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RETROFITTING EXISTING BUILDINGS BY COUPLING METHOD USING PASSIVE DEVICES

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2015
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______________________________

YANG Zhidong (Name of student)
Abstract

The objective of this study is to investigate the feasibility of retrofitting existing buildings by coupling method using passive devices.

The study begins with experimentally testing of a shear type fluid damper. It is found that the fractional derivative model can accurately represent the fluid damper. The equations of motion of buildings coupled by fluid dampers are then derived. Parametric studies show that when buildings with a substantial difference in the number of stories (or in the period of vibration) are linked by fluid dampers, significant seismic mitigation can be achieved. The response reduction is limited when the buildings have the same or similar number of stories.

The effect of the soil-structure interaction (SSI) on the response of buildings connected by fluid dampers is investigated. The soil-structure interaction or structure-soil-structure interaction (SSSI) has little effect on the response of coupled buildings on moderately soft or moderately stiff soil or rock site. For coupled buildings on soft soil, the response considering SSI is less than that without SSI. When buildings on soft soil are retrofitted by the coupling method, the analysis of coupled buildings without SSI or SSSI provides conservative examination. Thus subsequent analyses are conducted without SSI and SSSI.

An optimization procedure has been developed to optimize the position and size of fluid dampers in the frequency domain. It is found that the top floor of the building with lesser number of stories is the best location for the installation of fluid dampers. When there is a large difference in the number of stories (or in natural frequencies) between two buildings, the maximum standard deviation of drift can be substantially
reduced. Vice versa, response reduction diminishes when the natural frequencies of the buildings are close to each other.

Representing a nine and an eight-story building, two 1/15 scale models have been built and tested. When a fluid damper is installed between the two models in fixed-base conditions, the fundamental frequencies and damping ratios are slightly increased. There is a limited reduction in the maximum response. When one model in fixed-base condition is connected to the other model in base-isolated configuration by a visco-elastic damper, the response of both models is significantly reduced.

Theoretical studies on the response of a fixed-base building connected to a base-isolated building indicate that the properties of visco-elastic dampers are the dominant factor that influences the response of the fixed building. Analysis results show that the fundamental frequencies of both buildings are considerably reduced. The response of the coupled buildings is significantly decreased.

Supplementary studies have been conducted to retrofit a core-frame system. Base isolators are applied to protect the buildings from ground motion. Story isolators are employed to connect the cores and buildings. The retrofitting strategy can effectively reduce the maximum drift response of the cores and the frames.

To conclude, it is feasible to retrofit adjacent buildings by the coupling method using passive devices. For buildings with a different number of stories, it is recommended to connect them together using fluid dampers. Adjacent buildings with the same or similar number of stories can be strengthened by coupling them together with visco-elastic dampers with one of the buildings being base isolated.
Publications


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Chapter 1  Introduction

1.1 Background

In the last few decades, earthquakes have caused massive damage to building structures (Mitchell, et al. 1995; Maqsood and Schwarz 2010; Mimura, et al. 2011; Rathje, et al. 2011; Wilkinson, et al. 2012; Zhang, et al. 2012). Lessons from previous earthquakes have urged the update of seismic codes in many countries to avoid and/or to reduce seismic damage. While new buildings can comply with the updated seismic design standards, it is necessary to strengthen the existing buildings to provide sufficient seismic resistance to earthquakes. For instance, a lot of existing buildings in Hong Kong were designed without seismic consideration. As Hong Kong is now classified as a region with “low-to-moderate seismicity” (Chandler, et al. 2001; Lam, et al. 2002; GB50011-2010 2010), this prompts the need to strengthen the non-seismically designed buildings. Besides, structural integrity of existing buildings continually deteriorates over time which also increases the need to strengthen existing buildings.

Many strengthening methods have been studied to improve the performance of existing buildings when subjected to earthquakes. Conventionally, structural members are strengthened to resist the horizontal forces induced by earthquakes and to avoid collapse (Newman 2000). For instance, fiber-reinforced polymers have been applied to improve the properties of structural members (Hollaway 2010; Niroomandi, et al. 2010; Belarbi, et al. 2011; Dumas 2012). Conventional strengthening techniques often incorporate substantial demolition and lengthy
reconstruction which deters building owners from retrofitting existing buildings. Hence, new technologies using energy dissipation devices are introduced.

Structural control devices have been introduced to absorb earthquake energies and/or to counteract the earthquake motions in order to reduce structural response and possible damage. These devices include energy dissipation devices and seismic isolation devices. Energy dissipation devices add damping and energy dissipation ability to structures (Soong and Dargush 1997; Hanson and Soong 2001; Symans, et al. 2008). They are designed to limit or eliminate possible damage to the lateral load resisting system (Housner, et al. 1997; Constantinou, et al. 1998; Bozorgnia and Bertero 2004; Durucan and Dicleli 2010). Several types of energy dissipation devices including friction dampers, visco-elastic dampers, viscous dampers and MR dampers are commercially available. To improve the efficiency of energy dissipation devices, amplification devices may be necessary (Constantinou, et al. 2001; Chung, et al. 2009). However, these devices are invasive and can adversely affect the useable floor area. In addition, some energy dissipation devices may adversely affect the structural response by increasing building stiffness and consequently fundamental frequency of the buildings is increased.

Apart from the above, seismic isolation systems have become one of the most acceptable technologies in seismic strengthening (Kitagawa and Midorikawa 1998; Newman 2000). Seismic isolation systems decouple building structures from, primarily, the horizontal components of ground motion with isolators that have low horizontal stiffness (Kelly and Chitty 1980; Kelly and Beucke 1983). For instance, by incorporating a base isolation system between the foundation and the basement of a building, the superstructure can be isolated from ground shaking (Occhiuzzi, et al. 1994; Mokha, et al. 1996). As a result, the energy transmitted from ground to the

With shortage of land in big cities, it is a common practice for existing structures such as residential buildings to be adjacent to each other. In over-crowded areas, like Mong Kok district in Hong Kong, many mid-rise buildings are closely built due to this restriction. When subjected to severe earthquakes, damage due to the collision between adjacent buildings has been frequently observed. Post-earthquake investigation after the 1985 Mexico earthquake (Rosenblueth and Meli 1986), the 1989 Loma Prieta earthquake (Kasai and Maison 1997) and the 2010 Darfield earthquake (Cole, et al. 2011) have indicated the presence of massive pounding damage. One method to avoid pounding is to separate the buildings by an adequate distance. For instance, according to the International Building Code (2011), the minimum distance between two buildings in close proximity is the square root of the sum of the squares of the displacements of the two buildings. When retrofitting adjacent buildings, however, it may not be possible to provide the necessary minimum distance.

To improve the performance of existing buildings and to reduce the pounding between adjacent buildings, buildings in close proximity are proposed to be linked together by connecting devices, like the Triple Towers in Downtown Tokyo (Asano, et al. 2003). Researchers have proposed different types of connecting devices to connect adjacent buildings including passive dampers (Xu, et al. 1999a; Xu, et al. 1999b; Zhang and Xu 2000; Bhaskararao and Jangid 2006b), semi-active dampers (Christenson, et al. 2007a; Xu and Ng 2008; Bharti, et al. 2010) and active dampers (Christenson, et al. 2003; Zhang and Iwan 2003). It is now recognized that seismic response of adjacent buildings can be mitigated by connecting devices. In particular,
fluid dampers and visco-elastic dampers are now widely proposed to link adjacent buildings. Previous studies have shown that the response of buildings coupled by fluid dampers or visco-elastic dampers are reduced (Kim, et al. 2006b; Patel and Jangid 2010b). In general, damping of adjacent buildings can be increased by connecting buildings together using visco-elastic dampers or fluid dampers. Consequently, seismic response of adjacent buildings can be reduced (Zhang and Xu 1999). Further, there are some studies focused on optimizing the properties of connecting dampers to minimize the total dissipating energy (Zhu, et al. 2011). To minimize the vibration of adjacent buildings, connecting dampers are also proposed to be installed at several floor levels rather than at one floor level only (Huang and Zhu 2013). The above studies have demonstrated that with proper passive dampers, the response of coupled buildings can be mitigated. However, previous studies have not comprehensively investigated the coupling method.

1.2 Problems and directions

Firstly, previous studies have not comprehensively considered the influence of building properties on the effectiveness of connecting dampers. This is now studied in detail to reflect the performance of connecting dampers with the properties of adjacent buildings.

Secondly, dampers have not been accurately modeled in the theoretical analysis. Some studies on adjacent buildings coupled by fluid dampers have ignored the stiffness of fluid dampers and utilized the viscous dashpot model to computer structural response (Xu, et al. 1999b; Zhang and Xu 2000; Matsagar and Jangid 2006; Ge, et al. 2010). This idealization is acceptable when fluid dampers are installed in a building. When are added between adjacent buildings, however, the stiffness of fluid
dampers must be taken into account since it is not negligible in comparison with the
equivalent stiffness of a building. In this study, a fluid damper is tested. The
fractional derivative model with the best representation of the fluid damper is used to
predict the response of buildings coupled by fluid dampers. For visco-elastic
dampers, the linear Kelvin model has been adopted by most studies (Feng, et al. 2000;
Matsagar and Jangid 2005; Kim, et al. 2006b). Although performance of visco-
elastic dampers are strongly related to the excitation frequencies, this has been
ignored. In this study, the general mechanical model is utilized to represent the
frequency-dependent characteristics of visco-elastic dampers.

Thirdly, the effect of soil-structure interaction (SSI) has not been taken into account
in the analysis of adjacent buildings coupled by fluid dampers. A building on flexible
soil may behave differently from that on a rigid site (Wolf 1988; Stewart, et al. 1999a;
Stewart, et al. 1999b). Besides, there are interferences between adjacent buildings
due to structure-soil-structure interaction (SSSI) (Gueguen and Bard 2005; Lou, et al.
2011). In this study, the effect of SSI and SSSI on the response of coupled buildings
is investigated.

Fourthly, seldom experimental tests on models coupled by visco-elastic dampers
have been carried out. In the area of structural control, it is necessary to
experimentally verify the strategies of control. Therefore, in this study experimental
tests are carried out to explore the feasibility of mitigating the seismic response of
existing buildings by coupling method using visco-elastic dampers.

Fifthly, previous studies on adjacent buildings coupled visco-elastic dampers have
been carried out mostly in the time domain. Frequency domain analysis is a tool of
utmost importance in studying structural response to ground excitation (Humar and
Xia 1993). In this study, buildings coupled by visco-elastic dampers are analyzed in the frequency domain to assess the coupling method.

Lastly, previous studies are limited to elastic analysis. Under strong earthquakes, cracks develop in structural members and inter-story drift can be beyond elastic point. These lead to the reduction of lateral stiffness of the buildings. Inelastic response to strong earthquakes is different from elastic response. Therefore, this study investigates the effectiveness of connecting dampers in the control of adjacent buildings under strong earthquake excitations.

1.3 Objectives and scope

The objective of this study is to apply the retrofit strategy of connecting existing buildings together using passive devices to strengthen existing buildings. The performance of the retrofit strategy is to be assessed and verified in terms of reduction in inter-story drift. Details of the objective for this study are shown as following.

(1) To evaluate the effectiveness of fluid dampers in controlling the inter-story drift of adjacent buildings with the consideration of building properties.

(2) To evaluate the effect of soil structure interaction on the drift response of buildings coupled by fluid dampers.

(3) To evaluate the effectiveness of visco-elastic dampers together with base isolators in controlling the maximum drift of adjacent buildings with similar number of stories.

(4) To evaluate the effectiveness of base isolators and connecting isolators in reducing the maximum drift of core connected buildings.
To achieve these objectives, theoretical analysis and experimental studies have been carried out. Analytical models have been developed for parametric studies. Physical models have been constructed and tested on the shaking table to compare the response before and after the installation of connecting dampers.

The research tasks to accomplish the above mentioned objectives are detailed as follows:

Chapter 2 provides a literature review on the retrofit strategies which includes conventional methods, energy dissipation method, seismic isolation method and coupling method. The review is mainly focused on the use of coupling method to link adjacent buildings.

Chapter 3 compares different mathematical models for shear type fluid dampers. The use of fluid dampers to connect two buildings is explored. The effect of building properties on the performance of fluid dampers in connecting adjacent buildings is evaluated. Reduction in response is assessed.

In Chapter 4, the effect of soil-structure interaction and structure-soil-structure interaction on the response of adjacent buildings coupled by fluid dampers is investigated.

In Chapter 5, an optimization procedure is developed to adjust both the position and the size of connecting dampers. Effectiveness of fluid damper on response mitigation in the frequency domain is evaluated.

Chapter 6 presents the experimental study of retrofitting two building models by coupling method using passive devices. Scaled steel models have been constructed and tested on the shaking table. Response of the building models before and after coupling is compared.
In Chapter 7, analytical models have been then established to investigate the effectiveness of the coupling method at various damper sizes and different base isolation systems. Analyses in both the time domain and the frequency domain are carried out.

Chapter 8 focuses on analyzing a core connected building group retrofitted by partly isolated systems and connecting isolators.

Chapter 9 summarizes the research and major conclusions drawn from this study. Recommendations for future research have also been made.
2.1 Introduction

In recent years, many researchers have studied to use different strengthening techniques to improve the seismic performance of existing buildings. There are three approaches, namely conventional retrofitting technology, energy dissipation method and seismic isolation system. The first approach involves strengthening a building by allowing it to sustain larger ground excitation. This is the conventional method which includes strengthening existing members and adding new structural members. The second approach is enhancing the capability of energy dissipation by installing energy dissipation devices in a building. The additional devices include passive dampers, semi-active dampers or active dampers. The final approach is installing an isolation layer to reduce the transmission of earthquake energy to a building.

2.1.1 Conventional methods

Conventional retrofitting techniques include the addition and/or strengthening of existing structural members. For example, some researchers suggested adding shear walls or diagonal bracings to improve the lateral stiffness of existing buildings (Pincheira and Jirsa 1995; Perera, et al. 2004). Currently, advanced composite materials, such as fiber reinforced polymer (FRP), have been proposed to enhance the strength of structural elements. Figure 2.1 shows a T beam strengthened by FRP. FRP can be easily wrapped around structural elements, making it a popular alternative to retrofit existing buildings when subjected to seismic activity (Trianafillou 1998). The main purpose of using FRP is to confine structural members to improve their strength and ductility (Norris, et al. 1997; Yuksel, et al.
2006). This has been verified by analytical and experimental studies (Neale 2000; Zhao and Zhang 2007; Hollaway 2010). Drawbacks of the conventional method include severe demolition and lengthy reconstruction period, resulting in high expense.

### 2.1.2 Energy dissipation method


Traditional friction dampers (Pall and Marsh 1982; Lopez, et al. 2004; López-Almansa, et al. 2011), metallic dampers (Kurokawa, et al. 1998; Di Sarno and Manfredi 2010), visco-elastic dampers (Shen and Soong 1995; Aprile, et al. 1997; Asano, et al. 2001; Vasques, et al. 2010; Lewadowski, et al. 2012) and fluid dampers (Gluck, et al. 1996; Lee and Taylor 2001; Hwang, et al. 2008; Jiuhong, et al. 2008) all fall under the category of passive control system. Figure 2.2 and Figure 2.3 show two typical fluid dampers and a friction damper, respectively. Active control systems require an external power to control the actuators for the application of control forces (Datta 2003). Active tuned mass damper systems and active variable stiffness systems are two typical active control systems (Preumont and Seto 2008). For instance, variable stiffness systems shift structural frequencies away from the

Other researchers have proposed simultaneously adopting two or three control systems to improve the structural performance during an earthquake (Yang, et al. 1990; Johansson 2003). The combination of two or three control systems is defined as a hybrid control system (Antsaklis and Nerode 1998). Incorporating different control systems into one structure, the limitations when one control system acts alone is overcome (Kim and Adeli 2005a; Kim and Adeli 2005b). The most common hybrid control system is the integration of active devices and passive dampers (Fisco and Adeli 2011b).

2.1.3 Seismic isolation system

Seismic isolation systems isolate buildings from strong ground motions, resulting in a reduction of the energies transmitted from the ground (Syngellakis 2012), as well as reducing the dynamic response (Naeim and Kelly 1999). Many different types of
isolation devices or isolators have been proposed and designed (Kelly 1986). According to different control methods, seismic isolation systems can be categorized into passive (Komodromos and Phocas 2013), active (Chang and Spencer 2010) and semi-active seismic isolation systems (Nakajima, et al. 2012; Turan 2014). Passive seismic isolation systems are extensively applied to reduce the response of buildings from seismic activity. Figure 2.4 and Figure 2.5 show a typical lead rubber bearing and a friction pendulum bearing respectively. This type of isolation systems generally consists of flexible horizontal devices (e.g. rubber bearings, friction pendulum bearings) and energy dissipaters (e.g. metallic dampers and lead dampers as shown in Figure 2.6). The former take the superstructure away from resonance with ground motion, whilst the later absorb energies to decrease the horizontal deflection at isolation level. Active isolation systems are used to change the mechanical properties of isolation devices through active control forces based on the feed-back signal. Semi-active isolation systems are usually a combination of passive isolators and semi-active dampers (e.g. stiffness damper, semi-active electromagnetic friction damper, magneto-rheological dampers). Theoretical and experimental studies have demonstrated the efficiencies of active and semi-active seismic isolation systems. However, these two types of isolation systems are rarely used because of their high cost.

2.2 Coupling method using passive devices

With limited and restricted land use in big cities, it is a common practice for existing structures such as commercial buildings to be adjacent to each other. In over-crowded areas, like Mong Kok district in Hong Kong as shown in Figure 2.7, many mid-rise buildings are closely built due to this restriction. Seismic codes require a
gap or separation between buildings to accommodate large lateral displacement at the isolation level in case of an earthquake (ACI318; GB50011-2010). This can cause difficulties for buildings in big city since these buildings may be relatively close to each other. Recently, some researchers have proposed to couple adjacent buildings to mitigate the seismic response of these buildings. The advantage of the coupling method is that it is not necessary to significantly reconstruct the existing buildings. On the other hand, without adequate clear spacing between adjacent buildings, collisions can happen. For instance, survey data after the 1985 Mexico earthquake (Rosenblueth and Meli 1986), the 1989 Loma Prieta earthquake (Kasai and Maison 1997) and the 2010 Darfield earthquake (Cole, et al. 2011) have shown massive pounding damage. The connecting devices used in the coupling method can reduce the relative displacement between adjacent buildings to avoid pounding.

It is now widely accepted by structural engineers that the performance of passive energy dissipation devices are stable and reliable (Dolce, et al. 2005; Parulekar and Reddy 2009). As these devices are relatively inexpensive, a number of researchers have applied these devices to link adjacent buildings.

2.2.1 Coupling method using hinged links or steel dampers

Hinged links (Westermo 1989) were first recommended to link adjacent buildings to avoid pounding. Although this method can help to prevent pounding, it may increase the response of one of the coupled buildings.

Iwanami et al. (1996) have studied two buildings connected by a spring and a damper. Each building is represented by a single degree of freedom model. The P-Q theory is used to optimize tuning and damping parameters for suitable vibration modal modification.
In Japan, conical steel dampers have been developed to connect adjacent buildings (Kobori, et al. 1988). Through experimental studies and simulation research, conical steel dampers have been proved the ability to decrease the response of both buildings. As shown in Figure 2.8, building models connected by steel E-shaped dampers have been tested under several earthquake excitations (Cimellaro, et al. 2004). The E-shaped dampers have shown excellent energy absorbability.

2.2.2 Coupling method using friction dampers

Bhaskararao and Jangid (2006a) have studied the used of friction dampers to connect adjacent buildings. Both single-degree-of-freedom and multi-degree-of-freedom models have been used to estimate the efficiency of friction dampers. It is concluded that friction dampers can reduce the response of coupled buildings. Instead of installing connecting dampers at each and every floor level, the optimum position is where the largest relative displacement occurs, i.e. at the top floor. It is worth noticing that, in some cases, the maximum response of adjacent buildings connected by friction dampers may not be reduced.

2.2.3 Coupling method using fluid dampers

Luco et al. (1998a) have applied fluid dampers to connect beam type buildings. Fluid dampers are modeled by linear dashpots, and the first two natural modes of vibration are used to compute the optimum damping value of connecting dampers (Luco and De Barros 1998b). Their studies have shown that the effect of fluid dampers is related to the mass ratio, lateral stiffness ratio and height ratio of the two buildings. When the fluid dampers are optimized, they can significantly increase the first two modal damping ratios and consequently substantially decrease the seismic response. Xu et al. (1999a) have applied the Kelvin model to study the effectiveness of using
fluid dampers to connect adjacent buildings. Parametric studies have shown how the connecting dampers may affect the response of the adjacent buildings. It is concluded that fluid dampers with optimum properties can significantly reduce the response of both buildings. The Maxwell model has also been used to simulate fluid dampers which connected adjacent buildings (Zhang and Xu 2000; Zhu and Xu 2005; Zhu, et al. 2011). Parametric studies have been conducted to evaluate the reduction of seismic response. By increasing the modal damping ratios, fluid dampers can mitigate seismic response of coupled buildings. Besides theoretical studies, experimental studies on models coupled by fluid dampers have also been carried out. It has been shown that fluid dampers can mitigate the response to sinusoidal excitation (Xu, et al. 1999b).

2.2.4 Coupling method using viscoelastic dampers

Adjacent buildings coupled by viscoelastic dampers have been studied by Matsagar and Jangid (2005). The coupled systems are either an isolated building connected to a fixed-base building or two base isolated buildings linked by viscoelastic dampers. NEWMARK method has been used to compute the dynamic response. It is found that the connecting viscoelastic dampers are useful to reduce the relative displacement between adjacent buildings. In particular, when a base isolated building and a fixed base building are connected by viscoelastic dampers, there is a large difference in lateral stiffness between the two buildings. Its feasibility is verified from the results of the analysis of a six-story fixed-base building connected to a neighbor four-story base-isolated building. Kim et al. (2006b) have used the Kelvin model to investigate the effectiveness of viscoelastic dampers in reducing seismic response of neighboring structures. It is recommended to use the connecting dampers
when the neighboring buildings have different lateral resisting systems or different vibration modes.

### 2.3 Coupling method using active dampers

Employing different feedback strategies, active control devices have been proposed to couple adjacent buildings for seismic protection.

Mitsuta et al. (1994) have used reduced-order models to design the actuator between parallel buildings. A control system presented by Mitsuta et al. comprises (1) sensors which are used to measure the response of adjacent buildings; (2) controller which processes measured data and computes the control forces and (3) actuators which applies control forces to buildings. Seto, et al.(1995) studied using two active actuator to control buildings in parallel. It is demonstrated that the actuator sare effective. It is worth noticing that the position of connecting actuator must be carefully identified. Otherwise, spillover may occur which may amplify the response of adjacent buildings.

Employing acceleration feedback strategy, a control actuator is applied to link adjacent building models (as shown in Figure 2.9) for seismic protection (Christenson, et al. 2003). Based on experimental and theoretical studies, this strategy has shown to be effective in reducing the root mean square acceleration response. However, the peak response may not be reduced. Using genetic algorithms to obtain optimal controller gain, control force of active actuator has been specifically computed to effectively reduce the response of buildings coupled by active dampers (Hadi and Uz 2014). Further, to reduce the cost of active control actuators and measuring devices, position of actuators and sensors have been optimized to achieve the desired control performance (Gao, et al. 2013). In general,
active systems have two drawbacks. The first is that large external energy is necessary (Housner, et al. 1997). The second is that there is a time delay between the instant when response is measured and the instant when control forces are applied (Chung, et al. 1989). Under the category of active system, active coupling systems have the two drawbacks too.

Thus, active control system in combination with passive devices has been recommended to link adjacent buildings (Palacios Quiñ onero, et al. 2011). It is concluded that the hybrid control system can reduce the response, regardless if the active control system functions well or not.

2.4 Coupling method using semi-active dampers

2.4.1 Coupling method using semi-active magnetorheological dampers

Christenson et al. (2007b) have applied semi-active devices to joint adjacent buildings. It is recommended that the predominant vibration modes of coupled buildings are different and the dampers are not to be installed at a dominant vibration mode node. Magnetorheological dampers have also been claimed to be effective to reduce response of adjacent buildings (Bharti, et al. 2010). Based on LQR control, maximum response of buildings coupled by semi-active magnetorheological dampers can be reduced by around 20% (Motra, et al. 2011). If the magnetorheological dampers are installed at every two floor, desirable reduction in response can also be achieved.

As shown in Figure 2.10, models connected by semi-active magnetorheological dampers have been tested. It is demonstrated that semi-active magnetorheological dampers are slightly more effective in reducing the response of coupled buildings
than passive magnetorheological dampers (Basili, et al. 2013). However, peak response can not always be reduced if linked by magnetorheological dampers.

2.4.2 Coupling method using variable friction dampers

Besides magnetorheological dampers, variable friction dampers have also been experimentally investigated (Patel and Jangid 2010c). By changing the clamping force, a variable friction damper can modify its slip force to absorb energies as well as reduce the response of adjacent buildings. Performance of a variable friction damper is affected by the damper gain multiplier parameter (Shrimali and Bharti 2008). Experimental tests have indicated that variable friction dampers are effective in reducing the root mean square seismic response (Xu and Ng 2008). Yet, the maximum response may not be mitigated.

2.5 Adjacent buildings coupled by sky-bridges

Besides using dampers to connected buildings, sky-bridges with isolators and/or dampers have also been applied to couple neighboring buildings. In addition providing convenient passage between coupled buildings, sky-bridges can also partly mitigate the dynamic response of these buildings when subjected to wind or earthquake. In Japan, buildings connected by sky-bridges have been experimentally studied. As shown in Figure 2.11, Seto and Matsumoto (1999) studied a four-building group connected by several active control sky bridges. They carried out theoretical analysis and experimental tests to verify the method.

Well known examples of using sky bridges include the Petronas Towers 1 and 2 (as shown in Figure 2.12) in Kuala Lumpur, Malaysia (Thornton, et al. 1997). Another example of sky-bridge connected building groups is Triton Square Office Complex (as shown in Figure 2.13) which is composed of three buildings of 155m, 175m, and
195m tall, respectively. The actuators are around 35t each. Site tests have shown that the sky-bridge performs well to improve the damping of the building group (Asano, et al. 2003).

In China, building groups connected by sky-bridges have also studied. The MOMA building group (as shown in Figure 2.14) and the Hangzhou Citizen Center (as shown in Figure 2.15) are two typical examples. The MOMA building group consists of nine buildings connected by eight sky-bridges (Xu, et al. 2008). Six buildings are connected together by six sky-bridges to form the Hangzhou Citizen Center (Zheng, et al. 2009; Li and Li 2010). Parametric studies have been carried out to optimize the properties and the position of the sky-bridges (Sun, et al. 2014).

In South Korea, Lee et al. (2012) have applied finite element method to conduct three dimensional analysis of a building group coupled by sky-bridge through lead rubber bearings. The two buildings are shown in Figure 2.16. Based on the analysis results, damping in both directions is increased with a reduction of dynamic response.

2.6 Other types of dampers connected structures

A building with large horizontal dimensions requires the use of seismic joints or expansion joints to separate the building into several segments. For example, Rockwell Building 505 is separated into several sections by expansion gaps and joined together by fluid dampers. Fluid dampers are installed at seismic joints in T.F. Green Airport Parking Garage to reduce relative displacement between different sections and to reduce the size of seismic joints. Luoc and De Barros (1998a; 1998b) have studied the dynamic response of a composite structural system, which comprises of two parts, an external part which is stiffer and a flexible internal part. The two damper connected parts are simulated by two vertical beams. Based on their
studies, if the connection is strong, reduction in response could be obtained on the condition that lateral stiffness of the internal part is very small and damping ratio is close to the optimized value. On the other hand, when the flexible connection is adopted, the stiffer external part can have a large reduction in response at the sacrifice of a reduced reduction in the response of the internal part.

Qu and Xu (2001) have proposed to use electrorheological or magnetorheological dampers to couple a main tower and a podium, which is a common structural form in Hong Kong. Bingham model has been adopted to simulate the connecting dampers. Properly designed electrorheological or magnetorheological dampers can reduce building response. Experimental studies by Ng and Xu (2006) have shown that large response is expected when a main tower is rigidly connected to the podium. If they are connected by friction dampers, whipping effect can be avoided. Based on shaking table test of a building connected to a podium by piezoelectric friction dampers (acting as a semi-active control device), some reduction in the model response has been observed (Xu and Ng 2008).

2.7 Chapter summary

This chapter reviews various retrofitting strategies on enhancing the seismic performance of existing buildings. Conventional retrofitting techniques can improve the behavior of a building by strengthening so that the building can sustain large response. This includes strengthening the structural members and/or adding new structural members. Energy dissipation systems alleviate building response by enhancing the capability of energy dissipation. The energy dissipation system can be broken down into passive control systems, active control systems, semi-active control system and hybrid systems. Seismic isolation systems reduce the building
response by inserting an isolation layer between the ground and buildings to shift the principal period of vibration away from resonances.

Apart from the three approaches as mentioned above, some researchers have proposed to couple neighboring buildings together to achieve response reduction. The benefit of the coupling method is that minimal changes are required to the existing building. A number of theoretical and experimental studies have been conducted to investigate the feasibility of coupling buildings by dampers or sky-bridges. The connecting dampers include passive dampers, active dampers and semi-active dampers. The above studies have demonstrated that with properly designed connecting dampers or sky-bridges, response of coupled buildings can be mitigated.

Thus dampers are used to connect buildings to for retrofitting purpose in this study. Since effectiveness of connecting dampers has not been studied with the consideration of the influence of building properties, this is studied in the next chapter.
Figure 2.1 A T beam strengthened by FRP

Figure 2.2 Two piston-shape fluid dampers
Figure 2.3 A typical friction damper

Figure 2.4 A typical base isolator
Figure 2.5 A typical friction pendulum bearing

Figure 2.6 A steel damper (left) and some lead dampers (right) for adding damping to the isolation layer
Figure 2.7 Buildings in Hong Kong

(a) Buildings in Mong Kok district, Hong Kong

(b) Buildings in Ma Tau Wai district, Hong Kong
Figure 2.8 Steel models coupled by E-shape dampers (Cimellaro, et al. 2004)

Figure 2.9 Models coupled by active actuator (Christenson, et al. 2003)
Figure 2.10 Experimental models connected by semi-active magnetorheological dampers (Basili, et al. 2013)

Figure 2.11 A building group connected by sky-bridges (Seto and Matsumoto 1999)
Figure 2.12 Petronas Twin Towers
Figure 2.13 The Triton Square Office Complex coupled by skybridges

Figure 2.14 MOMA building group (Xiao 2008)
Figure 2.15 HongZhou Citizen Center (Zheng, et al. 2009)

Figure 2.16 A sky-bridge connected building group in Korea (Lee, et al. 2012)
Chapter 3  Retrofitting existing buildings by coupling method using fluid dampers: time domain analysis

3.1 Introduction

Fluid dampers are commonly used to increase the damping of buildings (Constantinou and Symans 1992; Cheng, et al. 2010). They are added between neighboring floors within a building to resist seismic forces or wind forces. Some researchers have used fluid dampers to join adjacent buildings to reduce the seismic response (Xu, et al. 1999a; Xu, et al. 1999b; Zhang and Xu 2000; Hejazi, et al. 2010). In these studies, the viscous model is applied to simulate the fluid dampers (Luco and De Barros 1998b; Matsagar and Jangid 2006; Ge, et al. 2010; Hejazi, et al. 2010; Patel and Jangid 2010b; Bigdeli, et al. 2013). When fluid dampers are installed within a building, stiffness of fluid dampers is insignificant as compared with the inter-story stiffness of a building and can be ignored in the analysis. When fluid dampers are used to link adjacent buildings, equivalent stiffness of a building can be smaller than the inter-story stiffness of a building. As a result, the stiffness of fluid dampers can not be ignored. Thus, it is necessary to use more accurate models to represent the fluid dampers. In addition, previous studies have not investigated the efficiency of using fluid dampers to link adjacent buildings with different number of stories(Xu, et al. 1999a; Xu, et al. 1999b; Zhang and Xu 2000; Ge, et al. 2010; Hejazi, et al. 2010). The performance of fluid dampers may be affected by the properties of adjacent buildings having different number of stories. Thus this is an
important area that needs further study.

