DESIGN OF URBAN RAIL CORRIDOR OVER TIME: FOR CITIES WITH HIGH POPULATION DENSITIES AND GROWTH UNCERTAINTIES

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Certificate of originality

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(Signed) ________________

(Name of student) ______Liu Ding___

I would like to dedicate this thesis to my loving parents and respectable supervisor Prof. W.H.K. Lam.

Abstract

To alleviate traffic congestion problems, urban rail transit lines are being built continuously in many metropolitan cities in China. These Chinese cities, like Hong Kong and Shanghai, have significantly higher population densities in urban areas, in contrast with European and American cities.

Traditional models for planning of urban rail transit lines are mainly static models and usually developed for European and American cities with low population densities. The research presented in this thesis aims to propose analytical models of optimising urban rail transit lines over time, for cities with high population densities.

This thesis contributes to the literature of urban rail transit lines design problems in the following several aspects but with focus on a linear urban transportation corridor for a monocentric city.

Firstly, an optimisation model is proposed to investigate over-year interaction between endogenous population densities and financial performance of a candidate rail transit line in Chapter 3. The proposed optimisation model is a bi-level programming model, with a lower level problem formulated as a user equilibrium model, and an upper level problem formulated as an urban rail transit line design model over time.

Traditional models for design of urban rail transit lines were developed with objectives of travel time or cost minimisation for a particular design year, while the planning data of the design year is given and fixed. By contrast, the optimisation objective of the mathematical programming model proposed in Chapter 3 is the social welfare maximisation for a period of time horizon.

It was found in Chapter 3 that a lack of integration of short-term decision variables, headway and fare, may result in excessive investments for the long-term rail construction, namely the increase of optimal rail line length. More population were attracted to live in vicinity of the candidate transit line while rail service was supplied. With extension of rail service over years, more population chose to move gradually from residential locations of CBD to residential locations of suburban areas. The candidate rail transit line can make

population densities more decentralised over time, namely more population distributed at residential locations of suburban areas.
Secondly, implementation adaptability of a candidate rail transit line is explored over years in Chapter 4. Adaptability is defined as the ability of the system to adapt to external changes, while maintaining satisfactory system performance. Implementation adaptability gives authorities and/or operators to fast-track or defer the future investment on the candidate rail transit line for several years, if necessary.

To obtain an adaptable candidate rail transit line suitable to accommodate external changes over the years, it is required that the candidate rail transit line can be fast-tracked or deferred several years. For example, if the total supply cost of the candidate rail transit line is lower than that predicted in the previous feasible study, the candidate rail transit line can be fast-tracked accordingly.

Nonetheless, less attention was given to examine the implementation adaptability of urban rail transit line in the literature. This is mainly due to the fact that most of the traditional models were static models and developed for a particular design year. To address the implementation adaptability issue for design of the candidate rail transit line over time, three alternatives are explored in Chapter 4: fast-tracking several years, deferring several years and do-nothing-alternative (DNA).

The analytical solutions of the optimal years, to be fast-tracked or deferred, are obtained by the proposed model in Chapter 4. Sensitivity tests for the optimal project start time are conducted with respect to the yearly variation of the total population and annual interest rate.

Thirdly, the effects of spatial and temporal correlation of population densities on the design of an urban rail transit line over years are investigated in Chapter 5. Population densities at different residential locations along a candidate rail transit line are correlated. Population growth in one location can positively influence the population growth in adjacent location. For instance, for a given total population along a linear rail transportation corridor, more households choose the central business district (CBD) to live, leaving fewer for suburban communities and new towns in the outlying locations. In this case, a negative spatial correlation exists between population densities at the CBD, and the suburban communities and new towns.

A closed-form mathematical programming model is proposed in Chapter 5 to investigate the effects of spatial and temporal correlation of population densities on the design of the candidate rail transit line. In the proposed model, the optimisation objective is the budget social welfare maximisation. The decision variables include rail line length, rail station number and project start time of the candidate rail transit line. The analytical solutions of the above decision variables are derived explicitly.

Finally, choices of alternative travel modes for households are incorporated in Chapter 6 to investigate the effects of households' prospect theory based travel behaviour over time for design of rail transit line in the liner transportation corridor. The available travel modes consist of car, bus, rail and park-and-ride.

The prospect theory based analytical mathematical model proposed in Chapter 6 is proved to be convex, and the existence and uniqueness of solutions are guaranteed. It was found that the population density within the linear transportation corridor was closely correlated with the modal split results of households and the financial performance of the candidate rail transit line. To certain extent, the park-and-ride may not be suitable for cities with high population densities such as Hong Kong.

This research appears to be the first devoted exclusively to the topic of using analytical models for investigation of urban rail design problems over time, with particular attention on the effects of population density by location along the linear transportation corridor over years. The transportation authorities could make use of the proposed models for proving useful insights in order to give the guidelines and/or strategies for design of the urban rail line over time in the urban area particularly in the corridor with potential for high population density but uncertainty in population growth in the future.

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Notations

$B(\text{Km})$: The length of the rail corridor.

$BFT(\text{HK\$})$: The benefits at full time of the planning time horizon.

$c(\text{HK\$})$: The variable operation cost for rail server supply of each passenger.

$c(x,t)(\text{HK\$})$: Yearly (nominal) generalised travel cost for travelling from residential location x to the central-business-district (CBD) in year t.

$c(x_1,x_2,t)(\text{HK\$})$: The generalised travel cost by rail from residential location x_1 to x_2 in year t.

$c_{i,x,n,t,\tau}(\text{HK\$})$: Households' actual generalised travel cost for arriving at the CBD from location x by travel mode i at departure time t in the n^{th} day of the τ period, and $i \in \{r,c,b,P\&R\}$, representing rail, car, bus and park-and-ride (P&R) mode respectively. $t \in \Gamma$, and Γ represents the morning peak hour, for example 8:30-10:00.

$c_{WIP,n,t,\tau}(\text{HK\$})$: Households' reference point to decide whether the generalised travel cost is high or low in the n^{th} day of the τ^{th} period, which is called as the households' willingness-to-pay (WIP) on the generalised travel cost.

$\hat{c}_{i,x,n,t,\tau}(\text{HK\$})$: Households' perceived generalised travel cost for arriving at the CBD from location x by travel mode i at departure time t in the n^{th} day of the τ period.

$C(\text{HK\$})$: The total supply cost of the candidate rail transit line.

$C_r(\text{HK\$})$: The daily construction cost of rail line.

$C_s(\text{HK\$})$: The daily fixed construction cost of each rail station.

$C_t(\text{HK\$})$: The fixed procurement cost of each train.

$C^t(\text{HK\$})$: The discount construction and operation cost in year t.

$C_{o,t}(\text{HK\$})$: The discount operation cost in year t.

$C_{L,t}(\text{HK\$})$: The discount construction cost of rail line in year t.

$C_{s,t}(\text{HK\$})$: The discount construction cost of rail stations in year t.

$CFT(\text{HK\$})$: The capital cost of the candidate rail transit line at full time of the planning and time horizon.

$CSR(\text{HK\$})$: The consumer surplus of the rail project.

CV_p: A standardised measure of dispersion of the probability distribution or frequency distribution of population density.

$D_s(t)(\text{Km})$: Average station spacing of the candidate rail transit line in year t. As $s=1$, D_1 represents the rail length of the candidate rail transit line.

$DCS_t(\text{HK\$})$: The discounted consumer surplus in year t.

$DP_t(\text{HK\$})$: The discounted profit of rail transit line in year t.

$E_s(\text{Km})$: The distance between the water threshold line and rail station s.

$E(PR)(\text{HK\$})$: The expected profit of rail operator.

$E(SW)(\text{HK\$})$: The expected social welfare.

$f_0(\text{HK\$})$: Fixed component of fare for using the rail service.

$f_1(\text{HK\$/km})$: The variable fare component per kilometre in year t.

$f_r(\text{HK\$})$: Rail fare.

$f_i^t(\text{HK\$})$: The distance-based fare of rail service for each passenger.

F: The fleet size of trains.

F^t: The fleet size of trains in year t.

$\Delta F^t(\text{HK\$})$: The increased fleet size of train in year t.

$h(q(x,t))(\text{Minutes})$: Headway of train during morning peak-hour in year t.

$h(x,t)/h_i(\text{HK\$})$: The consumption of housing in residential location x/i in year t.

$H^t(\text{Minutes})$: The average headway of trains in year t.

$H(x,t)(\text{Unit})$: Nominal housing supply at residential location x in year t.

i (%): Interest rate, converting future values to the present values.

$I(\text{HK\$})$: The average daily household income.

$L(t)/L^t(\text{Km})$: Rail line length in year t.

$\Delta L^t(\text{Km})$: The increased rail line length in year t.

$L_s(\text{Km})$: The distance between water threshold line located at the middle of rail station s and $s+1$.

$g_i^t(\text{HK\$})$: The daily consumption of non-housing goods for households in residential location i in year t.

$g(q(x,t))(\text{HK\$})$: The in-vehicle crowding cost per unit distance at location x in year t.

$G_s(\text{Km})$: The distance between the water threshold line and rail station $s+1$.

$m(\text{Years})$: The length of the planning and operation time horizon.

n_s^t: Rail station number in year t.

$P^0(\text{Persons})$: The total population in base year.

$P^t(\text{Persons})$: The total population along the candidate rail transit line in year t.

$P(x,t)/P^t(\text{Persons})$: (Nominal) population density at location x/i in year t.

$p_{i,ATIS,t,\tau}(\%)$: The predicted on-time arrival probability for arriving at the CBD by mode i on departure time t in the n^{th} day of the τ^{th} period given by ATIS.

$p_{i,n,t,\tau}(\%)$: The probability of obtaining the output of low generalised travel cost for arriving at the CBD by travel mode i on departure time t in the n^{th} day of the τ^{th} period.

$PR(\text{HK\$})$: Profit of the rail operator coming from fare revenue and subsidy.

$PV_{i,x,t,\tau}(\text{HK\$})$: Prospect value of choosing travel mode i from location x on departure time t in the τ period.

$q(x,t)(\text{Persons})$: (Nominal) travel demand of rail service at location x in year t.

$q_i^t(\text{Persons})$: Households' travel demand of rail service in rail station i in year t.

$q(x,t)(\text{Persons})$: Yearly varied travel demand of rail service at location x in year t.

$Q_s^t(\text{Persons})$: The travel demand of rail station s in year t.

$r_o^t(\text{HK\$})$: The rent per day in the CBD in year t.

$r_f^t(\text{HK\$})$: The fixed daily land rent for other uses.

$r(x,t)(\text{HK\$})$: Households' actual housing rent at location x in the τ^{th} period, $x \in \mathbf{X}$, and $\mathbf{X}$ is the choice set, i.e. many types of houses existed at location x.

$r_{WIP,x,\tau}(\text{HK\$})$: Households' reference point to decide whether the housing rent is high or low at location x in the τ^{th} period, which is called as households' willingness-to-pay (WTP) on housing rent.

$\tilde{r}(x,\tau)(\text{HK\$})$: Households' perceived housing rent at location x in the τ^{th} period.

$S\left(\text{HK\$}\right)$: Fare subsidy per passenger.

$SW\left(\text{HK\$}\right)$: Social welfare.

$\bar{t}\left(\text{Year}\right)$: The project start time of rail line.

$t_c\left(\text{Hour}\right)$: The average access time for households from residential locations to the rail stations.

$t_r^0\left(\text{Hour}\right)$: The movement time of the train per unit distance.

$t_i^t\left(\text{Hour}\right)$: The average in-vehicle time from rail station i to the CBD.

$t_{or}\left(\text{Hour}\right)$: The travel time except in-vehicle time by rail.

$t_r\left(\text{Hour}\right)$: The total travel time by rail.

$t_w\left(\text{Hour}\right)$: The households' average waiting time for rail service at stations.

$T\left(\text{Hour}\right)$: The cycle time of train operation.

$T_0\left(\text{Hour}\right)$: The constant terminal time on the circular line.

$T_{linehaul}\left(\text{Hour}\right)$: The total line-haul travel time for trains operations from station 1 to the CBD.

$T_{dwell}\left(\text{Hour}\right)$: The total dwelling delay for trains operations from station 1 to the CBD.

$T_R\left(\text{Hour}\right)$: The round journey time of the train.

$TDSW\left(\text{HK\$}\right)$: The total discount social welfare.

$U\left(x,t\right)\left(\text{HK\$}\right)$: (Nominal) disutility of households at residential location x in year t.

$U_i^t\left(g_i^t,h_i^t\right)\left(\text{HK\$}\right)$: The daily households' utility function for residential location i in year t.

$U\left(x,t\right)\left(\text{HK\$}\right)$: Yearly varied disutility of households at residential location x in year t.

$v^t\left(\text{Km/hour}\right)$: The average train speed in year t.

$V\left(\Delta c_{i,n,t,\tau}\right)\left(\text{HK\$}\right)$: Prospect value function of choosing travel mode i at departure time t in the n^{th} day of the τ^{th} period.

$Y_i^t\left(\text{Unit}\right)$: The housing supply in residential location i in year t.

$Z_F\left(\text{HK\$}\right)$: The total profit of the rail operator during the planning time horizon.

$\Delta rc\left(x,s,\tau\right)\left(\text{HK\$}\right)$: The difference between perceived living cost and reference points in terms of the generalised travel cost and housing rent at location x in the τ^{th} period.

$\Delta c_{i,n,t\tau}$ (HK$): The difference between perceived generalised travel cost and reference point for each travel mode i at departure time t in the n^{th} day of the τ^{th} period.

$\pi\left(p_{i,n,t\tau}\right)(\%)$: The probability weighting function.

α: A calibration parameter of waiting time, depending on the distributions of train headway and passenger arrival.

β_0 (Hour): The average train dwelling delay at each rail station.

α_r / β_r: Parameters of living cost function for house given by Ho and Wong (2007).

$\beta_\pi / \beta_P / \beta_c$: Parameters of over-year population density proposed by Dargay and Goodwin (1994).

γ_0 (HK$): The variable rail line cost, such as line overhead cost, maintenance cost and labor cost.

γ_1 (HK$): The fixed rail line cost per kilometre each year, such as land acquisition cost and line construction cost.

$\gamma^{'}$: Parameter of probability weighing function.

$\gamma(t)(\%)$: A compound growth factor of the total population in year t.

$\eta(\%)$: The inflation factor, implying that, for the same capacity enhancement, the fixed rail line cost increase by $(1+\eta)$ per year.

ρ: A parameter converting peak hour demand to yearly demand.

μ(HK$/hour): The value of time.

$\mu_c / \mu_w / \mu_i$ (HK$/hour): The values of time of access time, waiting time and in-vehicle time, respectively.

κ_0 (HK$): The fixed cost portion per rail station each year.

κ_1 (HK$): The operation cost per rail station each year.

$\kappa(t)(\%)$: A compound-account factor to discount the future amount of money to present values.

θ: The scale parameter of households for generalised travel cost.

$\pi(x,t)$(HK$): The perceived generalised travel cost from residential location i to the CBD in year t.

$\sigma_P(x,t)$(Persons): The standard deviation of population density in residential location x in year t.

$\sigma_P(x_1,t_1;x_2,t_2)$: The spatial and temporal covariance of population densities.

$\sigma(SW)$(HK$): The standard deviation of social welfare.

ς(Hour): The constant terminal time of trains.

$\varepsilon / \varepsilon_q$: A random term, with $E[\varepsilon] = 0$ and $E[\varepsilon_q] = 0$.

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Chapter 1 Introduction

1.1 Motivation

Hong Kong, a city of more than 7.0 million people with a land area of 1,104 square kilometres, is one of the most densely populated cities in the world. The population density of Hong Kong was approximately 34,000 persons per square kilometer of developed land in the year 2014. Over the past few decades, despite of the rapid economic growth and the drastic expansion of urban area, traffic congestion has been, to an extent, vastly alleviated in Hong Kong because of an efficient multi-modal transportation system.

Table 1.1 Travel modal split results of Hong Kong in August 2016.

Travel model	Bus	Rail	Private car	Taxi	Others
Percent	47.0%	42.5%	1.9%	7.5%	1.1%

(Source:td.gov.hk/filemanager/en/content_4744/1608.pdf)

Table 1 illustrates the travel modal split results of average daily passenger transport journeys of Hong Kong in August 2016. The travel mode of others includes tram and ferry. It can be seen that most people travel by the public travel modes of bus and rail. This accounts for 89.5% of the total passenger journeys. The high proportion of public travel modes usage can be partly attributed to the high population density stated above and an efficient bus and rail transit system.

In contrast with Hong Kong, there are many Asian cities suffering the congestion problem, for instance Beijing and Shanghai in China and Bangkok in Thailand (Ahmed, et al, 2008, Shan and Yai, 2011). The efficiency of the multi-modal transportation system in Hong Kong does not necessarily imply that a multi-modal transportation system can work well in the other Asian cities, because their population densities in urban area are often much lower than that in Hong Kong.

Compared with other travel modes, rail has higher travel capacity of supply and more reliable travel time. However, rail projects involve huge investment on constructions of rail lines and rail stations. To eliminate traffic congestions on roads, the Shanghai municipal government has recently commenced the project to extend Rail Transit Line 11 about 5.76 km westwards with a total of four stations. In Hong

Kong, a new rail transit line connecting Shatin new town to the Central with a total length of 17 km and ten stations is also under construction. This was started in 2011 and is expected to finish in 2019.

This thesis aims to introduce crucial elements to enrich the consideration of the development of efficient rail transit lines, particularly for fast-growing cities with high population densities, such as Beijing and Shanghai. The population densities in these high densely populated cities often own more significant uncertainties than in low densely populate cities. Thus, the uncertainties in population densities are the main concerns in this thesis. The new rail transit lines are to be built and then operated over a relatively long time horizon that lasts for years. Thus, rail transit line design problems in this thesis are investigated longitudinally over years.

1.2 Context

The time-dependent rail transit line design problem is essentially a network design problem (NDP). A vast and growing body of NDP studies has been presented in the past two decades. Many comprehensive NDP reviews have been conducted, for example those of Magnanti and Wong (1984), Yang and Bell (1998), and Xie and Levinson (2009).

The vast amount of NDP literature is not reviewed here, but key references are given to define the time-dependent rail transit line design problem. In terms of time dimensions, NDP can be classified into two types of single-period NDP and multi-period NDP. In most previous studies, the NDP is typically examined in a single period. Lo and Szeto (2004) extended a single-period NDP into a multi-period NDP. Szeto and Lo (2005) incorporated time-dependent tolling into a multi-period NDP. Szeto and Lo (2006) considered inter-generation equity into a multi-period NDP, whereas Lo and Szeto (2009) examined a multi-period NDP with cost-recovery constraints. Ma and Lo (2012) investigated time-dependent integrated transport supply and demand strategies and their impacts on land use patterns.

The time-dependent rail transit line design problem in this thesis is classified as the multi-period NDP. Specifically, the following problems are investigated in this thesis:

- The over-year interaction between endogenous population densities and financial performance of a candidate rail transit line is investigated and given in Chapter 3.

- The implementation adaptability of a candidate rail transit line over years is investigated with respect to changes in the total population and also annual interest rates in Chapter 4.
- The effects of spatial and temporal correlation of population densities on design of a rail transit line over years are investigated and described in Chapter 5.
- The effects of households' prospect theory based travel behaviour on the design of rail transit line over years are investigated and presented in Chapter 6.

1.3 Research objectives

The objectives of the study presented in this thesis are to propose models for urban rail transit line design over years, especially for fast-growing cities with uncertainties in population densities.

The detailed objectives of this thesis are listed as follows:

- Modelling over-year interaction between endogenous population densities and financial performance of a candidate rail transit line.
- Modelling implementation adaptability of a candidate rail transit line over years with respect to changes of the total population and annual interest rate.
- Modelling the effects of spatial and temporal correlation of population densities on design of a rail transit line over years.
- Modelling the effects of households' prospect theory based travel behaviour on the design of a candidate rail transit line over years.

1.4 Thesis outline

The thesis structure is shown in Figure 1.1. In Chapter 3, over-year interaction between endogenous population densities and financial performance of a candidate rail transit line is examined. In Chapter 4, the implementation adaptability of a candidate rail transit line over years is explored with respect to changes in the total population and annual interest rate. In Chapter 5, the effects of spatial and temporal correlation of population densities on the design of a rail transit line over years is investigated. In Chapter 6, the effects of households' prospect theory based travel behaviour on design of a candidate rail transit line over years are investigated. In Chapters 3, 4 and 5, the proposed models are based on utility theory for a linear transportation corridor with rail only. The model proposed in Chapter 6 is based on the prospect theory with consideration of multi-modal transportation corridor.

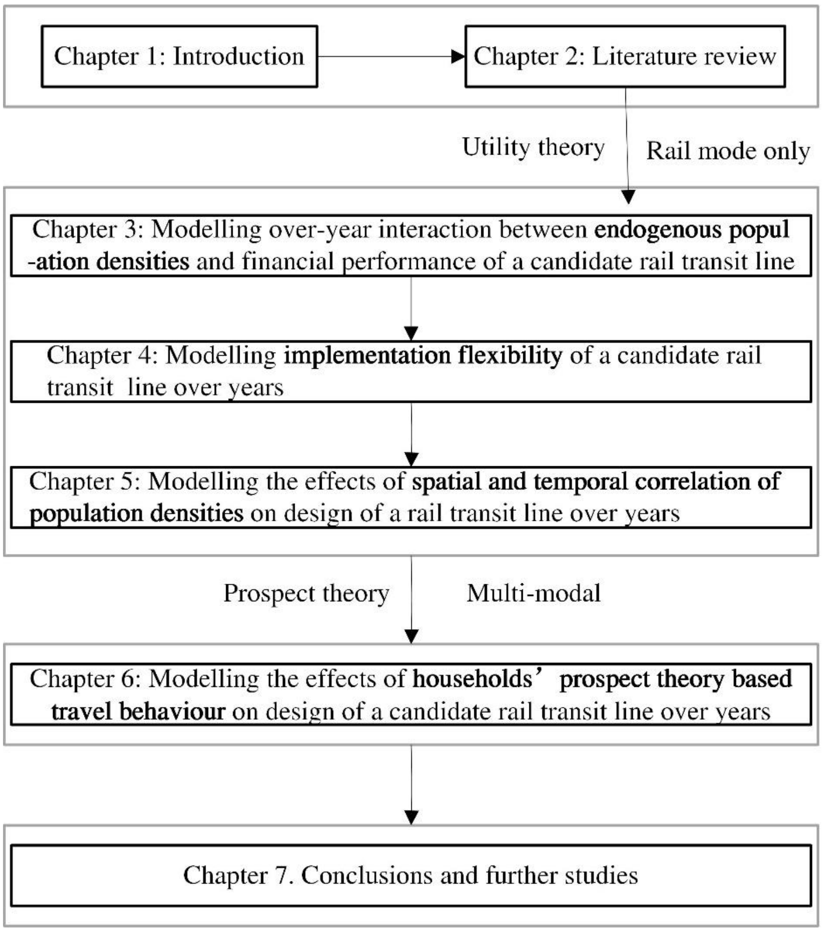


Figure 1.1: Thesis structure.

Chapter 2 Literature review

Network design problems (NDP) have been extensively investigated in previous studies. However, intensive research is rarely focused on the topic of urban rail transit line design problems over years.

A comprehensive review of such literature has been carried out to investigate research topics in connection with urban rail transit line design problems over years.

This chapter is organised as follows. A review of network design problem is given in Section 2.1. Some basic considerations in this thesis are presented in Section 2.2, including spatial and temporal correlation of population densities, prospect theory based travel behaviour.

2.1 Network design problem (NDP)

A vast and growing body of network design problem (NDP) studies has been presented in the past two decades. Many comprehensive NDP reviews have been conducted, for example those of Magnanti and Wong (1984), Yang and Bell (1998), and Xie and Levinson (2009). As stated in Section **1.2 Context,** the vast literature on this subject can be classified into single-modal NDP and multi-modal NDP with respect to travel modes, and single-period NDP and multi-period NDP with respect to time dimensions. Table 2.1 gives the detailed NDP classification. More detailed discussed are given in the following.

Table 2.1 NDP classification.

	Single modal NDP	Multi-modal NDP
Single-period NDP	LeBlanc (1975); Davis (1994)	Wirasinghe et al (1977); Si et al (2009)
Multi-period NDP	Lo and Szeto (2004); Szeto and Lo (2006); Ma and Lo (2012); Ukkusuri and Patil (2012); Chapters 3, 4, 5 of this thesis	Wong et al (2004); Liu et al (2009); Chapter 6 of this thesis

2.1.1 Single-modal and single-period NDP

The common investigation travel modes in single-modal NDP are auto, bus and rail train. NDP on auto can be generally divided into two categories: discrete and continuous, in terms with the types of design variables. Discrete-NDP (DNDP) deals with the addition of new roadway segments to a network (LeBlanc, 1975). Continuous-NDP (CNDP) addresses the capacity improvement or optimisation of existing network link (Davis, 1994).

In practice, one of the most common methods for DNDP was to specify several different network configurations and then to analyse each separately. Since the inherent number of limited alternative candidate networks, DNDP became a dominant tendency, i.e. the method of new links additional to existing networks. Mixed integer programming (MIP) is an extensively used technique. In a MIP, the 0-1 decision variables are typically assigned for each proposed new link (Abdulaal and Leblanc, 1979):

$$y_i = \begin{cases} 1 & \text{if link i is constructed;} \\ 0 & \text{if not.} \end{cases} \tag{2.1}$$

Constraints are expressed as

$$x_i \le My_i, \tag{2.2}$$

where M is a suitably large number. Then, DNDP could be formulated as

$$\min_{y_i=0,1,x_i\ge 0} T(x), \tag{2.3}$$

subject to flow conservation constraints, capacity constraints, budget constraints, emission constraints, and energy efficiency constraints if required. $T(x)$ is a performance measure of the designed network, for instance, traffic congestion, total user cost and operator cost.

For the solving methods of 0-1 mixed integer programming model (2.3), the computational difficulties would increase rapidly, as the numbers of 0-1 integer variables are added.

To avoid such inherent problems, CNDP was proposed (LeBlanc and Abdulaal, 1979; Davis, 1994). LeBlanc and Abdulaal (1979) addressed the optimal utilisation of an existing road transportation network and determined the improvement so that congestion in the network was minimised subject to budget constraints. Davis (1994) employed a stochastic user equilibrium approach to describe the travellers' route choice behaviours and proposed CNDP as a tractable, albeit large large-scale, differentiable nonlinear programming problem.

NDP on bus or train involves optimisation of route layouts, together with, the stop/station spacing and number, and associated operational characteristics such as departure frequency, fare, headway, vehicle scheduling. The formulation models of NDP with consideration of transit could also be classified as discrete and continuous. The model techniques used here are similar to those models for auto. However, the issues solved in NDP, regarding bus or train, are more complicated and by comparison unique. Some optimal variables design in NDP on bus or train could be recognised as a special research topic.

Take vehicle scheduling problems as an example. Vehicle scheduling problems (VSP) could be defined as the determination of optimal vehicle allocation to carry out all trips in a given transit timetable. A chain of trips are assigned to each vehicle although some, in order to reach optimality, may be deadheading or empty trips (Ceder, 2011). Generally, the previous research on VSP mainly considered only one type of transit vehicle. Dantzig and Fulkerson (1954) demonstrated that certain vehicle scheduling problems could be formulated as a minimum cost flow problem on a network. Freling et al (2001) investigated VSP in a case of single-depot and proposed a quasi-assignment formulations and corresponding auction algorithms. Haghani and Banihashemi (2002) presented the VSP model with multiple-depots. Regarding VSP with more than one type of vehicles, see the work of Diez-Canedo and Escalante (1977) and Ceder (2011). Diez-Canedo (1977) extended the work of Dantzig and Fulkerson (1954) to determine optimal vehicle fleet. Ceder (2011) addressed VSP with the consideration of trip characteristics and the particular vehicle requirements.

2.1.2 Multi-modal and multi-period NDP

An NDP in most previous studies is typically examined as a single-modal and single-period NDP (See discussions on page 2 of **1.2 Context**). Wirasinghe et al (1977) presented an approximate analytical model of a rail/ feeder bus system with a peak-period many to many type of demand. The underlying highway grid was assumed to be rectangular with the railway parallel to one axis. Moreover, this model was applied to the Galgary South Corridor LRT system. Si et al (2009) evaluated the revenue strategy of railway transport under multi-modal market competition by bi-level programming. It was concluded that railway agency should focus on not only the pricing but travel time and service level, in order to increase revenue. Mohaymany and Gholami (2010) adopted ant colony optimisation approach to conduct a discussion on multi-modal feeder network design problem. Tirachini et al (2010) compared three alternative forms of public transport- light rail, heavy rail and bus rapid transit. They established the conditions to determine which travel mode was preferred to another, in terms of the operator and user side offerings.

Lo and Szeto (2004) extended a single-period NDP into a multi-period NDP. Szeto and Lo (2005) incorporated time-dependent tolling into a multi-period NDP. Szeto and Lo (2006) considered intergeneration equity into a multi-period NDP, whereas Lo and Szeto (2009) examined a multi-period NDP with cost-recovery constraints. Ma and Lo (2012) investigated time-dependent integrated transport supply and demand strategies and their impact on land use patterns. Ukkusuri and Patil (2009) introduced flexibility into a multi-period NDP, in which future investment could be deferred or abandoned.

In terms of **1.3 Research objectives**, the objectives of this thesis are to propose models for efficient urban rail transit line design over years with consideration of uncertainties in population densities, especially for cities with high population densities. Some basic considerations in this thesis are presented as follows.

2.2 Some basic considerations in this thesis

2.2.1 Spatial and temporal correlation of population densities

Population densities in conventional travel choice models were commonly assumed to be a given and fixed value (See, e.g. Lam and Huang, 1992; Huang and Lam, 2002; Liu et al, 2009). Thus, the effects of population densities on travel mode choices were not explicitly investigated. These effects, however, are of some importance and significance.

The preferred travel mode in areas with low population density is private car, such as United Stated. The preferred travel mode in areas with high population density is transit, such as Hong Kong. One possible reason is that high congestion and long delay may occur on highways, during morning peak hour on working day at highly populated areas. The generalised travel cost of private car may be higher than transit in such areas.

In many areas, especially in cities with high population density like Shanghai and Hong Kong, people commonly make residential location choice and rail travel mode choice simultaneously (Yip et al, 2012; Li et al, 2012a; Ibeas, et al, 2013). In other words, population distributed at different residential locations along a candidate rail transit line is correlated with each other. For instance, with a given total population in a linear monocentric city, more persons live in the central business district (CBD), and then less live in suburban community and new towns. In such a case, negative spatial correlation exists between population densities at the CBD, and the suburban community and new towns.

In traditional transportation planning studies, population densities at each residential location are assumed to follow independence of irrelevant alter- natives (IIA). This assumption does not consider the spatial correlation given above, between population densities. Furthermore, population densities tend to vary year by year. If an increase of population density in the first year leads to an increase in the second year, positive temporal correlation then exists between population density in the first year and second year, and vice versa.

In Chapter 5, the effects of spatial and temporal correlation of population densities on design of a rail transit line over years are explored.

2.2.2 Prospect theory based travel behaviour

Prospect theory (PT) is regarded as a leading behaviour paradigm to understand decision-making under risks. In contrast with PT, utility theory deduces implications from normative preference axioms with the assumption of economic rational behaviour. However, PT can describe various choice behaviours with different risk preferences, i.e. risk preference, risk neutral, or risk aversion.

In previous studies, prospect theory has been applied on a number of transportation studies, especially on travellers' travel behaviour. Specific comprehensive reviews of prospect theory on travellers' behaviours have been provided by Van de kaa (2010), and Li and Henshier (2011).

Avineri (2006) examined the possibility of applying prospect theory on modelling stochastic network equilibrium. In this study, the effects of reference point value on such equilibrium were investigated. In a cumulative prospect theory framework, a simplified route choice decision problem was studied with two alternative routes, which had different travel time distributions. Travellers were assumed to know the distribution of travel time on each route and they behaved as if he/she was a prospect maximiser. The principle of Wardrop's User Equilibrium was extended to follow the risk-taking behaviour by prospect theory.

Van De Kaa (2010) yielded 106 studies that covered almost the whole range of domains and contexts of travel behaviours. They assessed the advantage of prospect theory for a better description of traveller's choice behaviour compared with utility theory. The comparison, on the descriptive ability of context-independent loss-neutral valuation and reference-dependent loss aversion, was made in terms of choice set composition, period and country, survey population and descriptive ability. It revealed that the extended prospect theory might appear a promising way to arrive at a better understanding, and prediction of human choice behaviours over the whole range of travel-related contexts.

Xu et al (2011) proposed a model to determine the traveller's reference points. As one application, based on the introduced prospect-based user equilibrium condition, an optimal pricing problem was taken into account. However, some original prospect theory components are not encapsulated completely. In the calculation of travellers' reference point, that kind of objective probability should be incorporated here, rather than that of subjective probability.

Li and Henshier (2011) summarised the problems existed in associated studies on travellers' behaviours modelling by prospect theory: (1) Lack on empirical estimates of prospect theoretic parameters; (2) Selective use of prospect theory components; (3) Reference point issues.

Prospect theory was originally designed to explain lotters' responses to a static situation involving risk, in the lacks of immediate feedback, repeated choices, the rigour, scope, behavioural principles or mechanisms (Timmermans, 2010). However, it has been applied to many dynamic situations, such as the asset allocation (Barberis and

Huang, 2001), intertemporal consumption (Attanasio and Paiella, 2011), equity premium puzzle (Benartzi and Thaler, 1995).

Prospect theory was also applied to dynamic situation in many previous studies on transportation research. For example, Liu et al (2004) uncovered the contribution of travel time reliability to dynamic route choice. The real-time loop data with coefficients was used in a mixed-logit model, representing individual traveller’s references or tastes towards travel time, reliability and cost. Jou et al (2008) investigated the dynamic commuter departure time choice under uncertainty, with a reference point hypothesis of prospect theory. Li and Hensher (2011) concluded that the interest in employing prospect theory is better to understand travel behaviour dynamics.

Prospect theory has been applied in many repeated travel choices, with feedback about the outcomes of each separate choice. For example, Erev and Barron (2005) demonstrated that reference-dependent loss aversion appeared extensively in repeated choice. Avineri and Praksher (2005) observed repeated choices in route choice experiments, so as to compare predictions of multinomial logit implementations of extended utility theory and (cumulative) prospect theory. In their work, route choice laboratory experiments and computer simulations were conducted in order to analyse route choice behaviour in iterative tasks with immediate feedback. Gao et al (2010) applied prospect theory in traffic equilibrium with repeated route choices in a congested network.

Prospect theory describes decision process in two stages, i.e. editing and evaluation. In editing phase, outcomes of the decision choice are calculated following some heuristic with a set of reference point, and then lesser outcomes are considered as losses and greater ones as gains. In evaluation phase, a prospect value would be computed based on the potential outcomes and their respective weighting probability function. Avineri (2006) extended Wardrop’s principle of user equilibrium to prospect theory based user equilibrium.

