A NON-SYSTEMATIC FACTOR AWAKENS: THE ROLE OF SENTIMENT IN THE HOUSING MARKET

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THE HOUSING MARKET

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CERTIFICATE OF ORIGINALITY

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ABSTRACT

Cities enjoy great progress in urban development until development hits a bottleneck. As various problems emerge in the progress, modern society appeals to a more harmonic development so as to reach a sustainable balance among three aspects: namely, society, economy and environment. Sustainable development is an important issue in enabling and balancing urban transformation in and through cities. Among those three aspects, the well-being of an economy is in the leading position as economic activities determine allocation of resources into the other two aspects, i.e. society and environment. A number of studies show that the basic and effective way to facilitate sustainable development is to attain the stability in economic growth. As property development makes up a large portion of urban development and housing markets take the majority of property markets, the housing market plays a prominent role in urban development and economic growth. Growing efforts are put into examining the critical factors that affect the stability of housing markets and investigating the market anomalies that disturb the stability. Most of these studies focus on systematic factors and some of them explore the puzzles that classical theories are less able to accurately explain.

These puzzles give rise to newly developed theories that turn to the effects of non-systematic risk. These new theories attempt to help classical models in enhancing elucidative power for yet-to-explain phenomena, based on some new notions and assumptions. One of these new notions is sentiment, which is derived from psychology theory. The new assumption related to this notion indicates that participants in a certain market are subject to sentiment (DeLong et al., 1990), which is defined as investor's

belief to market future trend. Evidence suggests that the property market is always affected by sentiment.

As the extant studies lack a comprehensive understanding of effects of sentiment in the housing market, this PhD study aims to address this issue. In line with this comprehensive goal, four specific research objectives have been undertaken in four chapters respectively to conduct a holistic investigation of the different roles of sentiment in the housing market.

Two studies conducted in Chapters 2 and 3 examined the role of sentiment at micro level in housing markets. More specifically, the first study explored effects of sentiment in optimal development for residential developers and the second investigated effects of sentiment in optimal housing consumption for households over the life-cycle. The other two chapters (Chapters 4 and 5) examined the macro effects of sentiment on the movement of housing markets. A new index developed in Chapter 3 was designed to measure changes in market sentiment for the housing market. Such an index based on trading intensity is able to capture changes in market sentiment more accurately and directly. Chapter 4 identified the short-term predictability of sentiment and gauged the long-term effect of sentiment, as well as verified the different roles of sentiment in sales and rental sectors, in the housing market.

This thesis has offered both theoretical and practical contributions. It bridges the knowledge gap due to the limited work in non-systematic risk in the housing market. In addition, this study sheds light on how the roles of sentiment differ at both micro and macro levels.

The participants on both sides of housing markets could benefit from the implications of this study, as follows. An optimal dynamic model associated with sentiment factors was developed to offer developers a more accurate approach to evaluating their proposed projects. This new model for project evaluation could assist developers in making optimal choices when facing the uncertainties of market fundamentals and sentiment. A life-cycle model was developed to explore the role of sentiment in households' housing choices including optimal tenure choice and housing demand. The model's implications gave an in-depth insight into how these choices changes with sentiment in housing markets.

On the other hand, a sentiment index was introduced to measure sentiment at macro level more precisely. Not only does the index provide a better understanding of the pricing of real estate as an asset, but also it benefits further macro studies on the dynamics of housing markets. This sentiment-index study provides important policy implications in two respects: that is (1) short-term predictability and (2) long-term effect of sentiment in housing markets. They can serve as references which benefit the relevant authorities, should they make policies to stabilize and improve the functioning of housing markets.

LIST OF PUBLICATIONS

Publications arising from the thesis

Hui, E. C. M., & **Wang, Z.** (2014). Market sentiment in private housing market. *Habitat International*, 44, 375-385.

Wang, Z., & Hui, E. C. M. (2017). Fundamentals and market sentiment in housing market. *Housing, Theory and Society*, 37(1), 57-78.

Working papers arising from the thesis

(2016) The role of sentiment in optimal development of residential projects.

(2017) The role of sentiment in household's optimal housing choices.

Other publications

Hui, E. C. M., Liang, C., **Wang, Z.**, Song, B. T., & Gu, Q. (2012). Real estate bubbles in China: a tale of two cities. *Construction Management and Economics*, 30(11), 951-961.

Hui, E. C. M., Lo, T. K., Chen, J., & **Wang, Z.** (2012). Housing and consumer markets in urban China. *Construction Management and Economics*, 30(2), 117-131.

Hui, E. C. M., **Wang, Z.**, Yiu, C. K. F., & Wong, H. (2013). Inside information of real estate developers. *Habitat International*, 40, 244-257.

Hui, E. C., & **Wang, Z.** (2014). Price anomalies and effectiveness of macro control policies: Evidence from Chinese housing markets. *Land Use Policy*, 39, 96-109.

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study of securitized property market. *Physica A: Statistical Mechanics and its Applications*, 426, 72-87.

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1 INTRODUCTION

1.1 Background and Problem Statement

1.1.1 Background

Due to the decreasing availability of land supply and increasing scarcity of natural sources, modern cities - especially metropolises - gradually shift to a compact mode with high population density and mobility (Hui et al., 2007). For instance, Hong Kong is a visible and well-known example of one of the most densely-populated cities in the world (Hui and Lam, 2005). The metropolis burdens itself with many complicated problems and conflicts in urban development. City managers are now required to develop a long-term plan to pursue the sustainability of urban development mainly with three aspects: economy, environment, and society. These three aspects interact with each other and form a cyclical system of city (O'Sullivan, 2003). As is known, economic actions are forms of social behaviour in terms of the allocation of scarce resources. Society makes decisions on an economy to determine the resources allocated to the other two domains. Among all branches of the economy, urban economics focuses not only on the location in decision making, but also on cities themselves "as cities represent centres of economic activity" (O'Sullivan, 2003). As such, urban economics plays the first part to drive the cyclical dynamics of the modern city (Quigley, 1998).

Scheffran (2000) suggests a plausible way to reach the balance between the three aspects in order to achieve sustainable growth. That way, which is basic and efficient, is to maintain stable growth in development. Stability of economic growth refers to avoiding excessive fluctuations in an economy's evolution (IMF, 2014). Instability brings uncertainty to the economy, frustrates investment, impedes trading and exchange, disturbs economic growth, and hurts other parts of society. Certainly, a market economy can be dynamic in the short term; a moderate degree of instability and structural change may happen in a healthy economy. Yet the real challenge for governments, academics, and policymakers is to control instability without dramatically harming the growth of an economy, social welfare, or social development (IMF, 2014). In this case, economic stability seeks to achieve (i) a stable relationship between quantity (availability) and quality, (ii) an adequate and sustainable rate of growth, and (iii) an acceptable degree of harmonic relationship between economy and ecology. It is imperative that sustainable development requires maintaining the stability of economic growth.

Urban city has changed the use of the land which it stands. In a city, land and buildings aggregate people within districts with different functions and thereby form the real estate market. Hui et al. (2012b) point out that real estate markets, especially housing markets, act as the mainstay of an economy. In the meantime, Hui et al. (2012) contend that real estate markets have considerable effects on the entire economy by contagiously affecting other sectors of the economy. In light of this, one of the keys to sustainable urban development is to maintain the stable growth of housing markets.

Residential property is not only a remarkable type of wealth in an urban economy, but also a main component of economy growth. Consider the case of Hong Kong, where the housing stock of the private housing market totalled 1.136 million units in 2013. In the same year, outstanding mortgage loans counted for 887.9 billion Hong Kong dollars in total and this counted for 41.5% of the total GDP (2,138 billion HKD). This ratio became higher: the size of the mortgage market in Hong Kong was equivalent to 43.6% of GDP in 2014 and 44.7% of GDP in 2015. On the other hand, property prices rose 7.7% (3.3% inflation-adjusted) while the economy growth was 3.1% in 2013, and property market boomed in 2014 as property prices rose 13.6% (8.2% inflation-adjusted) while the economy growth was 2.6%, according to official statistics.

It has been observed that the housing market rides the boom-bust cycles for several decades. However, many recent studies highlight the difficulty of identifying the cyclical pattern of these movements in housing markets. Along with drastic fluctuations, housing markets are deemed to be highly unpredictable (Hui and Wang, 2015).

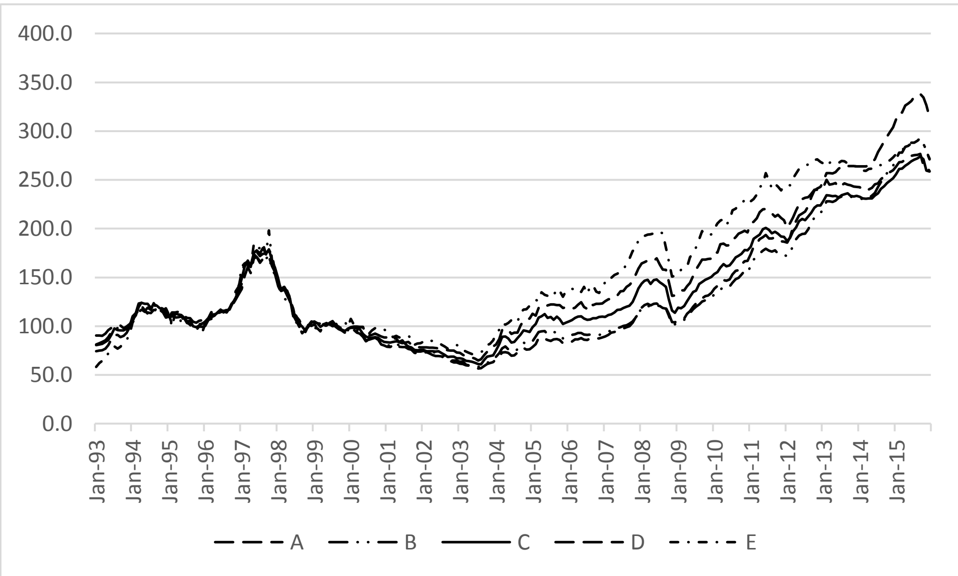

Figure 1.1. The charts of housing price indices by class

Notes: Class A refers to units sized less than 40m2; Class B refers to units between 40-69.9m2; Class C refers to units between 70-99.9m2; Class D refers to units between 100-159.9m2; and Class E refers to units that are 160m2 or greater in size. Data source: Rating and Valuation Department, HKSAR.

Housing prices in Hong Kong (or even around world) have become extremely volatile over the past two decades. Figure 1.1 gives an overview of the fluctuations of prices between 1993 and 2015. The integrated index for the five classes touched the bottom (58.4) in 2003 and climbed to the peak (306.1) in 2015, indicating 5.2 times the gap between the peak and the trough. The volatility (standard deviation) of housing price indices in the period of 1993-2015 is therefore recorded as 63.48 across these five classes. The growth of median household income, however, was not on the scale of that of housing prices. Growth of 33.1% was reported for the last 15 years (18,710 HKD per month in 2001 and 24,900 HKD per month in 2015[1]). Average home prices were 19 times median household income in 2015, rising from 12.6 times in 2013 and the highest level ever recorded in the last decade.

Classical theories allow a certain range of fluctuations around dynamic market equilibriums (Fama and French, 1993). Modelling asset prices based upon fundamental variables would infer that any anomalous component (e.g. bubbles) could be captured by the error term (White, 2015). This is contrast to the key assumption of such equilibrium model as the error term should be randomly distributed. There is increasing witness of price anomalies, which refer to extreme variations in housing prices that cannot be fully explained by market fundamentals (Stiglitz, 1990; Stevenson, 2008), thus challenging the classical theories and becoming a discordant note in the stability of housing markets. A lot of attention has been drawn to such anomalies. Recent studies (e.g. Jin et al., 2014; Ling et al., 2014) highlight the inefficiency of market mechanism which is in part responsible for the formation of those anomalies, in the presence of the effect of market participants' behaviour. To contribute to the knowledge of stability in the

[1] Data source: Census and Statistics Department, HKSAR

housing market, this study aims at a new angle from which we can have a better understanding of how market participants' behaviour affects housing markets. The following subsection introduces the specific problem that will be addressed by this study.

1.1.2 Problem Statement

Numerous recent studies have identified various problems pertaining to economic stability or instability. One of the most important problems is that classical theory attracts criticism, which is because market fundamentals cannot accurately explain the movements of the economy, especially sharp fluctuations, or the uncertainties attributed to non-systematic risks. Both of them bring instability of global and local economies. The criticism gives rise to the newly developed theory that attempts to help the classical models with enhancing the elucidative power of those models, based on some new notions and assumptions. At this point, the new theory, so called behavioural economics, derived from psychology theory, emerged as a possible guide to answering those questions.

One of these new notions developed and used in the behavioural studies is investor sentiment.) This concept indicates that investors in a certain market are subject to sentiment (DeLong et al., 1990; Barberis et al., 1998). A widely accepted **definition of investor sentiment** is the general propensity of investors towards markets which cannot be justified by the external information at hand (Backer and Wurgler, 2007). In other words, sentiment reflects investors' belief to anticipate price movement in a market.

DeLong et al. (1990) assume that sentiment is unpredictable in their study of noise trader. Barberis et al. (1998) advance that changes in investor sentiment are in part unpredictable. The reason behind this is that the movements of investor sentiment are subject to various idiosyncratic factors but also it is invariably connected with crowd psychology. Consequently, the movements can show a momentum trend or a huge fluctuation. Sentiment therefore cannot be fully justified by external information.

The empirical studies on investor sentiment and asset mispricing has largely focused on stock markets (e.g., Brown and Cliff, 2004, 2005; Baker and Wurgler 2006, 2007; Kumar and Lee 2006; Han 2008; Baker et al., 2012). Investor sentiment has a strong power in explaining abnormal changes in stock prices. Schmeling (2009) affirmed the impact of investor sentiment on expected stock returns across 18 industrialised countries. Shleifer and Vishny (1997) propose that the potential risk of betting against sentiment-based investors is high. As such, irrational investors' actions and reactions are highly possible to challenge classical theory by driving markets away from the fundamental level. The market itself is less able to fight against the effect of sentiment by forcing prices back to the equilibrium due to the market's defects. The short-term dynamics of asset markets become less predictable. The increasing market deviation can no longer be ignored, and the risks created by sentiment-driven activities have profound impacts on market equilibrium.

To date, after several crises in the real estate market, investor sentiment is now widely accepted as a key factor driving property prices (Clayton et al., 2009; Hui et al., 2013b), property securities (Lang and Schaefers, 2015) and REIT returns (Lin et al., 2009; Deng et al., 2014; Das et al., 2015; Freybote, 2016). The real estate market is affected by investor sentiment more significantly thanf the stock market, mainly due to the special nature of the market: illiquidity and limitations to short-selling.

Sentiment also affects the trading volume of the real estate market. Investors become more active when they are optimistic, and vice versa. In return, the volume may release a sign to the market as the volume reflects the change in housing demand. Clayton et al. (2008) discuss the possible presence of sentiment-based transactions by partially irrational investors, and find that the divergence of asset price from fundamental value can be attributed to the linkage of market-wide liquidity to investor sentiment.

The housing market is deemed as a highly incomplete market (Englund et al., 2002). Housing markets, as a perfect example of private investment markets, exhibit significant information asymmetries (Ling et al., 2014). In addition, some behavioural studies suggest investors always rely on noisy information (Froot et al., 1992; Welch 1992, Daniel et al., 1998). Such behaviour leads to momentum trading and thus drives prices away from fundamental values over short horizons. This would be especially enlarged by the shortage of continuous price revelation and suggests that the influence of sentiment on market values may last in a relatively long-term.

On the other hand, restrictions on short-selling make arbitrage costly and expedite the mispricing of economically equivalent assets (Lamont and Thaler, 2003; Scheinkman and Xiong, 2003). The housing (more generally real estate) market falls short of short-selling, which restricts sophisticated participants to eliminate mispricing (Clayton et al., 2009). As a result, the market affected by sentiment is more likely to exhibit a momentum pattern, along which the price drifts far away from the fundamental value. Brounen et al. (2013) document that public REITs with stricter short-selling constraints are susceptible to be overpriced. Furthermore, the efficiency of information in asset pricing is decayed in the presence of short-selling limitations (Diamond and Verrecchia, 1987).

Besides, the difference between the number of the studies on public (much more) and private markets gives a clue showing the difficulties of obtaining information on private equity investments (Kaplan and Schoar 2005). As pointed out by Ling et al. (2014) private markets supply an important testing ground for examining sentiment's pricing role.

In light of this, this study attempts to develop a better understanding of the role of sentiment in the housing market, and further to give in-depth insights into the effects of sentiment on the equilibrium of the housing market. This PhD research aims to conduct a comprehensive investigation from both macroeconomic and microeconomic perspectives, and is expected to make a contribution to the extant knowledge of stable growth of housing markets. The conceptual framework of this study, shown in Figure 1, will be introduced in the following section.

The effects of sentiment and related issues differ at two levels, i.e. the micro and macro levels. Thus, this study starts with investigations into the effect of sentiment on the supply and demand sides of the housing market. On the other hand, this study will introduce a novel approach[2] to establish an index for sentiment in the housing market. This index depicts sentiment by taking into account a group of factors regarding participants' behaviour, which captures the movement of market sentiment well. Using the new sentiment index, this study can do much better to understand how sentiment disturbs the market from equilibrium on a macro level. The research outcomes are expected to provide suggestions for future studies and to shed light on implications for the stable growth of housing markets.

[2] This approach has recently been published in our paper, Hui and Wang (2014a).

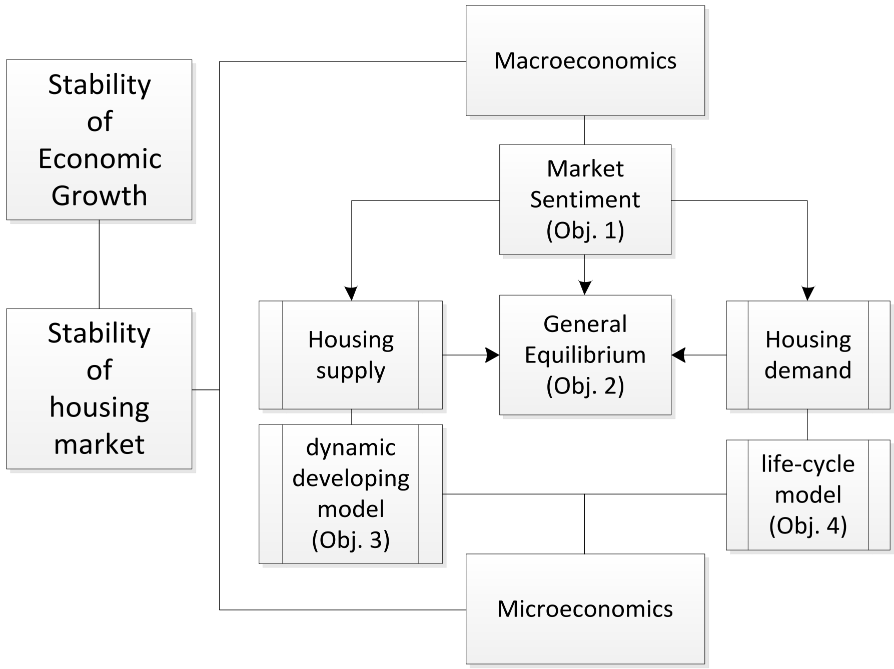

Figure 1.2. The framework of this PhD research

1.2 Research Objectives

There are four research objectives in this PhD study to contribute to the knowledge of sentiment in housing market, which can be summarised as follows:

(1) To develop an index of sentiment in the housing market. This objective is to provide a more accurate proxy for market sentiment which can benefit the investigation on the macro effect of sentiment on the dynamic market equilibrium. (See Chapter 3)

(2) To identify the short-term predictability and to examine long-term effects of market sentiment on dynamic equilibrium of the housing market. (See Chapter 4)

(3) To develop a dynamic optimisation model for housing development projects, taking account of market uncertainties and sentiment. This

objective is to examine how sentiment affects the supply side of the housing market. (See Chapter 5)

(4) To develop a life-cycle model for households' optimal housing choices, taking account of market uncertainties and sentiment. This objective is to examine how sentiment affects the demand side of the housing market. (See Chapter 6)

1.3 Two Angles of the Research

Stability of economic growth in the housing market requires sustainable dynamics of housing prices, efficient allocation of resources and sufficient management of market risks. Market/price movement fails to reach expected equilibrium defined by fundamentals gives rise to extensive research on market inefficiency. Following Case and Shiller (1989), issues of market inefficiency have been widely analysed in the real estate market (Marcato and Nanda, 2016).

Several studies looked at the sentiment of the housing sector (e.g. Goodman, 1994; Dua, 2008; Croce and Haurin, 2009; Ling et al., 2013; Marcato and Nanda, 2016); others concern the non-residential sector (e.g. Baker and Saltes, 2005; Clayton et al., 2009, Tsolacos et al., 2014; Ling et al., 2015). However, most of them are conducted only at macro level.

To be a holistic investigation, this study needs to be conducted from two levels: macro and micro levels. This PhD study begins at the macro level. This research attempts to examine the roles of sentiment in two aspects: the short-term prediction of sentiment on market indicators and long term relationship with market fundamentals, in the dynamics of the housing market. Market equilibrium reflects the steady state of the interaction between supply and demand. In this case, micro study will give in-depth insights into both sides. Two different theoretical analyses are carried out in order to examine the effects of sentiment in housing supply and housing demand respectively.

1.3.1 Macroeconomics: Market Sentiment

Irrational participants' sentiment aggregates and becomes a macro level effect on market movements. Following the discussion by Shiller (2000), there have been many studies of the role of specific biases in judgment that causes huge impacts on global economic and financial markets (e.g. Kahneman, 2003). After the investigations that were conducted at a micro level, this study turns to the structure and behaviour of the housing market and market sentiment.

Recently, there have been an increasing number of studies that inquire into the effect of sentiment among investors in the stock market (e.g. Baker and Wurgler, 2006, 2007). Some studies look into the relationship between investor sentiment and returns, both in time-variation and cross-section (Schmeling, 2009).

As for the real estate market, Clayton et al. (2008) suggest that sentiment is a cause to diverge property prices from fundamental values. Hui et al. (2013b) add that some of the property mispricing is attributed to the effect of sentiment. In particular, after real estate market crises, investor sentiment is now widely accepted as a critical indicator of market performance (Ling et al., 2014).

Since the housing (also real estate) market is characterised by its illiquid nature and limited short-selling, market sentiment should affect the housing market more significantly than other asset markets. Consequently, market sentiment needs to be cautiously measured to reflect market confidence. Hence, people can monitor the future expectations of participants and then make predictions on the trend of property prices. In this case, a novel approach is developed in Chapter 3 to indexing market sentiment in order to depict the dynamics behind the performance of the housing market.

On the other hand, the effect of sentiment on a participant's housing decision can be reflected by the intensity of trading activities, and trading volume affects the price movement in part (Clayton et al., 2009). Sentiment, in return, evolves and again is involved in participant decision making. Considering the endogeneity, I attempt to verify whether sentiment can be used to forecast trading volume in Chapter 3 as, to date, there has been no consensus on this issue.

1.3.2 Microeconomics: Supply and Demand

When considering stability in the housing market, market efficiency is the one of the most important issues in the context of economics. Since the notion relates to the rationality of market participants, this study aims to give an insight into this assumption by exploring the role of sentiment in market participants' decision making.

As is known, houses are a durable commodity. In particular, the two unique characteristics of the housing market (most of those characteristics can also feature in the real estate market) must be recognised through the fact that land is occupied by buildings. These characteristics are durability and heterogeneity. Firstly, houses can last for decades and even centuries, as the land underneath it is practically immobile. Due to this special feature, the housing market (also real estate market) can be considered a stock/flow market.

Housing stock comprises two parts: the majority of existing housing supply and subsequent inflows. The latter one arises from planned projects that are determined by the existing stock. With the nature of housing assets, that they are always of high worth, the value of incoming housing adds substantially to the growth of social wealth. In light of this, the stable growth of housing markets requires a sustainable evolution in which a balance between price and quantity is required.

This PhD study will look at the micro economics on two sides. The equilibrium between demand and supply in the housing market can price the real property in the long run (DiPasquale and Wheaton, 1996). The effects of sentiment on both supply and demand sides are therefore of interest. Since the developer's plan affects the supply, and households' housing choices affect the demand in the housing market, I restrict the focus to these two sides.

On the supply side, residential developers are concerned about the valuation of residential projects (including development and re-development) prior to other steps. In this case, optimal density and timing to commencement are the most important interests. Both of these interests exert influence on future housing supply, and subsequently on housing prices, in the private housing sector (Hui and Fung, 2009).

The residential developer has to estimate the expected housing demand at the time of sale. This is to determine future housing supply, by which the pricing strategy is governed. However, it is difficult to estimate the precise housing demand at a future time due to investment irreversibility and long development periods (Ott et al., 2012).

From historical data, Fallis (1985) estimates the long-run price elasticity of supply as 8.2, which is quite high. But the findings in his paper imply that supply tends to be very price inelastic in the short term. This disparity indicates that it is difficult to value property projections simply and directly on the basis of price dynamics because developers cannot find a constant estimation on supply due to the inconsistency in the price elasticity of supply.

With investment irreversibility, unprosperous project development would cause wanes in future cash flow or an enormous gap in capital stock. Residential projects always face a long development period which amplifies the effects of uncertainty with time. Holland et al. (2000) contend that market timing and investment irreversibility are the critical factors of success in residential projects. The valuation of residential projects becomes the foremost concern to the developers.

Berkovec and Goodman (1996) show that sentiment substantially affects the changes in turnover rates. They further indicate that sentiment exhibits a close link to housing demand. Such link makes the developer's estimation on housing demand more complicated. On the other hand, as sentiment is proven to affect market performance, the market participant's expectation on future returns is also disturbed by sentiment (Marcato and Nanda, 2016). As such, market sentiment plays a role in a developer's estimation on future project returns. Although the effects of sentiment on development decisions are undoubtedly important to residential developers, they have rarely been analysed by the extant real estate research. Due to this lack of literature, the present study aims to investigate the role of sentiment in decision making surrounding residential project development.

With locational immobility, houses have no physical marketplace, making evaluation difficult due to high searching costs, asymmetric information and constrained substitutability (Hui et al., 2012b). As such, immobility produces heterogeneity because of the distinction and sensitivity of location. It brings a move on the idea of housing from physical commodities to housing services (Muth, 1960). Olsen (1969) suggests a theoretical definition of a unit of housing service: that housing service is an unobservable construction. This concept helps to develop a better understanding of the household's choice associated with housing heterogeneity, which substantially affects housing demand.

Due to immobility and the high value of houses, consumption on housing services probably is the most significant expenditure in a household's budget (Han, 2008). Tenure choice refers to a household’s decision between owning and renting a house, which is one of the main determinants of housing demand. This topic has always attracted broad attention, involving different realms in economics, such as household income, consumption and savings during the life cycle, fluctuations in rental costs and house prices, homeowner tax policy, and so on. Hence, to model the demand for housing can show the aggregate implications of household behaviour (Attanasio et al., 2012).

It is worth noting that, after ownership of a house, a household is unlikely to switch their housing status back to tenant (Smith and Smith, 2007). In other words, a household’s first-time home purchase has a significant impact on housing demand. Once an owner decides to enter the housing market, his/her first-time home ownership brings immediate shock. Furthermore, a home owner can alter their demand on housing services by transacting, i.e. selling the current house and buying a new one. In addition, a household’s tenure choice and non-housing consumption would mutually affect each other (Henderson and Ioannides, 1983).

On the other hand, homeowner transaction behaviour largely explains the supply of the second-hand housing market. But as transaction cost is a considerable amount for households in housing transactions (Haurin and Gill, 2002), it curbs households to adjust their housing demand in the face of the uncertainty of housing prices. Han (2008) recognises that households hedge against future risk of housing price by changing their housing wealth and the hedging incentive would reduce the effect of transaction cost.

These two types of housing tenure choice constitute a cycle of housing mobility in urban cities. Unfortunately, no studies focus on the micro effect of sentiment on households' housing choices. Thus this study attempts to develop a life-cycle model of housing demand, by incorporating two choices into an integrated framework to examine the effect of sentiment on the household's behaviour pattern.

1.4 Significance of this Research

This PhD study focuses on the effects brought by a non-systematic factor, i.e. sentiment, in the housing market. As the study involves both theoretical and empirical analyses, it makes both theoretical and practical significance. This thesis contributes towards filling the knowledge gap that arises from the limited work in non-systematic risk in the housing market.

A novel framework is expected to establish a new index for gauging market sentiment. This approach is superior to traditional regression analysis (such as principal component analysis, in Baker and Wurgler, 2006). Such an index can explore the changes in the aggregate of investors' perception. The index is supposed to contribute both theoretically and practically. It could capture the sentiment-driven behavioural patterns and better describe the role of market sentiment in formation of transactions, which would enrich the theoretical understanding of non-systemic risk in the housing market. On the other hand, the index could benefit investors from its utility as a reference assisting portfolio selections for the purpose of maximising risk-adjusted returns. In addition, this index could benefit future macro studies on sentiment in the housing market. For instance, this index offers a more accurate index for the macro study (conducted in Chapter 4) on the effect of sentiment on the housing market movements.

The implication of the macro study using this new sentiment index may serve as a good basis and informative reference-point for the monitoring of the housing market for relevant authorities and policy makers. The macro study provides better insights into short-term dynamics and long-term movement of the market, which helps in regulating and other policy making in order to stabilise and improve the functioning of the housing market.

On the other hand, to address the lack of literature focusing on the micro effects of sentiment on the housing market, this study carries out two analyses of the roles of sentiment on the supply and demand sides of the housing market respectively. An optimal development model in the first analysis is expected to provide a more accurate project evaluation for developers. By incorporating market sentiment, this model can offer a better understanding of market and price uncertainties, in order to assist the residential developer in making optimal choices (e.g. density of project and timing of investment) in project development. The implication of this model helps explore the effects of sentiment on the supply in the housing market. In the second analysis, a life-cycle model is expected to investigate the role of sentiment in households' housing choice. This model can help distinguish the demand of housing investment from that of housing consumption. The implications of the model will contribute to the knowledge of how demand evolves with market sentiment in the housing market.

1.5 Structure of the study

This thesis is constituted of four studies to better and comprehensively understand the role of sentiment in the housing market. Each of the studies, examined in one of four chapters (Chapters 3-6), is intended to fulfil each of the four research objectives outlined in Section 1.2, in turn. The structure of the thesis is therefore as follows:

Chapter 1 introduces the research background and states the problem, and then derives four research objectives. These are followed by a discussion of the significance and value of this study.

Chapter 2 presents a comprehensive literature review for this research. It builds up on the literature regarding investor and market sentiments both in financial market and real estate market.

Chapter 3 is the study to achieve Objective 1. In this chapter, a novel approach is taken, based on transaction intensity. This is developed to capture investor sentiment on aggregate, i.e. market sentiment. This index provides a more accurate measure which will benefit further studies into the macro effects of sentiment on the housing market.

Chapter 4 is the study to achieve Objective 2. In this chapter, research is conducted to investigate the macro effects of sentiment on the short-term dynamics and long run movement of the housing market.

Chapter 5 is the study to achieve Objective 3. In this chapter, an option-based dynamic model is established to capture the optimal development decision that the residential developer should make with the consideration of market uncertainty and sentiment.

Chapter 6 is the study to achieve Objective 4. In this chapter, a life-cycle model associated with sentiment effect is established to capture the optimal housing choices that households would make when planning their housing and non-housing consumption over a life cycle.

Chapter 7 summaries the research findings arising from Chapters 3-6. The limitations of this study and directions for future research are also presented in this chapter.

Chapter

2 LITERATURE REVIEW

2.1 Introduction

This chapter reviews previous literature of sentiment study on both financial and real estate markets. The literature review mainly focuses on the following four areas:

- Behavioural and psychological biases
- Definition of sentiment
- Effects of sentiment in financial and real estate market
- Sentiment measurement and sentiment index

In addition, this chapter presents a summary of the knowledge gaps and the link between these gaps and research objectives.

2.2 Behavioural economics and psychological biases

Behavioural economics concerns the effects of psychological, cognitive, and cultural factors on the decision makings of individuals and institutions in economic activities. Furthermore, the consequences arising from these decisions for prices, returns, and resource allocation have been studied in the domain of behavioural economics.

Becker (1976), introducing his book The Economic Approach to Human Behavior, summarises a number of notions known as the pillars of so-called ‘rational choice’ theory in which human is assumed to show consistent preferences in maximizing behaviour. Then the rational choice theory has been applied to various academic fields ranging from crime to marriage.

Afterwards, Kahneman and Tversky won a reputation by publishing a number of papers for the development of prospect theory (firstly introduced in Kahneman & Tversky, 1979). This theory appears to undermine ideas about human nature held by mainstream economics by exploring that human decisions are not always optimal. Decision making is context-dependent as the way in which choices are framed can affect participant's willingness to take risk.

Decisions being not always optimal has another explanation by Simon (1982). Bounded rationality, a concept associated with Simon's 1950 work suggests that the environment in which human participants are evolved has compound impacts on their mind. Short of knowledge (or information) and computational capacities lead to limits to information processing (Simon, 1982; Kahneman, 2003).

A series of remarkable experiments conducted by Ariely reveals human "irrational" decision marking. For example, an experiment (Ariely et al., 2003) introducing anchoring effect, explains a process in which subsequent perceptions on evaluation can be affected by a non-conscious reference point determined by an initial value.

Some studies of behavioural economics also argue that humans don't make choices in isolation as they may share some common interests (Banerjee, 1992). Such as herding, people act as grouping together and maintaining the herd, or mimic other's behaviour pattern in the situation of lacking valid information. Hey and Morone (2004) establish a model of herd behaviour in a market context and summarise two strands of ideas regarding herd behaviours. The first is the one in a non-market context - Banerjee (1992) documents that the lack of sharing private information to public can cause a herding and is more likely to induce people to make decision socially undesirable to others. The second is that of information aggregation in market contexts: uninformed participants can become informed by the prices when collecting private information correctly and efficiently. Christie and Huang (1995), and Chang, Cheng and Khorana (2000) introduces empirical methodologies most commonly used in this kind of studies to test for herding.

DeLong et al. (1990) discuss irrational noise traders in their overlapping generations model. They find that irrational trading motived by erroneous beliefs affects asset pricing. Irrational trading creates disproportionate amount of risk in asset market due to the unpredictability of noise beliefs. In return, such risk yields a higher expected return for noise traders. For modern economy and financial market, following this 1990 work, increasing amount of studies on participants' psychology provide a bunch of evidence to support that, associating pricing theory with investors' psychology is necessary and reasonable (Hirshleifer, 2001). The growth of economics and finance literature employing behavioural approach in recognising the rationality of investors supplies increasing evidence that psychological biases of investors play important roles in decision-making, especially in a complicated and uncertain world with market frictions (e.g., Kahneman et al., 1982; Hirshleifer, 2001; Barberis and Thaler, 2003; Bokhari and Geltner, 2013; Ling et al., 2014).

Recently, researches in behavioural economics working on the amendments on the theory of asset pricing have brought up some new assumptions. Sentiment, as an indispensable part of the assumptions, reflects the role of psychology in modern economics and finance (DeLong et al., 1990). Some well-known studies imply that sentiment plays an important role in explaining the future movement of asset prices (see Farmer and Guo, 1994; Brown and Cliff, 2004; Baker and Wurgler, 2006; Baker et al., 2012). Given the definition, investor sentiment is the belief of an investor in relation to the part of anticipation of the price movement in a market which is not justified by market fundamentals (Baker and Wurgler, 2007). And this definition has been widely accepted and adopted in real estate studies (e.g. Marcato and Nanda, 2016). Also this definition is adopted in this study. Accordingly, market sentiment applied in this study indicates the aggregate of investor sentiment at market level, reflecting the mainstream attitude of investors (see Hui and Wang, 2014a).

2.3 Effect of sentiment in financial and real estate market

Traditional thinking of asset pricing in financial market points to the central task which is concerned with the relationship between market risks and the expected return awarded by bearing such risks. Hirshleifer (2001) argue that this central task should be extended to "*examine how expected returns are related to risk and to investor misevaluation*", and that it is reasonable to attach investors' psychology to pricing theory.

Recently, the boom and bust in the economic cycle has advocated a belief that investor sentiment induces a mispricing mechanism of assets in the financial market (Brown and Cliff, 2005). Baker and Wurgler (2006) carried out classic research on how investor sentiment affects returns in the U.S. stock market based on a unique sentiment index established in their paper. There is a noticeable trend, which is the adoption of the doctrine of psychology in economic studies in the last two decades. Growing literature (e.g. Brown and Cliff, 2004, 2005; Baker and Wurgler 2006, 2007; Kumar and Lee, 2006; Han, 2008; Baker et al., 2012) contribute to the knowledge of the role of investor sentiment. Brown and Cliff (2004), Lemmon and Portniaguina (2006), and other papers supply evidence for a role of investor sentiment in market returns of U.S. stocks. Yu and Yuan (2011) argue that the traditional trade-off relationship between risk and expected return can be found only in low sentiment periods. Stambaugh et al. (2012) give a discussion about predictive power of sentiment and argue that forecasting ability of sentiment is prominent in high sentiment period and is significant on stock in short legs. Bekaert et al. (2010) examine the effect of sentiment in the time-series relationships between government bond and stock market returns. Henderson et al. (2006) reveal that investor sentiment shows strong influence on aggregate financing patterns.

It is reasonable to believe that sentiment (or irrational trade, as it appears) involves the issue of bubbles (Baker and Wurgler, 2007; Stambaugh et al., 2012). Similar to financial markets, real estate markets have undergone several crises caused by widely accepted reasons, such as irrational pricing and trading. Tam et al. (2010) suggest that changes in the stock market, especially in real estate securities, can be regarded as a reflection of sentiment in the real estate market. Recently, an increasing number of studies (e.g. Clayton et al., 2009; Ling et al., 2014; Marcato and Nanda, 2016) have concentrated on the effect of sentiment on the return rate in the property market. Most of them have found a significant relationship between sentiment and market return.

A considerable body of studies on the forecasting of sentiment put efforts into real estate securitised market (e.g. Lin et al., 2009; Zhou and Anderson, 2013; Deng et al., 2014; Das et al., 2015; Freybote, 2016) and commercial real estate market (e.g. Ling, 2005; Newell and MacFarlane, 2006; McAllister et al., 2008; Clayton et al., 2009); but also indicates a lack of related findings in housing markets. In light of this, this study intends to supplement the knowledge of housing market. In addition, this study explores the predictability of the sentiment index on several factors, such as house price, rent and trading volume (liquidity), in order to show the importance of the role of sentiment in housing markets.

As the most prominent market indicator, property price is always affected by various fundamentals. Using cross sectional regressions, Case and Shiller (1990) found that construction cost, and changes in population and real income are the efficient determinants of housing price. Quigley (1999) studied the role of fundamentals in the US property market and found that the supply and demand sides of the property market are subject to specific economic factors, so are price movements. At the macro level, mutual effects between GDP and new residential projects are endogenously linked by the price (Crosthwaite, 2000). However, some other findings challenge this view: Clayton (1997) suggests that a sharp appreciation in housing prices can partly be attributed to investors' psychology. This is echoed by Case and Shiller (2003), suggesting that a rapid appreciation in housing price can be attributed in part to excessive expectation. Wong et al. (2005) point out that overconfidence can lead to a biased assessment in the evaluation of transactions.

Most of the studies express concern about the predictability of sentiment on the economic factors, especially the aggregate return rates. Using a non-linear causality tool, Dergiades (2012) found that sentiment shows significant predictive power with regard to stock returns. Baker et al. (2012) conclude that both local and global sentiment indices are useful predictors that index reversely affects the cross-sectional returns within markets. On the other hand, Baker and Stein (2004) built a model involving heterogeneous investors and defined liquidity as an indicator of investor sentiment. Liquidity change indicates the majority participator dominated by sentiment-based traders in the market place and therefore the mispricing of asset prices from fundamental values.

The relationship between rent and price has been widely discussed (e.g. Henderson and Ioanides, 1983; Poterba, 1992 and Gallin, 2008), and the rental market shows some specific features that are different from the sales market. Campbell and Cocco (2007) state that rental price is a crucial factor in households' decisions regarding housing or non-housing consumption. Some studies employ the rent–price ratio to study the dynamics and trends of housing markets. Wong et al. (2005) reveal that participants in housing markets show significantly different expectations regarding housing prices and rent. Since market sentiment arises from different factors including irrational expectations and limits of the market (such as limits to arbitrage), the degree of impact of sentiment will probably differ between sales and rental markets. It is justified to incorporate rent as a variable in our study with the purpose of examining whether the rental market is affected by sentiment. As Gallin (2008) admits the inefficiency of the rent-price ratio in predicting changes in rents, this chapter aims to investigate the forecasting power of sentiment with regard to future changes in rents.

