THREE STUDIES ON BEHAVIORAL OPERATIONS MANAGEMENT IN E-COMMERCE: RETURN POLICY, LABOR DELIVERY, AND NEW RETAIL

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Three Studies on Behavioral Operations Management in E-commerce: Return Policy, Labor Delivery, and New Retail

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CERTIFICATE OF ORIGINALITY

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Abstract

Behavior-based analysis and strategy are advantageous to firms by enabling them to improve the marketing effectiveness. Although Behavioral Operations Management (BOM) has attracted the attention of researchers for a decade, many aspects are still left for research due to the complexity of human behavior and the development of technology, especially in e-commerce. In this thesis, we conduct three studies on BOM in e-commerce, considering different behavioral aspects in terms of return policy, labor delivery, and new retail.

In the first study, we develop a series of consumer-valuation-based models to investigate the pricing and return policies of the sellers in a competitive e-commerce market. A novel two-dimensional valuation structure is built, which considers the valuations of a consumer on two products and the valuation differentiation of all consumers on each product. Besides, consumers are uncertain about the difference between their pre-purchase and post-purchase valuations, and are sensitive to the return policies. We consider both monopoly and duopoly (competitive) markets. In each market, two models are respectively developed, one with and one without the return policies. The monopoly models are formulated as nonlinear optimization programs, while the duopoly models are investigated using game theory. The return policy is characterized by the refund proportion to the consumer if a product is returned. We derive the optimal or Nash equilibrium solutions for the four models, and conduct some analytical and numerical investigations. The results show that return policy with a partial refund is always chosen by the sellers in both monopoly and duopoly markets. Return policy benefits the seller in a monopoly market, but may not benefit the sellers in a duopoly market. In the duopoly models, one seller can be considered as the seller in the monopoly model who meets a new competitor. The

seller's prices and revenues in the duopoly market are respectively lower than those in the monopoly market. Besides, the equilibrium prices in the duopoly models cannot be lower than 80% of the corresponding optimal prices in the monopoly models, which indicates that a monopoly seller will reduce its price by no more than 20% when there comes a competitor. Counter-intuitively, the monopoly seller will also reduce its refund proportion to consumers when it meets a competitor in the market.

In the second study, we focus on the labor participation behavior for the product delivery under fluctuating demand in e-commerce. We consider a model with peak and non-peak periods, where two wages are offered to the labors respectively. The labors are heterogeneous in their opportunity costs, and choose to participate in the product delivery or not by themselves. We first find the optimal wage decisions in the peak and non-peak periods to maximize the profit. Based on the optimal wages, we can determine the number of participating labors, their utilizations, and performance of the logistics system. Then we analyze the impact of the parameters, such as labor pool size, demand, labors' opportunity costs and consumer elasticity of delivery speed, on the optimal wage decisions.

In the third study, we consider "new retail" in e-commerce. In e-commerce, an online retail channel is traditionally offered to consumers for purchasing products to be delivered to them directly. The concept "new retail" is to establish an offline channel and integrate it with the online retail channel. The development of new retail encounters three main problems: locations of the offline stores, the price competition with the other traditional online retail, and the difficulty in consumer recognition in the two channels. We present a duopoly model consisting of a new retail firm and an online firm, which sell the same product in two periods. The two firms compete for the market share using the behavior-based pricing (BBP), which means that in the second period each firm offers different prices to consumers with different purchasing histories/behaviors in the first period. We also solve the benchmark model, where the histories/behaviors are not considered. The results provide valuable insights into the development of new retail in e-commerce. In the Nash equilibrium, prices of the new retail firm are higher than the corresponding prices of the online firm due to a higher channel cost for the offline stores and high-speed deliveries. Under certain condition, the new retail firm will establish an offline channel with a larger hassle cost (a measure of the easiness of reaching the offline stores by the consumers) in the BBP model than that in the benchmark model. Interestingly, the difficulty in consumer recognition results in that the new retail firm occupies more market share and may obtain a higher profit than that when the consumers are all recognized.

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Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction

Behavioral Operations Management (BOM), as a part of operations management, considers the human behaviors when people face decision problems. With the analysis of the human behaviors, firms can develop behavior-based strategies to improve the marketing effectiveness and benefits of e-commerce.

With the rapid development over the last 20 years, the current situation of e-commerce still attracts the attention of researchers. First, the competition of e-commerce is more and more intense. The reason is that the application and popularization of the information technology lower the threshold of participating in the e-commerce market. Second, the growth rate of the sales amount in e-commerce is continuously decreasing, from 50% in 2011 to 21.3% in 2017. Thus, it is crucial for the e-commerce firms to find a new way for further growth.

Considering human behaviors and the current situation of e-commerce, we conduct three studies on BOM in e-commerce, regarding different behavioral aspects in terms of return policy, labor delivery, and new retail. Our aim is to improve the operations management decisions of the e-commerce firms on the pricing and return policies in competition, the labor employment decisions with fluctuating demand, and the channel development of new retail in competition.

First, human behaviors in e-commerce mainly exist in the process of consumers' product purchasing and returns. The consumers make the purchasing and return decisions to benefit more. In the online purchasing channel, it is quite convenient for the consumers to find the products they want, observe the product information, valuate the products, compare the products from different retailers, and make their purchasing decisions. They probably also have the right to return the products if they want. For the e-commerce retailers in competition, it is vital to properly price their products and set return policies based on the consumers' valuations.

In Chapter 2, we investigate the pricing and return policies in a competitive market by developing a novel two-dimensional consumer-valuation-based model. We consider one consumer's valuations on two products and the differentiation of all consumers' valuations on each product. Besides, consumers are uncertain about the difference between their pre-purchase and post-purchase valuations, and are sensitive to the return policies. We consider both monopoly and duopoly (competitive) markets. In each market, two models are respectively developed, one with and one without the return policies. The monopoly models are formulated as nonlinear optimization programs, while the duopoly models are investigated using game theory. The return policy is characterized by the refund proportion to the consumer if a product is returned. The results show that return policy with a partial refund is always chosen by the sellers in both monopoly and duopoly markets. By comparing the monopoly and duopoly models, we conclude that a monopoly seller will reduce its price by no more than 20% when there comes a competitor, and counter-intuitively, the monopoly seller will also reduce its refund proportion to consumers when it meets a competitor in the market.

Second, human behaviors in e-commerce also exist in the labor employment market for product delivery. The labors choose to participate in the market based on the wages they earn. The main difficulty in e-commerce is the labor supply under fluctuating demand. Based on the demand, the e-commerce retailers decide the wages to labors, which determine the labor supply and the delivery performance. Additionally, the decisions should vary as the society develops. The human population may change and influence the pool size of potential labors. Also, new job opportunities may emerge and increase the opportunity cost to the labors. Thus, it is important to study how to make the labor employment decisions in e-commerce.

In Chapter 3, we address the challenge of delivery under fluctuating demand in the perspective of the labor participation behavior. We consider an e-commerce logistics system with alternately peak and non-peak demand periods. The logistics system decides the different wages to labors in the peak and non-peak periods. The labors are heterogeneous in their opportunity costs, and choose to participate in the product delivery or not by themselves. The labor supply in the peak periods, which indicates the delivery speed, will impact the consumers' purchasing behaviors. We first find the optimal wage decisions in the peak and non-peak periods to maximize the firm's profit. Based on the optimal wages, we can determine the number of participating labors, their utilizations, and performance of the logistics system. Then we analyze the impact of the parameters, such as labor pool size, demand, labors' opportunity costs and consumer elasticity of delivery speed, on the optimal wage decisions and logistics performance. We conclude that the increase in the labor pool size or the decrease in the labor opportunity cost, which causes more labors to participate, does not necessarily decrease the wages and increase the labor supply, and opposite impact may happen under different conditions. The increase in the part-time labor employment cost may result in more labors employed in the non-peak periods. However, the increased number of full-time labors results in lower labor utilization in the non-peak periods.

Third, as the growth rate of e-commerce decreases, the firms in practice are developing the "new retail" to achieve further growth. The concept of "new retail" is to establish an offline channel and integrate it with the online retail channel. The consumers can experience the product from the offline channel to eliminate the valuation uncertainty, which always exists in the traditional online channel. They can purchase from both the offline and online channels. However, the development of offline channel is not necessarily advantageous because of the higher channel cost than that of the online channel. Moreover, the price competition with the traditional online channel is also a challenge to the development of new retail.

In Chapter 4, we present a duopoly model consisting of a new retail firm and an online firm, which sell the same product in two periods. The two firms compete for the market share using the behavior-based pricing (BBP), which means that in the second period each firm offers different prices to consumers with different purchasing histories/behaviors in the first period. The development of new retail encounters three main problems: locations of the offline stores, the price competition with the other traditional online retail, and the difficulty in consumer recognition in the two channels. We also solve the benchmark model, where the histories/behaviors are not considered. We conclude that, in the BBP model, each price offered by the new retail firm is higher than the corresponding price of the online firm. Each firm offers lower prices to the competitor's consumers than its own consumers. Under certain condition, the new retail firm will establish an offline channel with a larger hassle cost (a measure of the easiness of reaching the offline stores by the consumers) in the BBP model than that in the benchmark model. Interestingly, the difficulty in consumer recognition results in that the new retail firm occupies more market share and may obtain a higher profit than that when the consumers are all recognized.

Chapter 2

Pricing and Return Policies in a Competitive Market: A Consumer-valuation Based Analysis with Valuation Uncertainties

2.1 Introduction

E-commerce has been rapidly developed in the recent 20 years. The most representative enterprises are eBay in America and Alibaba in China. Speaking of e-commerce, two phenomena are non-negligible. One is the intense competition of all e-sellers due to the low threshold for sellers to join the e-market. It only takes few hours to open one's own shop on the website. The other one is the great amount of consumer returns, because consumers can only observe the information of a product online and ascertain their valuations on the product after they receive it in an express box. This kind of products, whose valuations can only be ascertained after consumption, is called "experience goods" in economics. A return policy may be adopted by the sellers to manage consumer returns.

The intense competition and the return policy for consumer returns form a dilemma to the sellers. In e-commerce, consumers are sensitive to the severity of return policy because they want to return easily after purchase. A high level of severity of return policy may hold back the consumers from purchasing, which can be fatal to a seller in an intense competitive market. However, a lenient return policy may cause a loss to the seller because it leads to more returns and less revenue.

The purpose of our research is to investigate the pricing policy and return policy in a competitive market. Based on the background above, we summarize our research questions (RQs) as follows: (RQ1) Will a seller adopt return policy in a monopoly or duopoly market? (RQ2) What are the effects of return policy on a seller's price and revenue in a monopoly or duopoly market? (RQ3) What are the effects of competition on a seller's price, revenue and return policy, comparing the changes from a monopoly market to a duopoly market?

As a study in Behavioral Operations Management, we focus on the consumers' purchasing and return behaviors based on their product valuations. We develop a novel model involving a two-dimensional valuation structure, which considers one consumer's valuations on two products and the differentiation of all consumers' valuations on each product. To the best of our knowledge, there has been no research on consumer-valuation-based return policy under competition. Our study fills this gap.

To achieve our research aim, four models are developed: two monopoly models, one with and one without return policy; and two duopoly models, one with and one without return policy. The two monopoly models are formulated as nonlinear programming models, while the two duopoly models are investigated using game theory. The severity of return policy is characterized by the refund proportion to consumers if a product is returned.

We provide the optimal or Nash Equilibrium solutions for the four models and conduct some analytical and numerical studies to answer the RQs. Our results show that in both monopoly and duopoly markets, the sellers will adopt return policy (RQ1). In the monopoly market, adopting return policy does not affect the seller's price but brings the seller more revenue; in the duopoly market, return policy leads to lower prices and possibly lower revenues (RQ2). We consider one seller in the duopoly market as the seller in the monopoly market who meets a new competitor. The seller's price and revenue in the duopoly market are lower than those in the monopoly market; the price in the duopoly market cannot be lower than 80% of the optimal price in the monopoly market; additionally, the severity of return policy in the duopoly market is higher than that in the monopoly market (RQ3). These results provide management insights that a monopoly seller may reduce its price by no more than 20% when there comes a competitor, and the return policy can be severer in a competitive market than that in a monopoly market.

We present a literature review in Section 2.2. In Sections 2.3 and 2.4, we study the pricing and return policies in monopoly and duopoly markets, respectively. Analytical and numerical studies are conducted in Sections 2.5 and 2.6. The conclusions are given in Section 2.7.

2.2 Literature Review

Experience goods in e-commerce lead to consumers' post-purchase valuation uncertainty, so consumers may return the product after purchasing. Return policy is offered to the consumers to manage their returns. In the literature, there is a stream on consumer return policy considering money back (refund) in a monopoly market. Davis et al. (1995) treat the refund decision as dichotomous: whether or not to offer a full (100%) refund. Their research shows that offering a full refund can be more profitable than selling without returns under certain conditions. Che (1996) shows that full refund policy is more profitable when consumers are highly risk averse or selling costs are high. Davis et al. (1998) develop a model that allows the seller to reduce returns by altering the 'hassle' cost to the consumer for returning the product.

In these models, a seller offers either no refund or a full refund for a product. However, the seller may retain a portion of the price paid originally by consumers if returns are allowed. Hess et al. (1996) develop a model of partial refund to control inappropriate returns by opportunistic consumers. Based on this research, Chu et al. (1998) further study the distinction among 'no questions asked policy', 'no refunds' and 'verifiable problems only policy', and they find that the first policy is the optimal solution to handle consumer opportunism. In our study, the severity of return policy is measured by the proportion of refund to the consumers. A 100%

proportion represents a full refund policy and a proportion less than 100% represents a partial refund policy.

For the consumers, return policy means a protection for their purchasing decisions. Using a mechanism design approach, Akan et al. (2015) show that the optimal mechanism is a menu of expiring refund contracts when consumers observe their true valuations at different time epochs. However, Liu and Xiao (2008) show that with finite inventory, a menu of return policies that serves the entire population is no longer the optimal selling policy. Meanwhile, the seller is able to reduce the post-valuation uncertainty by providing information. Shulman et al. (2010) find that even if it is possible to eliminate returns without cost through the provision of information about the fit between consumer preferences and product characteristics, returns can nevertheless be part of the optimal product sales process. Our research analyses the impact of post-valuation uncertainty on the seller's revenue in monopoly and duopoly markets, by assuming a post-purchase uncertain component in consumers' valuations.

In addition to protecting consumers from post-valuation uncertainty, return policy is also useful to signal the product quality when consumers face a market with unknown product quality. Grossman (1981), Emons (1989), Moorthy and Srinivasan (1995) and Shieh (1996) comprise a stream of study demonstrating that return policy serves as providing an effective tool for high-quality sellers to distinguish themselves from the others and it costs more for the sellers whose product quality is lower to offer generous return policy. Another stream of study (Heal 1977, Matthews and Moore 1987, Welling 1989, Padmanabhan and Rao 1993) shows that a more generous return policy boosts consumer demand and increases seller's profit. To focus on our research aim, we assume that the seller provides no defective products that enable consumers to be satisfied in quality.

The literature above is on the return policy in a monopoly market. While in a competitive market, more studies are needed about how return policy works and whether the partial refund should be sustained. Guo (2009) builds on Xie and Gerstner's (2007) model to show that the sellers in a duopoly market should adopt identical partial refund policies for advanced sales only when the capacity is small such that the efficiency-improving effect is dominant. Shulman et al. (2011) use a Salop circle model to show that partial refund can be sustained in a competitive environment and the return policy is severer when the fit problem between products' attributes and consumers' preferences is more serious.

There is another stream of literature on the return policy throughout the supply chain. Cachon (2003) provides a review of the literature about return contracts between the retailer and the manufacturer including buy-back contract, quantity-flexibility contract and sales-rebate contract. Ferguson et al. (2006) propose a target rebate contract which is Pareto improving by providing an incentive to the retailer to increase its effort of reducing false failure returns. Xiao et al. (2010) investigate the adjustment of buyback contracts under different risk levels of consumers' valuations. Su (2009) studies the impacts of consumer returns on supply chain performance when the consumers exhibit valuation uncertainty and the manufacturer is confronted with demand uncertainty.

In sum, the main contribution of our work is the new competitive pricing and return policy models based on consumers' valuations under uncertainty. To the best of our knowledge, it is the first to incorporate one consumer's independent valuations on different products with all consumers' aggregate valuations on one product. It is worth mentioning that out of the four models we present, one is about pricing policy in a duopoly market without return policy. This model is a new pure price competition, which generates new insights on price decision of a monopolist facing a new joiner in the market.

2.3 Pricing Policy in a Monopoly Market

2.3.1 Assumptions and Notations

This section considers the pricing and return decisions of a monopolistic seller. The seller sells an experience good (product) to the consumers, and reserves the right to retain a portion of the price paid originally if a consumer returns the product. The seller makes two decisions: adopt a return policy or not (if he adopts the return policy, a refund proportion α will be given to the consumers if the product is returned, $0 \leq \alpha \leq 1$), and the price, p. The returned product has no salvage value to the seller. The seller's goal is to maximize the expected revenue.

The consumers are heterogeneous in their intrinsic preferences, which is represented by a pre-valuation v, and v is uniformly distributed over $[0, V]$. A post-purchase valuation uncertainty ϵ is observed only after a purchase is made, $\epsilon \sim U[-\delta, \delta]$. Therefore, the consumer's post-valuation is $v + \epsilon$. The value of δ is assumed to be less than $\frac{V}{2}$ following the assumption in Shulman et al. (2011). If one consumer purchases the product at price p, his utility is $v + \epsilon - p$.

Consumers make two decisions sequentially. Initially, they decide whether to purchase the product. If so, they decide whether to keep it after their valuation uncertainty is realized. Without loss of generality, we assume that the consumers have no hassle cost to make a return. They seek to maximize the expected utility of their own. The total volume of all consumers is normalized to one and one consumer will buy no more than one product.

The decision procedures are as follows. First, the seller determines price p, return policy W(with) or O(without), and refund proportion to consumers α if he adopts a return policy. Second, consumers make purchasing decisions based on their expected utility. Then the market demand D is realized. Third, each consumer who has bought the product decides whether to keep or return it. Till then, the revenue of the seller is finalized. Additionally, we focus on revenue management in the marketing process, so the production cost is out of our consideration. All the notations used in this chapter are presented in Table 2.1.

The next two subsections 2.3.2 and 2.3.3 are the studies on the monopoly models with and without return policy. Then Section 2.3.4 presents the comparison of the two monopoly models.

2.3.2 Monopoly Model without Return Policy

In this subsection, we first develop a simple monopoly model without return policy, which means the seller does not accept returned products. The seller sets the price

Notations	p	Price of product
	α	Refund proportion if return policy is adopted
	c	Return penalty to consumers, and $c = (1-\alpha)p$
	v	Pre-purchase valuation of consumer on product, $v \sim U[0, V]$
	ϵ	Valuation uncertainty of consumer after purchase, $\epsilon \sim U[-\delta, \delta]$
	$v+\epsilon$	Post-purchase valuation of consumer on product
Superscript and Subscript	i	Seller number, $i = 1$ for Seller 1 and $i = 2$ for Seller 2
	m, d	Market type, 'm' represents 'monopoly' and 'd' represents 'duopoly'
	O, W	Return policy, 'O' means 'without' and 'W' means 'with'
	j, k	Return policy of different seller, j for Seller 1 and k for Seller 2, $j, k = O$ or W

Table 2.1: Notations used throughout this study

p and only the consumers with non-negative expected utility will buy the product. With $\epsilon \sim U[-\delta, \delta]$, consumer's expected utility is $v + E(\epsilon) - p = v - p$, thus the threshold to buy the product is $v = p$. For description, we denote the probability density function of v as $f(v)$. The realized demand is $\int_p^V f(v)\,dv = \int_p^V \frac{1}{V}\,dv = \frac{V-p}{V}$. The seller's revenue without return policy is

$$\Pi_m^O = p\int_p^V f(v)\,dv = \frac{p(V-p)}{V}. \tag{2.1}$$

The following lemma characterizes the optimal price of the monopolistic seller without return policy.

Lemma 1. *The optimal price of monopolist without return policy is* $(p_m^O)^* = \frac{V}{2}$ *and the maximum revenue is* $(\pi_m^O)^* = \frac{V}{4}$.

2.3.3 Monopoly Model with Return Policy

In this subsection, we extend the analysis to the model with return policy, in which the seller offers to accept consumers' returns with a refund proportion α. Consumer's utility may be one of the following three cases. If they purchase and keep the product, the utility is the post-valuation minus price, that is, $v + \epsilon - p$. If they purchase but return the product, the utility is the returned money minus price, $\alpha p - p$. If they do not purchase, they receive zero utility.

To identify those consumers who may return products back after purchase, we choose one consumer with pre-valuation v'. It is clear that consumers keep products with $v' + \epsilon - p > \alpha p - p$, that is, $v' > \alpha p - \epsilon$. With $\epsilon \sim U[-\delta, \delta]$, a threshold v_r is defined on consumers' return and $v_r = \alpha p + \delta$.

When v' exceeds v_r, the consumer never returns because his pre-valuation is high enough to keep the product ($v' > \alpha p - \epsilon$ anyway), and return policy is not needed to them. The consumer's utility remains to be $v' + E(\epsilon) - p = v' - p$.

When v' is smaller than v_r, we rewrite $v' < \alpha p - \epsilon$ to $\epsilon < \alpha p - v'$. The consumers with $\epsilon < \alpha p - v'$ will return the product and gain $\alpha p - p$, and the consumers with $\epsilon \geq \alpha p - v'$ will keep the product and gain $v + \epsilon - p$. Thus, the expected utility in the return range 2δ is $\int_{\alpha p - v'}^{\delta} \frac{v' + \epsilon - p}{2\delta}\, d\epsilon + \int_{-\delta}^{\alpha p - v'} \frac{\alpha p - p}{2\delta}\, d\epsilon$.

Thus, if the purchase is made, the expected utility is

$$EU = \begin{cases} \alpha p - p, & \text{when } v' < \alpha p - \delta \\ \int_{\alpha p - v'}^{\delta} \frac{v' + \epsilon - p}{2\delta}\, d\epsilon + \int_{-\delta}^{\alpha p - v'} \frac{\alpha p - p}{2\delta}\, d\epsilon = \frac{(v' + \delta - \alpha p)^2}{4\delta} + \alpha p - p, & \text{when } \alpha p - \delta \leq v' < v_r \\ v' + E(\epsilon) - p = v' - p, & \text{when } v' \geq v_r. \end{cases} \tag{2.2}$$

Consumers purchase products when EU is non-negative. When $v' < \alpha p - \delta$, $EU = \alpha p - p < 0$, the consumers will not buy the product. When $\alpha p - \delta \leq v' < v_r$, we can establish a pre-valuation threshold to buy the product, v_b ($EU|_{v=v_b} = 0$). The realized buyers are consumers whose pre-valuations are not less than v_b, and $v_b = \alpha p - \delta + 2\sqrt{\delta p(1 - \alpha)}$. When $v' \geq v_r$, the threshold of consumers to buy the product is $v' = p$.

Moreover, if $p > v_r$, $EU|_{v'=v_r} = v_r - p < 0$. Only the consumers with $v' > p$ will buy the product, and they will not return the product because $p > v_r$. The model becomes the same with the model without return policy. We show in the proof for Lemma 2 that the seller will make $p \leq v_r$ to gain a higher revenue, and the consumers with $v' > v_b$ will buy the product.

Thus, the realized demand is $\int_{v_b}^{V} f(v)\, dv$. The expected returned amount of products is $\int_{v_b}^{\min\{v_r, V\}} \frac{v_r - v}{2\delta} f(v)\, dv$. The seller returns a refund proportion of αp to consumers who purchase and return. The seller's revenue with return policy is

$$\Pi_m^W = p \int_{v_b}^{V} f(v)\, dv - \alpha p \int_{v_b}^{\min\{v_r, V\}} \frac{v_r - v}{2\delta} f(v)\, dv. \tag{2.3}$$

The following lemma characterizes the optimal price and return policy (refund proportion) of a monopolistic seller considering return policy.

Lemma 2. *The optimal price and refund proportion of the monopolistic seller are* $(p_m^W)^* = \frac{V}{2}$ *and* $(\alpha_m^W)^* = 1 - \frac{\delta}{2V}$*. The maximum revenue is* $(\pi_m^W)^* = \frac{V}{4} + \frac{\delta^2}{16V}$*.*

2.3.4 Comparison of the Two Monopoly Models

By observing the optimal solutions of the two monopoly models, the following proposition demonstrates that in a monopoly market, return policy always benefits the seller.

