AN INTEGRATED APPLICATION OF
REAL OPTION THEORY
IN THE URBAN REDEVELOPMENT PROJECTS

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CERTIFICATE OF ORIGINALITY

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ZHONG Jiawei _____(Name of Student)

ABSTRACT

Urban redevelopment is a measure for handling urban decay problems. As an important component of urban renewal, urban redevelopment helps change original and outdated land use, replace severely dilapidated buildings, upgrade building structures, and rearrange communal facilities. Dilapidated buildings put many lives in danger; thus, finding a solution to this problem is imperative.

Undertaking redevelopment on multi-owner buildings usually takes a long time and requires an accurate valuation method for proper planning and scheduling. Real option approach is accepted as a superior framework that allows investors to choose their preferred scheduling to maximize project revenue. However, existing option pricing models fail to consider two special characteristics in the redevelopment of multi-owner buildings. Firstly, investors have to predict the values of old and new properties separately during the entire two-phase option period to reflect the difference between the properties in the same location; meanwhile, depreciation effect should be embedded in the pricing model. Secondly, when new buildings adopt vertical mixed-use, an improved pricing model should be required to consider the difference in design and lease restrictions against horizontal mixed-use developments. A literature review in Chapter 2 explains the reasons for considering these characteristics, which were not covered in extant studies.

This PhD study aims to bridge the two gaps in the valuation of redevelopment projects. Four research objectives are established to fill these gaps as follows: (1) to identify new parameters that capture the effect of depreciation on a building; (2) to examine the potential influences of

depreciation rate and other new parameters on project value; (3) to evaluate the expected waiting time to demolish and rebuild with depreciation effect; and (4) to establish an option pricing model for vertical mixed-use developments and examine the influence of different designs (horizontal or vertical) on project values.

The quantitative approaches and theoretical assumptions adopted in this study are demonstrated in Chapter 3. Three novel real option models are developed in Chapters 4, 5 and 6 to achieve the aforementioned four research objectives. A constant depreciation rate assumption and a new parameter of annual increase on the average building age are introduced to measure the depreciation effect. Chapter 4 explores the roles of depreciation rate and annual increase of average building age in the project valuation within a finite option period. The influences of the two factors are found greater than those of the price/cost volatility and interest rate. Chapter 5 examines the feasibility of the existing optimal exercise strategy when the length of the redevelopment period is unbounded. The optimal strategy is found to be considerably affected by depreciation rate and capital return rate. When the depreciation effect is small and capital return rate is high, investors can still start the demolition and rebuilding immediately as the optimal demolition price-to-cost ratio is reached. However, as the depreciation rate increases (e.g., 2% p.a. or above) and the capital return rate decreases (e.g., 8% p.a. or below), redevelopment should start when the optimal rebuilding price-to-cost ratio is reached at this point. Chapter 6 focuses on the differences of valuation models between horizontal and vertical mixed-use developments. A new parameter, critical height premium, is introduced to aid investors determine the building type with a higher value. This chapter also reveals how the volatilities in different markets influence the critical height premium and the choice of building type.

This study offers several theoretical and practical contributions. It provides an in-depth discussion why depreciation effect should be embedded in redevelopment option models and how to measure this effect properly when more than one property is involved in redevelopment. The findings from discrete- and continuous-time models for two-phase redevelopment projects have proved the importance of depreciation effect on project valuation and optimal timing. This study also identifies the differences of valuation models between horizontal and vertical mixed-use developments. Developers can apply the three novel models developed in this study to assist in their decision making on redevelopment projects and extend to more complicated projects, such as redevelopment of several vertical mixed-use buildings. Policy makers can predict developers' choices in redevelopment timing or building type accurately by incorporating the depreciation effect and vertical mixed-use forms into real option models.

LIST OF PUBLICATIONS

Conference papers arising from the thesis

Hui, E.C.M. and Zhong, J. (2016). A Compound Option Pricing Model in Urban Redevelopment Projects. *9th Global Chinese Real Estate Congress*. Hangzhou, China.

Working papers arising from the thesis

Zhong, J. and Hui, E.C.M. Depreciation, Building Age and Reference Market Price Statistics in Urban Redevelopment Option Pricing. (Revised and under second review by Journal of Urban Planning and Development)

Zhong, J. and Hui, E.C.M. Redevelopment strategies, building ages and acquisition standards.

Zhong, J. and Hui, E.C.M. Real option and vertical mixed-use development.

Other publications

Hui, E. C., **Zhong, J.**, & Yu, K. (2017). Property prices, housing policies for collateral and resale constraints. *International Journal of Strategic Property Management*, *21*(2), 115-128.

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Hui, E. C., Liang, C., **Zhong, J.**, & Ip, W. C. (2016). Capture the Abrupt Changes in Asian Residential Property Markets. *Habitat International*, *56*(2016), 235-244.

Hui, E. C., **Zhong, J.**, & Yu, K. (2015). Housing Policy, Work-residence Mismatch and Poverty Concentration. *Habitat International*, *48*(2015), 198-208.

Hui, E. C., Liang, C., **Zhong, J.**, & Ip, W. C. (2015). Structural and Policy Changes in the Chinese Housing Market. *Journal of Urban Planning and Development*, *142*(1), 04015012.

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CHAPTER 1 INTRODUCTION

1.1 Research background and problem statement

1.1.1 Introduction to redevelopment

Urban renewal is an efficient approach that addresses urban decay problems, enhances land values, develops a good living environment, reduces negative externality, improves hygiene conditions and achieves other desirable social and economic objectives (Adams & Hastings, 2001; Chan & Yung, 2004; Krieger & Higgins, 2002; G. K. L. Lee & Chan, 2008). Many scholars have discussed the relationship between urban renewal and sustainable development. Zheng, Shen, and Wang (2014) attributed the success of urban renewal to sustainable development with the planning subsystem (i.e. land, housing, infrastructure and heritage) and the social subsystem. Land and housing are more related to the economic benefits of urban renewal than the other components of planning subsystem.

Urban redevelopment, which includes demolitions of existing buildings and constructions, is generally a measure of urban renewal (De Sousa, 2008). In Hong Kong, the Urban Renewal Authority (URA) adopts four core strategies to achieve urban renewal, that is, to protect and utilise urban land potential effectively (Chan, Tang, & Yung, 2000). The four strategies are redevelopment, rehabilitation, revitalisation and heritage conservation. The URA's description properly differentiates the concepts of urban renewal and redevelopment.

The URA also studied the urban redevelopment programmes in several Asian cities (Law et al., 2009), including Singapore, Tokyo, Seoul, Taipei, Shanghai and Guangzhou. These redevelopment projects aimed to optimise the original function, revive idle lands, replace old factories with high-rise multiuse properties, clear squatters, remove dangerous and highly dilapidated properties, improve hygiene/air-conditioning/heating/fire-preservation conditions, upgrade housing structures, improve road systems, provide additional communal facilities and infrastructures, improve regional appearances, reduce local social conflicts, resist natural disasters and increase housing supplies.

Amongst these objectives, removing dangerous and dilapidated buildings has attracted the most attention since the collapse of a 55-year-old Chinese tenement building on Ma Tau Wai Road, which caused 4 dead and two injured people in 2010. Another balcony of a 61-year-old tenement building in the same district collapsed in 2017. Although the triggers of these collapses were illegal subdivision works and demolition of illegal structure, these accidents still alarmed people of the hidden safety problem of old buildings.

1.1.2 Current situations of dilapidated buildings in Hong Kong

This subsection further demonstrates the problem of dilapidated buildings in Hong Kong. By the end of 2017, Hong Kong had 2,773,600 living quarters in total (D. Wong, 2017). The government would announce the distribution of these domestic units by age annually. The most recent information is shown in Figure 1.1.

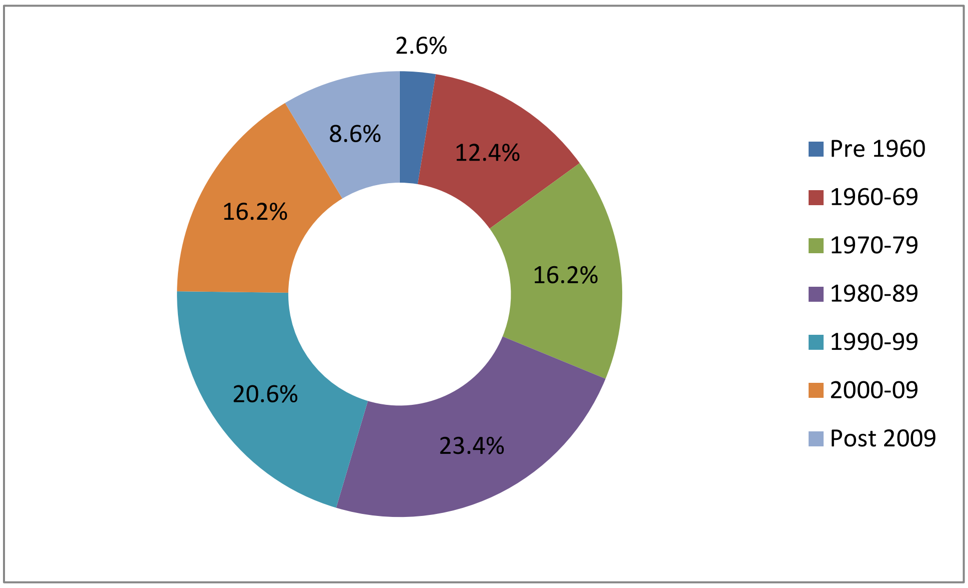


Figure 1.1 Stock distribution by age

(Source: Hong Kong Property Review 2018, Rating and Valuation Department (2018))

Approximately 15.0% of units were completed before 1970. In other words, more than 176,000 units will be over 50 years old in 2020. In 2030, this number will remarkably increase to more than 366,000. At that time, if the new construction speed in 2018–2029 is close to that in 2009–2017 and no old properties are replaced, then approximately 27.6% of private domestic units will be more than 50 years old. The government predicts that private domestic housing units over 70 years old will reach 326,000 in 2046 (Task Force on Land Supply, 2018).

The URA also reported the number of multi-owner buildings aged over 50 years. In 2010, this number was 4,000 and is expected to reach 16,000 in 2030 (Urban Renewal Authority, 2011). Amongst the 4,000 buildings aged over 50 years in 2010, approximately 2,200 properties were older than their design lives (D. C. W. Ho, Yau, Poon, & Liusman, 2012).

After the tragedy on Ma Tau Wai Road in 2010, the Buildings Department examined the overall maintenance situations of multi-owner buildings over 50 years in Hong Kong Island and Kowloon Peninsula. A total of 4,011 old properties in Kowloon City, Yau Tsim Mong, Shen Shui Po, Sheung Wan and North Point were investigated. From the investigation, 2 buildings required urgent maintenance, 1,030 needed comprehensive maintenance as soon as possible, and 1,270 showed slight physical damages (Buildings Department, 2010). Generally, these results indicate the unsatisfactory maintenance status of old multi-owner properties.

Insufficient maintenance treatment results in structural danger. Structural danger and unauthorised building works caused the Ma Tau Wai Road tragedy. The Buildings Department has recorded the annual/monthly reports received about dangerous buildings and unauthorised building works (Table 1.1). The high report frequencies reflect the public's concern about the safety of dilapidated properties, which may need to be redeveloped soon.

In addition, the outdated design for some of these old properties may not meet the present fire safety standards or other building ordinances. Common rehabilitation measures still may not help these properties comply with the present ordinances. For these old buildings, redevelopment is ultimately the optimal choice to improve the residential environment.

Year/month	Dangerous buildings	Unauthorised building work
2010	8,028	28,148
2011	7,533	38,538
2012	6,716	43,881
2013	7,342	43,366
2014	7,505	41,403
2015	6,933	40,595
2016	7,396	37,153
2017	7,188	31,710

2018 Jan	535	3,429
2018 Feb	385	2,243
2018 Mar	577	2,718
2018 Apr	481	2,621

Table 1.1 Reports received about dangerous buildings and unauthorised building works

(Source: Buildings Department, 2018
https://www.bd.gov.hk/english/documents/statistic/Md21e.pdf)

1.1.3 Two characteristics of redevelopment of multi-owner buildings

Redevelopment is a necessary measure for solving the problem of dilapidated buildings (Section 1.1.2). Redevelopment usually takes a long time because it involves many participants (Wang, Shen, Tang, & Skitmore, 2013). A superior valuation method is required to reflect such a complicated decision process.

Real option valuation can value redevelopment projects well. It estimates not only the project value when redevelopment is conducted at present but also when redevelopment is postponed to a more profitable timing within several years. This characteristic considers project valuation given that investors should have additional opportunities to determine redevelopment timings.

In Hong Kong, multi-owner buildings have become a commonplace for more than half a century. Hence, redevelopment projects have been usually conducted on old multi-owner properties. These projects have two special characteristics different from the redevelopment of single-detached houses. The first characteristic is that redevelopment of multi-owner buildings usually includes two phases. First is the demolition phase where the government/developers compensate and resettles the original residents/owners in old properties. Afterwards, the empty dilapidated buildings are demolished, and the land is cleared. Second is the rebuilding phase where a new property is rebuilt on the vacant land. To evaluate redevelopment projects, developers should predict the values of the old properties during demolition and those of newly built properties during the rebuilding. Therefore, a new real option approach that can adjust old and new property values during the entire option period while considering depreciation effect should be developed.

The second characteristic is the form of newly built buildings. To enhance the value of new properties, vertical mixed-use development is usually selected for high-density cities. Previous models focus on horizontal mixed-use development, which includes several buildings with different uses. The differences in construction forms and other lease restrictions for the two types indicate that the existing real option model for horizontal type may not be appropriate for vertical mixed-use development.

The present study aims to develop new real option approaches to reflect the two characteristics in the redevelopment of multi-owner buildings. Inappropriate choices of valuation approaches generate inaccurate project values and incorrect decisions in redevelopment. The remainder of Section 1.1 further demonstrates why the two characteristics should be emphasised.

1.1.4 Depreciation and property value in redevelopment

Redevelopment projects are usually conducted to replace dilapidated buildings, which have lower market values than the new properties at the same location. This value difference is usually explained by age or depreciation effect. In general cases, monetary compensation/acquisition in the demolition phase are not based directly on the market prices of new properties. Furthermore, the unit prices of old and new properties in a redevelopment project should not be equal in most cases due to depreciation effect.

This considerable value difference was estimated by Yiu (2007). In Yiu's study, the property value loss due to depreciation in 2005 was 34% of Hong Kong's gross domestic product. A 40-year-old building would depreciate approximately 45% of its initial value compared with a newly built one at the same location. Hence, depreciation effect should be measured for the valuation of redevelopment projects.

Depreciation is the value of a property that declines as the building age increases. This decline is the direct result of the ageing and out-of-date structure of the property (Bokhari & Geltner, 2016). A property deteriorates quickly, if no maintenance programme is offered (Margolis, 1981). Chau, Wong, and Yiu (2005) summarised three possible reasons for value depreciation: physical deterioration/outdated function, discount for the information asymmetry and limited residual time to lease maturity. As such, the property value in the long-term is considerably affected by depreciation/age effect. The project value is also determined by the difference between new and old property values. Hence, the depreciation effect should be considered in determining the redevelopment project value.

Another reason why depreciation effect is important in redevelopment is the length of the project period. Redevelopment projects, especially the ones on multi-owner buildings, usually take a long time from the acquisition of old buildings to the completion of new ones. Old and nearby properties during the entire option period are ageing. The ages of targeted old and nearby properties are varying. Hence, we cannot choose one price statistic to represent all these property values at the same time. Traditional project valuation approaches without depreciation adjustment leads to severe estimation bias in redevelopment project values.

The aforementioned reasons emphasise the importance of proper adjustments for depreciation effect. However, in Section 2.8.1, as a major valuation method in property development/redevelopment projects, the real option approaches usually do not consider depreciation effect because these approaches are nearly not developed for multi-owner buildings. This situation results in a serious bias in the estimation of project values and optimal redevelopment timings. This study investigates the manner in which the depreciation effect can be properly adjusted and how this effect influences the project value/optimal redevelopment strategy. Inappropriate treatment for depreciation effect will cause wrong project appraisals of and missed profitable timings for redevelopment. The outcomes of this research are expected to benefit society by providing effective approaches in decision making for redevelopment projects.

1.1.5 Mixed-use development

This subsection provides the background of mixed-use development, namely, horizontal/vertical mixed-use development and multiple intensive land use (MILU) development. Although single-use buildings are common, mixed-use designed properties also appear in urban renewal to achieve land use efficiency. A proper planned mixed-use project should enhance overall property value, reduce the investment risk by diversification, improve energy efficiency, release the traffic congestion, increase the satisfaction of residents and tenants, integrate public uses and increase municipal revenues (Planning Department, 2002; Rabianski, Gibler, Tidwell, & Clements, 2009; Rowley, 1996; Walker, 1997). Retail–residential and retail–office developments are two popular mixed-use structures.

Hoppenbrouwer and Louw (2005) classified mixed-use projects in four dimensions. Horizontal and vertical developments are classified in the spatial dimension. “Mixed-use development” in many studies usually refers to the horizontal type or mixed-use form-based zoning. The Planning Department incorporates different types of land use within the same administrative district. For example, if a district includes commercial and residential zones, then it will shorten some residents’ travel time to work. A comprehensive district with retail and residential properties can increase the accessibility and the potential values of both types of properties. By contrast, some zones are assigned as flexible land use. In these zones, developers can predetermine the actual proportions of different types of land use on the basis of the market environment. For example, the comprehensive development area and commercial/residential zone in Hong Kong, as well as the white site and business park–white site in Singapore since 1995, are representatives of flexible land use zoning.

Horizontal mixed-use development is applied in the urban renewal programmes or high-density cities in Europe. In the US, mixed-use development is combined with new types of urbanisation, such as smart growth and neotraditional neighbourhoods (Rabianski et al., 2009). Battery Park City in New York and Yebisu Garden Place in Tokyo are two representatives of horizontal mixed-use development (Cybriwsky, 1999). Commercial, recreational, retail and residential properties are included in these comprehensive development zones.

In many Asian cities, population pressure and insufficient land encourage the development of large-scale mixed-use structures (Lau, Giridharan, & Ganesan, 2003), which include many vertical mixed-use buildings. The high population pressures the government to support MILU developments (Zhu & Chiu, 2011). For example, two Asian metropolitans, Hong Kong and Singapore, have planned and built high-rise mixed-use properties for more than 30 years (Zhang, 2000). The integrated "rail–property" development model in Hong Kong has successfully achieved its objective to finance the subway construction and operation costs (Cervero & Murakami, 2009). Over 10 MILU developments have been constructed along the Mass Transit Railway (MTR) upon the stations. Different uses (e.g. residential units, retail units, public transportation facilities and carparks) are usually vertically distributed within these MILU developments. For retail and residential mixed properties, Mei Foo Sun Chuen, Metro City, Maritime Square and Whampoa Garden are famous large-scale vertical mixed-use estates. The tallest building in Hong Kong, namely, the International Commerce Centre, is a famous MILU skyscraper completed in 2010. According to its website, cutting-edge commercial offices, luxury residential units, modern retail shops and six-star hotels are located within this 118-storey development.

Amongst these developments, podium style is widely accepted to combine the multilevel shopping mall with residential buildings and green public space upon the podium. This structure can also be found in Shanghai, Singapore and Tokyo (Zhu & Chiu, 2011). In Singapore, Guoco Tower and South Beach are two new complex properties that comprise offices, hotels, retail space and luxury residential units. These properties contribute to the efficient use of common resources and are becoming new landmarks in the central business district (Jll Singapore, 2014). These vertical mixed-use properties can also attract renters, residents and customers and increase public space. They play an important role in the urban revitalisation.

Comprehensive analysis should be conducted in advance to achieve the synthesised goals in mixed-use development. Rabianski et al. (2009) summarised the following major issues in mixed-use developments: the demand of potential customers, the satisfaction of local residents and tenants, the support of public transportation facilities for the management of high passenger flow, sense of community, financial profitability and environmentally friendly issues. Although the successful performances of mixed-use development have been supported in empirical studies (Bookout, 1992; Frank & Pivo, 1994; Geoghegan, Wainger, & Bockstael, 1997; Kockelman, 1997; Levine & Frank, 2007; Nasar & Julian, 1995), mixed-use developments, especially vertical cases, require higher construction cost than single-use ones (Kettler, 2005; Koch, 2004). Complicated planning and architectural designs must satisfy the safety, economic efficiency and environmental requirements.

However, the popular vertical mixed-use development has considerable differences in the construction process compared with the traditional horizontal type. For example, units in different uses within the vertical type can only be constructed and sold simultaneously. The optimal development timing for a specific use is usually not the optimal timing for the entire project. Obtaining the optimal development timing in vertical type becomes more important than that in horizontal type, because the construction in the former is less flexible than that in the latter. Other differences will be further demonstrated in Section 2.8.2. These characteristics require a new model for development decisions, because traditional models are more appropriate for the horizontal type. Adopting an inappropriate pricing model will result in misleading development decisions for accepting/rejecting the project or for optimal development timing. In addition to handling depreciation effect mentioned in the previous section, this study aims address the issue to help investors achieve efficient land use goals.

1.2 Problem statement

As mentioned in Sections 1.1.1 and 1.1.2, the role of redevelopment is becoming increasingly important in high-density cities in Asia. The replacement of dilapidated buildings (Section 1.1.3) requires land use efficiency in redevelopment.

Redevelopment projects usually take a long time to be completed. This characteristic requires a valuation approach that is flexible in project decisions to handle future price uncertainties. Real option approaches are a commonly used valuation tool for redevelopment projects. Different from the discounted cash flow (DCF) method, real option approaches do not predetermine the investment timing. Instead, these approaches maximise the project value by applying an optimal exercise strategy. Titman (1985) and Capozza and Helsley (1990) emphasised that future price uncertainty, which was captured using real option approaches, made the immediate investment become less attractive. The disadvantage of DCF method or net present value (NPV) rule in redevelopment will be further discussed in Section 2.2.

Sections 1.1.4 and 1.1.5 have demonstrated the importance of depreciation effect and vertical mixed-use type in redevelopment project valuation. However, the existing literature in redevelopment option models (which will be presented in Chapter 2) has seldom focused on either of the aforementioned factors. Previous models have arguably failed to satisfy the demand of investors in project valuations, especially when the redevelopment occurs in high-density cities. This study will be based on the following assumptions:

1. The properties to be redeveloped and those to be rebuilt are multi-owner buildings. This is the most special assumption for this study.
2. The developer can purchase or demolish the old property, rebuild or sell the new property at any time during the related option period. Financial or other regulation restrictions are assumed to be already cleared by the developer. The market is frictionless. This is a common assumption in real option approach.
3. The risk-free interest rate, the capital return rate, market price and cost volatilities are constant during the option period.
4. The revenue of the redevelopment project is only from the sales or rents of new properties. This study only considers the major benefits in a redevelopment project. In actual projects, other revenues can be included in the calculation.
5. There is no arbitrage opportunity.

Compared to the financial asset in a financial option approach, the property market price does not change so frequently. For example, the price of a stock changes every minute or even every second. However, the price of a property does not. Property price indices only report the weekly or monthly trends. A discrete-time model can satisfy the developers' need. The results from a continuous-time model are adopted approximately in practice.

This study is performed to address the following fundamental research questions:

- What are the necessary adjustments that can properly capture the depreciation effect in redevelopment projects?
- How do the depreciation rate and other new factors influence the option value and expected waiting time to start?

- What components should be included to reflect the substantial difference in valuation models between horizontal and vertical mixed-use projects?

1.3 Research objectives

This study focuses on two related topics concerning option pricing for redevelopment. One is the measurement of depreciation effect, and the other is the characteristics of vertical mixed-use developments. The main research objectives are as follows.

Objective 1. To identify new parameters that capture the effect of depreciation on a building.

Objective 2. To examine the potential influences of depreciation rate and other new parameters on project value (in the discrete-time option model).

Objective 3. To evaluate the expected waiting time in demolishing and rebuilding (in a two-phase continuous-time redevelopment option) with depreciation effect.

Objective 4. To establish an option pricing model for vertical mixed-use developments and examine the influences of different design types (horizontal or vertical) on project values.

1.4 Significance of this study

With the aforementioned objectives, this PhD study investigates the influences of depreciation effect and vertical mixed-use development on project value and timing for redevelopment. As stated in Sections 1.1.4 and 1.1.5, the two topics are essentially related to multi-owner buildings, which is a common design feature of residential properties in high-density cities in Asia. However, previous studies on option pricing have focused only on the redevelopment of single-detached houses or horizontal mixed-use developments. This thesis fills the knowledge gaps in the two topics and benefits market participants and policy makers.

The theoretical contribution is the combination of the concepts of depreciation effect, vertical mixed-use development and real option approach. Three novel option pricing models are generated to provide effective estimations of option value and optimal decision strategies. The three models serve as good examples for the measurement of depreciation effect and the characteristic of simultaneous construction for vertical mixed-use development. In practical projects operated in different cities, the acquisition standards for old properties and the building designs for vertical mixed-use properties may be considerably diversified. This study develops a basic methodology that measures depreciation effect and vertical mixed-use development. Developers can apply the methodology in handling depreciation adjustments on the basis of different acquisition standards and particular adjustments for different vertical mixed-use structures.

This study also reveals the influences of depreciation effect on project values and optimal redevelopment timings. For investors/developers, these influences can assist them in exploring how project values and optimal timings change when they adopt different measurements of depreciation effect. They can gain insight into the influence of specific measurements from their development experiences. Moreover, policy makers can predict the trends of redevelopment timing for a given acquisition standard and/or a given maximum redevelopment period. Such given conditions alter the size of depreciation adjustments and the project values and optimal waiting time for redevelopment.

In addition, this study compares the difference in values between vertical- and horizontal-type of mixed-use developments. With this comparison, developers can re-examine whether the present project pricing model for vertical mixed-use development has included all potential characteristics. They can also obtain a good understanding of market price uncertainty for a specific use type. With the outcomes derived from the models, developers can decide well for multiphase vertical mixed-use developments. Furthermore, policy makers can predict the preference of investors between vertical and horizontal types under different market conditions.

1.5 Structure of this study

Chapter 1 introduces the study. It includes the research background, problem statement, research aim and objectives, research methodology and the significance of this study. The structure of this study is also included.

Chapter 2 reviews previous theoretical and empirical literature related to this study. The literature review includes discussions on the DCF method, the theory of real option in urban land valuation, the theory and application of real option in project valuation, the relationship between age/depreciation effect and real option in property pricing models and the existing real option pricing models for mixed-use developments. A summary of knowledge gaps and their links to research objectives is also provided in the chapter.

Chapter 3 outlines the quantitative techniques adopted in this study. The binomial tree model, stochastic differential equation, basket option and least square Monte Carlo (LSMC) method are introduced as basic mathematical tools for the real option approach. Moreover, an assumption of constant depreciation rate is presented and explained. This chapter introduces a new parameter named annual increase in average building age, followed by a discussion on average building age changes in different types of market price indices. The new parameter can aid in predicting future property values with accurate depreciation adjustments.

Chapter 4 presents the study to achieve Objectives 1 and 2. A discrete-time compound option model based on the mathematical expressions of depreciation effect in Chapter 3 is established to investigate how option value is affected by depreciation effect. This chapter then compares the various adjustments due to different types of market price indices using a case study on an actual redevelopment project.

Chapter 5 develops a continuous-time compound option model with depreciation adjustments to achieve Objective 3. The exercise strategy in the model is proved to be partially different from the traditional model without depreciation effect. Moreover, the influences on the optimal redevelopment timing from different acquisition standards are examined.

Chapter 6 establishes an option pricing model for vertical mixed-use developments to achieve Objective 4. The comparison between vertical and horizontal mixed-use developments derives a decision standard, that is, the critical height premium. This premium can aid developers in determining whether to develop a mixed-use project vertically or horizontally.

Chapter 7 summarises the findings of the study. The limitations of this study and directions for future research are also presented.

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

2.1.1 Structure of this chapter

This chapter theoretically and empirically presents a literature review on the real option approach in real estate markets. Section 2.1.2 briefly introduces compensation policies in redevelopment projects in different Asian cities and ensures that the real option approach can be applied to most of these projects. Section 2.2 begins from the disadvantage of DCF method in redevelopment to explain further the reasons to adopt the real option approach. Section 2.3 focuses on the existing real option theories in the urban land market. Section 2.4 reviews previous option pricing theories in development/redevelopment project valuation. In addition to the theoretical research, Section 2.5 contains the empirical applications in land and property valuations, including a few studies in Hong Kong and Singapore. Section 2.6 reviews previous literature on the relationship between the age/depreciation effect and real option in property pricing models. Section 2.7 discusses the existing real option pricing models for mixed-use developments. Finally, Section 2.8 summarises the knowledge gaps that this study attempts to fill. The links between these gaps and research objectives are also presented.

2.1.2 Compensation policies in redevelopment projects in different Asian cities

To operate a redevelopment project on multi-owner buildings, the government/developers should acquire the old properties from present residents or compensate the affected residents to help them find new accommodation. However, the development history, population density, land ownership system, economic conditions and building ageing conditions are varied in different cities and countries. Hence, the acquisition/compensation standards implemented by the governments are also diverse. Meanwhile, the developers should follow the corresponding standards in these cities. In Hong Kong, the URA has summarised related policies in several Asian cities (Law et al., 2009).

Most land in Japan is freehold. Therefore, in a public–private partnership model, the original landholders should initially surrender their own land and then receive the rights of new land, building or floor after the redevelopment. A large redevelopment project led by the private sector in Tokyo, named Roppongi Hills, included right conversion and monetary compensation. If the original residents' part of land right was not less than that after redevelopment, then they would receive a larger flat than before.

In South Korea, original residents in the Eunpyeong redevelopment in Seoul City could obtain three types of compensation, that is, other determined replacement lots/flats, monetary compensation or other residential houses with low prices. Some owners in other programmes were also prioritised to purchase a new flat after redevelopment.

In Mainland China, the urban land is usually state-owned, and the urban residents only have land-use rights. For the redevelopment in Shanghai, monetary compensation or a large flat in different regions are often offered to the affected residents, and the monetary compensation is based on the market price with an additional 25%–30% compensation. Although some options for nearby relocation are provided, the size of flats is usually smaller than that of the demolished one because of a higher unit market price. The Guangzhou government compensates original residents by providing cash. The standard is the value of their old properties plus the difference between "the average value of all old properties to be demolished" and "the average value of new flats in the neighbourhood region" (per m^2).

In Hong Kong, the URA compensates the affected residents with cash on the basis of

7-year-old property prices in the same region. A flat-for-flat scheme is also offered as an optional compensation. However, this scheme only provides a right for the residents to purchase in situ or Kai Tak flats after they accept the URA cash compensation instead of offering them new flats directly.

In Taiwan, the Taipei City government provides landowners with “an additional plot ratio” on their land after the redevelopment instead of cash compensation. Additional tax cuts are offered to stimulate the landowner-operated redevelopment.

As previously mentioned, monetary compensation is widely adopted in redevelopment projects in most Asian cities. In the following part of this study, the acquisition/compensation cost is assumed to be in cash, which can be quantified in the real option approach. The findings and indications from real option approach can be applied to many empirical cases based on cash compensation.

2.2 Disadvantages of DCF method in redevelopment

Before the development of real option approach, the DCF method is widely adopted as a major means for estimating project values. In the DCF approach, developers initially estimate the future case inflows and outflows. Cash inflow includes the rental income of retail properties/office and the sales income of residential properties. Meanwhile, cash outflow usually includes the construction cost of the building structure, the land conversion/auction cost and other management costs. Then, cash inflows and outflows are discounted with the pre-determined interest rate. The project with the largest positive net cash flow (i.e. inflow subtracted by outflow) is the optimal choice.

On the basis of DCF, Brueckner (1980), Wheaton (1982) and Munneke (1996) have suggested the “optimal redevelopment rule,” that is, a redevelopment should only occur when the land value in current use is smaller than its value in the redevelopment use, and the difference between the two values should exceed the cost of demolition and clearance. The land value here is the difference between property value and the construction cost of the building structure. DCF approach and the optimal redevelopment rule are intuitive and widely accepted in empirical valuations.

However, the redevelopment timing in the DCF approach is pre-determined. Developers should operate a redevelopment project immediately or at a fixed timing in the future. This assumption fails to consider the uncertainty of future market prices, because the real estate market usually has a high volatility (Kulatilaka & Marcus, 1992). In other words, the DCF approach ignores the irreversibility and flexibility in the real estate development. Irreversibility means that developers cannot retrieve their investment (i.e. construction cost) by demolishing the building, which is different from the financial asset. On the contrary, investors can sell many types of financial asset and retrieve their capital. As a result, developers should choose optimal construction timing. Flexibility means that developers can choose the construction timing. Although the maximum building period is bounded in some lease contract, developers can still choose the best timing within this period. However, the DCF approach only considers one or several predetermined construction timings and then compares the NPVs. When the future market price is uncertain, this DCF approach fails to look for the potential timing to achieve a high project value and ignores the time value of waiting. The two fundamental shortcomings in the DCF approach entail developers to look for appropriate construction timing on the basis of incoming future market information. Copeland and Keenan (1998) summarised that the NPV/DCF approach is a good tool only in two conditions, that is, a low uncertainty or an unchangeable investment plan even when new market information arrives.