Stein (1995) developed the down payment model to study the relationship between price and transaction volume. Clayton et al. (2008) provide an important finding that housing price and volume are positively correlated. They employed turnover as the indicator of sentiment to investigate the hypothesis that turnover should predict return reversals in the future. They mention that investor sentiment affects market-wide liquidity, causing property prices to deviate from their fundamental values. Agnello and Schuknecht (2011) also point out that liquidity has a significant impact on the probability of booms and busts occurring. Both anomalies in the pricing mechanism and illiquidity of the market are responsible for the issue of bubbles. The real estate market, which is relatively illiquid, is limited to hedging and high idiosyncratic risk and hence should be exposed to sentiment more significantly than the stock market. In addition, Clayton et al. (2009) found that high segmentation of private commercial real estate markets, accompanied by an asymmetry of information, lead to a substantial low level of liquidity, compared with public stock markets. Therefore, sentiment may somehow play a part in forecasting trading volume. However, these hypotheses are yet to be proven and there is still no direct conclusion on this issue. The macro study in this research shall attempt to fill this gap by studying the relationship between market sentiment and trading volume.

On the other hand, Hui and Wang (2014b) document that the real estate market has contagious effects on other industries and thus, has a significant influence on the whole economy. Hence, investor sentiment in the real estate market needs to be explored and several works have investigated it. Hui et al. (2013a) introduced a model based on option pricing models to measure investor sentiment by establishing a risk appetite indicator in the securitised real estate market. The empirical findings indicate that investors hold a lower risk appetite to the real estate market than to the general equity market, which implies that investors are less willing to bear risks when they invest in the real estate market. Zhou and Anderson (2013) empirically studied the market-wide herding behaviour in the U.S. equity REIT market. They found that REIT investors are prone to herding behaviour under turbulent market conditions. This indirectly implies that investors will be more sensitive to the market environment. As housing markets as a private market are more likely to show difference from public market (Marcato and Nanda, 2016) , this study aims to establish an index to capture sentiment movement in private market.

2.4 Sentiment measurement and sentiment index

Baker and Wurgler (2006) constructed an indirect investor sentiment index for financial market by using various proxies. This approach to measuring sentiment in such an indirect way has been adopted by subsequent works (e.g. Dergiades, 2012; Baker et al., 2012; Macrato and Nanda, 2016) on the stock market. Besides, their paper reveals that investor sentiment leads to a negative effect on cross-sectional stock returns such that higher sentiment induces relatively low subsequent returns on stocks. Such a situation is echoed in several empirical studies (e.g. Lemmon and Portniaguina, 2006; Schmeling, 2009), which have conducted detailed investigations on this issue in the global stock markets. Both Schmeling (2009) and Lemmon and Portniaguina (2006) carried out investigations using consumer confidence as an indicative measure of investor attitude. In particular, extending from Baker and Wurgler (2006), Baker et al. (2012) constructed a global sentiment index and six local indices. Their findings reveal the interrelationship between the local and global market through the channel of private capital flows.

Back on the real estate market, a large body of studies support the forecasting power of the confidence or sentiment index: Vuchelen (2004) in Belgium, Utaka (2003) in Japan, Chua and Tsiaplias (2009) in Australia, Parigi and Schlitzer (1997) and Malgarini and Margani (2009) in Italy, Easaw and Heravi (2004) and Hohenstatt and Kaesbauer (2014) in the UK, and Jin et al. (2014) and Marcato and Nanda (2016) in the US.

Marcato and Nanda (2016) summarize two prevailing methods to construct an index for sentiment in the real estate market: one is direct measurement based on a survey (e.g. Souleles, 2004; Baker and Saltes, 2005; Clayton et al., 2009; Hohenstatt and Kaesbauer, 2014) and the other is to form an indirect index (e.g. Baker and Wurgler, 2006, 2007; Baker et al., 2012; Marcato and Nanda) by selecting some underlying proxies to conduct a principal component analysis.

Nevertheless, the proxy selection for indexing sentiment embraces controversial standpoints of various angles. As such, a novel approach is developed and introduced in this study that is closer to the essence of market character; that is, the character of each transaction, which is on the microeconomics stage rather than the macroeconomics one.

2.5 Summary of the knowledge gaps

This sub-section intends to summarize the knowledge gaps mentioned above. The summary attempts to briefly describe connection between the research objectives and knowledge gaps recognized from literature review. Details are shown below.

Since the role of sentiment in stock markets is well-known nowadays, there is no straight study on the macro effect of sentiment on housing price or in housing market yet. This research aims to investigate how sentiment affects the movement of housing market, in order to fill this gap (that is objective 2). This macro study shall give in-depth insights into the difference in the effect of sentiment on different sectors of housing market (i.e. sale and rental markets). Prior to this objective, a tool or proxy is necessary to quantify the level of market sentiment. As such, the first step is to introduce an index which is newly designed to provide a reliable quantification of sentiment in housing market (that is objective 1).

Nowadays, economic model tends to be more accurate to capture future uncertainty for the sake of residential developer's decision making. Due to lack of literature providing implications of the role of sentiment in optimal decision on project development, therefore, objective 3 in this research is to investigate the role of market sentiment in valuation of residential project development.

Lack of studies addressing the problem of how sentiment affects the household's tenure choice reminds of that the dynamics demand of housing market hasn't been fully studied. As such, objective 4 aims at a deep study of the role of individual behavior pattern associated with sentiment in household's housing choices. Besides, owing to the shortage that previous literature concerns only from the perspective of buyer this research will integrate two groups of participants' (i.e. tenants and buyers) behavior into a multiple-process model.

Chapter[*]

3 A NEW INDEX OF MARKET SENTIMENT

The previous two chapters reported on two studies focusing on the micro effect of sentiment on the supply and demand sides of the housing market respectively. The studies in Chapters 3 and 4 focus on the macro effect of sentiment on the movement of the housing market. The study in this chapter establishes a novel index to gauge the aggregate of investor sentiment in the private housing market, in order to achieve Objective 1. This index provides a more accurate measure, which would benefit further studies on the macro effect of sentiment on the housing market.

3.1 Introduction

Rational investors, as the most important assumption in classical economic and finance theory, participate in all kinds of transactions across various markets. The assumption strictly prohibits sentiment (sensitive factor) from playing a role in modern economic or financial theory. Even if some irrational deals exist, classical theory appeases such a dispute by offset by arbitrageurs so that irrational deals have no long-term impact on asset prices. Recently, growing literature has paid attention to investigating the effect of investor sentiment on the stock market (e.g. Baker and Wurgler, 2006, 2007). Some studies provide evidence to argue that there is a relationship between investor sentiment and returns in terms of both time-variation and cross-section. Schmeling (2009) affirms the impact of

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[*] This chapter has been published in Hui and Wang (2014a). Market sentiment in private housing market. *Habitat International*, 44, 375-385.

investor sentiment on expected stock returns across 18 industrialized countries. Shleifer and Vishny (1997) propose that the potential risk of betting against sentiment-based investors is high. Rational investors are found to behave differently from what classical theory would suggest as they are not giving all the energy into forcing prices back to fundamental values.

The real estate market is the mainstay of the modern economy in many countries and thus sentiment investors who specifically invest in the real estate market cannot be ignored. The real estate market will be affected by sentiment more significantly than the stock market for two main reasons: illiquid nature and the limitation on short-selling. Clayton et al. (2008) discuss the possible presence of sentiment-based transactions by partially irrational investors and thus suggest that the deviation of asset prices from fundamentals can be attributed to the linkage of market-wide liquidity to investor sentiment. Moreover, the real estate market falls short of short-selling, which has restricted sophisticated participants to eliminate mispricing (Clayton et al., 2009).

In particular, in the presence of several crises in real estate markets, investor sentiment has been accepted to be a crucial factor driving property prices (Clayton et al., 2009; Ling et al., 2014; Marcato and Nanda, 2016) and REIT returns (Lin et al., 2009; Das et al., 2015; Freybote, 2016). In addition, Tam et al. (2010) contend that market sentiment is negatively correlated with the default rate of housing mortgages. In particular, the private housing market appeals to us for investigating the role of sentiment for several reasons. First, in the private housing market, the investors, who are mostly comprised of individual investors and households with a lower ability to obtain useful and complete information, are characterized by being more susceptible to sentiment. That is to say, compared with other sectors of the real estate market, the private housing market is sentiment-based with high potential. Second, Clayton et al. (2009) state that the liquidity of the private commercial real estate market has substantial disparity from that of public stock markets, which leads to high segmentation of the market and asymmetry of information. In contrast, the private housing market shows higher liquidity than other sectors in the real estate market, whose amount is close to the magnitude of the stock market. Moreover, the restriction on short-selling in the housing market confines the ability of market regulation to eliminate mispricing. Therefore, such a limitation could render a deviation in asset pricing in the market influenced by investor sentiment (Clayton et al., 2009; Hui et al., 2013b). However, there is very little literature that has addressed the issue of investor sentiment in the private housing market. Therefore, this study aims to investigate investor sentiment and provide a sophisticated approach to construct a sentiment index. Furthermore, the predictabilities of the sentiment index on price level, the return rate of price and trading volume (liquidity) are tested. More specifically, an equilibrium model of the dynamics of the housing market is established to capture both short and long-term sentiment relationships between sentiment and housing price, as well as other market fundamentals.

Accordingly, for the issue of sentiment indicator, some confidence indices have been released by various institutes and authorities across the international markets (Xu et al., 2010; Marcato and Nanda, 2016). In general, the index reflects the grade of the status shifting in a specific real estate market. The index is normally established by applying quantitative statistics to some fundamental economic indicators. Since the indicators are selected by experts to describe the situation in a certain market, it is inevitable that the variables will be either objective or subjective.

Consequently, investor sentiment needs to be effectively measured to reflect the confidence of investors in the market so that people can monitor the future expectations of investors, estimate the market perspective and make predictions on the trend of property prices. In this chapter, a model is established to measure investor sentiment. Firstly, the study follows the spirit of Tay et al. (2009) and extends it into an approach to form a sentiment index. This approach is different from the traditional method adopting regression on sentiment by some fundamental proxies. Distinctly, this approach is convenient for avoiding a controversial choice of sentiment proxies from fundamental market factors. Meanwhile, by using transaction records in a market, the investor behaviour in the market can be captured directly. In Hong Kong, the average number of transactions in the private housing market is more than 330 per day (Table 3.1). Data with such a high frequency can assist in the construction of a sentiment index. Furthermore, this study explores the predictability of sentiment to property price and trading volume. The former reveals the future trend in price level while the latter reflects the liquidity of the market.

The remainder of this chapter is structured as follows. Section 4.2 presents a literature review. Section 4.3 introduces the approach to constructing the sentiment index. Section 4.4 introduces the data and initial statistics, while Section 4.5 provides the empirical findings and implications. The final section summarizes the concluding remarks.

3.2 Framework of Index Construction

This section demonstrates the framework for establishing the sentiment index. According to the definition, investor sentiment, the feeling or tone of a certain market environment, can actually affect the transactions initiated by investors and thus the price movements of the assets. The market sentiment is therefore the aggregate of individual investor sentiment. Since sentiment is integrated into investors' decision-making process, sentiment plays an important role in the waiting time to the next transaction.

In the real estate market, the hedonic model/method is highly recommended in the construction of the price index (Goh et al., 2012), but it is important to realize that the hedonic pricing method is based on the fact that prices of properties are highly related to their attributes. However, nowadays sentiment plays an important role in property pricing in the real estate market (Clayton et al., 2009). Hence, hedonic regression is inappropriate for sentiment indexing because sentiment is an intangible idea rather than tangible property. Furthermore, the lack of literature focusing on index construction for sentiment has led to a knowledge gap. Therefore, this study erects a novel and feasible approach from a different perspective. Since sentiment is rooted in decision and eventually in transaction, the approach depends on transactions, such that, compared to the traditional approach (such as principal component analysis employed in Baker and Wurgler, 2006) this approach urges the sentiment index to come close to the market behaviour.

The key implication of this approach is that, among all of the participants in the market, some investors are driven by sentiment in their decision-making processes. Instead of distinguishing the investors, this study attempts to identify whether or not the transaction is driven by sentiment. Therefore, the approach aims to detect the likelihood of sentiment driving every transaction. Afterwards, the index explores the aggregate probability in a unit period (e.g. a month or a quarter) by summing up all of the probabilities of sentiment-based transaction. Accordingly, a general expression is given to describe the probability of sentiment-based transaction (PST) in a unit period. The transactions are categorized into two classes: trivial (rational) and sentiment-based trading, such that PST can be expressed as

$$PST = \frac{\text{Expected number of sentiment−based trades}}{\text{Expected total number of trades}} \quad (1)$$

Consequently, the sentiment index is compiled as a spread comprised of two opposite probabilities of sentiment (% of positive vs. % of negative).

$$PST = PST^P + PST^N$$

The equation reveals the sentiment-based trading induced by both positive and negative sentiments that arrive in a period. The spread comprises the two probabilities (PST^P vs. PST^N) that could reveal the investor sentiment. Simply, the sentiment index can be shown in the form of PST^P/PST as it shows the proportion of positive sentiment to the total. In this case, sentiment index fluctuates in the range of (0, 1).

Since transaction price and trading volume were not involved in the construction of the index, this study takes advantage of this to avoid possible co-linearity between the index and market fundamentals, including price, rent and trading volume. We may provide evidence for the predictability of index on price level, return rate of price and trading volume in the application section. Furthermore, the index and its implications may serve as a reference for the relevant authorities by providing an accurate and reliable measure of market sentiment changes in the private housing market, in order to stabilize and improve the environment of the housing market.

3.3 Model of Index Construction

This section introduces the model of the index in detail. Firstly, the model presents the measurement of the probability of a sentiment-based transaction. Applying the asymmetric autoregressive conditional duration (*AACD*), the model proposes to estimate the expectation of the inter-arrival time of the transaction. *AACD* is based on the model introduced by Bauwens and Giot (2003). Second, the model concerns aggregate numbers of two-side orders in a unit time to estimate the probability of sentiment-based trading (*PST*) based on the framework of Tay et al. (2009). The model extends to be closer to reality by allowing the probabilities of the sentimental period to vary in each period.

3.3.1 Probability of sentiment-based transaction

Initially, the sentiment in the unit period involved in each transaction is characterized by trivial (T), positive sentiment (P), and negative sentiment (N), so the set of states is defined by S = {T, P, N}. In terms of the perspective of the investor, the transactions can be divided into buy and sell. Following the idiomatic assumption in financial market that the arrival of a two-sided order in the unit period of trading follows independent Poisson distributions (e.g. Easley et al., 2002), some basic notations are introduced as follows:

Probability in unit period	Notation
Probability of trivial sentimental period	q^T
Probability of sentimental period	$1 - q^T$
Conditional on sentiment impact, the probability of negative sentiment	P^N
Conditional on sentiment impact, the probability of positive sentiment	$1 - P^N$
Probability of positive sentiment	$q^P = (1 - q^T)(1 - P^N)$
Probability of negative sentiment	$q^N = (1 - q^T)(P^N)$

Following the framework of Easley et al. (2002), the intensities of two-sided orders follow Poisson processes, which are denoted by λ_{-1} (for sell) and λ_1 (for buy) respectively, and each λ is constant throughout a unit period. When the investor has positive sentiment, the intensity of the buying order is stimulated and defined as λ_1^P, while the selling intensity remains as λ_{-1}. Likewise, with negative sentiment, the intensity of the selling order is stimulated and defined as λ_{-1}^{N} whereas the intensity of the buying order remains as λ_1. Based on these assumptions, the expected number of sentiment-based trades (*EST*) in period *d* is equal to

$$EST = \sum_i \left(q_P \lambda_{1,i}^P + q_N \lambda_{1,i}^N\right) x_i$$

where x_i denotes duration of the i-th trade, i.e. $x_i = t_i - t_{i-1}$, and the total expected number of trades (*TET*) in period *d* is

$$TET = \sum_i \left(q_P \lambda_{1,i}^P + q_N \lambda_{1,i}^N + \lambda_{1,i}^T + \lambda_{-1,i}^T\right) x_i$$

To sum up, in a specific period *d*, the probability of sentiment-based trading can be estimated as follows,

$$PST_d = EST/TET \qquad (2)$$

3.3.2 Conditional duration

The intensity of the latent Poisson process is structured by the parameter $\lambda_{j,i} = 1/\psi_{j,i}$. This section introduces the approach used to estimate the expectation of inter-arrival time ($\psi_{j,i}$) by using a multi-state duration model. As suggested by Tay et al. (2009), this model is associated with trading directions. To identify the direction of each transaction, this study employs the method introduced by Lee and Ready (1991). Let td_i denote the direction of the i-th trade such that $td_i = -1$ (1) denotes selling (buying) respectively. Accordingly, assume each buying/selling follows a stochastic process such that its waiting time follows an independent exponential distribution. Such an assumption of waiting time interval could be relaxed to more general distributional assumptions, such as member distributions from the exponential family. The information set ϕ_{i-1} consists of all of the records of trading direction, trading volume, and lagged duration until the (i-1)th trade. Given the information set ϕ_{i-1}, it is assumed that the expected mean waiting time is $\psi_{j,i}$ with j denoting the latent trading direction of the i-th trade. In addition, the duration of the i-th trade is denoted by $x_i = t_i - t_{i-1}$. Based on these assumptions, the model adopts the autoregressive conditional duration (*ACD*) model introduced by Engle and Russell (1998) and establishes a logarithmic conditional expectation of waiting time (c.f. Bauwens and Giot, 2003) as follows:

$$\log\psi_{j,i} = v_{j,-1}\mathbf{1}_{-1}(td_{i-1}) + v_{j,1}\mathbf{1}_{1}(td_{i-1}) + a_j\log\psi_{j,i-1} + b_j\log x_{i-1}$$

where j = -1, 1.

Furthermore, to incorporate the duration equation into the *PST* framework, the equation of expected duration is extended to be based on the state $s \in S$ = {P, T, N}, containing positive sentiment, trivial sentiment or negative sentiment. At this moment, two states (P and N) indicate the trades driven by the sentiment. If positive sentiment is involved, participants prefer a long position, while if negative sentiment is involved, participants prefer a short position. Consequently, each of these two states will decrease the duration and increase the trading intensity. Besides, the selling duration with positive sentiment and the buying duration with negative sentiment can be perceived as trivial trade.

Given the information set ϕ_{i-1}, $w_{j,i}^S$ denotes the conditional expected duration of trading direction j in state $s \in S$.

$$w_{j,i}^S = v_{j,-1}\mathbf{1}_{-1}(td_{i-1}) + v_{j,1}\mathbf{1}_{1}(td_{i-1}) + a_j\log\psi_{j,i-1} + b_j\log x_{i-1} + s_j\log PROXY_{i-1} \qquad (3)$$

for $j = -1$, 1 and $s \in S$. Obviously, for the housing market, the transaction direction, the lagged real and expected duration, as well as the lagged exogenous matrix (*PROXY*) play roles in $w_{j,i}^S$. A branch of studies on movements of the property market (e.g. Hui and Shen, 2006) suggest that the stock market always has a leading effect on short-term dynamics of the housing market. In addition, Leung et al. (2013) contend that the stock market is a significant indicator of sentiment in the housing market. Hence, the local stock index is a reasonable proxy in equation 3 for the regional market.

Intrinsically, following the sense of bias investors in the framework of Tay et al. (2009), asymmetric adjustment on sentiment-based transaction is meaningful. As such, s = T for a scenario of investor trading under trivial sentiment and thus, all trading is deemed as normal trading for the duration.

$$\log\psi_{j,i}^{T} = w_{j,i}^{T} \qquad j = -1,1$$

Conversely, for a scenario of investor trading under trivial sentiment, if s = P indicates a positive sentiment, there should be an adjustment in the long-order in the buy-initiated duration equation.

$$\log\psi_{j,i}^{P} = w_{j,i}^{P} - \mu\mathbf{1}_1(j) \qquad \mathbf{1}_1(j = 1) = 1$$

Similarly, is s = N for a negative sentiment, there should be an adjustment in the short-order in the sell-initiated duration equation.

$$\log\psi_{j,i}^{N} = w_{j,i}^{N} - \mu\mathbf{1}_{-1}(j) \qquad \mathbf{1}_{-1}(j = -1) = 1$$

The implication of an increase (decrease) in the conditional expected duration $\psi_{j,i}^{s}$ indicates a lower (higher) intensity as $\lambda_{j,i} = 1/\psi_{j,i}$, so as to depress (raise) the probability of the next transaction initiated by the investor in direction j.

3.3.3 Sentiment index

Returning to equation 2, a more precise inference regarding the probabilities of sentiment-based trading can be explored by decomposing *EST* into two sub-terms:

$$\sum_i\left(q_d^P\lambda_{1,i}^P + q_d^N\lambda_{-1,i}^N\right)x_i = \sum_i\left(q_d^P\lambda_{1,i}^P\right)x_i + \sum_i\left(q_d^N\lambda_{-1,i}^N\right)x_i$$

where PST_d^P is the probability of positive sentiment-based trading and PST_d^N is the probability of negative sentiment-based trading. Thus we have,

$$PST_d = PST_d^P + PST_d^N$$

The above equation shows the sentiment-based trading driven by both positive and negative sentiments that arrive in a period. With the time-varying probability of the sentimental period, the spread comprising the two probabilities (PST_d^P vs. PST_d^N) could reveal the investor sentiment.

3.4 Model Estimation

The parameters in the model of the index construction can be estimated by the maximum likelihood estimation (MLE) method. An improvement in the estimation of PST is to be made by a time-varying feature.

Given the information set Φ_{i-1}, the conditional joint density of (x_i, td_i) associated with states of sentiment is as follows:

$$p_i(x_i, td_i|\phi_{i-1}) = \prod_{j=-1,1}\left(\left(\frac{1}{\psi_{j,i}^s}\right)^{\mathbf{1}_j(td_i)} exp\left(\frac{-x_i}{\psi_{j,i}^s}\right)\right) \quad (4)$$

We transform equation 4 into the form of a Poisson process. With the parameter $\lambda_{j,i} = 1/\psi_{j,i}$ in the Poisson process, the conditional joint density, as in Bauwens and Giot (2003), can be also given by

$$p_i(x_i, td_i|\phi_{i-1}) = \lambda_{j=td,i} exp\left[-\left(\lambda_{-1,i} + \lambda_{1,i}\right)x_i\right]$$

Equation 4 indicates that x_i and td_i are independent in condition of Φ_{i-1}. Given N samples of observation (x_i, td_i), the parameter vector Θ in the model can be estimated by MLE as

$$L[\Theta|(x_i, td_i), \dots (x_n, td_n)] = p[(x_i, td_i), \dots (x_n, td_n)|\Theta] = \prod_{i=1} p(x_i, td_i|\Theta)$$

In this case, as q^s denotes the probability of the states mentioned above in period *d*, the total likelihood function (*TLF*) is written as

$$TLF = \prod_d^D\left(\sum_{s=P,T,N} q^s\left(\prod_{j=-1,1} p_i(x_i, td_i|\phi_{i-1})\right)\right) \quad (5)$$

Then the estimation of the parameters can be obtained by: $\max_{q,\psi|\phi_{i-1}} \{TLF\}$.

The model can be handled with time-varying probabilities of sentimental period (q^s) rather than constant probabilities. Such a feature of probability of sentiment can be modelled in several ways and then substituted into equation 5. As q^s is developed into time-varying probability as q_d^s, thus the extended *TLF* can be written as

$$TLF = \prod_d^D \left(\sum_{s=P,T,N} q_d^s \left(\prod_{j=-1,1} p_i(x_i, td_i | \phi_{i-1})\right)\right) \qquad (6)$$

Similar to equation 2, in period *d*, the *PST* with the probability of sentiment varying with *d* can be written as

$$PST_d = {EST(q_d^s)} \Big/ {TET(q_d^s)} \qquad (7)$$

Previous studies have discovered the effect of investor sentiment on trading volume (Baker et al., 2012). Recognizing this, it is reasonable that the time-varying probability of sentiment q^s can be perceived as a function of volume. Let V_d^B and V_d^S denote the amount of buying and selling lots in day *d* respectively. Then a Pareto distribution is constructed for the probability as follows.

The Pareto approach with only one parameter (*k*) has the advantage of reducing the complexity of the MLE method compared with other distributions (Hui et al., 2013a).

The Pareto distribution is firstly derived to describe the possession of aggregate wealth in a society. It is widely accepted that the distribution shows well that a minority of people control a large portion of the wealth in any society. Information as intangible wealth also conforms to the principle as the minority of traders are informed but crowds make their decision on the basis of an incomplete information set. Therefore, herd behaviour induces market sentiment.

Let *k* be the parameter of the Pareto distribution. In a unit period *d*, assume the probability of trivial sentimental period to be

$$q^T = 1 - \left[1 - \left(\frac{\min(V_d^B, V_d^S)}{V_d^B + V_d^S}\right)^k\right] = \left(\frac{\min(V_d^B, V_d^S)}{V_d^B + V_d^S}\right)^k \qquad (8)$$

The parameter constraint of the Pareto distribution requires $k > 0$. In addition, we have the following implication:

Remark 1. $k < 1$ implies that the transaction is prone to a high probability of being initiated without sentiment. In contrast, $k > 1$ implies that the investor is more susceptible to sentiment.

According to this remark, consider k as an indicator of sentiment and thus investor sentiment plays a significant role in the market when $k > 1$.

In period d, the probability of positive sentiment should be

$$q^P = (1 - q^T)\left(\frac{V_d^B}{V_d^B + V_d^S}\right) \quad (9)$$

Again, higher buying intention indicates a higher probability of positive sentiment, and contrarily, higher selling intention refers to a higher probability of negative sentiment. Meanwhile, the equation implies that if the number of buying orders is the same as that of selling orders, then the probabilities of the two sentiments are equal to a half.

3.5 Application

3.5.1 Empirical data

In the empirical application, using an enormous amount of data this study applies this approach to index the market sentiment for the private housing market in Hong Kong. First of all, the transaction data are from the EPRC database,[3] including every single private domestic transaction record between Jul 1991 and Dec 2011, as total of 2 077 957 observations. Two entries are involved in our model for each record: dealing time and dealing price. The former is used directly in the calculation of trading duration and the other is used to identify the trading direction according to the method in

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[3] The EPRC ltd. provides the EPRC database (http://eprc.com.hk), which contains all of the transaction records registered in Land Registry, HKSAR, since 1991. The EPRC ltd. is a member of HKET (Hong Kong Economic Times) Holdings, specializing in property information services in Hong Kong. The Land Registry is affiliated to the government of HKSAR and has a duty to maintain an efficient and effective land registration system to facilitate the orderly conduct of land transactions. Every transaction that occurs in HKSAR has to register a document of agreement at the Land Registry.

Lee and Ready (1991). The index has the capacity for sustainable update with time advance.

For the Hong Kong market, the Hang Seng Property Index (HSPI) is more concentrated than its mother index, i.e. the Hang Seng Index (HSI), on the property market. Therefore, equation 3 is attached by HSPI as a proper proxy for the last term ($PROXY_{i\text{-}1}$). Thus, the model of sentiment index involves the transaction data for private domestics and the Hang Seng Property Index (HSPI). HSPI is daily data collected from DataStream (provided by Thomson Reuters) so that the duration is consistent with the transaction data. Figure 3.1 displays the curve of HSPI in the period 1990-2011.

The price index (PI) of private domestics is issued by the Rating and Valuation Department (RVD), which is affiliated to the government of the Hong Kong Special Administrative Region (HKSAR). Both monthly and quarterly data were collected from 1991 to 2011 and used respectively in the two sections of the empirical study. The index actually includes three series; two are sub-indices and last one is an integral index. Each sub-index is sub-divided by reference to floor area. The Type 1 sub-index consists of units with a saleable area of less than $100m^2$. The Type 2 sub-index consists of units with a saleable area of $100m^2$ or above since these units are defined as luxury units. The integral index is compiled from two sub-indices. The price index measures the value changes to reflect the integral level of private domestic price at the time. The data were obtained from an open source published on the official website of RVD from 1993. Figure 3.2 shows the curve of the price index of private domestics.

Figure 3.1. The time-series chart of Hang Seng Property Index (Daily 1990-2011) Sources: DataStream

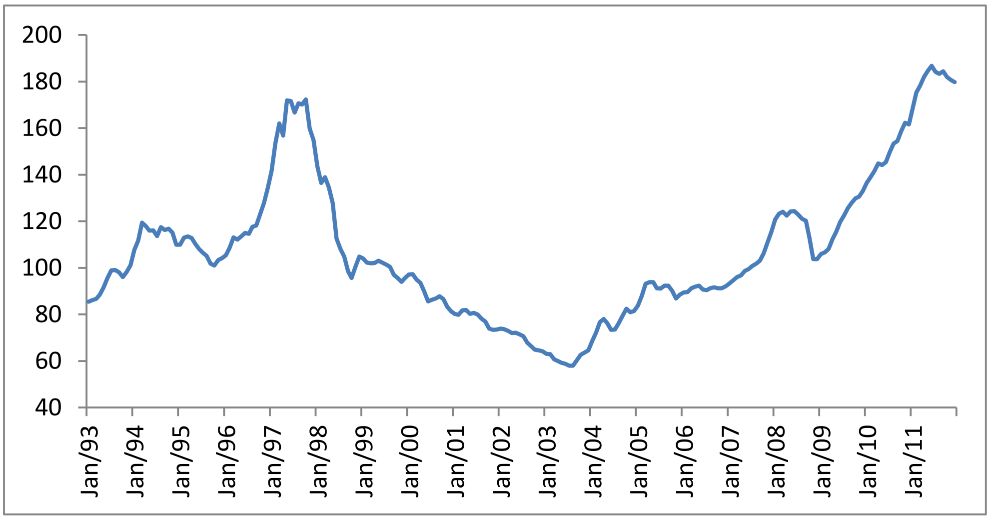

Figure 3.2. The time-series chart of price index of private domestic (Monthly 1993-2011) Sources: RVD (HKSAR).

Another four economic variables as market fundamentals are employed in our study. These were collected from the Census and Statistics Department (CSD, HKSAR) and are GDP, average income index (AII), Stock of permanent living quarters (STK) and Hong Kong interbank offered rates (HIBOR). It is reasonable to include all of these variables in the macroeconomic analysis of the tendency of housing prices according to some of the literature focusing on the Hong Kong market. The average income index (AII) is deflated by the composited consumer price index and is introduced as a descriptive indicator for the aggregate income level of households (Hui et al., 2012a). STK is an efficient index of housing stocks and it represents the total number of residential units in Hong Kong (Mesthrige and Hui, 2012). Hence it takes into account the unit numbers of newly completed residential housing and demolished housing at every time point. The HIBOR stands for the real riskless interest rate in Hong Kong (Hui et al., 2014).

Table 3.1 shows the summary statistics of the original data used in the establishment of the sentiment index. Panel A summarizes the statistics of the transaction data. The number of observations in the sample for buy-initiated trades is 1080275, slightly more than the number of sell-initiated trades. The average number of transactions in a day is mounted into 337.50. Clayton et al. (2009) suggest that the liquidity of the private commercial real estate market has substantial disparity from that of public stock markets. In contrast, the private housing market shows higher liquidity than other sectors in the real estate market. Panels B and C summarize the descriptive statistics of HSPI and PI of private domestics respectively. Two groups of descriptive statistics imply a lot of variation in the sample of HSPI and PI during the last two decades.

Table 3.1. Summary Statistics, 1991-2011

Panel A: transaction data of private domestic: 1991-2011			
Statistics	total	buy	sell
number of observations	2077956	1080275	997682
Statistics of trade			
average number of trades in a day	337.50	175.45	162.04
average number of trades in a month	8446.98	4391.36	4055.62
Statistics of duration (in seconds)			
average duration	423.23	423.42	423.06

Panel B: Hang Seng Property Index (HSPI): 1991-2011, daily.		
Statistics	Raw	return rate
Mean	18145.20	0.00009
Std.D.	6519.37	0.00824
Min	6287.01	-0.06199
Max	39540	0.08983

Panel C: Price Index of private domestic (PI): 1993-2011, monthly.						
	Integral PI		Type1 PI		Type2 PI	
Statistics	raw	return rate	raw	return rate	Raw	return rate
Mean	108.06	0.00335	107.32	0.00326	124.16	0.00499
Std.D.	31.02	0.02868	30.89	0.02877	40.61	0.03370
Min	58.4	-0.12595	57.9	-0.1275	67.4	-0.10660
Max	188.1	0.09299	186.8	0.09252	228.4	0.12073

Panel D: Macroeconomics Factors: 1993-2011, quarterly.					
Statistics	PI	GDP	AII	STK	HIBOR
Mean	108.54	348.39	98.27	2278.56	3.20
Std.D.	31.50	65.23	11.89	237.60	2.27
Min	59.3	209.71	74	1843.8	0.07
Max	188.1	517.78	117.9	2616.5	7.13

The following subsections outline the findings from the empirical application, with respective elaborations on their economic implications. Firstly, following this model, this study demonstrates the outcome of the pilot processes and establishes the sentiment index, to fulfil one of the main purposes of this study. Furthermore, to improve the economic theory proposed in the first section, some evidence from granger causality tests reveals that the sentiment index has significant predictability of future prices in the housing market.

3.5.2 The Sentiment Index

Table 3.2 reports the coefficient estimation using maximum likelihood estimation (MLE) in the model for estimating the conditional expected duration $\psi_{j,i}$ and establishing the sentiment index. In equation 3, there are four coefficients of constant adjustment on expected duration from the last transaction. At present, the four estimated coefficients can be ordered as $v_{-1,1} > v_{-1,-1} > v_{1,1} > v_{1,-1}$. The highest value is the buy-to-sell situation ($v_{-1,1}$) recorded as 0.24238, implying that switching trading positions from buy-to-sell shows the greatest intensity to enlarge the expected waiting time to the next transaction. In contrast, the sell-to-buy situation gives the lowest value, showing a relatively accelerating effect on the next transaction. Comparing the group ($v_{-1,1}$ and $v_{-1,-1}$) with group ($v_{1,1}$ and $v_{1,-1}$), the constants of to-buy are strictly less than the constants of to-sell. Such a situation implies a lower probability of the first arrival of selling purpose at the next transaction. In view of the sentiment, the investor expects the buying position to be stronger than the selling position.

The coefficients of proxy, i.e. the return rate of HSPI, at time i-1, have different impacts on the conditional expected duration equation. For both the buying and selling positions, s_j with a positive value indicates that a higher return rate leads to a longer expected duration, while a lower return rate causes a shorter expected duration.

Table 3.2. Summary Statistics on Coefficient Estimations

variables	parameters	coefficient
Constant adjustment from (*i*-1)th to (*i*)th trade		
Buy to buy	v_{11}	0.13729
Buy to sell	$v_{-1,1}$	0.24238
Sell to buy	$v_{1,-1}$	0.12030
Sell to sell	$v_{-1,-1}$	0.19712
Coefficient of (i-1)th expected duration		
For buy	a_1	0.91472
For sell	a_{-1}	0.86869
Coefficient of (i-1)th observed duration		
For buy	b_1	0.06365
For sell	b_{-1}	0.08493
Coefficient of (i-1)th Proxy: HSPI		
For buy	s_1	0.03988
For sell	s_{-1}	0.04011
Coefficient of adjustment by given sentiment		
coefficient	u	0.10948
Probability of sentiment (equation 8)		
Coefficient	k	1.34038

Since u has been assigned to denote the asymmetric adjustment based on sentiment, a positive value of u equals to 0.10984 as expected. This implies that for a given investor sentiment, sentiment affects the expected waiting time only on a specific side. More specifically, positive sentiment shortens the time in a buy-order and negative sentiment shortens the time in a sell-order. The coefficient k in equation 8 means that an investor is prone to a low probability of trading activity being initiated by sentiment when $k < 1$ and an inverse implication when $k > 1$. Therefore, the result that k is equal to 1.34038 implies that sentiment plays a significant role in investors' decision making in private domestic transactions.

Table 3.3. Summary Statistics on Sentiment Index
Monthly, 1991-2011

Statistics	PST_d	*PST*-positive	*PST*-negative	Sentiment index
Mean	0.33585	0.18774	0.14811	0.55587
Std.D.	0.02325	0.04919	0.03954	0.11824
Min	0.30266	0.10866	0.06514	0.29546
Max	0.40816	0.34302	0.25912	0.84042

Table 3.3 reports the statistics of the sentiment index generated by the model. The total *PST* varies from a low of 0.30266 to a high of 0.40816, which gives an overall view of investor sentiment affecting private domestic transactions during the period 1991-2011. Figure 3.3 illustrates the curve of probability of sentiment-based trade. As mentioned in the section "Model", since the sentiment spread includes two opposite probabilities: positive-sentiment (*PST*-positive) and negative-sentiment (*PST*-negative), the sentiment index is employed to be a ratio showing the proportion of positive-sentiment to total sentiment. In respect that the mean of the sentiment index (0.55587) over twenty years is larger than 0.5, this indicates that positive sentiment is incorporated in majority of transactions by investors. As the sentiment index varies with a large interval [0.2955, 0.8404], the standard deviation being 0.11824 reveals that there is a lot of fluctuation in the time series of the sentiment index, which is consistent with the view in Figure 3.4.

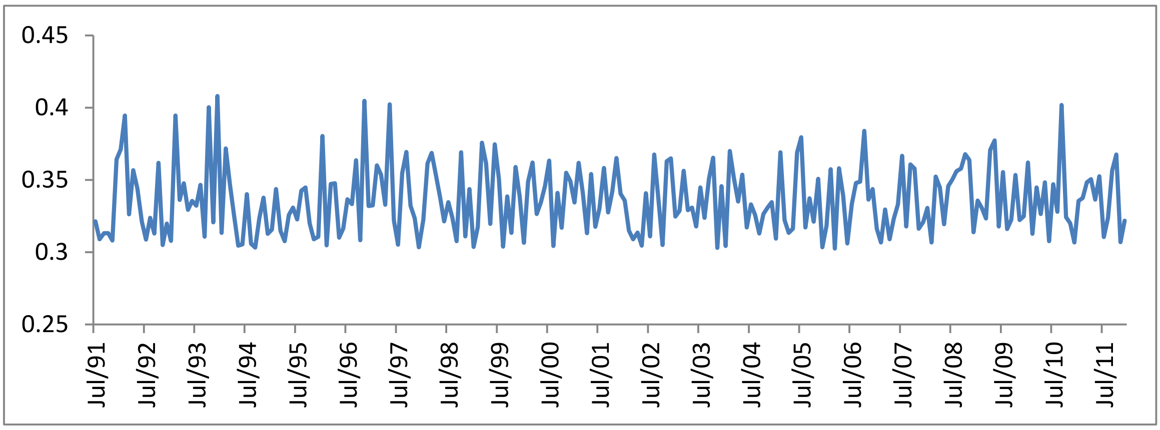

Figure 3.3. The time-series chart of PST_d (Monthly, 1991-2011)

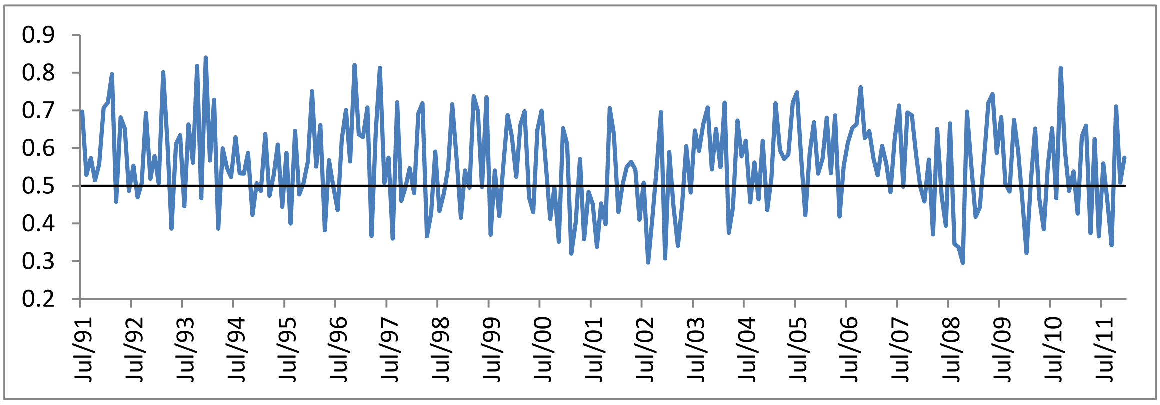

Figure 3.4. The time-series chart of sentiment index (Monthly, 1991-2011)

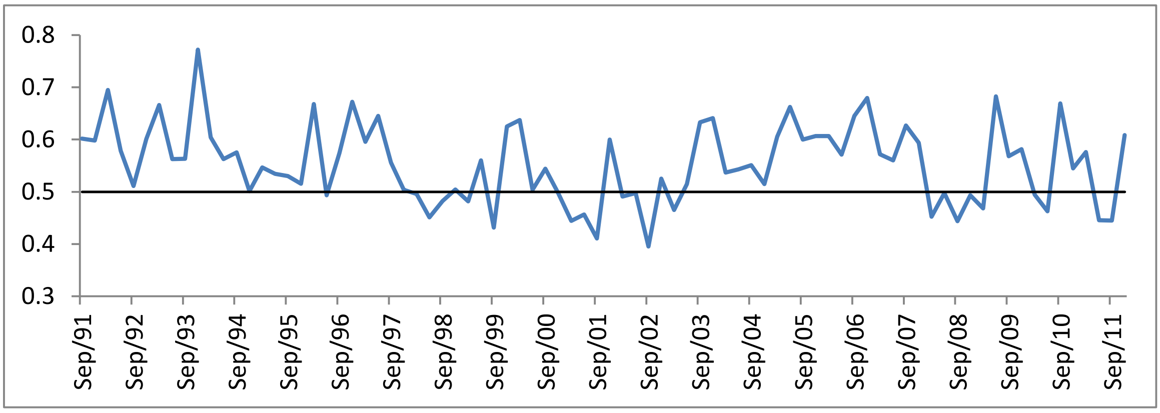

Figure 3.5. The time-series chart of sentiment index (Quarterly, 1991-2011)

Market sentiment is captured in both monthly (shown in Figure 3.4) and quarterly (shown in Figure 3.5) series. Figure 3.4 and Figure 3.5 reveals that sentiment fluctuates between positive and negative. Figure 3.6 combines

Figure 3.2 and Figure 3.4 to illustrate the correlation between sentiment index and price index.