In Chapter 6, a convex mathematical programming model is proposed, to investigate the effects of households' prospect theory based travel choice behaviour on design of a rail transit line over years.

2.3 Summary of this chapter

As stated in **1.2 of Chapter 1**, the time-dependent rail transit line design problem in this thesis is classified as the multi-period NDP. According to **1.4 of Chapter 1**, the time-dependent rail transit line design problems in Chapters 3, 4, and 5 are single-modal NDP with rail mode only. The time-dependent rail transit line design problem in Chapter 6 is multi-modal NDP with travel modes of rail, bus, private car and parking-and-riding.

In Chapter 3, taking into account the over-year interaction between endogenous population densities and financial performance of a candidate rail transit line, an optimal design of the candidate rail transit line is obtained.

In Chapter 4, implementation adaptability of the candidate rail transit line is explored over years, with consideration of growths of the total population and annual interest rates.

In Chapter 5, the effects of spatial and temporal correlation of population densities on the design of the candidate rail transit line are further investigated.

In Chapter 6, more travel modes and households' prospect theory based travel behaviour, are explored with a proposed model. In the proposed model, the households' learning process on travel modes choices over years is incorporated.

Chapter 3 Optimisation of a rail transit line design over years with consideration of endogenous population densities

In densely populated Asian cities, new candidate rail transit lines should be designed over a planning time horizon for strategic development in fast-growing corridors. On the other hand, the population densities can be significantly affected by the candidate rail transit line over the initial period particularly during the first few years after the opening of the rail transit lines concerned. To enjoy the travel convenience of rail service, households would like to move into residential locations in the candidate rail transportation corridor. In turn, over years, the annual revenue of the candidate rail transit line steadily increases. Interaction, therefore, exists between population densities and financial performance of the candidate rail transit line over the years particularly in the initial period.

As stated in **2.1** of Chapter 2, such over-year interaction between population densities and the financial performance of the candidate rail transit line has seldom been considered in the literature**.** The population densities in the previous studies have usually been assumed to be fixed or given exogenously. The work described in this chapter attempts to bridge this research gap and presents an optimisation model of a rail transit line over a period of several years after implementation. The population densities in the linear rail transit corridor are to be determined **endogenously** together with other design variables for the candidate rail transit line.

A bi-level mathematical programming model is proposed in this chapter. The lower level problem is formulated as a user equilibrium model, and the upper level problem formulated as a time-dependent rail transit line design model for a linear transportation corridor. Specifically, the following rail line design variables of the candidate rail transit line can affect population densities, including rail length, spacing (or number) of rail stations, headway and fare. For example, a longer length of the candidate rail transit line implies more coverage area of the rail service in the sub-urban area. More households may then, choose to live in residential locations with farther distances from the central business district (CBD) by minimising their own living and travel costs.

The remainder of this chapter is organised as follows. In the next section, a brief background of the research problem is presented. Some basic considerations, such as problem statement and model assumptions, are identified and given in Section 3.2. The proposed model formulation is given in Section 3.3. Section 3.4 gives a numerical example to show the application and contributions of the proposed model. Summary of this chapter is given in Section 3.5.

3.1 Background of the research problem

Many previous studies on the forecasting of population densities have been conducted under an existing transportation network with highways and rail transit lines. For example, studies of Levinson and Kumar (1997), Newman and Kenworthy (1989), and Malpezzi (1999) explored population densities with city-level observations data of households. These data of population densities were closely correlated with auto use. Bento et al (2005) investigated population densities for over 20,000 U.S. households living in the urbanized portion of 114 metropolitan statistical areas, with information on automobile ownership and annual miles. The above studies were all proposed for European or American cities located in developed countries.

For fast-growing cities in developing countries, however, population densities were found to be significantly affected by the existence of a rail transit line (Li et al, 2010; Li et al, 2012; Yip et al, 2014). Households were commonly found to have made residential location choices based on their accessibility to the rail service (Anas, 1981; Waddell, 1996; Sermons and Seredich, 2001). The prime reason for such households' choice behaviour could be a desire to make life more convenient as regards maneuverability. Additionally, when competing with road transport, in most cases rail service appears to be more reliable on travel time than other travel modes in Asian cities, especially during daily morning and evening peak-hour periods (Lam et al, 1998; Szeto et al, 2010).

Population densities in fast-growing cities of developing countries vary year by year. These yearly variations are more significant in a candidate rail transportation corridor, because the rail service supplied by the candidate rail transit line can attract more households to the catchment areas of the rail stations. In turn, the over-year financial performance of the candidate rail transit line also depends on yearly varied population densities. Namely, interaction exists between population densities and rail service of the candidate rail transit line over years. This over-year interaction between population densities and rail service of the candidate rail transit line has seldom been considered in the previous studies and is investigated in this chapter.

Many existing models for rail transit line design were focused on optimisation of the rail design variables, for example Vuchic and Newell (1968), Kocur and Hendrickson (1982), Spasovic et al (1994), Chien and Qin (2004), and Li et al (2010). For a detailed comparison, the studies are summarised in terms of rail design variables, as shown in Table 3.1. This table reveals that little attention has been paid to population densities. In the existing studies on rail transit line design, population densities were commonly assumed to be fixed or exogenous. This could be attributed to the fact that their models for rail transit lines were static models and conducted in a particular year.

Table 3.1 Some major analytical models for rail transit line design in a transportation corridor.

Citation	Design variables				Population densities
	Rail length	Spacing of rail stations	Headway	Fare	
Vuchi and Newell (1968)	√	×	×	√	Fixed
Kocur and Hendrickson (1982)		√	√	√	Fixed
Spasovic and Schonefeld (1993)	√	√	√	×	Fixed
Chien and Qin (2004)	√	√	√	×	Fixed
Li et al (2010)	√	√	√	√	Exogenous
This chapter	√	√	√	√	**Endogenous**

3.2 Basic considerations

3.2.1 Problem statement and contributions

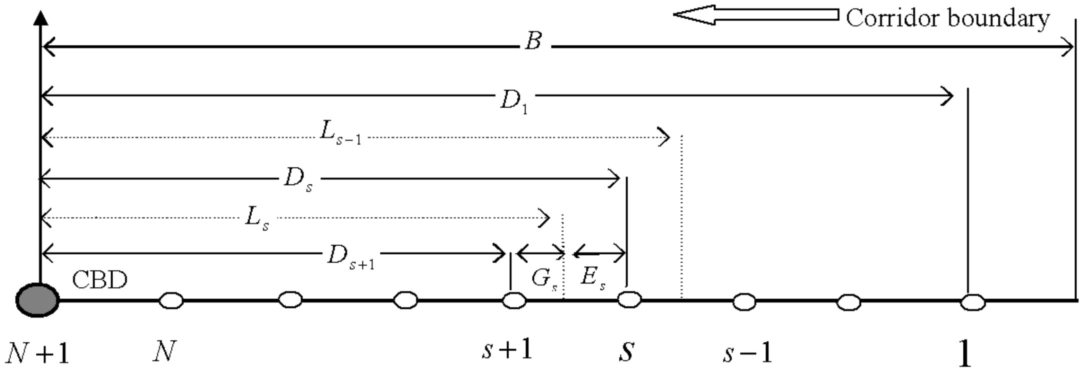


Figure 3.1: Rail transit line configuration over years in a transportation corridor

As shown in Figure 3.1, a candidate rail transit line is extended from CBD towards the boundary of the transportation corridor, where B is the length of this transportation corridor. There is an ordered sequence of rail stations $\{1, 2, \cdots, N+1\}$. The symbol D_s represents the distance between station s and CBD, and D_1 is the rail length of the candidate rail transit line. L_s is the distance between water threshold line located at the middle of rail station s and $s+1$, and G_s is the distance between the water threshold line and rail station $s+1$ and E_s is the distance between the water threshold line and rail station s. The design variables of the candidate rail transit line include rail line length D_1, number of rail stations N, headway H and fare f_s for rail service. (Li et al, 2010)

The major extension to the relevant literature in this chapter is: the incorporation of interaction, over a period of several years, between endogenous population densities and rail service of the candidate rail transit line over the years particularly during the initial period after opening.

3.2.2 Assumptions

To facilitate the presentation of the essential ideas, without loss of generality, some basic assumptions are made in this chapter, as follows.

A1 The characteristics of residential locations are represented solely by distances between CBD and rail stations around these residential locations in a linear transportation corridor or a linear city. Households are assumed to make residential location choice and travel mode choice along the candidate rail transit line simultaneously, so as to minimize the living cost and travelling disutility (Anas, 1981; Waddell, 1996; Sermons and Seredich, 2001).

A2 Households are assumed to board trains at the nearest rail stations, and the trains running along the candidate rail transit line stop at each rail station. This assumption has also been adopted by many previous related works, such as that of Wirasinghe and Ghoneim (1981), Kuah and Perl (1988), Chien and Qin (2004), which could be relaxed in the further study.

A3 The study period is assumed to be a peak-hour period, for instance the morning peak hour, which is usually the most critical period in the day (Li et al., 2012). Peak-hour data are commonly used to determine capacity in the planning of highway or rail transit line.

A4 Households' residential location choice can be affected by many design variables of a rail transit line, such as rail line length, rail station number, headway and fare. Specifically, rail line length is related with the coverage area of rail service. Rail station number has a direct effect on train operation speed and dwelling delays of trains. Headway can be used to determine the waiting time of households at rail stations and fare is a component of the generalised travel cost.

The above four design variables can be divided into two types: long-term and short-term design variables. Long-term design variables cannot be changed during operation stage, and short-term design variables can be updated from time to time. Rail line length and rail station number are long-term design variables, and headway and fare are short-term design variables. To fully explore the interaction between population densities and financial performance of the candidate rail transit line over years, both types of variables are considered in this chapter. These two types of design variables have also been considered in the works of Kocur and Hendrickson (1982) and Li et al (2010).

3.2.3 Population densities over years

The yearly varied population distribution is described by the following equation (Dargay and Goodwin, 1994):

$$\text{In}\, P(x,\tau) = \beta_\pi + \beta_c \,\text{In}\, c(x,\tau) + \beta_P \,\text{In}\, q_r(x,\tau) + \beta_q \,\text{In}\, q_r(x,\tau-1), \forall x \in [0,B], \quad (3.1)$$

where $P(x,\tau)$ is population density at residential location x in year τ, $c(x,\tau)$ is the equilibrium generalized travel cost from residential location x to CBD in year τ, $q_r(x,\tau)$ is travel demand of rail service at residential location x in year τ, and $q_r(x,\tau-1)$ is travel demand of rail service at residential location x in year $\tau-1$. β_π, β_P, β_c, and β_q are parameters depending on the speed of the adjustment process or the response pace, with β_c to be the short run elasticity with respect to the lowest travel cost and $\frac{\beta_c}{1-\beta_q}$ the long run elasticity.

3.3 Model formulation

In this chapter, a bi-level mathematical model is proposed, with a lower level problem formulated as a Wardrop's user equilibrium model, and an upper level problem formulated as an time-dependent rail transit line design model.

3.3.1Lower level problem – user equilibrium model

In terms of *A1*, households are assumed to make residential location choice and travel mode choice simultaneously, so as to minimize the living cost and traveling disutility. This living and traveling disutility can be expressed as:

$$U(x,\tau) = \tilde{c}(x,\tau) + \tilde{l}(x,t), \quad (3.2)$$

where $U(x,\tau)$ is living and traveling disutility for residential location x in year τ, $\tilde{c}(x,\tau)$ is generalized travel cost by rail from residential location x to CBD in year τ. $\tilde{l}(x,\tau)$ is living cost function for house, given by (Ho and Wong, 2007)

$$\tilde{l}(x,\tau) = \alpha_l \left(1 + \beta_l P(x,\tau) / \left(H(x,\tau) - P(x,\tau)\right)\right) \quad (3.3)$$

where $P(x,\tau)$ and $H(x,\tau)$ are population density and housing supply density at location x in year τ, and α_l and β_l are positive scalar parameters that represent the fixed and demand-dependent components of living cost function for house.

The generalised travel cost by rail from residential location x_1 to x_2, $x_1 \in [0,B]$, $x_2 \in [0,x_1]$, excluding the access time cost to rail stations and the fixed component of rail fare can be expressed as follows (Liu et al., 2009):

$$\tilde{c}(x_1,x_2,\tau) = ut_r^0(x_1 - x_2) + \int_{x_2}^{x_1} g(q(w,\tau))\,\mathrm{d}\,w + f_r\,, \tag{3.4}$$

where u is the value of time, t_r^0 is the movement time of the train per unit distance, f_r is rail fare, $q(x,\tau)$ is the elastic travel demand function of households for rail service at location x in year τ and $g(q(x,\tau))$ ($x \in [0,D_1]$) is the in-vehicle crowding cost per unit distance at location x in year τ, which is a strictly increasing function of $q(x,\tau)$, with $g^{'}(q(x,\tau)) > 0$ and $g(0) = 0$ (Huang, 2000). Then, the generalized travel cost by rail from location x to CBD in year τ is

$$\tilde{c}(x,\tau) = ut_{or} + c_r(x,0,\tau) \tag{3.5}$$

where t_{or} is the travel time except in-vehicle time by rail, and t_r denotes the total travel time by rail mode, i.e. $t_r = t_r^0 x + t_{or}$.

The travel time except in-vehicle time by rail t_{or} can be calculated by the sum of access time, waiting time:

$$t_{or} = t_w^0 + \alpha H \tag{3.6}$$

where t_w^0 is average walking time to the around rail stations, H is headway, and α is a calibration parameter which depends on the distributions of train headway and passenger arrival. The value $\alpha = 0.5$ is commonly used implying that a uniform random passenger arrival distribution and a constant headway between trains are assumed.

An equilibrium living and traveling disutility $U(x,\tau)$ can be obtained, while Wardrop's (1952) principle of user equilibrium reaches, "Equilibrium under the condition that no user can reduce his/her living and traveling disutility by unilaterally switching his/her residential location choice". Mathematically, this equilibrium condition can be expressed as:

$$P(x,\tau) > 0 \Leftrightarrow U(x,\tau) < U(y,\tau), \forall x,y \in [0,B], x \neq y \tag{3.7}$$

The above condition implies that at equilibrium, the living cost and traveling disutility at any residential location should be the minimal among all residential locations if the residential location is actually chosen by households at this residential location (Liu et al., 2009). Namely,

$$U(x,\tau) = \min\{U(x,\tau)\} \tag{3.8}$$

Population distribution $\mathbf{P}^* = \{P^*(x,\tau), x \in [0,B]\}$ is denoted as solutions of the above equilibrium condition of Eqs. (3.7) and (3.8). Substituted it into Eq. (3.1), and then $\mathbf{q}^* = \{q^*(x,\tau), x \in [0,B]\}$ can be obtained. With $\mathbf{q}^* = \{q^*(x,\tau), x \in [0,B]\}$ substituted into Eq. (3.5) and $\mathbf{P}^* = \{P^*(x,\tau), x \in [0,B]\}$ into Eq. (3.3), and then a new $\mathbf{P}^* = \{P^*(x,\tau), x \in [0,B]\}$ can be obtained. Namely, the equilibrium condition of Eqs. (3.7) and (3.8) is a fixed point problem. Mathematically, this equilibrium condition is equivalent to the following complementary slackness condition:

$$P(x,\tau)\Big(U(x,\tau) - U(x,\tau)\Big) = 0 \tag{3.9a}$$

$$U(x,\tau) - U(x,\tau) \geq 0\,. \tag{3.9b}$$

The most widely used solution algorithm for solving this user equilibrium problem is the Frank-Wolfe search algorithm. For solving the complementary slackness condition (3.9), this algorithm reduces to a sequence of shortest path computations and one-dimensional minimization (Sheffi, 1985).

3.3.2Upper level problem – time-dependent rail transit line design model

The rail transit line design problem is considered in a planning horizon of m years, namely $\tau \in [1,m]$. In the proposed time-dependent rail transit line design model, the variables include rail length, number of rail stations, headway and fare of rail service. The objective is social surplus maximization.

The time-dependent rail transit line design model is expressed as:

$$\text{Max } TDSW(D_1, N, H, f_r) \tag{3.10}$$

with constraints of Eqs. (3.7) and (3.8). $TDSW$ is denoted as the total discounted social welfare of the candidate rail transit line, given by:

$$TDSW = \sum_{\tau=1}^{m} \left(DP_\tau + DCS_\tau \right). \tag{3.11}$$

with DP_τ representing discounted profit of rail transit line in year τ, and DCS_τ discounted consumer surplus in year τ.

Discounted profit of rail transit line equals summation of discounted rail fare minus discounted construction and operation cost, namely

$$DP_\tau = \frac{\sum_{s=1}^{N} f_r Q_{s,\tau}}{(1+i)^{\tau-1}} - C_\tau \tag{3.12}$$

where $\frac{\sum_{s=1}^{N} f_r Q_{s,\tau}}{(1+i)^{\tau-1}}$ represents summation of discounted rail fare in year τ, $1/(1+i)^{\tau-1}$ is the discount factor for year τ, i is interest rate representing time cost of money, f_r is rail fare, $Q_{s,\tau}$ is travel demand of rail station s in year τ and C_τ is discounted construction and operation cost in year τ.

Travel demand of rail service at rail station s in year τ is calculated by

$$Q_{s,\tau} = \rho \int_{L_s}^{L_{s-1}} q(x,\tau)\,\mathrm{d}x, \forall s = 1,2,\cdots,N. \tag{3.13}$$

where ρ is a parameter converting peak hour demand to yearly demand. With the use of *A2*, households' water threshold line L_s is located at the middle point of line segment ($s, s+1$), and the distance of the passenger watershed line L_s from the CBD is given by

$$L_s = \frac{D_s + D_{s+1}}{2}, \forall s = 1,2,\cdots,N, \tag{3.14}$$

The discounted cost C_τ, which is incurred for construction and operations of the candidate rail transit line, consists of the following three cost components, expressed as (Chien and Schonfeld, 1998):

$$C_\tau = C_{o,\tau} + C_{L,\tau} + C_{s,\tau}. \tag{3.15}$$

where $C_{o,\tau}$, $C_{L,\tau}$ and $C_{s,\tau}$ are discounted operation cost, discounted construction cost of rail line, and discounted construction of rail stations.

The discounted operation cost $C_{o,\tau}$ is expressed as

$$C_{o,\tau} = \frac{\mu_o + \mu_1 F}{(1+i)^{\tau-1}}, \tag{3.16}$$

where μ_o is the fixed operation cost, μ_1 is the operation cost per train each year, and F is the fleet size (or the number of trains) . F equals the vehicle round journey time T_R divided by the headway H :

$$F = \frac{T_R}{H}, \tag{3.17}$$

where the round journey time T_R is composed of terminal time, line-haul travel time, and train dwelling delays at station (Lam et al., 2002; Li et al., 2009), expressed as:

$$T_R = \xi T_o + 2(T_{11} + T_{12}), \tag{3.18}$$

where T_o is the constant terminal time on the circular line and ξ is the number of terminal times of trains operating on the candidate rail transit line for each round journey. T_{11} and T_{12} are , respectively, the total line-haul travel time and total dwelling delay for train's operations from station 1 to CBD, defined as:

$$T_{11} = D_1 t_r^0 \tag{3.19}$$

$$T_{12} = \beta_0 \tag{3.20}$$

where t_r^0 is the movement time of the train per unit distance, D_1 is rail length, and β_0 is average train dwelling delay at each rail station (Lam et al., 1998; Zhang et al., 2010).

The discounted rail line cost $C_{L,\tau}$ is the sum of the fixed cost $\gamma_1 D_1$ (e.g. land acquisition cost, line construction cost) that is proportional to the rail length D_1 and the variable cost γ_0 (e.g. line overhead cost, maintenance cost and labor cost), discounted to present value terms (Li et al., 2012), namely

$$C_{L,\tau} = \gamma_1 D_1 (1+\eta)^{\tau-1} + \frac{\gamma_0}{(1+i)^{\tau-1}}, \tag{3.21}$$

where γ_1 is the fixed rail line cost per kilometer each year. The term $1/(1+\eta)^{\tau-1}$ represents the inflation factor. This inflation factor implies that, for the same capacity enhancement, the fixed rail line cost increases by η per period.

The discounted rail station cost $C_{s,\tau}$ includes a fixed cost (e.g. station land acquisition cost, design and construction cost) and a variable cost (e.g. station overhead cost, operating cost and maintenance cost), discounted to present value terms. Mathematically, $C_{s,\tau}$ is expressed as:

$$C_{s,\tau} = \kappa_0(1+\eta)^{\tau-1} + \frac{\kappa_1(N+1)}{(1+i)^{\tau-1}}, \tag{3.22}$$

where κ_0 is the fixed cost portion and κ_1 is the operation cost per rail station each year.

Consumer surplus measures the difference between what consumers are willing to pay and what they actually pay. The discounted consumer surplus for the candidate rail line in year τ, denoted as DCS_τ, can be expressed as:

$$DCS_\tau = \frac{\sum_{s=1}^{N}\left(\int_0^{Q_{s,\tau}} c^{-1}(x,\tau)\,\mathrm{d}x - f_r Q_{s,\tau}\right)}{(1+i)^{\tau-1}} \tag{3.23}$$

where $c^{-1}(x,\tau)$ is inverse demand function of households for rail service at residential location x in year τ, N is the number of rail stations, $Q_{s,\tau}$ is travel demand of rail service at rail station s in year τ given by Eq. (3.13). The first term in Eq. (3.23) is the total willing to pay of travel demand $Q_{s,\tau}$, and $f_r Q_{s,\tau}$ is their actual pay.

3.3.3Solution method

Without loss of generality, the time-dependent rail transit line design model Eq. (3.10) can be written as:

$$\text{Max TDSW}(\mathbf{x}) \tag{3.24a}$$

Subject to

$$\mathbf{h}(\mathbf{x}) = \mathbf{0} \tag{3.24b}$$

$$\mathbf{x} \geq \mathbf{0} \tag{3.24c}$$

where $\mathbf{x} = [D_1, N, H, f_r]^T$. $\mathbf{h}(\mathbf{x})$ is a vector function of $\mathbf{x}$ and $\mathbf{h}(\mathbf{x}) = \mathbf{0}$ is a vector equation representing user equilibrium condition Eq. (3.9). The model formulation Eq. (3.24) constitutes a constrained non-linear mathematical program. The step-by-step heuristic method procedure used in this chapter is given as follows (Li et al., 2010)

(1) *Initialization*. Choose an initial number of rail stations N and set

$\tau = 1$.

(2) *Update of the population distribution and travel demand of each rail station*. With a given rail length and the rail station number N, the average spacing of rail stations can be calculated. Given initial values of headway $H(\tau)$ and rail fare $f_r(\tau)$ in year τ, population distribution and travel demand of rail service can be obtained by Eqs. (3.1) - (3.9), which are sent to the lower level model in step (3).

(3) *Calculation and comparisons of TDSW, and update of rail length, headway, fare, and number of rail stations*. With the sent population distribution and travel demand of each rail station, TDSW can be calculated. Update rail length, headway, fare, and number of rail stations, which are sent to the upper level model in step (2).

(4) *Termination check*. If the gap between objective values in two times is smaller than zero, namely objective function value cannot be further increased, then stop. Then, population distribution and design variables of rail length, headway, fare and number of rail station are determined. Otherwise, update these design variables and go to Step (2).

3.4 Illustrative examples

3.4.1 Example 1

To facilitate the presentation of the essential ideas and contributions of the proposed model in this chapter, an illustrative example is employed. The rail transit line configuration is shown as Figure 3.1. The input parameters are listed in Table 3.2.

Table 3.2 Input parameters

Notation	Definition	value
D_1	Rail length (Km)	20 (initial value)
f_r	Rail fare (HK$/person)	6 ~ 10
H	Headway (Minutes)	6 (Initial value)
i	Interest rate	0.03
m	The planning time horizon (Years)	3
N	Number of rail stations	12 (Initial value)
t_r^0	The movement time of train per unit distance (Minutes)	60/80
t_w^0	Average walking time to the around station (Minutes)	10
α	Calibration parameter in travel time function	0.5

β_0	Average dwelling delay at each rail station (Minutes)	6
$\beta_\pi / \beta_c / \beta_P / \beta_q$	Parameters in yearly varied population distribution function	-0.5022/0.18/0.8/0.16
α_l / β_l	Parameters in living cost function for house	800/10
γ_0	Variable cost of rail line each year (Billion HK$)	1.8
γ_1	Fixed rail line cost per kilometer each year (Billion HK$)	10.08
η	Inflation factor	0.01
μ	Value of time (HK$/hour)	80
μ_0	The fixed operation cost of per train in each year (HK$)	3.6×10^5
μ_1	The operation cost per train in each year (HK$)	1.94×10^5
κ_0	Fixed cost portion of rail stations (Million HK$)	1.5
κ_1	Operation cost per rail station each year (Million HK$)	0.8
ξ	Number of terminal times of trains for each round journey	2
ρ	Parameter converting peak hour demand to yearly demand	6×365

The population densities, travel demand of rail service along the candidate rail transit line in initial year 1, are given in the following Table 3.3. The housing supply in each residential location is assumed to be 1.4×10^6 (housing units).

Table 3.3 Population distribution, travel demand of rail service and housing supply

Rail station	Population density (Persons/km)	Travel demand of rail service (Persons)	Housing supply (Housing units)
1	9870	800	1.4×10^6
2	8930	1088	
3	8080	1856	
4	8080	3040	
5	8080	2880	
6	6620	2000	
7	5990	3000	
8	5420	2500	
9	6620	4600	
10	5900	3700	

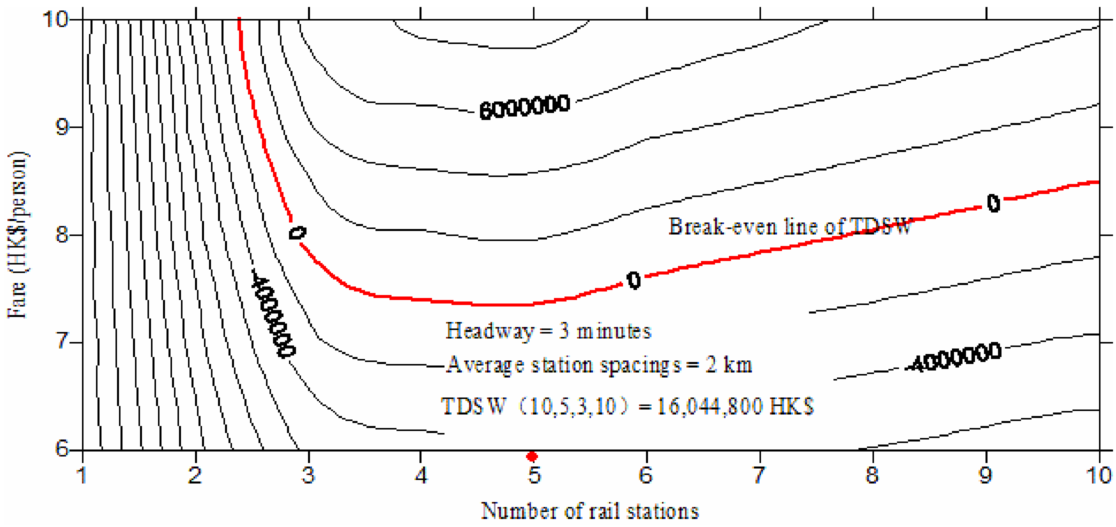


Figure 3.2: Optimal total discounted social welfare of the candidate rail transit line (TDSW) with respect to different design variables in the planning time horizon

Figure 3.2 presents the optimal total discounted social welfare of the candidate rail transit line (TDSW) with respect to different design variables in the planning time horizon, including number of rail stations (or average station spacing), rail length, fare and headway of rail service. It can be seen that the optimal TDSW is 16,044,800HK$ in this example. The corresponding solutions are as follows: number of rail stations is 5, rail length is 10 km, rail fare is 10 HK$/person, and headway is 3 minutes. Furthermore, the break-even line of TDSW, is described by the bolded red solid line as shown in Figure 3.2. With this, it is easier to show the solutions of rail design variables with a constraint of break-even TDSW.

Table 3.4 Population densities along the candidate rail transit line over years

Rail station number	Year1	Year 2	Year 3
1	**29.07%**	27.32%	**22.93%**
2	22.87%	22.77%	20.75%
3	16.95%	17.08%	18.77%
4	16.95%	16.70%	18.77%
5	**15.33%**	16.13%	**18.77%**
Total population (Persons)	430,400	476,700	527,000

The last line of Table 3.4 shows the total population in vicinity of the candidate transit line in the planning time horizon. The total population in vicinity of the candidate transit line increases from 430,400 persons in year 1 to 527,000 persons in year 3. This implies that more population is attracted to live in vicinity of the candidate transit line while such a rail service is supplied.

Table 3.4 presents the population densities along the candidate rail transit line over years in the planning time horizon. It can be seen that more population choose to move from CBD to residential locations of suburban areas from year 1 to year 3 with rail services supplied. For example, the population density at residential location around rail station 1 decreases from 29.70% in year 1 to 22.93% in year 3, while the population density at residential location around rail station 5 increases from 15.33% in year 1 to 18.77% in year 3. It implies that the candidate rail transit line makes population densities more decentralised, namely more population distributed at residential locations of suburban areas.

3.4.2 Example 2

The rail transit extension line in Central and Western District of Hong Kong, Western Island Line, is used for illustration of the proposed model in each chapter of this thesis.

As shown in Figure 3.3, this Western Island Line is started from Sheung Wan denoted numbered as ②. ① of Central is the Central Business District (CBD), which is connected to Sheung Wan by an existed railway line-Island Line. There are other residential locations along the Western Island Line in Central and Western District of Hong Kong Island, namely ③ Sai Ying Pun, ④ HKU, and ⑤ Kennedy Town. The layout scheme along the Western Island Line is ① ② ③ ④ ⑤.

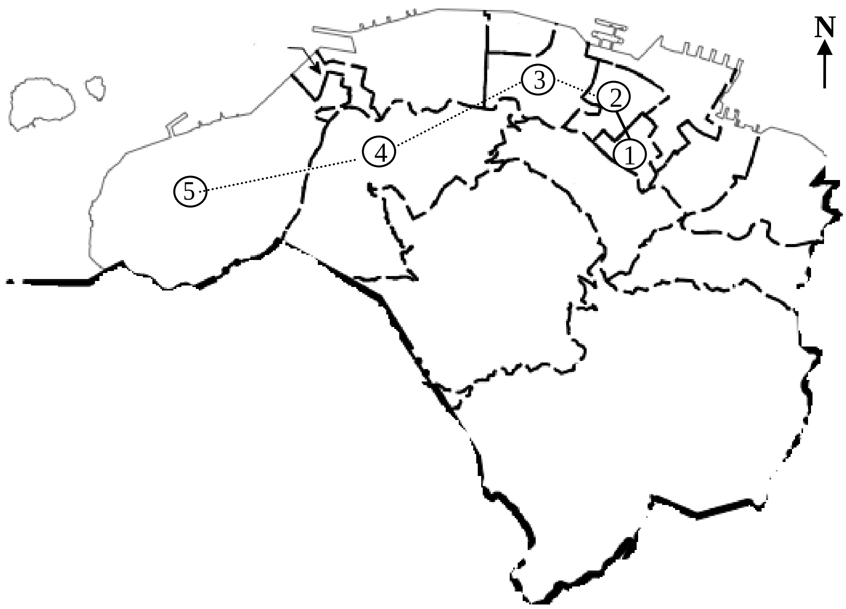


Figure 3.3: Western Island Line in Hong Kong Island

Table 3.5 Projected population densities in each residential location in year 2016

Residential location	District	Projected population densities in each residential location (Persons/km)
		Year of 2016
①Central (CBD)	1.2.2	6600
②Sheung Wan	1.1.4	10000
③Sai Ying Pun	1.1.3	26100
④HKU	1.4.1	20700
⑤Kennedy Town	1.1.1	**61500**

(Source:http://www.pland.gov.hk/pland_en/info_serv/statistic/tables/Lock_WGPD%20Report_2015-2024.pdf)

The data of population densities in each residential location along the Western Island Line are summarised in Table 1. It can be seen that the population density at Kennedy Town is highest, with 60500 persons/km in year 2011 and 61500 persons/km in year 2015.

Table 3.6 Housing supply in each residential location in base year 2016

Residential location	Public rental units	Private flats	Other quarters in private permanent housing	Non-domestic quarters	Temporary quarters	Total
①Central (CBD)	-	2440	130	61	-	2631
②Sheung Wan	-	3709	-	224	1	3934
③Sai Ying Pun	-	9531	-	237	19	9787
④HKU	-	5703	34	283	4	6024
⑤Kennedy Town	2825	19300	100	760	12	22997

(Sources: http://www.bycensus2016.gov.hk/en/bc-mt.html)

Table 3.5 gives the data of population densities in each residential location in year 2016 along the Western Island Line. Table 3.6 gives the housing supply in each residential location in base year 2016.

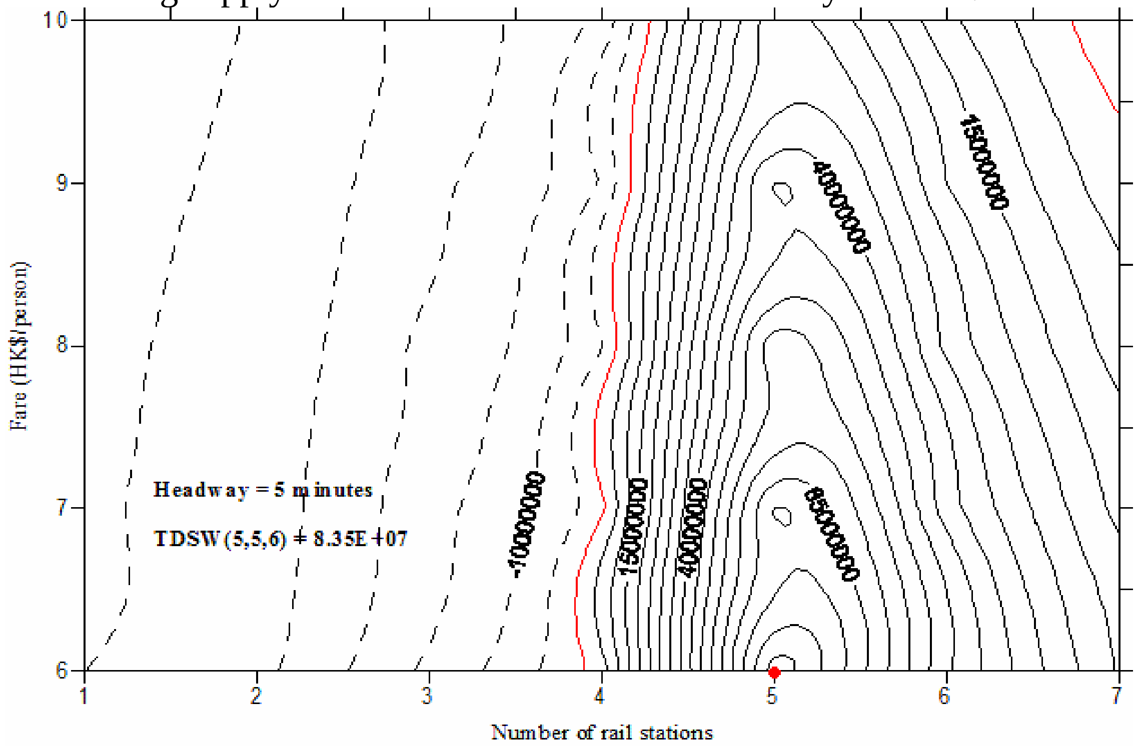


Figure 3.4: Optimal total discounted social welfare of the Western Island Line with respect to different design variables in the planning time horizon

Figure 3.4 presents the optimal total discounted social welfare of the candidate rail transit line (TDSW) with respect to different design variables in the planning time horizon, including number of rail stations, fare and headway of rail service. It can be seen that the optimal TDSW is 8.35E+07 HK$ for the Western Island Line. The corresponding solutions are as follows: number of rail stations is 5, rail fare is 6 HK$/person, and headway is 5 minutes. These numbers are consistent with the actual results of the Western Island Line: number of rail stations is 5, rail fare is 5.5 HK$/person, and headway is 1.9-5 minutes (www.mtr.com.hk).