In addition, the traditional DCF approach misses some profitable opportunities when the development plan is flexible. For example, developers can delay, expand, contract, or even abandon the development in some projects. Whether to choose these actions is usually based on future market information. Unfortunately, the DCF approach has a shortcoming in estimating the project value in these flexible cases.

Nonetheless, real option approach can well capture the market price uncertainty in the valuation process. Future market information is predicted on the basis of certain assumptions. Developers can predict the future revenues in different scenarios. Then, the average profit is estimated from predicted market prices and optimal actions taken by the development in these scenarios. Hence, the real option approach has a good performance when the investment is irreversible and flexible.

2.3 Real option and urban land value

Real option investigates the urban land value structure and potential impact factors. The introduction of price uncertainty leads to different implications on land value changes compared with the traditional DCF approach.

Titman (1985) first introduced the real option concept into the real estate area. Land value is estimated as an option to develop a property depending on market price changes. In this model, developers can choose to start or delay an investment until the next period. If the investment is delayed, then additional units are built when the market price increases but fewer units when the market price decreases. This one-period model is simple and straightforward. However, several important indications are derived in this simple model. As the market volatility increases, the vacant land value increases as an option. The construction is also delayed for good opportunities. High buildings with additional units are constructed when the height is not restricted.

Capozza and Helsley (1990) decomposed the urban land price into five components, namely, value of accessibility, conversion cost, uncertainty premium, net growth premium and the value of pure agricultural rents. Uncertainty premium and new growth premiums are replaced by the conversion option value of the agricultural land when the agricultural land is converted into urban land. In the aspect of urban extension, uncertainty is attributed to the delay of agricultural land conversion and city size extension and the increase in agricultural land value. This study was extended by Capozza and Sick (1994), who divided market uncertainty into systematic and unsystematic risks. Average urban land price is negatively related to systematic risk but positively related to unsystematic risk. The convertible agricultural land value is positively related to systematic risk. The relationship between convertible agricultural land value and unsystematic risk is uncertain.

Capozza and Li (1994) decomposed the converted land value into three components, that is, 1) the irreversible premium, which equals to the conversion option value for the agricultural land; 2) the intensity premium, which depends on the capital intensity on this land; and 3) the location premium. This decomposition is also an extension of the study by Capozza and Helsley (1990), in which development intensity is fixed. The variation of intensity delays the development and increases the hurdle rent. However, the varying intensity may increase or decrease the land value. A long waiting time for development increases the land value. The high hurdle rent, however, reduces the land value. The two changes are caused by the variable intensities, and have opposite influences on land price.

The studies after Titman have focused on the urban land value at the city level. The rent rate is a function of the distance to the CBD centre. These theoretical models have provided new macro descriptions of land value changes different from the traditional DCF approach. Findings have emphasised the importance of uncertainty in the valuation of vacant land. The uncertain factors are not limited to property income and construction cost. To describe market uncertainties, external economic factors (Hui & Wang, 2015), demand shock (Grenadier, 1996), supply shock (T. Lee & Jou, 2007), growth of land and building rent (Dale-Johnson & Brzeski, 2000) and other exogenous variables that determine the demand level (Guthrie, 2010) can represent market uncertainties. Hence, real option approaches have become an effective tool in the field of land economics.

2.4 Real option and project valuations

Another application of real option approach is for developing an empirical model to determine an optimal development/redevelopment strategy. Williams (1991) and Quigg (1993) built the most well-known continuous-time development models to determine the optimal construction timing. In their models, property price and construction cost follow geometric Brownian motions. Construction density is also determined by developers. The option period is assumed to be infinite. The optimal development strategy includes the hurdle price-to-cost ratio and the optimal building density. When the ratio of market price to construction cost reaches the hurdle price-to-cost ratio, developers should exercise the development option immediately to maximise its project profit.

The major difference between the two models is that in Quigg's model, the cash flow rate of undeveloped property depends on development density. Meanwhile, in Williams' model, the cash flow rate of undeveloped property is a constant (Hui & Fung, 2009). If the development density is pre-determined, then the formulas of the two models are the same. When the development cost becomes a constant and the cash flow rate of undeveloped property (i.e. the vacant land) is zero, Quigg's model is equivalent to Samuelson–McKean model (McKean, 1965; Samuelson, 1965).

In the literature on redevelopment, Capozza and Sick (1991) initially considered the redevelopment option as part of a long-term lease value, which is then divided into the value of the property that is initially built and that of a redevelopment option.

Redevelopment is defined as an opportunity to extend the property density only once. Generally, the freehold land value is consistently larger than the leased land value, when other parameters are the same. Prior to this study, this value difference is explained by the length of time to receive the property rent. However, Capozza and Sick suggested that the majority of this difference should be attributed to the length of redevelopment option period. The rent income difference is only a small part. The authors also conducted simulations to show how the conversion efficiency, conversion density, interest rate, rent growth and the remaining lease period influence the ratio of lease price to freehold land price.

Rosenthal and Helsley (1994) discussed the prices of single-family detached houses purchased for demolition and rebuilding. These prices are viewed as an estimator of the vacant land price. The houses to be demolished are considered valueless. The authors' empirical research supported the optimal redevelopment rule mentioned in Section 1.4.1 (Brueckner, 1980; Munneke, 1996; Wheaton, 1982). The demolition cost for single-family detached houses is extremely small and not included in this model. This empirical study did not suggest a separate model to generate the redevelopment option value from the market price and the construction cost. However, for specific developed land, the authors stated that the redevelopment probability could be estimated from the values of properties purchased for demolition and this specific developed land price.

Williams (1997) proposed a redevelopment model that allows multiple redevelopments by the property owner. In this model, the property quality influences the total rental income. When the quality falls below a certain level, the owner can redevelop the property, thereby improving the construction quality and increasing the land/house value. Property value is defined as a real option on the basis of quality and the unit's market price. Williams was first to consider property depreciation in the real option pricing. Redevelopment is owner led in this model, and the redevelopment decision depends on the residual value of the old property.

Downing and Wallace (2002) considered the renovation option for existing housing owners. A smaller spread between the investment return rate and the capital cost indicates a longer waiting time to add a new attribute to an existing house.

Sing and Lim (2004) applied the binomial tree model in estimating the optimal timing necessary to exercise the redevelopment options for the collective sale of apartments. This redevelopment is also an owner-led project. Hence, the interim rental income before the redevelopment should be included in the valuation. After deriving the option premium, the authors operated a regression for 30 collective sale sites in Singapore. The option premium is positively related to the land size and collective sale price.

As an extension to the binomial tree model (discrete-time model), McMillen and O'sullivan (2013) considered the optimal decision strategy whether to preserve or replace the old property. The possible decisions in the binomial tree model were extended in this study. In each node, the present values of three choices were compared in two steps to determine the optimal one. Developers compared the values of preserving the old property and replacing it with a new one. Then, the larger one was compared with the value of delaying the decision to the subsequent period. This study suggested a comprehensive decision process in the urban renewal programme. Shen and Pretorius (2013) also extended the binomial tree mode and included the length of lease contract, delay penalty, the financial status of the real estate company and capital cost.

The aforementioned real option models for development/redevelopment projects are one-stage options. These models assume that new buildings will be built immediately after the demolition of the old one. The two-stage compound option framework was initially introduced by Chen and Lai (2013) on the basis of the continuous-time real option models (Quigg, 1993; Williams, 1997). Their model was applied to the demolition and rebuild processes in Chinese cities. The original residents had to be compensated at the demolition stage on the basis of the average market level. As a result, the acquisition for the old building would occur when the market price decreased to a certain level. In the rebuilding stage, the new building would be constructed when the market price rose to a different level. The major finding concerns the time lag between the demolition and rebuilding decisions. If the redevelopment maturity is finite, then high price and cost volatilities, high price growth rate, low construction cost growth rate, low interest rate and high construction cost elasticity are factors for a large time lag between the two stages. Otherwise, if the redevelopment option has no time limit, then developers should rebuild the new property immediately after demolition.

2.5 Empirical project valuations based on real option approach

Many empirical analyses about the application of real option pricing models in land and project valuations have been published.

Schatzki (2003) compared the empirical results from real option model and traditional NPV approach in the land conversion process in Georgia State, US. In this study, the return volatility is high and the sunk cost is relatively large. The optimal conversion thresholds derived from the real option approach are higher than that from traditional DCF approach.

Cunningham (2006) examined the development timing and land price in Seattle and found considerable support for the positive relationships with market volatility. This work strongly proved that the land market is priced in accordance with the real option theory instead of traditional DCF approach. Later, (Cunningham, 2007) discussed the influence of urban growth controls in Seattle on land markets. After the urban growth boundary was imposed, the negative relationship between price volatility and development timing became considerably weaker.

Somerville (2001) used the panel data of the Canadian census metropolitan areas to investigate whether a common development should be valued as a compound option. Three possible decision points, namely, permit, start and completion, were discussed. In this study, a development process is consistently completed if it starts. Except for extreme market condition changes, developers start the construction immediately when the permit is obtained.

Yao and Pretorius (2014) emphasised the leasehold system in Hong Kong in the pricing of vacant land development. A long-term American call option model was adopted to compare with the results from a perpetual American option model due to the land leases in the vacant land transactions. Ten Hong Kong land conversion projects between 1990 and 1997 were selected within the same industry cycle. In eight of the ten cases, developers started the development close to (and earlier than) the optimal timing from the perpetual American calls (Quigg, 1993). The perpetual American call model is to some extent a proxy in finding the optimal development timing for the long-term American call option.

Real option approach is as an efficient tool in the land and property markets in Hong Kong and Singapore. The real option premium has been proved in Hong Kong's land market (Chiang, So, & Yeung, 2006; Yao & Pretorius, 2014). Leung and Hui (2005) suggested a new hybrid approach combined from the cost–benefit analysis and option pricing methods. The authors used this approach on public–private partnership (PPP) urban renewal projects. Multiple embedded options in the Hong Kong Disneyland project were also comprehensively analysed (Leung & Hui, 2002). Hui and Ng (2008) introduced the Samuelson–McKean closed-form option pricing model to value the Chelsea Court project, which was the highest service apartment in Tsuen Wan at that time. (Hui, Ng, & Lo, 2011) applied the same approach to Kwun Tong Town Centre, the largest urban redevelopment project in Hong Kong. The Samuelson–McKean closed-form model solution includes option elasticity, hurdle value (critical exercise timing) and option value. (D. K. H. Ho, Hui, & Ibrahim, 2009) introduced the binomial tree model and Samuelson–McKean real option model in pricing the upgrade option for Singapore's public housing under the Main Upgrading Programme Policy. The viability of two rehabilitation schemes, that is, Building Rehabilitation Materials Incentive Scheme and Building Rehabilitation Loan Scheme, were investigated by applying the binomial tree model on two buildings in Tai Kok Tsui (Hui & Lau, 2011). Li et al. (2014) considered the influence of an American deferred option in the pricing of privately owned public rental housing projects in China. These projects are PPP projects for improving the residential conditions of low-income families. This study was extended to a Building–Own–Operation–Concession (BOOC) mode with multiple options to attract developers (Li, Guo, You, & Hui, 2016). As the concession contract includes abandon, transfer and expansion options in this BOOC mode, developers have high flexibility in this programme. The financial burden of the local government in the public rental housing is also reduced.

2.6 Age effect and real option in property pricing models

In the aforementioned theoretical and empirical studies, the age effect of buildings is not emphasised as an important component. As buildings age, the depreciation of building structure leads to an increasing accumulated age effect on the commercial and residential property values. This effect has been proved in many theoretical and empirical studies (Baum & McElhinney, 2000; Bokhari & Geltner, 2016; Clapp & Giaccotto, 1998; Fisher & Martin, 2004; Francke & Minne, 2017; Hui, Chau, Pun, & Law, 2007; Hui, Wang, & Wong, 2014; Hui, Zhong, & Yu, 2012, 2016; Hulten & Wykoff, 1981). The age effect can be explained in two reasons. The renters and customer prefer to pay less for an old property compared with other new buildings in the neighbourhood. The owners should afford additional maintenance expenditures if they want to keep the transaction value of the old property consistent with the new property values.

However, except for the model developed by Williams (1997) in Section 2.3, the age or depreciation effect is seldom directly embedded in the real option pricing models. Instead, the widely accepted pricing model combined with age effect and real option was developed by (Clapp & Salavei, 2010). The authors built a new hedonic model with a redevelopment option value. This option value depends on the design differences between the old property and the new optimal one. The ratio of building structure value to land value (i.e. intensity in this study) was selected as a representative of the redevelopment option value. Intensity should decline as building age increases and as structure value decreases. This decline leads to an increase of redevelopment potential, because the property design is becoming outdated. The transaction records in Greenwich, Connecticut were adopted to test whether this new hedonic model was appropriate for the empirical cases. In addition to the age factors in the hedonic model, the intensity is negatively related to the transaction value. In other words, the redevelopment option value, which is positively related to the transaction value, is appropriately captured by the intensity.

Clapp and Salaveri's model was supported in the following studies. Clapp, Lindenthal, and Eichholtz (2010) found that the dynamics of property prices in West Berlin (1978–2007) were influenced by the redevelopment option. Clapp, Bardos, and Wong (2012) investigated how the drift rates and property taxes affect the redevelopment option value on the basis of transactions in 53 towns in Connecticut. A comprehensive study on West Berlin and Greenwich was also conducted by Clapp, Jou, and Lee (2012). The properties with high redevelopment potential have larger value changes during the market cycles than those with low redevelopment potential (Clapp, Eichholtz, & Lindenthal, 2013). (Munneke & Womack, 2013) further extended the model by Clapp and Salavei (2010) by combining spatial effect into the hedonic model.

2.7 Mixed-use development and real option

In addition to single-use development project valuations, the real option approach is also adopted in two scenarios, that is, two projects operated by two developers in the competitive market (Chu & Sing, 2007; Grenadier, 1996, 2005; Williams, 1993) and a mixed-use development with more than one function. This subsection focuses on the representative research in the latter scenario.

The positive price effects of other uses in mixed-use developments have been supported in many empirical studies. In a residential and retail mixed-use community in Kentland, single-family houses have positive premiums compared with houses in single-use communities (Tu & Eppli, 1999). Similar premiums also existed in Tucson (Cao & Cory, 1981) and Portland, Oregon (Song & Knaap, 2004). The diversity of commercial activities is a reason for high office rents in mixed-use developments (Liusman, Ho, Lo, & Lo, 2017; Vreeker, Groot, & Verhoef, 2004).

However, not all scholars agree with the economic benefit from mixed-use development. No considerable effect on housing prices from a neighbouring shopping centre in New Hampshire was found by Crafts (1998). Negative effect on residential prices from a close commercial region was observed (Mahan, Polasky, & Adams, 2000; Matthews & Turnbull, 2007). This negative effect is usually attributed to traffic congestion and noise pollution. Although the influence on prices varies in different studies, the popularity of mixed-use developments in the recent decades is still verified (Rabianski et al., 2009).

Existing valuation approaches for mixed-use projects are similar to those for single-use ones. The sales comparison, cost and income approaches are summarised as the major valuation approaches (Fisher & Martin, 2004; Rabianski et al., 2009; Ventolo, 2015). In these traditional approaches, developers can only choose to start the construction process immediately or never, thereby limiting the decisions that developers can make in actual programmes. Delaying the construction to wait for further market information may generate potential profits. This delay is usually ignored in the aforementioned approaches. DeLisle and Grissom (2013) attributed the difficulty of valuing mixed-use developments in empirical studies to data ambiguity. This ambiguity includes the heterogeneity of different projects, ambiguous classification for multiphase projects and insufficient data for small-scale mixed-use combinations. For example, the ground floors of some residential buildings are for retail use. However, the data for commercial market may not cover all these retail units.

Capozza and Li (1994) initially considered the decisions between two uses. They developed a one-time land conversion option from one land use to another. The conversion time and development density were determined from the land rents for both uses.

Geltner, Riddiough, and Stojanovic (1996) introduced the real option approach in generating the optimal decision in a mixed-use development zone. Two alternative land uses were available in this development. A function of two underlying assets was built to determine the project value. The completed building only contained one specific use. This option allows developers to choose one use that will delay the development compared with the development without a choice in use. If two alternative land uses have the same value, then the optimal choice of the developers is not to develop the vacant land in either use. The expected value to wait is greater than the revenue of either use when these uses have the same value. In short, the optimal strategy is to wait or choose the better one.

Childs, Riddiough, and Triantis (1996) extended the conclusions from the two studies (Capozza & Li, 1994; Geltner et al., 1996) to multi-conversion redevelopment cases. In this study, properties for two uses existed on the land. The redevelopment was to switch a land proportion from Use 1 to Use 2 (or vice versa) to optimise the land revenue on the basis of the two market prices. The authors proved that the optimal strategy in previous studies (i.e. wait or choose the better one) was under the assumption of constant marginal revenues to scale. In other words, the housing supply was assumed never to be influenced by the density of this targeted project. If the marginal revenues to scale were declining, then equal values for two alternative uses may not delay the development.

Hughen and Read (2017) organised an entire procedure for determining a mixed-use development with residential and commercial areas. This development was located in a form-based zoning structure. The first step was solving the optimisation problem to find the optimal combination of each use. The linear programming method was developed to solve this problem (Addae‐Dapaah, 2005). The second step occurred after the construction. At that time, developers still had a chance to convert a part of Type 1 property into Type 2 (or vice versa). This decision should be based on the new market information when the property was completed. This additional conversion option was priced from the conversion option pricing model (Childs et al., 1996). Although the zoning policy tended to encourage mixed-use projects, developers would only build mixed-use buildings under two situations, that is, 1) sufficiently low construction cost and the case where revenues were remarkably sensitive to residential/commercial property supply; and 2) developers had an option to convert the use after the construction. Hughen and Read emphasised the option to convert the use after the construction would promote mixed-use development when the marginal revenue was either constant or declining.

2.8 Summarisation of knowledge gaps

After the review of major contributions in the application of real option approach, this section summarises the research knowledge gap(s) of the existing studies in real estate valuation.

2.8.1 Depreciation effect in real option approach

The first major gap is about the depreciation or age effect. As mentioned in Section

1.1.4, depreciation measures the physical deterioration, functional obsolescence, information asymmetry between sellers and buyers and the length of residual time to the lease maturity. The amount of this depreciation should not be ignored, given that the entire redevelopment usually takes a long time. Section 2.6 has listed studies that support the negative relationship between building age and property value. These empirical studies enhance the depreciation effect. During the option period, dilapidated buildings depreciate, and the age difference between the new and old properties enlarges. These changes caused by the depreciation effect will underestimate the project profit in each decision period and the optimal strategy.

In traditional real option models, the depreciation effect does not appear separately. The future value of the new property is determined from a stochastic market price variable. Developers can choose the price statistics of newly built properties to derive the future value of new properties. In this case, the market price variable and the future value of new properties are the prices of buildings aged zero. Moreover, the depreciation effect does not exist during the option period.

However, in a redevelopment project, two property prices appear in different phases. The old property value is estimated in the demolition phase, whereas the new property value is estimated in the rebuilding phase. If the acquisition price of the old property is not equal to the newly built property value, then the two property values to be estimated are unequal.

To solve this problem, two choices can be adopted. The first one is to introduce two different stochastic variables to record the market prices of the newly built and old properties. Their drift rates and volatilities are derived separately. The two variables are based on different groups of properties; thus, they may be influenced by the sampling difference. For example, the transaction records for newly built properties in the past 5 years and properties over 50 years old may be distributed in different locations. Then, the different location characteristics in the two groups may lead to embedded bias in the estimation.

The second choice is to introduce one stochastic variable for recording the market price. Then, the new and old property values in the redevelopment project are adjusted from this variable by the depreciation effect. The drift rate and volatility should exclude the depreciation effect. Only one group of properties is sampled to generate this stochastic variable. The potential influences of location characteristics are excluded. The forthcoming issue is to define the depreciation effect properly, which will be discussed in Chapter 3.

Most studies in Sections 2.3–2.5 do not emphasise the above depreciation effect in real option pricing process, except for the works by Dixit and Pindyck (1994) and (Williams, 1997). The two extended models are excellent tools to value the option on real assets with depreciation effect. However, both models have their limitations when applied to redeveloping multi-owner properties. Dixit and Pindyck (1994) assumed a survival function for factory machines. The price changes during the option period were based on this survival function. However, the life expectancy of a property is usually considerably longer than machines in a factory. Only a small part of buildings should be replaced within a certain period. A redevelopment project should consider the resettlement of sitting residents, especially for the multi-owner buildings. Many redevelopment projects may be delayed, even when the building age of old properties are beyond their design life. As a result, the form of survival function may not properly capture the property value changes due to depreciation effect.

The strategy suggested by Williams (1997) implies that when the targeted property value decreases to a certain percentage of the value of a new property with the same structural, locational and other factors, it will be demolished and rebuilt. This strategy is appropriate for a one-stage model when the owner of the property chooses an optimal time to demolish the old property and rebuild a new one immediately. However, in a two-stage model, this strategy may not be optimal because the owner may delay the rebuilding process after the demolition depending on new market information.

Other new hedonic pricing models combined with redevelopment options can only investigate the existing influences caused by depreciation effect (Clapp, Bardos, et al., 2012; Clapp et al., 2013; Clapp, Jou, et al., 2012; Clapp et al., 2010; Clapp & Salavei, 2010; Munneke & Womack, 2013). Neither the redevelopment option value nor its expected exercise timing can be predicted from these models. Only the historical value of the redevelopment option can be derived.

To fill the gap, the present study initially examines the expression of depreciation effect in the prediction of property values (Objective 1). Then, discrete- and continuous-time models with embedded depreciation adjustments are developed (Objectives 2 and 3). The discrete-time model is based on the binomial tree model, whereas the continuous-time model is based on stochastic differential equation. Profit maximisation standard reflects the developers' rational behaviours. The influences of different factors on the redevelopment option value and expected exercise timing are investigated. To build an accurate pricing model, we discuss the important parameters for this study. These parameters are the building age of the reference properties used in the market price statistics, the building age of the old property to be acquired, and the building age of the new one to be rebuilt. Particularly, a new parameter named "average building age changes" is introduced to reflect the differences of the above parameters.

2.8.2 Model for vertical mixed-use developments

The second knowledge gap is that the mixed-use development option models at present are more appropriate for horizontal mixed-use projects than vertical ones (Addae‐Dapaah, 2005; Capozza & Li, 1994; Childs et al., 1996; Geltner et al., 1996; Hughen & Read, 2017). However, vertical- and horizontal-type developments have many differences in the pricing process. Traditional pricing methods (i.e. sales comparison, cost and income approaches) have shown the value differences in two types (Rabianski et al., 2009). Special designs in vertical developments should comply with the special safety standards for buildings with different uses. These designs lead to a high construction cost than horizontal development. The higher income of vertical developments usually results from the high value of the residential/office units when they are located on high floors.

In the real option approach, the models for horizontal projects consider the conversion of use. However, this conversion is usually difficult or expensive in the vertical type. For example, the podium structure typically consists of one podium and one/several high-rise building(s). The podium also contains retail stores. Some podiums also have public transportation facilities on the ground floor. Residential/office units are located in high-rise buildings (Lau, Giridharan, & Ganesan, 2005; Lau & Zhang, 2015; Zhu & Chiu, 2011). The two major uses are completely separated within the structure. No resident prefers living in the podium next to retail units. Few existing residents accept that lower floors in a residential building are for retail use. Hence, the conversion of uses in the podium is almost impossible. For other structures, if the building ordinances have different requirements for different uses, then the conversion expense is also high.

The second reason for high conversion cost is that the freehold assumption in previous models may not be consistent with the case in vertical mixed-use development, especially in high-density cities in Asia. These developments are usually bounded by land leases. If developers want to change the land use within the mixed-use project before the building process, then an official approval from the landlord is usually necessary. Additional land conversion premium may be required if the plot ratio for some high-unit-value use increases. In this study, land conversion" refers to the case where the redevelopment changes the original land use. Partial conversion after completion will be in conflict with the approval plan. Developers may have to pay a large amount of premium to the landowner (i.e. the government in the case of Hong Kong).

As previously discussed, the conversion option after completion should be excluded for vertical mixed-use developments. Furthermore, two additional characteristics should be considered in the new model. The first one is that the entire vertical mixed-use building must be constructed and sold simultaneously. Even when the retail units in the lower levels are completed before the residential units in the upper levels, developers will not rent out the retail units before the entire building is completed. The second one is that the land lease provides a building covenant period. Developers should complete the vertical mixed-use property within this period. Hence, a finite-time American basket option model written on two assets should be adopted to value this type of development.

The simultaneous construction for different uses in vertical mixed-use properties indicates that the optimal timing should maximise the revenue of the entire project but not the revenue for a specific use. Nonetheless, developers can optimise the development timings for different uses separately in horizontal mixed-use projects. This substantial difference emphasises that the optimal exercise timing in vertical types should be different from that in horizontal types. If we apply the traditional model for a horizontal type to estimate a vertical mixed-use project, then the optimal development timing and the project value will be remarkably biased. Furthermore, we can compare the results from models of vertical and horizontal types when the gross floor areas (GFAs) for different uses are similar. This comparison helps in determining the criteria for choosing a profitable type in the planning stage.

In view of the abovementioned concepts, Objective 4 is achieved by developing a new real option model that specifically caters for vertical mixed-use development.

CHAPTER 3 METHODOLOGY

3.1 Introduction

Before the development of real option models for specific development/redevelopment projects, this chapter introduces basic mathematical tools and some specific assumptions in two sections. Section 3.2 describes the basic mathematical tools for the real option approach, including the binomial tree model, stochastic differential equation, basket option and LSMC method. Section 3.3 discusses the constant depreciation rate assumption, as well as the different types of market price statistics and the average building age changes in these statistics. The discussion also includes the procedure to derive the average depreciation rate, and the reasons to adopt the constant depreciation rate assumption.

3.2 Basic mathematical tools

3.2.1 Binomial tree model

Cox, Ross, and Rubinstein (1979) initially suggested the Cox–Ross–Rubinstein binomial tree model for discrete-time option pricing. The entire option period is divided into a series of decision periods. At the end of each decision period, the stochastic state variable (i.e. the market price/construction cost) moves upwards or downwards. For example, for a specific decision period *t*, the initial market price *S* moves up to $\mathrm{S} * u_S$ with a probability of p_{uS} or moves down to $\mathrm{S} * d_S$ with a probability of p_{dS}, as shown as follows:

$$(3.1)\ S(t) = \begin{cases} \mathrm{S(t-1)} * u_S, probability = p_{uS}, u_S > 1 \\ \mathrm{S(t-1)} * d_S, probability = p_{dS}, d_S < 1 \end{cases}$$

where $S(t-1)$ is the market price at the end of period (t − 1) and the price at the beginning of period *t*.

The initial market price is $S(0)$, and the market price at the end of the last option period is $S(T)$. We can then derive $S(T)$ from Equation (3.1) step by step. The results can be described as a triangle tree. At the end of period *t*, this tree has *(t + 1)* notes with different values of $S(t)$.

On the basis of non-arbitrage assumption,

$$S(t) = S(t-1) * (1 + r_f) = S(t-1) * u_S * p_{uS} + S(t-1) * d_S * p_{dS}.$$

Note that

$$p_{uS} + p_{dS} = 1.$$

Then, the risk-neutral probability of p_{uS} can be defined as

$$p_{uS} = \frac{1+r_f-d_S}{u_S-d_S},$$

where r_f is the risk-free interest rate.

To simulate the American option value $V(0)$, the exercise value at the end of period *T* is initially calculated as follows:

$$V(T) = Max[S(T) - K, 0].$$

The exercise value $V(T)$ also has *(T + 1)* different values at each note.

Then, for the notes at the end of period *(T − 1)*, the option value is determined by the larger one between the exercise value and the value to delay the exercise, as shown as follows:

$$(3.2)\ V(T-1) = Max[S(T-1) - K, E[V(T)], 0],$$

where

$$(3.3)\ E[V(T)] = \{Max[S(T-1) * u_S - K, 0] * p_{uS} + Max[S(T-1) * d_S - K, 0] *$$

$$p_{dS}\} * (1 + r_f)^{-1}.$$

The option value $V(0)$ at the beginning of the option period is calculated backwards from Equations (3.2) and (3.3).

3.2.2 Stochastic differential equation

When one or several parameters in a differential equation is a stochastic process, this differential equation is called stochastic differential equation. In the real option area, this stochastic process is usually the derivative of geometric Brownian motion.

For the stochastic market price $S(t)$ at time *t*, Samuelson (1965) and McKean (1965) introduced a model to solve the stochastic differential equation in the following form:

$$dS(t) = \nu_S S dt + \sigma_S S dZ_S,$$

where ν_S is the risk-neutral growth rate (or drift rate), which is constant; σ_S is a constant variance; and Z_S is a Wiener process. Moreover, $\mathrm{E}(\mathrm{d}Z_S) = 0$, and $\mathrm{Var}(\mathrm{d}Z_S) = \mathrm{dt}$.

The Samuelson–McKean model is used to find the optimal timing *τ*, which can maximise

$$\mathrm{E}[e^{-r\tau}(S(\tau) - \mathrm{K})].$$

The option value *V* is a function of *S* and can be described by the following differential equation:

$$(3.4)\ \frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 V}{\partial S^2} + \nu_S S \frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} = rV.$$

Given the optimal exercise timing, the critical boundary of V is $V(S^*)$. Then,

$$(3.5)\ V\big(S^*(\tau)\big) = S^*(\tau) - K.$$

Equation (3.5) is the value-matching condition. The smooth-passing condition is

$$(3.6)\ \frac{\partial V}{\partial S}(S^*) = 1.$$

The general solution for the second-order differential equation (Equation (3.4)) consists of three components, that is,

the option elasticity λ:

$$\lambda = \frac{1}{\sigma_S^2} * [\frac{\sigma_S^2}{2} - v_S + \sqrt{\left(v_S - \frac{\sigma_S^2}{2}\right)^2 + 2r\sigma_S^2}];$$

the hurdle value S^*; if the market price S reaches or is larger than S^*, then we exercise the option immediately. Otherwise, we hold the option and wait;

$$S^* = \frac{K\lambda}{\lambda - 1}$$

and the option value V:

$$\mathrm{V} = (S^* - K)\left(\frac{S}{S^*}\right)^{\lambda}.$$

3.2.3 Basket option model

The unit market price for Land use 1 is $S_1(t)$, and the unit price for Land use 2 is $S_2(t)$ at time *t*. The respective unit construction costs are $K_1(t)$ and $K_2(t)$.

The basket option is an option written on two different assets. The exercise value is the sum of the exercise values in two normal options. The option value at the end of period *t* is derived from the market prices and construction costs at that time. The option value at the end of period *(t + 1)* is as follows:

$$V\big(S_1(t), S_2(t), K_1(t), K_2(t)\big) = Max\{0, S_1(t) * A_1 + S_2(t) * A_2 - K_1(t) * A_1 - K_2(t) * A_2, e^{-r}E\big[V\big(S_1(t+1), S_2(t+1), K_1(t+1), K_2(t+1)\big)\big]\} \quad (0 \le t \le T),$$

where A_1 and A_2 are the construction floor area (CFA) of Land uses 1 and 2, respectively. A_1 may not be equal to A_2.

Different from the normal call option, the analytical solution of the basket option does not exit. Instead, we adopt the LSMC method suggested by Longstaff and Schwartz (2001) to generate the expected option value by performing numerical simulations.

3.2.4 LSMC method

Longstaff and Schwartz (2001) developed an LSMC method to determine the value estimation of an American finite-time option on the basis of the optimal exercise strategy. The steps of the LSMC method are as follows.

(1) Generate *M* different paths for the asset price process $S(t)$. Suppose the option can be exercised only at the end of *N* equal time intervals.
(2) Calculate the cash flow of this option at the maturity for each path.
(3) For each path, only consider the paths with positive values at the end of the *(N − 1)*th time intervals. For these paths, denote the underlying asset prices $S(N-1)$ as a vector *X* and denote their corresponding DCFs in Step (2) as vector *Y*. *Y* is regressed on a function of *X* by least square regression method. The continuation value of the option at the end of the *(N − 1)*th time intervals is the estimated

conditional expectation of Y on X.

(4) Compare the early exercise value at the end of the $(N - 1)$th time intervals with the continuation value in Step (3). If the early exercise value is larger, then the cash flow at this time point is the early exercise value. If the continuation value is larger, then the cash flow at the end of the $(N - 1)$th time intervals is the DCF at the end of the Nth time intervals.

(5) Repeat Steps (3) and (4) backwards until the end of the 1st time intervals. To determine the option value at the end of period t, DCF vector Y should contain the cash flow at the end of period $(t + 1)$.

(6) The option value in a set of M paths is the mean of the DCFs at the end of the 1st time intervals for all paths.

(7) To increase the estimation accuracy, repeat Steps (1)–(6) to calculate the option values for several sets of path. The estimated option value is the average of these option values.

The algorithm applied in this study is adjusted on the basis of the algorithm by Hoyle (2016). The programming in Step (4) is revised when the continuation value is larger than the early exercise value.