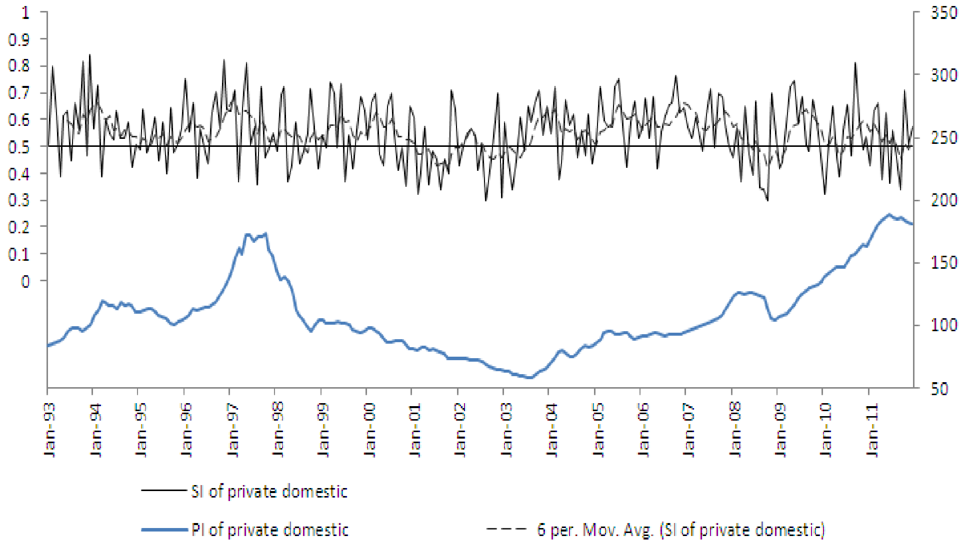

Figure 3.6. The time-series chart of sentiment index and price index (Monthly, 1993-2011). Notes: the dashed line approximates the trend of the sentiment index. The left y-axis measures the value of sentiment index and the right y-axis measures the value of price index.

The mainstream of investor sentiment changes around two financial crises and hence experiences five switches. The sentiment index going up indicates that a positive sentiment dominated investor transactions before 1997, and the boom in price led to prosperity in the housing market. The period 1990-1997 has always been considered a period of bubble formation in Hong Kong's real estate market. The sentiment index falls from 1997-2003; the whole housing market experienced a downturn in the aftermath of the Asian financial crisis. From 2003 to early 2008, positive sentiment strictly dominates in the market of private domestics. Immediately following this rebound, the subprime crisis destroyed the market and this was responsible for the negative sentiment in the year 2008.

The recovery of Hong Kong's economy pulled up the housing market after 2009 and resulted in the domination of positive sentiment. Referring to the statement in Baker and Wurgler (2007), an index does well with capturing sentiment if it succeeds in depicting the tracks of bubbles and crashes. Thus our sentiment index can be appraised as an efficient index.

3.5.3 Short-term predictability

Earlier studies appear to support the notion that investor sentiment has the possibility of predicting future aggregate market returns in the stock market (Baker and Wurgler, 2006, 2007; Baker et al., 2012). Accordingly, for the one of main purposes of this study, a hypothesis is proposed to test whether the sentiment index has predictability for several variables as economic factors in the private housing market.

Hypothesis: investor sentiment (sentiment index) has a forecasting force on price, return rate of price and trading volume (liquidity).

A fully-fledged way to investigate the function of the sentiment index in predicting housing prices is the Granger causality test. The Granger causality test is a statistical test with an alternative hypothesis for identifying whether one time-series is significant in forecasting another. Series X can be considered to Granger-cause Y if it can be shown by testing on lagged values of X (and with lagged values of Y also included), that those lagged X values provide statistically significant information about future values of Y. Table 3.4 shows the results of several pairs of Granger Causality tests. The lag, which was chosen to be 3 (in this case, it means a 3-month lag), is optimally determined by lag order selection criteria (Akaike information criterion, AIC and Schwarz information criterion, SC).

Table 3.4. The Results of Granger Causality Tests
Lag: 3

Null Hypothesis (H_0)	F-statistic	P-value	Reject H_0
SI *does not Granger cause* PI	17.9263	0.0000	**Y**
PI *does not Granger cause* SI	0.1072	0.8984	N
SI *does not Granger cause* PI_1	17.2105	0.0000	**Y**
PI_1 *does not Granger cause* SI	0.0836	0.9199	N
SI *does not Granger cause* PI_2	15.2143	0.0000	**Y**
PI_2 *does not Granger cause* SI	0.7317	0.4823	N
SI *does not Granger cause* ΔlnPI	22.0092	0.0000	**Y**
ΔlnPI *does not Granger cause* SI	0.2389	0.7877	N
SI *does not Granger cause* ΔlnPI_1	21.2535	0.0000	**Y**
ΔlnPI_1 *does not Granger cause* SI	0.2127	0.8085	N
SI *does not Granger cause* ΔlnPI_2	17.1384	0.0000	**Y**
ΔlnPI_2 *does not Granger cause* SI	0.8816	0.4156	N
SI *does not Granger cause* Volume	19.7550	0.0000	**Y**
Volume *does not Granger cause* SI	1.5036	0.2246	N
PI *does not Granger cause* Volume	1.7278	0.1622	N
Volume *does not Granger cause* PI	2.8415	0.0388	**Y**

Notes:
1. SI stands for sentiment index. PI stands for price index of private domestic. PI_1 and PI_2 stand for two sub-indices: Type1 and Type2. Volume stands for the number of transaction records in a period. All series are monthly data.
2. ΔlnPI is first differential series of logarithm PI. In other words, ΔlnPI is the continuous return rate of PI. So do ΔlnPI_1 and ΔlnPI_2.

All of the null hypotheses that the sentiment index does not Granger cause a certain indicator have been rejected. Such results suggest that the sentiment index is of informative in predicting three types of price index, three types of continuous return rates and trading volume. This could benefit either investors or policymaker in the housing market. First of all, the sentiment index exhibits its power of forecasting the return rate in the housing market, which is consistent with sentiment effect in the stock market. It is meaningful to expand the field in which the predictability of sentiment can be engaged. Meanwhile, the sentiment index has the ability to directly predict prices in the housing market. It is reasonable to state that investor sentiment is indeed a significant factor that cannot be ignored in asset pricing. On the other hand, the private housing market in Hong Kong is one of the most unpredictable property markets in the world due to its volatile features (Wong et al., 2005). More specifically, the fundamental analysis is less able to do well in explanation and prediction (Chan et al., 2001). Therefore, this study shows significant implications of the sentiment index, which can be employed for the analysis of the housing market in Hong Kong.

On the other hand, this study just complements the finding in Baker and Stein (2004), which confirms a predictive power of liquidity on investor sentiment. That is, the sentiment index is able to forecast the trading volume, which in turn, can forecast the price index. This explains the phenomenon that investor expectation precedes changes in trading volume, which leads to changes in price. Such a chain of actions can be summarized by conductivity among sentiment, liquidity and price in the dynamic of the housing market in Hong Kong.

Overall, due to the difficulty of identifying the useful information, or simply the asymmetric information, the investors attempt to herd behaviour, which seems not to be rational if it is judged by classical theory. In addition, the investors appear to herd more when facing fierce variation in housing prices (Hui et al., 2013b). Therefore, the sentiment (and its index) should be considered as requisite factors for economic analysis of the housing market and furthermore in all economics.

3.6 Conclusion

This chapter introduces a novel approach for establishing an index to capture investor sentiment in private housing markets. The approach focuses on the waiting time between every two transactions, and then uses econometric tools to estimate the expected duration of housing transactions in order to model the probability of sentiment-based trading. By using detailed transaction records over a period of twenty years (1991-2011) in the private housing market and applying the approach, the sentiment index is established for Hong Kong. The salient findings of this study are as follows. In Hong Kong, an overview of the sentiment index during the period describes the relative domination of positive sentiment in the private housing market. Generally, position-switching from buy to sell has a delaying effect on the expected waiting time (duration) for the next transaction. The proxy, HSPI, will delay (expedite) the next transaction when the return rate is positive (negative). In addition, the parameter k in the model is greater than 1, which implies that investors in the housing market are more likely to be sentiment-influenced. Our sentiment index is valid because of its efficiency in depicting the tracks of bubbles and crashes.

Additionally, the role of sentiment in the dynamics of housing prices proves the significance of short-term predictability. The analysis of the predictability of sentiment shows that the sentiment index is indicative of a host of key economic forces, such as price, return rate of price and trading volume (liquidity) in the private housing market.

The contribution of the study in this chapter is four-fold: (1) It introduces a new approach to forming a sentiment index by using detailed transaction records from relevant government sources. This approach is superior to traditional regression analysis (such as the principal component analysis in Baker and Wurgler, 2006), which may cause collinearity and is very sensitive to outliers. (2) This new sentiment index can provide significant findings to address our research objective, i.e. to identify the sentiment susceptibility of market (by reference to parameter k) and to investigate the effect of sentiment on the dynamic of private housing market (our application bases in Hong Kong). (3) Our new approach to indexing can be applied to other markets. For instance, it can be applied to the stock market by using high-frequency data. (4) It can be used to hypothesize, test and confirm the short-term predictability of a sentiment index to the price level and market liquidity.

On the one hand, this study enriches the literature on investor sentiment and can benefit investors who are concerned about the expectation of housing investment. On the other hand, the sentiment index may serve as a good basis for monitoring the housing market for relevant authorities and policy makers.

Chapter[*]

4 FUNDAMENTALS AND MARKET SENTIMENT

This chapter explores the macro effect of sentiment on the movement in the housing market, in order to achieve Objective 2. Specifically, the study first examines the predictability of sentiment on housing prices and rents in the short-term and then explores the exploratory power of sentiment on the long-term market equilibrium in both sales and rental sectors.

4.1 Introduction

The dynamics of housing prices have always drawn scholarly attention. Since houses are a durable product and their prices are fully determined as an asset, classic theory states that the equilibrium between demand and supply determines the price in the housing market (DiPasquale and Wheaton, 1996). Case and Shiller (2003) suggest that other than some market fundamentals, excessive market expectation also plays an important role in the rapid appreciation of housing prices. Expectation affects prospective housing demand and then affects the price. Irrational expectations can even lead to fluctuations that drive the price away from market fundamentals in property markets (Jin et al., 2014).

Behavioural economics had led to studies aiming at theoretical modifications for asset pricing based on new assumptions. Sentiment, as an indispensable part of those assumptions, reflects a reference to psychology in modern economics and finance (DeLong et al., 1990), and as a

[*] This chapter has been published in Wang and Hui (2017). Fundamentals and market sentiment in housing market. *Housing, Theory and Society*, 34(1), 57-78.

non-fundamental factor, cannot be justified by market fundamentals (Baker and Wurgler, 2007). Some previous efforts have attached great importance to sentiment in property market. For instance, Clayton et al. (2008) suggest that sentiment can cause the divergence of property prices from their fundamental value. Hui et al. (2013b) add to their findings that sentiment is even responsible for some property mispricing. Sentiment plays a more persistent role in driving prices away from fundamental values in private markets (Ling et al., 2014). Freybote (2016) and afterwards Freybote and Seagraves (2017) find that institutional investors refer their investment decisions to the sentiment of specialized real estate investors.

As the housing market rides the cycle, the variation in housing prices cannot be fully explained by fundamentals (Jin et al., 2014) and some models appeal to an autoregressive pattern. The previous literature is devoted to the investigation of the effect of non-fundamental factors in the housing market. It arouses the authors' interest in exploring whether sentiment (and its past value) contains informative content that explains the non-fundamental movements in the housing market.

Differences between investors' expectations of housing prices and rents are identified in Wong et al. (2005). As rent is more "fundamental" than housing price, the degree of the effect of sentiment should differ between the sales market and the rental market. In addition, due to no short-selling, switching between renting and owning a house is the only method that a household can use to hedge the future risk in housing prices. Thus, this chapter intends to examine whether there is a linkage through which the sentiment effect in the sales market can be transmitted into the rental market.

This chapter begins with an analysis of the statistical causality between market sentiment and three other market indicators: price, rent and trading volume. By taking advantage of the IRPDC method (developed by Hui and Chan, 2012), the tests provide evidence showing the predictability of sentiment in the dynamics of the housing market. This chapter further investigates the explanatory power of market sentiment in the long-term trends for the two housing sectors, i.e. sales and rental.

This chapter has meaningful implications in two ways. First, if sentiment has the power to predict other market indicators, the results can offer a better understanding of how sentiment, as a non-fundamental factor, drives fluctuations in the housing market. Conversely, if the market indicators are found to affect sentiment, this should help identify the causal factor for the formation of market expectation and explain the phenomenon of "herding" behaviour. Second, sentiment should have different roles in the sales and rental markets as housing prices are observed to be more volatile than rents. This chapter not only sheds light on the relationship between sentiment and market movements, but also provides insights into how the market actually works in terms of the influence of sentiment. These implications can benefit not only investors in terms of investment decision-making but also housing authorities with regard to policy-making.

The rest of this chapter will proceed as follows. Section 5.2 discusses the different roles of market sentiment and the advantage of sentiment index used in this chapter. Section 5.3 reviews some related literature. Section 5.4 outlines the IRPDC method and theoretical model, as well as the data description. Section 5.5 elaborates on results and implications of causality. Section 5.6 discusses the effect of sentiment on the long-term trend of the housing market. Section 5.7 presents the concluding remarks.

4.2 Different Roles of Market Sentiment

4.2.1 Different roles of market sentiment

Investors in the sales market may have various purposes including home ownership, investment for rent and price appreciation, or even speculation. In comparison, the rental market is simpler: tenants consume housing services through renting rather than owning. Landlords (investors) provide units to let and expect reasonable and stable cash flows, i.e. rental income. As such, the demand side has less critical determinants in decision-making and thus the rental market reaches a new equilibrium point with less friction compared to sales market. This implies less fluctuation in rents compared with those in housing prices. In reality, it can be observed that price fluctuations always exceed rental ones (Wong et al., 2005). The subsequent hypothesis is that sentiment can render more profound impacts on the sales market than on the rental market. Since little research has addressed this issue, this chapter attempts to reveal whether the degree of impact of sentiment differs between rental and sales markets.

To summarize, this chapter has several specific research goals derived from objective 4 regarding the role of sentiment in the housing market: 1) to find out whether the three indicators, i.e. house price, rent and liquidity (trading volume) can be predicted by market sentiment; 2) to explore the differences in the direction and relative strength of the statistical causality of sentiment; and 3) to study the long-term effect of sentiment in the rental and sales sectors of the housing market.

4.2.2 The advantage of new sentiment index

In this chapter, market sentiment is captured and proxied by the sentiment index introduced in the previous chapter (see Chapter 3). This index contains monthly and quarterly indices for the private housing market in Hong Kong. The sentiment index used in this chapter is based on detailed data from over 2 million records starting from 1991, which cover almost all sale and purchase agreements for private residential units registered in the Land Registry[4].

A large body of studies support the forecasting power of the confidence or sentiment index: Vuchelen (2004) in Belgium, Utaka (2003) in Japan, Chua and Tsiaplias (2009) in Australia, Parigi and Schlitzer (1997) and Malgarini and Margani (2009) in Italy, Easaw and Heravi (2004) and Hohenstatt and Kaesbauer (2014) in the UK, and Jin et al. (2014) and Marcato and Nanda (2016) in the US. Marcato and Nanda (2016) summarize two prevailing methods to construct an index for sentiment in the real estate market: one is direct measurement based on a survey (e.g. Souleles, 2004; Baker and Saltes, 2005; Clayton et al., 2009; Hohenstatt and Kaesbauer, 2014) and the other is to form an indirect index (e.g. Baker and Wurgler, 2006, 2007; Baker et al., 2012; Marcato and Nanda) by selecting some underlying proxies to conduct a principal component analysis. There are a few shortcomings embedded in these two methods that lead to inadequate measurement of the sentiment index. First, surveys are usually conducted online and respondents in such surveys are more likely to be certain kinds of individuals. This implies that the samples are not randomly selected and that bias might exist in the index derived from the survey data, which cannot fully reflect the average market expectations. Besides, the respondents in such a survey may come from either the supply-side or the demand-side. Due to information asymmetry in the housing market, there is sample heterogeneity and this may cause bias in the index compilation.

On the other hand, for indirect indices, contingent events, which may have a considerable and instant shock to proxies but obscure impacts on

[4] The Land Registry is an affiliated to government of HKSAR and has a duty to maintain an efficient and effective land registration system to facilitate the orderly conduct of land transactions. Every transaction that occurs in HKSAR has to register a document of agreement at the Land Registry.

sentiment or the other way round, could lead to mis-estimation of the indices. For instance, Hui and Liang (2015) examined the impacts of tax policy (Special Stamp Duty, SSD) on housing transactions and found that the policy caused a “venturi” effect and immediately shrank the transaction volume of the entire market. However, such policy intensified the transactions in the transaction clustering area. Due to the hidden biases in the house price index (see Hui and Liang, 2015), the SSD policy took an instant shock to the house price index but had a vague effect on sentiment in the short term. In addition, the composite measure that involves using underlying proxies (e.g. Baker and Wurgler, 2006, 2007) is inclined to find out which kind of asset is more likely to be affected by sentiment rather than to measure sentiment.

By contrast, the transaction-based sentiment index employed in this chapter addresses the shortcomings mentioned above. Noises can be embedded in survey-based data, but not in transactions as transactions are factual deals and every transaction reveals the participant’s decision, which indeed affects the spot prices of houses. Though sentiment is unobserved and difficult to measure directly, transactions are observable and contain the information regarding the current (rather than underlying) participants’ attitude towards the housing market. On the other hand, market liquidity is often regarded as an indicator of sentiment (e.g. Clayton et al., 2009). Among the indirect measures, the trade-based index explores the changes in probability of whether a transaction is driven by positive or negative sentiment, which makes this index more feasible to represent the changes in sentiment.

As strike price and trading volume are not directly involved in the construction of the index, our innovative approach can avoid collinearity between sentiment index and market indicators, i.e. price, rent and trading volume. In addition, our analysis shows there is only a slight correlation between sentiment and price. The detailed results are displayed in panel B in Table 4.1. Thus, this new index is preferable to the traditional sentiment index.

4.3 Methods and Data

The research framework consists of two stages. In the first stage, the causality analysis is adopted to investigate the statistical causality between market sentiment and the three market indicators (namely, price, rent and trading volume). Superior to the Granger Causality Test, the integrated renormalized PDC (IRPDC) method is employed in this chapter. This method is advanced to provide a detailed and rigorous statistic inference to verify the hypotheses. In the second stage, this chapter will examine the long-term effect of market sentiment on the movement in house prices and rents. The data for market indicators and fundamentals employed in this chapter are described in Section 5.4.3.

4.3.1 The method of causality analysis

The Granger Causality Test (GCT), first introduced by Granger (1969), is a widely used tool that establishes a quantitative model (based on the vector autoregressive model) for the analysis of causal relationships. The variables have a pairwise structure and are performed in estimated VAR models. However, GCT fails to obtain an accurate structure of covariance. That is, the causality of X to Y may also take the indirect effect (X to Z then to Y, where X, Y, Z are in multivariate process) into account. To overcome this drawback, new methods are developed to improve the ability to capture a multivariate process. One of those is the directed transfer function (DTF) introduced by Kaminski and Blinowska (1991). DTF introduces a more convenient process as it only requires one VAR model to identify the direct causal relationships among variables and it is compatible with the Granger causality test (Kaminski et al., 2001). DTF, however, may incorporate indirect relationships[5] into direct causality among variables.

The deficiency in DTF is resolved by the partial directed coherence (PDC) method introduced in Sameshima and Baccala (1999). Its statistical properties are summarized by Schelter et al. (2005). The PDC method only detects and presents direct impacts. Similar to the DTF and GCT, the PDC method has different statistical distributions for different relationships. In the early stage, PDC is used to examine the significance of a relationship and fails to further discuss the strength of any causality relationship. This limitation has been overcome by the renormalized PDC introduced by Schelter et al. (2009). RPDC renormalizes the statistics with the same distribution, whereas the critical value depends only on the number of observations, which is constant for a fixed dataset (Hui et al., 2012). Hui and Chan (2012) further improved the model by introducing the Integrated RPDC (IRPDC), which allows for more explicit viewing of the statistics.

This chapter employs the IRPDC method[6] to achieve our research objectives, which is to identify the casual relationship between sentiment and price, rent and volume in the housing market. It quantifies the degree of pairwise causality between any two of four variables and thus the results become comparable such that one can distinguish the most influencing factor for a certain variable from the others.

4.3.2 Dynamic equilibrium model

[5] For instance, variable A has an influence on B and B has an influence on C. Then the result by DTF may imply that variable A has a causal relationship with C, which is not the truth. Therefore, it is difficult to observe all of the true relationships among variables.

[6] The framework of IRPDC based on VAR model follows Hui and Chan (2012).

With regard to the statistical causality of sentiment, this chapter moves further to investigate the role of sentiment in the evolution of housing prices and rents. The traditional notion holds that prices in the housing market are well explained by market fundamentals, and that the relationship between prices and fundamentals is established based on the intersection of market supply and demand in the local economy. Following the framework of Quigley (1999) and Hui and Wang (2014b), housing price (*P*) can be represented by

$$P = f(H^D, H^S)$$

where H^D and H^S are housing demand and supply respectively. The demand of the housing market is a function of housing price, household affordability (household income[7] as a proxy) and the local economy (denoted by *Eco*), that is

$$H^D = D(P, INC, Eco)$$

The supply of the housing market is formulated by a function involving housing price, new completed flat[8] and local economy, and is shown as

$$H^S = S(P, New, Eco)$$

In this chapter, the GDP and real interest rate (denoted by *r*) are selected to represent the development of the local economy. Derived from the demand and supply equations, the basic reduced form of price function associated with market sentiment (*S*) is

$$P = f(INC, New, GDP, r, S)$$

Additionally, if the causality results suggest endogeneity between price and sentiment, it is necessary to extend the above model to an auto-correlated

[7] Housing demand at any time is always subject to household income (*INC*) (Quigley, 1999). Besides, a long-term correlation between house price and income is widely found (e.g. Holly et al., 2010).

[8] The New completed flat (*New*) is a significant indicator of housing supply in Hong Kong (Hui, 2003).

structure. A modified model can be expressed as

$$L(P,S) = f(INC, New, GDP, r) \qquad (1)$$

where $L_{(\,)}$ is the lag operator. The model will be enhanced by modification based on the causality results, which will be discussed in Section 5.6.

4.3.3 Market and Data Description

Despite the slowdown[9] in the recovery of the economy in 2012, Hong Kong's housing prices recorded an increase of 24%. Fuelled by low interest rates and strong non-local demand, property prices in Hong Kong have surged by 63.6% during 2011-2013. In comparison, increases in rents were milder at 36.8% (see Figure 4.1). However, there was an obvious decline in transaction volume, possibly due to government interventions such as the Special Stamp Duty[10] introduced in November 2010. Housing prices departed from the trend of the economy, indicating that conventional economic fundamentals are not effective enough to explain the dynamics of housing prices in Hong Kong (Case and Shiller, 2003; Hui and Wang, 2014a).

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[9] Referring to the figures issued by International Monetary Fund (IMF), GDP growth in Hong Kong slid sharply to 1.25% in 2012, compared with 5% in 2011 and 7.1% in 2010.

[10] The Stamp Duty (Amendment) Ordinance 2011 imposed Special Stamp Duty (SSD) on top of the ad valorem stamp duty on the disposal of residential properties with effect from 20 November 2010. Unless the transaction is exempted from SSD or SSD, any residential property acquired on or after 20 November 2010, either by an individual or a company (regardless of where it is incorporated), and resold within 24 months, will be subject to SSD. (Source: http://www.ird.gov.hk/eng/faq/index.htm#01)

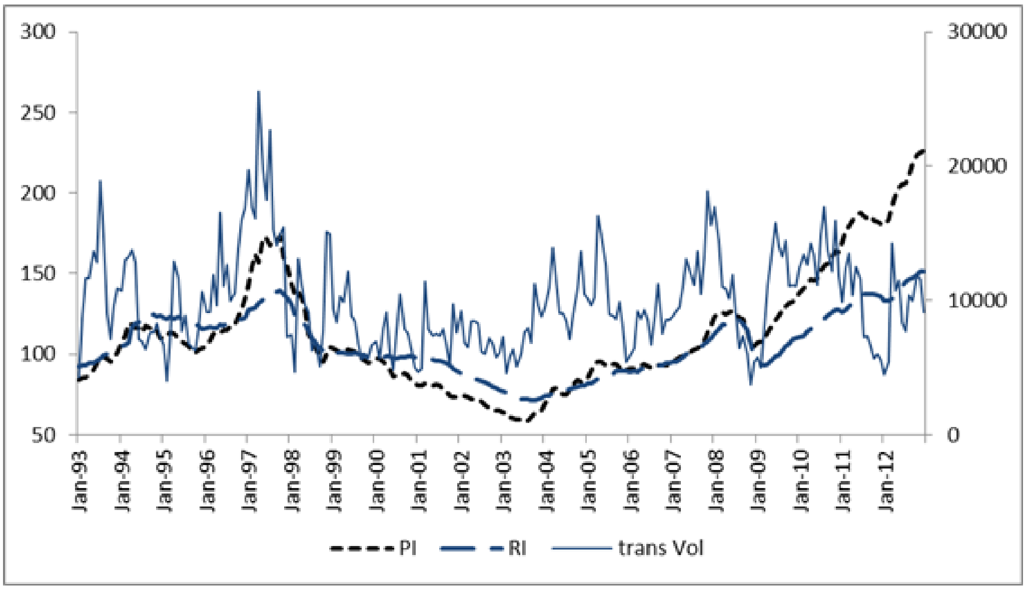


Figure 4.1. The chart of three monthly indices: Price Index (PI), Rental Index (RI) and Trading Volume (Vol) during 1993-2012.

The data contains two sets. Four variables in the first set for the causality tests are collected monthly. Apart from the aforementioned sentiment index compiled in Chapter 3, the price and rental indices, as well as trading volume, are collected over a span of twenty years from 1993-2012. The price return (PI) and rental return (RI) of private domestics are issued by the Rating and Valuation Department (RVD), the Hong Kong Special Administrative Region (HKSAR). These two indices measure the changes in value to reflect the integral level of performance of the housing market at any time. Trading Volume (Vol), defined as the aggregate number of sale and purchase agreements of residential units in a month, is also issued by RVD and announced by the Land Registry. The data for the three variables (price, rent and trading volume) are an open source and have been available from the official website of RVD since 1990. Figure 4.1 shows the price index of the housing market in Hong Kong.

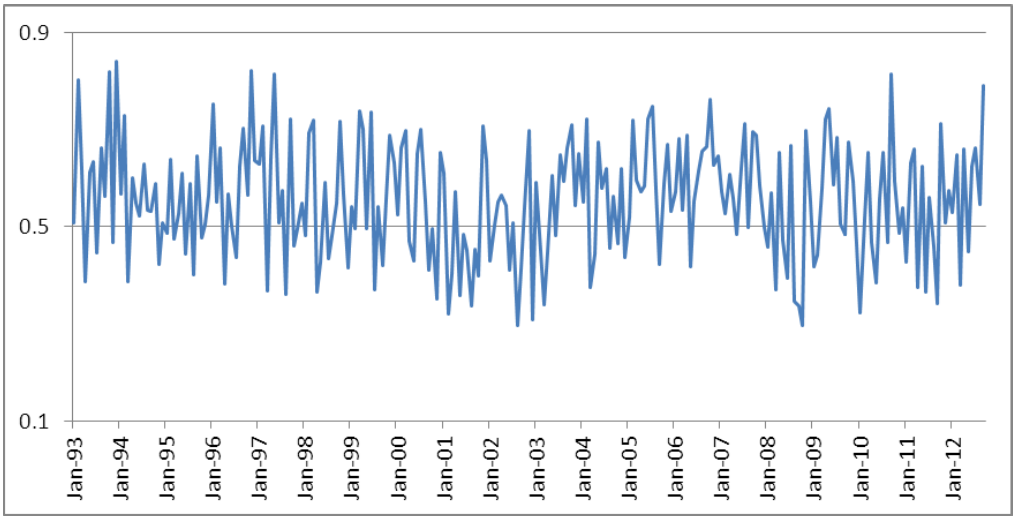


Figure 4.2. The chart of the monthly sentiment index during 1993-2012. When the curve is above (below) line 0.5, this indicates that positive (negative) sentiment dominates the market.

The original sentiment index (provided in Hui and Wang, 2014a) consists of pairwise sentiment measures (i.e., positive % vs. negative %). In this chapter, the index is transformed into a ratio of positive sentiment to total sentiment. An index value equal to 0.5 describes a neutral market sentiment where half of the sentiment is bullish and the other half is bearish. An index value above 0.5 indicates that positive sentiment dominates the housing market, and vice versa. The monthly data for the sentiment index from 1993 to 2012 are displayed in Figure 4.2 and the descriptive statistics of the sentiment index are given in Panel A of Table 4.1.

The second set includes the data for household income (INC), GDP, New completed flats (New), and real interest rate (r)[11]. All of these data are on a quarterly basis from 1993-2012 and were collected from the Census and Statistics Department, HKSAR. Prior to econometric analysis, the stationarity of the variables was verified in order to avoid mis-estimation.

[11] Real interest rate used in this paper is the mortgage rate that has been adjusted to remove the effects of inflation.

The non-stationary raw data are transformed by first order differencing as $y^*(i,t) = lny_{i,t} - lny_{i,t-1}$. The purpose is to transform the sample data into a return rate such that the mean of the transformed series is approximately zero. Among six variables, four (PI, RI, INC and GDP) are identified as non-stationary and thus are processed with the transformation. The new completed flats show an obvious pattern of seasonal fluctuations and thus are treated with de-trend adjustment.[12]

Table 4.1. Summary statistics of four variables

Panel A: Price Index (PI), Rental Index (RI), Trading Volume (Vol) and Sentiment Index (SI): 1993-2012 (monthly data with obs. = 237).

Statistics	PI	RI	Vol	SI
Mean	111.5	105.6	9912.5	0.5542
Std.D.	35.3	18.8	3642.5	0.1195
Min	58.4	71.3	3786	0.2955
Max	217.8	147.5	25572	0.8404

Panel B: The correlation analysis of variables in Panel A: monthly data with obs. = 237

	PI	RI	Vol	SI
PI	1.000			

RI	0.446*	1.000		
	[7.65]	---		
Vol	0.260*	0.061	1.000	
	[4.13]	[0.94]	---	
SI	-0.189*	-0.059	-0.057	1.000
	[-2.95]	[-0.91]	[-0.88]	---

Panel A in Table 4.1 is a summary of the descriptive statistics of the monthly data for four market indicators including sentiment. With reference to the standard deviation in Table 4.1, it is obvious that the volatilities in price are significantly greater than those in rent, which concurs with the findings of Hui and Zheng (2012). Panel B shows the correlation analysis of

[12] The Hodrick-Prescott filter (Hodrick and Prescott, 1980) is applied to de-trend the data.

four variables in the first data set. For the second data set, Panel A in Table 4.2 summarizes the descriptive statistics of quarterly data for the market fundamentals, followed by a correlation analysis of sentiment and the four exogenous variables as shown in Panel B.

Table 4.2. Summary statistics of market fundamentals

Panel A: Market fundamentals: 1993-2012 (quarterly data with obs. = 80).

Statistics	Household Income	GDP (million)	New completed flats	Real interest rate
Mean	16927.5	356493	4931.5	0.0627
Std.D.	1905.7	73570.7	2933.7	0.0216
Min	12300	209714	632	0.0115
Max	21100	557236	13425	0.1217

Panel B: The correlation analysis of variables in Panel C: quarterly data with obs. = 80

	INC	GDP	New	*r*	SI
Household Income (INC)	1.000 --				
GDP	-0.199 [-1.773]	1.000 --			
New completed flats (New)	-0.098 [-0.859]	-0.027 [-0.238]	1.000 --		
Real interest rate (*r*)	-0.014 [-0.118]	-0.040 [-0.349]	0.428* [4.127]	1.000 --	
Sentiment (SI)	0.100 [0.873]	0.193 [1.712]	-0.073 [-0.640]	0.033 [0.285]	1.000 --

Note: *t*-statistics are reported in brackets. Asterisk(*) denotes the significance at confidence level 5%.

4.4 Statistical Causality

This section gives an insight into the statistical causality among market sentiment (sentiment index) and the other three variables – house price return, rental return and trading volume using monthly data in the housing market in Hong Kong. A series of statistical tests were performed and the empirical findings are discussed below. Initially, the unit root test is adopted to verify the stationarity of the data as this is essential for the construction of the VAR model. The tests for lag selection according to several criteria are then carried out to determine the lag order in the VAR model. Afterwards, the VAR model of the four variables is estimated to fit the data. Both Granger causality test (GCT) and integrated renormalized PDC (IRPDC) are conducted. The latter provides more informative findings, compared to the GCT results.

4.4.1 Unit root test

Identifying the stationarity of variables through the unit root test is a preliminary step in econometric analysis. If the data are not stationary, the VAR model would be inefficient and then the IRPDC approach would be invalid. The unit root test based on the Schwarz-Bayesian Criterion was employed and the results are shown below in Table 4.3. The tests reject the null hypothesis that unit root exists, implying that multi-dimensional time series data are stationary to construct the VAR model.

Table 4.3. Summary of unit root tests on PI, RI, Vol and SI

Method	Statistic	Prob.	Cross-sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t-test	-15.1872	0.0000	4	940
Null: Unit root (assumes individual unit root process)				
ADF - Fisher Chi-square	354.474	0.0000	4	940
PP - Fisher Chi-square	296.307	0.0000	4	944

Note: Probabilities for Fisher tests are computed using an asymptotic Chi-square distribution. All other tests assume asymptotic normality.

4.4.2 Lag selection

Table 4.4 shows the results of the lag selection based on several criteria. Two options for lag orders are found to be acceptable, i.e. lag = 2 or 4. In general, it is better to follow the principle of parsimony in lag selection in the VAR model. In other words, the structure with lag 2 is much simpler than that with lag 4 (twice as many parameters in VAR[2] to estimate as in VAR[4]). Thus, VAR[2] is preferable in this case where all of these factors are considered.

Table 4.4. VAR lag Selection based on several criteria

Lag	LogL	LR	FPE	AIC	SC	HQ
1	1245.17	NA	2.56e-10	-10.73	-10.49	-10.63
2	1293.51	93.31	1.93e-10	-11.01	-10.53*	-10.82*
3	1315.72	42.08	1.83e-10	-11.07	-10.35	-10.78
4	1346.44	57.14*	1.61e-10*	-11.20*	-10.24	-10.81
5	1358.42	21.86	1.67e-10	-11.16	-9.96	-10.68
6	1369.55	19.92	1.74e-10	-11.12	-9.68	-10.54
7	1383.68	24.80	1.77e-10	-11.10	-9.42	-10.42
8	1398.40	25.32	1.80e-10	-11.09	-9.17	-10.32

Asterisk(*) indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

4.4.3 Traditional Granger causality

The results generated by the traditional Granger causality test (GCT) are provided for comparison with the IRPDC results. Table 4.5 exhibits the GCT results with lag 2 (months). Nine statistics are found significant to reject the null hypothesis of no causality. The results indicate that the price return (PI) and sentiment index (SI) Granger-cause other variables. In other words, PI and SI are likely to have a prediction power on other variables with pre lead lag of no more than 2 months. Meanwhile, the results reveal the predictability of RI on Vol and SI, i.e. changes in Vol and SI can be linked to the former terms of RI. However, no feedback from Vol to RI is found, implying that the changes in Vol might not necessarily have a significant impact on the performance of the rental market. Indeed, Vol only Granger-cause sentiment, which is somewhat consistent with the findings in Clayton et al. (2008) regarding the linkage between market liquidity and sentiment.

Table 4.5. Pairwise Granger Causality Tests
Lags: 2

Null Hypothesis:	Obs	F-Statistic	Prob.
RI does not Granger Cause PI	235	0.80472	0.4485
PI does not Granger Cause RI		55.1233	0.0000*
Vol does not Granger Cause PI	235	2.74942	0.0661
PI does not Granger Cause Vol		10.2650	0.0000*
SI does not Granger Cause PI	235	11.4791	0.0000*
PI does not Granger Cause SI		4.68517	0.0101*
Vol does not Granger Cause RI	235	7.28119	0.0009*
RI does not Granger Cause Vol		5.07206	0.0070*
SI does not Granger Cause RI	235	3.37529	0.0359*
RI does not Granger Cause SI		3.51382	0.0314*
SI does not Granger Cause Vol	235	8.24022	0.0003*
Vol does not Granger Cause SI		2.08484	0.1267

Note: Asterisk(*) denotes the significance at confidence level 5%.

As GCT may fail to capture all of the information for the covariance structure in the VAR model (Schelter et al., 2009), GCT can hardly reveal more useful and direct information for causality relationships between multi-dimensional data. Therefore, the more advanced method, IRPDC, is adopted to conduct a more precise investigation into the causal relationships and quantify the strengths of such relationships.

4.4.4 Estimated VAR model

The estimated VAR model incoporating four indicators (i.e. PI, RI, Vol, SI) with lag 2 to fit the monthly data for the IRPDC is shown as follows:

$$\begin{pmatrix} PI \\ RI \\ Vol \\ SI \end{pmatrix}_t = \begin{bmatrix} 0.5149 & 0.1968 & 0.0112 & 0.0284 \\ 0.2228 & 0.2738 & 0.0020 & 0.0028 \\ 3.2362 & -0.9466 & -0.2901 & 0.1267 \\ -1.3612 & -2.1417 & -0.0035 & -0.7703 \end{bmatrix} \begin{pmatrix} PI \\ RI \\ Vol \\ SI \end{pmatrix}_{t-1}$$

$$+ \begin{bmatrix} 0.0381 & -0.0420 & -0.0087 & 0.0151 \\ 0.1074 & 0.1493 & 0.0005 & 0.0060 \\ 0.6315 & -4.5265 & -0.2558 & 0.2049 \\ -0.0932 & 2.2284 & -0.0536 & -0.3846 \end{bmatrix} \begin{pmatrix} PI \\ RI \\ Vol \\ SI \end{pmatrix}_{t-2} + \begin{pmatrix} \epsilon_{PI} \\ \epsilon_{RI} \\ \epsilon_{Vol} \\ \epsilon_{SI} \end{pmatrix}$$

where the VAR[2] structure provides two coefficient matrices for the further steps, i.e. the Fourier transformation (refer to equation of $\boldsymbol{A}(\omega)$ in Hui and Chan, 2012). The error vector of $\boldsymbol{\epsilon}(t)$ is a 4-dimensional white noise or innovation process with covariance matrix $\boldsymbol{\Sigma}$. The IRPDC method will then be adopted based on this VAR model.

