CO-ORDINATION, WAREHOUSING, VEHICLE ROUTING AND DELIVERYMEN PROBLEMS OF SUPPLY CHAINS

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Certificate of Originality

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Abstract

Compared with traditional business management aiming to pursue single entity's own maximum benefit or minimum cost, supply chain management, which serves the market from a more systemic perspective, focuses its attention on achieving good responsiveness as well as economy performance through proper coordination of all the participants across the business functions of the supply chain, including procurement, manufacturing and distribution, etc. With the bloom of information evolution, more attentions have been given to increase the degree of coordination among multiple functions within the supply chain. Although, the more decision modules integrated in one co-ordination mathematical model, the better performance of the supply chain would be achieved, it is at the expense of computational time for searching the optimal solution due to the complexity of the model. Hence, our co-ordinated systems would focus on integrating two or more of the following decision levels: procurement, production, inventory, warehousing, vehicle routing and delivery men routing.

This thesis proposes and develops the mathematical models and solution methods for four supply chain coordinating systems.

(i) A synchronized cycles single-vendor multi-buyer supply chain model involving clustering of buyers with long and short cycles is proposed. The ordering cycle

of each buyer is either an integer multiple or an integer factor of the vendor's production cycle. The buyer-clustering mechanism, in which the ordering cycles adopted by buyers are allowed to be larger than the vendor's production cycle, increases the flexibility of the system, which reduces the total system cost. The effectiveness of this clustering synchronized cycles model is also analyzed.

(ii) An integrated production-warehouse location-inventory (PWLI) model is proposed. In this model, decision variables of warehouse location, production schedule and ordering frequency, are integrated in one model and determined simultaneously by minimizing the total system cost. Meanwhile, a synchronization mechanism is implemented to the system so as to coordinate inventory replenishment decisions. Numerical experiments have been carried out to illustrate the performance of this co-ordinated model.

(iii) An extension of the synchronized cycles PWLI model is proposed. In this extended model, deliveries are modeled by a set of heterogeneous vehicle routing problems instead of a fixed cost for each order. Numerical experiments have been carried out to illustrate the performance of this co-ordinated model.

(iv) An integrated model for multi-depot vehicle routing and delivery men problem is studied. This model incorporates a distribution network of multiple depots, multiple parking sites and multiple customers linked by trips of a fleet of homogeneous vehicles and a number of delivery men assigned to the vehicles. The objective of this model is to determine the number of delivery men assigned to each vehicle and the routing of vehicles and delivery men so as to minimize the total relevant costs involved in the two levels.

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Research Background

Supply chain, which is a network of entities linked by material, financial and information flows, performs the functions of raw material procurement, commodity manufacture and distribution (Ganeshan and Harrison (1995)). Compared with traditional business management aiming to pursue single entity's own maximum benefit or minimum cost, supply chain management, which serves the market from a more systemic perspective, focuses its attention on achieving good responsiveness as well as economy performance through proper coordination of all the participants across the supply chain.

In early development of supply chain management, vertical integration is proposed. In this inter-organizational model, an enterprise is engaged in several stages of a given supply chain and maximum benefit is obtained through economies of scale. However, in recent decades, with the bloom of information evolution and globalization, a high competitive market environment is emerging. Under this circumstance, each company is driven to focus on its core competence, which means only be engaged in the stage it does the best and outsource the rest of the stages to other companies. This intra-organizational supply chain model is called virtual integration and the most famous example is DELL Computers, which was found in 1984 and developed into a $ 12 billion company during its first 13 years (Magretta (1998)).

Today, innovative information technology provides an efficient way to help coorperation between isolated entities achieve in both type of integration models. But how to coordinate the material, information and financial flows such that the structure of a value chain will remain stable in the long run? The answer is to build a win-win situation to attract each participant to join the coordination. And the occurrence of a win-win situation is based on the premise that the supply chain system cost is reduced so that all parties can be benefited from the cost saving through some compensation schemes, such as quantity discount (Dada and Skikanth (1987), Weng (1995)), buybacks (Pasternack (1985)), two-part tariff (Moorthy (1987), Lariviere (1999)) and revenue-sharing (Cachon and Lariviere (2005)), etc..

In recent decades, various coordination models, aiming to reduce the total system cost, have been proposed for different functions and interfaces of supply chain (Arshinder et al. (2008)). The difficulties in supply chain planning is that hundreds of decisions have to be made in different areas like forecasting, location, inventory management, transportation planning, etc.. Based on decision modules' different features, decision making can be sorted into three levels, which are strategic, tactical and operational. However, decisions at different levels or in different areas usually are made independently, which might not be optimal for the whole supply chain (Shen and Qi (2007)). As a result, coordination at interfaces of those areas have been developed, including procurement-production coordination, production-inventory coordination and inventory-distribution coordination etc..

In the light of this thought, the main task of this research is to build a co-ordinated supply chain which integrates functional modules from three levels so as to improve the overall performance: obtaining a larger system cost saving when compared to the independent policy model. Although, the more decision modules integrated in one coordination mathematical model, the better performance of the supply chain would be achieved, it is at the expense of computational time for searching the optimal solution due to the complexity of the model. Hence, our co-ordinated system would focus on integrating the following three decision modules: inventory, location and routing.

The purpose of inventory management is buffering uncertainties and preventing stock-out with a minimum cost. This can be achieved by implementing certain production policy and replenishment policy to vendor and buyers, respectively, e.g. classical economic production quantity (EPQ) and economic order quantity (EOQ) models. In such policies, the vendor and buyers act separately to minimize their own cost but this is not optimal for the whole vendor-buyer system. Therefore, it is advantageous to the system if the production and ordering are planned in a collaborative way.

In addition to the challenge of the integration of inventory policies, the supply chain network design is also an issue that needs to be considered. Facility location is the most important decisions in the strategic level and has a great impact on the inventory decisions and routing decisions. However, in most cases, facility locations are planned at a strategic level without the consideration of inventory and routing cost, but in turn, inventory and routing decisions are greatly related to the location.

Thus, it is necessary to integrate tactical and operational decisions into the planning of facility locations.

The delivery in the last echelon of the supply chain is also worthy of study. Since the service level can be reflected by the delivery efficiency, how to make a distribution plan for the goods from depots to final customers at the minimum cost whilst at a high service level is a critical logistics problem.

An integrated optimization model combining two or more modules, which usually contains too many integer decision variables and constraints, is hard to be solved optimally and can only be solved by heuristics to obtain a near-optimal solution. One of the objectives of this research is to develop hybrid heuristics with an effort to find better near-optimal solutions for co-ordinated supply chain systems.

1.2 Research Objectives and Scope

In recent decades, various joint inventory policies have been established to minimize the total system cost. Chan and Kingsman (2007) proposed a coordination model for a single-vendor multi-buyer system, in which production cycle of vendor and ordering cycles of heterogeneous buyers are synchronized. Instead of forcing all the buyers to use a common cycle policy as described in Banerjee and Burton (1994), Chan and Kingsman (2007) allowed the buyers to select their own ordering cycles, which must be integer multiples of a basic time unit and integer factors of the vendor's production cycle. Intuitively, if a model can permit buyers to have their cycles larger than the vendor's production cycle, the system's flexibility can be increased and hence the total system cost can be improved.

In this research, a co-ordinated supply chain model is proposed for the coordination of production schedule, inventory policy, warehouse location and vehicle routing. Specifically, this research considers a three-echelon supply chain system consisting one manufacturer, a number of potential warehouses, a set of retailers with deterministic demand rate and a fleet of heterogeneous vehicles. Some of the potential warehouses will be open and each open warehouse is assigned to serve a group of the retailers. Products will be delivered to open warehouses from the manufacturer and then distributed to each retailer under the synchronized cycles policy. Using hybrid heuristics, this model attempts to achieve the following objectives at a minimum total system cost:

- *Inventory* - To determine production cycle for manufacturer and ordering cycles for open warehouses and retailers so that the inventory level at each echelon can be well controlled without stock-out.

- *Location* - To determine which potential warehouses should be open for most efficient inventory storage and product distribution. Subsequently, assign retailers to the nearest open warehouses.

- *Distribution* - To design delivery routes of a fleet of vehicles for scheduled deliveries of products from upstream echelon to downstream echelon.

Specifically, an integrated vehicle routing and delivery man problem is proposed for modeling the goods' distribution from the depots to the final customers, where extra delivery men can be assigned to the vehicles. In this model, customers are assumed to be in predefined clusters and the goods are shipped from depots to parking site near the clusters and then delivered to the customers by delivery men on foot. The objective of this two-tier vehicle routing problem is to determine the routing for the vehicles and delivery man at a minimum system cost in which total waiting time of the customers is incorporated.

1.3 Organization of the Thesis

This thesis is divided into seven chapters.

Chapter 1 introduces the background of supply chain cooperation and presents the research scope and outline of this thesis.

A literature review of the related work done is presented in Chapter 2. A literature review on supply chain models with respect to the areas of inventory management, warehouse location and delivery is presented.

A synchronized cycles single-vendor multi-buyer supply chain model with buyer-clustering is investigated in Chapter 3. In this model, the ordering cycles of the buyers are allowed to be larger than the vendor's production cycle instead of being restricted to an integer factor of the vendor's production cycle. This gives more flexibility to the system and hence the total system cost can be improved. Numerical experiments are carried out to test the performance of this model when compared with the independent policy model and the synchronized cycles model of Chan and Kingsman (2007).

Chapter 4 studies an integrated production-warehouse location-inventory model. Decision variables at different planning levels, i.e. warehouse location, production schedule and ordering frequency, are integrated in one model and determined simultaneously by minimizing the total system cost. Meanwhile, to coordinate inventory replenishment decisions, a synchronization mechanism is implemented to the system. Numerical experiments have been carried out to illustrate the performance of this co-ordinated model.

The co-ordinated model proposed in Chapter 4 is extended in Chapter 5. In this extended model, deliveries are modeled by a set of heterogenous vehicle routing problems instead of a fixed cost for each order regardless the order size. Some characteristics of the vehicle schedule are found from the solutions.

Chapter 6 proposes an integrated model for multi-depot vehicle routing and delivery men problem. This model incorporates a distribution network of multiple depots, multiple parking-sites and multiple customers linked by trips of a fleet of homogeneous vehicles and a number of delivery men assigned to the vehicles. The objective of this model is to determine the number of delivery men assigned to each vehicle and the routing of vehicles and delivery men so as to minimize the total relevant costs involved in the two levels. The solutions obtained by ALNS have revealed some characteristics of the routing schedule.

At last, Chapter 7 summarizes and concludes the whole thesis and suggests further possible research directions arising from the results of this thesis.

Chapter 2

Literature Review

2.1 Introduction

In today's high dynamic business environment, more and more companies are pushed to enhance their core competitiveness to survive in the industry. Cooperation between parties has been proven to be an effective way (Skjoett-Larsen et al. (2003)). In the existing literature, cooperation involving different supply chain areas, e.g. inventory management, facility location and distribution, etc., has been carried out for various purposes. A literature review on supply chain models with respect to the above areas is presented.

2.2 Integrated Inventory Problem

Traditionally, the vendor's inventory problem and buyer's inventory problem are treated independently. Therefore, EPQ and EOQ policies, which aim to minimize each individuals' own cost, are widely used. However, enterprises have recently realized that the inventories across the whole vendor-buyer system can be more efficiently managed through a co-ordinated inventory policy.

According to the layer numbers of the supply chain network, co-ordinated inventory models can be categorized into two-echelon models and multi-echelon models. Meanwhile, supply chain structure can be classified as convergent, divergent, conjoined or general based on the participant numbers in each echelon (Beamon and Chen (2001)).

2.2.1 Co-ordinated two-echelon supply chain models

One of the first two-echelon coordination models was proposed by Goyal (1977). The author developed a joint economic lot size (JELS) model to minimize the total relevant cost of a single-vendor single-buyer (SVSB) system. It assumed that the vendor has infinite production rate and replenishes the buyer by a lot-for-lot policy. Banerjee (1986a) generalized the JELS model in Goyal (1977) by relaxing the assumption of infinite production rate while still adopting lot-for-lot shipment policy, i.e. the production lot is transferred to the buyer in a single shipment only after the whole production process is finished. Banerjee's co-ordinated SVSB model was further generalized by Goyal (1988), which assumed the completed production lot is distributed to the purchaser in equal-sized shipments rather than one single shipment. Monahan (1984) proposed a co-ordinated SVSB model from vendor's perspective. In this model, vendor's profit is maximized by introducing a quantity discount scheme to entice the buyer to order in quantity. Monahan's model was extended by Banerjee (1986b), Joglekar (1988), Monahan (1988), Lee and Rosenblatt (1986) and Goyal (1987), etc.. Banerjee (1986b) generalized Monahan's model by incorporating vendor's inventory cost and demonstrated that the generalized version is equivalent to the JELS model proposed by Banerjee (1986a). Joglekar (1988) and Monahan (1988) discussed the shortcomings in assumptions of Monahan (1984), e.g. (a) vendor's production cycle is equal to buyer's ordering cycle, (b) vendor's holding cost is irrelevant to buyer's ordering cycle. Therefore, some modifications of the assumptions were proposed to make the model more practical. A comprehensive review of the JELS models up to 1989 was discussed in Goyal and Gupta (1989).

The models mentioned so far are all based on the assumption that the whole production lot is completed before shipping to the customer. Delivery structure without this constraint was extended by Lu (1995). The author suggested that one production lot is transported to the buyer in equal-size batches and partial delivery is allowed as long as a batch is completed. Hence, it is not necessary to wait for the completion of the whole production lot. With this flexibility, the new delivery policy reduces the inventory at the vendor, which leads to a lower total system cost. Equal-size shipment policies were also implemented in the models proposed by Banerjee and Kim (1995), Aderohunmu et al. (1995) and Ha and Kim (1997), etc..

In the light of this thought, different delivery structures for co-ordinated SVS-B system have been investigated. Goyal (1995) suggested that a lower total joint relevant cost can be obtained by a variant of equal-size shipment policy, called geometric shipment policy. In this model, the shipment size is progressively increased by the P/D ratio (P is the annual production rate of the vendor and D is the annual demand rate of the purchaser). Viswanathan (1998) named the equal-size policy proposed by Lu (1995) and geometric policy proposed by Goyal (1995) as the 'identical delivery quantity (IDQ)' policy and the 'delivery what is produced (DWP)' policy, respectively. Viswanathan (1998) also showed that neither of the policies dominates the other for all problem parameters. The geometric shipment policy was further generalized by Hill (1997), where the shipment size growth factor λ was treated as a decision variable under the constraint $1 \leqslant \lambda \leqslant P/D$. This generalized version outperforms the policies of Lu (1995) and Goyal (1995) since the latter two are the special cases of the former one, i.e. $\lambda = 1$ and $\lambda = P/D$. However, Goyal and Nebebe (2000) showed that a combination of the two special cases achieves a lower total cost than the generalized geometric policy. The combined policy assumes that the whole production lot will be delivered to the customer in n shipments where the first shipment is of size q and other $(n-1)$ shipments are of equal size qP/D. The combination of this 1-geometric and $(n-1)$-equal is extended to m-geometric and $(n-m)$-equal by Goyal (2000). The optimal solution to the geometric-then-equal delivery problem can be obtained by an exact iterative algorithm proposed by Hill (1999).

In a co-ordinated system, consignment stock (CS) is a simple way to enhance the flexibility of the inventory policy at both parties. A consignment stock mechanism implies that the vendor pays for the inventory cost held by the buyer until the products are used and the unused products at the buyer can be returned to the vendor at any time. Valentini and Zavanella (2003) investigated an industrial case and showed that CS outperforms the usual inventory policy. Braglia and Zavanella (2003) proposed an analytical model of CS policy and its numerical results were compared with Hill's (1999). This comparison showed that the CS model can be adopted in many situations successfully. An extensive review on CS policy can be found in Sarker (2014).

An extension of the co-ordinated single-vendor single-buyer system is to consider multi-buyer and/or multi-supplier, e.g. single-vendor multi-buyer (SVMB), multi-vendor single-buyer (MVSB) and multi-vendor multi-buyer (MVMB), which are closer to the reality.

Based on quantity discount policy, Lal and Staelin (1984) and Joglekar and Tharthare (1990) developed different coordination mechanisms to model SVMB systems. Lal and Staelin (1984) proposed a pricing structure aiming to optimize a single-vendor multi-identical-buyer system and then further generalized the model by relaxing the assumption of identical buyers. For heterogonous-buyer system, the authors developed an analytical pricing policy which achieves lower costs for both parties than EOQ policy. However, Joglekar and Tharthare (1990) pointed out that there were some shortcomings in Lal and Staelin (1984) and proposed an individually responsible and rational decision (IRRD) approach as a refined JELS model for SVMB systems. In the identical-buyer situation, Joglekar and Tharthare (1990) assumed an ordering cycle, which is an integer factor of the system planning horizon, to be applied to all the buyers. Nevertheless, the integer ratio-policy was not considered in the non-identical-buyer situation, as it might lead to shortages. To tackle this problem, Banerjee and Burton (1994) developed a replenishment mechanism for a supply chain consisting of one vendor and many heterogeneous buyers under deterministic conditions, aiming to minimize the total cost incurred by all parties. They suggested that a common ordering cycle, which is an integer factor of vendor's production cycle, is adopted by all buyers. The numerical results showed that this common cycle policy outperforms the independent policy, i.e. EPQ and EOQ policies. Banerjee and Banerjee (1994) developed a common cycle model incurring less inventory related costs by introducing electronic data interchange (EDI) technology so as to shorten the time lag in communication and shipment. An extension of this model was proposed by Woo et al. (2001). In this model, a single vendor purchases and processes raw materials and ships the finished products to the buyers adopting a common cycle policy and participants at both echelons are willing to invest in reducing the ordering cost (e.g. establishing EDI based inventory control system) so as to decrease the total system cost. Viswanathan and Piplani (2001) incorporated a price discount strategy into a common cycle model for the SVMB issue. However, vendor's inventory cost was not taken into account in the proposed model. Another extension of the common cycle model was provided by Siajadi et al. (2006). The author assumed that the order cycle of each buyer is equal to the production cycle of the vendor and the vendor will deliver a number of equal-size shipments to each buyer where the first deliveries to the buyers are scheduled at different time.

Hoque (2008) investigated three shipment policies for a SVMB system, two of which implement equal-size batches (part of a lot) and the third with geometric batches of the product. A comparative study of the results shows that the supply by unequal batches performs better. Hoque (2011a) proposed a co-ordinated model by delivering the vendors' lots with geometric and/or equal sized batches. Hoque (2011b) proposed a more realistic model that incorporated restriction on transportation and storage capacities as well as lead times and batch sizes. However, in all of the three papers, Hoque assumed that each buyer incurs the ordering cost only once, regardless the number of orders.

Similar to Lu (1995), Yao and Chiou (2004) studied a SVMB model to minimize the vendors total cost subject to the maximum cost that the buyer is willing to incur. Yao and Chiou (2004) explored the optimality structure of this integrated model and developed an efficient search algorithm to solve the optimal cost curve which turned out to be piece-wise convex.

Abdul-Jalbar et al. (2007) developed an integrated inventory model for the single-vendor two-buyer system, where integer-ratio policy was implemented to the buyers' ordering cycles. Chan and Kingsman (2007) proposed a coordinated model by synchronizing the ordering cycles and production cycle of a single-vendor multi-buyer supply chain. The synchronization is achieved by assuming a basic cycle time is implemented by the vendor and the order cycle for each buyer is an integer multiple of a basic time unit whilst being a factor of the production cycle. This model has a lower system cost when compared with common cycle policy. Chu and Leon (2008) considered the problem of coordinating the SVMB inventory system under different information policies, i.e. global information and private information. The production and ordering policies were determined such that the total related cost of the system is minimized. Under both information policies, a method for finding a 'common replenishment period policy' (CRPP) was proposed first and then the solution approach was extended to finding an 'asynchronous replenishment period policy' (ARPP) by relaxing the common cycle assumption.

Various coordination mechanisms for SVMB models have also been discussed by several researchers. Sarmah et al. (2008) implemented the credit option policy to the common cycle model. Chan et al. (2010) proposed a delayed payment method to guarantee that buyers' total relevant cost of synchronized cycle policy will not be increased when compared with independent policy. Under the vendor managed inventory & consignment stock (VMI & CS) partnership proposed by Ben-Daya et al. (2013a), the individual cost for each participant is always lower than that of no coordination between vendor and buyers.

So far, the literature on two-echelon integrated inventory models considering more than one vendor are scarce. Chen and Sarker (2010) studied a MVSB integrated procurement-production inventory system that incorporates delivery and shared transportation costs. Glock (2011) considered a buyer sourcing a product from heterogeneous suppliers and tackled both the supplier selection and lot size decision with the objective of minimizing total system costs. Another study of sourcing problem was provided by Glock (2012). The author investigated the case of multiple homogeneous suppliers delivering a product in equal-size shipments to a single buyer. Furthermore, Glock and Kim (2014) developed a shipment consolidation policy for the MVSB system, where the buyer has the option to group the vendors and each group delivers their batches jointly to the buyer.

2.2.2 Co-ordinated multi-echelon supply chain models

Due to rapid development in recent information technology, more attention has been given to the co-ordination among multiple organizations of a supply chain. One of the first integrated three-echelon models was proposed in Banerjee and Kim (1995), which considered a procurement-production model involving the raw material ordering. In this one supplier one buyer model, the supplier's production batch size is multiples of the buyer's ordering lot size while the length of each production is a multiple of the raw material ordering cycle. A more general model, which included multiple retailers in the last echelon, was analyzed in Banerjee et al. (2007). In this model, a common ordering cycle is adopted by all the retailers. Another extension of Banerjee and Kim (1995) was given by Lee (2005). The author studied an integrated supply chain consisting of one supplier, one manufacturer and one buyer. However, the raw material ordering cycle was set to be a decision variable chosen from the multiples or the factors of the production running time per setup.

Khouja (2003) formulated a three-stage supply chain model where a supplier ships raw materials to multiple manufacturers and in turn, each manufacturer distributes the finished items to multiple retailers. Three inventory coordination mechanisms among the chain members were analyzed: (a) Equal cycle time mechanism, a common cycle is implemented to all the participants in the supply chain. (b) Integer multipliers mechanism, the cycle time of each stage is an integer multiple of the cycle time of adjacent downstream stage and the members in the last stage have a common cycle. (c) Integer powers of two mechanism, the cycle time of each member in each stage is multiple of a basic cycle time where the multiple is an integer power of 2.

An extension of Khouja (2003) was developed by Ben-Daya and Al-Nassar (2008). This model includes multiple suppliers and allows shipment to be made before the whole lot is completed. The numerical results of the paper showed that, when both applying the integer multipliers inventory mechanism, the proposed shipment scheme outperformed the usual shipment policy, i.e., shipments are made only after the whole production is finished. This model was further developed by Ben-Daya et al. (2013b). The authors assumed that the number of raw material shipments to both parties in each cycle is a decision variable, as mentioned in Lee (2005). Under the same system structure and shipment policy, Abdelsalam and Elassal (2014) studied a three-layer supply chain with stochastic demand and varying holding and ordering costs. A consignment stock policy in a three-level supply chain was analyzed by Giri et al. (2015). A sensitivity analysis was presented in the paper to examine the effectiveness of the consignment stock policy.

Sarker and Diponegoro (2009) addressed an optimal policy for production and shipment in a supply chain system consisting of multiple suppliers, one manufacturer and multiple buyers. In contrast to the common cycle mechanism, the procurement and ordering policy is not a periodic process in the whole planning horizon since the production cycles can be of different lengths and the buyers do not have to follow a common order cycle. Therefore, the system cost was reduced due to the increased system flexibility.

One of the integrated inventory models which considered distributors as an echelon was analyzed by Wee and Yang (2004). The authors assumed replenishment cycles at each echelon are smaller than those of the next upstream echelon. Pourakbar et al. (2007) incorporated a distributor echelon in a four-stage system. In their paper, three possible situations of the relationship between supplier's and producers' replenishment cycles are considered. The cycle time of supplier's is not restricted to be an integer multiple of the next down-stream stage but can be a fraction of it.

In a three-level supply chain consisting of one supplier, one manufacturer and one retailer, Seifert et al. (2012) studied two kinds of sub-supply chain coordination, i.e., upstream coordination and downstream coordination, where the companies at the first two or last two echelons are coordinated and the third company acts alone. Jonrinaldi and Zhang (2013) considered a semi-centralized supply chain system with five echelons. In this system, the first two echelons are decentralized while the last three echelons are coordinated.

In some integrated three-stage supply chain models, shortages are allowed at the last stage and as backorders. Sajadieh et al. (2013) investigated a general supply chain including multiple-supplier, multi-manufacturer and multi-retailer. In this model, the retailers' lead times are stochastic and shortage is allowed. Backorder costs were also incorporated in the system cost in Sarker et al. (2014). The authors assumed that the demand rate and production rate are finite and constant and the deliveries between the stages are instantaneous. Another allowing shortage model was proposed by Guo and Li (2014). The model integrated supply-selection and inventory control as a three-layer system.

Chen and Chen (2005) investigated the multi-item problem for one manufacturer supplying a class of products to one retailer and the products belong to the same category and share the same production facility. Kim et al. (2006) also studied a three-layer supply chain involving multiple items, which were produced on a single facility. Thus, in addition to the cycle time of each participant, the production sequence of multiple items also need to be determined to minimize the total cost.

Jaber and Goyal (2008) analyzed a three-layer system consisting of multiple suppliers, one manufacturer and multiple non-identical retailers. The authors assumed that each supplier supplies multiple components to the manufacturer and a common ordering cycle is adopted by all buyers. A related model with the same chain structure of Jaber and Goyal (2008) was studied by Pal et al. (2012), where each supplier supplies only one type of raw material to the manufacturer. This model was further investigated by Raj et al. (2015), who assumed that each retailer has an exponential demand rate. An extension of Pal et al. (2012) was investigated by Sana et al. (2014), who considered multiple participants at each echelon and integrated product quality into the model.

Seliaman et al. (2009) proposed a general integrated n-echelon supply chain model consisting of one supplier and multiple members at each echelon. The paper assumed that the members at the same echelon share a common cycle time and the cycle time of each echelon adopts the integer multiplier policy. Another integrated n-echelon system was studied by Kim and Glock (2013), where different shipment policies between the stages, i.e., equal- and unequal- batches, were investigated.

2.3 Location-Inventory Problem

A location-inventory problem (LIP) integrates the supply chain strategic design decisions with tactical and operational management decisions so as to minimize the total supply chain cost from a systemic view.

Some basic location-inventory models have been studied from different perspectives. Eppen (1979) was the first one who demonstrated that the inventory costs in a decentralized location-inventory system exceeds those in a centralized system. Nozick and Turnquist (1998) discussed how to model safety stock requirements across a set of distribution centres (DCs) and how to integrate the safety stock cost into a location model. Barahona and Jensen (1998) proposed an accelerated Dantzig-Wolfe decomposition to solve the location model, where the inventory costs at the warehouses were explicitly incorporated. Teo et al. (2001) analyzed the impact on the inventory cost when several DCs were consolidated into a central one, but the shipment cost from the DCs to retailers was ignored in this model. Sourirajan et al. (2007) studied a two-stage supply chain network design problem, in which trade-offs between lead time and safety stock were considered. Yang et al. (2010) evaluated the effects of location decisions on the net profit of a vendor-managed inventory system. Ağralı et al. (2012) analyzed the cost of single-sourcing strategy in an uncapacitated

LIP with safety stock costs. Tancrez et al. (2012) introduced some new characteristics to the LIP by assuming multiple sourcing at each supply chain layer and direct shipments between factories and customers were allowed.

Extensions of the basic LIP with the replenishment taken place at the warehouse have also been proposed in the literature. Jayaraman (1998) analyzed the interdependence among the location decision, inventory policy and shipment policy in an integrated LIP. In their work, shipment frequency was first incorporated into an LIP and the inventory was modeled at the first two layers of the supply chain rather than the warehouse layer only. Both Miranda and Garrido (2008) and Silva and Gao (2013) proposed a joint location-distribution-inventory model with an object of determining the warehouse location, retailers' assignment and replenishment decisions of each warehouse. A power-of-two policy was implemented to coordinate replenishment intervals in ÜSter et al. (2008). However, the objective function of the LIP in ÜSter et al. (2008) did not include the warehouse location cost since the number of open warehouse was fixed as one and only the location of the warehouse was determined. Another coordinated LIP was proposed by Berman et al. (2012), who adopted periodic-review and order-up-to-level policies to control the inventories at DCs.

In contrast to the transportation costs incorporated in the above LIPs, which were all in a linear form, several models for location-inventory-routing problem (LIRP) have been proposed, where the shipment between the DCs and customers was modeled by a vehicle routing problem. Shen and Qi (2007) proposed a LIP model with the consideration of vehicle routing problem, however, the optimal routing cost in Shen and Qi (2007) was calculated by an approximation rather than exact routing.

Most of the LIRPs presented in the literature, where vehicle routes were determined exactly, were decomposed into subproblems during the search of optimal solution. Liu and Lee (2003) proposed a two-phase heuristic method for the LIRP, solving the decomposed routing-inventory problem and location problem sequentially to obtain an initial solution for the LIRP. Then a heuristic search was applied to find the set of open warehouses based on the initial solution. This two-phase heuristic was further improved by Liu and Lin (2005). The heuristic search in the second phase was decomposed into a location problem and a routing-inventory problem. Each of the sub-problem was solved iteratively by a hybrid heuristic combining tabu search (TS) with simulated annealing (SA) sharing the same tabu list. To solve an LIRP incorporating inventory costs in the first two layers, Ma and Davidrajuh (2005) proposed an iterative algorithm, which alternated between the location problem solved by a mixed integer programming and the routing-inventory problem solved by genetic algorithm and probability theory.

Javid and Azad (2010) studied an LIRP with the consideration of DCs' capacity levels. This made the problem more realistic and increases the capacity utilization of DCs to a high level. This extended model was solved by an iterative approach applied to the subproblems at strategic level and tactical level. Each subproblem was solved by a hybrid of SA and TS.

Guerrero et al. (2013) proposed an LIRP which assumed the inventory was managed at the last two layers of the supply chain. In addition to the general decision variables in LIRP, the quantities shipped from suppliers to depots and from depots to retailers per period were also determined. The authors decomposed the LIRP into two subproblems, i.e. the distribution problem and the routing problem, and proposed a hybrid approach to solve the LIRP, using exact methods to estimate the distribution cost and heuristic procedures to make the remaining routing decisions. Each subproblem is globally optimized by an iterative algorithm alternating between the two parts of decisions and the information sharing within each subproblems.

In the literature, there is little work which included production process in an LIP. Vidyarthi et al. (2007) proposed one, however, the production-related cost in the objective function of this model was calculated by a linear function of the produced quantity in the plant. In other words, the model does not consider all the production-related costs incurred at the plant, i.e. setup cost, holding cost, which are usually considered in a general production-inventory model.

2.4 Integrated Vehicle Routing and Delivery Man Problem

Since the Delivery Man Problem (DMP) is a variant of the Traveling Salesman Problem (TSP), the integrated vehicle routing and delivery man problem (VRDMP) can be treated as a two-tier vehicle routing problem, incorporating a distribution network of single or multiple depots, multiple parking-sites and multiple customers linked by the trips of a fleet of homogeneous vehicles and a number of delivery men assigned to the vehicles. In the first-level distribution, goods are transported from depot(s) to parking-sites by vehicles, while in second-level distribution, these goods are then delivered to customers by delivery men's multi-trips between the parking sites and customers' locations, which is known as 'last mile delivery problem'. The objective of this two-tier routing model is to determine the routes of the vehicles and the delivery men, and the number of delivery men assigned to each route so as to minimize the total relevant costs involved in the two levels whilst subject to the constraints of the model.

Pureza et al. (2012) proposed a model which incorporated the use of extra delivery men in a vehicle routing problem with time windows. The model was proposed for the distribution of tobacco and beverage in highly dense urban areas. Due to the heavy traffic and scarcity of parking slots, customers were predefined in clusters and each customer was served by a delivery man who walked from the vehicle's parking site for the cluster rather than served by a vehicle directly. This model consists of one depot, multiple customer clusters and a fleet of homogeneous capacitated vehicles with assigned delivery man not exceeding a maximum crew size. In this model, the routing decisions are only made in the first level of this system since the service time in each cluster is predetermined based on the number of delivery men assigned to the vehicle which serves this cluster. The objective function minimizes the fixed and variable costs of the required fleet and the cost of assigned delivery men. To obtain the minimum cost of this vehicle routing problem, the authors proposed two heuristics, i.e. a tabu search and an ant colony optimization. de Grancy and Reimann (2014) proposed a greedy randomized adaptive search procedure (GRASP) metaheuristic for the above vehicle routing problem and compared the results with that obtained from an ant colony optimization (ACO).

Some extensions have been made to the model of Pureza et al. (2012). de Grancy and Reimann (2015) introduced two cluster construction heuristics for the customers instead of the predefind clustering in Pureza et al. (2012). Ferreira and Pureza (2012) extended the model of Pureza et al. (2012) by adding the number of unserved clusters to the original objective function and developed a saving heuristic which iteratively performs an incremental search in the number of delivery men in each route.

Most of the literature on the integrated vehicle routing and delivery man problems did not consider the routing of the delivery men in customer clusters.

Chapter 3

Integrated Single-Vendor Multi-Buyer Synchronization Supply Chain Model with Buyer-Clustering

3.1 Introduction

With the growing focus on supply chain management, companies realize that besides competition, there is also opportunity for collaboration. The inventories across the entire supply chain can be more efficiently managed through greater cooperation and better coordination. As a consequence, companies are pushed not only towards integrating different decision processes within their operational borders, but also towards closer collaboration with their customers and suppliers. Restricting all buyers to have ordering cycles smaller than or equal to the vendor's production cycle is a common characteristic of all the co-ordination models (e.g. common cycle model, power-of-two model and synchronized cycles model). A synchronization model through the coordination of the delivery order cycles of the buyers with the production cycles of the vendor was developed by Chan and Kingsman (2007). The synchronization of the supply chain in this model is achieved by allowing the buyers to select their own ordering cycles, although these ordering cycles must be integer multiples of a basic time period and integer factors of the vendor's production cycle. Intuitively, if a model can permit buyers to have their cycles larger than the vendor's production cycle, the system's flexibility can be increased and hence the total system cost can be improved.

The purpose of this research is to develop a synchronization supply chain model involving a buyer-clustering mechanism, in which the ordering cycles adopted by buyers can either be smaller or larger than the vendor's production cycle so as to further reduce the coordinated system cost. As mentioned in the abstract, the collaboration mechanism of this new model is as follows:

(i) The buyers are classified either as short-cycle preferred (SCP) buyers or long-cycle preferred (LCP) buyers according to their optimal delivery periods obtained in the well-known economic order quantity (EOQ) model, i.e. independent optimization.

(ii) The delivery cycles of both groups of buyers are coordinated with the vendors production cycle such that the delivery periods of the SCP buyers and the LCP buyers must be integer factors and integer multiples, respectively, of the vendor's production periods. Two approaches, namely, the incremental-clustering approach and economic-cycle-clustering approach, are proposed for finding the optimal number of buyers in each group.

The organization of the rest of this chapter is as follows: Section 3.2 provides the assumptions and notations applied in this chapter. Section 3.3 and Section 3.4 provides a brief introduction of the economic order quantity model and the synchronized cycles model, respectively. Section 3.5 proposes a clustering synchronized cycle model with long and short ordering cycles. Section 3.6 and Section 3.7 discuss algorithms for finding the total minimum system cost of our model. Section 3.8 presents the numerical results. Conclusions are given in Section 3.9.

3.2 Assumptions and Notations

3.2.1 Assumptions

Throughout this research, the following assumptions are made.

- Constant and deterministic demand rate.
- Constant production rate larger than demand rate.
- Constant cost parameters.
- Finite planning horizon.
- No shortage allowed throughout the whole planning horizon.
- No lead time for each order delivery.
- No parallel production allowed.

3.2.2 Notations

Basic parameters:

n: The number of buyers.

d_i: Demand rate of buyer i, for $i = 1, \ldots, n$.

A_i: Ordering cost per order of buyer i, for $i = 1, \ldots, n$.

h_i: Holding cost per item per unit time of buyer i, for $i = 1, \ldots, n$.

C_i: Shipping cost per delivery to buyer i, for $i = 1, \ldots, n$.

P: Vendor's production rate.

S_v: Vendor's set-up cost per production.

h: Vendor's holding cost per item per unit time.

D: Total demand rate faced by the vendor.

α: Demand-Production ratio (DP ratio).

Independent policy:

T_i^*: Economic order cycle of buyer i, for $i = 1, \ldots, n$.

T_v^*: Economic production cycle for vendor.

B_i^{IND}: Minimum cost per unit time for buyer i, for $i = 1, \ldots, n$.

V^{IND}: Minimum cost per unit time for the vendor.

TC^{IND}: Total system cost per unit time.

β: Economic cycle ratio, $\beta = nT_v^* / \sum_i T_i^*$.

Synchronized cycles model:

T^{SYN}: Basic cycle time.

NT^{SYN}: Vendor's production cycle.

$k_i T^{SYN}$: Ordering cycle of buyer i, for $i = 1, \ldots, n$.

B_i^{SYN}: Total cost per unit time for buyer i, for $i = 1, \ldots, n$.

TC^{SYN}: Total system cost per unit time.

Clustering Synchronized cycles model:

SCP: Short-cycle preferred buyer group.

LCP: Long-cycle preferred buyer group.

n^s: Number of buyers in SCP group.

n^l: Number of buyers in LCP group.

d_i^s: Demand rate of ith SCP buyer, for $i = 1, \ldots, n^s$.

d_i^l: Demand rate of ith LCP buyer, for $i = 1, \ldots, n^l$.

A_i^s: Ordering cost per order of ith SCP buyer, for $i = 1, \ldots, n^s$.

A_i^l: Ordering cost per order of ith LCP buyer, for $i = 1, \ldots, n^l$.

h_i^s: Holding cost per item per unit time of ith SCP buyer, for $i = 1, \ldots, n^s$.

h_i^l: Holding cost per item per unit time of ith LCP buyer, for $i = 1, \ldots, n^l$.

C_i^s: Shipping cost per delivery to ith SCP buyer, for $i = 1, \ldots, n^s$.

C_i^l: Shipping cost per delivery to ith LCP buyer, for $i = 1, \ldots, n^l$.

D^s: Total demand rate of all SCP buyers faced by the vendor.

D^l: Total demand rate of all LCP buyers faced by the vendor.

T: Vendor's production cycle.

FT: Production running time to satisfy all SCP buyers demand in each production run.

MT: Whole planning horizon.

T/K_i: Ordering cycle of ith SCP buyer, for $i = 1, \ldots, n^s$.

γ_i: The first ordering time of ith SCP buyer, for $i = 1, \ldots, n^s$.

$k_i T$: Ordering cycle of ith LCP buyer, for $i = 1, \ldots, n^l$.

τ_i: The first ordering time of ith LCP buyer, for $i = 1, \ldots, n^l$.

OTS_i: Ordering time set of ith SCP buyer in the whole planning horizon, for $i = 1, \ldots, n^s$.

OTL_i: Ordering time set of ith LCP buyer in the whole planning horizon, for

$i = 1, \ldots, n^l$.

$Q_r(t)$: Quantity of goods produced in period $[(r-1)T, (r-1)T+t]$, for $r = 1, \ldots, M$ and $t \in (0, T]$.

q_i^s: Quantity of goods per order of ith SCP buyer, for $i = 1, \ldots, n^s$.

q_i^l: Quantity of goods per order of ith LCP buyer, for $i = 1, \ldots, n^l$.

$\hat{q}_{r,s}(t)$: Accumulated ordering quantity from all the SCP buyers in period $[(r-1)T, (r-1)T+t]$, for $r = 1, \ldots, M$ and $t \in (0, T]$.

$\hat{q}_{r,l}(t)$: Accumulated ordering quantity from all the LCP buyers in period $[(r-1)T, (r-1)T+t]$, for $r = 1, \ldots, M$ and $t \in (0, T]$.

$\hat{q}_r(t)$: Accumulated ordering quantity from all the buyers in period $[(r-1)T, (r-1)T+t]$, for $r = 1, \ldots, M$ and $t \in (0, T]$.

$short_r(t)$: Shortage at time $(r-1)T+t$ for $r = 1, \ldots, M$ and $t \in (0, T]$.

χ_r: Maximal shortage quantity in rth T, for $r = 1, \ldots, M$.

$\bar{D}_r^l$: Total demand from all LCP buyers in rth T, for $r = 1, \ldots, M$.

$\delta_{i,r}$: Binary variable, $\delta_{i,r} = 1$ indicates that ith LCP buyer orders in rth T; otherwise, $\delta_{i,r} = 0$, for $i = 1, \ldots, n^l$ and $r = 1, \ldots, M$.

$tstart_r$: Start time of rth production run, for $r = 1, \ldots, M$.

$tfinish_r$: Finish time of rth production run, for $r = 1, \ldots, M$.

TC^{CLU}: Total system cost per unit time.

3.3 The Independent Policy Model

During each vendor's production cycle, buyer i ($i = 1, \ldots, n$) faces a deterministic demand at d_i per unit time, incurs an ordering cost A_i per order and incurs an inventory holding cost h_i per unit item per unit time. If the buyers and the vendor operate independently, then buyer i will order a quantity Q_i every T_i units of time. The total cost per unit time for the buyer i, denoted by B_i, is as follows:

$$B_i = \frac{A_i}{T_i} + \frac{h_i d_i T_i}{2}. \tag{3.1}$$

The economic order interval and economic order quantity for buyer i, denoted by T_i^* and Q_i^* respectively, are

$$T_i^* = \sqrt{\frac{2A_i}{h_i d_i}} \tag{3.2}$$

and

$$Q_i^* = d_i T_i^* = \sqrt{\frac{2A_i d_i}{h_i}}. \tag{3.3}$$

The minimum total cost per unit time for buyer i, denoted by B_i^{IND}, is

$$B_i^{IND} = \sqrt{2A_i h_i d_i}. \tag{3.4}$$

Since the vendor faces orders from the n buyers with demand rates $d_1,\ d_2,\ \ldots,\ d_n$ per unit time respectively, the vendor has to satisfy a demand that occurs at an average rate of D per unit time, where

$$D = d_1 + d_2 + \ldots + d_n. \tag{3.5}$$

We assume that the vendor's production rate is P, where $P > D$. We also assume that the vendor incurs a set-up cost of S_v per production run, incurs a holding cost of h per unit item per unit time and an order processing and shipment cost of C_i per order received from buyer i $(i = 1, \ldots, n)$. If the vendor operates independently, he needs to carry a large safety stock to guarantee that there will not be any stock outs. The maximum demand may occur when all buyers order their goods at the same time. So, the vendor has to carry $\sum_{i=1}^{n} Q_i^*$ items as buffer stock. When the vendor's batch quantity is Q_v, the total cost per unit time for the vendor, denoted by $V^{IND}(Q_v)$, is

$$V^{IND}(Q_v) = \frac{S_v D}{Q_v} + \frac{hQ_v}{2}(1 - \frac{D}{P}) + \sum_{i=1}^{n} \frac{C_i d_i}{Q_i^*} + h\sum_{i=1}^{n} Q_i^*. \tag{3.6}$$

The vendor's economic batch quantity and economic production cycle, denoted by Q_v^* and T_v^* respectively, are

$$Q_v^* = \sqrt{\frac{2S_v D}{h(1 - \frac{D}{P})}}. \tag{3.7}$$

and

$$T_v^* = \sqrt{\frac{2S_v}{hD(1 - \frac{D}{P})}}. \tag{3.8}$$

When the order cycles and the order quantities for the vendor and the buyers are optimal, the total system cost for the independent policy, denoted by TC^{IND}, is

$$TC^{IND} = \sqrt{2S_v hD(1 - \frac{D}{P})} + \sum_{i=1}^{n} \left[\frac{(2A_i h_i + 2A_i h + C_i h_i)\sqrt{d_i}}{\sqrt{2A_i h_i}} \right]. \tag{3.9}$$

3.4 The Synchronized Cycles Model

By coordinating the delivery times of the buyers with the production cycle of the vendor, the vendor's safety stock can be reduced. Banerjee and Burton (1994) proposed that a common order cycle T^{COM} be adopted by all the buyers and the production cycle of the vendor be an integer multiple of T^{COM}. This system lacks some flexibility, especially when there are significant differences between the individual demand rates of the buyers, since low-demand buyers want to replenish more frequently than high-demand buyers.

Chan and Kingsman (2007) proposed a more general synchronized model which allows the buyers to choose their own lot sizes and order cycles. In this synchronized cycle model, given a basic cycle time T^{SYN}, the vendor's production cycle is an integer multiple of T^{SYN}, say NT^{SYN}. The ordering cycle of buyer i assumes the form k_iT^{SYN}, where k_i is an integer factor of N. For simplicity, they assume that the deliveries of goods to the buyers are instantaneous. More precisely, each buyer's orders are received and deducted from the vendor's inventory every k_iT^{SYN} units of time. As a consequence, there will be a set of demands $D_1,\ D_2,\ \ldots,\ D_N$ over every vendor's production cycle, where each demand is the summation of some buyers's order quantities. The vendor needs to determine the values of all the decision variables, namely, $N,\ k_1,\ k_2,\ \ldots,\ k_n$, so that the total system cost can be minimized, subject to the condition that all the demands are satisfied throughout his production cycle NT^{SYN}.

Under this synchronized cycles model that the buyer i orders every k_iT^{SYN} units of time, the total cost of buyer i, denoted by B_i^{SYN}, is

$$B_i^{SYN} = \frac{A_i}{k_iT^{SYN}} + \frac{h_id_ik_iT^{SYN}}{2}. \tag{3.10}$$

Let S_v denote the vendor's set up cost per production run, h the vendor's holding cost per item per unit time and C_i the vendor's processing and shipment cost per order received from buyer i ($i = 1, \ldots, n$). Then the vendor's set up cost and order processing and shipment cost per unit time can be expressed as

$$\frac{S_v}{NT^{SYN}} \tag{3.11}$$

and

$$\sum_{i=1}^{n} \frac{C_i}{k_i T^{SYN}} \tag{3.12}$$

respectively.

If every buyer places orders as early as possible in each of their order cycles, then the vendor's holding cost per unit time can be expressed as

$$\left[\frac{hD}{2} - \frac{hD^2}{2P}\right] NT^{SYN} + \sum_{i=1}^{n} d_i \left[\frac{h}{P} D - \frac{1}{2} h\right] k_i T^{SYN}. \tag{3.13}$$

Then the total relevant cost, denoted by TC^{SYN}, can be written as

$$\begin{aligned} TC^{SYN} =& \frac{S_v}{NT^{SYN}} + \left[\frac{hD}{2} - \frac{hD^2}{2P}\right] NT^{SYN} \\ &+ \sum_{i=1}^{n} \left\{ \frac{C_i + A_i}{k_i T^{SYN}} + d_i \left[\frac{h}{P} D - \frac{1}{2}(h - h_i)\right] k_i T^{SYN} \right\}. \end{aligned} \tag{3.14}$$

3.5 Clustering Synchronized Cycles Model

The numerical examples in Chan and Kingsman (2007) showed that the synchronized cycles model leads to a significant reduction in total system cost when compared to the situation that each buyer operates independently. However, this model assumes, as others, that each buyer's ordering cycle must be an integer factor of the vendor's cycle, i.e., the buyer's ordering cycle length must be less than or equal to that of the vendor's production cycle. Intuitively, if this constraint can be relaxed, the total system cost can be further reduced.

Following this line of thought, we develop a new synchronized cycles model in this section. In this model, the n buyers are divided into two groups, the 'long-cycle preferred' (LCP) buyers and the 'short-cycle preferred' (SCP) buyers, in accordance with their economic ordering cycles T^*. More precisely, we first assume that there are n^s buyers in the SCP group and n^l buyers in the LCP group, where

$$n^s + n^l = n \quad (n^s \geqslant 1,\ n^l \geqslant 1) \tag{3.15}$$

and n^s is an integer variable. Thus, the first n^s buyers with smaller T^* are classified as SCP buyers and the remaining n^l buyers with larger T^* are classified as LCP buyers. (Ties can be broken arbitrarily.)

The demand rates of an LCP buyer and an SCP buyer are denoted by d_i^l and d_i^s, respectively. Thus the demand rates of all the LCP buyers and all the SCP buyers, denoted by D^l and D^s, respectively, are

$$D^l = \sum_{i=1}^{n^l} d_i^l \tag{3.16}$$

and

$$D^s = \sum_{i=1}^{n^s} d_i^s. \tag{3.17}$$

The inventory holding cost per unit item per unit time for the LCP buyers and the SCP buyers are denoted by h_i^l and h_i^s respectively. The ordering cost per order for an LCP buyer and an SCP buyer are denoted by A_i^l and A_i^s, respectively. For each order received from an LCP buyer (respectively from an SCP buyer), the vendor's order processing and shipment cost is denoted by C_i^l (respectively C_i^s).

For this new synchronized cycles model, the vendor's production cycle T is an integer. The ordering cycle of buyer i from the SCP group assumes the form $\frac{T}{K_i}$, where K_i is an integer factor of T. Hence, buyer i from the SCP group orders the goods K_i times in one vendor's production cycle. The ordering cycle of buyer i from the LCP group assumes the form k_iT, where k_i is also an integer. Hence, buyer i from the LCP group orders goods once every k_iT units of time. The vendor's entire production horizon (consisting of M production cycles) is MT, where M is the least common multiple of $k_1, k_2, \ldots, k_{n^l}$. We assume that the maximum planning horizon is one year,

$$MT \leqslant 365. \tag{3.18}$$

Assume that at the beginning of a new vendor's production cycle, all the demands from his previous production cycle have been satisfied. Also assume that the vendor's production rate is P items per unit time. Then in order to satisfy the demands of all the SCP buyers (and only the SCP buyers) in this production cycle, the production needs to run for FT units of time, where

$$PFT = D^sT \ \Rightarrow \ F = \frac{D^s}{P} \tag{3.19}$$

We now need to find out the number of items produced in each vendor's production cycle in order to satisfy the total demands of all the SCP buyers and the LCP buyers. In other words, in any production cycle $((r-1)T, rT]$, $(r = 1, \ldots, M)$, the vendor's stock, after having satisfied the demands of both the SCP buyers and the LCP buyers, becomes zero at the end of the cycle.

For this purpose, we need to allocate the individual buyer's ordering times in the whole planning horizon. Assume that in any vendor's production cycle $((r-1)T, rT]$, $(r = 1, \ldots, M)$, the first ordering time of buyer i in the SCP group is $\gamma_i + (r-1)T$, (i.e. at γ_i units of time after the start of the production cycle) where γ_i is an integer such that $0 < \gamma_i \leqslant \frac{T}{K_i}$. Since buyer i orders goods once every $\frac{T}{K_i}$ units of time in each vendor's production cycle (and hence in the entire vendor's planning horizon), the ordering times of buyer i in the entire vendor's planning horizon can be completely specified by the vector OTS_i, where

$$OTS_i = \left[\gamma_i, \ldots, \gamma_i + \frac{T}{K_i}, \ldots, (M-1)T + \gamma_i, \ \ldots, \ (M-1)T + \gamma_i + \frac{(K_i - 1)T}{K_i}\right] \tag{3.20}$$

Let q_i^s be the ordering quantity at each ordering time of buyer i from the SCP group. Since buyer i from the SCP group orders goods K_i times in each vendor's production cycle, we have

$$q_i^s K_i = d_i^s T \ \Rightarrow \ q_i^s = \frac{d_i^s T}{K_i} \tag{3.21}$$

Assume that in the vendor's planning horizon $[0, MT]$, the first ordering time of buyer j in the LCP group is τ_j, where τ_j is an integer such that $0 < \tau_j \leqslant k_j T$. Since buyer j orders once every k_jT units of time, the ordering times of buyer j in the entire vendor's planning horizon can be completely specified by the vector OTL_j, where

$$OTL_j = [\tau_j, \tau_j + k_jT, \ldots, \tau_j + (M - k_j)T]. \tag{3.22}$$

Let q_j^l be the ordering quantity at each ordering time of buyer j from the LCP group. Also due to the fact that buyer j from the LCP group orders goods once in every k_jT units of time, we have

$$q_j^l = d_j^l \times k_jT \tag{3.23}$$

The total demand from all the LCP buyers in the period $((r-1)T, rT]$, $(r = 1, 2, \ldots, M)$, denoted by $\bar{D}_r^l$, can be expressed as follows:

$$\begin{aligned} \bar{D}_r^l &= \sum_{j=1}^{n^l} \delta_{j,r} q_j^l \\ &= \sum_{j=1}^{n^l} \delta_{j,r} d_j^l k_j T \end{aligned} \tag{3.24}$$

where

$$\delta_{j,r} = \begin{cases} 1, & \text{if buyer } j \text{ places an order in period } ((r-1)T, rT], \\ 0, & \text{otherwise.} \end{cases} \tag{3.25}$$

i.e.

$$\delta_{j,r} = \begin{cases} 1, & \text{if } \tau_j + \bar{n}k_jT \in ((r-1)T, rT] \text{ for some integer } \bar{n} \in \{0, \ldots, M-1\}, \\ 0, & \text{otherwise.} \end{cases} \tag{3.26}$$

For each production cycle $((r-1)T, rT]$, $(r = 1, 2, \ldots, M)$, the vendor must be able to produce sufficient stock in each production cycle to meet the demand of both the SCP and LCP buyers. Thus, we have

$$PFT + \bar{D}_r^l \leqslant PT \tag{3.27}$$

We now need to determine the optimal start time of the production run of the above production cycle. Let the optimal start time of the production run be denoted by $tstart_r$, $(r = 1, 2, \ldots, M)$. Then $tstart_r$ has to satisfy the following conditions:

(i) The production run can start at a time earlier than $(r-1)T$, but not later than $(r-1)T + 1$. That is,

$$tstart_r \leqslant (r-1)T + 1. \tag{3.28}$$

(ii) There should be no shortages of goods throughout the whole production cycle. The total amount of vendor's inventory in the production cycle should be kept at a minimum level, without incurring any shortage at any time during the entire production cycle.

(iii) The vendor cannot start a new production run when the previous production cycle has not finished.

In order to determine the start time of the production run to avoid shortage of goods and at the same time minimize the vendors total inventory (as stated in condition (ii)), we first need to find out the maximum amount of shortages that can occur if the production run starts at the very beginning of the production of the production cycle. Suppose that the production run starts at time $(r-1)T$. Since the vendor's production run is P items per unit time, in order to satisfy the demands of both the SCP and LCP buyers in the above production period, the production needs to run for $FT + \frac{\bar{D}_r^l}{P}$ units of time. Let $Q_r(t)$ $(t \in (0, T])$ be the amount of goods produced from time $(r-1)T$ to $(r-1)T + t$. Then

$$Q_r(t) = \begin{cases} Pt, & t \in (0, FT + \bar{D}_r^l/P] \\ PFT + \bar{D}_r^l, & t \in (FT + \bar{D}_r^l/P, T] \end{cases} . \tag{3.29}$$

Let $\hat{q}_{r,s}(t)$ be the accumulated ordering quantity from all the SCP buyers from time $(r-1)T$ to $(r-1)T + t$ inclusive. Since buyer i from the SCP group orders goods $\text{int}[\frac{t-\gamma_i}{T} K_i + 1]$ times over the period $((r-1)T, (r-1)T + t]$, we have

$$\begin{aligned} \hat{q}_{r,s}(t) &= \sum_{i=1}^{n^s} \text{int}\left[\frac{t-\gamma_i}{T} K_i + 1\right] \times q_i^s, \\ &= \sum_{i=1}^{n^s} \text{int}\left[\frac{t-\gamma_i}{T} K_i + 1\right] \times \frac{d_i^s T}{K_i}, \ t \in (0, T]. \end{aligned} \tag{3.30}$$

Let $\hat{q}_{r,l}(t)$ be the accumulated ordering quantity from all the LCP buyers from time $(r-1)T$ to $(r-1)T + t$ inclusive. To determine $\hat{q}_{r,l}(t)$, we use the following information derived from our model.

