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**EVOLUTIONARY OPTIMIZATION FOR SALES  
FORECASTING AND ORDER SCHEDULING IN  
FASHION SUPPLY CHAIN MANAGEMENT**

**DU WEI**

**Ph.D**

**The Hong Kong Polytechnic University**

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**The Hong Kong Polytechnic University**

**Institute of Textiles and Clothing**

**Evolutionary Optimization for Sales  
Forecasting and Order Scheduling in  
Fashion Supply Chain Management**

**Du Wei**

**A thesis submitted in partial fulfillment of the requirements**

**for the degree of Doctor of Philosophy**

**September 2015**

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TO MY FAMILY

For their constant love, support and encouragement

## Abstract

Facing with the increasingly fierce market competition, it is of paramount significance for apparel companies to establish effective and efficient fashion supply chains, which facilitate the reduction of costs, the improvement of service quality, and the enhancement of the competition ability of the companies. To build a competitive supply chain in fashion industry, it is necessary to improve its decision-making ability to manage problems, such as inventory problems, assembly line balancing problems, and so on. Among these problems, sales forecasting and order scheduling problems attract greater attention, because they largely influence the retailing and manufacturing in fashion supply chains. The primary purpose of this research is to solve sales forecasting and order scheduling problems in fashion supply chains via two hot branches of evolutionary optimization (multiobjective evolutionary optimization and robust evolutionary optimization) for the first time, and hence to establish a competitive and robust supply chain in fashion industry. The thesis consists of three main parts: algorithm development (Chapter 4), the application of the multiobjective optimization-based neural network model in fashion sales forecasting (Chapter 5), and the application of robust evolutionary optimization for fashion order scheduling (Chapter 6).

The algorithms developed in this thesis are evolutionary algorithms and they belong to a new branch of artificial intelligence. The first algorithm in this thesis is called nondominated sorting adaptive differential evolution (NSJADE), it is a part of the forecasting model for addressing the fashion sales forecasting problems. The second algorithm, known as event-triggered impulsive control scheme based differential evolution (ETI-DE), is developed as the optimization tool to get the robust schedules in fashion order scheduling. In detail, NSJADE is a new multiobjective evolutionary algorithm (MOEA), which is developed based on a classic MOEA, i.e., nondominated sorting genetic algorithm II (NSGA-II). NSJADE replaces the search engine of NSGA-II with adaptive differential evolution (JADE), and the proposed NSJADE shows better performance on multimodal problems according

to the experimental results. NSJADE is utilized for the fashion sales forecasting problems. In addition, a new scheme called ETI is presented in the framework of differential evolution (DE), and a powerful DE variant ETI-DE is obtained. The experimental results demonstrate that ETI can greatly enhance the performance of ten DE variants, and success-history based adaptive differential evolution with ETI (ETI-SHADE) has the best performance among all the variants. ETI-SHADE is modified to fit into the robust evolutionary optimization, and then to optimize the order scheduling problem in fashion supply chains after forecasting.

A multiobjective optimization-based neural network model (MOONN) is developed to handle a short-term replenishment forecasting problem in fashion supply chains. The model employs a new MOEA called NSJADE to optimize the input weights and hidden biases of NN for the short-term replenishment forecasting problem, which acquires the forecasting accuracy while alleviating the overfitting effect at the same time. Furthermore, the MOONN model also selects the appropriate number of hidden nodes of NN in terms of different replenishment forecasting cases. Experimental results demonstrate that the presented MOONN model can handle the short-term replenishment forecasting problem effectively, and show much superior performance to several popular forecasting models.

Robust ETI-SHADE is applied to develop robust order schedules in fashion supply chains. Unlike non-robust optimization, robust ETI-SHADE uses the mean effective objective value  $f^{\text{eff}}$  as the optimization objective. And the schedules obtained by ETI-SHADE are robust to the uncertain daily production quantity during the real production process. Experimental results show that schedules obtained by robust ETI-SHADE have uncertainty-tolerant ability, and benefit the real-world production in fashion supply chains.

The results of this research demonstrate that the utilization of multiobjective evolutionary optimization can offer satisfactory performance for the fashion sales forecasting problems, and the introduction of robust evolutionary optimization can generate robust schedules for fashion order scheduling

problems. It is revealed that these two branches of evolutionary optimization are of paramount significance to the establishment of an effective and efficient fashion supply chain.





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# Chapter 1

## Introduction

### 1.1 Background

Fashion supply chain is a system that is composed of people, organizations, technology, activities, resources, and information. An apparel product, from concept to customer, usually involves the following three steps: 1) the product is produced by manufacturers with raw materials from suppliers based on the ideas of designers; 2) the product is then distributed to retailers; 3) the product is finally delivered to the end customer. Facing with the increasingly fierce market competition, it is of great significance for apparel companies to establish an effective and efficient fashion supply chain, which facilitates the reduction of costs, the improvement of service quality, and the enhancement of the competition ability of the companies.

To build a competitive supply chain in fashion industry, there are many decision-making problems that should be solved beforehand, such as inventory problems, assembly line balancing problems, and so on. Among these problems, sales forecasting and order scheduling attract much attention, and they greatly influence the manufacturing and retailing in fashion supply chains. An accurate and timely sales forecasting helps retailers closely



match the demand and supply of their products, which benefits the control of inventory costs and the minimization of stockouts. In apparel manufacturing, the orders received by manufacturers have many product styles, different quantities, and different due dates. And an efficient and flexible order schedule is able to maximize the resource utilization and minimize the completion time of orders, which is also beneficial to the distribution and retailing in fashion supply chains.

### **1.1.1 Background of Fashion Sales Forecasting**

Fashion sales forecasting plays an indispensable role in retailing in fashion supply chains, and it estimates the future sales of an apparel product according to the historical data, market trends, and other related factors. Without fashion sales forecasting, there will be a great number of problems emerging in fashion supply chains: retroactive responses of operations, poor production planning, lost orders, inadequate customer service, poorly utilized resources, and so on [1]. Moreover, the fashion industry is characterized by short product life cycles, volatile customer demands, massive product varieties, and long supply processes [2]. And these features make the fashion sales forecasting very specific and complicated.

In the fashion industry, sales forecasting activities mostly depend on qualitative methods, like panel consensus and historical analogy. These methods are usually based on subjective assessment and experience of marketing personnel with simple statistical analysis of limited historical sales data. However, in order to get more flexible and robust methodology for fashion sales forecasting, it is necessary to design sophisticated forecasting models that are capable of considering both endogenous and exogenous factors. In the past two decades, a great number of models have been presented for the fashion sales forecasting. For example, a multivariate fuzzy forecasting

model was presented for forecasting the sales of women's apparel, which used historical sales, color and size as inputs [3]. The model showed superior forecasting performance to univariate models. An automatic forecasting system was developed for apparel sales forecasting, which consisted of two complementary models [4]. The first one obtained medium-term forecasting by using fuzzy logic to quantify the influence of explanatory variables, while the second fulfilled short-term forecasting by readjusting the medium-term forecasts. Their experimental results exhibited that the proposed model had better forecasting performance than three classical models. An evolutionary neural network (NN) model was developed to predict the future sales of apparel items with features of low demand uncertainty and weak seasonal trends [5]. The results revealed that the model performed better than the traditional autoregressive integrated moving average (ARIMA) model.

Among these models presented for the fashion sales forecasting, NN-based methods attracted much attention. As introduced above, NN provides promising performance of effective forecasts because of the capacities of nonlinearity, generalization and universal function approximation [6]. When NN was first introduced to the fashion sales forecasting problems, back-propagation (BP) algorithm was employed as the learning rule for NN. However, BP is a gradient-based learning algorithm, which has been criticized for a long time because of its slow convergence speed. In recent years, a novel learning algorithm called extreme learning machine (ELM) has been presented [7], which tends to provide good generalization performance and fast learning speed at the same time. Thereafter, ELM was introduced to fashion sales forecasting by Sun *et al.* [8], whose experiments demonstrated that ELM-based NN had superior forecasting performance BP-based NN. However, ELM may require far more hidden neurons due to the random determination of the input weights and hidden biases [9]. To handle this, Wong *et al.* optimized the input weights and hidden biases by

integrating ELM with harmony search algorithm, which is a newly developed evolutionary algorithm [10]. Their research in fashion sales forecasting exhibited the necessity of searching for the optimal values of input weights and hidden biases of ELM-based NN. In their research, training error was the only objective optimized by the evolutionary algorithm; the NN with the minimum training error was selected as the final network for the sales forecasting problem. In this case, the NN with the smallest training error is considered to have the best forecasting performance when it encounters the unseen data. Nevertheless, the features of the training samples do not represent the inherent underlying distribution of the new observations due to the existence of noise, which means the prediction effect of the “optimal” NN may be deteriorated by the overfitting phenomenon [11]. So it is not reasonable to merely minimize the training error of NN when executing the forecasting. While actually, for the forecasting problem solved by NN, there are other objectives that need to be optimized besides training error, like the number of hidden layer nodes or the sum of the absolute weights [12, 13].

### **1.1.2 Background of Fashion Order Scheduling**

In fashion supply chains, order scheduling is one of the decision-making problems in production planning. Fashion order scheduling aims to assign the orders that are received from retailers to the production lines, so that all the orders can be finished before the due dates. Schedules should be made by the planners before the production. Because of the characteristics of fashion industry, like labor-intensive industry, rising labor costs, short product life cycles, and massive product varieties [14], more attention needs to be paid to fashion order scheduling problems. For the past few decades, order scheduling problems in fashion supply chains have been investigated from different perspectives. For example, Chen and Pundoor [15] studied order

assignment and scheduling problems in three aspects: to determine which orders should be allocated to each plant, to schedule the allocated orders at each plant, and to schedule the shipping of finished orders from each plant to the distribution center. Leung *et al.* [16] addressed the multi-site order scheduling problem for a multinational lingerie company in Hong Kong. Guo *et al.* [17] investigated a multiobjective order allocation planning problem in a labor-intensive manufacturing company producing sportswear in Mainland China with the consideration of various real-world production features.

In recent years, as a powerful optimization tool [18], evolutionary algorithms (EAs) have been introduced to solve the order scheduling problems in fashion supply chains. For instance, Wong *et al.* utilized genetic algorithm (GA) to plan the production schedules in fabric-cutting department, which improved both makespan and cut-piece fulfilment rates [19]. Guo *et al.* adopted nondominated sorting genetic algorithm II (NSGA-II) to tackle the multiobjective scheduling problem for an apparel manufacturing company in China, which allowed for multiple plants, multiple production departments, and multiple production processes [20]. Wong *et al.* proposed a novel evolution strategy-based Pareto optimization algorithm (ESPO) to cope with the production planning problem in a labor-intensive manufacturing company producing knitwear products in China, which aimed at allocating production processes of each order to appropriate plants [21]. It has been widely recognized that EAs can find more efficient and flexible order schedules than traditional order scheduling methods in fashion industry, which largely depend on the experience of the planners. In the studies above, when the schedules were made before the real production, it was assumed that the daily production quantity of each order was fixed. However, in real production process, as a result of various uncertainties, like machine breakdown or operator absenteeism, the daily production quantity of each order is not always as expected. In this case, the schedules obtained in [19–21] need

to be updated frequently, which means these schedules are very sensitive. Moreover, when visiting some garment factories in Mainland China, it is found that most of the time, operators cannot finish the daily production quantity that was assigned to them and the production plans were shifted very often. After production starts, frequent modification of production plans will increase labor and time cost, which may reduce production efficiency and fail to complete the orders before their delivery dates.

## 1.2 Problem Statement

Recently, sales forecasting and order scheduling problems in fashion supply chains are modeled as evolutionary optimization problems, and evolutionary algorithms (EAs) have been widely utilized for these problems. In the framework of evolutionary optimization, this research solves sales forecasting and order scheduling problems in fashion supply chains via multiobjective evolutionary optimization and robust evolutionary optimization for the first time. Besides, a novel multiobjective evolutionary algorithm (MOEA) and a new scheme for differential evolution (DE) have been proposed, which is prepared for the optimization of fashion sales forecasting problem and fashion order scheduling problem.

(1) Newly proposed EAs: A new MOEA called nondominated sorting adaptive differential evolution (NSJADE) is developed based on a classic MOEA. NSJADE is utilized for the fashion sales forecasting problem in the following chapter. Besides, a new scheme called event-triggered impulsive control scheme (ETI) is presented in the framework of DE, and a powerful DE variant ETI-DE is obtained. ETI-DE is modified to fit into the robust optimization, and then to optimize the order scheduling problem in fashion industry later on.

(2) Fashion sales forecasting problem: A multiobjective optimization-based neural network model (MOONN) is developed to handle a short-term replenishment forecasting problem in fashion supply chains. The model employs a new MOEA called NSJADE to optimize the input weights and hidden biases of NN for the short-term replenishment forecasting problem, which acquires the forecasting accuracy while alleviating the overfitting effect at the same time. Furthermore, the MOONN model also selects the appropriate number of hidden nodes of NN in terms of different replenishment forecasting cases.

(3) Fashion order scheduling problem: A robust evolutionary algorithm called robust success-history based adaptive differential evolution with event-triggered impulsive control scheme (robust ETI-SHADE) is applied to develop robust order schedules in fashion supply chains. The schedules obtained do not have to be updated very often, because they have uncertainty-tolerant ability when facing with the uncertainty in real-world production.

### **1.3 Objectives**

The primary objective of this research is to solve sales forecasting and order scheduling problems in fashion supply chains via two hot branches of evolutionary optimization (multiobjective evolutionary optimization and robust evolutionary optimization) for the first time, and hence to establish a competitive and robust supply chain in fashion industry. Before that, a novel MOEA and a new scheme for DE have been proposed. The two developed EAs enrich the set of EAs and serve as the tools for the following optimization of fashion sales forecasting problem and fashion order scheduling problem:

(1) To develop the EAs for the two optimization problems in this research: sales forecasting problem and order scheduling problem in fashion supply

chains.

(2) To introduce multiobjective evolutionary optimization into fashion sales forecasting, which acquires the forecasting accuracy while alleviating the overfitting effect at the same time.

(3) To introduce robust evolutionary optimization into fashion order scheduling, which aims to obtain robust schedules with uncertainty-tolerant ability.

## 1.4 Methodology

This research solves two crucial decision-making problems in fashion supply chains: fashion sales forecasting and fashion order scheduling by means of evolutionary algorithms (EAs) or EA-based models. Two different methodologies are developed based on EA, and are described as follows:

(1) A multiobjective optimization-based neural network model (MOONN) is developed to handle a short-term replenishment forecasting problem in fashion supply chains. NN is responsible for detecting the underlying pattern of the training samples. A new MOEA called NSJADE is presented to optimize the input weights and hidden biases of NN for the forecasting problem.

(2) A robust EA is employed to search the robust schedules for the production planning in fashion supply chains. Unlike non-robust EAs, robust EAs evaluate the  $HN$  neighbouring points of the individual, and then calculate the average value of these  $HN$  values as the optimization objective of the order scheduling problem.

## 1.5 Significance of this Research

The significance of this research can be summarized as the following four aspects:

(1) The presented NSJADE enriches the set of the MOEAs, which derive from the idea of NSGA-II. Furthermore, the research of ETI-DE is an interdisciplinary one, which utilizes event-triggered mechanism (ETM) and impulsive control, two concepts in control theory, to improve the search performance of DE. The proposed ETI sheds light on the understandings of ETM and impulsive control in evolutionary computation, which broadens the applications of ETM and impulsive control in wider areas.

(2) A multiobjective optimization-based neural network model (MOONN) is proposed for the sales forecasting problems in fashion supply chains. It is the first work that investigates the sales forecasting problems in fashion supply chains by using MOO-based model. Different from other popular models, MOONN can ensure better forecasting performance and alleviate the overfitting effect at the same time.

(3) Order scheduling problems in fashion supply chains are investigated within the framework of robust evolutionary optimization for the first time. The order schedules obtained by robust evolutionary algorithms are robust to the perturbation of daily production quantities. When robust schedules are adopted, planners will reduce the times of modifying the order schedules during the production process, which increases the efficiency of the production in fashion supply chains. Besides, in this research, matching problem and learning effect are also considered in the optimization process, which makes the experimental environment more close to the real-world production environment.

(4) Two key decision-making problems in fashion supply chain management



are investigated within the framework of evolutionary optimization. According to the experimental results, multiobjective evolutionary optimization and robust evolutionary optimization exhibit effectiveness in sales forecasting and order scheduling problems in fashion supply chain management. Therefore, the performance of fashion supply chains can be greatly enhanced by introducing multiobjective evolutionary optimization and robust evolutionary optimization.

## **1.6 Structure of this Thesis**

The structure of this research can be summarized as follows:

In Chapter 2, a comprehensive literature review is provided including the existing research of fashion sales forecasting, order scheduling in fashion production planning, and evolutionary optimization.

In Chapter 3, the research methodology is introduced in detail, including the concepts and principles of multiobjective optimization, neural network, and extreme learning machine, robust evolutionary optimization, and differential evolution.

In Chapter 4, a novel multiobjective evolutionary algorithm called nondominated sorting adaptive differential evolution (NSJADE) is proposed. Then an event-triggered impulsive control scheme (ETI) is developed to improve the performance of differential evolution (DE), hence a new DE called ETI-DE can be obtained.

In Chapter 5, a short-term replenishment forecasting problem in fashion supply chains is handled by a NSJADE-based neural network model. And extensive experiments are conducted to show the effectiveness and superiority of the proposed model.

In Chapter 6, a robust evolutionary algorithm called robust success-history based adaptive differential evolution with event-triggered impulsive control scheme (robust ETI-SHADE) is proposed for developing robust order schedules in fashion supply chains. And two groups of experiments are carried out to display the effectiveness and superiority of introducing robust evolutionary optimization into fashion order scheduling problems.

In Chapter 7, the conclusions and the contributions of this research are summarized. Meanwhile, future work is also discussed in detail.

# Chapter 2

## Literature Review

Fashion supply chain is a system that consists of people, organizations, technology, activities, resources, and information. In this system, apparel products are produced by manufacturers with raw materials from suppliers, then distributed to retailers, and finally delivered to the end customer. It is of great importance for apparel companies to build an effective and efficient fashion supply chain, which can reduce costs, improve the service quality, and enhance the competition ability of the companies.

To establish a competitive supply chain in fashion industry, many decision-making problems should be solved beforehand. For instance, retailers have to control inventory costs and minimize stockouts, which are two main objectives of handling inventory problems. In the selling season, if some popular goods are out of stock, retailers may encounter loss of profit and decrease of customer satisfaction, which means temporary stockout has a significant downside on sales, profitability, and customer relationships. Therefore, for retailers, it is of paramount importance to make an accurate and timely forecasting, which helps them closely match the demand and supply in the competitive global market. While for manufacturers, during the manufacturing, they also need to cope with a number of decision-

making problems of production planning, such as assembly line balancing problems, order scheduling problems, and so on. Among these problems, order scheduling is a complicated and important task in fashion supply chains, since the orders received by manufacturers have massive product styles, different quantities, and different delivery dates. An efficient and flexible order schedule aims to maximize the resource utilization and minimize the completion time of orders, which also benefits the distribution and retailing in fashion supply chains.

For fashion sales forecasting, in the past few decades, a large number of classical or intelligent techniques have been proposed, and neural network (NN) is one of them. Recently, some researchers introduced single-objective evolutionary algorithms to optimize NN-based forecasting models, and training error is as the optimization objective. However, this operation may lead to the overfitting phenomenon of the forecasting model, which indicates more objectives are needed for the optimization. Therefore, multiobjective evolutionary algorithms are firstly utilized for the NN-based forecasting model in fashion supply chains. For order scheduling, evolutionary algorithms (EAs) are also a powerful tool for the optimization of effective order schedules in fashion supply chains. When the schedules are made by EAs before the real production, it is assumed that the daily production quantity of each order is fixed. However, in real production process, as a result of various uncertainties, like machine breakdown or operator absenteeism, the daily production quantity of each order is not always as expected. In this case, the schedules optimized by EAs need to be updated frequently, which means these schedules are very sensitive. Therefore, robust evolutionary algorithms are firstly introduced into the order scheduling problems in fashion supply chains.

This research applies evolutionary optimization to solve the two crucial decision-making problems, i.e., sales forecasting and order scheduling, in

fashion supply chains. In the following, the previous research in fashion sales forecasting and fashion order scheduling is reviewed respectively in Section 2.1 and Section 2.2. The previous applications of evolutionary optimization in fashion sales forecasting and fashion order scheduling are introduced in Section 2.3. Finally, the concluding remarks are provided in Section 2.4.

## **2.1 Fashion Sales Forecasting**

Fashion sales forecasting plays an indispensable role in retailing in fashion supply chains, and it estimates the future sales of an apparel product according to the historical data, market trends, and other related factors. Without fashion sales forecasting, there will be a great number of problems emerging in fashion supply chains: retroactive responses of operations, poor production planning, lost orders, inadequate customer service, poorly utilized resources, and so on [1].

### **2.1.1 Introduction**

The fashion industry is characterized by short product life cycles, volatile customer demands, tremendous product varieties, and long supply processes [2]. Uncertain customer demands in frequently changing market environment and numerous explanatory variables that influence fashion sales lead to an increase in irregularity or randomness of sales data [10]. Besides these features, there are other particularities of the clothing that should be considered [22]:

- (1) Sales are of strong seasonality because most garments are related to weather conditions. Seasonal data provide general trends, while unpre-

dictable variations of weather will result in significant peaks or hollows of the sales data.

- (2) Many external variables disturb the sales, such as end-of-season sale, sales promotion, purchasing power of consumers, and so on.
- (3) Sales are greatly depend on fashion trends, which means certain garments will only show up in one or two seasons. So historical sales of some items are not always available since they are ephemeral.
- (4) Each item may have many variations in sizes and colors.

All these constraints make the sales forecasts for fashion companies very specific and complex. In order to make more accurate forecasting, two main forecasting models are widely used: univariate forecasting model [5, 10] and multivariate forecasting model [8, 23]. In univariate forecasting model, researchers handle the sales forecasting problem relying on the historical sales data of the time series being predicted, which assumes the underlying variation of data is constant. For instance, Wong *et al.* utilized one-step-ahead sales data to predict the sales of medium-priced fashion products in Mainland China [10]. Au *et al.* predicted the sales of T-shirt and jeans from several shops with the previous time series data [5]. However, as mentioned above, the sales of fashion products are volatile, often influenced by fashion trends and weather conditions. So it is invalid to hypothesize that the trend of the time series sales data is unchanged during the forecasting period. To cope with this, researchers integrate other influencing factors as the inputs of forecasting models besides the historical time series data, which is known as multivariate forecasting. In the fashion industry, the following influencing factors are often taken into account when the multivariate forecasting is conducted:

- (1) Weather index: The weather in the selling season will influence the sales of the products. Usually, the average temperature during the selling

season is recorded for use.

- (2) Calendar data: These data often involve significant fluctuations in sales. Like in holidays, the sales of some items may increase along with more consumers.
- (3) Marketing strategy: It includes promotion and advertising strategy with or without the decrease of price. Different strategies have different impact on the sales volumes of retail products.
- (4) Features of items: Items' features indicate the number of colors and sizes, the material type, the style, as well as their match with the fashion trend, which have effects on the sales.
- (5) Retail information: It corresponds to the shop quantity, the location of stores. These factors might change year after year, which should be considered in the sales forecasts.
- (6) Economic indices: Various indices reflecting economic performance should be taken into consideration, like Consumer Confidence Index (CCI), Consumer Price Index (CPI), Gross Domestic Product (GDP), Producer Price Index (PPI), and so on.

Guo *et al.* predicted the future sales for a large fashion retail company by considering product attributes, climate index and economic indices [23]. Sun *et al.* achieved the forecasting according to the sales condition of a category of apparel with different sizes, colors and prices [8]. Price, the starting time of the sales and the life span of items were utilized to predict the future sales of certain products by Thomassey *et al.* [24].

From the perspective of fashion industry, the supply strategy of distributors or retailers is based on two steps: supply in a long-term or medium-term horizon and replenishment in a short-term horizon [22]. The first

step means distributing a certain number of products to the stores at the beginning of the sales season; the second step explains the replenishment for some fashion items during the sales season. Therefore, fashion sales forecasting is being studied according to these two horizons for clothing companies. Accurate long-term sales forecasting requires the company to be well-prepared before the selling season, which is basic for apparel companies; while successful short-term replenishment forecasting reflects the company's quick and efficient coping capacity. For example, Tanaka performed the long-term sales prediction based on the early sales and the correlations between short- and long-term accumulated sales within similar product groups [25]. Thomassey *et al.* achieved the medium-term sales forecasting at different sales aggregation levels [4]. And both long-term and short-term sales forecasts of women's sweater were conducted by Thomassey [22].

## **2.1.2 Techniques for sales forecasting**

As introduced before, sales forecasting problems can be classified into two categories according to the difference of the input variable number: univariate sales forecasting and multivariate sales forecasting. To generate these two kinds of sales forecasts, forecasting models need to be firstly established based on specific forecasting technique, which can approximate the future data based on available training samples.

Varieties of forecasting techniques have been widely employed in sales forecasting. In general, all of the forecasting methods can be divided into two categories: subjective and objective. For subjective forecasting methods, they rely most heavily on judgment and educated guesses, which include sales force composites, customer surveys, jury of executive opinion, Delphi method, and so on. Subjective forecasting methods often work when there is little data available for forecasting, especially for long-range forecasting. For



objective forecasting methods, historical data are of great significance. Time series methods belong to the objective forecasting methods, and they employ past data as the basis for forecasting future outcomes.

A time series is a sequence of data points, measured typically at successive points in time spaced at uniform time intervals. To predict time series data, historical data are collected to estimate future results. Time series forecasting methods can be divided into two groups: classical time series techniques based on mathematical and statistical models, and intelligent time series techniques. Classical time series forecasting techniques include Naïve, exponential smoothing [26, 27], autoregressive integrated moving average (ARIMA) [28], Kalman filtering [29], and so on. These techniques will be briefly introduced in the following paragraphs.

(1) *Naïve*. Naïve model is the simplest forecasting technique, and provides a benchmark against which more sophisticated models can be compared. A seasonal Naïve model was to predict the seasonal data, where all forecasts were equal to the most recent observation of the corresponding season [30]. For stationary time series data, Naïve model assumes that the forecast for any period equals the historical average.

(2) *Exponential smoothing*. Exponential smoothing models are widely used in forecasting time series. It makes an exponentially smoothing weighted average of past sales, trends, and seasonality to derive a forecast. A simple exponential smoothing was used to tackle the short-term sales forecasting, but it does not perform well when there is a trend in the data to be predicted [31]. While a robust Holt–Winters exponential smoothing method was developed for time series forecasting, where an easily implemented mechanism that automatically identifies outliers was presented [32]. The disadvantage of exponential smoothing is that it might smooth away important trends or cyclical changes within the data as well as the random variation, and thereby

distort the forecasting.

(3) *ARIMA*. ARIMA model is the generalization of autoregressive (AR), moving average (MA), and autoregressive moving average (ARMA) model [33]. The model is generally referred to as an  $ARIMA(p,d,q)$ , where  $p$ ,  $d$ ,  $q$  indicate the order of the autoregressive, integrated, and moving average parts of the model respectively. This technique is one of the most popular benchmark techniques [10, 28, 34]. However, the underlying theoretical model and structural relationships of ARIMA are not distinct.

(4) *Kalman filtering*. The standard Kalman filter is a state estimation technique. For example, a improved Kalman filter was proposed to estimate the new product diffusion models [29]. Kalman filtering is based on a probabilistic treatment of process and measurement noises, and it is a type of linear forecasting method.

These classical time series forecasting techniques approximate data generating process of the time series to be predicted based on the assumption that the future data imply the same or similar mathematical relationship with the classical technique. They are categorized as linear models that employ a linear functional form for time-series modeling. When it comes to predict the series featured by strong nonlinearity, these models often fail to work [35]. However, a series of nonlinear models, which are called intelligent forecasting techniques, have been proposed to handle these cases, like expert systems [36], fuzzy systems [37], neural network (NN) models [6, 38], and so on. Next, these techniques will be explained briefly.

(1) *Expert systems*. Expert systems utilize the knowledge of one or more forecasting experts to develop decision rules to form a forecast. It has been pointed out that an expert system could be easily developed to help executives in forecasting [39]. An application of expert systems for selecting techniques for demand forecasting was presented [36]. The expert system was built

to capture expert knowledge and acted as an advisor for choosing suitable demand forecasting techniques under various general business circumstances.

(2) *Fuzzy systems*. Fuzzy systems use fuzzy logic theory to tackle fuzzy and uncertain information in forecasting process. A fuzzy forecasting system was established based on fuzzy logic and a multiple regressive model, to predict the number of cans dispensed daily [40]. A multivariate fuzzy system was developed, which was used for forecasting women's casual sales [41]. The difficulty is how to express the knowledge of human experts in the form of fuzzy rules.

(3) *Neural networks*. NN is a mathematical model consisting of a group of artificial neurons connecting with each other; the strength of a connection between two nodes is called "weight". Learning rules adjust the weights of the network to better fit the underlying relation of the given data. NN techniques have been proved to have the potential to generate effective forecasts because of their capacities of nonlinearity, generalization and universal function approximation [6]. Back-propagation (BP) algorithm was employed as the learning rule to train the NN, where the model was to analyze the behavior of sales in a medium-size enterprise [42]. The forecasts generated by the NN model were found more accurate than those by ARIMA model. Recently, evolutionary algorithms have been introduced to NN models, which also exhibit impressive performance [10, 43–45].

Among the intelligent forecasting techniques, NN models are mostly utilized owing to their satisfactory ability of detecting and extracting nonlinear relationships of the given information; furthermore, many research works have shown that they exhibit much better performance than other traditional forecasting methods [5, 6, 10]. NN techniques can be employed to construct both univariate and multivariate forecasting models by using different number of input variables. For instance, if there are only historical observations

of sales time series as inputs, univariate NN models are obtained; while if there are other exogenous variables besides the historical data of time series, multivariate NN models are obtained. In addition, by setting different NN structures and parameters, different NN forecasting models can be built.

### **2.1.3 Techniques for fashion sales forecasting**

As discussed in 2.1.1, the fashion industry is characterized by short product life cycles, inconstant customer demands, massive product varieties, and long supply processes [2]. And these features make the fashion sales forecasting very specific and complicated. Therefore, it is of great importance to develop particular techniques for fashion sales forecasting.

In the fashion industry, sales forecasting activities mostly depend on qualitative methods, like panel consensus and historical analogy. These methods are usually based on subjective assessment and experience of marketing personnel with simple statistical analysis of limited historical sales data. However, in order to get more flexible and robust methodology for fashion sales forecasting, it is necessary to design sophisticated forecasting models that are capable of considering both endogenous and exogenous factors. In the past two decades, a great number of models have been presented for the fashion sales forecasting. For example, a multivariate fuzzy forecasting model was presented for forecasting the sales of women's apparel, which used historical sales, color and size as inputs [3]. The model showed superior forecasting performance to univariate models. An automatic forecasting system was developed for apparel sales forecasting, which consisted of two complementary models [4]. The first one obtained medium-term forecasting by using fuzzy logic to quantify the influence of explanatory variables, while the second fulfilled short-term forecasting by readjusting the medium-term forecasts. Their experimental results exhibited that the proposed model had

better forecasting performance than three classical models. An evolutionary NN model was developed to predict the future sales of apparel items with features of low demand uncertainty and weak seasonal trends [5]. The results revealed that the model performed better than the traditional ARIMA model.

Among these models presented for the fashion sales forecasting, NN-based methods attracted much attention. As introduced above, NN provides promising performance of effective forecasts because of the capacities of nonlinearity, generalization and universal function approximation [6]. When NN was first introduced to the fashion sales forecasting problems, back-propagation (BP) algorithm was employed as the learning rule for NN. However, BP is a gradient-based learning algorithm, which has been criticized for a long time because of its slow convergence speed. In recent years, a novel learning algorithm called extreme learning machine (ELM) has been presented [7], which tends to provide good generalization performance and fast learning speed at the same time. Thereafter, ELM was introduced to fashion sales forecasting by Sun *et al.* [8], whose experiments demonstrated that ELM-based NN had superior forecasting performance BP-based NN. However, ELM may require far more hidden neurons due to the random determination of the input weights and hidden biases [9]. To handle this, Wong *et al.* optimized the input weights and hidden biases by integrating ELM with harmony search algorithm, which is a newly developed evolutionary algorithm [10]. Their research in fashion sales forecasting exhibited the necessity of searching for the optimal values of input weights and hidden biases of ELM-based NN. In their research, training error was the only objective optimized by the evolutionary algorithm; the NN with the minimum training error was selected as the final network for the sales forecasting problem. In this case, the NN with the smallest training error is considered to have the best forecasting performance when it encounters the unseen data. Nevertheless, the features of the training samples do not

represent the inherent underlying distribution of the new observations due to the existence of noise, which means the prediction effect of the “optimal” NN may be deteriorated by the overfitting phenomenon [11]. So it is not reasonable to merely minimize the training error of NN when executing the forecasting. While actually, for the forecasting problem solved by NN, there are other objectives that need to be optimized besides training error, like the number of hidden layer nodes or the sum of the absolute weights [12, 13].

## **2.2 Order Scheduling in Fashion Production**

### **Planning**

The fashion industry is primarily concerned with the production of apparel products and accessories, which usually involves three stages: design, manufacturing and retailing. Among them, apparel manufacturing is the process of turning designers’ ideas into products and distributing to retailers, which includes three main operational processes: pre-production, production, and finishing. During the process of apparel manufacturing, there are many decision-making problems, such as assembly line balancing problem [46, 47], garment cutting problem [48, 49], multi-plant order tracking problem [50], seam quality evaluation problem [51], and so on; and order scheduling is one of them. In fashion supply chains, order scheduling considers the assignment of each order or its production process to proper production lines, with the purpose of making sure that orders can be finished before delivery dates.

#### **2.2.1 Decision-making Problems in Production Planning**

In manufacturing industry, production planning is of great importance to successful and efficient production management, since its performance large-

ly influences the supply chain performance of the company. During the process of production planning, various decision-making problems have been considered, such as manufacturing resources planning [52, 53], material requirements planning [54, 55], aggregate planning [56–58], and so on.

In detail, the production planning problems have been investigated as follows. Zhao *et al.* researched the key factors that affect the benefits from implementing manufacturing resources planning systems in China [53]. Jamalnia and Soukhakian modeled an aggregate planning problem in a fuzzy environment, which optimized several qualitative and quantitative objectives by genetic algorithm (GA) [56]. Man *et al.* proposed a multiobjective genetic algorithm (MOGA) to handle the earliness/tardiness production scheduling planning problems, which considered multi-product production and multi-process capacity balance [59]. Leung *et al.* studied order scheduling problems in an environment with dedicated resources in parallel, and two novel heuristics were developed to solve these problems [60]. Ashby and Uzsoy solved an order scheduling problem by considering order release, order sequencing, and group scheduling in a single-stage production system [61]. Guo *et al.* took multiple plants, multiple production departments, and multiple production processes into consideration when dealing with a multiobjective order scheduling problem; and Pareto optimization model was provided [20]. Chen and Pundoor researched a static and deterministic order assignment and scheduling problem, the objectives of which were: 1) assigning a set of orders to different plants; 2) delivering the completed orders to the distribution center in an intelligent and efficient way [15].

## **2.2.2 Order Scheduling in Fashion Supply Chains**

Because of the characteristics of fashion industry, like labor-intensive industry, rising labor costs, short product life cycles, and massive product

varieties [14], more attention needs to be paid to fashion order scheduling problems. For the past few decades, order scheduling problems in fashion supply chains have been investigated by many researchers from different perspectives. For example, Chen and Pundoor [15] studied order assignment and scheduling problems in three aspects: to determine which orders should be allocated to each plant, to schedule the allocated orders at each plant, and to schedule the shipping of finished orders from each plant to the distribution center. Leung *et al.* [16] addressed the multi-site order scheduling problem for a multinational lingerie company in Hong Kong. Guo *et al.* [17] investigated a multiobjective order allocation planning problem in a labor-intensive manufacturing company producing sportswear in Mainland China with the consideration of various real-world production features.

In recent years, as a powerful optimization tool [18], evolutionary algorithms (EAs) have been introduced to solve the order scheduling problems in fashion supply chains. For instance, Wong *et al.* utilized genetic algorithm (GA) to plan the production schedules in fabric-cutting department, which improved both makespan and cut-piece fulfilment rates [19]. Guo *et al.* adopted nondominated sorting genetic algorithm II (NSGA-II) to tackle the multiobjective scheduling problem for an apparel manufacturing company in China, which allowed for multiple plants, multiple production departments, and multiple production processes [20]. Wong *et al.* proposed a novel evolution strategy-based Pareto optimization algorithm (ESPO) to cope with the production planning problem in a labor-intensive manufacturing company producing knitwear products in China, which aimed at allocating production processes of each order to appropriate plants [21]. It has been widely recognized that EAs can find more efficient and flexible order schedules than traditional order scheduling methods in fashion industry, which largely depend on the experience of the planners. In the studies above, when the schedules were made before the real production, it was assumed that the daily



production quantity of each order was fixed. However, in real production process, as a result of various uncertainties, like machine breakdown or operator absenteeism, the daily production quantity of each order is not always as expected. In this case, the schedules obtained in [19–21] need to be updated frequently, which means these schedules are very sensitive. Moreover, when visiting some garment factories in Mainland China, it is found that most of the time, operators cannot finish the daily production quantity that was assigned to them and the production plans were shifted very often. It can be figured out that the production is in dynamic environments. And if static schedules are used, after production starts, frequent modification of production schedules will increase labor and time cost, which may reduce production efficiency and fail to complete the orders before their delivery dates.

In addition, in manufacturing systems, dynamic scheduling has been defined under three categories: completely reactive scheduling, predictive-reactive scheduling, and robust pro-active scheduling [62–66]. When uncertainty is taken into consideration, robustness is one of the key factors to maintain the stability of manufacturing systems. And so far, little research has been conducted to generate robust schedules [67]. Moreover, based on the discussions above, dynamic scheduling problems in fashion supply chain management also attracted little attention, and more research work is needed on the generation of robust order schedules.

## **2.3 Evolutionary Optimization**

Evolutionary optimization solves the optimization problems by a category of optimization algorithms that mimics the biological process of evolution [68]. This kind of optimization algorithms is called evolutionary algorithms

(EAs), which is a subset of evolutionary computation in artificial intelligence. The mechanism of EAs derives from biological evolution, which contains crossover, mutation, selection, and so on. EA is a population-based algorithm, and each individual in the population represents a candidate solution to the optimization problem. Fitness function of the optimization problem evaluates the quality of the solutions. At each generation, these individuals undergo crossover, mutation, and selection, and individuals with better fitness values can enter into the next generation. According to the different numbers of the optimization objectives, EAs can be divided into single-objective EAs and multiobjective EAs.

Since the 1970s and 1980s, some algorithmic implementations have been proposed based on the idea of natural evolution, like evolutionary strategies [69], evolutionary programming [70–72], genetic algorithms [73], and genetic programming [74]. In the past two decades, various EAs have been developed, such as differential evolution (DE) [75, 76], particle swarm optimization (PSO) [77, 78], ant colony optimization (ACO) [79, 80], and so on. Over the past years, EAs have proven to be highly efficient when solving complicated optimization problems in various application fields [81], such as engineering design [82–84], image processing [85], data mining [86], robot control [87, 88], supply chain management [20, 89], and so on.

