obtained from the transient stability assessment, dynamic stability assessment can be carried out using the Prony analysis. However, for on-line applications, the execution speed of Prony analysis has to be improved. The main drawback for the conventional Prony analysis is that the linear predictive matrix involved in the Prony calculation is an ill-conditioned matrix and a time consuming process called singular value decomposition (SVD) has to be used, rather than the conventional Gaussian elimination calculation, to overcome the numerical instability problem. In this thesis, a new Gaussian elimination based approach is proposed to significantly speedup the overall execution of the Prony analysis by eliminating the time consuming SVD process. A comprehensive study of the proposed method has been made to evaluate its performance in terms of both accuracy and execution speed [33,34] when compared with the conventional Singular Value Decomposition (SVD) approach.

Another enhancement which is valuable for on-line monitoring and control of a large-scale power system is an automatic damping classifier which could rank the oscillatory stability and severity of the power oscillations in each contingency case for the system operator. While algorithmic techniques of damping evaluation can produce erroneous conclusion when faced realistic data, engineers running power system stability studies will also be skeptical of results evaluated by computer. It was because the requirements corresponding to the opinions of the engineers are difficult to describe algorithmically. It was suggested that alternative approach using AI techniques could overcome the difficulties in incorporating “human judgments” into the classification
algorithm. In this thesis, an Adaptive Neuro Fuzzy Inference System (ANFIS) based AI technique is adopted to create a fast filter for automatic system damping classification [35]. Since the ANFIS is trained with system responses from a representative power system without the knowledge of either system models or topology, the resultant filter is system and network independent and can be applied to any practical power systems without retraining.

1.4 Specific contributions

The contributions of this thesis can be summarized as follows:

The bottleneck for the parallel piece-wise solution method has been identified, and a new innovative approach to accelerate the solution of the cut-node block is proposed and successfully applied to solve a large set of linear algebraic power system network equations. This new parallel approach can also be applied in other power system analysis calculations such as power-flow, transient and dynamic stability assessments and online applications.

A fast approach for the Prony analysis has proposed, implemented and fully tested for online power system oscillatory stability assessment. It is based on the Gaussian elimination with acceptance error (GEAE) to solve the linear predication matrix arisen from the Prony analysis. Tests on large power systems showed that a significant speed-up could be obtained when compared to the conventional SVD method with comparable accuracy. Test results also demonstrated that lower order model can be obtained with the proposed method for a given signal.
A simple but effective features extraction and ANFIS classifier for automatic power system damping classifications have been developed. The classifier has been trained to generally follow the Prony’s method but without the mis-ranking problem commonly found in algorithmic ranking approaches caused by the masking effect. The developed ANFIS system has three steps to output the solution which are fuzzification, inference, and defuzzification. Compared with conventional algorithmic based classifiers, ANFIS classifier has a very fast computational speed and is suitable for on-line applications as a ultra-fast screening filter.

1.5 Thesis organization

This thesis is organised as follows. Chapter 2 provides the literature review on the areas of transient stability assessment, and identifies the bottleneck of the existing methods. A novel approach for accelerating the parallel piece-wise solution method has been proposed with results fully presented and discussed. Chapter 3 briefly discusses the topic of dynamic stability assessment and presents a new fast solution approach to the Prony analysis called GEAE for on-line dynamic stability assessment. Chapter 4 shows how the power system damping could be classified using an artificial intelligence ANFIS classifier. Finally, based on the results presented in previous chapters, a discussion and conclusion for the complete thesis is provided in Chapter 5. In addition, the future work in the area of on-line stability assessment is also recommended.
1.6 References


Chapter 2  

**Fast transient stability simulation**

With the growth of electricity demand getting higher and higher, the electric power has become the essentials for the human being. The stability and reliability for the power system is a big challenge for power industry. The increasing complexity of electric power networks and interconnections to other infrastructures, vulnerabilities to cascading failures, interactive and large-scale nature of these networks, coupled with advances in modeling, computational methods, software technologies, simulations, control of networks and economic aspects, have stimulated the interest of the power industry and power engineer.

With the advent of deregulation, unbundling, and competition in the electric power industry, new ways are being sought to improve the efficiency of that network without seriously diminishing its reliability. The fast simulation tools can be very useful tools to help the power system engineers in planning, monitoring and controlling the system in high stability and reliability operation condition.

One of the most important studies in the design and operation of an electric power system is the transient stability assessment (TSA) [1-3]. This analysis entails the evaluation of the ability of a power system network to withstand large disturbances and survive the transient to a normal or acceptable operating condition.

### 2.1 Chapter outline

Section 2.2 describes the transient stability analysis simulation method. It briefly discusses the conventional time domain method and direct methods for power system transient stability assessment. Section 2.3 describes the parallel algorithms which could be applied to transient stability simulation for on-line transient stability
assessments. Early studies showed that solving the system algebraic network equations is the most computation demanding process for large-scale transient stability simulation and it is not obvious to parallelize the process. The re-ordering methods used in this thesis and various existing parallel solution methods are reviewed and discussed. Section 2.4 presents a new approach to accelerate the parallel piecewise solution method. Cases studies, implementation, performance evaluation and results comparison with the conventionally method are presented in Sections 2.5. Section 2.6 reports the results and discusses the performance of the new method. Finally, Section 2.7 concludes the chapter with a summary of major findings.

2.2 On-line transient stability analysis

Direct methods, which determine the transient stability of a power system without solving the system differential equations, have been received considerable attention in recent decade. Direct methods based on the transient energy function (TEF) and extended equal area criterion (EEAC) can all provide quantitative measures for fast transient stability assessment [3-5]. In spite of the many significant accomplishments in recent years in the application of the direct methods, modeling limitations and unreliability of computation techniques continue to be the major impediment to their widespread practical use. Whereas time-domain simulation, which is commonly used for multi-machine transient stability simulation, is still regarded as the best tool for transient stability analysis in terms of accuracy, reliability and modeling capability. However, the major drawback of it is that high computation power is required.

In order to achieve the goal of on-line transient stability simulation of large-scale power system networks, new technology to speed up the computation is necessary. The mathematical model of power system dynamic equations arisen from the transient stability simulation is a nonlinear dynamic system. It includes sets of high dimension
nonlinear differential equations and algebraic equations for describing the dynamics behavior of power system equipments, power flow equilibrium and network topology. As the processing power of serial computer cannot meet the demand for on-line transient stability simulation, the parallel approach by using parallel computer is another viable substitution.

The mathematics model of power system transient simulation can be described by a set of differential algebraic equations as below:

\[
\begin{align*}
\dot{X} &= f(X,V) \\
YV &= I
\end{align*}
\]  

(where \(X\) is a state variable vector, \(V\) is a voltage vector, \(Y\) is the admittance matrix of the power system network and \(I\) is a current injection vector. By separating the dependent variables into groups, the differential equation set (2.1) can be solved in a spatial parallel manner and each group is coordinated by the algebraic equation set (2.2).

At present, the practical available method of transient stability analysis is time domain simulation in which the nonlinear differential equations discussed above are solved by using step-by-step numerical integration techniques. There are two types of numerical integration techniques which are implicit and explicit integration method. The typical example for implicit and explicit integration methods are Trapezoidal Rule and Runge-Kutte (R-K) method [6-9] respectively. The basic principles for both the integration methods are detailed in Appendix A1.
2.3 Parallel computation algorithms for power system stability simulation

The models of a power system can be divided into two parts, namely generators and transmission network. For the generator part, inherent parallelism exists because each of generators is only affected by other generators through the transmission network. This can be exploited by a partitioned solution method such that each set of equations can be solved concurrently on a different processor. A complete solution is then obtained by solving the network equations. Exploiting concurrency in the network solution is less obvious and requires detailed analysis of the numerical algorithms. Fig 2.1 shows the basic algorithm for the transient stability simulation with parallel machine and parallel network solutions using eight processors.

![Diagram of basic parallel algorithm for the transient stability simulation](image_url)
2.3.1 Parallel computation of the power system algebraic network equation

There are two major classes of parallel computation of algebraic equation of network. One is to partition the network into several sub-networks. Each sub-network is first solved in parallel, and then the overall solution is obtained by combining the partial solution of each sub-network [10]. The other is to use spatial and time parallel calculation in the forward and backward substitution of the complete network matrix [11,12].

2.3.1.1 Gauss iteration for solving network algebraic equation

Implementation of a parallel method within a conventional dynamic stability program can be a very demanding task. Extensive modification of the original serial source code will often be needed. In this respect, the parallel Gauss iteration method might be the most simple to implement among all the available parallel methods. Only minor changes are needed to parallelize the conventional Gauss method. The following is the Gauss iteration equation of algebraic equation.

\[
U_i = \frac{1}{Y_{ii}} \left( I_i - \sum_{j=1 \atop j \neq i}^{n} Y_{ij} U_j \right) \quad i = 1, 2, \ldots, N
\]

Equation (2.3) is inherently parallel in nature. Because the diagonal element of matrix \( Y \) is dominating, the calculation can be converged quickly when the initial point is suitable. However, poor convergence can be resulted for large power system network without good guess of the initial solution.
2.3.1.2 W-matrix solution method

By taking the advantage of the sparsity of an admittance matrix, the effectiveness of the forward and backward substitution of the network algebraic equation solution can be improved remarkably. However, as the solution process is sequential, it is difficult to construct an efficient parallel method. In general, there are two strategies to construct the parallel computation. One is to develop the network equation

\[ YV = I \]  \hspace{1cm} (2.4)

into

\[ V = ZI \]  \hspace{1cm} (2.5)

where \( Z \) is the impedance matrix. Although the calculation of (2.5) can be easily parallelized, the matrix \( Z \) destroys the sparsity of admittance matrix completely and increases the computation load and memory usage to the extreme. The other parallel strategy \([13, 14]\) is to develop (2.4) into

\[ LDUV = I \]  \hspace{1cm} (2.6)