In this chapter, a shear type fluid damper is tested. Different mathematical models are compared to determine the best model to represent the shear type fluid dampers. The mathematical model is then used to derive the equations of motion of a left building connected to a right building by fluid dampers. To compare the efficiency of the fluid dampers in different cases, two adjacent buildings with different number of stories are connected together, in the range from 6 to 16 stories. The fundamental periods of the left and right building are from 0.608 s to 1.541 s and 0.586 s to 1.485 s respectively. After solving the equations of motions, the effectiveness of the fluid damper in reducing the maximum drift response is evaluated.

3.2 Fluid dampers

3.2.1 Damper configuration

Fluid dampers have been used extensively in buildings since 1990s. A fluid damper can have a piston-shape chamber in which silicone gel is forced to flow through the orifices of the piston head (Makris and Constantinou 1991). Alternatively, we have shear type fluid damper which consists of a tank that chambers the fluid and a steel plate (Arima, et al. 1988; Lu, et al. 2008b). Besides traditional passive fluid dampers, semi-active fluid dampers have also been developed. Semi-active fluid dampers can change their characteristic in response to the structural behavior, according to certain control algorithm. Effectiveness of fluid dampers has been demonstrated through experimental tests (Constantinou and Symans 1993; Lee and Taylor 2001). In general, it is not common to apply a semi-active fluid damper since performance of optimized passive fluid dampers are comparable to semi-active fluid dampers (Ahmadizadeh 2007).
In this study, a shear type fluid damper is fabricated and tested. As shown in Figure 3.1, it is composed of two external steel plates, which clamp the middle plate and a shear plate together. Sizes of the fluid damper are shown in Table 3.1. The shear plate is soaked in silicone gel which is housed by the two external plates and the middle plate. The contact area between the shear plate and the silicone gel is 80×80 mm² on both sides.

<table>
<thead>
<tr>
<th>Table 3.1 Sizes of the shear type fluid damper (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side length</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

3.2.2 Damper test

The fluid damper has been tested in a Material Testing System (MTS) machine under displacement control, as shown in Figure 3.2. Force applied to the ends of the fluid damper at any specific displacement is recorded by a data logger. To obtain the force-displacement response under sinusoidal excitation, the maximum shear displacement of the fluid damper is limited to 3 mm in this study. Table 3.2 shows the properties of fluid dampers studied by previous researchers. It is observed that the displacement amplitude is not a major concern.

<table>
<thead>
<tr>
<th>Table 3.2 Properties of fluid dampers studied by previous researchers</th>
</tr>
</thead>
<tbody>
<tr>
<td>References</td>
</tr>
<tr>
<td>Makris and Constantinou (1991)</td>
</tr>
<tr>
<td>Constantinou and Symans (1993)</td>
</tr>
<tr>
<td>Yun, et al.(2008)</td>
</tr>
<tr>
<td>Li (2006)</td>
</tr>
</tbody>
</table>

Figure 3.3 shows the force-displacement relationship of fluid dampers reported by
previous researchers (Li 2006). As shown, the hysteretic loop shape is not considerably affected by the maximum displacement. Thus, the dynamic properties of fluid dampers tested at 3 mm can be used to identify the parameters of fluid damper models.

Figure 3.4(a) shows the hysteresis loops of the fluid damper obtained from the experimental test. The size of the hysteresis loop increases with increasing excitation frequency. The hysteresis loop swells and rotates anticlockwise when the excitation frequency increases. It indicates that the dissipated energy and the force with respect to the maximum displacement increase with the increase of excitation frequency. Area of hysteresis loop illustrates the dissipated energy per cycle. Based on the test result, dissipated energy nonlinearly increases with the increase of excitation frequency. Results obtained from the tests are similar to that obtained from other studies on fluid damper under excitation at different frequencies (Park 2001; Lu, et al. 2012).

### 3.3 Damper models

Dynamic properties of fluid damper are sensitive to excitation frequency. Various rheological models have been proposed to simulate the characteristics of fluid dampers. The first mathematical model for fluid damper is a fractional derivative Maxwell model (Makris and Constantinou 1991; Makris, et al. 1993). The fractional derivative Maxwell model is in the following form:

\[
 f_{\text{damper}} + \lambda D^r \left( f_{\text{damper}} \right) = c D^q (u) \tag{3.1}
\]
where $f_{\text{damper}}$ is damper force, $\lambda$ and $c$ are coefficients, $D^r$ indicates the fractional derivative of order $r$ and $u$ is the relative displacement between the two ends of a fluid damper.

To reduce the analytical difficulties due to the complexity of the fractional derivative Maxwell model, the classical Maxwell model is used to simulate the behavior of the fluid dampers. The classic Maxwell model comprises stiffness and viscous elements connected in series (Hatada, et al. 2000; Zhang and Xu 2000; Singh, et al. 2003) and is in the following form:

$$f_{\text{damper}} + \lambda f_{\text{damper}} = cu$$

(3.2)

where $v$ is the relative velocity between the two ends of a fluid damper.

Taking Fourier transform on both sides of Equation (3.2) yields

$$F_d(\omega) + j\omega \lambda F_d(\omega) = j\omega c U(\omega)$$

(3.3)

where $F_d(\omega)$ and $U(\omega)$ are damper force and the relative displacement between the two ends of a fluid damper in the frequency domain, respectively and $j$ is the imaginary unit.

The imaginary part of the force in Equation (3.3) is equal to the imaginary part of the right side of Equation (3.3):

$$\text{Imag}[F_d(\omega)] + \omega \lambda \times \text{real}[F_d(\omega)] = \omega c \times \text{Real}[U(\omega)]$$

(3.4)

Equation (3.4) is used to identify the parameters $\lambda$ and $c$ through performing regression analysis using the test data.

As suggested by some researchers, the classical Maxwell model in some cases cannot accurately represent fluid dampers (Lu, et al. 2012). A generalized Maxwell
model is proposed to replace the classical Maxwell model. The model comprises a variable stiffness and a variable viscous element connected in series (Lu, et al. 2012). Due to the complexity of the generalized Maxwell model, popularity of this model is restricted.

It has been shown by some researchers that for structural applications the term $\lambda \hat{f}_{\text{damper}}$ in Equation (3.2) is insignificant below a certain frequency (Constantinou and Symans 1993). To simplify the structural analysis, the fluid dashpot model is substituted for the classic Maxwell model. The fluid force is assumed to be proportional to velocity to a power of $\alpha$ (Constantinou and Symans 1993; Constantinou, et al. 1998; Symans, et al. 2008):

$$f_{\text{damper}} = cv^\alpha$$  \hspace{1cm} (3.5)

where constant $\alpha$ is the exponent. When $\alpha = 1$, Equation (3.5) represents a linear fluid dashpot.

In the fluid dashpot model, a pseudo mass has been introduced to form a modified fluid dashpot model (Yun, et al. 2008). As the pseudo mass is not a well-accepted concept in modeling fluid dampers, the application of this mathematical model is constrained.

As compared with the above mentioned mathematical models, the fractional derivative model is capable of modeling the frequency-dependent feature of fluid dampers (Park 2001). The fractional derivative model can be expressed in the following form:

$$f_{\text{damper}} = 2A[G_0 u + G_1 D^\gamma (u)]$$  \hspace{1cm} (3.6)
where $A$ is the contact area of the fluid damper. $G_0$ and $G_1$ are the shear storage and shear loss moduli, respectively. As mentioned in Equation (3.1), $D^r$ indicates the fractional derivative of order $r$.

Expressing Equation (3.6) in the frequency domain, we have

$$F_d(\omega) = 2AG_0U(\omega) + 2A[\cos\left(\frac{\pi r}{2}\right) + j\sin\left(\frac{\pi r}{2}\right)]\omega^r G_1 U(\omega)$$

(3.7)

The imaginary part of Equation (3.7) can be used for parameter identification:

$$\text{Imag}[F_d(\omega)] = 2A[G_0 + \omega^r G_1 \cos\left(\frac{\pi r}{2}\right)]\text{Imag}[U(\omega)] + 2A\sin\left(\frac{\pi r}{2}\right)\omega^r G_1 \text{real}[U(\omega)]$$

(3.8)

To compare various models, a fluid damper has been fabricated and tested as mentioned in Section 3.2.2. Parameters of the Maxwell model, the viscous dashpot model and the fractional derivative model are identified through performing regression analysis using Equations (3.4), (3.5) and (3.8), respectively. The results are compared in Table 3.3.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters</th>
<th>$\rho$</th>
<th>RSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fractional derivative</td>
<td>$G_0 = 3.789 \times 10^5\text{N/m}^2$; $G_1 = 3.022 \times 10^6\text{Ns}^{0.87}/\text{m}^3$; $r = 0.87$</td>
<td>0.9958</td>
<td>1.3200$\times 10^7\text{N}$</td>
</tr>
<tr>
<td>model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maxwell model</td>
<td>$\lambda = 2.762 \times 10^{-6}\text{s}$, $c = 19349.2\text{Ns/m}$</td>
<td>0.9942</td>
<td>1.4347$\times 10^7\text{N}$</td>
</tr>
<tr>
<td>Viscous dashpot model</td>
<td>$c = 20045.1\text{Ns/m}$, $\alpha = 1$</td>
<td>0.9673</td>
<td>6.6026$\times 10^5\text{N}$</td>
</tr>
</tbody>
</table>

The correlation coefficient (Jackson 2011) $\rho$ between the predicted force and the test data is
\[
\rho = \frac{\sum_i [(f_i - \bar{f})(f_{\text{test},i} - \bar{f}_{\text{test}})]}{\sqrt{\sum_i (f_i - \bar{f})^2} \sqrt{\sum_i (f_{\text{test},i} - \bar{f}_{\text{test}})^2}}
\]  (3.9)

where \( f_i \) and \( f_{\text{test},i} \) are the \( i \)th data predicted by the various model and the \( i \)th test data, respectively. \( \bar{f} \) and \( \bar{f}_{\text{test}} \) are the average of the predicted force and applied force, respectively.

The comparison between the three correlation coefficients is shown in Table 3.3. They are close to 1. Best approximation is obtained using the fractional derivative model.

Residual sums of squares (Archdeacon 1994) (RSS) is

\[
\text{RSS} = \sum_i (f_i - f_{\text{test},i})^2
\]  (3.10)

As shown in Table 3.3, the fractional derivative model provides the smallest error.

Figure 3.4 compares the measured force-displacement and the predicted force-displacement relationships. The loops of the force against displacement predicted by the viscous dashpot model (Figure 3.4 (d)) is different from the experimental results (Figure 3.4 (a)).

The error between the maximum predicted force and testing force is calculated by

\[
\text{error}_{\text{max}} = \frac{(f_{\text{max, test}} - f_{\text{max, pred}})}{f_{\text{max, test}}} \times 100\%
\]  (3.11)

where \( f_{\text{max, test}} \) and \( f_{\text{max, pred}} \) are the maximum testing force and the maximum force predicted respectively.

Figure 3.5 compares the percentage error of the maximum force predicted by different models. It is observed that the percentage error of the maximum force predicted by the fractional derivative model is the smallest. Based on Table 3.3,
Figure 3.4 and Figure 3.5, it is concluded that the fractional derivative model is highly accurate in simulating a shear type fluid damper. Thus, the fractional derivative model is used to represent the fluid dampers in the following sections.

3.4 Buildings coupled by fluid dampers

Figure 3.6 shows two shear type buildings: a left building and a right building. The equations of motion (Humar 2012) of such a structural system are developed based on a two-dimensional formulation as follows:

\[
[M][\ddot{X}] + [C][\dot{X}] + [K][X] = -[M][I]\ddot{a}_g + [R]_{\text{dam}}
\]  
(3.12)

where \([R]_{\text{dam}}\) is the force contributed by the fluid dampers. Details of \([R]_{\text{dam}}\) are given in Section 3.4.1. \([I]\) is a unit vector indicating the earthquake force position. \(\ddot{a}_g\) is the ground acceleration.

\([M]\) = \([M]_l\), \([C]\) = \([C]_l\) and \([K]\) = \([K]_l\) are the mass matrix, damping matrix and stiffness matrix of the buildings respectively. Subscripts \(l\) and \(r\) represent the left building and the right building, respectively.

The mass matrix of the right building is \([M]_r = \begin{bmatrix} m_{r,1} & m_{r,2} & \cdots & m_{r,p} \end{bmatrix}\) where \(m_{r,1} \cdots m_{r,p}\) are the masses of the respective floors of the right building.
\[
[K]_r = \begin{bmatrix}
k_{r,1} + k_{r,2} & -k_{r,2} \\
-k_{r,2} & k_{r,p-1} + k_{r,p} \\
& \ddots \\
& & k_{r,p-1} + k_{r,p} \\
& & -k_{r,p} & k_{r,p}
\end{bmatrix}
\]

is the stiffness matrix of the right building. \(k_{r,1} \ldots k_{r,p}\) are the lateral stiffness of the respective floors of the right building.

Rayleigh damping (Chopra 2011) is used for both buildings. The damping matrix of the right building is

\[
[C]_r = \alpha_r [M]_r + \beta_r [K]_r
\]

where

\[
\alpha_r = \frac{2\zeta_r \omega_{r,1}}{(\omega_{r,1} + \omega_{r,2})} \quad \text{and} \quad \beta_r = \frac{2\zeta_r}{(\omega_{r,1} + \omega_{r,2})}
\] (Chopra 2011). The coefficients \(\omega_{r,1}\) and \(\omega_{r,2}\) are the circular frequencies of the first two vibration modes of the right building.

The mass matrix \([M]_l\), damping matrix \([C]_l\) and stiffness matrix \([K]_l\) of the left building are similar to \([M]_r\), \([C]_r\) and \([K]_r\) respectively.

### 3.4.1 Connection forces in vector form

Figure 3.7 shows a fluid damper which is used to connect adjacent buildings. The fractional derivative model for the fluid damper can be rewritten into the following form:

\[
f_{rk} = A_{\text{dam}} \left[ G_0 (x_{rk} - x_{lk}) + G_1 D^q (x_{rk} - x_{lk}) \right] \quad (3.13)
\]

where \(G_0\) and \(G_1\) are the storage modulus and loss modulus of the fluid damper respectively. \(A_{\text{dam}}\) is the contact area of the shear steel plate. As shown in Figure 3.7, \(x_{rk}\) and \(x_{lk}\) are the displacement of the two ends of the fluid damper respectively.

The connection force on the left side of the fluid damper can be similarly obtained:

\[
f_{lk} = A_{\text{dam}} \left[ G_0 (x_{lk} - x_{rk}) + G_1 D^q (x_{lk} - x_{rk}) \right] \quad (3.14)
\]
The corresponding connection forces applied to the building system are

\[
\begin{align*}
R_{rk} &= -f_{rk} \\
R_{lk} &= -f_{lk}
\end{align*}
\]  
(3.15)

Assembling all the connection forces on the neighboring buildings at each floor level gives the connection force vector,

\[
[R]_{\text{dam}} = -[K]_{\text{dam}} [X] - [N]_{\text{dam}}
\]  
(3.16)

where \([K]_{\text{dam}} = A_{\text{dam}} G_0 \begin{bmatrix} [E]_{\text{dam},l} & -[E]_{\text{dam},lr} \\ -[E]_{\text{dam},lr} & [E]_{\text{dam},r} \end{bmatrix}\) is the stiffness matrix related to the dampers. Both \([E]_{\text{dam},l}\) and \([E]_{\text{dam},r}\) are diagonal matrices. If a fluid damper is installed at the \(k\)th floor, then the \(k\)th diagonal elements of \([E]_{\text{dam},l}\) and \([E]_{\text{dam},r}\) are both equal to one. \([E]_{\text{dam},lr}\) and \([E]_{\text{dam},rl}\) are the diagonal matrices with the \(k\)th diagonal entry being unity to indicate a damper is installed at the \(k\)th floor.

\[
[N]_{\text{dam}} = A_{\text{dam}} G_1 \begin{bmatrix} [E]_{\text{dam},l} & -[E]_{\text{dam},lr} \\ -[E]_{\text{dam},lr} & [E]_{\text{dam},r} \end{bmatrix} \begin{bmatrix} D^q x_{11} \\ \vdots \\ D^q x_{ij} \\ \vdots \\ D^q x_{r1} \\ \vdots \\ D^q x_{rp} \end{bmatrix}
\]  
(3.17)

### 3.4.2 Approximation of fractional derivative

Equations (3.12) and (3.17) are solved by first converting the equations of motion into ordinary differential equations and numerically computing the differential equations by the NEWMARK method (Koh and Kelly 1990). It is assumed that the acceleration between two consecutive instants \(t_{n-1}\) and \(t_n\) is the average of the acceleration \(\ddot{x}(t_{n-1})\) at instant \(t_{n-1}\) and the acceleration \(\ddot{x}(t_{n})\) at instant \(t_{n}\). The above is consistent with the NEWMARK method. For instance, the acceleration from instant \(t_{n-1}\) to \(t_{n}\) at floor \(k\) of the left building is given by:
$$\ddot{x}_{lk,n} = \frac{1}{2} [\ddot{x}_{lk}(t_{n-1}) + \ddot{x}_{lk}(t_n)] \tag{3.18}$$

where $\ddot{x}_{lk}(t_{n-1})$ and $\ddot{x}_{lk}(t_n)$ are the accelerations of the $k$th floor of the left building at instant $t_{n-1}$ and instant $t_n$, respectively.

With the acceleration between two consecutive instants $t_{n-1}$ and $t_n$ being constant, the velocity between these two instants changes linearly. The velocity at floor $k$ of the left building at instant $\tau$ is,

$$\dot{x}_{lk}(\tau) = \dot{x}_{lk}(t_{n-1}) + (\tau - t_{n-1}) \ddot{x}_{lk,n} t_{n-1} < \tau < t_n \tag{3.19}$$

It is assumed that the displacement of both buildings at time $t = 0$ are 0. Applying the composition rule (Bagley and Torvik 1983; Press, et al. 2007) to $D^q$,

$$D^q x_{lk} \big|_{t=t_i} = \frac{1}{\Gamma(1-q)} \int_0^{t_i} \frac{\dot{x}_{lk}(\tau)}{(t_i-\tau)^q} d\tau \tag{3.20}$$

where $\Gamma$ is the Gamma function and $t_i$ is the time at step $i$.

$$D^q x_{lk} \big|_{t=t_i} = \frac{1}{\Gamma(1-q)} \sum_{n=1}^{i} \int_{t_{n-1}}^{t_n} \frac{\dot{x}_{lk}(\tau)}{(t_i-\tau)^q} d\tau \tag{3.21}$$

Substituting Equation (3.19) into the above equation gives

$$D^q x_{lk} \big|_{t=t_i} = \frac{1}{\Gamma(1-q)} \sum_{n=1}^{i} \int_{t_{n-1}}^{t_n} \frac{\dot{x}_{lk}(t_{n-1}) + (\tau - t_{n-1}) \ddot{x}_{lk,n}}{(t_i-\tau)^q} d\tau \tag{3.22}$$

After integration, Equation (3.23) is obtained,

$$D^q x_{lk} \big|_{t=t_i} = \dot{x}_{lk}(t_{i-1}) + \frac{h^{1-q}}{\Gamma(1-q)} \ddot{x}_{lk}(t_i) \tag{3.23}$$

where $h$ is the step size. $\dot{x}_{lk}(t_{i-1})$ is related to the response history. At the first step, $\dot{x}_{lk}(t_{i-1}) = 0$. 42
When the step number $i > 1$, \( \hat{x}_{lk}(t_{i-1}) = \frac{1}{\Gamma(1-q)} \sum_{n=1}^{i} \frac{(t_{i-1})^{2-q} - (t_i - t_n) x_{lk,n}}{t_{i-1} - t_n} \). \( \hat{x}_{lk}(t_i) \) is the unknown velocity which can be solved by the NEWMARK method.

Following the same approach, other fractional derivatives are simplified and \([N]_d\) in Equation (3.17) at time \( t_i \) can be divided into two parts as follows:

\[
[N]_{\text{dam},t_i} = [\hat{S}]_{\text{his},t_i} + \frac{h^{1-q}}{\Gamma(1-q)} [\hat{S}]_{t_i} \quad (3.24)
\]

where \([\hat{S}]_{t_i}\) is the velocity vector at instant \( t_i \) and \([\hat{S}]_{\text{his},t_i}\) is related to the response history. Each element of \([\hat{S}]_{\text{his},t_i}\) is in the form similar to \( \hat{x}_{lk}(t_{i-1}) \) in Equation (3.23).

The second part \( \frac{h^{1-q}}{\Gamma(1-q)} [\hat{S}]_{t_i} \) is related to the velocity response at instant \( t_i \).

Substituting Equation (3.16) into Equation (3.12) yields

\[
[M] [\ddot{X}] + [C] [\dot{X}] + [K]^{(2)} [X] = [M] [I] \ddot{a}_g + [N]_{\text{dam}} \quad (3.25)
\]

where \([K]^{(2)} = [K] + [K]_{\text{dam}}\). Equation (3.25) at instant \( t_i \) becomes

\[
[M] [\ddot{X}]_{t_i} + [C] [\dot{X}]_{t_i} + +[K]^{(2)} [X]_{t_i} = [M] [I] \ddot{a}_g,t_i + [N]_{\text{dam},t_i} \quad (3.26)
\]

By referring to Equation (3.23), Equation (3.26) can be rewritten into

\[
[M] [\ddot{X}]_{t_i} + [C]^{(2)} [\dot{X}]_{t_i} + +[K]^{(2)} [X]_{t_i} = [M] [I] \ddot{a}_g,t_i + [\hat{S}]_{\text{his},t_i} \quad (3.27)
\]

where \([C]^{(2)} = [C] + \frac{h^{1-q}}{\Gamma(1-q)} \begin{bmatrix} [E]_{\text{dam},t}^i \end{bmatrix} \). Equations (3.27) can be easily solved by the NEWMARK method.
3.5 Application study

3.5.1 Properties of the buildings

The example comprises a left building and a right building. To evaluate the efficiency of fluid dampers at different building properties, number of story of each building varies from six to sixteen floors. The left building has the same floor mass of $1.278 \text{ ton}$ and the same floor stiffness of $2.347 \times 10^3 \text{ MN/m}$. The floor mass of the right building is $1.143 \times 10^3 \text{ ton}$, with the same lateral floor stiffness of $2.258 \times 10^3 \text{ MN/m}$. Properties of the left building and right building are summarized in Table 3.4.

<table>
<thead>
<tr>
<th>Number of stories</th>
<th>Left building</th>
<th>Right building</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fundamental period (s)</td>
<td>Second mode period (s)</td>
</tr>
<tr>
<td>6</td>
<td>0.608</td>
<td>0.207</td>
</tr>
<tr>
<td>7</td>
<td>0.701</td>
<td>0.237</td>
</tr>
<tr>
<td>8</td>
<td>0.795</td>
<td>0.268</td>
</tr>
<tr>
<td>9</td>
<td>0.888</td>
<td>0.299</td>
</tr>
<tr>
<td>10</td>
<td>0.981</td>
<td>0.329</td>
</tr>
<tr>
<td>11</td>
<td>1.074</td>
<td>0.360</td>
</tr>
<tr>
<td>12</td>
<td>1.168</td>
<td>0.391</td>
</tr>
<tr>
<td>13</td>
<td>1.261</td>
<td>0.422</td>
</tr>
<tr>
<td>14</td>
<td>1.354</td>
<td>0.453</td>
</tr>
<tr>
<td>15</td>
<td>1.447</td>
<td>0.484</td>
</tr>
<tr>
<td>16</td>
<td>1.541</td>
<td>0.515</td>
</tr>
</tbody>
</table>

The two buildings are connected by fluid dampers at different floor levels. Since the properties and position of fluid dampers can significantly affect the seismic response, both damper position and damper area are optimized in the frequency domain. Details of the optimization procedure are presented in Chapter 5. Optimum values of fluid dampers are given in Table 3.5 to Table 3.15.
Earthquakes have different characteristics such as peak acceleration, duration and dominant frequency range. Accordingly, structural response varies differently to different earthquakes. The Chinese design code (GB50011-2010 2010) recommends applying seven earthquake records when performing the time-history analysis and using the average values obtained from the analyses. The seven earthquake records used in this chapter are shown in Table 3.16. The peak ground acceleration of the seven records are scaled to 1.5 m/s$^2$ and 3.1 m/s$^2$ representing moderate ground motion and rarely occurred ground motion respectively.

Table 3.5 Optimum areas (m$^2$) of fluid dampers for six-story left building and right buildings with varying number of stories

<table>
<thead>
<tr>
<th>Number of stories of the right building</th>
<th>Location of damper with respect to the left building</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3th floor</td>
</tr>
<tr>
<td>6</td>
<td>0.13</td>
</tr>
<tr>
<td>7</td>
<td>0.06</td>
</tr>
<tr>
<td>8</td>
<td>0.02</td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.6 Optimum areas (m$^2$) of fluid dampers for seven-story left building and right buildings with varying number of stories

<table>
<thead>
<tr>
<th>Number of stories of the right building</th>
<th>Location of damper with respect to the left building</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3th floor</td>
</tr>
<tr>
<td>6</td>
<td>0.03</td>
</tr>
<tr>
<td>7</td>
<td>0.13</td>
</tr>
<tr>
<td>8</td>
<td>0.06</td>
</tr>
<tr>
<td>9</td>
<td>0.03</td>
</tr>
<tr>
<td>10</td>
<td>0.03</td>
</tr>
<tr>
<td>11</td>
<td>0.06</td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.7 Optimum areas (m$^2$) of fluid dampers for eight-story left building and right buildings with varying number of stories

<table>
<thead>
<tr>
<th>Number of stories of the right building</th>
<th>Location of damper with respect to the left building</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3th floor</td>
</tr>
<tr>
<td>6</td>
<td>0.06</td>
</tr>
<tr>
<td>7</td>
<td>0.03</td>
</tr>
<tr>
<td>8</td>
<td>0.13</td>
</tr>
<tr>
<td>9</td>
<td>0.06</td>
</tr>
<tr>
<td>10</td>
<td>0.03</td>
</tr>
<tr>
<td>11</td>
<td>0.02</td>
</tr>
<tr>
<td>12</td>
<td>0.03</td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.8 Optimum areas (m$^2$) of fluid dampers for nine-story left building and right buildings with varying number of stories

<table>
<thead>
<tr>
<th>Number of stories of the right building</th>
<th>Location of damper with respect to the left building</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4th floor</td>
</tr>
<tr>
<td>6</td>
<td>0.06</td>
</tr>
<tr>
<td>7</td>
<td>0.03</td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.06</td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.9 Optimum areas (m$^2$) of fluid dampers for ten-story left building and right buildings with varying number of stories

<table>
<thead>
<tr>
<th>Number of stories of the right building</th>
<th>Location of damper with respect to the left building</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4th floor</td>
</tr>
<tr>
<td>6</td>
<td>0.63</td>
</tr>
<tr>
<td>7</td>
<td>0.03</td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.10 Optimum areas (m$^2$) of fluid dampers for eleven-story left building and right buildings with varying number of stories

<table>
<thead>
<tr>
<th>Number of stories of the right building</th>
<th>Location of damper with respect to the left building</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4th floor</td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.03</td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.03</td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.06</td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.11 Optimum areas (m$^2$) of fluid dampers for twelve-story left building and right buildings with varying number of stories

<table>
<thead>
<tr>
<th>Number of stories of the right building</th>
<th>Location of damper with respect to the left building</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4th floor</td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.06</td>
</tr>
<tr>
<td>9</td>
<td>0.03</td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0.06</td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.12 Optimum areas (m\(^2\)) of fluid dampers for thirteen-story left building and right buildings with varying number of stories

<table>
<thead>
<tr>
<th>Number of stories of the right building</th>
<th>Location of damper with respect to the left building</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3th floor</td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
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<tr>
<td>9</td>
<td>0.91</td>
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<tr>
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<td>0.03</td>
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<tr>
<td>11</td>
<td>0.03</td>
</tr>
<tr>
<td>12</td>
<td>0.03</td>
</tr>
<tr>
<td>13</td>
<td>0.13</td>
</tr>
<tr>
<td>14</td>
<td>0.13</td>
</tr>
<tr>
<td>15</td>
<td>0.06</td>
</tr>
<tr>
<td>16</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 3.13 Optimum areas (m\(^2\)) of fluid dampers for fourteen-story left building and right buildings with varying number of stories

<table>
<thead>
<tr>
<th>Number of stories of the right building</th>
<th>Location of damper with respect to the left building</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5th floor</td>
</tr>
<tr>
<td>6</td>
<td>7.78</td>
</tr>
<tr>
<td>7</td>
<td>7.63</td>
</tr>
<tr>
<td>8</td>
<td>0.03</td>
</tr>
<tr>
<td>9</td>
<td>0.72</td>
</tr>
<tr>
<td>10</td>
<td>0.03</td>
</tr>
<tr>
<td>11</td>
<td>0.03</td>
</tr>
<tr>
<td>12</td>
<td>0.03</td>
</tr>
<tr>
<td>13</td>
<td>0.06</td>
</tr>
<tr>
<td>14</td>
<td>0.13</td>
</tr>
<tr>
<td>15</td>
<td>0.13</td>
</tr>
<tr>
<td>16</td>
<td>0.06</td>
</tr>
</tbody>
</table>
Table 3.14 Optimum areas (m\(^2\)) of fluid dampers for fifteen-story left building and right buildings with varying number of stories

<table>
<thead>
<tr>
<th>Number of stories of the right building</th>
<th>Location of damper with respect to the left building</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6th floor</td>
</tr>
<tr>
<td>6</td>
<td>7.75</td>
</tr>
<tr>
<td>7</td>
<td>7.75</td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
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<tr>
<td>10</td>
<td>0.09</td>
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<td></td>
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<tr>
<td>13</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.15 Optimum areas (m\(^2\)) of fluid dampers for sixteen-story left building and right buildings with varying number of stories

<table>
<thead>
<tr>
<th>Number of stories of the right building</th>
<th>Location of damper with respect to the left building</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6th floor</td>
</tr>
<tr>
<td>6</td>
<td>8.31</td>
</tr>
<tr>
<td>7</td>
<td>7.66</td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.88</td>
</tr>
<tr>
<td>11</td>
<td></td>
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<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>
### Table 3.16 Earthquakes used for time-history analysis

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Station</th>
<th>Component</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imperial Valley</td>
<td>El Centro</td>
<td>180direction</td>
<td>1940</td>
</tr>
<tr>
<td>Tokachi-Oki</td>
<td>Hachinohe</td>
<td>EW</td>
<td>1968</td>
</tr>
<tr>
<td>Kobe</td>
<td>Takarazuka</td>
<td>TAZ090</td>
<td>1995</td>
</tr>
<tr>
<td>Michoacan</td>
<td>La Union, Mexico</td>
<td>180direction</td>
<td>1985</td>
</tr>
<tr>
<td>Hollister</td>
<td>Hollister City Hall</td>
<td>HCH715</td>
<td>1974</td>
</tr>
<tr>
<td>ChiChi</td>
<td>CHY036</td>
<td>EW</td>
<td>1999</td>
</tr>
<tr>
<td>Big Bear</td>
<td>San Bernardino-E &amp; Hospitality</td>
<td>HOS180</td>
<td>1992</td>
</tr>
</tbody>
</table>

#### 3.5.2 Response and comparison

The maximum drift is a useful indicator for evaluating the effect of fluid dampers on the overall response of the buildings. In order to compare the average maximum drift before and after the installation of connecting dampers under seven earthquakes, the percentage reduction in the average maximum drift is defined as shown in Table 3.17. Similarly, the percentage reduction in the average of the maximum RMS drift is defined as shown in Table 3.18. In the two tables, “Uncoupled” implies no connecting damper between the buildings and “Coupled” indicates that connecting dampers are installed between the buildings.