Proposition 1. *The optimal prices of the monopolistic seller with and without return policy are equal, that is,* $(p_m^O)^* = (p_m^W)^* = \frac{V}{2}$*. The optimal refund proportion to consumers* α *is smaller than* 100%*. Return policy brings the seller incremental revenue* $\frac{\delta^2}{16V}$*, which increases in* δ*.*

Proposition 1 indicates that the seller will always allow returns and provide a partial refund to the consumers. The seller's profit increases in the post-purchase uncertainty δ. Meanwhile, the refund proportion α decreases as δ increases according to Lemma 2. These results show that as the consumers' post-purchase uncertainty δ increases, the firm will decrease the refund proportion to the consumers and the firm's revenue increases consequently.

2.4 Pricing Policy in a Duopoly Market

Now, we turn to the duopoly models. These models are developed to study the prices and return policies in a competitive market. Two substitute products, which are experience goods, are respectively sold by two sellers. The two products are not perfect substitutes, which means one consumer may assess different valuations on them. One consumer's individual pre-valuations on these two substitutable are denoted by v_1, v_2, respectively. We assume that they are uniformly distributed and independent, that is

$$f(v_1, v_2) = \begin{cases} f(v_1)f(v_2) = \frac{1}{V_1 V_2}, & \text{when } 0 \le v_1 \le V_1 \quad \text{and} \quad 0 \le v_2 \le V_2 \\ 0, & \text{otherwise.} \end{cases} \tag{2.4}$$

Post-purchase valuation uncertainties exist, which are denoted by ϵ_1, ϵ_2, and $\epsilon_1 \sim U[-\delta_1, \delta_1]$ and $\epsilon_2 \sim U[-\delta_2, \delta_2]$. The same as in the monopoly model, the values of δ_1 and δ_2 are assumed to be less than $V_1/2$ and $V_2/2$, respectively. $0 < p_1 \leq V_1$ and $0 < p_2 \leq V_2$ are assumed to ensure the survivals of the two products in the market, for otherwise, no consumers will buy the product.

The two products are priced at p_1, p_2 by the two sellers. The sellers' decisions include the prices of products p_i $(i = 1, 2)$, return policies ("without" or "with") and refund proportions to consumers α_i $(i = 1, 2)$ if return policies are adopted. Both sellers seek to maximize their expected revenues.

Each consumer's decisions are as follows. Initially, he decides to buy from Seller 1, or buy from Seller 2, or leave the market. If he makes a purchase, he decides whether to keep the product based on his post-valuation. Each consumer wants to maximize individual expected utility. The total volume of consumers is normalized to one and one consumer will buy no more than one product.

This model captures both aggregate consumers' valuations on each product and one consumer's independent valuations on two products. It has a feature of two-dimension. Depending on the sellers' prices and return policies, consumers making purchase decisions allow us to define the market demands of both sellers. After consumers decide whether to keep the product or return it, the revenues of sellers are realized. Our aim is to find the Nash Equilibrium in the duopoly game.

2.4.1 Duopoly Model without return policy

In this subsection, we first investigate the duopoly model without return policy. Since prices are the only decision variables, the duopoly model is a pure price competition model. Based on the consumers' valuations, the two-dimensional valuation-based model is developed. Figure 2.1 demonstrates the demands realization in this model.

As mentioned before, each consumer gives two valuations v_1, v_2 to two experience goods (products) respectively. Observing the prices p_1, p_2 offered by two sellers, he has two expected utilities from purchasing the two products respectively: $v_1 + E(\epsilon_1) - p_1 = v_1 - p_1, v_2 + E(\epsilon_2) - p_2 = v_2 - p_2$ ($\epsilon_1 \sim U[-\delta_1, \delta_1]$ and $\epsilon_2 \sim U[-\delta_2, \delta_2]$). The consumer wants to maximize his utility, so he compares the three utilities: $v_1 - p_1$

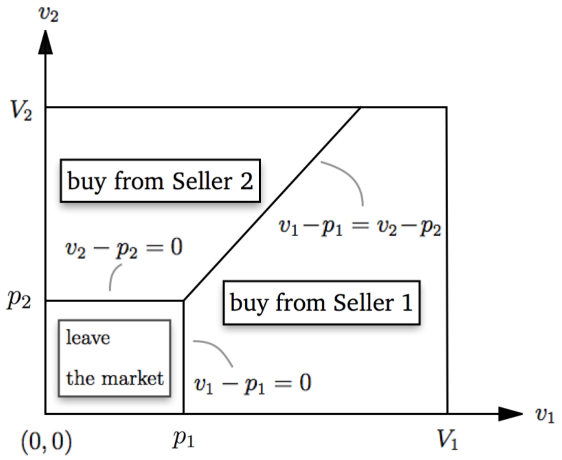


Figure 2.1: Market division in a duopoly market without return policy

(purchasing from Seller 1), $v_2 - p_2$ (purchasing from Seller 2) and 0 (leaving the market). This three utilities form three boundaries $v_1 - p_1 = v_2 - p_2$, $v_1 - p_1 = 0$ and $v_2 - p_2 = 0$ between consumers with different choices. Hence, from Figure 2.1, the demands of the two sellers are determined. Multiplying prices, the revenues of the two sellers are

$$\Pi_{d1}^{O} = p_1 \int_{p_1}^{V_1} \int_{0}^{\min\{v_1 - p_1 + p_2, V_2\}} f(v_1, v_2)\, dv_2\, dv_1, \tag{2.5}$$

$$\Pi_{d2}^{O} = p_2 \int_{p_2}^{V_2} \int_{0}^{\min\{v_2 - p_2 + p_1, V_1\}} f(v_1, v_2)\, dv_1\, dv_2. \tag{2.6}$$

By the calculations of giving any p_1 to find the optimal p_2 and the same in reverse, we find that the prices of two sellers in Nash Equilibrium are the solutions to the following equation set. The proof for equation set 2.7 is given in the Appendix A.

$$\begin{cases} (V_1 - 2p_1)\min\{V_1 - p_1 + p_2, V_2\} = \int_{p_2}^{\min\{V_1 - p_1 + p_2, V_2\}} (v_2 - p_2)\, dv_2 \\ (V_2 - 2p_2)\min\{V_2 - p_2 + p_1, V_1\} = \int_{p_1}^{\min\{V_2 - p_2 + p_1, V_1\}} (v_1 - p_1)\, dv_1 \end{cases} \tag{2.7}$$

We denote the equilibrium prices as p_{d1}^{O}, p_{d2}^{O}, whose close-form solutions are too complex to be given. However, we can find a range in which they exist. It is obvious that $V_i - 2p_{di}^{O} > 0$ $(i = 1, 2)$, so the equilibrium price p_{di}^{O} is smaller than $\frac{V_i}{2}$. Additionally, Figure 2.2 shows that given any p_1, there will be an optimal p_2 and the same in reverse p_1, which form the functions $p_2(p_1)$ and $p_1(p_2)$ (the two full lines).

It is proved that $\frac{\partial p_1(p_2)}{\partial p_2} > 0$, $\frac{\partial p_2(p_1)}{\partial p_1} > 0$, $\frac{\partial^2 p_1(p_2)}{\partial p_2^2} < 0$, and $\frac{\partial^2 p_2(p_1)}{\partial p_1^2} < 0$. The two functions $p_1(p_2)$ and $p_2(p_1)$ are concave, so we can draw the two dashed lines which cross at the point $(\frac{2V_1}{5}, \frac{2V_2}{5})$. Thus, we obtain that p_{di}^O is larger than $\frac{2V_i}{5}$ $(i = 1, 2)$.

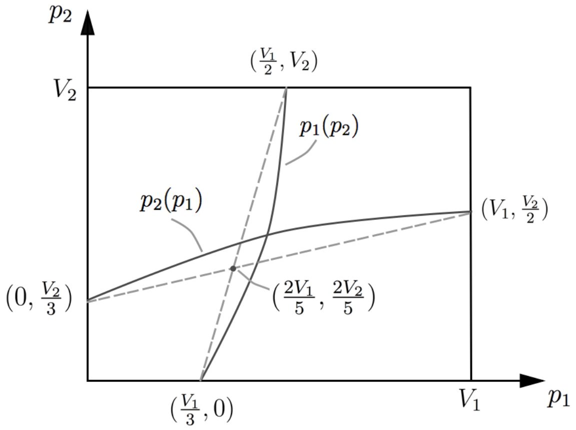


Figure 2.2: The ranges of the prices in Nash Equilibrium in the duopoly model

Lemma 3. *In a duopoly market, the equilibrium prices of two sellers without return policy are* p_{d1}^O *and* p_{d2}^O. p_{di}^O *is larger than* $\frac{2V_i}{5}$ *and smaller than* $\frac{V_i}{2}$, $i = 1, 2$.

Next, we consider the duopoly model with return policy. Then we will compare the prices and profits of the two sellers in the duopoly models with and without return polices, to reveal the impact of return policies.

2.4.2 Duopoly Model with return policy

In the duopoly model with return policy, both sellers not only decide their prices p_1, p_2, but also settle on whether to adopt return policies. They both may adopt or abandon the return policy. Hence the game shows up as in Table 2.2.

Seller 2\Seller 1	without	with
without	$(\Pi_{d1}^{OO}, \Pi_{d2}^{OO})$	$(\Pi_{d1}^{WO}, \Pi_{d2}^{WO})$
with	$(\Pi_{d1}^{OW}, \Pi_{d2}^{OW})$	$(\Pi_{d1}^{WW}, \Pi_{d2}^{WW})$

Table 2.2: The game in a duopoly market

If a seller adopts the return policy, he decides the refund proportion to consumers if they return. We still employ α_i $(i = 1, 2)$ to denote the two sellers' refund proportions to the consumers.

Each consumer has two valuations v_1, v_2 on the two experience goods. He observes two prices p_1, p_2, probably with return policy α_1, α_2. The consumer's expected utilities for "with" and "without" return policies are different. For the convenience of description, we denote the expected utility of the consumer with his valuation v_i without return policy as $u_i^O(v_i)$ and the expected utility with return policy as $u_i^W(v_i)$. The same as in Section 2.3, we have

$$u_i^O(v_i) = v_i + E(\epsilon_i) - p_i = v_i - p_i, \tag{2.8}$$

$$u_i^W(v_i) = \begin{cases} \frac{(v_i+\delta_i-\alpha_i p_i)^2}{4\delta_i} + \alpha_i p_i - p_i & \text{when } v_i < v_r, \\ v_i - p_i & \text{when } v_i \geq v_r. \end{cases} \tag{2.9}$$

The consumer maximizes his utility by comparing $u_1^j(v_1)$, $u_2^k(v_2)$ and 0 ($j, k = O$ or W). Again, the three utilities generate three boundaries that divide consumers into three parts: buy from Seller 1, buy from Seller 2, leave the market. The boundaries are $u_1^j(v_1) = u_2^k(v_2)$, $u_1^j(v_1) = 0$ and $u_2^k(v_2) = 0$ ($j, k = O$ or W).

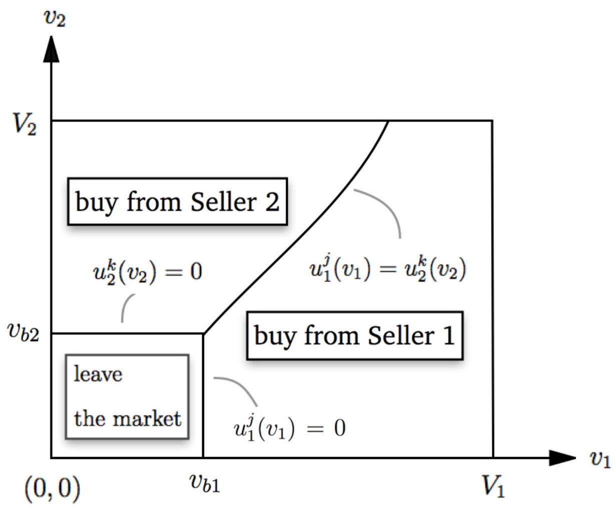


Figure 2.3: Market division in a duopoly market with return policy

From Figure 2.3, the points $v_i = v_{bi}$ ($i = 1, 2$) are derived from $u_1^j(v_1) = 0$ and $u_2^k(v_2) = 0$. Consumers whose pre-valuations are not less than v_{bi} will consider to buy from Seller i ($i = 1, 2$). In the same way as in Section 2.3.3, we have $v_{bi} = p_i$ when the seller abandons the return policy, and $v_{bi} = \alpha_i p_i - \delta_i + 2\sqrt{\delta_i p_i(1-\alpha_i)}$ when the seller adopts the return policy.

The boundary $u_1^j(v_1) = u_2^k(v_2)$ is re-written as $v_2 = (u_2^k)^{-1}(u_1^j(v_1))$ and $v_1 = (u_1^j)^{-1}(u_2^k(v_2))$ $(j, k = O$ or $W)$. From Figure 2.3, the demands for the two sellers are

$$D_{d1} = \int_{v_{b1}}^{V_1} \int_0^{\min\{(u_2^k)^{-1}(u_1^j(v_1)), V_2\}} f(v_1, v_2)\, dv_2\, dv_1, \tag{2.10}$$

$$D_{d2} = \int_{v_{b2}}^{V_2} \int_0^{\min\{(u_1^j)^{-1}(u_2^k(v_2)), V_1\}} f(v_1, v_2)\, dv_1\, dv_2. \tag{2.11}$$

When a seller does not accept returned products, his expected revenue is

$$\Pi_i^O = p_i D_{di}. \tag{2.12}$$

When the seller adopts a return policy, every consumer who purchases with valuation $v_i' < v_{ri}$ has probability $\frac{v_{ri} - v_i'}{2\delta_i}$ $(v_{ri} = \alpha_i p_i + \delta_i)$ to return the product. The return probability is derived in the same way as in Section 2.3.3. The expected numbers of consumers who return the products are respectively

$$R_{d1} = \int_{v_{b1}}^{min(v_{r1}, V_1)} \int_0^{\min\{(u_2^k)^{-1}(u_1^j(v_1)), V_2\}} \frac{v_{r1} - v_1}{2\delta_1} f(v_1, v_2)\, dv_2\, dv_1, \tag{2.13}$$

$$R_{d2} = \int_{v_{b2}}^{min(v_{r2}, V_2)} \int_0^{\min\{(u_1^j)^{-1}(u_2^k(v_2)), V_1\}} \frac{v_{r2} - v_2}{2\delta_2} f(v_1, v_2)\, dv_1\, dv_2. \tag{2.14}$$

The revenues of the two sellers consist of the gain from selling and the refund due to returns:

$$\Pi_{d1}^W = p_1 D_{d1} - (\alpha_1 p_1) R_{d1}, \tag{2.15}$$

$$\Pi_{d1}^W = p_2 D_{d2} - (\alpha_2 p_2) R_{d2}. \tag{2.16}$$

To find the Nash Equilibrium of this game, we first assume that Seller 2 holds strategy of a price p_2 and return policy j $(j = O$ or $W)$, and try to find out whether Seller 1 will adopt a return policy or not. We add a new notation $c_1 = (1 - \alpha_1)p_1$, which represents the return penalty (money not returned) to the consumer if he returns. By substituting $(1 - \alpha_1)p_1$ with c_1, Π_{d1}^W can be re-written as Π_{d1}^O plus a function of c_1 (See the proof in the Appendix A):

$$\begin{array}{rcl} \Pi_{d1}^{W} & = & \Pi_{d1}^{O} + R_{d1}(c_1), \\ \Pi_{d1}^{O} & = & p_1 D_{d1}^{W} = p_1 \int_{p_1}^{V_1} \min\{(u_2^k)^{-1}(v_1 - p_1), V_2\} \frac{1}{V_1 V_2}\, dv_1, \\ R(c_1) & = & c_1 \left(\int_{\sqrt{2\delta_1 c_1}}^{2\delta_1} \min\left\{ (u_2^k)^{-1} \left(\frac{v_1^2}{4\delta_1} - c_1 \right), V_2 \right\} \frac{1}{V_1 V_2}\, dv_1 \right) \\ & & -c_1 \left(\int_0^{\delta_1 - c_1} \min\left\{ (u_2^k)^{-1}(v_1), V_2 \right\} \frac{1}{V_1 V_2}\, dv_1 \right). \end{array} \tag{2.17}$$

We prove that $R_{d1}(c_1)$ is positive, and it can be viewed as the benefit brought by the return policy. Based on the formula of Π_{di}^{W}, it is clear that adopting a return policy is always valuable for Seller 1, no matter what strategy Seller 2 holds. In the same way, Seller 2 always prefers adopting return policy. So the game comes to an equilibrium, as in Table 2.3.

Seller 2\Seller 1	without	with
without	$(\Pi_{d1}^{OO}, \Pi_{d2}^{OO})$	$(\underline{\Pi_{d1}^{WO}}, \Pi_{d2}^{WO})$
with	$(\Pi_{d1}^{OW}, \underline{\Pi_{d2}^{OW}})$	$\underbrace{(\underline{\Pi_{d1}^{WW}}, \underline{\Pi_{d2}^{WW}})}_{\textit{Nash Equilibrium}}$

Table 2.3: The equilibrium in a duopoly market

Our next aim is to find the optimal prices and refunds when both sellers adopt a return policy. By calculations of given p_1, c_1 to find the optimal p_2, c_2 and the same in reverse, the prices in Nash Equilibrium p_{di}^{W} are the solutions to the following equation set. The equation set for $c_{di}^{W}, i = 1, 2$ is too complex so we put them in the Appendix A. The proof for equation set 2.18 is given in the Appendix A.

$$\begin{cases} (V_1 - 2p_1) \min\{(u_2^W)^{-1}(V_1 - p_1), V_2\} = \int_{v_{b2}^W}^{\min\{(u_2^W)^{-1}(V_1 - p_1), V_2\}} (u_2^W(v_2))\, dv_2 \\ (V_2 - 2p_2) \min\{(u_1^W)^{-1}(V_2 - p_2), V_1\} = \int_{v_{b1}^W}^{\min\{(u_1^W)^{-1}(V_2 - p_2), V_1\}} (u_1^W(v_1))\, dv_1 \end{cases} \tag{2.18}$$

Using the same way in Figure 2.2, we find the ranges of the equilibrium prices p_{di}^{W}. We also investigate the ranges of c_{di}^{W}. The results are given in the following proposition.

Proposition 2. *In the duopoly model with return policy, both sellers will adopt a return policy.* p_{di}^{W} *is larger than* $\frac{2V_i}{5}$ *and is smaller than* $\frac{V_i}{2}$ *(*$i = 1, 2$*).* c_{di}^{W} *is smaller than* $\frac{\delta_i}{4}$ *and the return proportion to consumers* α_{di}^{W} *is smaller than* 100% *(*$i = 1, 2$*).*

2.4.3 Comparison of the Two Duopoly Models

By comparing the two duopoly models with and without return policy, we can investigate the impact of return policies. We give the price comparison result in Lemma 4. However, the revenues in the two models are too complicated to be compared. Thus, we will make revenue comparison in the numerical study.

Lemma 4. *The equilibrium prices of the two sellers with return policy* p_{di}^{W} $(i = 1, 2)$ *are smaller than those without return policy* p_{di}^{O} $(i = 1, 2)$*, respectively.*

Lemma 4 indicates that the prices of the two sellers with return polices are lower than those without return policy. Note that when the sellers can adopt the return policy, both of them will choose the partial refund policy. When the other seller adopts a partial refund policy, the consumers' expected utilities of buying from it can be larger than those without return policy because the product can be returned. Thus, one seller's optimal price when the other seller adopts the return policy is smaller than that when the other seller does not adopt the return policy. Hence, both sellers' optimal prices with return policy are smaller than those without return policy, leading to that the equilibrium prices of the two sellers with return policy are lower than those without return policy.

2.5 Analytical Study

In the above section, we have given the solutions to the monopoly and duopoly models with and without return policy, respectively. We also have compared the two monopoly models and the two duopoly models, to investigate the impact of return policy. In this section, we will make the comparison of the monopoly and the duopoly models with return policy, and the comparison of the monopoly and duopoly models without return policy, to figure out the impact of competition.

2.5.1 Comparison of the Monopoly and Duopoly Models

We start with the comparison of the monopoly and duopoly models without return policy. We consider Seller 1 in the duopoly models as the monopoly seller who faces a competitor Seller 2, so the results of the two models are comparable.

The optimal price of the monopoly seller is $p_m^O = \frac{V}{2}$. From Lemma 3, the equilibrium price of Seller 1 in the duopoly market p_{d1}^O is larger than $\frac{2V_1}{5}$ and smaller than $\frac{V_1}{2}$. $\frac{2V_1}{5}$ is less than the monopoly seller's optimal price $\frac{V_1}{2}$ by 20%.

This result shows that the price reduction of the monopoly seller would be less than 20% when another competitive seller comes into the market. This result is coincident with the numerical study in Section 2.6.3, which provides a more precise result that the price reduction is smaller than 18%.

The revenue of Seller 1 in the duopoly model is proved to be less than that of the monopoly seller in the monopoly model. This result is quite reasonable because the market is shared by two sellers in the duopoly model.

Proposition 3. *One seller in the duopoly model can be considered as the seller in the monopoly model who meets a new competitor. Without return policy, the seller's equilibrium price and revenue in the duopoly model are less than those in the monopoly model, that is,* $p_{d1}^O < p_m^O$ *and* $\Pi_{d1}^O < \Pi_m^O$. *The percentage of price reduction is less than 20%.*

In the comparison of the monopoly and duopoly models with return policy, the results are similar to the results without return policy. The equilibrium price and revenue in the duopoly model are smaller than those in the monopoly model. Additionally, the return penalty cost c ($c = p(1-\alpha)$) is smaller in the duopoly model than that in the monopoly model, too. The results are given in the following proposition.

Proposition 4. *One seller in the duopoly model is considered as the seller in the monopoly model who meets a new competitor. With return policy, the seller's equilibrium price, revenue and return penalty cost in the duopoly model are less than those in the monopoly model, that is,* $p_{d1}^W < p_m^W$, $\Pi_{d1}^W < \Pi_m^W$, *and* $c_{d1}^W < c_m^W$. *The percentage of price reduction is less than 20%.*

As the severity of return policy is measured by the refund proportion α, we should also study the comparison of α of the monopoly and duopoly models with return policy. However, due to the complexity of the model, the study of refund proportion α is conducted by the following special case.

2.5.2 A Special Case

In this subsection, we consider a special case: $V_1 = V_2 = V$ and $\delta_1 = \delta_2 = \delta$, that is, the parameters of the two sellers are the same. Our aim is to see the severity of return policy is higher or lower in a duopoly market than that in a monopoly market. Although it is a special case, the result may be representative for all cases.

We denote the equilibrium decisions in the special case as p_d^W and c_d^W, and $c_d^W = p_d^W(1 - \alpha_d^W)$. The solutions of p_d^W and c_d^W are given by the following two equations (p_d^W and c_d^W are simplified to p and c):

$$p = \sqrt{2V^2 - \frac{\delta^2}{3} - \frac{8\delta^{\frac{1}{2}}c^{\frac{3}{2}}}{3} + 2\delta c + \frac{c^2}{2}} - V, \tag{2.19}$$

$$p(\sqrt{\delta} - \sqrt{c})(\sqrt{\delta} - 2\sqrt{c}) - \frac{\delta^2}{3} + 3c^{\frac{1}{2}}\delta^{\frac{3}{2}} - 5\delta c + \frac{13c^{\frac{3}{2}}\delta^{\frac{1}{2}}}{3} - 2c^2 - \delta c ln\frac{\delta}{c} = 0. \tag{2.20}$$

We have obtained that in the monopoly model without return policy, the solutions of price and return penalty are

$$p_m^W = \frac{V}{2}, c_m^W = \frac{\delta}{4}. \tag{2.21}$$

We prove in the Appendix A that

$$p_d^W / c_d^W < p_m^W / c_m^W. \tag{2.22}$$

Hence,

$$\alpha_d^W < \alpha_m^W. \tag{2.23}$$

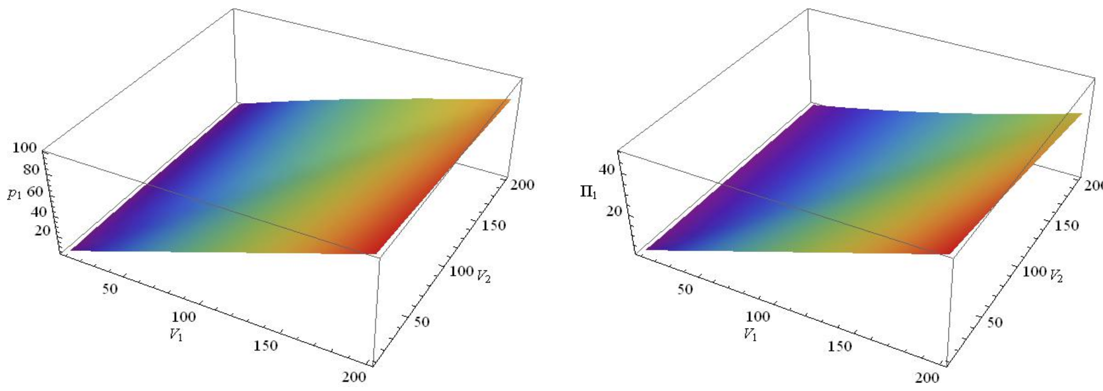
This result shows that the return policy is severer (refund proportion is larger) in the duopoly (competitive) market than that in the monopoly market, which is coincident with the numerical study in Section 2.6.4. Till now, we have gained some results analytically. However, the results are not complete. We supplement some results in the numerical study.