4.4.5 Integrated RPDC

Figure 4.3 shows the graph matrix for the results of the renormalized PDC derived from the estimated VAR[2] process. The sub-graph in the i-th row and j-th column displays the impact of process j on process i. The confidence interval at the 95% level for each RPDC in the sub-graph is highlighted by the shaded area. If the confidence interval is squeezed to a width of approximately zero, the directed causal relationship does not exist. Since the self-influenced causality is trivial and invalid under the Granger causality framework, the four sub-graphs in the diagonal are omitted in Figure 4.3. There are eight sub-graphs showing significant causality: PI to RI, PI to Vol; RI to SI, RI to Vol; SI to PI, SI to RI, SI to Vol and Vol to PI. These eight causal relationships are summarized in Figure 4.4(a).

Interestingly, three disparities are found when comparing the GCT results with IRPDC results. Contrary to the traditional GCT, the causalities of Vol to RI and of PI to SI are denied by the RPDC method. These three disparities might be due to the inherent shortcoming of GCT that is unlikely able to adequately capture the covariance among the multi-dimensional data. As such, market participants (including investors and authorities) should adopt IRPDC method in monitoring private housing market.

IRPDC is able to offer a quantified measure of the causality strengths among the four variables, however. These results (shown in Table 4.6) echo the sub-graphs in Figure 4.3. In Table 4.6, the statistics shown can be interpreted in two ways. First, such statistics can identify the significance of the predictability. Second, they can screen out the most important factor that predicts a specific variable (by comparing the results of IPRDC in the same row). The significance of the statistics can be identified by comparing them with the critical value given as $\theta_{0.05}$ = 0.1046. When IRPDC > $\theta_{0.005}$ (0.1849), the predictability can be regarded as being strong (see Hui et al., 2012). All of the 8 significant relationships are identified to exceed the threshold value.

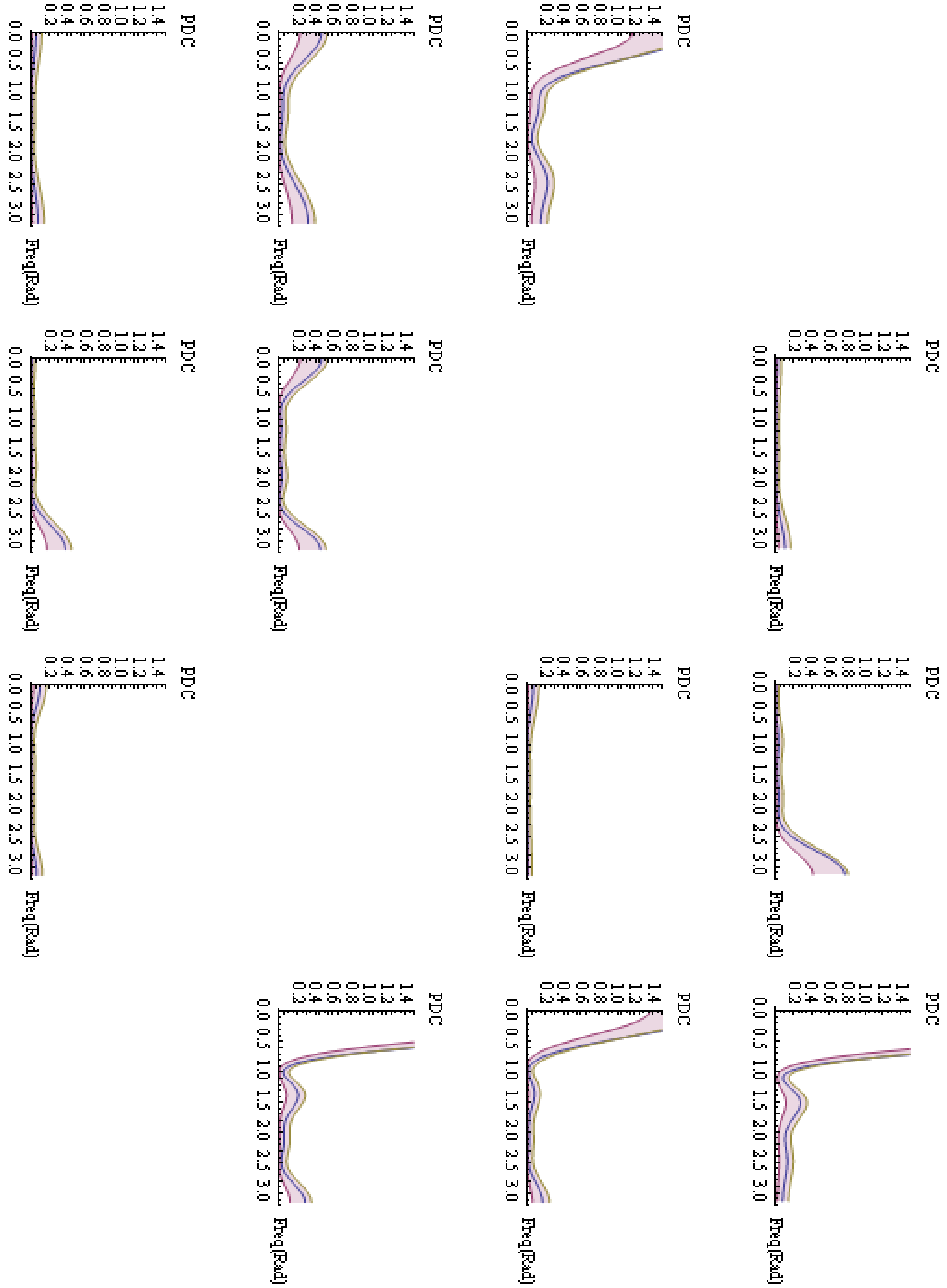


Figure 4.3. The graph matrix of the causal relationships using the RPDC method based on the estimated VAR[2] model.

Notes: Sub-graph in the *i*-th row and *j*-th column represents the RPDC causality of variable of $\boldsymbol{X}_j$ to $\boldsymbol{X}_i$ where *i*,*j* = 1,…4 represent price (PI), rent (RI), trading volume (Vol) and sentiment index (SI). The confidence interval at the 95% level for each RPDC in the sub-graph is highlighted by the shaded area. There are eight significant influenced patterns of causal relationships among the four variables, which correspond to Table 4.6.

Table 4.6. Test Statistics of Integrated Renormalized PDC

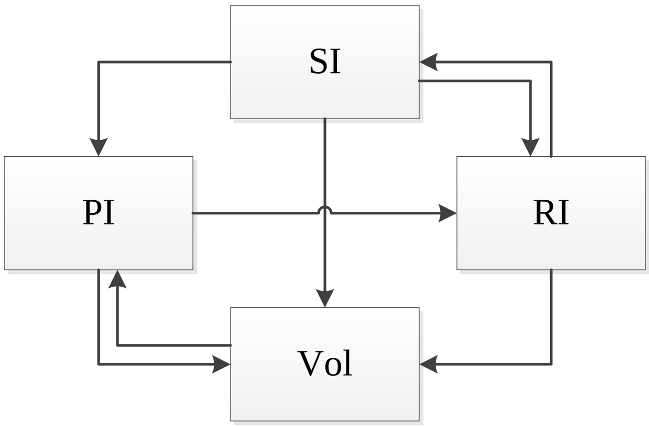
IRPDC	PI	RI	Vol	SI
PI	-	0.0699	0.4067*	**4.6454***
RI	**1.2619***	-	0.0451	1.0805*
Vol	0.5066*	0.4075*	-	**3.0045***
SI	0.0815	**0.1876***	0.0702	-

Note: statistic in i-th row and j-th volume represents the IRPDC causality of variable of $\boldsymbol{X}_j$ to $\boldsymbol{X}_i$. Critical values are given as: $\theta_{0.05} = 0.1046$; $\theta_{0.01} = 0.1068$; $\theta_{0.005} = 0.1849$. Asterisk (*) denotes the significant statistics. Number in bold indicates the biggest value in the same row.

In the last column of Table 4.6, three statistics are significant, suggesting that all three market indicators are affected by market sentiment. In other words, the housing market in Hong Kong is significantly affected by market sentiment. By comparing the values in this column, the strength of predictability of SI to PI (4.6454) is higher than the other two, i.e. SI to RI (1.0805) and SI to Vol (3.0045), indicating that SI has a greater impact on PI than on the others. This particularly suggests that the sales market is more likely to be affected by sentiment than the rental market. Such an implication can be attributed to the multiple demands, including demand for investment and speculation, in the sales market. This could contribute to the existing knowledge that no previous literature has ever addressed the difference in causal strengths of sentiment between rental and sales markets.

Furthermore, through a comparison among the four rows, it is found that the strongest levels of causality to a specific variable are: SI to PI, PI to RI, SI to Vol and RI to SI. These four causalities are stronger than the others and are summarized in Figure 4.4(b).

(a) The significant causal relationships among the four variables



(b) The most influential factor for a specific factor among the four variables

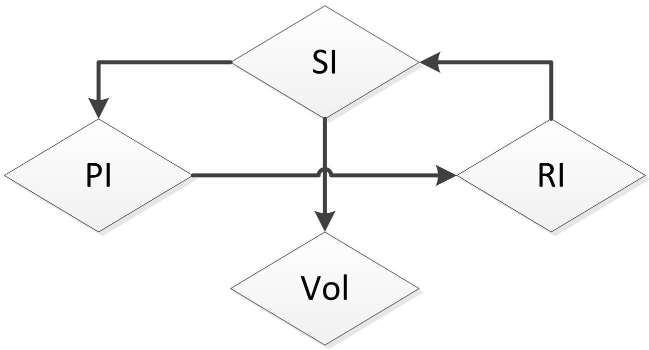


Figure 4.4. The most significant causal relationships among the four variables

Not surprisingly, the price return (PI) has the strongest power to predict the performance of rental return (RI) in the short term. As classical theory suggests that the housing price is fundamentally determined as the present value of future rental income (Case and Shiller, 1989; Gallin, 2008), changes in prices can predict future changes in rents. The first interesting finding for PI is that price and trading volume can predict each other in the short term though there is no agreement about the relationship between these two variables across global markets. By comparing the values in the first row of Table 4.6, the causal strength of SI to PI (4.6454) is much higher than that of Vol to PI (0.4067). By contrast, the statistic of RI to PI (0.0699) is insignificant. This indicates that PI is more likely to be affected by sentiment (SI) than by trading volume (Vol). Likewise, sentiment is found to be the strongest factor to predict volume, which is widely considered a proxy of market liquidity.

In sum, the findings above suggest that sentiment is the most significant factor to predict both price and volume in the short term for the housing market in Hong Kong, and this concurs with Clayton et al. (2008). Therefore, sentiment should be recognized as a significant variable in modelling the trend of the housing sales market.

On the other hand, RI is the only factor that can predict sentiment. Also, RI is capable of forecasting volume (0.4075*). These may give an insight into households' tenure choice in the housing market. Renters, as the most rational participants in the market, are sensitive to changes in rents. Renters may choose to hedge the future risk of renting/ownership by switching their tenure choice, especially in the presence of a low mortgage rate. Thus, households' tenure switching, i.e. renting to owning or owning to renting, is subject to rent levels and reflects their expectations of what future market trends will likely be. In sum, RI shows its power to predict trading volume and market sentiment.

Three causalities among PI, SI and RI form a one-way cycle as shown in Figure 4.4(b). This cycle reveals the indirect impact of rent to price and sheds light on how the rental market transmits its feedback indirectly towards the sales market.

4.4.6 Indirect impact of rent on price

DiPasquale and Wheaton (1996) suggest a short-term linkage between rent and price through a consistent channel ($P = R/i$) given a constant return rate (i). In this section, the empirical result of the short-term predictability of RI shows that RI is not a statistically significant indicator of PI, in terms of forecasting. However, the one-way cycle provides a possible explanation for how rent indirectly affects price in the housing market of Hong Kong in the short run.

In the rental market, if property prices rise, the short-term supply curves shift towards the left, which will boost up rents. When changes in rents are observed, tenants may switch their tenure choice between renting and owning. As a result, such switch represents a shock to the market that also affects sentiment. This explains the predictability of RI to SI. Then the changes in market sentiment affect housing prices, which can be justified by the predictability of SI to PI. In this chapter, market sentiment is considered to be an indicating variable to bridge the indirect linkage from RI to PI.

4.4.7 Asymmetry in causality strengths

Taking advantage of the IRPDC method, the analysis suggests asymmetries in pairs of causality between two variables. In Table 4.6, different strengths of predictabilities are found between PI and Vol: Vol to PI is 0.4067 and PI to Vol is 0.5066. Another pairwise causality between SI and RI shows a significant disparity at 1.0805 vs. 0.1876. Such a difference between the pairwise causalities reflects the market mechanism. Once the pairwise causalities are significant, the traditional GCT cannot capture the exact strengths of causality, but IRPDC can. Hence, the IRPDC results can provide more focused implications and important references to facilitate policy-makers or investors in their decision-making.

4.5 Using Sentiment to Explain House Prices and Rents

This section discusses the role of sentiment in the dynamics of the sales (housing price) and rental markets (rent) in Hong Kong. Several models are established for these two sectors. Primarily, a number of studies suggest controlling the effect of a financial crisis in modelling Asian housing markets. Hence a dummy variable (denoted by Fd[13]) is employed to proxy the crisis effect on Hong Kong's housing market.

Based on the IRPDC causality results of sentiment, the theoretical model suggested in Section 4 is specified. The result of IRPDC causality of SI to PI is mono-lateral, which indicates that the lag term of price shows insignificant power to predict sentiment. Based on equation 1, the VAR model is reduced to

$$P = f(L(P), SI, INC, New, GDP, r, Fd) \qquad (2)$$

where the operator $L(P)$ indicates a significant auto-correlation. More generally, to compare with model (2), a general VAR model is established as follows:

$$[P, SI]' = \mathbf{VAR}\{L[P, SI]', INC, New, GDP, r, Fd\} \qquad (2')$$

Model (2') is set to investigate whether housing price can explain the changes in sentiment, in order to verify the mono-lateral causality of SI to PI.

Second, in the light of the causality between sentiment and rent, this chapter also surveys the role of sentiment in the movement of rent. As mentioned, housing price affects rent and rent is also determined by the interactions between supply and demand. Similar to housing price, rent can be formulated by

$$RI = f(L(RI), P, INC, New, GDP, SI, r, Fd) \qquad (3)$$

Based on the pairwise causality between sentiment and rent, a VAR model is established and associated with sentiment to capture the dynamics of rent,

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[13] The dummy variable Fd indicates the effect of last two financial crises (i.e. two crises happened in 1997 and 2008 respectively). The two periods of taking effect are 1997Q4 – 1998Q3 and 2008Q3 – 2008Q4.

which is expressed as

$$[RI, SI]' = \mathbf{VAR}\{L[RI, SI]', P, INC, New, GDP, r, Fd\} \qquad (4)$$

Table 4.7. Estimation of models for sales market

Dependent variable	Benchmark model PI	Model 2 PI	Model 2' PI	SI
Lagged price	0.128	0.093	0.072	0.176
(PI_{-1})	[1.279]	[0.990]	[0.663]	[0.675]
Sentiment index		0.159*		
(SI)		[3.396]		
Lagged Sentiment index			0.074	0.060
(SI_{-1})			[1.362]	[0.455]
Household income	0.604*	0.530*	0.536*	0.412
(INC)	[3.127]	[2.922]	[2.699]	[0.859]
New completed flats	-0.033*	-0.027*	-0.032*	-0.034
(New)	[-2.038]	[-1.906]	[-2.008]	[-0.891]
GDP	-0.142	-0.202*	-0.134	0.388
	[-1.399]	[-2.105]	[-1.327]	[1.595]
Financial crisis	-0.133*	-0.119*	-0.131*	-0.086
(Fd)	[-5.485]	[-5.191]	[-5.459]	[-1.473]
Real interest rate	-0.261	-0.437	-0.356	1.032
(*r*)	[-0.752]	[-1.332]	[-1.011]	[1.213]
Intercept	0.304*	0.365*	0.350*	-0.348
(C)	[2.415]	[3.075]	[2.701]	[-1.111]
R^2	0.506	0.577	0.519	0.124
Log likelihood	122.108	128.060	123.130	55.217
Schwarz criterion (SC)	-2.777	-2.875	-2.747	-0.983

Note: *t*-statistics are reported in brackets. Asterisk (*) denotes the significance at confidence level 5%.

Table 4.7 presents the model estimations on the returns of housing prices in the sales market. The benchmark model in Table 4.7 excludes the sentiment variable, while sentiment models (2) and (2') do not. Firstly, the lagged term of price is insignificant in all three models, which implies a weak power of autocorrelation in explaining the housing price in Hong Kong. For the role of sentiment, the coefficient of the sentiment variable is significant in model (2), which indicates that sentiment affects changes in housing prices. As the coefficient of the sentiment variable in model (2) is positive, an increase in sentiment leads to a positive change in housing price. Meanwhile, when introducing sentiment into the benchmark model, the term GDP becomes significant. It indicates a bias of estimation induced by the lack of variables in the benchmark model, and the sentiment improves the model performance (measured by R^2) compared to the benchmark model.

Furthermore, the VAR model (2') reveals that the lagged term of the sentiment variable plays a dispensable role in explaining housing price. It indicates that only current sentiment affects the price return. In addition, model (2') offers no evidence to support the notion that the sentiment variable is autoregressive, or the sentiment is affected by the lagged price term. Thus model (2) is a better model that captures the housing price movement for Hong Kong as it has a higher R^2 and a lower value of the Schwarz criterion. Reveal of this long-term relationship between sentiment and price return can benefit market participants including investors and authorities.

Surprisingly, the coefficient of the term GDP in model (2) is negative, implying that the local economy inhibits housing price. Such a counter-intuitive situation can be explained by the extraneous housing demand. The housing market in Hong Kong features a combination of local and foreign demands. The inflow of demand (from the mainland and overseas) accounts for a sizeable proportion of the total housing demand[14] and the local economy has a small impact on this extraneous demand. Besides, the local real interest rate is insignificant in explaining housing

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[14] The ratio of individual home buyer from mainland was gradually increasing before 2013, was recorded as more than 20% (in trading volume) during 2010-2012, and reached the peak of 42% in 2011. Since the promulgation of special stamp duty in 2013, this ratio has fallen down to a figure around 10%.

price, showing that housing investment is mildly subject to the local capital cost. This implies that the investors come from the outside of Hong Kong and supports the above finding that a sizeable proportion of housing demand is not indigenous. Thus, the negative correlation between GDP and housing price and the insignificant coefficient of the real interest rate indicate the weakness of local housing demand.

Table 4.8. Estimation of models for rental market

	Benchmark model	Model 3	Model 4	
Dependent variable	RI	RI	RI	SI
Lagged rent	0.299*	0.301*	0.242*	0.162
(RI_{-1})	[3.322]	[3.321]	[2.836]	[0.436]
Sentiment index		-0.013		
(SI)		[-0.432]		
Lagged Sentiment index			0.093*	0.003
(SI_{-1})			[3.415]	[0.028]
Price	0.193*	0.204*	0.152*	0.908*
(PI)	[2.917]	[2.845]	[2.435]	[3.346]
Household income	0.099	0.098	0.040	-0.110
(INC)	[0.852]	[0.835]	[0.365]	[-0.229]
New completed flats	-0.009	-0.009	-0.009	-0.005
(New)	[-0.999]	[-1.000]	[-1.081]	[-0.137]
GDP	0.082	0.088	0.088	0.486
	[1.329]	[1.386]	[1.542]	[1.957]
Dummy variable	-0.058*	-0.058*	-0.058*	0.030
(Fd)	[-3.686]	[-3.634]	[-3.958]	[0.466]
Real interest rate	-0.006	0.011	-0.090	1.305
(*r*)	[-0.032]	[0.056]	[-0.505]	[1.694]
Intercept	0.081	0.072	0.144*	-0.657*
(C)	[1.114]	[0.955]	[2.051]	[-2.157]
R^2	0.615	0.616	0.672	0.247
Log likelihood	167.936	168.042	174.031	61.047
Schwarz criterion (SC)	-3.911	-3.857	-4.013	-1.078

Note: *t*-statistics are reported in brackets. Asterisk (*) denotes the significance at confidence level 5%.

Similarly, Table 4.8 reports the model estimations on rental return for the benchmark model, the linear model (3) and the VAR model (4). The benchmark model excludes the effect of sentiment, while the other two investigate the relationship of sentiment or the lagged term with rent. In contrast to price, the rent variable reveals an autocorrelation feature, as the coefficient of the lagged term of rent is significant for all three models. Note that the effect of sentiment in the linear model is identified as trivial and the sentiment variable does not help improve the model's performance. On the contrary, the coefficient of the lagged sentiment variable in the VAR model is significant. The positive correlation reveals that the sentiment elasticity of rent is 0.093.

It is interesting to note that the lagged term rather than current term of sentiment plays a prominent role in the dynamics of rent, which is different from the effect of sentiment on price. The lagged sentiment affects the previous housing price and the change in prices has a shock to rent that lasts longer than one period. It takes one period to transmit the effect of sentiment into the rental market. In addition, the coefficient of price in the sentiment equation in model (4) illustrates the significant impact of price on sentiment in the long run. This finding is consistent with those regarding the relationship between sentiment and price in model (2).

4.6 Conclusion

The first stage of this chapter aimed to examine the causal relationships among four indicators (price, rent, trading volume and sentiment) in Hong Kong's housing market. The causality results show the predictability of sentiment on other market indicators in the short run. With these implications, the second stage of this chapter investigates the role of market sentiment in the sales and rental markets. The empirical study spans over two decades: 1993-2012.

The causality analysis adopted in our empirical study, known as IRPDC, captures the predictability and the corresponding strength of economic factors. In the short term, market sentiment is a prominent indicator of forecasting price and trading volume. Compared to other factors, sentiment has an overwhelming power to predict housing prices, which implies that the housing market in Hong Kong is significantly affected by sentiment. The findings also show that rent is a significant predictor of sentiment. In addition, the one-way cycle of price, rent and sentiment implies that sentiment is an indicating factor in the indirect linkage of rent to price (see Figure 4.4b). Moreover, the asymmetry of causality strength between the four indicators can be verified by the IRPDC method. The analysis fills the knowledge gap as Granger causality is incapable of handling causality strength.

With the evidence of sentiment's causality, this chapter has confirmed the role of sentiment in the sales and rental markets. The findings show that sentiment has a significant effect on housing price and rent, but plays different roles in these two market sectors. House price is attributed to sentiment in part, while rent is affected by the lagged term of sentiment. This in turn provides some indirect evidence that supports the implication of a one-way cycle in causality investigation. These new findings will contribute to the knowledge in the field of housing studies.

The macro study in this chapter can benefit investors who are concerned about the predictability of market indicators and the movement of the housing market, as well as helping households in their housing choices. On the other hand, the implications of this chapter may serve as a useful reference for relevant authorities when they make policies to stabilize and improve the functioning of the housing market.

Chapter

5 THE ROLE OF SENTIMENT IN OPTIMAL DEVELOPMENT OF RESIDENTIAL PROJECTS

This chapter investigates the different roles of sentiment in residential development to achieve Objective 3. An option-based dynamic model incorporating different roles of sentiment is established to examine how sentiment affects developer's optimal development strategy in residential projects, and further has impacts on housing supply. Market sentiment affects optimal start time, development density and project value. It indicates that standard economic models should be revised to take into account this behavioural factor.

5.1 Introduction

Residential developers consider the profitability of housing projects and attempt to make a series of decisions (such as optimal timing and supply) over the course of those projects. Decisions at every stage are accompanied by opportunities and risks which affect the probability of success of the development projects. Recent years have witnessed instability in real estate markets across the world, including turbulence in property prices, waves of house mortgage defaults and a large amount of overbuilding (DeCoster and Strange, 2012). Overbuilding, which indicates oversupply, is more or less due to developers' inappropriate developing strategies in projects. As residential project development takes time and induces an investment lag (Bar-Ilan and Strange, 1996), the uncertainty and risks affect the return rate of the project. Thus developers have to understand, estimate, and control the risk in order to make optimal decisions prior to project development. Various factors stemming from external and internal aspects and the interactions between them complicate the decision making process.

Economic theory suggests that long-term prices can be determined by the equilibrium between demand and supply in the housing market (DiPasquale and Wheaton, 1996). In general, since fundamental demand fluctuates with the cyclicality of the housing market, developers as suppliers are faced with the challenge of determining the optimal time-varying supply. More specifically, the developer in the decision making process attempts to estimate local demand at the time of sale (or presale) in a certain area. With respect to the special characteristics of real estate projects such as long development periods and investment irreversibility, it is extremely difficult to foresee future demand (Ott et al., 2012). A long development period magnifies the influence of uncertainty as time progresses, and investment irreversibility leads to an enormous capital gap or shrinkage of cash flow. These two characteristics are of crucial importance for the success or failure of a real estate project (Holland et al., 2000). How to estimate the present value of a proposed development project and the scale of uncertainty embedded within the development therefore become the most prominent concerns for developers.

There are several types of uncertainty related to housing demand: land and house prices, housing stocks and stockholding costs, as well as government and political risks, which increase the project risks (Rocha et al., 2007). Generally, an enlargement of uncertainty brings greater risk, which can delay the project's development and lead to appreciation of land prices (Cunningham, 2006). On the other hand, by using real option analysis, uncertainty has been proven to benefit investors and developers (Grovenstein et al., 2011). Nowadays, an evaluation model employed at the stage of project planning is required to capture future uncertainty more accurately in order to benefit developers in the decision making process.

The classical evaluation model is based on the notion that only factors relating to systematic risk should be taken into consideration for project valuation (Holland et al., 2000). Some recent studies have paid attention to the role of non-systematic risk in real estate investment (e.g. Bulan et al., 2009). The channels through which factors of non-systematic risk affect the total uncertainty are diversified and thus it is difficult to fully understand the impact of these factors in option-based valuations. Nowadays, studies of housing market and demand intend to take into account the effects of non-systematic risk (e.g. Bourassa et al., 2009). Case and Shiller (1989) explain that market expectation is identified as a critical indicator for housing market performance. This remark is in line with the implications from behavioural economics, in which some studies have furnished an analysis of uncertainty with theoretical considerations based on psychological assumptions (e.g. DeLong et al., 1990). An essential part of these assumptions is the so-called sentiment factor, facilitating the role of psychology in modern economics and finance.

Drawing on the widely accepted definition presented by Baker and Wurgler (2007), sentiment is related to the investor's attitude with respect to anticipation of the market's movement, which cannot be justified by market fundamentals. Accordingly, market sentiment is the aggregate belief of investors towards the market trend (Hui and Wang, 2014). Sentiment originates from multiple factors. On one hand, irrationality in housing transactions can be attributed to imperfect information (Clapp et al., 1995). In the face of imperfect information, investors – especially individuals who have to undertake costly searches and bear heavy transaction costs – are likely to exhibit herding behaviour. This herding behaviour steers participants' belief in a future trend in the same direction, which could be manifested by market sentiment. On the other hand, the limitations of short-selling and shortage of liquidity in the housing market result in price momentum (Piazzesi and Schneider, 2009). Price inertia will amplify its impacts on and shift the housing demand and house prices. In addition, such inertia in price dynamics facilitates the herding behaviour and, in return, underpins the price momentum. Hence, market sentiment manifests the pattern of sentimental behaviour embedded in housing transactions.

Clayton et al. (2008) contend that sentiment takes credit for the divergence of property prices from market fundamentals. Hui et al. (2013) add that sentiment is even responsible for some property mispricing. Furthermore, due to lack of channels for short-selling in the real estate market, such mispricing cannot be eliminated or alleviated by rational transactions. Thus, market sentiment reports the irrationality in the housing market. It is therefore necessary to consider the effect of sentiment in the dynamics of the housing market.

Solid evidence supports the notion that sentiment is a key component driving the trends of real estate markets and even the recent financial crises (Clayton et al., 2009; Hui and Wang, 2014). Berkovec and Goodman (1996) introduce the link between consumer sentiment and housing demand: consumer sentiment plays an important role in changes in turnover rate, which have a positive correlation with changes in housing demand. As the sentiment factor cannot be justified by market fundamentals in the housing market, sentiment becomes an informative proxy for the trend of housing demand and thus it should not be neglected in the estimation of future housing demand. As such, the effects of sentiment on a developer's investment decisions in a residential project are undoubtedly prominent. However, such effects have rarely been analysed in the extant property research. Therefore, this study aims to investigate the role of sentiment in making investment decisions related to residential projects.

Titman's (1985) option-based model has become a prevailing method to investigate the feasibility of projects with an investment lag under uncertainty (Wang and Zhou, 2006). It can offer a better approach to evaluating profitability of an investment within the possibility of decisional flexibilities regarding information acquisition, deferral and abandonment in residential projects (Rocha et al., 2007). As the residential project is a typical example of a project with an investment lag, uncertainty and variation in the market conditions have a significant impact on these projects. The main and unique contributions of this model are ascribed to the capacity of exploring both the direct and indirect effects of market sentiment on the optimal investment decision for developers facing uncertainty and variation in market conditions. The option-based model developed by this study characterises the feature of expected house prices, in particular: how the house price forms based on the developer's sentiment, and the expected housing demand in the presence of uncertainty. On the other hand, as the investment has to focus on the present value of the future payoff, the model demonstrates how sensitive the expected market return is to sentiment.

The study of the role of market sentiment conveys a better understanding of the impact of non-systematic risk on decision-making for project investment. More specifically, the effects of sentiment on optimal density, expected waiting time and option-based project value are the most critical concerns to project development under uncertainty. This model sheds light on the pattern of developers' behaviour on the basis of sentiment, to explain their contributions to the dynamics of housing supply. The result also provides an insight with respect to the planning of land supply and urban development in the future, under different market conditions.

The rest of this chapter proceeds as follows: Section 2.2 introduces the theoretical model and Section 2.3 outlines the optimal choices inferred by the model. Section 2.4 discusses the expected waiting time to invest. This is followed by Section 2.5 that investigates the role of sentiment in affecting the optimal choices. Numerical analysis is carried out in Section 2.6 and the final section presents the concluding remarks.

5.2 Model Construction

With a tract of land in hand, it is imperative that a developer looks into the feasibility of project development. The developer aims at optimal decisions (e.g. timing and density) for project development under the market uncertainty. With investment irreversibility, it is worth delaying the investment in real property under the uncertain environment for the developer, which is first suggested by Arrow and Fisher (1974) based on the use of the concept of real options. An irreversible investment opportunity is similar to a call option in financial markets and an agreement to develop is deemed as the exercise of the option (Dixit and Pindyck, 1994). This study extends the classical option-based framework introduced in (Bar-Ilan and Stranger, 1996) to model irreversible investment with sentiment effect in residential development.

As the project valuation prior to the investment and investment takes time, the project valuation should be forward-looking. On the other hand, the final decision based on such a valuation will be determined relative to current expectations and business constraints (Atherton et al., 2008). As such, the valuation incorporates the developer's market expectation and, in return, reflects their expectation.

For real estate markets, the market sentiment has been found to contain forward-looking information for investors that has a predictability of real estate returns (Ling et al., 2014; Marcato and Nanda, 2016). After observing market sentiment (e.g. through monitoring a sentiment index), the developer forms their own view of the market trends and expectations (containing supply-side information) for the near future, as that developer's sentiment[15]. In this case, developer's sentiment containing supply-side response relies on the market sentiment (and a sentiment index) mainly capturing demand-side information. Since project valuation aims to benefit a developer's decision making, it is more straightforward to examine the effects of the developer's sentiment. In this study, an analytical framework is established by using the option-based modelling to investigate the effects of sentiment (through developer's sentiment) on investment decision-making for a residential project.

With the immobility of houses and diversifications in the characteristics of land lot, the heterogeneity of housing products makes the project unique. In this study, we assume the residential developer has a parcel of land seated in a certain district. The location uniqueness of the land and the developer's market positioning specify the market segments and thus lead to the local housing market being less perfectly competitive in this district. In other words, the developer is more likely to have monopoly power over the housing products delivered by this project. As such, the house price is a function of demand quantity (Ott et al., 2012) due to no supply curve for a monopoly producer. That is, $P = P(Q)$ in the short run, where Q denote the quantity of housing demand. The first derivative of the price function with respect to Q should be negative, i.e. $[P(Q)]' < 0$. As such, a simple form[16] derived with economic sense can be $P(Q) = Q^{-\varepsilon}$, where ε denotes the inverse of the elasticity of housing demand. In this study, the inverse elasticity of housing demand is assumed to be constant over the development period.

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[15] According to the evidence from the real estate development industry, professionals can be influenced by trends and exhibit habit persistence (Antwi and Henneberry, 1995).

[16] Alternative option of equation form for the relationship between price and quantity is a linear one such as $P(Q) = m - bQ$, where coefficient b captures the sensitivity of demand to price. The greater b indicates the lower sensitivity of demand to price.

In addition, price movement should conform to a long-term trend accompanied with stochastic fluctuations (see Quigg 1993; Bar-ilan and Strange, 1996). In this model, *X* denotes the state variable which describes the long-term trend incorporating stochastic fluctuations and thus indicates the long-term impacts of the external environment (such as population and economic growth, or political evolution) as well as any effects of external shocks (such as changes in inflowing demand) on housing prices. *X* at time *t* is assumed to follow a Geometric Brownian Motion (GBM) of the form

$$\frac{dX}{X} = \alpha_X dt + \sigma_X dw \qquad (1)$$

where α_X is the expected growth rate of *X*, σ_X is the standard deviation of growth rate, and *w* denotes the increment of a standard Wiener process. Capozza and Li (2002) stated that the parameters in the process can be observed by developers such that there are two theoretical cases - either perfect foresight if $\sigma = 0$, or rational expectations if $\sigma > 0$.

Accordingly, the house price is subject to the short-term housing demand *Q* associated with long-term adjustment *X* and in the mathematical form

$$P = XQ^{-\varepsilon} \qquad (2)$$

In the general case of residential projects, building density is required to be ascertained prior to the commencement of a project, which implies that the developer could hardly alter the supply arbitrarily since the start of the project, though the developer is operating in the face of varying market conditions. In light of this, the intrinsic value of the project development at any time *t* could be calculated if the developer decides to invest in the project with determined density *Q*. The development period is denoted by *δ*, which implies that the new completed house can be sold at time $t + \delta$. On the other hand, assume that the total cost of project development, denoted by $C(Q)$, consists of two parts, i.e. the fixed costs ($F > 0$) and variable costs depending on *Q* ($c > 0$, the unit cost of construction). That is, the total costs in such project can be formulated as $C(Q) = F + cQ$. With this total cost function, the intrinsic value denoted by **v** at time *t* is in the form of

$$\mathrm{v}(X_t) = E_t\left[e^{-\rho\delta} P_{t+\delta} Q - C(Q)\right] = e^{-\rho\delta} E_t[P_{t+\delta}]Q - C(Q) \qquad (3)$$

where *ρ* denotes the discount rate indicating capital cost. Equation (3) portrays a classic case of option-based analysis of project evaluation, implying that the developer waits for the optimal timing (denoted by T) to commence project development, in order to maximise the present value of expected earnings. Thus, at each time point t, they have two alternative choices in a waiting process: "to cease" means to stop waiting and invest at once, while "to continue" means to withhold an investment decision.

At any time t (t < T), the value of project (denoted by **V**) is determined by the expected value relying on the optimal timing (T) of development, i.e. in the form:

$$\mathrm{V}(X_t) = \max_T \{E_t[\mathrm{v}(X_T)]e^{-\rho(T-t)}\} \qquad (4)$$

Since the effect of sentiment is proved to be significant in housing asset pricing (Clayton et al., 2009) and housing transactions (Hui and Wang, 2014), this model is intended to be used to consider the roles of sentiment in project evaluation. As sentiment is derived from the belief basis and cannot be justified by market fundamentals, sentiment should be investigated as an independent factor in this model. Assume there is an available sentiment index (denoted by *SI*) measuring the market sentiment appropriately. Such index should reflect forward-looking expectation over a certain future period and this model would not be perplexed by the measurement.

On one hand, sentiment has an impact on future housing prices in project evaluation. That is, the expected house price at time $t + \delta$ should be adjusted by sentiment factor. To demonstrate this, take an instance of positive market sentiment. With positive sentiment in a local housing market, the majority of participants believe in a better future for the market and more people become willing to own properties. This causes the demand curve to shift to the right. On the other side, the supplier cannot increase supply quantity (Leishman, 2015) and the housing price rises from $E1$ to $E2$ (see Figure 5.1). The mechanism indicates that market sentiment can cause a price shifting in the short run.

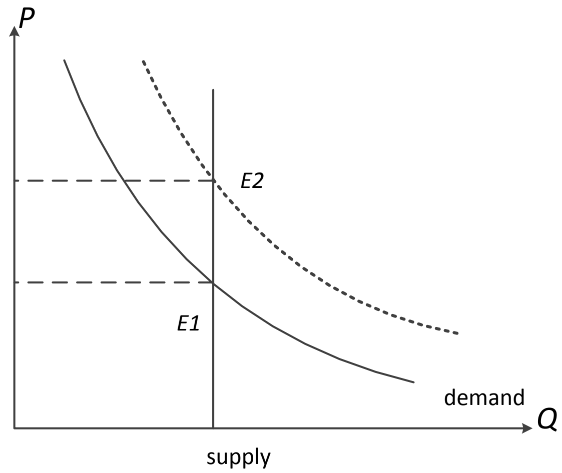


Figure 5.1. The effect of change in sentiment on the relationship between price and demand

As the project valuation is derived from the developer's perspective, it is more straight-forward to look at the effect of developers' sentiment, which should be taken into account for a calibration at time t for future house prices. Observing market sentiment, the developer forms their sentiment towards the market by combining their experiences and strategies with market sentiment. Assume the developer's sentiment is portrayed by a sentiment function[17] $f(SI)$. The sentiment function is not specified so that the variation of function form can capture the heterogeneity of developers. This sentiment function should be a monotonous increasing function with several properties: (i) $f'(SI) > 0$; (ii) $f(SI) = 1$ when market sentiment is neutral and thus the model reduces to a classical/baseline model without consideration of sentiment effect; (iii) $f(SI) > 1$ with positive (bullish) sentiment and $0 < f(SI) < 1$ with negative (bearish) sentiment. The last property implies that potential buyers would accept a higher price when market sentiment is positive and vice versa.

[17] More precisely, the developer's sentiment function should be a function of (Φ, SI), where Φ captures the idiosyncratic characteristics of developers. For instance, Cen et al. (2013) set up a specific structure to capture investor's belief (please see equation 4 in their paper) and Tse et al. (2011) suggest that developer's sentiment can be measured by developers' bid pattern at land auctions. For simple discussion, this study concentrates on sentiment effect rather than a whole picture of the effect of idiosyncratic characteristics.

More specifically, given the project's density, the expected house price at time $t+\delta$ in intrinsic value (equation 3) should be adjusted by the developer's sentiment $f(SI_t)$ as

$$E_t^s[P_{t+\delta}] = E_t[X_{t+\delta}Q^{-\varepsilon}]f(SI_t) = E_t[X_{t+\delta}]Q^{-\varepsilon}f(SI_t) \qquad (5)$$

On the other hand, a considerable number of studies indicate that the expected return rate of the real estate market is affected by sentiment (e.g. Marcato and Nanda, 2016). In this model, the short-term discount rate ρ in equation (3) is affected by the sentiment factor. Although developers' (or professional and expertise in real estate) forecasts show a high correlation with actual market return, the discrepancy between forecasts and real return cannot be ignored (McAllister et al., 2008). That is, ρ should be adjusted with the sentiment effect by a sentiment function, i.e. $\rho_s = g(SI_t)\rho$, where $g(SI_t)$ can be different from but has the same properties as $f(SI_t)$.

Therefore, concerning the effect of developer's sentiment on project evaluation, at the optimal time (when t = T) to commence the development, the intrinsic value (denoted by $\mathrm{v}(X_T)$) incorporating sentiment effect should be modified into the form of

$$\mathrm{v}(X_T) = E_T^s\left[e^{-\rho\delta}(P_{T+\delta}Q_T^{-\varepsilon})\right]\ f(SI_T) - C(Q_T)$$
$$= e^{-\rho_s\delta}E_T^s[(X_{T+\delta})]Q_T^{1-\varepsilon}f(SI_T) - (F + cQ_T)$$
$$= e^{-(\rho_s-\alpha_X)\delta}X_TQ_T^{1-\varepsilon}f(SI_T) - (F + cQ_T) \qquad (6)$$

5.3 Optimal Choices in Project Development

This subsection solves the model to obtain the optimal choices of development in a real estate project which is irreversible and confronted with uncertainty. The solution follows the framework of Dixit and Pindyck (1994) and consists of two parts, i.e. optimal density and optimal timing to develop.