### **2.3.1 Evolutionary Optimization for Fashion Sales**

#### **Forecasting**

As introduced above, neural networks (NNs) have shown their superior performance in fashion sales forecasting problems. Evolutionary optimization have not been introduced into fashion sales forecasting problems until extreme learning machine (ELM) [7] is developed as the learning algorithm for NN. Since then, “EA + ELM” has attracted increasing attentions of

researchers in the fashion sales forecasting area. For instance, a novel EA called harmony search was employed to optimize the ELM-based NN model for medium-term sales forecasting in fashion retail supply chains. And extensive experiments in terms of real fashion sales data demonstrated the effectiveness of their proposed model [10]. Besides, a multivariate intelligent decision-making model was developed by combining harmony search and ELM for a NN, which showed better performance than some non-EA models for the early sales-based retail forecasting problems in fashion industry [23].

It is worth noticing that in the literature above, the EAs used are single-objective, and the NN with the minimum training error is selected as the final network for the fashion sales forecasting problem. In this case, the NN with the smallest training error is considered to have the best forecasting performance when it encounters the unseen data. However, the features of the training samples do not represent the inherent underlying distribution of the new observations due to the existence of noise, which means the prediction effect of the “optimal” NN may be deteriorated by the overfitting phenomenon [11]. So it is not reasonable to merely minimize the training error of NN when executing the forecasting. While actually, for the forecasting problem solved by NN, there are other objectives that need to be optimized besides training error, like the number of hidden layer nodes or the sum of the absolute weights [12, 13]. Therefore, multiobjective EAs will be a promising candidate to replace single-objective EAs to optimize ELM-based NNs in the fashion sales forecasting problems.

### **2.3.2 Evolutionary Optimization for Fashion Order**

#### **Scheduling**

Recently, evolutionary algorithms (EAs) have been introduced to solve the order scheduling problems in fashion industry. For example, genetic

algorithm (GA) was utilized to plan the production schedules in fabric-cutting department, which improves both makespan and cut-piece fulfilment rates [19]. Nondominated sorting genetic algorithm II (NSGA-II) was adopted to tackle the multiobjective scheduling problem for an apparel manufacturing company in China, which allows for multiple plants, multiple production departments, and multiple production processes [20]. A novel evolution strategy-based Pareto optimization algorithm (ESPO) was proposed to cope with the production planning problem in a labor-intensive manufacturing company producing knitwear products in China, which aims at allocating production processes of each order to appropriate plants [21]. It has been widely recognized that EAs can find more efficient and flexible order schedules than traditional order scheduling methods in fashion industry, which largely depend on the experience of the planners.

In the studies above, when the schedules were made before the real production, it was assumed that the daily production quantity of each order was fixed. However, in real production process, as a result of various uncertainties, like machine breakdown or operator absenteeism, the daily production quantity of each order is not always as expected. In this case, the schedules obtained in [19–21] need to be updated frequently, which means these schedules are very sensitive. Moreover, when visiting some garment factories in Mainland China, it is found that most of the time, operators cannot finish the daily production quantity that was assigned to them and the production plans were shifted very often. After production begins, frequent modification of production plans will increase labor and time cost, which may reduce production efficiency and fail to complete the orders before their delivery dates. Therefore, it will solve the problem if planners can make a schedule that is optimized by EA, and shows robustness to the uncertainty at the same time.

## 2.4 Summary

According to the above literature review, the conclusions can be made as follows:

(1) In single-objective EA-based NN models for fashion sales forecasting, minimizing the training error is the only objective to be optimized, which means the NN with the smallest training error is used for predicting the future data. However, in order to alleviate the overfitting phenomenon, it is necessary to consider more optimization objectives when EA is used for fashion sales forecasting problems.

(2) For order scheduling problems in fashion supply chains, the schedules obtained by EA are sensitive to stochastic variation of daily production quantity during the process of real production. Therefore, it is necessary to develop robust order schedules in terms of EA for order scheduling problems in fashion industry.

In summary, based on the two conclusions, firstly, this research investigates the fashion sales forecasting problems by multiobjective EA-based models, the purpose of which is to alleviate the overfitting phenomenon in the forecasting and increase the prediction accuracy. Secondly, this research introduces robust evolutionary optimization into order scheduling problems in fashion supply chains, in order to obtain robust order schedules which are not sensitive to stochastic variation of daily production quantity during the process of real production, which means planners will reduce the times of modifying the order schedules during the production process.

# Chapter 3

## Research Methodology

In this research, sales forecasting and order scheduling problems in fashion supply chains are investigated. An accurate and timely forecasting helps retailers closely match the demand and supply in the competitive global apparel market. And an efficient and flexible order schedule benefits the distribution and retailing in fashion supply chains. These two decision-making problems are of paramount importance to manufacturing and retailing, which are two key components in fashion supply chains.

For fashion sales forecasting, neural network-based models have attracted much attention in recent years. In these NN-based models, extreme learning machine (ELM) and single-objective evolutionary algorithms (SOEAs) play a significant role in adjusting and optimizing the parameters of NN. ELM takes the place of back-propagation (BP) algorithm as the learning algorithm of NN, and is able to provide good generalization performance and fast learning speed for NN at the same time. While SOEAs are used to search the optimal values of input weights and hidden biases of ELM-based NN, where training error is set as the only optimization objective. However, the features of the training samples do not represent the inherent underlying distribution of the new observations due to the existence of noise. In order to alleviate

the overfitting phenomenon, we think that it is necessary to introduce more objectives when optimizing the ELM-based NN. Therefore, in this research, multiobjective evolutionary optimization is integrated into ELM-based NN model, which are utilized for the fashion sales forecasting problem.

For fashion order scheduling, evolutionary algorithms (EAs) have also been introduced as a powerful tool recently. The order scheduling problem is modeled as an optimization problem, and EAs are utilized to search the optimal schedules according to different predefined objectives. In the modelling, when the schedules are made before the real production, it is assumed that the daily production quantity of each order is fixed. However, in real-world production, as a result of various uncertainties, like machine breakdown or operator absenteeism, the daily production quantity of each order is not always as expected. In this case, the schedules need to be updated frequently, which means these schedules are very sensitive to perturbations. Therefore, in this research, robust evolutionary optimization is firstly introduced into the order scheduling problems in fashion supply chains, and robust EAs are adopted to search optimal and robust schedules for the problems.

Based on the discussions above, evolutionary optimization serves as the basis of the research methodologies in this research, and novel EAs or EA-based models are proposed to solve the sales forecasting and order scheduling problems in fashion supply chains. EAs belong to the optimization methods in artificial intelligence. The mechanism of EA derives from biological evolution, which contains crossover, mutation, selection, and so on. EA is a population-based algorithm, and each individual in the population represents a candidate solution to the optimization problem. Unlike traditional gradient-based optimization methods, EA is able to jump out of the local optima of the problem, which makes EA a popular and powerful tool when facing with the optimization problems in various research

fields. In this research, the methodologies can be divided two parts: 1) multiobjective evolutionary optimization-based NN model for fashion sales forecasting; 2) robust evolutionary optimization for fashion order scheduling. In the following, the research methodologies are introduced from these two aspects, including the concepts and principles of multiobjective optimization, neural network, extreme learning machine, differential evolution, and robust optimization.

### **3.1 Multiobjective Evolutionary Optimization-Based Neural Network Model**

In this research, a multiobjective evolutionary optimization-based neural network (MOONN) model is developed to solve the fashion sales forecasting problem. MOONN is a NN model, the learning algorithm of which is extreme learning machine (ELM). The optimization of the parameters in this NN model is modeled as a multiobjective optimization problem. And a novel multiobjective evolutionary algorithm (MOEA) is developed to search the optimal parameters for the NN model. In the following, we will briefly introduce the concepts of multiobjective optimization, neural network, and extreme learning machine.

#### **3.1.1 Multiobjective Optimization**

Nowadays, many real-world optimization problems involve various objectives that often conflict with each other. A multiobjective optimization problem can be formulated as follows (without any loss of generality, a minimization



problem is considered with a decision space  $\Omega$ ):

$$\begin{aligned} & \text{minimize} && F(x) = (f_1(x), \dots, f_n(x)) \\ & \text{s.t.} && x \in \Omega, \end{aligned} \tag{3.1}$$

where  $\Omega$  is a decision space and  $x \in \Omega$  is a decision vector.  $F(x) = (f_1(x), \dots, f_n(x))$  is the objective vector with  $n$  objectives to be minimized.

The objectives in (3.1) are conflicting pairs, which means that there is not a single solution optimizing all the objectives simultaneously. So it is necessary to seek a group of solutions that can balance all the objectives. Here the definitions of Pareto dominance, Pareto optimal solution, Pareto front and nondomination level are introduced below.

**Definition 3.1. (Pareto Dominance):** Given two objective vectors  $X_1, X_2 \in \mathbb{R}^n$ , then  $X_1$  dominates  $X_2$ , denoted as  $X_1 \prec X_2$ , iff  $x_{1i} \leq x_{2i}, \forall i \in \{1, 2, \dots, n\}$  and  $x_{1j} < x_{2j}, \exists j \in \{1, 2, \dots, n\}$ .

**Definition 3.2. (Pareto Optimal Solution):** A feasible solution  $x^* \in \Omega$  of (3.1) is called a Pareto optimal solution, iff  $\nexists x \in \Omega$  such that  $F(x) < F(x^*)$ .

**Definition 3.3. (Pareto Front):** The image of all the Pareto optimal solutions in the objective space is called the Pareto front (PF).

**Definition 3.4. (Nondomination Level):** For a group of vectors  $X_1, X_2, \dots, X_N \in \mathbb{R}^n$ , if none of the vectors in this group is dominated by the rest members of the group, we call this group of vectors belongs to the same nondomination level.

Figure 3.1 illustrates the dominance relationships between different solutions, where the solutions represented by closed blue circles are dominated by the solutions denoted by closed red squares. And all the blue solutions belong to the same nondomination level, while the whole red ones belong to another nondomination level.

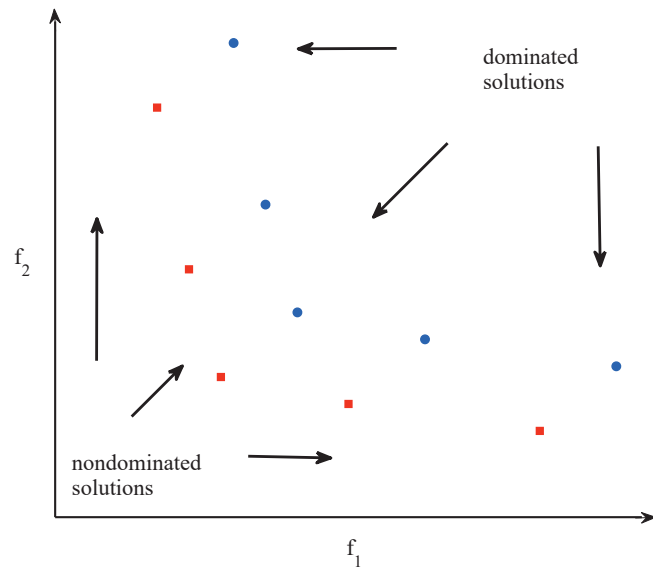


Figure 3.1: Illustration of the relationship between dominated and nondominated solutions.

### 3.1.2 Neural Network

Neural network, also called artificial neural network, is a mathematical model consisting of a group of artificial neurons connecting with each other; the strength of a connection between two nodes is called “weight”. Neural networks have been greatly employed in pattern classification, clustering, function approximation, forecasting, optimization, and so on [90]. An NN typically has three types of parameters: the connection pattern between the different layers of neurons; the learning process for updating the weights of the connections; and the activation function that converts a neuron’s weighted input to its output activation. According to the connection pattern, NNs can be grouped into two categories: feedforward networks and recurrent networks. In feedforward networks, graphs have no loops, while in recurrent networks; loops occur because of feedback connections. In terms of learning, there are three main learning paradigms: supervised, unsupervised, and hybrid. Under each paradigm, there are four basic types of learning rules: error-correction, Boltzmann, Hebbian, and competitive learning [90]. Based on different paradigm and learning rule, various learning algorithms have

been proposed, like back-propagation algorithm, linear discriminant analysis, principal component analysis, and so on. Activation function is crucial to NN, which determines the new level of activation based on the effective input and the current activation. There are several commonly-used activation functions: sigmoid, piecewise linear, and Gaussian function.

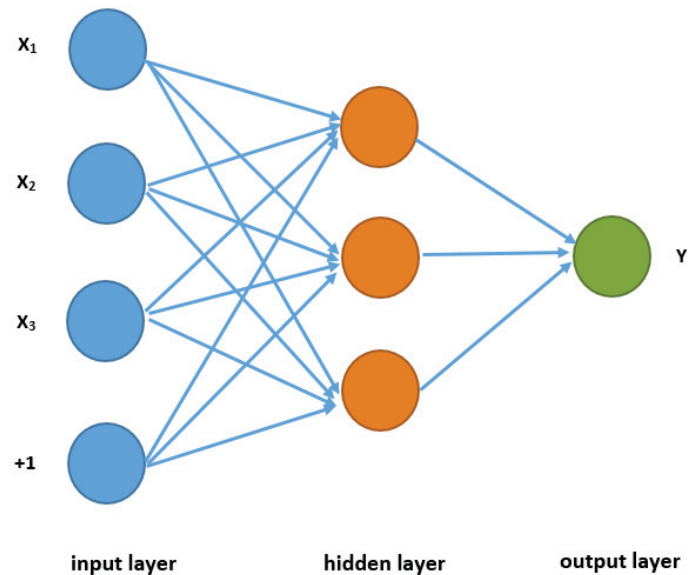


Figure 3.2: An example of SLFN.

Among all types of neural networks, feedforward neural networks have been extensively applied in many fields. Meanwhile, the universal approximation capability theorem [91] proves that a single hidden layer of neurons is totally enough for approximating any given function. Therefore, single hidden layer feedforward neural network (SLFN) is one of the most popular feedforward neural networks. Figure 3.2 shows an example of SLFN. In the figure,  $x_1$ ,  $x_2$ ,  $x_3$  are inputs of the network, “+1” is the bias, while  $y$  is the output.

### 3.1.3 Extreme Learning Machine

The extreme learning machine (ELM) algorithm was firstly proposed by Huang *et al.* [7], which aims to provide a better generalization performance

and much faster learning speed than gradient learning algorithms (like back-propagation algorithm). It makes use of the SLFN. The main concept behind ELM lies in the random initialization of the weights and biases in SLFN, from which the hidden layer output matrix and output weights can be calculated using the Moore-Penrose generalized inverse. The network is obtained with very few steps and very low computational cost. The summarized ELM algorithm is given below.

---

**Algorithm 1** Extreme learning machine (ELM)

---

- 1: Given a training set  $(\mathbf{x}_i, \mathbf{y}_i)$ ,  $\mathbf{x}_i \in \mathbf{R}^{d_1}$ ,  $\mathbf{y}_i \in \mathbf{R}^{d_2}$ , an activation function  $f: \mathbf{R} \rightarrow \mathbf{R}$ , and the number of hidden nodes  $N$ ;
  - 2: Randomly assign input weights  $\mathbf{w}_i$  and biases  $b_i$ ,  $1 \leq i \leq N$ ;
  - 3: Calculate the hidden layer output matrix  $\mathbf{H}$ ;
  - 4: Calculate the output weights matrix  $\beta = \mathbf{H}^\dagger \mathbf{Y}$ .
- 

In Algorithm 1,  $\mathbf{H}$  is the hidden layer output matrix, which can be calculated by the input, input weights and biases.  $\mathbf{H}^\dagger$  is the Moore-Penrose generalized inverse of matrix  $\mathbf{H}$ , which is the minimum norm least-square solution.

## 3.2 Robust Evolutionary Optimization

In this research, fashion order scheduling problems are solved via a robust differential evolution algorithm called robust success-history based adaptive differential evolution with event-triggered impulsive control scheme (ETI-SHADE). The fashion order scheduling problem is modeled as a single-objective robust optimization problem, in which perturbations are considered. By using robust ETI-SHADE, the scheduling process can be viewed as forward scheduling, because the optimization objective is to minimize the total tardiness of all the orders (schedule the orders as early as possible). In the following, we briefly introduce the definitions of robust optimization and differential evolution (DE). The details of ETI-SHADE will be explained in Chapters 4 and 6.

### 3.2.1 Robust optimization

Robust optimization deals with a type of optimization problems, the solutions of which are sensitive to variable perturbations. In robust optimization, practitioners may not be interested in searching the so-called global optimal solutions, especially when these solutions are sensitive to variable perturbations. In real-world optimization problems, practitioners prefer solutions that are less sensitive to small perturbations, and these are called *robust solutions*. According to the different number of optimization objectives, robust optimization can be categorized into: single-objective robust optimization and multiobjective robust optimization.

For a single-objective optimization problem:

$$\text{minimize } f(\mathbf{x}), \quad \mathbf{x} \in \Omega, \quad (3.2)$$

where  $\Omega$  is a decision space,  $\mathbf{x} = [x_1, x_2, \dots, x_D]^T$  is a decision vector, and  $D$  is the dimension size, representing the number of the decision variables involved in the problem. In Fig. 3.3, A and B are two minimum solutions of the problem. Of these two solutions, although B is the theoretical global best solution, it is quite sensitive to the variable perturbation, which means a small perturbation of B will alter the objective function value by a significant amount. While for A, we can find that the same perturbation of A only causes a small change to the objective function value, which indicates solution A is robust to the small perturbation. So for robust optimization, the target is to find solutions like A, while not like B.

For a robust optimization problem, there are several ways to find a robust solution. And one of the most popular and effective methods is to utilize a mean effective objective function for optimization, instead of using the original objective function. In the following, the definition of a robust solution

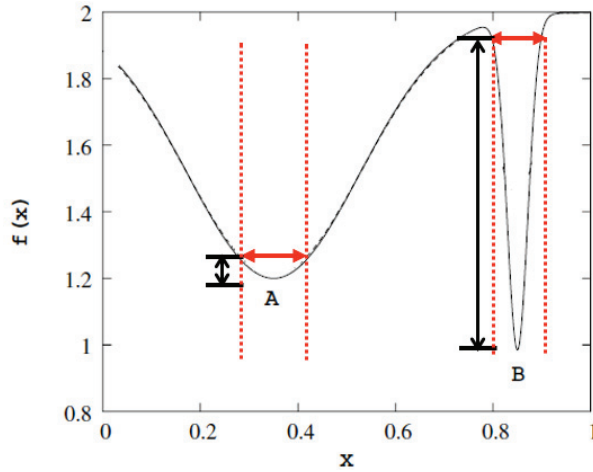


Figure 3.3: Illustration of global best solution (A) vs. robust solution (B) in a single-objective optimization problem.

is given.

**Definition 3.5. (Robust Solution):** For the minimization of an objective function  $f(\mathbf{x})$ , a solution  $\mathbf{x}^*$  is called a robust solution, if it is the global minimum of the mean effective function  $f^{\text{eff}}(\mathbf{x})$  defined with respect to a  $\delta$ -neighborhood as follows:

$$\text{minimize } f^{\text{eff}}(\mathbf{x}) = \frac{1}{|\mathcal{B}_\delta(\mathbf{x})|} \int_{\mathbf{y} \in \mathcal{B}_\delta(\mathbf{x})} f(\mathbf{y}) d(\mathbf{y}), \quad \mathbf{x} \in \Omega, \quad (3.3)$$

where  $\mathcal{B}_\delta(\mathbf{x})$  is the  $\delta$ -neighborhood of the solution  $\mathbf{x}$  and  $|\mathcal{B}_\delta(\mathbf{x})|$  is the hypervolume of the neighborhood.

According to Definition 3.5,  $f^{\text{eff}}(\mathbf{x})$  replaces  $f(\mathbf{x})$  to be the objective function of the optimization problem. For the past decade and more, robust optimization has gained increasing attention, and has been incorporated into the framework of EAs [92–97]. The target of robust EAs is to deal with uncertainties in optimization problems by EAs, especially in real-world optimization problems [96, 98–100].

### 3.2.2 Differential Evolution

Differential evolution (DE) is a population-based evolutionary algorithm for a numerical optimization problem. It initializes a population of  $NP$  individuals in a  $D$ -dimensional search space. Each individual represents a potential solution to the optimization problem. After initialization, at each generation, three operators: mutation, crossover and selection are employed to generate the offspring for the current population.

#### Mutation

Mutation is the most consequential operator in DE. Each vector  $\mathbf{x}_{i,G}$  in the population at the  $G$ th generation is called *target* vector. A mutant vector called *donor* vector is obtained through the differential mutation operation. For simplicity, the notation “DE/ $a/b$ ” is used to represent different mutation operators, where “DE” denotes the differential evolution, “ $a$ ” stands for the base vector, and “ $b$ ” indicates the number of difference vectors utilized. In DE, there are six mutation operators that are most widely used:

i) “DE/rand/1”

$$\mathbf{v}_{i,G} = \mathbf{x}_{r_1,G} + F \cdot (\mathbf{x}_{r_2,G} - \mathbf{x}_{r_3,G}), \quad (3.4)$$

ii) “DE/rand/2”

$$\mathbf{v}_{i,G} = \mathbf{x}_{r_1,G} + F \cdot (\mathbf{x}_{r_2,G} - \mathbf{x}_{r_3,G}) + F \cdot (\mathbf{x}_{r_4,G} - \mathbf{x}_{r_5,G}), \quad (3.5)$$

iii) “DE/best/1”

$$\mathbf{v}_{i,G} = \mathbf{x}_{best,G} + F \cdot (\mathbf{x}_{r_1,G} - \mathbf{x}_{r_2,G}), \quad (3.6)$$

iv) “DE/best/2”

$$\mathbf{v}_{i,G} = \mathbf{x}_{best,G} + F \cdot (\mathbf{x}_{r_1,G} - \mathbf{x}_{r_2,G}) + F \cdot (\mathbf{x}_{r_3,G} - \mathbf{x}_{r_4,G}), \quad (3.7)$$

v) “DE/current-to-best/1”

$$\mathbf{v}_{i,G} = \mathbf{x}_{i,G} + F \cdot (\mathbf{x}_{best,G} - \mathbf{x}_{i,G}) + F \cdot (\mathbf{x}_{r_1,G} - \mathbf{x}_{r_2,G}), \quad (3.8)$$

vi) “DE/current-to-rand/1”

$$\mathbf{u}_{i,G} = \mathbf{x}_{i,G} + K \cdot (\mathbf{x}_{r_1,G} - \mathbf{x}_{i,G}) + \hat{F} \cdot (\mathbf{x}_{r_2,G} - \mathbf{x}_{r_3,G}), \quad (3.9)$$

where  $\mathbf{x}_{best,G}$  specifies the best individual in the current population;  $r_1, r_2, r_3, r_4$  and  $r_5 \in \{1, 2, \dots, NP\}$ , and  $r_1 \neq r_2 \neq r_3 \neq r_4 \neq r_5 \neq i$ . The parameter  $F > 0$  is called *scaling factor*, which scales the difference vector. It is worth mentioning that (3.9) shows the rotation-invariant mutation [101].  $K$  is the combination coefficient, which should be selected with a uniform random distribution from  $[0, 1]$  and  $\hat{F} = K \cdot F$ . Since “DE/current-to-rand/1” contains both mutation and crossover, it is not necessary for the offspring to go through the crossover operation.

## Crossover

After mutation, a binomial crossover operation is implemented to generate the *trial* vector  $\mathbf{u}_i = [u_{i1}, u_{i2}, \dots, u_{iD}]^T$ :

$$u_{ij,G} = \begin{cases} v_{ij,G}, & \text{if } \text{rand}(0, 1) \leq CR \text{ or } j = j_{rand}, \\ x_{ij,G}, & \text{otherwise,} \end{cases} \quad (3.10)$$



where  $\text{rand}(0, 1)$  is a uniform random number in the range  $[0, 1]$ .  $CR \in [0, 1]$  is called *crossover probability*, which determines how much the trial vector is inherited from the mutant vector.  $j_{rand}$  is an integer randomly selected from 1 to  $D$  and newly generated for each  $i$ , which ensures at least one dimension of the trial vector will be different from the corresponding target vector. If  $u_{ij,G}$  is out of the boundary, it will be reinitialized in the range  $[L_j, U_j]$ .

## Selection

The selection operator employs a one-to-one swapping strategy, which picks the better one from each pair of  $\mathbf{x}_{i,G}$  and  $\mathbf{u}_{i,G}$  for the next generation:

$$\mathbf{x}_{i,G+1} = \begin{cases} \mathbf{u}_{i,G}, & \text{if } f(\mathbf{u}_{i,G}) \leq f(\mathbf{x}_{i,G}), \\ \mathbf{x}_{i,G}, & \text{otherwise.} \end{cases} \quad (3.11)$$

## Some variants of DE

In recent years, various DE variants have been proposed to solve the global numerical optimization problems. In this subsection, we choose several popular and powerful DE variants, and briefly describe the principles of them.

### 1) Differential evolution with self-adapting parameters (jDE)

A new version of DE called jDE was proposed recently, in which a self-adaptive approach are developed for parameters  $F$  and  $CR$  [102]. The parameters are calculated as follows:

$$F_{i,G+1} = \begin{cases} F_l + \text{rand}_1 * F_u, & \text{if } \text{rand}_2 < \tau_1, \\ F_{i,G}, & \text{otherwise,} \end{cases} \quad (3.12)$$

$$CR_{i,G+1} = \begin{cases} \text{rand}_3, & \text{if } \text{rand}_4 < \tau_2, \\ CR_{i,G}, & \text{otherwise,} \end{cases} \quad (3.13)$$

where  $rand_j, j \in \{1, 2, 3, 4\}$  are uniform random values from  $[0, 1]$ ;  $\tau_1$  and  $\tau_2$  are probabilities to adjust  $F$  and  $CR$ , respectively;  $F_l$  and  $F_u$  indicate the lower and upper bounds of  $F$ , respectively. The settings affect the mutation, crossover, and selection operations of the new vector.

## 2) Adaptive differential evolution with optional external archive (JADE)

JADE was developed by Zhang *et al.* [103], which is dedicated to solving single objective optimization problems. This algorithm shows its effectiveness in both unimodal and multimodal functions. The strategy “DE/current-to- $p$ best” with optional archive helps JADE achieve good balance between “greedy” and “diverse”, which is as follows:

$$\mathbf{v}_{i,G} = \mathbf{x}_{i,G} + F_i \cdot (\mathbf{x}_{best,G}^p - \mathbf{x}_{i,G}) + F_i \cdot (\mathbf{x}_{r_1,G} - \tilde{\mathbf{x}}_{r_2,G}), \quad (3.14)$$

where  $\mathbf{x}_{best,G}^p$  is randomly selected as one of the top  $100p\%$  individuals in the current population with  $p \in (0, 1]$ .  $\tilde{\mathbf{x}}_{r_2,G}$  is stochastically chosen from the current population and a set of archived inferior solutions.  $F_i$  denotes the scaling factor, which is updated independently in terms of a Cauchy distribution with location parameter  $\mu_F$  and scale parameter 0.1:

$$F_i = \text{randc}_i(\mu_F, 0.1), \quad (3.15)$$

$$\mu_F = (1 - c) \cdot \mu_F + c \cdot \text{mean}_L(S_F), \quad (3.16)$$

$$\text{mean}_L(S_F) = \frac{\sum_{F \in S_F} F^2}{\sum_{F \in S_F} F}, \quad (3.17)$$

where  $S_F$  stores all the successful scaling factors at the current generation;  $\mu_F$  is initialized as 0.5. Similarly, the crossover probability  $CR_i$  of each individual is generated according to a normal distribution of mean  $\mu_{CR}$  and

standard deviation 0.1:

$$CR_i = \text{randn}_i(\mu_{CR}, 0.1), \quad (3.18)$$

$$\mu_{CR} = (1 - c) \cdot \mu_{CR} + c \cdot \text{mean}_A(S_{CR}), \quad (3.19)$$

where  $S_{CR}$  is the set of all successful crossover probabilities at the current generation.

### 3) Composite differential evolution (CoDE)

Recently, CoDE was presented by Wang *et al.* [104], which combines three effective trial vector generation strategies with three groups of proper parameter settings. The three strategies and the corresponding parameter settings are listed as follows:

$$\text{DE/rand/1} \quad [F = 1.0, CR = 0.1]$$

$$\text{DE/rand/2} \quad [F = 1.0, CR = 0.9]$$

$$\text{DE/current-to-rand/1} \quad [F = 0.8, CR = 0.2]$$

In CoDE, three trial vectors are generated for each target vector; and the best one can enter into the next generation if it is better than the target vector.

### 4) Differential evolution with Strategy Adaptation (SaDE)

A self-adaptive DE called SaDE was developed, in which both trial vector generation strategies and control parameter settings are updated according to the historical experiences in breeding superior solutions [105]. The candidate strategy pool consists of four trial vector generation strategies: DE/rand/1, DE/rand/2, DE/rand-to-best/2, and DE/current-to-rand/1. The probability of choosing each strategy can be calculated as follows:

$$p_{k,G} = \frac{S_{k,G}}{\sum_{k=1}^4 S_{k,G}}, \quad (3.20)$$

$S_{k,G}$  denotes the success rate of the trial vector, which is generated by the  $k$ th strategy and successfully replaces its target vector.

### **3.3 Summary**

This chapter presents research methodologies from two aspects: 1) multiobjective evolutionary optimization-based NN model for fashion sales forecasting; 2) robust evolutionary optimization for fashion order scheduling. The concepts and principles of multiobjective optimization, differential evolution, robust optimization, neural network, and extreme learning machine are introduced in detail. The methodologies are utilized to solve the sales forecasting problems and order scheduling problems in fashion supply chains in the following chapters.

# **Chapter 4**

## **Newly Proposed Evolutionary Algorithms for Fashion Supply Chains**

In this chapter, first of all, a novel multiobjective evolutionary algorithm called nondominated sorting adaptive differential evolution (NSJADE) is proposed. Then a event-triggered impulsive control scheme (ETI) is developed to improve the performance of differential evolution (DE), hence a new DE called ETI-DE can be obtained. In the later chapters, NSJADE-based model is developed for fashion sales forecasting problems; ETI-DE is modified to solve the order scheduling problems in fashion supply chains.

### **4.1 Nondominated Sorting Adaptive Differential Evolution**

In this section, a new multiobjective evolutionary algorithm (MOEA) is proposed, which is called nondominated sorting adaptive differential evolution

(NSJADE). Based on NSJADE, a sales forecasting problem in fashion supply chains is handled in the next chapter. In the following, firstly, the motivation of presenting NSJADE is provided; secondly, the detailed framework of the algorithm is given; thirdly, a group of experiments demonstrate the effectiveness of NSJADE.

### **4.1.1 Motivation**

In the past two decades, MOEAs have attracted increasing attentions in the area of evolutionary computation. The objective of a MOEA is to search the Pareto front (see Definition 3.3) of the optimization problem as fast as possible. Recently, a variety of MOEAs have been proposed, such as nondominated sorting genetic algorithm II (NSGA-II) [106], a multiobjective evolutionary algorithm based on decomposition (MOEA/D) [107], a general indicator-based evolutionary algorithm (IBEA) [108], the strength Pareto evolutionary algorithm 2 (SPEA2) [109], and so on. Among them, NSGA-II is one of the most popular MOEAs. In NSGA-II, there are two critical operations: 1) fast nondominated sorting operation; 2) crowded-comparison operation. In fast nondominated sorting operation, all the solutions obtained at the current generation can be quickly sorted into different nondomination levels. While crowded-comparison operation differs individuals in the same nondomination level, which means the individual located in a less crowded area is preferred. The brief procedure of NSGA-II is shown in Fig. 4.1.

It is worth mentioning that NSGA-II fails to find the Pareto fronts of some multimodal problems (e.g., ZDT4 and ZDT6 of the ZDT test suite) in [106]. And we impute it to the genetic algorithm (GA), which serves as the search engine in NSGA-II. Although GA has proven to be an effective method for many optimization problems, its performance will deteriorate when facing complicated problems. The reason is that GA does not take advantage of

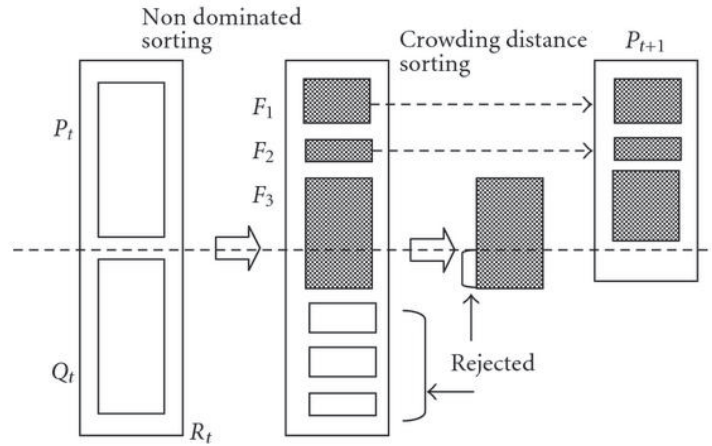


Figure 4.1: Illustration of the NSGA-II procedure.

the useful historical information to guide the search. Recently, a novel differential evolution (DE) algorithm called adaptive differential evolution (JADE) was proposed by Zhang and Sanderson [103]. JADE utilizes the top  $100p\%$  individuals in the current population with  $p \in (0, 1]$  to direct the search of the population, and forces the individuals away from the archived inferior solutions (see Section 3.2.2). And the experimental results in [103] demonstrate the superior performance of JADE in solving multimodal problems. Therefore, it is reasonable to replace the non-adaptive GA part in NSGA-II with JADE, and a nondominated sorting adaptive differential evolution (NSJADE) is proposed in the following.

#### 4.1.2 Nondominated Sorting Adaptive Differential Evolution

NSJADE is developed based on NSGA-II, and the purpose is to improve the search efficiency of the population by substituting GA with JADE as the search engine. The pseudocode of NSJADE is given in Algorithm 2.

Firstly, the settings of parameters are provided; and a population containing  $NP$  individuals is initialized. Then different levels of Pareto fronts are

generated by the *Nondominated\_Sort* function according to the objectives in the optimization problem, and the crowding-distance value is assigned to each individual  $\mathbf{x}$  [106]. Step 5 to step 28 indicate the search engine part of the proposed NSJADE, which is similar to JADE except for the selection operator (from step 22 to step 24). Three parent individuals are selected from 100p% best individuals, the current population  $\mathbf{P}$  and the current population  $\mathbf{P}$  combined with an archive  $\mathbf{A}$ , respectively. Then after mutation and crossover, a new offspring is generated. A parent individual will be added into the optional archive  $\mathbf{A}$  if it is dominated by its offspring. After checking the size of  $\mathbf{A}$  and updating the values of  $\mu_{CR}$  and  $\mu_F$ , the parent and the offspring individuals are combined by the *Combine* function. Then the new generation is generated after the *Nondominated\_Sort* and *Select\_Elitism* operations.

---

**Algorithm 2** Nondominated sorting adaptive differential evolution (NSJADE)

---

```

1: Begin
2:   Set  $\mu_{CR} = 0.5; \mu_F = 0.5; \mathbf{A} = \emptyset$ 
3:   Create a random initial population  $\{\mathbf{x}_{i,0} | i = 1, 2, \dots, NP\}$ 
4:    $\mathbf{x} = \text{Nondominated\_Sort}(\mathbf{x})$ 
5:   for  $g = 1$  to  $G$  do
6:      $S_F = \emptyset; S_{CR} = \emptyset$ 
7:     for  $i = 1$  to  $NP$  do
8:       Generate  $CR_i = \text{rand}_i(\mu_{CR}, 0.1), F_i = \text{rand}_i(\mu_F, 0.1)$ 
9:       Randomly select  $\mathbf{x}_{best,g}^p$  from 100p% best individuals
10:      Randomly select  $\mathbf{x}_{r1,g} \neq \mathbf{x}_{i,g}$  from current population  $\mathbf{P}$ 
11:      Randomly select  $\tilde{\mathbf{x}}_{r2,g} \neq \mathbf{x}_{r1,g} \neq \mathbf{x}_{i,g}$  from  $\mathbf{P} \cup \mathbf{A}$ 
12:       $\mathbf{v}_{i,g} = \mathbf{x}_{i,g} + F_i \cdot (\mathbf{x}_{best,g}^p - \mathbf{x}_{i,g}) + F_i \cdot (\mathbf{x}_{r1,g} - \tilde{\mathbf{x}}_{r2,g})$ 
13:      Check the boundary of  $\mathbf{v}_{i,g}$ 
14:      Generate  $j_{\text{rand}} = \text{randint}(1, D)$ 
15:      for  $j = 1$  to  $D$  do
16:        if  $j = j_{\text{rand}}$  or  $\text{rand}(0, 1) < CR_i$  then
17:           $u_{j,i,g} = v_{j,i,g}$ 
18:        else
19:           $u_{j,i,g} = x_{j,i,g}$ 
20:        end if
21:      end for
22:      if  $f(\mathbf{u}_{i,g}) < f(\mathbf{x}_{i,g})$  then
23:         $\mathbf{x}_{i,g} \rightarrow \mathbf{A}; CR_i \rightarrow S_{CR}, F_i \rightarrow S_F$ 
24:      end if
25:    end for
26:    Randomly remove individuals from  $\mathbf{A}$  to maintain  $\text{size}(\mathbf{A}) = NP$ 
27:     $\mu_{CR} = (1 - c) \cdot \mu_{CR} + c \cdot \text{mean}_A(S_{CR})$ 
28:     $\mu_F = (1 - c) \cdot \mu_F + c \cdot \text{mean}_L(S_F)$ 
29:     $\mathbf{x}_{\text{all}} = \text{Combine}(\mathbf{x}, \mathbf{u})$ 
30:     $\mathbf{x}_{\text{all}} = \text{Nondominated\_Sort}(\mathbf{x}_{\text{all}})$ 
31:     $\mathbf{x} = \text{Select\_Elitism}(\mathbf{x}_{\text{all}})$ 
32:  end for
33: End

```

---

It is worth noticing that from step 22 to step 24, an individual dominated by its offspring is dominated as a loser, and it will be stored into archive



**A.** Here a situation is left out: the parent has the same rank with its offspring, but has a smaller crowding distance. In terms of the comparison rules defined in [106], this parent individual is also a loser and should be kept in **A**. However, if this situation is considered in NSJADE, which means the crowding-distance values of these two individuals should be computed, the worst-case complexity of this operation is  $O(NP \log(2NP))$ . Therefore, in order to reduce the computation complexity of the algorithm, only the dominance relationship between these two individuals is calculated, instead of the crowding distance if they are in the same Pareto front.