The LSMC approach is appropriate for American put or American call options with dividends. If we choose the cost of carrying the underlying real asset as the asset drift rate, then the yield rate of developed properties is equivalent to the dividend in a financial American option (Merton, 1973; Yao & Pretorius, 2014). Hence, the LSMC approach is also applicable to the real options in this study.

The steps above are originally for the options for one asset. For the basket option written on S_1 and S_2, the cross items between S_1 and S_2 should be included as basis functions. Glasserman and Yu (2005) discussed the required number of paths and the number of basis functions comprehensively. Abbas-Turki and Lapeyre (2009) suggested that the degree of monomials for each asset should be constrained to no more than two.

The Laguerre polynomials chosen by Longstaff and Schwartz (2001) lead to a remarkably biased estimation in the basket option pricing. To minimise the underestimation in the LSMC approach, we adopt the set of basis functions suggested by (Coskan, 2008), that is,

$$1, S_1, S_2, {S_1}^2, {S_2}^2, S_1 S_2, {S_1}^2 S_2, S_1 {S_2}^2, {S_1}^2 {S_2}^2.$$

In each monomial, the degrees of S_1 and S_2 can be 0, 1, or 2. The number of basis function is 3^2 in a two-asset case and 3^3 in a three-asset case. This approach is appropriate for cases where the uses in a single building are no more than three different types. In some special projects where the uses are more than three, other advanced basis functions, such as Hermite, hyperbolic and Chebyshev polynomials, should be adopted.

3.3 Preparation for depreciation effect measurement

In the traditional study on property depreciation, the property value should be divided into land and building structure values. Land value does not depreciate over time. By contrast, building structure value will decline to zero (or an extremely small value) when the building age is beyond the useful age. Then, the proportion of land value in the entire property value gradually increases since its completion. However, in the real option approach, the land value is the redevelopment project value, that is, the difference between the new property value and the old property value excluding the demolition and clearance cost. The land value or the redevelopment project value is treated as the option value; thus, the land appraisal price is excluded in the model. In other words, the building structure value, which is usually estimated by the difference between the property value and the land appraisal price, does not appear in the real option model. The measurement of the depreciation effect in the real option approach is different from that in the traditional study of property depreciation. In addition, using structure value requires us to estimate the part of land value in each property transaction record. However, the low frequency of land transaction indicates that additional assumptions (e.g. land leverage and land value growth rate) must be made to estimate the component of land value for properties in all transactions. This will make the estimation of structure value subjective. Therefore, the following discussion assumes that the depreciation effect is measured on the entire property value but not the building structure value.

3.3.1 Different types of market statistics for reference

The traditional two-phase model by Chen and Lai (2013) assumes that acquisition price and new property price are based on the same property index directly. This assumption is only available when the acquisition standard is restricted to be similar to the newly built property value with other characteristics equivalent. Developers purchase the old property as a newly built one, demolish it and rebuild a new one. Unfortunately, this assumption is only satisfied in few empirical cases.

A rare compensation method is to provide some units in the new property for the original residents as a form of asset compensation. This method used to be adopted in Mainland China, but has been replaced by monetary compensation in recent years. If the original residents are compensated by new property units, the traditional two-phase model is still not appropriate. Instead, the compensation expense is delayed until the new property is completed. Developers are only concerned about the market price when the new property is completed and sold. Thus, a simple one-phase model is appropriate for the flat-to-flat compensation.

In a general two-phase redevelopment project, three building ages exist in the valuation process as follows: the building age of the old property to be demolished, the building age of the new property to be built and the building age of the properties used in the market price statistics for reference (i.e. price index or average price). The third one may even be different if the market price statistics are derived from different groups of property.

In an ideal case, all types of market price statistics are available. Developers can find the price statistics for newly built properties and for old properties aged over 50 years. Then, two different market price paths are generated from the two statistics. In this case, the two-phase real option is a compound option written on two different assets. The market price changes of newly built and old properties are different. Thus, the traditional two-phase model is not applicable to this ideal case.

However, this ideal case does not comply with the empirical situations. In many real estate markets, only the market price statistics from some groups of properties are available. Four usual scenarios are provided in terms of the availability of statistics, as follows.

Scenario 1. The price statistics for newly built properties are available. However, the price statistics for old properties are not based on the same group of properties.

In major cities of Mainland China, the government provides two types of market price statistics, that is, newly built and second-hand property markets. Developers can access the newly built property value from the first market statistics. However, the second-hand market price statistics do not emphasise the building age of the sample properties. The average building age in the sample set is changing during the entire sample period. Hence, the price statistics for second-hand properties is not a good representation of the values of those properties for academic research and commercial valuations. The new model adjusts the future value of old properties to be demolished automatically during the option period.

Scenario 2. The price statistics for old properties are available. Price statistics for newly built properties are unavailable.

In some mature cities, traditional repeat-sales price indices (e.g. S&P CoreLogic Case-Shiller Home Price Indices) or price indices based on a fixed group of properties (e.g. Centa-City Index in Hong Kong) have been adopted to measure the market changes. In these indices, the annual depreciation is embedded as the building ages of all the sample properties increase one year annually. No further adjustment is required for the old property price statistics.

Only a small proportion of the transaction records is from the newly completed properties. In comparison with the substantial differences in the location and structural characteristics of new property units, the number of transaction records is usually too small to generate a good market price index. Hence, the price statistics for newly built properties is not frequently used in these mature cities. To develop a new model, the future value for the new property to be built should be adjusted automatically during the option period.

However, some adjusted repeat-sales indices have combined the hedonic model or other methods to reflect the age effect [e.g. forward property repeat sales model (Chau, Wong, & Yiu, 2003; Leung, Hui, & Seabrooke, 2007) and age-adjust repeat sales model (S. Wong, Chau, Karato, & Shimizu, 2018)]. These adjusted indices are assumed to be based on a constant building age and exclude the age effect. They should also have the same treatment as the newly built property indices in Scenario 1 with a fixed depreciation adjustment term.

Scenario 3. Only the price statistics for a mixed group of old and new properties in this city are available.

This scenario occurs in small cities where the real estate market started to develop only a few years ago. For example, in Mainland China, the macro control policies on the property prices in large cities have become progressively strict over the past few years. Many developers have searched for good investment opportunities in small cities and some towns close to large cities. However, comprehensive market statistics are unavailable in those developing real estate markets. Developers can only find some mixed price indices derived from the transaction records of newly built and old properties during the sample period. These indices are diverse from those in Scenario 1 or 2.

In small cities where the transaction information is incomplete, the available price statistics are usually based on newly built units and units completed in most recent years. Suppose some of the completed units each year still appear in the transaction records in the following years. Then, developers can collect part of the previous transaction records for the completed units with their completion years and calculate the average building age in each year during the sample period. Subsequently, the annual change of average building age can be estimated. The new model should provide a reliable approximation of the future values for the new property to be built and the old one to be demolished. This approximation should be based on the price statistics for the entire market and the annual change of average building age.

Scenario 4. The acquisition standard is indirectly based on the market price statistics available.

This scenario applies to the case when the acquisition of old property is based on a predetermined valuation standard. In some cities, the price statistics for newly built properties or old ones are available. However, the acquisition standard for the old property is based on neither the price statistics for newly built properties nor the price statistics for nearby old properties. One example is the seven-year-old property acquisition standard provided by the URA in Hong Kong. The acquisition price is based on the seven -year-old property value in the same district. Given that the number of seven -year-old properties is small or these properties are located far from the targeted property, the acquisition price by area is usually estimated by professional surveyors. In this scenario, the new model should adjust the estimation of the future acquisition price and the value of newly built properties automatically. The building age of old properties is treated as seven years in this example.

The four scenarios delineate the possible classifications of availability of price statistics. In valuation, developers need to consider the depreciation effect of properties. A new, good model must adjust the future acquisition price, the newly built property value or both of them. The adjustment standard depends on the properties that are used to derive the price indices. Section 3.3.2 will explain the reasons for introducing a new parameter, that is, the annual increase in average building age. This parameter generalises the four scenarios into one model.

3.3.2 Average building age changes

In common real option pricing models, the future market price is estimated from current market price, market volatility and market drift rate from other models. In redevelopment projects, the values of old and newly built properties need to be estimated. However, the average building age of the properties used in the price indices is usually different from the new or old properties to be priced. These differences may even continue to change during the entire option period. As a result, an over- or under-depreciation bias will occur in the estimation. The market volatility and drift rate are usually derived from market price indices. The price indices are based on transaction records in the sample period. Hence, the average building age in the price indices in a specific sample period is only the average building age of the transacted units in this period. In this section, how this average building age changes in the four scenarios in Section 3.3.1 is discussed. A parameter, namely, annual increase in average building age, is introduced to compare the differences amongst scenarios. This parameter comes from the difference between average building age in year (t + 1) and average building age in year t.

In Scenario 1, the average building age of newly built properties each year are constantly equal to zero. The annual increase in average building age is also zero. Likewise, the new property to be built has a constant building age of zero. The future value of the new property is directly estimated from the available price indices without further adjustments. The old property to be demolished, however, depreciates annually in the option period. The future value of this old property must recover the under-depreciation part in the estimation when the available price indices are from newly built properties.

In Scenario 2, the average building age of properties in the traditional repeat-sales indices increases one year annually. This statement also applies to the indices from transaction records in a fixed group of properties. Hence, the annual increase in average building age for these indices is one year. The annual depreciation is embedded in these indices. The future value of the old property to be demolished can be directly derived from these available indices without further adjustments. The future value of the new property to be built, however, needs to recover the over-depreciation part in the estimation.

Scenario 3 is the most complicated. The sample period lasts for 10 years. A proportion of the completed units each year is transacted during the rest of the entire period. These assumptions are appropriate for immature property markets. In these markets, many new units are completed and sold annually. The majority of the transaction records are from the properties built within 10 years, which depreciate in less than 10 years. The minority of transactions are from properties older than 10 years. As a result, the annual increase in average building age should fall between 0 and 1 year. The future values of the old and new properties need to be adjusted on the basis of the annual increase in average building age. The derivation of annual increase in average building age has been mentioned in Section 3.3.1.

Scenario 4 is a special case of Scenario 1 or 2 depending on the price indices available. Developers only need to replace the building age of the old property by the predetermined building age defined in the acquisition standard. If the acquisition standard requires developers to purchase the old property as a new one and the available price index is for newly built properties, then the project can be priced by using the traditional two-stage model (Chen & Lai, 2013). The annual increase in average building is the same as that in Scenario 1 or 2.

In summary, the annual increase in average building age should be between 0 and 1 year. The preceding discussion transfers different scenarios into different values of one parameter in the same option pricing model. In a real option pricing model, this annual increase in average building age is a constant parameter in Scenarios 1, 2 and 4. In Scenario 3, developers should calculate the annual increase in average building age for each year during the sample period. Then, they should take the mean of these increases as the parameter value in the real option model. However, an assumption of constant depreciation rate is necessary to ensure that the building age is an additive parameter in the model. Section 3.3.3 will describe this assumption. Then, the reasons to adopt this assumption will be discussed in Section 3.3.4.

3.3.3 Constant depreciation rate assumption

Suppose a completed property depreciates at a constant annual rate of ξ ($0 < \xi < 1$). The market value of a specific property age in period *t* is defined as P_t^g, which satisfies

$$\ln\left(\frac{P_t^{g+z}}{P_t^g}\right) = \mathrm{z}\xi, \text{ for any integer t, } \mathrm{g} > 0, \mathrm{z} \geq 0,$$

or

$$P_t^{g+z} = P_t^g e^{z\xi}.$$

The constant depreciation rate assumption ensures the additive property of the depreciation effect. For example, if a property depreciates in *X* years in the first period and depreciates in *Y* years in the second period, then its total depreciation adjustment term should be based on *(X + Y)* years. The sum of the depreciation adjustments in the two periods is shown as follows:

$$P_t^g e^{X\xi} e^{Y\xi} = P_t^{g+X} e^{Y\xi} = P_t^g e^{(X+Y)\xi} = P_t^{g+X+Y}.$$

This additive property supports the reliability in calculating the average building age. If the depreciation effect is based on a quadratic function, then the above equation does not hold. The prediction of future property value relies on the building age in each decision period. In Section 3.3.4, the other reasons to choose this constant depreciation rate assumption will be demonstrated.

The constant depreciation rate is different from the straight-line depreciation. If the estimated useful life is L and the salvage value is zero, then the straight-line depreciation formula that is usually used in accounting records is as follows:

$$P_t^{g+z} = P_t^g - \frac{z}{L} P_0, \text{ for any integer t, } g > 0, z \geq 0, g + z \leq L.$$

Let $\xi = ln(1 - 1/L)$. The value of a property depreciates more slowly at a constant rate of ξ than that of the same property value under straight-line depreciation (Figure 3.1).

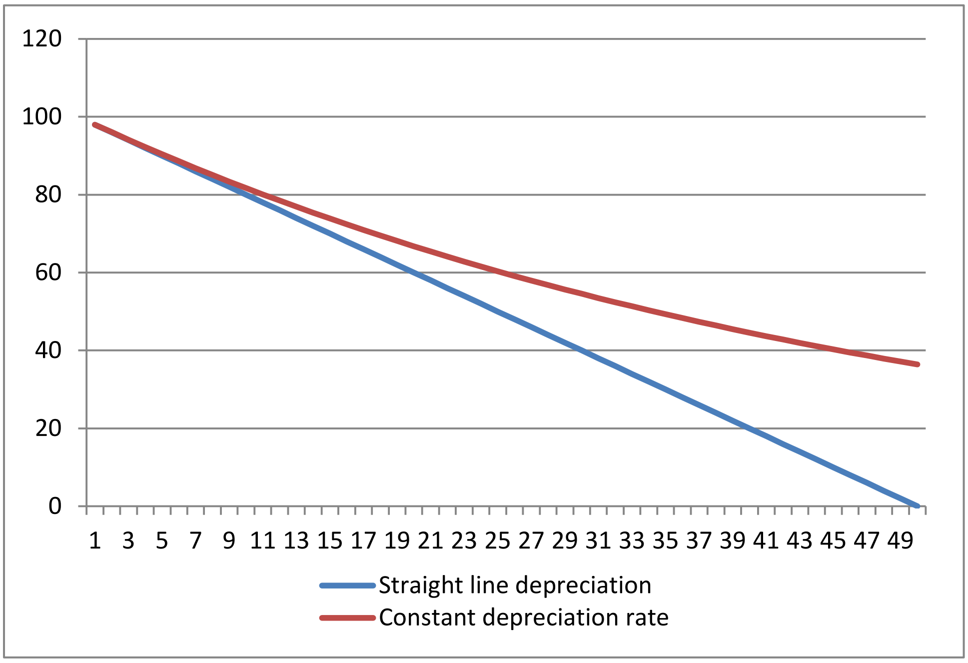


Figure 3.1 Comparison between two different depreciation assumptions

However, the straight-line depreciation assumption is not chosen in this study because we have to predetermine the maximum useful age of the property, which is subjective. Suppose that the design age is selected. If some old properties to be redeveloped are older than their design age, then the property value would become negative. Hence, the straight-line depreciation is not applicable to property valuation.

Here, the building age is the natural age since this property is completed. An argument is that maintenance and rehabilitation can improve the structural status and extend the useful (physical) life of the property. Hence, the natural age may be unreliable in estimating the depreciation effect. The influence of maintenance is substantial in single-detached houses. However, in the case of multi-owner buildings, the free-rider dilemma diminishes some homeowners' willingness to participate in maintenance programmes (Yau, 2011). This situation may reduce the rehabilitation frequency in multi-owner buildings compared with single-detached houses and then reduce the mismatch between natural age and property status. Many existing comprehensive studies about depreciation effect (Bokhari & Geltner, 2016; Clapp, Bardos, et al., 2012; Clapp et al., 2013; Clapp & Salavei, 2010; Francke & Minne, 2017; Williams, 1997) neither emphasise the difference between the natural and effective ages nor provide a good measure for the effective age[1].

In the empirical study, the value of this constant depreciation rate is an average depreciation rate for different building ages. To derive this average depreciation rate, a cross-sectional analysis based on the hedonic pricing model (Rosen, 1974) is conducted on nearby property transactions (Bokhari & Geltner, 2016; Francke & Minne, 2017; Jeffrey, Brent, Jerrold, & Webb, 2005).

Step 1 estimates the coefficients related to natural building age through the following equation:

$$(3.7)\ \ln(P_{it}) = \alpha_{1it} Age_{it} + \alpha_{2it} AgeSquare_{it} + \sum_{j=1}^{J} \beta_{jit} L_{jit} + \sum_{k=1}^{K} \beta_{kit} X_{kit} + \sum_{m=1}^{M} \beta_{mit} M_{mit} + \varepsilon_{it},$$

where P_{it} is the price of property transaction record i in year t; $\ln(P_{it})$ is the logarithm of this price; Age_{it} and $AgeSquare_{it}$ are the building age and the square of building age in the property transaction record i in year t, respectively; L_{jit} is a vector of J location characteristics in the property transaction record i in year t; X_{kit} is a vector of K structural characteristics (except building age) in the property transaction record i in year t; and M_{mit} is a set of M time-dummy variables representing the

[1] In this theoretical study, we do not focus on the measurement of effective age because it is another important issue in empirical property valuations. This measurement deviates from the objectives in this study. A local and consistent maintenance measurement standard should be used in the same sample set and the targeted old property to reduce the difference between the natural and effective ages. For example, the English House Condition Survey in UK (Department for Communities and Local Government, 2010), the biannual American Housing Survey in US (United States Census Bureau, 2008) and the dilapidation index in Hong Kong (D. C. W. Ho et al., 2012) provide reliable information for the property maintenance status. In an empirical study, the natural age can be adjusted on the basis of similar information. We admit that the measurement of effective age is important, but this is still beyond our research scope.

general market conditions in each year. These dummy variables are equal to 1 if $m = t$ or 0 otherwise. α_{1it} is expected to be negative, and α_{2it} is expected to be positive in the common case.

Step 2 calculates the annual price changes when the building age increases from T to $T + 1$ based on Equation (3.7). Here, the coefficients of the building age and the square of building age are two constants, which are defined as α_1 and α_2, respectively. Then,

$$(3.8)\quad \ln\left(\frac{P_{Age=T+1}}{P_{Age=T}}\right) = \alpha_1(T+1) + \alpha_2(T+1)^2 - \alpha_1 T - \alpha_2 T^2.$$

The location and structural characteristics are cancelled out. The general market conditions (time-dummy variables) are assumed to be constant when this property ages. As a result, the annual depreciation rate at age T can be determined by the following equation:

$$(3.9)\quad (Depreciation\ Rate)_{Age=T} = 1 - \frac{P_{Age=T+1}}{P_{Age=T}}.$$

The right-hand side of Equation (3.9) is derived from Equation (3.8).

Step 3 was suggested by Bokhari and Geltner (2016). They calculated the average depreciation rate from T_1 to T_2 by the mean of annual depreciation rates within these $T_2 - T_1$ years. Here, the equally weighted sum or the algebraic mean of $\left(1 - \frac{P_{Age=T+1}}{P_{Age=T}}\right)$ from T_1 to T_2 is adopted. We can also take the geometric mean of $\frac{P_{Age=T+1}}{P_{Age=T}}$ to achieve the value of $\left(1 - (Depreciation\ Rate)_{Age=T}\right)$; however, this method only uses the property values at T_1 and T_2. The values in the middle periods are cancelled out in the calculation, thereby wasting most market information. The algebraic mean of $\left(1 - \frac{P_{Age=T+1}}{P_{Age=T}}\right)$ is a good statistic for the constant depreciation rate.

If all the torn-down records of old buildings in the same region are available, then a survival probability for the old buildings can be generated. Hulten and Wykoff (1981) suggested a revised hedonic pricing model by multiplying the survival probability to replace Equation (3.7), as shown as follows:

$$\ln(P_{it} * p_T) = \alpha_{1it}Age_{it} + \alpha_{2it}AgeSquare_{it} + \sum_{j=1}^{J}\beta_{jit}L_{jit} + \sum_{k=1}^{K}\beta_{kit}X_{kit} + \sum_{m=1}^{M}\beta_{mit}M_{mit} + \varepsilon_{it},$$

where p_T is the survival probability until the building age reaches T.

In the remainder of this study, the term 'depreciation rate' represents the 'average depreciation rate' derived from the above steps.

3.3.4 Reasons for adopting the constant depreciation rate assumption

In addition to the additive property, three other reasons are provided to adopt the constant depreciation rate assumption.

Firstly, the constant depreciation rate assumption is consistent with the hedonic pricing model, which can be explained by Equations (3.8) and (3.9). If coefficient α_2 is insignificant, then the depreciation rate is the constant $(1 - e^{\alpha_1})$. If α_2 is significant, then the algebraic mean of $\left(1 - \frac{P_{Age=T+1}}{P_{Age=T}}\right)$ is chosen as the constant depreciation rate. Bokhari and Geltner (2016) proved that this approximation nearly equals the depreciation rate of the property at the median age within the sample properties.

Secondly, the adoption of quadratic depreciation effect will lead to a severe problem in the real option approach. For the real estate project valuation, the market volatility is assumed as a constant. This constant volatility, which is derived from the past market information, is independent of the average building age changes during the sample period. When the constant depreciation rate ξ is chosen, the average building age increases by *z* annually. The annual return for the first year is

$$\ln\left(\frac{P_{t+1}^{g+z}}{P_t^g}\right) = \ln\left(\frac{P_{t+1}^{g+z}}{P_{t+1}^g}\right) + \ln\left(\frac{P_{t+1}^g}{P_t^g}\right)$$

$$= \mathrm{z}\xi + \ln\left(\frac{P_{t+1}^g}{P_t^g}\right).$$

The average annual return in the following *n* years is

$$\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{P_{t+i}^{g+z}}{P_{t+i-1}^g}\right) = \mathrm{z}\xi + \frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{P_{t+i}^g}{P_{t+i-1}^g}\right).$$

The market volatility in this market is

$$\sqrt{\frac{1}{n-1}[\ln\left(\frac{P_{t+i}^{g+z}}{P_{t+i-1}^g}\right) - \frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{P_{t+i}^{g+z}}{P_{t+i-1}^g}\right)]^2} = \sqrt{\frac{1}{n-1}[\ln\left(\frac{P_{t+i}^g}{P_{t+i-1}^g}\right) - \frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{P_{t+i}^g}{P_{t+i-1}^g}\right)]^2},$$

which equals the market volatility in the same market if no depreciation occurs. Thus, the depreciation effect does not change the market volatility.

If the quadratic depreciation effect is chosen, then the annual return for the property aged *g* in the first year of the sample period is

$$\ln\left(\frac{P_{t+1}^{g+z}}{P_t^g}\right) = \ln\left(\frac{P_{t+1}^{g+z}}{P_{t+1}^g}\right) + \ln\left(\frac{P_{t+1}^g}{P_t^g}\right)$$

$$= (g+z)\xi_1 + (g+z)^2\xi_2 - g\xi_1 - g^2\xi_2 + \ln\left(\frac{P_{t+1}^g}{P_t^g}\right),$$

where *z* is the annual increase in average building age. $\xi_1 < 0$, and $\xi_2 > 0$.

The average annual return in the continuous *n* years is

$$\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{P_{t+i}^{g+iz}}{P_{t+i-1}^{g+(i-1)z}}\right)=\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{P_{t+i}^{g+iz}}{P_{t+i}^{g+(i-1)z}}\right)+\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{P_{t+i}^{g+(i-1)z}}{P_{t+i-1}^{g+(i-1)z}}\right)$$

$$=\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{\exp[(g+iz)\xi_1+(g+iz)^2\xi_2-g\xi_1-g^2\xi_2]}{\exp\{[g+(i-1)z]\xi_1+[g+(i-1)z]^2\xi_2-g\xi_1-g^2\xi_2\}}\right)$$

$$+\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{P_{t+i}^{g+(i-1)z}}{P_{t+i-1}^{g+(i-1)z}}\right)$$

$$=z\xi_1+2gz\xi_2-z^2\xi_2+\frac{1}{n}\sum_{i=1}^{n}(2\mathrm{i}\xi_2z^2)+\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{P_{t+i}^{g+(i-1)z}}{P_{t+i-1}^{g+(i-1)z}}\right)$$

$$=z\xi_1+2gz\xi_2-z^2\xi_2+\xi_2z^2(1+n)+\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{P_{t+i}^{g+(i-1)z}}{P_{t+i-1}^{g+(i-1)z}}\right).$$

The last term equals the average annual return of property price if no depreciation effect exists, as shown as follows:

$$\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{P_{t+i}^{g+(i-1)z}}{P_{t+i-1}^{g+(i-1)z}}\right)=\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{P_{t+i}^{g}\exp[(g+iz)\xi_1+(g+iz)^2\xi_2-g\xi_1-g^2\xi_2]}{P_{t+i-1}^{g}\exp[(g+iz)\xi_1+(g+iz)^2\xi_2-g\xi_1-g^2\xi_2]}\right).$$

The square of market volatility becomes

$$\frac{1}{n-1}\sum_{i=1}^{n}\left[\ln\left(\frac{P_{t+i}^{g+iz}}{P_{t+i-1}^{g+(i-1)z}}\right)-\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{P_{t+i}^{g+iz}}{P_{t+i-1}^{g+(i-1)z}}\right)\right]^2$$

$$=\frac{1}{n-1}\sum_{i=1}^{n}[\ln\left(\frac{P_{t+i}^{g}}{P_{t+i-1}^{g}}\right)\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{P_{t+i}^{g}}{P_{t+i-1}^{g}}\right)]^2+\frac{1}{n-1}\sum_{i=1}^{n}[\xi_2z^2(2i-1-n)]^2+$$

$$\frac{1}{n-1}\sum_{i=1}^{n}2[\ln\left(\frac{P_{t+i}^{g}}{P_{t+i-1}^{g}}\right)-\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{P_{t+i}^{g}}{P_{t+i-1}^{g}}\right)]\xi_2z^2(2i-1-n).$$

The first term is the square of market volatility when no depreciation effect exists. The second terms can be expressed as follows:

$$\frac{\xi_2z^2}{n-1}[\frac{2n(n+1)(2n+1)}{3}-3n(1+n)^2],$$

which is an increasing function of *n*.

The third term depends on the trend of $\frac{P_{t+i}^{g}}{P_{t+i-1}^{g}}$. For example, if the depreciation-adjusted market price increases/decreases at a constant speed, this term should be zero. If this market price increases at a growing speed, then the third term should be positive. If the depreciation-adjusted market price decreases at a growing speed, then the third term should be negative. However, the value of this term is not reduced as *n* increases.

In an extreme case, suppose that the depreciation-adjusted market price increases/decreases at a constant speed. Then, the market volatility should be zero. If the constant depreciation rate is chosen, then the market volatility is also zero. However, if the quadratic depreciation effect is chosen, the market volatility becomes

$$\frac{\xi_2 z^2}{n-1}\left[\frac{2n(n+1)(2n+1)}{3} - 3n(1+n)^2\right] > 0.$$

The preceding situation leads to an unreliable prediction in the future values of new and old properties. This bias cannot be reduced by increasing the sample size. Even if we consider shorter time intervals (e.g. quarterly or monthly depreciation), the above problem still exists. Hence, the quadratic depreciation effect is an inferior choice in the real option approach.

Thirdly, from the perspective of investors, an average depreciation rate is easy to compare with their expected capital return rate. They can also divide the observed market return into the depreciation-adjusted market return and the depreciation part. In addition, a time-varying depreciation rate will complicate explaining how the depreciation rate influences the exercise of redevelopment option and the option value in the real option approach.

Some empirical studies show that market volatility will increase if the average building age increases because land price becomes more volatile as land leverage becomes larger (Bostic, Longhofer, & Redfearn, 2007; Bourassa, Hoesli, Scognamiglio, & Zhang, 2011; Davis & Palumbo, 2008). However, in the real option approach, adopting the time-varying volatility assumption indicates that a long-term prediction of the market volatility curve must be made over time. No sufficient support is provided from existing theory for predicting the future time-varying volatility curve. As a result, we only choose the constant depreciation rate assumption with a constant market volatility in this study.

3.4 Chapter summary

This chapter consists of two major parts. The first part focuses on the basic mathematical models applied in this study, and the latter part suggests the constant depreciation rate assumption. After the discussions on different types of market price statistics, a parameter called annual increase in average building age is introduced to discriminate amongst the building age assumptions within these statistics. After describing the estimation procedures of the depreciation rate, we justify why the constant depreciation rate assumption should be adopted.

CHAPTER 4 DISCRETE-TIME REDEVELOPMENT OPTION

4.1 Introduction

Chapters 4 and 5 focus on the two-phase redevelopment projects on multi-owner buildings. In view of the discrete-time case in a finite period, this chapter discusses the manner in which the depreciation effect can be demonstrated properly in future value prediction, which is an important procedure in the redevelopment option model pricing. On the basis of the mathematical expressions of constant depreciation assumption and annual increase in average building age in Chapter 3, a discrete-time compound option model with depreciation is developed to achieve Objectives 1 and 2. The direct and indirect effects of depreciation on project value are tested and discussed by sensitivity tests. This new model is then applied to a real case, that is, Bailey Street/Wing Kwong Street Development Project (KC-009) in the URA in Hong Kong.

4.2 Discrete-time compound option model with depreciation

Discrete-time compound option model with depreciation is based on the binomial tree pricing model (Cox et al., 1979). Different from the traditional binomial tree model, the changes in the unit market price and the unit construction cost of the property are assumed to follow the binomial tree. The unit market price (per m^2) is defined as $S(t)$, and the unit construction cost (per m^2, CFA) is defined as $K(t)$. The following two-phase model is applicable to multi-owner buildings. The residents in these buildings do not have sufficient capital to redevelop building structures. In the first phase, or the demolition phase, developers can purchase the old property and demolish it during the option period. The maximum option period in the demolition phase is T_1 years. In the second phase, or the rebuilding phase, developers can rebuild a new property on the vacant land during the option period. The maximum option period in the rebuilding phase is T_2 years. If developers exercise the demolition option, then the vacant land with a rebuilding option will be obtained after the demolition process. Hence, this redevelopment option is a compound option with a maximum period of *(T_1 + T_2)* years. Other necessary parameters are denoted as follows.

During the *(t + 1)*th decision period, the unit market price $S(t+1)$ satisfies

$$(4.1)\quad S(t+1) = \begin{cases} \mathrm{S(t)} * u_S, probability = p_{uS}, u_S > 1 \\ \mathrm{S(t)} * d_S, probability = p_{dS}, d_S < 1 \end{cases}.$$

The unit construction cost $K(t+1)$ satisfies

$$(4.2)\quad K(t+1) = \begin{cases} \mathrm{K(t)} * u_K, probability = p_{uK}, u_K > 1 \\ \mathrm{K(t)} * d_K, probability = p_{dK}, d_K < 1 \end{cases}.$$

The annual increase in average building age is z $(0 \leq z \leq 1)$. T_{ave} is the average building age in the same neighbourhood in the beginning of the option; T_{old} is the building age of the old property in the beginning of the option; $PLOT_{new}$ and $PLOT_{old}$ are the plot ratios of the new and old properties, respectively; and r is the risk-free interest rate under the risk-neutral assumption. In each year, the option can be exercised at the end of *N* equal periods. Other interim variables will be defined in the pricing process.

Suppose that developers have exercised the demolition option at time *T* $(0 \leq T \leq T_1)$. In the second phase, they can exercise the rebuilding option at time t within [*T, T* + T_2].

The adjusted market value of the new property is

$$(4.3)\quad S^*(t) = S(t) * PLOT_{new} * e^{-T_{ave}\xi} * e^{-tz\xi}.$$

The above formula is important in the new model. $e^{-T_{ave}\xi}$ adjusts the over-depreciation part for the new property at the beginning of the compound option, whereas $e^{-tz\xi}$ adjusts the over-depreciation part for the new property since the beginning of the compound option to the end of period t.

The construction cost of this property is

$$K^*(t) = K(t) * PLOT_{new}.$$

The American rebuilding option value at the end of the maximum option period is

$$V_R(S(T+T_2), K(T+T_2)|S(T), K(T)) = Max[0, S^*(T+T_2) - K^*(T+T_2)].$$

The condition $(S(T), K(T))$ indicates that the demolition option is exercised at time *T*. For any period t within [*T, T + T₂*], the rebuilding option value at the end of this period is

$$(4.4)\quad V_R(S(t), K(t)|S(T), K(T)) = \text{Max}[0, S^*(t) - K^*(t), e^{-r}E[V_R(S(t+1), K(t+1)|S(T), K(T))]].$$

The expectation of the option value at the end of period *(t + 1)* {i.e. $E[V_R(S(t+1), K(t+1)|S(T), K(T))]$} is based on the probabilities in Equations (4.1) and (4.2).

The rebuilding option value is calculated backwards. The rebuilding option value at the end of the first period in the second phase is

$$V_R(S(T+1), K(T+1)|S(T), K(T)) =$$
$$\text{Max}[0, S^*(T+1) - K^*(T+1), e^{-r}E[V_R(S(T+2), K(T+2)|S(T), K(T))]].$$

Then,

$$V_R(S(T), K(T)|S(T), K(T)) = e^{-r}E[V_R(S(T+1), K(T+1)|S(T), K(T))].$$

$V_R(S(T), K(T)|S(T), K(T))$ is defined as $V_R\big(S(T), K(T)\big)$ for conciseness.