Buyer j from the LCP group orders goods once (and only once) in the period $((r-1)T, (r-1)T + t]$

$$\Leftrightarrow \delta_{j,r} = 1 \text{ and } t \geqslant \hat{\tau}_j, \text{ where } \hat{\tau}_j = \tau_j \mod T$$

From the above logical statement, we know that buyer j from the LCP group orders goods $\delta_{j,r} \times \text{int}\left[\frac{(t-\hat{\tau}_j)}{T} + 1\right]$ times over the period $((r-1)T, (r-1)T + t]$. Thus, we have

$$\begin{aligned}
\hat{q}_{r,l}(t) &= \sum_{j=1}^{n^l} \delta_{j,r} \times \text{int}\left[\frac{t-\hat{\tau}_j}{T}+1\right] \times q_j^l, \\
&= \sum_{j=1}^{n^l} \delta_{j,r} \times \text{int}\left[\frac{t-\hat{\tau}_j}{T}+1\right] \times d_j^l k_j T,\ t \in (0,T]. \qquad (3.31)
\end{aligned}$$

Let $\hat{q}_r(t)$ be the accumulated ordering quantity from all the buyers from time $(r-1)T$ to $(r-1)T+t$ inclusive. Then

$$\begin{aligned}
\hat{q}_r(t) &= \hat{q}_{r,s}(t) + \hat{q}_{r,l}(t) \\
&= \sum_{i=1}^{n^s} \text{int}\left[\frac{t-\gamma_i}{T}K_i+1\right] \times \frac{d_i^s T}{K_i} + \sum_{j=1}^{n^l} \delta_{j,r} \times \text{int}\left[\frac{t-\hat{\tau}_j}{T}+1\right] \times d_j^l k_j T.
\end{aligned} \qquad (3.32)$$

Let

$$short_r(t) = q_r(t) - Q_r(t). \qquad (3.33)$$

and

$$\chi_r = \max_{t \in (0,T]} short_r(t). \qquad (3.34)$$

Then $\chi_r \leqslant 0$ and $\chi_r \geqslant 0$ represent, respectively, the situation that there is no shortage of goods and the situation that there is no excessive storage of goods throughout the production period $((r-1)T, rT]$. Thus, in order to cancel out the maximum amount of shortage and at the same time minimize the vendor's total inventory, the vendor must start the production run at least $\frac{\chi_r}{P}$ units of time earlier than $(r-1)T$. In other words, the production run will start at a time $tstart_r$, where

$$tstart_r \leqslant (r-1)T - \frac{\chi_r}{P} \tag{3.35}$$

Thus, by combining (3.28) in condition (i) with (3.35), we get

$$tstart_r \leqslant \min\left[(r-1)T+1, (r-1)T - \frac{\chi_r}{P}\right]. \tag{3.36}$$

On the other hand, we must also take into consideration the fact that the vendor cannot start a new production run when the previous one has not finished (as stated in condition(iii)). From the previous discussion, we notice that for the production cycle $((r-1)T, rT]$, the production run will finish by the time $tfinish_r$, where

$$tfinish_r = tstart_r + FT + \frac{\bar{D}_r^l}{P}. \tag{3.37}$$

Thus, from (3.36) and (3.37), we have the following constraints

$$tstart_{r-1} + FT + \frac{\bar{D}_{r-1}^l}{P} \leqslant tstart_r \leqslant \min\left[(r-1)T+1, (r-1)T - \frac{\chi_r}{P}\right], \ r = 2, \ldots, M \tag{3.38}$$

and

$$tstart_1 \leqslant \min\left[1, -\frac{\chi_1}{P}\right]. \tag{3.39}$$

Furthermore, in order to minimize the vendor's total inventory, the starting time of any production cycle should be as late as possible. Thus, by setting

$$\begin{cases} tstart_M & = \min\left[(M-1)T+1, (M-1)T - \frac{\chi_M}{P}\right], \\ tstart_{M-1} & = \min\left[(M-2)T+1, (M-2)T - \frac{\chi_{M-1}}{P}, tstart_M - FT - \frac{\bar{D}^l_{M-1}}{P}\right], \\ & \dots \\ tstart_1 & = \min\left[1, -\frac{\chi_1}{P}, tstart_2 - FT - \frac{\bar{D}^l_1}{P}\right]. \end{cases} \tag{3.40}$$

both constraints (3.38) and (3.39) can be satisfied and at the same time, the vendor's inventory cost can be minimized.

The average stock held by the vendor is

$$\begin{aligned} &\frac{1}{MT}\left\{M\cdot\left(D^sT^2 - \frac{(D^s)^2}{2P}T^2 - \sum_{i=1}^{n^s}\sum_{u=1}^{K_i}(d_i^s\cdot\frac{T}{K_i})(T-\gamma_i-(u-1)\frac{T}{K_i})\right)\right. \\ &+\left(\sum_{r=1}^{M}T(1-F)\bar{D}^l_r - \frac{(\bar{D}^l_r)^2}{2P}\right) - \sum_{r=1}^{M}\sum_{j=1}^{n^l}\delta_{j,r}(d^l_j\cdot k_jT)(T-\hat{\tau}_j) \\ &\left.-\sum_{r=1}^{M}[(r-1)T - tstart_r](PFT+\bar{D}^l_r)\right\} \end{aligned} \tag{3.41}$$

where $tstart_r$ $(r = 1, \dots, M)$ is as defined in system (3.40).

Hence, the vendor's holding cost per unit time is given by

$$\begin{aligned} &\frac{h}{MT}\left\{M\cdot\left(D^sT^2 - \frac{(D^s)^2}{2P}T^2 - \sum_{i=1}^{n^s}\sum_{u=1}^{K_i}(d_i^s\cdot\frac{T}{K_i})(T-\gamma_i-(u-1)\frac{T}{K_i})\right)\right. \\ &+\left(\sum_{r=1}^{M}T(1-F)\bar{D}^l_r - \frac{(\bar{D}^l_r)^2}{2P}\right) - \sum_{r=1}^{M}\sum_{j=1}^{n^l}\delta_{j,r}(d^l_j\cdot k_jT)(T-\hat{\tau}_j) \\ &\left.-\sum_{r=1}^{M}[(r-1)T - tstart_r](PFT+\bar{D}^l_r)\right\}. \end{aligned} \tag{3.42}$$

The other relevant costs of our model are as follows:

$$\text{Vendor's setup cost per unit time} = \frac{S_v}{T}. \tag{3.43}$$

$$\text{Vendor's shipment cost per unit time} = \sum_{i=1}^{n^s} \frac{C_i^s K_i}{T} + \sum_{j=1}^{n^l} \frac{C_j^l}{k_j T}. \tag{3.44}$$

$$\text{Buyers' ordering cost per unit time} = \sum_{i=1}^{n^s} \frac{A_i^s K_i}{T} + \sum_{j=1}^{n^l} \frac{A_j^l}{k_j T}. \tag{3.45}$$

$$\text{Buyers' holding cost per unit time} = \frac{1}{2}\sum_{i=1}^{n^s} \frac{d_i^s h_i^s T}{K_i} + \frac{1}{2}\sum_{i=1}^{n^l} d_i^l h_i^l k_j T. \tag{3.46}$$

The total system cost per unit time of this clustering-synchronized cycles model, denoted by TC^{CLU}, is as follows:

$$\begin{aligned} TC^{CLU} = \quad & \frac{h}{T} \cdot \left(D^s T^2 - \frac{(D^s)^2}{2P} T^2 - \sum_{i=1}^{n^s} \sum_{u=1}^{K_i} (d_i^s \cdot \frac{T}{K_i})(T - \gamma_i - (u-1)\frac{T}{K_i}) \right) \\ & + \frac{h}{MT} \left\{ \left(\sum_{r=1}^{M} T(1-F)\bar{D}_r^l - \frac{(\bar{D}_r^l)^2}{2P} \right) - \sum_{r=1}^{M} \sum_{j=1}^{n^l} \delta_{j,r} (d_j^l \cdot k_j T)(T - \hat{\tau}_j) \right. \\ & \left. - \sum_{r=1}^{M} [(r-1)T - tstart_r](PFT + \bar{D}_r^l) \right\} + \frac{S_v}{T} + \sum_{i=1}^{n^s} \frac{C_i^s K_i}{T} + \sum_{j=1}^{n^l} \frac{C_j^l}{k_j T} \\ & + \sum_{i=1}^{n^s} \frac{A_i^s K_i}{T} + \sum_{j=1}^{n^l} \frac{A_j^l}{k_j T} + \frac{1}{2}\sum_{i=1}^{n^s} \frac{d_i^s h_i^s T}{K_i} + \frac{1}{2}\sum_{i=1}^{n^l} d_i^l h_i^l k_j T \end{aligned} \tag{3.47}$$

When the values of n^s, n^l have been determined, we can formulate our clustering-synchronized cycles model problem, denoted by **Problem CLU(n^l)**, as follows:

Subject to constraints (3.15), (3.18) and (3.27), find K_i $(i = 1, \ldots, n^s)$, γ_i $(i = 1, \ldots, n^s)$, k_i $(i = 1, \ldots, n^l)$, τ_i $(i = 1, \ldots, n^l)$ and T such that the total system cost given by (3.47) is minimized.

The above problem is a combinatorial optimization problem. Let the minimum total system cost of Problem CLU(n^l) (i.e. the minimum total system cost under the partition that there are n^l buyers in the LCP group) be denoted by $TC^{CLU,n^l,*}$, then the minimum total system cost of this clustering-synchronized cycles model, denoted by $TC^{CLU,*}$, is

$$TC^{CLU,*} = \min_{n^l \in \{1,\ldots,n-1\}} TC^{CLU,n^l,*} \tag{3.48}$$

3.6 Optimization of the Total System Cost of the Clustering Synchronized Cycles Model

This research proposes two clustering approaches for finding the optimal numbers of SCP and LCP buyers. The first approach is to perform an incremental searching to determine the number of LCP buyers n^l. The second approach is based on determining the number of LCP buyers n^l in advance (i.e. before the start of the optimization problem for minimizing the total system cost), which is based on each participants' T^* given by the EOQ model. Both of the two approaches are incorporated into Problem CLU(n^l). And the heuristic algorithms are presented in section 3.7.

3.6.1 Incremental-Clustering Approach

We first solve the problem of minimizing the total system cost of the model of Chan and Kingsman (2007) to obtain the optimal ordering cycles of all the buyers. All those buyers with optimal ordering cycles in that model equal to the vendor's production cycle are initially classified as LCP buyers, whilst all other buyers with optimal ordering cycles in that model less than the vendor's production cycle are initially classified as SCP buyers. If the ordering cycles of all the buyers are less than the vendor's production cycle, then the buyer with ordering cycle closest to that of the vendor's production cycle is initially classified as LCP buyers and the remaining buyers are classified as SCP buyers. (Ties can be broken arbitrarily.) Thus, we obtain the initial number of SCP buyers and LCP buyers. Let the initial number of LCP buyers be n^l and Algorithm 3.1 outlines the searching processing.

3.6.2 Economic-Cycle-Clustering Approach

In this approach, the number of LCP buyers n^l is predetermined based on each participants' T^* given by the EOQ model, i.e. buyer i is assigned to LCP group if his T^* is larger than T_v^* (if all the buyers' T^* are less than T_v^*, then the buyer with largest T^* is assigned to LCP group and the number of LCP buyers n^l is set as 1). Then solve Problem CLU(n^l), we have $TC_{best}^{CLU} = TC^{CLU,n^l}$.

3.7 Heuristics for Solving Problem CLU(n^l)

Three algorithms are presented for solving Problem CLU(n^l), including genetic algorithm (GA), simulated annealing (SA) and a hybrid of simulated annealing and genetic algorithm (SAGA).

3.7.1 Genetic Algorithm

Genetic algorithm (GA), introduced by Holland (1975), is one of the evolution algorithms mimics the mechanism of selection in biology nature. In GA, the decision variables of the optimization problem are encoded into a chromosome (string), and a solution pool is initialized by generating a number of chromosomes and this number

Algorithm 3.1: Incremental-Clustering Approach

1 ***Function*** Incremental-Clustering(n^l)

$n^l_{best} = n^l$

2 Solve Problem CLU(n^l_{best})

$TC^{CLU}_{best} = TC^{CLU,n^l_{best}}$

3 ***repeat***

4 Select neighborhood of n^l;

if $n^l = 1$ **then**

$\Omega(n^l) = \{2\}$;

elseif $n^l = n - 1$ **then**

$\Omega(n^l) = \{n-2\}$;

else

$\Omega(n^l) = \{n^l - 1, n^l + 1\}$.

5 Solve Problem CLU($\hat{n}^l$), $\hat{n}^l \in \Omega(n^l)$

$TC^{CLU,n^{l*}} = \min_{\hat{n}^l \in \Omega(n^l)} TC^{CLU,\hat{n}^l}$

6 Updating n^l_{best}, TC^{CLU}_{best} and n^l

if $TC^{CLU,n^{l*}} \leqslant TC^{CLU}_{best}$ **then**

$n^l_{best} = n^{l*}$;

$TC^{CLU}_{best} = TC^{CLU,n^{l*}}$;

$n^l = n^l_{best}$;

7 ***until*** one of the following conditions is met

$n^l_{best} = 1$

$n^l_{best} = n - 1$

$TC^{CLU,n^{l*}} > TC^{CLU}_{best}$

is called the population size. After the iteration process of selection, crossover and mutation, the population is evolved by exchanging the information on the alleles. A new chromosome may or may not outperforms the parents. The outlines of the genetic algorithm adopted in this chapter is shown in Algorithm 3.2.

Algorithm 3.2: Genetic Algorithm for Problem CLU(n^l)

Function (Genetic Algorithm)

Select an integer number for n^l, then $n^s = n - n^l$;

Population initialization: the length of each chromosome is $2n^s + 3n^l + 1$;

While stopping criteria is not achieved ***do***

  Crossover;

  Mutation;

  Updating population;

End While

The GA does not rely on analytic properties of the function to be optimized. The most important principles of GA are the re-iterativeness and randomness of the generated solutions. It makes GA widely suitable for finding near-optimal solutions with a reasonable computing time in many complex problems.

Representation of chromosome

Each chromosome represents a potential optimal solution, indicating the vendor's production cycle and buyers' delivery structures. A chromosome is of length $n^s * 2 + n^l * 3 + 1$, composed by K_i, γ_i, k_j, λ_j, η_j and T, such that $\tau_j = (\lambda_j - 1)T + \eta_j$, $\lambda_j \in \mathbb{Z}^+$, $\eta_j \in \mathbb{Z}^+$ and $\eta_j \leqslant T$ $(i = 1, \ldots, n^s, j = 1, \ldots, n^l)$.

In this optimization algorithm, the values of genes in segment I and II which have the same length of n^s represent the ordering time of SCP buyer. The segment III-V, Chromosome

	K_1	K_2	K_3	γ_1	γ_2	γ_3	k_1	k_2	λ_1	λ_2	η_1	η_2	T
Value	2	2	2	2	11	6	2	1	1	1	32	11	60
Gene location	1	2	3	4	5	6	7	8	9	10	11	12	13
Segment		I			II		III		IV		V		VI

Figure 3.1: A sample of chromosome structure of 3 SCP buyers and 2 LCP buyers

where the values represents the ordering time of LCP buyer, have the same length of n^l. And the last segment which includes the last gene is the value of vendor's production cycle. For example, in Figure 3.1, the vendor sets up a production run in every 60 time units. The whole planning horizon is 120 time units since the least common multiple of k_is is 2. The delivery time of the first SCP buyer in the first basic cycle can be calculated by the T value and the values of the genes in location 1 and 4, so the deliveries for this buyer are at time 2 and 32. Similarly, the delivery time for SCP buyer 2 is time 11 and 41, and time 6 and 36 for the third SCP buyer. The first LCP buyer's ordering time is determined by T and the values of genes in location 7, 9 and 11, so his ordering time in the whole planning horizon is $(\lambda_1 - 1)T + \eta_1 = 32$ which is the only delivery in the whole planning horizon. And the ordering time for LCP buyer 2 is time $(\lambda_2 - 1)T + \eta_2 = 11$ and next delivery is at time $11 + k_1T = 71$ which is in the second production cycle.

Population initialization

The population size in this research is 20. In the beginning of the optimization, the chromosome will be created arbitrarily, which implies that the delivery structure of each buyer will be randomly determined. Each chromosome will be subject to the constraints defined. An invalid chromosome is not allowed. In our algorithm, the genes are initialized in the following way:

T: a random integer, $T \in (1, 365]$.

K_i: a random integer factor of T , $i = 1, 2, \ldots, n^s$.

γ_i: a random real number, $\gamma_i \in (0, T/K_i]$, $i = 1, 2, \ldots, n^s$.

k_j: a random integer number, $k_j \in [1, \mathrm{bound}]$, the bound is given to satisfy the constraints $D_r \leqslant PT - PFT$ and $MT \leqslant 365$, $j = 1, 2, \ldots, n^l$.

λ_j: a random integer number, $\lambda_j \in [1, k_j]$, $j = 1, 2, \ldots, n^l$.

η_j: a random integer, $\eta_j \in [1, T]$, $j = 1, 2, \ldots, n^l$.

Fitness value

Fitness value defines the relative strength of a chromosome. It evaluates the chromosome structure and returns a value. In a minimization problem, the smaller the fitness value is, the stronger and more desirable the chromosome is. The fitness value is evaluated by an equation or equations. This equation of this research is the total system cost, calculated by Eq.(3.47) and a detailed demonstration for the calculation is shown in Appendix A.

Crossover

After selecting two parent chromosomes, a random number from 0 to 1 is generated to determine whether the crossover is conducted.

				Segment I					Segment II		
	Parent A	K_1^A	$\dots$	K_i^A	$\dots$	$K_{n^s}^A$	γ_1^A	$\dots$	γ_i^A	$\dots$	$\gamma_{n^s}^A$
①	Parent B	K_1^B	$\dots$	K_i^B	$\dots$	$K_{n^s}^B$	γ_1^B	$\dots$	γ_i^B	$\dots$	$\gamma_{n^s}^B$
				$\uparrow$					$\uparrow$		
	SCP Buyer			i					i		

$$\Downarrow$$

				Segment I					Segment II		
	Child A	K_1^A	$\dots$	$\boxed{K_i^B}$	$\dots$	$K_{n^s}^A$	γ_1^A	$\dots$	$\boxed{\gamma_i^B}$	$\dots$	$\gamma_{n^s}^A$
②	Child B	K_1^B	$\dots$	$\boxed{K_i^A}$	$\dots$	$K_{n^s}^B$	γ_1^B	$\dots$	$\boxed{\gamma_i^A}$	$\dots$	$\gamma_{n^s}^B$
				$\uparrow$					$\uparrow$		
	SCP Buyer			i					i		

Figure 3.2: Crossover for segments I and II when parents have same values of T

(i) Segment I, II and VI

- Two parent chromosomes have the same values of T

  Crossover is conducted at 2 random selected locations at most. The original strings are shown in Figure 3.2 ①. If buyer i's K_i is selected to crossover first, then the corresponding γ_i's are interchanged, and vice versa, as shown in Figure 3.2 ②. And the swap number is 2, so the crossover for segments I and II is completed.

- Two parent chromosomes have the different values of T

  In this case, all the values at segments I and VI are swapped, as shown in Figure 3.3 ②. Then check the feasibility of the two child chromosomes. The crossover process at segment I, II and VI is stopped at Figure 3.3 ②

		Segment I					Segment II					Segment VI
	Parent A	K_1^A	$\dots$	K_i^A	$\dots$	$K_{n^s}^A$	γ_1^A	$\dots$	γ_i^A	$\dots$	$\gamma_{n^s}^A$	T^A
①	Parent B	K_1^B	$\dots$	K_i^B	$\dots$	$K_{n^s}^B$	γ_1^B	$\dots$	γ_i^B	$\dots$	$\gamma_{n^s}^B$	T^B
				$\uparrow$					$\uparrow$			
	SCP Buyer			i					i			

$\Downarrow$

		Segment I					Segment II					Segment VI
	Child A	$\boxed{K_1^B}$	$\boxed{\dots}$	$\boxed{K_i^B}$	$\boxed{\dots}$	$\boxed{K_{n^s}^B}$	γ_1^A	$\dots$	γ_i^A	$\dots$	$\gamma_{n^s}^A$	$\boxed{T^B}$
②	Child B	$\boxed{K_1^A}$	$\boxed{\dots}$	$\boxed{K_i^A}$	$\boxed{\dots}$	$\boxed{K_{n^s}^A}$	γ_1^B	$\dots$	γ_i^B	$\dots$	$\gamma_{n^s}^B$	$\boxed{T^A}$
				$\uparrow$					$\uparrow$			
	SCP Buyer			i					i			

$\Downarrow$

		Segment I					Segment II					Segment VI
	Child A	$\boxed{K_1^B}$	$\boxed{\dots}$	$\boxed{K_i^B}$	$\boxed{\dots}$	$\boxed{K_{n^s}^B}$	γ_1^A	$\dots$	$\boxed{\gamma_i^B}$	$\dots$	$\gamma_{n^s}^A$	$\boxed{T^B}$
③	Child B	$\boxed{K_1^A}$	$\boxed{\dots}$	$\boxed{K_i^A}$	$\boxed{\dots}$	$\boxed{K_{n^s}^A}$	γ_1^B	$\dots$	$\boxed{\gamma_i^B}$	$\dots$	$\gamma_{n^s}^A$	$\boxed{T^A}$
				$\uparrow$					$\uparrow$			
	SCP Buyer			i					i			

Figure 3.3: Crossover for segments I, II and VI when parents have different values of T

if all the $K_i\gamma_i \leqslant T$ in children A and B, otherwise go to Figure 3.3 ③, i.e, if the constraint cannot be satisfied at buyer i, then the γs at buyer i are swapped.

(ii) Segment III and IV

For segments III and IV, crossover is conducted at 2 randomly selected locations at most. A candidate list is initialized as $(1, \dots, n^l)$. In Figure 3.4 ②, buyer i is selected to swap his ks, then i is removed from the candidate list. Then check the feasibility of children A and B of Figure 3.4 ②, (a) constraint $MT \leqslant 365$ or $PFT + \bar{D}_r^l \leqslant PT$ is not satisfied in either of the chromosomes, go back to Figure 3.4 ① and choose another buyer's ks for swap, i.e. buyer j, (b) only constraint $\lambda_i \leqslant k_i$ is not satisfied, then go to Figure 3.4 ③, exchange corresponding λs,

		Segment III							Segment IV						
	Parent A	k_1^A	$\dots$	k_i^A	$\dots$	k_j^A	$\dots$	$k_{n^l}^A$	λ_1^A	$\dots$	λ_i^A	$\dots$	λ_j^A	$\dots$	$\gamma_{n^l}^A$
①	Parent B	k_1^B	$\dots$	k_i^B	$\dots$	k_j^B	$\dots$	$k_{n^l}^B$	λ_1^B	$\dots$	λ_i^B	$\dots$	λ_j^B	$\dots$	$\gamma_{n^l}^B$
				$\uparrow$		$\uparrow$					$\uparrow$		$\uparrow$		
	LCP Buyer			i		j					i		j		

$\Downarrow$

		Segment III							Segment IV						
	Child A	k_1^A	$\dots$	$\boxed{k_i^B}$	$\dots$	k_j^A	$\dots$	$k_{n^l}^A$	λ_1^A	$\dots$	λ_i^A	$\dots$	λ_j^A	$\dots$	$\lambda_{n^l}^A$
②	Child B	k_1^B	$\dots$	$\boxed{k_i^A}$	$\dots$	k_j^B	$\dots$	$k_{n^l}^B$	λ_1^B	$\dots$	λ_i^B	$\dots$	λ_j^B	$\dots$	$\lambda_{n^l}^B$
				$\uparrow$		$\uparrow$					$\uparrow$		$\uparrow$		
	LCP Buyer			i		j					i		j		

$\Downarrow$

		Segment III							Segment IV						
	Child A	k_1^A	$\dots$	$\boxed{k_i^B}$	$\dots$	k_j^A	$\dots$	$k_{n^l}^A$	λ_1^A	$\dots$	$\boxed{\lambda_i^B}$	$\dots$	λ_j^A	$\dots$	$\lambda_{n^l}^A$
③	Child B	k_1^B	$\dots$	$\boxed{k_i^A}$	$\dots$	k_j^B	$\dots$	$k_{n^l}^B$	λ_1^B	$\dots$	$\boxed{\lambda_i^A}$	$\dots$	λ_j^B	$\dots$	$\gamma_{n^l}^B$
				$\uparrow$		$\uparrow$					$\uparrow$		$\uparrow$		
	LCP Buyer			i		j					i		j		

Figure 3.4: Crossover for segments III and IV

(c) both of them are feasible solutions. The crossover process for segments III and IV stops in case (b) and (c) or the candidate list becomes an empty set.

(iii) Segment V

For segment V, single-point crossover is adopted, as shown in Figure 3.5. Buyer i is selected randomly and the genes from η_i to η_{n^l} are swapped. Next, we check whether the constraint $\eta \leqslant T$ is satisfied. If not, the ηs at the corresponding locations, i.e. buyer j, are swapped again.

Mutation

Mutation is carried out in each chromosome following the steps in Algorithm 3.3. Note that when a gene is mutated, the related genes may also be mutated subject to the corresponding constraints. Invalid chromosomes are not allowed.

$$
\begin{array}{clccccccccc}
 & & & & & & \text{Segment V} & & & & \\
 & \text{Parent A} & \eta_1^A & \eta_2^A & \dots & \eta_i^A & \dots & \eta_j^A & \dots & \eta_{n^l}^A \\
① & \text{Parent B} & \eta_1^B & \eta_2^B & \dots & \eta_i^B & \dots & \eta_j^B & \dots & \eta_{n^l}^B \\
\hline
 & & & & & \uparrow & & \uparrow & & \\
 & \text{LCP Buyer} & & & & i & & j & &
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{clccccccccc}
 & & & & & & \text{Segment V} & & & & \\
 & \text{Child A} & \eta_1^A & \eta_2^A & \dots & \boxed{\eta_i^B} & \boxed{\dots} & \boxed{\eta_j^B} & \boxed{\dots} & \boxed{\eta_{n^l}^B} \\
② & \text{Child B} & \eta_1^B & \eta_2^A & \dots & \boxed{\eta_i^A} & \boxed{\dots} & \boxed{\eta_j^A} & \boxed{\dots} & \boxed{\eta_{n^l}^A} \\
\hline
 & & & & & \uparrow & & \uparrow & & \\
 & \text{LCP Buyer} & & & & i & & j & &
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{clccccccccc}
 & & & & & & \text{Segment V} & & & & \\
 & \text{Child A} & \eta_1^A & \eta_2^A & \dots & \boxed{\eta_i^B} & \boxed{\dots} & \eta_j^A & \boxed{\dots} & \boxed{\eta_{n^l}^B} \\
③ & \text{Child B} & \eta_1^B & \eta_2^A & \dots & \boxed{\eta_i^A} & \boxed{\dots} & \eta_j^B & \boxed{\dots} & \boxed{\eta_{n^l}^A} \\
\hline
 & & & & & \uparrow & & \uparrow & & \\
 & \text{LCP Buyer} & & & & i & & j & &
\end{array}
$$

Figure 3.5: Single-point Crossover for segment V

Population updating

After crossover and mutation, 20 child chromosomes are added to the previous population which includes 20 parent chromosomes. Sort these 40 chromosomes in a way that their total costs are in increasing order and the first 20 chromosomes are selected to form a new group of parents of next evolution. The idea of Elitist strategy (De Jong (1975)) is to bring the best of chromosomes from previous stage to the current stage without changing the gene structure. This ensures the best chromosomes can survive.

Stopping Criteria

In this research, the algorithm is stopped when the best chromosome has no improvement in successive 250 evolutions, which implies that a steady solution has

Algorithm 3.3: Mutation Operator

1 ***Function***(Mutation)

2 Generate a random number $rand_0$;

3 ***If*** $rand_0 < mut\_rate$, ***then***

mutate segment VI which has one gene T only, and mutate

Ks, γs and ηs, s.t. $K_i\gamma_i \leqslant T$ and $\eta_j \leqslant T$;

If not, go to Step 4;

4 ***For*** each SCP buyer i

generate a random number $rand_i$;

If $rand_i < mut\_rate$, ***then***

mutate K_i, s.t. $K_i \leqslant T/\gamma_i$;

Else

mutate γ_i, s.t. $\gamma_i \leqslant T/K_i$;

5 ***For*** each LCP buyer j

generate two random numbers $rand_j^1$ and $rand_j^2$;

If $rand_j^1 < mut\_rate$, ***then***

mutate k_j, s.t. $k_j \geqslant \delta_j$, $MT \leqslant 365$ and$PFT + \bar{D}_r^l \leqslant PT$;

If not, mutate δ_j, s.t. $\delta_j \leqslant k_j$;

7 ***If*** $rand_j^2 < mut\_rate$, ***then***

mutate η_j, s.t. $\eta_j \leqslant T$.

been achieved.

3.7.2 Simulated Annealing

Simulated Annealing (SA), which was first introduced by Metropolis et al. (1953) and then developed by Kirkpatrick et al. (1983) and Černỳ (1985), is a probabilistic algorithm for searching the global optimum of a given function, derived from the analogy with thermodynamics involving heating and cooling solids to obtain new crystals with better configuration.

The outlines of the simulated annealing is shown in Algorithm 3.4.

Algorithm 3.4: Simulated Annealing for Problem CLU(n^l)

Function (Simulated Annealing)
Select an integer number for n^l, then $n^s = n - n^l$;
Initialization: Set temperature $temp = temp_0$ (a large number), solution S_1, Metropolis length (iterations within each temperature) $= L$;
While Final temperature T_f is not achieved ***do***
    Set inner iteration $iter = 0$;
    While inner iteration is less than L ***do***
        $S_2 \leftarrow neighbor(S_1)$, $iter = iter + 1$;
        Update current solution by acceptance criterion;
    EndWhile
    Update temperature, $temp = r \cdot temp$.
End While

Solution Representation

The solution representation in SA is the same as in GA.

Temperature Setting

The initial temperature $temp_0$ is set to be a large number and the final temperature $temp_f$ is a positive number close to 0. The annealing rate (cooling rate) r is number from 0 to 1 (exclusive).

Neighbouring Solution

Searching neighbours of a solution is similar to the mutation process in GA. The difference is, in SA, we only choose one location to mutate with the probability of 100%. However, some related genes still need to be mutated subject to the corresponding constraints.

Acceptance Criterion

A change in energy ΔE occurs when conducting neighbouring move from S_1 to S_2. In a minimization problem, if ΔE is negative, S_2 is set as the current solution instead of S_1. Otherwise, S_2 may still be accepted according to the Boltzmann probability factor $e^{-\Delta \frac{E}{k_b \cdot temp}}$, where k_b is the Boltzmann constant and $temp$ is the current temperature. A random number is generated from the uniform distribution between 0 and 1 to compare with the Boltzmann probability. If the random number is smaller, S_2 is accepted. The acceptance of a worse solution prevents the algorithm being trapped in a local minimum. As the temperature tends to be zero, the probability of accepting a worse neighbouring solution becomes smaller.

Stopping Criterion

The SA stops when the final temperature is achieved or there is no neighbouring solution accepted in successive 250 SA iterations, whichever comes first.

3.7.3 A Hybrid of Simulated Annealing and Genetic Algorithm

Comparing to simulated annealing, genetic algorithm has a pool of solutions rather than just one solution. In addition, new solutions are generated not only by selfmutating (neighbouring moves in SA), but also by exchanging the information of two solutions from the pool (crossover process). Therefore, GA usually obtains better solutions due to high diversity of the population. Nevertheless, SA converges more quickly at the beginning of the searching process since it starts with a high temperature which yields a high probability of accepting the worse solutions such that the diversity of the solution structure is enhanced. In order to combine the optimality of GA and quick convergency of SA, a hybrid of GA and SA, i.e. SAGA, is applied to solve Problem CLU(n^l), and the outlines of SAGA is shown in Algorithm 3.5.

Algorithm 3.5: Hybrid SAGA for Problem CLU(n^l)

Function (SAGA)
Select an integer number for n^l, then $n^s = n - n^l$;
Run SA until the stopping criteria occurs;
Population initialization: half of the population comes from the converged solutions of SA, and the remaining half are random generated.
While stopping criteria is not achieved ***do***
    Crossover;
    Mutation;
    Updating population;
End While

3.8 Numerical Results

Numerical experiments have been conducted to illustrate the performance of our clustering synchronized cycles model (CLU). The results were obtained by the different combinations of the clustering approaches and heuristics for Problem CLU(n^l), namely, incremental-clustering approach with GA (denoted by IGA) and SA (denoted by ISA), and economic-cycle-clustering approach with GA (denoted by ECGA), SA (denoted by ECSA) and SAGA (denoted by ECSAGA). The performance of the CLU is also compared with those of the synchronized cycles model of Chan and Kingsman (2007) (denoted by SYN and solved by GA) and the independent optimization cycles model (denoted by IND).

3.8.1 Heuristic Parameters for Problem CLU(n^l)

The efficiency of the heuristics is greatly dependent on finding good parameters. The finding process shows that the following combinations of parameters work better than many other parameter combinations for the problem.

The parameters for Genetic Algorithm (GA) are as follows:

Crossover rate: 0.6

Mutation rate: 0.05

Population size: 20

Stopping criteria: There is no improvement for successive 250 iterations

The parameters for Simulated Annealing Algorithm (SA) are as follows:

Initial temperature: 2×number of buyers (i.e. $2n$)

Cooling rate: 0.95

Number of iterations (within each temperature): 2×number of buyers (i.e. $2n$)

Stopping criteria: There is no improvement for successive 250 SA iterations

The parameters for Hybrid Simulated Annealing and Genetic Algorithm (SAGA) are as follows:

Population size: 20

Initial temperature: 2×number of buyers (i.e. $2n$)

Cooling rate: 0.95

Number of SA iterations: 2×number of buyers (i.e. $2n$)

Number of iterations (within each temperature): 2×number of buyers (i.e. $2n$)

Crossover rate: 0.6

Mutation rate: 0.05

Stopping criteria: There is no improvement for successive 250 GA iterations

3.8.2 Performance of Clustering Synchronized Cycles Model for Different Problem Size

Six randomly generated datesets with different number of buyers (n) are used to test the performance of the CLU model, Examples 1 and 2 for $n = 10$, Examples 3 and 4 for $n = 30$ and Examples 5 and 6 for $n = 50$ (datasets of Examples 1-6 are given in Tables B.1-B.6). The buyers in each dataset are indexed by a decreasing order of their economic ordering cycles. And the D/P value, the ratio of the total demand rate of all the buyers to the vendor's production rate, equals to 0.5 for our experiments. All the algorithms are run in a computer with 3.40 GHz and 24 GB RAM.

Results for Examples 1-6

The performance comparison between different methods for the CLU and that of the IND and SYN are summarized in Tables 3.1-3.6 for Examples 1-6, respectively, in terms of total buyers' cost, vendor's cost, total system cost and CPU time. The average improvement of different CLU algorithms (i.e. IGA, ECGA, ISA, ECSA and ECSAGA), denoted by CLU(Ave.), are also showed in the last rows of Tables 3.1-3.6.

Method	Improvement over IND (%)			Improvement over SYN (%)			CPU Time (s')
	Buyers' cost	Vendor's cost	System cost	Buyers' cost	Vendor's cost	System cost	
SYN	-19.42	49.79	27.38	—	—	—	3.68
IGA	-20.42	57.37	33.54	-0.84	19.09	8.48	103.94
ECGA	-19.63	59.17	33.66	-0.18	18.69	8.64	63.36
ISA	-24.88	59.47	32.16	-4.57	19.28	6.58	85.52
ECSA	-20.28	56.99	31.98	-0.72	14.34	6.33	30.17
ECSAGA	-10.52	53.93	33.06	7.46	8.24	7.82	26.45
CLU(Ave.)	-19.15	57.39	32.88	0.23	15.93	7.57	61.89

Table 3.1: Comparison of Various Methods for Example 1

Method	Improvement over IND (%)			Improvement over SYN (%)			CPU Time (s')
	Buyers' cost	Vendor's cost	System cost	Buyers' cost	Vendor's cost	System cost	
SYN	-11.36	44.61	25.95	—	—	—	4.89
IGA	-15.65	47.72	26.59	-3.85	5.61	0.87	151.29
ECGA	-11.59	46.22	26.94	-0.20	2.90	1.35	6.93
ISA	-10.57	46.08	27.19	0.71	2.66	1.68	424.67
ECSA	-11.59	46.63	27.22	-0.20	3.65	1.72	4.38
ECSAGA	-14.81	46.08	25.78	3.09	2.66	0.23	5.23
CLU(Ave.)	-12.84	46.55	26.75	-0.09	3.50	1.17	118.50

Table 3.2: Comparison of Various Methods for Example 2

For Example 1, a 10-buyer dataset, Table 3.1 shows that the total system costs of clustering synchronized cycles model (CLU) obtained from five heuristics are always lower than the independent policy (IND) by an average of 32.9% whereas the total buyers' cost in the CLU has been increased by an average of 19.2% over IND. The CLU also outperforms the synchronized cycles model (SYN) by 6.33-8.64%, where the buyers' cost is decreased by 7.5% for ECSAGA but increased by 0.18-4.57% for other heuristics and the vendor's cost is saved by 16.0% on average. As shown in the last column of Table 3.1, among the five heuristics for the CLU, IGA obtains the minimum system cost but requires the maximum CPU time. In addition, when compared with the SAs (ISA and ECSA), GAs (IGA and ECGA) achieve 2.0% and 2.2% lower system costs while using 1.22 and 2.1 times of CPU time, respectively. The computational time required by the incremental-clustering (IGA and ISA) is longer than that of economic-cycle-clustering (ECGA and ECSA) and the results of these two clustering approach with the same heuristic algorithm are almost the same. ECSAGA has a better system cost than SAs and is faster than GAs.

For the results of another 10-buyer dataset, Example 2, Table 3.2 shows that the total relevant cost of the CLU model is also better than the IND by an average of 26.75% while only 0.23-1.72% better than the SYN. IGA and ISA are still the most time-consuming methods for the CLU model, but the other three economic-cycle-clustering methods save up to 80% CPU time than that of Example 1, which is of the same buyer size.

The results for Example 3, a 30-buyer dataset, presented in Table 3.3, also has the CLU model performing better than that of the IND. On average, the CLU model is better by 19.87%. And the improvement of the CLU model over the SYN model

Method	Improvement over IND (%)			Improvement over SYN (%)			CPU Time (s')
	Buyers' cost	Vendor's cost	System cost	Buyers' cost	Vendor's cost	System cost	
SYN	-24.17	26.87	16.00	—	—	—	21.61
IGA	-17.72	30.24	20.02	5.20	4.61	4.79	2303.09
ECGA	-21.99	30.77	19.53	1.76	5.33	4.21	352.96
ISA	-18.09	30.34	20.02	4.90	4.74	4.79	577.75
ECSA	-18.02	30.12	19.87	4.96	4.45	4.61	140.71
ECSAGA	-16.99	29.92	19.93	5.79	4.18	4.69	412.77
CLU(Ave.)	-18.56	30.28	19.87	4.52	4.66	4.62	757.46

Table 3.3: Comparison of Various Methods for Example 3

Method	Improvement over IND (%)			Improvement over SYN (%)			CPU Time (s')
	Buyers' cost	Vendor's cost	System cost	Buyers' cost	Vendor's cost	System cost	
SYN	-25.93	47.31	29.23	—	—	—	31.99
IGA	-24.38	47.18	29.52	1.24	-0.24	0.41	2827.71
ECGA	-21.34	47.26	30.33	3.64	-0.08	1.56	28.22
ISA	-28.92	48.72	29.55	-2.37	2.68	0.46	321.56
ECSA	-32.53	49.38	29.16	-5.24	3.94	-0.09	10.71
ECSAGA	-31.88	50.03	29.81	-4.72	5.17	0.82	36.69
CLU(Ave.)	-27.81	48.51	29.67	-1.49	2.29	0.63	644.98

Table 3.4: Comparison of Various Methods for Example 4

ranges from 4.21-4.79%, where not only the vendor's costs are reduced by an average of 4.66%, but also the total cost of the buyers' are reduced in all five CLU methods. As shown in the last two columns, the IGA method has the minimum system cost in this example, while spends up to 16 times of CPU time and obtains less than 1% cost improvement comparing to the other four CLU methods.

For the other 30-buyer dataset, Example 4, it can be seen from Table 3.4 that improvement of CLU over IND is 29.67% on average. However, CLU methods are not always better than SYN. The system cost obtained by ECSA is 0.09% higher than that of SYN and the cost savings by the other four CLU methods are up to 1.56%. And the CPU time required by the ECs (ECGA, ECSA and ECSAGA) are less than 10% of that in Example 3, which is similar to the situation in Examples 1 and 2.

Method	Improvement over IND (%)			Improvement over SYN (%)			CPU Time (s')
	Buyers' cost	Vendor's cost	System cost	Buyers' cost	Vendor's cost	System cost	
SYN	-24.03	28.77	15.95	—	—	—	69.19
IGA	-17.43	33.00	20.75	5.32	5.93	5.72	5738.61
ECGA	-21.00	34.07	20.70	2.45	7.44	5.65	2052.60
ISA	-24.07	33.75	19.71	-0.03	6.99	4.47	1140.68
ECSA	-21.35	34.44	20.89	2.16	7.96	5.88	335.48
ECSAGA	-21.22	34.16	20.72	2.27	7.57	5.67	1698.43
CLU(Ave.)	-21.01	33.89	20.55	2.43	7.18	5.48	2193.16

Table 3.5: Comparison of Various Methods for Example 5

Method	Improvement over IND (%)			Improvement over SYN (%)			CPU Time (s')
	Buyers' cost	Vendor's cost	System cost	Buyers' cost	Vendor's cost	System cost	
SYN	-23.93	49.49	27.24	—	—	—	103.04
IGA	-23.93	49.49	27.24	0.00	0.00	0.00	165.38
ECGA	-21.51	49.16	27.74	1.95	-0.65	0.69	60.97
ISA	-23.93	49.49	27.24	0.00	0.00	0.00	69.97
ECSA	-24.27	49.61	27.21	-0.27	0.23	-0.03	21.81
ECSAGA	-24.00	49.33	27.10	-0.06	-0.33	-0.19	91.12
CLU(Ave.)	-23.53	49.42	27.31	0.32	-0.15	0.09	81.85

Table 3.6: Comparison of Various Methods for Example 6

For the 50-buyer datasets, Examples 5 and 6, Table 3.5 and Table 3.6 also show that, when compared with the IND, the system costs in the CLU model are reduced by an average of 20.55% and 27.31%, respectively. When comparing CLU with SYN, the system costs decrease by 4.47%-5.88% in Example 5. However, the costs of CLU model and SYN model are almost the same in Example 6. And the CPU time used for each CLU method in Example 6 is only 2-7% of that in Example 5.

Cost breakdown for Examples 1-6

Cost	IND	SYN	CLU				
			IGA	ECGA	ISA	ECSA	ECSAGA
System	639.27	464.23	424.86	424.10	433.68	433.86	427.91
Buyers'	206.96	274.15	249.22	247.59	258.45	248.92	228.72
Vendor's	432.32	217.08	175.64	176.51	175.23	185.94	199.18
Vendor's set up	87.64	50.00	75.00	75.00	75.00	100.00	75.00
Vendor's processing	90.17	69.50	50.37	52.34	51.72	56.60	57.05
Vendor's holding	254.51	97.58	50.27	49.17	48.51	29.34	67.13
Buyer 1	4.39	7.24	4.59	4.42	4.42	4.78	4.59
Buyer 2	2.56	3.54	2.72	2.72	2.72	3.12	2.93
Buyer 3	6.02	8.30	6.42	6.08	6.08	6.22	6.08
Buyer 4	3.56	4.79	3.85	3.85	3.85	3.71	3.62
Buyer 5	7.24	7.29	8.46	7.34	7.34	7.53	8.46
Buyer 6	10.63	10.63	11.07	11.07	11.07	11.51	11.07
Buyer 7	32.35	46.32	36.36	36.36	36.36	33.03	36.36
Buyer 8	59.04	62.50	62.50	62.50	93.49	59.12	62.50
Buyer 9	53.88	55.35	61.36	61.36	61.36	78.71	61.36
Buyer 10	27.28	41.18	51.90	51.90	31.76	41.18	31.76

Table 3.7: Cost Breakdown of IND, SYN and CLU of Example 1

From the results shown in Tables 3.1-3.6, it can be seen that the CLU model allows buyers to have ordering cycles larger than the vendor's production cycle always outperforms the IND model and works better than the SYN model for Examples 1, 3 and 5 while performs almost the same for Examples 2, 4 and 6. In order to analyze how the CLU model works, the cost breakdown for different models, in terms of the cost of vendor and each individual buyer, are presented in Tables 3.7-3.12 for Examples 1-6, respectively.

The cost breakdown of Example 1, presented in Table 3.7, shows that the im-

Cost	IND	SYN	CLU				
			IGA	ECGA	ISA	ECSA	ECSAGA
System	162.20	120.12	119.07	118.50	118.09	118.05	120.39
Buyers'	54.08	60.23	62.55	60.35	59.80	60.35	62.09
Vendor's	108.12	59.89	56.52	58.15	58.29	57.70	58.30
Vendor's setup	24.70	20.00	19.05	22.22	22.22	22.22	25.00
Vendor's processing	31.38	20.55	18.45	19.54	20.22	19.54	19.05
Vendor's holding	52.04	19.34	19.03	16.39	15.85	15.94	14.25
Buyer 1	1.58	1.62	1.64	2.14	1.59	2.14	1.99
Buyer 2	3.08	3.17	3.22	3.11	3.11	3.11	3.08
Buyer 3	1.68	1.74	1.76	1.70	1.70	1.70	1.68
Buyer 4	4.70	4.91	4.98	4.78	4.78	4.78	4.71
Buyer 5	2.72	2.89	2.95	2.81	2.81	2.81	2.74
Buyer 6	3.21	3.57	3.66	3.43	3.43	3.43	3.31
Buyer 7	3.87	4.97	5.12	4.66	4.66	4.66	4.39
Buyer 8	6.86	8.94	9.24	8.39	8.39	8.39	7.88
Buyer 9	11.66	12.56	12.80	12.14	12.14	12.14	11.82
Buyer 10	14.73	15.85	17.19	17.19	17.19	17.19	20.49

Table 3.8: Cost Breakdown of IND, SYN and CLU of Example 2

provement of the vendor's cost of CLU over SYN is due to the large savings on processing and holding cost, which range from 17.9-27.52% and 31.2-69.9%, respectively. The breakdown of buyers' costs shows that buyer 1-4 have a large saving in CLU when compared with SYN. And buyer 1 always obtains the maximum cost reduction from different CLU heuristics, ranging from 33.97% to 38.94%.

Similar to Example 1, the cost breakdowns of Example 3 and Example 5, as shown in Table 3.9 and Table 3.11, imply that the savings on vendor's processing and holding cost lead to the improvement of CLU over SYN. For Examples 3 and 5, respectively, the average savings on processing cost are 38.79% and 32.02% while on holding cost are 12.55% and 14.01%. For the buyers' individual cost, the two tables show that the buyers with small index are most benefited from the CLU model,

Cost	IND	SYN	CLU				
			IGA	ECGA	ISA	ECSA	ECSAGA
System	725.42	609.38	580.16	583.75	580.19	581.29	580.83
Buyers'	154.53	191.38	181.90	188.51	182.49	182.37	180.77
Vendor's	570.89	417.50	398.26	395.24	397.70	398.93	400.05
Vendor's set up	196.19	166.67	200.00	166.67	200.00	200.00	200.00
Vendor's processing	59.58	55.83	33.86	33.58	33.33	34.55	35.56
Vendor's holding	315.12	195.00	164.40	194.99	164.37	164.38	164.50
Buyer 1	1.46	2.41	1.47	1.52	1.47	1.54	1.47
Buyer 2	1.25	2.05	1.26	1.31	1.26	1.33	1.26
Buyer 3	1.19	1.81	1.22	1.20	1.22	1.22	1.22
Buyer 4	0.93	1.35	0.97	1.04	0.97	1.06	0.97
Buyer 5	1.75	2.45	1.75	1.79	1.75	2.03	1.75
Buyer 6	1.12	1.57	1.44	1.27	1.44	1.30	1.44
Buyer 7	1.48	2.06	1.56	1.52	1.56	1.56	1.56
Buyer 8	0.85	1.18	0.85	0.87	0.85	0.90	0.85
Buyer 9	1.15	1.58	1.23	1.33	1.23	1.36	1.23
Buyer 10	0.99	1.24	1.11	1.22	1.11	1.25	1.11
Buyer 11	0.82	0.97	0.86	0.92	0.86	0.97	0.86
Buyer 12	1.08	1.23	1.33	1.25	1.33	1.33	1.33
Buyer 13	1.96	2.19	1.96	2.01	2.15	1.96	1.96
Buyer 14	1.25	1.39	1.61	1.51	1.61	1.61	1.61
Buyer 15	2.87	3.12	2.88	2.98	2.88	2.88	2.88
Buyer 16	1.39	1.48	1.61	1.78	1.61	1.91	1.61
Buyer 17	1.64	1.70	1.98	1.78	1.98	2.38	1.98
Buyer 18	0.64	0.66	0.78	0.87	0.78	0.94	0.78
Buyer 19	0.77	0.80	1.13	1.05	1.13	1.13	1.13
Buyer 20	0.95	0.97	1.20	1.36	1.47	1.47	1.20
Buyer 21	0.56	0.57	0.59	0.63	0.72	0.59	0.72
Buyer 22	0.56	0.56	0.88	0.75	0.88	0.67	0.88
Buyer 23	2.99	3.08	3.89	3.08	3.89	2.99	2.99
Buyer 24	0.45	0.47	0.81	0.47	0.81	0.45	0.45
Buyer 25	4.99	5.81	5.36	5.81	5.36	5.36	5.36
Buyer 26	6.01	8.91	7.85	8.91	7.85	7.85	7.85
Buyer 27	8.71	14.14	12.34	14.14	12.34	12.34	12.34
Buyer 28	15.43	29.63	25.48	29.63	25.48	25.48	25.48
Buyer 29	43.95	51.13	51.13	51.13	51.13	51.13	51.13
Buyer 30	45.37	45.38	45.38	45.38	45.38	45.38	45.38

Table 3.9: Cost Breakdown of IND, SYN and CLU of Example 3

Cost	IND	SYN	CLU				
			IGA	ECGA	ISA	ECSA	ECSAGA
System	299.76	212.15	211.28	208.85	211.18	212.35	210.41
Buyers’	74.00	93.19	92.04	89.79	95.40	98.07	97.59
Vendor’s	225.76	118.96	119.24	119.06	115.78	114.28	112.82
Vendor’s setup	59.69	66.67	62.50	55.56	62.50	62.50	55.56
Vendor’s processing	54.82	27.23	27.22	27.00	26.16	25.95	24.99
Vendor’s holding	111.25	25.06	29.52	36.50	27.12	25.82	32.27
Buyer 1	1.22	1.23	1.51	1.23	1.51	1.51	1.62
Buyer 2	0.98	0.98	0.99	1.00	1.27	0.99	1.00
Buyer 3	0.72	0.73	1.06	0.77	1.06	0.74	0.77
Buyer 4	1.13	1.15	1.68	1.21	1.68	1.17	1.21
Buyer 5	2.15	2.22	2.26	2.36	2.26	2.26	2.36
Buyer 6	2.14	2.23	2.27	2.38	2.27	2.27	2.38
Buyer 7	1.98	2.07	2.12	2.22	2.12	2.12	2.22
Buyer 8	1.82	1.91	1.95	2.05	1.95	1.95	2.05
Buyer 9	0.64	0.68	1.05	0.73	1.05	0.69	0.73
Buyer 10	0.87	0.97	1.00	1.07	1.00	1.00	1.07
Buyer 11	2.38	2.67	2.75	2.93	2.75	2.75	2.93
Buyer 12	0.77	0.87	0.90	0.96	0.90	0.90	0.96
Buyer 13	0.47	0.55	0.96	0.62	0.96	0.57	0.62
Buyer 14	0.74	0.88	0.92	0.99	0.92	0.92	0.99
Buyer 15	2.72	3.44	3.59	3.89	3.59	3.59	3.89
Buyer 16	1.37	1.75	1.82	1.40	1.82	1.82	1.97
Buyer 17	0.42	0.54	0.56	0.61	0.56	0.56	0.43
Buyer 18	2.58	3.38	3.53	3.84	3.53	3.53	3.84
Buyer 19	1.17	1.70	1.78	1.27	1.78	1.78	1.95
Buyer 20	0.37	0.58	0.61	0.67	0.61	0.61	0.67
Buyer 21	0.94	1.49	1.57	1.73	1.57	1.57	1.73
Buyer 22	4.04	6.52	6.86	4.66	6.86	6.86	7.56
Buyer 23	2.69	4.60	4.85	3.22	4.85	4.85	3.22
Buyer 24	4.55	5.00	5.14	5.46	8.22	5.14	5.46
Buyer 25	2.69	4.92	3.16	3.38	3.16	5.19	3.38
Buyer 26	0.72	1.39	1.47	1.63	1.47	1.47	1.63
Buyer 27	0.97	2.44	2.59	2.89	2.59	2.59	2.89
Buyer 28	7.69	11.54	8.12	9.89	8.12	12.12	13.31
Buyer 29	11.45	11.89	13.33	11.89	13.33	11.54	11.89
Buyer 30	11.61	12.86	11.63	12.86	11.63	15.01	12.86

Table 3.10: Cost Breakdown of IND, SYN and CLU of Example 4

Cost	IND	SYN	CLU				
			IGA	ECGA	ISA	ECSA	ECSAGA
System	991.06	832.98	785.38	785.89	795.71	784.01	785.74
Buyers'	240.64	298.47	282.58	291.17	298.56	292.03	291.70
Vendor's	750.41	534.52	502.79	494.72	497.15	491.99	494.04
Vendor's setup	231.98	178.57	250.00	208.33	178.57	208.33	208.33
Vendor's processing	131.83	114.71	78.06	79.44	77.39	77.14	77.92
Vendor's holding	386.61	241.23	174.74	206.95	241.19	206.51	207.79
Buyer 1	1.43	2.01	1.54	1.52	1.46	1.43	1.52
Buyer 2	0.87	1.19	1.05	1.05	1.01	1.05	0.94
Buyer 3	1.39	1.88	1.53	1.50	1.61	1.67	1.50
Buyer 4	1.15	1.54	1.28	1.25	1.20	1.16	1.25
Buyer 5	1.28	1.72	1.43	1.40	1.51	1.56	1.40
Buyer 6	1.16	1.49	1.33	1.30	1.24	1.47	1.30
Buyer 7	0.62	0.79	0.80	0.80	0.77	0.80	0.71
Buyer 8	0.86	1.07	1.01	0.99	0.94	0.89	0.89
Buyer 9	1.18	1.45	1.42	1.38	1.51	1.23	1.23
Buyer 10	1.12	1.36	1.12	1.12	1.12	1.16	1.12
Buyer 11	1.89	2.29	1.91	1.90	1.90	1.99	1.99
Buyer 12	1.84	2.21	1.86	2.20	1.85	1.94	1.94
Buyer 13	1.05	1.22	1.07	1.28	1.20	1.12	1.12
Buyer 14	1.29	1.51	1.62	1.58	1.47	1.38	1.58
Buyer 15	1.87	2.17	1.91	1.87	2.14	2.01	2.01
Buyer 16	1.32	1.52	1.68	1.64	1.52	1.42	1.64
Buyer 17	1.82	2.10	1.86	1.82	2.10	1.96	1.96
Buyer 18	0.67	0.77	0.86	0.98	0.93	0.98	0.84
Buyer 19	1.96	2.23	2.02	1.96	1.99	1.96	1.96
Buyer 20	1.77	1.97	1.84	1.77	2.11	1.96	2.29
Buyer 21	1.92	2.11	2.01	1.93	2.34	2.16	2.16
Buyer 22	1.20	1.32	1.26	1.21	1.24	1.36	1.36
Buyer 23	2.02	2.18	2.16	2.04	2.11	2.33	2.04
Buyer 24	2.56	2.74	2.75	2.59	2.69	2.98	2.98
Buyer 25	0.49	0.53	0.53	0.50	0.52	0.50	0.58
Buyer 26	2.11	2.24	2.30	2.15	2.24	2.50	2.15
Buyer 27	0.52	0.54	0.77	0.88	0.68	0.62	0.74
Buyer 28	1.93	2.01	2.15	1.99	2.09	1.99	1.99
Buyer 29	1.50	1.56	1.68	1.55	1.63	1.84	2.23
Buyer 30	0.76	0.79	0.86	1.14	1.04	0.94	0.79
Buyer 31	0.56	0.57	0.63	0.58	0.61	0.70	0.70
Buyer 32	1.79	1.84	2.03	1.87	1.97	2.24	2.24
Buyer 33	2.45	2.50	2.48	2.59	2.75	2.59	2.59
Buyer 34	0.60	0.61	0.99	0.96	0.86	0.78	0.78
Buyer 35	0.38	0.39	0.47	0.65	0.58	0.52	0.52
Buyer 36	2.51	2.52	3.13	2.79	3.01	2.79	2.79
Buyer 37	0.47	0.47	0.66	0.96	0.85	0.75	0.57
Buyer 38	4.05	4.09	4.54	4.99	4.09	4.99	4.99
Buyer 39	0.37	0.38	0.57	0.85	0.76	0.66	0.66
Buyer 40	0.50	0.58	0.99	0.54	0.94	0.54	0.54
Buyer 41	1.48	1.89	2.43	1.73	1.89	1.73	1.73
Buyer 42	5.41	7.54	6.17	6.81	7.54	6.81	6.81
Buyer 43	1.46	2.15	2.87	1.94	2.15	1.94	1.94
Buyer 44	6.60	7.72	9.68	7.19	7.72	7.19	7.19
Buyer 45	11.66	15.23	12.74	13.89	15.23	13.89	13.89
Buyer 46	26.62	59.50	44.65	51.97	59.50	51.97	51.97
Buyer 47	21.79	22.18	22.18	25.78	22.18	25.78	25.78
Buyer 48	33.55	38.14	38.14	38.14	38.14	38.14	38.14
Buyer 49	33.82	38.59	38.59	38.59	38.59	38.59	38.59
Buyer 50	43.04	43.05	43.05	43.05	43.05	43.05	43.05

Table 3.11: Cost Breakdown of IND, SYN and CLU of Example 5

Cost	IND	SYN	CLU				
			IGA	ECGA	ISA	ECSA	ECSAGA
System	388.18	282.45	282.45	280.50	282.45	282.54	282.98
Buyers'	117.66	145.81	145.81	142.97	145.81	146.21	145.90
Vendor's	270.52	136.64	136.64	137.53	136.64	136.32	137.08
Vendor's setup	65.95	55.56	55.56	62.50	55.56	57.69	68.18
Vendor's processing	76.91	37.44	37.44	38.35	37.44	37.81	38.75
Vendor's holding	127.65	43.64	43.64	36.67	43.64	40.83	30.15
Buyer 1	1.34	1.54	1.54	1.46	1.54	1.51	1.41
Buyer 2	1.67	1.93	1.93	1.83	1.93	1.89	1.77
Buyer 3	1.62	1.97	1.97	1.85	1.97	1.93	1.78
Buyer 4	1.19	1.48	1.48	1.39	1.48	1.45	1.21
Buyer 5	1.22	1.23	1.23	1.51	1.23	1.59	1.44
Buyer 6	1.31	1.74	1.74	1.61	1.74	1.69	1.54
Buyer 7	1.68	2.28	2.28	2.12	2.28	2.23	2.01
Buyer 8	2.11	2.87	2.87	2.66	2.87	2.79	2.53
Buyer 9	0.93	1.27	1.27	1.18	1.27	1.24	1.12
Buyer 10	2.16	2.99	2.99	2.77	2.99	2.91	2.63
Buyer 11	1.30	1.32	1.32	1.67	1.32	1.31	1.59
Buyer 12	0.98	1.39	1.39	1.28	1.39	1.35	1.22
Buyer 13	1.17	1.66	1.66	1.53	1.66	1.62	1.45
Buyer 14	1.02	1.47	1.47	1.36	1.47	1.43	1.28
Buyer 15	1.59	2.33	2.33	2.15	2.33	2.27	2.03
Buyer 16	0.85	1.25	1.25	1.15	1.25	1.22	1.09
Buyer 17	1.53	1.61	1.61	2.18	1.61	2.32	2.06
Buyer 18	1.14	1.80	1.80	1.65	1.80	1.75	1.55
Buyer 19	2.03	2.17	2.17	3.00	2.17	2.14	2.82
Buyer 20	0.99	1.64	1.64	1.50	1.64	1.59	1.41
Buyer 21	1.01	1.69	1.69	1.54	1.69	1.64	1.45
Buyer 22	1.59	1.74	1.74	2.47	1.74	2.62	2.31
Buyer 23	0.69	1.19	1.19	1.09	1.19	1.16	1.02
Buyer 24	2.63	2.95	2.95	2.81	2.95	2.90	3.97
Buyer 25	2.70	3.05	3.05	2.90	3.05	3.00	4.14
Buyer 26	0.62	1.12	1.12	1.02	1.12	1.09	0.95
Buyer 27	1.76	3.26	3.26	2.96	3.26	1.98	2.76
Buyer 28	2.73	3.18	3.18	2.74	3.18	3.12	2.91
Buyer 29	2.50	2.92	2.92	2.76	2.92	2.86	4.04
Buyer 30	1.54	1.87	1.87	1.76	1.87	1.83	2.63
Buyer 31	3.86	4.03	4.03	4.53	4.03	4.73	4.34
Buyer 32	1.64	2.19	2.19	2.03	2.19	2.14	1.94
Buyer 33	0.68	1.62	1.62	1.45	1.62	1.56	1.35
Buyer 34	2.68	2.99	2.99	2.69	2.99	3.69	3.32
Buyer 35	4.42	5.00	5.00	4.76	5.00	6.22	5.57
Buyer 36	3.04	3.49	3.49	3.32	3.49	3.11	3.05
Buyer 37	4.43	5.13	5.13	4.86	5.13	4.55	4.44
Buyer 38	3.14	4.75	4.75	3.46	4.75	3.23	4.12
Buyer 39	3.39	3.98	3.98	3.76	3.98	3.50	4.49
Buyer 40	2.21	2.61	2.61	2.47	2.61	3.31	2.95
Buyer 41	4.97	4.98	4.98	5.14	4.98	5.27	5.05
Buyer 42	0.44	1.34	1.34	1.20	1.34	1.30	1.11
Buyer 43	0.45	1.39	1.39	1.25	1.39	0.73	0.64
Buyer 44	5.48	5.48	5.48	5.75	5.48	5.91	5.62
Buyer 45	5.52	7.06	7.06	6.59	7.06	5.98	8.08
Buyer 46	5.15	6.64	6.64	6.19	6.64	5.60	5.31
Buyer 47	6.62	6.77	6.77	7.47	6.77	7.77	7.20
Buyer 48	4.39	4.78	4.78	5.53	4.78	5.82	5.26
Buyer 49	3.12	5.38	5.38	4.00	5.38	4.22	3.80
Buyer 50	6.40	7.30	7.30	8.64	7.30	9.13	8.16

Table 3.12: Cost Breakdown of IND, SYN and CLU of Example 6

i.e., the average cost saving of the first 10 buyers in Examples 3 and 5 are 24.92% and 14.09%, respectively. Similar to Example 1, buyer 1 also obtains the maximum saving percentage, with a range of 36.05-39.14% and 23.42-28.90% in Examples 3 and 5 respectively.