2.6 Numerical Study

2.6.1 Sensitivity Analysis

To improve the understanding of the impact of the parameters in our models, we conduct a sensitivity analysis. As the two monopoly models have been clearly discussed in Sections 2.3.2 and 2.3.3, the analysis in this section only involves the two duopoly models with and without return policy.

We begin with the duopoly model without return policy. There are two parameters: the maximum pre-purchase valuations that the consumers give to the two products V_1, V_2, which affect the decision variables p_1, p_2 and the expected revenues of the two sellers Π_{d1}^O, Π_{d2}^O. We make the sensitivity analysis for Seller 1 of V_1 and V_2 by increasing them from 10 to 200 with step size 10. The sensitivity analysis for Seller 2 of V_1 and V_2 is the same as that of Seller 1.



(Price) (Revenue)

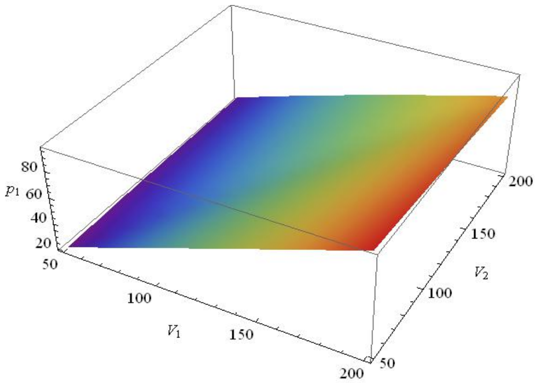
Figure 2.4: Sensitivity analysis for Seller 1 of V_1 and V_2 without return policy

Figure 2.4 illustrates that when V_2 remains unchanged, as V_1 increases, both the price and revenue of Seller 1 increase; when V_1 remains unchanged, as V_2 increases, the price of Seller 1 first decreases and then increases slightly, and the revenue of Seller 1 decreases slightly. This analysis indicates that, in a competitive market without return policy, as the maximum valuation of the seller's product increases, its revenue increases significantly, and its competitor's price and revenue are slightly influenced.

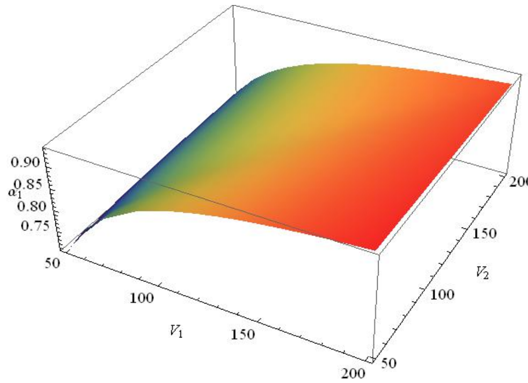
Next, in the duopoly model with return policy, two parameters are added: δ_1, δ_2.

Since the valuation uncertainties ϵ_1 and ϵ_2 are defined as $\epsilon_1 \sim U[-\delta_1, \delta_1]$ and $\epsilon_2 \sim U[-\delta_2, \delta_2]$, these two parameters represent the degree of valuation uncertainties. The two sellers' decision variables are the prices of the two products p_1, p_2, and refund proportions α_1, α_2 if the products are allowed to be returned.

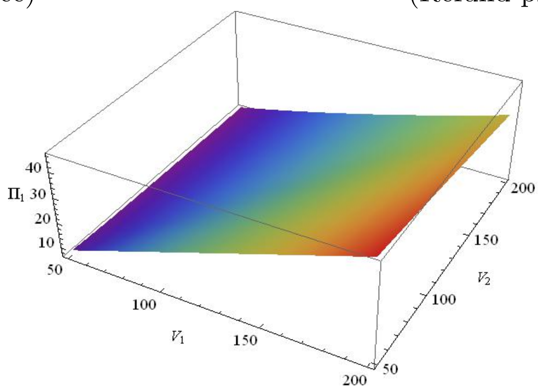
We first conduct the sensitivity analysis on the price, refund proportion, and profit of Seller 1 of the parameters V_1, V_2 by increasing them from 50 to 200 with step size 10. Note that we set $\delta_1 = \delta_2 = 25$ and we have $V_i \geq 2\delta_i, i = 1, 2$, so V_i must be no less than 50. The sensitivity analysis for Seller 2 of V_1 and V_2 is the same as that of Seller 1.



(Price)



(Refund proportion)

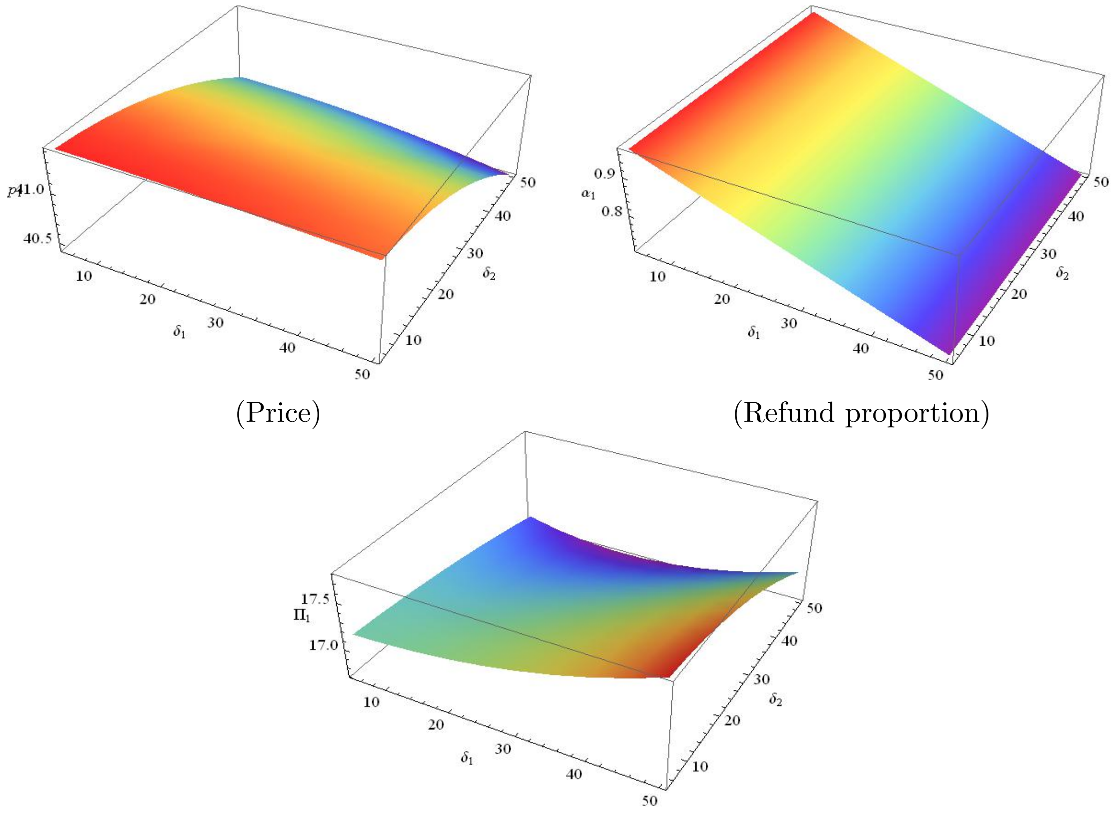


(Revenue)

Figure 2.5: Sensitivity analysis for Seller 1 of V_1 and V_2 with return policy

Figure 2.5 shows that when V_2 remains unchanged, as V_1 increases, the price, refund proportion and revenue of Seller 1 increase; when V_1 remains unchanged, as V_2 increases, the price and the refund proportion of Seller 1 first decrease and then increase slightly, and the revenue of Seller 1 decreases. This analysis indicates that, in a competitive market with return policy, as the maximum pre-purchase valuation of a product increases, both the revenue and the refund proportion increase significantly, but the impact on its competitor is mild.

Figure 2.6 presents the sensitivity analysis for Seller 1 of δ_1 and δ_2 by increasing them from 5 to 50 with step size 5, and we set $V_1 = V_2 = 100$. When δ_2 remains unchanged, as δ_1 increases, the price of Seller 1 decreases slightly and the refund proportion decreases, but the revenue increases; when δ_1 remains unchanged, as δ_2 increases, the price and revenue decrease, and the refund proportion is unaffected. This analysis suggests that, in a competitive market with return policy, as the uncertainty of one seller's consumer valuation after purchasing increases, its refund proportion decreases and the revenue increases, and its competitor's price and revenue decrease.

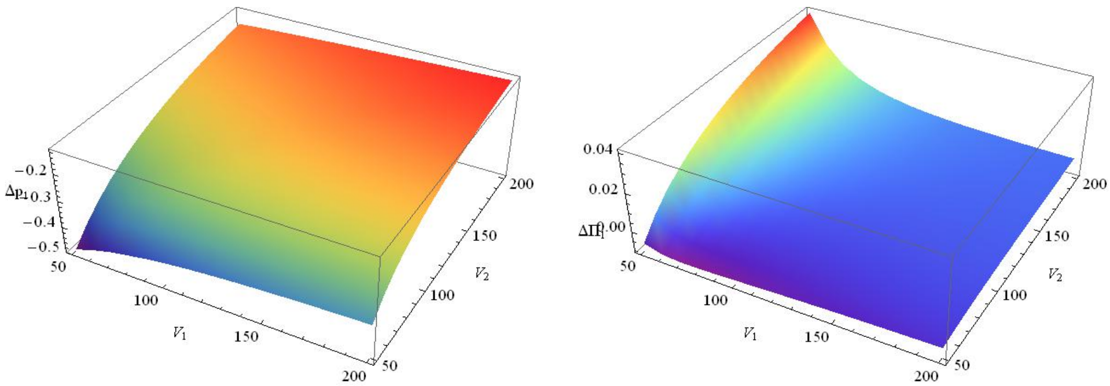


(Price) (Refund proportion)
(Revenue)

Figure 2.6: Sensitivity analysis for Seller 1 of δ_1 and δ_2 with return policy

2.6.2 Comparison of the Two Duopoly Models

In this subsection, we show the impact of return policy in a competitive market by comparing the two duopoly models with and without return policy. We compare the equilibrium prices and corresponding revenues of Seller 1 by the difference between the two models, that is, $\Delta p_{d1} = p_{d1}^W - p_{d1}^O$ and $\Delta \Pi_{d1} = \Pi_{d1}^W - \Pi_{d1}^O$, as V_1 and V_2 increase from 50 to 200 with step size 10 and $\delta_1 = 25, \delta_2 = 25$. The comparison of Seller 2 is the same as that of Seller 1.



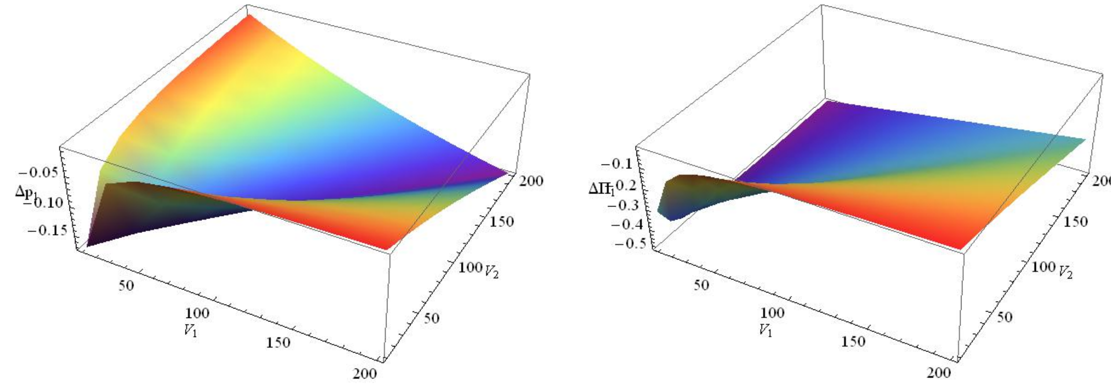
(Price) (Revenue)

Figure 2.7: The changes of Seller 1 from without return policy to with return policy

From Figure 2.7, we find that the price difference is negative, which indicates that the equilibrium price of Seller 1 with return policy is lower than that without return policy in a duopoly market. The revenue difference is sometimes positive and sometimes negative, which means that the revenue of Seller 1 with return policy may be higher or lower than that without return policy. When V_1 is small enough, the revenue of Seller 1 with return policy can be larger than that without return policy.

2.6.3 Comparison of the Monopoly and Duopoly Models without Return Policy

In this subsection, we numerically compare the monopoly and duopoly models without return policy. When return policy is not adopted, the only decision variable of the seller is the price. We consider the duopoly model as the monopoly seller in a monopoly model meets a new competitor in the market. By comparing the optimal price of the monopoly seller in the monopoly market and the equilibrium price in the duopoly market , we can investigate the price reduction of the seller when a new competitor shows up in the market in our consumer-valuation-based model.



(Price) (Revenue)

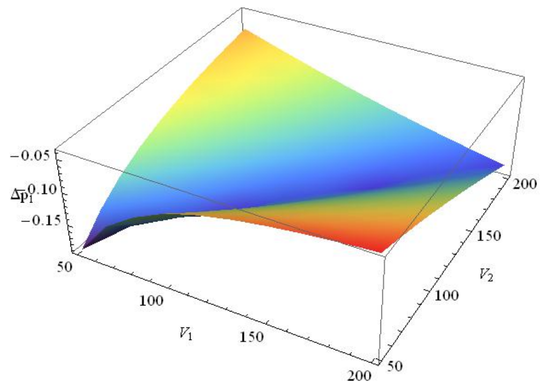
Figure 2.8: The changes of the monopoly seller facing a new seller in the market without return policy

From Figure 2.8, when one monopoly seller with V_1 meets a new seller with V_2 (V_1 and V_2 are set from 10 to 200 with step size 10) in the market, the monopoly seller reduces its price, and the reduction rate is less than 18%, which is consistent with our analytical result (the price reduction is less than 20%). The largest reduction is about 17.1572% when $V_1 = V_2$. The change of revenue is negative, so the monopoly seller's revenue is also reduced. When V_2 remains unchanged, as V_1 increases, the revenue reduction of the monopoly seller increases; when V_1 remains unchanged, as V_2 increases, the revenue reduction decreases.

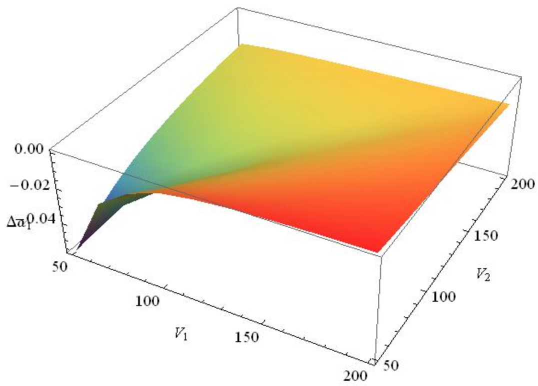
2.6.4 Comparison of the Monopoly and Duopoly Models with Return Policy

According to the Subsections 2.3.4 and 2.4.2, return policy is always chosen by a seller, no matter the market is monopoly or duopoly. In the special case, we obtain that the refund proportion is smaller in the duopoly model, which means that the return policy is severer in a competitive market than that in a monopoly market. Now, we study the general case numerically.

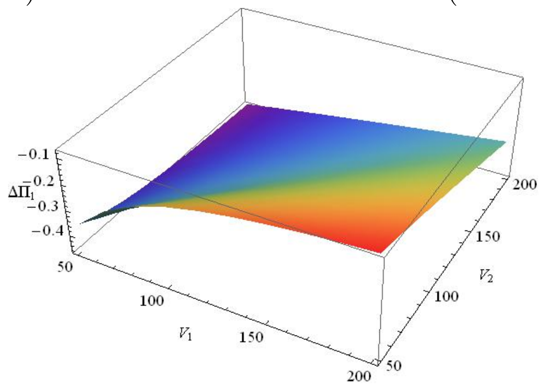
In Figure 2.9, one seller with V_1 changing from 50 to 200 with step size 10 and $\delta_1 = 25$ meets a new market participant with V_2 changing from 50 to 200 with step



(Price)



(Refund proportion)



(Revenue)

Figure 2.9: The changes of the monopoly seller facing a new competitor in the market with return policy

size 10 and $\delta_2 = 25$. The seller who is first in the market reduces the price by less than 18%. The difference of refund proportion before and after facing the competitor is negative, which indicates that the seller also reduces its refund proportion, that is, the seller provides a severer return policy to the consumers in a competitive market. The reason for this counter-intuitive result may be that the benefit of the un-refunded proportion extracted from consumer return overweighs the negative influence on the consumers' purchasing decisions due to the return policy with a lower refund.

2.7 Conclusions

This research is the first to study the pricing and return policy using two-dimensional valuation-based models. We consider one consumer's independent valuations on two products and the differentiation of all consumers' valuation on each product. To answer the three research questions, four models are developed: the monopoly models with and without return policy, and the duopoly models with and without return policy. The monopoly models are formulated as nonlinear optimization programs, while the duopoly models are investigated using game theory. The return policy is characterized by the refund proportion to the consumer if a product is returned.

We provide the optimal or Nash Equilibrium solutions for the four models and conduct some analytical and numerical studies. The results about the seller's decision on return policy are as follows: In a monopoly market, the seller will adopt return policy. In a duopoly market, both sellers adopt the return policy in the Nash Equilibrium.

The impact of return policy is captured by comparing the models with and without return policy. We find that in a monopoly market, return policy benefits the seller. However, in a duopoly market, return policy may not bring more revenues to the two sellers.

The impact of competition is shown in the comparison of the monopoly and duopoly models: In the duopoly market, one seller is considered as the monopoly seller facing a new competitor in the market. The seller's price and revenue in the duopoly market are both lower than those in the monopoly market. The equilibrium price in the duopoly models cannot be below than 80% of the optimal price in the monopoly model, which indicates that a monopoly seller will reduce its price by no more than 20% when there comes a competitor. One more interesting result is that the severity of the seller's return policy is higher in a duopoly (competitive) market than that in a monopoly market.

Chapter 3

The Role of Labor Market on the Performance of Logistics with Fluctuating Demand

3.1 Introduction

The demand for e-commerce fluctuates a lot over time. The sales amount rises significantly during promotion time. For example, the one-day sales amount of Alibaba reached 812 million on 11 Nov 2017 (Singles Day). By contrast, the average daily sales amount of Alibaba is about 33 million. It is observed that the daily sales amount in the promotion days (peak periods) can be ten times more than that in the normal days (non-peak periods).

The logistics system of the e-commerce implements the product deliveries to the consumers. In the peak periods, the amount increases sharply. Thus, the system in the peak periods faces high pressure. How to develop the system to handle the fluctuating demand and maximize the profit has become one of the "online-to-offline" (O2O) challenges in e-commerce operations management (EOM).

In this study, we address the O2O-EOM challenge in the perspective of the labor market. Nowadays, the deliveries are mainly implemented by labors. However, the labor shortage in the peak periods has continuously been a problem for the EOM, which arouses our attention. Also, even when the labors are replaced by unmanned aerial vehicle (UAV) to deliver goods in the near future, the challenge due to fluctuating demand still exists.

The shortage of labors in the peak periods and the resulting severe delivery delay will negatively impact the consumers' purchasing behaviors, leading to lower demand. One efficient way to avoid labor shortage in the peak periods is to directly increase the wage to the labors in the peak periods, as the labor market is a pool, in which the labors choose to participate or not. By increasing the wages, more labors are willing to participate as part-time workers.

Interestingly, the labor shortage is not solved simply, which still happen every year. The explanation is that the labors are not willing to participate because of the high opportunity cost, which is the loss of not choosing the optimal one of all the other choices when they choose to participate in the logistics system.

One alternative solution is to employ more full-time labors so that they can serve during the peak periods. The truth is that, in the non-peak periods, the increased number of labors will decrease the average workload of each labor, resulting in low utilization. Here, we assume that the income of a labor is proportional to his utilization. The low utilization and consequent low income will lead the labors to quit the job.

Hence, the labor participation behavior highly depends on the delivery wage, utilization, and opportunity cost. Behavior-based strategy for the labor employment will help improve the situation. For the logistics system, if the wages in the peak and non-peak periods to labors are settled optimally, the numbers of full-time and part-time labors can be at the optimal level to prepare for the services in both the peak and non-peak periods.

Based on the optimal wages, we can determine the number of participating labors, their utilizations, and performance of the logistics system. Then we analyze the impact of the parameters, such as labor pool size, demand, labors' opportunity costs and consumer elasticity of delivery speed, on the optimal wage decisions. We consider a model with non-peak and peak periods, in which two wages are offered to the labors respectively. The labors are heterogenous in their opportunity cost and choose to participate or not by themselves. We first find the optimal wage decisions and then analyze the impact of the parameters such as labor pool size, part-time labor employment cost, labors' opportunity cost and consumer elasticity of delivery speed.

We identify four conditions, under which different wages and the corresponding labor supply decisions are made. Hence, four cases are derived. In case (1), the full-time labors are fully utilized in the non-peak periods, and part-time labors are employed to fulfill the extra demand in the peak periods; in cases (2) and (3), the full-time labors are not fully utilized in the non-peak periods but are fully utilized in the peak periods, and part-time labors are also employed to fulfill the demand in the peak periods; and in case (4), the full-time labors are fully utilized in the peak and non-peak periods, and no part-time labors are employed in the peak periods.

In the analysis of the impact of the parameters, we conclude that the increase in the labor pool size or the decrease in the labor opportunity cost, which causes more participations of labors, does not necessarily decrease the wages and increase the labor supply. In other words, the opposite impact may happen under certain condition. When the wage in the peak periods is higher than that in the non-peak periods, the increase in the part-time labor employment cost leads to more labors employed in the non-peak periods. However, the increased number of full-time labors results in lower labor utilization in the non-peak periods. The consumer elasticity of delivery speed leads to lower demand in the peak periods. Under different conditions, the firm may decrease the numbers of labors in the peak and non-peak periods at the same time, or it will employ more labors to increase the supply and demand in the peak periods.

The rest of this chapter is organized as follows. We present the literature review following the Introduction. In Sections 3.3 and 3.4, we introduce the model setup and assumptions for model authenticity. In Section 3.5, we solve for the general solution of the optimal wages under different conditions. In Section 3.6, we analyze the impact of different parameters. We conclude the management insights and future research directions in Section 3.7. All proofs appear in the Appendix B.

3.2 Literature Review

Our work is related to a rich stream of literature in labor economics. Ehrenberg et al. (2016) comprehensively study the behavior of employers and employees, including the incentives of wages. Battalio et al. (1981) develop an experiment with leisure-work model and conclude that the workers prefer income to leisure when the price is right. Hirsch (2005) focuses on the wages of full-time and part-time workers and discovers the reason for the part-time wage penalty. Wolf (2000) considers a wage-hours model with labors' participating decisions, in which the impact of working hours on the wage rate is analyzed. Aaronson and French (2004) also study the negative effect of fewer part-time working hours on the wage. In our study, in the peak periods, the wages per unit of work for the full-time and part-time labors are the same, and the wages are the incentive to the labors to participate.

The wage-hours model indicates the elasticity of labors' participation. Camerer et al. (1997) show that elasticity may have a negative impact because the workers quit when they reach the daily income target. Chen (2016) develop a surge pricing model in which the drivers on Uber drive more when their earnings are high. Hall et al. (2017) imply that the drivers' supply in Uber is highly elastic because the drivers face no requirement on the supply hours. In our study, the labors have no elasticity of supply. The working time (utilization) is determined by the demand and supply.

Our work is also related to the study in a system with peak-period congestion. Arnott et al. (1993) extend a particular congestion model to consider the price-sensitive demand. Yang et al. (2013) study the electricity pricing with peak and non-peak periods with the consideration of consumer behavior, to shift some consumptions from the peak period to the non-peak period. Dong et al. (2017) build on Yang et al. (2013) to investigate the optimal capacity investment and pricing decisions in the non-peak and peak periods, and derive insights from real data. Gale and Holmes (1993) show that the peak-load pricing increases revenue by shifting demand from peak period to non-peak period. In our model, we do not incorporate the demand shift, but we do analyze the impact of the demands in peak and non-peak periods on the wage decisions and profit.