### 4.1.3 Experiments with Test Functions

Deb *et al.* used the ZDT test suite (ZDT1-ZDT4, ZDT6) to examine the performance of NSGA-II in [106]. Among them, ZDT4 is a complicated multimodal problem, which has  $21^9$  or  $7.94(10^{11})$  different local Pareto optimal fronts in the search space, merely one of which is the global best Pareto front. The experimental results in [106] show that real-coded NSGA-II gets stuck at different local Pareto optimal sets when solving ZDT4. For ZDT6, which is a nonconvex and nonuniform problem, NSGA-II also cannot converge to the true Pareto front. Next, we test the convergence effect of NSJADE and NSGA-II on ZDT4 and ZDT6. For the integrity of the experiments, we also list the results of these two algorithms on the rest ZDT functions. Both algorithms are run for 250 generations, and have the same population size  $NP = 100$ . For NSGA-II, we use the same distribution indices for crossover and mutation operators as in [106]:  $\eta_c = 20$ ,  $\eta_m = 20$ . For NSJADE, the parameters in JADE part are the same with [103]: the value of parameter  $c$  is 0.1; the size of the optional archive is the same as that of the population; the parameter  $p$  in  $p$ best mechanism is assigned to 0.05; other parameters are provided in Algorithm 2.

Fig. 4.2 shows the nondominated solutions obtained by NSJADE and NSGA-II on ZDT functions. From Fig. 4.2, we find that for ZDT1-ZDT3, NSJADE and NSGA-II have the similar convergence performance, which means the solutions obtained overlap the true Pareto front of each test function. While for ZDT4 and ZDT6, the solutions found by NSGA-II are a little bit far from the true Pareto front, which indicates that NSGA-II fails to find the Pareto front of these two functions. However, our proposed NSJADE can successfully help the population converge to the global Pareto optimal fronts of ZDT4 and ZDT6, which can be attributed to the powerful search performance of the modified search engine: JADE.

## **4.2 Differential Evolution with Event-triggered Impulsive Control Scheme**

In this section, an event-triggering-based impulsive control scheme (ETI) is introduced to improve the performance of differential evolution (DE). In the rest of this section, firstly, the motivation of proposing ETI is given; secondly, the four components of ETI are described in detail; thirdly, extensive experiments are conducted to display the effectiveness of the proposed ETI.

### **4.2.1 Motivation**

Differential evolution (DE), firstly proposed by Storn and Price [75, 76], has proven to be a reliable and powerful population-based evolutionary algorithm for global numerical optimization. Generally, DE employs three main operators: mutation, crossover, and selection at each generation for the population production [110–112]. The mutation operator provides the individuals with a sudden change or perturbation, which helps explore the

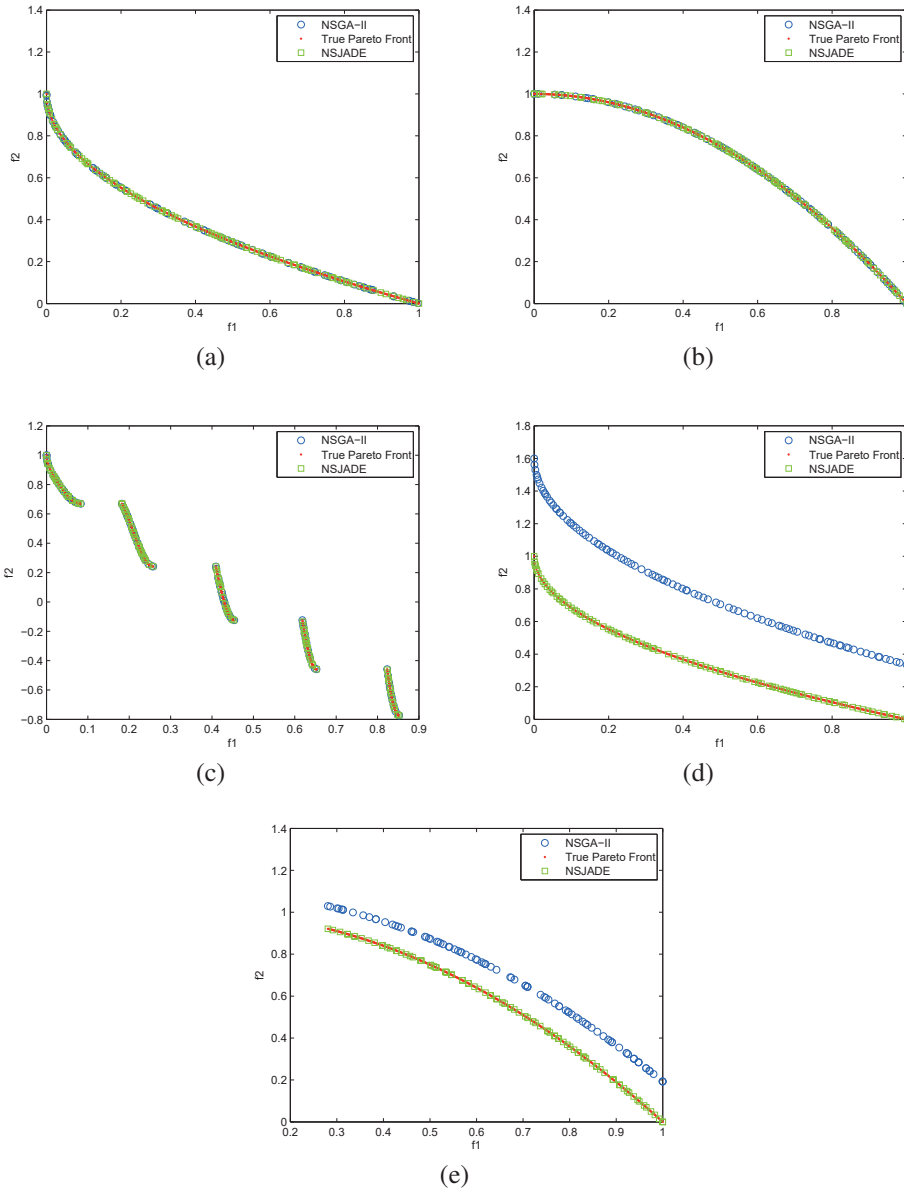


Figure 4.2: Comparison of nondominated solutions with NSGA-II & NSJADE on ZDT functions. (a) ZDT1; (b) ZDT2; (c) ZDT3; (d) ZDT4; (e) ZDT6.

search space. In order to increase the diversity of the population, the crossover operator is implemented after the mutation operation. The selection operator chooses the better one between a parent and its offspring, which guarantees that the population never deteriorates. In addition to these three basic operators, there are three control parameters which greatly influence the performance of DE: the mutation scale factor  $F$ , the crossover rate  $CR$ , and the population size  $NP$ . Most of the current research on DE has focused on four aspects to enhance the performance of DE: developing novel mutation operators [103, 113–120], designing new parameter control strategies [102, 103, 113, 114, 121–125], improving crossover operator [114, 126–128], and pooling multiple mutation strategies [104, 105, 129, 130].

Despite numerous efforts on improving DE from the above four aspects, there are some DE variants which take advantage of ideas from other disciplines. For instance, Rahnamayan *et al.* [131] presented opposition-based DE (ODE), which adopts opposition-based learning, a new scheme in machine intelligence, to speed up the convergence rate of DE. Laelo and Ali [132] made use of the attraction-repulsion concept in electromagnetism to boost the performance of the original DE. Vasile *et al.* [133] proposed a novel DE, which is inspired by discrete dynamical systems. These improvements on DE enlighten us to look through techniques in other areas, which might be introduced to the development of DE variants.

On another research frontier, as an important component in control theory, impulsive control has attracted much attention in recent years due to its high efficiency. As exemplified in [134, 135], impulsive effects can be detected in various dynamical system, like communication networks, electronic systems, biological networks, and so on. Besides, impulsive control is able to manipulate the states of a network to a desired value by adding impulsive signals to some specific nodes at certain instants. In addition, another effective technique, event-triggered mechanism (ETM), has also been widely

utilized [136–138]. In ETM, the state of the controller is updated when the system's state exceeds a given threshold. This way, ETM avoids the periodical execution of data updating, which efficiently saves communication resources.

Taking a look at how DE works in an optimization problem, the population in the evolution process can be treated as a complicated dynamical system, where individuals in the population can be regarded as nodes in complex networks or multi-agent systems. Furthermore, after initialization, the population begins to evolve by updating the positions in the search range. The update rate ( $UR$ ) of the population in each generation reflects the current search status. The decrease of  $UR$  indicates that the population gradually encounters stagnation. The worst situation is that  $UR$  drops to zero before the global best solution is found. Therefore, it is possible for us to embed impulsive control or ETM into DE so that the search performance of DE can be improved.

#### **4.2.2 An Event-Triggered Impulsive Control Scheme**

In this section, an event-triggered impulsive control scheme (ETI) for DE is proposed. In the following, firstly, the proposed ETI is introduced in detail, which involves four components, i.e., stabilizing impulses, destabilizing impulses, ranking assignment, and an adaptive mechanism. Afterwards, ETI is combined with DE to develop ETI-DE, the pseudo-code and the computational complexity analysis of which are also given.

In the proposed approach, ETM and impulsive control are integrated into the framework of DE algorithms. ETM identifies when the individuals should be injected with impulsive controllers, while impulsive control alters the positions of partial individuals when triggering conditions are violated. Specifically, two types of impulses, i.e., stabilizing impulses and destabilizing

impulses, are imposed on the selected individuals (sorted by an index according to the fitness value and the number of consecutive stagnation generation) when  $UR$  in the current generation decreases or equals to zero. On one hand, when  $UR$  begins to decrease, stabilizing impulses drive the individuals with lower rankings in the current population to approach the individuals with better fitness values. The purpose of stabilizing impulsive control is to help update some inferior individuals and to enhance the exploitation capability of the algorithm. On the other hand, when  $UR$  drops to zero or stabilizing impulses fail to take effect, destabilizing impulses randomly adjust the positions of the inferior individuals within the area of the current population. This operation improves the diversity of the population and hence improves the exploration ability of DE.

#### 1) Stabilizing Impulses

Stabilizing impulses are employed when  $UR$  begins to decrease. As mentioned before, in control theory, stabilizing impulses can be employed to regulate the states of a network to a desired value. Normally, the desired state is set as the reference state for the nodes to be injected with stabilizing impulsive controllers. In the framework of DE, stabilizing impulses mainly focus on improving the exploitation ability of DE. In DE algorithms, it is well known that good individuals (i.e., with smaller fitness values) usually contain useful information, which may be helpful to other individuals' evolution. Hence, these good individuals can be regarded as references. So when stabilizing impulsive control is triggered during the evolution, we set the individuals with smaller fitness values in the current generation as the reference states. The pseudo-code of stabilizing impulsive control is exhibited in Algorithm 3.

Assume that  $\mathbf{x}_{i,G}$  is one of the individuals at the  $G$ th generation that are chosen to undergo impulsive effects, where  $\mathbf{x}_{i,G} = [x_{i1,G}, x_{i2,G}, \dots, x_{iD,G}]^T$

---

**Algorithm 3** Stabilizing\_Impulsive\_Control ()

---

```
1: Begin
2:   /*  $\mathbf{x}_{i,G}$  is the individual that undergoes stabilizing impulsive control
3:   /*  $\zeta_{i,G}$  is a flag to indicate whether stabilizing impulsive control is to improve the fitness value
4:   /*  $\text{rand}(a,b)$  uniformly generate a random number belonging to the interval (a,b)
5:   /*  $DM$  is the number of dimensions selected to undergo stabilizing impulsive control in  $\mathbf{x}_{i,G}$ 
6:    $\mathbf{A} = \{1, 2, \dots, D\}$ ;  $\mathbf{B} = \emptyset$ ;  $\mathbf{C} = \emptyset$ 
7:   Randomly select an individual  $\mathbf{x}_{k,G}$  from the current population
8:   if  $f(\mathbf{x}_{i,G}) < f(\mathbf{x}_{k,G})$  then
9:     Set  $\mathbf{x}_{gbest,G}$  as the reference state  $\mathbf{s}_{i,G}$ 
10:    Generate  $\mathbf{B}$  by randomly selecting  $DM$  elements from  $\mathbf{A}$ 
11:    for  $j = 1$  to  $D$  do
12:      if  $j \in \mathbf{B}$  then
13:         $K_{ij,G} = \text{rand}(-1, 0)$ 
14:      else
15:         $K_{ij,G} = 0$ 
16:      end if
17:    end for
18:     $K_{i,G} = \text{diag}\{K_{i1,G}, K_{i2,G}, \dots, K_{iD,G}\}_{D \times D}$ ,  $i = 1, 2, \dots, NP$ 
19:     $\mathbf{e}_{i,G} = \mathbf{x}_{i,G} - \mathbf{s}_{i,G}$ 
20:     $\mathbf{x}_{i,G+} = \mathbf{x}_{i,G} + K_{i,G} \cdot \mathbf{e}_{i,G}$ 
21:  else
22:    Set  $\mathbf{x}_{k,G}$  as the reference state  $\mathbf{s}_{i,G}$ 
23:    Generate  $\mathbf{C}$  by randomly selecting  $DM$  elements from  $\mathbf{A}$ 
24:    for  $j = 1$  to  $D$  do
25:      if  $j \in \mathbf{C}$  then
26:         $K_{ij,G} = -1$ 
27:      else
28:         $K_{ij,G} = 0$ 
29:      end if
30:    end for
31:     $K_{i,G} = \text{diag}\{K_{i1,G}, K_{i2,G}, \dots, K_{iD,G}\}_{D \times D}$ ,  $i = 1, 2, \dots, NP$ 
32:     $\mathbf{e}_{i,G} = \mathbf{x}_{i,G} - \mathbf{s}_{i,G}$ 
33:     $\mathbf{x}_{i,G+} = \mathbf{x}_{i,G} + K_{i,G} \cdot \mathbf{e}_{i,G}$ 
34:  end if
35:  if  $f(\mathbf{x}_{i,G+}) \leq f(\mathbf{x}_{i,G})$  then
36:     $\mathbf{x}_{i,G} = \mathbf{x}_{i,G+}$ 
37:     $\zeta_{i,G} = 1$ 
38:  else
39:     $\zeta_{i,G} = 0$ 
40:  end if
41: End
```

---

and  $D$  is the dimension. We set  $\mathbf{s}_{i,G}$  as the reference state for  $\mathbf{x}_{i,G}$ , which is randomly selected from the best individual (*gbest*) or other individuals with smaller fitness values than  $\mathbf{x}_{i,G}$  in the current population. For each  $\mathbf{x}_{i,G}$ , a uniform random individual  $\mathbf{x}_{k,G}$  is firstly chosen from the current population. If  $f(\mathbf{x}_{i,G}) < f(\mathbf{x}_{k,G})$ , which means the randomly selected individual is worse than  $\mathbf{x}_{i,G}$ , then  $\mathbf{x}_{gbest,G}$  is the reference state for  $\mathbf{x}_{i,G}$ ; if  $f(\mathbf{x}_{i,G}) \geq f(\mathbf{x}_{k,G})$ , which means  $\mathbf{x}_{k,G}$  is better than  $\mathbf{x}_{i,G}$  in the current population, then  $\mathbf{x}_{k,G}$  is set as the reference state for  $\mathbf{x}_{i,G}$ .

The error between  $\mathbf{x}_{i,G}$  and its reference state  $\mathbf{s}_{i,G}$  at the  $G$ th generation can be obtained:

$$\mathbf{e}_{i,G} = \mathbf{x}_{i,G} - \mathbf{s}_{i,G}. \quad (4.1)$$

Then at the end of the  $G$ th generation, stabilizing impulses force the chosen individuals to approach their reference state. Here we get:

$$\mathbf{x}_{i,G^+} = \mathbf{x}_{i,G} + K_{i,G} \cdot \mathbf{e}_{i,G}, \quad (4.2)$$

where  $K_{i,G} = \text{diag}\{K_{i1,G}, K_{i2,G}, \dots, K_{iD,G}\}$  is the impulsive strength for individual  $\mathbf{x}_i$  at the  $G$ th generation.  $K_{ij,G} \in (-1, 0)$  shows that in the  $j$ th dimension,  $x_{i,G}$  lies on the line between the reference and the individual itself;  $K_{ij,G} = 0$  means that in the  $j$ th dimension,  $x_{i,G}$  is not injected with impulsive controllers;  $K_{ij,G} = -1$  indicates that in the  $j$ th dimension,  $x_{i,G}$  reaches the reference state,  $j = 1, 2, \dots, D$ .  $G^+$  denotes that stabilizing impulses are imposed on  $\mathbf{x}_{i,G}$  at the end of the  $G$ th generation. Every time,  $DM$  dimensions of  $\mathbf{x}_{i,G}$  are selected in a random way to be injected with impulsive controllers, where  $DM \in \{1, 2, \dots, D\}$ . When  $\mathbf{x}_{gbest,G}$  serves as the reference state, for the selected  $DM$  dimensions, the impulsive strength  $\hat{K}_{i,G} = \text{diag}\{K_{i1,G}, K_{i2,G}, \dots, K_{iDM,G}\}_{DM \times DM}$ , and  $K_{ij,G}$  is a uniform random number from  $-1$  to  $0$ ,  $j = 1, 2, \dots, D$ ; for the rest  $D - DM$  dimensions,  $\check{K}_{i,G} = \text{diag}\{0, 0, \dots, 0\}_{(D-DM) \times (D-DM)}$ . When  $\mathbf{x}_{k,G}$  is as the reference state, for the selected  $DM$  dimensions, the impulsive strength  $\hat{K}_{i,G} = \text{diag}\{-1, -1, \dots, -1\}_{DM \times DM}$ ; for the rest  $D - DM$  dimensions,  $\check{K}_{i,G} = \text{diag}\{0, 0, \dots, 0\}_{(D-DM) \times (D-DM)}$ .  $K_{i,G}$  is obtained from combining  $\hat{K}_{i,G}$  and  $\check{K}_{i,G}$ . It is noticed that  $\zeta$  is a flag to indicate whether stabilizing impulsive control is successful to improve the performance: when  $\zeta = 1$ , it means that the stabilizing impulsive control takes effect, and a new individual is introduced to the population by replacing an old one; when  $\zeta = 0$ , it shows that the stabilizing impulsive control fails to take effect.

## 2) Destabilizing Impulses

When  $UR$  drops to zero ( $UR = 0$ ) or stabilizing impulses fail to take effect ( $\zeta = 0$ ), destabilizing impulses are introduced to provide some



randomness during the evolution. When destabilizing impulses are triggered, the selected individuals can be moved to any position within the range of the current population. The pseudo-code of injecting destabilizing impulses is exhibited in Algorithm 4. Assume that  $\mathbf{x}_{i,G}$  is one of the individuals at the  $G$ th generation that are chosen to receive destabilizing impulses, where  $\mathbf{x}_{i,G} = [x_{i1,G}, x_{i2,G}, \dots, x_{iD,G}]^T$ .  $\min_{j,G}$  and  $\max_{j,G}$  are the minimum and maximum values of the  $j$ th dimension in the population at the  $G$ th generation,  $j = 1, 2, \dots, D$ . The lower and upper bounds of the range of the population at the  $G$ th generation are:

$$\mathbf{x}_{L,G} = [\min_{1,G}, \min_{2,G}, \dots, \min_{D,G}]^T, \quad (4.3)$$

$$\mathbf{x}_{U,G} = [\max_{1,G}, \max_{2,G}, \dots, \max_{D,G}]^T. \quad (4.4)$$

From Eqs. (4.3)-(4.4), we can obtain the error between  $\mathbf{x}_{U,G}$  and  $\mathbf{x}_{L,G}$  at the  $G$ th generation:

$$\mathbf{e}_{i,G} = \mathbf{x}_{U,G} - \mathbf{x}_{L,G}. \quad (4.5)$$

Then at the end of the  $G$ th generation, the positions of the chosen individuals are randomly updated in the specified range. Here we have:

$$\mathbf{x}_{i,G^+} = \mathbf{x}_{L,G} + K_{i,G} \cdot \mathbf{e}_{i,G}, \quad (4.6)$$

where  $K_{i,G} = \text{diag}\{K_{i1,G}, K_{i2,G}, \dots, K_{iD,G}\}$  is the impulsive strength for individual  $\mathbf{x}_i$  at the  $G$ th generation. Similarly,  $DM$  dimensions of  $\mathbf{x}_{i,G}$  are selected at random to be injected with impulses.  $K_{ij,G}$  is a uniform random number from 0 to 1,  $j = 1, 2, \dots, D$ .

### 3) Ranking Assignment

In the following, we need to consider which individuals should be injected with impulsive controllers. During the evolution process, we consider two

---

**Algorithm 4** Injecting Destabilizing Impulses ()

---

```
1: Begin
2:   /*  $\mathbf{x}_{i,G}$  is the individual that undergoes destabilizing impulses
3:   /*  $\min_{j,G}$  and  $\max_{j,G}$  are the minimum and maximum values of the  $j$ th dimension in the population at the  $G$ th
   generation
4:   /*  $\text{rand}(a,b)$  uniformly generate a random number belonging to the interval (a,b)
5:    $\mathbf{x}_{L,G} = [\min_{1,G}, \min_{2,G}, \dots, \min_{D,G}]^T$ 
6:    $\mathbf{x}_{U,G} = [\max_{1,G}, \max_{2,G}, \dots, \max_{D,G}]^T$ 
7:   for  $j = 1$  to  $D$  do
8:      $K_{j,G} = \text{rand}(0, 1)$ 
9:   end for
10:   $K_{i,G} = \text{diag}\{K_{i1,G}, K_{i2,G}, \dots, K_{iD,G}\}_{D \times D}, i = 1, 2, \dots, NP$ 
11:   $\mathbf{e}_{i,G} = \mathbf{x}_{U,G} - \mathbf{x}_{L,G}$ 
12:   $\mathbf{x}_{i,G^+} = \mathbf{x}_{L,G} + K_{i,G} \cdot \mathbf{e}_{i,G}$ 
13: End
```

---

measures to characterize the status of the individuals. The first one is the fitness value of each individual, while the second one is the number of each individual's consecutive stagnant generation. Fitness value is the most direct index to judge whether an individual should enter into the next generation or not. The number of consecutive stagnant generation reflects the degree of the activity of an individual in the evolution. If an individual does not evolve for a relatively long time, it might be necessary to introduce some additional operations to change its position. Based on these discussions, in this paper, we rank the population based on the fitness value and the number of consecutive stagnant generation, respectively.  $\tilde{R}$  is the ranking according to the fitness value, and  $\bar{R}$  is the ranking based on the number of consecutive stagnation generation. These two rankings are both ordered in an ascending way (i.e., from the best fitness value to the worst and from the smallest number of consecutive stagnation generation to the largest). Then we combine  $\tilde{R}$  and  $\bar{R}$  to get  $R$ , which indicates that the individuals are sorted according to both the fitness value and the number of consecutive stagnation generation.

$$R = \tilde{R} + \bar{R}. \quad (4.7)$$

$R$  not only reflects the fitness value of an individual but also delivers the degree of an individual's activity.

When impulsive control is triggered during the evolution, we select the

individuals with larger values of  $R$  from the population as the candidates to undergo stabilizing or destabilizing impulses. By specially displacing the individuals with higher rankings (i.e., larger  $R$ ), the evolution status of the population can be improved.

#### 4) An Adaptive Mechanism to Determine the Number of Individuals Taking Impulsive Control

Finally, in order to further improve the performance of ETI, an adaptive mechanism is proposed to determine the number of the individuals that should be injected with impulsive controllers. We firstly discuss the number of individuals ( $M$ ) with stabilizing impulses.  $LN$  and  $UN$  represent the lower and upper bound of  $M$ , respectively. When stabilizing impulsive control is triggered for the first time,  $M = LN$ . After  $\mathbf{x}_{i,G}$  experiences the stabilizing impulse, we get  $\mathbf{x}_{i,G^+}$ .  $\mathbf{x}_{i,G^+}$  can join the current population instead of  $\mathbf{x}_{i,G}$  if and only if  $f(\mathbf{x}_{i,G^+}) < f(\mathbf{x}_{i,G})$ . Every time  $\mathbf{x}_{i,G}$  is replaced with  $\mathbf{x}_{i,G^+}$  (i.e.,  $\zeta = 1$ , see step 36 in Algorithm 1),  $M$  keeps unchanged. If  $\zeta = 0$ ,  $M = M + 1$ . We aim to increase the success rate of stabilizing impulsive control by having more individuals to be injected with stabilizing impulsive controllers. Besides, if a new *gbest* is generated in the population,  $M$  drops to a random integer between  $[LN, M]$ . The reason for reducing  $M$  to a random integer between  $[LN, M]$  instead of  $LN$  is to increase the times of successful stabilizing impulsive control, especially in the later stage of the evolution. Next, we explain how to choose the number of individuals that undergo destabilizing impulses. As introduced above, destabilizing impulses are added in two cases: when  $UR = 0$  or  $\zeta = 0$ . Unlike stabilizing impulses, with the purpose of introducing some randomness, the selection operation (i.e., compare the fitness values of  $\mathbf{x}_{i,G}$  and  $\mathbf{x}_{i,G^+}$ ) will not be used after injecting destabilizing impulses, which means that  $\mathbf{x}_{i,G^+}$  replaces  $\mathbf{x}_{i,G}$  directly. Therefore, in order not to bring too many individuals with large fitness values into the population, we randomly select the individuals from  $M$  candidates to

---

**Algorithm 5** Random\_Selection\_of\_Individuals ()

---

```
1: Begin
2:   /*  $\{\mathbf{x}_{i,G} | i = 1, 2, \dots, M\}$  are the candidates that undergo destabilizing impulses
3:   /*  $pr$  is the probability for selecting individuals to be injected with destabilizing impulses
4:   /*  $\epsilon$  is a flag for judging whether it is necessary to increase  $pr$ 
5:   for  $i = 1$  to  $M$  do
6:      $r_i = \text{rand}$ 
7:   end for
8:    $pr = 0.2, \epsilon = 0$ 
9:   while  $\epsilon = 0$  do
10:    for  $i = 1$  to  $M$  do
11:      if  $r_i < pr$  then
12:         $\mathbf{x}_{i,G}$  will undergo destabilizing impulsive control later
13:         $\epsilon = 1$ 
14:      else
15:         $\epsilon = 0$ 
16:      end if
17:    end for
18:     $pr = pr + 0.2$ 
19:     $pr = \min(pr, 1.0)$ 
20:  end while
21: End
```

---

be injected with destabilizing impulses. The selection process is described in Algorithm 5.

#### 5) DE with An Event-Triggered Impulsive Control Scheme

Combining the developed event-triggered impulsive control scheme (ETI) with DE, the ETI-DE is proposed. The pseudo-code of ETI-DE with “DE/rand/1” mutation operator is given in Algorithm 6. From step 3 to step 24, it is the original DE algorithm with “DE/rand/1” mutation operator. The rest steps in Algorithm 6 illustrate the mechanism of ETI. ETM determines the moment to add impulses to the individuals, and impulsive control modifies the positions of partial individuals at the end of a certain generation. In detail, step 35 to step 42 and step 49 to step 56 describe the mechanism of destabilizing impulses, which are triggered when  $UR = 0$  or  $\zeta = 0$ . While step 43 to step 48 shows the details of stabilizing impulsive control, which is triggered when  $UR$  decreases and  $UR \neq 0$ . These two types of impulses are able to accelerate the convergence of the population by updating some inferior individuals, and improve the diversity of the population by introducing some randomness to the search. Furthermore, ETI is flexible to be integrated into other advanced DE variants, such as jDE [102], JADE [103], SaDE [105], and so on.

---

**Algorithm 6** DE with event-triggered impulsive control scheme

---

```
1: Begin
2:   /*  $UR$  is the update rate of the population in each generation
3:   /*  $UR_{tp}$  stores the temporary value of  $UR$ 
4:   /*  $gbest$  is the best individual of the population in the current generation
5:   /*  $gbest_{tp}$  stores the temporary value of  $gbest$ 
6:   /*  $rs$  records the number of individuals to be injected with destabilizing impulses
7:   Set  $LN = 1$ ;  $UN = NP$ ;  $M = LN$ ;  $UR = 0$ ;  $F = 0.5$ ;  $CR = 0.9$ 
8:   Create a random initial population  $\{\mathbf{x}_{i,0} | i = 1, 2, \dots, NP\}$ 
9:   Evaluate the fitness values of the population and record  $gbest$ 
10:  while the maximum evaluation number is not achieved do
11:    for  $G = 1$  to  $G_{max}$  do
12:       $UR_{tp} = UR$ ;  $gbest_{tp} = gbest$ 
13:      for  $i = 1$  to  $NP$  do
14:        Select randomly three individuals  $r_1 \neq r_2 \neq r_3 \neq i$ 
15:         $\mathbf{v}_{i,G} = \mathbf{x}_{r_1,G} + F \cdot (\mathbf{x}_{r_2,G} - \mathbf{x}_{r_3,G})$ 
16:        Check the boundary of  $\mathbf{v}_{i,G}$ 
17:        Generate  $j_{rand} = randi(D, 1)$ 
18:        for  $j = 1$  to  $D$  do
19:          if  $j = j_{rand}$  or  $rand < CR$  then
20:             $u_{ij,G} = v_{ij,G}$ 
21:          else
22:             $u_{ij,G} = x_{ij,G}$ 
23:          end if
24:        end for
25:        if  $f(\mathbf{u}_{i,G}) \leq f(\mathbf{x}_{i,G})$  then
26:           $\mathbf{x}_{i,G+1} = \mathbf{u}_{i,G}$ 
27:        end if
28:      end for
29:      Evaluate the fitness values of the population and record  $gbest$ 
30:      if  $gbest < gbest_{tp}$  then
31:         $M = randi([LN, M], 1)$ 
32:      end if
33:      Calculate  $\bar{R}$  and  $\bar{R}$  of the population,  $R = \bar{R} + \bar{R}$ 
34:      Calculate  $UR$  of the population
35:      if  $UR = 0$  then
36:         $M = \min(M, UN)$ 
37:        Select  $M$  individuals with the largest  $R$ -value as  $\{\mathbf{x}_{i,G} | i = 1, 2, \dots, M\}$ 
38:         $\{\mathbf{x}_{i,G} | i = 1, 2, \dots, rs\} = \mathbf{Random\_Selection\_of\_Individuals}()$ 
39:        for  $i = 1$  to  $rs$  do
40:           $\mathbf{x}_{i,G} = \mathbf{Injecting\_Destabilizing\_Impulsive}()$ 
41:          Evaluate the fitness value of  $\mathbf{x}_{i,G}$ 
42:        end for
43:      else if  $UR \neq 0$  and  $UR < UR_{tp}$  then
44:         $M = \min(M, UN)$ 
45:        Select  $M$  individuals with largest  $R$ -value as  $\{\mathbf{x}_{i,G} | i = 1, 2, \dots, M\}$ 
46:        for  $i = 1$  to  $M$  do
47:           $[\mathbf{x}_{i,G}, \zeta_{i,G}] = \mathbf{Stabilizing\_Impulsive\_Control}()$ 
48:        end for
49:        if  $\sum(\zeta_{1,G}, \zeta_{2,G}, \dots, \zeta_{M,G}) = 0$  then
50:           $\{\mathbf{x}_{i,G} | i = 1, 2, \dots, rs\} = \mathbf{Random\_Selection\_of\_Individuals}()$ 
51:          for  $i = 1$  to  $rs$  do
52:             $\mathbf{x}_{i,G} = \mathbf{Injecting\_Destabilizing\_Impulsive}()$ 
53:            Evaluate the fitness value of  $\mathbf{x}_{i,G}$ 
54:           $M = M + 1$ 
55:          end for
56:        end if
57:        Record the best individual of current population as  $gbest'$ 
58:        if  $gbest' < gbest$  then
59:           $M = randi([LN, M], 1)$ 
60:        end if
61:      end if
62:    end for
63:  end while
64: End
```

---

### 4.2.3 Experimental Results and Analysis

In this section, we carry out extensive experiments to evaluate the performance of our developed ETI-DE. The total 30 benchmark functions presented in the CEC 2014 competition on single objective real-parameter numerical optimization are selected as the test suite [139]. According to their characteristics, the functions can be divided into four groups: 1) unimodal functions (F01-F03); 2) simple multimodal functions (F04-F16); 3) hybrid functions (F17-F22); 4) composition functions (F23-F30). More details of these functions can be found in [139].

In the following experiments, the proposed event-triggered impulsive control scheme is incorporated with ten state-of-the-art DE variants. The parameters are set as follows:

- 1) DE/rand/1/bin with  $F = 0.5$ ,  $CR = 0.9$  [117];
- 2) DE/best/1/bin with  $F = 0.7$ ,  $CR = 0.5$  [116];
- 3) jDE with  $\tau_1 = 0.1$ ,  $\tau_2 = 0.1$  [102];
- 4) JADE with  $\mu_F = 0.5$ ,  $\mu_{CR} = 0.5$ ,  $c = 0.1$ ,  $p = 0.05$  [103];
- 5) CoDE with  $F = [1.0, 1.0, 0.8]$ ,  $CR = [0.1, 0.9, 0.2]$  [104];
- 6) SaDE with  $LP = 50$  [105];
- 7) ODE with  $F = 0.5$ ,  $CR = 0.9$ ,  $J_r = 0.3$  [131];
- 8) EPSDE with  $F = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9]$ ,  
 $CR = [0.4, 0.5, 0.6, 0.7, 0.8, 0.9]$  [129];
- 9) SHADE with initial  $M_F = 0.5$ ,  $M_{CR} = 0.5$ ,  $H = NP$  [140];
- 10) OXDE with  $F = 0.5$ ,  $CR = 0.9$  [126].

For the incorporated ETI-DE algorithms, the lower and upper bounds of the number of individuals that take impulsive control are set:  $LN = 1$ ,  $UN = NP$ . In order to make a fair comparison, like [116, 117, 119], the population size  $NP$  is set as 100 for all the algorithms. The maximum number of function evaluations (Max\_FES) is set to  $D \cdot 10000$ . We run each function optimized by each algorithm 51 times for the experiments [139]. The simulations are performed on an Intel Core i7 personal computer with 2.10-GHz central processing unit and 8-GB random access memory.

In the following, we assess the effectiveness of our developed scheme by comparing ten popular DE algorithms and their corresponding ETI-based variants on the test suite at  $D = [30, 50, 100]$ . The experimental results are provided in Tables A.1-A.9 in Appendix. “+/ $\approx$ /-” indicates that the performance of DE algorithms with ETI is significantly better than, equal to, and worse than those without ETI according to a Wilcoxon rank-sum test at 5% significance level. The better values compared between the DE variants and their corresponding ETI-based DEs are highlighted in **boldface**.

From Tables A.1-A.9, we can see that the ten ETI-DEs perform better than their corresponding original DE algorithms, which indicates the effectiveness of the proposed ETI. Next, the Holm-Bonferroni procedure with confidence level 0.05 is conducted to investigate the differences among the total twenty DE variants, i.e., ten DE algorithms and their corresponding ETI-based variants. The results of  $D = [30, 50, 100]$  are provided in Tables 4.1-4.3. According to these three tables, it can be found that ETI-SHADE has the highest rank among the twenty variants when  $D = 30, 50$  and 100, respectively, which means ETI-SHADE performs best for the test problems of different dimension sizes.

In summary, the presented ETI is very powerful and the ten ETI-DEs possess strong capabilities of rapid convergence and accurate search for the test

Table 4.1: Holm test on the fitness, reference algorithm = ETI-SHADE (rank=15.33) for functions F01-F30 at  $D = 30$ .

$j$	Optimizer	Rank	$z_j$	$p_j$	$\delta/j$	Hypothesis
1	ETI-OXDE	14.47	-5.67E-01	2.85E-01	2.63E-03	Accepted
2	ETI-DE/rand/1/bin	14.30	-6.76E-01	2.49E-01	2.50E-02	Accepted
3	ETI-jDE	13.70	-1.07E+00	1.42E-01	8.33E-03	Accepted
4	SHADE	13.37	-1.29E+00	9.90E-02	2.94E-03	Accepted
5	ETI-JADE	13.10	-1.46E+00	7.19E-02	6.25E-03	Accepted
6	ETI-EPSDE	12.23	-2.03E+00	2.12E-02	3.13E-03	Accepted
7	ETI-ODE	11.20	-2.71E+00	3.41E-03	3.57E-03	Accepted
8	ETI-SaDE	10.97	-2.86E+00	2.13E-03	4.17E-03	Accepted
9	JADE	10.37	-3.25E+00	5.74E-04	7.14E-03	Rejected
10	jDE	10.27	-3.32E+00	4.55E-04	1.00E-02	Rejected
11	OXDE	9.27	-3.97E+00	3.57E-05	2.78E-03	Rejected
12	SaDE	9.10	-4.08E+00	2.25E-05	4.55E-03	Rejected
13	DE/rand/1/bin	8.87	-4.23E+00	1.15E-05	5.00E-02	Rejected
14	EPSDE	8.83	-4.26E+00	1.04E-05	3.33E-03	Rejected
15	ETI-CoDE	8.80	-4.28E+00	9.47E-06	5.00E-03	Rejected
16	ODE	8.57	-4.43E+00	4.72E-06	3.85E-03	Rejected
17	ETI-DE/best/1/bin	7.17	-5.35E+00	4.49E-08	1.25E-02	Rejected
18	CoDE	6.03	-6.09E+00	5.71E-10	5.56E-03	Rejected
19	DE/best/1/bin	4.07	-7.38E+00	8.17E-14	1.67E-02	Rejected

Table 4.2: Holm test on the fitness, reference algorithm = ETI-SHADE (rank=15.10) for functions F01-F30 at  $D = 50$ .

$j$	Optimizer	Rank	$z_j$	$p_j$	$\delta/j$	Hypothesis
1	ETI-JADE	13.80	-8.51E-01	1.97E-01	6.25E-03	Accepted
2	ETI-OXDE	13.67	-9.38E-01	1.74E-01	2.63E-03	Accepted
3	ETI-DE/rand/1/bin	13.53	-1.03E+00	1.53E-01	2.50E-02	Accepted
4	ETI-EPSDE	13.27	-1.20E+00	1.15E-01	3.13E-03	Accepted
5	SHADE	13.10	-1.31E+00	9.52E-02	2.94E-03	Accepted
6	ETI-jDE	12.43	-1.75E+00	4.04E-02	8.33E-03	Accepted
7	ETI-CoDE	11.80	-2.16E+00	1.54E-02	5.00E-03	Accepted
8	JADE	10.87	-2.77E+00	2.79E-03	7.14E-03	Rejected
9	ETI-ODE	10.70	-2.88E+00	1.99E-03	3.57E-03	Rejected
10	jDE	10.53	-2.99E+00	1.40E-03	1.00E-02	Rejected
11	EPSDE	9.20	-3.86E+00	5.61E-05	3.33E-03	Rejected
12	DE/rand/1/bin	9.07	-3.95E+00	3.91E-05	5.00E-02	Rejected
13	OXDE	8.93	-4.04E+00	2.71E-05	2.78E-03	Rejected
14	CoDE	8.77	-4.15E+00	1.69E-05	5.56E-03	Rejected
15	ODE	8.53	-4.30E+00	8.58E-06	3.85E-03	Rejected
16	ETI-SaDE	8.40	-4.39E+00	5.77E-06	4.17E-03	Rejected
17	SaDE	7.40	-5.04E+00	2.32E-07	4.55E-03	Rejected
18	ETI-DE/best/1/bin	7.03	-5.28E+00	6.43E-08	1.25E-02	Rejected
19	DE/best/1/bin	3.87	-7.35E+00	9.62E-14	1.67E-02	Rejected



Table 4.3: Holm test on the fitness, reference algorithm = ETI-SHADE (rank=14.57) for functions F01-F30 at  $D = 100$ .