For the cost breakdowns of Examples 2, 4 and 6, Tables 3.8, 3.10 and 3.12 show that the cost performance of the CLU model and the SYN model are approximately the same, both in terms of the vendor's cost and the individual buyer's cost.

Optimal production and ordering cycles for Examples 1-6

The optimal production cycle of the vendor as well as the optimal ordering cycles of the buyers for Examples 1-6, are obtained by different CLU algorithms, e.g. IGA, ECGA and ISA, etc. The results are presented in Tables 3.13-3.18, respectively.

Cycle	IND	SYN	CLU				
			IGA	ECGA	ISA	ECSA	ECSAGA
Vendor	6.85	12	8	8	8	6	8
Buyer 1	35.53	12	48	40	40	54	48
Buyer 2	28.16	12	40	40	40	54	48
Buyer 3	27.89	12	40	32	32	36	32
Buyer 4	26.94	12	40	40	40	36	32
Buyer 5	13.54	12	24	16	16	18	24
Buyer 6	12.04	12	16	16	16	18	16
Buyer 7	4.88	12	8	8	8	6	8
Buyer 8	2.85	4	4	4	8	3	4
Buyer 9	2.38	3	4	4	4	6	4
Buyer 10	2.27	6	8	8	4	6	4

Table 3.13: Optimal Production Cycle of the Vendor and the Optimal Ordering Cycles of the Buyers in IND, SYN and CLU of Example 1

For Example 1, it can be seen from Table 3.13 that buyers 1-4 have large T_i^*s in the IND, at least three times of T_v^*, and the ordering cycles for buyers 1-4 in the

Cycle	IND	SYN	CLU				
			IGA	ECGA	ISA	ECSA	ECSAGA
Vendor	32.39	40	42	36	36	36	32
Buyer 1	31.65	40	42	72	36	72	64
Buyer 2	31.19	40	42	36	36	36	32
Buyer 3	30.95	40	42	36	36	36	32
Buyer 4	29.79	40	42	36	36	36	32
Buyer 5	27.96	40	42	36	36	36	32
Buyer 6	24.93	40	42	36	36	36	32
Buyer 7	19.14	40	42	36	36	36	32
Buyer 8	18.67	40	42	36	36	36	32
Buyer 9	13.55	20	21	18	18	18	16
Buyer 10	3.39	5	6	6	6	6	8

Table 3.14: Optimal Production Cycle of the Vendor and the Optimal Ordering Cycles of the Buyers in IND, SYN and CLU of Example 2

SYN are equal to the vendor's production cycle, while the corresponding ordering cycles in the CLU are several times of the vendor's production cycle, indicating that they are assigned to the LCP group in the CLU, which increases the flexibility of the system.

For another 10-buyer dataset, Example 2, in the IND column of Table 3.14, T_v^* is larger than the maximum T_i^*, indicating that the number of SCP buyers (n^s) is set as 9 in the economic-cycle-approach. Therefore, as shown in the ECGA, ECSA and ECSAGA columns, there is at most one buyer's ordering cycle larger than the vendor's production cycle.

The optimal cycles for Example 3, presented in Table 3.15, have the same cycle structure of Example 1. Near half of the buyers' T_i^*s are multiples of T_v^* and these buyers are all assigned to the LCP group in the CLU model. However, these buyers are restricted to adopt the same cycle length of the production cycle in the SYN model. As shown in the SYN column, only three buyers' ordering cycles are less

Cycle	IND	SYN	CLU				
			IGA	ECGA	ISA	ECSA	ECSAGA
Vendor	10.19	12	10	12	10	10	10
Buyer 1	35.66	12	40	48	40	50	40
Buyer 2	35.16	12	40	48	40	50	40
Buyer 3	31.96	12	40	36	40	40	40
Buyer 4	30.01	12	40	48	40	50	40
Buyer 5	28.63	12	30	36	30	50	30
Buyer 6	28.57	12	60	48	60	50	60
Buyer 7	28.45	12	40	36	40	40	40
Buyer 8	28.28	12	30	36	30	40	30
Buyer 9	27.74	12	40	48	40	50	40
Buyer 10	24.34	12	40	48	40	50	40
Buyer 11	22.03	12	30	36	30	40	30
Buyer 12	20.45	12	40	36	40	40	40
Buyer 13	19.36	12	20	24	30	20	20
Buyer 14	19.13	12	40	36	40	40	40
Buyer 15	18.12	12	20	24	20	20	20
Buyer 16	17.28	12	30	36	30	40	30
Buyer 17	15.90	12	30	24	30	40	30
Buyer 18	15.66	12	30	36	30	40	30
Buyer 19	15.62	12	40	36	40	40	40
Buyer 20	14.72	12	30	36	40	40	30
Buyer 21	14.36	12	20	24	30	20	30
Buyer 22	10.76	12	30	24	30	20	30
Buyer 23	9.37	12	20	12	20	10	10
Buyer 24	8.96	12	30	12	30	10	10
Buyer 25	6.81	12	10	12	10	10	10
Buyer 26	4.66	12	10	12	10	10	10
Buyer 27	4.13	12	10	12	10	10	10
Buyer 28	1.69	6	5	6	5	5	5
Buyer 29	1.14	2	2	2	2	2	2
Buyer 30	1.01	1	1	1	1	1	1

Table 3.15: Optimal Production Cycle of the Vendor and the Optimal Ordering Cycles of the Buyers in IND, SYN and CLU of Example 3

Cycle	IND	SYN	CLU				
			IGA	ECGA	ISA	ECSA	ECSAGA
Vendor	33.51	30	32	36	32	32	36
Buyer 1	32.75	30	64	36	64	64	72
Buyer 2	30.48	30	32	36	64	32	36
Buyer 3	25.00	30	64	36	64	32	36
Buyer 4	24.73	30	64	36	64	32	36
Buyer 5	23.22	30	32	36	32	32	36
Buyer 6	22.47	30	32	36	32	32	36
Buyer 7	22.18	30	32	36	32	32	36
Buyer 8	21.98	30	32	36	32	32	36
Buyer 9	21.78	30	64	36	64	32	36
Buyer 10	18.47	30	32	36	32	32	36
Buyer 11	18.47	30	32	36	32	32	36
Buyer 12	18.11	30	32	36	32	32	36
Buyer 13	16.87	30	64	36	64	32	36
Buyer 14	16.22	30	32	36	32	32	36
Buyer 15	14.70	30	32	36	32	32	36
Buyer 16	14.57	30	32	18	32	32	36
Buyer 17	14.36	30	32	36	32	32	18
Buyer 18	13.93	30	32	36	32	32	36
Buyer 19	11.96	30	32	18	32	32	36
Buyer 20	10.78	30	32	36	32	32	36
Buyer 21	10.65	30	32	36	32	32	36
Buyer 22	10.41	30	32	18	32	32	36
Buyer 23	9.67	30	32	18	32	32	18
Buyer 24	9.67	15	16	18	32	16	18
Buyer 25	8.93	30	16	18	16	32	18
Buyer 26	8.36	30	32	36	32	32	36
Buyer 27	6.20	30	32	36	32	32	36
Buyer 28	5.72	15	8	12	8	16	18
Buyer 29	4.54	6	8	6	8	4	6
Buyer 30	3.79	6	4	6	4	8	6

Table 3.16: Optimal Production Cycle of the Vendor and the Optimal Ordering Cycles of the Buyers in IND, SYN and CLU of Example 4

Cycle	IND	SYN	CLU				
			IGA	ECGA	ISA	ECSA	ECSAGA
Vendor	10.78	14	10	12	14	12	12
Buyer 1	33.65	14	50	48	42	36	48
Buyer 2	32.08	14	60	60	56	60	48
Buyer 3	31.77	14	50	48	56	60	48
Buyer 4	31.36	14	50	48	42	36	48
Buyer 5	31.23	14	50	48	56	60	48
Buyer 6	29.30	14	50	48	42	60	48
Buyer 7	28.87	14	60	60	56	60	48
Buyer 8	27.82	14	50	48	42	36	36
Buyer 9	27.03	14	50	48	56	36	36
Buyer 10	26.89	14	30	24	28	36	24
Buyer 11	26.41	14	30	24	28	36	36
Buyer 12	26.04	14	30	48	28	36	36
Buyer 13	24.82	14	30	48	42	36	36
Buyer 14	24.81	14	50	48	42	36	48
Buyer 15	24.60	14	30	24	42	36	36
Buyer 16	24.25	14	50	48	42	36	48
Buyer 17	24.22	14	30	24	42	36	36
Buyer 18	23.86	14	50	60	56	60	48
Buyer 19	23.49	14	30	24	28	24	24
Buyer 20	22.65	14	30	24	42	36	48
Buyer 21	21.90	14	30	24	42	36	36
Buyer 22	21.66	14	30	24	28	36	36
Buyer 23	20.79	14	30	24	28	36	24
Buyer 24	20.34	14	30	24	28	36	36
Buyer 25	20.29	14	30	24	28	24	36
Buyer 26	19.87	14	30	24	28	36	24
Buyer 27	19.35	14	50	60	42	36	48
Buyer 28	18.65	14	30	24	28	24	24
Buyer 29	18.64	14	30	24	28	36	48
Buyer 30	18.37	14	30	48	42	36	24
Buyer 31	17.93	14	30	24	28	36	36
Buyer 32	17.89	14	30	24	28	36	36
Buyer 33	17.17	14	20	24	28	24	24
Buyer 34	16.78	14	50	48	42	36	36
Buyer 35	15.68	14	30	48	42	36	36
Buyer 36	15.11	14	30	24	28	24	24
Buyer 37	12.70	14	30	48	42	36	24
Buyer 38	12.33	14	20	24	14	24	24
Buyer 39	10.89	14	30	48	42	36	36
Buyer 40	8.05	14	30	12	28	12	12
Buyer 41	6.77	14	20	12	14	12	12
Buyer 42	5.92	14	10	12	14	12	12
Buyer 43	5.47	14	20	12	14	12	12
Buyer 44	3.94	7	10	6	7	6	6
Buyer 45	3.26	7	5	6	7	6	6
Buyer 46	1.65	7	5	6	7	6	6
Buyer 47	1.65	2	2	3	2	3	3
Buyer 48	1.19	2	2	2	2	2	2
Buyer 49	1.18	2	2	2	2	2	2
Buyer 50	1.02	1	1	1	1	1	1

Table 3.17: Optimal Production Cycle of the Vendor and the Optimal Ordering Cycles of the Buyers in IND, SYN and CLU of Example 5

Cycle	IND	SYN	CLU				
			IGA	ECGA	ISA	ECSA	ECSAGA
Vendor	45.49	54	54	48	54	52	44
Buyer 1	31.40	54	54	48	54	52	44
Buyer 2	31.15	54	54	48	54	52	44
Buyer 3	28.36	54	54	48	54	52	44
Buyer 4	26.97	54	54	48	54	52	22
Buyer 5	24.52	27	27	48	27	52	44
Buyer 6	24.50	54	54	48	54	52	44
Buyer 7	23.77	54	54	48	54	52	44
Buyer 8	23.70	54	54	48	54	52	44
Buyer 9	23.60	54	54	48	54	52	44
Buyer 10	23.11	54	54	48	54	52	44
Buyer 11	23.00	27	27	48	27	26	44
Buyer 12	22.39	54	54	48	54	52	44
Buyer 13	22.23	54	54	48	54	52	44
Buyer 14	21.64	54	54	48	54	52	44
Buyer 15	21.32	54	54	48	54	52	44
Buyer 16	21.16	54	54	48	54	52	44
Buyer 17	19.61	27	27	48	27	52	44
Buyer 18	19.28	54	54	48	54	52	44
Buyer 19	18.71	27	27	48	27	26	44
Buyer 20	18.16	54	54	48	54	52	44
Buyer 21	17.86	54	54	48	54	52	44
Buyer 22	17.58	27	27	48	27	52	44
Buyer 23	17.32	54	54	48	54	52	44
Buyer 24	16.70	27	27	24	27	26	44
Buyer 25	16.32	27	27	24	27	26	44
Buyer 26	16.20	54	54	48	54	52	44
Buyer 27	15.88	54	54	48	54	26	44
Buyer 28	15.36	27	27	16	27	26	22
Buyer 29	15.22	27	27	24	27	26	44
Buyer 30	14.25	27	27	24	27	26	44
Buyer 31	13.46	18	18	24	18	26	22
Buyer 32	12.19	27	27	24	27	26	22
Buyer 33	11.84	54	54	48	54	52	44
Buyer 34	11.19	18	18	12	18	26	22
Buyer 35	10.86	18	18	16	18	26	22
Buyer 36	10.51	18	18	16	18	13	11
Buyer 37	10.37	18	18	16	18	13	11
Buyer 38	10.19	27	27	16	27	13	22
Buyer 39	10.03	18	18	16	18	13	22
Buyer 40	9.95	18	18	16	18	26	22
Buyer 41	9.25	9	9	12	9	13	11
Buyer 42	9.09	54	54	48	54	52	44
Buyer 43	8.93	54	54	48	54	26	22
Buyer 44	8.76	9	9	12	9	13	11
Buyer 45	8.69	18	18	16	18	13	22
Buyer 46	8.55	18	18	16	18	13	11
Buyer 47	7.25	9	9	12	9	13	11
Buyer 48	5.92	9	9	12	9	13	11
Buyer 49	5.76	18	18	12	18	13	11
Buyer 50	5.32	9	9	12	9	13	11

Table 3.18: Optimal Production Cycle of the Vendor and the Optimal Ordering Cycles of the Buyers in IND, SYN and CLU of Example 6

than the production cycle.

For Example 4, Table 3.16 shows that T_v^* is approximately the same as T_1^* in the IND model and most of the optimal cycles obtained in the CLU model are close to that of the SYN model.

From the optimal cycles of Example 5, as shown in Table 3.17, it can be seen that the length of vendor's economic production cycle is between the lengths of the economic ordering cycle of buyer 39 and buyer 40. And in the CLU model, the ordering cycles of buyer 1-39 obtained by different algorithms are all multiples of vendor's production cycle.

As for Example 6, the other 50-buyer dataset, the optimal cycles for different models are presented in Table 3.18. It can be seen that T_v^* is about 1.5 times of T_1^*. And as shown in the CLU columns, there is no buyer's ordering cycle larger than the vendor's production cycle.

3.8.3 Comparison of the Performances of Clustering Synchronized Cycles Model, Synchronized Cycles Model and Independent Model in Different Scenarios

From the performance of the CLU model for Examples 1-6, it can be seen that the CLU model is always better than the IND model by at least 20%. However, the improvement of CLU over SYN is not that stable, over the range of 4.21-8.64% for Examples 1, 3 and 5 whereas up to only 1.72% for the other three examples. Therefore, the performance of the CLU over SYN is data dependent, however, a large number of experiments need to be conducted so as to find the relation between the performance of the CLU model its problem parameters.

From the nature of the CLU model, its performance should depend on the structure of participants' ordering cycles given by the EOQ model. An economic cycle ratio is defined as

$$\beta = T_v^*/\overline{T}_i^*$$

to describe this structure, where $\overline{T}_i^* = \frac{1}{n}\sum_i T_i^*$.

For the 30-buyer and 50-buyer cases, we include a range of different values of β from 0.4 to 4.4 for comparisons, where the β range is sub-divided into groups of width 0.2 and 25 sets of data are randomly generated for each group of β. Hence our experiments consist of 1000 datasets, i.e. 500 sets for the 30-buyer cases and 500 sets for the 50-buyers cases. The D/P value, the ratio of the total demand rate of all the buyers to the vendor's production rate, is set as 0.5 for our experiments. All the experiments are run in a computer of speed 3.40 GHz.

β	SYN over IND (%)			IGA over IND (%)			ECGA over IND (%)			ISA over IND (%)			ECSA over IND (%)			ECSAGA over IND (%)		
	b	v	t	b	v	t	b	v	t	b	v	t	b	v	t	b	v	t
$[0.4, 0.6)$	29.83	-29.18	-16.91	26.73	-33.11	-20.65	25.65	-32.95	**-20.72**	27.83	-32.04	-19.60	28.36	-33.31	-20.48	27.31	-33.36	-20.71
$[0.6, 0.8)$	27.15	-31.18	-19.33	25.72	-34.30	**-22.06**	23.94	-33.75	-22.01	27.18	-33.85	-21.37	28.38	-34.63	-21.79	25.11	-33.95	-21.91
$[0.8, 1.0)$	23.37	-31.21	-20.33	25.65	-34.13	**-22.23**	24.12	-33.50	-22.06	25.32	-33.42	-21.78	24.16	-33.46	-21.97	23.68	-33.28	-21.93
$[1.0, 1.2)$	24.03	-33.70	-22.03	25.82	-35.59	**-23.18**	25.46	-35.19	-23.00	26.56	-34.82	-22.42	23.73	-34.63	-22.84	25.99	-35.20	-22.91
$[1.2, 1.4)$	24.22	-34.83	-22.90	26.38	-36.24	**-23.63**	23.73	-35.58	-23.57	26.69	-35.66	-23.06	24.62	-35.57	-23.39	24.00	-35.32	-23.34
$[1.4, 1.6)$	25.03	-37.27	-24.70	26.82	-38.26	**-25.15**	26.73	-38.18	-25.09	25.28	-37.45	-24.77	26.46	-37.93	-24.87	28.21	-38.13	-24.73
$[1.6, 1.8)$	27.89	-40.48	-26.27	28.37	-40.87	**-26.49**	26.36	-40.30	-26.46	28.99	-40.89	-26.36	28.19	-40.63	-26.33	29.12	-40.65	-26.23
$[1.8, 2.0)$	27.93	-41.06	-26.43	28.72	-41.39	-26.56	26.63	-40.97	**-26.66**	28.22	-41.21	-26.49	29.45	-41.41	-26.41	28.84	-41.18	-26.36
$[2.0, 2.2)$	28.88	-43.28	-27.72	28.30	-43.33	**-27.90**	28.69	-43.44	-27.89	28.80	-43.28	-27.74	30.32	-43.73	-27.75	30.78	-43.39	-27.40
$[2.2, 2.4)$	28.97	-45.37	-28.79	30.09	-45.82	**-28.90**	28.30	-45.16	-28.82	29.01	-45.37	-28.81	32.00	-46.13	-28.79	33.10	-45.93	-28.35
$[2.4, 2.6)$	30.81	-44.80	-28.12	29.64	-44.67	**-28.27**	29.94	-44.66	-28.16	30.25	-44.75	-28.21	30.13	-44.46	-27.99	30.36	-44.37	-27.83
$[2.6, 2.8)$	34.32	-46.43	-28.83	34.23	-46.48	-28.89	33.23	-46.17	-28.86	33.81	-46.39	**-28.92**	34.39	-46.40	-28.78	34.74	-46.11	-28.50
$[2.8, 3.0)$	32.46	-46.54	-29.19	32.80	-46.74	**-29.28**	33.18	-46.80	-29.22	32.78	-46.66	-29.22	34.46	-46.93	-29.07	35.95	-46.90	-28.67
$[3.0, 3.2)$	36.76	-47.98	-29.37	35.47	-47.85	**-29.53**	35.67	-47.71	-29.40	35.79	-47.80	-29.43	37.87	-48.45	-29.47	36.81	-47.48	-28.90
$[3.2, 3.4)$	37.24	-47.29	-28.85	36.98	-47.41	-29.01	36.19	-47.21	**-29.02**	36.50	-47.26	-28.99	38.52	-47.65	-28.87	37.38	-46.82	-28.45
$[3.4, 3.6)$	33.27	-48.13	-29.34	33.61	-48.48	**-29.53**	34.77	-48.59	-29.34	33.13	-48.12	-29.37	34.79	-48.55	-29.30	35.58	-48.19	-28.83
$[3.6, 3.8)$	35.84	-48.34	-29.49	35.47	-48.42	-29.64	34.80	-48.28	**-29.66**	35.48	-48.43	-29.64	38.17	-49.19	-29.63	39.68	-48.83	-29.03
$[3.8, 4.0)$	39.62	-48.93	-29.74	39.65	-49.01	-29.79	37.94	-48.43	-29.72	39.65	-49.10	**-29.85**	39.22	-48.80	-29.71	43.35	-49.26	-29.19
$[4.0, 4.2)$	39.96	-48.63	-29.31	38.57	-48.55	**-29.52**	36.98	-48.05	-29.49	39.03	-48.58	-29.43	40.36	-48.87	-29.41	41.37	-48.34	-28.76
$[4.2, 4.4)$	39.94	-48.76	-29.59	40.44	-49.00	-29.67	40.20	-48.89	-29.60	42.52	-48.96	-29.17	40.17	-49.01	**-29.72**	41.45	-48.41	-28.98

Table 3.19: Average Performance of CLU over IND for 30-buyer datasets

β	SYN over IND (%)			IGA over IND (%)			ECGA over IND (%)			ISA over IND (%)			ECSA over IND (%)			ECSAGA over IND (%)		
	b	v	t	b	v	t	b	v	t	b	v	t	b	v	t	b	v	t
[0.4, 0.6)	32.47	-31.64	-17.49	31.09	-37.03	-21.99	28.21	-36.48	**-22.20**	28.72	-35.46	-21.24	27.04	-35.75	-21.86	28.78	-36.62	-22.17
[0.6, 0.8)	23.74	-32.56	-19.67	24.96	-36.78	-22.68	22.22	-36.07	**-22.73**	25.10	-35.37	-21.53	24.35	-36.32	-22.42	23.02	-36.06	-22.55
[0.8, 1.0)	21.63	-34.93	-21.87	23.00	-37.90	**-23.84**	22.16	-37.53	-23.74	23.98	-36.79	-22.76	22.15	-36.96	-23.32	20.91	-36.90	-23.54
[1.0, 1.2)	20.62	-35.90	-22.92	22.80	-38.11	**-24.11**	21.61	-37.68	-24.07	23.14	-37.42	-23.53	23.81	-38.08	-23.84	21.26	-37.26	-23.83
[1.2, 1.4)	21.40	-37.40	-23.46	22.89	-38.86	**-24.22**	21.22	-38.07	-23.99	23.11	-38.41	-23.82	20.97	-38.02	-24.03	20.49	-37.61	-23.81
[1.4, 1.6)	20.65	-39.84	-24.98	22.25	-40.92	**-25.42**	22.10	-40.75	-25.30	20.84	-40.12	-25.15	22.55	-40.35	-24.96	20.98	-39.94	-24.96
[1.6, 1.8)	22.81	-40.50	-25.13	23.57	-40.97	-25.30	22.29	-40.66	**-25.39**	23.69	-40.87	-25.20	22.98	-40.78	-25.32	23.23	-40.53	-25.06
[1.8, 2.0)	24.46	-42.78	-26.26	23.95	-42.66	-26.31	23.33	-42.62	**-26.44**	24.71	-42.90	-26.30	24.88	-42.86	-26.28	24.67	-42.36	-25.93
[2.0, 2.2)	26.75	-44.85	-27.38	26.78	-44.96	**-27.47**	25.81	-44.62	-27.47	26.74	-44.89	-27.42	26.95	-44.83	-27.37	27.29	-44.43	-26.96
[2.2, 2.4)	26.53	-45.35	-27.56	26.15	-45.28	-27.62	26.27	-45.39	**-27.66**	27.03	-45.55	-27.59	27.48	-45.45	-27.43	27.31	-44.81	-26.99
[2.4, 2.6)	27.37	-45.13	-27.19	27.50	-45.28	-27.28	27.25	-45.42	**-27.44**	27.14	-45.16	-27.24	28.49	-45.55	-27.24	28.76	-44.95	-26.70
[2.6, 2.8)	28.76	-47.11	-28.13	28.57	-47.09	**-28.16**	28.52	-47.05	-28.14	28.75	-47.15	-28.16	28.57	-47.05	-28.11	30.46	-46.80	-27.48
[2.8, 3.0)	28.90	-47.16	-27.96	28.99	-47.29	-28.04	28.85	-47.28	**-28.08**	29.20	-47.30	-28.00	29.17	-47.36	-28.03	29.30	-46.56	-27.41
[3.0, 3.2)	29.30	-48.29	-28.43	29.92	-48.73	-28.60	29.20	-48.39	-28.57	29.12	-48.28	-28.49	29.87	-48.67	**-28.62**	30.47	-47.95	-27.90
[3.2, 3.4)	29.93	-48.08	-28.15	29.87	-48.25	**-28.29**	29.51	-48.10	-28.26	30.17	-48.24	-28.20	29.98	-48.06	-28.11	30.09	-47.47	-27.64
[3.4, 3.6)	32.32	-48.91	-28.40	31.50	-48.81	**-28.55**	32.04	-48.88	-28.45	32.32	-49.01	-28.49	33.02	-49.29	-28.54	33.44	-48.53	-27.83
[3.6, 3.8)	30.94	-48.37	-28.04	31.16	-48.61	-28.17	30.36	-48.35	**-28.17**	31.96	-46.94	-26.73	31.47	-48.64	-28.11	33.32	-48.41	-27.47
[3.8, 4.0)	29.92	-47.11	-27.46	30.64	-47.49	**-27.55**	30.98	-47.56	-27.54	30.33	-47.33	-27.52	30.22	-47.23	-27.48	32.24	-47.12	-26.89
[4.0, 4.2)	29.73	-47.31	-27.51	29.30	-47.33	**-27.62**	30.23	-47.58	-27.57	38.41	-46.58	-24.73	31.82	-48.06	-27.51	32.07	-47.31	-26.89
[4.2, 4.4)	31.86	-47.98	-28.06	32.09	-48.07	-28.08	31.18	-47.71	-28.06	39.38	-46.48	-25.11	32.16	-48.08	**-28.09**	34.19	-47.89	-27.41

Table 3.20: Average Performance of CLU over IND for 50-buyer datasets

β	IGA over SYN (%)			ECGA over SYN (%)			ISA over SYN (%)			ECSA over SYN (%)			ECSAGA over SYN (%)		
	b	v	t	b	v	t	b	v	t	b	v	t	b	v	t
[0.4, 0.6)	-2.10	-5.50	-4.50	-2.92	-5.26	-4.59	-1.33	-3.98	-3.24	-0.86	-5.75	-4.30	-1.48	-5.80	-4.57
[0.6, 0.8)	-0.94	-4.48	-3.39	-2.37	-3.69	-3.32	0.19	-3.90	-2.54	1.19	-4.97	-3.05	-1.48	-3.99	-3.19
[0.8, 1.0)	1.93	-4.24	-2.38	0.86	-3.26	-2.18	1.83	-3.14	-1.82	0.87	-3.22	-2.06	0.36	-3.01	-2.02
[1.0, 1.2)	1.52	-2.86	-1.48	1.21	-2.20	-1.24	2.07	-1.67	-0.49	-0.16	-1.39	-1.04	1.66	-2.21	-1.12
[1.2, 1.4)	1.71	-2.17	-0.95	-0.34	-1.18	-0.87	2.02	-1.29	-0.20	0.41	-1.13	-0.62	-0.14	-0.77	-0.56
[1.4, 1.6)	1.45	-1.56	-0.60	1.43	-1.45	-0.52	0.29	-0.28	-0.10	1.32	-1.06	-0.22	2.70	-1.36	-0.04
[1.6, 1.8)	0.40	-0.64	-0.30	-1.13	0.36	-0.26	0.90	-0.67	-0.12	0.36	-0.18	-0.08	1.06	-0.20	0.05
[1.8, 2.0)	0.70	-0.46	-0.18	-0.91	0.25	-0.30	0.27	-0.21	-0.08	1.32	-0.51	0.04	0.75	-0.15	0.10
[2.0, 2.2)	-0.44	-0.07	-0.25	-0.09	-0.24	-0.25	-0.07	0.02	-0.03	1.17	-0.77	-0.05	1.53	-0.18	0.43
[2.2, 2.4)	0.91	-0.81	-0.15	-0.44	0.44	-0.04	0.06	0.04	-0.02	2.39	-1.36	0.01	3.29	-0.97	0.62
[2.4, 2.6)	-0.85	0.25	-0.21	-0.55	0.22	-0.04	-0.42	0.09	-0.12	-0.36	0.69	0.19	-0.23	0.76	0.42
[2.6, 2.8)	-0.08	-0.11	-0.09	-0.76	0.51	-0.05	-0.38	0.09	-0.13	0.12	0.06	0.07	0.40	0.62	0.47
[2.8, 3.0)	0.25	-0.39	-0.12	0.60	-0.50	-0.04	0.24	-0.26	-0.04	1.61	-0.72	0.17	2.76	-0.70	0.74
[3.0, 3.2)	-0.88	0.26	-0.23	-0.72	0.59	-0.04	-0.68	0.35	-0.09	0.92	-0.83	-0.14	0.16	0.96	0.67
[3.2, 3.4)	-0.16	-0.22	-0.22	-0.65	0.27	-0.24	-0.47	0.07	-0.19	1.01	-0.63	-0.02	0.24	0.94	0.58
[3.4, 3.6)	0.28	-0.65	-0.27	1.21	-0.86	0.00	-0.09	0.04	-0.04	1.23	-0.72	0.06	1.84	-0.04	0.72
[3.6, 3.8)	-0.23	-0.13	-0.21	-0.65	0.13	-0.23	-0.19	-0.15	-0.20	1.81	-1.61	-0.19	2.87	-0.91	0.67
[3.8, 4.0)	0.02	-0.15	-0.07	-1.16	1.04	0.04	0.04	-0.32	-0.15	-0.23	0.29	0.04	2.67	-0.65	0.80
[4.0, 4.2)	-0.85	0.17	-0.30	-1.87	1.25	-0.25	-0.54	0.12	-0.18	0.45	-0.40	-0.15	1.16	0.60	0.77
[4.2, 4.4)	0.43	-0.43	-0.11	0.28	-0.22	-0.02	1.95	-0.34	0.61	0.30	-0.41	-0.18	1.22	0.76	0.87

Table 3.21: Average Performance of CLU over SYN for 30-buyer datasets

β	IGA over SYN (%)			ECGA over SYN (%)			ISA over SYN (%)			ECSA over SYN (%)			ECSAGA over SYN (%)		
	b	v	t	b	v	t	b	v	t	b	v	t	b	v	t
[0.4, 0.6)	-0.93	-7.88	-5.45	-3.12	-7.07	-5.71	-2.72	-5.59	-4.55	-4.01	-6.02	-5.30	-2.65	-7.25	-5.67
[0.6, 0.8)	1.14	-6.19	-3.73	-1.06	-5.19	-3.80	1.30	-4.09	-2.30	0.59	-5.58	-3.41	-0.42	-5.16	-3.58
[0.8, 1.0)	1.15	-4.58	-2.52	0.51	-4.00	-2.39	2.00	-2.87	-1.13	0.47	-3.11	-1.84	-0.49	-3.01	-2.14
[1.0, 1.2)	1.86	-3.41	-1.54	0.89	-2.79	-1.49	2.08	-2.41	-0.79	2.71	-3.38	-1.19	0.61	-2.09	-1.17
[1.2, 1.4)	1.26	-2.32	-0.99	-0.06	-1.05	-0.68	1.45	-1.58	-0.46	-0.27	-0.94	-0.72	-0.65	-0.29	-0.45
[1.4, 1.6)	1.33	-1.78	-0.58	1.21	-1.54	-0.42	0.15	-0.47	-0.21	1.57	-0.82	0.05	0.29	-0.17	0.04
[1.6, 1.8)	0.61	-0.81	-0.23	-0.35	-0.16	-0.34	0.73	-0.58	-0.09	0.15	-0.45	-0.25	0.40	0.05	0.10
[1.8, 2.0)	-0.42	0.21	-0.07	-0.88	0.31	-0.25	0.22	-0.19	-0.05	0.33	-0.08	-0.03	0.18	0.76	0.45
[2.0, 2.2)	0.03	-0.19	-0.12	-0.72	0.45	-0.12	-0.01	-0.09	-0.05	0.15	0.02	0.02	0.44	0.81	0.59
[2.2, 2.4)	-0.28	0.13	-0.08	-0.17	-0.08	-0.14	0.41	-0.37	-0.03	0.79	-0.18	0.18	0.64	0.97	0.80
[2.4, 2.6)	0.12	-0.26	-0.13	-0.04	-0.50	-0.35	-0.14	-0.05	-0.08	0.96	-0.70	-0.08	1.15	0.35	0.67
[2.6, 2.8)	-0.14	0.05	-0.04	-0.16	0.12	-0.01	0.01	-0.07	-0.04	-0.10	0.12	0.03	1.35	0.61	0.90
[2.8, 3.0)	0.08	-0.27	-0.11	0.00	-0.19	-0.15	0.24	-0.26	-0.04	0.29	-0.31	-0.08	0.37	1.15	0.77
[3.0, 3.2)	0.53	-0.78	-0.24	-0.03	-0.13	-0.20	-0.12	0.07	-0.09	0.48	-0.65	-0.27	0.98	0.76	0.74
[3.2, 3.4)	-0.04	-0.32	-0.20	-0.25	0.02	-0.16	0.20	-0.30	-0.07	0.11	0.08	0.05	0.21	1.25	0.70
[3.4, 3.6)	-0.57	0.22	-0.20	-0.12	0.12	-0.07	0.03	-0.19	-0.12	0.57	-0.77	-0.20	0.96	0.82	0.80
[3.6, 3.8)	0.19	-0.46	-0.17	-0.42	0.05	-0.18	0.79	2.75	1.82	0.43	-0.49	-0.10	1.83	-0.05	0.79
[3.8, 4.0)	0.56	-0.71	-0.13	0.81	-0.86	-0.10	0.32	-0.42	-0.09	0.24	-0.25	-0.03	1.79	0.00	0.79
[4.0, 4.2)	-0.32	-0.02	-0.16	0.43	-0.48	-0.09	6.67	1.44	3.83	1.65	-1.41	0.00	1.85	0.01	0.84
[4.2, 4.4)	0.20	-0.16	-0.02	-0.51	0.53	0.01	5.70	2.90	4.14	0.24	-0.16	-0.03	1.80	0.19	0.91

Table 3.22: Average Performance of CLU over SYN for 50-buyer datasets

Performance over IND

For the 30-buyer cases and the 50-buyer cases, the average performance of the CLU over the IND for each β intervals are shown in Tables 3.19 and 3.20, respectively. (Italic for b, v and t stand for the cost of the buyers, the vendor and the system, respectively.) And for each β interval, the CLU method with the best average performance is in bold type.

For the 500 datasets of 30 buyers, the system cost reduction percentage of the CLU model over the IND model increases from 20.72 to 29.72 when β increases from 0.4 to 4.4. And as shown in Table 3.19, the IGA has the best group-average performance of total system cost in 13 out of 20 β groups.

For another 500 datasets of 50 buyers, Table 3.20 shows that the improvement of the CLU over the IND increases from 22.20% to 28.09% as β increases from 0.4 to 4.4. And among different algorithms for CLU model, the GAs (IGA and ECGA) achieve the maximum average system cost reduction percentage in 18 β groups.

Performance over SYN

The average performance of CLU over SYN of the two cases are shown in Tables 3.21 and 3.22, respectively. In both tables, it can be seen that the CLU tends to have the same cost of SYN or even higher cost than that of SYN as β increases. As for the small values of β, i.e., in the first five groups, the CLU model works better than SYN by 0.95-4.59% and 0.99-5.71%, respectively. Especially in the first β group, where ECGA performs the best, not only the vendor's cost but also the buyers' cost has been reduced by an average of 2.92% for the 30-buyer cases and 3.12% for the 50-buyer cases, respectively.

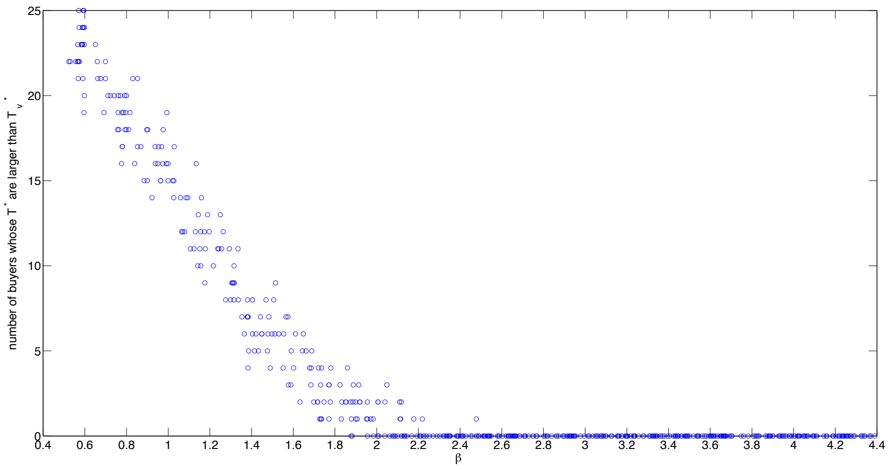

Figure 3.6: T^* structure for the 30-buyer cases

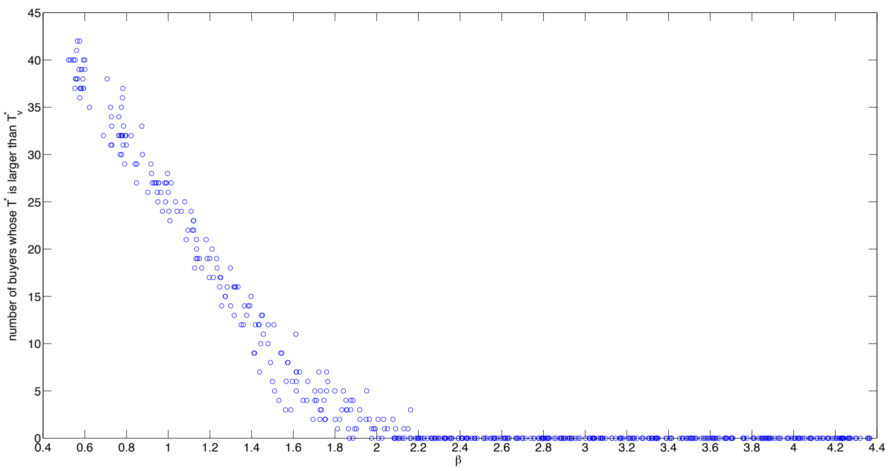

Figure 3.7: T^* structure for the 50-buyer cases

T^* structure for the 30-buyer cases and 50-buyer cases are shown in Figures 3.6 and 3.7, respectively. In both figures, the number of buyers whose T^* are larger than vendor's T_v^* gradually decreases when β increases. Especially, there is no buyer's T^* larger than T_v^* when β ranges from 2.6 to 4.4, implying that the buyers in these datasets can not be distinctly classified into SCP and LCP groups, which is the most important characteristics of CLU model differing from SYN model. As a result, the CLU only outperforms the SYN up to 0.3% for the last nine β groups, as shown in Tables 3.21 and 3.22. On the other hand, for the datasets with β from 0.4 to 1, where the CLU is significantly better than the SYN, it can be seen from Figures 3.6 and 3.7 that almost half of the buyers are assigned to LCP buyers, which has fully reflected the nature of CLU model, i.e. increasing the flexibility of the system so as to improve the system cost.

Computational time

As shown in the results of Examples 1-6, the CPU time required for the problems with same buyer size varies greatly. The reason for this is also related to the structure of problem data.

For the 30-buyer cases and 50-buyer cases, the average CPU time of SYN and CLU for each β interval are presented in Table 3.23 and Table 3.24. As β gradually increases, the computational time of SYN also tends to increase, while the computation time for all kinds of CLU algorithms decrease sharply due to the reduction in computation complexity, i.e. the gene length in CLU is $2n^s + 3n^l + 1 = 2n + n^l + 1$ and n^l becomes smaller when β gets larger.

β	SYN	CLU				
		IGA	ECGA	ISA	ECSA	ECSAGA
$[0.4, 0.6)$	18.10	1790.99	377.51	561.54	139.30	408.60
$[0.6, 0.8)$	20.05	2097.83	398.06	529.12	107.74	343.11
$[0.8, 1.0)$	21.67	1933.40	291.54	451.23	75.00	241.30
$[1.0, 1.2)$	24.67	1848.85	172.79	315.66	54.38	181.63
$[1.2, 1.4)$	23.95	2099.20	154.02	313.29	37.82	102.23
$[1.4, 1.6)$	26.51	1909.11	87.60	266.24	27.57	82.85
$[1.6, 1.8)$	28.44	2023.04	47.85	268.08	16.76	55.00
$[1.8, 2.0)$	28.32	1945.86	29.66	243.84	13.03	41.31
$[2.0, 2.2)$	30.36	1819.25	27.29	212.01	12.26	40.99
$[2.2, 2.4)$	32.40	1585.53	23.94	218.05	11.10	37.77
$[2.4, 2.6)$	29.25	1528.81	23.28	181.01	11.21	27.42
$[2.6, 2.8)$	37.37	1396.50	24.08	148.01	10.98	28.08
$[2.8, 3.0)$	35.52	854.82	23.49	105.56	11.54	33.13
$[3.0, 3.2)$	38.06	527.79	23.04	87.18	11.22	32.71
$[3.2, 3.4)$	36.15	678.10	23.76	103.83	11.62	32.68
$[3.4, 3.6)$	37.91	454.21	24.23	102.43	12.91	31.92
$[3.6, 3.8)$	38.58	267.20	24.76	90.95	12.33	31.74
$[3.8, 4.0)$	39.61	251.51	24.64	56.39	11.58	33.16
$[4.0, 4.2)$	38.66	192.15	24.37	49.24	11.69	35.42
$[4.2, 4.4)$	41.55	148.15	25.24	101.82	11.27	34.72

Table 3.23: CPU Time (s') of SYN and CLU for the 30-buyer cases

3.9 Conclusions

In this research, an integrated single-vendor multi-buyer synchronization supply chain model involving clustering of buyers with long and short ordering cycles is proposed. In this new model, the buyers are classified either as short-cycle preferred (SCP) buyers or long-cycle preferred (LCP). Then the ordering cycles of both groups of buyers are coordinated with the vendor's production cycle such that the ordering cycles of the SCP buyers and the LCP buyers must be integer factors and integer multiples of the vendor's production cycle, respectively. Three different heuristic algorithms are used for finding the optimal system cost of the model. Numerical

β	SYN	CLU				
		IGA	ECGA	ISA	ECSA	ECSAGA
$[0.4, 0.6)$	62.33	6890.83	1770.75	1599.42	344.00	1635.27
$[0.6, 0.8)$	73.72	6821.76	1281.86	1632.82	272.12	1174.99
$[0.8, 1.0)$	76.07	6755.43	1012.89	1143.89	218.64	765.76
$[1.0, 1.2)$	72.76	5806.12	765.88	1239.87	150.82	558.36
$[1.2, 1.4)$	83.84	6234.04	511.22	1110.16	101.78	382.74
$[1.4, 1.6)$	84.29	5894.23	314.11	998.51	57.83	251.41
$[1.6, 1.8)$	84.93	5610.54	153.97	626.46	31.91	178.51
$[1.8, 2.0)$	89.30	5210.36	86.12	638.55	22.45	116.89
$[2.0, 2.2)$	88.73	3536.92	62.88	408.02	20.84	93.64
$[2.2, 2.4)$	92.44	5038.68	58.74	416.40	22.08	90.77
$[2.4, 2.6)$	92.51	3691.89	51.80	366.81	21.38	94.56
$[2.6, 2.8)$	104.40	2666.53	48.34	280.04	20.99	94.47
$[2.8, 3.0)$	98.91	3219.44	56.41	254.49	21.81	92.78
$[3.0, 3.2)$	102.10	2613.26	58.25	228.02	21.16	85.75
$[3.2, 3.4)$	93.46	1177.44	60.52	137.16	21.00	85.20
$[3.4, 3.6)$	94.67	890.58	57.40	120.20	21.86	83.24
$[3.6, 3.8)$	102.20	471.17	57.20	151.33	20.41	82.78
$[3.8, 4.0)$	101.96	333.37	58.65	91.32	19.70	85.60
$[4.0, 4.2)$	99.95	296.41	59.37	158.35	20.14	76.14
$[4.2, 4.4)$	109.38	362.38	60.30	242.03	19.80	87.62

Table 3.24: CPU Time (s') of SYN and CLU for the 50-buyer cases

experiments are carried out to test the performance of this model when compared to the independent model and general synchronized model as well.

The overall performance of the clustering synchronized cycles model obtained by IGA, ECGA, ISA, ECSA and ECSAGA are always better than that of the independent model. For the comparison with the synchronized cycles model, the improvement obtained by the clustering model depends on the data structure. When β, the economic cycle ratio, is in the range of 0.4-1.4, the improvement of CLU over SYN varies from 1% to 5% for 30-buyer and 50-buyer problem, otherwise the solutions obtained from CLU model and SYN model are approximately the same. In the CLU model with small β, the vendor still gets a significant reduction over SYN due to large savings on the processing and holding cost while the total buyers' cost almost stays the same. And in the situation when β is small enough, both buyers' cost and vendor's cost can be reduced by the CLU model when compared with SYN since the ordering cycles of most buyers are closer to their economic ordering cycles. The numerical Examples 1, 3 and 5 indicate that the buyer with larger economic ordering cycle always benefits from the clustering mechanism, i.e. his cost gets closer to the minimum cost in IND. Furthermore, the LCP buyer whose economic ordering cycle is several times of the vendor's economic production cycle, almost achieves the same cost as in IND, while the costs for SCP buyers in CLU and SYN are approximately the same. Thus when β is small enough, the superiority for CLU model comes from three parts, (a) vendor's cost is significantly reduced when comparing both to IND and SYN, (b) LCP buyers' costs are closer to IND when comparing to SYN, (c) SCP buyers' costs is slightly changed when comparing to SYN.

Five algorithms are proposed to solve the CLU problem. For the datasets with small β, i.e., from 0.4 to 1.4, IGA usually obtains the minimum system cost but requires the maximum CPU time. In addition, comparing to SAs (ISA and ECSA), GAs (IGA and ECGA) achieve about 2% lower system costs while using multiples of CPU time. The multiple increases when the number of buyer increases. And the computational time required by the incremental-clustering (IGA and ISA) is longer than that of economic-cycle-clustering (ECGA and ECSA) and the results of these two clustering approach with the same heuristic algorithm are almost the same. So as a trade-off between the optimality and computational time, ECSAGA has a better system cost than SAs and is faster than GAs.

Chapter 4

Synchronized Co-ordination for an Integrated Production-Warehouse Location-Inventory Supply Chain

4.1 Introduction

Enterprises' competitiveness can be improved through a successful supply chain design, which can be roughly decomposed into three planning levels, i.e., strategic structural decisions, tactical inventory decisions and operational decisions on the shipments of goods in the network. In traditional supply chain management, the decisions variables at different levels are optimized sequentially. Since major capital often goes to the strategic location planning and the location decisions have a profound impact on the other two levels, optimizing the location of facilities can make a considerate improvement in the system cost. In recent decades, various coordination models, aimed to reduce the total system cost, have been proposed to integrate location problem and other supply chain functions: location-inventory models and location-routing models. So far, little attention have been paid to the integration of more than two functions due to the great complexity of the problem.

This research considers a co-ordinated three-layer production-warehouse location-inventory supply chain model, consisting of one manufacturer, multiple potential warehouses and multiple heterogenous retailers, where several warehouses will be open and each warehouse is served directly by the manufacturer and delivers products to the downstream retailers. The goal is to determine a subset of the potential warehouses to be open, an assignment of the retailers to these open warehouses and an replenishment policy for the supply chain so as to minimize the overall total system cost.

Although the objective functions of most of the three-layer location-inventory models have incorporated the location cost, shipment cost and inventory cost. The inventory cost only incurs at participants at the median layer rather than the whole system and the cost at the top layer is ignored. This may lead to sub-optimality, since the supply chain is not treated as a whole.

In contrast to other location-inventory models, the production process is incorporated into the proposed model when minimizing the total system cost. Furthermore, since the location decisions are involved, the proposed model is considered at the strategic level and the supply chain structure is not predetermined when comparing to the general integrated production-inventory models. Meanwhile, to coordinate inventory replenishment decisions, a synchronization mechanism is implemented: the ordering cycle of each open warehouse is an integer factor of the manufacturer's production cycle, and in turn, the ordering cycle of each retailer is an integer factor of the ordering cycle of the assigned warehouse.

The organization of the rest of this chapter is as follows: Section 4.2 provides the assumptions and notations applied in this research. Section 4.3 provides a production-warehouse location-inventory (PLI) supply chain model under independent policy, based on the classical economic order quantity (EOQ) model. Section 4.4 proposes a synchronized production-warehouse location-inventory model. Section 4.5 discusses algorithms for finding the total minimum system cost of the proposed model. Section 4.6 presents the numerical results. Conclusions are given in Section 4.7.