The solution to this problem proceeds in three steps: forward substitution, diagonal scaling and backward substitution. The computation of \( L \) and \( U \) is preceded by ordering of the rows and columns to minimize the number of new nonzero terms. Define:

\[ W^l = L^{-1} \]  \hspace{1cm} (2.7)
\[ W^u = L^{-1} \]  \hspace{1cm} (2.8)
\( W \) is lower triangular and sparse. By combining the (2.7) and (2.8) to (2.6), the solution to the original problem can be expressed as:

\[
V = W^u \cdot D^{-1} \cdot W^l \cdot I
\]  

(2.9)

Equation (2.9) can then be solved in three steps

Step (1) \( z = W^l \cdot I \)

Step (2) \( v = D^{-1} \cdot z \)

Step (3) \( v = W^u \cdot y \)

Within each step, all multiplications can be performed concurrently. The \( W \) matrix can be partitioned to enhance sparsity. The matrix \( L \) can be expressed as

\[
L = L_1 \cdot L_2 \cdots L_n
\]  

(2.10)

where each \( L_i \) is an identity matrix except for the i-th column, which contains column i of \( L \)

\[
W^l = L^{-1} = L_n^{-1} \cdots L_2^{-1} \cdot L_1^{-1} = W_n^l \cdots W_2^l \cdot W_1^l
\]  

(2.11)

where \( W_i^l \) is \( L_i \) with its signs reversed on the off-diagonal elements. Using this expanded form of \( W \) in (2.9):

\[
V = W_1^u \cdot W_2^u \cdots W_n^u \cdot D^{-1} \cdot W_n^l \cdots W_2^l \cdot W_1^l \cdot I
\]  

(2.12)

then, the adjacent \( W \) matrices can be further combined for instant, the \( W \) matrix is grouped into two parts, the solution for \( V \) can be expressed as
\[ V = W_a^u \cdot W_b^u \cdot D^{-1} \cdot W_b^l \cdot W_a^l \cdot I \]  \hspace{1cm} (2.13)

Equation (2.13) is processed from right to left in five serial steps. All multiplications within each step are amenable to parallel processing.

### 2.3.1.3 Parallel piecewise solution method

By examining the hierarchical structure of a power system, the power system network can be divided into several sub-networks. The node inside a sub-network are numbered and ordered one by one and the end result of this arrangement is that the admittance matrix will be in a bordered block diagonal form (BBDF) matrix as shown in Fig 2.2 for the case with 4 sub-networks and each part of sub-network connected by a cut-node block as shown in Fig 2.3.

![Fig 2.2 Bordered block diagonal form (BBDF) partitioning for 4 processors](image)
Fig 2.3 Cut-node block

The general parallel solution scheme for this BBDF matrix is that each diagonal block with its associated bordered block will be allocated to a processor and can be solved in a spatial parallel manner. After the partial solutions are calculated, the global solution can then be determined. This method is also called as the network piecewise method and was developed in the early sixties to overcome the limitation of small memory core in the mainframe at that time. The general diagram for parallel LU factorization and forward/backward substitution was shown in Fig 2.4 and Fig 2.5. The network partitioning can either be based on the node or branch tearing [15]. With the BBDF partitioning, the network algebraic equation will have the configuration as follow:

\[
\begin{bmatrix}
Y_{11} & Y_{1T} \\
Y_{22} & Y_{2T} \\
\vdots & \vdots \\
Y_{NN} & Y_{NT} \\
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N \\
\end{bmatrix}
=
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N \\
\end{bmatrix}
\]  

(2.14)

By using LU decomposition, admittance matrix becomes:
According to (2.4), the following can be derived:

\[
Y_{ii} = L_{ii} U_{ii} \quad (2.16)
\]

\[
Y_{iT} = L_{iT} U_{iT} \quad (2.17)
\]

\[
Y_{iT} = L_{iT} U_{iT} \quad i = 1, 2, \cdots N \quad (2.18)
\]

\[
Y_{iT} = L_{iT} U_{iT} + \sum_{i=1}^{N} L_{ii} U_{ii}
\]

where equations (2.16)-(2.18) are solved in parallel first, then after all \( L_{ii}, U_{iT} \) are calculated, equation (2.19) is solved to obtain the complete LU decomposition. As a result, equation (2.14) becomes:

\[
\begin{bmatrix}
L_{11} & U_{11} & U_{1T} \\
L_{22} & U_{22} & U_{2T} \\
\vdots & \vdots & \vdots \\
L_{NN} & U_{NN} & U_{NT} \\
L_{T1} & L_{T2} & \cdots & L_{TN} & L_{TT}
\end{bmatrix}
\begin{bmatrix}
U_{11} & U_{1T} \\
U_{22} & U_{2T} \\
\vdots & \vdots \\
U_{NN} & U_{NT} \\
U_{TT}
\end{bmatrix}
\begin{bmatrix}
V_{1} \\
V_{2} \\
\vdots \\
V_{N} \\
V_{T}
\end{bmatrix}
= \begin{bmatrix}
I_{1} \\
I_{2} \\
\vdots \\
I_{N} \\
I_{T}
\end{bmatrix}
\]

(2.20)

the parallel forward and backward substitution and the equation becomes:

\[
\begin{bmatrix}
U_{11} & U_{1T} \\
U_{22} & U_{2T} \\
\vdots & \vdots \\
U_{NN} & U_{NT} \\
U_{TT}
\end{bmatrix}
\begin{bmatrix}
V_{1} \\
V_{2} \\
\vdots \\
V_{N} \\
V_{T}
\end{bmatrix}
= \begin{bmatrix}
Z_{1} \\
Z_{2} \\
\vdots \\
Z_{N} \\
Z_{T}
\end{bmatrix}
\]

(2.21)

and
\[
\begin{bmatrix}
L_{11} &L_{22} & \cdots & L_{NN} \\
L_{12} &L_{22} & \cdots & L_{NN} \\
\vdots & \vdots & \ddots & \vdots \\
L_{1N} & L_{2N} & \cdots & L_{NN}
\end{bmatrix}
\begin{bmatrix}
Z_1 \\
Z_2 \\
\vdots \\
Z_N
\end{bmatrix}
= 
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N
\end{bmatrix}
\] (2.22)

where

\[L_i Z_i = I_i \quad \text{and} \quad I_i = L_i Z_i \] (2.23)
\[L_T Z_T = I_T - \sum_{k=1}^{N} I_{T,k} \] (2.24)

As there is no dependence existed in equation (2.23), \(Z_i\) and \(T_i\) can be solved in parallel for all \(i\). After all \(Z_i\) and \(T_i\) are solved, equation (2.24) can be solved to obtain \(Z_T\). Similarly for the backward substitution, equation (2.21) can be broken down into the following:

\[U_{TT} V_T = Z_T \] (2.25)
\[U_{ii} V_i + U_{iT} V_T = Z_i \] (2.26)

Equation (2.25) is solved to obtain \(V_T\) first, and then equation (2.26) is solved in parallel for all \(i\) to determine the sub-network voltage vector \(V_i\) in parallel.
Fig 2.4 Parallel LU factorization
2.4 Modified piecewise solution methods

This section illustrates the data add and exchange process and the new parallelized cut-node solution method. The implementation process, case study and the performance evaluation are discussed in the next section.

2.4.1 Data add and exchange

In the parallel LU factorization, the LU computation is distributed to different processors. The individual composed cut-node block of each part is sent back to one
processor (master processor), as shown in Fig 2.6. However, the summation process for the cut-node is still performed serially. To minimize the overhead of the data adding process, the technique of concurrent data add for composing cut-node process is adopted. The summation process for each cut-node is divided into intermediate sums between each processor instead of only one processor. The inter-transfer communication and summation can be divided into three stages as shown in Fig 2.7. The number of stages $m$ can be described in following equation (2.27). The composed cut-node is a dense matrix and its size increases as the number of partition increases. This also leads to increased performance of the parallelization to the cut-node. Further details are discussed in Section 2.4.2.

$$2^m = N \quad (2.27)$$

where

$m$ is number of summation and adding stage

$N$ is number partitions in Parallel LU factorization

![Fig 2.6  Conventional cut-node composing for 8 partitions](image.png)
2.4.2 Parallel cut-node solution

Referring to the piecewise method, the size of cut-node block is dependent on the number of partitions in the LU factorization. If the number of parts increases, the size of the cut-node block as well as the fills-in generated within the block increase and the cut-node block will eventually become a dense matrix. This is the main reason why the performance of the parallel algorithm drops rapidly as the number of partitions increases. Since the cut-node block is served as the synchronize point for the parallel solution, it is processed in serial manner. As a rough estimation, equation (2.27) can be used to show the parallel efficiency for the piecewise method against the number of partitions.

\[ \eta = \frac{E}{Np(Em + Ec)} \]  

(2.27)
where

$N_p$ is number processors

$E$ is the total operation count

$E_m$ is operation count in the largest part

$E_c$ is the operation count in cut-node

Equation (2.27) can be modified to become (2.28) where $n$ is number of processors for handling the cut-node part if the cut-node part could be parallelized. It speeds-up the solution time and efficiency. Fig 2.8 illustrated that by parallelizing the cut-node block, for instant, with $n = 2$, the efficiency can be sustained.

$$
\eta = \frac{E}{N_p(E_m + E_c / n)}
$$

(2.28)

Fig. 2.8 Comparison of equation 2.27 and equation 2.28 with $n=2$ by using table 2.2

As the number of partition increased, the cut-node block almost becomes a dense matrix. To parallelize the cut-node block, the parallel LU composition for dense matrix is adopted [16-18]. Because of its simplicity, there is no ordering technique required.
Hence, the ordering time is saved and it will not reduce the efficiency further by using this approach. Basically, the LU decomposition can be solved by two loops as for the dense matrix. An example code segment is included in Appendix A3.

