CAPACITY ALLOCATION OF LINER SHIPS WHEN CONSIDERING STRATEGICALLY THROWING AWAY CARGOES

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Capacity Allocation of Liner Ships when Considering Strategically Throwing away Cargoes

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CERTIFICATE OF ORIGINALITY

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______________________________ (Signed)

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Abstract: Nearly 90% of global trade is transported by ships. However, the increasing demand for high-quality shipping services and lower prices combined with the endless competition in the shipping market compel shipping companies to allocate the ship capacity more effectively. First, a benchmark capacity allocation model on a three-port (A , B and C) shipping line is established. In the model, the shipping company can observe the demand from the departure port A, but the demand from the intermediate port B is unobserved. When the shipping company allocates the capacity at the departure port A, the uncertainty from the intermediate port B must be considered. We find that the optimal capacity allocation strategy is related to a threshold parameter, which is decided by the shipping price of each shipping leg and the demand function of shipping leg $B-C$. When the threshold parameter is larger than the entire capacity of the ship, the shipping company should accept cargoes of shipping leg $A-B$ as many as possible; When the threshold parameter is relatively small, the profit function reaches the peak point when the confirmed cargo delivery amount of shipping leg $A-B$ is equal to the threshold parameter and the confirmed cargo delivery amount of shipping leg $A-C$ is equal to the gap between the entire capacity of the ship and the threshold parameter. The second model considers when strategically throwing away cargoes is allowed, how the shipping company should allocate the capacity. The optimal results show that strategically throwing away cargoes will increase the optimal amount of cargoes that should be accepted to shipping leg $A-C$ in some special cases. By comparing the expected revenues of the two models in the long run, the cases when throwing away cargoes should be allowed are shown. Lastly, a numerical example with exponential distribution demand is illustrated. The gap between the expected revenues of the two models shows that allowing the throwing away of cargoes can achieve higher expected revenues in all five cases.

Keywords: Liner Shipping, Capacity Allocation, Strategically Throwing Away Cargoes

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Chapter 1.

Introduction

1.1. Background and Motivation

It is well-known that nearly 90% of global trade is transported by ocean-going ships. Liner shipping helps more than million-ton of products travel to Europe and America [22]. Usually, costs for liner shipping companies to operate their services are stable when there are no large fluctuations of international oil prices. In recent years, benefiting from relatively low international oil prices, liner shipping companies could operate their services with relatively low costs. On the other hand, low international oil prices also cause the increase in demand for high quality shipping services and lower fees. As a result, Asian Shippers Association and Global Shippers Alliance were created to protect the rights of cargo owners [11]. In addition to the pressure from cargo owners, the competition from the shipping market has also been increasing. Six carriers, including Mitsui O.S.K. Lines et al., formed a new container carrier alliance in May 2016, named "THE Alliance", to compete with three other container carrier alliances: "CKYHE Alliance", "2M Alliance" and "Ocean Three" [24]. Figure 1.1 shows the present market share in different trade lanes. Endless and fierce competition also will prompt alliances to coordinate more effectively for the higher market share and revenue.

1.2. Research Question

Carrier alliances make it possible for shipping companies to share vessels and rent ship capacity from each other. Generally, if a shipping company accepts a booking, the cargoes will be delivered without throwing any away until reaching their destinations.

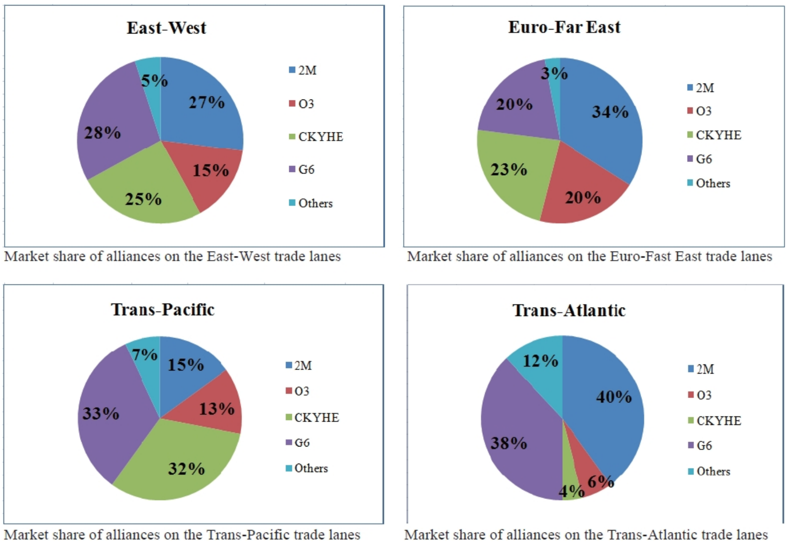


Figure 1.1.: Market share by 2M, O3, G6 and CKYHE alliances. [9]

Here, "throwing away" does not mean to abandon those cargoes but instead to find short-term storage until the next ship's arrival. The main reason for that is because a liner shipping company usually conducts shipping activity once every couple of days or couple of weeks. Throwing away cargoes at any port will lead to an extended delay, which may bring a higher delay cost or other invisible losses, such as reputation loss. However, if coordination within a carrier alliance can decrease the delay cost to an acceptable level, can strategically "throwing away" the cargoes result in a higher revenue?

This paper attempts to discuss how the behaviour of strategically throwing away cargoes affects a shipping company's decision regarding capacity allocation and revenue. For tractability, this paper simplified a liner shipping line to a simple three-port line.

Usually, about three weeks before departure, a shipping company begins to receive capacity bookings. At this time, reservations do not indicate the actual acceptance of bookings. Unlike the booking process in the airline, which confirms an order in a few seconds, a shipping company usually needs two days to decide whether to accept a booking and many shipping companies deal with consignors' reservations a week before the departure. For a liner ship on a long route, although the shipping company can observe the demands from the departure port in a line, the demand of the intermediate port is unknown. This conveys uncertainty for the shipping company's capacity allocation. Therefore, when the shipping company makes decisions regarding capacity allocation, the shipping company needs to take all demands from the shipping line into account, not only those from the departure port. The operating form of the throwing away is similar to transshipment, but their purposes are different. The shipping companies throw away cargoes in order to obtain higher revenues. Transshipment is usually used to describe the transshipping of cargoes between two types of ship in shipping because of different destinations and different shipping operating costs.

The long-haul shipping price usually reflects a discount effect compared with the sum of all sub-legs. When demands are large enough, accepting short distance orders can gain more for the shipping company than long haul. A liner ship's voyage usually requires a couple of weeks or more, the time sensitivity for shipping is lower for it than for other transport modes such as air. In addition, coordination between shipping companies of the same carrier alliance can shorten the delay. Both of that could make it possible for the shipping company to strategically throw away long-haul cargoes at the intermediate port. Of course, a penalty cost including the unloading and loading expenses, storage costs and delay costs will be applied to those thrown cargoes.

This paper formulates a shipping company capacity allocation decision model with the uncertainty of the demands of the intermediate port in a liner shipping line. The optimal capacity allocation strategy for the shipping company is proposed. More importantly, this paper provides the possibility of a fresh new coordination approach for carrier companies in a carrier alliance. A new capacity allocation model is set up where "throwing away cargoes" is allowed for the shipping company. The optimal strategy has a similar structure to the benchmark model. What's more, while not applying to all situations, this paper proves that "throwing away" cargoes could change the shipping company's capacity allocation and increase the revenue in certain cases. In the long run, allowing the "throwing away" of cargoes can also increase the expected revenue under special conditions. Conditions for allowing the throwing away of cargoes are also obtained.

1.3. Outlines

Related literatures are discussed in the second chapter. In the third chapter, we first discuss the benchmark model, which considers the uncertainty of the intermediate port's demand, how the shipping company should allocate the capacity. The model for strategically throwing away cargoes is discussed next. In the fourth chapter, managerial insights are provided through numerical examples. In the last chapter, we present the conclusions for this paper and the future works.

Chapter 2.

Literature Review

Few works of literature discussing the throwing away of shipping cargoes were found. However, similar literatures about capacity allocation, the newsboy model and overbooking were uncovered.

2.1. Capacity Allocation

Hetrakul et al. [17] analyzed ticket reservation data from a railway to search for optimal pricing and seat allocation in order to acquire the maximum revenues. By considering the taste heterogeneity of passengers, a customer choice model was developed. The results showed that satisfying short-haul demand achieved a greater revenue than long-haul demand. Amaruchkul et al. [2] considered how a single air-cargo carrier allocated the cargo capacity to multiple forwarders. The actual usage of each forwarder was modelled through a discrete Markov chain. The results were concluded with numerical examples. Dror et al. [10] modelled a seat allocation problem on flights as a network, in which passenger categorisation was considered on a multi-leg flight, but it was also assumed that no passenger would board at the second node. Only allocation principles were proposed. Wang et al. [28] analyzed how a liner container shipping carrier decided on the shipping route scheduling and cargo allocation scheme at each port when considering demurrage cost of cargoes that were incurred if cargoes needed to wait at ports. Zacharias et al. [35] studied the appointment scheduling problem at a medical facility when considering patients who displayed no-show behaviour. The difference between the usual studies of revenue management was that the objective included patients' waiting time and the doctor's idle time and overtime, but not the revenue. Xiao et al. [33] discussed another two-dimensional model, in which there were two demand types consuming different capacities in each dimension and demand arrivals follow a Poisson process. The optimal solution was discussed with an analytical form. Gallego et al. [14] studied one airline's capacity allocation problem. In this particular situation, two flights served the same market and the airline supplied a flexible service product, which supplied the flight service with a discount price and both flights were possible to be used to supply the flight service. The airline supplied the flexible service product only in the first period and supplied special products in two periods and with a higher price in the second period. The airline needed to decide the maximum number of each kind of product that would be accepted in each period to reach the maximum revenue depending on whether overbooking was allowed or not. Various properties of an optimal solution were discussed and an algorithm was designed to compute the optimal solution along with the expected revenue. The numerical results showed that more customers would like to purchase and the usage of capacity increases when the price of the flexible product is lower. Wilson et al. [32] discussed an airline that supplied two classes of ticket fares in two periods. Low fare customers always arrived before the high fare customers in each period and if the low fare tickets were sold out, a fraction of customers would purchase the high fare and the others would wait for the low fare in the second period. The optimal booking limits in the two periods were studied.

Wang et al. [29] studied how a shipping company should decide the optimal itinerary provision and the price to maximize its profit when customers, based on their utilities, could choose an itinerary among all itineraries that all shipping companies could supply among the ports. The utilities of customers included freight costs, inventory costs and a random item. Guericke et al. [15] studied a liner shipping cargo allocation problem in a shipping line network, in which the objective function considered the revenue from all services, the container depreciation, the container handling costs, transshipment costs of laden containers and empty equipment, the bunker cost, the fixed costs and a penalty part. Under the constraints of service levels, the optimal capacity allocation and the optimal sailing speed of each leg were discussed. Wang et al. [30] described the slot allocation in a shipping line network as resources allocation problem. The shipping services were described as a set of products. In their model, when the capacity of each resource that the freight operator supplied was a discrete random integer that was less than a maximum possible capacity and the demand was discretely distributed, the freight operator need to find the optimal resource allocation strategy to maximize its profit that included the expected sales revenue, the expected shortage costs and the expected salvage value. Another similar work of Wang [31] considered when there were three different products including normal delivering, reefer delivering and premium delivery, how the slots should be allocated. Fu et al. [13] studied when shipping line network faced the minimum quantity commitment uncertain demand, how the container slots should be allocated.