First, the developer can figure out the optimal density (Q_T) of development at time T on the basis of the first-order condition: $\partial\mathrm{v}(X_T)/\partial Q = 0$. The optimal density can be calculated as

$$Q_T = \left[\frac{ce^{(\rho-\alpha_X)\delta}}{f(SI)X_T(1-\varepsilon)}\right]^{-1/\varepsilon} = \left[\frac{f(SI)X_T(1-\varepsilon)}{ce^{(\rho-\alpha_X)\delta}}\right]^{1/\varepsilon} \qquad (7)$$

The above equation indicates that if Q_T exists, $Q_T > 0$ implies that $(1-\varepsilon)$

should be larger than 0, which means the inverse elasticity of housing demand $\varepsilon < 1$ and thus the elasticity of demand $1/\varepsilon$ should be greater than 1. This implication echoes with empirical evidence [18] from previous literature.

The optimal timing of project development is proved to be one of the most important concerns in decision making processes for a real estate project. In order to figure out the optimal timing, it is imperative to establish the value equation to study how the option value of the project changes. By solving this equation, the analytical solution of optimal timing is given in terms of expected waiting time to invest. According to Grenadier (1995), the expected growth of the option value should be equal to the expected return earned by investing the capital with the same amount as the option value into the market. Therefore, the differential equation is

$$\rho \mathrm{V} dt = E[d\mathrm{V}(X_t)]$$

Based on the model assumptions and applying the Ito's lemma, the instant change in option value is

$$d\mathrm{V}(X_t) = \left[\alpha_X X \mathrm{V}_X + \frac{1}{2}(\sigma_X X)^2 \mathrm{V}_{XX}\right] dt + [\sigma_X X \mathrm{V}_X] dw_X$$

Assume the trigger of optimal investment is determined by X^*, where X^* equals to X_T with Q^* or $Q_T > 0$. When $X < X^*$ or $t < T$, the value of project should satisfy the following second order ordinary differential equation:

$$\rho_s \mathrm{V} = \alpha_X X \mathrm{V}_X + \frac{1}{2}(\sigma_X X)^2 \mathrm{V}_{XX} \qquad (8)$$

given the initial-condition $\mathrm{V}(0) = 0$.

It is necessary to state that the value of project **V** should satisfy two boundary conditions to figure out the solution to the differential equation above. These two boundary conditions are value matching and smooth pasting. First, the value-matching condition required to fulfil the continuity of the value function is shown as:

$$\mathrm{V}(X^*) = \mathrm{v}(X_T)$$

Based on the project intrinsic value given by equation (6) and the optimal

[18] Theoretically, a developer with monopoly power will not run a business in housing markets if the housing demand is inelastic. Empirically, elasticity for newly completed residential property in some research is found to be larger than 2 according to previous literature such as Malpezzi and Maclennan (2001) and Ott et al. (2012).

density given by equation (7), the value of an optimal project development is calculated by

$$\mathrm{V}(X^*) = c^{\left(1-\frac{1}{\varepsilon}\right)}\left[e^{(-(\rho_s-\alpha_X)\delta)}f(SI)X_T\right]^{\frac{1}{\varepsilon}}(1-\varepsilon)^{\frac{1}{\varepsilon}}\left(\frac{1}{1-\varepsilon}-1\right)-F \qquad (9)$$

To pin down the free boundary (X*) of differential equation, the second boundary condition derived from optimization is called smooth pasting:

$$\mathrm{V}_X(X^*) \;=\; \mathrm{v}_X(X_T)$$

Based on equations (6) and (9), smooth pasting condition is given by

$$\mathrm{V}_X(X^*) = c^{\left(1-\frac{1}{\varepsilon}\right)}\left(e^{(-(\rho_s-\alpha_X)\delta)}f(SI)\right)^{\frac{1}{\varepsilon}}\left[\frac{1}{\varepsilon}X_T{}^{\left(\frac{1}{\varepsilon}-1\right)}\right](1-\varepsilon)^{\frac{1}{\varepsilon}}\left(\frac{1}{1-\varepsilon}-1\right) \quad (10)$$

In accordance with the solving process of real option (e.g. see Dixit and Pindyck, 1994), the project value at time *t* subject to the boundary conditions can be demonstrated as

$$\mathrm{V}(X_t, X^*) = \begin{cases} \mathrm{V}(X^*)\left(\frac{X_t}{X^*}\right)^{\Omega}, & X_t < X^* \\ \mathrm{V}(X_t), & X_t \geq X^* \end{cases} \qquad (11)$$

where $\Omega = \frac{1}{2} - \frac{\alpha_X}{\sigma_X{}^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha_X}{\sigma_X{}^2}\right)^2 + \frac{2\rho_s}{\sigma_X{}^2}}$ and it is obvious that $\Omega > 0$.

The above equation implies that term $(X/X^*)^{\Omega}$ is the stochastic discount factor in this option pricing (see also Wong, 2007). The existence of a stochastic discount factor infers the law of one price which ensures the uniqueness of option-based project valuation. In addition, the coefficient Ω describes the sensitivity of the project value to the exogenous adjuster of long-term trend *X*. Based on the transformation of equation (11), the nature of Ω can be revealed. Under the conditions t < T or $X < X^*$, equation (11) can be transformed into

$$\mathrm{V}(X)/\mathrm{V}(X^*) = (X/X^*)^{\Omega}$$

Set $L_X = X/X^*$ and $L_V = \mathrm{V}(X)/\mathrm{V}(X^*)$. The first ratio is to measure the growth of state variable *X* while the other is to measure the corresponding changes in project value $\mathrm{V}(X)$. Accordingly, Ω represents the state variable (the percentage L_X) elasticity of project value (the percentage L_V) as

$$\Omega = \frac{dln(L_V)}{dln(L_X)} = \frac{dL_V}{dL_X}\frac{L_X}{L_V}$$

In addition, the threshold (or the trigger) value of project development is critical for the developer's decision making. In this model, the threshold value of project development X* can be calculated based on equations

(9-11):

$$X^* = \Lambda \left[\frac{(1-\varepsilon)F\Omega}{\varepsilon\left(\Omega - \frac{1}{\varepsilon}\right)} \right]^{\varepsilon} \quad (12)$$

where $\Lambda = \frac{c^{(1-\varepsilon)} exp((\rho_S - \alpha_X)\delta)}{(1-\varepsilon)f(SI)}$.

The equation of threshold indicates that if X^* exists, $X^* > 0$ requires that $(\Omega - 1/\varepsilon)$ should be positive, which leads to the following proposition.

***Proposition* 1**. The necessary conditions for existence of the threshold value X^* is that the elasticity of project value (Ω) should be greater than the elasticity of housing demand $1/\varepsilon$ if housing demand is elastic ($1/\varepsilon > 1$). Furthermore, the existence of X^* means that $\rho_S > \alpha_X$, i.e. the adjusted expected return rate in the local housing market should be greater than the expected state growth rate.

The proof appears in Appendix A.

This proposition reveals two implications on the elasticity of project value (Ω). First, a higher elasticity of housing demand implies a higher elasticity of project value. Referring to equation (6), the project value is positively related to the state variable X in two ways, i.e. directly and indirectly through demand Q. It can be observed that demand is positively correlated with the state variable powered by $1/\varepsilon$, by transforming the price function (equation 2) into demand function. Hence, with the combination of two positive elasticity derived from direct and indirect effects, the elasticity of project value is larger than the elasticity of housing demand.

On the other hand, the definition of Ω implies that project value becomes elastic ($\Omega > 1$) when $\rho_S > \alpha_X$ and vice versa. This implication reflects that as the expected return rate of the housing market exceeds the growth rate of the state variable, indicating a risk premium, the fluctuations of project value are likely to be volatile and the project value becomes sensitive to the changes in state variable. In addition, due to the two inequalities: $\partial\Omega/\partial\rho_S > 0$ and $\partial\Omega/\partial\alpha_X < 0$, it is obvious that the increase in disparity (i.e. the risk premium) between the expected return rate of the housing market (ρ_S) and the growth rate of state variable (α_X) places Ω at a larger value.

Substituting X^* into equation (7), the optimal density becomes

$$Q^* = \frac{(1-\varepsilon)F\Omega}{c\varepsilon\left(\Omega-\frac{1}{\varepsilon}\right)} = \frac{(1-\varepsilon)F}{c\varepsilon}\frac{\Omega}{\left(\Omega-\frac{1}{\varepsilon}\right)} \quad (13)$$

while substituting X^* into equation (9), the optimal value becomes

$$\mathrm{V}^* = F\left[\frac{\Omega}{\left(\Omega-\frac{1}{\varepsilon}\right)} - 1\right] = \frac{F}{\varepsilon}\frac{1}{\left(\Omega-\frac{1}{\varepsilon}\right)} \quad (14)$$

By solving the model with the effects of uncertainty and sentiment, the implications of results show that the project developer should wait when the exogenous state is inferior to the threshold of X given by equation (12). Otherwise, the developer should commence the project development with the optimal density given by equation (13), while the intrinsic value of project development is determined by equation (14).

5.4 Probability of Investing

Generally, the traditional option-based analysis only concerns the existence of the trigger or the optimal timing of the project development. Although it is possible to find the threshold for optimal decision, the result is deemed to be trivial if the variable X cannot reach the threshold or it takes too long to reach the threshold. Therefore, in addition to existence of the threshold value, the possibility of optimal development, referring to the probability that the state variable X could reach the X^* (or the probability of investing, see Sarkar, 2000), is also prominent in the decision-making process of project development.

Given the initial condition of state variable X_0 at time t = 0 (suppose the process starts out at $X_0 < X^*$, otherwise the developer should invest immediately) and knowing that the state variable X follows the Geometric Brownian Motion, the probability density function of waiting time until the first arrival with respect to X^* is

$$f(t; X_0, X^*) = \frac{ln(X^*/X_0)}{\sigma_X\sqrt{2\pi t^3}} exp\left[\frac{\left(ln(X^*/X_0)-(\alpha_X-0.5\sigma_X^2)t\right)^2}{-2\sigma_X^2 t}\right]$$

Based on this density function, the expected waiting time for first reaching the threshold value can be derived as (see also Grenadier and Wang, 2005; and Wong, 2007)

$$E[t; X_0, X^*] = \frac{ln(X^*/X_0)}{\alpha_X-0.5\sigma_X^2} \quad (15)$$

From equation (15), it is clear that the expected waiting time is positively correlated to the threshold value, taking all other parameters $(X_0, \alpha_X, \sigma_X)$ as constants. In other words, the lower threshold value of the state variable will help shorten the waiting time and expedite the project investment. Note that this equation requires the presumption that $\alpha_X > 0.5\sigma_X^2$. In fact, Equation (1) implies that $\ln X$ follows a Brownian motion with drift $\alpha_X - 0.5\sigma_X^2$ and hence this presumption is guaranteed to have a positive drift. If the opposite condition, i.e. $\alpha_X < 0.5\sigma_X^2$, is set up, the probability of reaching the threshold in finite time could be given as $\left(\frac{X^*}{X_0}\right)^{2\alpha_X/\sigma_X^2-1}$, whilst the expected waiting time becomes infinite, which could be derived from Harrison (1985).

Furthermore, the developer can be in face of a penalty, even an adjudication of land losing, if the acquired land has been idle over a stipulated period[19]. This indicates that the length of waiting time for optimal project development is capped by the contract of land leasing. Such situation often happens in the cases of local governments selling land to developers. This consideration has practical significance of applying option-based analysis in optimal decision of project development. Accordingly, this scenario indicates that the developer holds an option with finite expiry *N* at time $t = 0$. According to equation (7) in Sarkar (2000), the probability of reaching the threshold value (X^N) within time period *N* is given by

$$Pr(X^N, N) = \Phi(d_1) + \left(\frac{X^N}{X_0}\right)^{2\alpha_X/\sigma_X^2-1} \Phi(d_2)$$

Where $d_1 = \frac{ln(X_0/X^N)+(\alpha_X-0.5\sigma_X^2)N}{\sigma_X\sqrt{N}}$ and $d_2 = \frac{ln(X_0/X^N)-(\alpha_X-0.5\sigma_X^2)N}{\sigma_X\sqrt{N}}$.

Then, the project value with limited expiry *N* is

$$\mathrm{V}_N(X_0, X^N, N) = \mathrm{V}(X_0, X^N)Pr(X^N, N)$$

Based on the above equation, the threshold value (X^N) of an optimal development project with finite expiry *N* can be determined through the first-order condition (FOC):

[19] This happens in some countries/regions, especially those areas are focusing on urbanization. For instance, mainland China and Hong Kong implement such policy in their land markets.

$\partial V_N(X_0, X^N, N) / \partial X^N = 0$

Note that there is no analytical solution to this FOC. But it is possible to have an asymptotic solution by using numerical calculation for the threshold value X^N. It takes no qualitative effect if the model ignores the limitation on expiry and thus it is convenient to consider the option value of project development a perpetual option. Since the main purpose of this study is to investigate the effects of market sentiment on optimal development strategy, the analysis and discussion in the sequel revolves around the model without finite expiry *N*.

5.5 Effects of Sentiment

To address the research question on the effects of sentiment on the supply side of the housing market, this section conducts a series of comparative static analysis in terms of critical indicators (expected waiting time, threshold value of state variable and optimal density) with respect to the sentiment factor, taking all other parameters ($X_0, c, F, \delta, \rho, \alpha_X, \sigma_X$) as constant. This analysis shall offer a solid footstone on which to build further studies.

The two functions f and g (as mentioned in Section 2.3) offer the sentiment adjustment to the price function and the expected market return rate respectively. Since $g(SI_t)$ has the same properties as $f(SI_t)$, we assume these two functions are equal ($f = g$) for simplicity and use f only in the sequel. Differentiate the equation of Ω with respect to sentiment (*SI*), which yields

$$\frac{\partial\Omega}{\partial SI} = \frac{\rho}{\sigma_X{}^2}\left(\left(\frac{1}{2} - \frac{\alpha_X}{\sigma_X{}^2}\right)^2 + \frac{2\rho f}{\sigma_X{}^2}\right)^{-1/2} f'(SI) \qquad (16)$$

It is obvious that $\frac{\partial\Omega}{\partial SI} > 0$ as $f'(SI) > 0$. The relationship implies that when the participants in the aggregate are optimistic that market tends to boom in the near future, the elasticity of project value rises and project value becomes more sensitive to exogenous impacts or shocks (proxied by state variable) and vice versa.

It is of interest to examine the effects of sentiment on the threshold *X** and expected waiting time for optimal development. Since these two terms are positively correlated, the sentiment effects on these two terms should have the same signs. Differentiating equation (15) with respect to sentiment (*SI*) yields

$$\frac{\partial E[t;X_0,X^*]}{\partial SI}=\frac{\partial E[t]}{\partial X^*}\frac{\partial X^*}{\partial SI}=\frac{1}{X^*(\alpha_X-0.5\sigma_X^2)}\frac{\partial X^*}{\partial SI} \tag{17}$$

Differentiating the threshold of state variable *X** with respect to sentiment (*SI*) yields

$$\frac{\partial X^*}{\partial SI}=\frac{\partial X^*(f,\rho,\Omega)}{\partial SI}=\frac{c^{(1-\varepsilon)}}{(1-\varepsilon)}\left[\frac{F(1-\varepsilon)}{\varepsilon}\right]^{\varepsilon}\frac{\partial M}{\partial SI} \tag{18}$$

where $M=\frac{e^{(\rho_S-\alpha_X)\delta}}{f(SI)}\left[\frac{\Omega}{\left(\Omega-\frac{1}{\varepsilon}\right)}\right]^{\varepsilon}$. In the above equation, as $\frac{c^{(1-\varepsilon)}}{(1-\varepsilon)}\left[\frac{F(1-\varepsilon)}{\varepsilon}\right]^{\varepsilon}>0$, this indicates that $\frac{\partial X^*}{\partial SI}$ (as well as $\frac{\partial E[t]}{\partial SI}$) has the same sign as that of $\frac{\partial M}{\partial SI}$. The expression of *M* indicates that the effects of sentiment on the threshold value *X** and expected waiting time ***E***(*t*) are not monotonic, as is shown in Proposition 2.

***Proposition* 2**. There exists a turning point for sentiment (denoted by SI_b), which is implicitly defined by

$\omega(f_b)=\delta$, where $f_b=f(SI_b)$

such that $\frac{\partial X^*}{\partial SI}>0$ and $\frac{\partial E[t]}{\partial SI}>0$ for every $SI>SI_b$ and vice versa.

The proof appears in the Appendix B.

Proposition 2 reveals the non-monotonicity in the effects of sentiment on the threshold value and expected waiting time. When market sentiment is low (less than SI_b), an increase in sentiment will reduce the threshold value and thus shorten the expected waiting time to invest. On the contrary, when market sentiment is high (greater than SI_b), an increase in sentiment will enlarge the threshold value and thus delay the investment. In other words, the effect of sentiment on expected waiting time shows a U-shape curve, i.e. downward at first and upward at last.

To see the intuition, this complex effect can be decomposed into three parts based on equation B.1 (see Appendix B). Note that, for sake of simplicity, we have set *g* = *f* at the beginning of this section. To offer a clearer insight, we redo equation B.1 without this simplification as

$$\frac{\partial M}{\partial SI}=-\frac{e^{(\rho_S-\alpha_X)\delta}}{f^2}f'\left[\frac{\Omega}{\left(\Omega-\frac{1}{\varepsilon}\right)}\right]^{\varepsilon}+\frac{e^{(\rho_S-\alpha_X)\delta}\rho\delta}{f}g'\left[\frac{\Omega}{\left(\Omega-\frac{1}{\varepsilon}\right)}\right]^{\varepsilon}$$

$$-\frac{e^{(\rho_S-\alpha_X)\delta}}{f}\frac{1}{\Omega\left(\Omega-\frac{1}{\varepsilon}\right)}\left[\frac{\Omega}{\left(\Omega-\frac{1}{\varepsilon}\right)}\right]^{\varepsilon}\frac{\partial\Omega}{\partial g}g' \qquad (19)$$

There are three channels through which sentiment affects the threshold value. The first term on the right hand side of the equation above shows the direct effect of sentiment, while the second and last terms indicate two indirect effects through discount rate (or expected return rate).

The direct effect demonstrated by the first term is negative, which reflects that an increase in sentiment will push down the threshold (X*) and thus shorten the waiting time. The second term implies that if market sentiment is higher, it provides a stronger adjusting power to pull up the capital cost, and reminds the developer of pursuing a higher expected return rate. As a greater expected return rate indicates a higher risk premium and larger systematic risk, the developer would wait for a higher threshold value[20] so as to diminish the possibility of unfavourable effects caused by systematic uncertainties in the future. The last term reveals a channel of the elasticity of project value through which sentiment affects the threshold. The negative sign indicates that a higher level of sentiment, which leads to a higher elasticity of project value, would reduce the threshold value. This makes project delay less beneficial.

In the aggregate, when sentiment is relatively low, the direct effect dominates among the three effects and acts as an accelerator for project development. When sentiment ascends, the two indirect effects become remarkable to determine the sign of the total impact of sentiment on the threshold value or the expected waiting time. In particular, when sentiment rises to a higher level, the indirect effect of sentiment through expected market return (second term) dominates, the total effect of sentiment comes out to be positive, increases the threshold value and thus causes a delay in project investment. Such result contributes to the knowledge found in previous studies of investment lags.

In addition to the shape of the curve, Proposition 3 provides a pair of lower and upper boundaries for the turning point of the curve, which are

[20] In traditional option-based analysis of project development, if discount rate is high enough, increase in discount rate will raise the threshold value of system uncertainty (such as *X* in this model).

easy to calculate and implement.

***Proposition* 3**. There exist two critical values of sentiment (denoted by SI_1 and SI_2), which are defined by

$$f_1 = f(SI_1) = \frac{1+\sqrt{1+\delta\sigma^2}}{2\rho\delta} \text{ and } f_2 = f(SI_2) = \frac{1}{\rho(\delta - A_2/\sigma^2)}$$

where $A_2 = \frac{1}{\Omega_2^2(\Omega_2 - 1/\varepsilon)}$ and $\Omega_2 = \frac{1}{2} - \frac{\alpha_X}{\sigma_X{}^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha_X}{\sigma_X{}^2}\right)^2 + \frac{2}{\delta\sigma_X{}^2}}$, such that $\frac{\partial X^*}{\partial SI} < 0$ and $\frac{\partial E[t]}{\partial SI} < 0$ for $SI < SI_1$, whilst $\frac{\partial X^*}{\partial SI} > 0$ and $\frac{\partial E[t]}{\partial SI} > 0$ for $SI > SI_2$.

The proof appears in Appendix C.

It is obvious that f_1 depends on the development period δ and f_2 is conditional on the term $\delta - A_2/\sigma^2$. In fact, taking sentiment and other factors constant, an increase in the development period will induce a rise in threshold value X^* (see equation 12), and will obviously deliver a delaying effect on the expected waiting time. Hence, it is of interest to investigate the role of development period δ in these two boundaries and the turning point. To this end, some implications are derived from the following corollary by applying statics exercise.

***Corollary* 1**. If the condition $\delta < \delta^*$ (δ^* is defined by equation D.3 in Appendix D) holds, then $f_2 = \frac{1}{\rho(\delta - A_2/\sigma^2)}$ in Proposition 3 exists. Furthermore, $\frac{df_2}{d\delta} < 0$. When $\delta > \delta^*$, f_2 does not exist. Moreover, $\frac{df_1}{d\delta} < 0$ and $\frac{df_b}{d\delta} < 0$.

The proof is given in Appendix D.

This corollary reveals a constraint on the development period δ to realize the non-trivial $f(SI_2)$. More generally, it explores that the first order derivative of three critical points, i.e. f_1, f_2 and f_b, with respect to the development period δ shows negative correlations, which implies that the turning point SI_b becomes lower if the development period is relatively long. In other words, the development period could trigger a delaying effect (on expected waiting time) when it becomes longer. Even when the sentiment is relatively low (the curve in the downward part), such delay effect would offset the accelerating effect arising from the rise in sentiment.

To receive the payoff in a shorter period, the developer aims to control the development period within a manageable length. The longer the development period is, the lower the net present value (NPV) of revenue is and thus the less lucrative the project is. That is, the developer is more likely to postpone the project and turn to another profitable project with a shorter development period.

Moreover, a longer development period indicates a higher risk embedded in the project rather than a profitable investment. Extending the development period could enlarge the expected variance of the state variable and thus magnify the underlying risks brought by the future uncertainty. Therefore, the developer is willing to wait for a higher threshold, as an effective strategy to hedge against future risk and uncertainty.

On the other hand, it is important to investigate the relationship between sentiment and optimal density (or the supply), and the relationship between sentiment and optimal project value based on the optimal density. The following discussion gives an insight into the effect of sentiment on housing supply in a project development.

Differentiating the equation of optimal density $Q*$ with respect to sentiment *SI*, and have

$$\frac{\partial Q^*}{\partial SI} = \frac{\partial Q^*}{\partial \Omega}\frac{\partial \Omega}{\partial SI} = \frac{F}{c\varepsilon\left(\frac{1}{1-\varepsilon}-1\right)}\frac{-1}{(\Omega-1/\varepsilon)^2}\frac{\partial \Omega}{\partial SI} \qquad (20)$$

With $\partial\Omega/\partial SI > 0$, it is evident from the above equation that $\partial Q^*/\partial SI < 0$, laying out a negative relationship between the optimal density and sentiment. In other words, the optimal density decided prior to the commencing of project development turns to be lower when the market sentiment is higher and vice versa. The reason for this is as follows. The home-buyer is more likely to take part in the housing market if the market sentiment is high (or positive). Since houses are heterogeneous and durable goods, it is difficult to increase the quantity of a certain property. In face of the housing stock being fixed in the short run, the home-buyer is willing to put more money on a property as housing demand rises. The supplier forms his attitude towards the market trend when he observes the demand curve shifts upwards. The reasonable choice for the developer is to provide high-end properties (certainly with high value) to meet the needs for more valuable property. High-end positioning can be partly justified by lower density of a project (lower density induces higher price). On the contrary, if market sentiment becomes negative, bullish enthusiasm recedes and owner-occupied (first-time buyer) demand dominates. Such demand can be fulfilled by the houses with the merit of being cost-effective. These economical houses are always the projects with a higher development density.

Differentiating the equation of value of optimal development with respect to sentiment *SI*, then obtain

$$\frac{\partial V^*}{\partial SI} = \frac{\partial V^*}{\partial \Omega}\frac{\partial \Omega}{\partial SI} = \frac{F}{\varepsilon}\frac{-1}{(\Omega - 1/\varepsilon)^2}\frac{\partial \Omega}{\partial SI} \qquad (21)$$

As $\partial\Omega/\partial SI > 0$, the above equation suggests $\partial V^*/\partial SI < 0$, and indicates that project value decreases when market sentiment becomes higher and vice versa. In fact, this negative effect can be decomposed into three parts. To see the intuition, differentiate equation 9 with respect to sentiment

$$\frac{\partial V^*}{\partial SI} = \frac{\partial V^*}{\partial \rho_s}\frac{\partial \rho_s}{\partial f}f' + \frac{\partial V^*}{\partial X^*}\frac{\partial X^*}{\partial f}f' + \frac{\partial V^*}{\partial f}f'$$

On the right hand side of the above equation, the first and second terms describe the indirect effects of sentiment and the last term measures the direct effect. The first term shows a negative correlation and reveals that higher (lower) level of sentiment induces an increase (decrease) in discount rate and makes investment lag more (less) costly, thus drives the present value of a project to fall (rise). The second term captures the effect of sentiment on the project value through the intermediate of threshold value. This effect is complicated due to $\partial X^*/\partial SI$ showing a U-shape pattern. The last term shows a positive correlation between project value and sentiment.

The negative total effect of sentiment on the project value indicates that the negative effect of discount rate (the first term) becomes more dominant along with the increase in sentiment. Especially when the sentiment is high (excess the turning point), the first term overwhelms the positive effects (of the second and last terms). Although a higher sentiment indicates that the developer anticipates higher house prices and a higher market return for the near future, due to a certain development period, the developer will experience investment lag and thus will miss the higher expected return. Furthermore, the longer the development period is, the lower the project value would be. At this moment, the developer may choose to wait and put money into an investment with a shorter lag. Hence, such expected lag of payoff plays a more significant role in determining the present value of the project in which developer's sentiment has a role.

5.6 Numerical Analysis

This section conducts a numerical analysis in order to provide a more straightforward demonstration. Taking the parameter set $\{\rho, \alpha_X, \sigma_X, c, F, \varepsilon, \delta, X_0\}$ as constant (given in Table 5.1), several scenarios are designed to explore the different patterns of sentiment effects on expected waiting time, optimal density and project value.

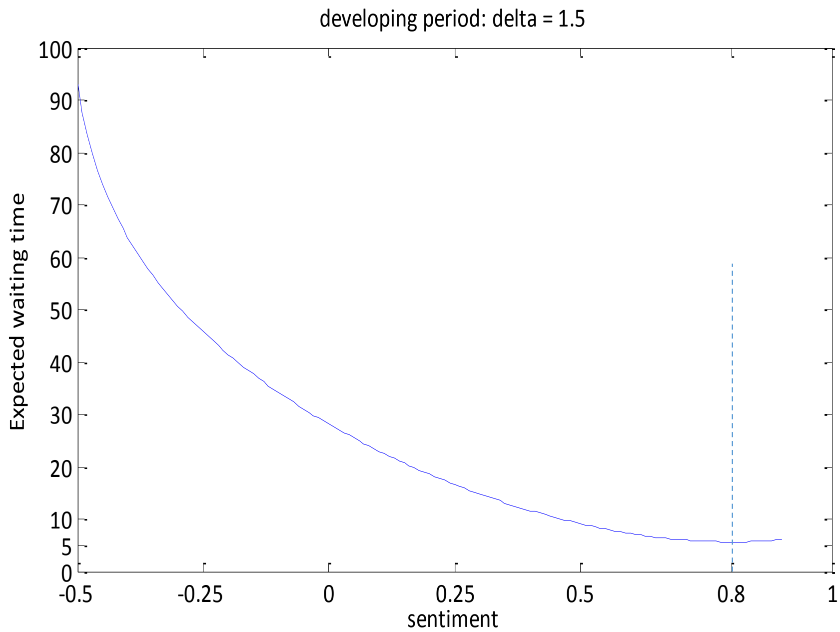
Prior to scenario studies, the descriptions of the market sentiment variable and the function of developer's sentiment are necessary. Assume a forward-looking index of market sentiment. Such index measures the participants' belief toward market movement in the near future at aggregate level and scaled within (-1, 1). If the variable approaches -1, the market is extremely pessimistic about the future; if the variable is equal to 0, the market sentiment can be interpreted as neutral; and if the variable approaches 1, the market is extremely optimistic. Moreover, the developer's sentiment in the project valuation can be depicted by the sentiment function *f*. In this study, the sentiment function reflects that the developer forms his sentiment based on the market sentiment and the function *f* is formulated in exponential form to fulfil the properties mentioned in the above section, so that $f = \upsilon \exp(SI)$ where v is the coefficient describing the extent to which the developer is sensitive to market sentiment. Set $v = 1$ in this analysis for simplifying discussion.

Table 5.1. Numerical analysis of sentiment effect

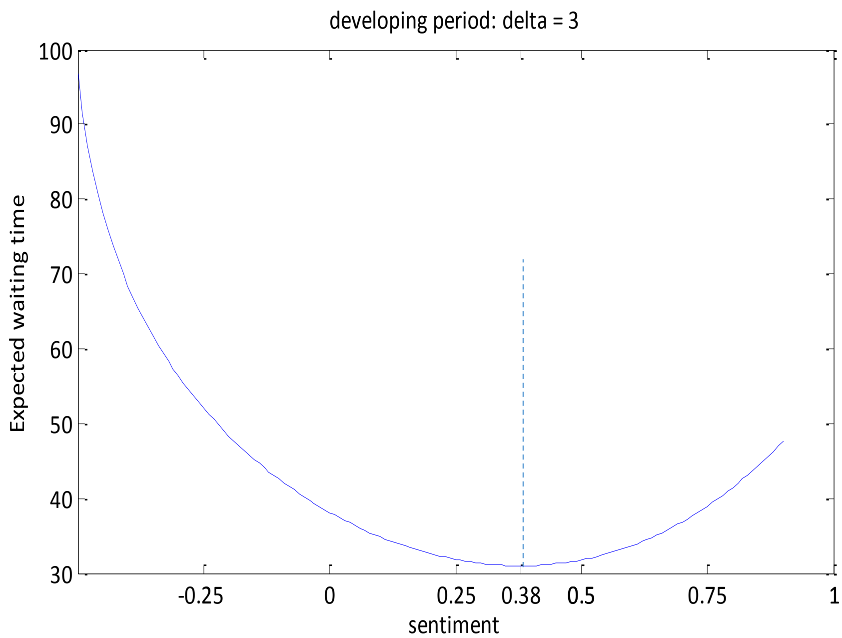
Panel A: Scenario 1 (base case)	
$\{\rho, \alpha_X, \sigma_X, \varepsilon, c, F, X_0,\}$ = {0.25, 0.05, 0.20, 1/1.8, 1, 1, 2} and δ = 1.5	
$\delta*$	4.72
SI_1	0.71
SI_2	0.85
SI_b	0.80
Panel B: Scenario 2	
$\{\rho, \alpha_X, \sigma_X, \varepsilon, c, F, X_0,\}$ = {0.25, 0.05, 0.20, 1/1.8, 1, 1, 2} and δ = 3	
$\delta*$	4.72
SI_1	0.23
SI_2	0.63
SI_b	0.38
Panel C: Scenario 3	
$\{\rho, \alpha_X, \sigma_X, \varepsilon, c, F, X_0,\}$ = {0.25, 0.05, 0.20, 1/1.8, 1, 1, 2} and δ = 6	
$\delta*$	4.72
SI_1	-0.25
SI_2	-
SI_b	0.02
Panel D: Scenario 4	
$\{\rho, \alpha_X, \sigma_X, \varepsilon, c, F, X_0,\}$ = {0.25, 0.05, 0.20, 1/1.8, 1, 1, 2} and δ = 8	
$\delta*$	4.72
SI_1	-0.44
SI_2	-
SI_b	-0.10

By applying equations (13)-(15), the optimal density, project value and expected waiting time are calculated respectively, for a series of values of sentiment index (*SI*). Preliminarily, $\delta*$ promulgated in Corollary 1 can be calculated according to equation 3 in Appendix D and is equal to 4.72. To study the role of δ in the effect of sentiment on expected waiting time, four scenario studies are designed with different values of δ: the first two scenarios are configured with $\delta < \delta*$ while the rest of scenarios are configured with $\delta > \delta*$. More specifically, four different values $\{1.5, 3, 6, 8\}$ are assigned to δ for a comparison to visually support the corollary. The details of each scenario and critical indicators derived from propositions are shown in Table 5.1, while Figure 5.2 displays four charts showing the different patterns of expected waiting time with respect to sentiment with different development periods.

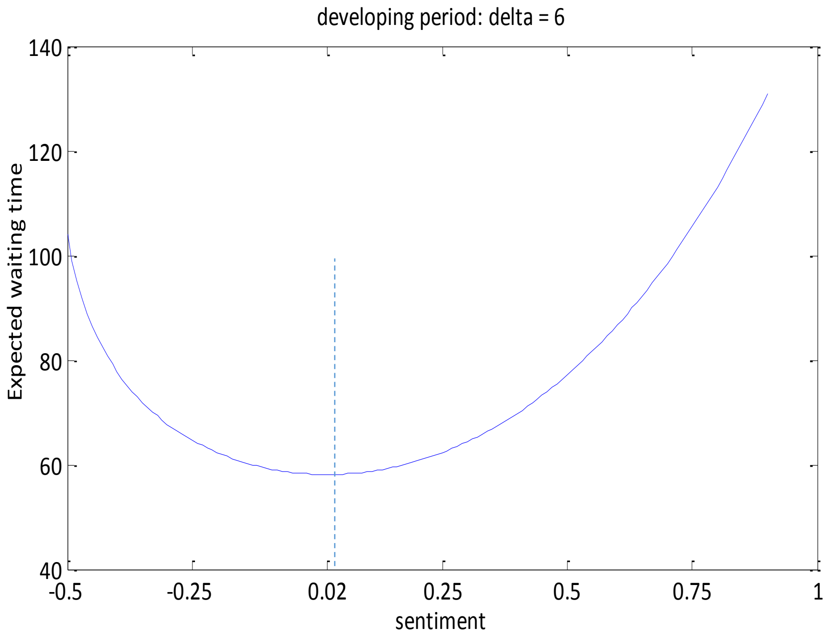
Generally, the expected waiting time decreases first and then increases with sentiment, showing convex U-shape patterns in four scenarios without exception. In the first two scenarios, as the development period is shorter than δ^* (4.72), we can easily find SI_1 and SI_2 suggested in Proposition 3 to identify the lower and upper bounds of the turning point SI_b. In addition, the turning point is identified with accuracy of 0.01. In the final two scenarios, SI_2 no longer exists as $\delta > \delta^*$. All the four turning points are marked on the x-axis in Figure 5.2. As evident from both Figure 5.2 and Table 5.1, the turning point emerges earlier when the development period δ extends, which echoes with $df_i/d\delta < 0$ where $i = b$, 1, 2. That is, the delaying effect of sentiment on the expected time to exercise the option of project development is more likely to be observed in projects with longer development periods than those with usual periods.



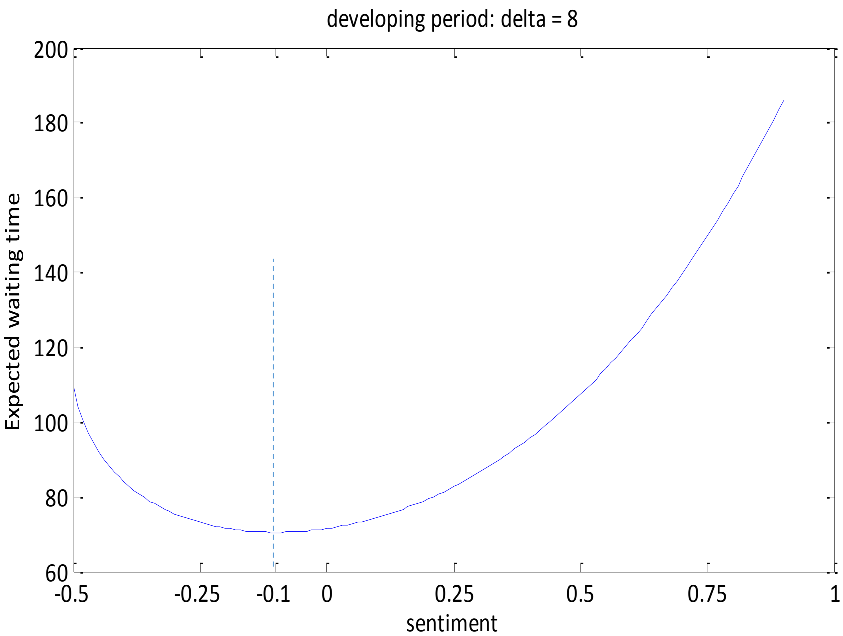
(a) Turning point of sentiment SI_b = 0.80 when δ = 1.5



(b) Turning point of sentiment SI_b = 0.38 when δ = 3



(c) Turning point of sentiment $SI_b = 0.02$ when $\delta = 6$



(d) Turning point of sentiment $SI_b = -0.10$ when $\delta = 8$

Figure 5.2. The relationship between expected waiting time and sentiment in four cases specified by different developing period ($\delta \in \{1.5, 3, 6, 8\}$)

For the charts of optimal density and project value with respect to sentiment, Figures 5.3 and 5.4 display two downward curves with different slopes. These two curves are consistent with the implications arising from equations (21) and (22) respectively, indicating that an increase in sentiment induces a decrease in optimal density or project value taking other factors as constant. In other words, developer should cut down the project density in the presence of high sentiment. A higher sentiment environment would lead to a residential project with lower value. Developer should take into account the sentiment effect in project evaluation.

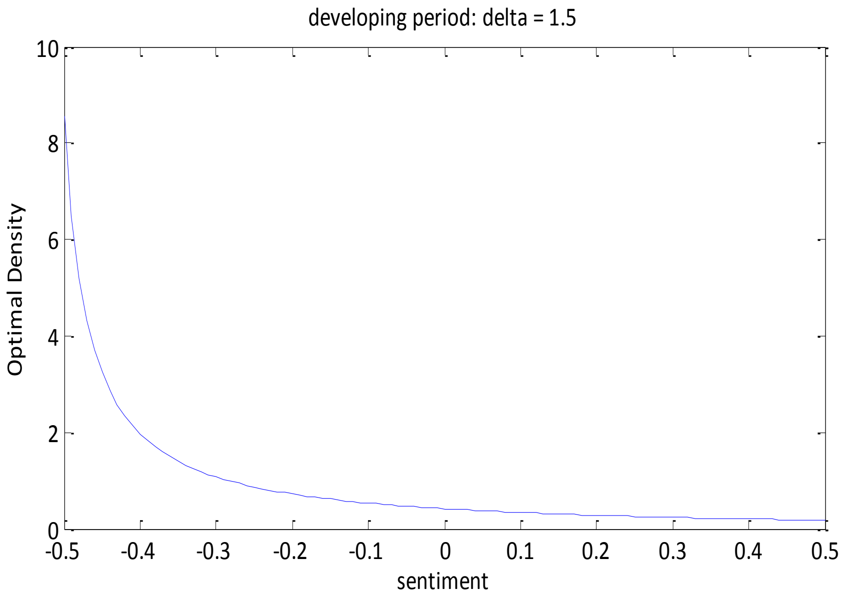


Figure 5.3. The relationship between optimal density and sentiment in the cases specified by development period $\delta = 1.5$)

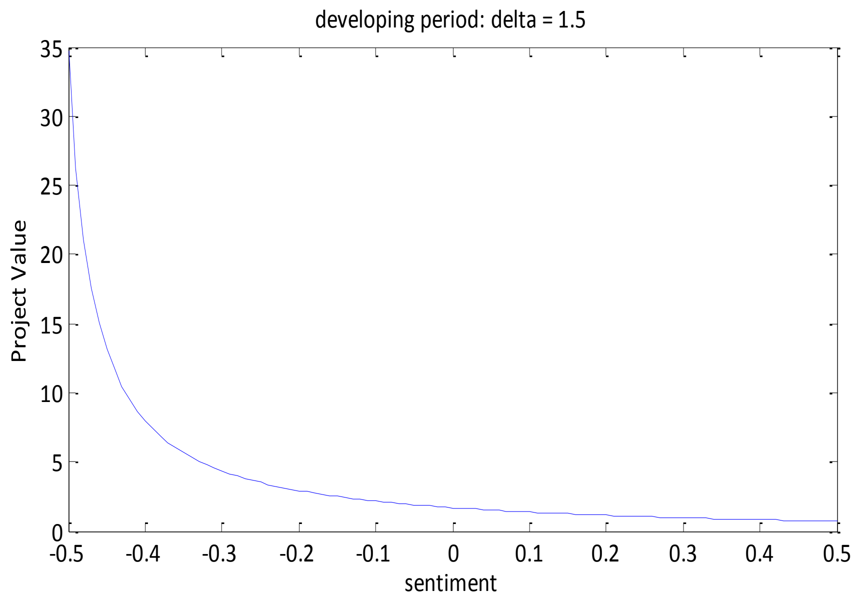


Figure 5.4. The relationship between project value and sentiment in the case specified by developing period $\delta = 1.5$

5.7 Conclusion

Previous studies focused mainly on project investment and development under uncertainty. Recently, several studies (e.g. Clayton et al., 2009) have suggested that market sentiment becomes a considerable factor which affects the performance of the property market. However, the problem of examining the role of market sentiment in project development, which eventually affects market supply, has yet to be addressed. Therefore, this study aims to investigate the effect of market sentiment on optimal decision related to project investment.