2.5 Implementation and case study

This section describes the implementation of the task partitioning method for the parallel piecewise solution. As mentioned in previous sections, the admittance matrix needs to be partitioned into a bordered block diagonal form (BBDF). Here, the IEEE 118 bus system is used as an example to illustrate the partitioning method. Together with a description of the parallel computer platform, a UK based 811 bus study is given to demonstrate the performance of the new approach when compared with the conventional one. The UK-811 System is the high-voltage transmission system covering England and Wales operated by the National Grid Company (NGC) plc which has an installed capacity of 57GW. It is linked to two major power systems: 275kV and 400kV AC links to the Scottish power system and a 2000MW DC link with France [19]. The physical characteristics of the UK-811 system are summarized in Table 2.1 [20].

<table>
<thead>
<tr>
<th>Network</th>
<th>UK-811</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Bus-bars</td>
<td>811</td>
</tr>
<tr>
<td>Number of lines</td>
<td>1476</td>
</tr>
</tbody>
</table>

Table 2.1 Physical characteristics of the UK-811 system
2.5.1 Task partitioning

For the parallel piecewise method outlined above, good task partitioning scheme is vitally important in making the parallel computation efficient. As shown in Fig 2.1, the diagonal and border blocks are distributed to a number of processing units for the parallel processing. While the sub-networks have to be approximately equal in size in order to minimize the load unbalance, the size of the cut-node block has to be as small as possible because it is directly related to the amount of data needed to be processed and exchanged among the processing units. An efficient multi-step heuristic method for partitioning an admittance matrix into a bordered block diagonal form (BBDF) with no restriction on the number of partitions has been developed to meet the above criteria. This method is referred as the factorization path graph partitioning method [20]. It takes advantage of the equivalent orderings obtained from the filled graph.

Fig 2.9 shows a path graph of the IEEE 118 bus system. Each branch of the path tree can be considered as a cluster of nodes. By splitting up the branches and then grouping them together into a predetermined number of even sized bunches, the corresponding matrix is effectively partitioned into a predetermined number of partitions. Each partition in turn is made up from a group of the node clusters, and the remaining trunk of the tree becomes the cut-node block $Y_{ij}$. Fig 2.9 to 2.12 shows the admittance matrix of the IEEE 118 bus system with various number of partitions.
Fig 2.9  Restructuring of IEEE 118-bus factorization path tree for 3 processors

Fig 2.10  IEEE 118 admittance matrix in BBDF without any partitioning
Fig 2.11  IEEE 118 admittance matrix in BBDF with 2 partitions

Fig 2.12  IEEE 118 admittance matrix in BBDF with 4 partitions
2.5.2 Parallel computer

The proposed parallel piecewise method has been successfully incorporated into a transient stability power system simulator. This simulator ran and tested on a SGI Origin 2100 parallel computer which has eight 350MHz processors with shared memory. To overcome the memory bandwidth limitation, the SN0 scalable shared memory architecture design was used in SGI Origin parallel computer series. The memory is distributed physically and there is no common bus that could become a bottleneck. Thus, the performance can be optimized by local memory access (LMA). The details for the SGI Origin 2100 memory architecture and the comparison of performance of local memory access (LMA) and remote memory access (RMA) are included in Appendix A2. Table 2.1 shows the speed up for the two typical cases class A and Class B for small and large problem size respectively for SGI Origin 2100 MIMD computer.
<table>
<thead>
<tr>
<th>No. of CPU</th>
<th>Class A</th>
<th>Speed-up</th>
<th>Class B</th>
<th>Speed-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55.43</td>
<td>1</td>
<td>810.41</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>29.53</td>
<td>1.88</td>
<td>419.72</td>
<td>1.93</td>
</tr>
<tr>
<td>4</td>
<td>15.56</td>
<td>3.56</td>
<td>215.95</td>
<td>3.75</td>
</tr>
<tr>
<td>8</td>
<td>8.05</td>
<td>6.89</td>
<td>108.42</td>
<td>7.47</td>
</tr>
</tbody>
</table>

Table 2.2 Performance on SN0 memory architecture

2.5.3 Case study

A 811-busbar power system derived from the UK National Grid system were selected to illustrate the effectiveness of the proposed algorithm. The resulting network admittance matrix has 2993 non-zero elements in total and was partitioned into up to 8 partitions, as detailed in Table 2.2, using a complementary BBDF partitioning algorithm.

<table>
<thead>
<tr>
<th>No of Partitions</th>
<th>Number of Partition Elements</th>
<th>Cut-node Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fill-ins</td>
<td>Min</td>
</tr>
<tr>
<td>1</td>
<td>1550</td>
<td>4543</td>
</tr>
<tr>
<td>2</td>
<td>1554</td>
<td>2003</td>
</tr>
<tr>
<td>4</td>
<td>1546</td>
<td>1013</td>
</tr>
<tr>
<td>8</td>
<td>1540</td>
<td>482</td>
</tr>
</tbody>
</table>

Table 2.3 BBDF partitioning on an 811node admittance matrix
2.5.3.1 Performance indices

Two indices are defined here in order to assess the parallel computation efficiency.

i. Accelerate ratio or Speed-up $S_p$

$$ S_p = \frac{T}{T_p} \quad (2.28) $$

where $T$ is serial program computation time, and $S_p$ is parallel computation run time with $p$ processing units.

ii. Efficiency $E_p$ for Parallel computation

$$ E_p = \frac{S_p}{P} \quad (2.29) $$

where $P$ is the number of processing units.

2.6 Results and comments

Both of the conventional method and new proposed method was implemented on the same parallel platform. Fig 2.14 and 2.15 show the comparison of computation speedup and efficiency of the LU decomposition for the conventional and new proposed parallel factorization using up to 8 processors respectively. The measurement for the single processor is taken from the best serial version with full optimization.

For the given test network, the best speedup for the conventional method achieved using 8 processors is 2.86. For this method, the overall efficiency drops rapidly as the number of processor units increased. Also, the unbalance of the computational load
contributes to the decreasing of efficiency. The speedup gained from the use of more processors is offset by the higher communication requirement and relatively larger serial bottleneck. The new proposed method which embedded the data exchange and parallel cut-node approach showed that the overall speed up and efficiency is improved. Especially, for the case with more partitions, the overall improvement is sharply increased. It is expected that for the 8 partitions case, the number of fill-ins and the matrix size is higher and bigger. The cut-node matrix is almost full and dense.

It is expected that the parallelization for dense LU decomposition will become more efficient as the matrix size gets larger.

In summary, the overall speed up and efficiency of the parallel piecewise solution method (included parallel LU decomposition and forward/backward substitution) is mainly dominated by the parallel LU factorization, as shown in Fig 2.16 and 2.17. For a larger or lower branch-to-bus ratio system, the speedup will be higher as there is a higher degree of parallelism available and the ratio of computation to communication is relatively higher.

![Fig 2.14 Speed up comparison of conventional and new parallel LU decomposition method](image)
Fig 2.15 Efficiency comparison of conventional and new Parallel LU decomposition method

Fig 2.16 Overall speed up comparison of conventional and new parallel piecewise solution method
2.7 Conclusion

The parallel processing techniques for matrix solution has been extensively used in very large systems which consist of hundreds of thousands of linear equations for analyzing real engineering problems. Block bordered diagonal form for LU factorization, forward and backward substitution is commonly adopted for parallel implementation. In the application to the power systems equations solution, the proposed add/exchange algorithm and parallel cut-node block for LU factorization can reduce the communication overhead and consequently gives a better overall performance and efficiency.

The results obtained showed that the speed up and efficiency drop rapidly for the conventional method as the number of partitions increases. It is because the number of element in the cut-node block is also increased and the cut-node block will
eventually become a dense matrix. The overall efficiency drops accordingly as cut-node block is the bottleneck of the parallel algorithm which has to be solved sequentially. In addition, the communication overhead is another factor for the efficiency drop, since all the local copy of cut-node in each part has to be collected before the summation process. The proposed method is specially designed to overcome the bottleneck caused by the cut-node block and hence improves the parallel piecewise solution method.

In conclusions, this chapter presented a new data add and exchange and parallel cut-node LU factorization approach for further speed-up the parallel piecewise solution method. The results were presented for the LU factorization of large algebra linear equations for the power system application. The new algorithm based on the proposed scheduling for summation and data exchange for composition the cut-node block instead of the traditional scheduling method. By using the parallelization techniques for the dense matrices, the large LU cut-node block can be solved concurrently. Hence, the bottleneck of the parallel piecewise solution method was identified and overcome by the use of the new data add/exchange algorithm for composing the cut-node block which consequently improved the efficiency.
2.8 References


Chapter 3  Accelerated Prony analysis

With the accelerated time-domain simulation discussed in previous chapter, on-line transient stability assessment could be carried out using a parallel computer. Any contingencies with transient instability would be detected and filtered out as a result of the assessment. In addition, the rotor responses of the contingency are also available as the output from the time-domain simulation. The next assessment following up would be the dynamic or oscillatory stability assessment and is the main focus of this chapter. For the dynamic stability assessment, early studies have shown that Prony analysis is a valuable and efficient tool for large power system dynamic analysis and can be used as a viable alternative to the conventional eigenvalue analysis. The main deficiency for limiting its application in the on-line environment is its high computation load since it has to solve the ill-conditioned Predictive Model Matrix using the Singular Value Decomposition (SVD) technique. In this chapter, the new algorithm, which is simple and fast, is presented to replace the time consuming SVD process.