2.2. Newsboy Model Related Literature

Li et al. [20] discussed a two-product newsboy model problem in which two demands are independently and exponentially distributed and an analytical solution procedure was presented. Bensoussan et al. [3] extended the classical newsboy model further. In their model, a target service level was committed first and the newsboy would observe the demand and update signals after an initial order. Lau [19] discussed the newsboy model problem with two products, while the objective was the probability of achieving a target. Khouja et al. [18] studied another two-product newsboy model, in which a customer could substitute a desired good with a certain amount of another kind of good. The optimal solutions' structures and a numerical example were presented. Tibben et al. [26] examined a long-term supply contract in which the order amount for every period was limited into a specific range and heuristic results were given. Chen et al. [8] developed a two-period supply chain model, in which the retailer ordered only at the beginning of the first period and returned a certain number of products to the manufacturer at the end of first period with a higher buy back price. The retailer would need to decide the proper inventory for the second period to maximize the expected profit. The optimal decisions in the centralized system were discussed. By comparing with the optimal decisions in the decentralized system, coordination strategies were also discussed. Reimann et al. [23] introduced a two-period newsboy model problem, in which the manufacturer offered new products to the market in the first period and offered new and remanufactured products in the second period. Independent distributed demands were assumed for the two periods. In addition, the situation of the inventory carryover from the first period was also considered. Ali et al. [7] studied a two-period inventory model, in which the newsboy could order before the selling period and also order at the beginning of the selling period. After ordering the second time, the newsboy could sell certain quantities of inventory to a client market with a lower price in comparison with the usual market. Moreover, this model also assumed that the unsatisfied orders could be backlogged until the next period and the demands at each period were independently and randomly distributed. Under some capacity constraints, the optimal solutions and a numerical example were discussed. The study from Tiwari et al. [27] found some differences compared with the previously mentioned study. In a supplier and retailer model, the usual assumption of the supplier was fulfilling all of the orders from the retailer, but suppliers may only fulfil partial orders in reality. Under this assumption, this paper discussed how the retailer should decide the optimal order amount when the newsboy could order one more time after the arrival of some demands. Finally, under the assumption of normal demand distribution, the structure of the optimal solution was discussed. Harrison et al. [16] noted that when a firm sells multiple products on the market, each of the products was manufactured using several resources and the demand for each product was uncertain. They next determined how the firm should decide its investment strategies and production plans. The results showed that the optimal investment strategy was a transformation of the multi-dimensional newsboy model determined by demand distributions, prices and marginal investment costs. A special case was analysed in a study by Bernstein et al. [4], in which a firm supplied two products with different priorities, and each of the products needed both a special resource and a common resource.

2.3. Overbooking and No-show Related Literature

Alstrup et al. [1] studied a capacity control model of a signal-leg flight with only two types of passengers, which allowed no-shows. Xu et al. [34] studied the optimal booking limits for a two-leg airline network model, while overbooking and cancellations were allowed. In their model, the customers' arrivals were modelled as a non-homogenous Markov process, and batch requests were allowed. Subramanian et al. [25] analysed the booking limits of a single-leg flight with multiple fare classes. Overbooking and customer cancellations were also allowed in their model. An interesting result showed that accepting a lower-fare class may obtain more revenues when cancellation refunds of the lower-fare class and higher-fare class were different. Chatwin [6] studied the booking limits of a single-leg flight allowing airline overbooking and customer cancellations. The difference for this study was that the customers' reserving process was modeled as a continuous-time birth-and-death process. Under the assumption that the fares and refunds were piecewise-constant functions of the time to flight, the optimal booking limits were discussed. A similar study was conducted by Chatwin [5], in which a discrete-time approach was use to describe customers' reserving process. Sharbel et al. [12] analysed a two-leg airline seat inventory control model with multiple fare classes, in which airline overbooking and customer cancellations were also allowed and each customer was allowed to reserve only one seat. The results were illustrated by a numerical example. Luo et al. [21] studied a two-dimensional (weight and volume) cargo overbooking model, in which an airline company needed to decide the overbooking level to minimize the sum of capacity spoilage and cargo offloading costs. The numerical result showed that an airline company can reduce about $5\% - 6\%$ of the cost when considering two-dimensional model.

Chapter 3.

Model

Table 3.1.: Parameters and definitions

Parameters	Definitions
K	Capacity of the ship
A	The departure port in the shipping line
B	The intermediate port in the shipping line
C	The final port in the shipping line
X_{AC}	The realized cargo delivery requirement for shipping leg $A-C$
X_{AB}	The realized cargo delivery requirement for shipping leg $A-B$
X_{BC}	The realized cargo delivery requirement for shipping leg $B-C$
x_{AC}	Random demand for the cargo delivery on the shipping leg $A-C$
x_{AB}	Random demand for the cargo delivery on the shipping leg $A-B$
x_{BC}	Random demand for the cargo delivery on the shipping leg $B-C$
$F_{AC}(x_{AC})$	Cumulative distribution function (cdf) of the demand for shipping leg $A-C$
$F_{AB}(x_{AB})$	Cdf of the demand for shipping leg $A-B$
$F_{BC}(x_{BC})$	Cdf of the demand for shipping leg $B-C$
$f_{AC}(x_{AC})$	Probability distribution function (pdf) of the demand for shipping leg $A-C$
$f_{AB}(x_{AB})$	Pdf of the demand for shipping leg $A-B$
$f_{BC}(x_{BC})$	Pdf of the demand for shipping leg $B-C$
S_{AC}	The confirmed cargo delivery amount for shipping leg $A-C$
S_{AB}	The confirmed cargo delivery amount for shipping leg $A-B$
S_{BC}	The confirmed cargo delivery amount for shipping leg $B-C$
T_{AC}	The amount of cargoes thrown away at port B
P_{AC}	The unit capacity price for shipping leg $A-C$
P_{AB}	The unit capacity price for shipping leg $A-B$
P_{BC}	The unit capacity price for shipping leg $B-C$
C_p	Unit penalty cost for the cargoes unloaded at port B
R_B	The revenue function of port B
$R_A(S_{AC}, S_{AB})$	The revenue function of the entire shipping line

In this section, a benchmark model, which considers the uncertain demand of the intermediate port, is set up. The optimal capacity allocation strategy is discussed and the expected revenues in the long run are shown. Then, a model for strategically throwing is established. The strategic throwing away model studies if the shipping company is allowed to throw away cargoes, how the shipping company should allocate the capacity. From a long-term view, the situations in which the shipping company should allow the throwing away of cargoes are discussed. The related parameters and definitions are shown in Table 3.1.

3.1. Benchmark Model

This section examines the situation where a shipping company operates a cargo ship with limited capacity K in a shipping line which starts from port A, passes port B and finally reaches port C. Usually, the unit price of long distance transportation has a discount effect compared with the sum of each sub-leg in a shipping line. Therefore, in this paper, $0 < P_{AC} < P_{AB} + P_{BC}$ is assumed first. After the arrivals of all cargoes of shipping leg $A - C$ and shipping leg $A - B$, the shipping company needs to decide how many cargoes should be accepted for shipping leg $A - C$ and shipping leg $A - B$, respectively. At this moment the shipping company has no specific information about the demand for the shipping leg $B - C$. We assumed that the cumulative distribution function of the demand for shipping leg $B - C$ is $F_{BC}(x_{BC})$. When the ship sails to port B, the company will set a deadline for receiving bookings of cargoes of shipping leg $B - C$. After the deadline, the company needs to decide how many cargoes should be accepted for shipping leg $B - C$. When the company decides the amount of cargoes that should be accepted for shipping leg $B - C$, the decision problem is expressed as follows:

$$
\begin{aligned}
&R_B = P_{BC} \cdot S_{BC} \\
&St. \\
&\begin{cases} 0 \le S_{BC} \le X_{BC}, \\ 0 \le S_{BC} \le K - S_{AC}. \end{cases}
\end{aligned}
$$

At port B, the shipping company confirms that S_{BC} units of cargos will be delivered from port B to port C with price of P_{BC}. Here, the confirmed cargo delivery amount of shipping leg $B - C$ S_{BC} should be no larger than the realized cargo delivery requirement for shipping leg $B - C$ and the remaining available capacity $K - S_{AC}$.

Proposition 3.1.1. *The optimal amount of cargoes for shipping leg $B - C$ has the following structure:*

$$
S_{BC}^{*} = \begin{cases} X_{BC}, & 0 \le X_{BC} < K - S_{AC} \\ K - S_{AC}, & K - S_{AC} \le X_{BC} \end{cases}
$$

Obviously, when the realized cargo delivery requirement for shipping leg $B - C$ is less than the remaining available capacity, the optimal amount of cargoes will be the realized cargo delivery requirement itself. Otherwise, the remaining available capacity will be the optimal amount.

The optimal revenue function can be easily described:

$$R_B^* = \begin{cases} P_{BC} \cdot X_{BC}, & 0 \leq X_{BC} < K - S_{AC} \\ P_{BC} \cdot K - P_{BC} \cdot S_{AC}, & K - S_{AC} \leq X_{BC} \end{cases}$$

At port A, the decision problem is shown as follows:

$$R_A(S_{AC}, S_{AB}) = P_{AC} \cdot S_{AC} + P_{AB} \cdot S_{AB} + E[R_B^*]$$

$$St.$$

$$\begin{cases} 0 \leq S_{AC} \leq X_{AC}, \\ 0 \leq S_{AB} \leq X_{AB}, \\ 0 \leq S_{AC} + S_{AB} \leq K. \end{cases}$$

At port A, the shipping company confirms that S_{AC} units of cargos will be delivered from port A to port C with price of P_{AC} and S_{AB} units of cargos will be delivered from port A to port B with price of P_{AB}. A expected revenue $E[R_B^*]$ will gain form port B. Here, the confirmed cargo delivery amount of shipping leg $A-C$ $(A-B)$ should be no larger than the realized cargo delivery requirement for shipping leg $A-C$ $(A-B)$. Besides, the sum of the confirmed cargo delivery amount of shipping leg $A-C$ and $A-B$ should not exceed the entire capacity of the ship K.

Proposition 3.1.2. *The optimal amount of cargoes for shipping leg $A-C$ and shipping leg $A-B$ have the structure shown in Table 3.2, where $\Lambda = F_{BC}^{-1}(\frac{P_{BC}-(P_{AC}-P_{AB})}{P_{BC}})$.*

Similar to the newsboy model, $\frac{P_{BC}-(P_{AC}-P_{AB})}{P_{BC}}$ is the critical fractile. In this model, we assume $P_{AC} - P_{AB} < P_{BC}$, which seems like the cost for shipping from port B to port C is $P_{AC} - P_{AB}$. We can easily find that Λ increases with P_{BC} and decreases with the gap between P_{AC} and P_{AB}. Thus, it can be observed that when $\Lambda > K$, which can be transformed to $F_{BC}(K) < (\frac{P_{BC}-(P_{AC}-P_{AB})}{P_{BC}})$, the optimal allocation strategy is satisfying the demand for shipping leg $A-B$ preferentially. That is to say, when the probability of the demand for shipping leg $B-C$ is less than the critical fractile, the shipping company should accept cargoes of shipping leg $A-B$ as much as possible.

Table 3.2.: Benchmark model: optimal allocation

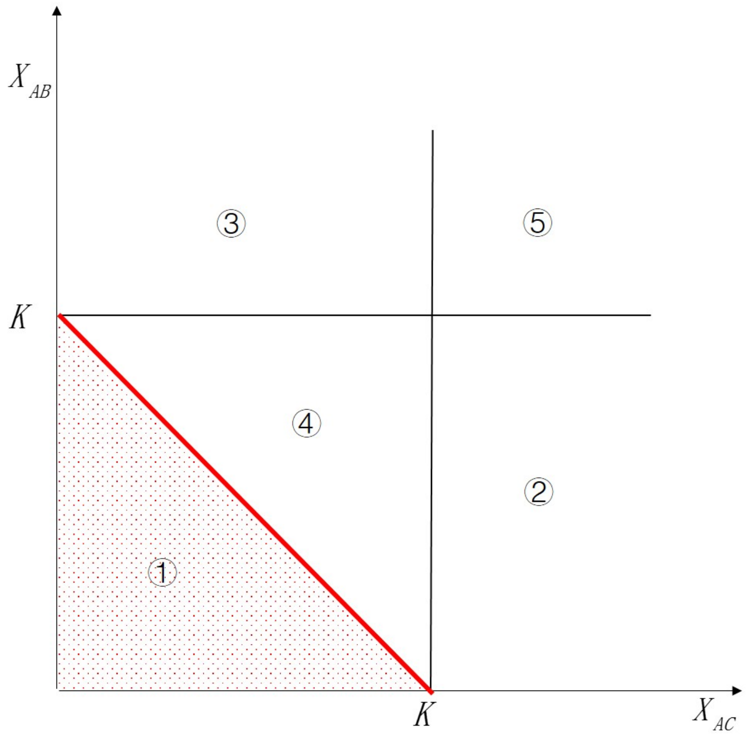
Λ	X_{AC}	X_{AB}	$X_{AC}+X_{AB}$	S^*_{AC}	S^*_{AB}
	$[0,K]$	$[0,K]$	$[0,K]$	X_{AC}	X_{AB}
	(K,∞)	$[0,K]$		$K-X_{AB}$	X_{AB}
(K,∞)	$[0,K]$	(K,∞)		0	K
	$[0,K]$	$[0,K]$	(K,∞)	$K-X_{AB}$	X_{AB}
	(K,∞)	(K,∞)		0	K
	$[0,K]$	$[0,K]$	$[0,K]$	X_{AC}	X_{AB}
	(K,∞)	$[0,\Lambda]$		$K-X_{AB}$	X_{AB}
	(K,∞)	$(\Lambda,K]$		$K-\Lambda$	Λ
	$[0,K-\Lambda]$	(K,∞)		X_{AC}	$K-X_{AC}$
$(0,K]$	$(K-\Lambda,K]$	(K,∞)		$K-\Lambda$	Λ
	$[K-\Lambda,K]$	$[0,\Lambda]$	(K,∞)	$K-X_{AB}$	X_{AB}
	$(K-\Lambda,K]$	$(\Lambda,K]$	(K,∞)	$K-\Lambda$	Λ
	$[0,K-\Lambda]$	$[\Lambda,K]$	(K,∞)	X_{AC}	$K-X_{AC}$
	(K,∞)	(K,∞)		$K-\Lambda$	Λ