$j$	Optimizer	Rank	$z_j$	$p_j$	$\delta/j$	Hypothesis
1	SHADE	13.77	-5.24E-01	3.00E-01	2.94E-03	Accepted
2	ETI-jDE	13.40	-7.64E-01	2.23E-01	8.33E-03	Accepted
3	ETI-CoDE	13.40	-7.64E-01	2.23E-01	5.00E-03	Accepted
4	ETI-JADE	13.00	-1.03E+00	1.53E-01	6.25E-03	Accepted
5	ETI-EPDSDE	12.67	-1.24E+00	1.07E-01	3.13E-03	Accepted
6	ETI-OXDE	12.60	-1.29E+00	9.90E-02	2.63E-03	Accepted
7	jDE	11.80	-1.81E+00	3.51E-02	1.00E-02	Accepted
8	ETI-DE/rand/1/bin	11.63	-1.92E+00	2.74E-02	2.50E-02	Accepted
9	JADE	10.93	-2.38E+00	8.69E-03	7.14E-03	Accepted
10	CoDE	10.20	-2.86E+00	2.13E-03	5.56E-03	Rejected
11	ETI-ODE	9.93	-3.03E+00	1.21E-03	3.57E-03	Rejected
12	ETI-SaDE	9.37	-3.40E+00	3.32E-04	4.17E-03	Rejected
13	EPDSDE	8.97	-3.67E+00	1.23E-04	3.33E-03	Rejected
14	OXDE	8.43	-4.02E+00	2.97E-05	2.78E-03	Rejected
15	DE/rand/1/bin	8.13	-4.21E+00	1.27E-05	5.00E-02	Rejected
16	SaDE	8.03	-4.28E+00	9.47E-06	4.55E-03	Rejected
17	ETI-DE/best/1/bin	7.57	-4.58E+00	2.30E-06	1.25E-02	Rejected
18	ODE	7.57	-4.58E+00	2.30E-06	3.85E-03	Rejected
19	DE/best/1/bin	4.03	-6.90E+00	2.68E-12	1.67E-02	Rejected

functions. The results of the Wilcoxon rank-sum test confirm that our scheme is of paramount importance to improve the performance of the considered DE algorithms. Besides, the results of the Holm-Bonferroni procedure denote that ETI-SHADE has the best performance on the test suite of different dimension sizes.

### 4.3 Summary

This chapter presents a novel MOEA (i.e., nondominated sorting adaptive differential evolution (NSJADE)) and a event-triggered impulsive control scheme (ETI) for DE. Firstly, NSJADE is developed based on a popular and classic MOEA: NSGA-II; and the experimental results demonstrate that the proposed NSJADE has better performance on multimodal problems than NSGA-II. Secondly, the idea of ETI derives from the concepts of event-triggered mechanism and impulsive control in control theory. ETI is developed to enhance both the exploration and exploitation abilities of DE. According to the experimental results, ETI greatly improves the performance of ten state-of-the-art DE variants.

# **Chapter 5**

## **Multiobjective Evolutionary Optimization for Sales Forecasting in Fashion Supply Chains**

This chapter handles a short-term replenishment forecasting problem in fashion supply chains by a neural network (NN)-based model, the parameters of which are optimized by a new multiobjective evolutionary algorithm (MOEA). Unlike the previous research, according to the different features of the datasets to be predicted, the developed model selects the optimal weights and the number of hidden nodes for NNs. Firstly, the mathematical model of the short-term replenishment forecasting problem is established. Secondly, a multiobjective optimization-based NN model is developed to deal with the forecasting problem. Thirdly, extensive experiments are conducted to show the effectiveness and superiority of our proposed model. Finally, the summary of this chapter is provided.

## 5.1 Problem Formulation

To solve the short-term replenishment forecasting problem in fashion supply chains, a NN-based forecasting model is proposed here. In previous NN-based models, only the training error is minimized, which will result in the overfitting phenomenon [11]. In this research, two objectives in the NN-based model are optimized: one is to minimize the average root-mean-square error of  $K$ -fold cross-validation ( $K$ -cv error) on the training data; the other is to minimize the sum of the absolute weights of NN. These two objectives are listed as follows:

1) Objective 1:

$$f_1 = \frac{1}{K} \sum_{k=1}^K E_k, \quad (5.1)$$

where  $K$  is the number that the training samples are divided into, and  $E_k$  is the root-mean-square error of predicting the  $k$ th part of the training data.

2) Objective 2:

$$f_2 = \sum_{i=1}^M |\omega_i|, \quad (5.2)$$

where  $\omega_i$  is a weight in the NN, and  $M$  is the total number of weights.

The first objective is related to the training error of the NN on the training samples. In Eq. 5.1, due to the limited number of the real data in the following experiments, the average root-mean-square error of  $K$ -cv error on the training data is used as the first objective. While under other circumstances that the training data is sufficient, it may be not necessary to carry out the  $K$ -fold cross-validation on the training set, which means any other accuracy measurement of the training can be selected as the objective. In Eq. 5.2, the second objective is concerned with the structure of the NN used in the forecasting model. On one hand,  $f_1$  is the training error of the model, which reflects the accuracy of the prediction, and a smaller  $f_1$  is desirable.

On the other hand, in order to increase the generalization of the model, the structure of the NN cannot be too complicated, which means a smaller sum of the absolute weights, i.e.,  $f_2$ , is preferred. These two objectives are conflicting with each other, and the improvement of one objective may cause the deterioration of the other one [141].

## **5.2 Multiobjective Optimization-Based Neural Network Model for the Problem**

A multiobjective optimization-based neural network model (MOONN) is proposed in this section for the short-term replenishment forecasting problem in fashion supply chains. For solving the forecasting problem, there are two processes: one is the forecasting process; the other is the optimization process. The flowcharts of these two processes are given in Figs. 5.1-5.2. In the forecasting process, two steps are involved: 1) In the first step, training data are utilized to train the proposed MOONN model, and the weights and the hidden node number of the model can be obtained. 2) In the second step, test data are used as the input of the MOONN model which has already been trained, and the output is the final sales forecast. The optimization process in Fig. 5.2 illustrates how the parameters of the forecasting model are optimized for the short-term replenishment forecasting problem. In the rest of this section, the optimization process is introduced in detail.

### **5.2.1 Representation**

The first step of the optimization process is to represent the solutions which will be optimized. For the short-term fashion replenishment forecasting problem, at the end of the optimization process, the NNs with the optimal

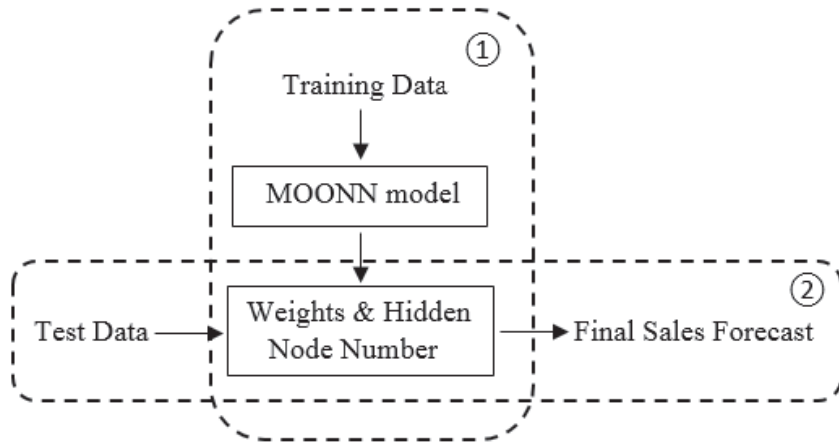


Figure 5.1: Flowchart of the forecasting process. ①: training model; ②: test model.

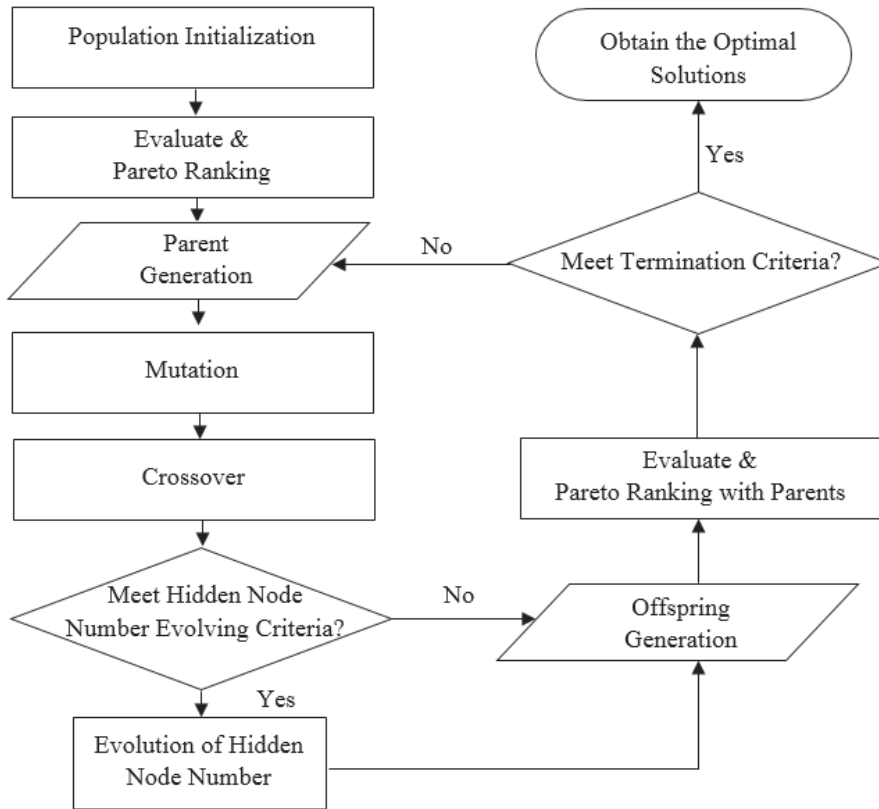


Figure 5.2: Flowchart of the optimization process.

values of weights and hidden biases, as well as the number of hidden nodes are obtained . Therefore, the representation of the solutions used in this research consists of two parts: one vector  $\Omega$  and one vector  $\Phi$ . The dimension of  $\Omega$  is  $(I + 1) \cdot H$ . Each element  $\omega_{ij} \in \Omega$  is the weight connecting the  $i$ th

input node and the  $j$ th hidden node.  $I$  and  $H$  denote the numbers of input nodes and hidden nodes, respectively.  $I + 1$  indicates that the hidden biases are also considered. The vector  $\Phi$  is of dimension  $H$ , where  $\phi_h \in \Phi$  is a binary value (i.e., 0 and 1) used to display the existence of each hidden node. In this research, each solution represents a NN candidate; and based on this representation, the training of weights and the selection of hidden nodes can be achieved at the same time.

It is worth pointing out that the learning algorithm of the NNs here is the extreme learning machine (ELM), so there is no need to assign the output weights in advance. The reason is that for ELM-based NNs, the output weights can be calculated based on the input weights and hidden biases, which is also an advantage of ELM-based NNs. So only the optimal values of input weights and hidden biases are searched for during the optimization process.

## 5.2.2 Population Initialization

The initialization process involves two parts: initializing vector  $\Omega$  and initializing vector  $\Phi$  for each individual in the population. For the initialization of  $\Omega$ , the real number in the specific range is randomly assigned to each dimension of the weight vector. For initializing vector  $\Phi$ , the maximum number of hidden nodes  $N_{max}$  is set firstly, then the number of hidden nodes is stochastically initialized from 1 to  $N_{max}$  for each individual.

## 5.2.3 Evolution of Vector $\Omega$

After the initialization process, the population has to evolve observing certain rules.  $\Omega$  is optimized among the specific range of real number, while  $\Phi$  is searched from 1 to the maximum number of hidden nodes  $N_{max}$ . In detail, two separate evolution schemes are developed for the evolution of these two

parts, respectively. After each generation of the evolution, the combination of  $\Omega$  and  $\Phi$  symbolizes a NN with particular parameters, which is used for the sales forecasting cases. Here, the rules of evolving weight vector  $\Omega$  are introduced firstly.

In Section 4.1, a new multiobjective optimization algorithm is proposed, which is called nondominated sorting adaptive differential evolution (NS-JADE). Here, NSJADE is selected as the optimization algorithm in the forecasting model MOONN. As explained before, JADE [103] is elected as the search engine of NSJADE; hence for  $\Omega$ , the mutation and crossover strategies of JADE are adopted. In the mutation stage, the strategy “DE/current-to-*p*best” guides the mutation of the parents:

$$\mathbf{v}_{i,g} = \mathbf{x}_{i,g} + F_i \cdot (\mathbf{x}_{best,g}^p - \mathbf{x}_{i,g}) + F_i \cdot (\mathbf{x}_{r1,g} - \tilde{\mathbf{x}}_{r2,g}) \quad (5.3)$$

At the  $g$ th generation, for individual  $\mathbf{x}_i$ , three other individuals  $\mathbf{x}_{best}^p$ ,  $\mathbf{x}_{r1}$ , and  $\tilde{\mathbf{x}}_{r2}$  are picked out in the mutation process. Among them,  $\mathbf{x}_{best}^p$  is chosen from the top 100*p*% ( $p \in (0, 1]$ ) individuals of the population;  $\mathbf{x}_{r1}$  is selected at random from the current population;  $\tilde{\mathbf{x}}_{r2}$  is from the current population along with an archive, which stores the inferior individuals during the evolution process. And  $\mathbf{x}_i$ ,  $\mathbf{x}_{best}^p$ ,  $\mathbf{x}_{r1}$  and  $\tilde{\mathbf{x}}_{r2}$  are different from each other. Moreover, at each generation, the mutation factor  $F_i$  of each individual is adaptively generated according to a Cauchy distribution.

After  $\mathbf{v}_i$  is generated, here we come to the crossover stage. In this stage, binomial crossover is performed on each dimension of vector  $\mathbf{v}_i$ :

$$u_{j,i,g} = \begin{cases} v_{j,i,g} & j = j_{\text{rand}} \text{ or } \text{rand}(0, 1) < CR_i \\ x_{j,i,g} & \text{otherwise} \end{cases} \quad (5.4)$$

In (5.4),  $j_{\text{rand}}$  is a randomly selected index, which ensures that  $\mathbf{u}_i$  gets at least one component from  $\mathbf{v}_i$ . And at each generation, the crossover probability

$CR_i$  is adaptively created based on a Normal distribution.

During the evolution process of  $\Omega$ , the mutation strategy “DE/current-to- $p$ best” and the crossover strategy guarantee a fast convergence speed and high accuracy for the evolution.

#### 5.2.4 Evolution of Vector $\Phi$

Vector  $\Phi$  stores a group of binary values, which indicates the existence of hidden nodes. For the evolution of  $\Phi$ , the search range is one-dimensional, from 1 to  $N_{max}$ . So only mutation operation is applied during the evolution of  $\Phi$ . In addition, mutation is adopted every few generations, which avoids the structure of NNs to be changed very frequently. This operation ensures that the weights of NN with specific structure can be trained for a longer time, which provides more opportunities to find the optimal parameters for the NN. The detailed mutation process is described as follows:

- Step 1. For individual  $i$ , another two individuals:  $r_1$  and  $r_2$  are randomly selected.
- Step 2. Perform XOR on each dimension of vectors  $\Phi_{r_1}$  and  $\Phi_{r_2}$ , and vector  $\Phi_{xor}$  can be generated.
- Step 3. Check each dimension of vector  $\Phi_{xor}$ : if the value of this dimension is 1, we also set 1 to the corresponding dimension of vector  $\Phi_i$ .
- Step 4. If there is no “1” in  $\Phi_i$ , we initialize  $\Phi_i$  again according to the procedure of Population Initialization.

It is noticed that XOR operation is involved in the evolution of vector  $\Phi$ , and the reasons are listed here. Unlike vector  $\Omega$ , vector  $\Phi$  consists of a sequence of binary values, the evolution of which should follow the rules of discrete



evolutionary algorithm. XOR operation calculates the distance between two discrete vectors, which is the essence of binary DE algorithm. So XOR is selected during the process of evolving vector  $\Phi$ .

To better explain the mutation process of  $\Phi$ , an example is provided in Fig. 5.3).  $\Phi_{r_1}$  and  $\Phi_{r_2}$  are two randomly selected vectors, each of which contains 10 dimensions.  $\Phi_{xor}$  is the vector obtained after the XOR operation on  $\Phi_{r_1}$  and  $\Phi_{r_2}$ , which means for the 2nd, 4th, 6th, 7th, 8th and 10th elements, the values of  $\Phi_{r_1}$  and  $\Phi_{r_2}$  are opposite. Moreover, combined with the value of the original vector  $\Phi_i$ , it is obvious that the values in the 2nd, 6th, 7th, 8th and 10th positions should be set to 1, hence the mutated one is obtained.

$\Phi_{r_1}$	1	0	1	1	0	1	0	0	0	1
$\Phi_{r_2}$	1	1	1	0	0	0	1	1	0	0
$\Phi_{xor}$	0	1	0	1	0	1	1	1	0	1
$\Phi_i$	0	0	1	1	0	0	0	0	0	0
$\Phi_i$ (mutated)	0	1	1	1	0	1	1	1	0	1

Figure 5.3: An example of mutation process of  $\Phi$ .

### 5.2.5 Selection

In a single evolution, after each individual (including  $\Omega$  and  $\Phi$ ) in the parent population goes through the mutation or crossover, a new generation is needed to select from the parent and the offspring population combined. Since there are two objectives involved in the optimization process, the Pareto optimal solutions of this problem need to be picked out. It is assumed that the parent population contains  $NP$  individuals; therefore, after a single evolution, there

are  $2NP$  individuals (each parent generates one offspring) in the candidate pool. The specific procedure of selection is given as follows:

Step 1. Calculate the values of two objectives  $f_1$  and  $f_2$  for each individual in the candidate pool.

Step 2. Rank the  $2NP$  individuals according to  $f_1$  and  $f_2$  based on the definition of Pareto dominance (see Definition 3.1 in Section 3.1.1). In addition, the labels “rank 1, rank 2, ...” are assigned to the individuals in different nondomination levels (see Definition 3.4 in Section 3.1.1).

Step 3. In each rank, the crowding distance [106] is calculated for each individual, which characterizes the degree of crowding where the individual is located.

Step 4. After both the rank and the crowding distance of each individual in the candidate pool are calculated, the individuals are sorted according to these two indices. To be specific, the individual with the lower rank and the larger crowding distance is preferred. Hence,  $NP$  individuals are chosen from the candidate pool as the population of a new generation.

The selection process needs to choose  $NP$  individuals from  $2NP$  candidates. These  $2NP$  individuals are firstly sorted based on their objective values, and then divided into different nondomination levels. This process is called nondominated sorting [106]. At different nondomination levels, crowding distance is computed and then assigned to each individual. Nondominated sorting and crowding-distance assignment help balance between exploration and exploitation for the algorithm.

At the end of one single evolution, there is another operation besides selection: comparing a parent individual  $\mathbf{x}_i$  and its offspring  $\mathbf{u}_i$ . If  $\mathbf{x}_i$  is

dominated by  $\mathbf{u}_i$ ,  $\mathbf{x}_i$  is recognized as a loser, which will be put into an archive (mentioned in Section 4.1). Accordingly, the mutation factor  $F_i$  and the crossover probability  $CR_i$  used in generating  $\mathbf{u}_i$  are marked “successful”, which helps produce the new  $F_{i+1}$  and  $CR_{i+1}$  for the next generation.

## 5.3 Experimental Results and Discussion

To evaluate the performance of the proposed forecasting model, extensive experiments were conducted in terms of the real-world sales data of apparel products, which predicted the amount of replenishment for certain fashion products during the selling season. In this section, the experimental setup is firstly introduced in detail, including fashion sales data we used, the accuracy measures, and the forecasting models used for comparison and their parameters. Later, two groups of experiments are conducted to demonstrate the effectiveness of the proposed model: 1) the short-term replenishment forecasting of several categories of products in the shops of separate cities are performed; 2) for the same categories of products, the sales of shops in different cities are combined, and then the amount of replenishment is predicted.

### 5.3.1 Fashion Sales Data

Real sales data were collected from one of the largest retailers in fashion industry in Hong Kong and Mainland China, which include the sales data of different types of T-shirts and pants in various cities from 01/1999 to 12/2009.

### 5.3.2 Accuracy Measures

Since there are various forecasting objectives, data scales and patterns, no accuracy measure is generally applicable to all forecasting tasks [142]. In this research, in order to reduce the possible bias generated by one single accuracy measure, three measures of prediction accuracy are utilized, including root mean absolute error (*RMSE*), mean absolute percentage error (*MAPE*) and mean absolute error (*MAE*). Define  $Y_t$  and  $F_t$  as the observation and forecast at time  $t$ , respectively, the specific formulas of these three accuracy measures are provided as follows:

$$RMSE = \sqrt{\text{mean}((Y_t - F_t)^2)} \quad (5.5)$$

$$MAPE = \text{mean}(|(Y_t - F_t)/Y_t|) \times 100\% \quad (5.6)$$

$$MAE = \text{mean}(|Y_t - F_t|) \quad (5.7)$$

### 5.3.3 Forecasting Models Used for Comparison and Their Parameters

In this research, 2 categories of experiments are performed: short-term replenishment forecasting of fashion products in separate cities and in combined cities. In each category, experiments are conducted from three aspects: (1) the forecasting performance of the proposed model is compared with two popular models and a variant of one of these two models: ELME [8], HI [10], and HI2; (2) to examine the effect of the module that helps choose the proper number of NN's hidden node, the performance of our proposed

models with or without this module are compared, and the models are marked as MOONN1 and MOONN2; (3) to observe the benefits brought by the  $K$ -cv error as one of the objectives, the performance of the presented model is compared with the similar MOONN model called MOONN3, which merely replaces  $K$ -cv error with  $RMSE$  as one of the two objectives. The models used in the experiments are summarized in Table 5.1.

Table 5.1: The models used in the experiments.

Model	Description
MOONN1	proposed in this research
MOONN2	variant of MOONN1 (without the module of optimizing hidden node number)
MOONN3	variant of MOONN1 ( $K$ -cv error substituting $RMSE$ as one objective)
ELME	proposed in [8]
HI	proposed in [10]
HI2	variant of HI ( $K$ -cv error substituting $RMSE$ as the objective)

After introducing the models used in the experiments, the parameters of each model are listed in Table 5.2. MOONN1 is the proposed model in this research, while the variants MOONN2 and MOONN3 are used for comparison, the main bodies of which are similar with MOONN1. Table 5.2 shows the parameters of these three MOONN models. For convenience, in the last four rows of Table 5.2, some parameters that are only used in part of these three models are also provided. *Range of hidden node number* is set for MOONN2, because in MOONN2, the number of hidden nodes is not picked intelligently, where the model is run repeatedly with different number of hidden nodes from 1 to 10. While for MOONN1 and MOONN3, *Searching Range of hidden node number* is set, as well as *Generation interval of evolving vector  $\Phi$* , because each individual of the population in MOONN1 and MOONN3 seeks the appropriate number of hidden nodes during the forecasting. The value of  $K$  for MOONN1 and MOONN2 is 10, because 10-fold cross-validation is most commonly used. While in MOONN3,  $K$ -cv error is substituted by  $RMSE$ , it is not necessary to configure the value of  $K$ .

For the other three models used in the experiment, i.e., ELME, HI and HI2,

Table 5.2: The Parameters of MOONN1, MOONN2 and MOONN3, where  $\mathbb{N}=\{1,2,3,\dots\}$ .

Parameter	Value
Population size	100
Generation	300
Initial range of the input weights	[-1, 1]
Activation function	Sigmoid
Range of hidden node number (for MOONN2)	$[1, 10] \cap \mathbb{N}$
Searching range of hidden node number (for MOONN1, MOONN3)	$[1, 10] \cap \mathbb{N}$
Generation interval of evolving vector $\rho$ (for MOONN1, MOONN3)	30
Value of $K$ (for MOONN1, MOONN2)	10

the parameters are the same as the original parameter settings [8, 10] (see Tables 5.3-5.4). In Table 5.3,  $P$  is the parameter that makes the ELME more stable, which is recommended to be selected from  $[100, 1000]$ . It has been demonstrated that the larger value of  $P$ , the better forecasting performance ELME can provide [8]; so here we choose 1000. In Table 5.4,  $PAR_{min}$  and  $PAR_{max}$  are the minimum and the maximum of pitch adjustment rate ( $PAR$ );  $bw_{min}$  and  $bw_{max}$  denote the minimum and the maximum of bandwidth ( $bw$ );  $HMCR$  is the harmony memory consideration rate;  $HMS$  represents the harmony memory size, like the population size in MOONN1, and is also set as 100;  $NI$  is the number of improvisations; *value of  $K$*  indicates in HI2,  $K$ -cv error substitutes RMSE as the objective to be optimized. Furthermore, for ELME, in the original paper, the number of hidden nodes is fixed. In order to reduce the randomness of output generated by ELME with a stationary number of hidden nodes, we set a parameter called *Range of hidden node number*, as shown in Table 5.3. The forecasting is carried out by running the ELME model repeatedly with different number of hidden nodes from 1 to 10, and then average these forecasts as the final result. For HI and HI2, the same operation also appears in the experiment. While for MOONN1, the model can select the appropriate number of hidden node from 1 to 10 intelligently according to different datasets.

It is noticed that besides HI, our proposed MOONN1 is also compared with a variant of HI, which is named HI2. The reason is as follows: In MOONN1,

Table 5.3: The Parameters of ELME, where  $\mathbb{N}=\{1,2,3,\dots\}$ .

Parameter	Value
Initial range of the input weights	$[-1, 1]$
Activation function	Sigmoid
Range of hidden node number	$[1, 10] \cap \mathbb{N}$
$P$	1000

Table 5.4: The Parameters of HI and HI2, where  $\mathbb{N}=\{1,2,3,\dots\}$ .

Parameter	Value
Initial range of the input weights	$[-1, 1]$
Activation function	Sigmoid
Range of hidden node number	$[1, 10] \cap \mathbb{N}$
Value of $K$ (for HI2)	10
$PAR_{min}$	0.45
$PAR_{max}$	0.99
$bw_{min}$	$1e-6$
$bw_{max}$	4
$HMCR$	0.95
$HMS$	100
$NI$	1000

$K$ -cv error of the training samples is as one of the objectives to be optimized. While in HI model, the objective is the  $RMSE$  of the training samples, but not  $K$ -cv error. Therefore, in order to fairly show the performance of both MOEA and the module exploring the hidden-node-number space in MOONN1, the variant of HI called HI2 is also provided in the comparison, which also utilizes  $K$ -cv error as the objective.

### 5.3.4 Experiment 1: Short-Term Replenishment

#### Forecasting of 7 Categories of Products in Cities $X \sim Z$

In this experiment, 7 categories (i.e., A, B, C, D, E, F and G) of products are selected from cities X, Y and Z. The task is to forecast the amount of replenishment for different categories of products separately in these cities. Table 5.5 gives the specific mapping of products and cities. For each category of products, there is a dataset which records the sales in all the shops in the corresponding city, so in Table 5.5, the numbers in the brackets are the samples in each dataset. Moreover, for each sample in city X, it includes

6 input variables, which are 3 early sales volumes (the first 3-, 7- and 14-day sales volumes), 1 weather index (average temperature during the selling season), and 2 economic indices (total retail sales index of consumer goods and consumer price index for clothing and footwear); while for city Y and Z, due to the lack of statistical data, total retail sales index of consumer goods is not contained. The input variables are determined according to their influence on fashion sales, which include weather data, economic data and historical sales data [22]. As shown in Table 5.2,  $K$ -cv error is one of the two objectives to be optimized in MOONN1 and MOONN2, and the value of  $K$  is 10, which indicates during the optimization, training samples are evaluated by 10-fold cross-validation. Therefore, the number of training samples needs to be appointed as integral multiples of 10, which means that the number of training samples should satisfy:  $N_{ts} = 10 * M$ , where  $M$  is a positive integer. In this experiment, to predict the amount of replenishment of product A:  $N_{ts}=40$ , and the other 8 samples are for the test; for product B and D:  $N_{ts} = 10$ , and the rest are test samples; for product C and product F and G in both cities Y and Z:  $N_{ts} = 20$ , and the rest are utilized as test samples; for product E:  $N_{ts} = 30$ , and the rest are for the test.

Table 5.5: Specific mapping of products and cities.

City	Category of product
X	A(48), B(17), C(25), D(21), E(39)
Y	F(24), G(32)
Z	F(30), G(31)

### Comparison of MOONN1, ELME, HI and HI2

First of all, the forecasting is performed by using four different models: the proposed MOONN1, ELME, HI and HI2. For MOONN1, HI and HI2, each model is run 25 times; while for ELME, the original paper suggests 100 to 1000 times (i.e., the parameter  $P$ ), and here the ELME model is run 1000 times. The comparison of forecasting results is shown in Tables 5.6-5.8.



Table 5.6: Forecasting results of MOONN1, ELME, HI, HI2, MOONN2 and MOONN3 in city X.

		MOONN1		ELME		HI		HI2		MOONN2		MOONN3	
		Training	Test	Training	Test	Training	Test	Training	Test	Training	Test	Training	Test
A	<i>RMSE</i>	1853.9	<b>1615.6</b>	1846.3	1973.5	1130.5	2054.2	1756.6	1985.3	1348.8	1783.6	1317.4	1713.3
	<i>MAPE</i>	15.4%	15.1%	14.4%	16.9%	9.2%	18.3%	13.8%	17.7%	10.0%	14.5%	11.0%	<b>14.3%</b>
	<i>MAE</i>	1210.2	1194.1	1158.9	1399.0	705.0	1440.6	1095.2	1433.5	788.4	<b>1181.3</b>	857.4	1186.6
B	<i>RMSE</i>	1185.9	<b>449.2</b>	866.1	2891.8	322.4	1296.2	778.4	1242.0	529.7	850.3	1010.9	511.1
	<i>MAPE</i>	112.7%	14.9%	73.7%	65.3%	28.2%	30.5%	59.2%	32.6%	42.2%	<b>14.0%</b>	98.1%	17.2%
	<i>MAE</i>	974.1	<b>371.7</b>	675.5	2286.2	250.7	1016.4	560.2	983.0	415.5	610.7	833.0	428.2
C	<i>RMSE</i>	1257.1	<b>826.4</b>	971.2	1224.4	510.7	1450.8	894.6	1545.6	643.2	881.3	863.3	2301.5
	<i>MAPE</i>	224.2%	<b>18.9%</b>	103.1%	25.8%	36.4%	31.1%	81.3%	30.6%	55.1%	22.4%	153.0%	33.5%
	<i>MAE</i>	988.7	<b>719.6</b>	676.4	1063.0	366.6	1263.6	612.3	1312.5	449.2	783.8	680.2	1698.3
D	<i>RMSE</i>	1320.1	<b>725.8</b>	931.8	1850.6	331.9	1611.3	835.7	5109.8	566.7	1339.4	1042.5	911.3
	<i>MAPE</i>	103.5%	<b>21.6%</b>	57.9%	43.6%	19.0%	31.2%	49.8%	110.4%	38.2%	29.9%	78.5%	24.2%
	<i>MAE</i>	1036.4	<b>608.5</b>	667.9	1372.1	240.4	1111.7	584.0	3858.6	408.1	984.6	784.8	722.1
E	<i>RMSE</i>	2017.9	<b>1426.7</b>	1722.0	1651.7	1431.0	2596.1	1673.9	1782.4	1495.7	2578.7	1379.0	3079.7
	<i>MAPE</i>	110.0%	<b>34.4%</b>	75.9%	41.1%	50.5%	64.1%	68.7%	44.5%	57.9%	63.2%	66.2%	70.9%
	<i>MAE</i>	1676.5	<b>1272.5</b>	1372.3	1529.2	1161.6	2391.1	1345.0	1660.3	1213.0	2362.2	1134.1	2633.2

Table 5.7: Forecasting results of MOONN1, ELME, HI, HI2, MOONN2 and MOONN3 in city Y.

		MOONN1		ELME		HI		HI2		MOONN2		MOONN3	
		Training	Test	Training	Test	Training	Test	Training	Test	Training	Test	Training	Test
F	<i>RMSE</i>	453.8	<b>306.5</b>	371.9	543.1	202.7	714.8	341.4	498.2	257.0	582.7	307.6	562.3
	<i>MAPE</i>	36.7%	<b>12.5%</b>	26.2%	20.3%	10.6%	28.6%	23.4%	24.2%	17.7%	24.9%	24.7%	18.9%
	<i>MAE</i>	385.2	<b>242.2</b>	302.8	444.9	142.5	627.7	277.5	454.7	208.8	530.0	254.4	465.2
G	<i>RMSE</i>	169.5	<b>96.9</b>	136.4	133.3	63.6	160.2	117.2	131.4	91.9	112.2	128.5	151.0
	<i>MAPE</i>	22.9%	<b>15.4%</b>	16.9%	19.2%	9.5%	20.5%	15.3%	18.6%	12.3%	16.5%	17.5%	21.1%
	<i>MAE</i>	128.4	<b>66.1</b>	102.2	111.7	53.8	132.9	91.5	109.9	71.1	96.5	97.6	121.0

Table 5.8: Forecasting results of MOONN1, ELME, HI, HI2, MOONN2 and MOONN3 in city Z.

		MOONN1		ELME		HI		HI2		MOONN2		MOONN3	
		Training	Test	Training	Test	Training	Test	Training	Test	Training	Test	Training	Test
F	<i>RMSE</i>	758.7	<b>847.1</b>	482.8	895.6	239.0	1158.8	419.3	1019.5	362.5	944.1	490.8	1389.1
	<i>MAPE</i>	75.8%	<b>18.2%</b>	39.6%	19.1%	15.9%	28.7%	33.9%	21.9%	29.4%	21.6%	48.6%	34.4%
	<i>MAE</i>	655.6	<b>618.5</b>	395.7	641.1	174.9	926.4	339.9	725.4	290.4	715.9	426.5	1119.2
G	<i>RMSE</i>	226.4	<b>135.4</b>	192.2	165.5	87.7	236.5	173.9	172.7	123.5	201.6	187.0	174.1
	<i>MAPE</i>	31.7%	<b>12.3%</b>	24.0%	14.5%	13.4%	18.5%	22.1%	15.4%	17.7%	17.2%	26.6%	13.5%
	<i>MAE</i>	173.4	<b>114.6</b>	144.1	141.2	72.0	194.7	133.1	149.9	98.9	175.3	142.7	139.5

Among these models, MOONN1 is based on multiobjective algorithm, HI and HI2 are grounded on single objective algorithm, while ELME does not process any optimization algorithm. As discussed in Section 3.1.1, the mechanism of multiobjective algorithms is different from single objective ones—that is, for multiobjective algorithms, a set of solutions are obtained at last, while we only choose the optimal solution when using single objective algorithms. Therefore, in order to unify the metrics, for MOONN1, as well as MOONN2 and MOONN3, the training results and test results obtained by the whole population are averaged as the final training and test results.

For all forecasting cases, from the *RMSE*, *MAPE* and *MAE* results of the three models, it can be figured out that HI performs the best on training samples. Compared with ELME, HI adopts the harmony search algorithm to optimize the NN, so it is reasonable that HI has the best results on training samples. However, due to the data insufficiency of the fashion sales forecasting, HI fails to extend the satisfying performance on training data to test data. From Tables 5.6-5.8, MOONN1 exhibits superior forecasting performance to ELME and HI on all the test samples, which can be attributed to MOONN1's strong ability of balancing the training and test accuracy.

As mentioned above, the proposed MOONN model (i.e., MOONN1) owns the module which evolves the number of hidden nodes, and this module effectively selects the appropriate size of neural network for the forecasting problem. In Table 5.9, the specific size of the evolved neural network in MOONN1 is listed. The mean and standard deviation of maximum, minimum and average number of hidden nodes during 25 runs are recorded in Table 5.9. According to the different characteristics of different cases, the sizes of the neural network used in the forecasting problems are also different. For instance, to predict the replenishment of product A in city X, the average size of the neural network in MOONN1 is 3.3; while for product E in city X, the average size of the neural network involved in the replenishment forecasting

problem is merely 1.4.

Table 5.9: Size of the evolved neural network in MOONN1.

	City X					City Y		City Z	
	A	B	C	D	E	F	G	F	G
max	7.7(1.2)	8.9(1.0)	6.6(1.5)	8.8(0.8)	2.2(0.4)	4.7(1.5)	8.1(1.5)	5.5(2.9)	5.7(1.2)
min	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)
avg	3.3(0.5)	2.7(0.4)	2.4(0.6)	2.4(0.2)	1.4(0.1)	2.3(0.6)	3.1(0.4)	2.5(1.1)	2.3(0.3)

### Comparison of MOONN1 and MOONN2

As introduced earlier, the number of hidden nodes can be evolved when using the proposed MOONN model (i.e., MOONN1). In this section, to validate its effectiveness, this module is removed from MOONN1, and the new model obtained is called MOONN2. The forecasting is performed by repeatedly running MOONN2 with different number of hidden nodes from 1 to 10, and then the results are averaged as the final result. Here, MOONN1 and MOONN2 are employed to forecast the amount of replenishment for the above products. Tables 5.6-5.8 exhibit the forecasting performance of MOONN1 and MOONN2. For all the categories of products, from the *RMSE*, *MAPE* and *MAE* results, MOONN2 behaves better on training samples, but MOONN1 shows excellent forecasting performance on test samples. It can be recognized that the module which selects the proper number of NN's hidden nodes helps MOONN1 perform better on 7 of 9 replenishment forecasting cases.

### Comparison of MOONN1 and MOONN3

In this subsection, the forecasting performance of MOONN1 and MOONN3 is compared. In MOONN1, the average error of committing  $K$ -fold cross-validation ( $K$ -cv error) on the training samples is set as one of the objectives to be optimized; while in MOONN3, the  $K$ -cv error is replaced by *RMSE* as one of the two objectives. Tables 5.6-5.8 show the forecasting performance

of MOONN1 and MOONN3. According to the results, it can be found that MOONN1 presents superior performance over MOONN3 on 8 of 9 forecasting cases, which indicates the effectiveness of selecting  $K$ -cv error as one of the two objectives.

In Table 5.10, the size of neural network after evolution in MOONN1 and MOONN3 is provided when forecasting the replenishment amount of the 7 categories of products above. The mean and standard deviation of maximum, minimum and average number of hidden nodes during 25 runs are recorded in Table 5.10. From the table, it can be found that for each forecasting problem, the maximum and average number of hidden nodes in MOONN3 is much larger than that in MOONN1. And it is well explained why the *RMSE*, *MAPE* and *MAE* generated by MOONN3 on training samples are much smaller than those by MOONN1; because larger-size neural networks have more accurate approximation ability.

Table 5.10: Size of the evolved neural network in MOONN1 and MOONN3.