3.1 Chapter outline

Section 3.2 introduces the Prony analysis method and its application to the power system stability analysis. Section 3.3 describes and reviews the solution steps of Prony analysis method with the conventional SVD approach. A new fast approach based on the Gaussian elimination with acceptance error is also presented. Section 3.4 discusses the implementation details and results obtained from the new solution approach. Two performance indices are introduced to facilitate the evaluation and validation of the new algorithm. Finally, Section 3.5 concludes with a brief summary of the major findings reported in this chapter.
3.2 Prony analysis and its application to the power system dynamic stability analysis

Power system dynamic stability has been a major concern in system planning and operation [1]. The dynamic behavior of an electric power system is described by the nature of the electro-mechanical oscillations between the machines and power network. Pole-slipping or out-of-step is typically a result of transient stability problems due to a large power imbalance between the mechanical power input to a generator and the available electric load. However, stability problems may also arise from a lack of machine damping torque in the post-contingency period by oscillatory instability.

Over the last several years, Prony analysis has been applied to several power system problems. Prony analysis is a method of fitting a linear combination of exponential terms to a finite number of samples of a signal spaced equally in time. Numerical advantages make it well suited for approximating high order signals with an optimum low order model [2]. The applications of Prony analysis to power system oscillations have been reported in previous studies [2-7]. It has been shown to be a valuable method for estimating the modal content of power oscillations from swing curves. The ability to extract the information from transient stability simulators and from large-scale system tests or disturbances is valuable to power system engineers [5]. There would be numerous applications of such tool in analysis, modeling, and control of power system dynamics.

The main advantage of Prony analysis is that both the size and complexity of the studied system are not limited as only the system responses are analyzed [5]. This means that standard transient stability study results, swing curve for example, obtained from a time domain simulation can be directly used for the assessment of
oscillatory stability.

In Prony analysis, the first step of calculation is to construct a linear prediction matrix from a measured or given signal. Since the linear prediction matrix is an ill-conditioned matrix, singular value decomposition (SVD) is traditionally applied to overcome the numerical instability problem arisen from solving the linear prediction matrix. However, SVD is a computational demanding process; and it will be a time consuming process for large-scale power systems. In this paper, a new approach will be introduced to supersede SVD so as to decrease the computational time required to solve the linear prediction matrix. Other advantages of Prony analysis are that it can be used to obtain system eigenvalues, transfer-function residues and initial condition residues.

Conventionally, eigenvalue analysis is widely adopted as a practical tool for studying and analyzing the dynamic stability of a power system [8-11] because it can provide a precise and clear indication of the oscillation frequency and damping of the network. Its main drawback is that it is very time consuming and takes up a lot of memory space during the process [1], especially when working on large-scale power system networks. As an illustration on the performance of eigenvalue analysis, Fig 3.1 shows the computational time of eigenvalue analysis running on a P4 2.4GHz computer with various system sizes. It is clear that the computational time increases dramatically as the system size increases, and it is not suitable for any online applications at all.
3.3 Solution steps for Prony analysis

Prony analysis is an emerging methodology that extends Fourier analysis. Suppose that a linear, time-invariant dynamic system is brought to an initial state $x_0 = x(0)$ at time $t_0$. It has a number of advantages; such as it can handle multi modal oscillations and it does not require filter parameters to be pre-set. To decompose the damping constant and oscillation frequency so as to predict the dominant mode in the power system, a system of differential equation of the following form is considered.

$$\dot{x} = A x$$

(1)
where $x$ is the state of the system. Let $\lambda_i, p_i, q_i^T$ be the eigenvalues, right eigenvectors, and left eigenvectors of $A$, respectively. The solution to (1) can be expressed as:

$$x(t) = \sum_{i=1}^{n} p_i q_i^T x_0 e^{\lambda_i t} = \sum_{i=1}^{n} R_i x_0 e^{\lambda_i t}$$

(2)

where $x_0 = x(0)$ is the initial state, $R_i = p_i q_i^T$ is an $n \times n$ residue matrix, $n$ is the number of components in $x$, and $q_i^T x_0$ is a scalar.

Prony analysis is a method of fitting a linear combination of exponential terms to a measured or simulated signal, say

$$y(t) = C x(t)$$

$$= \sum_{i=1}^{m} A_i \rho_i^j \cos(2\pi f_i t + \phi_i)$$

(3)

Prony method is designed to directly estimate the parameters for the exponential terms in (2) and (3), by fitting a function to an observed record of $y(t)$.

$$y(t) = \sum_{i=1}^{m} A_i e^{\sigma_i t} \cos(\omega_i t + \phi_i)$$

(4)

Let the record for $y(t)$ consist of N samples. The strategy for obtaining a Prony Solution as shown as follows:

Step 1: construct a discrete linear prediction model that fits the record.

Find $a_i$ for $i = 1$ to $n$.

$$\begin{bmatrix}
y_{n-1} & y_{n-2} & \cdots & y_0 \\
y_{n-0} & y_{n-1} & \cdots & y_1 \\
\vdots & \vdots & \ddots & \vdots \\
y_{N-2} & y_{N-3} & \cdots & y_{N-n-1}
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_n
\end{bmatrix}
= 
\begin{bmatrix}
y_{n+0} \\
y_{n+1} \\
\vdots \\
y_{N-1}
\end{bmatrix}$$

(5)
Step 2: find the roots $z_i$ of the characteristic polynomial associated with linear prediction model of previous step.

$$z^n - (a_1z^{n-1} + a_2z^{n-2} + \ldots + a_n) = 0$$

(6)

Step 3: using the roots of step 2 as a complex modal frequencies for the signal, determine the amplitude and initial phase for each mode. Let $y(t)$ is sampled at a constant period $T$, it can be rewritten in discrete-time form:

$$y_k = \sum_{i=1}^{n} B_i z_i^k$$

(7)

where $y_k = y(kT)$, $B_i = \frac{A_i}{2} e^{i\phi}$, and $z_i = e^{i\lambda T}$.

The objective is to find the $B_i$ and $z_i$ that produces $y_k = y(kT)$ for all $k$, which leads to the solving of the following ill-conditioned expression:

$$ZB = Y$$

(8)

3.3.1 New approach for solving the linear prediction matrix

There are some extensions and refinements required for the practical use of Prony analysis. The major problem for this method is that the true system dimension, $n$, may be unknown or may be so large that the fitted model would be a reduced-ordered approximation. The array dimensions in (5) will usually differ from those shown, and tend to over-fit the signal by using a generous number of samples and fitted modes. Also, the characteristic of linear prediction matrix extracted from (5) is an ill conditioned matrix, and as a result the rounding error could be very large. Singular-value analysis and other mechanisms are needed to adjust model features at each solution step.
3.3.1.1 Singular value decomposition (SVD)

Single value decomposition (SVD) method is based on the equation (9) of linear algebra, which any $M \times N$ matrix $A$ whose number of rows $M$ is greater than or equal to its number of columns $N$, can be written as the product of an $M \times N$ column-orthogonal matrix $U$, an $N \times N$ diagonal matrix $W$ with positive or zero elements (the singular values), and the transpose of an $N \times N$ orthogonal matrix $V$ [8].

$$
A = U \cdot W \cdot V^T
$$

(9)

The matrices $U$ and $V$ are orthogonal in the sense that their columns are orthonormal. That is, let:

$$
U = \{ U_1, U_2, \ldots, U_n \} \quad (10)
$$

$$
V = \{ V_1, V_2, \ldots, V_n \} \quad (11)
$$

Then

$$
U_i \cdot U_j = 0 \quad \text{for} \quad i \neq j \quad (12)
$$

and

$$
V_i \cdot V_j = 0 \quad \text{for} \quad i \neq j \quad (13)
$$

also,

$$
U^T U = U U^T = I \quad (14)
$$

and

$$
V^T V = V V^T = I \quad (15)
$$

Since the expression from Prony’s method yields an $N \times N$ square matrix, then $U$, $W$, and $V$ are all square matrices of the same size.
\[ A = [U][W][V^T] \]  
\[ (16) \]

The matrices \( U \) and \( V \) are each orthogonal in the sense that their columns are orthonormal. Since \( V \) is square, it is also row-orthonormal, which was shown in equation (10).

\[ [U][U^T] = [V^T][V] = 1 \]
\[ (17) \]

With the transformation in (8), the linear prediction matrix solution can be obtained with equation (10).

\[ x = V \cdot \left[ \text{diag}(1/W_j) \right] \cdot (U^T \cdot b) \]
\[ (12) \]

Singular value decomposition can be used to overcome the problems caused by ill-conditioned or singular matrices [11, 12] that the best lower rank approximation to the data matrix is obtained. Though SVD is a robust method for solving ill-conditioned matrix, it is a complex solution technique and could be very time-consuming for large system. On the other hand, the Gaussian elimination is probably the most efficient direct method for solving linear system (16), it cannot cope with ill-conditioned matrices due to excess rounding error. One common technique to improve the accuracy of Gaussian elimination is to adopt the total pivoting. It interchanges the row below as well as the column on the right of the working one to give the largest magnitude of the diagonal element, and the rounding error can be reduced. Having said that total pivoting alone cannot overcome the numerical difficulties raised from ill-conditioned matrices.

For the practical use of SVD, a threshold value has to be set to the diagonal matrix \( W \)
such that any diagonal elements which are smaller than this threshold will be set to zero. The effect of this is to get rid of the small terms which will give large round error. Alternatively, this can be interpreted as reducing the original matrix to a smaller full rank matrix.

3.3.1.2 Gaussian elimination with acceptance error (GEAE)

It is understood that the Gaussian elimination is an effective direct method to find the solution of a linear system $Ax = b$. It is an efficient method that it reduces $A$ to an upper triangular matrix by applying the row operations and then gives the solution of $x$ by backward substitution. The general Gaussian elimination method sometimes fails to give satisfactory results if $A$ is ill-conditioned because the elimination process involves a huge number of arithmetic operations.