To fulfil this objective, a new option-based dynamic model is established to capture how the developer makes the optimal decision with their sentiment under uncertainty. Specifically, this model implies that the relationship between expected waiting time to invest/develop and market sentiment shows a U-shape pattern. In other words, developer would have a shorter wait to develop at first and then a longer wait, along with the increase in sentiment. The U pattern can be attributed to a direct effect and two indirect effects brought out by sentiment. The direct effect implies that higher sentiment pulls down the threshold of development and thus shortens the waiting time. One indirect effect is that sentiment increases the expected return rate and makes waiting more beneficial. Meanwhile, the other indirect effect is that an increase in sentiment reduces the project value and thus makes a delay less worthy. On aggregate, when sentiment is relatively low, the direct effect tends to dominate the other two effects. This shows that an increase in sentiment leads to a shorter waiting time. When sentiment is substantially high, the indirect effect through expected return dominates among the three effects. In this case, an increase in sentiment leads to a project delay. Furthermore, the comparative analysis shows that extending development period shifts the turning point (SI_b) of the U-shape to a lower value. In other words, the positive relationship between expected waiting time and sentiment appears earlier for projects with a longer development period than those with a short duration.

On the other hand, this study reveals that the effects of sentiment on optimal density and project value are negative. The practice implication suggests that developer should cut down the project density in the presence of high sentiment. A higher sentiment environment would lead to a residential project with lower value. The policy implications arising from these results are worth discussing. The developer with different levels of sentiment plans the project development differently. With a higher level of sentiment, the developer aims to reduce supply by developing a project of lower density or delaying the project. This means that the land policy designed to supply more land to increase the housing supply would have smaller effects during the high sentiment period than the low sentiment one.

The housing market is more likely to be sentiment driven (Hui and Wang, 2014). This study conducts analysis of the effect of sentiment on optimal decision related to residential project investment and provides a more accurate model which could benefit developers. On the other hand, since this study provides better insights into the role of sentiment in housing supply, the implications of this study may serve as a useful reference for relevant authorities when considering their policies of land supply and urban planning at different levels of market sentiment.

5.8 Appendix

A. Proof of Proposition 1

This proof consists of two parts: (i) elasticity of project value (Ω) should be greater than elasticity of housing demand $1/\varepsilon$, which is the sufficient conditions for existence of the threshold vale *X**; and (ii) such condition infers that adjusted expected return rate in the local housing market is greater than the expected growth rate of state variable, i.e. $\rho_S > \alpha_X$.

The model assumption of elasticity of housing demand $(1/\varepsilon > 1)$ has already ensured that $1 - \varepsilon > 0$. The condition abovementioned aims to ensure that $\Omega - 1/\varepsilon > 0$.

According to $X^* = \Lambda\left[\frac{(1-\varepsilon)F\Omega}{\varepsilon(\Omega-1/\varepsilon)}\right]^{\varepsilon}$, it is clear that the condition leads to existence of *X**.

Furthermore, as this condition infers that elasticity of project value greater than 1 through $\Omega > 1/\varepsilon > 1$, we can have $\rho_S > \alpha_X$ through the following deduction. Looking at

$$\frac{1}{2} - \frac{\alpha_X}{\sigma_X{}^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha_X}{\sigma_X{}^2}\right)^2 + \frac{2\rho_S}{\sigma_X{}^2}} = \Omega > 1 = \frac{1}{2} - \frac{\alpha_X}{\sigma_X{}^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha_X}{\sigma_X{}^2}\right)^2 + \frac{2\alpha_X}{\sigma_X{}^2}}$$

The above inequality indicates $\frac{2\rho_S}{\sigma_X{}^2} > \frac{2\alpha_X}{\sigma_X{}^2}$ and thus $\rho_S > \alpha_X$.

Q.E.D.

B. Proof of Proposition 2

Differentiate *M* with respect to *SI* and we have

$$\frac{\partial M}{\partial SI} = \frac{e^{(\rho_S-\alpha_X)\delta}\rho\delta f'}{f}\left[\frac{\Omega}{\left(\Omega-\frac{1}{\varepsilon}\right)}\right]^{\varepsilon} - \frac{e^{(\rho_S-\alpha_X)\delta}f'}{f^2}\left[\frac{\Omega}{\left(\Omega-\frac{1}{\varepsilon}\right)}\right]^{\varepsilon}$$

$$-\frac{e^{(\rho_S-\alpha_X)\delta}}{f}\frac{1}{\left(\Omega-\frac{1}{\varepsilon}\right)^2}\frac{\partial\Omega}{\partial SI}\left[\frac{\Omega}{\left(\Omega-\frac{1}{\varepsilon}\right)}\right]^{\varepsilon-1} \qquad \text{(B.1)}$$

$$= \frac{e^{(\rho_S-\alpha_X)\delta}}{f^2}\left[\frac{\Omega}{\left(\Omega-\frac{1}{\varepsilon}\right)}\right]^{\varepsilon}\left(\delta\rho f f' - f' - \frac{f}{\Omega\left(\Omega-\frac{1}{\varepsilon}\right)}\frac{\partial\Omega}{\partial SI}\right)$$

$$= \frac{e^{(\rho_S-\alpha_X)\delta}}{f^2}\left[\frac{\Omega}{\left(\Omega-\frac{1}{\varepsilon}\right)}\right]^{\varepsilon} f'W$$

where $W = \delta\rho f - 1 - \frac{f}{\Omega\left(\Omega-\frac{1}{\varepsilon}\right)}\frac{\rho}{\sigma_X{}^2}\left(\left(\frac{1}{2} - \frac{\alpha_X}{\sigma_X{}^2}\right)^2 + \frac{2\rho f}{\sigma_X{}^2}\right)^{-1/2}$.

Since $\frac{e^{(\rho_S-\alpha_X)\delta}}{f^2}\left[\frac{\Omega}{\left(\Omega-\frac{1}{\varepsilon}\right)}\right]^{\varepsilon} f' > 0$, the sign of $\frac{\partial M}{\partial SI}$ depends on that of *W*.

After algebra transformation, we have $W = \rho f(\delta - \omega)$, where $\omega = \frac{1}{\rho f} + \frac{1}{\sigma^2 \Omega\left(\Omega - \frac{1}{\varepsilon}\right)\left(\Omega - \frac{1}{2} + \frac{\alpha}{\sigma^2}\right)}$. It can be seen that ω is a function of f (or sentiment *SI*), i.e. $\omega = \omega[f(SI)]$.

On the other hand, let $a = \frac{1}{2} - \frac{\alpha}{\sigma^2}$ and have

$$\frac{\partial \omega}{\partial f} = \frac{-1}{\rho f^2} + \frac{-1}{\sigma^2\left(\Omega^3 - \frac{1}{\varepsilon}\Omega^2 - a\Omega^2 + \frac{a}{\varepsilon}\Omega\right)^2}\left(3\Omega^2 - 2\left(\frac{1}{\varepsilon} + a\right)\Omega + \frac{a}{\varepsilon}\right)\frac{\partial \Omega}{\partial f}$$

Since $a < 0$ and $\Omega > \frac{1}{\varepsilon}$, we have $3\Omega^2 - 2\left(\frac{1}{\varepsilon} + a\right)\Omega + \frac{a}{\varepsilon} = \left(3\Omega - \frac{2}{\varepsilon}\right)\Omega - a\left(2\Omega - \frac{1}{\varepsilon}\right) > 0$. Since $\frac{\partial \Omega}{\partial f} > 0$, we obtain $\frac{\partial \omega}{\partial f} < 0$. That is, increase in f will bring a smaller ω. This remarks that function ω is monotonically descending but never be lower than zero and $\delta - \omega$ is monotonically ascending. As such, function W is monotonically increasing with respect to f (or sentiment *SI*).

Furthermore, there exists critical point of W (denoted by SI_b), which is implicitly defined by $\omega(f_b) = \delta$, where $f_b = f(SI_b)$, so that $\delta - \omega > 0$ and thus $W > 0$ for every $SI > SI_b$. As such, we have

$$\frac{\partial X^*}{\partial SI}, \frac{\partial E[t]}{\partial SI} \begin{cases} > 0 & SI > SI_b \\ < 0 & SI < SI_b \end{cases}$$

The uniqueness of critical point is guaranteed by the monotonicity of function W and thus SI_b is verified as the turning point of the curve of expected waiting time against sentiment. Hence, the curve is U-shape showing downward at first and upward at last.

Q.E.D.

C. Proof of Proposition 3

Recall that $W = \delta \rho f - 1 - \frac{f}{\Omega\left(\Omega - \frac{1}{\varepsilon}\right)} \frac{\rho}{\sigma_X{}^2}\left(\left(\frac{1}{2} - \frac{\alpha_X}{\sigma_X{}^2}\right)^2 + \frac{2\rho f}{\sigma_X{}^2}\right)^{-1/2}$ in Proposition 2. As $W = 0$ is a cubic equation of *f*, it is extremely complicated to find precise solutions to the equation. Instead of solving the equation, we provide some implications arising from two inequalities of *W*, which are more simple and direct. Note that the property $\rho_s = \rho f > \alpha_X > 0.5\sigma_X^2$ is utilized in deductions of these two inequalities.

On one hand, we aims to look at the case of $W < 0$. Let $A = \frac{1}{\Omega(\Omega - 1/\varepsilon)}$

and we have the first inequality of W as

$$W < \delta\rho f - 1 - A\frac{\rho f}{\sigma^2}\left(\left(\frac{1}{2} - \frac{\rho f}{\sigma^2}\right)^2 + \frac{2\rho f}{\sigma^2}\right)^{-1/2} = \delta\rho f - 1 - A\frac{\rho f}{\sigma^2}\left(\frac{1}{2} + \frac{\rho f}{\sigma^2}\right)^{-1}$$

$$< \delta\rho f - 1 - A\frac{\rho f}{\sigma^2}\left(\frac{2\rho f}{\sigma^2}\right)^{-1} = \delta\rho f - 1 - \frac{A}{2}$$

where $A = \frac{1}{\Omega(\Omega - 1/\varepsilon)} > \frac{1}{\Omega^2}$. As $\Omega < \sqrt{\frac{2\rho f}{\sigma^2}}$ leads to $A > \frac{\sigma^2}{2\rho f}$, the above inequality becomes $W < \delta\rho f - 1 - \frac{\sigma^2}{4\rho f}$, where the right hand side of the above inequality is monotonically increasing function with respect to *f*. Furthermore, two roots as $\frac{1 \pm \sqrt{1+\delta\sigma^2}}{2\rho\delta}$ for the equation $\delta\rho f - 1 - \frac{\sigma^2}{4\rho f} = 0$ exist, but one of them less than zero makes no sense. Since $\delta, \rho > 0$, $0 < f < f_1 = \frac{1+\sqrt{1+\delta\sigma^2}}{2\rho\delta}$ ensures that $W < 0$ and thus implies that $\frac{\partial X^*}{\partial SI} < 0$.

On the other hand, we establish an opposite inequality to reveal a positive relationship, i.e. $W > 0$. The inequality can be shown as follows.

Since $\left(\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2\rho f}{\sigma^2}\right)^{1/2} = \Omega - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right) > 1$, we have $W > \delta\rho f - 1 - \frac{\rho f}{\sigma^2}\frac{1}{\Omega(\Omega - 1/\varepsilon)}\frac{1}{\Omega}$

When $f > f_1 > 1/(\rho\delta)$ indicates an inequality of Ω showing that

$$\Omega = a + \sqrt{a^2 + \frac{2\rho f}{\sigma^2}} > a + \sqrt{a^2 + \frac{2\rho}{\sigma^2} f_1} = \Omega_1 > a + \sqrt{a^2 + \frac{2\rho}{\sigma^2}\frac{1}{\rho\delta}} = \Omega_2$$

where $a = \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)$ and thus the inequality of W becomes $W > \delta\rho f - 1 - \frac{\rho f}{\sigma^2}A_1 > \delta\rho f - 1 - \frac{\rho f}{\sigma^2}A_2$, where $A_i = \frac{1}{\Omega_i^2(\Omega_i - 1/\varepsilon)}$ and i = 1, 2.

Therefore, knowing that $\delta, \rho > 0$, the condition $f > \frac{1}{\rho(\delta - A_2/\sigma^2)} = f_2 > 0$ requiring $\delta > \frac{A_2}{\sigma^2}$, ensures that $W > 0$ and further implies that $\frac{\partial X^*}{\partial SI} > 0$.

Q.E.D.

D. Proof of Corollary 1

We aim to verify that $f_2 = \frac{1}{\rho(\delta - A_2/\sigma^2)}$ in proposition 2 do exist.

As existence of f_2 requires $\delta > A_2/\sigma^2$, it is necessary to the validity of inequality of δ for some $\delta \in R^+$. Because δ is embedded in A_2, $\delta >$

A_2/σ^2 is an implicit expression. The following discussion attempts to look for the condition for the existence of the inequality.

$$\delta - \frac{A_2}{\sigma^2} = \delta - \frac{1}{\sigma^2\Omega_2^2(\Omega_2-1/\varepsilon)} = \frac{\delta\sigma^2\Omega_2^2(\Omega_2-1/\varepsilon)-1}{\sigma^2\Omega_2^2(\Omega_2-1/\varepsilon)} = \frac{\left[\Omega_2^2(\Omega_2-1/\varepsilon)-1/(\delta\sigma^2)\right]}{\Omega_2^2(\Omega_2-1/\varepsilon)/\delta}$$

(D.1)

The dominator of equation C.1 is strictly positive and we only need to verify the sign of the numerator. For simplification, we set $x = 1/(\delta\sigma^2) > 0$, $a = (1/2 - \alpha/\sigma^2)$ and $b = 1/\varepsilon$. After substitutions, $\Omega_2 = a + \sqrt{a^2 + 2x}$ and the numerator of equation C.1 becomes

$$\Omega_2^2\left(\Omega_2 - \frac{1}{\varepsilon}\right) - \frac{1}{(\delta\sigma^2)} = \left(a + \sqrt{a^2+2x}\right)^2\left(a + \sqrt{a^2+2x} - b\right) - x$$

(D.2)

The right hand side of this expression is a cubic function with a positive coefficient of $x^{3/2}$. There exist three roots for the variable x listed as follows.

$$x_1 = \frac{1}{16}\left[y - \sqrt{y^2 - 32(8a^3 - 4a^2b)}\right]$$

$$x_2 = 0$$

$$x_3 = \frac{1}{16}\left[y + \sqrt{y^2 - 32(8a^3 - 4a^2b)}\right]$$

where $y = -8ab - 12a + 4b^2 + 4b + 1$. As $a < 0$ and $b > 0$, we definitely have $y > 0$ and $y^2 - 32(8a^3 - 4a^2b) > y^2 > 0$. With this, all the roots are real roots and follows an order that $x_1 < x_2 = 0 < x_3$. With the property of $x > 0$, then $x = 1/(\delta\sigma^2) > x_3$ ensures that equation B.1 > 0. Finally, this gives an upper boundary δ^* such that $\delta < \delta^*$ and

$$\delta^* = \frac{16}{\sigma^2\left[y + \sqrt{y^2 - 32(8a^3 - 4a^2b)}\right]} \qquad \text{(D.3)}$$

In addition, another condition $f_2 \geq f_1$ is reasonable to hold for existence of f2. For this consideration, a mathematical discussion is conducted and a consequent implication is given in the following.

According to $A_2 = \frac{1}{\Omega_2^2(\Omega_2-1/\varepsilon)} > \frac{1}{\Omega_1^2(\Omega_1-1/\varepsilon)} > \left(\frac{\sigma^2}{2\rho f_1}\right)^{3/2}$, we have

$$f_2 = \frac{1}{\rho\left(\delta - \frac{A_2}{\sigma^2}\right)} > \frac{1}{\rho\left(\delta - \frac{\sigma^3}{\sigma^2}\frac{1}{\sqrt{2\rho f_1}}\frac{1}{2\rho f_1}\right)}$$

Since $\frac{\sigma}{\sqrt{2\rho f_1}} = \frac{\sigma\sqrt{\delta}}{\sqrt{1+\sqrt{1+\delta\sigma^2}}} > \frac{\sigma\sqrt{\delta}}{1+\sqrt{1+\delta\sigma^2}} = \frac{\sqrt{1+\delta\sigma^2}-1}{\sigma\sqrt{\delta}}$ the above inequality becomes

$$f_2 > \frac{1}{\rho\left(\delta - \frac{1}{2\rho f_1}\left(\frac{\sqrt{1+\delta\sigma^2}-1}{\sigma\sqrt{\delta}}\right)\right)} = f_1 \frac{2}{z} \text{ where } z = 1 + \sqrt{1+\delta\sigma^2} - \frac{\sqrt{1+\delta\sigma^2}-1}{\sigma\sqrt{\delta}}$$

As such, 2 > z could guarantee $f_2 > f_1$. With some algebra transformation, this condition leads to another upper boundary of δ as $\delta < \frac{1}{\sigma^2}$. That is, $\delta < \frac{1}{\sigma^2}$ could guarantee $f_2 > f_1$. According to equation D.3, note that $\delta^* < \frac{16}{18\sigma^2} < \frac{1}{\sigma^2}$ due to a < 0 and b > 1. Therefore, $\delta < \delta^*$ has already satisfied $\delta < \frac{1}{\sigma^2}$ and thus guarantee $f_2 \geq f_1$.

Furthermore, differentiate f_2 with respect to δ and we have

$$\frac{df_2}{d\delta} = \frac{d\left\{\frac{1}{\rho(\delta - A_1/\sigma^2)}\right\}}{d\delta} = -{f_2}^2\rho\left[1 - \left(3\Omega_1^2 - \frac{2\Omega_1}{\varepsilon}\right)\frac{1}{\sqrt{\left(0.5-\frac{\alpha}{\sigma^2}\right)^2 + \frac{2}{\delta\sigma^2}}}\frac{-1}{\delta^2}\right]$$

Recall that $\Omega > \frac{1}{\varepsilon} > 1$, we have $1 - \left(3\Omega_1^2 - \frac{2\Omega_1}{\varepsilon}\right) < 0$. Hence the term in square bracket [ ] is positive and thus the right hand side is negative in total, which implies that $df_2/d\delta < 0$.

On the other hand, to investigate the role of developing period δ in f_1, differentiate f_1 with respect to δ and we have

$$\frac{df_1}{d\delta} = \frac{d\left\{\frac{1+\sqrt{1+\delta\sigma^2}}{2\rho\delta}\right\}}{d\delta} = \frac{\frac{1}{2\sigma^2\sqrt{1+\delta\sigma^2}} - 2\rho\left(1+\sqrt{1+\delta\sigma^2}\right)}{(2\rho\delta)^2} < \frac{\frac{1}{2\sigma^2} - 4\rho}{(2\rho\delta)^2} < 0$$

More generally, it is of interest to look at the effect of developing period in determining the turning point SI_b or f_b. Recall that $\delta = \omega(f_b)$ in Proposition 2 and take differentiation on both sides with respect to f_b we have

$$\frac{d\delta}{df_b} = \frac{d\omega(f_b)}{df_b} < 0$$

It indicates a negative relationship between δ and f_b. In other words, increase in developing period would induce a drop in the turning point f_b.

Q.E.D.

Chapter

6 LIFE CYCLE MODEL FOR SENTIMENT-BASED HOUSING DEMAND

In this chapter, a study is conducted to investigate the roles of sentiment in households' housing demand, in order to achieve Objective 4. A life-cycle model associated with different roles of sentiment is established to examine how sentiment wobbles households' life-cycle plans for housing and non-housing consumptions. In this model, sentiment affects households' decisions regarding tenure and housing service.

6.1 Introduction

Houses are durable and immobile products, usually with high values. In urban society, houses have been characterised by diversity and indivisibility that make housing a complex issue. Housing consumption not only takes up a substantial proportion of a household's budget, but also stores household wealth as an asset in a homeowner's portfolio, and further can be pledged for financial credit as the collateral. Housing is always, however, accompanied with various risks derived from market conditions and individual status. Changes in the housing price, interest rate, or household income flow can significantly affect the household's tenure choices and the housing wealth that the household would like to invest (Attanasio et al., 2012). In addition, lumpy transaction costs involved in housing transaction make deep impacts on housing decisions (Han, 2008; Attanasio et al., 2012) and bring about the difficulty in housing owning for households (Han, 2008).

A long-term plan of housing service for households has become a prominent topic in housing studies. Many studies are conducted within a life cycle framework. The life-cycle framework aims to provide insights into housing consumption patterns, while assuming households are rational enough to plan their consumption and saving over the life cycle. Within the framework a household attempts to optimise the lifetime utility by making decisions on the basis of the combined expectation of the market trend and household conditions, as well as various associated risks. Throughout such decision-making process, home ownership is determinant of the form of household utility (or utility function) in the life-time optimisation. Bottazzi et al. (2015) present some empirical evidence to confirm that home ownership shows considerable variation over the life cycle and cross section. The variation plays an important part in the process of household's wealth accumulation over the life cycle and even propels the evolution of business cycles (Attanasio et al., 2012). Housing demand consists of two aspects: housing tenure choice and housing service. Thus, one of the research objectives in this study is to establish an integrated model to account for the household's pattern of housing demand over the life cycle in terms of these two aspects.

Classical theory suggests that only the factors relative to systematic risk should be taken into modelling the demand for housing (Holland et al., 2000). Recently, some studies pay attention to non-systematic risk in real estate investment (e.g. Bulan et al., 2009). The channels, through which the factors of non-systematic risk affect the needs, are diversified and thus the roles of these factors are difficult to fully recognise. Some recent studies put some effort into the effects of these factors (e.g. Bourassa et al., 2009). Among such factors, market expectation, for example, is identified as a critical indicator for the performance of housing markets (Case and Shiller, 1989; Marcato and Nanda, 2016). This is echoed with behavioural economics, where some researches furnish the study of uncertainty and risk with theoretical considerations based on some psychological notions (e.g. DeLong et al., 1990). An essential part of these notions is so called the sentiment that facilitates the role of unsystematic or idiosyncratic factors in modern economics and finance.

Due to the information asymmetry in the housing market and to market imperfection, participants such as individual households can be less rational in reaching their decisions (Hui and Wang, 2014). Households' life experience and cognition, which are related to idiosyncratic characteristics, are decisive components in forming sentiment. As individual households are vulnerable to the access to quality information[21], households are prone to turn to sentiment, in addition to the available information, such as historical and current housing prices (sometimes there is a lack of such information), and professionals' advice, in order to reach judgement. As such, households are more likely to be sentiment-driven in the decision making process. In addition, Souleles (2004) suggests that an index capturing consumer sentiment significantly improves the forecasting of consumption growth.

As individual sentiment always shows herding behaviour and moves in one direction[22], market sentiment affects market performance in aggregate. Most of the recent studies focus on the two aspects to the effect of sentiment

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[21] The information economy, especially with the rise of Big Data has generated yet more information for participants. As such, research into the salience of information has become particularly relevant (Andreassen, 1990; Brown and Matysiak, 2000). An array of factors has been considered to take impacts on the salience of information, including inter alia, timing, presentation, and availability of alternatives (Andreassen, 1990; Bordalo et al., 2015).

[22] Isolated individuals always deviate from the canons of rational choice, but in the social context of exchange institutions can make decisions consistent with others (Lucas, 1987; Smith, 1991).

at the market level. From the first aspect, sentiment affects asset pricing. Clayton et al. (2008) contend that sentiment takes credit for the divergence of property prices from market fundamentals. Jin et al. (2014) study the role of sentiment in excess return of residential market and find that sentiment is a prominent exogenous variable in the pricing mechanism. Some empirical studies provide evidence to show the causality of sentiment for property mispricing (e.g. Hui et al., 2013) and sentiment becomes a significant indicator for the trend of housing prices (e.g. Hui and Wang, 2014). Tsolacos et al. (2014) conduct a study to examine the signals of rental growth in four key U.S. commercial rent series and discover the predicting power of sentiment is an indicator for the changes in the direction of rents. As the housing price is driven by sentiment, its variance could be enlarged by sentiment. As such, in the face of price uncertainty amplified by sentiment, households would unlikely get committed to a housing transaction when they are making a long-term plan to smoothen housing consumption.

From the second aspect, sentiment is found to play a considerable role in the changes in turnover rate which has a positive correlation with the changes in housing demand (Berkovec and Goodman, 1996). Baker and Stein (2004) suggest aggregate market liquidity serves as a proxy for sentiment. Hui and Wang (2014) add to their findings that sentiment in the housing market is not only an efficient predictor of housing price, but also has a strong forecasting power on transaction volume. As the housing market is characterised by being short-sale, sentiment-driven participants are more likely to take actions when participants are optimistic about future trends and thus increase the market liquidity during the over-confidence period (Ling et al., 2014). Thus the likelihood of house transactions substantially depends on the household's sentiment. As suggested by Han (2008), housing wealth risk derived from price uncertainty would reduce housing demand. Sentiment plays a different role at the micro level as it is involved in shaping a household's expectation and judgement. Then the household comes up with a hedging idea to alter their housing status and, more importantly, the proportion of housing wealth in the household's asset portfolio against uncertainty in future.

In this study, we will investigate how the household responds to the sentiment-driven housing market when sentiment plays different roles in household aspects and in market aspects. At the micro level, the effect of sentiment has rarely been analysed by the extant real estate research. Hence, this study aims to investigate such sentiment effect on household's life-cycle housing choices. The focus of this research can be split in two: First, it is aimed to create a better understanding of the effect of sentiment on housing tenure choice. That is, how the sentiment affects the optimal timing for a household's first-time home purchase. Second, the aim is to explore the effect of sentiment on optimal housing demand, when homeowners transact in the housing market.

The remainder of this chapter is organised as follows. Section 3.2 introduces the basic assumptions and establishes a life cycle model of a household's housing choice, and Section 3.3 derives the optimal solutions to this model. Section 3.4 conducts a series of simulations to numerically solve the model. Section 3.5 provides a summary and conclusion.

6.2 Life-cycle Model

This section introduces a life-cycle model considering housing and non-housing consumption. This model is developed with some specific features to characterise the role of sentiment in consumption choices that households make over their life cycle.

6.2.1 Basic assumptions and notations

This model is formed in discrete style and life cycle of a household contains T+1 periods. In every period $t \leq$ T, the household takes into consideration two consumptions, i.e. non-durable good consumption (c_t) and housing service consumption (h_t). The housing consumption is subject to the housing choices including tenure choice and house type. These two control variables reflect how a household moves along with housing ladder over the life cycle. For households, the tenure choice is to decide whether they transact or not. More specifically, transaction includes renting (d_t^1, for tenant) and moving to a new house (d_t^2, for owner).

The house type is a continuous and composite measurement of housing service in period *t* that should capture not only physical size but also the trait of house (representing hedonic features). The house type is denoted by the superscript j of h_t^j in this model and keeps constant for each type. Assume a household has already experienced j-1 housing transactions at the beginning of period *t* (the 1st transaction is house ownership) and will take *J* transactions over the life cycle. Note that the difference between the house types of two houses involved in any transaction (i.e., the household sells the current house and then buy a new house) indicates that the household either upgrades the quality of housing service or downgrades it.

Attanasio et al. (2012) suggest that, in the presence of increases in housing prices, homeowners would rather downsize the house than give up ownership. On the other hand, our life-cycle model allows the household to hold negative wealth[23] at the end of each period before time *T*+1 and thus

[23] The household is not allowed to raise their debt without limitations. As the final wealth has to be positive at the end of life cycle and the model will be solved backward, a dynamic limitation (negative value) as a lower bound will restrict the wealth at the end of each period.

the household would not default in housing mortgage, in the case of a fall in housing prices. As such, the model assumes that once the household owns a house, they would not turn to stop being an owner.

The dynamics of rent and house price are assumed to be exogenous. Furthermore, house prices and rents vary over time and across types. Thus, house price $\boldsymbol{P} \equiv (\boldsymbol{p_0}, \dots, \boldsymbol{p_T})$ and rent $\boldsymbol{R} \equiv (\boldsymbol{r_0}, \dots, \boldsymbol{r_T})$ are modeled by the first order Markov process with conditional density $\phi(\boldsymbol{p_t}|\boldsymbol{p_{t-1}})$ and $\psi(\boldsymbol{r_t}|\boldsymbol{r_{t-1}})$ respectively. Vector $\boldsymbol{p_t}$ and $\boldsymbol{r_t}$ are J dimensional as $(p_t^1, p_t^2, \dots, p_t^J)$ and $(r_t^1, r_t^2, \dots, r_t^J)$. The subscript (*t*) represents the time dimension and superscript (1, 2…*J*) corresponds to the type dimension. Although the transaction cost is charged on both sides, i.e. selling and buying, it can be implicitly transferred to the buyer from seller through the dealing price in reality. In view of this, consider the real transaction cost only happens in purchase of housing (either first-time home owning or the following transacting). In this model, transaction cost for buying is charged at the proportion (denoted by δ) of total house value.

For each period *t* prior to the last period, the household can observe the previous housing service h_{t-1}^{j-1}, financial wealth W_{t-1}, as well as the house price $\boldsymbol{P}$ and the rent $\boldsymbol{R}$ up to time *t*. In addition, the household is assumed to receive income flow (y_t) during the life time and the asset that the household holds pays an interest rate i_t in period *t*. The household attempts to maximize the life-time utility by choosing consumption c_t and housing service h_t^j in each period. The latter one is based on whether to make a transaction (represented by the status d_t^i). In the last period T+1, the utility is derived from the final wealth, which is imposed to be non-negative so as to guarantee that the household is able to pay off the debt eventually. In summary, the life-time utility objective can be defined as,

$$\max_{\left(c_t, d_t^i, h_t^j \right)} E_t \sum_{\tau=t}^{T} \beta^{\tau-t} u(c_\tau, h_\tau) + \beta^{T+1} \frac{W_{T+1}^{1-\gamma}}{1-\gamma} \qquad i = 1 \text{ or } 2 \qquad (1)$$

subject to

$$W_t = y_t + (1 + i_t)W_{t-1} - c_t - (1 - d_t^1) r_t^j h_{t-1}^{j-1}$$

$$+(d_{t-1}^1 - d_t^1)(1 + \delta) p_t^j h_t^j + d_t^1 d_t^2 \left[p_t^{j-1} h_{t-1}^{j-1} - (1 + \delta) p_t^j h_t^j \right] \qquad (2)$$

$$W_{T+1} = (1 + i_{T+1})W_T + p_{T+1}^J h_T^J \geq 0 \qquad (3)$$

where β indicates the discount rate and E_t denotes the expectation for remaining period subject to the information obtained up to time *t*. Equation 1 indicates a constraint on renting as the household will not change the type of rental house if he/she keeps renting. Equation 2 can also be interpreted as a bequest motivation, the household is assumed to realize his/her final wealth by selling his/her house, if he/she owns one, at the T+1 period.

The utility function employed in this model is increasing and bounded. It holds several properties such as being concave and second order differentiable. A constant relative risk aversion (CRRA) function is used to capture the utility derived from composite consumption in combination of non-durable goods c_t and housing service h_t, which is shown as

$$u(h_\tau, c_\tau; \theta_\tau) = \frac{\left(c_\tau^{1-\theta_\tau} h_\tau^{\theta_\tau} \right)^{1-\gamma}}{1-\gamma} \qquad (4)$$

The utility function is in Cobb-Douglas form so that non-durable consumption is augmented by housing service as housing is both consumption and an asset. In addition, this utility function captures housing preference θ_t to determine the utility weighting of housing service in the composite consumption. Housing preference varies over time as $(\theta_1, \theta_2, \ldots, \theta_T)$ to represent time-varying preferences among households during the life time.

6.2.2 The roles of market sentiment

Some studies document that market sentiment is an influential factor in the housing market. Sentiment mainly affects in two aspects, i.e. housing price and housing demand (Marcato and Nanda, 2016; Ling et al., 2014). This study intends to have in-depth insights into the effects of sentiment on households' tenure choices. Suppose that there is an available sentiment index (SI_t) which appropriately captures sentiment movement in the housing market. The index is negative ($SI_t < 0$) when market sentiment is bearish, and vice versa. As market sentiment is derived from participants' attitude toward market trends, the sentiment index should contain forward-looking information, which is able to reflect market expectations over a future period. According to the definition of sentiment, it cannot be justified by market fundamentals (Marcato and Nanda, 2016), and thus sentiment should be considered an independent factor in the model. Market sentiment (and sentiment index) is also assumed to be exogenous to individual households.

On one hand, market sentiment formed due to market and information incompleteness and can be reflected through transactions. As a result, at the market level, sentiment drives property prices to deviate from fundamental values (Ling et al., 2014). As market sentiment shows its forecasting power on housing prices (Hui and Wang, 2014), the lagged term of market sentiment acts a significant role in the dynamics of housing prices. In this model, a lagged term of market sentiment is incorporated into the pricing process, thus

$$E_t\boldsymbol{P}_{t+1} = \boldsymbol{\mu} g(SI_t) + \boldsymbol{\Psi}\boldsymbol{P}_t \quad (5)$$

where $g(SI_t)$ is a sentiment function and parameters in $\boldsymbol{\Psi}$ guarantee that the housing price processes are stationary AR(1). Function $g(SI_t)$ drives the first term on the right hand side to deviate a constant vector $\boldsymbol{\mu}$ according to SI_t. Hence this first term acts as a time-varying intercept adding a sentiment adjustment to the process.

The sentiment function (i.e. $g(SI_t)$ in the above equation) should be a monotonous increasing function with several properties: (i) $g'(SI) > 0$; (ii) $g(SI_t) = 1$ when market sentiment is neutral (i.e. $SI_t = 0$) and thus the model is reduced to a classical/baseline case without the concern of sentiment effect; (iii) $g(SI_t) > 1$ with bullish market sentiment ($SI_t > 0$) and $0 < f(SI_t) < 1$ with bearish market sentiment ($SI_t < 0$).

On the other hand, as the life cycle model is derived from the micro perspective, it is more straight-forward to look at the effect of sentiment on households' housing choices. Some theories such as "animal spirit," habit persistence, and forward-looking theories advocated in previous studies (e.g., Acemoglu and Scott, 1994; Shiller, 2000; Akerlof and Shiller, 2009), suggest that agents are likely to make future consumption according to their perceptions and sentiment (Marcato and Nanda, 2016).

Households are representative sentiment-based participants in housing markets. Rational responses to market fundamentals take part of but not whole credits in households' housing choices. Due to information incompleteness, their attitude towards markets, i.e. households' individual sentiment reflects their idiosyncratic attributes in understanding of the market and plays an important part as human capital in housing decision making. Thus the role of households' sentiment cannot be neglected in housing demand.

Demand is subject to sentiment, which affects the household's utility. Tenants are willing to own a house as ownership is more desirable to hedge the future appreciation of rent (Sinai and Souleles, 2005). Bullish sentiment strengthens such willingness to own a house while bearish sentiment delays the demand for home ownership. For home owners, bullish sentiment also intensifies the willingness to enjoy more housing service, and vice versa.

In light of this, sentiment modifies the utility premium derived from the changes in housing service in two aspects. The first is the sentiment-based owning effect (utility arising from owning a house), which is derived from house ownership. And the utility of housing service that the household enjoys is adjusted by multiplying the household's sentiment[24], i.e. $f_t(SI)$. Then, the household's utility function is extended to $u_{SI}\big(h_t, c_t, \theta_t, f(SI_t)\big)$, and the modified utility is given as

$$u_{SI}\big(h_t, c_t, \theta_t, f(SI_t)\big) = u\big(h_t^j, c_t, \theta_t\big)F[f(SI_t)] \tag{6}$$

where F is a function[25] of $f(SI_t)$, where households derive their own sentiment from their perception and cognition of market sentiment. $f(SI_t)$ is another sentiment function to capture the household's sentiment in favour of housing service. As This sentiment function $f(SI_t)$ can have a different form compared to $g(SI_t)$. Both sentiment functions, f and g hold the same properties of sentiment function.

The other one is an additive utility derived from sentiment-based house transactions. The additive term indicates that restriction by which housing and consumption are imposed to be homothetic is not necessary (see a discussion in Attanasio et al., 2012). This additive term shows an advantage of this model as it is capable to capture the utility derived from housing investment. This term comprises two components, i.e. the coefficient measured by sentiment function and the change in house service associated with another coefficient π. The sentiment coefficient indicates the

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[24] The usage of sentiment function instead of sentiment index is due to that it is admitted that the discrepancy between individual forecasts and consensus forecast (i.e. sentiment index) can be found (McAllister et al., 2008). Byrne and Lee (1999) argue that common tendency statistics do not robustly indicate the individual's agreement. As such, with the help of sentiment function, we allow idiosyncratic difference between household sentiment and market sentiment.

[25] A discussion on the form of function F is conducted in Lemma 0. Details are given in Appendix.

household's incentive to change the housing service as an investing activity. In doing so, the household can hedge against future fluctuations of housing prices. The sign of π implies whether housing is a luxury ($\pi > 0$) or a necessity ($\pi < 0$). In this model, set $\pi > 0$. Based on equation 6, the utility function can be further extended by this additive term. For the household with previous tenure choice $d_{t-1}^1 = 0$, the modified utility function becomes:

$$u(h_t, c_t, \theta_t) = \begin{cases} u(h_t^j, c_t, \theta_t) + (f(SI_t) - 1)\pi h_t^j & if d_t^1 = 1 \\ u(h_{t-1}^{j-1}, c_t, \theta_t) & if d_t^1 = 0 \end{cases} \tag{7}$$

The sentiment factor does not affect the utility of housing consumption when the household is a tenant. The additive term indicating a jump in utility is derived from first-time home ownership and this transformation of home ownership only happens once over the life time under the model assumption. In the case of tenants, since the housing transaction is affected by sentiment, the sentiment function appears only in the additive terms.

For home owners ($d_{t-1}^1 = 1$), the modified utility function becomes:

$$u(h_t, c_t, \theta_t; SI) = \begin{cases} u_{SI}\left(h_t^j, c_t, \theta_t, f(SI_t)\right) + (f(SI_t) - 1)\pi(h_t^j - h_{t-1}^{j-1}) & if d_t^2 = 1 \\ u_{SI}\left(h_{t-1}^{j-1}, c_t, \theta_t, f(SI_t)\right) & if d_t^2 = 0 \end{cases} \tag{8}$$

To ensure right continuity in utility function, set $u(d_t^1 = 1) = u(d_t^2 = 1)$. In the above utility functions, the additive utility term captures the additional utility derived from a housing transaction.

6.2.3 Bellman equation

To conduct an analytical analysis of optimal policy on housing choices, the lifetime utility can be transformed into Bellman equations. These Bellman equations can help in deriving the solution recursively and backwards. In each period, there are four different scenarios and four different bellman equations correspond to four scenarios respectively. Scenarios are classified into two categories as Scenarios 1 and 2 are for tenants while Scenarios 3 and 4 are for home owners.