In the labor participating decisions and wage decisions, we follow the studies about the on-demand service platforms with demand fluctuation. The logistics system is similar to the on-demand platform from the perspective that more labors lead to lower utilization and consequent lower income. The labors choose to participate when their payoffs are larger than the opportunity cost. Also, surge pricing is also used in our model, but the decision variables are wages instead. Cachon et al. (2017) study the surge pricing and conclude that the platform with self-scheduling benefits from it. Castillo et al. (2017) indicate that platform with uniform prices has to set very high price to avoid wild goose chase, but surge pricing can avoid both high prices and wild goose chase. More examples can be found in Banerjee et al. (2015), Tang et al. (2016), Taylor (2018), Cohen and Zhang (2017), Bimpikis et al. (2016), Chen and Hu (2017), Hu and Zhou (2016), and Benjaafar et al. (2018).

3.3 The Model

We consider an e-commerce logistics system with periodic demands to be delivered. The demand periods consist of alternately non-peak periods and peak periods, which are shown in Figure 3.1. The daily demands in the two periods are $d_j (j \in \{l, h\})$, where l represents the non-peak periods and h represents the peak periods. The lasting times of the non-peak and peak periods are t_l and t_h, respectively. The logistics system decides the wages to labors in peak and non-peak periods $w_j (j \in \{l, h\})$ per unit of work. In the peak periods, the wage is no less than that in the non-peak periods, that is, $w_l \leq w_h$.

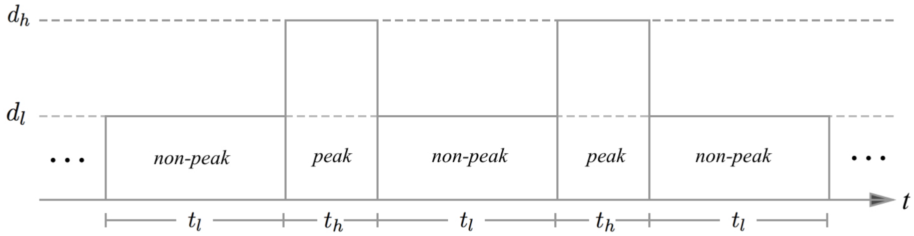


Figure 3.1: The delivery demands in peak and non-peak periods

Each labor can deliver a maximum of n units per day, so the maximum daily payoff of one labor is $w_j n$. In the peak period, adding one part-time labor causes a cost c_h for the logistics system, including the cost of employment, equipment, training, etc. Since the full-time labors serve as long-term employees, the cost of employment can be negligible.

The labors will choose if they participate or not. We assume that the opportunity cost of the labors to be available for work in one day is c, which follows density $f(\cdot)$ and distribution $F(\cdot)$ on $[0, \bar{c}]$. For the model efficiency, we assume that c follows a uniform distribution, that is $f(c) = \frac{1}{\bar{c}}, F(c) = \frac{c}{\bar{c}}$. The labor pool size (the number of potential labors in the market) is N_0.

Let u_j be the utilization of labors when they participate. The daily maximum supply is s_j. When the daily labor supply s_j exceeds the daily demand d_j, the labors will only be utilized for a proportion; when the total labor supply is less than or equal to the total demand ($s_j \leq d_j$), the labors are fully utilized. So, we have

$$u_j = \begin{cases} \frac{d_j}{s_j}, & s_j > d_j, \\ 1, & s_j \leq d_j. \end{cases} \tag{3.1}$$

All the notations used in this chapter are presented in Table 3.1.

d_j	The daily delivery demand in the system, $j \in [l, h]$
t_j	The lasting time of the peak and non-peak periods, $j \in [l, h]$
w_j	The wage to labors for one unit of delivery work, $j \in [l, h]$
s_j	The maximum daily supply, $j \in [l, h]$
N_j	The number of participating labors, $j \in [l, h]$
N_0	The pool size of labors who may participate in the market
c_h	The employment cost of one part-time labor in the peak periods
c	The opportunity cost of the labors, and $c \sim U[0, \bar{c}]$
n	The number of units that one labor can delivery in one day
a	The maximum demand in the peak periods
b	The elasticity of the delivery speed in the peak period

Table 3.1: Notations used throughout this study

The labors will participate when their payoffs are positive. As they may not be fully utilized, the daily payoff per labor is $u_j w_j n$. Thus, they will participate when $u_l w_l n - c > 0$ in the non-peak periods and $u_h w_h n - c > 0$ in the peak periods. Hence, the participating proportion of the labors is $F(u_j w_j n)$ in the non-peak or peak periods.

The labors who participate in the non-peak and peak periods are full-time workers and the labors who participate in the peak periods only serve as part-time workers. We assume that $u_l w_l \leq u_h w_h$ to guarantee that, in the peak periods, the full-time workers will not quit the job and the part-time labors will participate. The participating proportions are shown in Figure 3.2. The number N_j of labors who participate in the non-peak periods or peak periods is $N_0 F(u_j w_j n)$.

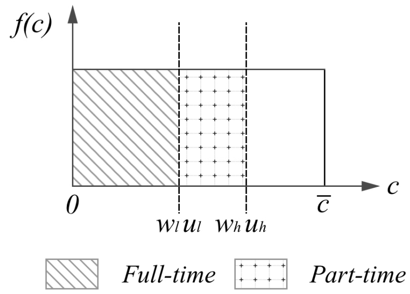


Figure 3.2: The participating proportions of the full-time and part-time labors

So the total labor supply s_j is

$$s_j = nN_j = N_0 nF(u_j w_j n). \tag{3.2}$$

Solving for the optimal daily supply s_j, we have:

(1) When $s_j > d_j$, $u_j = \frac{d_j}{s_j}$ and $s_j = N_0 nF(w_j u_j n)$. Thus, $s_j = N_0 nF(w_j \frac{d_j}{s_j} n) = \frac{N_0 n^2 w_j d_j}{\overline{c}} \frac{1}{s_j}$. We obtain

$$s_j = \sqrt{\frac{N_0 n^2 w_j d_j}{\overline{c}}}. \tag{3.3}$$

(2) When $s_j \leq d_j$, $u_j = 1$ and $s_j = N_0 nF(w_j u_j n)$. We obtain

$$s_j = \frac{N_0 n^2 w_j}{\overline{c}}. \tag{3.4}$$

In the peak periods, the demand is influenced by the delivery speed. A lower delivery speed in the peak period will decrease the demand. Let D_h be the total demand in one peak period, and $D_h = d_h t_h$. The delivery speed can be represented by $\frac{D_h}{s_h}$. Here, we adopt the format of traditional price elasticity to model the impact of delivery speed on demand. We assume that

$$D_h = a\left(\frac{D_h}{s_h}\right)^b, \tag{3.5}$$

where a is the maximum demand in the peak periods, $b \leq 0$ is the elasticity of the delivery speed in the peak period. Thus,

$$D_h = a^{\frac{1}{1-b}} s_h^{\frac{-b}{1-b}}. \tag{3.6}$$

Meanwhile, the logistics system charges price p per unit of work. The objective of the logistics system is to maximize its profit. Let Π be the system's profit, we have:

$$\max_{w_l, w_h} \quad \Pi = (p - w_l) d_l t_l + (p - w_h) d_h t_h - c_h (N_h - N_l)^+. \tag{3.7}$$

In the profit function, the first part is the profit from deliveries in the non-peak periods, the second part is the profit from deliveries in the peak periods, and the third part is the cost for hiring the part-time labors in the peak periods.

3.4 Model Authenticity

Before proceeding to solve the optimization solution, we now make some assumptions and discussions to guarantee the authenticity of our model, namely, to make the model more realistic. First, in the non-peak periods, the daily labor supply is not smaller than the daily demand, that is, $s_l \geq d_l$. The reason is that, if the daily labor supply is smaller than the daily demand, undelivered demand emerges every day and will accumulate until the logistics system breakdowns. In other words, the labor supply must satisfy the demand in the non-peak periods. Thus, we have

$$u_l = \begin{cases} \frac{d_l}{s_l}, & s_l > d_l, \\ 1, & s_l = d_l. \end{cases} \tag{3.8}$$

According to the definition of s_j in the last section, the daily labor supply s_l in the non-peak periods is

$$s_l = \begin{cases} \sqrt{\frac{N_0 n^2 w_l d_l}{\bar{c}}}, & s_l > d_l, \\ \frac{N_0 n^2 w_l}{\bar{c}}, & s_l = d_l. \end{cases} \tag{3.9}$$

Noted that when $s_l = \sqrt{\frac{N_0 n^2 w_l d_l}{\bar{c}}} > d_l$ and when $s_l = \frac{N_0 n^2 w_l}{\bar{c}} = d_l$, the condition to guarantee enough labor supply is

$$d_l \leq \frac{N_0 n^2 w_l}{\bar{c}}, \quad i.e. \quad w_l \geq \frac{d_l \bar{c}}{N_0 n^2}. \tag{3.10}$$

With $w_l \geq \frac{d_l \bar{c}}{N_0 n^2}$, the two cases of s_l can be combined to $s_l = \sqrt{\frac{N_0 n^2 w_l d_l}{\bar{c}}}$.

Second, in the peak periods, recall that the total demand D_h in one peak period is determined by the daily supply s_h, which implies that the daily supply s_h is assumed to be fully utilized. In other words, the total demand D_h in the peak periods is endogenous by the daily supply s_h. The lasting time of the peak periods t_h is also endogenous by s_h:

$$t_h = \frac{D_h}{s_h} = a^{\frac{1}{1-b}} s_h^{\frac{-1}{1-b}}. \tag{3.11}$$

As $d_h = \frac{D_h}{t_h}$, the daily delivery demand d_h is also endogenous, that is $d_h = s_h$. So, in the peak periods, the labor utilization $u_h = \frac{d_h}{s_h} = 1$, and the daily supply $s_h = \frac{N_0 n^2 w_h}{\bar{c}}$.

From $s_j = nN_j$, we have $N_j = \frac{s_j}{n}$, which will be used in the profit function. In addition, $w_l u_l \leq w_h$ ensures that $N_l \leq N_h$, so $(N_h - N_l)^+ = N_h - N_l$. The profit function is more specific now:

$$\begin{aligned}
\max_{w_l, w_h} \quad \Pi \quad &= (p - w_l) d_l t_l + (p - w_h) d_h t_h - c_h (N_h - N_l) \\
&= (p - w_l) d_l t_l + (p - w_h) a^{\frac{1}{1-b}} s_h^{\frac{-b}{1-b}} - c_h \left(\tfrac{s_h}{n} - \tfrac{s_l}{n}\right) \\
&= (p - w_l) d_l t_l + (p - w_h) a^{\frac{1}{1-b}} \left(\frac{N_0 n^2 w_h}{\bar{c}}\right)^{\frac{-b}{1-b}} - c_h \left(\frac{N_0 w_h n}{\bar{c}} - \sqrt{\frac{N_0 w_l d_l}{\bar{c}}}\right) \\
s.t. \qquad & \tfrac{d_l \bar{c}}{N_0 n^2} \leq w_l \leq w_h.
\end{aligned} \tag{3.12}$$

By solving this constrained optimization problem, we can obtain the optimal wage decisions, and the corresponding labor supply, logistics performance, and profit.

3.5 Optimal Wages

We use Lagrangian Multiplier Method to solve the constrained optimization problem. Before showing the solution, we now analyze the properties of the profit function, which will be helpful in presenting and understanding the optimal solution later.

First, we observe that without the constraint, the profit function can be written into two parts with w_l and w_h respectively:

$$\begin{aligned}
\Pi &= \Pi_l + \Pi_h \\
&= \left[(p - w_l)d_l t_l + c_h\sqrt{\frac{N_0 w_l d_l}{\bar{c}}}\right] \\
&+ \left[(p - w_h)a^{\frac{1}{1-b}}\left(\frac{N_0 n^2 w_h}{\bar{c}}\right)^{\frac{-b}{1-b}} - c_h\frac{N_0 n w_h}{\bar{c}}\right]
\end{aligned} \tag{3.13}$$

The following Lemma 5 shows the properties of Π_l and Π_h.

Lemma 5. *Let $\hat{w_l}$ and $\hat{w_h}$ be the optimal solution of the optimization of Π_l and Π_h, we have $\frac{\partial \Pi_l}{\partial w_l}|_{w_l=\hat{w_l}} = 0$ and $\frac{\partial \Pi_h}{\partial w_h}|_{w_h=\hat{w_h}} = 0$. $\hat{w_l} = \frac{c_h^2 N_0}{4\bar{c}d_l t_l^2}$ and $\hat{w_h}$ satisfies $g(\hat{w_h}) = \frac{-b}{1-b}pA\hat{w_h}^{\frac{-b}{1-b}-1} - (1 + \frac{-b}{1-b})A\hat{w_h}^{\frac{-b}{1-b}} - \frac{c_h N_0 n}{\bar{c}} = 0$, in which $A = a^{\frac{1}{1-b}}(\frac{N_0 n^2}{\bar{c}})^{\frac{-b}{1-b}}$.*

This lemma shows that, without the constraint, the profit Π_l and Π_h will increase and then decrease in w_l and w_h respectively, and the optimal solutions are obtained in the first order conditions. We now define the first order condition of w_h as $g(\cdot)$ for the convenience of expression later.

We also analyze the situation when $w_l = w_h = w$, which acts as a boundary in the constraint. Thus, the profit function will be

$$\Pi = (p - w)d_l t_l + (p - w)a^{\frac{1}{1-b}}\left(\frac{N_0 n^2 w}{\bar{c}}\right)^{\frac{-b}{1-b}} - c_h\left(\frac{N_0 n w}{\bar{c}} - \sqrt{\frac{N_0 w d_l}{\bar{c}}}\right).$$

Lemma 6 shows the properties of Π when $w_l = w_h = w$.

Lemma 6. *Let $\hat{w}$ be the optimal solution and we have $\frac{\partial \Pi}{\partial w}|_{w=\hat{w}} = 0$. $\hat{w}$ satisfies $h(\hat{w}) = \frac{-b}{1-b}pA\hat{w}^{\frac{-b}{1-b}-1} - (1 + \frac{-b}{1-b})A\hat{w}^{\frac{-b}{1-b}} + c_h\sqrt{\frac{N_0 d_l}{4\bar{c}\hat{w}}} - d_l t_l - \frac{c_h N_0 n}{\bar{c}} = 0$, in which $A = a^{\frac{1}{1-b}}(\frac{N_0 n^2}{\bar{c}})^{\frac{-b}{1-b}}$.*

Lemma 6 shows that when $w_l = w_h = w$ and without the constraint, the profit first increases and then decreases in w. We also define the first order condition of w as $h(\cdot)$ for the convenience of expression.

Now we directly give the optimal wage decisions in the constraint optimization problem, the solution is obtained by Lagrangian Multiplier Method. See the solution in Proposition 1, in which the definition of $g(\cdot)$ and $h(\cdot)$ are used.

Proposition 5. *The optimal wages of the logistics system are w_l^* and w_h^*.*

(1) If $d_l \geq \frac{c_h N_0 n}{2\bar{c}t_l}$ *and* $g(\frac{d_l\bar{c}}{N_0 n^2}) > 0$, *then* $w_l^* = \frac{d_l\bar{c}}{N_0 n^2}$, $w_h^* = \hat{w}_h$, $u_l^* = 1$ *and* $w_l^* < w_h^*$*;*

(2) If $d_l < \frac{c_h N_0 n}{2\bar{c}t_l}$ *and* $g(\frac{c_h^2 N_0}{4\bar{c}d_l t_l^2}) > 0$, *then* $w_l^* = \frac{c_h^2 N_0}{4\bar{c}d_l t_l^2}$, $w_h^* = \hat{w}_h$, $u_l^* = \frac{2\bar{c}d_l t_l}{c_h N_0 n} < 1$ *and* $w_l^* < w_h^*$*;*

(3) If $g(\frac{c_h^2 N_0}{4\bar{c}d_l t_l^2}) \leq 0$ *and* $h(\frac{d_l c}{N_0 n^2}) > 0$, *then* $u_l^* = \sqrt{\frac{d_l\bar{c}}{N_0 n^2 \hat{w}}} < 1$ *and* $w_l^* = w_h^* = \hat{w}$*;*

(4) If (i) $g(\frac{c_h^2 N_0}{4\bar{c}d_l t_l^2}) \leq 0$ *and* $h(\frac{d_l c}{N_0 n^2}) \leq 0$, *or (ii)* $g(\frac{c_h^2 N_0}{4\bar{c}d_l t_l^2}) > 0$ *and* $g(\frac{d_l\bar{c}}{N_0 n^2}) \leq 0$, *then* $w_l^* = w_h^* = \frac{d_l\bar{c}}{N_0 n^2}$ *and* $u_l^* = 1$.

Proposition 5 indicates that there are four cases, in which the wage decisions differ. Based on Proposition 5, we can analyze the labor supply choices in different cases. The results are given in the following proposition.

Proposition 6. *The daily labor supplies* s_l *and* s_h *in different cases are given as follows.*

- *In case (1), the daily labor supplies are* $d_l = s_l < s_h$, *the labor utilization in the non-peak period is* $u_l = 1$, *and part-time labors join in the peak periods, that is,* $N_l < N_h$*;*
- *In cases (2) and (3), the daily labor supplies are* $d_l < s_l < s_h$, *the labor utilization in the non-peak period is* $u_l < 1$, *and part-time labors join in the peak periods, that is,* $N_l < N_h$*;*
- *In case (4), the daily labor supplies are* $d_l = s_l = s_h$, *the labor utilization in the non-peak period is* $u_l = 1$, *and no part-time labors join in the peak periods, that is,* $N_l = N_h$.

The four cases indicate that under different demand situations, the daily labor supplies differ. In cases (1) and (4), the full-time labors are fully utilized in the non-peak periods; and in cases (2) and (3), the full-time labors are not fully utilized in the non-peak periods. In cases (1), (2) and (3), part-time labors are employed in the peak periods; and in case (4), no part-time labors are employed in the peak periods.

3.6 The Impact of Parameters on the Logistics Performance

In the above section, we have solved the optimal solutions of the wages w_j^* ($j \in [l, h]$), and the corresponding labor utilization u_j^* in the peak and non-peak periods. In this section, our aim is to investigate the impact of the parameters on the logistics performance. We focus on the impact on the labor supply s_j and the labor utilization u_j. Recall that the daily labor supplies in the peak and non-peak periods are $s_l^* = \sqrt{\frac{N_o n^2 d_l w_l^*}{\bar{c}}}$, and $s_h^* = \frac{N_0 n^2 w_h^*}{\bar{c}}$, and u_j^* ($j \in [l, h]$) are given in Proposition 5.

Among the parameters in the model, we focus on the impact of four parameters: the employment cost c_h, the labor market size N_0, the labor opportunity cost $\bar{c}$, and the delivery elasticity b, on the logistics performance.

We first discuss the impact of the employment cost c_h. The results are given in the following proposition.

Proposition 7. *The impacts of c_h differ in the four cases in Proposition 5.*

- *In case (1), in the non-peak periods, the wage and logistics performance are not influenced by c_h; and in the peak periods, w_h^* and s_h^* decrease in c_h.*
- *In case (2), in the non-peak periods, w_l^* and s_l^* increase in c_h, and u_l^* decreases in c_h; and in the peak periods, w_h^* and s_h^* decrease in c_h.*
- *In case (3), in the non-peak periods, w_l^* and s_l^* decrease in c_h, and u_l^* increases in c_h; and in the peak periods, w_h^* and s_h^* decrease in c_h.*
- *In case (4), the wages and logistics performance are not influenced by c_h.*

Note that in the non-peak periods of case (2), w_l^* and s_l^* increase as the employment cost c_h increases. This result is intuitive because when the cost of employing part-time labors increases, the logistics system may employ more full-time labors in the non-peak periods, who will help overcome the shortage of part-time labors in the peak periods. Meanwhile, the labor utilization in the non-peak periods may decrease, which implies that the increased number of full-time labors will decrease the average workload of each labor, leading to lower utilization. However, in case (3), the results are opposite because we have $w_l^* = w_h^*$. As the employment cost in peak periods c_h increases, the firms choose to decrease w_h^*, and decrease w_l^* at the same time. In the peak periods, the wages, daily supply, and total demand in one peak period may all decrease in c_h. The reason is that the higher cost of employing part-time labors results in the logistics system employing less part-time labors by decreasing the wage in the peak periods.

Next, we go to discuss the impacts of the labor pool size N_0 and the maximum opportunity cost $\bar{c}$. It is observed that $\frac{N_0}{\bar{c}}$ always exists in the formulas. Thus, we let $e = \frac{N_0}{\bar{c}}$, and investigate the impacts of e on the logistics performance. The increasing of e represents the increasing of N_0 or the decreasing of $\bar{c}$.

Proposition 8. *The impacts of e in the four cases in Proposition 5 are as follows.*

- *In case (1), in the non-peak periods, w_l^* decreases in e, and s_l^* is not influenced; and in the peak periods, w_h^* decreases in e, and s_h^* increases in e when $\frac{-b}{1-b} p a^{\frac{1}{1-b}} n^{\frac{-2b}{1-b}} \hat{w}_h^{\frac{-1}{1-b}} e^{\frac{-1}{1-b}} > nc_h$, and decreases otherwise.*
- *In case (2), in the non-peak periods, w_l^* and s_l^* increase in e, and u_l^* decreases in e; and in the peak periods, w_h^* decreases in e, and s_h^* increases in e when $\frac{-b}{1-b} p a^{\frac{1}{1-b}} n^{\frac{-2b}{1-b}} \hat{w}_h^{\frac{-1}{1-b}} e^{\frac{-1}{1-b}} > nc_h$, and decreases otherwise.*
- *In case (3), in the non-peak periods, w_l^* decreases in e when $c_h n(1 - \frac{-b}{1-b}) - c_h \sqrt{\frac{d_l}{16e\hat{w}}}(1 - \frac{-2b}{1-b}) - \frac{-b}{1-b}\frac{d_l t_l}{e} > 0$, and increases otherwise, s_l^* decreases in e, and u_l^* increases in e; in the peak periods, w_h^* decreases in e when $c_h n(1 - \frac{-b}{1-b}) - c_h \sqrt{\frac{d_l}{16e\hat{w}}}(1 - \frac{-2b}{1-b}) - \frac{-b}{1-b}\frac{d_l t_l}{e} > 0$, and increases otherwise, s_h^* decreases in e.*
- *In case (4), in the non-peak periods, w_l^* decreases in e, and s_l^* is not influenced; and in the peak periods, w_h^* decreases in e, and s_h^* is not influenced.*

Proposition 8 shows that w_j^* and s_j^* ($j \in [l, h]$) may increase or decrease in e. In common sense, as the labor pool size N_0 increases or the maximum opportunity cost $\bar{c}$ decreases, more labors are willing to participate, so the wages w_j^* probably decrease and the supplies s_j^* increase. However, the results indicate that the opposite impact may happen. The reason is that when e increases, under different conditions, the logistics system may employ more labors in the peak (non-peak) periods, and meanwhile employ less labors in the non-peak (peak) periods, resulting in the increase of w_j^* and the decrease of s_j^*. Moreover, as $e = \frac{N_0}{\bar{c}}$, the impact of the decrease of N_0 (increase of $\bar{c}$) can be weaken or even eliminated by the decrease of $\bar{c}$ (increase of N_0). With the development of the society, the changes of labor pool size and opportunity cost are foreseen. Thus, these results provide valuable insights into the labor market management in e-commerce logistics.

Finally, we focus on the delivery elasticity of the consumers. Given the maximum daily supply in the peak periods, the delivery elasticity b, which means the consumer sensitivity to the delivery speed, influences the total demand D_h in one peak period. When b increases ($b < 0$), D_h decreases. The impacts of b are given in the following proposition.

Proposition 9. *The impacts of b on the wages and logistics performance are as follows.*

- *In cases (1) and (2), in the non-peak periods, w_l^*, u_l^* and s_l^* are not influenced; and in the peak periods, w_h^* and s_h^* decrease in b when $(\frac{p}{\hat{w}_h} - 1) + \ln(ta^{-1}\hat{w}_h)[(\frac{p}{\hat{w}_h} - 1)\frac{-b}{1-b} - 1] > 0$, and increase otherwise.*

- *In case (3), in the non-peak periods, w_l^* and s_l^* decrease (u_l^* increases) in b when $(\frac{p}{\hat{w}} - 1) + \ln(ta^{-1}\hat{w})[(\frac{p}{\hat{w}} - 1)\frac{-b}{1-b} - 1] > 0$, and increase (decreases) otherwise; in the peak periods, w_h^* and s_h^* also decrease in b when $(\frac{p}{\hat{w}} - 1) + \ln(ta^{-1}\hat{w})[(\frac{p}{\hat{w}} - 1)\frac{-b}{1-b} - 1] > 0$, and increase otherwise.*

- *In case (4), the wages and logistics performance in the peak and non-peak periods are not influenced.*

This proposition reveals that as the delivery elasticity b increases, the wage w_h^* and supply s_h^* in the peak periods may decrease, because the demand in the peak periods decreases and less labors are needed. They may also increase in b because increasing the wage w_h^* and supply s_h^* will increase the demand in the peak periods, and increase the firm's profit consequently. What's more, in case (3), the wage w_l^*, labor utilization u_l^* and supply s_l^* may decrease for increase at the same time because we have $w_l^* = w_h^*$ and w_l^* changes with w_h^*.