The linear system expressing the power system generated by the Prony analysis is inevitably ill-conditioned. In order to reduce the possible computational error due to the effect of rounding, the Gaussian elimination algorithm is modified by carefully defining the appropriate form of the row operation together with the total pivoting technique. The effective row operation is such chosen that for a pivot entry $a_{ij}$

for $j = 1, 2, \ldots, n-1$:

$$ R_i - \left( \frac{a_{ij}}{a_{jj}} \right) R_j \rightarrow R_i $$

where $R_i$ and $R_j$ mean row $i$ and row $j$ from 1 to $n$ and $i > j$.

Rounding error can further be improved if higher precision is required by defining the row operation as:
\[ a_{ij} R_i - a_{ij} R_j \rightarrow R_i \] (14)

The precision is accomplished by noting that the division operation may cause larger computation error than multiplication and therefore is being postponed in the expenses of destroying some nice properties of the original coefficient matrix and possible, although slight, overflowing of the resulting triangular matrix. The total number of arithmetic operations in the two definitions of row operations is basically the same.

Total pivoting is a recognized but infrequently applied technique to reduce the rounding error. To determine the pivot entry and before row operations are applied in each column, the largest pivot \( a_{ij} \), for \( j = 1, 2, \ldots, n-1 \), is guaranteed in the sense that the magnitude of \( a_{ij} \) is greater than or equal to \( a_{rs} \) for all \( r \) and \( s > j \), interchange the row below as well as the column on the right whenever necessary. One of the biggest drawbacks of total pivoting is that the interchanging of columns needs to keep track of the permutation of the solution \( x_i \), hence requires extra effort to denote the corresponding positioning of the \( x_i \). Also, the searching of largest in magnitude of \( a_{ij} \) takes considerably computer time. Therefore it is not frequently used unless rounding error is critical.

The new approach purposed here is to borrow the idea of threshold from the SVD and apply it to the Gaussian elimination. In this approach, a new index called acceptance error (AE) is introduced. It is applied during the total pivoting and forms a stopping criterion for the Gaussian elimination. At each pivoting step, the selected pivot element is compared with AE. The elimination process will stop once the pivot element is smaller than the AE and the rest of columns and rows will be neglected. In other words, the order of the matrix will be reduced as a result. Fig 3.2 shows the programming flowchart of the Gaussian elimination with acceptance error (GEAE). The solution of predictive model matrix is obtained by backward substitution using
(15) with unknown variables corresponded to the neglected columns and rows set to zero.

\[ x_j = \frac{1}{a_{ii}} \left[ b' - \sum_{j=i+1}^{n} a_{ij} x_j \right] \]

Fig. 3.2 GEAE Programming Flowchart
3.4 Implementation and results

The proposed Prony analysis method with modified Gaussian elimination solver is implemented in C language and run on a SGI Origin 2100 server. For comparison purpose, a version for the standard method, which uses SVD instead of the propose GEAE method, has also been developed. Those two versions share the same code base and differ only in the solver for solving the linear prediction matrix. A timer is inserted in each of the SVD subroutine and the modified Gaussian elimination subroutine for time recording. The entire program is in-house made except the SVD subroutine which is adopted from the Numerical Recipes in C by Press [13]. This SVD routine is based on a routine by Forsythe et al., which is in turn based on the original routine of Golub and Reinsch and “the algorithm is very stable, and that it is very unusual for it ever to misbehave”. The Gaussian elimination routine needed to be in-house to implement the purposed row operation and the total pivot technique. The simple program flowchart was showed in Fig 3.2.

The testing cases used for validation is based the UK-811 National Grid System which operates at up to 400kV and is connected to the Scottish power system by two 400kV and two 275kV overhead lines and also inter-connected by a 2000MW DC link to France. The UK-811 System is the high-voltage transmission system covering England and Wales operated by the National Grid Company (NGC) plc which has an installed capacity of 57GW [14]. The system is composed of in excess of 7000km of overhead transmission lines and cables, 21600 towers, 280 sub-stations and up to 200 large generating units. 400 cases of the rotor angle responses from all the generators for various contingencies were recorded and used as a large set of testing cases for the evaluating the performance and robustness of the proposed methods. Each case lasts for 30 seconds with sampling period of 10 milliseconds, i.e. there are totally 3000 data points in each case. In Fig 3.3, two sets of data which show typical examples of a
realistic machine rotor angle are plotted.
Fig 3.3 Typical simulated machine rotor angle curves.

(a) Sample A

(b) Sample B
3.4.1 Evaluation indices

Two indices are defined here for accessing the performance of the proposed method.

1. Speed-up, $Sp$

\[
Sp = \frac{T_{SV D}}{T_{GEAE}}
\]  

(16)

where $T_{SV D}$ and $T_{GEAE}$ are the computation time for the SVD routine and GEAE respectively.

2. Signal to Noise Ratio, SNR (db)

As the reconstructed signal $\hat{y}(k)$ is only a fitted estimate of the original single $y(k)$, an appropriate measure for the quality of the fit is the signal to noise ratio, SNR as defined in (17).

\[
SNR = 20 \log \frac{\|\hat{y}(k) - y(k)\|}{\|y(k)\|}
\]  

(17)

where $\|\|$ denotes the root mean square norm and the SNR is in decibels (db).

3.4.2 Results

The results was shown in Table 3.1 for 10 randomly selected cases (out of 400 cases) for demonstrating the performance of the proposed method, GEAE, compared to the standard SVD method. The computation times of both methods are tabled together with the corresponding speedups, $Sp$, and signal to noise ratio, $SNR$. 

- 60 -
<table>
<thead>
<tr>
<th>Case no.</th>
<th>$T_{\text{SVD}}$ (s)</th>
<th>$T_{\text{GEAE}}$ (s)</th>
<th>Sp</th>
<th>SNR(db)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2175</td>
<td>0.0092</td>
<td>23.68</td>
<td>129.39</td>
</tr>
<tr>
<td>2</td>
<td>0.2157</td>
<td>0.0092</td>
<td>23.54</td>
<td>154.21</td>
</tr>
<tr>
<td>3</td>
<td>0.2138</td>
<td>0.0092</td>
<td>23.30</td>
<td>146.43</td>
</tr>
<tr>
<td>4</td>
<td>0.2184</td>
<td>0.0092</td>
<td>23.79</td>
<td>143.55</td>
</tr>
<tr>
<td>5</td>
<td>0.2121</td>
<td>0.0092</td>
<td>23.11</td>
<td>135.98</td>
</tr>
<tr>
<td>6</td>
<td>0.2191</td>
<td>0.0100</td>
<td>21.86</td>
<td>146.77</td>
</tr>
<tr>
<td>7</td>
<td>0.2094</td>
<td>0.0086</td>
<td>24.46</td>
<td>155.87</td>
</tr>
<tr>
<td>8</td>
<td>0.2096</td>
<td>0.0093</td>
<td>22.65</td>
<td>125.72</td>
</tr>
<tr>
<td>9</td>
<td>0.6052</td>
<td>0.0256</td>
<td>23.64</td>
<td>121.87</td>
</tr>
<tr>
<td>10</td>
<td>0.2029</td>
<td>0.0092</td>
<td>22.00</td>
<td>81.13</td>
</tr>
</tbody>
</table>

Table 3.1 Computational time and Speed-up

It shows that the GEAE method is much faster than the SVD method for solving the model predictive matrices. This is as expected due to the complexity involved for GEAE is much lower than that of SVD. The overall accuracy of the Prony analysis using either GEAE or SVD is found to be comparable with each other. As shown in Table 3.1 the signal to noise ratio, $SNR$, for the 10 selected cases between the raw data $y(k)$ and the reconstructed signal $\hat{y}(k)$ using the GEAE is mostly over 100 db. Fig 3.4 also shows that the speed-up of all ten cases is higher than twenty.
Fig 3.4 Speed-up index (SP=$T_{SVD}/T_{GEAE}$) for ten cases

Fig. 3.5.1 to 3.5.10 show the results of ten original power system swing curves (RAW data), and the swing curves reconstructed by SVD and GEAE respectively with acceptable accuracy. The estimated system orders for the reconstructed curves using SVD and GEAE in Fig 3.5.5 are 60 and 29 respectively. Similarly in Fig 3.5.7 but with higher accuracy, the estimated orders are 112 and 39 respectively. It is obvious that SVD based Prony analysis has a tendency of over-fit compared to the GEAE based method. For a comparable accuracy, the system order for the GEAE is generally smaller and hence faster solution time will be resulted.

The rank approximant to the data matrix determines the order of damping and function poles of the power system. A large order results in giving a higher order polynomial which increases the complexity of the approximation. Reversely, the error increases if the rank is being set too low. The best lower ranks settled by the Gaussian elimination in the ten cases are also smaller than that of the SVD algorithm and they are listed in Table 3.2.
Fig 3.5.1  Swing curve plot result for Case 1

Case 2

Fig 3.5.2  Swing curve plot result for Case 2
Fig 3.5.3  Swing curve plot result for Case 3

Fig 3.5.4  Swing curve plot result for Case 4
Fig 3.5.5  Swing curve plot result for Case 5

Fig 3.5.6  Swing curve plot result for Case 6
Fig 3.5.7  Swing curve plot result for Case 7

Fig 3.5.8  Swing curve plot result for Case 8
Fig 3.5.9  Swing curve plot result for Case 9

Fig 3.5.10  Swing curve plot result for Case 10
<table>
<thead>
<tr>
<th>Case no.</th>
<th>SVD (n)</th>
<th>GEAE (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>56</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>37</td>
<td>19</td>
</tr>
<tr>
<td>4</td>
<td>42</td>
<td>21</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
<td>29</td>
</tr>
<tr>
<td>6</td>
<td>49</td>
<td>25</td>
</tr>
<tr>
<td>7</td>
<td>112</td>
<td>39</td>
</tr>
<tr>
<td>8</td>
<td>55</td>
<td>29</td>
</tr>
<tr>
<td>9</td>
<td>111</td>
<td>36</td>
</tr>
<tr>
<td>10</td>
<td>60</td>
<td>27</td>
</tr>
</tbody>
</table>

Table 3.2  Model Order Comparison
3.5 Summaries and conclusions

This chapter presented a fast approach for solving the Prony analysis for online power system oscillatory stability assessment. Modern Prony analysis uses SVD to solve the normal equation resulting from the least-squares solution. The singular values less than some pre-specified threshold are neglected. That is the best lower rank approximant to the data matrix is obtained. A new approach to solve the linear predication matrix arisen from the Prony analysis was proposed.