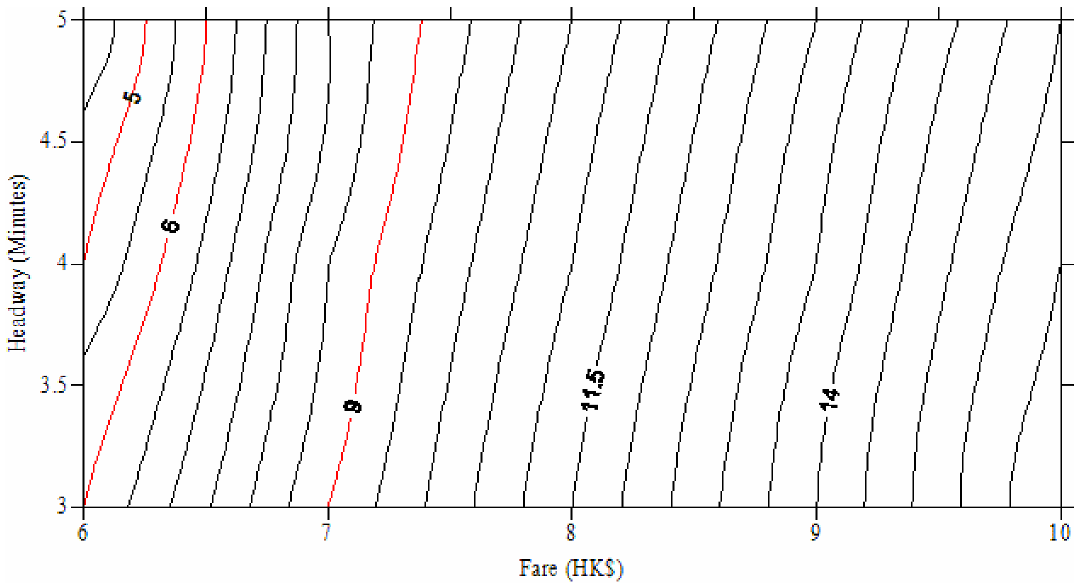


Figure 3.5: Optimal rail line length with respect to headway and fare

Figure 3.5 presents the impacts of headway and fare on the long-term design variable of rail line length with an assumption of average station spacing 1 km for the candidate rail line. It can be found that with a fixed fare of 6 HK$, the optimal rail line length increase from 5 km to 6 km while average headway decreases from 4 minutes to 3 minutes. With a fixed headway of 3 minutes, the optimal rail line length increases from 6 km to 8 km, while fare increases from 6 HK$ to 7 HK$.

These results imply that a lack of integration of short-term decision variables, headway and fare, may result in excessive investments for the long-term rail construction, namely the increase of optimal rail line length.

Figure 3.6 gives the total population along the Western Island Line over years. It can be seen that the total population along the Western Island Line increases from 124900 persons in year 2016to 128595 persons in year 2020. The total population in the above 5 years can be described by a regression equation:

$$y = -126.71x^2 + 1701.3x + 123332,$$

where y represents the total population in year x. The R^2=0.9859 of regression equation, is the coefficient of determination. It tells you how many points fall on the regression line. (http://www.statisticshowto.com/excel-regression-analysis-output-explained/)

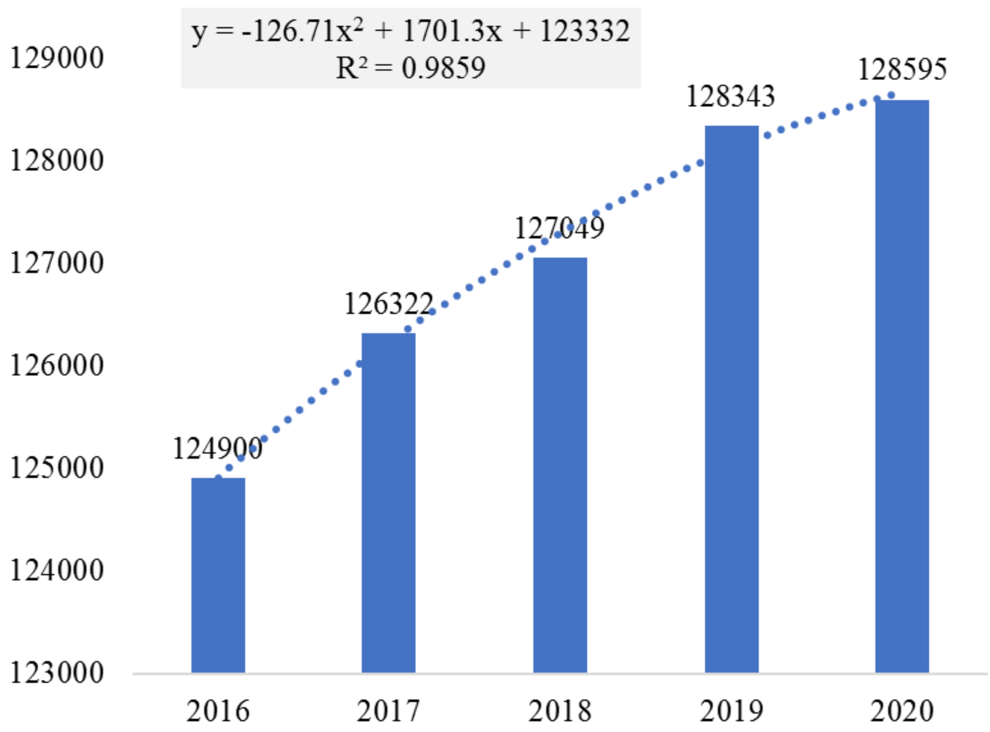
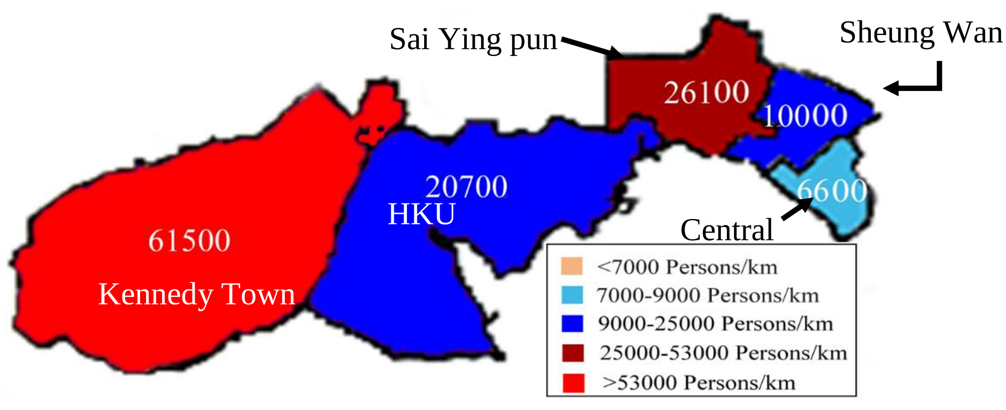
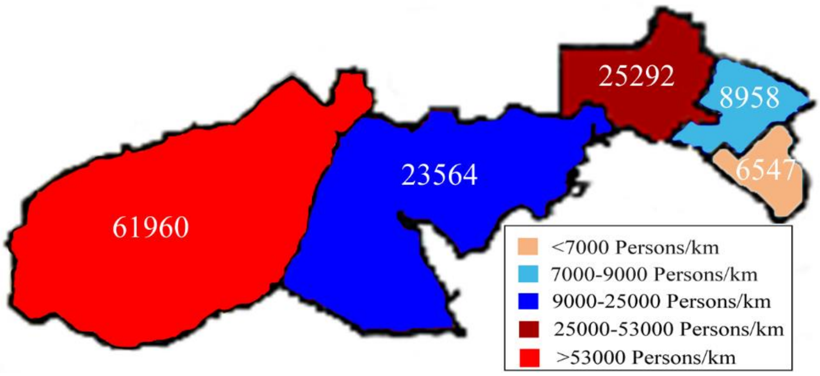


Figure 3.6: The total population along the Western Island Line over years

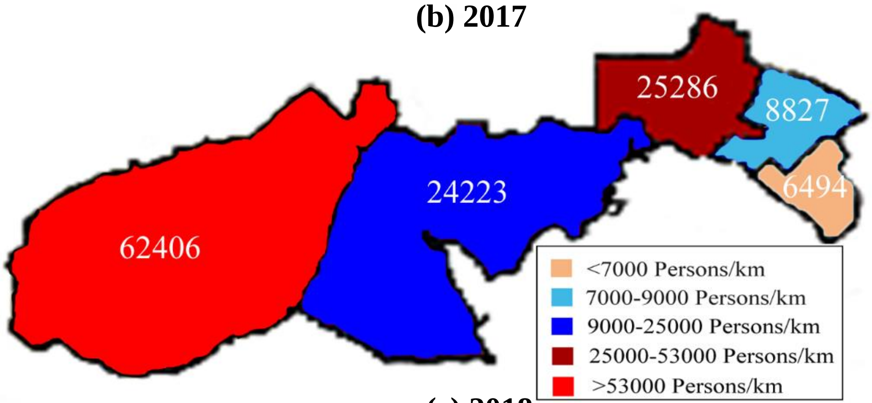
Figure 3.7 presents the forecasted population densities along the Western Island Line over the planning time horizon. It can be found population around rail stations HKU and Kennedy Town increases, while population around rail stations Central, Sheung Wan and Sai Ying Pun decreases. This implies that more population is attracted to live in suburban residential locations of the Western Island Line while the rail service of Western Island Line is provided.



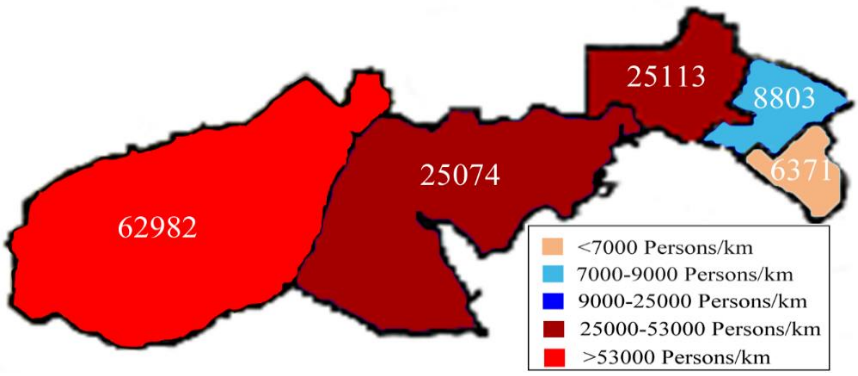
(a) 2016



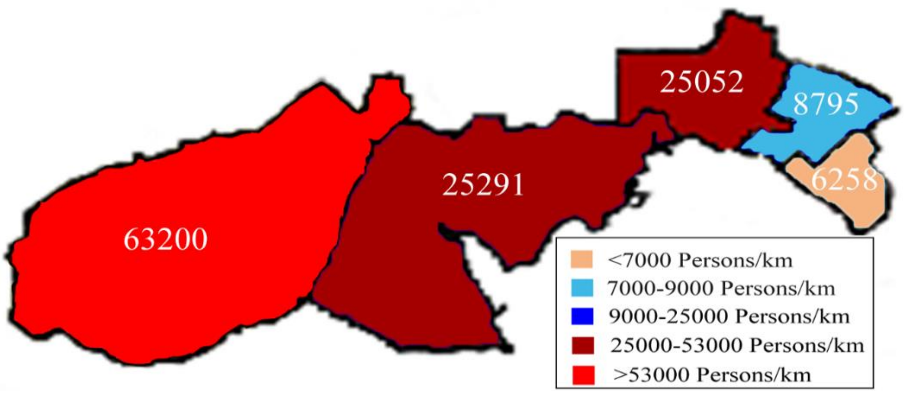
(b) 2017



(c) 2018



(d) 2019



(e) 2020

Figure 3.7: Population densities along the Western Island Line

3.5 Summary of this chapter

This chapter presents an optimisation model of a rail transit line with consideration of endogenous population densities. A bi-level mathematical model is proposed, with a lower level problem formulated as a user equilibrium model, and an upper level problem formulated as an time-dependent rail transit line design model. Distinguished from the traditional design models for rail transit lines, the interactions between yearly varied population densities and performance of the candidate rail transit line are incorporated endogenously and investigated explicitly in this chapter.

It was found that more people are attracted to live in the vicinity of the candidate rail transit line while rail services are provided. With rail service supplied, more people would choose to move from residential locations of CBD to the residential locations of suburban areas. The candidate rail transit line can contribute to the decentralization of the population densities in that it enables the greater distribution of people to the residential locations of suburban areas.

In this chapter, the yearly variations of population densities and annual average interest rate have not been fully explored. Yearly variations of population densities can directly affect travel demand of rail service and the further performance of the candidate rail transit line. The increase of annual average interest rate can lead to the increase of the total supply cost of the rail service particularly in the initial period. Hence, while population densities and annual average interest rates do not vary in accordance with prediction in the traditional feasible study, the construction of a candidate rail transit line may be deferred or fast-tracked several years. In Chapter 4, implementation adaptability of a candidate rail transit line is explored, with consideration of yearly variations of the total population and annual average interest rate.

In addition, the spatial and temporal correlation of population densities may significantly affect the design results of the candidate rail transit line over years. This effect will be investigated and presented in Chapter 5.

Only the rail transit mode has been taken account for model development in this chapter. However, more travel modes should be considered in reality. With this extension, the households' travel mode choice behaviour would be incorporated into the model proposed in

Chapter 6 (Chowdhury and Chien, 2002; Li, et al, 2006; Liu and Lam, 2013).

Chapter 4 Modelling implementation adaptability for design of rail transit line over years

As stated in Section **2.2.1** of Chapter 2, adaptability is defined as the ability of the system to adapt to external changes, while maintaining satisfactory system performance. The implementation adaptability of the rail transit line allows authorities and/or operators to fast-track or defer the future investment on the candidate rail transit line for several years, if necessary. For instance, while the capital cost of construction of the candidate rail transit line is lower than predicted in the previous feasibility study, the candidate rail transit line can be fast-tracked.

In this chapter, a closed-form mathematical programming model is proposed to explore implementation adaptability of a rail transit line over years. The analytical solutions of the optimal project start time and rail length of the candidate rail transit line are given explicitly.

The remainder of this chapter is organised as follows. In the next section, a brief background of the research problem is presented. Some basic considerations are given in Section 4.2. The model formulation and properties are then explored in Section 4.3. Section 4.4 gives two illustrative numerical examples to show the contributions of the proposed model. Summary of this chapter is given in Section 4.5.

4.1 Background of the research problem

Population density in previous rail design models was assumed to be given and fixed, such as Li et al (2012c). This assumption is generally acceptable, because their models were static models and proposed for one particular design year.

In reality, the yearly variations of population density, however, may be significant in fast growing cities, like Shanghai in China. The local capital investment costs for urban rail projects may vary over years. These variations are generally not taken into consideration in previous studies. This research gap is investigated to assess the implementation adaptability of the candidate rail transit line over years in this chapter.

4.2 Basic considerations

4.2.1 Problem statement and contributions

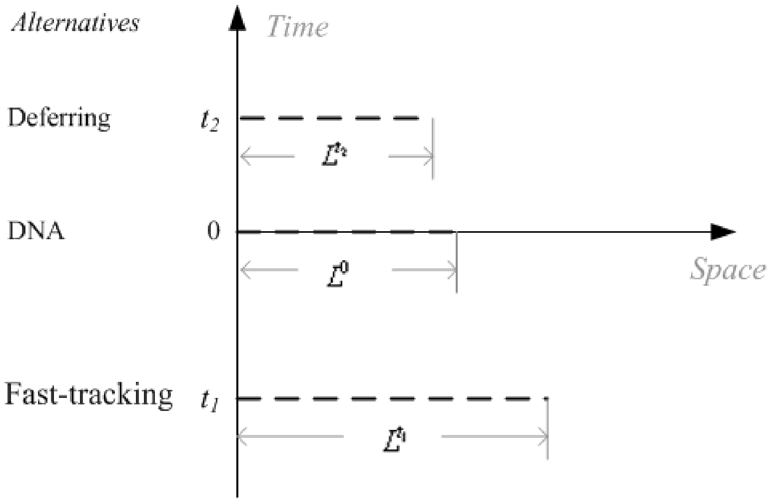


Figure 4.1: Configuration of a candidate rail transit line with consideration of implementation adaptability over years

As shown in Figure 4.1, L^0 is the rail length of the candidate rail transit line under alternative of do-nothing-alternative (DNA), namely the candidate rail transit line is under construction as suggested in the previous feasibility study.

L^{t_1} is the rail length of the candidate rail transit line under the alternative of fast-tracking it t_1 years. The fast-tracking reasons may be that actual population density until year t_1 is higher than predicted in the feasibility study due to a higher average annual growth rate of the total population $\gamma(t_1)$, or that the annual interest rate from year t_1, $i(t_1)$, is lower than that predicted in the feasibility study.

L^{t_2} is the rail length of the candidate rail transit line under the alternative of deferring it t_2 years. The deferring reasons could be that the actual population density until year 0 is lower than that predicted in the feasibility study due to a lower average annual growth rate of the total population. Until the year t_2 the actual population density is high enough to begin the construction of the candidate rail transit line, the average annual growth rate of the total population being

$\gamma(t_2)$, or possibly the annual interest rate from year 0 is higher than that predicted in the feasibility study, and until the year t_2 the actual average annual interest $i(t_2)$ is low enough to start the construction of the candidate rail transit line.

Regarding the implementation adaptability of the candidate rail transit line, specifically, the following research questions are examined:

- What is the optimal project start time of the candidate rail transit line, with respect to different annual growth rate of the total population and annual interest rate? In other words, for what period of time in year, should the candidate rail transit line be fast-tracked or deferred?

- Hong long the candidate rail transit line should be built in each year or extended in each design period?

- What are the effects of different alternatives of implementation adaptability on rail length of the candidate rail transit line? Do the alternatives of fast-tracking or deferring the candidate rail affect the length of candidate rail transit line?

Two major extensions to the related literature are presented in this chapter: (i) the effects of yearly varied rail service travel demand and yearly variation of the total supply cost of the candidate rail transit line on the implementation adaptability are investigated; (ii) the solutions of optimal project start time and year-by-year rail length are derived analytically.

4.2.2 Assumptions

To facilitate the presentation of the essential ideas, some basic assumptions are made in this chapter.

A1 The candidate rail transit line in this chapter is assumed to be linear and starting from the CBD and then built along a linear monocentric city, as shown in Figure 3.1 (Tsao et al, 2009; Liu et al, 2009). The implementation of the candidate rail transit line in each period is assumed to finish on time and the rail service is expected to be supplied at the end of each design period (Lo and Szeto, 2009).

A2 Households’ responses to the quality of rail service are measured by a generalised travel cost that is a weighted combination of in-vehicle time, access time, waiting time and the fare (Qian and Zhang, 2011). Households are assumed to be homogeneous and have the same preferred arrival time at the workplace located in the CBD. This chapter focuses mainly on households’ home-based work trips, which are compulsory activities, and thus the number of trips is not affected by a variety of factors, such as income level (Li et al, 2012a). The study period is assumed to be a peak hour, for instance the morning peak hour, which is usually the most critical period in the day (Li et al, 2012c). The model proposed in this chapter is mainly used for the purposes of strategic planning of a candidate rail transit line.

A3 The characteristics of residential locations are represented solely by distances between CBD and rail stations around these residential locations. Households are assumed to have a Cobb-Douglas form of utility function. Households' income is spent on transportation, housing rent and non-housing goods. Households are assumed to maximise the utility by choosing a residential location along the candidate rail transit line, and amount of other goods within its budget constraint (Anas, 1982; Li et al, 2014).

A4 This chapter focuses mainly on rail, and other types of travel modes such as buses and private car are not considered. This assumption is made because the construction and operation of the candidate rail transit line are not significantly affected by buses and private cars, and thus households' behaviours regarding travel mode choices are not considered in this chapter. This assumption however, is relaxed in Chapter 6, to consider households' behaviours on travel mode choice in multi-modal transportation network.

A5 Generally, land values are related to many factors. For instance, land values for residence and industry are different. The land value near the CBD is generally high, while the land value around the city boundary is comparatively low. The land value in areas with high population density is high, but low in areas with low population density. The land value in areas accommodating high income people is high, but low in low income areas. In this chapter, land use is classified into two types of residential land use and other uses. The land value for other uses is assumed to be constant in each design period of the planning and operation time horizon. (Li et al, 2012d)

A6 As stated in Chapter 3, the design variables of a candidate rail transit line can be divided into two types: long-term and short-term design variables. Long-term design variables cannot be changed during operation stage, and short-term design variables can be updated from time to time.
Long-term design variables include rail line length and rail station number. Short-term design variables consist of train headway and fare. To examine the implementation adaptability of a candidate rail transit line, the short-terms design variables of headway and fare are assumed to be fixed. However, the sensitivity analyses on headway and fare are carried out in the numerical example for assessing their impacts. Note that rail station number depends on rail line length and rail station spacing. To obtain analytical solutions of design variables, the rail station spacing is assumed to be distributed evenly. In other words, given a constant value of rail station spacing, once rail line length is optimised, rail station number is also determined. This assumption is also used in the works of Vuchi and Newell (1968) and Li et al (2012a).

4.2.3 Over-year travel demand of rail service and adjustment of annual interest rate

To describe the year-by-year variation of total population, a yearly growth factor is assumed and shown as below (Lo and Szeto, 2009)

$$P^t = \gamma(t) P^0 , \tag{4.1}$$

where P^t is the total population along the candidate rail transit line in year t, $\gamma(t)$ is a compound growth factor of the total population in year t, in contrast with the predicted total population in year 0. Specifically, with γ denoted as a constant year-by-year growth factor of the total population P^t and $\gamma > 0$, then $\gamma(t)$ can be defined as $(1+\gamma)^t$ while the candidate rail transit line is fast-tracked t years, and $\frac{1}{(1+\gamma)^t}$ while the candidate rail transit line is deferred t years. The reason for such definition of $\gamma(t)$ is that: $t = 0$ implies DNA alternative, which is the benchmark for comparison. Accordingly, the total population under alternatives of fast-tracking or deferring is calculated based on P^0.

The total supply cost of the candidate rail transit line includes yearly average fixed construction cost of the rail line, rail stations, yearly average fixed procurement cost of vehicles of trains, and yearly variable operation cost for rail service supply for each passenger during the planning and operation horizon. Mathematically, it can be expressed as follow

$$C = \sum_{t=\bar{t}}^{m+\bar{t}} \left(\sum_{i=1}^{n_s^t} \left(\Delta L^t C_r + \Delta n_s^t C_s + \Delta F^t C_t + c \right) \kappa(t) \right), \tag{4.2}$$

where $\bar{t}$ is the project start time of rail line. As $\bar{t} = 0$, the candidate rail transit line is implemented as planned, namely DNA alternative; as $\bar{t} > 0$, the rail line project is deferred $\bar{t}$ years; and as $\bar{t} < 0$, the rail line project is fast-tracked $\bar{t}$ years. m is the length of planning and operation time horizon. n_s^t is rail stations number in year t. $\Delta L^t = L^{t+1} - L^t$ is the increased rail line length in year t, $\Delta n_s^t = n_s^{t+1} - n_s^t$ is the increased rail station number in year t, and $\Delta F^t = F^{t+1} - F^t$ is the increased fleet size of train in year t. C_r is daily unit fixed construction cost of rail line, C_s is daily fixed construction cost of each rail station, C_t is fixed procurement cost of each train, and c is variable operation cost for rail service supply to each passenger.

$\kappa(t)$ is a compound-account factor to discount the future amount of money to present value. With κ is denoted as interest rate and $\kappa > 0$, $\kappa(t)$ is defined as $(1+\kappa)^t$ while the rail transit line is fast-tracked t years, and $\frac{1}{(1+\kappa)^t}$ while the rail transit line is deferred t years. This compound-account factor is also taken into account while considering the time cost of fare f_i^t, daily housing rent r_i^t at residential location i in year t.

4.2.4 Stakeholders of the candidate rail transit line

The stakeholders of the candidate rail transit line include the rail operator, households, and the government.

4.2.4.1 The rail operator

The optimisation objective of the rail operator is profit maximisation. Under fixed financial subsidy, profit of the rail operator comes from fare revenue and subsidy. Mathematically, it can be expressed as follows

$$PR = \sum_{t=\bar{t}}^{m+\bar{t}} \left(\sum_{i=1}^{n_s^t} q_i^t \gamma(t) \left(f_i^t - c \right) \kappa(t) \left(\Delta L^{tt} C_r + \Delta n_s^t C_s + \Delta F^t C_t + c \right) \kappa(t) \right), \quad (4.3)$$

where $\bar{t}$ is the project start time of rail line, m is the length of the planning and operation time horizon. n_s^t is rail stations number in year t, q_i^t is households' travel demand of rail service in rail station i in year t, f_i^t is distance-based fare of rail service for each passenger, c is the variable operation cost of rail transit service for each passenger, $\kappa(t)$ is compound-account factor to discount the future amount of money to the present value.

According to A3, an exponential elastic demand function is used and shown as below

$$q_i^t = P_i^t \exp\left(-\theta \pi_i^t\right), \forall i \in \left[1, n_s^t\right], \forall t \in [1, m], \quad (4.4)$$

where P_i^t is the population density in residential location i in year t, θ is the scale parameter of households for generalised travel cost, and π_i^t is the perceived generalised travel cost from residential location i to the CBD in year t.

The perceived generalised travel cost is a weighted combination of in-vehicle time, access time, waiting time and the fare, expressed as

$$\pi_i^t = f_i^t \kappa(t) + \mu_c t_c + \mu_w t_w + \mu_i t_i^t, \forall i \in \left[1, n_s^t\right], \forall t \in [1, m], \quad (4.5)$$

where μ_c, μ_w, μ_i are values of the access time, waiting time and in-vehicle time, respectively; f_i^t is distance-based fare for rail service, $\kappa(t)$ is the compound-account factor to discount the future amount of money to the present value, t_c is average access time for households from residential locations to the rail station, t_w is households' average waiting time for rail service at stations, and t_i^t is average in-vehicle time from rail station i to CBD. The distanced-based fare f_i^t is given by

$$f_i^t = f_0 + f_1^t D_i, \quad (4.6)$$

where f_0 is the fixed fare component, f_1^t is the variable fare component per kilometre in year t, and D_i is the distance between rail station i and the CBD. Waiting time t_w is affected by travel demand and rail service supply. For long-term planning, this value can be estimated using the following function

$$t_w = \lambda H^t, \tag{4.7}$$

where λ is a calibration parameter which depends on the distribution of train headway and passenger arrival time, and H^t is the average headway in year t.

The average headway H^t is dependent on the cycle time of train operation T and fleet size of trains F^t. It can be calculated by (Lam et al, 1998)

$$H^t = \frac{T}{F^t}. \tag{4.8}$$

The cycle time of train operation T is calculated by (Lam et al, 2002)

$$T = \frac{2L^t}{v^t} + 2\varsigma, \tag{4.9}$$

where L^t is the rail line length in year t, v^t is the average train speed in year t, ς is constant terminal time. On the basis of Eqs. (4.7) and (4.8), headway H^t is a function of rail line length L^t, and $\frac{\partial H^t}{\partial L^t} = \frac{2}{v^t F^t} > 0$.

The average in-vehicle time from rail station i to CBD, t_i^t, is given by the distance between rail station i and CBD D_i divided by the average train speed in year v^t, namely

$$t_i^t = \frac{D_i}{v^t}. \tag{4.10}$$

4.2.4.2 Households

Households are assumed to choose the residential locations to maximise their own utilities subject to budget constraint. According to A4, a Cobb-Douglas form of utility function is adopted, shown as follow

$$U_i^t\left(g_i^t, h_i^t\right) = \alpha \log g_i^t + \beta \log h_i^t, \forall i \in [1, N], \forall t \in [1, m], \tag{4.11}$$

where $U_i^t\left(g_i^t, h_i^t\right)$ represents the daily household utility function for residential location i in year t; g_i^t is the daily consumption of non-housing goods for households in residential location i in year t, of which the price is normalised to 1; h_i^t is the consumption of housing in residential location i in year t, measured in square meters of floor space; α and β are positive constraints, and $\alpha + \beta = 1$; N is residential locations number, and m is the length of time horizon.

The budget constraints for households are expressed as below

$$g_i^t + r_i^t h_i^t = I - \pi_i^t, \forall i \in [1, N], \forall t \in [1, m], \quad (4.12)$$

where r_i^t is the daily housing rent for per unit of housing in residential location i in year t, I is the average daily household income, and π_i^t is the daily generalised travel cost from residential location i to the CBD in year t.

According to Eqs. (4.10) and (4.11), households maximise their own utilities by trade-off between generalised travel cost and housing rent among all residential locations. Consequently, households' residential location choice equilibrium is reached. Under equilibrium condition, no households can increase his/her utility by unilaterally changing their locations choices. Mathematically, the utility maximisation for households can be expressed as

$$\max U_i^t\left(g_i^t, h_i^t\right) = \alpha \log g_i^t + \beta \log h_i^t. \quad (4.13)$$

Similar mathematical formulation has been formulated in Li et al (2012). According to the equilibrium condition proposed in their study, the equilibrium household utility is shown as follow

$$U_{equilibrium}^t = \alpha \log\left(\frac{I\alpha}{\alpha+\beta}\right) + \beta \log\left(\frac{I\beta}{r_0^t\left(\alpha+\beta\right)}\right), \quad (4.14)$$

with

$$r_i^t = r_0^t\left(1 - \frac{\pi_i^t}{I}\right)^{\frac{1}{\beta}}, \quad (4.15)$$

$$h_i^t = \frac{I\beta}{r_0^t}\left(1 - \frac{\pi_i^t}{I}\right)^{-\frac{\alpha}{\beta}}, \quad (4.16)$$

where $U_{equilibrium}^t$ is the equilibrium household utility in year t, and r_0^t is the housing rent in the CBD in year t. r_i^t in Eq. (4.15) is the daily housing rent function for per unit of housing in residential location i in year i, and h_i^t in Eq. (4.16) is the daily consumption function of housing for households in residential location i in year t. It can be seen that both r_i^t and h_i^t are functions of daily generalised travel cost from residential location i to the CBD in year t, π_i^t, and the average daily household income I.

To keep the balance of the supply and demand of housing, it requires that

$$h_i^t P_i^t = Y_i^t, \forall i \in [1, N], \quad (4.17)$$

where Y_i^t is housing supply in residential location i in year t. With Eq. (4.16) substituted into Eq. (4.17), the population density of households in residential location i in year t can be calculated by

$$P_i^t = \frac{r_0^t}{I\beta}\left(1-\frac{\pi_i^t}{I}\right)^{\frac{\alpha}{\beta}} Y_i^t, \forall i \in [1, N]. \tag{4.18}$$

In addition, at equilibrium, all households fit inside the supplied housing service

$$\sum_1^N P_i^t = \frac{r_0^t}{I\beta}\sum_1^N\left(1-\frac{\pi_i^t}{I}\right)^{\frac{\alpha}{\beta}} Y_i^t = P^t, \tag{4.19}$$

where P^t is the total number of households in the monocentric city.

On the basis of *A5*, land use is classified into two types of residential use and others use. The land value for others use is assumed to be constant in each design period of the planning and operation time horizon. At equilibrium, the housing rent for per unit of housing in residential location at city edge equals to the fixed daily land rent for other use. Mathematically, it can be expressed as follows

$$r_N^t = r_f^t, \tag{4.20}$$

Where N is the N^{th} residential location at city edge, and r_f^t is the fixed daily land rent for other uses.

4.2.4.3 The government

For the local government, more concerns are focused on the benefits and capital cost of the candidate rail transit line under alternatives of implementation adaptability over years. Consumer surplus is a component of benefits. The consumer surplus of the rail project (CSR) is given by

$$CSR = \sum_{t=\bar{t}}^{m+\bar{t}}\sum_i^{n_s^t}\left(\int_0^{q_i^t}\left(q_i^t\right)^{-1}(w)\,\mathrm{d}w - q_i^t\pi_i^t\right) = \sum_{t=\bar{t}}^{m+\bar{t}}\sum_i^{n_s^t}\frac{q_i^t}{\theta}, \tag{4.21}$$

where q_i^t is passenger travel demand of rail station i in year t, $\left(q_i^t\right)^{-1}$ is its inverse demand function with $\left(q_i^t\right)^{-1}\left(q_i^t\right) = \pi_i^t = \frac{1}{\theta}\ln\left(\frac{P_i^t}{q_i^t}\right)$ in terms of Eq. (4.6). In terms of Eq. (4.21), the benefits at full time are defined as (BFT) is

$$BFT = \sum_{t=\bar{t}}^{m+\bar{t}}\left(\sum_{i=1}^{n_s^t}\left(q_i^t\left(f_i^t + S - c\right)\kappa(t)\right)\right) + CSR. \tag{4.22}$$

The capital cost of the candidate rail transit line at full time of the planning time horizon CFT is calculated as

$$CFT = \sum_{t=\bar{t}}^{m+\bar{t}} \left(\sum_{i=1}^{n_s^t} \left(\left(\Delta L^t C_r + \Delta n_s^t C_s + \Delta F^t C_t \right) \kappa(t) \right) \right). \quad (4.23)$$

4.3 Model formulation and properties

In this section, a profit maximisation model is firstly formulated, followed by the presentation of some essential properties of the proposed model.