### Table 3.17 The ratio of the average maximum drift

<table>
<thead>
<tr>
<th>Building</th>
<th>The average of the maximum drift</th>
<th>The ratio of the average of the maximum drift</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncoupled</td>
<td>Coupled</td>
</tr>
<tr>
<td>m-story left building</td>
<td>$x_{\text{max}}^{\text{m, l, } r_0}$</td>
<td>$x_{\text{max}}^{\text{m, l, } r_n}$</td>
</tr>
<tr>
<td>n-story right building</td>
<td>$x_{\text{max}}^{\text{n, r, } l_0}$</td>
<td>$x_{\text{max}}^{\text{n, r, } l_n}$</td>
</tr>
</tbody>
</table>
Table 3.18 The ratio of the average of the maximum RMS drift

<table>
<thead>
<tr>
<th>Building</th>
<th>The average of the maximum RMS drift</th>
<th>The ratio of the average of the maximum RMS drift</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncoupled</td>
<td>Coupled</td>
</tr>
<tr>
<td>m-story left building</td>
<td>$x_{\text{RMS}}^{\text{Rh}m,\gamma_m}$</td>
<td>$x_{\text{RMS}}^{\text{Rh}m,j_m}$</td>
</tr>
<tr>
<td>n-story right building</td>
<td>$x_{\text{RMS}}^{\text{Rn}m,j_n}$</td>
<td>$x_{\text{RMS}}^{\text{Rn}m,j_m}$</td>
</tr>
</tbody>
</table>

$$r_{\text{dft}}^{\text{max},\text{Rn}m,j_m} = \frac{x_{\text{RMS}}^{\text{dft},\text{Rn}m,j_m} - x_{\text{RMS}}^{\text{dft},\text{Rn}m,j_0}}{x_{\text{RMS}}^{\text{dft},\text{Rn}m,j_0}} \times 100\%$$

Figure 3.8 and Figure 3.10 show the variation of the percentage reductions $r_{\text{dft}}^{\text{max},\text{Rn}m,j_m}$ and $r_{\text{dft}}^{\text{Rn}m,j_m}$ of the left building, respectively. When the difference in the period of vibration between the left building and the right building is small, the fluid dampers are ineffective in reducing the maximum inter-story drift. For instance, when a fifteen-story left building is connected to a fifteen story right building, the maximum RMS drift of the left building is amplified instead of reduced. When the difference in or in the period of vibration between the two buildings increases, the maximum drift of the left buildings is beneficially decreased. For instance, the maximum drift of the eight-story left building connected to the fifteen-story right building is reduced by more than 45%.

Variation of $r_{\text{dft}}^{\text{max},\text{Rn}m,j_m}$ and $r_{\text{dft}}^{\text{Rn}m,j_m}$ against the number of stories are shown in Figure 3.9 and Figure 3.11 respectively. The percentage reduction in the average response of the right building against the number of stories of the coupled buildings is similar to that of the left building. When the period of vibration of the right building is similar to that of the left building, limited reduction is achieved.

The black squares in Figure 3.12 and Figure 3.13 present the number of stories when the percentage reductions of both buildings are more than 40% and 30%, respectively.

As shown in these two figures, to simultaneously mitigate the response of both
buildings, it is necessary to have a large difference in the period of vibration between the adjacent buildings.

### 3.6 Inelastic response to rare earthquake excitations

The above studies are confined to elastic analysis. When buildings subjected to rare earthquake excitations, structural members may deform beyond the elastic range. The inelastic deformations are related to both the excitation intensity and the characteristics of the building. To approximate the inelastic behavior of buildings, models have been used in a variety of different ways. At the initial stage of nonlinear dynamic analysis, the elastic-perfectly plastic hysteretic model was used by many researchers due to the simplicity of the bi-linear model (Saul, et al. 1965; Borzi and Elnashai 2000). Load-displacement relationship is assumed to be linear before the yield point is reached (Wang 1994). After yielding, stiffness-hardening is considered by a post-yield stiffness (Wang 1994).

Many structural components and systems exhibit stiffness degradation when subjected to reverse loading (Saatcioglu and Humar 2003). Stiffness degradation in reinforced concrete components is usually the result of cracking, interaction with high stresses or loss of bonding. The level of stiffness degradation relies on the characteristics of the structure and on the loading history (Roufaiel and Meyer 1987; Hopper 2009). To overcome the drawback of the bilinear model which can not represent stiffness degradation, the Clough model is proposed to account for stiffness degradation by modifying the load reversal portion of the original elastic-plastic stiffness model (Clough, et al. 1965; Clough 1966). The unloading stiffness is assumed to be the initial elastic stiffness without the consideration of structural
damage. During reloading, the response point moves toward the last maximum response point (Chopra and Kan 1973).

Takeda et al. (1970) developed the degrading stiffness model with a trilinear primary curve. Later, the Takeda model was simplified by Otani (1974) and improved by Ozcebe and Saatcioglu (1989). Based on the trilinear model, a complex hysteretic model has also been developed (Ibarra, et al. 2005).

In this study, the Takeda model is used to represent the lateral stiffness of buildings. The Takeda model includes stiffness reduction after flexural cracking, stiffness deterioration after yielding and stiffness degradation with deformation. The reloading points moves toward the response point at previous maximum deformation (Takeda, et al. 1970; Borzi and Elnashai 2000). Unloading stiffness is a function of the previous maximum deformation.

As shown in Figure 3.14, line 1 indicates that the restoring force is linearly proportional to the interstory drift before the cracking point \((D_c, F_c)\) where \(D_c\) is the cracking displacement and \(F_c\) is the cracking force. The yield force and yield displacement at the yield point are \(F_y\) and \(D_y\), respectively. The cracking stiffness \(k_c\) (slope of line 2 in Figure 3.14) is 41.18\% of the elastic stiffness \(k_e\) (slope of line 1 in Figure 3.14). The cracking force \(F_c\) is equal to 30\% of the yield force \(F_y\) (Borzi and Elnashai 2000). According to the Chinese seismic design code (GB50011-2010 2010), the maximum inter-story drift angle for a reinforcement concrete frame is 1/550. Assuming the floor-to-floor height is 3m, the cracking displacement \(D_c\) (or the maximum elastic drift) is \(3000/550 = 5.45\) mm. Values of the parameters in the trilinear model are given in Table 3.19.
Table 3.19 Parameters of Takeda trilinear model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Left building</th>
<th>Right building</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial stiffness $k_e$ (MN/m)</td>
<td>$2.347 \times 10^3$</td>
<td>$2.258 \times 10^3$</td>
</tr>
<tr>
<td>Cracking displacement $D_c$ (mm)</td>
<td>5.45</td>
<td>5.45</td>
</tr>
<tr>
<td>Cracking force $F_c$ (MN)</td>
<td>12.79</td>
<td>12.31</td>
</tr>
<tr>
<td>Cracking stiffness $k_c$ (MN/m)</td>
<td>$9.665 \times 10^2$</td>
<td>$9.298 \times 10^2$</td>
</tr>
<tr>
<td>Yield displacement $D_y$ (mm)</td>
<td>36.36</td>
<td>36.36</td>
</tr>
<tr>
<td>Yield force $F_y$ (MN)</td>
<td>42.67</td>
<td>41.05</td>
</tr>
<tr>
<td>Yield stiffness $k_y$ (MN/m)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The percentage reduction, $r_{\Delta \text{max}, \ell_m, \ell_n}^{\text{max}}$, in the average of the maximum drift and the percentage reduction, $r_{\Delta \text{RMS}, \ell_m, \ell_n}^{\text{RMS}}$, in the average of the maximum RMS drift of the left building in the inelastic stage are shown in Figure 3.15 and Figure 3.17, respectively. Similarly, the inelastic analysis also shows that the response of the left building is not mitigated by fluid dampers when the two buildings have close periods of vibration. Significant reduction in drift can be achieved when the periods of vibration of adjacent buildings is substantially different. The percentage reductions $r_{\Delta \text{max}, \ell_m, \ell_n}^{\text{max}}$ and $r_{\Delta \text{RMS}, \ell_m, \ell_n}^{\text{RMS}}$ of the inelastic right buildings in Figure 3.16 and Figure 3.18 are analogous to that of the right building in the elastic stage, respectively.

The black squares in Figure 3.19 and Figure 3.20 present the number of stories when the percentage reductions in the average of the maximum drift of both buildings are more than 40% and 30%, respectively. To achieve response mitigation of both buildings at the same time, the period of vibration of the left building must be different from the period of vibration of stories of the right building.

As shown in the above, fluid dampers are more effective in controlling the response of two buildings with different periods of vibration. The larger difference in the period of vibration causes larger relative displacement and larger relative velocity.
between the buildings. As a result, a considerable reduction in response can be achieved.

3.7 Chapter summary

In this chapter, studies are carried out to evaluate the response of adjacent buildings connected by fluid dampers when subjected to earthquake excitations. Firstly, a fluid damper is fabricated and tested under displacement control. The fractional derivative model, the classic Maxwell model and the viscous dashpot model are applied to simulate the behavior of the fluid damper. After comparing the predictions by the three models with the test data, it is concluded that the fractional derivative model provides the best representation. Using the fractional derivative model, equations of motion of two buildings coupled by fluid dampers are derived. Further, an approximation method which is consistent with the traditional NEWMARK method is proposed to solve the equations of motion.

An application example has been conducted which involved analyzing two adjacent buildings with different number of stories, ranging from six to sixteen stories. It is shown that the performance of fluid dampers is affected by the properties of the buildings. When fluid dampers are used to couple two buildings having significantly different periods of vibration, the seismic response can be significantly controlled. When buildings have similar periods of vibration and are linked by fluid dampers, the reduction of the maximum response is limited.

Besides elastic analysis, inelastic response of buildings coupled by fluid dampers excited by rare earthquakes has also been studied. The Takeda model is used to represent the lateral stiffness of buildings. Similar to the results of elastic analysis,
under rare earthquakes the fluid dampers are more effective in reducing the response of two buildings with significantly different periods of vibration.

In this chapter, the response of adjacent buildings coupled by fluid damper is investigated without considering soil-structure interaction. As a building on flexible soil may behave differently from that on a rigid site, the response of buildings coupled by fluid dampers including soil-structure interaction is studied in Chapter 4. To further demonstrate the effectiveness of fluid dampers in mitigating the response of adjacent buildings by coupling them together, analysis in the frequency domain is carried out in Chapter 5. To verify the ineffectiveness of fluid dampers in controlling the response of two buildings with similar number of stories, a nine-story model and an eight-story model are built for experimental investigation. The results are presented in Chapter 6.
Figure 3.1 Fluid damper
Figure 3.2 Test setup of the fluid damper
Figure 3.3 Test results of fluid damper at different amplitude (Li 2006)
Figure 3.4 Force-displacement relationship of the fluid damper

Figure 3.5 Percentage error of the maximum force against frequency
Figure 3.6 Adjacent buildings and simplified analytical models

Figure 3.7 Displacement of fluid damper at floor $k$
Figure 3.8 Percentage reduction in the average maximum drift of the left buildings under moderate earthquakes

Figure 3.9 Percentage reduction in the average maximum drift of the right buildings under moderate earthquakes
Figure 3.10 Percentage reduction in the average of the maximum RMS drift of the left buildings under moderate earthquakes

Figure 3.11 Percentage reduction in the average of the maximum RMS drift of the right buildings under moderate earthquakes
Figure 3.12 Number of stories when the percentage reductions in the average of the maximum drift of both buildings are more than 40% under moderate earthquakes

Figure 3.13 Number of stories when the percentage reductions in the average of the maximum drift of both buildings are more than 30% under moderate earthquakes
Figure 3.14 Takeda model (Takeda, et al. 1970)
Figure 3.15 Percentage reduction in the average of the maximum drift of the left buildings under rare earthquakes

Figure 3.16 Percentage reduction in the average of the maximum drift of the right buildings under rare earthquakes
Figure 3.17 Percentage reduction in the average of the maximum RMS drift of the left buildings under rare earthquakes

Figure 3.18 Percentage reduction in the average of the maximum RMS drift of the right buildings under rare earthquakes
Figure 3.19 Number of stories when the percentage reductions in the average of the maximum drift of both buildings are more than 40% under rare earthquakes

Figure 3.20 Number of stories when the percentage reductions in the average of the maximum drift of both buildings are more than 30% under rare earthquakes
Chapter 4  

**Response of buildings coupled by fluid dampers including soil-structure interaction**

4.1 Introduction

In Chapter 3, the response of buildings connected by fluid dampers has been studied. It is concluded that fluid dampers are effective in controlling the response of adjacent buildings with different number of stories. The study is based on the assumption that the buildings have rigid foundations (i.e. fixed to the ground). Based on previous analytical studies on soil-structure interaction (SSI) (Jennings and Bielak 1973; Bielak 1974; Novak 1974; Veletsos and Meek 1974; Dutta, et al. 2004; Raychowdhury 2011) and experimental tests (Todorovska 2002; Pitilakis, et al. 2008; Trifunac 2008), the response of a building during an earthquake is affected by the interactions between the building and the soil media underlying the building (Wolf 1988; Gülkan and Clough 2012), named as structure-soil-structure interaction (SSSI). Thus, SSI is taken into account in the analysis of buildings coupled by fluid dampers in this chapter.

Firstly, an analytical model including SSI is presented and the equations of motion are derived. Considering neighboring buildings can interact with each other through the soil (Çelebi 1993; Gueguen and Bard 2005; Ghergu and Ionescu 2009), the analytical model is further developed to incorporate the structure-soil-structure interaction (SSSI). The effect of various soil types (from soft soil to rock) on the response of adjacent buildings coupled by fluid dampers is also discussed.
4.2 Equations of motion of buildings coupled by fluid dampers including SSI

The two common methods for modeling SSI are the direct method and the substructure method (Gutierrez and Chopra 1978; Chow, et al. 1988; Koh, et al. 1995). By implicitly considering the interaction, the preceding method fully analyses a building, its foundation and the soil under the foundation (Liu and Lu 1998; Pak and Guzina 1999). In the substructure method, virtual masses are added to the building models with springs and dashpots used to represent the SSI (Gutierrez and Chopra 1978; Stewart, et al. 1999). In this study, the substructure method is adopted to predict the response of coupled buildings including SSI due to its simplicity. The analytical models with and without SSI are shown in Figure 4.1.

4.2.1 Equations of motion

Figure 4.1(c) shows the analytical models of two buildings connected by fluid dampers involving SSI. The equations of motion (Wolf 1988; Chopra 2011; Humar 2012; Naserkhaki, et al. 2012) of such a structural system are developed based on a two-dimensional formulation,

\[
[M]_{ss} \ddot{X}_{ss} + [C]_{ss} \dot{X}_{ss} + [K]_{ss} X_{ss} = [M]_{ss} \ddot{a}_g + [R]_{dam,ss}
\]

(4.1)

where \( \ddot{a}_g \) is the ground acceleration. \([I] = \begin{bmatrix} 0 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} \) indicates the earthquake force position. \([X]_{ss}, [\dot{X}]_{ss}\) and \([\ddot{X}]_{ss}\) are the acceleration vector, the velocity vector and the displacement vector with respect to the base rock, respectively.
\[
[M]_{ss} = \begin{bmatrix} [M]_{l,ss} \\ [M]_{r,ss} \end{bmatrix}, \quad [C]_{ss} = \begin{bmatrix} [C]_{l,ss} \\ [C]_{r,ss} \end{bmatrix} \quad \text{and} \quad [K]_{ss} = \begin{bmatrix} [K]_{l,ss} \\ [K]_{r,ss} \end{bmatrix}
\]
are the mass matrix, damping matrix and stiffness matrix of the buildings with virtual soil masses respectively. The subscripts \(l\) and \(r\) represent the left and right building, respectively.

The mass matrix of the right building is

\[
[M]_{r,ss} = \begin{bmatrix}
I_{r,0} + \sum_{i=1}^{p} m_{r,i}h_i^2 & m_{r,1}h_1 & m_{r,2}h_2 & \cdots & m_{r,p}h_p \\
m_{r,1}h_1 & m_{r,0} & m_{r,1} & \cdots & m_{r,p} \\
m_{r,2}h_2 & m_{r,1} & m_{r,0} & \cdots & m_{r,p} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
m_{r,p}h_p & m_{r,1} & \cdots & m_{r,p} & m_{r,0}
\end{bmatrix}
\]

(4.2)

where \(h_i\) is the height of the \(i\)th story as shown in Figure 4.2. \(m_{r,1} \ldots m_{r,p}\) are the floor masses of the right building. The virtual soil mass (Wolf 1988) attached to the foundation of the right building for rocking motion is,

\[
I_{r,0} = \frac{1.24 \rho a^5}{1-\theta}
\]

(4.3)

where \(\theta\) is the Poisson’s ratio of the soil and \(2a\) is the width of the rectangular foundation of the right building. The virtual soil mass (Wolf 1988) attached to the foundation of the right building for horizontal motion is,

\[
m_{r,0} = \frac{4.37 (1-\theta) \rho a^3}{7-8\theta}
\]

(4.4)

where \(\rho = \frac{G}{v_s^2}\) is the mass density of the soil, with \(G\) and \(v_s\) being the shear modulus and shear wave velocity of the soil, respectively.

The stiffness matrix of the right building is,
\[
[K]_{r,ss} = \begin{bmatrix}
k_{r,\beta} & k_{r,0} + k_{r,1} & -k_{r,1} & \cdots & -k_{r,1} \\
-k_{r,1} & k_{r,1} + k_{r,2} & -k_{r,2} & \cdots & -k_{r,2} \\
-k_{r,2} & -k_{r,2} & \ddots & \vdots & \ddots \\
-k_{r,p-1} & k_{r,p} & -k_{r,p} & \cdots & -k_{r,p} \\
-k_{r,p} & -k_{r,p} & \cdots & \cdots & k_{r,p}
\end{bmatrix}
\] (4.5)

where \(k_{r,1} \ldots k_{r,p}\) are the lateral stiffness of the respective floors of the right building.

The rocking stiffness (Wolf 1988) of the soil attached to the foundation of the right building is,

\[
k_{r,\beta} = \frac{4.0 G a^3}{1 - \theta}
\] (4.6)

The horizontal stiffness (Wolf 1988) of the soil attached to the foundation of the right building is,

\[
k_{r,0} = \frac{9.2 G a}{2 - \theta}
\] (4.7)

The damping matrix of the right building with SSI is,

\[
[C]_{r,ss} = \begin{bmatrix}
c_{r,\beta} & c_{r,0} + c_{r,1} & -c_{r,1} & \cdots & -c_{r,1} \\
-c_{r,1} & c_{r,1} + c_{r,2} & -c_{r,2} & \cdots & -c_{r,2} \\
-c_{r,2} & -c_{r,2} & \ddots & \vdots & \ddots \\
c_{r,p-1} + c_{r,p} & -c_{r,p} & \cdots & \cdots & -c_{r,p} \\
-c_{r,p} & -c_{r,p} & \cdots & \cdots & c_{r,p}
\end{bmatrix}
\] (4.8)

The damping (Wolf 1988) of the soil attached to the foundation of the right building in the rocking direction is,

\[
c_{r,\beta} = \frac{2.40 \rho v a^4}{1 - \theta}
\] (4.9)

The horizontal damping (Wolf 1988) of the soil attached to the foundation of the right building is,
\[ c_{r,0} = \frac{1.50 \rho v_c a^2}{2 - \theta} \]  

(4.10)

Rayleigh damping is used for both buildings over the ground. Damping of the right building over the ground has been given in Chapter 3. The mass matrix \([M]_{l,ss}\), the damping matrix \([C]_{l,ss}\) and the stiffness matrix \([K]_{l,ss}\) of the left building are similar to \([M]_{r,ss}, [C]_{r,ss}\) and \([K]_{r,ss}\) respectively.

4.2.2 Inter-building connection forces

Figure 4.2 and Figure 4.3 show the total displacement of the adjacent buildings considering SSI and SSSI respectively. Different from the displacement vector \(X\) with respect to the ground in Chapter 3, the displacement vector \(X_{ss}\) in Equation (4.1) is relative to the base rock. The relative displacement between adjacent buildings at the \(k\)th floor is \((x_{rk} + h_k \theta_r) - (x_{lk} + h_k \theta_l)\). The connection force in Equation (3.13) becomes,

\[ f_{rk} = A_{dam} G_0 (x_{rk} + h_k \theta_r - x_{lk} - h_k \theta_l) + G_1 D^q (x_{rk} + h_k \theta_r - x_{lk} - h_k \theta_l) \]

(4.11)

Accordingly, the connection force vector in Equation (3.16) becomes,

\[ [R]_{dam} = -[K]_{dam} [X] - [N]_{dam} \]

(4.12)

where \([K]_{dam} = A_{dam} G_0 \begin{bmatrix} [D]_{l,T} & -[D]_{lr,T} \\ -[D]_{rl,T} & [D]_{r,T} \end{bmatrix} \begin{bmatrix} [D]_{l,T} & [D]_{lr,T} & [D]_{rl,T} & [D]_{r,T} \end{bmatrix} \) and \([D]_{l,T}\) and \([D]_{r,T}\) are four displacement transformation matrices. Dimensions of the four matrices are \(j \times j\), \(j \times p\), \(p \times j\) and \(p \times p\) respectively. As mentioned in Chapter 3, \(j\) and \(p\) are the number of stories of the left and right buildings respectively. When there is a connecting damper installed at the \(k\)th floor, the entry in row \(k + 2\) and column 1 and the entry in row \(k + 2\) and column \(k + 2\) of the matrix \([D]_{l,T}\) are \(h_k\) and 1,
respectively. $h_k$ is the height of the $k$th floor to the ground as shown in Figure 4.2.

The connection force related to damping in Equation (4.12) is

$$
[N]_{\text{dam}} = A_{\text{dam}} G_1 \begin{bmatrix}
[D]_{l,T} & -[D]_{r,T} \\
-[D]_{r,T} & [D]_{r,T}
\end{bmatrix}
\begin{bmatrix}
D^q x_{l1} \\
\vdots \\
D^q x_{ij} \\
\vdots \\
D^q x_{rp}
\end{bmatrix}
$$

(4.13)

4.2.3 Effect of through-soil coupling between adjacent building

When adjacent buildings are relatively close to each other, the vibration of one building may influence the response of the adjacent building through the soil (Mulliken and Karabalis 1998; Ghergu and Ionescu 2009). The seismic response of one building is not independent of the adjacent building (Lou, et al. 2011). Thus adjacent buildings connected by fluid dampers are also coupled through the underlying soil. The coupling or interaction is represented by the two elements as shown in Figure 4.1 (d). Besides that, there is a time lag between the response of one building and the influence on the response of the neighbouring building through the soil (Mulliken and Karabalis 1998). Table 4.1 presents the coefficients for SSSI.

<table>
<thead>
<tr>
<th>Table 4.1 Coefficients for SSSI (Mulliken and Karabalis 1998)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness</td>
</tr>
<tr>
<td>Damping</td>
</tr>
<tr>
<td>Time lag</td>
</tr>
</tbody>
</table>
When SSSI is considered, the equations of motion are,

\[ [M]_{SS} \ddot{X}_{SS} + [C]_{SS} \dot{X}_{SS} + [K]_{SS} X_{SS} = [M]_{SS} \ddot{a}_g + [R]_{dam,SS} + [F]_c \]  

(4.14)

where at any time \( t \) the SSSI force is,

\[ [F]_{c,t} = [K]_{c,h} [X]_{t-\Delta t_h} + [C]_{c,h} [\dot{X}]_{t-\Delta t_h} + [K]_{c,\theta} [X]_{t-\Delta t_\theta} + [C]_{c,\theta} [\dot{X}]_{t-\Delta t_\theta} \]  

(4.15)

\([K]_{c,h} , [C]_{c,h} , [K]_{c,\theta} \) and \([C]_{c,\theta} \) are the stiffness and damping matrices in the horizontal and rocking directions respectively.

\[ [K]_{c,h} = \begin{bmatrix} 1 & 2 & \cdots & j+4 \\ 2 & \ddots & \ddots \\ \vdots & \ddots & \ddots & \ddots \\ j+4 & \ddots & \ddots & k_{hc} \end{bmatrix} \]  

(4.16)

\([C]_{c,h} \) is similar to \([K]_{c,h} \) except that \( k_{hc} \) is replaced by \( c_{hc} \).

\[ [K]_{c,\theta} = \begin{bmatrix} 1 & 2 & \cdots & j+3 \\ 2 & \ddots & \ddots \\ \vdots & \ddots & \ddots & \ddots \\ j+3 & \ddots & \ddots & k_{\theta c} \end{bmatrix} \]  

(4.17)

\([C]_{c,\theta} \) is similar to \([K]_{c,\theta} \) except that \( k_{\theta c} \) is replaced by \( c_{\theta c} \).

4.3 Application study

4.3.1 Properties of buildings

The example comprises two buildings, a twelve-story left building and a seven-story right building. The properties of the two buildings have been provided in Chapter 3 and are restated here. The left building has the same floor mass, 1.278 ton and the
same floor stiffness, \(2.347 \times 10^3\) MN/m. The floor mass of the right building is \(1.143 \times 10^3\) ton, with the lateral floor stiffness, \(2.258 \times 10^3\) MN/m. The periods of the left and right buildings are shown in Table 4.2. Isolated piled foundation is assumed in this Chapter. Half-width of both buildings is assumed to be 20 m. In accordance with the Chinese Seismic Design Code (GB50011-2010 2010), the minimum width of seismic joint between the left and right building is 160 mm and this is selected as the distance between the two buildings. The connecting dampers between the left and right buildings are shown in Table 3.9.

Table 4.2 Properties of the buildings

<table>
<thead>
<tr>
<th>Building</th>
<th>Number of stories</th>
<th>Fundamental period (s)</th>
<th>Second mode period(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left building</td>
<td>12</td>
<td>1.168</td>
<td>0.391</td>
</tr>
<tr>
<td>Right building</td>
<td>7</td>
<td>0.676</td>
<td>0.229</td>
</tr>
</tbody>
</table>

Representing four types of soil conditions, four earthquake acceleration records are applied to excite the buildings to investigate their seismic response. The four acceleration time histories are shown in Figure 4.4. Details of the earthquake records are summarised in Table 4.3. A soil density of 1860 kg/m\(^3\) with a Poisson’s ratio of 0.33 is assumed. Table 4.4 gives the shear wave velocity of the ground used in this study.

Table 4.3 Earthquake records representing different types of soil conditions

<table>
<thead>
<tr>
<th>Earthquakes</th>
<th>Stations</th>
<th>Components</th>
<th>Years</th>
<th>Site condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Western Washington</td>
<td>Olympia Hwy Test Lab</td>
<td>N04W</td>
<td>1949</td>
<td>Soft soil</td>
</tr>
<tr>
<td>Imperial Valley</td>
<td>El Centro</td>
<td>180</td>
<td>1940</td>
<td>Moderately soft</td>
</tr>
<tr>
<td>Kern County</td>
<td>Taft Lincoln School</td>
<td>111</td>
<td>1952</td>
<td>Moderately stiff</td>
</tr>
<tr>
<td>Landers</td>
<td>Lucerne</td>
<td>000</td>
<td>1992</td>
<td>Rock</td>
</tr>
</tbody>
</table>

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Table 4.4 Shear wave velocity of the ground

<table>
<thead>
<tr>
<th>Site condition</th>
<th>Soft soil</th>
<th>Moderately soft</th>
<th>Moderately stiff</th>
<th>Rock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear wave velocity (m/s)</td>
<td>100</td>
<td>200</td>
<td>400</td>
<td>900</td>
</tr>
</tbody>
</table>

4.3.2 Response and comparison

Figure 4.5 and Figure 4.6 show the displacement envelopes of the left and right building at different soil sites respectively. As shown in Figure 4.5 (d) and Figure 4.6 (d), SSI or SSSI barely influence the maximum displacement of the buildings on the rock site. When the soil is soft, the displacement including SSI or SSSI is smaller as compared with the displacement of fixed-base buildings.

The percentage difference between the maximum drift \( x_{dft,SSI} \) including SSI and the maximum drift \( x_{dft,SSSI} \) including SSSI is,

\[
\rho_{SSI,SSSI} = \frac{x_{dft,SSSI} - x_{dft,SSI}}{x_{dft,SSI}} \times 100\%
\] \hspace{1cm} (4.20)

Table 4.5 shows the percentage difference between the maximum drift including SSI and that including SSSI. It is observed that the coupling interaction through the soil may cause a change by -3.12% to 6.78% in the maximum drift.

Table 4.5 Percentage difference between the maximum drift including SSI and that including SSSI

<table>
<thead>
<tr>
<th>Site condition</th>
<th>Building</th>
<th>Uncoupled</th>
<th>Coupled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft</td>
<td>Left</td>
<td>-1.24</td>
<td>1.33</td>
</tr>
<tr>
<td></td>
<td>Right</td>
<td>-3.12</td>
<td>6.78</td>
</tr>
<tr>
<td>Moderately soft</td>
<td>Left</td>
<td>0.02</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>Right</td>
<td>-0.19</td>
<td>-0.33</td>
</tr>
<tr>
<td>Moderately stiff</td>
<td>Left</td>
<td>0.28</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>Right</td>
<td>1.56</td>
<td>0.2</td>
</tr>
<tr>
<td>Rock</td>
<td>Left</td>
<td>-0.01</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Right</td>
<td>-0.02</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 4.6 and Table 4.7 show the maximum drift of the left and right buildings respectively. When resting on soft soil or rock site, the maximum drift of the left and right building is reduced by SSI or SSSI. When the soil is moderately soft or moderately stiff, the maximum drift may decrease or increase. However, the change is less than 1%.

Table 4.6 Comparison of the maximum drift of the left building

<table>
<thead>
<tr>
<th>Site condition</th>
<th>Uncoupled (mm)</th>
<th>Coupled (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed-base</td>
<td>SSI</td>
</tr>
<tr>
<td>Moderately stiff</td>
<td>29.316</td>
<td>28.961</td>
</tr>
<tr>
<td>Rock</td>
<td>8.390</td>
<td>8.354</td>
</tr>
</tbody>
</table>

Table 4.7 Comparison of the maximum drift of the right building

<table>
<thead>
<tr>
<th>Site condition</th>
<th>Uncoupled (mm)</th>
<th>Coupled (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed-base</td>
<td>SSI</td>
</tr>
<tr>
<td>Moderately soft</td>
<td>29.391</td>
<td>27.922</td>
</tr>
<tr>
<td>Moderately stiff</td>
<td>30.370</td>
<td>29.976</td>
</tr>
<tr>
<td>Rock</td>
<td>5.694</td>
<td>5.694</td>
</tr>
</tbody>
</table>

The percentage difference between the maximum drift $x_{dft}$ without SSI and the maximum drift $x_{dft,SSI}$ including SSI is defined as,

$$\rho_{SSI} = \frac{x_{dft,SSI} - x_{dft}}{x_{dft}} \times 100\%$$ (4.18)

Similarly, the percentage difference between the maximum drift $x_{dft}$ without SSSI and the maximum drift $x_{dft,SSSI}$ including SSSI is,

$$\rho_{SSSI} = \frac{x_{dft,SSSI} - x_{dft}}{x_{dft}} \times 100\%$$ (4.19)
Figure 4.7 and Figure 4.8 display the percentage difference in the maximum drift of the left and right building respectively. It is observed that resting on soft soil, before and after the adjacent buildings are retrofitted by connecting dampers, the maximum drift with SSI is lesser than that without SSI by more than 17% and 10% respectively. Regardless if the adjacent buildings are coupled or not, for buildings on moderately soft, moderately stiff or rock site, the change in the maximum drift caused by SSI or SSSI is less than 1%.

The application examples in Section 3.5 are also used as application examples in this chapter. The application buildings are excited by the earthquake records as shown in Table 4.8 and Table 4.9. Figure 4.9 and Figure 4.10 show the acceleration time histories representing soft soil and moderately stiff soil, respectively. Figure 4.11 and Figure 4.12 show the percentage difference $\rho_{SSI,SSSI}$ in the average of the maximum drift of the left and right buildings on soft soil and on moderately stiff soil respectively. It is observed that the coupling interaction through the soil may cause a change in the maximum drift by less than 5%.

<table>
<thead>
<tr>
<th>Earthquakes</th>
<th>Stations</th>
<th>Components</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Western Washington</td>
<td>Olympia Hwy Test Lab</td>
<td>N04W</td>
<td>1949</td>
</tr>
<tr>
<td>Loma Prieta</td>
<td>Apeel 2 Redwood</td>
<td>A02133</td>
<td>1989</td>
</tr>
<tr>
<td>Northridge</td>
<td>Montebelo-Bluff Rd.</td>
<td>BLF296</td>
<td>1994</td>
</tr>
<tr>
<td>Kobe</td>
<td>Kake</td>
<td>090</td>
<td>1995</td>
</tr>
<tr>
<td>Düzcce</td>
<td>Düzcce</td>
<td>180</td>
<td>1999</td>
</tr>
<tr>
<td>Tianjin</td>
<td>Tianjin</td>
<td>NS</td>
<td>1976</td>
</tr>
<tr>
<td>Shanghai</td>
<td>Artificial earthquake</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.13 and Figure 4.14 display the percentage difference $\rho_{SSI}$ in the average of the maximum drift of the left and right buildings respectively. It is observed that resting on soft soil, the maximum drift of the coupled buildings with SSI is lesser.
than that without SSI by 7% to 22%. For buildings on moderately stiff site, the change in the average of the maximum drift caused by SSI is less than 2%.