3.7 Conclusions

In this study, we consider the optimal wage decisions to the labors with peak and non-peak delivery demands. The optimal wages determine the labor supplies and the labor utilizations in the peak and non-peak periods. We identify four conditions, under which the labor employment decisions of the logistics system differ. The system may or may not fully utilize the full-time labors in the non-peak periods, and it may or may not employ part-time labors in the peak periods.

We also investigate the impacts of the parameters on the wages and logistics performance. With the development of society, the parameters, such as the labor pool size, the labor opportunity cost, the part-time labor employment cost, and the consumer's elasticity of delivery speed, will change significantly. With the understanding of how they impact the labor employment decisions and the corresponding logistics performance, the logistics system can adjust its decisions accordingly to maximize the profit.

We conclude that the increase in the labor pool size or the decrease in the labor opportunity cost, which causes more labors to participate, does not necessarily decrease the wages and increase the labor supply, and opposite impact may happen under different condition. The increase in the part-time labor employment cost may result in more labors employment in the non-peak periods, in the most realistic case when the wage in the peak periods is higher than that in the non-peak periods. However, the increased number of full-time labors results in lower labor utilization in the non-peak periods. The consumer elasticity of delivery speed leads to lower demand in the peak periods. Under different conditions, the firm may decrease the numbers of labors in the peak and non-peak periods at the same time, or it will employ more labors to increase the supply to increase the demand in the peak periods.

Chapter 4

New Retail versus Traditional Retail in E-commerce: Channel Establishment, Price Competition, and Consumer Recognition

4.1 Introduction

E-commerce has reached its development bottleneck in recent years. The growth of sales amount slows down all over the world. In China, the growth rate declined from 50% in 2011 to 21.3% in 2017 (see Table 4.1), and is forecasted to decrease continuously in the future years. E-commerce has to find a new way to achieve further growth.

Table 4.1: Growth rate of e-commerce sales

Year	2011	2012	2013	2014	2015	2016	2017	2018E	2019E
Sales (Trillion)	0.8	1.2	1.9	2.8	3.8	4.7	5.7	6.6	7.5
Growth Rate (%)	-	50.0	58.3	47.3	35.7	23.7	21.3	15.8	13.6

Key: 'E' means expected.

In response to this situation, a concept of "New Retail" was proposed in 2016 by Alibaba, the biggest e-commerce company in China. The "new retail" concept, which is taken as a threat to the other largest global player Amazon, is basically the integration of online and offline retailing channels. The two channels complement each other in three aspects: (1) touching the intangible online products in offline physical stores, (2) solving the offline showrooming problem by setting the same price in online and offline channels, (3) delivering the online orders from nearby offline stores to speed up the deliveries.

The consumers will have a refreshing purchasing experience buying from a firm with new retail. Experiencing the product offline eliminates their product value uncertainty. With the same price online and offline, they do not need to compare prices in the same firm's different channels. They can purchase online after experiencing, and wait for a shorter time for the product to come than from a traditional e-commerce firm. The excellent purchasing experience will definitely change the consumers' purchasing behaviors, and yield benefits to the new retail firms.

4.1.1 Real Cases Discussion

Some real cases are given below to show the practical actions of the e-commerce firms to establish new retail.

Amazon Books

Amazon Books is the physical extension of Amazon.com. It has 13 offline (physical) stores located in cities such as California and New York, and more offline stores are coming soon. Its products are classified into 35 categories, such as physical books and electronic devices. The consumers can buy the products directly from the offline stores, or buy them online and wait for the deliveries from the offline stores.

JD.com & Yonghui Supermarket

JD.com, which is the second largest e-commerce firm in China, cooperates with Yonghui Supermarket to transform parts of the supermarkets into its offline selling channel. Moreover, JD invites Tencent, the largest Internet service provider in China, to jointly support Yonghui Supermarket on the fresh foods new retail establishment. The offline stores named "Super Species" provide the consumers with the evaluation opportunity of the fresh foods. The consumers can buy the fresh foods directly from the offline stores, or buy them from the supermarket APP and wait for the deliveries from the offline stores.

Alibaba

Alibaba exhibits its online clothes in clothes shops named "Simple Style" to provide try-on service. The value uncertainty in online clothes purchasing is always very high, but in the offline shops, consumers can try the clothes on and eliminate the uncertainty. They can buy the clothes immediately from the offline shops, or buy them online using the QR codes on the clothes and wait for the deliveries at home for only a few hours. Alibaba also builds fresh food stores named "HEMA" for consumers to experience, and provides both the online and offline purchasing ways.

Although the cases above seem to go very well, the truth is that the offline stores in the real cases are very few. They are only trials for the later process of the complete development of new retail. There are three main difficulties that the e-commerce firms will encounter:

(1) There is a tradeoff between the channel building cost and the distances of the stores to the consumers. Closer distances to consumers will be easier for consumers to come, and consequently attract more demand. However, to cover a fixed market area, closer distances require more offline stores, leading to a higher channel building cost.

(2) The online competitors will compete with them for the market share. When the new retail firm establishes the offline channel, the well-developed or newly-built online competitors will not wait for it to finish.

(3) The combination of the online and offline channels gives rise to difficulty in recognizing consumers. Online consumers are easy to recognize by their accounts, but identifying a consumer in both channels needs high technology support, and should avoid the violation of consumer privacy at the same time.

In our model, we properly capture these three problems respectively and find the solutions to these problems, which will be elaborated in the next subsection.

4.1.2 The Model and Results

To investigate the problems above, we develop a duopoly model with a new retail firm and an online firm, who sell the same product during two selling periods. The new retail firm offers the same price in online and offline channels, a certain product value in offline stores and high-speed deliveries for online orders from offline stores. The online firm provides product with value uncertainty, and normal-speed deliveries.

The channel cost of the new retail firm is higher than that of the online firm, including the cost of channel establishment, logistics, etc. Here, we capture the problem of channel building tradeoff between the hassle cost to consumers and the channel cost. When the hassle cost decreases, the channel cost will quadratically increase.

The consumers of the new retail firm resolve the value uncertainty offline in the first period, and can purchase the product online in the second period. The consumers of the online firms buy the product with uncertainty for the first time, and if they buy the product for the second time, the product value is certain because of the first-time purchasing. The consumers are heterogenous in the sensitivity to the hassle cost of purchasing, which is measured by the average hassle distance between the consumers and the nearest offline stores. The consumers will compare the utilities from the two firms and maximize their overall utilities across the two selling periods.

The new retail firm and the online firm seek to occupy the market share, for which price competition is commonly used. We adopt the behavior-based pricing (BBP) competition model, in which the two firms offer different prices in the second period to consumers with different purchasing behaviors/histories (which firm they buy from) in the first period. In the BBP model, after the two firms announce prices in the first period, they will offer a pair of prices in the second period: one to its previous consumers for retaining them, and one to the competitor's previous consumers for attracting them. The consequence is that a part of consumers switch from their first-period chosen firm to the other firm. We solve for the Nash equilibrium of the two-period game. In addition, we also consider a benchmark pricing model, in which the consumer behaviors/histories are out of consideration.

One necessary matter of using BBP is the consumer recognition on their historical purchasing behaviors/histories. Consumers with different behaviors will be offered different prices from the firms. However, in new retail, consumer may not be completely recognized. To capture the problem of consumer recognition, we consider the situation that the new retail firm cannot recognize all its previous consumers, to investigate the influence of the difficulty of consumer recognition.

The BBP model is not new, but our model has unique novelties. First, we are the first to properly use it in a new retail and online competition problem. In our model, the second period is not the repeat of the first period, but fits the features of purchasing from the new retail firm and the online firm. We also consider the situations using behavior-based pricing (BBP) and without consideration of consumer behaviors/histories, to compare the new retail firm's decisions on channel building. Second, we consider different channel costs of the two firms. The new retail firm's choice on how well to build its offline stores to balance the channel cost and consumers' distances (or hassle costs) to offline stores is a significant problem. Third, considering the difficulty in consumer recognition, we analyze the situation when consumers are not completely recognized.

The results we obtain provide valuable insights to the new retail development in e-commerce. Each price of the new retail firm, which is the first-period price, the second-period price to own consumers or the second-period price to competitor's consumers, is higher than the corresponding price of the traditional online firm. The reason is that the new retail firm has a higher channel cost in building offline stores and high-speed deliveries from the stores. Under certain condition, the new retail firm will establish a larger hassle cost to consumers in the BBP model than that in the benchmark model. Interestingly, the difficulty of consumer recognition leads the new retail firm to occupying market share. Meanwhile, it may benefit the new retail firm.

The rest of the chapter is organized as follows. We present the literature review following the Introduction. In Sections 4.3 and 4.4, we introduce the model setup and present the benchmark model without consideration of consumer behaviors. In Section 4.5, we solve for the Nash equilibrium with BBP competition. In Sections 4.6 and 4.7, we analyze the impact of the BBP competition and consumer recognition. We conclude the managerial implications and directions for future research in Section

4.8. All proofs appear in the Appendix C.

4.2 Literature Review

The new retail is the integration of the online and offline channels. In the retail channel studies, there is a literature stream studying the competition between the manufacturer and its independent retailers. Chiang et al. (2003) construct a price setting game to show that direct marketing of the manufacturer helps improving its profit. Meanwhile, the participation of manufacturer may not hurt the retailer because of a lower wholesale price. Cattani et al. (2006) study the pricing matching between the manufacturer and the retailer. They find that when the direct channel of the manufacturer is not convenient enough for the consumers, the equal-pricing is preferred. Otherwise, the equal-pricing policy will be abandoned. Liu and Zhang (2006) conclude that the retailer is worse off when it can personalized pricing. Other aspects, e.g., sales effort (Tsay and Agrawal 2004), service decisions and competition (Hu and Li 2012, Chen et al. 2008), and drop-shipping (Netessine and Rudi 2006), are also studied.

Recently, Li et al. (2017) focus on the impact of the retailer's risk-averse behavior and the selling cost of manufacturer on the optimal supply chain decisions. Li et al. (2016) also consider a risk-averse retailer and present an improved risk-sharing contract for the supply chain cooperation. Amrouche and Yan (2016) investigate the wholesale pricing of the manufacturer when it simultaneously manages an online channel and a traditional offline channel. Soleimani et al. (2016) study the pricing strategies of dual-channel under centralization and decentralization. Ding et al. (2016) consider a hierarchical pricing decision process and find the optimal pricing strategy in a dual-channel supply chain with a manufacturer as the leader and a retailer as the follower. In the same setting, Huang et al. (2018) extend the investigation to consider an optimization problem under stochastic demand.

Omni-channel management is also broadly studied, the goal of which is to provide all available channels to consumers without barriers. Ansari et al. (2008) empirically study the consumer migration between channels. Ofek et al. (2011) explore the impact of product returns in the omni-channel setting. Gao and Su (2016) develop a theoretical framework to study the impact of Buy-Online-and-Pick-up-in-Store on the store operations. Unlike the concept of omni-channel, the concept of new retail in this chapter focuses on the integration of online and offline channels. The new retail does not consider all the available channels, and is also an improvement of omni-channel in the consumer data collection (recognition).

Our work is also closely related to a literature stream of the behavior-based pricing. Fudenberg and Villas-Boas (2006) present a comprehensive review. Villas-Boas (2004) considers a monopoly model in which the firms live infinitely and each consumer lives for two periods. He shows that the monopolist is worse off than if it could not recognize its previous consumers. Villas-Boas (1999) studies in the similar setup and shows that in a duopoly market, the firms offer lower prices to attract the competitor's consumers. Fudenberg and Tirole (2000) study the duopoly poaching under both short-term and long-term contracts when consumer's brand preferences are fixed or independent over time. Zhang (2011) shows that behavior-based personalization damages the product differentiation and intensifies the price competition. Shaffer and Zhang (2000) find that price discrimination leads to lower prices. When the demand is symmetric (asymmetric), charging lower prices to the competitor's (own) consumers is optimal. Gehrig et al. (2011) consider a model in which a new firm shows up in the market but does not have the access to consumer purchasing histories, and implies that the use of BBP is for exploitation, not exclusion.

The literature also examines when the firm should conduct the BBP. Acquisti and Varian (2005) study the situation when firms can commit to a pricing policy with price discrimination or not, and conclude that it is never optimal for the firm to distinguish the high-value and low-value consumers. Pazgal and Soberman (2008) examine the competitive effect when firms are able to commit about whether to conduct behavior-based pricing, and conclude that firms' profits of conducting BBP are always lower than those without BBP. Esteves (2010) extends this finding when the consumers are myopic. Shin and Sudhir (2010) attempts to answer the firms' dilemma that when firms should conduct behavior-based pricing, considering two features of consumers: heterogeneity in consumer value and changing preference. Chen (2008) studies the situation when the market consists of a stronger firm and a weaker firm, and concludes that when the weaker firm can exist in the market, and BBP benefits the consumers. Gehrig et al. (2012) develop asymmetric duopoly model to show that the uniform pricing is better off than history-based pricing, and the latter benefits the consumers.

Some studies focus on the impact on the BBP decisions of other factors. Li and Jain (2015) considers the consumer unfairness with behavior-based pricing, and studies the impact of fairness concerns on firms' prices, profits, consumer surplus and social welfare. Jing (2016) studies the firms' quality differentiation and profits in BBP model, when the product qualities are exogenous or endogenous. Rhee and Thomadsen (2016) study the asymmetric BBP model with vertical differentiation, and highlight the quality-adjusted cost difference between firms, and consumer discounting and firm discounting. Colombo (2016) develops a BBP model with incomplete information on consumers' purchasing histories and shows that the impact of information accuracy on profits is non-monotonic. Esteves and Cerqueira (2017) are the first considering the firms' advertising efforts on prices to target consumers in a BBP model. Caillaud and De Nijs (2014) consider the pricing discrimination with loyalty reward, which helps the firm in extracting more surplus from consumers who reveal strong preferences and in recognizing new consumers.

4.3 The Model

We consider a duopoly market, which consists of a new retail firm (n-firm) and an online firm (o-firm), selling the same product during two selling periods. The production cost of the product is c, which is assumed to be zero without loss of generality. The base value of the product is v. Following the literature (Li and Jain 2015, Jing 2016, Rhee and Thomadsen 2016), v is assumed to be constant and sufficiently high, so the market is fully covered.

The n-firm combines the online and offline channels, and offers the same price p_n in the two channels. We assume that one consumer to the new retail firm will first go to the offline stores to certain the value v with hassle cost t_n (transportation cost) and pay p_n. After that, he will buy the product online with the certain value v with a hassle cost h_n (paying shipping fees, waiting for the product to come). As the hassle costs in two periods are the costs of getting the product, by self immediately with t_n or by waiting for a high-speed delivery with h_n, the consumer experiences do not differ much. We assume that $t_n = h_n$ for the model efficiency. The combined channel cost of selling one product is c_n.

The online firm only provides online channel. One consumer to the online firm will buy with value uncertainty (discount) of αv at price p_o, and the hassle cost is h_o. We assume that all consumers are homogenous in the value uncertainty. After the first trial in the first period, the consumer will buy with certain value v with the same price p_o and hassle cost h_o for the second time. The channel cost of the online firm for selling one product is c_o.

With shorter distances to offline stores and higher speed deliveries, the n-firm is more convenient for consumers to purchase from than the o-firm. Hence, the consumer hassle cost of buying from the n-firm is less than that of the o-firm, that is, $h_n < h_o$. Meanwhile, the n-firm builds two combined channels and the o-firm has one online channel. Thus, the channel cost of the n-firm is higher than that of the o-firm, that is, $c_n > c_o$. Both firms seek to maximize their overall profits across the two periods.

We assume that consumers are heterogeneous in the sensitivity of hassle cost. The sensitivity level θ is assumed to be uniformly distributed at $[0, 1]$. The consumer utility of purchasing from the n-firm in each period is $v - p_n - \theta h_n$. For the o-firm, the utility of a first-time consumer is $\alpha v - p_o - \theta h_o$, and the utility of a second-time consumer is $v - p_o - \theta h_o$. The consumers compare the utilities of purchasing from the two firms and maximize their overall utility across the two periods.

We adopt the behavior-based pricing (BBP) model, in which both firms try to occupy the other firm's previous consumers with the tool of price. In the second period, they discriminate consumers by offering different prices to their previous consumers and its competitor's consumers. Thus, they decide a set of prices: the

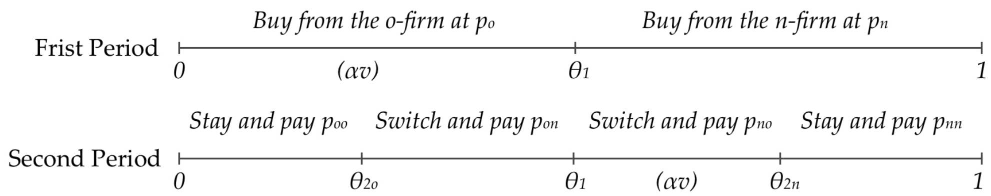


Figure 4.1: The market structure of the behavior-based pricing model.

first-period price $p_i, i \in [n, o]$, the second-period price to its first-period consumers p_{ii} and the second-period price to the competitor's first-period consumers $p_{ji}(j \neq i, j \in [n, o])$. The consequence is that in the second period, the two firms will poach the other firm's first-period market share.

The market structure in the two selling periods is illustrated in Fig. 4.1. In the first period, the consumers with $0 < \theta < \theta_1$ buy from the o-firm with product value αv, and the left consumers buy from the n-firm with certain product value v. In the second period, the o-firm's first-period market share is split into two market shares of two firms by θ_{2o}. The n-firm's first-period market share is also split by θ_{2n}, but the difference is that for consumers in $[\theta_1, \theta_{2n}]$, the product value is αv because it is the first time for them to purchase from the o-firm.

4.4 Benchmark without Consideration of Consumer Behaviors

Before proceeding, we present a benchmark model in which consumer behaviors/histories are not taken into account. In this case, the prices offered by one firm in two periods are the same.

The consumers make purchasing decisions to maximize their utilities across the two periods. As shown in Fig. 4.2, the marginal consumer who is indifferent in buying from the two firms locates at θ'. Consumers with $\theta < \theta'$ choose the o-firm because they are less sensitive to hassle cost, while consumers with $\theta > \theta'$ choose the n-firm. And in the first period, the o-firm's consumers purchase with uncertain value αv. Solving $(\alpha v - p_o' - \theta' h_o) + (v - p_o' - \theta' h_o) = 2(v - p_n' - \theta' h_n)$, we have

$$\theta' = \frac{2(p_n' - p_o') - v(1-\alpha)}{2(h_o - h_n)} \tag{4.1}$$

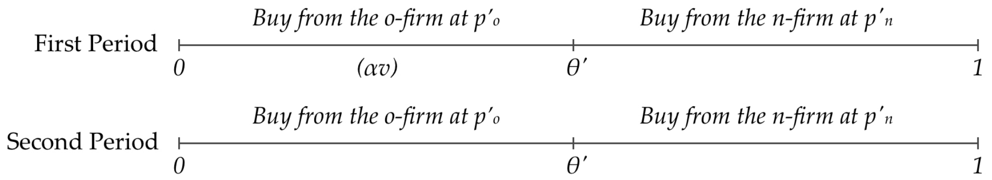


Figure 4.2: The market structure without consideration of consumer behaviors.

The o-firm maximizes its profit $\pi_o' = 2(p_o' - c_o)\theta'$, while the n-firm maximizes its profit $\pi_n' = 2(p_n' - c_n)(1 - \theta')$. The Nash equilibrium of the two firms are given in the following proposition.

Proposition 10. *(Equilibrium without consideration of consumer behaviors) The prices of the two firms are* $p_o' = [2(c_n + 2c_o) + 2(h_o - h_n) - v(1-\alpha)]/6, p_n' = [2(2c_n + c_o) + 4(h_o - h_n) + v(1-\alpha)]/6$. *It is obvious that* $p_n' > p_o'$. *The market cutoff* $\theta' = [2(c_n - c_o) + 2(h_o - h_n) - v(1-\alpha)]/[6(h_o - h_n)]$.
The market share of the o-firm is $s_o = \theta'$ *and the n-firm is* $s_n = 1 - \theta'$. $s_o < (\geq) s_n$ *when* $2(c_n - c_o) - (h_o - h_n) - v(1-\alpha) < (\geq) 0$.
The profits of the two firms are $\pi_o' = [2(c_n - c_o) + 2(h_o - h_n) - v(1-\alpha)]^2/[36(h_o - h_n)], and \pi_n' = [4(h_o - h_n) - 2(c_n - c_o) + v(1-\alpha)]^2/[36(h_o - h_n)]$. $\pi_o < (\geq) \pi_n$ *when* $2(c_n - c_o) - (h_o - h_n) - v(1-\alpha) < (\geq) 0$.
In addition, the n-firm can exist in the market only when $\theta' < 1$, *that is,* $2(c_n - c_o) - 4(h_o - h_n) - v(1-\alpha) < 0$. *The o-firm can exist in the market when* $\theta' > 0$, *that is* $2(c_n - c_o) + 2(h_o - h_n) - v(1-\alpha) > 0$.

The proposition shows that the n-firm provides a higher price than that of the o-firm. The reason is that the n-firm has a higher channel cost, and it provides the consumers with lower hassle cost and no value uncertainty. The market shares and profits of the two firms are influenced by the differences between the channel costs, hassle costs, and value uncertainties of the two firms: $c_n - c_o$, $h_o - h_n$, and

$v(1-\alpha)$. When $2(c_n - c_o) - (h_o - h_n) - v(1-\alpha) < (\geq)0$, the market share and the profit of the o-firm are smaller (larger) than those of the n-firm. When $2(c_n - c_o) - 4(h_o - h_n) - v(1-\alpha) < 0$, the n-firm can survive in the market, or the o-firm will occupy all the market share. When $2(c_n - c_o) + 2(h_o - h_n) - v(1-\alpha) > 0$, the o-firm can survive in the market, or the n-firm will occupy all the market share.

In this chapter, we emphasize the three problems faced by the new retail firm (n-firm): the choice on the distance of offline stores to consumers, the price decision in price competition, and the difficulty of consumer recognition. We now discuss the first problem: the distance of offline stores to consumers. As a shorter distance between the consumers and the offline stores indicates smaller transportation cost, shipping fee and waiting time, that is, a smaller hassle cost to the consumers. Therefore, the distance problem can be reflected by the hassle cost h_n. To provide a lower average hassle cost (shorter distance) to consumers in a market area, the n-firm must build more offline stores with a higher cost. Given the existing online hassle cost h_o and the corresponding channel cost c_o, the n-firm decreases the hassle cost to h_n with a higher channel cost c_n.

We assume that the relation between the reduced hassle cost and increased channel cost is quadratic: $c_n - c_o = A(h_o - h_n)^2$. Transforming it into $c_n = A(h_o - h_n)^2 + c_o$, this formula follows the commonly used quadratic cost function $c = aq^2 + bq + d$ in the operations management research with $b = 0$. $\frac{\partial c_n}{\partial (h_o - h_n)} = 2A(h_o - h_n) > 0$ satisfies the property that the marginal channel cost of reducing consumer hassle cost increases as $h_o - h_n$ increases, which corresponds to the reality.

Substituting $c_n - c_o = A(h_o - h_n)^2$ into the n-firm's profit $\pi'_n = [2(c_n - c_o) - 4(h_o - h_n) - v(1-\alpha)]^2/[36(h_o - h_n)]$, we solve for the optimal h_n^* and the result is given in the following proposition.

Proposition 11. *The optimal hassle cost $h_n'^*$ to the consumers of the n-firm is*

(1) if $v(1-\alpha) \leq \frac{2}{3A}$, $(h_o - h_n)^* = \frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}$, *then* $h_n'^* = \left(h_o - \frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}\right)^+$;

(2) if $v(1-\alpha) > \frac{2}{3A}$, $(h_o - h_n)^* = \frac{\sqrt{4+2Av(1-\alpha)}-2}{2A}$, *then* $h_n'^* = \left(h_o - \frac{\sqrt{4+2Av(1-\alpha)}-2}{2A}\right)^+$,

since the hassle cost $h_n'^*$ must be positive.