4.3.1 Model formulation

According to Li et al (2010, 2012a, 2012b), the profit of rail operator in cities with a high population density such as Hong Kong can be positive. With consideration of a fixed financial subsidy per rail passenger, the profit maximisation model for the rail operator can be formulated as follow

$$\max Z_F\left(L^t, \bar{t}\right) = PR + \sum_{t=\bar{t}}^{m+\bar{t}} \sum_{i=1}^{n_s^t} S\kappa(t) q_i^t$$
$$= \sum_{t=\bar{t}}^{m+\bar{t}} \left(\sum_{i=1}^{n_s^t} \left(q_i^t \left(f_i^t + S - c \right) \kappa(t) \right) \right) - \left(\left(\Delta L^t C_r + \Delta n_s^t C_s + \Delta F^t C_t \right) \kappa(t) \right) \quad (4.24)$$

where $Z_F\left(L^t, \bar{t}\right)$ is the total profit over years for the rail operator under fixed financial subsidy of each rail passenger, S is the fixed financial subsidy per rail passenger, n_s^t is rail station number. The design variables include the rail line length L^t, and rail project start time $\bar{t}$. The headway and fare are design values of rail operation, which are not design values while conducting the analysis of implementation adaptability.

4.3.2 Model properties

Proposition 1. For the profit maximisation model with fixed financial subsidy per rail passenger (4.24), at equilibrium condition of Eq. (4.13), the optimal rail length L^t is a decreasing function of unit fixed construction cost of rail line C_r.

Proof. To obtain the optimal solution for the rail line length, we set the partial derivative of objective function (4.24) with respect to L^t to zero. Then,

$$\frac{\partial Z_F\left(L^t,\bar{t}\right)}{\partial L^t}=\sum_{t=\bar{t}}^{m+\bar{t}}\left(\sum_{i=1}^{n_s^t}\left(\left(\frac{\partial q_i^t}{\partial L^t}\left(f_i^t+S-c\right)+C_r\right)\kappa\left(t\right)\right)\right)=0,$$

where

$$\frac{\partial q_i^t}{\partial L^t}=\frac{\partial P_i^t}{\partial L^t}\frac{P_i^t}{q_i^t}+P_i^t\frac{P_i^t}{q_i^t}\left(-\theta\frac{\partial \pi_i^t}{\partial L^t}\right),$$

$$\frac{\partial P_i^t}{\partial L^t}=-\frac{r_0^t\alpha}{I^2\beta^2}\left(1-\frac{\pi_i^t}{I}\right)^{\frac{\alpha}{\beta}-1}Y_i^t\frac{\partial \pi_i^t}{\partial L^t},$$

and q_i^t, π_i^t, and P_i^t, are given by Eqs. (4.4), (4.5), and (4.18) respectively.

In terms of Eqs. (4.7) and (4.8), we have

$$\frac{\partial \pi_i^t}{\partial L^t}=\mu_w\lambda\frac{\partial H^t}{\partial L^t}>0,$$

Substitute $\frac{\partial q_i^t}{\partial L^t}$ into $\frac{\partial Z_F\left(L^t,\bar{t}\right)}{\partial L^t}$, we can obtain

$$\sum_{i=1}^{n_s^t}\frac{P_i^t}{q_i^t}\left(P_i^t\theta+\frac{r_0^t}{I^2\beta^2}\left(1-\frac{\pi_i^t}{I}\right)^{\frac{\alpha}{\beta}-1}Y_i\right)\frac{\partial H^t}{\partial L^t}=\frac{C_r}{\mu_w\lambda}.$$

Since $\frac{r_0^t}{I^2\beta^2}>0$, $Y_i>0$, $\left(1-\frac{\pi_i^t}{I}\right)^{\frac{\alpha}{\beta}-1}>0$, P_i^t, q_i^t, and π_i^t are positive and constant for each location i at equilibrium condition of Eq. (4.13), therefore $\frac{\partial H^t}{\partial L^t}$ should be negative and constant for each residential location. Finally,

$$L^t=\frac{H^t\mu_w\lambda\sum_{i=1}^{n_s^t}\exp\left(\theta\pi_i^t\right)\left(P_i^tI^{3-\frac{\alpha}{\beta}}\beta^2\theta+r_0^t\alpha\left(1-\pi_i^t\right)^{\frac{\alpha}{\beta}-1}Y_i\right)}{C_rI^{3-\frac{\alpha}{\beta}}\beta^2}.$$

At equilibrium condition of Eq. (4.13), the part of $\sum_{i=1}^{n_s^t}\exp\left(\theta\pi_i^t\right)\left(P_i^tI^{3-\frac{\alpha}{\beta}}\beta^2\theta+r_0^t\alpha\left(1-\pi_i^t\right)^{\frac{\alpha}{\beta}-1}Y_i\right)$ is constant. It can be found that the optimal rail length for profit maximisation with fixed financial subsidy L^t is a decreasing function of daily unit fixed construction cost of rail line C_r.

Proposition 2. For the profit maximisation model with fixed financial subsidy per rail passenger (4.24), at equilibrium condition of Eq. (4.13), if the rail transit line is fast-tracked, the optimal rail line project start time (years) is

$$\left|\bar{t}\right| = \log_{1+\kappa} \frac{\sum_{t=\bar{t}}^{m+\bar{t}} \sum_{i=1}^{n_s^t} \left(f_i^t + S - c \right) \left(\prod_{y=\bar{t}}^{t} \gamma(y) \exp\left(-\theta \pi_i^t\right) + q_i^t t (1+\kappa)^{-1} \left(1 - \theta f_i^t \kappa(t)\right) \right)}{\sum_{t=\bar{t}}^{m+\bar{t}} t (1+\kappa)^{-1} \left(\Delta L^t C_r + \Delta n_s^t C_s + \Delta F^t C_t \right)}.$$

If the rail transit line is delayed, the optimal rail line project start time (years) is

$$\bar{t} = \log_{1+\kappa} \frac{\sum_{t=\bar{t}}^{m+\bar{t}} \sum_{i=1}^{n_s^t} \left(f_i^t + S - c \right) \left(-\prod_{y=\bar{t}}^{t} \gamma(y) \exp\left(-\theta \pi_i^t\right) + q_i^t t (1+\kappa)^{-1} \left(1 - \theta f_i^t \kappa(t)\right) \right)}{\sum_{t=\bar{t}}^{m+\bar{t}} t (1+\kappa)^{-1} \left(\Delta L^t C_r + \Delta n_s^t C_s + \Delta F^t C_t \right)}.$$

Proof. To obtain the optimal solution for the rail project start time, we set the partial derivative of objective function (4.24) with respect to $\bar{t}$ to zero. Then, we have

$$\frac{\partial Z_F\left(L^t, \bar{t}\right)}{\partial \bar{t}} = \sum_{t=\bar{t}}^{m+\bar{t}} \begin{pmatrix} \sum_{i=1}^{n_s^t} \left(\left(\frac{\partial q_i^t}{\partial \bar{t}} \left(f_i^t + S - c \right) + C_r \right) \kappa(t) \right) + q_i^t \left(f_i^t + S - c \right) \frac{\partial \kappa(t)}{\partial \bar{t}} \\ -\left(\Delta L^t C_r + \Delta n_s^t C_s + \Delta F^t C_t \right) \frac{\partial \kappa(t)}{\partial \bar{t}} \end{pmatrix} = 0,$$

where

$$\frac{\partial q_i^t}{\partial \bar{t}} = \exp\left(-\theta \pi_i^t\right) \left(\frac{\partial P_i^t}{\partial \bar{t}} - \theta P_i^t \frac{\partial \pi_i^t}{\partial \bar{t}} \right),$$

$$\frac{\partial P_i^t}{\partial \bar{t}} = \prod_{t=\bar{t}}^{t} \gamma(t),$$

$$\gamma(t) = \begin{cases} (1+\gamma)^{-|t|} & \text{if } t < 0; \\ (1+\gamma)^{t} & \text{if } t \geq 0. \end{cases}$$

$$\kappa(t) = \begin{cases} (1+\kappa)^{-|t|} & \text{if } t < 0; \\ (1+\kappa)^{t} & \text{if } t \geq 0. \end{cases}$$

$$\frac{\partial \kappa(t)}{\partial \bar{t}} = \begin{cases} |t| (1+\kappa)^{|t|-1} & \text{if } t < 0; \\ -t (1+\kappa)^{-(t+1)} & \text{if } t \geq 0. \end{cases}$$

and q_i^t, π_i^t, and P_i^t, are given by Eqs. (4.4), (4.5), and (4.18) respectively.

In terms of Eqs. (4.4) - (4.9), we have

$$\frac{\partial \pi_i^t}{\partial \bar{t}} = f_i^t \frac{\partial \kappa(t)}{\partial \bar{t}}.$$

Substitute $\frac{\partial q_i^t}{\partial \bar{t}}$ and $\frac{\partial \kappa(t)}{\partial \bar{t}}$ into $\frac{\partial Z_F\left(L^t, \bar{t}\right)}{\partial \bar{t}}$, we can obtain

$$(1+\kappa)^{\bar{t}} = \frac{\sum_{t=\bar{t}}^{m+\bar{t}} t(1+\kappa)^{-1}\left(\Delta L^t C_r + \Delta n_s^t C_s + \Delta F^t C_t\right)}{\sum_{t=\bar{t}}^{m+\bar{t}} \sum_{i=1}^{n_s^t} \left(f_i^t + S - c\right)\left(\prod_{y=\bar{t}}^{t} \gamma(y) \exp\left(-\theta \pi_i^t\right) + q_i^t t (1+\kappa)^{-1}\left(1 - \theta f_i^t \kappa(t)\right)\right)}$$

for $\bar{t} < 0$, and

$$(1+\kappa)^{-\bar{t}} = \frac{\sum_{t=\bar{t}}^{m+\bar{t}} t(1+\kappa)^{-1}\left(\Delta L^t C_r + \Delta n_s^t C_s + \Delta F^t C_t\right)}{\sum_{t=\bar{t}}^{m+\bar{t}} \sum_{i=1}^{n_s^t} \left(f_i^t + S - c\right)\left(-\prod_{y=\bar{t}}^{t} \gamma(y) \exp\left(-\theta \pi_i^t\right) + q_i^t t (1+\kappa)^{-1}\left(1 - \theta f_i^t \kappa(t)\right)\right)}$$

for $\bar{t} \geq 0$.

On the basis of Eq. (4.1), while $\bar{t} < 0$, the rail line project is fast-tracked $\left|\bar{t}\right|$ years. Specifically,

$$\left|\bar{t}\right| = \log_{1+\kappa} \frac{\sum_{t=\bar{t}}^{m+\bar{t}} \sum_{i=1}^{n_s^t} \left(f_i^t + S - c\right)\left(\prod_{y=\bar{t}}^{t} \gamma(y) \exp\left(-\theta \pi_i^t\right) + q_i^t t (1+\kappa)^{-1}\left(1 - \theta f_i^t \kappa(t)\right)\right)}{\sum_{t=\bar{t}}^{m+\bar{t}} t(1+\kappa)^{-1}\left(\Delta L^t C_r + \Delta n_s^t C_s + \Delta F^t C_t\right)}.$$

Similarly, while $\bar{t} \geq 0$, the rail line project is delayed $\bar{t}$ years. Specifically,

$$\bar{t} = \log_{1+\kappa} \frac{\sum_{t=\bar{t}}^{m+\bar{t}} \sum_{i=1}^{n_s^t} \left(f_i^t + S - c\right)\left(-\prod_{y=\bar{t}}^{t} \gamma(y) \exp\left(-\theta \pi_i^t\right) + q_i^t t (1+\kappa)^{-1}\left(1 - \theta f_i^t \kappa(t)\right)\right)}{\sum_{t=\bar{t}}^{m+\bar{t}} t(1+\kappa)^{-1}\left(\Delta L^t C_r + \Delta n_s^t C_s + \Delta F^t C_t\right)}.$$

4.4 Numerical examples

To facilitate the presentation of the essential ideas and contributions of this chapter, two illustrative examples are employed to illustrate the effects of the total population growth factor and also of the interest rate on the implementation adaptability of a candidate rail transit line. To certain extent, the numerical results are mainly used for illustration purposes. Therefore, some of them may not be realistic partly due to the input parameters and/or the model assumptions. For instance, it is assumed in the model proposed in this Chapter 4 that the length of the candidate rail transit line can be varied year by year. In practice, it may however be varied by each design year period.

In this Chapter, implementation adaptability of the candidate rail transit line is specifically implied by the project start time. If the candidate project start time is positive, the implication is that the candidate rail transit line should be deferred. If the candidate project start time is negative, it implies that the candidate rail transit line should be fast-tracked.

The configuration of the candidate rail transit line is shown in Figure 3.1of Chapter 3. For simplicity without loss of generality, the average station spacing is 1.1km (Li et al, 2012d). Other input notation parameters are summarised in Table 4.1, and shown as follows.

Table 4.1 Parameters.

Symbol	Definition	Value
$m(\text{Years})$	Planning and operation horizon	4
i	Interest rate	0~20%
γ	Growth rate of the total population along the transportation corridor	-0.1~0.3
$f_0(\text{HK\$})$	Fixed component of fare for rail service	4
$f_r(\text{HK\$/km})$	Variable component of fare per unit distance	0.1
$h_t(\text{Hour})$	Average headway of train during morning peak hour in year t	3/60
$L(0)$	Initial value of rail length in year 0	20
θ	Sensitivity parameter in travel demand function	0.02
α / β	Parameters of households' utility function	0.2/0.8
$I(\text{HK\$})$	Average daily household income	400
$r_0(\text{HK\$})$	Average daily housing rent in the CBD	300
$v^t(\text{Km/hour})$	Average train speed in year t	60
P^0	Initial value of total population in year 0	100000
Y_i^t	Housing supply at residential location i in year t	$P^t / L(t)$
λ	Parameter for waiting time	0.5
$\varsigma(\text{Hour})$	Constant terminal time	5/60
F^t	Fleet size for trains vehicles	5
$S(\text{HK\$})$	Fare subsidy	5
$c(\text{HK\$})$	Variable cost component of rail fare	7

$\mu_c / \mu_w / \mu_i$	Parameters for travel cost function	80/100/60
$C_r\,(\text{HK\$})$	Daily construction cost for rail line per kilometer	28000
$C_s\,(\text{Million HK\$})$	Daily construction cost of each rail station	5000

4.4.1 Example 1

In Figure 4.2, the solid lines are number (no.) of years to be deferred and the dashed lines are no. of years to be fast-tracked. For instance, the solid line of 2 implies that the candidate rail transit line should be deferred 2 years under profit maximisation. The red solid line of 0 implies the DNA alternative, namely implementing the candidate rail transit line as suggested in the feasibility study. The dashed line of -3 implies that the candidate rail transit line should be fast-tracked 3 years.

It can also be found that the fast-tracked years of the candidate rail transit line increase in line with the increases in the growth factor of the total population γ increases. For instance, as γ increases from 0.15 to 0.25 and κ decreases from 12.5% to 10%, the fast-tracked years increase from 2 years to 5 years. The deferred years of the candidate rail transit line increases as the growth factor of the total population decreases. For instance, the deferred years increase from 2.0 to 4.0, as the growth rate of the total population decreases from 0 to -0.05.

The dashed line of $\kappa/\gamma = 1$ can be used to analyse the effects of interest rate κ and the average growth factor of the total population γ on the implementation adaptability of the candidate rail transit line. For example, the intersection of lines $\kappa/\gamma = 1$ and $\bar{t} = 0$ is (1.095,9.5\%). However, on the right side of 1.095 and areas of $\kappa < 9.5\%$, $\gamma > \kappa$, the candidate rail transit line should be fast-tracked. In this area, $\kappa/\gamma < 1$, the alternative of fast-tracking the candidate rail transit line is better.

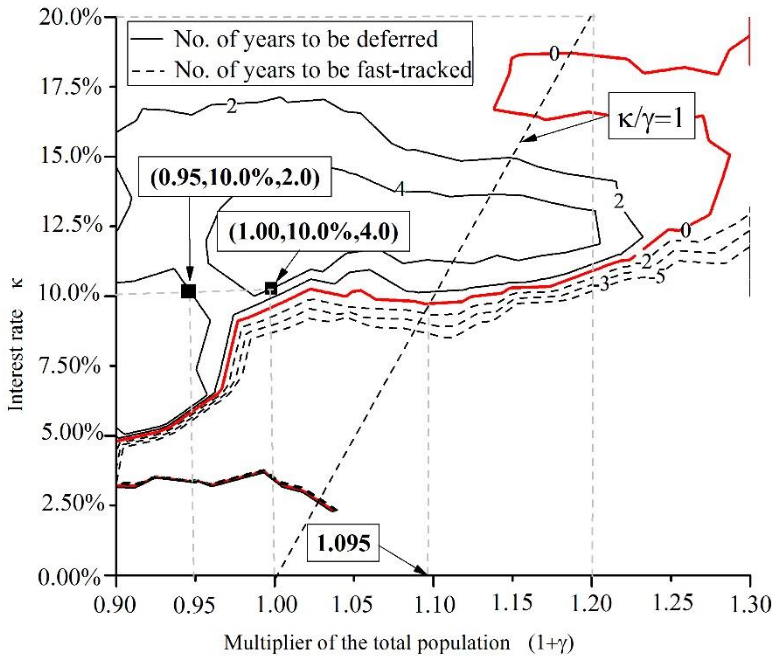


Figure 4.2: Implementation adaptability of the candidate rail transit line over years.

Table 4.2: Numerical results

γ	κ	$L^t\left(\text{Km}\right)$			$\bar{t}$	Capital Cost (HK$)	Benefits as full time (HK$)	Benefits at 1/3 time (HK$)	Return at full time (%)	Return at 1/3 time (%)
		$t=1$	$t=2$	$t=3$						
0.3	0.1	**22.50**	**23.26**	**24.16**	**-11**	8.69E+7	5.21E+7	1.75E+7	**60.0%**	20.2%
0.3	0.2	**21.08**	**21.89**	**22.91**	0.54	8.27E+7	1.92E+7	8.51E+7	23.2%	10.3%
0.3	0.3	**18.15**	**19.07**	**20.33**	0.81	7.42E+7	1.31E+7	4.47E+7	17.6%	6.0%
0.2	0.1	23.82	24.41	25.07	**-2.9**	9.03E+7	3.02E+7	1.01E+7	**33.4%**	11.2%
0.2	0.2	22.84	23.65	24.66	1.11	8.81E+7	1.46E+7	4.95E+7	16.6%	5.6%
0.2	0.3	22.26	23.36	24.87	0.88	8.75E+7	1.12E+7	3.82E+7	12.8%	4.4%
0.1	0.1	20.80	21.18	21.61	2.02	8.04E+7	1.60E+7	5.38E+7	19.9%	6.7%
0.1	0.2	21.63	22.32	23.18	1.56	8.40E+7	1.08E+7	3.65E+7	12.8%	4.4%
0.1	0.3	18.05	18.91	20.09	0.93	7.36E+7	9.50E+7	3.24E+7	12.9%	4.4%
-0.1	0.1	21.86	22.92	24.24	0.93	6.48E+7	2.21E+7	5.05E+7	12.8%	4.4%
-0.1	0.2	17.30	18.04	19.04	0.96	5.35E+7	1.82E+7	4.30E+7	12.5%	4.2%
-0.1	0.3	12.76	13.28	13.97	0.98	4.23E+7	1.44E+7	3.56E+7	11.9%	4.1%

Table 4.2 presents the detailed numerical results. It can be found that the rail length of the candidate rail transit line decrease as annual interest rate increases. For instance, as growth factor of the total population $\gamma = 0.3$, the optimal rail length in year 1 decreases from 22.50km to 18.15km, in year 2, it decreases from 23.26km to 19.07 km and in year 3, it decreases from 24.16km to 20.33km.

The benefits as full time in Table 4.2 are the summation of the financial flows of the candidate rail transit line in the planning and operation horizon, as defined by Eq. (4.22). The benefits at 1/3 time is the summation of the first year financial flows of the candidate rail transit line. The capital cost is the summation of the construction cost of the rail line and rail stations defined by Eq. (4.23). It can be found that the return at full time and return at 1/3 time under the alternative of fast-tracking the candidate rail transit line is larger than that under alternative of deferring the candidate rail transit line. For instance, the returns at full time are 60.0% under the alternative of fast-tracking the candidate rail transit line 11.0 years and 33.4% under the alternative of fast-tracking the candidate rail transit line 2.9 years respectively, which are larger than those under alternative of deferring the candidate rail transit line.

4.4.2 Example 2

In this numerical example, sensitivity analyses are conducted to explore the implementation adaptability of the candidate rail transit line over years in cities with high population densities, such as Shanghai, Hong Kong, and Taipei in China. Specifically, short-term design variables, namely headway and fare, on the implementation adaptability of the candidate transit line are examined. In Shanghai, the average annual interest rate κ for rail construction is 6.5% and average growth rate γ of the total population is 1.5% (Source: csj.sh.gov.cn). Similarly, in Hong Kong, the average annual interest rate κ for rail construction is 5.0% and average growth rate γ of the total population is 0.8% (Source: http://www.gov.hk/en/about/abouthk/factsheets/docs/population.pdf). In Taipei, the average annual interest rate κ for rail construction in Taipei is 8.0% and average growth rate γ of the total population is 0.092% (Source: pop.mof.gov.tw). Other input parameters are the same as in Example 1.

In Figure 4.3, the optimal project start time for Hong Kong is deferring 0.5 years in this numerical example. Similarly, the optimal project start time for Shanghai is fast-tracking 3.5 years. By contrast, the optimal project start time for Taipei is deferring 7.5 years.

It can also be found that the effect of average growth factor of the total population γ is more significant than the average annual interest rate. For instance, while the average growth factor of the total population increases from 0.8% to 1.5%, and the average annual interest rate κ increases from 5% to 6.5%, the optimal project start time varies from deferring 0.5 years to fast-tracking 3.5 years.

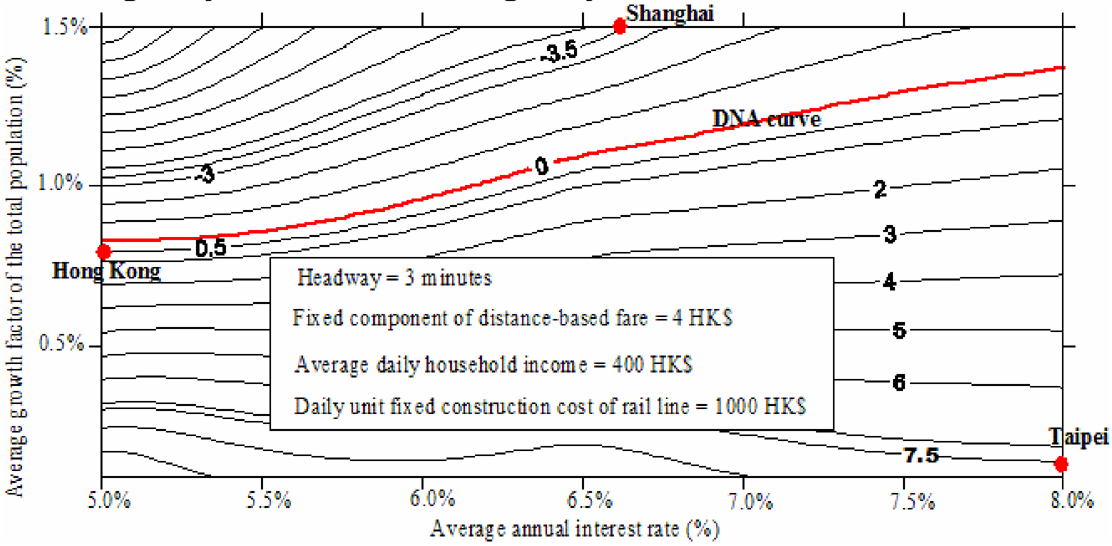


Figure 4.3: The implementation adaptability of the candidate rail transit line over years.

It can also be found that the effect of average growth factor of the total population γ is more significant than the average annual interest rate κ, on deferred years of the candidate rail transit line.

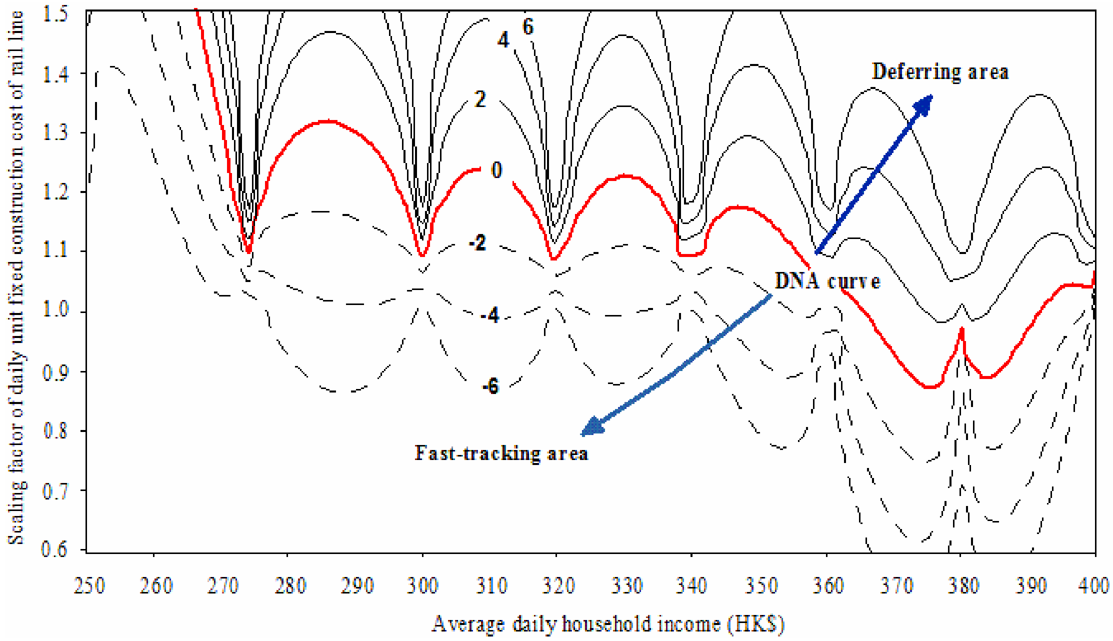


Figure 4.4: The effects of average daily household income and scaling factor of unit fixed construction cost of rail line on implementation adaptability of the candidate rail transit line

Figure 4.4 presents the effects of average daily household income and scaling factor of construction cost of rail line on implementation adaptability of the candidate rail transit line. The lines in the deferring area are number (no.) of years to be deferred, while the lines in the fast-tracking area are no. of years to be fast-tracked. The DNA (do-nothing-alternative) curve is the line of 0. For household average income of 400 HK$ and scaling factor of unit fixed construction cost of rail line 1.0, the optimal project start time is 0.5 years, namely deferring 0.5 years.

Table 4.3 Numerical results of with respect to I and scaling factor of C_r (Km)

Households Income I (HK$)	Rail length results of L^t for scaling factor of C_r (Km)								
	0.5			**1**			**1.5**		
	$t=1$	$t=2$	$t=3$	$t=1$	$t=2$	$t=3$	$t=1$	$t=2$	$t=3$
HK$ 163	59.1	59.5	59.9	18.1	18.3	18.4	10.5	10.6	10.6
HK$ 274	19.5	19.6	19.8	8.8	8.9	9.0	5.5	5.5	5.6
HK$ 400	**11.2**	**11.3**	**11.4**	**5.4**	**5.4**	**5.5**	3.4	3.4	3.5

Notes:

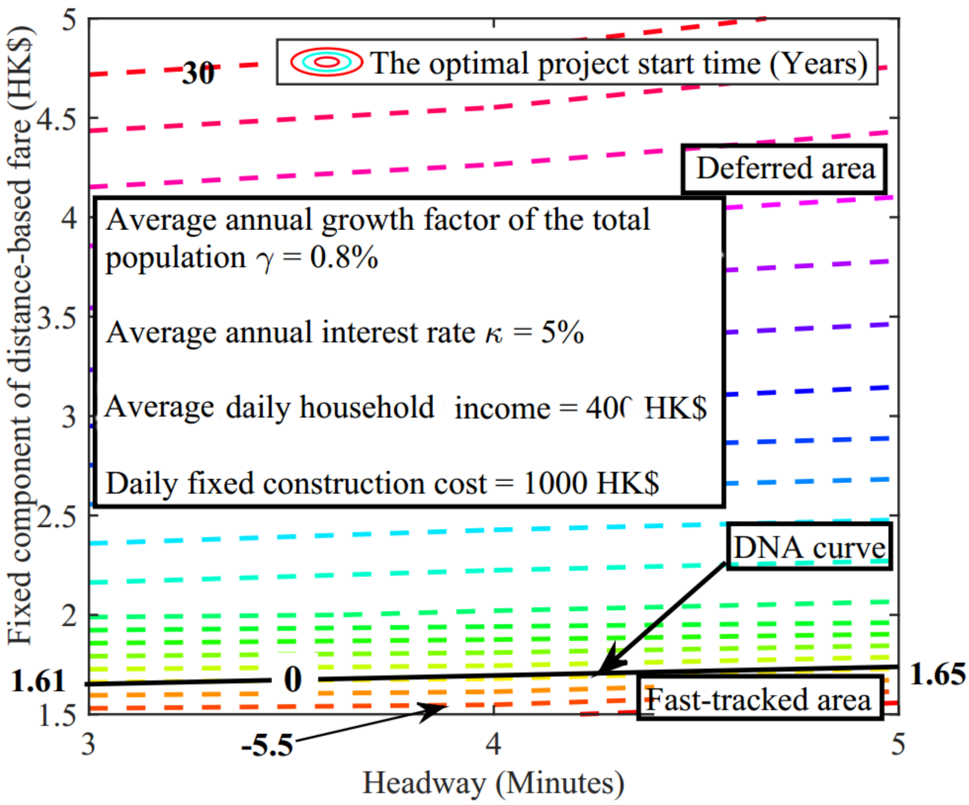
- $L^t\left(t \in \{1,2,3\}\right)$ represents new constructed rail length in year t;
- With loss of generality, average annual growth factor γ = 0.8%, average annual interest rate κ = 5%, and daily construction cost of rail line C_r = 1000 HK$/km.

Table 4.3 shows the effects of average daily household income and rail construction cost on rail line length. It can be found that the rail length of the candidate rail transit line decrease as the scaling factor of daily unit fixed construction cost increases from 0.5 to 1.5. For instance, as the scaling factor of daily unit fixed construction cost increases from 0.5 to 1, the optimal rail length decreases from 11.2 km to 5.4 km in year 1, from 11.3 km to 5.4km in year 2, and from 11.4 km to 5.5 km in year 3.

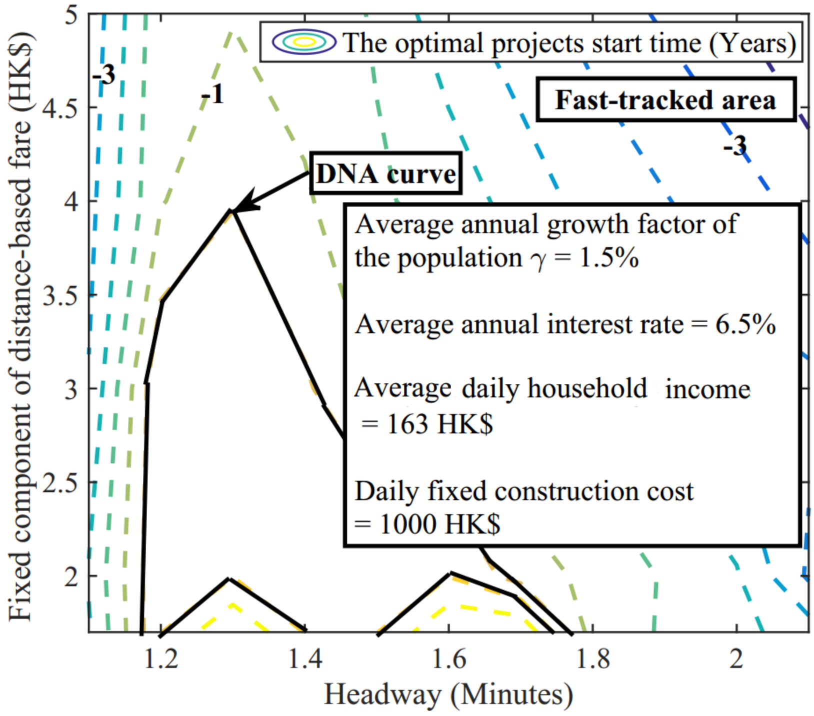
This numerical result is reasonable: since with a fixed budget, while the daily unit fixed construction cost increases, the maximal rail line length decreases. The less maximal rail line length may lead to shorter optimal rail line length. In terms of Proposition 1, the optimal rail line length decreases as well.

It can also be found that the rail length of the candidate rail transit line decreases as the average daily household income I increases from 163 HK$ to 400 HK$. For instance, as the average daily household income I increases from 163 HK$ to 274 HK$, the rail length decreases from 59.1 km to 19.5 km in year 1, from 59.5 km to 19.6 km in year 2, and from 59.9 km to 19.8 km in year 3. The above numerical results of household income and rail construction cost on rail line length are in accord with the Proposition 1.

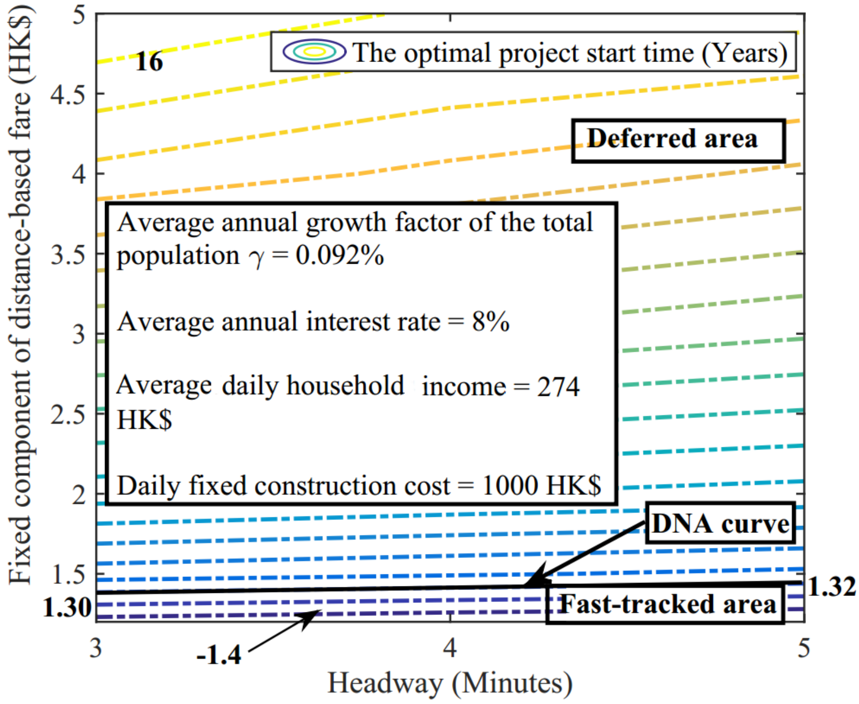
The reasons may be that households with high income level commonly have more travel mode choices, such as private car. This alternative multi-mode choice reduces their dependency on urban rail service. Thus, the rail length in areas with high household income may shorter than those areas with low household income. The other reason may be that, with the same budget, the maximal rail line length in areas with high household income is shorter than in areas with low household income. The Proposition 1 also indicates that the optimal rail line lengths in areas, with high household income, are shorter than those areas with low household income.