Testing cases were extracted from a power system transient stability simulator which discussed in chapter 2. The results show that a significant speed-up can be achieved when compared to the standard SVD method with comparable accuracy. In addition, the results also demonstrated that lower order model can be obtained with the proposed method for a given signal. The core of the proposed method is based on the Gaussian elimination with an acceptance error for reducing the dimension of the estimated system.

Furthermore, the Gaussian elimination based algorithm does not require much additional computer memory while the SVD requires up to three more $N \times N$ matrix memory for the matrices $U$, $W$ and $V$. Thus the Gaussian elimination can handle a much larger system and it can be concluded that the presented GEAE method is suitable for the on-line application of power system dynamic stability problem.
3.6 References


Chapter 4  Power system damping classification for dynamic stability assessment

Because of environmental concerns and financial constraints, operators need to run their systems ever closer to their stability limits. Detailed stability analysis is hence required to ensure both the quality and security of supply not being compromised. One of the most often used analysis tool is time domain simulation as it does have many advantages over other techniques of retaining all nonlinear characteristics of the system and plants. The disadvantage of time domain simulation is that the results generated in each run of simulation could be overwhelming for large-scale system studies. Proper interpretation of its outputs often also required experienced power system engineers. Also for the application of online oscillatory stability assessment, an automated approach for ranking the severity of the power oscillations in each contingency case is highly desirable. In this chapter, an automatic power system damping classifier using ANFIS techniques for dynamic stability assessment is presented.

4.1 Chapter outline

Section 4.2 and 4.3 introduce the needs for dynamic stability assessment and damping classification for the power system respectively. Section 4.4 first reviews the application of Artificial Neural Network (ANN) and Fuzzy Logic (FL) to the damping classification problem and discusses their limitations; then, the Adaptive Neural Fuzzy Inference System (ANFIS) is introduced and discussed. Section 4.5 and 4.6 present the feature extraction and the implementation method respectively. Section 4.7 reports the ranking results obtained by using ANFIS. Finally, section 4.8 summaries and concludes the chapter with the findings reported in this chapter.
4.2 Introduction to dynamic stability assessment and damping classification

For on-line contingency analysis using time-domain simulation approach, hundreds of machine swing curves may be generated per contingency, all of which must be classified automatically to yield the overall system stability within a short on-line timescale (10-30 minutes). Accurate classification of stability or instability is particularly important in applications which use time domain simulation analysis as an integral part of some greater analysis process. Algorithmic techniques include curve fitting [1], filtering [2] and Prony analysis [3] of machine rotor swing curves are not totally reliable when dealing with multi-mode oscillations as masking effects due to, say, slow governing actions can lead to mis-ranking and hence wrong conclusions.

While algorithmic techniques of damping evaluation can produce erroneous conclusion when faced realistic data, engineers running power system stability studies will also be skeptical of results evaluated by computer. It was because the requirements corresponding to the opinions of the engineers are difficult to describe algorithmically. It was suggested that alternative approach using AI techniques [4,5] could overcome the difficulties in incorporating “human judgments” into the classification algorithm. In this chapter, an Adaptive Neuro Fuzzy Inference System (ANFIS) based AI technique will be discuss and applied for automatic system damping classification.

4.3 Power system damping

Damping time is classically defined in relation to standard step response of a second order linear system and exponentially decaying sinusoid. It is based on the linear system and is second order. In practices many power system components have
non-linear characteristics because of their underlying nature and controller design. The power system will become highly non-linear, as shows in Fig 4.1, when these components are interconnected. The problem for high order system is that its responses cannot be assumed to be second order.

![Figure 4.1: Higher non-linear swing curve](image)

When faced with realistic data either recorded from real system or generated by simulation, simple damping evaluation techniques can produce erroneous conclusion. Engineers running power system stability studies will also be skeptical of results evaluated by computer, “checking by eye” will usually be required to ensure there is no mistakes. It was because the requirements corresponded to the opinions of the engineers are often contradictory and the problem of automatic damping ranking is proven to be difficult to solve algorithmically.
4.4 Adaptive neuro fuzzy inference system (ANFIS)

Firstly, this section reviews the possibilities of using the Artificial Neural Network (ANN) and Fuzzy Logic for damping classification; then, the Adaptive Neuro Fuzzy Inference System (ANFIS) is introduced and discussed.

4.4.1 Artificial neural networks (ANN)

Neural networks are composed of simple elements operating in parallel. These elements are inspired by biological nervous systems [3]. Conventionally, the design methods of controls systems require the construction of a mathematical model and analytical techniques to derive the control laws for the controller. However, the mathematical representation of a complex system may not be available or difficult to derive; ANN can then be applied to model the system performance and generalized through learning from a training set.

As in nature, the network function is determined largely by the connections between elements. To train a neural network to perform a particular function, it can adjust the values of the connections (weights) between elements. Commonly neural networks are adjusted, or trained, so that a particular input leads to a specific target output. It can be referred in Fig 4.2. The network is adjusted, based on a comparison of the output and the target, until the network output matches the target. Typically many
such input/target pairs will be used. It is called supervised learning or train a network. To increase the learning capability of the network, the neural network can have several layers that are combined the multiple layers of neuron. A typical practical three layers neural network was shown in Fig 4.3.

![Three layer Neural Network](Image)

Fig 4.3 Three layer Neural Network

The artificial neural network seems to be a good solution to the damping classification problem because it is able to learn the mapping function from a set of example curves for the learning process. The ANN has been applied and showed the ability for mapping function and classification for the power system application [6,7]. However, the major problem with the ANN approach is generating the sets of learning curves for training and test data in which each set of training data must be ranked or classified by an engineer in order for the network to learn the correct mapping. The training and test data sets must be representative of all types of curve that the classifier is likely to encounter and the ANN is able to generalize. It is not an easy task to creating such data sets by human. Also, for such large amount of data sets, it may not guarantee that the engineer always applies the same ranking criteria exactly
when making the judgment on these curves. Furthermore, the ANN solution has the disadvantage that it likes a black box which is unable to explain the reasoning behind any classification it makes. It means that when the engineer disagrees with the ANN classifier, it cannot be modified easily but the entire network has to be re-trained.

4.4.2 Fuzzy logic (FL)

Fuzzy logic is a superset of conventional (Boolean) logic [8] that has been extended to handle the concept of partial truth – truth values between "completely true" and "completely false". Fuzzy logic is able to perform the desired nonlinear mapping function. It allows the rule set to be specified using linguistic variables such as small, large and very large. There are three boxes shown in Fig 4.4. These three boxes show the basics of flow of information in fuzzy logic control.

![Fig 4.4 Basic flow of Fuzzy logic](image)

Fuzzification is the process to transform crisps data to fuzzy data. Fuzzy processing mainly based on table by look up approach (rule table look up) and finally defuzzification which has generalized and converted the fuzzy output into output data. The rule set can be defined as simply as “If the time constant is big, then damping is poor”. It may be possible to solve how the system reaches its conclusion. In addition, fuzzy logic is not only a rule set but also includes fuzzification and defuzzification [9]. Engineer needs to modify the input and output membership functions in order to ensure the classifier to give correct ranking results. However, it is totally relies on the
human experience and judgment only. It is not possible to incorporate the algorithmic method to the classifier for ranking.

4.4.3 Adaptive neuro fuzzy inference system (ANFIS)

The shortcoming of classical fuzzy logic system can be overcome by using an adaptive neuro fuzzy inference system (ANFIS) as shown in Fig 4.5, for example, for two inputs $x$ and $y$, and one output $z$. Architecturally, ANFIS consists of five layers.

Layer 1: Each node $i$ in this layer is an adaptive node with a node function where $x$ or $y$ are the inputs to node $i$, and $A_i$ or $B_i$ is a linguistic label associated with this node. $O_i$ is the membership grade of fuzzy set $A$ and it specifies the degree to which the given input $x$ or $y$ satisfies the quantifier $a$.

$$O_{ij} = \mu_a(x)$$
$$O_{ij} = \mu_a(y) \quad \text{for } i = 1, 2$$

Layer 2: Each node in this layer is a fixed node, whose output is the product of all incoming signals. Each node output represents the firing strength of a
Each node in this layer is a fixed node. The \( i^{\text{th}} \) node calculates the ratio of the \( i^{\text{th}} \) rule’s firing strength to the sum of all rules’ firing strength.

Layer 4: Each node in this layer is an adaptive node with a node function where weighting factor is normalized with firing strength from layer 3.

Layer 5: The signal node in this layer is a fixed node which computes overall output as the summation of all incoming signal [10].

### 4.5 Feature extraction

For the application of power system stability assessment, results from 30 seconds worth of simulation are available. For a 30 seconds time domain simulation with a step length of 10ms, results for each machine, swing curves for instance, are each represented by 3000 data points. For this amount of information, it would be impractical to feed all of them into the classifier. Instead, in practice, only important “features” should first be extracted and then applied to the classifier. Feature extraction is a common technique applied in the area of neural network to extract the important information from a given signal. The advantage of feature extraction is that it can reduce both the size and complexity of the neural networks.