4.2 Assumptions and Notations

4.2.1 Assumptions

Throughout this research, the following assumptions are made.

- Constant and deterministic demand rate.
- Constant production rate larger than demand rate.
- Constant cost parameters.
- Finite planning horizon.
- No shortage allowed throughout the whole planning horizon.
- No lead time for each order delivery.
- No parallel production allowed.
- Euclidean distance between participants.
- Unlimited warehouse capacity.
- Shipment cost per delivery proportional to the distance.

4.2.2 Notations

Basic parameters:

p: The number of potential warehouses.

n: The number of buyers.

M: Manufacturer.

W: Set of warehouses, consisting of $\{W_1,\ W_2,\ \dots,\ W_p\}$.

R: Set of retailers, consisting of $\{R_1,\ R_2,\ \dots,\ R_n\}$.

E: Nonnegative and symmetric distance matrix satisfying the triangle inequality, $E = \{e_{ij}\}$, $i, j \in M \bigcup W \bigcup R$.

ω: Transportation cost per unit distance.

q_i: Demand rate of retailer R_i, for $i = 1, \dots, n$.

A_i^R: Ordering cost per order of retailer R_i, for $i = 1, \dots, n$.

h_i^R: Holding cost per item per unit time of retailer R_i, for $i = 1, \dots, n$.

C_{ij}^W: Shipping cost per order from warehouse W_i to retailer R_j, for $i = 1, \dots, p$, $j = 1, \dots, n$.

q_i^W: Demand rate of warehouse W_i, for $i = 1, \dots, p$.

O_i^W: Fixed operation cost of open warehouse W_i per unit time, for $i = 1, \dots, p$.

h_i^W: Holding cost per item per unit time of warehouse W_i, for $i = 1, \dots, p$.

A_i^W: Ordering cost per order of warehouse W_i, for $i = 1, \dots, p$.

C_i^M: Shipping cost per delivery from manufacturer to warehouse W_i, for $i = 1, \dots, p$.

P: Manufacturer's production rate.

M_s: Manufacturer's set-up cost per production.

h: Manufacturer's holding cost per item per unit time.

q^M: Total demand rate faced by the manufacturer.

$Q(R_i)$: Order quantity for retailer R_i per order, for $i = 1, \ldots, n$.

$Q(W_i)$: Order quantity for warehouse W_i per order, for $i = 1, \ldots, p$.

$Q(M)$: Batch quantity per production run of the manufacturer.

α: Demand-Production ratio.

Independent policy:

z_i^*: Indicating warehouse W_i open or not under independent facility location problem, equals to one if warehouse W_i is open and zero otherwise.

y_{ij}^*: Indicating the assignment between retailers and warehouses under independent facility location problem, equals to one if retailer R_j is assigned to warehouse W_i, for $i = 1, \ldots, p$, $j = 1, \ldots, n$.

T_i^{R*}: Economic order cycle for retailer R_i, for $i = 1, \ldots, n$.

T_i^{W*}: Economic order cycle for warehouse W_i, for $i = 1, \ldots, p$.

T^{M*}: Economic production cycle for manufacturer.

$Q(R_i)^*$: Economic order quantity for retailer R_i, for $i = 1, \ldots, n$.

$Q(W_i)^*$: Economic order quantity for warehouse W_i, for $i = 1, \ldots, p$.

$Q(M)^*$: Economic batch quantity for manufacturer.

$C(R_i)^{IND}$: Minimum cost per unit time for retailer R_i, for $i = 1, \ldots, n$.

$C(W_i)^{IND}$: Minimum cost per unit time for warehouse W_i, for $i = 1, \ldots, p$.

$C(M)^{IND}$: Minimum cost per unit time for the manufacturer.

TC^{IND}: Total system cost per unit time.

Synchronized cycles model:

T: Basic time unit.

z_i: Equals to one if warehouse W_i is open and zero otherwise.

y_{ij}: Indicating the assignment between retailers and warehouses, equals to one if retailer R_j is assigned to warehouse W_i, for $i = 1, \ldots, p$, $j = 1, \ldots, n$.

NT: Manufacturer's production cycle.

$-ST$: The time that the production starts.

FT: The time that the production ends.

b: Nearest integer below F, where F is the F^{th} basic unit time.

k_iT: Ordering cycle of warehouse W_i, for $i = 1, \ldots, p$.

$\delta_{i,t}$: Equals to one if warehouse W_i is served in period tT and zero otherwise, for $i = 1, \ldots, p$.

s_iT: Ordering cycle of retailer R_i, for $i = 1, \ldots, n$.

$\eta_{i,t}$: Equals to one if retailer R_i is served in period tT and zero otherwise, for $i = 1, \ldots, p$.

Ψ: Surplus stock at the manufacturer when $t = 1$.

$D^W_{i,t}$: Ordering quantity at warehouse W_i in period tT, for $i = 1, \ldots, p$.

D^M_t: Ordering quantity at manufacturer in period tT.

π^W_i: Area under the inventory curve of warehouse W_i in one ordering cycle, $i = 1, \ldots, p$.

π^M: Area under the inventory curve of the manufacturer in one production cycle.

$C(R_i)^{SYN}$: Total cost per unit time for retailer R_i, for $i = 1, \ldots, n$.

$C(W_i)^{SYN}$: Total cost per unit time for warehouse W_i, for $i = 1, \ldots, p$.

$C(M)^{SYN}$: Total cost per unit time for the manufacturer.

TC^{SYN}: Total system cost per unit time.

4.3 Independent Policy Model

In this research, when warehouse W_i $(i = 1, \ldots, p)$ is selected to be open, a fixed operation cost O_i^W will be incurred and the downstream retailers of this warehouse are then determined based on the shortest distance policy, i.e., each retailer is assigned to the nearest open warehouse. During each manufacturer's production cycle, retailer R_i $(i = 1, \ldots, n)$ faces a deterministic demand at q_i per unit time, incurs an ordering cost A_i^R per order and incurs an inventory holding cost h_i^R per unit item per unit time. Each open warehouse W_i $(i = 1, \ldots, p)$ faces a combined demand q_i^W from the downstream retailers per unit time and orders from the manufacturer, incurs an ordering cost A_i^W per order and incurs an inventory holding cost h_i^W per unit item per unit time. Shipment cost per delivery from warehouse W_i to retailer R_j, denoted by C_{ij}^W, is proportional to the distance between W_i and R_j, i.e. $C_{ij}^W = \omega e_{W_i R_j}$, where ω is the transportation cost per unit distance. And each delivery from manufacturer to warehouse W_i incurs a shipment cost C_i^M, where $C_i^M = \omega e_{MW_i}$. In independent policy, this problem is solved sequentially by solving two sub-problems, i.e. warehouse location problem at the strategic level and inventory problem for each participant at the tactical level. The manufacturer has to make decisions at both the strategic level and the tactical level, i.e. (1) the warehouse location and the assignment of the customers are determined based on the operation cost for each warehouse, transportation cost per unit distance and the locations of all the participants in this supply chain system, (2) the production batch size and production cycle are determined by the economic production quantity model.

4.3.1 Warehouse Location Problem

The warehouse location problem we consider is as follows: Given one manufacturer M, a set of potential warehouses W and a set of retailers R, the objective is to decide which warehouses should be open and how to assign the retailers to the open warehouses so as to minimize the total system cost which consists of the warehouse operation cost and the shipment cost incurred when goods are delivered in the two echelons. A comprehensive introduction for the uncapacitated facility location problem can be found in Cornuéjols et al. (1983). The integer programming for the warehouse location (WL) problem in our model is formulated as follows:

$$\min \quad F = \sum_{i \in W} z_i O_i^W + \sum_{i \in W} z_i C_i^M + \sum_{j \in R} \sum_{i \in W} C_{ij}^W y_{ij} \tag{4.1}$$

$$\text{(WL) s.t.} \quad \sum_{i \in W} y_{ij} = 1 \qquad \forall j \in R \tag{4.2}$$

$$y_{ij} \leqslant z_i \qquad \forall i \in W,\ j \in R \tag{4.3}$$

$$y_{ij} = 0,\ 1 \qquad \forall i \in W,\ j \in R \tag{4.4}$$

$$z_i = 0,\ 1 \qquad \forall i \in W \tag{4.5}$$

The objective function in Eq.(4.1) is to minimize the sum of (1) warehouse operation cost, (2) distance cost from manufacturer to the open warehouses and, (3) distance cost from the open warehouses to their assigned retailers. Constraint (4.2) states that each retailer is only served by one warehouse. Constraint (4.3) ensures that each retailer can be assigned to an open warehouse. Constraints (4.4) and (4.5) ensure the decision variables z_i and y_{ij} to be binary. $z_i = 1$ indicates that warehouse W_i will be open and $y_{ij} = 1$ indicates that retailer R_j is assigned to warehouse W_i.

The optimal solution for this WL problem is denoted as z_i^* and y_{ij}^*.

4.3.2 Economic Cycles for Each Participant

If every participant in the supply chain operates independently, then retailer R_i at the bottom layer will order a quantity $Q(R_i)$ every T_i^R units of time. The total cost per unit time for the retailer R_i, denoted by $C(R_i)$, is as follows:

$$C(R_i) = \frac{A_i^R}{T_i^R} + \frac{h_i^R q_i T_i^R}{2} \tag{4.6}$$

The economic order interval and economic order quantity for retailer R_i, denoted by T_i^{R*} and $Q(R_i)^*$ respectively, are

$$T_i^{R*} = \sqrt{\frac{2A_i^R}{h_i^R q_i}} \tag{4.7}$$

and

$$Q(R_i)^* = q_i T_i^{R*} = \sqrt{\frac{2A_i^R q_i}{h_i^R}}. \tag{4.8}$$

The minimum total cost per unit time for retailer R_i, denoted by $C(R_i)^{IND}$, is

$$C(R_i)^{IND} = \sqrt{2A_i^R h_i^R q_i}. \tag{4.9}$$

Since each open warehouse faces a combined order from its downstream retailers, given the z_i^* and y_{ij}^* obtained from the WL problem, warehouse W_i has to satisfy a demand occurs at a rate of q_i^W per unit time, where

$$q_i^W = \sum_{j \in R} y_{ij}^* q_j. \tag{4.10}$$

Under the independent policy, warehouse W_i will order a quantity $Q(W_i)$ from the manufacturer every T_i^W units of time, and also need to carry a safety stock to guarantee there is no stock out. The maximum demand may occur when its retailers order their goods at the same time. So, the warehouse W_i has to carry $\sum_{j \in R} y_{ij}^* Q(R_i)^*$ items as buffer stock. And a shipment cost C_{ij}^W will be incurred when there is delivery from warehouse W_i to retailer R_j. The total cost per unit time for the warehouse W_i, denoted by $C(W_i)$, is as follows:

$$C(W_i) = \frac{A_i^W}{T_i^W} + \frac{h_i^W q_i^W T_i^W}{2} + h_i^w \sum_{j \in R} y_{ij}^* Q(R_i)^* + \sum_{j \in R} \frac{y_{ij}^* C_{ij}^W}{T_i^{R*}} \tag{4.11}$$

The economic order interval and economic order quantity for warehouse W_i, denoted by T_i^{W*} and $Q(W_i)^*$ respectively, are

$$T_i^{W*} = \sqrt{\frac{2A_i^W}{h_i^W q_i^W}} \tag{4.12}$$

and

$$Q(W_i)^* = q_i^W T_i^{W*} = \sqrt{\frac{2A_i^W q_i^W}{h_i^W}}. \tag{4.13}$$

The minimum total cost per unit time for warehouse W_i, denoted by $C(W_i)^{IND}$, is

$$C(W_i)^{IND} = \sqrt{2A_i^W h_i^W q_i^W} + h_i^w \sum_{j \in R} y_{ij}^* Q(R_i)^* + \sum_{j \in R} \frac{y_{ij}^* C_{ij}^W}{T_i^{R*}}. \tag{4.14}$$

The manufacturer at the top layer of the supply chain faces the orders from all the open warehouses with demand rates $q_1^W,\ q_2^W,\ \ldots,\ q_p^W$ per unit time respectively, the demand rate faced by the manufacturer, denoted by q^M, is

$$q^M = \sum_{i \in W} q_i^W. \tag{4.15}$$

We assume the manufacturer's production rate is P, where $P > q^M$. We also assume that the manufacturer incurs a set-up cost of M_s per production run, a holding cost of h per unit item per unit time and a shipment cost of C_i^M per order received from warehouse W_i ($\imath = 1, \ldots, p$). Similarly, if the manufacturer acts independently, a large safety stock $\sum_{i \in W} Q(W_i)^*$ has to be carried so as to guarantee that there will not be any stock outs. The manufacturer starts a production run every T^M units of time and produces a batch size $Q(M)$, where $Q(M) = q^M T^M$. The total cost per unit time for the manufacturer, denoted by $C(M)$, is

$$C(M) = \frac{M_s q^M}{Q(M)} + \frac{hQ(M)}{2}(1 - \frac{q^M}{P}) + \sum_{i \in W} \frac{C_i^M q_i^W}{Q(W_i)^*} + h \sum_{i \in W} Q(W_i)^*. \tag{4.16}$$

The manufacturer's economic batch quantity and economic production cycle, denoted by $Q(M)^*$ and T^{M*} respectively, are

$$Q(M)^* = \sqrt{\frac{2M_s q^M}{h(1 - \frac{q^M}{P})}} \tag{4.17}$$

and

$$T^{M*} = \sqrt{\frac{2M_s}{hq^M(1-\frac{q^M}{P})}}. \tag{4.18}$$

The minimum total cost per unit time for manufacturer, denoted by $C(M)^{IND}$, is

$$C(M)^{IND} = \sqrt{2M_s hq^M(1-\frac{q^M}{P})} + \sum_{i\in W} \frac{C_i^M q_i^W}{Q(W_i)^*} + h\sum_{i\in W} Q(W_i)^*. \tag{4.19}$$

4.3.3 Total System Cost

When the decisions for the WL problem and the replenishment cycle for each participant are optimal, the total system cost for the independent policy per unit time, including the cost for warehouse operation and the cost occurred in the production, ordering and shipment processes, denoted by TC^{IND}, is

$$\begin{aligned} TC^{IND} &= \sum_{i\in W} z_i^* O_i^W + C(M)^{IND} + \sum_{i\in W} C(W_i)^{IND} + \sum_{i\in R} C(R_i)^{IND} \\ &= \sum_{i\in W} z_i^* O_i^W + \sqrt{2M_s hq^M(1-\frac{q^M}{P})} + \sum_{i\in W} \frac{C_i^M}{T_i^{W*}} + h\sum_{i\in W} q_i^W T_i^{W*} \\ &\quad + \sum_{i\in W} \sqrt{2A_i^W h_i^W q_i^W} + \sum_{i\in W}\sum_{j\in R} \left(h_i^W y_{ij}^* q_i T_i^{R*} + \frac{y_{ij}^* C_{ij}^W}{T_i^{R*}} \right) \\ &\quad + \sum_{i\in R} 2A_i^R h_i^R q_i. \end{aligned} \tag{4.20}$$

4.4 Synchronized Production-Warehouse Location-Inventory Model

In the independent policy, where the decisions are made at a strategic level first, choosing a set of warehouses to be open, and then the production cycle or the ordering cycles for the participants at each layer of the supply chain are determined separately so as to minimize the total cost incurred at each individual, the impact of the location decisions on inventory and shipment costs are ignored, and both the warehouses and manufacturer have to carry a large safety stock along the whole planning horizon. Integrating the warehouse location problem and the production-inventory problem in a systemic way may improve the overall performance of the supply chain. Meanwhile, coordination implemented to the timing of shipments of the last two layers and the production of the manufacturer may significantly reduce the amount of stock held at the manufacturer and the warehouses, which would lead to a further reduction of the total system cost. It is shown in Chan and Kingsman (2007) that the system performance of forcing all participants at one layer adopting a common ordering cycle is not as good as that allowing them to have their own ordering cycles.

Following this line of thought, we develop a synchronized production-warehouse location-inventory model for a three-layer supply chain in this section. For each warehouse, there is a binary decision variable z_i indicating the warehouse is open or not and binary decision variables y_{ij} indicating the set of retailers assigned to this warehouse, where

$$z_i = \begin{cases} 1, & \text{if warehouse } W_i \text{ is open} \\ 0, & \text{otherwise} \end{cases}, \ \forall i \in W \tag{4.21}$$

and

$$y_{ij} = \begin{cases} 1, & \text{if retailer } R_j \text{ is assigned to warehouse } W_i \\ 0, & \text{otherwise} \end{cases}, \forall i \in W, \forall j \in R \tag{4.22}$$

Since each retailer can be served by one and only one open warehouse, constraints

(4.2) and (4.3) should be satisfied.

Let T be some basic time unit and production cycle of the manufacturer be NT, where $N \leqslant 365$ is an integer. For an open warehouse W_i, an ordering cycle k_iT $(i = 1, \ldots, p)$ is adopted, satisfying

$$k_i | N, \ \forall i \in \{i | i \in W, z_i = 1\} \tag{4.23}$$

And in turn, the ordering cycle for retailer R_j, denoted by $s_jT(j = 1, \ldots, n)$, is an integer factor of ordering cycle of the warehouse at the adjacent upstream stage, thus

$$s_j | \sum_{i \in W} y_{ij} k_i, \ \forall j \in R \tag{4.24}$$

So the the ordering quantity of retailer R_i and warehouse W_i per order, are

$$Q(R_i) = q_i s_i T, \ \forall i \in R \tag{4.25}$$

and

$$Q(W_i) = k_i T \sum_{j \in R} y_{ij} q_j, \ \forall i \in W \tag{4.26}$$

respectively. And the batch quantity of the manufacturer in one production run, is

$$Q(M) = NT \sum_{i \in R} q_i. \tag{4.27}$$

Assume that the manufacturer cycle starts at time 0 after satisfying all the demands from his previous cycle. Let a production run start at time $-ST$, a time ST

before the start of the manufacturer cycle, where S may be positive or negative. Also assume that the manufacturer's production rate is P items per unit time. Production stops at time FT, where F is not necessarily an integer. Let b be the nearest integer below F.

The manufacturer's stock, after having satisfied the demands in one production cycle, becomes zero at the end of the cycle. To give more flexibility to the system, it is not desirable to force all the orders to be made at the beginning of each cycle. Let

$$\eta_{i,t} = \begin{cases} 1, & \text{if retailer } R_i \text{ places an order in period } tT \\ 0, & \text{otherwise} \end{cases}, \quad i = 1, 2, \ldots, n \tag{4.28}$$

and

$$\delta_{i,t} = \begin{cases} 1, & \text{if warehouse } W_i \text{ places an order in period } tT \\ 0, & \text{otherwise} \end{cases}, \quad i = 1, 2, \ldots, p. \tag{4.29}$$

Only open warehouses can order from the manufacturer, so

$$\max_{t=1,\ldots,N} \delta_{i,t} = z_i, \ \forall i \in W \tag{4.30}$$

Since retailer R_i and open warehouse W_i order every s_iT and k_iT units of time, respectively, we have

$$\eta_{i,t+s_i} = \eta_{i,t}, \ \forall i \in R \tag{4.31}$$

and

$$\delta_{i,t+s_i} = \delta_{i,t},\ \forall i \in \{i|i \in W, z_i = 1\}. \tag{4.32}$$

Retailer R_i and open warehouse W_i order only once in each s_iT and k_iT units of time, respectively, so

$$\sum_{j=0}^{s_i-1} \eta_{i,t+j} = 1,\ \forall i \in R \tag{4.33}$$

and

$$\sum_{j=0}^{k_i-1} \delta_{i,t+j} = 1,\ \forall i \in \{i|i \in W, z_i = 1\}. \tag{4.34}$$

The first ordering time of retailer R_i and warehouse W_i are τ_iT and λ_iT time units away from the beginning of their cycles, respectively, satisfying

$$\tau_i \leqslant s_i,\ \forall i \in R \tag{4.35}$$

and

$$\lambda_i \leqslant k_i,\ \forall i \in \{i|i \in W, z_i = 1\}. \tag{4.36}$$

Since the ordering cycles for warehouses are synchronized with manufacturer's production cycle, which starts at time 0, the relationship between λ_i and $\delta_{i,t}$ is

$$\delta_{i,\lambda_i} = 1,\ \forall i \in \{i|i \in W, z_i = 1\}. \tag{4.37}$$

. However, since the goods can only be delivered to retailer after the upstream warehouse has received the goods from the manufacturer, to guarantee that there will not be any shortage at warehouse, we have

$$\eta_{j,\tau_j+\sum_{i\in W} y_{ij}\lambda_i} = 1, \ \forall j \in R. \tag{4.38}$$

With these notations, the ordering quantity at warehouse W_i at tT, denoted by $D^W_{i,t}$, is

$$D^W_{i,t} = \sum_{j\in R} y_{ij}\eta_{j,t}Q(R_i) = \sum_{j\in R} y_{ij}\eta_{j,t}s_jTq_j, \ \forall i \in W, \ t = 1, \ldots, N. \tag{4.39}$$

And the ordering quantity at the manufacturer at tT, denoted by D^M_t, is

$$D^M_t = \sum_{i\in W} \delta_{i,t}Q(W_i) = \sum_{i\in W}\sum_{j\in R} \delta_{i,t}k_iTy_{ij}q_j, \ t = 1, \ldots, N. \tag{4.40}$$

Let us define Ψ as the surplus stock above the demand D^M_1 at time T, then

$$(1 + S)PT = \Psi + D^M_1. \tag{4.41}$$

No shortages are allowed over the NT cycle, the total production must equal to the total demand over the NT cycle. Hence

$$(F + S)PT = \sum_{j=1}^{N} D^M_t. \tag{4.42}$$

Also, since no stock out is allowed, the production over the time $-ST$ to jT must be sufficient to meet the accumulated demand in this period, then

$$(S + j)PT \geqslant \sum_{t=1}^{j} D^M_t, j = 1, \ldots, \lfloor F \rfloor. \tag{4.43}$$

The above constraint is equivalent to the following:

$$\begin{cases} \Psi \geqslant 0 \\ \Psi + (j-1)P \geqslant \sum_{t=2}^{j} \sum_{j \in R} \delta_{i,t} k_i T y_{ij} q_j, \ j = 2, \ldots, \lfloor F \rfloor. \end{cases} \tag{4.44}$$

The inventory level for the manufacturer, open warehouse W_i and retailer R_i are shown in Figures 4.1-4.3, respectively. And the total inventory of one manufacturer cycle and one ordering cycle for W_i, are denoted by π^M and π_i^W, respectively. In order to calculate total inventory of the manufacturer and warehouses, the same methodology used by Section 3.5 (demonstrated in Appendix A) is implemented. First calculate the area under the inventory level of one cycle (NT for the manufacturer, $k_i T$ for warehouse W_i) when there is no order taken place. Then for any tT in the cycle, subtract an area of a rectangle whenever there is an order, i.e. (ordering quantity)$\times$ (NT-tT) for the manufacturer, (ordering quantity)$\times$ ($k_i T$-tT) for warehouse W_i, see Figure 4.4).

The average stock held by the manufacturer and warehouse W_i are

$$\frac{\pi^M}{NT} = T\left(N + S - \frac{N}{2P}\sum_{j \in R} q_j\right)\sum_{j \in R} q_j - \frac{T}{N}\sum_{t=1}^{N}\sum_{i \in W}\sum_{j \in R}(N-t)\delta_{i,t} k_i y_{ij} q_j \tag{4.45}$$

and

$$\frac{\pi_i^W}{k_i T} = k_i T \sum_{j \in R} y_{ij} q_j - \frac{T}{k_i}\sum_{t=1}^{k_i}\sum_{j \in R}(k_i - t) y_{ij} \eta_{j,t} s_j q_j \tag{4.46}$$

respectively. So the manufacturer's holding cost per unit time is given by

$$hT\left(N + S - \frac{N}{2P}\sum_{j \in R} q_j\right)\sum_{j \in R} q_j - \frac{hT}{N}\sum_{t=1}^{N}\sum_{i \in W}\sum_{j \in R}(N-t)\delta_{i,t} k_i y_{ij} q_j \tag{4.47}$$

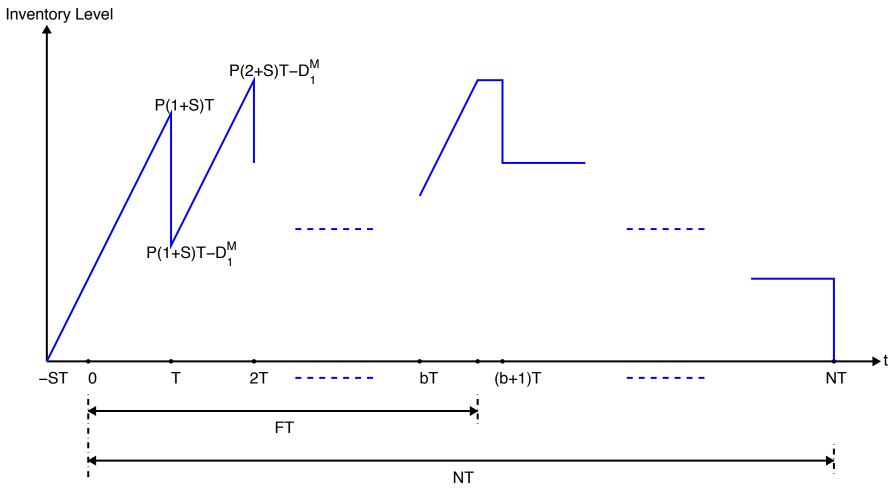

Figure 4.1: Inventory level of manufacturer

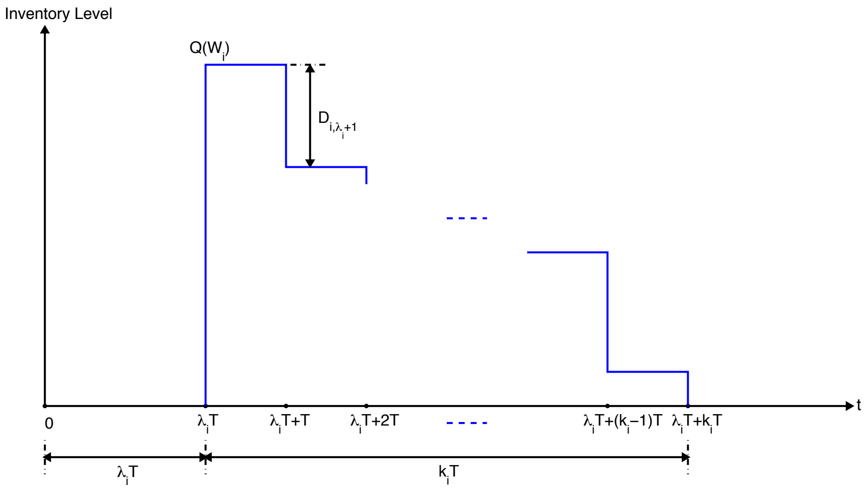

Figure 4.2: Inventory level of open warehouse W_i

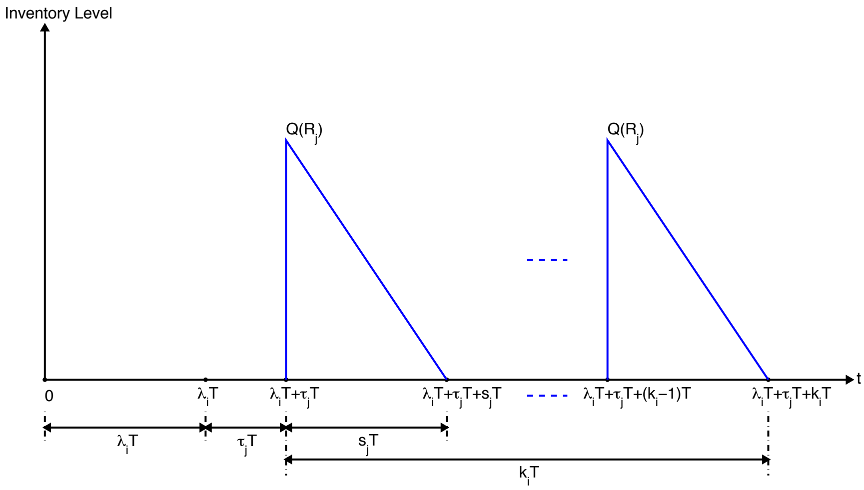

Figure 4.3: Inventory level of retailer R_j, which is assigned to warehouse W_i

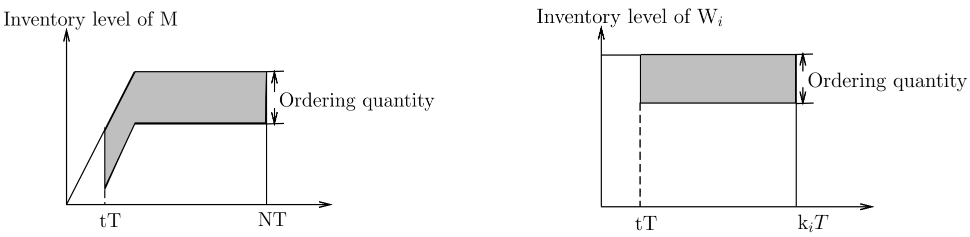

Figure 4.4: Inventory calculation for manufacturer and warehouse

and the holding cost per unit time incurred at warehouses is

$$\sum_{i\in W}\sum_{j\in R} h_i^W k_i T y_{ij} q_j - \sum_{i\in W}\sum_{t=1}^{k_i}\sum_{j\in R} \frac{h_i^W T}{k_i}(k_i - t) y_{ij} \eta_{j,t} s_j q_j. \tag{4.48}$$

The other relevant costs of our model are as follows:

$$\text{Warehouses operation cost per unit time} = \sum_{i\in W} z_i O_i^W. \tag{4.49}$$

$$\text{Manufacturer's setup cost per unit time} = \frac{M_s}{NT}. \tag{4.50}$$

$$\text{Warehouses' ordering cost per unit time} = \sum_{i \in W} z_i \frac{A_i^W}{k_i T}. \tag{4.51}$$

$$\text{Retailer' ordering cost per unit time} = \sum_{i \in R} \frac{A_i^R}{s_i T}. \tag{4.52}$$

Shipment cost from manufacturer to warehouses per unit time

$$= \sum_{i \in W} z_i \frac{C_i^M}{k_i T}. \tag{4.53}$$

Shipment cost from warehouses to retailers per unit time

$$= \sum_{i \in W} \sum_{j \in R} \frac{y_{ij} C_{ij}^W}{s_i T} \tag{4.54}$$

$$\text{Retailers' holding cost per unit time} = \frac{1}{2} \sum_{i \in R} h_i^R q_i s_i T \tag{4.55}$$

The total system cost per unit time of this synchronized production-warehouse location-inventory (PWLI(p,n)) model, denoted by TC^{SYN} , is as follows:

$$\begin{aligned} TC^{SYN} \quad = \quad & \frac{M_s}{NT} + \sum_{i \in W} z_i \left(O_i^W + \frac{A_i^W}{k_i T} + \frac{C_i^M}{k_i T} \right) + \sum_{i \in R} \left(\frac{A_i^R}{s_i T} + \frac{1}{2} h_i^R q_i s_i T \right) \\ & + \sum_{i \in W} \sum_{j \in R} \left(\frac{y_{ij} C_{ij}^W}{s_i T} + h_i^W k_i T y_{ij} q_j \right) - \sum_{i \in W} \sum_{t=1}^{k_i} \sum_{j \in R} \frac{h_i^W T}{k_i} (k_i - t) y_{ij} \eta_{j,t} s_j q_j \\ & + hT \left(N + S - \frac{N}{2P} \sum_{j \in R} q_j \right) \sum_{j \in R} q_j - \frac{hT}{N} \sum_{t=1}^{N} \sum_{i \in W} \sum_{j \in R} (N - t) \delta_{i,t} k_i y_{ij} q_j \end{aligned} \tag{4.56}$$

The objective is now to find the nonnegative values for N, z_i, y_{ij}, k_i, s_i, λ_i, τ_i and S that minimize the cost given by Eq.(4.56), subject to the constraints stated in this section. Since only the fifth term of the objective function, i.e. $-\sum_{i\in W}\sum_{t=1}^{k_i}\sum_{j\in R}\frac{h_i^W T}{k_i}(k_i-t)y_{ij}\eta_{j,t}s_jq_j$, is affected by the decision variables τ_i ($\eta_{i,t}$ is related to τ_i, see Eq.(4.38)). For given N, z_i, y_{ij}, k_i, s_i and λ_i, the value of the fifth term is smaller when τ_i is smaller, which means that the earlier the retailers order at the warehouses, the less holding cost will be incurred at the warehouses and thus the total system cost in Eq.(4.56) will be smaller. Hence, for given N, z_i, y_{ij}, k_i, s_i and λ_i, the cost given by Eq.(4.56) can be minimized when $\tau_i = 1$ ($i \in R$), i.e. all the retailers should order as soon as possible in their own cycle.

4.5 Heuristics

4.5.1 Heuristics for WL Problem

In the independent policy model, before calculating the economic cycles of each participant, a WL problem (Eq.(4.1)-(4.5)) at the strategic level for the manufacturer needs to be solved to obtain the optimal set of open warehouses, denoted by Ω^*. Since the uncapacitated location problem is NP-hard, which has been proved in Cornuéjols et al. (1983), heuristics are carried out to find the solutions. One of the first heuristics was proposed by Kuehn and Hamburger (1963), consisting of greedy heuristic and interchange heuristic, which have been widely used for a class of location problems in modern literature.

In this research, an exhaustive-interchange algorithm is adopted to solve the WL problem in the independent policy model, which is outlined in Algorithm 4.1.

We perform an exhaustive search on the initial set of Ω^i, which contains one warehouse. Given the initial set of Ω^i in Line 2, an interchange searching is implemented

Algorithm 4.1: Exhaustive-interchange algorithm for Problem WL

1 ***Function*** (Exhaustive-interchange algorithm)
2 ***Enumeration*** Initialize the set of open warehouse, $\Omega = \{i\}$
3   ***While*** stopping criteria is not achieved ***do***
4   Add a warehouse j to Ω
5   **if** $\min_{j \in \overline{\Omega}} \mathcal{F}(\Omega \bigcup \{j\}) < \mathcal{F}(\Omega)$ **then**
6                 $\Omega = \Omega \bigcup \{j\}$
7   Remove a warehouse j from Ω
8   **if** $\min_{j \in \Omega} \mathcal{F}(\Omega \backslash \{j\}) < \mathcal{F}(\Omega)$ **then**
9                 $\Omega = \Omega \backslash \{j\}$
10   ***End While***
11     $\Omega^i = \Omega$
12 **End Enumeration**
13 $\Omega^* = \{\Omega^i_{best} | \min_{i \in W} \mathcal{F}(\Omega^i)\}$.

in Lines 3-10. In each interchange iteration, we are allowed to open a warehouse, close a warehouse, or do both due to the warehouse number in Ω and the performance of the new set. The interchange searching stops when the Ω^i is same as the one at the beginning of that iteration, which means that there is no warehouse added to or removed from Ω, or the added one and removed one are same.

4.5.2 Genetic Algorithm for Synchronized PWLI Problem

Similar to Section 3.7.1 as mentioned, the outlines of the genetic algorithm adopted in this research is shown in Algorithm 4.2. In Line 2, an initial population of size 20 is created by a random process, where each chromosome is encoded with four segments, i.e. N, z_i, k_i, λ_i and s_j, where $i = 1, \ldots, p$ and $j = 1, \ldots, n$. The fitness values of the strings in the initial population are also calculated by Eq.(4.56). After

Algorithm 4.2: Genetic Algorithm for Synchronized PWLI(p, n)

1 ***Function*** (Genetic Algorithm)
2 Population initialization: the length of each chromosome is $1 + 3p + n$;
3 ***While*** stopping criteria is not achieved ***do***
4   Crossover;
5   Mutation;
6   Updating population;
7 ***End While***

the offsprings of the population being produced by crossover and by mutation of the segments in Lines 4 and 5, respectively, the population is updated in Line 6, by sorting the parent population and the offsprings in a way that their total costs are in increasing order and choosing the first 20 chromosomes to form a new group of parents of next evolution.

4.5.3 Simulated Annealing for Synchronized PWLI Problem

The outlines of the standard simulated annealing adopted in this research is shown in Algorithm 4.3. The solution in SA is of the same representation as in GA. Searching neighbours of a solution in Line 6 is similar to the mutation process in GA, choosing one location to mutate with the probability of 100% and, if necessary, mutating the related genes so as to satisfy the corresponding constraints.

4.5.4 Hybrid of Genetic Algorithm and Simulated Annealing

Due to the nature of the problem, the chromosome of the GA are of a particular structure, i.e. the k_i, λ_i and s_j have to match N and z_i. Hence any crossover or

Algorithm 4.3: Simulated Annealing for Synchronized PWLI(p, n)

Function (Simulated Annealing)

Initialization: Set temperature $temp = temp_0$ (a large number), solution S_1, Metropolis length (iterations within each temperature) = L;

While Final temperature T_f is not achieved ***do***

  Set inner iteration $iter = 0$;

  While inner iteration is less than L ***do***

    $S_2 \leftarrow neighbour(S_1)$, $iter = iter + 1$;

    Update current solution by acceptance criterion;

  EndWhile

  Update temperature, $temp = r \cdot temp$.

End While

mutation of N and z_i would require new values of k_i, λ_i and s_j. This means that the original k_i, λ_i and s_j of the parents cannot be kept in the new chromosome. Instead, they have to be re-generated randomly. Thus some optimality of chromosomes may be lost and the whole searching is more easily trapped in a local minimum. However, the acceptance criterion in SA, which allows a worse solution being accepted, prevents the algorithm being trapped in a local minimum. As the temperature tends to be zero, the probability of accepting a worse neighbouring solution becomes smaller.

In this research, a hybrid of genetic algorithm and simulated annealing is adopted to solve the synchronized $PWLI(p, n)$ model, where the simulated annealing is carried out after the genetic algorithm, i.e. the optimal solution obtained from the genetic algorithm is used as the initial solution S_1 in simulated annealing.

4.6 Numerical Results

Numerical experiments have been carried out to illustrate the performance of the synchronized cycles PWLI model. Five PWLI(p, n) examples are used in our experiments. Examples 1-4 for $(p, n) = (4, 10)$, $(8, 30)$, $(15, 50)$ and (20,100), respectively, Example 5 for $(p, n) = (2, 4)$. The data of the examples are randomly generated and are shown in Appendix C. The comparisons of the performances of synchronized cycles PWLI model, obtained by GA, SA and GASA, and independent policy (IND) are conducted on Examples 1-4. And the comparisons of the models under a full range of different values of α (Demand-Production ratio $\sum q_i^R/P$) from 0.1, 0.2, ..., up to 0.9 are also presented in our experiments. Example 5, with a small problem size of $(p, n) = (2, 4)$, has been tested for the quality of the heuristic solutions by comparing with the solutions obtained by exhaustive search. All the algorithms are run in a computer with 3.40 GHz and 24 GB RAM.

4.6.1 Heuristic Parameters

The efficiency of the heuristics is greatly dependent on finding good parameters. The finding process shows that the following combinations of parameters work better than many other parameter combinations for the problem. 450 trials (5 datasets *times* 9 α x 10 runs) are conducted with the following parameters.

The parameters for Genetic Algorithm (GA) are as follows:

Crossover rate: 0.8

Mutation rate: 0.01

Population size: 20

Stopping criteria: There is no improvement for successive $\min(1000, 100p)$ iterations or the iteration number comes to 10000

The parameters for Simulated Annealing (SA) are as follows:

Initial temperature: 2×sum number of potential warehouses and retailers (i.e. $2(p + n)$)

Final temperature:0.005

Cooling rate: $(\frac{0.005}{2(n+p)})^{\frac{1}{1000}}$

Number of iterations (within each temperature): 2×sum number of potential warehouses and retailers (i.e. $2(p + n)$)

Stopping criteria: There is no improvement for successive $\min(1000, 100p)$ SA iterations or the final temperature is achieved

The parameters for Hybrid Genetic Algorithm and Simulated Annealing (GASA) are as follows:

Population size: 20

Crossover rate: 0.6

Mutation rate: 0.05

Stopping criteria for GA process: There is no improvement for successive $\min(1000, 100p)$ iterations or the iteration number comes to $200 + 100 \times \lfloor p/10 \rfloor$;

Initial temperature: 2×sum number of potential warehouses and retailers (i.e. $2(p + n)$)

Final temperature:0.005

Cooling rate: $(\frac{0.005}{2(n+p)})^{\frac{1}{1000}}$

Number of iterations (within each temperature):2×number of potential warehouses (i.e. $2p$)

Stopping criteria for SA process: There is no improvement for successive $400 + 200 \times \lfloor p/10 \rfloor$ SA iterations or the final temperature is achieved.

4.6.2 Performance of the Synchronized PWLI Model

The average performance of the heuristics for Examples 1-4 are presented in Table 4.1 and the best results obtained are shown in Tables 4.2-4.5, respectively.

It can be seen from Table 4.1 that the improvement of GA, SA and GASA over IND decreases by 10.56%, 8.76% and 8.95%, respectively, when the problem size increases from (4,10) to (20,100), whereas the average CPU time used by the three heuristics have increased by 14, 19 and 13 times, respectively. And the improvement percentages are very close for the two medium problem size, $(p, n) = (8, 30)$ and $(p, n) = (15, 50)$.

(p,n)	GA		SA		GASA	
	TC over IND (%)	CPU Time (s')	TC over IND (%)	CPU Time (s')	TC over IND (%)	CPU Time (s')
(4,10)	30.81	10.46	30.95	17.24	30.70	6.12
(8,30)	26.13	36.58	26.56	61.32	25.69	16.53
(15,50)	26.69	72.07	27.98	155.84	27.34	39.55
(20,100)	20.25	159.09	22.19	358.95	21.76	90.41

Table 4.1: The average performance of the heuristics.

For the results of Example 1, Table 4.2 shows that the synchronized PWLI model outperforms the independent policy over the whole range of α. The total system cost of the synchronized PWLI model stays rather stable over the range of 88.91-92.57 with different α values. The improvement percentages of the best TC in the three heuristics over IND, which are shown in the last column, decreases from 33.22% to 29.54%, when α increases from 0.1 to 0.9.

α	GA		SA		GASA		IND	Improvement
	TC	CPU Time (s')	TC	CPU Time (s')	TC	CPU Time (s')	TC	over IND(%)
0.1	92.18	8.78	**91.78**	18.24	93.47	3.95	137.44	33.22
0.2	91.98	7.58	**91.97**	17.96	92.57	6.52	136.48	32.61
0.3	**92.57**	7.10	93.36	18.56	93.12	5.85	135.45	31.66
0.4	92.57	14.01	91.98	15.30	**91.97**	6.66	134.34	31.54
0.5	93.06	8.83	**91.97**	15.97	93.51	4.13	133.14	30.92
0.6	**91.69**	14.40	**91.69**	19.08	**91.69**	6.33	131.81	30.44
0.7	91.98	7.21	**91.22**	17.38	**91.22**	6.51	130.31	30.00
0.8	90.64	11.53	91.04	15.07	**90.30**	6.65	128.52	29.74
0.9	**88.91**	14.68	**88.91**	17.61	89.06	8.50	126.19	29.54

Table 4.2: Results for Example 1 ($(p, n) = (4, 10)$ case).

For Example 2, as shown in Table 4.3, when α increases from 0.1 to 0.9, the total cost of synchronized PWLI decreases by 7.30% and the improvement percentage of synchronized PWLI over IND decreases from 27.75% to 26.06%. The best solutions have been obtained by the SA in 7 out of 9 α values.

α	GA		SA		GASA		IND	Improvement
	TC	CPU Time (s')	TC	CPU Time (s')	TC	CPU Time (s')	TC	over IND(%)
0.1	290.89	25.68	**288.02**	68.61	301.92	15.65	398.63	27.75
0.2	292.00	24.76	**286.83**	68.78	289.97	16.02	395.41	27.46
0.3	286.96	49.86	**285.96**	67.28	286.57	16.96	391.99	27.05
0.4	287.53	45.63	287.53	64.83	**283.95**	16.01	388.30	26.87
0.5	286.06	39.58	**281.90**	59.67	282.84	18.64	384.30	26.65
0.6	280.58	32.01	**279.67**	52.03	**279.67**	16.44	379.87	26.38
0.7	279.70	45.77	**276.59**	54.62	276.81	15.99	374.84	26.21
0.8	272.72	28.63	**272.70**	57.02	277.03	15.33	368.87	26.07
0.9	**266.99**	37.28	268.86	59.00	279.04	17.69	361.10	26.06

Table 4.3: Results for Example 2 ($(p, n) = (8, 30)$ case).

The results for Example 3 with medium problem size $(p, n) = (15, 50)$, presented in Table 4.4, also have the synchronized PWLI performing better than that of the independent policy. The system cost of synchronized PWLI decreases by 8.59% as α increases from 0.1 to 0.9. In contrast to the results of Examples 1-2, the improvement over IND stays rather stable over the range of 27.54%-28.65%. SA also obtains the best solution in the three heuristics in 7 out of 9 α values, while its CPU times are multiples of that of GA when α ranges from 0.1-0.7.

α	GA		SA		GASA		IND	Improvement
	TC	CPU Time (s')	TC	CPU Time (s')	TC	CPU Time (s')	TC	over IND(%)
0.1	469.58	60.33	**448.90**	186.30	458.93	38.27	625.55	28.24
0.2	450.64	54.26	**443.00**	186.45	448.12	36.47	620.84	28.65
0.3	**440.49**	37.81	445.67	120.92	448.27	40.26	615.83	28.47
0.4	**437.98**	39.06	**437.98**	120.40	444.06	39.14	610.45	28.25
0.5	**434.24**	72.28	**434.24**	153.36	**434.24**	40.30	604.59	28.18
0.6	444.61	74.02	436.52	151.66	**433.38**	38.72	598.11	27.54
0.7	434.29	69.50	**426.09**	145.67	429.00	39.53	590.76	27.87
0.8	423.01	100.09	**419.46**	156.67	424.77	42.23	582.04	27.93
0.9	436.83	141.29	**410.35**	181.16	416.72	41.01	570.67	28.09

Table 4.4: Results for Example 3 ($(p, n) = (15, 50)$ case).

For the results of Example 4 with large problem size $(p, n) = (20, 100)$, Table 4.5 shows that the best improvement of synchronized PWLI over IND ranges from 21.66% to 23.61% over the whole range of α. And the total system cost of the synchronized model decreases by 5.70% as α increased from 0.1 to 0.9. In the three heuristics, SA outperforms the other two in 7 out of 9 α values at the expense of using multiples of CPU time.

α	GA		SA		GASA		IND	Improvement
	TC	CPU Time (s')	TC	CPU Time (s')	TC	CPU Time (s')	TC	over IND(%)
0.1	806.73	137.03	**755.70**	334.16	759.97	88.51	989.26	23.61
0.2	776.43	146.86	773.92	355.38	**756.10**	88.02	982.72	23.06
0.3	782.00	173.33	**756.32**	341.13	768.28	89.87	975.76	22.49
0.4	752.78	99.15	758.32	361.17	**744.49**	91.07	968.28	23.11
0.5	762.24	125.32	**747.60**	342.61	**747.60**	88.75	960.15	22.14
0.6	774.25	287.21	**733.85**	411.56	747.12	94.13	951.15	22.85
0.7	756.03	207.75	**737.12**	354.69	757.98	84.99	940.94	21.66
0.8	732.19	118.17	**723.90**	362.61	740.92	95.41	928.82	22.06
0.9	724.56	137.02	**712.62**	367.30	712.75	92.91	913.04	21.95

Table 4.5: Results for Example 4 ($(p, n) = (20, 100)$ case).

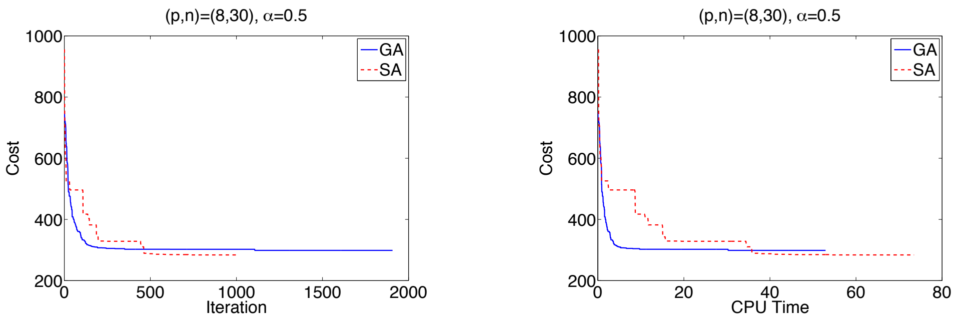

Figure 4.5: GA and SA procedure for $(p, n) = (8, 30)$ case with $\alpha = 0.5$

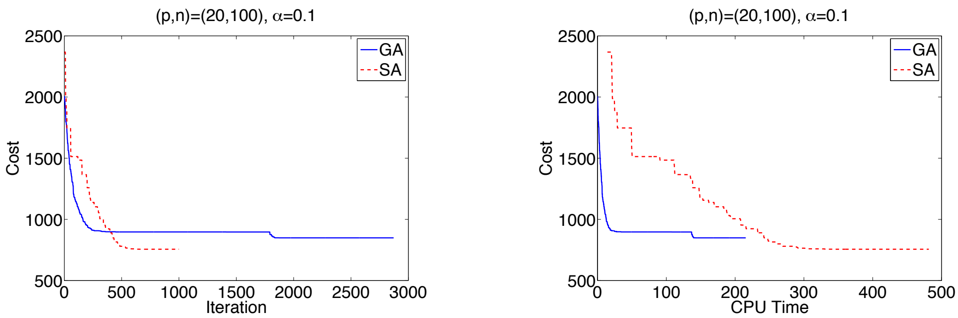

Figure 4.6: GA and SA procedure for $(p, n) = (20, 100)$ case with $\alpha = 0.1$

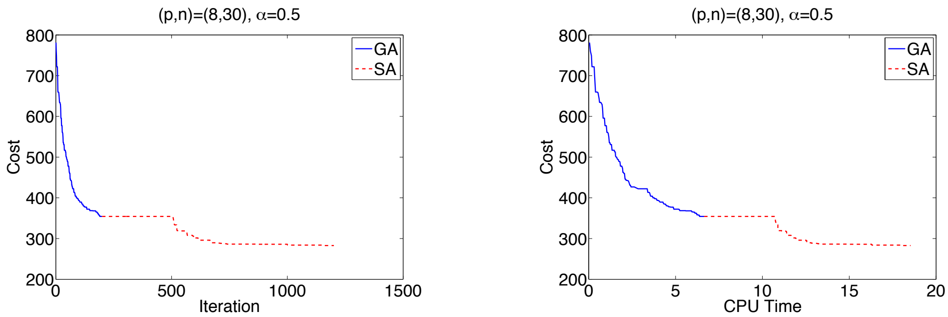

Figure 4.7: GASA procedure for $(p, n) = (8, 30)$ case with $\alpha = 0.5$

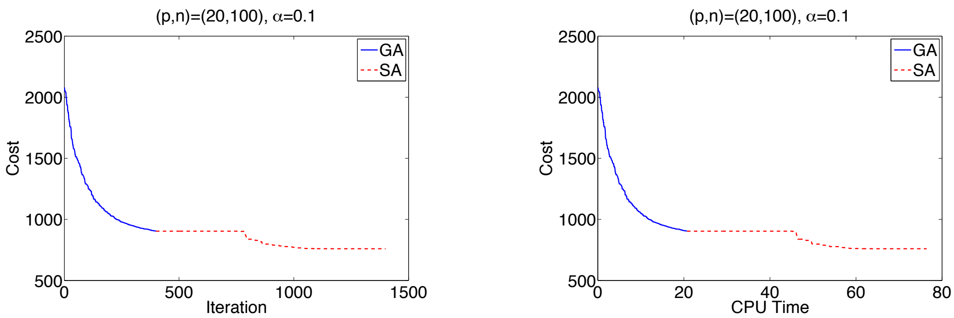

Figure 4.8: GASA procedure for $(p, n) = (20, 100)$ case with $\alpha = 0.1$

4.6.3 Performance of GASA

Figures 4.5-4.6 show the plots of the total system cost obtained by GA and SA vs. iteration steps of the heuristics and the CPU time. Figure 4.5 corresponds to Example 2 ($(p, n) = (8, 30)$ with $\alpha = 0.5$) and Figure 4.6 corresponds to Example 4 ($(p, n) = (20, 100)$ with $\alpha = 0.1$). For both cases, GA converges at about 200 iteration steps which is far less than that of SA. However, the CPU Time required by SA is longer than GA, indicating that an SA iteration is more time-consuming than a GA iteration. Both figures confirm that the GA for this synchronized PWLI model is more easily trapped in a local minimum.

Figures 4.7-4.8 show the plots of the total system cost obtained by GASA vs.

(p, n)	GA		SA	
	TC	CPU Time (s')	TC	CPU Time (s')
(4,10)	-0.16	38.05	-0.36	64.24
(8,30)	-0.59	52.32	-1.19	72.78
(15,50)	0.84	36.28	-0.90	73.95
(20,100)	1.88	37.64	-0.56	74.76

Table 4.6: The average percentage improvements of the GASA over GA and SA.

iteration steps and the CPU time for the above two cases, respectively. The GA and SA parts in this hybrid algorithm are plotted in solid line and dotted line, respectively. It can be seen that there is no improvement at the beginning of SA part due to the acceptance of the worse solution. And then the solution is further improved by SA.

The average percentage improvements of the GASA over GA and SA are summarized in Table 4.6. For Examples 1 and 2, the total system cost of GASA is worse than that of GA and SA within 0.36% and 1.19%, respectively, while the CPU time average reduction percentages have been achieved up to 64.24% and 72.78%, respectively. For Example 3, GASA outperforms GA in total cost and CPU time by 0.84% and 36.28%. And the cost performance of GASA is worse than that of SA by an average of 0.90% but the average CPU time is reduced by 73.95%. The performance of GASA in Example 4 is similar to Example 3, where the total cost obtained by GASA is better than GA by an average of 1.88% and still worse than SA by an average of 0.56%. The GASA in Example 4 uses 37.64% and 74.76% less of CPU time than that in GA and SA, respectively.

4.6.4 Quality of the Solutions Obtained by the Heuristics

In order to evaluate the performance of the heuristics applied in this research, 'exhaustive search' for the optimal solution is carried out for a synchronized PWLI$(2,4)$ model (dataset is given in Table C.5, Example 5). Since an exhaustive search for all the combinations of (N, z, k, λ, s) is impractical, a 'reduced exhaustive search' is developed for the comparison purpose. Therefore, the optimal solution obtained in this exhaustive search cannot be guaranteed as the actual optimal solution. The reduced exhaustive search is outlined in Algorithm 4.4.

Algorithm 4.4: Reduced exhaustive search for synchronized PWLI(p, n)

Function (Reduced exhaustive search)
Enumerate z_i, with $2^p - 1$ combinations
  For each combination $\vec{z}$ ***Do***
    Solution initialization: $z, \vec{k}, \vec{\lambda} = \vec{z},\ s_i = 1, \forall i \in R$
    Enumerate N from 1 to 365
      For each N ***Do***
        Solve $\min_{\forall \vec{k}, \forall \vec{s}} TC^{SYN}(\vec{k}, \vec{s})$
        $\vec{k}^*, \vec{s}^*$ are obtained for combination of $(\vec{z}, N, \vec{\lambda})$.
    End Enumeration on N; N^* is obtained for given $\vec{z}$;
    Do incremental search on λ_i, $\forall i \in W$
      $\vec{\lambda}^*$ is obtained for given $(\vec{z}, N^*, \vec{k}^*, \vec{s}^*)$
    End Do
End Enumeration on z_i, $\vec{z}^*$ is obtained for given $(N^*, \vec{k}^*, \vec{s}^*, \vec{\lambda}^*)$

The optimal solution for Example 5 obtained by the GA, SA, GASA and the reduced exhaustive search is shown in Table 4.7.