(a) Hong Kong



(b) Shanghai



(c) Taipei

Figure 4.5: The effects of headway and rail fare on implementation adaptability of the candidate rail transit line for Hong Kong, Shanghai, and Taipei.

Figure 4.5 presents the effects of headway and rail fare on the implementation adaptability of the candidate rail transit line for the cities of Hong Kong, Shanghai and Taipei.

In Figure 4.5(a), -5.5 means that the candidate rail transit line is fast-tracked 5.5 years in Hong Kong. DNA curve is the line of 0, which divides this figure into defer area and fast-track area.

In Figure 4.5(b), -1 implies that the candidate rail transit line is fast-tracked 1 year in Shanghai. DNA curve is the line of 0. For Shanghai, there is no alternative of deferring in this numerical example.

In Figure 4.5(c), -1.4represents that the candidate rail transit line is fast-tracked 1.4 years in Taipei. DNA curve is the line of 0, which divides this figure into defer area and fast-track area.

From Figure 4.5(a) and 4.5(b), it can also be found that the optimal project start time is more significantly affected by the fixed component of rail fare than the train headway in Hong Kong and Taipei. Figure 4.5(c) implies that the optimal project start time is less significantly affected by fixed component of rail fare than headway in Shanghai. In order words, the effects of fixed component of rail fare and headway on optimal project start time may have different effects for different cities.

This may be attributed to the complex effect mechanism of rail fare and headway on the optimal project start time. For instance, if the rail fare increases, the fare revenue of rail operator increases firstly, but decreases later. While the rail fare is too expensive for households, fewer households will travel with urban rail service. This result implies that the rail operator should determine the rail fare and headway, according to the development level of local city. It is not appropriate to use the schemes of rail fare and headway from other cities.

Figure 4.6 presents the implementation adaptability of the candidate rail transit line for cities of Hong Kong, Shanghai, and Taipei. The population densities in Hong Kong, Shanghai, and Taipei are 34,000, 13,400, and 9,951 persons/km, respectively.

The optimal project start time for Shanghai is -3.5 years, i.e. fast-tracking 3.5 years in this numerical example. The optimal project start time for Hong Kong is 0.5 years, i.e. deferring 0.5 years, while 7.5 years for Taipei, i.e. deferring 7.5 years.

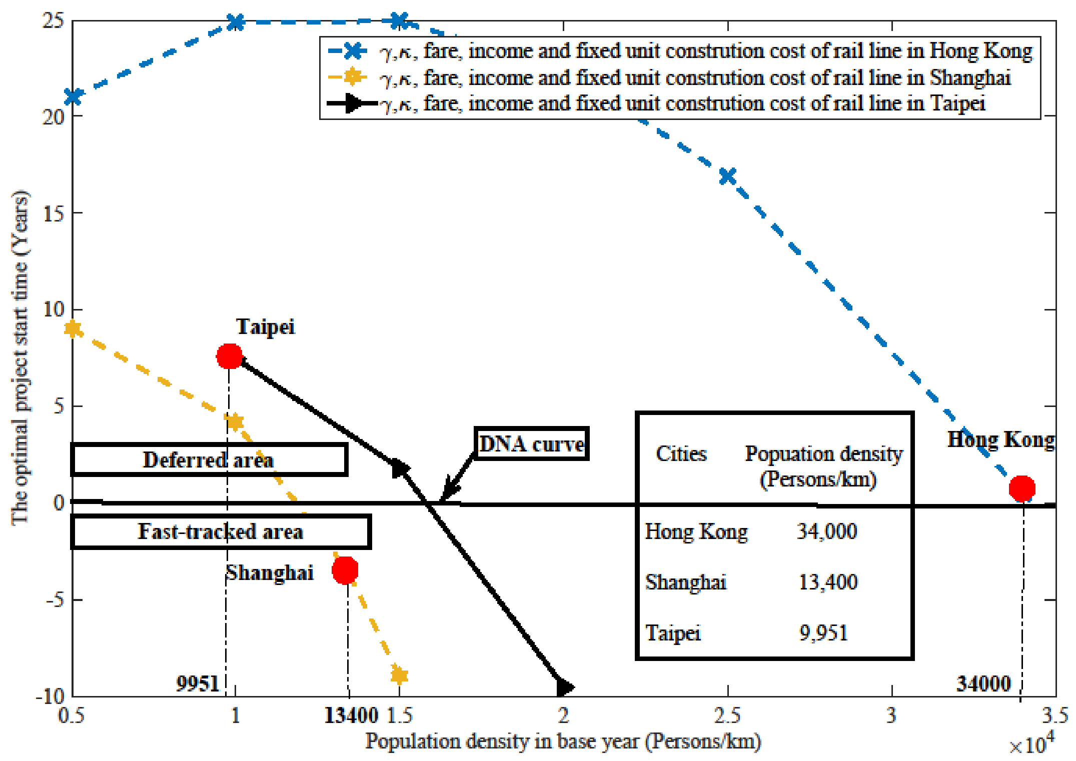


Figure 4.6: The implementation adaptability of the candidate rail transit line for Hong Kong, Shanghai, and Taipei

Table 4.4 Numerical results of CFT, BFT, $BF_{1/3}$,$RF_{1/3}$ with respect to γ and κ.

City	γ	κ	CFT	BFT	$BF_{1/3}$	RFT	$RF_{1/3}$
Hong Kong	0.8%	5%	4.25E+07	1.56E+07	5.19E+06	36.7%	12.2%
Shanghai	**1.5%**	6.5%	3.72E+07	1.58E+07	5.29E+06	**42.3%**	**14.2%**
Taipei	0.092%	8%	3.67E+07	1.15E+07	3.87E+06	31.3%	10.5%

Notes:

CFT represents capital cost as full time, measured in HK$; BFT represents benefit as full time, measured inn HK$; $BF_{1/3}$ represents benefit at 1/3 time, measured in HK$; RFT represents return at full time; $RF_{1/3}$ represents return at 1/3 time.

The benefits as full time in Table 4.4 are the summation of the financial flows of the candidate rail transit line in the planning and operation horizon, as defined by Eq. (4.22). The benefits at 1/3 time is the summation of the first year financial flows of the candidate rail transit line. The capital cost is the summation of the construction cost of the rail line and rail stations defined by Eq. (4.23). It can be found that the return at full time and return at 1/3 time of the candidate rail transit line in Shanghai is larger than in Hong Kong and Taipei. The returns at full time in Shanghai is 42.3% and the return at 1/3 time in Shanghai is 14.2%. This can be attributed to the high average growth factor of the total population in Shanghai, which is 1.5%, larger than Hong Kong and Taipei.

4.5 Summary of this chapter

This chapter proposes a closed-form mathematical programming model to explore implementation adaptability of a rail transit line over years. In contrast with the traditional single-period NDP models for rail transit line, the proposed models have the following merits: The candidate rail transit line can be fast-tracked or deferred; the detailed optimal fast-tracked or deferred time is given explicitly; the optimal rail transit line length in each year is given explicitly.

The proposed model explores the growth factor effects of the total population and annual interest rate effects on implementation adaptability of the candidate rail transit line. The proposed model offers several insights for over-year NDP. For instance, the fast-tracked years of the candidate rail transit line increases as the growth factor of the total population γ increases. The rail length of the candidate rail transit line decreases as the interest rate increases. The return at full time and return at 1/3 time under the alternative of fast-tracking the candidate rail transit line is larger than that under the alternative of deferring the candidate rail transit line.

In this chapter, the implementation adaptability analysis of the candidate rail transit line is conducted, with an assumption that the population density along the linear corridor is independent from space and time. The spatial and temporal correlation of population densities, however, may significantly affect the design results of the candidate rail transit line over years, which is investigated in Chapter 5.

Chapter 5 Modelling the effects of spatial and temporal correlation of population densities on the design of a rail transit line over years

As stated in **2.2.2** of Chapter 2, population densities at different residential locations along a rail line are spatially and temporally correlated.

For instance, population growth in one location can positively or negatively influence the population growth in another. Given the total population along a linear rail transportation corridor, more households may choose the central business district (CBD) to live, instead of choosing suburban communities and new towns with lower population density at the outlying areas. In such case, a negative spatial correlation relationship may exist between population densities at the CBD, and that at the suburban communities or new towns.

The existence of spatial and temporal correlation of population densities has been identified in many previous studies. In this chapter, a closed-form mathematical programming model is proposed to investigate the effects of spatial and temporal correlation of population densities on design of a candidate rail transit line over years.

In the proposed model, the optimisation objective is social welfare budget maximisation. The social welfare budget is defined as the summation of expected social welfare and social welfare margin. The model decision variables include rail line length, rail station number and project start time of the candidate rail transit line. Closed-form solutions for the proposed rail design model are given explicitly for different scenarios with various constraints.

The remainder of this chapter is organised as follows. In the next section, a brief background of the research problem is firstly presented, with the basic considerations given in Section 5.2. A rail design model is introduced in Section 5.3, taking account both of the spatial and temporal correlation of population densities along a linear transportation corridor. Section 5.4 gives two illustrative numerical examples to show the application and contributions of the proposed model. A summary of this chapter is given in Section 5.5.

5.1 Background of the research problem

5.1.1 Motivation

Traditional rail line design models aimed to minimise the total system cost or total passenger travel time, and have been proposed for use in many large European and American cities with relatively lower population densities in comparison with the one in Asia. For instance, Vuchic and Newell (1968) proposed analytical models to optimise rail station spacing with the objective of minimising total passenger travel time. Wirasinghe et al (2002) developed a methodology for optimising rail terminus locations in a cross-town corridor in Calgary, Canada with the objective of minimising total passenger travel cost.

In contrast, Lam and Zhou (2000), Zhou et al (2005), and Li et al (2012a) have suggested models for rail line design with the objectives of profit maximisation. Li et al (2012b) presented a model which was used for investigating the effects of integrated rail and property development on rail line design with the objective of social welfare maximisation. Their objectives of profit or social welfare maximisation are more appropriate for large Asian cities with higher population densities, such as Shanghai and Hong Kong.

Population densities in the above rail design models were generally assumed to given and fixed, and own the property of independence of irrelevant alternatives (IIA) spatially and/or temporally. This assumption was acceptable, because their rail design models were static and proposed for a particular future design year.

Although this IIA assumption of population densities is applicable in many contexts, it may not always be adequate for the long term planning for design of rail transit line. This IIA assumption does not taken into account the spatial and temporal correlation of population densities, which may significantly affect the design results for a candidate rail transit line over years.

In a linear rail transportation corridor, the population densities in each residential location vary year by year. If increase of population density in the first year leads to an increase in the second year, positive temporal correlations then exist between population densities in the first year and second year, and vice versa. If the increase of population density in one residential location leads to the increase of population density in another residential location, positive spatial temporal correlations then exist between population densities in these two residential locations.

To take account of the spatial and temporal correlation of population densities, the Nested logit model (e.g. Train et al, 1987; Bhat and Guo, 2004) and the C-logit model (e.g. Cascetta et al 1996; Zhou et al, 2012) have been proposed for modelling the residential location and/or travel choice behaviours. Correlations between choice alternatives are considered in these two types of choice models. Train et al (1987) presented nested logit model to investigate households' choice behaviour among local telephone service options with consideration of correlations between these choices. Bhat and Guo (2004) applied the Nestedlogit model to examine residential location choice taking account of spatial correlation of location choices. Cascetta et al (1996) proposed the C- logit model to investigate route choice with overlapping paths. Compared with the Nested logit model, the C-logit model has a simple closed-form probability expression and is simplerfor model calibration (Zhou et al, 2012).

It should be noted, however, that the Nested logit and C-logit models only use the correlations between alternatives to improve the estimated results in terms of the choice probability. Hence, for the application of Nested logit and C-logit models, the model results give the estimated values only and may not be useful for the explicit design of rail transit line.

To bridge the research gap, the effects of spatial and temporal correlation of population densities on design of rail transit line, are explored in this chapter. A closed-form programming model is proposed, with an optimisation objective of budget social welfare maximisation. The budget social welfare is defined as a summation of expected social welfare and social welfare margin. The model decision variables include rail line length, rail station number and the project start time.

5.1.2 Problem statement and contributions

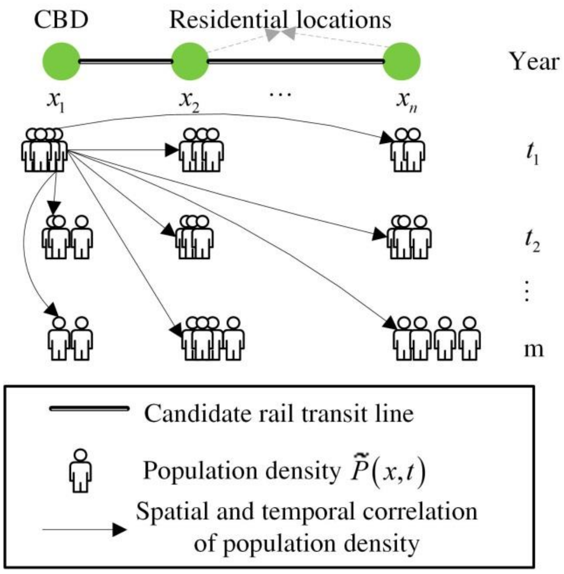


Figure 5.1: Spatial and temporal correlation of population densities in a candidate rail transit line over years

As stated above, in this chapter, the effects of spatial and temporal population density correlations on design of a candidate rail transit line over several years are investigated. The candidate rail transit line is developed for the case of the monocentric city in a linear transportation corridor.

As shown in Figure 5.1, the monocentric city is separated into n residential locations from the CBD to the city boundary, and the planning time horizon is divided into m equal time periods. n and m are positive integers. $L(t_1)$ and $L(t_2)$ are the lengths of candidate rail transit line with respect to project start time in year t_1 and t_2, which can be determined endogenously with the use of the proposed model. $P(x_1,t_1)$ and $P(x_2,t_2)$ are the population densities in residential location x_1 and x_2 in year t_1 and t_2 respectively, with $\forall x_1, x_2 \in [0,B]$ and $\forall t_1, t_2 \in [0,m]$. Spatial correlation exists between population densities $P(x_1,t_1)$ and $P(x_2,t_1)$, and $P(x_1,t_2)$ and $P(x_2,t_2)$, while temporal correlation exists between population densities of $P(x_1,t_1)$ and $P(x_1,t_2)$, and $P(x_2,t_1)$ and $P(x_2,t_2)$.

Two major extensions to the related literature are made in this chapter: (i) the effects of spatial and temporal correlation of population densities on design of a candidate rail transit line over years are investigated by a closed-form mathematical programming model; (ii) the analytical optimal solutions of design variables for the candidate rail transit line over years are obtained with the proposed model.

5.2 Basic considerations

5.2.1 Assumptions

To facilitate the presentation of the essential ideas, some basic assumptions are made in this chapter.

A1 The candidate rail transit line in this chapter is assumed to be linear and start from the CBD and then be built along a linear monocentric city (Tsao et al, 2009; Liu et al, 2009). The candidate rail transit line project in each period is assumed to finish on time and the rail service is expected to be supplied at the end of each design period (Lo and Szeto, 2009).

A2 The standard deviation (SD) of the population density is assumed to be an increasing function with respect to its mean value. This function is referred to as the stochastic population density function. In addition, the stochastic population density function is assumed to be a non-decreasing function with respect to its mean value. (Lam et al, 2008)

A3 Households' responses to the quality of the rail service provided are measured by a generalised travel cost that is a weighted combination of in-vehicle time, access time, waiting time and the fare (Qian and Zhang, 2011). Households are assumed to be homogeneous and have the same preferred arrival time at the workplace located in the CBD. This study focuses mainly on households' home-based work trips, which are compulsory activities. The number of trips, thus are not affected by other factors, such as income level (Li et al, 2012a).

A4 The study period is assumed to be a peak hour, for instance the morning peak hour, which is usually the most critical period in the day (Li et al, 2012c).

A5 As stated in Chapter 3, the design variables of a candidate rail transit line can be divided into two types: long-term and short-term design variables. Long-term design variables cannot be changed during operation stage, and short-term design variables can be updated from time to time.

In contrast with the design variables in Chapter 4, rail station number is one design variable in this chapter. This is because spatial correlation between population densities is directly related with rail station number. For instance, as rail station number increases, while population distribution is decentralised in more residential locations around rail stations, spatial correlations of population densities may become weak.

5.2.2 Spatial and temporal correlation of population densities

To take into account the spatial and temporal correlation of population densities, it is assumed that there exists a perturbation in population density. The yearly perturbed population density $P(x,t)$ is given by the following equation (Yin et al, 2009)

$$P(x,t) = P(x,t) + \varepsilon, \tag{5.1}$$

where $P(x,t)$ is the expected population density at location x in year t, $\mathrm{E}\left[P(x,t)\right] = P(x,t)$; ε is a random term, with $\mathrm{E}[\varepsilon] = 0$. It is noted that the expected population density $P(x,t)$ is a deterministic value. In terms of *A2*, the SD of population density can be expressed as (Lam et al, 2008)

$$\sigma\left[P(x,t)\right] = \sqrt{\mathrm{var}\left[P(x,t)\right]} = \varphi\left(P(x,t)\right), \tag{5.2}$$

where $\varphi(\bullet)$ is defined as the stochastic population density function, which represents the functional relationship between the mean value and the standard deviation of the stochastic population density. Specifically, a coefficient of variation of population density CV_P is defined as

$$CV_P = \frac{\varphi\left(P(x,t)\right)}{P(x,t)}, \tag{5.3}$$

where CV_P is a standardised measure of dispersion of the probability distribution or frequency distribution of population density.

To take spatial and temporal correlations between population densities into account, the following spatial and temporal covariance is defined as (Shao et al, 2014)

$$\sigma_P\left(x_1,t_1;x_2,t_2\right) = \mathrm{cov}\left[P\left(x_1,t_1\right),P\left(x_2,t_2\right)\right] = \rho_{x_1,t_1}^{x_2,t_2}\varphi\left(P\left(x_1,t_1\right)\right)\varphi\left(P\left(x_2,t_2\right)\right), \tag{5.4}$$

where $\rho_{x_1,t_1}^{x_2,t_2}\left(-1\le\rho_{x_1,t_1}^{x_2,t_2}\le 1\right)$ is the correlation coefficient, which is an important measurement reflecting the statistical correlation between $P(x_1,t_1)$ and $P(x_2,t_2)$. There are three correlation coefficient cases: negative, positive or zero, representing negative, positive statistical dependence or statistical independence of population densities. Specifically, with $x_1 = x_2$ and $t_1 = t_2$, the spatial and temporal covariance becomes the standard deviation value.

A similar equilibrium exists for households presented in Eqs. (4.13) - (4.15) of Chapter 4. The expected population density of households in residential location x in year t is expressed as below

$$P(x,t)=\frac{r_0^t}{I\beta}\left(1-\frac{\pi(x,t)}{I}\right)^{\frac{\alpha}{\beta}}Y(x,t), \tag{5.5}$$

where r_0^t represents the rent per day in the CBD in year t, I represents average daily household income, $\pi(x,t)$ is expected generalised travel cost from residential location x to the CBD in year t by rail, $Y(x,t)$ is house supply in residential location x in year t, and α and β are parameters for households utility function of Eq. (4.10), and $\alpha+\beta=1$ holds.

The balance between the supply and demand for housing holds, namely

$$h(x,t)P(x,t)=Y(x,t). \tag{5.6}$$

$$h(x,t)=\frac{I\beta}{r_0^t}\left(1-\frac{\pi(x,t)}{I}\right)^{-\frac{\alpha}{\beta}}, \tag{5.7}$$

where $h(x,t)$ represents the consumption of housing in residential location x in year t, measured in square meters of floor space.

The population conservation equation can be expressed by

$$\int_0^B P(w,t)\,\mathrm{d}w=P^t, \tag{5.8}$$

where P^t is the total population along the candidate rail transit line in year t, and B is the length of the rail corridor.

5.2.3 Social welfare budget

Social welfare is commonly used to assess the performance of a candidate rail transit line. Due to the yearly uncertainty associated with rail travel demand, the social welfare of the candidate rail transit line is also not a deterministic value. Because of the uncertainty of social welfare, an extra safety margin is assigned to ensure a higher probability of gaining a certain level of social welfare. In view of this, the concept of social welfare budget is proposed as follow

$$\phi(SW) = E[SW] + \lambda\sigma[SW], \tag{5.9}$$

where $\phi(SW)$ is the social welfare budget, $E[SW]$ is expected social welfare, λ is a negative parameter, $\sigma[SW]$ is the standard deviation of social welfare, and $\lambda\sigma[SW]$ is the social welfare margin.

λ relates to the requirement on ensuring a certain social welfare gain. A high value of λ implies a relatively high $\phi(SW)$, and a higher probability of social welfare gain. Formally, λ can be related mathematically to the probability that there is a gain in the budget social welfare, namely

$$p\left(SW \geq \phi(SW) = E[SW] + \lambda\sigma[SW]\right) = \delta, \tag{5.10}$$

where δ is the probability of a gain in the social welfare budget. Re-arranging terms in Eq. (5.11), then

$$p\left(\frac{SW - E[SW]}{\sigma[SW]} \geq \lambda\right) = \delta, \tag{5.11}$$

From Eq. (5.12), we can obtain

$$p\left(\frac{SW - E[SW]}{\sigma[SW]} < \lambda\right) = 1 - \delta, \tag{5.12}$$

Let $\Phi(\bullet)$ be the standard cumulative distribution function. Eq. (5.12) can be re-written as below

$$\Phi(\lambda) = 1 - \delta, \tag{5.13}$$

As $1 - \Phi(\lambda) = \Phi(-\lambda)$, Eq. (5.13) can be transformed as

$$\Phi(-\lambda) = \delta, \tag{5.14}$$

with Eq. (5.14), $\lambda = -\Phi^{-1}(\delta)$ can be obtained. Thus, the social welfare budget defined in Eq. (5.09) can be re-written as

$$\phi(SW) = E[SW] - \Phi^{-1}(\delta)\sigma[SW]. \tag{5.15}$$

The value of δ represents the government's or rail operator's attitudes toward social welfare gain. A larger δ implies a larger negative safety margin and a higher probability of a gain in social welfare budget.

Social welfare of the candidate rail transit line consists of the consumer surplus of households and the profit of rail operator. Mathematically, expected social welfare $E[SW]$ can be expressed as follow

$$E[SW] = E[CS] + E[PR], \tag{5.16}$$

where $E[CS]$ is expected consumer surplus of households, and $E[PR]$ is expected profit of rail operator.

The expected consumer surplus of households $E[CS]$ is given by

$$E[CS] = 365\left(\sum_{t=1}^{m}\int_0^B\int_0^{q(x,t)}\left(q(x,t)\right)^{-1}(w)\,\mathrm{d}w - q(x,t)\pi(x,t)\right), \tag{5.17}$$

where 365 is a parameter converting daily consumer surplus into yearly consumer surplus, $q(x,t) = E\left[q(x,t)\right]$ is expected travel demand of rail service from residential location x in year t, $\pi(x,t) = E\left[\pi(x,t)\right]$ is expected generalised travel cost from residential location x to the CBD in year t by rail, m is planning time horizon in years, B is the length of the rail corridor.

The expected profit of rail operator $E[PR]$ is given by

$$E[PR] = 365\left(\sum_{t=1}^{m}\int_0^B\left(q(x,t)\right)\left(f(x,t)-c\right)\mathrm{d}x - L(t)C_r - n_s^tC_s\right)\kappa(t), \tag{5.18}$$

where 365 is a parameter converting daily profit into yearly profit, $f(x,t)$ is rail fare, c is variable cost to supply rail service for each passenger, $L(t)$ is rail length in year t, C_r is yearly unit fixed maintenance cost of rail line, n_s^t is rail station number in year t, and C_s is yearly fixed operation cost of each rail station.

In term of *A3*, travel demand function of rail service from residential location x in year t, $q(x,t)$ is assumed to be given by an exponential function shown as below (Li et al, 2012c)

$$q(x,t) = P(x,t)\exp\left(-\theta\left(\pi(x,t)+\varepsilon_q\right)\right), \tag{5.19}$$

where θ is a positive constant, which responses the households sensitivity to the rail service level, and ε_q is a random term, with $E\left[\varepsilon_q\right] = 0$. The inverse function of travel demand can be obtained as follow

$$\left(q(x,t)\right)^{-1}\left(q(x,t)\right) = \pi(x,t) + \varepsilon_q = \frac{1}{\theta}\ln\frac{P(x,t)}{q(x,t)}, \tag{5.20}$$

Substitute it into Eq. (5.17), the following equation is obtained

$$E[CS] = 365\sum_{t=1}^{m}\int_0^B \frac{q(x,t)}{\theta}\,\mathrm{d}\,x. \tag{5.21}$$

The expected generalised travel cost consists of fare, access cost from residential locations to rail stations, waiting cost for rail service at stations, and in-vehicle cost from rail stations to the CBD, shown as below (Li et al, 2012d)

$$\pi(x,t) = f(x,t)\kappa(t) + \mu_c t_c(t) + \mu_w t_w(t) + \mu_i t_i(t), \tag{5.22}$$

where $\mu_c / \mu_w / \mu_i$ are values of access time, waiting time and in-vehicle time, respectively; $f(x,t)$ is distance-based fare for rail service, $\kappa(t)$ is a compound-account factor to convert future values to present values, $t_c(t)$ is average access time from residential locations to the rail station, with $\frac{\partial t_c(t)}{\partial n_s^t} < 0$, $t_w(t)$ is average waiting time for rail service at stations, and $t_i(t)$ is average in-vehicle cost from rail stations to CBD. The distanced-based fare $f(x,t)$ is given by

$$f(x,t) = f_0 + f_1 x, \tag{5.23}$$

where f_0 is the fixed fare component, and f_1 is the variable fare component per kilometre. Waiting time $t_w(t)$ is closely concerned with travel demand and supply of the rail service. For long-term planning, this value can be estimated using the following function

$$t_w(t) = 0.5h(t), \tag{5.24}$$

where 0.5 is a reasonable parameter for short train headway and passengers arrival time, and $h(t)$ is the average headway in year t (Guo et al, 2011).

The average headway in year t is closely concerned with cycle time of train operation $T(t)$ and fleet size of trains $F(t)$

$$h(t) = \frac{T(t)}{F(t)}. \tag{5.25}$$

The cycle time of train operation can be calculated by (Lam et al, 2002)

$$T(t) = \frac{2L(t)}{v(t)} + 2\varsigma, \tag{5.26}$$

where $L(t)$ is the rail line length in year t, $v(t)$ is the average train speed in year t, ς is average constant terminal time.

The average in-vehicle travel time from rail station i to the CBD, $t_i(t)$, is given by the distance between rail station i and the CBD $D_i(t)$, divided by the average train speed in year t, $v(t)$, namely

$$t_i(t) = \frac{D_i(t)}{v(t)}, \tag{5.27}$$

where $D_i(t)$ can be calculated as follow, if a constant station spacing is assumed

$$D_i(t) = \frac{L(t)}{n_s^t} \tag{5.28}$$

with $i \in [1, n_s^t]$.

In terms of Eqs. (5.1), (5.2), (5.19), (5.22)-(5.24), the expected travel demand of rail service $q(x,t)$ is

$$q(x,t) = P(x,t)\exp\left(-\theta\left(\pi(x,t)+\varepsilon_q\right)\right), \tag{5.29}$$

and substitute Eqs. (5.18), (5.21), and (5.26) into Eq. (5.16), we obtain the expected budget social welfare shown as below

$$E[SW] = 365\left(\sum_{t=1}^{m}\int_0^B \left(q(x,t)\right)\left(f(x,t)+\frac{1}{\theta\kappa(t)}-c\right)\mathrm{d}x - L(t)C_r - n_s^t C_s\right)\kappa(t). \tag{5.30}$$

The standard deviation of travel demand for rail service $\sigma[q(x,t)]$ is

$$\sigma[q(x,t)] = \varphi\left(P(x,t)\right)\exp\left(-\theta\left(\pi(x,t)+\varepsilon_q\right)\right), \tag{5.31}$$

and the standard deviation of budget social welfare can be calculated as follow

$$\sigma[SW] = 365\left(\sum_{t=1}^{m}\int_0^B CV_P \rho_{x_1,t_1}^{x_2,t_2} q(x,t)\left(f(x,t)+\frac{1}{\theta}-c\kappa(t)\right)\right)\mathrm{d}x. \tag{5.32}$$

5.3 Model formulation and properties

As stated above, the government or rail operator aims to maximise the social welfare budget of the candidate rail transit line by determining the optimum rail line length, rail station number and project start time of the candidate rail transit line.

5.3.1 Model formulation

In terms of Eqs. (5.15), (5.29), and (5.31), the social welfare budget maximisation model is formulated as below

$$\max_{L(t),n_s^t,\hat{t}} \phi(SW) = 365\sum_{t=1}^{m}\left(\begin{array}{l}\int_0^B q(x,t)\left(f(x,t)+\dfrac{1}{\theta\kappa(t)}-c\right)\left(1-\Phi^{-1}(\alpha)CV_P\rho_{x_1,t_1}^{x_2,t_2}\right)\mathrm{d}x \\ -L(t)C_r - n_s^t C_s\end{array}\right)\kappa(t), \tag{5.33}$$

where $\phi(SW)$ is the social welfare budget, $L(t)$ is rail line length, n_s^t is rail station number, and $\hat{t}$ is project start time of the candidate rail transit line.

5.3.2 Model properties

Proposition 1. For the budget social welfare maximisation problem (5.33), the social welfare budget $\phi(SW)$ is a decreasing function of the spatial and temporal correlation coefficient $\rho_{x_1,t_1}^{x_2,t_2}$.

Proof. In terms of Eq. (5.33), it can be found that the variation of budget social welfare $\phi(SW)$ with respect to spatial and temporal correlation coefficient $\rho_{x_1,t_1}^{x_2,t_2}$ depends on the variation of $\left(1-\Phi^{-1}(\alpha)CV_P\rho_{x_1,t_1}^{x_2,t_2}\right)$ with respect to $\rho_{x_1,t_1}^{x_2,t_2}$. It is easy to find that $\left(1-\Phi^{-1}(\alpha)CV_P\rho_{x_1,t_1}^{x_2,t_2}\right)$ is a decreasing function of $\rho_{x_1,t_1}^{x_2,t_2}$.

Proposition 2. For the social welfare budget maximisation problem (5.33), at the equilibrium of Eqs. (5.5)-(5.7), the optimal rail length $L(t)$, rail stations number n_s^t, and project start time $\hat{t}$ can be obtained by the following equations

$$L(t) = \frac{\int_0^B q(x,t)\left(f(x,t)+\dfrac{1}{\theta\kappa(t)}-c\right)\left(1-\Phi^{-1}(\alpha)CV_P\rho_{x_1,t_1}^{x_2,t_2}\right)\mathrm{d}x}{C_r},$$

$$n_s^t = \frac{\int_0^B q(x,t)\left(f(x,t)+\dfrac{1}{\theta\kappa(t)}-c\right)\left(1-\Phi^{-1}(\alpha)CV_P\rho_{x_1,t_1}^{x_2,t_2}\right)\mathrm{d}x}{C_s},$$

$$\hat{t} = \log_{1+k}\left|\frac{(C_r+C_s)(1+\kappa)}{1-\Phi^{-1}(\alpha)CV_P\rho_{x_1,t_1}^{x_2,t_2}} - \sum_{t=1}^{m}\int_0^B\left(\frac{1}{\theta}\frac{\partial q(x,t)}{\partial t}+q(x,t)\big(f(x,t)-c\big)\right)\mathrm{d}\,x\right|$$
$$-\sum_{t=1}^{m}\log_{1+\kappa}\left|\int_0^B\left(\frac{\partial q(x,t)}{\partial t}\big(f(x,t)-c\big)\right)\mathrm{d}\,x\right|,$$

with

$$\frac{\partial q(x,t)}{\partial t} = P(x,t)\,f(x,t)(1+\kappa)\left(\frac{1}{\beta\big(I-\pi(x,t)\big)}-\theta\right).$$

Proof. To obtain the optimal solution of the rail line length, the partial derivative of objective function Eq. (5.33) with respect to $L(t)$ was set to zero. Then,

$$\frac{\partial\phi(SW)}{\partial L(t)} = 365\sum_{t=1}^{m}\left(\int_0^B\frac{\partial q(x,t)}{\partial L(t)}\left(f(x,t)+\frac{1}{\theta\kappa(t)}-c\right)\left(1-\Phi^{-1}(\alpha)CV_P\rho_{x_1,t_1}^{x_2,t_2}\right)\mathrm{d}\,x - C_r\right)\kappa(t)=0,$$

where

$$\frac{\partial q(x,t)}{\partial L(t)} = \exp\Big(-\theta\big(\pi(x,t)+\varepsilon_q\big)\Big)\left(\frac{\partial P(x,t)}{\partial L(t)}-\theta P(x,t)\frac{\partial\pi(x,t)}{\partial L(t)}\right),$$

$$\frac{\partial P(x,t)}{\partial L(t)} = \frac{\alpha r_0^t}{I^2\beta^2}\left(1-\frac{\pi(x,t)}{I}\right)^{\frac{\alpha-\beta}{\beta}}Y(x,t)\frac{\partial\pi(x,t)}{\partial L(t)},$$

$$\frac{\partial\pi(x,t)}{\partial L(t)} = \mu_w\eta\frac{\partial h(x,t)}{\partial L(t)},$$

$$\frac{\partial h(x,t)}{\partial L(t)} = \frac{2}{v(t)F(t)} > 0,$$

then, the optimal rail line length can be obtained as follows

$$L(t) = \frac{\int_0^B q(x,t)\left(f(x,t)+\frac{1}{\theta\kappa(t)}-c\right)\left(1-\Phi^{-1}(\alpha)CV_P\rho_{x_1,t_1}^{x_2,t_2}\right)\mathrm{d}\,x}{C_r}.$$

Similarly, to obtain the optimal solution of the rail station number, the partial derivative of objective function Eq. (5.33) with respect to n_s^t was set to zero, namely

$$\frac{\partial\phi(SW)}{\partial n_s^t} = 365\sum_{t=1}^{m}\left(\int_0^B\frac{\partial q(x,t)}{\partial n_s^t}\left(f(x,t)+\frac{1}{\theta\kappa(t)}-c\right)\left(1-\Phi^{-1}(\alpha)CV_P\rho_{x_1,t_1}^{x_2,t_2}\right)\mathrm{d}\,x - C_s\right)\kappa(t)=0,$$

then, the optimal rail station number can be obtained as follows

$$n_s^t = \frac{\int_0^B q(x,t)\left(f(x,t)+\frac{1}{\theta\kappa(t)}-c\right)\left(1-\Phi^{-1}(\alpha)CV_P\rho_{x_1,t_1}^{x_2,t_2}\right)\mathrm{d}\,x}{C_s}.$$

To obtain the optimal solution of project start time of the candidate rail transit line, the partial derivative of objective function Eq. (5.33) with respect to t was set to zero, namely

$$\frac{\partial \phi(SW)}{\partial t}=365\sum_{t=1}^{m}\left(\begin{array}{l}\int_0^B \frac{\partial q(x,t)}{\partial t}\left(f(x,t)\kappa(t)+\frac{1}{\theta}-c\kappa(t)\right)\left(1-\Phi^{-1}(\alpha)CV_P\rho_{x_1,t_1}^{x_2,t_2}\right)\\ +q(x,t)\left(f(x,t)-c\right)\left(1-\Phi^{-1}(\alpha)CV_P\rho_{x_1,t_1}^{x_2,t_2}\right)\frac{\partial \kappa(t)}{\partial t}\mathrm{d}x-C_r\frac{\partial \kappa(t)}{\partial t}-C_s\frac{\partial \kappa(t)}{\partial t}\end{array}\right)=0,$$

where

$$\frac{\partial q(x,t)}{\partial t}=\exp\left(-\theta\left(\pi(x,t)+\varepsilon_q\right)\right)\left(\frac{\partial P(x,t)}{\partial t}-P(x,t)\frac{\partial \pi(x,t)}{\partial t}\right),$$

$$\frac{\partial P(x,t)}{\partial L(t)}=-\frac{\alpha r_0^t}{I^2\beta^2}\left(1-\frac{\pi(x,t)}{I}\right)^{\frac{\alpha-\beta}{\beta}}Y(x,t)\frac{\partial \pi(x,t)}{\partial t},$$

$$\frac{\partial \pi(x,t)}{\partial t}=f(x,t)\frac{\partial \kappa(t)}{\partial t},$$

$$\frac{\partial \kappa(t)}{\partial t}=1+\kappa,$$

then, the optimal project start time of the candidate rail transit line can be obtained as follows

$$\hat{t}=\log_{1+k}\left|\frac{(C_r+C_s)(1+\kappa)}{1-\Phi^{-1}(\alpha)CV_P\rho_{x_1,t_1}^{x_2,t_2}}-\sum_{t=1}^{m}\int_0^B\left(\frac{1}{\theta}\frac{\partial q(x,t)}{\partial t}+q(x,t)\left(f(x,t)-c\right)\right)\mathrm{d}x\right|$$
$$-\sum_{t=1}^{m}\log_{1+\kappa}\left|\int_0^B\left(\frac{\partial q(x,t)}{\partial t}\left(f(x,t)-c\right)\right)\mathrm{d}x\right|,$$

with

$$\frac{\partial q(x,t)}{\partial t}=P(x,t)\,f(x,t)(1+\kappa)\left(\frac{1}{\beta\left(I-\pi(x,t)\right)}-\theta\right).$$

5.4 Numerical examples

To facilitate the presentation of the essential ideas and contributions of this study, two illustrative examples are given below.