If $0<\Lambda\le K$, which means the probability of the demand for shipping leg $B-C$ is no less than the critical fractile, the situation becomes more complicated. When the arrival demands of shipping leg $A-C$ and shipping leg $A-B$ are both relatively low and their sum is lower than the whole ship capacity, the optimal delivery quantities of cargoes for shipping leg $A-C$ and shipping leg $A-B$ are the quantities of arrival cargoes only. When the arrival demands of shipping leg $A-C$ and shipping leg $A-B$ are both relatively lower than the whole ship capacity but the sum of them is higher than the whole ship capacity, the revenue function reaches the peak point at ($S_{AC} = K-\Lambda, S_{AB} = \Lambda$). When the arrival demand of shipping leg $A-C$ is larger than the capacity of ship K, the revenue function reaches the peak point at $S_{AB} = \Lambda$. In the case that the arrival demand of shipping leg $A-B$ is less than Λ, the optimal delivery quantity for shipping leg $A-B$ should be equal to the arrival demand. If the arrival demand of shipping leg $A-B$ is higher than Λ, the optimal delivery quantity for shipping leg $A-B$ is just Λ. When the arrival demand of shipping leg $A-C$ is larger than the capacity of ship K, the revenue function reaches the peak point at $S_{AC} = K-\Lambda$. In the case that the arrival demand of shipping leg $A-C$ is less than $K-\Lambda$, the optimal delivery quantity for shipping leg $A-C$ should be the arrival demand. If the arrival demand of shipping leg $A-C$ is higher than $K-\Lambda$, the optimal delivery quantity for shipping leg $A-B$ is $K-\Lambda$. Figure 3.1 shows the shipping company's optimal capacity allocation in two cases.

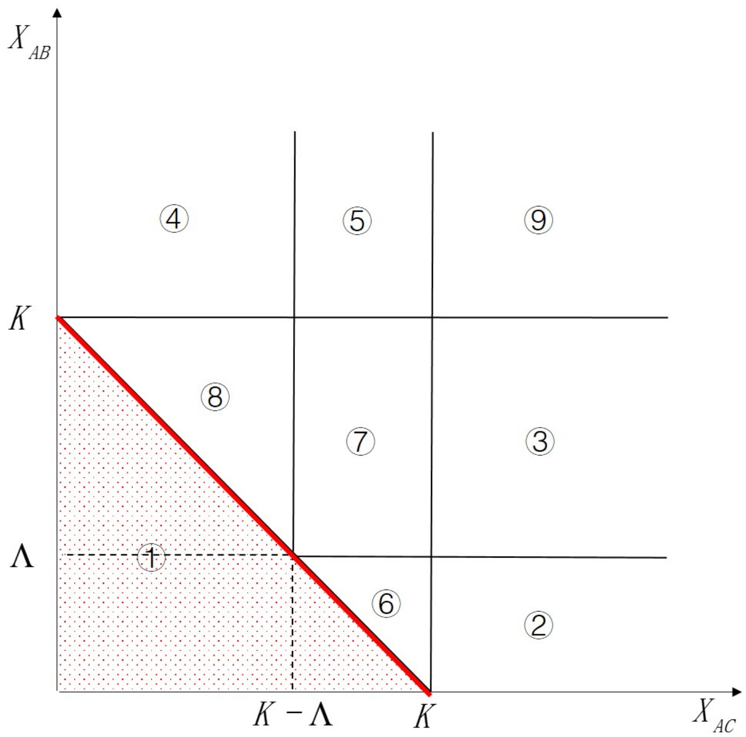
We assume that the cumulative distribution functions of arrival cargoes for shipping leg $A-C$ and arrival cargoes for shipping leg $A-B$ follow $F_{AC}(x_{AC})$ and $F_{AB}(x_{AB})$ in the long run. Thus, in the case $\Lambda > K$, when the shipping company completes one shipping service, they can get the expected revenue in the long run as follows:

$$
\begin{aligned}
E[R_{BLA}] = &\int_0^K \int_0^{K-x_{AC}} \Bigg(\int_{K-x_{AC}}^{\infty} P_{BC}(K \\
&- x_{AC})\, dF_{BC}(x_{BC}) + \int_0^{K-x_{AC}} P_{BC}x_{BC}\, dF_{BC}(x_{BC}) + P_{AB}x_{AB} + \\
&P_{AC}x_{AC}\Bigg)\, dF_{AB}(x_{AB}) dF_{AC}(x_{AC}) + \int_K^{\infty} \int_0^K \Bigg(P_{AC}(K \\
&- x_{AB}) + P_{AB}x_{AB} + \int_{x_{AB}}^{\infty} P_{BC}x_{AB}\, dF_{BC}(x_{BC}) + \int_0^{x_{AB}} P_{BC}x_{BC} \\
&dF_{BC}(x_{BC})\Bigg)\, dF_{AB}(x_{AB}) dF_{AC}(x_{AC}) + \int_0^K \int_{K-x_{AC}}^{K} \Bigg(P_{AC}(K - x_{AB}) + \\
&P_{AB}x_{AB} + \int_{x_{AB}}^{\infty} P_{BC}x_{AB}\, dF_{BC}(x_{BC}) \\
&+ \int_0^{x_{AB}} P_{BC}x_{BC}\, dF_{BC}(x_{BC})\Bigg)\, dF_{AB}(x_{AB}) dF_{AC}(x_{AC}) + \int_K^{\infty} \Bigg(KP_{AB} + \\
&\int_K^{\infty} KP_{BC}\, dF_{BC}(x_{BC}) + \int_0^K P_{BC}x_{BC}\, dF_{BC}(x_{BC})\Bigg)\, dF_{AB}(x_{AB})
\end{aligned}
\tag{3.1}
$$

When $0 < \Lambda \leq K$, in the long run, the shipping company completing one shipping



(a) $\Lambda > K$



(b) $0 < \Lambda \leq K$

Figure 3.1.: Benchmark model: optimal capacity allocation

service can get the expected revenue:

$$E[R_{BLE}] = \int_{K-\Lambda}^{K} \int_{K-x_{AC}}^{\Lambda} \Big(P_{AC}(K - x_{AB}) + P_{AB}x_{AB} + \int_{x_{AB}}^{\infty} P_{BC}x_{AB}\, dF_{BC}(x_{BC}) + \int_{0}^{x_{AB}} P_{BC}x_{BC}\, dF_{BC}(x_{BC}) \Big) dF_{AB}(x_{AB}) dF_{AC}(x_{AC}) + \int_{K}^{\infty} \int_{0}^{\Lambda} \Big(P_{AC}(K - x_{AB}) + P_{AB}x_{AB} + \int_{x_{AB}}^{\infty} P_{BC}x_{AB}\, dF_{BC}(x_{BC}) + \int_{0}^{x_{AB}} P_{BC}x_{BC}\, dF_{BC}(x_{BC}) \Big) dF_{AB}(x_{AB}) dF_{AC}(x_{AC}) + \int_{0}^{K-\Lambda} \int_{K-x_{AC}}^{\infty} \Big(P_{AB}(K - x_{AC}) + P_{AC}x_{AC} + \int_{K-x_{AC}}^{\infty} P_{BC}(K - x_{AC})\, dF_{BC}(x_{BC}) + \int_{0}^{K-x_{AC}} P_{BC}x_{BC}\, dF_{BC}(x_{BC}) \Big) dF_{AB}(x_{AB}) dF_{AC}(x_{AC}) + \int_{K-\Lambda}^{\infty} \int_{\Lambda}^{\infty} \Big(\Lambda P_{AB} + P_{AC}(K - \Lambda) + \int_{\Lambda}^{\infty} (\Lambda P_{BC})\, dF_{BC}(x_{BC}) + \int_{0}^{\Lambda} P_{BC}x_{BC}\, dF_{BC}(x_{BC}) \Big) dF_{AB}(x_{AB}) dF_{AC}(x_{AC}) + \int_{0}^{K} \int_{0}^{K-x_{AC}} \Big(P_{AB}x_{AB} + P_{AC}x_{AC} + \int_{K-x_{AC}}^{\infty} P_{BC}(K - x_{AC})\, dF_{BC}(x_{BC}) + \int_{0}^{K-x_{AC}} P_{BC}x_{BC}\, dF_{BC}(x_{BC}) \Big) dF_{AB}(x_{AB}) dF_{AC}(x_{AC}) \tag{3.2}$$

3.2. Strategically Throwing Away Model

Similar to the benchmark model, a shipping company operates a cargo ship with limited capacity K in a shipping line which starts from port A, passes port B and finally reaches port C. At port A, after the arrivals of all cargoes of shipping leg $A - C$ and shipping leg $A - B$, the shipping company needs to decide how many cargoes should be accepted for shipping leg $A - C$ and shipping leg $A - B$, respectively. At this time point, the demand for capacity of shipping leg $B - C$ is still unobserved, so the capacity allocation of shipping leg $A - C$ and shipping leg $A - B$ needs to consider the uncertainty regarding shipping leg $B - C$. At port B, after the arrival of cargoes from shipping leg $B - C$, the shipping company needs to decide whether the cargoes of shipping leg $A - C$ need to be thrown away or not and if needed, how many cargoes should be thrown away.

At port B, the shipping company can obtain revenue from accepting cargoes of shipping leg $B - C$ and needs to pay a penalty cost for the thrown away cargoes of shipping leg

$A - C$ if the throwing away behavior exists. The revenue of port B can be expressed as follows:

$$
\begin{aligned}
&R_B = P_{BC} \cdot S_{BC} - C_p \cdot T_{AC} \\
&St. \\
&\begin{cases}
0 \le S_{BC} \le X_{BC}, \\
0 \le S_{BC} \le K - S_{AC} + T_{AC}, \\
0 \le T_{AC} \le S_{AC}.
\end{cases}
\end{aligned}
$$

At port B, the shipping company decides to throw away T_{AC} units of cargoes with unit penalty cost C_p and confirms that S_{BC} units of cargos will be delivered from port B to port C with price of P_{BC}. Here, the amount of thrown away cargoes T_{AC} should be no larger than the confirmed cargo delivery amount of shipping leg $A - C$. The confirmed cargo delivery amount of shipping leg $B - C$ should be no larger than the realized cargo delivery requirement for shipping leg $B - C$ and the remaining available capacity $K - S_{AC} + T_{AC}$.

Obviously, when the shipping price of shipping leg $B - C$ is less than the unit penalty cost: $P_{BC} < C_p$, the shipping company will never throw away cargoes from shipping leg $A - C$. This problem then degenerates to the benchmark model.

Proposition 3.2.1. *When the shipping price of shipping leg $B - C$ is no less than the unit penalty cost: $P_{BC} \ge C_p$, the optimal amount of cargoes thrown away at port B has the following structure:*

$$
T_{AC}^{*} = \begin{cases}
0, & 0 \le X_{BC} < K - S_{AC} \\
S_{AC} + X_{BC} - K, & K - S_{AC} \le X_{BC} < K \\
S_{AC}, & K \le X_{BC}
\end{cases}
$$

Obviously, from the structure of the optimal amount of cargoes thrown away at port B, we can see that: (1) If the arrival cargo demand for shipping leg $B - C$ is less than the available capacity before throwing away any cargoes of shipping leg $A - C$, the shipping company has no need to throw away cargoes. Thus, the optimal amount of cargoes thrown is 0; (2) If the arrival cargo demand for shipping leg $B - C$ is larger than the available capacity before throwing away any cargoes from shipping leg $A - C$ but less than the capacity of the ship, the shipping company needs to satisfy the demand for shipping leg $B - C$ first. The optimal amount of cargoes thrown away should be the gap between the sum of accepted cargoes from shipping leg $A - C$ and the arrival cargoes from shipping leg $B - C$ and the whole capacity of the ship. (3) If the arrival cargo demand for shipping leg $B - C$ is larger than the entire capacity of the ship, the shipping company should throw away all accepted cargoes from shipping leg $A - C$. Figure 3.2 shows the optimal amount that should be thrown away at the intermediate port with the change of arrival demand.