		City X				City Y		City Z		
		A	B	C	D	E	F	G	F	G
MOONN1	max	7.7(1.2)	8.9(1.0)	6.6(1.5)	8.8(0.8)	2.2(0.4)	4.7(1.5)	8.1(1.5)	5.5(2.9)	5.7(1.2)
	min	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)
	avg	3.3(0.5)	2.7(0.4)	2.4(0.6)	2.4(0.2)	1.4(0.1)	2.3(0.6)	3.1(0.4)	2.5(1.1)	2.3(0.3)
MOONN3	max	10.0(0.2)	10.0(0)	10.0(0.2)	10.0(0)	9.8(0.5)	9.8(0.4)	10.0(0)	9.9(0.3)	9.9(0.3)
	min	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)	1.0(0)
	avg	5.3(0.3)	3.2(0.2)	4.8(0.3)	3.1(0.3)	5.7(0.4)	5.3(0.3)	4.9(0.3)	5.0(0.3)	4.8(0.3)

### 5.3.5 Experiment 2: Short-Term Replenishment

#### Forecasting of 2 Categories of Products in Cities Y~Z

In Experiment 1, the amount of replenishment for 7 categories of products in city X, Y and Z is predicted. For instance, like product F and G, although they were sold in both city Y and Z, the replenishment of F and G in city Y (or Z) is predicted by means of the sales data merely in city Y (or Z). While in this experiment, the sales data of the same category of products are combined in different cities for the forecasting. According to the sales data

provided by the retailer, two categories of products are collected, which were sold in two cities from 1999 to 2009. These two categories are F and G, and their amounts of replenishment in city Y and Z are predicted in Experiment 1. Now, for product F and G, the sales data in city Y and Z are added; so there are 54 and 63 samples of each product respectively. Like the previous experiment, each sample includes 5 input variables, which are 3 early sales volumes (the first 3-, 7- and 14-day sales volumes), 1 weather index (average temperature during the selling season), and 1 economic index (consumer price index for clothing and footwear). Like experiment 1, the number of training samples should satisfy:  $N_{ts} = 10 * M$ , where  $M$  is a positive integer. So for category F, with the help of total 54 samples, the amount of replenishment in city Y and Z is predicted respectively, where  $N_{ts} = 40$ , and the rest 14 samples in either city are test data; similarly, to forecast the replenishment amount of category G in city Y and Z,  $N_{ts} = 50$ , and the rest 13 samples are test data. In this experiment, the forecasting performance of MOONN1 and other models is also compared, and the results are given in the following three subsections.

### **Comparison of MOONN1, ELME, HI and HI2**

First of all, as in Experiment 1, the forecasting performance of four different models is compared: MOONN1, ELME, HI and HI2. For MOONN1, HI and HI2, each model is run 25 times; while for ELME, the original paper suggests 100 to 1000 times (i.e., the parameter  $P$ ), and here the ELME model is run 1000 times. The forecasting results are given in Table 5.11. Firstly, the forecasting performance among the four models is compared. For both product F and product G in city Y, MOONN1 shows the superiority of forecasting test samples on all of the three accuracy measures; for both product F and product G in city Z, ELME and MOONN1 have better

forecasting performance than HI and HI2.

Table 5.11: Forecasting results of MOONN1, ELME, HI, HI2, MOONN2 and MOONN3.

		MOONN1		ELME		HI		HI2		MOONN2		MOONN3		
		Training	Test	Training	Test	Training	Test	Training	Test	Training	Test	Training	Test	
F	city Y	<i>RMSE</i>	758.2	495.9	548.7	490.3	380.5	661.1	518.0	486.8	443.5	<b>473.9</b>	524.8	626.4
		<i>MAPE</i>	68.8%	13.3%	42.4%	13.8%	21.9%	16.0%	38.7%	14.3%	30.7%	<b>13.0%</b>	43.8%	18.2%
		<i>MAE</i>	632.6	<b>334.6</b>	445.7	370.0	286.3	453.9	418.8	376.6	350.2	367.3	440.0	474.2
	city Z	<i>RMSE</i>	755.8	<b>647.2</b>	504.0	687.7	344.8	917.6	475.0	741.5	394.0	812.7	503.5	945.9
		<i>MAPE</i>	70.5%	17.0%	39.1%	<b>16.2%</b>	20.6%	21.1%	36.5%	18.0%	28.9%	18.3%	44.4%	20.9%
		<i>MAE</i>	638.8	473.4	417.0	<b>465.9</b>	262.8	618.2	388.0	510.2	323.2	542.1	427.4	587.7
G	city Y	<i>RMSE</i>	202.5	<b>113.6</b>	188.2	120.7	121.4	207.9	173.6	133.8	134.4	169.9	175.2	141.0
		<i>MAPE</i>	25.2%	17.6%	20.0%	<b>17.4%</b>	12.73%	18.57%	18.8%	17.8%	14.8%	18.2%	21.8%	18.3%
		<i>MAE</i>	146.5	<b>93.5</b>	128.0	102.3	86.1	143.5	121.7	112.2	93.7	130.1	125.8	109.6
	city Z	<i>RMSE</i>	214.6	<b>134.9</b>	192.8	144.5	136.3	172.0	178.2	138.3	142.7	166.7	178.9	175.2
		<i>MAPE</i>	26.8%	<b>11.5%</b>	21.3%	12.1%	15.5%	13.1%	19.5%	12.3%	17.3%	11.8%	23.2%	12.8%
		<i>MAE</i>	150.2	<b>105.4</b>	131.3	112.8	100.9	130.6	124.6	112.2	105.0	121.4	129.0	131.0

In addition, as mentioned above, the proposed MOONN model (i.e., MOONN1) owns the module which evolves the number of hidden nodes, and this module effectively selects the appropriate size of neural network for the forecasting problem. In Table 5.12, the specific size of the evolved neural network in MOONN1 is listed. The mean and standard deviation of maximum, minimum and average number of hidden nodes during 25 runs are recorded in Table 5.12.

Table 5.12: Size of the evolved neural network in MOONN1.

	City Y		City Z	
	F	G	F	G
max	3.2(1.4)	7.3(1.8)	3.0(1.3)	6.3(1.4)
min	1.0(0)	1.0(0)	1.0(0)	1.0(0)
avg	1.7(0.4)	3.3(0.6)	1.6(0.2)	3.0(0.8)

### Comparison of MOONN1 and MOONN2

As introduced earlier, the number of hidden nodes can be evolved when employing MOONN1. In this section, to validate its effectiveness, this module is removed from MOONN1 as in Experiment 1, and the new model obtained is called MOONN2. The forecasting is performed by repeatedly running MOONN2 with different number of hidden nodes from 1 to 10, and then the results are averaged as the final result. Here, MOONN1 and MOONN2 are employed to forecast the amount of replenishment for the

above products. Table 5.11 exhibits the forecasting performance of MOONN1 and MOONN2. According to Table 5.11, MOONN1 has better forecasting accuracy for 3 cases, which are the replenishment of product F in city Z, product G in both city Y and city Z.

### Comparison of MOONN1 and MOONN3

In this subsection, the forecasting performance of MOONN1 and MOONN3 is compared. In MOONN1, the average error of committing  $K$ -fold cross-validation ( $K$ -cv error) on the training samples is set as one of the objectives to be optimized; while in MOONN3, the  $K$ -cv error is replaced by RMSE as one of the two objectives. Table 5.11 shows the forecasting performance of MOONN1 and MOONN3. According to the 3 accuracy measures, it can be found that MOONN1 beats MOONN3 on all the forecasting cases.

In Table 5.13, the size of neural network after evolution in MOONN1 and MOONN3 is offered when forecasting the replenishment amount of the categories of products above. The mean and standard deviation of maximum, minimum and average number of hidden nodes during 25 runs are recorded in Table 5.13.

Table 5.13: Size of the evolved neural network in MOONN1 and MOONN3.

		City Y		City Z	
		F	G	F	G
MOONN1	max	3.2(1.4)	7.3(1.8)	3.0(1.3)	6.3(1.4)
	min	1.0(0)	1.0(0)	1.0(0)	1.0(0)
	avg	1.7(0.4)	3.3(0.6)	1.6(0.2)	3.0(0.8)
MOONN3	max	10.0(0.2)	9.8(0.4)	10.0(0)	10.0(0)
	min	1.0(0)	1.0(0)	1.0(0)	1.0(0)
	avg	5.1(0.4)	4.9(0.3)	5.5(0.3)	5.2(0.3)

### 5.3.6 Performance Comparison of the Models

Based on the two experiments presented in the previous two subsections, here the forecasting performance of the models used in the former experiments

is further compared. Figures 5.4-5.5 illustrate the comparison of forecasting performance generated by different models. In Figure 5.4, each bar indicates the number of the best forecasting performance generated by its corresponding model according to a specified accuracy measure; while in Figure 5.5, each bar represents the number of the worst forecasting performance obtained by the corresponding model in terms of different accuracy measure. From these two figures, MOONN1 generates the best forecasting performance for 12, 8 and 11 cases when *RMSE*, *MAPE* and *MAE* are used as the accuracy measure respectively. In addition, MOONN1 never has the worst forecasting performance for all the cases when three different accuracy measures are used. It is proved that the proposed model is able to provide much superior forecasting performance to other models.

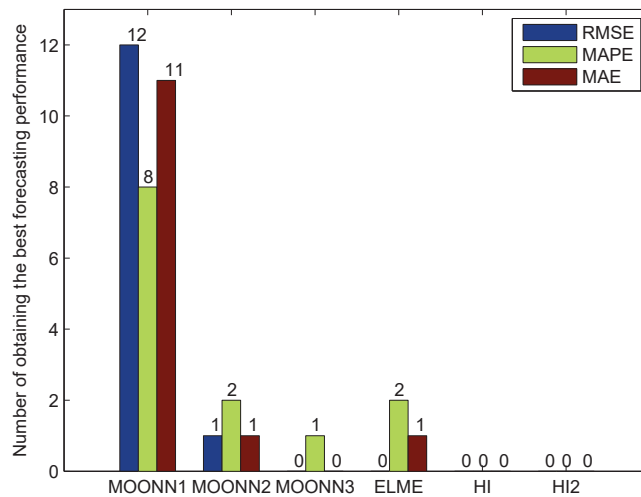


Figure 5.4: Comparison of the best forecasting performance of different models.

### 5.3.7 Summary of the Experiments

From the above results, the proposed MOONN1 can not only outperform its variants MOONN2 and MOONN3, but also show superior performance to several popular models, like ELME, HI and HI2, when facing the short-term



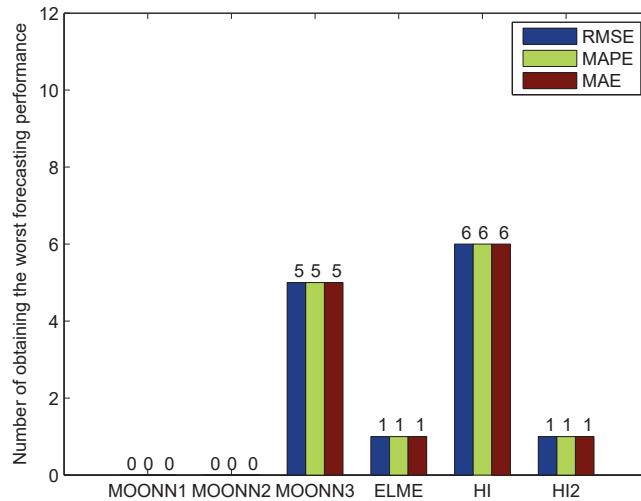


Figure 5.5: Comparison of the worst forecasting performance of different models.

replenishment forecasting problem in fashion industry. It can be attributed to the innovations and improvements of MOONN1, which are summarized as follows:

- (1) MOONN is introduced to solve the short-term replenishment forecasting problem in fashion industry, where JADE is selected as the search engine to improve the search performance of the proposed MOONN1;
- (2) A comprehensive module of exploring the hidden-node-number space is developed in the proposed MOONN model;
- (3) Instead of the common training error,  $K$ -cv error on the training samples is set as one of the objectives we intend to optimize.

Based on these operations, more than one objective can be optimized when making the short-term replenishment forecasting in fashion industry. In addition, the appropriate number of hidden nodes of the neural network can be found according to different problems. Therefore, MOONN1 is guaranteed to acquire the forecasting accuracy while alleviating the overfitting effect at the same time.

## 5.4 Summary

In this chapter, the short-term replenishment forecasting problem is investigated based on the real-world sales data in fashion industry, which facilitates related fashion retail companies to make reasonable replenishment prediction when facing the increasingly fierce market competition. An effective MOONN model was proposed to deal with the investigated problem. The model employs a new multiobjective evolutionary algorithm named NSJADE to optimize the input weights and hidden biases of NN for the short-term replenishment forecasting problem, which acquires the forecasting accuracy while alleviating the overfitting effect at the same time. Furthermore, the MOONN model also selects the appropriate number of hidden nodes of NN in terms of different replenishment forecasting cases. Meanwhile, in order to reduce the side effects of insufficient historical data, the average error of committing  $K$ -fold cross-validation on the training samples is substituted for the general root mean square error as one of the two objectives in the optimization process.

Extensive experiments in terms of the real-world fashion retail data were conducted to validate the proposed MOONN model. The results of the experiments indicate that the presented MOONN model can handle the short-term replenishment forecasting problem effectively, and show much superior performance to several popular forecasting models.

## **Chapter 6**

# **Robust Evolutionary Optimization for Order Scheduling in Fashion Supply Chains**

In this chapter, the order scheduling problems in fashion supply chains are solved via robust evolutionary optimization for the first time. In detail, a robust evolutionary algorithm called robust success-history based adaptive differential evolution with event-triggered impulsive control scheme (robust ETI-SHADE) is proposed for developing robust order schedules in fashion supply chains. In the following, the mathematical model of the fashion order scheduling problem is established at first. Secondly, robust ETI-SHADE is presented to handle the fashion order scheduling problem. Thirdly, two groups of experiments are carried out to display the effectiveness and superiority of introducing robust evolutionary optimization into fashion order scheduling problems. Lastly, the summary of this chapter is given.

## 6.1 Problem Formulation

At the manufacturing stage of fashion supply chains, manufacturers receive orders from customers firstly, and then assign them to multiple factories. In each factory, according to different production processes of producing different product, orders are assigned to different production departments. Usually, it involves several production processes when producing an apparel product, such as cutting, sewing, pressing, tagging, and so on. Among these processes, sewing is the most complicated one, which costs a large amount of time during the manufacturing. Therefore, to simplify the problem, only sewing process is considered when modeling the optimization problem in this chapter. The notations used in developing the mathematical model of the problem are listed in Table 6.1.

Table 6.1: Notations used in the mathematical model.

Notation	Description
$n$	the number of orders
$m$	the number of production lines
$O_i$	$i$ th order, $i = 1, 2, \dots, n$
$L_k$	$i$ th order, $k = 1, 2, \dots, m$
$T_i$	time (days) of producing $i$ th order, $i = 1, 2, \dots, n$
$q_{ij}$	production quantity of $i$ th order on $j$ th day, $i = 1, 2, \dots, n, j = 1, 2, \dots, T_i$
$D_i$	due date of $i$ th order, $i = 1, 2, \dots, n$
$F_i$	finishing date of $i$ th order, $i = 1, 2, \dots, n$

The order scheduling problem investigated in this chapter considers  $m$  production lines, which is numbered as 1 to  $m$ . Different production lines have a group of operators with different machines and skills that are suited to specific product types. And a mismatch of product to production line will lead to the lower performance of the production. Additionally, for each production line, when a new type of product is introduced into production, it takes a period of time for operators to reach the highest efficiency. The variation of efficiency when producing a new type of product can be illustrated by the learning curve. For order  $O_i$ ,  $q_{ij}$  indicates the production quantity of order  $O_i$  on the  $j$ th day. The value of  $q_{ij}$  depends on the following three

factors: 1) the production line  $L_k$  that  $O_i$  is assigned to; 2) which order is processed on  $L_k$  before  $O_i$ ; 3) the learning curve of producing  $O_i$ . The first factor reflects whether  $O_i$  matches  $L_k$ , and a match of product to production line will result in larger  $q_{ij}$ . The second factor reflects the proficiency of the operators on  $L_k$  when producing  $O_i$ . If the same type of product has been processed on  $L_k$  before  $O_i$ , the operators will be more proficient when processing  $O_i$ . The third factor indicates that the value of  $q_{ij}$  also depends on the operator's learning curve of processing  $O_i$ . In the previous studies, it was assumed that the daily production quantity of each order was fixed. However, in real production process, as a result of various uncertainties, like machine breakdown or operator absenteeism, the daily production quantity of each order is not always as expected. In this case, the schedules need to be updated frequently, which means these schedules are very sensitive. While in real-world manufacturing, operators often fail to finish the daily production quantity that was assigned to them and the production plans were shifted very often. After production starts, frequent modification of production plans will increase labor and time cost, which may reduce production efficiency and fail to complete the orders before their delivery dates. Therefore, in this chapter, the daily production quantity of each order varies in a certain interval, which means the schedules obtained are not sensitive to stochastic variation of daily production quantity during the process of real production.

The schedules obtained in this chapter achieve two objectives: 1) the schedules can minimize the total tardiness of all orders; 2) the schedules are robust to random variation of daily production quantity during the process of real production. The first objective is set as the preliminary optimization objective of the order scheduling problem, and is expressed as:

$$\text{minimize } f = \sum_{i=1}^n (F_i - D_i), \quad (6.1)$$

where  $D_i$  is the due date of  $i$ th order, and  $F_i$  is the finishing date of  $i$ th order,  $i = 1, 2, \dots, n$ .

In order to achieve the second objective,  $f$  needs to be converted into  $f^{\text{eff}}$  (see Definition 3.5). It is noticed that  $f$  is related to the production quantity of  $i$ th order on  $j$ th day, i.e.,  $q_{ij}$ ,  $i = 1, 2, \dots, n$ ,  $j = 1, 2, \dots, T_i$ . We assume that  $q_{ij}$  varies in a certain interval  $[(1 - \sigma_1)q_{ij}, (1 + \sigma_2)q_{ij}]$ . When calculating  $f^{\text{eff}}$ ,  $HN$  neighbouring points will be selected around  $q_{ij}$  for each order on each production day. And in the following,  $f^{\text{eff}}$  will be optimized by a robust evolutionary algorithm, the purpose of which is to obtain robust order schedules in the fashion supply chains.

## 6.2 Robust Evolutionary Algorithm for the Problem

In this section, a robust evolutionary algorithm called robust success-history based adaptive differential evolution with event-triggered impulsive control scheme (robust ETI-SHADE) is proposed for developing robust order schedules in fashion supply chains. SHADE is a DE variant, which is recently proposed by Tanabe and Fukunaga [140]. SHADE is an enhancement to JADE (see Section 3.2.2), and a historical memory of successful control parameter settings is utilized to guide the selection of future control parameter values. In Section 4.2, ETI is proposed to improve the explorative and exploitative performance of DE. From the results of the experiments in Section 4.2, ETI-SHADE exhibits the best performance when dealing with the CEC 2014 test suite at  $D = [30, 50, 100]$  (see Tables 4.1-4.3). Therefore, robust ETI-SHADE is selected as the optimization algorithm to search the optimal and robust schedules for fashion order scheduling problems. In the rest of this section, the optimization process is described in detail.

## 6.2.1 Representation

The first step of the optimization is to encode the potential solutions of the problem into chromosomes. For order scheduling problems, the potential solutions are order schedules, which denotes the assignment of the orders on the whole production lines. In this research, order split is considered, and it is assumed that the orders can be at most divided into 2 sub-orders. And each chromosome has two parts: part a and part b. In part a, every two bits represent which production lines the orders will be assigned to; and part b denotes the split percentage of each order. The chromosome representation is illustrated in Fig. 6.1. The length of the chromosome is  $3n$ , where  $n$  is the number of the orders.

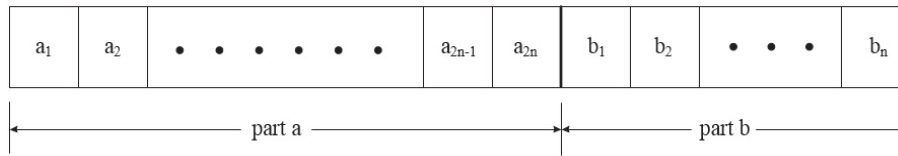


Figure 6.1: Chromosome representation.

Fig. 6.2 shows an example of the representation, which considers an order scheduling problem of allocating 4 orders to 3 production lines. Part a indicates which production lines these 4 orders are assigned to. In detail, the first two bits show that order 1 is processed on line 1; the second two bits denote that order 2 is produced on lines 2 and 3, respectively; the third two bits signifies that order 3 is processed on lines 1 and 2, respectively; and the last two bits in part a mean that order 4 is manufactured on lines 3 and 1, respectively. Part b shows the split percentage of each order during the production. And part b is meaningful only when the orders are split into sub-orders (e.g., orders 2,3 and 4). For example, 0.3 means that order 2 is split into 2 sub-orders, the sizes of which are 30% and 70% of the original order size; and these two sub-orders are processed on lines 2 and 3, respectively. And there is no use for 0.9 since order 1 is not divided into sub-orders.

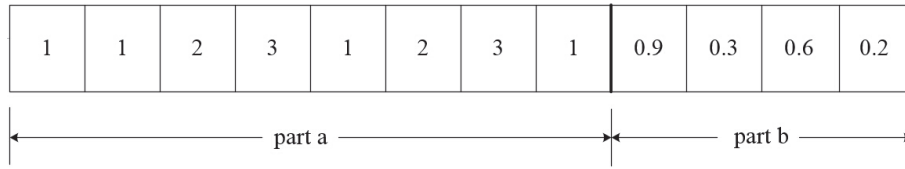


Figure 6.2: An example of chromosome representation.

## 6.2.2 Population Initialization

Since the chromosome contains two parts: part a and part b, the initialization process needs to initialize these two parts respectively. For the initialization of part a, integers in the range  $[1, m]$  are randomly assigned to each dimension of part a, where  $m$  is the number of the production line. Part b shows the split percentage of each order. And to simplify the optimization process, the percentage value is selected from 0.1 0.9, with the interval of 0.1. So for the initialization of part b, values from  $[0.1, 0.9]$  with the interval of 0.1 are stochastically assigned to each dimension of part b.

## 6.2.3 Evaluation of the Population

After the population initialization, the fitness value  $f^{\text{eff}}$  of each individual needs to be evaluated.  $f^{\text{eff}}$  is the mean effective objective value in the robust optimization. To calculate  $f^{\text{eff}}$ ,  $HN$  neighbouring points will be randomly selected around the individual within a predefined range. Then these  $HN$  points will be evaluated, hence we will get  $f_1, f_2, \dots, f_{HN}$ . And  $f_i (i = 1, 2, \dots, HN)$  is the total tardiness of all orders, which is expressed by late days. Next, we explain how to calculate  $f_i (i = 1, 2, \dots, HN)$  in detail, which involves the following five steps:

Step 1. Split the orders according to the split percentage in part b of the chromosome if the orders will be processed on different production lines based on the representation of part a.



- Step 2. According to part a of the chromosome, assign the orders or sub-orders to all the production lines.
- Step 3. On each production line, calculate the daily production quantity  $q_{ij}$  of each order according to the following three factors. Take order  $i$  on production line  $j$  for example: 1) the efficiency of processing order  $i$  on production line  $j$ ; 2) the order processed on production line  $j$  before order  $i$ ; 3) the learning curve of order  $i$ .
- Step 4. Generate a new value of  $q_{ij}$  by randomly select a value in  $[(1 - \sigma_1)q_{ij}, (1 + \sigma_2)q_{ij}]$ .
- Step 5. According to the newly generated value of  $q_{ij}$ , compare the finishing date and the due date of each order, and calculate the total late days.

The minimum value of  $f_i(i = 1, 2, \dots, HN)$  is 0, which means all the orders can be completed before their due dates. If  $f_i(i = 1, 2, \dots, HN)$  is larger than 0, it indicates that some orders cannot be finished before the deadline. After computing  $f_i(i = 1, 2, \dots, HN)$  of the individual's  $H$  neighbouring points, the fitness value  $f^{\text{eff}}$  of the individual can be obtained by average  $f_i(i = 1, 2, \dots, HN)$ . Besides  $f_i(i = 1, 2, \dots, HN)$ , another value  $g_i(i = 1, 2, \dots, HN)$  will be computed during the evaluation process.  $g_i(i = 1, 2, \dots, HN)$  records the early days of all the orders, and  $g^{\text{eff}}$  can be obtained by averaging  $g_i(i = 1, 2, \dots, HN)$ . In the later stage,  $g^{\text{eff}}$  helps the selection of the population.

#### 6.2.4 Mutation

In this research, robust ETI-SHADE is used to optimize the order scheduling problem. Therefore, after the population initialization, the population will undergo the mutation operation of robust ETI-SHADE. The mutation strategy

in robust ETI-SHADE is the same as that in JADE [103]. In the mutation stage, the strategy “DE/current-to- $p$ best” guides the mutation of the parents:

$$\mathbf{v}_{i,g} = \mathbf{x}_{i,g} + F_i \cdot (\mathbf{x}_{best,g}^p - \mathbf{x}_{i,g}) + F_i \cdot (\mathbf{x}_{r1,g} - \tilde{\mathbf{x}}_{r2,g}) \quad (6.2)$$

At the  $g$ th generation, for individual  $\mathbf{x}_i$ , three other individuals  $\mathbf{x}_{best}^p$ ,  $\mathbf{x}_{r1}$ , and  $\tilde{\mathbf{x}}_{r2}$  are picked out in the mutation process. Among them,  $\mathbf{x}_{best}^p$  is chosen from the top  $100p\%$  ( $p \in (0, 1]$ ) individuals of the population;  $\mathbf{x}_{r1}$  is selected at random from the current population;  $\tilde{\mathbf{x}}_{r2}$  is from the current population along with an archive, which stores the inferior individuals during the evolution process. And  $\mathbf{x}_i$ ,  $\mathbf{x}_{best}^p$ ,  $\mathbf{x}_{r1}$  and  $\tilde{\mathbf{x}}_{r2}$  are different from each other. Moreover, at each generation, the mutation factor  $F_i$  of each individual is adaptively generated according to a Cauchy distribution.

### 6.2.5 Crossover

After  $\mathbf{v}_i$  is generated, the population will go through the crossover stage. In this stage, binomial crossover is performed on each dimension of vector  $\mathbf{v}_i$ :

$$u_{j,i,g} = \begin{cases} v_{j,i,g} & j = j_{\text{rand}} \text{ or } \text{rand}(0, 1) < CR_i \\ x_{j,i,g} & \text{otherwise} \end{cases} \quad (6.3)$$

In (6.3),  $j_{\text{rand}}$  is a randomly selected index, which ensures that  $\mathbf{u}_i$  gets at least one component from  $\mathbf{v}_i$ . And at each generation, the crossover probability  $CR_i$  is adaptively created based on a Normal distribution.

## 6.2.6 Selection

After mutation and crossover,  $\mathbf{u}_i$  will be compared with the target vector  $\mathbf{x}_i$  to determine which individual can survive in the next generation:

$$\mathbf{x}_{i,g+1} = \begin{cases} \mathbf{u}_{i,g}, & \text{if } f^{\text{eff}}(\mathbf{u}_{i,g}) < f^{\text{eff}}(\mathbf{x}_{i,g}), \\ \mathbf{u}_{i,g}, & \text{if } f^{\text{eff}}(\mathbf{u}_{i,g}) = f^{\text{eff}}(\mathbf{x}_{i,g}) \text{ and } g^{\text{eff}}(\mathbf{u}_{i,g}) \geq g^{\text{eff}}(\mathbf{x}_{i,g}), \\ \mathbf{x}_{i,g}, & \text{otherwise.} \end{cases} \quad (6.4)$$

In selection operation, we select the individual with the smaller  $f^{\text{eff}}$  value (fewer late days). If two individuals have the same  $f^{\text{eff}}$  value, the individual with the larger  $g^{\text{eff}}$  value (more early days) will be chosen. This operation ensures that the schedules obtained have the fewest late days and the most early days.

## 6.2.7 Parameter Adaptation

For a DE algorithm, there are two critical parameters: scaling factor  $F$  and crossover probability  $CR$ . In this research, robust ETI-SHADE is used as the optimization algorithm, which is based on SHADE [140]. Therefore, the update of  $F$  and  $CR$  conforms to the evolution rules proposed in SHADE, which is a history-based parameter adaptation. The mechanism of the parameter adaptation is described as follows:

1) For  $F$  and  $CR$ , a historical memory with  $H$  entries is established:  $[M_{F,1}, \dots, M_{F,H}]$ ,  $[M_{CR,1}, \dots, M_{CR,H}]$ . In the beginning, the values of  $M_{F,i}$  and  $M_{CR,i}$ ,  $i = 1, 2, \dots, H$ , are initialized as 0.5.

2) At each generation,  $F_i$  and  $CR_i$  of each individual  $\mathbf{x}_i$  are generated by Eqs. 6.5-6.6:

$$F_i = \text{randc}_i(M_{F,k}, 0.1), \quad (6.5)$$

$$CR_i = \text{randn}_i(M_{CR,k}, 0.1), \quad (6.6)$$

where  $\text{randc}()$  and  $\text{randn}()$  are applied to generate random numbers of Cauchy distribution and normal distribution, and  $k$  is randomly chosen from  $[1, H]$ .

3) After selection, if the individual  $\mathbf{x}_i$  is successfully replaced by  $\mathbf{u}_i$ , its parameters  $F$  and  $CR$  will be recorded in  $S_F$  and  $S_{CR}$ . And the elements in  $[M_{F,1}, \dots, M_{F,H}]$  and  $[M_{CR,1}, \dots, M_{CR,H}]$  will be updated as below:

$$M_{F,r,g+1} = \begin{cases} \text{mean}_L(S_F), & \text{if } S_F \neq \emptyset, \\ M_{F,r,g}, & \text{otherwise,} \end{cases} \quad (6.7)$$

$$M_{CR,r,g+1} = \begin{cases} \text{mean}_A(S_{CR}), & \text{if } S_{CR} \neq \emptyset, \\ M_{CR,r,g}, & \text{otherwise.} \end{cases} \quad (6.8)$$

where  $\text{mean}_A()$  is the arithmetic mean operation, and  $\text{mean}_L()$  is the Lehmer mean operation.  $r$  ( $1 \leq r \leq H$ ) determines which element in  $[M_{F,1}, \dots, M_{F,H}]$  and  $[M_{CR,1}, \dots, M_{CR,H}]$  will be updated.

## 6.2.8 Further Exploration and Exploitation

Unlike the original SHADE, an event-triggering-based impulsive control scheme (ETI) is introduced to improve the performance of SHADE, and the new variant is called ETI-SHADE.

ETI contains two concepts of the control theory: event-triggered mechanism (ETM) and impulsive control. ETM and impulsive control are integrated into the framework of DE algorithms. ETM identifies when the individuals should be injected with impulsive controllers, while impulsive control alters the positions of partial individuals when triggering conditions are violated. Specifically, two types of impulses, i.e., stabilizing impulses and destabilizing impulses, are imposed on the selected individuals (sorted by an index

according to the fitness value and the number of consecutive stagnation generation) when the update rate ( $UR$ ) in the current generation decreases or equals to zero. On one hand, when  $UR$  begins to decrease, stabilizing impulses drive the individuals with lower rankings in the current population to approach the individuals with better fitness values. The purpose of stabilizing impulsive control is to help update some inferior individuals and to enhance the exploitation capability of the algorithm. On the other hand, when  $UR$  drops to zero or stabilizing impulses fail to take effect, destabilizing impulses randomly adjust the positions of the inferior individuals within the area of the current population. This operation improves the diversity of the population and hence improves the exploration ability of DE.

The experimental results in Section 4.2 reveal that ETI can enhance the exploration and exploitation performance of the original DE algorithm. For more details of ETI, please refer to Section 4.2.

### **6.3 Experimental Results and Discussion**

To demonstrate the effectiveness and superiority of introducing robust evolutionary optimization into fashion order scheduling problems, two groups of experiments were conducted based on the test data from Fast React, a powerful system for apparel production planning. In this section, Fast React is firstly introduced. Secondly, the test data are provided in detail, which contain the information of the production lines and the orders. Thirdly, two groups of experiments with different number of orders are conducted, in which robust ETI-SHADE is used to search the robust order schedules in fashion supply chains.

### 6.3.1 Fast React

Fast React is a production planning system for fashion industry, which is in use in a number of real businesses ranging from carpet, cloth and lace weavers to shoe manufacturers, clothing companies and so on. The system is for production planning in real-world production, which involves mismatch problem of production lines and orders, learning effect of operators, and so on. By using Fast React, the companies can make the planning much simpler and more efficient.

### 6.3.2 Test Data

The test data are collected from Fast React, and include 6 production lines and 30 orders. For the 6 production lines, the efficiencies of processing different type of orders are listed in Table 6.2. Each order is characterized by five attributes: order number, product type, due date, order size, and standard minutes per piece. The details of the orders are listed in Table 6.3.

Table 6.2: The details of the 6 production lines.

Production Line No.	Efficiency for Blouse (%)	Efficiency for Skirts/Pants (%)	Efficiency for Jackets (%)	Capacity (mins/day)
1	100	80	80	6240
2	100	80	80	6240
3	80	100	80	6720
4	80	100	80	6720
5	80	80	100	6720
6	80	80	100	6720

As shown in Table 6.3, there are four types of products in the production process: blouse, skirts, pants, and jackets. According to the learning curve of producing each product, the efficiency of the operators gradually reaches the highest level (100%) as time goes on. In the following, the learning curves of these four types of products are provided in Figs. 6.3-6.5.

In the next two subsections, two groups of experiments are conducted

Table 6.3: The details of the 30 orders.

Order No.	Product Type	Due Date (days)	Order Size (pieces)	Standard Mins/Piece
1	Skirts	4	500	18.2
2	Skirts	4	700	18.2
3	Skirts	6	1000	16.7
4	Blouse	7	800	32.2
5	Skirts	8	800	16.7
6	Jackets	9	850	54.6
7	Skirts	9	1000	16.7
8	Skirts	7	400	16.7
9	Blouse	10	1250	15
10	Jackets	14	1000	53.78
11	Skirts	11	800	20.55
12	Blouse	13	1000	15
13	Pants	8	800	34
14	Blouse	20	3000	32.2
15	Skirts	15	800	20.55
16	Blouse	16	650	32.2
17	Skirts	14	1000	16.7
18	Jackets	16	500	53.78
19	Jackets	18	800	53.78
20	Skirts	10	870	14.2
21	Blouse	21	860	32.2
22	Skirts	19	1000	16.7
23	Skirts	18	800	20.55
24	Skirts	18	500	16.7
25	Blouse	20	700	32.2
26	Skirts	22	400	16.7
27	Jackets	26	850	54.6
28	Pants	22	600	34
29	Blouse	28	5000	12.6
30	Blouse	25	700	32.2

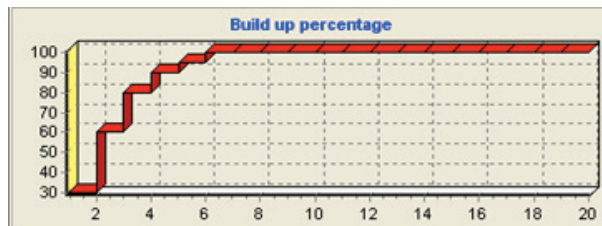


Figure 6.3: The learning curve of the blouse production.

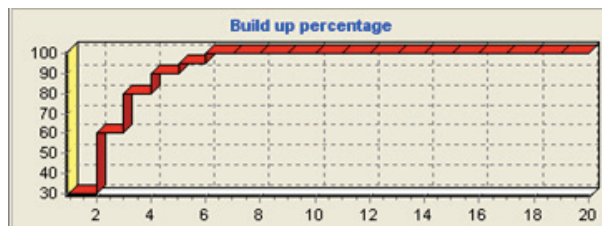


Figure 6.4: The learning curve of the skirts/pants production.

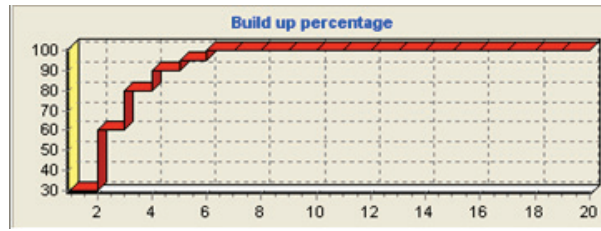


Figure 6.5: The learning curve of the jackets production.

according to the test data. In the first experiment, 20 orders are assigned to 6 production lines with the consideration of uncertain daily production quantities. In the second experiment, we increase the number of orders to 30, and these orders are also allocated to 6 production lines with the consideration of uncertain daily production quantities.

### 6.3.3 Experiment 1: Order Scheduling Problems of 20 Orders in Fashion Supply Chains

In the section, 20 orders are assigned to total 6 production lines in terms of robust ETI-SHADE in fashion supply chains. The details of the 20 orders are listed in Table 6.3, and the order number is from 1 to 20. The parameters of ETI-SHADE are the same as the original work:  $M_F = 0.5$ ,  $M_{CR} = 0.5$ ,  $H = NP$ ,  $LN = 1$ ,  $UN = NP$ , where  $NP$  is the population size and is set as 100 in this research. The number of neighbouring points  $HN = 10$ . The extent of neighbourhood  $\sigma = [-\sigma_1, \sigma_2]$ , where  $\sigma_1 = 0.15$ ,  $\sigma_2 = 0.05$ . This setting means that if the daily production quantity is set as 100, in real production process, the quantity varies in the range of  $[85, 105]$ . The maximum number of function evaluations (Max\_FES) is set to  $D \cdot 10000$ , where  $D$  is the dimension size of the problem.

Non-robust ETI-SHADE and robust ETI-SHADE are utilized to search the optimal order schedules of the total 20 orders, respectively. The schedule obtained by non-robust ETI-SHADE is called schedule A, and the one



achieved by robust ETI-SHADE is called schedule B. In Table 6.4, we record the due date, finish date in schedule A and in schedule B respectively, which are represented by the days from the beginning of the production. According to the results in Table 6.4, it can be found that in both schedules, all the 20 orders can be finished before the due dates. While due to the uncertainty introduced in schedule B, it costs more days to process 9 orders in schedule B than that in schedule A, and these orders are highlighted in **boldface**. The detailed order assignments on 6 production lines are provided in Tables 6.5-6.6. The figures in the brackets denote the specific order size after order split.

Table 6.4: The finish dates of the 20 orders in schedule A and schedule B.

Order No.	Due Date (days)	Finish Date in Schedule A (days)	Finish Date in Schedule B (days)
1	4	3	3
2	4	4	4
3	6	6	6
4	7	6	6
5	8	7	7
6	9	9	8
7	9	9	9
8	7	3	<b>4</b>
9	10	9	<b>10</b>
10	14	13	<b>14</b>
11	11	10	<b>11</b>
12	13	12	12
13	8	7	<b>8</b>
14	20	20	20
15	15	13	<b>15</b>
16	16	15	<b>16</b>
17	14	11	<b>12</b>
18	16	15	15
19	18	18	18
20	10	9	<b>10</b>

Table 6.5: The details of the order assignments on 6 production lines in schedule A.