5.4.1 Example 1

The input parameters are summarised in Table 5.1.

Table 5.1 Parameters.

Symbol	Definition	Value
c(HK$)	Variable cost for rail service	3

C_r (HK$/km)	Daily unit fixed maintenance cost of rail line	10^6
C_s (HK$)	Daily fixed operation cost of each rail station	1.1×10^6
CV_P	Coefficient of variation of population density	0.3
f_0 (HK$)	Fixed component of fare for using the rail service	4
f_r (HK$/km)	Variable component of fare per unit distance	0.1
$F(0)$	Fleet size of trains in base year 0	5
I (HK$)	Average daily household income	400
$L(0)$(Km)	Initial value of rail length in year 0	20
m(Years)	The planning and operation time horizon	3
n_s^0	Initial value of rail station number in year 0	20
θ	Sensitivity parameter in travel demand function	0.02
P^0	Initial value of total population in year 0	100000
α / β	Parameters of households' utility function	0.2/0.8
r_0 (HK$)	Average daily housing rent in the CBD	300
t_t (Hour)	Average access time	400
$v(0)$(Km/hour)	Average train speed in year 0	60
$Y(x,0)$(Unit)	Housing supply in year 0	P^0/B
κ	Interest rate	0.01
γ	Growth rate of the total population along the transportation corridor	0.1
δ(%)	Probability of gaining budget social welfare	95
ς(Hour)	Constant terminal time	5/60
η	Parameter of waiting time function	0.5
$\mu_c / \mu_w / \mu_i$	Parameters for travel cost function	80/100/60

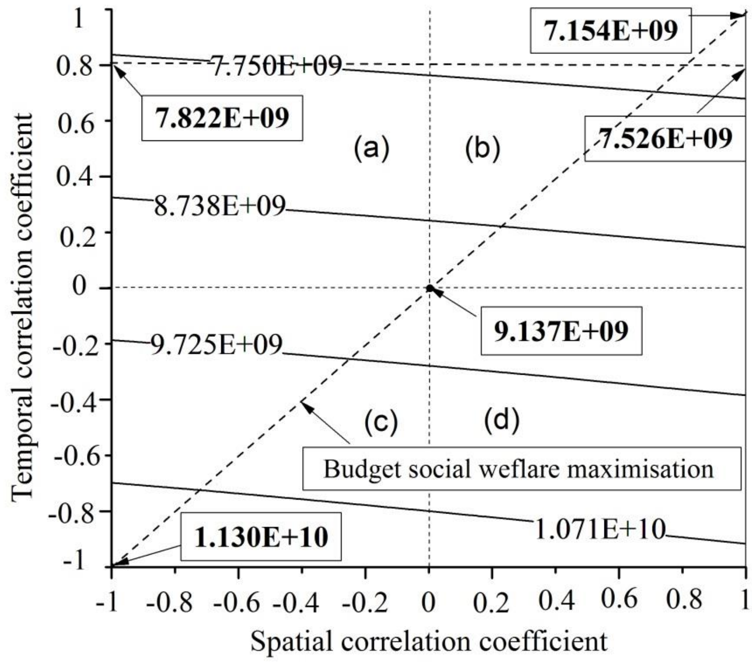


Figure 5.2: The effects of spatial and temporal correlation of population density on social welfare budget.

Figure 5.2 plots the contour of optimal social welfare budget in the space of spatial and temporal correlation coefficient (cc) with the objective of social welfare budget maximisation.

It can be seen that for a particular spatial cc, as temporal cc increases, the optimal social welfare budget of the candidate rail transit line decreases. For instance, for spatial cc 0, as temporal cc increases from -1 to 1, the optimal social welfare budget decreases from level of 1.071×10^{10} HK$ to 7.750×10^{9} HK$.

For a given total population, positive temporal cc means that the increase of population density in the first year leads to the increases of population density in the next year. As a result, households are distributed to limited residential locations and the total population has a centralised distribution.

In summary, as temporal cc increases, the total population has a more centralised distribution, the social welfare budget of the candidate rail transit line decreases. More centralised population distribution can leads to lower social welfare budget of the candidate rail transit line. Decentralised population distribution takes high social welfare budget of the candidate rail transit line.

Similarly, for given temporal cc, as spatial cc increases, the optimal social welfare budget decreases. For instance, for temporal cc 0.8, as spatial cc increases from -1 to 1, the optimal social welfare budget decreases from 7.822×10^{9} HK$ to 7.526×10^{9} HK$.

Positive spatial cc implies the increase from population density in a residential location and leads to the increase of population density in another residential location. A type of cooperation relationship may exist between these two adjacent residential locations. For instance, the population growth in anew town can lead to the increase of population density in residential locations of the adjacent suburban city.

In summary, as spatial cc increases, the residential locations are more correlated with each other, the optimal budget social welfare decreases. More correlated residential locations can lead to lower budget social welfare for the candidate rail transit line. Conversely, a competitive relationship between residential locations leads to the availability of a high budget social welfare for the candidate rail transit line.

It is also noted that the effects of temporal cc on the optimal social welfare budget is more significant than spatial. For instance, as temporal cc increases from -1 to 1, the optimal social welfare budget decreases from level of 1.071×10^{10} HK$ to 7.750×10^{9}. As spatial cc increases from -1 to 1 and temporal cc of 0.8, the optimal social welfare budget decreases from 7.822×10^{9} HK$ to 7.526×10^{9} HK$.

Compared with traditional studies assuming a spatial and temporal cc of 0, the optimal social welfare is overestimated in parts of (a) and (b) in Figure 5.2 and underestimated in parts of (c) and (d). For instance, in part of (b), the results in traditional studies are overestimated from 7.154×10^{9} HK$ to 9.137×10^{9} HK$ with spatial and temporal cc of 1. In part (c), the results in traditional studies are underestimated from 1.130×10^{10} HK$ to 9.137×10^{9} HK$ with spatial and temporal cc of -1.

Table 5.2 Numerical results

Temporal cc	Spatial cc	$L(t)$ (Km)			n_s^t			$\hat{t}$
		$t=1$	$t=2$	$t=3$	$t=1$	$t=2$	$t=3$	
-1	-1	**30.98**	**17.49**	**9.59**	**28**	**16**	**9**	**11.19**
-1	-0.8	29.95	16.91	9.27	27	15	8	11.20
-1	-0.6	28.93	16.34	8.95	26	15	8	11.21
-1	-0.4	17.91	15.76	8.64	25	14	8	11.22
-1	-0.2	26.88	15.18	8.32	24	14	8	11.22

-1	0	15.86	14.60	8.00	24	13	7	11.23
-1	0.2	14.84	14.02	7.68	23	13	7	11.24
-1	0.4	13.81	13.45	7.37	22	12	7	11.25
-1	0.6	22.79	12.87	7.05	21	12	6	11.26
-1	0.8	22.77	12.29	6.73	20	11	6	11.27
-1	1	**20.74**	11.71	6.42	19	11	6	11.29
1	-1	**22.09**	**14.36**	**9.21**	20	13	8	**8.60**
1	-0.8	21.36	13.88	8.90	19	13	8	8.60
1	-0.6	20.63	13.41	8.60	19	12	8	8.61
1	-0.4	19.90	12.93	8.29	18	12	8	8.61
1	-0.2	19.17	12.46	7.99	17	11	7	8.62
1	0	18.44	11.99	7.68	17	11	7	8.62
1	0.2	17.71	11.51	7.38	16	10	7	8.63
1	0.4	16.98	11.04	7.08	15	10	6	8.64
1	0.6	16.25	10.56	6.77	15	10	6	8.65
1	0.8	15.52	10.09	6.47	14	9	6	8.66
1	1	14.79	9.61	6.16	13	9	6	8.67

Notes:

- “cc” represents covariance coefficient
- $L(t)$ represents optimal rail length in year t ($t \in \{1,2,3\}$), n_s^t represents optimal rail station number in year t, $\hat{t}$ represents optimal project start time of the candidate rail transit line with respect to base year

Table 5.2 shows numerical results of optimal rail line length $L(t)$,optimal rail station number n_s^t, and optimal project start time of the candidate rail transit line in terms of base year with respect to temporal correlation coefficient (cc) of -1 and 1, and spatial cc from -1 to 1. It can be seen that with temporal cc of -1 the optimal rail line length $L(t)$ in each year is longer than that of a temporal cc of 1. For instance, with temporal and spatial cc of -1, the optimal rail line length in year 1 is 30.98 km, 17.49 km in year 2 and 9.59 km in year 3, while the optimal rail line length in year 1 is 22.09 km, 14.36 km in year 2 and 9.21 in year 3 with temporal cc of 1 and spatial cc of -1. It implies that the optimal rail line length is longer with decentralised population distribution than that with centralised population distribution.

It can also be seen that the optimal rail line length $L(t)$ in each year decreases as spatial cc increases from -1 to 1. For instance, with temporal cc of -1 and spatial cc increases from -1 to 1, the optimal rail line length in year 1, decreases from 30.98 km to 20.74 km. It implies that as cooperation between residential locations becomes strong and competition between residential locations becomes week, the optimal rail line length in year 1 decreases.

From Table 5.2, it can be found that the optimal project start time of the candidate rail transit line $\hat{t}$ is fast-tracked when temporal cc increases from -1 to 1. For instance, with temporal and spatial cc of -1, the optimal project start time of the candidate rail transit line is year 11.19 in terms of base year, while the optimal project start time of the candidate rail transit line, with temporal cc of 1 and spatial cc of -1 is 8.60. It implies that the optimal project start time of the candidate rail transit line is earlier under centralised population distribution than that under decentralised population distribution.

5.4.2 Example 2

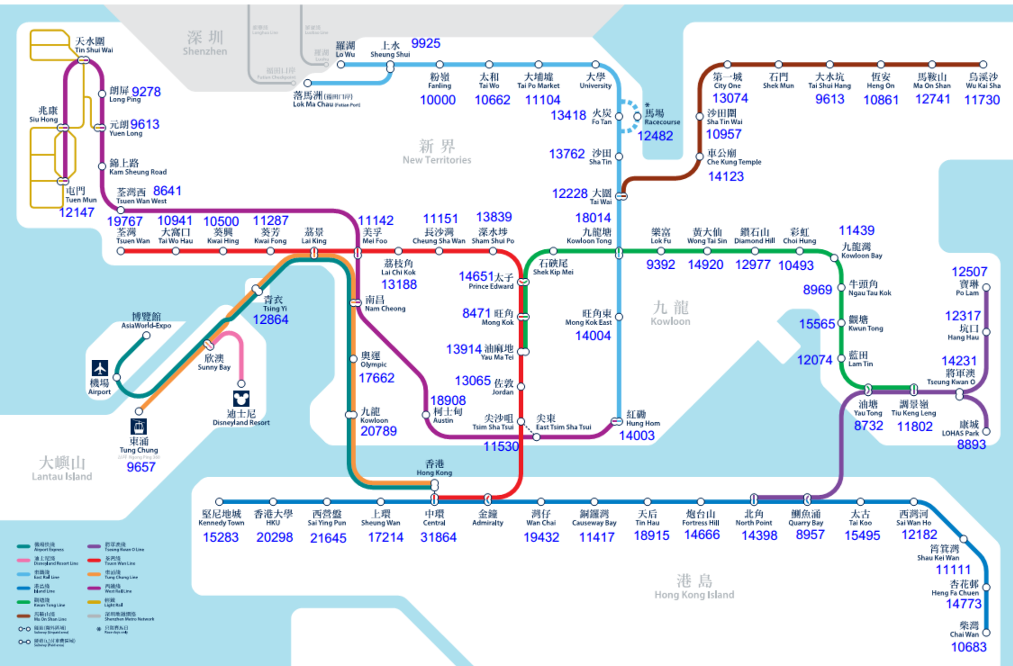


Figure 5.3: The housing unit prices map around MTR stations for Hong Kong in the first half year of 2015.

Figure 5.3 gives the housing unit prices map around MTR stations for Hong Kong in the first half year of 2015. The housing unit prices, within the range of two hundred meters at each MTR station, are the transaction prices of representing housing estates. The representing housing estates are in residential locations, which have the largest transaction numbers of housing estates in the past six months. The housing unit prices are measured in HK$/square feet (Sqft).The data come from Centra data, linked by hk.centranet.com/eng/ehome.htm.

Table 5.3 The housing rent list along the Western Island Line

Station	Representing housing estates	Housing price (HK$/Sqft)	Daily housing rent (HK$)
①Central (CBD)	Winner Building	31864	676.34
②Sheung Wan	Hollywood Terrace	17214	366.52
③Sai Ying Pun	Island Grest	21645	460.88
④HKU	The Belcher's	20298	432.88
⑤Kennedy Town	Smithfield Terrace	15283	325.43

Notes:

- The housing rent price ratio is around 3% in Hong Kong at year 2015. The housing rent price ratio is a measure of the relative affordability of renting and buying in a given housing market. It is calculated as the ratio of home prices to annual rental rates. This data comes from Chiefgroup of Hong Kong, linked by www.chiefgroup.com.hk_upload/pdf1_20140226170953_BJ_20131230.pdf
- The average flat size is 36.5 sqft, according to Housing Authority Annual Report 2014-2015 (www.housing.wa.gov.au/housingDocuments).
- Daily housing rent = (36.5*7*Housing price*3%)/(12*30). For instance, 676.34 = (31864*7*3%*36.5)/(12*30).

Table 5.3 gives the housing rent list of representing housing estates at each rail stations of the West Island Line in first half year of 2015. The housing rent price ratio is around 3% in Hong Kong at year 2015. This data comes from Chiefgroup of Hong Kong (www.chiefgroup.com.hk). The average flat size is 36.5 sqft, according to Housing Authority Annual Report 2014-2015 (www.housing.wa.gov.au/housingDocuments). The daily housing rents around rail stations of Western Island Line are calculated based on the housing prices, housing rent price ratio, the average flat size, and a constant parameter. For instance, in this example, Daily housing rent = (36.5*7*Housing price*3%)/(12*30), and 676.34 = (31864*3%*36.5*7)/(12*30).

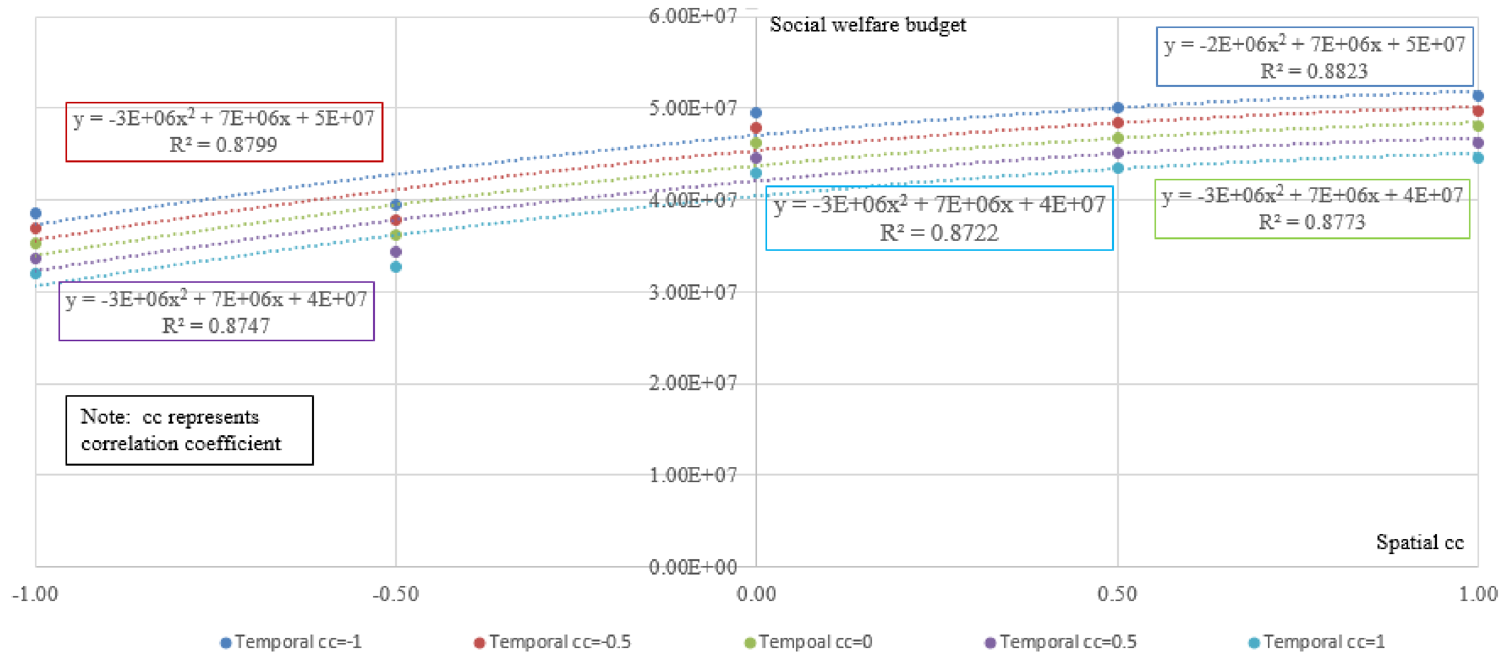


Figure 5.4: The effects of spatial and temporal correlations of population densities on social welfare budget for the Western Island Line

Figure 5.4 shows the effects of spatial and temporal correlation of population densities on social welfare budget for the Western Island Line. It can be found that with a given spatial correlation coefficient (cc) of population densities, as temporal cc increases from -1 to 1, the social welfare budget for the Western Island Line decreases. It also can be found that the effect of temporal cc of population densities is more significant than spatial cc of population densities on the social welfare budget. The results are in accord with the results of numerical example 1 of this chapter.

5.5 Summary of this chapter

This chapter proposes a closed-form model to investigate the effects of spatial and temporal correlation of population densities on design of a candidate rail transit line over years. The traditional studies with an assumption of IIA population densities, namely spatial and temporal correlation of population densities of 0, are special cases of the proposed model in this chapter.

The proposed model offers several new insights. For example, decentralised population distribution takes high social welfare budget of the candidate rail transit line. Competition between residential locations takes high social welfare budget of the candidate rail transit line. The effect of temporal correlation coefficient (cc) on the optimal social welfare budget is more significant than spatial correlation coefficient. The optimal rail line length $L(t)$ in each year is longer in contrast temporal cc -1 with temporal cc 1. The optimal project start time of the candidate rail transit line $\hat{t}$ is fast-tracked as temporal cc increase from -1 to 1.

This chapter provides a new avenue of research for modelling and analysis of spatial and temporal correlation of population densities on design of a candidate rail transit line over years.

In this chapter, population is assumed to be homogenous with trips commuting only from residences to CBD. The proposed model is extended to incorporate the effects of households' risk preference on early and late arrival to CBD on design of a candidate rail transit line over years in Chapter 6.

Only rail mode is considered in this chapter. This assumption is extended to a multi-modal situation in Chapter 6. With more travel modes being considered, households' travel mode choice behaviour can be incorporated into the extended models (Chowdhury and Chien, 2002; Li, et al, 2006; Liu and Lam, 2013) for design of a candidate rail transit line in a linear transportation corridor.

Chapter 6 Modelling the effects of population density on households' travel mode choice behaviour over a period of years for design of rail transit line

As stated in Section **2.2.3** of Chapter 2, the conventional travel choice models in transportation research are commonly formulated with an assumption of a given and fixed population density. The effects of population density on travel choice could not, therefore, be explicitly incorporated into the conventional models.

Travel choice models in previous studies are usually developed by using discrete choice approach or user equilibrium principles based on the utility theory. Thus, many significant characteristics of travellers' behaviours, such as risk preference and learning process over time, cannot be captured in these existing models.

In this chapter, a convex prospect theory based model is proposed to investigate the effects of population density on the households' travel mode choice behaviour over time for design of rail transit line in a multimodal transportation corridor. It is shown that population density is closely related to the modal split results and the financial performance of the rail system in the linear corridor. In general, the park-and-ride mode may not be suitable for areas with high population density, and vice versa. The determination of travellers' reference points on the generalised travel cost is also explored in this chapter. A numerical example is given to illustrate the application of the proposed model together with some insightful findings.

The remainder of this Chapter 6 is organised as follows. In the following section, a brief background of the research problem is presented. The problem statement, basic assumptions and notations are given in Section 6.2. Section 6.3 presents the actual generalised travel cost by travel mode concerned, i.e. car, bus, rail and park-and-ride modes within the linear transportation corridor. In Section 6.4, the prospect theory based user equilibrium condition is firstly investigated for the problem concerned. It follows with the proposed model for the day-to-day mode-choice equilibrium problem. Some equilibrium properties, with given restrictive assumptions, are discussed in Section 6.5. The determination of travellers' reference points over time are investigated in Section 6.6. A numerical example is used to illustrate the application of the proposed model and is presented in Section 6.7 together with some insightful findings. Section 6.8 gives a summary of this chapter.

6.1 Background of the research problem

Population density in conventional travel choice models was commonly assumed to be a given and fixed value (See, e.g. Lam and Huang, 1992; Huang and Lam, 2002; Liu et al, 2009). Thus, the effects of population density on travel modes choice cannot be explicitly considered. These effects, however, are quite important and relatively significant, for long-term strategic planning of a rail transit line.

The preferred travel mode in areas with low population density is the private car, such as United Stated, whereas the preferred travel mode in Asian cities with high population density such as Hong Kong is the public transit. One possible reason is that high congestion and long delays may occur on the road networks in highly populated areas during the morning peak hour and particularly on working days. Therefore in such areas the generalised travel cost (i.e. operating cost and travel time) of private cars may be higher than that of using transit mode.

The travel mode choice problem in previous studies is usually tackled in the step of traffic assignment with discrete choice approach of using various logit-type models or probit-type models, or the user equilibrium principles originating from the work of Wardrop (1952). Both the above two methods comply with the utility theory (UT) paradigm, thus preventing some significant characteristics of travellers' behaviours, such as risk preference and learning process over time, from being considered.

Prospect theory (PT) is regarded as a leading behaviour paradigm to understand decision-making under risk. Compared with UT, which deduces implications from normative preference axioms with the assumption of economic rational behaviour of travellers, PT can be used to describe travellers' various choice behaviours with different risk preferences, such as risk preference, risk neutral, or risk aversion and capture travellers learning process of various choice behaviours over time. Furthermore, PT can be seen as an extension of UT.

Prospect theory was originally designed to explain lotters' responses to a static situation involving risk, in the lacks of immediate feedback, repeated choices, the rigor, scope, behavioural principles or mechanisms (Timmermans, 2010). Prospect theory, however, has been applied to many dynamic situations, such as the asset allocation (Barberis and Huang, 2001), intertemporal consumption (Attanasio and Paiella, 2011), and equity premium puzzle (Benartzi and Thaler, 1995).

Prospect theory has also been applied to the dynamic situation in many previous transportation research studies. For example, Liu et al (2004) extended the concept of travel time reliability to dynamic route choice based on real-time loop data with coefficients in a mixed-logit model representing individual traveller's references or tastes towards travel time, reliability and cost. Jou et al (2008) investigated the dynamic commuter departure time choice under uncertainty in network with a reference point hypothesis of prospect theory. Li and Hensher (2011) concluded that prospect theory can be used to better understand the travel behaviour dynamics.

Prospect theory has been applied in many repeated travel choices with feedback about the outcomes of each separate choice. For example, Erev and Barron (2005) demonstrated that reference-dependent loss aversion appeared extensively in repeated choice. Avineri and Praksher (2005) observed repeated choices in route choice experiments so as to compare predictions of multinomial logit implementations of extended utility theory and (cumulative) prospect theory. In their work, route choice laboratory experiments and computer simulations were conducted in order to analyse route choice behaviour in iterative tasks with immediate feedback. Gao et al (2010) applied prospect theory in traffic equilibrium with repeated route choices for congested network.

Table 6.1 Prospect theory based recurrent choice investigation in some previous studies

Citations	Recurrent choice	Survey population
Mahmassani, 1990	Departure times	400
Hensher, 2001	Car trips	439
De Palma and Picard, 2005	Routes	2387

Lam and Small, 2001	Free and tolled lanes	533
Small *et al*., 2005	Free and tolled lanes	603
Bogers et al., 2005	Route changing alternatives	1515

To illustrate this issue better, the prospect theory based recurrent choice problems investigated in some previous related studies have been summarised in Table 6.1. It is noted that prospect theory has been applied to different recurrent choice problems, including car trips, routes, free and tolled lanes, departure time, and route changing alternatives.

Prospect theory describes the decision process in two stages, i.e. editing and evaluation. In the editing phase, outcomes of the decision choice are calculated following some heuristic with a set of reference point. Then lesser outcomes are then considered as losses and the greater ones as gains. In the evaluation phase, a prospect value is computed based on the potential outcomes and their respective weighting probability function. Avineri (2006) extended Wardrop's principle of user equilibrium to prospect theory based user equilibrium. In this chapter, a prospect theory based mode-choice equilibrium model is proposed under an advanced transportation information system (ATIS) in a multimodal transportation corridor. The effects of population density on modal split results and the travel modes of rail and park-and-ride are investigated in a linear transportation corridor for design of rail transit line over time.

The advanced transportation information system (ATIS), which has been widely adopted (Chen et al, 2012), provides day-to-day travel time information to travellers pre-trip planning. Travellers, who have not complied with purchased professional ATIS equipment, can now acquire ATIS information from smart phone.

Yang (1998) investigated endogenous market penetration of professional ATIS equipment assuming perfect road information for equipped travellers. Lo and Szeto (2001) explored the effects of ATIS on time savings and congestion reduction, and analysed whether and to what extent ATIS services can substitute network capacity expansion. Szeto and Lo (2005) examined the effects of ATIS on passenger transportation performance based on travellers' departure time, route choices and their propensity to subscribe the service at different charge levels. Huang et al (2006) explored the evolution of daily path travel time, daily ATIS compliance rate and yearly ATIS adoption with stochastic user equilibrium.

Only the private car mode was considered in the above studies. Thus, no mode choice information was provided. In contrast, car, bus, rail train and park-and-ride travel modes are considered in this chapter to better investigate the effects of travellers' mode choice behaviour over a period of years for design of rail transit line in a linear transportation corridor.

6.2 Assumptions and notations

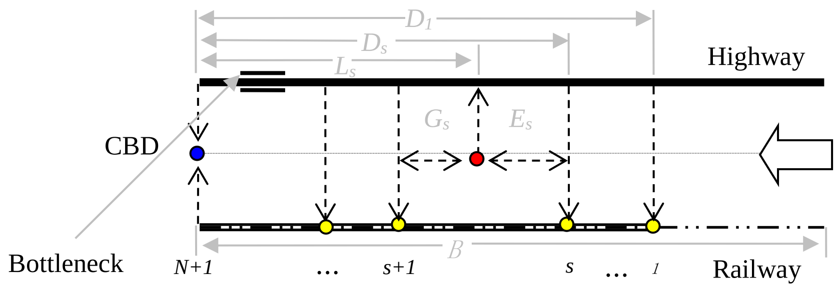


Figure 6.1: A small multi-modal transportation corridor network.

As shown in Figure 1, a simple multimodal transportation corridor network is designed to describe the travelers' day-to-day morning peak hour trips between different locations of the corridor and the central business district (CBD). There are two congestible road-based travel modes on the highway, namely bus and private car. The bottleneck exists in the highway, like submarine tunnel. In order to alleviate the road congestion problem, railway service has been supplied. There is an ordered sequence of stations numbered as $\{1,2,\cdots,N+1\}$ from locations to the CBD. The symbol B is the corridor length, D_s represents the distance between station s and the CBD, and D_1 is the length of the railway line. To attract more travellers to train from car, a park-and-ride site is assumed to be located at the bottleneck, and a relatively lower parking fee is charged than that at the CBD.

To facilitate the presentation of the essential ideas, some basic assumptions are made in this chapter, as follows.

A1 All travellers are assumed to be homogeneous and they have the identical preferred arrival time to the CBD, t^*, say, 9:30 a.m. at the workplace in CBD. On each working day, the travellers could choose different travel modes among rail train, private car, bus and park-and-ride during morning peak hour in each considered horizon period over years. This assumption could be extended in further studies, such as multi-class travellers. (Yang and Huang, 2004; Han and Yang, 2008)

A2 For the travel mode of train, travellers are assumed to board the vehicles of trains at their nearest rail station, and the trains stop at every station along the railway line. This assumption has also been adopted by many previous works, such as that of Wirasinghe and Ghoneim (1981), which could however be relaxed in further study.

A3 For private car mode, it is assumed that travellers prefer to solo driving (Qian and Zhang, 2011). This assumption could be relaxed to consider carpools in a further study if necessary (Huang, 2000a).

A4 Toll, if exists, is charged to the travellers by private car mode at the CBD and park-and ride site. (Liu et al., 2009; Qian and Zhang, 2011)

A5 In reality, travellers do remember their previous travel cost but have delays in their responses to the generalised travel cost changes of each travel mode over time. To capture this travel behavior characteristic, a method based on the prospect theory method is used to capture the travellers' response to such process of learning and adjustment. Besides, the travellers are assumed to behave as if he/she is a prospect maximiser (Avineri, 2006).

Notations are made as below：

$c_{i,x,n,t,\tau}$: The travellers' actual generalised travel cost for arriving at the CBD from location x by travel mode i at departure time t in the n^{th} day of the τ^{th} period, and $i \in r \cup c \cup b \cup P\&R$, representing rail train, car, bus and park-and-ride (P&R) mode respectively. $t \in \Gamma$, and Γ represents the morning peak hour, for example 8:30-10:00.

$\hat{c}_{i,x,n,t,\tau}$*:* The travellers' perceived generalised travel cost for arriving at the CBD from location x by travel mode i at departure time t in the n^{th} day of the τ^{th} period, and

$$\hat{c}_{i,x,n,t,\tau} = c_{i,x,n,t,\tau} + \psi , \tag{6.1}$$

where ψ is the travellers' perceived error for the actual generalised travel cost, which is assumed to follow normal distribution.

$c_{WIP,n,t,\tau}$: The travellers' subject reference point to decide whether the generalised travel cost is high or low in the n^{th} day of the τ^{th} period, which is called as the travellers' willingness-to-pay (WIP).

$p_{i,\text{AITS},t,\tau}$: The predicted on-time arrival probability for arriving at the CBD by mode i on departure time t in the n^{th} day of the τ^{th} period given by ATIS. This predicted on-time arrival probability is assumed to be calculated after the proposed multi-modal prospect theory user equilibrium has been reached. This predicted on-time arrival probability is fundamentally determined by the facilities of considered transportation system. Therefore, it would be higher if the rail customer service level were improved in each period. To present the improvement of rail customer service level period by period, it is assumed to be given by (Xu et al., 2012)

$$p_{i,AITS,t,\tau} = p_{i,AITS,t,1} + (\tau - 1)\upsilon, \left(0 < p_{i,AITS,t,\tau} \le 1\right) \tag{6.2}$$

where $p_{i,AITS,t,1}$ represents the predicted on-time arrival probability in the initial period, and υ represents the improvement rate of predicted on-time arrival probability period by period.

$\hat{p}_{i,n,t,\tau}$: The probability of obtaining the output of low generalised travel cost for arriving at the CBD by travel mode i at departure time t in the n^{th} day of the τ^{th} period, and

$$\hat{p}_{i,n,t,\tau} = \delta \hat{p}_{i,n-1,t,\tau} + (1-\delta) p_{i,AITS,t,\tau}. \tag{6.3}$$

where δ represents travellers learning behavior from the experience of yesterday and $1-\delta$ represents his/her trust on ATIS.