According to Table 4.7, SA achieves the same solution as the exhaustive search in

8 out of 9 α values, while GASA and GA achieve in 4 and 0, respectively. However, the exhaustive search is time-consuming, with an average of more than 2 hours for each α value. Therefore, the heuristics adopted in this research are capable of obtaining the optimal solution, and require far less CPU time compared to the exhaustive search.

α	GA		SA		GASA		Reduced Exhaustive Search	
	TC	CPU Time (s')	TC	CPU Time (s')	TC	CPU Time (s')	TC	CPU Time (s')
0.1	64.00	2.45	**63.30**	2.96	63.67	3.26	**63.30**	7773.49
0.2	63.14	3.84	63.09	1.39	**63.06**	3.73	**63.06**	5692.38
0.3	62.70	2.68	**62.60**	2.11	**62.60**	3.12	**62.60**	7728.52
0.4	62.34	3.42	**62.11**	3.90	**62.11**	2.84	**62.11**	6614.85
0.5	61.99	4.49	**61.53**	1.33	**61.53**	3.09	**61.53**	7905.25
0.6	61.03	2.51	**60.87**	2.06	61.09	2.59	**60.87**	6781.76
0.7	60.19	2.31	**60.05**	2.11	60.76	2.87	**60.05**	8058.65
0.8	59.49	4.82	**59.02**	6.19	59.25	2.98	**59.02**	6710.43
0.9	58.13	3.00	**57.60**	5.65	57.61	4.41	**57.60**	8930.43

Table 4.7: Comparison of minimum total cost obtained by GA, SA, GASA and the reduced enumerative search for Example 5 ($(p, n) = (2, 4)$ case).

4.6.5 System Cost Divisions

The system cost division can be conducted in two ways, i.e. the costs incurred at different decision levels and different supply chain layers. Based on the decision levels, i.e. strategic, tactical and operational, the system cost can be divided into (a) warehouse operating cost, (b) inventory cost, which includes the production cost at the manufacturer, ordering costs of the warehouses and retailers and the holding cost of the system, (c) shipment cost incurred in the two echelon delivery. Based on the supply chain layers, the system cost can be divided into (a) cost of the manufacturer, denoted by $C(M)$, including production cost, holding cost, shipment cost when products delivered to warehouses, (b) total cost incurred at the warehouse, denoted by $C(W)$, including the warehouse operating cost, holding cost, ordering cost when orders from the manufacturer and shipment cost when delivers the products to the retailers, (c) total cost of the retailers, denoted by $C(R)$, including the holding cost and the ordering cost when orders from the warehouse.

The system cost divisions of the synchronized PWLI model solved by GASA and the independent policy for Examples 1-4 are presented in Tables 4.8-4.11, respectively.

For Example 1, as shown in Table 4.8, the operating cost, inventory cost and the shipment cost in the synchronized model is 50%, 19.24% and 30.82% less than that of the independent policy, respectively. And the cost incurred at the manufacturer and the warehouses have been reduced in the synchronized model by an average of 36.24% and 45.91% when comparing to the IND, respectively, and the total cost of the retailers is increased by 14.09% on average.

For Example 2, Table 4.9 shows that the cost incurred in all three decision levels are improved by an average of 66.67%, 13.38% and 11.87% in synchronized model when compared with IND, respectively. The shipment cost of the synchronized model outperforms the IND only by 0.68%, and 3.63% when $\alpha = 0.5$ and 0.8, respectively. The average cost performance at the layers in the synchronized model is similar to Example 1, the cost of the manufacturer and warehouses are reduced by 33.54% and 49.89%, respectively, and the cost of the retailers is increased by 16.46%. The improvement percentage of the manufacturer's cost decreases from 50.06% to 18.25% as α increases from 0.1 to 0.9.

For Example 3, according to Table 4.10, the shipment cost performance of the synchronized model, which varies by the α, for α = 0.1, 0.2, 0.3, 0.8 and 0.9, is worse than IND by a range of 3.02-20.34%, and for other α values, is improved by a range of 0.62-10.13%. The improvement percentage of the inventory cost ranges from 9.57% to 16.52%. For the manufacturer, the cost improvement is 26.73% on average with a range from 23.60% to 30.61%. The average cost performance of the synchronized model over IND is 54.14% better for the warehouse layer while 17.02% worse for the retailer layer.

For Example 4, the costs at the strategic and tactical level are improved by an average of 69.44% and 14.09% over the whole range of α, respectively, while the cost at the operational level increased by an average of 11.50%, which always has a smaller value in the IND except when α = 0.3 and 0.8. The cost of the manufacturer and the warehouses in the synchronized model are reduced, respectively, by 21.73% and 47.73% on average, and the cost of the retailers increases by an average of 13.92%.

α	GASA							IND						
	TC	Decision levels			System layers			TC	Decision levels			System layers		
		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$
0.1	93.47	20	58.97	14.51	14.12	42.48	36.87	137.44	40	72.37	25.07	27.19	79.86	30.39
0.2	92.57	20	57.46	15.11	12.87	44.47	35.23	136.48	40	71.41	25.07	26.23	79.86	30.39
0.3	93.12	20	54.45	18.67	15.77	43.23	34.12	135.45	40	70.38	25.07	25.20	79.86	30.39
0.4	91.97	20	56.00	15.97	14.19	42.57	35.22	134.34	40	69.28	25.07	24.10	79.86	30.39
0.5	93.51	20	55.59	17.91	16.23	43.14	34.14	133.14	40	68.07	25.07	22.90	79.86	30.39
0.6	91.69	20	53.03	18.67	14.34	43.23	34.12	131.81	40	66.75	25.07	21.57	79.86	30.39
0.7	91.22	20	52.55	18.67	13.87	43.23	34.12	130.31	40	65.24	25.07	20.06	79.86	30.39
0.8	90.30	20	51.64	18.67	12.96	43.23	34.12	128.52	40	63.45	25.07	18.27	79.86	30.39
0.9	89.06	20	51.15	17.91	11.79	43.14	34.14	126.19	40	61.12	25.07	15.94	79.86	30.39

Table 4.8: System cost division for Example 1 ($(p, n) = (4, 10)$ case).

α	GASA							IND						
	TC	Decision levels			System layers			TC	Decision levels			System layers		
		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$
0.1	301.92	30	221.26	50.66	39.12	113.33	149.48	398.63	90	245.99	62.64	78.32	195.18	125.13
0.2	289.97	30	205.10	54.87	49.95	95.51	144.51	395.41	90	242.77	62.64	75.10	195.18	125.13
0.3	286.57	30	203.86	52.71	45.28	95.31	145.98	391.99	90	239.34	62.64	71.68	195.18	125.13
0.4	283.95	30	199.06	54.90	44.81	92.85	146.30	388.30	90	235.66	62.64	67.99	195.18	125.13
0.5	282.84	30	190.62	62.22	42.92	99.52	140.41	384.30	90	231.65	62.64	63.99	195.18	125.13
0.6	279.67	30	194.77	54.90	40.52	92.85	146.30	379.87	90	227.22	62.64	59.56	195.18	125.13
0.7	276.81	30	191.92	54.90	37.66	92.85	146.30	374.84	90	222.19	62.64	54.53	195.18	125.13
0.8	277.03	30	186.66	60.37	32.38	95.44	149.20	368.87	90	216.23	62.64	48.56	195.18	125.13
0.9	279.04	30	197.71	51.33	33.35	102.64	143.05	361.10	90	208.46	62.64	40.79	195.18	125.13

Table 4.9: System cost division for Example 2 ($(p, n) = (8, 30)$ case).

α	GASA							IND						
	TC	Decision levels			System layers			TC	Decision levels			System layers		
		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$
0.1	458.93	40	321.21	97.73	73.97	154.12	230.84	625.55	160	373.62	91.93	106.61	323.80	195.14
0.2	448.12	40	313.41	94.70	71.55	149.13	227.44	620.84	160	368.91	91.93	101.90	323.80	195.14
0.3	448.27	40	312.34	95.94	70.44	150.23	227.60	615.83	160	363.90	91.93	96.89	323.80	195.14
0.4	444.06	40	316.69	87.37	66.26	150.30	227.50	610.45	160	358.51	91.93	91.50	323.80	195.14
0.5	434.24	40	302.88	91.36	63.37	145.53	225.35	604.59	160	352.66	91.93	85.65	323.80	195.14
0.6	433.38	40	310.76	82.62	59.55	139.21	234.62	598.11	160	346.18	91.93	79.17	323.80	195.14
0.7	429.00	40	306.39	82.62	55.17	139.21	234.62	590.76	160	338.83	91.93	71.82	323.80	195.14
0.8	424.77	40	290.07	94.70	48.21	149.13	227.44	582.04	160	330.11	91.93	63.10	323.80	195.14
0.9	416.72	40	266.08	110.63	37.42	159.54	219.76	570.67	160	318.74	91.93	51.73	323.80	195.14

Table 4.10: System cost division for Example 3 ($(p,n) = (15, 50)$ case).

α	GASA							IND						
	TC	Decision levels			System layers			TC	Decision levels			System layers		
		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$
0.1	759.97	50	556.01	153.96	101.37	243.17	415.43	989.26	200	644.56	144.70	137.58	492.52	359.16
0.2	756.10	50	552.14	153.96	97.50	243.17	415.43	982.72	200	638.02	144.70	131.04	492.52	359.16
0.3	768.28	100	533.40	134.88	94.56	267.26	406.46	975.76	200	631.06	144.70	124.08	492.52	359.16
0.4	744.49	50	523.65	170.84	91.41	244.96	408.12	968.28	200	623.58	144.70	116.60	492.52	359.16
0.5	747.60	50	548.33	149.27	85.49	247.44	414.67	960.15	200	615.45	144.70	108.47	492.52	359.16
0.6	747.12	50	547.48	149.64	84.66	247.84	414.61	951.15	200	606.45	144.70	99.47	492.52	359.16
0.7	757.98	50	492.00	215.98	71.84	289.09	397.05	940.94	200	596.24	144.70	89.26	492.52	359.16
0.8	740.92	100	505.49	135.43	60.37	277.33	403.23	928.82	200	584.12	144.70	77.15	492.52	359.16
0.9	712.75	50	474.59	188.16	48.51	256.69	407.55	913.04	200	568.34	144.70	61.36	492.52	359.16

Table 4.11: System cost division for Example 4 ($(p,n) = (20, 100)$ case).

4.7 Conclusions

In this research, an integrated production-warehouse location-inventory model is proposed. This model determines the warehouse location, production schedule and ordering frequencies by minimizing the total system cost. In this model, the production cycle of the manufacturer and the ordering cycles of the warehouses and the retailers are synchronized, i.e. the ordering cycle for each warehouse or retailer is an integer factor of the his adjacent upstream participant's cycle. Three different heuristic algorithms are used for finding the optimal system cost of the model. Numerical experiments are carried out to test the performance of this integrated model when compared to the independent policy, where the decisions at different levels are made separately so as to minimize the cost involved in each subproblem.

The overall performance of the synchronized PWLI model obtained by GA, SA and GASA are always better than that of the independent policy model. The average improvement percentage on the total system cost decreases from 31% to 22% as the problem size increases from 4 potential warehouses and 10 retailers to 20 potential warehouses and 100 retailers.

In the cost division for different decision levels, the cost incurred at the strategic level is always improved by a large percentage, more than 50%, when compared to the independent policy model. This indicates that the warehouse location decisions have a big impact on the system cost and can be benefited from the co-ordinated model. The reason is that the number of open warehouses obtained by the WL problem is always larger than that in the co-ordinated one. In addition, about 49-73% of the total system cost saving come from the cost reduction achieved by the co-ordinated warehouse location. This applies to different problem sizes. The shipment cost in our model is better than that of the IND in the problems of size (4,10) and (8,30), but worse in the other two examples which have larger problem sizes. This implies more open warehouses, as in the IND, can reduce the total shipment cost for cases of large problem size. The inventory cost in the co-ordinated model also outperforms the IND by a range of 13-19%.

In the cost division for different supply chain layers, as the problem size increases from (4,10) to (20,100), the manufacturer's cost reduction percentage decreases from 36% to 21%, and the contribution of manufacturer saving to the the system saving decreases from 21% to 11%. The warehouses cost in the co-ordinated model is improved by a range of 46-54%, and reduction on the warehouses counts for about 100% of the system saving for different problem size, indicating that the almost all system cost savings come from the warehouse layer, and the cost augment at the retailers can be offset by the cost reduction at the manufacturer. The cost incurred at the retailers is always larger in the co-ordinated model, which is 14-17% more than that of the IND.

Three heuristic algorithms are proposed to solve the co-ordinated model. The quality of the solutions obtained by the heuristics have been confirmed valid by comparing their solutions with that obtained by an exhaustive search. For most of the cases, SA usually performs the best but with a longer computational time. The searching procedure of GA shows that GA converges relatively fast but more easily trapped into a local optimal due to the characteristic of the solution structure. Combining the above two algorithms, GASA has a more than 60% saving on CPU time while the solution is within 0.56% inferior of the solution obtained by SA.

Chapter 5

Incorporating Vehicle Routing Problem in a Synchronized Production-Warehouse Location-Inventory Model

5.1 Introduction

In Chapter 4, a synchronized production-warehouse location-inventory (PWLI) model, which consists of a single manufacturer, a set of potential warehouses and a set of customers with deterministic demand is proposed. The objective of the synchronized PWLI model is to determine the set of open warehouses and the production and ordering schedules for the participants so as to minimize the total cost which includes the warehouse operation cost, system inventory cost and shipment cost incurred in two echelons. The shipment cost for each order is ωe, where ω is the transportation cost per unit distance and e is the distance between the vendor and the buyer.

Facility location and vehicle routing are the two most crucial elements in logistics. However, in most cases, facility locations are planned at the strategic level without the consideration of routing cost.

As routing decisions are greatly related to warehouse location, this research proposes to consider the shipment cost in the PWLI model is obtained from a vehicle routing model instead of a fixed cost for each order. This new model, denoted as PWLIR, integrates the routing decisions, the location decisions and inventory decisions with an objective to improve the total system cost. Since the number of deliveries in each echelon varies at every time point of the planning horizon, this model is different from the other location-inventory-routing models. In other words, a vehicle routing problem has to be solved in each time point for each echelon. Furthermore, in the vehicle routing problem incorporated in the PWLI, an unlimited fleet of heterogeneous vehicles is considered and split deliveries are also allowed.

5.2 Notations

Parameters:

Depot: Depot set.

Customer: Set of demand points served by depot.

Demand: Customer demand vector, $Demand = (q_1, q_2, \ldots, q_n)$.

I: Set of vehicle type, $\{1, 2, \ldots, u\}$.

V_i^c: Capacity for the ith type of vehicle, $i \in I$.

V_i^w: Weight for the ith type of vehicle, $i \in I$.

V_i^f: Fixed cost for the ith type of vehicle per route, $i \in I$.

V_i^v: Variable cost for the ith type of vehicle per unit weight per unit distance, $i \in I$.

V: Parameter matrix for the heterogeneous vehicles, where the ith row is $V^i =$

$[i, V_i^c, V_i^w, V_i^f, V_i^v]$.

ϕ: Weight for per unit product.

G: Graph $G = (N, A)$, where $N = (0, 1, 2, \ldots, n)$ is the set of nodes, node $\{0\}$ corresponds to the depot, node $\{i\}$ corresponds to customer i $(i = 1, \ldots, n)$, and A=$\{(i, j) : 0 \leqslant i, j \leqslant n, i \neq j\}$ denotes the set of arcs.

e_{ij}: The distance between nodes i and j, $(i, j) \in A$.

N_0: Set of customer points, $N_0 = N \backslash \{0\}$.

$f_i^{k,r}$: Quantity of the ith customer's demand served by rth vehicle of type k, $k \in I,\ j \in N_0$.

Θ: Optimal total cost of the vehicles' fixed costs and variable cost in a fleet size and mix vehicle routing problem with split delivery. $\Theta = FSMVRPSD(Depot, Customer, Demand, V)$

n^k: Number of vehicles of type k available, assumed to be unlimited, $k \in I$.

Decision Variables:

$x_{ij}^{k,r}$: Binary variable. It is equal to 1 if the rth vehicle of type k travels from i to j, $(i, j) \in A,\ k \in I,\ r = 1, \ldots, n^k$.

$\rho_{ij}^{k,r}$: Goods quantity loaded on the rth vehicle of type k when travels on arc (i, j), $(i, j) \in A,\ k \in I,\ r = 1, \ldots, n^k$.

5.3 Fleet Size and Mix Vehicle Routing Problem with Split Deliveries

The heterogeneous vehicle routing problems (HVRP) generally consider a fleet of capacitated vehicles to serve a set of customers with known demands. The objective of these problems is to determine the fleet composition and the vehicle routes by minimizing the relevant vehicle cost including the fixed and variable cost. The HVRPs can be divided into two major classes, i.e. (1) the fleet size and mix vehicle routing problem (introduced by Golden et al. (1984)) where the number of available vehicles of each type is unlimited, and (2) the heterogeneous fixed fleet vehicle routing problem (introduced by Taillard (1999)) where the fleet size is predetermined. In this research, the fleet size and mix vehicle routing problem is adopted to model the shipment between the echelons. Moreover, split deliveries are also allowed in this vehicle routing problem.

The problem is given by a central *Depot*, denoted by node $\{0\}$ and a set of demand points, $Customer$, $\{1, 2, \ldots, n\}$, residing at n different locations. $A{=}\{(i,j) : 0 \leqslant i, j \leqslant n, i \neq j\}$ denotes the set of arcs. The *Demand* at node $\{i\}$ is q_i, $i = 1, \ldots, n$. The customers are served from the depot by an unlimited fleet of heterogeneous vehicles, which traverses the customers' nodes starting and ending at the depot. We have a set $I = \{1, \ldots, u\}$ vehicle types. And the ith type of vehicle has a capacity V_i^c, net weight v_i^w, a fixed cost V_i^f per route and a variable cost V_i^v per unit load per unit distance. Define a parameter matrix V for this heterogenous fleet, where the ith row is $V^i = [i, V_i^c, V_i^w, V_i^f, V_i^v]$. The loading on the rth vehicle of type k travelling on arc (i,j) is denoted by $\rho_{ij}^{k,r}$. The cost of a vehicle of type k traversing the arc (i,j) is obtained by multiplying the distance between node i and j, denoted by e_{ij}, and the weight of vehicle, which equals to the vehicle net weight plus the loading weight, and the variable unit cost V_k^v, i.e. $e_{ij}(V_k^w + loading)V_k^v$. The number of vehicles of type k available, denoted by n^k, is assumed to be unlimited, $k \in I$.

The demand at a customer is allowed to be served by more than one vehicle, since the demand may exceed the vehicle capacity. The decision variables of this fleet size and mix vehicle routing problem with split deliveries (FSMVRPSD) are route succeeding variables $x_{ij}^{k,r}$, which equals to 1 if the rth vehicle of type k travels from i to j, the loading variables $\rho_{ij}^{k,r}$, the loading on the rth vehicle of type k when travels on arc (i,j), $(i,j) \in A$, $k \in I$, $r = 1, \ldots, n^k$, and the demand splitting variables $f_i^{k,r}$, the quantity of the ith customer's demand served by the rth vehicle of type k, $k \in I$, $j \in N_0$, $r = 1, \ldots, n^k$.

The FSMVRPSD model is formulated as follows:

$$\min \quad \sum_{k\in I} V_k^f \sum_{j=1}^{n} \sum_{r=1}^{n^k} x_{0,j}^{k,r} + \sum_{(i,j)\in A} \sum_{k\in I} \sum_{r=1}^{n^k} V_k^v x_{ij}^{k,r} (\phi \rho_{ij}^{k,r} + V_k^w) e_{ij} \tag{5.1}$$

$$\text{s.t.} \quad \sum_{k\in I} \sum_{r=1}^{n^k} \sum_{i\in N} x_{ij}^{k,r} \geqslant 1, \quad j \in N_0 \tag{5.2}$$

$$\sum_{j\in N_0} x_{0,j}^{k,s} = 1, \quad k \in I,\ s \leqslant \sum_{r=1}^{n^k} \sum_{j\in N_0} x_{0,i}^{k,r} \tag{5.3}$$

$$\sum_{i\in N\backslash\{j\}} x_{ij}^{k,r} - \sum_{i\in N\backslash\{j\}} x_{ji}^{k,r} = 0, \quad j \in N,\ k \in I,\ r \leqslant n^k \tag{5.4}$$

$$\sum_{k\in I} \sum_{r=1}^{n^k} f_i^{k,r} = q_i, \quad i \in N_0 \tag{5.5}$$

$$\sum_{i\in N_0} f_i^{k,r} \leqslant V_k^c, \quad k \in I, r = 1, \ldots, n^k \tag{5.6}$$

$$\sum_{k\in I} \sum_{r=1}^{n^k} \sum_{j\in N\backslash\{i\}} \rho_{ji}^{k,r} - \sum_{k\in I} \sum_{r=1}^{n^k} \sum_{j\in N\backslash\{i\}} \rho_{ij}^{k,r} = q_i, \quad i \in N_0 \tag{5.7}$$

$$f_j^{k,r} x_{ij}^{k,r} \leqslant \rho_{ij}^{k,r} \leqslant (V_k^c - f_i^{k,r}) x_{ij}^{k,r}, \quad i,j \in N_0,\ k \in I,\ r = 1, \ldots, n^k \tag{5.8}$$

$$\sum_{i\in N_0} \rho_{i0}^{k,r} = 0, \quad k \in I,\ r = 1, \ldots, n^k \tag{5.9}$$

$$x_{ij}^{k,r} = \{0, 1\}, \quad (i, j) \in A, \; k \in I, r = 1, \ldots, n^k \tag{5.10}$$

$$f_i^{k,r} \geqslant 0, \quad i \in N_0, \; k \in I, \; r = 1, \ldots, n^k \tag{5.11}$$

$$\rho_{ij}^{k,r} \geqslant 0, \quad (i, j) \in A, \; k \in I, \; r = 1, \ldots, n^k \tag{5.12}$$

In the above formulation, the objective function in Eq.(5.1) is to minimize the sum of fixed cost and variable cost of the heterogeneous fleet. Constraint (5.2) ensures that each customer is visited at least once. Constraint (5.3) guarantees that each selected vehicle is only sent out once and multi-trip is not allowed. Constraint (5.4) states that the vehicles flow on each node, i.e. the numbers of vehicle comes to and leaves from each node are the same. Constraint (5.5) guarantees that each buyer's demand is satisfied. Constraint (5.6) guarantees that the capacity of each vehicle will not be exceeded. Constraints (5.7) and (5.8) guarantee the commodity flow on each customer node. Constraint (5.9) guarantees that there is no commodity in the vehicle when it returns to the depot. Constraint (5.10) guarantees that the decision variable $x_{ij}^{k,r}$ is binary. Constraints (5.11) and (5.12) guarantee the decision variables $f_i^{k,r}$ and $\rho_{ij}^{k,r}$ are non-negative. The optimal solution for this model is denoted by $\Theta = FSMVRPSD(Depot, Customer, Demand, V)$.

5.4 The Synchronized Cycles Model for the P-WLIR Problem

In this section, the synchronized cycles PWLI model of Chapter 4 is integrated with the vehicle routing problem introduced in Section 5.3. For the production, warehouse location and inventory parts of the PWLIR model, we use the same notations as those of Chapter 4.

5.4.1 Shipment Cost from Warehouses to Retailers

At time tT, the shipping quantity from the warehouse W_i to retailer R_j is denoted by $Q_{i,j}^{WR}(t)$, i.e.

$$Q_{i,j}^{WR}(t) = y_{ij}\eta_{j,t}Q(R_i) = y_{ij}\eta_{j,t}s_jTq_j,\ \ t = 1, \ldots, N. \tag{5.13}$$

Warehouse W_i faces a demand vector

$$Q_i^{WR}(t) = [Q_{i,1}^{WR}(t), Q_{i,2}^{WR}(t), \ldots, Q_{i,n}^{WR}(t)],\ \ \forall i \in W,\ \ t = 1, \ldots, N. \tag{5.14}$$

from the retailers placing orders at tT. Therefore, the shipping cost from the warehouses to the retailers per unit time expressed in Eq.(4.54) can be replaced by the following:

$$\frac{1}{NT}\sum_{t=1}^{NT}\sum_{i\in W} FSMVRPSD(W_i, R, Q_i^{WR}(t), V) \tag{5.15}$$

5.4.2 Shipment Cost from Manufacturer to Warehouses

Similarly, at time tT, the shipping quantity from the manufacturer to warehouse W_i is denoted by $Q_i^{MW}(t)$, i.e.

$$Q_i^{MW}(t) = \delta_{i,t}Q(W_i) = \sum_{j\in R}\delta_{i,t}k_iTy_{ij}q_j,\ \ t = 1, \ldots, N. \tag{5.16}$$

The manufacturer faces a demand vector

$$Q^{MW}(t) = [Q_1^{MW}(t), Q_2^{MW}(t), \ldots, Q_p^{MW}(t)],\ \ t = 1, \ldots, N. \tag{5.17}$$

from the warehouses at time tT. Therefore, the shipping cost from the manufacturer to the warehouses per unit time expressed in Eq.(4.53) can be replaced by the following:

$$\frac{1}{NT}\sum_{t=1}^{N} FSMVRPSD(M, W, Q^{MW}(t), V) \tag{5.18}$$

5.4.3 Total System Cost

The total system cost of this synchronized cycles PWLIR model is given as follows:

$$\begin{aligned}
TC^{SYN} = {} & \frac{M_s}{NT} + \sum_{i\in W} z_i\left(O_i^W + \frac{A_i^W}{k_i T}\right) + \sum_{i\in R}\left(\frac{A_i^R}{s_i T} + \frac{1}{2}h_i^R q_i s_i T\right) \\
& + \sum_{i\in W}\sum_{j\in R}\left(h_i^W k_i T y_{ij} q_j\right) - \sum_{i\in W}\sum_{t=1}^{k_i}\sum_{j\in R}\frac{h_i^W T}{k_i}(k_i - t) y_{ij}\eta_{j,t} s_j q_j \\
& + hT\left(N + S - \frac{N}{2P}\sum_{j\in R} q_j\right)\sum_{j\in R} q_j - \frac{hT}{N}\sum_{t=1}^{N}\sum_{i\in W}\sum_{j\in R}(N-t)\delta_{i,t} k_i y_{ij} q_j \\
& + \frac{1}{NT}\sum_{t=1}^{N} FSMVRPSD(M, W, Q^{MW}(t), V) \\
& + \frac{1}{NT}\sum_{t=1}^{NT}\sum_{i\in W} FSMVRPSD(W_i, R, Q_i^{WR}(t), V)
\end{aligned} \tag{5.19}$$

5.5 The Independent Policy for the PWLIR Problem

Under the independent policy, the optimal ordering quantity for each retailer is $Q(R_i)^*$ every T_i^{R*} units of time and for each warehouse is $Q(W_i)^*$ every T_i^{W*} units of time. The optimal ordering cycles for the warehouse and retailers may not be integer multiples of the unit time, or there may be several deliveries to one warehouse or retailer in one unit of time. Thus, for the deliveries in each unit of time, the upstream participant may combine the deliveries which have the same destination and have a routing plan for all the orders within this time unit.

In order to make a fair comparison with the synchronized cycles PWLIR model, the following assumptions are made for the independent policy:

(i) In the independent policy, all the orders within each time unit, T, are combined for the deliveries at the end of this time unit, i.e., if retailer R_1 and R_2 have the ordering cycle of $0.3T$ and $0.7T$, respectively, then the vehicle planning for T is to make two deliveries of $3Q(R_1)^*$ and $Q(R_2)^*$ units of quantity to retailer R_1 and R_2, respectively, since the three orders from R_1 at time $t = 0.3T, 0.6T$ and $0.9T$ are combined as one delivery.

(ii) To compare with the synchronized cycles model, the final time $\widetilde{N^R}T$ and $\widetilde{N^W}T$ of the independent policy are required. For the synchronized cycles model, the shipping costs in Eqs. (5.15) and (5.18) can be interpreted as the average routing cost per unit time in delivering a total of Nq^MT units of item in NT units of time to the retailers and warehouses, respectively. To match this situation of the synchronized cycles model, two final times $\widetilde{N^R}T$ and $\widetilde{N^W}T$ are set for the independent policy as the time the Nq^MTth item is delivered to the retailer and the warehouse, respectively. Therefore, the routing cost per unit time obtained for the independent policy can be interpreted as the average cost per unit time in delivering a total of Nq^MT units of item to the warehouses and retailers, which has the same interpretation as the synchronized cycles model.

5.5.1 Shipment Cost from Warehouses to Retailers

At time tT, the shipping quantity from the warehouse W_i to retailer R_j, denoted by $\widetilde{Q}_{i,j}^{WR}(t)$, is

$$\widetilde{Q}_{i,j}^{WR}(t) = \sum_{j \in R} y_{ij} Q(R_j)^* \left(\left\lfloor \frac{t}{T_j^{R*}} \right\rfloor - \left\lfloor \frac{t-1}{T_j^{R*}} \right\rfloor \right). \tag{5.20}$$

Warehouse W_i faces a demand vector

$$\widetilde{Q}_i^{WR}(t) = [\widetilde{Q}_{i,1}^{WR}(t), \widetilde{Q}_{i,2}^{WR}(t), \ldots, \widetilde{Q}_{i,n}^{WR}(t)], \ \forall i \in W. \tag{5.21}$$

from the retailers at tT. Therefore, the shipping cost from the warehouses to the retailers per unit time in the independent policy model can be expressed as:

$$\frac{1}{\widetilde{N^R} T} \sum_{t=1}^{\widetilde{N^R}} \sum_{i \in W} FSMVRPSD(W_i, R, \widetilde{Q}^{WR}(t), V) \tag{5.22}$$

where $\widetilde{N^R}$ is given by

$$\widetilde{N^R} = \arg\min_{\tilde{N}} \left\{ \sum_{t=1}^{\tilde{N}} \sum_{i \in W} \sum_{j \in R} \widetilde{Q}_{i,j}^{WR}(t) \geqslant N q^M T \right\}. \tag{5.23}$$

5.5.2 Shipment Cost from Manufacturer to Warehouses

At time tT, the order quantity for an open warehouse W_i, can be expressed as

$$\widetilde{Q}_i^{MW}(t) = Q(W_i)^* \left(\left\lfloor \frac{t}{T_i^{W*}} \right\rfloor - \left\lfloor \frac{t-1}{T_i^{W*}} \right\rfloor \right). \tag{5.24}$$

The manufacturer faces a demand vector

$$\widetilde{Q}^{MW}(t) = [\widetilde{Q}_1^{MW}(t), \widetilde{Q}_2^{MW}(t), \ldots, \widetilde{Q}_p^{MW}(t)] \tag{5.25}$$

from the warehouses at tT. Therefore, the shipping cost from the manufacturer to the warehouses per unit time in the independent policy can be expressed by the following:

$$\frac{1}{\widetilde{N^W} T} \sum_{t=1}^{\widetilde{N^W}} FSMVRPSD(M, W, \widetilde{Q}^{MW}(t), V) \tag{5.26}$$

where $\widetilde{N^W}$ is given by

$$\widetilde{N^W} = \arg\min_{\tilde{N}} \left\{ \sum_{t=1}^{\tilde{N}} \sum_{i \in W} \widetilde{Q}_i^{MW}(t) \geqslant N q^M T \right\}. \tag{5.27}$$

5.5.3 Total System Cost

With the above shipment costs, the total system cost per unit time of the independent PWLIR model becomes:

$$\begin{aligned} TC^{IND} = & \sum_{i \in W} z_i^* O_i^W + \sqrt{2 M_s h q^M (1 - \frac{q^M}{P})} + h \sum_{i \in W} q_i^W T_i^{W*} \\ & + \sum_{i \in W} \sqrt{2 A_i^W h_i^W q_i^W} + \sum_{i \in W} \sum_{j \in R} h_i^W y_{ij}^* q_i T_i^{R*} + \sum_{i \in R} 2 A_i^R h_i^R q_i \\ & + \frac{1}{\widetilde{N^W} T} \sum_{t=1}^{\widetilde{N^W}} FSMVRPSD(M, W, \widetilde{Q}^{MW}(t), V) \\ & + \frac{1}{\widetilde{N^R} T} \sum_{t=1}^{\widetilde{N^R}} \sum_{i \in W} FSMVRPSD(W_i, R, \widetilde{Q}^{WR}(t), V) \end{aligned} \tag{5.28}$$

5.6 Heuristics

5.6.1 Heuristics for Solving FSMVRPSD

With the production and ordering schedules given by the independent policy model and synchronized cycles model, the shipment costs incorporated in the above models can then be obtained by solving a fleet size and mix vehicle routing problem with split deliveries (FSMVRPSD). The vehicle routing problem, which is a well-known NP-hard problem, can only be solved by heuristics to obtain a near-optimal solution in most situations, let alone taking the considerations of heterogeneous vehicle and split deliveries into account. Therefore, heuristics are designed to obtain a near-optimal solution of the FSMVRPSD.

In this research, a heuristic based on the adaptive large neighbourhood search (ALNS), proposed by Ropke and Pisinger (2006), is applied to solve the FSMVRPSD. The outlines of the heuristics are described in Algorithm 5.1.

Algorithm 5.1: Heuristics for FSMVRPSD

Function (FSMVRPSD)
Initial solution: Generate a feasible solution S_1 by applying the constructive heuristics (described in Algorithm 5.2);
While stopping criterion is not achieved ***do***
    Set inner iteration number $iter = 0$ and set the score for all the neighbourhood as zero
    While $iter$ is less than L ***do***
        Choose a neighbourhood i, $S_2 \leftarrow neighbour_i(S_1)$, $iter = iter + 1$;
        Update the score of neighbourhood i;
    End While
    Update the weight w_i for each neighbourhood
End While

There are two main procedures in the heuristics depicted in Algorithm 5.1, starting with a constructive heuristic to find a feasible initial solution (Line 2), followed by an adaptive neighbourhood search (Line 3-10) aiming at the solution improvement. In Line 6, $neighbour_i$ is chosen by a roulette wheel scheme based on the weight of each neighbourhood. We choose $neighbour_i$ with the probability $\frac{w_i}{\sum_{i=1}^{\ell} w_i}$, where ℓ is the total number of candidate neighbourhoods. The weight of $neighbour_i$ is updated for every L ALNS iterations, calculated by the accumulated scores obtained by this neighbourhood in this segment (L iterations).

Algorithm 5.2: Initial Solution

1 ***Function*** (Initial Solution for FSMVRPSD)
2 Convert the customer's locations to radians w.r.t. the origin point of the depot's location to polar coordinate (in radian).
3 Sorting the customers by the ascending order of the angle of their polar coordinates, denote the customer sequence as Γ.
4 Route set $\Re = \varnothing$, current route $r = \varnothing$, current vacancy $\Delta = \max V_c$
5 ***While*** $\Gamma \neq \varnothing$
6   Current demand q is the unserved demand of the first customer in Γ
7   **If** $q \leqslant \Delta$, **then**
8   insert this customer to the last of route r with the loading of q;
9   update current vacancy $\Delta = \Delta - q$;
10   delete this customer from Γ.
11   **Else**
12   insert this this customer to the last of route r with the loading of Δ
13   update the unserved demand of this customer
14   put r into route set $\Re$
15   reset $r = \varnothing$ and $\Delta = \max V_c$
16   **END If**
17 ***End While***
18 Vehicle selection: for each route in $\Re$, choose the type of vehicle which achieves the minimum idle capacity.

In the constructive heuristic shown in Algorithm 5.2, the customers are assigned by a 'scanning' scheme, i.e. setting the depot as origin point and the customers are allocated anticlockwise. For each customer insertion i, we deliver an amount which is the minimum of the idle capacity of the largest vehicle and the unserved demand of customer i, as shown in Lines 7-16. At last, in Line 18, a vehicle selection is implemented to each route aiming to maximize the utilization of vehicle capacity.

Neighbourhoods

Based on some common operators, the following neighbourhoods are applied in our heuristics:

(1) Intra-route 2-opt. Randomly select one route serving more than 2 customers, randomly swap the order of any two customers from that route.

(2) Intra-route relocate. Randomly select one route serving more than 1 customer, remove one customer from this route and insert it to a random position of the route.

(3) Inter-route 2-opt. Randomly swap two customers from two different routes.

(4) Customer relocate-1. Randomly select one customer, relocate all his demand to the current routes.

(5) Customer relocate-2. Randomly select two customers, relocate all their demands to the current routes.

(6) Route deletion. Reassign the demands of a selected route.

(7) Route addition. Add a new route for the customers who have split deliveries.

In the above neighbourhoods, the 'customer relocate' and 'route deletion' may generate new split deliveries, while the 'inter-route 2-opt' and 'Route addition' may eliminate the split deliveries. The 'route deletion' is aiming to eliminate the vehicles with small capacity. After the new routes obtained from neighbouring move, vehicle selection procedure is implemented again for all the routes.

5.6.2 Heuristics for the Synchronized Cycles PWLIR Model

In Chapter 4, several heuristics for finding the solutions for warehouse location and system inventory policy simultaneously have been developed. For the synchronized cycle PWLIR model proposed in this chapter, where a series of FSMVRPSD problems is embedded, decisions on the warehouse location, system inventory policy and shipment policy aiming to minimize the total system cost given by Eq.(5.19) have to be made. However, since the input for the FSMVRPSD are solutions of the location and inventory problem, for each combination of $(z_i, N, k_i, \lambda_i, s_j)$, it is needed to solve a set of FSMVRPSD. Each time unit when there is at least one order, a FSMVRPSD is then formulated. Therefore, if the routing decision variables are encoded into the chromosome with the location and inventory decisions and solved simultaneously, the computational time for running a full heuristic would increase exponentially with the problem size. In this research, the FSMVRPSD is incorporated in the PWLIR model with a purpose of examining the impact on the cost performance in different decision levels and system layers as well as the shipment components in each echelon via coordination. The impact should be similar as long as the same methodology is applied to solve the FSMVRPSD no matter whether the routing schedule is solved simultaneously. Therefore, the synchronized cycles model and independent policy model can still be compared when we are not using heuristics to determine all the decision variables simultaneously.

For the above reasons, this model is decomposed into two sub-models, i.e. a production-warehouse location-inventory (PWLI) problem and a set of FSMVRPS-D, and solved sequentially. Thus, the solution searching for the synchronized cycles PWLIR can be called a PWLI-first and FSMVRPSD-second procedure. The values of z_i, N, k_i, λ_i, and s_j in the PWLI problem can be obtained by the heuristics proposed in Sections 4.5.2-4.5.4 without considerations of shipment cost. Then the $(z_i, N, k_i, \lambda_i, s_j)$ obtained are input for the second sub-model and its the routing cost can be obtained by Algorithm 5.1.

5.7 Numerical Results

Numerical experiments have been carried out to illustrate the performance of the synchronized cycles PWLIR model, where the shipment cost is modeled by a vehicle routing problem. In this chapter, the four data sets (Tables C.1-C.4) presented in Chapter 4 are used. The problem sizes of the four data sets are $(p, n) = (4, 10)$, $(8, 30)$, $(15, 50)$ and (20,100), respectively. The same set of vehicle parameters (Table D.1) for all the examples is used. For the synchronized cycles PWLIR model, the results are obtained by choosing the best one in 10 runs of the PWLI-first and FSMVRPSD-second procedure.

5.7.1 System Cost Divisions

The total system cost divisions of the synchronized cycles PWLIR model solved by GASA and independent policy for Examples1-4 are presented in Tables 5.1-4.11, respectively. Comparing with the cost divisions (See Section 4.6.5) of the PWLI model, which does not include the routing problem, the sum of the warehouse operation cost and inventory cost in the PWLIR is always less than that in PWLI. This is due to the first-step heuristic applied in the PWLIR to minimize the total system cost excluding the shipment cost.

For Example 1, as shown in Table 5.1, the system cost of the synchronized cycles model (SYN) is 34.28% less than that of the independent policy (IND) on average. The costs incurred in all the three decision levels are improved by an average of 50%, 24.97% and 34.18% when compared with the IND, respectively. The cost incurred at the manufacturer and the warehouses in the synchronized cycles model are reduced by an average of 21.21% and 53.05%, respectively, and the total cost of the retailers is increased by a range from 3.81% to 12.11%. The improvement percentage of the manufacturer's cost decreases from 26.87% to 15.96% as α increases from 0.1 to 0.9.

For Example 2, according to Table 5.2, the improvement percentage of the SYN over IND stays rather stable within the range of 32.77-34.42% with different α values. The cost performance of different decisions levels in the SYN are 66.66%, 20.25% and 34.50% better than those of the IND on average. The average cost performance of different layers in the SYN is similar to Example 1 when compared the with IND. The cost of the manufacturer and the warehouses are reduced by 13.57% and 63.60%, respectively, while the total cost of the retailers is increased by 5.09%. And the augment percentage of the retails' cost increases from 2.05% to 8.99% as α increases from 0.1 to 0.9.

α	GASA							IND						
	TC	Decision levels			System layers			TC	Decision levels			System layers		
		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$
0.1	95.54	20.00	51.94	23.60	24.48	39.51	31.55	148.32	40.00	72.37	35.95	33.47	84.46	30.39
0.2	95.54	20.00	51.94	23.60	24.48	39.51	31.55	148.07	40.00	71.41	36.66	32.50	85.18	30.39
0.3	94.87	20.00	51.77	23.10	23.67	39.65	31.55	148.28	40.00	70.38	37.90	32.22	85.67	30.39
0.4	100.00	20.00	50.79	29.21	24.18	43.93	31.90	147.64	40.00	69.28	38.36	31.12	86.13	30.39
0.5	97.40	20.00	52.34	25.06	23.55	39.78	34.07	145.89	40.00	68.07	37.81	29.92	85.58	30.39
0.6	96.70	20.00	51.90	24.80	22.51	40.11	34.07	144.14	40.00	66.75	37.39	27.27	86.48	30.39
0.7	95.41	20.00	50.35	25.06	21.56	39.78	34.07	143.38	40.00	65.24	38.15	26.37	86.62	30.39
0.8	91.95	20.00	48.85	23.10	20.75	39.65	31.55	142.92	40.00	63.45	39.47	25.18	87.36	30.39
0.9	93.33	20.00	45.87	27.46	19.25	42.18	31.90	141.23	40.00	61.12	40.11	22.91	87.92	30.39

Table 5.1: System cost division for Example 1 ($(p, n) = (4, 10)$ case).

α	SYN							IND						
	TC	Decision levels			System layers			TC	Decision levels			System layers		
		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$
0.1	291.13	30.00	191.95	69.18	79.17	84.27	127.69	438.29	90.00	245.99	102.30	98.19	214.98	125.13
0.2	289.46	30.00	189.21	70.25	78.71	81.93	128.82	434.39	90.00	242.77	101.62	94.97	214.29	125.13
0.3	285.87	30.00	188.24	67.64	78.59	75.40	131.88	434.72	90.00	239.34	105.38	94.01	215.58	125.13
0.4	287.81	30.00	189.86	67.95	77.83	79.73	130.25	431.96	90.00	235.66	106.31	89.81	217.03	125.13
0.5	286.04	30.00	186.87	69.17	74.09	84.26	127.69	428.22	90.00	231.65	106.57	85.44	217.65	125.13
0.6	281.58	30.00	183.47	68.11	71.08	76.06	134.45	418.83	90.00	227.22	101.60	79.42	214.28	125.13
0.7	278.26	30.00	178.57	69.69	67.43	76.38	134.45	420.02	90.00	222.19	107.83	75.99	218.91	125.13
0.8	272.46	30.00	172.43	70.02	64.51	76.06	131.88	415.48	90.00	216.23	109.25	71.51	218.84	125.13
0.9	267.72	30.00	168.90	68.81	56.01	75.33	136.37	406.03	90.00	208.46	107.58	62.95	217.96	125.13

Table 5.2: System cost division for Example 2 ($(p, n) = (8, 30)$ case).

α	SYN							IND						
	TC	Decision levels			System layers			TC	Decision levels			System layers		
		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$
0.1	455.66	40.00	298.78	116.88	133.16	114.51	207.99	704.83	160.00	373.62	171.22	154.52	355.17	195.14
0.2	458.03	40.00	297.01	121.02	128.24	121.12	208.68	699.72	160.00	368.91	170.81	149.80	354.78	195.14
0.3	447.78	40.00	289.04	118.74	125.64	117.52	204.62	697.83	160.00	363.90	173.93	146.03	356.66	195.14
0.4	449.26	40.00	289.24	120.02	120.17	120.40	208.68	690.15	160.00	358.51	171.63	140.13	354.87	195.14
0.5	440.96	40.00	281.25	119.71	120.37	114.93	205.67	686.38	160.00	352.66	173.73	134.79	356.45	195.14
0.6	434.89	40.00	276.67	118.22	113.59	116.68	204.62	683.40	160.00	346.18	177.22	129.07	359.18	195.14
0.7	433.41	40.00	269.50	123.91	111.58	117.21	204.62	676.14	160.00	338.83	177.31	121.72	359.28	195.14
0.8	428.29	40.00	266.72	121.57	99.89	119.71	208.68	668.09	160.00	330.11	177.98	113.69	359.26	195.14
0.9	413.50	40.00	252.12	121.38	88.72	120.17	204.62	658.06	160.00	318.74	179.31	102.68	360.23	195.14

Table 5.3: System cost division for Example 3 ($(p, n) = (15, 50)$ case).

α	SYN							IND						
	TC	Decision levels			System layers			TC	Decision levels			System layers		
		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$		Operating	Inventory	Shipment	$C(M)$	$C(W)$	$C(R)$
0.1	804.52	50.00	519.23	235.29	218.28	195.56	390.68	1124.10	200.00	644.56	279.53	239.43	525.50	359.16
0.2	795.17	50.00	509.05	236.12	214.28	197.80	383.09	1121.31	200.00	638.02	283.29	233.45	528.70	359.16
0.3	793.90	50.00	510.21	233.70	208.11	195.11	390.68	1113.39	200.00	631.06	282.32	226.30	527.93	359.16
0.4	792.16	50.00	497.43	244.73	208.92	200.14	383.09	1105.13	200.00	623.58	281.55	218.84	527.12	359.16
0.5	781.35	50.00	490.37	240.98	196.91	208.54	375.91	1097.37	200.00	615.45	281.93	211.04	527.16	359.16
0.6	776.75	50.00	484.18	242.57	194.50	199.16	383.09	1091.33	200.00	606.45	284.88	202.16	530.01	359.16
0.7	764.72	50.00	477.69	237.03	183.15	198.48	383.09	1078.88	200.00	596.24	282.64	191.94	527.78	359.16
0.8	761.27	50.00	471.28	240.00	173.03	216.63	371.62	1072.39	200.00	584.12	288.27	180.56	532.67	359.16
0.9	738.19	50.00	452.33	235.86	157.50	197.60	383.09	1062.96	200.00	568.34	294.62	166.59	537.21	359.16

Table 5.4: System cost division for Example 4 ($(p, n) = (20, 100)$ case).

For Example 3, Table 5.3 shows that the SYN outperforms the IND by a stable percentage of 35.75% on average. And the improvement percentages of the inventory cost and shipment cost also stay stable over a range of 19.20-20.57% and 29.15-33.29%, respectively. For the manufacturer, the cost improvement is 12.58% on average with a range from 8.33% to 14.39%. For the warehouse layer, the improvement percentage stays rather stable over a range of 65.86-67.76%. The total cost of the retailer layer in the SYN is worse by an average of 5.80% when compared with the IND.

For Example 4, as shown in Table 5.4, the improvement percentage of the total system cost in the SYN over the IND ranges from 28.43% to 30.55%, and the average improvement percentage for the three decisions levels are 75%, 19.90% and 16.11%, respectively. The average cost performance of the SYN over the IND are 6.03% and 62.03% better for the first two layers while 6.66% worse for the retailer layer.

Figures 5.1-5.4 illustrate directly the percentages of the cost components in the three decision levels in the SYN and the IND for Examples 1-4, respectively, it is observed that the coordination of different decision levels, fewer warehouses would be open, and hence the proportion of warehouse operation cost decreases.

5.7.2 Shipment Components

Since one of the purposes of this research is to analyze the impact of coordination on the shipment components, the following notations are made: To deliver Nq^MT items to both the warehouse layer and the retailer layer, the number of the ith type vehicle required ($\max_t$\{the number of ith type vehicle used in tT\}) for the manufacturer-warehouse (M-W) echelon, the warehouse-retailer (W-R) echelon and

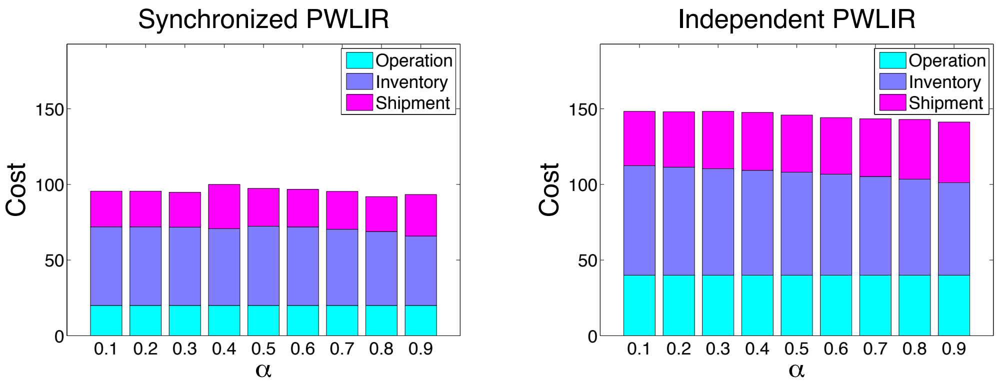

Figure 5.1: Cost divisions for Example 1

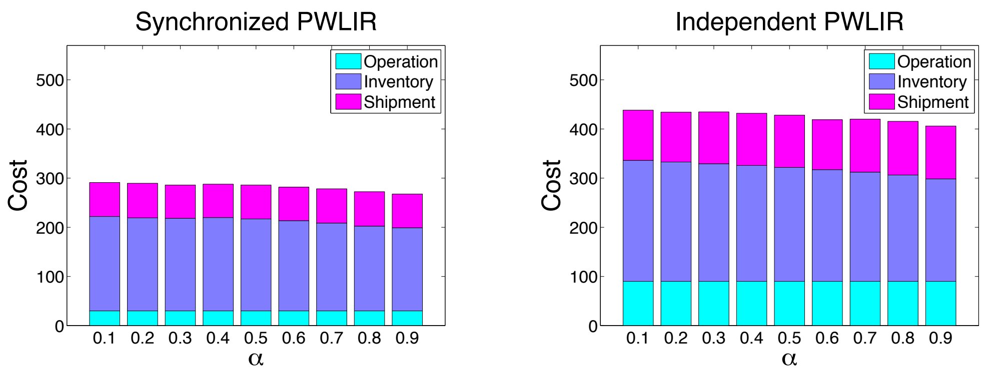

Figure 5.2: Cost divisions for Example 2

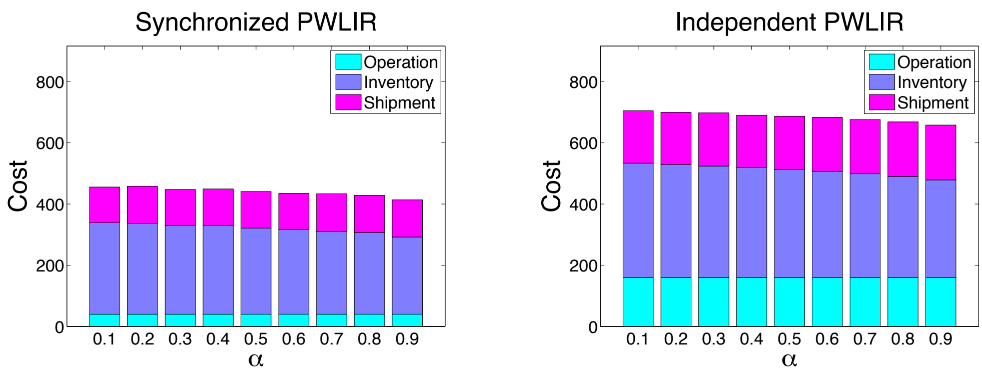

Figure 5.3: Cost divisions for Example 3

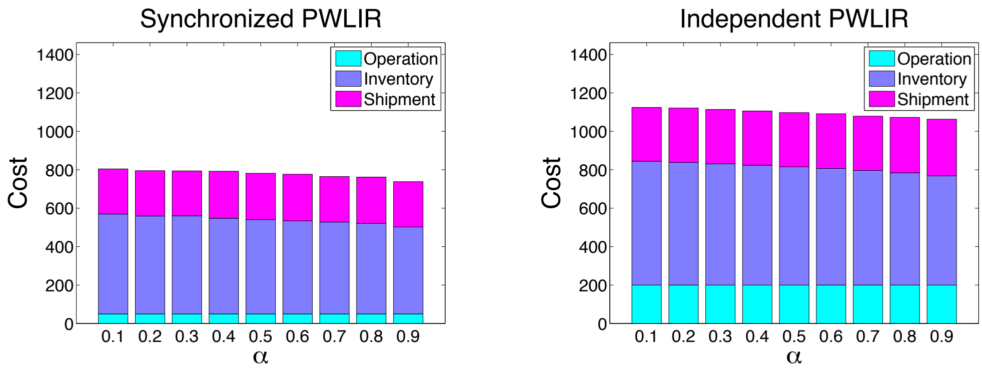

Figure 5.4: Cost divisions for Example 4

the whole system in the SYN, are denoted by m_i^M, m_i^W and m_i^S, respectively, and the corresponding notations for the IND are $\widetilde{m_i}^M$, $\widetilde{m_i}^W$ and $\widetilde{m_i}^S$, respectively. Similarly, the total number of routes delivered by ith type vehicle ($\sum_t\{$the number of ith type vehicle used in $tT\}$) in the M-W echelon, W-R echelon and the whole system in the SYN, are denoted by U_i^M, U_i^W and U_i^S, respectively, and the corresponding notations for the IND are $\widetilde{U_i}^M$, $\widetilde{U_i}^W$ and $\widetilde{U_i}^S$, respectively. The shipment cost per unit time in the M-W echelon, W-R echelon and the system in SYN are denoted as ξ^M, ξ^W and ξ^S, respectively, while the corresponding notations for the IND are $\widetilde{\xi}^M$, $\widetilde{\xi}^W$ and $\widetilde{\xi}^S$, respectively.

The details of the shipment components for Examples 1-4 when α equals to 0.5 are presented in Tables 5.5-5.7.

For the M-W echelon, the results are shown in Table 5.5. The time needed to deliver Nq^MT item to the warehouse layer in the IND is always larger than that in the SYN, i.e. $\widetilde{N^W} > N$. And the shipment cost per unit time in the SYN is improved by an average of 5.22% when compared with the IND. As seen from the 'operation' column in Tables 5.1-5.4, the number of open warehouses is only one in the SYN for all of the four examples and the corresponding numbers range from 2 to 4 in the IND for the four problem sizes. However, as the ordering quantity of the warehouse should be very large which may need to be split into several vehicles, most of the vehicles in the SYN and the IND may only deliver to one warehouse. Thusthe total number of routes for each type of vehicle in the two models (U_i^M and $\widetilde{U_i}^M$) are nearly the same. Similarly, the shipment cost per unit time of the two models (ξ^M and $\widetilde{\xi}^M$) are also very close to each other. In the case of problem size $(20, 100)$, the vehicles of small capacity are also used in the IND. The reason for this may lie in the ordering schedule of the warehouse, i.e. the number of open warehouse is 4 in this case and the warehouses are not synchronized and hence, the warehouse which has the smallest ordering quantity may order alone for some time units and can be served by a vehicle of small capacity. Though the total number of routes for each type of vehicle in the SYN is similar to that in IND, the required number of each type of vehicle in the SYN is larger than that in the IND due to the difference between the maximum ordering quantity in each time unit of the whole planning horizon for the two models.