For different sets of $\big(T, S(T), K(T)\big)$, the corresponding $V_R\big(S(T), K(T)\big)$ is valued as the asset price in the first phase. In the empirical pricing process, a three-dimensional matrix is used to store the values of $V_R\big(S(T), K(T)\big)$.

In the first phase, developers can exercise the rebuilding option at time *t* within [*0, T_1*].

The adjusted market value of the old property is

$$(4.5)\ \ C^*(t) = S(t) * PLOT_{old} * e^{(T_{old}-T_{ave})\xi} * e^{(1-z)t\xi}.$$

The above formula is also important in the new model. $e^{(T_{old}-T_{ave})\xi}$ adjusts the under-depreciation part for the old property at the beginning of the compound option, and $e^{(1-z)t\xi}$ adjusts the under-depreciation part for the old property since the beginning of the compound option to the end of period t.

The demolition cost is positively related to the construction cost as follows:

$$D^*(T) = K(T) * PLOT_{old} * DEM,$$

where *DEM* is a constant ratio for deriving the demolition cost from the construction cost in the same period.

The value of the compound option at the end of period T_1 is

$$V_C\big(S(\mathrm{T}_1), K(\mathrm{T}_1)\big) = Max[V_R\big(S(\mathrm{T}_1), K(\mathrm{T}_1)\big) - C^*(\mathrm{T}_1) - D^*(\mathrm{T}_1), 0].$$

For any period t within [$0,\ T_1$], the compound option value at the end of this period is

$$(4.6)\ \ V_C\big(S(\mathrm{t}), K(\mathrm{t})\big) = \ Max[0, V_R\big(S(\mathrm{t}), K(\mathrm{t})\big) - C^*(\mathrm{t}) - D^*(\mathrm{t}), e^{-r}E[V_C\big(S(\mathrm{t}+1), K(\mathrm{t}+1)\big)]].$$

Similarly, the compound option value at the end of the first period is

$$V_C\big(S(1), K(1)\big) = Max[0, V_R\big(S(1), K(1)\big) - C^*(1) - D^*(1), e^{-r}E[V_C\big(S(2), K(2)\big)]].$$

Then,

$$V_C\big(S(0), K(0)\big) = e^{-r}E[V_C\big(S(1), K(1)\big)],$$

which is the desired compound option value.

4.3 Model properties

The depreciation term is the major difference between this new model and the traditional two-phase model; thus, the following properties focus on the influence of this depreciation effect. The following propositions are initially discussed theoretically and then tested empirically.

Proposition 1. The depreciation term reduces the option price changes caused by the changes of market price volatility and construction cost volatility.

Assume that $S(\mathrm{t})$ and $K(\mathrm{t})$ are known. In the second phase, for a specific *t*, if all the tree nodes satisfy

$$S(\mathrm{t}+1) - K(\mathrm{t}+1) \geq 0$$

and

$$S(\mathrm{t}) - K(\mathrm{t}) \geq 0,$$

then the risk-neutral probability assumption indicates that

$$Max[0, S(\mathrm{t}) - K(\mathrm{t})] = S(\mathrm{t}) - K(\mathrm{t}) = e^{-r}E[S(\mathrm{t}+1) - K(\mathrm{t}+1)] =$$
$$e^{-r}E[Max[0, S(\mathrm{t}+1) - K(\mathrm{t}+1)]].$$

In this extreme example, the option value exercised at the end of period *t* equals the discounted option value when it is exercised at the end of period *(t + 1)*. Developers can exercise the option at the end of either one of the two periods. Hence, the value of delaying the exercise originates from the proportion of the tree nodes when $S(\mathrm{t}+1) - K(\mathrm{t}+1) < 0$ or $S(\mathrm{t}) - K(\mathrm{t}) < 0$. When the volatilities of market price and/or construction cost increase, the proportion of the tree nodes when $S(\mathrm{t}+1) - K(\mathrm{t}+1) \geq 0$ also changes significantly. The value of delaying the exercise will increase. Thus, the option value increases with volatility.

As buildings age between new properties and nearby properties increase, the depreciation adjustment term increases the market value of the new property in the second phase. The adjustment also decreases the acquisition value of old properties in the first phase. Hence, the inequality $S(\mathrm{t}+1) - K(\mathrm{t}+1) \geq 0$ holds considerably easier than the traditional model without depreciation. The depreciation term decreases the value of delaying the exercise.

On the basis of the preceding discussions, we compare two scenarios. The depreciation rates are different, whereas other parameters are the same. Assume that the price volatilities in two scenarios increase in the same amount. In a higher depreciation environment, the proportion of the tree nodes when $S(\mathrm{t}+1)-K(\mathrm{t}+1)\geq 0$ will be less influenced in comparison with the case in a lower depreciation environment. Hence, Proposition 1 holds.

Proposition 2. As the depreciation rate becomes larger, the option value will be less influenced by the changes in interest rates.

Assume that $S^*(T)$ and $K^*(T)$ are known. If a depreciation term is ignored, then the rebuild option exercise value in period *(t + 1)* when discounted to period *t* is:

$$e^{-r}Max[p_{uS}p_{uK}\big(u_S S^*(t)-u_K K^*(t)\big)+p_{uS}p_{dK}\big(u_S S^*(t)-d_K K^*(t)\big)+ \\ p_{dS}p_{uK}\big(d_S S^*(t)-u_K K^*(t)\big)+p_{dS}p_{dK}\big(d_S S^*(t)-d_K K^*(t)\big),0].$$

If a depreciation term exists, then

$$e^{-r}Max[p_{uS}p_{uK}\left(u_S e^{-z\xi}S^*(t)-u_K K^*(t)\right)+p_{uS}p_{dK}\left(u_S e^{-z\xi}S^*(t)-d_K K^*(t)\right)+ \\ p_{dS}p_{uK}\left(d_S e^{-z\xi}S^*(t)-u_K K^*(t)\right)+p_{dS}p_{dK}\left(d_S e^{-z\xi}S^*(t)-d_K K^*(t)\right),0],$$

where $e^{-z\xi}>1$; the value of the second equation is larger than that of the first one. The first-order derivative is taken with respect to *r*.

$$(-r)e^{-r}Max[p_{uS}p_{uK}\left(u_S e^{-z\xi}S^*(t)-u_K K^*(t)\right)+p_{uS}p_{dK}\left(u_S e^{-z\xi}S^*(t)-d_K K^*(t)\right) \\ +p_{dS}p_{uK}\left(d_S e^{-z\xi}S^*(t)-u_K K^*(t)\right) \\ +p_{dS}p_{dK}\left(d_S e^{-z\xi}S^*(t)-d_K K^*(t)\right),0]\leq 0$$

The first-order derivative becomes larger (although still negative) if the depreciation rate becomes larger (e^{ξ} becomes smaller and then $e^{-z\xi}$ becomes larger). Thus, the option value decreases more slowly when the interest rate increases.

After a similar proof in the first phase, the redevelopment option value becomes less influenced by the interest rate changes as the depreciation rate becomes higher.

Proposition 3. When the depreciation rate and the other parameters are fixed, the rebuilding option value is an increasing function of *z*, which is the annual increase in average building age. The adjusted market value of the old property and the compound redevelopment option value are an increasing function of *z*.

When $S(t)$ and $K(t)$ are known, the first-order derivative of the rebuild option exercise value with respect to *z* at Period *T* is

$$(-t\xi)Max[e^{-zt\xi}e^{-T_{ave}\xi}PLOT_{new}S(t),0] \geq 0,(\xi < 0),$$

which is a decreasing function of ξ and an increasing function of the depreciation rate.

The first-order derivative of the old property market value with respect to z is

$$(-t\xi)S(t) * PLOT_{old} * e^{(T_{old}-T_{ave})\xi} * e^{(1-z)t\xi} \geq 0,(\xi < 0),$$

which is a decreasing function of ξ and an increasing function of the depreciation rate.

Three reasons are provided to prove that the $e^{-zT\xi}e^{-T_{ave}\xi}PLOT_{new}S(T)$ increases considerably faster than $S(T) * PLOT_{old} * e^{(T_{old}-T_{ave})\xi} * e^{(1-z)T\xi}$ as *z* increases.

(1) $PLOT_{new} \geq PLOT_{old}$;

(2) *t* in the second phase is larger than that in the first phase;

(3) $e^{-T_{ave}\xi} > 1 > e^{(T_{old}-T_{ave})\xi}$.

The redevelopment option value is an increasing function of *z*.

4.4 Empirical data description

To test the model properties and explain the model application procedures when different types of market statistics are available, empirical data were collected from the Hong Kong property market.

For the sensitivity analysis, the market price volatility was generated from the quarterly price index for Class B market in Kowloon from Q1 of 2005 to Q3 of 2015. The construction cost volatility was generated from the quarterly Building Works Tender Price Index by Architectural Services Department from Q1 of 2005 to Q3 of 2015 (Arcadis, 2016). The reason to choose Class B market is that the unit sizes in the transaction records adopted in the case study in Section 4.6 mainly belong to this market. Most units in new properties are also expected to be in Class B.

The property price in Q4 of 2015 was chosen from Rating and Valuation Department. The quarterly average price for the Class B market in Kowloon was 110,560 HKD per m^2 GFA.

The construction cost in Q4 of 2015 was chosen from the ARCADIS Construction Cost Handbook. The building cost of high-rise average-standard domestic apartments was 20,700–23,500 HKD per m^2 CFA. The cost of servicing these apartments was 3,800–5,300 HKD per m^2 CFA for a total cost of 24,500–28,800 HKD per m^2 CFA. The average value of 26,650 HKD was determined as the construction cost when the first phase of the compound redevelopment option starts. The CFA of an apartment was defined as 1.2 times its GFA according to the general case in Hong Kong market.

The interest rate was the annualised 10-year exchange fund note monthly yield rate given by the Hong Kong Monetary Authority. Other statistics were from the Census and Statistics Department. Table 4.1 shows a summary of the parameters applied in this chapter.

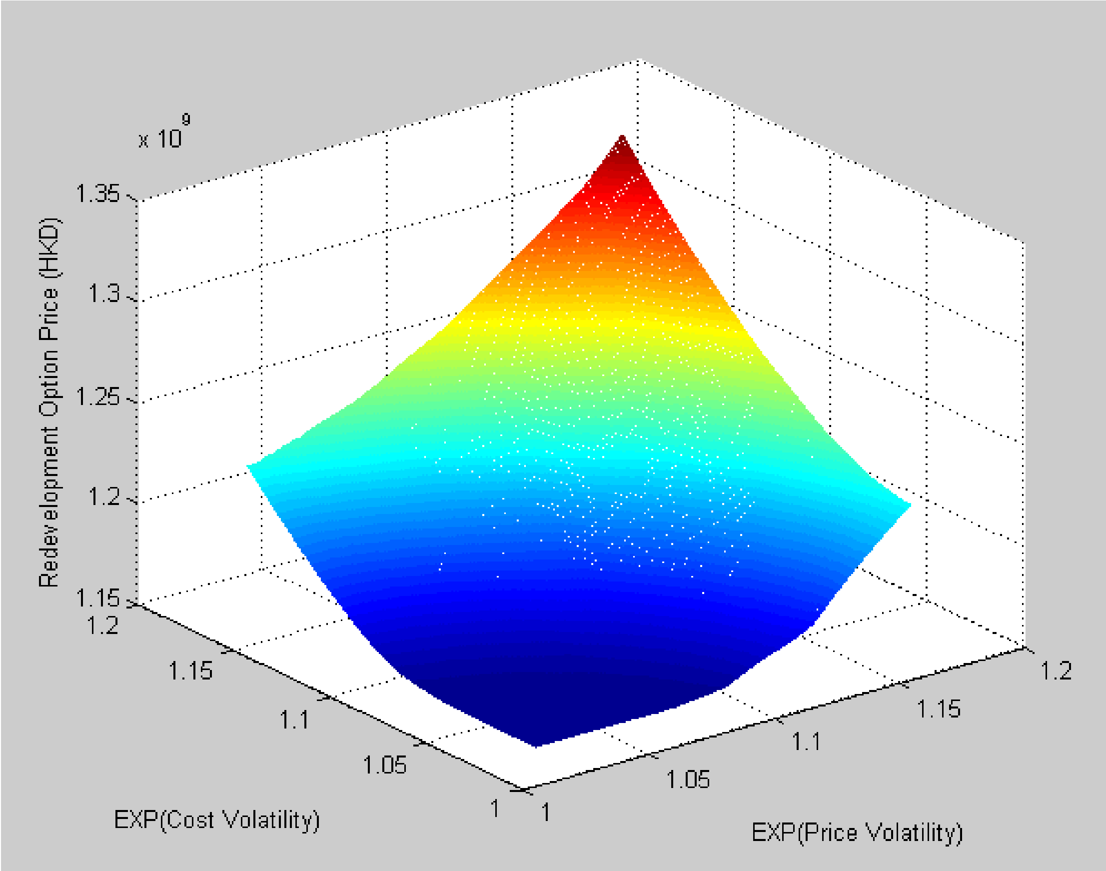
Parameters and symbols	Values
Annualised construction cost volatility (basic scenario), σ_K	10.67%
Annualised property price volatility (basic scenario), σ_S	8.06%
Construction cost (per m^2 CFA), $K(0)$	26,650 HKD
Property price (per m^2 GFA), $S(0)$	110,560 HKD
Demolition cost (per m^2 CFA), DEM* $K(0)$	3% * 26,650 HKD
Consultant fee (per m^2 CFA), only occurs once as the rebuild option is exercised, not listed in the model	10% * $K(t)$ at period *t*
Interest rate (basic scenario), which is equal to $1/e^{-r} - 1$	1.85%
Plot ratio for Class B site buildings (15–18 m; old property to be demolished), a minimum value for $PLOT_{old}$ in this study	4.0
Plot ratio for Class B site buildings (over 61 m; new property to be built), a maximum value for $PLOT_{new}$ in this study	9.0
Redevelopment land area, not listed in the model and only used when calculating the total value of this project	5,000 m^2

Table 4.1 Model parameters in Chapter 4

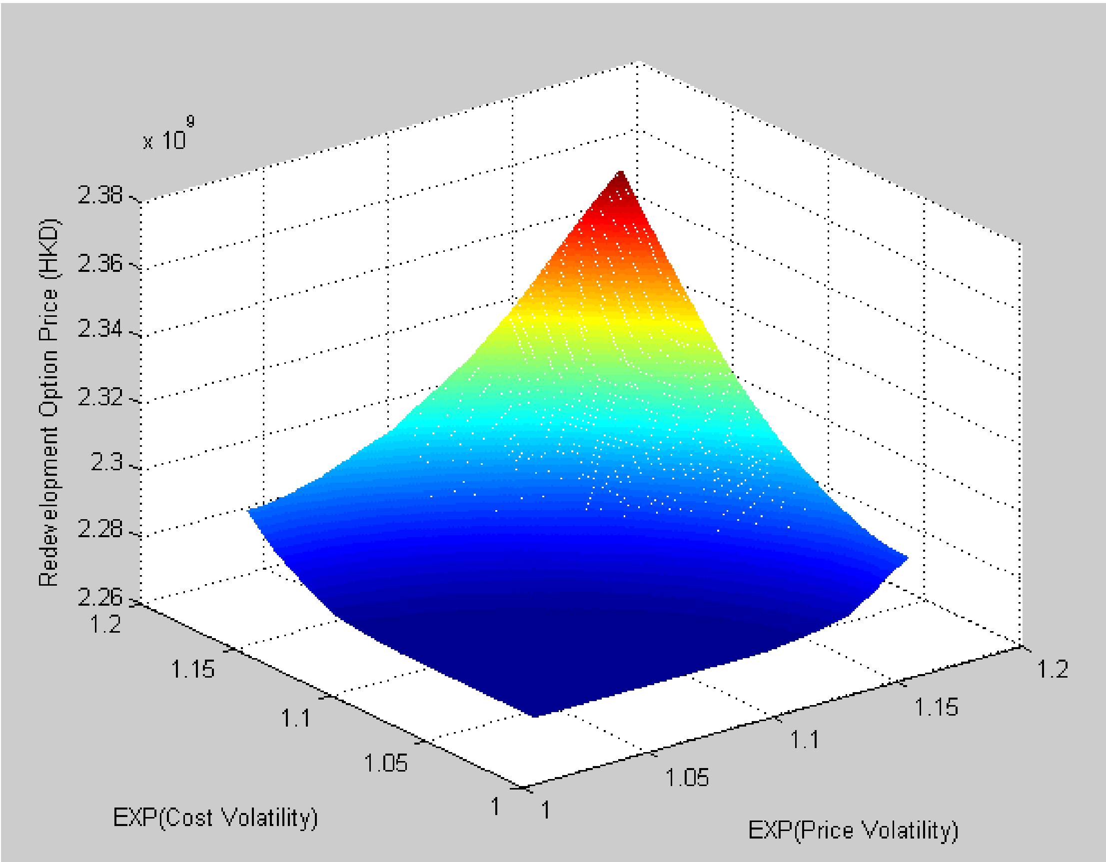
An additional case study on KC-009 in the URA) was also conducted. The average market price for the acquisition was estimated in July 2017, and the market and cost volatility were generated from the history data between 2007 and 2017. Construction cost was based on the information in Q2 of 2017 in the ARCADIS Construction Cost Handbook (Arcadis, 2017). Interest rate was also from the same period. To estimate the annual depreciation rate, necessary property information was obtained from Centa-data and transaction data in EPRC property database. Centa-data is owned by the Centaline Property company, which is the largest property agent company in Hong Kong. EPRC is owned by Hong Kong Economic Times Holdings. The database has all the transaction records in the Land Registry since 1991.

4.5 Sensitivity analysis for depreciation rate and other parameters

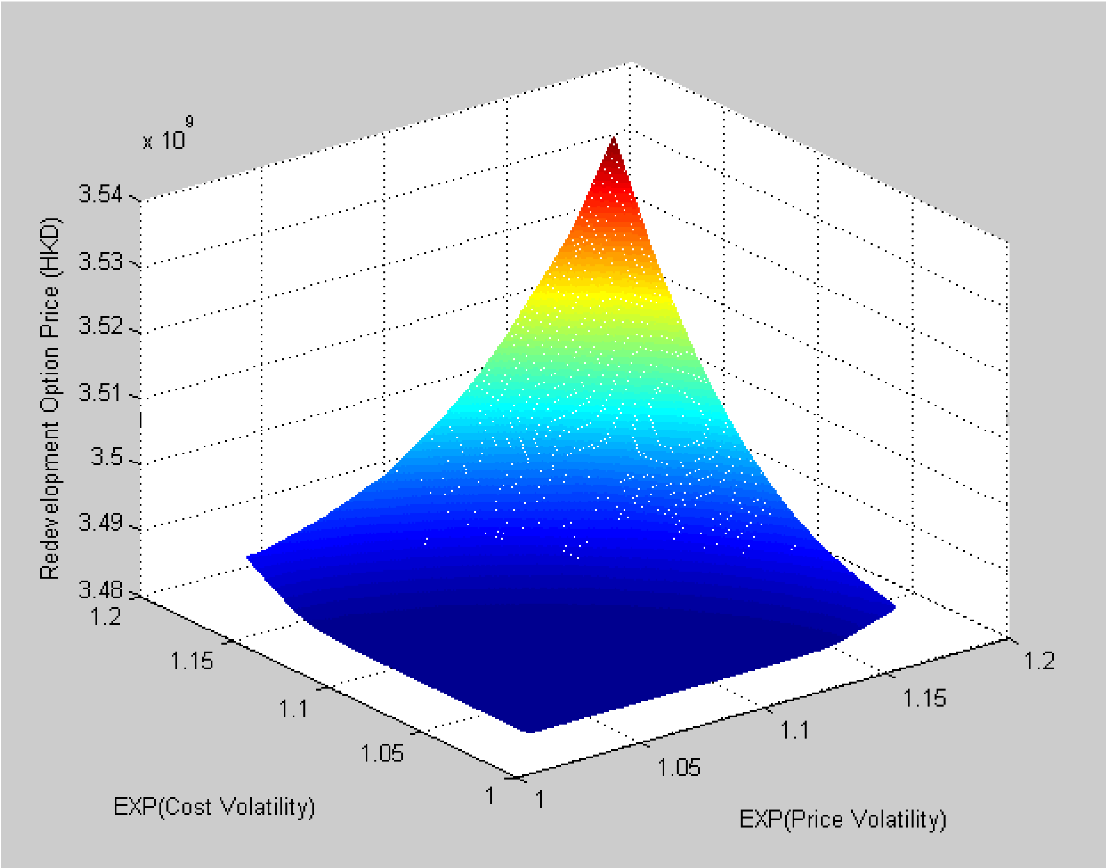
The original influences of volatilities and interest rate on the option value do not change due to the depreciation effect. Higher market price volatility, higher construction cost volatility and lower interest rate still indicate a larger redevelopment option value. However, the sensitivity to these factors is indirectly influenced by the depreciation rate. Figures 4.1 and 4.2 provide two sets of graphs to illustrate how the option value changes due to these factors if the depreciation rate is different. The annual increase in average building age is set as 0 in these simulations (i.e. Scenario 1 in Section 3.3). When the compound option becomes available, the average building age in the same neighbourhood is 30 years and the targeted property is 50 years old.



(a) Depreciation = 0%



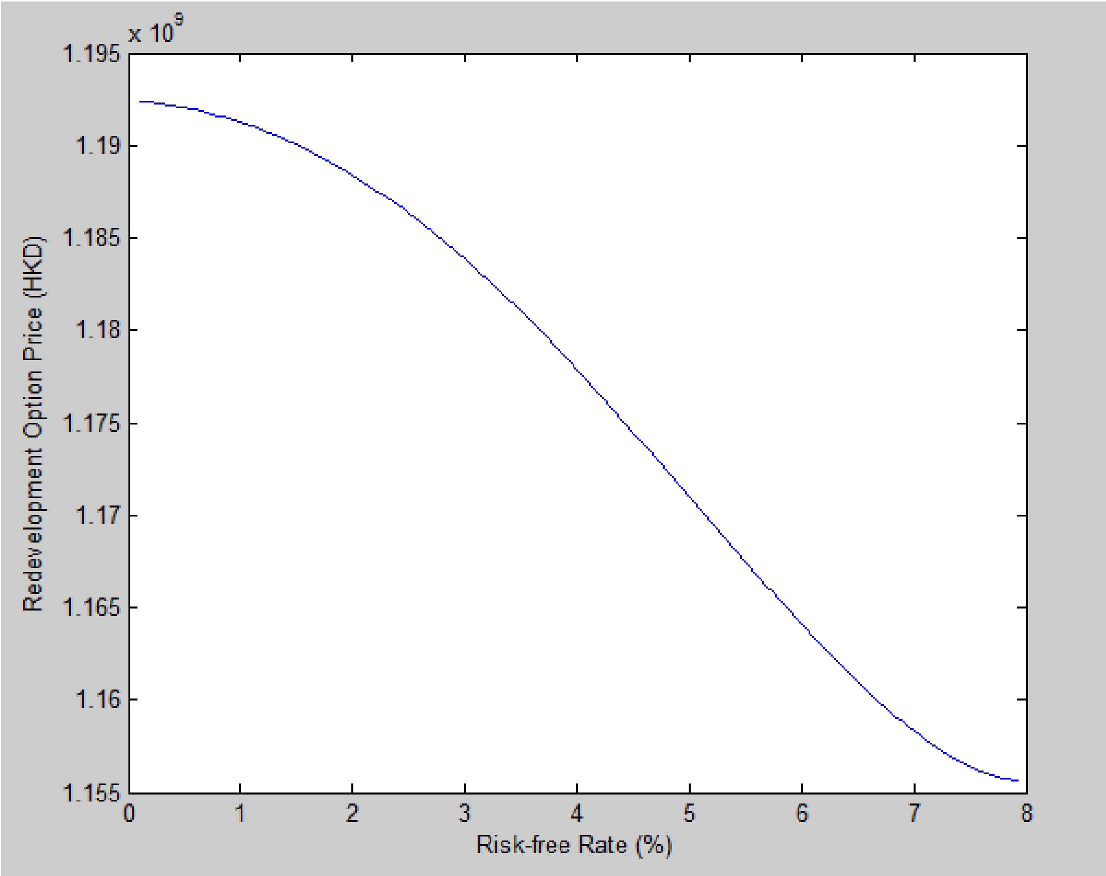
(b) Depreciation = 0.5%



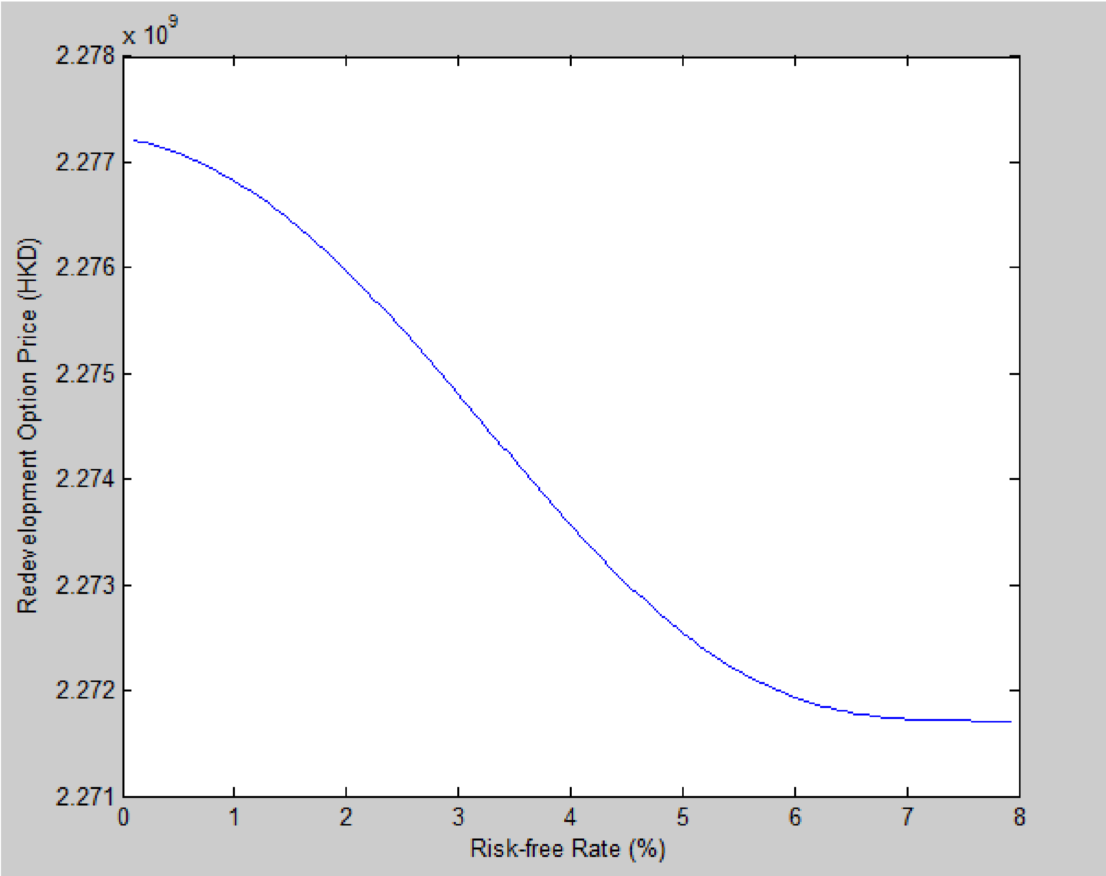
(c) Depreciation = 1%

Figure 4.1 Option value sensitivity testing against price and cost volatilities

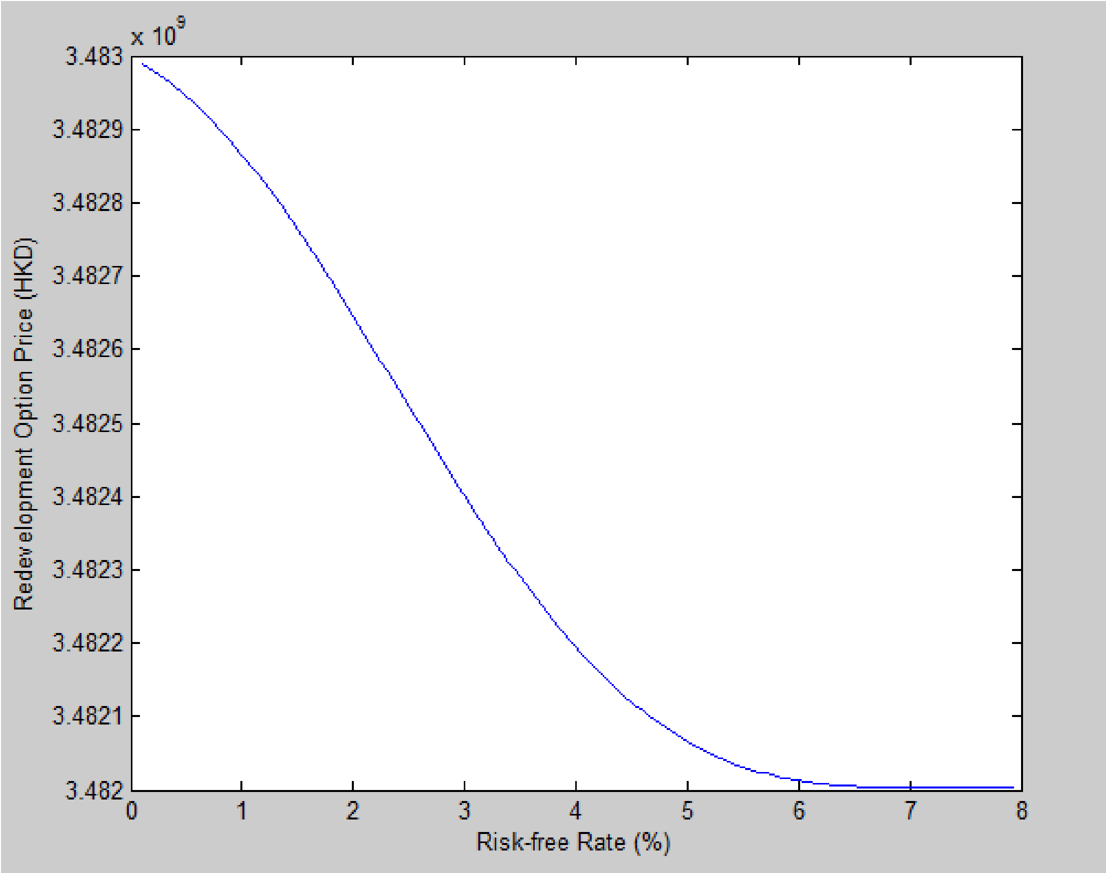
The z-axis, whose scales represent the redevelopment option value, shows the significant effect of the depreciation rate. Generally, the option value is positively related to two different volatilities. A high depreciation rate also indicates the high redevelopment potential of the targeted project. However, in the high depreciation environment (Figure 4.1(c)), the option value only increases from 3.48 to 3.53 billion, reflecting approximately 1.4% growth. In comparison, in the zero-depreciation environment (Figure 4.1(a)), the option value increases from approximately 1.15 to approximately 1.32 billion, which is more than 10% growth. Proposition 1 is supported. The depreciation effect reduces the potential value to delay the option exercise. The option value changes caused by volatility also diminish.



(a) Depreciation = 0%



(b) Depreciation = 0.5%



(c) Depreciation = 1%

Figure 4.2 Option value sensitivity testing against interest rate

A similar phenomenon occurs in the sensitivity test against the interest rate. The option makes having a positive intrinsic value considerably easier as the depreciation rate becomes larger. During the option period, making the exercise profit in each node under zero is more difficult. Proposition 2 is also supported. The developers are less concerned about interest rate changes if the depreciation effect becomes larger in the region where the project is located.

4.6 Influence of annual increase in average building age: A case study on KC-009

The importance of the newly introduced parameter (i.e. annual increase in average building age) is discussed based on a case study conducted on KC-009, a typical redevelopment project for residential buildings in the downtown. The targeted buildings are surrounded by many old residential properties and a few new ones at different ages. A proper estimation of depreciation effect can be obtained from these properties. In July 2017, the URA provided the average market price of the 7-year-old property in the same region, which was valued by surveyors at 15,383 HKD per ft^2 (approximately 165,581 HKD per m^2). This estimated price was based on the GFA standard. According to the Arcadis Construction Cost Handbook: China and Hong Kong 2017, the average total cost is 26,650 HKD per m^2 CFA in this case study.

The Centa-City Leading Index (in Kowloon) was selected as the market statistics for reference. The reasons for adopting this index are twofold. Firstly, it has a detailed description about the type of sample properties. The index is based on a fixed group of completed properties, which belongs to Scenario 2 in Section 3.3. Moreover, the mostly used Hong Kong property market index by the Rating and Valuation Department does not provide sufficient information about the building age of sample properties. Secondly, the Centa-City Leading Index (CCL Index) was the most recent index when this study was conducted. The market volatility is also estimated from the same index to reflect the newest market information.

The interest rate was generated from the annualised monthly yield rate of 10-year government bonds issued under the Institutional Bond Issuance Programme by the Hong Kong Monetary Authority. Table 4.2 summarises the relevant parameters.