Production Line No.	Order Assignments
1	$O_1(350), O_3(800), O_5(640), O_7(500), O_{20}(348), O_{17}(500), O_{15}(800)$
2	$O_1(150), O_2(420), O_3(200), O_5(160), O_{13}(400), O_7(500), O_{11}(560), O_{12}(300), O_{14}(1200)$
3	$O_4(800), O_9(1250), O_{12}(700), O_{16}(65), O_{14}(1800)$
4	$O_8(320), O_{13}(400), O_{20}(522), O_{11}(240), O_{17}(500), O_{16}(585)$
5	$O_2(280), O_8(80), O_6(170), O_{10}(700), O_{19}(560)$
6	$O_6(680), O_{10}(300), O_{18}(500), O_{19}(240)$

In the following, we introduce uncertainty into schedule A, and the results are given in Table 6.7. From the results, we can find that if uncertainty is

Table 6.6: The details of the order assignments on 6 production lines in schedule B.

Production Line No.	Order Assignments
1	$O_2(350), O_3(400), O_5(480), O_{13}(320), O_{20}(870), O_{17}(1000), O_{15}(800)$
2	$O_1(250), O_2(350), O_3(600), O_5(320), O_7(600), O_{11}(800), O_{14}(1200)$
3	$O_8(400), O_{13}(480), O_7(400), O_{12}(700), O_{16}(650)$
4	$O_4(800), O_9(1250), O_{12}(300), O_{14}(1800)$
5	$O_1(250), O_6(255), O_{10}(300), O_{18}(500), O_{19}(400)$
6	$O_6(595), O_{10}(700), O_{19}(400)$

brought into schedule A, more days will be needed for 13 of the 20 orders, which are highlighted in **boldface**. More seriously, for order 14 and order 19 (in boldface with gray background), they cannot be completed before their due dates if uncertainty is considered in schedule A. Therefore, it can be concluded that schedule A is sensitive to the perturbation, while schedule B is robust to the perturbation. In practice, planners will update their schedule during the production. If we consider the schedule like A, which is with fixed daily production quantities, the schedule will be updated very often because of the randomness in reality. While schedule B has uncertainty-tolerant ability. So schedule B is recommended for the real-world production.

### 6.3.4 Experiment 2: Order Scheduling Problems of 30 Orders in Fashion Supply Chains

In this section, the total 30 orders are assigned to 6 production lines in fashion supply chains. The optimization algorithm is robust ETI-SHADE, which aims to search the optimal and robust order schedules for the production. The details of the orders and the production lines are listed in Tables 6.2-6.3. The parameters of ETI-SHADE and the settings of the experiment are the same as that in Experiment 1.

Non-robust ETI-SHADE and robust ETI-SHADE are applied to search the optimal order schedules of the total 30 orders, respectively. The schedule obtained by non-robust ETI-SHADE is called schedule C, and the one

Table 6.7: The finish dates of the 20 orders in schedule A when uncertainty is introduced.

Order No.	Due Date (days)	Finish Date in Schedule A	Finish Date in Schedule A (With Uncertainty)
1	4	3	3
2	4	4	4
3	6	6	6
4	7	6	6
5	8	7	<b>8</b>
6	9	9	9
7	9	9	9
8	7	3	<b>4</b>
9	10	9	<b>10</b>
10	14	13	<b>14</b>
11	11	10	<b>11</b>
12	13	12	<b>13</b>
13	8	7	<b>8</b>
14	20	20	<b>21</b>
15	15	13	<b>14</b>
16	16	15	<b>16</b>
17	14	11	11
18	16	15	<b>16</b>
19	18	18	<b>19</b>
20	10	9	<b>10</b>

achieved by robust ETI-SHADE is called schedule D. In Table 6.8, we record the due date, finish date in schedule C and in schedule D respectively, which are represented by the days from the beginning of the production. According to the results in Table 6.8, it can be found that in schedule C, all the orders can be completed before their due dates. While due to the uncertainty introduced in schedule D, it costs more days to process 16 orders in schedule D than that in schedule C, and these orders are highlighted in **boldface**. More seriously, in schedule D, four orders (i.e., orders 6, 7, 14, and 28) cannot be finished before the due dates, which are highlighted in boldface with gray background. The results indicate that uncertainty (i.e., varying daily production quantity) has a great impact on the actual delivery date of the orders during the production process. For these 4 four orders in schedule D, manufacturers can negotiate earlier with the retailers who place these orders about the delay in delivery or arrange operators to work extra hours. The detailed order assignments on 6 production lines are provided in Tables 6.9-6.10. The figures in the brackets

denote the specific order size after order split.

Table 6.8: The finish dates of the 30 orders in schedule C and schedule D.

Order No.	Due Date (days)	Finish Date in Schedule C (days)	Finish Date in Schedule D (days)
1	4	3	<b>4</b>
2	4	4	4
3	6	6	6
4	7	6	6
5	8	8	8
6	9	9	<b>10</b>
7	9	9	<b>10</b>
8	7	6	6
9	10	9	9
10	14	11	11
11	11	11	11
12	13	11	11
13	8	8	8
14	20	20	<b>22</b>
15	15	15	14
16	16	14	<b>15</b>
17	14	12	<b>14</b>
18	16	13	<b>15</b>
19	18	16	<b>18</b>
20	10	10	10
21	21	21	21
22	19	17	<b>19</b>
23	18	15	<b>16</b>
24	18	15	<b>17</b>
25	20	20	20
26	22	18	<b>20</b>
27	26	23	<b>25</b>
28	22	21	<b>23</b>
29	28	26	<b>28</b>
30	25	24	<b>25</b>

Table 6.9: The details of the order assignments on 6 production lines in schedule C.

Production Line No.	Order Assignments
1	$O_1(250), O_{13}(800), O_7(600), O_{20}(522), O_{11}(160), O_{17}(600), O_{23}(800), O_{24}(250), O_{22}(600), O_{26}(400), O_{28}(600), O_{30}(280)$
2	$O_2(560), O_3(400), O_8(400), O_5(800), O_7(400), O_{11}(640), O_{17}(400), O_{15}(800), O_{24}(250), O_{22}(400), O_{25}(140), O_{21}(430), O_{30}(420)$
3	$O_1(250), O_2(140), O_3(600), O_{20}(348), O_{12}(200), O_{14}(2100)$
4	$O_4(800), O_9(1250), O_{12}(800), O_{16}(650), O_{14}(900), O_{21}(430), O_{29}(2500)$
5	$O_6(85), O_{10}(800), O_{18}(250), O_{19}(480), O_{27}(850)$
6	$O_6(765), O_{10}(200), O_{18}(250), O_{19}(320), O_{25}(560), O_{29}(2500)$

In the following, we introduce uncertainty into schedule C, and the results are given in Table 6.11. From the results, we can find that if uncertainty is brought into schedule C, more days will be needed for 25 of the 30 orders, which are highlighted in **boldface**. More seriously, for 10 orders (in boldface

Table 6.10: The details of the order assignments on 6 production lines in schedule D.

Production Line No.	Order Assignments
1	$O_1(50), O_2(420), O_3(200), O_5(320), O_{13}(400), O_{20}(696), O_{11}(560), O_{17}(900), O_{15}(240), O_{23}(560), O_{24}(450), O_{22}(700), O_{26}(400), O_{28}(420), O_{30}(140)$
2	$O_2(280), O_3(800), O_{13}(400), O_7(1000), O_{11}(240), O_{17}(100), O_{15}(560), O_{23}(240), O_{24}(50), O_{22}(300), O_{25}(490), O_{28}(180), O_{27}(85)$
3	$O_1(450), O_8(400), O_5(480), O_{20}(174), O_{12}(700), O_{16}(650), O_{25}(210), O_{21}(860), O_{30}(560), O_{29}(2000)$
4	$O_4(800), O_9(1250), O_{12}(300), O_{14}(2100), O_{29}(3000)$
5	$O_{10}(900), O_{18}(500), O_{14}(900)$
6	$O_6(850), O_{10}(100), O_{19}(800), O_{27}(765)$

with gray background), they cannot be completed before their due dates if uncertainty is considered in schedule C. Look back at Table 6.8, in schedule D, there are only total 4 orders that will violate the due dates. Therefore, it can be concluded that schedule C is sensitive to the perturbation, while schedule D is more robust to the perturbation.

### 6.3.5 Summary of the Experiments

In this section, non-robust and robust evolutionary algorithms (i.e., non-robust and robust ETI-SHADE) are used for order scheduling problems in fashion supply chains, respectively. According to the results shown above, it can be concluded that order schedules obtained by non-robust ETI-SHADE are sensitive to the perturbation in the real production, i.e., shifting daily production quantity. And schedules obtained robust ETI-SHADE benefits the real-world production in the following two ways: 1) if in the schedules, all the orders can be finished before their due dates, the production can follow these schedules; 2) if in the schedules, there are some orders that cannot be completed before the due dates, manufacturers may be fully prepared according to these schedules by negotiating earlier with the retailers who place these orders about the delay in delivery or arranging operators to work extra hours.

Table 6.11: The finish dates of the 30 orders in schedule C when uncertainty is introduced.

<b>Order No.</b>	<b>Due Date (days)</b>	<b>Finish Date in Schedule C</b>	<b>Finish Date in Schedule C (With Uncertainty)</b>
1	4	3	3
2	4	4	4
3	6	6	7
4	7	6	6
5	8	8	9
6	9	9	10
7	9	9	10
8	7	6	7
9	10	9	9
10	14	11	12
11	11	11	12
12	13	11	12
13	8	8	8
14	20	20	21
15	15	15	16
16	16	14	15
17	14	12	13
18	16	13	14
19	18	16	17
20	10	10	11
21	21	21	22
22	19	17	18
23	18	15	16
24	18	15	16
25	20	20	22
26	22	18	19
27	26	23	24
28	22	21	22
29	28	26	28
30	25	24	25

## 6.4 Summary

In this chapter, robust evolutionary optimization is introduced to solve the order scheduling problems in fashion supply chains. A robust evolutionary algorithm called robust success-history based adaptive differential evolution with event-triggered impulsive control scheme (robust ETI-SHADE) is proposed for developing robust order schedules in fashion supply chains. Robust ETI-SHADE aims to search optimal and robust schedules for the real-world production. In addition, two groups of experiments are carried out to display the effectiveness and superiority of introducing robust evolutionary optimization into fashion order scheduling problems. The experimental results show that schedules obtained by robust ETI-SHADE have uncertainty-tolerant ability, and benefit the real-world production in fashion supply chains.

# Chapter 7

## Conclusions

This research is set out to solve sales forecasting and order scheduling problems in fashion supply chains via two hot branches of evolutionary optimization (multiobjective evolutionary optimization and robust evolutionary optimization) for the first time, and hence to establish a competitive and robust supply chain in fashion industry. This research has provided satisfactory strategies for fashion sales forecasting and order scheduling problems by means of multiobjective evolutionary optimization and robust evolutionary optimization; meanwhile, the proposed models has also broadened the applications of evolutionary optimization in wider areas. In this chapter, firstly, the concluding remarks of this research are summarized, including the summaries of the development of novel evolutionary algorithms, multiobjective evolutionary optimization for fashion sales forecasting, and robust evolutionary optimization for fashion order scheduling. Secondly, the contributions of this research are given. Then the future work is presented based on the limitations of this research. Lastly, the related publications are listed at the end of this chapter.



## 7.1 Development of Novel Evolutionary

### Algorithms

In Chapter 4, two variants of the evolutionary algorithms have been proposed, one is a new multiobjective evolutionary algorithm (MOEA) called nondominated sorting adaptive differential evolution (NSJADE), and the other one is a novel differential evolution (DE) variant called differential evolution with event-triggered impulsive control scheme (ETI-DE). Firstly, NSJADE modifies the search engine of the classic MOEA called NSGA-II by replacing genetic algorithm (GA) with an adaptive differential evolution (JADE). This operation enhances the efficiency of the search, in which the useful historical information of the population can be utilized. The experimental results reveal that the presented NSJADE performs better than NSGA-II on multimodal problems, which makes NSJADE a promising candidate for solving the fashion sales forecasting in the later chapter. Secondly, ETI-DE integrates two popular concepts in control theory, event-triggered mechanism and impulsive control, into the design of DE. By means of ETI, both exploitation and exploration abilities of the whole population in DE can be meliorated. The experimental results show that ETI-DEs greatly improve the performance of their original DEs on the test function at  $D = [30, 50, 100]$ . In addition, from the results of the experiments in Section 4.2, ETI-SHADE exhibits the best performance when dealing with the CEC 2014 test suite at  $D = [30, 50, 100]$ . Due to the superiority of ETI-SHADE in solving high-dimensional problems, it is modified to fit into the robust optimization, and then to optimize the order scheduling problem in fashion industry later on.

## **7.2 Multiobjective Evolutionary Optimization for Fashion Sales Forecasting**

In Chapter 5, fashion sales forecasting problems are investigated by a multiobjective evolutionary optimization-based neural network model for the first time. In detail, a multiobjective optimization-based neural network model (MOONN) is developed to handle a short-term replenishment forecasting problem in fashion supply chains. The model employs a new multiobjective evolutionary algorithm called NSJADE to optimize the input weights and hidden biases of NN for the short-term replenishment forecasting problem, which acquires the forecasting accuracy while alleviating the overfitting effect at the same time. Furthermore, the MOONN model also selects the appropriate number of hidden nodes of NN in terms of different replenishment forecasting cases. Meanwhile, in order to reduce the side effects of insufficient historical data, the general root mean square error is replaced with the average error of committing  $K$ -fold cross-validation on the training samples as one of the two objectives in the optimization process. Extensive experiments in terms of the real-world fashion retail data have been carried out to validate the proposed MOONN model. The results of the experiments indicate that the presented MOONN model can handle the short-term replenishment forecasting problem effectively, and show much superior performance to several popular forecasting models.

## **7.3 Robust Evolutionary Optimization for Fashion Order Scheduling**

In Chapter 6, fashion order scheduling problems are solved within the framework of robust evolutionary optimization for the first time. In detail,

a robust evolutionary algorithm called robust success-history based adaptive differential evolution with event-triggered impulsive control scheme (robust ETI-SHADE) is employed to develop robust order schedules in fashion supply chains. During the optimization process, robust ETI-SHADE evaluates the  $HN$  neighbouring points of the individual, and then calculate the average value of these  $HN$  values as the optimization objective of the order scheduling problem. The intention is to obtain robust order schedules in fashion supply chains. Two groups of experiments have been performed to show the effectiveness of the introduction of robust evolutionary algorithms in fashion order scheduling. It is revealed that the schedule got by non-robust ETI-SHADE (schedule A) is sensitive to the perturbation, while the schedule obtained by robust ETI-SHADE (schedule B) is robust to the perturbation. In practice, planners will update their schedule during the production. If we consider the schedule like A, which is with fixed daily production quantities, the schedule will be updated very often because of the randomness in reality. While schedule B has uncertainty-tolerant ability. So schedule B is recommended for the real-world production.

## **7.4 Contributions of this Research**

The contributions of this research include: 1) the contributions of proposing the newly proposed evolutionary algorithms for fashion supply chains; 2) the contributions of solving sales forecasting problems in fashion supply chains by a multiobjective evolutionary optimization based neural network model; 3) the contributions of solving order scheduling problems in fashion supply chains within the framework of robust evolutionary optimization; 4) the performance of fashion supply chains can be greatly improved by using multiobjective evolutionary optimization and robust evolutionary optimization. In the following, these four contributions are discussed in detail.

Firstly, NSJADE is proposed based on a popular and classic MOEA: NSGA-II. In recent years, many works have focused on the improvement of NSGA-II, and our research belongs to one of them. The presented NSJADE enriches the set of the MOEAs, which derive from the idea of NSGA-II. Furthermore, a new scheme ETI is developed for the DE algorithms. Unlike other research, this research is an interdisciplinary one, which utilizes event-triggered mechanism (ETM) and impulsive control, two concepts in control theory, to improve the search performance of DE. The development of ETI reveal that it is promising to develop powerful evolutionary algorithms by knowledge in other disciplines. The proposed ETI also sheds light on the understandings of ETM and impulsive control in evolutionary computation, which broadens the applications of ETM and impulsive control in wider areas.

Secondly, a multiobjective optimization-based neural network model (MOONN) is proposed for the sales forecasting problems in fashion supply chains. It is the first work that investigates the sales forecasting problems in fashion supply chains by using MOO-based model. Different from other popular models, MOONN can ensure better forecasting performance and alleviate the overfitting effect at the same time. In addition, the MOONN model also selects the appropriate number of hidden nodes of NN in terms of different forecasting cases; while in other models, the number of hidden nodes of NN is not adaptive.

Thirdly, order scheduling problems in fashion supply chains are solved in the framework of robust evolutionary optimization for the first time. When visiting some garment factories in Mainland China, it is found that most of the time, operators cannot finish the daily production quantity that was assigned to them and the production plans were shifted very often. After production starts, frequent modification of production plans will increase labor and time cost, which may reduce production efficiency and fail to complete the orders before their delivery dates. While the order schedules obtained by robust

evolutionary algorithms are robust to the perturbation of daily production quantities. When robust schedules are adopted, planners will reduce the times of modifying the order schedules during the production process, which increases the efficiency of the production in fashion supply chains. Besides, in this research, matching problem and learning effect are also considered in the optimization process, which makes the experimental environment more close to the real-world production environment.

Fourthly, two key decision-making problems in fashion supply chain management are investigated within the framework of evolutionary optimization. According to the experimental results, multiobjective evolutionary optimization and robust evolutionary optimization exhibit effectiveness in sales forecasting and order scheduling problems in fashion supply chain management. Therefore, the performance of fashion supply chains can be greatly enhanced by introducing multiobjective evolutionary optimization and robust evolutionary optimization.

## **7.5 Limitations and Future Work**

In Chapter 5, MOONN model is used to solve the short-term replenishment forecasting problem in fashion supply chains, which belongs to a multivariate forecasting problem. As introduced before, forecasting problems can be divided into two categories: univariate and multivariate. Therefore, the performance of MOONN on univariate forecasting problems needs to be examined in the future research. Furthermore, it is noticed that to solve the forecasting problem in this research, the inputs of the MOONN model include weather data, economic data and historical sales data. However, these inputs may not be optimal, which means some inputs may be redundant. And irrelevant input variables probably decrease the accuracy of the forecasting

model. So it is necessary to examine the redundancy of the input variables. Since ELM was proposed by Huang *et al.* [7], it has been widely used in fashion sales forecasting. While recently, a new ELM, called optimally pruned extreme learning machine (OP-ELM), has been presented, which can delete the irrelevant inputs for the forecasting problems [143, 144]. Hence, in the future, we are interested to investigate whether OP-ELM is effective for the fashion sales forecasting problems.

In Chapter 6, a robust evolutionary algorithm is employed to optimize the order scheduling problems in fashion supply chains. In this work, the main purpose is to introduce robust evolutionary algorithms into the optimization of fashion order scheduling problems. So in order to simplify the problem, we set several assumptions, such as only considering the sewing process, the order can be at most split into two sub-orders, and so on. In the future research, it is necessary to consider more processes and more sub-orders. Moreover, in this research, it is assumed that all the orders are ready for production when the production process starts. However, in real-world apparel production, a series of activities need to be carried out before an order can be put into production. And these activities are known as pre-production events. In future research, pre-production events can be considered into the robust order scheduling problems in the fashion industry, and the problem can be modelled as a multiobjective optimization problem.

## **7.6 Related Publications**

The author demonstrated the originality of this research through the following publications:

[1] W. Du, S. Y. S. Leung, Y. Tang, A. V. Vasilakos. Differential evolution with event-triggered impulsive control. *IEEE Transactions on Cybernetics*,

accepted, 2016.

[2] W. Du, S. Y. S. Leung, C. K. Kwong. Time series forecasting by neural networks: A knee point-based multiobjective evolutionary algorithm approach. *Expert Systems with Applications*, 2014, 41(18): 8049-8061.

[3] W. Du, S. Y. S. Leung, C. K. Kwong. A multiobjective optimization-based neural network model for short-term replenishment forecasting in fashion industry. *Neurocomputing*, 2015, 151: 342-353.

[4] W. Du, S. Y. S. Leung, C. K. Kwong, Y. Tang, Improving differential evolution with impulsive control framework, in 2015 IEEE Congress on Evolutionary Computation (CEC), pp. 3080-3087, IEEE, 2015.

# Appendix

Table A.1: Experimental results of DE/rand/1/bin, DE/best/1/bin, jDE, JADE and the related ETI-based variants for functions F01-F30 at  $D = 30$ .

Function	DE/rand/1/bin	ETI-DE/rand/1/bin	DE/best/1/bin	ETI-DE/best/1/bin
F01	5.731E+04 ± 4.288E+04	9.916E+04 ± 6.514E+04	2.395E+07 ± 8.307E+06	1.943E+06 ± 1.009E+06
F02	2.229E-15 ± 7.717E-15	0.000E+00 ± 0.000E+00	2.786E-14 ± 3.980E-15	2.341E-14 ± 1.094E-14
F03	5.573E-15 ± 1.707E-14	0.000E+00 ± 0.000E+00	2.003E-08 ± 1.465E-08	3.864E-09 ± 2.710E-09
F04	2.340E-01 ± 1.217E-01	1.638E+00 ± 8.823E+00	6.658E+01 ± 2.845E+01	9.530E+00 ± 2.228E+01
F05	2.094E+01 ± 5.340E-02	2.036E+01 ± 9.584E-02	2.092E+01 ± 5.139E-02	2.037E+01 ± 1.520E-01
F06	4.206E-01 ± 7.135E-01	6.459E-01 ± 9.502E-01	1.136E+00 ± 1.056E+00	1.491E+00 ± 1.247E+00
F07	3.383E-04 ± 1.709E-03	0.000E+00 ± 0.000E+00	5.068E-03 ± 8.609E-03	4.299E-03 ± 5.928E-03
F08	1.256E+02 ± 2.355E+01	2.705E+01 ± 7.498E+00	8.643E+01 ± 1.907E+01	1.676E+01 ± 4.181E+00
F09	1.780E+02 ± 7.943E+00	2.620E+01 ± 1.143E+01	1.830E+02 ± 1.006E+01	3.922E+01 ± 1.097E+01
F10	5.153E+03 ± 7.884E+02	4.790E+02 ± 2.054E+02	1.447E+03 ± 1.012E+03	2.492E+02 ± 1.397E+02
F11	6.817E+03 ± 2.739E+02	1.659E+03 ± 6.172E+02	6.411E+03 ± 2.731E+02	2.042E+03 ± 6.790E+02
F12	2.426E+00 ± 2.209E-01	1.539E-01 ± 7.075E-02	2.061E+00 ± 2.656E-01	1.609E-01 ± 9.056E-02
F13	3.525E-01 ± 4.635E-02	1.294E-01 ± 4.294E-02	3.814E-01 ± 4.366E-02	2.266E-01 ± 5.200E-02
F14	2.834E-01 ± 3.250E-02	2.418E-01 ± 4.008E-02	3.352E-01 ± 1.816E-01	3.145E-01 ± 1.507E-01
F15	1.548E+01 ± 9.788E-01	3.726E+00 ± 1.152E+00	1.653E+01 ± 1.111E+00	4.421E+00 ± 1.688E+00
F16	1.242E+01 ± 2.280E-01	8.754E+00 ± 9.531E-01	1.215E+01 ± 3.340E-01	9.596E+00 ± 8.715E-01
F17	1.331E+03 ± 2.044E+02	4.381E+02 ± 3.364E+02	5.478E+05 ± 3.288E+05	6.073E+04 ± 5.213E+04
F18	5.407E+01 ± 5.476E+00	1.061E+01 ± 4.898E+00	9.291E+02 ± 5.305E+02	1.979E+03 ± 2.396E+03
F19	4.529E+00 ± 3.228E-01	2.279E+00 ± 6.371E-01	6.547E+00 ± 1.325E+00	4.723E+00 ± 1.589E+00
F20	3.354E+01 ± 7.184E+00	7.475E+00 ± 2.768E+00	9.802E+01 ± 1.407E+01	3.668E+01 ± 1.679E+01
F21	6.480E+02 ± 1.565E+02	2.353E+02 ± 2.406E+02	2.051E+04 ± 9.870E+03	1.143E+04 ± 1.013E+04
F22	3.902E+01 ± 3.470E+01	1.268E+02 ± 1.332E+02	1.570E+02 ± 1.108E+02	1.964E+02 ± 1.157E+02
F23	3.152E+02 ± 4.019E-13	3.152E+02 ± 4.019E-13	3.152E+02 ± 4.019E-13	3.152E+02 ± 4.019E-13
F24	2.185E+02 ± 8.658E+00	2.182E+02 ± 8.527E+00	2.238E+02 ± 7.113E+00	2.228E+02 ± 8.438E+00
F25	2.027E+02 ± 1.438E-01	2.027E+02 ± 1.201E-01	2.089E+02 ± 2.326E+00	2.030E+02 ± 4.116E-01
F26	1.003E+02 ± 4.955E-02	1.001E+02 ± 4.295E-02	1.004E+02 ± 5.113E-02	1.002E+02 ± 4.926E-02
F27	3.592E+02 ± 4.864E+01	3.542E+02 ± 4.772E+01	3.793E+02 ± 5.099E+01	3.804E+02 ± 5.092E+01
F28	8.037E+02 ± 2.643E+01	8.111E+02 ± 2.479E+01	7.832E+02 ± 5.699E+01	8.085E+02 ± 4.622E+01
F29	6.838E+02 ± 1.331E+02	6.930E+02 ± 1.155E+02	1.775E+05 ± 1.249E+06	1.707E+05 ± 1.210E+06
F30	5.817E+02 ± 2.335E+02	6.597E+02 ± 2.440E+02	1.985E+03 ± 7.121E+02	1.723E+03 ± 7.477E+02
+ / ≈ / -	-	18/75	-	20/91
Function	jDE	ETI-jDE	JADE	ETI-JADE
F01	7.607E+04 ± 7.289E+04	6.028E+04 ± 5.206E+04	6.786E+02 ± 1.513E+03	7.397E+02 ± 1.392E+03
F02	1.672E-15 ± 6.754E-15	0.000E+00 ± 0.000E+00	2.062E-14 ± 1.281E-14	0.000E+00 ± 0.000E+00
F03	1.895E-14 ± 2.706E-14	0.000E+00 ± 0.000E+00	7.013E-03 ± 4.473E-02	6.664E-04 ± 1.563E-03
F04	4.419E+00 ± 1.263E+01	1.510E+00 ± 8.925E-01	9.140E-14 ± 5.094E-14	4.570E-14 ± 3.599E-14
F05	2.031E+01 ± 3.685E-02	2.018E+01 ± 7.198E-02	2.029E+01 ± 3.142E-02	2.000E+01 ± 6.736E-03
F06	9.958E+00 ± 5.049E+00	1.196E+00 ± 1.339E+00	9.776E+00 ± 2.184E+00	5.091E-01 ± 6.621E-01
F07	8.694E-14 ± 5.375E-14	0.000E+00 ± 0.000E+00	1.933E-04 ± 1.380E-03	1.450E-04 ± 1.036E-03
F08	0.000E+00 ± 0.000E+00	0.000E+00 ± 0.000E+00	0.000E+00 ± 0.000E+00	0.000E+00 ± 0.000E+00
F09	4.833E+01 ± 7.541E+00	3.024E+01 ± 7.435E+00	2.661E+01 ± 4.899E+00	2.215E+01 ± 6.311E+00
F10	8.164E-04 ± 5.831E-03	3.375E-02 ± 1.584E-01	5.715E-03 ± 1.027E-02	4.450E-02 ± 2.533E-02
F11	2.467E+03 ± 2.574E+02	1.635E+03 ± 3.810E+02	1.609E+03 ± 1.860E+02	1.161E+03 ± 3.315E+02
F12	4.379E-01 ± 6.200E-02	1.086E-01 ± 3.883E-02	2.617E-01 ± 3.054E-02	7.452E-02 ± 2.892E-02
F13	2.983E-01 ± 4.292E-02	1.418E-01 ± 4.445E-02	2.108E-01 ± 3.266E-02	1.333E-01 ± 3.113E-02
F14	2.849E-01 ± 3.398E-02	2.312E-01 ± 3.912E-02	2.363E-01 ± 2.931E-02	1.653E-01 ± 2.401E-02
F15	5.760E+00 ± 7.449E-01	3.138E+00 ± 8.144E-01	3.071E+00 ± 3.652E-01	2.498E+00 ± 4.397E-01
F16	9.988E+00 ± 2.805E-01	8.749E+00 ± 6.316E-01	9.387E+00 ± 4.264E-01	8.034E+00 ± 5.731E-01
F17	2.421E+03 ± 2.324E+03	1.644E+03 ± 1.330E+03	2.156E+04 ± 3.901E+02	7.018E+03 ± 4.133E+04
F18	1.624E+01 ± 7.817E+00	4.974E+01 ± 2.227E+02	1.108E+02 ± 1.900E+02	5.322E+01 ± 2.940E+01
F19	4.667E+00 ± 7.450E-01	3.187E+00 ± 7.709E-01	4.712E+00 ± 9.733E-01	3.990E+00 ± 7.751E-01
F20	1.183E+01 ± 4.053E+00	9.888E+00 ± 3.417E+00	3.197E+03 ± 2.827E+03	1.503E+02 ± 1.329E+02
F21	2.704E+02 ± 1.978E+02	2.701E+02 ± 2.337E+02	1.818E+04 ± 1.829E+04	1.010E+04 ± 3.108E+04
F22	1.385E+02 ± 7.209E+01	8.190E+01 ± 7.052E+01	1.574E+02 ± 5.828E+01	1.250E+02 ± 8.933E+01
F23	3.152E+02 ± 4.158E-13	3.152E+02 ± 3.591E-13	3.152E+02 ± 4.019E-13	3.152E+02 ± 4.019E-13
F24	2.244E+02 ± 1.327E+00	2.246E+02 ± 1.698E+00	2.247E+02 ± 2.685E+00	2.257E+02 ± 3.520E+00
F25	2.034E+02 ± 6.756E-01	2.033E+02 ± 5.580E-01	2.039E+02 ± 1.286E+00	2.036E+02 ± 9.330E-01
F26	1.003E+02 ± 4.065E-02	1.001E+02 ± 4.171E-02	1.002E+02 ± 3.378E-02	1.001E+02 ± 2.824E-02
F27	3.719E+02 ± 4.554E+01	3.615E+02 ± 4.769E+01	3.479E+02 ± 5.006E+01	3.296E+02 ± 4.276E+01
F28	7.949E+02 ± 2.382E+01	8.093E+02 ± 2.925E+01	7.939E+02 ± 3.596E+01	7.811E+02 ± 2.611E+01
F29	8.195E+02 ± 9.126E+01	8.497E+02 ± 1.694E+02	7.382E+02 ± 1.024E+02	8.034E+02 ± 2.778E+02
F30	1.594E+03 ± 7.014E+02	1.586E+03 ± 7.090E+02	1.595E+03 ± 6.113E+02	1.599E+03 ± 5.335E+02
+ / ≈ / -	-	15/13/2	-	22/7/1



Table A.2: Experimental results of CoDE, SaDE, ODE, EPSDE and the related ETI-based variants for functions F01-F30 at  $D = 30$ .

Function	CoDE	ETI-CoDE	SaDE	ETI-SaDE
F01	<b>2.184E+04</b> ± <b>1.332E+04</b>	3.804E+04 ± 3.551E+04	<b>2.154E+05</b> ± <b>1.393E+05</b>	3.172E+05 ± 2.056E+05
F02	5.669E+00 ± 2.394E+00	<b>2.080E+00</b> ± <b>9.575E-01</b>	<b>0.000E+00</b> ± <b>0.000E+00</b>	1.115E-15 ± 5.572E-15
F03	1.333E-04 ± 6.273E-05	<b>7.439E-05</b> ± <b>3.463E-05</b>	<b>6.353E-14</b> ± <b>1.997E-13</b>	3.745E-13 ± 9.110E-13
F04	2.204E+01 ± 2.401E+01	<b>1.509E+01</b> ± <b>2.130E+01</b>	<b>4.273E+01</b> ± <b>3.633E+01</b>	5.201E+01 ± 3.374E+01
F05	2.061E+01 ± 5.216E-02	<b>2.042E+01</b> ± <b>1.322E-01</b>	2.057E+01 ± 5.161E-02	<b>2.001E+01</b> ± <b>1.948E-02</b>
F06	2.124E+01 ± 1.822E+00	<b>6.675E+00</b> ± <b>3.235E+00</b>	<b>1.719E+00</b> ± <b>1.057E+00</b>	2.080E+00 ± 1.352E+00
F07	5.371E-05 ± 5.881E-05	<b>1.358E-05</b> ± <b>3.937E-05</b>	<b>3.183E-03</b> ± <b>8.174E-03</b>	3.955E-03 ± 8.490E-03
F08	1.824E+01 ± 1.636E+00	<b>6.334E+00</b> ± <b>1.895E+00</b>	0.000E+00 ± 0.000E+00	0.000E+00 ± 0.000E+00
F09	1.399E+02 ± 9.160E+00	<b>8.767E+01</b> ± <b>2.032E+01</b>	5.952E+01 ± 1.560E+01	<b>3.110E+01</b> ± <b>9.134E+00</b>
F10	7.704E+02 ± 1.002E+02	<b>2.423E+02</b> ± <b>4.743E+01</b>	1.510E-01 ± 2.273E-01	<b>1.282E-01</b> ± <b>2.626E-01</b>
F11	4.811E+03 ± 2.151E+02	<b>3.511E+03</b> ± <b>6.444E+02</b>	3.892E+03 ± 2.947E+02	<b>1.619E+03</b> ± <b>4.800E+02</b>
F12	9.812E-01 ± 1.267E-01	<b>5.455E-01</b> ± <b>1.853E-01</b>	8.832E-01 ± 1.056E-01	<b>8.407E-02</b> ± <b>5.259E-02</b>
F13	4.681E-01 ± 4.819E-02	<b>3.347E-01</b> ± <b>4.548E-02</b>	2.622E-01 ± 4.002E-02	<b>1.299E-01</b> ± <b>3.373E-02</b>
F14	2.747E-01 ± 3.585E-02	<b>2.568E-01</b> ± <b>3.001E-02</b>	2.321E-01 ± 2.717E-02	<b>2.048E-01</b> ± <b>3.540E-02</b>
F15	1.364E+01 ± 1.053E+00	<b>1.104E+01</b> ± <b>1.529E+00</b>	8.765E+00 ± 1.110E+00	<b>2.943E+00</b> ± <b>7.349E-01</b>
F16	1.166E+01 ± 2.212E-01	<b>1.072E+01</b> ± <b>4.441E-01</b>	1.115E+01 ± 3.483E-01	<b>8.856E+00</b> ± <b>6.375E-01</b>
F17	1.459E+03 ± 1.392E+02	<b>1.111E+03</b> ± <b>2.613E+02</b>	1.115E+03 ± 8.520E+02	<b>1.057E+03</b> ± <b>6.486E+02</b>
F18	4.886E+01 ± 6.302E+00	<b>3.932E+01</b> ± <b>8.687E+00</b>	<b>6.130E+01</b> ± <b>1.971E+01</b>	6.963E+01 ± 2.024E+01
F19	7.146E+00 ± 9.217E-01	<b>5.521E+00</b> ± <b>4.447E-01</b>	4.416E+00 ± 6.744E-01	<b>3.247E+00</b> ± <b>5.728E-01</b>
F20	3.122E+01 ± 4.810E+00	<b>1.868E+01</b> ± <b>4.791E+00</b>	2.907E+01 ± 1.633E+01	<b>2.618E+01</b> ± <b>1.286E+01</b>
F21	7.434E+02 ± 1.231E+02	<b>4.671E+02</b> ± <b>1.650E+02</b>	<b>3.048E+02</b> ± <b>1.844E+02</b>	3.479E+02 ± 3.775E+02
F22	1.379E+02 ± 5.940E+01	<b>6.633E+01</b> ± <b>6.230E+01</b>	<b>1.271E+02</b> ± <b>6.225E+01</b>	1.336E+02 ± 7.198E+01
F23	3.152E+02 ± 5.604E-07	<b>3.152E+02</b> ± <b>2.420E-07</b>	<b>3.152E+02</b> ± <b>2.267E-13</b>	3.152E+02 ± 3.591E-13
F24	2.259E+02 ± 5.965E-01	<b>2.236E+02</b> ± <b>1.858E+00</b>	<b>2.245E+02</b> ± <b>9.550E-01</b>	2.250E+02 ± 1.819E+00
F25	2.029E+02 ± 1.372E-01	2.029E+02 ± 1.698E-01	<b>2.054E+02</b> ± <b>3.269E+00</b>	2.063E+02 ± 2.749E+00
F26	1.004E+02 ± 5.399E-02	<b>1.003E+02</b> ± <b>5.619E-02</b>	1.003E+02 ± 3.863E-02	<b>1.001E+02</b> ± <b>3.469E-02</b>
F27	4.006E+02 ± 2.169E-01	4.006E+02 ± 2.300E-01	<b>3.622E+02</b> ± <b>3.573E+01</b>	3.665E+02 ± 3.922E+01
F28	9.339E+02 ± 2.252E+01	<b>8.144E+02</b> ± <b>3.637E+01</b>	<b>8.567E+02</b> ± <b>2.882E+01</b>	8.638E+02 ± 3.121E+01
F29	6.527E+02 ± 1.649E+02	<b>5.619E+02</b> ± <b>2.108E+02</b>	<b>7.988E+02</b> ± <b>1.499E+02</b>	8.626E+02 ± 1.532E+02
F30	1.177E+03 ± 1.224E+02	<b>8.947E+02</b> ± <b>1.353E+02</b>	<b>8.989E+02</b> ± <b>2.985E+02</b>	9.272E+02 ± 2.869E+02
+ / ≈ / -	-	26/3/1	-	10/15/5
Function	ODE	ETI-ODE	EPSDE	ETI-EPSDE
F01	<b>1.170E+05</b> ± <b>8.746E+04</b>	1.309E+05 ± 1.044E+05	5.591E+04 ± 1.059E+05	<b>3.511E+04</b> ± <b>4.930E+04</b>
F02	1.112E+03 ± 2.697E+03	<b>1.003E+03</b> ± <b>2.884E+03</b>	<b>3.065E-14</b> ± <b>1.783E-14</b>	3.734E-14 ± 5.068E-14
F03	2.786E-14 ± 2.870E-14	<b>7.802E-15</b> ± <b>1.976E-14</b>	2.848E-12 ± 1.696E-11	<b>4.815E-13</b> ± <b>1.081E-12</b>
F04	6.494E+00 ± 1.809E+01	<b>4.981E+00</b> ± <b>1.565E+01</b>	<b>6.990E-04</b> ± <b>3.563E-03</b>	8.933E-02 ± 1.970E-01
F05	2.079E+01 ± 1.377E-01	<b>2.041E+01</b> ± <b>1.190E-01</b>	2.040E+01 ± 3.218E-02	<b>2.019E+01</b> ± <b>7.086E-02</b>
F06	7.431E-01 ± 9.961E-01	<b>6.091E-01</b> ± <b>9.048E-01</b>	2.127E+01 ± 1.216E+00	<b>7.191E+00</b> ± <b>2.588E+00</b>
F07	<b>7.247E-04</b> ± <b>2.928E-03</b>	9.662E-04 ± 4.326E-03	9.140E-14 ± 5.094E-14	<b>0.000E+00</b> ± <b>0.000E+00</b>
F08	4.823E+01 ± 1.830E+01	<b>3.366E+01</b> ± <b>1.211E+01</b>	0.000E+00 ± 0.000E+00	0.000E+00 ± 0.000E+00
F09	3.137E+01 ± 2.547E+01	<b>2.273E+01</b> ± <b>6.578E+00</b>	5.620E+01 ± 7.875E+00	<b>2.564E+01</b> ± <b>6.439E+00</b>
F10	3.028E+03 ± 6.371E+02	<b>1.250E+03</b> ± <b>3.774E+02</b>	4.952E+01 ± 7.321E+01	<b>7.127E-01</b> ± <b>7.791E-01</b>
F11	3.015E+03 ± 1.585E+03	<b>1.856E+03</b> ± <b>5.142E+02</b>	4.026E+03 ± 2.773E+02	<b>1.785E+03</b> ± <b>5.206E+02</b>
F12	9.704E-01 ± 4.947E-01	<b>2.141E-01</b> ± <b>1.401E-01</b>	5.957E-01 ± 7.183E-02	<b>1.541E-01</b> ± <b>6.239E-02</b>
F13	3.238E-01 ± 5.479E-02	<b>2.322E-01</b> ± <b>5.860E-02</b>	2.641E-01 ± 3.122E-02	<b>1.081E-01</b> ± <b>2.398E-02</b>
F14	2.564E-01 ± 4.047E-02	<b>2.407E-01</b> ± <b>3.660E-02</b>	2.957E-01 ± 6.696E-02	<b>2.462E-01</b> ± <b>5.273E-02</b>
F15	6.645E+00 ± 3.582E+00	<b>3.555E+00</b> ± <b>1.108E+00</b>	6.666E+00 ± 6.908E-01	<b>2.658E+00</b> ± <b>5.487E-01</b>
F16	1.190E+01 ± 7.073E-01	<b>9.955E+00</b> ± <b>8.098E-01</b>	1.136E+01 ± 3.455E-01	<b>9.872E+00</b> ± <b>7.322E-01</b>
F17	1.499E+03 ± 1.747E+02	<b>5.678E+02</b> ± <b>2.914E+02</b>	1.483E+04 ± 2.057E+04	<b>7.063E+03</b> ± <b>7.119E+03</b>
F18	1.117E+01 ± 1.004E+01	<b>9.229E+00</b> ± <b>3.693E+00</b>	7.573E+01 ± 6.570E+01	<b>6.578E+01</b> ± <b>6.310E+01</b>
F19	3.214E+00 ± 8.941E-01	<b>2.381E+00</b> ± <b>5.002E-01</b>	1.391E+01 ± 7.427E-01	<b>1.046E+01</b> ± <b>2.087E+00</b>
F20	3.754E+01 ± 5.030E+00	<b>9.498E+00</b> ± <b>4.159E+00</b>	2.419E+01 ± 7.229E+00	<b>1.411E+01</b> ± <b>5.658E+00</b>
F21	7.235E+02 ± 1.666E+02	<b>2.892E+02</b> ± <b>2.336E+02</b>	2.403E+03 ± 5.676E+03	<b>8.514E+02</b> ± <b>1.585E+03</b>
F22	<b>2.172E+02</b> ± <b>1.255E+02</b>	2.274E+02 ± 1.251E+02	2.758E+02 ± 7.885E+01	<b>1.719E+02</b> ± <b>1.085E+02</b>
F23	3.152E+02 ± 4.019E-13	3.152E+02 ± 4.019E-13	3.140E+02 ± 5.705E-13	<b>3.140E+02</b> ± <b>2.034E-13</b>
F24	<b>2.152E+02</b> ± <b>1.083E+01</b>	2.158E+02 ± 1.031E+01	2.254E+02 ± 4.600E+00	<b>2.252E+02</b> ± <b>3.434E+00</b>
F25	<b>2.026E+02</b> ± <b>1.114E-01</b>	2.027E+02 ± 1.049E-01	2.003E+02 ± 2.107E-01	2.003E+02 ± 1.150E-01
F26	1.003E+02 ± 3.882E-02	<b>1.002E+02</b> ± <b>5.584E-02</b>	1.003E+02 ± 4.216E-02	<b>1.001E+02</b> ± <b>3.363E-02</b>
F27	<b>3.928E+02</b> ± <b>2.734E+01</b>	3.929E+02 ± 2.736E+01	8.960E+02 ± 1.116E+02	<b>4.750E+02</b> ± <b>7.544E+01</b>
F28	<b>7.981E+02</b> ± <b>2.702E+01</b>	8.028E+02 ± 2.941E+01	3.805E+02 ± 5.494E+00	<b>3.801E+02</b> ± <b>5.766E+00</b>
F29	6.989E+02 ± 9.394E+01	<b>6.985E+02</b> ± <b>1.229E+02</b>	2.150E+02 ± 1.084E+00	<b>2.121E+02</b> ± <b>1.643E+00</b>
F30	<b>5.822E+02</b> ± <b>1.637E+02</b>	5.900E+02 ± 2.006E+02	5.793E+02 ± 1.399E+02	<b>4.034E+02</b> ± <b>1.060E+02</b>
+ / ≈ / -	-	16/13/1	-	21/8/1

Table A.3: Experimental results of SHADE, OXDE and the related ETI-based variants for functions F01-F30 at  $D = 30$ .