Table 4.9 Earthquake records representing moderately stiff soil

<table>
<thead>
<tr>
<th>Earthquakes</th>
<th>Stations</th>
<th>Components</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kern County</td>
<td>Taft Lincoln School</td>
<td>111</td>
<td>1952</td>
</tr>
<tr>
<td>Whittier narrows</td>
<td>Alhambra Fremont</td>
<td>270</td>
<td>1987</td>
</tr>
<tr>
<td>Nelchina</td>
<td>Valdez, AK</td>
<td>090</td>
<td>2004</td>
</tr>
<tr>
<td>Northridge</td>
<td>Castaic Old Ridge RT.</td>
<td>360</td>
<td>1994</td>
</tr>
<tr>
<td>Obsidian Butte</td>
<td>Ocotillo Wells</td>
<td>360</td>
<td>2005</td>
</tr>
<tr>
<td>Parkfield</td>
<td>Coalinga, CA</td>
<td>360</td>
<td>2004</td>
</tr>
<tr>
<td>Lafayette</td>
<td>Vallejo Fire Station</td>
<td>000</td>
<td>2007</td>
</tr>
</tbody>
</table>

Figure 4.15 and Figure 4.16 show the percentage difference $\rho_{SSI}$ in the average of the maximum drift of the left and right buildings respectively. It is observed that resting on soft soil, the average of the maximum drift of the coupled buildings with SSI is lesser than that without SSI by 7.2% to 24.3%. For buildings on moderately stiff site, the change in the average of the maximum drift caused by SSI is less than 2%.

The above analysis shows that the SSI or SSSI have positive contribution to the seismic response of buildings on soft soil. This is consistent with some seismic codes. For instance, both the Chinese Seismic Design Code (GB50011-2010 2010) and NEHRP-2003 Seismic Code (2003) indicate that SSI leads to an increase in the period of vibration of the building (due to the incorporation of the horizontal stiffness and rocking stiffness of the soil under the building) and thus response acceleration and base shear are decreased according to the seismic design spectra.

4.4 Chapter summary

In this chapter, studies have been carried out to evaluate the soil-structure interaction on the response of adjacent buildings connected by fluid dampers. Firstly, equations
of motions of adjacent buildings involving soil-structure interaction are formed using the substructure method. The equations are further developed to include the coupling effect between adjacent buildings through the soil.

As an application, a twelve-story left building connected to a seven-story right building are excited by four earthquake records representing four types of site conditions. The change in the maximum drift due to SSI or SSSI is less than 1% for buildings on moderately soft, moderately stiff or rock sites. Regardless if the adjacent buildings are coupled or not, SSI and SSSI can decrease the maximum drift by 10.9% to 26.6% for buildings on soft soil. Based on the above analysis, for buildings on soft soil, the response including the effect of SSI or SSSI is lesser than that without SSI and SSSI. When buildings on soft soil are coupled by connecting devices, analysis of the response of coupled buildings without considering the effect of SSI or SSSI provides a safety margin. Therefore, SSI and SSSI are ignored in the subsequent analysis in this study.
Figure 4.1 Adjacent buildings and simplified analytical models
Figure 4.2 Total floor displacement of adjacent buildings including soil-structure interaction

Figure 4.3 Total floor displacement of adjacent buildings including structure-soil-structure interaction
Figure 4.4 Ground accelerations
Figure 4.5 Displacement envelope of the left building at different soil sites
Figure 4.6 Displacement envelope of the right building at different soil sites
Figure 4.7 Percentage difference in the maximum drift of the left building

Figure 4.8 Percentage difference in the maximum drift of the right building
Figure 4.9 Seven ground accelerations representing soft soil
Figure 4.10 Seven ground accelerations representing moderately stiff soil
Figure 4.11 Percentage difference $\rho_{SSI,SSI}$ of the left and right buildings on soft soil
Figure 4.12 Percentage difference $\rho_{SSI, SSI}$ of the left and right buildings on moderately stiff soil
Figure 4.13 Percentage difference $\rho_{SSI}$ of the left and right buildings on soft soil
Figure 4.14 Percentage difference $\rho_{SS1}$ of the left and right buildings on moderately stiff soil.
Figure 4.15 Percentage difference $\rho_{SSSI}$ of the left and right buildings on soft soil
Figure 4.16 Percentage difference $\rho_{SSSI}$ of the left and right buildings on moderately stiff soil
Chapter 5  Retrofitting existing buildings by
coupling method using fluid dampers:
frequency domain analysis

5. 1 Introduction

In Chapter 3, the response of adjacent buildings connected by fluid dampers to earthquake excitations has been studied. Analyses have been carried out in the time domain. It is well demonstrated that fluid dampers are useful to mitigate seismic response of two buildings by coupling them together. When two adjacent buildings with similar number of stories are linked by fluid dampers, the maximum response is slightly reduced. Hence, structural response to the ground excitation is strongly dependent on the earthquake input in the time domain. As the nature of ground motion is random, the ground excitations can be described in a non-deterministic way such as in the form of a frequency spectrum. Thus, this chapter carries out the frequency domain analysis of adjacent buildings coupled by fluid dampers.

Firstly, two adjacent buildings are modeled by two single-degree-of-freedom (SDOF) systems to study the effectiveness of applying fluid dampers with different properties in reducing the standard deviation of the displacement of the adjacent buildings. The position of the fluid dampers between two adjacent buildings can also significantly affect the response of adjacent buildings. Thus, analysis of two multi-degree-of-freedom (MDOF) systems is performed to simultaneously optimize the location and size of the connecting dampers.
5.2 Analysis of two SDOF systems

5.2.1 Spectral density function of the ground acceleration

Several spectral density functions of the ground acceleration have been proposed in the past decades. This chapter uses the Kanai-Tajimi spectral density function which is commonly used to excite the coupled buildings (Tajimi 1960). The Kanai-Tajimi spectral density function is in the following form

\[ S_g(\omega) = \frac{1+4\zeta_g^2(\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2+4\zeta_g^2(\omega/\omega_g)^2]S_0} \]  

(5.1)

where \( \zeta_g \) and \( \omega_g \) are the filter natural frequency and damping ratio of the site, respectively. These two parameters have values of 16.9 rad and 0.94 respectively (Sues, et al. 1985). \( S_0 \), defined as the amplitude of the bedrock excitation spectrum, is approximated (Key 1988) by Equation (5.2).

\[ S_0 = \frac{0.141\ddot{x}_{g,\text{max}}\zeta_g}{\omega_g(1+4\zeta_g^2)^{1/2}} \]  

(5.2)

where \( \ddot{x}_{g,\text{max}} \) is the maximum ground acceleration.

5.2.2 Response of two SDOF systems coupled by fluid damper in the frequency domain

Figure 5.1 displays the analytical model of two SDOF systems which represent two adjacent buildings connected by dampers at particular stories. The advantage of using fluid dampers is that they can significantly improve the energy absorption capacity. Without considering the spatial variation of the ground motion, it is assumed that both buildings are excited by the same seismic acceleration. The equations of motion of the coupled two SDOF systems are
\[
[M]_{2\text{sdof}} \begin{bmatrix}
\ddot{x}_l \\
\ddot{x}_r
\end{bmatrix} + [C]_{2\text{sdof}} \begin{bmatrix}
\dot{x}_l \\
\dot{x}_r
\end{bmatrix} + [K]_{2\text{sdof}} \begin{bmatrix}
x_l \\
x_r
\end{bmatrix} = - \begin{bmatrix}
m_l \\
m_r
\end{bmatrix} \ddot{x}_g + \begin{bmatrix}
f_{\text{dam}, l} \\
f_{\text{dam}, r}
\end{bmatrix} \tag{5.3}
\]

where \([M]_{2\text{sdof}} = \begin{bmatrix} m_l & 0 \\ 0 & m_r \end{bmatrix}\), \([C]_{2\text{sdof}} = \begin{bmatrix} c_l & 0 \\ 0 & c_r \end{bmatrix}\) and \([K]_{2\text{sdof}} = \begin{bmatrix} k_l & 0 \\ 0 & k_r \end{bmatrix}\) are the mass matrix, damping matrix and stiffness matrix of the coupled system respectively.

The connection forces are

\[
\begin{bmatrix}
f_{\text{dam}, l} \\
f_{\text{dam}, r}
\end{bmatrix} = A_{\text{dam}} G_0 \begin{bmatrix} x_r - x_l \\ x_l - x_r \end{bmatrix} + A_{\text{dam}} G_1 \begin{bmatrix} D_q (x_r - x_l) \\ D_q (x_l - x_r) \end{bmatrix} \tag{5.4}
\]

Equation (5.3) in the frequency domain is

\[
\begin{bmatrix}
X_l(\omega) \\
X_r(\omega)
\end{bmatrix} = - \begin{bmatrix} m_l & m_r \end{bmatrix} X_g(\omega) + \begin{bmatrix} F_{\text{dam}, l}(\omega) \\ F_{\text{dam}, r}(\omega) \end{bmatrix} \tag{5.5}
\]

Equation (5.4) in the frequency domain is

\[
\begin{bmatrix}
F_{\text{dam}, l}(\omega) \\
F_{\text{dam}, r}(\omega)
\end{bmatrix} = A_{\text{dam}} \left\{ G_0 + G_1 \left[ \cos \left(\frac{\pi q}{2}\right) + j \sin \left(\frac{\pi q}{2}\right) \right] \omega^q \right\} \begin{bmatrix} X_r(\omega) - X_l(\omega) \\ X_l(\omega) - X_r(\omega) \end{bmatrix} \tag{5.6}
\]

Substituting Equation (5.6) into Equation (5.5) yields

\[
\begin{bmatrix}
X_l(\omega) \\
X_r(\omega)
\end{bmatrix} = - \begin{bmatrix} m_l & m_r \end{bmatrix} X_g(\omega) \tag{5.7}
\]

where

\[
\begin{aligned}
[C]' &= [C]_{2\text{sdof}} + [C]_{\text{dam}} \\
[K]' &= [K]_{2\text{sdof}} + [K]_{\text{dam}}^{(1)} + [K]_{\text{dam}}^{(2)}
\end{aligned}
\tag{5.8}
\]

where \([C]_{\text{dam}} = A_{\text{dam}} G_1 \omega^q \begin{bmatrix} \frac{\pi q}{2} \\ -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\)

\[
[K]_{\text{dam}}^{(1)} = A_{\text{dam}} G_0 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}
\]
\[
[K]^{(2)}_{\text{dam}} = A_{\text{dam}} G_1 \omega^q \cos \left( \frac{\pi q}{2} \right) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}
\]

From Equation (5.7), the following equation can be obtained:

\[
\begin{bmatrix} X_l(\omega) \\ X_r(\omega) \end{bmatrix} = [H_{\text{dsp}}(\omega)] X_g(\omega)
\]

(5.9)

where the transfer function from the ground acceleration to the structural displacement response is

\[
[H_{\text{dsp}}(\omega)] = \frac{-[m^1]_{\text{mr}}}{-\omega^2 [M] + j \omega [C] + [K]}
\]

(5.10)

As a result, the displacement power spectral density (Newland 2012) is

\[
S_{XX}(\omega) = [H_{\text{dsp}}(\omega)]^T [H_{\text{dsp}}(\omega)]^T S_g(\omega)
\]

(5.11)

where the superscript T and * are the matrix transpose and complex conjugate respectively.

Further, the standard deviation of the displacement (Lutes and Sarkani 2003) can be derived as

\[
\sigma_X = \left[ \int_{-\infty}^{+\infty} S_{XX}(\omega) \, d\omega \right]^{\frac{1}{2}}
\]

(5.12)

In this study, the trapezoidal rule has been used to integrate the standard deviation in Equation (5.12). Since the ground motion is not rich in high-frequency content, the negative infinity and positive infinity in Equation (5.12) are replaced by 0 rad/s and 60π rad/s, respectively. The step size is selected as 0.002 rad/s.

5.2.3 Application examples

The properties of the left building and the right building are displayed in Table 5.1. The mass and story number of the left building are equal to that of the left building in
Section 3.5.1. To compare the efficiency of the fluid dampers, the number of stories of the right building varies from 6 to 16. The equivalent stiffness in Table 5.1 is calculated by \( \frac{4\pi^2 m}{T_1^2} \), where \( T_1 \) represents the period of fundamental mode and \( m \) is the total mass of the building. Proportional damping is adopted. Equivalent damping is assumed to be 3% of the corresponding equivalent stiffness.

Percentage difference in the period of vibration of the left and the right buildings is defined as:

\[
\rho_{\text{period}} = \frac{T_r - T_l}{T_l} \times 100\% \quad (5.13)
\]

where \( T_l \) and \( T_r \) are the fundamental periods of vibration of the left building and the right building respectively.

The percentage reduction in the maximum power spectral density of the displacement of the right building is

\[
\rho_{S_{X,X,r,p,A}} = \frac{S_{X,X,r,p,0} - S_{X,X,r,p,A}}{S_{X,X,r,p,0}} \times 100\% \quad (5.14)
\]

where \( S_{X,X,r,p,0} \) and \( S_{X,X,r,p,A} \) are the maximum power spectral density of the displacement of the right building in the case of without connecting damper and with connecting damper of area \( A \), respectively. \( p \) is the total number of stories of the right building, ranging between 6 to 16.

The percentage reduction in the standard deviation of the displacement of the right building is

\[
\rho_{\sigma_{X,r,p,A}} = \frac{\sigma_{X,r,p,0} - \sigma_{X,r,p,A}}{\sigma_{X,r,p,0}} \times 100\% \quad (5.15)
\]
where $\sigma_{X,r,p,0}$ and $\sigma_{X,r,p,A}$ are the standard deviation of the displacement of the right building in case of without connecting damper and with connecting damper of area $A$, respectively. Similarly the percentage reduction in the maximum power spectral density and the standard deviation of the displacement of the right building are obtained accordingly.

Table 5.1 Properties of the left and right buildings

<table>
<thead>
<tr>
<th>Building</th>
<th>Equivalent mass ($\times 10^6$ kg)</th>
<th>Equivalent damping ($\times 10^6$ Ns/m)</th>
<th>Equivalent stiffness ($\times 10^6$ N/m)</th>
<th>Period (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left</td>
<td>11.502</td>
<td>4.885</td>
<td>576.180</td>
<td>0.888</td>
</tr>
<tr>
<td></td>
<td>6.858</td>
<td>4.409</td>
<td>787.361</td>
<td>0.586</td>
</tr>
<tr>
<td></td>
<td>8.001</td>
<td>4.461</td>
<td>690.798</td>
<td>0.676</td>
</tr>
<tr>
<td></td>
<td>9.144</td>
<td>4.500</td>
<td>615.148</td>
<td>0.766</td>
</tr>
<tr>
<td></td>
<td>10.287</td>
<td>4.531</td>
<td>554.331</td>
<td>0.856</td>
</tr>
<tr>
<td></td>
<td>11.430</td>
<td>4.556</td>
<td>504.400</td>
<td>0.946</td>
</tr>
<tr>
<td>Right</td>
<td>12.573</td>
<td>4.576</td>
<td>462.685</td>
<td>1.036</td>
</tr>
<tr>
<td></td>
<td>13.716</td>
<td>4.593</td>
<td>427.320</td>
<td>1.126</td>
</tr>
<tr>
<td></td>
<td>14.859</td>
<td>4.608</td>
<td>396.963</td>
<td>1.216</td>
</tr>
<tr>
<td></td>
<td>16.002</td>
<td>4.621</td>
<td>370.622</td>
<td>1.306</td>
</tr>
<tr>
<td></td>
<td>17.145</td>
<td>4.632</td>
<td>347.552</td>
<td>1.396</td>
</tr>
<tr>
<td></td>
<td>18.288</td>
<td>4.641</td>
<td>327.181</td>
<td>1.485</td>
</tr>
</tbody>
</table>

Figure 5.2 to Figure 5.5 give the percentage reduction in the maximum displacement power spectral density and the percentage reduction in the standard deviation of the displacement of both buildings. It can be observed that the effectiveness of the connecting dampers is strongly related to the difference in the fundamental period of the left building and the right building. Regardless of the size of the connecting damper, reduction diminished when the fundamental period of the right building is approaching the fundamental period of the left building. When the difference in the fundamental period of the connected buildings increases, more reduction in displacement can be achieved. Moreover, the response reduction increases with increasing damper size up to a certain damper size. Then, the response reduction
reduces with a further increase in the damper size. From these observations, it can be concluded that there exists a threshold damper area which minimizes the response.

5.3 Analysis of two multi-degree-of-freedom systems

5.3.1 Spectral density of drift response

For multi-degree-of-freedom systems coupled by fluid dampers, Equation (5.3) and Equation (5.8) are replaced by the following two equations:

\[
[M][\ddot{X}] + [C][\dot{X}] + [K][X] = -[M][I] \ddot{d}_g + [R]_{\text{dam}} \tag{5.16}
\]

\[
\begin{align*}
([C])' &= [C] + [C]_{\text{dam}} \\
([K])' &= [K] + [K]^{(1)}_{\text{dam}} + [K]^{(2)}_{\text{dam}}
\end{align*}
\tag{5.17}
\]

where \([C]_{\text{dam}} = A_{\text{dam}} G_1 \omega^{q-1} \sin \left(\frac{\pi q}{2}\right) \begin{bmatrix} [E]_{\text{dam},l} & -[E]_{\text{dam},br} \\ -[E]_{\text{dam},rl} & [E]_{\text{dam},r} \end{bmatrix}\]

\[
[K]^{(1)}_{\text{dam}} = A_{\text{dam}} G_0 \begin{bmatrix} [E]_{\text{dam},l} & -[E]_{\text{dam},br} \\ -[E]_{\text{dam},rl} & [E]_{\text{dam},r} \end{bmatrix}
\]

\[
[K]^{(2)}_{\text{dam}} = A_{\text{dam}} G_1 \omega^{q-1} \cos \left(\frac{\pi q}{2}\right) \begin{bmatrix} [E]_{\text{dam},l} & -[E]_{\text{dam},br} \\ -[E]_{\text{dam},rl} & [E]_{\text{dam},r} \end{bmatrix}
\]

Details of the mass matrix \([M]\), damping matrix \([C]\), stiffness matrix \([K]\) and damper position matrices \([E]_{\text{dam},l}\), \([E]_{\text{dam},br}\), \([E]_{\text{dam},rl}\) and \([E]_{\text{dam},r}\) are given in Chapter 3. Accordingly, the transfer function from ground acceleration to displacement in Equation (5.10) becomes

\[
[H_{\text{dsp}}(\omega)] = \frac{-[M][I]}{(-\omega^2[M] + j \omega [C] + [K])} \tag{5.18}
\]

The transfer function from ground acceleration to drift is

\[
[H_{\text{drf}}(\omega)] = [E]_{\text{drift}} [H_{\text{dsp}}(\omega)] \tag{5.19}
\]
where \([E]_{\text{drift}} = \begin{bmatrix} 1 & \cdots & 1 & \cdots & 1 \\ -1 & 1 & -1 & 1 & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \end{bmatrix} \begin{bmatrix} j+1-j+p \\ 1-j \\ \end{bmatrix} \).}

As a result, the drift power spectral density (Newland 2012) is

\[
S_{X_{\text{drift}}} X_{\text{drift}} (\omega) = [H_{\text{drift}} (\omega)]^* [H_{\text{drift}} (\omega)]^T S_g (\omega)
\]

(5.20)

Further, the standard deviation of drift (Lutes and Sarkani 2003) can be derived as

\[
\sigma_{X_{\text{drift}}} = \left[ \int_{-\infty}^{\infty} S_{X_{\text{drift}}} X_{\text{drift}} (\omega) d\omega \right]^{\frac{1}{2}}
\]

(5.21)

### 5.3.2 Optimal damper position and damper size

To increase the efficiency of connecting dampers, some researchers have assumed that the optimal damping coefficients of connecting dampers are proportional to the relative velocity between adjacent buildings (Patel and Jangid 2010a; Patel and Jangid 2010b). Therefore, dampers connected on the top stories were designed to have large damping coefficients (Patel and Jangid 2010a; Patel and Jangid 2010b).

Optimization of connecting dampers has also been investigated through experimental studies (Xu, et al. 1999; Yang, et al. 2003).

In this chapter, an optimization procedure has been developed to optimize the damper position and damper size as shown in Figure 5.6. The main objective is to reduce the maximum standard deviation of the drift of a \( j \)-story left building and a \( p \)-story right building. To achieve this objective, a vector of size \( n_{\text{min}} \times 1 \) is defined as \( A_{\text{dam}} = [A_1 \cdots A_k \cdots A_{n_{\text{min}}}]^T \) to represent the area of connecting dampers. \( n_{\text{min}} \) is
equal to $j$ if $j$ is lesser than $p$. Otherwise, $n_{\min}$ is equal to $p$. The $k$th entry of vector $A_{\text{dam}}$ is the damper area at the $k$th floor.

(I) The optimization procedure begins with the left building completely unconnected with the right building. The damper area vector $A_{\text{dam}} = [0, 0, \cdots, 0]^T$. The standard deviation of drift is computed using Equation (5.21). The maximum standard deviation $\sigma_{\text{dft, max}}^0$ and the floor number $p_{\text{max}}^0$ where the maximum standard deviation occurs are then identified.

(II) At the $i$th iteration ($i$ starts from 1), the connecting damper area is increased at floor $k$ (starts from 1 to $n_{\min}$) by a small area $\Delta$. The maximum incremental area $\Delta$ is not larger than the area that can reduce the maximum standard deviation of drift by 1%. The damper area vector becomes $[A_1, i-1 \cdots A_{k, i-1} + \Delta \cdots A_{n_{\min}, i-1}]^T$ where $A_{k, i-1}$ is the area of the damper at floor $k$ at iteration $i - 1$. Using Equation (5.21) to compute the standard deviation $\sigma_{\text{dft, max}}^{i, k}$ of the drift at floor $p_{\text{max}}^{i-1}$. The superscript $k$ indicates where the damper is increased.

(III) After computing $n_{\min}$ simulations, $\sigma_{\text{dft, max}}^{i, 1}, \sigma_{\text{dft, max}}^{i, k}, \cdots, \sigma_{\text{dft, max}}^{i, n_{\min}}$ are obtained. By comparing these values with the standard deviation $\sigma_{\text{dft, max}}^{i-1}$, effect of increasing the damper area at different location on the maximum standard deviation of drift is obtained.

(IV) The best location to increase the size of damper is where the incremental area $\Delta$ causes maximally reduction in the standard deviation of the maximum drift. After fixing the damper size at this location, the standard deviation of drift is computed using Equation (5.21). The maximum standard deviation $\sigma_{\text{dft, max}}^i$ and the floor number $p_{\text{max}}^i$ where the maximum standard deviation occurs are then identified.
(V) The above procedure (i.e. steps (II) to (IV)) repeats until the reduction in the maximum standard deviation of drift cannot be achieved. Alternatively, the procedure terminates when the total damper area is larger than the allowable damper area.

The flowchart presenting this procedure is depicted in Figure 5.6.

After ending the above optimization procedure, different dampers are allocated at different floor levels. Suppose the damper area at floor \( k \) is \( A_k \), then the damper distribution factor is defined as:

\[
\rho_{A_k} = \frac{A_k}{A_{\text{total}}} \times 100\% \quad (5.22)
\]

where \( A_{\text{total}} \) is the total area of all connecting dampers between two buildings. The damper distribution factor \( \rho_{A_k} \) can represent the vertical distribution of the connecting dampers between two adjacent buildings.

Suppose at the \( i \)th iteration, the connecting damper is increased at floor \( k \) by \( \Delta_{ik} \) and the maximum standard deviation of the drift of the left building is reduced by \( \Delta_{\sigma_{\text{drf}},i,k} \). The contribution factor of the \( \Delta_{ik} \) to the response reduction of the left building is defined as

\[
\rho_{\sigma_{\text{drf}},i,k} = \frac{\Delta_{\sigma_{\text{drf}},i,k}}{\Delta_{\sigma_{\text{drf}},\text{opt},i}} \times 100\% \quad (5.23)
\]

where \( \Delta_{\sigma_{\text{drf}},\text{opt},i} \) is the total reduction in the maximum standard deviation of the drift of the left building when the optimization procedure ends.

Summing all \( \rho_{\sigma_{\text{drf}},i,k} \) yields the contribution factor of the damper at floor \( k \) to the response reduction of the left building.
\[ \rho_{ik} = \sum_{i=1}^{N} \rho_{\sigma_{ik}} \]  

(5.24)

where \( N \) is total number of iteration. The contribution factor of the damper at floor \( k \) to the response reduction of the right building is calculated in the same way.

### 5.3.3 Application study

In the analysis of multi-degree-of-freedom system, the left and the right buildings are connected with varying number of stories, between six to sixteen stories. The mass and stiffness of the left building at each floor level are \( 1.278 \times 10^3 \) ton and \( 2.347 \times 10^3 \) MN respectively. For the right building, the floor mass is \( 1.143 \times 10^3 \) ton and the floor stiffness is \( 2.258 \times 10^3 \) MN. The fundamental periods of the right buildings are in Table 5.1. The fundamental periods and the second mode periods of the buildings are shown in Figure 5.51 and Figure 5.53 respectively.

Performing the optimization procedure in Figure 5.6, the optimal damper position and damper distribution factor are shown in Figure 5.7 to Figure 5.50. It is observed that most of the connecting dampers are assigned at the top floor of the lower building except when the difference between the number of stories of two buildings is small. When the number of stories of the right building is similar to that of the left building, a damper with large area is required to be installed at a lower floor. The dampers at the top floor of the lower building, however, still provide the largest contribution to control the response of both buildings.

To evaluate the efficiency of the connecting dampers on the seismic response of adjacent buildings, the percentage reduction in the maximum standard deviation of the drift of the left building is defined as
\[
\rho_{\sigma_{dft,J}} = \frac{\sigma_{dft,J,\text{max},0} - \sigma_{dft,J,\text{max},\text{opt}}}{\sigma_{dft,J,\text{max},0}} \times 100\%
\]

(5.25)

where \(\sigma_{dft,J,\text{max},0}\) and \(\sigma_{dft,J,\text{max},\text{opt}}\) are the maximum standard deviation of the drift of the left building without and with connecting dampers respectively.

Figure 5.51 gives the percentage reduction in the maximum standard deviation of the drift of the left building. It is observed that when the difference in the fundamental period of vibration of the buildings is small (or the number of stories is similar), seismic mitigation cannot be achieved. For instance, when a left building is coupled with a right building, both with six stories, less than 5% reduction is made by the fluid dampers. Increasing the difference in the fundamental period of vibration increases the effectiveness of the fluid dampers in seismic mitigation. As shown in Figure 5.52, when connecting dampers are installed between a six-story left building and right buildings with different number of stories, reduction in response increases with increasing the fundamental period of vibration of the right building. Maximum reduction is achieved when connected to a thirteen-story right building. Afterwards, the maximum reduction, with the second mode period of vibration of the right buildings approaches the fundamental period of vibration of the six-story left building and the reduction in response reduces accordingly. This is probably because the resonance between the two buildings affects the performance of the fluid dampers.

Figure 5.53 shows the percentage reduction in the maximum standard deviation of the drift of the right buildings. The effectiveness of the connecting dampers on the seismic response reduction of the right building is similar to the observations in Figure 5.51.
Table 5.2 presents the percentage reduction in the maximum standard deviation of drift when a nine-story left building is connected to a right building with varying stories. When the difference in the fundamental period of vibration of the buildings (calculated by Equation (5.13)) is less than 13.74%, the reduction of the maximum standard deviation of the drift of the left building and the right buildings are reduced by less than 17.3% and 20.9%, respectively. Vice versa, significant improvement is observed. Similar to the results from the analysis of two single-degree-of-freedom systems, the analysis of the two multi-degree-of-freedom systems has also demonstrated that fluid dampers are more effective.

Table 5.2 Percentage reduction in the maximum standard deviation of drift

<table>
<thead>
<tr>
<th>Number of stories of the right building</th>
<th>Percentage period difference (%)</th>
<th>Percentage reduction in the maximum standard deviation of drift (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Left building</td>
</tr>
<tr>
<td>6</td>
<td>-34.01</td>
<td>37.8</td>
</tr>
<tr>
<td>7</td>
<td>-23.87</td>
<td>29.4</td>
</tr>
<tr>
<td>8</td>
<td>-13.74</td>
<td>17.3</td>
</tr>
<tr>
<td>9</td>
<td>-3.60</td>
<td>1.90</td>
</tr>
<tr>
<td>10</td>
<td>6.53</td>
<td>8.50</td>
</tr>
<tr>
<td>11</td>
<td>16.67</td>
<td>23.3</td>
</tr>
<tr>
<td>12</td>
<td>26.80</td>
<td>32.5</td>
</tr>
<tr>
<td>13</td>
<td>36.94</td>
<td>38.7</td>
</tr>
<tr>
<td>14</td>
<td>47.07</td>
<td>43.4</td>
</tr>
<tr>
<td>15</td>
<td>57.21</td>
<td>46.4</td>
</tr>
<tr>
<td>16</td>
<td>67.23</td>
<td>48.7</td>
</tr>
</tbody>
</table>

5.4 Chapter summary

This chapter studies the response of adjacent buildings connected by fluid dampers to ground excitations in the frequency domain. Firstly, analysis of two single-degree-of-freedom systems is carried out. The effect of the size of fluid dampers on the standard deviation of displacement response of adjacent buildings coupled by fluid
dampers has been investigated. It is found that the effectiveness of fluid dampers is strongly related to the difference in the number of stories (or in the period of vibration) of the adjacent buildings. Little reduction can be achieved when the difference in the number of stories of the adjacent buildings are small. In case the difference is large, standard deviation of displacement can be significantly reduced. Moreover, the response reduction reaches a peak at a certain damper size. It is concluded that there exists a critical damper area which minimizes the response of the adjacent buildings.

Besides size, the position of fluid dampers between two adjacent buildings also influences the control of adjacent buildings. To optimize the position and size of the connecting dampers at the same time, an optimization procedure has been developed. At each step, the damper position is optimized and the damper at the optimal position is increased by a small amount. Similar to the results of the analysis of two SDOF systems, the analysis results of two MDOF systems also demonstrates that little response reduction can be achieved when fluid dampers are installed between two buildings with small percentage period difference (or small difference in the number of stories). When two adjacent buildings with large percentage period difference (or large difference in the number of stories), the top floor of the lower building between adjacent buildings is the best place for fluid dampers and the fluid dampers with optimal size at optimal position can effectively reduce the maximum standard deviation of the drift of the adjacent buildings.
Figure 5.1 Simplified analytical model

Figure 5.2 The maximum displacement power spectral density of the left building
Figure 5.3 The maximum displacement power spectral density of the right building

Figure 5.4 Percentage reduction in the standard deviation of the displacement of the left building
Figure 5.5 Percentage reduction in the standard deviation of the displacement of the right building
Use Equation (5.21) to compute the standard deviation of the drift of adjacent buildings without connecting dampers. Set $A_{\text{dam}} = [0 \ldots 0 \ldots 0]^T$

Identify the maximum standard deviation $\sigma^0_{\text{dft, max}}$ of drift and the floor number $p^0_{\text{max}}$, where $\sigma^0_{\text{dft, max}}$ occurs.

Step $i = 1$

Floor number $k = 1$

Increase damper area at floor $k$, $A_{\text{dam}} = [A_{1,i-1} \ldots A_{k,i-1} + \Delta \ldots]$

Use Equation (5.21) to compute the standard deviation $\sigma^{i}_{\text{dft, max}}$ of drift at floor $p^i_{\text{max}}$

If $k$ is less than the smaller number of stories

Compare $\sigma^{i}_{\text{dft, max}} \ldots \sigma^{i-1}_{\text{dft, max}} \ldots$ with $\sigma^{i-1}_{\text{dft, max}}$

Identify the best damper position where the incremental area $\Delta$ maximally reduce the standard deviation of drift at floor $p^{i-1}_{\text{max}}$.