Parameters and symbols	Values
Annualised construction cost volatility (basic scenario), σ_K	11.45%
Annualised property price volatility (basic scenario), σ_S	11.36%
Construction cost (per m^2 CFA), $K(0)$	26,650 HKD
Property price (per m^2 GFA), $S(0)$	165,581 HKD

Demolition cost (per m^2 CFA), DEM* $K(0)$	3% * 26,650 HKD
Consultant fee (per m^2 CFA), only occurs once as the rebuild option is exercised, not listed in the model	10% * $K(t)$ at period t
Interest rate (basic scenario), which is equal to $1/e^{-r} - 1$	1.492%
Total GFA for old properties (estimated from transaction records in Centa-data, only including the residential area)	39,840 m^2
Total GFA for new properties (provided by URA, only including the residential area)	55,000 m^2

Table 4.2 Parameters in the case study on KC-009

The property transaction records between August 2015 and July 2017 in the same region were collected from the EPRC database. The sample region includes the building blocks which are no more than 5 min walking distance from the redevelopment region. The sample region (the large polygon) and the redevelopment region (the small polygon) are shown in Figure 4.3. The location characteristics in this sample region are equal because the walking distance from the redevelopment region is limited to 5 min.

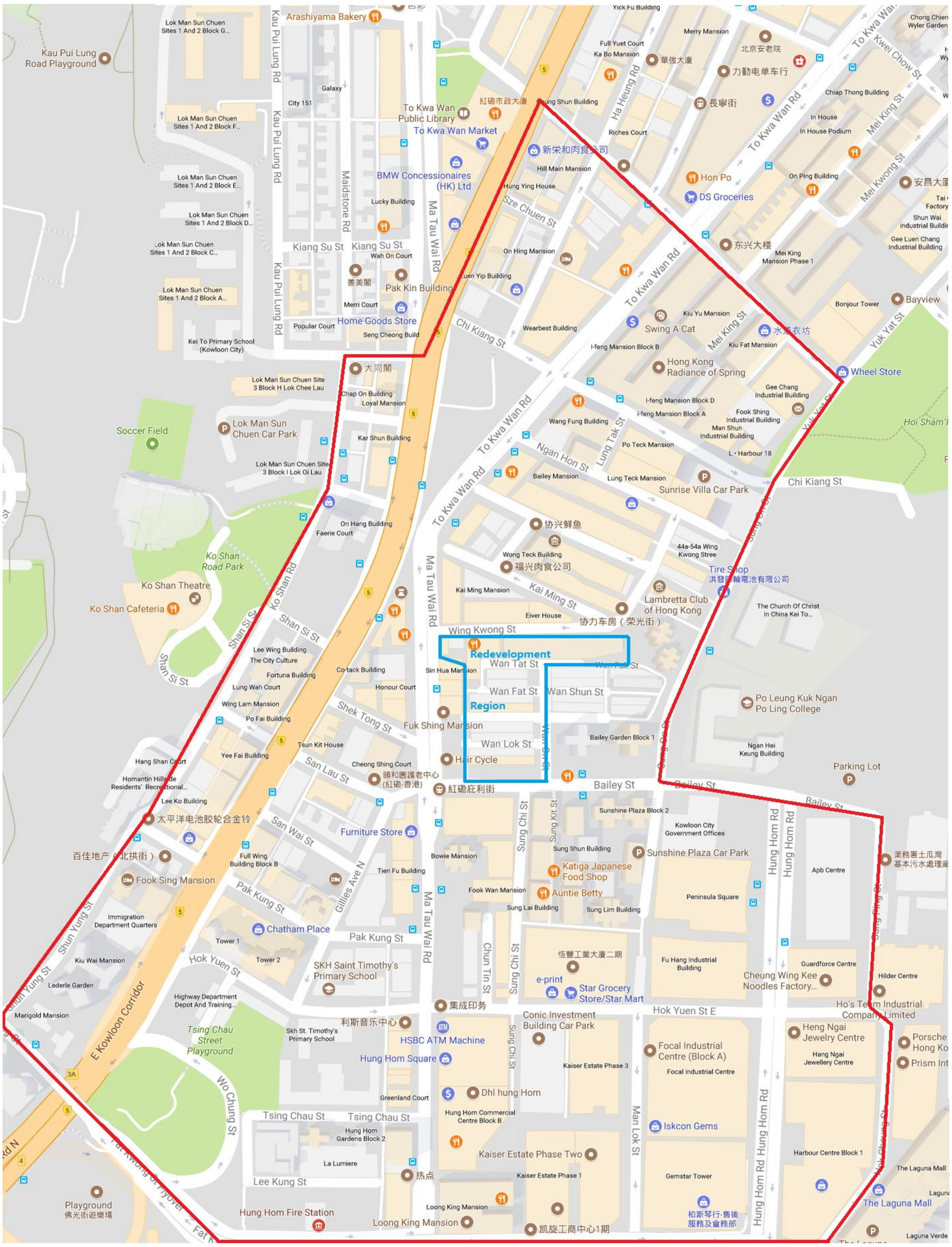


Figure 4.3 Redevelopment region (small polygon) and sampling region (large polygon) for the KC-009 programme

(Source: Google Map. Accessed in August 2017)

A preliminary study was conducted to estimate the annual depreciation rate in the redevelopment project using the traditional hedonic pricing model (Rosen, 1974).

Quantitative variables include floor level, age, square of age, area (saleable area) and weekly CCL index. A small proportion of records only have gross area information; thus, their areas were transformed into saleable area based on the median efficiency ratio of 74% in this region. Two dummy variables were added to describe the location statistics. One is whether the apartment is within 5 min walk from Ho Man Tin MTR station, and the other is whether the apartment has a sea view. Table 4.3 summarises the descriptive statistics for quantitative variables. A total of 472 available transaction records with complete information are used in the regression.

	Min	Max	Median	Mean	Std. Deviation
Price (million HKD)	0.98	14.00	3.95	4.32	1.85
Floor	1	32	10	11.65	7.61
Area (ft^2)	195	1051	360	371.56	125.15
Age (year)	0	59	35	29.73	17.57
CCL index	122.97	157.46	142.43	140.72	9.38

Table 4.3 Descriptive statistics for the quantitative variables in the case study on KC-009

SPSS16 was chosen to generate the regression results. Model 1 includes linear and quadratic terms of age effect (Table 4.4) to test the significance of quadratic depreciation. The transaction price, floor level, area and CCL index are taken as logarithms in the regression model.

	Model 1	Model 2
Ln Floor	0.007 (0.631)	0.006 (0.675)
Ln Area	0.581 (0.000)	0.591 (0.000)

Age	−0.016 (0.002)	−0.013 (0.000)
Age^2	$6.203*10^{-5}$ (0.087)	-
Ln CCL	1.228 (0.000)	1.262 (0.000)
MTR	0.083 (0.035)	0.082 (0.039)
Seaview	0.250 (0.000)	0.253 (0.000)
Constant	1.818 (0.195)	1.444 (0.299)
R^2	0.691	0.689
Adjusted R^2	0.686	0.685

Table 4.4 Regression results (coefficients and p values) on KC-009

However, the regression coefficient of the square of age is insignificant. Model 2 removes this term (Age^2). It can slightly better estimate the coefficient of the linear depreciation effect. In Model 2, the average annual depreciation effect is obtained as 1.3%.

On the basis of the depreciation rate, the compound option values in different scenarios can be estimated from the new discrete-time model (Table 4.5). Except for the Scenario Traditional with zero depreciation rate, the other scenarios have the same depreciation rate at 1.3% annually. Scenarios 1(a), 2(a) and 3(a) are simulated from Scenarios 1, 2 and 3 in Section 3.3.1, respectively. Scenarios 1(b), 2(b) and 3(b) are simulated with a special acquisition standard suggested by the URA to compare with the normal cases where the acquisition is based on the actual building age of the old property. The old property is acquired as a 7-year-old building in the neighbourhood. Scenarios 1(b), 2(b) and 3(b) refer to Scenario 4 in Section 3.3.1. The last scenario, namely, Scenario Traditional, is simulated without any depreciation effect. Four combinations of phase periods are offered (10 + 5, 5 + 5, 5 + 10 and 10 + 10). All the other parameters are based on Table 4.1 and are equal, except for the listed variables in Table 4.5. Several indications are derived from the comparison amongst these scenarios.

i. Phase 1 = 10 years, Phase 2 = 5 years		
Scenario 1(a) z = 0, T = 60	Scenario 2(a) z = 1, T = 60	Scenario 3(a) z = 0.6, T = 60
$5.1048*10^9$	$6.8632*10^9$	$6.1147*10^9$
Scenario 1(b) z = 0, T = 7	Scenario 2(b) z = 1, T = 7	Scenario 3(b) z = 0.6, T = 7
$2.2422*10^9$	$3.5711*10^9$	$2.9967*10^9$
Scenario Traditional (depreciation = 0%)	$7.3370*10^8$	
ii. Phase 1 = 5 years, Phase 2 = 5 years		
Scenario 1(a) z = 0, T = 60	Scenario 2(a) z = 1, T = 60	Scenario 3(a) z = 0.6, T = 60
$4.9070*10^9$	$6.0934*10^9$	$5.5985*10^9$
Scenario 1(b) z = 0, T = 7	Scenario 2(b) z = 1, T = 7	Scenario 3(b) z = 0.6, T = 7
$1.8227*10^9$	$2.7943*10^9$	$2.3869*10^9$
Scenario Traditional (depreciation = 0%)	$6.2934*10^8$	
iii. Phase 1 = 5 years, Phase 2 = 10 years		
Scenario 1(a) z = 0, T = 60	Scenario 2(a) z = 1, T = 60	Scenario 3(a) z = 0.6, T = 60
$4.9086*10^9$	$6.8631*10^9$	$6.0314*10^9$
Scenario 1(b) z = 0, T = 7	Scenario 2(b) z = 1, T = 7	Scenario 3(b) z = 0.6, T = 7
$1.8238*10^9$	$3.5636*10^9$	$2.8175*10^9$
Scenario Traditional (depreciation = 0%)	$6.2959*10^8$	
iv. Phase 1 = 10 years, Phase 2 = 10 years		
Scenario 1(a) z = 0, T = 60	Scenario 2(a) z = 1, T = 60	Scenario 3(a) z = 0.6, T = 60
$5.1098*10^9$	$7.6858*10^9$	$6.5666*10^9$
Scenario 1(b) z = 0, T = 7	Scenario 2(b) z = 1, T = 7	Scenario 3(b) z = 0.6, T =

		7
$2.2436*10^9$	$4.3888*10^9$	$3.4426*10^9$
Scenario Traditional (depreciation = 0%)	$7.3389*10^8$	

Table 4.5. Estimated compound option values (HKD) in different scenarios

Note: The values of different price indices must be the same to facilitate comparison of estimation variations. Z is the annual increase in average building age. T is the building age standard of the old property in the acquisition.

Scenario 1(a). The newly built property price indices or age-adjusted repeat-sales price indices are available. The property is acquired by private developers as a 60-year-old property.

Scenario 1(b). The newly built property price indices or age-adjusted repeat-sales price indices are available. The property is acquired by the URA as a 7-year-old property.

Scenario 2(a). The traditional repeat-sales price indices or indices from pre-determined group of properties are available. The property is acquired by private developers as a 60-year-old property.

Scenario 2(b). The traditional repeat-sales price indices or indices from pre-determined group of properties are available. The property is acquired by the URA as a 7-year-old property.

Scenario 3(a). Only an immature mixed price index from different types of properties is available. The property is acquired by private developers as a 60-year-old property.

Scenario 3(b). Only an immature mixed price index from different types of properties is available. The property is acquired by the URA as a 7-year-old property.

Scenario Traditional. The depreciation effect is ignored.

Firstly, several consistent trends are revealed about the option values amongst these scenarios within each combination of phase period. The traditional method underestimated the option value (over 70% in this study) compared with other scenarios. Within each scenario number, Scenario (a) consistently has a larger option value than Scenario (b). If the values of these different price indices are assumed to be the same, the largest option value will be observed in Scenario 2 amongst the three scenarios.

The first two phenomena can be explained by the reduction of acquisition cost due to a large depreciation adjustment. For the third phenomenon, this difference comes from the large over-depreciation adjustment for the new building in Scenario 2. In addition, no under-depreciation adjustment is needed in Scenario 2. In most redevelopment projects, the GFA in the newly built properties is larger than that in the old ones. Hence, the newly built property in Scenario 2 has the largest value, which makes the compound option value the largest one.

The findings suggest significant differences in option values amongst Scenarios 1, 2 and 3. Hence, the annual increase in average building age is worth investigating. The option value is found an increasing function of the (Age) factor. To avoid a significant underestimation of redevelopment option value, we should not simply adjust the initial values of new and old properties properly with depreciation term. Annual depreciation effect may still be different between the market price statistics for reference and the new/old property values. For the mature cities with many dilapidated buildings, the market price statistics are based on completed properties for a long period of time (e.g. Scenario 2). Thus, the traditional method has the most significant underestimation in these cities, where redevelopment is an important measure in the urban renewal.

Secondly, with a fixed maximum period in Phase 1, a longer maximum period in Phase 2 will lead to a larger increase in the option value in Scenario 2 compared with those in Scenarios 1 and 3. The option values in two combinations, ii and iii, are compared. Scenario 2(a) has a 12.6% increase, which is higher than in Scenario 1(a) (0.03%) and in Scenario 3(a) (7.7%). For the option values in the remaining combinations (i and iv), Scenario 2(a) increases by 12.0%, whereas Scenarios 1(a) and 3(a) only increase by

0.09% and 7.4%, respectively. If the developers have a longer period in the rebuilding phase, they should make a larger over-depreciation adjustment for the new property at the end of the option period. As a result, the option value estimated from the traditional repeat-sales indices (or other indices from a fixed group of properties) should have a larger upward adjustment compared with that estimated from newly built property indices.

Thirdly, with a fixed maximum period in Phase 2, a longer maximum period in Phase 1 will lead to a larger increase in the option value in Scenario 2 compared with those in Scenarios 1 and 3. For i and ii, Scenario 2(a) has a 12.6% increase, whereas Scenarios 1(a) and 3(a) increase by 4.0% and 9.2%, respectively. Similar results are observed in the comparison between iii and iv. An extended demolition phase not only enlarges the range of under-depreciation adjustment for the old property but also enlarges the range of over-depreciation adjustment for the new property. For example, assume the maximum demolition phase is extended from 5 to 10 years and the rebuilding phase is still 5 years. Then, the maximum adjustment for the old property is extended to 10 years, and that for the new property is extended to 15 years. The GFA of the new property is usually larger than that of the old one. The compound option in Scenario 2 will achieve a larger value than that in Scenarios 1 and 3.

Finally, if the total redevelopment period is fixed, then a longer period of demolition (and a shorter period of rebuilding) will cause a larger option value increase in Scenario 1 than in Scenario 2, as indicated by the comparison between i and iii. The maximum adjustments in the two combinations are both 15 years for the new property. The maximum under-depreciation adjustment for the old property is larger in Scenario 1 than that in Scenario 2.

In summary, a comprehensive view about the depreciation adjustments is provided in Table 4.5 when different types of market price indices are adopted. If only the demolition period or the rebuilding period becomes longer and other factors remain constant, then the option value based on traditional repeat-sales indices (or other indices from a fixed group of properties) will have a larger potential to increase compared with other indices. Meanwhile, if the total maximum option period is fixed, then a longer demolition period will lead to a larger growth in the option value estimated from newly built property indices than those estimated from other indices. These new findings provide help in understanding the relationship between redevelopment option value and the length of option period in each phase. Particularly, the annual increase in average building age is the key factor to explain the differences between market indices. Before applying a specific market index on option valuation, developers should know how to generate the index from the transaction records.

4.7 Chapter summary

A two-phase compound option pricing model is developed in this chapter to estimate the redevelopment project values for multi-owner buildings. This model introduces two novel features, namely, constant depreciation rate assumption and annual increase in average building age. Given the two factors, the discrete-time option model properly adjusts the depreciation effects for properties in demolition and rebuilding phases. The findings suggest that the two factors show greater influences on option value than volatilities and interest rate. However, the amounts of depreciation adjustments depend on the types of market indices for reference. To support this statement, a case study on a Hong Kong URA redevelopment project was conducted to compare the estimated option values based on different assumptions about the market indices for reference. If the traditional repeat-sales indices are adopted, then a longer decision period in each phase will lead to greater increase in the option value than the cases based on other types of indices.

As mentioned in Section 4.1, this chapter focuses on the redevelopment project within a finite period. Demolition and rebuilding phases are bounded in a fixed maturity. In the next chapter, these conditions will be released to find an optimal exercise strategy.

CHAPTER 5 CONTINUOUS-TIME REDEVELOPMENT OPTION

5.1 Introduction

This chapter combines the two-phase continuous-time compound option model with depreciation adjustments to achieve Objective 3; that is, to investigate the expected waiting time to demolish and rebuild in a two-phase continuous-time redevelopment option, in which the depreciation effect is considered. The direct influence of depreciation effect on the optimal redevelopment timing is investigated. In addition, the optimal exercise strategy in the new model is derived and discussed. On the basis of this strategy, whether the depreciation effect influences the optimal redevelopment timing is investigated. Finally, this chapter will discuss the effects of different acquisition standards for old properties on optimal redevelopment timing. In comparison with the discrete-time compound option model in Chapter 4, the model in this chapter is appropriate for different scenarios. The scope of application is described in the next section.

5.2 Basic assumptions and the scope of application

Similar to the discrete-time model in Chapter 4, the continuous-time model in this chapter is a two-phase compound option model. On the basis of the traditional continuous-time real option frameworks (Dixit & Pindyck, 1994; Quigg, 1993; Williams, 1991), the unit market price $S(t)$ and the unit construction cost $K(t)$ follow the geometric Brownian motion during the entire compound option period.

The maximum exercise period in both phases are assumed to be infinite. The market demand is assumed not to be influenced by the demolition of the old property and the rebuilding of the new one. In the following derivation, the length of time of demolishing the old building and rebuilding the new one is assumed to be zero. To relax this assumption, some adjustments are added to the results. For example, the length of time of demolishing the old building and rebuilding the new one is defined as T_a and T_b, respectively. The revenue from the new property is delayed for a period of $(T_a + T_b)$, and the construction of this new property is delayed for a period of T_b compared with the optimal exercise strategy when $T_a = T_b = 0$. Chen and Lai (2013) discussed the relationship between the lengths of T_a and T_b and the optimal exercise timing and thus will no longer be elaborated in the present work.

Assume the unit market price $S(t)$ is a stochastic variable, which has a constant and risk-neutral growth rate (or drift rate) ν_S and a constant variance σ_S. Then, the geometric Brownian motion is expressed as

$$dS(t) = \nu_S S dt + \sigma_S S dZ_S,$$

where Z_S is the Wiener process, which satisfies $\mathrm{E}(\mathrm{d}Z_S) = 0$ and $\mathrm{Var}(\mathrm{d}Z_S) = \mathrm{dt}$. Similarly, the unit construction cost $K(t)$ follows a geometric Brownian motion, as shown as follows:

$$dK(t) = \nu_K K dt + \sigma_K K dZ_K,$$

where Z_K is the Wiener process, which satisfies $\mathrm{E}(\mathrm{d}Z_K) = 0$ and $\mathrm{Var}(\mathrm{d}Z_K) = \mathrm{dt}$.

The correlation coefficient between Z_S and Z_K is ρ. $\mathrm{Cov}(Z_S, Z_K) = \rho\sigma_S\sigma_K$.

The risk-adjusted capital return rate is defined as r $(r > 0)$. $\nu_S < r$ and $\nu_K < r$; otherwise, the option value diverges to infinity.

The annual increase in average building age in the neighbourhood is defined as z $(0 \leq \mathrm{z} \leq 1)$. The constant annual depreciation rate is ξ $(0 < e^{\xi} \leq 1$, or $\xi \leq 0)$. On the basis of the assumption in Section 3.3.3,

$$\ln\left(\frac{S(t)^{g+z}}{S(t)^{g}}\right) = \mathrm{z}\xi, \text{ for any integer t, } \mathrm{g} > 0, \mathrm{z} \geq 0.$$

Given the depreciation effect, the convergence of option value will be even slower than the traditional model. The convergence condition is $r - v_S + \xi > 0$.

The solution procedures in this chapter are based on an infinite option period. In Chapter 4, we have discussed the case where the demolition and rebuilding phases are finite. This binomial tree model is appropriate if the correlation between market price and construction cost can be ignored, or the construction cost is not a stochastic process. This chapter instead focuses on the case where both phases are infinite. Many multi-owner buildings are built on leased land; however, this period assumption is still applicable to the following situations.

Firstly, the land lease of the old property can be extended automatically or by paying an extension fee, which is sufficiently small to encourage the homeowners to continue to live in the old property. This situation is not rare in many countries, including Mainland China, where the government is the landowner. The demolition phase is not bounded by the length of present land lease contract. The developers can wait for an optimal timing in an infinite maturity.

Secondly, the rebuilding of a new property generally requires the developers to sign a new land lease. The redevelopment project usually increases the land use density or changes the residential use into residential–commercial mixed use. Both cases require a new agreement between the developers and landowner. The new land lease period will start only if it is agreed between both parties. In some cases, if no new lease is required within a fixed period after the demolition, then the rebuilding phase can be viewed as infinite.

Thirdly, in some lease contracts, only the rebuilding phase has a finite period. This requirement indicates that the landowner does not want the land to be vacant for a long time. If the theoretical optimal exercise strategy of the two-phase compound option is starting the rebuilding as soon as the demolition is completed, then the length of the rebuilding phase is unimportant. The developers can wait for the best timing to demolish the old building and then rebuild the new one immediately. Chen and Lai (2013) proved that this optimal exercise strategy is applicable to the case when depreciation effect does not exist. If we prove that the optimal exercise strategy in our new model is also the same strategy, then we can apply this strategy even when the rebuilding phase is finite.

However, the discrete- and continuous-time models still do not cover all possible situations. If the construction cost is a stochastic process and the correlation between market price and construction cost is significantly away from zero, then the discrete-time model is inapplicable. If the landowner requires the developers to demolish the old property and rebuild a new property within a finite period, then the infinite period assumption for both phases in the continuous-time model is inapplicable. The solution to this finite-time two-phase redevelopment option model will be discussed in Chapter 6 when the basket option is introduced.

Except for the specific case above, the two models cover the rest of the situations. They provide a comprehensive discussion on the influence of depreciation effect on option value and optimal exercise timing in the majority of cases.

5.3 Derivation of solutions to optimal exercise timing

In the rebuilding phase, the adjusted market price for the new property to be built is

$$S(t) * L_2 * e^{-T_{ave}\xi} * e^{-\xi z t},$$

where L_2 is the plot ratio of the new property to be built, and T_{ave} is the average building age at the beginning of the compound option. This form is similar to the adjusted market price of the new property in Chapter 4.

If the rebuilding option is exercised at time t, the option value is

(5.1) $$V_2\big(S(t), K(t)\big) = S(t) * L_2 * e^{-T_{ave}\xi} * e^{-\xi \mathrm{zt}} - K(t) * L_2.$$

Define $S_2(t) = S(t) * e^{-\xi \mathrm{zt}}$. Then,

$$dS_2(t) = d\big[S(t) * e^{-\xi \mathrm{zt}}\big]$$

$$= e^{-\xi \mathrm{zt}} dS(t) - S(t)\xi \mathrm{z} e^{-\xi \mathrm{zt}} dt$$

$$= e^{-\xi \mathrm{zt}}(\nu_S S(t)dt + \sigma_S S(t) dZ_S) - S(t)\xi \mathrm{z} e^{-\xi \mathrm{zt}} dt$$

$$= S_2(t)[\nu_S - \xi \mathrm{z}]\mathrm{dt} + S_2(t)\sigma_S dZ_S.$$

Therefore, $S_2(t)$ follows a geometric Brownian motion, with a constant and risk-neutral growth rate $(\nu_S - \xi \mathrm{z})$ and a constant variance σ_S. The remaining part $L_2 * e^{-T_{ave}\xi}$ is a constant.

The value of rebuilding option $V_2\big(S(t), K(t)\big)$ is obtained from the partial differential equation as follows:

(5.2) $$\frac{1}{2}\left[\sigma_S^2 S_2^2 \frac{\partial^2 V_2}{\partial S_2^2} + 2\rho\sigma_S\sigma_K S_2 K \frac{\partial^2 V_2}{\partial S_2 \partial K} + \sigma_K^2 K^2 \frac{\partial^2 V_2}{\partial K^2}\right] + S_2[\nu_S - \xi \mathrm{z}]\frac{\partial V_2}{\partial S_2} + K\nu_K \frac{\partial V_2}{\partial K} - rV_2 = 0.$$

Define the critical boundary of V_2 at the optimal exercise timing as $V_2(S_2^*, K^*)$. Then, Equation (5.1) becomes

$$V_2(S_2^*, K^*) = L_2 * e^{-T_{ave}\xi} S_2^* - L_2 K^*,$$

which is the value-matching condition. The smooth-pasting conditions are

$$\frac{\partial V_2}{\partial S_2}(S_2^*) = L_2 * e^{-T_{ave}\xi};$$

$$\frac{\partial V_2}{\partial K}(K^*) = -L_2.$$

To solve the partial differential equation with two stochastic processes, the usual method is to introduce a new stochastic variable that is the ratio of the two processes (Dixit & Pindyck, 1994; McDonald & Siegel, 1986; Williams, 1991).

Define $\varphi_2 = \frac{S_2}{K}$. Then, the time-depending depreciation term is embedded in S_2. Let

$$f_2(\varphi_2) = V_2(S_2, K)/K;$$

$$d\varphi_2 = (\nu_S - \xi z - \nu_K + \sigma_K^2 - \rho\sigma_S\sigma_K)\varphi_2 dt + \sqrt{\sigma_S^2 + \sigma_K^2 - 2\rho\sigma_S\sigma_K}\varphi_2 dZ.$$

Then,

(5.3) $$f_2(\varphi_2^*) = \frac{V_2(S_2^*, K^*)}{K^*} = L_2 * e^{-T_{ave}\xi}\varphi_2^* - L_2;$$

(5.4) $$f_2(0) = 0.$$

The smooth-pasting condition requires the following:

(5.5) $$f_2'(\varphi_2^*) = L_2 * e^{-T_{ave}\xi};$$

(5.6) $$f_2(\varphi_2^*) - \varphi_2^* f_2'(\varphi_2^*) = -L_2.$$

The stochastic equation (Equation (5.2)) becomes

(5.7) $$\frac{1}{2}(\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2)\varphi_2^2 f_2''(\varphi_2) + [\nu_S - \xi z - \nu_K]\varphi_2 f_2'(\varphi_2) - (r - \nu_K)f_2(\varphi_2) = 0.$$

The general solution to this second-order differential equation (Equation (5.7)) is

$$f_2(\varphi_2) = C_2\varphi_2^\lambda.$$

Combining the value-matching condition (Equation (5.3)), boundary condition (Equation (5.4)) and smooth-pasting conditions (Equations (5.5) and (5.6)) yields

$$f_2(\varphi_2) = \begin{cases} C_2\varphi_2^\lambda & , if\ \varphi_2 \le \varphi_2^* \\ L_2 * e^{-T_{ave}\xi}\varphi_2^* - L_2 & , if\ \varphi_2 > \varphi_2^* \end{cases},$$

where

$$C_2 = \frac{L_2 * e^{-T_{ave}\xi}\varphi_2^* - L_2}{\varphi_2^{*\lambda}}.$$

The optimal rebuilding ratio is

(5.8) $$\varphi_2^* = \frac{\lambda}{\lambda - 1}e^{T_{ave}\xi},$$

where the rebuilding option elasticity λ is

$$\lambda = \left[\frac{1}{2} - \frac{\nu_S - \xi z - \nu_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}\right] + \sqrt{\left[\frac{1}{2} - \frac{\nu_S - \xi z - \nu_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}\right]^2 + \frac{2(r - \nu_K)}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}}.$$

The condition $\lambda > 1$ must hold. Otherwise, the rebuilding option is never exercised.

In the demolition phase, $f_2(\varphi_2)$ and φ_2^* are treated as known parameters. Define the building age of old property at the beginning of compound option as T_{old}. Then, the adjusted market value of the old property at time t is

$$S(t) * L_1 * e^{(T_{old} - T_{ave})\xi} * e^{\xi(1-z)t},$$

where L_1 is the plot ratio of the old property. This expression is similar to the adjusted market value of the old property in Chapter 4.

Suppose the demolition option is exercised at time t, the option value is

$$(5.9)\, V_1\big(S(t), K(t)\big) = Max[V_2\big(S(t), K(t)\big)] - S(t) * L_1 * e^{(T_{old} - T_{ave})\xi} * e^{\xi(1-z)t} - K(t) * L_1 * DEM,$$

where DEM represents a constant ratio of the demolition cost to the construction cost for the same building area in the same period.

Similar to Equation (5.2), the value of $V_1\big(S(t), K(t)\big)$ should satisfy the following partial differential equation:

$$(5.10) \quad \frac{1}{2}\left[\sigma_S^2 S_1^2 \frac{\partial^2 V_1}{\partial S_1^2} + 2\rho\sigma_S\sigma_K S_1 K \frac{\partial^2 V_1}{\partial S_1 \partial K} + \sigma_K^2 K^2 \frac{\partial^2 V_1}{\partial K^2}\right] + S_1[\nu_S + \xi(1 - z)]\frac{\partial V_1}{\partial S_1} + K\nu_K \frac{\partial V_1}{\partial K} - rV_1 = 0,$$

where S_1 is defined as $S(t) * e^{\xi(1-y)t}$. $S_1(t)$ also follows a geometric Brownian motion, with a constant and risk-neutral growth rate $[\nu_S + \xi(1 - z)]$ and a constant variance σ_S.

Define the critical boundary of V_1 at the optimal exercise timing as $V_1(S_1^*, K^*)$. Then, Equation (5.9) becomes

$$V_1(S_1^*, K^*) = V_2(S_2^*, K^*) - L_1 * e^{(T_{old}-T_{ave})\xi} S_1^* - L_1 * DEM * K^*.$$

Define $\varphi_1 = \frac{S_1}{K}$. Let

$$f_1(\varphi_1) = V_1(S_1, K)/K;$$

$$d\varphi_1 = (\nu_S + \xi(1-\mathrm{z}) - \nu_K + \sigma_K^2 - \rho\sigma_S\sigma_K)\varphi_1 dt + \sqrt{\sigma_S^2 + \sigma_K^2 - 2\rho\sigma_S\sigma_K}\varphi_1 dZ.$$

Then, the value-matching condition becomes

(5.11) $$f_1(\varphi_1^*) = \frac{V_1(S_1^*, K^*)}{K^*} = f_2(\varphi_2(\varphi_1^*)) - L_1 * e^{(T_{old}-T_{ave})\xi}\varphi_1^* - L_1 * DEM;$$

(5.12) $$f_1(0) = 0.$$

φ_2 is a linear function of φ_1 because

$$\frac{\varphi_1(t)}{\varphi_2(t)} = \frac{S_1(t)}{S_2(t)} = \frac{e^{\xi(1-\mathrm{z})\mathrm{t}}}{e^{-\xi\mathrm{zt}}} = e^{\xi\mathrm{t}} \leq 1.$$

The smooth-pasting condition includes

(5.13) $$f_1'(\varphi_1^*) = f_2'(\varphi_2(\varphi_1^*)) - L_1 * e^{(T_{old}-T_{ave})\xi};$$

(5.14) $$f_1(\varphi_1^*) - \varphi_1^* f_1'(\varphi_1^*) = -L_1.$$

From the definition of φ_1, the stochastic equation (Equation (5.9)) becomes

(5.15) $$\frac{1}{2}(\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2)\varphi_1^2 f_1''(\varphi_1) + [\nu_S + \xi(1-\mathrm{y}) - \nu_K]\varphi_1 f_1'(\varphi_1) - (r - \nu_K) f_1(\varphi_1) = 0.$$

The general solution to this second-order differential equation (Equation (5.15)) is

$$f_1(\varphi_1) = C_1\varphi_1^{\theta}.$$

To derive the formulas for C_1 and θ, the comparison between two critical values φ_1^* and φ_2^* is necessary. The relationship between the two values will determine the values of $f_2(\varphi_1^* e^{-\xi\mathrm{t}})$ and φ_1^*. Three possible cases are listed to avoid missing any possible solutions.

Case 1. Assume $\varphi_1^* < \varphi_2^*$, and $\varphi_1^* e^{-\xi t} < \varphi_2^*$.

Equation (5.11) becomes

$$C_1 \varphi_1^{*\theta} = C_2(\varphi_1^* e^{-\xi t})^\lambda - L_1 * e^{(T_{old}-T_{ave})\xi} \varphi_1^* - L_1 * DEM. \tag{5.16}$$

Equation (5.13) becomes

$$\theta C_1 \varphi_1^{*\theta-1} = C_2 \lambda (\varphi_1^*)^{\lambda-1} (e^{-\xi t})^\lambda - L_1 * e^{(T_{old}-T_{ave})\xi}. \tag{5.17}$$

Thus,

$$C_1 = \frac{C_2(\varphi_1^* e^{-\xi t})^\lambda - L_1 * e^{(T_{old}-T_{ave})\xi} \varphi_1^* - L_1 * DEM}{\varphi_1^{*\theta}}.$$

Consider $(5.17) * \varphi_1^* - (5.16) * \theta$.

$$(\theta - \lambda) C_2 (\varphi_1^* e^{-\xi t})^\lambda = (\theta - 1) L_1 * e^{(T_{old}-T_{ave})\xi} \varphi_1^* + \theta L_1 * DEM \tag{5.18}$$

φ_1^*, as a function of *t*, satisfies the above equation.