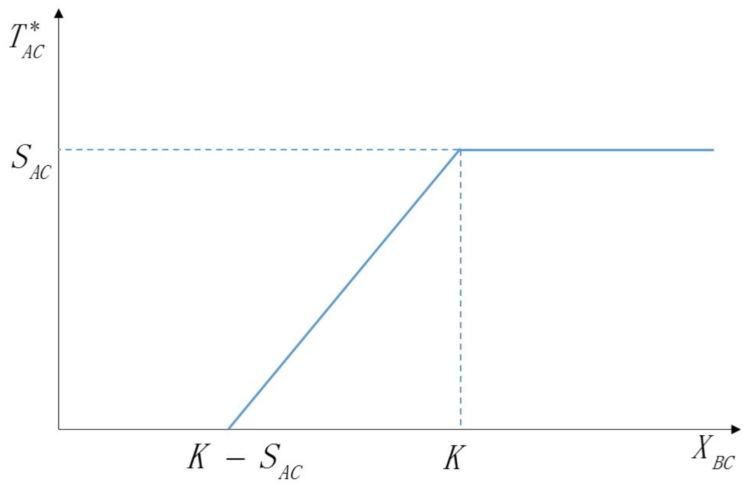


Figure 3.2.: The optimal throwing away cargoes

The optimal revenues also can be easily expressed:

$$R_B^* = \begin{cases} P_{BC} \cdot X_{BC}, & 0 \leq X_{BC} < K - S_{AC} \\ P_{BC} \cdot X_{BC} - C_p \cdot (S_{AC} + X_{BC} - K), & K - S_{AC} \leq X_{BC} < K \\ P_{BC} \cdot K - C_p \cdot S_{AC}, & K \leq X_{BC} \end{cases}$$

Figure 3.3 displays the revenue that the shipping company can gain under the optimal capacity allocation strategy for throwing away cargoes.

At port A, the demands for the capacity of shipping leg $A - C$ and shipping leg $A - B$ are known, but the demand for the capacity of shipping leg $B - C$ is unknown. Along with considering the capacity allocation problem between shipping leg $A - C$ and shipping leg $A - B$, the uncertainty of the demands for the capacity of shipping leg $B - C$ should be considered too. The entire revenue of this shipping line includes the revenue from

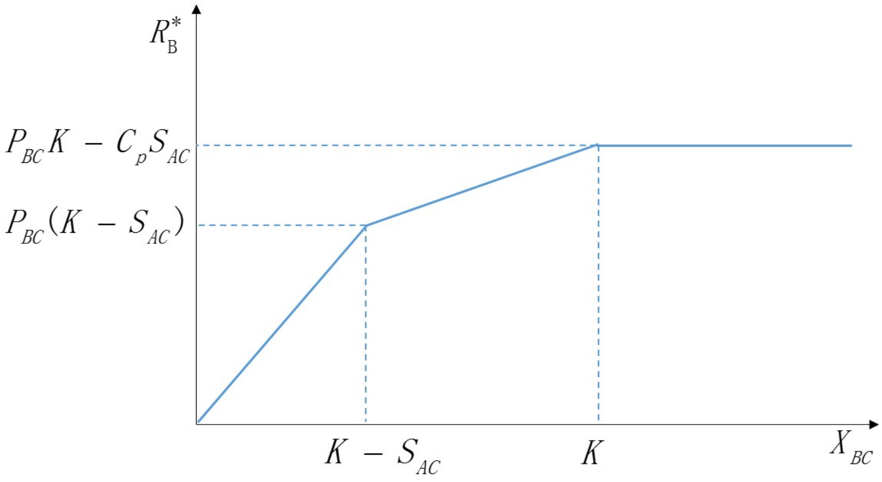


Figure 3.3.: The optimal revenue

accepting the cargoes of shipping leg $A - C$ and shipping leg $A - B$ and the revenue at port B. The revenue of port A can be expressed as follows:

$$R_A(S_{AC}, S_{AB}) = P_{AC} \cdot S_{AC} + P_{AB} \cdot S_{AB} + E[R_B^*]$$

$$St.$$

$$\begin{cases} 0 \leq S_{AC} \leq X_{AC}, \\ 0 \leq S_{AB} \leq X_{AB}, \\ 0 \leq S_{AC} + S_{AB} \leq K. \end{cases}$$

At port A, the shipping company confirms that S_{AC} units of cargos will be delivered from port A to port C with price of P_{AC} and S_{AB} units of cargos will be delivered from port A to port B with price of P_{AB}. An expected revenue $E[R_B^*]$ will gain form port B. Here, the confirmed cargo delivery amount of shipping leg $A-C$ $(A-B)$ should be no larger than the realized cargo delivery requirement for shipping leg $A-C$ $(A-B)$. Besides, the sum of the confirmed cargo delivery amount of shipping leg $A - C$ and $A - B$ should not exceed the entire capacity of the ship K.

Proposition 3.2.2. *(1)When $P_{AC} - P_{AB} < C_p < P_{BC}$, the optimal capacities allocated to shipping leg $A-C$ and shipping leg $A-B$ have a similar structure to the benchmark model shown in Table 3.3, where $\Theta = F_{BC}^{-1}(\frac{C_p-(P_{AC}-P_{AB})}{C_p})$; (2)When $0 < C_p \leq P_{AC} - P_{AB}$, the optimal capacity allocated to shipping leg $A-C$ and shipping leg $A-B$ have the structure shown in Table 3.4.*

Table 3.3.: Strategically throwing away model: $P_{AC} - P_{AB} < C_p < P_{BC}$, optimal allocation

Θ	X_{AC}	X_{AB}	$X_{AC} + X_{AB}$	S^*_{AC}	S^*_{AB}
(K, ∞)	$[0, K]$	$[0, K]$	$[0, K]$	X_{AC}	X_{AB}
	(K, ∞)	$[0, K]$		$K - X_{AB}$	X_{AB}
	$[0, K]$	(K, ∞)		0	K
	$[0, K]$	$[0, K]$	(K, ∞)	$K - X_{AB}$	X_{AB}
	(K, ∞)	(K, ∞)		0	K
$(0, K]$	$[0, K]$	$[0, K]$	$[0, K]$	X_{AC}	X_{AB}
	(K, ∞)	$[0, \Theta]$		$K - X_{AB}$	X_{AB}
	(K, ∞)	$(\Theta, K]$		$K - \Theta$	Θ
	$[0, K - \Theta]$	(K, ∞)		X_{AC}	$K - X_{AC}$
	$(K - \Theta, K]$	(K, ∞)		$K - \Theta$	Θ
	$[K - \Theta, K]$	$[0, \Theta]$	(K, ∞)	$K - X_{AB}$	X_{AB}
	$(K - \Theta, K]$	$(\Theta, K]$	(K, ∞)	$K - \Theta$	Θ
	$[0, K - \Theta]$	$[\Theta, K]$	(K, ∞)	X_{AC}	$K - X_{AC}$
	(K, ∞)	(K, ∞)		$K - \Theta$	Θ

Table 3.4.: Strategically throwing away model: $0 < C_p \leq P_{AC} - P_{AB}$, optimal allocation

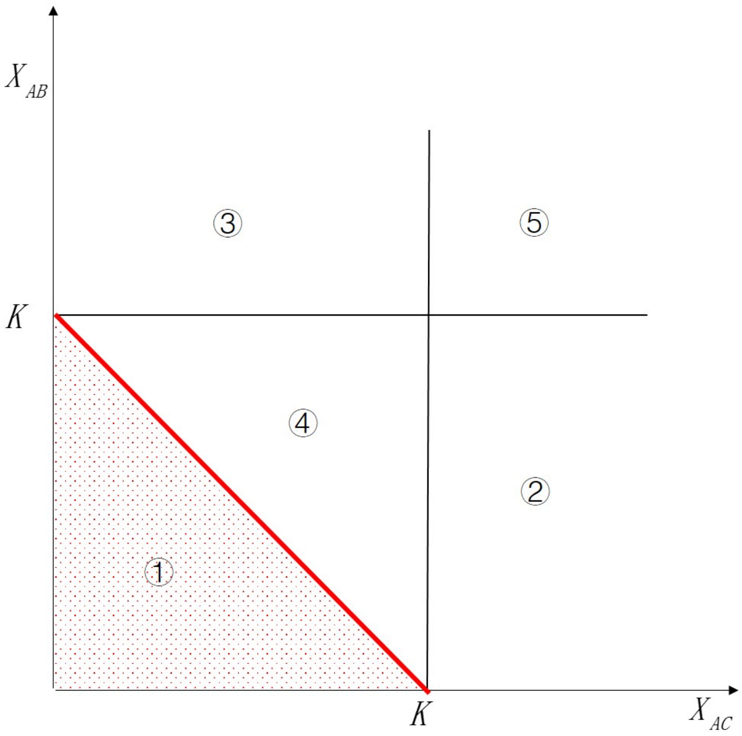
X_{AC}	X_{AB}	$X_{AC} + X_{AB}$	S^*_{AC}	S^*_{AB}
$[0, K]$	$[0, K]$	$[0, K]$	X_{AC}	X_{AB}
(K, ∞)	$[0, K]$		K	0
$[0, K]$	(K, ∞)		X_{AC}	$K - X_{AC}$
$[0, K]$	$[0, K]$	(K, ∞)	X_{AC}	$K - X_{AC}$
(K, ∞)	(K, ∞)		K	0

We can easily prove $\Lambda > \Theta$. Based on Table 3.3, if strategically throwing away cargoes is allowed, the optimal quantities of cargoes that should be accepted for shipping leg $A - C$ increases and shipping leg $A - B$ decreases in some special cases. Figure 3.4 shows the shipping company's optimal capacity allocation in two cases.

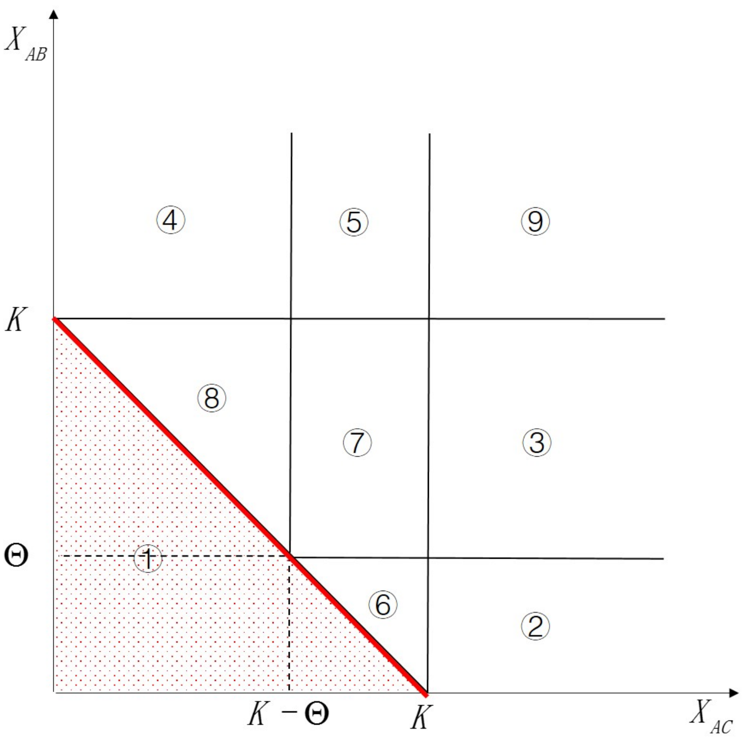
In the long run, cumulative distribution functions of the amount of arrival cargoes for shipping leg $A - C$ and the amount of arrival cargoes for shipping leg $A - B$ follow $F_{AC}(x_{AC})$ and $F_{AB}(x_{AB})$. Thus, in the case of $P_{AC} - P_{AB} < C_p < P_{BC}$ and $\Theta > K$, the shipping company completing one shipping service can obtain the expected revenue in the long run as follows:

$$
\begin{aligned}
E[R_{SLA}] = &\int_K^\infty \int_0^K \Big(P_{AB}x_{AB} + P_{AC}(K - x_{AB}) + \int_K^\infty (KP_{BC} - C_p(K - x_{AB})) \\
&dF_{BC}(x_{BC}) + \int_{x_{AB}}^K (P_{BC}x_{BC} - C_p(x_{BC} - x_{AB}))\, dF_{BC}(x_{BC}) + \int_0^{x_{AB}} \\
&P_{BC}x_{BC}\, dF_{BC}(x_{BC}) \Big)\, dF_{AB}(x_{AB})dx_{AC} + \int_0^K \int_{K-x_{AC}}^K \Big(P_{AB}x_{AB} + P_{AC}(K \\
&- x_{AB}) + \int_K^\infty (KP_{BC} - C_p(K - x_{AB}))\, dF_{BC}(x_{BC}) + \int_{x_{AB}}^K (P_{BC}x_{BC} - \\
&C_p(x_{BC} - x_{AB}))\, dF_{BC}(x_{BC}) \\
&+ \int_0^{x_{AB}} P_{BC}x_{BC}\, dF_{BC}(x_{BC}) \Big)\, dF_{AB}(x_{AB})dx_{AC} + \int_0^K \int_0^{K-x_{AC}} \Big(P_{AB}x_{AB} \\
&+ P_{AC}x_{AC} + \int_K^\infty (KP_{BC} - C_p x_{AC})\, dF_{BC}(x_{BC}) \\
&+ \int_{K-x_{AC}}^K (P_{BC}x_{BC} - C_p(-K + x_{AC} + x_{BC}))\, dF_{BC}(x_{BC}) + \int_0^{K-x_{AC}} \\
&P_{BC}x_{BC}\, dF_{BC}(x_{BC}) \Big)\, dF_{AB}(x_{AB})dF_{AC}(x_{AC}) + \int_0^\infty \int_K^\infty \Big(KP_{AB} + \int_K^\infty \\
&KP_{BC}\, dF_{BC}(x_{BC}) + \int_0^K P_{BC}x_{BC}\, dF_{BC}(x_{BC}) \Big)\, dF_{AB}(x_{AB})dF_{AC}(x_{AC})
\end{aligned}
\tag{3.3}
$$