For the W-R echelon, Table 5.6 shows that the time needed to deliver Nq^MT item to the retailer layer in the IND is also always larger than that in the SYN, i.e. $\widetilde{N^R} > N$. The shipment cost per unit time in the SYN is 40.62% better than that in the IND on average, with a range of 45.06-49.47% for the first three cases and 22.51% for the case of large problem size ($(p, n) = (20, 100)$). The reason for the lower improvement percentage in the fourth case is that the number of open warehouse in the IND is obtained by the independent warehouse location model which aims to minimize the warehouse operation cost and the distance cost, and is 4 times of the number in the SYN. This leads to a sharp decrease of the total distances between the retailers and their corresponding warehouses. The total number of routes in the IND is far higher than the one obtained in the SYN, i.e. $\left(\sum_i \widetilde{U_i}^W > \sum_i U_i^W\right)$. For the cases of problem size (p, n) =(4,10), (8,30) and (15,50), there are more routes served by the vehicle of small capacity in the IND than in the SYN, i.e. $\widetilde{U_i}^W > U_i^W$, i =1, 2 and 3, while the number of routes for vehicle of large capacity in the IND is much less than that in SYN, i.e. $\widetilde{U_4}^W < U_4^W$. For the IND, the orders from the retailers are not synchronized thus resulting in an increase in the number of routes of single-individual, where each vehicle only delivers one order to a single retailer. To deliver $Nq^M T$ items to the retailer layer, the required number of small vehicles in the IND is much higher than that in SYN ($\widetilde{m_i}^W > m_i^W$, i = 1, 2 and 3), while the required number of big vehicles is much less ($\widetilde{m_4}^W < m_4^W$). This is also due to the separated orders from the retailers of the IND. The delivery planning in the SYN combines the orders from several customers and thus the travelling cost is decreased by the routes of multi-individual, where each vehicle serves more than one retailer.

The detail of the shipment components for the whole system is presented in Table 5.7. Benefited from the synchronized orders, the total number of routes in the SYN is reduced by 54.55%, 52.84%, 50.70% and 28.77% for the four examples, respectively, when compared with the IND. At least 60% of the routes in the SYN are served by the vehicles with the largest capacity while the vehicle assignment for the IND is more diversified, i.e. all types of vehicles are scheduled in the four cases. Similarly, the percentage of the vehicles with the largest capacity over the total required number of vehicles is more than 75% in the SYN, while it is 52.63% at most in the IND. Due to a high utilization of the large vehicles for the synchronized orders, the improvement of the shipment cost per unit time of a synchronized cycles system is reduced by 33.74%, 35.09%, 31.09% and 14.52% for the four examples, respectively, when compared with the independent policy system.

(p,n)	SYN										IND									
	m_1^M	m_2^M	m_3^M	m_4^M	U_1^M	U_2^M	U_3^M	U_4^M	N	ξ^M	$\widetilde{m_1}^M$	$\widetilde{m_2}^M$	$\widetilde{m_3}^M$	$\widetilde{m_4}^M$	$\widetilde{U_1}^M$	$\widetilde{U_2}^M$	$\widetilde{U_3}^M$	$\widetilde{U_4}^M$	$\widetilde{N^W}$	$\tilde{\xi}^M$
$(4,10)$	0	0	0	2	0	0	0	12	48	10.96	0	0	0	2	0	0	0	14	50	12.16
$(8,30)$	0	0	0	12	0	0	0	48	64	31.86	0	0	0	6	0	0	0	52	68	32.73
$(15,50)$	0	0	0	11	0	0	0	88	64	58.88	0	0	1	9	0	0	10	93	68	62.24
$(20,100)$	0	0	0	27	0	0	0	189	70	111.68	1	1	1	12	2	2	16	192	74	115.06

Table 5.5: Shipment components of Manufacturer-Warehouse echelon when $\alpha = 0.5$.

(p,n)	SYN										IND									
	m_1^W	m_2^W	m_3^W	m_4^W	U_1^W	U_2^W	U_3^W	U_4^W	N	ξ^W	$\widetilde{m_1}^W$	$\widetilde{m_2}^W$	$\widetilde{m_3}^W$	$\widetilde{m_4}^W$	$\widetilde{U_1}^W$	$\widetilde{U_2}^W$	$\widetilde{U_3}^W$	$\widetilde{U_4}^W$	$\widetilde{N^R}$	$\tilde{\xi}^W$
$(4,10)$	0	0	1	1	0	0	12	6	48	14.09	2	2	2	1	5	21	24	2	56	25.65
$(8,30)$	2	0	1	6	8	0	4	48	64	37.31	3	3	3	4	18	44	93	22	70	73.84
$(15,50)$	0	0	0	9	0	0	0	88	64	60.83	6	4	4	4	26	44	125	59	70	111.48
$(20,100)$	0	1	0	21	0	28	0	189	70	129.30	6	4	5	8	30	22	105	201	76	166.87

Table 5.6: Shipment components of Warehouse-Retailer echelon when $\alpha = 0.5$.

(p,n)	SYN										IND									
	m_1^S	m_2^S	m_3^S	m_4^S	U_1^S	U_2^S	U_3^S	U_4^S	$\sum_i U_i^S$	ξ^S	$\widetilde{m_1}^S$	$\widetilde{m_2}^S$	$\widetilde{m_3}^S$	$\widetilde{m_4}^S$	$\widetilde{U_1}^S$	$\widetilde{U_2}^S$	$\widetilde{U_3}^S$	$\widetilde{U_4}^S$	$\sum_i \widetilde{U_i}^S$	$\tilde{\xi}^S$
$(4,10)$	0	0	1	3	0	0	12	18	30	25.06	2	2	2	3	5	21	24	16	66	37.81
$(8,30)$	2	0	1	18	8	0	4	96	108	69.17	3	3	3	10	18	44	93	74	229	106.57
$(15,50)$	0	0	0	20	0	0	0	176	176	119.71	6	4	5	13	26	44	135	152	357	173.73
$(20,100)$	0	1	0	48	0	28	0	378	406	240.98	7	5	6	20	32	24	121	393	570	281.93

Table 5.7: Shipment components of the whole system when $\alpha = 0.5$.

5.8 Conclusions

In this research, an extension of the synchronized cycles production-warehouse location-inventory (PWLI) model is proposed. In this extended model (PWLIR), the deliveries are modeled by a set of fleet size and mix vehicle routing problems with split deliveries (FSMVRPSD). This model aims to determine the warehouse location, production and ordering frequencies and the routes for the shipment so as to minimize the total system relevant cost. The synchronized cycles PWLIR is solved by a PWLI-first and FSMVRPSD-second procedure. The PWLIR is then compared with the independent policy model. At last, the impact on the total cost and shipment components via coordination has been analyzed.

The overall performance of the synchronized cycles PWLIR model is always better than that of the independent policy model by an average of 33%, ranging from 29% to 36%. The average improvement percentages of the shipment cost of the synchronized model over the independent policy are 34%, 35%, 31% and 16% for Examples 1-4, respectively.

From the comparisons of the shipment components of the synchronized cycles model and the independent policy model, the following conclusions are drawn:

(1) There are more routes served by the vehicles of large capacity in the synchronized model than in the independent model.

(2) Most of the required vehicles in the synchronized model are large vehicles, while the required vehicles in the independent policy model are more diversified.

(3) The shipment cost can be reduced by adopting the synchronized cycles model,

where more deliveries can be fulfilled by routes of multi-individual rather than routes of single-individual as in the independent policy model. The cost reduction is due to the fact that using a large vehicle for a route of multi-individual is usually cheaper than using several small vehicles for the split routes of single-individual.

(4) There is a trade-off between the number of open warehouse and the shipment cost. For the cases where the number of open warehouse in the IND is much higher than that of the SYN, the improvement percentage for the shipment cost in the SYN over the IND is smaller when comparing with the cases where the numbers of open warehouse in the SYN and the IND are close to each other.

Chapter 6

Integrated Multi-Depot Vehicle Routing and Delivery Men Problem

6.1 Introduction

With the bloom of information technology and e-commerce, the 'last-mile' delivery problem becomes a more practical and challenging issue in nowadays logistics management since the deliveries are usually required to be made in one day and meanwhile the total relevant cost is minimized. For the 'last-mile' deliveries in high-density populated area, where accessibility constraints may arise for the vehicles' parking due to the heavy traffic and scarcity of parking slots, some hybrid delivery models which incorporate two types of urban transport modes have been proposed, i.e. (1) the truck and trailer routing problem (Villegas et al. (2013), Belenguer et al. (2015)) where some of the customers can be served by a trailer detached from a truck in the second-level delivery and, (2) the flying and sidekick traveling salesman problem (Murray and Chu (2015)) where the customers can also be served in the second-level delivery fulfilled by a set of unmanned aircraft coordinated with the driver-operated truck. This is similar to the Amazon's Prime Air UAV system (UAV stands for the unmanned aerial vehicles).

In this research, an integrated two-echelon vehicle routing model is proposed for the 'last-mile' delivery problem. This model integrates a distribution network of multiple depots, multiple parking-sites and multiple customers linked by the trips of a fleet of homogeneous vehicles and a number of delivery men assigned to the vehicles. In the first-level distribution, goods are transported from depots to parking-sites by vehicles, while in the second-level distribution, the goods are then delivered to customers by delivery men's multi-trips between the parking sites and customers' locations. The objective of this model is to determine the number of delivery men assigned to each vehicle and the routing for the vehicles and delivery men so as to minimize the total relevant costs involved in the two levels. Since the delivery efficiency is also an important element in the 'last-mile' delivery, the cost of the customers' goodwill is also considered in the proposed model besides the cost for vehicle and delivery men. The cost of the customers' goodwill is reflected by the total waiting time for the customers.

The organization of the rest of this chapter is as follows: Section 6.2 provides the assumptions and notations adopted. Section 6.3 presents the formulation of the integrated multi-depot vehicle routing and delivery men problem. Section 6.4 discusses heuristics for finding the optimal solution of our model. Section 6.5 presents the numerical results. Conclusion are given in Section 6.6.

6.2 Assumptions and Notations

6.2.1 Assumptions

The following assumptions are made throughout this research.

- All the customers are pre-clustered.
- The assignment between the parking site and customer cluster is pre-defined and each cluster can only be assigned to one parking site.
- Each parking site is visited exactly once by one vehicle.
- Each customer is served exactly once by one delivery man.
- Each vehicle has one route at most.
- Each delivery has one vehicle route at most.
- Multiple trips are allowed for a delivery man.
- The total demand of each customer cluster is smaller than the vehicle capacity.
- The delivery system starts at time 0.

6.2.2 Notations

Parameters:

V: Set of all the nodes, $V = V_C \bigcup V_P \bigcup V_D$.

V_C: Set of n customers.

V_P: Set of p parking sites.

V_D: Set of m depots.

V_C^k: Set of customers who would be served by the delivery man from parking site k.

d_{ij}: Distance between node i and j, $i, j \in V, i \neq j$.

q_i: Demand of customer $i \in V_C$.

q_i^P: Unloading quantity at parking site i.

Q: Capacity of each vehicle.

s: Average speed of each vehicle.

Q_0: Capacity of each delivery man.

s_0: Average speed of each delivery man.

L: Maximum number of delivery men can be assigned to a vehicle. The vehicle assigned with l delivery men is called mode-l vehicle, $1 \leqslant l \leqslant L$.

M: Maximum number of delivery men can be assigned to the whole fleet of vehicles.

T: Maximum time duration for the system delivering all the demands to the customer.

α: Fixed cost of each vehicle.

β: Variable cost of vehicle for per unit distance .

γ: Personnel cost for each delivery man.

λ: Goodwill cost for customers' per unit waiting time, which is the elapsed time from the delivery system starting to the customer being visited by a delivery man.

Variables:

x_{ijlk}: Binary decision variable. Equals to 1 if the pair of nodes i and j is served by a mode-l vehicle started from depot k, and 0 otherwise, $i, j \in V_D \bigcup V_P, i \neq j$,

$1 \leqslant l \leqslant L$, $k \in V_D$.

$y_{ijrw}^{k,l}$: Binary decision variable. Equals to 1 if a mode-l vehicle is parking at parking site k and the arc $(i,\ j)$ is traversed by the wth route of rth delivery man, $i, j \in V_C^k \bigcup \{k\}, i \neq j$, $1 \leqslant l \leqslant L$, $k \in V_D$, $1 \leqslant r \leqslant l$, $w \in \mathbf{Z}^+$.

u_{kl}: The loading on the mode-l vehicle when arrives at node k, $k \in V_D \bigcup V_P$.

$\widetilde{u}_{ikl}$ the loading on the delivery man when arrives at node $i \in V_C^k \bigcup \{k\}$ associated with a mode-l vehicle, $k \in V_P$.

t_p^{kl}: The parking time for a mode-l vehicle at parking site k.

T_a^k: The time when the last vehicle arrives at node k, $k \in V_D \bigcup V_P$.

T_b^i: The time when customer i is visited by a delivery man.

T_{rw}^{kl}: The starting time of the wth route of the rth delivery man when delivering the demands of the customers in V_C^k associated with a mode-l vehicle.

6.3 Model Formulation

The integrated multi-depot vehicle routing and delivery men routing problem (MD-VRDMP) can be formulated in a mathematical model as follows. Let $G = (V, E)$ be a directed graph, where V is the set of all the nodes in the system and $E = \{(i, j) : i, j \in V\}$ is the set of arcs connecting each pair of the nodes. The nodes set V can be further divided into three sub-sets: $V_C = \{v_1, \ldots, v_n\}$ which is the customer set to be served; $V_P = \{v_{n+1}, \ldots, v_{n+p}\}$ which is the set of parking sites; and $V_D = \{v_{n+p+1}, \ldots, v_{n+p+m}\}$ is the set of depots. The customers are all pre-clustered and each customer cluster is pre-assigned to one parking site. Denote V_C^i as the set of customers to be served by parking site $i \in V_P$. Each customer $i \in V_C$ has a nonnegative demand q_i and hence the total unloading quantity at parking site i, denoted by q_i^P, can be expressed as $q_i^P = \sum_{j \in V_C^i} q_j$. The length of arc $(i, j) \in E$ is d_{ij}. There is a fleet of homogeneous vehicles with capacity Q and speed s. Each vehicle can be assigned with a set of delivery men and the number of delivery men assigned to each vehicle, denoted by l, can not be larger than L. The vehicle with l delivery men is called a mode-l vehicle. Each delivery man has a loading capacity Q_0 and with a speed of s_0.

In the first echelon, each vehicle route starts from a depot and then stops at a set of parking site waiting for all the assigned delivery men delivering the demands to the corresponding customers on foot and coming back to the parking site, and ends at the same depot. In the second echelon, a delivery man leaves the parking site carrying the demands to the customers and returns to the same parking site. Note that the delivery man is allowed to have multiple trips from any parking site since the demands may be larger than his loading capacity.

The parking time for the vehicle at node $k \in V_P$, denoted by t_p^{kl}, where l is the number of delivery men assigned to this vehicle. Denote T_a^k as the latest arrival time of the vehicles at node $k \in V_D \bigcup V_P$, i.e. since each parking site can only be visited exactly once, when $k \in V_P$, this notation means the arrival time of the vehicle at the parking site. For each route, there is a constraint on the time duration, which can not exceed T, i.e. $T_a^k \leqslant T,\ k \in V_D$. Notations T_b^i are introduced to represent the time when delivery man arrives at customer $i \in V_C$. When a mode-l vehicle arrives at node $k \in V_D \bigcup V_P$, the loading on the vehicle is denoted by u_{kl}, and when a delivery man arrives at the node $i \in V_C^k \bigcup \{k\}$, the loading at the delivery man is denoted by $\widetilde{u}_{ikl}$.

The binary decision variable x_{ijlk} equals to 1 if the pair of nodes i and j is served by a mode-l vehicle started from depot k, and 0 otherwise. The binary decision variable y_{ijrw}^{kl} equals to 1 if a mode-l vehicle is parking at node $k \in V_P$ and the arc $\{(i,j) : i,j \in V_C^k\}$ is traversed by the wth route of the rth delivery man (the starting time for this route is denoted by T_{rw}^{kl}) of this vehicle ($r \leqslant l$), and 0 otherwise. With these notations, the expression of the arrival time for vehicle at parking site $j \in V_P$ is

$$T_a^j = \sum_{i \in V_D \bigcup V_P \setminus \{j\}} \sum_{l=1}^{L} \sum_{k \in V_D} (T_a^i + t_p^{il} + \frac{d_{ij}}{s}) x_{ijlk}, j \in V_P. \tag{6.1}$$

And the last time for depot $k \in V_D$ being visited is at time

$$T_a^k = \max_{i \in V_P} \sum_{l=1}^{L} \sum_{k \in V_D} (T_i^a + t_p^{il} + \frac{d_{ik}}{s}) x_{iklk}, k \in V_D. \tag{6.2}$$

When a mode-l vehicle stops at parking site $k \in V_P$, the wth route of the rth delivery man starts at time

$$T_{rw}^{kl} = \begin{cases} T_a^k, & \text{when } w = 1 \\ T_a^k + \frac{1}{s_0} \sum\limits_{h=1}^{w-1} \sum\limits_{i,j \in \{v_k\} \bigcup V_C^k} y_{ijrh}^{kl} \cdot d_{ij}, & \text{when } w > 1 \end{cases}. \tag{6.3}$$

Then the time when service begins at customer $i \in V_C^k$ can be expressed as

$$T_b^i = \begin{cases} T_{rw}^{kl} + \frac{d_{ki}}{s_0}, & \text{when } i \text{ is the first stop of the route} \\ \sum\limits_{j \in V_P \bigcup V_C \setminus \{i\}} [y_{jirw}^{kl} \cdot (T_b^j + \frac{d_{ji}}{s_0})], & \text{otherwise} \end{cases}. \tag{6.4}$$

The waiting time for a mode-l vehicle at parking site $k \in V_P$ is

$$t_p^{kl} = \max_{\{r=1,\ldots,l\}} \frac{1}{s_0} \sum_{i,j \in \{v_k\} \bigcup V_C^k,\ i \neq j} \sum_h y_{ijrh}^{kl} \tag{6.5}$$

Then the MDVRDMP with a fleet of homogeneous vehicles can be formulated as follows:

$$\begin{aligned} \min \qquad z \;=\; & \alpha \sum_{i \in V_D} \sum_{j \in V_P} \sum_{l=1}^{L} \sum_{k \in V_D} x_{ijlk} + \beta \sum_{i,j \in V_D \bigcup V_P, (i \neq j)} \sum_{l=1}^{L} \sum_{k \in V_D} d_{ij} \cdot x_{ijlk} \\ & + \gamma \sum_{i \in V_D} \sum_{j \in V_P} \sum_{l=1}^{L} \sum_{k \in V_D} l \cdot x_{ijlk} + \lambda \sum_{i \in V_C} T_b^i \end{aligned} \tag{6.6}$$

$$\text{s.t.} \qquad \sum_{i \in V_D \bigcup V_P \setminus \{j\}} \sum_{l=1}^{L} \sum_{k \in V_D} x_{ijlk} = 1,\ \ j \in V_P \tag{6.7}$$

$$\sum_{j \in V_D \bigcup V_P \setminus \{i\}} \sum_{l=1}^{L} \sum_{k \in V_D} x_{ijlk} = 1,\ \ i \in V_P \tag{6.8}$$

$$\sum_{i \in V_P \bigcup V_C \setminus \{j\}} \sum_{l=1}^{L} \sum_{r=1}^{l} \sum_{w} y_{ijrw}^{kl} = 1,\ \ j \in V_C,\ \ k \in V_P \tag{6.9}$$

$$\sum_{j \in V_P \bigcup V_C \setminus \{i\}} \sum_{l=1}^{L} \sum_{r=1}^{l} \sum_{w} y_{ijrw}^{kl} = 1,\ \ i \in V_C,\ \ k \in V_P \tag{6.10}$$

$$\sum_{j \in V_D \bigcup V_P \setminus \{i\}} \sum_{l=1}^{L} \sum_{k \in V_D} x_{ijlk} = \sum_{j \in V_D \bigcup V_P \setminus \{i\}} \sum_{l=1}^{L} \sum_{k \in V_D} x_{jilk},\ \ i \in V_D \bigcup V_P \tag{6.11}$$

$$\sum_{j \in V_C^k \bigcup \{k\} \setminus \{i\}} \sum_{r=1}^{l} \sum_{w} y_{ijrw}^{kl} = \sum_{j \in V_C^k \bigcup \{k\} \setminus \{i\}} \sum_{r=1}^{l} \sum_{w} y_{jirw}^{kl},\ \ i \in V_C^k \bigcup \{k\}, 1 \leqslant l \leqslant L \tag{6.12}$$

$$u_{jl} \leqslant u_{il} - x_{ijlk} \sum_{r \in V_C^k} q_r + Q(1 - x_{ijlk}),\ \ i,j \in V_P, i \neq j, 1 \leqslant l \leqslant L, k \in V_D \tag{6.13}$$

$$\widetilde{u}_{jkl} \leqslant \widetilde{u}_{ikl} - q_i \sum_{r=1}^{l} \sum_{w} y_{ijrw}^{kl} + Q_0(1 - \sum_{r=1}^{l} \sum_{w} y_{ijrw}^{kl}),\ i, j \in V_C^k, i \neq j, 1 \leqslant l \leqslant L, k \in V_D \tag{6.14}$$

$$T_a^i + (t_p^{il} + \frac{d_{ij}}{s}) x_{ijlk} - T(1 - x_{ijlk}) \leqslant T_a^j,\ i \in V_P, j \in V_D \bigcup V_P, 1 \leqslant l \leqslant L, k \in V_D \tag{6.15}$$

$$\sum_{i \in V_D} \sum_{j \in V_P} \sum_{l=1}^{L} \sum_{k \in V_D} l \cdot x_{ijlk} \leqslant M \tag{6.16}$$

$$\sum_{r \in V_C^j} q_r \leqslant \sum_{i \in V_D \bigcup V_P \backslash \{j\}} \sum_{l=1}^{L} \sum_{k \in V_D} u_{jl} \cdot x_{ijlk},\ j \in V_P, 1 \leqslant l \leqslant L \tag{6.17}$$

$$u_{jl} \leqslant \sum_{i \in V_D \bigcup V_P \backslash \{j\}} \sum_{k \in V_D} Q \cdot x_{ijlk},\ j \in V_P, 1 \leqslant l \leqslant L \tag{6.18}$$

$$\sum_{i,j \in V_C^k \bigcup \{k\}} \sum_{r=1}^{l} \sum_{w} y_{ijrw}^{kl} \leqslant \sum_{i' \in V_D \bigcup V_P \backslash \{k\}} \sum_{k' \in V_D} x_{i'klk'},\ k \in V_P, 1 \leqslant l \leqslant L \tag{6.19}$$

$$0 \leqslant T_a^i \leqslant T,\ i \in V_D \bigcup V_P \tag{6.20}$$

$$x_{ijli'} = 0,\ i, i' \in V_D, i \neq i', j \in V_P, 1 \leqslant l \leqslant L \tag{6.21}$$

$$x_{ijlj'} = 0,\ j, j' \in V_D, j \neq j', i \in V_P, 1 \leqslant l \leqslant L \tag{6.22}$$

$$u_{kl} = 0,\ k \in V_D, 1 \leqslant l \leqslant L \tag{6.23}$$

$$\widetilde{u}_{kkl} = 0,\ k \in V_P, 1 \leqslant l \leqslant L \tag{6.24}$$

$$x_{ijlk} = \{0, 1\},\ i, j \in V_D \bigcup V_P, i \neq j, 1 \leqslant l \leqslant L, k \in V_D \tag{6.25}$$

$$y_{ijrw}^{kl} = \{0, 1\},\ i, j \in V_C^k \bigcup \{k\}, i \neq j, 1 \leqslant l \leqslant L, 1 \leqslant r \leqslant l, k \in V_D, w \in \mathbf{Z}^+ \tag{6.26}$$

The objective function in Eq. (6.6) minimizes the total relevant cost z, which consists of the cost of the following four components with a cost coefficient for each of them, i.e. α, β, γ and λ, respectively.

- The total number of assigned vehicles to the delivery system, given by the term $\sum_{i\in V_D}\sum_{j\in V_P}\sum_{l=1}^{L}\sum_{k\in V_D} x_{ijlk}$.
- The total distance travelled by the vehicles, given by the term $\sum_{i,j\in V_D\bigcup V_P,(i\neq j)}\sum_{l=1}^{L}\sum_{k\in V_D} d_{ij}\cdot x_{ijlk}$.
- The total number of assigned delivery men, given by the term $\sum_{i\in V_D}\sum_{j\in V_P}\sum_{l=1}^{L}\sum_{k\in V_D} l\cdot x_{ijlk}$.
- The sum of arrival times at the customers, given by $\sum_{i\in V_C} T_b^i$.

The first two terms are widely used in the literature of vehicle routing problems, which aims to minimize the fixed and variable cost of the vehicles. The third term can be seen as the personnel cost of the delivery men, which also has been discussed in some of the literatures(Pureza et al. (2012), de Grancy and Reimann (2014)). The fourth term, a sum of the arrival time at each nodes, which is usually used as the objective of the following two variants of the vehicle routing problem. (1) The cumulative capacitated vehicle routing problem (Ribeiro and Laporte (2012), Rivera et al. (2016)). This type of routing problem arises when the priority of the customers are considered, i.e. vital goods need to be delivered to or picked up from a set of nodes after a natural disaster. (2) The delivery man problem (Heilporn et al. (2010), Bjelić et al. (2013), Luo et al. (2014)). For some variants of the vehicle routing problem, both the performance of time and profits are incorporated in one objective function (Coene and Spieksma (2008), Dewilde et al. (2013)). And a generalized consistent vehicle routing problem was proposed in Kovacs et al. (2014), where the objective function is to minimize the weighted average of the total travel time and the maximum arrival time difference. In this research, a sum of the vehicle fixed and variable cost, delivery men cost and customers' goodwill cost is adopted as the objective function to illustrate the performance of this model.

Constraints (6.7)-(6.8) guarantee that each parking site is visited exactly once, and similarly, constraints (6.9)-(6.10) ensure that each customer is served by one delivery man exactly once. Constraints (6.11)-(6.12) are the flow conservation equations for the routes of vehicles and delivery men, respectively, stating that for each node, the vehicles or delivery men leaving from this node are exactly the same as the ones entering the node. Constraint (6.13) depicts the relation between the routing flow variables and the load variables for the vehicle routing echelon, to guarantee that there is no sub-tours without the depot. Similarly, constraint (6.14) prevents the sub-tours for the delivery men's routing. Constraint (6.15) defines the relation between the routing flow variables and the time variable for each node in the first echelon. This eliminates the formation of sub-tours without depots. Constraint (6.16) ensures that the total number of assigned delivery men does not exceed the available number M. Constraints (6.17)-(6.18) state that when a vehicle enters parking site j, the loading on this vehicle is larger than the total demand of the customers who are assigned to this parking site, and meanwhile this loading is less than or equal to the vehicle capacity. Constraint (6.19) requires that if there is no mode-l vehicle stopping at parking site k, then the flow variables for the delivery men routing with the upper index kl would be zero. Constraint (6.20) limits the maximum duration of the whole system. This means that for each depot, the returning time for the last vehicle can not be later than time T. Constraints (6.21)-(6.22) ensure that each vehicle can not visit two different depots. Constraint (6.23) states that the loading on each vehicle is zero when it returns to the depot, and similarly, (6.24) imposes that there would be no load for the delivery man when he comes back to the parking site. Constraints (6.25)-(6.26) refer to the binary of the decision variables.

6.4 Heuristics

For the special delivery structure of the model, a delivery man problem DMP(k, l) is formulated for a given parking site k and a visiting vehicle of mode-l, i.e. the number of delivery men assigned to this vehicle. Since only the fourth term in Eq.(6.6) is affected by the second-echelon's routing, for routes of the l delivery men, they need to be determined with the objective of minimizing the sum of the waiting time of the customers whom are assigned to the parking site k. Since the routing of the delivery men is based on the input value of l assigned in the vehicle routing level, the procedure of searching for the solutions of MDVRDMP needs to be divided into two parts, i.e. determine (1) the vehicle routing and delivery men assignment first and (2) determine the routes for the delivery men. However, the solutions from both of the two parts affect each other, i.e. the delivery efficiency of the second-echelon is largely dependent on the number of delivery men assigned to the vehicle and in turn, the routing of delivery men may influence the time that the vehicle returns to the depot. Thus the maximum duration time for the vehicle may be violated. Hence, the whole searching procedure can not be simply decomposed into two parts and solved sequentially.

Since the model for MDVRDMP embeds a set of delivery man problems (DMP), which is NP-hard, two heuristics, i.e. genetic algorithm (GA) and adaptive large neighbourhood search (ALNS), are applied to find the optimal solutions of this model.

The routing schedule of the two echelons are solved iteratively, and the routing information for each echelon is exchanged when moving between solution spaces. The detailed process for finding the optimal solutions of MDVRDMP is as follows (also shown in Figure 6.1).

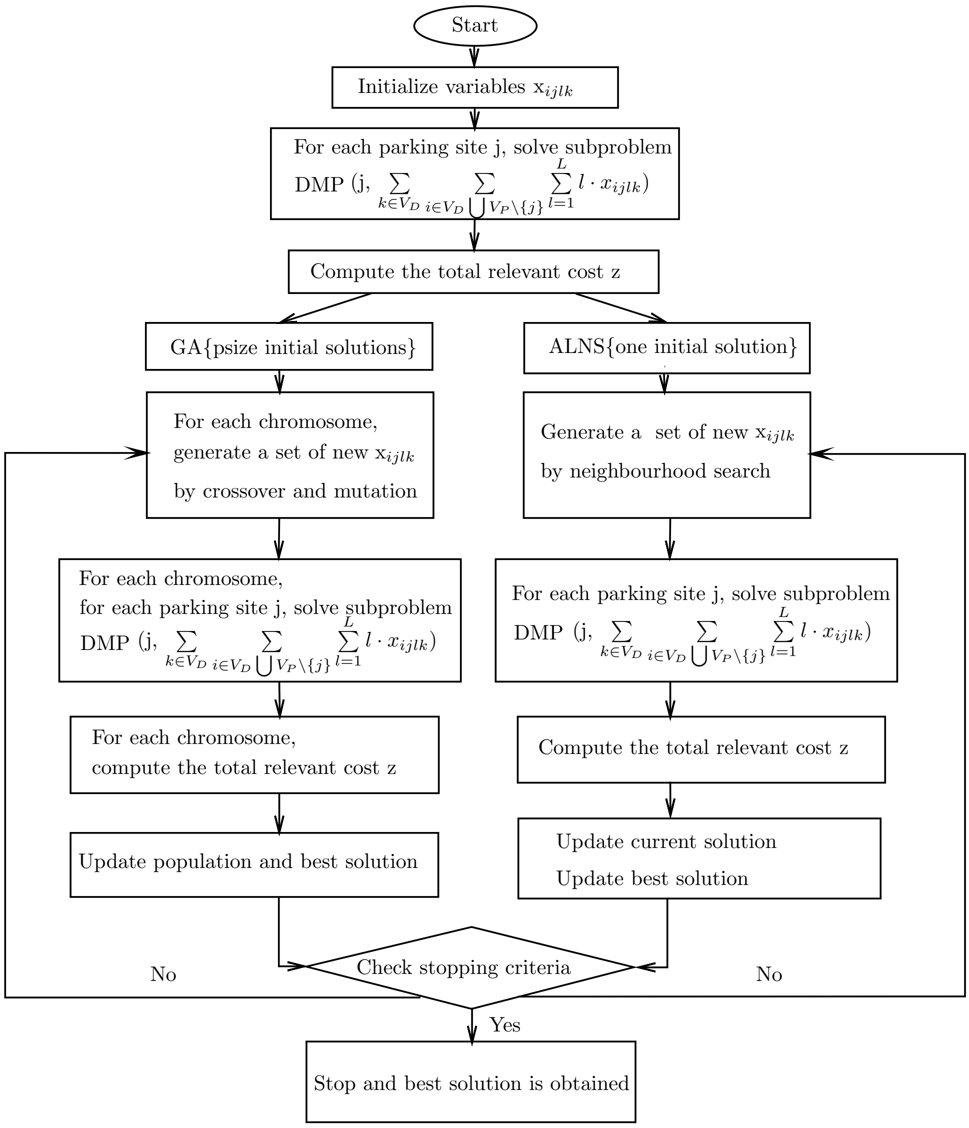

Figure 6.1: Flowchart for the heuristics

Phase 1: Finding the initial solution.

Step 1: Initialize the variables of x_{ijlk}, then the routing schedule of the first-level delivery is determined.

Step 2: The number of delivery men assigned to parking site j can be calculated as $\sum_{k\in V_D}\sum_{i\in V_D\bigcup V_P\backslash\{j\}}\sum_{l=1}^{L} l\cdot x_{ijlk}$. Then for each parking site, a subproblem DMP(j, $\sum_{k\in V_D}\sum_{i\in V_D\bigcup V_P\backslash\{j\}}\sum_{l=1}^{L} l\cdot x_{ijlk}$) is formulated. The routing schedule of the second-level delivery can be obtained by solving a set of p DMP problems.

Step 3: The sum of the delivery cost of two levels of this initial solution can be calculated by Eq.6.6.

Step 4: The number of initial solutions in GA is $psize$, where $psize$ is the population size of the GA. The number of initial solution for ALNS is one.

Phase 2: Improving of the initial solution by GA or ALNS

Step 5: Searching new values for the variables of x_{ijlk} for the first-level delivery. In GA, this process is conducted by crossover and mutation of the current solution. In ALNS, it is conducted by moving to a neighbour of the current solution.

Step6: Based on the new solutions of x_{ijlk}, a set of p DMP problems is formulated. In GA, there are a number of $psize\times p$ DMPs need to be solved. In ALNS, the number of DMPs to be solved is p.

Step 7: Based on the first-level routing schedule obtain from step 5 and second-level routing schedule from step 6, the fitness of the population in GA and the objective value of the new solution in ALNS can be calculated.

Step 8: In GA, update the survive population in this iteration and record the best solution found so far. In ALNS, update the current solution and the best solution found so far.

Step 9: Check the stoping criteria is achieved or not. If achieved, then the algorithm stops and the best solution for the MDVRDMP is found. Otherwise, go to step 5.

6.4.1 Genetic Algorithm for Solving the MDVRDMP

A survey of genetic algorithms (GA) that are designed for solving the multi-depot vehicle routing problem was presented in Karakatič and Podgorelec (2015). In this survey, the most frequent operators for selection, crossover and mutation were reviewed and tested on five standard benchmark problems. Results showed that the solutions obtained by GA did not achieve the optimal solutions for all the problems but the their deviations from the optimal solutions were within 10%. Karakatič and Podgorelec (2015) concluded that GA could find good enough solutions efficiently. The comparison for different GA operators was also conducted and some operators were found to be quite good for the standard multi-depot vehicle routing problem. Based on the good operators found in Karakatič and Podgorelec (2015), a GA method is developed to solve the MDVRDMP in this research. The main processes for the GA are described in Algorithm 6.1.

Gene Representation

The shipments in the vehicle routing echelon are made from m depots to p parking sites. A chromosome is encoded by four segments: (I) an array of the p parking sites, $\{a_1, a_2, \ldots, a_p\}$, which is a permutation of the index of parking sites, (II) the index of depots for the p parking sites, $\{b_1, b_2, \ldots, b_p\}$, i.e. parking site a_i is assigned to

Algorithm 6.1: GA for MDVRDMP

```
Function (GA)
Population initialization: the length of each chromosome is 4×p;
While stopping criteria is not achieved do
    Crossover;
    Mutation;
    Updating population;
End While
```

depot b_i, $i = 1, \ldots, p$, (III) the index of route for the p parking sites, $\{c_1, c_2, \ldots, c_p\}$, parking site a_i is served by the c_ith route, $i = 1, \ldots, p$, (IV) the number of delivery men for the p parking sites, $\{d_1, d_2, \ldots, d_p\}$, parking site a_i is visited by a vehicle with d_i delivery men, $i = 1, \ldots, p$.

Fitness Value

Whenever a permutation of parking site index is generated, it is needed to map it to a solution so as to obtain the fitness value calculated by Eq.(6.6). The outlines of the mapping method, i.e. gene mapping, are shown in Algorithm 6.2. Lines 3-13 is to assign a depot and a route to each parking site. The process is started with the vehicle from the first depot and visit the first unscheduled customer in the permutation array. This process continues until the vehicle's vacancy is not enough for the customer's demand or a randomly generated number is larger than or equal to the ratio of current vacancy over the vehicle capacity. This vehicle then returns to its home depot, and a vehicle from next depot starts with the following unscheduled customer in the permutation array. If this depot is already the last one, then the next assigning depot is the first one again. After all the parking site being assigned to a depot and a route, the assignment of delivery man to the vehicles begins from Line 14 to Line 20. In Line 14, the initial number of delivery man for each vehicle is set as one. Then an extra number of delivery men can be assigned to the vehicle (Line 17), making the total assigned number in this vehicle not exceeding L and the total number assigned number in the system not exceeding M.

Crossover

A random value from (0,1) is generated to determine whether the crossover process is performed. A 2-point crossover is applied to Segment I of two chromosomes if the random value is less than the crossover rate. Two points, i and j are then chosen from the segment, where $1 \leqslant i < j \leqslant p$. For each child's Segment I, the middle part (from point i to point j) is the same as the one of their parent's Segment I, and the other parent is swept from point $j+1$ circularly onward to complete the child. A demonstration can be found in Figure 6.2, $p = 12$, $i = 4$, $j = 9$. Child 1 has the same Segment I as Parent 1. A candidate list 1 is then made for child 1. Candidate list 1 is formed by the $j+1$th to the pth gene of Parent 2 followed by the first to the jth node of Parent 2. Then candidate list 1' is generated from candidate list 1 by deleting the genes which can be found in Parent 1's Segment I. Then the genes of candidate list 1' are put into Child 1's blank positions. The first gene of candidate list 1' is put into the $j+1$th position of Child 1. Then the next gene of candidate list 1' is put into the $j+2$th position of Child 1. This process continues in a circular manner until Child 1 is completed. The process for Child 2 is similar. After the crossover process is conducted for Segment I, the vehicle capacity is checked to see whether each child gene satisfies the capacity constraint. If not, gene mapping process of the Segment

Algorithm 6.2: Gene Mapping

Function (Gene Mapping $\{a_1, a_2, \ldots, a_p\}$)

Set gene position i=1, assigning depot J=1, assigning parking site a_i, route index $R = 1$, vehicle vacancy $\Delta = Q$;

While $i \leqslant p$ ***do***

  Generate a random number $0 < \widetilde{r} < 1$

  If $\widetilde{r} < \frac{\Delta}{Q}$ and $\Delta > q^P_{a_i}$ **then**

    $b_i = J,\ c_i = R,\ \Delta = \Delta - q^P_{a_i}$

  Else

    Update assigning depot. If $J = m$, then $J = 1$, otherwise $J = J + 1$;

    Add a new route, $R = R + 1$ and $\Delta = Q$;

    $b_i = J,\ c_i = R,\ \Delta = \Delta - q^P_{a_i}$

  End If

  Update gene position $i = i + 1$;

End While

For the rth route, initialize the the number of delivery men assigned to this route as $l(r) = 1$. Then the number of available delivery men $S = M - R$;

Set current assigning route $R = 1$;

While The delivery man reassignment is not conducted for all the routes ***do***

  Choose a random number l' from $[0, 1, \ldots, \min\{L - 1, S\}]$

  Update $l(r) = 1 + l'$, $S = S - l'$;

  Choose another unassigned route

End While

Update $d_i = l(c_i)$ for all i;

I of this child gene will be performed.

Parent 1	8	10	11	**6**	**9**	**4**	**5**	**1**	**7**	3	2	12
Parent 2	4	9	2	**6**	**5**	**11**	**10**	**12**	**1**	7	8	3

Child 1				**6**	**9**	**4**	**5**	**1**	**7**			
Candidate list 1	$\boxed{7}$	8	3	$\boxed{4}$	$\boxed{9}$	2	$\boxed{6}$	$\boxed{5}$	11	10	12	$\boxed{1}$
						$\Downarrow$						
Child 1				**6**	**9**	**4**	**5**	**1**	**7**			
Candidate list 1’		8	3			2			11	10	12	
						$\Downarrow$						
Child 1	11	10	12	**6**	**9**	**4**	**5**	**1**	**7**	8	3	2

Child 2				**6**	**5**	**11**	**10**	**12**	**1**			
Candidate list 2	3	2	$\boxed{12}$	8	$\boxed{10}$	$\boxed{11}$	$\boxed{6}$	9	4	$\boxed{5}$	$\boxed{1}$	7
						$\Downarrow$						
Child 2				**6**	**5**	**11**	**10**	**12**	**1**			
Candidate list 2’	3	2		8				9	4			7
						$\Downarrow$						
Child 2	8	9	4	**6**	**5**	**11**	**10**	**12**	**1**	7	3	2

Figure 6.2: 2-point crossover for segment I

Mutation

Seven mutation operators are used in this research and at most one of them is chosen to be performed in each iteration. Each operator is selected with the same probability.

1. Randomly choose two points i and j $(1 \leqslant i < j \leqslant p)$ from Segment I. Perform an inversion mutation to the points and gene mapping to the new Segment I if

capacity constraint is not satisfied.

2. Randomly choose two points i and j $(1 \leqslant i < j \leqslant p)$ from Segment I. Swap the two points and perform gene mapping if necessary.

3. Randomly choose one route i $(1 \leqslant i \leqslant \max_j c_j)$ and change the number of delivery men for this route.

4. Randomly choose one route i $(1 \leqslant i \leqslant \max_j c_j)$ and change the assigned depot for this route.

5. Randomly choose one route i $(1 \leqslant i \leqslant \max_j c_j)$ and split it to two routes if this route has at least two stops.

6. Randomly choose two routes i and j $(1 \leqslant i \leqslant j \leqslant \max_j c_j)$. Swap the assigned depots for two routes.

7. Randomly choose two points i and j $(1 \leqslant i < j \leqslant p)$. Swap a_i and a_j if they are in the same route $c_i = c_j$.

6.4.2 Adaptive Large Neighbourhood Search for Solving the MDVRDMP

Adaptive large neighbourhood search (ALNS) (Ropke and Pisinger (2006)) has been widely used for a class of vehicle routing problems in recent years. In this research, a heuristic based on ALNS is proposed to solve the MDVRDMP. The main process for this ALNS is similar to the one in Section 5.6.1 and the outlines are shown in Algorithm 6.3. In this ALNS, whenever a new solution is generated (Line 2 and Line 6), a local search is performed based on a greedy algorithm. Based on

Algorithm 6.3: ALNS for MDVRDMP

Function (MDVRDMP)

Initial solution: Generate an initial solution S_1 by applying the heuristics described in Algorithm 6.4;

While stopping criteria is not achieved ***do***

  Set inner iteration number $iter = 0$ and set the score for all the neighbourhood as zero

  While $iter$ is less than $ITER$ ***do***

    Choose a neighbourhood i, $S_2 \leftarrow neighbour_i(S_1)$, $iter = iter + 1$;

    Update the score of neighbourhood i;

  End While

  Update the weight w_i for each neighbourhood

End While

the performance (score) of each neighbour in every $ITER$ iterations, the weight of each neighbourhood is calclulated in the end of every $ITER$ iteration. Hence the probability of selecting each neighbourhood is dynamic over the ALNS process.

In the heuristics for generating an initial solution (Algorithm 6.4), the parking sites are clustered first, i.e. assigned to the nearest depot. When the vacancy of the current vehicle is larger than the total delivery quantity to the parking site, the parking site will be inserted to the this route with a probability Δ/Q, which is the ratio of vehicle vacancy over the vehicle capacity. When the vacancy decreases, a new route is more likely to be added.

The following neighbourhoods are used in the ALNS:

1. Randomly swap two parking sites from two different routes.

2. Randomly remove one parking site from one route and insert it to another

Algorithm 6.4: Initial Solution for MDVRDMP

Function (Initial Solution)

Assign parking site i to the nearest depot, $1 \leqslant i \leqslant p$. Then the vehicles from depot j would visit a set of parking sites, denoted V_j^P;

Representation for the rth route: $[d_r, l_r, a_1, \ldots, a_{i_r}]$, indicating this route starts from depot d_r with l_r delivery men on it and visit the parking site with the index order of $[a_1, \ldots, a_{i_r}]$.

For each depot j

    if $V_j^P \neq \varnothing$, **then**

        Current route for parking site j, $R(j) = 1$, vacancy of the vehicle $\Delta = Q$, the set of unrouted parking site $\Gamma = V_j^P$.

        While $\Gamma \neq \varnothing$, ***Do***

            Generate a random value $0 < rand < 1$;

            The first parking site in Γ, parking site i is to be assigned;

            If $rand < \frac{\Delta}{Q}$ and $\Delta > q_i^P$ **Then**

                Insert the parking site to the $R(j)$th route of depot j ;

                Update current vacancy $\Delta = \Delta - q_i^P$;

                Delete this parking site from Γ.

            Else

                Add a new route for depot j, $R(j) = R(j) + 1$, $\Delta = Q$;

                Insert this parking site to the $R(j)$th route of depot j;

                Update current vacancy $\Delta = \Delta - q_i^P$;

                Delete this parking site from Γ.

            END If

        End While

    End If

End For

route.

3. Randomly select one parking site to form a new route.

4. Randomly select one route and change the assigned depot.

5. Randomly select one route and change the number of assigned deliverymen.

6.4.3 Adaptive Large Neighbourhood Search for Solving the DMP

For each total cost evaluation, there would be p DMPs to be solved (see Figure 6.1). Thus, in the whole searching process, the number of DMP to be solved may be extremely large, i.e. $p\times$ (the number of iterations) $\times$ (population size) in GA and $p\times$ (the number of iterations) for ALNS. However, the total number of all the possible DMPs is only $p \times L$ (p parking sites and for each parking site, the number of delivery man assigned to this parking site has L choices, i.e. $1, \ldots, L$), which is far less than the number of times being solved in the searching procedure of GA and ALNS. To save the CPU time, the $p \times L$ DMPs can be solved in advance, and the obtained solutions can be called in Algorithms 6.1 and 6.3 directly rather than solved repeatedly.

The DMP(k, l) is to determine the routes of the l delivery men when they deliver the demands to the customers in V_k^C so as to minimize the total waiting time of these customers, i.e. $\sum_{i \in V_k^C}(T_b^i - T_a^k)$, where T_b^i is the time of customer i being visited and T_a^k is the time of the vehicle arriving at parking site k. An ALNS is applied to solve the DMP(k, l). The main process is similar to Algorithm 6.3. Whenever a new route is generated, a local search is performed. Since the objective is to minimize the total waiting time of all the customers, the order of the routes is also need to be optimized in the local search.

Assume that the delivery man has two routes which serve customers I:(0,1,2,3,0) and II:(0,4,0). Also assumes that the traveling time of each arc in the two routes is 1 unit of time.

If the order of the routes is (I,II)=(0,1,2,3,0,4,0), then the waiting time for the customer g_i ($i = 1, \ldots, 4$) are 1, 2, 3 and 5 units of time respectively. Hence, the total waiting time for the 4 customers is 11 units of time.

If the order of the routes is (II,I)=(0,4,0,1,2,3,0), the waiting time for the five customers are 1, 3, 4 and 5 units of time, respectively. Hence, the total waiting time for the 4 customers is 13 units of time.

Therefore the total waiting time of the customers for routes order (I,II) and (II,I) are 11 and 13 units of time, respectively. As a conclusion, even the routes have the same customer visiting order, the order of the routes still affects the total waiting time.

The following neighbourhoods are used for solving DMP(k, l).

1. Randomly swap two customers in two different route.
2. Randomly remove one customer from the route and insert it to another route.
3. Randomly select one customer to form a new route.
4. When there is more than one delivery man, $l > 1$, randomly swap two routes for two delivery men.

6.5 Numerical Results

Numerical experiments have been carried out to illustrate the performance of the integrated vehicle routing and delivery man model. The numerical results are presented for Examples 1-3, with sizes (m, p, n) of $(2, 12, 60)$, $(3, 20, 200)$ and $(6, 40, 400)$, respectively. The data of examples are randomly generated and are shown in Appendix E. The nodes in each example are scattered in a 50×50 square area. For each parking site, there is a set of clustered customers to be served by this parking site.

6.5.1 Heuristic Parameters

The numerical experiments are conducted with the following parameters.

The parameters for Genetic Algorithm (GA) are as follows:

Crossover rate: 0.6

Mutation rate: 0.01

Population size: 50

Stopping Criteria: The total amount of time is limited to one hour if no feasible solution is found. Otherwise, the algorithm stops when 10000 iterations are completed.

The parameters for Adaptive Large neighbourhood search (ALNS) are as follows:

Number of iteration for weight updating: The weight of each neighbour is updated in every 100 iterations.

Initial neighbourhood scores for every 100 iterations: Equals to 1 for each neighbourhood.

Stopping criteria: The total amount of time is limited to one hour if no feasible solution is found. Otherwise, the algorithm stops when 10000 iterations are completed.

6.5.2 Comparison of the Performance of ALNS and GA

The performance of the two heuristics for Examples 1-3 are presented in Table 6.1. The values for the number of vehicle (No.V.), total distance (Dis.), the number of assigned delivery man (No.D.), the total waiting time (W.T.) for the customers and the total cost (TC) obtained by the ALNS are depicted in columns 4-8, respectively. The corresponding values obtained by the GA are shown in columns 10-14, respectively. Note that the CPU time shown in Table 6.1 does not include the time for solving the $P \times L$ DMPs. When the algorithm were being run both ALNS and GA can call the routing solutions of the $P \times L$ DMPs. Hence it is unnecessary to take the time for solving DPMs into consideration when comparison is conducted between the ALNS and the GA.

From Table 6.1, it can be seen that the overall performance of the ALNS is always better than that of the GA. For Examples 1 and 2, both the ALNS and the GA have completed 10000 iterations. The total costs obtained by the ALNS are 6.44% and 20.9% less than that in the GA, respectively. But the CPU time used by ALNS is 60% of that in the GA at most. For Example 3, the GA fails to find a feasible solution in one hour. The GA performs worse as the problem size gets larger.

The routing components obtained by the ALNS are also better than the GA. For Example 1, the GA and the ALNS find the solutions with the same No.V., but the other components in the GA are all larger than those in the ALNS. For Example 2,

Ex.	(m,p,n)	ALNS(Best run out of 10)						GA(Best run out of 10)					
		CPU Time (s')	No.V	Dis.	No.D.	W.T.	TC	CPU Time (s')	No.V.	Dis.	No.D.	W.T.	TC
1	(2,12,60)	87	5	273	9	81	407	190	5	300	11	87	435
2	(3,20,200)	151	7	293	14	298	878	263	8	545	21	324	1110
3	(6,40,400)	364	11	417	22	385	1471	3600	17	1135	44	645	infeasible

Table 6.1: Results for Examples 1-3

the value of No.D. in the GA is 1.5 times of the value in the ALNS, but the W.T. in the GA is not benefited from the large value of No.D.

6.5.3 The Routes for the MDVRDMP

Vehicle Routing from Depots to Parking Sites

The routes in the first echelon is conducted by the vehicles from multiple depots to a set parking sites. The solutions for the first echelon routing for Examples 1-3 are illustrated in Figures 6.3-6.8.

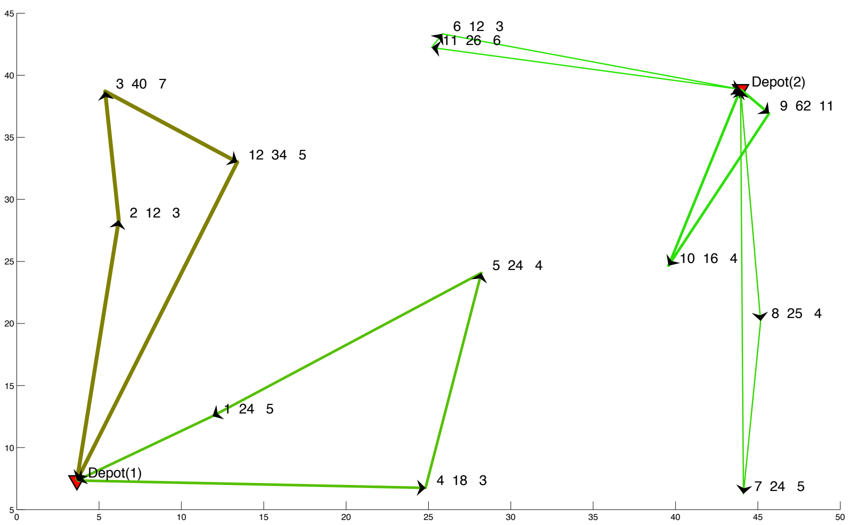

Figure 6.3: Vehicle routes obtained by ALNS for Example 1

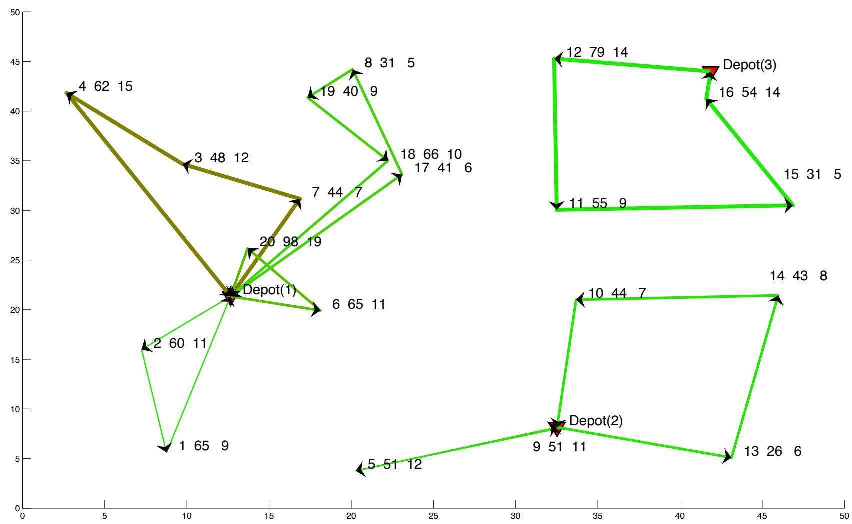

Figure 6.4: Vehicle routes obtained by ALNS for Example 2

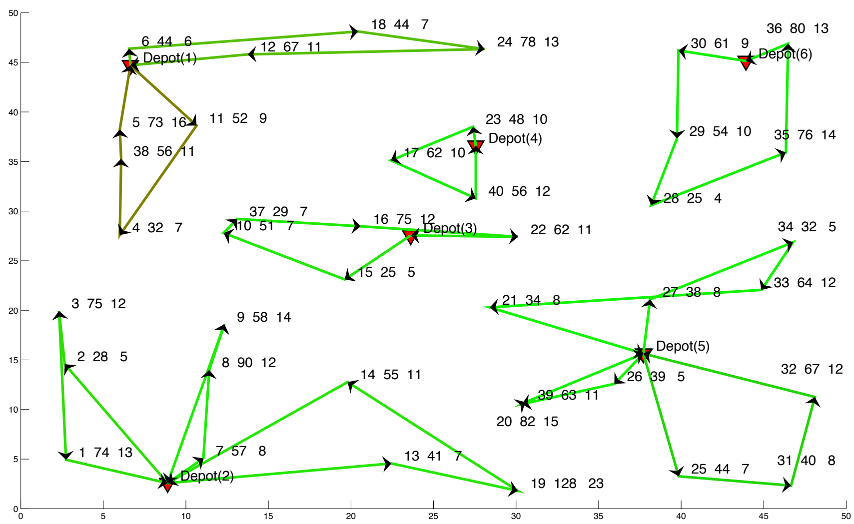

Figure 6.5: Vehicle routes obtained by ALNS for Example 3

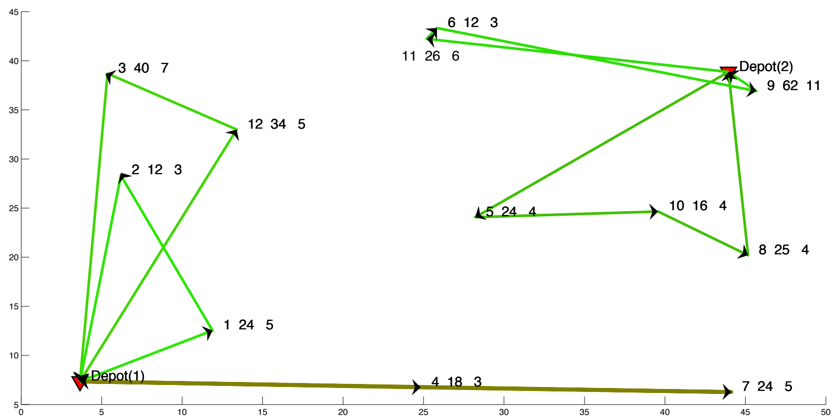

Figure 6.6: Vehicle routes obtained by GA for Example 1

In each figure, the depots are marked by red triangles. The thickness of the lines of each route indicates the number of delivery men assigned to this route. The thicker the line is, the more delivery men are assigned. For each parking site node, there is a triple of values, i.e. the parking site index, the unloading quantity at this parking site and the number of customers assigned to this parking site.