Function	SHADE	ETI-SHADE	OXDE	ETI-OXDE
F01	<b>3.906E+02</b> ± <b>7.084E+02</b>	8.130E+02 ± 1.615E+03≈	<b>6.857E+04</b> ± <b>4.661E+04</b>	1.223E+05 ± 1.088E+05−
F02	1.505E-14 ± 1.433E-14	<b>0.000E+00</b> ± <b>0.000E+00</b> +	1.672E-15 ± 6.754E-15	<b>0.000E+00</b> ± <b>0.000E+00</b> ≈
F03	5.016E-14 ± 1.850E-14	<b>0.000E+00</b> ± <b>0.000E+00</b> +	7.802E-15 ± 1.976E-14	<b>0.000E+00</b> ± <b>0.000E+00</b> +
F04	7.691E-14 ± 3.909E-14	<b>3.232E-14</b> ± <b>3.062E-14</b> +	<b>3.840E-01</b> ± <b>1.739E-01</b>	1.790E+00 ± 8.803E+00−
F05	2.012E+01 ± 1.870E-02	<b>2.001E+01</b> ± <b>1.381E-02</b> +	2.094E+01 ± 5.352E-02	<b>2.035E+01</b> ± <b>1.120E-01</b> +
F06	2.020E-01 ± 4.562E-01	<b>1.535E-01</b> ± <b>3.487E-01</b> ≈	<b>2.653E-01</b> ± <b>5.908E-01</b>	4.959E-01 ± 7.787E-01≈
F07	1.450E-04 ± 1.036E-03	<b>0.000E+00</b> ± <b>0.000E+00</b> +	1.933E-04 ± 1.380E-03	<b>0.000E+00</b> ± <b>0.000E+00</b> +
F08	0.000E+00 ± 0.000E+00	0.000E+00 ± 0.000E+00≈	7.314E+01 ± 1.018E+01	<b>2.262E+01</b> ± <b>6.532E+00</b> +
F09	<b>1.716E+01</b> ± <b>2.587E+00</b>	2.319E+01 ± 7.531E+00−	1.680E+02 ± 1.024E+01	<b>2.331E+01</b> ± <b>8.349E+00</b> +
F10	<b>6.123E-03</b> ± <b>1.199E-02</b>	5.062E-02 ± 3.067E-02−	3.536E+03 ± 3.991E+02	<b>4.691E+02</b> ± <b>1.807E+02</b> +
F11	1.466E+03 ± 2.063E+02	<b>1.073E+03</b> ± <b>3.004E+02</b> +	6.778E+03 ± 2.403E+02	<b>1.580E+03</b> ± <b>6.373E+02</b> +
F12	1.708E-01 ± 2.257E-02	<b>9.390E-02</b> ± <b>2.741E-02</b> +	2.318E+00 ± 2.597E-01	<b>1.542E-01</b> ± <b>8.034E-02</b> +
F13	1.939E-01 ± 3.422E-02	<b>8.990E-02</b> ± <b>3.152E-02</b> +	3.048E-01 ± 4.257E-02	<b>1.168E-01</b> ± <b>3.644E-02</b> +
F14	<b>2.316E-01</b> ± <b>3.143E-02</b>	2.330E-01 ± 4.701E-02≈	2.786E-01 ± 2.713E-02	<b>2.413E-01</b> ± <b>4.478E-02</b> +
F15	<b>2.648E+00</b> ± <b>3.546E-01</b>	2.723E+00 ± 5.139E-01≈	1.497E+01 ± 9.149E-01	<b>3.153E+00</b> ± <b>7.595E-01</b> +
F16	9.123E+00 ± 3.900E-01	<b>8.287E+00</b> ± <b>9.698E-01</b> +	1.220E+01 ± 3.029E-01	<b>8.626E+00</b> ± <b>9.342E-01</b> +
F17	1.026E+03 ± 3.523E+02	<b>6.763E+02</b> ± <b>3.557E+02</b> +	1.369E+03 ± 1.878E+02	<b>3.940E+02</b> ± <b>2.234E+02</b> +
F18	5.098E+01 ± 2.850E+01	<b>2.632E+01</b> ± <b>1.758E+01</b> +	5.112E+01 ± 6.109E+00	<b>9.004E+00</b> ± <b>3.580E+00</b> +
F19	4.387E+00 ± 8.805E-01	<b>2.796E+00</b> ± <b>9.481E-01</b> +	4.370E+00 ± 4.166E-01	<b>2.149E+00</b> ± <b>6.486E-01</b> +
F20	<b>9.799E+00</b> ± <b>4.687E+00</b>	1.013E+01 ± 5.200E+00≈	3.386E+01 ± 6.464E+00	<b>7.226E+00</b> ± <b>3.777E+00</b> +
F21	2.749E+02 ± 1.287E+02	<b>2.048E+02</b> ± <b>9.481E+01</b> +	6.908E+02 ± 1.468E+02	<b>2.036E+02</b> ± <b>2.099E+02</b> +
F22	1.298E+02 ± 6.434E+01	<b>1.127E+02</b> ± <b>6.322E+01</b> ≈	<b>3.157E+01</b> ± <b>2.178E+01</b>	8.262E+01 ± 9.952E+01≈
F23	3.152E+02 ± 4.019E-13	3.152E+02 ± 4.019E-13≈	3.152E+02 ± 4.019E-13	3.152E+02 ± 4.019E-13≈
F24	2.248E+02 ± 2.067E+00	<b>2.245E+02</b> ± <b>1.932E+00</b> ≈	<b>2.201E+02</b> ± <b>7.656E+00</b>	2.222E+02 ± 3.318E+00≈
F25	2.035E+02 ± 7.346E-01	<b>2.033E+02</b> ± <b>6.748E-01</b> ≈	2.027E+02 ± 2.035E-01	2.027E+02 ± 1.681E-01≈
F26	1.002E+02 ± 2.962E-02	<b>1.001E+02</b> ± <b>2.649E-02</b> +	1.003E+02 ± 4.333E-02	<b>1.001E+02</b> ± <b>4.341E-02</b> +
F27	<b>3.118E+02</b> ± <b>3.272E+01</b>	3.169E+02 ± 3.601E+01≈	<b>3.442E+02</b> ± <b>4.998E+01</b>	3.449E+02 ± 4.755E+01≈
F28	8.333E+02 ± 3.827E+01	<b>8.170E+02</b> ± <b>3.752E+01</b> +	<b>7.990E+02</b> ± <b>2.660E+01</b>	8.057E+02 ± 2.462E+01≈
F29	7.241E+02 ± 1.024E+01	<b>7.177E+02</b> ± <b>3.016E+01</b> ≈	<b>6.001E+02</b> ± <b>2.283E+02</b>	6.650E+02 ± 1.651E+02≈
F30	<b>1.342E+03</b> ± <b>4.891E+02</b>	1.383E+03 ± 4.858E+02≈	<b>5.173E+02</b> ± <b>1.192E+02</b>	5.986E+02 ± 2.165E+02−
+ / ≈ / −	-	15/13/2	-	18/9/3

Table A.4: Experimental results of DE/rand/1/bin, DE/best/1/bin, jDE, JADE and the related ETI-based variants for functions F01-F30 at  $D = 50$ .

Function	DE/rand/1/bin	ETI-DE/rand/1/bin	DE/best/1/bin	ETI-DE/best/1/bin
F01	1.399E+06 ± 5.347E+05	<b>8.417E+05 ± 2.873E+05+</b>	1.002E+08 ± 2.370E+07	<b>4.095E+06 ± 1.553E+06+</b>
F02	<b>1.913E+02 ± 7.531E+02</b>	3.045E+03 ± 4.719E+03-	<b>4.361E+02 ± 7.096E+02</b>	7.788E+03 ± 8.179E+03-
F03	<b>2.863E-01 ± 1.144E+00</b>	3.588E-01 ± 7.479E-01≈	2.509E+04 ± 3.903E+03	<b>2.343E+03 ± 8.583E+02+</b>
F04	<b>7.729E+01 ± 2.730E+01</b>	8.793E+01 ± 8.976E+00≈	9.695E+01 ± 3.682E+00	<b>8.948E+01 ± 1.293E+01+</b>
F05	2.113E+01 ± 3.349E-02	<b>2.042E+01 ± 1.022E-01+</b>	2.111E+01 ± 3.342E-02	<b>2.021E+01 ± 1.193E-01+</b>
F06	<b>1.215E+00 ± 1.384E+00</b>	1.786E+00 ± 1.671E+00≈	3.805E+00 ± 2.336E+00	<b>3.646E+00 ± 1.741E+00≈</b>
F07	1.450E-04 ± 1.036E-03	<b>1.450E-04 ± 1.036E-03+</b>	<b>1.498E-03 ± 3.975E-03</b>	2.560E-03 ± 6.209E-03-
F08	1.942E+02 ± 4.580E+01	<b>4.596E+01 ± 1.224E+01+</b>	2.521E+02 ± 1.859E+01	<b>3.865E+01 ± 8.400E+00+</b>
F09	3.517E+02 ± 1.609E+01	<b>4.258E+01 ± 1.185E+01+</b>	3.856E+02 ± 1.613E+01	<b>8.326E+01 ± 2.356E+01+</b>
F10	9.467E+03 ± 1.349E+03	<b>1.007E+03 ± 4.188E+02+</b>	7.469E+03 ± 7.172E+02	<b>6.763E+02 ± 3.118E+02+</b>
F11	1.299E+04 ± 3.968E+02	<b>3.912E+03 ± 1.003E+03+</b>	1.291E+04 ± 3.944E+02	<b>4.712E+03 ± 1.271E+03+</b>
F12	3.243E+00 ± 2.661E-01	<b>1.181E-01 ± 4.673E-02+</b>	3.135E+00 ± 3.174E-01	<b>1.791E-01 ± 1.430E-01+</b>
F13	4.575E-01 ± 4.489E-02	<b>2.078E-01 ± 4.905E-02+</b>	5.184E-01 ± 6.684E-02	<b>3.322E-01 ± 6.814E-02+</b>
F14	3.369E-01 ± 1.081E-01	<b>2.968E-01 ± 8.065E-02+</b>	6.317E-01 ± 3.148E-01	<b>5.299E-01 ± 2.338E-01+</b>
F15	3.150E+01 ± 1.147E+00	<b>6.090E+00 ± 1.801E+00+</b>	3.449E+01 ± 1.561E+00	<b>8.260E+00 ± 3.338E+00+</b>
F16	2.211E+01 ± 3.114E-01	<b>1.735E+01 ± 1.212E+00+</b>	2.205E+01 ± 1.885E-01	<b>1.882E+01 ± 1.163E+00+</b>
F17	<b>1.419E+04 ± 9.317E+03</b>	1.726E+04 ± 1.479E+04≈	5.585E+06 ± 1.832E+06	<b>2.510E+05 ± 1.288E+05+</b>
F18	1.342E+02 ± 1.028E+01	<b>3.080E+01 ± 2.020E+01+</b>	2.104E+03 ± 1.681E+03	<b>1.942E+03 ± 1.779E+03≈</b>
F19	1.191E+01 ± 6.751E-01	<b>5.682E+00 ± 1.329E+00+</b>	1.453E+01 ± 1.209E+00	<b>1.021E+01 ± 2.430E+00+</b>
F20	9.887E+01 ± 1.069E+01	<b>3.173E+01 ± 2.098E+01+</b>	6.342E+03 ± 2.346E+03	<b>5.123E+02 ± 1.482E+02+</b>
F21	2.630E+03 ± 5.177E+02	<b>1.980E+03 ± 3.372E+03+</b>	2.042E+06 ± 9.419E+05	<b>1.297E+05 ± 6.660E+04+</b>
F22	<b>7.133E+02 ± 4.126E+02</b>	7.907E+02 ± 3.243E+02≈	1.038E+03 ± 1.927E+02	<b>6.956E+02 ± 2.660E+02+</b>
F23	3.440E+02 ± 4.168E-13	<b>3.440E+02 ± 2.870E-13+</b>	3.440E+02 ± 4.593E-13	3.440E+02 ± 4.502E-13≈
F24	2.704E+02 ± 2.504E+00	2.704E+02 ± 2.061E+00≈	2.685E+02 ± 4.189E+00	<b>2.682E+02 ± 3.990E+00≈</b>
F25	<b>2.054E+02 ± 4.166E-01</b>	2.055E+02 ± 4.956E-01≈	2.207E+02 ± 5.461E+00	<b>2.071E+02 ± 1.360E+00+</b>
F26	1.005E+02 ± 5.418E-02	<b>1.002E+02 ± 5.262E-02+</b>	1.416E+02 ± 7.129E+01	<b>1.362E+02 ± 6.714E+01+</b>
F27	<b>3.662E+02 ± 3.538E+01</b>	3.765E+02 ± 3.819E+01≈	<b>4.513E+02 ± 6.521E+01</b>	4.613E+02 ± 5.697E+01≈
F28	<b>1.064E+03 ± 4.712E+01</b>	1.086E+03 ± 3.219E+01-	<b>1.117E+03 ± 7.357E+01</b>	1.132E+03 ± 1.084E+02≈
F29	<b>9.907E+02 ± 2.505E+02</b>	1.003E+03 ± 2.532E+02≈	<b>1.552E+04 ± 1.291E+04-</b>	2.235E+06 ± 9.030E+06-
F30	<b>8.296E+03 ± 3.421E+02</b>	8.355E+03 ± 3.688E+02≈	<b>8.856E+03 ± 5.345E+02</b>	9.202E+03 ± 7.778E+02-
+ / ≈ / -	-	18/10/2	-	20/6/4
Function	jDE	ETI-jDE	JADE	ETI-JADE
F01	<b>4.750E+05 ± 2.246E+05</b>	5.481E+05 ± 2.832E+05≈	1.491E+04 ± 1.146E+04	<b>1.478E+04 ± 1.049E+04≈</b>
F02	<b>1.903E-08 ± 5.204E-08</b>	2.933E-04 ± 1.823E-03-	1.109E-13 ± 4.765E-14	<b>4.291E-14 ± 2.300E-14+</b>
F03	<b>2.975E-09 ± 8.334E-09</b>	3.788E-08 ± 1.505E-07-	3.642E+03 ± 2.669E+03	<b>2.410E+02 ± 1.470E+02+</b>
F04	<b>8.501E+01 ± 1.683E+01</b>	9.510E+01 ± 2.684E+00-	<b>1.934E+01 ± 3.929E+01</b>	2.059E+01 ± 3.956E+01≈
F05	2.043E+01 ± 2.793E-02	<b>2.020E+01 ± 7.078E-02+</b>	2.036E+01 ± 3.189E-02	<b>2.000E+01 ± 1.558E-03+</b>
F06	1.879E+01 ± 1.075E+01	<b>5.189E+00 ± 3.469E+00+</b>	1.516E+01 ± 7.311E+00	<b>1.815E+00 ± 1.428E+00+</b>
F07	2.876E-13 ± 1.352E-13	<b>1.115E-13 ± 1.592E-14+</b>	1.546E-03 ± 3.815E-03	<b>1.497E-03 ± 4.123E-03+</b>
F08	8.917E-14 ± 4.722E-14	<b>1.783E-14 ± 4.176E-14+</b>	0.000E+00 ± 0.000E+00	0.000E+00 ± 0.000E+00≈
F09	9.724E+01 ± 1.169E+01	<b>6.678E+01 ± 1.425E+01+</b>	5.273E+01 ± 7.892E+00	<b>4.823E+01 ± 9.089E+00+</b>
F10	<b>4.899E-04 ± 2.449E-03</b>	9.954E-02 ± 1.997E-01-	<b>9.797E-03 ± 1.306E-02</b>	8.401E-02 ± 3.171E-02-
F11	5.118E+03 ± 3.803E+02	<b>4.076E+03 ± 7.188E+02+</b>	3.755E+03 ± 2.801E+02	<b>2.762E+03 ± 5.427E+02+</b>
F12	4.531E-01 ± 4.543E-02	<b>1.307E-01 ± 5.968E-02+</b>	2.568E-01 ± 3.108E-02	<b>6.556E-02 ± 1.964E-02+</b>
F13	3.836E-01 ± 4.738E-02	<b>1.941E-01 ± 5.446E-02+</b>	3.155E-01 ± 4.368E-02	<b>2.170E-01 ± 3.634E-02+</b>
F14	3.334E-01 ± 6.863E-02	<b>2.803E-01 ± 7.508E-02+</b>	2.997E-01 ± 6.446E-02	<b>2.058E-01 ± 6.257E-02+</b>
F15	1.206E+01 ± 1.151E+00	<b>5.837E+00 ± 1.207E+00+</b>	7.148E+00 ± 8.931E-01	<b>5.201E+00 ± 9.448E-01+</b>
F16	1.842E+01 ± 3.976E-01	<b>1.713E+01 ± 1.026E+00+</b>	1.769E+01 ± 4.703E-01	<b>1.627E+01 ± 1.213E+00+</b>
F17	2.468E+04 ± 1.684E+04	<b>1.812E+04 ± 1.050E+04+</b>	<b>2.281E+03 ± 5.218E+02</b>	2.419E+03 ± 7.839E+02≈
F18	4.649E+02 ± 4.976E+02	<b>3.494E+02 ± 4.942E+02+</b>	1.767E+02 ± 4.464E+01	<b>1.711E+02 ± 6.012E+01≈</b>
F19	1.365E+01 ± 5.279E+00	<b>1.038E+01 ± 2.366E+00+</b>	1.256E+01 ± 4.826E+00	<b>8.597E+00 ± 2.887E+00+</b>
F20	5.216E+01 ± 2.186E+01	<b>4.423E+01 ± 1.838E+01+</b>	7.550E+03 ± 6.804E+03	<b>5.982E+02 ± 3.716E+02+</b>
F21	1.094E+04 ± 1.149E+04	<b>8.675E+03 ± 7.983E+03≈</b>	<b>1.287E+03 ± 3.467E+02</b>	6.242E+03 ± 3.599E+04≈
F22	5.653E+02 ± 1.379E+02	<b>4.841E+02 ± 1.741E+02+</b>	5.167E+02 ± 1.434E+02	<b>4.666E+02 ± 1.790E+02≈</b>
F23	3.440E+02 ± 4.601E-13	<b>3.440E+02 ± 2.870E-13+</b>	3.440E+02 ± 5.291E-13	<b>3.440E+02 ± 3.178E-13+</b>
F24	<b>2.686E+02 ± 2.335E+00</b>	2.684E+02 ± 2.794E+00≈	<b>2.741E+02 ± 2.603E+00</b>	2.742E+02 ± 2.340E+00≈
F25	<b>2.078E+02 ± 2.195E+00</b>	2.082E+02 ± 2.204E+00≈	2.167E+02 ± 6.147E+00	<b>2.149E+02 ± 6.860E+00≈</b>
F26	<b>1.004E+02 ± 3.596E-02</b>	1.061E+02 ± 2.374E+01-	<b>1.023E+02 ± 1.396E+01-</b>	1.042E+02 ± 1.955E+01-
F27	4.371E+02 ± 6.160E+01	<b>4.212E+02 ± 5.894E+01≈</b>	4.562E+02 ± 7.799E+01	<b>4.361E+02 ± 4.814E+01≈</b>
F28	<b>1.115E+03 ± 4.899E+01</b>	1.117E+03 ± 4.666E+01≈	1.131E+03 ± 5.225E+01	<b>1.091E+03 ± 4.202E+01+</b>
F29	<b>1.041E+03 ± 1.626E+02</b>	1.197E+03 ± 2.403E+02-	<b>8.795E+02 ± 6.936E+01</b>	8.893E+02 ± 6.632E+01≈
F30	<b>8.580E+03 ± 4.519E+02</b>	8.882E+03 ± 4.267E+02-	9.827E+03 ± 6.665E+02	<b>9.484E+03 ± 6.525E+02+</b>
+ / ≈ / -	-	17/6/7	-	17/11/2

Table A.5: Experimental results of CoDE, SaDE, ODE, EPSDE and the related ETI-based variants for functions F01-F30 at  $D = 50$ .

Function	CoDE	ETI-CoDE	SaDE	ETI-SaDE
F01	<b>7.877E+05</b> ± <b>3.093E+05</b>	8.691E+05 ± 5.128E+05≈	<b>7.958E+05</b> ± <b>2.986E+05</b>	8.604E+05 ± 2.710E+05≈
F02	3.873E+03 ± 2.258E+03	<b>3.161E+03</b> ± <b>1.944E+03</b> ≈	<b>3.436E+03</b> ± <b>3.353E+03</b>	4.227E+03 ± 3.214E+03≈
F03	<b>5.828E-02</b> ± <b>7.029E-02</b>	1.334E-01 ± 1.667E-01-	<b>7.512E+02</b> ± <b>6.082E+02</b>	1.328E+03 ± 1.112E+03-
F04	9.092E+01 ± 6.637E+00	<b>8.934E+01</b> ± <b>4.282E+00</b> +	<b>8.122E+01</b> ± <b>4.663E+01</b>	9.818E+01 ± 3.930E+01-
F05	2.086E+01 ± 4.480E-02	<b>2.042E+01</b> ± <b>1.358E-01</b> +	2.077E+01 ± 4.449E-02	<b>2.001E+01</b> ± <b>1.835E-02</b> +
F06	2.794E+01 ± 1.217E+01	<b>4.142E+00</b> ± <b>2.397E+00</b> +	1.182E+01 ± 2.452E+00	<b>1.155E+01</b> ± <b>2.673E+00</b> ≈
F07	7.102E-06 ± 3.582E-06	<b>2.480E-06</b> ± <b>1.727E-06</b> +	<b>1.000E-02</b> ± <b>8.789E-03</b>	1.004E-02 ± 1.193E-02≈
F08	7.428E+01 ± 4.396E+00	<b>3.287E+01</b> ± <b>3.368E+00</b> +	<b>0.000E+00</b> ± <b>0.000E+00</b>	5.853E-02 ± 2.364E-01≈
F09	2.973E+02 ± 2.075E+01	<b>1.387E+02</b> ± <b>3.861E+01</b> +	9.482E+01 ± 3.865E+01	<b>7.402E+01</b> ± <b>1.222E+01</b> +
F10	3.335E+03 ± 2.386E+02	<b>8.018E+02</b> ± <b>1.169E+02</b> +	2.764E+01 ± 1.080E+01	<b>9.854E-01</b> ± <b>6.320E-01</b> +
F11	1.033E+04 ± 3.671E+02	<b>6.082E+03</b> ± <b>1.048E+03</b> +	8.556E+03 ± 3.558E+02	<b>3.737E+03</b> ± <b>3.745E+02</b> +
F12	1.577E+00 ± 1.478E-01	<b>5.616E-01</b> ± <b>1.854E-01</b> +	1.240E+00 ± 1.188E-01	<b>7.364E-02</b> ± <b>2.528E-02</b> ≈
F13	5.543E-01 ± 5.591E-02	<b>4.068E-01</b> ± <b>5.768E-02</b> +	4.209E-01 ± 4.735E-02	<b>2.137E-01</b> ± <b>5.112E-02</b> +
F14	3.228E-01 ± 3.682E-02	<b>2.830E-01</b> ± <b>3.709E-02</b> +	3.064E-01 ± 2.593E-02	<b>2.544E-01</b> ± <b>3.891E-02</b> +
F15	2.825E+01 ± 1.545E+00	<b>1.948E+01</b> ± <b>3.975E+00</b> +	2.112E+01 ± 6.862E+00	<b>8.273E+00</b> ± <b>1.870E+00</b> +
F16	2.118E+01 ± 2.221E-01	<b>1.929E+01</b> ± <b>5.568E-01</b> +	2.052E+01 ± 2.447E-01	<b>1.732E+01</b> ± <b>9.682E-01</b> +
F17	2.964E+03 ± 3.952E+02	<b>2.730E+03</b> ± <b>9.646E+02</b> +	3.006E+04 ± 2.625E+04	<b>2.970E+04</b> ± <b>1.570E+04</b> ≈
F18	1.027E+02 ± 1.810E+01	<b>5.411E+01</b> ± <b>2.544E+01</b> +	<b>3.981E+02</b> ± <b>2.776E+02</b>	4.904E+02 ± 3.299E+02≈
F19	1.330E+01 ± 1.182E+00	<b>1.240E+01</b> ± <b>3.398E-01</b> +	2.502E+01 ± 1.880E+01	<b>1.377E+01</b> ± <b>1.107E+01</b> +
F20	6.251E+01 ± 1.339E+01	<b>3.067E+01</b> ± <b>7.307E+00</b> +	<b>2.069E+02</b> ± <b>9.249E+01</b>	2.397E+02 ± 2.002E+02≈
F21	1.882E+03 ± 2.204E+02	<b>1.047E+03</b> ± <b>3.777E+02</b> +	<b>1.563E+04</b> ± <b>1.586E+04</b>	2.222E+04 ± 2.260E+04≈
F22	5.863E+02 ± 1.418E+02	<b>4.275E+02</b> ± <b>1.660E+02</b> +	<b>4.098E+02</b> ± <b>1.439E+02</b>	4.653E+02 ± 2.154E+02≈
F23	3.440E+02 ± 4.093E-08	<b>3.440E+02</b> ± <b>1.359E-08</b> +	3.440E+02 ± 2.852E-13	3.440E+02 ± 2.852E-13≈
F24	<b>2.627E+02</b> ± <b>3.220E+00</b>	2.638E+02 ± 2.756E+00≈	<b>2.726E+02</b> ± <b>4.279E+00</b>	2.731E+02 ± 3.494E+00≈
F25	2.059E+02 ± 6.269E-01	<b>2.058E+02</b> ± <b>5.001E-01</b> ≈	<b>2.055E+02</b> ± <b>8.294E+00</b>	2.076E+02 ± 9.299E+00≈
F26	1.006E+02 ± 4.751E-02	<b>1.004E+02</b> ± <b>5.514E-02</b> +	<b>1.727E+02</b> ± <b>4.493E+01</b>	1.746E+02 ± 4.396E+01≈
F27	4.291E+02 ± 6.612E+01	<b>3.766E+02</b> ± <b>4.335E+01</b> +	6.089E+02 ± 6.989E+01	<b>6.078E+02</b> ± <b>6.185E+01</b> ≈
F28	1.423E+03 ± 4.798E+01	<b>1.122E+03</b> ± <b>4.648E+01</b> +	<b>1.330E+03</b> ± <b>8.201E+01</b>	1.331E+03 ± 8.932E+01≈
F29	<b>7.542E+02</b> ± <b>1.529E+02</b>	7.855E+02 ± 1.798E+02≈	<b>1.236E+03</b> ± <b>2.526E+02</b>	1.305E+03 ± 2.803E+02≈
F30	8.309E+03 ± 2.630E+02	<b>8.299E+03</b> ± <b>2.357E+02</b> ≈	<b>1.066E+04</b> ± <b>1.091E+03</b>	1.085E+04 ± 1.159E+03≈
+ / ≈ / -	-	23/6/1	-	10/18/2
Function	ODE	ETI-ODE	EPSDE	ETI-EPSDE
F01	2.160E+06 ± 8.911E+05	<b>1.264E+06</b> ± <b>5.074E+05</b> +	5.228E+06 ± 1.114E+07	<b>1.267E+05</b> ± <b>8.445E+04</b> ≈
F02	5.682E+03 ± 4.416E+03	<b>4.956E+03</b> ± <b>3.944E+03</b> ≈	1.221E-09 ± 3.981E-09	<b>1.136E-09</b> ± <b>2.786E-09</b> +
F03	<b>6.914E-01</b> ± <b>1.059E+00</b>	1.361E+00 ± 4.410E+00≈	5.613E-04 ± 1.803E-03	<b>2.138E-04</b> ± <b>9.474E-04</b> +
F04	9.261E+01 ± 5.486E+00	<b>9.068E+01</b> ± <b>1.452E+01</b> ≈	<b>2.824E+01</b> ± <b>1.121E+01</b>	3.209E+01 ± 8.036E+00-
F05	2.104E+01 ± 1.054E-01	<b>2.049E+01</b> ± <b>1.107E-01</b> +	2.065E+01 ± 3.530E-02	<b>2.025E+01</b> ± <b>9.256E-02</b> +
F06	2.120E+00 ± 1.297E+00	<b>1.888E+00</b> ± <b>1.558E+00</b> ≈	4.897E+01 ± 2.112E+00	<b>1.242E+01</b> ± <b>6.929E+00</b> +
F07	4.724E-03 ± 9.449E-03	<b>4.492E-03</b> ± <b>6.837E-03</b> +	<b>2.657E-03</b> ± <b>5.233E-03</b>	3.334E-03 ± 5.139E-03-
F08	8.073E+01 ± 4.452E+01	<b>6.488E+01</b> ± <b>2.528E+01</b> ≈	8.917E-14 ± 4.722E-14	<b>0.000E+00</b> ± <b>0.000E+00</b> +
F09	5.367E+01 ± 3.510E+01	<b>5.051E+01</b> ± <b>1.268E+01</b> ≈	1.743E+02 ± 1.928E+01	<b>5.662E+01</b> ± <b>1.497E+01</b> +
F10	6.508E+03 ± 1.215E+03	<b>2.694E+03</b> ± <b>5.742E+02</b> +	1.759E+03 ± 7.739E+02	<b>2.316E+01</b> ± <b>2.284E+01</b> +
F11	7.749E+03 ± 3.408E+03	<b>3.917E+03</b> ± <b>1.057E+03</b> +	9.345E+03 ± 4.413E+02	<b>3.916E+03</b> ± <b>7.314E+02</b> +
F12	1.791E+00 ± 8.211E-01	<b>1.560E-01</b> ± <b>1.042E-01</b> +	9.649E-01 ± 1.050E-01	<b>1.623E-01</b> ± <b>7.933E-02</b> +
F13	4.409E-01 ± 4.890E-02	<b>3.321E-01</b> ± <b>6.496E-02</b> +	3.327E-01 ± 4.012E-02	<b>1.672E-01</b> ± <b>3.822E-02</b> +
F14	3.229E-01 ± 3.573E-02	<b>2.972E-01</b> ± <b>3.398E-02</b> +	3.462E-01 ± 6.003E-02	<b>2.970E-01</b> ± <b>5.311E-02</b> +
F15	2.003E+01 ± 9.295E+00	<b>6.188E+00</b> ± <b>1.467E+00</b> +	1.868E+01 ± 1.505E+00	<b>5.094E+00</b> ± <b>9.808E-01</b> +
F16	2.164E+01 ± 8.085E-01	<b>1.858E+01</b> ± <b>9.307E-01</b> +	2.104E+01 ± 2.848E-01	<b>1.881E+01</b> ± <b>7.622E-01</b> +
F17	<b>1.654E+04</b> ± <b>1.223E+04</b>	2.280E+04 ± 1.954E+04≈	1.582E+05 ± 1.004E+05	<b>8.766E+04</b> ± <b>8.386E+04</b> +
F18	8.829E+01 ± 7.113E+01	<b>7.208E+01</b> ± <b>5.898E+01</b> ≈	<b>1.677E+03</b> ± <b>2.241E+03</b>	3.388E+03 ± 4.670E+03-
F19	9.496E+00 ± 1.918E+00	<b>7.733E+00</b> ± <b>1.228E+00</b> +	2.568E+01 ± 9.916E-01	<b>1.020E+01</b> ± <b>3.064E+00</b> +
F20	1.006E+02 ± 1.232E+01	<b>3.740E+01</b> ± <b>1.743E+01</b> +	1.838E+02 ± 1.937E+02	<b>7.176E+01</b> ± <b>5.689E+01</b> +
F21	2.500E+03 ± 4.791E+02	<b>1.814E+03</b> ± <b>1.365E+03</b> +	4.540E+04 ± 3.883E+04	<b>2.011E+04</b> ± <b>2.559E+04</b> +
F22	<b>7.028E+02</b> ± <b>2.918E+02</b>	7.894E+02 ± 2.928E+02≈	9.084E+02 ± 1.552E+02	<b>7.126E+02</b> ± <b>2.173E+02</b> +
F23	3.440E+02 ± 4.362E-13	<b>3.440E+02</b> ± <b>4.799E-13</b> +	3.370E+02 ± 1.247E-12	<b>3.370E+02</b> ± <b>3.563E-13</b> +
F24	2.706E+02 ± 2.302E+00	<b>2.697E+02</b> ± <b>2.725E+00</b> ≈	2.677E+02 ± 5.315E+00	<b>2.666E+02</b> ± <b>4.163E+00</b> ≈
F25	<b>2.051E+02</b> ± <b>1.334E+00</b>	2.055E+02 ± 5.588E-01≈	2.016E+02 ± 3.273E+00	<b>2.003E+02</b> ± <b>3.397E-02</b> ≈
F26	<b>1.082E+02</b> ± <b>2.707E+01</b>	1.101E+02 ± 2.996E+01-	1.003E+02 ± 4.575E-02	<b>1.002E+02</b> ± <b>2.880E-02</b> +
F27	3.914E+02 ± 3.605E+01	<b>3.894E+02</b> ± <b>4.092E+01</b> ≈	1.631E+03 ± 4.400E+01	<b>6.501E+02</b> ± <b>1.363E+02</b> +
F28	1.085E+03 ± 3.171E+01	<b>1.080E+03</b> ± <b>4.628E+01</b> ≈	<b>3.759E+02</b> ± <b>8.858E+00</b>	3.789E+02 ± 9.615E+00-
F29	<b>9.656E+02</b> ± <b>2.225E+02</b>	1.099E+03 ± 2.646E+02-	2.269E+02 ± 3.544E+00	<b>2.208E+02</b> ± <b>1.998E+00</b> +
F30	8.421E+03 ± 4.141E+02	<b>8.390E+03</b> ± <b>3.966E+02</b> ≈	1.227E+03 ± 1.994E+02	<b>7.671E+02</b> ± <b>2.439E+02</b> +
+ / ≈ / -	14/14/2	-	23/3/4	-

Table A.6: Experimental results of SHADE, OXDE and the related ETI-based variants for functions F01-F30 at  $D = 50$ .