In Equation (5.18),

$$\theta = \left[\frac{1}{2} - \frac{\nu_S + \xi(1-z) - \nu_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}\right] + \sqrt{\left[\frac{1}{2} - \frac{\nu_S + \xi(1-z) - \nu_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}\right]^2 + \frac{2(r - \nu_K)}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}},$$

where $\theta > 1$. Otherwise, the option value will diverge.

Case 2. Assume $\varphi_1^* < \varphi_2^* \leq \varphi_1^* e^{-\xi t}$.

Equation (5.11) becomes

$$C_1 \varphi_1^{*\theta} = L_2 * e^{-T_{ave}\xi} \varphi_1^* e^{-\xi t} - L_2 - L_1 * e^{(T_{old}-T_{ave})\xi} \varphi_1^* - L_1 * DEM. \tag{5.19}$$

Equation (13) becomes

(5.20) $$\theta C_1 \varphi_1^{*\theta-1} = L_2 * e^{-T_{ave}\xi} e^{-\xi t} - L_1 * e^{(T_{old}-T_{ave})\xi}.$$

Combining the value-matching condition (Equation (5.11)), boundary condition (Equation (5.12)) and smooth-pasting conditions (Equations (5.13) and (5.14)) yields

$$f_1(\varphi_1) = \begin{cases} C_1 \varphi_1^{\theta} & , if\ \varphi_1 \leq \varphi_1^* \\ L_2 * e^{-T_{ave}\xi} e^{-\xi t} \varphi_1 - L_2 - L_1 * e^{(T_{old}-T_{ave})\xi} \varphi_1 - L_1 * DEM, & if\ \varphi_1 > \varphi_1^* \end{cases},$$

where

$$C_1 = \frac{L_2 * e^{-T_{ave}\xi} e^{-\xi t} \varphi_1^* - L_2 - L_1 * e^{(T_{old}-T_{ave})\xi} \varphi_1^* - L_1 * DEM}{\varphi_1^{*\theta}}.$$

The optimal ratio is

(5.21) $$\varphi_1^* = \frac{\theta}{\theta-1} * \frac{L_2 + L_1 * DEM}{L_2 e^{-T_{ave}\xi} e^{-\xi t} - L_1 e^{(T_{old}-T_{ave})\xi}};$$

$$\theta = \left[\frac{1}{2} - \frac{v_S + \xi(1-z) - v_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}\right] + \sqrt{\left[\frac{1}{2} - \frac{v_S + \xi(1-z) - v_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}\right]^2 + \frac{2(r - v_K)}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}}.$$

The optimal exercise ratio $\frac{S(t) * e^{\xi(1-y)t}}{K(t)}$ is also a function of t.

$e^{-\xi} \geq 1$ indicates that φ_1^* is a decreasing function of *t*. The upper bound value is achieved if $t \to 0$.

$$\varphi_1^* < \frac{\theta}{\theta-1} * \frac{L_2 + L_1 * DEM}{L_2 e^{-T_{ave}\xi} - L_1 e^{(T_{old}-T_{ave})\xi}}$$

The lower-bound value is 0 if $t \to \infty$.

Case 3. Assume $\varphi_1^* \geq \varphi_2^*$.

The value-matching and smooth-pasting conditions are still Equations (5.19) and (5.20), respectively. The solution is the same as Case 2 when $\varphi_1^* < \varphi_2^* \leq \varphi_1^* e^{-\xi t}$.

A further mathematical discussion on φ_1^* and φ_2^* is presented in Appendix A.

On the basis of the work by Chen and Lai (2013), the conditions in Case 1 will lead to a sequential exercise strategy. In the following discussions, this strategy is called Type

1 strategy. Meanwhile, the conditions in Cases 2 and 3 will lead to a simultaneous exercise strategy, called Type 2 strategy. In both strategies, φ_1^* is a function of *t*, which indicates that the solution of φ_1^* may not exist in some time intervals for some combinations of parameters. In those intervals, the optimal exercise strategies do not exist either in Type 1 or 2 strategy.

In this model, the appreciation of property value resulting from urban expansion and/or the increasing housing demand is measured in the term of ν_S in the geometric Brownian motion of $S(t)$. The formula of $S(t)$ also assumes that the appreciation of property value is based on the total property value (i.e. building structure value and land value). On the basis of the implication of stochastic differential equation and the explanation by Bokhari and Geltner (2016), the land value appreciation rate will not be estimated separately.

5.4 Influences of different factors on optimal exercise ratio

To investigate the potential factors on the optimal exercise strategy, $\varphi_1^*(t)$ is rewritten as:

$$\frac{S(t)}{K(t)} e^{\xi(1-z)t} = \frac{\theta}{\theta-1} * \frac{L_2+L_1*DEM}{L_2 e^{-T_{ave}\xi} e^{-\xi t} - L_1 e^{(T_{old}-T_{ave})\xi}}.$$

S(0) and K(0) are known and fixed. Define $\varphi_3(t) = \frac{S(t)}{K(t)}$. Then, $\varphi_3^*(t)$ is the optimal exercise strategy for

(5.22) $$\varphi_3(t) = \frac{\theta}{\theta-1} * \frac{L_2+L_1*DEM}{L_2 e^{-T_{ave}\xi} e^{-\xi t} - L_1 e^{(T_{old}-T_{ave})\xi}} * e^{-\xi(1-z)t},$$

where $\theta = \left[\frac{1}{2} - \frac{\nu_S+\xi(1-z)-\nu_K}{\sigma_S^2-2\rho\sigma_S\sigma_K+\sigma_K^2}\right] + \sqrt{\left[\frac{1}{2} - \frac{\nu_S+\xi(1-z)-\nu_K}{\sigma_S^2-2\rho\sigma_S\sigma_K+\sigma_K^2}\right]^2 + \frac{2(r-\nu_K)}{\sigma_S^2-2\rho\sigma_S\sigma_K+\sigma_K^2}}.$

The optimal strategies of $\varphi_1^*(t)$ and $\varphi_3^*(t)$ are equivalent.

On the basis of Equation (5.22), the potential factors on $\varphi_3^*(t)$ have the following properties.

Property 1. The value of $\frac{\theta}{\theta-1}$ increases with the decrease in the value of $(\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2)$, which results in a high trigger value $\varphi_3^*(t)$.

Property 2. The capital return rate r, which only appears in the formula of θ, is negatively related to $\varphi_3^*(t)$.

Property 3. The value of $\frac{\theta}{\theta-1}$ and the trigger value $\varphi_3^*(t)$ decrease with the increase of the correlation coefficient ρ.

Property 4. A strong depreciation effect (ξ decreases) indicates a large θ but small $\frac{\theta}{\theta-1}$. $\frac{L_2+L_1*DEM}{L_2e^{-T_{ave}\xi}e^{-\xi t}-L_1e^{(T_{old}-T_{ave})\xi}} * e^{-\xi(1-z)t}$ is also small. As a result, a small $\varphi_3^*(t)$ is derived.

Property 5. $\frac{\theta}{\theta-1}$ increases but $\frac{L_2+L_1*DEM}{L_2e^{-T_{ave}\xi}e^{-\xi t}-L_1e^{(T_{old}-T_{ave})\xi}} * e^{-\xi(1-z)t}$ decreases with the rise of the annual increase in average building age z. The effect of z on $\varphi_3^*(t)$ is not a monotonic trend.

5.5 Influential factors on expected exercise time

Monte Carlo simulation is adopted to discuss the potential influences of the model parameters on the option expected exercise time. Table 5.1 presents the baseline values of these parameters. When one or two specific parameters are investigated, the remaining are set as the baseline values. These baseline values are adopted as the same values for the parameters in Chen and Lai's study (2013) to examine the influence of depreciation effect. The property market and construction conditions are described by these parameters.

Parameters	Baseline values
Drift rate of market price (ν_S)	4%
Drift rate of construction cost (ν_K)	2%
Volatility of market price (σ_S)	10%
Volatility of construction cost (σ_K)	10%
Correlation coefficient between S and K (ρ)	0
Depreciation rate (ξ)	Log(0.99) (1% annual depreciation rate)
Annual expected capital return rate (r)	10%
Annual increase in average building age (y)	0.5
Average building age in the same region at the beginning of the option (T_{ave})	30
Building age of the targeted property at the beginning of the option (T_{old})	50
Plot ratio of the new property to be built (L_2)	9.0*
Plot ratio of the old property to be demolished (L_1)	5.2*
Initial ratio of market price to construction cost [S(0)/K(0)]	1.5
Ratio of demolition cost to construction cost (DEM)	0.03

Table 5.1 Baseline for different factors in Chapter 5

**Arcadis Annual Construction Cost Handbook (Arcadis, 2016). In Hong Kong, when a Class B site building height is below 30 m (but over 27 m), the maximum domestic plot ratio is 5.2. When the Class B site building is over 61 m, the maximum domestic plot ratio is 9.0. 'Class B Site means a corner site that borders on two specified streets, neither of which is less than 4.5 m wide, as defined under B(P)R' (Planning Department, 2016).*

Although the option period is infinite, the time-dependent optimal exercise ratio makes generating the expected exercise timing for an actual infinite period as usual cases difficult (Chen & Lai, 2013; Øksendal, 2003). The simulations instead operate on a sufficiently long period of time, which is defined as 100 years in this study. In each minimum time interval in Monte Carlo simulations, variable φ_1 is compared with the optimal exercise ratio φ_1^*. Even when $\varphi_1 > \varphi_1^*$, the compound option value must be positive.

Three statistics, namely, conditional expected exercise time, percentage of exercised paths and censored expected exercise time, are introduced to demonstrate the exercise time within this long period of time comprehensively. Only the simulated paths in which the compound option is exercised within 100 years are included in the calculation of conditional expected exercise time. The percentage of exercised paths within 100 years measures the coverage of this conditional expected exercise time. The censored expected exercise time also considers the unexercised paths within this period of time. The exercise time of these unexercised paths are recorded as (100 + *dt*) years, where *dt* is a minimum time interval. Although the conditional expected exercise time < the censored expected exercise time < the actual expected exercise time, the two statistics still provide important information about the expected exercise time.

The major findings about the influences of different factors are as follows.

(1) Both types of expected exercise time will increase with the market price volatility

(Table 5.2). However, if the market price volatility is extremely high (20% in Table 5.2), both types of expected exercise time will increase as the construction volatility increases and then declines.

Parameters	$\sigma_S = 0.02$	$\sigma_S = 0.05$	$\sigma_S = 0.10$	$\sigma_S = 0.15$	$\sigma_S = 0.20$
	<0.01	0.7848	5.3663	10.2758	13.3490
$\sigma_K = 0.02$	100%	100%	99.85%	97.49%	90.99%
	<0.01	0.7848	5.5045	12.5281	21.1607
	0.7347	2.0353	5.9823	10.4824	13.4440
$\sigma_K = 0.05$	100%	100%	99.81%	97.44%	91.52%
	0.7347	2.0353	6.1610	12.7744	20.7813
	3.9466	4.7722	7.5086	10.9764	13.3941
$\sigma_K = 0.10$	99.99%	99.98%	99.63%	97.65%	92.52%
	3.9562	4.7950	7.8527	13.0687	19.8712
	6.3045	6.8667	8.6346	11.1341	13.3716
$\sigma_K = 0.15$	99.92%	99.88%	99.48%	98.10%	94.15%
	6.3776	6.9748	9.1079	12.8227	18.4382
	7.6878	8.0791	9.2796	11.2258	13.1954
$\sigma_K = 0.20$	99.86%	99.74%	99.43%	98.35%	95.75%
	7.8189	8.3218	9.7968	12.6907	16.8833

Table 5.2 Conditional expected exercise time (in years), percentage of exercised paths (in %) and censored expected exercise time (in years) with market price volatility (σ_S) and construction cost volatility (σ_K). The other parameters are set as the baseline values.

Note: All paths are exercised in Type 2 strategy in all scenarios. Type 2 strategy is the optimal choice for all t in all scenarios.

This interesting phenomenon can be explained as the different effects of market price volatility, construction cost volatility and depreciation. Φksendal's optimal stopping time theory (Øksendal, 2003) states that if the depreciation effect does not exist, the expected exercise time should be positively related to market price volatility and positively related to $(\sigma_S^2 - \sigma_K^2)$. The latter relationship means that a high construction cost volatility also accelerates the demolition process. Meanwhile, the depreciation effect decreases the time-dependent optimal exercise ratio as time increases and accelerates the decision to start the project. The influence of depreciation effect is stronger to the right side of Table 5.2 because the expected exercise time is larger in the right side. These different effects lead to an inconsistent relationship between the construction cost volatility and expected exercise time.

(2) A high capital return rate increases the opportunity cost to postpone the redevelopment and hence reduces the expected waiting time for redevelopment (Table 5.3). A small capital return rate which is extremely close to the market price drift rate will even impede the exercise of the option.

(3) The correlation coefficient measures the possibility when the market price and construction cost move in the same direction. A high coefficient decreases the uncertainty between both factors, thereby reducing the expected waiting time (Table 5.3).

The two phenomena can be explained by Properties 2 and 3 in Section 5.4 earlier.

Parameters	$r = 0.06$	$r = 0.08$	$r = 0.10$	$r = 0.12$	$r = 0.14$
$\rho = -0.5$	1.0821 0.04% 99.9724	11.8885 97.06% 14.4793	9.4412 98.90% 10.4357	7.1519 99.33% 7.7778	5.5680 99.50% 6.0421

$\rho = -0.25$	0.8200 0.01% >100	11.2681 97.79% 13.2311	8.5146 99.39% 9.0726	6.2338 99.51% 6.6933	4.8270 99.70% 5.1088
$\rho = 0$	>100 0.00% >100 *	10.4196 98.35% 11.8942	7.5086 99.63% 7.8527	5.3828 99.80% 5.5702	3.9009 99.84% 4.0527
$\rho = 0.25$	>100 0.00% >100 *	9.1169 99.20% 9.8440	6.1279 99.84% 6.2819	4.0710 99.93% 4.1404	2.6830 99.97% 2.7161
$\rho = 0.5$	>100 0.00% >100 *	7.5814 99.73% 7.8310	4.4243 99.99% 4.4376	2.4177 99.98% 2.4392	1.1335 100% 1.1354

Table 5.3 Conditional expected exercise time (in years), percentage of exercised paths (in %) and censored expected exercise time (in years) with various capital return rates (r) and correlation coefficients (ρ). The other parameters are set as the baseline values.

Note: Except for three unexercised scenarios with '', all paths are exercised in Type 2 strategy because Type 1 strategy is never the optimal choice for any t when* $r = 0.06$. *Type 2 strategy is the optimal choice for all t in the remaining scenarios.*

(4) The depreciation rate is the most influential factor in the redevelopment option model (Table 5.4). A high depreciation rate remarkably shortens the expected waiting time. As the depreciation rate increases, the future optimal exercise ratio declines faster over time.

Parameters	$\xi = \log(0.98)$ 2% p.a.	$\xi = \log(0.985)$ 1.5% p.a.	$\xi = \log(0.99)$ 1% p.a.	$\xi = \log(0.995)$ 0.5% p.a.	$\xi = \log(1)$ $= 0$
$z = 0$	<0.01 100.00% <0.01	<0.01 100.00% <0.01	6.6467 99.32% 7.2816	19.8786 96.99% 22.2921	36.5919 87.94% 44.2414
$z = 0.25$	<0.01 100.00% <0.01	<0.01 100.00% <0.01	7.0696 99.50% 7.5380	19.8957 97.45% 21.9354	36.8157 87.95% 44.4281
$z = 0.5$	39.8202 99.92% 39.8708 *	<0.01 100.00% <0.01	7.4919 99.62% 7.8454	19.7384 97.56% 21.6954	36.7821 87.86% 44.4554
$z = 0.75$	53.1990 99.91% 53.2402 *	0.0243 99.99% 0.0303	7.9559 99.76% 8.1408	19.6025 98.03% 21.1882	36.5239 87.93% 44.1879
$z = 1$	>100 0.00% >100	0.5980 99.59% 1.0017	8.3386 99.82% 8.5054	19.6489 98.25% 21.0537	36.6858 87.89% 44.3544

Table 5.4 Conditional expected exercise time (in years), percentage of exercised paths (in %) and censored expected exercise time (in years) with various depreciation rates (ξ) and annual increase in average building ages (z). The other parameters are set as the baseline values.

Note: The two scenarios with '' cannot be exercised in Type 2 strategy. The expected exercise time is obtained from Type 1 strategy, which represents sequential exercise. The option is never exercised when* $\xi = log(0.98)$ *and* $z = 1$. *The rest of the scenarios are all exercised in Type 2 strategy, which represents simultaneous exercise.*

In extreme scenarios where the depreciation rate and annual increase in average building age are high, the option cannot be optimally exercised in Type 2 strategy. However, a time interval generally exists in which the option can be exercised in Type 2 strategy. If the depreciation rate is zero, then this interval equals the entire option period. However, the depreciation effect limits the length of time interval for Type 2 strategy. In the next section, the available time intervals for Types 1 and 2 strategies will be derived and discussed in a high depreciation environment.

5.6 Available time intervals for different exercise strategies

This section focuses on a special issue that only exists in the depreciation-adjusted model. In Equation (18), if the depreciation rate is 0, then $\theta = \lambda$. No solution of φ_1^* satisfies the following:

$$(\theta - \lambda)C_2(\varphi_1^* e^{-\xi t})^{\lambda} = (\theta - 1)L_1 * e^{(T_{old} - T_{ave})\xi}\varphi_1^* + \theta L_1 * DEM.$$

If no depreciation effect exists, the compound option should never be exercised in Type 1 strategy. The optimal exercise strategy must be the Type 2 strategy, which is consistent with the work by Chen and Lai (2013). In addition, when the depreciation rate is zero, φ_1^* and φ_2^* are independent of *t*. If $\varphi_1^* > \varphi_2^*$ when $t = 0$, then $\varphi_1^* > \varphi_2^*$ holds for all *t*.

The depreciation effect changes the above two conditions. Firstly, Type 1 strategy is feasible for some specific scenarios. Secondly, the relationship between $\varphi_1^* e^{-\xi t}$ and φ_2^* depends on t. When Type 2 strategy is feasible, $\varphi_1^* e^{-\xi t}$ is a decreasing function of *t*. When Type 1 strategy is feasible, $\varphi_1^* e^{-\xi t}$ is initially a decreasing function of *t* and then becomes an increasing function of *t*. However, the turning point is usually extremely large, which makes $\varphi_1^* e^{-\xi t}$ appear as a monotone decreasing function in a long period of time. Given the two conditions, whether the option can be exercised sequentially and whether the simultaneous exercise strategy is better than the sequential one at all times must be confirmed.

The simulation results in Section 5.5 show that Type 2 strategy is the best choice for all t in most scenarios. The available time intervals for both strategies change because of the values of three parameters, namely, capital return rate, depreciation rate and annual increase in average building age. To further demonstrate the effects of the three factors on the available time intervals, 45 scenarios (5*3*3) are discussed in Table 5.5.

		$\xi = \log(0.98)$ 2% p.a.	$\xi = \log(0.985)$ 1.5% p.a.	$\xi = \log(0.99)$ 1% p.a.
r = 0.06	y = 0	Type 1: [0, 100] Type 2: Null [b]	Type 1: [18.72, 100] Type 2: Null [b]	Type 1: Null Type 2: [0, 25.98]
	y = 0.5	Type 1: [0, 100] Type 2: Null [b]	Type 1: [0, 100] Type 2: Null [b]	Type 1: Null Type 2: [0, 0.41]
	y = 1	$\lambda < 1$	Type 1: [0, 100] Type 2: Null [b]	Type 1: [0, 100] Type 2: Null
r = 0.07	y = 0	Type 1: [2.44, 100] Type 2: Null [b]	Type 1: Null Type 2: [0, 4.83]	Type 1: Null Type 2: [0, 61.83]
	y = 0.5	Type 1: [0, 100] Type 2: Null [b]	Type 1: Null Type 2: Null [a]	Type 1: Null Type 2: [0, 41.05]
	y = 1	Type 1: [0, 100] Type 2: Null [b]	Type 1: [0, 100] Type 2: Null [b]	Type 1: Null Type 2: [0, 19.45]
r = 0.08	y = 0	Type 1: [19.34, 100] Type 2: Null [b]	Type 1: [90.16, 100] Type 2: [0, 21.75]	Type 1: Null Type 2: [0, 90.32]
	y = 0.5	Type 1: [6.39,	Type 1: Null	Type 1: Null

		100] Type 2: Null [b]	Type 2: [0, 5.02]	Type 2: [0, 72.29]
	y = 1	Type 1: [0, 100] Type 2: Null [b]	Type 1: Null Type 2: Null [a]	Type 1: Null Type 2: [0, 54.16]
r = 0.09	y = 0	Type 1: [23.62, 100] Type 2: [0, 3.59]	Type 1: [79.06, 100] Type 2: [0, 35.99]	Type 1: Null Type 2: [0, 100]
	y = 0.5	Type 1: [34.36, 100] Type 2: Null [b]	Type 1: Null Type 2: [0, 20.89]	Type 1: Null Type 2: [0, 97.93]
	y = 1	Type 1: [15.00, 81.61] Type 2: Null [b]	Type 1: Null Type 2: [0, 6.05]	Type 1: Null Type 2: [0, 81.90]
r = 0.10	y = 0	Type 1: [23.83, 100] Type 2: [0, 12.13]	Type 1: [70.45, 100] Type 2: [0, 48.34]	Type 1: Null Type 2: [0, 100]
	y = 0.5	Type 1: [39.42, 100] Type 2: Null [b]	Type 1: Null Type 2: [0, 34.42]	Type 1: Null Type 2: [0, 100]
	y = 1	Type 1: Null Type 2: Null [a]	Type 1: Null Type 2: [0, 20.96]	Type 1: Null Type 2: [0, 100]

Table 5.5 Available time intervals of two types of exercise in 45 selected scenarios

Note: The time periods are the available time intervals when Type 1/2 strategy is feasible.

a. In these scenarios, the option cannot be exercised in either Type 1 or 2 strategy.

b. In these scenarios, the option cannot be exercised in Type 2 strategy.

The option is exercised in Type 2 strategy in the rest of the scenarios.

Several trends in the ranges of available time intervals can be observed in Table 5.5.

(1) Suppose that the capital return rate and annual increase in average building age are constant. The lower bound of available time intervals for Type 1 strategy and the upper bound of available time intervals for Type 2 strategy increase with the decrease in depreciation rate. As a result, the range of available time intervals for Type 1 strategy will be smaller; however, the range of available time intervals for Type 2 strategy will be larger.

(2) Assume capital return rate and depreciation rate as constant. The upper bound of available time intervals for Type 2 strategy decreases with the rise of the annual increase in average building age. The range of available time intervals for Type 2 strategy will be smaller. No consistent trend is observed between the annual increase in average building age and the range of available time intervals for Type 1 strategy.

(3) Suppose that the annual increase in average building age and the depreciation rate are constant. The upper bound of available time intervals for Type 2 strategy increases with the capital return rate. The range of available time intervals for Type 2 strategy will be larger. No consistent trend is observed between the capital return rate and the range of available time intervals for Type 1 strategy.

Table 5.5 reveals that Type 2 strategy may not be feasible when high depreciation rate is high (1.5% p.a. or above) and/or capital return rate is low (10% p.a. or below) during the observation period. For these cases, Type 1 strategy is the alternative choice. However, whether the sequential exercise strategy is the optimal choice in these cases remains uncertain. A new comparison strategy is introduced to determine a superior strategy. In this comparison strategy, the compound option is exercised simultaneously if the inequality

$$\frac{S(t)}{K(t)} * e^{-\xi y t} > \varphi_2^*$$

holds for the first *t*. If the optimal rebuilding ratio in Phase 2 is reached at the first time, the two phases in the compound option are exercised simultaneously. This condition is another simultaneous exercise strategy, although its critical ratio is lower than that in Type 2 strategy.

As shown in Table 5.5, 9 out of 15 scenarios where annual depreciation rate is 1.5% are chosen for the comparison of the performances of the three strategies. For Types 1 and 2 strategies, the numbers of exercised paths within 100,000 simulated paths are recorded, as well as the numbers of paths in which the option values based on the comparison strategy are higher than those in Type 1 (and 2) strategy. The conditional expected option values for all the feasible strategies in each scenario are calculated. Here, the term 'conditional' indicates that this option value is only generated from the paths exercised within 100 years. Otherwise, the option value in this path is viewed as 0. The initial price-to-cost ratio is defined as 1.2 to avoid immediate exercise at time 0. Table 5.6 lists the related statistics. The divisions of sets (I), (II) and (III) are based on the length of the available time intervals for two strategies during [0, 100].

Set (I)			
Scenarios	r = 0.06 y = 0	r = 0.06 y = 0.5	r = 0.07 y = 1
Type 1 boundary	[18.72, 100]	[0, 100]	[0, 100]
# of paths exercised in Type 1 strategy	96,750	98,642	99,439
# of paths exercised in Type 1 strategy (positive option value)	96,265	92,045	96,730

# of paths where the intrinsic option value in the comparison strategy is larger than that in Type 1 strategy	45,961	65,111	59,055
Conditional expected option value in Type 1 strategy	572.8155	670.3719	682.7291
Conditional expected option value in the comparison strategy	544.2152	731.4694	718.9531
Set (II)			
Scenarios	r = 0.07 y = 0	r = 0.09 y = 0.5	r = 0.09 y = 1
Type 2 boundary	[0, 4.83]	[0, 20.89]	[0, 6.05]
# of paths exercised in Type 2 strategy	62,671	93,116	61,275
# of paths exercised in Type 2 strategy (positive option value)	62,671	93,116	61,275
# of paths where the intrinsic option value in the comparison strategy is larger than that in	36,425	6,748	38,589

Type 2 strategy			
Conditional expected option value in Type 2 strategy	370.8524	597.5184	457.3479
Conditional expected option value in the comparison strategy	437.6559	391.3815	479.9780
Set (III)			
Scenarios	r = 0.08 y = 0	r = 0.09 y = 0	r = 0.10 y = 0
Type 1 boundary	[90.16, 100]	[79.06, 100]	[70.45, 100]
# of paths exercised in Type 1 strategy	94,672	96,398	97,546
# of paths exercised in Type 1 strategy (positive option value)	94,534	96,059	97,045
# of paths where the intrinsic option value in the comparison strategy is larger than that in Type 1 strategy	99,110	99,891	99,988
Type 2 boundary	[0, 21.75]	[0, 35.99]	[0, 48.34]
# of paths exercised in Type 2 strategy	92,992	97,292	98,711
# of paths exercised	92,992	97,292	98,711

in Type 2 strategy (positive option value)			
# of paths where the intrinsic option value in the comparison strategy is larger than that in Type 2 strategy	6,710	2,754	1,400
Conditional expected option value in Type 1 strategy	38.8775	26.6141	19.4356
Conditional expected option value in Type 2 strategy	523.7142	503.5282	468.9839
Conditional expected option value in the comparison strategy	364.4314	323.0678	322.8327

Table 5.6 Numbers of exercised paths and conditional expected option value based on different strategies (within 100,000 simulations in each scenario)

Note: The depreciation rate is 1.5% p.a. for all these scenarios.

Set (I) consists of three scenarios when Type 2 strategy is completely unfeasible during the entire observation period. The available time intervals for Type 1 strategy are wide.

Many paths can be exercised positively in the optimal Type 1 strategy. However, a significant percentage of paths (46%–65%) have higher intrinsic values when they are exercised in the comparison strategy. The conditional expected option value in the comparison strategy is also higher than that in Type 1 strategy in two of the three scenarios. This phenomenon suggests that even when Type 2 strategy is unfeasible, the comparison strategy, which is a type of simultaneous exercise strategy, is still usually a superior choice. Type 1 strategy should be adopted only when the capital return rate is extremely low and the annual increase in average building age is extremely small.

Set (II) includes three scenarios when Type 1 strategy is completely unfeasible during the entire observation period. Unfortunately, the available time intervals for Type 2 strategy are narrow. If these time intervals are less than 10 years, 1/3 of the paths will not be exercised in Type 2 strategy. The percentage of paths exercised in the comparison strategy is even smaller. However, the conditional expected option values are larger in the comparison strategy than those in Type 2 strategy in two of the three scenarios. If the percentage of paths exercised in Type 2 strategy is not sufficiently high, then the comparison strategy may still be a superior choice.

Set (III) is an interesting group, where Types 1 and 2 strategies are partially feasible during the observation period. The conditional expected option values in Type 2 strategy are the most valuable. The option value in Type 1 strategy is considerably smaller than that in the comparison strategy. When the available time intervals in Type 2 strategy is sufficiently wide, this strategy is the optimal choice.

The results in Tables 5.5 and 5.6 show that if the depreciation rate is high (1.5% p.a. or above) and/or the capital return rate is low (in this study, 10% p.a. or below), then the optimal exercise strategy should be derived in any of the following ways.

(1) Find the available time intervals in Type 2 strategy. If the compound option can be exercised in Type 2 strategy during the entire observation period, then this strategy is

the optimal one. This case is the most general.

(2) If the compound option can only be exercised in Type 2 strategy in part of the observation period, then the conditional expected option value in Type 2 strategy and that in the comparison strategy should be compared, in which the one larger is the optimal choice.

(3) If the compound option will not be exercised in Type 2 strategy in any of the observation period, the available time intervals in Type 1 strategy should be determined. The conditional expected option value in Type 1 strategy and that in the comparison strategy should be compared, in which the one larger is the optimal choice.

The developers can continuously calculate the conditional expected option value in all three strategies in any scenario and compare them. Type 2 strategy is generally the best choice in most scenarios. Type 1 strategy is only considered optimal in few cases. The comparison strategy, which suggests that the compound option should be simultaneously exercised if the price-to-cost ratio reaches the optimal rebuilding ratio, is an alternative choice when Type 2 strategy is unfeasible during the observation period.

A high depreciation rate, a low capital return rate and a large annual increase in average building age will cause a small range of available time intervals for Type 2 strategy and thus a low probability to choose Type 2 strategy as the optimal. The three factors increase the new property value when it is built in the future. On the basis of Equation (5.21), that is,

$$\varphi_1^* = \frac{\theta}{\theta-1} * \frac{L_2+L_1*DEM}{L_2 e^{-T_{ave}\xi} e^{-\xi t} - L_1 e^{(T_{old}-T_{ave})\xi}}$$

$$\theta = \left[\frac{1}{2} - \frac{\nu_S+\xi(1-z)-\nu_K}{\sigma_S^2-2\rho\sigma_S\sigma_K+\sigma_K^2}\right] + \sqrt{\left[\frac{1}{2} - \frac{\nu_S+\xi(1-z)-\nu_K}{\sigma_S^2-2\rho\sigma_S\sigma_K+\sigma_K^2}\right]^2 + \frac{2(r-\nu_K)}{\sigma_S^2-2\rho\sigma_S\sigma_K+\sigma_K^2}},$$

the optimal rebuilding ratio is

(5.8)
$$\varphi_2^* = \frac{\lambda}{\lambda-1} e^{T_{ave}\xi}$$

$$\lambda = \left[\frac{1}{2} - \frac{\nu_S - \xi z - \nu_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}\right] + \sqrt{\left[\frac{1}{2} - \frac{\nu_S - \xi z - \nu_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}\right]^2 + \frac{2(r-\nu_K)}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}}.$$

Consider the depreciation rate (ξ). When ξ increases, $\varphi_1^*(t)$ decreases considerably faster than φ_2^*. Thus, the optimal demolition ratio declines remarkably faster than the optimal rebuilding ratio, and the condition of $\varphi_2^* \leq \varphi_1^*(t)e^{-\xi t}$ becomes more difficult to hold. As a result, a high depreciation rate leads to a small range of available time interval in Type 2 strategy. Similarly, if the capital return rate or the annual increase in average building age increases, then φ_2^* increases considerably faster than $\varphi_1^*(t)$, and the inequality $\varphi_2^* \leq \varphi_1^*(t)e^{-\xi t}$ may not hold. Volatilities and the correlation coefficient do not have a similar influence. When $\varphi_2^* \leq \varphi_1^*(t)e^{-\xi t}$ is not satisfied, the developers can take the optimal rebuilding ratio φ_2^* as the trigger of the optimal simultaneous exercise strategy.

5.7 Influences of different acquisition standards

Two parameters, namely, average building age in the neighbourhood and the building age of the old property, are not discussed in the previous sections because they are not economic factors. However, if the predetermined acquisition standard is not directly based on the actual building age of the old property, then the importance of the two parameters will increase. In Section 3.3.1, we have described Scenario 4 when the acquisition standard is not based on the actual building age of the old property. A high acquisition standard or considerable compensation to the original residents will reduce the project value. However, the manner in which the expected redevelopment timing will be affected by the acquisition standard must be investigated. This section discusses different acquisition standards, which are based on the relationship between T_{old} and T_{ave}.