After fix the damper size at the best position, use Equation (5.21) to compute the standard deviation of drift and identify $\sigma^{i}_{\text{dft, max}}$ and $p^i_{\text{max}}$

If the reduction caused by the incremental area $\Delta$ at the best damper position is larger than 1% of $\sigma^{i-1}_{\text{dft, max}}$

No

If the reduction is positive

Yes

If the total damper area is less than the allowable area

Yes

No

End

Figure 5.6 Flow chart for the optimization of damper position and damper size
Figure 5.7 Distribution factor of the connecting dampers between six-story left buildings and different right buildings.

Figure 5.8 Contribution factor of the connecting dampers between six-story left buildings and different right buildings.
Figure 5.9 Distribution factor of the connecting dampers between seven-story left buildings and different right buildings

Figure 5.10 Contribution factor of the connecting dampers between seven-story left buildings and different right buildings
Figure 5.11 Distribution factor of the connecting dampers between eight-story left buildings and different right buildings

Figure 5.12 Contribution factor of the connecting dampers between eight-story left buildings and different right buildings
Figure 5.13 Distribution factor of the connecting dampers between nine-story left buildings and different right buildings

Figure 5.14 Contribution factor of the connecting dampers between nine-story left buildings and different right buildings
Figure 5.15 Distribution factor of the connecting dampers between ten-story left buildings and different right buildings.

Figure 5.16 Contribution factor of the connecting dampers between ten-story left buildings and different right buildings.
Figure 5.17 Distribution factor of the connecting dampers between eleven-story left buildings and different right buildings

Figure 5.18 Contribution factor of the connecting dampers between eleven-story left buildings and different right buildings
Figure 5.19 Distribution factor of the connecting dampers between twelve-story left buildings and different right buildings

Figure 5.20 Contribution factor of the connecting dampers between twelve-story left buildings and different right buildings
Figure 5.21 Distribution factor of the connecting dampers between thirteen-story left buildings and different right buildings

Figure 5.22 Contribution factor of the connecting dampers between thirteen-story left buildings and different right buildings
Figure 5.23 Distribution factor of the connecting dampers between fourteen-story left buildings and different right buildings

Figure 5.24 Contribution factor of the connecting dampers between fourteen-story left buildings and different right buildings
Figure 5.25 Distribution factor of the connecting dampers between fifteen-story left buildings and different right buildings

Figure 5.26 Contribution factor of the connecting dampers between fifteen-story left buildings and different right buildings
Figure 5.27 Distribution factor of the connecting dampers between sixteen-story left buildings and different right buildings

Figure 5.28 Contribution factor of the connecting dampers between sixteen-story left buildings and different right buildings
Figure 5.29 Distribution factor of the connecting dampers between different left buildings and six-story right building

Figure 5.30 Contribution factor of the connecting dampers between different left buildings and six-story right building
Figure 5.31 Distribution factor of the connecting dampers between different left buildings and seven-story right building

Figure 5.32 Contribution factor of the connecting dampers between different left buildings and seven-story right building
Figure 5.33 Distribution factor of the connecting dampers between different left buildings and eight-story right building

Figure 5.34 Contribution factor of the connecting dampers between different left buildings and eight-story right building
Figure 5.35 Distribution factor of the connecting dampers between different left buildings and nine-story right building

Figure 5.36 Contribution factor of the connecting dampers between different left buildings and nine-story right building
Figure 5.37 Distribution factor of the connecting dampers between different left buildings and ten-story right building

Figure 5.38 Contribution factor of the connecting dampers between different left buildings and ten-story right building
Figure 5.39 Distribution factor of the connecting dampers between different left buildings and eleven-story right building

Figure 5.40 Contribution factor of the connecting dampers between different left buildings and eleven-story right building
Figure 5.41 Distribution factor of the connecting dampers between different left buildings and twelve-story right building

Figure 5.42 Contribution factor of the connecting dampers between different left buildings and twelve-story right building
Figure 5.43 Distribution factor of the connecting dampers between different left buildings and thirteen-story right building

Figure 5.44 Contribution factor of the connecting dampers between different left buildings and thirteen-story right building
Figure 5.45 Distribution factor of the connecting dampers between different left buildings and fourteen-story right building

Figure 5.46 Contribution factor of the connecting dampers between different left buildings and fourteen-story right building
Figure 5.47 Distribution factor of the connecting dampers between different left buildings and fifteen-story right building

Figure 5.48 Contribution factor of the connecting dampers between different left buildings and fifteen-story right building
Figure 5.49 Distribution factor of the connecting dampers between different left buildings and sixteen-story right building

Figure 5.50 Contribution factor of the connecting dampers between different left buildings and sixteen-story right building
Figure 5.51 Percentage reduction in the maximum standard deviation of the drift of the left building

Figure 5.52 Percentage reduction in the maximum standard deviation of the drift of the left six-story building
Figure 5.53 Percentage reduction in the maximum standard deviation of the drift of the right building
Chapter 6  Experimental study

6.1 Introduction

As demonstrated in Chapter 3 and Chapter 5, when the difference in the number of stories (or in the period of vibration) of two adjacent buildings is large, fluid dampers can effectively reduce the seismic response of both buildings by connecting them together. Test have been conducted on physical models connected by fluid dampers with large difference in the number of stories (or lateral stiffness) (Xu, et al. 1999b; Roh, et al. 2011). In this study, two models with similar number of stories have been constructed to test the effectiveness of fluid dampers in controlling the response of adjacent models. The fluid damper is installed at the eighth floor between the two models.

In this chapter, the building models are initially discussed. Characteristics of the building models are identified to provide fundamental insight into the building models. The seismic response is then assessed by carrying out tests on the shaking table in uncoupled and coupled configurations.

Besides the fluid damper, a visco-elastic damper is also used to couple the two models. Two special cases are considered. In the first case, both models are fixed to the shaking table. The visco-elastic damper connecting the test models is used for energy dissipation. In the second case, the left model is fixed to the shaking table, while the right model is isolated from the shaking table. The visco-elastic damper is used to change the frequency of the left model by connecting it to the right model in base-isolated configuration. The effect of the visco-elastic damper in controlling
structural response is evaluated by comparing the response result of the models in uncoupled configuration with that in coupled configuration.

### 6.2 Building models

The buildings considered in this chapter comprise a left building and a right building. The left building is a nine-story frame with the fundamental period at 0.89s. The right building has eight stories. It has a fundamental period at 0.77s when its base is fixed to the ground. Floor stiffnesses of the prototype buildings are $2.347 \times 10^3$ MN/m and $2.258 \times 10^3$ MN/m respectively. The two buildings are scaled with a geometry ratio of 1/15. Considering the difficulties in full compliance with the simulation law (Harris and Sabnis 2010), the mass and time ratios are assigned to be 1/27100 and 3/10, respectively. The scale factor of lateral stiffness is 1/2439. Details of the scale factors are shown in Table 6.1. Such an approach has been successfully used by other researchers (Li, et al. 2006; Ng and Xu 2006; Roh, et al. 2011).

<table>
<thead>
<tr>
<th>Scale factors</th>
<th>Left building</th>
<th>Left model</th>
<th>Right building</th>
<th>Right model</th>
<th>Scale factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor mass</td>
<td>$1.278 \times 10^3$ ton</td>
<td>47.15 kg</td>
<td>$1.143 \times 10^3$ ton</td>
<td>42.18 kg</td>
<td>1/27100</td>
</tr>
<tr>
<td>Time step</td>
<td>0.1 s</td>
<td>0.03 s</td>
<td>0.1 s</td>
<td>0.03 s</td>
<td>3/10</td>
</tr>
<tr>
<td>Displacement</td>
<td>300 cm</td>
<td>20 cm</td>
<td>300 cm</td>
<td>20 cm</td>
<td>1/15</td>
</tr>
<tr>
<td>Acceleration</td>
<td>$1.35 \text{ m/s}^2$</td>
<td>$1 \text{ m/s}^2$</td>
<td>$1.35 \text{ m/s}^2$</td>
<td>$1 \text{ m/s}^2$</td>
<td>$100/135$</td>
</tr>
</tbody>
</table>

The two models are shown in Figure 6.1. On the left is the left model representing the left building, having a height of 1.8 m with a mass of 442.6 kg. 840 mm x 440 mm x 16 mm steel plates and 13mm diameter steel bars are used to represent the floor slabs and columns respectively. On the right is the right model for the right building with a height and total mass of 1.6 m and 364.7kg, respectively.750 mm x
440 mm x 15 mm steel plates and 11 mm diameter steel bars are used to simulate the floor slabs and columns respectively.

Ambient response of the two models was measured by accelerometers installed at each and every floor. The data were analyzed by the Frequency Domain Decomposition Method (Brincker, et al. 2001; Lamarche, et al. 2008) to estimate the natural frequencies and damping ratios of the models. The results are shown in Table 6.3.

6.3 Testing of models coupled by fluid dampers

6.3.1 Earthquake simulator and data acquisition system

The unidirectional earthquake simulator in the Structural Dynamics Laboratory, The Hong Kong Polytechnic University was designed and built by MTS Corporation. It has a 3 m × 3 m table driven by a hydraulic actuator. Maximum displacement of the shaking table is ±100 mm. Acceleration excited by the shaking table can reach as much as ±1 g. Operational frequency ranges from 0 to 50 Hz.

Data acquisition system consisted of accelerometers, charge amplifiers, NI PCI-6052E data acquisition board and a LabVIEW package. Accelerometers (model 4370) are used to measure the accelerations of the models and shaking table excitations. Charge amplifiers 2635 produced by Brüel & Kjær connect the accelerometers to the A/D converter. The A/D converter with 16-bit resolution and ±0.05 to ±10 V input range converts amplified signal to digital representation. Lower and upper cut-off frequencies are 0.1 Hz and 500 Hz, respectively.
6.3.2 Test results

The two models were first bolted to the shaking table. Response of the models when subjected to random excitations was measured to estimate modal frequencies and modal damping ratios. The power spectral densities of the acceleration under ambient vibration are shown in Figure 6.2 and Figure 6.3. It is observed that in the case that the two models are fixed to the shaking table, the modal frequency is slightly increased by the installation of the fluid damper.

Three earthquake records as shown in Table 6.2 were then used to excite the models to provide the control data in the absence of connecting dampers between the two models. A fluid damper was installed at the eighth floor to connect the two models, as shown in Figure 6.4. The coupled system is excited by the three earthquake records to evaluate the response of the coupled system.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Station</th>
<th>Component</th>
<th>Year</th>
<th>Characteristic frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imperial Valley</td>
<td>EI Centro</td>
<td>Impvalli-Elc180</td>
<td>1940</td>
<td>6.25 Hz~25 Hz</td>
</tr>
<tr>
<td>Tokachi-Oki</td>
<td>Hachinohe harbor</td>
<td>West-East</td>
<td>1968</td>
<td>4.17 Hz~25 Hz</td>
</tr>
<tr>
<td>Kobe</td>
<td>Takarazuka</td>
<td>TAZ090</td>
<td>1995</td>
<td>4.17 Hz~25 Hz</td>
</tr>
</tbody>
</table>

The three earthquake records were scaled with peak accelerations being at 1.0 m/s² and time intervals reduced to 0.3 of the original data.

Based on the accelerations recorded before and after the installation of the fluid damper, the fundamental frequencies \( f \) and damping ratios \( \zeta \) are estimated. The results are given in Table 6.3. In the table, “Uncoupled” implies no connecting damper between the two models. “Coupled” indicates that the left model and the right model are coupled by the fluid damper respectively. The fundamental
frequencies are increased by 1.06% and 0.92% for the left and right models respectively by the installation of the fluid damper. The damping ratios for the first two modes of both models are slightly increased.

Table 6.3 Frequencies and damping ratios of the experimental models

<table>
<thead>
<tr>
<th>Mode</th>
<th>Left Model</th>
<th>Right model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncoupled</td>
<td>Coupled</td>
</tr>
<tr>
<td></td>
<td>$f$ (Hz)</td>
<td>$\zeta$ (%)</td>
</tr>
<tr>
<td>1st</td>
<td>3.75</td>
<td>0.75</td>
</tr>
<tr>
<td>2nd</td>
<td>11.31</td>
<td>0.79</td>
</tr>
</tbody>
</table>

The amplification factor $\rho_x$ of the displacement is defined as

$$\rho_x = \frac{x_{\text{max}}}{a_s} \quad (6.1)$$

where $x_{\text{max}}$ is the maximum displacement and $a_s$ is the maximum acceleration generated by the shaking table.

Similar to the amplification factor $\rho_x$ of displacement, $\rho_a = \frac{\ddot{x}_{\text{max}}}{a_s}$ is defined as the amplification factor of acceleration where $\ddot{x}_{\text{max}}$ is the maximum acceleration of the test model. The amplification factor of the root-mean-square (RMS) response is calculated in the same way. Table 6.4 compares the amplification factors of the displacement and acceleration of the left model under different earthquake records before and after the installation of the fluid damper. It is observed that the amplification factor of the RMS response can be reduced by 20.49% to 30.21%. However, under the Hachinohe earthquake, the maximum response of the left model can be reduced only by 2.30%.
Table 6.4 Amplification factors of the left model

<table>
<thead>
<tr>
<th>Amplification factor</th>
<th>Earthquake</th>
<th>Uncoupled</th>
<th>Coupled</th>
<th>Percentage reduction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplification factor of the acceleration</td>
<td>El Centro</td>
<td>3.121</td>
<td>2.502</td>
<td>19.85</td>
</tr>
<tr>
<td></td>
<td>Hachinohe</td>
<td>3.662</td>
<td>3.578</td>
<td>2.29</td>
</tr>
<tr>
<td></td>
<td>Kobe</td>
<td>4.325</td>
<td>3.602</td>
<td>16.71</td>
</tr>
<tr>
<td>Amplification factor of the displacement ($10^{-3}$ s$^2$)</td>
<td>El Centro</td>
<td>5.693</td>
<td>4.645</td>
<td>18.41</td>
</tr>
<tr>
<td></td>
<td>Hachinohe</td>
<td>5.863</td>
<td>5.825</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>Kobe</td>
<td>7.479</td>
<td>6.702</td>
<td>10.39</td>
</tr>
<tr>
<td>Amplification factor of the RMS acceleration</td>
<td>El Centro</td>
<td>0.559</td>
<td>0.390</td>
<td>30.21</td>
</tr>
<tr>
<td></td>
<td>Hachinohe</td>
<td>0.598</td>
<td>0.475</td>
<td>20.49</td>
</tr>
<tr>
<td></td>
<td>Kobe</td>
<td>0.622</td>
<td>0.456</td>
<td>26.71</td>
</tr>
<tr>
<td>Amplification factor of the RMS displacement ($10^{-3}$ s$^2$)</td>
<td>El Centro</td>
<td>0.993</td>
<td>0.710</td>
<td>28.51</td>
</tr>
<tr>
<td></td>
<td>Hachinohe</td>
<td>1.034</td>
<td>0.795</td>
<td>23.08</td>
</tr>
<tr>
<td></td>
<td>Kobe</td>
<td>1.120</td>
<td>0.834</td>
<td>25.54</td>
</tr>
</tbody>
</table>

Table 6.5 compares the amplification factor of the right model (in fixed-base condition) before and after the installation of the fluid damper. The reduction of the RMS response is 12.36% to 41.65%. The reduction in the maximum response to the El Centro earthquake and Kobe earthquake is less than 7.1% and 12.3%, respectively.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Earthquake</th>
<th>Uncoupled</th>
<th>Coupled</th>
<th>Percentage reduction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplification factor of the acceleration</td>
<td>El Centro</td>
<td>3.157</td>
<td>2.944</td>
<td>6.76</td>
</tr>
<tr>
<td></td>
<td>Hachinohe</td>
<td>7.844</td>
<td>5.869</td>
<td>25.17</td>
</tr>
<tr>
<td></td>
<td>Kobe</td>
<td>4.282</td>
<td>3.815</td>
<td>10.92</td>
</tr>
<tr>
<td>Amplification factor of the displacement ($10^{-3}$ s$^2$)</td>
<td>El Centro</td>
<td>4.129</td>
<td>3.840</td>
<td>7.02</td>
</tr>
<tr>
<td></td>
<td>Hachinohe</td>
<td>9.028</td>
<td>7.008</td>
<td>22.38</td>
</tr>
<tr>
<td></td>
<td>Kobe</td>
<td>6.495</td>
<td>5.702</td>
<td>12.21</td>
</tr>
<tr>
<td>Amplification factor of the RMS acceleration</td>
<td>El Centro</td>
<td>0.475</td>
<td>0.337</td>
<td>29.21</td>
</tr>
<tr>
<td></td>
<td>Hachinohe</td>
<td>1.467</td>
<td>0.856</td>
<td>41.65</td>
</tr>
<tr>
<td></td>
<td>Kobe</td>
<td>0.509</td>
<td>0.429</td>
<td>15.74</td>
</tr>
<tr>
<td>Amplification factor of the RMS displacement ($10^{-3}$ s$^2$)</td>
<td>El Centro</td>
<td>0.653</td>
<td>0.499</td>
<td>23.61</td>
</tr>
<tr>
<td></td>
<td>Hachinohe</td>
<td>1.951</td>
<td>1.218</td>
<td>37.60</td>
</tr>
<tr>
<td></td>
<td>Kobe</td>
<td>0.714</td>
<td>0.626</td>
<td>12.36</td>
</tr>
</tbody>
</table>

The top floor displacement of the left model before and after the installation of the fluid damper is shown in Figure 6.5 to Figure 6.7. Figure 6.8 to Figure 6.10 compare
the top floor displacement of the right model (in fixed condition) before and after the installation of the fluid damper. The two figures have demonstrated that the fluid damper can reduce the RMS response of both buildings. However, the reduction in the maximum response is small. This is consistent with the simulation results as shown in Chapter 3.

6.4 Prediction by numerical models

As shown in Section 6.3, the fluid damper is ineffective in reducing the response of the models with similar properties. In this section, analytical studies on the prototype buildings coupled by fluid dampers are performed using Equation (3.12). Simulation results are compared with the test results. Using Equation (3.8), the stiffness and damping of the fluid damper are 69.6 N/m and 209.92 Ns/m, respectively. According to the similitude law, the stiffness and damping of the fluid damper for the prototype buildings are 0.170 MN/m and 0.512 MNs/m, respectively.

The response of the models and that of the prototype buildings are compared in Figure 6.5 to Figure 6.10. It is found the response of the prototype buildings is similar to that of the models.

Table 6.6 and Table 6.7 show the difference between the displacement of the left model and that of the left building. The maximum difference is between -8.0% and 0.53%.

Table 6.8 and Table 6.9 show the difference between the displacement of the right model and that of the right building. The maximum difference is between -13.62% and 7.55%.
Table 6.6 Difference between the response of the left model (uncoupled with the right model) and that of the left prototype building (uncoupled with the right building)

<table>
<thead>
<tr>
<th>Earthquakes</th>
<th>Response</th>
<th>(A) Test</th>
<th>(B) Simulation</th>
<th>( \frac{(A) - (B)}{15 \times 100%} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro</td>
<td>Maximum displacement</td>
<td>4.71 mm</td>
<td>72.79 mm</td>
<td>-2.92%</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-5.31 mm</td>
<td>-80.71 mm</td>
<td>-1.30%</td>
</tr>
<tr>
<td>Hachinohe</td>
<td>Maximum displacement</td>
<td>6.53 mm</td>
<td>101.57 mm</td>
<td>-3.71%</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-6.64 mm</td>
<td>-103.06 mm</td>
<td>-3.53%</td>
</tr>
<tr>
<td>Kobe</td>
<td>Maximum displacement</td>
<td>8.64 mm</td>
<td>134.62 mm</td>
<td>-3.83%</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-9.18 mm</td>
<td>-142.44 mm</td>
<td>-3.49%</td>
</tr>
</tbody>
</table>

Table 6.7 Difference between the response of the left model (coupled with the right model by fluid damper) and that of the left prototype building (coupled with the right building by fluid damper)

<table>
<thead>
<tr>
<th>Earthquakes</th>
<th>Response</th>
<th>(A) Test</th>
<th>(B) Simulation</th>
<th>( \frac{(A) - (B)}{15 \times 100%} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro</td>
<td>Maximum displacement</td>
<td>4.65 mm</td>
<td>75.29 mm</td>
<td>-8.00%</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-4.82 mm</td>
<td>-72.85 mm</td>
<td>-0.70%</td>
</tr>
<tr>
<td>Hachinohe</td>
<td>Maximum displacement</td>
<td>6.54 mm</td>
<td>104.05 mm</td>
<td>-6.08%</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-6.83 mm</td>
<td>-105.40 mm</td>
<td>-2.85%</td>
</tr>
<tr>
<td>Kobe</td>
<td>Maximum displacement</td>
<td>8.71 mm</td>
<td>129.93 mm</td>
<td>0.53%</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-9.15 mm</td>
<td>-137.75 mm</td>
<td>-0.35%</td>
</tr>
</tbody>
</table>

Table 6.8 Difference between the response of the right model (uncoupled with the left model) and that of the right prototype building (uncoupled with the left building)

<table>
<thead>
<tr>
<th>Earthquakes</th>
<th>Response</th>
<th>(A) Test</th>
<th>(B) Simulation</th>
<th>( \frac{(A) - (B)}{15 \times 100%} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro</td>
<td>Maximum displacement</td>
<td>2.78 mm</td>
<td>40.73 mm</td>
<td>2.41%</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-3.85 mm</td>
<td>-57.06 mm</td>
<td>1.26%</td>
</tr>
<tr>
<td>Hachinohe</td>
<td>Maximum displacement</td>
<td>10.22 mm</td>
<td>151.63 mm</td>
<td>1.07%</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-10.17 mm</td>
<td>-148.93 mm</td>
<td>2.40%</td>
</tr>
<tr>
<td>Kobe</td>
<td>Maximum displacement</td>
<td>7.47 mm</td>
<td>106.54 mm</td>
<td>4.96%</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-7.97 mm</td>
<td>-114.13 mm</td>
<td>4.51%</td>
</tr>
</tbody>
</table>
Table 6.9 Difference between the response of the right model (coupled with the left model by fluid damper) and that of the right prototype building (coupled with the left building by fluid damper)

<table>
<thead>
<tr>
<th>Earthquakes</th>
<th>Response</th>
<th>(A) Test</th>
<th>(B) Simulation</th>
<th>( \frac{(A) - (B)}{15} \times 100% )</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro</td>
<td>Maximum displacement</td>
<td>3.11 mm</td>
<td>43.17 mm</td>
<td>7.55 %</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-3.77 mm</td>
<td>-55.44 mm</td>
<td>1.91 %</td>
</tr>
<tr>
<td>Hachinohe</td>
<td>Maximum displacement</td>
<td>7.78 mm</td>
<td>126.68 mm</td>
<td>-8.62 %</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-7.24 mm</td>
<td>-123.37 mm</td>
<td>-13.62 %</td>
</tr>
<tr>
<td>Kobe</td>
<td>Maximum displacement</td>
<td>7.36 mm</td>
<td>105.67 mm</td>
<td>4.23 %</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-7.39 mm</td>
<td>-107.60 mm</td>
<td>2.95 %</td>
</tr>
</tbody>
</table>

### 6.5 Visco-elastic damper

Visco-elastic dampers are fabricated using 20 layers of visco-elastic damping pads sandwiched by 19 layers of steel pads as shown in Figure 6.12(a). Dimensions of both the visco-elastic damping pads and steel pads are 20mm x 30mm x 0.8mm. The total thickness of the visco-elastic dampers is 16 mm.

#### 6.5.1 Properties of visco-elastic dampers

Properties of visco-elastic dampers are assessed via a MTS testing machine as shown in Figure 6.12(b). A small capacity load cell is used to measure the applied load. Sinusoidal actions at different frequencies (1 Hz to 5 Hz) and different amplitudes (1 mm to 4 mm) were applied. The maximum excitation frequency was limited to 5 Hz because the fundamental frequencies of the two test models are less than 5 Hz. The applied loads are aligned with the centre line of the damper. Visco-elastic dampers perform well at different excitation frequencies and different displacement amplitudes. Test results have shown similar hysteretic loops at different excitation frequencies and different displacement amplitudes. Figure 6.13 displays the force-displacement relationships at different displacement amplitude and different frequencies.
6.5.2 Visco-elastic damper model

As shown in Figure 6.13, the performance of the visco-elastic dampers is strongly influenced by the excitation frequency. The traditional Kelvin model does not provide accurate representation of the visco-elastic dampers. After comparing various mathematical models, it is found that the general mechanical model can accurately model visco-elastic dampers (Park 2001). As shown in Figure 6.14, the visco-elastic damper is modeled using four parameters, i.e. \( k_1, c_1, k_2 \) and \( c_2 \). The parameters are computed by performing regression analysis on the measured data in Section 6.5.1.

The total damper force is expressed by the following equation

\[
f = f_k + f_m
\]  

where \( f_k \) is the force in the Kelvin component.

\[
f_k = k_1 \Delta + c_1 \dot{\Delta}
\]  

\( \Delta \) and \( \dot{\Delta} \) are the respective relative displacement and relative velocity between the two ends of the visco-elastic damper.

\( f_m \) is the Maxwell component (Soong and Dargush 1997).

\[
\frac{f_m}{k_2} + \frac{f_m}{c_2} = \dot{\Delta}
\]  

Taking Fourier transform on both sides of Equation (6.3) gives

\[
F_k(\omega) = k_1 \Delta(\omega) + j \omega c_1 \Delta(\omega)
\]

\[
F_k(\omega) = (k_1 + j \omega c_1) \Delta(\omega)
\]  

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where $F_k(\omega)$ and $\Delta(\omega)$ are the damper force and displacement in the Kelvin component in the frequency domain respectively. $j$ is the imaginary unit.

Taking Fourier transform on both sides of Equation (6.4) gives

$$j\omega F_m(\omega) + \frac{F_m(\omega)}{c_2} = j\omega \Delta(\omega)$$

$$F_m(\omega) = \frac{j\omega k^2 c_2 + \omega^2 k_2 c_2^2}{k_2^2 + \omega^2 c_2^2} \Delta(\omega)$$  \hspace{1cm} (6.6)

where $F_m(\omega)$ is the damper force in the Maxwell component in the frequency domain.

Taking Fourier transform on both sides of Equation (6.2) gives

$$F(\omega) = F_k(\omega) + F_m(\omega)$$

$$F(\omega) = (k_1 + j\omega c_1)\Delta(\omega) + \frac{j\omega k^2 c_2 \Delta(\omega) + \omega^2 k_2 c_2^2 \Delta(\omega)}{k_2^2 + \omega^2 c_2^2}$$  \hspace{1cm} (6.7)

The imaginary part of the connection force on the left side of in Equation (6.7) is equal to the imaginary part of the right side of Equation (6.7):

$$\text{Imag}[F(\omega)] = k_1 \text{Imag}[\Delta(\omega)] + \omega c_1 \text{Real}[\Delta(\omega)] + \alpha_1 \text{Real}[\Delta(\omega)] + \alpha_2 \text{Imag}[\Delta(\omega)]$$  \hspace{1cm} (6.8)

where $\alpha_1 = \frac{\omega k^2 c_2}{k_2^2 + \omega^2 c_2^2}$ and $\alpha_2 = \frac{\omega^2 k_2 c_2^2}{k_2^2 + \omega^2 c_2^2}$.

Equation (6.8) is used to identify the parameters of the general mechanical model based on the test data. The results are given in Table 6.10. The calculation of correlation coefficient between the test data and predicted data is provided in Chapter 3.
Table 6.10 Properties of the visco-elastic damper

<table>
<thead>
<tr>
<th></th>
<th>Stiffness $k_1$ (kN/m)</th>
<th>Damping $c_1$ (Ns/m)</th>
<th>Stiffness $k_2$ (kN/m)</th>
<th>Damping $c_2$ (Ns/m)</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>34.264</td>
<td>763.7</td>
<td>18.809</td>
<td>1666.8</td>
<td>0.9998</td>
</tr>
</tbody>
</table>

6.6 Base isolation system

The base isolators are specifically fabricated as shown in Figure 6.15. Each base isolator comprises two steel plates and three rubber bearings. Each rubber bearing is consisted of 9 layers of cylindrical polyurethane pieces providing small lateral stiffness and 8 layers of steel washers to enhance the bearing capacity. The outer diameter, inner diameter and thickness of cylindrical polyurethane pieces are 27 mm, 10 mm and 3 mm, respectively. The outer diameter, inner diameter and thickness of the steel washers are 25 mm, 10 mm and 0.2 mm respectively. Cylindrical polyurethane pieces and steel washers were glued together by epoxy. The thickness of each rubber bearing is 32 mm.

To estimate the lateral stiffness of the base isolators, four isolators were installed between two large steel plates. They were fixed to the shaking table and loaded with 360 kg steel plates. Accelerations under impulse force were measured to estimate the frequency and the damping ratio of the isolators using the Frequency Domain Decomposition Method (Brincker, et al. 2001; Lamarche, et al. 2008). The equivalent lateral stiffness and equivalent damping ratio of the 4 isolators are $k_{iso} = 135.329$ kN/m and $\zeta_{iso} = 5.0\%$ respectively.
6.7 Testing of models coupled by the visco-elastic damper

6.7.1 Test arrangement

The models were tested on the shaking table. The test was divided into two stages.

(1) At stage one, the visco-elastic dampers is installed at the 8th floor between the models which are fixed to the shaking table as shown in Figure 6.11. The response of the models to the three earthquakes is recorded.

(2) At stage two, base isolators are added between the right model and the shaking table. The visco-elastic damper is incorporated between the models at the 8th floor where the maximum response occurred. Figure 6.2(c) and Figure 6.3(c) show the power spectral density of the acceleration of the models after the installation of base isolators and the visco-elastic damper under ambient vibration, respectively. It is observed that the fundamental frequency of the two models is decreased. This may beneficially reduce the response of the two models. The coupled models are tested under the three earthquake records as shown in Table 6.2.

6.7.2 Test results of models coupled by visco-elastic damper

Based on the results obtained from stage one, the fundamental frequency $f$ and damping ratios $\zeta$ are estimated. The results are given in Table 6.11 and Table 6.12. In the two tables, “Uncoupled” implies no connecting damper between the two models. The damping ratios of the two models (in fixed-base conditions) are slightly increased after the installation of the visco-elastic damper between the two models. However, the fundamental frequency of the left model is adversely increased.

Properties of the models at stage 2 are also shown in Table 6.11 and Table 6.12. In comparison with the two models in fixed-base conditions, the fundamental
frequencies of the two models are reduced and the damping ratios of the two models are increased after the right model is isolated from the shaking table and the visco-elastic damper is installed between the two models.

Table 6.11 The modal frequencies and damping ratios of the left model

<table>
<thead>
<tr>
<th>Modes</th>
<th>Uncoupled</th>
<th>Coupled with the right model in fixed-base condition</th>
<th>Coupled with the right model in base isolated configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f$ (Hz)</td>
<td>$\zeta$ (%)</td>
<td>$f$ (Hz)</td>
</tr>
<tr>
<td>1st</td>
<td>3.75</td>
<td>0.75</td>
<td>3.90</td>
</tr>
<tr>
<td>2nd</td>
<td>11.31</td>
<td>0.79</td>
<td>11.40</td>
</tr>
</tbody>
</table>

Table 6.12 The modal frequencies and damping ratios of the right model

<table>
<thead>
<tr>
<th>Modes</th>
<th>Fixed-base</th>
<th>Base isolated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncoupled</td>
<td>Coupled</td>
</tr>
<tr>
<td></td>
<td>$f$ (Hz)</td>
<td>$\zeta$ (%)</td>
</tr>
<tr>
<td>1st</td>
<td>4.35</td>
<td>0.60</td>
</tr>
<tr>
<td>2nd</td>
<td>14.45</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Figure 6.16 to Figure 6.18 compare the top floor displacement of the left model when subjected to the three earthquake records. According to the displacement time history, when the two models in fixed-base conditions are connected by the visco-elastic damper, limited reduction is achieved. When the left model is coupled with the right model in base-isolated configuration by the visco-elastic damper, significant reduction in the displacement can be observed.

Figure 6.19 to Figure 6.21 display the displacement of the right model when subjected to the three earthquake records. The response of the right model in base-isolated configuration is considerably reduced regardless if connected to the left model or not. The displacement of the right model at isolation layer is also presented in Figure 6.19 to Figure 6.21. Comparison between the top floor displacement of the
right model (in base-isolated configuration) and that at isolation layer indicates that the total displacement is mainly contributed by the base isolators.