Function	SHADE	ETI-SHADE	OXDE	ETI-OXDE
F01	<b>1.374E+04</b> ± <b>9.334E+03</b>	3.188E+04 ± 1.375E+04−	1.047E+06 ± 4.357E+05	<b>8.382E+05</b> ± <b>3.255E+05</b> +
F02	9.585E-14 ± 5.361E-14	<b>4.625E-14</b> ± <b>5.209E-14</b> +	<b>6.528E+02</b> ± <b>1.547E+03</b>	4.519E+03 ± 4.998E+03−
F03	2.808E-12 ± 7.062E-12	<b>2.508E-13</b> ± <b>1.592E-13</b> +	<b>9.332E-02</b> ± <b>3.010E-01</b>	2.155E-01 ± 5.330E-01≈
F04	1.450E+01 ± 3.446E+01	<b>1.245E+01</b> ± <b>3.183E+01</b> ≈	8.687E+01 ± 1.146E+01	<b>8.564E+01</b> ± <b>1.452E+01</b> ≈
F05	2.014E+01 ± 1.705E-02	<b>2.000E+01</b> ± <b>2.948E-03</b> +	2.113E+01 ± 3.902E-02	<b>2.038E+01</b> ± <b>1.104E-01</b> +
F06	3.527E+00 ± 1.365E+00	<b>3.000E+00</b> ± <b>1.702E+00</b> ≈	<b>1.110E+00</b> ± <b>1.314E+00</b>	1.305E+00 ± 1.351E+00≈
F07	2.512E-03 ± 4.937E-03	<b>1.643E-03</b> ± <b>3.439E-03</b> +	1.014E-03 ± 3.311E-03	<b>4.833E-04</b> ± <b>1.973E-03</b> +
F08	1.070E-13 ± 2.702E-14	<b>6.687E-15</b> ± <b>3.531E-14</b> +	1.075E+02 ± 2.251E+01	<b>3.073E+01</b> ± <b>1.170E+01</b> +
F09	<b>3.409E+01</b> ± <b>6.201E+00</b>	4.269E+01 ± 9.900E+00−	3.336E+02 ± 1.278E+01	<b>4.453E+01</b> ± <b>1.010E+01</b> +
F10	<b>2.939E-03</b> ± <b>5.906E-03</b>	6.417E-02 ± 2.691E-02−	6.362E+03 ± 7.719E+02	<b>7.054E+02</b> ± <b>2.974E+02</b> +
F11	3.430E+03 ± 3.403E+02	<b>2.724E+03</b> ± <b>5.724E+02</b> +	1.300E+04 ± 3.803E+02	<b>3.631E+03</b> ± <b>1.096E+03</b> +
F12	1.585E-01 ± 1.990E-02	<b>7.260E-02</b> ± <b>2.223E-02</b> +	3.222E+00 ± 2.889E-01	<b>1.025E-01</b> ± <b>4.692E-02</b> +
F13	3.024E-01 ± 4.534E-02	<b>1.554E-01</b> ± <b>3.601E-02</b> +	4.219E-01 ± 4.219E-02	<b>2.090E-01</b> ± <b>4.733E-02</b> +
F14	2.982E-01 ± 6.684E-02	<b>2.823E-01</b> ± <b>6.032E-02</b> ≈	3.320E-01 ± 9.822E-02	<b>2.774E-01</b> ± <b>3.406E-02</b> +
F15	5.860E+00 ± 7.329E-01	<b>5.137E+00</b> ± <b>9.464E-01</b> +	3.014E+01 ± 9.878E-01	<b>5.631E+00</b> ± <b>1.421E+00</b> +
F16	1.732E+01 ± 4.542E-01	<b>1.672E+01</b> ± <b>1.933E+00</b> +	2.202E+01 ± 2.116E-01	<b>1.724E+01</b> ± <b>1.148E+00</b> +
F17	<b>2.599E+03</b> ± <b>6.941E+02</b>	2.635E+03 ± 7.490E+02≈	<b>1.604E+04</b> ± <b>1.455E+04</b>	2.193E+04 ± 2.142E+04≈
F18	1.633E+02 ± 4.539E+01	<b>1.439E+02</b> ± <b>4.042E+01</b> +	1.472E+02 ± 6.802E+01	<b>2.830E+01</b> ± <b>1.472E+01</b> +
F19	1.096E+01 ± 2.903E+00	<b>6.579E+00</b> ± <b>1.467E+00</b> +	1.214E+01 ± 1.011E+00	<b>6.334E+00</b> ± <b>1.322E+00</b> +
F20	2.019E+02 ± 5.724E+01	<b>1.076E+02</b> ± <b>5.150E+01</b> +	1.008E+02 ± 1.603E+01	<b>2.687E+01</b> ± <b>1.081E+01</b> +
F21	1.358E+03 ± 3.495E+02	<b>1.051E+03</b> ± <b>3.090E+02</b> +	2.725E+03 ± 1.230E+03	<b>1.702E+03</b> ± <b>1.088E+03</b> +
F22	<b>3.838E+02</b> ± <b>1.475E+02</b>	4.002E+02 ± 1.615E+02≈	<b>4.735E+02</b> ± <b>3.008E+02</b>	6.570E+02 ± 2.852E+02−
F23	3.440E+02 ± 4.362E-13	<b>3.440E+02</b> ± <b>2.870E-13</b> +	3.440E+02 ± 5.112E-13	<b>3.440E+02</b> ± <b>2.870E-13</b> +
F24	2.733E+02 ± 2.020E+00	2.733E+02 ± 2.386E+00≈	2.710E+02 ± 2.307E+00	<b>2.709E+02</b> ± <b>2.414E+00</b> ≈
F25	2.156E+02 ± 5.659E+00	<b>2.109E+02</b> ± <b>5.524E+00</b> +	<b>2.057E+02</b> ± <b>6.160E-01</b>	2.058E+02 ± 6.393E-01≈
F26	1.004E+02 ± 1.243E-01	<b>1.002E+02</b> ± <b>1.095E-01</b> +	1.004E+02 ± 5.200E-02	<b>1.002E+02</b> ± <b>4.163E-02</b> +
F27	<b>4.145E+02</b> ± <b>5.602E+01</b>	4.202E+02 ± 4.984E+01≈	<b>3.733E+02</b> ± <b>4.540E+01</b>	3.797E+02 ± 4.113E+01≈
F28	<b>1.128E+03</b> ± <b>5.885E+01</b>	1.142E+03 ± 4.453E+01≈	<b>1.086E+03</b> ± <b>3.947E+01</b>	1.100E+03 ± 4.380E+01≈
F29	9.038E+02 ± 5.998E+01	<b>8.617E+02</b> ± <b>8.029E+01</b> +	<b>9.716E+02</b> ± <b>2.646E+02</b>	1.083E+03 ± 2.910E+02−
F30	9.578E+03 ± 5.958E+02	<b>9.140E+03</b> ± <b>5.998E+02</b> +	<b>8.453E+03</b> ± <b>4.336E+02</b>	8.495E+03 ± 4.222E+02≈
+ / ≈ / −	-	19/8/3	-	18/9/3

Table A.7: Experimental results of DE/rand/1/bin, DE/best/1/bin, jDE, JADE and the related ETI-based variants for functions F01-F30 at  $D = 100$ .

Function	DE/rand/1/bin	ETI-DE/rand/1/bin	DE/best/1/bin	ETI-DE/best/1/bin
F1	4.492E+06 ± 1.385E+06	<b>3.839E+06 ± 1.121E+06+</b>	8.224E+08 ± 1.595E+08	<b>1.907E+07 ± 5.201E+06+</b>
F2	2.156E+04 ± 2.371E+04	<b>1.706E+04 ± 1.555E+04≈</b>	<b>1.348E+03 ± 1.893E+03</b>	3.751E+04 ± 4.295E+04−
F3	<b>1.399E+03 ± 1.081E+03</b>	1.571E+03 ± 1.199E+03≈	1.150E+05 ± 1.502E+04	<b>7.867E+03 ± 2.525E+03+</b>
F4	<b>1.781E+02 ± 3.805E+01</b>	1.795E+02 ± 3.485E+01≈	1.980E+02 ± 3.322E+01	<b>1.806E+02 ± 3.210E+01+</b>
F5	2.131E+01 ± 2.829E-02	<b>2.046E+01 ± 1.129E-01+</b>	2.131E+01 ± 2.563E-02	<b>2.008E+01 ± 4.757E-02+</b>
F6	<b>1.186E+01 ± 3.390E+00</b>	1.426E+01 ± 3.402E+00−	1.875E+01 ± 4.783E+00	<b>1.602E+01 ± 4.572E+00+</b>
F7	4.833E-04 ± 1.973E-03	<b>4.351E-04 ± 1.758E-03+</b>	<b>5.025E-04 ± 2.462E-03</b>	6.280E-04 ± 2.646E-03−
F8	1.126E+02 ± 9.111E+01	<b>8.837E+01 ± 1.537E+01≈</b>	7.333E+02 ± 2.221E+01	<b>1.181E+02 ± 1.735E+01+</b>
F9	8.142E+02 ± 2.583E+01	<b>1.106E+02 ± 2.601E+01+</b>	9.598E+02 ± 3.435E+01	<b>2.072E+02 ± 3.442E+01+</b>
F10	1.589E+04 ± 4.471E+03	<b>3.285E+03 ± 6.956E+02+</b>	2.246E+04 ± 1.054E+03	<b>2.814E+03 ± 4.876E+02+</b>
F11	2.989E+04 ± 5.597E+02	<b>1.100E+04 ± 2.049E+03+</b>	3.011E+04 ± 5.930E+02	<b>1.184E+04 ± 1.966E+03+</b>
F12	3.952E+00 ± 2.244E-01	<b>1.007E-01 ± 5.807E-02+</b>	3.963E+00 ± 2.473E-01	<b>1.840E-01 ± 1.052E-01+</b>
F13	5.664E-01 ± 5.154E-02	<b>3.129E-01 ± 6.513E-02+</b>	7.062E-01 ± 6.178E-02	<b>5.128E-01 ± 6.996E-02+</b>
F14	3.467E-01 ± 2.972E-02	<b>3.192E-01 ± 6.130E-02+</b>	5.375E-01 ± 3.111E-01	<b>4.503E-01 ± 2.716E-01+</b>
F15	7.304E+01 ± 2.655E+00	<b>1.257E+01 ± 3.170E+00+</b>	8.612E+01 ± 3.042E+00	<b>1.735E+01 ± 4.187E+00+</b>
F16	4.645E+01 ± 2.328E-01	<b>4.098E+01 ± 1.297E+00+</b>	4.637E+01 ± 3.278E-01	<b>4.256E+01 ± 1.462E+00+</b>
F17	4.617E+05 ± 1.868E+05	<b>3.286E+05 ± 1.602E+05+</b>	6.061E+07 ± 1.914E+07	<b>1.462E+06 ± 5.263E+05+</b>
F18	<b>1.160E+03 ± 1.304E+03</b>	1.570E+03 ± 1.970E+03≈	3.796E+03 ± 3.163E+03	<b>2.775E+03 ± 2.967E+03+</b>
F19	9.509E+01 ± 3.930E+00	<b>9.061E+01 ± 3.084E+00+</b>	9.717E+01 ± 4.583E+00	<b>9.553E+01 ± 3.520E+00+</b>
F20	<b>9.451E+02 ± 3.634E+02</b>	1.144E+03 ± 5.922E+02≈	3.399E+04 ± 9.231E+03	<b>1.847E+03 ± 4.719E+02+</b>
F21	1.204E+05 ± 5.725E+04	<b>9.581E+04 ± 4.694E+04+</b>	2.593E+07 ± 7.782E+06	<b>6.532E+05 ± 2.852E+05+</b>
F22	3.755E+03 ± 5.746E+02	<b>2.535E+03 ± 5.856E+02+</b>	4.160E+03 ± 2.416E+02	<b>2.291E+03 ± 4.868E+02+</b>
F23	3.482E+02 ± 2.611E-12	<b>3.482E+02 ± 7.560E-13+</b>	3.482E+02 ± 7.366E-05	<b>3.482E+02 ± 8.509E-10+</b>
F24	<b>3.869E+02 ± 4.720E+00</b>	3.878E+02 ± 4.670E+00≈	<b>3.769E+02 ± 3.873E+00−</b>	3.813E+02 ± 4.991E+00−
F25	<b>2.245E+02 ± 3.278E+00</b>	2.285E+02 ± 4.325E+00−	3.357E+02 ± 2.576E+01	<b>2.291E+02 ± 4.170E+00+</b>
F26	<b>1.886E+02 ± 3.247E+01</b>	1.919E+02 ± 4.533E+01≈	2.734E+02 ± 1.697E+01	<b>2.004E+02 ± 2.706E+01+</b>
F27	<b>5.679E+02 ± 8.023E+01</b>	6.012E+02 ± 8.001E+01−	7.793E+02 ± 1.096E+02	<b>7.523E+02 ± 1.213E+02≈</b>
F28	<b>2.033E+03 ± 2.420E+02</b>	2.054E+03 ± 1.984E+02≈	<b>2.168E+03 ± 2.775E+02</b>	2.261E+03 ± 3.294E+02≈
F29	1.776E+03 ± 1.798E+02	<b>1.759E+03 ± 1.776E+02≈</b>	1.086E+04 ± 6.422E+03	<b>2.339E+03 ± 2.936E+02+</b>
F30	<b>5.861E+03 ± 1.089E+03</b>	6.303E+03 ± 1.138E+03≈	<b>7.969E+03 ± 1.091E+03</b>	8.473E+03 ± 1.015E+03−
+ / ≈ / −	-	16/11/3	-	24/2/4
Function	jDE	ETI-jDE	JADE	ETI-JADE
F1	<b>1.878E+06 ± 6.322E+05</b>	2.053E+06 ± 6.477E+05≈	<b>1.078E+05 ± 6.721E+04</b>	1.324E+05 ± 5.954E+04−
F2	<b>1.259E-07 ± 6.179E-07</b>	1.378E+04 ± 1.571E+04−	5.697E-10 ± 1.756E-09	<b>1.977E-10 ± 3.712E-10≈</b>
F3	3.568E-05 ± 1.022E-04	<b>1.381E-05 ± 3.120E-05≈</b>	5.928E+03 ± 3.432E+03	<b>1.725E+02 ± 1.471E+02+</b>
F4	<b>1.746E+02 ± 3.132E+01</b>	1.756E+02 ± 3.369E+01≈	<b>7.111E+01 ± 5.730E+01</b>	9.431E+01 ± 5.167E+01≈
F5	2.066E+01 ± 2.576E-02	<b>2.021E+01 ± 6.850E-02+</b>	2.047E+01 ± 1.092E-01	<b>2.000E+01 ± 1.460E-02+</b>
F6	6.800E+01 ± 1.182E+01	<b>3.359E+01 ± 1.119E+01+</b>	4.683E+01 ± 1.551E+01	<b>2.456E+01 ± 9.721E+00+</b>
F7	1.482E-12 ± 6.223E-13	<b>7.334E-13 ± 3.539E-13+</b>	<b>1.059E-03 ± 6.311E-03</b>	1.932E-03 ± 4.706E-03−
F8	<b>1.137E-13 ± 0.000E+00</b>	1.951E-02 ± 1.393E-01≈	1.137E-13 ± 0.000E+00	<b>1.115E-13 ± 1.592E-14≈</b>
F9	2.463E+02 ± 2.905E+01	<b>1.858E+02 ± 2.431E+01+</b>	1.466E+02 ± 1.881E+01	<b>1.412E+02 ± 2.340E+01≈</b>
F10	<b>9.391E-03 ± 4.757E-02</b>	2.652E-01 ± 2.648E-01−	<b>1.249E-02 ± 7.495E-03</b>	5.414E-01 ± 4.185E-01−
F11	1.345E+04 ± 6.212E+02	<b>1.160E+04 ± 1.315E+03+</b>	1.045E+04 ± 5.475E+02	<b>9.279E+03 ± 1.231E+03+</b>
F12	6.235E-01 ± 6.179E-02	<b>2.581E-01 ± 8.278E-02+</b>	3.323E-01 ± 2.376E-02	<b>9.579E-02 ± 4.241E-02+</b>
F13	4.881E-01 ± 4.725E-02	<b>2.954E-01 ± 5.444E-02+</b>	4.116E-01 ± 4.119E-02	<b>3.080E-01 ± 3.904E-02+</b>
F14	3.491E-01 ± 2.215E-02	<b>2.945E-01 ± 7.411E-02+</b>	3.189E-01 ± 3.032E-02	<b>2.257E-01 ± 2.380E-02+</b>
F15	3.077E+01 ± 2.450E+00	<b>1.611E+01 ± 2.556E+00+</b>	2.890E+01 ± 3.965E+00	<b>2.006E+01 ± 4.605E+00+</b>
F16	4.062E+01 ± 3.575E-01	<b>4.009E+01 ± 1.275E+00+</b>	4.000E+01 ± 5.816E-01	<b>3.867E+01 ± 1.812E+00+</b>
F17	1.568E+05 ± 6.784E+04	<b>1.239E+05 ± 4.630E+04+</b>	<b>1.129E+04 ± 5.790E+03</b>	1.238E+04 ± 4.222E+03≈
F18	<b>6.959E+02 ± 8.169E+02</b>	8.355E+02 ± 1.221E+03≈	1.220E+03 ± 1.088E+03	<b>9.313E+02 ± 8.386E+02≈</b>
F19	9.250E+01 ± 2.114E+00	<b>9.171E+01 ± 1.841E+00≈</b>	9.952E+01 ± 1.499E+01	<b>9.546E+01 ± 1.560E+01+</b>
F20	3.267E+02 ± 1.272E+02	<b>2.686E+02 ± 7.477E+01+</b>	8.725E+03 ± 1.411E+04	<b>1.397E+03 ± 1.585E+03≈</b>
F21	6.641E+04 ± 3.030E+04	<b>5.097E+04 ± 2.589E+04+</b>	<b>3.670E+03 ± 1.247E+03</b>	3.799E+03 ± 1.626E+03≈
F22	1.799E+03 ± 2.215E+02	<b>1.636E+03 ± 3.086E+02+</b>	1.542E+03 ± 2.369E+02	<b>1.453E+03 ± 3.498E+02≈</b>
F23	3.482E+02 ± 6.059E-13	<b>3.482E+02 ± 3.326E-13+</b>	3.482E+02 ± 8.632E-13	<b>3.482E+02 ± 1.783E-13+</b>
F24	3.741E+02 ± 3.152E+00	<b>3.736E+02 ± 3.478E+00≈</b>	<b>3.982E+02 ± 5.171E+00</b>	3.997E+02 ± 6.048E+00≈
F25	<b>2.674E+02 ± 1.097E+01</b>	2.687E+02 ± 1.088E+01≈	2.723E+02 ± 6.593E+00	<b>2.709E+02 ± 7.059E+00≈</b>
F26	1.963E+02 ± 1.955E+01	1.963E+02 ± 1.957E+01≈	2.001E+02 ± 5.518E-03	2.001E+02 ± 4.206E-03≈
F27	9.959E+02 ± 3.216E+02	<b>7.754E+02 ± 2.203E+02+</b>	<b>1.040E+03 ± 1.195E+02</b>	1.057E+03 ± 1.180E+02≈
F28	<b>2.248E+03 ± 1.736E+02</b>	2.353E+03 ± 3.026E+02≈	<b>2.336E+03 ± 2.438E+02</b>	2.383E+03 ± 3.300E+02≈
F29	1.630E+03 ± 2.145E+02	<b>1.543E+03 ± 1.427E+02+</b>	<b>1.317E+03 ± 1.968E+02</b>	1.371E+03 ± 1.514E+02≈
F30	<b>8.366E+03 ± 7.740E+02</b>	8.883E+03 ± 7.220E+02−	<b>8.442E+03 ± 1.316E+03</b>	8.987E+03 ± 1.141E+03−
+ / ≈ / −	-	17/10/3	-	11/15/4

Table A.8: Experimental results of CoDE, SaDE, ODE, EPSDE and the related ETI-based variants for functions F01-F30 at  $D = 100$ .

Function	CoDE	ETI-CoDE	SaDE	ETI-SaDE
F1	7.302E+06 ± 2.209E+06	<b>6.582E+06 ± 1.825E+06</b> ≈	<b>4.668E+06 ± 8.056E+05</b>	5.645E+06 ± 1.094E+06-
F2	2.087E+03 ± 1.182E+03	<b>1.327E+03 ± 1.043E+03</b> +	<b>1.182E+04 ± 6.929E+03</b>	1.277E+04 ± 6.811E+03≈
F3	<b>5.783E+01 ± 6.644E+01</b>	9.812E+01 ± 1.027E+02≈	<b>4.495E+03 ± 2.815E+03</b>	6.066E+03 ± 2.713E+03-
F4	1.658E+02 ± 3.064E+01	<b>1.643E+02 ± 2.806E+01</b> ≈	2.453E+02 ± 5.654E+01	<b>2.331E+02 ± 5.522E+01</b> ≈
F5	2.119E+01 ± 2.729E-02	<b>2.044E+01 ± 1.738E-01</b> +	2.105E+01 ± 2.378E-02	<b>2.000E+01 ± 1.347E-02</b> +
F6	1.819E+01 ± 5.910E+00	<b>1.467E+01 ± 3.570E+00</b> +	<b>6.068E+01 ± 4.398E+00</b>	6.069E+01 ± 3.848E+00≈
F7	1.413E-07 ± 8.612E-08	<b>6.173E-08 ± 4.626E-08</b> +	4.245E-03 ± 8.185E-03	<b>2.026E-03 ± 5.645E-03</b> +
F8	3.398E+02 ± 1.358E+01	<b>1.053E+02 ± 8.744E+00</b> +	<b>1.034E+00 ± 1.014E+00</b>	4.936E+00 ± 2.379E+00-
F9	5.033E+02 ± 1.730E+02	<b>1.318E+02 ± 2.517E+01</b> +	2.475E+02 ± 2.351E+01	<b>2.373E+02 ± 2.792E+01</b> +
F10	1.216E+04 ± 5.379E+02	<b>2.151E+03 ± 2.908E+02</b> +	1.785E+02 ± 5.005E+01	<b>7.752E+00 ± 1.672E+01</b> +
F11	2.643E+04 ± 7.001E+02	<b>1.041E+04 ± 1.901E+03</b> +	2.200E+04 ± 5.744E+02	<b>1.079E+04 ± 1.098E+03</b> +
F12	2.634E+00 ± 2.068E-01	<b>5.697E-01 ± 2.227E-01</b> +	1.839E+00 ± 1.530E-01	<b>1.298E-01 ± 6.013E-02</b> +
F13	6.183E-01 ± 5.293E-02	<b>4.461E-01 ± 6.450E-02</b> +	4.711E-01 ± 4.466E-02	<b>3.083E-01 ± 5.450E-02</b> +
F14	3.486E-01 ± 3.203E-02	<b>3.086E-01 ± 3.041E-02</b> +	3.160E-01 ± 1.909E-02	<b>2.786E-01 ± 2.310E-02</b> +
F15	6.540E+01 ± 6.259E+00	<b>1.910E+01 ± 6.459E+00</b> +	4.542E+01 ± 1.324E+01	<b>2.972E+01 ± 6.004E+00</b> +
F16	4.568E+01 ± 3.572E-01	<b>4.116E+01 ± 8.476E-01</b> +	4.416E+01 ± 3.856E-01	<b>3.993E+01 ± 1.101E+00</b> +
F17	2.973E+05 ± 1.670E+05	<b>2.505E+05 ± 1.402E+05</b> ≈	<b>2.503E+05 ± 9.490E+04</b>	2.927E+05 ± 1.190E+05≈
F18	<b>4.032E+02 ± 4.443E+02</b>	5.427E+02 ± 7.248E+02≈	6.416E+02 ± 4.535E+02	<b>6.066E+02 ± 3.753E+02</b> ≈
F19	9.428E+01 ± 1.070E+00	<b>9.291E+01 ± 1.249E+00</b> +	8.265E+01 ± 2.814E+01	<b>7.044E+01 ± 2.566E+01</b> +
F20	<b>2.753E+02 ± 9.865E+01</b>	2.968E+02 ± 1.159E+02≈	<b>2.463E+03 ± 1.301E+03</b>	2.597E+03 ± 1.152E+03≈
F21	7.940E+04 ± 4.935E+04	<b>6.949E+04 ± 3.607E+04</b> ≈	<b>1.574E+05 ± 6.988E+04</b>	1.624E+05 ± 7.355E+04≈
F22	1.919E+03 ± 5.006E+02	<b>1.637E+03 ± 4.148E+02</b> +	<b>1.251E+03 ± 2.801E+02</b>	1.536E+03 ± 3.443E+02-
F23	3.482E+02 ± 2.969E-07	<b>3.482E+02 ± 1.305E-07</b> +	<b>3.482E+02 ± 1.964E-08</b>	3.482E+02 ± 1.200E-05-
F24	<b>3.684E+02 ± 3.191E+00</b>	3.692E+02 ± 3.532E+00≈	<b>3.857E+02 ± 5.626E+00</b>	3.859E+02 ± 4.935E+00-
F25	<b>2.017E+02 ± 8.472E+00</b>	2.081E+02 ± 1.707E+01≈	<b>2.043E+02 ± 1.206E+01</b>	2.076E+02 ± 1.573E+01≈
F26	<b>1.806E+02 ± 3.991E+01</b>	1.884E+02 ± 3.245E+01-	2.001E+02 ± 2.022E-02	2.001E+02 ± 2.581E-02≈
F27	<b>4.591E+02 ± 5.613E+01</b>	4.813E+02 ± 6.371E+01≈	<b>1.431E+03 ± 1.158E+02</b>	1.435E+03 ± 1.117E+02≈
F28	2.563E+03 ± 2.465E+02	<b>2.131E+03 ± 1.056E+02</b> +	2.795E+03 ± 2.382E+02	<b>2.759E+03 ± 1.942E+02</b> ≈
F29	1.833E+03 ± 4.927E+02	<b>1.682E+03 ± 6.330E+02</b> ≈	<b>1.943E+03 ± 2.238E+02</b>	2.428E+03 ± 3.840E+02-
F30	5.147E+03 ± 1.760E+03	<b>4.950E+03 ± 1.669E+03</b> ≈	<b>9.534E+03 ± 3.439E+03</b>	9.997E+03 ± 2.429E+03≈
+/≈/-	-	17/12/1	-	11/13/6
Function	ODE	ETI-ODE	EPSDE	ETI-EPSDE
F1	3.109E+06 ± 8.105E+05	<b>2.925E+06 ± 9.690E+05</b> ≈	2.388E+06 ± 1.041E+07	<b>3.222E+05 ± 1.336E+05</b> ≈
F2	<b>2.718E+05 ± 1.819E+06</b>	2.902E+05 ± 1.964E+06≈	<b>1.967E-05 ± 8.542E-05</b>	2.241E-01 ± 1.021E+00-
F3	1.644E+03 ± 1.183E+03	<b>1.442E+03 ± 1.395E+03</b> ≈	2.672E-01 ± 6.444E-01	<b>2.124E-01 ± 6.074E-01</b> ≈
F4	<b>1.964E+02 ± 4.831E+01</b>	2.039E+02 ± 5.111E+01≈	<b>1.376E+02 ± 3.623E+01</b>	1.488E+02 ± 3.984E+01≈
F5	2.130E+01 ± 4.209E-02	<b>2.057E+01 ± 9.523E-02</b> +	2.109E+01 ± 4.071E-02	<b>2.037E+01 ± 1.453E-01</b> +
F6	2.449E+01 ± 4.682E+00	<b>2.386E+01 ± 4.307E+00</b> ≈	1.331E+02 ± 4.369E+00	<b>3.725E+01 ± 1.683E+01</b> +
F7	1.389E-02 ± 2.352E-02	<b>9.160E-03 ± 1.550E-02</b> ≈	3.088E-03 ± 6.560E-03	<b>2.170E-03 ± 6.250E-03</b> +
F8	<b>1.132E+02 ± 5.360E+01</b>	1.271E+02 ± 4.461E+01≈	1.823E+02 ± 2.928E+01	<b>6.159E+00 ± 2.611E+00</b> +
F9	2.004E+02 ± 1.027E+02	<b>1.698E+02 ± 3.374E+01</b> ≈	6.493E+02 ± 3.099E+01	<b>1.601E+02 ± 2.894E+01</b> +
F10	1.244E+04 ± 5.061E+03	<b>6.249E+03 ± 1.206E+03</b> +	1.234E+04 ± 1.270E+03	<b>5.827E+02 ± 1.901E+02</b> +
F11	1.526E+04 ± 6.792E+03	<b>1.039E+04 ± 1.727E+03</b> +	2.684E+04 ± 1.085E+03	<b>1.087E+04 ± 1.280E+03</b> +
F12	2.882E+00 ± 1.119E+00	<b>8.405E-02 ± 7.344E-02</b> +	2.118E+00 ± 2.683E-01	<b>2.295E-01 ± 7.314E-02</b> +
F13	5.335E-01 ± 5.856E-02	<b>4.391E-01 ± 5.738E-02</b> +	4.275E-01 ± 4.395E-02	<b>2.456E-01 ± 4.422E-02</b> +
F14	3.420E-01 ± 2.794E-02	<b>2.974E-01 ± 2.983E-02</b> +	3.516E-01 ± 2.494E-02	<b>3.202E-01 ± 3.074E-02</b> +
F15	5.464E+01 ± 2.483E+01	<b>2.206E+01 ± 5.405E+00</b> +	6.995E+01 ± 6.923E+00	<b>1.867E+01 ± 4.223E+00</b> +
F16	4.593E+01 ± 1.042E+00	<b>4.138E+01 ± 1.323E+00</b> +	4.612E+01 ± 4.921E-01	<b>4.174E+01 ± 1.044E+00</b> +
F17	7.537E+05 ± 2.784E+05	<b>4.813E+05 ± 1.775E+05</b> +	3.851E+06 ± 1.048E+07	<b>4.034E+05 ± 3.911E+05</b> ≈
F18	<b>7.654E+02 ± 1.111E+03</b>	8.292E+02 ± 8.662E+02≈	<b>2.540E+03 ± 2.848E+03</b>	3.339E+03 ± 5.001E+03≈
F19	9.844E+01 ± 9.179E+00	<b>9.084E+01 ± 6.942E+00</b> +	6.088E+01 ± 1.880E+01	<b>3.555E+01 ± 3.227E+00</b> +
F20	1.110E+03 ± 4.718E+02	<b>1.074E+03 ± 5.110E+02</b> ≈	1.612E+03 ± 5.652E+03	<b>5.726E+02 ± 4.694E+02</b> +
F21	1.522E+05 ± 6.776E+04	<b>1.242E+05 ± 6.395E+04</b> +	3.146E+05 ± 1.481E+05	<b>1.107E+05 ± 1.434E+05</b> +
F22	2.015E+03 ± 6.075E+02	<b>1.993E+03 ± 5.868E+02</b> ≈	2.467E+03 ± 2.859E+02	<b>1.943E+03 ± 4.307E+02</b> +
F23	3.482E+02 ± 1.116E-10	3.482E+02 ± 7.178E-11≈	3.450E+02 ± 3.636E-11	<b>3.450E+02 ± 4.482E-12</b> +
F24	<b>3.852E+02 ± 5.792E+00</b>	3.853E+02 ± 6.142E+00≈	<b>3.912E+02 ± 4.836E+00</b>	3.915E+02 ± 4.675E+00≈
F25	<b>2.105E+02 ± 1.927E+01</b>	2.127E+02 ± 2.030E+01≈	<b>2.220E+02 ± 2.864E+01</b>	2.414E+02 ± 2.829E+01-
F26	2.001E+02 ± 2.257E-02	<b>2.001E+02 ± 2.617E-02</b> +	<b>1.239E+02 ± 4.268E+01</b>	1.257E+02 ± 4.393E+01-
F27	<b>8.551E+02 ± 1.064E+02</b>	8.769E+02 ± 8.050E+01≈	3.790E+03 ± 5.534E+01	<b>1.004E+03 ± 1.944E+02</b> +
F28	<b>2.009E+03 ± 2.382E+02</b>	2.023E+03 ± 2.378E+02≈	<b>6.921E+02 ± 7.096E+01</b>	7.101E+02 ± 8.571E+01≈
F29	<b>1.814E+03 ± 1.657E+02</b>	1.852E+03 ± 1.419E+02≈	2.581E+02 ± 1.353E+01	<b>2.495E+02 ± 1.622E+01</b> +
F30	<b>5.772E+03 ± 8.938E+02</b>	6.403E+03 ± 1.033E+03-	3.142E+03 ± 3.861E+02	<b>2.069E+03 ± 4.035E+02</b> +
+/≈/-	-	12/17/1	-	21/6/3



Table A.9: Experimental results of SHADE, OXDE and the related ETI-based variants for functions F01-F30 at  $D = 100$ .

Function	SHADE	ETI-SHADE	OXDE	ETI-OXDE
F01	1.154E+05 ± 6.252E+04	<b>1.038E+05</b> ± 5.440E+04≈	<b>2.280E+06</b> ± 5.801E+05	2.588E+06 ± 6.312E+05-
F02	<b>1.667E-10</b> ± <b>5.476E-10</b>	1.757E-10 ± 5.859E-10≈	<b>1.104E+04</b> ± <b>1.402E+04</b>	1.056E+04 ± 1.126E+04≈
F03	2.262E-02 ± 8.214E-02	<b>7.922E-04</b> ± <b>1.708E-03</b> +	<b>9.700E+02</b> ± <b>7.702E+02</b>	1.438E+03 ± 1.488E+03≈
F04	8.451E+01 ± 5.945E+01	<b>6.752E+01</b> ± <b>5.021E+01</b> ≈	1.796E+02 ± 3.941E+01	<b>1.739E+02</b> ± <b>4.484E+01</b> ≈
F05	2.020E+01 ± 1.718E-02	<b>2.001E+01</b> ± <b>1.784E-02</b> +	2.132E+01 ± 2.440E-02	<b>2.038E+01</b> ± <b>1.537E-01</b> +
F06	3.444E+01 ± 5.045E+00	<b>3.174E+01</b> ± <b>4.229E+00</b> +	<b>1.817E+01</b> ± <b>3.616E+00</b>	1.888E+01 ± 3.213E+00≈
F07	5.899E-03 ± 3.069E-02	<b>1.160E-03</b> ± <b>3.283E-03</b> +	1.834E-03 ± 5.939E-03	<b>1.111E-03</b> ± <b>3.663E-03</b> +
F08	<b>1.404E-12</b> ± <b>8.454E-14</b>	9.754E-02 ± 5.717E-01-	7.543E+01 ± 1.541E+01	<b>7.464E+01</b> ± <b>1.095E+01</b> ≈
F09	<b>1.099E+02</b> ± <b>1.274E+01</b>	1.199E+02 ± 2.039E+01-	7.734E+02 ± 1.031E+02	<b>1.134E+02</b> ± <b>1.943E+01</b> +
F10	<b>5.021E-03</b> ± <b>6.122E-03</b>	1.312E-01 ± 1.011E-01-	7.227E+03 ± 4.329E+03	<b>1.251E+03</b> ± <b>4.636E+02</b> +
F11	9.749E+03 ± 5.968E+02	<b>8.588E+03</b> ± <b>1.120E+03</b> +	2.981E+04 ± 4.731E+02	<b>1.021E+04</b> ± <b>1.767E+03</b> +
F12	2.244E-01 ± 2.403E-02	<b>1.160E-01</b> ± <b>6.441E-02</b> +	3.987E+00 ± 2.189E-01	<b>1.035E-01</b> ± <b>5.049E-02</b> +
F13	3.855E-01 ± 4.297E-02	<b>2.983E-01</b> ± <b>4.113E-02</b> +	5.358E-01 ± 4.709E-02	<b>2.940E-01</b> ± <b>5.152E-02</b> +
F14	<b>3.033E-01</b> ± <b>2.806E-02</b>	3.159E-01 ± 4.296E-02-	3.388E-01 ± 2.319E-02	<b>3.052E-01</b> ± <b>2.711E-02</b> +
F15	2.456E+01 ± 3.324E+00	<b>1.692E+01</b> ± <b>2.631E+00</b> +	6.797E+01 ± 9.027E+00	<b>1.247E+01</b> ± <b>2.364E+00</b> +
F16	<b>3.960E+01</b> ± <b>5.692E-01</b>	3.996E+01 ± 2.544E+00-	4.631E+01 ± 2.927E-01	<b>4.073E+01</b> ± <b>1.573E+00</b> +
F17	<b>1.139E+04</b> ± <b>3.952E+03</b>	1.427E+04 ± 5.114E+03-	3.879E+05 ± 1.678E+05	<b>2.814E+05</b> ± <b>1.055E+05</b> +
F18	<b>6.302E+02</b> ± <b>4.772E+02</b>	6.563E+02 ± 6.080E+02≈	1.277E+03 ± 1.480E+03	<b>1.014E+03</b> ± <b>1.362E+03</b> ≈
F19	9.419E+01 ± 1.562E+01	<b>9.258E+01</b> ± <b>2.034E+01</b> ≈	9.541E+01 ± 3.704E+00	<b>9.089E+01</b> ± <b>6.182E+00</b> +
F20	4.802E+02 ± 1.106E+02	<b>4.537E+02</b> ± <b>1.283E+02</b> ≈	<b>8.213E+02</b> ± <b>2.415E+02</b>	1.097E+03 ± 4.562E+02-
F21	<b>3.463E+03</b> ± <b>9.418E+02</b>	4.037E+03 ± 1.824E+03≈	1.041E+05 ± 4.218E+04	<b>8.985E+04</b> ± <b>3.736E+04</b> ≈
F22	<b>1.359E+03</b> ± <b>2.151E+02</b>	1.417E+03 ± 3.207E+02≈	3.839E+03 ± 4.987E+02	<b>2.068E+03</b> ± <b>5.016E+02</b> +
F23	3.482E+02 ± 8.168E-13	<b>3.482E+02</b> ± <b>1.783E-13</b> +	3.482E+02 ± 1.027E-12	<b>3.482E+02</b> ± <b>9.326E-13</b> +
F24	3.954E+02 ± 4.165E+00	<b>3.949E+02</b> ± <b>5.089E+00</b> ≈	<b>3.873E+02</b> ± <b>4.418E+00</b>	3.876E+02 ± 4.741E+00≈
F25	<b>2.581E+02</b> ± <b>5.652E+00</b>	2.593E+02 ± 5.942E+00≈	<b>2.381E+02</b> ± <b>1.186E+01</b>	2.403E+02 ± 1.048E+01≈
F26	2.001E+02 ± 2.648E-02	2.001E+02 ± 7.752E-03≈	<b>1.963E+02</b> ± <b>1.954E+01</b>	1.982E+02 ± 1.400E+01-
F27	9.514E+02 ± 8.835E+01	<b>9.450E+02</b> ± <b>9.438E+01</b> ≈	<b>7.261E+02</b> ± <b>8.961E+01</b>	7.491E+02 ± 1.013E+02≈
F28	2.304E+03 ± 2.278E+02	<b>2.281E+03</b> ± <b>2.121E+02</b> ≈	<b>2.048E+03</b> ± <b>2.318E+02</b>	2.099E+03 ± 1.926E+02≈
F29	1.309E+03 ± 1.976E+02	<b>1.297E+03</b> ± <b>1.715E+02</b> ≈	<b>1.602E+03</b> ± <b>1.992E+02</b>	1.688E+03 ± 1.616E+02-
F30	8.480E+03 ± 1.309E+03	<b>8.363E+03</b> ± <b>1.015E+03</b> ≈	6.470E+03 ± 1.173E+03	<b>6.387E+03</b> ± <b>1.143E+03</b> ≈
+ / ≈ / -	-	9/15/6	-	14/12/4



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