In the case of $P_{AC} - P_{AB} < C_p < P_{BC}$ and $0 < \Theta \leq K$, in the long run, the shipping



(a) $\Theta > K$



(b) $0 < \Theta \leq K$

Figure 3.4.: Strategically throwing away model: optimal capacity allocation

company completing one shipping service can obtain the expected revenue as follows:

$$
\begin{aligned}
E[R_{SLE}] = \int_K^{\infty} \int_0^{\Theta} \Bigg(& P_{AB}x_{AB} + P_{AC}(K - x_{AB}) + \int_K^{\infty} (KP_{BC} - C_p(K - x_{AB})) \\
& dF_{BC}(x_{BC}) + \int_{x_{AB}}^{K} (P_{BC}x_{BC} - C_p(x_{BC} - x_{AB}))\, dF_{BC}(x_{BC}) + \int_0^{x_{AB}} \\
& P_{BC}x_{BC}\, dF_{BC}(x_{BC}) \Bigg)\, dF_{AB}(x_{AB}) dF_{AC}(x_{AC}) + \int_{K-\Theta}^{K} \int_{K-x_{AC}}^{\Theta} \Bigg(P_{AB}x_{AB} + \\
& P_{AC}(K - x_{AB}) + \int_K^{\infty} (KP_{BC} - C_p(K - x_{AB}))\, dF_{BC}(x_{BC}) \\
& + \int_{x_{AB}}^{K} (P_{BC}x_{BC} - C_p(x_{BC} - x_{AB}))\, dF_{BC}(x_{BC}) + \int_0^{x_{AB}} P_{BC}x_{BC} \\
& dF_{BC}(x_{BC}) \Bigg)\, dF_{AB}(x_{AB}) dF_{AC}(x_{AC}) + \int_0^{K-\Theta} \int_{K-x_{AC}}^{\infty} \Bigg(P_{AB}(K - x_{AC}) + \\
& P_{AC}x_{AC} + \int_K^{\infty} (KP_{BC} - C_p x_{AC})\, dF_{BC}(x_{BC}) \\
& + \int_{K-x_{AC}}^{K} (P_{BC}x_{BC} - C_p(-K + x_{AC} + x_{BC}))\, dF_{BC}(x_{BC}) + \int_0^{K-x_{AC}} \\
& P_{BC}x_{BC}\, dF_{BC}(x_{BC}) \Bigg)\, dF_{AB}(x_{AB}) dF_{AC}(x_{AC}) + \int_{K-\Theta}^{\infty} \int_{\Theta}^{\infty} \Bigg(\Theta P_{AB} \\
& + \int_{\Theta}^{K} (P_{BC}x_{BC} - C_p(x_{BC} - \Theta))\, dF_{BC}(x_{BC}) \\
& + \int_K^{\infty} (KP_{BC} - C_p(K - \Theta))\, dF_{BC}(x_{BC}) + P_{AC}(K - \Theta) + \int_0^{\Theta} P_{BC}x_{BC} \\
& dF_{BC}(x_{BC}) \Bigg)\, dF_{AB}(x_{AB}) dF_{AC}(x_{AC}) + \int_0^{K} \int_0^{K-x_{AC}} \Bigg(P_{AB}x_{AB} + P_{AC}x_{AC} + \\
& \int_K^{\infty} (KP_{BC} - C_p x_{AC})\, dF_{BC}(x_{BC}) + \int_{K-x_{AC}}^{K} (P_{BC}x_{BC} - C_p(-K + x_{AC} + \\
& x_{BC}))\, dF_{BC}(x_{BC}) + \int_0^{K-x_{AC}} P_{BC}x_{BC}\, dF_{BC}(x_{BC}) \Bigg)\, dF_{AB}(x_{AB}) dF_{AC}(x_{AC})
\end{aligned}
\tag{3.4}
$$

In the case of $0 < C_p \leq P_{AC} - P_{AB}$, the shipping company completing one shipping service can get the expected revenue in the long run:

$$E[R_{SCL}] = \int_0^K \int_0^{K-x_{AC}} \Big(\int_K^\infty (KP_{BC} - C_p x_{AC})\, dF_{BC}(x_{BC}) + \int_{K-x_{AC}}^K (P_{BC}x_{BC} - C_p(-K + x_{AC} + x_{BC}))\, dF_{BC}(x_{BC}) + \int_0^{K-x_{AC}} P_{BC}x_{BC}\, dF_{BC}(x_{BC}) + P_{AB}x_{AB} + P_{AC}x_{AC} \Big) dF_{AB}(x_{AB}) dF_{AC}(x_{AC}) + \int_0^K \int_0^\infty \Big(\int_K^\infty (KP_{BC} - C_p x_{AC})\, dF_{BC}(x_{BC}) + \int_{K-x_{AC}}^K (P_{BC}x_{BC} - C_p(-K + x_{AC} + x_{BC}))\, dF_{BC}(x_{BC}) + \int_0^{K-x_{AC}} P_{BC}x_{BC}\, dF_{BC}(x_{BC}) + P_{AB}(K - x_{AC}) + P_{AC}x_{AC} \Big) dF_{AB}(x_{AB}) dF_{AC}(x_{AC}) + \int_K^\infty \int_0^\infty \Big(\int_K^\infty (KP_{BC} - C_p K)\, dF_{BC}(x_{BC}) + \int_0^K (P_{BC}x_{BC} - C_p x_{BC})\, dF_{BC}(x_{BC}) + KP_{AC} \Big) dF_{AB}(x_{AB}) dF_{AC}(x_{AC}) \tag{3.5}$$

Through comparing the expected revenues in different cases, the cases in which the shipping company should allow throwing away of cargoes can be determined.

Proposition 3.2.3. *Only in the following cases, allowing throwing away cargoes can achieve higher expected revenues in the long run:*

When $P_{AC} - P_{AB} < C_p < P_{BC}$,

(1) $F_{BC}(K) < 1 - (P_{AC} - P_{AB}/C_p)$ *and* $E[R_{SLA}] > E[R_{BLA}]$*;*

(2) $1 - (P_{AC} - P_{AB}/C_p) \leq F_{BC}(K) < 1 - (P_{AC} - P_{AB}/P_{BC})$ *and* $E[R_{SLE}] > E[R_{BLA}]$*;*

(3) $F_{BC}(K) \geq 1 - (P_{AC} - P_{AB}/P_{BC})$ *and* $E[R_{SLE}] > E[R_{BLE}]$.

When $0 < C_p \leq P_{AC} - P_{AB}$,

(4) $F_{BC}(K) < 1 - (P_{AC} - P_{AB}/P_{BC})$ *and* $E[R_{SCL}] > E[R_{BLA}]$*;*

(5) $F_{BC}(K) \geq 1 - (P_{AC} - P_{AB}/P_{BC})$ *and* $E[R_{SCL}] > E[R_{BLE}]$*;*

When the unit penalty cost is larger than the gap between the price of shipping leg $A - C$ and the price of shipping leg $A - B$ and less than the price of shipping leg $A-B$, the shipping company chooses the optimal capacity allocation strategy depending on the relationship of K, Λ and Θ. In the benchmark model, when $\Lambda > K$, which is $F_{BC}(K) < 1 - (P_{AC} - P_{AB}/P_{BC})$, the shipping company can gain the expected revenue $E[R_{BLA}]$ in the long run and when $0 < \Lambda \leq K$, which is $F_{BC}(K) \leq 1 - (P_{AC} - P_{AB}/P_{BC})$, the shipping company can gain the expected revenue $E[R_{BLE}]$

in the long run. In the strategically throwing away model, when $\Theta > K$, which is $F_{BC}(K) < 1-(P_{AC}-P_{AB}/C_p)$, the shipping company can gain the expected revenue $E[R_{SLA}]$ in the long run and when $0 < \Theta \leq K$, which is $F_{BC}(K) \geq 1-(P_{AC}-P_{AB}/C_p)$, the shipping company can gain the expected revenue $E[R_{SLE}]$ in the long run. Because $1-(P_{AC}-P_{AB}/C_p) < 1-(P_{AC}-P_{AB}/P_{BC})$, the first three cases describe the three conditions that allowing throwing away cargoes is better in the long run. However, when the unit penalty cost is no larger than the gap between the price of shipping leg $A-C$ and the price of shipping leg $A-B$, in the strategically throwing away model, the shipping company chooses a different capacity allocation strategy and gain the expected revenue $E[R_{SCL}]$ in the long run. The last two cases illustrate the two conditions that allowing throwing away cargoes is better in the long run.

Chapter 4.

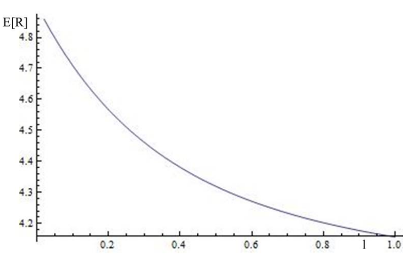
Numerical Studies

According to assumptions, the arrival cargoes follow the distributions $F_{AC}(x_{AC})$, $F_{AB}(x_{AB})$ and $F_{BC}(x_{BC})$. Collection of the arrival data to estimate the distribution of arrival cargoes is difficult. In this chapter, a common demand distribution, exponential distribution, was applied in two models to derive the shipping company's best decisions. The assumed cumulative distribution functions of the cargo arrivals for each shipping leg $F_{AC}(x_{AC})$, $F_{AB}(x_{AB})$ and $F_{BC}(x_{BC})$ follow exponential distribution, that is, $F_{AC}(x_{AC}) = 1 - e^{-mx_{AC}}$, $F_{AB}(x_{AB}) = 1 - e^{-lx_{AB}}$ and $F_{BC}(x_{BC}) = 1 - e^{-nx_{BC}}$, where $0 < m$, $0 < l$ and $0 < n$. Based on the property of exponential distribution, we know that $E[X_{AC}] = \frac{1}{m}$, $E[X_{AB}] = \frac{1}{l}$ and $E[X_{BC}] = \frac{1}{n}$. In addition: $P_{AC} = 0.8$, $P_{AB} = 0.6$, $P_{BC} = 0.4$ and $K = 5$.

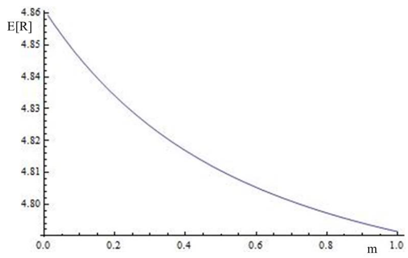
In this chapter, the change of the expected revenue with the exponential parameters was illustrated in each case. In each special case, the expected revenue of the strategically throwing away model was compared with the benchmark model.