Table 6.13 and Table 6.14 compare the amplification factors of the displacement and acceleration of the left model under different earthquake records. When the left model is connected to the right model in fixed-base condition, maximum response of the left model is not reduced. When connected to the right model in base-isolated configuration, the maximum displacement and acceleration of the left model are reduced by more than 24.8% and 26.4%, respectively.

Table 6.13 Amplification factors of the acceleration the left model

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Uncoupled</th>
<th>Coupled with the right model in fixed-base condition</th>
<th>Coupled with the right model in base-isolated configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro</td>
<td>3.12</td>
<td>3.16</td>
<td>2.11</td>
</tr>
<tr>
<td>Hachinohe</td>
<td>3.66</td>
<td>5.29</td>
<td>2.75</td>
</tr>
<tr>
<td>Kobe</td>
<td>4.32</td>
<td>4.34</td>
<td>2.47</td>
</tr>
</tbody>
</table>

Table 6.14 Amplification factors of the displacement the left model

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Uncoupled</th>
<th>Coupled with the right model in fixed-base condition</th>
<th>Coupled with the right model in base-isolated configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro</td>
<td>5.69</td>
<td>6.08</td>
<td>4.19</td>
</tr>
<tr>
<td>Hachinohe</td>
<td>5.86</td>
<td>7.97</td>
<td>3.50</td>
</tr>
<tr>
<td>Kobe</td>
<td>7.48</td>
<td>7.93</td>
<td>4.52</td>
</tr>
</tbody>
</table>

The amplification factors of the RMS acceleration and displacement of the left model are shown in Table 6.15 and Table 6.16 respectively. The response of the left model is not always decreased when coupled with the right model in fixed-base condition. When connected to the right model in base-isolated configuration, the visco-elastic damper is capable of reducing the RMS response of the left model by more than 49.5%.
Table 6.17 and Table 6.18 compare the response of the right model to three earthquake records. When connected to the fixed left model, the maximum acceleration and RMS acceleration of the right model in base-isolated configuration are reduced by more than 34.6% and 28.7%, respectively.

Table 6.15 Amplification factor of the RMS acceleration of the left model

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Uncoupled</th>
<th>Coupled with the right model in fixed-base condition</th>
<th>Coupled with the right model in base-isolated configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro</td>
<td>0.56</td>
<td>0.41</td>
<td>0.19</td>
</tr>
<tr>
<td>Hachinohe</td>
<td>0.60</td>
<td>0.51</td>
<td>0.26</td>
</tr>
<tr>
<td>Kobe</td>
<td>0.62</td>
<td>0.57</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 6.16 Amplification factor of the RMS displacement ($\times 10^{-3} \text{ s}^2$) of the left model

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Uncoupled</th>
<th>Coupled with the right model in fixed-base condition</th>
<th>Coupled with the right model in base-isolated configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro</td>
<td>0.99</td>
<td>0.90</td>
<td>0.50</td>
</tr>
<tr>
<td>Hachinohe</td>
<td>1.03</td>
<td>1.10</td>
<td>0.46</td>
</tr>
<tr>
<td>Kobe</td>
<td>1.12</td>
<td>1.22</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table 6.17 Amplification factor of the acceleration of the right model

<table>
<thead>
<tr>
<th>Amplification factor</th>
<th>Earthquake</th>
<th>Fixed-base, uncoupled</th>
<th>Fixed-base, coupled</th>
<th>Base-isolated, coupled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplification factor of the acceleration</td>
<td>El Centro</td>
<td>3.16</td>
<td>2.62</td>
<td>1.71</td>
</tr>
<tr>
<td></td>
<td>Hachinohe</td>
<td>7.84</td>
<td>4.24</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td>Kobe</td>
<td>4.28</td>
<td>3.67</td>
<td>1.92</td>
</tr>
<tr>
<td>Amplification factor of the RMS acceleration</td>
<td>El Centro</td>
<td>0.48</td>
<td>0.32</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>Hachinohe</td>
<td>1.47</td>
<td>0.36</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>Kobe</td>
<td>0.51</td>
<td>0.43</td>
<td>0.19</td>
</tr>
</tbody>
</table>

The amplification factors of the displacement of the right model are given in Table 6.18. In some cases the maximum displacement of the right model in base-isolated configuration is larger than the displacement in fixed-base condition. This is because the top floor displacement of the right model in base-isolated configuration includes
the displacement of the isolation layer. Test results have shown that around 71.5%, 63.4% and 77.7% of the peak displacement of the right model are contributed by the base isolators under the three earthquake records.

Table 6.18 Amplification factor ($\times 10^3$ s$^2$) of the displacement of the right model

<table>
<thead>
<tr>
<th>Amplification factor of the displacement</th>
<th>Earthquake</th>
<th>Fixed-base, uncoupled</th>
<th>Fixed-base, coupled</th>
<th>Base-isolated, coupled</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro</td>
<td>4.13</td>
<td>4.72</td>
<td>5.41</td>
<td></td>
</tr>
<tr>
<td>Hachinohe</td>
<td>9.03</td>
<td>5.92</td>
<td>3.52</td>
<td></td>
</tr>
<tr>
<td>Kobe</td>
<td>6.49</td>
<td>6.03</td>
<td>6.29</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Amplification factor of the RMSdisplacement</th>
<th>Earthquake</th>
<th>Fixed-base, uncoupled</th>
<th>Fixed-base, coupled</th>
<th>Base-isolated, coupled</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro</td>
<td>0.65</td>
<td>0.68</td>
<td>0.81</td>
<td></td>
</tr>
<tr>
<td>Hachinohe</td>
<td>1.95</td>
<td>0.75</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td>Kobe</td>
<td>0.71</td>
<td>0.89</td>
<td>0.70</td>
<td></td>
</tr>
</tbody>
</table>

6.8 Prediction by numerical models

As shown in Section 6.7, visco-elastic dampers are effective in controlling the response of the two adjacent models in combination with base isolators. In this section, the prototype buildings coupled by visco-elastic dampers are analyzed using Equation (7.1). The results are compared with the models. According to previous research, properties of visco-elastic dampers are proportional to damper size (Soong and Dargush 1997; Parulekar and Reddy 2009). In this study, size of visco-elastic damper connecting the test models is scaled by a factor of 2439 (the stiffness ratio of the prototype building to that of the model) to comply with the similitude law. The stiffness and damping of the visco-elastic damper are shown in Table 6.19. Properties of the base isolators are similarly scaled as shown in Table 6.20.
Table 6.19 Properties of the visco-elastic damper

<table>
<thead>
<tr>
<th>Properties</th>
<th>Visco-elastic damper connecting two models</th>
<th>Visco-elastic damper connecting two buildings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness $k_1$ (kN/m)</td>
<td>$3.4264 \times 10^4$</td>
<td>$8.3521 \times 10^4$</td>
</tr>
<tr>
<td>Damping $c_1$ (Ns/m)</td>
<td>$7.637 \times 10^6$</td>
<td>$1.8626 \times 10^9$</td>
</tr>
<tr>
<td>Stiffness $k_2$ (kN/m)</td>
<td>$1.8809 \times 10^4$</td>
<td>$4.5875 \times 10^4$</td>
</tr>
<tr>
<td>Damping $c_2$ (Ns/m)</td>
<td>$1.6668 \times 10^4$</td>
<td>$4.0653 \times 10^6$</td>
</tr>
</tbody>
</table>

Table 6.20 Properties of the base isolator

<table>
<thead>
<tr>
<th>Properties</th>
<th>Base isolator under the right model</th>
<th>Base isolator under the right building</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness $k_1$ (kN/m)</td>
<td>$1.35329 \times 10^2$</td>
<td>$3.301 \times 10^3$</td>
</tr>
<tr>
<td>Damping ratio</td>
<td>5%</td>
<td>5%</td>
</tr>
</tbody>
</table>

The response of the models and that of the prototype buildings are compared in Figure 6.21 to Figure 6.27. It is found the response of the prototype buildings is similar to that of the models.

Table 6.21 shows the difference between the displacement of the left model tested in Section 6.7 and that of the left building simulated using Equation (7.1). The maximum difference is 7.64%.

Table 6.21 Difference between the response of the left model (coupled with the right model in base-isolated configuration) and that of the left prototype building (coupled with right building base-isolated configuration)

<table>
<thead>
<tr>
<th>Earthquakes</th>
<th>Response</th>
<th>(A) Test</th>
<th>(B) Simulation</th>
<th>$\frac{(A) - (B)}{15} \times 100%$ (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro</td>
<td>Maximum displacement</td>
<td>3.04 mm</td>
<td>45.42 mm</td>
<td>0.30 %</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-4.15 mm</td>
<td>-61.36 mm</td>
<td>1.33 %</td>
</tr>
<tr>
<td>Hachinohe</td>
<td>Maximum displacement</td>
<td>3.92 mm</td>
<td>56.19 mm</td>
<td>4.49 %</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-3.59 mm</td>
<td>-54.30 mm</td>
<td>-0.87 %</td>
</tr>
<tr>
<td>Kobe</td>
<td>Maximum displacement</td>
<td>5.92 mm</td>
<td>81.96 mm</td>
<td>7.64 %</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-4.10 mm</td>
<td>-63.10 mm</td>
<td>-2.67 %</td>
</tr>
</tbody>
</table>

Table 6.22 shows the difference between the displacement of the right model tested in Section 6.7 and that of the right building simulated using Equation (7.1). The
difference is between -12.53% and 5.50%.

Table 6.22 Difference between the response of the right model (coupled with the left model) in base-isolated configuration and that of the right prototype building (coupled with left building) in base-isolated configuration

<table>
<thead>
<tr>
<th>Earthquakes</th>
<th>Response</th>
<th>(A) Test</th>
<th>(B) Simulation</th>
<th>((A) - \frac{(B)}{15} \times 100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro</td>
<td>Maximum displacement</td>
<td>5.36 mm</td>
<td>81.63 mm</td>
<td>-1.62%</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-4.51 mm</td>
<td>-69.05 mm</td>
<td>-2.14%</td>
</tr>
<tr>
<td>Hachinohe</td>
<td>Maximum displacement</td>
<td>3.68 mm</td>
<td>54.05 mm</td>
<td>1.98%</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-3.95 mm</td>
<td>-61. mm</td>
<td>-3.18%</td>
</tr>
<tr>
<td>Kobe</td>
<td>Maximum displacement</td>
<td>8.24 mm</td>
<td>139.10 mm</td>
<td>-12.53%</td>
</tr>
<tr>
<td></td>
<td>Minimum displacement</td>
<td>-6.75 mm</td>
<td>-95.67 mm</td>
<td>5.50%</td>
</tr>
</tbody>
</table>

6.9 Discussion

Table 6.23 and Table 6.24 show the change in the frequency and damping ratio of the left model and the right model after the installation of the connecting damper (either the fluid damper or the visco-elastic damper), respectively. When the models are fixed to the shaking table, the damping ratio is beneficially increased by the installation of the connecting damper. However, the modal frequency is adversely increased. Thus, the main mechanism of reducing the response of the two models by coupling them together is increasing their damping to improve capability of energy absorption.

When the left model is connected to the right model in base-isolated configuration by the visco-elastic damper, the fundamental frequency of the two models is decreased and the damping ratio is beneficially increased as well. Both the shift in the frequency and the increase in the damping ratio contribute to the reduction in the response of the two models. Therefore, significant reduction is achieved.
Table 6.23 Change in the frequency and damping ratio of the left model after the installation of the connecting damper

<table>
<thead>
<tr>
<th>Mode</th>
<th>Coupled with the right model in fixed-base condition</th>
<th>Coupled with the right model in base-isolated configuration by the visco-elastic damper</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>By the fluid damper</td>
<td>By the visco-elastic damper</td>
</tr>
<tr>
<td>Frequency</td>
<td>Damping ratio</td>
<td>Frequency</td>
</tr>
<tr>
<td>1st</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>2nd</td>
<td>−</td>
<td>↑</td>
</tr>
</tbody>
</table>

Note: ↑: increase
↑: decrease
−: unchanged

Table 6.24 Change in the frequency and damping ratio of the right model after the installation of the connecting damper

<table>
<thead>
<tr>
<th>Mode</th>
<th>Fixed-base</th>
<th>Base isolated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coupled by the fluid damper</td>
<td>Coupled by the visco-elastic damper</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>Damping ratio</td>
</tr>
<tr>
<td>1st</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>2nd</td>
<td>↑</td>
<td>↑</td>
</tr>
</tbody>
</table>

6.10 Chapter summary

To investigate the feasibility of mitigating the response of the buildings with similar number of stories (or period of vibration) by coupling method using passive dampers, two scaled steel models have been constructed for experimental investigation. The two models were firstly fixed to the shaking table and were subjected to random excitation to identify the properties of the models. The two models were then excited by three earthquake records in the absence of connecting dampers, coupled by a fluid damper and coupled by a visco-elastic damper separately. After the installation of the fluid damper, both the fundamental frequencies and the first mode damping ratios of
the two models (in fixed-base conditions) are slightly increased. Reduction in the maximum response to shaking table excitations is limited.

When a visco-elastic damper is installed between two models in fixed-base conditions, the response of the two models to shaking table excitation is not considerably controlled.

When one of the models is base-isolated, significant reduction can be achieved by the installation of the visco-elastic damper between the two models. The maximum dynamic response and the root-mean-square response of the left model are reduced by at least 24.8% and 49.5%, respectively. The acceleration response of the right model is mitigated by more than 34.6%. Like typical base isolated buildings, the displacement response of the isolated right model may not be decreased. In this study, the base isolators produce more than 63.4% of the total displacements of the right model under the respective earthquake records.

Numerical models will be further developed in the next chapter to evaluate the performance of the proposed seismic mitigation system.
Figure 6.1 Experimental models
Figure 6.2 The power spectral density of the acceleration of the left model
Figure 6.3 The power spectral density of the acceleration of the right model

(a) Right model in fixed-base condition, uncoupled

(b) Right model in fixed-base condition coupled with the left model by fluid damper

(c) Right model in base isolated configuration coupled with the left model by visco-elastic damper
(a) Models coupled by the fluid damper

(b) The fluid damper between two test models

Figure 6.4 Models coupled by the fluid damper
Figure 6.5 Comparison between the top floor displacement of the left model and that of the left building under El Centro earthquake before and after the installation of the fluid damper
Figure 6.6 Comparison between the top floor displacement of the left model and that of the left building under Hachinohe earthquake before and after the installation of the fluid damper
Figure 6.7 Comparison between the top floor displacement of the left model and that of the left building under Kobe earthquake before and after the installation of the fluid damper
Figure 6.8 Comparison between the top floor displacement of the right model and that of the right building (in fixed-base condition) under El Centro earthquake before and after the installation of the fluid damper.
Figure 6.9 Comparison between the top floor displacement of the right model and that of the right building (in fixed-base condition) under Hachinohe earthquake before and after the installation of the fluid damper
Figure 6.10 Comparison between the top floor displacement of the right model and that of the right building (in fixed-base condition) under Kobe earthquake before and after the installation of the fluid damper
Figure 6.11 The left model is coupled with the right model in base-isolated configuration by visco-elastic damper

(a) Damper fabrication  (b) Test setup  (c) Damper installed between models

Figure 6.12 Visco-elastic dampers
Figure 6.13 Force-displacement relationship of the visco-elastic damper at different frequencies

Figure 6.14 Visco-elastic damper model
Figure 6.15 Base isolator (mm)

Figure 6.16 Comparison of the top floor displacement of the left model under El Centro earthquake

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Figure 6.17 Comparison of the top floor displacement of the left model under Hachinohe earthquake
Figure 6.18 Comparison of the top floor displacement of the left model under Kobe earthquake
Figure 6.19 Comparison of the displacement of the right model under El Centro earthquake
Figure 6.20 Comparison of the displacement of the right model under Hachinohe earthquake
Figure 6.21 Comparison of the displacement of the right model under Kobe earthquake
Figure 6.22 Comparison between the top floor displacement of the left model and that of the left building under El Centro earthquake before and after the installation of the visco-elastic damper
Figure 6.23 Comparison between the top floor displacement of the left model and that of the left building under Hachinohe earthquake before and after the installation of the visco-elastic damper
Figure 6.24 Comparison between the top floor displacement of the left model and that of the left building under Kobe earthquake before and after the installation of the visco-elastic damper
Figure 6.25 Comparison between the top floor displacement of the right model and that of the right building under El Centro earthquake before and after the installation of the visco-elastic damper and base isolators
Figure 6.26 Comparison between the top floor displacement of the right model and that of the right building under Hachinohe earthquake before and after the installation of the visco-elastic damper and base isolators.
Figure 6.27 Comparison between the top floor displacement of the right model and that of the right building under Kobe earthquake before and after the installation of the visco-elastic damper and base isolators
7.1 Introduction

The experimental studies of retrofitting existing buildings by visco-elastic dampers and base isolators have been presented in Chapter 6. When a fixed model is connected to a base isolated model, the visco-elastic dampers have positive contribution in reducing the response. As the properties of the visco-elastic dampers can have significant effect on the response, it is necessary to thoroughly investigate the effectiveness of the visco-elastic dampers through extensive numerical studies.

In this chapter, numerical studies are conducted to supplement the previous investigations in Chapter 6. The study starts with the establishment of the analytical model of buildings coupled by visco-elastic dampers. Equations of motions are derived. Parametric studies are carried out to investigate the effect of varying the properties of the visco-elastic dampers and base isolators on response of adjacent buildings retrofitted by visco-elastic dampers and base isolators. To further investigate the feasibility of the retrofitting strategy, the commonly used Kanai-Tajimi spectral density function is used to excite the coupled buildings in the frequency domain. The effect of the visco-elastic dampers on the power spectral density (PSD) function of the response of both buildings is evaluated. The standard deviation of the response of the adjacent buildings with and without retrofitting is then compared to estimate the effectiveness of the visco-elastic dampers.


7.2 Selection of base isolators

A building can be protected by different seismic isolation systems which include passive, active and semi-active isolation systems. A passive seismic isolation system usually means a combination of passive isolators and passive dampers. An active or a semi-active seismic isolation system indicates either a set of isolators with changeable characteristics (e.g. stiffness controllable or friction controllable isolators) or a combination of passive isolators and active and semi-active control devices (e.g. electro-rheological dampers, magneto-rheological dampers) which can provide mitigation forces according to feedback signals.

Passive isolation systems have been applied in newly constructed buildings (e.g. Nursing Tower of San Bernardino County Medical Center (Asher, et al. 1996)) or to retrofit existing buildings (e.g. Los Angeles City Hall (Youssef 1996)). They usually consist of one or several different types of isolators with passive energy dissipation devices (e.g. steel hysteretic dampers and viscous fluid dampers). There are many applications in Japan, United States and China. For instance, the base isolation system in the Nursing Tower of San Bernardino County Medical Center comprises high damping rubber bearings, natural rubber bearings and viscous dampers installed at the base of the Medical Center (Asher, et al. 1996). The lateral stiffness of the high damping and linear rubber bearings is relatively small and thus the fundamental period of vibration of the Medical Center is relatively large. The high damping rubber bearings and viscous dampers increase the damping of the base isolation system. The latter serves to control the overall building displacement. Another example of a building retrofitted by passive isolation systems is Los Angeles City Hall (Youssef 1996). In this building, a passive seismic isolation system (comprising
high damping rubber bearings, viscous fluid dampers and sliding bearings) is applied to protect the superstructure from strong ground motions.

Although passive seismic isolation can protect structures from severe seismic forces, an isolation system designed for a certain earthquake will not be optimal for another earthquake and vice versa (Yoshioka, et al. 2002). On the other hand, active seismic isolation systems, due to its adaptive nature offering a wide range of characteristics, are generally able to provide desired seismic performance when subjected to earthquakes (Chang and Spencer 2010). An active isolation system comprises three types of elements: sensors, actuators, and a controller with a predetermined control algorithm (Feng 1993; Yang, et al. 1996; Madden, et al. 2002). Feng (1993) introduced a friction-controllable sliding system to protect buildings. A variable friction force is produced by the change of the pressure according to control signals in response to displacement and acceleration measurements. Numerical analysis indicates that this system is effective for earthquake protection with a broad range of intensity. Active seismic isolation system with variable stiffness has also been studied (Yang, et al. 1996). Simulation results have indicated that the control method is viable and is effective in reducing the inter-storey drifts of seismic excited buildings.

In order to control an active isolation system, a large amount of energy is required. This drawback confines the application of active isolation systems. Besides, the control of an active isolation system is not stable if the control method is properly adopted or if control system malfunctions (Lu, et al. 2008a). As an alternative, semi-active isolation systems have been proposed (e.g. passive isolator in combination with semi-active stiffness damper or semi-active electromagnetic friction damper) to reduce structural response. Lu, et al. (2008a) proposed a sliding base isolation system with variable stiffness which can control the restoring force provided by the system.
by a semi-active control strategy. Simulation results have shown that the low-frequency resonance is significantly mitigated by the proposed system. Thus, both displacement and acceleration response can be mitigated. Kim, et al. (2006a) combined friction pendulum bearings with magnetorheological dampers into hybrid isolation system. Their study has shown that the hybrid system can robustly control the vibration of structures undergoing a wide variety of seismic loads.

Although theoretical and experimental studies have demonstrated the efficiencies of active and semi-active seismic isolation systems, these two types of isolation systems are expensive. Thanks to their popularity, natural rubber bearings (NRBs) and lead rubber bearings (LRBs) are selected as isolation devices in this chapter.

7.3 Equations of motion

As shown in Figure 7.1, the two buildings considered in this chapter are a left building with fixed base and a right building with base isolators. While the story number of the left building is \( j \), the right building has \( p \) stories. The two buildings are connected by visco-elastic dampers at each and every floor. The base isolators are designed to protect the right building and provide large difference in lateral stiffness between the adjacent buildings. The base isolation system consists of NRBs, LRBs or a combination of these two types of bearings.

7.3.1 Equations of motion of the coupled system

Figure 7.1(b) shows the simplified shear models of the building group. It is assumed that the ground acceleration applied to both buildings is the same and that spatial variations of the ground motion are neglected. The equations of motion (Chopra 2011) of such a structural system are developed in the form of:
\[ [M][\ddot{X}] + [C][\dot{X}] + [K][X] = -[M][I]g + [R]_{\text{dam}} + [R]_{\text{iso}} \]  
\text{(7.1)}

where \([\ddot{X}] = \begin{bmatrix} \ddot{X} \end{bmatrix}_l, [\dot{X}] = \begin{bmatrix} \dot{X} \end{bmatrix}_l\) and \([X] = \begin{bmatrix} X \end{bmatrix}_l\) are the respective acceleration vector, velocity vector and displacement vector.

\([M] = \begin{bmatrix} [M]_l \\ [M]_r \end{bmatrix}, [C] = \begin{bmatrix} [C]_l \\ [C]_r \end{bmatrix} \) and \([K] = \begin{bmatrix} [K]_l \\ [K]_r \end{bmatrix}\) are the respective mass matrix, damping matrix and stiffness matrix of the connected buildings. Subscripts \(l\) and \(r\) represent the left building and the right building, respectively.

\[ [M]_r = \begin{bmatrix} m_{r,\text{isolation}} \\ m_{r,1} \\ m_{r,2} \\ \vdots \\ m_{r,p} \end{bmatrix} \] is the mass matrix of the right building. \(m_{r,\text{isolation}}, m_{r,1}, \ldots m_{r,p}\) are the masses of the respective floors of the right building.

The mass matrix of the left building is \([M]_l = \begin{bmatrix} m_{l,1} \\ m_{l,2} \\ \vdots \\ m_{l,i} \end{bmatrix}\) where \(m_{l,1}, m_{l,2}, \ldots m_{l,i}\) are the masses of the left building.

\[ [K]_l = \begin{bmatrix} k_{l,1} + k_{l,2} & -k_{l,2} \\ -k_{l,2} & k_{l,2} + k_{l,3} \\ \vdots & \vdots & \ddots \\ k_{l,j-1} + k_{l,j} & -k_{l,j} & \cdots \\ -k_{l,j} & k_{l,j} \end{bmatrix} \] is the stiffness matrix of the left building. \(k_{l,1}, \ldots k_{l,j}\) are the lateral stiffness of the respective floors of the left building.
\[
[K]_r = \begin{bmatrix}
k_{r,1} & -k_{r,1} \\
-k_{r,1} & k_{r,1} + k_{r,2} \\
& \ddots \\
-k_{r,p-1} + k_{r,p} & -k_{r,p} \\
-k_{r,p} & k_{r,p}
\end{bmatrix}
\]
is the stiffness matrix of the right building. \( k_{r,1} \ldots k_{r,p} \) are the lateral stiffness of the respective floors of the right building. Note that lateral stiffness at isolation layer is 0.

As for the damping matrix \([C]_l\) of the left building, Rayleigh damping is assumed. Damping ratios of the first and second mode are assumed to be \( \zeta_l = 3\% \) for the left building. So the damping matrix of the left building is \([C]_l = \alpha_l[M]_l + \beta_l[K]_l\) in which
\[
\alpha_l = \frac{2\zeta_l \omega_{l,1} \omega_{l,2}}{(\omega_{l,1} + \omega_{l,2})}, \quad \beta_l = \frac{2\zeta_l}{(\omega_{l,1} + \omega_{l,2})},
\]
\(\omega_{l,1}\) and \(\omega_{l,2}\) are the circular frequencies of the first two modes of the left building. Rayleigh damping is also applied to right building above the isolation layer.

\([I]\) is a unit vector indicating the earthquake force location.

\(\ddot{a}_g\) is ground acceleration.

\([R]_{\text{dam}}\) is the force across connecting dampers.

\([R]_{\text{iso}}\) is the restoring forces contributed by base isolators.

### 7.3.2 Restoring forces at base isolation layer

Consistent with design specifications (Occhiuzzi, et al. 1994; GB50011-2010 2010), NRBs are modeled by the Kelvin model with equivalent stiffness \(k_{\text{NRB}}\) and equivalent damping ratio \(\zeta_{\text{NRB}}\). The restoring forces contributed by NRBs are
\[
f_{\text{NRB}} = -(k_{\text{NRB}} x_{\text{NRB}} + c_{\text{NRB}} \dot{x}_{\text{NRB}})
\]
(7.2)
where equivalent damping \( c_{\text{NRB}} = \frac{2\zeta_{\text{NRB}} k_{\text{NRB}}}{\omega_{\text{isolation}}} \) in which \( \omega_{\text{isolation}} \) is the 1st circular modal frequency of the isolated right building. \( \zeta_{\text{NRB}} \) is assumed to be 5%.

\( \dot{x}_{\text{NRB}} \) and \( x_{\text{NRB}} \) are the respective velocity and displacement of the natural rubber bearings.

LRBs are also applied to protect the right building. Lead cores of LRBs are designed to reduce the lateral displacement at isolation layer by adding energy absorbing capacity. The force-deformation relationship is represented by Bouc-Wen model (Nagarajaiah, et al. 1993; Deb 2004) which gives the restoring force of LRBs:

\[
f_{\text{LRB}} = - [k_{y,\text{LR}} x_{\text{LR}} + (1 - \alpha_{\text{LR}})k_{0,\text{LR}} z]
\]  

(7.3)

where \( k_{y,\text{LR}} \) and \( k_0 \) are the respective yield stiffness and initial stiffness of LRBs.

\( \alpha_{\text{LR}} = k_{y,\text{LR}} / k_{0,\text{LR}} \) is the stiffness ratio.

\( x_{\text{LR}} \) is the displacement of the lead rubber bearings.

\( z \) is a hysteretic dimensionless quantity. It satisfies the following relation (Nagarajaiah, et al. 1991):

\[
z = \frac{1}{d^y} [A_{\text{LR}} \dot{x}_{\text{LR}} - \gamma_{\text{LR}} z^2 \dot{x}_{\text{LR}} \text{sgn}(\dot{x}_{\text{LR}} z) - \beta_{\text{LR}} z^2 \dot{x}_{\text{LR}}]
\]  

(7.4)

where \( d^y \) is yield displacement and \( A_{\text{LR}} = 1, \gamma_{\text{LR}} = 0.9 \) and \( \beta_{\text{LR}} = 0.1 \) (Nagarajaiah, et al. 1991).

The restoring force vector is

\[
[R]_{\text{iso}} = [R]_{\text{NRB}} + [R]_{\text{LR}}
\]  

(7.5)

where \([R]_{\text{LR}}\) represents the total restoring force provided by LRBs:

\[
[R]_{\text{LR}} = [0 \ 0 \ \cdots \ 0 \ \sum f_{\text{LR}} \ 0 \ \cdots \ 0]^T
\]  

(7.6)
$[R]_{NRB}$ is the total restoring force contributed by NRBs:

$$[R]_{NRB} = [0 \ 0 \ \cdots \ 0 \ \sum f_{NRB} \ 0 \ \cdots \ 0]^T \quad (7.7)$$

All elements of vector $[R]_{isolator}$ are zeros except the element $j + 1$ equals to the total isolator restoring force.

Substitute Equation (7.2) into the above equation, Equation (7.7) becomes

$$[R]_{NRB} = -[C]_{NRB} [\dot{S}] - [K]_{NRB} [S] \quad (7.8)$$

where the damping matrix related to NRBs is

$$[C]_{NRB} =$$

$$\begin{bmatrix}
0 & 0 & \cdots & 0 \\
\vdots & & \ddots & \vdots \\
0 & \cdots & \ddots & \sum c_{NRB} \\
\vdots & \cdots & \ddots & 0 \\
0 & \cdots & \ddots & 0
\end{bmatrix} \quad (7.9)$$

Similarly, the stiffness matrix related to NRBs is

$$[K]_{NRB} =$$

$$\begin{bmatrix}
0 & 0 & \cdots & 0 \\
\vdots & & \ddots & \vdots \\
0 & \cdots & \ddots & \sum k_{NRB} \\
\vdots & \cdots & \ddots & 0 \\
0 & \cdots & \ddots & 0
\end{bmatrix} \quad (7.10)$$

Substitute Equation (7.5) and Equation (7.8) into Equation (7.1) gives

$$[M][\ddot{X}] + [C]^{(1)}[\dot{X}] + [K]^{(1)}[X] = -[M][I] \ddot{a}_g + [R]_{dam} + [R]_{LR} \quad (7.11)$$

where $[C]^{(1)} = [C] + [C]_{NRB}$ and $[K]^{(1)} = [K] + [K]_{NRB}$. 

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7.3.3 Connection forces

As demonstrated in previous chapter, a general mechanical model is used to simulate the visco-elastic dampers. Figure 7.2 shows the general mechanical model which comprises a Kelvin model and a Maxwell model. The connection force of the visco-elastic damper is

\[
\begin{align*}
    f_{rk} &= f_{rk,K} + f_{rk,M} \\
    f_{lk} &= f_{lk,K} + f_{lk,M}
\end{align*}
\]  

(7.12)

where \( f_{rk,K} = -f_{lk,K} \) are the forces in the Kelvin component.

In equation (7.12), the subscripts \( K \) and \( M \) represent the Kelvin model and the Maxwell model, respectively. The subscript \( k \) means floor number.

\[
\begin{align*}
    f_{rk,K} &= k_{k,K}(x_{rk} - x_{lk}) + c_{k,K}(\dot{x}_{rk} - \dot{x}_{lk}) \\
    f_{lk,K} &= k_{k,K}(x_{lk} - x_{rk}) + c_{k,K}(\dot{x}_{lk} - \dot{x}_{rk})
\end{align*}
\]  

(7.13)

where \( x_{rk} \) and \( x_{lk} \) are the respective displacement of the right building and the left building at floor \( k \). \( \dot{x}_{rk} \) and \( \dot{x}_{lk} \) are the velocity of the right building and the left building at floor \( k \), respectively.