This proposition indicates that, when the consumer value uncertainty of the o-firm $v(1-\alpha)$ is smaller or larger than $\frac{2}{3A}$, the n-firm will provide different optimal hassle costs to consumers. We can observe that when $h_n < h_n'^*$, as the hassle cost h_n increases, the profit of the n-firm increases. This counter-intuitive result is because although the higher hassle cost has a negative impact on the consumers' purchasing decisions, the benefit due to the quadratical decrease of the channel cost will overweigh the negative impact, and consequently increase the profit.

4.5 Competition with Behavior-based Pricing

In this section, we study the price competition with behavior-based pricing. The two firms try to poach the competitor's market share in the second period, by offering different prices to consumers with different purchasing behaviors in the first period.

We solve for the subgame-perfect equilibrium of the two-period game backwards. We will first analyze the consumer choices and the firms' pricing decisions in the second period, taken the first-period market share as given.

4.5.1 Competition in the Second Period

In the second period, the market share in the first period is taken as settled. We assume a cutoff value θ_1 such that consumers with $\theta < \theta_1$ purchase from the o-firm in the first period, while consumers with $\theta > \theta_1$ purchase from the n-firm in the first period. Thus, θ_1 also represents the market share of the o-firm in the first period and $1-\theta_1$ represents the market share of the n-firm.

The competitions in the second period occur separately in the market share of the n-firm and the o-firm in the first period. As shown in Fig. 4.3, consumers who purchase from the o-firm are divided by a new indifferent cutoff value θ_{2o}. Consumers with $\theta < \theta_{2o}$ buy from the o-firm for the second time in the second period, and consumers with $\theta_{2o} < \theta < \theta_1$ switch and buy from the n-firm for the first time in the second period. Thus, θ_{2o} can be found by setting $v - p_{oo} - \theta_{2o}h_o = v - p_{on} - \theta_{2o}h_n$. Solving for θ_{2o} yields

$$\theta_{2o} = \frac{p_{on} - p_{oo}}{h_o - h_n} \tag{4.2}$$

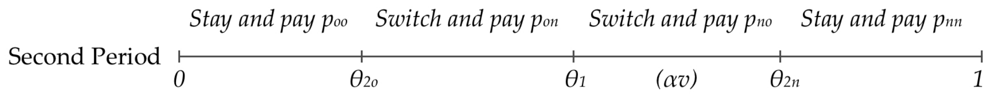


Figure 4.3: The two competitive regions in the second period: $\theta < \theta_1$ and $\theta > \theta_1$.

The o-firm sets its price to maximize its profit $\pi^o_{2o} = (p_{oo} - c_o)\theta_{2o}$, while the n-firm maximizes $\pi^n_{2o} = (p_{on} - c_n)(\theta_1 - \theta_{2o})$. The equilibrium is summarized in Lemma 7.

Lemma 7. *(Competition on the o-firm's market share) The equilibrium can be expressed as a function of* θ_1. *The prices are* $p^*_{oo} = [(c_n + 2c_o) + \theta_1(h_o - h_n)]/3$, *and* $p^*_{on} = [(2c_n + c_o) + 2\theta_1(h_o - h_n)]/3$. *The n-firm's market share is split by two firms with* $\theta_{2o} = [(c_n - c_o) + \theta_1(h_o - h_n)]/[3(h_o - h_n)]$.

We then consider the competition in the n-firm's market share in the first period. The consumers with $\theta_1 < \theta < \theta_{2n}$ switch and buy from the o-firm for the first time with value αv, and consumers with $\theta_{2n} < \theta < 1$ buy from the n-firm for the second time. The new cutoff value θ_{2n} is solved by setting $\alpha v - p_{no} - \theta_{2o} h_o = v - p_{nn} - \theta_{2o} h_n$. Thus,

$$\theta_{2n} = \frac{p_{nn} - p_{no} - v(1-\alpha)}{h_o - h_n}. \tag{4.3}$$

In such a case, the o-firm maximizes its profit $\pi^o_{2n} = (p_{no} - c_o)(\theta_{2n} - \theta_1)$ and the o-firm maximizes $\pi^n_{2n} = (p_{nn} - c_n)(1 - \theta_{2n})$. The equilibrium is summarized in Lemma 8.

Lemma 8. *(Competition on the n-firm's market share) The equilibrium can be expressed as a function of* θ_1. *The prices are* $p^*_{no} = [(c_n + 2c_o) + (1 - 2\theta_1)(h_o - h_n) - v(1 - \alpha)]/3$, *and* $p^*_{nn} = [(2c_n + c_o + (2 - \theta_1)(h_o - h_n) + v(1 - \alpha)]/3$. *The o-firm's market share is split by the two firms with* $\theta_{2n} = [(c_n - c_o) + (1 + \theta_1)(h_o - h_n) - v(1 - \alpha)]/[3(h_o - h_n)]$.

Lemma 7 and 8 present the equilibrium results in the second period in terms of θ_1. Next, we go back to the first period.

4.5.2 Competition in the First Period

In the first period, the consumers are clear that their choices in the first period will affect their prices in the second period. Thus, their decisions are based on the utility of two periods. As shown in Fig. 4.4, at the cutoff value θ_1, the marginal consumer is indifferent between buying from the o-firm in the first period and the n-firm in the second period, and from the n-firm in the first period and o-firm in the second period, so

$$(\alpha v - p_o - \theta_1 h_o) + (v - p_{on} - \theta_1 h_n) = (v - p_n - \theta_1 h_n) + (\alpha v - p_{no} - \theta_1 h_o). \quad (4.4)$$

From equation (4.4) we have

$$p_o - p_n = p_{no} - p_{on}. \quad (4.5)$$

In Lemma 7 and Lemma 8, we have obtained p^*_{no} and p^*_{on} in the function of θ_1. By substituting p^*_{no} and p^*_{on} , θ_1 can be represented as a function of $p_o - p_n$:

$$\theta_1 = \frac{(c_o - c_n) + (h_o - h_n) - 3(p_o - p_n) - v(1-\alpha)}{4(h_o - h_n)}. \quad (4.6)$$

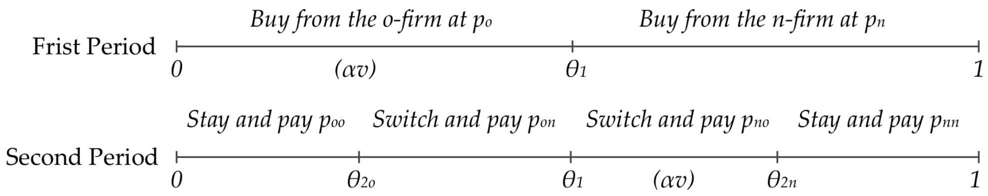


Figure 4.4: The competition in the first period.

The profits of the o-firm and n-firm are the overall profits across the two periods, which can be specified using the first-period market shares determined by θ_1 and the second-period equilibrium prices and market shares in Lemma 7 and Lemma 8. Thus, the profits of the o-firm and the n-firm are

$$\pi_o = (p_o - c_o)\theta_1 + (p_{oo} - c_o)\theta_{2o} + (p_{no} - c_o)(\theta_{2n} - \theta_1), \quad (4.7)$$

$$\pi_n = (p_n - c_n)(1 - \theta_1) + (p_{on} - c_n)(\theta_1 - \theta_{2o}) + (p_{nn} - c_n)(1 - \theta_{2n}). \quad (4.8)$$

4.5.3 Equilibrium

The pure-strategy equilibrium of π_o and π_n is found by solving the Nash game between the o-firm and n-firm. $p_{oo}, p_{on}, p_{no}, p_{nn}$ are written by Lemmas 7 and 8, θ_{2o}

and θ_{2n} by equations (4.2) and (4.3), π_o and π_n by equations (4.7) and (4.8). All the decision variables in equilibrium are solved. The results are given in the following proposition.

Proposition 12. *The equilibrium prices p_o and p_n, and market cutoff θ_1 in the first period are*

$$\begin{aligned} p_o =& [(6c_n + 18c_o) + 13(h_o - h_n) - 11v(1-\alpha)]/24, \\ p_n =& [(18c_n + 6c_o) + 19(h_o - h_n) - 5v(1-\alpha)]/24, \\ \theta_1 =& [2(c_n - c_o) + 7(h_o - h_n) - v(1-\alpha)]/[16(h_o - h_n)]. \end{aligned}$$

The prices and market cutoffs in the second period are

$$\begin{aligned} p_{oo} =& [(18c_n + 30c_o) + 7(h_o - h_n) - v(1-\alpha)]/48, \\ p_{on} =& [(18c_n + 6c_o) + 7(h_o - h_n) - v(1-\alpha)]/24, \\ p_{no} =& [(6c_n + 18c_o) + (h_o - h_n) - 7v(1-\alpha)]/24, \\ p_{nn} =& [(30c_n + 18c_o) + 25(h_o - h_n) + 17v(1-\alpha)]/48, \\ \theta_{2o} =& [18(c_n - c_o) + 7(h_o - h_n) - v(1-\alpha)]/[48(h_o - h_n)], \\ \theta_{2n} =& [18(c_n - c_o) + 23(h_o - h_n) - 17v(1-\alpha)]/[48(h_o - h_n)]. \end{aligned}$$

The equilibrium profits of the two firms are

$$\begin{aligned} \pi_o =& [(540(c_n - c_o)^2 + 599(h_o - h_n)^2 - 610(h_o - h_n)v(1-\alpha) + 263v^2(1-\alpha)^2 \\ & +708(h_o - h_n)(c_n - c_o) - 540(c_n - c_o)v(1-\alpha)]/[2304(h_o - h_n)], \\ \pi_n =& [(540(c_n - c_o)^2 + 1847(h_o - h_n)^2 + 638(h_o - h_n)v(1-\alpha) + 263v^2(1-\alpha)^2 \\ & -1788(h_o - h_n)(c_n - c_o) - 540(c_n - c_o)v(1-\alpha)]/[2304(h_o - h_n)]. \end{aligned}$$

In addition, from $\theta_{2o} < \theta_1 < \theta_{2n}$, we have $7(h_n - h_o) - 6(c_n - c_o) - v(1-\alpha) > 0$ and $(h_o - h_n) + 6(c_n - c_o) - 7v(1-\alpha) > 0$.

The analysis of the equilibrium will be presented in the next section.

4.6 BBP Equilibrium Analysis

In this section, we analyze the equilibrium with behavior-based pricing. First, we compare the prices offered by the two firms in the two periods and the results are given in the following proposition.

Proposition 13. *The price of the n-firm is larger than that of the o-firm in the first period, $p_n > p_o$. In the second period, the n-firm also offers higher prices than the o-firm for one firm's first-period consumers, $p_{on} > p_{oo}$ and $p_{nn} > p_{no}$. Meanwhile, each firm offers a lower price to the competitor's consumers than that to its previous consumers, $p_{on} < p_{nn}$ and $p_{no} < p_{oo}$.*

This proposition indicates that, each price offered by the n-firm is larger than the corresponding price of the o-firm in both periods. The reason is that each price of the n-firm minuses the corresponding price of the o-firm is determined to be positive because $c_n > c_o$ (the n-firm has a larger channel cost than that of the o-firm) and $h_n < h_o$ (the n-firm provides a smaller average consumer hassle cost than that of the o-firm). In addition, both firms offer a lower price to its competitor's consumers than that to its previous consumers, which means that they treat its competitor's consumers better than their own consumers. This result is in accord with the literature.

The market share of the o-firm is $so_1 = \theta_1$ in the first period and $so_2 = \theta_{2o} + \theta_{2n} - \theta_1$ in the second period. The market share of the n-firm is $sn_1 = 1 - \theta_1$ in the first period and $sn_2 = \theta_1 - \theta_{2o} + 1 - \theta_{2n}$ in the second period. We compare the market shares and profits of the two firms and give the condition when one firm obtains more market share and profit than the other.

Proposition 14. *When $2(c_n - c_o) - (h_o - h_n) - v(1-\alpha) < (\geq) 0$, the o-firm's market shares in two periods and profit are all smaller (larger) than those of the n-firm, and vice versa.*

The condition $2(c_n - c_o) - (h_o - h_n) - v(1-\alpha) < (\geq) 0$ determines the comparison results of the two firms on market shares in two periods and the profits. Recall that in the benchmark model (see Section 4.4), the condition for the o-firm's market share and profit to be larger (no larger) than those of the n-firm is also $2(c_n - c_o) - (h_o - h_n) - v(1-\alpha) < (\geq) 0$. The conditions which determine the comparison result of the two firms are the same in the BBP model and benchmark model.

However, the optimal decision of hassle cost h_n in BBP competition will be different from the benchmark model. We still assume that $c_n - c_o = A(h_o - h_n)^2$. By substituting it into the profit of n-firm, we have

$$\begin{aligned}\pi_n = &(h_o - h_n)[540A^2(h_o - h_n)^2 - 1788A(h_o - h_n) + 1847] \\ &- [540A(h_o - h_n) - 638]v(1-\alpha) + \frac{263v^2(1-\alpha)^2}{2304(h_o - h_n)}\end{aligned} \tag{4.9}$$

We calculate the optimal h_n^* and give the result in the following proposition.

Proposition 15. *The optimal hassle cost choice h_n^* to the n-firm with BBP competition is:*

$h_n^* = (h_o - h^*)^+$, where h^* satisfies $(1620A^2h^{*2} - 3576Ah^* + 1847) - 540Av(1-\alpha) - \frac{263v^2(1-\alpha)^2}{h^{*2}} = 0$ and is unique.

Compared with the optimal hassle cost $h_n'^*$ in the benchmark model without consideration of consumer behaviors, we have the following proposition.

Proposition 16. *The comparison result of the hassle costs in the BBP model and benchmark model is:*

(1) if $v(1-\alpha) \leq \frac{53(\sqrt{6817}-2619)}{4096A}$, then $h^* \geq \frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}$, so $h_n^* \leq h_n'^*$;

(2) if $v(1-\alpha) > \frac{53(\sqrt{6817}-2619)}{4096A}$, then $h^* < \frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}$ and $h^* < \frac{\sqrt{4+2Av(1-\alpha)}-2}{2A}$, so $h_n^* > h_n'^*$.

This proposition indicates that when the value uncertainty (discount) $v(1-\alpha)$ of the o-firm is small enough, the n-firm will establish an offline channel with a smaller hassle cost than that in the benchmark model. When the value uncertainty of the o-firm is large, the n-firm will establish the offline channel with a larger hassle cost than that in the benchmark model. This result reveals the insight that in the BBP competition, the new retail firm who chooses a product with large enough value uncertainty can build a smaller number of online stores than that without the BBP competition. In practice, this insight is applicative to the the new retail firms who choose the products with large value uncertainty, such as fresh foods and clothes.

4.7 Consumer Recognition

We have solved the problems of the n-firm on the store distance to consumers (represented by consumer hassle cost) and price competition, now we focus on the difficulty of consumer recognition. We assume that the n-firm cannot recognize all its previous consumers. For the un-recognized consumers, the difference is that when they purchase from the n-firm in the second period, they are taken as the o-firm's first-period consumers. The price to the un-recognized consumers are p_{on} instead of p_{nn} (noted that $p_{on} < p_{nn}$), which leads to different consumer purchasing decisions.

As shown in Fig. 4.5, a part of consumers, who would switch to the o-firm in the second period, now stay with n-firm at un-recognized price p_{on}. A new cutoff value θ_u shows up, at which the utility of purchasing from the o-firm $\alpha v - p_{no} - \theta_u h_o$ and the utility from the n-firm $v - p_{on} - \theta_u h_n$ are equal. So, we have

$$\theta_u = \frac{p_{on} - p_{no} - v(1-\alpha)}{h_o - h_n} = \frac{2(c_n - c_o) + (h_o - h_n) - 3v(1-\alpha)}{4(h_o - h_n)}. \tag{4.10}$$

Moreover, as shown in Fig. 4.6, if θ_u is smaller than θ_1, then the market structure is affected more by consumer un-recognition. As all the n-firm's first-period consumers stay with the n-firm because they are un-recognized, their purchasing decisions in the first period will exclude the consideration of the second period. They directly choose between buying from the o-firm at price p_o and from the n-firm at price p_n. Solving $\alpha v - p_o - \theta h_o = v - p_n - \theta h_n$ and based on equation (4.5) $p_o - p_n = p_{no} - p_{on}$, we obtain

$$\theta = \frac{p_n - p_o - v(1-\alpha)}{h_o - h_n} = \frac{p_{on} - p_{no} - v(1-\alpha)}{h_o - h_n} = \theta_u. \tag{4.11}$$

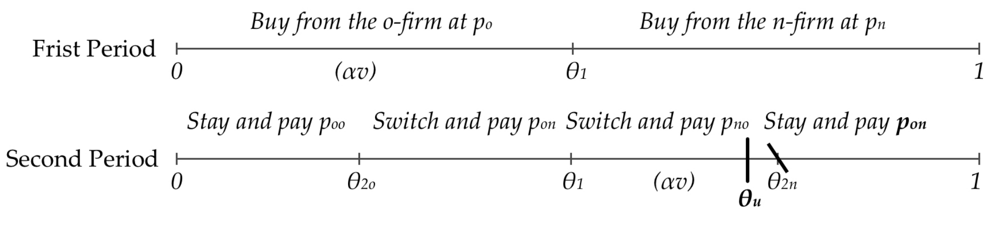


Figure 4.5: The market structure with consumer un-recognition ($\theta_u \geq \theta_1$).

Our aim is to explore the impact of consumer recognition on the n-firm's profit. If $\theta_u \geq \theta_1$ (see Fig. 4.5), the n-firm retains extra consumers in $[\theta_u, \theta_{2n}]$ in the second period. This part of extra market share is $\Delta\theta n = \theta_{2n} - \theta_u$. Although the n-firm retains more consumers, it offers the un-recognized consumers a lower price. The n-firm's profit difference because of consumer un-recognition is $\Delta\pi_n = (p_{on} - c_n)(1 - \theta_u) - (p_{nn} - c_n)(1 - \theta_{2n})$.

If $\theta_u < \theta_1$ (see Fig. 4.6), the n-firm retains all its first-period consumers, and it also occupies more market share in the first period. The extra market share because of consumer recognition is $\Delta\theta n_1 = \theta_1 - \theta_u$ in the first period and $\Delta\theta n_2 = \theta_{2n} - \theta_1$ in the second period. The profit difference due to consumer recognition is $\Delta\pi_n = (p_n - c_n)(\theta_1 - \theta_u) + (p_{on} - c_n)(1 - \theta_1) - (p_{nn} - c_n)(1 - \theta_{2n})$.

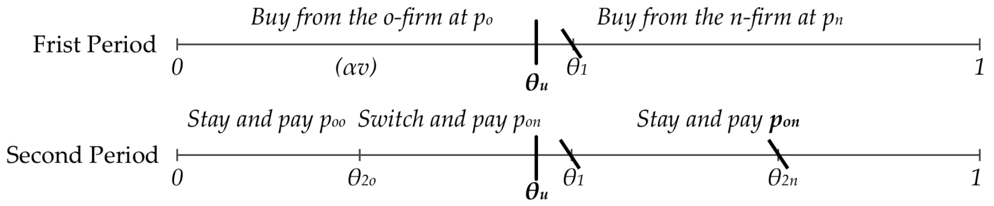


Figure 4.6: The market structure with consumer un-recognition ($\theta_u < \theta_1$).

The impact of consumer un-recognition to the n-firm is summarized in the following proposition.

Proposition 17. *(The impact of consumer un-recognition to the n-firm)*

(1) if $6(c_n - c_o) - 3(h_o - h_n) - 11v(1 - \alpha) \leq 0$*, then* $\theta_u \geq \theta_1$*. The n-firm occupies extra market share* $\Delta\theta n = \frac{11(h_o - h_n) - 6(c_n - c_o) + 19v(1-\alpha)}{48(h_o - h_n)}$ *than that with complete recognition in the second period. The profit difference of the n-firm because of consumer un-recognition is* $\Delta\pi_n = -\frac{[11(h_o - h_n) - 6(c_n - c_o) + 19v(1-\alpha)]^2}{2304(h_o - h_n)} < 0$*, which indicates that the profit of the n-firm decreases because of the consumer un-recognition.*

(2) if $6(c_n - c_o) - 3(h_o - h_n) - 11v(1 - \alpha) < 0$*, then* $\theta_u < \theta_1$*. The n-firm occupies extra market share* $\Delta\theta n_1 = \frac{-[6(c_n - c_o) - 3(h_o - h_n) - 11v(1-\alpha)]}{16(h_o - h_n)}$ *in the first period, and* $\Delta\theta n_2 = \frac{(h_o - h_n) + 6(c_n - c_o) - 7v(1-\alpha)}{24(h_o - h_n)}$ *in the second period. The profit difference due*

to consumer un-recognition is $\Delta\pi_n$*, and* $\Delta\pi_n > 0$ *when* $95(h_o - h_n)^2 + 372(c_n - c_o)v(1-\alpha) + 302(h_o - h_n)v(1-\alpha) - 36(c_n - c_o)^2 - 300(c_n - c_o)(h_o - h_n) - 625v^2(1-\alpha)^2 > 0$.

The consumer un-recognition problem results in that the new retail occupies more market share in the BBP model. Under certain condition, it even benefits the n-firm. However, to the o-firm, the consumer un-recognition of the n-firm is harmful. The n-firm's extra market share is the loss of the o-firm, and the o-firm obviously loses the profit on the market share.

4.8 Conclusions

Some companies in e-commerce have been developing the new retail in recent 2-3 years. The three main challenges are the building of offline stores, the prices in a competition environment, and the difficulty in recognizing consumers. This chapter seeks to provide management insights to help the e-commerce in developing its new retail mode. We employ the behavior-based pricing (BBP) model, to characterize the price competition between the new retail firm and the traditional online firm. We capture the building of offline stores by the consumer hassle cost, which is a measure of the easiness of reaching the offline stores by the consumers.

We conclude that, in the BBP model, each price offered by the n-firm is higher than the corresponding price of the o-firm. Each firm offers a lower price to the competitor's consumers than its own consumers. This result is in accordance with the common sense and literature. We also provide the optimal decision on the new retail firm's consumer hassle cost. When the consumers' value uncertainty to the online firm is large enough, the n-firm will provide a larger hassle cost to the consumers in the BBP model than that in the benchmark model.

As for the consumer un-recognition to the new retail firm, it is not a weakness in the BBP competition, but leads the new retail firm to occupy more market share. The profit difference because of consumer un-recognition may not be positive. Moreover, there exists a condition where the new retail firm benefits from the consumer un-recognition.

Chapter 5

Summary and Future Research

In this thesis, we conduct three studies on Behavioral Operations Management in e-commerce, considering different behavioral aspects in terms of return policy, labor delivery, and new retail. We first point out some future research directions for each study respectively, and then summarize the overall contributions and future development of the three studies.

In the first study, we develop a series of consumer-valuation-based models to investigate the pricing and return policies of the sellers in a competitive e-commerce market. The results show that return policy with a partial refund is always chosen by the sellers in both monopoly and duopoly markets. By comparing the monopoly and duopoly models, we conclude that a monopoly seller will reduce its price by no more than 20% when there comes a competitor, and counter-intuitively, the monopoly seller will also reduce its refund proportion to consumers when it meets a competitor in the market.

This study provides management insights about how a monopoly seller should react to a new competitor in the market, and the return policy decisions of the sellers in a competitive market. We assume that the consumers' valuations on the two products are uniformly distributed and the consumers' valuations on two products are independent. In the future work, more general assumptions on the consumers' valuations should be considered. Additionally, more studies are expected concerning the inventory and quality decisions of the sellers, or the exchange policy instead of the return policy.

In the second study, we focus on the labor wages and the corresponding labor participation behavior for the product delivery with peak and non-peak periods in e-commerce. We find the optimal wage decisions in the peak and non-peak periods to maximize the profit, and analyze the impact of parameters on the wage decisions and logistics performance.

This study provides insights into how the logistics system should make labor employment decisions. In the future work, one direction is the labor replacement from human labors to unmanned aerial vehicles. The decisions of a sharing economic delivery system is another interesting direction following this study.

In the third study, we consider the concept of "new retail", which means to establish an offline channel and integrate it with the online retail channel. We present a BBP model and conclude that the new retail firm will establish an offline channel with a larger hassle cost in the BBP model than that in the benchmark model under certain condition. Interestingly, the difficulty in consumer recognition results in that the new retail firm occupies more market share and may obtain a higher profit than that when the consumers are all recognized.