4.1. Benchmark Model

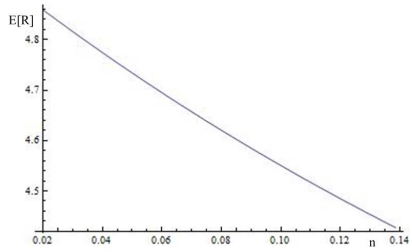
For the benchmark model, $\Lambda = -\frac{\log\left(\frac{P_{AC}-P_{AB}}{P_{BC}}\right)}{n} = -\frac{\log\left(\frac{1}{2}\right)}{n}$. Thus, for $\Lambda > K$, that is $n < -\frac{1}{5}\log\left(\frac{1}{2}\right)$, the expected revenue that the shipping company can obtain in the long run for each shipping service is shown in Figure 4.1. From the figure, we can see that the expected revenue decreases with l, m and n or increases with the expected value of demand for each shipping leg. When $0 < \Lambda \leq K$, that is $n \geq -\frac{1}{5}\log\left(\frac{1}{2}\right)$, the expected revenue that the shipping company can obtain in the long run for each shipping service is shown in Figure 4.2. From the figure, we can see that the expected revenue decreases



(a) m=0.01; n=0.02



(b) l=0.02; n=0.02



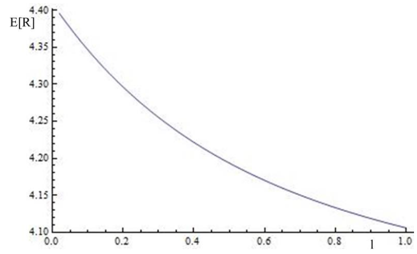
(c) m=0.01; l=0.02

Figure 4.1.: Benchmark model: the expected revenue for $\Lambda > K$

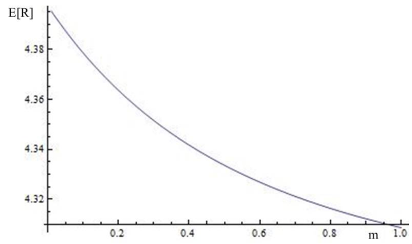
with l, m and n or increases with the expected value of demand for each shipping leg.

4.2. Strategically throwing away model

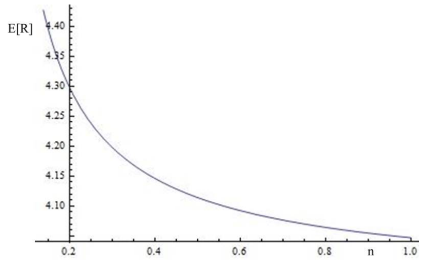
In this section, we need to discuss the location of penalty cost C_p. When $P_{AC} - P_{AB} < C_p < P_{BC}$, that is, $0.2 < C_p < 0.4$ in our settings, the shipping company makes decisions following Table 3.3. We fix $C_p = 0.3$ in this case. Then $\Theta = -\frac{\log\left(\frac{P_{AC}-P_{AB}}{C_p}\right)}{n} = -\frac{\log\left(\frac{2}{3}\right)}{n}$. Next, when $n < -\frac{\log\left(\frac{2}{3}\right)}{5}$, the expected revenue that the shipping company can obtain in the long run completing one shipping service is shown in Figure 4.3. When $n \geq -\frac{\log\left(\frac{2}{3}\right)}{5}$, the situation is similar to the benchmark model, Figure 4.4 shows the expected revenue that the shipping company can obtain with l, m and n. When $0 < C_p \leq P_{AC} - P_{AB}$, using our settings, the shipping company makes decisions following Table 3.4. We fix $C_p = 0.1$. The expected revenue that the shipping company can gain in the long run for each shipping service is shown in Figure 4.5. From Figure 4.3, Figure 4.4 and Figure 4.5, we can see that the expected revenue decreases with l, m and n in three cases.



(a) m=0.01; n=0.15

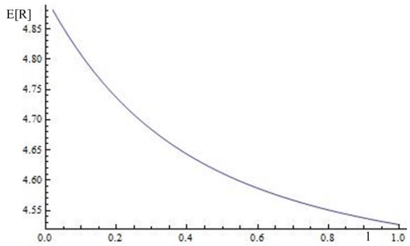


(b) l=0.02; n=0.15

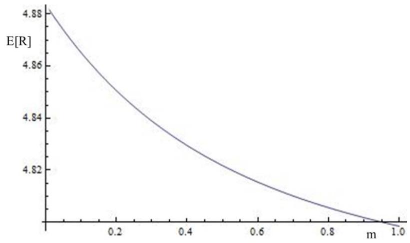


(c) l=0.02; m=0.01

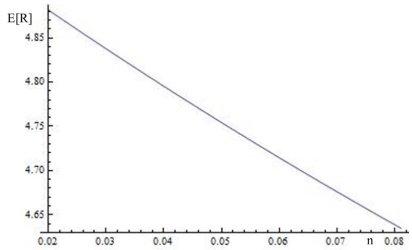
Figure 4.2.: Benchmark model: the expected revenue for $0 < \Lambda \leq K$



(a) n=0.02; m=0.01

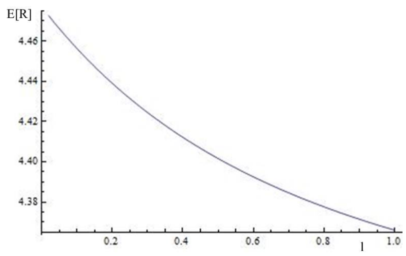


(b) l=0.02; n=0.02

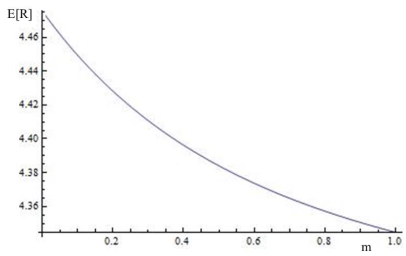


(c) l=0.02; m=0.01

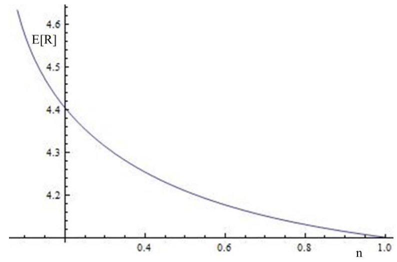
Figure 4.3.: Strategically throwing away model: $C_p = 0.3$, the expected revenue changing with l, m and n for $\Theta > K$



(a) m=0.01; n=0.15

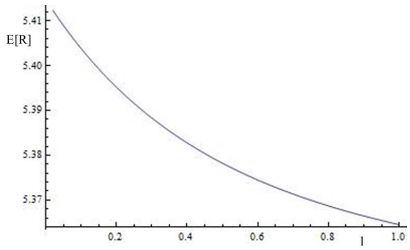


(b) l=0.02; n=0.15

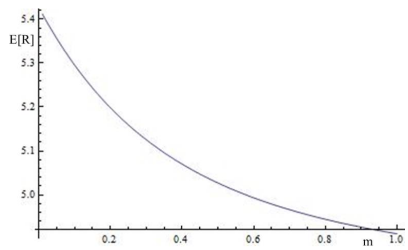


(c) l=0.02; m=0.01

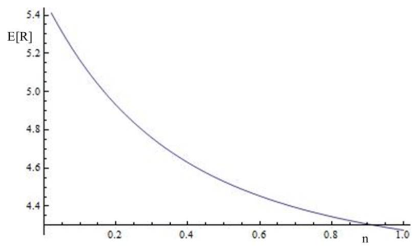
Figure 4.4.: Strategically throwing away model: $C_p = 0.3$, the expected revenue changing with l, m and n for $0 < \Theta \leq K$



(a) m=0.01; n=0.02



(b) l=0.02; n=0.02



(c) l=0.02; m=0.01

Figure 4.5.: Strategically throwing away model: $C_p = 0.1$, the expected revenue changing with l, m and n.

4.3. Comparison

First, the expected revenue of the strategically throwing away model and the benchmark model when $P_{AC} - P_{AB} < C_p < P_{BC}$ ($C_p = 0.3$ in our settings) were compared. Considering that the expected revenue in each model includes different cases when n, the parameter of the demand function of shipping leg $B-C$, locates in different ranges, the expected revenues in each range needs to be compared. Details are displayed in Table 4.1.

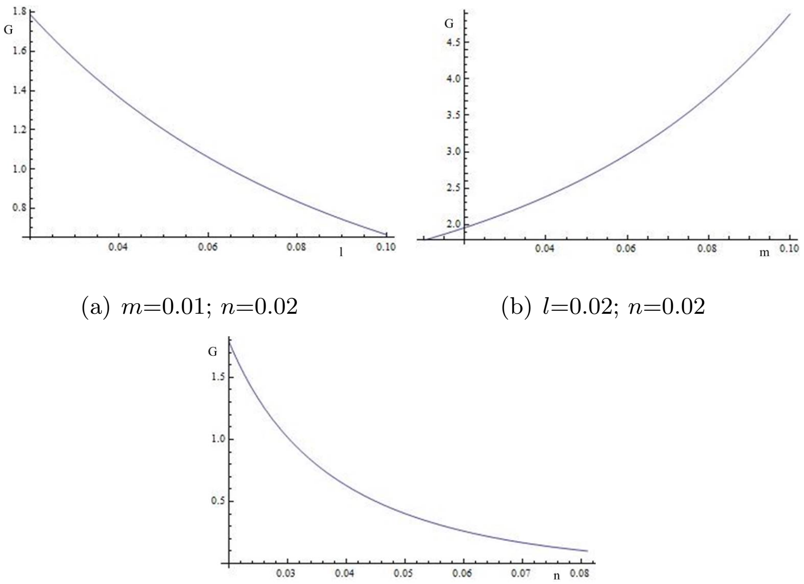
Table 4.1.: The expected revenues

n	$E[R]$	
	Benchmark Model	Strategically throwing away model
$(0, -\frac{1}{5}\log\left(\frac{2}{3}\right))$	$E[R_{BLA}]$	$E[R_{SLA}]$
$[-\frac{1}{5}\log\left(\frac{2}{3}\right), -\frac{1}{5}\log\left(\frac{1}{2}\right))$	$E[R_{BLA}]$	$E[R_{SLE}]$
$[-\frac{1}{5}\log\left(\frac{1}{2}\right), \infty)$	$E[R_{BLE}]$	$E[R_{SLE}]$

When n is located in the range $(0, -\frac{1}{5}\log\left(\frac{2}{3}\right))$, Figure 4.6 shows the gap (G) between $E[R_{SLA}]$ and $E[R_{BLA}]$. From the figures, we can see that the gap is positive. Thus, when the expected value of the demand for shipping leg $B-C$ is high, the shipping company would always choose to allow strategically throwing away cargoes. In addition, the gap decreases with l and n but increases with m. When the expected values of the demands for shipping leg $A-B$ and $B-C$ increase, the gap between two expected revenues increase. The increasing of the expected value of the demand for shipping leg $A-C$ weakens the advantage of allowing strategically throwing away cargoes.

When n is located in the range $[-\frac{1}{5}\log\left(\frac{2}{3}\right), -\frac{1}{5}\log\left(\frac{1}{2}\right))$, Figure 4.7 shows the gap between $E[R_{SLE}]$ and $E[R_{BLA}]$. In this case, the gap is positive. The shipping company should always choose to allow strategically throwing away cargoes. In addition, the gap increases with l and n but decreases with m. Thus, when the expected value of the demands for shipping leg $A-C$ increases, the gap between two expected revenues increases. The increasing of the expected values of the demand for shipping leg $A-B$ and $B-C$ weakens the advantage of allowing strategically throwing away cargoes.