$\Delta c_{i,n,t,\tau}$: The deviation between perceived generalised travel cost and reference point for each travel mode i at departure time t in the τ^{th} period, and

$$\Delta c_{i,n,t,\tau} = c_{WIP,n,t,\tau} - \hat{c}_{i,n,t,\tau}, \tag{6.4}$$

$\pi\left(\hat{p}_{i,n,t,\tau}\right)$: The probability weighting function, which is given by

$$\pi\left(\hat{p}_{i,n,t,\tau}\right) = \frac{\hat{p}_{i,n,t,\tau}^{\gamma'}}{\left(\hat{p}_{i,n,t,\tau}^{\gamma'} + \left(1 - \hat{p}_{i,n,t,\tau}\right)^{\gamma'}\right)^{\frac{1}{\gamma'}}}, \tag{6.5}$$

where γ'=0.61 for $\Delta c_{i,t,n,\tau} \ge 0$ and γ'=0.69 for $\Delta c_{i,t,n,\tau} < 0$.

$V\left(\Delta c_{i,n,t,\tau}\right)$: The value function of choosing travel mode i at departure time t in the n^{th} day of the τ^{th} period, and

$$V\left(\Delta c_{i,n,t,\tau}\right)=\begin{cases}\Delta c_{i,n,t,\tau}{}^{\alpha'} & ,\Delta c_{i,n,t,\tau}\geq 0\\ -\lambda'\left(-\Delta c_{i,n,t,\tau}\right)^{\beta'} & ,\Delta c_{i,n,t,\tau}<0\end{cases},\alpha'=\beta'=0.88,\ \lambda'=2.25\,. \quad (6.6)$$

While the values of α',β', γ' and λ' equal 1, prospect theory would become utility theory.

$PS_{i,x,t,\tau}$: The prospect value of choosing travel mode i from location x on departure time t in the τth period, and

$$PS_{i,x,t,\tau}=\sum_{n\in\tau}\pi\left(\hat{p}_{i,n,t,\tau}\right)V\left(\Delta c_{i,n,t,\tau}\right). \quad (6.7)$$

6.3 The actual generalised travel cost by each travel mode

According to *A5*, travellers are assumed to behave as if he/she is a prospect maximiser, which is calculated on basis of the actual generalised travel costs by different travel modes. The generalised travel cost by each travel mode is defined as the monetary cost plus the travel time weighted by the value of time. For simplicity the capacity of the highway and the speed of the trains are both assumed to be constant and fixed in the study corridor. The value of time for all travellers is assumed to be identical (Liu et al, 2009). The expressions of location-dependent actual generalised travel cost of the travellers over time by rail, car, bus, and park-and-ride mode are respectively given as follows.

6.3.1 The actual generalised travel cost by rail

The time-dependent actual travel cost by rail from location x_1 to x_2, $x_1\in[0,B]$, $x_2\in[0,x_1]$, excluding the access time cost for using rail and the fixed component of the rail fare could be expressed as below (Liu et al, 2009)

$$c_r\left(x_1,x_2,t,\tau\right)=ut_r^0\left(x_1-x_2\right)+\int_0^t\int_{x_2}^{x_1}g_r\left(q_r(w,t,\tau)\right)\mathrm{d}w\mathrm{d}t+f_r\left(x_1-x_2\right), \quad (6.8)$$

where u is the value of time, t_r^0 is the running time of train per unit distance, f_r is the variable component of the rail fare, $q_r(x,t,\tau)$ is the elastic demand function of travellers for rail train mode at location x on time t in period τ and $\int_0^t g_r\left(q_r(x,t,\tau)\right)\mathrm{d}t$ (0 in the subscript represent the train departure time) is the in-vehicle crowding cost per unit distance at location x , $x \in [0, B]$, which is a strictly increasing function of $q_r(x,t,\tau)$, with $g_r^{'}\left(q_r(x,t,\tau)\right) > 0$, $g_r^{''}\left(q_r(x,t,\tau)\right) < 0$,and $g_r(0) = 0$ (Huang, 2000b). The total actual generalised travel cost by rail train mode from location x to the CBD at departure time t in period τ is then,

$$c_r\left(x,t,\tau\right) = ut_{or} + f_r^0 + c_r\left(x,0,t,\tau\right) + \max\left\{\gamma\left(t + t_r - t^*\right), \beta\left(t^* - t - t_r\right)\right\} ,(6.9)$$

where t_{or} is the travel time apart from in-vehicle time by rail, which will be defined later, and f_r^0 is the fixed component of the rail fare. $\max\left\{\gamma\left(t + t_r - t^*\right), \beta\left(t^* - t - t_r\right)\right\}$ is the early arrival or late arrival penalty, where β and γ measure the generalised cost of one extra unit time of early schedule delay and late schedule delay, respectively. t_r denotes the total travel time by rail, i.e. $t_r = t_r^0 x + t_{or}$. The travel time except in-vehicle time by rail can be calculated by the summation of access time and waiting time, i.e.

$$t_{or} = t_w^0 + \alpha H, \qquad (6.10)$$

where t_w^0 is the walking time to the station around. α is a calibration parameter which depends on the distributions of train headway and travellers arrival. The value $\alpha = 0.5$ is commonly used, implying the assumption of a uniform random traveller arrival distribution and a constant time headway in each considered design period between trains.

6.3.2 The actual generalised travel cost by car

The actual generalised travel cost of the travellers by car at location x on departure time t in period τ could be given by

$$\begin{aligned} c_c\left(x,t,\tau\right) &= ut_{oc} + f_c x + u\int_0^x t_c\left(q_h(w,t,\tau)\right)\mathrm{d}w \\ &\quad + \max\left\{\gamma\left(t + t_c - t^*\right), \beta\left(t^* - t - t_c\right)\right\} + P_c, \end{aligned} \qquad (6.11)$$

where t_{oc} is the access time for driving a car from home to the highway, f_c is the operating cost per unit distance (including such as the petrol cost, insurance and highway toll), $t_c\left(q_h(x,t,\tau)\right)$ is the travel time for driving unit distance around location x departing at time t in period τ, $q_h(x,t,\tau)$ is the elastic demand function of travellers on highway at location x on time t in period τ ,

$\max\left\{\gamma\left(t+t_c-t^*\right),\beta\left(t^*-t-t_c\right)\right\}$ is the early arrival or late arrival penalty as mentioned above, and P_c is the parking charge at the CBD. t_c is the total travel time by private car mode, i.e. $t_c=\int_0^x t_c\left(q_h(w,t,\tau)\right)\mathrm{d}w+t_{oc}$.

6.3.3 The actual generalised travel cost by bus

The actual generalised travel cost of the travellers by bus mode at location x on departure time t in period τ could be given by

$$c_b\left(x,t,\tau\right)=ut_{ob}^0+f_b^0+f_bx+\int_0^t\int_{x_2}^{x_1}g_b\left(q_b(w,t,\tau)\right)\mathrm{d}w\,\mathrm{d}t \\ +\max\left\{\gamma\left(t+t_b-t^*\right),\beta\left(t^*-t-t_b\right)\right\} \quad (6.12)$$

where t_{ob}^0 is the access time from home to the bus stop, t_b is the total travel time by bus mode, which would be defined later, and f_b^0 is the fixed component of the railway fare, f_b is the variable component of the bus fare. Similarly, $q_b(x,t,\tau)$ is the elastic demand function of travellers for bus mode at location x on time t in period τ and $\int_0^t g_b\left(q_b(x,t,\tau)\right)\mathrm{d}t$ $(0$ in the subscript represents the bus offset time) is the in-vehicle crowding cost per unit distance at location x, $x\in[0,B]$. The same function attributes exist as those in the rail mode given above.

The total travel time by bus mode can be calculated by

$$t_b=t_{ob}^0+\alpha_bH_b+\int_0^x t_h\left(q_h(w,t,\tau)\right)\mathrm{d}w \quad (6.13)$$

where H_b is the time headway of the bus service, α_b is a calibration parameter which depends on the distributions of bus headway and traveller arrival. $\int_0^x t_h\left(q_h(w,t,\tau)\right)\mathrm{d}w$ is the bus operating time from location x to the CBD.

6.3.4 The actual generalised travel cost by P&R mode

Since the parking charge at the P&R site is relatively lower than that at the CBD, travellers could park their cars at this site and continue their trips by trains to the CBD. Similar to Wong et al (2004) and Liu et al (2009), the actual generalised travel cost of travellers at location

x on departure time t who park his/her car at P&R site located at $x_{p\&R}$ in period τ can be given by

$$c_{p\&R}\left(x,t,\tau\right)=ut_{oc}+f_{c}\left(x-x_{P\&R}\right)+u\int_{x_{P\&R}}^{x}t_{c}\left(q_{h}(w,t,\tau)\right)\mathrm{d}\,w+ut_{P\&R}+f_{r}^{0}+t_{or}$$
$$+c_{r}\left(x_{P\&R},0,t,\tau\right)+\max\left\{\gamma\left(t+t_{P\&R}-t^{*}\right),\beta\left(t^{*}-t-t_{P\&R}\right)\right\}+P_{P\&R} \quad (6.14)$$

where $t_{P\&R}$ is the required time from mode car to train, $P_{P\&R}$ is the parking charge at P&R site, $t_{P\&R}$ is the total travel time by P&R mode, and

$$t_{P\&R}=\int_{x_{P\&R}}^{x}t_{c}\left(q_{h}(w,t,\tau)\right)\mathrm{d}\,w+t_{oc}+t_{P\&R}+t_{r}^{0}x_{P\&R}+t_{or}. \quad (6.15)$$

6.4 The prospect theory based mode-choice equilibrium

As in the study of Avineri (2006), the Wardrop (1952) principle of user equilibrium could be extended to prospect theory based mode-choice user equilibrium (PTMUE), "Equilibrium under the condition that no user can increase his/her mode choice prospect value by unilaterally switching his/her mode choice behaviour".

Definition: The conditions of the above PTMUE condition could be expressed as

$$q_{i}(x,t,\tau)>0\ \Rightarrow PS_{i,x,t,\tau}=\min\left\{PS_{i,x,t,\tau}\right\},i\in r\cup c\cup b\cup P\,\&\,R,\ x\in[0,B],\left(6.16\right)$$

where $q_i(x,t,\tau)$ are the travel demand densities (the number of travellers per unit distance) who choose the rail train, private car, bus, and P&R mode at location x on departure time t in the n^{th} of the τ^{th} period, respectively. $PS_{i,x,t,\tau}$ are the prospect values for the train, car, bus, and P&R mode respectively and the prospect values are calculated by Eqs. (6.1)-(6.7). The above definition implies that at equilibrium, the prospect value of the travellers by a travel mode at any location should be the minimal among the four modes if that mode is actually used by travellers on that departure time during morning peak hour at that location.

Given the travel demand density $q_0(x,t,\tau)$, $x\in[0,B]$, it could be proved that the prospect theory based user equilibrium mode choice $\mathbf{q}^{*}=(\mathbf{q}_{r},\mathbf{q}_{c},\mathbf{q}_{b},\mathbf{q}_{P\&R})^{\mathrm{T}}$ is the solution of the following maximisation problem

$$\max L\left(\mathbf{q},n,t,\tau\right)=q_r\left(x,t,\tau\right)PS_{or}+q_c\left(x,t,\tau\right)PS_{oc}+q_b\left(x,t,\tau\right)PS_{ob}+q_{p\&R}\left(x,t,\tau\right)PS_{op\&R}$$

$$+\int_0^{v_r(x,t,\tau)}PS_r(v)\,\mathrm{d}v+\int_0^{v_c(x,t,\tau)}PS_c(v)\,\mathrm{d}v+\int_0^{v_b(x,t,\tau)}PS_b(v)\,\mathrm{d}v+\int_0^{v_{P\&R}(x,t,\tau)}PS_{P\&R}(v)\,\mathrm{d}v, \qquad \left(6.17a\right)$$

Subject to

$$q_r(x,t,\tau)+q_c(x,t,\tau)+q_b(x,t,\tau)+q_{P\&R}(x,t,\tau)=q_0(x,t,\tau) \qquad \left(6.17b\right)$$

$$PS_{r,x,t,\tau}=\sum_{n\in\tau}\Big\{\pi\left(\hat{p}_{r,n,t,\tau}\right)\cdot\Big[c_{WIP,x,t,\tau}-ut_w^0-\alpha uH-t_r^0x-f_rx-\int_0^t\int_{x_2}^{x_1}g_r\left(q_r(w,t,\tau)\right)\mathrm{d}w\,\mathrm{d}t$$
$$-f_r^0-\beta(t^*-t-t_r)\Big]^{\alpha'}-\pi(1-\hat{p}_{r,n,t,\tau})\cdot\lambda'\cdot\Big[-c_{WIP,x,t,\tau}+ut_w^0+\alpha uH+t_r^0x$$
$$+f_r^0+f_rx+\int_0^t\int_{x_2}^{x_1}g_r\left(q_r(w,t,\tau)\right)\mathrm{d}w\,\mathrm{d}t+\gamma\left(t+t_r-t^*\right)\Big]^{\beta'}\Big\}, \qquad (6.17c)$$

$$PS_{c,x,t,\tau}=\sum_{n\in\tau}\Big\{\pi\left(\hat{p}_{c,n,t,\tau}\right)\cdot\Big[c_{WIP,x,t,\tau}-ut_{oc}-P_c-f_cx-u\int_0^x t_c\left(v_h(w,t,\tau)\right)\mathrm{d}w$$
$$-\beta(t^*-t-t_c)\Big]^{\alpha'}-\pi\left(1-\hat{p}_{c,n,t,\tau}\right)\cdot\lambda'\cdot\Big[-c_{WIP,x,t,\tau}+ut_{oc}+P_c$$
$$+f_cx+u\int_0^x t_c\left(v_h(w,t,\tau)\right)\mathrm{d}w+\gamma(t+t_c-t^*)\Big]^{\beta'}\Big\}, \qquad \left(6.17d\right)$$

$$PS_{b,x,t,\tau}=\sum_{n\in\tau}\Big\{\pi\left(\hat{p}_{b,n,t,\tau}\right)\cdot\Big[c_{WIP,x,t,\tau}-ut_{ob}-f_b^0-f_bx-\int_0^t\int_{x_2}^{x_1}g_b\left(q_b(w,t,\tau)\right)\mathrm{d}w\,\mathrm{d}t$$
$$-\beta(t^*-t-t_b)\Big]^{\alpha'}-\pi(1-\hat{p}_{b,n,t,\tau})\cdot\lambda'\cdot\Big[-c_{WIP,x,t,\tau}+ut_{ob}+f_b^0$$
$$+f_bx+\int_0^t\int_{x_2}^{x_1}g_b\left(q_b(w,t,\tau)\right)\mathrm{d}w\,\mathrm{d}t+\gamma(t+t_b-t^*)\Big]^{\beta'}\Big\}, \qquad \left(6.17e\right)$$

$$PS_{P\&R,x,t,\tau}=\sum_{n\in\tau}\Big\{\pi\left(\hat{p}_{p\&R,n,t,\tau}\right)\Big[c_{WIP,x,t,\tau}-ut_{oc}-f_c\left(x-x_{p\&R}\right)-u\int_0^x t_c\left(v_h(w,t,\tau)\right)\mathrm{d}w$$
$$-ut_{P\&R}-f_r^0-c_r\left(x_{P\&R},0,t,\tau\right)-P_{P\&R}-\beta\left(t^*-t-t_{P\&R}\right)\Big]^{\alpha'}-\pi\left(1-\hat{p}_{P\&R,n,t,\tau}\right)$$
$$\cdot\lambda'\Big[-c_{WIP,x,t,\tau}+ut_{oc}+P_{P\&R}+f_c\left(x-x_{p\&R}\right)+u\int_0^x t_c\left(v_h(w,t,\tau)\right)\mathrm{d}w+ut_{P\&R}$$
$$+f_r^0+c_r\left(x_{P\&R},0,t,\tau\right)+\gamma\left(t+t_{P\&R}-t^*\right)\Big]^{\beta'}\Big\}, \qquad (6.17f)$$

$$PS_{or}=\sum_{n\in\tau}\Big[\pi\left(\hat{p}_{i,n,t,\tau}\right)\left(c_{WIP,x,t,\tau}-ut_w^0-\alpha uH-f_r^0\right)^{\alpha'}$$
$$-\pi\left(1-\hat{p}_{i,n,t,\tau}\right)\lambda'\left(-c_{WIP,x,t,\tau}+ut_w^0+\alpha uH+f_r^0\right)^{\beta'}\Big], \qquad (6.17g)$$

$$PS_{oc} = \sum_{n\in\tau}\left[\pi\left(\hat{p}_{c,n,t,\tau}\right)\left(c_{WIP,x,t,\tau} - ut_{oc} - P_c\right)^{\alpha'} - \pi\left(1-\hat{p}_{c,n,t,\tau}\right)\lambda'\left(-c_{WIP,x,t,\tau} + ut_{oc} + P_c\right)^{\beta'}\right], \qquad (6.17h)$$

$$PS_{ob} = \sum_{n\in\tau}\left[\pi\left(\hat{p}_{b,n,t,\tau}\right)\left(c_{WIP,x,t,\tau} - ut_{ob} - f_b^0\right)^{\alpha'} - \pi\left(1-\hat{p}_{b,n,t,\tau}\right)\lambda'\left(-c_{WIP,x,t,\tau} + ut_{ob} + f_b^0\right)^{\beta'}\right], \qquad (6.17\text{i})$$

$$PS_{op\&R} = \sum_{n\in\tau}\left[\pi\left(\hat{p}_{P\&R,n,t,\tau}\right)\left(c_{WIP,x,t,\tau} - ut_{oc} - P_{P\&R} - ut_{P\&R} - ut_w^0 - \alpha uH - f_r^0\right)^{\alpha'}\right.$$
$$\left. -\pi\left(1-\hat{p}_{p\&R,n,t,\tau}\right)\lambda'\left(-c_{WIP,x,t,\tau} + ut_{oc} + P_{P\&R} + ut_{P\&R} + ut_w^0 + \alpha uH + f_r^0\right)^{\beta'}\right], \qquad (6.17\text{j})$$

$$PS_i\left(v_i(x,t,\tau)\right) = PS_{i,x,t,\tau} - PS_{oi}, \qquad (6.17\text{k})$$

$$v_r(x,t,\tau) = \int_0^t\int_{x_2}^{x_1} q_r(w,t,\tau)\,\mathrm{d}w\,\mathrm{d}t = \int_x^B q_r(w,t,\tau)\,\mathrm{d}w \qquad (6.17\text{j})$$

$$v_h(x,t,\tau) = \int_0^x q_h(w,t,\tau)\,\mathrm{d}w \qquad (6.17k)$$

$$v_b(x,t,\tau) = \int_0^t\int_{x_2}^{x_1} q_b(w,t,\tau)\,\mathrm{d}w\,\mathrm{d}t = \int_x^B q_b(w,t,\tau)\,\mathrm{d}w \qquad (6.17l)$$

$$q_h(x,t,\tau) = \begin{cases} q_c(x,t,\tau) + \dfrac{q_b(x,t,\tau)}{K} + q_{P\&R}(x,t,\tau), x\in[x_{P\&R}, B] \\ q_c(x,t,\tau) + \dfrac{q_b(x,t,\tau)}{K}, x\in[0, x_{P\&R}] \end{cases} \qquad (6.17m)$$

$$q_i(x,t,\tau) \ge 0,\ \forall i \in r\bigcup c\bigcup b\bigcup P\&R;\ x\in[0.B], \qquad (6.17n)$$

$$q_h(x,t,\tau) = \begin{cases} q_c(x,t,\tau) + \dfrac{q_b(x,t,\tau)}{K} + q_{P\&R}(x,t,\tau), x\in[x_{P\&R}, B] \\ q_c(x,t,\tau) + \dfrac{q_b(x,t,\tau)}{K}, x\in[0, x_{P\&R}] \end{cases} \qquad (6.17o)$$

$$q_i(x,t,\tau) \ge 0,\ \forall i \in r\bigcup c\bigcup b\bigcup P\&R;\ x\in[0.B], \qquad (6.17p)$$

where K is the average passenger load in each bus vehicle during morning peak hour $t\in\Gamma$.

According to the Karush-Kuhn-Tucker condition of the above maximisation problem, the solution of the problem Eq. (6.17) satisfies the following complementary slackness condition

$$q_i(x,t,\tau)\left(PS_{i,x,t,\tau} - \lambda(x,t,\tau)\right) = 0, \qquad (6.18\text{a})$$

$$PS_{i,t,\tau}(x) - \lambda(x,t,\tau) \ge 0, \qquad (6.18\text{b})$$

$$q_r(x,t,\tau) + q_p(x,t,\tau) + q_b(x,t,\tau) + q_{P\&R}(x,t,\tau) = q_0(x,t,\tau), \qquad (6.18\text{c})$$

$$q_i(x,t,\tau) \ge 0,\ \forall i \in r\bigcup c\bigcup b\bigcup P\&R;\ x\in[0,B]. \qquad (6.18\text{d})$$

where $\lambda(x,t,\tau)$ is the Lagrange multiplier associated with constraint Eq. (6.17b). The above conditions show that $\lambda(x,t,\tau)$ is the travellers' equilibrium prospect value for leaving home to the CBD from location x on departure time t in period τ. Thus, all travellers leaving from location x on departure time t in period τ have the identical or maximal prospect value at equilibrium, regardless of the mode chosen.

Since the objective function Eq. (6.17a) is strictly convex with respect to the flow density $\mathbf{q}^{*} = (\mathbf{q}_r, \mathbf{q}_c, \mathbf{q}_b, \mathbf{q}_{P\&R})^{\mathrm{T}}$, the solution of the above model is unique.

Proof. The second-order derivative of $L(\mathbf{q},n,t,\tau)$ with regard to $q_i(x,t,\tau)$ is derived as below

$$\frac{\partial L(\mathbf{q},n,t,\tau)}{\partial q_i(x,t,\tau)} = PS_{i,x,t,\tau}.$$

$$\frac{\partial L(\mathbf{q},n,t,\tau)}{\partial^2 q_r(x,t,\tau)} = \sum_{n\in\tau}\left(g_r^{'}\left(q_r(B,0,\tau)\right) - g_r^{'}\left(q_r(x,t,\tau)\right)\right)\left\{\pi(\hat{p}_{r,n,t,\tau})\cdot\alpha^{'}\left[c_{WIP,x,t,\tau} - ut_w^0 - \alpha uH\right.\right.$$
$$\left. -t_r^0 x - f_r x - \int_0^t\int_{x_2}^{x_1} g_r\left(q_r(w,t,\tau)\right)\mathrm{d}w\,\mathrm{d}t - f_r^0 - \beta\left(t^* - t - t_r\right)\right]^{\alpha^{'}-1} + \pi\left(1-\hat{p}_{r,n,t,\tau}\right)\cdot\lambda^{'}\cdot\beta^{'}\cdot$$
$$\left.\left[-c_{WIP,x,t,\tau} + ut_w^0 + \alpha uH + t_r^0 x + f_r^0 + f_r x + \int_0^t\int_{x_2}^{x_1} g_r\left(q_r(w,t,\tau)\right)\mathrm{d}w\,\mathrm{d}t + \gamma\left(t + t_r - t^*\right)\right]^{\beta^{'}-1}\right\}$$

From $g_r^{''}\left(q_r(x,t,\tau)\right) < 0$ and $q_r(x,t,\tau) > q_r(B,0,\tau)$, we know

$$g_r^{'}\left(q_r(B,0,\tau)\right) - g_r^{'}\left(q_r(x,t,\tau)\right) > 0.$$

$c_{WIP,x,t,\tau}$ represents the generalised travel cost arriving at CBD on time, which is more than the generalised travel cost with respect to early arrival and less than that with respect to late arrival, and $\alpha^{'} = \beta^{'} = 0.88$ as given above, thus

$$\left[c_{WIP,x,t,\tau} - ut_w^0 - \alpha uH - t_r^0 x - f_r x - \int_0^t\int_{x_2}^{x_1} g_r\left(q_r(w,t,\tau)\right)\mathrm{d}w\,\mathrm{d}t - f_r^0 - \beta(t^* - t - t_r)\right]^{\alpha^{'}-1} > 0,$$

$$\left[-c_{WIP,x,t,\tau} + ut_w^0 + \alpha uH + t_r^0 x + f_r^0 + f_r x + \int_0^t\int_{x_2}^{x_1} g_r\left(q_r(w,t,\tau)\right)\mathrm{d}w\,\mathrm{d}t + \gamma(t + t_r - t^*)\right]^{\beta^{'}-1} > 0.$$

Since weighting probability function $\pi\left(\hat{p}_{r,n,t,\tau}\right) \geq 0$, and $\lambda^{'} = 2.25$ as given above, we have

$$\frac{\partial L(\mathbf{q},n,t,\tau)}{\partial^2 q_r(x,t,\tau)} > 0.$$

Similarly, we could have $\dfrac{\partial L(\mathbf{q},n,t,\tau)}{\partial^2 q_c(x,t,\tau)} > 0$ and $\dfrac{\partial L(\mathbf{q},n,t,\tau)}{\partial^2 q_b(x,t,\tau)} > 0$.

Since all constraints (6.17b)-(6.17p) are convex functions of $q_{r,x,t,\tau}, q_{b,x,t,\tau}, q_{c,x,t,\tau},$, therefore, there exists a unique solution with respect to $q_{i,x,t,\tau}$, $\forall i \in r \cup c \cup b$. The total demand density $q_0(x,t,\tau)$ is assumed to be constant in each period τ. Hence the solution of

$q_{P\&R,x,t,\tau}$ is also unique. This completes the proof of the existence and uniqueness of the solution of the above optimization problem (6.17). The most widely used solution algorithm for solving the convex problem is the Frank-Wolfe searching algorithm. For solving the PTMUE problem considered here, the proposed solution reduces to a sequence of shortest path computations and one-dimensional minimizations (Sheffi, 1985).

6.5 Equilibrium properties

With some restrictive assumptions, the equilibrium properties are examined by looking into the relationship between the travellers' mode choices.

6.5.1 Equilibrium without rail service improvement over years

When a toll in the bottleneck is not implemented, the settings of the model proposed in this Chapter 6 can be deduced to that used in the study of Jehiel (1993). In his study, only two travel modes are taken into account. In this chapter, the settings of the proposed model are similar to that examined in the study of Wong et al (2004) and Liu et al (2009) in the absence of P&R facilities. In their models, the concept of generalised travel cost was used. Jehiel (1993) demonstrated that, the user equilibrium state was reached by a "simple solution" pattern in which both modes were used between the CBD and a location. Only one mode was used from the chosen location to the corridor boundary. Wong et al (2004) further examined various possible mode choice patterns under an assumption that a congested highway had a higher fixed cost than the congestion-free railway. Liu et al (2009) extended the study of Wong et al (2004) to the situation with in-carriage crowding. Readers may refer to the above relevant papers for details.

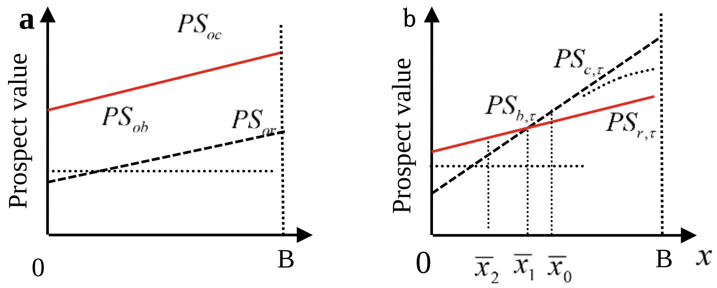


Figure 6.2: Possible equilibrium mode-choice patterns under free-flow conditions.

Without loss of generality, similar to the assumption used in the study of Wong et al. (2004), using the notations of (6.17g), (6.17h) and (6.17i), the following assumptions are made (shown in Figure6.2(a))

$$PS_{oc} > PS_{ob} > PS_{or}. \qquad (6.19)$$

The analysis made by Wong et al. (2004) is similar to the situation shown in Figure 6.2(b), but bus mode is not considered there. We assume that travel on train from the corridor boundary takes least prospect value, and that of bus is less than car through highway if the highway is free-flow in period τ (shown in Figure6.2(b)), i.e.

$$PS_{or} + PS_{r,t,\tau}(0)B < PS_{ob} + PS_{b,t,\tau}(0)B < PS_{oc} + PS_{c,t,\tau}(0)B. \quad (6.20)$$

Obviously, from assumptions (6.19) and (6.20), we could obtain

$$PS_{r,\tau}(0) < PS_{b,\tau}(0) < PS_{c,\tau}(0). \qquad (6.21)$$

In other words, under free-flow conditions, the car mode takes along with most prospect value than the other modes in period τ .

6.5.2 Equilibrium with rail service improvement over years for single travel mode

Eqs. (6.19) and (6.21) imply that the rail train mode generally has a lower fixed prospect value and a lower variable prospect value than that of the other two modes at free-flow state, resulting in congestion on the highway. Therefore, it is feasible for governments to improve the rail customer service level over the years to encourage some travellers transfer to train travel. Hence with fewer cars, the highway speed should improve. The total prospect value of all travellers may increase by means of the implementation of this policy.

With rail service improvement over years, prospect value for each travel mode will change. The travellers, who travelled by car and bus before, may choose rail mode instead. This change could be illustrated using the diagram of Figure 6.3. In period τ, a particular watershed phenomenon may appear (i.e. travellers between location $\bar{x}_7$ and the corridor boundary drive private car, and those between $\bar{x}_5$ and $\bar{x}_7$ travel by bus, while others between the CBD and $\bar{x}_3$ choose rail mode), whereas a general case will be that one or more travel mode are used simultaneously between these locations at PTMUE.

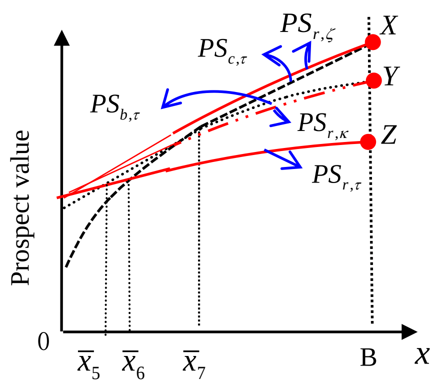


Figure 6.3: Prospect value changes with the railway improvement over years.

If rail service level is improved to $PS_{r,t,\kappa}$ in period κ as shown in Figure 6.3, some travellers from the corridor boundary Y will change their travel mode from bus to rail. Most travellers between location $\overline{x}_7$ and the corridor boundary will drive car and most those travellers between the CBD and $\overline{x}_7$ will take rail. If rail service level is improved to $PS_{r,t,\zeta}$ in period ζ, some travellers from the corridor boundary X will change their travel mode from car to rail. Moreover, if high toll are implemented at the bottleneck, car may never be chosen by the travellers.

The improvement of rail service level between $PS_{r,t,\kappa}$ and $PS_{r,t,\zeta}$ could make the shift of the above curves in Figure 6.3. The travel speed of highway will increase and so as the in-vehicle crowding in rail. The new point $\overline{x}$ of intersection in period ξ could be solved by

$$PS_{r,t,\xi}(\overline{x}) = PS_{c,t,\xi}(\overline{x}),\ \left(PS_{r,t,\kappa}(0) < PS_{r,t,\xi}(0) < PS_{r,t,\zeta}(0)\right),\ \overline{x} \in [x_7, B]. \quad (6.22)$$

The influence of rail service improvement over years to the travellers' prospect value could be calculated after the solution of $\overline{x}$ is obtained. Furthermore, the speed change could also be calculated and sensitivity analysis could be done as well.

6.5.3 Equilibrium property of P&R mode

Since rail generally has lower fixed cost and higher variable cost at the free-flow state than car, it is possible for travellers to choose P&R mode so as to enjoy the superiority of the two modes. Liu et al. (2009) investigated the optimal site selection of multiple P&R facilities. Similar to the assumption used there, to assure the travellers located very close to the corridor boundary to take P&R mode, the following condition is necessary to guarantee

$$PS_{oP\&R} + PS_{P\&R,t,\tau}(0)B < PS_{or} + PS_{r,t,\tau}(0)B. \tag{6.23}$$

In order to facilitating transfer of travel mode from car to rail conveniently, P&R site is normally located near rail station. The travellers starting from stations before the station with P&R site will not take P&R mode. The intersection exits between the P&R station and the one before it, which could be determined by

$$PS_{r,t,\xi}(x) = PS_{P\&R,t,\xi}(x),\ x \in [0, x_{P\&R}]. \tag{6.24}$$

Therefore, it is possible for travellers to travel backward from locations between CBD and P&R site to P&R facility so as to take P&R mode.

6.6 The travellers' reference points over years

As in the study of Tian et al. (2007) and Qian and Zhang (2011), we label the train which is scheduled to arrive at the CBD exactly on the preferred arrival time as the 0 run, the k^{th} run before the preferred arrival time t^* as k^+ and the k^{th} run after the preferred arrival time t^* as k^-. For the traveler travelling to the CBD departing at time t, the total travel time cost and fare are fixed. Actually, travellers make travel decision by tradeoff between in-vehicle crowding cost and schedule early/late cost, which could be expressed as below

$$C_r^{'}(x,t,\tau) = \int_0^t \int_{x_2}^{x_1} g_r\left(q_r(w,t,\tau)\right) \mathrm{d}w\,\mathrm{d}t + \max\left\{\gamma\left(t + t_r - t^*\right), \beta\left(t^* - t - t_r\right)\right\}. \tag{6.25}$$

Given a simple crowding function (Qian and Zhang, 2011),

$$g_r\left(Q_k(x,t,\tau)\right) = \eta \cdot Q_k(x,t,\tau) \cdot H(\tau), \tag{6.26}$$

where $Q_k(x,t,\tau) = \int_0^t \int_x^B q_r(w,t,\tau)\,\mathrm{d}w\,\mathrm{d}t$ is the traveller number in the train arriving at location x on time t in period τ, and η is a function parameter. Combined with $t^* - t = k^+ H$ and $t - t^* = k^- H$, then

$$C_r^{'}(x,t,\tau) = \begin{cases} \eta \cdot Q_{k^+}(x,t,\tau) \cdot H(\tau) + \beta k^+(\tau) H(\tau), & \text{early arrival} \\ \eta \cdot Q_{k^-}(x,t,\tau) \cdot H(\tau) + \gamma k^-(\tau) H(\tau), & \text{late arrival} \\ \eta \cdot Q_0(x,t,\tau) \cdot H(\tau), & \text{punctual arrival} \end{cases} \tag{6.27}$$

At the equilibrium point, a traveller would experience the same in-vehicle crowding cost and schedule early/late cost, no matter which run he/she takes. Thus,

$$Q_{i^+}(x,t,\tau) - Q_{j^+}(x,t,\tau) = \frac{\beta(j^+(\tau) - j^-(\tau))}{\eta} \quad (6.28a)$$

$$Q_0(x,t,\tau) - Q_{j^+}(x,t,\tau) = \frac{\beta j^+(\tau)}{\eta} \quad (6.28b)$$

$$Q_{i^-}(x,t,\tau) - Q_{j^-}(x,t,\tau) = \frac{\gamma(j^-(\tau) - i^-(\tau))}{\eta} \quad (6.28c)$$

$$Q_0(x,t,\tau) - Q_{j^-}(x,t,\tau) = \frac{\gamma j^-(\tau)}{\eta} \quad (6.28d)$$

From Eqs. (6.28a) - (6.28d), we could know that the travel demand for the run before the preferred arrival time is as follows (Qian and Zhang, 2011)

$$\left(Q_0(x,t,\tau) - \frac{\beta}{\eta}\right), \left(Q_{1^+}(x,t-H,\tau) - \frac{2\beta}{\eta}\right), \cdots, \left(Q_{n_w{}^+}(x,t-n_2H,\tau) - \frac{n_2\beta}{\eta}\right), \quad (6.29)$$

where $n_2 = \mathrm{int}\left[\frac{Q_0(x,t,\tau)\eta}{\beta}\right]$. Similarly, the travel demand for the run after the preferred arrival time could be obtained as

$$\left(Q_0(x,t,\tau) - \frac{\gamma}{\eta}\right), \left(Q_{1^-}(x,t+H,\tau) - \frac{2\gamma}{\eta}\right), \cdots, \left(Q_{n_1^-}(x,t+n_1H,\tau) - \frac{n_1\gamma}{\eta}\right), \quad (6.30)$$

where $n_1 = \mathrm{int}\left[\frac{Q_0(x,t,\tau)\eta}{\left(1-p_{r,AITS,t,\tau}\right)\cdot\gamma}\right]$.