This study is motivated by real case applications of BBP. We consider the competition of a new retail firm and an online firm, as the new retail mainly influences the consumer purchasing behaviors online. However, the new retail also has an impact on the offline market because the offline stores occupy a part of the offline market. This impact can be a future research direction. In addition, the BBP model works on the premise that consumers can clearly observe the prices they are offered. However, consumers need efforts to make prices visible. The impact of this situation is also worth investigating.

In summary, the three studies capture the human behaviors in product purchasing and returns, employment, and channel choice in e-commerce. The three studies will contribute to the decision making on pricing and return policy, labor wages, and channel development for the e-commerce companies. In fact, the interactions between the consumers and the e-commerce companies are far more complex. First, the aspects that influence the consumers' purchasing and returns decisions are many, including the reviews from previous consumers, discount, bundle selling. Second, the human employment in e-commerce occurs not only in the product delivery, but also in every link of the operations. Third, there are multiple purchasing ways for the consumers, for example, Buy Online and Pick Up In Store (BOPS), subscription, or crowd-funding, etc. Thus, a lot of research directions in behavioral operations management in e-commerce remain to be explored.

Appendix A

Proofs for Chapter 2

Proof for Lemma 2 in Section 2.3.3

We consider two cases because there is $\min\{v_r, V\}$ in Equation (2.3):

(1)when $v_r = \alpha p + \delta \geq V$:

$\Pi_m^W = p\frac{V-v_b}{V} - \alpha p \int_{v_b}^{V} \frac{\alpha p+\delta-v}{2\delta V}\,dv = \frac{1}{4\delta V}(V-v_b)[4\delta p + \alpha p(V+v_b) - 2\alpha p(\alpha p+\delta)]$, where $v_b = \alpha p - \delta + 2\sqrt{\delta(p-\alpha p)}$. Let $\alpha p = t$, and $p = t + \frac{(v_b-t+\delta)^2}{4\delta}$, then Π_m^W is a function of t and v_b, and $t \geq V - \delta$.

$\frac{\partial \Pi_m^W}{\partial(t)} = \frac{V-v_b}{4\delta V}(V - v_b - 2t) \leq \frac{V-v_b}{4\delta V}[V - v_b - 2(V-\delta)] = \frac{V-v_b}{4\delta V}(-V + 2\delta - v_b)$. With $\delta < \frac{V}{2}$, $\frac{\partial \Pi_m^W}{\partial(t)} < 0$, so the optimal solution in case (1) is at the smallest $\alpha p = V - \delta$, that is, $\alpha p + \delta = v_r = V$.

(2)when $v_r = \alpha p + \delta \leq V$:

$\Pi_m^W = p\frac{V-v_b}{V} - \alpha p \int_{v_b}^{v_r} \frac{\alpha p+\delta-v}{2\delta V}\,dv = \frac{1}{V}[p(V-p) + [\sqrt{c}(\sqrt{\delta}-\sqrt{c})]^2] = \Pi_m^O + R_m(c)$.

The optimal solution is $p^* = \frac{V}{2}$ and $\sqrt{c} = \frac{\sqrt{\delta}}{2}$, so $\alpha^* = 1 - \frac{\delta}{2V}$.

The optimal solution in case (1) is included in case (2), so the optimal solution is $p^* = \frac{V}{2}$ and $\alpha^* = 1 - \frac{\delta}{2V}$ in case (2).

Proof for Equations (2.7) and (2.18) in Sections 2.4.1 and 2.4.2

The equilibrium prices in the duopoly models with and without return policy are given in Equation Sets (2.7) and (2.18). We first give the calculations for obtaining them.

$\Pi_{d1}^k = p_1\frac{1}{V_1V_2}\int_{p_1}^{V_1} \min\{(u_2^k)^{-1}(v_1-p_1), V_2\}\,dv_1$, $k = O, W$. We consider two cases:

(1) when $(u_2^k)^{-1}(V_1-p_1) < V_2$: $\Pi_{d1}^k = p_1\frac{1}{V_1V_2}\int_{p_1}^{V_1}(u_2^k)^{-1}(v_1-p_1)\,dv_1$

$= p_1\int_{p_1}^{V_1} x\,d(u_2^k + p_1) = p_1[(V_1-p_1)(u_2^k)^{-1}(V_1-p_1) - \int_{(u_2^k)^{-1}(0)}^{(u_2^k)^{-1}(V_1-p_1)} u_2^k(x)\,dx]$.

(2) when $(u_2^k)^{-1}(V_1-p_1) \geq V_2$: $\Pi_{d1}^k = p_1 \int_{p_1}^{u_2^k(V_2)+p_1} (u_2^k)^{-1}(v_1-p_1)\, dv_1 + p_1 \int_{u_2^k(V_2)+p_1}^{V_1} (V_2)\, dv_1$.

For the two cases, we have

$\frac{\partial \Pi_{d1}^k}{\partial p_1} = (V_1 - 2p_1)\min\{(u_2^k)^{-1}(V_1-p_1), V_2\} - \int_{(u_2^k)^{-1}(0)}^{\min\{(u_2^k)^{-1}(V_1-p_1),V_2\}} (u_2^k(v_2))\, dv_2$.

And $\frac{\partial^2 \Pi_{d1}^k}{\partial p_1^2} = -2\min\{(u_2^k)^{-1}(V_1-p_1), V_2\} - p_1 \frac{\partial \min\{(u_2^k)^{-1}(V_1-p_1),V_2\}}{\partial p_1} < 0$.

$\frac{\partial \Pi_{d1}^k}{\partial p_1}|_{p_1=0} > 0$ and $\frac{\partial \Pi_{d1}^k}{\partial p_1}|_{p_1=V_1} < 0$, so the optimal p_1 is at $\frac{\partial \Pi_{d1}^k}{\partial p_1} = 0$. It is the same for p_2.

Proof for $\Pi_{d1}^W = \Pi_{d1}^O + R_{d1}(c_1)$ in Section 2.4.2

We first prove that $p_i \leq v_{ri} < V_i$.Three cases are considered:

(1)$v_{ri} < p_i(< V_i)$: all consumers' valuation will be high enough to not return the product, that is, the revenue will be the same as the revenue without return policy. $\Pi_{di}^W = \Pi_{d1}^O$.

(2)$p_i \leq v_{ri} < V_i$: consumers with valuation lower than v_{ri} have the probability to return and consumers with valuation higher than v_{ri} will keep the product. $\Pi_{di}^W = \Pi_{di}^O + R_{di}(c_i)$.

(3)$(p_i <)V_i \leq v_{ri}$: all consumers have the probability to return the product. $\Pi_{di}^W = \Pi_{di}^O + R'_{di}(c_i)$. $R'_{di}(c_i) = R_{di}(c_i) - c_i \int_{V_i}^{v_{ri}} \left(\frac{(v_i+\delta_i+c_i-p_i)^2}{4\delta_i} - c_i\right) dv_i$.

It is clear that the optimal choice happens in the second case.

$\Pi_{d1}^W = p_1 D_{d1} - (\alpha_1 p_1) R_{d1}$

$= p_1 \int_{v_{b1}}^{V_1}\int_0^{\min\{(u_2^k)^{-1}(u_1^W(v_1)),V_2\}} \frac{1}{V_1V_2}\, dv_2\, dv_1$

$-(\alpha_1 p_1) \int_{v_{b1}}^{\alpha_1 p_1+\delta_1}\int_0^{\min\{(u_2^k)^{-1}(u_1^W(v_1)),V_2\}} \frac{v_{r1}-v_1}{2\delta_1} \frac{1}{V_1V_2}\, dv_2\, dv_1$

$= p_1 \int_{p_1}^{V_1} \min\{(u_2^k)^{-1}(v_1-p_1), V_2\} \frac{1}{V_1V_2}\, dv_1$

$-p_1 \int_{p_1}^{\alpha_1 p_1+\delta_1} \min\{(u_2^k)^{-1}(v_1-p_1), V_2\} \frac{1}{V_1V_2}\, dv_1$

$+p_1 \int_{v_{b1}}^{\alpha_1 p_1+\delta_1} \min\{(u_2^k)^{-1}((\frac{(v_1+\delta_1-\alpha_1 p_1)^2}{4\delta_1} + \alpha_1 p_1 - p_1), V_2\} \frac{1}{V_1V_2}\, dv_1$

$-(\alpha_1 p_1) \int_{v_{b1}}^{\alpha_1 p_1+\delta_1} \min\{(u_2^k)^{-1}(\frac{(v_1+\delta_1-\alpha_1 p_1)^2}{4\delta_1} + \alpha_1 p_1 - p_1), V_2\} \frac{1}{V_1V_2}\, dv_1$

$+(\alpha_1 p_1) \int_{p_1}^{\alpha_1 p_1+\delta_1} \min\{(u_2^k)^{-1}(v_1-p_1), V_2\} \frac{1}{V_1V_2}\, dv_1$

$= \Pi_{d1}^O + R_{d1}(c_1)$,

because the first part is Π_{d1}^O; the second and fifth parts are combined to be second part in $R_{d1}(c_1)$; the third and fourth parts are combined to be first part in $R_{d1}(c_1)$.

Proof for Lemma 3 and Proposition 2 in Sections 2.4.1 and 2.4.2

For the equation $(V_1-2p_1)\min\{(u_2^k)^{-1}(V_1-p_1),V_2\} = \int_{(u_2^k)^{-1}(0)}^{\min\{u_2^k(V_1-p_1),V_2\}}(u_2^k(v_2))\,dv_2$, we first have $V_1-2p_1>0$, so $p_{d1}^k<\frac{V_1}{2}, k=O,W$. $(u_2^k)^{-1}(v_2)$ are given in Equations (2.8) and (2.9). Additionally, $(u_2^O)^{-1}(v_2)=v_2+p_2$; $(u_2^W)^{-1}(v_2)=\alpha_2p_2-\delta_2+2\sqrt{\delta_2(v_2+c_2)}$ when $v_2\le\delta_2-c_2$; $(u_2^W)^{-1}(v_2)=v_2+p_2$ when $v_2>\delta_2-c_2$. We consider two cases:

(1) when $(u_2^k)^{-1}(v_1-p_1)<V_2$: $(V_1-2p_1)(u_2^k)^{-1}(V_1-p_1)=\int_{(u_2^k)^{-1}(0)}^{(u_2^k)^{-1}(V_1-p_1)}((u_2^k(v_2))\,dv_2$.

In this case, $-2(u_2^k)^{-1}(V_1-p_1)\frac{\partial p_1}{\partial p_2}=\int_{(u_2^k)^{-1}(0)}^{(u_2^k)^{-1}(V_1-p_1)}\frac{\partial u_2^k(v_2)}{\partial p_2}\,dv_2+p_1\frac{\partial(u_2^k)^{-1}(V_1-p_1)}{\partial p_2}<0.$

and $-2(u_2^k)^{-1}(V_1-p_1)\frac{\partial^2 p_1}{\partial p_2^2}=\int_{(u_2^k)^{-1}(0)}^{(u_2^k)^{-1}(V_1-p_1)}\frac{\partial^2 u_2^k(v_2)}{\partial p_2^2}\,dv_2+(V_1-p_1+3\frac{\partial p_1}{\partial p_2})\frac{\partial(u_2^k)^{-1}(V_1-p_1)}{\partial p_2}+p_1\frac{\partial^2(u_2^k)^{-1}(V_1-p_1)}{\partial^2 p_2}-\frac{\partial u_2^k(v_2)}{\partial p_2}|_{v_2=(u_2^k)^{-1}(0)}\frac{\partial(u_2^k)^{-1}(0)}{\partial p_2}>0.$

(2) when $(u_2^k)^{-1}(v_1-p_1)\ge V_2$: $(V_1-2p_1)V_2=\int_{(u_2^k)^{-1}(0)}^{V_2}(u_2^k(v_2))\,dv_2$.

In this case, $-\frac{V_2}{2}\frac{\partial p_1}{\partial p_2}=\int_{(u_2^k)^{-1}(0)}^{V_2}(\frac{\partial u_2^k(v_2)}{\partial p_2})\,dv_2<0$

and $-\frac{V_2}{2}\frac{\partial^2 p_1}{\partial p_2^2}=\int_{(u_2^k)^{-1}(0)}^{V_2}(\frac{\partial^2 u_2^k(v_2)}{\partial p_2^2})\,dv_2-\frac{\partial u_2^k(v_2)}{\partial p_2}|_{v_2=(u_2^k)^{-1}(0)}\frac{\partial(u_2^k)^{-1}(0)}{\partial p_2}>0.$

For all $u_2^k(v_2$, we have $\frac{\partial p_1(p_2)}{\partial p_2}>0$ and $\frac{\partial^2 p_1(p_2)}{\partial p_2^2}<0$. From Figure 2.2, we have $p_{d1}^k>\frac{2V_1}{5}$.

Because $\Pi_{d1}^W=\Pi_{d1}^O+R_{d1}(c_1)$, the sellers will adopt return policy in duopoly market. As c_1 increases, $\frac{\partial R_{d1}(c_1)}{\partial c_1}$ is positive and then negative. The optimal c_{d1}^W is at $\frac{\partial R_{d1}(c_1)}{\partial c_1}=0$, which satisfies $v_{b2}^W(\sqrt{\delta_1}-\sqrt{c_1})(\sqrt{\delta_1}-2\sqrt{c_1})+\int_{v_{b2}^W}^{\min\{(u_2^W)^{-1}(\delta_1-c_1),V_2\}}(u_2^W(v_2)+\delta_1+2c_1-2\sqrt{\delta_1(u_2^W(v_2)+c_1)}-c_1\sqrt{\frac{\delta_1}{u_2^W(v_2)+c_1}})\,dv_2$=0

$\frac{\partial R_{d1}(c_1)}{\partial c_1}|_{c_1=\frac{\delta_1}{4}}<0$, so $c_{d1}^W<\frac{\delta_1}{4}$. Also, $\frac{\partial R_{d1}(c_1)}{\partial c_1}|_{c_1=0}>0$, so $c_{d1}^W>0$, and $\alpha_{di}^W<100\%$.

Proof for Lemma 4 in Section 2.4.3

The equilibrium prices in the duopoly models with and without return policy are given in Equation Sets (2.7) and (2.18). The two sets are in similar form.

Without return policy: $(V_1-2p_1)\min\{V_1-p_1+p_2,V_2\}=\int_{p_2}^{\min\{V_1-p_1+p_2,V_2\}}(v_2-p_2)\,dv_2$.

With return policy: $(V_1-2p_1)\min\{(u_2^W)^{-1}(V_1-p_1),V_2\}=\int_{v_{b2}^W}^{\min\{(u_2^W)^{-1}(V_1-p_1),V_2\}}(u_2^W(v_2))\,dv_2$.

p_{d1}^O and p_{d1}^W are the solutions to the above two equations. We first employ a $\hat{p_1}$ who satisfies: $(V_1-2p_1)\min\{(u_2^W)^{-1}(V_1-p_1),V_2\}=\int_{p_2}^{\min\{(u_2^W)^{-1}(V_1-p_1),V_2\}}(v_2-p_2))\,dv_2$.

Since $u_2^W(v_2)>v_2-p_2$, it is obvious that $p_{d1}^W<\hat{p_1}$.

Let M satisfies $(V_1-2p_1)M=\int_{p_2}^{M}(v_2-p_2)\,dv_2$, and $M>p_2$.

$\frac{\partial((V_1-2p_1)M)}{\partial M} = V_1 - 2p_1 = \frac{\partial(\int_{p_2}^{M}(v_2-p_2)\,dv_2)}{\partial M} = \frac{M}{2} - p_2 + \frac{p_2^2}{2M}$ and $\frac{\partial(V_1-2p_1)}{\partial M} = \frac{1}{2}(1-\frac{p_2^2}{M^2}) > 0$

Since $\min\{V_1 - p_1 + p_2, V_2\} \geq \min\{(u_2^W)^{-1}(V_1 - p_1), V_2\}$, $V_1 - 2p_{d1}^O \leq V_1 - 2\hat{p_1}$, so $p_{d1}^O \geq \hat{p_1}$.

Thus, $p_{d1}^O \geq \hat{p_1} > p_{d1}^W$.

Proof for Proposition 3 in Section 2.5.1

In Lemma 3, we have $\frac{2V_1}{5} < p_{d1}^O < \frac{V_1}{2}$, and $p_m^O = \frac{V_1}{2}$. So, $\frac{4V_1}{5} < p_{d1}^O < p_m^O$.

Π_{d1}^O is given in Equation (2.5), which is price multiplying demand. See from Figure 2.1, the demand is smaller than $\frac{V_1-p_1}{V_1}$, so $\Pi_{d1}^O < \frac{p_{d1}^O(V_1-p_{d1}^O)}{V_1}$. $\Pi_m^O = \frac{p_m^O(V_1-p_m^O)}{V_1}$. With $p_{d1}^O < p_m^O = \frac{V_1}{2}$, it is obvious that $\Pi_{d1}^O < \Pi_m^O$.

Proof for Proposition 4 in Section 2.5.1

In Proposition 2, we have $\frac{2V_1}{5} < p_{d1}^W < \frac{V_1}{2}$, and $p_m^W = \frac{V_1}{2}$. So, $\frac{4V_1}{5} < p_{d1}^W < p_m^W$.

Also, since $c_{d1}^W < \frac{\delta}{4}$ and $c_m^W = \frac{\delta_1}{4}$, $c_{d1}^W < c_m^W$.

Noted that $\Pi_{d1}^W = \Pi_{d1}^O + R_{d1}(c_1)$, $\Pi_m^W = \Pi_m^O + R_m(c)$(from proof for Lemma 2) and $\Pi_{d1}^O < \Pi_m^O$, if $R_{d1}(c_1) < R_m(c)$, then $\Pi_{d1}^W < \Pi_m^W$.

$R_{d1}(c_1) = c_1\left(\int_{\sqrt{2\delta_1 c_1}}^{2\delta_1} \min\left\{(u_2^k)^{-1}\left(\frac{v_1^2}{4\delta_1} - c_1\right), V_2\right\} \frac{1}{V_1 V_2}\, dv_1\right)$
$-c_1\left(\int_0^{\delta_1 - c_1} \min\left\{(u_2^k)^{-1}(v_1), V_2\right\} \frac{1}{V_1 V_2}\, dv_1\right)$
$= c_1\left(\int_{\sqrt{2\delta_1 c_1}}^{2\delta_1} \min\left\{(u_2^k)^{-1}\left(\frac{v_1^2}{4\delta_1} - c_1\right), V_2\right\} \frac{1}{V_1 V_2}\, dv_1\right)$
$-c_1\left(\int_{\sqrt{2\delta_1 c_1}}^{2\delta_1} \min\left\{(u_2^k)^{-1}\left(\frac{v_1^2}{4\delta_1} - c_1\right), V_2\right\} \frac{1}{V_1 V_2}\, d\left(\frac{v_1^2}{4\delta_1} - c_1\right)\right)$
$= c_1\left(\int_{\sqrt{2\delta_1 c_1}}^{2\delta_1} \min\left\{(u_2^k)^{-1}\left(\frac{v_1^2}{4\delta_1} - c_1\right), V_2\right\} \frac{1}{V_1 V_2}\left(1 - \frac{v_1}{2\delta_1}\right) dv_1\right)$
$< c_1\left(\int_{\sqrt{2\delta_1 c_1}}^{2\delta_1} \frac{1}{V_1}\left(1 - \frac{v_1}{2\delta_1}\right) dv_1\right) = c_1(2\delta_1 - 2\sqrt{\delta_1 c_1}) - c_1(\delta_1 - c_1) = [\sqrt{c_1}(\sqrt{\delta_1} - \sqrt{c_1})]^2.$

So, $R_{d1}(c_1) < [\sqrt{c_{d1}^W}(\sqrt{\delta_1} - \sqrt{c_{d1}^W})]^2$. $R_m(c_1) = [\sqrt{c_m^W}(\sqrt{\delta_1} - \sqrt{c_m^W})]^2$.

Since $c_{d1}^W < \frac{\delta}{4} = (c_m^W)^*$, it is obvious that $R_{d1}(c_1) < R_m(c)$.

Proof for $p_d^W/c_d^W < p_m^W/c_m^W$ in Section 2.5.2

Let $f(c) = -\frac{\delta^2}{3} - \frac{8\sqrt{\delta c^3}}{3} + 2\delta c + \frac{c^2}{2}$. Then $p_d^W = \sqrt{2V^2 + f(c)} - V$. $\frac{\partial f(c)}{\partial c} = c + 2\delta - 4\sqrt{\delta c}$. $\frac{\partial^2 f(c)}{\partial c^2} = 1 - 2\sqrt{\frac{\delta}{c}} < 0$. $\frac{\partial f(c)}{\partial c}|_{c=\frac{\delta}{4}} = \frac{\delta}{4} > 0$, so $f(c)$ increases in c. $f(c)|_{c=\frac{\delta}{4}} = -\frac{13\delta}{96} < 0$.

So, $-\frac{\delta^2}{3} < f(c) < 0$ when $c < \frac{\delta}{4}$, and $\sqrt{2V^2 - \frac{\delta^2}{3}} - V < p_d^W < (\sqrt{2} - 1)V$.

Let $\frac{\partial R_d(c)}{\partial c} = p(\sqrt{\delta} - \sqrt{c})(\sqrt{\delta} - 2\sqrt{c}) - \frac{\delta^2}{3} + 3c^{\frac{1}{2}}\delta^{\frac{3}{2}} - 5\delta c + \frac{13c^{\frac{3}{2}}\delta^{\frac{1}{2}}}{3} - 2c^2 - \delta c ln\frac{\delta}{c}$.

We have $\frac{\partial R_d(c)}{\partial c}$ is positive and then negative. The optimal c_d^W is at $\frac{\partial R_d(c)}{\partial c} = 0$.

Also, with $p > \sqrt{2V^2 - \frac{\delta^2}{3}} - V > (\sqrt{\frac{23}{3}} - 2)\delta$, we have $\frac{\partial R_d(c)}{\partial c}|_{c=\frac{\delta}{2(\sqrt{2}+1)}} > 0$, so $c_d^W > \frac{\delta}{2(\sqrt{2}+1)}$. Thus, $p_d^W / c_d^W < \frac{\sqrt{2}-1)V}{\frac{\delta}{2(\sqrt{2}+1)}} = \frac{2V}{\delta} = p_m^W / c_m^W$.

Appendix B

Proofs for Chapter 3

Proof of Lemma 5.

$\frac{\partial^2\Pi_l}{\partial w_l^2} = -c_h\sqrt{\frac{N_0 d_l}{16n\bar{c}\hat{w}^3}} < 0$ and $\frac{\partial^2\Pi_h}{\partial w_h^2} = \frac{-b}{1-b}(\frac{-b}{1-b}-1)pA\hat{w_h}^{\frac{-b}{1-b}-2} - (1+\frac{-b}{1-b})\frac{-b}{1-b}A\hat{w_h}^{\frac{-b}{1-b}-1} < 0$, in which $A = a^{\frac{1}{1-b}}(\frac{N_0 n^2}{\bar{c}})^{\frac{-b}{1-b}}$. Thus, Π_l and Π_h are both concave in w_l and w_h respectively, and the optimal solutions are given in the first order conditions.

Proof of Lemma 6.

$\frac{\partial^2\Pi}{\partial w^2} = \frac{-b}{1-b}(\frac{-b}{1-b}-1)pA\hat{w}^{\frac{-b}{1-b}-2} - (1+\frac{-b}{1-b})\frac{-b}{1-b}A\hat{w}^{\frac{-b}{1-b}-1} - c_h\sqrt{\frac{N_0 d_l}{16\bar{c}\hat{w}^3}} < 0$. Π is concave in w. Thus, the optimal solution is given in the first order condition.

Proof of Proposition 5.