As shown in Figure 7.2, \( f_{rk,M} = -f_{lk,M} \) is the force in Maxwell component at floor \( k \) (Soong and Dargush 1997):

\[
\begin{align*}
    \frac{\ddot{x}_{rk}}{k_{k,M}} + \frac{\ddot{x}_{rk}}{c_{k,M}} &= \dot{x}_{rk} - \dot{x}_{lk} \\
    \frac{\ddot{x}_{lk}}{k_{k,M}} + \frac{\ddot{x}_{lk}}{c_{k,M}} &= \dot{x}_{lk} - \dot{x}_{rk}
\end{align*}
\]  

(7.14)

Referring to Equation (7.12), assembling all the connection forces on the neighboring buildings at each floor level gives the connection force vector:

\[
[R]_{\text{dam}} = -[R]_K - [R]_M
\]  

(7.15)

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where \([R]_K\) and \([R]_M\) are the vector representing the forces contributed by the Kelvin component and Maxwell component.

\[
[R]_K = \begin{bmatrix} f_{1_1,K} & f_{1_2,K} & \cdots & f_{ij,K} & f_{r_1,K} & f_{r_2,K} & \cdots & f_{rp,K} \end{bmatrix}^T \quad (7.16-1)
\]

\[
[R]_M = \begin{bmatrix} f_{1_1,M} & f_{1_2,M} & \cdots & f_{ij,M} & f_{r_1,M} & f_{r_2,M} & \cdots & f_{rp,M} \end{bmatrix}^T \quad (7.16-2)
\]

According to Equation (7.13), Equation (7.16-1) can be rewritten as:

\[
[R]_K = -[K]_K [X] - [C]_K \dot{X} \quad (7.17)
\]

where \([K]_K\) and \([C]_K\) are respectively stiffness matrix and damping matrix related to Kelvin component.

\[
[K]_K = \begin{bmatrix} k_{1,1} & -k_{1,2} & \cdots & -k_{1,p} \\ k_{2,1} & k_{2,2} & \cdots & -k_{2,p} \\ \vdots & \vdots & \ddots & \vdots \\ -k_{p,1} & -k_{p,2} & \cdots & k_{p,1} \end{bmatrix} \quad (7.18)
\]

\[
[C]_K = \begin{bmatrix} c_{1,1} & -c_{1,2} & \cdots & -c_{1,p} \\ c_{2,1} & c_{2,2} & \cdots & -c_{2,p} \\ \vdots & \vdots & \ddots & \vdots \\ -c_{p,1} & -c_{p,2} & \cdots & c_{p,1} \end{bmatrix} \quad (7.19)
\]

Substituting Equation (7.15) and Equation (7.17) into Equation (7.11) yields

\[
[M][\ddot{X}] + [C]’[\dot{X}] + [K]'[X] = -[M][I]\ddot{a}_g - [R]_M + [R]_{LR} \quad (7.20)
\]

where \([C]’ = [C] = [C]_K + [C]_{NRB}\) and \([K]’ = [K] = [K]_K + [K]_{NRB}\).
7.3.4 Solution of equations of motion

The above equation may be rewritten in an incremental form:

\[
[M]\Delta \ddot{X} + [C]' \Delta \dot{\dot{X}} + [K]' \Delta \dot{X} = -[M][I] \Delta \ddot{a}_g - [R]_M + [R]_{LR} \tag{7.21}
\]

where \(\Delta \ddot{X} = \ddot{X}_{t_i} - \ddot{X}_{t_{i-1}}\), \(\Delta \dot{X} = \dot{X}_{t_i} - \dot{X}_{t_{i-1}}\), and \(\Delta X = X_{t_i} - X_{t_{i-1}}\) are the respective acceleration increment, velocity increment and displacement increment vector.

\(\Delta \ddot{a}_g = \ddot{a}_{g,t_i} - \ddot{a}_{g,t_{i-1}}\) is the ground acceleration increment vector between instants \(t_i\) and \(t_{i-1}\).

\(\Delta [R]_M = [R]_{M,t_i} - [R]_{M,t_{i-1}}\) is the Maxwell force increment vector.

\(\Delta [R]_{LR} = [R]_{LR,t_i} - [R]_{LR,t_{i-1}}\) is the LRB force increment vector.

To solve Equation (7.21), it is necessary to approximate the Maxwell forces and the LRB restoring forces which are followed below.

7.3.4.1 Approximation of Maxwell force

At instant \(t_{i-1}\) and instant \(t_i\), Equation (7.14) becomes

\[
\begin{align*}
\frac{\dot{f}_{rk,M,t_{i-1}}}{k_{rk,M}} + \frac{\dot{f}_{rk,M,t_{i-1}}}{c_{rk,M}} &= \ddot{x}_{rk,t_{i-1}} - \ddot{x}_{lk,t_{i-1}} \\
\frac{\dot{f}_{lk,M,t_{i-1}}}{k_{lk,M}} + \frac{\dot{f}_{lk,M,t_{i-1}}}{c_{lk,M}} &= \ddot{x}_{lk,t_{i-1}} - \ddot{x}_{rk,t_{i-1}} \\
\end{align*}
\tag{7.22}
\]

\[
\begin{align*}
\frac{\ddot{f}_{rk,M,t_i}}{k_{rk,M}} + \frac{\ddot{f}_{rk,M,t_i}}{c_{rk,M}} &= \dddot{x}_{rk,t_i} - \dddot{x}_{lk,t_i} \\
\frac{\ddot{f}_{lk,M,t_i}}{k_{lk,M}} + \frac{\ddot{f}_{lk,M,t_i}}{c_{lk,M}} &= \dddot{x}_{lk,t_i} - \dddot{x}_{rk,t_i} \\
\end{align*}
\tag{7.23}
\]

Combining Equation (7.22) and Equation (7.23) gives
\[
\begin{align*}
\frac{f_{rk,M,t_i-1} + f_{rk,M,t_i}}{k_{k,M}} + f_{rk,M,t_i-1} + f_{rk,M,t_i} \quad & = \left( \ddot{x}_{rk,t_i} - \ddot{x}_{lk,t_i} \right) + \left( \ddot{x}_{rk,t_i-1} - \ddot{x}_{lk,t_i-1} \right) \\
\frac{f_{ik,M,t_i-1} + f_{ik,M,t_i}}{k_{k,M}} + f_{ik,M,t_i-1} + f_{ik,M,t_i} \quad & = \left( \ddot{x}_{lk,t_i} - \ddot{x}_{rk,t_i} \right) + \left( \ddot{x}_{lk,t_i-1} - \ddot{x}_{rk,t_i-1} \right)
\end{align*}
\] (7.24)

Supposing the Maxwell force changes linearly between two small consecutive steps:

\[
\begin{align*}
\frac{f_{rk,M,t_i-1} + f_{rk,M,t_i}}{2} \quad & = \frac{f_{rk,M,t_i} - f_{rk,M,t_i}}{\Delta t} \\
\frac{f_{ik,M,t_i-1} + f_{ik,M,t_i}}{2} \quad & = \frac{f_{ik,M,t_i} - f_{ik,M,t_i}}{\Delta t}
\end{align*}
\] (7.25)

Substituting equation (7.25) into equation (7.24) yields

\[
\begin{align*}
\{ f_{rk,M,t_i} = p_1 f_{rk,M,t_i-1} + p_2 \left[ (\ddot{x}_{rk,t_i} - \ddot{x}_{lk,t_i}) + (\ddot{x}_{rk,t_i-1} - \ddot{x}_{lk,t_i-1}) \right] \\
\{ f_{ik,M,t_i} = p_1 f_{ik,M,t_i-1} + p_2 \left[ (\ddot{x}_{lk,t_i} - \ddot{x}_{rk,t_i}) + (\ddot{x}_{lk,t_i-1} - \ddot{x}_{rk,t_i-1}) \right]
\end{align*}
\] (7.26)

where \( p_1 = \frac{2c_{k,M} - \Delta t k_{k,M}}{2c_{k,M} + \Delta t k_{k,M}} \) and \( p_2 = \frac{\Delta t k_{k,M} c_{k,M}}{2c_{k,M} + \Delta t k_{k,M}} \).

### 7.3.4.2 Approximation of LRB restoring force

At time \( t_i \), Equation (7.4) becomes

\[
\ddot{z}_{t_i} = \frac{1}{\gamma} \left[ A_{LRB} \ddot{x}_{LRB,t_i} - \gamma_{LRB} z_{t_i}^2 \ddot{x}_{LRB,t_i} \text{sgn}(\ddot{x}_{LRB,t_i}) - \beta_{LRB} z_{t_i}^2 \ddot{x}_{LRB,t_i} \right] - \gamma_{LRB} z_{t_i}^2 \ddot{x}_{LRB,t_i}
\] (7.27)

where the subscript \( t_i \) represents time.

The Fourth Order Runge-Kutta Method (Press, et al. 2007) is used to estimate \( z_{t_i} \):

\[
z_{t_i} = z_{t_{i-1}} + \left[ \dot{z}_{t_i}^{(1)} + 2\dot{z}_{t_i}^{(2)} + 2\dot{z}_{t_i}^{(3)} + \dot{z}_{t_i}^{(4)} \right] \frac{\Delta t}{6}
\] (7.28)

where

\[
\begin{align*}
\dot{z}_{t_i}^{(1)} & = \frac{1}{\gamma} \left[ A_{LRB} \ddot{x}_{LRB,t_i} - \gamma_{LRB} z_{t_i}^2 \ddot{x}_{LRB,t_i} \text{sgn}(\ddot{x}_{LRB,t_i}) - \beta_{LRB} z_{t_i}^2 \ddot{x}_{LRB,t_i} \right] \\
\dot{z}_{t_i}^{(2)} & = \frac{1}{\gamma} \left[ A_{LRB} \ddot{x}_{LRB,t_i} - \gamma_{LRB} z_{t_i}^2 \ddot{x}_{LRB,t_i} \text{sgn}(\ddot{x}_{LRB,t_i}) - \beta_{LRB} z_{t_i}^2 \ddot{x}_{LRB,t_i} \right]
\end{align*}
\] (7.29)

\[
\begin{align*}
\{ \dot{z}_{t_i} & = z_{t_{i-1}} + \frac{\Delta t}{2} \dot{z}_{t_i}^{(1)} \\
\{ \ddot{x}_{LR} & = \ddot{x}_{LR,t_{i-1}} + (\ddot{x}_{LR,t_i} - \ddot{x}_{LR,t_{i-1}}) / 2 \\
\{ \dot{z}_{t_i}^{(2)} & = \frac{1}{\gamma} \left[ A_{LRB} \ddot{x}_{LR,t_i} - \gamma_{LR} (\dot{z}_{t_{i-1}})^2 \ddot{x}_{LR,t_{i-1}} \text{sgn}(\ddot{x}_{LR,t_{i-1}}) - \beta_{LR} (\dot{z}_{t_{i-1}})^2 \ddot{x}_{LR,t_{i-1}} \right]
\end{align*}
\] (7.30)
\[
\begin{align*}
\begin{cases}
z_{t_i}'' = z_{t_i-1} + \frac{\Delta t}{2} \ddot{z}_{t_i} \\
\dot{x}_{LR}^{(2)} = \dot{x}_{LR,t_i-1} + (\dot{x}_{LR,t_i} - \dot{x}_{LR,t_i-1})/2 \\
\ddot{z}_{t_i} = \frac{1}{d_y} \left[ A_{LR} \ddot{x}_{LR}^{(2)} - \gamma_{LR} (z_{t_i-1}'' \dot{x}_{LR}^{(2)}) \text{sgn}(\dot{x}_{LR}^{(2)} z_{t_i-1}'') - \beta_{LR} (z_{t_i-1}'')^{2} \dot{x}_{LR}^{(2)} \right]
\end{cases}
\end{align*}
\] (7.31)

\[
\begin{align*}
\begin{cases}
z_{t_i-1} = z_{t_i-1} + \Delta t \ddot{z}_{t_i}^{(3)} \\
\dot{z}_{t_i}^{(4)} = \frac{1}{d_y} \left[ A_{LR} \dot{x}_{LR,t_i} - \gamma_{LR} (z_{t_i-1}'' \dot{x}_{LR,t_i}) \text{sgn}(\ddot{x}_{LR}^{(2)} z_{t_i-1}'') - \beta_{LR} (z_{t_i-1}'')^{2} \dot{x}_{LR,t_i} \right]
\end{cases}
\end{align*}
\] (7.32)

With Equation (7.26) and Equation (7.28), Equation (7.20) can be solved by NEWMARK method in combination with NEWTON-RAPHSON method (Newmark 1959; Datta 2010) to compute the response of the coupled system. Details of the iteration are not stated in this study since this combination is commonly used. During the NEWTON-RAPHSON iteration, assume the force error at iteration \( j \) is \( e_t = \{ [R]_{M,j} - [R]_{M,j-1} \}^{T} \times \{ [R]_{M,j} - [R]_{M,j-1} \} + (f_{LRB,j} - f_{LRB,j-1})^2 \) where \([R]_{M,j-1}\) and \([R]_{M,j-1}\) are the forces in the Maxwell component at iteration \( j \) and iteration \( j - 1 \), respectively. \( f_{LRB,j} \) and \( f_{LRB,j-1} \) are the restoring forces provided by LRBs at iteration \( j \) and iteration \( j - 1 \), respectively. Convergence is achieved when the force error is less than \( 1 \times 10^{-5} \) N.

### 7.4 An example of application

#### 7.4.1 Properties of buildings

The example comprises a nine-story moment resisting frame (the left building) and a nine-story moment resisting frame (the right building), both of which are common in Hong Kong. When the two buildings are not connected by dampers and the right building is without seismic isolation, fundamental periods of the left building and the right building are 0.886 s and 0.766 s respectively. Details of the two buildings are shown in Table 7.1. These two buildings are connected by visco-elastic dampers at
every floor level of the right building, as shown in Figure 7.1.

Table 7.1 Floor mass and floor stiffness of the two buildings

<table>
<thead>
<tr>
<th>Building</th>
<th>Mass (ton)</th>
<th>Stiffness (MN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left building</td>
<td>1.278×10^3</td>
<td>2.347×10^3</td>
</tr>
<tr>
<td>Right building</td>
<td>1.143×10^3</td>
<td>2.258×10^3</td>
</tr>
</tbody>
</table>

In respective of the base isolation system, three different cases as shown in Table 7.2 are considered. In case 1 and case 2, only NRBs or LRBS are used, respectively. In case 3, the base isolation system consists of 18 NRBs and 18 LRBS. In all three cases, capacities of the base isolation systems are larger than the weight of the right building at 99.305 MN.

Table 7.2 Mechanical properties of isolators

<table>
<thead>
<tr>
<th>Isolator type</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isolator No.</td>
<td>RB-700</td>
<td>RB-700</td>
<td>LRB-700</td>
</tr>
<tr>
<td>Equivalent stiffness (MN/m)</td>
<td>0.742</td>
<td>1.205</td>
<td>1.205</td>
</tr>
<tr>
<td>Initial stiffness (MN/m)</td>
<td>-</td>
<td>-</td>
<td>9.060</td>
</tr>
<tr>
<td>Post-elastic stiffness (MN/m)</td>
<td>-</td>
<td>-</td>
<td>0.755</td>
</tr>
<tr>
<td>Yield force (kN)</td>
<td>-</td>
<td>-</td>
<td>90.0</td>
</tr>
<tr>
<td>Capacity (MN)</td>
<td>3.839</td>
<td>3.839</td>
<td>3.848</td>
</tr>
<tr>
<td>Total capacity (MN)</td>
<td>138.2</td>
<td>138.4</td>
<td>138.5</td>
</tr>
</tbody>
</table>

7.4.2 Ground motion

The adjacent buildings are assumed to be located in a medium-stiff soil area. As shown in Table 7.3, seven earthquake records are applied in accordance with the Chinese Seismic Design Code (GB50011-2010 2010). Peak ground accelerations of the earthquake records are scaled to 1.5 m/s^2 and 3.1 m/s^2 representing moderate earthquakes and rare earthquakes, respectively.
Table 7.3 Earthquake records

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Station</th>
<th>Component</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imperial Valley</td>
<td>El Centro</td>
<td>180direction</td>
<td>1940</td>
</tr>
<tr>
<td>Tokachi-Oki</td>
<td>Hachinohe</td>
<td>EW</td>
<td>1968</td>
</tr>
<tr>
<td>Kobe</td>
<td>Takarazuka</td>
<td>TAZ090</td>
<td>1995</td>
</tr>
<tr>
<td>Michoacan</td>
<td>La Union, Mexico</td>
<td>180direction</td>
<td>1985</td>
</tr>
<tr>
<td>Hollister</td>
<td>Hollister City Hall</td>
<td>HCH271direction</td>
<td>1974</td>
</tr>
<tr>
<td>ChiChi</td>
<td>CHY036</td>
<td>EW</td>
<td>1999</td>
</tr>
<tr>
<td>Big Bear</td>
<td>San Bernardino-E &amp;Hospitality</td>
<td>HOS180</td>
<td>1992</td>
</tr>
</tbody>
</table>

7.4.3 Effect of visco-elastic dampers

The main purpose of applying connecting dampers is to improve the response of existing buildings from seismic behaviour. The level of response reduction depends on the performance of the connecting dampers. According to previous research, damper properties are proportional to $r_{VE} = A_{VE}/t_{VE}$ which is the ratio of the visco-elastic damper area $A_{VE}$ to the thickness $t_{VE}$ of the visco-elastic damper (Soong and Dargush 1997; Parulekar and Reddy 2009). By varying the ratio $r_{VE}$, the stiffness and damping of the connecting dampers can be changed. Referring to Table 6.5, the four parameters of the visco-elastic damper at any ratio $r_{VE}$ are shown in Table 7.4.

Table 7.4 Properties of connecting dampers

<table>
<thead>
<tr>
<th>Stiffness $k_1$ (MN/m)</th>
<th>Damping $c_1$ (kNs/m)</th>
<th>Stiffness $k_2$ (MN/m)</th>
<th>Damping $c_2$ (kNs/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9137$r_{VE}$</td>
<td>20.37$r_{VE}$</td>
<td>0.5016$r_{VE}$</td>
<td>44.45$r_{VE}$</td>
</tr>
</tbody>
</table>

The maximum inter-story drifts at various damper areas are used to calculate the optimum damper. Figure 7.4 and Figure 7.5 show the variation of the average of the maximum inter-story drifts and the average of the maximum RMS inter-story drifts against $r_{VE}$. The following are observed:

198
(I) In Case 1 (i.e. the base isolation system comprises 36 NR-700s, as shown in Table 7.2), when the coupled buildings are excited by seven earthquake records, if $r_{VE} < 25.2$ m, the maximum drift of the left building decreases by increasing $r_{VE}$. Vice versa, if $r_{VE} > 25.2$ m, increasing $r_{VE}$ significantly increases the maximum drift of the left building. The optimum ratio $r_{VE}$ for the left building is 25.2 m. The variation of the average maximum RMS drifts against $r_{VE}$ is similar to the maximum drifts against $r_{VE}$. The optimum ratio $r_{VE}$ is 25.2m.

(II) In Case 1, the connecting dampers have an insignificant effect on the maximum drifts of the right isolated building when $r_{VE}$ is less than the threshold value at 31.8m. Further increasing the ratio $r_{VE}$ above the threshold value will increase the maximum inter-story drift of the right isolated building.

(III) In Case 2 and Case 3 (i.e. the base isolation system comprises LRBs, as mentioned in Table 7.2), the variation of the maximum drift against $r_{VE}$ is similar to the preceding case. As shown in Figure 7.4, the maximum drifts of the right isolated building in Case 2 and Case 3 are larger than the corresponding maximum drifts in Case 1. This indicates that the LRBs in Case 2 and in Case 3 induce larger structural response. This is probably due to the equivalent stiffness of the LRBs being larger than that of the same size NRs. Accordingly, the fundamental period of the middle frame in Case 2 is smaller than that in Case 1.

As the use of LRBs causes larger seismic response, the isolation system is designed to comprise of 36 NRs (i.e. Case 1) with $r_{VE}$ chosen to be 25.2m. The use of NRs is economic since a LRB is more expensive than a NRB. The response of the right isolated building is barely affected by the connecting damper at the optimum ratio.
7.4.4 Response comparison

Table 7.5 and Table 7.6 compare the average of the maximum response of the left building before and after being connected to the right building in base isolated configuration by visco-elastic dampers. Under moderate earthquakes, the average maximum response and the average of the maximum RMS response of the left building are mitigated by at least 30% and 42%, respectively. Excited by rare earthquakes, the visco-elastic dampers are more effective in reducing seismic response of the left building. More than 40% and 51% reduction in the average of the maximum drift and the maximum RMS drift can be achieved respectively.

Figure 7.6 (a) shows the displacement time history of the left building without being connected to the right building. Figure 7.6 (b) presents the displacement of the left building when connected to the right building in base isolated configuration. It is observed that massive reduction in the displacement of the left building is achieved by connecting to the right building in base isolated configuration. This indicates that the connecting dampers are effective in improving the performances of the left building.

Table 7.5 Response of the left building under moderate earthquakes

<table>
<thead>
<tr>
<th>Average response</th>
<th>Uncoupled (mm)</th>
<th>Coupled (mm)</th>
<th>Response ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum displacement</td>
<td>71.79</td>
<td>42.13</td>
<td>58.68</td>
</tr>
<tr>
<td>Maximum RMS displacement</td>
<td>15.42</td>
<td>7.58</td>
<td>49.16</td>
</tr>
<tr>
<td>Maximum drift</td>
<td>12.09</td>
<td>8.42</td>
<td>69.63</td>
</tr>
<tr>
<td>Maximum RMS drift</td>
<td>2.57</td>
<td>1.48</td>
<td>57.60</td>
</tr>
</tbody>
</table>
Table 7.6 Response of the left building under rare earthquakes

<table>
<thead>
<tr>
<th>Average response</th>
<th>Uncoupled (mm)</th>
<th>Coupled (mm)</th>
<th>Response ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum displacement</td>
<td>95.32</td>
<td>49.43</td>
<td>51.86</td>
</tr>
<tr>
<td>Maximum RMS displacement</td>
<td>17.28</td>
<td>9.56</td>
<td>55.34</td>
</tr>
<tr>
<td>Maximum drift</td>
<td>20.22</td>
<td>12.03</td>
<td>59.48</td>
</tr>
<tr>
<td>Maximum RMS drift</td>
<td>4.68</td>
<td>2.25</td>
<td>48.08</td>
</tr>
</tbody>
</table>

Table 7.7 and Table 7.8 give the average of the maximum response of the right building under moderate earthquakes and rare earthquakes respectively. The average of the maximum drift of the base isolated right building (connected to the left building) is mitigated by more than 91% in comparison with that of the right building in fixed-base condition. When the base isolated right building is connected to the left building, the response of the isolated right building is also mitigated compared with that of the isolated right building without being connected to the left building. Table 7.7 shows that the displacement of the right building in base isolated configuration is larger than that of the right building in fixed-base condition. However, Figure 7.7 shows that the displacement of the right building in base isolated configuration mainly occurs at isolation layer which is normal for base isolated buildings.

Table 7.7 Response of the right building under moderate earthquakes

<table>
<thead>
<tr>
<th>Average response</th>
<th>(A) Fixed, uncoupled (mm)</th>
<th>(B) Isolated, coupled (mm)</th>
<th>(\frac{(B)}{(A)} \times 100%)</th>
<th>Isolated, uncoupled (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum displacement</td>
<td>59.00</td>
<td>74.52</td>
<td>126.30 %</td>
<td>100.27</td>
</tr>
<tr>
<td>Maximum RMS displacement</td>
<td>10.84</td>
<td>14.40</td>
<td>132.88 %</td>
<td>23.57</td>
</tr>
<tr>
<td>Maximum drift</td>
<td>10.76</td>
<td>0.96</td>
<td>8.88 %</td>
<td>1.06</td>
</tr>
<tr>
<td>Maximum RMS drift</td>
<td>1.94</td>
<td>0.15</td>
<td>7.89 %</td>
<td>0.24</td>
</tr>
</tbody>
</table>
### Table 7.8 Response of the right building under rare earthquakes

<table>
<thead>
<tr>
<th>Average response</th>
<th>Fixed, uncoupled (mm)</th>
<th>Isolated, coupled (mm)</th>
<th>Response ratio (%)</th>
<th>Isolated, uncoupled (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum displacement</td>
<td>96.19</td>
<td>85.57</td>
<td>88.96</td>
<td>207.23</td>
</tr>
<tr>
<td>Maximum RMS displacement</td>
<td>19.76</td>
<td>17.52</td>
<td>88.66</td>
<td>48.71</td>
</tr>
<tr>
<td>Maximum drift</td>
<td>17.90</td>
<td>1.00</td>
<td>5.59</td>
<td>2.18</td>
</tr>
<tr>
<td>Maximum RMS drift</td>
<td>3.33</td>
<td>0.17</td>
<td>5.12</td>
<td>0.50</td>
</tr>
</tbody>
</table>

### 7.5 Response assessment by power spectral density functions

The power spectral density function of response can indicate the distribution of response at different frequencies (Lutes and Sarkani 2003). Using the power spectral density function, further insight into the efficiency of the coupling strategy is provided. In the past decades, several spectral density functions of the ground acceleration have been proposed. This study adopts the Kanai-Tajimi spectral density function which has been commonly used to in the study of buildings (Tajimi 1960). Details of the Kanai-Tajimi spectral density function is given in Chapter 5.

#### 7.5.1 Power spectral density function of displacement

Taking Fourier transform on both sides of Equation (7.14) yields

\[
\begin{align*}
F_{l,k,M}(\omega) &= \frac{j\omega k^2_{k,M}c_{k,M}^2 + \omega^2 k_{k,M} \xi^2_{l,M}}{k^2_{l,M} + \omega^2 \xi^2_{l,M}} [X_{l,k}(\omega) - X_{r,k}(\omega)] \\
F_{r,k,M}(\omega) &= \frac{j\omega k^2_{k,M}c_{k,M}^2 + \omega^2 k_{k,M} \xi^2_{l,M}}{k^2_{r,M} + \omega^2 \xi^2_{r,M}} [X_{r,k}(\omega) - X_{l,k}(\omega)]
\end{align*}
\]

(7.33)

Referring to Equation (7.16-2), the Maxwell force vector \([R]_M\) in the frequency domain is

\[
F_M(\omega) = [F_{11,M}(\omega) \ldots F_{l,k,M}(\omega) \ldots F_{r1,M}(\omega) \ldots F_{rq,M}(\omega)]^T
\]

(7.34)
Substituting Equation (7.33) into the above equation yeilds

\[ F_M(\omega) = [K]_M X(\omega) \]  \hspace{1cm} (7.35)

where \( X(\omega) = [X_{11}(\omega) \ldots X_{lk}(\omega) \ldots X_{r1}(\omega) \ldots X_{rk}(\omega) \ldots X_{rp}(\omega)]^T \)

is the displacement response in the frequency domain.

\[ [K]_M = \begin{bmatrix}
  k_{1,M} & \cdots & -k_{1,M} \\
  \vdots & \ddots & \vdots \\
  -k_{1,M} & \cdots & k_{k,M} \\
  \vdots & \ddots & \vdots \\
  -k_{k,M} & \cdots & -k_{p,M} \\
  -k_{p,M} & \cdots & k_{p,M}
\end{bmatrix} \]

is the stiffness matrix related the Maxwell component in the frequency domain. The

stiffness \( k_{k,M} = \frac{i\omega k_{k,M}c_{k,M} + \omega^2 k_{k,M}c_{k,M}^2}{k_{k,M} + \omega^2 c_{k,M}^2} \) at story \( k \) is related to excitation frequencies.

When all base isolators are NRBs, Equation (7.21) may be written into the following

form:

\[ [M][\ddot{X}] + [C] '[X] + [K]' [X] = -[M][I]\ddot{a}_g - [R]_M \]  \hspace{1cm} (7.36)

The Fourier transform of Equation (7.36) is

\[ \{ -\omega^2[M] + i\omega[C]' + [K]' \} X(\omega) = -[M][I]A_g(\omega) - F_M(\omega) \]  \hspace{1cm} (7.37)

where \( i \) is the imaginary unit and \( A_g(\omega) \) is ground acceleration in the frequency

domain.

Substituting Equation (7.35) into Equation (7.37) leads to

\[ X(\omega) = [H_0(\omega)]A_g(\omega) \]  \hspace{1cm} (7.38)

where the transfer function from ground acceleration to displacement is
\[ [H_0(\omega)] = \frac{-[M][i]}{-\omega^2[M]+i\omega[C]+[\kappa]^T+[\kappa]M} \]  

(7.39)

As a result, the displacement power spectral density (Newland 2012) is

\[ S_{XX}(\omega) = [H_0(\omega)]^T[H_0(\omega)] S_g(\omega) \]  

(7.40)

where the superscript T and * are the respective matrix transpose and complex conjugate. Autospectral displacement response densities are the diagonal elements of \( S_{XX}(\omega) \). Under the Kanai-Tajimi spectral density excitation, the autospectral density of the displacement response of the left building at its top floor level is shown in Figure 7.8(a).

### 7.5.2 Power spectral density function of inter-story drift

Inter-story drift response in the frequency domain is

\[
[D(\omega)] = \begin{bmatrix} X_{l1}(\omega) \\
X_{l2}(\omega) \\
\vdots \\
X_{l(k-1)}(\omega) \\
X_{r1}(\omega) \\
X_{r2}(\omega) \\
\vdots \\
X_{r(k-1)}(\omega) \\
X_{rp}(\omega) \end{bmatrix} - \begin{bmatrix} 0 \\
X_{l1}(\omega) \\
\vdots \\
X_{l(k-1)}(\omega) \\
0 \\
X_{r1}(\omega) \\
\vdots \\
X_{r(k-1)}(\omega) \\
X_{rp}(\omega) \end{bmatrix} \]  

(7.41)

According to Equation (7.38),

\[ [D(\omega)] = [H_1(\omega)] A_g(\omega) \]  

(7.42)
\[ [E]_{\text{drift}} = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
-1 & 1 & \cdots & 1 \\
\vdots & \vdots & \ddots & \vdots \\
-1 & 1 & \cdots & 1 \\
\end{bmatrix} \quad \text{and} \quad [H_1(\omega)] = [E]_{\text{drift}} [H_0(\omega)].
\]

Accordingly, the interstory drift power spectral density matrix of the coupled system is
\[ [S]_{D,D} = [H_1(\omega)]^*[H_1(\omega)]^T S_g \quad (7.43) \]

### 7.5.3 Auto-spectral density of the story shear

Inter-story shear force in time domain is
\[ [R]_{\text{shear}} = [K]_{\text{shear}} [X] + [C]_{\text{shear}} [\dot{X}] \quad (7.44) \]

where \([K]_{\text{shear}} = \begin{bmatrix}
k_{l1} & \cdots & k_{lj} & \sum k_{\text{NRB}} \\
k_{l2} & \cdots & k_{lj} & k_{r1} \\
\vdots & \ddots & \vdots & \vdots \\
-k_{l1} & \cdots & -k_{lj} & -k_{r1} \end{bmatrix}\). Similar to

\[ [K]_{\text{shear}} , [C]_{\text{shear}} = \begin{bmatrix}
c_{l1} & \cdots & c_{lj} & \sum c_{\text{NRB}} \\
c_{l2} & \cdots & c_{lj} & c_{r1} \\
\vdots & \ddots & \vdots & \vdots \\
-c_{l1} & \cdots & -c_{lj} & -c_{r1} \end{bmatrix}. \]

Thus, the inter-story shear force in the frequency domain can be calculated by the
following equations:

\[ F(\omega)_{\text{shear}} = ([K]_{\text{shear}} + i\omega[C]_{\text{shear}})X(\omega) \]  

(7.45)

Substituting Equation (7.38) into Equation (7.45) yeilds

\[ F(\omega)_{\text{shear}} = [H_2(\omega)]X(\omega) \]  

(7.46)

where the transfer function from ground acceleration to interstory shear response is

\[ [H_2(\omega)] = ([K]_{\text{shear}} + i\omega[C]_{\text{shear}})[H_0(\omega)] \] . Consequently, the power spectral density matrix (Lutes and Sarkani 2003; Newland 2012) of story shear is

\[ [S]_{s,s} = [H_2(\omega)]^*[H_2(\omega)]^T S_g \]  

(7.47)

Under the Kanai-Tajimi spectral density excitation, the auto-spectral densities of the response of the left building and the right building are shown in Figure 7.8 and Figure 7.9, respectively. The following can be observed:

(1) In the case without connecting dampers, the predominant frequency of the left building is around 7.071 rad/s (the corresponding period is 0.888 s). The maximum spectral density of top floor displacement, the maximum spectral density of the first story drift and the maximum spectral density of base shear are 1.89\times 10^{-3} \text{ m}^2/\text{rad} \cdot \text{s}, 5.16\times 10^{-5} \text{ m}^2/\text{rad} \cdot \text{s} and 2.841\times 10^{14} S_0 N^2/\text{rad} \cdot \text{s}, respectively.