In case of power swings, both oscillation magnitude and frequency can be quantified by the ‘string’ feature – the total length of the swing curve. Longer string represented larger oscillation magnitude and frequency. Alternatively, the magnitude and the “energy” of the curve can be quantified by the “envelope area” feature – the area enclosed by the envelope of the curve.
4.6 Implementation

The testing data generated was based on the UK National Grid System. This system operates at up to 400kV and linked to two main power systems via the 275kV and 400kV AC links to Scottish power system and 2000MW DC link to France. In total, 864 sets of data were generated from a number of contingencies with various power system operating conditions. Among all the testing data, 90% of the cases were used for the ANFIS training.

The ANFIS classifier was trained against the results obtained using the “dominant mode” approach. The dominant mode of each case was determined analytically using the Prony method. First, all oscillation modes in a swing curve were extracted, and then the envelope $a_i e^{\sigma_i}$ of each oscillation mode as show in Fig 4.6 was used to estimate the oscillation energy of the mode by calculating the area enclosed by the envelope for the first minute.

![Fig 4.6 Envelope found by Prony method](image)
The one with the highest oscillation energy is considered as the dominant mode and the amount of energy of this dominant mode is used as an oscillation index for the ranking of the swing curves.

![Diagram](image.png)

Fig 4.7 Overview of the ANFIS classifier

Fig 4.7 shows the overview of an ANFIS classifier. The inputs to this classifier are the rotor angle data, i.e. swing curve, which was divided into 3 parts, 10 seconds each. The ‘string’ feature of each part of the rotor angle data are extracted to feed into the ANFIS classifier. Each input of the ANFIS classifier consists of 5 membership functions which were defined as a generalized bell function.

\[
\text{Generalized bell } (x; a, b, c) = \frac{1}{1 + \left| \frac{x - c}{a} \right|^{2b}} \quad (4.1)
\]

where the parameter ‘b’ is usually positive. Specifically, ‘c’ is represented the position of center, ‘a’ is quantified width and b is represented the slopes at the
crossover points of the generalized bell membership function. As result, this classifier has 125 \((5^3)\) rules in the rule table. The number of output membership functions is also 125 and each output membership functions is a linear function. The ANFIS classifier was trained with the backpropagation training method until there was no further significant improvement after 1,000,000 iterations. Table 4.1 shows the parameters of membership function for three ANFIS inputs after the backpropagation training.

<table>
<thead>
<tr>
<th>Input 1</th>
<th>Input 2</th>
<th>Input 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>0.06958</td>
<td>4.874</td>
<td>0.1</td>
</tr>
<tr>
<td>-1.26E-05</td>
<td>2.246</td>
<td>0.2212</td>
</tr>
<tr>
<td>0.08931</td>
<td>2.847</td>
<td>0.4186</td>
</tr>
<tr>
<td>0.0407</td>
<td>5.093</td>
<td>0.255</td>
</tr>
<tr>
<td>0.7819</td>
<td>1.072</td>
<td>1.85</td>
</tr>
</tbody>
</table>

Table 4.1 Trained parameters of membership function for the three ANFIS inputs

### 4.7 Classification results

Eight cases were selected randomly for testing the relative performance of the ANFIS classifier. Rankings obtained with the Prony’s method and ANFIS classifier were presented in Fig 4.8 and 4.9, respectively. The ranking sequence was arranged from the best to worst from the top to bottom. Obviously, the result of ANFIS classifier is more preferred than the Prony based dominant mode approach as there was one mis-rank (rank 8, case.208) in the ranking by the dominant mode approach. Since the computation speed for ANFIS system is about 10 times faster than the dominant mode approach in calculating the oscillation index (for one case), the speedup could
be higher if number of ranking case increases. Table 4.2 details the ranking indices for each case by both the analytical dominant mode approach and AI based ANFIS approach.

<table>
<thead>
<tr>
<th></th>
<th>Ranking (Prony)</th>
<th>Index (Prony)</th>
<th>Ranking (ANFIS)</th>
<th>Index (ANFIS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Case.087</td>
<td>333.28</td>
<td>Case.087</td>
<td>365.85</td>
</tr>
<tr>
<td>2</td>
<td>Case.040</td>
<td>498.87</td>
<td>Case.208</td>
<td>952.13</td>
</tr>
<tr>
<td>3</td>
<td>Case.075</td>
<td>1513.12</td>
<td>Case.040</td>
<td>1267.56</td>
</tr>
<tr>
<td>4</td>
<td>Case.047</td>
<td>1563.05</td>
<td>Case.075</td>
<td>1538.82</td>
</tr>
<tr>
<td>5</td>
<td>Case.038</td>
<td>1628.06</td>
<td>Case.047</td>
<td>1563.12</td>
</tr>
<tr>
<td>6</td>
<td>Case.029</td>
<td>1646.40</td>
<td>Case.038</td>
<td>1630.23</td>
</tr>
<tr>
<td>7</td>
<td>Case.028</td>
<td>1916.50</td>
<td>Case.029</td>
<td>1635.41</td>
</tr>
<tr>
<td>8</td>
<td>Case.208</td>
<td>2186.25</td>
<td>Case.028</td>
<td>1903.75</td>
</tr>
</tbody>
</table>

Table 4.2 Ranking results
Fig 4.8  Ranking by the Prony method
Fig 4.9  Ranking by the ANFIS classifier
4.8 Summaries and conclusions

This chapter presented a new method based on simple features extraction and ANFIS classifier. Test results showed that the classifier has been trained to follow the Prony method. The main drawback for the Prony method is that it is a computation demanding analytical process and ranking algorithm based on it is time consuming and suffers from the masking effects. As the ANFIS system has only three steps to output the solution, which are fuzzification, inference and defuzzification. The computation time is much shorter. The total classifying time could be greatly reduced by using the ANFIS system. More importantly, analytical approaches, such as the Prony method based “dominant mode” approach, may mis-rank contingencies and could lead to wrong conclusion. As demonstrated in the above results, the influences of masking effects could be minimized by using AI techniques. Based on the testing results, ANFIS based classifier is well suit to this ranking application. Both the classifying speed and accuracy are better than the traditional analytical approaches.
4.9 References


Chapter 5  Conclusions and future work

This thesis has investigated the acceleration techniques for online transient and dynamic stability assessment. Major contributions included the design and implementation of a new approach of data adding and exchange scheme for removing the bottleneck of parallel piecewise solution method, a novel algorithm for fast Prony analysis, and an ANFIS based automatic damping classifier. In this chapter a brief summary of all major findings will be presented as the conclusion of this thesis.

5.1 The new approach

This thesis presented and discussed three major acceleration techniques for on-line power system transient and dynamic stability assessments. Firstly, an accelerated parallel piecewise solution was implemented for transient stability simulation. The generator swing curves obtained as the results from the simulation can be further processed for dynamic stability assessment. Secondly, a novel algorithm called GEAE for fast Prony analysis was proposed and developed. GEAE is based on Gaussian elimination and is designed to replace the time consuming singular value decomposition (SVD) needed for solving the ill-conditioned model predictive matrix. GEAE is simple and fast. It has the advantage that lower order model estimation can be obtained with compatible accuracy. Based on the fast Prony analysis, on-line dynamic and oscillatory stability assessment for large power system can be realized. The last but no the least, an ANFIS classification system for the power system damping ranking was presented. Because of its flexibility and simplicity, the solution is much faster than other algorithmic classifiers. In addition, it can be easily modified to avoid the mis-rankings.
5.1.1 New Approach for parallel piecewise solution method

Chapter 2 described a new data add-exchange and parallel cut-node approach for further speed-up the parallel piece-wise solution method. The presented approach is successfully implemented and incorporated into power system simulator for solving large algebraic linear equations application. The UK Grid 811 bus-bars power system network were tested in 8 processors SGI Origin 2100 IRIX platform.

The conventional approach is to first order and partition the network admittance matrix into the block bordered diagonal form and then solve them using LU decomposition in parallel. The bottleneck of the conventional parallel LU factorization was identified and overcome by the use of the new data add/exchange algorithm for composing the cut-node block which consequently improves the efficiency. This approach can effectively reduce the communication and management overhead and consequently able to obtain a higher overall performance and it can also be adapted to other engineering application. The parallel piecewise solution is further speeded up by parallelizing the cut-node solution.

5.1.2 Accelerated Prony analysis for dynamic stability assessment

Chapter 3 presented a fast approach for solving the Prony analysis for online power system oscillatory stability assessment. A new approach, called GEAE, to solve the linear predication matrix arisen from the Prony analysis was proposed and presented. Testing cases (swing curves) were extracted from the results obtained from the transient stability simulation as presented in Chapter 2. Because of the simplicity of this algorithm, its computational speed is significantly faster than that of the conventional SVD method. The results show that a significant speed-up can be achieved when compared to the standard SVD method without compromising
accuracy. In addition, the results also demonstrated that lower order model can be obtained with the proposed method for a given signal. The core of the proposed method is based on the Gaussian elimination with an acceptance error for reducing the dimension of the estimated system. The Gaussian elimination does not require much extra computer memory whilst the traditional SVD method which required up to three extra N by N matrix memory for storing U, V and W. Therefore, the presented method is possible to handle a much larger power system for dynamic stability assessment.

5.1.3 Power System damping classification by ANFIS system

Chapter 4 presented a new method based on simple features extraction and ANFIS classifier for damping classification in power system dynamic stability assessment. The result has been shown that the classifier has been trained to follow the Prony’s method. The major drawback for the Prony’s method is that it is a computation demanding analytical process and suffers from the masking effects. As the ANFIS system has only three steps to output the solution, which are fuzzification, inference and defuzzification. The computation time is much shorter, and the total classifying time could be greatly reduced by using the ANFIS system. More importantly, analytical approaches, such as the Prony’s method based “dominant mode” approach, may mis-rank contingencies and could lead to wrong conclusion. As demonstrated in this thesis, the influences of masking effects could be minimized by using AI techniques. Based on the testing results, ANFIS based classifier is well suit to this ranking application. Both the classifying speed and accuracy are better than the traditional analytical approaches.
5.2 Future work

The future work goes to incorporate the discussed algorithms into an on-line dynamic security assessment (DSA) system. The on-line DSA function must be capable of assessing hundreds of credible transient and dynamic contingencies in the time frame of the execution cycles of the real-time sequence of the Energy Management System (EMS) which is in the order of 15 min to 20 min. Currently, time domain simulation is the only reliable means of assessing the system stability; however, it is a computational demanding process and is not well suited for on-line propose.