Using the Lagrangian Multiplier Method, we first have:

$$\begin{aligned}\Pi(w_l, w_h, \lambda_1, \lambda_2) &= (p - w_l)d_l t_l + (p - w_h)a^{\frac{1}{1-b}}\left(\frac{N_0 n^2 w_h}{\bar{c}}\right)^{\frac{-b}{1-b}} - c_h\left(\frac{N_0 w_h n}{\bar{c}} - \sqrt{\frac{N_0 w_l d_l}{\bar{c}}}\right)\\ &+\lambda_1(\frac{d_l\bar{c}}{N_0 n^2} - w_l + \eta_1) + \lambda_2(w_l - w_h + \eta_2)\end{aligned}$$

The Extreme Value will be found at

$$\frac{\partial\Pi(w_l,w_h,\lambda_1,\lambda_2)}{\partial w_l} = 0, \quad \frac{\partial\Pi(w_l,w_h,\lambda_1,\lambda_2)}{\partial w_h} = 0,$$
$$\frac{\partial\Pi(w_l,w_h,\lambda_1,\lambda_2)}{\partial \lambda_1} = 0, \quad \frac{\partial\Pi(w_l,w_h,\lambda_1,\lambda_2)}{\partial \lambda_2} = 0.$$

We find four possible conditions, which determine the four cases in Proposition 5:

$$\begin{aligned}&(1)\lambda_1 = 0, \lambda_2 = 0;\\ &(2)\lambda_1 \neq 0, \eta_1 = 0, \lambda_2 = 0;\\ &(3)\lambda_1 = 0, \lambda_2 \neq 0, \eta_2 = 0;\\ &(4)\lambda_1 \neq 0, \eta_1 = 0, \lambda_2 \neq 0, \eta_2 = 0.\end{aligned}$$

In cases (1) and (4), $w_l^* = \frac{d_l\bar{c}}{N_0 n^2}$, so $u_l^* = \frac{d_l}{s_l} = \frac{d_l\bar{c}}{N_o n^2 w_l^*} = 1$. In case (2), as $d_l < \frac{c_h N_0 n}{2\bar{c}t_l}$, $u_l^* = \frac{2\bar{c}t_l}{c_h N_0 n}d_l < 1$. In case (3), as $\hat{w} > \frac{d_l\bar{c}}{N_0 n^2}$, $u_l^* = \sqrt{\frac{d_l\bar{c}}{N_0 n^2\hat{w}}} < 1$.

Proof of Proposition 6.

It is obvious that when $w_l^* = 1$, $d_l = s_l$; when $w_l^* < 1$, $d_l < s_l$. When $w_l^* = w_h^*$ and $u_l^* = 1$, $N_l = N_h$; when $w_l^* u_l^* < w_h^*$, $N_l < N_h$.

Proof of Proposition 7.

In case (1), it is obvious that the non-peak periods are not influenced by c_h. In the peak periods, $\frac{-b}{1-b}(\frac{-b}{1-b}-1)pA\hat{w}_h{}^{\frac{-b}{1-b}-2}\frac{\partial \hat{w}_h}{\partial c_h} - (1+\frac{-b}{1-b})\frac{-b}{1-b}A\hat{w}_h{}^{\frac{-b}{1-b}-1}\frac{\partial \hat{w}_h}{\partial c_h} - \frac{N_0 n}{\bar{c}} = 0$, in which $A = a^{\frac{1}{1-b}}(\frac{N_0 n^2}{\bar{c}})^{\frac{-b}{1-b}}$. Thus, $\frac{\partial \hat{w}_h}{\partial c_h} = \frac{\partial w_h^*}{\partial c_h} < 0$. $s_h^* = \frac{N_o n^2 w_h^*}{\bar{c}}$, so $\frac{\partial s_h^*}{\partial c_h} < 0$.
In case (2), it is obvious that in the non-peak periods, $\frac{\partial w_l^*}{\partial c_h} = \frac{2c_h N_0}{4\bar{c} d_l t_l^2} > 0$. $s_l^* = \sqrt{\frac{N_o n^2 w_l^* d_l}{\bar{c}}}$, so $\frac{\partial s_l^*}{\partial c_h} > 0$. $\frac{\partial u_l^*}{\partial c_h} = -\frac{2\bar{c} d_l t_l}{c_h^2 N_0 n} < 0$. In the peak periods, the analysis is the same with that in case (1).
In case (3), $\frac{-b}{1-b}(\frac{-b}{1-b}-1)pA\hat{w}^{\frac{-b}{1-b}-2}\frac{\partial \hat{w}}{\partial c_h} - (1+\frac{-b}{1-b})\frac{-b}{1-b}A\hat{w}^{\frac{-b}{1-b}-1}\frac{\partial \hat{w}}{\partial c_h} - c_h\sqrt{\frac{N_0 d_l}{16\bar{c}\hat{w}^3}}\frac{\partial \hat{w}}{\partial c_h} + \sqrt{\frac{N_0 d_l}{4\bar{c}\hat{w}}} - \frac{N_0 n}{\bar{c}} = 0$. With $\hat{w} > \frac{d_l \bar{c}}{N_0 n^2}$, we have $\frac{\partial \hat{w}}{\partial c_h} < 0$.
In case (4), it is obvious that the solutions are not influenced by c_h.

Proof of Proposition 8.

In case (1), in the non-peak periods, $\frac{\partial w_l^*}{\partial e} = -\frac{d_l}{e^2 n^2} < 0$, $s_l^* = d_l$. In the peak periods, $[\frac{-b}{1-b}(\frac{-b}{1-b}-1)pa^{\frac{1}{1-b}}(en^2)^{\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-b}{1-b}-2}\frac{\partial \hat{w}_h}{\partial e} - (1+\frac{-b}{1-b})\frac{-b}{1-b}a^{\frac{1}{1-b}}(en^2)^{\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-b}{1-b}-1}\frac{\partial \hat{w}_h}{\partial e}] + [\frac{-b}{1-b}(\frac{-b}{1-b})pa^{\frac{1}{1-b}}e^{\frac{-b}{1-b}-1}n^{2\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-b}{1-b}-1} - (1+\frac{-b}{1-b})\frac{-b}{1-b}a^{\frac{1}{1-b}}e^{\frac{-b}{1-b}-1}n^{2\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-b}{1-b}}] - c_h n = 0$. With $\frac{-b}{1-b}pa^{\frac{1}{1-b}}e^{\frac{-b}{1-b}}n^{2\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-b}{1-b}-1} - (1+\frac{-b}{1-b})a^{\frac{1}{1-b}}e^{\frac{-b}{1-b}}n^{2\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-b}{1-b}} = c_h n e$, we have $[\frac{-b}{1-b}(\frac{-b}{1-b})pa^{\frac{1}{1-b}}e^{\frac{-b}{1-b}-1}n^{2\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-b}{1-b}-1} - (1+\frac{-b}{1-b})\frac{-b}{1-b}a^{\frac{1}{1-b}}e^{\frac{-b}{1-b}-1}n^{2\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-b}{1-b}}] = \frac{-b}{1-b}c_h n$. Thus, $[\frac{-b}{1-b}(\frac{-b}{1-b}-1)pa^{\frac{1}{1-b}}(en^2)^{\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-b}{1-b}-2} - (1+\frac{-b}{1-b})\frac{-b}{1-b}a^{\frac{1}{1-b}}(en^2)^{\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-b}{1-b}-1}]\frac{\partial \hat{w}_h}{\partial e} = c_h n(1-\frac{-b}{1-b})$, and $\frac{\partial \hat{w}_h}{\partial e} < 0$. Let E$=[\frac{-b}{1-b}(\frac{-b}{1-b}-1)pa^{\frac{1}{1-b}}(en^2)^{\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-b}{1-b}-2} - (1+\frac{-b}{1-b})\frac{-b}{1-b}a^{\frac{1}{1-b}}(en^2)^{\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-b}{1-b}-1}]$, and $\frac{\partial \hat{w}_h}{\partial e} = \frac{c_h n(1-\frac{-b}{1-b})}{E}$.
$s_h^* = en^2 w_h^*$, $\frac{\partial s_h^*}{\partial e} = n^2 e(\frac{\hat{w_h^*}}{e} + \frac{\partial \hat{w_h^*}}{\partial e}) = n^2 e(\frac{\hat{w_h^*}}{e} + \frac{c_h n(1-\frac{-b}{1-b})}{E}) = n^2\frac{1}{E}[Ew_h^* + ec_h n(1-\frac{-b}{1-b})] = n^2\frac{1}{E}[\frac{-b}{1-b}ec_h n - \frac{-b}{1-b}pa^{\frac{1}{1-b}}n^{2\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-1}{1-b}}e^{\frac{-b}{1-b}} + ec_h n(1-\frac{-b}{1-b})]$. $\frac{\partial s_h^*}{\partial e} > 0$ when $\frac{-b}{1-b}pa^{\frac{1}{1-b}}n^{\frac{-2b}{1-b}}\hat{w}_h{}^{\frac{-1}{1-b}}e^{\frac{-1}{1-b}} > nc_h$.
In case (2), $\frac{\partial w_l^*}{\partial e} = \frac{c_h^2}{4d_l t_l^2} > 0$. $s_l^* = en^2 w_l^*$, $\frac{\partial s_l^*}{\partial e} = n^2 w_l^* + n^2\frac{\partial w_l^*}{\partial e} > 0$. $\frac{\partial u_l^*}{\partial e} = -\frac{2d_l t_l}{c_h e^2 n} < 0$.
In the peak periods, the solutions are the same with those in case (1).
In case (3), $[\frac{-b}{1-b}(\frac{-b}{1-b}-1)pa^{\frac{1}{1-b}}(en^2)^{\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-b}{1-b}-2}\frac{\partial \hat{w}}{\partial e} - (1+\frac{-b}{1-b})\frac{-b}{1-b}a^{\frac{1}{1-b}}(en^2)^{\frac{-b}{1-b}}\hat{w}^{\frac{-b}{1-b}-1}\frac{\partial \hat{w}}{\partial e}] +$

$[\frac{-b}{1-b}(\frac{-b}{1-b})pa^{\frac{1}{1-b}}e^{\frac{-b}{1-b}-1}n^{2\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-b}{1-b}-1} - (1+\frac{-b}{1-b})\frac{-b}{1-b}a^{\frac{1}{1-b}}e^{\frac{-b}{1-b}-1}n^{2\frac{-b}{1-b}}\hat{w}_h{}^{\frac{-b}{1-b}}] + [c_h\sqrt{\frac{d_l}{16e\hat{w}}} - c_h\sqrt{\frac{ed_l}{16\hat{w}^3}}\frac{\partial\hat{w}}{\partial e}] - c_h n = 0$. $F\frac{\partial\hat{w}}{\partial e} + \frac{-b}{1-b}(c_h n + \frac{d_l t_l}{e} - c_h\sqrt{\frac{d_l}{4e\hat{w}}}) + c_h\sqrt{\frac{d_l}{16e\hat{w}}} - c_h n = 0$, $F < 0$.
Thus, $\frac{\partial\hat{w}}{\partial e} < 0$ when $c_h n(1 - \frac{-b}{1-b}) - c_h\sqrt{\frac{d_l}{16e\hat{w}}}(1 - \frac{-2b}{1-b}) - \frac{-b}{1-b}\frac{d_l t_l}{e} > 0$.
$s_l^* = \sqrt{en^2\hat{w}d_l}$, $\frac{\partial s_l^*}{\partial e} = \frac{1}{2}\sqrt{\frac{en^2}{\hat{w}d_l}}(\frac{\partial\hat{w}}{\partial e} + \frac{\hat{w}}{e}) = \frac{1}{2}\sqrt{\frac{en^2}{\hat{w}d_l}}\frac{1}{eF}\{F\hat{w} + e[c_h(1-\frac{-b}{1-b}) - c_h\sqrt{\frac{d_l}{16e\hat{w}}}(1-\frac{-2b}{1-b}) - \frac{-b}{1-b}\frac{d_l t_l}{e}]\} = \frac{1}{2}\sqrt{\frac{en^2}{\hat{w}d_l}}\frac{1}{eF}\{ec_h n - c_h\sqrt{\frac{ed_l}{4\hat{w}}} - \frac{-b}{1-b}pa^{\frac{1}{1-b}}(en^2)^{\frac{-b}{1-b}}\hat{w}^{\frac{-b}{1-b}-1}\} < 0$ with $\hat{w} \le w_l^* = \frac{c_h^2 e}{4d_l t_l^2}$. $u_l^* = \frac{d_l}{s_l}$, so $\frac{\partial u_l^*}{\partial e} > 0$. $s_h^* = en^2\hat{w}$, $\frac{\partial s_h^*}{\partial e} = n^2(\frac{\partial\hat{w}}{\partial e} + \frac{\hat{w}}{e}) < 0$.
In case (4), $\frac{\partial w_l^*}{\partial e} = \frac{\partial w_h^*}{\partial e} = -\frac{d_l}{n^2e^2} < 0$. $s_l^* = s_h^* = d_l$, which are not influenced.

Proof of Proposition 9.

In cases (1) and (2), it is obvious that the non-peak periods are not influenced.
In the peak periods, let $\frac{-b}{1-b} = b_0$, and $\frac{N_0 n^2}{\bar{c}} = t$, we have $b_0 pa^{1-b_0}t^{b_0}\hat{w}_h{}^{b_0-1} - (1+b_0)a^{1-b_0}t^{b_0}\hat{w}_h{}^{b_0} - \frac{c_h N_0 n}{\bar{c}} = b_0 p\frac{a}{\hat{w}_h}(ta^{-1}\hat{w}_h)^{b_0} - b_0 a(ta^{-1}\hat{w}_h)^{b_0} - a(ta^{-1}\hat{w}_h)^{b_0} - \frac{c_h N_0 n}{\bar{c}} = 0$. With $\frac{\partial(ta^{-1}\hat{w}_h)^{b_0}}{\partial b_0} = \frac{\partial e^{b_0\ln(ta^{-1}\hat{w}_h)}}{\partial b_0} = e^{b_0\ln(ta^{-1}\hat{w}_h)}[\ln(ta^{-1}\hat{w}_h) + \frac{b_0}{\hat{w}_h}\frac{\partial\hat{w}_h}{\partial b_0}] = (ta^{-1}\hat{w}_h)^{b_0}[\ln(ta^{-1}\hat{w}_h) + \frac{b_0}{\hat{w}_h}\frac{\partial\hat{w}_h}{\partial b_0}]$, we have $\frac{\partial\hat{w}_h}{\partial b_0}(ta^{-1}\hat{w}_h)^{b_0}\frac{ab_0}{\hat{w}_h}[(b_0-1)\frac{p}{\hat{w}_h} - b_0 - 1] + a(ta^{-1}\hat{w}_h)^{b_0}[(\frac{p}{\hat{w}_h} - 1) + \ln(ta^{-1}\hat{w}_h)(\frac{pb_0}{\hat{w}_h} - b_0 - 1)]$. $[(b_0-1)\frac{p}{\hat{w}_h} - b_0 - 1] < 0$, $\ln(ta^{-1}\hat{w}_h) < 0$, $(\frac{pb_0}{\hat{w}_h} - b_0 - 1) > 0$, so $\frac{\partial\hat{w}_h}{\partial b_0} > 0$ when $[(\frac{p}{\hat{w}_h} - 1) + \ln(ta^{-1}\hat{w}_h)(\frac{pb_0}{\hat{w}_h} - b_0 - 1)] > 0$. $b_0 = \frac{1}{1+\frac{1}{-b}}$, b_0 decreases in b. Thus, $\frac{\partial\hat{w}_h}{\partial b} < 0$ when $[(\frac{p}{\hat{w}_h} - 1) + \ln(ta^{-1}\hat{w}_h)(\frac{pb_0}{\hat{w}_h} - b_0 - 1)] > 0$.
In case (3), $\frac{\partial\hat{w}}{\partial b_0}\{(ta^{-1}\hat{w})^{b_0}\frac{ab_0}{\hat{w}}[(b_0-1)\frac{p}{\hat{w}} - b_0 - 1] - c_h\sqrt{\frac{N_0 d_l}{16\bar{c}\hat{w}^3}}\} + a(ta^{-1}\hat{w})^{b_0}[(\frac{p}{\hat{w}} - 1) + \ln(ta^{-1}\hat{w})(\frac{pb_0}{\hat{w}} - b_0 - 1)]$, so $\frac{\partial\hat{w}_h}{\partial b_0} > 0$ and $\frac{\partial\hat{w}_h}{\partial b} < 0$ when $[(\frac{p}{\hat{w}_h} - 1) + \ln(ta^{-1}\hat{w}_h)(\frac{pb_0}{\hat{w}_h} - b_0 - 1)] > 0$.
In case (4), it is obvious that all the solutions are not influenced by b.

Appendix C

Proofs for Chapter 4

Proof of Proposition 10.

The o-firm's profit is $\pi'_o = (p'_o - c_o)\frac{2(p'_n - p'_o) - v(1-\alpha)}{2(h_o - h_n)}$. The optimal p'_o is given by the first order condition: $p'_o = \frac{2c_o + 2p'_n - v(1-\alpha)}{4}$. The n-firm's profit is $\pi'_n = (p'_n - c_n)(1 - \frac{2(p'_n - p'_o) - v(1-\alpha)}{2(h_o - h_n)})$. The optimal p'_n is given by the first order condition: $p'_n = \frac{2c_n - 2h_n + 2h_o + 2p'_o + v(1-\alpha)}{4}$. Solve the two equations we obtain p'_o and p'_n. The cutoff value θ' and the profits of the two firms are obtained by substituting p'_o and p'_n.

Proof of Proposition 11.

Substituting $c_n - c_o = A(h_o - h_n)^2$, conditions $2(c_n - c_o) - 4(h_o - h_n) - v(1-\alpha) < 0$ and $2(c_n - c_o) + 2(h_o - h_n) - v(1-\alpha) > 0$ determine that $\frac{\sqrt{4+2Av(1-\alpha)}-2}{2A} \leq (h_o - h_n) \leq \frac{2+\sqrt{4+2Av(1-\alpha)}}{2A}$.

Substituting $c_n - c_o = A(h_o - h_n)^2$ into the n-firm's profit, we have:

$\pi'_n = \frac{[-2A(h_o-h_n)^2 + 4(h_o-h_n) + v(1-\alpha)]^2}{36(h_o-h_n)}$. $\frac{\partial \pi'_n}{\partial(h_o-h_n)} = \frac{[-(2(c_n-c_o)-4(h_o-h_n)-v(1-\alpha)]}{36(h_o-h_n)^2}[-6A(h_o - h_n)^2 + 4(h_o - h_n) - v(1-\alpha)]$, and $\frac{[-(2(c_n-c_o)-4(h_o-h_n)-v(1-\alpha)]}{36(h_o-h_n)^2} > 0$. $-6A(h_o - h_n)^2 + 4(h_o - h_n) - v(1-\alpha)$ is positive and then negative as $h_o - h_n$ increases when $v(1-\alpha) \leq \frac{2}{3A}$. Thus, π'_n increases then decreases and $(h_o - h_n)^*$ is at $\frac{\partial \pi'_n}{\partial(h_o-h_n)} = 0$, that is, $(h_o - h_n)^* = \frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}$.

When $v(1-\alpha) > \frac{2}{3A}$, $-6A(h_o - h_n)^2 + 4(h_o - h_n) - v(1-\alpha)$ is negative, so π'_n decreases as $(h_o - h_n)$ increases, $(h_o - h_n)^* = \frac{\sqrt{4+2Av(1-\alpha)}-2}{2A}$.

Proof of Lemma 7.

The o-firm's profit in the o-firm's first-period market share is $\pi_{2o}^{o} = (p_o - c_o)\frac{p_{on}-p_{oo}}{h_o-h_n}$. The optimal p_{oo} is given by the first order condition: $p_{oo} = \frac{c_o+p_{on}}{2}$. The n-firm's profit is $\pi_{2o}^{n} = (p_{on} - c_n)(\theta_1 - \frac{p_{on}-p_{oo}}{h_o-h_n})$. The optimal p_{on} is given by the first order condition: $p_{on} = \frac{c_n-(h_n-h_o)\theta_1+p_{oo}}{2}$. Solving the two equations, we obtain p_{oo}^* and p_{on}^*.

Proof of Lemma 8.

The o-firm's profit in the n-firm's first-period market share is $\pi_{2n}^{o} = (p_{no}-c_o)(\frac{p_{nn}-p_{no}-v(1-\alpha)}{h_o-h_n} - \theta_1)$. The optimal p_{no} is given by the first order condition: $p_{no} = \frac{c_o+p_{nn}-(h_o-h_n)\theta_1-v(1-\alpha)}{2}$. The n-firm's profit is $\pi_{2n}^{n} = (p_{nn} - c_n)(1 - \frac{p_{nn}-p_{no}-v(1-\alpha)}{h_o-h_n})$. The optimal p_{nn} is given by the first order condition: $p_{nn} = \frac{c_n-(h_n-h_o)+p_{no}+v(1-\alpha)}{2}$. Solving the two equations, we obtain p_{no}^* and p_{nn}^*.

Proof of Proposition 12.

We substitute $\theta_1, p_{oo}^*, p_{on}^*, p_{no}^*, p_{nn}^*$ into the two firms' profits. The two variables are p_o and p_n. The profit functions are concave. The first order conditions give: $p_o = \frac{c_n+5c_o+3(h_o-h_n)+p_n-3v(1-\alpha)}{7}$ and $p_n = \frac{5c_n+c_o+5(h_o-h_n)+p_o-v(1-\alpha)}{7}$. Solving the two equations, we obtain p_o and p_n.

Proof of Proposition 13.

$c_n > c_o$ and $h_o > h_n$. So, $p_n-p_o = \frac{2(c_n-c_o)+(h_o-h_n)+v(1-\alpha)}{4} > 0$. $v(1-\alpha) < (h_o-h_n)$, so $p_{on} - p_{oo} = \frac{18(c_n-c_o)+7(h_o-h_n)-v(1-\alpha)}{48} > 0$, $p_{nn} - p_{no} = \frac{18(c_n-c_o)+23(h_o-h_n)+31v(1-\alpha)}{48} > 0$, $p_{no} - p_{oo} = \frac{6(c_n-c_o)+5(h_o-h_n)+13v(1-\alpha)}{48} > 0$, $p_{on} - p_{nn} = \frac{6(c_n-c_o)+11(h_o-h_n)+19v(1-\alpha)}{48} > 0$.

Proof of Proposition 14.

The differences between the market shares of the o-firm and the n-firm in the two periods are: $so_1-sn_1 = \frac{2(c_n-c_o)-(h_o-h_n)-v(1-\alpha)}{8(h_o-h_n)}$, and $so_2-sn_2 = \frac{5[2(c_n-c_o)-(h_o-h_n)-v(1-\alpha)]}{8(h_o-h_n)}$. The difference between the profits of the two firms is $\pi_o-\pi_n = \frac{13[2(c_n-c_o)-(h_o-h_n)-v(1-\alpha)]}{24}$.

Proof of Proposition 15.

$\frac{\partial\pi_n}{\partial(h_o-h_n)} = [1620A^2(h_o-h_n)^2 - 3576A(h_o-h_n) + 1847] - 540Av(1-\alpha) - \frac{263v^2(1-\alpha)^2}{(h_o-h_n)^2}$. $\frac{\partial^4\pi_n}{\partial(h_o-h_n)^4} > 0$. $\frac{\partial^3\pi_n}{\partial(h_o-h_n)^3}$ changes from negative to positive as $(h_o - h_n)$ increases. $\frac{\partial^2\pi_n}{\partial(h_o-h_n)^2}$ first decreases from a positive value, then increases, however, we cannot judge the value is positive or negative. We use the compel method. $\frac{\partial\pi_n}{\partial(h_o-h_n)}$ is larger than $\frac{135A^2(h_o-h_n)^2-343A(h_o-h_n)+132}{192}$, and smaller than $\frac{1620A^2(h_o-h_n)^2-3576A(h_o-h_n)+1847}{2304}$. From the monotonicity of the two functions, $\frac{\partial\pi_n}{\partial(h_o-h_n)}$ increases then decreases. Thus, $(h_o - h_n)^*$ satisfies $\frac{\partial\pi_n}{\partial(h_o-h_n)} = 0$.

Proof of Proposition 16.

When $v(1-\alpha) \leq \frac{2}{3A}$, we substitute $h_o - h_n = \frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}$ into $\frac{\partial\pi_n}{\partial(h_o-h_n)}$.
If $\frac{\partial\pi_n}{\partial(h_o-h_n)} \leq 0$, that is, $v(1-\alpha) \leq \frac{53(\sqrt{6817}-2619)}{4096A}$, $h^* = (h_o - h_n)^* \geq \frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}$.
If $\frac{\partial\pi_n}{\partial(h_o-h_n)} > 0$, that is, $v(1-\alpha) > \frac{53(\sqrt{6817}-2619)}{4096A}$, $h^* = (h_o - h_n)^* < \frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}$.
When $v(1-\alpha) > \frac{2}{3A}$, we substitute $h_o - h_n = \frac{\sqrt{4+2Av(1-\alpha)}-2}{2A}$ into $\frac{\partial\pi_n}{\partial(h_o-h_n)}$. $\frac{\partial\pi_n}{\partial(h_o-h_n)} \leq 0$, $h^* = (h_o - h_n)^* < \frac{\sqrt{4+2Av(1-\alpha)}-2}{2A}$.

Proof of Proposition 17.

$\Delta\pi_n = \frac{[v(1-\alpha)-3(h_o-h_n)][6(c_n-c_o)-3(h_o-h_n)-11v(1-\alpha)]}{96(h_o-h_n)} - \frac{[11(h_o-h_n)-6(c_n-c_o)+19v(1-\alpha)]^2}{2304(h_o-h_n)}$
> 0, which can be transformed to the form in the proposition.

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