Let $N_r(t-kH,\tau) = \int_0^B Q_k(w,t-kH,\tau)\mathrm{d}w$ denotes the travel demand by train on departure time $t-kH \in \Gamma$ in period τ. Given the total travel demand $\sum_{i=-n_2}^{n_1} N_r(t-KH,\tau) = N_r(\tau)$ by train mode, we could calculate the travel flow who takes the 0 run train at in period τ as

$$Q_0(t,\tau) = \frac{N_r(\tau) + \frac{\beta}{\eta}\frac{n_2^2+n_2}{2} + \frac{\gamma}{\eta}\frac{n_1^2+n_1}{2}}{(1+n_1+n_2)}, \quad (6.31)$$

where n_1 represents the number of trains after the 0 run train during the morning peak hour Γ, n_2 represents the number of trains before the 0 run train during the morning peak hour Γ, and $N_r(\tau)$ represents the total passenger demand for rail service during morning peak hour in period τ. For the calibration of $N_r(\tau)$, observation method could be adopted (Tian *et al.*, 2007; Jou *et al.*, 2008). Tian *et al.* (2007) observed the passengers departure pattern of Beijing No. 13 rail line during morning peak hour. Jou *et al.* (2008) observed 454 passenger departure time decision samples, with 51 from one-day samples, 153 from three-day samples and 250 from the five-day samples.

For those travellers who take the 0 run train and arrive at CBD on time, the perceived generalised travel cost and their willing-to-pay equals, i.e. $\Delta c_{r,n,t,\tau} = 0$. It implies that no loss or gain would exit. Thus, the perceived generalised travel cost for travellers who take the 0 run train is equivalent to their willing-to-pay, i.e. reference point of the generalised travel cost, therefore we have

$$c_{WIP,x,t,\tau} = ut_r + f_r^0 + f_r x + Q_0(x,t,\tau)\cdot\eta\cdot H. \tag{6.32}$$

6.7 Numerical example

To facilitate the presentation of the essential ideas and contributions of this chapter, one illustrative example is employed. The mode split results are presented firstly, and the effects of population density on the performance of rail mode then are examined.

6.7.1 Preliminary

Consider a linear city with travellers uniformly distributed with respect to location, x, from the CBD at $x = 0$ out to the fixed city boundary $x = B$. A rail improvement project will be implemented over years so as to ease traffic congestion existing on the highway. The network is shown as Figure 6.1. The input data for this example are given as follows:
$B = 40$ km, $q_0(x) = 10000/20000/30000/40000/50000$ persons/km, $x_{P\&R} = 10$ km, u = 60 HK$/h, t_r^0 = 0.125 h/km, $f_r^0 = 3$ HK$/km/person, t_w^0 = 0.2h, f_r = 0.2 HK$/km/person, α = 0.5, H = 0.05/0.075/0.1/0.125 h, $\beta = 30$, $\gamma = 90$, t_{oc} = 0.2/6.0 h, f_c^0 = 9 HK$/vehicle, f_c = 0.4 HK$/km, $t^* = 9:30$ A.M., P_c = 20 HK$/vehicle, f_b^0 = 1 HK$/km/person, f_b = 0.03 HK$/km/person, t_{ob}^0 = 0.01 h, H_b = 0.2 h, α_b = 0.5, η = 0.02, $\hat{p}_{i,1,t,\tau}$ = 0, $p_{i,AITS,t,1} = 0.85$, $\upsilon = 0.025$, $\delta = 0.8$, $\psi \sim N(0,3)$, μ_0 = 100 HK$, μ_1 = 540 HK$/frequency, ξ = 2,

$T_0 = 0.1$ h, $D_1 = 30/35/40$ km, $\beta_0 = 0.1$ h, $D_s - D_{s-1} = 1$ km, $\gamma_0 = 5000$ HK\$, $\gamma_1 =$ 28000 HK\$/km, $\kappa_0 = 1.5\times10^3$ HK\$, $r = 0.01\ \text{period}^{-1}$, $\kappa_1 = 8\times10^3$ HK\$.

The in-vehicle crowding function for rail train mode is $g_r(v) = 0.5\ln(1+v_r/20000)$ HK\$/km . The in-vehicle crowding function for bus mode is $g_b(v) = 0.5\ln(1+v/12000)$ HK\$/km and the travel time function on highway is $v_a(w) = 1+0.5(w/20000)^4$ hour.

6.7.2 The effects of population density on the performance of rail service improvement

Profit could be used to measure the effects of population density on the performance of rail service improvement. The revenue of rail operator comes from fare income can be calculated by

$$R_f(\tau) = N_r(\tau) f_r^0 + f_r \sum_{s=1}^{N} \int_{t-n_2H}^{t+n_1H} Q_k(s,t,\tau) D_s \,\mathrm{d}t, \qquad (6.33)$$

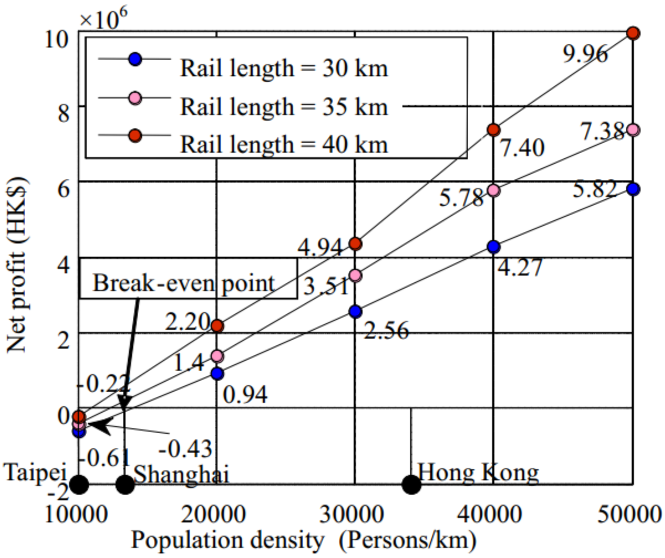
The net profit of railway service improvement could be calculated by

$$Z = R_r(\tau) - C, \qquad (6.34)$$

where

$$C = \gamma_1 D_1 + \gamma_0 D_1 + (\kappa_1 + \kappa_0)\frac{D_1}{D_s - D_{s-1}}, \qquad (6.35)$$

Where γ_1 represents the fixed railway line cost per kilometer in each period, γ_0 represents the variable operation cost for railway line per kilometer in each period, κ_0 represents the fixed railway station cost in each period, and κ_0 represents the variable railway station cost in each period.



Cities	Average population density (Persons/km)
Hong Kong	34,000
Shanghai	1,3400
Taipei	9,650

Figure 6.4: The net profit with respect to population density and rail length.

Figure 6.4 makes clear the net profit when the population density along the corridor changes from 10000 persons/km to 50000 persons/km. The train headway is set to be 0.1 h here. It was found that, the value of the total net profit increased with the increase of rail length and would be negative if the population density was 10000 persons/km, no matter how long the rail length would be. This means that rail improvement projects suffered a deficit in areas with a population density of 10000 persons/km.

Table 6.2 The estimated net profit with respect to population density and on-time arrival probability improvement in **each period (or year)** τ

Average population density (Persons/km)	The net profit in each period (10^6 HK$)					Annual Increase
	1	**2**	**3**	**4**	**5**	
34,000 (Hong Kong)	6.1462	6.1513	6.1590	6.1661	6.1751	**0.094%**
13,400 (Shanghai)	0.4960	0.5005	0.5018	0.5047	0.5120	**0.647%**
9,650(Taipei)	-1.1035	-1.0432	-0.8946	-0.8159	-0.4346	**12.108%**

Table 6.2 presents the net profit with respect to population density and on-time arrival probability improvement **over years**. It could been seen that, the annual increase of net profit decreased sharply as the population density increased from 9650 persons/km to 34000 persons/km, i.e. 12.108% to 0.094%. This could be attributed to the different demand elasticity of train service for different population densities. In areas with lower population density, the demand elasticity of train service was high, thus, any improvement in on-time arrival probability would greatly improve the net profit. However, in areas with high population density, since the demand elasticity of train

service is low, the net profit increase from on-time arrival probability improvement is correspondingly relatively low.

6.7.3 Modal split results without P&R mode

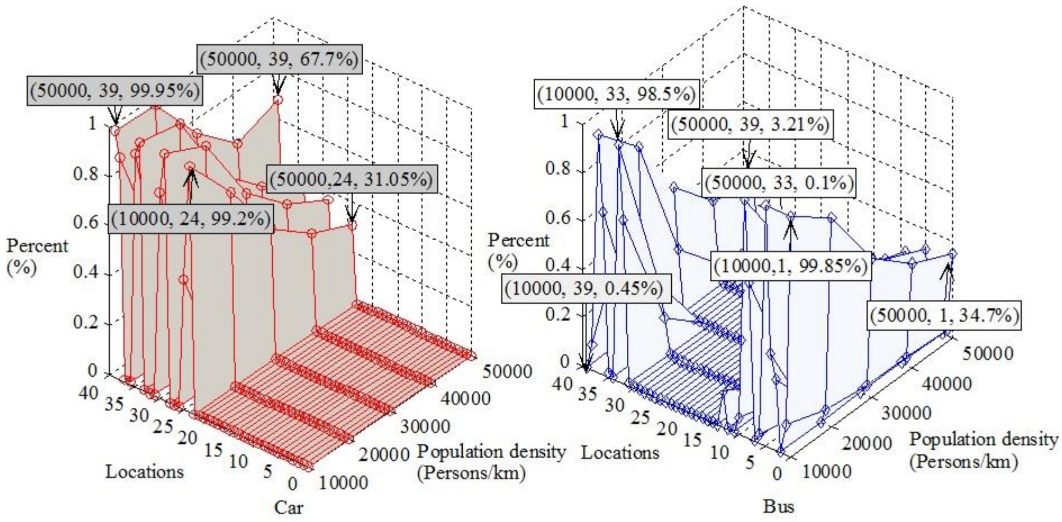


Figure 6.5: Modal split between different travel modes at the PTMUE during morning peak hour.

Given the above input data, Figure 6.5 shows the modal split results between different travel modes at the PTMUE during morning peak hour, i.e. train, car, and bus during the morning peak hour 8:30 am-10:00 am. The travel mode decision is made every 0.1h at every location by travellers. The preferred arrival time to the CBD is 9:30 am. The total demand is assumed to be varied from 10000 persons/km to 50000 persons/km. The parking charge is set to be 20 HK$ at the CBD. The rail length is 40 km and the headway of train is 0.1 h.

From Figure 6.5, it could be seen that the car mode dominates near the boundary of the corridor city when population density is 10000 persons/km, and the mode share of travel demand by car decreases as population density increases from 10000 to 50000 persons/km. As for the bus mode, the mode share decreases near the locations of CBD as population density increases from 10000 to 50000 persons/km, whereas its mode share of travel demand increases near the boundary of the corridor city as population density increases. This appears to be mainly due to the emerging traffic congestion on the highway. Such congestion leads to the travel demand shift from car and bus to train for higher population density. The rail mode shares also part of the total travel demand. This mode share is very high in the middle area of the corridor city, i.e. from locations 10 to 20, but is low near the CBD and the boundary of the corridor city. Due to the travel demand shift from car and bus, the rail mode share increases with the increase of population density.

6.7.4 Modal split results with P&R mode

Park-and-ride (P&R) facilities are being used in many metropolitan cities, such as London and New York in western countries, and Hong Kong, Beijing, Singapore, and Bangkok in Eastern Asia. The principal objective of P&R is to encourage modal shifts from low occupancy mode, i.e. private car, to higher occupancy mode, such as rail and bus, so as to reduce traffic congestion in urban areas (Noel, 1988; Lam et al, 2001; Li et al, 2006; Liu et al, 2009). Noel (1988) appears to be the first to investigate the effect of P&R facilities, finding that trip makers experienced increased travel comfort and reduced travel costs with the introduction of P&R schemes. Lam et al (2001) examined the application of P&R facilities in Hong Kong. Li et al (2006) explored the P&R services modelling in a multi-modal transportation network with elastic demand. Liu et al (2009) optimised the locations choice of P&R facilities in a multimodal linear monocentric city.

The above studies revealed that P&R facilities were widely assumed to be an effective way to alleviate traffic congestion and were demonstrated to be effective in western countries, for instance London (Niblett and Palmer, 1993) and New York (New York State Departure of Transportation, 2012). Further studies, however, are necessary to confirm the suitability of P&R facilities in high population density area. In this example, a P&R facility is assumed to be sited at the bottleneck as shown in Figure 6.1, and the parking charges is assumed to be relatively lower than that at CBD. The population density in each location varies from 10000 persons/km to 50000 persons/km so as to investigate its effects on modal split result of P&R mode.

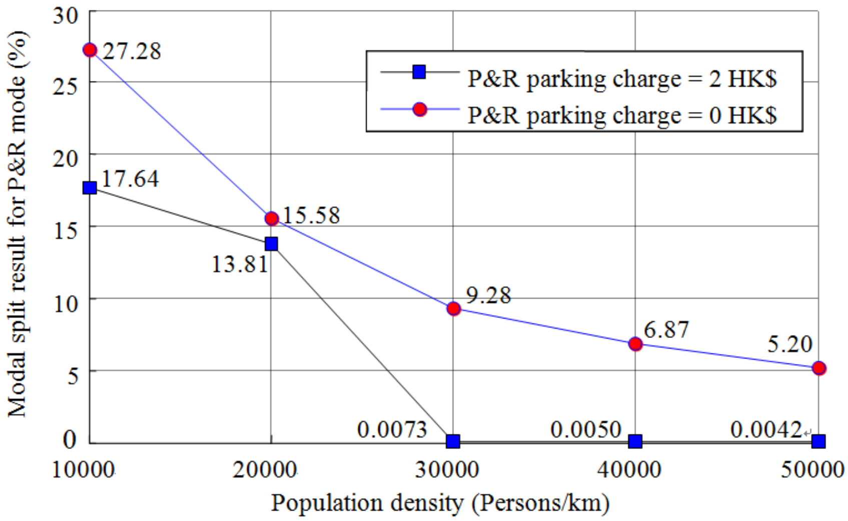


Figure 6.6: Modal split result for P&R with respect to parking charges of 0 and 2 HK$

Figure 6.6 presents the modal split result for P&R mode with respect to parking charge of 0 and 2 HK$ at P&R site. It could be seen that the travellers who choose to take P&R mode decrease sharply as the population density increases to more than 30000 persons/km. In the numerical example given, there are only 0.0073% travellers traveling by P&R mode in area with the population density of 30000 persons/km with respect to parking charge of 2 HK$. This result could be attributed to the increase of generalised travel cost due to huge demand for train service during morning peak hour at most high densely populated area. In contrast, P&R facilities perform better at areas with a population density of no more than 20000 persons/km. In such areas, travellers could enjoy the advantage of P&R mode. For instance, in the given numerical example, there are 13.81% travellers traveling by P&R mode in area with the population density of 20000 persons/km with respect to parking charge of 2 HK$. This result accords with experience reflected from Hong Kong and Shanghai, with population density of 34000 persons/ km and 13400 persons/km, respectively. The P&R facilities were found to be not satisfactory for Hong Kong, but more so for Shanghai.

6.8 Summary of this chapter

In this Chapter6, a convex programming model was proposed for design of a rail transit line in a multi-modal linear transportation corridor with road-based and non-road based transit modes. In the proposed model, the day-to-day traveller's mode choice behavior was considered explicitly together with the effects of population density on the rail transit line design over time. It was assumed that there are two existing road-based travel modes (i.e. car and bus modes) from different locations of the corridor to the Central Business District. These two road-based travel modes would obviously influence each other in operation when congestion appears in the road-based corridor.

In the proposed model, travellers were assumed to behave as prospect maximisers over time. It was shown that under a stable intermodal prospect-based mode-choice user equilibrium (PTMUE) condition, population density has significant impacts on the modal split results and the financial performance of rail train and Parking-and-ride modes. Parking and ride mode may not be suitable for areas with high population density.

This chapter proposes a robust prospect theory based mode-choice equilibrium model under ATIS and investigates the effects of population density on traveller's mode choice behaviour. The proposed model could be used to improve the existing ATIS so as to supply the mode choice information required for travellers. Further studies on the following issues can be carried out in future:

- The network uncertainty was not considered in this chapter. Although perceived error of generalised travel cost is considered, the network uncertainty is not taken into account. The travel cost of rail train, bus, car, and P&R mode should be stochastic in practice. The predicted on-time arrival probability by ATIS is given exogenously in this chapter, which could however be incorporated endogenously into the proposed model for further extension.

- Energy efficiency, traffic emission and equity should also be investigated. In the given example of this chapter, efficiency gained in most locations but lost in some locations, therefore, equity is an important issue for study in future.

Chapter 7 Conclusions and further studies

The objectives of this thesis are to propose analytical models for optimal design of urban rail transit line over time, especially for fast-growing Asian cities with high population densities and uncertainties in population growth along the potential rail transit corridor. The study objectives are laid out in details in Section **1.3** of Chapter 1.

The findings based on the study objectives, shown in each chapter, are presented accordingly in this thesis. In Section 7.2below, the relationships between the study objectives and findings detailed in the different chapters are identified and discussed.

The implications of the study findings for future research on this topic are then given together with general insights to rail planners, operators and local governments for design of a candidate rail transit line in a linear transportation corridor. Section 7.3 finally summarises some new interesting research questions based on the work presented in this thesis.

7.1 Study Findings

As indicated above, the study findings are presented chapter by chapter together with addressing the research problems formulated in Section **1.2** in Chapter 1. How these findings can be connected to the research problems concerned that are to be described in Section 7.2.

In Chapter 3, an analytical mathematical model is formulated to investigate the over-year interaction between population densities and financial performance of a candidate rail transit line. It was found that more people were attracted to live in vicinity of the candidate transit line if rail service was provided. With rail service in the linear transportation corridor, more population chose to move from residential locations of CBD to residential locations of suburban areas. The candidate rail transit line can make population densities more decentralized. As a result, more population distributed at residential locations of the suburban areas.

Chapter 3 explores the justification for the construction of a candidate rail transit line in a linear transportation corridor for a monocentric city.

In other words, under what circumstances, it is necessary to build or extend a rail transit line? Primarily, it may be partly due to the effects of rail transit line on the adjacent land use development over time. As a matter of fact, more decentralized population distribution along the rail transit corridor, enables more comfortable living environment. For instance, more efficient use of CBD and spaces of suburban area and new towns. The analytical model proposed in Chapter 3 has the potential to help authorities and/or operators in dealing with construction decisions of a candidate rail transit line over time.

For instance, the new rail project can lead to more decentralized population distribution together with the change of land use development in the rail transit corridor. It is interesting to investigate the **transit oriental development** (TOD) in the suburban area and new towns along the rail transit line. In the past, a large number of studies have discussed TOD projects, such as enhancing development density to increase transit ridership (Cervero, 1994), and assessing the impacts of rail projects on property projects (McDonald and Osuji, 1995), and sustainable urban development (Lin and Gau, 2006). Closed-form analytical models for TOD planning remain very fewin literature (Lin and Gau, 2006; Li et al, 2012), which can be further developed based on the model proposed in Chapter 3.

In Chapter 4, **implementation adaptability** of a candidate rail transit line is explored over years. Implementation adaptability gives authorities and/or operators to fast-track or defer the future investment on the candidate rail transit line for several years, if necessary.

Several insights for network design problem (NDP) over years are offered by the model proposed in Chapter 4. Examples include, the fast-tracked period of the candidate rail transit line would be shortened as the growth factor of the total population γ increases and interest rate κ decreases. The length of the candidate rail transit line decreases as interest rate increases. The returns at full time and return at 1/3 time under the alternative of fast-tracking the candidate rail transit line are larger than that under the alternative of deferring the candidate rail transit line.

The analytical model proposed in Chapter 4 is useful for many fast-growing cities in China. For large metropolitan cities, such as Shenzhen and Shanghai, the proposed model can be applied to determine the candidate rail project start time in an urban transportation corridor. For smaller-size fast-growing cities, the

proposed model can help to determine the required growth rate of total population and annual interest rate of capital investments for timely construction of a rail transit line in the corridor concerned.
In Chapter 5, **the effects of spatial and temporal correlation of population densities** on the design of a rail transit line over years are investigated. With the use of the model proposed in Chapter 5, some insightful findings are shown in the numerical results particularly on the effects of spatial and temporal correlation of population densities in the linear transportation corridor. For example, decentralized population distribution along the corridor would lead to higher social welfare budget of the candidate rail transit line. The higher social welfare budget of the candidate rail transit line may be due to the spatial competition between adjacent residential locations in the corridor. However, it was found in the numerical example that the effect of temporal correlation coefficient (cc) on the optimal social welfare budget is more significant than that of the spatial correlation coefficient. The optimal rail line length $L(t)$ in each year is longer compared temporal cc of -1 with that with temporal cc of 1. The optimal project start time of the candidate rail transit line $\hat{t}$ is fast-tracked as temporal cc increase from -1 to 1.

In Chapter 6, a convex mathematical model is proposed to examine the effects of travellers' prospect theory based travel mode choice behaviours over time on design of a candidate rail transit line, with special attention given to the effects of population density over period of years. The considered travel modes include auto, bus, train, and park-and-ride in a linear transportation corridor. It is concluded that under a stable intermodal prospect theory based mode-choice user equilibrium condition, population density is closely concerned with the modal split results and the performance of train and parking-and-ride modes. The park-and-ride mode may not be suitable for areas with high population density. It was found that, the value of the total net profit increased with the increase of rail length and would be negative if the population density was 10000 persons/km, no matter how long the rail length would be. This means that rail improvement projects suffered a deficit in areas with a population density of 10000 persons/km.

7.2 Relationship, methodology and implications of different chapters

7.2.1 Relationship of different chapters

The previous section gives a summary of the findings based on the analyses given in each chapter of this thesis. This section firstly presents the uncertainties of population densities in each chapter in Table 7.1. The relationships of different chapters are then discussed based on a monocentric city with one Central Business District (CBD) in a linear transportation corridor. Table 7.2summarises the design variables of the model proposed in each chapter.

In the monocentric city as shown in Figure 7.1, if the increase of population density $P(x_1,t_1)$ in residential location x_1 in year t_1 leads to the increase of population density $P(x_1,t_2)$ in residential location x_1 in year t_2, then a positive temporal correlation of population density exists between population densities $P(x_1,t_1)$ and $P(x_1,t_2)$. Similarly, if the increase of population density $P(x_1,t_1)$ in residential location x_1 in year t_1 leads to the increase of population density $P(x_2,t_1)$ in residential location x_2 in year t_1, then a positive spatial correlation of population densities exists between population densities $P(x_1,t_1)$ and $P(x_2,t_1)$.

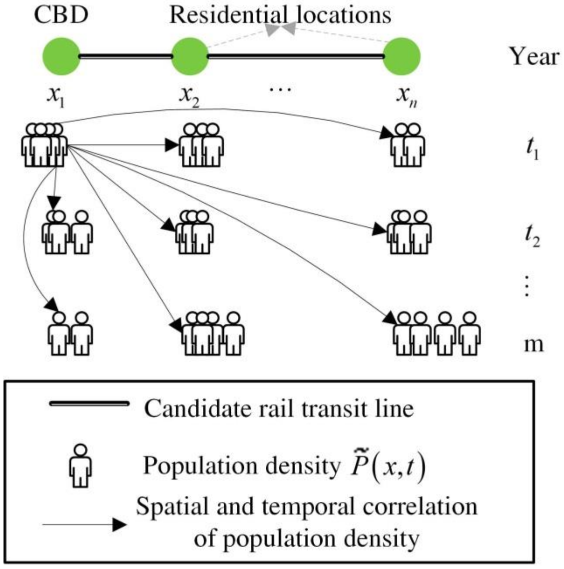


Figure 7.1: Uncertainties of population densities in a monocentric city with linear transportation corridor

Table 7.1 Design variables of the model proposed in each chapter.

Chapter	Design variables of the proposed model				
	Rail length	Numbers of rail stations	Headway	Fare	Project start time
3	√	√	√	√	Given and fixed
4	√	√	×	×	√
5	√	√	×	×	√
6	Sensitivity analysis				Given and fixed

Table 7.1 presents the design variables of the models proposed in different chapters. As stated in assumptions of Chapter 3, long-term design variables of rail length and number of rail stations and short-term design variables of headway and fare are taken into account simultaneously in Chapter 3. The project start time is assumed to be given in Chapter 3, which is relaxed and optimised in Chapters 4 and 5.

In practice, the short-term design variables of headway and fare are not taken into account while conducting the optimisation of long-term design variables and project start time of the candidate rail transit line. It is noted, however, that these short-term design variables of headway and fare can affect the long-term design variables and project start time. For instance, in Chapter 4, the optimal rail line length is

$$L^t = \frac{H^t \mu_w \lambda \sum_{i=1}^{n_s^t} \exp\left(\theta \pi_i^t\right) \left(P_i^t I^{3-\frac{\alpha}{\beta}} \beta^2 \theta + r_0^t \alpha \left(1-\pi_i^t\right)^{\frac{\alpha}{\beta}-1} Y_i \right)}{C_r I^{3-\frac{\alpha}{\beta}} \beta^2}. \quad (7.1)$$

where L^t is rail line length, and H^t is headway. π_i^t is generalised travel cost, which is a function of fare given by Eq. (4.5). Namely, rail line length L^t is a function of headway and fare.

In Chapter 6, sensitivity analysis on rail length is conducted, namely from 30km to 40 km as shown in Figure 6.6. More attentions are paid on the modelling of travellers' learning behaviour over years together with the effects of population densities for design of rail transit line over time.

Table 7.2 Uncertainties of population densities in each chapter of this thesis

Chapter	Uncertainties of population densities
3	Population density function Eq. (3.1) proposed by Dargay and Goodwin (1994). This function considers the relationship between population density in year t and $t-1$
4	Yearly variation of the population given by a constant growth factor
5	Spatial and temporal correlation between population densities
6	Variation from low population density to high population density

Table 7.2 summarises the uncertainties of population densities in each chapter. Table 7.3 presents the correlation of population densities in different chapters.

Table 7.3 Modelling correlation of population densities in different chapters.

Chapter	Spatial correlation of population densities	Temporal correlation of population densities	Multi-modal; travel behaviour over years
3	×	√	×
4	√	√	×
5	√	√	×
6	×	×	√

It can be seen in Table 7.3 that based on a detailed population density function proposed by Daygay and Goodwin (1994), the model proposed in Chapter 3 only considers the temporal correlation of population densities in two conjoint years. The spatial correlation of population densities is not considered. To use this model, parameters calibrations are required, for instance $\beta_c / \beta_p / \beta_\pi$.

In Chapter 4, the temporal correlation of population densities is considered explicitly. With a growth factor of the total population, in terms of Eqs. (4.1), (4.18), and (4.19), the following equation can be obtained

$$P_i^t = \left(1+\gamma\left(t\right)\right)P_i^{t-1}, \tag{7.2}$$

while $\gamma > 0$, it implies that population density in residential location i positively increases from year $t-1$ to year t. Due to the constraint of Eq. (4.19), the total population is distributed to different residential locations. The spatial correlation of population densities are considered, but not explicitly. For the proposed model in Chapter 4, parameters calibrations are required, for instance the constant year-by-year growth factor of the total population γ and annual average interest rate κ.

In Chapter 5, a closed-form mathematical programming model is proposed to investigate the effects of spatial and temporal correlation of population densities on design of a candidate rail transit line over years. Only rail mode is considered in this chapter. This assumption is extended to a multi-modal situation in Chapter 6.

In Chapter 6, sensitivity analysis on population density is conducted from low population density to high population density, which is ranged from 10000 persons/km to 50000 persons/km. The focus of this chapter is on the effects of population density on households' travel mode choice behaviour over a period of years for design of rail transit line. To use the proposed model in this chapter, calibrations are required, for instance parameters of prospect theory $\alpha' / \beta' / \gamma'$.

7.2.2 Research methodology

Table 7.1 shows how uncertainties of population densities are considered. These uncertainties of population densities are incorporated into the proposed models and related to the research problems proposed in **Section 1.2** of Chapter 1.

In Chapter 3, to investigate the over-year interaction between population densities and financial performance of the candidate rail transit line, the population densities function Eq. (3.1) proposed by Dargay and Goodwin (1994) is used. In this function, the population density in year t is a function of generalised travel cost in year t and travel demand in year $t-1$. The travel demand in year $t-1$ is a function of population density in year $t-1$. Thus, population density in year t is a function of population density in year $t-1$. The financial performance of the candidate rail transit line is a function of travel demand and further is, a function of population density over years.

In Chapter 4, yearly variation of the total population density is given by a constant yearly growth factor. This growth factor can be used to describe travel demand variation of rail service over time. To investigate the implementation adaptability of the candidate rail transit line, travel supply cost variation of rail service is taken into account by an annual interest rate.

In Chapter 5, the spatial and temporal correlation of population densities are measured by spatial and temporal covariance. To obtain a closed-form solution, a social welfare budget is introduced, which is a function of the spatial and temporal covariance of population densities along the linear transportation corridor.

In Chapter 6, to investigate the over-year effects of travel behaviour on design of a candidate rail transit line, a prospect theory based multimodal user equilibrium model is proposed. In the numerical results, population density is varied from low population density to high population density in order to show their effects on the modal split results and subsequently on the design the candidate rail transit line over time.

7.2.3 Implication for rail planners, operators and local governments

This section presents the research implication of this thesis and gives some general insights to rail planners, operators and local governments for design of a candidate rail transit line in monocentric city with one CBD in a linear transportation corridor.

In Chapter 3, the insights for construction of a candidate rail transit line are investigated. Primarily, it was found that a lack of integration of short-term decision variables, headway and fare, may result in excessive investments for the long-term rail construction, namely the increase of optimal rail line length. It was found the candidate rail transit line can make population densities more decentralized. As a result, more population distributed at residential locations of the suburban areas.

In Chapter 4, with consideration of yearly variation of the total population density along the linear transportation corridor, the closed-form solutions are derived for the design variables of the candidate rail transit line over years. Rail planners, operators and local governments can make use of the proposed analytical solutions to determine whether to fast-track or defer the candidate rail transit line.

Chapter 5 examines the spatial and temporal relationships of population densities together with their impacts on the closed-form solutions for the rail transit design variables. With the use of the model proposed in this chapter, the rail planners, operators and local governments can better understand the effects of spatial and temporal correlation of population densities on the design of a rail transit line over years.

The closed-form mathematical programming model proposed in Chapter 5, can be used to analyse the effects of a candidate rail transit line on the correlations of different residential locations along the candidate rail transit line over time. It can also be extended to assess the impacts of the high speed rail on the development correlations of cities along the high speed rail route in China. The high speed rail in China experiences rapid development in recent years. The Chinese government is very concerned about the effects of high speed rail on correlations of cities' development along the high speed rail route. This is an important research topic for further extension of this study.

Chapter 6 gives, by numerical example, the break-even population density for the candidate rail transit line in a multi-modal transportation corridor over period of years. The effects of population density on modal split results are presented using sensitivity analysis from low population density to high population density over time. A further interesting finding was that that parking-and-ride may not suitable for cities with a high population density of more than 20000 persons/km.

The prospect theory based analytical model proposed in Chapter 6, can be applied to help the authorities to determine the development of multi-modal transportation systems for high-density developed cities in China. For instance, rail transit lines are under construction in many fast-growing cities of China. Whether it is appropriate to promote the park-and-ride mode simultaneously with the rail transit project development? With the use of the model proposed in Chapter 6, it was found in the numerical example results that provision of park-and-ride mode may not be suitable for rail transit corridor with population densities of more than 30,000 persons/km, if the charge for using park-and-ride facility is 2 HK$/hour only. Further case studies in some fast-growing Chinese cities should be carried out so as to provide empirical evidence and support for development of urban rail transit line with or without park-and-ride facilities under uncertainties in population densities along the linear multi-modal transportation corridor.

7.3 Further research

New avenues for research on modelling and analysis of rail transit line design problems over time, especially for fast-growing cities with high population densities, are presented in this thesis. Related questions for further research, however, are also raised. Although the following list suggested for further research is not exhaustive, an outline of these potential research extensions is suggested below:

A monocentric city is assumed in this thesis, that is, only with one CBD and several other residential locations in a linear transportation corridor. The city boundary is not explicitly considered. Therefore, it is necessary to consider explicitly the city boundary in order to extend the proposed models to polycentric CBD corridor for further study.

In this thesis, the investment risk investigated for design of a candidate rail transit line in a linear transportation corridor is based on the interest rates, temporal and spatial population covariance and population variation over the years. However, there are many other investment risk sources considered by different investors for alternative transportation projects under uncertainties over time. For instance, the client investment risk related to private transit operators are different for the one considered by government sector. A detailed multi-modal transportation network equilibrium model should be developed to take into account the effects of alternative transportation projects with both demand and supply uncertainties, since the performance of these related alternative projects are influenced considerably.

The decision to extend a candidate rail transit line involves consideration of technological, social, environmental, and economic factors. The key factor could be social or in other words a desire to make life more convenient as regards maneuverability for a specific group of people, namely those living in the vicinity of the urban rail line and new stations to be constructed. However, only pressing economic factor is considered in this thesis. More detailed social and environmental factors and even the technology advancement can also be taken into account for further studies, such as appreciation of social equity and environmental sustainability along the candidate rail transit line for smart mobility in the multi-modal transportation system.

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