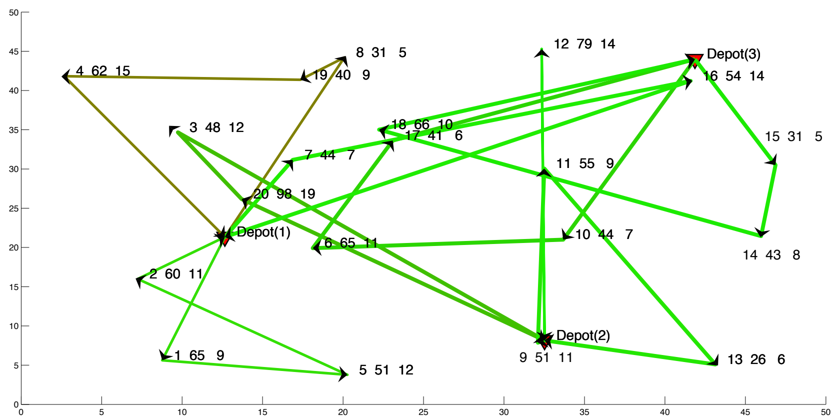

Figure 6.7: Vehicle routes obtained by GA for Example 2

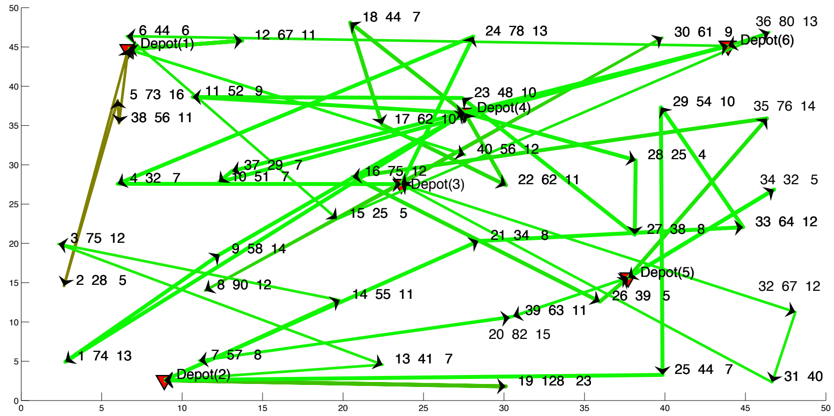

Figure 6.8: Vehicle routes obtained by GA for Example 3

The routing schedule for the routes in Figure 6.3 is given as follows:

- $l = 3$, Depot(1) - 2 - 3 - 12 - Depot(1)
- $l = 2$, Depot(1) - 4 - 5 - 1 - Depot(1)
- $l = 1$, Depot(2) - 8 - 7 - Depot(2)
- $l = 2$, Depot(2) - 9 - 10 - Depot(2)
- $l = 1$, Depot(2) - 11 - 6 - Depot(2)

From the routes in Figures 6.3-6.5, the following can be observed:

- In each route, the vehicles are more likely to visit the nearest parking site first and the last stop of the route is usually very far away from the depot. This confirms that the term of goodwill cost in the objective function really affects the route planning.

- If a route has a large loading or includes more parking sites than other routes, then this route may be assigned with more than one delivery man so as to shorten the total parking time of the route. Otherwise, the extra parking time at one parking site would be accumulated to the waiting time of all the nodes after this parking site. This is similar to the case mentioned in Section 6.4.3.

Delivery Man Routing from Parking Sites to Customers

For each set of data, there are a total of $p \times L$ combinations of the DMP. Each combination is a multi-trip routing problem. An illustration of the second-echelon routing is shown in Figure 6.9.

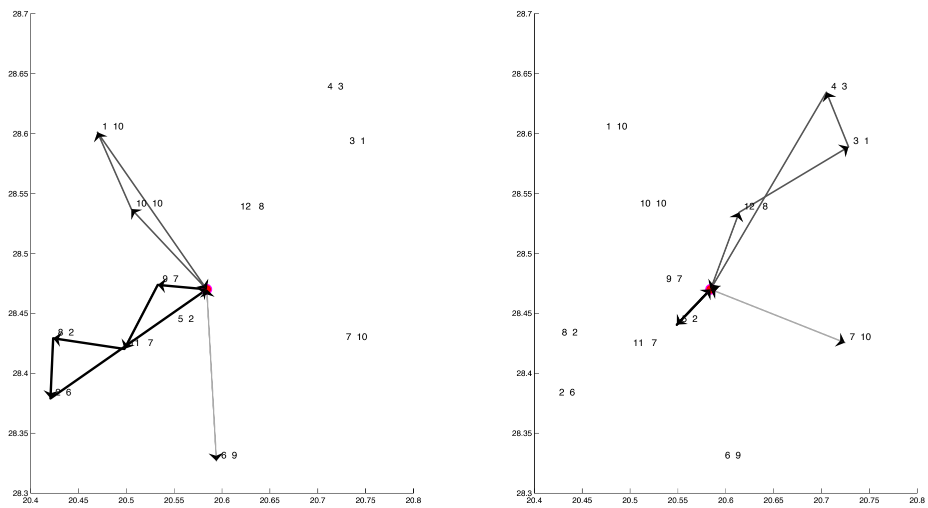

Figure 6.9: ALNS solution for DMP(16,2) in Example 3

The routes provided in Figure 6.9 are obtained by solving the DMP(16,2) in Example 3, i.e. determine the routes for two delivery men who serves the customers assigned to parking site 16. The two sub-figures are the routes for the two delivery men. The location of the parking site is marked by a red circle. For each customer assigned to this parking site, there is a pair of values indicating the customer index and customer demand (Note that the customers in this figure are re-indexed from 1). Since the DMP is a multi-trip problem, the sequence of the route will also effect the waiting time of the customers. The thickness of the line is used to depict the route sequence. The thicker the line is, the earlier this route is assigned. The routes for the two delivery men are $\{0, 9, 11, 8, 2, 0, 10, 1, 0, 6, 0\}$ and $\{0, 5, 0, 12, 3, 4, 0, 7, 0\}$ (node 0 is for the parking site), respectively. From Figure 6.9, the following can be observed:

- The isolated customer is usually visited at the end and is the only customer in that route.
- The customers in one route are more likely to be visited in an order with increasing distance.

6.5.4 Sensitivity Analysis

To investigate the impact of the cost coefficients on the four routing components, sensitivity analysis is performed on α, β, γ and λ, and the maximum duration T. For each coefficient, a multiplier μ is used. The values of μ $\{0.5, 0.6, \ldots, 1.4, 1.5\}$ are tested, i.e. each coefficient has a percentage change ranging from [-50%, +50%].

Table 6.2 shows the solutions of Examples 1-3 with different goodwill cost coefficient $\lambda' = \mu * \lambda$. As μ decreases from 1.5 to 0.5, the solution obtained in Example 1 stays rather stable. For Example 2, when the λ' drops from 1.5λ to 0.5λ, the No.D. decreases. For the solution obtained in Example 3, the No.D is reduced by one and the W.T. is increased by 20% while λ' decreases. Small values of λ' would reduce the priority of minimizing the W.T. in Eq.(6.6), thus the W.T. increases when μ decreases from 1 to 0.5.

μ	Ex.1				Ex.2				Ex.3			
	No.V.	Dis.	No.D.	W.T.	No.V.	Dis.	No.D.	W.T.	No.V.	Dis.	No.D.	W.T.
1.5	5	270	9	80	8	309	18	235	11	419	23	383
1.4	5	270	10	73	7	294	13	292	11	426	24	377
1.3	5	269	8	88	8	310	13	267	11	412	22	396
1.2	6	309	9	75	8	307	12	266	11	422	22	379
1.1	5	273	9	82	8	326	11	275	10	415	20	438
1.0	5	273	9	81	7	293	14	298	11	417	22	385
0.9	5	270	9	80	7	304	13	297	11	396	22	395
0.8	5	271	9	81	7	294	12	298	10	401	20	432
0.7	6	271	8	87	8	317	11	282	10	423	20	451
0.6	5	271	8	87	9	321	12	268	10	407	19	439
0.5	5	271	8	87	8	307	11	276	10	397	19	460

Table 6.2: Sensitivity Analysis for λ for Examples 1-3

Table 6.3 shows the solutions of Examples 1-3 with different maximum durations $T' = \mu * T$. Note that for the cases $\mu = 0.7 \sim 0.5$ in Example 2 and $\mu = 0.5$ in Example 3, the time durations are too short to find feasible solutions for those cases. As μ decreases from 1.2 to 0.5, the No.V. and No.D. are almost doubled and the the W.T. is decreased sharply in all the examples.

μ	Ex.1				Ex.2				Ex.3			
	No.V.	Dis.	No.D.	W.T.	No.V.	Dis.	No.D.	W.T.	No.V.	Dis.	No.D.	W.T.
1.5	4	250	7	85	6	320	10	325	10	384	20	430
1.4	4	250	6	95	6	278	11	320	9	395	20	450
1.3	4	243	6	97	6	288	12	303	10	404	21	427
1.2	4	261	6	97	6	310	9	339	10	391	20	438
1.1	5	280	8	93	7	309	11	292	10	418	21	417
1.0	5	273	9	81	7	293	14	298	11	417	22	385
0.9	7	322	9	77	8	311	17	256	11	401	23	386
0.8	7	320	12	80	10	357	17	238	12	453	26	369
0.7	**11**	**451**	**16**	**58**	13	396	19	219	14	461	29	313
0.6	**11**	**451**	**18**	**56**	17	452	24	191	18	541	35	273
0.5	**10**	**387**	**15**	**60**	**19**	**513**	**28**	**179**	29	652	41	274

Table 6.3: Sensitivity Analysis for T for Examples 1-3

Tables 6.4-6.6 show the solutions of the examples with different cost coefficient α,

β and γ, respectively. The results show that the solutions are not sensitive to these cost coefficients given that their percentage change is in the range of [-50%,50%].

μ	Ex.1				Ex.2				Ex.3			
	No.V.	Dis.	No.D.	W.T.	No.V.	Dis.	No.D.	W.T.	No.V.	Dis.	No.D.	W.T.
1.5	5	270	9	80	8	326	13	278	9	385	20	469
1.4	5	271	8	87	8	314	15	252	10	399	20	418
1.3	5	269	8	88	7	302	13	309	10	390	21	415
1.2	5	271	8	87	8	328	12	282	10	410	22	455
1.1	5	270	10	73	8	308	13	276	11	412	22	388
1.0	5	273	9	81	7	293	14	298	11	417	22	385
0.9	5	265	9	83	8	298	14	254	11	401	23	401
0.8	5	269	8	88	8	314	15	252	10	408	20	433
0.7	5	267	9	79	7	318	15	272	9	392	20	476
0.6	5	297	8	82	7	330	13	270	10	399	21	433
0.5	5	269	8	88	8	313	11	276	11	405	22	379

Table 6.4: Sensitivity Analysis for α for Examples 1-3

μ	Ex.1				Ex.2				Ex.3			
	No.V.	Dis.	No.D.	W.T.	No.V.	Dis.	No.D.	W.T.	No.V.	Dis.	No.D.	W.T.
1.5	5	269	8	88	7	321	12	304	10	381	20	426
1.4	6	271	8	87	7	312	13	294	9	384	19	476
1.3	5	267	9	79	8	300	13	269	10	418	20	424
1.2	6	271	8	87	7	312	14	281	11	392	23	386
1.1	5	267	9	79	7	319	12	303	11	408	22	391
1.0	5	273	9	81	7	293	14	298	11	417	22	385
0.9	5	269	8	88	8	316	12	269	10	407	21	420
0.8	5	269	8	88	8	303	14	255	10	414	20	435
0.7	5	271	9	81	7	323	13	295	11	410	22	372
0.6	5	267	9	79	6	286	13	300	11	417	23	380
0.5	5	267	9	79	8	294	13	263	11	439	22	384

Table 6.5: Sensitivity Analysis for β for Examples 1-3

6.6 Conclusions

In this research, an integrated model for multi-depot vehicle routing and delivery men problem (MDVRDMP) is developed. The model schedules routes for both of the vehicles and delivery men with an objective to minimize the total relevant cost which consists of fixed and variable costs of vehicles, personnel cost of delivery men and goodwill cost for the total waiting time of the customers.

μ	Ex.1				Ex.2				Ex.3			
	No.V.	Dis.	No.D.	W.T.	No.V.	Dis.	No.D.	W.T.	No.V.	Dis.	No.D.	W.T.
1.5	6	271	8	87	7	297	13	289	10	407	20	431
1.4	6	271	8	87	8	315	13	263	11	418	22	385
1.3	5	269	8	88	8	304	11	299	10	409	20	424
1.2	5	271	8	87	7	309	12	289	11	404	22	389
1.1	5	271	8	87	7	293	13	293	10	406	21	437
1.0	5	273	9	81	7	293	14	298	11	417	22	385
0.9	6	271	9	81	8	307	13	261	10	421	21	425
0.8	5	301	10	75	8	314	12	265	10	387	20	420
0.7	5	289	10	72	8	314	11	272	10	413	22	422
0.6	4	252	10	83	8	317	14	259	11	423	23	388
0.5	5	290	10	68	7	313	14	286	9	383	22	470

Table 6.6: Sensitivity Analysis for γ for Examples 1-3

Two heuristics have been designed to solve the MDVRDMP, i.e. adaptive large neighbourhood search (ALNS) and genetic algorithm (GA). The results obtained from the two heuristics show that ALNS outperforms GA in all aspects. After completing the same iteration numbers, ALNS always finds a better solution within a shorter CPU time when compared with GA. The components provided by the ALNS solution are all less than those of GA. Hence, the GA adopted in this research does not work well. Moreover, when the problem size equals to (6,40,400), GA fails to find a feasible solution in one hour.

From the routing solutions obtained by ALNS, some characteristics of the solutions of this model can be observed. Both delivery man and vehicle are likely to visit the nearest node. If a route has a large total demand or the number of parking sites assigned to this route is more than the other routes, more than one delivery man would be assigned to this route. If a delivery man is assigned multiple trips, the sequence of trips would affect the system objective value. The route for an isolated buyer is usually scheduled for the last route.

In order to investigate how the goodwill cost affects the system solution, a sensitivity analysis has been conducted to various values of λ. Results show that when the new λ' decreases from 1.5λ to 0.5λ, the problems of medium size (Example 2) and large size (Example 3) are sensitive to the fluctuation of this coefficient. When the coefficient increases, more delivery men would be assigned to the vehicles so as to enhance the delivery efficiency at the second-echelon and reduce the total waiting time of the customers. The sensitivity analysis on the time duration T shows that when T decreases, the system would assign more vehicles and delivery men to satisfy the time constraint of the system. However, the solutions of the model are not sensitive to the new values of α, β and γ. The new values of these coeffecients are ranging from 0.5-1.5 times of the original values. As a conclusion, this model is sensitive to the time-related coefficients, i.e. the cost coefficient of goodwill, λ, and the maximum duration T.

Chapter 7

Conclusions and Suggestions for Future Research

This thesis investigates mathematical models and solution methods for four supply chain coordinating systems. Chapters 3-5 explore possible enhancements and applications regarding the synchronized cycles model of Chan and Kingsman (2007). Chapter 3 proposes a synchronized cycles supply chain model involving clustering of buyers. Chapter 4 incorporates the synchronized cycles policy into an integrated production-warehouse location-inventory (PWLI) model. Chapter 5 further investigates the synchronized cycles PWLI model by integrating it with a vehicle routing problem. Chapter 6 proposes an integrated model for multi-depot vehicle routing and delivery men problem.

In Chapter 3, a synchronized cycles single-vendor multi-buyer supply chain model involving clustering of buyers with long and short ordering cycles is proposed. In this new model, the buyers are classified either as short-cycle preferred (SCP) buyers or long-cycle preferred (LCP). The ordering cycles of both groups of buyers are coordinated with the vendor's production cycle such that the ordering cycles of the SCP buyers and the LCP buyers must be integer factors and integer multiples of the vendor's production cycle, respectively. The overall performance of this model is always better than that of the independent policy model. When the model is compared with the synchronized cycles model developed by Chan and Kingsman (2007), the improvement obtained by this model is data dependent. When β ($\beta = T_v^*/\overline{T}_i^*$), the economic cycle ratio, is small, i.e. 0.4-1.4, the better performance of the model comes three parts, (a) vendor's cost is significantly reduced, (b) LCP buyers' costs are closer to the values of the independent policy model, (c) SCP buyers' costs is slightly changed.

In Chapter 4, an integrated production-warehouse location-inventory model is proposed. This model determines the warehouse location, production schedule and ordering frequencies simultaneously by minimizing the total system cost. The production cycle of the manufacturer and the ordering cycles of the warehouses and the retailers are synchronized. The overall performance of the synchronized cycles PWLI model obtained by GA, SA and GASA are always better than that of the independent policy model. The warehouse location decisions have a big impact on the system cost and can be benefited from the co-ordinated model. In the cost division for different supply chain layers, almost all system cost savings come from the warehouse layer, and the cost augment at the retailers can be offset by the cost savings at the manufacturer.

Most of the three-echelon supply chain models in the past assumed that the shipment cost is fixed regardless of order size. Furthermore, vehicle cost and vehicle capacity were not taken into consideration. In Chapter 5, an extension of the synchronized cycles PWLI model is proposed. In this extended model (PWLIR), deliveries are modeled by a set of fleet size and mix vehicle routing problem with split deliveries (FSMVRPSD). The synchronized cycles PWLIR model always outperforms the independent policy model. The shipment cost can also be reduced by adopting the synchronized cycles PWLIR model, in which more shipments are fulfilled by routes containing multiple buyers rather than routes with only one buyer each as in the independent policy model. In the synchronized cycles PWLIR model, more routes are served by the vehicles of large capacity and most of the fleet size required in the model are large vehicles. However, the required vehicles in the independent policy model are more diversified. A trade-off is found between the number of open warehouse and the shipmen cost. For the cases where the number of open warehouse in the independent policy model is much higher than that of the synchronized cycles PWLIR model, the improvement percentage of the shipment cost achieved by the model is smaller when compared with the cases where the number of open warehouses in the two models are close to each other.

In Chapter 6, an integrated model for multi-depot vehicle routing and delivery men problem is studied. This model incorporates a distribution network of multiple depots, multiple parking-sites and multiple customers linked by trips of a fleet of homogeneous vehicles and a number of delivery men assigned to the vehicles. The objective of this model is to determine the number of delivery men assigned to each vehicle and the routing of vehicles and delivery men so as to minimize the total relevant costs involved in the two levels. The solutions obtained by ALNS have revealed some characteristics of the routing schedule. The solutions suggest that more than 1 delivery man is assigned to a route with large demand or the number of parking sites in this route is larger than that of the others. The deliveries of each route is very dependent on the distance between the demand points and start point, i.e. the customer at a nearer location would by served earlier. Both of these two characteristics have confirmed that there are impacts on the routing schedule when total waiting time is incorporated into the objective function. To investigate the impact of the total waiting time on the objective value, a sensitivity analysis is conducted for the goodwill cost coefficient. In general, the higher the coefficient value, the more delivery men would be assigned to enhance the delivery efficiency.

For further research, the synchronized cycles PWLI and PWLIR models can be compared with other existing coodinated models. For another direction of the future research, the study on the integrated vehicle routing and delivery men problem is very limited. For further research, more explorations can be considered regarding this issue in integrated routing models. Relaxing the assumption for predetermined clusters can make this model more general.

Appendix A

Total System Cost of the Clustering Synchronized Cycles Model

Suppose that there are altogether 5 buyers, which consist of 2 LCP buyers and 3 SCP buyers. The vendor's basic production cycle $T = 60$ time units.

Let $\alpha = \frac{D}{P} = 0.7$, where D is the total demand rate from all the buyers and P is the production rate.

Vendor's setup cost per production run $S_v = 250$.

Vendor's holding cost per unit item per unit time $h = 0.005$.

The values of all the parameters and decision variables of the SCP buyers and the LCP buyers are given in Table A.1 and Table A.2, respectively.

SCP Buyer i	d_i^s	C_i^s	A_i^s	h_i^s	K_i	γ_i
1	20	40	18	0.007	2	2
2	15	40	15	0.009	2	11
3	10	40	6	0.010	2	6

Table A.1: Parameters and Decision Variables of the SCP Buyers

LCP Buyer j	d^l_j	C^l_j	A^l_j	h^l_j	k_j	τ_j
1	8	40	20	0.008	2	32
2	5	40	10	0.010	1	11

Table A.2: Parameters and Decision Variables of the LCP Buyers

The total demand rates from all the SCP buyers D^s

$$= \sum_{i=1}^{n^s} d^s_i = 20 + 15 + 10 = 45.$$

The production rate P

$$= \frac{D}{\alpha} = \frac{20 + 15 + 10 + 8 + 5}{0.7} = 82.8571.$$

$$F = \frac{D^s}{P} = 0.5431.$$

$$M = \text{LCM}(k_1, k_2) = 2.$$

Hence, the planning horizon MT

$$= 2 \times 60 = 120.$$

Hence we have

$$\begin{aligned} D_1 &= \delta_{1,1} d^l_1 k_1 T + \delta_{2,1} d^l_2 k_2 T \\ &= 1 \times 8 \times 2 \times 60 + 1 \times 5 \times 1 \times 60 \\ &= 1260 \end{aligned}$$

and

$$D_2 = \delta_{1,2} d^l_1 k_1 T + \delta_{2,2} d^l_2 k_2 T$$

$$
\begin{aligned}
&= 0 + 1 \times 5 \times 1 \times 60 \\
&= 300.
\end{aligned}
$$

The vector representing the ordering times of buyer 1, buyer 2 and buyer 3 from the SCP group, denoted by OTS_1, OTS_2 and OTS_3, respectively, are as follows:

$$
\begin{aligned}
OTS_1 &= \left[\gamma_1, \gamma_1 + \frac{T}{K_1}, \gamma_1 + T, \gamma_1 + T + \frac{T}{K_1}\right] \\
&= [2, 2 + 30, 2 + 60, 2 + 60 + 30] \\
&= [2, 32, 62, 92].
\end{aligned}
$$

$$
\begin{aligned}
OTS_2 &= \left[\gamma_2, \gamma_2 + \frac{T}{K_2}, \gamma_2 + T, \gamma_2 + T + \frac{T}{K_2}\right] \\
&= [11, 11 + 30, 11 + 60, 11 + 60 + 30] \\
&= [11, 41, 71, 101].
\end{aligned}
$$

$$
\begin{aligned}
OTS_3 &= \left[\gamma_3, \gamma_3 + \frac{T}{K_3}, \gamma_3 + T, \gamma_3 + T + \frac{T}{K_3}\right] \\
&= [6, 6 + 30, 6 + 60, 6 + 60 + 30] \\
&= [6, 36, 66, 96].
\end{aligned}
$$

The vector representing the ordering times of buyer 1 and buyer 2 from the LCP group, denoted by OTL_1 and OTL_2, respectively, are as follows:

$$OTL_1 = [\tau_1] = [32].$$

$$OTL_2 = [\tau_2, \tau_2 + k_2 T] = [11, 11 + 60] = [11, 71].$$

We first assume that the production run of each of the vendor's prodcution cycle starts at the beginning of the cycle. If the vendor has to satisfy the demand of the SCP buyers only, then his inventory levels for the first and the second production cycles are exactly the same. The inventory curve for these two production cycles is as shown in Figure A.1. The area under his inventory curve for both the first and the second production cycles, denoted by $Area_1$, is as follows:

$$\begin{aligned} Area_1 &= M \times \frac{[(1-F)T + T] \times PFT}{2} = 2 \times \frac{P(2-F)FT^2}{2} \\ &= 2 \times \frac{D^s \left(2 - \frac{D^s}{P} T^2\right)}{2} = 2 \times ((D^s)^2 T^2 - \frac{(D^s)^2}{2P} T^2) \\ &= 236017. \end{aligned}$$

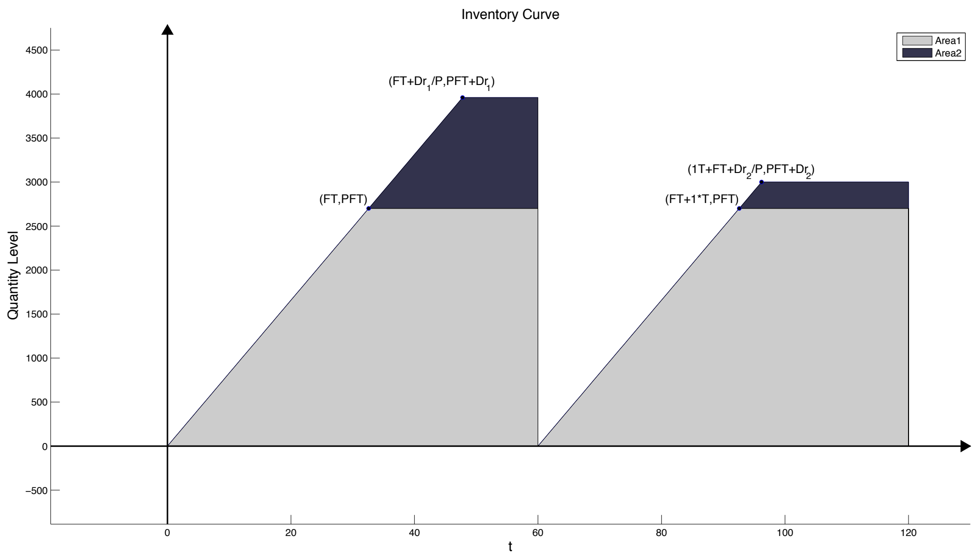

Figure A.1: Illustration for $Area_1$ and $Area_2$

However, if the vendor has to satisfy the demand of the LCP buyers as well as the SCP buyers in the production cycle $((r-1)T, rT]$, $(r = 1, 2)$, the vendor needs to produce additional D_r^l items. His new inventory curves for all the two production cycles are as shown in Figure A.1. The additional area under the inventory curve due to the demands from the LCP buyers in the two production cycles, denoted by $Area_2$, is as follows:

$$\begin{aligned} Area_2 &= \sum_{r=1}^{M} \left[\left(T - FT - \frac{D_r^l}{P} \right) + (T - FT) \right] \times \frac{D_r^l}{2} \\ &= \sum_{r=1}^{M} \left[2T - 2FT - \frac{D_r^l}{P} \right] \times \frac{D_r^l}{2} \\ &= 32642. \end{aligned}$$

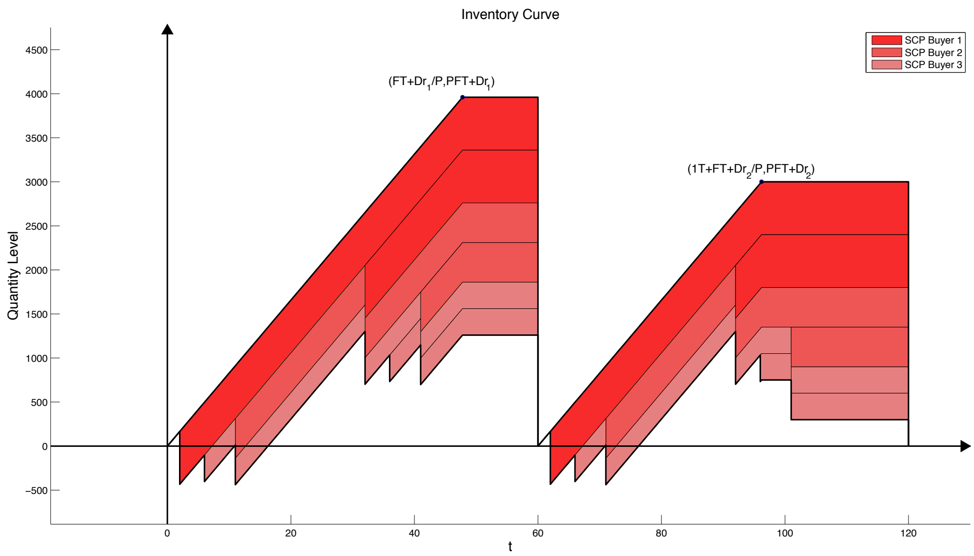

Figure A.2: Illustration for $Area_3$

Due to the changes in the inventory level at the times goods are delivered to the SCP buyers, the area under the vendor's inventory curve will be decreased. His new inventory curve is as shown in Figure A.2. The shaded region of Figure A.2

represents the reduction in area under the inventory curve caused by the deliveries of goods to the SCP buyers. The reduction in area in the first and second production cycle, denoted by $Area_3$, is as follows:

$$
\begin{aligned}
Area_3 &= M \times \sum_{i=1}^{n^s} \sum_{u=1}^{K_i} (d_i^s \cdot \frac{T}{K_i})(T - \gamma_i - (u-1)\frac{T}{K_i}) \\
&= 2 \times 20 \times \frac{60}{2} \times [(60-2) + (60-2-30)] \\
&\quad +2 \times 15 \times \frac{60}{2} \times [(60-11) + (60-11-30)] \\
&\quad +2 \times 10 \times \frac{60}{2} \times [(60-6) + (60-6-30)] \\
&= 211200.
\end{aligned}
$$

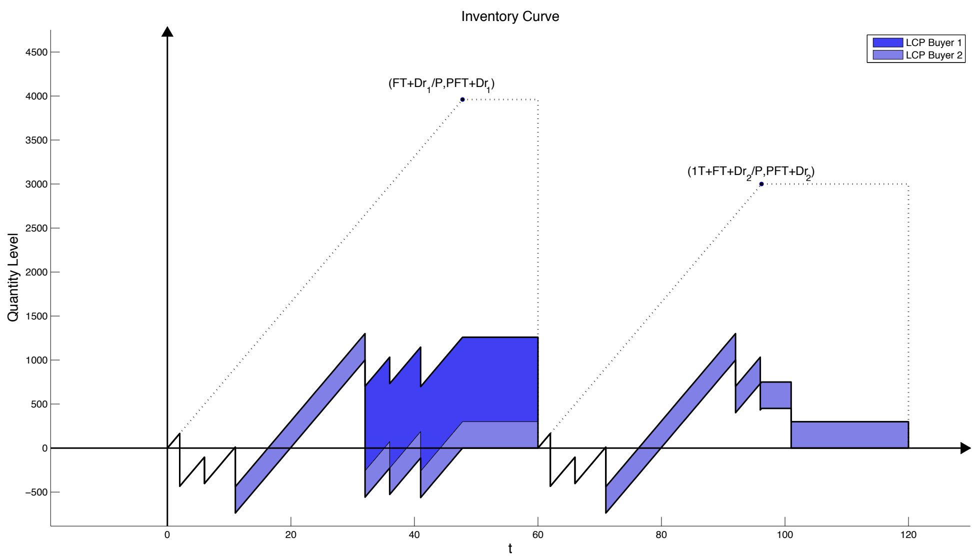

Figure A.3: Illustration for $Area_4$

Due to the changes in the inventory levels at the times goods are delivered to the LCP buyers, the area under the vendor's inventory curve will be further decreased.

His new inventory curve for the first and second production cycles is shown in Figure A.3. The shaded regions of Figure A.3 represent the reduction in area under the inventory curve caused by the sudden changes in the inventory levels in the two production cycles. The total reduction in area in the two prodcution cycles, denoted by $Area_4$, is as follows:

$$\begin{aligned}
Area_4 &= \sum_{r=1}^{M}\sum_{j=1}^{n^l} \delta_{j,r}(d_j^l k_j T)(T - \hat{\tau}_j) \\
&= 8 \times 120 \times (60 - 32) + 5 \times 60 \times (60 - 11) + 5 \times 60 \times (60 - 11) \\
&= 56280
\end{aligned}$$

After goods have been delivered to both the SCP buyers and the LCP buyers, the vendor's inventory level is as shown in Figure A.4. Since any shortage of goods is not allowed, the production run of each vendor's production cycle should start earlier than the beginning of the production cycle. From Figure A.4, we observe that the maximum amount of shortages in the vendor's first and second production cycles both occur at $t = 11$ from the start of the production cycles and the amount of shortages are the same for both cycles. From (3.31), the accumulated ordering quantity from all the buyers at $t = 11$ from the start of the production cycles is as follows:

$$\begin{aligned}
\hat{q}_1(11) &= \hat{q}_{1,s}(11) + \hat{q}_{1,l}(11) \\
&= \sum_{i=1}^{3} \text{int}\left[\frac{11 - \gamma_i}{T} K_i + 1\right] \frac{d_i^s T}{K_i} + \sum_{j=1}^{2} \delta_{j,1} \times \text{int}\left[\frac{(11 - \hat{\tau}_j)}{T} + 1\right] \times d_j^l k_j T \\
&= \text{int}\left[\frac{11 - 2}{60} \times 2 + 1\right] \times \frac{20 \times 60}{2} + \text{int}\left[\frac{11 - 11}{60} \times 2 + 1\right] \times \frac{15 \times 60}{2}
\end{aligned}$$

$$
\begin{aligned}
& +\text{int}\left[\frac{11-11}{60}\times 2+1\right]\times\frac{10\times 60}{2}+1\times\text{int}\left[\frac{11-32}{60}+1\right]\times 8\times 120 \\
& +1\times\text{int}\left[\frac{11-11}{60}+1\right]\times 5\times 60 \\
= \;& 600+450+300+0+300 \\
= \;& 1650.
\end{aligned}
$$

The amount of goods produced from time $(r-1)T$ to $(r-1)T+11$, $(r=1,2)$

$$= P\times 11 = 82.8571\times 11 = 911.4281.$$

Thus,

$$
\begin{aligned}
\chi_1 &= \max_{t\in(0,60]} short_1(t) = short_1(11) \\
&= 1650-911.4281 = 738.5719.
\end{aligned}
$$

Similarly,

$$\chi_2 = 738.5719.$$

From (3.40), we have

$$
\begin{aligned}
tstart_2 &= \min\left[(2-1)T+1,(2-1)T-\frac{\chi_2}{P}\right] = \min[61, 51.0862] \\
&= 51.0862.
\end{aligned}
$$

and

$$
\begin{aligned}
tstart_1 &= \min\left[1,-\frac{\chi_1}{P}, tstart_2-FT-\frac{D_1}{P}\right] = \min[1,-8.9138, 4.2933] \\
&= -8.9138.
\end{aligned}
$$

By letting $tstart_1 = -8.9138$ and $tstart_2 = 51.0862$, both the constraints (3.38) and (3.39) are satisfied and at the same time, the vendor's total inventory will be minimized. Thus, the start times of the production run of both production cycles should shift 8.9138 time units from the beginning of the production cycles. The

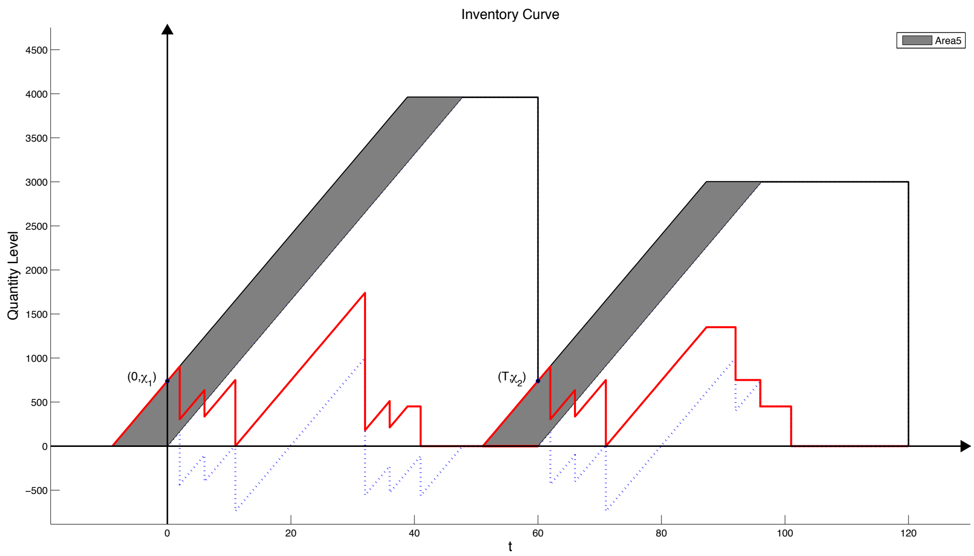

Figure A.4: Illustration for $Area_5$

dotted line and the solid line in Figure A.4 show the vendor's inventory levels of the two production cycles before and after shifting of the start times of the production runs, respectively. The shaded regions in Figure A.4 show the increase in area under the inventory curve in the two production cycles due to the shifting of the start times. This increase in area in the two production cycles, denoted by $Area_5$, is as follows:

$$\begin{aligned} Area_5 &= \sum_{r=1}^{M} [(r-1)T - tstart_r] \times (PFT + D_r^l) \\ &= [0 - (-8.9138)] \times (82.8571 \times 0.5431 \times 60 + 1260) \end{aligned}$$

$$
\begin{aligned}
& +(60-51.0862)\times(82.8571\times 0.5431\times 60+300) \\
= \quad & 62040.
\end{aligned}
$$

So the total area under the vendor's inventory curve, denoted by $Area$, is as follows:

$$
\begin{aligned}
Area &= Area_1 + Area_2 - Area_3 - Area_4 + Area_5 \\
& M \cdot \left(D^s T^2 - \frac{(D^s)^2}{2P} T^2 - \sum_{i=1}^{n^s} \sum_{u=1}^{K_i} (d_i^s \cdot \frac{T}{K_i})(T - \gamma_i - (u-1)\frac{T}{K_i}) \right) \\
& + \left(\sum_{r=1}^{M} T(1-F)\bar{D}_r^l - \frac{(\bar{D}_r^l)^2}{2P} \right) - \sum_{r=1}^{M} \sum_{j=1}^{n^l} \delta_{j,r} (d_j^l \cdot k_j T)(T - \hat{\tau}_j) \\
& - \sum_{r=1}^{M} [(r-1)T - tstart_r](PFT + \bar{D}_r^l) \\
&= 236017 + 32642 - 211200 - 56280 + 62040 \\
&= 63219.
\end{aligned}
$$

Vendor's inventory cost per unit time

$$= h \cdot \frac{Area}{MT} = 2.6341.$$

Vendor's setup cost per unit time

$$= \frac{S_v}{T} = 4.1667.$$

Vendor's order processing and shipment cost per unit time

$$= \sum_{i=1}^{n^s} \frac{C_i^s}{T/K_i} + \sum_{j=1}^{n^l} \frac{C_j^l}{k_j T} = 5.$$

Buyers' ordering cost per unit time

$$= \sum_{i=1}^{n^s} \frac{A_i^s}{T/K_i} + \sum_{j=1}^{n^l} \frac{A_j^l}{k_j T} = 1.6333.$$

Buyers' holding cost per unit time is

$$\frac{1}{2}\sum_{i=1}^{n^s} d_i^s h_i^s (T/K_i) + \frac{1}{2}\sum_{i=1}^{n^l} d_i^l h_i^l (k_j T) = 10.9650.$$

The total system cost per unit time is

$$\begin{aligned}
TC^{CLU} \quad &= \quad \frac{h}{T}\cdot\left(D^s T^2 - \frac{(D^s)^2}{2P}T^2 - \sum_{i=1}^{n^s}\sum_{u=1}^{K_i}(d_i^s \cdot \frac{T}{K_i})(T - \gamma_i - (u-1)\frac{T}{K_i})\right) \\
&\quad + \frac{h}{MT}\left\{\left(\sum_{r=1}^{M} T(1-F)\bar{D}_r^l - \frac{(\bar{D}_r^l)^2}{2P}\right) - \sum_{r=1}^{M}\sum_{j=1}^{n^l}\delta_{j,r}(d_j^l \cdot k_j T)(T - \hat{\tau}_j)\right. \\
&\quad \left. - \sum_{r=1}^{M}[(r-1)T - tstart_r](PFT + \bar{D}_r^l)\right\} + \frac{S_v}{T} + \sum_{i=1}^{n^s}\frac{C_i^s K_i}{T} + \sum_{j=1}^{n^l}\frac{C_j^l}{k_j T} \\
&\quad + \sum_{i=1}^{n^s}\frac{A_i^s K_i}{T} + \sum_{j=1}^{n^l}\frac{A_j^l}{k_j T} + \frac{1}{2}\sum_{i=1}^{n^s}\frac{d_i^s h_i^s T}{K_i} + \frac{1}{2}\sum_{i=1}^{n^l} d_i^l h_i^l k_j T \\
&= \quad 2.6341 + 4.1667 + 5 + 1.6333 + 10.9650 \\
&= \quad 24.3991.
\end{aligned}$$

Appendix B

Datasets for Clustering Synchronized Cycles Model

$S_v = 600,\ h = 0.035$				
Buyer i	d_i	C_i	A_i	h_i
1	4	52	78	0.031
2	2	63	36	0.045
3	4	51	84	0.054
4	3	40	48	0.044
5	11	69	49	0.049
6	19	47	64	0.047
7	154	66	79	0.043
8	570	56	84	0.036
9	420	44	64	0.054
10	276	51	31	0.044

Table B.1: Data for Example 1

$S_v = 800,\ h = 0.025$				
Buyer i	d_i	C_i	A_i	h_i
1	1	49	25	0.050
2	3	63	48	0.033
3	1	30	26	0.054
4	3	23	70	0.053
5	2	65	38	0.049
6	3	40	40	0.043
7	4	53	37	0.051
8	8	59	64	0.046
9	16	32	79	0.054
10	81	47	25	0.054

Table B.2: Data for Example 2

$S_v = 2000,\ h = 0.030$				
Buyer i	d_i	C_i	A_i	h_i
1	1	9	26	0.041
2	1	10	22	0.036
3	1	12	19	0.037
4	1	17	14	0.031
5	2	6	25	0.031
6	1	33	16	0.039
7	1	16	21	0.052
8	1	5	12	0.030
9	1	26	16	0.042
10	1	29	12	0.041
11	1	10	9	0.037
12	1	24	11	0.053
13	2	8	19	0.051
14	2	30	12	0.033
15	3	12	26	0.053
16	2	31	12	0.040
17	3	21	13	0.034
18	1	17	5	0.041
19	1	30	6	0.049
20	2	32	7	0.032
21	1	8	4	0.039
22	1	15	3	0.052
23	6	18	14	0.053
24	1	14	2	0.050
25	14	18	17	0.052
26	43	15	14	0.030
27	49	12	18	0.043
28	239	12	13	0.038
29	933	16	25	0.041
30	1250	6	23	0.036

Table B.3: Data for Example 3

$S_v = 2000,\ h = 0.025$				
Buyer i	d_i	C_i	A_i	h_i
1	1	21	20	0.037
2	1	18	15	0.032
3	1	30	9	0.029
4	1	33	14	0.046
5	3	19	25	0.031
6	3	23	24	0.032
7	2	6	22	0.045
8	3	8	20	0.028
9	1	28	7	0.030
10	1	33	8	0.047
11	3	29	22	0.043
12	1	15	7	0.043
13	1	26	4	0.028
14	1	16	6	0.046
15	4	34	20	0.046
16	2	10	10	0.047
17	1	12	3	0.029
18	6	32	18	0.031
19	3	6	7	0.033
20	1	8	2	0.034
21	2	19	5	0.044
22	14	25	21	0.028
23	7	28	13	0.040
24	10	25	22	0.047
25	7	25	12	0.043
26	2	25	3	0.043
27	4	32	3	0.039
28	46	33	22	0.029
29	59	13	26	0.043
30	94	15	22	0.033

Table B.4: Data for Example 4

$S_v = 2500,\ h = 0.020$				
Buyer i	d_i	C_i	A_i	h_i
1	1	16	24	0.042
2	1	27	14	0.027
3	2	29	22	0.022
4	1	20	18	0.037
5	2	29	20	0.021
6	1	26	17	0.040
7	1	26	9	0.022
8	1	21	12	0.031
9	2	24	16	0.022
10	1	7	15	0.042
11	3	12	25	0.024
12	2	17	24	0.035
13	2	14	13	0.021
14	2	29	16	0.026
15	2	22	23	0.038
16	2	29	16	0.027
17	3	22	22	0.025
18	1	30	8	0.028
19	2	7	23	0.042
20	3	28	20	0.026
21	3	28	21	0.029
22	2	18	13	0.028
23	3	19	21	0.032
24	3	33	26	0.042
25	1	5	5	0.024
26	4	26	21	0.027
27	1	17	5	0.027
28	5	17	18	0.021
29	2	32	14	0.040
30	1	20	7	0.042
31	1	13	5	0.031
32	4	28	16	0.025
33	5	7	21	0.029
34	1	22	5	0.036
35	1	13	3	0.024
36	4	31	19	0.042
37	1	24	3	0.037
38	12	33	25	0.027
39	1	20	2	0.034
40	3	31	2	0.021
41	6	22	5	0.036
42	45	24	16	0.020
43	6	32	4	0.045
44	48	12	13	0.035
45	80	14	19	0.045
46	742	15	22	0.022
47	320	16	18	0.041
48	858	23	20	0.033
49	722	23	20	0.040
50	1385	10	22	0.030

Table B.5: Data for Example 5

$S_v = 3000,\ h = 0.025$				
Buyer i	d_i	C_i	A_i	h_i
1	1	22	21	0.043
2	1	15	26	0.054
3	1	34	23	0.057
4	1	8	16	0.044
5	1	10	15	0.050
6	1	31	16	0.053
7	2	8	20	0.035
8	2	9	25	0.045
9	1	32	11	0.040
10	2	16	25	0.047
11	1	11	15	0.057
12	1	22	11	0.044
13	1	33	13	0.053
14	1	12	11	0.047
15	2	26	17	0.037
16	1	23	9	0.040
17	2	10	15	0.039
18	1	23	11	0.059
19	2	19	19	0.054
20	1	33	9	0.055
21	1	34	9	0.056
22	2	16	14	0.045
23	1	11	6	0.040
24	3	25	22	0.053
25	3	29	22	0.055
26	1	34	5	0.038
27	2	31	14	0.056
28	4	7	21	0.045
29	3	12	19	0.055
30	3	6	11	0.036
31	5	11	26	0.057
32	3	14	10	0.045
33	1	34	4	0.057
34	6	12	15	0.040
35	8	27	24	0.051
36	5	12	16	0.058
37	9	9	23	0.048
38	8	21	16	0.039
39	6	20	17	0.056
40	4	16	11	0.056
41	10	8	23	0.054
42	1	20	2	0.048
43	1	17	2	0.050
44	11	10	24	0.057
45	17	27	24	0.037
46	11	32	22	0.055
47	16	15	24	0.057
48	18	27	13	0.041
49	11	34	9	0.049
50	32	29	17	0.038

Table B.6: Data for Example 6

Appendix C

Datasets for Synchronized PWLI Model

Example 1 with 4 potential warehouses and 10 retailers.

$S_v = 600,\ h = 0.035,\ Location(M) = (2.0196, 0.0732)$					
Potential Warehouse W_i	XCOOR.	YCOOR.	O_i^W	A_i^W	h_i^W
1	2.3107	1.0016	20	30	0.0050
2	3.4931	0.2102	20	30	0.0050
3	0.7807	3.3530	20	30	0.0050
4	2.9833	3.7999	20	30	0.0050

Retailer R_i	XCOOR.	YCOOR.	q_i	A_i^R	h_i^R
1	1.9768	4.1673	41	13	0.0131
2	2.7817	3.4471	16	10	0.0393
3	0.2890	3.9066	19	8	0.0355
4	3.6267	2.2832	27	17	0.0012
5	1.0086	1.0971	5	14	0.0392
6	2.3457	2.9663	42	7	0.0416
7	1.8854	0.0938	6	16	0.0261
8	4.3166	4.6513	31	8	0.0496
9	2.7277	3.0029	5	20	0.0366
10	3.3992	0.4240	11	15	0.0076

Table C.1: Data for Example 1

Example 2 with 8 potential warehouses and 30 retailers.

$S_v = 1200,\ h = 0.0022,\ Location(M) = (3.7122, 3.7806)$					
Potential Warehouse W_i	XCOOR.	YCOOR.	O_i^W	A_i^W	h_i^W
1	3.6809	1.0136	30	30	0.0050
2	3.3055	2.7667	30	30	0.0050
3	4.4864	0.5503	30	30	0.0050
4	3.1521	0.1703	30	30	0.0050
5	4.7098	4.9986	30	30	0.0050
6	0.2523	3.8684	30	30	0.0050
7	1.8757	0.6501	30	30	0.0050
8	4.1095	1.3418	30	30	0.0050

Retailer R_i	XCOOR.	YCOOR.	q_i	A_i^R	h_i^R
1	0.7975	4.3390	13	23	0.0266
2	3.1250	1.9439	12	24	0.0095
3	2.1087	4.6056	36	24	0.0056
4	3.6292	2.6097	13	10	0.0293
5	4.6335	0.0612	12	11	0.0454
6	3.8862	0.4905	35	21	0.0268
7	0.4656	2.7167	13	17	0.0370
8	0.0337	0.1487	30	8	0.0211
9	2.9247	2.5842	7	18	0.0123
10	0.9737	4.1560	16	16	0.0429
11	3.1720	1.1125	24	6	0.0211
12	1.4138	4.6171	20	23	0.0491
13	4.8125	4.2341	38	20	0.0182
14	0.6098	3.5498	22	22	0.0083
15	2.3554	2.9354	7	25	0.0488
16	3.8819	4.5031	15	13	0.0481
17	1.0303	2.8151	35	21	0.0232
18	2.8499	0.0110	23	10	0.0352
19	2.0760	4.6144	44	12	0.0266
20	0.1234	3.4088	15	12	0.0369
21	0.7831	4.3745	6	21	0.0209
22	0.8440	2.2664	8	19	0.0119
23	0.2517	0.3963	23	25	0.0357
24	4.5934	4.8491	37	24	0.0446
25	3.2900	1.1724	16	22	0.0096
26	2.8025	2.2165	35	13	0.0476
27	3.4005	2.9071	25	9	0.0017
28	4.6173	3.2885	12	14	0.0067
29	2.0397	0.8451	33	23	0.0167
30	0.4780	2.4597	42	19	0.0496

Table C.2: Data for Example 2

Example 3 with 15 potential warehouses and 50 retailers.

$S_v = 2100,\ h = 0.0015,\ Location(M) = (2.3163, 4.6161)$					
Potential Warehouse W_i	XCOOR.	YCOOR.	O_i^W	A_i^W	h_i^W
1	3.1533	1.0736	40	30	0.0050
2	3.5101	3.0147	40	30	0.0050
3	3.3188	4.2594	40	30	0.0050
4	0.4355	1.0217	40	30	0.0050
5	1.9352	4.7132	40	30	0.0050
6	3.9828	4.9185	40	30	0.0050
7	1.0224	2.8920	40	30	0.0050
8	3.7633	1.0569	40	30	0.0050
9	3.1861	0.3343	40	30	0.0050
10	2.7298	4.1491	40	30	0.0050
11	1.7239	4.4870	40	30	0.0050
12	2.1786	1.8171	40	30	0.0050
13	2.0812	0.2698	40	30	0.0050
14	4.9072	4.4284	40	30	0.0050
15	3.9904	3.4200	40	30	0.0050

Retailer R_i	XCOOR.	YCOOR.	q_i	A_i^R	h_i^R
1	1.0977	3.4390	13	20	0.0019
2	1.8501	0.3608	6	7	0.0177
3	1.0812	2.2619	7	10	0.0373
4	0.3155	3.8958	10	8	0.0240
5	3.1294	4.7844	16	18	0.0179
6	3.2659	1.6368	17	12	0.0207
7	2.4948	3.9980	36	23	0.0494
8	3.3696	1.6631	13	20	0.0354
9	4.5818	4.0307	20	21	0.0159
10	0.2183	1.0574	18	10	0.0445
11	4.1510	2.7703	5	6	0.0185
12	2.9057	3.9559	26	23	0.0286
13	2.3942	2.9805	44	17	0.0292
14	4.5489	1.1981	22	8	0.0241
15	2.4094	2.6181	37	18	0.0458
16	0.2353	3.6016	26	8	0.0449
17	2.3979	0.1121	6	22	0.0377
18	0.2712	2.1909	34	9	0.0450
19	0.9844	1.2278	9	13	0.0328
20	2.6000	0.6385	26	13	0.0423
21	3.8094	1.3731	21	8	0.0121
22	1.9737	2.1780	37	7	0.0303
23	0.4432	0.8507	30	24	0.0330
24	0.2405	2.4983	18	24	0.0369
25	0.9590	4.4752	23	14	0.0295
26	4.9401	1.9551	39	6	0.0390
27	3.5494	4.7111	16	7	0.0150
28	4.1146	3.1679	40	11	0.0468
29	0.2350	4.7531	31	22	0.0126
30	2.9083	3.4990	42	6	0.0116

Retailer R_i	XCOOR.	YCOOR.	q_i	A_i^R	h_i^R
31	0.1954	2.0640	37	13	0.0158
32	0.5354	3.5648	18	24	0.0135
33	4.8995	1.1202	27	21	0.0311
34	2.7838	2.4720	39	9	0.0097
35	1.9380	4.6726	33	22	0.0500
36	2.7262	3.3876	14	23	0.0231
37	1.1076	1.7949	6	11	0.0342
38	0.5965	1.2699	38	16	0.0156
39	0.2793	3.0460	38	13	0.0456
40	2.0186	0.1865	34	24	0.0016
41	4.7431	4.4277	26	15	0.0062
42	1.4709	1.6361	33	10	0.0470
43	2.9233	3.9730	9	6	0.0375
44	4.1939	2.8546	19	6	0.0404
45	3.2050	4.4255	13	19	0.0115
46	3.2679	1.6315	35	19	0.0165
47	1.5632	2.8875	28	24	0.0115
48	3.4301	2.1357	28	19	0.0109
49	3.4094	3.2132	21	10	0.0078
50	3.9735	2.0753	11	9	0.0305

Table C.3: Data for Example 3

Example 4 with 20 potential warehouses and 100 retailers.