(2) In the case with connecting dampers, the predominant frequency of the left building reduces to 4.029 rad/s. The maximum spectral density of top floor displacement, the maximum spectral density of the first story drift and the maximum spectral density of base shear are around 1.469\times 10^{-4} \text{ m}^2/\text{rad} \cdot \text{s}, 6.594\times 10^{-6} \text{ m}^2/\text{rad} \cdot \text{s} and 3.633\times 10^{13} S_0 N^2/\text{rad} \cdot \text{s}, respectively. By comparing these values with the corresponding values in the case without connecting damper, it can be concluded that the connecting dampers can significantly reduce the dynamic response of the left
building in the frequency domain.

(3) Compared with the maximum PSD of the response of the right building in fixed-base condition, that of the right building in base isolated configuration (connected to the left building) is much smaller. Especially, the maximum PSD of the drift and base shear of the right building are reduced by more than 99%, respectively.

7.5.4 Standard deviation of response

After the response power spectral density functions have been computed, the standard deviation of response can be numerically approximated. For example, the standard deviation of displacement, standard deviation of drift and standard deviation of story shear (Lutes and Sarkani 2003) of the left building at $k$th floor are

\[
\begin{align*}
\sigma_{x_{nlk}} &= \left[ \int_{-\infty}^{+\infty} S_{X_kX_k}(\omega) \, d\omega \right]^{\frac{1}{2}} \\
\sigma_{d_{lk}} &= \left[ \int_{-\infty}^{+\infty} S_{D_kD_k}(\omega) \, d\omega \right]^{\frac{1}{2}} \\
\sigma_{s_{lk}} &= \left[ \int_{-\infty}^{+\infty} S_{S_kS_k}(\omega) \, d\omega \right]^{\frac{1}{2}}
\end{align*}
\]  

(7.48)

where $S_{X_kX_k}(\omega)$, $S_{D_kD_k}(\omega)$ and $S_{S_kS_k}(\omega)$ are auto spectral density function of the displacement, inter-story drift and inter-story shear of the left building at floor $k$, respectively.

With a step size of 0.002 rad/s, the trapezoidal rule has been used to approximate the standard deviation in Equation (7.48). The lower limit and upper limit in Equation (7.48) are respective 0 rad/s and $60\pi$ rad/s.

Figure 7.10 and Figure 7.11 display the standard deviation of the response of the two buildings. It can be found that all the standard deviation of the response of the left building at all the stories are significantly mitigated after connecting to the right building by visco-elastic dampers. In particular, the standard deviation of the first
story drift of the left building is reduced from 5.93 mm to 3.69 mm with the percentage reduction of 37.77%, because of the installation of the visco-elastic dampers. For the right building, although the standard deviation of displacement is increased after the addition of the base isolators (which is normal for base isolated building to experience larger displacement with respective to the ground (Naeim and Kelly 1999)), the maximum standard deviation of the story drift of the right building with and without retrofit are respectively 5.41mm and 0.20mm, leading to a 96.3% reduction. This indicates that the right building is well protected by the base isolators.

7.6 Chapter summary

In this chapter, adjacent buildings is proposed to be retrofitted by coupling method using visco-elastic dampers with one of the buildings being protected by traditional passive base isolation system. Equations of motion are formulated to assess the feasibility of the proposed retrofit strategy.

The impact of the variation of base isolator properties on the dynamic response of the connected buildings is evaluated. As compared with natural rubber bearings, lead rubber bearings may increase the response of the base-isolated building. The effect of applying different seismic isolation systems on the response of the fixed-base building (the one connected to the base isolated building) is limited. Parametric studies concerning properties of the visco-elastic dampers are carried out. The numerical results indicate that the proposed method makes significantly reduction in structural response to both moderate earthquakes and rare earthquakes.

Further, the Kanai-Tajimi spectral density function is used to excite the coupled buildings in the frequency domain. The effect of the connecting dampers on the response power spectral density function of both buildings and on the standard
deviation of the response of the buildings has been evaluated. The analysis in the frequency domain shows that the fundamental frequencies of both buildings are reduced and the peaks of the power spectral densities are considerably decreased. The standard deviation of the drift of the coupled building is reduced by 39.76% and 93.09%, respectively.
Figure 7.1 Adjacent buildings and analytical models

Figure 7.2 Connection force at floor $k$
Figure 7.3 Response spectra of the seven earthquakes used
Figure 7.4 The average of the maximum drift against ratio $r_{VE}$

Figure 7.5 The average of the maximum RMS drift against ratio $r_{VE}$
Figure 7.6 Top floor displacement response of the left building
Figure 7.7 Displacement envelope of the right building
Figure 7.8 Spectral density of the response of the left building
Figure 7.9 Spectral density of the response of the right building
Figure 7.10 Standard deviation of the response of the left building

Figure 7.11 Standard deviation of the response of the right building


Chapter 8  Strengthening core connected building groups by base isolators and connecting isolators

8.1 Introduction

As Hong Kong is now classified as a region with “low-to-moderate seismicity” (Chandler, et al. 2001; Lam, et al. 2002; GB50011-2010 2010), it may be necessary to retrofit the existing buildings which have been designed without seismic consideration (Pun and Ambraseys 1992; Lee, et al. 1998; Naeim and Kelly 1999; Chandler, et al. 2001). Over years, different retrofit strategies have been proposed to improve the behaviors of existing buildings (Lam, et al. 2002; Ibrahim 2008). It is common in Hong Kong that some of the existing buildings are connected to each other to form building groups. When subjected to earthquakes, a building group may suffer damage caused by large lateral forces as well as collision between the buildings. For example, collapse of structures has been observed because of pounding during Loma Prieta earthquake (Kasai and Maison 1997). Therefore, this chapter extends the studies in previous chapters to focus on a building group consisting of buildings supported by traditional concrete cores. Buildings of The Hong Kong Polytechnic University are examples of this type of building group. Figure 8.1 schematically shows two frames connected by a middle core. Corbels of the cores are used as the connection to support the floors of the two frames. Coupling technologies are used to retrofit such a building group.

Conventionally, a seismic isolation system isolates the whole superstructure above
the isolation layer from ground motion. However, it is impractical to horizontally cut through the cores to install seismic isolation devices. Further, concrete cores are commonly used as the primary lateral force resisting system (Fanella 2010). As a result, this study applies a practical base isolation system to the frames together with story isolators to strengthen the original core-frame building group to reduce the seismic response to earthquakes.

8.2 Retrofit scheme

The retrofit scheme is shown in Figure 8.2. As compared with the original building group, (a) base isolators are installed at ground level to protect the frames above; (b) connection between the cores and ground remains intact; and (c) the cores and the frames are connected at each floor level by story isolators. The story isolators are replaceable and act as energy dissipation devices when subjected to wind or earthquake. The story isolators can absorb energies and transmit part of the weight of the frames to the cores.

The load transferring mechanism is as follows:

(1) Both the cores and the frames are designed to transfer the gravity load. Part of the weight of the frame is transferred to the cores by corbels. This benefits the cores by reducing the vertical tensile stress in the cores when the cores are subjected to lateral forces.

(2) Under wind loading, the cores provide the necessary lateral stiffness to limit the lateral deflection and to prevent possible wind induced oscillation.

(3) Under earthquake motion, the cores are the principal lateral load resisting system and the frames are protected by base isolators. Story isolators are used to dissipate
energy to reduce response.

8.3 Analytical model

Assuming the building group is subjected to one-direction ground motion and without torsional effects, then the building group is simplified to a two dimensional model. In this analysis, the cores and the frame are assumed to be linear elastic throughout the loading history.

8.3.1 Core model

Cores can be modeled by beam elements (Ramamurty 2010). The two commonly-used beam elements are the classic beam element and the Timoshenko beam element. The classical beam model can simulate bending response without transverse shear effect. The Timoshenko beam model is developed from the classical beam theory and incorporated with first-order shear deformation effects by allowing the core axis not acting normal to the deformed longitudinal axis (Karnovskii and Lebed 2004). In this study, the Timoshenko beam element is used. The cores have both flexural deformation and shear deformation. As shown in Figure 8.3, at each floor level the cores are divided into a number of segments connected at the nodal points.

8.3.1.1 Timoshenko beam element

Supposing the Timoshenko beam element has a length \( l \), transverse displacement \( u \), bending rotation \( \theta \) at \( x \) place (as shown in Figure 8.4 (a)), simple linear shape functions \( N_1 = 1 - \frac{x}{l} \) and \( N_2 = \frac{x}{l} \) are used, the consistent mass matrix of the Timoshenko element is

\[
\begin{bmatrix}
\frac{IA}{3} & 0 & \frac{IA}{6} & 0 \\
0 & \frac{H}{3} & 0 & \frac{H}{6} \\
\frac{IA}{6} & 0 & \frac{IA}{3} & 0 \\
0 & \frac{H}{6} & 0 & \frac{H}{3}
\end{bmatrix}
\]

in which \( \rho \) is the density of the core,
A is the sectional area and I is the moment of inertia (Karnovskii and Lebed 2004).

Element stiffness matrix is $K^e = K_s^e + K_b^e$. $K_s^e = \frac{EI}{l} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$ and $K_b^e = \frac{\mu GA}{l} \begin{bmatrix} 1 & l/2 & \cdots & 1 & l/2 \\ l/2 & l^2/4 & \cdots & -l/2 & l^2/4 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & -l/2 & \cdots & 1 & -l/2 \\ l/2 & l^2/4 & \cdots & -l/2 & l^2/4 \end{bmatrix}$ in which $E$ is Young’s modulus, $G$ is shear modulus and $\mu$ is the shear coefficient of the Timoshenko beam (Hutchinson 2001).

For a thin walled hollow circular section, the shear coefficient $\mu = \frac{6(1+\nu)(1+m^2)^2}{(7+6\nu)(1+m^2)^2+(20+12\nu)m^2}$ in which $\nu$ is the Poisson’s ratio (Hutchinson 2001).

The coefficient $m = \frac{d-t}{d+t}$ where $d$ and $t$ sectional dimensions as shown in Figure 8.4 (b).

8.3.1.2 Equations of motion of core

Using the Timoshenko beam elements to model the cores, the equations of motion of the cores are as follows (Chopra 2011):

$$[M]_c [\ddot{X}]_c + [C]_c [\dot{X}]_c + [K]_c [X]_c = -[M]_c [I]_c \ddot{a}_g$$ (8.1)

where superscript $c$ denotes the cores. $[\ddot{X}]_c$, $[\dot{X}]_c$ and $[X]_c$ are the acceleration vector, the velocity vector and the displacement vector, respectively. $[M]_c$, $[C]_c$ and $[K]_c$ are the mass matrix, the damping matrix and the stiffness matrix, respectively. $\ddot{a}_g$ is the ground acceleration. Position vector $[I]_c = [1,0,1,0 \cdots 1,0]^T$. Rayleigh damping (Chopra 2011) is adopted with the first and second modal damping ratios $\xi_c$ at 0.03 and $[C]_c = \alpha_c [M]_c + \beta_c [K]_c$.

$$\alpha_c = \frac{2\xi_c \omega_c \omega_c \omega_c}{\omega_c \omega_c + \omega_c \omega_c}, \quad \beta_c = \frac{2\xi_c}{\omega_c \omega_c + \omega_c \omega_c}$$ (8.2)
where $\omega_{1,c}$ and $\omega_{c,2}$ are the first and the second modal circular frequency of the cores.

Seismic response of the cores (e.g. acceleration, velocity, and displacement at any time $t$) can be obtained by solving the equations of motion numerically using the NEWMARK method (Krenk 2009; Humar 2012). The numerical procedure has been programmed using MATLAB software.

### 8.3.1.4 Model validation

To verify the proposed model, dynamic response of a 36 m high concrete core subjected to El Centro earthquake is predicted using the above equations. The diameter and thickness of the core are respectively 6 m and 0.4 m. To assess the accuracy of the proposed model, the core is also analyzed using ANSYS. Shell elements are used as shown in Figure 8.3 (c). The maximum shell element size is 0.5 m×0.5 m. Table 8.1 compares the fundamental periods and the maximum displacements computed by the two methods. The maximum difference is less than 5%. Figure 8.5 compares the displacement response of the cores estimated by ANSYS and present study when subjected to El Centro earthquake. The two response are similar to each other. This indicates that the Timoshenko beam elements provide reasonable prediction of the response of cores when subjected to earthquake action.

<table>
<thead>
<tr>
<th></th>
<th>(A) ANSYS</th>
<th>(B) Timoshenko beam model</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental period (s)</td>
<td>0.3226</td>
<td>0.3217</td>
<td>0.27%</td>
</tr>
<tr>
<td>Second mode period (s)</td>
<td>0.0627</td>
<td>0.0617</td>
<td>1.59%</td>
</tr>
<tr>
<td>Maximum displacement at top floor (mm)</td>
<td>43.8426</td>
<td>44.9287</td>
<td>-2.48%</td>
</tr>
<tr>
<td>Minimum displacement at top floor (mm)</td>
<td>41.1024</td>
<td>42.9928</td>
<td>-4.60%</td>
</tr>
</tbody>
</table>
8.3.2 Model of connecting isolators

Lead rubber bearings are used as story isolators. Lead rubber bearings can be modeled by the Bouc-Wen model (Buckle and Mayes 1990; Constantinou, et al. 1990; Ni, et al. 1998). As shown in Figure 8.6 (a), properties of lead rubber bearings are defined by three parameters: total lead rubber bearings yield force $f_y$, total initial shear stiffness $k_1$ and post-yield shear stiffness $k_2$. Total post-yield stiffness of lead rubber bearings is assumed to be 1/10 of the total initial stiffness (i.e. $k_1=10k_2$) (Naeim and Kelly 1999). At each floor level, two LRB300 isolators are designed to connect the cores and the frames. Lateral post-elastic stiffness of a LRB300 is 0.435 MN/m and the corresponding yield force is 25.48 kN.

8.3.3 Base isolated frame model

8.3.3.1 Base isolator model

Two types of base isolators are installed between the ground and the frames including natural rubber bearing and lead rubber bearings. Figure 8.6 (b) shows a typical hysteresis loop of rubber bearings subjected to sinusoidal force. Equivalent stiffness and equivalent damping ratio are used to model natural rubber bearing (Spencer and Nagarajaiah 2003; Higashino and Okamoto 2006) and the equivalent damping ratio is assumed to be 5%.

8.3.3.2 Frame model

Multi-degree-of-freedom shear models are used to represent the frames. Masses are assumed to be lumped at each floor level. Table 8.2 shows the masses and the lateral stiffness of the two frames. Total mass of the left frame and the right frame are respectively $7.667 \times 10^3$ ton and $6.835 \times 10^3$ ton. When these frames are fixed to the ground, their fundamental periods are 1.1326s and 1.1917s, respectively.
Table 8.2 Masses and lateral stiffness at each floor level

<table>
<thead>
<tr>
<th>Floor</th>
<th>Left frame</th>
<th>Right frame</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mass (ton)</td>
<td>Lateral stiffness (MN/m)</td>
</tr>
<tr>
<td>Ground floor</td>
<td>638.8</td>
<td>1.8462×10^1</td>
</tr>
<tr>
<td>Top floor</td>
<td>638.8</td>
<td>1.1286×10^3</td>
</tr>
</tbody>
</table>

The mass matrix of the base isolated left frame is

\[
[M]_{lf} = \begin{bmatrix}
m_{l,1} & \cdots & m_{l,2} \\
\vdots & \ddots & \vdots \\
m_{l,12} & \cdots & m_{l,12}
\end{bmatrix}
\]  \hspace{1cm} (8.3)

where \( m_{l,1}, m_{l,2}, \ldots, m_{l,12} \) are masses at each floor level as shown in Figure 8.7.

Stiffness matrix of the base isolated left frame is

\[
[K]_{lf} = \begin{bmatrix}
k_{l,iso} + k_{l,2} & -k_{l,2} & \cdots & -k_{l,12} \\
-k_{l,2} & k_{l,2} + k_{l,3} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
-k_{l,12} & \cdots & \cdots & k_{l,12}
\end{bmatrix}
\]  \hspace{1cm} (8.4)

where \( k_{l,iso}, k_{l,2}, \ldots, k_{l,12} \) are lateral stiffness at each floor level as shown in Figure 8.7.

Rayleigh damping is adopted for the frame over isolation layer. Both the first and second modal damping ratios, \( \varphi \), are 3%. The damping matrix of the left base isolated frame is

\[
[C]_{lf} = \alpha_{lf} \begin{bmatrix}
0 & m_{l,2} & \cdots & m_{l,12}
\end{bmatrix} + \beta_{lf} [K]_{lf}^{(1)} + \frac{2\zeta_{l,iso}}{\omega_{l,iso}} \begin{bmatrix}
k_{l,iso} & 0 & \cdots & 0
\end{bmatrix}
\]  \hspace{1cm} (8.5)
where $\zeta_{1,iso}$ is the damping ratio of base isolation system and is assumed to be 5%. $\omega_{1,iso}$ is the fundamental period of vibration of the left frame in base-isolated configuration. $[K]_{lf}^{(1)} = \begin{bmatrix} k_{l,2} & -k_{l,2} \\ -k_{l,2} & k_{l,2} + k_{l,3} \\ \ddots & \ddots \\ -k_{l,12} & k_{l,12} \end{bmatrix}$.

$$\alpha_{lf} = \frac{2\zeta \omega_{lf,1} \omega_{lf,2}}{\omega_{lf,1} + \omega_{lf,2}}, \quad \beta_{lf} = \frac{2\zeta}{\omega_{lf,1} + \omega_{lf,2}}$$ (8.6)

where $\omega_{lf,1}$ and $\omega_{lf,2}$ are the first and the second modal circular frequency of the left frame in fixed-base condition. $\zeta = 3\%$ is the damping ratio of the left frame.

The mass matrix, the stiffness matrix and the damping matrix of the right base isolated frame are similar to that of the left isolated frame.

8.3.4 Model of the new building group and Equations of motion

Figure 8.7 shows the simplified analytical model of the building group. Three cores are simulated by three Timoshenko beams. Two shear models represent the left frame and the right frame. The Bouc-Wen model is used to simulate the connecting isolators between the cores and the frames.

Equations of motion (Chopra 2011) of the structural system as shown in Figure 8.7 are developed in the form of:

$$[M]_{gr} \ddot{\mathbf{x}}_{gr} + [C]_{gr} \dot{\mathbf{x}}_{gr} + [K]_{gr} \mathbf{x}_{gr} + [R]_{bt} + [R]_{si} = -[M]_{gr} \dot{I}_{gr} \ddot{a}_g \quad (8.7)$$

where the mass matrix $[M]_{gr} = \begin{bmatrix} [M]_{c} & [M]_{lf} \\ [M]_{c} & [M]_{rf} \end{bmatrix}$ in which $[M]_{c}$,

$[M]_{lf}$ and $[M]_{rf}$ are respectively the mass matrix of the core, the mass matrix of the
left frame and the mass matrix of the right frame.

\[
[K]_{gr} = \begin{bmatrix}
[K]_c & [K]_{lf} \\
[K]_c & [K]_{rf} & [K]_c
\end{bmatrix}
\]

The stiffness matrix \([K]_{gr}\) is similar to the stiffness matrix \([K]_{lf}\) and \([K]_{rf}\). The damping matrix \([C]_{gr}\) is similar to the stiffness matrix \([K]_{gr}\).

The restoring force of lead rubber bearings at base is

\[
[R]_{bi} = \begin{bmatrix}
0 & \cdots & 0 & [R]_{bi,lf} & 0 & \cdots & 0 & [R]_{bi,rf} & 0 & \cdots & 0
\end{bmatrix}^T
\text{ (8.8)}
\]

where \(R_{bi,lf}\) and \(R_{bi,rf}\) are respectively the restoring force of the lead rubber bearings under the left frame and the right frame.

The restoring force vector contributed by connecting lead rubber bearings is

\[
[R]_{sl} = \begin{bmatrix}
[R]_{si,lf} & [R]_{si,lf} & [R]_{si,lf} & [R]_{si,rf} & [R]_{si,rf} & [R]_{si,rf}
\end{bmatrix}
\text{ (8.9)}
\]

where the subscripts \(si, lc, lf, mc, rf\) and \(rc\) respectively represent story isolators, the left core, the left frame, the middle core, the right frame and the right core.

\[
\begin{cases}
R_{si,lf} = \begin{bmatrix}
1 & \cdots & 1 & 0 & \cdots & 0
\end{bmatrix} \\
R_{si,lf} = \begin{bmatrix}
R_{1,lf} & R_{2,lf} & \cdots & R_{12,lf}
\end{bmatrix}
\end{cases}
\text{ (8.10)}
\]

where \(R_{1,lf}\) is the restoring force contributed by the connecting lead rubber bearings at the first floor. Detail of the connecting force \(R_{1,lf}\) can be calculated by Equation (7.3) in Chapter 7. Other connecting force in Equation (8.9) can be calculated in similar way.

The position matrix \([I]_{gr}\) is

\[
[I]_{gr} = \begin{bmatrix}
[I]_{c}^T & [I]_{tr}^T & [I]_{c}^T & [I]_{rf}^T & [I]_{c}^T
\end{bmatrix}^T
\text{ in which } [I]_{lf} \text{ and}
\]
Equation (8.7) is solved using the NEWMARK method (Krenk 2009; Humar 2012), and response (e.g. acceleration, velocity and displacement) at any time $t$ is obtained numerically. In the analysis, both the cores and the frames are assumed to behave linear elastically throughout the loading history.

8.4 Earthquakes input and simulation results

8.4.1 Site condition and earthquakes input

The building group is assumed to be located in a medium soft area with seismic intensity at the VII degree in accordance with the Chinese code (GB50011-2010 2010). The cores and frames are subjected to seven earthquakes. Peak ground accelerations of the earthquake records are scaled to 3.1 m/s$^2$ representing rare earthquakes. Details of the earthquake records are shown in Chapter 7.

8.4.2 Simulation results

In consideration of the nonlinear properties of the base isolators, small time interval $\Delta t = 0.01/100 = 1\times10^{-4}$s is used to solve Equation (8.11). The force displacement relationship of the connecting isolators is shown in Figure 8.8 to Figure 8.14.

Figure 8.15 shows the average of the maximum drift of the cores at each floor. Like a deformed cantilever beam subjected to transverse force, the maximum drifts of the cores occur at the top floor level. Table 8.3 compares the average of the maximum drift of the cores. In comparison with the drifts without retrofitting, the retrofitting strategy can effectively reduce the maximum drift of the cores by more than 77%.

The displacement envelops of the frames are shown in Figure 8.16. The maximum displacement occurs at the top floor. Most of the lateral displacement is contributed
by base isolators. The maximum drifts of the frames are 2.59 mm and 2.77 mm. As compared with the corresponding response before retrofit, the maximum drift can be reduced by more than 81%. These results show that the two frames are well isolated from ground motions.

Figure 8.17 shows the average of the maximum relative displacement between the cores and frames at each floor. The maximum relative displacement between the left core and the left frame, between the left frame and the middle core, between the middle core and the right frame, between the right core and right frame are 68.6 mm, 65.5 mm, 61.2 mm and 72.9 mm, respectively. These values are less than the minimum seismic joint width which is 210 mm according to the Chinese Code for Seismic Design of Buildings (GB50011-2010 2010). The relative displacement between the cores and frames comply with the Chinese Seismic Design Code.

<table>
<thead>
<tr>
<th>Story No.</th>
<th>Left core Without retrofit</th>
<th>Left core With retrofit</th>
<th>Middle core Without retrofit</th>
<th>Middle core With retrofit</th>
<th>Right core Without retrofit</th>
<th>Right core With retrofit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.191</td>
<td>0.532</td>
<td>3.191</td>
<td>0.475</td>
<td>3.191</td>
<td>0.531</td>
</tr>
<tr>
<td>2</td>
<td>6.154</td>
<td>1.135</td>
<td>6.154</td>
<td>0.999</td>
<td>6.154</td>
<td>1.133</td>
</tr>
<tr>
<td>3</td>
<td>8.663</td>
<td>1.647</td>
<td>8.663</td>
<td>1.452</td>
<td>8.663</td>
<td>1.644</td>
</tr>
<tr>
<td>4</td>
<td>10.664</td>
<td>2.074</td>
<td>10.664</td>
<td>1.834</td>
<td>10.664</td>
<td>2.069</td>
</tr>
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8.5 Chapter summary

This chapter applies base isolation technologies and story isolators to retrofit core connected buildings. An application is studied to verify the feasibility of the retrofit strategy. Base isolators are designed to protect the frame from earthquakes. Story isolators provide lateral resistance and ability to dissipate energy. The Timoshenko beam elements are used to model the cores. Analytical model has been presented to simulate the core connected buildings installed with base isolators and story isolators. Equations of motion are formulated to study the response of the building system to ground motions. Simulation results have shown that the maximum drift of the cores and the maximum drift of the frames can be reduced by more than 77% and 81% respectively.
Figure 8.1A two building group connected by cores

(a) Story isolators connected to core wall

(b) Elevation

Figure 8.2 Retrofit scheme
Figure 8.3 Core and simulation model

(a) Core tube
(b) Cantilevered beam
(c) Model in Ansys

Figure 8.4 Timoshenko beam element

(a) Timoshenko beam element
(b) Core section

Diameter (d) Thickness (t)
Figure 8.5 Comparison of displacement response of cores to El Centro earthquake

Figure 8.6 Models of natural rubber bearings and lead rubber bearings
Figure 8.7 Analytical model

Figure 8.8 Restoring force against displacement of story isolators at top floor under El Centro earthquake
Figure 8.9 Restoring force against displacement of story isolators at top floor under Hachinohe earthquake

Figure 8.10 Restoring force against displacement of story isolators at top floor under Kobe earthquake
Figure 8.11 Restoring force against displacement of story isolators at top floor under Hollister earthquake

Figure 8.12 Restoring force against displacement of story isolators at top floor under Mexico earthquake
Figure 8.13 Restoring force against displacement of story isolators at top floor under Chichi earthquake

Figure 8.14 Restoring force against displacement of story isolators at top floor under Big Bear earthquake
Figure 8.15 The average of the maximum story drifts of the cores under seven earthquakes

Figure 8.16 The average of the maximum displacement of the frames at each floor level
Figure 8.17 The average of the maximum relative displacement between cores and frames
Chapter 9  Conclusions

9.1 Conclusions

A method of retrofitting existing buildings by coupling method using passive devices has been studied. Both theoretical and experimental approaches have been employed to evaluate the efficiency of the coupling method in mitigating the seismic response of adjacent buildings. Theoretical studies have been carried out, in both the time and frequency domain. Experimental tests have verified and complemented the theoretical studies. Main results and conclusions obtained in this study can be summarized as follows.

(1) A shear type fluid damper has been fabricated and tested under cyclic loading. The fractional derivative model, the Maxwell model and the viscous model have been applied to represent the fluid damper. It is found that the fractional derivative model provides an accurate representation of the shear type fluid damper. The fractional derivative model is then used to derive the equations of motion of two buildings coupled by fluid dampers in the time-domain. Parametric studies have been carried out by varying the number of stories of the buildings from six to sixteen and subjected to seven different ground motions. When buildings with a substantial difference in the period of vibration are linked by fluid dampers, significant seismic mitigation can be achieved. The reduction in the maximum response is limited when the buildings have similar periods of vibration. Similar to the results of elastic analysis, under rare earthquakes the fluid dampers are more effective in reducing the response of adjacent buildings with significantly different periods of vibration.
(2) To further study the effectiveness of fluid dampers, analysis of two single-degree-of-freedom systems is carried out to predict the response of adjacent buildings connected by fluid dampers in the frequency domain. It is observed that the response of adjacent buildings is significantly affected by the size of fluid dampers and the difference in the fundamental period of vibration of adjacent buildings. Only minimal reduction can be achieved when the difference in the period of vibration of adjacent buildings is small. When there is a large difference in the period of vibration of adjacent buildings, the standard deviation of displacement can be considerably reduced. Further, an optimization procedure has been developed to optimize the position and size of connecting dampers between two multistory buildings. Based on the numerical studies, the top floor of the building with lesser number of stories is the best location for the installation of the fluid dampers. With large difference in the fundamental period of vibration between the buildings, the maximum standard deviation of the drift of the buildings can be substantially reduced. Vice versa, response reduction diminishes when the natural frequencies of the buildings are close to each other.

(3) The effect of soil-structure interaction on the response of adjacent buildings connected by fluid dampers has been investigated. Analytical models of damper coupled buildings including soil-structure interaction or structure-soil-structure interaction have been established using the substructure method. The change in the maximum drift due to soil-structure interaction or structure-soil-structure interaction is less than 1% for buildings on moderately soft, moderately stiff or rock sites. Regardless if the adjacent buildings are coupled or not, SSI and SSSI can decrease the maximum drift by 10.9% to 26.6% for buildings on soft soil. Therefore, without
considering the effect of SSI or SSSI, the analysis of the response of buildings coupled by connecting devices on soft soil provides a safety margin.

(4) To verify the theoretical studies and to explore means of seismic mitigation for buildings having similar periods of vibration, shaking table tests are carried out on two steel frames connected by a damper. The steel frames are in 1/15 scale and represent one nine-story building and one eight-story building. The latter includes either fixed base or base-isolated conditions. When a fluid damper is installed, the buildings respond with slight increase in fundamental frequencies and damping ratios, and there is a limited reduction in the maximum response. To improve the efficiency of seismic mitigation, the steel frames are connected by a visco-elastic damper and with one of the steel frame being base-isolated. In such a case, the maximum dynamic response of the models is reduced by 24.8% to 49.5%. Thus, for buildings having the same or similar number of stories, it is recommended to connect them using visco-elastic dampers and to isolate one of the buildings from the ground for seismic mitigation.

(5) Influence of visco-elastic dampers and base isolation systems on the response of coupled building is numerically evaluated. Parametric studies indicate that the properties of visco-elastic dampers are the dominant factor that influences the response of the fixed-base building (connected to the base-isolated building by visco-elastic dampers). Properly designed visco-elastic dampers can significantly reduce the response of the fixed-base building. Base isolation systems have little impact on the fixed-base building. However, the use of lead rubber bearings may increase the response of the base-isolated building in comparison with the use of natural rubber bearings. To further investigate the effectiveness of the visco-elastic dampers, the coupled buildings with the best performance dampers and base isolation system are
excited by the Kanai-Tajimi spectral density function in the frequency domain. Analysis in the frequency domain has shown that the fundamental frequencies of both buildings are considerably reduced. Peaks of the response power spectral densities of the coupled buildings are significantly decreased. The standard deviations of the drift of the coupled buildings are reduced by 39.76% and 93.09%, respectively. The results in the frequency domain further demonstrate that the effectiveness of the coupling method in mitigating adjacent buildings with similar number of stories (or similar periods) using visco-elastic dampers and base isolators.

(6) Supplementary studies have been conducted to retrofit a core-frame system resembling the structural form of the buildings in The Hong Kong Polytechnic University. Base isolators are applied to protect the buildings from ground motion. Story isolators are employed to connect the cores and buildings for energy dissipation and relative displacement control. Simulation results have shown that the retrofitting strategy can effectively reduce the maximum drift response of the cores by more than 77%. As with the corresponding response without retrofitting, the maximum response of the frames can be reduced by more than 81%. These results show the two frames are well isolated from ground motions.

9.2 Future studies

Some important results and findings of retrofitting existing buildings by coupling method using passive device have been achieved. To establish a complete design philosophy for the application of coupling method, further research is required. The following presents some important issues that need further studies:

(1) Earthquakes in this study are applied in one direction. Future studies should consider bidirectional and torsional motion of the coupled buildings. Hence, the
performance of passive control for asymmetric buildings or building clusters can be investigated.

(2) Other devices (in particular semi-active dampers which only need limited power) can be employed to link adjacent buildings for seismic mitigation. Comparison of different control systems to couple adjacent buildings is meaningful for practical application.

(3) Linear frequency domain analysis has been carried out in this study. Future study may perform nonlinear analysis to investigate the response of the coupled buildings under strong ground excitations in the frequency domain.

(4) The coupling method can be extended to connect several buildings together for seismic mitigation. When several buildings are coupled together, the effect of spatial variation of the ground motion needs to be taken into account.

(5) The coupling method can be extended to connect tall buildings with different structural forms (for instance: shear wall building or wall-frame buildings). Lateral wind loading may be the governing factor in the design. The coupling method can be used to reduce the structural response to the wind load.
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