Scenario 1: for a household who is a tenant continues renting the same house in period *t*, the value function based on $\{d_{t-1}^{1}, d_{t}^{1}\}$ = {0, 0} can be defined as

$$V_t^R(W_{t-1}, h_{t-1}^{j-1}, \boldsymbol{r_t}) = \max_{(C_t)} u(h_{t-1}^{j-1}, c_t, \theta_\tau) + \beta E_t V_{t+1}(W_t^R, h_{t-1}^{j-1}, \boldsymbol{r_{t+1}})$$

(9)

Subject to:

$$W_t^R = y_t + (1+i_t)W_{t-1} - c_t - r_t^{j-1} h_{t-1}^{j-1}$$

Scenario 2: for a household switches their tenure status from tenant to home owner (this is the first-time house purchasing) at the end of period *t*, the value function based on $\{d_{t-1}^{1}, d_{t}^{1}\}$ = {0, 1} is in the form of

$$V_t^O(W_{t-1}, h_{t-1}^{j-1}, \boldsymbol{p_t}) = \max_{(c_t, h_t^j)} u(h_t^j, c_t, \theta_\tau) + \beta E_t V_{t+1}(W_t^O, h_t^j, \boldsymbol{p_{t+1}})$$

(10)

Subject to:

$$W_t^O = y_t + (1+i_t)W_{t-1} - c_t - r_t^{j-1} h_{t-1}^{j-1} - (1+\delta) p_t^j h_t^j$$

Scenario 3: for a household who has already owned a house and decides to stay in the house in period *t*, the value function based on $\{d_{t-1}^{1}, d_{t}^{2}\}$ = {1, 0} can be shown as

$$V_t^N(W_{t-1}, h_{t-1}^{j-1}, \boldsymbol{p_t}) = \max_{(c_t)} u(h_{t-1}^{j-1}, c_t, \theta_\tau, SI_t) + \beta E_t V_{t+1}(W_t^N, h_{t-1}^{j-1}, \boldsymbol{p_{t+1}})$$

(11)

Subject to:

$$W_t^N = y_t + (1+i_t)W_{t-1} - c_t$$

Scenario 4: for a household who has already owned a house and decides to transact their house in period *t*, the value function based on

$\{d_{t-1}^1, d_t^2\}$ = {1, 1} is defined as

$$V_t^T\left(W_{t-1}, h_{t-1}^{j-1}, \boldsymbol{p_t}\right) = \max_{\left(h_t^j, c_t\right)} u\left(h_t^j, c_t; \theta_\tau, SI_t\right) + \beta E_t V_{t+1}\left(W_t^T, h_t^j, \boldsymbol{p_{t+1}}\right)$$

(12)

Subject to:

$$W_t^T = y_t + (1 + i_t)W_{t-1} - c_t + \left[p_t^{j-1} h_{t-1}^{j-1} - (1 + \delta) p_t^j h_t^j\right]$$

Prior to the end of the life cycle, households who are tenants ($d_{t-1}^1 = 0$) or home owners ($d_{t-1}^1 = 1$) have to make a decision whether they will make a housing transaction (first-time house owning for tenants and changing houses for home owners) or not, in order to maximize the expected value function in each period. First, for households who are tenants, the expected value function is shown as

$$V_t\left(W_{t-1}, h_{t-1}^{j-1}, \boldsymbol{p_t}, \boldsymbol{r_t}, \boldsymbol{s}_t\right) = \max_{\left(d_t^1, d_t^2, h_t^j, c_t\right)} \{V_t^R, V_t^O\}$$

Based on the comparison between costs of staying in the current house and the costs of transaction, the household's tenure choice is made on whether to start home ownership in period *t*. Similarly, for households who are homeowners, the expected value function is shown as

$$V_t\left(W_{t-1}, h_{t-1}^{j-1}, \boldsymbol{p_t}, \boldsymbol{s}_t\right) = \max_{\left(h_t^j, c_t\right)} \{V_t^N, V_t^T\}$$

If a household is expected to benefit from a transaction (accompanied with transaction costs), he/she will choose to sell the current house and buy a new one.

This model is to capture the different roles of sentiment in housing demand. Specifically, by using this model, this study is to explore the effect of sentiment in households' housing choices including tenure and housing service. The model implications in the following section consist of two components: 1) the optimal policy whether to transact in two aspects, i.e. the optimal time *t* for first-time home purchase ($d_{t-1}^1 = 0$ and $d_t^1 = 1$) for tenants, and for changing the house ($d_t^2 = 1$) for home owners; and 2) the optimal housing demand conditional on the household's status.

6.3 Optimal Housing Choices

In the previous section, the theoretical model presented the mixed decision problem, which is difficult to solve directly. Such mixture of discrete and continuous choices (or control variables) may result in a stochastic Euler equation (Pakes, 1994). Yet such equation may fail to give marginal conditions for optimisation. In addition, the transaction costs complicate this situation (Han, 2008). To solve this problem, this study adopts the spirit of the option value of waiting, as in the classical investment literature, and derives a function to smoothen the mixed value function.

Under the presumption of future decisions being optimised, the Bellman equation adds the expected time *t*+1 utility, conditional on the time *t* consumption decisions to the current utility. The difference between two expectations of time *t*+1 value function arising from two opposite scenarios indicates that it is optimal to transact when the expected benefit from the transaction is larger than that from remaining. On the other hand, the transaction cost embedded in every transaction makes sense of waiting. No transaction cost leads to lossless change in total wealth and makes waiting worthless. Hence, the conditions on the information set at time *t* including conditional densities $\phi(\boldsymbol{p_{t+1}}|\boldsymbol{p_t})$ and $\psi(\boldsymbol{r_{t+1}}|\boldsymbol{r_t})$, the expected option value of waiting can be expressed as

$$E_t O_{t+1}^1(W_t, h_t) = E_t[\mathbf{1}(D_{t+1}^1)(D_{t+1}^1)]$$

for households who are tenants, where $D_{t+1}^1 \equiv V_{t+1}^R(W_t, h_t) - V_{t+1}^O(W_t, h_t)$ and $\mathbf{1}\left(D_{t+1}^i\right)$ is an indicator function and equals to 1 if the variable is positive, and

$$E_t O_{t+1}^2(W_t, h_t) = E_t[\mathbf{1}(D_{t+1}^2)(D_{t+1}^2)]$$

for homeowners, where $D_{t+1}^{2} \equiv V_{t+1}^{N}(W_t, h_t) - V_{t+1}^{T}(W_t, h_t)$. In this case, the probability of remaining at time *t*+1 can be expressed as

$$Pr_t\left(d_{t+1}^{i} = 0\right) = Pr_t\left(D_{t+1}^{i} > 0\right), \qquad i = 1,2$$

Thus,

$$E_t V_{t+1}(W_t, h_t) = E_t[V_{t+1}^{O} + O_{t+1}^{1}] \quad \text{or} \quad E_t V_{t+1}(W_t, h_t) = E_t[V_{t+1}^{T} + O_{t+1}^{2}]$$

(13)

The value of waiting embodies all the opportunity costs from a housing transaction and thus equations of option value of waiting smoothens the Bellman equation by replacing the second term with the above equation. For households who are tenants in period *t*, if they choose to launch their home ownership at the end of this period, they will enjoy the utility which is the same as the home owner's utility. Meanwhile, the discounted expectation of value function in the next period can be expressed as the combination of value of transacting and option value of waiting. That is,

$$V_t^{O}\left(W_{t-1}, h_{t-1}^{j-1}, SI_t\right) = \max_{(h_t, c_t)} u(h_t, c_t) + \beta E_t[V_{t+1}^{T}(W_t^{O}, h_t, SI_t) + O_{t+1}^{2}(W_t^{O}, h_t, SI_t)] \qquad (14)$$

Similarly, for home owners, we have

$$V_t^{T}\left(W_{t-1}, h_{t-1}^{j-1}, SI_t\right) = \max_{(h_t, c_t)} u(h_t, c_t; SI_t) + \beta E_t[V_{t+1}^{T}(W_t^{T}, h_t, SI_t) + O_{t+1}^{2}(W_t^{T}, h_t, SI_t)] \qquad (15)$$

With this substitution, this study is able to have model implications in two aspects: (1) optimal transacting time, i.e. whether to transact or not is conditional on the current housing service and (2) optimal value of housing service (h_t^{j}) is conditional on transacting.

6.3.1 Optimal transacting timing

To explore the effect of sentiment in optimal transacting timing, this study starts with analysis of transacting boundaries. The boundaries define an interval within which it is optimal for households to make transactions. Then this study attempts to discuss the role of sentiment in affecting the boundaries.

In the presence of transaction costs, a household has a reason to suspend a transaction. As the housing price varies over time and transaction cost is proportional to the price, the effect of transaction cost becomes time-varying. As such, the likelihood of transacting becomes volatile and the transacting boundaries are no longer fixed over time. As discussed above, the option value of waiting smooth the mixed value function which reflects the household's tenure choice. By using this approach, the model is able to have an optimal policy in a simple and clear form.

This discussion begins with the case of home owners and then turns to the case of households who are tenants as the two cases are similar but the latter is a bit more complex. Following the spirit of Proposition 4.3 in Han (2008), the following Lemmas define the optimal interval for transacting. Then the model implications regarding the effect of sentiment on the likelihood of transacting are summarised in Proposition 1.

Lemma 1.1 (*Optimal policy on housing transaction*). There exists an optimal policy that is subject to an optimal interval defined by $\left[b^l_{2,t}, b^u_{2,t}\right]$, where $b^l_{2,t}$ and $b^u_{2,t}$ are solutions to a quadratic equation of $(hs^T_t - TC^T_t)$ as $D^2_t = -\frac{1}{2}\frac{\partial^2 u}{\partial c^2}(hs^T_t - TC^T_t)^2 - \left(\frac{\partial^2 u}{\partial c^2}(c^N_t - c^T_t) + \frac{1}{2}\frac{\partial^2 u}{\partial c \partial h}\left(h^{j-1}_{t-1} - h^j_t\right) + \frac{\partial u}{\partial c}\right)(hs^T_t - TC^T_t) + C^T = 0$

Where $hs^T_t = p^{j-1}_t h^{j-1}_{t-1} - p^j_t h^j_t$ representing the change in housing asset value, $TC^T_t = \delta p^j_t h^j_t$ representing the transaction cost and $C^T = \frac{\partial u}{\partial c}(1+\delta)p^j_t\left(h^{j-1}_{t-1} - h^j_t\right) + \frac{1}{2}\frac{\partial^2 u}{\partial c \partial h}(1+\delta)p^j_t\left(h^{j-1}_{t-1} - h^j_t\right)^2$.

When $hs^T_t - TC^T_t \in \left[b^l_{2,t}, b^u_{2,t}\right]$, $D^2_t \leq 0$ indicating that the household should transact at time *t*.

The proof is given in Appendix A.

Lemma 1.1 indicates that the household's optimal choice depends on the combination of the difference in housing asset value between current house and former house, and the transaction cost, i.e. $(hs_t - TC_t)$ in total. If the combination falls within the interval, the benefit from waiting becomes negative and thus the household should choose to transact.

Lemma 1.2 (*Optimal policy on home ownership*). There exists an optimal interval defined by $\left[b_{1,t}^{l}, b_{1,t}^{u}\right]$, where $b_{1,t}^{l}$ and $b_{1,t}^{u}$ are solutions to a quadratic equation of $(hs_t^O - TC_t^O)$ as $D_t^1 = -\frac{1}{2}\frac{\partial^2 u}{\partial c^2}(hs_t^O - TC_t^O)^2 - \left(\frac{\partial^2 u}{\partial c^2}(c_t^R - c_t^O) - \frac{1}{2}\frac{\partial^2 u}{\partial c \partial h}\left(h_{t-1}^{j-1} - h_t^j\right) + \frac{\partial u}{\partial c}\right)(hs_t^O - TC_t^O) + C^O = 0$

Where $hs_t^O = -p_t^j h_t^j$ representing the difference in housing asset value between current house and former house (as the tenant didn't own a house in previous periods, the value of former house equals to 0), $TC_t^O = \delta p_t^j h_t^j$ representing the transaction cost and $C^O = \frac{\partial u}{\partial c}(1+\delta)p_t^j\left(h_{t-1}^{j-1} - h_t^j\right) + \frac{1}{2}\frac{\partial^2 u}{\partial c \partial h}(1+\delta)p_t^j\left(h_{t-1}^{j-1} - h_t^j\right)^2 - (f(SI_t) - 1)\pi h_t^j$.

When $hs_t^O - TC_t^O \in \left[b_{1,t}^{l}, b_{1,t}^{u}\right]$, $D_t^1 < 0$ indicating that the household should transact at time *t*.

The proof is given in Appendix B.

Lemma 1.1 and Lemma 1.2 remark the differences in two aspects: (i) the difference in housing asset value and (ii) constant C in quadratic equations. Since the case refers to the first-time home owning, the difference in housing asset value is fully determined by the asset value of the house that the household aims for. The utility jump derived from the first-time home owning adds an additional term into C^O and such term affects the range of interval of transacting.

Based on the implications shown in the above Lemmas, an analysis is conducted as follows to study the effect of sentiment on the optimal policy of transacting. As the optimal policy depends on the optimal interval of transacting, the range of this interval denoted by $\left[b_{i,t}^{u} - b_{i,t}^{l}\right]$, is a reasonable estimate of the likelihood of transacting and thus substantially affects the optimal policy. As such, it is of interest to investigate how sentiment affects optimal interval of transacting, i.e. $\left[b_{i,t}^{l}, b_{i,t}^{u}\right]$ and model implication is summarised in Proposition 1.

Proposition 1 (*Effect of sentiment on optimal transacting*). Sentiment plays different roles in affecting the likelihood of housing transacting at time *t* as

$$\frac{\partial\left(b_{1,t}^{u} - b_{1,t}^{l}\right)}{\partial SI_t} > 0 \text{ and } \frac{\partial\left(b_{2,t}^{u} - b_{2,t}^{l}\right)}{\partial SI_t} = 0$$

The proof is given in Appendix C.

Proposition 1 shows that sentiment positively affects the likelihood of first-time ownership while only trivially affects the likelihood of transacting. For households who are tenants, sentiment affects their lifetime housing choice through the first-time home purchase only and such choice depends on the current sentiment. In a certain period, sentiment stimulates households' hedging demand against the future uncertainty about rents (or house prices) and thus a rise in current sentiment could increase the likelihood of first-time home purchase.

The situation for housing transactions is different. A transaction consists of selling the current house and buying a new one in the same period and thus the effect of sentiment can be considered to be separated into two parts. The first part is the sentiment effect in de-ownership and second is the sentiment effect in an ownership that is conditional on de-ownership. Although the utility change and hedging demand depend on the household's sentiment, sentiment has opposite impacts on these two parts. Proposition 1 implies that the combination of opposite trades offsets the effects of sentiment in these two parts.

6.3.2 Optimal housing service

Prior to the analysis of optimal housing service conditional on transacting[26], the first order condition (FOC) may give a first insight into the optimality of the problem. The mixed optimisation problem can be reduced to a continuous one by using equations 14 or 15 above. Then the first order condition can be taken as a normal case:

$$FOC\colon c_t\colon \quad \frac{\partial u_t}{\partial c_t} + \beta E_t\left(\frac{\partial V_{t+1}^T}{\partial W_t} + \frac{\partial O_{t+1}^2}{\partial W_t}\right)\frac{\partial W_t}{\partial c_t} = \frac{\partial u_t}{\partial c_t} - \beta E_t\left(\frac{\partial V_{t+1}^T}{\partial W_t} + \frac{\partial O_{t+1}^2}{\partial W_t}\right) = 0$$

$$FOC\colon h_t\colon \quad \frac{\partial u_t}{\partial h_t} + \beta E_t\left(\frac{\partial V_{t+1}^T}{\partial h_t} + \frac{\partial O_{t+1}^2}{\partial h_t}\right) + \beta E_t\left(\frac{\partial V_{t+1}^T}{\partial W_t} + \frac{\partial O_{t+1}^2}{\partial W_t}\right)\frac{\partial W_t}{\partial h_t} = 0$$

The FOCs reveal the inter-temporal relationship between current and future consumption, which can be reflected by the marginal utility of the current period and the marginal value of future periods. As the non-durable consumption only affects the current utility, the marginal utility derived from the additional consumption on non-durable goods should be equal to the expected future value derived from the additional *ex ante* wealth (as the same amount of non-durable consumption) in the sense of optimality. As housing is a durable good, the situation is relatively complex. Based on the additional housing consumption in period *t*, the combination of marginal consumption utility and expected marginal future value (including consumption utility and asset value) should be equal to the expected marginal value derived from the additional *ex ante* wealth (as the same amount of housing consumption).

The marginal rate of substitution (*MRS*) between housing and

[26] No matter this transaction is either first owning or changing a house, the household will be the home owner in the next period. Thus, two V_{t+1} in the value function are the same.

non-housing consumption is introduced here. The *MRS* captures the exchange rate between the two consumptions under optimality. It indicates that the marginal utility of housing service is measured in terms of the amount of non-housing consumption being given up. Conditional on transacting, the optimal housing demand in period *t* therefore can be implicitly determined by the *MRS* as given in Lemma 2.

Lemma 2 (*Optimal housing service*). The optimal housing demand in period *t* is implicitly determined by the *MRS* equation as

$MRS = m_{t+1}E_t p_{t+1}^{j} - (1+\delta)p_t + \frac{\beta E_t \frac{\partial O_{t+1}^{2}}{\partial h_t}}{\frac{\partial u_t}{\partial c_t}}$, where $m_{t+1} = \frac{\beta \frac{\partial u_{t+1}}{\partial c_{t+1}}}{\frac{\partial u_t}{\partial c_t}}$ is a stochastic discount factor (*SDF*).

The proof is given in Appendix D.

Lemma 2 indicates that *MRS* is subject to three terms which represent the user costs of housing service and this result is consistent with Han (2008)'s. The first term is expected unit price at time *t*+1 discounted by *SDF* and the second one is the present unit price adjusted by the proportion of transaction cost. The combination of the first two terms reveals the present value of inter-temporal change in the asset value of housing service purchased at time *t*.

The last term indicates the marginal substitution rate between value of option of waiting at time *t*+1 and non-housing consumption. $E_t \frac{\partial O_{t+1}^{2}}{\partial h_t}$ reflects the expected marginal benefit of waiting at time *t*+1 if a household obtained more housing service (h_t) at time *t*. If the household already enjoys more housing service, the value function of staying in the current house becomes greater and the likelihood of waiting would rise. As such, this derivative should be positive. As a result, *MRS* should compensate for the benefit that households cannot enjoy from waiting. The model has implications on the role of sentiment in affecting the optimal housing service (proxied by *MRS*) are summarised in Proposition 2.

Proposition 2 (*Effect of sentiment on housing demand*). The effect of sentiment on *MRS* is determined by the relationship between sentiment functions f' and g' as

$$\frac{\partial MRS}{\partial SI_t} \geq 0 \quad if \;\; SI_t \in \{SI_t: g' \geq K(lnf)'\}$$

$$\frac{\partial MRS}{\partial SI_t} < 0 \quad if \;\; SI_t \in \{SI_t: g' < K(lnf)'\}$$

where f and g are sentiment functions and

$$K = \frac{\beta}{\mu^j}\left(E_t p_{t+1}^j + E_t \frac{\partial O_{t+1}^2}{\partial h_t}\left(\frac{\partial u_{t+1}}{\partial c_{t+1}}\right)^{-1}\right) > 0.$$

The proof is given in Appendix E.

Proposition 2 provides a better understanding of how sentiment affects the optimal housing service (implicitly represented by *MRS*). It implies an inconsistent effect of sentiment on *MRS*. The forms of the sentiment functions (f, g) may complicate the situation as there may exist several real roots of the equation $g' = K(lnf)'$. The inequalities indicate that the discrepancy in the role of sentiment between the macro-level (market) and the micro-level (household) could lead to completely different implications. In other words, a household's sentiment cannot fully predict the effect of sentiment on market behaviour.

A corollary is derived from Proposition 2 to illustrate a special case which helps simplify the situation and thus provides some informative implications. Assume the household can always make a good estimation on the market trend. This means that the household's sentiment function f and market price sentiment function g have a persistent correlation(denoted by ω). Recall that $f(0) = g(0) = 1$ and thus such specific linear relationship are set as $\omega(g-1) = (f-1)$. The details of Corollary 1 are shown below.

Corollary 1 (*A simple case*). Assume a linear correlation between two sentiment functions as $\omega(g-1)=(f-1)$, where $\omega>0$ is a constant coefficient . Then a turning point si_t is defined by $f(si_t)=\omega K$ and Proposition 2 can be reduced to

$\frac{\partial MRS}{\partial SI_t} \geq 0 \quad if\ SI_t \geq si_t$ and $\frac{\partial MRS}{\partial SI_t} < 0 \quad if\ SI_t < si_t$.

Proof.

Under the new assumption the relationship becomes $\omega g' = f'$ and thus $MRS'_{SI} = m_{t+1} f'\{1/\omega - K/f\}$. As $f>0$, it leads to $MRS'_{SI}>0$ if $f>\omega K$ and vice versa. In other words, the sentiment positively affect the *MRS* if $SI_t \in \{SI_t\colon f(SI_t) > \omega K\}$.

Q.E.D.

Corollary 1 suggests that the effect of sentiment on the marginal rate of substitution (*MRS*) exhibits a U-shape pattern along with the increase in market sentiment. That is, the *MRS* curve goes downwards at first and then upwards. If market sentiment is relatively low ($SI_t < si_t$), a rise in market sentiment will diminish the *MRS*. If market sentiment is relatively high ($SI_t > si_t$), a rise in such sentiment will enlarge the *MRS*.

This complex relationship is attributed to the combined impact of owning effect and hedging demand. The hedging demand for housing service evolves from negatively strong (i.e. households would like to reduce their housing with strong intentions) to positively strong (i.e. households would like to augment their housing with strong intentions) with sentiment, while the owning effect impels households to increase their housing service. When market sentiment is extremely bearish, hedging demand drives households to enjoy less housing, though the owning effect has an opposite impact. The owning effect is overwhelmed by hedging demand at this moment. Households have to give up a lot of non-housing consumption if they would like to enjoy additional housing service.

With a rise in sentiment, the hedging demand for degrading becomes lower while the marginal benefit from owning effect of housing gradually elevates. As a result, the *MRS* decreases due to the combined impact of owning effect and hedging demand becoming moderate. Households need to sacrifice non-housing consumption that is less than the previous amount for additional housing service. When sentiment gradually rises, the *MRS* reaches the trough of the curve and the hedging demand reverses to the demand for upgrading. Then the hedging demand aligns with the owning effect, and has a positive effect on overall housing demand. As a result, the amount of non-housing consumption that households would like to give up for additional housing service increases. In a sum, sentiment affects hedging demand and owning effect in different ways: sentiment determines the direction of hedging while sentiment can just adjust the scale of owning effect.

6.4 Numerical Analysis

Since the model established in this study is mixed and complex, it is difficult to explicitly show practical implications. A mature strategy is to conduct a numerical analysis in which we simulate a household's optimal housing choices over the life cycle. This section will run hundreds of simulations to verify the implications that are derived from the analytical solutions to the model. To achieve this, a series of mimic economies, in which a household is confronted with different circumstances, are set up. In each economy, two different cases are built up to simulate: 1) the optimal timing and housing service of first-time home owning for tenants and 2) the optimal housing demand for home owners over the life cycle.

The first case is to optimise the non-housing consumption over the life cycle and the housing service the household who is the tenant first purchases a house at the end of period *t.* Since the life cycle covers T periods, there are T values of total utility (refer to Equation 1) conditional on the first-time home purchase in period *t*. As such, the optimal timing for the first-time home owning can be explored by the maximum of T total utilities. The second case is to optimise the non-housing consumption and the housing demand over the life cycle while the home owner is assumed to transact in every period. Each case compares the sentiment model with the benchmark model[27] to explore the effects of sentiment on housing tenure choices and housing demand among households.

6.4.1 Simulation design

Assume a typical household makes a housing plan of 30-years long, at the age of 30 and retires at 61. That is, the household's life cycle covers 30 individual periods if one period is assumed to last one year. For each simulation, the household profile and the artificial economy are determined by several exogenous state variables, i.e. household income, housing taste, house prices, in addition to rent and market sentiment.

The household's income is assumed to have an autoregressive process as suggested by Hall (1978) that "income is a distributed lag of past actual income". Households' income and their housing tastes are given exogenously and are with age. Income process follows an AR(1) process as

$$inc_t = \mu_{inc} + \eta inc_{t-1} + e_t$$

where μ_{inc} denotes the constant and η denotes the autoregressive parameter in the income process. The error term e_t follows the standard

[27] The benchmark model is the standard model without sentiment effect. In other words, if sentiment functions are strictly set to be equal to 1, the sentiment model will be reduced to benchmark model.

normal distribution. In addition, housing taste (and preferences) is assumed to be rising during young- to middle-age and declining after the age of 50: a young family gradually increases the propensity of housing as the size of the family becomes bigger and then decreases the propensity as the children grow up and move out. As such, household taste and preferences for housing are given as

$$hst_t = \frac{1}{500}[450 - (t - 20)^2]$$

where $t = 1 \ldots 30$ and $t = 20$ reaches the peak of housing taste.

To focus on our objective and to ease the computational burden, we assume there is a process describing the dynamics of house prices and rents. In that process, housing price (hp) and rent ($rent$) follow the 1st order Markov process:

$$\binom{hp}{rent}_t = \boldsymbol{\mu_p} + A\binom{hp}{rent}_{t-1} + e_t$$

where $\boldsymbol{\mu_p}$ is a 2×1 vector indicating the constant in this process and A is a 2×2 matrix representing the autoregressive and cross-sectional coefficients. The error term is $e_t \sim N\left(0, diag(\sigma_1^2, \sigma_2^2)\right)$, where the standard deviations for price and rent are different: rent has a much smaller standard deviation as house price always show greater fluctuations than rent (Wang and Hui, 2016).

The sentiment effect emerges in two aspects. The first is derived from market sentiment into the housing price formation process. In accordance with Equation 5, the housing price formation process adjusted by sentiment (shp_t) can be expressed as

$$shp_t = hp_t + \mu_p[g(SI_{t-1}) - 1]$$

where $g(SI_{t-1})$ denotes the function of market sentiment. Second is derived from households' sentiment into households' tenure choice by $f(SI_t)$. Meanwhile, assume a linear correlation ω between two sentiment functions as mentioned in Corollary 1.

For the two cases, 500 simulations have been conducted based on the stochastic specification in the above-mentioned setting[28] for households, whereby this study can examine the life-cycle pattern of households' housing choices. The average of the optimal choices over 500 simulations is taken to reveal the average housing behaviour that the household exhibits over a life cycle.

6.4.2 Key findings from simulation

Simulations are carried out for two cases with different aims. The first case focuses on the optimal timing for households to transform their role from tenants to home owners while the second case emphasizes on the optimal housing demand among homeowners. In both cases, the results of the sentiment model is compared to that of the benchmark model (sentiment is set to be neutral) and it is of interest to examine the role of sentiment in households' housing choice.

Figures 6.1-3 are to verify the model's implications for the first case with respect to housing choices among households who are tenant. Figures 6.1 and 6.2 plot the averages and standard deviations (over 500 simulations) of total utility values conditional on tenants who purchase their first house at the end of period *t*. In both figures, there are three curves with three paths resulted from different model specifications. First, the benchmark model is the one with trivial effects arising from sentiment. This means that the price formation process and household decision making are not affected by sentiment. The path of the benchmark model is displayed by the dotted line in each figure. The solid line and dashed line exhibit the results derived from the sentiment models. The solid line represents a standard sentiment

[28] Details on the model specification are given in Appendix.

model while the dashed line shows the sentiment model in which the sentiment function is adjusted by a positive translation on the sentiment index. In other words, the household in the positive translatory model faces the sentiment index (and thus the sentiment function) slightly higher at every time point than in the standard sentiment model for each simulation.

In Figure 5.1, the maximum of total utility value on each path indicates the average of an optimal time for households to purchase a first house, in each model. Overall, households prefer to launch their first-time home purchase in the first few years or the last few years due to the effects of housing taste and time value. Another interesting finding is that the dotted line lies above the other two curves most of the time, implying that the average total utility derived from non-sentimental life cycle is higher than that from the life-cycle influenced by sentiment. Thus, sentiment plays a significant role in affecting life-cycle utility. In other words, on average the consumption choices optimised by the fully rational household in a rational market bring more happiness than that by sentimental household in a sentiment-based market. The reason is as follows. At a market level, sentiment leads to a greater price risk as it amplifies the price fluctuations. It implies that the risk of housing wealth becomes higher and thus households may reduce housing consumption. From the household perspective, sentiment gives rise to fluctuations of utility and brings about uncertainty of utility. Thus the total utility in the presence of the sentiment effect is less than the total utility under certainty. The gaps of utility between benchmark model and sentiment models have gradually narrowed over the last 10 periods. This is because the impact of the first-time home purchase tapers off due to the decreasing housing taste and time value.

Figure 6.1. The curve of total utility values of the first-time home owning if the first-time home purchase happens in period *t*.

Notes: x-axis shows periods 1-30 and y-axis shows total utility values.

Figure 6.2 displays the standard deviation of total utility value, which indirectly reflects the likelihood of transaction at each time point. As each simulation experiences different artificial economies and sentiment paths, this analysis cannot give a precise indication regarding the effect of sentiment on the likelihood of transaction at a certain time point. Instead, this analysis examines the model's implication and sentiment effect over the life cycle. A high standard deviation indicates a higher possibility of a certain time point becoming the optimal timing. In Figure 5.2, solid and dash lines are found to lie above the dot line before Period 24[29]. This indicates that the standard deviations of total utility value derived from sentiment models are higher than that from the benchmark model and sentiment does affect the likelihood of transacting. Furthermore, almost

[29] The deviation of total utility in benchmark model rebounds in last few periods mainly because of the decreasing housing taste and accumulative wealth. As such, this study could restrict the attention on the young to middle age periods, i.e. periods before 21.

every point on the dashed line is found to be slightly higher than that on the solid line, which confirms the positive effect of sentiment on the likelihood of transacting.

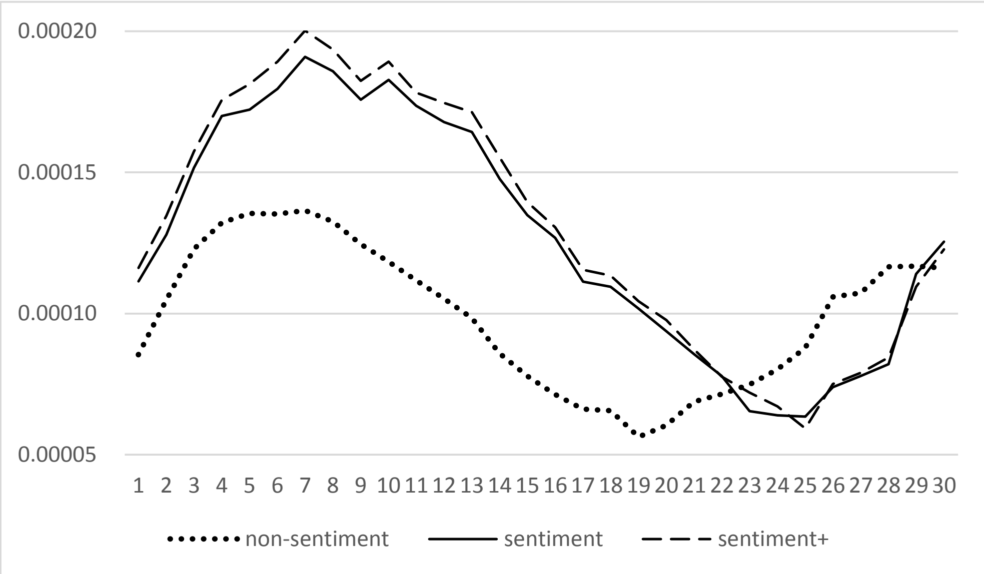

Figure 6.2. The curve of the standard deviation of total utility values if the household's first-time home purchase happens in period *t*.

Notes: x-axis shows periods 1-30 and y-axis shows the standard deviation of total utility values.

Figure 6.3 explores the averages (over 500 simulations) of optimal housing service at time *t* if a household purchases the first home in the period. Overall, households at young and middle ages narrow their choices similar to the time $t = 0$ housing service. However, senior households pursue much more housing service but much less when the time approaches the last period. This situation is mainly attributed to the total utility affected by the finite horizon of the life cycle.

According to the corollary, the relationship between housing demand and sentiment can switch from negative to positive with rises in sentiment. As such, this complicated relationship would impel the path of sentiment-based housing service to wave around the path of benchmark housing service. Figure 5.3 confirms this implication as the solid line keeps crossing over the dotted line. On the other hand, the dashed line represents the path of optimal housing service in the positive translatory sentiment model also oscillates around the solid line, which again confirms the idea of the corollary.

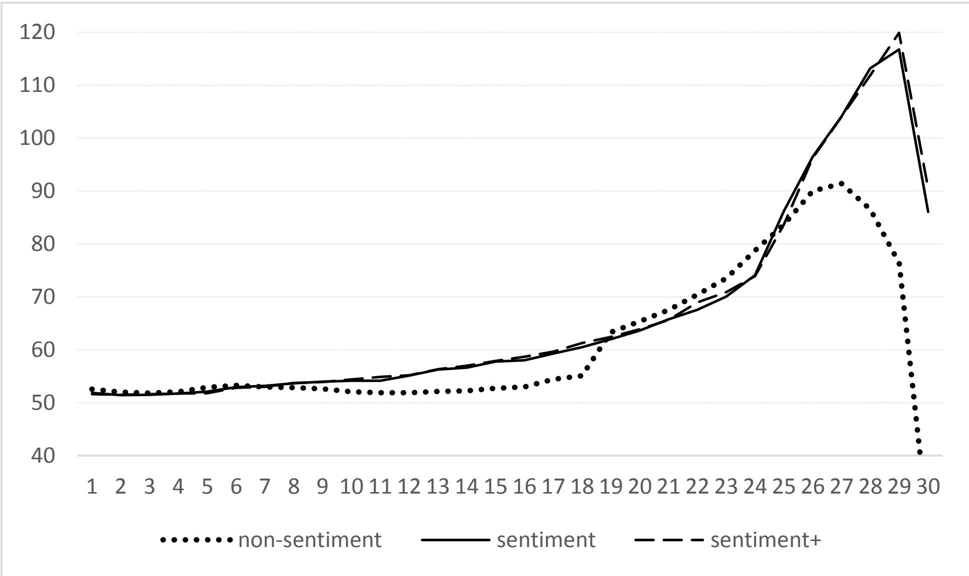

Figure 6.3. The curve of optimal values of housing service the household should take for a first-time home owning if the first-time home purchase happens in period *t*.
Notes: x-axis shows periods 1-30 and y-axis shows optimal housing service.

Figure 6.4 is to verify the model's implications for the second case regarding home owners' housing demand and plots three curves of optimal housing service if the home owner is forced to transact in each period. Due to the arched shape of the time-varying housing taste over the life cycle, three paths of optimal housing demand also exhibit hump shapes. The path of the benchmark model is smoother than the other two. In the benchmark model, households with future price uncertainty would like to smooth the path of housing consumption over the life cycle. Although market sentiment increases the volatility of house prices, the household's ex ante judgement based on household's sentiment helps them reduce the future uncertainty. In this case, they could take advantage of this to make a housing hedge against future wealth variations as the household is forced to transact in each period. Thus, in Figure 6.4 it is obvious that two paths of housing demand in sentiment models deviate from the smoother path in the benchmark model. On the other hand, the dashed line again goes up and down along the solid line.

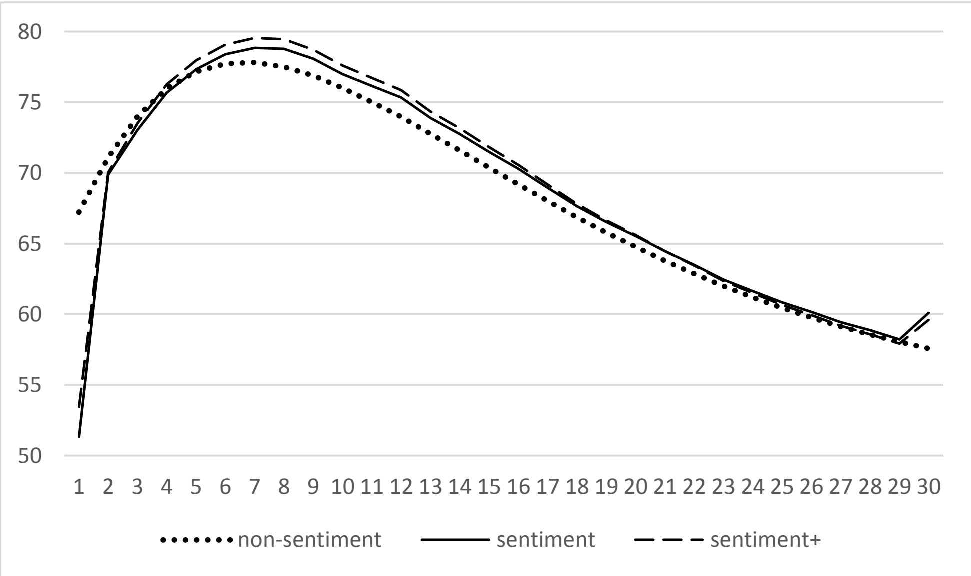

Figure 6.4. The life-cycle curve of optimal values of housing service if the household transacts in each period.

Notes: x-axis shows periods 1-30 and y-axis shows optimal housing service.

However, the sentiment also brings uncertainty to the utility derived from the combination of non-housing consumption and housing service. The comparison among three average values of total utility in three models shows that the total utility of a classical household is slightly higher than the other two[30] indicating that the effect of sentiment on utility uncertainty overwhelms the effect of sentiment on the housing hedge.

6.5 Conclusion

Nowadays the housing market is supposed to be affected by sentiment. Although recent studies pay much attention to the sentiment effect on the market level, the lack of literature focusing on the role of sentiment at the micro level spurs this study, which aims to have a better understanding of the effect of sentiment on households' housing choice over the life cycle. More specifically, this study has two objectives in terms of the tenure and housing service choices. The first is to better understand how sentiment affects the optimal timing for households' first-time home purchases. The other is to explore the effect of sentiment on the optimal housing service when home owner transacts in the housing market.

To achieve the goals, a theoretical life-cycle model is established to investigate the different roles of sentiment. This model has several advantages that could make a knowledge contribution to the existing life-cycle study. First, this model integrates two different types of decision making, i.e. housing tenure choice and housing demand over the life cycle. Second, this model is able to distinguish the utility of housing consumption and the utility of housing investment. Third, this model incorporates different roles of sentiment at both market and micro level, and thus enables us to explore more accurate implications. Fourth, this model can be reduced to the standard model without sentiment effect by using a trivial sentiment function (set sentiment function equal to 1) and thus allows us to compare

[30] The simulation shows that the average value of total utility in the three different models are -0.00083 for benchmark model, -0.00085 for standard sentiment model and -0.00093 for positive translatory sentiment model.

the pathway of housing consumption between sentiment-driven and rational households.

The key theoretical implications derived from optimal solutions to the model are two-fold: (i) sentiment positively affects the likelihood of first-time home purchase while trivially affects the likelihood of transacting and (ii) the effect of sentiment on the marginal rate of substitution (*MRS*) between housing and non-housing consumption reverses with an increase in sentiment. Sentiment negatively affects *MRS* when sentiment is lower than the threshold and then positively affects *MRS* when sentiment exceeds the threshold.

Simulation has been conducted 500 times to numerically verify the model's implications. It is worth noting that the life-cycle utility in the benchmark model (without sentiment effect) is higher than those in sentiment models. In addition, the comparison between standard sentiment model and positive translatory model indicates that sentiment-based life-cycle utility decreases with sentiment.

As this study provides better insights into the role of sentiment in housing demand, its implications may serve as a useful reference for relevant authorities in monitoring and policy making regarding housing demand under different situations of market sentiment.

6.6 Appendix

Lemma 0 (*Function F*).

Recall that $u(d_t^1 = 1) = u(d_t^2 = 1)$ and thus

$$u\left(h_t^j, c_t, \theta_t\right) + (f(SI_t) - 1)\pi h_t^j = u^{SI}\left(h_t^j, c_t, \theta_t, f(SI_t)\right) + (f(SI_t) - 1)\pi\left(h_t^j - h_{t-1}^{j-1}\right)$$

After simplification, we have $\pi h_{t-1}^{j-1} = \frac{u_{SI} - u}{(f(SI_t) - 1)}$

Since $u_{SI} = uF$, take derivative on both sides w.r.t. SI_t and we have

$$0 = \frac{\left(\frac{\partial u_{SI}}{\partial f} f'\right)(f-1) - (u_{SI} - u)f'}{(f-1)^2} = uf' \frac{\frac{dF}{df}(f-1) - (F-1)}{(f-1)^2}$$

Since $f' > 0$, we have $\frac{dF}{(F-1)} = \frac{df}{(f-1)}$. Hence, $F = f$ and $u_{SI} = uf$.