When n is located in the range $[-\frac{1}{5}\log\left(\frac{1}{2}\right), \infty)$, Figure 4.8 shows the gap between $E[R_{SLE}]$ and $E[R_{BLE}]$. From the figures, we can see that the gap is positive. Thus, the shipping company would always allow strategically throwing away cargoes. In addition, the gap increases with l or decreases with the expected value of demands for

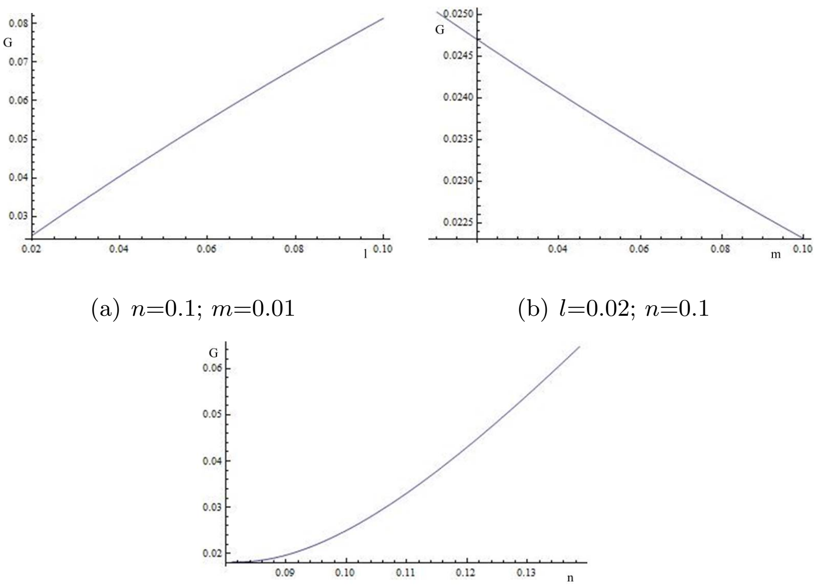


(a) m=0.01; n=0.02

(b) l=0.02; n=0.02

(c) l=0.02; m=0.01

Figure 4.6.: $E[R_{SLA}] - E[R_{BLA}]\,:\;0 < n < -\frac{1}{5}\log\left(\frac{2}{3}\right)$



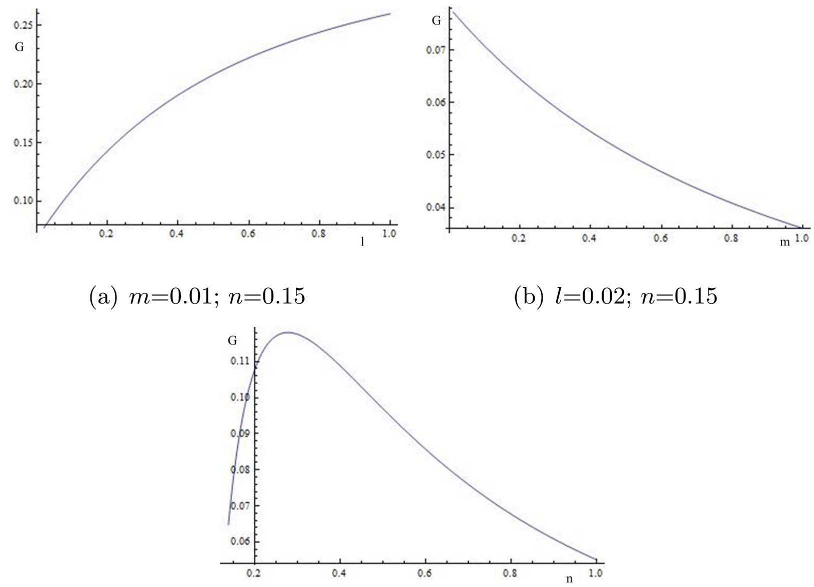
(a) n=0.1; m=0.01

(b) l=0.02; n=0.1

(c) l=0.02; m=0.01

Figure 4.7.: $E[R_{SLE}] - E[R_{BLA}]\,:\;-\frac{1}{5}\log\left(\frac{2}{3}\right) \le n < -\frac{1}{5}\log\left(\frac{1}{2}\right)$

shipping leg $A - B$. The gap decreases with the increasing of m or decreasing of the expected value of demands for shipping leg $A - C$. The gap increases with n first, and then decreases. The increasing of the expected value of demands for shipping leg $B - C$ weakens the advantage of allowing strategically throwing away cargoes first, then strengthens the advantage.



(a) m=0.01; n=0.15

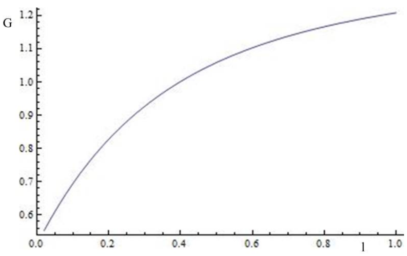
(b) l=0.02; n=0.15

(c) l=0.02; m=0.01

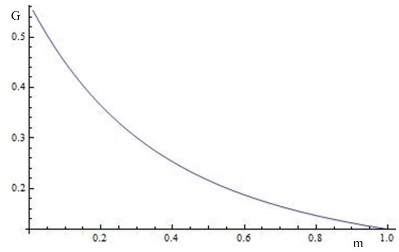
Figure 4.8.: $E[R_{SLE}] - E[R_{BLE}] \,:\, n \geq -\frac{1}{5}\log\left(\frac{1}{2}\right)$

When $0 < C_p \leq P_{AC} - P_{AB}$, the shipping company allocates the capacity as shown in Table 4.1 when "throwing away cargoes" is allowed. When n is located in the range $(0, -\frac{1}{5}\log\left(\frac{1}{2}\right))$, Figure 4.9 shows the gap between $E[R_{SCL}]$ and $E[R_{BLA}]$. From the figures, we can see that the gap is positive. Thus, the shipping company would always allow strategically throwing away cargoes. From the figure, we can see that the gap increases with l and n but decreases with m. It means that the expected revenue decreases with the expected values of demand for shipping leg $A - B$ and $B - C$ but increases with the the expected value of demand for shipping leg $A - C$.

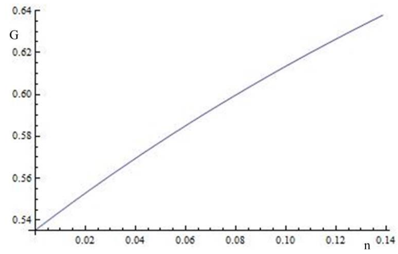
When n is located in the range $[-\frac{1}{5}\log\left(\frac{1}{2}\right), \infty)$, Figure 4.10 shows the gap between $E[R_{SCL}]$ and $E[R_{BLE}]$. From the figures, we can see that the gap is positive. Thus, the shipping company would always allow strategically throwing away cargoes. In addition, the gap increases with l or decreases with the expected value of demands for shipping leg $A - B$. The gap decreases with the increasing of m or decreasing of the expected value of demands for shipping leg $A - C$. The gap increases with n first, and then



(a) m=0.01; n=0.02



(b) l=0.02; n=0.02



(c) l=0.02; m=0.01

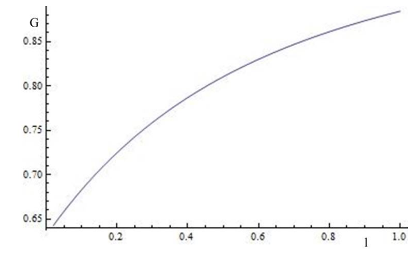
Figure 4.9.: $E[R_{SCL}] - E[R_{BLA}]$: $0 < n < -\frac{1}{5}\log\left(\frac{1}{2}\right)$

decreases.

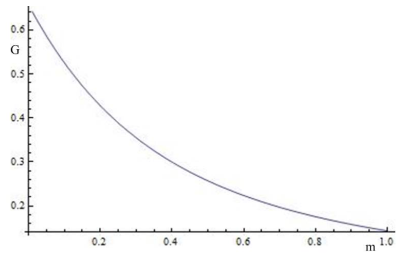
Table 4.2 illustrates the five cases whether the shipping company should allow throwing away cargoes or not. In the C_p part, LA represents $P_{AC} - P_{AB} < C_p < P_{BC}$ and LE represents $0 < C_p \leq P_{AC} - P_{AB}$. From the table, we can see that the shipping company should allow throwing away cargoes in all five cases. In addition, the gap increases with n in two cases, decreases with n in one case and increases with n first then decreases in two cases.

Table 4.2.: The cases that strategically throwing away cargoes should be allowed

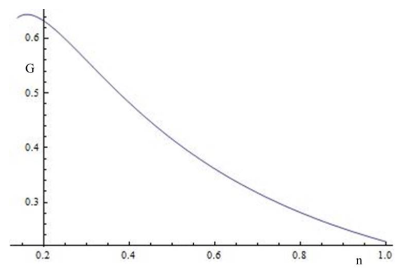
C_p	n	G	$PositiveorNegative$	$IncreaseorDecrease$	$AlloworNot$
LA	$(0, -\frac{1}{5}\log\left(\frac{2}{3}\right))$	$E[R_{SLA}] - E[R_{BLA}]$	P	D	A
	$[-\frac{1}{5}\log\left(\frac{2}{3}\right), -\frac{1}{5}\log\left(\frac{1}{2}\right)$	$E[R_{SLE}] - E[R_{BLA}]$	P	I	A
	$[-\frac{1}{5}\log\left(\frac{1}{2}\right), \infty)$	$E[R_{SLE}] - E[R_{BLE}]$	P		A
LE	$(0, -\frac{1}{5}\log\left(\frac{1}{2}\right))$	$E[R_{SCL}] - E[R_{BLA}]$	P	I	A
	$[-\frac{1}{5}\log\left(\frac{1}{2}\right), \infty)$	$E[R_{SCL}] - E[R_{BLE}]$	P		A



(a) m=0.01; n=0.15



(b) l=0.02; n=0.15



(c) l=0.02; m=0.01

Figure 4.10.: $E[R_{SCL}] - E[R_{BLE}] : \; n \geq -\frac{1}{5}\log\left(\frac{1}{2}\right)$

Chapter 5.

Conclusion and Suggestions for Future Research

For this paper, a shipping company, which supplies liner shipping services in a three-port shipping line, is considered. This company needs to decide how many cargoes should be accepted at the departure port when only the demands of the departure port are known and the demands of the intermediate port is unknown. We find the optimal capacity allocation strategy is decided by a threshold parameter, which increases with the shipping price of shipping leg $B-C$ and decreases with the gap between the shipping price of shipping leg $A-C$ and the shipping price of shipping leg $A-B$. When the threshold parameter is larger than the entire capacity of the ship, the shipping company should first accept cargoes from shipping leg $A-B$ as many as possible, and then accept the cargoes from shipping leg $A-C$. When the threshold parameter is smaller than the capacity, the situation becomes more complicated. The revenue function reaches the peak point when the confirmed cargo delivery amount of shipping leg $A-B$ equals the threshold parameter and the confirmed cargo delivery amount of shipping leg $A-C$ equals the gap between the entire capacity of the ship and the threshold parameter. Thus, if the arrival demand of shipping leg $A-C$ is larger than the threshold parameter and the arrival demand of shipping leg $A-B$ is larger than the gap between the whole capacity of the ship and the threshold parameter, the optimal strategy for the shipping company would be accepting the cargo of shipping leg $A-B$ and $A-C$ with the amount of the threshold value and the gap between the entire capacity of the ship and the threshold parameter, respectively. If the arrival demands of shipping leg $A-C$ is less than the threshold parameter and the arrival demands of shipping leg $A-B$ is less than the gap between the whole capacity of the ship and the threshold parameter, the optimal strategy for the shipping company would be accepting all the arrival demands.

Furthermore, how the shipping company decides the optimal cargoes' amount when strategically throwing away cargoes is allowed was also examined. We find the optimal capacity allocation choices of the shipping company have a similar structure to that of the benchmark model but with a different threshold parameter, in which the shipping price of shipping leg $B - C$ in the threshold parameter of benchmark model is now replaced by the penalty cost of the thrown away cargoes. It has been proven that through strategically throwing away cargoes, the optimal amount of cargoes of shipping leg $A - C$ can be increased in some special cases. Finally, allowing the throwing away of cargoes will always increase the expected revenues. In addition, the increasing of the expected value of demands for shipping leg $B - C$ will not always strengthens the advantage of allowing the throwing away of cargoes.

This paper provides the theoretical proof that "throwing away cargoes" can achieve a higher expected revenue. This paper considers a shipping company supplies shipping service with only one ship, so no ship deployment problem needs to be considered here. The choice of the capacity of the ship is related with the distribution of the demand for the cargo delivery of each shipping leg. However, no any mechanism or details of cooperation between shipping companies in carrier alliance were suggested. For example, it is important to know how shipping companies price the rent when capacity renting is allowed. The penalty has been simplified to a unit fixed cost in this paper. However, the real structure of the penalty cost is much more complicated, for example, uploading and unloading costs, delay costs, storage costs and so on. If we incorporate empty container repositioning in our model, it will increases the penalty cost of the thrown cargoes. Consequently, the optimal capacity allocation strategy under that when throwing away cargoes is allowed will less differ from that when throwing away cargoes is not allowed. The incorporation of empty container repositioning thus weakens the advantages of allowing throwing away cargoes. Additionally, in reality, shipping companies supply liner services either on a shipping circle or on a line. This paper considers the capacity allocation problem for a 3-port shipping line but not for an N-port line. That is because even only considering a 3-port shipping line, there are already 4 decision variables. If we consider the capacity allocation problem on an N-port line, the decision problem will become very complicated because of the huge number of decision variables. The shipping prices for different kinds of cargoes are also different due to their different properties, even those at the same port. even those at the same port. Our model only considers the decision problem in a line and supposes cargoes are shipped at the same shipping price. Future studies could be conducted that also consider the mechanism and details of cooperation, the impact of penalty cost and distinguishing shipping prices of different kinds of cargoes.

Appendix A.

Proposition 3.1.2.