$S_v = 3000,\ h = 0.001,\ Location(M) = (1.5452, 2.7520)$					
Potential Warehouse W_i	XCOOR.	YCOOR.	O_i^W	A_i^W	h_i^W
1	0.0091	1.0191	50	30	0.0050
2	1.4046	3.2659	50	30	0.0050
3	1.0806	1.9235	50	30	0.0050
4	3.8009	3.5875	50	30	0.0050
5	2.9868	1.1448	50	30	0.0050
6	3.5894	0.4651	50	30	0.0050
7	4.3356	3.7313	50	30	0.0050
8	2.1873	3.9488	50	30	0.0050
9	0.1012	2.4176	50	30	0.0050
10	2.2167	0.4317	50	30	0.0050
11	4.6803	3.8626	50	30	0.0050
12	4.2460	2.5156	50	30	0.0050
13	4.1644	3.8635	50	30	0.0050
14	1.9808	2.6567	50	30	0.0050
15	4.6031	0.7779	50	30	0.0050
16	3.4756	1.5355	50	30	0.0050
17	3.1440	3.7149	50	30	0.0050
18	1.8630	2.4747	50	30	0.0050
19	3.5403	0.8086	50	30	0.0050
20	4.6112	4.7171	50	30	0.0050

Retailer R_i	XCOOR.	YCOOR.	q_i	A_i^R	h_i^R
1	0.9595	4.1350	39	17	0.0375
2	1.1032	0.8104	37	8	0.0062
3	0.7239	0.8181	43	16	0.0089
4	3.3274	2.6060	17	11	0.0219
5	4.0205	1.6106	41	8	0.0080
6	2.5864	3.0746	33	14	0.0193
7	2.7215	1.4109	21	12	0.0005
8	2.8103	2.4558	17	7	0.0010
9	0.0110	1.7156	31	21	0.0094
10	1.9638	3.0803	26	6	0.0137
11	4.8764	0.7504	17	12	0.0388
12	2.9759	4.0406	23	10	0.0212
13	1.5075	3.8098	9	21	0.0189
14	2.5334	4.6219	16	16	0.0318
15	3.9506	1.8439	37	7	0.0026
16	3.2775	4.1259	36	12	0.0240
17	1.9638	1.0283	29	25	0.0231
18	4.1290	1.7386	18	14	0.0068
19	4.3207	3.5023	13	8	0.0432
20	1.9953	1.9650	25	10	0.0026
21	3.5951	4.5855	24	8	0.0193
22	1.2733	4.7715	19	24	0.0343
23	2.8152	4.5775	9	16	0.0175
24	1.3063	3.3813	19	18	0.0131
25	4.4332	1.7938	42	24	0.0032
26	4.1505	1.3261	16	11	0.0066
27	1.6671	2.6284	24	9	0.0431
28	2.0336	4.3447	14	22	0.0136
29	1.5965	4.4561	41	17	0.0130
30	3.6062	1.1694	21	25	0.0278
31	1.6744	4.9122	44	23	0.0149
32	2.4090	0.6548	40	8	0.0331
33	3.1778	2.7792	32	17	0.0059
34	2.8214	1.8745	29	9	0.0326
35	0.6177	1.7192	35	11	0.0178
36	1.4781	1.9843	34	7	0.0120
37	4.8047	1.5043	36	23	0.0357
38	0.9265	1.3484	9	6	0.0290
39	4.0266	3.2307	37	17	0.0197
40	0.3890	0.5253	5	15	0.0042
41	3.4781	2.4039	34	7	0.0294
42	4.8003	2.9603	31	19	0.0388
43	4.6338	0.4479	12	10	0.0265
44	0.2377	3.7439	12	7	0.0210
45	0.2475	1.9247	20	16	0.0450
46	3.6132	3.0586	28	8	0.0212
47	3.0590	3.0321	12	14	0.0188
48	4.6438	4.9648	11	17	0.0311
49	1.1816	3.3945	33	6	0.0307
50	2.0146	4.2702	28	11	0.0046

Retailer R_i	XCOOR.	YCOOR.	q_i	A_i^R	h_i^R
51	1.9705	2.2853	34	13	0.0381
52	1.8584	3.5672	18	17	0.0286
53	4.7109	4.6033	13	17	0.0152
54	1.5081	1.7068	11	12	0.0063
55	2.0033	2.2494	5	14	0.0207
56	3.9764	3.1502	14	10	0.0053
57	2.2753	0.5099	7	21	0.0391
58	3.4438	4.2283	40	13	0.0269
59	1.3118	2.5500	17	9	0.0194
60	2.9314	4.2593	24	9	0.0467
61	1.7999	1.5509	44	23	0.0435
62	0.6951	0.8150	14	7	0.0344
63	0.1001	4.1113	39	8	0.0328
64	4.8140	0.6427	22	8	0.0219
65	2.8414	4.5636	6	17	0.0426
66	4.4857	2.4523	17	12	0.0413
67	1.2578	1.0160	37	18	0.0181
68	0.5266	4.3815	27	10	0.0139
69	3.0729	0.9988	15	10	0.0006
70	3.9628	3.6469	34	11	0.0117
71	1.6589	3.9404	11	8	0.0022
72	3.1770	2.7156	33	18	0.0384
73	3.3114	3.9889	27	17	0.0318
74	1.6720	0.7847	31	18	0.0249
75	1.6786	2.0680	35	10	0.0402
76	4.2837	1.8129	25	6	0.0249
77	1.6008	3.9015	7	7	0.0304
78	1.8218	3.1625	33	10	0.0397
79	4.2572	3.1592	32	9	0.0225
80	2.3785	4.3769	17	25	0.0058
81	1.2459	2.6106	19	22	0.0259
82	1.5914	1.2872	33	23	0.0497
83	4.4163	2.6735	5	16	0.0280
84	1.8757	2.1993	28	19	0.0312
85	0.1882	3.1558	34	10	0.0250
86	0.3500	1.9264	37	23	0.0441
87	1.9328	1.3121	15	7	0.0073
88	0.9336	0.3249	11	8	0.0431
89	0.9030	3.0624	25	7	0.0481
90	1.7687	0.1344	34	25	0.0036
91	0.9793	1.5196	8	9	0.0006
92	3.0691	2.1029	17	23	0.0091
93	1.7600	4.0636	38	11	0.0430
94	0.5276	0.9563	44	6	0.0465
95	1.5289	1.6044	20	25	0.0225
96	3.1575	1.1095	24	20	0.0426
97	1.8002	1.0274	6	22	0.0189
98	2.8729	2.7918	18	13	0.0296
99	3.4966	3.0280	17	14	0.0211
100	4.6237	1.3657	20	21	0.0319

Table C.4: Data for Example 4

Example 5 with 2 potential warehouses and 4 retailers.

$S_v = 300,\ h = 0.0028,\ Location(M) = (0.7465, 1.7499)$					
Potential Warehouse W_i	XCOOR.	YCOOR.	O_i^W	A_i^W	h_i^W
1	1.2875	0.9830	15	30	0.0050
2	4.2036	1.2554	15	30	0.0050

Retailer R_i	XCOOR.	YCOOR.	q_i	A_i^R	h_i^R
1	1.2714	3.0802	35	11	0.0223
2	4.0714	2.3664	36	19	0.0323
3	1.2176	1.7583	12	19	0.0355
4	4.6463	4.1541	24	9	0.0377

Table C.5: Data for Example 5

Appendix D

Vehicle Parameters for Synchronized PWLIR Model

$\phi = 0.001$				
Vehicle Type i	V_i^c	V_i^f	V_i^v	V_i^w
1	50	10	0.30	0.5
2	150	25	0.40	0.6
3	400	30	0.45	0.8
4	900	40	0.50	1

Table D.1: The vehicle parameters for the PWLIR model.

Appendix E

Datasets for the Integrated Multi-Depot Vehicle Routing and Delivery Men Problem

Example 1:
Q=150, s=30, Q_0=25, s_0=2, L=3, M=24, T=4
α=40, β=0.2, γ=8, λ=1

Depot i	Xcoor.	Ycoor.	Depot i	Xcoor.	Ycoor.	Depot i	Xcoor.	Ycoor.
1	3.66	7.35	2	43.95	38.87			
Parking site i	Xcoor.	Ycoor.	Parking site i	Xcoor.	Ycoor.	Parking site i	Xcoor.	Ycoor.
1	11.89	12.53	5	28.21	24.07	9	45.68	36.92
2	6.20	28.33	6	25.84	43.34	10	39.57	24.65
3	5.37	38.70	7	44.12	6.27	11	25.19	42.23
4	24.82	6.75	8	45.17	20.20	12	13.39	33.00

Customer i	Xcoor.	Ycoor.	q_i	Parking site	Customer i	Xcoor.	Ycoor.	q_i	Parking site
1	5.04	38.67	7	3	16	24.85	6.59	8	4
2	45.21	20.35	10	8	17	45.38	36.69	8	9
3	6.24	28.05	3	2	18	27.90	24.03	5	5
4	26.07	43.66	4	6	19	13.59	32.78	10	12
5	25.64	43.62	2	6	20	5.51	38.77	1	3
6	25.37	42.44	1	11	21	5.19	38.53	8	3
7	44.42	6.34	1	7	22	39.27	24.39	1	10
8	45.75	37.19	4	9	23	13.70	33.31	9	12
9	25.06	42.31	10	11	24	45.21	20.35	4	8
10	11.87	12.23	4	1	25	45.48	37.10	9	9
11	13.38	33.10	7	12	26	5.44	38.59	6	3
12	39.35	24.91	7	10	27	44.90	20.38	5	8
13	13.06	33.22	6	12	28	45.72	36.77	4	9
14	45.59	37.10	7	9	29	39.82	24.54	6	10
15	45.93	36.87	7	9	30	24.68	6.67	3	4

Customer i	Xcoor.	Ycoor.	q_i	parking site	Customer i	Xcoor.	Ycoor.	q_i	parking site
31	45.47	36.76	3	9	46	5.89	28.45	1	2
32	43.99	6.18	4	7	47	25.27	42.54	3	11
33	25.34	42.54	9	11	48	13.64	33.00	2	12
34	24.95	6.46	7	4	49	5.52	38.59	6	3
35	43.81	6.05	2	7	50	5.57	38.45	4	3
36	25.17	41.91	1	11	51	11.94	12.82	2	1
37	11.57	12.62	9	1	52	44.28	6.01	8	7
38	11.66	12.20	5	1	53	45.84	37.23	7	9
39	5.46	38.89	8	3	54	45.85	36.93	3	9
40	39.56	24.84	2	10	55	43.94	6.33	9	7
41	6.42	28.49	8	2	56	26.09	43.09	6	6
42	11.98	12.85	4	1	57	25.15	42.35	2	11
43	27.97	24.00	9	5	58	45.79	36.73	6	9
44	28.21	24.11	4	5	59	45.87	36.84	4	9
45	45.50	20.24	6	8	60	28.47	24.02	6	5

Table E.1: Data for Example 1

Example 2:
Q=300, s=30, Q_0=25, s_0=2, L=3, M=40, T=4
α=50, β=0.4, γ=8, λ=1

Depot i	Xcoor.	Ycoor.	Depot i	Xcoor.	Ycoor.	Depot i	Xcoor.	Ycoor.
1	12.66	21.34	2	32.52	8.16	3	41.87	43.99
Parking site i	Xcoor.	Ycoor.	Parking site i	Xcoor.	Ycoor.	Parking site i	Xcoor.	Ycoor.
1	8.76	5.62	8	20.08	44.20	15	46.87	30.51
2	7.24	15.99	9	32.11	7.86	16	41.60	41.23
3	9.74	34.67	10	33.68	21.00	17	23.09	33.63
4	2.67	41.83	11	32.50	30.06	18	22.24	35.01
5	20.27	3.77	12	32.34	45.29	19	17.35	41.37
6	18.12	19.91	13	43.13	5.10	20	13.70	26.17
7	16.88	31.16	14	45.97	21.44			

Customer i	Xcoor.	Ycoor.	q_i	parking site	Customer i	Xcoor.	Ycoor.	q_i	parking site
1	20.05	44.33	8	8	21	20.27	3.56	1	5
2	32.37	30.02	5	11	22	32.28	30.27	3	11
3	41.59	41.14	3	16	23	8.94	5.60	9	1
4	13.57	26.10	7	20	24	2.61	41.66	3	4
5	22.11	34.89	7	18	25	41.71	41.37	2	16
6	13.65	26.20	10	20	26	2.82	41.78	10	4
7	13.74	26.31	3	20	27	41.50	41.19	7	16
8	7.08	16.21	8	2	28	22.20	35.06	3	18
9	20.29	3.91	2	5	29	16.69	30.97	9	7
10	46.88	30.43	4	15	30	23.02	33.71	7	17
11	18.28	20.11	5	6	31	22.48	34.98	10	18
12	8.68	5.84	7	1	32	20.38	3.73	10	5
13	17.46	41.28	6	19	33	17.43	41.28	8	19
14	9.98	34.66	1	3	34	18.23	19.74	10	6
15	9.62	34.77	1	3	35	7.25	15.81	3	2
16	23.09	33.71	10	17	36	8.84	5.67	9	1
17	13.56	26.11	10	20	37	13.89	26.24	10	20
18	17.33	41.28	3	19	38	32.37	45.43	10	12
19	7.34	15.75	7	2	39	17.14	41.25	1	19
20	32.30	45.44	8	12	40	33.63	20.98	9	10

Customer i	Xcoor.	Ycoor.	q_i	parking site	Customer i	Xcoor.	Ycoor.	q_i	parking site
41	32.14	45.25	5	12	101	46.04	21.22	6	14
42	16.90	30.91	6	7	102	32.41	30.00	1	11
43	20.39	3.63	4	5	103	46.84	30.72	4	15
44	46.83	30.71	5	15	104	45.79	21.41	1	14
45	2.63	41.94	1	4	105	9.66	34.51	2	3
46	13.72	25.99	1	20	106	32.50	45.37	9	12
47	41.45	41.34	4	16	107	41.48	41.23	5	16
48	41.37	41.08	1	16	108	2.84	41.90	5	4
49	32.32	7.82	8	9	109	32.32	45.40	3	12
50	2.59	41.84	2	4	110	42.90	4.94	9	13
51	13.93	26.30	7	20	111	43.12	5.14	3	13
52	41.74	41.36	3	16	112	20.24	3.97	2	5
53	31.89	7.73	4	9	113	7.34	16.00	2	2
54	2.73	42.03	3	4	114	18.03	19.72	9	6
55	32.66	29.98	9	11	115	17.17	41.46	9	19
56	7.30	15.93	8	2	116	8.56	5.70	2	1
57	9.91	34.50	3	3	117	20.10	3.59	5	5
58	46.85	30.53	10	15	118	9.76	34.70	2	3
59	32.16	45.04	3	12	119	23.33	33.79	8	17
60	17.97	19.90	7	6	120	17.93	19.79	2	6
61	2.66	41.70	1	4	121	20.31	44.07	3	8
62	13.68	26.01	3	20	122	32.27	45.42	8	12
63	13.56	26.29	2	20	123	17.97	19.86	8	6
64	13.91	26.29	4	20	124	43.10	5.19	5	13
65	33.62	20.80	8	10	125	31.92	7.70	2	9
66	33.45	21.11	10	10	126	32.09	45.13	6	12
67	32.15	45.51	2	12	127	41.79	41.09	3	16
68	2.72	41.58	2	4	128	32.32	7.81	4	9
69	45.82	21.42	9	14	129	9.60	34.64	4	3
70	32.73	29.85	8	11	130	45.91	21.22	3	14
71	31.89	7.91	3	9	131	8.77	5.40	5	1
72	32.63	29.97	10	11	132	17.90	20.07	6	6
73	22.47	34.82	6	18	133	18.29	20.02	5	6
74	20.50	3.55	4	5	134	13.82	26.34	3	20
75	32.24	45.28	1	12	135	32.05	8.07	1	9
76	46.20	21.28	7	14	136	42.94	4.90	2	13
77	32.23	7.73	9	9	137	2.92	41.78	8	4
78	19.87	44.14	6	8	138	13.76	26.05	9	20
79	22.10	35.13	6	18	139	7.22	15.98	2	2
80	8.91	5.62	9	1	140	20.29	3.82	2	5
81	9.64	34.75	10	3	141	41.81	41.41	5	16
82	9.56	34.77	9	3	142	46.82	30.45	8	15
83	22.09	34.88	8	18	143	2.67	41.83	4	4
84	7.28	16.04	7	2	144	8.69	5.61	10	1
85	9.74	34.69	7	3	145	20.38	3.55	4	5
86	2.68	41.73	1	4	146	32.31	30.15	10	11
87	8.59	5.67	6	1	147	45.76	21.67	5	14
88	20.35	3.78	3	5	148	22.37	34.95	8	18
89	22.97	33.80	7	17	149	7.15	15.96	3	2
90	33.52	21.18	7	10	150	9.79	34.43	4	3
91	41.84	41.10	4	16	151	13.49	26.33	4	20
92	9.68	34.76	2	3	152	16.98	31.22	1	7
93	20.32	3.90	5	5	153	33.91	20.77	1	10
94	2.80	41.95	6	4	154	8.99	5.73	8	1
95	32.04	8.06	4	9	155	13.88	26.10	6	20
96	18.33	19.91	9	6	156	20.04	44.43	9	8
97	41.70	41.33	5	16	157	7.20	15.88	7	2
98	16.65	31.35	8	7	158	23.07	33.80	6	17
99	16.81	31.07	7	7	159	46.21	21.63	8	14
100	32.32	7.77	3	9	160	2.90	41.89	6	4

Customer i	Xcoor.	Ycoor.	q_i	parking site	Customer i	Xcoor.	Ycoor.	q_i	parking site
161	2.50	41.81	5	4	181	7.22	16.18	5	2
162	7.11	16.05	8	2	182	41.40	41.16	9	16
163	22.21	35.19	5	18	183	32.67	30.18	8	11
164	33.62	21.02	2	10	184	13.62	26.04	5	20
165	2.49	41.60	5	4	185	9.66	34.79	3	3
166	43.30	5.25	3	13	186	33.70	20.86	7	10
167	20.11	44.22	5	8	187	13.68	26.14	8	20
168	41.49	41.42	2	16	188	32.03	8.07	5	9
169	13.56	26.36	2	20	189	32.50	30.22	1	11
170	32.52	45.17	7	12	190	32.38	45.05	2	12
171	17.50	41.42	1	19	191	17.35	41.34	3	19
172	22.18	34.81	4	18	192	13.67	26.35	3	20
173	32.43	45.27	10	12	193	23.00	33.86	3	17
174	32.20	8.03	8	9	194	13.61	25.99	1	20
175	17.89	19.92	3	6	195	17.41	41.38	8	19
176	16.95	31.27	9	7	196	32.42	45.05	5	12
177	18.22	19.67	1	6	197	16.86	31.02	4	7
178	17.28	41.43	1	19	198	41.57	41.01	1	16
179	43.35	4.92	4	13	199	20.41	3.67	9	5
180	45.97	21.63	4	14	200	22.07	35.12	9	18

Table E.2: Data for Example 2

Example 3:
Q=600, s=30, Q_0=25, s_0=2, L=3, M=80, T=4
α=60, β=0.6, γ=8, λ=1

Depot i	Xcoor.	Ycoor.	Depot i	Xcoor.	Ycoor.	Depot i	Xcoor.	Ycoor.
1	6.64	44.70	3	23.62	27.55	5	37.71	15.63
2	8.89	2.58	4	27.56	36.59	6	43.93	45.14
Parking site i	Xcoor.	Ycoor.	Parking site i	Xcoor.	Ycoor.	Parking site i	Xcoor.	Ycoor.
1	2.74	4.93	15	19.63	23.11	29	39.75	37.33
2	2.66	14.58	16	20.58	28.47	30	39.84	46.19
3	2.32	19.93	17	22.42	35.12	31	46.66	2.33
4	6.00	27.58	18	20.44	48.11	32	48.11	11.26
5	6.00	38.23	19	30.11	1.80	33	44.86	22.06
6	6.56	46.38	20	30.36	10.55	34	46.78	26.85
7	11.06	5.08	21	28.41	20.29	35	46.34	35.91
8	11.42	13.99	22	30.13	27.41	36	46.48	46.88
9	12.34	18.60	23	27.40	38.51	37	13.10	29.21
10	12.29	27.79	24	28.10	46.33	38	6.08	35.28
11	10.65	38.63	25	39.85	3.26	39	30.57	10.74
12	13.74	45.80	26	35.96	12.59	40	27.56	31.29
13	22.45	4.56	27	38.12	21.10			
14	19.80	12.77	28	38.18	30.59			

Customer i	Xcoor.	Ycoor.	q_i	Parking site	Customer i	Xcoor.	Ycoor.	q_i	Parking site
1	44.91	22.07	9	33	61	2.47	19.92	7	3
2	48.20	11.19	6	32	62	46.65	26.69	5	34
3	46.31	35.87	2	35	63	27.64	31.33	10	40
4	39.92	46.17	10	30	64	6.10	27.75	4	4
5	5.85	38.33	5	5	65	46.63	2.44	3	31
6	39.79	46.12	6	30	66	46.63	26.97	3	34
7	30.25	10.70	9	20	67	2.39	19.81	10	3
8	28.37	20.40	5	21	68	20.31	48.15	7	18
9	11.04	5.14	10	7	69	6.14	38.36	1	5
10	30.26	27.38	7	22	70	29.98	1.80	1	19
11	5.88	27.63	6	4	71	30.56	10.63	5	39
12	20.47	28.60	10	16	72	28.23	46.47	10	24
13	30.25	1.73	1	19	73	38.21	21.05	9	27
14	29.99	1.72	5	19	74	20.73	28.59	1	16
15	2.83	4.86	6	1	75	28.01	46.32	6	24
16	46.57	46.72	7	36	76	30.49	10.68	2	39
17	27.36	38.56	2	23	77	46.43	46.76	2	36
18	6.58	46.34	8	6	78	6.24	35.26	8	38
19	2.22	20.07	3	3	79	11.51	14.13	9	8
20	27.71	31.27	2	40	80	48.22	11.38	1	32
21	5.88	38.08	4	5	81	28.24	46.32	2	24
22	20.42	28.38	6	16	82	38.17	21.25	3	27
23	30.25	10.69	9	20	83	12.18	27.91	2	10
24	38.17	30.55	5	28	84	27.47	38.45	2	23
25	44.99	22.06	5	33	85	6.49	46.44	9	6
26	46.67	2.24	8	31	86	12.50	18.45	8	9
27	2.78	4.95	7	1	87	30.73	10.66	7	39
28	12.28	18.74	4	9	88	39.90	46.09	2	30
29	22.57	35.09	2	17	89	12.25	18.43	6	9
30	46.40	46.80	9	36	90	38.23	21.16	2	27
31	39.68	37.27	6	29	91	20.71	28.63	3	16
32	13.60	45.84	4	12	92	10.95	5.12	10	7
33	27.97	46.42	3	24	93	38.12	21.17	10	27
34	28.44	20.17	5	21	94	6.02	38.21	1	5
35	46.55	2.32	4	31	95	12.21	18.70	6	9
36	2.80	4.83	3	1	96	22.35	4.49	4	13
37	28.02	46.40	8	24	97	46.30	35.97	3	35
38	2.53	14.67	2	2	98	22.49	35.27	9	17
39	28.55	20.42	2	21	99	27.70	31.38	6	40
40	12.17	27.78	10	10	100	30.14	27.41	6	22
41	30.43	10.71	8	20	101	19.62	23.18	1	15
42	2.88	4.76	9	1	102	35.85	12.63	9	26
43	39.71	3.31	7	25	103	38.02	30.57	4	28
44	30.04	1.90	6	19	104	38.32	30.70	6	28
45	2.24	20.01	9	3	105	48.12	11.20	5	32
46	36.08	12.60	6	26	106	12.97	29.12	3	37
47	38.14	20.98	1	27	107	10.64	38.49	8	11
48	27.61	31.44	5	40	108	46.74	2.32	8	31
49	2.86	4.89	6	1	109	30.57	10.67	5	39
50	2.74	14.71	10	2	110	46.71	26.82	7	34
51	19.60	23.18	6	15	111	35.85	12.50	10	26
52	39.65	37.42	3	29	112	46.60	2.28	8	31
53	39.78	3.18	1	25	113	22.33	35.00	10	17
54	10.55	38.65	8	11	114	27.33	38.53	7	23
55	12.40	18.56	3	9	115	30.20	27.38	5	22
56	22.31	35.25	9	17	116	20.41	48.22	9	18
57	30.64	10.70	9	39	117	30.19	1.76	2	19
58	22.38	4.60	6	13	118	5.89	38.12	9	5
59	12.40	18.49	4	9	119	30.32	10.55	1	20
60	30.15	1.85	10	19	120	30.27	1.92	1	19

Customer i	Xcoor.	Ycoor.	q_i	Parking site	Customer i	Xcoor.	Ycoor.	q_i	Parking site
121	30.38	10.47	8	20	181	39.77	3.33	3	25
122	2.79	5.06	9	1	182	11.16	5.03	6	7
123	20.56	48.27	2	18	183	19.66	23.01	8	15
124	30.32	10.47	2	20	184	28.23	46.36	10	24
125	27.51	31.13	1	40	185	39.64	37.24	9	29
126	2.23	20.04	4	3	186	12.43	18.72	4	9
127	46.38	46.87	10	36	187	13.64	45.66	7	12
128	20.41	48.02	10	18	188	30.20	1.90	5	19
129	6.12	27.54	3	4	189	28.25	46.30	4	24
130	6.57	46.49	7	6	190	2.42	20.07	7	3
131	30.05	27.43	3	22	191	22.38	35.27	6	17
132	6.03	27.67	4	4	192	30.22	1.73	8	19
133	35.94	12.70	10	26	193	39.92	46.33	7	30
134	27.99	46.34	10	24	194	12.41	27.83	9	10
135	46.60	2.49	1	31	195	13.74	45.73	10	12
136	6.55	46.39	3	6	196	27.41	38.43	6	23
137	20.55	28.44	2	16	197	46.38	35.93	3	35
138	2.30	20.04	9	3	198	44.86	22.13	1	33
139	28.23	46.17	2	24	199	46.64	26.96	9	34
140	48.12	11.13	9	32	200	5.98	27.46	9	4
141	39.90	3.30	8	25	201	10.78	38.67	2	11
142	27.57	31.43	5	40	202	10.67	38.64	1	11
143	20.47	48.13	4	18	203	6.04	38.10	1	5
144	19.79	23.09	8	15	204	5.95	35.29	7	38
145	30.58	10.68	3	39	205	27.70	31.28	1	40
146	30.04	27.35	6	22	206	6.11	38.22	1	5
147	44.84	22.08	5	33	207	30.44	10.66	7	20
148	19.77	12.77	2	14	208	44.87	22.00	9	33
149	20.59	28.33	9	16	209	30.31	10.50	5	20
150	12.30	18.51	5	9	210	27.68	31.43	3	40
151	22.54	34.96	6	17	211	39.60	37.26	1	29
152	46.49	46.97	9	36	212	22.47	4.43	6	13
153	5.93	38.37	5	5	213	11.35	13.91	10	8
154	46.79	2.23	1	31	214	46.46	46.75	3	36
155	35.87	12.65	4	26	215	39.76	37.49	7	29
156	22.51	4.57	6	13	216	5.89	38.23	4	5
157	2.74	4.92	4	1	217	22.46	35.15	9	17
158	46.53	46.94	5	36	218	27.35	38.52	8	23
159	2.76	14.63	4	2	219	30.16	1.89	3	19
160	30.06	1.94	8	19	220	13.64	45.88	10	12
161	39.89	37.40	9	29	221	27.67	31.44	4	40
162	11.07	5.18	8	7	222	46.46	36.00	10	35
163	6.04	38.27	3	5	223	46.55	46.94	6	36
164	12.17	27.85	9	10	224	27.55	31.40	7	40
165	2.90	4.86	8	1	225	46.18	35.87	8	35
166	30.41	10.91	3	39	226	20.41	48.20	5	18
167	12.38	18.43	1	9	227	5.91	38.25	8	5
168	48.15	11.21	4	32	228	22.55	35.15	2	17
169	19.64	12.67	1	14	229	48.03	11.26	5	32
170	2.71	14.60	2	2	230	44.91	22.20	2	33
171	30.58	10.58	8	39	231	13.86	45.73	9	12
172	39.98	46.33	9	30	232	13.14	29.29	6	37
173	46.28	35.85	5	35	233	30.25	27.26	7	22
174	22.40	4.53	7	13	234	46.32	46.99	2	36
175	27.31	38.39	2	23	235	6.20	35.11	7	38
176	12.12	27.66	8	10	236	39.64	37.20	10	29
177	2.61	4.94	8	1	237	30.60	10.59	5	39
178	46.50	35.79	9	35	238	30.11	1.82	8	19
179	30.40	10.57	4	20	239	19.85	12.77	10	14
180	47.94	11.17	4	32	240	2.85	4.86	8	1

Customer i	Xcoor.	Ycoor.	q_i	Parking site	Customer i	Xcoor.	Ycoor.	q_i	Parking site
241	39.80	37.35	1	29	301	22.48	35.04	4	17
242	38.28	20.99	1	27	302	6.09	35.12	6	38
243	12.96	29.37	9	37	303	30.32	10.46	9	20
244	6.62	46.46	10	6	304	5.99	38.21	5	5
245	12.37	18.73	7	9	305	11.18	5.00	2	7
246	22.58	34.99	5	17	306	46.61	26.87	8	34
247	22.49	4.41	10	13	307	6.05	35.44	6	38
248	2.67	4.80	1	1	308	2.76	4.81	3	1
249	28.30	20.32	4	21	309	13.80	45.82	2	12
250	6.64	46.53	7	6	310	39.74	37.49	7	29
251	13.88	45.82	3	12	311	20.72	28.43	10	16
252	39.70	3.25	10	25	312	30.07	27.32	4	22
253	30.49	10.69	4	20	313	45.02	22.14	5	33
254	27.28	38.58	2	23	314	13.06	29.11	2	37
255	12.49	18.74	2	9	315	39.97	3.31	10	25
256	39.86	46.33	10	30	316	39.75	3.42	5	25
257	2.24	19.81	10	3	317	2.50	14.50	10	2
258	46.31	35.84	7	35	318	19.76	12.87	6	14
259	2.48	19.78	3	3	319	11.51	13.90	9	8
260	6.03	35.23	4	38	320	27.58	31.42	6	40
261	12.45	18.75	4	9	321	48.22	11.37	5	32
262	39.82	46.18	1	30	322	12.31	18.73	1	9
263	30.17	1.90	7	19	323	19.72	12.92	2	14
264	11.29	13.90	9	8	324	11.12	4.99	9	7
265	30.49	10.39	3	20	325	10.48	38.60	6	11
266	30.07	27.33	7	22	326	12.12	27.77	9	10
267	29.95	1.85	6	19	327	10.54	38.62	1	11
268	28.25	46.48	3	24	328	46.41	46.86	1	36
269	29.95	1.96	7	19	329	22.54	4.58	2	13
270	39.70	46.22	9	30	330	12.31	18.48	3	9
271	13.09	29.23	7	37	331	11.40	14.08	4	8
272	19.91	12.79	2	14	332	13.77	45.64	5	12
273	6.03	35.36	1	38	333	28.23	46.28	4	24
274	46.40	46.98	8	36	334	20.42	28.43	2	16
275	28.17	46.19	10	24	335	30.67	10.61	6	39
276	13.66	45.73	1	12	336	27.40	38.66	1	23
277	45.02	22.16	8	33	337	28.01	46.23	6	24
278	46.29	35.77	4	35	338	5.92	38.18	7	5
279	19.96	12.87	10	14	339	10.48	38.53	8	11
280	28.24	20.41	1	21	340	2.48	19.89	8	3
281	5.86	38.13	4	5	341	30.01	1.78	7	19
282	13.25	29.14	1	37	342	45.02	22.14	6	33
283	30.22	27.47	4	22	343	19.96	12.81	8	14
284	6.15	35.25	4	38	344	27.40	38.37	9	23
285	30.02	1.82	9	19	345	11.44	14.02	4	8
286	48.13	11.24	6	32	346	46.34	35.75	10	35
287	10.78	38.48	9	11	347	30.33	10.56	2	20
288	13.04	29.09	1	37	348	6.16	35.19	3	38
289	30.21	27.55	5	22	349	28.29	20.39	9	21
290	27.29	38.44	9	23	350	30.29	10.50	4	20
291	45.00	22.15	8	33	351	11.58	13.83	7	8
292	10.90	5.22	8	7	352	11.57	13.93	10	8
293	30.72	10.73	10	39	353	5.99	35.29	2	38
294	30.26	1.65	4	19	354	10.99	5.23	4	7
295	11.30	13.95	9	8	355	6.05	35.14	8	38
296	19.87	12.74	4	14	356	20.60	48.13	7	18
297	46.31	35.75	1	35	357	38.29	30.55	10	28
298	2.58	4.97	2	1	358	20.53	28.47	7	16
299	44.85	21.95	2	33	359	20.51	28.54	10	16
300	2.40	20.10	4	3	360	11.32	14.05	8	8

Customer i	Xcoor.	Ycoor.	q_i	Parking site	Customer i	Xcoor.	Ycoor.	q_i	Parking site
361	20.50	28.42	7	16	381	28.55	20.41	7	21
362	12.35	27.85	4	10	382	30.18	1.94	4	19
363	48.14	11.24	8	32	383	46.46	46.87	9	36
364	29.99	1.91	5	19	384	2.28	19.89	1	3
365	13.74	45.73	10	12	385	38.08	21.04	8	27
366	39.85	37.43	1	29	386	20.61	28.53	8	16
367	30.36	10.46	7	20	387	28.50	20.31	1	21
368	11.40	14.08	4	8	388	19.96	12.79	8	14
369	44.94	22.04	4	33	389	38.22	21.07	4	27
370	46.61	46.73	9	36	390	27.44	31.13	6	40
371	48.00	11.31	4	32	391	5.85	38.39	7	5
372	30.16	1.68	10	19	392	19.49	23.11	2	15
373	6.05	27.67	1	4	393	6.03	27.63	5	4
374	30.18	27.51	8	22	394	19.76	12.62	2	14
375	46.58	2.40	7	31	395	6.06	38.07	8	5
376	46.23	35.90	1	35	396	30.15	1.93	9	19
377	39.88	46.16	7	30	397	46.35	35.75	5	35
378	47.97	11.36	10	32	398	30.17	1.74	2	19
379	46.27	35.79	8	35	399	10.50	38.48	9	11
380	11.48	14.07	7	8	400	13.84	45.67	6	12

Table E.3: Data for Example 3

Bibliography

Abdelsalam, H. M. and Elassal, M. M. (2014), "Joint economic lot sizing problem for a threeXLayer supply chain with stochastic demand," *International Journal of Production Economics*, 155, 272–283.

Abdul-Jalbar, B., Gutierrez, J. M., and Sicilia, J. (2007), "An integrated inventory model for the single-vendor two-buyer problem," *International Journal of Production Economics*, 108, 246–258.

Aderohunmu, R., Mobolurin, A., and Bryson, N. (1995), "Joint Vendor-Buyer Policy in JIT Manufacturing," *Journal of The Operational Research Society*, pp. 375–385.

Ağralı, S., Geunes, J., and Taşkın, Z. C. (2012), "A facility location model with safety stock costs: analysis of the cost of single-sourcing requirements," *Journal of Global Optimization*, 54, 551–581.

Arshinder, Kanda, A., and Deshmukh, S. G. (2008), "Supply Chain Coordination: Perspectives, Empirical Studies and Research Directions," *International Journal of Production Economics*, 115, 316–335.

Banerjee, A. (1986a), "A Joint Economic-Lot-Size Model for Purchaser and Vendor," *Decision Sciences*, 17, 292–311.

Banerjee, A. (1986b), "On A Quantity Discount Pricing Model to Increase Vendor Profits," *Management Science*, 32, 1513–1517.

Banerjee, A. and Banerjee, S. (1994), "A Coordinated Order-up-to Inventory Control Policy for a Single Supplier and Multiple Buyers Using Electronic Data Interchange," *International Journal of Production Economics*, 35, 85–91.

Banerjee, A. and Burton, J. S. (1994), "Coordinated vs. Independent Inventory Replenishment Policies for a Vendor and Multiple Buyers," *International Journal of Production Economics*, 35, 215–222.

Banerjee, A. and Kim, S. L. (1995), “An Integrated JIT Inventory Model,” *International Journal of Operations & Production Management*, 15, 237–244.

Banerjee, A., Kim, S.-L., and Burton, J. (2007), “Supply chain coordination through effective multi-stage inventory linkages in a JIT environment,” *International Journal of Production Economics*, 108, 271–280.

Barahona, F. and Jensen, D. (1998), “Plant location with minimum inventory,” *Mathematical Programming*, 83, 101–111.

Beamon, B. M. and Chen, V. C. P. (2001), “Performance Analysis of Conjoined Supply Chains,” *International Journal of Production Research*, 39, 3195–3218.

Belenguer, J. M., Benavent, E., Martínez, A., Prins, C., Prodhon, C., and Villegas, J. G. (2015), “A branch-and-cut algorithm for the single truck and trailer routing problem with satellite depots,” *Transportation Science*.

Ben-Daya, M. and Al-Nassar, A. (2008), “An integrated inventory production system in a three-layer supply chain,” *Production Planning and Control*, 19, 97–104.

Ben-Daya, M., Hassini, E., Hariga, M., and AlDurgam, M. M. (2013a), “Consignment and Vendor Managed Inventory in Single-Vendor Multiple Buyers Supply Chains,” *International Journal of Production Research*, 51, 1347–1365.

Ben-Daya, M., Asad, R., and Seliaman, M. (2013b), “An integrated production inventory model with raw material replenishment considerations in a three layer supply chain,” *International Journal of Production Economics*, 143, 53–61.

Berman, O., Krass, D., and Tajbakhsh, M. M. (2012), “A coordinated location-inventory model,” *European Journal of Operational Research*, 217, 500–508.

Bjelić, N., Vidović, M., and Popović, D. (2013), “Variable neighborhood search algorithm for heterogeneous traveling repairmen problem with time windows,” *Expert Systems with Applications*, 40, 5997–6006.

Braglia, M. and Zavanella, L. (2003), “Modelling an Industrial Strategy for Inventory Management in Supply Chains: The ’Consignment Stock’ Case,” *International Journal of Production Research*, 41, 3793–3808.

Cachon, G. P. and Lariviere, M. A. (2005), "Supply Chain Coordination with Revenue-Sharing Contracts: Strengths and Limitations," *Management Science*, 51, 30–44.

Černỳ, V. (1985), "Thermodynamical approach to the traveling salesman problem: An efficient simulation algorithm," *Journal of optimization theory and applications*, 45, 41–51.

Chan, C. K. and Kingsman, B. G. (2007), "Coordination in a Single-Vendor Multi-Buyer Supply Chain by Synchronizing Delivery and Production Cycles," *Transportation Research Part E: Logistics and Transportation Review*, 43, 90–111.

Chan, C. K., Lee, Y. C. E., and Goyal, S. K. (2010), "A Delayed Payment Method in Co-ordinating a Single-Vendor Multi-Buyer Supply Chain," *International Journal of Production Economics*, 127, 95–102.

Chen, T.-H. and Chen, J.-M. (2005), "Optimizing supply chain collaboration based on joint replenishment and channel coordination," *Transportation Research Part E: Logistics and Transportation Review*, 41, 261–285.

Chen, Z. X. and Sarker, B. R. (2010), "Multi-Vendor Integrated Procurement-Production System under Shared Transportation and Just-In-Time Delivery System," *Journal of the Operational Research Society*, 61, 1654–1666.

Chu, C.-L. and Leon, V. J. (2008), "Single-Vendor Multi-Buyer Inventory Coordination under Private Information," *European Journal of Operational Research*, 191, 485–503.

Coene, S. and Spieksma, F. C. (2008), "Profit-based latency problems on the line," *Operations Research Letters*, 36, 333–337.

Cornuéjols, G., Nemhauser, G. L., and Wolsey, L. A. (1983), "The uncapacitated facility location problem," Tech. rep., DTIC Document.

Dada, M. and Skikanth, K. N. (1987), "Pricing Policies for Quantity Discounts," *Management Science*, 33, 1247–1252.

de Grancy, G. S. and Reimann, M. (2014), "Vehicle routing problems with time windows and multiple service workers: a systematic comparison between ACO and GRASP," *Central European Journal of Operations Research*, pp. 1–20.

de Grancy, G. S. and Reimann, M. (2015), "Evaluating two new heuristics for constructing customer clusters in a VRPTW with multiple service workers," *Central European Journal of Operations Research*, 23, 479–500.

De Jong, K. A. (1975), "Analysis of the behavior of a class of genetic adaptive systems," .

Dewilde, T., Cattrysse, D., Coene, S., Spieksma, F. C., and Vansteenwegen, P. (2013), "Heuristics for the traveling repairman problem with profits," *Computers & Operations Research*, 40, 1700–1707.

Eppen, G. D. (1979), "Note-effects of centralization on expected costs in a multi-location newsboy problem," *Management Science*, 25, 498–501.

Ferreira, V. d. O. and Pureza, V. (2012), "Some experiments with a savings heuristic and a tabu search approach for the vehicle routing problem with multiple deliverymen," *Pesquisa Operacional*, 32, 443–463.

Ganeshan, R. and Harrison, T. P. (1995), "An Introduction to Supply Chain Management," pp. Department of Management Science and Information Systems, The Pennsylvania State University, University Park, PA.

Giri, B., Chakraborty, A., and Maiti, T. (2015), "Effectiveness of consignment stock policy in a three-level supply chain," *Operational Research*, pp. 1–28.

Glock, C. H. (2011), "A Multiple-Vendor Single-Buyer Integrated Inventory Model with a Variable Number of Vendors," *Computers & Industrial Engineering*, 60, 173–182.

Glock, C. H. (2012), "Coordination of a Production Network with a Single Buyer and Multiple Vendors," *International Journal of Production Economics*, 135, 771–780.

Glock, C. H. and Kim, T. (2014), "Shipment Consolidation in a Multiple-Vendor–Single-Buyer Integrated Inventory Model," *Computers & Industrial Engineering*, 70, 31–42.

Golden, B., Assad, A., Levy, L., and Gheysens, F. (1984), "The fleet size and mix vehicle routing problem," *Computers & Operations Research*, 11, 49–66.

Goyal, S. K. (1977), "An Integrated Inventory Model for a Single Supplier-Single Sustomer Problem," *The International Journal of Production Research*, 15, 107–111.

Goyal, S. K. (1987), "Comment on: A Generalized Quantity Discount Pricing Model to Increase Supplier's Profits," *Management Science*, 33, 1635–1636.

Goyal, S. K. (1988), "A Joint Economic-Lot-Size Model for Purchaser and Vendor: A Comment," *Decision Sciences*, 19, 236–241.

Goyal, S. K. (1995), "A One-Vendor Multi-Buyer Integrated Inventory Model: A Comment," *European Journal of Operational Research*, 82, 209–210.

Goyal, S. K. (2000), "On Improving the Single-Vendor Single-Buyer Integrated Production Inventory Model with a Generalized Policy," *European Journal of Operational Research*, 125, 429–430.

Goyal, S. K. and Gupta, Y. P. (1989), "Integrated Inventory Models: The Buyer-Vendor Coordination," *European Journal of Operational Research*, 41, 261–269.

Goyal, S. K. and Nebebe, F. (2000), "Determination of Economic Production–Shipment Policy for a Single-Vendor-Single–Buyer System," *European Journal of Operational Research*, 121, 175–178.

Guerrero, W. J., Prodhon, C., Velasco, N., and Amaya, C. A. (2013), "Hybrid heuristic for the inventory location-routing problem with deterministic demand," *International Journal of Production Economics*, 146, 359–370.

Guo, C. and Li, X. (2014), "A multi-echelon inventory system with supplier selection and order allocation under stochastic demand," *International Journal of Production Economics*, 151, 37–47.

Ha, D. and Kim, S.-L. (1997), "Implementation of JIT Purchasing: An Integrated Approach," *Production Planning & Control*, 8, 152–157.

Heilporn, G., Cordeau, J.-F., and Laporte, G. (2010), "The delivery man problem with time windows," *Discrete Optimization*, 7, 269–282.

Hill, R. M. (1997), "The Single-Vendor Single-Buyer Integrated Production-Inventory Model with a Generalised Policy," *European Journal of Operational Research*, 97, 493–499.

Hill, R. M. (1999), "The Optimal Production and Shipment Policy for the Single-Vendor Single-Buyer Integrated Production-Inventory Problem," *International Journal of Production Research*, 37, 2463–2475.

Holland, J. H. (1975), "Adaptation in natural and artificial system: an introduction with application to biology, control and artificial intelligence," *Ann Arbor, University of Michigan Press.*

Hoque, M. A. (2008), "Synchronization in the Single-Manufacturer Multi-Buyer Integrated Inventory Supply Chain," *European Journal of Operational Research*, 188, 811–825.

Hoque, M. A. (2011a), "Generalized Single-Vendor Multi-Buyer Integrated Inventory Supply Chain Models with a Better Synchronization," *International Journal of Production Economics*, 131, 463–472.

Hoque, M. A. (2011b), "An Optimal Solution Technique to the Single-Vendor Multi-Buyer Integrated Inventory Supply Chain by Incorporating Some Realistic Factors," *European Journal of Operational Research*, 215, 80–88.

Jaber, M. and Goyal, S. (2008), "Coordinating a three-level supply chain with multiple suppliers, a vendor and multiple buyers," *International Journal of Production Economics*, 116, 95–103.

Javid, A. A. and Azad, N. (2010), "Incorporating location, routing and inventory decisions in supply chain network design," *Transportation Research Part E: Logistics and Transportation Review*, 46, 582–597.

Jayaraman, V. (1998), "Transportation, facility location and inventory issues in distribution network design: An investigation," *International Journal of Operations & Production Management*, 18, 471–494.

Joglekar, P. and Tharthare, S. (1990), "The Individually Responsible and Rational Decision Approach to Economic Lot Sizes for One Vendor and Many Purchasers," *Decision Sciences*, 21, 492–506.

Joglekar, P. N. (1988), "Comments on A Quantity Discount Pricing Model to Increase Vendor Profits," *Management Science*, 34, 1391–1398.

Jonrinaldi and Zhang, D. (2013), "An integrated production and inventory model for a whole manufacturing supply chain involving reverse logistics with finite horizon period," *Omega*, 41, 598–620.

Karakatič, S. and Podgorelec, V. (2015), "A survey of genetic algorithms for solving multi depot vehicle routing problem," *Applied Soft Computing*, 27, 519–532.

Khouja, M. (2003), "Optimizing inventory decisions in a multi-stage multi-customer supply chain," *Transportation Research Part E: Logistics and Transportation Review*, 39, 193–208.

Kim, T. and Glock, C. H. (2013), "A multi-stage joint economic lot size model with lead time penalty costs," *Computers & Industrial Engineering*, 66, 133–146.

Kim, T., Hong, Y., and Chang, S. Y. (2006), "Joint economic procurement-production-delivery policy for multiple items in a single-manufacturer, multiple-retailer system," *International Journal of Production Economics*, 103, 199–208.

Kirkpatrick, S., Gelatt, C. D., Vecchi, M. P., et al. (1983), "Optimization by simulated annealing," *science*, 220, 671–680.

Kovacs, A. A., Golden, B. L., Hartl, R. F., and Parragh, S. N. (2014), "The generalized consistent vehicle routing problem," *Transportation Science*, 49, 796–816.

Kuehn, A. A. and Hamburger, M. J. (1963), "A heuristic program for locating warehouses," *Management science*, 9, 643–666.

Lal, R. and Staelin, R. (1984), "An Approach for Developing an Optimal Discount Pricing Policy," *Management Science*, 30, 1524–1539.

Lariviere, M. A. (1999), "Supply Chain Contracting and Coordination with Stochastic Demand," in *Quantitative Models for Supply Chain Management*, pp. 233–268, Springer.

Lee, H. L. and Rosenblatt, M. J. (1986), "A Generalized Quantity Discount Pricing Model to Increase Supplier's Profits," *Management Science*, 32, 1177–1185.

Lee, W. (2005), "A joint economic lot size model for raw material ordering, manufacturing setup, and finished goods delivering," *Omega*, 33, 163–174.

Liu, S. and Lee, S. (2003), "A two-phase heuristic method for the multi-depot location routing problem taking inventory control decisions into consideration," *The International Journal of Advanced Manufacturing Technology*, 22, 941–950.

Liu, S. and Lin, C. (2005), "A heuristic method for the combined location routing and inventory problem," *The International Journal of Advanced Manufacturing Technology*, 26, 372–381.

Lu, L. (1995), "A One-Vendor Multi-Buyer Integrated Inventory Model," *European Journal of Operational Research*, 81, 312–323.

Luo, Z., Qin, H., and Lim, A. (2014), "Branch-and-price-and-cut for the multiple traveling repairman problem with distance constraints," *European Journal of Operational Research*, 234, 49–60.

Ma, H. and Davidrajuh, R. (2005), "An iterative approach for distribution chain design in agile virtual environment," *Industrial Management & Data Systems*, 105, 815–834.

Magretta, J. (1998), "The Power of Virtual Integration: An Interview with Dell Computers, Michael Dell," *Harvard Business Review*, 76, 72–84.

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., and Teller, E. (1953), "Equation of state calculations by fast computing machines," *The journal of chemical physics*, 21, 1087–1092.

Miranda, P. A. and Garrido, R. A. (2008), "Valid inequalities for Lagrangian relaxation in an inventory location problem with stochastic capacity," *Transportation Research Part E: Logistics and Transportation Review*, 44, 47–65.

Monahan, J. P. (1984), "A Quantity Discount Pricing Model to Increase Vendor Profits," *Management Science*, 30, 720–726.

Monahan, J. P. (1988), "On Comments on a Quantity Discount Pricing Model to Increase Vendor Profits," *Management Science*, 34, 1398–1400.

Moorthy, K. S. (1987), "Comment-Managing Channel Profits: Comment," *Marketing Science*, 6, 375–379.

Murray, C. C. and Chu, A. G. (2015), "The flying sidekick traveling salesman problem: Optimization of drone-assisted parcel delivery," *Transportation Research Part C: Emerging Technologies*, 54, 86–109.

Nozick, L. K. and Turnquist, M. A. (1998), "Integrating inventory impacts into a fixed-charge model for locating distribution centers," *Transportation Research Part E: Logistics and Transportation Review*, 34, 173–186.

Pal, B., Sana, S. S., and Chaudhuri, K. (2012), "A three layer multi-item production–inventory model for multiple suppliers and retailers," *Economic Modelling*, 29, 2704–2710.

Pasternack, B. A. (1985), "Optimal Pricing and Return Policies for Perishable Commodities," *Marketing Science*, 4, 166–176.

Pourakbar, M., Farahani, R. Z., and Asgari, N. (2007), "A joint economic lot-size model for an integrated supply network using genetic algorithm," *Applied mathematics and Computation*, 189, 583–596.

Pureza, V., Morabito, R., and Reimann, M. (2012), "Vehicle routing with multiple deliverymen: modeling and heuristic approaches for the VRPTW," *European Journal of Operational Research*, 218, 636–647.

Raj, R., Kaliraman, N., Chandra, S., and Chaudhry, H. (2015), "Integrated production inventory model: multi-item, multiple suppliers and retailers, exponential demand rate," *Uncertain Supply Chain Management*, 3, 213–224.

Ribeiro, G. M. and Laporte, G. (2012), "An adaptive large neighborhood search heuristic for the cumulative capacitated vehicle routing problem," *Computers & Operations Research*, 39, 728–735.

Rivera, J. C., Afsar, H. M., and Prins, C. (2016), "Mathematical formulations and exact algorithm for the multitrip cumulative capacitated single-vehicle routing problem," *European Journal of Operational Research*, 249, 93–104.

Ropke, S. and Pisinger, D. (2006), "An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows," *Transportation science*, 40, 455–472.

Sajadieh, M. S., Fallahnezhad, M. S., and Khosravi, M. (2013), "A joint optimal policy for a multiple-suppliers multiple-manufacturers multiple-retailers system," *International Journal of Production Economics*, 146, 738–744.

Sana, S. S., Chedid, J. A., and Navarro, K. S. (2014), "A three layer supply chain model with multiple suppliers, manufacturers and retailers for multiple items," *Applied Mathematics and Computation*, 229, 139–150.

Sarker, B. R. (2014), "Consignment Stocking Policy Models for Supply Chain Systems: A Critical Review and Comparative Perspectives," *International Journal of Production Economics*, 155, 52 – 67, Celebrating a century of the economic order quantity model.

Sarker, B. R. and Diponegoro, A. (2009), "Optimal Production Plans and Shipment Schedules in a Supply-Chain System with Multiple Suppliers and Multiple Buyers," *European Journal of Operational Research*, 194, 753–773.

Sarker, B. R., Rochanaluk, R., Yi, H., and Egbelu, P. J. (2014), "An operational policy for a three-stage distributive supply chain system with retailers backorders," *International Journal of Production Economics*, 156, 332–345.

Sarmah, S., Acharya, D., and Goyal, S. (2008), "Coordination of a Single-Manufacturer/Multi-Buyer Supply Chain with Credit Option," *International Journal of Production Economics*, 111, 676–685.

Seifert, R. W., Zequeira, R. I., and Liao, S. (2012), "A three-echelon supply chain with price-only contracts and sub-supply chain coordination," *International Journal of Production Economics*, 138, 345–353.

Seliaman, M. et al. (2009), "A generalized algebraic model for optimizing inventory decisions in a multi-stage complex supply chain," *Transportation Research Part E: Logistics and Transportation Review*, 45, 409–418.

Shen, Z. J. M. and Qi, L. (2007), "Incorporating Inventory and Routing Costs in Strategic Location Models," *European Journal of Operational Research*, 179, 372–389.

Siajadi, H., Ibrahim, R. N., and Lochert, P. B. (2006), "A Single-Vendor Multiple-Buyer Inventory Model with a Multiple-Shipment Policy," *The International Journal of Advanced Manufacturing Technology*, 27, 1030–1037.

Silva, F. and Gao, L. (2013), “A joint replenishment inventory-location model,” *Networks and Spatial Economics*, 13, 107–122.

Skjoett-Larsen, T., Thernøe, C., and Andresen, C. (2003), “Supply Chain Collaboration: Theoretical Perspectives and Empirical Evidence,” *International Journal of Physical Distribution & Logistics Management*, 33, 531–549.

Sourirajan, K., Ozsen, L., and Uzsoy, R. (2007), “A single-product network design model with lead time and safety stock considerations,” *IIE Transactions*, 39, 411–424.

Taillard, É. D. (1999), “A heuristic column generation method for the heterogeneous fleet VRP,” *Revue française d'automatique, d'informatique et de recherche opérationnelle. Recherche opérationnelle*, 33, 1–14.

Tancrez, J.-S., Lange, J.-C., and Semal, P. (2012), “A location-inventory model for large three-level supply chains,” *Transportation Research Part E: Logistics and Transportation Review*, 48, 485–502.

Teo, C. P., Ou, J., and Goh, M. (2001), “Impact on inventory costs with consolidation of distribution centers,” *Iie Transactions*, 33, 99–110.

ÜSter, H., Keskin, B. B., and ÇEtinkaya, S. (2008), “Integrated warehouse location and inventory decisions in a three-tier distribution system,” *IIE Transactions*, 40, 718–732.

Valentini, G. and Zavanella, L. (2003), “The Consignment Stock of Inventories: Industrial Case and Performance Analysis,” *International Journal of Production Economics*, 81, 215–224.

Vidyarthi, N., Çelebi, E., Elhedhli, S., and Jewkes, E. (2007), “Integrated production-inventory-distribution system design with risk pooling: model formulation and heuristic solution,” *Transportation Science*, 41, 392–408.

Villegas, J. G., Prins, C., Prodhon, C., Medaglia, A. L., and Velasco, N. (2013), “A matheuristic for the truck and trailer routing problem,” *European Journal of Operational Research*, 230, 231–244.

Viswanathan, S. (1998), “Optimal Strategy for the Integrated Vendor-Buyer Inventory Model,” *European Journal of Operational Research*, 105, 38–42.

Viswanathan, S. and Piplani, R. (2001), "Coordinating Supply Chain Inventories through Common Replenishment Epochs," *European Journal of Operational Research*, 129, 277–286.

Wee, H.-M. and Yang, P.-C. (2004), "The optimal and heuristic solutions of a distribution network," *European Journal of Operational Research*, 158, 626–632.

Weng, Z. K. (1995), "Channel Coordination and Quantity Discount," *Management Science*, 41, 1509–1522.

Woo, Y. Y., Hsu, S.-L., and Wu, S. (2001), "An Integrated Inventory Model for a Single Vendor and Multiple Buyers with Ordering Cost Reduction," *International Journal of Production Economics*, 73, 203–215.

Yang, L., Ng, C., and Cheng, T. E. (2010), "Evaluating the effects of distribution centres on the performance of vendor-managed inventory systems," *European Journal of Operational Research*, 201, 112–122.

Yao, M.-J. and Chiou, C.-C. (2004), "On a Replenishment Coordination Model in an Integrated Supply Chain with One Vendor and Multiple Buyers," *European Journal of Operational Research*, 159, 406–419.