Q.E.D.

Lemma 1.1 (*Optimal policy on housing transaction*).

Proof.

Following the proof framework of proposition 4.3 in Han (2008), we result in a similar implication for household who owns a house. It is optimal for household not to transact if household benefits more from waiting as $D_t^2 > 0$.

$$D_t^2 \equiv V_t^N(W_{t-1}, h_{t-1}) - V_t^T(W_{t-1}, h_{t-1})$$
$$= u\left(c_t^N, h_{t-1}^{j-1}; SI_t\right) + \beta E_t\left[V_{t+1}^T\left(W_t^N, h_{t-1}^{j-1}\right) + O_{t+1}^2\left(W_t^N, h_{t-1}^{j-1}\right)\right]$$
$$-\left\{u\left(c_t^T, h_t^j; SI_t\right) + +\beta E_t\left[V_{t+1}^T\left(W_t^T, h_t^j\right) + O_{t+1}^2\left(W_t^T, h_t^j\right)\right]\right\}$$

By using second order Tyler expansion at point $\left(c_t^T, W_t^T, h_t^j\right)$, we have

$$D_t^2 = \left(\frac{\partial u}{\partial c}\right)_{c=c_t^T}(c_t^N - c_t^T) + \frac{1}{2}\left(\frac{\partial^2 u}{\partial c^2}\right)_{c=c_t^T}(c_t^N - c_t^T)^2$$
$$+\left(\frac{\partial u}{\partial h}\right)_{h=h_t^j}\left(h_{t-1}^{j-1} - h_t^j\right) + \frac{1}{2}\left(\frac{\partial^2 u}{\partial h^2}\right)_{h=h_t^j}\left(h_{t-1}^{j-1} - h_t^j\right)^2$$
$$+\frac{1}{2}\left(\frac{\partial^2 u}{\partial c \partial h}\right)_{c=c_t^T, h=h_t^j}(c_t^N - c_t^T)\left(h_{t-1}^{j-1} - h_t^j\right)$$
$$+\beta\left(\frac{\partial E_t V_{t+1}^T}{\partial h} + \frac{\partial E_t O_{t+1}^2}{\partial h}\right)_{h=h_t^j}\left(h_{t-1}^{j-1} - h_t^j\right)$$
$$+\beta\left(\frac{\partial E_t V_{t+1}^T}{\partial W} + \frac{\partial E_t O_{t+1}^2}{\partial W}\right)_{W=W_t^T}(W_t^N - W_t^T)$$
$$+\frac{\beta}{2}\left(\frac{\partial^2 E_t V_{t+1}^T}{\partial h^2} + \frac{\partial^2 E_t O_{t+1}^2}{\partial h^2}\right)_{h=h_t^j}\left(h_{t-1}^{j-1} - h_t^j\right)^2$$
$$+\frac{\beta}{2}\left(\frac{\partial^2 E_t V_{t+1}^T}{\partial W^2} + \frac{\partial^2 E_t O_{t+1}^2}{\partial W^2}\right)_{W=W_t^T}(W_t^N - W_t^T)^2$$
$$+\frac{\beta}{2}\left(\frac{\partial^2 E_t V_{t+1}^T}{\partial W \partial h} + \frac{\partial^2 E_t O_{t+1}^2}{\partial W \partial h}\right)_{W=W_t^T, h=h_t^j}\left(h_{t-1}^{j-1} - h_t^j\right)(W_t^N - W_t^T)$$

According to the budget constraints, $c_t^N - c_t^T = -(W_t^N - W_t^T) - hs_t^T + TC_t^T$, where $hs_t^T = p_t^{j-1}h_{t-1}^{j-1} - p_t^j h_t^j$ and $TC_t^T = \delta p_t^j h_t^j$, and thus we have

$$D_t^2 = \left(-\frac{\partial u}{\partial c} + \beta\left(\frac{\partial E_t V_{t+1}^T}{\partial W} + \frac{\partial E_t O_{t+1}^2}{\partial W}\right)\right)(W_t^N - W_t^T)$$
$$+\frac{1}{2}\left(\frac{\partial^2 u}{\partial c^2} + \beta\left(\frac{\partial^2 E_t V_{t+1}^T}{\partial W^2} + \frac{\partial^2 E_t O_{t+1}^2}{\partial W^2}\right)\right)(W_t^N - W_t^T)^2$$
$$+\left(\frac{\partial u}{\partial h} + \beta\left(\frac{\partial E_t V_{t+1}^T}{\partial h} + \frac{\partial E_t O_{t+1}^2}{\partial h}\right)\right)\left(h_{t-1}^{j-1} - h_t^j\right)$$
$$+\frac{1}{2}\left(\frac{\partial^2 u}{\partial h^2} + \beta\left(\frac{\partial^2 E_t V_{t+1}^T}{\partial h^2} + \frac{\partial^2 E_t O_{t+1}^2}{\partial h^2}\right)\right)\left(h_{t-1}^{j-1} - h_t^j\right)^2$$
$$+\left\{\frac{1}{2}\left(\frac{\partial^2 u}{\partial c \partial h}\right)_{c=c_t^T, h=h_t^j}(c_t^N - c_t^T) + \frac{\beta}{2}\left(\frac{\partial^2 E_t V_{t+1}^T}{\partial W \partial h} + \frac{\partial^2 E_t O_{t+1}^2}{\partial W \partial h}\right)_{W=W_t^T, h=h_t^j}(W_t^N - W_t^T)\right\}\left(h_{t-1}^{j-1} - h_t^j\right)$$
$$+\left[\frac{1}{2}\frac{\partial^2 u}{\partial c^2}(hs_t - TC_t) + \frac{\partial^2 u}{\partial c^2}(W_t^N - W_t^T) - \frac{\partial u}{\partial c}\right](hs_t - TC_t)$$

After applying the first order condition and Envelop theorem to simplify the above equation, we have

$$D_t^2 = \frac{\partial u}{\partial c}(1+\delta)p_t^j\left(h_{t-1}^{j-1} - h_t^j\right) + \frac{1}{2}\frac{\partial^2 u}{\partial c \partial h}(1+\delta)p_t^j\left(h_{t-1}^{j-1} - h_t^j\right)^2$$
$$+\left[\frac{1}{2}\frac{\partial^2 u}{\partial c^2}(hs_t - TC_t) - \frac{\partial^2 u}{\partial c^2}(c_t^N - c_t^T + hs_t - TC_t) - \frac{1}{2}\frac{\partial^2 u}{\partial c \partial h}\left(h_{t-1}^{j-1} - h_t^j\right) - \frac{\partial u}{\partial c}\right](hs_t - TC_t)$$

and further

$$D_t^2 = -\frac{1}{2}\frac{\partial^2 u}{\partial c^2}(hs_t^T - TC_t^T)^2 - \left(\frac{\partial^2 u}{\partial c^2}(c_t^N - c_t^T) + \frac{1}{2}\frac{\partial^2 u}{\partial c \partial h}\left(h_{t-1}^{j-1} - h_t^j\right) + \frac{\partial u}{\partial c}\right)(hs_t^T - TC_t^T) + C^T$$

where $C^T = \frac{\partial u}{\partial c}(1+\delta)p_t^j\left(h_{t-1}^{j-1} - h_t^j\right) + \frac{1}{2}\frac{\partial^2 u}{\partial c \partial h}(1+\delta)p_t^j\left(h_{t-1}^{j-1} - h_t^j\right)^2$.

On the right hand side, it is a classical quadric equation of $(hs_t^T - TC_t^T)$ and there exists a pair of solutions (i.e. $b_{2,t}^l, b_{2,t}^u$ in Lemma 1.1) to this equation if and only if $\left(\frac{\partial^2 u}{\partial c^2}(c_t^N - c_t^T) + \frac{1}{2}\frac{\partial^2 u}{\partial c \partial h}\left(h_{t-1}^{j-1} - h_t^j\right) + \frac{\partial u}{\partial c}\right)^2 + 2\frac{\partial^2 u}{\partial c^2}C^T > 0$. As we know that $-\frac{1}{2}\frac{\partial^2 u}{\partial c^2} > 0$, $hs_t^T - TC_t^T \in \left[b_{2,t}^l, b_{2,t}^u\right]$ indicates $D_t^2 \leq 0$ indicating that the household should transact at time *t*.

Q.E.D.

Lemma 1.2 (*Optimal policy on home ownership*).

Proof.

Lemma 1.2 is different from Lemma 1.1in part. It results in a similar but different implication for household who rents a house. It is optimal for household not to launch home ownership if household benefits more from waiting as $D_t^1 > 0$.

$$D_t^1 \equiv V_t^R(W_{t-1}, h_{t-1}) - V_t^O(W_{t-1}, h_{t-1})$$
$$= u\left(c_t^R, h_{t-1}^{j-1}\right) + \beta E_t\left[V_{t+1}^O\left(W_t^R, h_{t-1}^{j-1}\right) + O_{t+1}^1\left(W_t^R, h_{t-1}^{j-1}\right)\right]$$
$$-\left\{u\left(c_t^O, h_t^j\right) + (f_t(SI) - 1)\pi^1 h_t^j + \beta E_t\left[V_{t+1}^T\left(W_t^O, h_t^j\right) + O_{t+1}^2\left(W_t^O, h_t^j\right)\right]\right\}$$

The situation is more complicated in the case of first ownership. Referring to value equation (9 and 11), it remarks that $V_{t+1}^T\left(W_t^O, h_t^j\right) = V_{t+1}^O\left(W_t^O, h_t^j\right)$ due to the right continuity of utility function. On the other hand, the two option values of waiting should be consistent if the transaction costs are the same and thus, $O_{t+1}^2\left(W_t^O, h_t^j\right) = O_{t+1}^1\left(W_t^O, h_t^j\right)$. Then we have

$$D_t^1 \equiv V_t^R(W_{t-1}, h_{t-1}) - V_t^O(W_{t-1}, h_{t-1})$$
$$= u\left(c_t^R, h_{t-1}^{j-1}\right) + \beta E_t\left[V_{t+1}^O\left(W_t^R, h_{t-1}^{j-1}\right) + O_{t+1}^1\left(W_t^R, h_{t-1}^{j-1}\right)\right]$$
$$-\left\{u\left(c_t^O, h_t^j\right) + \beta E_t\left[V_{t+1}^O\left(W_t^O, h_t^j\right) + O_{t+1}^1\left(W_t^O, h_t^j\right)\right]\right\}$$

where $u\left(c_t^O, h_t^j\right) = u\left(c_t^O, h_t^j\right) + (f(SI_t) - 1)\pi h_t^j$. Hence, we have

$$D_t^1 \equiv V_t^R(W_{t-1}, h_{t-1}) - V_t^O(W_{t-1}, h_{t-1})$$
$$= \left\{u\left(c_t^R, h_{t-1}^{j-1}\right) + (f(SI_t) - 1)\pi h_{t-1}^{j-1} - (f_t(SI) - 1)\pi h_{t-1}^{j-1}\right\} +$$
$$\beta E_t\left[V_{t+1}^O\left(W_t^R, h_{t-1}^{j-1}\right) + O_{t+1}^1\left(W_t^R, h_{t-1}^{j-1}\right)\right] - \left\{u\left(c_t^O, h_t^j\right) + \right.$$
$$\left.\beta E_t\left[V_{t+1}^O\left(W_t^O, h_t^j\right) + O_{t+1}^1\left(W_t^O, h_t^j\right)\right]\right\}$$

By using second order Tyler expansion at point $\left(c_t^O, W_t^O, h_t^j\right)$, we have

$$D_t^1 = \left(\frac{\partial u}{\partial c}\right)_{c=c_t^O}(c_t^R - c_t^O) + \frac{1}{2}\left(\frac{\partial^2 u}{\partial c^2}\right)_{c=c_t^O}(c_t^R - c_t^O)^2$$
$$+ \left(\frac{\partial u}{\partial h}\right)_{h=h_t^j}\left(h_{t-1}^{j-1} - h_t^j\right) + \frac{1}{2}\left(\frac{\partial^2 u}{\partial h^2}\right)_{h=h_t^j}\left(h_{t-1}^{j-1} - h_t^j\right)^2$$

$$+\frac{1}{2}\left(\frac{\partial^2 u}{\partial c\partial h}\right)_{c=c_t^O,h=h_t^j}(c_t^R - c_t^O)\left(h_{t-1}^{j-1} - h_t^j\right)$$

$$-(f(SI_t) - 1)\pi h_{t-1}^{j-1}$$

$$+\beta\left(\frac{\partial E_t V_{t+1}^O}{\partial h} + \frac{\partial E_t O_{t+1}^1}{\partial h}\right)_{h=h_t^j}\left(h_{t-1}^{j-1} - h_t^j\right)$$

$$+\beta\left(\frac{\partial E_t V_{t+1}^O}{\partial W} + \frac{\partial E_t O_{t+1}^1}{\partial W}\right)_{W=W_t^O}(W_t^R - W_t^O)$$

$$+\frac{\beta}{2}\left(\frac{\partial^2 E_t V_{t+1}^O}{\partial h^2} + \frac{\partial^2 E_t O_{t+1}^1}{\partial h^2}\right)_{h=h_t^j}\left(h_{t-1}^{j-1} - h_t^j\right)^2$$

$$+\frac{\beta}{2}\left(\frac{\partial^2 E_t V_{t+1}^O}{\partial W^2} + \frac{\partial^2 E_t O_{t+1}^1}{\partial W^2}\right)_{W=W_t^O}(W_t^R - W_t^O)^2$$

$$+\frac{\beta}{2}\left(\frac{\partial^2 E_t V_{t+1}^O}{\partial W\partial h} + \frac{\partial^2 E_t O_{t+1}^1}{\partial W\partial h}\right)_{W=W_t^O,h=h_t^j}\left(h_{t-1}^{j-1} - h_t^j\right)(W_t^R - W_t^O)$$

According to the budget constraints, $c_t^R - c_t^O = -(W_t^R - W_t^O) - hs_t^O + TC_t^O$, where $hs_t^O = -p_t^j h_t^j$ and $TC_t^O = \delta p_t^j h_t^j$. Thus we have

$$D_t^1 = \left(-\frac{\partial u}{\partial c} + \beta\left(\frac{\partial E_t V_{t+1}^O}{\partial W} + \frac{\partial E_t O_{t+1}^1}{\partial W}\right)\right)(W_t^R - W_t^O)$$
$$+\frac{1}{2}\left(\frac{\partial^2 u}{\partial c^2} + \beta\left(\frac{\partial^2 E_t V_{t+1}^O}{\partial W^2} + \frac{\partial^2 E_t O_{t+1}^1}{\partial W^2}\right)\right)(W_t^R - W_t^O)^2$$
$$+\left(\frac{\partial u}{\partial h} + \beta\left(\frac{\partial E_t V_{t+1}^O}{\partial h} + \frac{\partial E_t O_{t+1}^1}{\partial h}\right)\right)\left(h_{t-1}^{j-1} - h_t^j\right)$$
$$+\frac{1}{2}\left(\frac{\partial^2 u}{\partial h^2} + \beta\left(\frac{\partial^2 E_t V_{t+1}^O}{\partial h^2} + \frac{\partial^2 E_t O_{t+1}^1}{\partial h^2}\right)\right)\left(h_{t-1}^{j-1} - h_t^j\right)^2$$
$$+\left\{\frac{1}{2}\left(\frac{\partial^2 u}{\partial c\partial h}\right)_{c=c_t^O,h=h_t^j}(c_t^R - c_t^O) + \frac{\beta}{2}\left(\frac{\partial^2 E_t V_{t+1}^O}{\partial W\partial h} + \frac{\partial^2 E_t O_{t+1}^1}{\partial W\partial h}\right)_{W=W_t^O,h=h_t^j}(W_t^R - W_t^O)\right\}\left(h_{t-1}^{j-1} - h_t^j\right)$$
$$+\left[\frac{1}{2}\frac{\partial^2 u}{\partial c^2}(hs_t^O - TC_t^O) + \frac{\partial^2 u}{\partial c^2}(W_t^R - W_t^O) - \frac{\partial u}{\partial c}\right](hs_t^O - TC_t^O) - (f(SI_t) - 1)\pi h_{t-1}^{j-1}$$

After applying the first order condition and Envelop theorem to simplify the above equation, we have

$$D_t^1 = \frac{\partial u}{\partial c}(1+\delta)p_t^j\left(h_{t-1}^{j-1} - h_t^j\right) + \frac{1}{2}\frac{\partial^2 u}{\partial c\partial h}(1+\delta)p_t^j\left(h_{t-1}^{j-1} - h_t^j\right)^2$$

$$-(f(SI_t) - 1)\pi h_{t-1}^{j-1}$$

$+\left[\frac{1}{2}\frac{\partial^2 u}{\partial c^2}(hs_t^O - TC_t^O) - \frac{\partial^2 u}{\partial c^2}(c_t^R - c_t^O + hs_t^O - TC_t^O) - \frac{1}{2}\frac{\partial^2 u}{\partial c \partial h}\left(h_{t-1}^{j-1} - h_t^j\right) - \frac{\partial u}{\partial c}\right](hs_t^O - TC_t^O)$

and further

$$D_t^1 = -\frac{1}{2}\frac{\partial^2 u}{\partial c^2}(hs_t^O - TC_t^O)^2 - \left(\frac{\partial^2 u}{\partial c^2}(c_t^R - c_t^O) - \frac{1}{2}\frac{\partial^2 u}{\partial c \partial h}\left(h_{t-1}^{j-1} - h_t^j\right) + \frac{\partial u}{\partial c}\right)(hs_t^O - TC_t^O) + C^O$$

where $C^O = \frac{\partial u}{\partial c}(1+\delta)p_t^j\left(h_{t-1}^{j-1} - h_t^j\right) + \frac{1}{2}\frac{\partial^2 u}{\partial c \partial h}(1+\delta)p_t^j\left(h_{t-1}^{j-1} - h_t^j\right)^2 - (f(SI_t) - 1)\pi h_{t-1}^{j-1}$.

On the right hand side, it is a classical quadric equation of $(hs_t^O - TC_t^O)$ and there exists a pair of solutions (i.e. $b_{1,t}^l, b_{1,t}^u$ in Lemma 1.2) to this equation if and only if $\left(\frac{\partial^2 u}{\partial c^2}(c_t^N - c_t^T) + \frac{\partial u}{\partial c}\right)^2 + 2\frac{\partial^2 u}{\partial c^2}C^O > 0$. As $-\frac{1}{2}\frac{\partial^2 u}{\partial c^2} > 0$, $hs_t^O - TC_t^O \in \left[b_{1,t}^l, b_{1,t}^u\right]$ indicates $D_t^1 \leq 0$ indicating that the household should transact at time *t*.

Q.E.D.

Proposition 1 (*Effect of sentiment on housing transaction*).

Proof.

Recall two quadric equations $D_t^1 = 0$ and $D_t^2 = 0$. Here define $A = -\frac{1}{2}\frac{\partial^2 u}{\partial c^2}$ and $B = -\left(\frac{\partial^2 u}{\partial c^2}\Delta c + \frac{1}{2}\frac{\partial^2 u}{\partial c \partial h}\Delta h + \frac{\partial u}{\partial c}\right)$, where $\Delta c = c_t^R - c_t^O$ for $D_t^1 = 0$ or $\Delta c = c_t^N - c_t^T$ for $D_t^2 = 0$ and $\Delta h = \left(h_{t-1}^{j-1} - h_t^j\right)$. Thus we have $b_{i,t}^u - b_{i,t}^l = \frac{\sqrt{B^2 - 4AC}}{A}$.

For home owner, recall the utility function (7) and some derivatives are shown as

$$\frac{\partial^2 u}{\partial c^2} = \frac{\partial u}{\partial c}\frac{\theta_t \gamma - \theta_t - \gamma}{c_t} \text{ and } \frac{\partial^2 u}{\partial c \partial h} = \frac{\partial u}{\partial c}\frac{\theta_t(1-\gamma)}{h_t}$$

Substitute the above equations into $D_t^2 = 0$ and we have

$-\frac{1}{2}\frac{\partial u}{\partial c}\frac{\theta_t\gamma-\theta_t-\gamma}{c_t}(hs_t^T-TC_t^T)^2-\left(\frac{\partial u}{\partial c}\frac{\theta_t\gamma-\theta_t-\gamma}{c_t}(c_t^N-c_t^T)+\frac{1}{2}\frac{\partial u}{\partial c}\frac{\theta_t(1-\gamma)}{h_t}+\right.$

$\left.\frac{\partial u}{\partial c}\right)(hs_t^T-TC_t^T)+C^T=0$

where $C^T=\frac{\partial u}{\partial c}(1+\delta)p_t^j\left(h_{t-1}^{j-1}-h_t^j\right)+\frac{1}{2}\frac{\partial u}{\partial c}\frac{\theta_t(1-\gamma)}{h_t}(1+\delta)p_t^j\left(h_{t-1}^{j-1}-\right.$

$\left.h_t^j\right)^2$.

After simplification in D_t^2 and we have new

$D_t^2=-\frac{1}{2}\frac{\theta_t\gamma-\theta_t-\gamma}{c_t}(hs_t^T-TC_t^T)^2-\left(\frac{\theta_t\gamma-\theta_t-\gamma}{c_t}(c_t^N-c_t^T)+\frac{1}{2}\frac{\theta_t(1-\gamma)}{h_t}+\right.$

$\left.1\right)(hs_t^T-TC_t^T)+C^T=0$

Where new $C^T=(1+\delta)p_t^j\left(h_{t-1}^{j-1}-h_t^j\right)+\frac{1}{2}\frac{\theta_t(1-\gamma)}{h_t}(1+\delta)p_t^j\left(h_{t-1}^{j-1}-\right.$

$\left.h_t^j\right)^2$. As only $\frac{\partial u}{\partial c}$ involves sentiment term, the new $D_t^2=0$ does not involve sentiment term anymore and thus the effect of sentiment on the timing of housing transaction by home owner is trivial.

On the other hand, situation is different for tenant household. Recall the utility function (6) that is $u(h_t,c_t)=u(h_t,c_t,\theta_t)+(f(SI_t)-1)\pi h_t^j$ but $\frac{\partial u}{\partial c}$ does not involve sentiment term. As such, effect of sentiment emerges in D_t^1 only through C^O. Hence we have

$\frac{\partial(b_{1,t}^u-b_{1,t}^l)}{\partial C}=\left(\frac{\sqrt{B^2-4AC}}{2A}\right)_C'=\frac{-1}{\sqrt{B^2-4AC}}<0$ and $\frac{\partial C^O}{\partial SI}=-\pi h_{t-1}^{j-1}f'<0$

and thus $\frac{\partial(b_{1,t}^u-b_{1,t}^l)}{\partial SI}=\frac{\partial(b_{1,t}^u-b_{1,t}^l)}{\partial C}\frac{\partial C}{\partial SI}>0$. It indicates that sentiment positively affects the likelihood of first ownership.

Q.E.D.

Lemma 2 (*Effect of sentiment on housing demand*)

Proof.

Based on the FOCs, marginal rate of substitution (*MRS*) is

$$MRS = -\frac{\frac{\partial u_t}{\partial h_t}}{\frac{\partial u_t}{\partial c_t}} = \frac{\beta E_t\left(\frac{\partial V_{t+1}^T}{\partial h_t}+\frac{\partial O_{t+1}^2}{\partial h_t}\right)+\beta E_t\left(\frac{\partial V_{t+1}^T}{\partial W_t}+\frac{\partial O_{t+1}^2}{\partial W_t}\right)\frac{\partial W_t}{\partial h_t}}{\frac{\partial u_t}{\partial c_t}} =$$

$$\frac{\beta E_t\left(\frac{\partial V_{t+1}^T}{\partial h_t}+\frac{\partial O_{t+1}^2}{\partial h_t}\right)-\frac{\partial u_t}{\partial c_t}(1+\delta)p_t}{\frac{\partial u_t}{\partial c_t}}$$

Note that when applying the Envelop Theorem with regard to h_{t-1}, we have

$$\frac{\partial V_t^T}{\partial h_{t-1}} = \beta E_t\left(\frac{\partial V_{t+1}^T}{\partial W_t} + \frac{\partial O_{t+1}^2}{\partial W_t}\right)\frac{\partial W_t}{\partial h_{t-1}}$$

By combining with FOC of of c_t, we have

$$\frac{\partial V_t^T}{\partial h_{t-1}} = \frac{\partial u_t}{\partial c_t}\frac{\partial W_t}{\partial h_{t-1}}$$

and updating the above equation one period and take time *t* expectation, we have

$$E_t\frac{\partial V_{t+1}^T}{\partial h_t} = \frac{\partial u_{t+1}}{\partial c_{t+1}}E_t\frac{\partial W_{t+1}}{\partial h_t} = \frac{\partial u_{t+1}}{\partial c_{t+1}}E_t p_{t+1}^j$$

Substituting the above equation into *MRS*, we finally have

$$MRS = -\frac{\frac{\partial u_t}{\partial h_t}}{\frac{\partial u_t}{\partial c_t}} = \frac{\beta E_t\left(\frac{\partial V_{t+1}^T}{\partial h_t}+\frac{\partial O_{t+1}^2}{\partial h_t}\right)-\frac{\partial u_t}{\partial c_t}(1+\delta)p_t}{\frac{\partial u_t}{\partial c_t}} = \frac{\beta\frac{\partial u_{t+1}}{\partial c_{t+1}}}{\frac{\partial u_t}{\partial c_t}}E_t p_{t+1}^j - (1+\delta)p_t + \frac{\beta E_t\frac{\partial O_{t+1}^2}{\partial h_t}}{\frac{\partial u_t}{\partial c_t}}$$

or $MRS = m_{t+1}E_t p_{t+1}^j - (1+\delta)p_t + \frac{\beta E_t\frac{\partial O_{t+1}^2}{\partial h_t}}{\frac{\partial u_t}{\partial c_t}}$ where $m_{t+1} = \frac{\beta\frac{\partial u_{t+1}}{\partial c_{t+1}}}{\frac{\partial u_t}{\partial c_t}}$ is a stochastic discount factor.

Q.E.D.

Proposition 2 (*Effect of sentiment on housing demand*)

Proof.

Differentiate *MRS* with regard to sentiment factor SI_t,

$$\frac{\partial MRS}{\partial SI_t} = \frac{\partial m_{t+1}}{\partial SI_t}E_t p_{t+1}^j + m_{t+1}\frac{\partial E_t p_{t+1}^j}{\partial SI_t} + \beta E_t\frac{\partial O_{t+1}^2}{\partial h_t}\left(\frac{\partial u_t}{\partial c_t}^{-1}\right)'_{SI_t}$$

Note that $u_t = u_{SI} + (f(SI_t) - 1)\pi\left(h_t^j - h_{t-1}^{j-1}\right)$ and $\frac{\partial u_t}{\partial c_t} = \frac{\partial u_{SI}}{\partial c_t} = f(SI_t)\frac{\partial u}{\partial c_t}$ where $u = u\left(h_t^j, c_t, \theta_t\right)$. Then we have

$\left(\frac{\partial u_t}{\partial c_t}^{-1}\right)'_{SI_t} = \left(\frac{\partial u}{\partial c_t}\right)^{-1}\frac{-f'}{f^2}$, $\frac{\partial m_{t+1}}{\partial SI_t} = \beta\frac{\partial u_{t+1}}{\partial c_{t+1}}\left(\frac{\partial u_t}{\partial c_t}^{-1}\right)'_{SI_t}$ and $\frac{\partial E_t p_{t+1}^j}{\partial SI_t} = \mu^j g'_{SI_t}$

Hence we have

$$\frac{\partial MRS}{\partial SI_t} = \beta\frac{\partial u_{t+1}}{\partial c_{t+1}}\left(\frac{\partial u}{\partial c_t}\right)^{-1}\frac{-f'}{f^2}E_t p_{t+1}^j + m_{t+1}\mu^j g'_{SI_t} + \beta E_t\frac{\partial O_{t+1}^2}{\partial h_t}\left(\frac{\partial u}{\partial c_t}\right)^{-1}\frac{-f'}{f^2}$$

$$= \beta\frac{\partial u_{t+1}}{\partial c_{t+1}}\left(\frac{\partial u}{\partial c_t}\right)^{-1}\frac{-f'}{f^2}E_t p_{t+1}^j + m_{t+1}\mu^j g'_{SI_t} + \beta E_t\frac{\partial O_{t+1}^2}{\partial h_t}\left(\frac{\partial u}{\partial c_t}\right)^{-1}\frac{-f'}{f^2}$$

$$= m_{t+1}\left\{\mu^j g' - \beta\left(E_t p_{t+1}^j + E_t\frac{\partial O_{t+1}^2}{\partial h_t}\left(\frac{\partial u_{t+1}}{\partial c_{t+1}}\right)^{-1}\right)\frac{f'}{f}\right\}$$

Note that $m_{t+1}, \mu^j, E_t p_{t+1}^j > 0$ and $\frac{\partial O_{t+1}^2}{\partial h_t} > 0$. Since all the terms in the above equation are positive, the sign of $\frac{\partial MRS}{\partial SI_t}$ depends on $g' - K(lnf)'$, where $K = \frac{\beta}{\mu^j}\left(E_t p_{t+1}^j + E_t\frac{\partial O_{t+1}^2}{\partial h_t}\left(\frac{\partial u_{t+1}}{\partial c_{t+1}}\right)^{-1}\right) > 0$.

Thus, the relationship between sentiment functions f' and g' determines the sign of effect of sentiment on *MRS*. It indicates that $\frac{\partial MRS}{\partial SI_t} > 0$ if $SI_t \in \{SI_t : g' > K(lnf)'\}$.

Q.E.D.

Simulation parameterization

Utility function and wealth constraint: $\beta = 0.9,\ \gamma = 3,\ \delta = 0.1,\ T = 30$.

Household's initial states: $h_0 = 50$, $W_0 = 0$ in case 1 and $h_0 = 50$, $W_0 = 4000$ in case 2.

Housing taste function: $hst_t = \frac{1}{500}[450 - (t - 20)^2]$ where t = 1 …30.

Income function: $\mu_{inc} = 28,\ \eta = 0.9,\ e_t \sim N(0,1)$.

Price function: $\boldsymbol{\mu_p} = \begin{pmatrix} 12 \\ 0.2 \end{pmatrix}$, $A = \begin{pmatrix} 0.91 & 0 \\ 0 & 0.91 \end{pmatrix}$, and

$$e_t \sim N\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}0.24 & 0\\0 & 0.04\end{pmatrix}\right).$$

Sentiment function: linear correlation $\omega = 0.9$

For positive translatory model, unit of translation on sentiment = 0.05.

Chapter

7 CONCLUSION

This chapter presents a summary of the major research findings and highlights the contributions. It also appreciates the limitations of this study, followed by the future directions for research.

7.1 Summary of the Major Findings

This study aimed to investigate the effects of sentiment in the housing market and to explore the necessity of incorporating sentiment factors into standard economic models and analyses. By successfully accomplishing the four research objectivities established in Chapter 1, the major findings in Chapter 3-6 are summarised as follows.

6.1.1 Findings in Chapter 3

To investigate the role of market sentiment in the housing market, a sentiment index was developed in the third study to capture investor behaviour on aggregate. The index was compiled based on transaction intensity. Assume that some participants are driven by sentiment in the housing market. Sentiment-based investors would wait for less time before reaching a deal, compared with rational investors. That is, sentimental investors will take action more quickly.

The sentiment index essentially comprises two indicators arising from two aggregate probabilities. These two probabilities are derived from positive and negative sentiment-based transactions respectively. The index reports monthly measurement on market sentiment.

In empirical analysis, by using elaborate transaction records from 1991 to 2011 in Hong Kong (more than two million registrations which covered almost all sale and purchase agreements for private residential units registered in the Land Registry), the sentiment index was established. Generally, evidence shows that participants in housing markets are more likely to be sentiment-driven. On the other hand, the study suggests that, in the trading process, there is a delaying effect on the expected waiting time (duration) from buying to selling.

6.1.2 Findings in Chapter 4

In Chapter 5, the study explored the macro effects of sentiment on the housing market in two stages. The empirical study paid attention to the housing market in Hong Kong over two decades, from 1993-2012.

The first purpose of this chapter is to examine the power of sentiment in predicting three market indicators (price, rent and transaction volume) in the housing market. By using an advanced causality analysis called Integrated Renormalised Partial Directed Coherence (IRPDC), the study targeted at the private housing market. In the short-run, market sentiment is a prominent indicator of forecasting prices and trading volumes. Compared to other factors, sentiment has an overwhelming power to predict housing prices. This implies that the private housing market in Hong Kong is significantly affected by sentiment. The findings also show that rent is a significant predictor of sentiment. In addition, the one-way cycle of price, rent, and sentiment implies that sentiment is an indicating factor in the indirect linkage of rent to price. Moreover, the asymmetry of causality strength between the four indicators can be verified by the IRPDC method. The analytical study fills the gap of knowledge, as Granger causality – the most common approach – is incapable of handling causality strength.

In the second stage, this study investigated the long-run relationships of market sentiment in the transaction market and the rental market. The findings show that sentiment has a significant effect on housing prices and rents, but plays different roles within the two separate market sectors. Changes in house prices are attributed in part to sentiment, while rent is affected by the lagged term of sentiment. The findings provide some indirect evidence that supports the implication of a one-way cycle in causality investigation. Such new findings in turn contribute to the knowledge of how sentiment affects the performance of housing markets.

6.1.3 Findings in Chapter 5

In Chapter 5, an option-based dynamic model was developed to address the first research objective. It showed, by both mathematical derivation and numerical analysis, that the expected waiting time to invest/develop exhibits a U-shape pattern against sentiment; and the turning point of the U-shape pattern is more likely to appear earlier in projects with longer development periods. In other words, an increase in sentiment shortens the wait to develop at first and then reverse to delay the development with the rise in sentiment. This practical implication can benefit the developer in optimal decision making. In addition, as market sentiment intensifies, the optimal density and project value declines, ceteris paribus. Standard economic models should be revised to take into account this behavioural factor.

The policy implications arising from these results are worth discussing. Developers with different levels of sentiment plans project development differently. With a higher level of sentiment, developers tend to reduce supply by developing projects of lower densities or even delaying the completion of a project. This indicates that policies designed to increase housing supply by offering more land have limited effects during the high sentiment period than otherwise.

6.1.4 Findings in Chapter 6

In Chapter 6, a life-cycle model was established to capture the different roles of sentiment in optimal housing choices for households. The theoretical solutions to the model shed light on the effects of sentiment in households' housing demand. The main theoretical implications derived from optimal solutions are two-fold: (i) sentiment positively affects the likelihood of first-time home ownership while barely affects the likelihood of transacting; and (ii) the effect of sentiment on the marginal rate of substitution (MRS) between housing and non-housing consumption varies from negative to positive with an increase in sentiment level. More specifically, MRS decreases with sentiment when sentiment is below the threshold (or the turning point), and then increases with sentiment when sentiment exceeds the threshold.

Simulation was conducted 500 times to numerically verify theoretical implications arising from the model and explore practical implications. It is worth noting that the life-cycle utility in the benchmark model (without sentiment effect) is higher than that in sentiment models. This gap of life-cycle utility indicates that the benefit from households' sentiment-driven hedging is overwhelmed by the premium of uncertainty arising from the macro effects of sentiment on the market. It also implies that sentiment induces households to make sub-optimal choices. In addition, the comparison of life-cycle utility between the standard sentiment model and the positive-translatory sentiment model (which has a higher sentiment in every period) suggests that the gap becomes larger if the household is consistently more affirmative than average. That is, optimistic agent always makes a less optimal decision.

7.2 Contribution

This study has offered theoretical and practical contributions, by virtue of a better understanding of the effects of sentiment on the housing market it provided. Compared to classical studies (e.g. Clayton et al., 2008; Ling et al., 2014) into the effects of sentiment on the real estate market, this study also sheds light on how the roles of sentiment differ at both macro and micro levels. Specific contributions of this research are four-fold:

(1) A novel approach is developed to establish a series of indices for gauging market sentiment, as the approach is superior to traditional regression analysis (such as principal component analysis in Baker and Wurgler, 2006). Such index can reflect the trend of aggregate investors' perception and offer useful implications which may provide assistance to addressing the abovementioned research objectives, such as the investigation of the effect of sentiment at the micro level in an empirical study.

The index is expected to contribute both theoretically and practically. From a theoretical standpoint, it provides a new approach to measuring sentiment in housing markets from a different perspective. This transaction-based index sets up a good example to demonstrate how to dig out behavioural information from behaviour patterns. Besides, it can help in better understanding the pricing of real estate as an asset, and the role of market sentiment in explaining property prices (and returns). From a practical standpoint, it could be used as a reference for possible future price changes in the residential sub-markets. In other words, it sheds some light for various stakeholders in different aspects. For (potential homebuyers as) investors the indices could be used as a reference, assisting their portfolio selections for the purpose of maximising their risk-adjusted returns. For (potential homebuyers as) users, these indices could assist them in their decisions on the timing when to enter the market. In addition, this newly established index could benefit studies at macro level (such as the study carried out in Chapter 4) on the movements of housing markets.

(2) The macro study upgrades the knowledge regarding the aggregate effect of sentiment on the housing market. The implications of this study benefit investors not only by confirming the predictability of sentiment on other market indicators, but also by exploring the role of sentiment in long-term market movement. In addition, this study sheds light on the different roles of sentiment in rental and transaction markets. On the other hand, this study can assist relevant regulatory authorities. It may serve as a good basis and informative reference-point for the monitoring of the housing market. This macro analysis offers an important insight into market mechanisms in the long-run and the implications of this study help policy makers stabilise and improve the functioning of the housing market.

(3) An optimal development model is established in association with sentiment factor to provide a more accurate approach to project valuation for developers. This model can be applied in value appraisal, to which an investment decision on residential development or re-development projects can be referred. In Hong Kong, applications of this model include, but are not limited to: optimal development proposed on a parcel of land, re-development of an existing project, as well as re-development cases initiated by the Urban Renewal Authority (URA). By incorporating market sentiment, the model can offer a better understanding of the effect of market and price uncertainties, in order to assist in making optimal choices (e.g. density of project and timing of investment) in project development. On the other hand, the policy implications derived from this model are helpful to policy making regarding housing market supply. Policy makers should

adjust housing market regulations according to different levels of sentiment. Overall, the analysis based on this model highlights the importance of considering market sentiment in residential development decisions, and illustrates the complexity of project valuation prior to the start of a project.

(4) A life-cycle model is established to investigate the role of sentiment in housing choices and, further, to show how the sentiment factor affects the demand for housing service. This model incorporates two decision-making processes, i.e. housing tenure choice and housing transaction choice, into one integrated framework. Compared to the framework in Han (2008), this model captures the full path along which a household decides its housing consumption over a life cycle. This can benefit future research into housing tenure choices. Meanwhile, the model helps distinguish the utility of housing investment from the utility of housing consumption. The implications of the model and its simulations enrich the knowledge of how housing demand evolves with sentiment in the housing market.

7.3 Limitations and Future Research Directions

The scope of this study is limited to the private housing sector of the real estate market. The implications arising from this study do not befit the public housing system. But it is relatively easy to apply the research method and analysis developed in this study to other sectors of the real estate market, such as the office market. These issues could be considered by future research.

This study provided fundamental research to explore the effects of sentiment and thus the theoretical models only consider general situations for representative agents. Perhaps in future, this study could be expanded to investigate the impacts of external shocks stemming from sudden events and policies within the framework developed in this study.

Theoretical analysis has been conducted in the first two chapters. The analytical models built are novel and ground-breaking, but are only numerical. It would be preferable in future studies to have such models to be calibrated by appropriate historical or census data, if they are available.

Deficiencies in public data could be another limitation to this research. This can be reflected in two aspects: (1) Availability of data: some of the public data shows the predicament in which only partial information/data for recent years (e.g. last decade) are available. Although the data are publicly provided by relevant authorities, it is difficult to request the full length of the data. (2) Accuracy of data: some public data might be adjusted a few years after they have been published (commonly 2-3 years). This case mainly applies to macroeconomic data, such as GDP, household income, and inflation rates. Certainly, theoretical analysis does not require any data and thus would not be impeded by those obstacles. Yet, such shortcomings may curb simulations and empirical studies.

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