Based on the structure of the optimal revenue of port B, the expected revenue of port B at port A when the cargos' demand for shipping leg $B - C$ is unknown is shown as follow:

$$E[R_B^*] = \int_0^{K-S_{AC}} P_{BC} \cdot x_{BC} \cdot f_{BC}(x_{BC}) dx_{BC} +$$
$$\int_{K-S_{AC}}^{\infty} [P_{BC} \cdot K - P_{BC} \cdot S_{AC}] \cdot f_{BC}(x_{BC}) dx_{BC}$$

The first order derivative of the expected revenue function is:

$$\frac{\partial E[R_B^*]}{\partial S_{AC}} = P_{BC} F_{BC}(K - S_{AC}) - P_{BC} \leq 0$$

The second order derivative of the expected revenue function is:

$$\frac{\partial^2 E[R_B^*]}{\partial^2 S_{AC}} = -f_{BC}(K - S_{AC}) \leq 0$$

Then we go back to the revenue function at port A. The first order partial derivative of the expected revenue with respect to the confirmed cargo delivery amount of shipping leg $A - C$ S_{AC} is:

$$\frac{\partial R_A(S_{AC}, S_{AB})}{\partial S_{AC}}$$
$$= P_{AC} + \frac{\partial E[R_B^*]}{\partial S_{AC}}$$
$$= P_{AC} + P_{BC} F_{BC}(K - S_{AC}) - P_{BC} > 0$$

The second order partial derivative of the expected revenue with respect to S_{AC} is:

$$\frac{\partial^2 R_A(S_{AC}, S_{AB})}{\partial^2 S_{AC}} = -f_{BC}(K - S_{AC}) \leq 0$$

The first order partial derivative of the expectation revenue with respect to the confirmed cargo delivery amount of shipping leg $A-B$ S_{AB} is: $\frac{\partial R_A(S_{AC},S_{AB})}{\partial S_{AB}} = P_{AB} > 0$.

So, if the amount of whole arrival cargos $X_{AC}+X_{AB}$ does not exceed the whole capacity K, the optimal delivery quantities would be the arrival cargos' amount: $S^*_{AC} = X_{AC}$; $S^*_{AB} = X_{AB}$.

If the arrival cargos for both shipping leg $A-C$ and $A-B$ exceed the whole capacity of the ship, the optimal delivery quantities for shipping leg $A-C$ S^*_{AC} and the optimal delivery quantities for shipping leg $A-B$ S^*_{AB} must be achieved on the line $S_{AC}+S_{AB} = K$. That means the optimal solution must satisfy: $S^*_{AB} = K - S^*_{AC}$. So, the maximum revenue can be described as: $R^*_A(S^*_{AC}, S^*_{AB}(S^*_{AC})) = P_{AC} * S^*_{AC} + P_{AB} * (K - S^*_{AC}) + E[R^*_B]$. This problem actually becomes a one decision variable problem. The first order derivative of this problem becomes:

$$\frac{\partial R^*_A(S^*_{AC}, S^*_{AB}(S^*_{AC}))}{\partial S^*_{AC}} = P_{AC} - P_{AB} + P_{BC}F_{BC}(K - S^*_{AC}) - P_{BC}$$

Then, through discussing the location of demands, we can get the optimal strategy of the shipping company. A similar location discussion is shown in the Proof of Proposition 3.2.2.

Appendix B.

Proof of Proposition 3.2.2

Based on the structure of the optimal revenue of port B under the condition of $P_{BC} \geq C_p$, we can easily get the expected revenue of port B at port A when the cargos' demand for shipping leg $B-C$ is unknown as follow:

$$E[R_B^*] = \int_0^{K-S_{AC}} P_{BC} \cdot x_{BC} \cdot f_{BC}(x_{BC}) dx_{BC} +$$
$$\int_{K-S_{AC}}^{K} [P_{BC} \cdot x_{BC} - C_p \cdot (x_{BC} + S_{AC} - K)] \cdot f_{BC}(x_{BC}) dx_{BC}$$
$$+ \int_K^{\infty} (P_{BC} \cdot K - C_p \cdot S_{AC}) \cdot f_{BC}(x_{BC}) dx_{BC}$$

Firstly, we show this function is concave and decreasing with S_{AC}. The first order derivative of the expected revenue function is:

$$\begin{aligned}
\frac{\partial E[R_B^*]}{\partial S_{AC}} &= -P_{BC}(K - S_{AC}) \cdot f_{BC}(K - S_{AC}) \\
&- C_p F_{BC}(K) + C_p \int_0^{K-S_{AC}} f_{BC}(x_{BC}) dx_{BC} \\
&+ [P_{BC}(K - S_{AC}) - C_p(K - S_{AC} + S_{AC} - K)] f_{BC}(K - S_{AC}) \\
&- C_p \int_K^{\infty} f_{BC}(x_{BC}) dx_{BC} \\
&= C_p F_{BC}(K - S_{AC}) - C_p \leq 0
\end{aligned}$$

The second order derivative of the expected revenue function is:

$$\frac{\partial^2 E[R_B^*]}{\partial^2 S_{AC}} = -f_{BC}(K - S_{AC}) \leq 0$$

Then we go back to the revenue function at port A. The first order partial derivative of the expected revenue with respect to the confirmed cargo delivery amount of shipping leg $A-C$ S_{AC} is:

$$\begin{aligned}&\frac{\partial R_A(S_{AC},S_{AB})}{\partial S_{AC}}\\&=P_{AC}+\frac{\partial E[R_B^*]}{\partial S_{AC}}\\&=P_{AC}+C_pF_{BC}(K-S_{AC})-C_p\end{aligned}$$

Under the condition of $P_{AC}>P_{BC}$, and $P_{BC}\geq C_p$, we can know $\frac{\partial R_A(S_{AC},S_{AB})}{\partial S_{AC}}>0$.

The second order partial derivative of the expected revenue with respect to S_{AC} is:

$$\frac{\partial^2 R_A(S_{AC},S_{AB})}{\partial^2 S_{AC}}=-f_{BC}(K-S_{AC})\leq 0$$

The first order partial derivative of the expectation revenue with respect to the confirmed cargo delivery amount of shipping leg $A-B$ S_{AB} is: $\frac{\partial R_A(S_{AC},S_{AB})}{\partial S_{AB}}=P_{AB}>0$.

So, if the amount of whole arrival cargos $X_{AC}+X_{AB}$ does not exceed the whole capacity K, the optimal delivery quantities would be the arrival cargos' amount: $S_{AC}^*=X_{AC}$; $S_{AB}^*=X_{AB}$.

If the arrival cargos for both shipping leg $A-C$ and $A-B$ exceed the whole capacity of the ship, the optimal delivery quantities for shipping leg $A-C$ S_{AC}^* and the optimal delivery quantities for shipping leg $A-B$ S_{AB}^* must be achieved on the line $S_{AC}+S_{AB}=K$. That means the optimal solution must satisfy: $S_{AB}^*=K-S_{AC}^*$. So, the maximum revenue can be described as: $R_A^*(S_{AC}^*,S_{AB}^*(S_{AC}^*))=P_{AC}*S_{AC}^*+P_{AB}*(K-S_{AC}^*)+E[R_B^*]$. This problem actually becomes a one decision variable problem. The first order derivative of this problem becomes:

$$\frac{\partial R_A^*(S_{AC}^*,S_{AB}^*(S_{AC}^*))}{\partial S_{AC}^*}=P_{AC}-P_{AB}+C_pF_{BC}(K-S_{AC}^*)-C_p$$

Case 1: $X_{AC}+X_{AB}\leq K$

In this case, the capacity of the ship is over supplied. Thus:

$S_{AC}^*=X_{AC}$; $S_{AB}^*=X_{AB}$.

Case 2: $X_{AC}>K$ and $X_{AB}\leq K$

Under this case, we first consider when $P_{AC}-P_{AB}-C_p\geq 0$, $\frac{\partial R_A^*(S_{AC}^*,S_{AB}^*(S_{AC}^*))}{\partial S_{AC}^*}\geq 0$.

$S_{AC}^* = K$; $S_{AB}^* = 0$.

Then we consider when $P_{AC} - P_{AB} - C_p < 0$, The first order derivative of the revenue function at port A is no larger than 0 when $K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p}) \leq S_{AC} \leq K$ and larger than 0 when $0 \leq S_{AC} < K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p})$. It means the revenue function at port A increases in $[0, K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p}))$ and decreases in $[K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p}), K]$. We need to discuss the location of the point $K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p})$ on $[0, K]$ to find the optimal amount of cargoes for shipping leg $A - C$ S_{AC}^*:

(1) When $K - X_{AB} < K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p}) \leq K$, $S_{AC}^* = K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p})$; $S_{AB}^* = F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p})$.

(2) When $0 \leq K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p}) \leq K - X_{AB}$, $S_{AC}^* = K - X_{AB}$; $S_{AB}^* = X_{AB}$.

Case 3: $X_{AC} \leq K$ and $X_{AB} > K$

Under this case, we first consider when $P_{AC} - P_{AB} - C_p \geq 0$, $\frac{\partial R_A^*(S_{AC}^*, S_{AB}^*(S_{AC}^*))}{\partial S_{AC}^*} \geq 0$.
$S_{AC}^* = X_{AC}$; $S_{AB}^* = K - X_{AC}$.

Then we consider when $P_{AC} - P_{AB} - C_p < 0$, there exist three situations:

(1) when $K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p}) < 0$, $S_{AC}^* = 0$; $S_{AB}^* = K$;

(2) when $0 \leq K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p}) \leq X_{AC}$, $S_{AC}^* = K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p})$; $S_{AB}^* = F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p})$;

(3) when $X_{AC} < K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p}) \leq K$, $S_{AC}^* = X_{AC}$; $S_{AB}^* = K - X_{AC}$.

Case 4: $X_{AC} < K, X_{AB} < K$ and $X_{AC} + X_{AB} > K$

Under this case, we first consider when $P_{AC} - P_{AB} - C_p \geq 0$, $\frac{\partial R_A^*(S_{AC}^*, S_{AB}^*(S_{AC}^*))}{\partial S_{AC}^*} \geq 0$.
$S_{AC}^* = X_{AC}$; $S_{AB}^* = K - X_{AC}$.

Then we consider when $P_{AC} - P_{AB} - C_p < 0$, there exist three situations:

(1) when $K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p}) < K - X_{AB}$, $S_{AC}^* = K - X_{AB}$; $S_{AB}^* = X_{AB}$;

(2) when $K - X_{AB} \leq K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p}) < X_{AC}$, $S_{AC}^* = K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p})$; $S_{AB}^* = F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p})$;

(3) when $X_{AC} \leq K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p}) \leq K$, $S_{AC}^* = X_{AC}$; $S_{AB}^* = K - X_{AC}$.

Case 5: $X_{AC} \geq K$ and $X_{AB} \geq K$

Under this case, we first consider when $P_{AC} - P_{AB} - C_p \geq 0$, $\frac{\partial R_A^*(S_{AC}^*, S_{AB}^*(S_{AC}^*))}{\partial S_{AC}^*} \geq 0$.
$S_{AC}^* = K$; $S_{AB}^* = 0$.

Then we consider when $P_{AC} - P_{AB} - C_p < 0$, there exist two situations:

(1) when $K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p}) < 0$, $S_{AC}^* = 0$; $S_{AB}^* = K$.

(2) when $0 \leq K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p}) \leq K$, $S_{AC}^* = K - F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p})$; $S_{AB}^* = F_{BC}^{-1}(\frac{P_{AB}+C_p-P_{AC}}{C_p})$.

Appendix C.

Proposition 3.2.3.

In the benchmark model, the shipping company decides differently based on K and Λ. $K > \Lambda$ can be transformed to $F_{BC}(K) > 1 - (P_{AC} - P_{AB}/P_{BC})$. When $F_{BC}(K) > 1-(P_{AC}-P_{AB}/P_{BC})$, the shipping company can obtain the expected revenue $E[R_{BLA}]$ in the long run. When $F_{BC}(K) \leq 1 - (P_{AC} - P_{AB}/P_{BC})$ the shipping company can obtain the expected revenue $E[R_{BLE}]$.

In the strategically throwing away model, when $P_{AC} - P_{AB} < C_p < P_{BC}$, the shipping company decides differently based on K and Θ. $K > \Theta$ can be transformed to $F_{BC}(K) > 1 - (P_{AC} - P_{AB}/C_p)$. When $F_{BC}(K) > 1 - (P_{AC} - P_{AB}/C_p)$, the shipping company can obtain the expected revenue $E[R_{SLA}]$ in the long run. When $F_{BC}(K) \leq 1-(P_{AC}-P_{AB}/C_p)$ the shipping company can obtain the expected revenue $E[R_{SLE}]$.

When $0 < C_p \leq P_{AC} - P_{AB}$, the shipping company can obtain the expected revenue $E[R_{SCL}]$.

Because $C_p < P_{BC}$ is assumed, $\Lambda > \Theta$. Thus, through comparing the expected revenue in each case, we can get the five cases that throwing away cargoes should be allowed.

$$S^*_{BC} = \begin{cases} X_{BC}, & 0 \leq X_{BC} < K \\ K, & K \leq X_{BC} \end{cases}$$

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