On the basis of the formula of $\varphi_1^*(t)$,

(21) $$\varphi_1^* = \frac{\theta}{\theta-1} * \frac{L_2+L_1*DEM}{L_2 e^{-T_{ave}\xi} e^{-\xi t} - L_1 e^{(T_{old}-T_{ave})\xi}}.$$

As $T_{old} \geq 0$, $e^{-T_{ave}\xi} \geq e^{(T_{old}-T_{ave})\xi}$ constantly holds. A smaller T_{old} indicates a longer expected exercise time. Lower depreciation adjustments to the old property value increase the range of variation in the future value of this old property and then increases the expected waiting time for redevelopment. Table 5.7 shows the expected exercise time for four different acquisition standards with the same average building age. Nine scenarios where Type 2 strategy is available during the entire observation period are discussed.

Parameters	(I) $T_{old} = 50 > T_{ave}$			(II) $T_{old} = T_{ave} = 30$		
	ξ = log(0.985)	ξ = log(0.99)	ξ = log(0.995)	ξ = log(0.985)	ξ = log(0.99)	ξ = log(0.995)
z = 0.25	<0.01 100.00% <0.01	7.0696 99.50% 7.5380	19.8957 97.45% 21.9354	1.4615 99.94% 1.5246	10.8156 99.26% 11.4791	22.4704 96.80% 24.9532
z = 0.5	<0.01 100.00% <0.01	7.4919 99.62% 7.8454	19.7384 97.56% 21.6954	2.2420 99.69% 2.5432	11.0533 99.45% 11.5443	22.0273 97.20% 24.2124
z = 0.75	0.0243 99.99% 0.0303	7.9559 99.76% 8.1408	19.6025 98.03% 21.1882	2.9653 99.60% 3.3516	11.2303 99.60% 11.5819	22.0989 97.69% 23.9002

	(III) $0 < T_{old} = 7 < T_{ave}$			(IV) $T_{old} = 0$		
	ξ = log(0.985)	ξ = log(0.99)	ξ = log(0.995)	ξ = log(0.985)	ξ = log(0.99)	ξ = log(0.995)
$z = 0.25$	8.6085 99.66% 8.9157	16.3704 98.81% 17.3624	25.6240 96.32% 28.3645	11.2467 99.60% 11.6053	18.3078 98.56% 19.4827	26.7408 95.97% 29.6907
$z = 0.5$	8.5328 99.59% 8.9060	16.1839 99.18% 16.8729	25.3612 96.69% 27.8350	11.0205 99.56% 11.4120	18.0281 99.02% 18.8283	26.5608 96.47% 29.1536
$z = 0.75$	8.7895 99.62% 9.1380	15.8492 99.34% 16.4080	25.2924 97.09% 27.4666	11.0364 99.65% 11.3460	17.6729 99.29% 18.2592	26.3693 97.06% 28.5328

Table 5.7 Conditional expected exercise time (in years), percentage of exercised paths (in %) and censored expected exercise time (in years) with various depreciation rates and annual increase in average building ages under different acquisition standards. The other parameters are set as the baseline values.

Note: In all the above scenarios, Type 2 strategy is confirmed as the optimal. The difference of expected exercise time is unrelated to the exercise strategy.

Standard (I) is the simplest. It determines the acquisition value of the old property based on its actual age. Standard (II) is adopted in many countries. It requires the developers to purchase the old property based on the average price in the neighbourhood. Standard (III) is suggested by the Hong Kong URA. It requires the URA to purchase the old property based on the price of a 7-year-old building in the same region. This price is usually valued by professional surveyors. Standard (IV) is the highest compensation for the original residents. It requires the old property to be acquired as a new building in the same area/region.

When the annual depreciation rate is 1.5%, the expected exercise time increases from 0 to 11.0 years as T_{old} declines to zero. However, if the depreciation rate is adjusted downwards to only 0.5%, this expected exercise time will increase only 7.0 years, which emphasises the importance of an accurately estimated depreciation rate. The depreciation rate is the only parameter that enlarges the influence of T_{old} on the expected exercise time in Equation (21). Moreover, the changes of z show minimal effects on the waiting time for redevelopment, that is, only 1–2 years.

This study proves that the expected exercise time is more affected by the depreciation rate than the acquisition standard. We compare two cases in Table 5.7. In one case, the acquisition is based on a new property value with a depreciation rate of 1%. In the other case, the acquisition is based on the original building age of the old property (50 years) with a depreciation rate of 1.5%. The waiting time for redevelopment remains longer when the depreciation rate is 1.5%. If the depreciation effect is not strong, raising the acquisition standard does not severely delay the urban renewal projects as expected.

The determination of acquisition standard is not only an economic but also a public issue. A proper acquisition standard can resolve a conflict between the developers (or the government) and original residents and encourage the efficient land use in redevelopment. An actual redevelopment considers the time for negotiation between the developers and residents and the time for resettlement before demolition. A high acquisition standard can decisively reduce the negotiation time and the overall redevelopment project period.

5.8 Chapter summary

This chapter combines the two-phase continuous-time option pricing model for multi-owner properties, with an emphasis on depreciation adjustments. The depreciation effect shows a strong and direct effect on the optimal redevelopment timing because it can decrease the time-dependent optimal exercise ratio. This declining optimal exercise ratio also indirectly influences the relationship between expected exercise timing and volatilities.

This chapter analyses the feasibility of sequential exercise strategy and simultaneous exercise strategy when the conditions of depreciation rate, annual increase in average building age and capital return rate are different. In cases with high depreciation (1.5% p.a. or above) and/or low capital return rate (10% p.a. or below), the traditional optimal exercise strategy (i.e. exercising two-phase option simultaneously if the optimal demolition price-to-cost ratio is reached) is found unfeasible after several years. In this study, if the depreciation rate reaches 2% p.a. and the capital return rate is no more than 8% p.a., this traditional strategy is unfeasible in the entire period. An alternative simultaneous strategy is suggested to achieve superior revenue than the traditional optimal exercise strategy in these cases. This alternative strategy suggests exercising two-phase compound option simultaneously if the optimal rebuilding price-to-cost ratio is reached. When the depreciation rate is higher and the capital return rate is lower (e.g. depreciation rate is 2% p.a. and capital return rate is 7% p.a.), the sequential exercise strategy may be the optimal choice. A new and comprehensive decision rule is suggested to choose the best exercise strategy to cover all possible cases. The analysis of the strategy feasibility is the major contribution in this chapter. Finally, this chapter finds a positive relationship between the acquisition price of old properties and the expected waiting time for redevelopment.

In summary, Chapters 4 and 5 provide a comprehensive discussion on how the depreciation effect influences an entire two-phase (i.e. demolition and rebuilding) redevelopment project. The next chapter will focus on a specific form of construction in the rebuilding phase, namely, the vertical mixed-use development. We will analyse its influences on project value and waiting time for development.

Appendix A. Further discussion on $\boldsymbol{\varphi_1^*}$ and $\boldsymbol{\varphi_2^*}$

Note that

$$\lambda = \left[\frac{1}{2} - \frac{\nu_S - \xi z - \nu_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}\right] + \sqrt{\left[\frac{1}{2} - \frac{\nu_S - \xi z - \nu_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}\right]^2 + \frac{2(r - \nu_K)}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}};$$

$$\theta = \left[\frac{1}{2} - \frac{\nu_S + \xi(1 - z) - \nu_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}\right] + \sqrt{\left[\frac{1}{2} - \frac{\nu_S + \xi(1 - z) - \nu_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}\right]^2 + \frac{2(r - \nu_K)}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}}.$$

Clearly,

$$\nu_S - \xi z - \nu_K \geq \nu_S + \xi(1 - z) - \nu_K, (\xi \leq 0).$$

Hence, $\theta \geq \lambda$, $\frac{\theta}{\theta - 1} \leq \frac{\lambda}{\lambda - 1}$ $(\theta, \lambda > 1)$.

a. For Case 1, consider the following:

$$(\theta - \lambda)C_2(\varphi_1^* e^{-\xi t})^\lambda = (\theta - 1)L_1 * e^{(T_{old} - T_{ave})\xi}\varphi_1^* + \theta L_1 * DEM. \quad (20)$$

Equation (20) never holds if $\theta = \lambda$ $(\xi = 0)$ or no depreciation exists. The difference between θ and λ increases with the decrease of ξ (and increase of depreciation effect).

The solution should be the intersection of the curve $Y_1 = (\theta - \lambda)C_2(e^{-\xi t})^\lambda(\varphi_1^*)^\lambda$ and the line $Y_2 = (\theta - 1)L_1 * e^{(T_{old} - T_{ave})\xi}\varphi_1^* + \theta L_1 * DEM$.

Y_1 and Y_2 are monotonic increasing functions, and $Y_1 < Y_2$ when $\varphi_1^* = 0$. The solution φ_1^* should also satisfy $\varphi_1^* < \varphi_1^* e^{-\xi t} < \varphi_2^*$.

λ is not an integer; thus, achieving the analytical solution of Equation (20) is difficult. However, we can apply the monotonic increasing properties of Y_1 and Y_2 and the requirement of $\varphi_1^* e^{-\xi t} < \varphi_2^*$ to find possible solutions in simulations. Matlab is adopted to find the numerical solution of φ_1^* for each available t.

b. For Cases 2 and 3, we can derive that

$$\frac{L_2 + L_1 * DEM}{L_2 e^{-T_{ave}\xi} - L_1 e^{(T_{old} - T_{ave})\xi}} > e^{T_{ave}\xi}$$

and $L_2 e^{-T_{ave}\xi} - L_1 e^{(T_{old} - T_{ave})\xi} > 0$ $(L_2 > L_1)$.

The upper bound of φ_1^* is larger than φ_2^*. The lower bound of φ_1^* is 0 and smaller than φ_2^*.

φ_1^* is a decreasing function of t, and a unique t satisfies the following:

$$\varphi_1^*(t) = \varphi_2^*(t).$$

Here, $\varphi_2^*(t) = \frac{\lambda}{\lambda - 1} e^{T_{ave}\xi}$ for any t.

We aim to examine whether the optimal ratios φ_1^* and φ_2^* correspond to the relationship conditions in Cases 2 and 3.

On the basis of the solution of φ_1^* and φ_2^*,

$$\frac{\varphi_1^* e^{-\xi t}}{\varphi_2^*}$$

$$= \frac{\theta}{\theta - 1} * \frac{\lambda - 1}{\lambda} \frac{L_2 + L_1 * DEM}{L_2 e^{-T_{ave}\xi} e^{-\xi t} - L_1 e^{(T_{old} - T_{ave})\xi}} * \frac{e^{-\xi t}}{e^{T_{ave}\xi}}$$

$$= \frac{\theta}{\theta - 1} * \frac{\lambda - 1}{\lambda} \frac{L_2 e^{-\xi t} + L_1 e^{-\xi t} * DEM}{L_2 e^{-\xi t} - L_1 e^{T_{old}\xi}}.$$

$e^{-\xi t} \geq 1$ and $\frac{\theta}{\theta - 1} > \frac{\lambda}{\lambda - 1}$. Thus,

$$\frac{\varphi_1^* e^{-\xi t}}{\varphi_2^*} \geq 1.$$

Although

$$\varphi_1^*(t) < \varphi_2^*(t)$$

for some t, $\varphi_1^* e^{-\xi t} \geq \varphi_2^*$ still holds, which corresponds to the conditions in Cases 2 and 3.

The solution of $f_1(\varphi_1)$ and φ_1^* is reasonable.

This inequality indicates a looser condition to exercise the compound option. In Chen and Lai's (2013) model without depreciation effect, φ_1^* and φ_2^* are constants. $\varphi_1^* \geq \varphi_2^*$ must hold. φ_1 is constantly equal to φ_2 when no depreciation exists; thus, the two options should be exercised together. Our new model supports an extended assumption and allows $\varphi_1^*(t) < \varphi_2^*(t)$ for some t. However, the solution of φ_1^* guarantees that when $\varphi_1 \geq \varphi_1^*$ in the first phase, we constantly have $\varphi_2 = \varphi_1 e^{-\xi t} \geq \varphi_2^*$, which satisfies the optimal exercise ratio in the second phase. The two options in the two phases should still be exercised simultaneously.

CHAPTER 6 OPTION FOR VERTICAL MIXED-USE DEVELOPMENT

6.1 Introduction

To achieve Objective 4 (i.e. to develop an option pricing model for vertical mixed-use developments in high-density cities), this chapter introduces the finite-time American basket option model to obtain the optimal development for a building with two or more uses. The characteristics of vertical mixed-use development in Section 2.8.2 should be included in the new model. In addition to the sensitivity tests about the effects of different factors on option value and expected exercise timing, this chapter discusses the decision criteria to choose between the vertical and horizontal types in the planning of a mixed-use development.

6.2 Option pricing model for vertical mixed-use development

The option pricing model for vertical mixed-use development is based on the basket option model for two different assets. Suppose two types of land use are included in a mixed-use development, denoted as Types L and H. The explanation of the initials L and H will be provided in Section 6.3.1. Then, for Types L and H, the unit market prices are $S_L(t)$ and $S_H(t)$, and the unit construction costs are $K_L(t)$ and $K_H(t)$, respectively. For the unit market prices, assume that they follow the geometric Brownian motions, as shown as follows:

$$dS_L(t) = \nu_{S_L} S_L dt + \sigma_{S_L} S_L dZ_{S_L};$$

$$dS_H(t) = \nu_{S_H} S_2 dt + \sigma_{S_H} S_H dZ_{S_H};$$

$$Cov(Z_{S_L}, Z_{S_H}) = \rho \sigma_{S_L} \sigma_{S_H}.$$

The fluctuation of construction cost is assumed to be smaller than the market price. Assume the construction costs increase at a constant rate annually.

$$dK_L(t) = \mu_L K_L dt,\ \ \mu_L \geq 0$$
$$dK_H(t) = \mu_H K_H dt,\ \mu_H \geq 0$$

$K_L(t)$ and $K_H(t)$ are not stochastic processes. In this chapter, we focus on the effect of market price uncertainty. The construction cost uncertainty is not viewed as a major factor in the mixed-use development pricing. This assumption can be relaxed by considering the construction cost as a 'negative payoff asset.' The basket option model remains appropriate when the construction cost is stochastic. The following discussion only assumes the market price as stochastic variables.

Conditions $\mu_L \geq 0$ and $\mu_H \geq 0$ are set to avoid the acceleration in option exercise due to the declining construction cost.

As mentioned in Section 2.8.2, developers are usually involved in a land lease contract in the vertical mixed-use development. This contract becomes valid after the demolition and clearance processes. Hence, it is a one-phase redevelopment project on a vacant land. In each period, the intrinsic value of this option is

$$(6.1)\ \ V\big(S_L(t), S_H(t), K_L(t), K_H(t)\big) = Max\{0, S_L(t) * A_L + S_H(t) * A_H - K_L(t) * A_L - K_H(t) * A_H - C, e^{-r\Delta t} E\big[V\big(S_L(t+\Delta t), S_H(t+\Delta t), K_L(t+\Delta t), K_H(t+\Delta t)\big)\big]\}$$
$$(0 \leq t \leq T),$$

where C is other fixed costs during the construction process, which are independent of the CFA; A_L and A_H are the GFA of Types L and H, respectively; $e^{-r\Delta t}E\big[V\big(S_L(t+\Delta t), S_H(t+\Delta t), K_L(t+\Delta t), K_H(t+\Delta t)\big)\big]$ represents the present value of this option if the exercise is delayed at time *t*; and *r* is the risk-free interest rate.

As a boundary condition, the option value at the end of the maximum construction period is

$$(6.2) V\big(S_L(t), S_H(t), K_L(t), K_H(t)\big) = Max\{0, S_L(T) * A_L + S_H(T) * A_H - K_L(T) * A_L - K_H(T) * A_H - C].$$

To maximise the revenue, the developers will exercise the option when the present value of $V\big(S_L(t), S_H(t), K_L(t), K_H(t)\big)$ is the largest. As mentioned in Section 3.2.4, the following discussions are based on the LSMC method.

The conversion option is excluded in this pricing model because the lease contract in a vertical mixed-use development does not contain this option. In such an option (without changes in GFAs), the developers can change the use of some properties. However, for redevelopment, the GFAs for two uses must be agreed to by the landowners and developers in the lease contract (when the development option becomes valid). These GFAs will not change in the construction process. Hence, no conversion option is embedded in this case. Instead of the conversion option, this chapter investigates the influence of GFAs for different uses on option value.

6.3 Sensitivity tests

6.3.1 Market characteristics

To emphasise the difference between two types of properties in a mixed-use development, a preliminary study is conducted before sensitivity tests are conducted. The characteristics of Hong Kong's residential and retail property markets (e.g. current prices, price volatilities, rental yields, construction costs and market correlation) are collected from various sources. Price data are taken from the Rating and Valuation Department and the Census and Statistics Department (Census and Statistics Department, 2017). Construction cost data are taken from the Buildings Department and the Arcadis Construction Cost Handbook (Arcadis, 2018). Interest rate is obtained from the Government Bond Programme in HKSAR. These parameters are summarised in Table 6.1

Parameters	Values
Risk-free interest rate	1.75% p.a.
Rental yield for residential properties	3.73% p.a.
Rental yield for retail properties	4.73% p.a.
HKD prime rate	5.00% p.a.
Volatility of residential properties	13.16% p.a.
Volatility of retail properties	41.91% p.a.
Residential unit price (per m^2)	126,679 HKD
Retail unit price (per m^2, based on transaction data)	363,328 HKD
Residential unit cost (per m^2)	26,650 HKD
Retail unit cost (per m^2)	35,650 HKD
Increase rate of construction cost	4.35% p.a.

Note: The retail unit price is estimated based on the monthly retail rent.

Table 6.1 Characteristics of Hong Kong property markets

The risk-free interest rate is extremely low. In the past decade, the low interest rate environment has attracted developers to build new properties as soon as possible. Low interest rate is also believed to stimulate the real estate prices (Tse, 1996; T. Y. J. Wong, Hui, & Seabrooke, 2003). However, this environment is expected to change because the US Federal Reserve continues to raise its interest rates three times in 2017. The overheated property market is also believed to be cooled down because the mortgage rate is also increasing. A high interest rate decreases the housing demand (Follain, 1982; Kau & Keenan, 1980), and the significantly high property prices are expected to be reduced, as well as the considerable difference between market price and construction cost.

The remarkable difference between market price and construction cost indicates that the development option has a good positive intrinsic value, which will result in an immediate exercise of options in most scenarios. To investigate the option in a longer period, the difference between price and cost, the risk-free interest rate and the irregular high volatility of the retail market will be adjusted in sensitivity tests. The parameter values in Table 6.1 reveal the differences between residential and retail property markets. For the former market, the unit price, unit construction cost, rental yield and market volatility are all lower than those for the latter market. In the simulations, the prices of property units in two uses are based on two different markets (e.g. residential and retail markets). We define the two markets, low- and high-unit-price markets, as Type L and Type H markets, respectively. The sensitivity tests are indirectly related to the ways of use. Hence, we only apply the data to the simulations.

How the potential factors affect a mixed-use development will be examined in the following section. In each scenario, the expected exercise time (year), expected option value (million HKD) and percentage of exercised paths (%) in a 10-year period are reported. The total GFA in the simulation refers to the Kwun Tong Town Centre Project (K7)
(https://www.ura.org.hk/en/project/redevelopment/kwun-tong-town-centre-project).
The land area covers 53,500 m^2, the total GFA is approximately 401,250 m^2, the number of residential flats is 2,298, the residential GFA is approximately 151,232 m^2, the commercial GFA is approximately 209,640 m^2, government, institution and community GFA is approximately 14,300 m^2, open-space GFA is approximately 9,348 m^2 and the GFA of other uses accounts for approximately 16.700 m^2. This redevelopment affects 3,139 residents in 1,290 households. Only residential and commercial/retail uses are considered in this simulation.

6.3.2 Market volatilities

Table 6.2(a) shows that the project is conducted based on its original GFA combination. High volatilities of Type L and/or H property increase the option value and the expected waiting time. If volatilities are high, then the real option is less likely to be exercised before the expiration. High market uncertainty entails a high probability to wait for a high development profit. However, a high risk also implies a great chance of the failure to exercise. When both volatilities are high (v_L = 20%, v_H = 30%), over 25% of 100,000 simulated paths are not exercised in this study.

	$v_L = 0.06$	$v_L = 0.13$	$v_L = 0.20$
$v_H = 0.10$	0.0421 30,646.40 99.99%	0.2240 30,685.17 99.64%	0.3007 30,739.67 99.23%
$v_H = 0.20$	2.3589 31,978.49 87.27%	2.8106 32,430.36 85.41%	3.0821 32,957.13 82.58%
$v_H = 0.30$	3.7217 35,217.45 76.10%	3.4041 35,454.99 79.04%	3.9499 36,368.89 74.12%

(a) Original design: $GFA_L = 151{,}232\ m^2$, $GFA_H = 209{,}640\ m^2$

GFA_L and GFA_H are the GFA for Types L and H uses, respectively.

	$v_L = 0.06$	$v_L = 0.13$	$v_L = 0.20$
$v_H = 0.10$	0.0182 28,881.47 100.00%	0.1198 28,911.63 99.81%	0.4395 28,988.24 99.09%

$v_H = 0.20$	2.3549 29,845.42 90.86%	2.5835 30,302.69 87.38%	2.7171 30,793.61 82.20%
$v_H = 0.30$	3.3849 32,131.14 81.16%	3.7512 32,932.54 78.25%	3.5738 33,571.82 75.75%

(b) $GFA_L = GFA_H$ = 180,436 m^2 (50% GFA)

	$v_L = 0.06$	$v_L = 0.13$	$v_L = 0.20$
$v_H = 0.10$	0.0145 22,380.88 100.00%	0.3375 22,444.16 99.56%	1.7488 22,756.89 96.05%
$v_H = 0.20$	0.3607 22,445.05 99.83%	1.8711 22,801.95 94.85%	2.6471 23,426.48 88.12%
$v_H = 0.30$	0.7884 22,562.01 99.25%	2.3298 23,272.16 90.59%	3.0406 24,378.43 82.69%

(c) GFA_L = 288,698 m^2 (80% GFA), GFA_H = 72,174 m^2 (20% GFA)

Table 6.2 Expected waiting time for development, expected option value and percentage of exercised paths in different volatility and GFA combinations

A comparison of Tables 6.2(b) and 6.2(a) shows that the major difference is about the option value and expected exercise time. Both values in Table 6.2(b) are smaller than those in Table 6.2(a). In Table 6.2(a), the high-unit-value property has larger GFA and development value than the low-unit-value property. The reduction of GFA for high-unit-value property in Table 6.2(b) leads to a lower total project value and lower waiting time to start, compared with low-unit-value property. To maximise the total project value, the developers will attempt to obtain the maximum permitted proportion for high-unit-value property. Hence, the optimal building strategy is to maximise the GFA for high-unit-value property. This strategy will also imply a long expected waiting time. The next set of simulations will also support this strategy.

In the third set of simulations, 80% of the total GFA is adopted to develop the low-unit-value property. The option value and expected waiting time for development are lower than the scenarios in Tables 6.2(a) and 6.2(b), respectively. This finding supports the strategy of maximising the entire mixed-use development value. The second finding comes from the effect of volatility in Table 6.2(c). When v_H is at the median level (20%), the option value increases by approximately 4.4% if v_L grows from 6% to 20%. This change becomes 2.9% if v_L is high (20%) and v_H increases from 10% to 20%. This result indicates that v_L has a larger effect on option value than v_H. The volatility of low-unit-value property becomes the more important factor compared with that of high-unit-value property. Moreover, v_H has a larger effect on option value than v_L in Tables 6.2(a) and 6.2(b). These phenomena suggest that if the initial total development value of a special use is larger than that of the other use, its volatility will have a more considerable effect on option value. The influence on expected exercise time also depends on the initial total development value.

These findings suggest that neither the unit market price nor the GFA is the major factor for determining which volatility has a larger effect on the option value. The importance of initial total development value, which is the product of unit market price and GFA, should be noticed in a mixed-use development. This indication also derives some interesting implications. For a residential–retail mixed-use development, the GFA of residential properties is usually considerably larger than that of retail property. K7 is a special case. If the initial total development value of residential property exceeds that of retail property, then the developers will mainly focus on the historical volatility of residential property instead of that of retail property to determine the timing to start. Similar explanations can be achieved if the mixed-use development contains normal and luxury residential units (with a large GFA in each unit). If the initial total development value of normal units is the major part, then the market volatility shows a greater effect on the entire project value and development timing.

6.3.3 Rental yield and risk-free interest rate

Rental yield rate is the opportunity cost of deferring the building decision. The difference between risk-free interest rate and rental yield rate is the drift rate of the stochastic differential process of each property. As a result, the option value is expected to be negatively correlated to the rental yield rate. In this part, the default values for two rental yield rates are assumed to be 3.73%.

Under the assumption of a pre-determined rental yield rate, the option value is positively influenced by the risk-free interest rate. A high interest rate not only works as a large discount factor but also indicates a high drift rate of the stochastic process.

Tables 6.3(a) and 6.3(b) show the option value and expected exercise time change with respect to the risk-free interest rate and the rental yield rates of two types of properties. Table 6.3 shows that the increase in the drift rate exceeds that of the discount factor. The expected exercise time and option value increase with the interest rate. If the rental rate is constant, then an interest rate increase will enlarge the future exercise profit, thereby increasing the overall value of the option. A higher potential profit also induces the developer to wait longer for a better exercise timing if the additional profit is expected to exceed the time opportunity cost (i.e. the discount factor).

	$ren_L = 0.0173$	$ren_L = 0.0373$	$ren_L = 0.0573$

r = 0.07	1.4556 313.4995 88.65%	0.8089 309.6810 92.05%	0.2397 307.2838 97.63%
r = 0.10	5.2301 358.8336 79.66%	4.7498 345.7120 81.79%	3.6312 334.1074 83.14%
r = 0.13	7.6450 428.1847 87.96%	6.9387 406.3722 85.02%	6.4904 392.3894 86.78%
	ren_H = 0.0173	ren_H = 0.0373	ren_H = 0.0573
r = 0.07	4.0366 338.9535 79.44%	0.6098 309.2726 94.05%	0.0070 306.1911 100.00%
r = 0.10	8.0203 419.1857 77.35%	4.7805 345.4761 83.52%	1.3296 312.5612 91.47%
r = 0.13	8.9602 496.0224 87.64%	6.9261 407.5214 81.27%	4.6605 349.2549 87.59%

(a) Original design: GFA_L = 151,232 m^2, GFA_H = 209,640 m^2

	ren_L = 0.0173	ren_L = 0.0373	ren_L = 0.0573
r = 0.07	0.6195 225.2713 97.30%	0.0099 223.8231 100.00%	0.0069 223.7468 100.00%
r = 0.10	7.2884 264.0541 90.09%	3.2709 233.4266 93.93%	0.1199 224.2415 99.67%

r = 0.13	8.5593 313.8523 83.54%	6.9287 271.0200 93.73%	4.4784 242.5161 96.85%
	ren_H = 0.0173	ren_H = 0.0373	ren_H = 0.0573
r = 0.07	0.2059 224.4946 99.17%	0.0078 223.7578 100.00%	0.0064 223.7333 100.00%
r = 0.10	6.1366 252.4889 90.86%	3.2709 233.4266 93.93%	0.5253 225.0277 98.58%
r = 0.13	8.4653 300.5902 91.10%	6.9635 271.5241 93.11%	5.2615 249.7380 92.90%

(b) GFA_L = 288,698 m^2 (80% GFA), GFA_H = 72,174 m^2 (20% GFA)

Table 6.3 Expected waiting time for development, expected option value and percentage of exercised paths in different combinations of rental yield rates, risk-free interest rate and GFA

When the interest rate is kept constant, the change of the rental yield rate is reflected in the change of the drift rate. Similar to the case of volatilities, if Type L (or H) property has a larger initial total value, then its rental yield rate has a more significant effect on the option value and expected exercise time. The reason is that a large initial total value indicates a large opportunity cost if the construction process is delayed. The large opportunity cost would encourage the developers to start the project in a shorter term.

6.3.4 Solution to the finite-time two-phase redevelopment option model

Section 5.2 has mentioned a specific case for the redevelopment option model. In this case, the maximum demolition period and the maximum rebuilding period are T_1 and T_2, respectively. Market price and construction cost are stochastic processes with a correlation coefficient ρ. The binomial tree model in Chapter 4 and the stochastic differential equation in Chapter 5 are inappropriate for this case. The basket option model and LSMC method in this chapter can instead be applied to determine the project value and expected exercise timing.

According to this equation:

(6.1) $$V\big(S_L(t), S_H(t), K_L(t), K_H(t)\big) = Max\{0, S_L(t) * A_L + S_H(t) * A_H - K_L(t) * A_L - K_H(t) * A_H - C, e^{-r\Delta t} E\big[V\big(S_L(t+\Delta t), S_H(t+\Delta t), K_L(t+\Delta t), K_H(t+\Delta t)\big)\big]\ \}$$
$(0 \leq t \leq T)$

For the rebuilding phase, $S_L(t)$ is replaced by the depreciation-adjusted new property value. $S_H(t)$ is replaced by the additive inverse of the construction cost. Let $K_L(t) = K_H(t) = 0$. $A_L = A_H$ and equals the GFA of new properties. The rebuilding option value can be derived in the same manner in this section.

For the demolition phase, $S_L(t)$ is replaced by the rebuilding option value from time *t* to time (t + T_2). $S_H(t)$ is replaced by the additive inverse of the sum of demolition cost and the depreciation-adjusted old property value. $K_L(t) = K_H(t) = 0$. $A_L = A_H$ and equals the GFA of old properties. The compound option value can be estimated in the same manner as the rebuilding option value.

This solution is provided to achieve the option value and expected redevelopment timing for the case which were not covered in Chapters 4 and 5. The major findings about the depreciation effect in these chapters still hold when the correlation coefficient between market price and construction cost is non-zero.

6.4 Vertical and horizontal developments

6.4.1 Basic assumptions

As mentioned in Section 1.2.6, horizontal mixed-use development is another type of popular planning. Before the development, developers should determine to choose between horizontal and vertical type in the planning stage. In an empirical case, social need, building design, environment aspects and project profitability are comprehensively analysed to choose the superior type. In this chapter, we only focus on the aspect of project profitability by applying the real option approach to the pricing procedures. Furthermore, the developer is assumed to be allowed to choose between two types of mixed-use developments within the maximum plot ratio. If the land lease contract or the zoning plan has already restricted the type of development, the developer should just follow the required type in the contract or the zoning plan.

Several additional necessary assumptions are made before simulations for the comparability of the values between horizontal and vertical types.

Assumption 1. For two different uses in the mixed-use development, the GFA and CFA for each use in the vertical type are equal to those in the horizontal type. The construction cost per square meter of CFA (and then GFA) for each use in the vertical type is also equal to that in the horizontal type.

This assumption is about the building area and building cost in two types. The construction cost measured in CFA is transferred into that measured in GFA. The differences in GFA and CFA are excluded in the pricing of two different types of mixed-use development. In an empirical case, vertical project may include an additional fixed cost for fire services and other safety measures. This additional cost is not proportional to the GFA. In this chapter, this cost is assumed to be zero for simplification, which is a minor adjustment in the empirical pricing process.

Assumption 2. The advantage of choosing horizontal type is the larger flexibility in determining the building timing for separate buildings in different uses.

The real option approach has a major difference from other traditional valuation approaches (e.g. sales comparison, cost and income approaches). The building timing is changeable in the real option approach instead of pre-determined. The optimal development timing is the timing to achieve the greatest profit.

In the horizontal development, two different uses are separately located in two different buildings. The developers can choose separate construction timings. Although the construction process of the latter property may have negative externality on the former completed one, this externality is viewed as temporary and even trivial if the construction period is assumed to be extremely short. The separation of construction timings is not a reason to reduce the horizontal development value. However, properties for two uses must be constructed and sold simultaneously in the vertical development. Asking the purchasers to move in the lower floors when the upper floors are still waiting for construction is impossible. As a result, the vertical mixed-use project only has a single optimal building timing.

If the fixed cost is ignored, then the option value of the entire horizontal mixed-use project at time t is

$$V\big(S_L(t), K_L(t)\big) + V\big(S_H(t), K_H(t)\big) = Max\big\{0, S_L(t) * A_L - K_L(t) * A_L, e^{-r\Delta t} E\big[V\big(S_L(t+\Delta t), K_L(t+\Delta t)\big)\big]\big\} + Max\big\{0, S_H(t) * A_H - K_H(t) * A_H, e^{-r\Delta t} E\big[V\big(S_H(t+\Delta t), K_H(t+\Delta t)\big)\big]\big\} \quad (0 \le t \le T).$$

The option value of the entire vertical mixed-use project at the same time is

$$V\big(S_L(t), S_H(t), K_L(t), K_H(t)\big) = Max\{0, S_L(t) * A_L + S_H(t) * A_H - K_L(t) * A_L -$$

$K_H(t) * A_H - C, e^{-r\Delta t} E\left[V\left(S_L(t+\Delta t), S_H(t+\Delta t), K_L(t+\Delta t), K_H(t+\Delta t)\right)\right] \quad \}$

$(0 \leq t \leq T)$.