In order to achieve the on-line DSA, more investigations on the simulation techniques and algorithms should be performed. Furthermore, it can be applied to online Preventive and Emergency control for power system operation.

To incorporate the findings obtained in this research into on-line transient stability and dynamic stability assessment practically, the next step is to interface the simulation engine to EMS system. The EMS distributes the contingency case to DSA data base. The transient stability assessment will be performed to obtain the stability index. For the transient stability assessment, the newly developed parallel piece-wise solution method can be applied here to reduce the computation time for each contingency case. The unstable case will be extracted and the numerical sensitivity analysis for each case for each generator will also be calculated.

Based on the results obtained from transient stability calculation, the dynamic stability assessment will also be performed by the Fast Prony analysis based algorithm and hence, the small signal stability index can be obtained for short time dynamic assessment.

After the stability analysis, the stability computation engine exports the each
contingency case to an AI based fast automatic damping classifier for oscillatory stability ranking for system operator for on-line monitoring. The overall functional diagram which applied the research work was illustrated as shown in Fig 5.1.

Fast Power system stability computation engine

Since, all the critical unstable cases are screened out by the stability analysis engine; the computation demand for numerical sensitivity analysis is significantly decreased. It further provides the opportunity for preventive and emergency control. For the preventive control, the actions are mainly on moving the current operating point to obtain the larger security margin with the economical optimization. The emergency control is applied to enlarge a security limit by changing power system condition when a fault or severe disturbance occurs. The simulation flow chart has shown in Fig. 5.2.
EMS

Contingency

Simulation

Stability Assessment

Numerical sensitivity analysis (for each gen.)

Generation / Power transfer $\rho I/\rho P$ or ... Limit or constraints

Fig. 5.2 Simulation flow chart
5.3 Conclusions

This thesis has attempted to investigate the acceleration technique for on-line transient stability and dynamic stability assessment experiment and implementation of proposed algorithms.

In particular, this thesis has proposed two new algorithms. Implementation of above two methods demonstrated that the algorithms can produce fast assessment for transient and dynamic stability assessment. A parallel cut-node with new add and exchange method improved parallel piece-wise solution method for solving power system network equations in transient stability analysis. The fast Prony analysis can be used to assess the oscillatory stability instead of eigen-values analysis for on-line dynamic stability assessment. In additions, a fast and reliable damping classifier is valuable tools to system control engineer to have good knowledge and alert of power system damping. This thesis presented an ANFIS classifier which demonstrated the mis-ranking can be avoided and draws an acceptable conclusion.

In conclusion, the results of this thesis demonstrate the value in the presented acceleration technique for on-line security assessment and it is the author’s firm belief that future research in this area will be profitable to on-line dynamic security assessment, power system engineering and the scientific community in general.
Appendix

A1 Numerical integration method

The implicit integration and explicit integration is discussed in following section:

A1.1 Implicit integration methods

Implicit integration methods use interpolation functions for the expression under the integral. Interpolation implies that the functions must pass through the yet unknown points at time $t_1$.

Consider the differential equation

$$\frac{dx}{dt} = f(x, t) \quad \text{with} \quad x = x_0 \quad \text{at} \quad t = t_0$$

The solution for $x$ at $t = t_1 = t_0 + \Delta t$ may be expressed in integral form as

$$x_1 = x_0 + \int_{t_0}^{t_1} f(x, \tau) d\tau \quad (A1)$$

Trapezoidal rule is the simplest implicit integration method by using the linear interpolation. Refer to Fig A1, the name – trapezoidal means that the area under the integral of equation A1 is approximated by trapezoids.
The trapezoidal rule for Equation 13.19 is given by

\[ x_1 = x_0 + \frac{\Delta t}{2} \left[ f(x_0 + t_0) + f(x_1 + t_1) \right] \]  

(A2)

A general formula giving the value of \( x \) at \( t = t_{n+1} \) is

\[ x_{n+1} = x_n + \frac{\Delta t}{2} \left[ f(x_n + t_n) + f(x_{n+1} + t_{n+1}) \right] \]  

(A3)

Referring to equation A3 that \( x_{n+1} \) appears on both sides of the equation. It implies that the variable \( x \) is computed as a function of its value at the previous time step as well as the current unknown value. Therefore, an implicit equation must be solved.

For the power system applications, the trapezoidal rule which is a second-order method has been widely applied. However, there are some other higher-order implicit integration methods which proposed in literature on numerical methods. Due to the programming difficulties and numerical stability, they have not been widely used for power system applications.
A1.2 Runge-kutta (R-K) methods

The R-K methods have been widely applied in different scientific and engineering applications. It approximates the Taylor series but do not require explicit evaluation of higher derivative than the first. By evaluating several first derivations, the effects of higher derivatives are incorporated. There are different orders R-K methods depending on the number of terms retained in Taylor series.

A1.2.1 Second-order R-K method

Considering the first-order differential equation as follows:

\[
\frac{dx}{dt} = f(x,t)
\]

the second-order R-K formula for the value of \(x\) at \( t = t_0 + \Delta t \) is

\[
x_1 = x_0 + \Delta x = x_0 + \frac{k_1 + k_2}{2}
\]

where

\[
k_1 = f(x_0, t_0) \Delta t
\]

\[
k_1 = f(x_0 + k_1, t_0 + \Delta t) \Delta t
\]

For the second-order R-K method, it considered the first and second derivative terms in the Taylor series, thus, the error is on the order of \(\Delta t^3\).
A general formula giving the value of $x$ for the $(n+1)^{st}$ step is

$$x_{n+1} = x_n + \frac{k_1 + k_2}{2}$$

where

$$k_1 = f(x_n, t_n)\Delta t$$
$$k_1 = f(x_n + k_1, t_n + \Delta t)\Delta t$$

### A1.2.2 Fourth-order R-K method

By considering up to fourth derivative terms in the Taylor series expansion which will has fourth-order R-K method.

The general formula giving the value of $x$ for the $(n+1)^{st}$ step is

$$x_{n+1} = x_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$k_1 = f(x_n, t_n)\Delta t$$
$$k_2 = f\left(x_n + \frac{k_1}{2}, t_n + \frac{\Delta t}{2}\right)\Delta t$$
$$k_3 = f\left(x_n + \frac{k_2}{2}, t_n + \frac{\Delta t}{2}\right)\Delta t$$
$$k_4 = f(x_n + k_3, t_n + \Delta t)\Delta t$$

The physical interpretation of the above solution is as follows:

$k_1$ = (slope at the beginning of time step) $\Delta t$
$k_2$ = (first approximation to slope at midstep) $\Delta t$
$k_3$ = (second approximation to slope at midstep) $\Delta t$
$k_4$ = (slope at the end of step) $\Delta t$
\[ \Delta x = \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) \]

\( \Delta x \) is the incremental value of \( x \) which is the weighted average of estimates based on slopes at the beginning, midpoint and the end of time step and it has an error on the order of \( \Delta t^5 \).

**A2 SGI Architecture**

Since, the program has been run in SGI origin 2100 Multi-Processor machine. The first step of investigation is to examine the basic architecture of SGI system which has been shown in Fig A2.

The SGI system is using NUMA: Non-Uniform Memory Access. There was a major different between x86 PC systems. It has a Global switch interconnected to other Rack (CPU board with local memory). But for the x86 PC, it was a shared memory for global access. The program was written for x86 PC system. All of the data structure was accessed in global manner. It was an opposite performance for running
on SGI system due to the all the processor has to wait or race to others when accessing the shared data structure.

### A2.1 Performance of RMA and LMA in SGI

For the SGI system, Local Memory Accesses (LMA) is much faster than remote memory accesses (RMA). The correct attitude was that all the data structure has to be defined in its local processor board. For this issue, a parallel matrix multiplication testing program has been developed to illustrate the result between LMA and RMA in SGI system. The result has been shown that for different data size and different number of processors used, the performance of LMA is much better than the RMA which was shown in Fig. A3 and Fig A5.
Fig. A3 Comparison the RMA and LMA in speed up for different data size and processors

For the parallel computation efficiency, the Fig A4 has been shown that it can be viewed as two bands. The efficiency of all the LMA cases was higher than RMA cases. Also, the different between each case can be demonstrated the remote memory access bandwidth of SGI.
Fig A4 Comparison the efficiency in different data size and processors

Fig A5 Comparison the speed up in different data size and processors
A3 LU Method for dense matrix

A3.1 A example code segment for Traditional LU method

for(i=k+1;i<n;i++) {
    for(i=k+1;i<n;i++) {
        a[i][k]=a[i][k]/a[k][k];
        for(j=k+1;j<n;j++) {
            a[i][j]=a[i][j]-a[i][k]*a[k][j];
        }
    }
}

A3.2 A example code segment for Parallel LU method

for (k=0;k<n-1;k++) {
    processor1( )
    {
        int i, j;
        double d,c;
        for (i=k+1;i<n;i+=2){
            A[i][k]=A[i][k]/A[k][k];
            for(j=k+1;j<n;j++){
            }
        }
    }
}
Processor2( )
{
    int i, j;
    for (i=k+2;i<n;i+=2)
    {
        A[i][k]=A[i][k]/A[k][k];
        for(j=k+1;j<n;j++)
        {
        }
    }
}