The following inequality is true at all times:

$$Max(A+B) \leq Max(A) + Max(B).$$

Hence,

$$Max[0, S_L(t) * A_L + S_H(t) * A_H - K_L(t) * A_L - K_H(t) * A_H]$$
$$\leq Max[0, S_L(t) * A_L - K_L(t) * A_L]$$
$$+ Max[0, S_H(t) * A_H - K_H(t) * A_H]$$
$$(0 \leq t \leq T).$$

Then,

$$Max\left\{e^{-r\Delta t} E\left[V\left(S_L(t+\Delta t), S_H(t+\Delta t), K_L(t+\Delta t), K_H(t+\Delta t)\right)\right]\right\} \leq$$
$$Max\left\{e^{-r\Delta t} E\left[V\left(S_L(t+\Delta t), K_L(t+\Delta t)\right)\right]\right\} +$$
$$Max\left\{e^{-r\Delta t} E\left[V\left(S_H(t+\Delta t), K_H(t+\Delta t)\right)\right]\right\}.$$

We prove that:

$$V\left(S_L(t), S_H(t), K_L(t), K_H(t)\right) \leq V\left(S_L(t), K_L(t)\right) + V\left(S_H(t), K_H(t)\right) \quad (0 \leq t \leq T),$$

which indicates the advantage of choosing the horizontal type in the valuation process.

Assumption 3. The advantage of choosing vertical type is the higher market value of the upper structure for a different use compared with the value of an individual building in the horizontal type.

To compare the project values between two types, the availability of two types should be confirmed. The land size should be insufficiently large to build two separate buildings without violating the building regulations. If both types are available, the vertical type can increase the height of the upper structure for a different use. For some specific use (e.g. residential use), the market value is positively related to the height.

The residents' preference of living in higher floors has been proved in empirical studies (Chau, Wong, Yau, & Yeung, 2007; Choy, Mak, & Ho, 2007; Hui et al., 2007; Hui et al., 2016; Jim & Chen, 2009; Lau et al., 2005; Mok, Chan, & Cho, 1995). Residents can enjoy better scenic views, avoid unnecessary disturbances from the street and suffer less air and noise pollution in higher floor units.

A podium model has been promoted in Hong Kong since the 1980s to increase the benefits from lower residential units. This model consists of two structures in one property. The lower structure, called the podium, is usually for retail use. Dozens of different stores, supermarkets or even a cinema are arranged in an indoor podium structure. The upper structure is one or several residential buildings located on the top of the podium. The podium becomes an artificial ground floor for the residential buildings. The heights of the residential units, especially for the lower floor units, are significantly increased. Hence, the market values of the residential units are expected to be higher than those in a horizontal project.

Assumption 4. The height premium of the upper structure in the vertical development is derived from the hedonic pricing model.

The design differences between two types are excluded in the comparison. The upper structure of the vertical development is assumed to be designed in the same manner as the separate building of the horizontal project. If the upper structure contains multiple residential buildings, then they are compared with the same number of buildings on the ground in the horizontal project. Not all the space on the top of the podium is used for the buildings on top.

6.4.2 Model descriptions

The height of the podium is assumed to be equivalent to the height of X floors in a residential building. For example, the minimum height in one storey should be 2.5 m for residential use according to the Building (Planning) Regulations (Cap. 123, Section 38). In Hong Kong, a podium is usually 15 m high, which translates to six floors for residential use. The other structural attributes are assumed to be the same. The value of a residential unit on the Nth floor in the vertical development should equal the value of a residential unit on the N + X floor in the horizontal one. The relationship between property value (P), floor number (N) and other potential attributes (A) can be derived from the hedonic pricing model (Freeman, 1979; Hui et al., 2007; Hui & Liang, 2016; Jim & Chen, 2010; Rosen, 1974) as follows:

$$\mathrm{LnP} = \alpha \times \mathrm{N} + \beta \times \mathrm{A}$$

or

$$\mathrm{P} = \mathrm{e}^{\alpha \times \mathrm{N}} \times \mathrm{e}^{\beta \times \mathrm{A}},$$

where α is the coefficient of the floor level on property value, and β is a coefficient vector that represents the effects of other attributes A on property value.

Define the value of a residential unit on the Nth floor in the vertical development as P_1 and the value of another unit on the Nth floor in the horizontal development as P_2. Then,

$$\mathrm{P}_1 = \mathrm{e}^{\alpha \times \mathrm{X}} \times \mathrm{P}_2.$$

Meanwhile, the other attributes are equal.

$\mathrm{e}^{\alpha \times \mathrm{X}}$ is the height premium in the vertical development. For example, the Ordinary Least Squares coefficient of floor level in Chapter 4 was estimated as 0.006. If this coefficient is adopted, the height premium becomes $\mathrm{e}^{0.006*6} = 1.0367$.

Define the unit market price of the residential (low unit price) part in the vertical project as $S_L(t)$ and the unit price of the retail/commercial (high unit price) part as

$S_H(t)$. Then, on the basis of Assumptions 1 and 4, the unit price of a residential building in the horizontal project becomes $S_L(t) \times e^{-\alpha \times X}$. The unit price of a commercial building is still $S_H(t)$ because the heights of commercial buildings are the same in two types.

The total value of the vertical development at time t is

(6.3) $V(S_L(t), S_H(t), K_L(t), K_H(t)) = Max\{0, S_L(t) * A_L + S_H(t) * A_H - K_L(t) * A_L - K_H(t) * A_H - C, e^{-r\Delta t} E[V(S_L(t+\Delta t), S_H(t+\Delta t), K_L(t+\Delta t), K_H(t+\Delta t))]\}$ $(0 \leq t \leq T)$,

Where C is the additional construction fixed cost in the vertical development compared with the horizontal one. The other symbols are the same as Equation (6.1).

The total value of the horizontal development at time t is

(6.4)

$$V(S_L(t), K_L(t)) + V(S_H(t), K_H(t)) = Max\{0, S_L(t) \times e^{-\alpha \times X} \times A_L - K_L(t) \times A_L, e^{-r\Delta t} E[V(S_L(t+\Delta t), K_L(t+\Delta t))]\} + Max\{0, S_H(t) \times A_H - K_H(t) \times A_H, e^{-r\Delta t} E[V(S_H(t+\Delta t), K_H(t+\Delta t))]\}.$$

However, the analytical solution of the hurdle value for $e^{-\alpha \times X}$ does not exist. Monte Carlo simulations are adopted to achieve the estimation as follows.

Step 1. Estimate the option values of $V(S_L(t), S_H(t), K_L(t), K_H(t))$, $V(S_L(t), K_L(t))$ and $V(S_H(t), K_H(t))$.

Step 2. Calculate the value of $V(S_L(t), K_L(t))$, which satisfies

$$V(S_L(t), S_H(t), K_L(t), K_H(t)) = V(S_L(t), K_L(t)) + V(S_H(t), K_H(t))$$

and denote it as $\hat{V}(S_L(t), K_L(t))$.

Step 3. Calculate the ratio $V\big(S_L(t),K_L(t)\big)/\hat{V}\big(S_L(t),K_L(t)\big)$. Substitute

$$\left(\frac{V\big(S_L(t),K_L(t)\big)}{\hat{V}\big(S_L(t),K_L(t)\big)}-1\right)\times 0.5$$

into $\mathrm{e}^{\alpha\times\mathrm{X}}$ and estimate the new option value of $V\big(S_L(t),K_L(t)\big)$. Denote this option value as $\tilde{V}\big(S_L(t),K_L(t)\big)$.

Step 4. Compare $\tilde{V}\big(S_L(t),K_L(t)\big)$ and $\hat{V}\big(S_L(t),K_L(t)\big)$. If $\tilde{V}\big(S_L(t),K_L(t)\big) > \hat{V}\big(S_L(t),K_L(t)\big)$, then increase the value of $\mathrm{e}^{\alpha\times\mathrm{X}}$ and replace the value of $\tilde{V}\big(S_L(t),K_L(t)\big)$. If $\tilde{V}\big(S_L(t),K_L(t)\big) < \hat{V}\big(S_L(t),K_L(t)\big)$, then decrease the value of $\mathrm{e}^{\alpha\times\mathrm{X}}$ and replace the value of $\tilde{V}\big(S_L(t),K_L(t)\big)$. The amount of increase or decrease depends on the accuracy of $\mathrm{e}^{\alpha\times\mathrm{X}}$.

Step 5. Repeat Step 4 until the difference between $\tilde{V}\big(S_L(t),K_L(t)\big)$ and $\hat{V}\big(S_L(t),K_L(t)\big)$ is sufficiently small.

In the empirical study, the estimation of option value by Monte Carlo simulations changes in a small scale if the sample paths are changed. In addition, changing the value of $\mathrm{e}^{-\alpha\times\mathrm{X}}$ continuously is difficult. The two boundaries are calculated as

$$\widehat{V_1}\big(S_L(t),K_L(t)\big) = 99.5\% \times \hat{V}\big(S_L(t),K_L(t)\big);$$

$$\widehat{V_2}\big(S_L(t),K_L(t)\big) = 100.5\% \times \hat{V}\big(S_L(t),K_L(t)\big).$$

In this chapter, the value of $\mathrm{e}^{\alpha\times\mathrm{X}}$ changes at a minimum scale of 0.1%. We can find a maximum α_1 which satisfies

$$\tilde{V}\big(S_L(t),K_L(t)|\mathrm{e}^{\alpha_1\times\mathrm{X}}\big) > \widehat{V_2}\big(S_L(t),K_L(t)\big).$$

For any α smaller than α_1, $\tilde{V}\big(S_L(t),K_L(t)|\mathrm{e}^{\alpha\times\mathrm{X}}\big) > \widehat{V_2}\big(S_L(t),K_L(t)\big)$. In this case, the value of horizontal development should be larger than that of vertical one, that is,

$$\tilde{V}\big(S_L(t),K_L(t)|\mathrm{e}^{\alpha\times\mathrm{X}}\big) + V\big(S_H(t),K_H(t)\big) > \widehat{V_2}\big(S_L(t),K_L(t)\big) + V\big(S_H(t),K_H(t)\big)$$
$$> V\big(S_L(t),S_H(t),K_L(t),K_H(t)\big).$$

If the height premium is sufficiently small, then the horizontal type is the more profitable choice.

Similarly, a minimum α_2 exists and satisfies

$$\tilde{V}(S_L(t), K_L(t)|e^{\alpha_2 \times X}) < \widehat{V_1}(S_L(t), K_L(t)).$$

For any α larger than α_2, $\tilde{V}(S_L(t), K_L(t)|e^{\alpha \times X}) < \widehat{V_1}(S_L(t), K_L(t))$. The value of the vertical project is larger.

$$\tilde{V}(S_L(t), K_L(t)|e^{\alpha \times X}) + V(S_H(t), K_H(t)) < \widehat{V_1}(S_L(t), K_L(t)) + V(S_H(t), K_H(t)) < V(S_L(t), S_H(t), K_L(t), K_H(t))$$

A large height premium attracts the developer to adopt the vertical type.

6.4.3 Simulation results and discussions

This section discusses the effects of the volatility and proportion of each use on the critical value. The initial market price and construction cost in the horizontal type are set to be the same as those in the vertical type in Section 6.3. The estimated option values of two uses, $V(S_L(t), K_L(t))$ and $V(S_H(t), K_H(t))$, are generated and listed in Table 6.4.

Case 1 (original design): GFA_L = 151,232 m^2, GFA_H = 209,640 m^2			
	$v_L = 0.06$	$v_L = 0.13$	$v_L = 0.20$
$V(S_L(t), K_L(t))$	7,562.47	7,636.39	7,956.29
	$v_H = 0.10$	$v_H = 0.20$	$v_H = 0.30$
$V(S_H(t), K_H(t))$	23,092.92	25,625.93	30,005.88

Case 2: GFA_L = 288,698 m^2 (80% GFA), GFA_H = 72,174 m^2 (20% GFA)			
	v_L = 0.06	v_L = 0.13	v_L = 0.20
$V(S_L(t), K_L(t))$	14,436.54	14,577.65	15,188.33
	v_H = 0.10	v_H = 0.20	v_H = 0.30
$V(S_H(t), K_H(t))$	7,950.38	8,822.44	10,330.36

Table 6.4 Estimated option value for individual buildings in horizontal development

For each combination of GFA and volatilities, three sets of statistics are achieved, namely, the hurdle option value $\hat{V}(S_L(t), K_L(t))$, the hurdle ratio $\left(\frac{V(S_L(t),K_L(t))}{\hat{V}(S_L(t),K_L(t))} - 1\right)$ and the pairs of critical height premium $[e^{\alpha_1 \times X} - 1, e^{\alpha_2 \times X} - 1]$. The trends of the three parameters are investigated in Table 6.5 based on the results in Tables 6.2 and 6.4.

The critical height premium is the most important parameter in determining the type of mixed-use development. Suppose the critical height premium becomes higher. Then, a higher height premium is required to comprise the loss of flexibility in the vertical type compared with the horizontal type where the structures for two uses can be built separately. The horizontal type tends to be chosen if the critical height premium is difficult to reach. The other two parameters do not influence the decision directly. Their values are introduced to describe how the critical height premium changes due to volatilities.

Case 1 (original design): GFA_L = 151,232 m^2, GFA_H = 209,640 m^2			
	v_L = 0.06	v_L = 0.13	v_L = 0.20
v_H = 0.10	7,553.48 0.12% (0, 0.3%)	7,592.25 0.58% (0, 0.1%)	7,646.75 4.05% (1.7%, 2.4%)
v_H = 0.20	6,352.56	6,804.43	7,331.20

	19.05% (6.9%, 7.4%)	12.23% (5.0%, 5.5%)	8.53% (4.0%, 4.8%)
$v_H = 0.30$	5,211.57 45.11% (15.2%, 15.7%)	5,449.11 40.14% (15.6%, 16.2%)	6,363.01 25.04% (11.7%, 12.4%)

Case 2: $GFA_L = 288{,}698\ m^2$ (80% GFA), $GFA_H = 72{,}174\ m^2$ (20% GFA)			
	$v_L = 0.06$	$v_L = 0.13$	$v_L = 0.20$
$v_H = 0.10$	14,430.50 0.04% (0, 0.3%)	14,493.78 0.58% (0, 0.4%)	14,806.51 2.58% (1.0%, 1.7%)
$v_H = 0.20$	13,622.61 5.97% (2.2%, 2.7%)	13,979.51 4.28% (1.4%, 2.1%)	14,604.04 4.00% (1.7%, 2.4%)
$v_H = 0.30$	12,231.65 18.03% (6.6%, 7.1%)	12,941.80 12.64% (5.1%, 5.8%)	14,048.07 8.12% (3.8%, 4.6%)

Table 6.5 Hurdle option value, hurdle ratio and critical height premium in different scenarios

The lower boundary in some scenarios is zero. This phenomenon indicates that any positive height premium can discourage the developers to choose the horizontal type.

Three major findings are observed.

(1) The trends of hurdle option values and the hurdle ratio $\left(\frac{V(S_L(t),K_L(t))}{\hat{V}(S_L(t),K_L(t))} - 1\right)$ are opposite.

A higher residential market volatility (a lower retail market volatility) results in higher hurdle option value. However, the hurdle ratio is negatively related to the residential market volatility except when the retail market volatility is extremely low.

This phenomenon can be explained by the comparison between Tables 6.2 and 6.4. As the residential/retail market volatility increases, the separate building values in the horizontal type appreciate more than the total project value in the vertical type. The reason is that the simultaneous construction of the entire vertical project reduces the potential profit compared with separate buildings. As a result, the hurdle ratio decreases if the residential market volatility increases or the retail market volatility decreases.

(2) The critical height premium is bounded by the hurdle ratio. This premium increases with the retail market volatility but does not have a consistent relationship with the residential market volatility.

The first sentence can be explained by the following inequality:

$Max[0, S_L(t) \times \mathrm{e}^{-\alpha \times X} \times A_L - K_L(t) \times A_L] < \mathrm{e}^{-\beta \times X} \times Max[0, S_L(t) \times A_L - K_L(t) \times A_L]$ (if $\alpha = \beta$),

where $\mathrm{e}^{\alpha \times X} - 1$ is the critical height premium, and $\mathrm{e}^{\beta \times X} - 1$ is the hurdle ratio. For the left side to equal the right side, $\mathrm{e}^{\alpha \times X}$ should decrease. Hence $\alpha < \beta$.

The inconsistent relationship between the critical height premium and the residential market volatility is also explained on the basis of the preceding inequality. The increase of residential market volatility reduces $\mathrm{e}^{\beta \times X}$ and increases the residential building value simultaneously. As discussed in the first finding, the total vertical project value increases more slowly than the residential building value in the horizontal project. A higher critical height premium is then required to keep the vertical project as a more profitable one. Hence, the critical height premium should be positively related to the residential market volatility. However, this premium is still bounded by the hurdle ratio, which is negatively related to the residential market volatility. The two opposite trends cause an inconsistent relationship between this premium and the residential market volatility.

If the retail market is expected to be more volatile, then this finding indicates that the horizontal type is a better choice. Moreover, the developers must run detailed simulations to determine the type of mixed-use development if the residential market becomes more volatile.

(3) If the major income of the project is from retail property, then the horizontal type is considerably easier to be adopted. By contrast, if the major income is from residential units, then the vertical type is more likely to be chosen.

When the GFA for residential use becomes larger, the hurdle option value and the hurdle ratio are reduced. The critical height premium, as bounded by the hurdle ratio, is also reduced. Hence, if a high-rise residential building is contained in the mixed-use project, then the vertical type is preferred because the property value for residential use is usually higher than that for retail use.

6.4.4 Extension to the real option model for vertical mixed-use developments

In Section 6.2, the height premium for the residential use was not emphasised. The basic model is applicable in the cases where the height premium is trivial or previously embedded in the reference market value. The former case is observed when the value of the upper structure is insensitive to its height. The latter case requires the developers to choose the proper completed vertical mixed-use developments in the neighbourhood when determining the reference market price for the upper structure. For example, if the podium of the new development is expected to be 15 m high, then the developers should choose the sample properties with a 15-m-high podium. The drift rate and the volatility are not affected by the height premium. The sampling process only influences the initial estimated value of the upper structure.

However, if the vertical development with the same height conditions is insufficient, then a revised option pricing model with the height premium should be adopted. Suppose Type L use is in the upper structure and Type H use is in the lower structure. Then, the unit market prices $S_L(t)$ and $S_H(t)$ are determined from the horizontal mixed-use or single-use development. Assume that the height of the lower structure is the same as the height of X floors in the upper structure. Then, Equation (6.1) should be rewritten as

(6.5) $$V\big(S_L(t), S_H(t), K_L(t), K_H(t)\big) = Max\{0, S_L(t) * e^{\alpha \times X} * A_L + S_H(t) * A_H - K_L(t) * A_L - K_H(t) * A_H - C, e^{-r\Delta t} E\big[V\big(S_L(t+\Delta t), S_H(t+\Delta t), K_L(t+\Delta t), K_H(t+\Delta t)\big)\big] \ (0 \le t \le T).$$

The coefficient of the height premium α is predetermined by hedonic regression on transaction records. This equation is a generalised vertical mixed-use development pricing model.

6.5 Chapter summary

This chapter establishes a 'new' real option approach to value the vertical mixed-use developments. In view of the characteristics of vertical mixed-use developments, the effects of different factors on option value and expected exercise timing are examined.

The first finding in a vertical mixed-use development relates to the influences of market uncertainty (i.e. volatility) in different uses. The initial total development value in a specific use, instead of the unit market value or GFA, determines the influence of market volatility in this use on the project value and expected exercise timing. In a retail–residential mixed-use project, if the revenue is primarily from the residential units, then the residential market volatility has a greater effect on project decisions. This statement holds true even when the unit market price in the retail market is higher.

The second and the more important finding is about the valuation criteria to determine the building type in a mixed-use development. A parameter called critical height premium is introduced to assist the developers' decisions. If the estimated height premium in this area is larger than the critical height premium, then the property should be vertically developed. Otherwise, it should be horizontally built. In a retail–residential mixed-use project, the property tends to be horizontally developed if the retail market volatility becomes higher. If the major project revenue is from the residential part, then the vertical type is more likely to be adopted.

Finally, an adjustment to the vertical mixed-use development option model is provided to contain a height premium if similar vertical mixed-use properties are insufficient to estimate the market value of the upper structure.

CHAPTER 7 CONCLUSIONS

7.1 Introduction

This chapter summarises the major research findings in previous chapters and emphasises the contributions in this study. Four research objectives were achieved completely. We introduced the constant depreciation rate assumption and a parameter, namely, annual increase in average building age, to describe the depreciation effect in different market conditions (Objective 1). Then, the influence of depreciation effect on project value and expected waiting time to demolish and rebuild were found and discussed based on real option approach (Objectives 2 and 3). In addition, the option pricing model for vertical type of mixed-use buildings in the rebuilding phase was developed. The differences of option values between vertical and horizontal types were compared (Objective 4). The limitations of the current study are also discussed in this chapter along with suggestions for future research directions.

7.2 Summary of major research findings

7.2.1 Importance and measurement of depreciation effect in redevelopment options

The first significance of this study is the emphasis on the necessity of considering the depreciation effect in redevelopment projects in the real option approach. Depreciation adjustments can help investors simultaneously predict future values of old properties to be demolished and new properties to be built. Otherwise, investors must predict the two values from new and old property price indices with considerably great efforts to exclude the differences in locational characteristics between the two indices.

In the measurement of depreciation effect, four advantages were provided to support the constant depreciation rate assumption as follows: 1) The additive property supports the reliability in the calculation of average building age. 2) On the basis of this assumption, the depreciation rate can be directly derived from the coefficients in the hedonic pricing model. 3) If constant depreciation rate is adopted, the then market volatility is the same as the case without depreciation effect. Meanwhile, the adoption of quadratic depreciation effect will lead to a biased market volatility and unreliable prediction in the property values. 4) An average depreciation rate in this assumption can be conveniently compared with investors' expected capital return rate.

A new innovative parameter, namely, annual increase in average building age, was introduced to reflect the building age changes for properties used in the market price statistics (e.g. price indices). This new parameter increases the accuracy in depreciation adjustments for future property values in the long-run option. The relationship between this parameter and the types of market indices was also discovered. The value of this parameter should generally fall between 0 and 1 year.

7.2.2 Influence of depreciation effect on project value

A finite discrete-time model with depreciation adjustments was developed to investigate how the depreciation effect influences project value. Two major relationships were explored from the simulations results. Firstly, constant depreciation rate and the annual increase in average building age contribute to diminish the underestimation of option value in traditional redevelopment option models. They have the most significant effects on option value compared with other traditional factors. Secondly, the effects of volatility and interest rate on project value decrease if the depreciation rate becomes larger. If the depreciation effect becomes stronger, then the market uncertainty from volatilities and interest rates would have weaker influences on project decisions.

In a case study on KC-009, the influences of period length in each phase and different types of market indices on project value were discovered. If the project value is estimated based on indices from a fixed group of completed properties, then ignoring the depreciation effect will lead to a 34.4%–39.8% value difference in a 15-year redevelopment project, which is larger than the differences from other types of indices. In addition, when the indices from a fixed group of completed properties are adopted, a longer maximum period in the demolition phase or in the rebuilding phase (from 5 years to 10 years) will result in a larger increase (approximately 12.7%) in project value compared with the cases based on other types of price indices (less than 9.2% in this case study). However, if the total redevelopment period is fixed, a longer period of demolition will lead to a smaller increase (less than 0.01%) in project value when the traditional repeat-sales indices are adopted compared with the cases based on other types of price indices (more than 1.4% in this case study).

7.2.3 Influence of depreciation effect on optimal redevelopment strategy

A novel continuous-time redevelopment option model with depreciation adjustments was developed to reveal the influence of depreciation effect on optimal redevelopment strategy. Monte Carlo simulation results indicated that ignoring the depreciation effect would cause severe overestimation of expected exercise timing. When 1% p.a. depreciation rate is adopted, the conditional expected exercise time is about 6.6–8.3 years, depending on the annual increase in average building age. If the depreciation rate is set as zero, this expected exercise time significantly increases to approximately 36.7 years. Depreciation rate is the most significant factor in determining the expected exercise timing in this long-term option pricing model.

Furthermore, in high-depreciation scenarios, the traditional optimal simultaneous redevelopment strategy suggested by Chen and Lai (2013) only remains optimal in four of nine simulated cases. A high depreciation rate (1.5% p.a. or above) and/or low capital return rate (10% p.a. or below) makes this strategy unfeasible after several years or even unfeasible in the entire period. An alternative strategy was suggested by the authors. This alternative strategy chooses the optimal rebuilding ratio as the trigger of demolition and rebuilding phases simultaneously. It is the optimal choice in four of nine cases. In some extreme scenario (depreciation rate = 1.5% p.a. and capital return rate = 6% p.a. in this study), the developers should wait for another optimal rebuilding timing when the demolition is completed. These findings reject that the traditional simultaneous redevelopment strategy is consistently optimal when the option period is infinite.

Finally, we found that a higher acquisition standard, which assumes a lower building age of the old property, results in a longer waiting time for redevelopment. For example, suppose the depreciation rate is 0.5% p.a. If a 50-year-old property is acquired based on a 7-year-old property value, the expected waiting time for redevelopment will be extended from 19.7 to 25.4 years. This effect will be amplified by the depreciation rate. Assume that the depreciation rate becomes 1% p.a. For the same acquisition, the expected waiting time for redevelopment will be extended from 7.5 to 16.2 years. The relationship between acquisition standard and waiting time for redevelopment has yet to be discussed in previous real option models.

7.2.4 Findings in vertical mixed-use development

On the basis of the specific characteristics in the construction process, Chapter 6 presented a newly developed option pricing model for vertical mixed-use developments, such as building forms in some Asian metropolitans. The simultaneous construction of property units in multiple uses lead to considerable differences in project values and optimal development timing compared with horizontal mixed-use development.

The most innovative finding is about how the market uncertainty influences the developers' decision on developing a mixed-use project horizontally or vertically in a revenue aspect. The key is to compare the flexibility premium in the horizontal type with the height premium in the vertical type. The flexibility premium is the value premium when property units in two uses can be constructed at different timings compared with the case where these units must be built simultaneously. The height premium is the value premium when height-sensitive property units are built on an artificial podium compared with the cases where these units are built on the ground. If the market volatility in relation to the property type of the lower structure becomes higher, then the vertical type is generally less likely to be adopted. However, the market volatility in relation to the property type of the upper structure can either increase or decrease the chance to adopt vertical type because this volatility influences the threshold of vertical type in two opposite ways.

In addition, if the revenue of a project is largely from its upper structure, then the vertical-type development is usually a superior choice. According to project information in Kwun Tong Town Centre, if the height premium is based on the case study in Chapter 4 ($e^{0.006*6} - 1 = 3.67\%$), then the vertical type should be adopted in half of simulated scenarios. However, if the GFA for residential use is smaller than that for retail use, then the vertical type is superior in only less than 1/3 of scenarios.

The authors also introduced an extended option model that includes the height premium term in the intrinsic option value, which will aid investors avoid the estimation bias in height premium due to the different heights in the lower structure of other properties.

7.3 Contributions

This study has important theoretical and practical contributions to the decision making in redevelopment projects. In comparison with traditional models adopted to achieve the project value and optimal decisions, this study sheds light on the manner in which redevelopment project value and redevelopment timing are affected by the depreciation adjustments and the vertical type of mixed-use structures.

(1) A comprehensive discussion is conducted to explain why the depreciation effect should be considered in the real option approach. The necessity of measuring the depreciation effect comes from: a) the increasing difference in building ages between old and new properties and b) the concentrated location characteristics in transaction records for newly completed properties. Previous redevelopment real option models have failed to emphasise the depreciation effect, especially when the future values of more than one property must be estimated. This gap is filled comprehensively in this study.

To describe the depreciation effect, two parameters, namely, constant depreciation rate and annual increase in average building age, are introduced. Both components bridge the gaps in the measurement of depreciation effect, thereby enabling the improved prediction of the future values of new and old properties in the redevelopment option and assisting in developing new option pricing models for two-phase redevelopment projects. The discussions on the two components establish a good example to demonstrate how to handle two or more properties at different ages in an option pricing model for redevelopment. The results in this study are not only applicable to redevelopment projects in Hong Kong. In many other Asian cities, the conditions and policies for redevelopment projects for multi-owner buildings may significant differ (as mentioned in Section 1.2.4). The new models in this study may need further revisions for other practical cases. However, this study can be used as a reference in adjusting the depreciation effect appropriately in redevelopment project decisions, by choosing proper values for the building age of the target old property (based on the acquisition standard), for the annual increase in average building age and for the depreciation rate. To increase the accuracy of decisions in a specific project, these values should be estimated from transaction records in the nearby properties, instead of the records in the whole city.

(2) The findings in two real option models with depreciation adjustments extend our

knowledge about the importance of depreciation effect in real option valuation. The depreciation effect influences the project value and redevelopment timing directly and indirectly. Hence, market participants should be aware of depreciation adjustment, which is not only conducted at the beginning of the option to adjust the initial value but also during the entire pricing process in a project with more than one property.

The discussions on the real option approach with depreciation effect benefits investors and governments in three ways. Firstly, new reliable models are suggested to predict the profitability of new redevelopment projects. In addition, the investor behaviours, such as the change of redevelopment timings, can be explained more comprehensively by incorporating the depreciation effect in real option models. Secondly, convincing pieces of evidence are provided to require the developers/government to increase the estimation accuracy of the depreciation rate. The empirical study in Chapter 4 sets a good example for estimating the depreciation rate in a specific redevelopment project with limited transaction records. The estimation process can be further improved if the developers/government has sufficient and detailed transaction data. Thirdly, the new models can be used to predict effectively the effect of changes in land lease condition on a redevelopment project, such as the length of phase period and the acquisition standard for old properties.

(3) The real option model for vertical mixed-use development provides an appropriate approach for appraisal in Asian cities. In this type of mixed-use development, the developer should focus on the optimal timing for simultaneous construction of property units in different uses rather than solely the proportion of each use. This model offers a better understanding of the influence of market price uncertainties on different types of uses. Developers should also be aware of the lease conditions and building regulations, which will cause considerable differences in the pricing process compared with the horizontal type.

This comparison can be applied in the appraisal of different designs to enable optimal decisions in project planning. Policy makers can also predict the investor preference for vertical or horizontal type if the uncertainty in volatility or interest rate changes. The importance of flexibility premium (and height premium, as explained in Section 7.2.4) are emphasised in mixed-use development decisions.

The vertical and horizontal development models can be further applied to complicated projects. For example, assume that a mixed-use project is developed in multiple phases. Then each phase is a vertical mixed-use building with a flexible amount of property units. The flexibility premium in horizontal type and height premium in vertical type should be considered in this multiphase project. Market participants can benefit from this study by evaluating the different premiums embedded in an actual mixed-use development and adopting the optimal development strategy to maximise the revenue of an entire project.

7.4 Limitations and future research directions

This study still has some limitations. Firstly, similar to other research works about redevelopment pricing model, this study focuses on model construction and simulations. To test the influence of depreciation effect, sufficient market information from at least two cities must be collected based on consistent standards in the same sampling method. The dataset will become even smaller when the transaction records are limited to multi-owner residential buildings. However, the establishment of a multi-city transaction dataset for multi-owner residential buildings is outside the scope of this study. Further empirical tests can be supplemented to enhance the findings if sufficient market data are available.

Secondly, the measure of effective ages based on maintenance status (see Section 3.3.3) is not covered in this study. The depreciation here is based on natural building age. The major reason is that maintenance history for properties is not provided in either EPRC or Centa-data, which are official data sources. Developers should pay attention to the maintenance status and its influence on effective age to estimate the amount of depreciation accurately. However, the measure standard is also outside the scope of this study. Public organisations or private companies are recommended to apply their own criterion to determine the effective ages of related properties. The depreciation rate is derived based on these effective ages. Similarly, this study has not discussed the functional depreciation, which points to the value loss from an outdated design or an outdated use. Some adjustments should be embedded in the models in practice. For the old building, the value decrease due to functional obsolescence can be included by the initial depreciation adjustment of $e^{(T_{old}-T_{ave})\xi}$. If the new property has some special new functions which can increase the property value, a functional premium can be added to the estimated price. The adjustment of annual depreciation would be treated the same as the case when effective ages are used. Furthermore, other mature Geographic-Information-System-based decision supporting systems [e.g. models suggested by Chrysochoou et al. (2012), Oh (2001) and Thomas (2002)] can be introduced to provide reliable information for the status of property maintenance.

Thirdly, the influence of acquisition standards on expected waiting time for redevelopment can be further investigated. Future studies can quantify the relationship between acquisition standards and lengths of negotiation periods. The negotiation period can then be added to the expected waiting time to demolish old properties. This adjustment will make the relationship between acquisition standards and expected waiting time become closer to the actual situation. Policy makers can also have accurate predictions about the redevelopment participant behaviours if the minimum acquisition standard is amended. However, this limitation is a willingness-to-accept problem for original residents in the redevelopment area, which is not the focus of this study.

7.5 Chapter summary

After a comprehensive review of the major findings in Chapters 4–6, the contributions in this study are summarised. Three limitations and future research directions are then discussed.

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